Computer Algebra
Independent Integration Tests

Summer 2024

2-Exponentials/155-2-Exponential-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**710**]. This is test number [155].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.73 (701)	1.27 (9)
Mathematica	97.46 (692)	2.54 (18)
Fricas	88.59 (629)	11.41 (81)
Maple	82.54 (586)	17.46 (124)
Mupad	77.61 (551)	22.39 (159)
Maxima	66.06 (469)	33.94 (241)
Reduce	51.83 (368)	48.17 (342)
Giac	48.45 (344)	51.55 (366)
Sympy	47.61 (338)	52.39 (372)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

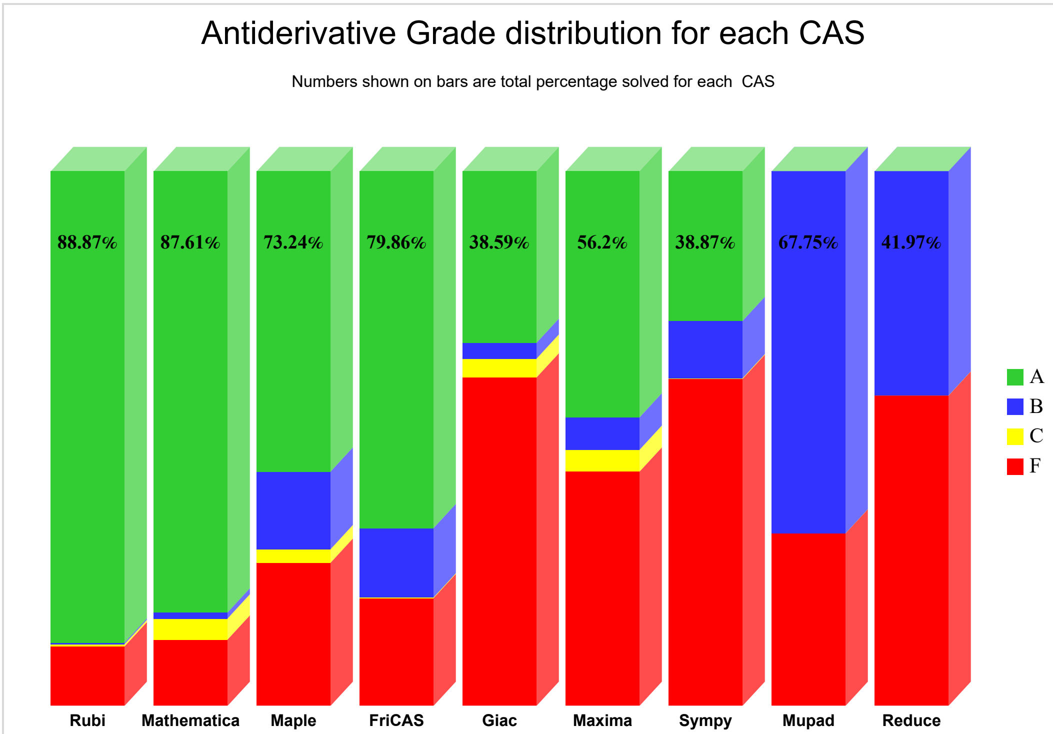
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

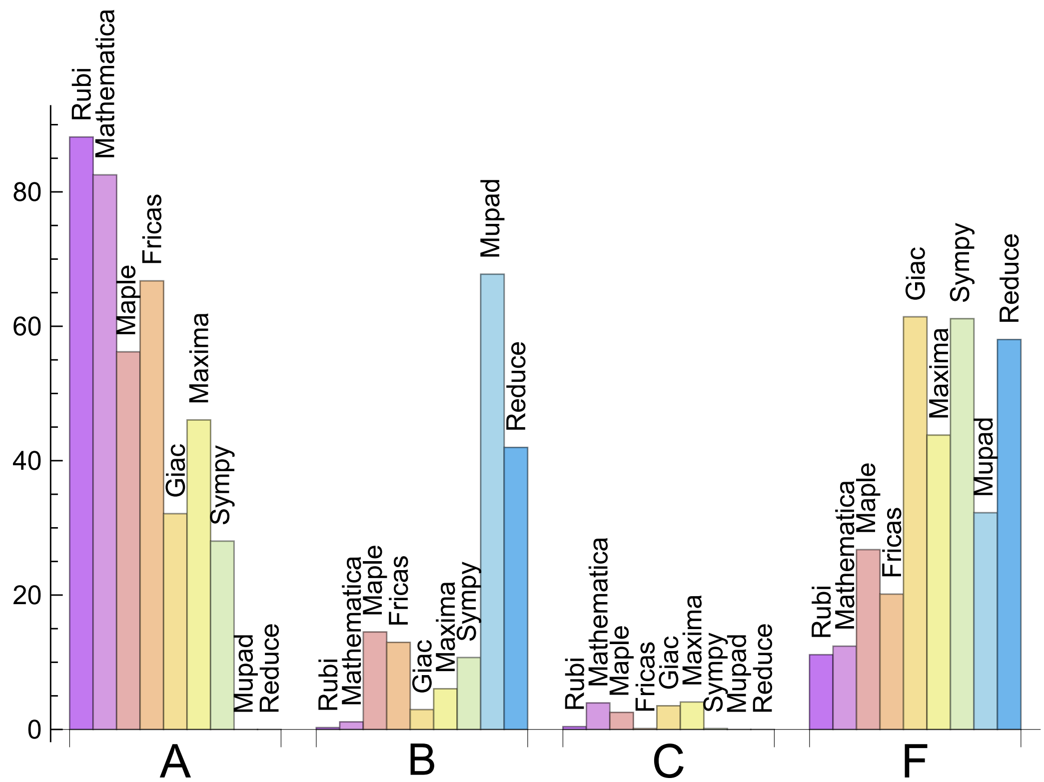
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.169	0.282	0.423	11.127
Mathematica	82.535	1.127	3.944	12.394
Fricas	66.761	12.958	0.141	20.141
Maple	56.197	14.507	2.535	26.761
Maxima	46.056	6.056	4.085	43.803
Giac	32.113	2.958	3.521	61.408
Sympy	28.028	10.704	0.141	61.127
Mupad	0.000	67.746	0.000	32.254
Reduce	0.000	41.972	0.000	58.028

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	9	100.00	0.00	0.00
Mathematica	18	100.00	0.00	0.00
Fricas	81	76.54	0.00	23.46
Maple	124	87.90	12.10	0.00
Mupad	159	0.00	100.00	0.00
Maxima	241	90.87	0.00	9.13
Reduce	342	100.00	0.00	0.00
Sympy	372	89.52	9.14	1.34
Giac	366	99.18	0.00	0.82

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.10
Maxima	0.11
Giac	0.18
Reduce	0.21
Mupad	0.22
Maple	0.31
Mathematica	0.35
Rubi	0.57
Sympy	5.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	60.07	0.94	37.00	1.00
Mupad	70.54	1.36	43.00	1.00
Rubi	72.93	1.04	46.00	1.00
Sympy	87.12	1.52	28.00	1.00
Giac	113.27	2.16	29.00	1.00
Fricas	116.69	1.79	55.00	1.07
Maple	128.53	1.92	45.00	1.00
Maxima	139.15	2.17	25.00	0.93
Reduce	7688.87	422.54	38.00	1.03

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

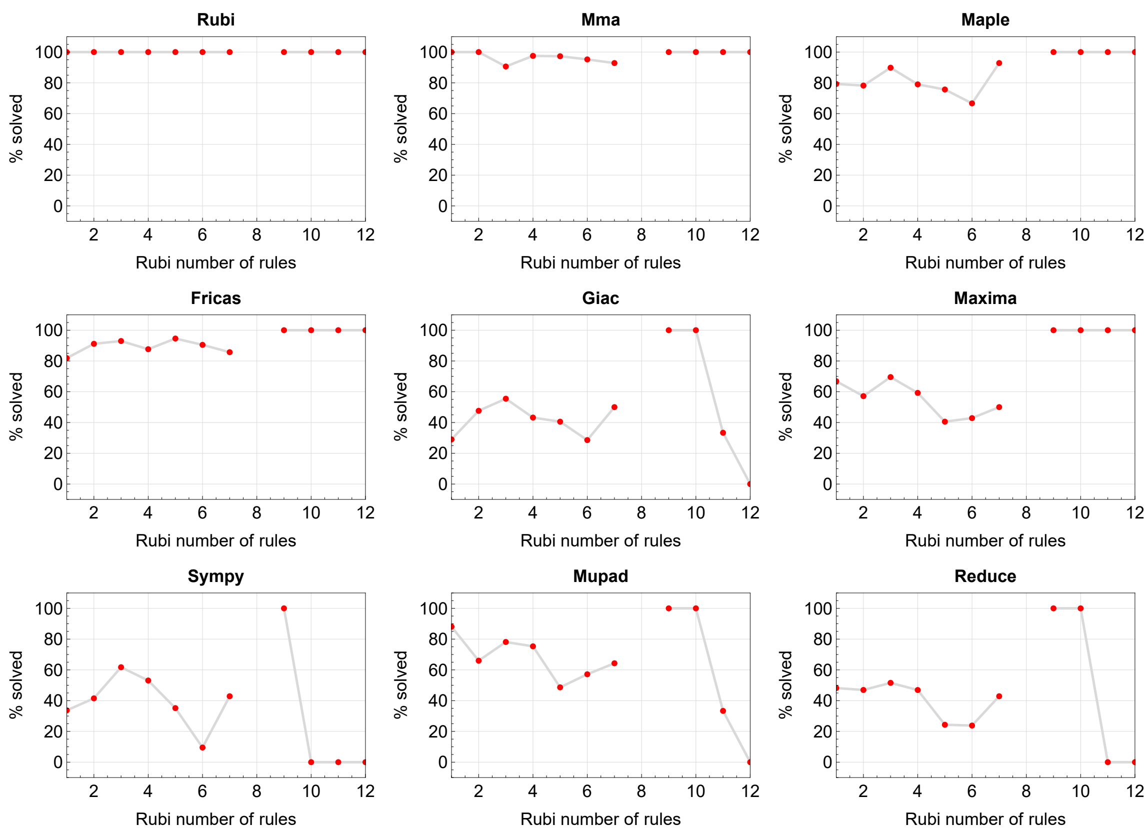


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

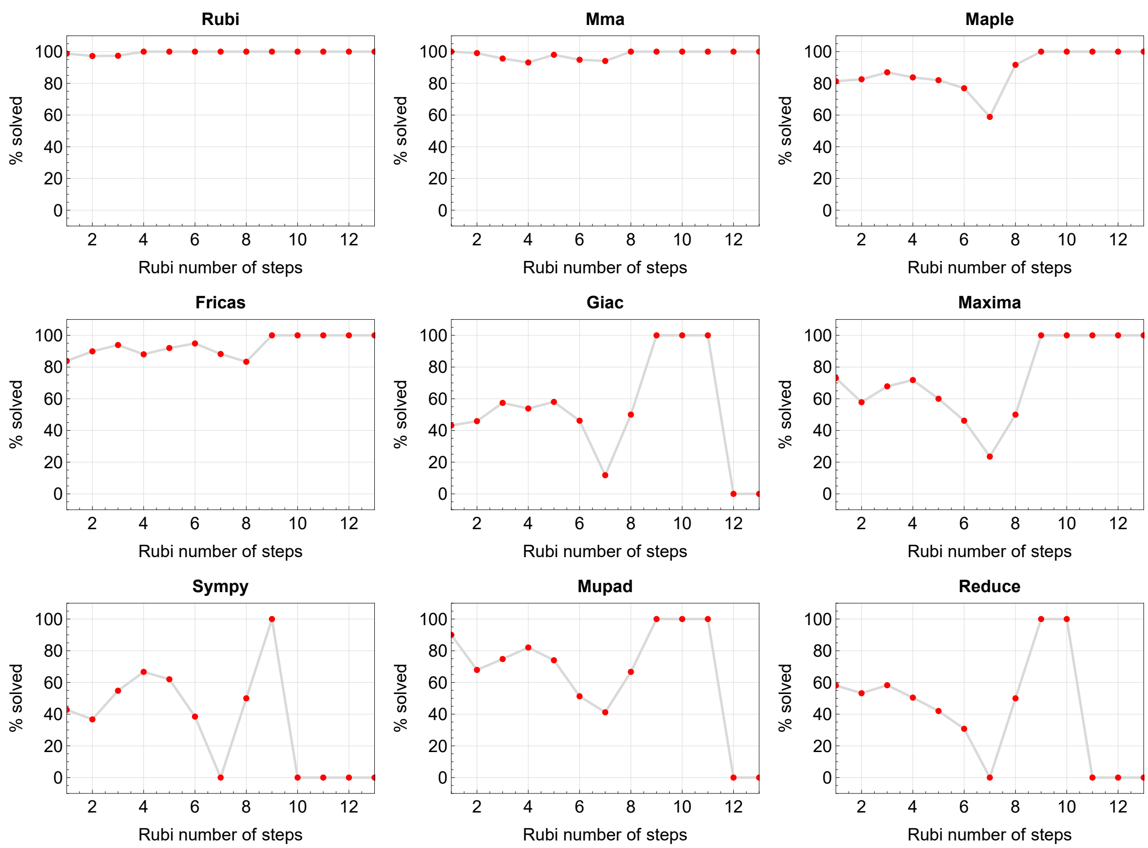


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

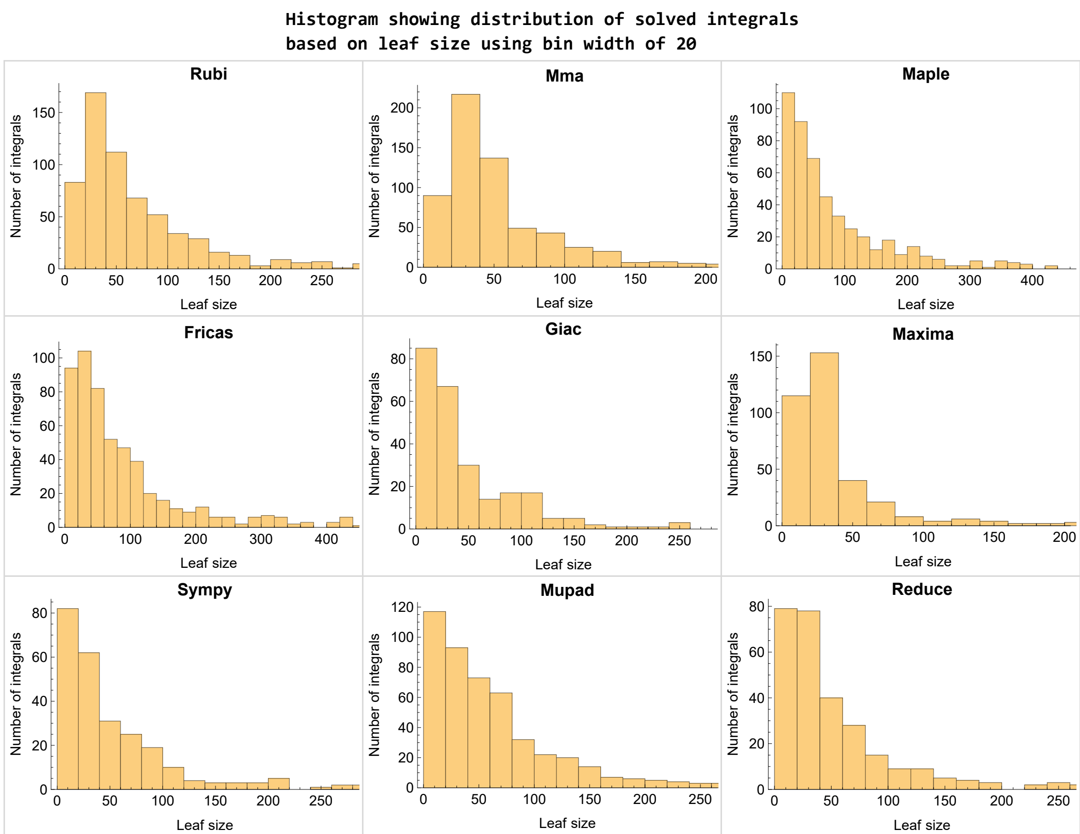


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

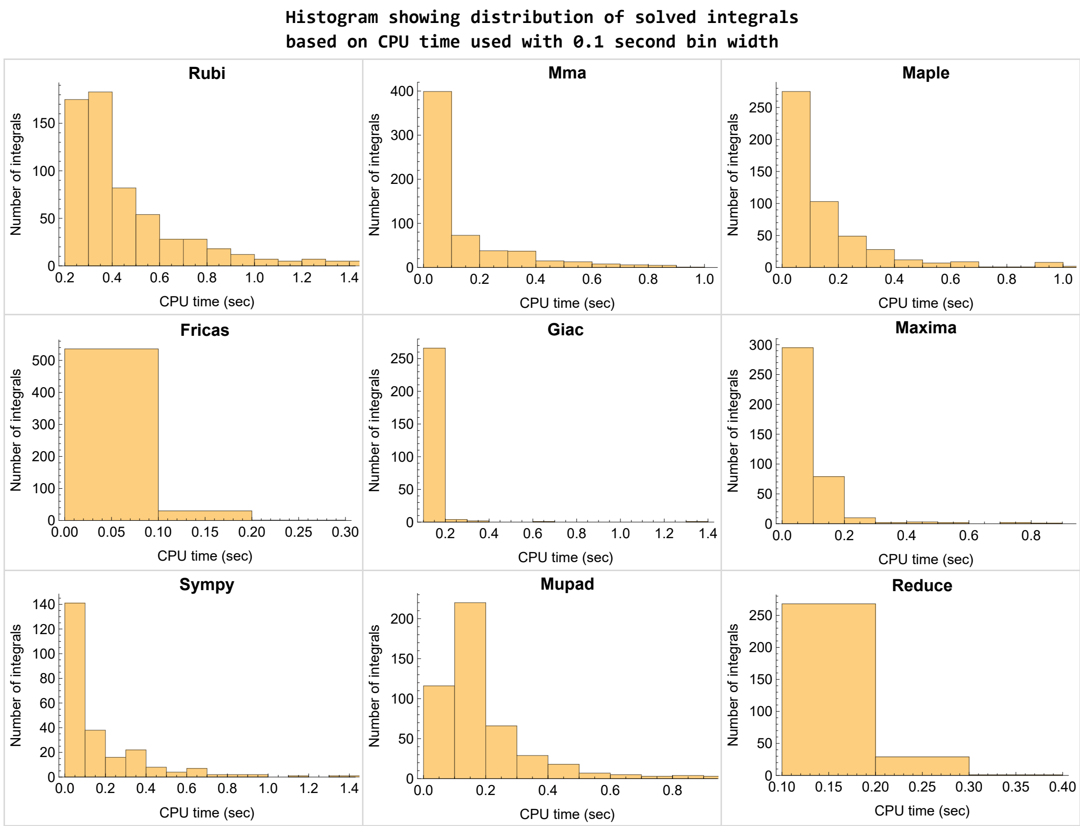


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

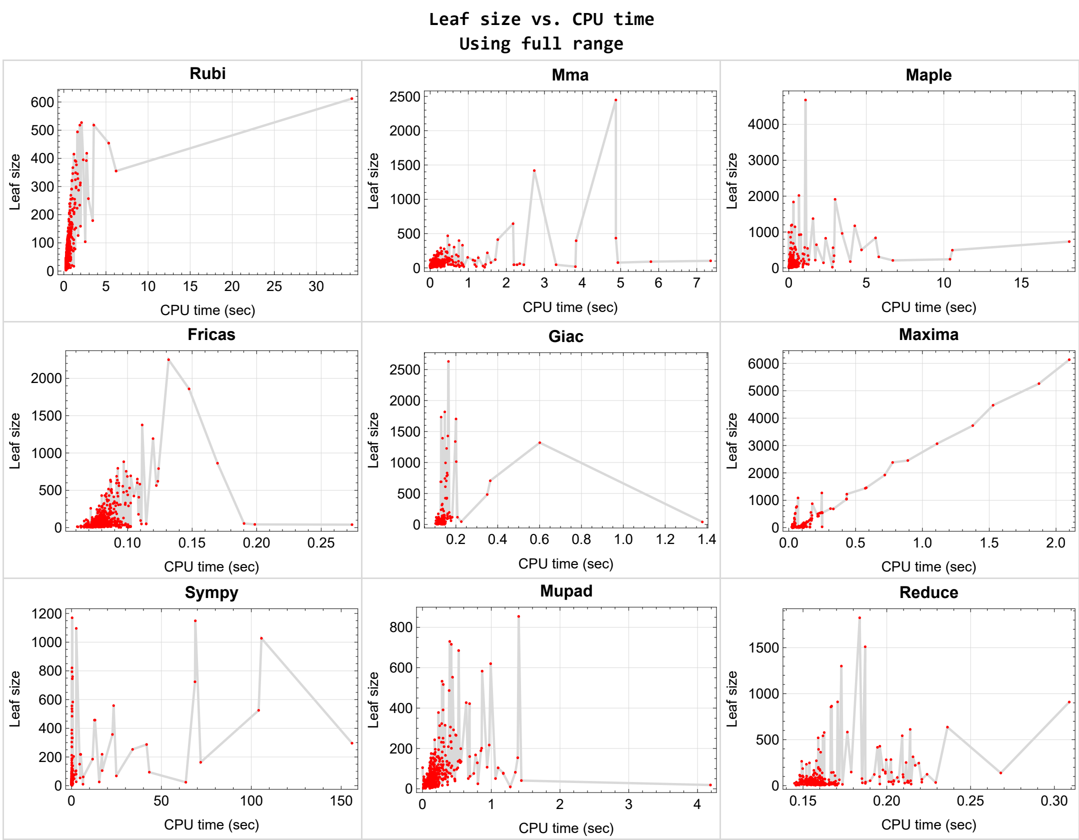


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{138, 139, 140, 144, 145, 146, 147, 162, 163, 164, 170, 171, 172, 173, 174, 176, 177, 178, 183, 184, 185, 322, 323, 324, 329, 330, 347, 348, 349, 354, 355, 364, 365, 370, 371, 376, 377, 381, 382, 383, 466, 470, 471, 472, 477, 478, 498, 499, 517, 522, 523, 524, 532, 537, 538, 539, 670, 671, 672, 674, 677, 681, 682, 683, 684, 685, 686, 687, 688, 701}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {635, 636, 637, 640, 646}

Mathematica $\{\}$

Maple $\{186, 514, 529\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

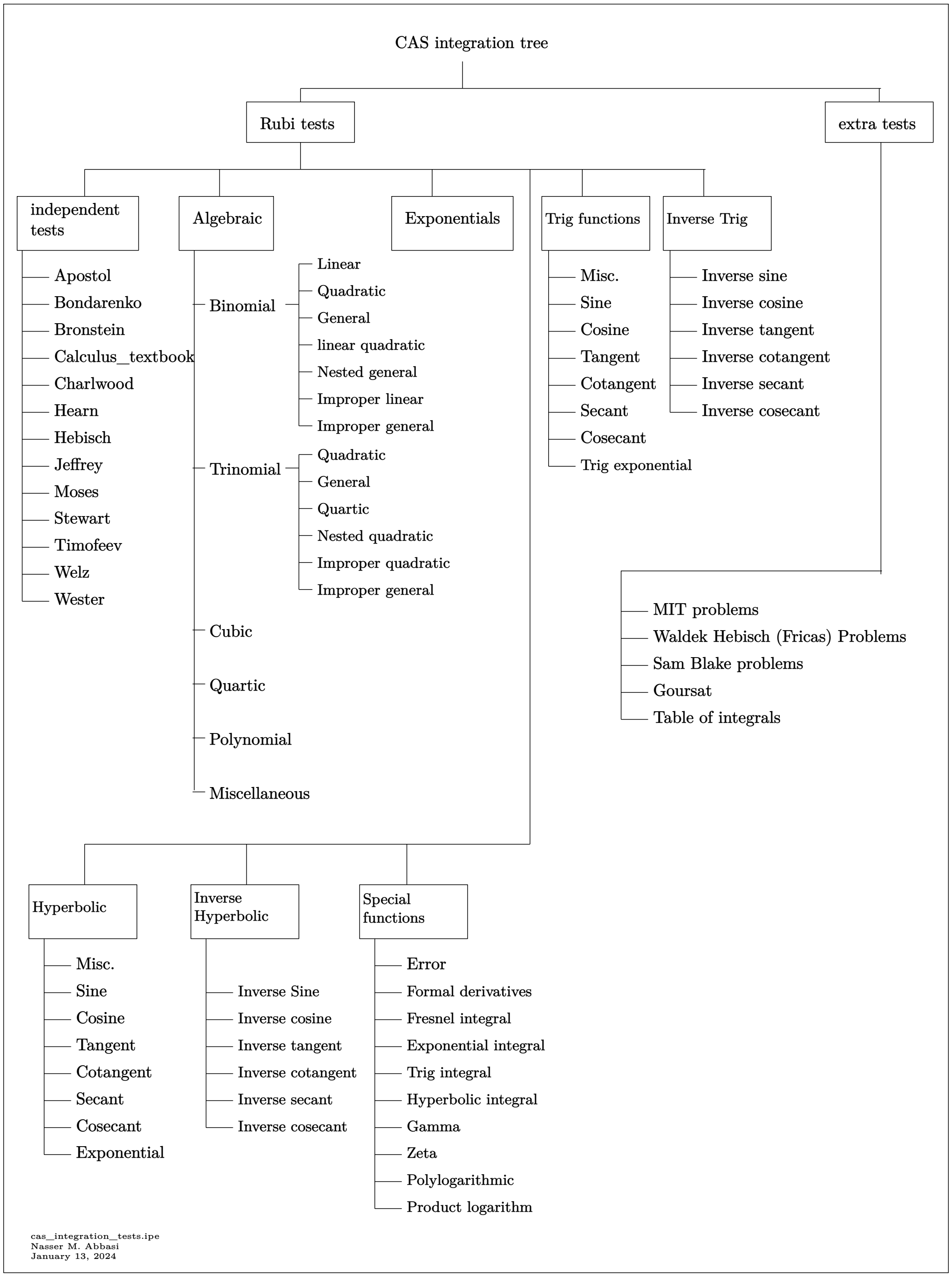
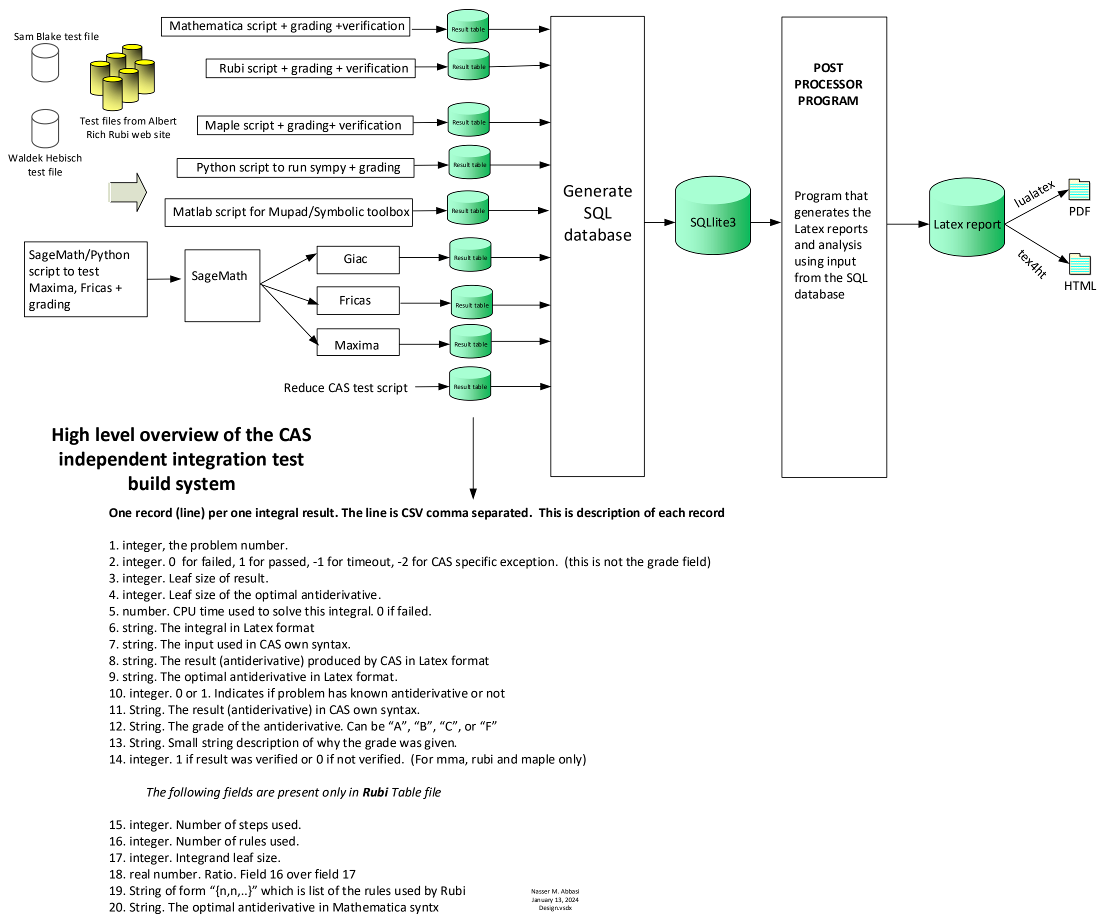


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 175, 179, 180, 181, 182, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 350, 351, 352, 353, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 378, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 468, 469, 473, 474, 475, 476, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 673, 675, 676, 678, 679, 680, 689, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 702, 703, 704, 705, 706, 708, 709 }

B grade { 577, 578 }

C grade { 1, 2, 3 }

F normal fail { 4, 5, 6, 508, 615, 617, 698, 707, 710 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 175, 179, 180, 181, 182, 186, 187, 188, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 328, 331, 332, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 350, 351, 352, 353, 356, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 378, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 476, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 491, 492, 493, 496, 497, 501, 502, 503, 504, 505, 506, 507, 508, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 572, 573, 574, 575, 576, 577, 578, 581, 582, 583, 584, 585, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 673, 675, 676, 678, 679, 680, 689, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710 }

B grade { 494, 495, 500, 554, 559, 571, 579, 580 }

C grade { 7, 8, 32, 33, 61, 62, 73, 74, 98, 99, 123, 124, 189, 190, 215, 216, 246, 247, 259, 260, 285, 286, 302, 303, 304, 305, 306, 586 }

F normal fail { 333, 334, 342, 357, 358, 359, 467, 468, 469, 473, 474, 475, 489, 490, 509, 510, 525, 698 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 94, 95, 96, 97, 98, 99, 117, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 191, 192, 193, 194, 195, 196, 197, 198, 207, 208, 209, 210, 211, 212, 213, 218, 219, 220, 239, 240, 241, 242, 243, 244, 245, 253, 254, 255, 256, 257, 258, 265, 266, 267, 268, 269, 270, 271, 272, 273, 281, 282, 283, 298, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 321, 331, 332, 333, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 405, 407, 409, 410, 411, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 443, 446, 448, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 476, 485, 486, 487, 491, 492, 493, 540, 541, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 589, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 673, 675, 676, 678, 679, 680, 689, 698, 699, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710 }

B grade { 17, 18, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 63, 64, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 175, 189, 190, 199, 200, 201, 202, 203, 204, 205, 206, 215, 216, 217, 236, 237, 238, 246, 247, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 284, 285, 286, 312, 313, 316, 317, 318, 319, 320, 334, 335, 336, 356, 357, 358, 359, 378, 404, 406, 408, 425, 427, 444, 445, 447, 450, 451, 463, 464, 496, 497, 500, 559, 567, 588, 627 }

C grade { 113, 114, 115, 116, 118, 119, 120, 122, 186, 488, 489, 490, 514, 529, 690, 691, 692, 693 }

F normal fail { 141, 142, 143, 165, 166, 167, 168, 169, 179, 180, 181, 182, 187, 188, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 248, 274, 275, 276, 277, 278, 279, 280, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 325, 326, 327, 328, 350, 351, 352, 353, 412, 414, 424, 426, 440, 441, 442, 449, 462, 465, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 494, 495, 501, 502, 503, 504, 505, 506, 507, 508, 509, 526, 527, 528, 530, 531, 533, 534, 535, 536, 542, 590, 649, 694, 695, 696, 697 }

F(-1) timedout fail { 510, 511, 512, 513, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 117, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 141, 142, 143, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 175, 186, 188, 191, 192, 193, 194, 195, 201, 202, 204, 205, 206, 207, 208, 209, 214, 218, 219, 220, 228, 229, 233, 234, 238, 239, 240, 241, 242, 243, 244, 245, 251, 252, 253, 254, 256, 263, 264, 265, 266, 267, 268, 272, 273, 290, 291, 298, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 328, 331, 332, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 350, 351, 356, 357, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 378, 379, 380, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 496, 497, 500, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 540, 541, 543, 544, 545, 546, 547, 548, 549, 560, 563, 564, 565, 566, 568, 569, 570, 572, 573, 574, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 650, 652, 653, 654, 656, 657, 658, 660, 661, 662, 663, 664, 665, 666, 680, 689, 696, 698, 699, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710 }

B grade { 17, 18, 42, 43, 51, 52, 63, 64, 88, 89, 169, 189, 190, 196, 197, 198, 199, 200, 203, 210, 211, 212, 213, 215, 216, 217, 221, 222, 223, 224, 225, 226, 227, 230, 231, 232, 236, 237, 246, 247, 249, 250, 255, 257, 258, 259, 260, 261, 262, 269, 270, 271, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 312, 313, 333, 334, 342, 352, 353, 358, 359, 494, 495, 559, 561, 562, 567, 571, 575, 580, 588, 604, 607, 627, 643, 651 }

C grade { 542 }

F normal fail { 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 122, 133, 179, 180, 181, 182, 187, 235, 248, 274, 293, 294, 295, 296, 297, 299, 300, 301, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 501, 502, 503, 504, 505, 506, 507, 550, 551, 552, 553, 554, 555, 556, 557, 558, 690, 691, 692, 693, 694, 695 }

F(-1) timedout fail { }

F(-2) exception fail { 466, 472, 655, 659, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 682, 697 }

Maxima

A grade { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 148, 175, 194, 207, 220, 233, 234, 242, 255, 256, 281, 302, 303, 304, 305, 306, 307, 321, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 386, 392, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 409, 411, 413, 415, 416, 417, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 433, 434, 438, 439, 443, 446, 448, 452, 453, 454, 455, 456, 457, 458, 459, 485, 486, 488, 489, 490, 493, 540, 541, 546, 559, 560, 562, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 644, 645, 646, 647, 648, 650, 651, 653, 655, 656, 657, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 673, 675, 676, 678, 679, 680, 689, 690, 691, 692, 693, 697, 698, 699, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710 }

B grade { 136, 201, 202, 203, 204, 205, 206, 215, 216, 217, 218, 219, 257, 258, 259, 260, 282, 283, 284, 285, 286, 316, 317, 318, 319, 320, 378, 379, 380, 385, 391, 406, 407, 410, 476, 561, 567, 575, 588, 627, 641, 643, 654 }

C grade { 1, 2, 3, 58, 59, 60, 61, 62, 70, 71, 72, 73, 74, 95, 96, 97, 98, 99, 189, 190, 191, 192, 193, 384, 390, 543, 544, 545, 658 }

F normal fail { 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 179, 180, 181, 182, 186, 187, 188, 195, 196, 197, 198, 199, 200, 208, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 315, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 350, 351, 352, 353, 356, 357, 358, 359, 387, 388, 389, 393, 394, 395, 412, 414, 424, 426, 435, 436, 440, 441, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 542, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 649, 652, 694, 695, 696 }

F(-1) timedout fail { }

F(-2) exception fail { 432, 437, 442, 444, 445, 447, 449, 450, 451, 460, 461, 462, 463, 464, 465, 487, 491, 492, 494, 495, 496, 497 }

Giac

A grade { 1, 2, 3, 6, 7, 8, 9, 10, 12, 19, 20, 21, 22, 23, 24, 25, 33, 34, 35, 37, 50, 57, 69, 94, 121, 125, 131, 134, 135, 136, 137, 148, 186, 189, 190, 191, 192, 194, 201, 202, 203, 204, 205, 206, 207, 220, 242, 307, 314, 315, 316, 317, 318, 319, 320, 321, 337, 338, 339, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 378, 379, 380, 385, 386, 391, 392, 397, 399, 401, 403, 405, 407, 409, 411, 413, 415, 417, 419, 421, 423, 425, 427, 428, 429, 430, 431, 432, 443, 444, 445, 450, 451, 452, 453, 455, 458, 459, 460, 463, 497, 508, 513, 521, 528, 536, 540, 541, 543, 544, 545, 546, 547, 559, 560, 563, 564, 565, 566, 568, 569, 570, 571, 572, 573, 574, 576, 577, 578, 579, 581, 582, 583, 584, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 640, 641, 642, 644, 645, 646, 650, 651, 653, 654, 655, 656, 658, 660, 661, 662, 663, 664, 665, 666, 675, 680, 689, 700, 702, 703, 704, 706, 707, 709, 710 }

B grade { 4, 5, 217, 218, 255, 281, 335, 336, 340, 341, 342, 476, 561, 562, 567, 575, 580, 588, 627, 643, 652 }

C grade { 11, 36, 193, 219, 233, 234, 257, 384, 390, 485, 486, 487, 491, 492, 493, 509, 550, 551, 552, 553, 554, 659, 699, 705, 708 }

F normal fail { 13, 14, 15, 16, 17, 18, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 126, 127, 128, 129, 130, 132, 133, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 175, 179, 180, 181, 182, 187, 188, 195, 196, 197, 198, 199, 200, 208, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 325, 326, 327, 328, 331, 332, 333, 334, 343, 344, 345, 346, 350, 351, 352, 353, 356, 357, 358, 359, 387, 388, 389, 393, 394, 395, 396, 398, 400, 402, 404, 406, 408, 410, 412, 414, 416, 418, 420, 422, 424, 426, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 446, 447, 448, 449, 454, 456, 457, 461, 462, 464, 465, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 488, 489, 490, 494, 495, 496, 500, 501, 502, 503, 504, 505, 506, 507, 510, 511, 512, 514, 515, 516, 518, 519, 520, 525, 526, 527, 529, 530, 531, 533, 534, 535, 542, 548, 549, 555, 556, 557, 558, 639, 647, 648, 649, 657, 667, 668, 669, 673, 676, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

F(-1) timedout fail { }

F(-2) exception fail { 32, 215, 216 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 125, 134, 135, 136, 137, 148, 151, 152, 153, 161, 169, 175, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 299, 300, 301, 307, 314, 315, 316, 317, 318, 319, 320, 321, 337, 338, 339, 346, 353, 360, 361, 362, 363, 366, 367, 368, 369, 372, 373, 374, 375, 378, 379, 380, 384, 385, 386, 388, 390, 391, 392, 394, 396, 397, 398, 399, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 428, 429, 430, 431, 432, 443, 444, 445, 450, 451, 452, 453, 455, 456, 458, 459, 460, 463, 476, 485, 486, 487, 488, 489, 490, 491, 492, 493, 497, 503, 514, 529, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 673, 675, 676, 678, 679, 680, 692, 693, 694, 695, 697, 698, 699, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710 }

C grade { }

F normal fail { }

F(-1) timedout fail { 46, 47, 115, 116, 117, 123, 124, 126, 127, 128, 129, 130, 131, 132, 133, 141, 142, 143, 149, 150, 154, 155, 156, 157, 158, 159, 160, 165, 166, 167, 168, 179, 180, 181, 182, 186, 187, 228, 229, 296, 297, 298, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 340, 341, 342, 343, 344, 345, 350, 351, 352, 356, 357, 358, 359, 387, 389, 393, 395, 400, 402, 424, 425, 426, 427, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 446, 447, 448, 449, 454, 457, 461, 462, 464, 465, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 494, 495, 496, 500, 501, 502, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 515, 516, 518, 519, 520, 521, 525, 526, 527, 528, 530, 531, 533, 534, 535, 536, 559, 590, 628, 648, 654, 689, 690, 691, 696 }

F(-2) exception fail { }

Sympy

A grade { 3, 7, 8, 9, 10, 11, 12, 32, 33, 34, 35, 36, 37, 50, 57, 58, 59, 60, 61, 62, 69, 70, 71, 72, 73, 74, 94, 95, 96, 97, 98, 99, 121, 123, 148, 194, 233, 242, 243, 244, 245, 256, 304, 369, 386, 392, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 420, 421, 422, 423, 428, 429, 430, 431, 432, 443, 444, 445, 450, 451, 452, 453, 455, 456, 458, 459, 460, 463, 497, 515, 540, 541, 543, 544, 545, 546, 547, 548, 552, 553, 555, 556, 559, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 577, 578, 579, 580, 581, 582, 583, 584, 586, 587, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 623, 624, 625, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 642, 644, 645, 646, 647, 648, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 675, 680, 702, 703, 704, 709 }

B grade { 1, 2, 4, 5, 6, 124, 125, 189, 190, 191, 192, 193, 215, 216, 217, 218, 219, 220, 246, 247, 255, 257, 258, 259, 260, 281, 282, 283, 284, 285, 286, 302, 303, 305, 306, 307, 384, 390, 476, 485, 486, 487, 492, 493, 511, 512, 513, 519, 520, 521, 527, 528, 530, 535, 536, 554, 560, 561, 572, 575, 576, 585, 588, 589, 621, 622, 626, 627, 641, 643, 651, 689, 699, 705, 706, 708 }

C grade { 692 }

F normal fail { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 75, 76, 77, 78, 79, 80, 81, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 175, 179, 180, 181, 182, 186, 187, 188, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 248, 249, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 274, 275, 276, 277, 278, 279, 280, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 350, 351, 352, 353, 356, 357, 360, 361, 362, 363, 366, 367, 368, 372, 373, 374, 375, 378, 379, 380, 385, 387, 388, 389, 391, 393, 394, 395, 416, 417, 418, 419, 424, 425, 426, 427, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 446, 447, 448, 449, 454, 457, 461, 462, 464, 465, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 488, 489, 490, 494, 495, 496, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 514, 525, 529, 607, 649, 667, 668, 669, 673, 676, 678, 679, 690, 691, 693, 694, 695, 696, 697, 698, 700 }

F(-1) timedout fail { 82, 83, 84, 85, 86, 87, 106, 268, 269, 270, 271, 272, 273, 292, 358, 359, 478, 499, 516, 518, 523, 524, 526, 531, 533, 534, 538, 539, 542, 549, 550, 551, 557, 558 }

F(-2) exception fail { 491, 517, 532, 707, 710 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 117, 121, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137, 148, 186, 189, 190, 191, 192, 193, 194, 201, 202, 203, 204, 205, 206, 207, 215, 216, 217, 218, 219, 220, 233, 234, 242, 243, 244, 245, 246, 247, 255, 256, 257, 258, 259, 260, 281, 282, 283, 284, 285, 286, 302, 303, 304, 305, 306, 307, 316, 317, 318, 319, 320, 321, 360, 361, 362, 363, 366, 367, 368, 369, 378, 379, 380, 384, 385, 386, 390, 391, 392, 397, 399, 401, 403, 405, 407, 409, 411, 417, 419, 421, 423, 428, 429, 430, 431, 432, 443, 444, 445, 450, 451, 452, 453, 455, 456, 458, 459, 460, 463, 476, 485, 486, 487, 491, 492, 493, 497, 540, 541, 543, 544, 545, 546, 560, 561, 562, 564, 567, 568, 569, 570, 572, 573, 574, 575, 576, 577, 578, 579, 581, 585, 586, 588, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 613, 614, 615, 616, 617, 618, 619, 620, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 680, 689, 699, 700, 702, 703, 704, 705, 706, 707, 708, 709, 710 }

C grade { }

F normal fail { 26, 27, 28, 29, 30, 31, 44, 45, 46, 47, 48, 49, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 122, 129, 130, 131, 132, 133, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 175, 179, 180, 181, 182, 187, 188, 195, 196, 197, 198, 199, 200, 208, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 235, 236, 237, 238, 239, 240, 241, 248, 249, 250, 251, 252, 253, 254, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 308, 309, 310, 311, 312, 313, 314, 315, 325, 326, 327, 328, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 350, 351, 352, 353, 356, 357, 358, 359, 372, 373, 374, 375, 387, 388, 389, 393, 394, 395, 396, 398, 400, 402, 404, 406, 408, 410, 412, 413, 414, 415, 416, 418, 420, 422, 424, 425, 426, 427, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 446, 447, 448, 449, 454, 457, 461, 462, 464, 465, 467, 468, 469, 473, 474, 475, 479, 480, 481, 482, 483, 484, 488, 489, 490, 494, 495, 496, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 518, 519, 520, 521, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 542, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 563, 565, 566, 571, 580, 582, 583, 584, 587, 589, 590, 610, 611, 612, 621, 622, 623, 624, 632, 636, 647, 648, 649, 654, 657, 673, 675, 676, 678, 679, 690, 691, 692, 693, 694, 695, 696, 697, 698 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	324	35	35	446	35	87	48	35	49
N.S.	1	8.76	0.95	0.95	12.05	0.95	2.35	1.30	0.95	1.32
time (sec)	N/A	1.365	0.044	0.090	0.173	0.088	0.097	0.133	0.229	0.332

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	349	41	41	558	40	92	66	40	53
N.S.	1	7.76	0.91	0.91	12.40	0.89	2.04	1.47	0.89	1.18
time (sec)	N/A	1.390	0.347	0.072	0.248	0.081	0.077	0.137	0.156	0.162

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	377	48	52	527	52	112	91	51	52
N.S.	1	5.71	0.73	0.79	7.98	0.79	1.70	1.38	0.77	0.79
time (sec)	N/A	1.494	1.172	0.092	0.228	0.091	0.114	0.144	0.156	0.185

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	0	39	41	52	40	124	95	40	69
N.S.	1	0.00	0.93	0.98	1.24	0.95	2.95	2.26	0.95	1.64
time (sec)	N/A	0.000	0.076	0.123	0.110	0.274	0.106	0.158	0.154	0.121

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	B	B	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	0	43	43	55	43	121	98	43	71
N.S.	1	0.00	0.96	0.96	1.22	0.96	2.69	2.18	0.96	1.58
time (sec)	N/A	0.000	0.063	0.101	0.113	0.199	0.125	0.161	0.149	0.094

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	B	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	50	53	68	55	158	132	52	57
N.S.	1	0.00	0.72	0.77	0.99	0.80	2.29	1.91	0.75	0.83
time (sec)	N/A	0.000	0.076	0.116	0.110	0.190	0.219	0.172	0.158	0.209

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	24	76	92	75	94	79	75	76
N.S.	1	1.00	0.31	0.97	1.18	0.96	1.21	1.01	0.96	0.97
time (sec)	N/A	0.349	0.056	0.161	0.028	0.080	0.068	0.131	0.156	0.144

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	24	64	77	63	80	67	63	63
N.S.	1	1.00	0.37	0.98	1.18	0.97	1.23	1.03	0.97	0.97
time (sec)	N/A	0.326	0.052	0.095	0.033	0.082	0.062	0.142	0.154	0.100

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	110	53	52	62	51	66	55	51	52
N.S.	1	1.28	0.62	0.60	0.72	0.59	0.77	0.64	0.59	0.60
time (sec)	N/A	0.555	0.055	0.065	0.033	0.087	0.062	0.121	0.153	0.106

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	77	41	40	47	39	53	43	39	39
N.S.	1	1.24	0.66	0.65	0.76	0.63	0.85	0.69	0.63	0.63
time (sec)	N/A	0.441	0.051	0.049	0.035	0.085	0.051	0.117	0.154	0.097

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	32	27	39	689	27	27
N.S.	1	1.00	0.66	0.64	0.73	0.61	0.89	15.66	0.61	0.61
time (sec)	N/A	0.335	0.048	0.035	0.029	0.079	0.049	0.127	0.160	0.079

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	22	18	18	18
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.10	0.90	0.90	0.90
time (sec)	N/A	0.259	0.035	0.028	0.025	0.083	0.039	0.107	0.158	0.062

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	13	13	0	0	13	13
N.S.	1	1.00	1.00	1.07	0.87	0.87	0.00	0.00	0.87	0.87
time (sec)	N/A	0.259	0.041	0.039	0.074	0.080	0.000	0.000	0.161	0.086

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	35	18	35	0	0	33	32
N.S.	1	1.00	0.91	1.00	0.51	1.00	0.00	0.00	0.94	0.91
time (sec)	N/A	0.327	0.049	0.047	0.072	0.079	0.000	0.000	0.160	0.108

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	59	48	52	22	48	0	0	52	57
N.S.	1	1.02	0.83	0.90	0.38	0.83	0.00	0.00	0.90	0.98
time (sec)	N/A	0.412	0.061	0.072	0.081	0.085	0.000	0.000	0.161	0.098

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	59	72	22	59	0	0	71	69
N.S.	1	1.02	0.73	0.89	0.27	0.73	0.00	0.00	0.88	0.85
time (sec)	N/A	0.500	0.073	0.099	0.075	0.083	0.000	0.000	0.154	0.099

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	91	22	71	0	0	90	90
N.S.	1	1.00	1.00	3.79	0.92	2.96	0.00	0.00	3.75	3.75
time (sec)	N/A	0.269	0.047	0.160	0.070	0.081	0.000	0.000	0.160	0.113

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	110	22	83	0	0	109	102
N.S.	1	1.00	1.00	4.58	0.92	3.46	0.00	0.00	4.54	4.25
time (sec)	N/A	0.267	0.049	0.234	0.068	0.084	0.000	0.000	0.160	0.110

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	123	127	113	0	116	167	154
N.S.	1	1.00	1.00	3.62	3.74	3.32	0.00	3.41	4.91	4.53
time (sec)	N/A	0.267	0.075	0.297	0.027	0.076	0.000	0.106	0.156	0.155

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	111	112	101	0	104	143	139
N.S.	1	1.00	1.00	3.26	3.29	2.97	0.00	3.06	4.21	4.09
time (sec)	N/A	0.263	0.067	0.169	0.029	0.082	0.000	0.109	0.167	0.125

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	164	95	99	97	89	0	92	119	116
N.S.	1	1.28	0.74	0.77	0.76	0.70	0.00	0.72	0.93	0.91
time (sec)	N/A	0.662	0.110	0.112	0.027	0.083	0.000	0.112	0.156	0.111

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	129	83	87	82	77	0	80	95	98
N.S.	1	1.23	0.79	0.83	0.78	0.73	0.00	0.76	0.90	0.93
time (sec)	N/A	0.541	0.096	0.080	0.039	0.095	0.000	0.115	0.150	0.113

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	94	71	75	67	65	0	68	71	75
N.S.	1	1.15	0.87	0.91	0.82	0.79	0.00	0.83	0.87	0.91
time (sec)	N/A	0.423	0.088	0.056	0.035	0.080	0.000	0.116	0.149	0.100

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	54	53	49	0	57	50	54
N.S.	1	1.00	1.00	0.92	0.90	0.83	0.00	0.97	0.85	0.92
time (sec)	N/A	0.321	0.072	0.040	0.032	0.111	0.000	0.114	0.151	0.084

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	26	25	32	0	26	26	26
N.S.	1	1.00	1.00	0.70	0.68	0.86	0.00	0.70	0.70	0.70
time (sec)	N/A	0.244	0.053	0.026	0.032	0.096	0.000	0.135	0.145	0.062

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	28	40	0	0	17	44
N.S.	1	1.00	1.00	0.90	0.57	0.82	0.00	0.00	0.35	0.90
time (sec)	N/A	0.312	0.080	0.042	0.074	0.088	0.000	0.000	0.165	0.094

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	67	28	57	0	0	17	70
N.S.	1	1.00	0.85	0.92	0.38	0.78	0.00	0.00	0.23	0.96
time (sec)	N/A	0.396	0.112	0.063	0.072	0.082	0.000	0.000	0.157	0.121

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	77	86	28	73	0	0	17	109
N.S.	1	1.01	0.80	0.90	0.29	0.76	0.00	0.00	0.18	1.14
time (sec)	N/A	0.483	0.106	0.079	0.075	0.091	0.000	0.000	0.159	0.120

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	89	98	28	85	0	0	17	131
N.S.	1	1.02	0.75	0.82	0.24	0.71	0.00	0.00	0.14	1.10
time (sec)	N/A	0.573	0.118	0.127	0.077	0.078	0.000	0.000	0.168	0.107

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	110	28	97	0	0	17	153
N.S.	1	1.00	1.00	3.24	0.82	2.85	0.00	0.00	0.50	4.50
time (sec)	N/A	0.262	0.081	0.185	0.074	0.084	0.000	0.000	0.163	0.119

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	122	28	109	0	0	17	175
N.S.	1	1.00	1.00	3.59	0.82	3.21	0.00	0.00	0.50	5.15
time (sec)	N/A	0.261	0.077	0.285	0.076	0.084	0.000	0.000	0.159	0.127

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	24	76	92	75	94	0	75	76
N.S.	1	1.00	0.31	0.97	1.18	0.96	1.21	0.00	0.96	0.97
time (sec)	N/A	0.331	0.081	0.425	0.034	0.088	0.075	0.000	0.156	0.134

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	24	64	77	63	80	105	63	63
N.S.	1	1.00	0.37	0.98	1.18	0.97	1.23	1.62	0.97	0.97
time (sec)	N/A	0.317	0.065	0.250	0.033	0.085	0.066	0.137	0.157	0.100

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	110	53	52	62	51	66	83	51	51
N.S.	1	1.31	0.63	0.62	0.74	0.61	0.79	0.99	0.61	0.61
time (sec)	N/A	0.572	0.062	0.140	0.031	0.083	0.062	0.122	0.161	0.103

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	41	40	47	39	53	61	39	39
N.S.	1	1.15	0.61	0.60	0.70	0.58	0.79	0.91	0.58	0.58
time (sec)	N/A	0.454	0.054	0.079	0.027	0.086	0.054	0.105	0.160	0.078

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	32	27	39	689	27	27
N.S.	1	1.00	0.66	0.64	0.73	0.61	0.89	15.66	0.61	0.61
time (sec)	N/A	0.346	0.047	0.046	0.028	0.093	0.053	0.130	0.157	0.066

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	18	22	18	18	18
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.10	0.90	0.90	0.90
time (sec)	N/A	0.264	0.021	0.033	0.029	0.084	0.040	0.128	0.153	0.052

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	13	13	0	0	13	13
N.S.	1	1.00	1.00	2.73	0.87	0.87	0.00	0.00	0.87	0.87
time (sec)	N/A	0.258	0.042	0.033	0.076	0.087	0.000	0.000	0.156	0.063

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	97	18	35	0	0	33	32
N.S.	1	1.00	0.91	2.77	0.51	1.00	0.00	0.00	0.94	0.91
time (sec)	N/A	0.328	0.052	0.049	0.078	0.080	0.000	0.000	0.154	0.093

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	59	48	141	22	48	0	0	52	57
N.S.	1	1.02	0.83	2.43	0.38	0.83	0.00	0.00	0.90	0.98
time (sec)	N/A	0.412	0.067	0.090	0.075	0.083	0.000	0.000	0.157	0.095

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	59	177	22	59	0	0	71	69
N.S.	1	1.02	0.73	2.19	0.27	0.73	0.00	0.00	0.88	0.85
time (sec)	N/A	0.504	0.073	0.174	0.081	0.074	0.000	0.000	0.162	0.091

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	213	22	71	0	0	90	90
N.S.	1	1.00	1.00	8.88	0.92	2.96	0.00	0.00	3.75	3.75
time (sec)	N/A	0.270	0.048	0.332	0.071	0.072	0.000	0.000	0.159	0.106

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	249	22	83	0	0	109	102
N.S.	1	1.00	1.00	10.38	0.92	3.46	0.00	0.00	4.54	4.25
time (sec)	N/A	0.273	0.048	0.593	0.074	0.076	0.000	0.000	0.169	0.098

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	106	28	49	0	0	37	71
N.S.	1	1.00	1.00	3.12	0.82	1.44	0.00	0.00	1.09	2.09
time (sec)	N/A	0.278	0.082	0.053	0.072	0.114	0.000	0.000	0.161	0.070

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	109	28	47	0	0	33	75
N.S.	1	1.00	1.00	3.21	0.82	1.38	0.00	0.00	0.97	2.21
time (sec)	N/A	0.273	0.085	0.047	0.068	0.082	0.000	0.000	0.161	0.064

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	75	28	29	0	0	15	0
N.S.	1	1.00	1.00	2.21	0.82	0.85	0.00	0.00	0.44	0.00
time (sec)	N/A	0.265	0.077	0.035	0.069	0.089	0.000	0.000	0.150	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	78	26	29	0	0	13	0
N.S.	1	1.00	1.00	2.44	0.81	0.91	0.00	0.00	0.41	0.00
time (sec)	N/A	0.245	0.084	0.027	0.073	0.092	0.000	0.000	0.149	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	100	28	38	0	0	17	63
N.S.	1	1.00	1.00	2.94	0.82	1.12	0.00	0.00	0.50	1.85
time (sec)	N/A	0.263	0.102	0.039	0.068	0.098	0.000	0.000	0.154	0.090

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	102	28	41	0	0	17	70
N.S.	1	1.00	1.00	3.00	0.82	1.21	0.00	0.00	0.50	2.06
time (sec)	N/A	0.263	0.097	0.043	0.071	0.092	0.000	0.000	0.149	0.088

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	8	7	8	9	8
N.S.	1	1.00	1.00	0.82	0.73	0.73	0.64	0.73	0.82	0.73
time (sec)	N/A	0.228	0.019	0.031	0.026	0.082	0.042	0.114	0.155	0.051

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	121	22	80	0	0	103	99
N.S.	1	1.00	1.00	5.50	1.00	3.64	0.00	0.00	4.68	4.50
time (sec)	N/A	0.256	0.008	0.122	0.075	0.077	0.000	0.000	0.148	0.143

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	99	21	68	0	0	84	87
N.S.	1	1.00	1.00	4.71	1.00	3.24	0.00	0.00	4.00	4.14
time (sec)	N/A	0.253	0.006	0.098	0.075	0.074	0.000	0.000	0.161	0.124

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	76	53	77	22	56	0	0	65	66
N.S.	1	0.96	0.67	0.97	0.28	0.71	0.00	0.00	0.82	0.84
time (sec)	N/A	0.443	0.029	0.083	0.079	0.079	0.000	0.000	0.165	0.108

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	52	40	55	21	43	0	0	46	54
N.S.	1	0.93	0.71	0.98	0.38	0.77	0.00	0.00	0.82	0.96
time (sec)	N/A	0.361	0.022	0.076	0.081	0.096	0.000	0.000	0.150	0.111

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	31	18	30	0	0	27	27
N.S.	1	1.00	1.00	1.11	0.64	1.07	0.00	0.00	0.96	0.96
time (sec)	N/A	0.291	0.011	0.063	0.074	0.081	0.000	0.000	0.147	0.095

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	13	13	0	0	13	13
N.S.	1	1.00	1.00	1.15	1.00	1.00	0.00	0.00	1.00	1.00
time (sec)	N/A	0.247	0.005	0.064	0.075	0.080	0.000	0.000	0.151	0.074

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	20	20	20	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.11	1.11	1.11	1.00
time (sec)	N/A	0.248	0.007	0.032	0.027	0.076	0.047	0.131	0.149	0.063

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	27	32	21	31	22	0	29	27
N.S.	1	1.00	0.69	0.82	0.54	0.79	0.56	0.00	0.74	0.69
time (sec)	N/A	0.323	0.010	0.039	0.079	0.075	0.047	0.000	0.156	0.063

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	70	41	44	22	43	39	0	43	45
N.S.	1	1.15	0.67	0.72	0.36	0.70	0.64	0.00	0.70	0.74
time (sec)	N/A	0.419	0.013	0.051	0.076	0.090	0.060	0.000	0.152	0.087

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	101	53	56	21	55	53	0	55	57
N.S.	1	1.23	0.65	0.68	0.26	0.67	0.65	0.00	0.67	0.70
time (sec)	N/A	0.530	0.014	0.059	0.082	0.074	0.063	0.000	0.153	0.096

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	22	68	22	67	66	0	67	69
N.S.	1	1.00	0.34	1.05	0.34	1.03	1.02	0.00	1.03	1.06
time (sec)	N/A	0.315	0.006	0.072	0.077	0.084	0.066	0.000	0.148	0.110

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	21	80	21	79	80	0	79	81
N.S.	1	1.00	0.27	1.04	0.27	1.03	1.04	0.00	1.03	1.05
time (sec)	N/A	0.340	0.006	0.086	0.080	0.073	0.066	0.000	0.152	0.126

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	123	22	84	0	0	105	102
N.S.	1	1.00	1.00	5.12	0.92	3.50	0.00	0.00	4.38	4.25
time (sec)	N/A	0.273	0.008	0.323	0.071	0.072	0.000	0.000	0.149	0.230

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	101	22	72	0	0	86	90
N.S.	1	1.00	1.00	4.21	0.92	3.00	0.00	0.00	3.58	3.75
time (sec)	N/A	0.274	0.005	0.201	0.071	0.088	0.000	0.000	0.157	0.182

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	57	79	22	60	0	0	67	69
N.S.	1	1.02	0.70	0.98	0.27	0.74	0.00	0.00	0.83	0.85
time (sec)	N/A	0.503	0.024	0.125	0.078	0.080	0.000	0.000	0.156	0.163

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	59	44	57	22	47	0	0	48	57
N.S.	1	1.02	0.76	0.98	0.38	0.81	0.00	0.00	0.83	0.98
time (sec)	N/A	0.412	0.021	0.085	0.079	0.076	0.000	0.000	0.161	0.141

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	35	18	35	0	0	30	33
N.S.	1	1.00	0.91	1.00	0.51	1.00	0.00	0.00	0.86	0.94
time (sec)	N/A	0.319	0.009	0.056	0.075	0.080	0.000	0.000	0.156	0.146

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	13	13	0	0	13	13
N.S.	1	1.00	1.00	1.07	0.87	0.87	0.00	0.00	0.87	0.87
time (sec)	N/A	0.258	0.005	0.046	0.075	0.078	0.000	0.000	0.161	0.074

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	22	27	22	22	18
N.S.	1	1.00	1.00	0.95	0.90	1.10	1.35	1.10	1.10	0.90
time (sec)	N/A	0.257	0.007	0.043	0.028	0.076	0.050	0.115	0.158	0.063

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	35	22	34	29	0	34	36
N.S.	1	1.00	0.73	0.80	0.50	0.77	0.66	0.00	0.77	0.82
time (sec)	N/A	0.338	0.012	0.062	0.077	0.074	0.064	0.000	0.167	0.066

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	77	45	47	22	47	44	0	48	47
N.S.	1	1.24	0.73	0.76	0.35	0.76	0.71	0.00	0.77	0.76
time (sec)	N/A	0.438	0.014	0.095	0.074	0.077	0.063	0.000	0.158	0.087

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	110	58	59	22	60	58	0	60	60
N.S.	1	1.28	0.67	0.69	0.26	0.70	0.67	0.00	0.70	0.70
time (sec)	N/A	0.543	0.015	0.142	0.076	0.072	0.069	0.000	0.155	0.102

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	24	71	22	71	71	0	72	72
N.S.	1	1.00	0.35	1.03	0.32	1.03	1.03	0.00	1.04	1.04
time (sec)	N/A	0.320	0.006	0.213	0.081	0.074	0.098	0.000	0.163	0.116

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	24	83	22	84	85	0	84	84
N.S.	1	1.00	0.29	1.01	0.27	1.02	1.04	0.00	1.02	1.02
time (sec)	N/A	0.341	0.006	0.299	0.078	0.094	0.102	0.000	0.155	0.131

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	124	28	110	0	0	196	173
N.S.	1	1.00	1.00	3.65	0.82	3.24	0.00	0.00	5.76	5.09
time (sec)	N/A	0.274	0.008	0.378	0.083	0.077	0.000	0.000	0.159	0.160

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	112	28	98	0	0	171	151
N.S.	1	1.00	1.00	3.29	0.82	2.88	0.00	0.00	5.03	4.44
time (sec)	N/A	0.271	0.007	0.239	0.075	0.077	0.000	0.000	0.160	0.129

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	86	100	28	86	0	0	146	129
N.S.	1	1.02	0.72	0.84	0.24	0.72	0.00	0.00	1.23	1.08
time (sec)	N/A	0.643	0.044	0.155	0.075	0.076	0.000	0.000	0.165	0.116

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	74	88	28	74	0	0	121	107
N.S.	1	1.01	0.77	0.92	0.29	0.77	0.00	0.00	1.26	1.11
time (sec)	N/A	0.547	0.035	0.100	0.075	0.079	0.000	0.000	0.156	0.111

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	67	28	56	0	0	95	71
N.S.	1	1.00	0.82	0.92	0.38	0.77	0.00	0.00	1.30	0.97
time (sec)	N/A	0.452	0.028	0.065	0.075	0.077	0.000	0.000	0.150	0.180

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	26	42	0	0	69	44
N.S.	1	1.00	1.00	0.90	0.53	0.86	0.00	0.00	1.41	0.90
time (sec)	N/A	0.355	0.015	0.042	0.076	0.077	0.000	0.000	0.172	0.144

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	28	34	34	0	0	45	28
N.S.	1	1.00	1.00	0.72	0.87	0.87	0.00	0.00	1.15	0.72
time (sec)	N/A	0.305	0.009	0.054	0.084	0.081	0.000	0.000	0.167	0.097

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	58	28	58	0	0	19	58
N.S.	1	1.00	1.00	0.92	0.44	0.92	0.00	0.00	0.30	0.92
time (sec)	N/A	0.402	0.023	0.083	0.249	0.080	0.000	0.000	0.158	0.135

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	98	74	79	28	76	0	0	19	79
N.S.	1	1.14	0.86	0.92	0.33	0.88	0.00	0.00	0.22	0.92
time (sec)	N/A	0.518	0.061	0.132	0.134	0.083	0.000	0.000	0.159	0.151

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	133	86	91	28	88	0	0	19	102
N.S.	1	1.22	0.79	0.83	0.26	0.81	0.00	0.00	0.17	0.94
time (sec)	N/A	0.632	0.068	0.207	0.084	0.088	0.000	0.000	0.159	0.175

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	168	100	103	28	100	0	0	19	121
N.S.	1	1.27	0.76	0.78	0.21	0.76	0.00	0.00	0.14	0.92
time (sec)	N/A	0.770	0.096	0.315	0.091	0.087	0.000	0.000	0.154	0.174

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	115	28	112	0	0	19	142
N.S.	1	1.00	1.00	3.38	0.82	3.29	0.00	0.00	0.56	4.18
time (sec)	N/A	0.274	0.009	0.477	0.077	0.083	0.000	0.000	0.155	0.195

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	127	28	124	0	0	19	159
N.S.	1	1.00	1.00	3.74	0.82	3.65	0.00	0.00	0.56	4.68
time (sec)	N/A	0.276	0.010	0.690	0.081	0.075	0.000	0.000	0.156	0.210

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	249	22	84	0	0	105	102
N.S.	1	1.00	1.00	10.38	0.92	3.50	0.00	0.00	4.38	4.25
time (sec)	N/A	0.272	0.008	0.934	0.078	0.088	0.000	0.000	0.150	0.210

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	213	22	72	0	0	86	90
N.S.	1	1.00	1.00	8.88	0.92	3.00	0.00	0.00	3.58	3.75
time (sec)	N/A	0.273	0.006	0.532	0.080	0.070	0.000	0.000	0.154	0.177

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	57	177	22	60	0	0	67	69
N.S.	1	1.02	0.70	2.19	0.27	0.74	0.00	0.00	0.83	0.85
time (sec)	N/A	0.508	0.025	0.293	0.080	0.074	0.000	0.000	0.151	0.145

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	59	44	141	22	47	0	0	48	57
N.S.	1	1.02	0.76	2.43	0.38	0.81	0.00	0.00	0.83	0.98
time (sec)	N/A	0.430	0.018	0.153	0.081	0.073	0.000	0.000	0.149	0.130

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	97	18	35	0	0	30	33
N.S.	1	1.00	0.91	2.77	0.51	1.00	0.00	0.00	0.86	0.94
time (sec)	N/A	0.333	0.008	0.072	0.077	0.078	0.000	0.000	0.155	0.126

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	13	13	0	0	13	13
N.S.	1	1.00	1.00	2.73	0.87	0.87	0.00	0.00	0.87	0.87
time (sec)	N/A	0.260	0.005	0.051	0.075	0.074	0.000	0.000	0.159	0.074

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	22	27	22	22	18
N.S.	1	1.00	1.00	0.95	0.90	1.10	1.35	1.10	1.10	0.90
time (sec)	N/A	0.263	0.007	0.060	0.026	0.068	0.048	0.130	0.158	0.057

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	35	22	34	29	0	34	36
N.S.	1	1.00	0.73	0.80	0.50	0.77	0.66	0.00	0.77	0.82
time (sec)	N/A	0.343	0.010	0.113	0.077	0.078	0.081	0.000	0.157	0.063

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	45	47	22	47	44	0	48	48
N.S.	1	1.15	0.67	0.70	0.33	0.70	0.66	0.00	0.72	0.72
time (sec)	N/A	0.436	0.013	0.212	0.082	0.068	0.066	0.000	0.158	0.091

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	110	58	59	22	60	58	0	60	60
N.S.	1	1.33	0.70	0.71	0.27	0.72	0.70	0.00	0.72	0.72
time (sec)	N/A	0.552	0.015	0.364	0.075	0.082	0.076	0.000	0.156	0.119

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	24	71	22	71	71	0	72	72
N.S.	1	1.00	0.35	1.03	0.32	1.03	1.03	0.00	1.04	1.04
time (sec)	N/A	0.325	0.006	0.604	0.078	0.073	0.082	0.000	0.163	0.123

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	24	83	22	84	85	0	84	84
N.S.	1	1.00	0.29	1.01	0.27	1.02	1.04	0.00	1.02	1.02
time (sec)	N/A	0.340	0.006	0.921	0.076	0.079	0.096	0.000	0.157	0.136

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	120	28	55	0	0	96	88
N.S.	1	1.00	1.00	3.53	0.82	1.62	0.00	0.00	2.82	2.59
time (sec)	N/A	0.276	0.007	0.121	0.074	0.068	0.000	0.000	0.160	0.119

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	115	28	51	0	0	98	80
N.S.	1	1.00	1.00	3.38	0.82	1.50	0.00	0.00	2.88	2.35
time (sec)	N/A	0.270	0.007	0.100	0.077	0.079	0.000	0.000	0.162	0.113

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	105	28	41	0	0	70	70
N.S.	1	1.00	1.00	3.09	0.82	1.21	0.00	0.00	2.06	2.06
time (sec)	N/A	0.261	0.007	0.066	0.074	0.082	0.000	0.000	0.157	0.117

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	98	26	38	0	0	73	48
N.S.	1	1.00	1.00	3.06	0.81	1.19	0.00	0.00	2.28	1.50
time (sec)	N/A	0.244	0.006	0.054	0.072	0.070	0.000	0.000	0.168	0.099

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	82	28	29	0	0	45	46
N.S.	1	1.00	1.00	2.41	0.82	0.85	0.00	0.00	1.32	1.35
time (sec)	N/A	0.263	0.009	0.070	0.073	0.086	0.000	0.000	0.169	0.095

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	78	28	29	0	0	48	33
N.S.	1	1.00	1.00	2.29	0.82	0.85	0.00	0.00	1.41	0.97
time (sec)	N/A	0.264	0.008	0.087	0.077	0.073	0.000	0.000	0.157	0.084

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	112	28	53	0	0	19	77
N.S.	1	1.00	1.00	3.29	0.82	1.56	0.00	0.00	0.56	2.26
time (sec)	N/A	0.268	0.009	0.132	0.070	0.098	0.000	0.000	0.158	0.109

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	140	38	40	0	0	17	56
N.S.	1	1.00	1.00	3.04	0.83	0.87	0.00	0.00	0.37	1.22
time (sec)	N/A	0.281	0.035	0.069	0.080	0.090	0.000	0.000	0.155	0.090

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	140	38	40	0	0	17	49
N.S.	1	1.00	1.00	3.04	0.83	0.87	0.00	0.00	0.37	1.07
time (sec)	N/A	0.282	0.036	0.069	0.076	0.071	0.000	0.000	0.159	0.153

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	104	32	33	0	0	38	34
N.S.	1	1.00	1.00	3.06	0.94	0.97	0.00	0.00	1.12	1.00
time (sec)	N/A	0.276	0.030	0.071	0.073	0.075	0.000	0.000	0.160	0.078

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	136	35	0	0	0	17	52
N.S.	1	1.00	1.00	3.89	1.00	0.00	0.00	0.00	0.49	1.49
time (sec)	N/A	0.278	0.009	0.095	0.078	0.000	0.000	0.000	0.160	0.077

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	169	38	0	0	0	107	54
N.S.	1	1.00	1.00	3.67	0.83	0.00	0.00	0.00	2.33	1.17
time (sec)	N/A	0.290	0.012	0.178	0.073	0.000	0.000	0.000	0.161	0.093

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	169	38	0	0	0	112	54
N.S.	1	1.00	1.00	3.67	0.83	0.00	0.00	0.00	2.43	1.17
time (sec)	N/A	0.282	0.012	0.312	0.074	0.000	0.000	0.000	0.161	0.076

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	212	41	0	0	0	17	54
N.S.	1	1.00	1.00	5.44	1.05	0.00	0.00	0.00	0.44	1.38
time (sec)	N/A	0.289	0.012	0.099	0.089	0.000	0.000	0.000	0.157	0.160

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	212	41	0	0	0	17	54
N.S.	1	1.00	1.00	5.44	1.05	0.00	0.00	0.00	0.44	1.38
time (sec)	N/A	0.289	0.010	0.080	0.087	0.000	0.000	0.000	0.160	0.145

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	212	41	0	0	0	15	0
N.S.	1	1.00	1.00	5.44	1.05	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.278	0.010	0.056	0.090	0.000	0.000	0.000	0.162	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	201	35	0	0	0	13	0
N.S.	1	1.00	1.00	5.74	1.00	0.00	0.00	0.00	0.37	0.00
time (sec)	N/A	0.257	0.008	0.037	0.084	0.000	0.000	0.000	0.160	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	19	15	15	0	0	15	0
N.S.	1	1.00	1.00	1.27	1.00	1.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.258	0.006	0.079	0.085	0.070	0.000	0.000	0.159	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	195	37	0	0	0	17	52
N.S.	1	1.00	1.00	5.27	1.00	0.00	0.00	0.00	0.46	1.41
time (sec)	N/A	0.280	0.007	0.069	0.077	0.000	0.000	0.000	0.162	0.080

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	212	39	0	0	0	17	48
N.S.	1	1.00	1.00	5.44	1.00	0.00	0.00	0.00	0.44	1.23
time (sec)	N/A	0.274	0.007	0.095	0.076	0.000	0.000	0.000	0.158	0.077

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	212	39	0	0	0	17	52
N.S.	1	1.00	1.00	5.44	1.00	0.00	0.00	0.00	0.44	1.33
time (sec)	N/A	0.276	0.008	0.161	0.078	0.000	0.000	0.000	0.160	0.084

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	13	11	9	9	10	9	14	9
N.S.	1	1.00	0.65	0.55	0.45	0.45	0.50	0.45	0.70	0.45
time (sec)	N/A	0.315	0.074	0.088	0.028	0.069	0.029	0.115	0.156	0.014

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	280	47	0	0	0	17	79
N.S.	1	1.00	1.00	6.09	1.02	0.00	0.00	0.00	0.37	1.72
time (sec)	N/A	0.288	0.014	0.100	0.088	0.000	0.000	0.000	0.166	0.214

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	80	24	44	51	47	100	0	43	0
N.S.	1	1.13	0.34	0.62	0.72	0.66	1.41	0.00	0.61	0.00
time (sec)	N/A	0.490	0.007	0.074	0.035	0.078	2.234	0.000	0.154	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	25	30	34	33	73	0	29	0
N.S.	1	1.00	0.56	0.67	0.76	0.73	1.62	0.00	0.64	0.00
time (sec)	N/A	0.356	0.007	0.090	0.034	0.087	1.368	0.000	0.158	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	24	44	20	20	20
N.S.	1	1.00	1.00	1.05	1.00	1.20	2.20	1.00	1.00	1.00
time (sec)	N/A	0.264	0.007	0.091	0.030	0.073	0.916	0.133	0.163	0.099

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	19	15	15	0	0	15	0
N.S.	1	1.00	1.00	1.27	1.00	1.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.259	0.001	0.000	0.080	0.087	0.000	0.000	0.149	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	20	43	20	43	0	0	37	0
N.S.	1	1.00	0.53	1.13	0.53	1.13	0.00	0.00	0.97	0.00
time (sec)	N/A	0.346	0.007	0.128	0.084	0.083	0.000	0.000	0.149	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	67	25	70	25	61	0	0	61	0
N.S.	1	0.94	0.35	0.99	0.35	0.86	0.00	0.00	0.86	0.00
time (sec)	N/A	0.448	0.007	0.136	0.085	0.086	0.000	0.000	0.150	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	113	39	82	33	82	0	0	22	0
N.S.	1	1.09	0.38	0.79	0.32	0.79	0.00	0.00	0.21	0.00
time (sec)	N/A	0.590	0.014	0.103	0.080	0.082	0.000	0.000	0.152	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	39	67	33	64	0	0	22	0
N.S.	1	1.00	0.53	0.91	0.45	0.86	0.00	0.00	0.30	0.00
time (sec)	N/A	0.431	0.013	0.096	0.077	0.077	0.000	0.000	0.152	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	32	38	42	0	33	22	0
N.S.	1	1.00	1.00	0.74	0.88	0.98	0.00	0.77	0.51	0.00
time (sec)	N/A	0.317	0.013	0.103	0.080	0.094	0.000	0.143	0.157	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	39	59	35	83	0	0	24	0
N.S.	1	1.00	0.59	0.89	0.53	1.26	0.00	0.00	0.36	0.00
time (sec)	N/A	0.418	0.012	0.117	0.084	0.091	0.000	0.000	0.166	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	97	39	79	35	0	0	0	24	0
N.S.	1	1.01	0.41	0.82	0.36	0.00	0.00	0.00	0.25	0.00
time (sec)	N/A	0.535	0.011	0.108	0.079	0.000	0.000	0.000	0.167	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	96	249	264	113	0	136	233	171
N.S.	1	1.00	0.47	1.23	1.30	0.56	0.00	0.67	1.15	0.84
time (sec)	N/A	0.639	0.313	0.073	0.170	0.092	0.000	0.134	0.152	0.180

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	83	168	218	95	0	107	148	121
N.S.	1	1.00	0.59	1.20	1.56	0.68	0.00	0.76	1.06	0.86
time (sec)	N/A	0.493	0.255	0.052	0.143	0.078	0.000	0.137	0.179	0.143

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	63	80	131	72	0	77	83	66
N.S.	1	1.00	0.93	1.18	1.93	1.06	0.00	1.13	1.22	0.97
time (sec)	N/A	0.356	0.095	0.043	0.126	0.073	0.000	0.134	0.157	0.112

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	41	40	45	0	33	42	45
N.S.	1	1.00	1.00	1.00	0.98	1.10	0.00	0.80	1.02	1.10
time (sec)	N/A	0.254	0.046	0.031	0.035	0.074	0.000	0.120	0.151	0.051

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	12	17	31	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.80	1.13	2.07	1.13
time (sec)	N/A	0.260	0.192	0.008	0.115	0.074	0.844	0.118	0.155	0.059

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	17	31	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	1.13	2.07	1.13
time (sec)	N/A	0.449	0.788	0.009	0.113	0.077	0.514	0.124	0.159	0.075

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	17	31	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	1.13	2.07	1.13
time (sec)	N/A	0.638	0.611	0.010	0.114	0.075	0.577	0.124	0.166	0.074

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	111	0	0	155	0	0	131	0
N.S.	1	1.00	0.92	0.00	0.00	1.29	0.00	0.00	1.09	0.00
time (sec)	N/A	0.448	0.502	0.000	0.000	0.079	0.000	0.000	0.157	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	114	0	0	41	0
N.S.	1	1.00	0.93	0.00	0.00	1.24	0.00	0.00	0.45	0.00
time (sec)	N/A	0.369	0.328	0.000	0.000	0.077	0.000	0.000	0.156	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	60	0	0	39	0
N.S.	1	1.00	1.00	0.00	0.00	1.36	0.00	0.00	0.89	0.00
time (sec)	N/A	0.254	0.084	0.000	0.000	0.080	0.000	0.000	0.157	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	40	12	17	43	17
N.S.	1	1.00	1.13	1.00	1.13	2.67	0.80	1.13	2.87	1.13
time (sec)	N/A	0.255	0.513	0.007	0.134	0.074	0.981	0.153	0.158	0.063

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	40	14	17	43	17
N.S.	1	1.00	1.13	1.00	1.13	2.67	0.93	1.13	2.87	1.13
time (sec)	N/A	0.807	1.915	0.010	0.134	0.073	0.548	0.173	0.154	0.072

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	40	14	17	43	17
N.S.	1	1.00	1.13	1.00	1.13	2.67	0.93	1.13	2.87	1.13
time (sec)	N/A	1.127	1.488	0.010	0.133	0.076	0.619	0.152	0.157	0.073

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	35	32	34	34	39	34	38	34
N.S.	1	1.00	1.06	0.97	1.03	1.03	1.18	1.03	1.15	1.03
time (sec)	N/A	0.438	0.217	0.020	0.129	0.082	100.309	0.125	0.174	0.070

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	26	29	19	19	34	19	18	19
N.S.	1	0.98	0.65	0.72	0.48	0.48	0.85	0.48	0.45	0.48
time (sec)	N/A	0.321	0.043	0.011	0.025	0.078	0.090	0.108	0.163	0.051

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	241	517	0	243	0	0	1136	0
N.S.	1	1.00	0.83	1.78	0.00	0.84	0.00	0.00	3.90	0.00
time (sec)	N/A	0.927	0.186	0.222	0.000	0.079	0.000	0.000	0.165	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	179	359	0	171	0	0	854	0
N.S.	1	1.00	0.67	1.33	0.00	0.64	0.00	0.00	3.17	0.00
time (sec)	N/A	0.831	0.143	0.170	0.000	0.080	0.000	0.000	0.168	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	128	227	0	114	0	0	601	209
N.S.	1	1.00	0.56	0.99	0.00	0.50	0.00	0.00	2.62	0.91
time (sec)	N/A	0.739	0.101	0.141	0.000	0.084	0.000	0.000	0.161	0.313

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	82	126	0	71	0	0	380	136
N.S.	1	1.00	0.68	1.05	0.00	0.59	0.00	0.00	3.17	1.13
time (sec)	N/A	0.494	0.070	0.112	0.000	0.082	0.000	0.000	0.160	0.180

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	52	0	40	0	0	190	50
N.S.	1	1.00	1.00	1.27	0.00	0.98	0.00	0.00	4.63	1.22
time (sec)	N/A	0.332	0.023	0.084	0.000	0.079	0.000	0.000	0.163	0.112

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	47	0	41	0	0	17	0
N.S.	1	1.00	1.00	1.15	0.00	1.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.628	0.039	0.110	0.000	0.100	0.000	0.000	0.156	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	75	68	80	0	60	0	0	1123	0
N.S.	1	1.10	1.00	1.18	0.00	0.88	0.00	0.00	16.51	0.00
time (sec)	N/A	0.938	0.123	0.123	0.000	0.092	0.000	0.000	0.162	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	153	115	174	0	110	0	0	0	0
N.S.	1	0.92	0.69	1.05	0.00	0.66	0.00	0.00	0.00	0.00
time (sec)	N/A	1.423	0.189	0.151	0.000	0.079	0.000	0.000	0.163	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	195	343	0	201	0	0	0	0
N.S.	1	1.00	0.47	0.83	0.00	0.48	0.00	0.00	0.00	0.00
time (sec)	N/A	1.192	0.221	0.293	0.000	0.083	0.000	0.000	0.178	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	148	228	0	156	0	0	0	0
N.S.	1	1.00	0.51	0.78	0.00	0.54	0.00	0.00	0.00	0.00
time (sec)	N/A	0.912	0.147	0.204	0.000	0.079	0.000	0.000	0.172	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	131	175	0	128	0	0	0	0
N.S.	1	1.00	0.64	0.85	0.00	0.62	0.00	0.00	0.00	0.00
time (sec)	N/A	0.711	0.120	0.164	0.000	0.084	0.000	0.000	0.174	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	89	93	0	107	0	0	0	0
N.S.	1	1.00	0.80	0.84	0.00	0.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.088	0.120	0.000	0.079	0.000	0.000	0.174	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	65	0	68	0	0	966	53
N.S.	1	1.00	1.00	1.05	0.00	1.10	0.00	0.00	15.58	0.85
time (sec)	N/A	0.383	0.040	0.105	0.000	0.080	0.000	0.000	0.158	0.195

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	12	17	28	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.80	1.13	1.87	1.13
time (sec)	N/A	0.258	0.167	0.005	0.102	0.075	1.412	0.169	0.164	0.092

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	17	3627	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	1.13	241.80	1.13
time (sec)	N/A	0.333	0.468	0.010	0.105	0.082	1.588	0.154	0.186	0.130

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	17	3628	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	1.13	241.87	1.13
time (sec)	N/A	0.322	0.317	0.012	0.105	0.080	2.423	0.159	0.194	0.126

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	219	0	0	248	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	1.04	0.00	0.00	0.00	0.00
time (sec)	N/A	0.675	0.218	0.000	0.000	0.090	0.000	0.000	0.207	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	167	0	0	221	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.20	0.00	0.00	0.00	0.00
time (sec)	N/A	0.591	0.403	0.000	0.000	0.082	0.000	0.000	0.200	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	127	0	0	194	0	0	0	0
N.S.	1	1.00	0.89	0.00	0.00	1.37	0.00	0.00	0.00	0.00
time (sec)	N/A	0.500	0.078	0.000	0.000	0.077	0.000	0.000	0.187	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	154	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	1.67	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	0.076	0.000	0.000	0.078	0.000	0.000	0.174	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	94	0	0	0	68
N.S.	1	1.00	1.00	0.00	0.00	2.14	0.00	0.00	0.00	1.55
time (sec)	N/A	0.261	0.012	0.000	0.000	0.069	0.000	0.000	0.176	0.338

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	39	12	17	39	17
N.S.	1	1.00	1.13	1.00	1.13	2.60	0.80	1.13	2.60	1.13
time (sec)	N/A	0.260	0.155	0.006	0.125	0.068	2.131	0.197	0.146	0.097

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	39	14	17	10349	17
N.S.	1	1.00	1.13	1.00	1.13	2.60	0.93	1.13	689.93	1.13
time (sec)	N/A	0.333	0.514	0.010	0.123	0.070	2.857	0.216	0.203	0.255

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	39	14	17	12757	17
N.S.	1	1.00	1.13	1.00	1.13	2.60	0.93	1.13	850.47	1.13
time (sec)	N/A	0.333	0.044	0.011	0.131	0.075	3.973	0.193	0.230	0.266

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	40	14	17	43	17
N.S.	1	1.00	1.13	1.00	1.13	2.67	0.93	1.13	2.87	1.13
time (sec)	N/A	0.348	0.270	0.015	0.135	0.074	1.027	0.143	0.168	0.092

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	17	31	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	1.13	2.07	1.13
time (sec)	N/A	0.358	0.192	0.015	0.117	0.079	1.063	0.116	0.158	0.098

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	36	117	36	39	0	0	45	41
N.S.	1	1.00	0.88	2.85	0.88	0.95	0.00	0.00	1.10	1.00
time (sec)	N/A	0.294	0.037	0.073	0.077	0.070	0.000	0.000	0.159	0.080

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	12	17	77721	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.80	1.13	5181.40	1.13
time (sec)	N/A	0.341	0.052	0.017	0.082	0.078	3.440	0.120	0.239	0.089

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	28	14	17	441986	17
N.S.	1	1.00	1.13	1.00	1.13	1.87	0.93	1.13	29465.73	1.13
time (sec)	N/A	0.345	0.136	0.020	0.106	0.081	80.207	0.179	0.492	0.118

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.349	0.066	0.007	0.079	0.074	3.388	0.188	0.166	0.055

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	183	0	0	0	0	0	17	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.645	0.169	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	136	0	0	0	0	0	17	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.530	0.078	0.000	0.000	0.000	0.000	0.000	0.147	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	91	0	0	0	0	0	15	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.402	0.047	0.000	0.000	0.000	0.000	0.000	0.154	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	0	0	13	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.267	0.011	0.000	0.000	0.000	0.000	0.000	0.151	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	12	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.80	1.13	1.13	1.13
time (sec)	N/A	0.263	0.044	0.009	0.073	0.079	0.418	0.152	0.158	0.063

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.334	0.041	0.024	0.074	0.072	0.425	0.157	0.156	0.061

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13	1.13
time (sec)	N/A	0.343	0.043	0.005	0.069	0.073	0.518	0.168	0.171	0.059

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	189	0	34	0	41	38	0
N.S.	1	1.00	1.00	3.94	0.00	0.71	0.00	0.85	0.79	0.00
time (sec)	N/A	0.287	0.045	0.168	0.000	0.075	0.000	1.378	0.166	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	0	0	0	0	0	18	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00
time (sec)	N/A	0.306	0.020	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	59	0	0	37	75
N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	0.61	1.23
time (sec)	N/A	0.370	0.203	0.000	0.000	0.075	0.000	0.000	0.162	0.289

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	C	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	31	569	5261	468	794	145	578	553
N.S.	1	1.00	0.30	5.42	50.10	4.46	7.56	1.38	5.50	5.27
time (sec)	N/A	0.487	0.319	1.026	1.872	0.088	0.236	0.146	0.163	0.437

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	C	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	31	386	3727	324	556	124	395	391
N.S.	1	1.00	0.35	4.39	42.35	3.68	6.32	1.41	4.49	4.44
time (sec)	N/A	0.443	0.297	0.550	1.377	0.082	0.188	0.160	0.159	0.299

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	150	72	239	2452	208	364	103	248	253
N.S.	1	1.19	0.57	1.90	19.46	1.65	2.89	0.82	1.97	2.01
time (sec)	N/A	0.864	0.324	0.292	0.891	0.083	0.139	0.137	0.154	0.241

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	106	56	128	1438	120	212	82	137	142
N.S.	1	1.16	0.62	1.41	15.80	1.32	2.33	0.90	1.51	1.56
time (sec)	N/A	0.651	0.290	0.145	0.573	0.089	0.111	0.141	0.157	0.177

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	53	683	60	99	1227	62	67
N.S.	1	1.00	0.65	0.85	11.02	0.97	1.60	19.79	1.00	1.08
time (sec)	N/A	0.458	0.264	0.079	0.334	0.078	0.082	0.156	0.152	0.136

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	35	34	35	35	25
N.S.	1	1.00	1.00	0.96	0.93	1.30	1.26	1.30	1.30	0.93
time (sec)	N/A	0.298	0.035	0.038	0.025	0.077	0.066	0.117	0.148	0.100

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	32	0	0	37	20
N.S.	1	1.00	1.00	1.05	0.00	1.45	0.00	0.00	1.68	0.91
time (sec)	N/A	0.330	0.282	0.084	0.000	0.074	0.000	0.000	0.156	0.147

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	53	0	100	0	0	0	51
N.S.	1	1.00	0.89	1.00	0.00	1.89	0.00	0.00	0.00	0.96
time (sec)	N/A	0.473	0.311	0.125	0.000	0.079	0.000	0.000	0.171	0.666

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	88	64	86	0	183	0	0	0	76
N.S.	1	1.01	0.74	0.99	0.00	2.10	0.00	0.00	0.00	0.87
time (sec)	N/A	0.625	0.344	0.211	0.000	0.079	0.000	0.000	0.171	1.175

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	123	79	119	0	292	0	0	0	104
N.S.	1	1.02	0.65	0.98	0.00	2.41	0.00	0.00	0.00	0.86
time (sec)	N/A	0.796	0.366	0.375	0.000	0.078	0.000	0.000	0.183	0.272

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	152	0	430	0	0	0	120
N.S.	1	1.00	1.00	4.90	0.00	13.87	0.00	0.00	0.00	3.87
time (sec)	N/A	0.334	0.279	0.615	0.000	0.080	0.000	0.000	0.191	0.270

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	185	0	596	0	0	0	136
N.S.	1	1.00	1.00	5.97	0.00	19.23	0.00	0.00	0.00	4.39
time (sec)	N/A	0.337	0.294	0.999	0.000	0.091	0.000	0.000	0.201	0.354

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	1911	6135	617	0	195	1825	209
N.S.	1	1.00	1.00	39.00	125.20	12.59	0.00	3.98	37.24	4.27
time (sec)	N/A	0.343	0.860	2.985	2.099	0.087	0.000	0.168	0.184	0.376

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	1374	4471	456	0	174	1300	730
N.S.	1	1.00	1.00	28.04	91.24	9.31	0.00	3.55	26.53	14.90
time (sec)	N/A	0.350	0.739	1.564	1.529	0.084	0.000	0.160	0.173	0.396

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	215	153	929	3066	323	0	153	865	533
N.S.	1	1.20	0.85	5.19	17.13	1.80	0.00	0.85	4.83	2.98
time (sec)	N/A	1.032	0.984	0.786	1.110	0.089	0.000	0.144	0.167	0.285

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	169	126	576	1922	218	0	132	520	378
N.S.	1	1.17	0.87	3.97	13.26	1.50	0.00	0.91	3.59	2.61
time (sec)	N/A	0.802	0.502	0.401	0.719	0.083	0.000	0.141	0.159	0.232

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	123	90	315	1037	141	0	111	265	243
N.S.	1	1.11	0.81	2.84	9.34	1.27	0.00	1.00	2.39	2.19
time (sec)	N/A	0.594	0.380	0.192	0.432	0.080	0.000	0.147	0.149	0.177

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	146	413	88	0	91	115	130
N.S.	1	1.00	1.00	1.90	5.36	1.14	0.00	1.18	1.49	1.69
time (sec)	N/A	0.415	0.292	0.101	0.216	0.079	0.000	0.129	0.148	0.154

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	58	58	48	0	36	45	48
N.S.	1	1.00	1.00	1.32	1.32	1.09	0.00	0.82	1.02	1.09
time (sec)	N/A	0.258	0.096	0.037	0.035	0.076	0.000	0.116	0.154	0.022

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	62	0	83	0	0	48	86
N.S.	1	1.00	0.94	0.93	0.00	1.24	0.00	0.00	0.72	1.28
time (sec)	N/A	0.407	0.376	0.109	0.000	0.087	0.000	0.000	0.152	0.355

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	96	0	163	0	0	0	201
N.S.	1	1.00	0.79	0.94	0.00	1.60	0.00	0.00	0.00	1.97
time (sec)	N/A	0.550	0.452	0.157	0.000	0.079	0.000	0.000	0.168	0.860

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	137	97	129	0	288	0	0	0	168
N.S.	1	1.01	0.71	0.95	0.00	2.12	0.00	0.00	0.00	1.24
time (sec)	N/A	0.717	0.501	0.274	0.000	0.080	0.000	0.000	0.219	0.772

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	112	162	0	429	0	0	0	201
N.S.	1	1.01	0.66	0.95	0.00	2.52	0.00	0.00	0.00	1.18
time (sec)	N/A	0.899	0.579	0.482	0.000	0.086	0.000	0.000	0.251	0.421

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	195	0	598	0	0	0	234
N.S.	1	1.00	1.00	3.98	0.00	12.20	0.00	0.00	0.00	4.78
time (sec)	N/A	0.341	0.474	0.807	0.000	0.093	0.000	0.000	0.199	0.416

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	228	0	795	0	0	0	267
N.S.	1	1.00	1.00	4.65	0.00	16.22	0.00	0.00	0.00	5.45
time (sec)	N/A	0.340	0.567	1.301	0.000	0.092	0.000	0.000	0.214	0.482

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	71	0	0	49	75
N.S.	1	1.00	1.00	0.00	0.00	1.16	0.00	0.00	0.80	1.23
time (sec)	N/A	0.359	0.231	0.000	0.000	0.079	0.000	0.000	0.160	0.202

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	31	835	1268	688	1170	0	856	685
N.S.	1	1.00	0.30	7.95	12.08	6.55	11.14	0.00	8.15	6.52
time (sec)	N/A	0.455	0.367	5.595	0.247	0.092	0.322	0.000	0.167	0.526

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	31	562	874	474	821	0	583	487
N.S.	1	1.00	0.35	6.39	9.93	5.39	9.33	0.00	6.62	5.53
time (sec)	N/A	0.457	0.327	2.855	0.176	0.097	0.251	0.000	0.176	0.388

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	150	75	343	555	302	536	1320	364	323
N.S.	1	1.21	0.60	2.77	4.48	2.44	4.32	10.65	2.94	2.60
time (sec)	N/A	0.927	0.372	1.237	0.166	0.084	0.204	0.601	0.161	0.279

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	106	56	178	308	172	304	705	199	196
N.S.	1	1.10	0.58	1.85	3.21	1.79	3.17	7.34	2.07	2.04
time (sec)	N/A	0.691	0.337	0.500	0.164	0.082	0.131	0.364	0.159	0.186

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	40	67	133	84	143	1014	88	95
N.S.	1	1.00	0.65	1.08	2.15	1.35	2.31	16.35	1.42	1.53
time (sec)	N/A	0.492	0.293	0.171	0.161	0.088	0.097	0.202	0.159	0.174

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	47	44	47	47	25
N.S.	1	1.00	1.00	0.96	0.93	1.74	1.63	1.74	1.74	0.93
time (sec)	N/A	0.336	0.047	0.073	0.030	0.087	0.067	0.135	0.153	0.091

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	44	0	0	49	20
N.S.	1	1.00	1.00	0.00	0.00	2.00	0.00	0.00	2.23	0.91
time (sec)	N/A	0.330	0.325	0.000	0.000	0.079	0.000	0.000	0.163	0.106

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	0	0	147	0	0	0	51
N.S.	1	1.00	0.89	0.00	0.00	2.77	0.00	0.00	0.00	0.96
time (sec)	N/A	0.474	0.338	0.000	0.000	0.076	0.000	0.000	0.195	0.310

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	88	64	0	0	269	0	0	0	76
N.S.	1	1.01	0.74	0.00	0.00	3.09	0.00	0.00	0.00	0.87
time (sec)	N/A	0.650	0.373	0.000	0.000	0.080	0.000	0.000	0.215	0.746

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	123	80	0	0	431	0	0	0	104
N.S.	1	1.02	0.66	0.00	0.00	3.56	0.00	0.00	0.00	0.86
time (sec)	N/A	0.797	0.399	0.000	0.000	0.082	0.000	0.000	0.242	0.285

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	636	0	0	0	120
N.S.	1	1.00	1.00	0.00	0.00	20.52	0.00	0.00	0.00	3.87
time (sec)	N/A	0.336	0.327	0.000	0.000	0.092	0.000	0.000	0.306	0.359

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	883	0	0	0	136
N.S.	1	1.00	1.00	0.00	0.00	28.48	0.00	0.00	0.00	4.39
time (sec)	N/A	0.337	0.341	0.000	0.000	0.097	0.000	0.000	0.331	0.518

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	118	0	0	119	112
N.S.	1	1.00	1.00	0.00	0.00	2.41	0.00	0.00	2.43	2.29
time (sec)	N/A	0.350	0.364	0.000	0.000	0.088	0.000	0.000	0.255	0.293

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	63	0	0	79	0
N.S.	1	1.00	1.00	0.00	0.00	1.29	0.00	0.00	1.61	0.00
time (sec)	N/A	0.314	0.271	0.000	0.000	0.085	0.000	0.000	0.211	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	63	0	0	41	0
N.S.	1	1.00	1.00	0.00	0.00	1.34	0.00	0.00	0.87	0.00
time (sec)	N/A	0.263	0.147	0.000	0.000	0.076	0.000	0.000	0.220	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	110	0	0	60	74
N.S.	1	1.00	1.00	0.00	0.00	2.24	0.00	0.00	1.22	1.51
time (sec)	N/A	0.337	0.524	0.000	0.000	0.080	0.000	0.000	0.185	0.223

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	135	0	0	71	87
N.S.	1	1.00	1.00	0.00	0.00	2.76	0.00	0.00	1.45	1.78
time (sec)	N/A	0.332	0.553	0.000	0.000	0.080	0.000	0.000	0.183	0.246

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	226	0	0	0	130
N.S.	1	1.00	1.00	0.00	0.00	4.61	0.00	0.00	0.00	2.65
time (sec)	N/A	0.336	0.598	0.000	0.000	0.080	0.000	0.000	0.239	0.550

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	C	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	42	0	43	42	73	781	36	38
N.S.	1	0.97	0.66	0.00	0.67	0.66	1.14	12.20	0.56	0.59
time (sec)	N/A	0.354	0.072	0.000	0.035	0.084	0.181	0.156	0.212	0.115

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	C	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	60	0	62	58	0	1338	54	54
N.S.	1	1.04	0.60	0.00	0.62	0.58	0.00	13.38	0.54	0.54
time (sec)	N/A	0.449	0.062	0.000	0.034	0.087	0.000	0.197	0.214	0.127

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	0	73
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.46
time (sec)	N/A	0.338	0.025	0.000	0.000	0.000	0.000	0.000	0.197	0.184

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	534	0	244	0	0	827	181
N.S.	1	1.00	1.00	18.41	0.00	8.41	0.00	0.00	28.52	6.24
time (sec)	N/A	0.315	0.010	0.431	0.000	0.076	0.000	0.000	0.195	0.186

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	368	0	175	0	0	594	148
N.S.	1	1.00	1.00	13.14	0.00	6.25	0.00	0.00	21.21	5.29
time (sec)	N/A	0.310	0.010	0.297	0.000	0.080	0.000	0.000	0.190	0.179

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	116	76	234	0	120	0	0	402	89
N.S.	1	0.97	0.64	1.97	0.00	1.01	0.00	0.00	3.38	0.75
time (sec)	N/A	0.623	0.084	0.224	0.000	0.084	0.000	0.000	0.192	0.257

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	81	58	133	0	77	0	0	248	82
N.S.	1	0.95	0.68	1.56	0.00	0.91	0.00	0.00	2.92	0.96
time (sec)	N/A	0.505	0.060	0.131	0.000	0.081	0.000	0.000	0.191	1.351

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	61	0	51	0	0	230	47
N.S.	1	1.00	0.91	1.33	0.00	1.11	0.00	0.00	5.00	1.02
time (sec)	N/A	0.367	0.033	0.099	0.000	0.085	0.000	0.000	0.191	0.471

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	22	0	20	0	0	29	20
N.S.	1	1.00	1.00	1.10	0.00	1.00	0.00	0.00	1.45	1.00
time (sec)	N/A	0.300	0.009	0.151	0.000	0.078	0.000	0.000	0.192	0.150

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	31	34	31	31	25
N.S.	1	1.00	1.00	1.04	1.00	1.24	1.36	1.24	1.24	1.00
time (sec)	N/A	0.303	0.012	0.118	0.027	0.074	0.098	0.121	0.185	0.806

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	41	51	0	51	44	0	47	41
N.S.	1	1.00	0.72	0.89	0.00	0.89	0.77	0.00	0.82	0.72
time (sec)	N/A	0.422	0.025	0.167	0.000	0.077	0.081	0.000	0.204	1.438

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	99	60	79	0	95	102	0	89	104
N.S.	1	1.10	0.67	0.88	0.00	1.06	1.13	0.00	0.99	1.16
time (sec)	N/A	0.563	0.034	0.236	0.000	0.078	0.104	0.000	0.206	1.100

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	141	76	125	0	150	177	0	146	161
N.S.	1	1.16	0.62	1.02	0.00	1.23	1.45	0.00	1.20	1.32
time (sec)	N/A	0.727	0.045	0.347	0.000	0.080	0.123	0.000	0.203	0.227

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	29	189	0	219	272	0	221	231
N.S.	1	1.00	0.32	2.05	0.00	2.38	2.96	0.00	2.40	2.51
time (sec)	N/A	0.402	0.011	0.458	0.000	0.083	0.143	0.000	0.217	0.276

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	28	271	0	302	388	0	314	315
N.S.	1	1.00	0.26	2.51	0.00	2.80	3.59	0.00	2.91	2.92
time (sec)	N/A	0.420	0.010	0.643	0.000	0.086	0.169	0.000	0.216	0.329

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0	73
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.20
time (sec)	N/A	0.340	0.042	0.000	0.000	0.000	0.000	0.000	0.194	0.223

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	961	0	465	0	0	0	136
N.S.	1	1.00	1.00	31.00	0.00	15.00	0.00	0.00	0.00	4.39
time (sec)	N/A	0.314	0.013	3.441	0.000	0.092	0.000	0.000	0.204	0.295

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	646	0	331	0	0	0	120
N.S.	1	1.00	1.00	20.84	0.00	10.68	0.00	0.00	0.00	3.87
time (sec)	N/A	0.310	0.012	1.783	0.000	0.092	0.000	0.000	0.216	0.231

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	123	96	395	0	225	0	0	0	92
N.S.	1	1.02	0.79	3.26	0.00	1.86	0.00	0.00	0.00	0.76
time (sec)	N/A	0.693	0.180	0.908	0.000	0.087	0.000	0.000	0.215	0.220

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	88	71	208	0	145	0	0	0	76
N.S.	1	1.01	0.82	2.39	0.00	1.67	0.00	0.00	0.00	0.87
time (sec)	N/A	0.546	0.051	0.393	0.000	0.085	0.000	0.000	0.216	0.173

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	86	0	96	0	0	0	51
N.S.	1	1.00	0.89	1.62	0.00	1.81	0.00	0.00	0.00	0.96
time (sec)	N/A	0.400	0.030	0.157	0.000	0.090	0.000	0.000	0.221	1.062

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	31	0	0	52	20
N.S.	1	1.00	1.00	1.05	0.00	1.41	0.00	0.00	2.36	0.91
time (sec)	N/A	0.298	0.010	0.153	0.000	0.083	0.000	0.000	0.204	0.204

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	54	53	54	54	37
N.S.	1	1.00	1.00	0.96	0.93	2.00	1.96	2.00	2.00	1.37
time (sec)	N/A	0.300	0.014	0.260	0.025	0.074	0.136	0.136	0.190	0.112

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	47	74	101	100	82	0	93	97
N.S.	1	1.00	0.76	1.19	1.63	1.61	1.32	0.00	1.50	1.56
time (sec)	N/A	0.427	0.031	0.490	0.045	0.082	0.112	0.000	0.210	0.181

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	106	64	127	208	180	189	1703	171	183
N.S.	1	1.16	0.70	1.40	2.29	1.98	2.08	18.71	1.88	2.01
time (sec)	N/A	0.576	0.042	0.988	0.042	0.081	0.149	0.201	0.198	0.287

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	150	81	216	349	287	333	0	282	292
N.S.	1	1.19	0.64	1.71	2.77	2.28	2.64	0.00	2.24	2.32
time (sec)	N/A	0.751	0.053	1.709	0.041	0.086	0.174	0.000	0.200	0.452

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	31	341	526	420	518	0	429	427
N.S.	1	1.00	0.32	3.55	5.48	4.38	5.40	0.00	4.47	4.45
time (sec)	N/A	0.410	0.011	2.923	0.049	0.093	0.300	0.000	0.196	0.636

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	31	502	740	583	745	0	612	583
N.S.	1	1.00	0.27	4.44	6.55	5.16	6.59	0.00	5.42	5.16
time (sec)	N/A	0.436	0.012	4.691	0.056	0.110	0.412	0.000	0.214	0.867

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	1173	0	561	0	0	0	265
N.S.	1	1.00	1.00	23.94	0.00	11.45	0.00	0.00	0.00	5.41
time (sec)	N/A	0.331	0.037	4.255	0.000	0.093	0.000	0.000	0.192	0.488

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	826	0	413	0	0	0	232
N.S.	1	1.00	1.00	16.86	0.00	8.43	0.00	0.00	0.00	4.73
time (sec)	N/A	0.328	0.034	2.378	0.000	0.089	0.000	0.000	0.202	0.414

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	172	113	543	0	293	0	0	0	199
N.S.	1	1.01	0.66	3.19	0.00	1.72	0.00	0.00	0.00	1.17
time (sec)	N/A	0.925	0.164	1.210	0.000	0.091	0.000	0.000	0.196	0.557

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	137	97	324	0	201	0	0	0	166
N.S.	1	1.01	0.71	2.38	0.00	1.48	0.00	0.00	0.00	1.22
time (sec)	N/A	0.735	0.123	0.612	0.000	0.086	0.000	0.000	0.191	0.334

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	79	169	0	130	0	0	0	97
N.S.	1	1.00	0.77	1.66	0.00	1.27	0.00	0.00	0.00	0.95
time (sec)	N/A	0.565	0.100	0.294	0.000	0.086	0.000	0.000	0.213	0.497

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	63	74	0	91	0	0	1286	62
N.S.	1	1.00	0.94	1.10	0.00	1.36	0.00	0.00	19.19	0.93
time (sec)	N/A	0.419	0.055	0.110	0.000	0.079	0.000	0.000	0.200	0.691

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	45	0	0	1242	35
N.S.	1	1.00	1.00	0.76	0.00	0.98	0.00	0.00	27.00	0.76
time (sec)	N/A	0.340	0.015	0.265	0.000	0.084	0.000	0.000	0.191	0.205

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	76	0	117	0	0	1118	76
N.S.	1	1.00	1.00	0.94	0.00	1.44	0.00	0.00	13.80	0.94
time (sec)	N/A	0.495	0.059	0.561	0.000	0.093	0.000	0.000	0.196	0.326

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	95	109	0	199	0	0	1610	105
N.S.	1	1.10	0.83	0.95	0.00	1.73	0.00	0.00	14.00	0.91
time (sec)	N/A	0.654	0.112	1.143	0.000	0.089	0.000	0.000	0.205	0.416

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	173	111	142	0	305	0	0	0	142
N.S.	1	1.16	0.74	0.95	0.00	2.05	0.00	0.00	0.00	0.95
time (sec)	N/A	0.835	0.156	2.238	0.000	0.093	0.000	0.000	0.210	0.541

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	219	127	175	0	439	0	0	0	160
N.S.	1	1.20	0.69	0.96	0.00	2.40	0.00	0.00	0.00	0.87
time (sec)	N/A	1.030	0.202	3.967	0.000	0.100	0.000	0.000	0.206	0.687

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	208	0	601	0	0	0	189
N.S.	1	1.00	1.00	4.24	0.00	12.27	0.00	0.00	0.00	3.86
time (sec)	N/A	0.317	0.054	6.711	0.000	0.108	0.000	0.000	0.210	0.850

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	241	0	791	0	0	0	217
N.S.	1	1.00	1.00	4.92	0.00	16.14	0.00	0.00	0.00	4.43
time (sec)	N/A	0.324	0.035	10.398	0.000	0.124	0.000	0.000	0.242	0.971

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	0	73
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.20
time (sec)	N/A	0.342	0.053	0.000	0.000	0.000	0.000	0.000	0.212	0.245

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	686	0	0	0	136
N.S.	1	1.00	1.00	0.00	0.00	22.13	0.00	0.00	0.00	4.39
time (sec)	N/A	0.309	0.016	0.000	0.000	0.100	0.000	0.000	0.255	0.356

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	0	0	487	0	0	0	120
N.S.	1	1.00	1.00	0.00	0.00	15.71	0.00	0.00	0.00	3.87
time (sec)	N/A	0.308	0.014	0.000	0.000	0.099	0.000	0.000	0.229	0.302

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	123	96	0	0	330	0	0	0	92
N.S.	1	1.02	0.79	0.00	0.00	2.73	0.00	0.00	0.00	0.76
time (sec)	N/A	0.702	0.233	0.000	0.000	0.090	0.000	0.000	0.218	0.308

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	88	71	0	0	213	0	0	0	76
N.S.	1	1.01	0.82	0.00	0.00	2.45	0.00	0.00	0.00	0.87
time (sec)	N/A	0.571	0.086	0.000	0.000	0.091	0.000	0.000	0.197	0.225

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	47	0	0	141	0	0	0	51
N.S.	1	1.00	0.89	0.00	0.00	2.66	0.00	0.00	0.00	0.96
time (sec)	N/A	0.418	0.041	0.000	0.000	0.084	0.000	0.000	0.204	0.182

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	0	42	0	0	75	20
N.S.	1	1.00	1.00	0.00	0.00	1.91	0.00	0.00	3.41	0.91
time (sec)	N/A	0.295	0.013	0.000	0.000	0.083	0.000	0.000	0.169	0.165

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	77	65	77	77	48
N.S.	1	1.00	1.00	0.96	0.93	2.85	2.41	2.85	2.85	1.78
time (sec)	N/A	0.303	0.018	0.499	0.025	0.081	0.188	0.143	0.185	0.147

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	47	97	144	148	114	0	138	136
N.S.	1	1.00	0.76	1.56	2.32	2.39	1.84	0.00	2.23	2.19
time (sec)	N/A	0.429	0.036	1.263	0.041	0.080	0.142	0.000	0.268	0.247

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	106	64	175	300	265	270	0	252	263
N.S.	1	1.10	0.67	1.82	3.12	2.76	2.81	0.00	2.62	2.74
time (sec)	N/A	0.576	0.055	2.883	0.044	0.087	0.194	0.000	0.211	0.399

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	150	73	307	507	423	484	0	417	422
N.S.	1	1.22	0.59	2.50	4.12	3.44	3.93	0.00	3.39	3.43
time (sec)	N/A	0.733	0.042	5.802	0.047	0.105	0.267	0.000	0.194	0.685

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	31	493	770	621	760	0	636	620
N.S.	1	1.00	0.32	5.14	8.02	6.47	7.92	0.00	6.62	6.46
time (sec)	N/A	0.408	0.015	10.549	0.060	0.123	0.568	0.000	0.236	0.991

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	31	733	1085	863	1096	0	909	854
N.S.	1	1.00	0.27	6.49	9.60	7.64	9.70	0.00	8.04	7.56
time (sec)	N/A	0.452	0.013	18.087	0.069	0.170	2.625	0.000	0.309	1.399

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	178	0	0	0	128
N.S.	1	1.00	1.00	0.00	0.00	3.63	0.00	0.00	0.00	2.61
time (sec)	N/A	0.324	0.035	0.000	0.000	0.108	0.000	0.000	0.283	0.390

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	142	0	0	0	107
N.S.	1	1.00	1.00	0.00	0.00	2.90	0.00	0.00	0.00	2.18
time (sec)	N/A	0.302	0.029	0.000	0.000	0.082	0.000	0.000	0.271	0.940

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	129	0	0	0	71
N.S.	1	1.00	1.00	0.00	0.00	2.74	0.00	0.00	0.00	1.51
time (sec)	N/A	0.263	0.017	0.000	0.000	0.081	0.000	0.000	0.299	0.341

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	59	0	0	0	58
N.S.	1	1.00	1.00	0.00	0.00	1.20	0.00	0.00	0.00	1.18
time (sec)	N/A	0.317	0.035	0.000	0.000	0.082	0.000	0.000	0.241	0.251

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	58	0	0	0	48
N.S.	1	1.00	1.00	0.00	0.00	1.18	0.00	0.00	0.00	0.98
time (sec)	N/A	0.317	0.035	0.000	0.000	0.080	0.000	0.000	0.269	0.229

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	155	0	0	0	114
N.S.	1	1.00	1.00	0.00	0.00	3.16	0.00	0.00	0.00	2.33
time (sec)	N/A	0.315	0.041	0.000	0.000	0.083	0.000	0.000	0.176	0.427

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	0	0	0	0	0	25	93
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.41	1.52
time (sec)	N/A	0.331	0.027	0.000	0.000	0.000	0.000	0.000	0.169	0.460

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	87	73
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.61	1.35
time (sec)	N/A	0.325	0.022	0.000	0.000	0.000	0.000	0.000	0.168	0.306

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	62	73
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.15	1.35
time (sec)	N/A	0.324	0.021	0.000	0.000	0.000	0.000	0.000	0.164	0.297

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	37	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.69	0.00
time (sec)	N/A	0.307	0.017	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	0	0	0	0	0	17	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.34	0.00
time (sec)	N/A	0.281	0.014	0.000	0.000	0.000	0.000	0.000	0.162	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	0	22	0	0	25	0
N.S.	1	1.00	1.00	1.18	0.00	1.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.288	0.008	0.193	0.000	0.073	0.000	0.000	0.154	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	36	71
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.69	1.37
time (sec)	N/A	0.318	0.017	0.000	0.000	0.000	0.000	0.000	0.155	0.174

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	47	67
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.87	1.24
time (sec)	N/A	0.312	0.015	0.000	0.000	0.000	0.000	0.000	0.152	0.173

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	58	71
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.07	1.31
time (sec)	N/A	0.312	0.014	0.000	0.000	0.000	0.000	0.000	0.155	0.178

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	32	113	129	116	287	0	112	0
N.S.	1	1.00	0.28	0.99	1.13	1.02	2.52	0.00	0.98	0.00
time (sec)	N/A	0.427	0.011	0.328	0.043	0.078	41.756	0.000	0.156	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	31	95	108	98	253	0	94	0
N.S.	1	1.00	0.33	1.01	1.15	1.04	2.69	0.00	1.00	0.00
time (sec)	N/A	0.404	0.012	0.307	0.041	0.074	33.969	0.000	0.155	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	155	32	77	87	80	219	0	76	0
N.S.	1	1.13	0.23	0.56	0.64	0.58	1.60	0.00	0.55	0.00
time (sec)	N/A	0.778	0.011	0.312	0.042	0.082	16.993	0.000	0.160	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	109	31	59	66	62	185	0	58	0
N.S.	1	1.09	0.31	0.59	0.66	0.62	1.85	0.00	0.58	0.00
time (sec)	N/A	0.560	0.011	0.304	0.043	0.076	11.721	0.000	0.156	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	32	41	45	44	150	0	40	0
N.S.	1	1.00	0.51	0.65	0.71	0.70	2.38	0.00	0.63	0.00
time (sec)	N/A	0.411	0.011	0.328	0.044	0.084	4.499	0.000	0.152	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	27	31	104	27	27	27
N.S.	1	1.00	1.00	1.04	1.00	1.15	3.85	1.00	1.00	1.00
time (sec)	N/A	0.290	0.014	0.309	0.025	0.074	2.517	0.139	0.154	0.145

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	0	22	0	0	25	0
N.S.	1	1.00	1.00	1.18	0.00	1.00	0.00	0.00	1.14	0.00
time (sec)	N/A	0.287	0.001	0.000	0.000	0.073	0.000	0.000	0.154	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	27	61	0	62	0	0	40	0
N.S.	1	1.00	0.48	1.09	0.00	1.11	0.00	0.00	0.71	0.00
time (sec)	N/A	0.410	0.011	0.234	0.000	0.078	0.000	0.000	0.157	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	96	32	99	0	84	0	0	44	0
N.S.	1	0.96	0.32	0.99	0.00	0.84	0.00	0.00	0.44	0.00
time (sec)	N/A	0.541	0.010	0.373	0.000	0.080	0.000	0.000	0.201	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	136	31	137	0	101	0	0	44	0
N.S.	1	0.98	0.22	0.99	0.00	0.73	0.00	0.00	0.32	0.00
time (sec)	N/A	0.681	0.012	0.364	0.000	0.074	0.000	0.000	0.153	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	145	0	119	0	0	44	0
N.S.	1	1.00	1.00	4.53	0.00	3.72	0.00	0.00	1.38	0.00
time (sec)	N/A	0.298	0.011	0.356	0.000	0.075	0.000	0.000	0.212	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	174	0	137	0	0	44	0
N.S.	1	1.00	1.00	5.61	0.00	4.42	0.00	0.00	1.42	0.00
time (sec)	N/A	0.301	0.011	0.362	0.000	0.073	0.000	0.000	0.160	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	0	50	0	37	30	39
N.S.	1	1.00	1.00	0.77	0.00	1.06	0.00	0.79	0.64	0.83
time (sec)	N/A	0.340	0.017	0.388	0.000	0.090	0.000	0.132	0.158	0.473

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	34	0	47	0	35	42	35
N.S.	1	1.00	1.00	0.72	0.00	1.00	0.00	0.74	0.89	0.74
time (sec)	N/A	0.344	0.012	0.206	0.000	0.082	0.000	0.124	0.159	0.364

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	412	1837	1456	531	0	704	1510	716
N.S.	1	1.00	0.80	3.55	2.81	1.03	0.00	1.36	2.92	1.38
time (sec)	N/A	1.901	1.774	0.303	0.580	0.102	0.000	0.154	0.187	0.420

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	220	1198	1052	364	0	418	911	517
N.S.	1	1.00	0.57	3.08	2.70	0.94	0.00	1.07	2.34	1.33
time (sec)	N/A	1.436	1.501	0.201	0.432	0.084	0.000	0.155	0.171	0.302

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	148	707	695	208	0	248	542	313
N.S.	1	1.00	0.57	2.74	2.69	0.81	0.00	0.96	2.10	1.21
time (sec)	N/A	1.022	1.263	0.138	0.312	0.087	0.000	0.135	0.162	0.250

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	105	384	422	135	0	152	271	194
N.S.	1	1.00	0.62	2.26	2.48	0.79	0.00	0.89	1.59	1.14
time (sec)	N/A	0.743	1.174	0.107	0.223	0.083	0.000	0.159	0.158	0.234

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	74	162	195	85	0	93	123	96
N.S.	1	1.00	0.91	2.00	2.41	1.05	0.00	1.15	1.52	1.19
time (sec)	N/A	0.474	0.823	0.057	0.132	0.086	0.000	0.140	0.158	0.168

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	58	58	48	0	36	45	48
N.S.	1	1.00	1.00	1.32	1.32	1.09	0.00	0.82	1.02	1.09
time (sec)	N/A	0.269	0.004	0.023	0.037	0.080	0.000	0.133	0.154	0.020

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	33	17	23	37	23
N.S.	1	1.00	1.10	1.00	1.10	1.57	0.81	1.10	1.76	1.10
time (sec)	N/A	0.333	1.223	0.028	0.124	0.075	0.587	0.144	0.157	0.214

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	44	19	23	48	23
N.S.	1	1.00	1.10	1.00	1.10	2.10	0.90	1.10	2.29	1.10
time (sec)	N/A	0.702	2.134	0.030	0.126	0.081	0.729	0.244	0.164	0.875

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	23	21	23	55	19	23	9784	23
N.S.	1	1.00	1.10	1.00	1.10	2.62	0.90	1.10	465.90	1.10
time (sec)	N/A	1.190	2.058	0.030	0.123	0.075	1.480	0.151	0.197	2.478

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	167	0	0	239	0	0	416	0
N.S.	1	1.00	0.94	0.00	0.00	1.35	0.00	0.00	2.35	0.00
time (sec)	N/A	0.605	0.573	0.000	0.000	0.078	0.000	0.000	0.164	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	117	0	0	175	0	0	214	0
N.S.	1	1.00	0.93	0.00	0.00	1.39	0.00	0.00	1.70	0.00
time (sec)	N/A	0.501	0.427	0.000	0.000	0.079	0.000	0.000	0.162	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	86	0	0	110	0	0	77	0
N.S.	1	1.00	0.93	0.00	0.00	1.20	0.00	0.00	0.84	0.00
time (sec)	N/A	0.394	0.364	0.000	0.000	0.073	0.000	0.000	0.148	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	52	0	0	39	0
N.S.	1	1.00	1.00	0.00	0.00	1.30	0.00	0.00	0.98	0.00
time (sec)	N/A	0.261	0.075	0.000	0.000	0.076	0.000	0.000	0.159	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	43	48	20	47	20
N.S.	1	1.00	1.11	0.95	1.05	2.26	2.53	1.05	2.47	1.05
time (sec)	N/A	0.275	0.632	0.030	0.144	0.070	42.032	0.136	0.160	0.102

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	54	60	1	58	20
N.S.	1	1.00	1.11	0.95	1.05	2.84	3.16	0.05	3.05	1.05
time (sec)	N/A	0.911	3.026	0.033	0.136	0.078	138.238	0.238	0.161	0.142

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	66	106	0	89	0	0	29	0
N.S.	1	1.00	0.93	1.49	0.00	1.25	0.00	0.00	0.41	0.00
time (sec)	N/A	1.117	0.142	0.236	0.000	0.085	0.000	0.000	0.165	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	126	116	191	0	179	0	0	0	0
N.S.	1	1.09	1.00	1.65	0.00	1.54	0.00	0.00	0.00	0.00
time (sec)	N/A	1.695	0.366	0.319	0.000	0.084	0.000	0.000	0.299	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	257	0	506	0	555	0	0	0	0
N.S.	1	0.96	0.00	1.90	0.00	2.08	0.00	0.00	0.00	0.00
time (sec)	N/A	2.914	0.000	0.450	0.000	0.096	0.000	0.000	0.520	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	454	0	922	0	1376	0	0	23	0
N.S.	1	0.99	0.00	2.00	0.00	2.99	0.00	0.00	0.05	0.00
time (sec)	N/A	5.326	0.000	0.675	0.000	0.111	0.000	0.000	200.042	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	468	1146	0	638	0	1428	0	0
N.S.	1	1.00	1.35	3.31	0.00	1.84	0.00	4.13	0.00	0.00
time (sec)	N/A	1.058	0.461	0.400	0.000	0.087	0.000	0.161	0.229	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	292	682	0	377	0	831	0	0
N.S.	1	1.00	0.91	2.13	0.00	1.18	0.00	2.60	0.00	0.00
time (sec)	N/A	0.957	0.277	0.289	0.000	0.084	0.000	0.158	0.211	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	170	356	0	197	0	425	1145	306
N.S.	1	1.00	0.67	1.40	0.00	0.77	0.00	1.67	4.49	1.20
time (sec)	N/A	0.810	0.172	0.234	0.000	0.089	0.000	0.150	0.196	0.407

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	150	0	83	0	172	532	153
N.S.	1	1.00	0.73	1.20	0.00	0.66	0.00	1.38	4.26	1.22
time (sec)	N/A	0.522	0.096	0.172	0.000	0.092	0.000	0.116	0.217	0.237

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	42	0	35	0	50	176	44
N.S.	1	1.00	1.00	1.14	0.00	0.95	0.00	1.35	4.76	1.19
time (sec)	N/A	0.337	0.021	0.039	0.000	0.075	0.000	0.118	0.218	0.142

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	65	0	71	0	483	21	0
N.S.	1	1.00	0.90	1.05	0.00	1.15	0.00	7.79	0.34	0.00
time (sec)	N/A	0.791	0.074	0.132	0.000	0.079	0.000	0.350	0.203	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	111	105	97	0	154	0	330	0	0
N.S.	1	1.04	0.98	0.91	0.00	1.44	0.00	3.08	0.00	0.00
time (sec)	N/A	1.177	0.181	0.203	0.000	0.076	0.000	0.135	0.712	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	234	0	240	0	517	0	1733	0	0
N.S.	1	0.98	0.00	1.00	0.00	2.15	0.00	7.22	0.00	0.00
time (sec)	N/A	1.881	0.000	0.297	0.000	0.085	0.000	0.129	3.342	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	243	553	0	311	0	0	0	0
N.S.	1	1.00	0.75	1.72	0.00	0.97	0.00	0.00	0.00	0.00
time (sec)	N/A	0.956	0.303	0.369	0.000	0.087	0.000	0.000	0.243	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	176	313	0	196	0	0	0	0
N.S.	1	1.00	0.82	1.46	0.00	0.91	0.00	0.00	0.00	0.00
time (sec)	N/A	0.727	0.207	0.238	0.000	0.079	0.000	0.000	0.219	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	85	140	0	122	0	0	0	0
N.S.	1	1.00	0.77	1.26	0.00	1.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.144	0.132	0.000	0.084	0.000	0.000	0.212	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	48	0	63	0	0	0	43
N.S.	1	1.00	1.00	0.96	0.00	1.26	0.00	0.00	0.00	0.86
time (sec)	N/A	0.386	0.030	0.081	0.000	0.076	0.000	0.000	0.216	0.174

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	31	26	20	32	20
N.S.	1	1.00	1.11	0.95	1.05	1.63	1.37	1.05	1.68	1.05
time (sec)	N/A	0.273	0.046	0.046	0.120	0.073	1.210	0.147	0.211	0.138

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	42	27	1	318316	20
N.S.	1	1.00	1.11	0.95	1.05	2.21	1.42	0.05	16753.47	1.05
time (sec)	N/A	0.364	0.296	0.049	0.108	0.076	4.419	0.192	2.346	0.451

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	53	27	20	771854	20
N.S.	1	1.00	1.11	0.95	1.05	2.79	1.42	1.05	40623.89	1.05
time (sec)	N/A	0.366	0.088	0.074	0.121	0.076	167.230	0.152	5.940	0.691

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	195	0	0	349	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	1.69	0.00	0.00	0.00	0.00
time (sec)	N/A	0.679	0.178	0.000	0.000	0.090	0.000	0.000	0.344	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	136	0	0	259	0	0	0	0
N.S.	1	1.00	0.90	0.00	0.00	1.72	0.00	0.00	0.00	0.00
time (sec)	N/A	0.539	0.097	0.000	0.000	0.071	0.000	0.000	0.259	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	85	0	0	169	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	1.84	0.00	0.00	0.00	0.00
time (sec)	N/A	0.386	0.101	0.000	0.000	0.079	0.000	0.000	0.242	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	89	0	0	0	61
N.S.	1	1.00	1.00	0.00	0.00	2.22	0.00	0.00	0.00	1.52
time (sec)	N/A	0.258	0.009	0.000	0.000	0.066	0.000	0.000	0.206	0.335

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	42	37	20	43	20
N.S.	1	1.00	1.11	0.95	1.05	2.21	1.95	1.05	2.26	1.05
time (sec)	N/A	0.274	0.042	0.055	0.207	0.070	1.961	0.160	0.199	0.123

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	18	20	53	39	1	1139170	20
N.S.	1	1.00	1.11	0.95	1.05	2.79	2.05	0.05	59956.32	1.05
time (sec)	N/A	0.363	0.369	0.046	0.154	0.079	9.007	0.371	7.250	0.368

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	436	0	135	0	0	35	0
N.S.	1	1.00	0.98	4.19	0.00	1.30	0.00	0.00	0.34	0.00
time (sec)	N/A	2.538	0.337	0.329	0.000	0.082	0.000	0.000	0.191	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	179	0	582	0	220	0	0	0	0
N.S.	1	1.13	0.00	3.66	0.00	1.38	0.00	0.00	0.00	0.00
time (sec)	N/A	3.428	0.000	0.399	0.000	0.083	0.000	0.000	1.149	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	355	0	2019	0	755	0	0	0	0
N.S.	1	0.97	0.00	5.52	0.00	2.06	0.00	0.00	0.00	0.00
time (sec)	N/A	6.203	0.000	0.655	0.000	0.099	0.000	0.000	3.838	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	612	0	4680	0	2250	0	0	28	0
N.S.	1	0.97	0.00	7.38	0.00	3.55	0.00	0.00	0.04	0.00
time (sec)	N/A	34.206	0.000	1.080	0.000	0.132	0.000	0.000	200.026	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	295	122	218	201	114	0	137	245	153
N.S.	1	1.36	0.56	1.00	0.93	0.53	0.00	0.63	1.13	0.71
time (sec)	N/A	1.451	0.369	0.085	0.135	0.081	0.000	0.136	0.219	0.283

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	170	104	163	166	95	0	108	165	111
N.S.	1	1.04	0.63	0.99	1.01	0.58	0.00	0.66	1.01	0.68
time (sec)	N/A	0.744	0.292	0.062	0.123	0.110	0.000	0.140	0.160	0.205

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	79	107	73	0	80	98	71
N.S.	1	1.00	1.00	0.98	1.32	0.90	0.00	0.99	1.21	0.88
time (sec)	N/A	0.390	0.171	0.050	0.110	0.075	0.000	0.143	0.196	0.169

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	50	50	55	0	50	56	49
N.S.	1	1.00	1.00	0.89	0.89	0.98	0.00	0.89	1.00	0.88
time (sec)	N/A	0.298	0.060	0.038	0.032	0.073	0.000	0.134	0.204	0.089

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	21	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.31	1.12
time (sec)	N/A	0.270	0.273	0.008	0.090	0.086	0.394	0.133	0.197	0.106

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	21	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.31	1.12
time (sec)	N/A	0.572	0.578	0.010	0.094	0.075	0.307	0.134	0.208	0.161

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	242	91	154	181	89	0	104	151	112
N.S.	1	1.34	0.50	0.85	1.00	0.49	0.00	0.57	0.83	0.62
time (sec)	N/A	1.334	0.463	0.106	0.112	0.077	0.000	0.133	0.199	0.165

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	140	79	111	151	72	0	80	122	80
N.S.	1	1.04	0.59	0.83	1.13	0.54	0.00	0.60	0.91	0.60
time (sec)	N/A	0.693	0.310	0.052	0.106	0.074	0.000	0.124	0.193	0.178

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	53	98	57	0	58	69	58
N.S.	1	1.00	1.03	0.80	1.48	0.86	0.00	0.88	1.05	0.88
time (sec)	N/A	0.375	0.165	0.044	0.095	0.076	0.000	0.135	0.221	0.083

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	34	32	36	41	38	38	40
N.S.	1	1.00	1.05	0.77	0.73	0.82	0.93	0.86	0.86	0.91
time (sec)	N/A	0.289	0.064	0.020	0.031	0.076	0.303	0.130	0.221	0.018

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	17	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.00	1.06	1.41	1.06
time (sec)	N/A	0.274	0.293	0.014	0.096	0.065	2.531	0.131	0.210	0.087

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	18	19	18	24	18
N.S.	1	1.00	1.12	0.94	1.06	1.06	1.12	1.06	1.41	1.06
time (sec)	N/A	0.559	0.549	0.011	0.092	0.066	2.149	0.120	0.219	0.137

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	392	191	368	267	212	0	250	348	230
N.S.	1	1.32	0.64	1.24	0.90	0.71	0.00	0.84	1.17	0.77
time (sec)	N/A	2.670	0.687	0.115	0.154	0.077	0.000	0.137	0.223	0.304

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	225	144	212	221	148	0	152	178	150
N.S.	1	1.04	0.67	0.98	1.02	0.69	0.00	0.70	0.82	0.69
time (sec)	N/A	1.255	0.495	0.058	0.135	0.081	0.000	0.144	0.204	0.163

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	116	102	143	107	0	104	77	95
N.S.	1	1.00	1.08	0.95	1.34	1.00	0.00	0.97	0.72	0.89
time (sec)	N/A	0.565	0.247	0.042	0.113	0.068	0.000	0.136	0.213	0.183

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	57	58	74	0	68	25	60
N.S.	1	1.00	1.00	0.84	0.85	1.09	0.00	1.00	0.37	0.88
time (sec)	N/A	0.361	0.044	0.021	0.033	0.075	0.000	0.136	0.226	0.019

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	26	29	18	29	18
N.S.	1	1.00	1.12	0.94	1.06	1.53	1.71	1.06	1.71	1.06
time (sec)	N/A	0.437	0.502	0.018	0.107	0.064	7.045	0.121	0.211	0.109

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	26	31	18	29	18
N.S.	1	1.00	1.12	0.94	1.06	1.53	1.82	1.06	1.71	1.06
time (sec)	N/A	1.120	0.973	0.011	0.110	0.067	9.357	0.130	0.220	0.171

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	345	169	550	539	204	0	240	543	251
N.S.	1	1.30	0.64	2.07	2.03	0.77	0.00	0.90	2.04	0.94
time (sec)	N/A	1.619	0.739	0.161	0.242	0.082	0.000	0.137	0.209	0.299

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	195	123	307	332	130	0	143	284	153
N.S.	1	1.03	0.65	1.62	1.76	0.69	0.00	0.76	1.50	0.81
time (sec)	N/A	0.825	0.538	0.100	0.173	0.090	0.000	0.136	0.202	0.241

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	96	131	160	83	0	89	134	80
N.S.	1	1.00	1.07	1.46	1.78	0.92	0.00	0.99	1.49	0.89
time (sec)	N/A	0.410	0.350	0.059	0.115	0.076	0.000	0.143	0.199	0.179

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	25	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.25	1.10
time (sec)	N/A	0.287	0.557	0.030	0.087	0.075	0.371	0.151	0.195	0.225

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	33	19	22	36	22
N.S.	1	1.00	1.10	1.00	1.10	1.65	0.95	1.10	1.80	1.10
time (sec)	N/A	0.667	1.152	0.029	0.090	0.077	0.677	0.216	0.212	0.771

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	44	19	22	8588	22
N.S.	1	1.00	1.10	1.00	1.10	2.20	0.95	1.10	429.40	1.10
time (sec)	N/A	1.025	1.575	0.031	0.090	0.071	0.769	0.164	0.237	1.875

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	45	539	41	85	798	44	44
N.S.	1	1.00	0.69	1.00	11.98	0.91	1.89	17.73	0.98	0.98
time (sec)	N/A	0.379	0.635	0.090	0.251	0.079	0.080	0.160	0.207	0.225

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	86	99	332	74	0	88	124	92
N.S.	1	1.00	1.10	1.27	4.26	0.95	0.00	1.13	1.59	1.18
time (sec)	N/A	0.411	0.604	0.107	0.173	0.090	0.000	0.135	0.224	0.197

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	24	17	17	17
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.41	1.00	1.00	1.00
time (sec)	N/A	0.256	0.117	0.037	0.027	0.076	0.051	0.126	0.215	0.144

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	40	0	47	0	0	26	0
N.S.	1	1.00	1.00	1.03	0.00	1.21	0.00	0.00	0.67	0.00
time (sec)	N/A	0.302	0.299	0.099	0.000	0.078	0.000	0.000	0.207	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	96	101	0	85	0	0	37	76
N.S.	1	1.00	1.14	1.20	0.00	1.01	0.00	0.00	0.44	0.90
time (sec)	N/A	0.412	0.743	0.112	0.000	0.073	0.000	0.000	0.208	0.405

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	79	88	0	106	0	0	0	0
N.S.	1	1.00	1.14	1.28	0.00	1.54	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	0.673	0.142	0.000	0.073	0.000	0.000	0.216	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	29	42	536	40	83	742	43	43
N.S.	1	1.00	0.67	0.98	12.47	0.93	1.93	17.26	1.00	1.00
time (sec)	N/A	0.355	0.254	0.103	0.236	0.077	0.072	0.147	0.201	0.157

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	84	90	329	68	0	77	121	86
N.S.	1	1.00	1.12	1.20	4.39	0.91	0.00	1.03	1.61	1.15
time (sec)	N/A	0.409	0.242	0.105	0.165	0.079	0.000	0.144	0.210	0.164

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	16	16	22	16	16	16
N.S.	1	1.00	1.00	0.94	1.00	1.00	1.38	1.00	1.00	1.00
time (sec)	N/A	0.257	0.052	0.052	0.030	0.072	0.085	0.117	0.204	0.119

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	33	0	42	0	0	22	0
N.S.	1	1.00	1.00	0.89	0.00	1.14	0.00	0.00	0.59	0.00
time (sec)	N/A	0.288	0.081	0.120	0.000	0.081	0.000	0.000	0.207	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	94	87	0	79	0	0	33	73
N.S.	1	1.00	1.16	1.07	0.00	0.98	0.00	0.00	0.41	0.90
time (sec)	N/A	0.399	0.248	0.108	0.000	0.074	0.000	0.000	0.215	0.218

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	77	74	0	100	0	0	0	0
N.S.	1	1.00	1.17	1.12	0.00	1.52	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.223	0.125	0.000	0.077	0.000	0.000	0.231	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	27	27	33	30	25	41	0	15	26
N.S.	1	0.90	0.90	1.10	1.00	0.83	1.37	0.00	0.50	0.87
time (sec)	N/A	0.306	0.176	0.092	0.106	0.075	0.146	0.000	0.226	0.163

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	27	27	28	30	25	31	31	25	26
N.S.	1	0.90	0.90	0.93	1.00	0.83	1.03	1.03	0.83	0.87
time (sec)	N/A	0.308	0.002	0.046	0.031	0.074	0.077	0.124	0.187	0.132

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	29	26	35	33	27	44	0	19	27
N.S.	1	0.91	0.81	1.09	1.03	0.84	1.38	0.00	0.59	0.84
time (sec)	N/A	0.321	0.197	0.056	0.107	0.082	0.150	0.000	0.217	0.151

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	29	26	30	33	27	34	34	28	27
N.S.	1	0.91	0.81	0.94	1.03	0.84	1.06	1.06	0.88	0.84
time (sec)	N/A	0.321	0.003	0.041	0.031	0.077	0.071	0.114	0.211	0.130

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	50	59	39	90	0	32	0
N.S.	1	1.00	0.62	0.86	1.02	0.67	1.55	0.00	0.55	0.00
time (sec)	N/A	0.366	0.189	0.066	0.114	0.068	0.186	0.000	0.192	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	41	59	39	66	48	39	47
N.S.	1	1.00	0.62	0.71	1.02	0.67	1.14	0.83	0.67	0.81
time (sec)	N/A	0.359	0.003	0.044	0.026	0.074	0.091	0.122	0.187	0.155

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	38	50	58	41	90	0	33	0
N.S.	1	1.03	0.66	0.86	1.00	0.71	1.55	0.00	0.57	0.00
time (sec)	N/A	0.363	0.191	0.054	0.112	0.078	0.188	0.000	0.216	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	60	38	42	58	41	66	50	41	47
N.S.	1	1.03	0.66	0.72	1.00	0.71	1.14	0.86	0.71	0.81
time (sec)	N/A	0.363	0.002	0.043	0.031	0.070	0.093	0.116	0.203	0.149

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	53	21	86	29	0	15	22
N.S.	1	1.00	1.00	1.77	0.70	2.87	0.97	0.00	0.50	0.73
time (sec)	N/A	0.283	0.073	0.061	0.112	0.081	0.166	0.000	0.155	0.172

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	22	21	86	24	21	29	22
N.S.	1	1.00	1.00	0.73	0.70	2.87	0.80	0.70	0.97	0.73
time (sec)	N/A	0.284	0.002	0.053	0.106	0.077	0.087	0.127	0.157	0.128

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	49	45	86	29	0	19	22
N.S.	1	1.00	1.00	1.63	1.50	2.87	0.97	0.00	0.63	0.73
time (sec)	N/A	0.281	0.074	0.067	0.103	0.082	0.174	0.000	0.154	0.237

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	22	45	86	24	24	44	22
N.S.	1	1.00	1.00	0.73	1.50	2.87	0.80	0.80	1.47	0.73
time (sec)	N/A	0.284	0.003	0.053	0.103	0.091	0.085	0.139	0.158	0.214

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	40	40	74	68	102	54	0	32	35
N.S.	1	0.93	0.93	1.72	1.58	2.37	1.26	0.00	0.74	0.81
time (sec)	N/A	0.309	0.073	0.068	0.115	0.078	0.291	0.000	0.178	0.160

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	40	40	36	39	102	44	38	34	35
N.S.	1	0.93	0.93	0.84	0.91	2.37	1.02	0.88	0.79	0.81
time (sec)	N/A	0.312	0.002	0.057	0.112	0.079	0.130	0.135	0.213	0.066

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	40	40	70	88	103	54	0	33	35
N.S.	1	0.93	0.93	1.63	2.05	2.40	1.26	0.00	0.77	0.81
time (sec)	N/A	0.312	0.074	0.065	0.121	0.081	0.220	0.000	0.211	0.168

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	40	40	36	65	103	44	39	52	35
N.S.	1	0.93	0.93	0.84	1.51	2.40	1.02	0.91	1.21	0.81
time (sec)	N/A	0.319	0.002	0.059	0.110	0.087	0.116	0.119	0.213	0.153

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	33	0	0	82	61	0	23	28
N.S.	1	1.06	1.06	0.00	0.00	2.65	1.97	0.00	0.74	0.90
time (sec)	N/A	0.296	0.112	0.000	0.000	0.080	0.406	0.000	0.190	0.244

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	33	33	29	22	82	61	31	27	28
N.S.	1	1.06	1.06	0.94	0.71	2.65	1.97	1.00	0.87	0.90
time (sec)	N/A	0.299	0.002	0.022	0.028	0.081	0.419	0.138	0.215	0.187

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	34	0	0	81	66	0	28	33
N.S.	1	1.06	1.06	0.00	0.00	2.53	2.06	0.00	0.88	1.03
time (sec)	N/A	0.302	0.145	0.000	0.000	0.080	0.428	0.000	0.203	0.240

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	34	34	29	22	81	66	36	32	33
N.S.	1	1.06	1.06	0.91	0.69	2.53	2.06	1.12	1.00	1.03
time (sec)	N/A	0.295	0.003	0.026	0.103	0.083	0.433	0.140	0.199	0.186

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	40	19	30	0	0	28	24
N.S.	1	1.00	1.46	1.67	0.79	1.25	0.00	0.00	1.17	1.00
time (sec)	N/A	0.298	0.096	0.062	0.159	0.078	0.000	0.000	0.222	0.133

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	35	40	24	30	0	29	18	24
N.S.	1	1.00	1.46	1.67	1.00	1.25	0.00	1.21	0.75	1.00
time (sec)	N/A	0.291	0.002	0.047	0.033	0.079	0.000	0.117	0.196	0.110

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	38	44	21	32	0	0	32	25
N.S.	1	1.00	1.52	1.76	0.84	1.28	0.00	0.00	1.28	1.00
time (sec)	N/A	0.300	0.095	0.048	0.156	0.079	0.000	0.000	0.203	0.136

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	38	44	25	32	0	33	20	25
N.S.	1	1.00	1.52	1.76	1.00	1.28	0.00	1.32	0.80	1.00
time (sec)	N/A	0.299	0.002	0.041	0.032	0.084	0.000	0.140	0.162	0.116

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	41	29	29	68	27	56	0	23	28
N.S.	1	0.93	0.66	0.66	1.55	0.61	1.27	0.00	0.52	0.64
time (sec)	N/A	0.316	0.107	0.030	0.103	0.076	0.308	0.000	0.168	0.181

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	41	29	29	38	27	58	31	26	28
N.S.	1	0.93	0.66	0.66	0.86	0.61	1.32	0.70	0.59	0.64
time (sec)	N/A	0.314	0.003	0.040	0.029	0.073	0.315	0.129	0.155	0.137

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	30	29	71	28	58	0	28	28
N.S.	1	0.93	0.65	0.63	1.54	0.61	1.26	0.00	0.61	0.61
time (sec)	N/A	0.324	0.107	0.027	0.107	0.074	0.316	0.000	0.168	0.171

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	43	30	29	40	28	60	33	28	28
N.S.	1	0.93	0.65	0.63	0.87	0.61	1.30	0.72	0.61	0.61
time (sec)	N/A	0.318	0.003	0.038	0.025	0.080	0.322	0.133	0.156	0.139

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	89	111	0	0	175	0	0	28	0
N.S.	1	0.96	1.19	0.00	0.00	1.88	0.00	0.00	0.30	0.00
time (sec)	N/A	0.356	0.506	0.000	0.000	0.087	0.000	0.000	0.162	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	89	111	182	124	175	0	92	77	0
N.S.	1	0.96	1.19	1.96	1.33	1.88	0.00	0.99	0.83	0.00
time (sec)	N/A	0.371	0.005	0.093	0.119	0.086	0.000	0.160	0.169	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	93	115	0	0	185	0	0	32	0
N.S.	1	0.97	1.20	0.00	0.00	1.93	0.00	0.00	0.33	0.00
time (sec)	N/A	0.369	0.490	0.000	0.000	0.085	0.000	0.000	0.172	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	93	115	198	130	185	0	96	85	0
N.S.	1	0.97	1.20	2.06	1.35	1.93	0.00	1.00	0.89	0.00
time (sec)	N/A	0.378	0.004	0.101	0.110	0.092	0.000	0.158	0.164	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	20	18	16	15	25	14	15	39	15
N.S.	1	1.18	1.06	0.94	0.88	1.47	0.82	0.88	2.29	0.88
time (sec)	N/A	0.284	0.031	0.032	0.027	0.074	0.039	0.123	0.155	0.043

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	27	27	19	18	18	17	18	20	18
N.S.	1	1.12	1.12	0.79	0.75	0.75	0.71	0.75	0.83	0.75
time (sec)	N/A	0.313	0.047	0.039	0.027	0.102	0.047	0.108	0.158	0.043

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	75	54	35	43	53	22	46	60	32
N.S.	1	1.04	0.75	0.49	0.60	0.74	0.31	0.64	0.83	0.44
time (sec)	N/A	0.441	0.115	0.043	0.108	0.084	0.062	0.134	0.157	0.191

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	52	47	37	34	34	24	34	36	34
N.S.	1	1.18	1.07	0.84	0.77	0.77	0.55	0.77	0.82	0.77
time (sec)	N/A	0.366	0.062	0.030	0.108	0.078	0.060	0.126	0.155	0.130

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	69	66	0	219	63	63	104	63
N.S.	1	1.06	1.03	0.99	0.00	3.27	0.94	0.94	1.55	0.94
time (sec)	N/A	0.417	0.211	0.106	0.000	0.087	0.152	0.134	0.152	0.202

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	47	38	38	37	49	0	0	18	0
N.S.	1	1.07	0.86	0.86	0.84	1.11	0.00	0.00	0.41	0.00
time (sec)	N/A	0.817	0.095	0.037	0.029	0.076	0.000	0.000	0.149	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	52	49	41	38	38	0	0	18	0
N.S.	1	0.96	0.91	0.76	0.70	0.70	0.00	0.00	0.33	0.00
time (sec)	N/A	0.582	0.090	0.033	0.035	0.077	0.000	0.000	0.157	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	165	120	183	0	86	0	0	16	0
N.S.	1	0.92	0.67	1.02	0.00	0.48	0.00	0.00	0.09	0.00
time (sec)	N/A	0.869	0.130	0.027	0.000	0.077	0.000	0.000	0.164	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	193	144	235	0	90	0	0	18	0
N.S.	1	0.95	0.71	1.15	0.00	0.44	0.00	0.00	0.09	0.00
time (sec)	N/A	0.880	0.135	0.029	0.000	0.077	0.000	0.000	0.155	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	255	205	376	0	280	0	0	20	0
N.S.	1	0.92	0.74	1.36	0.00	1.01	0.00	0.00	0.07	0.00
time (sec)	N/A	1.273	0.292	0.048	0.000	0.080	0.000	0.000	0.155	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	57	65	62	76	0	0	20	0
N.S.	1	1.08	0.79	0.90	0.86	1.06	0.00	0.00	0.28	0.00
time (sec)	N/A	1.330	0.130	0.041	0.028	0.075	0.000	0.000	0.156	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	82	77	62	59	59	0	0	20	0
N.S.	1	1.06	1.00	0.81	0.77	0.77	0.00	0.00	0.26	0.00
time (sec)	N/A	0.837	0.117	0.037	0.033	0.072	0.000	0.000	0.157	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	259	203	172	0	0	138	0	0	18	0
N.S.	1	0.78	0.66	0.00	0.00	0.53	0.00	0.00	0.07	0.00
time (sec)	N/A	1.206	0.187	0.000	0.000	0.077	0.000	0.000	0.153	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	237	216	0	0	142	0	0	20	0
N.S.	1	0.81	0.74	0.00	0.00	0.48	0.00	0.00	0.07	0.00
time (sec)	N/A	1.300	0.264	0.000	0.000	0.075	0.000	0.000	0.153	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	313	278	0	0	415	0	0	22	0
N.S.	1	0.80	0.71	0.00	0.00	1.06	0.00	0.00	0.06	0.00
time (sec)	N/A	1.964	0.368	0.000	0.000	0.083	0.000	0.000	0.158	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	44	46	48	59	51	59	77	50
N.S.	1	1.00	1.10	1.15	1.20	1.48	1.28	1.48	1.92	1.25
time (sec)	N/A	0.314	0.083	0.049	0.029	0.081	0.063	0.177	0.155	0.173

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	96	92	547	0	309	112	110	139	96
N.S.	1	1.02	0.98	5.82	0.00	3.29	1.19	1.17	1.48	1.02
time (sec)	N/A	0.451	0.244	0.109	0.000	0.098	0.219	0.136	0.161	0.237

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	96	92	546	0	309	104	110	139	96
N.S.	1	1.02	0.98	5.81	0.00	3.29	1.11	1.17	1.48	1.02
time (sec)	N/A	0.452	0.259	0.118	0.000	0.094	0.240	0.118	0.157	0.239

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	98	88	143	95	143	0	0	27	0
N.S.	1	1.02	0.92	1.49	0.99	1.49	0.00	0.00	0.28	0.00
time (sec)	N/A	1.152	0.230	0.072	0.045	0.080	0.000	0.000	0.164	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	299	236	855	0	497	0	0	126	0
N.S.	1	0.88	0.70	2.53	0.00	1.47	0.00	0.00	0.37	0.00
time (sec)	N/A	1.452	0.416	0.114	0.000	0.087	0.000	0.000	0.164	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	159	123	232	159	210	0	0	29	0
N.S.	1	1.10	0.85	1.60	1.10	1.45	0.00	0.00	0.20	0.00
time (sec)	N/A	1.977	0.322	0.089	0.049	0.080	0.000	0.000	0.155	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	484	395	335	0	0	694	0	0	132	0
N.S.	1	0.82	0.69	0.00	0.00	1.43	0.00	0.00	0.27	0.00
time (sec)	N/A	2.302	0.501	0.000	0.000	0.102	0.000	0.000	0.169	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	118	993	0	330	139	119	186	105
N.S.	1	1.01	1.15	9.64	0.00	3.20	1.35	1.16	1.81	1.02
time (sec)	N/A	0.530	0.339	0.181	0.000	0.098	0.619	0.140	0.150	0.241

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	118	993	0	330	139	119	186	105
N.S.	1	1.01	1.15	9.64	0.00	3.20	1.35	1.16	1.81	1.02
time (sec)	N/A	0.499	0.015	0.000	0.000	0.102	0.609	0.117	0.159	0.001

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	11	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	1.22	0.89
time (sec)	N/A	0.250	0.018	0.023	0.037	0.068	0.032	0.110	0.155	0.104

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	23	20	19	18	23	14	28	33	18
N.S.	1	1.15	1.00	0.95	0.90	1.15	0.70	1.40	1.65	0.90
time (sec)	N/A	0.555	0.057	0.030	0.032	0.075	0.039	0.131	0.156	0.035

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	41	33	32	30	38	0	0	82	0
N.S.	1	1.21	0.97	0.94	0.88	1.12	0.00	0.00	2.41	0.00
time (sec)	N/A	0.821	0.089	0.032	0.028	0.070	0.000	0.000	0.154	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	22	20	19	22	26	20
N.S.	1	1.00	1.00	1.15	1.10	1.00	0.95	1.10	1.30	1.00
time (sec)	N/A	0.272	0.041	0.036	0.026	0.072	0.045	0.116	0.158	0.132

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	51	44	64	57	61	42	0	63	52
N.S.	1	1.02	0.88	1.28	1.14	1.22	0.84	0.00	1.26	1.04
time (sec)	N/A	0.782	0.147	0.053	0.034	0.082	0.060	0.000	0.156	0.150

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	63	134	74	114	0	0	155	0
N.S.	1	1.08	0.84	1.79	0.99	1.52	0.00	0.00	2.07	0.00
time (sec)	N/A	1.210	0.192	0.092	0.043	0.093	0.000	0.000	0.159	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	14	13	12	13	15	13
N.S.	1	1.00	1.00	1.08	1.08	1.00	0.92	1.00	1.15	1.00
time (sec)	N/A	0.254	0.050	0.051	0.031	0.068	0.039	0.116	0.152	0.120

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	25	18	16	19	17	17	18	19	17
N.S.	1	1.09	0.78	0.70	0.83	0.74	0.74	0.78	0.83	0.74
time (sec)	N/A	0.307	0.051	0.033	0.031	0.072	0.059	0.132	0.161	0.140

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	40	36	0	126	36	35	46	35
N.S.	1	1.00	1.11	1.00	0.00	3.50	1.00	0.97	1.28	0.97
time (sec)	N/A	0.349	0.079	0.047	0.000	0.082	0.116	0.131	0.159	0.113

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	165	123	171	0	214	0	0	23	0
N.S.	1	1.04	0.77	1.08	0.00	1.35	0.00	0.00	0.14	0.00
time (sec)	N/A	1.013	0.128	0.036	0.000	0.078	0.000	0.000	0.159	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	244	223	185	0	0	316	0	0	25	0
N.S.	1	0.91	0.76	0.00	0.00	1.30	0.00	0.00	0.10	0.00
time (sec)	N/A	1.572	0.105	0.000	0.000	0.081	0.000	0.000	0.159	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	51	135	0	189	66	48	57	47
N.S.	1	1.00	1.09	2.87	0.00	4.02	1.40	1.02	1.21	1.00
time (sec)	N/A	0.373	0.199	0.064	0.000	0.088	0.169	0.138	0.159	0.189

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	209	149	433	0	353	0	0	126	0
N.S.	1	1.03	0.73	2.13	0.00	1.74	0.00	0.00	0.62	0.00
time (sec)	N/A	1.208	0.453	0.106	0.000	0.084	0.000	0.000	0.169	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	305	236	0	0	489	0	0	132	0
N.S.	1	0.98	0.76	0.00	0.00	1.58	0.00	0.00	0.43	0.00
time (sec)	N/A	1.939	0.440	0.000	0.000	0.087	0.000	0.000	0.170	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	46	48	0	36	48	47	48
N.S.	1	1.00	1.04	0.92	0.96	0.00	0.72	0.96	0.94	0.96
time (sec)	N/A	0.475	0.689	0.089	0.518	0.000	74.712	1.290	0.169	0.299

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	124	0	0	0	0	0	0	328	0
N.S.	1	0.81	0.00	0.00	0.00	0.00	0.00	0.00	2.13	0.00
time (sec)	N/A	0.704	0.000	0.000	0.000	0.000	0.000	0.000	0.194	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	112	92	0	0	0	0	0	0	224	0
N.S.	1	0.82	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00
time (sec)	N/A	0.646	0.000	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	60	0	0	0	0	0	0	120	0
N.S.	1	0.86	0.00	0.00	0.00	0.00	0.00	0.00	1.71	0.00
time (sec)	N/A	0.473	0.000	0.000	0.000	0.000	0.000	0.000	0.186	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	46	48	69	36	48	119	48
N.S.	1	1.00	1.04	0.92	0.96	1.38	0.72	0.96	2.38	0.96
time (sec)	N/A	0.483	3.844	0.083	0.114	0.082	7.549	1.041	0.171	0.500

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	52	46	204	133	37	48	240	48
N.S.	1	1.00	1.04	0.92	4.08	2.66	0.74	0.96	4.80	0.96
time (sec)	N/A	0.474	1.361	0.091	0.142	7.126	66.549	5.069	0.190	0.442

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	43	48	0	41	47	46	46
N.S.	1	1.00	1.04	0.91	1.02	0.00	0.87	1.00	0.98	0.98
time (sec)	N/A	0.557	0.945	0.025	0.450	0.000	115.984	1.185	0.173	0.277

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	135	0	0	0	0	0	0	179	0
N.S.	1	0.89	0.00	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.744	0.000	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	99	0	0	0	0	0	0	128	0
N.S.	1	0.90	0.00	0.00	0.00	0.00	0.00	0.00	1.16	0.00
time (sec)	N/A	0.704	0.000	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	63	0	0	0	0	0	0	75	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	1.10	0.00
time (sec)	N/A	0.515	0.000	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	26	31	25	20	33	27	14
N.S.	1	1.00	1.00	1.86	2.21	1.79	1.43	2.36	1.93	1.00
time (sec)	N/A	0.224	0.004	0.069	0.025	0.073	0.062	0.116	0.163	0.029

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	43	48	72	78	47	79	46
N.S.	1	1.00	1.04	0.91	1.02	1.53	1.66	1.00	1.68	0.98
time (sec)	N/A	0.546	1.749	0.023	0.110	0.080	11.736	0.800	0.177	0.193

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	43	187	134	0	47	152	46
N.S.	1	1.00	1.04	0.91	3.98	2.85	0.00	1.00	3.23	0.98
time (sec)	N/A	0.528	1.840	0.026	0.128	1.898	0.000	1.346	0.189	0.712

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.697	1.594	0.000	0.000	0.000	0.000	0.000	0.170	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.410	0.670	0.000	0.000	0.000	0.000	0.000	0.167	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.411	0.681	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0	34	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.21	0.00
time (sec)	N/A	0.393	1.387	0.000	0.000	0.000	0.000	0.000	0.180	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	0	0	52	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.79	0.00
time (sec)	N/A	0.407	0.771	0.000	0.000	0.000	0.000	0.000	0.178	0.000

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	0	0	0	0	0	54	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.86	0.00
time (sec)	N/A	0.407	0.690	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	58	35	52	67	71	583	2631	71	35
N.S.	1	1.18	0.71	1.06	1.37	1.45	11.90	53.69	1.45	0.71
time (sec)	N/A	0.558	0.066	0.106	0.032	0.078	0.692	0.164	0.154	0.105

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	26	25	37	34	190	994	34	23
N.S.	1	1.00	0.84	0.81	1.19	1.10	6.13	32.06	1.10	0.74
time (sec)	N/A	0.390	0.037	0.053	0.027	0.076	0.410	0.150	0.153	0.012

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	14	31	237	14	14
N.S.	1	1.00	1.00	1.07	0.00	1.00	2.21	16.93	1.00	1.00
time (sec)	N/A	0.270	0.022	0.036	0.000	0.070	0.283	0.129	0.156	0.123

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	10	56	8	10	0	0	12	8
N.S.	1	1.00	1.25	7.00	1.00	1.25	0.00	0.00	1.50	1.00
time (sec)	N/A	0.310	0.039	0.052	0.083	0.072	0.000	0.000	0.154	0.016

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	A	A	F	F	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	0	160	16	34	0	0	45	28
N.S.	1	1.00	0.00	6.15	0.62	1.31	0.00	0.00	1.73	1.08
time (sec)	N/A	0.435	0.000	0.057	0.081	0.071	0.000	0.000	0.155	0.101

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	C	A	A	F	F	F	B
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	49	0	225	19	61	0	0	97	59
N.S.	1	0.96	0.00	4.41	0.37	1.20	0.00	0.00	1.90	1.16
time (sec)	N/A	0.564	0.000	0.094	0.080	0.076	0.000	0.000	0.154	0.106

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-2)	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	18	0	216	18	18
N.S.	1	1.00	1.00	1.06	0.00	1.00	0.00	12.00	1.00	1.00
time (sec)	N/A	0.277	0.029	0.046	0.000	0.072	0.000	0.142	0.154	0.133

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	20	0	19	49	313	19	19
N.S.	1	1.00	1.11	1.05	0.00	1.00	2.58	16.47	1.00	1.00
time (sec)	N/A	0.340	0.042	0.157	0.000	0.076	0.635	0.133	0.161	0.114

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	72	43	57	72	75	333	1817	75	43
N.S.	1	1.18	0.70	0.93	1.18	1.23	5.46	29.79	1.23	0.70
time (sec)	N/A	0.593	0.065	0.061	0.030	0.094	0.462	0.147	0.152	0.104

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	770	518	2448	0	0	1859	0	0	736	0
N.S.	1	0.67	3.18	0.00	0.00	2.41	0.00	0.00	0.96	0.00
time (sec)	N/A	3.555	4.875	0.000	0.000	0.148	0.000	0.000	0.202	0.000

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	599	418	1419	0	0	1193	0	0	544	0
N.S.	1	0.70	2.37	0.00	0.00	1.99	0.00	0.00	0.91	0.00
time (sec)	N/A	2.715	2.734	0.000	0.000	0.120	0.000	0.000	0.219	0.000

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	328	644	1182	0	651	0	0	348	0
N.S.	1	0.77	1.50	2.76	0.00	1.52	0.00	0.00	0.81	0.00
time (sec)	N/A	1.666	2.176	0.156	0.000	0.108	0.000	0.000	0.179	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	100	100	176	0	291	116	95	178	91
N.S.	1	1.05	1.05	1.85	0.00	3.06	1.22	1.00	1.87	0.96
time (sec)	N/A	0.522	0.330	0.139	0.000	0.101	0.526	0.137	0.160	0.173

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	41	43	56	42	43	134	43
N.S.	1	1.00	1.05	0.93	0.98	1.27	0.95	0.98	3.05	0.98
time (sec)	N/A	1.591	1.402	0.072	0.085	0.084	54.711	0.160	0.187	0.551

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	46	41	43	92	0	43	240	43
N.S.	1	1.00	1.05	0.93	0.98	2.09	0.00	0.98	5.45	0.98
time (sec)	N/A	1.455	12.057	0.087	0.103	0.083	0.000	0.405	0.204	0.412

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	287	398	318	0	251	0	0	78	0
N.S.	1	1.91	2.65	2.12	0.00	1.67	0.00	0.00	0.52	0.00
time (sec)	N/A	1.749	0.757	0.152	0.000	0.086	0.000	0.000	0.174	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	85	0	0	0	0	0	139	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	2.14	0.00
time (sec)	N/A	0.377	0.249	0.000	0.000	0.000	0.000	0.000	0.173	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	85	0	0	0	0	0	135	0
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	2.08	0.00
time (sec)	N/A	0.349	0.163	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	83	0	0	0	0	0	124	58
N.S.	1	1.00	1.48	0.00	0.00	0.00	0.00	0.00	2.21	1.04
time (sec)	N/A	0.318	0.159	0.000	0.000	0.000	0.000	0.000	0.163	0.367

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	50	0	0	0	0	0	60	0
N.S.	1	0.98	0.86	0.00	0.00	0.00	0.00	0.00	1.03	0.00
time (sec)	N/A	0.344	0.187	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	81	0	0	0	0	0	158	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	2.39	0.00
time (sec)	N/A	0.364	0.190	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	85	0	0	0	0	0	162	0
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	2.38	0.00
time (sec)	N/A	0.371	0.192	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	94	0	0	0	0	0	213	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	2.77	0.00
time (sec)	N/A	0.400	0.282	0.000	0.000	0.000	0.000	0.000	0.163	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	F	F	A	F	A	F	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	40	0	40	0	0	34	0	37	8	0
N.S.	1	0.00	1.00	0.00	0.00	0.85	0.00	0.92	0.20	0.00
time (sec)	N/A	0.000	0.110	0.000	0.000	0.079	0.000	0.120	0.155	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	C	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	59	0	54	22	0
N.S.	1	1.00	0.00	0.00	0.00	0.78	0.00	0.71	0.29	0.00
time (sec)	N/A	0.455	0.000	0.000	0.000	0.081	0.000	0.124	0.156	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F(-1)	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	143	0	0	36	0
N.S.	1	1.00	0.00	0.00	0.00	1.04	0.00	0.00	0.26	0.00
time (sec)	N/A	0.700	0.000	0.000	0.000	0.083	0.000	0.000	0.157	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	123	0	0	119	525	0	85	0
N.S.	1	1.00	1.00	0.00	0.00	0.97	4.27	0.00	0.69	0.00
time (sec)	N/A	0.613	0.416	0.000	0.000	0.082	104.103	0.000	0.163	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	0	0	112	357	0	52	0
N.S.	1	1.00	1.00	0.00	0.00	0.97	3.10	0.00	0.45	0.00
time (sec)	N/A	0.581	1.124	0.000	0.000	0.077	22.752	0.000	0.167	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	116	218	101	25	0
N.S.	1	1.00	1.00	0.00	0.00	0.98	1.85	0.86	0.21	0.00
time (sec)	N/A	0.540	0.126	0.000	0.000	0.073	5.119	0.173	0.162	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	377	0	57	0	0	36	49
N.S.	1	1.00	1.00	5.63	0.00	0.85	0.00	0.00	0.54	0.73
time (sec)	N/A	0.380	0.147	0.996	0.000	0.076	0.000	0.000	0.163	0.226

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	121	0	0	119	163	0	47	0
N.S.	1	1.00	1.00	0.00	0.00	0.98	1.35	0.00	0.39	0.00
time (sec)	N/A	0.584	0.379	0.000	0.000	0.078	71.859	0.000	0.162	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	117	0	0	114	0	0	58	0
N.S.	1	1.00	0.99	0.00	0.00	0.97	0.00	0.00	0.49	0.00
time (sec)	N/A	0.572	0.381	0.000	0.000	0.080	0.000	0.000	0.172	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	0	30	31	0	30	33	31
N.S.	1	1.00	1.07	0.00	1.07	1.11	0.00	1.07	1.18	1.11
time (sec)	N/A	0.287	4.799	180.000	0.421	0.080	0.000	0.237	0.163	0.132

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	494	396	0	0	513	0	0	113	0
N.S.	1	0.98	0.79	0.00	0.00	1.02	0.00	0.00	0.23	0.00
time (sec)	N/A	1.616	3.832	180.000	0.000	0.088	0.000	0.000	0.170	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	372	367	303	0	0	367	1027	0	82	0
N.S.	1	0.99	0.81	0.00	0.00	0.99	2.76	0.00	0.22	0.00
time (sec)	N/A	1.071	0.628	180.000	0.000	0.085	105.667	0.000	0.172	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	242	240	204	0	0	231	558	0	51	0
N.S.	1	0.99	0.84	0.00	0.00	0.95	2.31	0.00	0.21	0.00
time (sec)	N/A	0.742	0.309	180.000	0.000	0.081	23.416	0.000	0.162	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F(-1)	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	0	0	116	218	101	25	0
N.S.	1	1.00	1.00	0.00	0.00	0.98	1.85	0.86	0.21	0.00
time (sec)	N/A	0.533	0.077	180.000	0.000	0.082	4.895	0.175	0.159	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	0	30	31	24	30	33	31
N.S.	1	1.00	1.07	0.00	1.07	1.11	0.86	1.07	1.18	1.11
time (sec)	N/A	0.296	2.238	180.000	0.383	0.087	9.062	0.262	0.163	0.135

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	0	30	42	0	30	44	31
N.S.	1	1.00	1.07	0.00	1.07	1.50	0.00	1.07	1.57	1.11
time (sec)	N/A	0.295	5.440	180.000	0.413	0.077	0.000	1.075	0.166	0.137

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	F(-1)	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	0	30	53	0	30	55	31
N.S.	1	1.00	1.07	0.00	1.07	1.89	0.00	1.07	1.96	1.11
time (sec)	N/A	0.296	5.617	180.000	0.405	0.085	0.000	0.309	0.170	0.149

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F(-1)	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	169	0	0	0	169	0	0	56	0
N.S.	1	1.10	0.00	0.00	0.00	1.10	0.00	0.00	0.37	0.00
time (sec)	N/A	0.965	0.000	180.000	0.000	0.082	0.000	0.000	0.166	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	129	0	0	134	0	0	141	0
N.S.	1	1.00	0.97	0.00	0.00	1.01	0.00	0.00	1.06	0.00
time (sec)	N/A	0.850	0.510	0.000	0.000	0.081	0.000	0.000	0.185	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	120	0	0	128	724	0	90	0
N.S.	1	1.14	0.98	0.00	0.00	1.05	5.93	0.00	0.74	0.00
time (sec)	N/A	0.836	1.710	0.000	0.000	0.082	68.679	0.000	0.170	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	0	0	131	457	117	45	0
N.S.	1	1.00	0.98	0.00	0.00	1.04	3.63	0.93	0.36	0.00
time (sec)	N/A	0.733	0.112	0.000	0.000	0.078	12.746	0.207	0.159	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	A	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	70	59	59	125	0	66	0	0	56	63
N.S.	1	0.84	0.84	1.79	0.00	0.94	0.00	0.00	0.80	0.90
time (sec)	N/A	0.548	0.178	0.941	0.000	0.076	0.000	0.000	0.168	0.166

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	126	0	0	134	296	0	67	0
N.S.	1	1.00	0.98	0.00	0.00	1.05	2.31	0.00	0.52	0.00
time (sec)	N/A	0.811	0.457	0.000	0.000	0.080	156.042	0.000	0.178	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	142	121	0	0	129	0	0	78	0
N.S.	1	1.13	0.96	0.00	0.00	1.02	0.00	0.00	0.62	0.00
time (sec)	N/A	0.862	0.449	0.000	0.000	0.097	0.000	0.000	0.172	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	50	0	30	53	31
N.S.	1	1.00	1.07	1.00	1.07	1.79	0.00	1.07	1.89	1.11
time (sec)	N/A	0.296	2.140	0.036	0.679	0.083	0.000	0.326	0.188	0.135

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	535	527	434	0	0	564	0	0	187	0
N.S.	1	0.99	0.81	0.00	0.00	1.05	0.00	0.00	0.35	0.00
time (sec)	N/A	2.096	4.875	0.000	0.000	0.122	0.000	0.000	0.219	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	397	392	331	0	0	407	0	0	138	0
N.S.	1	0.99	0.83	0.00	0.00	1.03	0.00	0.00	0.35	0.00
time (sec)	N/A	1.342	0.848	0.000	0.000	0.109	0.000	0.000	0.199	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	257	255	221	0	0	259	1149	0	89	0
N.S.	1	0.99	0.86	0.00	0.00	1.01	4.47	0.00	0.35	0.00
time (sec)	N/A	0.917	0.448	0.000	0.000	0.082	68.876	0.000	0.181	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	0	0	131	457	117	45	0
N.S.	1	1.00	0.98	0.00	0.00	1.04	3.63	0.93	0.36	0.00
time (sec)	N/A	0.713	0.066	0.000	0.000	0.077	13.065	0.182	0.167	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	50	24	30	53	31
N.S.	1	1.00	1.07	1.00	1.07	1.79	0.86	1.07	1.89	1.11
time (sec)	N/A	0.296	2.295	0.032	0.640	0.074	15.824	0.350	0.181	0.141

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	61	0	30	64	31
N.S.	1	1.00	1.07	1.00	1.07	2.18	0.00	1.07	2.29	1.11
time (sec)	N/A	0.301	8.054	0.031	0.653	0.074	0.000	1.554	0.204	0.154

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	30	28	30	72	0	30	75	31
N.S.	1	1.00	1.07	1.00	1.07	2.57	0.00	1.07	2.68	1.11
time (sec)	N/A	0.298	8.504	0.032	0.660	0.077	0.000	0.461	0.189	0.157

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	17	17	24	17	17	17
N.S.	1	1.00	1.00	1.06	1.00	1.00	1.41	1.00	1.00	1.00
time (sec)	N/A	0.355	0.081	0.104	0.026	0.087	0.052	0.122	0.165	0.133

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	20	32	20	20	20
N.S.	1	1.00	0.95	1.05	1.00	1.00	1.60	1.00	1.00	1.00
time (sec)	N/A	0.501	0.882	0.903	0.030	0.076	0.359	0.115	0.168	0.366

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	0	0	24	0	0	114	49
N.S.	1	1.00	0.90	0.00	0.00	0.49	0.00	0.00	2.33	1.00
time (sec)	N/A	0.566	0.173	0.000	0.000	0.076	0.000	0.000	0.173	0.209

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	94	49	87	2381	109	160	53	145	145
N.S.	1	1.04	0.54	0.97	26.46	1.21	1.78	0.59	1.61	1.61
time (sec)	N/A	0.691	0.290	0.616	0.777	0.073	0.118	0.132	0.168	0.252

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	36	62	1223	55	68	42	64	64
N.S.	1	1.03	0.56	0.97	19.11	0.86	1.06	0.66	1.00	1.00
time (sec)	N/A	0.610	0.264	0.522	0.434	0.074	0.083	0.135	0.172	0.157

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	23	24	501	23	22	23	24	23
N.S.	1	1.00	0.61	0.63	13.18	0.61	0.58	0.61	0.63	0.61
time (sec)	N/A	0.480	0.189	0.199	0.217	0.074	0.057	0.129	0.171	0.052

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	10	11	12	13
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.83	0.92	1.00	1.08
time (sec)	N/A	0.264	0.085	0.046	0.032	0.102	0.041	0.114	0.152	0.042

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	19	0	11	10	11	63	11
N.S.	1	1.00	0.91	1.73	0.00	1.00	0.91	1.00	5.73	1.00
time (sec)	N/A	0.530	1.430	0.264	0.000	0.071	6.365	0.136	0.159	0.226

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	45	0	49	24	0	117	44
N.S.	1	1.00	0.92	1.18	0.00	1.29	0.63	0.00	3.08	1.16
time (sec)	N/A	0.596	0.828	0.289	0.000	0.073	63.573	0.000	0.173	0.297

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	71	50	70	0	111	0	0	199	62
N.S.	1	0.99	0.69	0.97	0.00	1.54	0.00	0.00	2.76	0.86
time (sec)	N/A	0.695	1.453	0.365	0.000	0.077	0.000	0.000	0.176	0.426

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	148	46	119	0	0	0	91	32	135
N.S.	1	1.04	0.32	0.84	0.00	0.00	0.00	0.64	0.23	0.95
time (sec)	N/A	1.262	3.305	0.328	0.000	0.000	0.000	0.135	200.017	0.561

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	46	94	0	0	0	77	299	117
N.S.	1	1.04	0.41	0.84	0.00	0.00	0.00	0.69	2.67	1.04
time (sec)	N/A	1.027	2.459	0.267	0.000	0.000	0.000	0.125	3.727	0.378

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	84	46	69	0	0	94	63	158	102
N.S.	1	1.02	0.56	0.84	0.00	0.00	1.15	0.77	1.93	1.24
time (sec)	N/A	0.861	2.282	0.248	0.000	0.000	43.269	0.138	0.360	0.292

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	44	0	0	78	45	61	76
N.S.	1	1.00	0.88	0.85	0.00	0.00	1.50	0.87	1.17	1.46
time (sec)	N/A	0.677	2.197	0.244	0.000	0.000	1.918	0.139	0.234	0.231

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	F	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	46	18	0	0	49	20	65	49
N.S.	1	1.00	2.19	0.86	0.00	0.00	2.33	0.95	3.10	2.33
time (sec)	N/A	0.672	2.200	0.252	0.000	0.000	1.474	0.135	0.198	0.279

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	62	45	0	0	80	0	131	79
N.S.	1	1.00	1.22	0.88	0.00	0.00	1.57	0.00	2.57	1.55
time (sec)	N/A	0.763	2.351	0.252	0.000	0.000	2.565	0.000	0.228	0.382

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	86	77	70	0	0	105	0	251	104
N.S.	1	1.01	0.91	0.82	0.00	0.00	1.24	0.00	2.95	1.22
time (sec)	N/A	0.840	4.927	0.244	0.000	0.000	17.035	0.000	0.273	0.513

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	121	91	95	0	0	0	0	421	129
N.S.	1	1.05	0.79	0.83	0.00	0.00	0.00	0.00	3.66	1.12
time (sec)	N/A	0.923	5.794	0.254	0.000	0.000	0.000	0.000	0.372	0.797

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	156	103	120	0	0	0	0	641	154
N.S.	1	1.08	0.71	0.83	0.00	0.00	0.00	0.00	4.42	1.06
time (sec)	N/A	0.978	7.362	0.250	0.000	0.000	0.000	0.000	0.502	1.385

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	42	37	14	18	7	9	20	0
N.S.	1	1.00	5.25	4.62	1.75	2.25	0.88	1.12	2.50	0.00
time (sec)	N/A	0.273	0.048	0.139	0.105	0.073	0.460	0.140	0.168	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	8	7	7	15	7	8	7
N.S.	1	1.00	1.00	0.67	0.58	0.58	1.25	0.58	0.67	0.58
time (sec)	N/A	0.256	0.024	0.041	0.109	0.072	0.045	0.136	0.161	0.129

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	15	15	15	16	17	15
N.S.	1	1.00	1.00	1.00	3.75	3.75	3.75	4.00	4.25	3.75
time (sec)	N/A	0.251	0.043	0.031	0.026	0.072	0.043	0.114	0.162	0.072

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	26	32	15	30	27	13
N.S.	1	1.00	1.00	0.70	1.30	1.60	0.75	1.50	1.35	0.65
time (sec)	N/A	0.264	0.060	0.033	0.107	0.077	0.052	0.127	0.161	0.084

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	53	26	25	34	32	25	45	24
N.S.	1	1.00	1.47	0.72	0.69	0.94	0.89	0.69	1.25	0.67
time (sec)	N/A	0.276	0.121	0.043	0.111	0.098	0.358	0.138	0.159	0.053

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	11	11	10	11	12	11
N.S.	1	1.00	0.64	0.55	0.50	0.50	0.45	0.50	0.55	0.50
time (sec)	N/A	0.294	0.029	0.047	0.032	0.074	0.029	0.114	0.154	0.021

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	45	21	20	35	20	20	43	20
N.S.	1	1.00	1.55	0.72	0.69	1.21	0.69	0.69	1.48	0.69
time (sec)	N/A	0.277	0.094	0.033	0.103	0.074	0.325	0.115	0.162	0.117

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	26	11	12	21	15	21	29	15
N.S.	1	1.00	1.86	0.79	0.86	1.50	1.07	1.50	2.07	1.07
time (sec)	N/A	0.297	0.096	0.046	0.106	0.073	0.324	0.140	0.165	0.156

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	15	15	15	16	17	15
N.S.	1	1.00	1.00	1.33	1.25	1.25	1.25	1.33	1.42	1.25
time (sec)	N/A	0.261	0.038	0.033	0.035	0.072	0.058	0.103	0.157	0.075

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.85	0.77	0.77	0.62	0.77	0.92	0.77
time (sec)	N/A	0.246	0.015	0.040	0.030	0.069	0.034	0.113	0.160	0.112

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	17	23	14	17	21	16
N.S.	1	1.00	1.00	1.06	1.06	1.44	0.88	1.06	1.31	1.00
time (sec)	N/A	0.243	0.014	0.092	0.031	0.073	0.033	0.125	0.156	0.126

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	27	21	14	13	13	14	13	14	13
N.S.	1	1.50	1.17	0.78	0.72	0.72	0.78	0.72	0.78	0.72
time (sec)	N/A	0.305	0.044	0.030	0.027	0.068	0.036	0.145	0.174	0.042

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	24	4	3	20	3	3	27	3
N.S.	1	1.00	6.00	1.00	0.75	5.00	0.75	0.75	6.75	0.75
time (sec)	N/A	0.258	0.059	0.029	0.108	0.071	0.313	0.133	0.154	0.107

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	17	7	35	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	1.70	0.70	3.50	0.70
time (sec)	N/A	0.264	0.031	0.036	0.108	0.077	0.050	0.109	0.148	0.110

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	30	24	18	17	21	17	18	25	17
N.S.	1	1.11	0.89	0.67	0.63	0.78	0.63	0.67	0.93	0.63
time (sec)	N/A	0.296	0.037	0.033	0.032	0.073	0.046	0.133	0.154	0.042

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	10	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.83	0.83	1.00	0.83
time (sec)	N/A	0.314	0.024	0.030	0.028	0.073	0.035	0.128	0.154	0.029

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	15	15	15	16	17	15
N.S.	1	1.00	1.00	1.00	2.50	2.50	2.50	2.67	2.83	2.50
time (sec)	N/A	0.254	0.037	0.029	0.025	0.074	0.046	0.116	0.159	0.049

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	15	3	4	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	3.75	0.75	1.00	0.75
time (sec)	N/A	0.255	0.024	0.032	0.105	0.093	0.045	0.136	0.160	0.109

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	25	17	11	10	11	8	12	16	11
N.S.	1	2.08	1.42	0.92	0.83	0.92	0.67	1.00	1.33	0.92
time (sec)	N/A	0.331	0.046	0.037	0.030	0.075	0.039	0.135	0.156	0.031

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	23	15	9	8	11	8	11	12	11
N.S.	1	2.30	1.50	0.90	0.80	1.10	0.80	1.10	1.20	1.10
time (sec)	N/A	0.334	0.042	0.036	0.038	0.073	0.035	0.112	0.159	0.103

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	25	39	12	14	13	10	14	30	22
N.S.	1	1.39	2.17	0.67	0.78	0.72	0.56	0.78	1.67	1.22
time (sec)	N/A	0.336	0.070	0.053	0.033	0.075	0.040	0.118	0.154	0.134

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	20	4	3	16	3	16	23	3
N.S.	1	1.00	5.00	1.00	0.75	4.00	0.75	4.00	5.75	0.75
time (sec)	N/A	0.256	0.040	0.030	0.104	0.071	0.261	0.114	0.152	0.044

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	8	8	8	8	8
N.S.	1	1.00	1.00	0.82	0.73	0.73	0.73	0.73	0.73	0.73
time (sec)	N/A	0.264	0.007	0.026	0.032	0.070	0.079	0.128	0.153	0.188

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	13	13	14	13	25	13
N.S.	1	1.00	1.00	0.78	0.72	0.72	0.78	0.72	1.39	0.72
time (sec)	N/A	0.404	0.048	0.105	0.106	0.074	0.666	0.118	0.153	0.197

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	21	20	29	24	29	39	20
N.S.	1	1.00	1.32	0.68	0.65	0.94	0.77	0.94	1.26	0.65
time (sec)	N/A	0.276	0.064	0.035	0.105	0.069	0.289	0.124	0.156	0.123

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	41	19	18	29	20	29	39	18
N.S.	1	1.00	1.52	0.70	0.67	1.07	0.74	1.07	1.44	0.67
time (sec)	N/A	0.269	0.059	0.031	0.107	0.072	0.276	0.125	0.149	0.034

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	17	7	8	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	1.70	0.70	0.80	0.70
time (sec)	N/A	0.458	0.040	0.052	0.110	0.079	0.055	0.113	0.156	0.039

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	30	13	16	11	11	24	11	14	16
N.S.	1	1.07	0.46	0.57	0.39	0.39	0.86	0.39	0.50	0.57
time (sec)	N/A	0.349	0.004	0.021	0.028	0.073	0.335	0.128	0.150	0.109

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	20	13	16	16	17	16	23	12
N.S.	1	1.00	1.25	0.81	1.00	1.00	1.06	1.00	1.44	0.75
time (sec)	N/A	0.271	0.043	0.030	0.033	0.085	0.255	0.129	0.155	0.226

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	15	15	15	16	17	15
N.S.	1	1.00	1.00	1.33	1.25	1.25	1.25	1.33	1.42	1.25
time (sec)	N/A	0.260	0.045	0.034	0.027	0.070	0.045	0.134	0.168	0.168

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	17	7	17	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	1.70	0.70	1.70	0.70
time (sec)	N/A	0.252	0.277	0.037	0.105	0.072	0.047	0.131	0.161	0.031

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	0	9	18	8	18	25	0
N.S.	1	1.00	1.71	0.00	0.64	1.29	0.57	1.29	1.79	0.00
time (sec)	N/A	0.265	0.046	0.000	0.107	0.101	0.341	0.118	0.156	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	39	28	26	29	29	32	25	26	25
N.S.	1	1.11	0.80	0.74	0.83	0.83	0.91	0.71	0.74	0.71
time (sec)	N/A	0.535	0.095	0.421	0.030	0.079	0.255	0.134	0.158	0.138

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	8	7	8	8	8
N.S.	1	1.00	1.00	0.82	0.73	0.73	0.64	0.73	0.73	0.73
time (sec)	N/A	0.232	0.023	0.037	0.031	0.073	0.029	0.131	0.155	0.028

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	8	7	8	8	8
N.S.	1	1.00	1.00	0.82	0.73	0.73	0.64	0.73	0.73	0.73
time (sec)	N/A	0.243	0.016	0.034	0.026	0.071	0.031	0.113	0.148	0.020

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	7	6	6	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.78	0.67	0.67	0.67
time (sec)	N/A	0.231	0.005	0.018	0.030	0.101	0.075	0.132	0.170	0.115

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	7	6	7	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.78	0.67	0.78	0.67
time (sec)	N/A	0.231	0.005	0.019	0.032	0.071	0.111	0.109	0.152	0.142

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	31	59	31	31	31	32	31
N.S.	1	1.00	0.50	0.46	0.87	0.46	0.46	0.46	0.47	0.46
time (sec)	N/A	0.448	0.127	0.066	0.034	0.067	0.042	0.138	0.162	0.033

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	19	19	20	19	24	21
N.S.	1	1.00	0.93	0.75	0.68	0.68	0.71	0.68	0.86	0.75
time (sec)	N/A	0.324	0.052	0.090	0.031	0.069	0.042	0.141	0.169	0.111

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	20	19	18	22	18	23	18
N.S.	1	1.00	0.88	0.77	0.73	0.69	0.85	0.69	0.88	0.69
time (sec)	N/A	0.301	0.027	0.096	0.034	0.079	0.036	0.114	0.163	0.118

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	8	8	7	8	11	8
N.S.	1	1.00	1.00	1.00	0.89	0.89	0.78	0.89	1.22	0.89
time (sec)	N/A	0.261	0.003	0.036	0.031	0.080	0.045	0.132	0.161	0.035

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	18	17	21	20	17	20	17
N.S.	1	1.00	0.74	0.67	0.63	0.78	0.74	0.63	0.74	0.63
time (sec)	N/A	0.272	0.058	0.181	0.027	0.074	0.161	0.106	0.164	0.017

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	15	17	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.88	1.00	0.88
time (sec)	N/A	0.318	0.048	0.050	0.031	0.071	0.055	0.113	0.166	0.120

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.278	0.025	0.029	0.033	0.073	0.036	0.132	0.155	0.102

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	20	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.74	0.70
time (sec)	N/A	0.268	0.056	0.182	0.027	0.074	0.086	0.113	0.158	0.017

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	7	5	4	16	8	6	8	6
N.S.	1	1.00	1.40	1.00	0.80	3.20	1.60	1.20	1.60	1.20
time (sec)	N/A	0.297	0.010	0.180	0.026	0.080	0.312	0.129	0.166	0.027

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	24	24	18	16	24	17	19	27	19
N.S.	1	1.14	1.14	0.86	0.76	1.14	0.81	0.90	1.29	0.90
time (sec)	N/A	0.291	0.035	0.030	0.028	0.079	0.043	0.131	0.166	0.040

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	24	19	20	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.89	0.70	0.74	0.70
time (sec)	N/A	0.270	0.085	0.266	0.028	0.071	0.088	0.114	0.163	0.122

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	28	39	52	0	41	89	78
N.S.	1	1.00	1.00	0.82	1.15	1.53	0.00	1.21	2.62	2.29
time (sec)	N/A	0.437	0.026	0.529	0.033	0.114	0.000	0.119	0.161	1.175

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	20	18	16	18	14	16	25	10
N.S.	1	1.00	0.77	0.69	0.62	0.69	0.54	0.62	0.96	0.38
time (sec)	N/A	0.291	0.086	0.062	0.031	0.074	0.041	0.132	0.159	0.034

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	10	14	13	13	12	13	15	13
N.S.	1	1.00	0.67	0.93	0.87	0.87	0.80	0.87	1.00	0.87
time (sec)	N/A	0.307	0.040	0.041	0.027	0.070	0.053	0.105	0.167	0.128

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	15	17	14	22	17	15	14
N.S.	1	1.00	0.81	0.56	0.63	0.52	0.81	0.63	0.56	0.52
time (sec)	N/A	0.291	0.037	0.036	0.031	0.070	0.399	0.112	0.158	0.037

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	22	15	17	14	22	17	15	14
N.S.	1	1.00	0.81	0.56	0.63	0.52	0.81	0.63	0.56	0.52
time (sec)	N/A	0.291	0.039	0.033	0.033	0.073	0.510	0.138	0.165	0.118

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	50	48	62	58	49	57	49
N.S.	1	1.00	0.90	0.81	0.77	1.00	0.94	0.79	0.92	0.79
time (sec)	N/A	0.369	0.226	0.110	0.105	0.076	0.877	0.135	0.162	0.468

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	19	11	10	11	12	11
N.S.	1	1.00	0.63	0.63	1.00	0.58	0.53	0.58	0.63	0.58
time (sec)	N/A	0.332	0.037	0.035	0.028	0.100	0.033	0.119	0.162	0.111

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	16	28	16	15	16	17	16
N.S.	1	1.00	0.59	0.50	0.88	0.50	0.47	0.50	0.53	0.50
time (sec)	N/A	0.352	0.040	0.035	0.033	0.066	0.035	0.132	0.164	0.113

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	11	11	10	10	8	10	11	10
N.S.	1	0.00	1.00	1.00	0.91	0.91	0.73	0.91	1.00	0.91
time (sec)	N/A	0.000	0.050	0.159	0.080	0.076	0.158	0.111	0.155	0.156

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	6	6	5	6	7	6
N.S.	1	1.00	1.00	0.78	0.67	0.67	0.56	0.67	0.78	0.67
time (sec)	N/A	0.261	0.010	0.049	0.029	0.074	0.043	0.110	0.161	0.021

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	9	15	9	18	12	16	9	9
N.S.	1	0.00	1.00	1.67	1.00	2.00	1.33	1.78	1.00	1.00
time (sec)	N/A	0.000	0.025	0.062	0.073	0.091	0.075	0.149	0.162	0.186

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	28	26	22	21	21	22	21	23	21
N.S.	1	1.12	1.04	0.88	0.84	0.84	0.88	0.84	0.92	0.84
time (sec)	N/A	0.287	0.027	0.068	0.026	0.070	0.044	0.132	0.154	0.032

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	40	34	33	33	37	33	36	33
N.S.	1	1.08	1.00	0.85	0.82	0.82	0.92	0.82	0.90	0.82
time (sec)	N/A	0.302	0.035	0.122	0.027	0.091	0.053	0.129	0.162	0.041

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	53	46	45	45	51	45	49	45
N.S.	1	1.06	1.00	0.87	0.85	0.85	0.96	0.85	0.92	0.85
time (sec)	N/A	0.320	0.037	0.237	0.026	0.068	0.062	0.136	0.169	0.035

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	26	45	80	60	29	16	25
N.S.	1	1.00	1.00	0.81	1.41	2.50	1.88	0.91	0.50	0.78
time (sec)	N/A	0.293	0.064	0.060	0.112	0.078	0.593	0.120	0.167	0.163

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	28	27	85	58	27	18	31
N.S.	1	1.00	1.00	0.82	0.79	2.50	1.71	0.79	0.53	0.91
time (sec)	N/A	0.296	0.064	0.053	0.117	0.076	0.617	0.137	0.169	0.164

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	50	42	63	107	78	44	14	43
N.S.	1	0.96	0.94	0.79	1.19	2.02	1.47	0.83	0.26	0.81
time (sec)	N/A	0.303	0.071	0.033	0.111	0.077	0.985	0.116	0.161	0.156

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	55	54	46	47	115	75	45	16	47
N.S.	1	0.96	0.95	0.81	0.82	2.02	1.32	0.79	0.28	0.82
time (sec)	N/A	0.308	0.067	0.037	0.116	0.073	0.855	0.135	0.155	0.135

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	20	18	17	21	24	17	20	19
N.S.	1	1.00	0.74	0.67	0.63	0.78	0.89	0.63	0.74	0.70
time (sec)	N/A	0.272	0.050	0.165	0.031	0.078	0.090	0.115	0.157	0.100

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	19	8	10	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.90	0.80	1.00	0.80
time (sec)	N/A	0.262	0.034	0.033	0.108	0.075	0.051	0.114	0.158	0.045

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	18	17	17	19	18	20	17
N.S.	1	1.00	1.00	1.80	1.70	1.70	1.90	1.80	2.00	1.70
time (sec)	N/A	0.261	0.047	0.034	0.026	0.074	0.052	0.126	0.156	0.138

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	10	14	21	15	0
N.S.	1	1.00	1.00	0.83	0.78	0.56	0.78	1.17	0.83	0.00
time (sec)	N/A	0.272	3.813	0.029	0.031	0.069	0.767	0.110	0.170	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	19	18	26	42	17	33	25	17
N.S.	1	1.00	0.40	0.38	0.55	0.89	0.36	0.70	0.53	0.36
time (sec)	N/A	0.359	0.078	0.042	0.107	0.074	0.058	0.145	0.162	0.288

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	54	24	21	21	21	20	21	22	21
N.S.	1	1.23	0.55	0.48	0.48	0.48	0.45	0.48	0.50	0.48
time (sec)	N/A	0.391	0.023	0.037	0.031	0.071	0.034	0.130	0.163	0.016

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	57	26	23	30	23	24	23	27	23
N.S.	1	1.10	0.50	0.44	0.58	0.44	0.46	0.44	0.52	0.44
time (sec)	N/A	0.411	0.038	0.044	0.031	0.064	0.036	0.123	0.157	0.113

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	45	23	22	35	24	22	43	22
N.S.	1	1.00	1.36	0.70	0.67	1.06	0.73	0.67	1.30	0.67
time (sec)	N/A	0.284	0.100	0.037	0.106	0.080	0.321	0.139	0.161	0.132

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	54	33	39	37	32	36	53	35	32
N.S.	1	1.08	0.66	0.78	0.74	0.64	0.72	1.06	0.70	0.64
time (sec)	N/A	0.321	0.055	0.025	0.033	0.070	0.699	0.127	0.157	0.126

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	48	62	61	50	61	75	61	38
N.S.	1	1.00	0.59	0.77	0.75	0.62	0.75	0.93	0.75	0.47
time (sec)	N/A	0.362	0.067	0.056	0.032	0.073	6.508	0.134	0.159	0.109

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	46	54	45	44	44	49	44	49	44
N.S.	1	0.84	0.98	0.82	0.80	0.80	0.89	0.80	0.89	0.80
time (sec)	N/A	0.451	0.054	0.622	0.033	0.070	0.061	0.126	0.158	0.122

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	55	27	26	43	26	26	54	26
N.S.	1	1.00	1.57	0.77	0.74	1.23	0.74	0.74	1.54	0.74
time (sec)	N/A	0.521	0.116	0.087	0.107	0.077	15.408	0.120	0.166	0.148

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	24	29	24	23	23	24	23	27	24
N.S.	1	0.75	0.91	0.75	0.72	0.72	0.75	0.72	0.84	0.75
time (sec)	N/A	0.602	0.094	0.161	0.033	0.072	0.062	0.137	0.162	0.127

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	3	3	3	3	5	3
N.S.	1	1.00	1.00	0.80	0.60	0.60	0.60	0.60	1.00	0.60
time (sec)	N/A	0.253	0.008	0.021	0.032	0.073	0.236	0.145	0.169	0.017

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	5	4	4	5	0	7	4
N.S.	1	1.00	1.00	0.71	0.57	0.57	0.71	0.00	1.00	0.57
time (sec)	N/A	0.308	0.018	0.032	0.027	0.066	0.393	0.000	0.163	0.025

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	25	23	17	16	19	17	24	24	8
N.S.	1	1.14	1.05	0.77	0.73	0.86	0.77	1.09	1.09	0.36
time (sec)	N/A	0.297	0.036	0.066	0.032	0.067	0.039	0.140	0.163	0.116

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	7	3	15	3	4	3
N.S.	1	1.00	1.00	1.00	1.75	0.75	3.75	0.75	1.00	0.75
time (sec)	N/A	0.243	0.005	0.021	0.110	0.095	0.042	0.138	0.157	0.011

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	11	10	10	10	10	17	12
N.S.	1	1.00	1.00	0.85	0.77	0.77	0.77	0.77	1.31	0.92
time (sec)	N/A	0.253	0.024	0.031	0.033	0.080	0.028	0.130	0.153	0.039

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	19	15	15	16	17	15
N.S.	1	1.00	1.00	1.00	3.17	2.50	2.50	2.67	2.83	2.50
time (sec)	N/A	0.246	0.002	0.025	0.027	0.070	0.046	0.127	0.161	0.031

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	11	11	10	10	10	10	18	12
N.S.	1	1.00	0.73	0.73	0.67	0.67	0.67	0.67	1.20	0.80
time (sec)	N/A	0.256	0.054	0.033	0.028	0.062	0.028	0.107	0.161	0.125

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	18	21	18	15	17	21	17
N.S.	1	1.00	1.05	0.82	0.95	0.82	0.68	0.77	0.95	0.77
time (sec)	N/A	0.316	0.056	0.092	0.031	0.068	0.044	0.119	0.160	0.031

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	27	30	23	24	25	26	30	31	22
N.S.	1	0.87	0.97	0.74	0.77	0.81	0.84	0.97	1.00	0.71
time (sec)	N/A	0.316	0.051	0.159	0.039	0.070	0.056	0.139	0.161	0.120

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	21	21	23	22	21	29	0	34	21
N.S.	1	0.95	0.95	1.05	1.00	0.95	1.32	0.00	1.55	0.95
time (sec)	N/A	0.303	0.123	0.075	0.103	0.073	0.123	0.000	0.149	0.204

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	26	23	39	40	25	39	0	44	0
N.S.	1	0.76	0.68	1.15	1.18	0.74	1.15	0.00	1.29	0.00
time (sec)	N/A	0.310	0.143	0.052	0.104	0.075	0.145	0.000	0.154	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	0	28	0	0	42	27
N.S.	1	1.00	1.00	0.00	0.00	1.22	0.00	0.00	1.83	1.17
time (sec)	N/A	0.306	0.371	0.000	0.000	0.072	0.000	0.000	0.153	0.143

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	41	51	74	35	54	35	39	50
N.S.	1	1.00	0.62	0.77	1.12	0.53	0.82	0.53	0.59	0.76
time (sec)	N/A	0.568	0.242	0.075	0.033	0.069	4.743	0.115	0.146	0.173

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	20	19	25	20	20	30	19
N.S.	1	1.00	1.00	1.67	1.58	2.08	1.67	1.67	2.50	1.58
time (sec)	N/A	0.262	0.050	0.035	0.028	0.073	0.050	0.128	0.162	0.036

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	12	0	11	8	54	12	11
N.S.	1	1.00	0.81	0.75	0.00	0.69	0.50	3.38	0.75	0.69
time (sec)	N/A	0.355	0.081	0.065	0.000	0.068	0.049	0.129	0.159	0.130

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	13	13	10	13	15	14
N.S.	1	1.00	1.00	0.88	0.81	0.81	0.62	0.81	0.94	0.88
time (sec)	N/A	0.306	0.151	0.057	0.029	0.061	0.035	0.127	0.165	0.134

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	23	74	33	27	33	45	0
N.S.	1	1.00	1.00	0.70	2.24	1.00	0.82	1.00	1.36	0.00
time (sec)	N/A	0.284	0.055	0.049	0.031	0.076	0.379	0.126	0.163	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	0	8	8	8	8
N.S.	1	1.00	1.00	0.82	0.73	0.00	0.73	0.73	0.73	0.73
time (sec)	N/A	0.304	0.030	0.057	0.028	0.000	0.069	0.131	0.147	0.154

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	5	5	6	5
N.S.	1	1.00	1.00	1.00	0.83	0.83	0.83	0.83	1.00	0.83
time (sec)	N/A	0.293	0.020	0.023	0.031	0.064	0.038	0.130	0.146	0.020

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	19	18	14	0	11	21
N.S.	1	1.00	1.00	0.89	1.00	0.95	0.74	0.00	0.58	1.11
time (sec)	N/A	0.276	0.030	0.031	0.051	0.094	0.147	0.000	0.147	0.104

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	36	15	12	20	16	15
N.S.	1	1.00	0.84	0.84	1.89	0.79	0.63	1.05	0.84	0.79
time (sec)	N/A	0.442	0.134	0.050	0.052	0.071	0.036	0.135	0.153	0.121

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	41	60	63	0	97	1392	59	62
N.S.	1	1.09	0.73	1.07	1.12	0.00	1.73	24.86	1.05	1.11
time (sec)	N/A	0.405	0.140	0.174	0.034	0.000	0.066	0.136	0.149	0.195

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	12	13	13	10	13	11	11
N.S.	1	1.00	0.80	0.80	0.87	0.87	0.67	0.87	0.73	0.73
time (sec)	N/A	0.250	0.042	0.056	0.033	0.076	0.040	0.113	0.160	0.038

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	12	11	10	11	10	11
N.S.	1	1.00	1.00	0.86	0.86	0.79	0.71	0.79	0.71	0.79
time (sec)	N/A	0.249	0.006	0.055	0.026	0.070	0.059	0.135	0.163	0.123

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	8	11	11	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.00	1.00	1.00
time (sec)	N/A	0.245	0.005	0.050	0.025	0.085	0.039	0.108	0.161	0.119

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	20	17	17	20	17	17
N.S.	1	1.00	1.00	0.85	1.00	0.85	0.85	1.00	0.85	0.85
time (sec)	N/A	0.245	0.021	0.062	0.024	0.067	0.059	0.130	0.159	0.122

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	19	16	34	16	15	16	19	16
N.S.	1	1.00	0.59	0.50	1.06	0.50	0.47	0.50	0.59	0.50
time (sec)	N/A	0.325	0.092	0.053	0.031	0.065	0.036	0.110	0.158	0.114

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	40	36	27	45	26
N.S.	1	1.00	1.00	0.85	0.82	1.21	1.09	0.82	1.36	0.79
time (sec)	N/A	0.254	0.025	0.201	0.030	0.084	0.040	0.132	0.162	0.165

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	12	11	11	10	11	10	11
N.S.	1	1.00	1.00	0.57	0.52	0.52	0.48	0.52	0.48	0.52
time (sec)	N/A	0.251	0.005	0.066	0.026	0.070	0.058	0.132	0.160	0.125

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	0	0	0	8	8
N.S.	1	1.00	1.00	0.82	0.73	0.00	0.00	0.00	0.73	0.73
time (sec)	N/A	0.284	0.003	0.029	0.063	0.000	0.000	0.000	0.157	0.135

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	16	0	0	0	9	9
N.S.	1	1.00	1.00	0.83	1.33	0.00	0.00	0.00	0.75	0.75
time (sec)	N/A	0.603	0.178	0.070	0.060	0.000	0.000	0.000	0.163	0.150

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	9	0	0	0	9	9
N.S.	1	1.00	1.00	0.83	0.75	0.00	0.00	0.00	0.75	0.75
time (sec)	N/A	0.434	0.014	0.033	0.061	0.000	0.000	0.000	0.162	0.139

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	12	14	0	15	14	55	14
N.S.	1	1.00	1.12	0.75	0.88	0.00	0.94	0.88	3.44	0.88
time (sec)	N/A	0.583	0.257	0.000	0.059	0.000	0.776	0.157	0.168	0.152

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	28	19	21	0	15	21	55	21
N.S.	1	1.00	1.08	0.73	0.81	0.00	0.58	0.81	2.12	0.81
time (sec)	N/A	0.424	0.024	0.000	0.061	0.000	0.233	0.156	0.167	0.118

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	10	12	0	14	12	55	12
N.S.	1	1.00	1.14	0.71	0.86	0.00	1.00	0.86	3.93	0.86
time (sec)	N/A	0.499	0.245	0.002	0.064	0.000	0.354	0.147	0.175	0.113

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	23	0	0	0	63	15
N.S.	1	1.00	1.00	0.80	1.15	0.00	0.00	0.00	3.15	0.75
time (sec)	N/A	0.973	1.211	0.105	0.074	0.000	0.000	0.000	0.168	0.232

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	18	0	19	18	21	18
N.S.	1	1.00	1.10	0.80	0.90	0.00	0.95	0.90	1.05	0.90
time (sec)	N/A	0.601	0.678	0.000	0.067	0.000	0.470	0.134	0.155	0.132

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	9	8	0	12	8	27	8
N.S.	1	1.00	1.00	0.69	0.62	0.00	0.92	0.62	2.08	0.62
time (sec)	N/A	0.313	0.029	0.025	0.025	0.000	0.092	0.117	0.153	0.139

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	9	8	0	0	0	27	8
N.S.	1	1.00	1.00	0.69	0.62	0.00	0.00	0.00	2.08	0.62
time (sec)	N/A	0.329	0.003	0.039	0.060	0.000	0.000	0.000	0.164	0.128

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	8	10	0	10	10	11	10
N.S.	1	1.00	1.18	0.73	0.91	0.00	0.91	0.91	1.00	0.91
time (sec)	N/A	0.387	0.081	0.000	0.055	0.000	0.237	0.136	0.160	0.109

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	16	0	0	0	45	9
N.S.	1	1.00	1.00	0.83	1.33	0.00	0.00	0.00	3.75	0.75
time (sec)	N/A	0.790	0.188	0.042	0.063	0.000	0.000	0.000	0.169	0.153

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	16	0	0	0	45	9
N.S.	1	1.00	1.00	0.83	1.33	0.00	0.00	0.00	3.75	0.75
time (sec)	N/A	0.442	0.016	0.046	0.071	0.000	0.000	0.000	0.177	0.146

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	34	31	23	24	27	22	28	31	22
N.S.	1	1.10	1.00	0.74	0.77	0.87	0.71	0.90	1.00	0.71
time (sec)	N/A	0.368	0.036	0.161	0.032	0.073	0.055	0.124	0.158	0.124

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	8	10	10	7	10	19	10
N.S.	1	1.00	1.22	0.89	1.11	1.11	0.78	1.11	2.11	1.11
time (sec)	N/A	0.310	5.719	0.005	0.055	0.068	0.209	0.127	0.164	0.100

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	10	12	0	12	12	19	12
N.S.	1	1.00	1.15	0.77	0.92	0.00	0.92	0.92	1.46	0.92
time (sec)	N/A	0.352	0.107	0.004	0.059	0.000	0.240	0.127	0.154	0.118

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	9	14	11	8	11	18	11
N.S.	1	1.00	1.18	0.82	1.27	1.00	0.73	1.00	1.64	1.00
time (sec)	N/A	0.318	0.025	0.003	0.061	0.065	0.210	0.105	0.156	0.115

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	18	13	10	13	24	13
N.S.	1	1.00	1.15	0.85	1.38	1.00	0.77	1.00	1.85	1.00
time (sec)	N/A	0.334	0.104	0.004	0.073	0.070	0.249	0.144	0.156	0.115

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	9	11	11	8	11	11	11
N.S.	1	1.00	1.22	1.00	1.22	1.22	0.89	1.22	1.22	1.22
time (sec)	N/A	0.356	0.078	0.003	0.052	0.061	0.200	0.107	0.155	0.102

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	11	13	13	10	13	13	13
N.S.	1	1.00	1.18	1.00	1.18	1.18	0.91	1.18	1.18	1.18
time (sec)	N/A	0.390	0.089	0.003	0.053	0.064	0.244	0.122	0.156	0.105

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	11	9	11	20	10	11	11	11
N.S.	1	1.00	1.22	1.00	1.22	2.22	1.11	1.22	1.22	1.22
time (sec)	N/A	0.377	0.031	0.003	0.050	0.070	0.397	0.110	0.161	0.114

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	13	11	13	22	12	13	13	13
N.S.	1	1.00	1.18	1.00	1.18	2.00	1.09	1.18	1.18	1.18
time (sec)	N/A	0.419	0.033	0.004	0.052	0.067	0.476	0.130	0.152	0.113

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	37	58	46	78	46	37	0
N.S.	1	1.00	1.00	1.03	1.61	1.28	2.17	1.28	1.03	0.00
time (sec)	N/A	0.472	0.122	0.488	0.075	0.076	0.388	0.226	0.160	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	241	36	0	0	0	21	0
N.S.	1	1.00	1.00	6.51	0.97	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.378	0.027	0.059	0.083	0.000	0.000	0.000	0.159	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	298	50	0	0	0	24	0
N.S.	1	1.00	1.00	5.96	1.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.372	0.036	0.283	0.129	0.000	0.000	0.000	0.158	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	219	38	0	105	0	11	58
N.S.	1	1.00	1.00	5.92	1.03	0.00	2.84	0.00	0.30	1.57
time (sec)	N/A	0.280	0.013	0.085	0.083	0.000	0.704	0.000	0.161	0.240

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	267	42	0	0	0	11	71
N.S.	1	1.00	1.00	6.51	1.02	0.00	0.00	0.00	0.27	1.73
time (sec)	N/A	0.279	0.013	0.141	0.089	0.000	0.000	0.000	0.169	0.291

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	0	0	19	77
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.37	1.48
time (sec)	N/A	0.310	0.020	0.000	0.000	0.000	0.000	0.000	0.160	0.284

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	19	94
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.34	1.68
time (sec)	N/A	0.305	0.019	0.000	0.000	0.000	0.000	0.000	0.159	0.342

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	74	0	0	89	0	0	36	0
N.S.	1	1.00	0.92	0.00	0.00	1.11	0.00	0.00	0.45	0.00
time (sec)	N/A	0.360	0.318	0.000	0.000	0.073	0.000	0.000	0.161	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	0	21	0	0	0	49	14
N.S.	1	1.00	1.00	0.00	1.24	0.00	0.00	0.00	2.88	0.82
time (sec)	N/A	1.182	0.769	0.000	0.058	0.000	0.000	0.000	0.157	0.178

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	A	A	A	F	F	F	B
verified	N/A	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	0	0	16	21	18	0	0	64	18
N.S.	1	0.00	0.00	0.89	1.17	1.00	0.00	0.00	3.56	1.00
time (sec)	N/A	0.000	0.000	0.106	0.073	0.093	0.000	0.000	0.167	0.195

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	10	13	27	76	11	10
N.S.	1	1.00	1.00	1.00	0.91	1.18	2.45	6.91	1.00	0.91
time (sec)	N/A	0.398	0.005	0.234	0.034	0.069	2.598	0.122	0.158	1.277

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	20	21	0	21	19	19
N.S.	1	1.00	1.00	1.00	1.00	1.05	0.00	1.05	0.95	0.95
time (sec)	N/A	0.528	0.004	2.811	0.035	0.077	0.000	0.137	0.155	4.188

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	24	26	28	31	26	26	26
N.S.	1	1.00	1.08	0.96	1.04	1.12	1.24	1.04	1.04	1.04
time (sec)	N/A	0.529	0.008	0.011	0.113	0.072	16.862	0.750	0.153	0.149

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	15	15	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.239	0.023	0.032	0.032	0.081	0.037	0.135	0.157	0.137

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	15	15	16	15
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.00	1.00	1.07	1.00
time (sec)	N/A	0.244	0.023	0.035	0.024	0.071	0.037	0.111	0.162	0.035

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	12	12	13	13	10	13	11	11
N.S.	1	1.00	0.80	0.80	0.87	0.87	0.67	0.87	0.73	0.73
time (sec)	N/A	0.236	0.022	0.040	0.026	0.069	0.036	0.129	0.159	0.044

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	30	26	53	427	26	26
N.S.	1	1.00	1.00	1.04	1.15	1.00	2.04	16.42	1.00	1.00
time (sec)	N/A	0.331	0.067	0.096	0.044	0.073	0.351	0.157	0.152	0.155

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	27	30	26	82	32	28	26
N.S.	1	1.00	1.00	1.04	1.15	1.00	3.15	1.23	1.08	1.00
time (sec)	N/A	0.332	0.072	0.099	0.111	0.067	0.345	0.147	0.154	0.154

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	F(-2)	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	19	27	25	26	0	32	23	16
N.S.	1	0.00	1.00	1.42	1.32	1.37	0.00	1.68	1.21	0.84
time (sec)	N/A	0.000	0.052	0.085	0.106	0.072	0.000	0.136	0.167	0.047

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	42	37	204	609	37	37
N.S.	1	1.00	1.00	1.03	1.14	1.00	5.51	16.46	1.00	1.00
time (sec)	N/A	0.753	0.133	0.251	0.036	0.075	1.161	0.150	0.168	0.151

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	38	42	37	68	45	40	37
N.S.	1	1.00	1.00	1.03	1.14	1.00	1.84	1.22	1.08	1.00
time (sec)	N/A	0.767	0.133	0.283	0.114	0.073	24.933	0.151	0.149	0.153

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	A	A	A	A	F(-2)	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	0	34	38	34	37	0	45	32	21
N.S.	1	0.00	0.97	1.09	0.97	1.06	0.00	1.29	0.91	0.60
time (sec)	N/A	0.000	0.081	0.342	0.103	0.071	0.000	0.130	0.163	0.158

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [433] had the largest ratio of [.6875000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	3	3	8.76	53	0.057
2	C	3	3	7.76	55	0.055
3	C	3	3	5.71	52	0.058
4	F	0	0	N/A	0.000	N/A
5	F	0	0	N/A	0.000	N/A
6	F	0	0	N/A	0.000	N/A
7	A	1	1	1.00	13	0.077
8	A	1	1	1.00	13	0.077
9	A	4	4	1.28	13	0.308
10	A	3	3	1.24	13	0.231
11	A	2	2	1.00	13	0.154
12	A	1	1	1.00	11	0.091
13	A	1	1	1.00	13	0.077
14	A	2	2	1.00	13	0.154
15	A	3	3	1.02	13	0.231
16	A	4	4	1.02	13	0.308
17	A	1	1	1.00	13	0.077
18	A	1	1	1.00	13	0.077
19	A	1	1	1.00	13	0.077
20	A	1	1	1.00	13	0.077
21	A	5	5	1.28	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	4	1.23	13	0.308
23	A	3	3	1.15	13	0.231
24	A	2	2	1.00	13	0.154
25	A	1	1	1.00	9	0.111
26	A	2	2	1.00	13	0.154
27	A	3	3	1.00	13	0.231
28	A	4	4	1.01	13	0.308
29	A	5	5	1.02	13	0.385
30	A	1	1	1.00	13	0.077
31	A	1	1	1.00	13	0.077
32	A	1	1	1.00	13	0.077
33	A	1	1	1.00	13	0.077
34	A	4	4	1.31	13	0.308
35	A	3	3	1.15	13	0.231
36	A	2	2	1.00	13	0.154
37	A	1	1	1.00	13	0.077
38	A	1	1	1.00	13	0.077
39	A	2	2	1.00	13	0.154
40	A	3	3	1.02	13	0.231
41	A	4	4	1.02	13	0.308
42	A	1	1	1.00	13	0.077
43	A	1	1	1.00	13	0.077
44	A	1	1	1.00	13	0.077
45	A	1	1	1.00	13	0.077
46	A	1	1	1.00	11	0.091
47	A	1	1	1.00	9	0.111
48	A	1	1	1.00	13	0.077
49	A	1	1	1.00	13	0.077
50	A	1	1	1.00	11	0.091
51	A	1	1	1.00	13	0.077
52	A	1	1	1.00	13	0.077
53	A	4	4	0.96	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	3	3	0.93	11	0.273
55	A	2	2	1.00	9	0.222
56	A	1	1	1.00	13	0.077
57	A	1	1	1.00	13	0.077
58	A	2	2	1.00	13	0.154
59	A	3	3	1.15	13	0.231
60	A	4	4	1.23	13	0.308
61	A	1	1	1.00	13	0.077
62	A	1	1	1.00	13	0.077
63	A	1	1	1.00	13	0.077
64	A	1	1	1.00	13	0.077
65	A	4	4	1.02	13	0.308
66	A	3	3	1.02	13	0.231
67	A	2	2	1.00	11	0.182
68	A	1	1	1.00	13	0.077
69	A	1	1	1.00	13	0.077
70	A	2	2	1.00	13	0.154
71	A	3	3	1.24	13	0.231
72	A	4	4	1.28	13	0.308
73	A	1	1	1.00	13	0.077
74	A	1	1	1.00	13	0.077
75	A	1	1	1.00	13	0.077
76	A	1	1	1.00	13	0.077
77	A	7	6	1.02	13	0.462
78	A	6	5	1.01	13	0.385
79	A	5	4	1.00	13	0.308
80	A	4	3	1.00	9	0.333
81	A	3	2	1.00	13	0.154
82	A	4	3	1.00	13	0.231
83	A	5	4	1.14	13	0.308
84	A	6	5	1.22	13	0.385
85	A	7	6	1.27	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.00	13	0.077
87	A	1	1	1.00	13	0.077
88	A	1	1	1.00	13	0.077
89	A	1	1	1.00	13	0.077
90	A	4	4	1.02	13	0.308
91	A	3	3	1.02	13	0.231
92	A	2	2	1.00	13	0.154
93	A	1	1	1.00	13	0.077
94	A	1	1	1.00	13	0.077
95	A	2	2	1.00	13	0.154
96	A	3	3	1.15	13	0.231
97	A	4	4	1.33	13	0.308
98	A	1	1	1.00	13	0.077
99	A	1	1	1.00	13	0.077
100	A	1	1	1.00	13	0.077
101	A	1	1	1.00	13	0.077
102	A	1	1	1.00	11	0.091
103	A	1	1	1.00	9	0.111
104	A	1	1	1.00	13	0.077
105	A	1	1	1.00	13	0.077
106	A	1	1	1.00	13	0.077
107	A	1	1	1.00	13	0.077
108	A	1	1	1.00	13	0.077
109	A	1	1	1.00	11	0.091
110	A	1	1	1.00	13	0.077
111	A	1	1	1.00	13	0.077
112	A	1	1	1.00	13	0.077
113	A	1	1	1.00	13	0.077
114	A	1	1	1.00	13	0.077
115	A	1	1	1.00	11	0.091
116	A	1	1	1.00	9	0.111
117	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	1	1	1.00	13	0.077
119	A	1	1	1.00	13	0.077
120	A	1	1	1.00	13	0.077
121	A	2	2	1.00	9	0.222
122	A	1	1	1.00	13	0.077
123	A	3	3	1.13	17	0.176
124	A	2	2	1.00	17	0.118
125	A	1	1	1.00	15	0.067
126	A	1	1	1.00	13	0.077
127	A	2	2	1.00	17	0.118
128	A	3	3	0.94	17	0.176
129	A	5	4	1.09	19	0.211
130	A	4	3	1.00	19	0.158
131	A	3	2	1.00	19	0.105
132	A	4	3	1.00	19	0.158
133	A	5	4	1.01	19	0.211
134	A	2	2	1.00	15	0.133
135	A	2	2	1.00	15	0.133
136	A	2	2	1.00	13	0.154
137	A	1	1	1.00	11	0.091
138	N/A	1	0	1.00	15	0.000
139	N/A	3	0	1.00	15	0.000
140	N/A	4	0	1.00	15	0.000
141	A	2	2	1.00	15	0.133
142	A	2	2	1.00	13	0.154
143	A	1	1	1.00	11	0.091
144	N/A	1	0	1.00	15	0.000
145	N/A	3	0	1.00	15	0.000
146	N/A	3	0	1.00	15	0.000
147	N/A	1	0	1.00	33	0.000
148	A	4	3	0.98	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	A	2	2	1.00	15	0.133
150	A	2	2	1.00	15	0.133
151	A	2	2	1.00	15	0.133
152	A	2	2	1.00	13	0.154
153	A	2	2	1.00	11	0.182
154	A	5	4	1.00	15	0.267
155	A	3	3	1.10	15	0.200
156	A	3	3	0.92	15	0.200
157	A	2	2	1.00	15	0.133
158	A	2	2	1.00	15	0.133
159	A	2	2	1.00	15	0.133
160	A	2	2	1.00	13	0.154
161	A	4	3	1.00	11	0.273
162	N/A	1	0	1.00	15	0.000
163	N/A	1	0	1.00	15	0.000
164	N/A	1	0	1.00	15	0.000
165	A	2	2	1.00	15	0.133
166	A	2	2	1.00	15	0.133
167	A	2	2	1.00	15	0.133
168	A	2	2	1.00	13	0.154
169	A	1	1	1.00	11	0.091
170	N/A	1	0	1.00	15	0.000
171	N/A	1	0	1.00	15	0.000
172	N/A	1	0	1.00	15	0.000
173	N/A	1	0	1.00	15	0.000
174	N/A	2	0	1.00	15	0.000
175	A	1	1	1.00	13	0.077
176	N/A	1	0	1.00	15	0.000
177	N/A	1	0	1.00	15	0.000
178	N/A	1	0	1.00	15	0.000
179	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	2	2	1.00	15	0.133
181	A	2	2	1.00	13	0.154
182	A	1	1	1.00	11	0.091
183	N/A	1	0	1.00	15	0.000
184	N/A	1	0	1.00	15	0.000
185	N/A	1	0	1.00	15	0.000
186	A	1	1	1.00	17	0.059
187	A	1	1	1.00	15	0.067
188	A	1	1	1.00	21	0.048
189	A	1	1	1.00	21	0.048
190	A	1	1	1.00	21	0.048
191	A	4	4	1.19	21	0.190
192	A	3	3	1.16	21	0.143
193	A	2	2	1.00	21	0.095
194	A	1	1	1.00	19	0.053
195	A	1	1	1.00	21	0.048
196	A	2	2	1.00	21	0.095
197	A	3	3	1.01	21	0.143
198	A	4	4	1.02	21	0.190
199	A	1	1	1.00	21	0.048
200	A	1	1	1.00	21	0.048
201	A	1	1	1.00	21	0.048
202	A	1	1	1.00	21	0.048
203	A	5	5	1.20	21	0.238
204	A	4	4	1.17	21	0.190
205	A	3	3	1.11	21	0.143
206	A	2	2	1.00	21	0.095
207	A	1	1	1.00	13	0.077
208	A	2	2	1.00	21	0.095
209	A	3	3	1.00	21	0.143
210	A	4	4	1.01	21	0.190
211	A	5	5	1.01	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
212	A	1	1	1.00	21	0.048
213	A	1	1	1.00	21	0.048
214	A	1	1	1.00	21	0.048
215	A	1	1	1.00	21	0.048
216	A	1	1	1.00	21	0.048
217	A	4	4	1.21	21	0.190
218	A	3	3	1.10	21	0.143
219	A	2	2	1.00	21	0.095
220	A	1	1	1.00	21	0.048
221	A	1	1	1.00	21	0.048
222	A	2	2	1.00	21	0.095
223	A	3	3	1.01	21	0.143
224	A	4	4	1.02	21	0.190
225	A	1	1	1.00	21	0.048
226	A	1	1	1.00	21	0.048
227	A	1	1	1.00	21	0.048
228	A	1	1	1.00	19	0.053
229	A	1	1	1.00	13	0.077
230	A	1	1	1.00	21	0.048
231	A	1	1	1.00	21	0.048
232	A	1	1	1.00	21	0.048
233	A	4	3	0.97	15	0.200
234	A	5	4	1.04	15	0.267
235	A	1	1	1.00	21	0.048
236	A	1	1	1.00	21	0.048
237	A	1	1	1.00	21	0.048
238	A	4	4	0.97	21	0.190
239	A	3	3	0.95	19	0.158
240	A	2	2	1.00	13	0.154
241	A	1	1	1.00	21	0.048
242	A	1	1	1.00	21	0.048
243	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	3	3	1.10	21	0.143
245	A	4	4	1.16	21	0.190
246	A	1	1	1.00	21	0.048
247	A	1	1	1.00	21	0.048
248	A	1	1	1.00	21	0.048
249	A	1	1	1.00	21	0.048
250	A	1	1	1.00	21	0.048
251	A	4	4	1.02	21	0.190
252	A	3	3	1.01	21	0.143
253	A	2	2	1.00	19	0.105
254	A	1	1	1.00	21	0.048
255	A	1	1	1.00	21	0.048
256	A	2	2	1.00	21	0.095
257	A	3	3	1.16	21	0.143
258	A	4	4	1.19	21	0.190
259	A	1	1	1.00	21	0.048
260	A	1	1	1.00	21	0.048
261	A	1	1	1.00	21	0.048
262	A	1	1	1.00	21	0.048
263	A	7	6	1.01	21	0.286
264	A	6	5	1.01	21	0.238
265	A	5	4	1.00	21	0.190
266	A	4	3	1.00	13	0.231
267	A	3	2	1.00	21	0.095
268	A	4	3	1.00	21	0.143
269	A	5	4	1.10	21	0.190
270	A	6	5	1.16	21	0.238
271	A	7	6	1.20	21	0.286
272	A	1	1	1.00	21	0.048
273	A	1	1	1.00	21	0.048
274	A	1	1	1.00	21	0.048
275	A	1	1	1.00	21	0.048

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
276	A	1	1	1.00	21	0.048
277	A	4	4	1.02	21	0.190
278	A	3	3	1.01	21	0.143
279	A	2	2	1.00	21	0.095
280	A	1	1	1.00	21	0.048
281	A	1	1	1.00	21	0.048
282	A	2	2	1.00	21	0.095
283	A	3	3	1.10	21	0.143
284	A	4	4	1.22	21	0.190
285	A	1	1	1.00	21	0.048
286	A	1	1	1.00	21	0.048
287	A	1	1	1.00	21	0.048
288	A	1	1	1.00	19	0.053
289	A	1	1	1.00	13	0.077
290	A	1	1	1.00	21	0.048
291	A	1	1	1.00	21	0.048
292	A	1	1	1.00	21	0.048
293	A	1	1	1.00	21	0.048
294	A	1	1	1.00	21	0.048
295	A	1	1	1.00	21	0.048
296	A	1	1	1.00	19	0.053
297	A	1	1	1.00	13	0.077
298	A	1	1	1.00	21	0.048
299	A	1	1	1.00	21	0.048
300	A	1	1	1.00	21	0.048
301	A	1	1	1.00	21	0.048
302	A	1	1	1.00	25	0.040
303	A	1	1	1.00	25	0.040
304	A	4	4	1.13	25	0.160
305	A	3	3	1.09	25	0.120
306	A	2	2	1.00	25	0.080
307	A	1	1	1.00	23	0.043

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	1	1	1.00	21	0.048
309	A	2	2	1.00	25	0.080
310	A	3	3	0.96	25	0.120
311	A	4	4	0.98	25	0.160
312	A	1	1	1.00	25	0.040
313	A	1	1	1.00	25	0.040
314	A	3	2	1.00	25	0.080
315	A	3	2	1.00	26	0.077
316	A	2	2	1.00	21	0.095
317	A	2	2	1.00	21	0.095
318	A	2	2	1.00	21	0.095
319	A	2	2	1.00	21	0.095
320	A	2	2	1.00	19	0.105
321	A	1	1	1.00	13	0.077
322	N/A	1	0	1.00	21	0.000
323	N/A	3	0	1.00	21	0.000
324	N/A	4	0	1.00	21	0.000
325	A	2	2	1.00	19	0.105
326	A	2	2	1.00	19	0.105
327	A	2	2	1.00	17	0.118
328	A	1	1	1.00	11	0.091
329	N/A	1	0	1.00	19	0.000
330	N/A	3	0	1.00	19	0.000
331	A	5	4	1.00	21	0.190
332	A	3	3	1.09	21	0.143
333	A	3	3	0.96	21	0.143
334	A	3	3	0.99	21	0.143
335	A	2	2	1.00	19	0.105
336	A	2	2	1.00	19	0.105
337	A	2	2	1.00	19	0.105
338	A	2	2	1.00	17	0.118
339	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
340	A	5	4	1.00	19	0.211
341	A	3	3	1.04	19	0.158
342	A	3	3	0.98	19	0.158
343	A	2	2	1.00	19	0.105
344	A	2	2	1.00	19	0.105
345	A	2	2	1.00	17	0.118
346	A	4	3	1.00	11	0.273
347	N/A	1	0	1.00	19	0.000
348	N/A	1	0	1.00	19	0.000
349	N/A	1	0	1.00	19	0.000
350	A	2	2	1.00	19	0.105
351	A	2	2	1.00	19	0.105
352	A	2	2	1.00	17	0.118
353	A	1	1	1.00	11	0.091
354	N/A	1	0	1.00	19	0.000
355	N/A	1	0	1.00	19	0.000
356	A	6	5	1.00	26	0.192
357	A	3	3	1.13	26	0.115
358	A	3	3	0.97	26	0.115
359	A	7	7	0.97	26	0.269
360	A	10	10	1.36	16	0.625
361	A	6	6	1.04	16	0.375
362	A	3	3	1.00	14	0.214
363	A	2	2	1.00	12	0.167
364	N/A	1	0	1.00	16	0.000
365	N/A	4	0	1.00	16	0.000
366	A	10	10	1.34	17	0.588
367	A	6	6	1.04	17	0.353
368	A	3	3	1.00	15	0.200
369	A	2	2	1.00	13	0.154
370	N/A	1	0	1.00	17	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
371	N/A	4	0	1.00	17	0.000
372	A	11	11	1.32	17	0.647
373	A	7	7	1.04	17	0.412
374	A	4	4	1.00	15	0.267
375	A	3	3	1.00	13	0.231
376	N/A	2	0	1.00	17	0.000
377	N/A	5	0	1.00	17	0.000
378	A	10	10	1.30	20	0.500
379	A	6	6	1.03	20	0.300
380	A	3	3	1.00	18	0.167
381	N/A	1	0	1.00	20	0.000
382	N/A	4	0	1.00	20	0.000
383	N/A	5	0	1.00	20	0.000
384	A	2	2	1.00	21	0.095
385	A	3	3	1.00	21	0.143
386	A	1	1	1.00	19	0.053
387	A	1	1	1.00	21	0.048
388	A	3	3	1.00	21	0.143
389	A	2	2	1.00	21	0.095
390	A	2	2	1.00	20	0.100
391	A	3	3	1.00	20	0.150
392	A	1	1	1.00	18	0.056
393	A	1	1	1.00	20	0.050
394	A	3	3	1.00	20	0.150
395	A	2	2	1.00	20	0.100
396	A	4	3	0.90	13	0.231
397	A	4	3	0.90	15	0.200
398	A	4	3	0.91	14	0.214
399	A	4	3	0.91	16	0.188
400	A	4	3	1.00	15	0.200
401	A	4	3	1.00	17	0.176
402	A	4	3	1.03	16	0.188

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
403	A	4	3	1.03	18	0.167
404	A	3	2	1.00	13	0.154
405	A	3	2	1.00	15	0.133
406	A	3	2	1.00	14	0.143
407	A	3	2	1.00	16	0.125
408	A	5	4	0.93	15	0.267
409	A	5	4	0.93	15	0.267
410	A	5	4	0.93	16	0.250
411	A	5	4	0.93	16	0.250
412	A	4	3	1.06	15	0.200
413	A	4	3	1.06	17	0.176
414	A	4	3	1.06	16	0.188
415	A	4	3	1.06	18	0.167
416	A	3	2	1.00	17	0.118
417	A	3	2	1.00	17	0.118
418	A	3	2	1.00	18	0.111
419	A	3	2	1.00	18	0.111
420	A	4	3	0.93	15	0.200
421	A	4	3	0.93	17	0.176
422	A	4	3	0.93	16	0.188
423	A	4	3	0.93	18	0.167
424	A	6	5	0.96	17	0.294
425	A	6	5	0.96	19	0.263
426	A	6	5	0.97	18	0.278
427	A	6	5	0.97	20	0.250
428	A	4	3	1.18	14	0.214
429	A	4	3	1.12	14	0.214
430	A	5	4	1.04	12	0.333
431	A	8	7	1.18	14	0.500
432	A	8	7	1.06	16	0.438
433	A	12	11	1.07	16	0.688
434	A	7	6	0.96	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
435	A	6	5	0.92	14	0.357
436	A	6	5	0.95	16	0.312
437	A	6	5	0.92	18	0.278
438	A	13	12	1.08	18	0.667
439	A	8	7	1.06	18	0.389
440	A	7	6	0.78	16	0.375
441	A	7	6	0.81	18	0.333
442	A	7	6	0.80	20	0.300
443	A	4	3	1.00	23	0.130
444	A	8	7	1.02	25	0.280
445	A	8	7	1.02	24	0.292
446	A	12	11	1.02	25	0.440
447	A	6	5	0.88	27	0.185
448	A	13	12	1.10	27	0.444
449	A	7	6	0.82	29	0.207
450	A	4	3	1.01	37	0.081
451	A	4	3	1.01	36	0.083
452	A	3	2	1.00	12	0.167
453	A	8	7	1.15	14	0.500
454	A	8	7	1.21	16	0.438
455	A	3	2	1.00	21	0.095
456	A	8	7	1.02	23	0.304
457	A	8	7	1.08	25	0.280
458	A	3	2	1.00	12	0.167
459	A	4	3	1.09	16	0.188
460	A	5	4	1.00	16	0.250
461	A	6	5	1.04	18	0.278
462	A	7	6	0.91	20	0.300
463	A	5	4	1.00	25	0.160
464	A	6	5	1.03	27	0.185
465	A	7	6	0.98	29	0.207
466	N/A	1	0	1.00	50	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
467	A	4	3	0.81	50	0.060
468	A	4	3	0.82	50	0.060
469	A	4	3	0.86	48	0.062
470	N/A	1	0	1.00	50	0.000
471	N/A	1	0	1.00	50	0.000
472	N/A	1	0	1.00	47	0.000
473	A	4	3	0.89	47	0.064
474	A	4	3	0.90	47	0.064
475	A	4	3	0.93	45	0.067
476	A	1	1	1.00	14	0.071
477	N/A	1	0	1.00	47	0.000
478	N/A	1	0	1.00	47	0.000
479	A	4	3	1.00	37	0.081
480	A	3	2	1.00	36	0.056
481	A	3	2	1.00	36	0.056
482	A	3	2	1.00	35	0.057
483	A	3	2	1.00	36	0.056
484	A	3	2	1.00	36	0.056
485	A	6	6	1.18	10	0.600
486	A	4	4	1.00	8	0.500
487	A	2	2	1.00	7	0.286
488	A	2	2	1.00	10	0.200
489	A	4	4	1.00	10	0.400
490	A	6	6	0.96	10	0.600
491	A	2	2	1.00	9	0.222
492	A	3	3	1.00	10	0.300
493	A	6	6	1.18	12	0.500
494	A	8	7	0.67	44	0.159
495	A	7	6	0.70	44	0.136
496	A	6	5	0.77	42	0.119
497	A	4	3	1.05	37	0.081
498	N/A	2	0	1.00	44	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
499	N/A	2	0	1.00	44	0.000
500	A	7	6	1.91	47	0.128
501	A	4	4	1.00	18	0.222
502	A	4	4	1.00	16	0.250
503	A	4	4	1.00	14	0.286
504	A	5	4	0.98	18	0.222
505	A	4	4	1.00	18	0.222
506	A	4	4	1.00	18	0.222
507	A	4	4	1.00	20	0.200
508	F	0	0	N/A	0.000	N/A
509	A	4	3	1.00	20	0.150
510	A	5	4	1.00	31	0.129
511	A	5	4	1.00	31	0.129
512	A	5	4	1.00	29	0.138
513	A	5	4	1.00	20	0.200
514	A	3	2	1.00	31	0.065
515	A	5	4	1.00	31	0.129
516	A	5	4	1.00	31	0.129
517	N/A	1	0	1.00	28	0.000
518	A	3	2	0.98	28	0.071
519	A	3	2	0.99	28	0.071
520	A	3	2	0.99	26	0.077
521	A	5	4	1.00	20	0.200
522	N/A	1	0	1.00	28	0.000
523	N/A	1	0	1.00	28	0.000
524	N/A	1	0	1.00	28	0.000
525	A	6	5	1.10	31	0.161
526	A	6	5	1.00	31	0.161
527	A	6	5	1.14	29	0.172
528	A	6	5	1.00	20	0.250
529	A	5	4	0.84	31	0.129
530	A	6	5	1.00	31	0.161

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
531	A	6	5	1.13	31	0.161
532	N/A	1	0	1.00	28	0.000
533	A	3	2	0.99	28	0.071
534	A	3	2	0.99	28	0.071
535	A	3	2	0.99	26	0.077
536	A	6	5	1.00	20	0.250
537	N/A	1	0	1.00	28	0.000
538	N/A	1	0	1.00	28	0.000
539	N/A	1	0	1.00	28	0.000
540	A	1	1	1.00	21	0.048
541	A	1	1	1.00	33	0.030
542	A	3	2	1.00	31	0.065
543	A	6	5	1.04	31	0.161
544	A	5	4	1.03	31	0.129
545	A	4	3	1.00	29	0.103
546	A	1	1	1.00	19	0.053
547	A	3	2	1.00	31	0.065
548	A	4	3	1.00	31	0.097
549	A	5	4	0.99	31	0.129
550	A	8	7	1.04	33	0.212
551	A	7	6	1.04	33	0.182
552	A	6	5	1.02	33	0.152
553	A	5	4	1.00	33	0.121
554	A	4	3	1.00	33	0.091
555	A	5	4	1.00	33	0.121
556	A	6	5	1.01	33	0.152
557	A	7	6	1.05	33	0.182
558	A	8	7	1.08	33	0.212
559	A	3	2	1.00	19	0.105
560	A	3	2	1.00	13	0.154
561	A	3	2	1.00	15	0.133
562	A	3	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
563	A	4	3	1.00	17	0.176
564	A	2	2	1.00	9	0.222
565	A	4	3	1.00	17	0.176
566	A	4	3	1.00	18	0.167
567	A	3	2	1.00	13	0.154
568	A	1	1	1.00	11	0.091
569	A	1	1	1.00	9	0.111
570	A	5	4	1.50	17	0.235
571	A	3	2	1.00	17	0.118
572	A	3	2	1.00	15	0.133
573	A	5	4	1.11	13	0.308
574	A	5	4	1.00	16	0.250
575	A	3	2	1.00	13	0.154
576	A	3	2	1.00	13	0.154
577	B	6	5	2.08	23	0.217
578	B	6	5	2.30	23	0.217
579	A	6	5	1.39	27	0.185
580	A	3	2	1.00	15	0.133
581	A	1	1	1.00	17	0.059
582	A	5	4	1.00	15	0.267
583	A	4	3	1.00	15	0.200
584	A	4	3	1.00	15	0.200
585	A	4	3	1.00	18	0.167
586	A	4	3	1.07	11	0.273
587	A	4	3	1.00	15	0.200
588	A	3	2	1.00	15	0.133
589	A	3	2	1.00	15	0.133
590	A	3	2	1.00	17	0.118
591	A	3	2	1.11	17	0.118
592	A	1	1	1.00	9	0.111
593	A	1	1	1.00	11	0.091
594	A	1	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
595	A	1	1	1.00	13	0.077
596	A	2	2	1.00	16	0.125
597	A	2	2	1.00	7	0.286
598	A	4	3	1.00	20	0.150
599	A	3	2	1.00	18	0.111
600	A	1	1	1.00	10	0.100
601	A	4	3	1.00	20	0.150
602	A	4	3	1.00	13	0.231
603	A	1	1	1.00	10	0.100
604	A	4	3	1.00	8	0.375
605	A	4	3	1.14	15	0.200
606	A	1	1	1.00	10	0.100
607	A	6	5	1.00	14	0.357
608	A	2	2	1.00	11	0.182
609	A	4	3	1.00	18	0.167
610	A	4	3	1.00	15	0.200
611	A	4	3	1.00	15	0.200
612	A	6	5	1.00	32	0.156
613	A	3	3	1.00	11	0.273
614	A	3	3	1.00	13	0.231
615	F	0	0	N/A	0.000	N/A
616	A	3	2	1.00	23	0.087
617	F	0	0	N/A	0.000	N/A
618	A	4	3	1.12	9	0.333
619	A	4	3	1.08	9	0.333
620	A	4	3	1.06	9	0.333
621	A	4	3	1.00	15	0.200
622	A	4	3	1.00	17	0.176
623	A	5	4	0.96	15	0.267
624	A	5	4	0.96	17	0.235
625	A	1	1	1.00	10	0.100
626	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
627	A	5	4	1.00	15	0.267
628	A	3	2	1.00	17	0.118
629	A	4	3	1.00	18	0.167
630	A	4	4	1.23	9	0.444
631	A	4	4	1.10	11	0.364
632	A	4	3	1.00	17	0.176
633	A	4	3	1.08	19	0.158
634	A	4	3	1.00	17	0.176
635	A	5	4	0.84	15	0.267
636	A	5	4	1.00	22	0.182
637	A	5	4	0.75	22	0.182
638	A	3	2	1.00	7	0.286
639	A	4	3	1.00	12	0.250
640	A	5	4	1.14	11	0.364
641	A	3	2	1.00	11	0.182
642	A	3	2	1.00	11	0.182
643	A	3	2	1.00	13	0.154
644	A	3	2	1.00	13	0.154
645	A	6	5	1.00	17	0.294
646	A	6	5	0.87	17	0.294
647	A	4	3	0.95	13	0.231
648	A	4	3	0.76	15	0.200
649	A	1	1	1.00	25	0.040
650	A	3	3	1.00	18	0.167
651	A	5	4	1.00	13	0.308
652	A	2	2	1.00	15	0.133
653	A	1	1	1.00	16	0.062
654	A	4	3	1.00	19	0.158
655	A	1	1	1.00	15	0.067
656	A	1	1	1.00	13	0.077
657	A	2	2	1.00	9	0.222
658	A	2	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
659	A	3	3	1.09	15	0.200
660	A	1	1	1.00	9	0.111
661	A	1	1	1.00	13	0.077
662	A	1	1	1.00	9	0.111
663	A	1	1	1.00	9	0.111
664	A	2	2	1.00	14	0.143
665	A	1	1	1.00	15	0.067
666	A	1	1	1.00	13	0.077
667	A	1	1	1.00	23	0.043
668	A	1	1	1.00	28	0.036
669	A	1	1	1.00	37	0.027
670	N/A	2	0	1.00	16	0.000
671	N/A	1	0	1.00	26	0.000
672	N/A	3	0	1.00	14	0.000
673	A	1	1	1.00	50	0.020
674	N/A	2	0	1.00	20	0.000
675	A	2	2	1.00	16	0.125
676	A	1	1	1.00	34	0.029
677	N/A	2	0	1.00	11	0.000
678	A	2	2	1.00	23	0.087
679	A	1	1	1.00	39	0.026
680	A	5	4	1.10	26	0.154
681	N/A	1	0	1.00	9	0.000
682	N/A	1	0	1.00	13	0.000
683	N/A	1	0	1.00	11	0.000
684	N/A	1	0	1.00	13	0.000
685	N/A	2	0	1.00	9	0.000
686	N/A	2	0	1.00	11	0.000
687	N/A	2	0	1.00	9	0.000
688	N/A	2	0	1.00	11	0.000
689	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	2	2	1.00	15	0.133
691	A	2	2	1.00	19	0.105
692	A	1	1	1.00	9	0.111
693	A	1	1	1.00	9	0.111
694	A	1	1	1.00	17	0.059
695	A	1	1	1.00	17	0.059
696	A	2	2	1.00	11	0.182
697	A	2	2	1.00	43	0.047
698	F	0	0	N/A	0.000	N/A
699	A	4	4	1.00	23	0.174
700	A	2	2	1.00	26	0.077
701	N/A	5	0	1.00	25	0.000
702	A	1	1	1.00	7	0.143
703	A	1	1	1.00	7	0.143
704	A	1	1	1.00	7	0.143
705	A	2	2	1.00	15	0.133
706	A	2	2	1.00	15	0.133
707	F	0	0	N/A	0.000	N/A
708	A	9	9	1.00	22	0.409
709	A	9	9	1.00	22	0.409
710	F	0	0	N/A	0.000	N/A

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx$

Optimal result

Integrand size = 53, antiderivative size = 37

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx = \frac{F^{a+bx+cx^2}(2ce-bf+2cfx)}{4c^2\log(F)}$$

output
```
1/4*F^(c*x^2+b*x+a)*(2*c*f*x-b*f+2*c*e)/c^2/ln(F)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx = \frac{F^{a+x(b+cx)}(-bf+2c(e+fx))}{4c^2\log(F)}$$

input
```
Integrate[F^(a + b*x + c*x^2)*(e*x + f*x^2 + (2*c*f + 2*b*c*e*Log[F] - b^2
*f*Log[F])/(4*c^2*Log[F])),x]
```

output
```
(F^(a + x*(b + c*x))*(-(b*f) + 2*c*(e + f*x)))/(4*c^2*Log[F])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.37 (sec) , antiderivative size = 324, normalized size of antiderivative = 8.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx+cx^2}\left(\frac{b^2(-f)\log(F)+2bce\log(F)+2cf}{4c^2\log(F)}+ex+fx^2\right)dx$$
$$\Big\downarrow \text{7292}$$
$$\int F^{a+bx+cx^2}\left(\frac{b\log(F)(2ce-bf)+2cf}{4c^2\log(F)}+ex+fx^2\right)dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{F^{a+bx+cx^2}(b\log(F)(2ce-bf)+2cf)}{4c^2\log(F)}+exF^{a+bx+cx^2}+fx^2F^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{\pi}F^{a-\frac{b^2}{4c}}(b\log(F)(2ce-bf)+2cf)\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\log^{\frac{3}{2}}(F)}-\frac{\sqrt{\pi}beF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(F)}}-\frac{\sqrt{\pi}fF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(F)}+\frac{\sqrt{\pi}b^2fF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\sqrt{\log(F)}}-\frac{bfF^{a+bx+cx^2}}{4c^2\log(F)}+\frac{eF^{a+bx+cx^2}}{2c\log(F)}+\frac{fxF^{a+bx+cx^2}}{2c\log(F)}$$

input
```
Int[F^(a + b*x + c*x^2)*(e*x + f*x^2 + (2*c*f + 2*b*c*e*Log[F] - b^2*f*Log
[F])/(4*c^2*Log[F])),x]
```

output
```
-1/4*(f*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt
[c])])/(c^(3/2)*Log[F]^(3/2)) + (e*F^(a + b*x + c*x^2))/(2*c*Log[F]) - (b*
f*F^(a + b*x + c*x^2))/(4*c^2*Log[F]) + (f*F^(a + b*x + c*x^2)*x)/(2*c*Log
[F]) - (b*e*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*
Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[F]]) + (b^2*f*F^(a - b^2/(4*c))*Sqrt[Pi]*Er
fi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])])/(8*c^(5/2)*Sqrt[Log[F]]) + (F^
(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])]*(2*c
*f + b*(2*c*e - b*f)*Log[F]))/(8*c^(5/2)*Log[F]^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{F^{c\,x^{2}+b x +a} \left(-2 c f x +b f -2 e c \right)}{4 c^{2} \ln \left(F \right)}$	35
risch	$-\frac{F^{c\,x^{2}+b x +a} \left(-2 c f x +b f -2 e c \right)}{4 c^{2} \ln \left(F \right)}$	35
norman	$\frac{-\frac{\left(b f -2 e c \right) {\mathrm e}^{\left(c\,x^{2}+b x +a \right) \ln \left(F \right)}}{4 c \ln \left(F \right)}+\frac{f x \,{\mathrm e}^{\left(c\,x^{2}+b x +a \right) \ln \left(F \right)}}{2 \ln \left(F \right)}}{c}$	59
parallelrisch	$\frac{2 c f x \,F^{c\,x^{2}+b x +a}-F^{c\,x^{2}+b x +a} b f +2 F^{c\,x^{2}+b x +a} c e}{4 c^{2} \ln \left(F \right)}$	60

input
```
int(F^(c*x^2+b*x+a)*(e*x+f*x^2+1/4*(2*c*f+2*b*c*e*ln(F)-b^2*f*ln(F))/c^2/l
n(F)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*F^(c*x^2+b*x+a)*(-2*c*f*x+b*f-2*c*e)/c^2/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)dx = \frac{(2\,cfx+2\,ce-bf)F^{cx^2+bx+a}}{4\,c^2\log(F)}$$

input
```
integrate(F^(c*x^2+b*x+a)*(e*x+f*x^2+1/4*(2*c*f+2*b*c*e*log(F)-b^2*f*log(F
))/c^2/log(F)),x, algorithm="fricas")
```

output
```
1/4*(2*c*f*x + 2*c*e - b*f)*F^(c*x^2 + b*x + a)/(c^2*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(34) = 68.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.35

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)dx = \begin{cases}\frac{F^{a+bx+cx^2}(-bf+2ce+2cfx)}{4c^2\log(F)} & \text{for } c^2\log(F)\neq 0\\ \frac{ex^2}{2}+\frac{fx^3}{3}+\frac{x(-b^2f\log(F)+2bce\log(F)+2cf)}{4c^2\log(F)} & \text{otherwise}\end{cases}$$

input
```
integrate(F**(c*x**2+b*x+a)*(e*x+f*x**2+1/4*(2*c*f+2*b*c*e*ln(F)-b**2*f*ln
(F))/c**2/ln(F)),x)
```

output
```
Piecewise((F**(a + b*x + c*x**2)*(-b*f + 2*c*e + 2*c*f*x)/(4*c**2*log(F)),
 Ne(c**2*log(F), 0)), (e*x**2/2 + f*x**3/3 + x*(-b**2*f*log(F) + 2*b*c*e*l
og(F) + 2*c*f)/(4*c**2*log(F)), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 446, normalized size of antiderivative = 12.05

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*x^2+b*x+a)*(e*x+f*x^2+1/4*(2*c*f+2*b*c*e*log(F)-b^2*f*log(F
))/c^2/log(F)),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F)/c)) - 1)*
log(F)^2/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(3/2)) - 2*F^(1/4*(2*c*
x + b)^2/c)*c*log(F)/(c*log(F))^(3/2))*F^(a - 1/4*b^2/c)*e/sqrt(c*log(F))
+ 1/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F)/c)) -
1)*log(F)^3/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(5/2)) - 4*(2*c*x +
b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(F)/c)*log(F)^3/((-(2*c*x + b)^2*log
(F)/c)^(3/2)*(c*log(F))^(5/2)) - 4*F^(1/4*(2*c*x + b)^2/c)*b*c*log(F)^2/(c
*log(F))^(5/2))*F^(a - 1/4*b^2/c)*f/sqrt(c*log(F)) + 1/4*sqrt(pi)*F^a*b*e*
erf(sqrt(-c*log(F))*x - 1/2*b*log(F)/sqrt(-c*log(F)))/(sqrt(-c*log(F))*F^(
1/4*b^2/c)*c) - 1/8*sqrt(pi)*F^a*b^2*f*erf(sqrt(-c*log(F))*x - 1/2*b*log(F
)/sqrt(-c*log(F)))/(sqrt(-c*log(F))*F^(1/4*b^2/c)*c^2) + 1/4*sqrt(pi)*F^a*
f*erf(sqrt(-c*log(F))*x - 1/2*b*log(F)/sqrt(-c*log(F)))/(sqrt(-c*log(F))*F
^(1/4*b^2/c)*c*log(F))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx = \frac{\left(cf\left(2\,x+\frac{b}{c}\right)+2\,ce-2\,bf\right)e^{\left(cx^2\log(F)+bx\log(F)+a\log(F)\right)}}{4\,c^2\log\left(F\right)}$$

input
```
integrate(F^(c*x^2+b*x+a)*(e*x+f*x^2+1/4*(2*c*f+2*b*c*e*log(F)-b^2*f*log(F
))/c^2/log(F)),x, algorithm="giac")
```

output
```
1/4*(c*f*(2*x + b/c) + 2*c*e - 2*b*f)*e^(c*x^2*log(F) + b*x*log(F) + a*log
(F))/(c^2*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx$$
$$=-\frac{F^{c\,x^2+b\,x+a}\,\left(\frac{b\,f}{4}-\frac{c\,e}{2}\right)-\frac{F^{c\,x^2+b\,x+a}\,c\,f\,x}{2}}{c^2\,\ln\left(F\right)}$$

input
```
int(F^(a + b*x + c*x^2)*(e*x + f*x^2 + ((c*f)/2 - (b^2*f*log(F))/4 + (b*c*
e*log(F))/2)/(c^2*log(F))),x)
```

output
```
-(F^(a + b*x + c*x^2)*((b*f)/4 - (c*e)/2) - (F^(a + b*x + c*x^2)*c*f*x)/2)
/(c^2*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int F^{a+bx+cx^2}\left(ex+fx^2+\frac{2cf+2bce\log(F)-b^2f\log(F)}{4c^2\log(F)}\right)\,dx$$
$$=\frac{f^{c\,x^2+bx+a}(2cfx-bf+2ce)}{4\log\left(f\right)c^2}$$

input
```
int(F^(c*x^2+b*x+a)*(e*x+f*x^2+1/4*(2*c*f+2*b*c*e*log(F)-b^2*f*log(F))/c^2
/log(F)),x)
```

output
```
(f**(a + b*x + c*x**2)*( - b*f + 2*c*e + 2*c*f*x))/(4*log(f)*c**2)
```

3.2 $\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx$

Optimal result

Integrand size = 55, antiderivative size = 45

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx = \frac{F^{a+bx+cx^2}\left(bfx-\frac{f-2cd\log(F)}{\log(F)}\right)}{2bc\log(F)}$$

output
```
1/2*F^(c*x^2+b*x+a)*(b*f*x-(f-2*c*d*ln(F))/ln(F))/b/c/ln(F)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx = \frac{F^{a+x(b+cx)}(-f+(2cd+bfx)\log(F))}{2bc\log^2(F)}$$

input
```
Integrate[F^(a + b*x + c*x^2)*(d + f*x^2 + (x*(-2*c*f + 4*c^2*d*Log[F] + b
^2*f*Log[F]))/(2*b*c*Log[F])),x]
```

output
```
(F^(a + x*(b + c*x))*(-f + (2*c*d + b*f*x)*Log[F]))/(2*b*c*Log[F]^2)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.39 (sec) , antiderivative size = 349, normalized size of antiderivative = 7.76, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.055, Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx+cx^2}\left(\frac{x\left(b^2f\log(F)+4c^2d\log(F)-2cf\right)}{2bc\log(F)}+d+fx^2\right)dx$$

$$\Big\downarrow \text{7292}$$

$$\int F^{a+bx+cx^2}\left(-\frac{x\left(2cf-\log(F)\left(b^2f+4c^2d\right)\right)}{2bc\log(F)}+d+fx^2\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{xF^{a+bx+cx^2}\left(\log(F)\left(b^2f+4c^2d\right)-2cf\right)}{2bc\log(F)}+dF^{a+bx+cx^2}+fx^2F^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}fF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(F)}+\frac{\sqrt{\pi}b^2fF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\sqrt{\log(F)}}-\frac{F^{a+bx+cx^2}\left(2cf-\log(F)\left(b^2f+4c^2d\right)\right)}{4bc^2\log^2(F)}+\frac{\sqrt{\pi}F^{a-\frac{b^2}{4c}}\left(2cf-\log(F)\left(b^2f+4c^2d\right)\right)\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{8c^{5/2}\log^{\frac{3}{2}}(F)}+\frac{\sqrt{\pi}dF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(F)}}-\frac{bfF^{a+bx+cx^2}}{4c^2\log(F)}+\frac{fxF^{a+bx+cx^2}}{2c\log(F)}$$

input
```
Int[F^(a + b*x + c*x^2)*(d + f*x^2 + (x*(-2*c*f + 4*c^2*d*Log[F] + b^2*f*L
og[F]))/(2*b*c*Log[F])),x]
```

output
```
-1/4*(f*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt
[c])])/(c^(3/2)*Log[F]^(3/2)) - (b*f*F^(a + b*x + c*x^2))/(4*c^2*Log[F]) +
 (f*F^(a + b*x + c*x^2)*x)/(2*c*Log[F]) + (d*F^(a - b^2/(4*c))*Sqrt[Pi]*Er
fi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])])/(2*Sqrt[c]*Sqrt[Log[F]]) + (b^
2*f*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])
])/(8*c^(5/2)*Sqrt[Log[F]]) - (F^(a + b*x + c*x^2)*(2*c*f - (4*c^2*d + b^2
*f)*Log[F]))/(4*b*c^2*Log[F]^2) + (F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2
*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])]*(2*c*f - (4*c^2*d + b^2*f)*Log[F]))/(8*c^
(5/2)*Log[F]^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

method	result	size
gosper	$\frac{F^{c\,x^{2}+b x+a} \left(b f x \ln \left(F \right)+2 c d \ln \left(F \right)-f \right)}{2 c b \ln \left(F \right)^{2}}$	41
risch	$\frac{F^{c\,x^{2}+b x+a} \left(b f x \ln \left(F \right)+2 c d \ln \left(F \right)-f \right)}{2 c b \ln \left(F \right)^{2}}$	41
norman	$\frac{\left(2 c d \ln \left(F \right)-f \right) {\mathrm e}^{\left(c\,x^{2}+b x+a \right) \ln \left(F \right)}}{2 c b \ln \left(F \right)^{2}}+\frac{f x\,{\mathrm e}^{\left(c\,x^{2}+b x+a \right) \ln \left(F \right)}}{2 c \ln \left(F \right)}$	63
parallelrisch	$\frac{b f x\,F^{c\,x^{2}+b x+a} \ln \left(F \right)+2 \ln \left(F \right) F^{c\,x^{2}+b x+a} c d -F^{c\,x^{2}+b x+a} f}{2 c b \ln \left(F \right)^{2}}$	65

input
```
int(F^(c*x^2+b*x+a)*(d+f*x^2+1/2*x*(-2*c*f+4*c^2*d*ln(F)+b^2*f*ln(F))/b/c/
ln(F)),x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(c*x^2+b*x+a)*(b*f*x*ln(F)+2*c*d*ln(F)-f)/c/b/ln(F)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx = \frac{{\left({\left(b f x + 2 \, c d\right)} \log\left(F\right) - f\right)} F^{c x^{2} + b x + a}}{2 \, b c \log\left(F\right)^{2}}$$

input
```
integrate(F^(c*x^2+b*x+a)*(d+f*x^2+1/2*x*(-2*c*f+4*c^2*d*log(F)+b^2*f*log(
F))/b/c/log(F)),x, algorithm="fricas")
```

output
```
1/2*((b*f*x + 2*c*d)*log(F) - f)*F^(c*x^2 + b*x + a)/(b*c*log(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(37) = 74.

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.04

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)dx$$
$$=\begin{cases}\frac{F^{a+bx+cx^2}(bfx\log{(F)}+2cd\log{(F)}-f)}{2bc\log{(F)}^2} & \text{for } bc\log{(F)}^2\neq 0\\ dx+\frac{fx^3}{3}+\frac{x^2(b^2f\log{(F)}+4c^2d\log{(F)}-2cf)}{4bc\log{(F)}} & \text{otherwise}\end{cases}$$

input
```
integrate(F**(c*x**2+b*x+a)*(d+f*x**2+1/2*x*(-2*c*f+4*c**2*d*ln(F)+b**2*f*
ln(F))/b/c/ln(F)),x)
```

output
```
Piecewise((F**(a + b*x + c*x**2)*(b*f*x*log(F) + 2*c*d*log(F) - f)/(2*b*c*
log(F)**2), Ne(b*c*log(F)**2, 0)), (d*x + f*x**3/3 + x**2*(b**2*f*log(F) +
 4*c**2*d*log(F) - 2*c*f)/(4*b*c*log(F)), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 558, normalized size of antiderivative = 12.40

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)dx = \text{Too large to display}$$

input
```
integrate(F^(c*x^2+b*x+a)*(d+f*x^2+1/2*x*(-2*c*f+4*c^2*d*log(F)+b^2*f*log(
F))/b/c/log(F)),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F)/c)) - 1)*
log(F)^2/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(3/2)) - 2*F^(1/4*(2*c*
x + b)^2/c)*c*log(F)/(c*log(F))^(3/2))*F^(a - 1/4*b^2/c)*c*d/(sqrt(c*log(F
))*b) + 1/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F)/
c)) - 1)*log(F)^3/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(5/2)) - 4*(2*
c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(F)/c)*log(F)^3/((-(2*c*x + b)
^2*log(F)/c)^(3/2)*(c*log(F))^(5/2)) - 4*F^(1/4*(2*c*x + b)^2/c)*b*c*log(F
)^2/(c*log(F))^(5/2))*F^(a - 1/4*b^2/c)*f/sqrt(c*log(F)) - 1/8*(sqrt(pi)*(
2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F)/c)) - 1)*log(F)^2/(sqrt(-
(2*c*x + b)^2*log(F)/c)*(c*log(F))^(3/2)) - 2*F^(1/4*(2*c*x + b)^2/c)*c*lo
g(F)/(c*log(F))^(3/2))*F^(a - 1/4*b^2/c)*b*f/(sqrt(c*log(F))*c) + 1/2*sqrt
(pi)*F^a*d*erf(sqrt(-c*log(F))*x - 1/2*b*log(F)/sqrt(-c*log(F)))/(sqrt(-c*
log(F))*F^(1/4*b^2/c)) + 1/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x
 + b)^2*log(F)/c)) - 1)*log(F)^2/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))
^(3/2)) - 2*F^(1/4*(2*c*x + b)^2/c)*c*log(F)/(c*log(F))^(3/2))*F^(a - 1/4*
b^2/c)*f/(sqrt(c*log(F))*b*log(F))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx = \frac{\left(bcf\left(2\,x+\frac{b}{c}\right)\log\left(F\right)+4\,c^2d\log\left(F\right)-b^2f\log\left(F\right)-2\,cf\right)e^{\left(cx^2\log(F)+bx\log(F)+a\log(F)\right)}}{4\,bc^2\log\left(F\right)^2}$$

input
```
integrate(F^(c*x^2+b*x+a)*(d+f*x^2+1/2*x*(-2*c*f+4*c^2*d*log(F)+b^2*f*log(
F))/b/c/log(F)),x, algorithm="giac")
```

output
```
1/4*(b*c*f*(2*x + b/c)*log(F) + 4*c^2*d*log(F) - b^2*f*log(F) - 2*c*f)*e^(
c*x^2*log(F) + b*x*log(F) + a*log(F))/(b*c^2*log(F)^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx$$
$$=\frac{F^{c\,x^2+b\,x+a}\,\left(c\,d-\frac{f}{2\,\ln(F)}\right)+\frac{F^{c\,x^2+b\,x+a}\,b\,f\,x}{2}}{b\,c\,\ln\left(F\right)}$$

input
```
int(F^(a + b*x + c*x^2)*(d + f*x^2 + (x*(4*c^2*d*log(F) - 2*c*f + b^2*f*lo
g(F)))/(2*b*c*log(F))),x)
```

output
```
(F^(a + b*x + c*x^2)*(c*d - f/(2*log(F))) + (F^(a + b*x + c*x^2)*b*f*x)/2)
/(b*c*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int F^{a+bx+cx^2}\left(d+fx^2+\frac{x(-2cf+4c^2d\log(F)+b^2f\log(F))}{2bc\log(F)}\right)\,dx$$
$$=\frac{f^{c\,x^2+bx+a}(\log(f)\,bfx+2\log(f)\,cd-f)}{2\log\left(f\right)^2bc}$$

input
```
int(F^(c*x^2+b*x+a)*(d+f*x^2+1/2*x*(-2*c*f+4*c^2*d*log(F)+b^2*f*log(F))/b/
c/log(F)),x)
```

output
```
(f**(a + b*x + c*x**2)*(log(f)*b*f*x + 2*log(f)*c*d - f))/(2*log(f)**2*b*c
)
```

3.3 $\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2\left(2c^2d\log(F)-bce\log(F)\right)}{2c-b^2\log(F)}\right)\,dx$

Optimal result

Integrand size = 52, antiderivative size = 66

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)\,dx = \frac{F^{a+bx+cx^2}\left(\frac{(2cd-be)x\log(F)}{2c-b^2\log(F)}+\frac{e-bd\log(F)}{2c-b^2\log(F)}\right)}{\log(F)}$$

output
```
F^(c*x^2+b*x+a)*((-b*e+2*c*d)*x*ln(F)/(2*c-b^2*ln(F))+(e-b*d*ln(F))/(2*c-b
^2*ln(F)))/ln(F)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)\,dx = \frac{F^{a+x(b+cx)}(-e+(-2cdx+b(d+ex))\log(F))}{\log(F)\left(-2c+b^2\log(F)\right)}$$

input
```
Integrate[F^(a + b*x + c*x^2)*(d + e*x + (2*x^2*(2*c^2*d*Log[F] - b*c*e*Lo
g[F]))/(2*c - b^2*Log[F])),x]
```

output
```
(F^(a + x*(b + c*x))*(-e + (-2*c*d*x + b*(d + e*x))*Log[F]))/(Log[F]*(-2*c
 + b^2*Log[F]))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.49 (sec) , antiderivative size = 377, normalized size of antiderivative = 5.71, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.058, Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx+cx^2}\left(\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}+d+ex\right)dx$$

$$\Big\downarrow \text{7292}$$

$$\int F^{a+bx+cx^2}\left(\frac{2cx^2\log(F)(2cd-be)}{2c-b^2\log(F)}+d+ex\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{2cx^2\log(F)(2cd-be)F^{a+bx+cx^2}}{2c-b^2\log(F)}+dF^{a+bx+cx^2}+exF^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}b^2\sqrt{\log(F)}F^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}(2c-b^2\log(F))}-\frac{\sqrt{\pi}beF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(F)}}-\frac{\sqrt{\pi}F^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(F)}(2c-b^2\log(F))}-\frac{b(2cd-be)F^{a+bx+cx^2}}{2c(2c-b^2\log(F))}+\frac{x(2cd-be)F^{a+bx+cx^2}}{2c-b^2\log(F)}+\frac{\sqrt{\pi}dF^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(F)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(F)}}+\frac{eF^{a+bx+cx^2}}{2c\log(F)}$$

input
```
Int[F^(a + b*x + c*x^2)*(d + e*x + (2*x^2*(2*c^2*d*Log[F] - b*c*e*Log[F]))
/(2*c - b^2*Log[F])),x]
```

output
```
(e*F^(a + b*x + c*x^2))/(2*c*Log[F]) + (d*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[
((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])])/(2*Sqrt[c]*Sqrt[Log[F]]) - (b*e*F
^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c])])/(4
*c^(3/2)*Sqrt[Log[F]]) - (b*(2*c*d - b*e)*F^(a + b*x + c*x^2))/(2*c*(2*c -
 b^2*Log[F])) + ((2*c*d - b*e)*F^(a + b*x + c*x^2)*x)/(2*c - b^2*Log[F]) -
 ((2*c*d - b*e)*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])
/(2*Sqrt[c])])/(2*Sqrt[c]*Sqrt[Log[F]]*(2*c - b^2*Log[F])) + (b^2*(2*c*d -
 b*e)*F^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[F]])/(2*Sqrt[c
])]*Sqrt[Log[F]])/(4*c^(3/2)*(2*c - b^2*Log[F]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

method	result	size
gosper	$\frac{F^{c\,x^{2}+b x +a} \left(\ln \left(F \right) b e x -2 \ln \left(F \right) d c x +b d \ln \left(F \right)-e \right)}{\left(b^{2} \ln \left(F \right)-2 c \right) \ln \left(F \right)}$	52
risch	$\frac{F^{c\,x^{2}+b x +a} \left(\ln \left(F \right) b e x -2 \ln \left(F \right) d c x +b d \ln \left(F \right)-e \right)}{\left(b^{2} \ln \left(F \right)-2 c \right) \ln \left(F \right)}$	52
norman	$\frac{\left(b d \ln \left(F \right)-e \right) {\mathrm e}^{\left(c\,x^{2}+b x +a \right) \ln \left(F \right)}}{\left(b^{2} \ln \left(F \right)-2 c \right) \ln \left(F \right)}+\frac{\left(b e -2 c d \right) x \,{\mathrm e}^{\left(c\,x^{2}+b x +a \right) \ln \left(F \right)}}{b^{2} \ln \left(F \right)-2 c}$	78
parallelrisch	$\frac{\ln \left(F \right) x \,F^{c\,x^{2}+b x +a} b e -2 \ln \left(F \right) x \,F^{c\,x^{2}+b x +a} c d +\ln \left(F \right) F^{c\,x^{2}+b x +a} b d -F^{c\,x^{2}+b x +a} e}{\left(b^{2} \ln \left(F \right)-2 c \right) \ln \left(F \right)}$	88

input
```
int(F^(c*x^2+b*x+a)*(d+e*x+2*x^2*(2*c^2*d*ln(F)-b*c*e*ln(F))/(2*c-b^2*ln(F
))),x,method=_RETURNVERBOSE)
```

output
```
F^(c*x^2+b*x+a)*(ln(F)*b*e*x-2*ln(F)*d*c*x+b*d*ln(F)-e)/(b^2*ln(F)-2*c)/ln
(F)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)dx = \frac{((bd-(2\,cd-be)x)\log{(F)}-e)F^{cx^2+bx+a}}{b^2\log{(F)}^2-2\,c\log{(F)}}$$

input
```
integrate(F^(c*x^2+b*x+a)*(d+e*x+2*x^2*(2*c^2*d*log(F)-b*c*e*log(F))/(2*c-
b^2*log(F))),x, algorithm="fricas")
```

output
```
((b*d - (2*c*d - b*e)*x)*log(F) - e)*F^(c*x^2 + b*x + a)/(b^2*log(F)^2 - 2
*c*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.70

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)dx = \begin{cases} \frac{F^{a+bx+cx^2}(bd\log{(F)}+bex\log{(F)}-2cdx\log{(F)}-e)}{b^2\log{(F)}^2-2c\log{(F)}} & \text{for } b^2\log{(F)}^2-2c\log{(F)}\neq 0 \\ dx+\frac{ex^2}{2}+\frac{x^3\cdot(2bce\log{(F)}-4c^2d\log{(F)})}{3b^2\log{(F)}-6c} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*x**2+b*x+a)*(d+e*x+2*x**2*(2*c**2*d*ln(F)-b*c*e*ln(F))/(2*
c-b**2*ln(F))),x)
```

output
```
Piecewise((F**(a + b*x + c*x**2)*(b*d*log(F) + b*e*x*log(F) - 2*c*d*x*log(
F) - e)/(b**2*log(F)**2 - 2*c*log(F)), Ne(b**2*log(F)**2 - 2*c*log(F), 0))
, (d*x + e*x**2/2 + x**3*(2*b*c*e*log(F) - 4*c**2*d*log(F))/(3*b**2*log(F)
 - 6*c), True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 527, normalized size of antiderivative = 7.98

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)\,dx=\text{Too large to display}$$

input
```
integrate(F^(c*x^2+b*x+a)*(d+e*x+2*x^2*(2*c^2*d*log(F)-b*c*e*log(F))/(2*c-
b^2*log(F))),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F)/c)) - 1
)*log(F)^3/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(5/2)) - 4*(2*c*x + b
)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(F)/c)*log(F)^3/((-(2*c*x + b)^2*log(
F)/c)^(3/2)*(c*log(F))^(5/2)) - 4*F^(1/4*(2*c*x + b)^2/c)*b*c*log(F)^2/(c*
log(F))^(5/2))*F^(a - 1/4*b^2/c)*c^2*d*log(F)/((b^2*log(F) - 2*c)*sqrt(c*l
og(F))) + 1/4*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(F
)/c)) - 1)*log(F)^3/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(5/2)) - 4*(
2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(F)/c)*log(F)^3/((-(2*c*x +
b)^2*log(F)/c)^(3/2)*(c*log(F))^(5/2)) - 4*F^(1/4*(2*c*x + b)^2/c)*b*c*log
(F)^2/(c*log(F))^(5/2))*F^(a - 1/4*b^2/c)*b*c*e*log(F)/((b^2*log(F) - 2*c)
*sqrt(c*log(F))) - 1/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^
2*log(F)/c)) - 1)*log(F)^2/(sqrt(-(2*c*x + b)^2*log(F)/c)*(c*log(F))^(3/2)
) - 2*F^(1/4*(2*c*x + b)^2/c)*c*log(F)/(c*log(F))^(3/2))*F^(a - 1/4*b^2/c)
*e/sqrt(c*log(F)) + 1/2*sqrt(pi)*F^a*d*erf(sqrt(-c*log(F))*x - 1/2*b*log(F
)/sqrt(-c*log(F)))/(sqrt(-c*log(F))*F^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.38

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)\,dx=$$
$$-\frac{\left(2\,c^2d\left(2\,x+\frac{b}{c}\right)\log\left(F\right)-bce\left(2\,x+\frac{b}{c}\right)\log\left(F\right)-4\,bcd\log\left(F\right)+b^2e\log\left(F\right)+2\,ce\right)e^{\left(cx^2\log(F)+bx\log(F)+\right.}}{2\left(b^2\log\left(F\right)-2\,c\right)c\log\left(F\right)}$$

input
```
integrate(F^(c*x^2+b*x+a)*(d+e*x+2*x^2*(2*c^2*d*log(F)-b*c*e*log(F))/(2*c-
b^2*log(F))),x, algorithm="giac")
```

output
```
-1/2*(2*c^2*d*(2*x + b/c)*log(F) - b*c*e*(2*x + b/c)*log(F) - 4*b*c*d*log(
F) + b^2*e*log(F) + 2*c*e)*e^(c*x^2*log(F) + b*x*log(F) + a*log(F))/((b^2*
log(F) - 2*c)*c*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)\,dx$$
$$=\frac{F^{c\,x^2+b\,x+a}\,\left(e-b\,d\,\ln\left(F\right)-b\,e\,x\,\ln\left(F\right)+2\,c\,d\,x\,\ln\left(F\right)\right)}{\ln\left(F\right)\,\left(2\,c-b^2\,\ln\left(F\right)\right)}$$

input
```
int(F^(a + b*x + c*x^2)*(d + e*x + (2*x^2*(2*c^2*d*log(F) - b*c*e*log(F)))
/(2*c - b^2*log(F))),x)
```

output
```
(F^(a + b*x + c*x^2)*(e - b*d*log(F) - b*e*x*log(F) + 2*c*d*x*log(F)))/(lo
g(F)*(2*c - b^2*log(F)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

$$\int F^{a+bx+cx^2}\left(d+ex+\frac{2x^2(2c^2d\log(F)-bce\log(F))}{2c-b^2\log(F)}\right)\,dx$$
$$=\frac{f^{c\,x^2+bx+a}(\log(f)\,bd+\log(f)\,bex-2\log(f)\,cdx-e)}{\log\left(f\right)\left(\log\left(f\right)b^2-2c\right)}$$

input
```
int(F^(c*x^2+b*x+a)*(d+e*x+2*x^2*(2*c^2*d*log(F)-b*c*e*log(F))/(2*c-b^2*lo
g(F))),x)
```

output
```
(f**(a + b*x + c*x**2)*(log(f)*b*d + log(f)*b*e*x - 2*log(f)*c*d*x - e))/(
log(f)*(log(f)*b**2 - 2*c))
```

3.4 $\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)x}{9d^2}+gx^2+hx^3+\frac{3dh+3b}{}\right.$

Optimal result

Integrand size = 84, antiderivative size = 42

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)dx=\frac{F^{a+bx+cx^2+dx^3}(3dg-2ch+3dhx)}{9d^2\log(F)}$$

output
```
1/9*F^(d*x^3+c*x^2+b*x+a)*(3*d*h*x-2*c*h+3*d*g)/d^2/ln(F)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)dx=\frac{F^{a+x(b+x(c+dx))}(-2ch+3d(g+hx))}{9d^2\log(F)}$$

input
```
Integrate[F^(a + b*x + c*x^2 + d*x^3)*(((6*c*d*g - 4*c^2*h + 3*b*d*h)*x)/(
9*d^2) + g*x^2 + h*x^3 + (3*d*h + 3*b*d*g*Log[F] - 2*b*c*h*Log[F])/(9*d^2*
Log[F])),x]
```

output
```
(F^(a + x*(b + x*(c + d*x)))*(-2*c*h + 3*d*(g + h*x)))/(9*d^2*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{x\left(3bdh-4c^2h+6cdg\right)}{9d^2}+\frac{-2bch\log(F)+3bdg\log(F)+3dh}{9d^2\log(F)}+gx^2+hx^3\right)dx$$

$$\Big\downarrow \ 7292$$

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{x\left(3bdh-4c^2h+6cdg\right)}{9d^2}+\frac{b\log(F)(3dg-2ch)+3dh}{9d^2\log(F)}+gx^2+hx^3\right)dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{x\left(3bdh-4c^2h+6cdg\right)F^{a+bx+cx^2+dx^3}}{9d^2}+\frac{F^{a+bx+cx^2+dx^3}(b\log(F)(3dg-2ch)+3dh)}{9d^2\log(F)}+gx^2F^{a+bx+cx^2+dx^3}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(3bdh-4c^2h+6cdg\right)\int F^{dx^3+cx^2+bx+a}xdx}{9d^2}+\frac{(b\log(F)(3dg-2ch)+3dh)\int F^{dx^3+cx^2+bx+a}dx}{9d^2\log(F)}+g\int F^{dx^3+cx^2+bx+a}x^2dx+h\int F^{dx^3+cx^2+bx+a}x^3dx$$

input
```
Int[F^(a + b*x + c*x^2 + d*x^3)*(((6*c*d*g - 4*c^2*h + 3*b*d*h)*x)/(9*d^2)
 + g*x^2 + h*x^3 + (3*d*h + 3*b*d*g*Log[F] - 2*b*c*h*Log[F])/(9*d^2*Log[F]
)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
gosper	$-\frac{F^{d\,x^3+c\,x^2+bx+a}(-3dhx+2ch-3dg)}{9d^2\ln(F)}$	41
risch	$-\frac{F^{d\,x^3+c\,x^2+bx+a}(-3dhx+2ch-3dg)}{9d^2\ln(F)}$	41
norman	$\frac{-\frac{(2ch-3dg)\,\mathrm{e}^{\left(d\,x^3+c\,x^2+bx+a\right)\ln(F)}}{9d\ln(F)}+\frac{hx\,\mathrm{e}^{\left(d\,x^3+c\,x^2+bx+a\right)\ln(F)}}{3\ln(F)}}{d}$	70
parallelrisch	$\frac{3dh\,F^{d\,x^3+c\,x^2+bx+a}x-2F^{d\,x^3+c\,x^2+bx+a}ch+3F^{d\,x^3+c\,x^2+bx+a}dg}{9d^2\ln(F)}$	75

input
```
int(F^(d*x^3+c*x^2+b*x+a)*(1/9*(3*b*d*h-4*c^2*h+6*c*d*g)*x/d^2+g*x^2+h*x^3
+1/9*(3*d*h+3*b*d*g*ln(F)-2*b*c*h*ln(F))/d^2/ln(F)),x,method=_RETURNVERBOS
E)
```

output
```
-1/9*F^(d*x^3+c*x^2+b*x+a)*(-3*d*h*x+2*c*h-3*d*g)/d^2/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)\,x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)\,dx=\frac{(3\,dhx+3\,dg-2\,ch)F^{dx^3+cx^2+bx+a}}{9\,d^2\log{(F)}}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(1/9*(3*b*d*h-4*c^2*h+6*c*d*g)*x/d^2+g*x^2
+h*x^3+1/9*(3*d*h+3*b*d*g*log(F)-2*b*c*h*log(F))/d^2/log(F)),x, algorithm=
"fricas")
```

output
```
1/9*(3*d*h*x + 3*d*g - 2*c*h)*F^(d*x^3 + c*x^2 + b*x + a)/(d^2*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(41) = 82.

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.95

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)\,x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)\,dx=\begin{cases}\frac{F^{a+bx+cx^2+dx^3}(-2ch+3dg+3dhx)}{9d^2\log{(F)}} & \text{for}\ d^2\log{(F)}\neq 0\\ \frac{gx^3}{3}+\frac{hx^4}{4}+\frac{x^2\cdot(3bdh-4c^2h+6cdg)}{18d^2}+\frac{x(-2bch\log{(F)}+3bdg\log{(F)}+3dh)}{9d^2\log{(F)}} & \text{otherwise}\end{cases}$$

input
```
integrate(F**(d*x**3+c*x**2+b*x+a)*(1/9*(3*b*d*h-4*c**2*h+6*c*d*g)*x/d**2+
g*x**2+h*x**3+1/9*(3*d*h+3*b*d*g*ln(F)-2*b*c*h*ln(F))/d**2/ln(F)),x)
```

output
```
Piecewise(((F**(a + b*x + c*x**2 + d*x**3)*(-2*c*h + 3*d*g + 3*d*h*x)/(9*d*
*2*log(F)), Ne(d**2*log(F), 0)), (g*x**3/3 + h*x**4/4 + x**2*(3*b*d*h - 4*
c**2*h + 6*c*d*g)/(18*d**2) + x*(-2*b*c*h*log(F) + 3*b*d*g*log(F) + 3*d*h)
/(9*d**2*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)dx = \frac{(3F^a dhx+(3dg-2ch)F^a)e^{(dx^3\log(F)+cx^2\log(F)+bx\log(F))}}{9d^2\log(F)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(1/9*(3*b*d*h-4*c^2*h+6*c*d*g)*x/d^2+g*x^2
+h*x^3+1/9*(3*d*h+3*b*d*g*log(F)-2*b*c*h*log(F))/d^2/log(F)),x, algorithm=
"maxima")
```

output
```
1/9*(3*F^a*d*h*x + (3*d*g - 2*c*h)*F^a)*e^(d*x^3*log(F) + c*x^2*log(F) + b
*x*log(F))/(d^2*log(F))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(40) = 80.

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.26

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)dx = \frac{3F^a dhxe^{(dx^3\log(F)+cx^2\log(F)+bx\log(F))}+3F^a dge^{(dx^3\log(F)+cx^2\log(F)+bx\log(F))}-2F^a che^{(dx^3\log(F)+cx^2\log(F)+}}{9d^2\log(F)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(1/9*(3*b*d*h-4*c^2*h+6*c*d*g)*x/d^2+g*x^2
+h*x^3+1/9*(3*d*h+3*b*d*g*log(F)-2*b*c*h*log(F))/d^2/log(F)),x, algorithm=
"giac")
```

output
```
1/9*(3*F^a*d*h*x*e^(d*x^3*log(F) + c*x^2*log(F) + b*x*log(F)) + 3*F^a*d*g*
e^(d*x^3*log(F) + c*x^2*log(F) + b*x*log(F)) - 2*F^a*c*h*e^(d*x^3*log(F) +
 c*x^2*log(F) + b*x*log(F)))/(d^2*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)\,x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)\,dx = -\frac{F^a\,F^{c\,x^2}\,F^{d\,x^3}\,F^{b\,x}\,\left(2\,c\,h-3\,d\,g\right)-3\,F^a\,F^{c\,x^2}\,F^{d\,x^3}\,F^{b\,x}\,d\,h\,x}{9\,d^2\,\ln\left(F\right)}$$

input
```
int(F^(a + b*x + c*x^2 + d*x^3)*(g*x^2 + h*x^3 + ((d*h)/3 - (2*b*c*h*log(F
))/9 + (b*d*g*log(F))/3)/(d^2*log(F)) + (x*(3*b*d*h - 4*c^2*h + 6*c*d*g))/
(9*d^2)),x)
```

output
```
-(F^a*F^(c*x^2)*F^(d*x^3)*F^(b*x)*(2*c*h - 3*d*g) - 3*F^a*F^(c*x^2)*F^(d*x
^3)*F^(b*x)*d*h*x)/(9*d^2*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{(6cdg-4c^2h+3bdh)\,x}{9d^2}+gx^2+hx^3+\frac{3dh+3bdg\log(F)-2bch\log(F)}{9d^2\log(F)}\right)\,dx = \frac{f^{d\,x^3+c\,x^2+bx+a}(3dhx-2ch+3dg)}{9\log\left(f\right)d^2}$$

input
```
int(F^(d*x^3+c*x^2+b*x+a)*(1/9*(3*b*d*h-4*c^2*h+6*c*d*g)*x/d^2+g*x^2+h*x^3
+1/9*(3*d*h+3*b*d*g*log(F)-2*b*c*h*log(F))/d^2/log(F)),x)
```

output
```
(f**(a + b*x + c*x**2 + d*x**3)*( - 2*c*h + 3*d*g + 3*d*h*x))/(9*log(f)*d*
*2)
```

3.5 $\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)x^2}{6cd}+hx^3+\frac{3dh-2}{}\right.$

Optimal result

Integrand size = 109, antiderivative size = 45

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx=\frac{F^{a+bx+cx^2+dx^3}(3df-bh+2chx)}{6cd\log(F)}$$

output
```
1/6*F^(d*x^3+c*x^2+b*x+a)*(2*c*h*x-b*h+3*d*f)/c/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx=\frac{F^{a+x(b+x(c+dx))}(3df-bh+2chx)}{6cd\log(F)}$$

input
```
Integrate[F^(a + b*x + c*x^2 + d*x^3)*(f*x + ((9*d^2*f + 4*c^2*h - 3*b*d*h
)*x^2)/(6*c*d) + h*x^3 + (3*d*h - 2*b*c*h*Log[F] + (b*(9*d^2*f + 4*c^2*h -
 3*b*d*h)*Log[F])/(2*c))/(9*d^2*Log[F])),x]
```

output
```
(F^(a + x*(b + x*(c + d*x)))*(3*d*f - b*h + 2*c*h*x))/(6*c*d*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{\frac{b\log(F)\left(-3bdh+4c^2h+9d^2f\right)}{2c}-2bch\log(F)+3dh}{9d^2\log(F)}+\frac{x^2\left(-3bdh+4c^2h+9d^2f\right)}{6cd}+fx+hx^3\right)dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{x^2\left(-3bdh+4c^2h+9d^2f\right)}{6cd}+\frac{b\log(F)(3df-bh)+2ch}{6cd\log(F)}+fx+hx^3\right)dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{x^2\left(-3bdh+4c^2h+9d^2f\right)F^{a+bx+cx^2+dx^3}}{6cd}+\frac{F^{a+bx+cx^2+dx^3}(b\log(F)(3df-bh)+2ch)}{6cd\log(F)}+fxF^{a+bx+cx^2+dx^3}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(-3bdh+4c^2h+9d^2f\right)\int F^{dx^3+cx^2+bx+a}x^2dx}{6cd}+\frac{(b\log(F)(3df-bh)+2ch)\int F^{dx^3+cx^2+bx+a}dx}{6cd\log(F)}+f\int F^{dx^3+cx^2+bx+a}xdx+h\int F^{dx^3+cx^2+bx+a}x^3dx$$

input
```
Int[F^(a + b*x + c*x^2 + d*x^3)*(f*x + ((9*d^2*f + 4*c^2*h - 3*b*d*h)*x^2)
/(6*c*d) + h*x^3 + (3*d*h - 2*b*c*h*Log[F] + (b*(9*d^2*f + 4*c^2*h - 3*b*d
*h)*Log[F])/(2*c))/(9*d^2*Log[F])),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
gosper	$-\frac{F^{d\,x^{3}+c\,x^{2}+bx+a}\left(-2chx+bh-3df\right)}{6dc\ln\left(F\right)}$	43
risch	$-\frac{F^{d\,x^{3}+c\,x^{2}+bx+a}\left(-2chx+bh-3df\right)}{6dc\ln\left(F\right)}$	43
norman	$\frac{\frac{hx\,{\mathrm e}^{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\ln\left(F\right)}}{3\ln\left(F\right)}-\frac{\left(bh-3df\right){\mathrm e}^{\left(d\,x^{3}+c\,x^{2}+bx+a\right)\ln\left(F\right)}}{6\ln\left(F\right)c}}{d}$	69
parallelrisch	$\frac{2ch\,F^{d\,x^{3}+c\,x^{2}+bx+a}x-F^{d\,x^{3}+c\,x^{2}+bx+a}bh+3F^{d\,x^{3}+c\,x^{2}+bx+a}df}{6dc\ln\left(F\right)}$	78

input
```
int(F^(d*x^3+c*x^2+b*x+a)*(f*x+1/6*(-3*b*d*h+4*c^2*h+9*d^2*f)*x^2/c/d+h*x^
3+1/9*(3*d*h-2*b*c*h*ln(F)+1/2*b*(-3*b*d*h+4*c^2*h+9*d^2*f)*ln(F)/c)/d^2/l
n(F)),x,method=_RETURNVERBOSE)
```

output
```
-1/6/d*F^(d*x^3+c*x^2+b*x+a)*(-2*c*h*x+b*h-3*d*f)/c/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx=\frac{(2\,chx+3\,df-bh)F^{dx^3+cx^2+bx+a}}{6\,cd\log(F)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(f*x+1/6*(-3*b*d*h+4*c^2*h+9*d^2*f)*x^2/c/
d+h*x^3+1/9*(3*d*h-2*b*c*h*log(F)+1/2*b*(-3*b*d*h+4*c^2*h+9*d^2*f)*log(F)/
c)/d^2/log(F)),x, algorithm="fricas")
```

output
```
1/6*(2*c*h*x + 3*d*f - b*h)*F^(d*x^3 + c*x^2 + b*x + a)/(c*d*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(39) = 78.

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.69

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx=\begin{cases}\frac{F^{a+bx+cx^2+dx^3}(-bh+2chx+3df)}{6cd\log(F)} & \text{for}\ cd\log(F)\neq 0\\ \frac{fx^2}{2}+\frac{hx^4}{4}+\frac{x^3(-3bdh+4c^2h+9d^2f)}{18cd}+\frac{x(-b^2h\log(F)+3bdf\log(F)+2ch)}{6cd\log(F)} & \text{otherwise}\end{cases}$$

input
```
integrate(F**(d*x**3+c*x**2+b*x+a)*(f*x+1/6*(-3*b*d*h+4*c**2*h+9*d**2*f)*x
**2/c/d+h*x**3+1/9*(3*d*h-2*b*c*h*ln(F)+1/2*b*(-3*b*d*h+4*c**2*h+9*d**2*f)
*ln(F)/c)/d**2/ln(F)),x)
```

output
```
Piecewise((F**(a + b*x + c*x**2 + d*x**3)*(-b*h + 2*c*h*x + 3*d*f)/(6*c*d*
log(F)), Ne(c*d*log(F), 0)), (f*x**2/2 + h*x**4/4 + x**3*(-3*b*d*h + 4*c**
2*h + 9*d**2*f)/(18*c*d) + x*(-b**2*h*log(F) + 3*b*d*f*log(F) + 2*c*h)/(6*
c*d*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx =\frac{(2\,F^a chx+(3\,df-bh)F^a)e^{\left(dx^3\log(F)+cx^2\log(F)+bx\log(F)\right)}}{6\,cd\log(F)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(f*x+1/6*(-3*b*d*h+4*c^2*h+9*d^2*f)*x^2/c/
d+h*x^3+1/9*(3*d*h-2*b*c*h*log(F)+1/2*b*(-3*b*d*h+4*c^2*h+9*d^2*f)*log(F)/
c)/d^2/log(F)),x, algorithm="maxima")
```

output
```
1/6*(2*F^a*c*h*x + (3*d*f - b*h)*F^a)*e^(d*x^3*log(F) + c*x^2*log(F) + b*x
*log(F))/(c*d*log(F))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(43) = 86.

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx =\frac{2\,F^a chxe^{\left(dx^3\log(F)+cx^2\log(F)+bx\log(F)\right)}+3\,F^a dfe^{\left(dx^3\log(F)+cx^2\log(F)+bx\log(F)\right)}-F^a bhe^{\left(dx^3\log(F)+cx^2\log(F)+bx\log(F)\right)}}{6\,cd\log(F)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(f*x+1/6*(-3*b*d*h+4*c^2*h+9*d^2*f)*x^2/c/
d+h*x^3+1/9*(3*d*h-2*b*c*h*log(F)+1/2*b*(-3*b*d*h+4*c^2*h+9*d^2*f)*log(F)/
c)/d^2/log(F)),x, algorithm="giac")
```

output
```
1/6*(2*F^a*c*h*x*e^(d*x^3*log(F) + c*x^2*log(F) + b*x*log(F)) + 3*F^a*d*f*
e^(d*x^3*log(F) + c*x^2*log(F) + b*x*log(F)) - F^a*b*h*e^(d*x^3*log(F) + c
*x^2*log(F) + b*x*log(F)))/(c*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx = -\frac{F^a\,F^{c\,x^2}\,F^{d\,x^3}\,F^{b\,x}\,(b\,h-3\,d\,f)-2\,F^a\,F^{c\,x^2}\,F^{d\,x^3}\,F^{b\,x}\,c\,h\,x}{6\,c\,d\,\ln(F)}$$

input
```
int(F^(a + b*x + c*x^2 + d*x^3)*(f*x + h*x^3 + ((d*h)/3 + (b*log(F)*(9*d^2
*f + 4*c^2*h - 3*b*d*h))/(18*c) - (2*b*c*h*log(F))/9)/(d^2*log(F)) + (x^2*
(9*d^2*f + 4*c^2*h - 3*b*d*h))/(6*c*d)),x)
```

output
```
-(F^a*F^(c*x^2)*F^(d*x^3)*F^(b*x)*(b*h - 3*d*f) - 2*F^a*F^(c*x^2)*F^(d*x^3
)*F^(b*x)*c*h*x)/(6*c*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int F^{a+bx+cx^2+dx^3}\left(fx+\frac{(9d^2f+4c^2h-3bdh)\,x^2}{6cd}+hx^3+\frac{3dh-2bch\log(F)+\frac{b(9d^2f+4c^2h-3bdh)\log(F)}{2c}}{9d^2\log(F)}\right)dx = \frac{f^{d\,x^3+c\,x^2+bx+a}(2chx-bh+3df)}{6\log(f)\,cd}$$

input
```
int(F^(d*x^3+c*x^2+b*x+a)*(f*x+1/6*(-3*b*d*h+4*c^2*h+9*d^2*f)*x^2/c/d+h*x^
3+1/9*(3*d*h-2*b*c*h*log(F)+1/2*b*(-3*b*d*h+4*c^2*h+9*d^2*f)*log(F)/c)/d^2
/log(F)),x)
```

output
```
(f**(a + b*x + c*x**2 + d*x**3)*( - b*h + 2*c*h*x + 3*d*f))/(6*log(f)*c*d)
```

3.6 $\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2}{4c^2-3b}}{}\right.$

Optimal result

Integrand size = 132, antiderivative size = 69

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx = \frac{F^{a+bx+cx^2+dx^3}\left(\frac{2cf-bg}{4c^2-3bd}-\frac{(3df-2cg)x}{4c^2-3bd}\right)}{\log(F)}$$

output
```
F^(d*x^3+c*x^2+b*x+a)*((-b*g+2*c*f)/(-3*b*d+4*c^2)-(-2*c*g+3*d*f)*x/(-3*b*
d+4*c^2))/ln(F)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx=\frac{F^{a+x(b+x(c+dx))}(-bg-3dfx+2c(f+gx))}{(4c^2-3bd)\log(F)}$$

input
```
Integrate[F^(a + b*x + c*x^2 + d*x^3)*(f*x + g*x^2 - (3*(3*d^2*f - 2*c*d*g
)*x^3)/(4*c^2 - 3*b*d) + ((-9*d*(3*d^2*f - 2*c*d*g))/(4*c^2 - 3*b*d) + 3*b
*d*g*Log[F] + (6*b*c*(3*d^2*f - 2*c*d*g)*Log[F])/(4*c^2 - 3*b*d))/(9*d^2*L
og[F])),x]
```

output
```
(F^(a + x*(b + x*(c + d*x)))*(-(b*g) - 3*d*f*x + 2*c*(f + g*x)))/((4*c^2 -
 3*b*d)*Log[F])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx+cx^2+dx^3}\left(\frac{\frac{6bc\log(F)(3d^2f-2cdg)}{4c^2-3bd}-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)}{9d^2\log(F)}-\frac{3x^3(3d^2f-2cdg)}{4c^2-3bd}+fx+gx^2\right)dx$$

$$\Big\downarrow \text{7292}$$

$$\int F^{a+bx+cx^2+dx^3}\left(-\frac{-b\log(F)(2cf-bg)-2cg+3df}{\log(F)\,(4c^2-3bd)}-\frac{3dx^3(3df-2cg)}{4c^2-3bd}+fx+gx^2\right)dx$$

$$\Big\downarrow \text{7293}$$

$$\int\left(\frac{3dx^3(3df-2cg)F^{a+bx+cx^2+dx^3}}{3bd-4c^2}+\frac{F^{a+bx+cx^2+dx^3}(b\log(F)(2cf-bg)+2cg-3df)}{\log(F)\,(4c^2-3bd)}+fxF^{a+bx+cx^2+dx^3}+ga\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d(3df-2cg)\int F^{dx^3+cx^2+bx+a}x^3dx}{4c^2-3bd}-\frac{(-b\log(F)(2cf-bg)-2cg+3df)\int F^{dx^3+cx^2+bx+a}dx}{\log(F)\,(4c^2-3bd)}+f\int F^{dx^3+cx^2+bx+a}xdx+g\int F^{dx^3+cx^2+bx+a}x^2dx$$

input
```
Int[F^(a + b*x + c*x^2 + d*x^3)*(f*x + g*x^2 - (3*(3*d^2*f - 2*c*d*g)*x^3)
/(4*c^2 - 3*b*d) + ((-9*d*(3*d^2*f - 2*c*d*g))/(4*c^2 - 3*b*d) + 3*b*d*g*L
og[F] + (6*b*c*(3*d^2*f - 2*c*d*g)*Log[F])/(4*c^2 - 3*b*d))/(9*d^2*Log[F])
),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{F^{d\,x^3+c\,x^2+bx+a}(-2cgx+3dfx+bg-2cf)}{(3bd-4c^2)\ln(F)}$	53
risch	$\frac{F^{d\,x^3+c\,x^2+bx+a}(-2cgx+3dfx+bg-2cf)}{(3bd-4c^2)\ln(F)}$	53
norman	$\frac{\frac{d(bg-2cf)\mathrm{e}^{\left(d\,x^3+c\,x^2+bx+a\right)\ln(F)}}{\left(3bd-4c^2\right)\ln(F)}-\frac{d(2cg-3df)x\,\mathrm{e}^{\left(d\,x^3+c\,x^2+bx+a\right)\ln(F)}}{\ln(F)\left(3bd-4c^2\right)}}{d}$	99
parallelrisch	$\frac{-2x\,F^{d\,x^3+c\,x^2+bx+a}cg+3x\,F^{d\,x^3+c\,x^2+bx+a}df+F^{d\,x^3+c\,x^2+bx+a}bg-2F^{d\,x^3+c\,x^2+bx+a}cf}{(3bd-4c^2)\ln(F)}$	104

input
```
int(F^(d*x^3+c*x^2+b*x+a)*(f*x+g*x^2-3*(-2*c*d*g+3*d^2*f)*x^3/(-3*b*d+4*c^
2)+1/9*(-9*d*(-2*c*d*g+3*d^2*f)/(-3*b*d+4*c^2)+3*b*d*g*ln(F)+6*b*c*(-2*c*d
*g+3*d^2*f)*ln(F)/(-3*b*d+4*c^2))/d^2/ln(F)),x,method=_RETURNVERBOSE)
```

output
```
F^(d*x^3+c*x^2+b*x+a)*(-2*c*g*x+3*d*f*x+b*g-2*c*f)/(3*b*d-4*c^2)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx=\frac{(2\,cf-bg-(3\,df-2\,cg)x)F^{dx^3+cx^2+bx+a}}{(4\,c^2-3\,bd)\log{(F)}}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(f*x+g*x^2-3*(-2*c*d*g+3*d^2*f)*x^3/(-3*b*
d+4*c^2)+1/9*(-9*d*(-2*c*d*g+3*d^2*f)/(-3*b*d+4*c^2)+3*b*d*g*log(F)+6*b*c*
(-2*c*d*g+3*d^2*f)*log(F)/(-3*b*d+4*c^2))/d^2/log(F)),x, algorithm="fricas
")
```

output
```
(2*c*f - b*g - (3*d*f - 2*c*g)*x)*F^(d*x^3 + c*x^2 + b*x + a)/((4*c^2 - 3*
b*d)*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(60) = 120$.

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.29

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx$$
$$=\begin{cases}\frac{F^{a+bx+cx^2+dx^3}(bg-2cf-2cgx+3dfx)}{3bd\log(F)-4c^2\log(F)} & \text{for } 3bd\log(F)-4c^2\log(F)\neq 0\\ \frac{fx^2}{2}+\frac{gx^3}{3}+\frac{x^4(-6cdg+9d^2f)}{12bd-16c^2}+\frac{x(b^2g\log(F)-2bcf\log(F)-2cg+3df)}{3bd\log(F)-4c^2\log(F)} & \text{otherwise}\end{cases}$$

input
```
integrate(F**(d*x**3+c*x**2+b*x+a)*(f*x+g*x**2-3*(-2*c*d*g+3*d**2*f)*x**3/
(-3*b*d+4*c**2)+1/9*(-9*d*(-2*c*d*g+3*d**2*f)/(-3*b*d+4*c**2)+3*b*d*g*ln(F
)+6*b*c*(-2*c*d*g+3*d**2*f)*ln(F)/(-3*b*d+4*c**2))/d**2/ln(F)),x)
```

output
```
Piecewise((F**(a + b*x + c*x**2 + d*x**3)*(b*g - 2*c*f - 2*c*g*x + 3*d*f*x
)/(3*b*d*log(F) - 4*c**2*log(F)), Ne(3*b*d*log(F) - 4*c**2*log(F), 0)), (f
*x**2/2 + g*x**3/3 + x**4*(-6*c*d*g + 9*d**2*f)/(12*b*d - 16*c**2) + x*(b*
*2*g*log(F) - 2*b*c*f*log(F) - 2*c*g + 3*d*f)/(3*b*d*log(F) - 4*c**2*log(F
)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx=$$
$$-\frac{((3\,df-2\,cg)F^ax-(2\,cf-bg)F^a)e^{(dx^3\log(F)+cx^2\log(F)+bx\log(F))}}{(4\,c^2-3\,bd)\log(F)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(f*x+g*x^2-3*(-2*c*d*g+3*d^2*f)*x^3/(-3*b*
d+4*c^2)+1/9*(-9*d*(-2*c*d*g+3*d^2*f)/(-3*b*d+4*c^2)+3*b*d*g*log(F)+6*b*c*
(-2*c*d*g+3*d^2*f)*log(F)/(-3*b*d+4*c^2))/d^2/log(F)),x, algorithm="maxima
")
```

output
```
-((3*d*f - 2*c*g)*F^a*x - (2*c*f - b*g)*F^a)*e^(d*x^3*log(F) + c*x^2*log(F
) + b*x*log(F))/((4*c^2 - 3*b*d)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.91

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx = -\frac{3\,F^a dfxe^{\left(dx^3\log(F)+cx^2\log(F)+bx\log(F)\right)}-2\,F^a cgxe^{\left(dx^3\log(F)+cx^2\log(F)+bx\log(F)\right)}-2\,F^a cfe^{\left(dx^3\log(F)+cx^2\log(\right.}}{4\,c^2\log\left(F\right)-3\,bd\log\left(F\right)}$$

input
```
integrate(F^(d*x^3+c*x^2+b*x+a)*(f*x+g*x^2-3*(-2*c*d*g+3*d^2*f)*x^3/(-3*b*
d+4*c^2)+1/9*(-9*d*(-2*c*d*g+3*d^2*f)/(-3*b*d+4*c^2)+3*b*d*g*log(F)+6*b*c*
(-2*c*d*g+3*d^2*f)*log(F)/(-3*b*d+4*c^2))/d^2/log(F)),x, algorithm="giac")
```

output
```
-(3*F^a*d*f*x*e^(d*x^3*log(F) + c*x^2*log(F) + b*x*log(F)) - 2*F^a*c*g*x*e
^(d*x^3*log(F) + c*x^2*log(F) + b*x*log(F)) - 2*F^a*c*f*e^(d*x^3*log(F) +
c*x^2*log(F) + b*x*log(F)) + F^a*b*g*e^(d*x^3*log(F) + c*x^2*log(F) + b*x*
log(F)))/(4*c^2*log(F) - 3*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx=\frac{F^a\,F^{c\,x^2}\,F^{d\,x^3}\,F^{b\,x}\,(b\,g-2\,c\,f-2\,c\,g\,x+3\,d\,f\,x)}{\ln(F)\,(3\,b\,d-4\,c^2)}$$

input
```
int(F^(a + b*x + c*x^2 + d*x^3)*(f*x + g*x^2 + ((d*(3*d^2*f - 2*c*d*g))/(3
*b*d - 4*c^2) + (b*d*g*log(F))/3 - (2*b*c*log(F)*(3*d^2*f - 2*c*d*g))/(3*(
3*b*d - 4*c^2)))/(d^2*log(F)) + (3*x^3*(3*d^2*f - 2*c*d*g))/(3*b*d - 4*c^2
)),x)
```

output
```
(F^a*F^(c*x^2)*F^(d*x^3)*F^(b*x)*(b*g - 2*c*f - 2*c*g*x + 3*d*f*x))/(log(F
)*(3*b*d - 4*c^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int F^{a+bx+cx^2+dx^3}\left(fx+gx^2-\frac{3(3d^2f-2cdg)\,x^3}{4c^2-3bd}+\frac{-\frac{9d(3d^2f-2cdg)}{4c^2-3bd}+3bdg\log(F)+\frac{6bc(3d^2f-2cdg)\log(F)}{4c^2-3bd}}{9d^2\log(F)}\right)dx=\frac{f^{d\,x^3+c\,x^2+bx+a}(-2cgx+3dfx+bg-2cf)}{\log(f)\,(3bd-4c^2)}$$

input
```
int(F^(d*x^3+c*x^2+b*x+a)*(f*x+g*x^2-3*(-2*c*d*g+3*d^2*f)*x^3/(-3*b*d+4*c^
2)+1/9*(-9*d*(-2*c*d*g+3*d^2*f)/(-3*b*d+4*c^2)+3*b*d*g*log(F)+6*b*c*(-2*c*
d*g+3*d^2*f)*log(F)/(-3*b*d+4*c^2))/d^2/log(F)),x)
```

output
```
(f**(a + b*x + c*x**2 + d*x**3)*(b*g - 2*c*f - 2*c*g*x + 3*d*f*x))/(log(f)
*(3*b*d - 4*c**2))
```

3.7 $\int f^{a+bx^2} x^{11}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int f^{a+bx^2} x^{11}\, dx = -\frac{f^{a+bx^2}\left(120-120bx^2\log(f)+60b^2x^4\log^2(f)-20b^3x^6\log^3(f)+5b^4x^8\log^4(f)-b^5x^{10}\log^5(f)\right)}{2b^6\log^6(f)}$$

output
```
-1/2*f^(b*x^2+a)*(120-120*b*x^2*ln(f)+60*b^2*x^4*ln(f)^2-20*b^3*x^6*ln(f)^
3+5*b^4*x^8*ln(f)^4-b^5*x^10*ln(f)^5)/b^6/ln(f)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int f^{a+bx^2} x^{11}\, dx = -\frac{f^a\Gamma(6,-bx^2\log(f))}{2b^6\log^6(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^11,x]
```

output
```
-1/2*(f^a*Gamma[6, -(b*x^2*Log[f])])/(b^6*Log[f]^6)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} f^{a+bx^2} \, dx$$

$$\Big\downarrow \ 2647$$

$$-\frac{f^{a+bx^2} (-b^5 x^{10} \log^5(f) + 5 b^4 x^8 \log^4(f) - 20 b^3 x^6 \log^3(f) + 60 b^2 x^4 \log^2(f) - 120 b x^2 \log(f) + 120)}{2 b^6 \log^6(f)}$$

input
```
Int[f^(a + b*x^2)*x^11,x]
```

output
```
-1/2*(f^(a + b*x^2)*(120 - 120*b*x^2*Log[f] + 60*b^2*x^4*Log[f]^2 - 20*b^3
*x^6*Log[f]^3 + 5*b^4*x^8*Log[f]^4 - b^5*x^10*Log[f]^5))/(b^6*Log[f]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{\left(b^{5} x^{10} \ln(f)^{5}-5 b^{4} x^{8} \ln(f)^{4}+20 b^{3} x^{6} \ln(f)^{3}-60 b^{2} x^{4} \ln(f)^{2}+120 b\,x^{2} \ln(f)-120\right) f^{b\,x^{2}+a}}{2 \ln(f)^{6} b^{6}}$
risch	$\frac{\left(b^{5} x^{10} \ln(f)^{5}-5 b^{4} x^{8} \ln(f)^{4}+20 b^{3} x^{6} \ln(f)^{3}-60 b^{2} x^{4} \ln(f)^{2}+120 b\,x^{2} \ln(f)-120\right) f^{b\,x^{2}+a}}{2 \ln(f)^{6} b^{6}}$
orering	$\frac{\left(b^{5} x^{10} \ln(f)^{5}-5 b^{4} x^{8} \ln(f)^{4}+20 b^{3} x^{6} \ln(f)^{3}-60 b^{2} x^{4} \ln(f)^{2}+120 b\,x^{2} \ln(f)-120\right) f^{b\,x^{2}+a}}{2 \ln(f)^{6} b^{6}}$
meijerg	$\frac{f^{a} \left(120-\frac{\left(-6 b^{5} x^{10} \ln(f)^{5}+30 b^{4} x^{8} \ln(f)^{4}-120 b^{3} x^{6} \ln(f)^{3}+360 b^{2} x^{4} \ln(f)^{2}-720 b\,x^{2} \ln(f)+720\right) {\mathrm e}^{b\,x^{2} \ln(f)}}{6}\right)}{2 b^{6} \ln(f)^{6}}$
parallelrisch	$\frac{f^{b\,x^{2}+a} x^{10} \ln(f)^{5} b^{5}-5 f^{b\,x^{2}+a} x^{8} \ln(f)^{4} b^{4}+20 f^{b\,x^{2}+a} x^{6} \ln(f)^{3} b^{3}-60 f^{b\,x^{2}+a} x^{4} \ln(f)^{2} b^{2}+120 f^{b\,x^{2}+a} x^{2} \ln(f) b-120 f^{b\,x^{2}+a}}{2 \ln(f)^{6} b^{6}}$
norman	$-\frac{60\,{\mathrm e}^{\left(b\,x^{2}+a\right) \ln(f)}}{b^{6} \ln(f)^{6}}+\frac{60 x^{2} {\mathrm e}^{\left(b\,x^{2}+a\right) \ln(f)}}{b^{5} \ln(f)^{5}}+\frac{x^{10} {\mathrm e}^{\left(b\,x^{2}+a\right) \ln(f)}}{2 b \ln(f)}-\frac{30 x^{4} {\mathrm e}^{\left(b\,x^{2}+a\right) \ln(f)}}{\ln(f)^{4} b^{4}}+\frac{10 x^{6} {\mathrm e}^{\left(b\,x^{2}+a\right) \ln(f)}}{\ln(f)^{3} b^{3}}-\frac{5 x^{8}}{}$

input
```
int(f^(b*x^2+a)*x^11,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^5*x^10*ln(f)^5-5*b^4*x^8*ln(f)^4+20*b^3*x^6*ln(f)^3-60*b^2*x^4*ln(f
)^2+120*b*x^2*ln(f)-120)*f^(b*x^2+a)/ln(f)^6/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int f^{a+bx^2} x^{11}\,dx = \frac{\left(b^5 x^{10} \log(f)^5 - 5\,b^4 x^8 \log(f)^4 + 20\,b^3 x^6 \log(f)^3 - 60\,b^2 x^4 \log(f)^2 + 120\,b x^2 \log(f) - 120\right) f^{bx^2+a}}{2\,b^6 \log(f)^6}$$

input
```
integrate(f^(b*x^2+a)*x^11,x, algorithm="fricas")
```

output
```
1/2*(b^5*x^10*log(f)^5 - 5*b^4*x^8*log(f)^4 + 20*b^3*x^6*log(f)^3 - 60*b^2
*x^4*log(f)^2 + 120*b*x^2*log(f) - 120)*f^(b*x^2 + a)/(b^6*log(f)^6)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int f^{a+bx^2}x^{11}\,dx$$
$$= \begin{cases} \frac{f^{a+bx^2}\left(b^5x^{10}\log{(f)}^5-5b^4x^8\log{(f)}^4+20b^3x^6\log{(f)}^3-60b^2x^4\log{(f)}^2+120bx^2\log{(f)}-120\right)}{2b^6\log{(f)}^6} & \text{for } b^6\log{(f)}^6\neq 0 \\ \frac{x^{12}}{12} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x**2+a)*x**11,x)
```

output
```
Piecewise((f**(a + b*x**2)*(b**5*x**10*log(f)**5 - 5*b**4*x**8*log(f)**4 +
 20*b**3*x**6*log(f)**3 - 60*b**2*x**4*log(f)**2 + 120*b*x**2*log(f) - 120
)/(2*b**6*log(f)**6), Ne(b**6*log(f)**6, 0)), (x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int f^{a+bx^2}x^{11}\,dx$$
$$= \frac{(b^5f^ax^{10}\log{(f)}^5-5\,b^4f^ax^8\log{(f)}^4+20\,b^3f^ax^6\log{(f)}^3-60\,b^2f^ax^4\log{(f)}^2+120\,bf^ax^2\log{(f)}-120}{2\,b^6\log{(f)}^6}$$

input
```
integrate(f^(b*x^2+a)*x^11,x, algorithm="maxima")
```

output
```
1/2*(b^5*f^a*x^10*log(f)^5 - 5*b^4*f^a*x^8*log(f)^4 + 20*b^3*f^a*x^6*log(f
)^3 - 60*b^2*f^a*x^4*log(f)^2 + 120*b*f^a*x^2*log(f) - 120*f^a)*f^(b*x^2)/
(b^6*log(f)^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int f^{a+bx^2}x^{11}\,dx = \frac{\left(b^5x^{10}\log\left(f\right)^5 - 5\,b^4x^8\log\left(f\right)^4 + 20\,b^3x^6\log\left(f\right)^3 - 60\,b^2x^4\log\left(f\right)^2 + 120\,bx^2\log\left(f\right) - 120\right)e^{\left(bx^2\log(f)+a\right.}}{2\,b^6\log\left(f\right)^6}$$

input
```
integrate(f^(b*x^2+a)*x^11,x, algorithm="giac")
```

output
```
1/2*(b^5*x^10*log(f)^5 - 5*b^4*x^8*log(f)^4 + 20*b^3*x^6*log(f)^3 - 60*b^2
*x^4*log(f)^2 + 120*b*x^2*log(f) - 120)*e^(b*x^2*log(f) + a*log(f))/(b^6*l
og(f)^6)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int f^{a+bx^2}x^{11}\,dx = -\frac{f^{b\,x^2+a}\left(-\frac{b^5\,x^{10}\,\ln(f)^5}{2} + \frac{5\,b^4\,x^8\,\ln(f)^4}{2} - 10\,b^3\,x^6\,\ln\left(f\right)^3 + 30\,b^2\,x^4\,\ln\left(f\right)^2 - 60\,b\,x^2\,\ln\left(f\right) + 60\right)}{b^6\,\ln\left(f\right)^6}$$

input
```
int(f^(a + b*x^2)*x^11,x)
```

output
```
-(f^(a + b*x^2)*(30*b^2*x^4*log(f)^2 - 10*b^3*x^6*log(f)^3 + (5*b^4*x^8*lo
g(f)^4)/2 - (b^5*x^10*log(f)^5)/2 - 60*b*x^2*log(f) + 60))/(b^6*log(f)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int f^{a+bx^2}x^{11}\,dx$$
$$= \frac{f^{b\,x^2+a}\left(\log(f)^5\,b^5x^{10}-5\log(f)^4\,b^4x^8+20\log(f)^3\,b^3x^6-60\log(f)^2\,b^2x^4+120\log(f)\,b\,x^2-120\right)}{2\log(f)^6\,b^6}$$

input
```
int(f^(b*x^2+a)*x^11,x)
```

output
```
(f**(a + b*x**2)*(log(f)**5*b**5*x**10 - 5*log(f)**4*b**4*x**8 + 20*log(f)
**3*b**3*x**6 - 60*log(f)**2*b**2*x**4 + 120*log(f)*b*x**2 - 120))/(2*log(
f)**6*b**6)
```

3.8 $\int f^{a+bx^2} x^9 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int f^{a+bx^2} x^9 \, dx = \frac{f^{a+bx^2}(24 - 24bx^2 \log(f) + 12b^2x^4 \log^2(f) - 4b^3x^6 \log^3(f) + b^4x^8 \log^4(f))}{2b^5 \log^5(f)}$$

output
```
1/2*f^(b*x^2+a)*(24-24*b*x^2*ln(f)+12*b^2*x^4*ln(f)^2-4*b^3*x^6*ln(f)^3+b^
4*x^8*ln(f)^4)/b^5/ln(f)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.37

$$\int f^{a+bx^2} x^9 \, dx = \frac{f^a \Gamma(5, -bx^2 \log(f))}{2b^5 \log^5(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^9,x]
```

output
```
(f^a*Gamma[5, -(b*x^2*Log[f])])/(2*b^5*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 f^{a+bx^2} \, dx$$

$$\Big\downarrow \; 2647$$

$$\frac{f^{a+bx^2}\left(b^4x^8\log^4(f)-4b^3x^6\log^3(f)+12b^2x^4\log^2(f)-24bx^2\log(f)+24\right)}{2b^5\log^5(f)}$$

input
```
Int[f^(a + b*x^2)*x^9,x]
```

output
```
(f^(a + b*x^2)*(24 - 24*b*x^2*Log[f] + 12*b^2*x^4*Log[f]^2 - 4*b^3*x^6*Log
[f]^3 + b^4*x^8*Log[f]^4))/(2*b^5*Log[f]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result	size
gosper	$\frac{f^{b\,x^2+a}\left(24-24b\,x^2\ln(f)+12b^2x^4\ln(f)^2-4b^3x^6\ln(f)^3+b^4x^8\ln(f)^4\right)}{2b^5\ln(f)^5}$	64
risch	$\frac{f^{b\,x^2+a}\left(24-24b\,x^2\ln(f)+12b^2x^4\ln(f)^2-4b^3x^6\ln(f)^3+b^4x^8\ln(f)^4\right)}{2b^5\ln(f)^5}$	64
orering	$\frac{f^{b\,x^2+a}\left(24-24b\,x^2\ln(f)+12b^2x^4\ln(f)^2-4b^3x^6\ln(f)^3+b^4x^8\ln(f)^4\right)}{2b^5\ln(f)^5}$	64
meijerg	$-\frac{f^a\left(24-\frac{\left(5b^4x^8\ln(f)^4-20b^3x^6\ln(f)^3+60b^2x^4\ln(f)^2-120b\,x^2\ln(f)+120\right)e^{b\,x^2\ln(f)}}{5}\right)}{2b^5\ln(f)^5}$	71
parallelrisch	$\frac{f^{b\,x^2+a}x^8\ln(f)^4b^4-4f^{b\,x^2+a}x^6\ln(f)^3b^3+12f^{b\,x^2+a}x^4\ln(f)^2b^2-24f^{b\,x^2+a}x^2\ln(f)b+24f^{b\,x^2+a}}{2\ln(f)^5b^5}$	101
norman	$\frac{12\,e^{\left(b\,x^2+a\right)\ln(f)}}{b^5\ln(f)^5}+\frac{x^8e^{\left(b\,x^2+a\right)\ln(f)}}{2b\ln(f)}-\frac{12x^2e^{\left(b\,x^2+a\right)\ln(f)}}{\ln(f)^4b^4}+\frac{6x^4e^{\left(b\,x^2+a\right)\ln(f)}}{\ln(f)^3b^3}-\frac{2x^6e^{\left(b\,x^2+a\right)\ln(f)}}{\ln(f)^2b^2}$	114

input
```
int(f^(b*x^2+a)*x^9,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^(b*x^2+a)*(24-24*b*x^2*ln(f)+12*b^2*x^4*ln(f)^2-4*b^3*x^6*ln(f)^3+b^
4*x^8*ln(f)^4)/b^5/ln(f)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int f^{a+bx^2}x^9\,dx = \frac{\left(b^4x^8\log(f)^4-4\,b^3x^6\log(f)^3+12\,b^2x^4\log(f)^2-24\,bx^2\log(f)+24\right)f^{bx^2+a}}{2\,b^5\log(f)^5}$$

input
```
integrate(f^(b*x^2+a)*x^9,x, algorithm="fricas")
```

output
```
1/2*(b^4*x^8*log(f)^4 - 4*b^3*x^6*log(f)^3 + 12*b^2*x^4*log(f)^2 - 24*b*x^
2*log(f) + 24)*f^(b*x^2 + a)/(b^5*log(f)^5)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int f^{a+bx^2} x^9 \, dx = \begin{cases} \frac{f^{a+bx^2}\left(b^4x^8\log{(f)}^4 - 4b^3x^6\log{(f)}^3 + 12b^2x^4\log{(f)}^2 - 24bx^2\log{(f)} + 24\right)}{2b^5\log{(f)}^5} & \text{for } b^5\log{(f)}^5 \neq 0 \\ \frac{x^{10}}{10} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x**2+a)*x**9,x)
```

output
```
Piecewise((f**(a + b*x**2)*(b**4*x**8*log(f)**4 - 4*b**3*x**6*log(f)**3 +
12*b**2*x**4*log(f)**2 - 24*b*x**2*log(f) + 24)/(2*b**5*log(f)**5), Ne(b**
5*log(f)**5, 0)), (x**10/10, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int f^{a+bx^2} x^9 \, dx = \frac{\left(b^4 f^a x^8 \log{(f)}^4 - 4\, b^3 f^a x^6 \log{(f)}^3 + 12\, b^2 f^a x^4 \log{(f)}^2 - 24\, b f^a x^2 \log{(f)} + 24\, f^a\right) f^{bx^2}}{2\, b^5 \log{(f)}^5}$$

input
```
integrate(f^(b*x^2+a)*x^9,x, algorithm="maxima")
```

output
```
1/2*(b^4*f^a*x^8*log(f)^4 - 4*b^3*f^a*x^6*log(f)^3 + 12*b^2*f^a*x^4*log(f)
^2 - 24*b*f^a*x^2*log(f) + 24*f^a)*f^(b*x^2)/(b^5*log(f)^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int f^{a+bx^2}x^9\,dx = \frac{\left(b^4x^8\log(f)^4 - 4\,b^3x^6\log(f)^3 + 12\,b^2x^4\log(f)^2 - 24\,bx^2\log(f) + 24\right)e^{\left(bx^2\log(f)+a\log(f)\right)}}{2\,b^5\log(f)^5}$$

input
```
integrate(f^(b*x^2+a)*x^9,x, algorithm="giac")
```

output
```
1/2*(b^4*x^8*log(f)^4 - 4*b^3*x^6*log(f)^3 + 12*b^2*x^4*log(f)^2 - 24*b*x^
2*log(f) + 24)*e^(b*x^2*log(f) + a*log(f))/(b^5*log(f)^5)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int f^{a+bx^2}x^9\,dx = \frac{f^{b\,x^2+a}\left(\frac{b^4\,x^8\,\ln(f)^4}{2} - 2\,b^3\,x^6\ln(f)^3 + 6\,b^2\,x^4\ln(f)^2 - 12\,b\,x^2\,\ln(f) + 12\right)}{b^5\ln(f)^5}$$

input
```
int(f^(a + b*x^2)*x^9,x)
```

output
```
(f^(a + b*x^2)*(6*b^2*x^4*log(f)^2 - 2*b^3*x^6*log(f)^3 + (b^4*x^8*log(f)^
4)/2 - 12*b*x^2*log(f) + 12))/(b^5*log(f)^5)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int f^{a+bx^2}x^9\,dx$$
$$= \frac{f^{b\,x^2+a}\left(\log(f)^4\,b^4x^8-4\log(f)^3\,b^3x^6+12\log(f)^2\,b^2x^4-24\log(f)\,b\,x^2+24\right)}{2\log\left(f\right)^5b^5}$$

input
```
int(f^(b*x^2+a)*x^9,x)
```

output
```
(f**(a + b*x**2)*(log(f)**4*b**4*x**8 - 4*log(f)**3*b**3*x**6 + 12*log(f)*
*2*b**2*x**4 - 24*log(f)*b*x**2 + 24))/(2*log(f)**5*b**5)
```

3.9 $\int f^{a+bx^2} x^7 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int f^{a+bx^2} x^7 \, dx = -\frac{3f^{a+bx^2}}{b^4 \log^4(f)} + \frac{3f^{a+bx^2}x^2}{b^3 \log^3(f)} - \frac{3f^{a+bx^2}x^4}{2b^2 \log^2(f)} + \frac{f^{a+bx^2}x^6}{2b \log(f)}$$

output
```
-3*f^(b*x^2+a)/b^4/ln(f)^4+3*f^(b*x^2+a)*x^2/b^3/ln(f)^3-3/2*f^(b*x^2+a)*x
^4/b^2/ln(f)^2+1/2*f^(b*x^2+a)*x^6/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.62

$$\int f^{a+bx^2} x^7 \, dx = \frac{f^{a+bx^2}(-6 + 6bx^2 \log(f) - 3b^2x^4 \log^2(f) + b^3x^6 \log^3(f))}{2b^4 \log^4(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^7,x]
```

output
```
(f^(a + b*x^2)*(-6 + 6*b*x^2*Log[f] - 3*b^2*x^4*Log[f]^2 + b^3*x^6*Log[f]^
3))/(2*b^4*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 f^{a+bx^2} dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^6 f^{a+bx^2}}{2b\log(f)}-\frac{3\int f^{bx^2+a}x^5dx}{b\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^6 f^{a+bx^2}}{2b\log(f)}-\frac{3\left(\frac{x^4 f^{a+bx^2}}{2b\log(f)}-\frac{2\int f^{bx^2+a}x^3dx}{b\log(f)}\right)}{b\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^6 f^{a+bx^2}}{2b\log(f)}-\frac{3\left(\frac{x^4 f^{a+bx^2}}{2b\log(f)}-\frac{2\left(\frac{x^2 f^{a+bx^2}}{2b\log(f)}-\frac{\int f^{bx^2+a}xdx}{b\log(f)}\right)}{b\log(f)}\right)}{b\log(f)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{x^6 f^{a+bx^2}}{2b\log(f)}-\frac{3\left(\frac{x^4 f^{a+bx^2}}{2b\log(f)}-\frac{2\left(\frac{x^2 f^{a+bx^2}}{2b\log(f)}-\frac{f^{a+bx^2}}{2b^2\log^2(f)}\right)}{b\log(f)}\right)}{b\log(f)}$$

input
```
Int[f^(a + b*x^2)*x^7,x]
```

output
```
(f^(a + b*x^2)*x^6)/(2*b*Log[f]) - (3*((f^(a + b*x^2)*x^4)/(2*b*Log[f]) -
(2*(-1/2*f^(a + b*x^2)/(b^2*Log[f]^2) + (f^(a + b*x^2)*x^2)/(2*b*Log[f])))
/(b*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\left(b^{3} x^{6} \ln \left(f\right)^{3}-3 b^{2} x^{4} \ln \left(f\right)^{2}+6 b \,x^{2} \ln \left(f\right)-6\right) f^{b \,x^{2}+a}}{2 \ln \left(f\right)^{4} b^{4}}$	52
risch	$\frac{\left(b^{3} x^{6} \ln \left(f\right)^{3}-3 b^{2} x^{4} \ln \left(f\right)^{2}+6 b \,x^{2} \ln \left(f\right)-6\right) f^{b \,x^{2}+a}}{2 \ln \left(f\right)^{4} b^{4}}$	52
orering	$\frac{\left(b^{3} x^{6} \ln \left(f\right)^{3}-3 b^{2} x^{4} \ln \left(f\right)^{2}+6 b \,x^{2} \ln \left(f\right)-6\right) f^{b \,x^{2}+a}}{2 \ln \left(f\right)^{4} b^{4}}$	52
meijerg	$\frac{f^{a} \left(6-\frac{\left(-4 b^{3} x^{6} \ln \left(f\right)^{3}+12 b^{2} x^{4} \ln \left(f\right)^{2}-24 b \,x^{2} \ln \left(f\right)+24\right) {\mathrm e}^{b \,x^{2} \ln \left(f\right)}}{4}\right)}{2 b^{4} \ln \left(f\right)^{4}}$	59
parallelrisch	$\frac{f^{b \,x^{2}+a} x^{6} \ln \left(f\right)^{3} b^{3}-3 f^{b \,x^{2}+a} x^{4} \ln \left(f\right)^{2} b^{2}+6 f^{b \,x^{2}+a} x^{2} \ln \left(f\right) b-6 f^{b \,x^{2}+a}}{2 \ln \left(f\right)^{4} b^{4}}$	80
norman	$-\frac{3 \,{\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f\right)}}{\ln \left(f\right)^{4} b^{4}}+\frac{x^{6} {\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f\right)}}{2 b \ln \left(f\right)}+\frac{3 x^{2} {\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f\right)}}{\ln \left(f\right)^{3} b^{3}}-\frac{3 x^{4} {\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f\right)}}{2 \ln \left(f\right)^{2} b^{2}}$	91

input
```
int(f^(b*x^2+a)*x^7,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^3*x^6*ln(f)^3-3*b^2*x^4*ln(f)^2+6*b*x^2*ln(f)-6)*f^(b*x^2+a)/ln(f)^
4/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int f^{a+bx^2}x^7\,dx=\frac{\left(b^3x^6\log(f)^3-3\,b^2x^4\log(f)^2+6\,bx^2\log(f)-6\right)f^{bx^2+a}}{2\,b^4\log(f)^4}$$

input
```
integrate(f^(b*x^2+a)*x^7,x, algorithm="fricas")
```

output
```
1/2*(b^3*x^6*log(f)^3 - 3*b^2*x^4*log(f)^2 + 6*b*x^2*log(f) - 6)*f^(b*x^2
+ a)/(b^4*log(f)^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int f^{a+bx^2}x^7\,dx=\begin{cases}\frac{f^{a+bx^2}\left(b^3x^6\log(f)^3-3b^2x^4\log(f)^2+6bx^2\log(f)-6\right)}{2b^4\log(f)^4} & \text{for } b^4\log(f)^4\neq 0\\ \frac{x^8}{8} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x**2+a)*x**7,x)
```

output
```
Piecewise((f**(a + b*x**2)*(b**3*x**6*log(f)**3 - 3*b**2*x**4*log(f)**2 +
6*b*x**2*log(f) - 6)/(2*b**4*log(f)**4), Ne(b**4*log(f)**4, 0)), (x**8/8,
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int f^{a+bx^2}x^7\,dx = \frac{\left(b^3f^ax^6\log\left(f\right)^3 - 3\,b^2f^ax^4\log\left(f\right)^2 + 6\,bf^ax^2\log\left(f\right) - 6\,f^a\right)f^{bx^2}}{2\,b^4\log\left(f\right)^4}$$

input
```
integrate(f^(b*x^2+a)*x^7,x, algorithm="maxima")
```

output
```
1/2*(b^3*f^a*x^6*log(f)^3 - 3*b^2*f^a*x^4*log(f)^2 + 6*b*f^a*x^2*log(f) -
6*f^a)*f^(b*x^2)/(b^4*log(f)^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.64

$$\int f^{a+bx^2}x^7\,dx = \frac{\left(b^3x^6\log\left(f\right)^3 - 3\,b^2x^4\log\left(f\right)^2 + 6\,bx^2\log\left(f\right) - 6\right)e^{\left(bx^2\log(f) + a\log(f)\right)}}{2\,b^4\log\left(f\right)^4}$$

input
```
integrate(f^(b*x^2+a)*x^7,x, algorithm="giac")
```

output
```
1/2*(b^3*x^6*log(f)^3 - 3*b^2*x^4*log(f)^2 + 6*b*x^2*log(f) - 6)*e^(b*x^2*
log(f) + a*log(f))/(b^4*log(f)^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int f^{a+bx^2}x^7\,dx = -\frac{f^{b\,x^2+a}\,\left(-\frac{b^3\,x^6\,\ln(f)^3}{2} + \frac{3\,b^2\,x^4\,\ln(f)^2}{2} - 3\,b\,x^2\,\ln\left(f\right) + 3\right)}{b^4\,\ln\left(f\right)^4}$$

input
```
int(f^(a + b*x^2)*x^7,x)
```

output
```
-(f^(a + b*x^2)*((3*b^2*x^4*log(f)^2)/2 - (b^3*x^6*log(f)^3)/2 - 3*b*x^2*l
og(f) + 3))/(b^4*log(f)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int f^{a+bx^2}x^7\,dx = \frac{f^{b\,x^2+a}\left(\log(f)^3\,b^3x^6 - 3\log(f)^2\,b^2x^4 + 6\log(f)\,b\,x^2 - 6\right)}{2\log(f)^4\,b^4}$$

input
```
int(f^(b*x^2+a)*x^7,x)
```

output
```
(f**(a + b*x**2)*(log(f)**3*b**3*x**6 - 3*log(f)**2*b**2*x**4 + 6*log(f)*b
*x**2 - 6))/(2*log(f)**4*b**4)
```

3.10 $\int f^{a+bx^2}x^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int f^{a+bx^2}x^5\,dx=\frac{f^{a+bx^2}}{b^3\log^3(f)}-\frac{f^{a+bx^2}x^2}{b^2\log^2(f)}+\frac{f^{a+bx^2}x^4}{2b\log(f)}$$

output
```
f^(b*x^2+a)/b^3/ln(f)^3-f^(b*x^2+a)*x^2/b^2/ln(f)^2+1/2*f^(b*x^2+a)*x^4/b/
ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

$$\int f^{a+bx^2}x^5\,dx=\frac{f^{a+bx^2}\left(2-2bx^2\log(f)+b^2x^4\log^2(f)\right)}{2b^3\log^3(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^5,x]
```

output
```
(f^(a + b*x^2)*(2 - 2*b*x^2*Log[f] + b^2*x^4*Log[f]^2))/(2*b^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 f^{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^4 f^{a+bx^2}}{2b\log(f)}-\frac{2\int f^{bx^2+a}x^3dx}{b\log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^4 f^{a+bx^2}}{2b\log(f)}-\frac{2\left(\frac{x^2 f^{a+bx^2}}{2b\log(f)}-\frac{\int f^{bx^2+a}xdx}{b\log(f)}\right)}{b\log(f)}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{x^4 f^{a+bx^2}}{2b\log(f)}-\frac{2\left(\frac{x^2 f^{a+bx^2}}{2b\log(f)}-\frac{f^{a+bx^2}}{2b^2\log^2(f)}\right)}{b\log(f)}$$

input
```
Int[f^(a + b*x^2)*x^5,x]
```

output
```
(f^(a + b*x^2)*x^4)/(2*b*Log[f]) - (2*(-1/2*f^(a + b*x^2)/(b^2*Log[f]^2) +
 (f^(a + b*x^2)*x^2)/(2*b*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{\left(b^{2} x^{4} \ln \left(f \right)^{2}-2 b \,x^{2} \ln \left(f \right)+2\right) f^{b \,x^{2}+a}}{2 \ln \left(f \right)^{3} b^{3}}$	40
risch	$\frac{\left(b^{2} x^{4} \ln \left(f \right)^{2}-2 b \,x^{2} \ln \left(f \right)+2\right) f^{b \,x^{2}+a}}{2 \ln \left(f \right)^{3} b^{3}}$	40
orering	$\frac{\left(b^{2} x^{4} \ln \left(f \right)^{2}-2 b \,x^{2} \ln \left(f \right)+2\right) f^{b \,x^{2}+a}}{2 \ln \left(f \right)^{3} b^{3}}$	40
meijerg	$-\frac{f^{a} \left(2-\frac{\left(3 b^{2} x^{4} \ln \left(f \right)^{2}-6 b \,x^{2} \ln \left(f \right)+6\right) {\mathrm e}^{b \,x^{2} \ln \left(f \right)}}{3}\right)}{2 b^{3} \ln \left(f \right)^{3}}$	47
parallelrisch	$\frac{f^{b \,x^{2}+a} x^{4} \ln \left(f \right)^{2} b^{2}-2 f^{b \,x^{2}+a} x^{2} \ln \left(f \right) b +2 f^{b \,x^{2}+a}}{2 \ln \left(f \right)^{3} b^{3}}$	59
norman	$\frac{{\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f \right)}}{\ln \left(f \right)^{3} b^{3}}+\frac{x^{4} {\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f \right)}}{2 b \ln \left(f \right)}-\frac{x^{2} {\mathrm e}^{\left(b \,x^{2}+a \right) \ln \left(f \right)}}{\ln \left(f \right)^{2} b^{2}}$	67

input
```
int(f^(b*x^2+a)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*x^4*ln(f)^2-2*b*x^2*ln(f)+2)*f^(b*x^2+a)/ln(f)^3/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int f^{a+bx^2}x^5\,dx=\frac{\left(b^2x^4\log\left(f\right)^2-2\,bx^2\log\left(f\right)+2\right)f^{bx^2+a}}{2\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(b*x^2+a)*x^5,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^4*log(f)^2 - 2*b*x^2*log(f) + 2)*f^(b*x^2 + a)/(b^3*log(f)^3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int f^{a+bx^2}x^5\,dx=\begin{cases}\frac{f^{a+bx^2}\left(b^2x^4\log\left(f\right)^2-2bx^2\log\left(f\right)+2\right)}{2b^3\log\left(f\right)^3} & \text{for } b^3\log\left(f\right)^3\neq 0\\ \frac{x^6}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x**2+a)*x**5,x)
```

output
```
Piecewise((f**(a + b*x**2)*(b**2*x**4*log(f)**2 - 2*b*x**2*log(f) + 2)/(2*
b**3*log(f)**3), Ne(b**3*log(f)**3, 0)), (x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int f^{a+bx^2}x^5\,dx=\frac{\left(b^2f^ax^4\log\left(f\right)^2-2\,bf^ax^2\log\left(f\right)+2\,f^a\right)f^{bx^2}}{2\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(b*x^2+a)*x^5,x, algorithm="maxima")
```

output
```
1/2*(b^2*f^a*x^4*log(f)^2 - 2*b*f^a*x^2*log(f) + 2*f^a)*f^(b*x^2)/(b^3*log
(f)^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int f^{a+bx^2}x^5\,dx=\frac{\left(b^2x^4\log\left(f\right)^2-2\,bx^2\log\left(f\right)+2\right)e^{\left(bx^2\log\left(f\right)+a\log\left(f\right)\right)}}{2\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(b*x^2+a)*x^5,x, algorithm="giac")
```

output
```
1/2*(b^2*x^4*log(f)^2 - 2*b*x^2*log(f) + 2)*e^(b*x^2*log(f) + a*log(f))/(b
^3*log(f)^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int f^{a+bx^2}x^5\,dx=\frac{f^{b\,x^2+a}\,\left(\frac{b^2\,x^4\,\ln\left(f\right)^2}{2}-b\,x^2\,\ln\left(f\right)+1\right)}{b^3\,\ln\left(f\right)^3}$$

input
```
int(f^(a + b*x^2)*x^5,x)
```

output
```
(f^(a + b*x^2)*((b^2*x^4*log(f)^2)/2 - b*x^2*log(f) + 1))/(b^3*log(f)^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int f^{a+bx^2}x^5\,dx = \frac{f^{b\,x^2+a}\left(\log(f)^2\,b^2x^4-2\log(f)\,b\,x^2+2\right)}{2\log\left(f\right)^3 b^3}$$

input
```
int(f^(b*x^2+a)*x^5,x)
```

output
```
(f**(a + b*x**2)*(log(f)**2*b**2*x**4 - 2*log(f)*b*x**2 + 2))/(2*log(f)**3
*b**3)
```

3.11 $\int f^{a+bx^2} x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int f^{a+bx^2} x^3 \, dx = -\frac{f^{a+bx^2}}{2b^2 \log^2(f)} + \frac{f^{a+bx^2} x^2}{2b \log(f)}$$

output
```
-1/2*f^(b*x^2+a)/b^2/ln(f)^2+1/2*f^(b*x^2+a)*x^2/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int f^{a+bx^2} x^3 \, dx = \frac{f^{a+bx^2} (-1 + bx^2 \log(f))}{2b^2 \log^2(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^3,x]
```

output
```
(f^(a + b*x^2)*(-1 + b*x^2*Log[f]))/(2*b^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^2 f^{a+bx^2}}{2b \log(f)} - \frac{\int f^{bx^2+a} x dx}{b \log(f)}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{x^2 f^{a+bx^2}}{2b \log(f)} - \frac{f^{a+bx^2}}{2b^2 \log^2(f)}$$

input
```
Int[f^(a + b*x^2)*x^3,x]
```

output
```
-1/2*f^(a + b*x^2)/(b^2*Log[f]^2) + (f^(a + b*x^2)*x^2)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(b\,x^{2} \ln \left(f \right)-1\right) f^{b\,x^{2}+a}}{2 \ln \left(f \right)^{2} b^{2}}$	28
risch	$\frac{\left(b\,x^{2} \ln \left(f \right)-1\right) f^{b\,x^{2}+a}}{2 \ln \left(f \right)^{2} b^{2}}$	28
orering	$\frac{\left(b\,x^{2} \ln \left(f \right)-1\right) f^{b\,x^{2}+a}}{2 \ln \left(f \right)^{2} b^{2}}$	28
meijerg	$\frac{f^{a} \left(1-\frac{\left(-2 b\,x^{2} \ln \left(f \right)+2\right) {\mathrm e}^{b\,x^{2} \ln \left(f \right)}}{2}\right)}{2 b^{2} \ln \left(f \right)^{2}}$	35
parallelrisch	$\frac{f^{b\,x^{2}+a} x^{2} \ln \left(f \right) b -f^{b\,x^{2}+a}}{2 \ln \left(f \right)^{2} b^{2}}$	38
norman	$-\frac{{\mathrm e}^{\left(b\,x^{2}+a \right) \ln \left(f \right)}}{2 \ln \left(f \right)^{2} b^{2}}+\frac{x^{2} {\mathrm e}^{\left(b\,x^{2}+a \right) \ln \left(f \right)}}{2 b \ln \left(f \right)}$	45

input
```
int(f^(b*x^2+a)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2*ln(f)-1)*f^(b*x^2+a)/ln(f)^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int f^{a+bx^2}x^3\,dx = \frac{(bx^2\log(f)-1)f^{bx^2+a}}{2\,b^2\log(f)^2}$$

input
```
integrate(f^(b*x^2+a)*x^3,x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(f) - 1)*f^(b*x^2 + a)/(b^2*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int f^{a+bx^2}x^3\,dx = \begin{cases} \frac{f^{a+bx^2}(bx^2\log(f)-1)}{2b^2\log(f)^2} & \text{for } b^2\log(f)^2 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x**2+a)*x**3,x)
```

output
```
Piecewise((f**(a + b*x**2)*(b*x**2*log(f) - 1)/(2*b**2*log(f)**2), Ne(b**2
*log(f)**2, 0)), (x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int f^{a+bx^2}x^3\,dx = \frac{(bf^ax^2\log(f)-f^a)f^{bx^2}}{2\,b^2\log(f)^2}$$

input
```
integrate(f^(b*x^2+a)*x^3,x, algorithm="maxima")
```

output
```
1/2*(b*f^a*x^2*log(f) - f^a)*f^(b*x^2)/(b^2*log(f)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 689, normalized size of antiderivative = 15.66

$$\int f^{a+bx^2}x^3\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x^2+a)*x^3,x, algorithm="giac")
```

output
```
1/2*(2*((pi*b*x^2*sgn(f) - pi*b*x^2)*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*l
og(abs(f)))/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*
b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) + (pi^2*b^2*sgn(f) - pi^2*
b^2 + 2*b^2*log(abs(f))^2)*(b*x^2*log(abs(f)) - 1)/((pi^2*b^2*sgn(f) - pi^
2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log
(abs(f)))^2))*cos(-1/2*pi*b*x^2*sgn(f) + 1/2*pi*b*x^2 - 1/2*pi*a*sgn(f) +
1/2*pi*a) + ((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)*(pi*b*x^2*
sgn(f) - pi*b*x^2)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 +
 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) - 4*(pi*b^2*log(abs
(f))*sgn(f) - pi*b^2*log(abs(f)))*(b*x^2*log(abs(f)) - 1)/((pi^2*b^2*sgn(f
) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*
b^2*log(abs(f)))^2))*sin(-1/2*pi*b*x^2*sgn(f) + 1/2*pi*b*x^2 - 1/2*pi*a*sg
n(f) + 1/2*pi*a))*e^(b*x^2*log(abs(f)) + a*log(abs(f))) - 1/4*I*((pi*b*x^2
*sgn(f) - pi*b*x^2 - 2*I*b*x^2*log(abs(f)) + 2*I)*e^(1/2*I*pi*b*x^2*sgn(f)
 - 1/2*I*pi*b*x^2 + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(pi^2*b^2*sgn(f) + 2*I
*pi*b^2*log(abs(f))*sgn(f) - pi^2*b^2 - 2*I*pi*b^2*log(abs(f)) + 2*b^2*log
(abs(f))^2) + (pi*b*x^2*sgn(f) - pi*b*x^2 + 2*I*b*x^2*log(abs(f)) - 2*I)*e
^(-1/2*I*pi*b*x^2*sgn(f) + 1/2*I*pi*b*x^2 - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a
)/(pi^2*b^2*sgn(f) - 2*I*pi*b^2*log(abs(f))*sgn(f) - pi^2*b^2 + 2*I*pi*b^2
*log(abs(f)) + 2*b^2*log(abs(f))^2))*e^(b*x^2*log(abs(f)) + a*log(abs(f...
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int f^{a+bx^2}x^3\,dx=\frac{f^{b\,x^2+a}\,\left(\frac{b\,x^2\,\ln(f)}{2}-\frac{1}{2}\right)}{b^2\,\ln\left(f\right)^2}$$

input
```
int(f^(a + b*x^2)*x^3,x)
```

output
```
(f^(a + b*x^2)*((b*x^2*log(f))/2 - 1/2))/(b^2*log(f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int f^{a+bx^2}x^3\,dx=\frac{f^{b\,x^2+a}(\log(f)\,b\,x^2-1)}{2\log\left(f\right)^2b^2}$$

input
```
int(f^(b*x^2+a)*x^3,x)
```

output
```
(f**(a + b*x**2)*(log(f)*b*x**2 - 1))/(2*log(f)**2*b**2)
```

3.12 $\int f^{a+bx^2} x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int f^{a+bx^2} x \, dx = \frac{f^{a+bx^2}}{2b\log(f)}$$

output
```
1/2*f^(b*x^2+a)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^2} x \, dx = \frac{f^{a+bx^2}}{2b\log(f)}$$

input
```
Integrate[f^(a + b*x^2)*x,x]
```

output
```
f^(a + b*x^2)/(2*b*Log[f])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{a+bx^2} dx$$
$$\Big\downarrow \; 2638$$
$$\frac{f^{a+bx^2}}{2b\log(f)}$$

input
```
Int[f^(a + b*x^2)*x,x]
```

output
```
f^(a + b*x^2)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{f^{b\,x^2+a}}{2b\ln(f)}$	19
derivativedivides	$\frac{f^{b\,x^2+a}}{2b\ln(f)}$	19
default	$\frac{f^{b\,x^2+a}}{2b\ln(f)}$	19
risch	$\frac{f^{b\,x^2+a}}{2b\ln(f)}$	19
parallelrisch	$\frac{f^{b\,x^2+a}}{2b\ln(f)}$	19
orering	$\frac{f^{b\,x^2+a}}{2b\ln(f)}$	19
norman	$\frac{\mathrm{e}^{\left(b\,x^2+a\right)\ln(f)}}{2b\ln(f)}$	21
meijerg	$-\frac{f^a\left(1-\mathrm{e}^{b\,x^2\ln(f)}\right)}{2b\ln(f)}$	25

input
```
int(f^(b*x^2+a)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^(b*x^2+a)/b/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2}x\,dx = \frac{f^{bx^2+a}}{2\,b\log\left(f\right)}$$

input
```
integrate(f^(b*x^2+a)*x,x, algorithm="fricas")
```

output
```
1/2*f^(b*x^2 + a)/(b*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int f^{a+bx^2}x\,dx=\begin{cases}\frac{f^{a+bx^2}}{2b\log(f)} & \text{for } b\log(f)\neq 0\\ \frac{x^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x**2+a)*x,x)
```

output
```
Piecewise((f**(a + b*x**2)/(2*b*log(f)), Ne(b*log(f), 0)), (x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2}x\,dx=\frac{f^{bx^2+a}}{2\,b\log(f)}$$

input
```
integrate(f^(b*x^2+a)*x,x, algorithm="maxima")
```

output
```
1/2*f^(b*x^2 + a)/(b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2}x\,dx=\frac{f^{bx^2+a}}{2\,b\log(f)}$$

input
```
integrate(f^(b*x^2+a)*x,x, algorithm="giac")
```

output
```
1/2*f^(b*x^2 + a)/(b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2} x\,dx = \frac{f^{b\,x^2+a}}{2\,b\,\ln\left(f\right)}$$

input
```
int(f^(a + b*x^2)*x,x)
```

output
```
f^(a + b*x^2)/(2*b*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2} x\,dx = \frac{f^{b\,x^2+a}}{2\log\left(f\right)b}$$

input
```
int(f^(b*x^2+a)*x,x)
```

output
```
f**(a + b*x**2)/(2*log(f)*b)
```

3.13 $\int \frac{f^{a+bx^2}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{f^{a+bx^2}}{x}\,dx = \frac{1}{2}f^a \operatorname{ExpIntegralEi}\left(bx^2\log(f)\right)$$

output
```
1/2*f^a*Ei(b*x^2*ln(f))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x}\,dx = \frac{1}{2}f^a \operatorname{ExpIntegralEi}\left(bx^2\log(f)\right)$$

input
```
Integrate[f^(a + b*x^2)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^2*Log[f]])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x} \, dx$$
$$\Big\downarrow \text{ 2639}$$
$$\frac{1}{2} f^a \text{ExpIntegralEi}\left(bx^2 \log(f)\right)$$

input
```
Int[f^(a + b*x^2)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^2*Log[f]])/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$-\frac{f^{a} \operatorname{expIntegral}_{1}\left(-b \,x^{2} \ln \left(f \right)\right)}{2}$	16
meijerg	$\frac{f^{a} \left(2 \ln \left(x \right)+\ln \left(-b \right)+\ln \left(\ln \left(f \right)\right)-\ln \left(-b \,x^{2} \ln \left(f \right)\right)-\operatorname{expIntegral}_{1}\left(-b \,x^{2} \ln \left(f \right)\right)\right)}{2}$	41

input
```
int(f^(b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*Ei(1,-b*x^2*ln(f))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^2}}{x}\,dx = \frac{1}{2}\,f^a\mathrm{Ei}\left(bx^2\log\left(f\right)\right)$$

input
```
integrate(f^(b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/2*f^a*Ei(b*x^2*log(f))
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x}\,dx = \int \frac{f^{a+bx^2}}{x}\,dx$$

input
```
integrate(f**(b*x**2+a)/x,x)
```

output
```
Integral(f**(a + b*x**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^2}}{x}\,dx = \frac{1}{2}\,f^a\text{Ei}\left(bx^2\log\left(f\right)\right)$$

input
```
integrate(f^(b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/2*f^a*Ei(b*x^2*log(f))
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x}\,dx = \int \frac{f^{bx^2+a}}{x}\,dx$$

input
```
integrate(f^(b*x^2+a)/x,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^2}}{x}\,dx = \frac{f^a\,\mathrm{ei}\left(b\,x^2\,\ln\left(f\right)\right)}{2}$$

input
```
int(f^(a + b*x^2)/x,x)
```

output
```
(f^a*ei(b*x^2*log(f)))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^2}}{x}\,dx = \frac{f^a ei(\log(f)\,b\,x^2)}{2}$$

input
```
int(f^(b*x^2+a)/x,x)
```

output
```
(f**a*ei(log(f)*b*x**2))/2
```

3.14 $\int \frac{f^{a+bx^2}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{f^{a+bx^2}}{x^3} dx = -\frac{f^{a+bx^2}}{2x^2} + \frac{1}{2} b f^a \text{ExpIntegralEi}\left(bx^2 \log(f)\right) \log(f)$$

output
```
-1/2*f^(b*x^2+a)/x^2+1/2*b*f^a*Ei(b*x^2*ln(f))*ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{f^{a+bx^2}}{x^3} dx = \frac{1}{2} f^a \left(-\frac{f^{bx^2}}{x^2} + b \text{ExpIntegralEi}\left(bx^2 \log(f)\right) \log(f)\right)$$

input
```
Integrate[f^(a + b*x^2)/x^3,x]
```

output
```
(f^a*(-(f^(b*x^2)/x^2) + b*ExpIntegralEi[b*x^2*Log[f]]*Log[f]))/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^3} dx$$

$$\Big\downarrow \text{2643}$$

$$b\log(f)\int \frac{f^{bx^2+a}}{x}dx - \frac{f^{a+bx^2}}{2x^2}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2}bf^a\log(f)\,\text{ExpIntegralEi}\left(bx^2\log(f)\right) - \frac{f^{a+bx^2}}{2x^2}$$

input
```
Int[f^(a + b*x^2)/x^3,x]
```

output
```
-1/2*f^(a + b*x^2)/x^2 + (b*f^a*ExpIntegralEi[b*x^2*Log[f]]*Log[f])/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{f^{a} f^{b\,x^{2}}}{2x^{2}}-\frac{f^{a} \ln\left(f\right) b \,\mathrm{expIntegral}_{1}\left(-b\,x^{2} \ln\left(f\right)\right)}{2}$
meijerg	$-\frac{f^{a} b \ln\left(f\right) \left(\frac{1}{b\,x^{2} \ln\left(f\right)}+1-2\ln\left(x\right)-\ln\left(-b\right)-\ln\left(\ln\left(f\right)\right)-\frac{2b\,x^{2} \ln\left(f\right)+2}{2b\,x^{2} \ln\left(f\right)}+\frac{\mathrm{e}^{b\,x^{2} \ln\left(f\right)}}{b\,x^{2} \ln\left(f\right)}+\ln\left(-b\,x^{2} \ln\left(f\right)\right)+\mathrm{expIntegral}_{1}\left(-b\,x^{2} \ln\left(f\right)\right)\right)}{2}$

input
```
int(f^(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/x^2*f^(b*x^2)-1/2*f^a*ln(f)*b*Ei(1,-b*x^2*ln(f))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x^3}\,dx = \frac{bf^{a}x^{2}\mathrm{Ei}\left(bx^{2}\log\left(f\right)\right)\log\left(f\right) - f^{bx^{2}+a}}{2\,x^{2}}$$

input
```
integrate(f^(b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(b*f^a*x^2*Ei(b*x^2*log(f))*log(f) - f^(b*x^2 + a))/x^2
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^3}\,dx = \int \frac{f^{a+bx^2}}{x^3}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**3,x)
```

output
```
Integral(f**(a + b*x**2)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{f^{a+bx^2}}{x^3}\,dx = \frac{1}{2}bf^a\Gamma\left(-1, -bx^2\log\left(f\right)\right)\log\left(f\right)$$

input
```
integrate(f^(b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*b*f^a*gamma(-1, -b*x^2*log(f))*log(f)
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^3}\,dx = \int \frac{f^{bx^2+a}}{x^3}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{f^{a+bx^2}}{x^3}\,dx = -\frac{f^a\left(f^{b\,x^2}+b\,x^2\,\ln\left(f\right)\,\mathrm{expint}\left(-b\,x^2\,\ln\left(f\right)\right)\right)}{2\,x^2}$$

input
```
int(f^(a + b*x^2)/x^3,x)
```

output
```
-(f^a*(f^(b*x^2) + b*x^2*log(f)*expint(-b*x^2*log(f))))/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{f^{a+bx^2}}{x^3} \, dx = \frac{f^a \left(ei(\log(f) \, b \, x^2) \log(f) \, b \, x^2 - f^{b \, x^2} \right)}{2x^2}$$

input
```
int(f^(b*x^2+a)/x^3,x)
```

output
```
(f**a*(ei(log(f)*b*x**2)*log(f)*b*x**2 - f**(b*x**2)))/(2*x**2)
```

3.15 $\int \frac{f^{a+bx^2}}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{f^{a+bx^2}}{x^5} dx = -\frac{f^{a+bx^2}}{4x^4} - \frac{bf^{a+bx^2}\log(f)}{4x^2} + \frac{1}{4}b^2f^a \operatorname{ExpIntegralEi}\left(bx^2\log(f)\right)\log^2(f)$$

output
```
-1/4*f^(b*x^2+a)/x^4-1/4*b*f^(b*x^2+a)*ln(f)/x^2+1/4*b^2*f^a*Ei(b*x^2*ln(f
))*ln(f)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{f^{a+bx^2}}{x^5} dx = \frac{f^a\left(b^2x^4 \operatorname{ExpIntegralEi}\left(bx^2\log(f)\right)\log^2(f) - f^{bx^2}(1+bx^2\log(f))\right)}{4x^4}$$

input
```
Integrate[f^(a + b*x^2)/x^5,x]
```

output
```
(f^a*(b^2*x^4*ExpIntegralEi[b*x^2*Log[f]]*Log[f]^2 - f^(b*x^2)*(1 + b*x^2*
Log[f])))/(4*x^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^5} \, dx$$
$$\Big\downarrow \text{2643}$$
$$\frac{1}{2} b \log(f) \int \frac{f^{bx^2+a}}{x^3} dx - \frac{f^{a+bx^2}}{4x^4}$$
$$\Big\downarrow \text{2643}$$
$$\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{bx^2+a}}{x} dx - \frac{f^{a+bx^2}}{2x^2} \right) - \frac{f^{a+bx^2}}{4x^4}$$
$$\Big\downarrow \text{2639}$$
$$\frac{1}{2} b \log(f) \left(\frac{1}{2} b f^a \log(f) \text{ExpIntegralEi}\left(bx^2 \log(f)\right) - \frac{f^{a+bx^2}}{2x^2} \right) - \frac{f^{a+bx^2}}{4x^4}$$

input
```
Int[f^(a + b*x^2)/x^5,x]
```

output
```
-1/4*f^(a + b*x^2)/x^4 + (b*Log[f]*(-1/2*f^(a + b*x^2)/x^2 + (b*f^a*ExpInt
egralEi[b*x^2*Log[f]]*Log[f])/2))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result
risch	$-\frac{f^{a}\left(\ln(f)^{2} \operatorname{expIntegral}_{1}\left(-b\,x^{2} \ln(f)\right) b^{2} x^{4}+\ln(f) f^{b\,x^{2}} b\,x^{2}+f^{b\,x^{2}}\right)}{4 x^{4}}$
meijerg	$\frac{f^{a} b^{2} \ln(f)^{2}\left(-\frac{1}{2 b^{2} x^{4} \ln(f)^{2}}-\frac{1}{b\,x^{2} \ln(f)}-\frac{3}{4}+\ln(x)+\frac{\ln(-b)}{2}+\frac{\ln(\ln(f))}{2}+\frac{9 b^{2} x^{4} \ln(f)^{2}+12 b\,x^{2} \ln(f)+6}{12 b^{2} x^{4} \ln(f)^{2}}-\frac{\left(3 b\,x^{2} \ln(f)+3\right) \mathrm{e}^{b\,x^{2} \ln(f)}}{6 b^{2} x^{4} \ln(f)^{2}}-\ln\left(-b\right.}{2}$

input
```
int(f^(b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*f^a*(ln(f)^2*Ei(1,-b*x^2*ln(f))*b^2*x^4+ln(f)*f^(b*x^2)*b*x^2+f^(b*x^
2))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{f^{a+bx^2}}{x^5}\,dx = \frac{b^2 f^a x^4 \mathrm{Ei}\left(bx^2\log\left(f\right)\right)\log\left(f\right)^2 - \left(bx^2\log\left(f\right)+1\right)f^{bx^2+a}}{4\,x^4}$$

input
```
integrate(f^(b*x^2+a)/x^5,x, algorithm="fricas")
```

output
```
1/4*(b^2*f^a*x^4*Ei(b*x^2*log(f))*log(f)^2 - (b*x^2*log(f) + 1)*f^(b*x^2 +
 a))/x^4
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^5}\,dx = \int \frac{f^{a+bx^2}}{x^5}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**5,x)
```

output
```
Integral(f**(a + b*x**2)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.38

$$\int \frac{f^{a+bx^2}}{x^5}\,dx = -\frac{1}{2}\,b^2 f^a \Gamma\left(-2, -bx^2\log\left(f\right)\right)\log\left(f\right)^2$$

input
```
integrate(f^(b*x^2+a)/x^5,x, algorithm="maxima")
```

output
```
-1/2*b^2*f^a*gamma(-2, -b*x^2*log(f))*log(f)^2
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^5}\,dx = \int \frac{f^{bx^2+a}}{x^5}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^5,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{f^{a+bx^2}}{x^5} \, dx = -\frac{b^2 \, f^a \ln(f)^2 \left(f^{b\,x^2} \left(\frac{1}{2\,b\,x^2\,\ln(f)} + \frac{1}{2\,b^2\,x^4\ln(f)^2}\right) + \frac{\mathrm{expint}(-b\,x^2\,\ln(f))}{2}\right)}{2}$$

input
```
int(f^(a + b*x^2)/x^5,x)
```

output
```
-(b^2*f^a*log(f)^2*(f^(b*x^2)*(1/(2*b*x^2*log(f)) + 1/(2*b^2*x^4*log(f)^2)
) + expint(-b*x^2*log(f))/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+bx^2}}{x^5} \, dx = \frac{f^a \left(ei(\log(f)\,b\,x^2)\log(f)^2\,b^2x^4 - f^{b\,x^2}\log(f)\,b\,x^2 - f^{b\,x^2}\right)}{4x^4}$$

input
```
int(f^(b*x^2+a)/x^5,x)
```

output
```
(f**a*(ei(log(f)*b*x**2)*log(f)**2*b**2*x**4 - f**(b*x**2)*log(f)*b*x**2 -
 f**(b*x**2)))/(4*x**4)
```

3.16 $\int \frac{f^{a+bx^2}}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{f^{a+bx^2}}{x^7}\,dx = -\frac{f^{a+bx^2}}{6x^6} - \frac{bf^{a+bx^2}\log(f)}{12x^4} - \frac{b^2f^{a+bx^2}\log^2(f)}{12x^2} + \frac{1}{12}b^3f^a\,\text{ExpIntegralEi}\left(bx^2\log(f)\right)\log^3(f)$$

output
```
-1/6*f^(b*x^2+a)/x^6-1/12*b*f^(b*x^2+a)*ln(f)/x^4-1/12*b^2*f^(b*x^2+a)*ln(
f)^2/x^2+1/12*b^3*f^a*Ei(b*x^2*ln(f))*ln(f)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+bx^2}}{x^7}\,dx = \frac{f^a\left(b^3x^6\,\text{ExpIntegralEi}\,(bx^2\log(f))\log^3(f) - f^{bx^2}\left(2+bx^2\log(f)+b^2x^4\log^2(f)\right)\right)}{12x^6}$$

input
```
Integrate[f^(a + b*x^2)/x^7,x]
```

output
```
(f^a*(b^3*x^6*ExpIntegralEi[b*x^2*Log[f]]*Log[f]^3 - f^(b*x^2)*(2 + b*x^2*
Log[f] + b^2*x^4*Log[f]^2)))/(12*x^6)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^7}dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(f)\int \frac{f^{bx^2+a}}{x^5}dx-\frac{f^{a+bx^2}}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(f)\left(\frac{1}{2}b\log(f)\int \frac{f^{bx^2+a}}{x^3}dx-\frac{f^{a+bx^2}}{4x^4}\right)-\frac{f^{a+bx^2}}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(f)\left(\frac{1}{2}b\log(f)\left(b\log(f)\int \frac{f^{bx^2+a}}{x}dx-\frac{f^{a+bx^2}}{2x^2}\right)-\frac{f^{a+bx^2}}{4x^4}\right)-\frac{f^{a+bx^2}}{6x^6}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3}b\log(f)\left(\frac{1}{2}b\log(f)\left(\frac{1}{2}bf^a\log(f)\text{ExpIntegralEi}\left(bx^2\log(f)\right)-\frac{f^{a+bx^2}}{2x^2}\right)-\frac{f^{a+bx^2}}{4x^4}\right)-\frac{f^{a+bx^2}}{6x^6}$$

input
```
Int[f^(a + b*x^2)/x^7,x]
```

output
```
-1/6*f^(a + b*x^2)/x^6 + (b*Log[f]*(-1/4*f^(a + b*x^2)/x^4 + (b*Log[f]*(-1
/2*f^(a + b*x^2)/x^2 + (b*f^a*ExpIntegralEi[b*x^2*Log[f]]*Log[f])/2))/2))/
3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{f^{a}\left(\ln(f)^{3}\operatorname{expIntegral}_{1}\left(-b\,x^{2}\ln(f)\right)b^{3}x^{6}+\ln(f)^{2}f^{b\,x^{2}}b^{2}x^{4}+\ln(f)f^{b\,x^{2}}b\,x^{2}+2f^{b\,x^{2}}\right)}{12x^{6}}$
meijerg	$-\frac{f^{a}b^{3}\ln(f)^{3}\left(\frac{1}{3b^{3}x^{6}\ln(f)^{3}}+\frac{1}{2b^{2}x^{4}\ln(f)^{2}}+\frac{1}{2b\,x^{2}\ln(f)}+\frac{11}{36}-\frac{\ln(x)}{3}-\frac{\ln(-b)}{6}-\frac{\ln(\ln(f))}{6}-\frac{22b^{3}x^{6}\ln(f)^{3}+36b^{2}x^{4}\ln(f)^{2}+36b\,x^{2}\ln(f)+24}{72b^{3}x^{6}\ln(f)^{3}}+\right.}{2}$

input
```
int(f^(b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/12*f^a*(ln(f)^3*Ei(1,-b*x^2*ln(f))*b^3*x^6+ln(f)^2*f^(b*x^2)*b^2*x^4+ln
(f)*f^(b*x^2)*b*x^2+2*f^(b*x^2))/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+bx^2}}{x^7} dx = \frac{b^3 f^a x^6 \text{Ei}(bx^2 \log(f)) \log(f)^3 - \left(b^2 x^4 \log(f)^2 + bx^2 \log(f) + 2\right) f^{bx^2+a}}{12 x^6}$$

input
```
integrate(f^(b*x^2+a)/x^7,x, algorithm="fricas")
```

output
```
1/12*(b^3*f^a*x^6*Ei(b*x^2*log(f))*log(f)^3 - (b^2*x^4*log(f)^2 + b*x^2*lo
g(f) + 2)*f^(b*x^2 + a))/x^6
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^7} dx = \int \frac{f^{a+bx^2}}{x^7} dx$$

input
```
integrate(f**(b*x**2+a)/x**7,x)
```

output
```
Integral(f**(a + b*x**2)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+bx^2}}{x^7} dx = \frac{1}{2} b^3 f^a \Gamma\left(-3, -bx^2 \log(f)\right) \log(f)^3$$

input
```
integrate(f^(b*x^2+a)/x^7,x, algorithm="maxima")
```

output
```
1/2*b^3*f^a*gamma(-3, -b*x^2*log(f))*log(f)^3
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^7}\,dx = \int \frac{f^{bx^2+a}}{x^7}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^7,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{f^{a+bx^2}}{x^7}\,dx = -\frac{b^3\,f^a\ln(f)^3\left(f^{b\,x^2}\left(\frac{1}{6\,b\,x^2\,\ln(f)}+\frac{1}{6\,b^2\,x^4\,\ln(f)^2}+\frac{1}{3\,b^3\,x^6\,\ln(f)^3}\right)+\frac{\mathrm{expint}\left(-b\,x^2\,\ln(f)\right)}{6}\right)}{2}$$

input
```
int(f^(a + b*x^2)/x^7,x)
```

output
```
-(b^3*f^a*log(f)^3*(f^(b*x^2)*(1/(6*b*x^2*log(f)) + 1/(6*b^2*x^4*log(f)^2)
 + 1/(3*b^3*x^6*log(f)^3)) + expint(-b*x^2*log(f))/6))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{f^{a+bx^2}}{x^7}\,dx = \frac{f^a\left(ei(\log(f)\,b\,x^2)\log(f)^3\,b^3x^6 - f^{b\,x^2}\log(f)^2\,b^2x^4 - f^{b\,x^2}\log(f)\,b\,x^2 - 2f^{b\,x^2}\right)}{12x^6}$$

input
```
int(f^(b*x^2+a)/x^7,x)
```

output
```
(f**a*(ei(log(f)*b*x**2)*log(f)**3*b**3*x**6 - f**(b*x**2)*log(f)**2*b**2*
x**4 - f**(b*x**2)*log(f)*b*x**2 - 2*f**(b*x**2)))/(12*x**6)
```

3.17 $\int \frac{f^{a+bx^2}}{x^9} dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{f^{a+bx^2}}{x^9} dx = -\frac{1}{2} b^4 f^a \Gamma\left(-4, -bx^2 \log(f)\right) \log^4(f)$$

output
```
-1/2*f^a/x^8*Ei(5,-b*x^2*ln(f))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x^9} dx = -\frac{1}{2} b^4 f^a \Gamma\left(-4, -bx^2 \log(f)\right) \log^4(f)$$

input
```
Integrate[f^(a + b*x^2)/x^9,x]
```

output
```
-1/2*(b^4*f^a*Gamma[-4, -(b*x^2*Log[f])]*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^9} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{1}{2} b^4 f^a \log^4(f) \Gamma\left(-4, -bx^2 \log(f)\right)$$

input
```
Int[f^(a + b*x^2)/x^9,x]
```

output
```
-1/2*(b^4*f^a*Gamma[-4, -(b*x^2*Log[f])]*Log[f]^4)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(18) = 36.

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.79

method	result
risch	$-\frac{f^{a}\left(\ln(f)^{4} \operatorname{expIntegral}_{1}\left(-b\,x^{2} \ln(f)\right) b^{4} x^{8}+\ln(f)^{3} f^{b\,x^{2}} b^{3} x^{6}+\ln(f)^{2} f^{b\,x^{2}} b^{2} x^{4}+2 \ln(f) f^{b\,x^{2}} b\,x^{2}+6 f^{b\,x^{2}}\right)}{48 x^{8}}$
meijerg	$f^{a} b^{4} \ln(f)^{4}\left(-\frac{1}{4 b^{4} x^{8} \ln(f)^{4}}-\frac{1}{3 b^{3} x^{6} \ln(f)^{3}}-\frac{1}{4 b^{2} x^{4} \ln(f)^{2}}-\frac{1}{6 b\,x^{2} \ln(f)}-\frac{25}{288}+\frac{\ln(x)}{12}+\frac{\ln(-b)}{24}+\frac{\ln(\ln(f))}{24}+\frac{125 b^{4} x^{8} \ln(f)^{4}+240 b^{3} x^{6} \ln(f)^{3}+}{1440 b^{4}}\right.$

input
```
int(f^(b*x^2+a)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/48*f^a*(ln(f)^4*Ei(1,-b*x^2*ln(f))*b^4*x^8+ln(f)^3*f^(b*x^2)*b^3*x^6+ln
(f)^2*f^(b*x^2)*b^2*x^4+2*ln(f)*f^(b*x^2)*b*x^2+6*f^(b*x^2))/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{f^{a+bx^2}}{x^9}\,dx = \frac{b^4 f^a x^8 \mathrm{Ei}(bx^2 \log{(f)}) \log{(f)}^4 - \left(b^3 x^6 \log{(f)}^3 + b^2 x^4 \log{(f)}^2 + 2\,bx^2 \log{(f)} + 6\right) f^{bx^2+a}}{48\,x^8}$$

input
```
integrate(f^(b*x^2+a)/x^9,x, algorithm="fricas")
```

output
```
1/48*(b^4*f^a*x^8*Ei(b*x^2*log(f))*log(f)^4 - (b^3*x^6*log(f)^3 + b^2*x^4*
log(f)^2 + 2*b*x^2*log(f) + 6)*f^(b*x^2 + a))/x^8
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^9}\,dx = \int \frac{f^{a+bx^2}}{x^9}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**9,x)
```

output
```
Integral(f**(a + b*x**2)/x**9, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+bx^2}}{x^9}\,dx = -\frac{1}{2}\,b^4 f^a \Gamma\left(-4, -bx^2 \log\left(f\right)\right) \log\left(f\right)^4$$

input
```
integrate(f^(b*x^2+a)/x^9,x, algorithm="maxima")
```

output
```
-1/2*b^4*f^a*gamma(-4, -b*x^2*log(f))*log(f)^4
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^9}\,dx = \int \frac{f^{bx^2+a}}{x^9}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^9,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^9, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.75

$$\int \frac{f^{a+bx^2}}{x^9}\,dx = -\frac{b^4\,f^a\ln(f)^4\,\mathrm{expint}(-b\,x^2\,\ln(f))}{48} - \frac{b^4\,f^a\,f^{b\,x^2}\ln(f)^4\left(\frac{1}{24\,b\,x^2\,\ln(f)}+\frac{1}{24\,b^2\,x^4\ln(f)^2}+\frac{1}{12\,b^3\,x^6\ln(f)^3}+\frac{1}{4\,b^4\,x^8\ln(f)^4}\right)}{2}$$

input
```
int(f^(a + b*x^2)/x^9,x)
```

output
```
- (b^4*f^a*log(f)^4*expint(-b*x^2*log(f)))/48 - (b^4*f^a*f^(b*x^2)*log(f)^
4*(1/(24*b*x^2*log(f)) + 1/(24*b^2*x^4*log(f)^2) + 1/(12*b^3*x^6*log(f)^3)
 + 1/(4*b^4*x^8*log(f)^4)))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.75

$$\int \frac{f^{a+bx^2}}{x^9}\,dx = \frac{f^a\left(ei(\log(f)\,b\,x^2)\log(f)^4\,b^4x^8 - f^{b\,x^2}\log(f)^3\,b^3x^6 - f^{b\,x^2}\log(f)^2\,b^2x^4 - 2f^{b\,x^2}\log(f)\,b\,x^2 - 6f^{b\,x^2}\right)}{48x^8}$$

input
```
int(f^(b*x^2+a)/x^9,x)
```

output
```
(f**a*(ei(log(f)*b*x**2)*log(f)**4*b**4*x**8 - f**(b*x**2)*log(f)**3*b**3*
x**6 - f**(b*x**2)*log(f)**2*b**2*x**4 - 2*f**(b*x**2)*log(f)*b*x**2 - 6*f
**(b*x**2)))/(48*x**8)
```

3.18 $\int \frac{f^{a+bx^2}}{x^{11}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \frac{1}{2}b^5f^a\Gamma\left(-5,-bx^2\log(f)\right)\log^5(f)$$

output
```
-1/2*f^a/x^10*Ei(6,-b*x^2*ln(f))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \frac{1}{2}b^5f^a\Gamma\left(-5,-bx^2\log(f)\right)\log^5(f)$$

input
```
Integrate[f^(a + b*x^2)/x^11,x]
```

output
```
(b^5*f^a*Gamma[-5, -(b*x^2*Log[f])]*Log[f]^5)/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^{11}} \, dx$$

$$\Big\downarrow \; 2648$$

$$\frac{1}{2} b^5 f^a \log^5(f) \Gamma\left(-5, -bx^2 \log(f)\right)$$

input
```
Int[f^(a + b*x^2)/x^11,x]
```

output
```
(b^5*f^a*Gamma[-5, -(b*x^2*Log[f])]*Log[f]^5)/2
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(18) = 36.

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 4.58

method	result
risch	$-\frac{f^{a}\left(\ln(f)^{5} \operatorname{expIntegral}_{1}\left(-b\,x^{2}\ln(f)\right) b^{5}x^{10}+\ln(f)^{4} f^{b\,x^{2}} b^{4}x^{8}+\ln(f)^{3} f^{b\,x^{2}} b^{3}x^{6}+2\ln(f)^{2} f^{b\,x^{2}} b^{2}x^{4}+6\ln(f) f^{b\,x^{2}} b\,x^{2}+24 f^{b\,x^{2}}\right)}{240x^{10}}$
meijerg	$-\frac{f^{a}b^{5}\ln(f)^{5}\left(\frac{1}{5b^{5}x^{10}\ln(f)^{5}}+\frac{1}{4b^{4}x^{8}\ln(f)^{4}}+\frac{1}{6b^{3}x^{6}\ln(f)^{3}}+\frac{1}{12b^{2}x^{4}\ln(f)^{2}}+\frac{1}{24b\,x^{2}\ln(f)}+\frac{137}{7200}-\frac{\ln(x)}{60}-\frac{\ln(-b)}{120}-\frac{\ln(\ln(f))}{120}-\frac{137b^{5}x^{10}\ln}{}\right.}{}$

input
```
int(f^(b*x^2+a)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/240*f^a*(ln(f)^5*Ei(1,-b*x^2*ln(f))*b^5*x^10+ln(f)^4*f^(b*x^2)*b^4*x^8+
ln(f)^3*f^(b*x^2)*b^3*x^6+2*ln(f)^2*f^(b*x^2)*b^2*x^4+6*ln(f)*f^(b*x^2)*b*
x^2+24*f^(b*x^2))/x^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.46

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \frac{b^5f^ax^{10}\text{Ei}(bx^2\log(f))\log(f)^5-\left(b^4x^8\log(f)^4+b^3x^6\log(f)^3+2\,b^2x^4\log(f)^2+6\,bx^2\log(f)+24\right)f^{bx^2}}{240\,x^{10}}$$

input
```
integrate(f^(b*x^2+a)/x^11,x, algorithm="fricas")
```

output
```
1/240*(b^5*f^a*x^10*Ei(b*x^2*log(f))*log(f)^5 - (b^4*x^8*log(f)^4 + b^3*x^
6*log(f)^3 + 2*b^2*x^4*log(f)^2 + 6*b*x^2*log(f) + 24)*f^(b*x^2 + a))/x^10
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \int \frac{f^{a+bx^2}}{x^{11}}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**11,x)
```

output
```
Integral(f**(a + b*x**2)/x**11, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \frac{1}{2}\,b^5 f^a \Gamma\left(-5, -bx^2 \log\left(f\right)\right) \log\left(f\right)^5$$

input
```
integrate(f^(b*x^2+a)/x^11,x, algorithm="maxima")
```

output
```
1/2*b^5*f^a*gamma(-5, -b*x^2*log(f))*log(f)^5
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \int \frac{f^{bx^2+a}}{x^{11}}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^11,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^11, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = -\frac{b^5\,f^a\ln{(f)}^5\operatorname{expint}(-b\,x^2\,\ln{(f)})}{240} - \frac{b^5\,f^a\,f^{b\,x^2}\ln{(f)}^5\left(\frac{1}{120\,b\,x^2\,\ln(f)}+\frac{1}{120\,b^2\,x^4\ln(f)^2}+\frac{1}{60\,b^3\,x^6\ln(f)^3}+\frac{1}{20\,b^4\,x^8\ln(f)^4}+\frac{1}{5\,b^5\,x^{10}\ln(f)^5}\right)}{2}$$

input
```
int(f^(a + b*x^2)/x^11,x)
```

output
```
- (b^5*f^a*log(f)^5*expint(-b*x^2*log(f)))/240 - (b^5*f^a*f^(b*x^2)*log(f)
^5*(1/(120*b*x^2*log(f)) + 1/(120*b^2*x^4*log(f)^2) + 1/(60*b^3*x^6*log(f)
^3) + 1/(20*b^4*x^8*log(f)^4) + 1/(5*b^5*x^10*log(f)^5)))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int \frac{f^{a+bx^2}}{x^{11}}\,dx = \frac{f^a\Big(ei(\log(f)\,b\,x^2)\log(f)^5\,b^5x^{10} - f^{b\,x^2}\log(f)^4\,b^4x^8 - f^{b\,x^2}\log(f)^3\,b^3x^6 - 2f^{b\,x^2}\log(f)^2\,b^2x^4 - 6f^{b\,x^2}\log(f}{240x^{10}}$$

input
```
int(f^(b*x^2+a)/x^11,x)
```

output
```
(f**a*(ei(log(f)*b*x**2)*log(f)**5*b**5*x**10 - f**(b*x**2)*log(f)**4*b**4
*x**8 - f**(b*x**2)*log(f)**3*b**3*x**6 - 2*f**(b*x**2)*log(f)**2*b**2*x**
4 - 6*f**(b*x**2)*log(f)*b*x**2 - 24*f**(b*x**2)))/(240*x**10)
```

3.19 $\int f^{a+bx^2}x^{12}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+bx^2}x^{12}\,dx=-\frac{f^a x^{13}\Gamma\left(\frac{13}{2},-bx^2\log(f)\right)}{2\left(-bx^2\log(f)\right)^{13/2}}$$

output
```
-1/2*f^a*x^13*(524288/56215335686336962052386218 75*GAMMA(51/2,-b*x^2*ln(f)
)-524288/5621533568633696205238621875*(-b*x^2*ln(f))^(49/2)*exp(b*x^2*ln(f
))-262144/114725174870075432759971875*(-b*x^2*ln(f))^(47/2)*exp(b*x^2*ln(f
))-131072/2440961167448413462978125*(-b*x^2*ln(f))^(45/2)*exp(b*x^2*ln(f))
-65536/54243581498853632510625*(-b*x^2*ln(f))^(43/2)*exp(b*x^2*ln(f))-3276
8/1261478639508224011875*(-b*x^2*ln(f))^(41/2)*exp(b*x^2*ln(f))-16384/3076
7771695322536875*(-b*x^2*ln(f))^(39/2)*exp(b*x^2*ln(f))-8192/7889172229569
88125*(-b*x^2*ln(f))^(37/2)*exp(b*x^2*ln(f))-4096/21322087106945625*(-b*x^
2*ln(f))^(35/2)*exp(b*x^2*ln(f))-2048/609202488769875*(-b*x^2*ln(f))^(33/2
)*exp(b*x^2*ln(f))-1024/18460681477875*(-b*x^2*ln(f))^(31/2)*exp(b*x^2*ln(
f))-512/595505854125*(-b*x^2*ln(f))^(29/2)*exp(b*x^2*ln(f))-256/2053468462
5*(-b*x^2*ln(f))^(27/2)*exp(b*x^2*ln(f))-128/760543875*(-b*x^2*ln(f))^(25/
2)*exp(b*x^2*ln(f))-64/30421755*(-b*x^2*ln(f))^(23/2)*exp(b*x^2*ln(f))-32/
1322685*(-b*x^2*ln(f))^(21/2)*exp(b*x^2*ln(f))-16/62985*(-b*x^2*ln(f))^(19
/2)*exp(b*x^2*ln(f))-8/3315*(-b*x^2*ln(f))^(17/2)*exp(b*x^2*ln(f))-4/195*(
-b*x^2*ln(f))^(15/2)*exp(b*x^2*ln(f))-2/13*(-b*x^2*ln(f))^(13/2)*exp(b*x^2
*ln(f)))/(-b*x^2*ln(f))^(13/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx^2} x^{12} \, dx = -\frac{f^a x^{13} \Gamma\left(\frac{13}{2}, -bx^2 \log(f)\right)}{2 \left(-bx^2 \log(f)\right)^{13/2}}$$

input
```
Integrate[f^(a + b*x^2)*x^12,x]
```

output
```
-1/2*(f^a*x^13*Gamma[13/2, -(b*x^2*Log[f])])/(-(b*x^2*Log[f]))^(13/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{12} f^{a+bx^2} \, dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{x^{13} f^a \Gamma\left(\frac{13}{2}, -bx^2 \log(f)\right)}{2 \left(-bx^2 \log(f)\right)^{13/2}}$$

input
```
Int[f^(a + b*x^2)*x^12,x]
```

output
```
-1/2*(f^a*x^13*Gamma[13/2, -(b*x^2*Log[f])])/(-(b*x^2*Log[f]))^(13/2)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 3.62

method	result
meijerg	$\frac{f^{a}\left(-\frac{x(-b)^{\frac{13}{2}}\sqrt{\ln(f)}\left(-416b^{5}x^{10}\ln(f)^{5}+2288b^{4}x^{8}\ln(f)^{4}-10296b^{3}x^{6}\ln(f)^{3}+36036b^{2}x^{4}\ln(f)^{2}-90090b\,x^{2}\ln(f)+135135\right)e^{b\,x^{2}\ln(f)}}{416b^{6}}+\frac{10395}{}\right.}{2b^{6}\ln(f)^{\frac{13}{2}}\sqrt{-b}}$
risch	$\frac{f^{a}f^{b\,x^{2}}x^{11}}{2\ln(f)b}-\frac{11f^{a}x^{9}f^{b\,x^{2}}}{4\ln(f)^{2}b^{2}}+\frac{99f^{a}x^{7}f^{b\,x^{2}}}{8\ln(f)^{3}b^{3}}-\frac{693f^{a}x^{5}f^{b\,x^{2}}}{16\ln(f)^{4}b^{4}}+\frac{3465f^{a}x^{3}f^{b\,x^{2}}}{32\ln(f)^{5}b^{5}}-\frac{10395f^{a}x\,f^{b\,x^{2}}}{64\ln(f)^{6}b^{6}}+\frac{10395f^{a}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-}\right.}{128\ln(f)^{6}b^{6}\sqrt{-b}}$

input
```
int(f^(b*x^2+a)*x^12,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/b^6/ln(f)^(13/2)/(-b)^(1/2)*(-1/416*x*(-b)^(13/2)*ln(f)^(1/2)*(-41
6*b^5*x^10*ln(f)^5+2288*b^4*x^8*ln(f)^4-10296*b^3*x^6*ln(f)^3+36036*b^2*x^
4*ln(f)^2-90090*b*x^2*ln(f)+135135)/b^6*exp(b*x^2*ln(f))+10395/64*(-b)^(13
/2)/b^(13/2)*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.32

$$\int f^{a+bx^2}x^{12}\,dx =$$

$$-\frac{10395\sqrt{\pi}\sqrt{-b\log\left(f\right)}f^{a}\,\mathrm{erf}\left(\sqrt{-b\log\left(f\right)}x\right)-2\left(32\,b^{6}x^{11}\log\left(f\right)^{6}-176\,b^{5}x^{9}\log\left(f\right)^{5}+792\,b^{4}x^{7}\log\left(\right.\right.}{128\,b^{7}\log\left(f\right)^{7}}$$

input
```
integrate(f^(b*x^2+a)*x^12,x, algorithm="fricas")
```

output
```
-1/128*(10395*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x) - 2*(32*
b^6*x^11*log(f)^6 - 176*b^5*x^9*log(f)^5 + 792*b^4*x^7*log(f)^4 - 2772*b^3
*x^5*log(f)^3 + 6930*b^2*x^3*log(f)^2 - 10395*b*x*log(f))*f^(b*x^2 + a))/(
b^7*log(f)^7)
```

Sympy [F]

$$\int f^{a+bx^2}x^{12}\,dx=\int f^{a+bx^2}x^{12}\,dx$$

input
```
integrate(f**(b*x**2+a)*x**12,x)
```

output
```
Integral(f**(a + b*x**2)*x**12, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.74

$$\int f^{a+bx^2}x^{12}\,dx = \frac{\left(32\,b^5f^ax^{11}\log(f)^5-176\,b^4f^ax^9\log(f)^4+792\,b^3f^ax^7\log(f)^3-2772\,b^2f^ax^5\log(f)^2+6930\,bf^ax^3\log(\right.}{64\,b^6\log(f)^6} + \frac{10395\,\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-b\log(f)}x\right)}{128\,\sqrt{-b\log(f)}b^6\log(f)^6}$$

input
```
integrate(f^(b*x^2+a)*x^12,x, algorithm="maxima")
```

output
```
1/64*(32*b^5*f^a*x^11*log(f)^5 - 176*b^4*f^a*x^9*log(f)^4 + 792*b^3*f^a*x^
7*log(f)^3 - 2772*b^2*f^a*x^5*log(f)^2 + 6930*b*f^a*x^3*log(f) - 10395*f^a
*x)*f^(b*x^2)/(b^6*log(f)^6) + 10395/128*sqrt(pi)*f^a*erf(sqrt(-b*log(f))*
x)/(sqrt(-b*log(f))*b^6*log(f)^6)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.41

$$\int f^{a+bx^2}x^{12}\,dx = -\frac{10395\,\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-b\log(f)}x\right)}{128\,\sqrt{-b\log(f)}b^6\log(f)^6} + \frac{(32\,b^5x^{11}\log(f)^5 - 176\,b^4x^9\log(f)^4 + 792\,b^3x^7\log(f)^3 - 2772\,b^2x^5\log(f)^2 + 6930\,bx^3\log(f) - 1039}{64\,b^6\log(f)^6}$$

input
```
integrate(f^(b*x^2+a)*x^12,x, algorithm="giac")
```

output
```
-10395/128*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^6*log(f
)^6) + 1/64*(32*b^5*x^11*log(f)^5 - 176*b^4*x^9*log(f)^4 + 792*b^3*x^7*log
(f)^3 - 2772*b^2*x^5*log(f)^2 + 6930*b*x^3*log(f) - 10395*x)*e^(b*x^2*log(
f) + a*log(f))/(b^6*log(f)^6)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 4.53

$$\int f^{a+bx^2}x^{12}\,dx$$
$$= \frac{\frac{f^a\left(\frac{10395\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)}{128} - \frac{10395\,f^{b\,x^2}\,x\,\sqrt{b\,\ln(f)}}{64}\right)}{\sqrt{b\,\ln(f)}} - \frac{693\,b^2\,f^{b\,x^2+a}\,x^5\,\ln(f)^2}{16} + \frac{99\,b^3\,f^{b\,x^2+a}\,x^7\,\ln(f)^3}{8} - \frac{11\,b^4\,f^{b\,x^2+a}\,x^9\,\ln(f)^4}{4} + \frac{b}{}}{b^6\ln(f)^6}$$

input
```
int(f^(a + b*x^2)*x^12,x)
```

output
```
((f^a*((10395*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2)))/128 - (10395*f
^(b*x^2)*x*(b*log(f))^(1/2))/64))/(b*log(f))^(1/2) - (693*b^2*f^(a + b*x^2
)*x^5*log(f)^2)/16 + (99*b^3*f^(a + b*x^2)*x^7*log(f)^3)/8 - (11*b^4*f^(a
+ b*x^2)*x^9*log(f)^4)/4 + (b^5*f^(a + b*x^2)*x^11*log(f)^5)/2 + (3465*b*f
^(a + b*x^2)*x^3*log(f))/32)/(b^6*log(f)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 4.91

$$\int f^{a+bx^2}x^{12}\,dx$$

$$= \frac{f^a\left(-10395\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\,ix\right)i + 64f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\log(f)^5\,b^5x^{11} - 352f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\log(f)^4$$

input
```
int(f^(b*x^2+a)*x^12,x)
```

output
```
(f**a*( - 10395*sqrt(pi)*erf(sqrt(b)*sqrt(log(f))*i*x)*i + 64*f**(b*x**2)*
sqrt(b)*sqrt(log(f))*log(f)**5*b**5*x**11 - 352*f**(b*x**2)*sqrt(b)*sqrt(l
og(f))*log(f)**4*b**4*x**9 + 1584*f**(b*x**2)*sqrt(b)*sqrt(log(f))*log(f)*
*3*b**3*x**7 - 5544*f**(b*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2*x**5 +
 13860*f**(b*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*x**3 - 20790*f**(b*x**2)*
sqrt(b)*sqrt(log(f))*x))/(128*sqrt(b)*sqrt(log(f))*log(f)**6*b**6)
```

3.20 $\int f^{a+bx^2} x^{10}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+bx^2} x^{10}\, dx = -\frac{f^a x^{11} \Gamma\left(\frac{11}{2}, -bx^2 \log(f)\right)}{2\left(-bx^2 \log(f)\right)^{11/2}}$$

output

```
-1/2*f^a*x^11*(1048576/6183686925497065825762484062 5*GAMMA(51/2,-b*x^2*ln(
f))-1048576/61836869254970658257624840625*(-b*x^2*ln(f))^(49/2)*exp(b*x^2*
ln(f))-524288/1261976923570829760359690625*(-b*x^2*ln(f))^(47/2)*exp(b*x^2
*ln(f))-262144/26850572841932548092759375*(-b*x^2*ln(f))^(45/2)*exp(b*x^2*
ln(f))-131072/596679396487389957616875*(-b*x^2*ln(f))^(43/2)*exp(b*x^2*ln(
f))-65536/13876265034590464130625*(-b*x^2*ln(f))^(41/2)*exp(b*x^2*ln(f))-3
2768/338445488648547905625*(-b*x^2*ln(f))^(39/2)*exp(b*x^2*ln(f))-16384/86
78089452526869375*(-b*x^2*ln(f))^(37/2)*exp(b*x^2*ln(f))-8192/234542958176
401875*(-b*x^2*ln(f))^(35/2)*exp(b*x^2*ln(f))-4096/6701227376468625*(-b*x^
2*ln(f))^(33/2)*exp(b*x^2*ln(f))-2048/203067496256625*(-b*x^2*ln(f))^(31/2
)*exp(b*x^2*ln(f))-1024/6550564395375*(-b*x^2*ln(f))^(29/2)*exp(b*x^2*ln(f
))-512/225881530875*(-b*x^2*ln(f))^(27/2)*exp(b*x^2*ln(f))-256/8365982625*
(-b*x^2*ln(f))^(25/2)*exp(b*x^2*ln(f))-128/334639305*(-b*x^2*ln(f))^(23/2)
*exp(b*x^2*ln(f))-64/14549535*(-b*x^2*ln(f))^(21/2)*exp(b*x^2*ln(f))-32/69
2835*(-b*x^2*ln(f))^(19/2)*exp(b*x^2*ln(f))-16/36465*(-b*x^2*ln(f))^(17/2)
*exp(b*x^2*ln(f))-8/2145*(-b*x^2*ln(f))^(15/2)*exp(b*x^2*ln(f))-4/143*(-b*
x^2*ln(f))^(13/2)*exp(b*x^2*ln(f))-2/11*(-b*x^2*ln(f))^(11/2)*exp(b*x^2*ln
(f)))/(-b*x^2*ln(f))^(11/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx^2}x^{10}\,dx=-\frac{f^a x^{11}\Gamma\left(\frac{11}{2},-bx^2\log(f)\right)}{2\left(-bx^2\log(f)\right)^{11/2}}$$

input
```
Integrate[f^(a + b*x^2)*x^10,x]
```

output
```
-1/2*(f^a*x^11*Gamma[11/2, -(b*x^2*Log[f])])/(-(b*x^2*Log[f]))^(11/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10}f^{a+bx^2}\,dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{x^{11}f^a\Gamma\left(\frac{11}{2},-bx^2\log(f)\right)}{2\left(-bx^2\log(f)\right)^{11/2}}$$

input
```
Int[f^(a + b*x^2)*x^10,x]
```

output
```
-1/2*(f^a*x^11*Gamma[11/2, -(b*x^2*Log[f])])/(-(b*x^2*Log[f]))^(11/2)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.26

method	result
meijerg	$-\frac{f^{a}\left(\frac{x(-b)^{\frac{11}{2}}\sqrt{\ln(f)}\left(176b^{4}x^{8}\ln(f)^{4}-792b^{3}x^{6}\ln(f)^{3}+2772b^{2}x^{4}\ln(f)^{2}-6930b\,x^{2}\ln(f)+10395\right)e^{b\,x^{2}\ln(f)}}{176b^{5}}-\frac{945(-b)^{\frac{11}{2}}\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{b}\,x\sqrt{\ln(f)}\right.}{32b^{\frac{11}{2}}}}{2b^{5}\ln(f)^{\frac{11}{2}}\sqrt{-b}}$
risch	$\frac{f^{a}x^{9}f^{b\,x^{2}}}{2\ln(f)b}-\frac{9f^{a}x^{7}f^{b\,x^{2}}}{4\ln(f)^{2}b^{2}}+\frac{63f^{a}x^{5}f^{b\,x^{2}}}{8\ln(f)^{3}b^{3}}-\frac{315f^{a}x^{3}f^{b\,x^{2}}}{16\ln(f)^{4}b^{4}}+\frac{945f^{a}x\,f^{b\,x^{2}}}{32\ln(f)^{5}b^{5}}-\frac{945f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b\ln(f)}\,x\right)}{64\ln(f)^{5}b^{5}\sqrt{-b\ln(f)}}$

input
```
int(f^(b*x^2+a)*x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/b^5/ln(f)^(11/2)/(-b)^(1/2)*(1/176*x*(-b)^(11/2)*ln(f)^(1/2)*(176
*b^4*x^8*ln(f)^4-792*b^3*x^6*ln(f)^3+2772*b^2*x^4*ln(f)^2-6930*b*x^2*ln(f)
+10395)/b^5*exp(b*x^2*ln(f))-945/32*(-b)^(11/2)/b^(11/2)*Pi^(1/2)*erfi(b^(
1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.97

$$\int f^{a+bx^2}x^{10}\,dx$$
$$=\frac{945\sqrt{\pi}\sqrt{-b\log(f)}f^{a}\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)+2\left(16\,b^{5}x^{9}\log(f)^{5}-72\,b^{4}x^{7}\log(f)^{4}+252\,b^{3}x^{5}\log(f)^{3}-\right.}{64\,b^{6}\log(f)^{6}}$$

input
```
integrate(f^(b*x^2+a)*x^10,x, algorithm="fricas")
```

output
```
1/64*(945*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x) + 2*(16*b^5*
x^9*log(f)^5 - 72*b^4*x^7*log(f)^4 + 252*b^3*x^5*log(f)^3 - 630*b^2*x^3*lo
g(f)^2 + 945*b*x*log(f))*f^(b*x^2 + a))/(b^6*log(f)^6)
```

Sympy [F]

$$\int f^{a+bx^2}x^{10}\,dx=\int f^{a+bx^2}x^{10}\,dx$$

input
```
integrate(f**(b*x**2+a)*x**10,x)
```

output
```
Integral(f**(a + b*x**2)*x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.29

$$\int f^{a+bx^2}x^{10}\,dx = \frac{\left(16\,b^4f^ax^9\log\left(f\right)^4-72\,b^3f^ax^7\log\left(f\right)^3+252\,b^2f^ax^5\log\left(f\right)^2-630\,bf^ax^3\log\left(f\right)+945\,f^ax\right)f^{bx^2}}{32\,b^5\log\left(f\right)^5} - \frac{945\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-b\log\left(f\right)}x\right)}{64\,\sqrt{-b\log\left(f\right)}b^5\log\left(f\right)^5}$$

input
```
integrate(f^(b*x^2+a)*x^10,x, algorithm="maxima")
```

output
```
1/32*(16*b^4*f^a*x^9*log(f)^4 - 72*b^3*f^a*x^7*log(f)^3 + 252*b^2*f^a*x^5*
log(f)^2 - 630*b*f^a*x^3*log(f) + 945*f^a*x)*f^(b*x^2)/(b^5*log(f)^5) - 94
5/64*sqrt(pi)*f^a*erf(sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^5*log(f)^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

$$\int f^{a+bx^2}x^{10}\,dx = \frac{945\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-b\log(f)}x\right)}{64\sqrt{-b\log(f)}b^5\log(f)^5} + \frac{\left(16\,b^4x^9\log(f)^4 - 72\,b^3x^7\log(f)^3 + 252\,b^2x^5\log(f)^2 - 630\,bx^3\log(f) + 945\,x\right)e^{\left(bx^2\log(f)+a\log(f)\right)}}{32\,b^5\log(f)^5}$$

input
```
integrate(f^(b*x^2+a)*x^10,x, algorithm="giac")
```

output
```
945/64*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^5*log(f)^5)
 + 1/32*(16*b^4*x^9*log(f)^4 - 72*b^3*x^7*log(f)^3 + 252*b^2*x^5*log(f)^2
- 630*b*x^3*log(f) + 945*x)*e^(b*x^2*log(f) + a*log(f))/(b^5*log(f)^5)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 4.09

$$\int f^{a+bx^2}x^{10}\,dx = -\frac{\frac{f^a\left(945\sqrt{\pi}\operatorname{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)-1890\,f^{b\,x^2}\,x\,\sqrt{b\,\ln(f)}\right)}{64\sqrt{b\,\ln(f)}} - \frac{63\,b^2\,f^a\,f^{b\,x^2}\,x^5\ln(f)^2}{8} + \frac{9\,b^3\,f^a\,f^{b\,x^2}\,x^7\ln(f)^3}{4} - \frac{b^4\,f^a\,f^{b\,x^2}\,x^9\ln(f)^4}{2} + 31}{b^5\ln(f)^5}$$

input
```
int(f^(a + b*x^2)*x^10,x)
```

output
```
-((f^a*(945*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2)) - 1890*f^(b*x^2)*
x*(b*log(f))^(1/2)))/(64*(b*log(f))^(1/2)) - (63*b^2*f^a*f^(b*x^2)*x^5*log
(f)^2)/8 + (9*b^3*f^a*f^(b*x^2)*x^7*log(f)^3)/4 - (b^4*f^a*f^(b*x^2)*x^9*l
og(f)^4)/2 + (315*b*f^a*f^(b*x^2)*x^3*log(f))/16)/(b^5*log(f)^5)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.21

$$\int f^{a+bx^2}x^{10}\,dx$$
$$= \frac{f^a\left(945\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\,ix\right)i + 32f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\,\log(f)^4\,b^4x^9 - 144f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\,\log(f)^3\,b^3x^7}{64\sqrt{b}\,\sqrt{\log(f)}}$$

input
```
int(f^(b*x^2+a)*x^10,x)
```

output
```
(f**a*(945*sqrt(pi)*erf(sqrt(b)*sqrt(log(f))*i*x)*i + 32*f**(b*x**2)*sqrt(
b)*sqrt(log(f))*log(f)**4*b**4*x**9 - 144*f**(b*x**2)*sqrt(b)*sqrt(log(f))
*log(f)**3*b**3*x**7 + 504*f**(b*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2
*x**5 - 1260*f**(b*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*x**3 + 1890*f**(b*x
**2)*sqrt(b)*sqrt(log(f))*x))/(64*sqrt(b)*sqrt(log(f))*log(f)**5*b**5)
```

3.21 $\int f^{a+bx^2} x^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 128

$$\int f^{a+bx^2} x^8\,dx = \frac{105 f^a \sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{32 b^{9/2}\log^{\frac{9}{2}}(f)} - \frac{105 f^{a+bx^2} x}{16 b^4 \log^4(f)} + \frac{35 f^{a+bx^2} x^3}{8 b^3 \log^3(f)} - \frac{7 f^{a+bx^2} x^5}{4 b^2 \log^2(f)} + \frac{f^{a+bx^2} x^7}{2 b \log(f)}$$

output
```
105/32*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))/b^(9/2)/ln(f)^(9/2)-105/16
*f^(b*x^2+a)*x/b^4/ln(f)^4+35/8*f^(b*x^2+a)*x^3/b^3/ln(f)^3-7/4*f^(b*x^2+a
)*x^5/b^2/ln(f)^2+1/2*f^(b*x^2+a)*x^7/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.74

$$\int f^{a+bx^2} x^8\,dx = \frac{f^a\left(105\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right) + 2\sqrt{b} f^{bx^2} x\sqrt{\log(f)}\left(-105 + 70bx^2\log(f) - 28b^2x^4\log^2(f) + 8b^3x^6\log^3(\right.\right.}{32 b^{9/2}\log^{\frac{9}{2}}(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^8,x]
```

output
```
(f^a*(105*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]] + 2*Sqrt[b]*f^(b*x^2)*x*Sq
rt[Log[f]]*(-105 + 70*b*x^2*Log[f] - 28*b^2*x^4*Log[f]^2 + 8*b^3*x^6*Log[f
]^3)))/(32*b^(9/2)*Log[f]^(9/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.28, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2641, 2641, 2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 f^{a+bx^2} dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^7 f^{a+bx^2}}{2b\log(f)} - \frac{7\int f^{bx^2+a}x^6dx}{2b\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^7 f^{a+bx^2}}{2b\log(f)} - \frac{7\left(\frac{x^5 f^{a+bx^2}}{2b\log(f)} - \frac{5\int f^{bx^2+a}x^4dx}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^7 f^{a+bx^2}}{2b\log(f)} - \frac{7\left(\frac{x^5 f^{a+bx^2}}{2b\log(f)} - \frac{5\left(\frac{x^3 f^{a+bx^2}}{2b\log(f)} - \frac{3\int f^{bx^2+a}x^2dx}{2b\log(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^7 f^{a+bx^2}}{2b\log(f)} - \frac{7\left(\frac{x^5 f^{a+bx^2}}{2b\log(f)} - \frac{5\left(\frac{x^3 f^{a+bx^2}}{2b\log(f)} - \frac{3\left(\frac{x f^{a+bx^2}}{2b\log(f)} - \frac{\int f^{bx^2+a}dx}{2b\log(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{x^7 f^{a+bx^2}}{2b\log(f)} - \frac{7\left(\frac{x^5 f^{a+bx^2}}{2b\log(f)} - \frac{5\left(\frac{x^3 f^{a+bx^2}}{2b\log(f)} - \frac{3\left(\frac{x f^{a+bx^2}}{2b\log(f)} - \frac{\sqrt{\pi} f^a \operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}$$

input
```
Int[f^(a + b*x^2)*x^8,x]
```

output
```
(f^(a + b*x^2)*x^7)/(2*b*Log[f]) - (7*((f^(a + b*x^2)*x^5)/(2*b*Log[f]) -
(5*((f^(a + b*x^2)*x^3)/(2*b*Log[f]) - (3*(-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[b]
*x*Sqrt[Log[f]]])/(b^(3/2)*Log[f]^(3/2)) + (f^(a + b*x^2)*x)/(2*b*Log[f]))
)/(2*b*Log[f])))/(2*b*Log[f])))/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\frac{f^a\left(-\frac{x(-b)^{\frac{9}{2}}\sqrt{\ln(f)}\left(-72b^3x^6\ln(f)^3+252b^2x^4\ln(f)^2-630bx^2\ln(f)+945\right)e^{bx^2\ln(f)}}{72b^4}+\frac{105(-b)^{\frac{9}{2}}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}\,x\sqrt{\ln(f)}\right)}{16b^{\frac{9}{2}}}\right)}{2\ln(f)^{\frac{9}{2}}b^4\sqrt{-b}}$	99
risch	$\frac{f^a x^7 f^{b x^2}}{2\ln(f)b}-\frac{7f^a x^5 f^{b x^2}}{4\ln(f)^2 b^2}+\frac{35 f^a x^3 f^{b x^2}}{8\ln(f)^3 b^3}-\frac{105 f^a x f^{b x^2}}{16\ln(f)^4 b^4}+\frac{105 f^a\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-b\ln(f)}\,x\right)}{32\ln(f)^4 b^4\sqrt{-b\ln(f)}}$	120

input
```
int(f^(b*x^2+a)*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/ln(f)^(9/2)/b^4/(-b)^(1/2)*(-1/72*x*(-b)^(9/2)*ln(f)^(1/2)*(-72*b^
3*x^6*ln(f)^3+252*b^2*x^4*ln(f)^2-630*b*x^2*ln(f)+945)/b^4*exp(b*x^2*ln(f)
)+105/16*(-b)^(9/2)/b^(9/2)*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int f^{a+bx^2}x^8\,dx = -\frac{105\sqrt{\pi}\sqrt{-b\log(f)}f^a\,\mathrm{erf}\left(\sqrt{-b\log(f)}x\right)-2\left(8b^4x^7\log(f)^4-28b^3x^5\log(f)^3+70b^2x^3\log(f)^2-\right.}{32b^5\log(f)^5}$$

input
```
integrate(f^(b*x^2+a)*x^8,x, algorithm="fricas")
```

output
```
-1/32*(105*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x) - 2*(8*b^4*
x^7*log(f)^4 - 28*b^3*x^5*log(f)^3 + 70*b^2*x^3*log(f)^2 - 105*b*x*log(f))
*f^(b*x^2 + a))/(b^5*log(f)^5)
```

Sympy [F]

$$\int f^{a+bx^2}x^8\,dx = \int f^{a+bx^2}x^8\,dx$$

input
```
integrate(f**(b*x**2+a)*x**8,x)
```

output
```
Integral(f**(a + b*x**2)*x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int f^{a+bx^2}x^8\,dx = \frac{\left(8\,b^3f^ax^7\log\left(f\right)^3 - 28\,b^2f^ax^5\log\left(f\right)^2 + 70\,bf^ax^3\log\left(f\right) - 105\,f^ax\right)f^{bx^2}}{16\,b^4\log\left(f\right)^4} + \frac{105\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-b\log\left(f\right)}x\right)}{32\,\sqrt{-b\log\left(f\right)}b^4\log\left(f\right)^4}$$

input
```
integrate(f^(b*x^2+a)*x^8,x, algorithm="maxima")
```

output
```
1/16*(8*b^3*f^a*x^7*log(f)^3 - 28*b^2*f^a*x^5*log(f)^2 + 70*b*f^a*x^3*log(
f) - 105*f^a*x)*f^(b*x^2)/(b^4*log(f)^4) + 105/32*sqrt(pi)*f^a*erf(sqrt(-b
*log(f))*x)/(sqrt(-b*log(f))*b^4*log(f)^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int f^{a+bx^2}x^8\,dx$$
$$= -\frac{105\,\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-b\log(f)}x\right)}{32\,\sqrt{-b\log(f)}b^4\log(f)^4} + \frac{\left(8\,b^3x^7\log(f)^3-28\,b^2x^5\log(f)^2+70\,bx^3\log(f)-105\,x\right)e^{\left(bx^2\log(f)+a\log(f)\right)}}{16\,b^4\log(f)^4}$$

input
```
integrate(f^(b*x^2+a)*x^8,x, algorithm="giac")
```

output
```
-105/32*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^4*log(f)^4
) + 1/16*(8*b^3*x^7*log(f)^3 - 28*b^2*x^5*log(f)^2 + 70*b*x^3*log(f) - 105
*x)*e^(b*x^2*log(f) + a*log(f))/(b^4*log(f)^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int f^{a+bx^2}x^8\,dx$$
$$= \frac{\frac{f^a\left(105\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)-210\,f^{b\,x^2}\,x\,\sqrt{b\,\ln(f)}\right)}{32\,\sqrt{b\,\ln(f)}}-\frac{7\,b^2\,f^a\,f^{b\,x^2}\,x^5\,\ln(f)^2}{4}+\frac{b^3\,f^a\,f^{b\,x^2}\,x^7\,\ln(f)^3}{2}+\frac{35\,b\,f^a\,f^{b\,x^2}\,x^3\,\ln(f)}{8}}{b^4\ln(f)^4}$$

input
```
int(f^(a + b*x^2)*x^8,x)
```

output
```
((f^a*(105*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2)) - 210*f^(b*x^2)*x*
(b*log(f))^(1/2)))/(32*(b*log(f))^(1/2)) - (7*b^2*f^a*f^(b*x^2)*x^5*log(f)
^2)/4 + (b^3*f^a*f^(b*x^2)*x^7*log(f)^3)/2 + (35*b*f^a*f^(b*x^2)*x^3*log(f
))/8)/(b^4*log(f)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93

$$\int f^{a+bx^2}x^8\,dx$$
$$= \frac{f^a\left(-105\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\,ix\right)i+16f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\,\log(f)^3\,b^3x^7-56f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\,\log(f)^2\,b^2x^5}{32\sqrt{b}\,\sqrt{\log(f)}\,\log(f)^4\,b^4}$$

input
```
int(f^(b*x^2+a)*x^8,x)
```

output
```
(f**a*( - 105*sqrt(pi)*erf(sqrt(b)*sqrt(log(f))*i*x)*i + 16*f**(b*x**2)*sq
rt(b)*sqrt(log(f))*log(f)**3*b**3*x**7 - 56*f**(b*x**2)*sqrt(b)*sqrt(log(f
))*log(f)**2*b**2*x**5 + 140*f**(b*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*x**
3 - 210*f**(b*x**2)*sqrt(b)*sqrt(log(f))*x))/(32*sqrt(b)*sqrt(log(f))*log(
f)**4*b**4)
```

3.22 $\int f^{a+bx^2} x^6 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 105

$$\int f^{a+bx^2} x^6 \, dx = -\frac{15 f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} x \sqrt{\log(f)}\right)}{16 b^{7/2} \log^{\frac{7}{2}}(f)} + \frac{15 f^{a+bx^2} x}{8 b^3 \log^3(f)} - \frac{5 f^{a+bx^2} x^3}{4 b^2 \log^2(f)} + \frac{f^{a+bx^2} x^5}{2 b \log(f)}$$

output
```
-15/16*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))/b^(7/2)/ln(f)^(7/2)+15/8*f
^(b*x^2+a)*x/b^3/ln(f)^3-5/4*f^(b*x^2+a)*x^3/b^2/ln(f)^2+1/2*f^(b*x^2+a)*x
^5/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.79

$$\int f^{a+bx^2} x^6 \, dx = \frac{f^a \left(-15 \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} x \sqrt{\log(f)}\right) + 2 \sqrt{b} f^{bx^2} x \sqrt{\log(f)} \left(15 - 10 b x^2 \log(f) + 4 b^2 x^4 \log^2(f)\right)\right)}{16 b^{7/2} \log^{\frac{7}{2}}(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^6,x]
```

output
```
(f^a*(-15*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]] + 2*Sqrt[b]*f^(b*x^2)*x*Sq
rt[Log[f]]*(15 - 10*b*x^2*Log[f] + 4*b^2*x^4*Log[f]^2)))/(16*b^(7/2)*Log[f
]^(7/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2641, 2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 f^{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^5 f^{a+bx^2}}{2b\log(f)}-\frac{5\int f^{bx^2+a}x^4dx}{2b\log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^5 f^{a+bx^2}}{2b\log(f)}-\frac{5\left(\frac{x^3 f^{a+bx^2}}{2b\log(f)}-\frac{3\int f^{bx^2+a}x^2dx}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^5 f^{a+bx^2}}{2b\log(f)}-\frac{5\left(\frac{x^3 f^{a+bx^2}}{2b\log(f)}-\frac{3\left(\frac{xf^{a+bx^2}}{2b\log(f)}-\frac{\int f^{bx^2+a}dx}{2b\log(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{x^5 f^{a+bx^2}}{2b\log(f)}-\frac{5\left(\frac{x^3 f^{a+bx^2}}{2b\log(f)}-\frac{3\left(\frac{xf^{a+bx^2}}{2b\log(f)}-\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}\right)}{2b\log(f)}\right)}{2b\log(f)}$$

input
```
Int[f^(a + b*x^2)*x^6,x]
```

output
```
(f^(a + b*x^2)*x^5)/(2*b*Log[f]) - (5*((f^(a + b*x^2)*x^3)/(2*b*Log[f]) -
(3*(-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]])/(b^(3/2)*Log[f]^(3/2)
) + (f^(a + b*x^2)*x)/(2*b*Log[f])))/(2*b*Log[f])))/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.83

method	result	size
meijerg	$-\frac{f^{a}\left(\frac{x(-b)^{\frac{7}{2}}\sqrt{\ln(f)}\left(28b^{2}x^{4}\ln(f)^{2}-70b\,x^{2}\ln(f)+105\right)e^{b\,x^{2}\ln(f)}}{28b^{3}}-\frac{15(-b)^{\frac{7}{2}}\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{b}\,x\sqrt{\ln(f)}\right)}{8b^{\frac{7}{2}}}\right)}{2\ln(f)^{\frac{7}{2}}b^{3}\sqrt{-b}}$	87
risch	$\frac{f^{a}x^{5}f^{b\,x^{2}}}{2\ln(f)b}-\frac{5f^{a}x^{3}f^{b\,x^{2}}}{4\ln(f)^{2}b^{2}}+\frac{15f^{a}x\,f^{b\,x^{2}}}{8\ln(f)^{3}b^{3}}-\frac{15f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b\ln(f)}\,x\right)}{16\ln(f)^{3}b^{3}\sqrt{-b\ln(f)}}$	98

input
```
int(f^(b*x^2+a)*x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/ln(f)^(7/2)/b^3/(-b)^(1/2)*(1/28*x*(-b)^(7/2)*ln(f)^(1/2)*(28*b^2
*x^4*ln(f)^2-70*b*x^2*ln(f)+105)/b^3*exp(b*x^2*ln(f))-15/8*(-b)^(7/2)/b^(7
/2)*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.73

$$\int f^{a+bx^2}x^6\,dx = \frac{15\,\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right) + 2\left(4\,b^3x^5\log(f)^3 - 10\,b^2x^3\log(f)^2 + 15\,bx\log(f)\right)f^{bx^2+a}}{16\,b^4\log(f)^4}$$

input
```
integrate(f^(b*x^2+a)*x^6,x, algorithm="fricas")
```

output
```
1/16*(15*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x) + 2*(4*b^3*x^
5*log(f)^3 - 10*b^2*x^3*log(f)^2 + 15*b*x*log(f))*f^(b*x^2 + a))/(b^4*log(
f)^4)
```

Sympy [F]

$$\int f^{a+bx^2}x^6\,dx = \int f^{a+bx^2}x^6\,dx$$

input
```
integrate(f**(b*x**2+a)*x**6,x)
```

output
```
Integral(f**(a + b*x**2)*x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

$$\int f^{a+bx^2}x^6\,dx = \frac{\left(4\,b^2f^ax^5\log(f)^2 - 10\,bf^ax^3\log(f) + 15\,f^ax\right)f^{bx^2}}{8\,b^3\log(f)^3} - \frac{15\,\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)}{16\,\sqrt{-b\log(f)}b^3\log(f)^3}$$

input
```
integrate(f^(b*x^2+a)*x^6,x, algorithm="maxima")
```

output
```
1/8*(4*b^2*f^a*x^5*log(f)^2 - 10*b*f^a*x^3*log(f) + 15*f^a*x)*f^(b*x^2)/(b
^3*log(f)^3) - 15/16*sqrt(pi)*f^a*erf(sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*
b^3*log(f)^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.76

$$\int f^{a+bx^2}x^6\,dx = \frac{15\,\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-b\log(f)}x\right)}{16\,\sqrt{-b\log(f)}b^3\log(f)^3} + \frac{\left(4\,b^2x^5\log(f)^2 - 10\,bx^3\log(f) + 15\,x\right)e^{\left(bx^2\log(f)+a\log(f)\right)}}{8\,b^3\log(f)^3}$$

input
```
integrate(f^(b*x^2+a)*x^6,x, algorithm="giac")
```

output
```
15/16*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^3*log(f)^3)
+ 1/8*(4*b^2*x^5*log(f)^2 - 10*b*x^3*log(f) + 15*x)*e^(b*x^2*log(f) + a*lo
g(f))/(b^3*log(f)^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int f^{a+bx^2}x^6\,dx = \frac{15\,f^a\,f^{b\,x^2}\,x}{8\,b^3\ln(f)^3} + \frac{f^a\,f^{b\,x^2}\,x^5}{2\,b\,\ln(f)} - \frac{5\,f^a\,f^{b\,x^2}\,x^3}{4\,b^2\ln(f)^2} - \frac{15\,f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)}{16\,b^3\ln(f)^3\,\sqrt{b\,\ln(f)}}$$

input
```
int(f^(a + b*x^2)*x^6,x)
```

output
```
(15*f^a*f^(b*x^2)*x)/(8*b^3*log(f)^3) + (f^a*f^(b*x^2)*x^5)/(2*b*log(f)) -
 (5*f^a*f^(b*x^2)*x^3)/(4*b^2*log(f)^2) - (15*f^a*pi^(1/2)*erfi((b*x*log(f
))/(b*log(f))^(1/2)))/(16*b^3*log(f)^3*(b*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2} x^6 \, dx = \frac{f^a \left(15\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\,ix\right) i + 8f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\log(f)^2 b^2 x^5 - 20f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\log(f)\,b\,x^3 + 30\right.}{16\sqrt{b}\,\sqrt{\log(f)}\log(f)^3 b^3}$$

input
```
int(f^(b*x^2+a)*x^6,x)
```

output
```
(f**a*(15*sqrt(pi)*erf(sqrt(b)*sqrt(log(f))*i*x)*i + 8*f**(b*x**2)*sqrt(b)
*sqrt(log(f))*log(f)**2*b**2*x**5 - 20*f**(b*x**2)*sqrt(b)*sqrt(log(f))*lo
g(f)*b*x**3 + 30*f**(b*x**2)*sqrt(b)*sqrt(log(f))*x))/(16*sqrt(b)*sqrt(log
(f))*log(f)**3*b**3)
```

3.23 $\int f^{a+bx^2} x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int f^{a+bx^2} x^4 \, dx = \frac{3f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{8b^{5/2}\log^{\frac{5}{2}}(f)} - \frac{3f^{a+bx^2}x}{4b^2\log^2(f)} + \frac{f^{a+bx^2}x^3}{2b\log(f)}$$

output
```
3/8*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))/b^(5/2)/ln(f)^(5/2)-3/4*f^(b*
x^2+a)*x/b^2/ln(f)^2+1/2*f^(b*x^2+a)*x^3/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int f^{a+bx^2} x^4 \, dx = \frac{f^a\left(3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right) + 2\sqrt{b}f^{bx^2}x\sqrt{\log(f)}(-3+2bx^2\log(f))\right)}{8b^{5/2}\log^{\frac{5}{2}}(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^4,x]
```

output
```
(f^a*(3*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]] + 2*Sqrt[b]*f^(b*x^2)*x*Sqrt
[Log[f]]*(-3 + 2*b*x^2*Log[f])))/(8*b^(5/2)*Log[f]^(5/2))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{a+bx^2} \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^3 f^{a+bx^2}}{2b\log(f)} - \frac{3\int f^{bx^2+a}x^2dx}{2b\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{x^3 f^{a+bx^2}}{2b\log(f)} - \frac{3\left(\frac{xf^{a+bx^2}}{2b\log(f)} - \frac{\int f^{bx^2+a}dx}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{x^3 f^{a+bx^2}}{2b\log(f)} - \frac{3\left(\frac{xf^{a+bx^2}}{2b\log(f)} - \frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}\right)}{2b\log(f)}$$

input
```
Int[f^(a + b*x^2)*x^4,x]
```

output
```
(f^(a + b*x^2)*x^3)/(2*b*Log[f]) - (3*(-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*S
qrt[Log[f]]])/(b^(3/2)*Log[f]^(3/2)) + (f^(a + b*x^2)*x)/(2*b*Log[f])))/(2
*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

method	result	size
meijerg	$\frac{f^{a}\left(-\frac{x(-b)^{\frac{5}{2}}\sqrt{\ln(f)}\left(-10bx^{2}\ln(f)+15\right)e^{b\,x^{2}\ln(f)}}{10b^{2}}+\frac{3(-b)^{\frac{5}{2}}\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{b}\,x\sqrt{\ln(f)}\right)}{4b^{\frac{5}{2}}}\right)}{2\ln(f)^{\frac{5}{2}}b^{2}\sqrt{-b}}$	75
risch	$\frac{f^{a}x^{3}f^{b\,x^{2}}}{2\ln(f)b}-\frac{3f^{a}x\,f^{b\,x^{2}}}{4\ln(f)^{2}b^{2}}+\frac{3f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b\ln(f)}\,x\right)}{8\ln(f)^{2}b^{2}\sqrt{-b\ln(f)}}$	76

input
```
int(f^(b*x^2+a)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/ln(f)^(5/2)/b^2/(-b)^(1/2)*(-1/10*x*(-b)^(5/2)*ln(f)^(1/2)*(-10*b*
x^2*ln(f)+15)/b^2*exp(b*x^2*ln(f))+3/4*(-b)^(5/2)/b^(5/2)*Pi^(1/2)*erfi(b^
(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int f^{a+bx^2}x^4\,dx$$
$$= -\frac{3\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right) - 2\left(2b^2x^3\log(f)^2 - 3bx\log(f)\right)f^{bx^2+a}}{8b^3\log(f)^3}$$

input
```
integrate(f^(b*x^2+a)*x^4,x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x) - 2*(2*b^2*x^3
*log(f)^2 - 3*b*x*log(f))*f^(b*x^2 + a))/(b^3*log(f)^3)
```

Sympy [F]

$$\int f^{a+bx^2}x^4\,dx = \int f^{a+bx^2}x^4\,dx$$

input
```
integrate(f**(b*x**2+a)*x**4,x)
```

output
```
Integral(f**(a + b*x**2)*x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

$$\int f^{a+bx^2}x^4\,dx = \frac{(2bf^ax^3\log(f) - 3f^ax)f^{bx^2}}{4b^2\log(f)^2} + \frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)}{8\sqrt{-b\log(f)}b^2\log(f)^2}$$

input
```
integrate(f^(b*x^2+a)*x^4,x, algorithm="maxima")
```

output
```
1/4*(2*b*f^a*x^3*log(f) - 3*f^a*x)*f^(b*x^2)/(b^2*log(f)^2) + 3/8*sqrt(pi)
*f^a*erf(sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^2*log(f)^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int f^{a+bx^2}x^4\,dx = -\frac{3\,\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-b\log\left(f\right)}x\right)}{8\,\sqrt{-b\log\left(f\right)}b^2\log\left(f\right)^2} + \frac{(2\,bx^3\log\left(f\right) - 3\,x)e^{\left(bx^2\log(f)+a\log(f)\right)}}{4\,b^2\log\left(f\right)^2}$$

input
```
integrate(f^(b*x^2+a)*x^4,x, algorithm="giac")
```

output
```
-3/8*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b^2*log(f)^2) +
 1/4*(2*b*x^3*log(f) - 3*x)*e^(b*x^2*log(f) + a*log(f))/(b^2*log(f)^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int f^{a+bx^2}x^4\,dx = \frac{f^a\,\left(3\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right) - 6\,f^{b\,x^2}\,x\,\sqrt{b\,\ln\left(f\right)}\right)}{8\,b^2\,\ln\left(f\right)^2\,\sqrt{b\,\ln\left(f\right)}} + \frac{f^a\,f^{b\,x^2}\,x^3}{2\,b\,\ln\left(f\right)}$$

input
```
int(f^(a + b*x^2)*x^4,x)
```

output
```
(f^a*(3*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2)) - 6*f^(b*x^2)*x*(b*lo
g(f))^(1/2)))/(8*b^2*log(f)^2*(b*log(f))^(1/2)) + (f^a*f^(b*x^2)*x^3)/(2*b
*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int f^{a+bx^2} x^4 \, dx$$
$$= \frac{f^a \left(-3\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\,ix\right) i + 4 f^{b\,x^2} \sqrt{b}\,\sqrt{\log(f)}\,\log(f)\,b\,x^3 - 6 f^{b\,x^2} \sqrt{b}\,\sqrt{\log(f)}\,x\right)}{8\sqrt{b}\,\sqrt{\log(f)}\,\log(f)^2\,b^2}$$

input

```
int(f^(b*x^2+a)*x^4,x)
```

output

```
(f**a*( - 3*sqrt(pi)*erf(sqrt(b)*sqrt(log(f))*i*x)*i + 4*f**(b*x**2)*sqrt(
b)*sqrt(log(f))*log(f)*b*x**3 - 6*f**(b*x**2)*sqrt(b)*sqrt(log(f))*x))/(8*
sqrt(b)*sqrt(log(f))*log(f)**2*b**2)
```

3.24 $\int f^{a+bx^2} x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int f^{a+bx^2} x^2 \, dx = -\frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} x \sqrt{\log(f)}\right)}{4 b^{3/2} \log^{\frac{3}{2}}(f)} + \frac{f^{a+bx^2} x}{2 b \log(f)}$$

output
```
-1/4*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))/b^(3/2)/ln(f)^(3/2)+1/2*f^(b
*x^2+a)*x/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int f^{a+bx^2} x^2 \, dx = -\frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} x \sqrt{\log(f)}\right)}{4 b^{3/2} \log^{\frac{3}{2}}(f)} + \frac{f^{a+bx^2} x}{2 b \log(f)}$$

input
```
Integrate[f^(a + b*x^2)*x^2,x]
```

output
```
-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]])/(b^(3/2)*Log[f]^(3/2)) +
(f^(a + b*x^2)*x)/(2*b*Log[f])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x f^{a+bx^2}}{2b\log(f)} - \frac{\int f^{bx^2+a}dx}{2b\log(f)}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{x f^{a+bx^2}}{2b\log(f)} - \frac{\sqrt{\pi} f^a \text{erfi}\left(\sqrt{b} x \sqrt{\log(f)}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}$$

input
```
Int[f^(a + b*x^2)*x^2,x]
```

output
```
-1/4*(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]])/(b^(3/2)*Log[f]^(3/2)) +
(f^(a + b*x^2)*x)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{f^{a} x \,f^{b \,x^{2}}}{2 \ln \left(f \right) b}-\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln \left(f \right)}\, x \right)}{4 \ln \left(f \right) b \sqrt{-b \ln \left(f \right)}}$	54
meijerg	$-\frac{f^{a} \left(\frac{x \left(-b \right)^{\frac{3}{2}} \sqrt{\ln \left(f \right)}\, {\mathrm e}^{b \,x^{2} \ln \left(f \right)}}{b}-\frac{\left(-b \right)^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln \left(f \right)}\right)}{2 b^{\frac{3}{2}}}\right)}{2 b \ln \left(f \right)^{\frac{3}{2}} \sqrt{-b}}$	64

input
```
int(f^(b*x^2+a)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/ln(f)/b*x*f^(b*x^2)-1/4*f^a/ln(f)/b*Pi^(1/2)/(-b*ln(f))^(1/2)*erf(
(-b*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int f^{a+bx^2}x^2\,dx=\frac{2\,bf^{bx^2+a}x\log\left(f\right)+\sqrt{\pi}\sqrt{-b\log\left(f\right)}f^a\,\mathrm{erf}\left(\sqrt{-b\log\left(f\right)}x\right)}{4\,b^2\log\left(f\right)^2}$$

input
```
integrate(f^(b*x^2+a)*x^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*f^(b*x^2 + a)*x*log(f) + sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b
*log(f))*x))/(b^2*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx^2}x^2\,dx=\int f^{a+bx^2}x^2\,dx$$

input
```
integrate(f**(b*x**2+a)*x**2,x)
```

output
```
Integral(f**(a + b*x**2)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int f^{a+bx^2}x^2\,dx=\frac{f^{bx^2}f^ax}{2\,b\log\left(f\right)}-\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-b\log\left(f\right)}x\right)}{4\,\sqrt{-b\log\left(f\right)}b\log\left(f\right)}$$

input
```
integrate(f^(b*x^2+a)*x^2,x, algorithm="maxima")
```

output
```
1/2*f^(b*x^2)*f^a*x/(b*log(f)) - 1/4*sqrt(pi)*f^a*erf(sqrt(-b*log(f))*x)/(
sqrt(-b*log(f))*b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int f^{a+bx^2}x^2\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-b\log(f)}x\right)}{4\sqrt{-b\log(f)}b\log(f)} + \frac{xe^{\left(bx^2\log(f)+a\log(f)\right)}}{2\,b\log(f)}$$

input
```
integrate(f^(b*x^2+a)*x^2,x, algorithm="giac")
```

output
```
1/4*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/(sqrt(-b*log(f))*b*log(f)) + 1/2*
x*e^(b*x^2*log(f) + a*log(f))/(b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int f^{a+bx^2}x^2\,dx = \frac{f^a\,f^{b\,x^2}\,x}{2\,b\,\ln(f)} - \frac{f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)}{4\,b\,\ln(f)\,\sqrt{b\,\ln(f)}}$$

input
```
int(f^(a + b*x^2)*x^2,x)
```

output
```
(f^a*f^(b*x^2)*x)/(2*b*log(f)) - (f^a*pi^(1/2)*erfi((b*x*log(f))/(b*log(f)
)^(1/2)))/(4*b*log(f)*(b*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int f^{a+bx^2}x^2\,dx = \frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\,ix\right)i + 2f^{b\,x^2}\sqrt{b}\,\sqrt{\log(f)}\,x\right)}{4\sqrt{b}\,\sqrt{\log(f)}\log(f)\,b}$$

input
```
int(f^(b*x^2+a)*x^2,x)
```

output
```
(f**a*(sqrt(pi)*erf(sqrt(b)*sqrt(log(f))*i*x)*i + 2*f**(b*x**2)*sqrt(b)*sq
rt(log(f))*x))/(4*sqrt(b)*sqrt(log(f))*log(f)*b)
```

3.25 $\int f^{a+bx^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 37

$$\int f^{a+bx^2} dx = \frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

output
```
1/2*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))/b^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int f^{a+bx^2} dx = \frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x^2),x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]])/(2*Sqrt[b]*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx^2}\,dx$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x^2),x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]])/(2*Sqrt[b]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
meijerg	$\frac{f^{a}\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{b}\,x\sqrt{\ln(f)}\right)}{2\sqrt{b}\,\sqrt{\ln(f)}}$	26
risch	$\frac{f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b\ln(f)}\,x\right)}{2\sqrt{-b\ln(f)}}$	26

input
```
int(f^(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))/b^(1/2)/ln(f)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int f^{a+bx^2}\,dx=-\frac{\sqrt{\pi}\sqrt{-b\log{(f)}}f^a\,\mathrm{erf}\left(\sqrt{-b\log{(f)}}x\right)}{2\,b\log{(f)}}$$

input
```
integrate(f^(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x)/(b*log(f))
```

Sympy [F]

$$\int f^{a+bx^2}\,dx=\int f^{a+bx^2}\,dx$$

input
```
integrate(f**(b*x**2+a),x)
```

output
```
Integral(f**(a + b*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int f^{a+bx^2}\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-b\log\left(f\right)}x\right)}{2\sqrt{-b\log\left(f\right)}}$$

input
```
integrate(f^(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*f^a*erf(sqrt(-b*log(f))*x)/sqrt(-b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int f^{a+bx^2}\,dx=-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-b\log\left(f\right)}x\right)}{2\sqrt{-b\log\left(f\right)}}$$

input
```
integrate(f^(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*x)/sqrt(-b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int f^{a+bx^2}\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)}{2\sqrt{b\,\ln\left(f\right)}}$$

input
```
int(f^(a + b*x^2),x)
```

output
```
(f^a*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2)))/(2*(b*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int f^{a+bx^2} dx = -\frac{\sqrt{\pi}\, f^a \mathrm{erf}\left(\sqrt{b}\,\sqrt{\log(f)}\, ix\right) i}{2\sqrt{b}\,\sqrt{\log(f)}}$$

input
```
int(f^(b*x^2+a),x)
```

output
```
( - sqrt(pi)*f**a*erf(sqrt(b)*sqrt(log(f))*i*x)*i)/(2*sqrt(b)*sqrt(log(f))
)
```

3.26 $\int \frac{f^{a+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = -\frac{f^{a+bx^2}}{x} + \sqrt{b}f^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\sqrt{\log(f)}$$

output
```
-f^(b*x^2+a)/x+b^(1/2)*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))*ln(f)^(1/2
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = -\frac{f^{a+bx^2}}{x} + \sqrt{b}f^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\sqrt{\log(f)}$$

input
```
Integrate[f^(a + b*x^2)/x^2,x]
```

output
```
-(f^(a + b*x^2)/x) + Sqrt[b]*f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Sqr
t[Log[f]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^2} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$2b\log(f)\int f^{bx^2+a}dx - \frac{f^{a+bx^2}}{x}$$

$$\Big\downarrow \text{ 2633}$$

$$\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{x}$$

input
```
Int[f^(a + b*x^2)/x^2,x]
```

output
```
-(f^(a + b*x^2)/x) + Sqrt[b]*f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Sqr
t[Log[f]]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
risch	$-\frac{f^{a} f^{b \,x^{2}}}{x}+\frac{f^{a} \ln \left(f \right) b \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln \left(f \right)}\, x \right)}{\sqrt{-b \ln \left(f \right)}}$	44
meijerg	$-\frac{f^{a} b \sqrt{\ln \left(f \right)}\, \left(-\frac{2 \,{\mathrm e}^{b \,x^{2} \ln \left(f \right)}}{x \sqrt{-b}\, \sqrt{\ln \left(f \right)}}+\frac{2 \sqrt{b}\, \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln \left(f \right)}\right)}{\sqrt{-b}}\right)}{2 \sqrt{-b}}$	62

input
```
int(f^(b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-f^a/x*f^(b*x^2)+f^a*ln(f)*b*Pi^(1/2)/(-b*ln(f))^(1/2)*erf((-b*ln(f))^(1/2
)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = -\frac{\sqrt{\pi}\sqrt{-b\log(f)}f^a x\,\mathrm{erf}\left(\sqrt{-b\log(f)}x\right) + f^{bx^2+a}}{x}$$

input
```
integrate(f^(b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
-(sqrt(pi)*sqrt(-b*log(f))*f^a*x*erf(sqrt(-b*log(f))*x) + f^(b*x^2 + a))/x
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = \int \frac{f^{a+bx^2}}{x^2}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**2,x)
```

output
```
Integral(f**(a + b*x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = -\frac{\sqrt{-bx^2\log(f)}f^a\Gamma\left(-\frac{1}{2}, -bx^2\log(f)\right)}{2\,x}$$

input
```
integrate(f^(b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(-b*x^2*log(f))*f^a*gamma(-1/2, -b*x^2*log(f))/x
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = \int \frac{f^{bx^2+a}}{x^2}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = \frac{b\,f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)\,\ln(f)}{\sqrt{b\,\ln(f)}} - \frac{f^a\,f^{b\,x^2}}{x}$$

input
```
int(f^(a + b*x^2)/x^2,x)
```

output
```
(b*f^a*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2))*log(f))/(b*log(f))^(1/
2) - (f^a*f^(b*x^2))/x
```

Reduce [F]

$$\int \frac{f^{a+bx^2}}{x^2}\,dx = f^a\left(\int \frac{f^{b\,x^2}}{x^2}dx\right)$$

input
```
int(f^(b*x^2+a)/x^2,x)
```

output
```
f**a*int(f**(b*x**2)/x**2,x)
```

3.27 $\int \frac{f^{a+bx^2}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{f^{a+bx^2}}{x^4} dx = -\frac{f^{a+bx^2}}{3x^3} - \frac{2bf^{a+bx^2}\log(f)}{3x} + \frac{2}{3}b^{3/2}f^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\log^{\frac{3}{2}}(f)$$

output
```
-1/3*f^(b*x^2+a)/x^3-2/3*b*f^(b*x^2+a)*ln(f)/x+2/3*b^(3/2)*f^a*Pi^(1/2)*er
fi(b^(1/2)*x*ln(f)^(1/2))*ln(f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{f^{a+bx^2}}{x^4} dx = \frac{1}{3}f^a\left(2b^{3/2}\sqrt{\pi}\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\log^{\frac{3}{2}}(f) - \frac{f^{bx^2}(1+2bx^2\log(f))}{x^3}\right)$$

input
```
Integrate[f^(a + b*x^2)/x^4,x]
```

output
```
(f^a*(2*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Log[f]^(3/2) - (f^(b
*x^2)*(1 + 2*b*x^2*Log[f]))/x^3))/3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^4} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{3} b \log(f) \int \frac{f^{bx^2+a}}{x^2} dx - \frac{f^{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{3} b \log(f) \left(2b \log(f) \int f^{bx^2+a} dx - \frac{f^{a+bx^2}}{x}\right) - \frac{f^{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{3} b \log(f) \left(\sqrt{\pi} \sqrt{b} f^a \sqrt{\log(f)} \text{erfi}\left(\sqrt{b} x \sqrt{\log(f)}\right) - \frac{f^{a+bx^2}}{x}\right) - \frac{f^{a+bx^2}}{3x^3}$$

input
```
Int[f^(a + b*x^2)/x^4,x]
```

output
```
-1/3*f^(a + b*x^2)/x^3 + (2*b*(-(f^(a + b*x^2)/x) + Sqrt[b]*f^a*Sqrt[Pi]*E
rfi[Sqrt[b]*x*Sqrt[Log[f]]]*Sqrt[Log[f]])*Log[f])/3
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{f^{b\,x^{2}} f^{a}}{3 x^{3}}-\frac{2 f^{a} f^{b\,x^{2}} b \ln\left(f\right)}{3 x}+\frac{2 f^{a} \ln\left(f\right)^{2} b^{2} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b\ln\left(f\right)}\, x\right)}{3 \sqrt{-b\ln\left(f\right)}}$	67
meijerg	$\frac{f^{a} \ln\left(f\right)^{\frac{3}{2}} b^{2} \left(-\frac{2 \left(2 b\,x^{2} \ln\left(f\right)+1\right) {\mathrm e}^{b\,x^{2} \ln\left(f\right)}}{3 x^{3} \left(-b\right)^{\frac{3}{2}} \ln\left(f\right)^{\frac{3}{2}}}+\frac{4 b^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln\left(f\right)}\right)}{3 \left(-b\right)^{\frac{3}{2}}}\right)}{2 \sqrt{-b}}$	74

input
```
int(f^(b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^(b*x^2)*f^a/x^3-2/3*f^a/x*f^(b*x^2)*b*ln(f)+2/3*f^a*ln(f)^2*b^2*Pi^
(1/2)/(-b*ln(f))^(1/2)*erf((-b*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{f^{a+bx^2}}{x^4}\,dx = -\frac{2\,\sqrt{\pi}\sqrt{-b\log\left(f\right)}bf^{a}x^{3}\operatorname{erf}\left(\sqrt{-b\log\left(f\right)}x\right)\log\left(f\right)+\left(2\,bx^{2}\log\left(f\right)+1\right)f^{bx^{2}+a}}{3\,x^{3}}$$

input
```
integrate(f^(b*x^2+a)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(pi)*sqrt(-b*log(f))*b*f^a*x^3*erf(sqrt(-b*log(f))*x)*log(f) +
 (2*b*x^2*log(f) + 1)*f^(b*x^2 + a))/x^3
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^4}\,dx = \int \frac{f^{a+bx^2}}{x^4}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**4,x)
```

output
```
Integral(f**(a + b*x**2)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int \frac{f^{a+bx^2}}{x^4}\,dx = -\frac{\left(-bx^2\log\left(f\right)\right)^{\frac{3}{2}} f^a\Gamma\left(-\frac{3}{2}, -bx^2\log\left(f\right)\right)}{2\,x^3}$$

input
```
integrate(f^(b*x^2+a)/x^4,x, algorithm="maxima")
```

output
```
-1/2*(-b*x^2*log(f))^(3/2)*f^a*gamma(-3/2, -b*x^2*log(f))/x^3
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^4}\,dx = \int \frac{f^{bx^2+a}}{x^4}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^4,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int \frac{f^{a+bx^2}}{x^4}\,dx = \frac{2\,b^2\,f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(f)}{\sqrt{b\,\ln(f)}}\right)\,\ln\left(f\right)^2}{3\,\sqrt{b\,\ln\left(f\right)}} - \frac{\frac{f^a\,f^{b\,x^2}}{3}+\frac{2\,b\,f^a\,f^{b\,x^2}\,x^2\,\ln(f)}{3}}{x^3}$$

input
```
int(f^(a + b*x^2)/x^4,x)
```

output
```
(2*b^2*f^a*pi^(1/2)*erfi((b*x*log(f))/(b*log(f))^(1/2))*log(f)^2)/(3*(b*lo
g(f))^(1/2)) - ((f^a*f^(b*x^2))/3 + (2*b*f^a*f^(b*x^2)*x^2*log(f))/3)/x^3
```

Reduce [F]

$$\int \frac{f^{a+bx^2}}{x^4}\,dx = f^a\left(\int \frac{f^{b\,x^2}}{x^4}dx\right)$$

input
```
int(f^(b*x^2+a)/x^4,x)
```

output
```
f**a*int(f**(b*x**2)/x**4,x)
```

3.28 $\int \frac{f^{a+bx^2}}{x^6} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 96

$$\int \frac{f^{a+bx^2}}{x^6} \, dx = -\frac{f^{a+bx^2}}{5x^5} - \frac{2bf^{a+bx^2}\log(f)}{15x^3} - \frac{4b^2f^{a+bx^2}\log^2(f)}{15x} + \frac{4}{15}b^{5/2}f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\log^{\frac{5}{2}}(f)$$

output
```
-1/5*f^(b*x^2+a)/x^5-2/15*b*f^(b*x^2+a)*ln(f)/x^3-4/15*b^2*f^(b*x^2+a)*ln(
f)^2/x+4/15*b^(5/2)*f^a*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2))*ln(f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{f^{a+bx^2}}{x^6} \, dx = \frac{f^a\left(4b^{5/2}\sqrt{\pi}x^5\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\log^{\frac{5}{2}}(f) - f^{bx^2}\left(3+2bx^2\log(f)+4b^2x^4\log^2(f)\right)\right)}{15x^5}$$

input
```
Integrate[f^(a + b*x^2)/x^6,x]
```

output
```
(f^a*(4*b^(5/2)*Sqrt[Pi]*x^5*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Log[f]^(5/2) - f
^(b*x^2)*(3 + 2*b*x^2*Log[f] + 4*b^2*x^4*Log[f]^2)))/(15*x^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2643, 2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^6}dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(f)\int \frac{f^{bx^2+a}}{x^4}dx-\frac{f^{a+bx^2}}{5x^5}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\int \frac{f^{bx^2+a}}{x^2}dx-\frac{f^{a+bx^2}}{3x^3}\right)-\frac{f^{a+bx^2}}{5x^5}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(2b\log(f)\int f^{bx^2+a}dx-\frac{f^{a+bx^2}}{x}\right)-\frac{f^{a+bx^2}}{3x^3}\right)-\frac{f^{a+bx^2}}{5x^5}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\text{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)-\frac{f^{a+bx^2}}{x}\right)-\frac{f^{a+bx^2}}{3x^3}\right)-\frac{f^{a+bx^2}}{5x^5}$$

input
```
Int[f^(a + b*x^2)/x^6,x]
```

output
```
-1/5*f^(a + b*x^2)/x^5 + (2*b*Log[f]*(-1/3*f^(a + b*x^2)/x^3 + (2*b*(-(f^(
a + b*x^2)/x) + Sqrt[b]*f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Sqrt[Log
[f]])*Log[f])/3))/5
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

method	result	size
meijerg	$-\frac{f^{a} \ln\left(f\right)^{\frac{5}{2}} b^{3} \left(-\frac{2 \left(\frac{4 b^{2} x^{4} \ln\left(f\right)^{2}}{3}+\frac{2 b \,x^{2} \ln\left(f\right)}{3}+1\right) {\mathrm e}^{b \,x^{2} \ln\left(f\right)}}{5 x^{5} \left(-b\right)^{\frac{5}{2}} \ln\left(f\right)^{\frac{5}{2}}}+\frac{8 b^{\frac{5}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln\left(f\right)}\right)}{15 \left(-b\right)^{\frac{5}{2}}}\right)}{2 \sqrt{-b}}$	86
risch	$-\frac{f^{a} f^{b \,x^{2}}}{5 x^{5}}-\frac{2 f^{a} \ln\left(f\right) b \,f^{b \,x^{2}}}{15 x^{3}}-\frac{4 f^{a} \ln\left(f\right)^{2} b^{2} f^{b \,x^{2}}}{15 x}+\frac{4 f^{a} \ln\left(f\right)^{3} b^{3} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln\left(f\right)}\, x\right)}{15 \sqrt{-b \ln\left(f\right)}}$	89

input
```
int(f^(b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*ln(f)^(5/2)*b^3/(-b)^(1/2)*(-2/5/x^5/(-b)^(5/2)/ln(f)^(5/2)*(4/3*
b^2*x^4*ln(f)^2+2/3*b*x^2*ln(f)+1)*exp(b*x^2*ln(f))+8/15/(-b)^(5/2)*b^(5/2
)*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+bx^2}}{x^6}\,dx = -\frac{4\sqrt{\pi}\sqrt{-b\log(f)}b^2f^ax^5\,\mathrm{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^2 + \left(4\,b^2x^4\log(f)^2 + 2\,bx^2\log(f) + 3\right)f^{bx^2+a}}{15\,x^5}$$

input
```
integrate(f^(b*x^2+a)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(4*sqrt(pi)*sqrt(-b*log(f))*b^2*f^a*x^5*erf(sqrt(-b*log(f))*x)*log(f
)^2 + (4*b^2*x^4*log(f)^2 + 2*b*x^2*log(f) + 3)*f^(b*x^2 + a))/x^5
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^6}\,dx = \int \frac{f^{a+bx^2}}{x^6}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**6,x)
```

output
```
Integral(f**(a + b*x**2)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.29

$$\int \frac{f^{a+bx^2}}{x^6}\,dx = -\frac{\left(-bx^2\log(f)\right)^{\frac{5}{2}}\,f^a\Gamma\left(-\frac{5}{2}, -bx^2\log(f)\right)}{2\,x^5}$$

input
```
integrate(f^(b*x^2+a)/x^6,x, algorithm="maxima")
```

output
```
-1/2*(-b*x^2*log(f))^(5/2)*f^a*gamma(-5/2, -b*x^2*log(f))/x^5
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^6}\,dx = \int \frac{f^{bx^2+a}}{x^6}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^6,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14

$$\int \frac{f^{a+bx^2}}{x^6}\,dx = \frac{4\,f^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,x^2\,\ln(f)}\right)\,\left(-b\,x^2\,\ln(f)\right)^{5/2}}{15\,x^5} - \frac{4\,f^a\,\sqrt{\pi}\,\left(-b\,x^2\,\ln(f)\right)^{5/2}}{15\,x^5} - \frac{f^a\,f^{b\,x^2}}{5\,x^5} - \frac{4\,b^2\,f^a\,f^{b\,x^2}\,\ln(f)^2}{15\,x} - \frac{2\,b\,f^a\,f^{b\,x^2}\,\ln(f)}{15\,x^3}$$

input
```
int(f^(a + b*x^2)/x^6,x)
```

output
```
(4*f^a*pi^(1/2)*erfc((-b*x^2*log(f))^(1/2))*(-b*x^2*log(f))^(5/2))/(15*x^5
) - (4*f^a*pi^(1/2)*(-b*x^2*log(f))^(5/2))/(15*x^5) - (f^a*f^(b*x^2))/(5*x
^5) - (4*b^2*f^a*f^(b*x^2)*log(f)^2)/(15*x) - (2*b*f^a*f^(b*x^2)*log(f))/(
15*x^3)
```

Reduce [F]

$$\int \frac{f^{a+bx^2}}{x^6} \, dx = f^a \left(\int \frac{f^{b\,x^2}}{x^6} dx \right)$$

input
```
int(f^(b*x^2+a)/x^6,x)
```

output
```
f**a*int(f**(b*x**2)/x**6,x)
```

3.29 $\int \frac{f^{a+bx^2}}{x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = -\frac{f^{a+bx^2}}{7x^7} - \frac{2bf^{a+bx^2}\log(f)}{35x^5} - \frac{4b^2f^{a+bx^2}\log^2(f)}{105x^3} - \frac{8b^3f^{a+bx^2}\log^3(f)}{105x} + \frac{8}{105}b^{7/2}f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\log^{\frac{7}{2}}(f)$$

output
```
-1/7*f^(b*x^2+a)/x^7-2/35*b*f^(b*x^2+a)*ln(f)/x^5-4/105*b^2*f^(b*x^2+a)*ln
(f)^2/x^3-8/105*b^3*f^(b*x^2+a)*ln(f)^3/x+8/105*b^(7/2)*f^a*Pi^(1/2)*erfi(
b^(1/2)*x*ln(f)^(1/2))*ln(f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = \frac{f^a\left(8b^{7/2}\sqrt{\pi}x^7\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)\log^{\frac{7}{2}}(f) - f^{bx^2}\left(15+6bx^2\log(f)+4b^2x^4\log^2(f)+8b^3x^6\log^3(f)\right)\right)}{105x^7}$$

input
```
Integrate[f^(a + b*x^2)/x^8,x]
```

output
```
(f^a*(8*b^(7/2)*Sqrt[Pi]*x^7*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Log[f]^(7/2) - f
^(b*x^2)*(15 + 6*b*x^2*Log[f] + 4*b^2*x^4*Log[f]^2 + 8*b^3*x^6*Log[f]^3)))
/(105*x^7)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2643, 2643, 2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^8} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \int \frac{f^{bx^2+a}}{x^6} dx - \frac{f^{a+bx^2}}{7x^7}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \left(\frac{2}{5} b \log(f) \int \frac{f^{bx^2+a}}{x^4} dx - \frac{f^{a+bx^2}}{5x^5}\right) - \frac{f^{a+bx^2}}{7x^7}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \left(\frac{2}{5} b \log(f) \left(\frac{2}{3} b \log(f) \int \frac{f^{bx^2+a}}{x^2} dx - \frac{f^{a+bx^2}}{3x^3}\right) - \frac{f^{a+bx^2}}{5x^5}\right) - \frac{f^{a+bx^2}}{7x^7}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \left(\frac{2}{5} b \log(f) \left(\frac{2}{3} b \log(f) \left(2b \log(f) \int f^{bx^2+a} dx - \frac{f^{a+bx^2}}{x}\right) - \frac{f^{a+bx^2}}{3x^3}\right) - \frac{f^{a+bx^2}}{5x^5}\right) - \frac{f^{a+bx^2}}{7x^7}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{7}b\log(f)\left(\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\operatorname{erfi}\left(\sqrt{b}x\sqrt{\log(f)}\right)-\frac{f^{a+bx^2}}{x}\right)-\frac{f^{a+bx^2}}{3x^3}\right)-\frac{f^{a+bx^2}}{5x^5}\right)-\frac{f^{a+bx^2}}{7x^7}$$

input
```
Int[f^(a + b*x^2)/x^8,x]
```

output
```
-1/7*f^(a + b*x^2)/x^7 + (2*b*Log[f]*(-1/5*f^(a + b*x^2)/x^5 + (2*b*Log[f]
*(-1/3*f^(a + b*x^2)/x^3 + (2*b*(-(f^(a + b*x^2)/x) + Sqrt[b]*f^a*Sqrt[Pi]
*Erfi[Sqrt[b]*x*Sqrt[Log[f]]]*Sqrt[Log[f]])*Log[f])/3))/5))/7
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{f^{a} b^{4} \ln\left(f\right)^{\frac{7}{2}} \left(-\frac{2 \left(\frac{8 b^{3} x^{6} \ln\left(f\right)^{3}}{15}+\frac{4 b^{2} x^{4} \ln\left(f\right)^{2}}{15}+\frac{2 b \,x^{2} \ln\left(f\right)}{5}+1\right) {\mathrm e}^{b \,x^{2} \ln\left(f\right)}}{7 x^{7} \left(-b\right)^{\frac{7}{2}} \ln\left(f\right)^{\frac{7}{2}}}+\frac{16 b^{\frac{7}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln\left(f\right)}\right)}{105 \left(-b\right)^{\frac{7}{2}}}\right)}{2 \sqrt{-b}}$	98
risch	$-\frac{f^{b \,x^{2}} f^{a}}{7 x^{7}}-\frac{2 f^{a} \ln\left(f\right) b \,f^{b \,x^{2}}}{35 x^{5}}-\frac{4 f^{a} \ln\left(f\right)^{2} b^{2} f^{b \,x^{2}}}{105 x^{3}}-\frac{8 f^{a} \ln\left(f\right)^{3} b^{3} f^{b \,x^{2}}}{105 x}+\frac{8 f^{a} \ln\left(f\right)^{4} b^{4} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln\left(f\right)}\, x\right)}{105 \sqrt{-b \ln\left(f\right)}}$	111

input
```
int(f^(b*x^2+a)/x^8,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*b^4*ln(f)^(7/2)/(-b)^(1/2)*(-2/7/x^7/(-b)^(7/2)/ln(f)^(7/2)*(8/15*
b^3*x^6*ln(f)^3+4/15*b^2*x^4*ln(f)^2+2/5*b*x^2*ln(f)+1)*exp(b*x^2*ln(f))+1
6/105/(-b)^(7/2)*b^(7/2)*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = -\frac{8\sqrt{\pi}\sqrt{-b\log(f)}b^3f^ax^7\operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^3+\left(8\,b^3x^6\log(f)^3+4\,b^2x^4\log(f)^2+6\,bx^2\log(f$$

$$105\,x^7$$

input
```
integrate(f^(b*x^2+a)/x^8,x, algorithm="fricas")
```

output
```
-1/105*(8*sqrt(pi)*sqrt(-b*log(f))*b^3*f^a*x^7*erf(sqrt(-b*log(f))*x)*log(
f)^3 + (8*b^3*x^6*log(f)^3 + 4*b^2*x^4*log(f)^2 + 6*b*x^2*log(f) + 15)*f^(
b*x^2 + a))/x^7
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = \int \frac{f^{a+bx^2}}{x^8}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**8,x)
```

output
```
Integral(f**(a + b*x**2)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.24

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = -\frac{\left(-bx^2\log\left(f\right)\right)^{\frac{7}{2}} f^a\Gamma\left(-\frac{7}{2}, -bx^2\log\left(f\right)\right)}{2\,x^7}$$

input
```
integrate(f^(b*x^2+a)/x^8,x, algorithm="maxima")
```

output
```
-1/2*(-b*x^2*log(f))^(7/2)*f^a*gamma(-7/2, -b*x^2*log(f))/x^7
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = \int \frac{f^{bx^2+a}}{x^8}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^8,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = \frac{8\,f^a\,\sqrt{\pi}\,\left(-b\,x^2\,\ln\left(f\right)\right)^{7/2}}{105\,x^7} - \frac{f^a\,f^{b\,x^2}}{7\,x^7} - \frac{8\,f^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,x^2\,\ln\left(f\right)}\right)\,\left(-b\,x^2\,\ln\left(f\right)\right)^{7/2}}{105\,x^7} - \frac{4\,b^2\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^2}{105\,x^3} - \frac{8\,b^3\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^3}{105\,x} - \frac{2\,b\,f^a\,f^{b\,x^2}\,\ln\left(f\right)}{35\,x^5}$$

input
```
int(f^(a + b*x^2)/x^8,x)
```

output
```
(8*f^a*pi^(1/2)*(-b*x^2*log(f))^(7/2))/(105*x^7) - (f^a*f^(b*x^2))/(7*x^7)
 - (8*f^a*pi^(1/2)*erfc((-b*x^2*log(f))^(1/2))*(-b*x^2*log(f))^(7/2))/(105
*x^7) - (4*b^2*f^a*f^(b*x^2)*log(f)^2)/(105*x^3) - (8*b^3*f^a*f^(b*x^2)*lo
g(f)^3)/(105*x) - (2*b*f^a*f^(b*x^2)*log(f))/(35*x^5)
```

Reduce [F]

$$\int \frac{f^{a+bx^2}}{x^8}\,dx = f^a\left(\int \frac{f^{b\,x^2}}{x^8}dx\right)$$

input
```
int(f^(b*x^2+a)/x^8,x)
```

output
```
f**a*int(f**(b*x**2)/x**8,x)
```

3.30 $\int \frac{f^{a+bx^2}}{x^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+bx^2}}{x^{10}} \, dx = -\frac{f^a\Gamma\left(-\frac{9}{2}, -bx^2\log(f)\right)\left(-bx^2\log(f)\right)^{9/2}}{2x^9}$$

output
```
-1/2*f^a*(-32/945*Pi^(1/2)*erfc((-b*x^2*ln(f))^(1/2))+32/945/(-b*x^2*ln(f)
)^(1/2)*exp(b*x^2*ln(f))-16/945/(-b*x^2*ln(f))^(3/2)*exp(b*x^2*ln(f))+8/31
5/(-b*x^2*ln(f))^(5/2)*exp(b*x^2*ln(f))-4/63/(-b*x^2*ln(f))^(7/2)*exp(b*x^
2*ln(f))+2/9/(-b*x^2*ln(f))^(9/2)*exp(b*x^2*ln(f)))*(-b*x^2*ln(f))^(9/2)/x
^9
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x^{10}} \, dx = -\frac{f^a\Gamma\left(-\frac{9}{2}, -bx^2\log(f)\right)\left(-bx^2\log(f)\right)^{9/2}}{2x^9}$$

input
```
Integrate[f^(a + b*x^2)/x^10,x]
```

output
```
-1/2*(f^a*Gamma[-9/2, -(b*x^2*Log[f])]*(-(b*x^2*Log[f]))^(9/2))/x^9
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{f^a\left(-bx^2\log(f)\right)^{9/2}\Gamma\left(-\frac{9}{2},-bx^2\log(f)\right)}{2x^9}$$

input
```
Int[f^(a + b*x^2)/x^10,x]
```

output
```
-1/2*(f^a*Gamma[-9/2, -(b*x^2*Log[f])]*(-(b*x^2*Log[f]))^(9/2))/x^9
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.24

method	result
meijerg	$-\frac{f^{a} b^{5} \ln(f)^{\frac{9}{2}} \left(-\frac{2\left(\frac{16 b^{4} x^{8} \ln(f)^{4}}{105}+\frac{8 b^{3} x^{6} \ln(f)^{3}}{105}+\frac{4 b^{2} x^{4} \ln(f)^{2}}{35}+\frac{2 b\,x^{2} \ln(f)}{7}+1\right) \mathrm{e}^{b\,x^{2} \ln(f)}}{9 x^{9} (-b)^{\frac{9}{2}} \ln(f)^{\frac{9}{2}}}+\frac{32 b^{\frac{9}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln(f)}\right)}{945 (-b)^{\frac{9}{2}}}\right)}{2 \sqrt{-b}}$
risch	$-\frac{f^{b\,x^{2}} f^{a}}{9 x^{9}}-\frac{2 f^{a} \ln(f) b\, f^{b\,x^{2}}}{63 x^{7}}-\frac{4 f^{a} \ln(f)^{2} b^{2} f^{b\,x^{2}}}{315 x^{5}}-\frac{8 f^{a} \ln(f)^{3} b^{3} f^{b\,x^{2}}}{945 x^{3}}-\frac{16 f^{a} \ln(f)^{4} b^{4} f^{b\,x^{2}}}{945 x}+\frac{16 f^{a} \ln(f)^{5} b^{5} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-\right.}{945 \sqrt{-b \ln(f)}}$

input
```
int(f^(b*x^2+a)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*b^5*ln(f)^(9/2)/(-b)^(1/2)*(-2/9/x^9/(-b)^(9/2)/ln(f)^(9/2)*(16/1
05*b^4*x^8*ln(f)^4+8/105*b^3*x^6*ln(f)^3+4/35*b^2*x^4*ln(f)^2+2/7*b*x^2*ln
(f)+1)*exp(b*x^2*ln(f))+32/945/(-b)^(9/2)*b^(9/2)*Pi^(1/2)*erfi(b^(1/2)*x*
ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.85

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx =$$
$$-\frac{16\sqrt{\pi}\sqrt{-b\log(f)}b^4 f^a x^9 \operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^4+\left(16\,b^4x^8\log(f)^4+8\,b^3x^6\log(f)^3+12\,b^2x^4\log\right.}{945\,x^9}$$

input
```
integrate(f^(b*x^2+a)/x^10,x, algorithm="fricas")
```

output
```
-1/945*(16*sqrt(pi)*sqrt(-b*log(f))*b^4*f^a*x^9*erf(sqrt(-b*log(f))*x)*log
(f)^4 + (16*b^4*x^8*log(f)^4 + 8*b^3*x^6*log(f)^3 + 12*b^2*x^4*log(f)^2 +
30*b*x^2*log(f) + 105)*f^(b*x^2 + a))/x^9
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx = \int \frac{f^{a+bx^2}}{x^{10}}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**10,x)
```

output
```
Integral(f**(a + b*x**2)/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx = -\frac{\left(-bx^2\log\left(f\right)\right)^{\frac{9}{2}} f^a\Gamma\left(-\frac{9}{2}, -bx^2\log\left(f\right)\right)}{2\,x^9}$$

input
```
integrate(f^(b*x^2+a)/x^10,x, algorithm="maxima")
```

output
```
-1/2*(-b*x^2*log(f))^(9/2)*f^a*gamma(-9/2, -b*x^2*log(f))/x^9
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx = \int \frac{f^{bx^2+a}}{x^{10}}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^10,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 4.50

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx = \frac{16\,f^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,x^2\,\ln\left(f\right)}\right)\,\left(-b\,x^2\,\ln\left(f\right)\right)^{9/2}}{945\,x^9} - \frac{16\,f^a\,\sqrt{\pi}\,\left(-b\,x^2\,\ln\left(f\right)\right)^{9/2}}{945\,x^9} - \frac{f^a\,f^{b\,x^2}}{9\,x^9} - \frac{4\,b^2\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^2}{315\,x^5} - \frac{8\,b^3\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^3}{945\,x^3} - \frac{16\,b^4\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^4}{945\,x} - \frac{2\,b\,f^a\,f^{b\,x^2}\,\ln\left(f\right)}{63\,x^7}$$

input
```
int(f^(a + b*x^2)/x^10,x)
```

output
```
(16*f^a*pi^(1/2)*erfc((-b*x^2*log(f))^(1/2))*(-b*x^2*log(f))^(9/2))/(945*x
^9) - (16*f^a*pi^(1/2)*(-b*x^2*log(f))^(9/2))/(945*x^9) - (f^a*f^(b*x^2))/
(9*x^9) - (4*b^2*f^a*f^(b*x^2)*log(f)^2)/(315*x^5) - (8*b^3*f^a*f^(b*x^2)*
log(f)^3)/(945*x^3) - (16*b^4*f^a*f^(b*x^2)*log(f)^4)/(945*x) - (2*b*f^a*f
^(b*x^2)*log(f))/(63*x^7)
```

Reduce [F]

$$\int \frac{f^{a+bx^2}}{x^{10}}\,dx = f^a\left(\int \frac{f^{b\,x^2}}{x^{10}}dx\right)$$

input
```
int(f^(b*x^2+a)/x^10,x)
```

output
```
f**a*int(f**(b*x**2)/x**10,x)
```

3.31 $\int \frac{f^{a+bx^2}}{x^{12}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = -\frac{f^a\Gamma\left(-\frac{11}{2},-bx^2\log(f)\right)\left(-bx^2\log(f)\right)^{11/2}}{2x^{11}}$$

output
```
-1/2*f^a*(64/10395*Pi^(1/2)*erfc((-b*x^2*ln(f))^(1/2))-64/10395/(-b*x^2*ln
(f))^(1/2)*exp(b*x^2*ln(f))+32/10395/(-b*x^2*ln(f))^(3/2)*exp(b*x^2*ln(f))
-16/3465/(-b*x^2*ln(f))^(5/2)*exp(b*x^2*ln(f))+8/693/(-b*x^2*ln(f))^(7/2)*
exp(b*x^2*ln(f))-4/99/(-b*x^2*ln(f))^(9/2)*exp(b*x^2*ln(f))+2/11/(-b*x^2*l
n(f))^(11/2)*exp(b*x^2*ln(f)))*(-b*x^2*ln(f))^(11/2)/x^11
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = -\frac{f^a\Gamma\left(-\frac{11}{2},-bx^2\log(f)\right)\left(-bx^2\log(f)\right)^{11/2}}{2x^{11}}$$

input
```
Integrate[f^(a + b*x^2)/x^12,x]
```

output
```
-1/2*(f^a*Gamma[-11/2, -(b*x^2*Log[f])]*(-(b*x^2*Log[f]))^(11/2))/x^11
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^2}}{x^{12}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{f^a \left(-bx^2 \log(f)\right)^{11/2} \Gamma\left(-\frac{11}{2}, -bx^2 \log(f)\right)}{2x^{11}}$$

input
```
Int[f^(a + b*x^2)/x^12,x]
```

output
```
-1/2*(f^a*Gamma[-11/2, -(b*x^2*Log[f])]*(-(b*x^2*Log[f]))^(11/2))/x^11
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 3.59

method	result
meijerg	$\frac{f^{a} b^{6} \ln(f)^{\frac{11}{2}} \left(-\frac{2\left(\frac{32 b^{5} x^{10} \ln(f)^{5}}{945}+\frac{16 b^{4} x^{8} \ln(f)^{4}}{945}+\frac{8 b^{3} x^{6} \ln(f)^{3}}{315}+\frac{4 b^{2} x^{4} \ln(f)^{2}}{63}+\frac{2 b x^{2} \ln(f)}{9}+1\right) e^{b x^{2} \ln(f)}}{11 x^{11} (-b)^{\frac{11}{2}} \ln(f)^{\frac{11}{2}}}+\frac{64 b^{\frac{11}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x \sqrt{\ln(f)}\right)}{10395 (-b)^{\frac{11}{2}}}\right.}{2\sqrt{-b}}$
risch	$-\frac{f^{b x^{2}} f^{a}}{11 x^{11}}-\frac{2 f^{a} \ln(f) b f^{b x^{2}}}{99 x^{9}}-\frac{4 f^{a} \ln(f)^{2} b^{2} f^{b x^{2}}}{693 x^{7}}-\frac{8 f^{a} \ln(f)^{3} b^{3} f^{b x^{2}}}{3465 x^{5}}-\frac{16 f^{a} \ln(f)^{4} b^{4} f^{b x^{2}}}{10395 x^{3}}-\frac{32 f^{a} \ln(f)^{5} b^{5} f^{b x^{2}}}{10395 x}+\frac{32}{}$

input
```
int(f^(b*x^2+a)/x^12,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*b^6*ln(f)^(11/2)/(-b)^(1/2)*(-2/11/x^11/(-b)^(11/2)/ln(f)^(11/2)*(
32/945*b^5*x^10*ln(f)^5+16/945*b^4*x^8*ln(f)^4+8/315*b^3*x^6*ln(f)^3+4/63*
b^2*x^4*ln(f)^2+2/9*b*x^2*ln(f)+1)*exp(b*x^2*ln(f))+64/10395/(-b)^(11/2)*b
^(11/2)*Pi^(1/2)*erfi(b^(1/2)*x*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.21

$$\int \frac{f^{a+bx^2}}{x^{12}}\, dx = -\frac{32\sqrt{\pi}\sqrt{-b\log(f)} b^5 f^a x^{11} \operatorname{erf}\left(\sqrt{-b\log(f)}x\right)\log(f)^5 + \left(32 b^5 x^{10}\log(f)^5 + 16 b^4 x^8 \log(f)^4 + 24 b^3 x\right.}{10395\, x^{11}}$$

input
```
integrate(f^(b*x^2+a)/x^12,x, algorithm="fricas")
```

output
```
-1/10395*(32*sqrt(pi)*sqrt(-b*log(f))*b^5*f^a*x^11*erf(sqrt(-b*log(f))*x)*
log(f)^5 + (32*b^5*x^10*log(f)^5 + 16*b^4*x^8*log(f)^4 + 24*b^3*x^6*log(f)
^3 + 60*b^2*x^4*log(f)^2 + 210*b*x^2*log(f) + 945)*f^(b*x^2 + a))/x^11
```

Sympy [F]

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = \int \frac{f^{a+bx^2}}{x^{12}}\,dx$$

input
```
integrate(f**(b*x**2+a)/x**12,x)
```

output
```
Integral(f**(a + b*x**2)/x**12, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = -\frac{\left(-bx^2\log\left(f\right)\right)^{\frac{11}{2}} f^a\Gamma\left(-\frac{11}{2}, -bx^2\log\left(f\right)\right)}{2\,x^{11}}$$

input
```
integrate(f^(b*x^2+a)/x^12,x, algorithm="maxima")
```

output
```
-1/2*(-b*x^2*log(f))^(11/2)*f^a*gamma(-11/2, -b*x^2*log(f))/x^11
```

Giac [F]

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = \int \frac{f^{bx^2+a}}{x^{12}}\,dx$$

input
```
integrate(f^(b*x^2+a)/x^12,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)/x^12, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 175, normalized size of antiderivative = 5.15

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = \frac{32\,f^a\,\sqrt{\pi}\,\left(-b\,x^2\,\ln\left(f\right)\right)^{11/2}}{10395\,x^{11}} - \frac{f^a\,f^{b\,x^2}}{11\,x^{11}} - \frac{32\,f^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,x^2\,\ln\left(f\right)}\right)\,\left(-b\,x^2\,\ln\left(f\right)\right)^{11/2}}{10395\,x^{11}} - \frac{4\,b^2\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^2}{693\,x^7} - \frac{8\,b^3\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^3}{3465\,x^5} - \frac{16\,b^4\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^4}{10395\,x^3} - \frac{32\,b^5\,f^a\,f^{b\,x^2}\,\ln\left(f\right)^5}{10395\,x} - \frac{2\,b\,f^a\,f^{b\,x^2}\,\ln\left(f\right)}{99\,x^9}$$

input
```
int(f^(a + b*x^2)/x^12,x)
```

output
```
(32*f^a*pi^(1/2)*(-b*x^2*log(f))^(11/2))/(10395*x^11) - (f^a*f^(b*x^2))/(1
1*x^11) - (32*f^a*pi^(1/2)*erfc((-b*x^2*log(f))^(1/2))*(-b*x^2*log(f))^(11
/2))/(10395*x^11) - (4*b^2*f^a*f^(b*x^2)*log(f)^2)/(693*x^7) - (8*b^3*f^a*
f^(b*x^2)*log(f)^3)/(3465*x^5) - (16*b^4*f^a*f^(b*x^2)*log(f)^4)/(10395*x^
3) - (32*b^5*f^a*f^(b*x^2)*log(f)^5)/(10395*x) - (2*b*f^a*f^(b*x^2)*log(f)
)/(99*x^9)
```

Reduce [F]

$$\int \frac{f^{a+bx^2}}{x^{12}}\,dx = f^a\left(\int \frac{f^{b\,x^2}}{x^{12}}dx\right)$$

input
```
int(f^(b*x^2+a)/x^12,x)
```

output
```
f**a*int(f**(b*x**2)/x**12,x)
```

3.32 $\int f^{a+bx^3}x^{17}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int f^{a+bx^3}x^{17}\,dx = -\frac{f^{a+bx^3}\left(120-120bx^3\log(f)+60b^2x^6\log^2(f)-20b^3x^9\log^3(f)+5b^4x^{12}\log^4(f)-b^5x^{15}\log^5(f)\right)}{3b^6\log^6(f)}$$

output
```
-1/3*f^(b*x^3+a)*(120-120*b*x^3*ln(f)+60*b^2*x^6*ln(f)^2-20*b^3*x^9*ln(f)^
3+5*b^4*x^12*ln(f)^4-b^5*x^15*ln(f)^5)/b^6/ln(f)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int f^{a+bx^3}x^{17}\,dx = -\frac{f^a\Gamma(6,-bx^3\log(f))}{3b^6\log^6(f)}$$

input
```
Integrate[f^(a + b*x^3)*x^17,x]
```

output
```
-1/3*(f^a*Gamma[6, -(b*x^3*Log[f])])/(b^6*Log[f]^6)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{17} f^{a+bx^3} \, dx$$

$$\Big\downarrow \; 2647$$

$$-\frac{f^{a+bx^3} \left(-b^5 x^{15} \log^5(f) + 5b^4 x^{12} \log^4(f) - 20b^3 x^9 \log^3(f) + 60b^2 x^6 \log^2(f) - 120bx^3 \log(f) + 120\right)}{3b^6 \log^6(f)}$$

input
```
Int[f^(a + b*x^3)*x^17,x]
```

output
```
-1/3*(f^(a + b*x^3)*(120 - 120*b*x^3*Log[f] + 60*b^2*x^6*Log[f]^2 - 20*b^3
*x^9*Log[f]^3 + 5*b^4*x^12*Log[f]^4 - b^5*x^15*Log[f]^5))/(b^6*Log[f]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

method	result
gosper	$\frac{\left(b^{5} x^{15} \ln \left(f\right)^{5}-5 b^{4} x^{12} \ln \left(f\right)^{4}+20 b^{3} x^{9} \ln \left(f\right)^{3}-60 b^{2} x^{6} \ln \left(f\right)^{2}+120 b \,x^{3} \ln \left(f\right)-120\right) f^{b \,x^{3}+a}}{3 b^{6} \ln \left(f\right)^{6}}$
risch	$\frac{\left(b^{5} x^{15} \ln \left(f\right)^{5}-5 b^{4} x^{12} \ln \left(f\right)^{4}+20 b^{3} x^{9} \ln \left(f\right)^{3}-60 b^{2} x^{6} \ln \left(f\right)^{2}+120 b \,x^{3} \ln \left(f\right)-120\right) f^{b \,x^{3}+a}}{3 b^{6} \ln \left(f\right)^{6}}$
orering	$\frac{\left(b^{5} x^{15} \ln \left(f\right)^{5}-5 b^{4} x^{12} \ln \left(f\right)^{4}+20 b^{3} x^{9} \ln \left(f\right)^{3}-60 b^{2} x^{6} \ln \left(f\right)^{2}+120 b \,x^{3} \ln \left(f\right)-120\right) f^{b \,x^{3}+a}}{3 b^{6} \ln \left(f\right)^{6}}$
meijerg	$\frac{f^{a} \left(120-\frac{\left(-6 b^{5} x^{15} \ln \left(f\right)^{5}+30 b^{4} x^{12} \ln \left(f\right)^{4}-120 b^{3} x^{9} \ln \left(f\right)^{3}+360 b^{2} x^{6} \ln \left(f\right)^{2}-720 b \,x^{3} \ln \left(f\right)+720\right) {\mathrm e}^{b \,x^{3} \ln \left(f\right)}}{6}\right)}{3 b^{6} \ln \left(f\right)^{6}}$
parallelrisch	$\frac{f^{b \,x^{3}+a} x^{15} b^{5} \ln \left(f\right)^{5}-5 f^{b \,x^{3}+a} x^{12} b^{4} \ln \left(f\right)^{4}+20 f^{b \,x^{3}+a} x^{9} b^{3} \ln \left(f\right)^{3}-60 f^{b \,x^{3}+a} x^{6} b^{2} \ln \left(f\right)^{2}+120 f^{b \,x^{3}+a} x^{3} b \ln \left(f\right)-120 f^{b \,x^{3}+}}{3 b^{6} \ln \left(f\right)^{6}}$
norman	$-\frac{40 \,{\mathrm e}^{\left(b \,x^{3}+a \right) \ln \left(f\right)}}{b^{6} \ln \left(f\right)^{6}}+\frac{40 x^{3} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln \left(f\right)}}{b^{5} \ln \left(f\right)^{5}}+\frac{x^{15} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln \left(f\right)}}{3 b \ln \left(f\right)}-\frac{20 x^{6} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln \left(f\right)}}{\ln \left(f\right)^{4} b^{4}}+\frac{20 x^{9} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln \left(f\right)}}{3 \ln \left(f\right)^{3} b^{3}}-5x^{1}$

input
```
int(f^(b*x^3+a)*x^17,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^5*x^15*ln(f)^5-5*b^4*x^12*ln(f)^4+20*b^3*x^9*ln(f)^3-60*b^2*x^6*ln(
f)^2+120*b*x^3*ln(f)-120)*f^(b*x^3+a)/b^6/ln(f)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int f^{a+bx^3} x^{17}\,dx = \frac{{\left(b^{5} x^{15} \log \left(f\right)^{5} - 5 \, b^{4} x^{12} \log \left(f\right)^{4} + 20 \, b^{3} x^{9} \log \left(f\right)^{3} - 60 \, b^{2} x^{6} \log \left(f\right)^{2} + 120 \, b x^{3} \log \left(f\right) - 120\right)} f^{b x^{3} + a}}{3 \, b^{6} \log \left(f\right)^{6}}$$

input
```
integrate(f^(b*x^3+a)*x^17,x, algorithm="fricas")
```

output
```
1/3*(b^5*x^15*log(f)^5 - 5*b^4*x^12*log(f)^4 + 20*b^3*x^9*log(f)^3 - 60*b^
2*x^6*log(f)^2 + 120*b*x^3*log(f) - 120)*f^(b*x^3 + a)/(b^6*log(f)^6)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.21

$$\int f^{a+bx^3}x^{17}\,dx$$
$$=\begin{cases} \frac{f^{a+bx^3}\left(b^5x^{15}\log(f)^5-5b^4x^{12}\log(f)^4+20b^3x^9\log(f)^3-60b^2x^6\log(f)^2+120bx^3\log(f)-120\right)}{3b^6\log(f)^6} & \text{for } b^6\log(f)^6\neq 0 \\ \frac{x^{18}}{18} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x**3+a)*x**17,x)
```

output
```
Piecewise((f**(a + b*x**3)*(b**5*x**15*log(f)**5 - 5*b**4*x**12*log(f)**4
+ 20*b**3*x**9*log(f)**3 - 60*b**2*x**6*log(f)**2 + 120*b*x**3*log(f) - 12
0)/(3*b**6*log(f)**6), Ne(b**6*log(f)**6, 0)), (x**18/18, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int f^{a+bx^3}x^{17}\,dx$$
$$=\frac{\left(b^5f^ax^{15}\log(f)^5-5\,b^4f^ax^{12}\log(f)^4+20\,b^3f^ax^9\log(f)^3-60\,b^2f^ax^6\log(f)^2+120\,bf^ax^3\log(f)-120\right.}{3\,b^6\log(f)^6}$$

input
```
integrate(f^(b*x^3+a)*x^17,x, algorithm="maxima")
```

output
```
1/3*(b^5*f^a*x^15*log(f)^5 - 5*b^4*f^a*x^12*log(f)^4 + 20*b^3*f^a*x^9*log(
f)^3 - 60*b^2*f^a*x^6*log(f)^2 + 120*b*f^a*x^3*log(f) - 120*f^a)*f^(b*x^3)
/(b^6*log(f)^6)
```

Giac [F(-2)]

Exception generated.

$$\int f^{a+bx^3}x^{17}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(f^(b*x^3+a)*x^17,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error
: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int f^{a+bx^3}x^{17}\,dx = -\frac{f^{b\,x^3+a}\left(-\frac{b^5\,x^{15}\ln(f)^5}{3}+\frac{5\,b^4\,x^{12}\ln(f)^4}{3}-\frac{20\,b^3\,x^9\ln(f)^3}{3}+20\,b^2\,x^6\ln\left(f\right)^2-40\,b\,x^3\,\ln\left(f\right)+40\right)}{b^6\ln\left(f\right)^6}$$

input
```
int(f^(a + b*x^3)*x^17,x)
```

output
```
-(f^(a + b*x^3)*(20*b^2*x^6*log(f)^2 - (20*b^3*x^9*log(f)^3)/3 + (5*b^4*x^
12*log(f)^4)/3 - (b^5*x^15*log(f)^5)/3 - 40*b*x^3*log(f) + 40))/(b^6*log(f
)^6)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int f^{a+bx^3} x^{17} \, dx$$
$$= \frac{f^{b\,x^3+a} \left(\log(f)^5 b^5 x^{15} - 5\log(f)^4 b^4 x^{12} + 20\log(f)^3 b^3 x^9 - 60\log(f)^2 b^2 x^6 + 120\log(f)\, b\, x^3 - 120\right)}{3\log(f)^6 b^6}$$

input
```
int(f^(b*x^3+a)*x^17,x)
```

output
```
(f**(a + b*x**3)*(log(f)**5*b**5*x**15 - 5*log(f)**4*b**4*x**12 + 20*log(f
)**3*b**3*x**9 - 60*log(f)**2*b**2*x**6 + 120*log(f)*b*x**3 - 120))/(3*log
(f)**6*b**6)
```

3.33 $\int f^{a+bx^3} x^{14}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int f^{a+bx^3} x^{14}\,dx = \frac{f^{a+bx^3}\left(24-24bx^3\log(f)+12b^2x^6\log^2(f)-4b^3x^9\log^3(f)+b^4x^{12}\log^4(f)\right)}{3b^5\log^5(f)}$$

output
```
1/3*f^(b*x^3+a)*(24-24*b*x^3*ln(f)+12*b^2*x^6*ln(f)^2-4*b^3*x^9*ln(f)^3+b^
4*x^12*ln(f)^4)/b^5/ln(f)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.37

$$\int f^{a+bx^3} x^{14}\,dx = \frac{f^a\Gamma(5,-bx^3\log(f))}{3b^5\log^5(f)}$$

input
```
Integrate[f^(a + b*x^3)*x^14,x]
```

output
```
(f^a*Gamma[5, -(b*x^3*Log[f])])/(3*b^5*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14} f^{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 2647}$$

$$\frac{f^{a+bx^3} \left(b^4 x^{12} \log^4(f) - 4b^3 x^9 \log^3(f) + 12b^2 x^6 \log^2(f) - 24bx^3 \log(f) + 24\right)}{3b^5 \log^5(f)}$$

input
```
Int[f^(a + b*x^3)*x^14,x]
```

output
```
(f^(a + b*x^3)*(24 - 24*b*x^3*Log[f] + 12*b^2*x^6*Log[f]^2 - 4*b^3*x^9*Log
[f]^3 + b^4*x^12*Log[f]^4))/(3*b^5*Log[f]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result	size
gosper	$\frac{f^{b\,x^3+a}\left(24-24b\,x^3\ln(f)+12b^2x^6\ln(f)^2-4b^3x^9\ln(f)^3+b^4x^{12}\ln(f)^4\right)}{3b^5\ln(f)^5}$	64
risch	$\frac{f^{b\,x^3+a}\left(24-24b\,x^3\ln(f)+12b^2x^6\ln(f)^2-4b^3x^9\ln(f)^3+b^4x^{12}\ln(f)^4\right)}{3b^5\ln(f)^5}$	64
orering	$\frac{f^{b\,x^3+a}\left(24-24b\,x^3\ln(f)+12b^2x^6\ln(f)^2-4b^3x^9\ln(f)^3+b^4x^{12}\ln(f)^4\right)}{3b^5\ln(f)^5}$	64
meijerg	$-\frac{f^a\left(24-\frac{\left(5b^4x^{12}\ln(f)^4-20b^3x^9\ln(f)^3+60b^2x^6\ln(f)^2-120b\,x^3\ln(f)+120\right)e^{b\,x^3\ln(f)}}{5}\right)}{3b^5\ln(f)^5}$	71
parallelrisch	$\frac{f^{b\,x^3+a}x^{12}b^4\ln(f)^4-4f^{b\,x^3+a}x^9b^3\ln(f)^3+12f^{b\,x^3+a}x^6b^2\ln(f)^2-24f^{b\,x^3+a}x^3b\ln(f)+24f^{b\,x^3+a}}{3\ln(f)^5b^5}$	101
norman	$\frac{8\,e^{\left(b\,x^3+a\right)\ln(f)}}{b^5\ln(f)^5}+\frac{x^{12}e^{\left(b\,x^3+a\right)\ln(f)}}{3b\ln(f)}-\frac{8x^3e^{\left(b\,x^3+a\right)\ln(f)}}{\ln(f)^4b^4}+\frac{4x^6e^{\left(b\,x^3+a\right)\ln(f)}}{\ln(f)^3b^3}-\frac{4x^9e^{\left(b\,x^3+a\right)\ln(f)}}{3\ln(f)^2b^2}$	114

input
```
int(f^(b*x^3+a)*x^14,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^(b*x^3+a)*(24-24*b*x^3*ln(f)+12*b^2*x^6*ln(f)^2-4*b^3*x^9*ln(f)^3+b^
4*x^12*ln(f)^4)/b^5/ln(f)^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int f^{a+bx^3}x^{14}\,dx = \frac{\left(b^4x^{12}\log(f)^4-4\,b^3x^9\log(f)^3+12\,b^2x^6\log(f)^2-24\,bx^3\log(f)+24\right)f^{bx^3+a}}{3\,b^5\log(f)^5}$$

input
```
integrate(f^(b*x^3+a)*x^14,x, algorithm="fricas")
```

output
```
1/3*(b^4*x^12*log(f)^4 - 4*b^3*x^9*log(f)^3 + 12*b^2*x^6*log(f)^2 - 24*b*x
^3*log(f) + 24)*f^(b*x^3 + a)/(b^5*log(f)^5)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int f^{a+bx^3}x^{14}\,dx = \begin{cases} \frac{f^{a+bx^3}\left(b^4x^{12}\log(f)^4-4b^3x^9\log(f)^3+12b^2x^6\log(f)^2-24bx^3\log(f)+24\right)}{3b^5\log(f)^5} & \text{for } b^5\log(f)^5\neq 0\\ \frac{x^{15}}{15} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x**3+a)*x**14,x)
```

output
```
Piecewise((f**(a + b*x**3)*(b**4*x**12*log(f)**4 - 4*b**3*x**9*log(f)**3 +
 12*b**2*x**6*log(f)**2 - 24*b*x**3*log(f) + 24)/(3*b**5*log(f)**5), Ne(b*
*5*log(f)**5, 0)), (x**15/15, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int f^{a+bx^3}x^{14}\,dx = \frac{\left(b^4f^ax^{12}\log(f)^4-4\,b^3f^ax^9\log(f)^3+12\,b^2f^ax^6\log(f)^2-24\,bf^ax^3\log(f)+24\,f^a\right)f^{bx^3}}{3\,b^5\log(f)^5}$$

input
```
integrate(f^(b*x^3+a)*x^14,x, algorithm="maxima")
```

output
```
1/3*(b^4*f^a*x^12*log(f)^4 - 4*b^3*f^a*x^9*log(f)^3 + 12*b^2*f^a*x^6*log(f
)^2 - 24*b*f^a*x^3*log(f) + 24*f^a)*f^(b*x^3)/(b^5*log(f)^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int f^{a+bx^3}x^{14}\,dx = \frac{b^4 f^{bx^3} f^a x^{12}\log(f)^4 - 4\,b^3 f^{bx^3} f^a x^9 \log(f)^3 + 12\,b^2 f^{bx^3} f^a x^6 \log(f)^2 - 24\,b f^{bx^3} f^a x^3 \log(f) + 24\,f^{bx^3} f^a}{3\,b^5 \log(f)^5}$$

input
```
integrate(f^(b*x^3+a)*x^14,x, algorithm="giac")
```

output
```
1/3*(b^4*f^(b*x^3)*f^a*x^12*log(f)^4 - 4*b^3*f^(b*x^3)*f^a*x^9*log(f)^3 +
12*b^2*f^(b*x^3)*f^a*x^6*log(f)^2 - 24*b*f^(b*x^3)*f^a*x^3*log(f) + 24*f^(
b*x^3)*f^a)/(b^5*log(f)^5)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int f^{a+bx^3}x^{14}\,dx = \frac{f^{b\,x^3+a}\left(\frac{b^4\,x^{12}\,\ln(f)^4}{3} - \frac{4\,b^3\,x^9\,\ln(f)^3}{3} + 4\,b^2\,x^6\,\ln(f)^2 - 8\,b\,x^3\,\ln(f) + 8\right)}{b^5\,\ln(f)^5}$$

input
```
int(f^(a + b*x^3)*x^14,x)
```

output
```
(f^(a + b*x^3)*(4*b^2*x^6*log(f)^2 - (4*b^3*x^9*log(f)^3)/3 + (b^4*x^12*lo
g(f)^4)/3 - 8*b*x^3*log(f) + 8))/(b^5*log(f)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int f^{a+bx^3}x^{14}\,dx = \frac{f^{b\,x^3+a}\left(\log(f)^4\,b^4x^{12}-4\log(f)^3\,b^3x^9+12\log(f)^2\,b^2x^6-24\log(f)\,b\,x^3+24\right)}{3\log\left(f\right)^5b^5}$$

input
```
int(f^(b*x^3+a)*x^14,x)
```

output
```
(f**(a + b*x**3)*(log(f)**4*b**4*x**12 - 4*log(f)**3*b**3*x**9 + 12*log(f)
**2*b**2*x**6 - 24*log(f)*b*x**3 + 24))/(3*log(f)**5*b**5)
```

3.34 $\int f^{a+bx^3}x^{11}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int f^{a+bx^3}x^{11}\,dx=-\frac{2f^{a+bx^3}}{b^4\log^4(f)}+\frac{2f^{a+bx^3}x^3}{b^3\log^3(f)}-\frac{f^{a+bx^3}x^6}{b^2\log^2(f)}+\frac{f^{a+bx^3}x^9}{3b\log(f)}$$

output
```
-2*f^(b*x^3+a)/b^4/ln(f)^4+2*f^(b*x^3+a)*x^3/b^3/ln(f)^3-f^(b*x^3+a)*x^6/b
^2/ln(f)^2+1/3*f^(b*x^3+a)*x^9/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

$$\int f^{a+bx^3}x^{11}\,dx=\frac{f^{a+bx^3}\left(-6+6bx^3\log(f)-3b^2x^6\log^2(f)+b^3x^9\log^3(f)\right)}{3b^4\log^4(f)}$$

input
```
Integrate[f^(a + b*x^3)*x^11,x]
```

output
```
(f^(a + b*x^3)*(-6 + 6*b*x^3*Log[f] - 3*b^2*x^6*Log[f]^2 + b^3*x^9*Log[f]^
3))/(3*b^4*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} f^{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^9 f^{a+bx^3}}{3b\log(f)} - \frac{3\int f^{bx^3+a}x^8dx}{b\log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^9 f^{a+bx^3}}{3b\log(f)} - \frac{3\left(\frac{x^6 f^{a+bx^3}}{3b\log(f)} - \frac{2\int f^{bx^3+a}x^5dx}{b\log(f)}\right)}{b\log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^9 f^{a+bx^3}}{3b\log(f)} - \frac{3\left(\frac{x^6 f^{a+bx^3}}{3b\log(f)} - \frac{2\left(\frac{x^3 f^{a+bx^3}}{3b\log(f)} - \frac{\int f^{bx^3+a}x^2dx}{b\log(f)}\right)}{b\log(f)}\right)}{b\log(f)}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{x^9 f^{a+bx^3}}{3b\log(f)} - \frac{3\left(\frac{x^6 f^{a+bx^3}}{3b\log(f)} - \frac{2\left(\frac{x^3 f^{a+bx^3}}{3b\log(f)} - \frac{f^{a+bx^3}}{3b^2\log^2(f)}\right)}{b\log(f)}\right)}{b\log(f)}$$

input
```
Int[f^(a + b*x^3)*x^11,x]
```

output
```
(f^(a + b*x^3)*x^9)/(3*b*Log[f]) - (3*((f^(a + b*x^3)*x^6)/(3*b*Log[f]) -
(2*(-1/3*f^(a + b*x^3)/(b^2*Log[f]^2) + (f^(a + b*x^3)*x^3)/(3*b*Log[f])))
/(b*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{\left(b^{3} x^{9} \ln\left(f\right)^{3}-3 b^{2} x^{6} \ln\left(f\right)^{2}+6 b \,x^{3} \ln\left(f\right)-6\right) f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{4} b^{4}}$	52
risch	$\frac{\left(b^{3} x^{9} \ln\left(f\right)^{3}-3 b^{2} x^{6} \ln\left(f\right)^{2}+6 b \,x^{3} \ln\left(f\right)-6\right) f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{4} b^{4}}$	52
orering	$\frac{\left(b^{3} x^{9} \ln\left(f\right)^{3}-3 b^{2} x^{6} \ln\left(f\right)^{2}+6 b \,x^{3} \ln\left(f\right)-6\right) f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{4} b^{4}}$	52
meijerg	$\frac{f^{a} \left(6-\frac{\left(-4 b^{3} x^{9} \ln\left(f\right)^{3}+12 b^{2} x^{6} \ln\left(f\right)^{2}-24 b \,x^{3} \ln\left(f\right)+24\right) {\mathrm e}^{b \,x^{3} \ln\left(f\right)}}{4}\right)}{3 b^{4} \ln\left(f\right)^{4}}$	59
parallelrisch	$\frac{f^{b \,x^{3}+a} x^{9} b^{3} \ln\left(f\right)^{3}-3 f^{b \,x^{3}+a} x^{6} b^{2} \ln\left(f\right)^{2}+6 f^{b \,x^{3}+a} x^{3} b \ln\left(f\right)-6 f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{4} b^{4}}$	80
norman	$-\frac{2 \,{\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{\ln\left(f\right)^{4} b^{4}}+\frac{x^{9} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{3 b \ln\left(f\right)}+\frac{2 x^{3} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{\ln\left(f\right)^{3} b^{3}}-\frac{x^{6} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{\ln\left(f\right)^{2} b^{2}}$	91

input
```
int(f^(b*x^3+a)*x^11,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^3*x^9*ln(f)^3-3*b^2*x^6*ln(f)^2+6*b*x^3*ln(f)-6)*f^(b*x^3+a)/ln(f)^
4/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3}x^{11}\,dx=\frac{\left(b^3x^9\log\left(f\right)^3-3\,b^2x^6\log\left(f\right)^2+6\,bx^3\log\left(f\right)-6\right)f^{bx^3+a}}{3\,b^4\log\left(f\right)^4}$$

input
```
integrate(f^(b*x^3+a)*x^11,x, algorithm="fricas")
```

output
```
1/3*(b^3*x^9*log(f)^3 - 3*b^2*x^6*log(f)^2 + 6*b*x^3*log(f) - 6)*f^(b*x^3
+ a)/(b^4*log(f)^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.79

$$\int f^{a+bx^3}x^{11}\,dx=\begin{cases}\frac{f^{a+bx^3}\left(b^3x^9\log\left(f\right)^3-3b^2x^6\log\left(f\right)^2+6bx^3\log\left(f\right)-6\right)}{3b^4\log\left(f\right)^4} & \text{for}\ b^4\log\left(f\right)^4\neq 0\\ \frac{x^{12}}{12} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x**3+a)*x**11,x)
```

output
```
Piecewise((f**(a + b*x**3)*(b**3*x**9*log(f)**3 - 3*b**2*x**6*log(f)**2 +
6*b*x**3*log(f) - 6)/(3*b**4*log(f)**4), Ne(b**4*log(f)**4, 0)), (x**12/12
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.74

$$\int f^{a+bx^3}x^{11}\,dx = \frac{\left(b^3f^ax^9\log\left(f\right)^3 - 3\,b^2f^ax^6\log\left(f\right)^2 + 6\,bf^ax^3\log\left(f\right) - 6\,f^a\right)f^{bx^3}}{3\,b^4\log\left(f\right)^4}$$

input
```
integrate(f^(b*x^3+a)*x^11,x, algorithm="maxima")
```

output
```
1/3*(b^3*f^a*x^9*log(f)^3 - 3*b^2*f^a*x^6*log(f)^2 + 6*b*f^a*x^3*log(f) -
6*f^a)*f^(b*x^3)/(b^4*log(f)^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int f^{a+bx^3}x^{11}\,dx = \frac{b^3f^{bx^3}f^ax^9\log\left(f\right)^3 - 3\,b^2f^{bx^3}f^ax^6\log\left(f\right)^2 + 6\,bf^{bx^3}f^ax^3\log\left(f\right) - 6\,f^{bx^3}f^a}{3\,b^4\log\left(f\right)^4}$$

input
```
integrate(f^(b*x^3+a)*x^11,x, algorithm="giac")
```

output
```
1/3*(b^3*f^(b*x^3)*f^a*x^9*log(f)^3 - 3*b^2*f^(b*x^3)*f^a*x^6*log(f)^2 + 6
*b*f^(b*x^3)*f^a*x^3*log(f) - 6*f^(b*x^3)*f^a)/(b^4*log(f)^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3}x^{11}\,dx = -\frac{f^{b\,x^3+a}\,\left(-\frac{b^3\,x^9\,\ln(f)^3}{3} + b^2\,x^6\,\ln\left(f\right)^2 - 2\,b\,x^3\,\ln\left(f\right) + 2\right)}{b^4\,\ln\left(f\right)^4}$$

input
```
int(f^(a + b*x^3)*x^11,x)
```

output
```
-(f^(a + b*x^3)*(b^2*x^6*log(f)^2 - (b^3*x^9*log(f)^3)/3 - 2*b*x^3*log(f)
+ 2))/(b^4*log(f)^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3}x^{11}\,dx=\frac{f^{b\,x^3+a}\left(\log(f)^3\,b^3x^9-3\log(f)^2\,b^2x^6+6\log(f)\,b\,x^3-6\right)}{3\log\left(f\right)^4b^4}$$

input
```
int(f^(b*x^3+a)*x^11,x)
```

output
```
(f**(a + b*x**3)*(log(f)**3*b**3*x**9 - 3*log(f)**2*b**2*x**6 + 6*log(f)*b
*x**3 - 6))/(3*log(f)**4*b**4)
```

3.35 $\int f^{a+bx^3} x^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int f^{a+bx^3} x^8 \, dx = \frac{2f^{a+bx^3}}{3b^3 \log^3(f)} - \frac{2f^{a+bx^3} x^3}{3b^2 \log^2(f)} + \frac{f^{a+bx^3} x^6}{3b \log(f)}$$

output
```
2/3*f^(b*x^3+a)/b^3/ln(f)^3-2/3*f^(b*x^3+a)*x^3/b^2/ln(f)^2+1/3*f^(b*x^3+a
)*x^6/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3} x^8 \, dx = \frac{f^{a+bx^3} \left(2 - 2bx^3 \log(f) + b^2 x^6 \log^2(f)\right)}{3b^3 \log^3(f)}$$

input
```
Integrate[f^(a + b*x^3)*x^8,x]
```

output
```
(f^(a + b*x^3)*(2 - 2*b*x^3*Log[f] + b^2*x^6*Log[f]^2))/(3*b^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 f^{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^6 f^{a+bx^3}}{3b\log(f)} - \frac{2\int f^{bx^3+a}x^5dx}{b\log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{x^6 f^{a+bx^3}}{3b\log(f)} - \frac{2\left(\frac{x^3 f^{a+bx^3}}{3b\log(f)} - \frac{\int f^{bx^3+a}x^2dx}{b\log(f)}\right)}{b\log(f)}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{x^6 f^{a+bx^3}}{3b\log(f)} - \frac{2\left(\frac{x^3 f^{a+bx^3}}{3b\log(f)} - \frac{f^{a+bx^3}}{3b^2\log^2(f)}\right)}{b\log(f)}$$

input
```
Int[f^(a + b*x^3)*x^8,x]
```

output
```
(f^(a + b*x^3)*x^6)/(3*b*Log[f]) - (2*(-1/3*f^(a + b*x^3)/(b^2*Log[f]^2) +
 (f^(a + b*x^3)*x^3)/(3*b*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.60

method	result	size
gosper	$\frac{\left(b^{2} x^{6} \ln\left(f\right)^{2}-2 b \,x^{3} \ln\left(f\right)+2\right) f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{3} b^{3}}$	40
risch	$\frac{\left(b^{2} x^{6} \ln\left(f\right)^{2}-2 b \,x^{3} \ln\left(f\right)+2\right) f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{3} b^{3}}$	40
orering	$\frac{\left(b^{2} x^{6} \ln\left(f\right)^{2}-2 b \,x^{3} \ln\left(f\right)+2\right) f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{3} b^{3}}$	40
meijerg	$-\frac{f^{a} \left(2-\frac{\left(3 b^{2} x^{6} \ln\left(f\right)^{2}-6 b \,x^{3} \ln\left(f\right)+6\right) {\mathrm e}^{b \,x^{3} \ln\left(f\right)}}{3}\right)}{3 b^{3} \ln\left(f\right)^{3}}$	47
parallelrisch	$\frac{f^{b \,x^{3}+a} x^{6} b^{2} \ln\left(f\right)^{2}-2 f^{b \,x^{3}+a} x^{3} b \ln\left(f\right)+2 f^{b \,x^{3}+a}}{3 \ln\left(f\right)^{3} b^{3}}$	59
norman	$\frac{2 \,{\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{3 \ln\left(f\right)^{3} b^{3}}+\frac{x^{6} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{3 b \ln\left(f\right)}-\frac{2 x^{3} {\mathrm e}^{\left(b \,x^{3}+a \right) \ln\left(f\right)}}{3 \ln\left(f\right)^{2} b^{2}}$	68

input
```
int(f^(b*x^3+a)*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^2*x^6*ln(f)^2-2*b*x^3*ln(f)+2)*f^(b*x^3+a)/ln(f)^3/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int f^{a+bx^3}x^8\,dx=\frac{\left(b^2x^6\log\left(f\right)^2-2\,bx^3\log\left(f\right)+2\right)f^{bx^3+a}}{3\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(b*x^3+a)*x^8,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^6*log(f)^2 - 2*b*x^3*log(f) + 2)*f^(b*x^3 + a)/(b^3*log(f)^3)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int f^{a+bx^3}x^8\,dx=\begin{cases}\frac{f^{a+bx^3}\left(b^2x^6\log\left(f\right)^2-2bx^3\log\left(f\right)+2\right)}{3b^3\log\left(f\right)^3} & \text{for } b^3\log\left(f\right)^3\neq 0\\ \frac{x^9}{9} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x**3+a)*x**8,x)
```

output
```
Piecewise((f**(a + b*x**3)*(b**2*x**6*log(f)**2 - 2*b*x**3*log(f) + 2)/(3*
b**3*log(f)**3), Ne(b**3*log(f)**3, 0)), (x**9/9, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int f^{a+bx^3}x^8\,dx=\frac{\left(b^2f^ax^6\log\left(f\right)^2-2\,bf^ax^3\log\left(f\right)+2\,f^a\right)f^{bx^3}}{3\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(b*x^3+a)*x^8,x, algorithm="maxima")
```

output
```
1/3*(b^2*f^a*x^6*log(f)^2 - 2*b*f^a*x^3*log(f) + 2*f^a)*f^(b*x^3)/(b^3*log
(f)^3)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int f^{a+bx^3}x^8\,dx=\frac{b^2f^{bx^3}f^ax^6\log(f)^2-2\,bf^{bx^3}f^ax^3\log(f)+2\,f^{bx^3}f^a}{3\,b^3\log(f)^3}$$

input
```
integrate(f^(b*x^3+a)*x^8,x, algorithm="giac")
```

output
```
1/3*(b^2*f^(b*x^3)*f^a*x^6*log(f)^2 - 2*b*f^(b*x^3)*f^a*x^3*log(f) + 2*f^(
b*x^3)*f^a)/(b^3*log(f)^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int f^{a+bx^3}x^8\,dx=\frac{f^{b\,x^3+a}\,\left(\frac{b^2\,x^6\,\ln(f)^2}{3}-\frac{2\,b\,x^3\,\ln(f)}{3}+\frac{2}{3}\right)}{b^3\ln(f)^3}$$

input
```
int(f^(a + b*x^3)*x^8,x)
```

output
```
(f^(a + b*x^3)*((b^2*x^6*log(f)^2)/3 - (2*b*x^3*log(f))/3 + 2/3))/(b^3*log
(f)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.58

$$\int f^{a+bx^3} x^8 \, dx = \frac{f^{b\,x^3+a} \left(\log(f)^2\, b^2 x^6 - 2\log(f)\, b\, x^3 + 2\right)}{3\log\left(f\right)^3 b^3}$$

input
```
int(f^(b*x^3+a)*x^8,x)
```

output
```
(f**(a + b*x**3)*(log(f)**2*b**2*x**6 - 2*log(f)*b*x**3 + 2))/(3*log(f)**3
*b**3)
```

3.36 $\int f^{a+bx^3} x^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int f^{a+bx^3} x^5\,dx = -\frac{f^{a+bx^3}}{3b^2\log^2(f)} + \frac{f^{a+bx^3}x^3}{3b\log(f)}$$

output
```
-1/3*f^(b*x^3+a)/b^2/ln(f)^2+1/3*f^(b*x^3+a)*x^3/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int f^{a+bx^3} x^5\,dx = \frac{f^{a+bx^3}(-1+bx^3\log(f))}{3b^2\log^2(f)}$$

input
```
Integrate[f^(a + b*x^3)*x^5,x]
```

output
```
(f^(a + b*x^3)*(-1 + b*x^3*Log[f]))/(3*b^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 f^{a+bx^3} \, dx$$

$$\Big\downarrow \; 2641$$

$$\frac{x^3 f^{a+bx^3}}{3b\log(f)} - \frac{\int f^{bx^3+a} x^2 dx}{b\log(f)}$$

$$\Big\downarrow \; 2638$$

$$\frac{x^3 f^{a+bx^3}}{3b\log(f)} - \frac{f^{a+bx^3}}{3b^2\log^2(f)}$$

input
```
Int[f^(a + b*x^3)*x^5,x]
```

output
```
-1/3*f^(a + b*x^3)/(b^2*Log[f]^2) + (f^(a + b*x^3)*x^3)/(3*b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\left(b\,x^{3} \ln \left(f \right)-1\right) f^{b\,x^{3}+a}}{3 \ln \left(f \right)^{2} b^{2}}$	28
risch	$\frac{\left(b\,x^{3} \ln \left(f \right)-1\right) f^{b\,x^{3}+a}}{3 \ln \left(f \right)^{2} b^{2}}$	28
orering	$\frac{\left(b\,x^{3} \ln \left(f \right)-1\right) f^{b\,x^{3}+a}}{3 \ln \left(f \right)^{2} b^{2}}$	28
meijerg	$\frac{f^{a} \left(1-\frac{\left(-2 b\,x^{3} \ln \left(f \right)+2\right) {\mathrm e}^{b\,x^{3} \ln \left(f \right)}}{2}\right)}{3 b^{2} \ln \left(f \right)^{2}}$	35
parallelrisch	$\frac{f^{b\,x^{3}+a} x^{3} b \ln \left(f \right)-f^{b\,x^{3}+a}}{3 \ln \left(f \right)^{2} b^{2}}$	38
norman	$-\frac{{\mathrm e}^{\left(b\,x^{3}+a \right) \ln \left(f \right)}}{3 \ln \left(f \right)^{2} b^{2}}+\frac{x^{3} {\mathrm e}^{\left(b\,x^{3}+a \right) \ln \left(f \right)}}{3 b \ln \left(f \right)}$	45

input
```
int(f^(b*x^3+a)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^3*ln(f)-1)*f^(b*x^3+a)/ln(f)^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3}x^5\,dx = \frac{(bx^3\log(f)-1)f^{bx^3+a}}{3\,b^2\log(f)^2}$$

input
```
integrate(f^(b*x^3+a)*x^5,x, algorithm="fricas")
```

output
```
1/3*(b*x^3*log(f) - 1)*f^(b*x^3 + a)/(b^2*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int f^{a+bx^3}x^5\,dx = \begin{cases} \frac{f^{a+bx^3}(bx^3\log(f)-1)}{3b^2\log(f)^2} & \text{for } b^2\log(f)^2\neq 0 \\ \frac{x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x**3+a)*x**5,x)
```

output
```
Piecewise((f**(a + b*x**3)*(b*x**3*log(f) - 1)/(3*b**2*log(f)**2), Ne(b**2
*log(f)**2, 0)), (x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int f^{a+bx^3}x^5\,dx = \frac{(bf^ax^3\log(f)-f^a)f^{bx^3}}{3\,b^2\log(f)^2}$$

input
```
integrate(f^(b*x^3+a)*x^5,x, algorithm="maxima")
```

output
```
1/3*(b*f^a*x^3*log(f) - f^a)*f^(b*x^3)/(b^2*log(f)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 689, normalized size of antiderivative = 15.66

$$\int f^{a+bx^3} x^5\, dx = \text{Too large to display}$$

input
```
integrate(f^(b*x^3+a)*x^5,x, algorithm="giac")
```

output
```
1/3*(2*((b*x^3*log(abs(f)) - 1)*(pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(ab
s(f))^2)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2
*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) + (pi*b*x^3*sgn(f) - pi*b*x^3
)*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))/((pi^2*b^2*sgn(f) - pi^
2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log
(abs(f)))^2))*cos(-1/2*pi*b*x^3*sgn(f) + 1/2*pi*b*x^3 - 1/2*pi*a*sgn(f) +
1/2*pi*a) + ((pi*b*x^3*sgn(f) - pi*b*x^3)*(pi^2*b^2*sgn(f) - pi^2*b^2 + 2*
b^2*log(abs(f))^2)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 +
 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) - 4*(b*x^3*log(abs(
f)) - 1)*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))/((pi^2*b^2*sgn(f
) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*
b^2*log(abs(f)))^2))*sin(-1/2*pi*b*x^3*sgn(f) + 1/2*pi*b*x^3 - 1/2*pi*a*sg
n(f) + 1/2*pi*a))*e^(b*x^3*log(abs(f)) + a*log(abs(f))) - 1/6*I*((pi*b*x^3
*sgn(f) - pi*b*x^3 - 2*I*b*x^3*log(abs(f)) + 2*I)*e^(1/2*I*pi*b*x^3*sgn(f)
 - 1/2*I*pi*b*x^3 + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(pi^2*b^2*sgn(f) + 2*I
*pi*b^2*log(abs(f))*sgn(f) - pi^2*b^2 - 2*I*pi*b^2*log(abs(f)) + 2*b^2*log
(abs(f))^2) + (pi*b*x^3*sgn(f) - pi*b*x^3 + 2*I*b*x^3*log(abs(f)) - 2*I)*e
^(-1/2*I*pi*b*x^3*sgn(f) + 1/2*I*pi*b*x^3 - 1/2*I*pi*a*sgn(f) + 1/2*I*pi*a
)/(pi^2*b^2*sgn(f) - 2*I*pi*b^2*log(abs(f))*sgn(f) - pi^2*b^2 + 2*I*pi*b^2
*log(abs(f)) + 2*b^2*log(abs(f))^2))*e^(b*x^3*log(abs(f)) + a*log(abs(f...
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3} x^5 \, dx = \frac{f^{b\,x^3+a}\left(\frac{b\,x^3\,\ln(f)}{3}-\frac{1}{3}\right)}{b^2\ln\left(f\right)^2}$$

input
```
int(f^(a + b*x^3)*x^5,x)
```

output
```
(f^(a + b*x^3)*((b*x^3*log(f))/3 - 1/3))/(b^2*log(f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int f^{a+bx^3} x^5 \, dx = \frac{f^{b\,x^3+a}(\log(f)\,b\,x^3-1)}{3\log\left(f\right)^2 b^2}$$

input
```
int(f^(b*x^3+a)*x^5,x)
```

output
```
(f**(a + b*x**3)*(log(f)*b*x**3 - 1))/(3*log(f)**2*b**2)
```

3.37 $\int f^{a+bx^3}x^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int f^{a+bx^3}x^2\,dx=\frac{f^{a+bx^3}}{3b\log(f)}$$

output
```
1/3*f^(b*x^3+a)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^3}x^2\,dx=\frac{f^{a+bx^3}}{3b\log(f)}$$

input
```
Integrate[f^(a + b*x^3)*x^2,x]
```

output
```
f^(a + b*x^3)/(3*b*Log[f])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+bx^3} \, dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{f^{a+bx^3}}{3b\log(f)}$$

input
```
Int[f^(a + b*x^3)*x^2,x]
```

output
```
f^(a + b*x^3)/(3*b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{f^{b\,x^{3}+a}}{3b\ln(f)}$	19
derivativedivides	$\frac{f^{b\,x^{3}+a}}{3b\ln(f)}$	19
default	$\frac{f^{b\,x^{3}+a}}{3b\ln(f)}$	19
risch	$\frac{f^{b\,x^{3}+a}}{3b\ln(f)}$	19
parallelrisch	$\frac{f^{b\,x^{3}+a}}{3b\ln(f)}$	19
orering	$\frac{f^{b\,x^{3}+a}}{3b\ln(f)}$	19
norman	$\frac{\mathrm{e}^{\left(b\,x^{3}+a\right)\ln(f)}}{3b\ln(f)}$	21
meijerg	$-\frac{f^{a}\left(1-\mathrm{e}^{b\,x^{3}\ln(f)}\right)}{3b\ln(f)}$	25

input
```
int(f^(b*x^3+a)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^(b*x^3+a)/b/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^3}x^2\,dx=\frac{f^{bx^3+a}}{3\,b\log{(f)}}$$

input
```
integrate(f^(b*x^3+a)*x^2,x, algorithm="fricas")
```

output
```
1/3*f^(b*x^3 + a)/(b*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int f^{a+bx^3}x^2\,dx = \begin{cases} \frac{f^{a+bx^3}}{3b\log(f)} & \text{for } b\log(f) \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x**3+a)*x**2,x)
```

output
```
Piecewise((f**(a + b*x**3)/(3*b*log(f)), Ne(b*log(f), 0)), (x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^3}x^2\,dx = \frac{f^{bx^3+a}}{3\,b\log(f)}$$

input
```
integrate(f^(b*x^3+a)*x^2,x, algorithm="maxima")
```

output
```
1/3*f^(b*x^3 + a)/(b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^3}x^2\,dx = \frac{f^{bx^3+a}}{3\,b\log(f)}$$

input
```
integrate(f^(b*x^3+a)*x^2,x, algorithm="giac")
```

output
```
1/3*f^(b*x^3 + a)/(b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^3}x^2\,dx=\frac{f^{b\,x^3+a}}{3\,b\,\ln\left(f\right)}$$

input
```
int(f^(a + b*x^3)*x^2,x)
```

output
```
f^(a + b*x^3)/(3*b*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int f^{a+bx^3}x^2\,dx=\frac{f^{b\,x^3+a}}{3\log\left(f\right)b}$$

input
```
int(f^(b*x^3+a)*x^2,x)
```

output
```
f**(a + b*x**3)/(3*log(f)*b)
```

3.38 $\int \frac{f^{a+bx^3}}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{f^{a+bx^3}}{x} \, dx = \frac{1}{3} f^a \text{ExpIntegralEi}\left(bx^3 \log(f)\right)$$

output
```
1/3*f^a*Ei(b*x^3*ln(f))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^3}}{x} \, dx = \frac{1}{3} f^a \text{ExpIntegralEi}\left(bx^3 \log(f)\right)$$

input
```
Integrate[f^(a + b*x^3)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^3*Log[f]])/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x} \, dx$$

$$\Big\downarrow \ 2639$$

$$\frac{1}{3} f^a \operatorname{ExpIntegralEi}\left(bx^3 \log(f)\right)$$

input
```
Int[f^(a + b*x^3)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^3*Log[f]])/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(13) = 26.

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

method	result	size
meijerg	$\frac{f^{a} \left(3 \ln \left(x \right)+\ln \left(-b \right)+\ln \left(\ln \left(f \right)\right)-\ln \left(-b \,x^{3} \ln \left(f \right)\right)-\operatorname{expIntegral}_{1}\left(-b \,x^{3} \ln \left(f \right)\right)\right)}{3}$	41

input
```
int(f^(b*x^3+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*(3*ln(x)+ln(-b)+ln(ln(f))-ln(-b*x^3*ln(f))-Ei(1,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^3}}{x}\,dx = \frac{1}{3}\,f^a\text{Ei}\big(bx^3\log\left(f\right)\big)$$

input
```
integrate(f^(b*x^3+a)/x,x, algorithm="fricas")
```

output
```
1/3*f^a*Ei(b*x^3*log(f))
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x}\,dx = \int \frac{f^{a+bx^3}}{x}\,dx$$

input
```
integrate(f**(b*x**3+a)/x,x)
```

output
```
Integral(f**(a + b*x**3)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^3}}{x}\,dx = \frac{1}{3}\,f^a \mathrm{Ei}\big(bx^3\log{(f)}\big)$$

input
```
integrate(f^(b*x^3+a)/x,x, algorithm="maxima")
```

output
```
1/3*f^a*Ei(b*x^3*log(f))
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x}\,dx = \int \frac{f^{bx^3+a}}{x}\,dx$$

input
```
integrate(f^(b*x^3+a)/x,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^3}}{x}\,dx = \frac{f^a\,\mathrm{ei}(b\,x^3\,\ln{(f)})}{3}$$

input
```
int(f^(a + b*x^3)/x,x)
```

output
```
(f^a*ei(b*x^3*log(f)))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+bx^3}}{x}\,dx = \frac{f^a\,ei(\log(f)\,b\,x^3)}{3}$$

input
```
int(f^(b*x^3+a)/x,x)
```

output
```
(f**a*ei(log(f)*b*x**3))/3
```

3.39 $\int \frac{f^{a+bx^3}}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = -\frac{f^{a+bx^3}}{3x^3} + \frac{1}{3}bf^a \operatorname{ExpIntegralEi}\left(bx^3\log(f)\right)\log(f)$$

output
```
-1/3*f^(b*x^3+a)/x^3+1/3*b*f^a*Ei(b*x^3*ln(f))*ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = \frac{1}{3}f^a\left(-\frac{f^{bx^3}}{x^3} + b\operatorname{ExpIntegralEi}\left(bx^3\log(f)\right)\log(f)\right)$$

input
```
Integrate[f^(a + b*x^3)/x^4,x]
```

output
```
(f^a*(-(f^(b*x^3)/x^3) + b*ExpIntegralEi[b*x^3*Log[f]]*Log[f]))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^4} dx$$

$$\Big\downarrow \ 2643$$

$$b\log(f)\int \frac{f^{bx^3+a}}{x}dx - \frac{f^{a+bx^3}}{3x^3}$$

$$\Big\downarrow \ 2639$$

$$\frac{1}{3}bf^a\log(f)\,\text{ExpIntegralEi}\left(bx^3\log(f)\right) - \frac{f^{a+bx^3}}{3x^3}$$

input
```
Int[f^(a + b*x^3)/x^4,x]
```

output
```
-1/3*f^(a + b*x^3)/x^3 + (b*f^a*ExpIntegralEi[b*x^3*Log[f]]*Log[f])/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(31) = 62.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

method	result
meijerg	$-\frac{f^{a} b \ln \left(f \right) \left(\frac{1}{x^{3} b \ln \left(f \right)}+1-3 \ln \left(x \right)-\ln \left(-b \right)-\ln \left(\ln \left(f \right)\right)-\frac{2 b \,x^{3} \ln \left(f \right)+2}{2 b \,x^{3} \ln \left(f \right)}+\frac{{\mathrm e}^{b \,x^{3} \ln \left(f \right)}}{x^{3} b \ln \left(f \right)}+\ln \left(-b \,x^{3} \ln \left(f \right)\right)+\operatorname{expIntegral}_{1}\left(-b \,x^{3} \ln \left(f \right)\right)\right)}{3}$

input
```
int(f^(b*x^3+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*b*ln(f)*(1/x^3/b/ln(f)+1-3*ln(x)-ln(-b)-ln(ln(f))-1/2/b/x^3/ln(f)
*(2*b*x^3*ln(f)+2)+1/x^3/b/ln(f)*exp(b*x^3*ln(f))+ln(-b*x^3*ln(f))+Ei(1,-b
*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = \frac{bf^a x^3 \mathrm{Ei}\left(bx^3 \log\left(f\right)\right) \log\left(f\right) - f^{bx^3+a}}{3\,x^3}$$

input
```
integrate(f^(b*x^3+a)/x^4,x, algorithm="fricas")
```

output
```
1/3*(b*f^a*x^3*Ei(b*x^3*log(f))*log(f) - f^(b*x^3 + a))/x^3
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = \int \frac{f^{a+bx^3}}{x^4}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**4,x)
```

output
```
Integral(f**(a + b*x**3)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = \frac{1}{3}\,bf^a\Gamma\left(-1, -bx^3\log\left(f\right)\right)\log\left(f\right)$$

input
```
integrate(f^(b*x^3+a)/x^4,x, algorithm="maxima")
```

output
```
1/3*b*f^a*gamma(-1, -b*x^3*log(f))*log(f)
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = \int \frac{f^{bx^3+a}}{x^4}\,dx$$

input
```
integrate(f^(b*x^3+a)/x^4,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = -\frac{f^a\,\left(f^{b\,x^3}+b\,x^3\,\ln\left(f\right)\,\mathrm{expint}(-b\,x^3\,\ln\left(f\right))\right)}{3\,x^3}$$

input
```
int(f^(a + b*x^3)/x^4,x)
```

output
```
-(f^a*(f^(b*x^3) + b*x^3*log(f)*expint(-b*x^3*log(f))))/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{f^{a+bx^3}}{x^4}\,dx = \frac{f^a\left(ei(\log(f)\,b\,x^3)\log(f)\,b\,x^3 - f^{b\,x^3}\right)}{3x^3}$$

input
```
int(f^(b*x^3+a)/x^4,x)
```

output
```
(f**a*(ei(log(f)*b*x**3)*log(f)*b*x**3 - f**(b*x**3)))/(3*x**3)
```

3.40 $\int \frac{f^{a+bx^3}}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{f^{a+bx^3}}{x^7} dx = -\frac{f^{a+bx^3}}{6x^6} - \frac{bf^{a+bx^3}\log(f)}{6x^3} + \frac{1}{6}b^2 f^a \operatorname{ExpIntegralEi}\left(bx^3\log(f)\right)\log^2(f)$$

output
```
-1/6*f^(b*x^3+a)/x^6-1/6*b*f^(b*x^3+a)*ln(f)/x^3+1/6*b^2*f^a*Ei(b*x^3*ln(f
))*ln(f)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{f^{a+bx^3}}{x^7} dx = \frac{f^a\left(b^2x^6 \operatorname{ExpIntegralEi}\left(bx^3\log(f)\right)\log^2(f) - f^{bx^3}(1+bx^3\log(f))\right)}{6x^6}$$

input
```
Integrate[f^(a + b*x^3)/x^7,x]
```

output
```
(f^a*(b^2*x^6*ExpIntegralEi[b*x^3*Log[f]]*Log[f]^2 - f^(b*x^3)*(1 + b*x^3*
Log[f])))/(6*x^6)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^7} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \int \frac{f^{bx^3+a}}{x^4} dx - \frac{f^{a+bx^3}}{6x^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{bx^3+a}}{x} dx - \frac{f^{a+bx^3}}{3x^3} \right) - \frac{f^{a+bx^3}}{6x^6}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2} b \log(f) \left(\frac{1}{3} b f^a \log(f) \text{ExpIntegralEi}\left(bx^3 \log(f)\right) - \frac{f^{a+bx^3}}{3x^3} \right) - \frac{f^{a+bx^3}}{6x^6}$$

input
```
Int[f^(a + b*x^3)/x^7,x]
```

output
```
-1/6*f^(a + b*x^3)/x^6 + (b*Log[f]*(-1/3*f^(a + b*x^3)/x^3 + (b*f^a*ExpInt
egralEi[b*x^3*Log[f]]*Log[f])/3))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(52) = 104$.

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

method	result
meijerg	$\frac{f^a b^2 \ln(f)^2 \left(-\frac{1}{2b^2x^6\ln(f)^2}-\frac{1}{x^3b\ln(f)}-\frac{3}{4}+\frac{3\ln(x)}{2}+\frac{\ln(-b)}{2}+\frac{\ln(\ln(f))}{2}+\frac{9b^2x^6\ln(f)^2+12bx^3\ln(f)+6}{12b^2x^6\ln(f)^2}-\frac{\left(3bx^3\ln(f)+3\right)e^{bx^3\ln(f)}}{6b^2x^6\ln(f)^2}-\frac{\ln\left(-\right.}{}\right.}{3}$

input
```
int(f^(b*x^3+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*b^2*ln(f)^2*(-1/2/b^2/x^6/ln(f)^2-1/x^3/b/ln(f)-3/4+3/2*ln(x)+1/2*
ln(-b)+1/2*ln(ln(f))+1/12/b^2/x^6/ln(f)^2*(9*b^2*x^6*ln(f)^2+12*b*x^3*ln(f
)+6)-1/6/b^2/x^6/ln(f)^2*(3*b*x^3*ln(f)+3)*exp(b*x^3*ln(f))-1/2*ln(-b*x^3*
ln(f))-1/2*Ei(1,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{f^{a+bx^3}}{x^7}\,dx = \frac{b^2f^ax^6\text{Ei}(bx^3\log(f))\log(f)^2-(bx^3\log(f)+1)f^{bx^3+a}}{6\,x^6}$$

input
```
integrate(f^(b*x^3+a)/x^7,x, algorithm="fricas")
```

output
```
1/6*(b^2*f^a*x^6*Ei(b*x^3*log(f))*log(f)^2 - (b*x^3*log(f) + 1)*f^(b*x^3 +
 a))/x^6
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^7}\,dx = \int \frac{f^{a+bx^3}}{x^7}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**7,x)
```

output
```
Integral(f**(a + b*x**3)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.38

$$\int \frac{f^{a+bx^3}}{x^7}\,dx = -\frac{1}{3}\,b^2f^a\Gamma\left(-2,-bx^3\log\left(f\right)\right)\log\left(f\right)^2$$

input
```
integrate(f^(b*x^3+a)/x^7,x, algorithm="maxima")
```

output
```
-1/3*b^2*f^a*gamma(-2, -b*x^3*log(f))*log(f)^2
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^7}\,dx = \int \frac{f^{bx^3+a}}{x^7}\,dx$$

input
```
integrate(f^(b*x^3+a)/x^7,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{f^{a+bx^3}}{x^7}\,dx = -\frac{b^2\,f^a\ln\left(f\right)^2\,\left(f^{b\,x^3}\,\left(\frac{1}{2\,b\,x^3\,\ln\left(f\right)}+\frac{1}{2\,b^2\,x^6\,\ln\left(f\right)^2}\right)+\frac{\mathrm{expint}\left(-b\,x^3\,\ln\left(f\right)\right)}{2}\right)}{3}$$

input
```
int(f^(a + b*x^3)/x^7,x)
```

output
```
-(b^2*f^a*log(f)^2*(f^(b*x^3)*(1/(2*b*x^3*log(f)) + 1/(2*b^2*x^6*log(f)^2)
) + expint(-b*x^3*log(f))/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+bx^3}}{x^7}\,dx = \frac{f^a\left(ei(\log(f)\,b\,x^3)\log(f)^2\,b^2x^6-f^{b\,x^3}\log(f)\,b\,x^3-f^{b\,x^3}\right)}{6x^6}$$

input
```
int(f^(b*x^3+a)/x^7,x)
```

output
```
(f**a*(ei(log(f)*b*x**3)*log(f)**2*b**2*x**6 - f**(b*x**3)*log(f)*b*x**3 -
 f**(b*x**3)))/(6*x**6)
```

3.41 $\int \frac{f^{a+bx^3}}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = -\frac{f^{a+bx^3}}{9x^9} - \frac{bf^{a+bx^3}\log(f)}{18x^6} - \frac{b^2f^{a+bx^3}\log^2(f)}{18x^3} + \frac{1}{18}b^3f^a\,\text{ExpIntegralEi}\left(bx^3\log(f)\right)\log^3(f)$$

output
```
-1/9*f^(b*x^3+a)/x^9-1/18*b*f^(b*x^3+a)*ln(f)/x^6-1/18*b^2*f^(b*x^3+a)*ln(
f)^2/x^3+1/18*b^3*f^a*Ei(b*x^3*ln(f))*ln(f)^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = \frac{f^a\left(b^3x^9\,\text{ExpIntegralEi}\left(bx^3\log(f)\right)\log^3(f) - f^{bx^3}\left(2+bx^3\log(f)+b^2x^6\log^2(f)\right)\right)}{18x^9}$$

input
```
Integrate[f^(a + b*x^3)/x^10,x]
```

output
```
(f^a*(b^3*x^9*ExpIntegralEi[b*x^3*Log[f]]*Log[f]^3 - f^(b*x^3)*(2 + b*x^3*
Log[f] + b^2*x^6*Log[f]^2)))/(18*x^9)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^{10}}dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(f)\int \frac{f^{bx^3+a}}{x^7}dx-\frac{f^{a+bx^3}}{9x^9}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(f)\left(\frac{1}{2}b\log(f)\int \frac{f^{bx^3+a}}{x^4}dx-\frac{f^{a+bx^3}}{6x^6}\right)-\frac{f^{a+bx^3}}{9x^9}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(f)\left(\frac{1}{2}b\log(f)\left(b\log(f)\int \frac{f^{bx^3+a}}{x}dx-\frac{f^{a+bx^3}}{3x^3}\right)-\frac{f^{a+bx^3}}{6x^6}\right)-\frac{f^{a+bx^3}}{9x^9}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3}b\log(f)\left(\frac{1}{2}b\log(f)\left(\frac{1}{3}bf^a\log(f)\,\text{ExpIntegralEi}\left(bx^3\log(f)\right)-\frac{f^{a+bx^3}}{3x^3}\right)-\frac{f^{a+bx^3}}{6x^6}\right)-\frac{f^{a+bx^3}}{9x^9}$$

input
```
Int[f^(a + b*x^3)/x^10,x]
```

output
```
-1/9*f^(a + b*x^3)/x^9 + (b*Log[f]*(-1/6*f^(a + b*x^3)/x^6 + (b*Log[f]*(-1
/3*f^(a + b*x^3)/x^3 + (b*f^a*ExpIntegralEi[b*x^3*Log[f]]*Log[f])/3))/2))/
3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(73) = 146$.

Time = 0.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

method	result
meijerg	$-\frac{f^a b^3 \ln(f)^3 \left(\frac{1}{3b^3x^9\ln(f)^3}+\frac{1}{2b^2x^6\ln(f)^2}+\frac{1}{2x^3b\ln(f)}+\frac{11}{36}-\frac{\ln(x)}{2}-\frac{\ln(-b)}{6}-\frac{\ln(\ln(f))}{6}-\frac{22b^3x^9\ln(f)^3+36b^2x^6\ln(f)^2+36bx^3\ln(f)+24}{72b^3x^9\ln(f)^3}+ \right.}{3}$

input
```
int(f^(b*x^3+a)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*b^3*ln(f)^3*(1/3/b^3/x^9/ln(f)^3+1/2/b^2/x^6/ln(f)^2+1/2/x^3/b/ln
(f)+11/36-1/2*ln(x)-1/6*ln(-b)-1/6*ln(ln(f))-1/72/b^3/x^9/ln(f)^3*(22*b^3*
x^9*ln(f)^3+36*b^2*x^6*ln(f)^2+36*b*x^3*ln(f)+24)+1/24/b^3/x^9/ln(f)^3*(4*
b^2*x^6*ln(f)^2+4*b*x^3*ln(f)+8)*exp(b*x^3*ln(f))+1/6*ln(-b*x^3*ln(f))+1/6
*Ei(1,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = \frac{b^3 f^a x^9 \mathrm{Ei}(bx^3\log(f))\log(f)^3 - \left(b^2x^6\log(f)^2 + bx^3\log(f) + 2\right)f^{bx^3+a}}{18\,x^9}$$

input
```
integrate(f^(b*x^3+a)/x^10,x, algorithm="fricas")
```

output
```
1/18*(b^3*f^a*x^9*Ei(b*x^3*log(f))*log(f)^3 - (b^2*x^6*log(f)^2 + b*x^3*lo
g(f) + 2)*f^(b*x^3 + a))/x^9
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = \int \frac{f^{a+bx^3}}{x^{10}}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**10,x)
```

output
```
Integral(f**(a + b*x**3)/x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = \frac{1}{3}b^3 f^a \Gamma\left(-3, -bx^3\log(f)\right)\log(f)^3$$

input
```
integrate(f^(b*x^3+a)/x^10,x, algorithm="maxima")
```

output
```
1/3*b^3*f^a*gamma(-3, -b*x^3*log(f))*log(f)^3
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = \int \frac{f^{bx^3+a}}{x^{10}}\,dx$$

input
```
integrate(f^(b*x^3+a)/x^10,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = -\frac{b^3\,f^a\,\ln(f)^3\,\left(f^{b\,x^3}\,\left(\frac{1}{6\,b\,x^3\,\ln(f)}+\frac{1}{6\,b^2\,x^6\,\ln(f)^2}+\frac{1}{3\,b^3\,x^9\,\ln(f)^3}\right)+\frac{\mathrm{expint}\left(-b\,x^3\,\ln(f)\right)}{6}\right)}{3}$$

input
```
int(f^(a + b*x^3)/x^10,x)
```

output
```
-(b^3*f^a*log(f)^3*(f^(b*x^3)*(1/(6*b*x^3*log(f)) + 1/(6*b^2*x^6*log(f)^2)
 + 1/(3*b^3*x^9*log(f)^3)) + expint(-b*x^3*log(f))/6))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{f^{a+bx^3}}{x^{10}}\,dx = \frac{f^a\left(ei(\log(f)\,b\,x^3)\log(f)^3\,b^3x^9-f^{b\,x^3}\log(f)^2\,b^2x^6-f^{b\,x^3}\log(f)\,b\,x^3-2f^{b\,x^3}\right)}{18x^9}$$

input
```
int(f^(b*x^3+a)/x^10,x)
```

output

```
(f**a*(ei(log(f)*b*x**3)*log(f)**3*b**3*x**9 - f**(b*x**3)*log(f)**2*b**2*
x**6 - f**(b*x**3)*log(f)*b*x**3 - 2*f**(b*x**3)))/(18*x**9)
```

3.42 $\int \frac{f^{a+bx^3}}{x^{13}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{f^{a+bx^3}}{x^{13}} \, dx = -\frac{1}{3} b^4 f^a \Gamma\left(-4, -bx^3 \log(f)\right) \log^4(f)$$

output
```
-1/3*f^a/x^12*Ei(5,-b*x^3*ln(f))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^3}}{x^{13}} \, dx = -\frac{1}{3} b^4 f^a \Gamma\left(-4, -bx^3 \log(f)\right) \log^4(f)$$

input
```
Integrate[f^(a + b*x^3)/x^13,x]
```

output
```
-1/3*(b^4*f^a*Gamma[-4, -(b*x^3*Log[f])]*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^{13}} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{1}{3} b^4 f^a \log^4(f) \Gamma\left(-4, -bx^3 \log(f)\right)$$

input
```
Int[f^(a + b*x^3)/x^13,x]
```

output
```
-1/3*(b^4*f^a*Gamma[-4, -(b*x^3*Log[f])]*Log[f]^4)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(18) = 36$.

Time = 0.33 (sec) , antiderivative size = 213, normalized size of antiderivative = 8.88

method	result
meijerg	$f^a b^4 \ln(f)^4 \left(-\frac{1}{4b^4x^{12}\ln(f)^4} - \frac{1}{3b^3x^9\ln(f)^3} - \frac{1}{4b^2x^6\ln(f)^2} - \frac{1}{6x^3b\ln(f)} - \frac{25}{288} + \frac{\ln(x)}{8} + \frac{\ln(-b)}{24} + \frac{\ln(\ln(f))}{24} + \frac{125b^4x^{12}\ln(f)^4+240b^3x^9\ln(f)^3}{1440b^4}\right.$

input
```
int(f^(b*x^3+a)/x^13,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*b^4*ln(f)^4*(-1/4/b^4/x^12/ln(f)^4-1/3/b^3/x^9/ln(f)^3-1/4/b^2/x^6
/ln(f)^2-1/6/x^3/b/ln(f)-25/288+1/8*ln(x)+1/24*ln(-b)+1/24*ln(ln(f))+1/144
0/b^4/x^12/ln(f)^4*(125*b^4*x^12*ln(f)^4+240*b^3*x^9*ln(f)^3+360*b^2*x^6*l
n(f)^2+480*b*x^3*ln(f)+360)-1/120/b^4/x^12/ln(f)^4*(5*b^3*x^9*ln(f)^3+5*b^
2*x^6*ln(f)^2+10*b*x^3*ln(f)+30)*exp(b*x^3*ln(f))-1/24*ln(-b*x^3*ln(f))-1/
24*Ei(1,-b*x^3*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{f^{a+bx^3}}{x^{13}}\,dx = \frac{b^4 f^a x^{12}\mathrm{Ei}(bx^3\log(f))\log(f)^4 - \left(b^3x^9\log(f)^3 + b^2x^6\log(f)^2 + 2\,bx^3\log(f) + 6\right)f^{bx^3+a}}{72\,x^{12}}$$

input
```
integrate(f^(b*x^3+a)/x^13,x, algorithm="fricas")
```

output
```
1/72*(b^4*f^a*x^12*Ei(b*x^3*log(f))*log(f)^4 - (b^3*x^9*log(f)^3 + b^2*x^6
*log(f)^2 + 2*b*x^3*log(f) + 6)*f^(b*x^3 + a))/x^12
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^{13}}\,dx = \int \frac{f^{a+bx^3}}{x^{13}}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**13,x)
```

output
```
Integral(f**(a + b*x**3)/x**13, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+bx^3}}{x^{13}}\,dx = -\frac{1}{3}\,b^4 f^a \Gamma\left(-4, -bx^3 \log\left(f\right)\right) \log\left(f\right)^4$$

input
```
integrate(f^(b*x^3+a)/x^13,x, algorithm="maxima")
```

output
```
-1/3*b^4*f^a*gamma(-4, -b*x^3*log(f))*log(f)^4
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^{13}}\,dx = \int \frac{f^{bx^3+a}}{x^{13}}\,dx$$

input
```
integrate(f^(b*x^3+a)/x^13,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^13, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.75

$$\int \frac{f^{a+bx^3}}{x^{13}}\,dx = -\frac{b^4\, f^a \ln\left(f\right)^4 \operatorname{expint}\left(-b\, x^3\, \ln\left(f\right)\right)}{72} - \frac{b^4\, f^a\, f^{b\, x^3} \ln\left(f\right)^4 \left(\frac{1}{24\, b\, x^3\, \ln(f)} + \frac{1}{24\, b^2\, x^6\, \ln(f)^2} + \frac{1}{12\, b^3\, x^9\, \ln(f)^3} + \frac{1}{4\, b^4\, x^{12}\, \ln(f)^4}\right)}{3}$$

input
```
int(f^(a + b*x^3)/x^13,x)
```

output
```
- (b^4*f^a*log(f)^4*expint(-b*x^3*log(f)))/72 - (b^4*f^a*f^(b*x^3)*log(f)^
4*(1/(24*b*x^3*log(f)) + 1/(24*b^2*x^6*log(f)^2) + 1/(12*b^3*x^9*log(f)^3)
 + 1/(4*b^4*x^12*log(f)^4)))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.75

$$\int \frac{f^{a+bx^3}}{x^{13}}\,dx = \frac{f^a\left(ei(\log(f)\,b\,x^3)\log(f)^4\,b^4x^{12} - f^{b\,x^3}\log(f)^3\,b^3x^9 - f^{b\,x^3}\log(f)^2\,b^2x^6 - 2f^{b\,x^3}\log(f)\,b\,x^3 - 6f^{b\,x^3}\right)}{72x^{12}}$$

input
```
int(f^(b*x^3+a)/x^13,x)
```

output
```
(f**a*(ei(log(f)*b*x**3)*log(f)**4*b**4*x**12 - f**(b*x**3)*log(f)**3*b**3
*x**9 - f**(b*x**3)*log(f)**2*b**2*x**6 - 2*f**(b*x**3)*log(f)*b*x**3 - 6*
f**(b*x**3)))/(72*x**12)
```

3.43 $\int \frac{f^{a+bx^3}}{x^{16}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{f^{a+bx^3}}{x^{16}} \, dx = \frac{1}{3} b^5 f^a \Gamma\left(-5, -bx^3 \log(f)\right) \log^5(f)$$

output
```
-1/3*f^a/x^15*Ei(6,-b*x^3*ln(f))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^3}}{x^{16}} \, dx = \frac{1}{3} b^5 f^a \Gamma\left(-5, -bx^3 \log(f)\right) \log^5(f)$$

input
```
Integrate[f^(a + b*x^3)/x^16,x]
```

output
```
(b^5*f^a*Gamma[-5, -(b*x^3*Log[f])]*Log[f]^5)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{1}{3}b^5 f^a \log^5(f)\Gamma\left(-5,-bx^3\log(f)\right)$$

input
```
Int[f^(a + b*x^3)/x^16,x]
```

output
```
(b^5*f^a*Gamma[-5, -(b*x^3*Log[f])]*Log[f]^5)/3
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(18) = 36.

Time = 0.59 (sec) , antiderivative size = 249, normalized size of antiderivative = 10.38

method	result
meijerg	$-\frac{f^a b^5 \ln(f)^5 \left(\frac{1}{5b^5x^{15}\ln(f)^5}+\frac{1}{4b^4x^{12}\ln(f)^4}+\frac{1}{6b^3x^9\ln(f)^3}+\frac{1}{12b^2x^6\ln(f)^2}+\frac{1}{24x^3b\ln(f)}+\frac{137}{7200}-\frac{\ln(x)}{40}-\frac{\ln(-b)}{120}-\frac{\ln(\ln(f))}{120}-\frac{137b^5x^{15}\ln}{}\right.}{}$

input
```
int(f^(b*x^3+a)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*b^5*ln(f)^5*(1/5/b^5/x^15/ln(f)^5+1/4/b^4/x^12/ln(f)^4+1/6/b^3/x^
9/ln(f)^3+1/12/b^2/x^6/ln(f)^2+1/24/x^3/b/ln(f)+137/7200-1/40*ln(x)-1/120*
ln(-b)-1/120*ln(ln(f))-1/7200/b^5/x^15/ln(f)^5*(137*b^5*x^15*ln(f)^5+300*b
^4*x^12*ln(f)^4+600*b^3*x^9*ln(f)^3+1200*b^2*x^6*ln(f)^2+1800*b*x^3*ln(f)+
1440)+1/720/b^5/x^15/ln(f)^5*(6*b^4*x^12*ln(f)^4+6*b^3*x^9*ln(f)^3+12*b^2*
x^6*ln(f)^2+36*b*x^3*ln(f)+144)*exp(b*x^3*ln(f))+1/120*ln(-b*x^3*ln(f))+1/
120*Ei(1,-b*x^3*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 3.46

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx = \frac{b^5 f^a x^{15} \text{Ei}(bx^3 \log(f)) \log(f)^5 - \left(b^4 x^{12} \log(f)^4 + b^3 x^9 \log(f)^3 + 2\,b^2 x^6 \log(f)^2 + 6\,bx^3 \log(f) + 24\right) f^{bx}}{360\,x^{15}}$$

input
```
integrate(f^(b*x^3+a)/x^16,x, algorithm="fricas")
```

output
```
1/360*(b^5*f^a*x^15*Ei(b*x^3*log(f))*log(f)^5 - (b^4*x^12*log(f)^4 + b^3*x
^9*log(f)^3 + 2*b^2*x^6*log(f)^2 + 6*b*x^3*log(f) + 24)*f^(b*x^3 + a))/x^1
5
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx = \int \frac{f^{a+bx^3}}{x^{16}}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**16,x)
```

output
```
Integral(f**(a + b*x**3)/x**16, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx = \frac{1}{3}\,b^5 f^a \Gamma\left(-5, -bx^3 \log(f)\right) \log(f)^5$$

input
```
integrate(f^(b*x^3+a)/x^16,x, algorithm="maxima")
```

output
```
1/3*b^5*f^a*gamma(-5, -b*x^3*log(f))*log(f)^5
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx = \int \frac{f^{bx^3+a}}{x^{16}}\,dx$$

input
```
integrate(f^(b*x^3+a)/x^16,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^16, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx = -\frac{b^5\,f^a\,\ln(f)^5\,\mathrm{expint}(-b\,x^3\,\ln(f))}{360} - \frac{b^5\,f^a\,f^{b\,x^3}\,\ln(f)^5\,\left(\frac{1}{120\,b\,x^3\,\ln(f)}+\frac{1}{120\,b^2\,x^6\,\ln(f)^2}+\frac{1}{60\,b^3\,x^9\,\ln(f)^3}+\frac{1}{20\,b^4\,x^{12}\,\ln(f)^4}+\frac{1}{5\,b^5\,x^{15}\,\ln(f)^5}\right)}{3}$$

input
```
int(f^(a + b*x^3)/x^16,x)
```

output
```
- (b^5*f^a*log(f)^5*expint(-b*x^3*log(f)))/360 - (b^5*f^a*f^(b*x^3)*log(f)
^5*(1/(120*b*x^3*log(f)) + 1/(120*b^2*x^6*log(f)^2) + 1/(60*b^3*x^9*log(f)
^3) + 1/(20*b^4*x^12*log(f)^4) + 1/(5*b^5*x^15*log(f)^5)))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int \frac{f^{a+bx^3}}{x^{16}}\,dx$$
$$= \frac{f^a\Big(ei(\log(f)\,b\,x^3)\log(f)^5\,b^5x^{15} - f^{b\,x^3}\log(f)^4\,b^4x^{12} - f^{b\,x^3}\log(f)^3\,b^3x^9 - 2f^{b\,x^3}\log(f)^2\,b^2x^6 - 6f^{b\,x^3}\log(}{360x^{15}}$$

input
```
int(f^(b*x^3+a)/x^16,x)
```

output
```
(f**a*(ei(log(f)*b*x**3)*log(f)**5*b**5*x**15 - f**(b*x**3)*log(f)**4*b**4
*x**12 - f**(b*x**3)*log(f)**3*b**3*x**9 - 2*f**(b*x**3)*log(f)**2*b**2*x*
*6 - 6*f**(b*x**3)*log(f)*b*x**3 - 24*f**(b*x**3)))/(360*x**15)
```

3.44 $\int f^{a+bx^3} x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+bx^3} x^4 \, dx = -\frac{f^a x^5 \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3\left(-bx^3 \log(f)\right)^{5/3}}$$

output
```
-1/3*f^a*x^5*GAMMA(5/3,-b*x^3*ln(f))/(-b*x^3*ln(f))^(5/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx^3} x^4 \, dx = -\frac{f^a x^5 \Gamma\left(\frac{5}{3}, -bx^3 \log(f)\right)}{3\left(-bx^3 \log(f)\right)^{5/3}}$$

input
```
Integrate[f^(a + b*x^3)*x^4,x]
```

output
```
-1/3*(f^a*x^5*Gamma[5/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(5/3)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{a+bx^3}\,dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{x^5 f^a \Gamma\left(\frac{5}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{5/3}}$$

input
```
Int[f^(a + b*x^3)*x^4,x]
```

output
```
-1/3*(f^a*x^5*Gamma[5/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(5/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(28) = 56.

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.12

method	result	size
meijerg	$\frac{f^a\left(-\frac{2x^2(-b)^{\frac{5}{3}}\ln(f)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}{3b\left(-b\,x^3\ln(f)\right)^{\frac{2}{3}}}+\frac{x^2(-b)^{\frac{5}{3}}\ln(f)^{\frac{2}{3}}\mathrm{e}^{b\,x^3\ln(f)}}{b}+\frac{2x^2(-b)^{\frac{5}{3}}\ln(f)^{\frac{2}{3}}\Gamma\left(\frac{2}{3},-b\,x^3\ln(f)\right)}{3b\left(-b\,x^3\ln(f)\right)^{\frac{2}{3}}}\right)}{3(-b)^{\frac{5}{3}}\ln(f)^{\frac{5}{3}}}$	106

input
```
int(f^(b*x^3+a)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a/(-b)^(5/3)/ln(f)^(5/3)*(-2/3*x^2*(-b)^(5/3)*ln(f)^(2/3)/b*GAMMA(2/
3)/(-b*x^3*ln(f))^(2/3)+x^2*(-b)^(5/3)*ln(f)^(2/3)/b*exp(b*x^3*ln(f))+2/3*
x^2*(-b)^(5/3)*ln(f)^(2/3)/b/(-b*x^3*ln(f))^(2/3)*GAMMA(2/3,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int f^{a+bx^3}x^4\,dx = \frac{3\,bf^{bx^3+a}x^2\log(f) - 2\left(-b\log(f)\right)^{\frac{1}{3}}\,f^a\Gamma\left(\frac{2}{3}, -bx^3\log(f)\right)}{9\,b^2\log(f)^2}$$

input
```
integrate(f^(b*x^3+a)*x^4,x, algorithm="fricas")
```

output
```
1/9*(3*b*f^(b*x^3 + a)*x^2*log(f) - 2*(-b*log(f))^(1/3)*f^a*gamma(2/3, -b*
x^3*log(f)))/(b^2*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx^3}x^4\,dx = \int f^{a+bx^3}x^4\,dx$$

input
```
integrate(f**(b*x**3+a)*x**4,x)
```

output
```
Integral(f**(a + b*x**3)*x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+bx^3}x^4\,dx=-\frac{f^ax^5\Gamma\left(\frac{5}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{\frac{5}{3}}}$$

input
```
integrate(f^(b*x^3+a)*x^4,x, algorithm="maxima")
```

output
```
-1/3*f^a*x^5*gamma(5/3, -b*x^3*log(f))/(-b*x^3*log(f))^(5/3)
```

Giac [F]

$$\int f^{a+bx^3}x^4\,dx=\int f^{bx^3+a}x^4\,dx$$

input
```
integrate(f^(b*x^3+a)*x^4,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)*x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.09

$$\int f^{a+bx^3}x^4\,dx=\frac{2\,f^a\,x^5\,\Gamma\left(\frac{2}{3}\right)}{9\left(-b\,x^3\,\ln(f)\right)^{5/3}}-\frac{2\,f^a\,x^5\,\Gamma\left(\frac{2}{3},-b\,x^3\,\ln(f)\right)}{9\left(-b\,x^3\,\ln(f)\right)^{5/3}}+\frac{f^a\,f^{b\,x^3}\,x^2}{3\,b\,\ln(f)}$$

input
```
int(f^(a + b*x^3)*x^4,x)
```

output
```
(2*f^a*x^5*gamma(2/3))/(9*(-b*x^3*log(f))^(5/3)) - (2*f^a*x^5*igamma(2/3,
-b*x^3*log(f)))/(9*(-b*x^3*log(f))^(5/3)) + (f^a*f^(b*x^3)*x^2)/(3*b*log(f
))
```

Reduce [F]

$$\int f^{a+bx^3}x^4\,dx = \frac{f^a\left(f^{b\,x^3}x^2-2\left(\int f^{b\,x^3}xdx\right)\right)}{3\log\left(f\right)b}$$

input
```
int(f^(b*x^3+a)*x^4,x)
```

output
```
(f**a*(f**(b*x**3)*x**2 - 2*int(f**(b*x**3)*x,x)))/(3*log(f)*b)
```

3.45 $\int f^{a+bx^3}x^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+bx^3}x^3\,dx=-\frac{f^a x^4\Gamma\left(\frac{4}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{4/3}}$$

output
```
-1/3*f^a*x^4*GAMMA(4/3,-b*x^3*ln(f))/(-b*x^3*ln(f))^(4/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx^3}x^3\,dx=-\frac{f^a x^4\Gamma\left(\frac{4}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{4/3}}$$

input
```
Integrate[f^(a + b*x^3)*x^3,x]
```

output
```
-1/3*(f^a*x^4*Gamma[4/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(4/3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{a+bx^3} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{x^4 f^a \Gamma\left(\frac{4}{3}, -bx^3 \log(f)\right)}{3\left(-bx^3 \log(f)\right)^{4/3}}$$

input
```
Int[f^(a + b*x^3)*x^3,x]
```

output
```
-1/3*(f^a*x^4*Gamma[4/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(4/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(28) = 56$.

Time = 0.05 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.21

method	result	size
meijerg	$-\frac{f^{a}\left(-\frac{2x(-b)^{\frac{4}{3}}\ln(f)^{\frac{1}{3}}\pi\sqrt{3}}{9b\Gamma\left(\frac{2}{3}\right)\left(-b\,x^{3}\ln(f)\right)^{\frac{1}{3}}}+\frac{x(-b)^{\frac{4}{3}}\ln(f)^{\frac{1}{3}}\mathrm{e}^{b\,x^{3}\ln(f)}}{b}+\frac{x(-b)^{\frac{4}{3}}\ln(f)^{\frac{1}{3}}\Gamma\left(\frac{1}{3},-b\,x^{3}\ln(f)\right)}{3b\left(-b\,x^{3}\ln(f)\right)^{\frac{1}{3}}}\right)}{3b\ln(f)^{\frac{4}{3}}(-b)^{\frac{1}{3}}}$	109

input
```
int(f^(b*x^3+a)*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a/b/ln(f)^(4/3)/(-b)^(1/3)*(-2/9*x*(-b)^(4/3)*ln(f)^(1/3)/b*Pi*3^(1
/2)/GAMMA(2/3)/(-b*x^3*ln(f))^(1/3)+x*(-b)^(4/3)*ln(f)^(1/3)/b*exp(b*x^3*l
n(f))+1/3*x*(-b)^(4/3)*ln(f)^(1/3)/b/(-b*x^3*ln(f))^(1/3)*GAMMA(1/3,-b*x^3
*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int f^{a+bx^3}x^3\,dx=\frac{3\,bf^{bx^3+a}x\log{(f)}-(-b\log{(f)})^{\frac{2}{3}}\,f^a\Gamma\left(\frac{1}{3},-bx^3\log{(f)}\right)}{9\,b^2\log{(f)}^2}$$

input
```
integrate(f^(b*x^3+a)*x^3,x, algorithm="fricas")
```

output
```
1/9*(3*b*f^(b*x^3 + a)*x*log(f) - (-b*log(f))^(2/3)*f^a*gamma(1/3, -b*x^3*
log(f)))/(b^2*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx^3}x^3\,dx=\int f^{a+bx^3}x^3\,dx$$

input
```
integrate(f**(b*x**3+a)*x**3,x)
```

output
```
Integral(f**(a + b*x**3)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+bx^3}x^3\,dx=-\frac{f^ax^4\Gamma\left(\frac{4}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{\frac{4}{3}}}$$

input
```
integrate(f^(b*x^3+a)*x^3,x, algorithm="maxima")
```

output
```
-1/3*f^a*x^4*gamma(4/3, -b*x^3*log(f))/(-b*x^3*log(f))^(4/3)
```

Giac [F]

$$\int f^{a+bx^3}x^3\,dx=\int f^{bx^3+a}x^3\,dx$$

input
```
integrate(f^(b*x^3+a)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int f^{a+bx^3}x^3\,dx=\frac{f^a\,f^{b\,x^3}\,x}{3\,b\,\ln(f)}-\frac{f^a\,x^4\,\Gamma\left(\frac{1}{3},-b\,x^3\,\ln(f)\right)}{9\left(-b\,x^3\,\ln(f)\right)^{4/3}}+\frac{2\,\pi\,\sqrt{3}\,f^a\,x^4}{27\,\Gamma\left(\frac{2}{3}\right)\left(-b\,x^3\,\ln(f)\right)^{4/3}}$$

input
```
int(f^(a + b*x^3)*x^3,x)
```

output
```
(f^a*f^(b*x^3)*x)/(3*b*log(f)) - (f^a*x^4*igamma(1/3, -b*x^3*log(f)))/(9*(
-b*x^3*log(f))^(4/3)) + (2*3^(1/2)*f^a*x^4*pi)/(27*gamma(2/3)*(-b*x^3*log(
f))^(4/3))
```

Reduce [F]

$$\int f^{a+bx^3}x^3\,dx = \frac{f^a\left(f^{b\,x^3}x-\left(\int f^{b\,x^3}dx\right)\right)}{3\log\left(f\right)b}$$

input
```
int(f^(b*x^3+a)*x^3,x)
```

output
```
(f**a*(f**(b*x**3)*x - int(f**(b*x**3),x)))/(3*log(f)*b)
```

3.46 $\int f^{a+bx^3}x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int f^{a+bx^3}x\,dx=-\frac{f^ax^2\Gamma\left(\frac{2}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{2/3}}$$

output
```
-1/3*f^a*x^2*GAMMA(2/3,-b*x^3*ln(f))/(-b*x^3*ln(f))^(2/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx^3}x\,dx=-\frac{f^ax^2\Gamma\left(\frac{2}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{2/3}}$$

input
```
Integrate[f^(a + b*x^3)*x,x]
```

output
```
-1/3*(f^a*x^2*Gamma[2/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(2/3)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xf^{a+bx^3}\,dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{x^2f^a\Gamma\left(\frac{2}{3},-bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{2/3}}$$

input
```
Int[f^(a + b*x^3)*x,x]
```

output
```
-1/3*(f^a*x^2*Gamma[2/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(2/3)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

method	result	size
meijerg	$\frac{f^{a}\left(\frac{x^{2}(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}{\left(-b\,x^{3}\ln(f)\right)^{\frac{2}{3}}}-\frac{x^{2}(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}\Gamma\left(\frac{2}{3},-b\,x^{3}\ln(f)\right)}{\left(-b\,x^{3}\ln(f)\right)^{\frac{2}{3}}}\right)}{3(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}}$	75

input
```
int(f^(b*x^3+a)*x,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a/(-b)^(2/3)/ln(f)^(2/3)*(x^2*(-b)^(2/3)*ln(f)^(2/3)*GAMMA(2/3)/(-b*
x^3*ln(f))^(2/3)-x^2*(-b)^(2/3)*ln(f)^(2/3)/(-b*x^3*ln(f))^(2/3)*GAMMA(2/3
,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int f^{a+bx^3}x\,dx=\frac{(-b\log(f))^{\frac{1}{3}}\,f^a\Gamma\left(\frac{2}{3},-bx^3\log(f)\right)}{3\,b\log(f)}$$

input
```
integrate(f^(b*x^3+a)*x,x, algorithm="fricas")
```

output
```
1/3*(-b*log(f))^(1/3)*f^a*gamma(2/3, -b*x^3*log(f))/(b*log(f))
```

Sympy [F]

$$\int f^{a+bx^3}x\,dx=\int f^{a+bx^3}x\,dx$$

input
```
integrate(f**(b*x**3+a)*x,x)
```

output
```
Integral(f**(a + b*x**3)*x, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+bx^3} x\,dx = -\frac{f^a x^2 \Gamma\left(\frac{2}{3}, -bx^3 \log(f)\right)}{3 \left(-bx^3 \log(f)\right)^{\frac{2}{3}}}$$

input
```
integrate(f^(b*x^3+a)*x,x, algorithm="maxima")
```

output
```
-1/3*f^a*x^2*gamma(2/3, -b*x^3*log(f))/(-b*x^3*log(f))^(2/3)
```

Giac [F]

$$\int f^{a+bx^3} x\,dx = \int f^{bx^3+a} x\,dx$$

input
```
integrate(f^(b*x^3+a)*x,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^3} x\,dx = \int f^{b\,x^3+a}\,x\,dx$$

input
```
int(f^(a + b*x^3)*x,x)
```

output
```
int(f^(a + b*x^3)*x, x)
```

Reduce [F]

$$\int f^{a+bx^3} x\,dx = f^a \left(\int f^{b\,x^3} x dx \right)$$

input
```
int(f^(b*x^3+a)*x,x)
```

output
```
f**a*int(f**(b*x**3)*x,x)
```

3.47 $\int f^{a+bx^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int f^{a+bx^3} dx = -\frac{f^a x \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3\sqrt[3]{-bx^3 \log(f)}}$$

output
```
-1/3*f^a*x*GAMMA(1/3,-b*x^3*ln(f))/(-b*x^3*ln(f))^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int f^{a+bx^3} dx = -\frac{f^a x \Gamma\left(\frac{1}{3}, -bx^3 \log(f)\right)}{3\sqrt[3]{-bx^3 \log(f)}}$$

input
```
Integrate[f^(a + b*x^3),x]
```

output
```
-1/3*(f^a*x*Gamma[1/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(1/3)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx^3}\,dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{xf^a\Gamma\left(\frac{1}{3},-bx^3\log(f)\right)}{3\sqrt[3]{-bx^3\log(f)}}$$

input
```
Int[f^(a + b*x^3),x]
```

output
```
-1/3*(f^a*x*Gamma[1/3, -(b*x^3*Log[f])])/(-(b*x^3*Log[f]))^(1/3)
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. 2(26) = 52.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.44

method	result	size
meijerg	$\frac{f^{a}\left(\frac{2x(-b)^{\frac{1}{3}}\ln(f)^{\frac{1}{3}}\pi\sqrt{3}}{3\Gamma\left(\frac{2}{3}\right)\left(-b\,x^{3}\ln(f)\right)^{\frac{1}{3}}}-\frac{x(-b)^{\frac{1}{3}}\ln(f)^{\frac{1}{3}}\Gamma\left(\frac{1}{3},-b\,x^{3}\ln(f)\right)}{\left(-b\,x^{3}\ln(f)\right)^{\frac{1}{3}}}\right)}{3(-b)^{\frac{1}{3}}\ln(f)^{\frac{1}{3}}}$	78

input
```
int(f^(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a/(-b)^(1/3)/ln(f)^(1/3)*(2/3*x*(-b)^(1/3)*ln(f)^(1/3)*Pi*3^(1/2)/GA
MMA(2/3)/(-b*x^3*ln(f))^(1/3)-x*(-b)^(1/3)*ln(f)^(1/3)/(-b*x^3*ln(f))^(1/3
)*GAMMA(1/3,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int f^{a+bx^3}\,dx = \frac{(-b\log{(f)})^{\frac{2}{3}}\,f^a\Gamma\left(\frac{1}{3}, -bx^3\log{(f)}\right)}{3\,b\log{(f)}}$$

input
```
integrate(f^(b*x^3+a),x, algorithm="fricas")
```

output
```
1/3*(-b*log(f))^(2/3)*f^a*gamma(1/3, -b*x^3*log(f))/(b*log(f))
```

Sympy [F]

$$\int f^{a+bx^3}\,dx = \int f^{a+bx^3}\,dx$$

input
```
integrate(f**(b*x**3+a),x)
```

output
```
Integral(f**(a + b*x**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int f^{a+bx^3}\,dx = -\frac{f^a x\Gamma\left(\frac{1}{3}, -bx^3\log(f)\right)}{3\left(-bx^3\log(f)\right)^{\frac{1}{3}}}$$

input
```
integrate(f^(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/3*f^a*x*gamma(1/3, -b*x^3*log(f))/(-b*x^3*log(f))^(1/3)
```

Giac [F]

$$\int f^{a+bx^3}\,dx = \int f^{bx^3+a}\,dx$$

input
```
integrate(f^(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^3}\,dx = \int f^{b\,x^3+a}\,dx$$

input
```
int(f^(a + b*x^3),x)
```

output
```
int(f^(a + b*x^3), x)
```

Reduce [F]

$$\int f^{a+bx^3}\,dx = f^a\left(\int f^{b\,x^3}dx\right)$$

input
```
int(f^(b*x^3+a),x)
```

output
```
f**a*int(f**(b*x**3),x)
```

3.48 $\int \frac{f^{a+bx^3}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+bx^3}}{x^2} dx = -\frac{f^a \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right) \sqrt[3]{-bx^3 \log(f)}}{3x}$$

output
```
-1/3*f^a*GAMMA(-1/3,-b*x^3*ln(f))*(-b*x^3*ln(f))^(1/3)/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^3}}{x^2} dx = -\frac{f^a \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right) \sqrt[3]{-bx^3 \log(f)}}{3x}$$

input
```
Integrate[f^(a + b*x^3)/x^2,x]
```

output
```
-1/3*(f^a*Gamma[-1/3, -(b*x^3*Log[f])]*(-(b*x^3*Log[f]))^(1/3))/x
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^2} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{f^a \sqrt[3]{-bx^3 \log(f)}\Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right)}{3x}$$

input
```
Int[f^(a + b*x^3)/x^2,x]
```

output
```
-1/3*(f^a*Gamma[-1/3, -(b*x^3*Log[f])]*(-(b*x^3*Log[f]))^(1/3))/x
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.94

method	result	size
meijerg	$\frac{f^{a} (-b)^{\frac{1}{3}} \ln(f)^{\frac{1}{3}} \left(\frac{3 x^{2} \ln(f)^{\frac{2}{3}} b \Gamma\left(\frac{2}{3}\right)}{(-b)^{\frac{1}{3}} \left(-b\,x^{3} \ln(f)\right)^{\frac{2}{3}}}-\frac{3\,{\mathrm e}^{b\,x^{3} \ln(f)}}{x (-b)^{\frac{1}{3}} \ln(f)^{\frac{1}{3}}}-\frac{3 x^{2} \ln(f)^{\frac{2}{3}} b \Gamma\left(\frac{2}{3}, -b\,x^{3} \ln(f)\right)}{(-b)^{\frac{1}{3}} \left(-b\,x^{3} \ln(f)\right)^{\frac{2}{3}}}\right)}{3}$	100

input
```
int(f^(b*x^3+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*(-b)^(1/3)*ln(f)^(1/3)*(3*x^2/(-b)^(1/3)*ln(f)^(2/3)*b*GAMMA(2/3)/
(-b*x^3*ln(f))^(2/3)-3/x/(-b)^(1/3)/ln(f)^(1/3)*exp(b*x^3*ln(f))-3*x^2/(-b
)^(1/3)*ln(f)^(2/3)*b/(-b*x^3*ln(f))^(2/3)*GAMMA(2/3,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx^3}}{x^2}\,dx = \frac{(-b\log(f))^{\frac{1}{3}}\,f^a x\Gamma\left(\frac{2}{3}, -bx^3\log(f)\right) - f^{bx^3+a}}{x}$$

input
```
integrate(f^(b*x^3+a)/x^2,x, algorithm="fricas")
```

output
```
((-b*log(f))^(1/3)*f^a*x*gamma(2/3, -b*x^3*log(f)) - f^(b*x^3 + a))/x
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^2}\,dx = \int \frac{f^{a+bx^3}}{x^2}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**2,x)
```

output
```
Integral(f**(a + b*x**3)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+bx^3}}{x^2} dx = -\frac{(-bx^3 \log(f))^{\frac{1}{3}} f^a \Gamma\left(-\frac{1}{3}, -bx^3 \log(f)\right)}{3x}$$

input
```
integrate(f^(b*x^3+a)/x^2,x, algorithm="maxima")
```

output
```
-1/3*(-b*x^3*log(f))^(1/3)*f^a*gamma(-1/3, -b*x^3*log(f))/x
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^2} dx = \int \frac{f^{bx^3+a}}{x^2} dx$$

input
```
integrate(f^(b*x^3+a)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int \frac{f^{a+bx^3}}{x^2} dx = \frac{f^a \Gamma\left(\frac{2}{3}, -b x^3 \ln(f)\right) \left(-b x^3 \ln(f)\right)^{1/3}}{x} - \frac{f^a \Gamma\left(\frac{2}{3}\right) \left(-b x^3 \ln(f)\right)^{1/3}}{x} - \frac{f^a f^{b x^3}}{x}$$

input
```
int(f^(a + b*x^3)/x^2,x)
```

output
```
(f^a*igamma(2/3, -b*x^3*log(f))*(-b*x^3*log(f))^(1/3))/x - (f^a*gamma(2/3)
*(-b*x^3*log(f))^(1/3))/x - (f^a*f^(b*x^3))/x
```

Reduce [F]

$$\int \frac{f^{a+bx^3}}{x^2}\,dx = f^a\left(\int \frac{f^{b\,x^3}}{x^2}dx\right)$$

input
```
int(f^(b*x^3+a)/x^2,x)
```

output
```
f**a*int(f**(b*x**3)/x**2,x)
```

3.49 $\int \frac{f^{a+bx^3}}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+bx^3}}{x^3} \, dx = -\frac{f^a \Gamma\left(-\frac{2}{3}, -bx^3 \log(f)\right) \left(-bx^3 \log(f)\right)^{2/3}}{3x^2}$$

output
```
-1/3*f^a*GAMMA(-2/3,-b*x^3*ln(f))*(-b*x^3*ln(f))^(2/3)/x^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^3}}{x^3} \, dx = -\frac{f^a \Gamma\left(-\frac{2}{3}, -bx^3 \log(f)\right) \left(-bx^3 \log(f)\right)^{2/3}}{3x^2}$$

input
```
Integrate[f^(a + b*x^3)/x^3,x]
```

output
```
-1/3*(f^a*Gamma[-2/3, -(b*x^3*Log[f])]*(-(b*x^3*Log[f]))^(2/3))/x^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^3}}{x^3} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{f^a \left(-bx^3 \log(f)\right)^{2/3} \Gamma\left(-\frac{2}{3}, -bx^3 \log(f)\right)}{3x^2}$$

input
```
Int[f^(a + b*x^3)/x^3,x]
```

output
```
-1/3*(f^a*Gamma[-2/3, -(b*x^3*Log[f])]*(-(b*x^3*Log[f]))^(2/3))/x^2
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(28) = 56.

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.00

method	result	size
meijerg	$-\frac{f^{a} b \ln\left(f\right)^{\frac{2}{3}} \left(\frac{x \ln\left(f\right)^{\frac{1}{3}} b \pi \sqrt{3}}{\left(-b\right)^{\frac{2}{3}} \Gamma\left(\frac{2}{3}\right) \left(-b\,x^{3} \ln\left(f\right)\right)^{\frac{1}{3}}}-\frac{3\,{\mathrm e}^{b\,x^{3} \ln\left(f\right)}}{2 x^{2} \left(-b\right)^{\frac{2}{3}} \ln\left(f\right)^{\frac{2}{3}}}-\frac{3 x \ln\left(f\right)^{\frac{1}{3}} b \Gamma\left(\frac{1}{3}, -b\,x^{3} \ln\left(f\right)\right)}{2 \left(-b\right)^{\frac{2}{3}} \left(-b\,x^{3} \ln\left(f\right)\right)^{\frac{1}{3}}}\right)}{3 \left(-b\right)^{\frac{1}{3}}}$	102

input
```
int(f^(b*x^3+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*b*ln(f)^(2/3)/(-b)^(1/3)*(x/(-b)^(2/3)*ln(f)^(1/3)*b*Pi*3^(1/2)/G
AMMA(2/3)/(-b*x^3*ln(f))^(1/3)-3/2/x^2/(-b)^(2/3)/ln(f)^(2/3)*exp(b*x^3*ln
(f))-3/2*x/(-b)^(2/3)*ln(f)^(1/3)*b/(-b*x^3*ln(f))^(1/3)*GAMMA(1/3,-b*x^3*
ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{f^{a+bx^3}}{x^3}\,dx = \frac{\left(-b\log\left(f\right)\right)^{\frac{2}{3}} f^{a} x^{2} \Gamma\left(\frac{1}{3}, -bx^{3}\log\left(f\right)\right) - f^{bx^{3}+a}}{2\,x^{2}}$$

input
```
integrate(f^(b*x^3+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*((-b*log(f))^(2/3)*f^a*x^2*gamma(1/3, -b*x^3*log(f)) - f^(b*x^3 + a))/
x^2
```

Sympy [F]

$$\int \frac{f^{a+bx^3}}{x^3}\,dx = \int \frac{f^{a+bx^3}}{x^3}\,dx$$

input
```
integrate(f**(b*x**3+a)/x**3,x)
```

output
```
Integral(f**(a + b*x**3)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+bx^3}}{x^3}\,dx = -\frac{(-bx^3\log(f))^{\frac{2}{3}}\,f^a\Gamma\left(-\frac{2}{3}, -bx^3\log(f)\right)}{3\,x^2}$$

input
```
integrate(f^(b*x^3+a)/x^3,x, algorithm="maxima")
```

output
```
-1/3*(-b*x^3*log(f))^(2/3)*f^a*gamma(-2/3, -b*x^3*log(f))/x^2
```

Giac [F]

$$\int \frac{f^{a+bx^3}}{x^3}\,dx = \int \frac{f^{bx^3+a}}{x^3}\,dx$$

input
```
integrate(f^(b*x^3+a)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int \frac{f^{a+bx^3}}{x^3}\,dx = \frac{f^a\,\Gamma\left(\frac{1}{3}, -b\,x^3\,\ln(f)\right)\,\left(-b\,x^3\,\ln(f)\right)^{2/3}}{2\,x^2} - \frac{f^a\,f^{b\,x^3}}{2\,x^2} - \frac{\pi\,\sqrt{3}\,f^a\,\left(-b\,x^3\,\ln(f)\right)^{2/3}}{3\,x^2\,\Gamma\left(\frac{2}{3}\right)}$$

input
```
int(f^(a + b*x^3)/x^3,x)
```

output
```
(f^a*igamma(1/3, -b*x^3*log(f))*(-b*x^3*log(f))^(2/3))/(2*x^2) - (f^a*f^(b
*x^3))/(2*x^2) - (3^(1/2)*f^a*pi*(-b*x^3*log(f))^(2/3))/(3*x^2*gamma(2/3))
```

Reduce [F]

$$\int \frac{f^{a+bx^3}}{x^3} \, dx = f^a \left(\int \frac{f^{b\,x^3}}{x^3} dx \right)$$

input
```
int(f^(b*x^3+a)/x^3,x)
```

output
```
f**a*int(f**(b*x**3)/x**3,x)
```

3.50 $\int e^{4x^3} x^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int e^{4x^3} x^2 \, dx = \frac{e^{4x^3}}{12}$$

output
```
1/12*exp(4*x^3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{4x^3} x^2 \, dx = \frac{e^{4x^3}}{12}$$

input
```
Integrate[E^(4*x^3)*x^2,x]
```

output
```
E^(4*x^3)/12
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4x^3} x^2 \, dx$$
$$\Big\downarrow \; 2638$$
$$\frac{e^{4x^3}}{12}$$

input
```
Int[E^(4*x^3)*x^2,x]
```

output
```
E^(4*x^3)/12
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{e^{4x^3}}{12}$	9
derivativedivides	$\frac{e^{4x^3}}{12}$	9
default	$\frac{e^{4x^3}}{12}$	9
norman	$\frac{e^{4x^3}}{12}$	9
risch	$\frac{e^{4x^3}}{12}$	9
parallelrisch	$\frac{e^{4x^3}}{12}$	9
orering	$\frac{e^{4x^3}}{12}$	9
meijerg	$-\frac{1}{12}+\frac{e^{4x^3}}{12}$	11

input
```
int(exp(4*x^3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*exp(4*x^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{4x^3}x^2\,dx=\frac{1}{12}\,e^{\left(4\,x^3\right)}$$

input
```
integrate(exp(4*x^3)*x^2,x, algorithm="fricas")
```

output
```
1/12*e^(4*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{4x^3}x^2\,dx=\frac{e^{4x^3}}{12}$$

input
```
integrate(exp(4*x**3)*x**2,x)
```

output
```
exp(4*x**3)/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{4x^3}x^2\,dx=\frac{1}{12}\,e^{\left(4\,x^3\right)}$$

input
```
integrate(exp(4*x^3)*x^2,x, algorithm="maxima")
```

output
```
1/12*e^(4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{4x^3}x^2\,dx=\frac{1}{12}\,e^{\left(4\,x^3\right)}$$

input
```
integrate(exp(4*x^3)*x^2,x, algorithm="giac")
```

output
```
1/12*e^(4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{4x^3} x^2 \, dx = \frac{\mathrm{e}^{4\,x^3}}{12}$$

input
```
int(x^2*exp(4*x^3),x)
```

output
```
exp(4*x^3)/12
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{4x^3} x^2 \, dx = \frac{e^{4x^3}}{12}$$

input
```
int(exp(4*x^3)*x^2,x)
```

output
```
e**(4*x**3)/12
```

3.51 $\int f^{a+\frac{b}{x}} x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int f^{a+\frac{b}{x}} x^4 \, dx = -b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x}\right) \log^5(f)$$

output
```
f^a*x^5*Ei(6,-b*ln(f)/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}} x^4 \, dx = -b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x}\right) \log^5(f)$$

input
```
Integrate[f^(a + b/x)*x^4,x]
```

output
```
-(b^5*f^a*Gamma[-5, -((b*Log[f])/x)]*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{a+\frac{b}{x}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$-b^5 f^a \log^5(f)\Gamma\left(-5,-\frac{b\log(f)}{x}\right)$$

input
```
Int[f^(a + b/x)*x^4,x]
```

output
```
-(b^5*f^a*Gamma[-5, -((b*Log[f])/x)]*Log[f]^5)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(17) = 34.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 5.50

method	result
risch	$\frac{f^{a}\,\mathrm{expIntegral}_{1}\left(-\frac{b\ln(f)}{x}\right) b^{5}\ln(f)^{5}}{120}+\frac{f^{a} f^{\frac{b}{x}} x\, b^{4}\ln(f)^{4}}{120}+\frac{f^{a} f^{\frac{b}{x}} x^{2} b^{3}\ln(f)^{3}}{120}+\frac{f^{a} f^{\frac{b}{x}} x^{3} b^{2}\ln(f)^{2}}{60}+\frac{f^{a} f^{\frac{b}{x}} x^{4} b\ln(f)}{20}+\frac{f^{a} f^{\frac{b}{x}}}{5}$
meijerg	$f^{a} b^{5}\ln(f)^{5}\left(\frac{x^{5}}{5b^{5}\ln(f)^{5}}+\frac{x^{4}}{4b^{4}\ln(f)^{4}}+\frac{x^{3}}{6b^{3}\ln(f)^{3}}+\frac{x^{2}}{12b^{2}\ln(f)^{2}}+\frac{x}{24b\ln(f)}+\frac{137}{7200}+\frac{\ln(x)}{120}-\frac{\ln(-b)}{120}-\frac{\ln(\ln(f}{120}\right.$

input
```
int(f^(a+b/x)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/120*f^a*Ei(1,-b*ln(f)/x)*b^5*ln(f)^5+1/120*f^a*f^(b/x)*x*b^4*ln(f)^4+1/1
20*f^a*f^(b/x)*x^2*b^3*ln(f)^3+1/60*f^a*f^(b/x)*x^3*b^2*ln(f)^2+1/20*f^a*f
^(b/x)*x^4*b*ln(f)+1/5*f^a*f^(b/x)*x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int f^{a+\frac{b}{x}} x^4\,dx = -\frac{1}{120}\,b^5 f^a \mathrm{Ei}\left(\frac{b\log(f)}{x}\right)\log(f)^5 + \frac{1}{120}\left(b^4 x\log(f)^4 + b^3x^2\log(f)^3 + 2\,b^2x^3\log(f)^2 + 6\,bx^4\log(f) + 24\,x^5\right)f^{\frac{ax+b}{x}}$$

input
```
integrate(f^(a+b/x)*x^4,x, algorithm="fricas")
```

output
```
-1/120*b^5*f^a*Ei(b*log(f)/x)*log(f)^5 + 1/120*(b^4*x*log(f)^4 + b^3*x^2*l
og(f)^3 + 2*b^2*x^3*log(f)^2 + 6*b*x^4*log(f) + 24*x^5)*f^((a*x + b)/x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x}}x^4\,dx=\int f^{a+\frac{b}{x}}x^4\,dx$$

input
```
integrate(f**(a+b/x)*x**4,x)
```

output
```
Integral(f**(a + b/x)*x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}}x^4\,dx=-b^5f^a\Gamma\left(-5,-\frac{b\log(f)}{x}\right)\log(f)^5$$

input
```
integrate(f^(a+b/x)*x^4,x, algorithm="maxima")
```

output
```
-b^5*f^a*gamma(-5, -b*log(f)/x)*log(f)^5
```

Giac [F]

$$\int f^{a+\frac{b}{x}}x^4\,dx=\int f^{a+\frac{b}{x}}x^4\,dx$$

input
```
integrate(f^(a+b/x)*x^4,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)*x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.50

$$\int f^{a+\frac{b}{x}} x^4\,dx = \frac{b^5\,f^a\ln\left(f\right)^5\,\mathrm{expint}\left(-\frac{b\,\ln(f)}{x}\right)}{120}+b^5\,f^a\,f^{b/x}\ln\left(f\right)^5\left(\frac{x^2}{120\,b^2\ln\left(f\right)^2}+\frac{x^3}{60\,b^3\ln\left(f\right)^3}+\frac{x^4}{20\,b^4\ln\left(f\right)^4}+\frac{x^5}{5\,b^5\ln\left(f\right)^5}+\frac{x}{120\,b\,\ln\left(f\right)}\right)$$

input
```
int(f^(a + b/x)*x^4,x)
```

output
```
(b^5*f^a*log(f)^5*expint(-(b*log(f))/x))/120 + b^5*f^a*f^(b/x)*log(f)^5*(x
^2/(120*b^2*log(f)^2) + x^3/(60*b^3*log(f)^3) + x^4/(20*b^4*log(f)^4) + x^
5/(5*b^5*log(f)^5) + x/(120*b*log(f)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68

$$\int f^{a+\frac{b}{x}} x^4\,dx = \frac{f^a\left(-ei\left(\frac{\log(f)b}{x}\right)\log(f)^5b^5+f^{\frac{b}{x}}\log(f)^4b^4x+f^{\frac{b}{x}}\log(f)^3b^3x^2+2f^{\frac{b}{x}}\log(f)^2b^2x^3+6f^{\frac{b}{x}}\log(f)\,b\,x^4+24f$$

$$120$$

input
```
int(f^(a+b/x)*x^4,x)
```

output
```
(f**a*( - ei((log(f)*b)/x)*log(f)**5*b**5 + f**(b/x)*log(f)**4*b**4*x + f*
*(b/x)*log(f)**3*b**3*x**2 + 2*f**(b/x)*log(f)**2*b**2*x**3 + 6*f**(b/x)*l
og(f)*b*x**4 + 24*f**(b/x)*x**5))/120
```

3.52 $\int f^{a+\frac{b}{x}} x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int f^{a+\frac{b}{x}} x^3 \, dx = b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x}\right) \log^4(f)$$

output
```
f^a*x^4*Ei(5,-b*ln(f)/x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}} x^3 \, dx = b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x}\right) \log^4(f)$$

input
```
Integrate[f^(a + b/x)*x^3,x]
```

output
```
b^4*f^a*Gamma[-4, -((b*Log[f])/x)]*Log[f]^4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{a+\frac{b}{x}} \, dx$$

$$\Big\downarrow \; 2648$$

$$b^4 f^a \log^4(f)\Gamma\left(-4,-\frac{b\log(f)}{x}\right)$$

input
```
Int[f^(a + b/x)*x^3,x]
```

output
```
b^4*f^a*Gamma[-4, -((b*Log[f])/x)]*Log[f]^4
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.71

method	result
risch	$\frac{f^{a}\,\mathrm{expIntegral}_1\left(-\frac{b\ln(f)}{x}\right)b^4\ln(f)^4}{24}+\frac{f^a f^{\frac{b}{x}}x\,b^3\ln(f)^3}{24}+\frac{f^a f^{\frac{b}{x}}x^2b^2\ln(f)^2}{24}+\frac{f^a f^{\frac{b}{x}}x^3b\ln(f)}{12}+\frac{f^a f^{\frac{b}{x}}x^4}{4}$
meijerg	$-f^a\ln(f)^4\,b^4\Bigg(-\frac{x^4}{4b^4\ln(f)^4}-\frac{x^3}{3b^3\ln(f)^3}-\frac{x^2}{4b^2\ln(f)^2}-\frac{x}{6b\ln(f)}-\frac{25}{288}-\frac{\ln(x)}{24}+\frac{\ln(-b)}{24}+\frac{\ln(\ln(f))}{24}+\frac{x^4\big(12}{}$

input
```
int(f^(a+b/x)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24*f^a*Ei(1,-b*ln(f)/x)*b^4*ln(f)^4+1/24*f^a*f^(b/x)*x*b^3*ln(f)^3+1/24*
f^a*f^(b/x)*x^2*b^2*ln(f)^2+1/12*f^a*f^(b/x)*x^3*b*ln(f)+1/4*f^a*f^(b/x)*x
^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(21) = 42$.

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int f^{a+\frac{b}{x}}x^3\,dx = -\frac{1}{24}\,b^4f^a\mathrm{Ei}\left(\frac{b\log(f)}{x}\right)\log(f)^4 + \frac{1}{24}\left(b^3x\log(f)^3 + b^2x^2\log(f)^2 + 2\,bx^3\log(f) + 6\,x^4\right)f^{\frac{ax+b}{x}}$$

input
```
integrate(f^(a+b/x)*x^3,x, algorithm="fricas")
```

output
```
-1/24*b^4*f^a*Ei(b*log(f)/x)*log(f)^4 + 1/24*(b^3*x*log(f)^3 + b^2*x^2*log
(f)^2 + 2*b*x^3*log(f) + 6*x^4)*f^((a*x + b)/x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x}}x^3\,dx = \int f^{a+\frac{b}{x}}x^3\,dx$$

input
```
integrate(f**(a+b/x)*x**3,x)
```

output
```
Integral(f**(a + b/x)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}}x^3\,dx = b^4 f^a \Gamma\left(-4, -\frac{b\log(f)}{x}\right)\log(f)^4$$

input
```
integrate(f^(a+b/x)*x^3,x, algorithm="maxima")
```

output
```
b^4*f^a*gamma(-4, -b*log(f)/x)*log(f)^4
```

Giac [F]

$$\int f^{a+\frac{b}{x}}x^3\,dx = \int f^{a+\frac{b}{x}}x^3\,dx$$

input
```
integrate(f^(a+b/x)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.14

$$\int f^{a+\frac{b}{x}}x^3\,dx=\frac{b^4\,f^a\ln\left(f\right)^4\operatorname{expint}\left(-\frac{b\,\ln(f)}{x}\right)}{24}+b^4\,f^a\,f^{b/x}\ln\left(f\right)^4\left(\frac{x^2}{24\,b^2\ln\left(f\right)^2}+\frac{x^3}{12\,b^3\ln\left(f\right)^3}+\frac{x^4}{4\,b^4\ln\left(f\right)^4}+\frac{x}{24\,b\,\ln\left(f\right)}\right)$$

input
```
int(f^(a + b/x)*x^3,x)
```

output
```
(b^4*f^a*log(f)^4*expint(-(b*log(f))/x))/24 + b^4*f^a*f^(b/x)*log(f)^4*(x^
2/(24*b^2*log(f)^2) + x^3/(12*b^3*log(f)^3) + x^4/(4*b^4*log(f)^4) + x/(24
*b*log(f)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.00

$$\int f^{a+\frac{b}{x}}x^3\,dx=\frac{f^a\left(-ei\left(\frac{\log(f)b}{x}\right)\log(f)^4b^4+f^{\frac{b}{x}}\log(f)^3b^3x+f^{\frac{b}{x}}\log(f)^2b^2x^2+2f^{\frac{b}{x}}\log(f)\,b\,x^3+6f^{\frac{b}{x}}x^4\right)}{24}$$

input
```
int(f^(a+b/x)*x^3,x)
```

output
```
(f**a*( - ei((log(f)*b)/x)*log(f)**4*b**4 + f**(b/x)*log(f)**3*b**3*x + f*
*(b/x)*log(f)**2*b**2*x**2 + 2*f**(b/x)*log(f)*b*x**3 + 6*f**(b/x)*x**4))/
24
```

3.53 $\int f^{a+\frac{b}{x}} x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int f^{a+\frac{b}{x}} x^2 \, dx = \frac{1}{3} f^{a+\frac{b}{x}} x^3 + \frac{1}{6} b f^{a+\frac{b}{x}} x^2 \log(f) + \frac{1}{6} b^2 f^{a+\frac{b}{x}} x \log^2(f) - \frac{1}{6} b^3 f^a \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x}\right) \log^3(f)$$

output
```
1/3*f^(a+b/x)*x^3+1/6*b*f^(a+b/x)*x^2*ln(f)+1/6*b^2*f^(a+b/x)*x*ln(f)^2-1/
6*b^3*f^a*Ei(b*ln(f)/x)*ln(f)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

$$\int f^{a+\frac{b}{x}} x^2 \, dx = \frac{1}{6} f^a \left(-b^3 \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x}\right) \log^3(f) + f^{b/x} x \left(2x^2 + bx \log(f) + b^2 \log^2(f)\right)\right)$$

input
```
Integrate[f^(a + b/x)*x^2,x]
```

output
```
(f^a*(-(b^3*ExpIntegralEi[(b*Log[f])/x]*Log[f]^3) + f^(b/x)*x*(2*x^2 + b*x
*Log[f] + b^2*Log[f]^2)))/6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2643, 2643, 2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+\frac{b}{x}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \int f^{a+\frac{b}{x}} x dx + \frac{1}{3} x^3 f^{a+\frac{b}{x}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \int f^{a+\frac{b}{x}} dx + \frac{1}{2} x^2 f^{a+\frac{b}{x}}\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x}}$$

$$\Big\downarrow \text{2635}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{a+\frac{b}{x}}}{x} dx + x f^{a+\frac{b}{x}}\right) + \frac{1}{2} x^2 f^{a+\frac{b}{x}}\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \left(x f^{a+\frac{b}{x}} - b f^a \log(f) \text{ExpIntegralEi}\left(\frac{b \log(f)}{x}\right)\right) + \frac{1}{2} x^2 f^{a+\frac{b}{x}}\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x}}$$

input
```
Int[f^(a + b/x)*x^2,x]
```

output
```
(f^(a + b/x)*x^3)/3 + (b*Log[f]*((f^(a + b/x)*x^2)/2 + (b*Log[f]*(f^(a + b
/x)*x - b*f^a*ExpIntegralEi[(b*Log[f])/x]*Log[f]))/2))/3
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

method	result
risch	$\frac{f^{a} \operatorname{expIntegral}_{1}\left(-\frac{b \ln(f)}{x}\right) b^{3} \ln(f)^{3}}{6}+\frac{f^{a} f^{\frac{b}{x}} x\, b^{2} \ln(f)^{2}}{6}+\frac{f^{a} f^{\frac{b}{x}} x^{2} b \ln(f)}{6}+\frac{f^{a} f^{\frac{b}{x}} x^{3}}{3}$
meijerg	$f^{a} \ln\left(f\right)^{3} b^{3} \left(\frac{x^{3}}{3 b^{3} \ln(f)^{3}}+\frac{x^{2}}{2 b^{2} \ln(f)^{2}}+\frac{x}{2 b \ln(f)}+\frac{11}{36}+\frac{\ln(x)}{6}-\frac{\ln(-b)}{6}-\frac{\ln(\ln(f))}{6}-\frac{x^{3}\left(\frac{22 b^{3} \ln(f)^{3}}{x^{3}}+\frac{36 b^{2} \ln(f)^{2}}{x^{2}}\right.}{72 b^{3} \ln(f)^{3}}\right.$

input
```
int(f^(a+b/x)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*f^a*Ei(1,-b*ln(f)/x)*b^3*ln(f)^3+1/6*f^a*f^(b/x)*x*b^2*ln(f)^2+1/6*f^a
*f^(b/x)*x^2*b*ln(f)+1/3*f^a*f^(b/x)*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int f^{a+\frac{b}{x}}x^2\,dx$$
$$= -\frac{1}{6}\,b^3f^a\mathrm{Ei}\left(\frac{b\log(f)}{x}\right)\log(f)^3+\frac{1}{6}\left(b^2x\log(f)^2+bx^2\log(f)+2\,x^3\right)f^{\frac{ax+b}{x}}$$

input
```
integrate(f^(a+b/x)*x^2,x, algorithm="fricas")
```

output
```
-1/6*b^3*f^a*Ei(b*log(f)/x)*log(f)^3 + 1/6*(b^2*x*log(f)^2 + b*x^2*log(f)
+ 2*x^3)*f^((a*x + b)/x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x}}x^2\,dx=\int f^{a+\frac{b}{x}}x^2\,dx$$

input
```
integrate(f**(a+b/x)*x**2,x)
```

output
```
Integral(f**(a + b/x)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.28

$$\int f^{a+\frac{b}{x}}x^2\,dx=-b^3f^a\Gamma\left(-3,-\frac{b\log(f)}{x}\right)\log(f)^3$$

input
```
integrate(f^(a+b/x)*x^2,x, algorithm="maxima")
```

output
```
-b^3*f^a*gamma(-3, -b*log(f)/x)*log(f)^3
```

Giac [F]

$$\int f^{a+\frac{b}{x}} x^2 \, dx = \int f^{a+\frac{b}{x}} x^2 \, dx$$

input
```
integrate(f^(a+b/x)*x^2,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int f^{a+\frac{b}{x}} x^2 \, dx = b^3 f^a \ln(f)^3 \left(f^{b/x} \left(\frac{x^2}{6 b^2 \ln(f)^2} + \frac{x^3}{3 b^3 \ln(f)^3} + \frac{x}{6 b \ln(f)} \right) + \frac{\operatorname{expint}\left(-\frac{b \ln(f)}{x}\right)}{6} \right)$$

input
```
int(f^(a + b/x)*x^2,x)
```

output
```
b^3*f^a*log(f)^3*(f^(b/x)*(x^2/(6*b^2*log(f)^2) + x^3/(3*b^3*log(f)^3) + x
/(6*b*log(f))) + expint(-(b*log(f))/x)/6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x}} x^2 \, dx = \frac{f^a \left(-ei\left(\frac{\log(f) b}{x}\right) \log(f)^3 b^3 + f^{\frac{b}{x}} \log(f)^2 b^2 x + f^{\frac{b}{x}} \log(f) b x^2 + 2 f^{\frac{b}{x}} x^3 \right)}{6}$$

input
```
int(f^(a+b/x)*x^2,x)
```

output
```
(f**a*( - ei((log(f)*b)/x)*log(f)**3*b**3 + f**(b/x)*log(f)**2*b**2*x + f*
*(b/x)*log(f)*b*x**2 + 2*f**(b/x)*x**3))/6
```

3.54 $\int f^{a+\frac{b}{x}} x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int f^{a+\frac{b}{x}} x\,dx = \frac{1}{2}f^{a+\frac{b}{x}}x^2 + \frac{1}{2}bf^{a+\frac{b}{x}}x\log(f) - \frac{1}{2}b^2 f^a \text{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)\log^2(f)$$

output
```
1/2*f^(a+b/x)*x^2+1/2*b*f^(a+b/x)*x*ln(f)-1/2*b^2*f^a*Ei(b*ln(f)/x)*ln(f)^
2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

$$\int f^{a+\frac{b}{x}} x\,dx = \frac{1}{2}f^a\left(-b^2 \text{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)\log^2(f) + f^{b/x}x(x+b\log(f))\right)$$

input
```
Integrate[f^(a + b/x)*x,x]
```

output
```
(f^a*(-(b^2*ExpIntegralEi[(b*Log[f])/x]*Log[f]^2) + f^(b/x)*x*(x + b*Log[f
])))/2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2643, 2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{a+\frac{b}{x}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \int f^{a+\frac{b}{x}} dx + \frac{1}{2} x^2 f^{a+\frac{b}{x}}$$

$$\Big\downarrow \text{2635}$$

$$\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{a+\frac{b}{x}}}{x} dx + x f^{a+\frac{b}{x}} \right) + \frac{1}{2} x^2 f^{a+\frac{b}{x}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2} b \log(f) \left(x f^{a+\frac{b}{x}} - b f^a \log(f) \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x} \right) \right) + \frac{1}{2} x^2 f^{a+\frac{b}{x}}$$

input
```
Int[f^(a + b/x)*x,x]
```

output
```
(f^(a + b/x)*x^2)/2 + (b*Log[f]*(f^(a + b/x)*x - b*f^a*ExpIntegralEi[(b*Lo
g[f])/x]*Log[f]))/2
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

method	result
risch	$\frac{f^{a} \operatorname{expIntegral}_{1}\left(-\frac{b \ln(f)}{x}\right) b^{2} \ln(f)^{2}}{2}+\frac{f^{a} f^{\frac{b}{x}} x b \ln(f)}{2}+\frac{f^{a} f^{\frac{b}{x}} x^{2}}{2}$
meijerg	$-f^{a} b^{2} \ln\left(f\right)^{2} \left(-\frac{x^{2}}{2 b^{2} \ln(f)^{2}}-\frac{x}{b \ln(f)}-\frac{3}{4}-\frac{\ln(x)}{2}+\frac{\ln(-b)}{2}+\frac{\ln(\ln(f))}{2}+\frac{x^{2}\left(\frac{9 b^{2} \ln(f)^{2}}{x^{2}}+\frac{12 b \ln(f)}{x}+6\right)}{12 b^{2} \ln(f)^{2}}-\frac{x^{2}\left(3\right.}{}\right.$

input
```
int(f^(a+b/x)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*Ei(1,-b*ln(f)/x)*b^2*ln(f)^2+1/2*f^a*f^(b/x)*x*b*ln(f)+1/2*f^a*f^(
b/x)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int f^{a+\frac{b}{x}} x \, dx = -\frac{1}{2} b^{2} f^{a} \operatorname{Ei}\left(\frac{b \log\left(f\right)}{x}\right) \log\left(f\right)^{2} + \frac{1}{2} \left(b x \log\left(f\right) + x^{2}\right) f^{\frac{a x + b}{x}}$$

input
```
integrate(f^(a+b/x)*x,x, algorithm="fricas")
```

output
```
-1/2*b^2*f^a*Ei(b*log(f)/x)*log(f)^2 + 1/2*(b*x*log(f) + x^2)*f^((a*x + b)
/x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x}}x\,dx=\int f^{a+\frac{b}{x}}x\,dx$$

input
```
integrate(f**(a+b/x)*x,x)
```

output
```
Integral(f**(a + b/x)*x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.38

$$\int f^{a+\frac{b}{x}}x\,dx=b^2f^a\Gamma\left(-2,-\frac{b\log(f)}{x}\right)\log(f)^2$$

input
```
integrate(f^(a+b/x)*x,x, algorithm="maxima")
```

output
```
b^2*f^a*gamma(-2, -b*log(f)/x)*log(f)^2
```

Giac [F]

$$\int f^{a+\frac{b}{x}}x\,dx=\int f^{a+\frac{b}{x}}x\,dx$$

input
```
integrate(f^(a+b/x)*x,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int f^{a+\frac{b}{x}}x\,dx = b^2\,f^a\ln\left(f\right)^2\left(f^{b/x}\left(\frac{x^2}{2\,b^2\ln\left(f\right)^2}+\frac{x}{2\,b\,\ln\left(f\right)}\right)+\frac{\mathrm{expint}\left(-\frac{b\,\ln(f)}{x}\right)}{2}\right)$$

input
```
int(f^(a + b/x)*x,x)
```

output
```
b^2*f^a*log(f)^2*(f^(b/x)*(x^2/(2*b^2*log(f)^2) + x/(2*b*log(f))) + expint
(-(b*log(f))/x)/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x}}x\,dx = \frac{f^a\left(-ei\left(\frac{\log(f)b}{x}\right)\log(f)^2\,b^2+f^{\frac{b}{x}}\log(f)\,bx+f^{\frac{b}{x}}x^2\right)}{2}$$

input
```
int(f^(a+b/x)*x,x)
```

output
```
(f**a*( - ei((log(f)*b)/x)*log(f)**2*b**2 + f**(b/x)*log(f)*b*x + f**(b/x)
*x**2))/2
```

3.55 $\int f^{a+\frac{b}{x}} dx$

Optimal result

Integrand size = 9, antiderivative size = 28

$$\int f^{a+\frac{b}{x}} dx = f^{a+\frac{b}{x}}x - bf^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)\log(f)$$

output
```
f^(a+b/x)*x-b*f^a*Ei(b*ln(f)/x)*ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}} dx = f^{a+\frac{b}{x}}x - bf^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)\log(f)$$

input
```
Integrate[f^(a + b/x),x]
```

output
```
f^(a + b/x)*x - b*f^a*ExpIntegralEi[(b*Log[f])/x]*Log[f]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+\frac{b}{x}}\,dx$$

$$\Big\downarrow \ 2635$$

$$b\log(f)\int \frac{f^{a+\frac{b}{x}}}{x}dx+xf^{a+\frac{b}{x}}$$

$$\Big\downarrow \ 2639$$

$$xf^{a+\frac{b}{x}}-bf^a\log(f)\,\text{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)$$

input
```
Int[f^(a + b/x),x]
```

output
```
f^(a + b/x)*x - b*f^a*ExpIntegralEi[(b*Log[f])/x]*Log[f]
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

method	result
risch	$f^{a} \operatorname{expIntegral}_{1}\left(-\frac{b \ln(f)}{x}\right) b \ln \left(f\right)+f^{a} f^{\frac{b}{x}} x$
meijerg	$f^{a} b \ln \left(f\right) \left(\frac{x}{b \ln(f)}+1+\ln \left(x\right)-\ln \left(-b\right)-\ln \left(\ln \left(f\right)\right)-\frac{x \left(\frac{2 b \ln(f)}{x}+2\right)}{2 b \ln(f)}+\frac{x \,{\mathrm e}^{\frac{b \ln(f)}{x}}}{b \ln(f)}+\ln \left(-\frac{b \ln(f)}{x}\right)+$

input
```
int(f^(a+b/x),x,method=_RETURNVERBOSE)
```

output
```
f^a*Ei(1,-b*ln(f)/x)*b*ln(f)+f^a*f^(b/x)*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int f^{a+\frac{b}{x}}\,dx=-bf^{a}{\rm Ei}\left(\frac{b\log\left(f\right)}{x}\right)\log\left(f\right)+f^{\frac{ax+b}{x}}x$$

input
```
integrate(f^(a+b/x),x, algorithm="fricas")
```

output
```
-b*f^a*Ei(b*log(f)/x)*log(f) + f^((a*x + b)/x)*x
```

Sympy [F]

$$\int f^{a+\frac{b}{x}}\,dx=\int f^{a+\frac{b}{x}}\,dx$$

input
```
integrate(f**(a+b/x),x)
```

output
```
Integral(f**(a + b/x), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

$$\int f^{a+\frac{b}{x}}\,dx = -bf^a\Gamma\left(-1,-\frac{b\log(f)}{x}\right)\log(f)$$

input
```
integrate(f^(a+b/x),x, algorithm="maxima")
```

output
```
-b*f^a*gamma(-1, -b*log(f)/x)*log(f)
```

Giac [F]

$$\int f^{a+\frac{b}{x}}\,dx = \int f^{a+\frac{b}{x}}\,dx$$

input
```
integrate(f^(a+b/x),x, algorithm="giac")
```

output
```
integrate(f^(a + b/x), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int f^{a+\frac{b}{x}}\,dx = f^a\left(f^{b/x}\,x + b\,\ln(f)\,\mathrm{expint}\left(-\frac{b\,\ln(f)}{x}\right)\right)$$

input
```
int(f^(a + b/x),x)
```

output
```
f^a*(f^(b/x)*x + b*log(f)*expint(-(b*log(f))/x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int f^{a+\frac{b}{x}}\,dx = f^{a}\left(-ei\left(\frac{\log(f)\,b}{x}\right)\log(f)\,b+f^{\frac{b}{x}}x\right)$$

input
```
int(f^(a+b/x),x)
```

output
```
f**a*( - ei((log(f)*b)/x)*log(f)*b + f**(b/x)*x)
```

3.56 $\int \frac{f^{a+\frac{b}{x}}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{f^{a+\frac{b}{x}}}{x} dx = -f^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)$$

output
```
-f^a*Ei(b*ln(f)/x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x} dx = -f^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x}\right)$$

input
```
Integrate[f^(a + b/x)/x,x]
```

output
```
-(f^a*ExpIntegralEi[(b*Log[f])/x])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x} \, dx$$

$$\Big\downarrow \text{2639}$$

$$-f^a \text{ExpIntegralEi}\left(\frac{b \log(f)}{x}\right)$$

input
```
Int[f^(a + b/x)/x,x]
```

output
```
-(f^a*ExpIntegralEi[(b*Log[f])/x])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
risch	$f^a \operatorname{expIntegral}_1\left(-\frac{b \ln(f)}{x}\right)$	15
meijerg	$-f^a\left(-\ln(x)+\ln(-b)+\ln(\ln(f))-\ln\left(-\frac{b \ln(f)}{x}\right)-\operatorname{expIntegral}_1\left(-\frac{b \ln(f)}{x}\right)\right)$	41

input
```
int(f^(a+b/x)/x,x,method=_RETURNVERBOSE)
```

output
```
f^a*Ei(1,-b*ln(f)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x}\,dx = -f^a \mathrm{Ei}\left(\frac{b\log(f)}{x}\right)$$

input
```
integrate(f^(a+b/x)/x,x, algorithm="fricas")
```

output
```
-f^a*Ei(b*log(f)/x)
```

Sympy [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x}\,dx$$

input
```
integrate(f**(a+b/x)/x,x)
```

output
```
Integral(f**(a + b/x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x}\,dx = -f^a \operatorname{Ei}\left(\frac{b\log(f)}{x}\right)$$

input
```
integrate(f^(a+b/x)/x,x, algorithm="maxima")
```

output
```
-f^a*Ei(b*log(f)/x)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x}\,dx$$

input
```
integrate(f^(a+b/x)/x,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x}\,dx = -f^a \operatorname{ei}\left(\frac{b\ln(f)}{x}\right)$$

input
```
int(f^(a + b/x)/x,x)
```

output
```
-f^a*ei((b*log(f))/x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x} \, dx = -f^a ei\left(\frac{\log(f) \, b}{x}\right)$$

input
```
int(f^(a+b/x)/x,x)
```

output
```
- f**a*ei((log(f)*b)/x)
```

3.57 $\int \frac{f^{a+\frac{b}{x}}}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{f^{a+\frac{b}{x}}}{x^2} \, dx = -\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

output
```
-f^(a+b/x)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x^2} \, dx = -\frac{f^{a+\frac{b}{x}}}{b \log(f)}$$

input
```
Integrate[f^(a + b/x)/x^2,x]
```

output
```
-(f^(a + b/x)/(b*Log[f]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x^2} \, dx$$
$$\Big\downarrow \text{2638}$$
$$-\frac{f^{a+\frac{b}{x}}}{b\log(f)}$$

input
```
Int[f^(a + b/x)/x^2,x]
```

output
```
-(f^(a + b/x)/(b*Log[f]))
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$-\frac{f^{a+\frac{b}{x}}}{b\ln(f)}$	19
default	$-\frac{f^{a+\frac{b}{x}}}{b\ln(f)}$	19
parallelrisch	$-\frac{f^{a+\frac{b}{x}}}{b\ln(f)}$	19
norman	$-\frac{\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln(f)}}{b\ln(f)}$	21
risch	$-\frac{f^{\frac{ax+b}{x}}}{b\ln(f)}$	21
meijerg	$\frac{f^{a}\left(1-\mathrm{e}^{\frac{b\ln(f)}{x}}\right)}{\ln(f)b}$	24

input
```
int(f^(a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-f^(a+b/x)/b/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{f^{a+\frac{b}{x}}}{x^2}\,dx = -\frac{f^{\frac{ax+b}{x}}}{b\log(f)}$$

input
```
integrate(f^(a+b/x)/x^2,x, algorithm="fricas")
```

output
```
-f^((a*x + b)/x)/(b*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{f^{a+\frac{b}{x}}}{x^2}\,dx = \begin{cases} -\frac{f^{a+\frac{b}{x}}}{b\log(f)} & \text{for } b\log(f) \neq 0 \\ -\frac{1}{x} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(a+b/x)/x**2,x)
```

output
```
Piecewise((-f**(a + b/x)/(b*log(f)), Ne(b*log(f), 0)), (-1/x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x^2}\,dx = -\frac{f^{a+\frac{b}{x}}}{b\log(f)}$$

input
```
integrate(f^(a+b/x)/x^2,x, algorithm="maxima")
```

output
```
-f^(a + b/x)/(b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{f^{a+\frac{b}{x}}}{x^2}\,dx = -\frac{f^{\frac{ax+b}{x}}}{b\log(f)}$$

input
```
integrate(f^(a+b/x)/x^2,x, algorithm="giac")
```

output
```
-f^((a*x + b)/x)/(b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x}}}{x^2}\,dx = -\frac{f^{a+\frac{b}{x}}}{b\ln(f)}$$

input
```
int(f^(a + b/x)/x^2,x)
```

output
```
-f^(a + b/x)/(b*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{f^{a+\frac{b}{x}}}{x^2}\,dx = -\frac{f^{\frac{ax+b}{x}}}{\log(f)\,b}$$

input
```
int(f^(a+b/x)/x^2,x)
```

output
```
( - f**((a*x + b)/x))/(log(f)*b)
```

3.58 $\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx=\frac{f^{a+\frac{b}{x}}}{b^2\log^2(f)}-\frac{f^{a+\frac{b}{x}}}{bx\log(f)}$$

output
```
f^(a+b/x)/b^2/ln(f)^2-f^(a+b/x)/b/x/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx=\frac{f^{a+\frac{b}{x}}(x-b\log(f))}{b^2x\log^2(f)}$$

input
```
Integrate[f^(a + b/x)/x^3,x]
```

output
```
(f^(a + b/x)*(x - b*Log[f]))/(b^2*x*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x^3} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{f^{a+\frac{b}{x}}}{x^2}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x}}}{bx\log(f)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{f^{a+\frac{b}{x}}}{b^2\log^2(f)}-\frac{f^{a+\frac{b}{x}}}{bx\log(f)}$$

input
```
Int[f^(a + b/x)/x^3,x]
```

output
```
f^(a + b/x)/(b^2*Log[f]^2) - f^(a + b/x)/(b*x*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{(b\ln(f)-x)f^{\frac{ax+b}{x}}}{b^2\ln(f)^2 x}$	32
meijerg	$-\frac{f^a\left(1-\frac{\left(-\frac{2b\ln(f)}{x}+2\right)\mathrm{e}^{\frac{b\ln(f)}{x}}}{2}\right)}{\ln(f)^2 b^2}$	35
parallelrisch	$\frac{-f^{a+\frac{b}{x}}b\ln(f)+f^{a+\frac{b}{x}}x}{x\,b^2\ln(f)^2}$	38
norman	$\frac{\frac{x^2\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln(f)}}{b^2\ln(f)^2}-\frac{x\,\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln(f)}}{b\ln(f)}}{x^2}$	49

input
```
int(f^(a+b/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(f)-x)/b^2/ln(f)^2/x*f^((a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx = -\frac{(b\log(f)-x)f^{\frac{ax+b}{x}}}{b^2x\log(f)^2}$$

input
```
integrate(f^(a+b/x)/x^3,x, algorithm="fricas")
```

output
```
-(b*log(f) - x)*f^((a*x + b)/x)/(b^2*x*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.56

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx = \frac{f^{a+\frac{b}{x}}(-b\log(f)+x)}{b^2x\log(f)^2}$$

input
```
integrate(f**(a+b/x)/x**3,x)
```

output
```
f**(a + b/x)*(-b*log(f) + x)/(b**2*x*log(f)**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.54

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx = \frac{f^a\Gamma\left(2,-\frac{b\log(f)}{x}\right)}{b^2\log(f)^2}$$

input
```
integrate(f^(a+b/x)/x^3,x, algorithm="maxima")
```

output
```
f^a*gamma(2, -b*log(f)/x)/(b^2*log(f)^2)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx$$

input
```
integrate(f^(a+b/x)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx = \frac{f^{a+\frac{b}{x}}\,(x - b\,\ln(f))}{b^2\,x\,\ln(f)^2}$$

input
```
int(f^(a + b/x)/x^3,x)
```

output
```
(f^(a + b/x)*(x - b*log(f)))/(b^2*x*log(f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{f^{a+\frac{b}{x}}}{x^3}\,dx = \frac{f^{\frac{ax+b}{x}}(-\log(f)\,b + x)}{\log(f)^2\,b^2x}$$

input
```
int(f^(a+b/x)/x^3,x)
```

output
```
(f**((a*x + b)/x)*( - log(f)*b + x))/(log(f)**2*b**2*x)
```

3.59 $\int \frac{f^{a+\frac{b}{x}}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{f^{a+\frac{b}{x}}}{x^4} dx = -\frac{2f^{a+\frac{b}{x}}}{b^3\log^3(f)} + \frac{2f^{a+\frac{b}{x}}}{b^2x\log^2(f)} - \frac{f^{a+\frac{b}{x}}}{bx^2\log(f)}$$

output
```
-2*f^(a+b/x)/b^3/ln(f)^3+2*f^(a+b/x)/b^2/x/ln(f)^2-f^(a+b/x)/b/x^2/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+\frac{b}{x}}}{x^4} dx = -\frac{f^{a+\frac{b}{x}}(2x^2 - 2bx\log(f) + b^2\log^2(f))}{b^3x^2\log^3(f)}$$

input
```
Integrate[f^(a + b/x)/x^4,x]
```

output
```
-((f^(a + b/x)*(2*x^2 - 2*b*x*Log[f] + b^2*Log[f]^2))/(b^3*x^2*Log[f]^3))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x^4} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2 \int \frac{f^{a+\frac{b}{x}}}{x^3}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2 \left(-\frac{\int \frac{f^{a+\frac{b}{x}}}{x^2}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{bx \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{2 \left(\frac{f^{a+\frac{b}{x}}}{b^2 \log^2(f)}-\frac{f^{a+\frac{b}{x}}}{bx \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{bx^2 \log(f)}$$

input
```
Int[f^(a + b/x)/x^4,x]
```

output
```
-(f^(a + b/x)/(b*x^2*Log[f])) - (2*(f^(a + b/x)/(b^2*Log[f]^2) - f^(a + b/
x)/(b*x*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{\left(\ln\left(f\right)^{2} b^{2}-2 b x \ln\left(f\right)+2 x^{2}\right) f^{\frac{a x+b}{x}}}{\ln\left(f\right)^{3} b^{3} x^{2}}$	44
meijerg	$\frac{f^{a}\left(2-\frac{\left(\frac{3 b^{2} \ln\left(f\right)^{2}}{x^{2}}-\frac{6 b \ln\left(f\right)}{x}+6\right) {\mathrm e}^{\frac{b \ln\left(f\right)}{x}}}{3}\right)}{\ln\left(f\right)^{3} b^{3}}$	46
parallelrisch	$\frac{-\ln\left(f\right)^{2} f^{a+\frac{b}{x}} b^{2}+2 b\, f^{a+\frac{b}{x}} x \ln\left(f\right)-2 f^{a+\frac{b}{x}} x^{2}}{x^{2} \ln\left(f\right)^{3} b^{3}}$	60
norman	$\frac{-\frac{x\,{\mathrm e}^{\left(a+\frac{b}{x}\right) \ln\left(f\right)}}{b \ln\left(f\right)}+\frac{2 x^{2} {\mathrm e}^{\left(a+\frac{b}{x}\right) \ln\left(f\right)}}{b^{2} \ln\left(f\right)^{2}}-\frac{2 x^{3} {\mathrm e}^{\left(a+\frac{b}{x}\right) \ln\left(f\right)}}{b^{3} \ln\left(f\right)^{3}}}{x^{3}}$	73

input
```
int(f^(a+b/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-(ln(f)^2*b^2-2*b*x*ln(f)+2*x^2)/ln(f)^3/b^3/x^2*f^((a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx = -\frac{\left(b^2\log\left(f\right)^2 - 2\,bx\log\left(f\right) + 2\,x^2\right)f^{\frac{ax+b}{x}}}{b^3x^2\log\left(f\right)^3}$$

input
```
integrate(f^(a+b/x)/x^4,x, algorithm="fricas")
```

output
```
-(b^2*log(f)^2 - 2*b*x*log(f) + 2*x^2)*f^((a*x + b)/x)/(b^3*x^2*log(f)^3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx = \frac{f^{a+\frac{b}{x}}\left(-b^2\log\left(f\right)^2 + 2bx\log\left(f\right) - 2x^2\right)}{b^3x^2\log\left(f\right)^3}$$

input
```
integrate(f**(a+b/x)/x**4,x)
```

output
```
f**(a + b/x)*(-b**2*log(f)**2 + 2*b*x*log(f) - 2*x**2)/(b**3*x**2*log(f)**
3)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx = -\frac{f^a\Gamma\left(3, -\frac{b\log(f)}{x}\right)}{b^3\log\left(f\right)^3}$$

input
```
integrate(f^(a+b/x)/x^4,x, algorithm="maxima")
```

output
```
-f^a*gamma(3, -b*log(f)/x)/(b^3*log(f)^3)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx$$

input
```
integrate(f^(a+b/x)/x^4,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.74

$$\int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx = -\frac{f^{a+\frac{b}{x}}\left(\frac{1}{b\ln(f)}+\frac{2x^2}{b^3\ln(f)^3}-\frac{2x}{b^2\ln(f)^2}\right)}{x^2}$$

input
```
int(f^(a + b/x)/x^4,x)
```

output
```
-(f^(a + b/x)*(1/(b*log(f)) + (2*x^2)/(b^3*log(f)^3) - (2*x)/(b^2*log(f)^2
)))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x}}}{x^4}\,dx = \frac{f^{\frac{ax+b}{x}}\left(-\log(f)^2 b^2+2\log(f)\,bx-2x^2\right)}{\log(f)^3 b^3x^2}$$

input
```
int(f^(a+b/x)/x^4,x)
```

output
```
(f**((a*x + b)/x)*( - log(f)**2*b**2 + 2*log(f)*b*x - 2*x**2))/(log(f)**3*
b**3*x**2)
```

3.60 $\int \frac{f^{a+\frac{b}{x}}}{x^5} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{f^{a+\frac{b}{x}}}{x^5} \, dx = \frac{6f^{a+\frac{b}{x}}}{b^4 \log^4(f)} - \frac{6f^{a+\frac{b}{x}}}{b^3 x \log^3(f)} + \frac{3f^{a+\frac{b}{x}}}{b^2 x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x}}}{b x^3 \log(f)}$$

output
```
6*f^(a+b/x)/b^4/ln(f)^4-6*f^(a+b/x)/b^3/x/ln(f)^3+3*f^(a+b/x)/b^2/x^2/ln(f
)^2-f^(a+b/x)/b/x^3/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{f^{a+\frac{b}{x}}}{x^5} \, dx = \frac{f^{a+\frac{b}{x}}(6x^3 - 6bx^2 \log(f) + 3b^2 x \log^2(f) - b^3 \log^3(f))}{b^4 x^3 \log^4(f)}$$

input
```
Integrate[f^(a + b/x)/x^5,x]
```

output
```
(f^(a + b/x)*(6*x^3 - 6*b*x^2*Log[f] + 3*b^2*x*Log[f]^2 - b^3*Log[f]^3))/(
b^4*x^3*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x^5} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \int \frac{f^{a+\frac{b}{x}}}{x^4}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^3 \log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \left(-\frac{2 \int \frac{f^{a+\frac{b}{x}}}{x^3}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^2 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^3 \log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \left(-\frac{2 \left(-\frac{\int \frac{f^{a+\frac{b}{x}}}{x^2}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^2 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^3 \log(f)}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3 \left(-\frac{2 \left(\frac{f^{a+\frac{b}{x}}}{b^2 \log^2(f)}-\frac{f^{a+\frac{b}{x}}}{b x \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^2 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x}}}{b x^3 \log(f)}$$

input
```
Int[f^(a + b/x)/x^5,x]
```

output
```
-(f^(a + b/x)/(b*x^3*Log[f])) - (3*(-(f^(a + b/x)/(b*x^2*Log[f])) - (2*(f^
(a + b/x)/(b^2*Log[f]^2) - f^(a + b/x)/(b*x*Log[f])))/(b*Log[f])))/(b*Log[
f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{\left(\ln(f)^3 b^3-3\ln(f)^2 b^2 x+6b\,x^2\ln(f)-6x^3\right) f^{\frac{ax+b}{x}}}{\ln(f)^4 b^4 x^3}$	56
meijerg	$-\frac{f^a\left(6-\frac{\left(-\frac{4b^3\ln(f)^3}{x^3}+\frac{12b^2\ln(f)^2}{x^2}-\frac{24b\ln(f)}{x}+24\right)e^{\frac{b\ln(f)}{x}}}{4}\right)}{b^4\ln(f)^4}$	59
parallelrisch	$\frac{-\ln(f)^3 f^{a+\frac{b}{x}} b^3+3b^2 f^{a+\frac{b}{x}} x\ln(f)^2-6b\,f^{a+\frac{b}{x}} x^2\ln(f)+6f^{a+\frac{b}{x}} x^3}{x^3\ln(f)^4 b^4}$	81
norman	$\frac{-\frac{x\,e^{\left(a+\frac{b}{x}\right)\ln(f)}}{b\ln(f)}+\frac{3x^2 e^{\left(a+\frac{b}{x}\right)\ln(f)}}{b^2\ln(f)^2}-\frac{6x^3 e^{\left(a+\frac{b}{x}\right)\ln(f)}}{b^3\ln(f)^3}+\frac{6x^4 e^{\left(a+\frac{b}{x}\right)\ln(f)}}{b^4\ln(f)^4}}{x^4}$	96

input
```
int(f^(a+b/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-(ln(f)^3*b^3-3*ln(f)^2*b^2*x+6*b*x^2*ln(f)-6*x^3)/ln(f)^4/b^4/x^3*f^((a*x
+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+\frac{b}{x}}}{x^5}\,dx = -\frac{\left(b^3\log\left(f\right)^3 - 3\,b^2x\log\left(f\right)^2 + 6\,bx^2\log\left(f\right) - 6\,x^3\right)f^{\frac{ax+b}{x}}}{b^4x^3\log\left(f\right)^4}$$

input
```
integrate(f^(a+b/x)/x^5,x, algorithm="fricas")
```

output
```
-(b^3*log(f)^3 - 3*b^2*x*log(f)^2 + 6*b*x^2*log(f) - 6*x^3)*f^((a*x + b)/x
)/(b^4*x^3*log(f)^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

$$\int \frac{f^{a+\frac{b}{x}}}{x^5}\,dx = \frac{f^{a+\frac{b}{x}}\left(-b^3\log\left(f\right)^3 + 3b^2x\log\left(f\right)^2 - 6bx^2\log\left(f\right) + 6x^3\right)}{b^4x^3\log\left(f\right)^4}$$

input
```
integrate(f**(a+b/x)/x**5,x)
```

output
```
f**(a + b/x)*(-b**3*log(f)**3 + 3*b**2*x*log(f)**2 - 6*b*x**2*log(f) + 6*x
**3)/(b**4*x**3*log(f)**4)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.26

$$\int \frac{f^{a+\frac{b}{x}}}{x^5}\,dx = \frac{f^a\Gamma\left(4, -\frac{b\log(f)}{x}\right)}{b^4\log(f)^4}$$

input
```
integrate(f^(a+b/x)/x^5,x, algorithm="maxima")
```

output
```
f^a*gamma(4, -b*log(f)/x)/(b^4*log(f)^4)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x^5}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x^5}\,dx$$

input
```
integrate(f^(a+b/x)/x^5,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x}}}{x^5}\,dx = -\frac{f^{a+\frac{b}{x}}\left(\frac{1}{b\ln(f)} + \frac{6x^2}{b^3\ln(f)^3} - \frac{6x^3}{b^4\ln(f)^4} - \frac{3x}{b^2\ln(f)^2}\right)}{x^3}$$

input
```
int(f^(a + b/x)/x^5,x)
```

output
```
-(f^(a + b/x)*(1/(b*log(f)) + (6*x^2)/(b^3*log(f)^3) - (6*x^3)/(b^4*log(f)
^4) - (3*x)/(b^2*log(f)^2)))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+\frac{b}{x}}}{x^5}\, dx = \frac{f^{\frac{ax+b}{x}} \left(-\log(f)^3 b^3 + 3\log(f)^2 b^2 x - 6\log(f)\, b\, x^2 + 6x^3\right)}{\log(f)^4 b^4 x^3}$$

input
```
int(f^(a+b/x)/x^5,x)
```

output
```
(f**((a*x + b)/x)*( - log(f)**3*b**3 + 3*log(f)**2*b**2*x - 6*log(f)*b*x**
2 + 6*x**3))/(log(f)**4*b**4*x**3)
```

3.61 $\int \frac{f^{a+\frac{b}{x}}}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{f^{a+\frac{b}{x}}}{x^6} dx = -\frac{f^{a+\frac{b}{x}}(24x^4 - 24bx^3\log(f) + 12b^2x^2\log^2(f) - 4b^3x\log^3(f) + b^4\log^4(f))}{b^5x^4\log^5(f)}$$

output
```
-f^(a+b/x)*(24*x^4-24*b*x^3*ln(f)+12*b^2*x^2*ln(f)^2-4*b^3*x*ln(f)^3+b^4*l
n(f)^4)/b^5/x^4/ln(f)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.34

$$\int \frac{f^{a+\frac{b}{x}}}{x^6} dx = -\frac{f^a\Gamma\left(5, -\frac{b\log(f)}{x}\right)}{b^5\log^5(f)}$$

input
```
Integrate[f^(a + b/x)/x^6,x]
```

output
```
-((f^a*Gamma[5, -((b*Log[f])/x)])/(b^5*Log[f]^5))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2647\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x^6} \, dx$$

$$\Big\downarrow \text{2647}$$

$$-\frac{f^{a+\frac{b}{x}} \left(b^4 \log^4(f) - 4b^3 x \log^3(f) + 12b^2 x^2 \log^2(f) - 24bx^3 \log(f) + 24x^4\right)}{b^5 x^4 \log^5(f)}$$

input
```
Int[f^(a + b/x)/x^6,x]
```

output
```
-((f^(a + b/x)*(24*x^4 - 24*b*x^3*Log[f] + 12*b^2*x^2*Log[f]^2 - 4*b^3*x*L
og[f]^3 + b^4*Log[f]^4))/(b^5*x^4*Log[f]^5))
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{\left(24x^{4}-24b\,x^{3}\ln\left(f\right)+12b^{2}x^{2}\ln\left(f\right)^{2}-4b^{3}x\ln\left(f\right)^{3}+b^{4}\ln\left(f\right)^{4}\right)f^{\frac{ax+b}{x}}}{b^{5}\ln\left(f\right)^{5}x^{4}}$	68
meijerg	$\frac{f^{a}\left(24-\frac{\left(\frac{5b^{4}\ln\left(f\right)^{4}}{x^{4}}-\frac{20b^{3}\ln\left(f\right)^{3}}{x^{3}}+\frac{60b^{2}\ln\left(f\right)^{2}}{x^{2}}-\frac{120b\ln\left(f\right)}{x}+120\right)\mathrm{e}^{\frac{b\ln\left(f\right)}{x}}}{5}\right)}{b^{5}\ln\left(f\right)^{5}}$	70
parallelrisch	$\frac{-\ln\left(f\right)^{4}f^{a+\frac{b}{x}}b^{4}+4\ln\left(f\right)^{3}x\,f^{a+\frac{b}{x}}b^{3}-12\ln\left(f\right)^{2}x^{2}f^{a+\frac{b}{x}}b^{2}+24\ln\left(f\right)x^{3}f^{a+\frac{b}{x}}b-24f^{a+\frac{b}{x}}x^{4}}{x^{4}b^{5}\ln\left(f\right)^{5}}$	102
norman	$\frac{-\frac{x\,\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln\left(f\right)}}{b\ln\left(f\right)}+\frac{4x^{2}\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln\left(f\right)}}{b^{2}\ln\left(f\right)^{2}}-\frac{12x^{3}\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln\left(f\right)}}{b^{3}\ln\left(f\right)^{3}}+\frac{24x^{4}\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln\left(f\right)}}{b^{4}\ln\left(f\right)^{4}}-\frac{24x^{5}\mathrm{e}^{\left(a+\frac{b}{x}\right)\ln\left(f\right)}}{b^{5}\ln\left(f\right)^{5}}}{x^{5}}$	119

input
```
int(f^(a+b/x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-(24*x^4-24*b*x^3*ln(f)+12*b^2*x^2*ln(f)^2-4*b^3*x*ln(f)^3+b^4*ln(f)^4)/b^
5/ln(f)^5/x^4*f^((a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x}}}{x^{6}}\,dx = -\frac{\left(b^{4}\log\left(f\right)^{4}-4\,b^{3}x\log\left(f\right)^{3}+12\,b^{2}x^{2}\log\left(f\right)^{2}-24\,bx^{3}\log\left(f\right)+24\,x^{4}\right)f^{\frac{ax+b}{x}}}{b^{5}x^{4}\log\left(f\right)^{5}}$$

input
```
integrate(f^(a+b/x)/x^6,x, algorithm="fricas")
```

output
```
-(b^4*log(f)^4 - 4*b^3*x*log(f)^3 + 12*b^2*x^2*log(f)^2 - 24*b*x^3*log(f)
+ 24*x^4)*f^((a*x + b)/x)/(b^5*x^4*log(f)^5)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x}}}{x^6}\,dx = \frac{f^{a+\frac{b}{x}}\left(-b^4\log\left(f\right)^4+4b^3x\log\left(f\right)^3-12b^2x^2\log\left(f\right)^2+24bx^3\log\left(f\right)-24x^4\right)}{b^5x^4\log\left(f\right)^5}$$

input
```
integrate(f**(a+b/x)/x**6,x)
```

output
```
f**(a + b/x)*(-b**4*log(f)**4 + 4*b**3*x*log(f)**3 - 12*b**2*x**2*log(f)**
2 + 24*b*x**3*log(f) - 24*x**4)/(b**5*x**4*log(f)**5)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.34

$$\int \frac{f^{a+\frac{b}{x}}}{x^6}\,dx = -\frac{f^a\Gamma\left(5,-\frac{b\log(f)}{x}\right)}{b^5\log\left(f\right)^5}$$

input
```
integrate(f^(a+b/x)/x^6,x, algorithm="maxima")
```

output
```
-f^a*gamma(5, -b*log(f)/x)/(b^5*log(f)^5)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x^6}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x^6}\,dx$$

input
```
integrate(f^(a+b/x)/x^6,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{f^{a+\frac{b}{x}}}{x^6}\,dx = -\frac{f^{a+\frac{b}{x}}\left(\frac{1}{b\ln(f)}+\frac{12x^2}{b^3\ln(f)^3}-\frac{24x^3}{b^4\ln(f)^4}+\frac{24x^4}{b^5\ln(f)^5}-\frac{4x}{b^2\ln(f)^2}\right)}{x^4}$$

input
```
int(f^(a + b/x)/x^6,x)
```

output
```
-(f^(a + b/x)*(1/(b*log(f)) + (12*x^2)/(b^3*log(f)^3) - (24*x^3)/(b^4*log(
f)^4) + (24*x^4)/(b^5*log(f)^5) - (4*x)/(b^2*log(f)^2)))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x}}}{x^6}\,dx = \frac{f^{\frac{ax+b}{x}}\left(-\log(f)^4b^4+4\log(f)^3b^3x-12\log(f)^2b^2x^2+24\log(f)\,b\,x^3-24x^4\right)}{\log(f)^5b^5x^4}$$

input
```
int(f^(a+b/x)/x^6,x)
```

output
```
(f**((a*x + b)/x)*( - log(f)**4*b**4 + 4*log(f)**3*b**3*x - 12*log(f)**2*b
**2*x**2 + 24*log(f)*b*x**3 - 24*x**4))/(log(f)**5*b**5*x**4)
```

3.62 $\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = \frac{f^{a+\frac{b}{x}}\left(120x^5-120bx^4\log(f)+60b^2x^3\log^2(f)-20b^3x^2\log^3(f)+5b^4x\log^4(f)-b^5\log^5(f)\right)}{b^6x^5\log^6(f)}$$

output
```
f^(a+b/x)*(120*x^5-120*b*x^4*ln(f)+60*b^2*x^3*ln(f)^2-20*b^3*x^2*ln(f)^3+5
*b^4*x*ln(f)^4-b^5*ln(f)^5)/b^6/x^5/ln(f)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = \frac{f^a\Gamma\left(6,-\frac{b\log(f)}{x}\right)}{b^6\log^6(f)}$$

input
```
Integrate[f^(a + b/x)/x^7,x]
```

output
```
(f^a*Gamma[6, -((b*Log[f])/x)])/(b^6*Log[f]^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x}}}{x^7} \, dx$$

$$\Big\downarrow \text{2647}$$

$$\frac{f^{a+\frac{b}{x}} \left(-b^5 \log^5(f) + 5b^4 x \log^4(f) - 20b^3 x^2 \log^3(f) + 60b^2 x^3 \log^2(f) - 120 b x^4 \log(f) + 120 x^5\right)}{b^6 x^5 \log^6(f)}$$

input
```
Int[f^(a + b/x)/x^7,x]
```

output
```
(f^(a + b/x)*(120*x^5 - 120*b*x^4*Log[f] + 60*b^2*x^3*Log[f]^2 - 20*b^3*x^
2*Log[f]^3 + 5*b^4*x*Log[f]^4 - b^5*Log[f]^5))/(b^6*x^5*Log[f]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{\left(b^{5} \ln \left(f\right)^{5}-5 b^{4} x \ln \left(f\right)^{4}+20 b^{3} x^{2} \ln \left(f\right)^{3}-60 b^{2} x^{3} \ln \left(f\right)^{2}+120 b \,x^{4} \ln \left(f\right)-120 x^{5}\right) f^{\frac{a x +b}{x}}}{b^{6} \ln \left(f\right)^{6} x^{5}}$	80
meijerg	$-\frac{f^{a} \left(120-\frac{\left(-\frac{6 b^{5} \ln \left(f\right)^{5}}{x^{5}}+\frac{30 b^{4} \ln \left(f\right)^{4}}{x^{4}}-\frac{120 b^{3} \ln \left(f\right)^{3}}{x^{3}}+\frac{360 b^{2} \ln \left(f\right)^{2}}{x^{2}}-\frac{720 b \ln \left(f\right)}{x}+720\right) {\mathrm e}^{\frac{b \ln \left(f\right)}{x}}}{6}\right)}{b^{6} \ln \left(f\right)^{6}}$	83
parallelrisch	$\frac{-\ln \left(f\right)^{5} f^{a +\frac{b}{x}} b^{5}+5 \ln \left(f\right)^{4} x \,f^{a +\frac{b}{x}} b^{4}-20 \ln \left(f\right)^{3} x^{2} f^{a +\frac{b}{x}} b^{3}+60 \ln \left(f\right)^{2} x^{3} f^{a +\frac{b}{x}} b^{2}-120 \ln \left(f\right) x^{4} f^{a +\frac{b}{x}} b +120 f^{a +\frac{b}{x}} x^{5}}{x^{5} b^{6} \ln \left(f\right)^{6}}$	123
norman	$\frac{\frac{120 x^{6} {\mathrm e}^{\left(a +\frac{b}{x}\right) \ln \left(f\right)}}{b^{6} \ln \left(f\right)^{6}}-\frac{x \,{\mathrm e}^{\left(a +\frac{b}{x}\right) \ln \left(f\right)}}{b \ln \left(f\right)}+\frac{5 x^{2} {\mathrm e}^{\left(a +\frac{b}{x}\right) \ln \left(f\right)}}{b^{2} \ln \left(f\right)^{2}}-\frac{20 x^{3} {\mathrm e}^{\left(a +\frac{b}{x}\right) \ln \left(f\right)}}{b^{3} \ln \left(f\right)^{3}}+\frac{60 x^{4} {\mathrm e}^{\left(a +\frac{b}{x}\right) \ln \left(f\right)}}{b^{4} \ln \left(f\right)^{4}}-\frac{120 x^{5} {\mathrm e}^{\left(a +\frac{b}{x}\right) \ln \left(f\right)}}{b^{5} \ln \left(f\right)^{5}}}{x^{6}}$	142

input
```
int(f^(a+b/x)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-(b^5*ln(f)^5-5*b^4*x*ln(f)^4+20*b^3*x^2*ln(f)^3-60*b^2*x^3*ln(f)^2+120*b*
x^4*ln(f)-120*x^5)/b^6/ln(f)^6/x^5*f^((a*x+b)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = -\frac{{\left(b^{5} \log \left(f\right)^{5} - 5 \, b^{4} x \log \left(f\right)^{4} + 20 \, b^{3} x^{2} \log \left(f\right)^{3} - 60 \, b^{2} x^{3} \log \left(f\right)^{2} + 120 \, b x^{4} \log \left(f\right) - 120 \, x^{5}\right)} f^{\frac{a x + b}{x}}}{b^{6} x^{5} \log \left(f\right)^{6}}$$

input
```
integrate(f^(a+b/x)/x^7,x, algorithm="fricas")
```

output
```
-(b^5*log(f)^5 - 5*b^4*x*log(f)^4 + 20*b^3*x^2*log(f)^3 - 60*b^2*x^3*log(f
)^2 + 120*b*x^4*log(f) - 120*x^5)*f^((a*x + b)/x)/(b^6*x^5*log(f)^6)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = \frac{f^{a+\frac{b}{x}}\left(-b^5\log(f)^5+5b^4x\log(f)^4-20b^3x^2\log(f)^3+60b^2x^3\log(f)^2-120bx^4\log(f)+120x^5\right)}{b^6x^5\log(f)^6}$$

input
```
integrate(f**(a+b/x)/x**7,x)
```

output
```
f**(a + b/x)*(-b**5*log(f)**5 + 5*b**4*x*log(f)**4 - 20*b**3*x**2*log(f)**
3 + 60*b**2*x**3*log(f)**2 - 120*b*x**4*log(f) + 120*x**5)/(b**6*x**5*log(
f)**6)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = \frac{f^a\Gamma\left(6,-\frac{b\log(f)}{x}\right)}{b^6\log(f)^6}$$

input
```
integrate(f^(a+b/x)/x^7,x, algorithm="maxima")
```

output
```
f^a*gamma(6, -b*log(f)/x)/(b^6*log(f)^6)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = \int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx$$

input
```
integrate(f^(a+b/x)/x^7,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = -\frac{f^{a+\frac{b}{x}}\left(\frac{1}{b\ln(f)}+\frac{20\,x^2}{b^3\ln(f)^3}-\frac{60\,x^3}{b^4\ln(f)^4}+\frac{120\,x^4}{b^5\ln(f)^5}-\frac{120\,x^5}{b^6\ln(f)^6}-\frac{5\,x}{b^2\ln(f)^2}\right)}{x^5}$$

input
```
int(f^(a + b/x)/x^7,x)
```

output
```
-(f^(a + b/x)*(1/(b*log(f)) + (20*x^2)/(b^3*log(f)^3) - (60*x^3)/(b^4*log(
f)^4) + (120*x^4)/(b^5*log(f)^5) - (120*x^5)/(b^6*log(f)^6) - (5*x)/(b^2*l
og(f)^2)))/x^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x}}}{x^7}\,dx = \frac{f^{\frac{ax+b}{x}}\left(-\log(f)^5 b^5+5\log(f)^4 b^4x-20\log(f)^3 b^3x^2+60\log(f)^2 b^2x^3-120\log(f)\,b\,x^4+120x^5\right)}{\log(f)^6 b^6x^5}$$

input
```
int(f^(a+b/x)/x^7,x)
```

output

```
(f**((a*x + b)/x)*( - log(f)**5*b**5 + 5*log(f)**4*b**4*x - 20*log(f)**3*b
**3*x**2 + 60*log(f)**2*b**2*x**3 - 120*log(f)*b*x**4 + 120*x**5))/(log(f)
**6*b**6*x**5)
```

3.63 $\int f^{a+\frac{b}{x^2}} x^9 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int f^{a+\frac{b}{x^2}} x^9 \, dx = -\frac{1}{2} b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right) \log^5(f)$$

output
```
1/2*f^a*x^10*Ei(6,-b*ln(f)/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}} x^9 \, dx = -\frac{1}{2} b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right) \log^5(f)$$

input
```
Integrate[f^(a + b/x^2)*x^9,x]
```

output
```
-1/2*(b^5*f^a*Gamma[-5, -((b*Log[f])/x^2)]*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{1}{2} b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)*x^9,x]
```

output
```
-1/2*(b^5*f^a*Gamma[-5, -((b*Log[f])/x^2)]*Log[f]^5)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(18) = 36.

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 5.12

method	result
risch	$\frac{f^{a} x^{10} f^{\frac{b}{x^{2}}}}{10}+\frac{f^{a} \ln(f) b\, x^{8} f^{\frac{b}{x^{2}}}}{40}+\frac{f^{a} \ln(f)^{2} b^{2} x^{6} f^{\frac{b}{x^{2}}}}{120}+\frac{f^{a} \ln(f)^{3} b^{3} x^{4} f^{\frac{b}{x^{2}}}}{240}+\frac{f^{a} \ln(f)^{4} b^{4} x^{2} f^{\frac{b}{x^{2}}}}{240}+\frac{f^{a} \ln(f)^{5} b^{5} \operatorname{expIntegral}}{240}$
meijerg	$\frac{f^{a} b^{5} \ln(f)^{5} \left(\frac{x^{10}}{5 b^{5} \ln(f)^{5}}+\frac{x^{8}}{4 b^{4} \ln(f)^{4}}+\frac{x^{6}}{6 b^{3} \ln(f)^{3}}+\frac{x^{4}}{12 b^{2} \ln(f)^{2}}+\frac{x^{2}}{24 b \ln(f)}+\frac{137}{7200}+\frac{\ln(x)}{60}-\frac{\ln(-b)}{120}-\frac{\ln(\ln(f))}{120}-\frac{x^{10}\left(\frac{137 b^{5} \ln(f)^{5}}{x^{10}}+\frac{300 b^{4} \ln(}{x^{8}}\right.}{}\right.}{}$

input
```
int(f^(a+b/x^2)*x^9,x,method=_RETURNVERBOSE)
```

output
```
1/10*f^a*x^10*f^(b/x^2)+1/40*f^a*ln(f)*b*x^8*f^(b/x^2)+1/120*f^a*ln(f)^2*b
^2*x^6*f^(b/x^2)+1/240*f^a*ln(f)^3*b^3*x^4*f^(b/x^2)+1/240*f^a*ln(f)^4*b^4
*x^2*f^(b/x^2)+1/240*f^a*ln(f)^5*b^5*Ei(1,-b*ln(f)/x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int f^{a+\frac{b}{x^2}} x^9 \, dx$$
$$= -\frac{1}{240} b^5 f^a \text{Ei}\left(\frac{b \log(f)}{x^2}\right) \log(f)^5$$
$$+ \frac{1}{240} \left(24 x^{10} + 6 b x^8 \log(f) + 2 b^2 x^6 \log(f)^2 + b^3 x^4 \log(f)^3 + b^4 x^2 \log(f)^4\right) f^{\frac{a x^2 + b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^9,x, algorithm="fricas")
```

output
```
-1/240*b^5*f^a*Ei(b*log(f)/x^2)*log(f)^5 + 1/240*(24*x^10 + 6*b*x^8*log(f)
 + 2*b^2*x^6*log(f)^2 + b^3*x^4*log(f)^3 + b^4*x^2*log(f)^4)*f^((a*x^2 + b
)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^9\,dx = \int f^{a+\frac{b}{x^2}}x^9\,dx$$

input
```
integrate(f**(a+b/x**2)*x**9,x)
```

output
```
Integral(f**(a + b/x**2)*x**9, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int f^{a+\frac{b}{x^2}}x^9\,dx = -\frac{1}{2}\,b^5f^a\Gamma\left(-5,-\frac{b\log(f)}{x^2}\right)\log(f)^5$$

input
```
integrate(f^(a+b/x^2)*x^9,x, algorithm="maxima")
```

output
```
-1/2*b^5*f^a*gamma(-5, -b*log(f)/x^2)*log(f)^5
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^9\,dx = \int f^{a+\frac{b}{x^2}}x^9\,dx$$

input
```
integrate(f^(a+b/x^2)*x^9,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^9, x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int f^{a+\frac{b}{x^2}} x^9 \, dx = \frac{b^5 f^a \ln(f)^5 \operatorname{expint}\left(-\frac{b \ln(f)}{x^2}\right)}{240} + \frac{b^5 f^a f^{\frac{b}{x^2}} \ln(f)^5 \left(\frac{x^2}{120 b \ln(f)} + \frac{x^4}{120 b^2 \ln(f)^2} + \frac{x^6}{60 b^3 \ln(f)^3} + \frac{x^8}{20 b^4 \ln(f)^4} + \frac{x^{10}}{5 b^5 \ln(f)^5}\right)}{2}$$

input
```
int(f^(a + b/x^2)*x^9,x)
```

output
```
(b^5*f^a*log(f)^5*expint(-(b*log(f))/x^2))/240 + (b^5*f^a*f^(b/x^2)*log(f)
^5*(x^2/(120*b*log(f)) + x^4/(120*b^2*log(f)^2) + x^6/(60*b^3*log(f)^3) +
x^8/(20*b^4*log(f)^4) + x^10/(5*b^5*log(f)^5)))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.38

$$\int f^{a+\frac{b}{x^2}} x^9 \, dx = \frac{f^a \left(-ei\left(\frac{\log(f) b}{x^2}\right) \log(f)^5 b^5 + f^{\frac{b}{x^2}} \log(f)^4 b^4 x^2 + f^{\frac{b}{x^2}} \log(f)^3 b^3 x^4 + 2 f^{\frac{b}{x^2}} \log(f)^2 b^2 x^6 + 6 f^{\frac{b}{x^2}} \log(f) b x^8 + \right.}{240}$$

input
```
int(f^(a+b/x^2)*x^9,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**2)*log(f)**5*b**5 + f**(b/x**2)*log(f)**4*b**4*
x**2 + f**(b/x**2)*log(f)**3*b**3*x**4 + 2*f**(b/x**2)*log(f)**2*b**2*x**6
 + 6*f**(b/x**2)*log(f)*b*x**8 + 24*f**(b/x**2)*x**10))/240
```

3.64 $\int f^{a+\frac{b}{x^2}} x^7 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int f^{a+\frac{b}{x^2}} x^7 \, dx = \frac{1}{2} b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x^2}\right) \log^4(f)$$

output
```
1/2*f^a*x^8*Ei(5,-b*ln(f)/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}} x^7 \, dx = \frac{1}{2} b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x^2}\right) \log^4(f)$$

input
```
Integrate[f^(a + b/x^2)*x^7,x]
```

output
```
(b^4*f^a*Gamma[-4, -((b*Log[f])/x^2)]*Log[f]^4)/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \ 2648$$

$$\frac{1}{2} b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)*x^7,x]
```

output
```
(b^4*f^a*Gamma[-4, -((b*Log[f])/x^2)]*Log[f]^4)/2
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(18) = 36$.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.21

method	result
risch	$\frac{f^{a} f^{\frac{b}{x^{2}}} x^{8}}{8}+\frac{f^{a} \ln (f) b x^{6} f^{\frac{b}{x^{2}}}}{24}+\frac{f^{a} \ln (f)^{2} b^{2} x^{4} f^{\frac{b}{x^{2}}}}{48}+\frac{f^{a} \ln (f)^{3} b^{3} x^{2} f^{\frac{b}{x^{2}}}}{48}+\frac{f^{a} \ln (f)^{4} b^{4} \operatorname{expIntegral}_{1}\left(-\frac{b \ln (f)}{x^{2}}\right)}{48}$
meijerg	$-\frac{f^{a} b^{4} \ln (f)^{4}\left(-\frac{x^{8}}{4 b^{4} \ln (f)^{4}}-\frac{x^{6}}{3 b^{3} \ln (f)^{3}}-\frac{x^{4}}{4 b^{2} \ln (f)^{2}}-\frac{x^{2}}{6 b \ln (f)}-\frac{25}{288}-\frac{\ln (x)}{12}+\frac{\ln (-b)}{24}+\frac{\ln (\ln (f))}{24}+\frac{x^{8}\left(\frac{125 b^{4} \ln (f)^{4}}{x^{8}}+\frac{240 b^{3} \ln (f)^{3}}{x^{6}}+\frac{360 b^{2}}{x}\right.}{1440 b^{4} \ln (f)^{4}}\right.}{2}$

input
```
int(f^(a+b/x^2)*x^7,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^a*f^(b/x^2)*x^8+1/24*f^a*ln(f)*b*x^6*f^(b/x^2)+1/48*f^a*ln(f)^2*b^2*
x^4*f^(b/x^2)+1/48*f^a*ln(f)^3*b^3*x^2*f^(b/x^2)+1/48*f^a*ln(f)^4*b^4*Ei(1
,-b*ln(f)/x^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int f^{a+\frac{b}{x^2}} x^7\,dx = -\frac{1}{48}\,b^4 f^a \mathrm{Ei}\left(\frac{b\log(f)}{x^2}\right)\log(f)^4 + \frac{1}{48}\left(6\,x^8 + 2\,bx^6\log(f) + b^2x^4\log(f)^2 + b^3x^2\log(f)^3\right)f^{\frac{ax^2+b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^7,x, algorithm="fricas")
```

output
```
-1/48*b^4*f^a*Ei(b*log(f)/x^2)*log(f)^4 + 1/48*(6*x^8 + 2*b*x^6*log(f) + b
^2*x^4*log(f)^2 + b^3*x^2*log(f)^3)*f^((a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^7\,dx=\int f^{a+\frac{b}{x^2}}x^7\,dx$$

input
```
integrate(f**(a+b/x**2)*x**7,x)
```

output
```
Integral(f**(a + b/x**2)*x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int f^{a+\frac{b}{x^2}}x^7\,dx=\frac{1}{2}\,b^4f^a\Gamma\left(-4,-\frac{b\log\left(f\right)}{x^2}\right)\log\left(f\right)^4$$

input
```
integrate(f^(a+b/x^2)*x^7,x, algorithm="maxima")
```

output
```
1/2*b^4*f^a*gamma(-4, -b*log(f)/x^2)*log(f)^4
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^7\,dx=\int f^{a+\frac{b}{x^2}}x^7\,dx$$

input
```
integrate(f^(a+b/x^2)*x^7,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.75

$$\int f^{a+\frac{b}{x^2}}x^7\,dx = \frac{b^4\,f^a\ln\left(f\right)^4\,\mathrm{expint}\left(-\frac{b\,\ln(f)}{x^2}\right)}{48} + \frac{b^4\,f^a\,f^{\frac{b}{x^2}}\ln\left(f\right)^4\left(\frac{x^2}{24\,b\,\ln(f)}+\frac{x^4}{24\,b^2\ln(f)^2}+\frac{x^6}{12\,b^3\ln(f)^3}+\frac{x^8}{4\,b^4\ln(f)^4}\right)}{2}$$

input
```
int(f^(a + b/x^2)*x^7,x)
```

output
```
(b^4*f^a*log(f)^4*expint(-(b*log(f))/x^2))/48 + (b^4*f^a*f^(b/x^2)*log(f)^
4*(x^2/(24*b*log(f)) + x^4/(24*b^2*log(f)^2) + x^6/(12*b^3*log(f)^3) + x^8
/(4*b^4*log(f)^4)))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.58

$$\int f^{a+\frac{b}{x^2}}x^7\,dx = \frac{f^a\left(-ei\left(\frac{\log(f)b}{x^2}\right)\log(f)^4\,b^4+f^{\frac{b}{x^2}}\log(f)^3\,b^3x^2+f^{\frac{b}{x^2}}\log(f)^2\,b^2x^4+2f^{\frac{b}{x^2}}\log(f)\,b\,x^6+6f^{\frac{b}{x^2}}x^8\right)}{48}$$

input
```
int(f^(a+b/x^2)*x^7,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**2)*log(f)**4*b**4 + f**(b/x**2)*log(f)**3*b**3*
x**2 + f**(b/x**2)*log(f)**2*b**2*x**4 + 2*f**(b/x**2)*log(f)*b*x**6 + 6*f
**(b/x**2)*x**8))/48
```

3.65 $\int f^{a+\frac{b}{x^2}} x^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int f^{a+\frac{b}{x^2}} x^5 \, dx = \frac{1}{6} f^{a+\frac{b}{x^2}} x^6 + \frac{1}{12} b f^{a+\frac{b}{x^2}} x^4 \log(f) + \frac{1}{12} b^2 f^{a+\frac{b}{x^2}} x^2 \log^2(f) - \frac{1}{12} b^3 f^a \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right) \log^3(f)$$

output
```
1/6*f^(a+b/x^2)*x^6+1/12*b*f^(a+b/x^2)*x^4*ln(f)+1/12*b^2*f^(a+b/x^2)*x^2*
ln(f)^2-1/12*b^3*f^a*Ei(b*ln(f)/x^2)*ln(f)^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int f^{a+\frac{b}{x^2}} x^5 \, dx = \frac{1}{12} f^a \left(-b^3 \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right) \log^3(f) + f^{\frac{b}{x^2}} x^2 \left(2x^4 + b x^2 \log(f) + b^2 \log^2(f)\right)\right)$$

input
```
Integrate[f^(a + b/x^2)*x^5,x]
```

output
```
(f^a*(-(b^3*ExpIntegralEi[(b*Log[f])/x^2]*Log[f]^3) + f^(b/x^2)*x^2*(2*x^4
 + b*x^2*Log[f] + b^2*Log[f]^2)))/12
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \int f^{a+\frac{b}{x^2}} x^3 dx + \frac{1}{6} x^6 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \int f^{a+\frac{b}{x^2}} x dx + \frac{1}{4} x^4 f^{a+\frac{b}{x^2}}\right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{a+\frac{b}{x^2}}}{x} dx + \frac{1}{2} x^2 f^{a+\frac{b}{x^2}}\right) + \frac{1}{4} x^4 f^{a+\frac{b}{x^2}}\right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \left(\frac{1}{2} x^2 f^{a+\frac{b}{x^2}} - \frac{1}{2} b f^a \log(f) \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right)\right) + \frac{1}{4} x^4 f^{a+\frac{b}{x^2}}\right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^2}}$$

input
```
Int[f^(a + b/x^2)*x^5,x]
```

output
```
(f^(a + b/x^2)*x^6)/6 + (b*Log[f]*((f^(a + b/x^2)*x^4)/4 + (b*Log[f]*((f^(
a + b/x^2)*x^2)/2 - (b*f^a*ExpIntegralEi[(b*Log[f])/x^2]*Log[f])/2))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

method	result
risch	$\frac{f^{a} x^{6} f^{\frac{b}{x^{2}}}}{6}+\frac{f^{a} \ln(f) b\, x^{4} f^{\frac{b}{x^{2}}}}{12}+\frac{f^{a} \ln(f)^{2} b^{2} x^{2} f^{\frac{b}{x^{2}}}}{12}+\frac{f^{a} \ln(f)^{3} b^{3} \operatorname{expIntegral}_{1}\left(-\frac{b \ln(f)}{x^{2}}\right)}{12}$
meijerg	$\frac{f^{a} b^{3} \ln(f)^{3} \left(\frac{x^{6}}{3 b^{3} \ln(f)^{3}}+\frac{x^{4}}{2 b^{2} \ln(f)^{2}}+\frac{x^{2}}{2 b \ln(f)}+\frac{11}{36}+\frac{\ln(x)}{3}-\frac{\ln(-b)}{6}-\frac{\ln(\ln(f))}{6}-\frac{x^{6}\left(\frac{22 b^{3} \ln(f)^{3}}{x^{6}}+\frac{36 b^{2} \ln(f)^{2}}{x^{4}}+\frac{36 b \ln(f)}{x^{2}}+24\right)}{72 b^{3} \ln(f)^{3}}+\right.}{2}$

input
```
int(f^(a+b/x^2)*x^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*f^a*x^6*f^(b/x^2)+1/12*f^a*ln(f)*b*x^4*f^(b/x^2)+1/12*f^a*ln(f)^2*b^2*
x^2*f^(b/x^2)+1/12*f^a*ln(f)^3*b^3*Ei(1,-b*ln(f)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int f^{a+\frac{b}{x^2}}x^5\,dx = -\frac{1}{12}\,b^3f^a\mathrm{Ei}\left(\frac{b\log\left(f\right)}{x^2}\right)\log\left(f\right)^3 + \frac{1}{12}\left(2\,x^6+bx^4\log\left(f\right)+b^2x^2\log\left(f\right)^2\right)f^{\frac{ax^2+b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^5,x, algorithm="fricas")
```

output
```
-1/12*b^3*f^a*Ei(b*log(f)/x^2)*log(f)^3 + 1/12*(2*x^6 + b*x^4*log(f) + b^2
*x^2*log(f)^2)*f^((a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^5\,dx = \int f^{a+\frac{b}{x^2}}x^5\,dx$$

input
```
integrate(f**(a+b/x**2)*x**5,x)
```

output
```
Integral(f**(a + b/x**2)*x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int f^{a+\frac{b}{x^2}}x^5\,dx = -\frac{1}{2}\,b^3f^a\Gamma\left(-3,-\frac{b\log\left(f\right)}{x^2}\right)\log\left(f\right)^3$$

input
```
integrate(f^(a+b/x^2)*x^5,x, algorithm="maxima")
```

output
```
-1/2*b^3*f^a*gamma(-3, -b*log(f)/x^2)*log(f)^3
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^5\,dx=\int f^{a+\frac{b}{x^2}}x^5\,dx$$

input
```
integrate(f^(a+b/x^2)*x^5,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int f^{a+\frac{b}{x^2}}x^5\,dx=\frac{b^3\,f^a\ln(f)^3\left(f^{\frac{b}{x^2}}\left(\frac{x^2}{6\,b\,\ln(f)}+\frac{x^4}{6\,b^2\ln(f)^2}+\frac{x^6}{3\,b^3\ln(f)^3}\right)+\frac{\mathrm{expint}\left(-\frac{b\,\ln(f)}{x^2}\right)}{6}\right)}{2}$$

input
```
int(f^(a + b/x^2)*x^5,x)
```

output
```
(b^3*f^a*log(f)^3*(f^(b/x^2)*(x^2/(6*b*log(f)) + x^4/(6*b^2*log(f)^2) + x^
6/(3*b^3*log(f)^3)) + expint(-(b*log(f))/x^2)/6))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int f^{a+\frac{b}{x^2}}x^5\,dx=\frac{f^a\left(-ei\left(\frac{\log(f)b}{x^2}\right)\log(f)^3b^3+f^{\frac{b}{x^2}}\log(f)^2b^2x^2+f^{\frac{b}{x^2}}\log(f)\,b\,x^4+2f^{\frac{b}{x^2}}x^6\right)}{12}$$

input
```
int(f^(a+b/x^2)*x^5,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**2)*log(f)**3*b**3 + f**(b/x**2)*log(f)**2*b**2*
x**2 + f**(b/x**2)*log(f)*b*x**4 + 2*f**(b/x**2)*x**6))/12
```

3.66 $\int f^{a+\frac{b}{x^2}} x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int f^{a+\frac{b}{x^2}} x^3 \, dx = \frac{1}{4} f^{a+\frac{b}{x^2}} x^4 + \frac{1}{4} b f^{a+\frac{b}{x^2}} x^2 \log(f) - \frac{1}{4} b^2 f^a \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right) \log^2(f)$$

output
```
1/4*f^(a+b/x^2)*x^4+1/4*b*f^(a+b/x^2)*x^2*ln(f)-1/4*b^2*f^a*Ei(b*ln(f)/x^2
)*ln(f)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int f^{a+\frac{b}{x^2}} x^3 \, dx = \frac{1}{4} f^a \left(-b^2 \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right) \log^2(f) + f^{\frac{b}{x^2}} x^2 \left(x^2 + b \log(f)\right) \right)$$

input
```
Integrate[f^(a + b/x^2)*x^3,x]
```

output
```
(f^a*(-(b^2*ExpIntegralEi[(b*Log[f])/x^2]*Log[f]^2) + f^(b/x^2)*x^2*(x^2 +
 b*Log[f])))/4
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \int f^{a+\frac{b}{x^2}} x dx + \frac{1}{4} x^4 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{a+\frac{b}{x^2}}}{x} dx + \frac{1}{2} x^2 f^{a+\frac{b}{x^2}} \right) + \frac{1}{4} x^4 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2} b \log(f) \left(\frac{1}{2} x^2 f^{a+\frac{b}{x^2}} - \frac{1}{2} b f^a \log(f) \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^2} \right) \right) + \frac{1}{4} x^4 f^{a+\frac{b}{x^2}}$$

input
```
Int[f^(a + b/x^2)*x^3,x]
```

output
```
(f^(a + b/x^2)*x^4)/4 + (b*Log[f]*((f^(a + b/x^2)*x^2)/2 - (b*f^a*ExpInteg
ralEi[(b*Log[f])/x^2]*Log[f])/2))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result
risch	$\frac{f^{a} x^{4} f^{\frac{b}{x^{2}}}}{4}+\frac{f^{a} \ln(f) b\, x^{2} f^{\frac{b}{x^{2}}}}{4}+\frac{f^{a} \ln(f)^{2} b^{2} \operatorname{expIntegral}_{1}\left(-\frac{b \ln(f)}{x^{2}}\right)}{4}$
meijerg	$-\frac{f^{a} b^{2} \ln(f)^{2}\left(-\frac{x^{4}}{2 b^{2} \ln(f)^{2}}-\frac{x^{2}}{b \ln(f)}-\frac{3}{4}-\ln(x)+\frac{\ln(-b)}{2}+\frac{\ln(\ln(f))}{2}+\frac{x^{4}\left(\frac{9 b^{2} \ln(f)^{2}}{x^{4}}+\frac{12 b \ln(f)}{x^{2}}+6\right)}{12 b^{2} \ln(f)^{2}}-\frac{x^{4}\left(\frac{3 b \ln(f)}{x^{2}}+3\right) e^{\frac{b \ln(f)}{x^{2}}}}{6 b^{2} \ln(f)^{2}}-\frac{\ln\left(-\frac{b \ln ...}{...}\right)}{2} ...\right.}{2}$

input
```
int(f^(a+b/x^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*f^a*x^4*f^(b/x^2)+1/4*f^a*ln(f)*b*x^2*f^(b/x^2)+1/4*f^a*ln(f)^2*b^2*Ei
(1,-b*ln(f)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int f^{a+\frac{b}{x^2}} x^3\, dx = -\frac{1}{4}\, b^2 f^a \mathrm{Ei}\left(\frac{b \log{(f)}}{x^2}\right) \log{(f)}^2 + \frac{1}{4}\left(x^4 + b x^2 \log{(f)}\right) f^{\frac{a x^2 + b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^3,x, algorithm="fricas")
```

output
```
-1/4*b^2*f^a*Ei(b*log(f)/x^2)*log(f)^2 + 1/4*(x^4 + b*x^2*log(f))*f^((a*x^
2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^3\,dx=\int f^{a+\frac{b}{x^2}}x^3\,dx$$

input
```
integrate(f**(a+b/x**2)*x**3,x)
```

output
```
Integral(f**(a + b/x**2)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.38

$$\int f^{a+\frac{b}{x^2}}x^3\,dx=\frac{1}{2}\,b^2f^a\Gamma\left(-2,-\frac{b\log\left(f\right)}{x^2}\right)\log\left(f\right)^2$$

input
```
integrate(f^(a+b/x^2)*x^3,x, algorithm="maxima")
```

output
```
1/2*b^2*f^a*gamma(-2, -b*log(f)/x^2)*log(f)^2
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^3\,dx=\int f^{a+\frac{b}{x^2}}x^3\,dx$$

input
```
integrate(f^(a+b/x^2)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int f^{a+\frac{b}{x^2}} x^3 \, dx = \frac{b^2 f^a \ln(f)^2 \left(f^{\frac{b}{x^2}} \left(\frac{x^2}{2 b \ln(f)} + \frac{x^4}{2 b^2 \ln(f)^2}\right) + \frac{\mathrm{expint}\left(-\frac{b \ln(f)}{x^2}\right)}{2}\right)}{2}$$

input
```
int(f^(a + b/x^2)*x^3,x)
```

output
```
(b^2*f^a*log(f)^2*(f^(b/x^2)*(x^2/(2*b*log(f)) + x^4/(2*b^2*log(f)^2)) + e
xpint(-(b*log(f))/x^2)/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int f^{a+\frac{b}{x^2}} x^3 \, dx = \frac{f^a \left(-ei\left(\frac{\log(f) b}{x^2}\right) \log(f)^2 b^2 + f^{\frac{b}{x^2}} \log(f) \, b \, x^2 + f^{\frac{b}{x^2}} x^4\right)}{4}$$

input
```
int(f^(a+b/x^2)*x^3,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**2)*log(f)**2*b**2 + f**(b/x**2)*log(f)*b*x**2 +
 f**(b/x**2)*x**4))/4
```

3.67 $\int f^{a+\frac{b}{x^2}} x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 35

$$\int f^{a+\frac{b}{x^2}} x\,dx = \frac{1}{2} f^{a+\frac{b}{x^2}} x^2 - \frac{1}{2} b f^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x^2}\right)\log(f)$$

output
```
1/2*f^(a+b/x^2)*x^2-1/2*b*f^a*Ei(b*ln(f)/x^2)*ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int f^{a+\frac{b}{x^2}} x\,dx = \frac{1}{2} f^a \left(f^{\frac{b}{x^2}} x^2 - b \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x^2}\right)\log(f)\right)$$

input
```
Integrate[f^(a + b/x^2)*x,x]
```

output
```
(f^a*(f^(b/x^2)*x^2 - b*ExpIntegralEi[(b*Log[f])/x^2]*Log[f]))/2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$b \log(f) \int \frac{f^{a+\frac{b}{x^2}}}{x} dx + \frac{1}{2} x^2 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{2} x^2 f^{a+\frac{b}{x^2}} - \frac{1}{2} b f^a \log(f) \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)*x,x]
```

output
```
(f^(a + b/x^2)*x^2)/2 - (b*f^a*ExpIntegralEi[(b*Log[f])/x^2]*Log[f])/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{f^{a} f^{\frac{b}{x^{2}}} x^{2}}{2}+\frac{f^{a} \ln \left(f\right) b \,\operatorname{expIntegral}_{1}\left(-\frac{b \ln \left(f\right)}{x^{2}}\right)}{2}$	35
meijerg	$\frac{f^{a} b \ln \left(f\right) \left(\frac{x^{2}}{b \ln \left(f\right)}+1+2 \ln \left(x\right)-\ln \left(-b\right)-\ln \left(\ln \left(f\right)\right)-\frac{x^{2} \left(\frac{2 b \ln \left(f\right)}{x^{2}}+2\right)}{2 b \ln \left(f\right)}+\frac{x^{2} {\mathrm e}^{\frac{b \ln \left(f\right)}{x^{2}}}}{b \ln \left(f\right)}+\ln \left(-\frac{b \ln \left(f\right)}{x^{2}}\right)+\operatorname{expIntegral}_{1}\left(-\frac{b \ln \left(f\right)}{x^{2}}\right)\right)}{2}$	97

input
```
int(f^(a+b/x^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*f^(b/x^2)*x^2+1/2*f^a*ln(f)*b*Ei(1,-b*ln(f)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}} x\,dx = -\frac{1}{2}\,b f^{a} \operatorname{Ei}\left(\frac{b \log\left(f\right)}{x^{2}}\right) \log\left(f\right) + \frac{1}{2}\,f^{\frac{a x^{2} + b}{x^{2}}} x^{2}$$

input
```
integrate(f^(a+b/x^2)*x,x, algorithm="fricas")
```

output
```
-1/2*b*f^a*Ei(b*log(f)/x^2)*log(f) + 1/2*f^((a*x^2 + b)/x^2)*x^2
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}} x\,dx = \int f^{a + \frac{b}{x^{2}}} x\, dx$$

input
```
integrate(f**(a+b/x**2)*x,x)
```

output
```
Integral(f**(a + b/x**2)*x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int f^{a+\frac{b}{x^2}} x\,dx = -\frac{1}{2} b f^a \Gamma\left(-1, -\frac{b \log(f)}{x^2}\right) \log(f)$$

input
```
integrate(f^(a+b/x^2)*x,x, algorithm="maxima")
```

output
```
-1/2*b*f^a*gamma(-1, -b*log(f)/x^2)*log(f)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}} x\,dx = \int f^{a+\frac{b}{x^2}} x\,dx$$

input
```
integrate(f^(a+b/x^2)*x,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int f^{a+\frac{b}{x^2}} x\,dx = \frac{f^a f^{\frac{b}{x^2}} x^2}{2} + \frac{b f^a \ln(f) \operatorname{expint}\left(-\frac{b \ln(f)}{x^2}\right)}{2}$$

input
```
int(f^(a + b/x^2)*x,x)
```

output
```
(f^a*f^(b/x^2)*x^2)/2 + (b*f^a*log(f)*expint(-(b*log(f))/x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int f^{a+\frac{b}{x^2}} x \, dx = \frac{f^a \left(-ei\left(\frac{\log(f) b}{x^2}\right) \log(f) \, b + f^{\frac{b}{x^2}} x^2\right)}{2}$$

input
```
int(f^(a+b/x^2)*x,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**2)*log(f)*b + f**(b/x**2)*x**2))/2
```

3.68 $\int \frac{f^{a+\frac{b}{x^2}}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{f^{a+\frac{b}{x^2}}}{x} dx = -\frac{1}{2} f^a \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right)$$

output
```
-1/2*f^a*Ei(b*ln(f)/x^2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^2}}}{x} dx = -\frac{1}{2} f^a \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right)$$

input
```
Integrate[f^(a + b/x^2)/x,x]
```

output
```
-1/2*(f^a*ExpIntegralEi[(b*Log[f])/x^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x} \, dx$$

$$\Big\downarrow \; 2639$$

$$-\frac{1}{2} f^a \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)/x,x]
```

output
```
-1/2*(f^a*ExpIntegralEi[(b*Log[f])/x^2])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
risch	$\frac{f^{a} \operatorname{expIntegral}_{1}\left(-\frac{b \ln\left(f\right)}{x^{2}}\right)}{2}$	16
meijerg	$-\frac{f^{a} \left(-2 \ln\left(x\right)+\ln\left(-b\right)+\ln\left(\ln\left(f\right)\right)-\ln\left(-\frac{b \ln\left(f\right)}{x^{2}}\right)-\operatorname{expIntegral}_{1}\left(-\frac{b \ln\left(f\right)}{x^{2}}\right)\right)}{2}$	41

input
```
int(f^(a+b/x^2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*Ei(1,-b*ln(f)/x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx = -\frac{1}{2}\,f^a\mathrm{Ei}\left(\frac{b\log\left(f\right)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)/x,x, algorithm="fricas")
```

output
```
-1/2*f^a*Ei(b*log(f)/x^2)
```

Sympy [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx$$

input
```
integrate(f**(a+b/x**2)/x,x)
```

output
```
Integral(f**(a + b/x**2)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx = -\frac{1}{2}\,f^a \mathrm{Ei}\left(\frac{b\log\left(f\right)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)/x,x, algorithm="maxima")
```

output
```
-1/2*f^a*Ei(b*log(f)/x^2)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx$$

input
```
integrate(f^(a+b/x^2)/x,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx = -\frac{f^a\,\mathrm{ei}\left(\frac{b\,\ln(f)}{x^2}\right)}{2}$$

input
```
int(f^(a + b/x^2)/x,x)
```

output
```
-(f^a*ei((b*log(f))/x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^2}}}{x}\,dx = -\frac{f^a ei\left(\frac{\log(f)b}{x^2}\right)}{2}$$

input
```
int(f^(a+b/x^2)/x,x)
```

output
```
( - f**a*ei((log(f)*b)/x**2))/2
```

3.69 $\int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx = -\frac{f^{a+\frac{b}{x^2}}}{2b\log(f)}$$

output
```
-1/2*f^(a+b/x^2)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3} dx = -\frac{f^{a+\frac{b}{x^2}}}{2b\log(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^3,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*Log[f])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3} \, dx$$

$$\Big\downarrow \text{2638}$$

$$-\frac{f^{a+\frac{b}{x^2}}}{2b\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^3,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{f^{a+\frac{b}{x^2}}}{2b\ln(f)}$	19
default	$-\frac{f^{a+\frac{b}{x^2}}}{2b\ln(f)}$	19
parallelrisch	$-\frac{f^{a+\frac{b}{x^2}}}{2b\ln(f)}$	19
norman	$-\frac{e^{\left(a+\frac{b}{x^2}\right)\ln(f)}}{2b\ln(f)}$	21
risch	$-\frac{f^{\frac{a\,x^2+b}{x^2}}}{2b\ln(f)}$	23
meijerg	$\frac{f^a\left(1-e^{\frac{b\ln(f)}{x^2}}\right)}{2b\ln(f)}$	25

input
```
int(f^(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^(a+b/x^2)/b/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{f^{\frac{ax^2+b}{x^2}}}{2\,b\log\left(f\right)}$$

input
```
integrate(f^(a+b/x^2)/x^3,x, algorithm="fricas")
```

output
```
-1/2*f^((a*x^2 + b)/x^2)/(b*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3}\,dx = \begin{cases} -\frac{f^{a+\frac{b}{x^2}}}{2b\log{(f)}} & \text{for}\ b\log{(f)} \neq 0 \\ -\frac{1}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(a+b/x**2)/x**3,x)
```

output
```
Piecewise((-f**(a + b/x**2)/(2*b*log(f)), Ne(b*log(f), 0)), (-1/(2*x**2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{f^{a+\frac{b}{x^2}}}{2\,b\log{(f)}}$$

input
```
integrate(f^(a+b/x^2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*f^(a + b/x^2)/(b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{f^{\frac{ax^2+b}{x^2}}}{2\,b\log{(f)}}$$

input
```
integrate(f^(a+b/x^2)/x^3,x, algorithm="giac")
```

output
```
-1/2*f^((a*x^2 + b)/x^2)/(b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{f^{a+\frac{b}{x^2}}}{2\,b\,\ln\left(f\right)}$$

input
```
int(f^(a + b/x^2)/x^3,x)
```

output
```
-f^(a + b/x^2)/(2*b*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^3}\,dx = -\frac{f^{\frac{a\,x^2+b}{x^2}}}{2\log\left(f\right)b}$$

input
```
int(f^(a+b/x^2)/x^3,x)
```

output
```
( - f**((a*x**2 + b)/x**2))/(2*log(f)*b)
```

3.70 $\int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx = \frac{f^{a+\frac{b}{x^2}}}{2b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^2 \log(f)}$$

output
```
1/2*f^(a+b/x^2)/b^2/ln(f)^2-1/2*f^(a+b/x^2)/b/x^2/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5} dx = \frac{f^{a+\frac{b}{x^2}}(x^2 - b\log(f))}{2b^2x^2 \log^2(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^5,x]
```

output
```
(f^(a + b/x^2)*(x^2 - b*Log[f]))/(2*b^2*x^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^3}dx}{b\log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^2\log(f)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{f^{a+\frac{b}{x^2}}}{2b^2\log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^2\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^5,x]
```

output
```
f^(a + b/x^2)/(2*b^2*Log[f]^2) - f^(a + b/x^2)/(2*b*x^2*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
meijerg	$-\frac{f^{a}\left(1-\frac{\left(-\frac{2 b \ln\left(f\right)}{x^{2}}+2\right) {\mathrm e}^{\frac{b \ln\left(f\right)}{x^{2}}}}{2}\right)}{2 b^{2} \ln\left(f\right)^{2}}$	35
risch	$-\frac{\left(b \ln\left(f\right)-x^{2}\right) f^{\frac{a \,x^{2}+b}{x^{2}}}}{2 \ln\left(f\right)^{2} b^{2} x^{2}}$	36
parallelrisch	$\frac{-f^{a+\frac{b}{x^{2}}} b \ln\left(f\right)+f^{a+\frac{b}{x^{2}}} x^{2}}{2 x^{2} \ln\left(f\right)^{2} b^{2}}$	41
norman	$\frac{-\frac{x^{2} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln\left(f\right)}}{2 b \ln\left(f\right)}+\frac{x^{4} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln\left(f\right)}}{2 b^{2} \ln\left(f\right)^{2}}}{x^{4}}$	52

input
```
int(f^(a+b/x^2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/b^2/ln(f)^2*(1-1/2*(-2*b*ln(f)/x^2+2)*exp(b*ln(f)/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5}\, dx = \frac{(x^2 - b\log(f))f^{\frac{ax^2+b}{x^2}}}{2\,b^2x^2\log(f)^2}$$

input
```
integrate(f^(a+b/x^2)/x^5,x, algorithm="fricas")
```

output
```
1/2*(x^2 - b*log(f))*f^((a*x^2 + b)/x^2)/(b^2*x^2*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5}\, dx = \frac{f^{a+\frac{b}{x^2}}(-b\log(f) + x^2)}{2b^2x^2\log(f)^2}$$

input
```
integrate(f**(a+b/x**2)/x**5,x)
```

output
```
f**(a + b/x**2)*(-b*log(f) + x**2)/(2*b**2*x**2*log(f)**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5}\, dx = \frac{f^a\Gamma\left(2, -\frac{b\log(f)}{x^2}\right)}{2\,b^2\log(f)^2}$$

input
```
integrate(f^(a+b/x^2)/x^5,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(2, -b*log(f)/x^2)/(b^2*log(f)^2)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^5}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^5,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5}\,dx = -\frac{f^{a+\frac{b}{x^2}}\left(\frac{1}{2\,b\,\ln(f)}-\frac{x^2}{2\,b^2\,\ln(f)^2}\right)}{x^2}$$

input
```
int(f^(a + b/x^2)/x^5,x)
```

output
```
-(f^(a + b/x^2)*(1/(2*b*log(f)) - x^2/(2*b^2*log(f)^2)))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^5}\,dx = \frac{f^{\frac{a\,x^2+b}{x^2}}\left(-\log(f)\,b+x^2\right)}{2\log\left(f\right)^2 b^2 x^2}$$

input
```
int(f^(a+b/x^2)/x^5,x)
```

output
```
(f**((a*x**2 + b)/x**2)*( - log(f)*b + x**2))/(2*log(f)**2*b**2*x**2)
```

3.71 $\int \frac{f^{a+\frac{b}{x^2}}}{x^7} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7} \, dx = -\frac{f^{a+\frac{b}{x^2}}}{b^3 \log^3(f)} + \frac{f^{a+\frac{b}{x^2}}}{b^2 x^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^4 \log(f)}$$

output
```
-f^(a+b/x^2)/b^3/ln(f)^3+f^(a+b/x^2)/b^2/x^2/ln(f)^2-1/2*f^(a+b/x^2)/b/x^4
/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7} \, dx = -\frac{f^{a+\frac{b}{x^2}} \left(2 x^4 - 2 b x^2 \log(f) + b^2 \log^2(f)\right)}{2 b^3 x^4 \log^3(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^7,x]
```

output
```
-1/2*(f^(a + b/x^2)*(2*x^4 - 2*b*x^2*Log[f] + b^2*Log[f]^2))/(b^3*x^4*Log[
f]^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\int \frac{f^{a+\frac{b}{x^2}}}{x^5}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^4\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\left(-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^3}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^2\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^4\log(f)}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{2\left(\frac{f^{a+\frac{b}{x^2}}}{2b^2\log^2(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^2\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^4\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^7,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*x^4*Log[f]) - (2*(f^(a + b/x^2)/(2*b^2*Log[f]^2) - f
^(a + b/x^2)/(2*b*x^2*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

method	result	size
meijerg	$\frac{f^{a}\left(2-\frac{\left(\frac{3b^{2}\ln\left(f\right)^{2}}{x^{4}}-\frac{6b\ln\left(f\right)}{x^{2}}+6\right){\mathrm e}^{\frac{b\ln\left(f\right)}{x^{2}}}}{3}\right)}{2b^{3}\ln\left(f\right)^{3}}$	47
risch	$-\frac{\left(\ln\left(f\right)^{2}b^{2}-2b\,x^{2}\ln\left(f\right)+2x^{4}\right)f^{\frac{a\,x^{2}+b}{x^{2}}}}{2\ln\left(f\right)^{3}b^{3}x^{4}}$	48
parallelrisch	$\frac{-f^{a+\frac{b}{x^{2}}}\ln\left(f\right)^{2}b^{2}+2b\,f^{a+\frac{b}{x^{2}}}x^{2}\ln\left(f\right)-2f^{a+\frac{b}{x^{2}}}x^{4}}{2x^{4}\ln\left(f\right)^{3}b^{3}}$	63
norman	$\frac{\frac{x^{4}{\mathrm e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{b^{2}\ln\left(f\right)^{2}}-\frac{x^{2}{\mathrm e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{2b\ln\left(f\right)}-\frac{x^{6}{\mathrm e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{b^{3}\ln\left(f\right)^{3}}}{x^{6}}$	74

input
```
int(f^(a+b/x^2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/b^3/ln(f)^3*(2-1/3*(3*b^2*ln(f)^2/x^4-6*b*ln(f)/x^2+6)*exp(b*ln(f)
/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx = -\frac{\left(2\,x^4-2\,bx^2\log\left(f\right)+b^2\log\left(f\right)^2\right)f^{\frac{ax^2+b}{x^2}}}{2\,b^3x^4\log\left(f\right)^3}$$

input
```
integrate(f^(a+b/x^2)/x^7,x, algorithm="fricas")
```

output
```
-1/2*(2*x^4 - 2*b*x^2*log(f) + b^2*log(f)^2)*f^((a*x^2 + b)/x^2)/(b^3*x^4*
log(f)^3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.71

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx = \frac{f^{a+\frac{b}{x^2}}\left(-b^2\log\left(f\right)^2+2bx^2\log\left(f\right)-2x^4\right)}{2b^3x^4\log\left(f\right)^3}$$

input
```
integrate(f**(a+b/x**2)/x**7,x)
```

output
```
f**(a + b/x**2)*(-b**2*log(f)**2 + 2*b*x**2*log(f) - 2*x**4)/(2*b**3*x**4*
log(f)**3)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.35

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx = -\frac{f^a\Gamma\left(3,-\frac{b\log(f)}{x^2}\right)}{2\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(a+b/x^2)/x^7,x, algorithm="maxima")
```

output
```
-1/2*f^a*gamma(3, -b*log(f)/x^2)/(b^3*log(f)^3)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx=\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^7,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx=-\frac{f^{a+\frac{b}{x^2}}\left(\frac{1}{2\,b\,\ln(f)}-\frac{x^2}{b^2\ln(f)^2}+\frac{x^4}{b^3\ln(f)^3}\right)}{x^4}$$

input
```
int(f^(a + b/x^2)/x^7,x)
```

output
```
-(f^(a + b/x^2)*(1/(2*b*log(f)) - x^2/(b^2*log(f)^2) + x^4/(b^3*log(f)^3))
)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^7}\,dx=\frac{f^{\frac{a\,x^2+b}{x^2}}\left(-\log(f)^2\,b^2+2\log(f)\,b\,x^2-2x^4\right)}{2\log(f)^3\,b^3x^4}$$

input
```
int(f^(a+b/x^2)/x^7,x)
```

output
```
(f**((a*x**2 + b)/x**2)*( - log(f)**2*b**2 + 2*log(f)*b*x**2 - 2*x**4))/(2
*log(f)**3*b**3*x**4)
```

3.72 $\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx=\frac{3f^{a+\frac{b}{x^2}}}{b^4\log^4(f)}-\frac{3f^{a+\frac{b}{x^2}}}{b^3x^2\log^3(f)}+\frac{3f^{a+\frac{b}{x^2}}}{2b^2x^4\log^2(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^6\log(f)}$$

output
```
3*f^(a+b/x^2)/b^4/ln(f)^4-3*f^(a+b/x^2)/b^3/x^2/ln(f)^3+3/2*f^(a+b/x^2)/b^
2/x^4/ln(f)^2-1/2*f^(a+b/x^2)/b/x^6/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx=\frac{f^{a+\frac{b}{x^2}}\left(6x^6-6bx^4\log(f)+3b^2x^2\log^2(f)-b^3\log^3(f)\right)}{2b^4x^6\log^4(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^9,x]
```

output
```
(f^(a + b/x^2)*(6*x^6 - 6*b*x^4*Log[f] + 3*b^2*x^2*Log[f]^2 - b^3*Log[f]^3
))/(2*b^4*x^6*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \int \frac{f^{a+\frac{b}{x^2}}}{x^7}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^6 \log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \left(-\frac{2 \int \frac{f^{a+\frac{b}{x^2}}}{x^5}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^4 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^6 \log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \left(-\frac{2 \left(-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^3}dx}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^2 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^4 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^6 \log(f)}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3 \left(-\frac{2 \left(\frac{f^{a+\frac{b}{x^2}}}{2 b^2 \log^2(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^2 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^4 \log(f)}\right)}{b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^6 \log(f)}$$

input
```
Int[f^(a + b/x^2)/x^9,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*x^6*Log[f]) - (3*(-1/2*f^(a + b/x^2)/(b*x^4*Log[f])
- (2*(f^(a + b/x^2)/(2*b^2*Log[f]^2) - f^(a + b/x^2)/(2*b*x^2*Log[f])))/(b
*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

method	result	size
meijerg	$-\frac{f^{a}\left(6-\frac{\left(-\frac{4b^{3}\ln\left(f\right)^{3}}{x^{6}}+\frac{12b^{2}\ln\left(f\right)^{2}}{x^{4}}-\frac{24b\ln\left(f\right)}{x^{2}}+24\right)\mathrm{e}^{\frac{b\ln\left(f\right)}{x^{2}}}}{4}\right)}{2b^{4}\ln\left(f\right)^{4}}$	59
risch	$-\frac{\left(\ln\left(f\right)^{3}b^{3}-3b^{2}x^{2}\ln\left(f\right)^{2}+6b\,x^{4}\ln\left(f\right)-6x^{6}\right)f^{\frac{a\,x^{2}+b}{x^{2}}}}{2\ln\left(f\right)^{4}b^{4}x^{6}}$	60
parallelrisch	$\frac{-f^{a+\frac{b}{x^{2}}}\ln\left(f\right)^{3}b^{3}+3b^{2}f^{a+\frac{b}{x^{2}}}x^{2}\ln\left(f\right)^{2}-6b\,f^{a+\frac{b}{x^{2}}}x^{4}\ln\left(f\right)+6f^{a+\frac{b}{x^{2}}}x^{6}}{2x^{6}\ln\left(f\right)^{4}b^{4}}$	84
norman	$\frac{-\frac{x^{2}\mathrm{e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{2b\ln\left(f\right)}+\frac{3x^{4}\mathrm{e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{2b^{2}\ln\left(f\right)^{2}}-\frac{3x^{6}\mathrm{e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{b^{3}\ln\left(f\right)^{3}}+\frac{3x^{8}\mathrm{e}^{\left(a+\frac{b}{x^{2}}\right)\ln\left(f\right)}}{b^{4}\ln\left(f\right)^{4}}}{x^{8}}$	98

input
```
int(f^(a+b/x^2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/b^4/ln(f)^4*(6-1/4*(-4*b^3*ln(f)^3/x^6+12*b^2*ln(f)^2/x^4-24*b*ln
(f)/x^2+24)*exp(b*ln(f)/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx = \frac{\left(6\,x^6 - 6\,bx^4\log\left(f\right) + 3\,b^2x^2\log\left(f\right)^2 - b^3\log\left(f\right)^3\right)f^{\frac{ax^2+b}{x^2}}}{2\,b^4x^6\log\left(f\right)^4}$$

input
```
integrate(f^(a+b/x^2)/x^9,x, algorithm="fricas")
```

output
```
1/2*(6*x^6 - 6*b*x^4*log(f) + 3*b^2*x^2*log(f)^2 - b^3*log(f)^3)*f^((a*x^2
 + b)/x^2)/(b^4*x^6*log(f)^4)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx = \frac{f^{a+\frac{b}{x^2}}\left(-b^3\log\left(f\right)^3 + 3b^2x^2\log\left(f\right)^2 - 6bx^4\log\left(f\right) + 6x^6\right)}{2b^4x^6\log\left(f\right)^4}$$

input
```
integrate(f**(a+b/x**2)/x**9,x)
```

output
```
f**(a + b/x**2)*(-b**3*log(f)**3 + 3*b**2*x**2*log(f)**2 - 6*b*x**4*log(f)
 + 6*x**6)/(2*b**4*x**6*log(f)**4)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.26

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx = \frac{f^a\Gamma\left(4, -\frac{b\log(f)}{x^2}\right)}{2\,b^4\log\left(f\right)^4}$$

input
```
integrate(f^(a+b/x^2)/x^9,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(4, -b*log(f)/x^2)/(b^4*log(f)^4)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^9,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^9, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx = -\frac{f^{a+\frac{b}{x^2}}\left(\frac{1}{2\,b\,\ln(f)} - \frac{3\,x^2}{2\,b^2\ln(f)^2} + \frac{3\,x^4}{b^3\ln(f)^3} - \frac{3\,x^6}{b^4\ln(f)^4}\right)}{x^6}$$

input
```
int(f^(a + b/x^2)/x^9,x)
```

output
```
-(f^(a + b/x^2)*(1/(2*b*log(f)) - (3*x^2)/(2*b^2*log(f)^2) + (3*x^4)/(b^3*
log(f)^3) - (3*x^6)/(b^4*log(f)^4)))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^9}\,dx = \frac{f^{\frac{a x^2+b}{x^2}}\left(-\log(f)^3 b^3+3\log(f)^2 b^2x^2-6\log(f)\,b\,x^4+6x^6\right)}{2\log\left(f\right)^4 b^4x^6}$$

input
```
int(f^(a+b/x^2)/x^9,x)
```

output
```
(f**((a*x**2 + b)/x**2)*( - log(f)**3*b**3 + 3*log(f)**2*b**2*x**2 - 6*log
(f)*b*x**4 + 6*x**6))/(2*log(f)**4*b**4*x**6)
```

3.73 $\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} dx = -\frac{f^{a+\frac{b}{x^2}}\left(24x^8-24bx^6\log(f)+12b^2x^4\log^2(f)-4b^3x^2\log^3(f)+b^4\log^4(f)\right)}{2b^5x^8\log^5(f)}$$

output
```
-1/2*f^(a+b/x^2)*(24*x^8-24*b*x^6*ln(f)+12*b^2*x^4*ln(f)^2-4*b^3*x^2*ln(f)
^3+b^4*ln(f)^4)/b^5/x^8/ln(f)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} dx = -\frac{f^a\Gamma\left(5,-\frac{b\log(f)}{x^2}\right)}{2b^5\log^5(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^11,x]
```

output
```
-1/2*(f^a*Gamma[5, -((b*Log[f])/x^2)])/(b^5*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}} \, dx$$

$$\Big\downarrow \ 2647$$

$$-\frac{f^{a+\frac{b}{x^2}} \left(b^4 \log^4(f) - 4b^3x^2 \log^3(f) + 12b^2x^4 \log^2(f) - 24bx^6 \log(f) + 24x^8\right)}{2b^5x^8 \log^5(f)}$$

input
```
Int[f^(a + b/x^2)/x^11,x]
```

output
```
-1/2*(f^(a + b/x^2)*(24*x^8 - 24*b*x^6*Log[f] + 12*b^2*x^4*Log[f]^2 - 4*b^
3*x^2*Log[f]^3 + b^4*Log[f]^4))/(b^5*x^8*Log[f]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{f^{a}\left(24-\frac{\left(\frac{5b^{4}\ln(f)^{4}}{x^{8}}-\frac{20b^{3}\ln(f)^{3}}{x^{6}}+\frac{60b^{2}\ln(f)^{2}}{x^{4}}-\frac{120b\ln(f)}{x^{2}}+120\right)e^{\frac{b\ln(f)}{x^{2}}}}{5}\right)}{2b^{5}\ln(f)^{5}}$	71
risch	$-\frac{\left(24x^{8}-24b\,x^{6}\ln(f)+12b^{2}x^{4}\ln(f)^{2}-4b^{3}x^{2}\ln(f)^{3}+b^{4}\ln(f)^{4}\right)f^{\frac{a\,x^{2}+b}{x^{2}}}}{2b^{5}\ln(f)^{5}x^{8}}$	72
parallelrisch	$\frac{-f^{a+\frac{b}{x^{2}}}b^{4}\ln(f)^{4}+4f^{a+\frac{b}{x^{2}}}x^{2}b^{3}\ln(f)^{3}-12f^{a+\frac{b}{x^{2}}}x^{4}b^{2}\ln(f)^{2}+24f^{a+\frac{b}{x^{2}}}x^{6}b\ln(f)-24f^{a+\frac{b}{x^{2}}}x^{8}}{2x^{8}b^{5}\ln(f)^{5}}$	105
norman	$\frac{-\frac{x^{2}e^{\left(a+\frac{b}{x^{2}}\right)\ln(f)}}{2b\ln(f)}+\frac{2x^{4}e^{\left(a+\frac{b}{x^{2}}\right)\ln(f)}}{b^{2}\ln(f)^{2}}-\frac{6x^{6}e^{\left(a+\frac{b}{x^{2}}\right)\ln(f)}}{b^{3}\ln(f)^{3}}+\frac{12x^{8}e^{\left(a+\frac{b}{x^{2}}\right)\ln(f)}}{b^{4}\ln(f)^{4}}-\frac{12x^{10}e^{\left(a+\frac{b}{x^{2}}\right)\ln(f)}}{b^{5}\ln(f)^{5}}}{x^{10}}$	121

input
```
int(f^(a+b/x^2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/b^5/ln(f)^5*(24-1/5*(5*b^4*ln(f)^4/x^8-20*b^3*ln(f)^3/x^6+60*b^2*l
n(f)^2/x^4-120*b*ln(f)/x^2+120)*exp(b*ln(f)/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx = -\frac{\left(24\,x^{8}-24\,bx^{6}\log\left(f\right)+12\,b^{2}x^{4}\log\left(f\right)^{2}-4\,b^{3}x^{2}\log\left(f\right)^{3}+b^{4}\log\left(f\right)^{4}\right)f^{\frac{ax^{2}+b}{x^{2}}}}{2\,b^{5}x^{8}\log\left(f\right)^{5}}$$

input
```
integrate(f^(a+b/x^2)/x^11,x, algorithm="fricas")
```

output
```
-1/2*(24*x^8 - 24*b*x^6*log(f) + 12*b^2*x^4*log(f)^2 - 4*b^3*x^2*log(f)^3
+ b^4*log(f)^4)*f^((a*x^2 + b)/x^2)/(b^5*x^8*log(f)^5)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx = \frac{f^{a+\frac{b}{x^2}}\left(-b^4\log(f)^4+4b^3x^2\log(f)^3-12b^2x^4\log(f)^2+24bx^6\log(f)-24x^8\right)}{2b^5x^8\log(f)^5}$$

input
```
integrate(f**(a+b/x**2)/x**11,x)
```

output
```
f**(a + b/x**2)*(-b**4*log(f)**4 + 4*b**3*x**2*log(f)**3 - 12*b**2*x**4*lo
g(f)**2 + 24*b*x**6*log(f) - 24*x**8)/(2*b**5*x**8*log(f)**5)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx = -\frac{f^a\Gamma\left(5,-\frac{b\log(f)}{x^2}\right)}{2\,b^5\log(f)^5}$$

input
```
integrate(f^(a+b/x^2)/x^11,x, algorithm="maxima")
```

output
```
-1/2*f^a*gamma(5, -b*log(f)/x^2)/(b^5*log(f)^5)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^11,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^11, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx = -\frac{f^{a+\frac{b}{x^2}}\left(\frac{1}{2\,b\,\ln(f)}-\frac{2\,x^2}{b^2\,\ln(f)^2}+\frac{6\,x^4}{b^3\,\ln(f)^3}-\frac{12\,x^6}{b^4\,\ln(f)^4}+\frac{12\,x^8}{b^5\,\ln(f)^5}\right)}{x^8}$$

input
```
int(f^(a + b/x^2)/x^11,x)
```

output
```
-(f^(a + b/x^2)*(1/(2*b*log(f)) - (2*x^2)/(b^2*log(f)^2) + (6*x^4)/(b^3*lo
g(f)^3) - (12*x^6)/(b^4*log(f)^4) + (12*x^8)/(b^5*log(f)^5)))/x^8
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{11}}\,dx = \frac{f^{\frac{a\,x^2+b}{x^2}}\left(-\log(f)^4 b^4+4\log(f)^3 b^3x^2-12\log(f)^2 b^2x^4+24\log(f)\,b\,x^6-24x^8\right)}{2\log(f)^5 b^5x^8}$$

input
```
int(f^(a+b/x^2)/x^11,x)
```

output
```
(f**((a*x**2 + b)/x**2)*( - log(f)**4*b**4 + 4*log(f)**3*b**3*x**2 - 12*lo
g(f)**2*b**2*x**4 + 24*log(f)*b*x**6 - 24*x**8))/(2*log(f)**5*b**5*x**8)
```

3.74 $\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} dx = \frac{f^{a+\frac{b}{x^2}}\left(120x^{10}-120bx^8\log(f)+60b^2x^6\log^2(f)-20b^3x^4\log^3(f)+5b^4x^2\log^4(f)-b^5\log^5(f)\right)}{2b^6x^{10}\log^6(f)}$$

output
```
1/2*f^(a+b/x^2)*(120*x^10-120*b*x^8*ln(f)+60*b^2*x^6*ln(f)^2-20*b^3*x^4*ln
(f)^3+5*b^4*x^2*ln(f)^4-b^5*ln(f)^5)/b^6/x^10/ln(f)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.29

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}} dx = \frac{f^a\Gamma\left(6,-\frac{b\log(f)}{x^2}\right)}{2b^6\log^6(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^13,x]
```

output
```
(f^a*Gamma[6, -((b*Log[f])/x^2)])/(2*b^6*Log[f]^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx$$

$$\Big\downarrow \text{2647}$$

$$\frac{f^{a+\frac{b}{x^2}}\left(-b^5\log^5(f)+5b^4x^2\log^4(f)-20b^3x^4\log^3(f)+60b^2x^6\log^2(f)-120bx^8\log(f)+120x^{10}\right)}{2b^6x^{10}\log^6(f)}$$

input
```
Int[f^(a + b/x^2)/x^13,x]
```

output
```
(f^(a + b/x^2)*(120*x^10 - 120*b*x^8*Log[f] + 60*b^2*x^6*Log[f]^2 - 20*b^3
*x^4*Log[f]^3 + 5*b^4*x^2*Log[f]^4 - b^5*Log[f]^5))/(2*b^6*x^10*Log[f]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

method	result
meijerg	$-\frac{f^{a}\left(120-\frac{\left(-\frac{6 b^{5} \ln \left(f\right)^{5}}{x^{10}}+\frac{30 b^{4} \ln \left(f\right)^{4}}{x^{8}}-\frac{120 b^{3} \ln \left(f\right)^{3}}{x^{6}}+\frac{360 b^{2} \ln \left(f\right)^{2}}{x^{4}}-\frac{720 b \ln \left(f\right)}{x^{2}}+720\right) {\mathrm e}^{\frac{b \ln \left(f\right)}{x^{2}}}}{6}\right)}{2 b^{6} \ln \left(f\right)^{6}}$
risch	$-\frac{\left(b^{5} \ln \left(f\right)^{5}-5 b^{4} x^{2} \ln \left(f\right)^{4}+20 b^{3} x^{4} \ln \left(f\right)^{3}-60 b^{2} x^{6} \ln \left(f\right)^{2}+120 b \,x^{8} \ln \left(f\right)-120 x^{10}\right) f^{\frac{a \,x^{2}+b}{x^{2}}}}{2 b^{6} \ln \left(f\right)^{6} x^{10}}$
parallelrisch	$\frac{-f^{a+\frac{b}{x^{2}}} b^{5} \ln \left(f\right)^{5}+5 f^{a+\frac{b}{x^{2}}} x^{2} b^{4} \ln \left(f\right)^{4}-20 f^{a+\frac{b}{x^{2}}} x^{4} b^{3} \ln \left(f\right)^{3}+60 f^{a+\frac{b}{x^{2}}} x^{6} b^{2} \ln \left(f\right)^{2}-120 f^{a+\frac{b}{x^{2}}} x^{8} b \ln \left(f\right)+120 f^{a+\frac{b}{x^{2}}} x^{10}}{2 x^{10} b^{6} \ln \left(f\right)^{6}}$
norman	$\frac{\frac{60 x^{12} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln \left(f\right)}}{b^{6} \ln \left(f\right)^{6}}-\frac{x^{2} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln \left(f\right)}}{2 b \ln \left(f\right)}+\frac{5 x^{4} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln \left(f\right)}}{2 b^{2} \ln \left(f\right)^{2}}-\frac{10 x^{6} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln \left(f\right)}}{b^{3} \ln \left(f\right)^{3}}+\frac{30 x^{8} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln \left(f\right)}}{b^{4} \ln \left(f\right)^{4}}-\frac{60 x^{10} {\mathrm e}^{\left(a+\frac{b}{x^{2}}\right) \ln \left(f\right)}}{b^{5} \ln \left(f\right)^{5}}}{x^{12}}$

input
```
int(f^(a+b/x^2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/b^6/ln(f)^6*(120-1/6*(-6*b^5*ln(f)^5/x^10+30*b^4*ln(f)^4/x^8-120*
b^3*ln(f)^3/x^6+360*b^2*ln(f)^2/x^4-720*b*ln(f)/x^2+720)*exp(b*ln(f)/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx = \frac{\left(120\,x^{10} - 120\,bx^{8}\log\left(f\right) + 60\,b^{2}x^{6}\log\left(f\right)^{2} - 20\,b^{3}x^{4}\log\left(f\right)^{3} + 5\,b^{4}x^{2}\log\left(f\right)^{4} - b^{5}\log\left(f\right)^{5}\right)f^{\frac{ax^{2}+b}{x^{2}}}}{2\,b^{6}x^{10}\log\left(f\right)^{6}}$$

input
```
integrate(f^(a+b/x^2)/x^13,x, algorithm="fricas")
```

output
```
1/2*(120*x^10 - 120*b*x^8*log(f) + 60*b^2*x^6*log(f)^2 - 20*b^3*x^4*log(f)
^3 + 5*b^4*x^2*log(f)^4 - b^5*log(f)^5)*f^((a*x^2 + b)/x^2)/(b^6*x^10*log(
f)^6)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx = \frac{f^{a+\frac{b}{x^2}}\left(-b^5\log(f)^5+5b^4x^2\log(f)^4-20b^3x^4\log(f)^3+60b^2x^6\log(f)^2-120bx^8\log(f)+120x^{10}\right)}{2b^6x^{10}\log(f)^6}$$

input
```
integrate(f**(a+b/x**2)/x**13,x)
```

output
```
f**(a + b/x**2)*(-b**5*log(f)**5 + 5*b**4*x**2*log(f)**4 - 20*b**3*x**4*lo
g(f)**3 + 60*b**2*x**6*log(f)**2 - 120*b*x**8*log(f) + 120*x**10)/(2*b**6*
x**10*log(f)**6)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx = \frac{f^a\Gamma\left(6,-\frac{b\log(f)}{x^2}\right)}{2\,b^6\log(f)^6}$$

input
```
integrate(f^(a+b/x^2)/x^13,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(6, -b*log(f)/x^2)/(b^6*log(f)^6)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^13,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^13, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx = -\frac{f^{a+\frac{b}{x^2}}\left(\frac{1}{2\,b\,\ln(f)}-\frac{5\,x^2}{2\,b^2\ln(f)^2}+\frac{10\,x^4}{b^3\ln(f)^3}-\frac{30\,x^6}{b^4\ln(f)^4}+\frac{60\,x^8}{b^5\ln(f)^5}-\frac{60\,x^{10}}{b^6\ln(f)^6}\right)}{x^{10}}$$

input
```
int(f^(a + b/x^2)/x^13,x)
```

output
```
-(f^(a + b/x^2)*(1/(2*b*log(f)) - (5*x^2)/(2*b^2*log(f)^2) + (10*x^4)/(b^3
*log(f)^3) - (30*x^6)/(b^4*log(f)^4) + (60*x^8)/(b^5*log(f)^5) - (60*x^10)
/(b^6*log(f)^6)))/x^10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{13}}\,dx = \frac{f^{\frac{a x^2+b}{x^2}}\left(-\log(f)^5 b^5+5\log(f)^4 b^4x^2-20\log(f)^3 b^3x^4+60\log(f)^2 b^2x^6-120\log(f)\,b\,x^8+120x^{10}\right)}{2\log(f)^6 b^6x^{10}}$$

input
```
int(f^(a+b/x^2)/x^13,x)
```

output
```
(f**((a*x**2 + b)/x**2)*( - log(f)**5*b**5 + 5*log(f)**4*b**4*x**2 - 20*lo
g(f)**3*b**3*x**4 + 60*log(f)**2*b**2*x**6 - 120*log(f)*b*x**8 + 120*x**10
))/(2*log(f)**6*b**6*x**10)
```

3.75 $\int f^{a+\frac{b}{x^2}} x^{10}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+\frac{b}{x^2}} x^{10}\,dx = \frac{1}{2} f^a x^{11} \Gamma\left(-\frac{11}{2}, -\frac{b\log(f)}{x^2}\right)\left(-\frac{b\log(f)}{x^2}\right)^{11/2}$$

output
```
1/2*f^a*x^11*(64/10395*Pi^(1/2)*erfc((-b*ln(f)/x^2)^(1/2))-64/10395/(-b*ln
(f)/x^2)^(1/2)*exp(b*ln(f)/x^2)+32/10395/(-b*ln(f)/x^2)^(3/2)*exp(b*ln(f)/
x^2)-16/3465/(-b*ln(f)/x^2)^(5/2)*exp(b*ln(f)/x^2)+8/693/(-b*ln(f)/x^2)^(7
/2)*exp(b*ln(f)/x^2)-4/99/(-b*ln(f)/x^2)^(9/2)*exp(b*ln(f)/x^2)+2/11/(-b*l
n(f)/x^2)^(11/2)*exp(b*ln(f)/x^2))*(-b*ln(f)/x^2)^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}} x^{10}\,dx = \frac{1}{2} f^a x^{11} \Gamma\left(-\frac{11}{2}, -\frac{b\log(f)}{x^2}\right)\left(-\frac{b\log(f)}{x^2}\right)^{11/2}$$

input
```
Integrate[f^(a + b/x^2)*x^10,x]
```

output
```
(f^a*x^11*Gamma[-11/2, -((b*Log[f])/x^2)]*(-((b*Log[f])/x^2))^(11/2))/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{10} f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \; 2648$$

$$\frac{1}{2} x^{11} f^a \left(-\frac{b \log(f)}{x^2}\right)^{11/2} \Gamma\left(-\frac{11}{2},-\frac{b \log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)*x^10,x]
```

output
```
(f^a*x^11*Gamma[-11/2, -((b*Log[f])/x^2)]*(-((b*Log[f])/x^2))^(11/2))/2
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

method	result
meijerg	$\frac{f^a b^5 \ln(f)^{\frac{11}{2}} \sqrt{-b}\left(-\frac{2x^{11}\left(\frac{32b^5\ln(f)^5}{945x^{10}}+\frac{16b^4\ln(f)^4}{945x^8}+\frac{8b^3\ln(f)^3}{315x^6}+\frac{4b^2\ln(f)^2}{63x^4}+\frac{2b\ln(f)}{9x^2}+1\right)e^{\frac{b\ln(f)}{x^2}}}{11(-b)^{\frac{11}{2}}\ln(f)^{\frac{11}{2}}}+\frac{64b^{\frac{11}{2}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{\ln(f)}}{x}\right)}{10395(-b)^{\frac{11}{2}}}\right)}{2}$
risch	$\frac{f^a x^{11} f^{\frac{b}{x^2}}}{11}+\frac{2f^a\ln(f)b\,x^9 f^{\frac{b}{x^2}}}{99}+\frac{4f^a\ln(f)^2 b^2 x^7 f^{\frac{b}{x^2}}}{693}+\frac{8f^a\ln(f)^3 b^3 x^5 f^{\frac{b}{x^2}}}{3465}+\frac{16f^a\ln(f)^4 b^4 x^3 f^{\frac{b}{x^2}}}{10395}+\frac{32f^a\ln(f)^5 b^5 x\, f^{\frac{b}{x^2}}}{10395}$

input
```
int(f^(a+b/x^2)*x^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*b^5*ln(f)^(11/2)*(-b)^(1/2)*(-2/11*x^11/(-b)^(11/2)/ln(f)^(11/2)*(
32/945*b^5*ln(f)^5/x^10+16/945*b^4*ln(f)^4/x^8+8/315*b^3*ln(f)^3/x^6+4/63*
b^2*ln(f)^2/x^4+2/9*b*ln(f)/x^2+1)*exp(b*ln(f)/x^2)+64/10395/(-b)^(11/2)*b
^(11/2)*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.24

$$\int f^{a+\frac{b}{x^2}}x^{10}\,dx=\frac{32}{10395}\sqrt{\pi}\sqrt{-b\log(f)}b^5 f^a\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)\log(f)^5$$
$$+\frac{1}{10395}\left(945\,x^{11}+210\,bx^9\log(f)+60\,b^2x^7\log(f)^2+24\,b^3x^5\log(f)^3+16\,b^4x^3\log(f)^4+32\,b^5x\log(f)^5\right)f^{\frac{ax^2+b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^10,x, algorithm="fricas")
```

output
```
32/10395*sqrt(pi)*sqrt(-b*log(f))*b^5*f^a*erf(sqrt(-b*log(f))/x)*log(f)^5
+ 1/10395*(945*x^11 + 210*b*x^9*log(f) + 60*b^2*x^7*log(f)^2 + 24*b^3*x^5*
log(f)^3 + 16*b^4*x^3*log(f)^4 + 32*b^5*x*log(f)^5)*f^((a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^{10}\,dx=\int f^{a+\frac{b}{x^2}}x^{10}\,dx$$

input
```
integrate(f**(a+b/x**2)*x**10,x)
```

output
```
Integral(f**(a + b/x**2)*x**10, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x^2}}x^{10}\,dx=\frac{1}{2}f^a x^{11}\left(-\frac{b\log(f)}{x^2}\right)^{\frac{11}{2}}\Gamma\left(-\frac{11}{2},-\frac{b\log(f)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)*x^10,x, algorithm="maxima")
```

output
```
1/2*f^a*x^11*(-b*log(f)/x^2)^(11/2)*gamma(-11/2, -b*log(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^{10}\,dx=\int f^{a+\frac{b}{x^2}}x^{10}\,dx$$

input
```
integrate(f^(a+b/x^2)*x^10,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 5.09

$$\int f^{a+\frac{b}{x^2}} x^{10}\,dx = \frac{f^a\,f^{\frac{b}{x^2}}\,x^{11}}{11} - \frac{32\,f^a\,x^{11}\,\sqrt{\pi}\,\left(-\frac{b\,\ln(f)}{x^2}\right)^{11/2}}{10395} + \frac{32\,f^a\,x^{11}\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln(f)}{x^2}}\right)\,\left(-\frac{b\,\ln(f)}{x^2}\right)^{11/2}}{10395} + \frac{32\,b^5\,f^a\,f^{\frac{b}{x^2}}\,x\ln(f)^5}{10395} + \frac{4\,b^2\,f^a\,f^{\frac{b}{x^2}}\,x^7\ln(f)^2}{693} + \frac{8\,b^3\,f^a\,f^{\frac{b}{x^2}}\,x^5\ln(f)^3}{3465} + \frac{16\,b^4\,f^a\,f^{\frac{b}{x^2}}\,x^3\ln(f)^4}{10395} + \frac{2\,b\,f^a\,f^{\frac{b}{x^2}}\,x^9\,\ln(f)}{99}$$

input
```
int(f^(a + b/x^2)*x^10,x)
```

output
```
(f^a*f^(b/x^2)*x^11)/11 - (32*f^a*x^11*pi^(1/2)*(-(b*log(f))/x^2)^(11/2))/
10395 + (32*f^a*x^11*pi^(1/2)*erfc((-(b*log(f))/x^2)^(1/2))*(-(b*log(f))/x
^2)^(11/2))/10395 + (32*b^5*f^a*f^(b/x^2)*x*log(f)^5)/10395 + (4*b^2*f^a*f
^(b/x^2)*x^7*log(f)^2)/693 + (8*b^3*f^a*f^(b/x^2)*x^5*log(f)^3)/3465 + (16
*b^4*f^a*f^(b/x^2)*x^3*log(f)^4)/10395 + (2*b*f^a*f^(b/x^2)*x^9*log(f))/99
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}} x^{10}\,dx = \frac{-64f^{\frac{a\,x^2+b}{x^2}}\log(f)^6 b^6 + 32f^{\frac{a\,x^2+b}{x^2}}\log(f)^5 b^5x^2 + 16f^{\frac{a\,x^2+b}{x^2}}\log(f)^4 b^4x^4 + 24f^{\frac{a\,x^2+b}{x^2}}\log(f)^3 b^3x^6 + 60f^{\frac{a\,x^2+b}{x^2}}}{10395x}$$

input
```
int(f^(a+b/x^2)*x^10,x)
```

output
```
( - 64*f**((a*x**2 + b)/x**2)*log(f)**6*b**6 + 32*f**((a*x**2 + b)/x**2)*l
og(f)**5*b**5*x**2 + 16*f**((a*x**2 + b)/x**2)*log(f)**4*b**4*x**4 + 24*f*
*((a*x**2 + b)/x**2)*log(f)**3*b**3*x**6 + 60*f**((a*x**2 + b)/x**2)*log(f
)**2*b**2*x**8 + 210*f**((a*x**2 + b)/x**2)*log(f)*b*x**10 + 945*f**((a*x*
*2 + b)/x**2)*x**12 - 128*int(f**((a*x**2 + b)/x**2)/x**4,x)*log(f)**7*b**
7*x)/(10395*x)
```

3.76 $\int f^{a+\frac{b}{x^2}} x^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+\frac{b}{x^2}} x^8 \, dx = \frac{1}{2} f^a x^9 \Gamma\left(-\frac{9}{2}, -\frac{b\log(f)}{x^2}\right) \left(-\frac{b\log(f)}{x^2}\right)^{9/2}$$

output
```
1/2*f^a*x^9*(-32/945*Pi^(1/2)*erfc((-b*ln(f)/x^2)^(1/2))+32/945/(-b*ln(f)/
x^2)^(1/2)*exp(b*ln(f)/x^2)-16/945/(-b*ln(f)/x^2)^(3/2)*exp(b*ln(f)/x^2)+8
/315/(-b*ln(f)/x^2)^(5/2)*exp(b*ln(f)/x^2)-4/63/(-b*ln(f)/x^2)^(7/2)*exp(b
*ln(f)/x^2)+2/9/(-b*ln(f)/x^2)^(9/2)*exp(b*ln(f)/x^2))*(-b*ln(f)/x^2)^(9/2
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}} x^8 \, dx = \frac{1}{2} f^a x^9 \Gamma\left(-\frac{9}{2}, -\frac{b\log(f)}{x^2}\right) \left(-\frac{b\log(f)}{x^2}\right)^{9/2}$$

input
```
Integrate[f^(a + b/x^2)*x^8,x]
```

output
```
(f^a*x^9*Gamma[-9/2, -((b*Log[f])/x^2)]*(-((b*Log[f])/x^2))^(9/2))/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 f^{a+\frac{b}{x^2}}\,dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{1}{2}x^9 f^a\left(-\frac{b\log(f)}{x^2}\right)^{9/2}\Gamma\left(-\frac{9}{2},-\frac{b\log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)*x^8,x]
```

output
```
(f^a*x^9*Gamma[-9/2, -((b*Log[f])/x^2)]*(-((b*Log[f])/x^2))^(9/2))/2
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.29

method	result
meijerg	$-\frac{f^a b^4 \ln(f)^{\frac{9}{2}} \sqrt{-b}\left(-\frac{2x^9\left(\frac{16b^4\ln(f)^4}{105x^8}+\frac{8b^3\ln(f)^3}{105x^6}+\frac{4b^2\ln(f)^2}{35x^4}+\frac{2b\ln(f)}{7x^2}+1\right)e^{\frac{b\ln(f)}{x^2}}}{9(-b)^{\frac{9}{2}}\ln(f)^{\frac{9}{2}}}+\frac{32b^{\frac{9}{2}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{\ln(f)}}{x}\right)}{945(-b)^{\frac{9}{2}}}\right)}{2}$
risch	$\frac{f^a f^{\frac{b}{x^2}} x^9}{9}+\frac{2f^a\ln(f)bx^7f^{\frac{b}{x^2}}}{63}+\frac{4f^a\ln(f)^2b^2x^5f^{\frac{b}{x^2}}}{315}+\frac{8f^a\ln(f)^3b^3x^3f^{\frac{b}{x^2}}}{945}+\frac{16f^a\ln(f)^4b^4xf^{\frac{b}{x^2}}}{945}-\frac{16f^a\ln(f)^5b^5\sqrt{\pi}\,\mathrm{er}}{945\sqrt{-b\mathrm{l}}}$

input
```
int(f^(a+b/x^2)*x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*b^4*ln(f)^(9/2)*(-b)^(1/2)*(-2/9*x^9/(-b)^(9/2)/ln(f)^(9/2)*(16/1
05*b^4*ln(f)^4/x^8+8/105*b^3*ln(f)^3/x^6+4/35*b^2*ln(f)^2/x^4+2/7*b*ln(f)/
x^2+1)*exp(b*ln(f)/x^2)+32/945/(-b)^(9/2)*b^(9/2)*Pi^(1/2)*erfi(b^(1/2)*ln
(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.88

$$\int f^{a+\frac{b}{x^2}}x^8\,dx=\frac{16}{945}\sqrt{\pi}\sqrt{-b\log(f)}b^4f^a\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)\log(f)^4$$
$$+\frac{1}{945}\left(105x^9+30bx^7\log(f)+12b^2x^5\log(f)^2+8b^3x^3\log(f)^3+16b^4x\log(f)^4\right)f^{\frac{ax^2+b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^8,x, algorithm="fricas")
```

output
```
16/945*sqrt(pi)*sqrt(-b*log(f))*b^4*f^a*erf(sqrt(-b*log(f))/x)*log(f)^4 +
1/945*(105*x^9 + 30*b*x^7*log(f) + 12*b^2*x^5*log(f)^2 + 8*b^3*x^3*log(f)^
3 + 16*b^4*x*log(f)^4)*f^((a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^8\,dx = \int f^{a+\frac{b}{x^2}}x^8\,dx$$

input
```
integrate(f**(a+b/x**2)*x**8,x)
```

output
```
Integral(f**(a + b/x**2)*x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x^2}}x^8\,dx = \frac{1}{2}f^a x^9\left(-\frac{b\log(f)}{x^2}\right)^{\frac{9}{2}}\Gamma\left(-\frac{9}{2}, -\frac{b\log(f)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)*x^8,x, algorithm="maxima")
```

output
```
1/2*f^a*x^9*(-b*log(f)/x^2)^(9/2)*gamma(-9/2, -b*log(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^8\,dx = \int f^{a+\frac{b}{x^2}}x^8\,dx$$

input
```
integrate(f^(a+b/x^2)*x^8,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 4.44

$$\int f^{a+\frac{b}{x^2}}x^8\,dx=\frac{f^a\,f^{\frac{b}{x^2}}\,x^9}{9}+\frac{16\,f^a\,x^9\,\sqrt{\pi}\,\left(-\frac{b\,\ln(f)}{x^2}\right)^{9/2}}{945}-\frac{16\,f^a\,x^9\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln(f)}{x^2}}\right)\,\left(-\frac{b\,\ln(f)}{x^2}\right)^{9/2}}{945}+\frac{16\,b^4\,f^a\,f^{\frac{b}{x^2}}\,x\ln\left(f\right)^4}{945}+\frac{4\,b^2\,f^a\,f^{\frac{b}{x^2}}\,x^5\ln\left(f\right)^2}{315}+\frac{8\,b^3\,f^a\,f^{\frac{b}{x^2}}\,x^3\ln\left(f\right)^3}{945}+\frac{2\,b\,f^a\,f^{\frac{b}{x^2}}\,x^7\,\ln\left(f\right)}{63}$$

input
```
int(f^(a + b/x^2)*x^8,x)
```

output
```
(f^a*f^(b/x^2)*x^9)/9 + (16*f^a*x^9*pi^(1/2)*(-(b*log(f))/x^2)^(9/2))/945
- (16*f^a*x^9*pi^(1/2)*erfc((-(b*log(f))/x^2)^(1/2))*(-(b*log(f))/x^2)^(9/
2))/945 + (16*b^4*f^a*f^(b/x^2)*x*log(f)^4)/945 + (4*b^2*f^a*f^(b/x^2)*x^5
*log(f)^2)/315 + (8*b^3*f^a*f^(b/x^2)*x^3*log(f)^3)/945 + (2*b*f^a*f^(b/x^
2)*x^7*log(f))/63
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}}x^8\,dx=\frac{-32f^{\frac{a\,x^2+b}{x^2}}\log(f)^5\,b^5+16f^{\frac{a\,x^2+b}{x^2}}\log(f)^4\,b^4x^2+8f^{\frac{a\,x^2+b}{x^2}}\log(f)^3\,b^3x^4+12f^{\frac{a\,x^2+b}{x^2}}\log(f)^2\,b^2x^6+30f^{\frac{a\,x^2+b}{x^2}}\mathrm{lo}}{945x}$$

input
```
int(f^(a+b/x^2)*x^8,x)
```

output
```
( - 32*f**((a*x**2 + b)/x**2)*log(f)**5*b**5 + 16*f**((a*x**2 + b)/x**2)*l
og(f)**4*b**4*x**2 + 8*f**((a*x**2 + b)/x**2)*log(f)**3*b**3*x**4 + 12*f**
((a*x**2 + b)/x**2)*log(f)**2*b**2*x**6 + 30*f**((a*x**2 + b)/x**2)*log(f)
*b*x**8 + 105*f**((a*x**2 + b)/x**2)*x**10 - 64*int(f**((a*x**2 + b)/x**2)
/x**4,x)*log(f)**6*b**6*x)/(945*x)
```

3.77 $\int f^{a+\frac{b}{x^2}} x^6 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 119

$$\int f^{a+\frac{b}{x^2}} x^6 \, dx = \frac{1}{7} f^{a+\frac{b}{x^2}} x^7 + \frac{2}{35} b f^{a+\frac{b}{x^2}} x^5 \log(f) + \frac{4}{105} b^2 f^{a+\frac{b}{x^2}} x^3 \log^2(f) + \frac{8}{105} b^3 f^{a+\frac{b}{x^2}} x \log^3(f) - \frac{8}{105} b^{7/2} f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \log^{\frac{7}{2}}(f)$$

output
```
1/7*f^(a+b/x^2)*x^7+2/35*b*f^(a+b/x^2)*x^5*ln(f)+4/105*b^2*f^(a+b/x^2)*x^3
*ln(f)^2+8/105*b^3*f^(a+b/x^2)*x*ln(f)^3-8/105*b^(7/2)*f^a*Pi^(1/2)*erfi(b
^(1/2)*ln(f)^(1/2)/x)*ln(f)^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int f^{a+\frac{b}{x^2}} x^6 \, dx = \frac{1}{105} f^a \left(-8 b^{7/2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \log^{\frac{7}{2}}(f) + f^{\frac{b}{x^2}} x \left(15 x^6 + 6 b x^4 \log(f) + 4 b^2 x^2 \log^2(f) + 8 b^3 \log^3(f)\right)\right)$$

input
```
Integrate[f^(a + b/x^2)*x^6,x]
```

output
```
(f^a*(-8*b^(7/2)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x]*Log[f]^(7/2) + f^
(b/x^2)*x*(15*x^6 + 6*b*x^4*Log[f] + 4*b^2*x^2*Log[f]^2 + 8*b^3*Log[f]^3))
)/105
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2643, 2643, 2643, 2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \int f^{a+\frac{b}{x^2}} x^4 dx + \frac{1}{7} x^7 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \left(\frac{2}{5} b \log(f) \int f^{a+\frac{b}{x^2}} x^2 dx + \frac{1}{5} x^5 f^{a+\frac{b}{x^2}}\right) + \frac{1}{7} x^7 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7} b \log(f) \left(\frac{2}{5} b \log(f) \left(\frac{2}{3} b \log(f) \int f^{a+\frac{b}{x^2}} dx + \frac{1}{3} x^3 f^{a+\frac{b}{x^2}}\right) + \frac{1}{5} x^5 f^{a+\frac{b}{x^2}}\right) + \frac{1}{7} x^7 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2635}$$

$$\frac{2}{7} b \log(f) \left(\frac{2}{5} b \log(f) \left(\frac{2}{3} b \log(f) \left(2 b \log(f) \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx + x f^{a+\frac{b}{x^2}}\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x^2}}\right) + \frac{1}{5} x^5 f^{a+\frac{b}{x^2}}\right) + \frac{1}{7} x^7 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2640}$$

$$\frac{2}{7}b\log(f)\left(\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(xf^{a+\frac{b}{x^2}}-2b\log(f)\int f^{a+\frac{b}{x^2}}d\frac{1}{x}\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}\right)+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}\right)+\frac{1}{7}x^7f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{7}b\log(f)\left(\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(xf^{a+\frac{b}{x^2}}-\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}\right)+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}\right)+\frac{1}{7}x^7f^{a+\frac{b}{x^2}}$$

input
```
Int[f^(a + b/x^2)*x^6,x]
```

output
```
(f^(a + b/x^2)*x^7)/7 + (2*b*Log[f]*((f^(a + b/x^2)*x^5)/5 + (2*b*Log[f]*(
(f^(a + b/x^2)*x^3)/3 + (2*b*(f^(a + b/x^2)*x - Sqrt[b]*f^a*Sqrt[Pi]*Erfi[
(Sqrt[b]*Sqrt[Log[f]])/x]*Sqrt[Log[f]])*Log[f])/3))/5))/7
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.84

method	result	size
meijerg	$\frac{f^{a} \ln(f)^{\frac{7}{2}} b^{3} \sqrt{-b}\left(-\frac{2 x^{7}\left(\frac{8 b^{3} \ln(f)^{3}}{15 x^{6}}+\frac{4 b^{2} \ln(f)^{2}}{15 x^{4}}+\frac{2 b \ln(f)}{5 x^{2}}+1\right) \mathrm{e}^{\frac{b \ln(f)}{x^{2}}}}{7(-b)^{\frac{7}{2}} \ln(f)^{\frac{7}{2}}}+\frac{16 b^{\frac{7}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\frac{\sqrt{b}\, \sqrt{\ln(f)}}{x}\right)}{105(-b)^{\frac{7}{2}}}\right)}{2}$	100
risch	$\frac{f^{a} f^{\frac{b}{x^{2}}} x^{7}}{7}+\frac{2 f^{a} \ln(f) b\, x^{5} f^{\frac{b}{x^{2}}}}{35}+\frac{4 f^{a} \ln(f)^{2} b^{2} x^{3} f^{\frac{b}{x^{2}}}}{105}+\frac{8 f^{a} \ln(f)^{3} b^{3} x\, f^{\frac{b}{x^{2}}}}{105}-\frac{8 f^{a} \ln(f)^{4} b^{4} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln(f)}}{x}\right)}{105 \sqrt{-b \ln(f)}}$	111

input
```
int(f^(a+b/x^2)*x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*ln(f)^(7/2)*b^3*(-b)^(1/2)*(-2/7*x^7/(-b)^(7/2)/ln(f)^(7/2)*(8/15*
b^3*ln(f)^3/x^6+4/15*b^2*ln(f)^2/x^4+2/5*b*ln(f)/x^2+1)*exp(b*ln(f)/x^2)+1
6/105/(-b)^(7/2)*b^(7/2)*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.72

$$\int f^{a+\frac{b}{x^2}} x^6\,dx = \frac{8}{105}\,\sqrt{\pi}\sqrt{-b\log\left(f\right)}b^3 f^a \operatorname{erf}\left(\frac{\sqrt{-b\log\left(f\right)}}{x}\right)\log\left(f\right)^3 + \frac{1}{105}\left(15\,x^7 + 6\,bx^5\log\left(f\right) + 4\,b^2x^3\log\left(f\right)^2 + 8\,b^3x\log\left(f\right)^3\right)f^{\frac{ax^2+b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^6,x, algorithm="fricas")
```

output
```
8/105*sqrt(pi)*sqrt(-b*log(f))*b^3*f^a*erf(sqrt(-b*log(f))/x)*log(f)^3 + 1
/105*(15*x^7 + 6*b*x^5*log(f) + 4*b^2*x^3*log(f)^2 + 8*b^3*x*log(f)^3)*f^(
(a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^6\,dx=\int f^{a+\frac{b}{x^2}}x^6\,dx$$

input
```
integrate(f**(a+b/x**2)*x**6,x)
```

output
```
Integral(f**(a + b/x**2)*x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.24

$$\int f^{a+\frac{b}{x^2}}x^6\,dx=\frac{1}{2}\,f^a x^7\left(-\frac{b\log\left(f\right)}{x^2}\right)^{\frac{7}{2}}\Gamma\left(-\frac{7}{2},-\frac{b\log\left(f\right)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)*x^6,x, algorithm="maxima")
```

output
```
1/2*f^a*x^7*(-b*log(f)/x^2)^(7/2)*gamma(-7/2, -b*log(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^6\,dx=\int f^{a+\frac{b}{x^2}}x^6\,dx$$

input
```
integrate(f^(a+b/x^2)*x^6,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int f^{a+\frac{b}{x^2}}x^6\,dx = \frac{f^a\,f^{\frac{b}{x^2}}\,x^7}{7} - \frac{8\,f^a\,x^7\,\sqrt{\pi}\,\left(-\frac{b\,\ln(f)}{x^2}\right)^{7/2}}{105} + \frac{8\,f^a\,x^7\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln(f)}{x^2}}\right)\,\left(-\frac{b\,\ln(f)}{x^2}\right)^{7/2}}{105} + \frac{8\,b^3\,f^a\,f^{\frac{b}{x^2}}\,x\ln\left(f\right)^3}{105} + \frac{4\,b^2\,f^a\,f^{\frac{b}{x^2}}\,x^3\ln\left(f\right)^2}{105} + \frac{2\,b\,f^a\,f^{\frac{b}{x^2}}\,x^5\,\ln\left(f\right)}{35}$$

input
```
int(f^(a + b/x^2)*x^6,x)
```

output
```
(f^a*f^(b/x^2)*x^7)/7 - (8*f^a*x^7*pi^(1/2)*(-(b*log(f))/x^2)^(7/2))/105 +
 (8*f^a*x^7*pi^(1/2)*erfc((-(b*log(f))/x^2)^(1/2))*(-(b*log(f))/x^2)^(7/2)
)/105 + (8*b^3*f^a*f^(b/x^2)*x*log(f)^3)/105 + (4*b^2*f^a*f^(b/x^2)*x^3*lo
g(f)^2)/105 + (2*b*f^a*f^(b/x^2)*x^5*log(f))/35
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}}x^6\,dx = \frac{-16f^{\frac{a\,x^2+b}{x^2}}\log(f)^4\,b^4+8f^{\frac{a\,x^2+b}{x^2}}\log(f)^3\,b^3x^2+4f^{\frac{a\,x^2+b}{x^2}}\log(f)^2\,b^2x^4+6f^{\frac{a\,x^2+b}{x^2}}\log(f)\,b\,x^6+15f^{\frac{a\,x^2+b}{x^2}}x^8-}{105x}$$

input
```
int(f^(a+b/x^2)*x^6,x)
```

output
```
( - 16*f**((a*x**2 + b)/x**2)*log(f)**4*b**4 + 8*f**((a*x**2 + b)/x**2)*lo
g(f)**3*b**3*x**2 + 4*f**((a*x**2 + b)/x**2)*log(f)**2*b**2*x**4 + 6*f**((
a*x**2 + b)/x**2)*log(f)*b*x**6 + 15*f**((a*x**2 + b)/x**2)*x**8 - 32*int(
f**((a*x**2 + b)/x**2)/x**4,x)*log(f)**5*b**5*x)/(105*x)
```

3.78 $\int f^{a+\frac{b}{x^2}} x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 96

$$\int f^{a+\frac{b}{x^2}} x^4 \, dx = \frac{1}{5} f^{a+\frac{b}{x^2}} x^5 + \frac{2}{15} b f^{a+\frac{b}{x^2}} x^3 \log(f) + \frac{4}{15} b^2 f^{a+\frac{b}{x^2}} x \log^2(f) - \frac{4}{15} b^{5/2} f^a \sqrt{\pi} \text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \log^{\frac{5}{2}}(f)$$

output
```
1/5*f^(a+b/x^2)*x^5+2/15*b*f^(a+b/x^2)*x^3*ln(f)+4/15*b^2*f^(a+b/x^2)*x*ln
(f)^2-4/15*b^(5/2)*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)*ln(f)^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int f^{a+\frac{b}{x^2}} x^4 \, dx = \frac{1}{15} f^a \left(-4 b^{5/2} \sqrt{\pi} \text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \log^{\frac{5}{2}}(f) + f^{\frac{b}{x^2}} x \left(3x^4 + 2bx^2 \log(f) + 4b^2 \log^2(f)\right)\right)$$

input
```
Integrate[f^(a + b/x^2)*x^4,x]
```

output
```
(f^a*(-4*b^(5/2)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x]*Log[f]^(5/2) + f^
(b/x^2)*x*(3*x^4 + 2*b*x^2*Log[f] + 4*b^2*Log[f]^2)))/15
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2643, 2643, 2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{a+\frac{b}{x^2}}\,dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(f)\int f^{a+\frac{b}{x^2}}x^2dx+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\int f^{a+\frac{b}{x^2}}dx+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}\right)+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2635}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(2b\log(f)\int \frac{f^{a+\frac{b}{x^2}}}{x^2}dx+xf^{a+\frac{b}{x^2}}\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}\right)+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2640}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(xf^{a+\frac{b}{x^2}}-2b\log(f)\int f^{a+\frac{b}{x^2}}d\frac{1}{x}\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}\right)+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{5}b\log(f)\left(\frac{2}{3}b\log(f)\left(xf^{a+\frac{b}{x^2}}-\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)\right)+\frac{1}{3}x^3f^{a+\frac{b}{x^2}}\right)+\frac{1}{5}x^5f^{a+\frac{b}{x^2}}$$

input
```
Int[f^(a + b/x^2)*x^4,x]
```

output
```
(f^(a + b/x^2)*x^5)/5 + (2*b*Log[f]*((f^(a + b/x^2)*x^3)/3 + (2*b*(f^(a +
b/x^2)*x - Sqrt[b]*f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x]*Sqrt[Log[f]
])*Log[f])/3))/5
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

method	result	size
meijerg	$-\frac{f^{a} \ln\left(f\right)^{\frac{5}{2}} b^{2} \sqrt{-b}\, \left(-\frac{2 x^{5} \left(\frac{4 b^{2} \ln\left(f\right)^{2}}{3 x^{4}}+\frac{2 b \ln\left(f\right)}{3 x^{2}}+1\right) {\mathrm e}^{\frac{b \ln\left(f\right)}{x^{2}}}}{5 \left(-b\right)^{\frac{5}{2}} \ln\left(f\right)^{\frac{5}{2}}}+\frac{8 b^{\frac{5}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\frac{\sqrt{b}\, \sqrt{\ln\left(f\right)}}{x}\right)}{15 \left(-b\right)^{\frac{5}{2}}}\right)}{2}$	88
risch	$\frac{f^{a} f^{\frac{b}{x^{2}}} x^{5}}{5}+\frac{2 f^{a} \ln\left(f\right) b\, x^{3} f^{\frac{b}{x^{2}}}}{15}+\frac{4 f^{a} \ln\left(f\right)^{2} b^{2} x\, f^{\frac{b}{x^{2}}}}{15}-\frac{4 f^{a} \ln\left(f\right)^{3} b^{3} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln\left(f\right)}}{x}\right)}{15 \sqrt{-b \ln\left(f\right)}}$	89

input
```
int(f^(a+b/x^2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*ln(f)^(5/2)*b^2*(-b)^(1/2)*(-2/5*x^5/(-b)^(5/2)/ln(f)^(5/2)*(4/3*
b^2*ln(f)^2/x^4+2/3*b*ln(f)/x^2+1)*exp(b*ln(f)/x^2)+8/15/(-b)^(5/2)*b^(5/2
)*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int f^{a+\frac{b}{x^2}} x^4 \, dx = \frac{4}{15} \sqrt{\pi} \sqrt{-b \log(f)} b^2 f^a \operatorname{erf}\left(\frac{\sqrt{-b \log(f)}}{x}\right) \log(f)^2 + \frac{1}{15} \left(3 x^5 + 2 b x^3 \log(f) + 4 b^2 x \log(f)^2\right) f^{\frac{a x^2 + b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^4,x, algorithm="fricas")
```

output
```
4/15*sqrt(pi)*sqrt(-b*log(f))*b^2*f^a*erf(sqrt(-b*log(f))/x)*log(f)^2 + 1/
15*(3*x^5 + 2*b*x^3*log(f) + 4*b^2*x*log(f)^2)*f^((a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^4\,dx = \int f^{a+\frac{b}{x^2}}x^4\,dx$$

input
```
integrate(f**(a+b/x**2)*x**4,x)
```

output
```
Integral(f**(a + b/x**2)*x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.29

$$\int f^{a+\frac{b}{x^2}}x^4\,dx = \frac{1}{2}f^a x^5\left(-\frac{b\log(f)}{x^2}\right)^{\frac{5}{2}}\Gamma\left(-\frac{5}{2},-\frac{b\log(f)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)*x^4,x, algorithm="maxima")
```

output
```
1/2*f^a*x^5*(-b*log(f)/x^2)^(5/2)*gamma(-5/2, -b*log(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^4\,dx = \int f^{a+\frac{b}{x^2}}x^4\,dx$$

input
```
integrate(f^(a+b/x^2)*x^4,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int f^{a+\frac{b}{x^2}}x^4\,dx=\frac{f^a\,f^{\frac{b}{x^2}}\,x^5}{5}+\frac{4\,f^a\,x^5\,\sqrt{\pi}\,{\left(-\frac{b\,\ln(f)}{x^2}\right)}^{5/2}}{15}-\frac{4\,f^a\,x^5\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln(f)}{x^2}}\right)\,{\left(-\frac{b\,\ln(f)}{x^2}\right)}^{5/2}}{15}+\frac{4\,b^2\,f^a\,f^{\frac{b}{x^2}}\,x\,{\ln\left(f\right)}^2}{15}+\frac{2\,b\,f^a\,f^{\frac{b}{x^2}}\,x^3\,\ln\left(f\right)}{15}$$

input
```
int(f^(a + b/x^2)*x^4,x)
```

output
```
(f^a*f^(b/x^2)*x^5)/5 + (4*f^a*x^5*pi^(1/2)*(-(b*log(f))/x^2)^(5/2))/15 -
(4*f^a*x^5*pi^(1/2)*erfc((-(b*log(f))/x^2)^(1/2))*(-(b*log(f))/x^2)^(5/2))
/15 + (4*b^2*f^a*f^(b/x^2)*x*log(f)^2)/15 + (2*b*f^a*f^(b/x^2)*x^3*log(f))
/15
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}}x^4\,dx=\frac{-8f^{\frac{a\,x^2+b}{x^2}}\log(f)^3\,b^3+4f^{\frac{a\,x^2+b}{x^2}}\log(f)^2\,b^2x^2+2f^{\frac{a\,x^2+b}{x^2}}\log(f)\,b\,x^4+3f^{\frac{a\,x^2+b}{x^2}}x^6-16\left(\int\frac{f^{\frac{a\,x^2+b}{x^2}}}{x^4}dx\right)\log(f)^4}{15x}$$

input
```
int(f^(a+b/x^2)*x^4,x)
```

output
```
( - 8*f**((a*x**2 + b)/x**2)*log(f)**3*b**3 + 4*f**((a*x**2 + b)/x**2)*log
(f)**2*b**2*x**2 + 2*f**((a*x**2 + b)/x**2)*log(f)*b*x**4 + 3*f**((a*x**2
+ b)/x**2)*x**6 - 16*int(f**((a*x**2 + b)/x**2)/x**4,x)*log(f)**4*b**4*x)/
(15*x)
```

3.79 $\int f^{a+\frac{b}{x^2}} x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int f^{a+\frac{b}{x^2}} x^2 \, dx = \frac{1}{3} f^{a+\frac{b}{x^2}} x^3 + \frac{2}{3} b f^{a+\frac{b}{x^2}} x \log(f) - \frac{2}{3} b^{3/2} f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \log^{\frac{3}{2}}(f)$$

output
```
1/3*f^(a+b/x^2)*x^3+2/3*b*f^(a+b/x^2)*x*ln(f)-2/3*b^(3/2)*f^a*Pi^(1/2)*erf
i(b^(1/2)*ln(f)^(1/2)/x)*ln(f)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x^2}} x^2 \, dx = \frac{1}{3} f^a \left(-2 b^{3/2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) \log^{\frac{3}{2}}(f) + f^{\frac{b}{x^2}} x \left(x^2 + 2 b \log(f)\right)\right)$$

input
```
Integrate[f^(a + b/x^2)*x^2,x]
```

output
```
(f^a*(-2*b^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x]*Log[f]^(3/2) + f^
(b/x^2)*x*(x^2 + 2*b*Log[f])))/3
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2643, 2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+\frac{b}{x^2}} \, dx$$

$$\Big\downarrow \; 2643$$

$$\frac{2}{3} b \log(f) \int f^{a+\frac{b}{x^2}} dx + \frac{1}{3} x^3 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \; 2635$$

$$\frac{2}{3} b \log(f) \left(2 b \log(f) \int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx + x f^{a+\frac{b}{x^2}}\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \; 2640$$

$$\frac{2}{3} b \log(f) \left(x f^{a+\frac{b}{x^2}} - 2 b \log(f) \int f^{a+\frac{b}{x^2}} d\frac{1}{x}\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \; 2633$$

$$\frac{2}{3} b \log(f) \left(x f^{a+\frac{b}{x^2}} - \sqrt{\pi} \sqrt{b} f^a \sqrt{\log(f)} \text{erfi}\left(\frac{\sqrt{b} \sqrt{\log(f)}}{x}\right)\right) + \frac{1}{3} x^3 f^{a+\frac{b}{x^2}}$$

input
```
Int[f^(a + b/x^2)*x^2,x]
```

output
```
(f^(a + b/x^2)*x^3)/3 + (2*b*(f^(a + b/x^2)*x - Sqrt[b]*f^a*Sqrt[Pi]*Erfi[
(Sqrt[b]*Sqrt[Log[f]])/x]*Sqrt[Log[f]])*Log[f])/3
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{f^{a} f^{\frac{b}{x^{2}}} x^{3}}{3}+\frac{2 f^{a} \ln\left(f\right) b x\, f^{\frac{b}{x^{2}}}}{3}-\frac{2 f^{a} \ln\left(f\right)^{2} b^{2} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln\left(f\right)}}{x}\right)}{3 \sqrt{-b \ln\left(f\right)}}$	67
meijerg	$\frac{f^{a} b \ln\left(f\right)^{\frac{3}{2}} \sqrt{-b}\, \left(-\frac{2 x^{3} \left(\frac{2 b \ln\left(f\right)}{x^{2}}+1\right) {\mathrm e}^{\frac{b \ln\left(f\right)}{x^{2}}}}{3 \left(-b\right)^{\frac{3}{2}} \ln\left(f\right)^{\frac{3}{2}}}+\frac{4 b^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\frac{\sqrt{b}\, \sqrt{\ln\left(f\right)}}{x}\right)}{3 \left(-b\right)^{\frac{3}{2}}}\right)}{2}$	74

input
```
int(f^(a+b/x^2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*f^(b/x^2)*x^3+2/3*f^a*ln(f)*b*x*f^(b/x^2)-2/3*f^a*ln(f)^2*b^2*Pi^(
1/2)/(-b*ln(f))^(1/2)*erf((-b*ln(f))^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int f^{a+\frac{b}{x^2}}x^2\,dx = \frac{2}{3}\,\sqrt{\pi}\sqrt{-b\log\left(f\right)}bf^a\,\mathrm{erf}\left(\frac{\sqrt{-b\log\left(f\right)}}{x}\right)\log\left(f\right) + \frac{1}{3}\left(x^3+2\,bx\log\left(f\right)\right)f^{\frac{ax^2+b}{x^2}}$$

input
```
integrate(f^(a+b/x^2)*x^2,x, algorithm="fricas")
```

output
```
2/3*sqrt(pi)*sqrt(-b*log(f))*b*f^a*erf(sqrt(-b*log(f))/x)*log(f) + 1/3*(x^
3 + 2*b*x*log(f))*f^((a*x^2 + b)/x^2)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^2\,dx = \int f^{a+\frac{b}{x^2}}x^2\,dx$$

input
```
integrate(f**(a+b/x**2)*x**2,x)
```

output
```
Integral(f**(a + b/x**2)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int f^{a+\frac{b}{x^2}}x^2\,dx = \frac{1}{2}\,f^a x^3\left(-\frac{b\log(f)}{x^2}\right)^{\frac{3}{2}}\Gamma\left(-\frac{3}{2},-\frac{b\log(f)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)*x^2,x, algorithm="maxima")
```

output
```
1/2*f^a*x^3*(-b*log(f)/x^2)^(3/2)*gamma(-3/2, -b*log(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}x^2\,dx = \int f^{a+\frac{b}{x^2}}x^2\,dx$$

input
```
integrate(f^(a+b/x^2)*x^2,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int f^{a+\frac{b}{x^2}}x^2\,dx = x^3\left(\frac{f^a\,f^{\frac{b}{x^2}}}{3}+\frac{2\,b\,f^a\,f^{\frac{b}{x^2}}\,\ln(f)}{3\,x^2}\right)-\frac{2\,b^2\,f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(f)}{x\,\sqrt{b\,\ln(f)}}\right)\,\ln(f)^2}{3\,\sqrt{b\,\ln(f)}}$$

input
```
int(f^(a + b/x^2)*x^2,x)
```

output
```
x^3*((f^a*f^(b/x^2))/3 + (2*b*f^a*f^(b/x^2)*log(f))/(3*x^2)) - (2*b^2*f^a*
pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2)))*log(f)^2)/(3*(b*log(f))^(1/
2))
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}}x^2\,dx$$
$$= \frac{-4f^{\frac{a\,x^2+b}{x^2}}\log(f)^2\,b^2+2f^{\frac{a\,x^2+b}{x^2}}\log(f)\,b\,x^2+f^{\frac{a\,x^2+b}{x^2}}x^4-8\left(\int\frac{f^{\frac{a\,x^2+b}{x^2}}}{x^4}dx\right)\log(f)^3\,b^3x}{3x}$$

input
```
int(f^(a+b/x^2)*x^2,x)
```

output
```
( - 4*f**((a*x**2 + b)/x**2)*log(f)**2*b**2 + 2*f**((a*x**2 + b)/x**2)*log
(f)*b*x**2 + f**((a*x**2 + b)/x**2)*x**4 - 8*int(f**((a*x**2 + b)/x**2)/x*
*4,x)*log(f)**3*b**3*x)/(3*x)
```

3.80 $\int f^{a+\frac{b}{x^2}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 49

$$\int f^{a+\frac{b}{x^2}}\,dx=f^{a+\frac{b}{x^2}}x-\sqrt{b}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)\sqrt{\log(f)}$$

output
```
f^(a+b/x^2)*x-b^(1/2)*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)*ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}}\,dx=f^{a+\frac{b}{x^2}}x-\sqrt{b}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)\sqrt{\log(f)}$$

input
```
Integrate[f^(a + b/x^2),x]
```

output
```
f^(a + b/x^2)*x - Sqrt[b]*f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x]*Sqrt
[Log[f]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+\frac{b}{x^2}}\,dx$$

$$\Big\downarrow \text{2635}$$

$$2b\log(f)\int \frac{f^{a+\frac{b}{x^2}}}{x^2}dx+xf^{a+\frac{b}{x^2}}$$

$$\Big\downarrow \text{2640}$$

$$xf^{a+\frac{b}{x^2}}-2b\log(f)\int f^{a+\frac{b}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{2633}$$

$$xf^{a+\frac{b}{x^2}}-\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)$$

input
```
Int[f^(a + b/x^2),x]
```

output
```
f^(a + b/x^2)*x - Sqrt[b]*f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x]*Sqrt
[Log[f]]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
risch	$f^{a} f^{\frac{b}{x^{2}}} x -\frac{f^{a} \ln \left(f \right) b \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln \left(f \right)}}{x}\right)}{\sqrt{-b \ln \left(f \right)}}$	44
meijerg	$-\frac{f^{a} \sqrt{-b}\, \sqrt{\ln \left(f \right)}\, \left(-\frac{2 x \,{\mathrm e}^{\frac{b \ln \left(f \right)}{x^{2}}}}{\sqrt{-b}\, \sqrt{\ln \left(f \right)}}+\frac{2 \sqrt{b}\, \sqrt{\pi}\, \operatorname{erfi}\left(\frac{\sqrt{b}\, \sqrt{\ln \left(f \right)}}{x}\right)}{\sqrt{-b}}\right)}{2}$	61

input
```
int(f^(a+b/x^2),x,method=_RETURNVERBOSE)
```

output
```
f^a*f^(b/x^2)*x-f^a*ln(f)*b*Pi^(1/2)/(-b*ln(f))^(1/2)*erf((-b*ln(f))^(1/2)
/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int f^{a+\frac{b}{x^2}}\,dx=\sqrt{\pi}\sqrt{-b\log\left(f\right)}f^{a}\operatorname{erf}\left(\frac{\sqrt{-b\log\left(f\right)}}{x}\right)+f^{\frac{ax^{2}+b}{x^{2}}}x$$

input
```
integrate(f^(a+b/x^2),x, algorithm="fricas")
```

output
```
sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))/x) + f^((a*x^2 + b)/x^2)*
x
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}\,dx=\int f^{a+\frac{b}{x^2}}\,dx$$

input
```
integrate(f**(a+b/x**2),x)
```

output
```
Integral(f**(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int f^{a+\frac{b}{x^2}}\,dx=\frac{1}{2}\,f^{a}x\sqrt{-\frac{b\log\left(f\right)}{x^2}}\Gamma\left(-\frac{1}{2},-\frac{b\log\left(f\right)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2),x, algorithm="maxima")
```

output
```
1/2*f^a*x*sqrt(-b*log(f)/x^2)*gamma(-1/2, -b*log(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}}\,dx=\int f^{a+\frac{b}{x^2}}\,dx$$

input
```
integrate(f^(a+b/x^2),x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int f^{a+\frac{b}{x^2}}\,dx = f^a\,f^{\frac{b}{x^2}}\,x - \frac{b\,f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(f)}{x\,\sqrt{b\,\ln(f)}}\right)\,\ln\left(f\right)}{\sqrt{b\,\ln\left(f\right)}}$$

input
```
int(f^(a + b/x^2),x)
```

output
```
f^a*f^(b/x^2)*x - (b*f^a*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2)))*lo
g(f))/(b*log(f))^(1/2)
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}}\,dx = \frac{-2f^{\frac{a\,x^2+b}{x^2}}\log(f)\,b + f^{\frac{a\,x^2+b}{x^2}}x^2 - 4\left(\int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^4}dx\right)\log(f)^2\,b^2x}{x}$$

input
```
int(f^(a+b/x^2),x)
```

output
```
( - 2*f**((a*x**2 + b)/x**2)*log(f)*b + f**((a*x**2 + b)/x**2)*x**2 - 4*in
t(f**((a*x**2 + b)/x**2)/x**4,x)*log(f)**2*b**2*x)/x
```

3.81 $\int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx = -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

output
```
-1/2*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)/b^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx = -\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b/x^2)/x^2,x]
```

output
```
-1/2*(f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(Sqrt[b]*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx$$

$$\Big\downarrow \text{2640}$$

$$-\int f^{a+\frac{b}{x^2}} d\frac{1}{x}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\sqrt{\pi} f^a \text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{2\sqrt{b}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b/x^2)/x^2,x]
```

output
```
-1/2*(f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(Sqrt[b]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
meijerg	$-\frac{f^a\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{b}\,\sqrt{\ln(f)}}{x}\right)}{2\sqrt{b}\,\sqrt{\ln(f)}}$	28
risch	$-\frac{f^a\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b\ln(f)}}{x}\right)}{2\sqrt{-b\ln(f)}}$	28

input
```
int(f^(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)/b^(1/2)/ln(f)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx = \frac{\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)}{2\,b\log(f)}$$

input
```
integrate(f^(a+b/x^2)/x^2,x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))/x)/(b*log(f))
```

Sympy [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx$$

input
```
integrate(f**(a+b/x**2)/x**2,x)
```

output
```
Integral(f**(a + b/x**2)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx = -\frac{\sqrt{\pi}\,f^a\left(\operatorname{erf}\left(\sqrt{-\frac{b\log(f)}{x^2}}\right)-1\right)}{2\,x\sqrt{-\frac{b\log(f)}{x^2}}}$$

input
```
integrate(f^(a+b/x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(pi)*f^a*(erf(sqrt(-b*log(f)/x^2)) - 1)/(x*sqrt(-b*log(f)/x^2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx = -\frac{f^a\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{b\,\ln(f)}{x\,\sqrt{b\,\ln(f)}}\right)}{2\,\sqrt{b\,\ln\left(f\right)}}$$

input
```
int(f^(a + b/x^2)/x^2,x)
```

output
```
-(f^a*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2))))/(2*(b*log(f))^(1/2))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^2}\,dx = \frac{-f^{\frac{a\,x^2+b}{x^2}}-2\left(\int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^4}dx\right)\log(f)\,bx}{x}$$

input
```
int(f^(a+b/x^2)/x^2,x)
```

output
```
( - f**((a*x**2 + b)/x**2) - 2*int(f**((a*x**2 + b)/x**2)/x**4,x)*log(f)*b
*x)/x
```

3.82 $\int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx = \frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2} \log^{\frac{3}{2}}(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)}$$

output
```
1/4*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)/b^(3/2)/ln(f)^(3/2)-1/2*f^(a+
b/x^2)/b/x/ln(f)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx = \frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2} \log^{\frac{3}{2}}(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx \log(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^4,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(4*b^(3/2)*Log[f]^(3/2)) - f
^(a + b/x^2)/(2*b*x*Log[f])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^2}dx}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}$$

$$\Big\downarrow \text{2640}$$

$$\frac{\int f^{a+\frac{b}{x^2}}d\frac{1}{x}}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}f^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^4,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(4*b^(3/2)*Log[f]^(3/2)) - f
^(a + b/x^2)/(2*b*x*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
risch	$-\frac{f^{a} f^{\frac{b}{x^{2}}}}{2 x b \ln\left(f\right)}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln\left(f\right)}}{x}\right)}{4 \ln\left(f\right) b \sqrt{-b \ln\left(f\right)}}$	58
meijerg	$-\frac{f^{a} \sqrt{-b}\, \left(\frac{\left(-b\right)^{\frac{3}{2}} \sqrt{\ln\left(f\right)}\, \mathrm{e}^{\frac{b \ln\left(f\right)}{x^{2}}}}{x b}-\frac{\left(-b\right)^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\frac{\sqrt{b}\, \sqrt{\ln\left(f\right)}}{x}\right)}{2 b^{\frac{3}{2}}}\right)}{2 \ln\left(f\right)^{\frac{3}{2}} b^{2}}$	68

input
```
int(f^(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*f^(b/x^2)/x/b/ln(f)+1/4*f^a/ln(f)/b*Pi^(1/2)/(-b*ln(f))^(1/2)*erf
((-b*ln(f))^(1/2)/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4}\,dx = -\frac{\sqrt{\pi}\sqrt{-b\log(f)}f^a x\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right) + 2\,bf^{\frac{ax^2+b}{x^2}}\log(f)}{4\,b^2x\log(f)^2}$$

input
```
integrate(f^(a+b/x^2)/x^4,x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-b*log(f))*f^a*x*erf(sqrt(-b*log(f))/x) + 2*b*f^((a*x^
2 + b)/x^2)*log(f))/(b^2*x*log(f)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**2)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.44

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4}\,dx = \frac{f^a\Gamma\left(\frac{3}{2}, -\frac{b\log(f)}{x^2}\right)}{2\,x^3\left(-\frac{b\log(f)}{x^2}\right)^{\frac{3}{2}}}$$

input
```
integrate(f^(a+b/x^2)/x^4,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(3/2, -b*log(f)/x^2)/(x^3*(-b*log(f)/x^2)^(3/2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} \, dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^4} \, dx$$

input
```
integrate(f^(a+b/x^2)/x^4,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} \, dx = \frac{f^a \sqrt{\pi} \operatorname{erfi}\left(\frac{b \ln(f)}{x \sqrt{b \ln(f)}}\right)}{4 b \ln(f) \sqrt{b \ln(f)}} - \frac{f^a f^{\frac{b}{x^2}}}{2 b x \ln(f)}$$

input
```
int(f^(a + b/x^2)/x^4,x)
```

output
```
(f^a*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2))))/(4*b*log(f)*(b*log(f)
)^(1/2)) - (f^a*f^(b/x^2))/(2*b*x*log(f))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^4} \, dx = \int \frac{f^{\frac{a x^2+b}{x^2}}}{x^4} dx$$

input
```
int(f^(a+b/x^2)/x^4,x)
```

output
```
int(f**((a*x**2 + b)/x**2)/x**4,x)
```

3.83 $\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = -\frac{3f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{8b^{5/2}\log^{\frac{5}{2}}(f)} + \frac{3f^{a+\frac{b}{x^2}}}{4b^2x\log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

output
```
-3/8*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)/b^(5/2)/ln(f)^(5/2)+3/4*f^(a
+b/x^2)/b^2/x/ln(f)^2-1/2*f^(a+b/x^2)/b/x^3/ln(f)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = -\frac{3f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{8b^{5/2}\log^{\frac{5}{2}}(f)} + \frac{f^{a+\frac{b}{x^2}}(3x^2-2b\log(f))}{4b^2x^3\log^2(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^6,x]
```

output
```
(-3*f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(8*b^(5/2)*Log[f]^(5/2))
+ (f^(a + b/x^2)*(3*x^2 - 2*b*Log[f]))/(4*b^2*x^3*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2641, 2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\int \frac{f^{a+\frac{b}{x^2}}}{x^4}dx}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\left(-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^2}dx}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

$$\Big\downarrow \text{2640}$$

$$-\frac{3\left(\frac{\int f^{a+\frac{b}{x^2}}d\frac{1}{x}}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{3\left(\frac{\sqrt{\pi}f^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^6,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*x^3*Log[f]) - (3*((f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[L
og[f]])/x])/(4*b^(3/2)*Log[f]^(3/2)) - f^(a + b/x^2)/(2*b*x*Log[f])))/(2*b
*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

method	result	size
meijerg	$\frac{f^{a}\sqrt{-b}\left(-\frac{(-b)^{\frac{5}{2}}\sqrt{\ln(f)}\left(-\frac{10b\ln(f)}{x^{2}}+15\right){\mathrm e}^{\frac{b\ln(f)}{x^{2}}}}{10x\,b^{2}}+\frac{3(-b)^{\frac{5}{2}}\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\sqrt{b}\,\sqrt{\ln(f)}}{x}\right)}{4b^{\frac{5}{2}}}\right)}{2\ln(f)^{\frac{5}{2}}b^{3}}$	79
risch	$-\frac{f^{a}f^{\frac{b}{x^{2}}}}{2x^{3}b\ln(f)}+\frac{3f^{a}f^{\frac{b}{x^{2}}}}{4\ln(f)^{2}b^{2}x}-\frac{3f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b\ln(f)}}{x}\right)}{8\ln(f)^{2}b^{2}\sqrt{-b\ln(f)}}$	80

input
```
int(f^(a+b/x^2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/ln(f)^(5/2)/b^3*(-b)^(1/2)*(-1/10/x*(-b)^(5/2)*ln(f)^(1/2)*(-10*b*
ln(f)/x^2+15)/b^2*exp(b*ln(f)/x^2)+3/4*(-b)^(5/2)/b^(5/2)*Pi^(1/2)*erfi(b^
(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.88

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = \frac{3\sqrt{\pi}\sqrt{-b\log(f)}f^a x^3 \operatorname{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right) + 2\left(3\,bx^2\log(f) - 2\,b^2\log(f)^2\right)f^{\frac{ax^2+b}{x^2}}}{8\,b^3x^3\log(f)^3}$$

input
```
integrate(f^(a+b/x^2)/x^6,x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(pi)*sqrt(-b*log(f))*f^a*x^3*erf(sqrt(-b*log(f))/x) + 2*(3*b*x^
2*log(f) - 2*b^2*log(f)^2)*f^((a*x^2 + b)/x^2))/(b^3*x^3*log(f)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**2)/x**6,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.33

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = \frac{f^a\Gamma\left(\frac{5}{2}, -\frac{b\log(f)}{x^2}\right)}{2\,x^5\left(-\frac{b\log(f)}{x^2}\right)^{\frac{5}{2}}}$$

input
```
integrate(f^(a+b/x^2)/x^6,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(5/2, -b*log(f)/x^2)/(x^5*(-b*log(f)/x^2)^(5/2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^6,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = -\frac{f^a\left(3\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(f)}{x\,\sqrt{b\,\ln(f)}}\right)-\frac{6\,f^{\frac{b}{x^2}}\,\sqrt{b\,\ln(f)}}{x}\right)}{8\,b^2\ln\left(f\right)^2\sqrt{b\,\ln\left(f\right)}}-\frac{f^a\,f^{\frac{b}{x^2}}}{2\,b\,x^3\,\ln\left(f\right)}$$

input
```
int(f^(a + b/x^2)/x^6,x)
```

output
```
- (f^a*(3*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2))) - (6*f^(b/x^2)*(b
*log(f))^(1/2))/x))/(8*b^2*log(f)^2*(b*log(f))^(1/2)) - (f^a*f^(b/x^2))/(2
*b*x^3*log(f))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^6}\,dx = \int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^6}dx$$

input
```
int(f^(a+b/x^2)/x^6,x)
```

output
```
int(f**((a*x**2 + b)/x**2)/x**6,x)
```

3.84 $\int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx = \frac{15f^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{16b^{7/2}\log^{\frac{7}{2}}(f)} - \frac{15f^{a+\frac{b}{x^2}}}{8b^3x\log^3(f)} + \frac{5f^{a+\frac{b}{x^2}}}{4b^2x^3\log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}$$

output
```
15/16*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)/b^(7/2)/ln(f)^(7/2)-15/8*f^
(a+b/x^2)/b^3/x/ln(f)^3+5/4*f^(a+b/x^2)/b^2/x^3/ln(f)^2-1/2*f^(a+b/x^2)/b/
x^5/ln(f)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8} dx = \frac{15f^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{16b^{7/2}\log^{\frac{7}{2}}(f)} - \frac{f^{a+\frac{b}{x^2}}\left(15x^4 - 10bx^2\log(f) + 4b^2\log^2(f)\right)}{8b^3x^5\log^3(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^8,x]
```

output
```
(15*f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(16*b^(7/2)*Log[f]^(7/2))
 - (f^(a + b/x^2)*(15*x^4 - 10*b*x^2*Log[f] + 4*b^2*Log[f]^2))/(8*b^3*x^5*
Log[f]^3)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {2641, 2641, 2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{5 \int \frac{f^{a+\frac{b}{x^2}}}{x^6} dx}{2 b \log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^5 \log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{5 \left(-\frac{3 \int \frac{f^{a+\frac{b}{x^2}}}{x^4} dx}{2 b \log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^3 \log(f)}\right)}{2 b \log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^5 \log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{5 \left(-\frac{3 \left(-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^2} dx}{2 b \log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x \log(f)}\right)}{2 b \log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^3 \log(f)}\right)}{2 b \log(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^5 \log(f)}$$

$$\Big\downarrow \text{ 2640}$$

$$-\frac{5\left(-\frac{3\left(\frac{\int f^{a+\frac{b}{x^2}}d\frac{1}{x}}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{5\left(-\frac{3\left(\frac{\sqrt{\pi}f^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^8,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*x^5*Log[f]) - (5*(-1/2*f^(a + b/x^2)/(b*x^3*Log[f])
- (3*((f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x])/(4*b^(3/2)*Log[f]^(3/2
)) - f^(a + b/x^2)/(2*b*x*Log[f])))/(2*b*Log[f])))/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result	size
meijerg	$-\frac{f^{a}\sqrt{-b}\left(\frac{(-b)^{\frac{7}{2}}\sqrt{\ln(f)}\left(\frac{28b^{2}\ln(f)^{2}}{x^{4}}-\frac{70b\ln(f)}{x^{2}}+105\right)e^{\frac{b\ln(f)}{x^{2}}}}{28x\,b^{3}}-\frac{15(-b)^{\frac{7}{2}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\,\sqrt{\ln(f)}}{x}\right)}{8b^{\frac{7}{2}}}\right)}{2\ln(f)^{\frac{7}{2}}b^{4}}$	91
risch	$-\frac{f^{a}f^{\frac{b}{x^{2}}}}{2x^{5}b\ln(f)}+\frac{5f^{a}f^{\frac{b}{x^{2}}}}{4\ln(f)^{2}b^{2}x^{3}}-\frac{15f^{a}f^{\frac{b}{x^{2}}}}{8\ln(f)^{3}b^{3}x}+\frac{15f^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b\ln(f)}}{x}\right)}{16\ln(f)^{3}b^{3}\sqrt{-b\ln(f)}}$	102

input
```
int(f^(a+b/x^2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/ln(f)^(7/2)/b^4*(-b)^(1/2)*(1/28/x*(-b)^(7/2)*ln(f)^(1/2)*(28*b^2
*ln(f)^2/x^4-70*b*ln(f)/x^2+105)/b^3*exp(b*ln(f)/x^2)-15/8*(-b)^(7/2)/b^(7
/2)*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx = -\frac{15\sqrt{\pi}\sqrt{-b\log(f)}f^{a}x^{5}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)+2\left(15\,bx^{4}\log(f)-10\,b^{2}x^{2}\log(f)^{2}+4\,b^{3}\log(f)^{3}\right)f^{\frac{ax^{2}+b}{x^{2}}}}{16\,b^{4}x^{5}\log(f)^{4}}$$

input
```
integrate(f^(a+b/x^2)/x^8,x, algorithm="fricas")
```

output
```
-1/16*(15*sqrt(pi)*sqrt(-b*log(f))*f^a*x^5*erf(sqrt(-b*log(f))/x) + 2*(15*
b*x^4*log(f) - 10*b^2*x^2*log(f)^2 + 4*b^3*log(f)^3)*f^((a*x^2 + b)/x^2))/
(b^4*x^5*log(f)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**2)/x**8,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.26

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx = \frac{f^a\Gamma\left(\frac{7}{2}, -\frac{b\log(f)}{x^2}\right)}{2\,x^7\left(-\frac{b\log(f)}{x^2}\right)^{\frac{7}{2}}}$$

input
```
integrate(f^(a+b/x^2)/x^8,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(7/2, -b*log(f)/x^2)/(x^7*(-b*log(f)/x^2)^(7/2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^8,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx = \frac{5\,f^a\,f^{\frac{b}{x^2}}}{4\,b^2\,x^3\ln\left(f\right)^2} - \frac{f^a\,f^{\frac{b}{x^2}}}{2\,b\,x^5\,\ln\left(f\right)} - \frac{15\,f^a\,f^{\frac{b}{x^2}}}{8\,b^3\,x\ln\left(f\right)^3} + \frac{15\,f^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(f)}{x\,\sqrt{b\,\ln(f)}}\right)}{16\,b^3\ln\left(f\right)^3\,\sqrt{b\,\ln\left(f\right)}}$$

input
```
int(f^(a + b/x^2)/x^8,x)
```

output
```
(5*f^a*f^(b/x^2))/(4*b^2*x^3*log(f)^2) - (f^a*f^(b/x^2))/(2*b*x^5*log(f))
- (15*f^a*f^(b/x^2))/(8*b^3*x*log(f)^3) + (15*f^a*pi^(1/2)*erfi((b*log(f))
/(x*(b*log(f))^(1/2))))/(16*b^3*log(f)^3*(b*log(f))^(1/2))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^8}\,dx = \int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^8}dx$$

input
```
int(f^(a+b/x^2)/x^8,x)
```

output
```
int(f**((a*x**2 + b)/x**2)/x**8,x)
```

3.85 $\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 132

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = -\frac{105 f^a\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{32 b^{9/2}\log^{\frac{9}{2}}(f)} + \frac{105 f^{a+\frac{b}{x^2}}}{16 b^4 x\log^4(f)} - \frac{35 f^{a+\frac{b}{x^2}}}{8 b^3 x^3\log^3(f)} + \frac{7 f^{a+\frac{b}{x^2}}}{4 b^2 x^5\log^2(f)} - \frac{f^{a+\frac{b}{x^2}}}{2 b x^7\log(f)}$$

output
```
-105/32*f^a*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x)/b^(9/2)/ln(f)^(9/2)+105/1
6*f^(a+b/x^2)/b^4/x/ln(f)^4-35/8*f^(a+b/x^2)/b^3/x^3/ln(f)^3+7/4*f^(a+b/x^
2)/b^2/x^5/ln(f)^2-1/2*f^(a+b/x^2)/b/x^7/ln(f)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = \frac{f^a\left(-105\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right) + \frac{2\sqrt{b} f^{\frac{b}{x^2}}\sqrt{\log(f)}\left(105x^6 - 70bx^4\log(f) + 28b^2x^2\log^2(f) - 8b^3\log^3(f)\right)}{x^7}\right)}{32 b^{9/2}\log^{\frac{9}{2}}(f)}$$

input
```
Integrate[f^(a + b/x^2)/x^10,x]
```

output
```
(f^a*(-105*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[f]])/x] + (2*Sqrt[b]*f^(b/x^2)*
Sqrt[Log[f]]*(105*x^6 - 70*b*x^4*Log[f] + 28*b^2*x^2*Log[f]^2 - 8*b^3*Log[
f]^3))/x^7))/(32*b^(9/2)*Log[f]^(9/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {2641, 2641, 2641, 2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{7 \int \frac{f^{a+\frac{b}{x^2}}}{x^8}dx}{2 b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^7 \log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{7 \left(-\frac{5 \int \frac{f^{a+\frac{b}{x^2}}}{x^6}dx}{2 b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^5 \log(f)}\right)}{2 b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^7 \log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{7 \left(-\frac{5 \left(-\frac{3 \int \frac{f^{a+\frac{b}{x^2}}}{x^4}dx}{2 b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^3 \log(f)}\right)}{2 b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^5 \log(f)}\right)}{2 b \log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2 b x^7 \log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{7\left(-\frac{5\left(-\frac{3\left(-\frac{\int \frac{f^{a+\frac{b}{x^2}}}{x^2}dx}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^7\log(f)}$$

$$\Big\downarrow \ 2640$$

$$-\frac{7\left(-\frac{5\left(-\frac{3\left(\frac{\int f^{a+\frac{b}{x^2}}d\frac{1}{x}}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^7\log(f)}$$

$$\Big\downarrow \ 2633$$

$$-\frac{7\left(-\frac{5\left(-\frac{3\left(\frac{\sqrt{\pi}f^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(f)}}{x}\right)}{4b^{3/2}\log^{\frac{3}{2}}(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^3\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^5\log(f)}\right)}{2b\log(f)}-\frac{f^{a+\frac{b}{x^2}}}{2bx^7\log(f)}$$

input
```
Int[f^(a + b/x^2)/x^10,x]
```

output
```
-1/2*f^(a + b/x^2)/(b*x^7*Log[f]) - (7*(-1/2*f^(a + b/x^2)/(b*x^5*Log[f])
- (5*(-1/2*f^(a + b/x^2)/(b*x^3*Log[f]) - (3*((f^a*Sqrt[Pi]*Erfi[(Sqrt[b]*
Sqrt[Log[f]])/x])/(4*b^(3/2)*Log[f]^(3/2)) - f^(a + b/x^2)/(2*b*x*Log[f]))
)/(2*b*Log[f])))/(2*b*Log[f])))/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.78

method	result	size
meijerg	$\frac{f^{a} \sqrt{-b}\, \left(-\frac{\left(-b \right)^{\frac{9}{2}} \sqrt{\ln \left(f \right)}\, \left(-\frac{72 b^{3} \ln \left(f \right)^{3}}{x^{6}}+\frac{252 b^{2} \ln \left(f \right)^{2}}{x^{4}}-\frac{630 b \ln \left(f \right)}{x^{2}}+945\right) {\mathrm e}^{\frac{b \ln \left(f \right)}{x^{2}}}}{72 x \,b^{4}}+\frac{105 \left(-b \right)^{\frac{9}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\frac{\sqrt{b}\, \sqrt{\ln \left(f \right)}}{x}\right)}{16 b^{\frac{9}{2}}}\right)}{2 b^{5} \ln \left(f \right)^{\frac{9}{2}}}$	103
risch	$-\frac{f^{a} f^{\frac{b}{x^{2}}}}{2 x^{7} b \ln \left(f \right)}+\frac{7 f^{a} f^{\frac{b}{x^{2}}}}{4 \ln \left(f \right)^{2} b^{2} x^{5}}-\frac{35 f^{a} f^{\frac{b}{x^{2}}}}{8 \ln \left(f \right)^{3} b^{3} x^{3}}+\frac{105 f^{a} f^{\frac{b}{x^{2}}}}{16 \ln \left(f \right)^{4} b^{4} x}-\frac{105 f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln \left(f \right)}}{x}\right)}{32 \ln \left(f \right)^{4} b^{4} \sqrt{-b \ln \left(f \right)}}$	124

input
```
int(f^(a+b/x^2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/b^5/ln(f)^(9/2)*(-b)^(1/2)*(-1/72/x*(-b)^(9/2)*ln(f)^(1/2)*(-72*b^
3*ln(f)^3/x^6+252*b^2*ln(f)^2/x^4-630*b*ln(f)/x^2+945)/b^4*exp(b*ln(f)/x^2
)+105/16*(-b)^(9/2)/b^(9/2)*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx$$
$$= \frac{105\,\sqrt{\pi}\sqrt{-b\log\left(f\right)}f^a x^7\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right) + 2\left(105\,bx^6\log\left(f\right) - 70\,b^2x^4\log\left(f\right)^2 + 28\,b^3x^2\log\left(f\right)^3 - 8\,b^4\,\mathrm{l}}{32\,b^5x^7\log\left(f\right)^5}$$

input
```
integrate(f^(a+b/x^2)/x^10,x, algorithm="fricas")
```

output
```
1/32*(105*sqrt(pi)*sqrt(-b*log(f))*f^a*x^7*erf(sqrt(-b*log(f))/x) + 2*(105
*b*x^6*log(f) - 70*b^2*x^4*log(f)^2 + 28*b^3*x^2*log(f)^3 - 8*b^4*log(f)^4
)*f^((a*x^2 + b)/x^2))/(b^5*x^7*log(f)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**2)/x**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.21

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = \frac{f^a \Gamma\left(\frac{9}{2}, -\frac{b\log(f)}{x^2}\right)}{2\,x^9 \left(-\frac{b\log(f)}{x^2}\right)^{\frac{9}{2}}}$$

input
```
integrate(f^(a+b/x^2)/x^10,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(9/2, -b*log(f)/x^2)/(x^9*(-b*log(f)/x^2)^(9/2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^10,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = -\frac{\frac{f^a \left(105\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(f)}{x\,\sqrt{b\,\ln(f)}}\right) - \frac{210\,f\,^{\frac{b}{x^2}}\,\sqrt{b\,\ln(f)}}{x}\right)}{32\,\sqrt{b\,\ln(f)}} - \frac{7\,b^2\,f^a\,f^{\frac{b}{x^2}}\,\ln(f)^2}{4\,x^5} + \frac{b^3\,f^a\,f^{\frac{b}{x^2}}\,\ln(f)^3}{2\,x^7} + \frac{35\,b\,f^a\,f^{\frac{b}{x^2}}\,\ln(f)}{8\,x^3}}{b^4\,\ln\left(f\right)^4}$$

input
```
int(f^(a + b/x^2)/x^10,x)
```

output
```
-((f^a*(105*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2))) - (210*f^(b/x^2
)*(b*log(f))^(1/2))/x))/(32*(b*log(f))^(1/2)) - (7*b^2*f^a*f^(b/x^2)*log(f
)^2)/(4*x^5) + (b^3*f^a*f^(b/x^2)*log(f)^3)/(2*x^7) + (35*b*f^a*f^(b/x^2)*
log(f))/(8*x^3))/(b^4*log(f)^4)
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{10}}\,dx = \int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^{10}}dx$$

input
```
int(f^(a+b/x^2)/x^10,x)
```

output
```
int(f**((a*x**2 + b)/x**2)/x**10,x)
```

3.86 $\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \frac{f^a\Gamma\left(\frac{11}{2},-\frac{b\log(f)}{x^2}\right)}{2x^{11}\left(-\frac{b\log(f)}{x^2}\right)^{11/2}}$$

output
```
1/2*f^a*(1048576/61836869254970658257624840625*GAMMA(51/2,-b*ln(f)/x^2)-10
48576/61836869254970658257624840625*(-b*ln(f)/x^2)^(49/2)*exp(b*ln(f)/x^2)
-524288/1261976923570829760359690625*(-b*ln(f)/x^2)^(47/2)*exp(b*ln(f)/x^2
)-262144/26850572841932548092759375*(-b*ln(f)/x^2)^(45/2)*exp(b*ln(f)/x^2)
-131072/596679396487389957616875*(-b*ln(f)/x^2)^(43/2)*exp(b*ln(f)/x^2)-65
536/13876265034590464130625*(-b*ln(f)/x^2)^(41/2)*exp(b*ln(f)/x^2)-32768/3
38445488648547905625*(-b*ln(f)/x^2)^(39/2)*exp(b*ln(f)/x^2)-16384/86780894
52526869375*(-b*ln(f)/x^2)^(37/2)*exp(b*ln(f)/x^2)-8192/234542958176401875
*(-b*ln(f)/x^2)^(35/2)*exp(b*ln(f)/x^2)-4096/6701227376468625*(-b*ln(f)/x^
2)^(33/2)*exp(b*ln(f)/x^2)-2048/203067496256625*(-b*ln(f)/x^2)^(31/2)*exp(
b*ln(f)/x^2)-1024/6550564395375*(-b*ln(f)/x^2)^(29/2)*exp(b*ln(f)/x^2)-512
/225881530875*(-b*ln(f)/x^2)^(27/2)*exp(b*ln(f)/x^2)-256/8365982625*(-b*ln
(f)/x^2)^(25/2)*exp(b*ln(f)/x^2)-128/334639305*(-b*ln(f)/x^2)^(23/2)*exp(b
*ln(f)/x^2)-64/14549535*(-b*ln(f)/x^2)^(21/2)*exp(b*ln(f)/x^2)-32/692835*(
-b*ln(f)/x^2)^(19/2)*exp(b*ln(f)/x^2)-16/36465*(-b*ln(f)/x^2)^(17/2)*exp(b
*ln(f)/x^2)-8/2145*(-b*ln(f)/x^2)^(15/2)*exp(b*ln(f)/x^2)-4/143*(-b*ln(f)/
x^2)^(13/2)*exp(b*ln(f)/x^2)-2/11*(-b*ln(f)/x^2)^(11/2)*exp(b*ln(f)/x^2))/
x^11/(-b*ln(f)/x^2)^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \frac{f^a\Gamma\left(\frac{11}{2},-\frac{b\log(f)}{x^2}\right)}{2x^{11}\left(-\frac{b\log(f)}{x^2}\right)^{11/2}}$$

input
```
Integrate[f^(a + b/x^2)/x^12,x]
```

output
```
(f^a*Gamma[11/2, -((b*Log[f])/x^2)])/(2*x^11*(-((b*Log[f])/x^2))^(11/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{f^a \Gamma\left(\frac{11}{2},-\frac{b\log(f)}{x^2}\right)}{2x^{11}\left(-\frac{b\log(f)}{x^2}\right)^{11/2}}$$

input
```
Int[f^(a + b/x^2)/x^12,x]
```

output
```
(f^a*Gamma[11/2, -((b*Log[f])/x^2)])/(2*x^11*(-((b*Log[f])/x^2))^(11/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.38

method	result
meijerg	$-\frac{f^{a}\sqrt{-b}\left(\frac{(-b)^{\frac{11}{2}}\sqrt{\ln(f)}\left(\frac{176b^{4}\ln(f)^{4}}{x^{8}}-\frac{792b^{3}\ln(f)^{3}}{x^{6}}+\frac{2772b^{2}\ln(f)^{2}}{x^{4}}-\frac{6930b\ln(f)}{x^{2}}+10395\right)\mathrm{e}^{\frac{b\ln(f)}{x^{2}}}}{176x\,b^{5}}-\frac{945(-b)^{\frac{11}{2}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\,\sqrt{\ln(f)}}{x}\right)}{32b^{\frac{11}{2}}}\right)}{2b^{6}\ln(f)^{\frac{11}{2}}}$
risch	$-\frac{f^{a}f^{\frac{b}{x^{2}}}}{2x^{9}b\ln(f)}+\frac{9f^{a}f^{\frac{b}{x^{2}}}}{4\ln(f)^{2}b^{2}x^{7}}-\frac{63f^{a}f^{\frac{b}{x^{2}}}}{8\ln(f)^{3}b^{3}x^{5}}+\frac{315f^{a}f^{\frac{b}{x^{2}}}}{16\ln(f)^{4}b^{4}x^{3}}-\frac{945f^{a}f^{\frac{b}{x^{2}}}}{32\ln(f)^{5}b^{5}x}+\frac{945f^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b\ln(f)}}{x}\right)}{64\ln(f)^{5}b^{5}\sqrt{-b\ln(f)}}$

input
```
int(f^(a+b/x^2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a/b^6/ln(f)^(11/2)*(-b)^(1/2)*(1/176/x*(-b)^(11/2)*ln(f)^(1/2)*(176
*b^4*ln(f)^4/x^8-792*b^3*ln(f)^3/x^6+2772*b^2*ln(f)^2/x^4-6930*b*ln(f)/x^2
+10395)/b^5*exp(b*ln(f)/x^2)-945/32*(-b)^(11/2)/b^(11/2)*Pi^(1/2)*erfi(b^(
1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.29

$$\int \frac{f^{a+\frac{b}{x^{2}}}}{x^{12}}\,dx = -\frac{945\sqrt{\pi}\sqrt{-b\log(f)}f^{a}x^{9}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)+2\left(945\,bx^{8}\log(f)-630\,b^{2}x^{6}\log(f)^{2}+252\,b^{3}x^{4}\log(f)^{3}-\right.}{64\,b^{6}x^{9}\log(f)^{6}}$$

input
```
integrate(f^(a+b/x^2)/x^12,x, algorithm="fricas")
```

output
```
-1/64*(945*sqrt(pi)*sqrt(-b*log(f))*f^a*x^9*erf(sqrt(-b*log(f))/x) + 2*(94
5*b*x^8*log(f) - 630*b^2*x^6*log(f)^2 + 252*b^3*x^4*log(f)^3 - 72*b^4*x^2*
log(f)^4 + 16*b^5*log(f)^5)*f^((a*x^2 + b)/x^2))/(b^6*x^9*log(f)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**2)/x**12,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \frac{f^a\Gamma\left(\frac{11}{2}, -\frac{b\log(f)}{x^2}\right)}{2\,x^{11}\left(-\frac{b\log(f)}{x^2}\right)^{\frac{11}{2}}}$$

input
```
integrate(f^(a+b/x^2)/x^12,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(11/2, -b*log(f)/x^2)/(x^11*(-b*log(f)/x^2)^(11/2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^12,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^12, x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 4.18

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \frac{\frac{f^a\left(945\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\ln(f)}{x\sqrt{b\ln(f)}}\right)-\frac{1890\,f^{\frac{b}{x^2}}\sqrt{b\ln(f)}}{x}\right)}{64\sqrt{b\ln(f)}}-\frac{63\,b^2\,f^a\,f^{\frac{b}{x^2}}\ln(f)^2}{8\,x^5}+\frac{9\,b^3\,f^a\,f^{\frac{b}{x^2}}\ln(f)^3}{4\,x^7}-\frac{b^4\,f^a\,f^{\frac{b}{x^2}}\ln(f)^4}{2\,x^9}+\frac{315\,b\,f^a\,f^{\frac{b}{x^2}}\ln(f}{16\,x^3}}{b^5\ln(f)^5}$$

input
```
int(f^(a + b/x^2)/x^12,x)
```

output
```
((f^a*(945*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2))) - (1890*f^(b/x^2
)*(b*log(f))^(1/2))/x))/(64*(b*log(f))^(1/2)) - (63*b^2*f^a*f^(b/x^2)*log(
f)^2)/(8*x^5) + (9*b^3*f^a*f^(b/x^2)*log(f)^3)/(4*x^7) - (b^4*f^a*f^(b/x^2
)*log(f)^4)/(2*x^9) + (315*b*f^a*f^(b/x^2)*log(f))/(16*x^3))/(b^5*log(f)^5
)
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{12}}\,dx = \int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^{12}}dx$$

input
```
int(f^(a+b/x^2)/x^12,x)
```

output
```
int(f**((a*x**2 + b)/x**2)/x**12,x)
```

3.87 $\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx=\frac{f^a\Gamma\left(\frac{13}{2},-\frac{b\log(f)}{x^2}\right)}{2x^{13}\left(-\frac{b\log(f)}{x^2}\right)^{13/2}}$$

output
```
1/2*f^a*(524288/5621533568633696205238621875*GAMMA(51/2,-b*ln(f)/x^2)-5242
88/5621533568633696205238621875*(-b*ln(f)/x^2)^(49/2)*exp(b*ln(f)/x^2)-262
144/114725174870075432759971875*(-b*ln(f)/x^2)^(47/2)*exp(b*ln(f)/x^2)-131
072/2440961167448413462978125*(-b*ln(f)/x^2)^(45/2)*exp(b*ln(f)/x^2)-65536
/54243581498853632510625*(-b*ln(f)/x^2)^(43/2)*exp(b*ln(f)/x^2)-32768/1261
478639508224011875*(-b*ln(f)/x^2)^(41/2)*exp(b*ln(f)/x^2)-16384/3076777169
5322536875*(-b*ln(f)/x^2)^(39/2)*exp(b*ln(f)/x^2)-8192/788917222956988125*
(-b*ln(f)/x^2)^(37/2)*exp(b*ln(f)/x^2)-4096/21322087106945625*(-b*ln(f)/x^
2)^(35/2)*exp(b*ln(f)/x^2)-2048/609202488769875*(-b*ln(f)/x^2)^(33/2)*exp(
b*ln(f)/x^2)-1024/18460681477875*(-b*ln(f)/x^2)^(31/2)*exp(b*ln(f)/x^2)-51
2/595505854125*(-b*ln(f)/x^2)^(29/2)*exp(b*ln(f)/x^2)-256/20534684625*(-b*
ln(f)/x^2)^(27/2)*exp(b*ln(f)/x^2)-128/760543875*(-b*ln(f)/x^2)^(25/2)*exp
(b*ln(f)/x^2)-64/30421755*(-b*ln(f)/x^2)^(23/2)*exp(b*ln(f)/x^2)-32/132268
5*(-b*ln(f)/x^2)^(21/2)*exp(b*ln(f)/x^2)-16/62985*(-b*ln(f)/x^2)^(19/2)*ex
p(b*ln(f)/x^2)-8/3315*(-b*ln(f)/x^2)^(17/2)*exp(b*ln(f)/x^2)-4/195*(-b*ln(
f)/x^2)^(15/2)*exp(b*ln(f)/x^2)-2/13*(-b*ln(f)/x^2)^(13/2)*exp(b*ln(f)/x^2
))/x^13/(-b*ln(f)/x^2)^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx = \frac{f^a\Gamma\left(\frac{13}{2},-\frac{b\log(f)}{x^2}\right)}{2x^{13}\left(-\frac{b\log(f)}{x^2}\right)^{13/2}}$$

input
```
Integrate[f^(a + b/x^2)/x^14,x]
```

output
```
(f^a*Gamma[13/2, -((b*Log[f])/x^2)])/(2*x^13*(-((b*Log[f])/x^2))^(13/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}} \, dx$$

$$\Big\downarrow \; 2648$$

$$\frac{f^a \Gamma\left(\frac{13}{2}, -\frac{b\log(f)}{x^2}\right)}{2x^{13}\left(-\frac{b\log(f)}{x^2}\right)^{13/2}}$$

input
```
Int[f^(a + b/x^2)/x^14,x]
```

output
```
(f^a*Gamma[13/2, -((b*Log[f])/x^2)])/(2*x^13*(-((b*Log[f])/x^2))^(13/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.74

method	result
meijerg	$\frac{f^{a}\sqrt{-b}\left(-\frac{(-b)^{\frac{13}{2}}\sqrt{\ln(f)}\left(-\frac{416b^{5}\ln(f)^{5}}{x^{10}}+\frac{2288b^{4}\ln(f)^{4}}{x^{8}}-\frac{10296b^{3}\ln(f)^{3}}{x^{6}}+\frac{36036b^{2}\ln(f)^{2}}{x^{4}}-\frac{90090b\ln(f)}{x^{2}}+135135\right)\mathrm{e}^{\frac{b\ln(f)}{x^{2}}}}{416x\,b^{6}}+\frac{10395(-b)^{\frac{13}{2}}}{\cdots}\right.}{2b^{7}\ln(f)^{\frac{13}{2}}}$
risch	$-\frac{f^{a}f^{\frac{b}{x^{2}}}}{2x^{11}b\ln(f)}+\frac{11f^{a}f^{\frac{b}{x^{2}}}}{4\ln(f)^{2}b^{2}x^{9}}-\frac{99f^{a}f^{\frac{b}{x^{2}}}}{8\ln(f)^{3}b^{3}x^{7}}+\frac{693f^{a}f^{\frac{b}{x^{2}}}}{16\ln(f)^{4}b^{4}x^{5}}-\frac{3465f^{a}f^{\frac{b}{x^{2}}}}{32\ln(f)^{5}b^{5}x^{3}}+\frac{10395f^{a}f^{\frac{b}{x^{2}}}}{64\ln(f)^{6}b^{6}x}-\frac{10395f^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b}\cdots}{x}\right.}{128\ln(f)^{6}b^{6}\sqrt{-b\ln\cdots}}$

input
```
int(f^(a+b/x^2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a/b^7/ln(f)^(13/2)*(-b)^(1/2)*(-1/416/x*(-b)^(13/2)*ln(f)^(1/2)*(-41
6*b^5*ln(f)^5/x^10+2288*b^4*ln(f)^4/x^8-10296*b^3*ln(f)^3/x^6+36036*b^2*ln
(f)^2/x^4-90090*b*ln(f)/x^2+135135)/b^6*exp(b*ln(f)/x^2)+10395/64*(-b)^(13
/2)/b^(13/2)*Pi^(1/2)*erfi(b^(1/2)*ln(f)^(1/2)/x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.65

$$\int\frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx$$

$$=\frac{10395\sqrt{\pi}\sqrt{-b\log(f)}f^{a}x^{11}\,\mathrm{erf}\left(\frac{\sqrt{-b\log(f)}}{x}\right)+2\left(10395\,bx^{10}\log(f)-6930\,b^{2}x^{8}\log(f)^{2}+2772\,b^{3}x^{6}\log(\cdots\right.}{128\,b^{7}x^{11}\log(f)^{7}}$$

input
```
integrate(f^(a+b/x^2)/x^14,x, algorithm="fricas")
```

output
```
1/128*(10395*sqrt(pi)*sqrt(-b*log(f))*f^a*x^11*erf(sqrt(-b*log(f))/x) + 2*
(10395*b*x^10*log(f) - 6930*b^2*x^8*log(f)^2 + 2772*b^3*x^6*log(f)^3 - 792
*b^4*x^4*log(f)^4 + 176*b^5*x^2*log(f)^5 - 32*b^6*log(f)^6)*f^((a*x^2 + b)
/x^2))/(b^7*x^11*log(f)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**2)/x**14,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx = \frac{f^a\Gamma\left(\frac{13}{2}, -\frac{b\log(f)}{x^2}\right)}{2\,x^{13}\left(-\frac{b\log(f)}{x^2}\right)^{\frac{13}{2}}}$$

input
```
integrate(f^(a+b/x^2)/x^14,x, algorithm="maxima")
```

output
```
1/2*f^a*gamma(13/2, -b*log(f)/x^2)/(x^13*(-b*log(f)/x^2)^(13/2))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx = \int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx$$

input
```
integrate(f^(a+b/x^2)/x^14,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)/x^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 159, normalized size of antiderivative = 4.68

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx =$$

$$-\frac{\frac{f^a\left(\frac{10395\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(f)}{x\sqrt{b\,\ln(f)}}\right)}{128}-\frac{10395\,f^{\frac{b}{x^2}}\sqrt{b\,\ln(f)}}{64\,x}\right)}{\sqrt{b\,\ln(f)}}-\frac{693\,b^2\,f^{a+\frac{b}{x^2}}\,\ln(f)^2}{16\,x^5}+\frac{99\,b^3\,f^{a+\frac{b}{x^2}}\,\ln(f)^3}{8\,x^7}-\frac{11\,b^4\,f^{a+\frac{b}{x^2}}\,\ln(f)^4}{4\,x^9}+\frac{b^5\,f^{a+\frac{b}{x^2}}}{2\,x}}{b^6\ln(f)^6}$$

input
```
int(f^(a + b/x^2)/x^14,x)
```

output
```
-((f^a*((10395*pi^(1/2)*erfi((b*log(f))/(x*(b*log(f))^(1/2))))/128 - (1039
5*f^(b/x^2)*(b*log(f))^(1/2))/(64*x)))/(b*log(f))^(1/2) - (693*b^2*f^(a +
b/x^2)*log(f)^2)/(16*x^5) + (99*b^3*f^(a + b/x^2)*log(f)^3)/(8*x^7) - (11*
b^4*f^(a + b/x^2)*log(f)^4)/(4*x^9) + (b^5*f^(a + b/x^2)*log(f)^5)/(2*x^11
) + (3465*b*f^(a + b/x^2)*log(f))/(32*x^3))/(b^6*log(f)^6)
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^2}}}{x^{14}}\,dx = \int \frac{f^{\frac{a\,x^2+b}{x^2}}}{x^{14}}dx$$

input
```
int(f^(a+b/x^2)/x^14,x)
```

output
```
int(f**((a*x**2 + b)/x**2)/x**14,x)
```

3.88 $\int f^{a+\frac{b}{x^3}} x^{14} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int f^{a+\frac{b}{x^3}} x^{14} \, dx = -\frac{1}{3} b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right) \log^5(f)$$

output
```
1/3*f^a*x^15*Ei(6,-b*ln(f)/x^3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x^{14} \, dx = -\frac{1}{3} b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right) \log^5(f)$$

input
```
Integrate[f^(a + b/x^3)*x^14,x]
```

output
```
-1/3*(b^5*f^a*Gamma[-5, -((b*Log[f])/x^3)]*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14} f^{a+\frac{b}{x^3}} dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{1}{3} b^5 f^a \log^5(f) \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x^14,x]
```

output
```
-1/3*(b^5*f^a*Gamma[-5, -((b*Log[f])/x^3)]*Log[f]^5)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(18) = 36.

Time = 0.93 (sec) , antiderivative size = 249, normalized size of antiderivative = 10.38

method	result
meijerg	$\frac{f^a b^5 \ln(f)^5 \left(\frac{x^{15}}{5b^5\ln(f)^5}+\frac{x^{12}}{4b^4\ln(f)^4}+\frac{x^9}{6b^3\ln(f)^3}+\frac{x^6}{12b^2\ln(f)^2}+\frac{x^3}{24b\ln(f)}+\frac{137}{7200}+\frac{\ln(x)}{40}-\frac{\ln(-b)}{120}-\frac{\ln(\ln(f))}{120}-\frac{x^{15}\left(\frac{137b^5\ln(f)^5}{x^{15}}+\frac{300b^4\ln(}{x^{12}}\right.}{}\right.}{}$

input
```
int(f^(a+b/x^3)*x^14,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*b^5*ln(f)^5*(1/5*x^15/b^5/ln(f)^5+1/4*x^12/b^4/ln(f)^4+1/6*x^9/b^3
/ln(f)^3+1/12*x^6/b^2/ln(f)^2+1/24*x^3/b/ln(f)+137/7200+1/40*ln(x)-1/120*l
n(-b)-1/120*ln(ln(f))-1/7200/b^5/ln(f)^5*x^15*(137*b^5*ln(f)^5/x^15+300*b^
4*ln(f)^4/x^12+600*b^3*ln(f)^3/x^9+1200*b^2*ln(f)^2/x^6+1800*b*ln(f)/x^3+1
440)+1/720/b^5/ln(f)^5*x^15*(6*b^4*ln(f)^4/x^12+6*b^3*ln(f)^3/x^9+12*b^2*l
n(f)^2/x^6+36*b*ln(f)/x^3+144)*exp(b*ln(f)/x^3)+1/120*ln(-b*ln(f)/x^3)+1/1
20*Ei(1,-b*ln(f)/x^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.50

$$\int f^{a+\frac{b}{x^3}} x^{14}\,dx$$
$$= -\frac{1}{360}\,b^5 f^a \mathrm{Ei}\left(\frac{b\log(f)}{x^3}\right)\log(f)^5 + \frac{1}{360}\left(24\,x^{15} + 6\,bx^{12}\log(f) + 2\,b^2x^9\log(f)^2 + b^3x^6\log(f)^3 + b^4x^3\log(f)^4\right) f^{\frac{ax^3+b}{x^3}}$$

input
```
integrate(f^(a+b/x^3)*x^14,x, algorithm="fricas")
```

output
```
-1/360*b^5*f^a*Ei(b*log(f)/x^3)*log(f)^5 + 1/360*(24*x^15 + 6*b*x^12*log(f
) + 2*b^2*x^9*log(f)^2 + b^3*x^6*log(f)^3 + b^4*x^3*log(f)^4)*f^((a*x^3 +
b)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}} x^{14} \, dx = \int f^{a+\frac{b}{x^3}} x^{14} \, dx$$

input
```
integrate(f**(a+b/x**3)*x**14,x)
```

output
```
Integral(f**(a + b/x**3)*x**14, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int f^{a+\frac{b}{x^3}} x^{14} \, dx = -\frac{1}{3} b^5 f^a \Gamma\left(-5, -\frac{b \log(f)}{x^3}\right) \log(f)^5$$

input
```
integrate(f^(a+b/x^3)*x^14,x, algorithm="maxima")
```

output
```
-1/3*b^5*f^a*gamma(-5, -b*log(f)/x^3)*log(f)^5
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}} x^{14} \, dx = \int f^{a+\frac{b}{x^3}} x^{14} \, dx$$

input
```
integrate(f^(a+b/x^3)*x^14,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.25

$$\int f^{a+\frac{b}{x^3}}x^{14}\,dx = \frac{b^5\,f^a\ln(f)^5\,\mathrm{expint}\left(-\frac{b\ln(f)}{x^3}\right)}{360} + \frac{b^5\,f^a\,f^{\frac{b}{x^3}}\ln(f)^5\left(\frac{x^3}{120\,b\,\ln(f)}+\frac{x^6}{120\,b^2\ln(f)^2}+\frac{x^9}{60\,b^3\ln(f)^3}+\frac{x^{12}}{20\,b^4\ln(f)^4}+\frac{x^{15}}{5\,b^5\ln(f)^5}\right)}{3}$$

input
```
int(f^(a + b/x^3)*x^14,x)
```

output
```
(b^5*f^a*log(f)^5*expint(-(b*log(f))/x^3))/360 + (b^5*f^a*f^(b/x^3)*log(f)
^5*(x^3/(120*b*log(f)) + x^6/(120*b^2*log(f)^2) + x^9/(60*b^3*log(f)^3) +
x^12/(20*b^4*log(f)^4) + x^15/(5*b^5*log(f)^5)))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 4.38

$$\int f^{a+\frac{b}{x^3}}x^{14}\,dx = \frac{f^a\left(-ei\left(\frac{\log(f)b}{x^3}\right)\log(f)^5b^5+f^{\frac{b}{x^3}}\log(f)^4b^4x^3+f^{\frac{b}{x^3}}\log(f)^3b^3x^6+2f^{\frac{b}{x^3}}\log(f)^2b^2x^9+6f^{\frac{b}{x^3}}\log(f)\,b\,x^{12}+\right.}{360}$$

input
```
int(f^(a+b/x^3)*x^14,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**3)*log(f)**5*b**5 + f**(b/x**3)*log(f)**4*b**4*
x**3 + f**(b/x**3)*log(f)**3*b**3*x**6 + 2*f**(b/x**3)*log(f)**2*b**2*x**9
 + 6*f**(b/x**3)*log(f)*b*x**12 + 24*f**(b/x**3)*x**15))/360
```

3.89 $\int f^{a+\frac{b}{x^3}} x^{11} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int f^{a+\frac{b}{x^3}} x^{11} \, dx = \frac{1}{3} b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right) \log^4(f)$$

output
```
1/3*f^a*x^12*Ei(5,-b*ln(f)/x^3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x^{11} \, dx = \frac{1}{3} b^4 f^a \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right) \log^4(f)$$

input
```
Integrate[f^(a + b/x^3)*x^11,x]
```

output
```
(b^4*f^a*Gamma[-4, -((b*Log[f])/x^3)]*Log[f]^4)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11} f^{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{1}{3} b^4 f^a \log^4(f) \Gamma\left(-4, -\frac{b \log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x^11,x]
```

output
```
(b^4*f^a*Gamma[-4, -((b*Log[f])/x^3)]*Log[f]^4)/3
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(18) = 36.

Time = 0.53 (sec) , antiderivative size = 213, normalized size of antiderivative = 8.88

method	result
meijerg	$-\frac{f^a b^4 \ln(f)^4 \left(-\frac{x^{12}}{4b^4 \ln(f)^4}-\frac{x^9}{3b^3 \ln(f)^3}-\frac{x^6}{4b^2 \ln(f)^2}-\frac{x^3}{6b\ln(f)}-\frac{25}{288}-\frac{\ln(x)}{8}+\frac{\ln(-b)}{24}+\frac{\ln(\ln(f))}{24}+\frac{x^{12}\left(\frac{125b^4 \ln(f)^4}{x^{12}}+\frac{240b^3 \ln(f)^3}{x^9}+\frac{360b^2}{}\right.}{1440b^4 \ln(f)^4}\right.}{3}$

input
```
int(f^(a+b/x^3)*x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*b^4*ln(f)^4*(-1/4*x^12/b^4/ln(f)^4-1/3*x^9/b^3/ln(f)^3-1/4*x^6/b^
2/ln(f)^2-1/6*x^3/b/ln(f)-25/288-1/8*ln(x)+1/24*ln(-b)+1/24*ln(ln(f))+1/14
40/b^4/ln(f)^4*x^12*(125*b^4*ln(f)^4/x^12+240*b^3*ln(f)^3/x^9+360*b^2*ln(f
)^2/x^6+480*b*ln(f)/x^3+360)-1/120/b^4/ln(f)^4*x^12*(5*b^3*ln(f)^3/x^9+5*b
^2*ln(f)^2/x^6+10*b*ln(f)/x^3+30)*exp(b*ln(f)/x^3)-1/24*ln(-b*ln(f)/x^3)-1
/24*Ei(1,-b*ln(f)/x^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int f^{a+\frac{b}{x^3}} x^{11}\,dx = -\frac{1}{72}\,b^4 f^a \mathrm{Ei}\left(\frac{b\log\left(f\right)}{x^3}\right)\log\left(f\right)^4 + \frac{1}{72}\left(6\,x^{12}+2\,bx^9\log\left(f\right)+b^2x^6\log\left(f\right)^2+b^3x^3\log\left(f\right)^3\right)f^{\frac{ax^3+b}{x^3}}$$

input
```
integrate(f^(a+b/x^3)*x^11,x, algorithm="fricas")
```

output
```
-1/72*b^4*f^a*Ei(b*log(f)/x^3)*log(f)^4 + 1/72*(6*x^12 + 2*b*x^9*log(f) +
b^2*x^6*log(f)^2 + b^3*x^3*log(f)^3)*f^((a*x^3 + b)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}x^{11}\,dx=\int f^{a+\frac{b}{x^3}}x^{11}\,dx$$

input
```
integrate(f**(a+b/x**3)*x**11,x)
```

output
```
Integral(f**(a + b/x**3)*x**11, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int f^{a+\frac{b}{x^3}}x^{11}\,dx=\frac{1}{3}\,b^4f^a\Gamma\left(-4,-\frac{b\log{(f)}}{x^3}\right)\log{(f)}^4$$

input
```
integrate(f^(a+b/x^3)*x^11,x, algorithm="maxima")
```

output
```
1/3*b^4*f^a*gamma(-4, -b*log(f)/x^3)*log(f)^4
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x^{11}\,dx=\int f^{a+\frac{b}{x^3}}x^{11}\,dx$$

input
```
integrate(f^(a+b/x^3)*x^11,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^11, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.75

$$\int f^{a+\frac{b}{x^3}}x^{11}\,dx=\frac{b^4\,f^a\ln\left(f\right)^4\,\mathrm{expint}\left(-\frac{b\,\ln(f)}{x^3}\right)}{72}+\frac{b^4\,f^a\,f^{\frac{b}{x^3}}\,\ln\left(f\right)^4\left(\frac{x^3}{24\,b\,\ln(f)}+\frac{x^6}{24\,b^2\ln(f)^2}+\frac{x^9}{12\,b^3\ln(f)^3}+\frac{x^{12}}{4\,b^4\ln(f)^4}\right)}{3}$$

input
```
int(f^(a + b/x^3)*x^11,x)
```

output
```
(b^4*f^a*log(f)^4*expint(-(b*log(f))/x^3))/72 + (b^4*f^a*f^(b/x^3)*log(f)^
4*(x^3/(24*b*log(f)) + x^6/(24*b^2*log(f)^2) + x^9/(12*b^3*log(f)^3) + x^1
2/(4*b^4*log(f)^4)))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.58

$$\int f^{a+\frac{b}{x^3}}x^{11}\,dx=\frac{f^a\left(-ei\left(\frac{\log(f)b}{x^3}\right)\log(f)^4\,b^4+f^{\frac{b}{x^3}}\log(f)^3\,b^3x^3+f^{\frac{b}{x^3}}\log(f)^2\,b^2x^6+2f^{\frac{b}{x^3}}\log(f)\,b\,x^9+6f^{\frac{b}{x^3}}x^{12}\right)}{72}$$

input
```
int(f^(a+b/x^3)*x^11,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**3)*log(f)**4*b**4 + f**(b/x**3)*log(f)**3*b**3*
x**3 + f**(b/x**3)*log(f)**2*b**2*x**6 + 2*f**(b/x**3)*log(f)*b*x**9 + 6*f
**(b/x**3)*x**12))/72
```

3.90 $\int f^{a+\frac{b}{x^3}} x^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int f^{a+\frac{b}{x^3}} x^8 \, dx = \frac{1}{9} f^{a+\frac{b}{x^3}} x^9 + \frac{1}{18} b f^{a+\frac{b}{x^3}} x^6 \log(f) + \frac{1}{18} b^2 f^{a+\frac{b}{x^3}} x^3 \log^2(f) - \frac{1}{18} b^3 f^a \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^3}\right) \log^3(f)$$

output
```
1/9*f^(a+b/x^3)*x^9+1/18*b*f^(a+b/x^3)*x^6*ln(f)+1/18*b^2*f^(a+b/x^3)*x^3*
ln(f)^2-1/18*b^3*f^a*Ei(b*ln(f)/x^3)*ln(f)^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int f^{a+\frac{b}{x^3}} x^8 \, dx = \frac{1}{18} f^a \left(-b^3 \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^3}\right) \log^3(f) + f^{\frac{b}{x^3}} x^3 \left(2x^6 + b x^3 \log(f) + b^2 \log^2(f)\right)\right)$$

input
```
Integrate[f^(a + b/x^3)*x^8,x]
```

output
```
(f^a*(-(b^3*ExpIntegralEi[(b*Log[f])/x^3]*Log[f]^3) + f^(b/x^3)*x^3*(2*x^6
 + b*x^3*Log[f] + b^2*Log[f]^2)))/18
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 f^{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \int f^{a+\frac{b}{x^3}} x^5 dx + \frac{1}{9} x^9 f^{a+\frac{b}{x^3}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \int f^{a+\frac{b}{x^3}} x^2 dx + \frac{1}{6} x^6 f^{a+\frac{b}{x^3}}\right) + \frac{1}{9} x^9 f^{a+\frac{b}{x^3}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{a+\frac{b}{x^3}}}{x} dx + \frac{1}{3} x^3 f^{a+\frac{b}{x^3}}\right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^3}}\right) + \frac{1}{9} x^9 f^{a+\frac{b}{x^3}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3} b \log(f) \left(\frac{1}{2} b \log(f) \left(\frac{1}{3} x^3 f^{a+\frac{b}{x^3}} - \frac{1}{3} b f^a \log(f) \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^3}\right)\right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^3}}\right) + \frac{1}{9} x^9 f^{a+\frac{b}{x^3}}$$

input
```
Int[f^(a + b/x^3)*x^8,x]
```

output
```
(f^(a + b/x^3)*x^9)/9 + (b*Log[f]*((f^(a + b/x^3)*x^6)/6 + (b*Log[f]*((f^(
a + b/x^3)*x^3)/3 - (b*f^a*ExpIntegralEi[(b*Log[f])/x^3]*Log[f])/3))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(73) = 146$.

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.19

method	result
meijerg	$\frac{f^{a} b^{3} \ln(f)^{3} \left(\frac{x^{9}}{3 b^{3} \ln(f)^{3}}+\frac{x^{6}}{2 b^{2} \ln(f)^{2}}+\frac{x^{3}}{2 b \ln(f)}+\frac{11}{36}+\frac{\ln(x)}{2}-\frac{\ln(-b)}{6}-\frac{\ln(\ln(f))}{6}-\frac{x^{9}\left(\frac{22 b^{3} \ln(f)^{3}}{x^{9}}+\frac{36 b^{2} \ln(f)^{2}}{x^{6}}+\frac{36 b \ln(f)}{x^{3}}+24\right)}{72 b^{3} \ln(f)^{3}}+\frac{x^{9}\left(\frac{4 b^{2} \ln\right.}{x}\right.}{}\right.}{3}$

input
```
int(f^(a+b/x^3)*x^8,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*b^3*ln(f)^3*(1/3*x^9/b^3/ln(f)^3+1/2*x^6/b^2/ln(f)^2+1/2*x^3/b/ln(
f)+11/36+1/2*ln(x)-1/6*ln(-b)-1/6*ln(ln(f))-1/72/b^3/ln(f)^3*x^9*(22*b^3*l
n(f)^3/x^9+36*b^2*ln(f)^2/x^6+36*b*ln(f)/x^3+24)+1/24/b^3/ln(f)^3*x^9*(4*b
^2*ln(f)^2/x^6+4*b*ln(f)/x^3+8)*exp(b*ln(f)/x^3)+1/6*ln(-b*ln(f)/x^3)+1/6*
Ei(1,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int f^{a+\frac{b}{x^3}}x^8\,dx = -\frac{1}{18}\,b^3f^a\mathrm{Ei}\left(\frac{b\log(f)}{x^3}\right)\log(f)^3 + \frac{1}{18}\left(2\,x^9+bx^6\log(f)+b^2x^3\log(f)^2\right)f^{\frac{ax^3+b}{x^3}}$$

input
```
integrate(f^(a+b/x^3)*x^8,x, algorithm="fricas")
```

output
```
-1/18*b^3*f^a*Ei(b*log(f)/x^3)*log(f)^3 + 1/18*(2*x^9 + b*x^6*log(f) + b^2
*x^3*log(f)^2)*f^((a*x^3 + b)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}x^8\,dx = \int f^{a+\frac{b}{x^3}}x^8\,dx$$

input
```
integrate(f**(a+b/x**3)*x**8,x)
```

output
```
Integral(f**(a + b/x**3)*x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int f^{a+\frac{b}{x^3}}x^8\,dx = -\frac{1}{3}\,b^3f^a\Gamma\left(-3,-\frac{b\log(f)}{x^3}\right)\log(f)^3$$

input
```
integrate(f^(a+b/x^3)*x^8,x, algorithm="maxima")
```

output
```
-1/3*b^3*f^a*gamma(-3, -b*log(f)/x^3)*log(f)^3
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}} x^8 \, dx = \int f^{a+\frac{b}{x^3}} x^8 \, dx$$

input
```
integrate(f^(a+b/x^3)*x^8,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int f^{a+\frac{b}{x^3}} x^8 \, dx = \frac{b^3 f^a \ln(f)^3 \left(f^{\frac{b}{x^3}} \left(\frac{x^3}{6 b \ln(f)} + \frac{x^6}{6 b^2 \ln(f)^2} + \frac{x^9}{3 b^3 \ln(f)^3}\right) + \frac{\mathrm{expint}\left(-\frac{b \ln(f)}{x^3}\right)}{6}\right)}{3}$$

input
```
int(f^(a + b/x^3)*x^8,x)
```

output
```
(b^3*f^a*log(f)^3*(f^(b/x^3)*(x^3/(6*b*log(f)) + x^6/(6*b^2*log(f)^2) + x^
9/(3*b^3*log(f)^3)) + expint(-(b*log(f))/x^3)/6))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int f^{a+\frac{b}{x^3}} x^8 \, dx = \frac{f^a \left(-ei\left(\frac{\log(f) b}{x^3}\right) \log(f)^3 b^3 + f^{\frac{b}{x^3}} \log(f)^2 b^2 x^3 + f^{\frac{b}{x^3}} \log(f) b x^6 + 2 f^{\frac{b}{x^3}} x^9\right)}{18}$$

input
```
int(f^(a+b/x^3)*x^8,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**3)*log(f)**3*b**3 + f**(b/x**3)*log(f)**2*b**2*
x**3 + f**(b/x**3)*log(f)*b*x**6 + 2*f**(b/x**3)*x**9))/18
```

3.91 $\int f^{a+\frac{b}{x^3}} x^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int f^{a+\frac{b}{x^3}} x^5\,dx = \frac{1}{6} f^{a+\frac{b}{x^3}} x^6 + \frac{1}{6} b f^{a+\frac{b}{x^3}} x^3 \log(f) - \frac{1}{6} b^2 f^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x^3}\right)\log^2(f)$$

output
```
1/6*f^(a+b/x^3)*x^6+1/6*b*f^(a+b/x^3)*x^3*ln(f)-1/6*b^2*f^a*Ei(b*ln(f)/x^3
)*ln(f)^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int f^{a+\frac{b}{x^3}} x^5\,dx = \frac{1}{6} f^a\left(-b^2 \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x^3}\right)\log^2(f) + f^{\frac{b}{x^3}} x^3 (x^3 + b\log(f))\right)$$

input
```
Integrate[f^(a + b/x^3)*x^5,x]
```

output
```
(f^a*(-(b^2*ExpIntegralEi[(b*Log[f])/x^3]*Log[f]^2) + f^(b/x^3)*x^3*(x^3 +
 b*Log[f])))/6
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 f^{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \int f^{a+\frac{b}{x^3}} x^2 dx + \frac{1}{6} x^6 f^{a+\frac{b}{x^3}}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{a+\frac{b}{x^3}}}{x} dx + \frac{1}{3} x^3 f^{a+\frac{b}{x^3}} \right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^3}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2} b \log(f) \left(\frac{1}{3} x^3 f^{a+\frac{b}{x^3}} - \frac{1}{3} b f^a \log(f) \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^3} \right) \right) + \frac{1}{6} x^6 f^{a+\frac{b}{x^3}}$$

input
```
Int[f^(a + b/x^3)*x^5,x]
```

output
```
(f^(a + b/x^3)*x^6)/6 + (b*Log[f]*((f^(a + b/x^3)*x^3)/3 - (b*f^a*ExpInteg
ralEi[(b*Log[f])/x^3]*Log[f])/3))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(52) = 104$.

Time = 0.15 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.43

method	result
meijerg	$-\frac{f^a b^2 \ln(f)^2 \left(-\frac{x^6}{2b^2\ln(f)^2}-\frac{x^3}{b\ln(f)}-\frac{3}{4}-\frac{3\ln(x)}{2}+\frac{\ln(-b)}{2}+\frac{\ln(\ln(f))}{2}+\frac{x^6\left(\frac{9b^2\ln(f)^2}{x^6}+\frac{12b\ln(f)}{x^3}+6\right)}{12b^2\ln(f)^2}-\frac{x^6\left(\frac{3b\ln(f)}{x^3}+3\right)e^{\frac{b\ln(f)}{x^3}}}{6b^2\ln(f)^2}-\ln\left(-b\right.\right.}{3}$

input
```
int(f^(a+b/x^3)*x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*b^2*ln(f)^2*(-1/2*x^6/b^2/ln(f)^2-x^3/b/ln(f)-3/4-3/2*ln(x)+1/2*l
n(-b)+1/2*ln(ln(f))+1/12/b^2/ln(f)^2*x^6*(9*b^2*ln(f)^2/x^6+12*b*ln(f)/x^3
+6)-1/6/b^2/ln(f)^2*x^6*(3*b*ln(f)/x^3+3)*exp(b*ln(f)/x^3)-1/2*ln(-b*ln(f)
/x^3)-1/2*Ei(1,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int f^{a+\frac{b}{x^3}} x^5 \, dx = -\frac{1}{6} b^2 f^a \text{Ei}\left(\frac{b \log(f)}{x^3}\right) \log(f)^2 + \frac{1}{6} \left(x^6 + b x^3 \log(f)\right) f^{\frac{a x^3 + b}{x^3}}$$

input
```
integrate(f^(a+b/x^3)*x^5,x, algorithm="fricas")
```

output
```
-1/6*b^2*f^a*Ei(b*log(f)/x^3)*log(f)^2 + 1/6*(x^6 + b*x^3*log(f))*f^((a*x^
3 + b)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}x^5\,dx=\int f^{a+\frac{b}{x^3}}x^5\,dx$$

input
```
integrate(f**(a+b/x**3)*x**5,x)
```

output
```
Integral(f**(a + b/x**3)*x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.38

$$\int f^{a+\frac{b}{x^3}}x^5\,dx=\frac{1}{3}b^2f^a\Gamma\left(-2,-\frac{b\log(f)}{x^3}\right)\log(f)^2$$

input
```
integrate(f^(a+b/x^3)*x^5,x, algorithm="maxima")
```

output
```
1/3*b^2*f^a*gamma(-2, -b*log(f)/x^3)*log(f)^2
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x^5\,dx=\int f^{a+\frac{b}{x^3}}x^5\,dx$$

input
```
integrate(f^(a+b/x^3)*x^5,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int f^{a+\frac{b}{x^3}}x^5\,dx=\frac{b^2\,f^a\ln\left(f\right)^2\,\left(f^{\frac{b}{x^3}}\,\left(\frac{x^3}{2\,b\,\ln(f)}+\frac{x^6}{2\,b^2\ln(f)^2}\right)+\frac{\mathrm{expint}\left(-\frac{b\,\ln(f)}{x^3}\right)}{2}\right)}{3}$$

input
```
int(f^(a + b/x^3)*x^5,x)
```

output
```
(b^2*f^a*log(f)^2*(f^(b/x^3)*(x^3/(2*b*log(f)) + x^6/(2*b^2*log(f)^2)) + e
xpint(-(b*log(f))/x^3)/2))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int f^{a+\frac{b}{x^3}}x^5\,dx=\frac{f^a\left(-ei\left(\frac{\log(f)b}{x^3}\right)\log(f)^2\,b^2+f^{\frac{b}{x^3}}\log(f)\,b\,x^3+f^{\frac{b}{x^3}}x^6\right)}{6}$$

input
```
int(f^(a+b/x^3)*x^5,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**3)*log(f)**2*b**2 + f**(b/x**3)*log(f)*b*x**3 +
 f**(b/x**3)*x**6))/6
```

3.92 $\int f^{a+\frac{b}{x^3}} x^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int f^{a+\frac{b}{x^3}} x^2 \, dx = \frac{1}{3} f^{a+\frac{b}{x^3}} x^3 - \frac{1}{3} b f^a \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^3}\right) \log(f)$$

output
```
1/3*f^(a+b/x^3)*x^3-1/3*b*f^a*Ei(b*ln(f)/x^3)*ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int f^{a+\frac{b}{x^3}} x^2 \, dx = \frac{1}{3} f^a \left(f^{\frac{b}{x^3}} x^3 - b \operatorname{ExpIntegralEi}\left(\frac{b \log(f)}{x^3}\right) \log(f)\right)$$

input
```
Integrate[f^(a + b/x^3)*x^2,x]
```

output
```
(f^a*(f^(b/x^3)*x^3 - b*ExpIntegralEi[(b*Log[f])/x^3]*Log[f]))/3
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$b\log(f)\int \frac{f^{a+\frac{b}{x^3}}}{x}dx + \frac{1}{3}x^3 f^{a+\frac{b}{x^3}}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3}x^3 f^{a+\frac{b}{x^3}} - \frac{1}{3}bf^a\log(f)\,\text{ExpIntegralEi}\left(\frac{b\log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x^2,x]
```

output
```
(f^(a + b/x^3)*x^3)/3 - (b*f^a*ExpIntegralEi[(b*Log[f])/x^3]*Log[f])/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(31) = 62.

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.77

method	result	size
meijerg	$\frac{f^{a} b \ln \left(f \right) \left(\frac{x^{3}}{b \ln \left(f \right)}+1+3 \ln \left(x \right)-\ln \left(-b \right)-\ln \left(\ln \left(f \right)\right)-\frac{x^{3} \left(\frac{2 b \ln \left(f \right)}{x^{3}}+2\right)}{2 b \ln \left(f \right)}+\frac{x^{3} {\mathrm e}^{\frac{b \ln \left(f \right)}{x^{3}}}}{b \ln \left(f \right)}+\ln \left(-\frac{b \ln \left(f \right)}{x^{3}}\right)+\operatorname{expIntegral}_{1}\left(-\frac{b \ln \left(f \right)}{x^{3}}\right)\right)}{3}$	97

input
```
int(f^(a+b/x^3)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*b*ln(f)*(x^3/b/ln(f)+1+3*ln(x)-ln(-b)-ln(ln(f))-1/2/b/ln(f)*x^3*(2
*b*ln(f)/x^3+2)+1/b/ln(f)*x^3*exp(b*ln(f)/x^3)+ln(-b*ln(f)/x^3)+Ei(1,-b*ln
(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x^2 \, dx = \frac{1}{3} \, f^{\frac{a x^{3} + b}{x^{3}}} x^{3} - \frac{1}{3} \, b f^{a} {\rm Ei}\left(\frac{b \log\left(f\right)}{x^{3}}\right) \log\left(f\right)$$

input
```
integrate(f^(a+b/x^3)*x^2,x, algorithm="fricas")
```

output
```
1/3*f^((a*x^3 + b)/x^3)*x^3 - 1/3*b*f^a*Ei(b*log(f)/x^3)*log(f)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}} x^2 \, dx = \int f^{a + \frac{b}{x^{3}}} x^{2}\, dx$$

input
```
integrate(f**(a+b/x**3)*x**2,x)
```

output
```
Integral(f**(a + b/x**3)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.51

$$\int f^{a+\frac{b}{x^3}}x^2\,dx = -\frac{1}{3}\,bf^a\Gamma\left(-1,-\frac{b\log\left(f\right)}{x^3}\right)\log\left(f\right)$$

input
```
integrate(f^(a+b/x^3)*x^2,x, algorithm="maxima")
```

output
```
-1/3*b*f^a*gamma(-1, -b*log(f)/x^3)*log(f)
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x^2\,dx = \int f^{a+\frac{b}{x^3}}x^2\,dx$$

input
```
integrate(f^(a+b/x^3)*x^2,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int f^{a+\frac{b}{x^3}}x^2\,dx = \frac{f^a\,f^{\frac{b}{x^3}}\,x^3}{3} + \frac{b\,f^a\,\ln\left(f\right)\,\mathrm{expint}\left(-\frac{b\,\ln(f)}{x^3}\right)}{3}$$

input
```
int(f^(a + b/x^3)*x^2,x)
```

output
```
(f^a*f^(b/x^3)*x^3)/3 + (b*f^a*log(f)*expint(-(b*log(f))/x^3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int f^{a+\frac{b}{x^3}} x^2\,dx = \frac{f^a\left(-ei\left(\frac{\log(f)b}{x^3}\right)\log(f)\,b + f^{\frac{b}{x^3}} x^3\right)}{3}$$

input
```
int(f^(a+b/x^3)*x^2,x)
```

output
```
(f**a*( - ei((log(f)*b)/x**3)*log(f)*b + f**(b/x**3)*x**3))/3
```

3.93 $\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = -\frac{1}{3}f^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x^3}\right)$$

output
```
-1/3*f^a*Ei(b*ln(f)/x^3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = -\frac{1}{3}f^a \operatorname{ExpIntegralEi}\left(\frac{b\log(f)}{x^3}\right)$$

input
```
Integrate[f^(a + b/x^3)/x,x]
```

output
```
-1/3*(f^a*ExpIntegralEi[(b*Log[f])/x^3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x} \, dx$$

$$\Big\downarrow \ 2639$$

$$-\frac{1}{3} f^a \text{ExpIntegralEi}\left(\frac{b \log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)/x,x]
```

output
```
-1/3*(f^a*ExpIntegralEi[(b*Log[f])/x^3])
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(13) = 26.

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.73

method	result	size
meijerg	$-\frac{f^{a}\left(-3\ln(x)+\ln(-b)+\ln(\ln(f))-\ln\left(-\frac{b\ln(f)}{x^{3}}\right)-\text{expIntegral}_{1}\left(-\frac{b\ln(f)}{x^{3}}\right)\right)}{3}$	41

input
```
int(f^(a+b/x^3)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*(-3*ln(x)+ln(-b)+ln(ln(f))-ln(-b*ln(f)/x^3)-Ei(1,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = -\frac{1}{3}\,f^a\mathrm{Ei}\left(\frac{b\log\left(f\right)}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)/x,x, algorithm="fricas")
```

output
```
-1/3*f^a*Ei(b*log(f)/x^3)
```

Sympy [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx$$

input
```
integrate(f**(a+b/x**3)/x,x)
```

output
```
Integral(f**(a + b/x**3)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = -\frac{1}{3}\,f^a \mathrm{Ei}\left(\frac{b\log(f)}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)/x,x, algorithm="maxima")
```

output
```
-1/3*f^a*Ei(b*log(f)/x^3)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx$$

input
```
integrate(f^(a+b/x^3)/x,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = -\frac{f^a\,\mathrm{ei}\left(\frac{b\,\ln(f)}{x^3}\right)}{3}$$

input
```
int(f^(a + b/x^3)/x,x)
```

output
```
-(f^a*ei((b*log(f))/x^3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+\frac{b}{x^3}}}{x}\,dx = -\frac{f^a ei\left(\frac{\log(f)b}{x^3}\right)}{3}$$

input
```
int(f^(a+b/x^3)/x,x)
```

output
```
( - f**a*ei((log(f)*b)/x**3))/3
```

3.94 $\int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx = -\frac{f^{a+\frac{b}{x^3}}}{3b\log(f)}$$

output
```
-1/3*f^(a+b/x^3)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx = -\frac{f^{a+\frac{b}{x^3}}}{3b\log(f)}$$

input
```
Integrate[f^(a + b/x^3)/x^4,x]
```

output
```
-1/3*f^(a + b/x^3)/(b*Log[f])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} \, dx$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{f^{a+\frac{b}{x^3}}}{3b\log(f)}$$

input
```
Int[f^(a + b/x^3)/x^4,x]
```

output
```
-1/3*f^(a + b/x^3)/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{f^{a+\frac{b}{x^3}}}{3b\ln(f)}$	19
default	$-\frac{f^{a+\frac{b}{x^3}}}{3b\ln(f)}$	19
parallelrisch	$-\frac{f^{a+\frac{b}{x^3}}}{3b\ln(f)}$	19
norman	$-\frac{e^{\left(a+\frac{b}{x^3}\right)\ln(f)}}{3b\ln(f)}$	21
risch	$-\frac{f^{\frac{a\,x^3+b}{x^3}}}{3b\ln(f)}$	23
meijerg	$\frac{f^a\left(1-e^{\frac{b\ln(f)}{x^3}}\right)}{3b\ln(f)}$	25

input
```
int(f^(a+b/x^3)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^(a+b/x^3)/b/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{f^{\frac{ax^3+b}{x^3}}}{3\,b\log\left(f\right)}$$

input
```
integrate(f^(a+b/x^3)/x^4,x, algorithm="fricas")
```

output
```
-1/3*f^((a*x^3 + b)/x^3)/(b*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} \, dx = \begin{cases} -\frac{f^{a+\frac{b}{x^3}}}{3b\log(f)} & \text{for } b\log(f) \neq 0 \\ -\frac{1}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(a+b/x**3)/x**4,x)
```

output
```
Piecewise((-f**(a + b/x**3)/(3*b*log(f)), Ne(b*log(f), 0)), (-1/(3*x**3),
True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} \, dx = -\frac{f^{a+\frac{b}{x^3}}}{3\,b\log(f)}$$

input
```
integrate(f^(a+b/x^3)/x^4,x, algorithm="maxima")
```

output
```
-1/3*f^(a + b/x^3)/(b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4} \, dx = -\frac{f^{\frac{ax^3+b}{x^3}}}{3\,b\log(f)}$$

input
```
integrate(f^(a+b/x^3)/x^4,x, algorithm="giac")
```

output
```
-1/3*f^((a*x^3 + b)/x^3)/(b*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{f^{a+\frac{b}{x^3}}}{3\,b\,\ln\left(f\right)}$$

input
```
int(f^(a + b/x^3)/x^4,x)
```

output
```
-f^(a + b/x^3)/(3*b*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^4}\,dx = -\frac{f^{\frac{a\,x^3+b}{x^3}}}{3\log\left(f\right)b}$$

input
```
int(f^(a+b/x^3)/x^4,x)
```

output
```
( - f**((a*x**3 + b)/x**3))/(3*log(f)*b)
```

3.95 $\int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx = \frac{f^{a+\frac{b}{x^3}}}{3b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^3 \log(f)}$$

output
```
1/3*f^(a+b/x^3)/b^2/ln(f)^2-1/3*f^(a+b/x^3)/b/x^3/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx = \frac{f^{a+\frac{b}{x^3}}(x^3 - b\log(f))}{3b^2x^3 \log^2(f)}$$

input
```
Integrate[f^(a + b/x^3)/x^7,x]
```

output
```
(f^(a + b/x^3)*(x^3 - b*Log[f]))/(3*b^2*x^3*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{f^{a+\frac{b}{x^3}}}{x^4}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^3\log(f)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{f^{a+\frac{b}{x^3}}}{3b^2\log^2(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^3\log(f)}$$

input
```
Int[f^(a + b/x^3)/x^7,x]
```

output
```
f^(a + b/x^3)/(3*b^2*Log[f]^2) - f^(a + b/x^3)/(3*b*x^3*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

method	result	size
meijerg	$-\frac{f^{a}\left(1-\frac{\left(-\frac{2b\ln\left(f\right)}{x^{3}}+2\right){\mathrm e}^{\frac{b\ln\left(f\right)}{x^{3}}}}{2}\right)}{3b^{2}\ln\left(f\right)^{2}}$	35
risch	$-\frac{\left(-x^{3}+b\ln\left(f\right)\right)f^{\frac{a\,x^{3}+b}{x^{3}}}}{3\ln\left(f\right)^{2}b^{2}x^{3}}$	36
parallelrisch	$\frac{f^{a+\frac{b}{x^{3}}}x^{3}-f^{a+\frac{b}{x^{3}}}b\ln\left(f\right)}{3x^{3}\ln\left(f\right)^{2}b^{2}}$	41
norman	$\frac{-\frac{x^{3}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln\left(f\right)}}{3b\ln\left(f\right)}+\frac{x^{6}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln\left(f\right)}}{3b^{2}\ln\left(f\right)^{2}}}{x^{6}}$	52

input
```
int(f^(a+b/x^3)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a/b^2/ln(f)^2*(1-1/2*(-2*b*ln(f)/x^3+2)*exp(b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx = \frac{(x^3 - b\log(f))f^{\frac{ax^3+b}{x^3}}}{3\,b^2x^3\log(f)^2}$$

input
```
integrate(f^(a+b/x^3)/x^7,x, algorithm="fricas")
```

output
```
1/3*(x^3 - b*log(f))*f^((a*x^3 + b)/x^3)/(b^2*x^3*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx = \frac{f^{a+\frac{b}{x^3}}(-b\log(f)+x^3)}{3b^2x^3\log(f)^2}$$

input
```
integrate(f**(a+b/x**3)/x**7,x)
```

output
```
f**(a + b/x**3)*(-b*log(f) + x**3)/(3*b**2*x**3*log(f)**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx = \frac{f^a\Gamma\left(2,-\frac{b\log(f)}{x^3}\right)}{3\,b^2\log(f)^2}$$

input
```
integrate(f^(a+b/x^3)/x^7,x, algorithm="maxima")
```

output
```
1/3*f^a*gamma(2, -b*log(f)/x^3)/(b^2*log(f)^2)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^7,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx = -\frac{f^{a+\frac{b}{x^3}}\left(\frac{1}{3\,b\,\ln(f)}-\frac{x^3}{3\,b^2\,\ln(f)^2}\right)}{x^3}$$

input
```
int(f^(a + b/x^3)/x^7,x)
```

output
```
-(f^(a + b/x^3)*(1/(3*b*log(f)) - x^3/(3*b^2*log(f)^2)))/x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^7}\,dx = \frac{f^{\frac{a\,x^3+b}{x^3}}\left(-\log(f)\,b+x^3\right)}{3\log\left(f\right)^2 b^2 x^3}$$

input
```
int(f^(a+b/x^3)/x^7,x)
```

output
```
(f**((a*x**3 + b)/x**3)*( - log(f)*b + x**3))/(3*log(f)**2*b**2*x**3)
```

3.96 $\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{2f^{a+\frac{b}{x^3}}}{3b^3\log^3(f)} + \frac{2f^{a+\frac{b}{x^3}}}{3b^2x^3\log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3bx^6\log(f)}$$

output
```
-2/3*f^(a+b/x^3)/b^3/ln(f)^3+2/3*f^(a+b/x^3)/b^2/x^3/ln(f)^2-1/3*f^(a+b/x^
3)/b/x^6/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{f^{a+\frac{b}{x^3}}\left(2x^6 - 2bx^3\log(f) + b^2\log^2(f)\right)}{3b^3x^6\log^3(f)}$$

input
```
Integrate[f^(a + b/x^3)/x^10,x]
```

output
```
-1/3*(f^(a + b/x^3)*(2*x^6 - 2*b*x^3*Log[f] + b^2*Log[f]^2))/(b^3*x^6*Log[
f]^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{2 \int \frac{f^{a+\frac{b}{x^3}}}{x^7} dx}{b \log(f)} - \frac{f^{a+\frac{b}{x^3}}}{3 b x^6 \log(f)}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{2 \left(-\frac{\int \frac{f^{a+\frac{b}{x^3}}}{x^4} dx}{b \log(f)} - \frac{f^{a+\frac{b}{x^3}}}{3 b x^3 \log(f)}\right)}{b \log(f)} - \frac{f^{a+\frac{b}{x^3}}}{3 b x^6 \log(f)}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{2 \left(\frac{f^{a+\frac{b}{x^3}}}{3 b^2 \log^2(f)} - \frac{f^{a+\frac{b}{x^3}}}{3 b x^3 \log(f)}\right)}{b \log(f)} - \frac{f^{a+\frac{b}{x^3}}}{3 b x^6 \log(f)}$$

input
```
Int[f^(a + b/x^3)/x^10,x]
```

output
```
-1/3*f^(a + b/x^3)/(b*x^6*Log[f]) - (2*(f^(a + b/x^3)/(3*b^2*Log[f]^2) - f
^(a + b/x^3)/(3*b*x^3*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

method	result	size
meijerg	$\frac{f^{a}\left(2-\frac{\left(\frac{3b^{2}\ln(f)^{2}}{x^{6}}-\frac{6b\ln(f)}{x^{3}}+6\right){\mathrm e}^{\frac{b\ln(f)}{x^{3}}}}{3}\right)}{3b^{3}\ln(f)^{3}}$	47
risch	$-\frac{\left(2x^{6}-2b\,x^{3}\ln(f)+\ln(f)^{2}b^{2}\right)f^{\frac{a\,x^{3}+b}{x^{3}}}}{3\ln(f)^{3}b^{3}x^{6}}$	48
parallelrisch	$\frac{-2f^{a+\frac{b}{x^{3}}}x^{6}+2b\,f^{a+\frac{b}{x^{3}}}x^{3}\ln(f)-f^{a+\frac{b}{x^{3}}}\ln(f)^{2}b^{2}}{3x^{6}\ln(f)^{3}b^{3}}$	63
norman	$\frac{-\frac{x^{3}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{3b\ln(f)}+\frac{2x^{6}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{3b^{2}\ln(f)^{2}}-\frac{2x^{9}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{3b^{3}\ln(f)^{3}}}{x^{9}}$	75

input
```
int(f^(a+b/x^3)/x^10,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a/b^3/ln(f)^3*(2-1/3*(3*b^2*ln(f)^2/x^6-6*b*ln(f)/x^3+6)*exp(b*ln(f)
/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{\left(2\,x^6 - 2\,bx^3\log\left(f\right) + b^2\log\left(f\right)^2\right)f^{\frac{ax^3+b}{x^3}}}{3\,b^3x^6\log\left(f\right)^3}$$

input
```
integrate(f^(a+b/x^3)/x^10,x, algorithm="fricas")
```

output
```
-1/3*(2*x^6 - 2*b*x^3*log(f) + b^2*log(f)^2)*f^((a*x^3 + b)/x^3)/(b^3*x^6*
log(f)^3)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx = \frac{f^{a+\frac{b}{x^3}}\left(-b^2\log\left(f\right)^2 + 2bx^3\log\left(f\right) - 2x^6\right)}{3b^3x^6\log\left(f\right)^3}$$

input
```
integrate(f**(a+b/x**3)/x**10,x)
```

output
```
f**(a + b/x**3)*(-b**2*log(f)**2 + 2*b*x**3*log(f) - 2*x**6)/(3*b**3*x**6*
log(f)**3)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.33

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx = -\frac{f^a\Gamma\left(3, -\frac{b\log(f)}{x^3}\right)}{3\,b^3\log\left(f\right)^3}$$

input
```
integrate(f^(a+b/x^3)/x^10,x, algorithm="maxima")
```

output
```
-1/3*f^a*gamma(3, -b*log(f)/x^3)/(b^3*log(f)^3)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx=\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^10,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx=-\frac{f^{a+\frac{b}{x^3}}\left(\frac{1}{3\,b\,\ln(f)}-\frac{2\,x^3}{3\,b^2\,\ln(f)^2}+\frac{2\,x^6}{3\,b^3\,\ln(f)^3}\right)}{x^6}$$

input
```
int(f^(a + b/x^3)/x^10,x)
```

output
```
-(f^(a + b/x^3)*(1/(3*b*log(f)) - (2*x^3)/(3*b^2*log(f)^2) + (2*x^6)/(3*b^
3*log(f)^3)))/x^6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}\,dx=\frac{f^{\frac{a\,x^3+b}{x^3}}\left(-\log(f)^2 b^2+2\log(f)\,b\,x^3-2x^6\right)}{3\log(f)^3 b^3x^6}$$

input
```
int(f^(a+b/x^3)/x^10,x)
```

output
```
(f**((a*x**3 + b)/x**3)*( - log(f)**2*b**2 + 2*log(f)*b*x**3 - 2*x**6))/(3
*log(f)**3*b**3*x**6)
```

3.97 $\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx=\frac{2f^{a+\frac{b}{x^3}}}{b^4\log^4(f)}-\frac{2f^{a+\frac{b}{x^3}}}{b^3x^3\log^3(f)}+\frac{f^{a+\frac{b}{x^3}}}{b^2x^6\log^2(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^9\log(f)}$$

output
```
2*f^(a+b/x^3)/b^4/ln(f)^4-2*f^(a+b/x^3)/b^3/x^3/ln(f)^3+f^(a+b/x^3)/b^2/x^
6/ln(f)^2-1/3*f^(a+b/x^3)/b/x^9/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx=\frac{f^{a+\frac{b}{x^3}}\left(6x^9-6bx^6\log(f)+3b^2x^3\log^2(f)-b^3\log^3(f)\right)}{3b^4x^9\log^4(f)}$$

input
```
Integrate[f^(a + b/x^3)/x^13,x]
```

output
```
(f^(a + b/x^3)*(6*x^9 - 6*b*x^6*Log[f] + 3*b^2*x^3*Log[f]^2 - b^3*Log[f]^3
))/(3*b^4*x^9*Log[f]^4)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3 \int \frac{f^{a+\frac{b}{x^3}}}{x^{10}}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^9\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\left(-\frac{2\int \frac{f^{a+\frac{b}{x^3}}}{x^7}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^6\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^9\log(f)}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \frac{f^{a+\frac{b}{x^3}}}{x^4}dx}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^3\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^6\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^9\log(f)}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3\left(-\frac{2\left(\frac{f^{a+\frac{b}{x^3}}}{3b^2\log^2(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^3\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^6\log(f)}\right)}{b\log(f)}-\frac{f^{a+\frac{b}{x^3}}}{3bx^9\log(f)}$$

input
```
Int[f^(a + b/x^3)/x^13,x]
```

output
```
-1/3*f^(a + b/x^3)/(b*x^9*Log[f]) - (3*(-1/3*f^(a + b/x^3)/(b*x^6*Log[f])
- (2*(f^(a + b/x^3)/(3*b^2*Log[f]^2) - f^(a + b/x^3)/(3*b*x^3*Log[f])))/(b
*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.71

method	result	size
meijerg	$-\frac{f^{a}\left(6-\frac{\left(-\frac{4b^{3}\ln\left(f\right)^{3}}{x^{9}}+\frac{12b^{2}\ln\left(f\right)^{2}}{x^{6}}-\frac{24b\ln\left(f\right)}{x^{3}}+24\right){\mathrm e}^{\frac{b\ln\left(f\right)}{x^{3}}}}{4}\right)}{3b^{4}\ln\left(f\right)^{4}}$	59
risch	$-\frac{\left(-6x^{9}+6b\,x^{6}\ln\left(f\right)-3b^{2}x^{3}\ln\left(f\right)^{2}+\ln\left(f\right)^{3}b^{3}\right)f^{\frac{a\,x^{3}+b}{x^{3}}}}{3\ln\left(f\right)^{4}b^{4}x^{9}}$	60
parallelrisch	$\frac{6f^{a+\frac{b}{x^{3}}}x^{9}-6b\,f^{a+\frac{b}{x^{3}}}x^{6}\ln\left(f\right)+3b^{2}f^{a+\frac{b}{x^{3}}}x^{3}\ln\left(f\right)^{2}-f^{a+\frac{b}{x^{3}}}\ln\left(f\right)^{3}b^{3}}{3x^{9}\ln\left(f\right)^{4}b^{4}}$	84
norman	$\frac{\frac{x^{6}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln\left(f\right)}}{b^{2}\ln\left(f\right)^{2}}-\frac{x^{3}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln\left(f\right)}}{3b\ln\left(f\right)}-\frac{2x^{9}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln\left(f\right)}}{b^{3}\ln\left(f\right)^{3}}+\frac{2x^{12}{\mathrm e}^{\left(a+\frac{b}{x^{3}}\right)\ln\left(f\right)}}{b^{4}\ln\left(f\right)^{4}}}{x^{12}}$	97

input
```
int(f^(a+b/x^3)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a/b^4/ln(f)^4*(6-1/4*(-4*b^3*ln(f)^3/x^9+12*b^2*ln(f)^2/x^6-24*b*ln
(f)/x^3+24)*exp(b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{\left(6\,x^9 - 6\,bx^6\log\left(f\right) + 3\,b^2x^3\log\left(f\right)^2 - b^3\log\left(f\right)^3\right)f^{\frac{ax^3+b}{x^3}}}{3\,b^4x^9\log\left(f\right)^4}$$

input
```
integrate(f^(a+b/x^3)/x^13,x, algorithm="fricas")
```

output
```
1/3*(6*x^9 - 6*b*x^6*log(f) + 3*b^2*x^3*log(f)^2 - b^3*log(f)^3)*f^((a*x^3
 + b)/x^3)/(b^4*x^9*log(f)^4)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{f^{a+\frac{b}{x^3}}\left(-b^3\log\left(f\right)^3 + 3b^2x^3\log\left(f\right)^2 - 6bx^6\log\left(f\right) + 6x^9\right)}{3b^4x^9\log\left(f\right)^4}$$

input
```
integrate(f**(a+b/x**3)/x**13,x)
```

output
```
f**(a + b/x**3)*(-b**3*log(f)**3 + 3*b**2*x**3*log(f)**2 - 6*b*x**6*log(f)
 + 6*x**9)/(3*b**4*x**9*log(f)**4)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{f^a\Gamma\left(4,-\frac{b\log(f)}{x^3}\right)}{3\,b^4\log(f)^4}$$

input
```
integrate(f^(a+b/x^3)/x^13,x, algorithm="maxima")
```

output
```
1/3*f^a*gamma(4, -b*log(f)/x^3)/(b^4*log(f)^4)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^13,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^13, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx = -\frac{f^{a+\frac{b}{x^3}}\left(\frac{1}{3\,b\,\ln(f)}-\frac{x^3}{b^2\ln(f)^2}+\frac{2\,x^6}{b^3\ln(f)^3}-\frac{2\,x^9}{b^4\ln(f)^4}\right)}{x^9}$$

input
```
int(f^(a + b/x^3)/x^13,x)
```

output
```
-(f^(a + b/x^3)*(1/(3*b*log(f)) - x^3/(b^2*log(f)^2) + (2*x^6)/(b^3*log(f)
^3) - (2*x^9)/(b^4*log(f)^4)))/x^9
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{13}}\,dx = \frac{f^{\frac{a\,x^3+b}{x^3}}\left(-\log(f)^3\,b^3+3\log(f)^2\,b^2x^3-6\log(f)\,b\,x^6+6x^9\right)}{3\log\left(f\right)^4 b^4x^9}$$

input
```
int(f^(a+b/x^3)/x^13,x)
```

output
```
(f**((a*x**3 + b)/x**3)*( - log(f)**3*b**3 + 3*log(f)**2*b**2*x**3 - 6*log
(f)*b*x**6 + 6*x**9))/(3*log(f)**4*b**4*x**9)
```

3.98 $\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = -\frac{f^{a+\frac{b}{x^3}}\left(24x^{12}-24bx^9\log(f)+12b^2x^6\log^2(f)-4b^3x^3\log^3(f)+b^4\log^4(f)\right)}{3b^5x^{12}\log^5(f)}$$

output
```
-1/3*f^(a+b/x^3)*(24*x^12-24*b*x^9*ln(f)+12*b^2*x^6*ln(f)^2-4*b^3*x^3*ln(f
)^3+b^4*ln(f)^4)/b^5/x^12/ln(f)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.35

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = -\frac{f^a\Gamma\left(5,-\frac{b\log(f)}{x^3}\right)}{3b^5\log^5(f)}$$

input
```
Integrate[f^(a + b/x^3)/x^16,x]
```

output
```
-1/3*(f^a*Gamma[5, -((b*Log[f])/x^3)])/(b^5*Log[f]^5)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}} \, dx$$

$$\Big\downarrow \text{ 2647}$$

$$-\frac{f^{a+\frac{b}{x^3}} \left(b^4 \log^4(f) - 4b^3x^3 \log^3(f) + 12b^2x^6 \log^2(f) - 24bx^9 \log(f) + 24x^{12}\right)}{3b^5x^{12} \log^5(f)}$$

input
```
Int[f^(a + b/x^3)/x^16,x]
```

output
```
-1/3*(f^(a + b/x^3)*(24*x^12 - 24*b*x^9*Log[f] + 12*b^2*x^6*Log[f]^2 - 4*b
^3*x^3*Log[f]^3 + b^4*Log[f]^4))/(b^5*x^12*Log[f]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{f^{a}\left(24-\frac{\left(\frac{5b^{4}\ln(f)^{4}}{x^{12}}-\frac{20b^{3}\ln(f)^{3}}{x^{9}}+\frac{60b^{2}\ln(f)^{2}}{x^{6}}-\frac{120b\ln(f)}{x^{3}}+120\right)e^{\frac{b\ln(f)}{x^{3}}}}{5}\right)}{3b^{5}\ln(f)^{5}}$	71
risch	$-\frac{\left(24x^{12}-24b\,x^{9}\ln(f)+12b^{2}x^{6}\ln(f)^{2}-4b^{3}x^{3}\ln(f)^{3}+b^{4}\ln(f)^{4}\right)f^{\frac{a\,x^{3}+b}{x^{3}}}}{3b^{5}\ln(f)^{5}x^{12}}$	72
parallelrisch	$\frac{-24f^{a+\frac{b}{x^{3}}}x^{12}+24f^{a+\frac{b}{x^{3}}}x^{9}b\ln(f)-12f^{a+\frac{b}{x^{3}}}x^{6}b^{2}\ln(f)^{2}+4f^{a+\frac{b}{x^{3}}}x^{3}b^{3}\ln(f)^{3}-f^{a+\frac{b}{x^{3}}}b^{4}\ln(f)^{4}}{3x^{12}b^{5}\ln(f)^{5}}$	105
norman	$\frac{-\frac{x^{3}e^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{3b\ln(f)}+\frac{4x^{6}e^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{3b^{2}\ln(f)^{2}}-\frac{4x^{9}e^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{b^{3}\ln(f)^{3}}+\frac{8x^{12}e^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{b^{4}\ln(f)^{4}}-\frac{8x^{15}e^{\left(a+\frac{b}{x^{3}}\right)\ln(f)}}{b^{5}\ln(f)^{5}}}{x^{15}}$	121

input
```
int(f^(a+b/x^3)/x^16,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a/b^5/ln(f)^5*(24-1/5*(5*b^4*ln(f)^4/x^12-20*b^3*ln(f)^3/x^9+60*b^2*
ln(f)^2/x^6-120*b*ln(f)/x^3+120)*exp(b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x^{3}}}}{x^{16}}\,dx = -\frac{\left(24\,x^{12}-24\,bx^{9}\log\left(f\right)+12\,b^{2}x^{6}\log\left(f\right)^{2}-4\,b^{3}x^{3}\log\left(f\right)^{3}+b^{4}\log\left(f\right)^{4}\right)f^{\frac{ax^{3}+b}{x^{3}}}}{3\,b^{5}x^{12}\log\left(f\right)^{5}}$$

input
```
integrate(f^(a+b/x^3)/x^16,x, algorithm="fricas")
```

output
```
-1/3*(24*x^12 - 24*b*x^9*log(f) + 12*b^2*x^6*log(f)^2 - 4*b^3*x^3*log(f)^3
 + b^4*log(f)^4)*f^((a*x^3 + b)/x^3)/(b^5*x^12*log(f)^5)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = \frac{f^{a+\frac{b}{x^3}}\left(-b^4\log\left(f\right)^4+4b^3x^3\log\left(f\right)^3-12b^2x^6\log\left(f\right)^2+24bx^9\log\left(f\right)-24x^{12}\right)}{3b^5x^{12}\log\left(f\right)^5}$$

input
```
integrate(f**(a+b/x**3)/x**16,x)
```

output
```
f**(a + b/x**3)*(-b**4*log(f)**4 + 4*b**3*x**3*log(f)**3 - 12*b**2*x**6*lo
g(f)**2 + 24*b*x**9*log(f) - 24*x**12)/(3*b**5*x**12*log(f)**5)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.32

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = -\frac{f^a\Gamma\left(5,-\frac{b\log(f)}{x^3}\right)}{3\,b^5\log\left(f\right)^5}$$

input
```
integrate(f^(a+b/x^3)/x^16,x, algorithm="maxima")
```

output
```
-1/3*f^a*gamma(5, -b*log(f)/x^3)/(b^5*log(f)^5)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^16,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^16, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = -\frac{f^{a+\frac{b}{x^3}}\left(\frac{1}{3\,b\,\ln(f)}-\frac{4\,x^3}{3\,b^2\ln(f)^2}+\frac{4\,x^6}{b^3\ln(f)^3}-\frac{8\,x^9}{b^4\ln(f)^4}+\frac{8\,x^{12}}{b^5\ln(f)^5}\right)}{x^{12}}$$

input
```
int(f^(a + b/x^3)/x^16,x)
```

output
```
-(f^(a + b/x^3)*(1/(3*b*log(f)) - (4*x^3)/(3*b^2*log(f)^2) + (4*x^6)/(b^3*
log(f)^3) - (8*x^9)/(b^4*log(f)^4) + (8*x^12)/(b^5*log(f)^5)))/x^12
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{16}}\,dx = \frac{f^{\frac{a\,x^3+b}{x^3}}\left(-\log(f)^4 b^4+4\log(f)^3 b^3x^3-12\log(f)^2 b^2x^6+24\log(f)\,b\,x^9-24x^{12}\right)}{3\log(f)^5 b^5x^{12}}$$

input
```
int(f^(a+b/x^3)/x^16,x)
```

output
```
(f**((a*x**3 + b)/x**3)*( - log(f)**4*b**4 + 4*log(f)**3*b**3*x**3 - 12*lo
g(f)**2*b**2*x**6 + 24*log(f)*b*x**9 - 24*x**12))/(3*log(f)**5*b**5*x**12)
```

3.99 $\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = \frac{f^{a+\frac{b}{x^3}}\left(120x^{15}-120bx^{12}\log(f)+60b^2x^9\log^2(f)-20b^3x^6\log^3(f)+5b^4x^3\log^4(f)-b^5\log^5(f)\right)}{3b^6x^{15}\log^6(f)}$$

output
```
1/3*f^(a+b/x^3)*(120*x^15-120*b*x^12*ln(f)+60*b^2*x^9*ln(f)^2-20*b^3*x^6*l
n(f)^3+5*b^4*x^3*ln(f)^4-b^5*ln(f)^5)/b^6/x^15/ln(f)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.29

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = \frac{f^a\Gamma\left(6,-\frac{b\log(f)}{x^3}\right)}{3b^6\log^6(f)}$$

input
```
Integrate[f^(a + b/x^3)/x^19,x]
```

output
```
(f^a*Gamma[6, -((b*Log[f])/x^3)])/(3*b^6*Log[f]^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}} \, dx$$

$$\Big\downarrow \ 2647$$

$$\frac{f^{a+\frac{b}{x^3}} \left(-b^5 \log^5(f)+5 b^4 x^3 \log^4(f)-20 b^3 x^6 \log^3(f)+60 b^2 x^9 \log^2(f)-120 b x^{12} \log(f)+120 x^{15}\right)}{3 b^6 x^{15} \log^6(f)}$$

input
```
Int[f^(a + b/x^3)/x^19,x]
```

output
```
(f^(a + b/x^3)*(120*x^15 - 120*b*x^12*Log[f] + 60*b^2*x^9*Log[f]^2 - 20*b^
3*x^6*Log[f]^3 + 5*b^4*x^3*Log[f]^4 - b^5*Log[f]^5))/(3*b^6*x^15*Log[f]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

method	result
meijerg	$-\frac{f^{a}\left(120-\frac{\left(-\frac{6 b^{5} \ln\left(f\right)^{5}}{x^{15}}+\frac{30 b^{4} \ln\left(f\right)^{4}}{x^{12}}-\frac{120 b^{3} \ln\left(f\right)^{3}}{x^{9}}+\frac{360 b^{2} \ln\left(f\right)^{2}}{x^{6}}-\frac{720 b \ln\left(f\right)}{x^{3}}+720\right) {\mathrm e}^{\frac{b \ln\left(f\right)}{x^{3}}}}{6}\right)}{3 b^{6} \ln\left(f\right)^{6}}$
risch	$-\frac{\left(-120 x^{15}+120 b \,x^{12} \ln\left(f\right)-60 b^{2} x^{9} \ln\left(f\right)^{2}+20 b^{3} x^{6} \ln\left(f\right)^{3}-5 b^{4} x^{3} \ln\left(f\right)^{4}+b^{5} \ln\left(f\right)^{5}\right) f^{\frac{a \,x^{3}+b}{x^{3}}}}{3 \ln\left(f\right)^{6} b^{6} x^{15}}$
parallelrisch	$\frac{120 f^{a+\frac{b}{x^{3}}} x^{15}-120 f^{a+\frac{b}{x^{3}}} x^{12} \ln\left(f\right) b+60 f^{a+\frac{b}{x^{3}}} x^{9} \ln\left(f\right)^{2} b^{2}-20 f^{a+\frac{b}{x^{3}}} x^{6} \ln\left(f\right)^{3} b^{3}+5 f^{a+\frac{b}{x^{3}}} x^{3} \ln\left(f\right)^{4} b^{4}-f^{a+\frac{b}{x^{3}}} \ln\left(f\right)^{5} b^{5}}{3 x^{15} \ln\left(f\right)^{6} b^{6}}$

input
```
int(f^(a+b/x^3)/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a/b^6/ln(f)^6*(120-1/6*(-6*b^5*ln(f)^5/x^15+30*b^4*ln(f)^4/x^12-120
*b^3*ln(f)^3/x^9+360*b^2*ln(f)^2/x^6-720*b*ln(f)/x^3+720)*exp(b*ln(f)/x^3)
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = \frac{\left(120\,x^{15}-120\,bx^{12}\log\left(f\right)+60\,b^{2}x^{9}\log\left(f\right)^{2}-20\,b^{3}x^{6}\log\left(f\right)^{3}+5\,b^{4}x^{3}\log\left(f\right)^{4}-b^{5}\log\left(f\right)^{5}\right)f^{\frac{ax^{3}+b}{x^{3}}}}{3\,b^{6}x^{15}\log\left(f\right)^{6}}$$

input
```
integrate(f^(a+b/x^3)/x^19,x, algorithm="fricas")
```

output
```
1/3*(120*x^15 - 120*b*x^12*log(f) + 60*b^2*x^9*log(f)^2 - 20*b^3*x^6*log(f
)^3 + 5*b^4*x^3*log(f)^4 - b^5*log(f)^5)*f^((a*x^3 + b)/x^3)/(b^6*x^15*log
(f)^6)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx$$
$$= \frac{f^{a+\frac{b}{x^3}}\left(-b^5\log(f)^5+5b^4x^3\log(f)^4-20b^3x^6\log(f)^3+60b^2x^9\log(f)^2-120bx^{12}\log(f)+120x^{15}\right)}{3b^6x^{15}\log(f)^6}$$

input
```
integrate(f**(a+b/x**3)/x**19,x)
```

output
```
f**(a + b/x**3)*(-b**5*log(f)**5 + 5*b**4*x**3*log(f)**4 - 20*b**3*x**6*lo
g(f)**3 + 60*b**2*x**9*log(f)**2 - 120*b*x**12*log(f) + 120*x**15)/(3*b**6
*x**15*log(f)**6)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.27

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = \frac{f^a\Gamma\left(6,-\frac{b\log(f)}{x^3}\right)}{3\,b^6\log(f)^6}$$

input
```
integrate(f^(a+b/x^3)/x^19,x, algorithm="maxima")
```

output
```
1/3*f^a*gamma(6, -b*log(f)/x^3)/(b^6*log(f)^6)
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^19,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^19, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = -\frac{f^{a+\frac{b}{x^3}}\left(\frac{1}{3\,b\,\ln(f)} - \frac{5\,x^3}{3\,b^2\ln(f)^2} + \frac{20\,x^6}{3\,b^3\ln(f)^3} - \frac{20\,x^9}{b^4\ln(f)^4} + \frac{40\,x^{12}}{b^5\ln(f)^5} - \frac{40\,x^{15}}{b^6\ln(f)^6}\right)}{x^{15}}$$

input
```
int(f^(a + b/x^3)/x^19,x)
```

output
```
-(f^(a + b/x^3)*(1/(3*b*log(f)) - (5*x^3)/(3*b^2*log(f)^2) + (20*x^6)/(3*b
^3*log(f)^3) - (20*x^9)/(b^4*log(f)^4) + (40*x^12)/(b^5*log(f)^5) - (40*x^
15)/(b^6*log(f)^6)))/x^15
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^{19}}\,dx = \frac{f^{\frac{a x^3+b}{x^3}}\left(-\log(f)^5 b^5 + 5\log(f)^4 b^4x^3 - 20\log(f)^3 b^3x^6 + 60\log(f)^2 b^2x^9 - 120\log(f)\,b\,x^{12} + 120x^{15}\right)}{3\log(f)^6 b^6x^{15}}$$

input
```
int(f^(a+b/x^3)/x^19,x)
```

output
```
(f**((a*x**3 + b)/x**3)*( - log(f)**5*b**5 + 5*log(f)**4*b**4*x**3 - 20*lo
g(f)**3*b**3*x**6 + 60*log(f)**2*b**2*x**9 - 120*log(f)*b*x**12 + 120*x**1
5))/(3*log(f)**6*b**6*x**15)
```

3.100 $\int f^{a+\frac{b}{x^3}} x^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+\frac{b}{x^3}} x^4 \, dx = \frac{1}{3} f^a x^5 \Gamma\left(-\frac{5}{3}, -\frac{b \log(f)}{x^3}\right) \left(-\frac{b \log(f)}{x^3}\right)^{5/3}$$

output
```
1/3*f^a*x^5*GAMMA(-5/3,-b*ln(f)/x^3)*(-b*ln(f)/x^3)^(5/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x^4 \, dx = \frac{1}{3} f^a x^5 \Gamma\left(-\frac{5}{3}, -\frac{b \log(f)}{x^3}\right) \left(-\frac{b \log(f)}{x^3}\right)^{5/3}$$

input
```
Integrate[f^(a + b/x^3)*x^4,x]
```

output
```
(f^a*x^5*Gamma[-5/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(5/3))/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{a+\frac{b}{x^3}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{1}{3}x^5 f^a\left(-\frac{b\log(f)}{x^3}\right)^{5/3}\Gamma\left(-\frac{5}{3},-\frac{b\log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x^4,x]
```

output
```
(f^a*x^5*Gamma[-5/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(5/3))/3
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(28) = 56.

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.53

method	result	size
meijerg	$-\frac{f^{a}(-b)^{\frac{5}{3}} \ln(f)^{\frac{5}{3}}\left(\frac{3\ln(f)^{\frac{1}{3}} b^{2}\pi\sqrt{3}}{5x(-b)^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{1}{3}}}-\frac{3x^{5}\left(\frac{3b\ln(f)}{2x^{3}}+1\right)\mathrm{e}^{\frac{b\ln(f)}{x^{3}}}}{5(-b)^{\frac{5}{3}}\ln(f)^{\frac{5}{3}}}-\frac{9\ln(f)^{\frac{1}{3}} b^{2}\Gamma\left(\frac{1}{3},-\frac{b\ln(f)}{x^{3}}\right)}{10x(-b)^{\frac{5}{3}}\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{1}{3}}}\right)}{3}$	120

input
```
int(f^(a+b/x^3)*x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*(-b)^(5/3)*ln(f)^(5/3)*(3/5/x/(-b)^(5/3)*ln(f)^(1/3)*b^2*Pi*3^(1/
2)/GAMMA(2/3)/(-b*ln(f)/x^3)^(1/3)-3/5*x^5/(-b)^(5/3)/ln(f)^(5/3)*(3/2*b*l
n(f)/x^3+1)*exp(b*ln(f)/x^3)-9/10/x/(-b)^(5/3)*ln(f)^(1/3)*b^2/(-b*ln(f)/x
^3)^(1/3)*GAMMA(1/3,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.62

$$\int f^{a+\frac{b}{x^3}}x^4\,dx = -\frac{3}{10}\left(-b\log\left(f\right)\right)^{\frac{2}{3}}bf^{a}\Gamma\left(\frac{1}{3},-\frac{b\log\left(f\right)}{x^{3}}\right)\log\left(f\right)+\frac{1}{10}\left(2\,x^{5}+3\,bx^{2}\log\left(f\right)\right)f^{\frac{ax^{3}+b}{x^{3}}}$$

input
```
integrate(f^(a+b/x^3)*x^4,x, algorithm="fricas")
```

output
```
-3/10*(-b*log(f))^(2/3)*b*f^a*gamma(1/3, -b*log(f)/x^3)*log(f) + 1/10*(2*x
^5 + 3*b*x^2*log(f))*f^((a*x^3 + b)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}x^4\,dx = \int f^{a+\frac{b}{x^{3}}}x^{4}\,dx$$

input
```
integrate(f**(a+b/x**3)*x**4,x)
```

output
```
Integral(f**(a + b/x**3)*x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x^3}}x^4\,dx=\frac{1}{3}\,f^a x^5\left(-\frac{b\log\left(f\right)}{x^3}\right)^{\frac{5}{3}}\Gamma\left(-\frac{5}{3},-\frac{b\log\left(f\right)}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)*x^4,x, algorithm="maxima")
```

output
```
1/3*f^a*x^5*(-b*log(f)/x^3)^(5/3)*gamma(-5/3, -b*log(f)/x^3)
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x^4\,dx=\int f^{a+\frac{b}{x^3}}x^4\,dx$$

input
```
integrate(f^(a+b/x^3)*x^4,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.59

$$\int f^{a+\frac{b}{x^3}}x^4\,dx=\frac{f^a\,f^{\frac{b}{x^3}}\,x^5}{5}+\frac{3\,f^a\,x^5\,\Gamma\left(\frac{1}{3},-\frac{b\,\ln(f)}{x^3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{5/3}}{10}+\frac{3\,b\,f^a\,f^{\frac{b}{x^3}}\,x^2\,\ln\left(f\right)}{10}-\frac{\pi\,\sqrt{3}\,f^a\,x^5\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{5/3}}{5\,\Gamma\left(\frac{2}{3}\right)}$$

input
```
int(f^(a + b/x^3)*x^4,x)
```

output
```
(f^a*f^(b/x^3)*x^5)/5 + (3*f^a*x^5*igamma(1/3, -(b*log(f))/x^3)*(-(b*log(f
))/x^3)^(5/3))/10 + (3*b*f^a*f^(b/x^3)*x^2*log(f))/10 - (3^(1/2)*f^a*x^5*p
i*(-(b*log(f))/x^3)^(5/3))/(5*gamma(2/3))
```

Reduce [F]

$$\int f^{a+\frac{b}{x^3}}x^4\,dx = \frac{-9f^{\frac{a\,x^3+b}{x^3}}\log(f)^2\,b^2+3f^{\frac{a\,x^3+b}{x^3}}\log(f)\,b\,x^3+2f^{\frac{a\,x^3+b}{x^3}}x^6-27\left(\int\frac{f^{\frac{a\,x^3+b}{x^3}}}{x^5}dx\right)\log(f)^3\,b^3x}{10x}$$

input
```
int(f^(a+b/x^3)*x^4,x)
```

output
```
( - 9*f**((a*x**3 + b)/x**3)*log(f)**2*b**2 + 3*f**((a*x**3 + b)/x**3)*log
(f)*b*x**3 + 2*f**((a*x**3 + b)/x**3)*x**6 - 27*int(f**((a*x**3 + b)/x**3)
/x**5,x)*log(f)**3*b**3*x)/(10*x)
```

3.101 $\int f^{a+\frac{b}{x^3}} x^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int f^{a+\frac{b}{x^3}} x^3 \, dx = \frac{1}{3} f^a x^4 \Gamma\left(-\frac{4}{3}, -\frac{b \log(f)}{x^3}\right) \left(-\frac{b \log(f)}{x^3}\right)^{4/3}$$

output
```
1/3*f^a*x^4*GAMMA(-4/3,-b*ln(f)/x^3)*(-b*ln(f)/x^3)^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x^3 \, dx = \frac{1}{3} f^a x^4 \Gamma\left(-\frac{4}{3}, -\frac{b \log(f)}{x^3}\right) \left(-\frac{b \log(f)}{x^3}\right)^{4/3}$$

input
```
Integrate[f^(a + b/x^3)*x^3,x]
```

output
```
(f^a*x^4*Gamma[-4/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(4/3))/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{a+\frac{b}{x^3}} \, dx$$
$$\Big\downarrow \; 2648$$
$$\frac{1}{3} x^4 f^a \left(-\frac{b \log(f)}{x^3}\right)^{4/3} \Gamma\left(-\frac{4}{3}, -\frac{b \log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x^3,x]
```

output
```
(f^a*x^4*Gamma[-4/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(4/3))/3
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 3.38

method	result	size
meijerg	$\frac{f^{a} b \ln(f)^{\frac{4}{3}} (-b)^{\frac{1}{3}} \left(\frac{9 \ln(f)^{\frac{2}{3}} b^{2} \Gamma\left(\frac{2}{3}\right)}{4 x^{2} (-b)^{\frac{4}{3}} \left(-\frac{b \ln(f)}{x^{3}}\right)^{\frac{2}{3}}}-\frac{3 x^{4} \left(\frac{3 b \ln(f)}{x^{3}}+1\right) {\mathrm e}^{\frac{b \ln(f)}{x^{3}}}}{4 (-b)^{\frac{4}{3}} \ln(f)^{\frac{4}{3}}}-\frac{9 \ln(f)^{\frac{2}{3}} b^{2} \Gamma\left(\frac{2}{3}, -\frac{b \ln(f)}{x^{3}}\right)}{4 x^{2} (-b)^{\frac{4}{3}} \left(-\frac{b \ln(f)}{x^{3}}\right)^{\frac{2}{3}}}\right)}{3}$	115

input
```
int(f^(a+b/x^3)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*b*ln(f)^(4/3)*(-b)^(1/3)*(9/4/x^2/(-b)^(4/3)*ln(f)^(2/3)*b^2*GAMMA
(2/3)/(-b*ln(f)/x^3)^(2/3)-3/4*x^4/(-b)^(4/3)/ln(f)^(4/3)*(3*b*ln(f)/x^3+1
)*exp(b*ln(f)/x^3)-9/4/x^2/(-b)^(4/3)*ln(f)^(2/3)*b^2/(-b*ln(f)/x^3)^(2/3)
*GAMMA(2/3,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int f^{a+\frac{b}{x^3}} x^3 \, dx = -\frac{3}{4} \left(-b \log\left(f\right)\right)^{\frac{1}{3}} b f^{a} \Gamma\left(\frac{2}{3}, -\frac{b \log\left(f\right)}{x^{3}}\right) \log\left(f\right) + \frac{1}{4} \left(x^{4} + 3 \, b x \log\left(f\right)\right) f^{\frac{a x^{3} + b}{x^{3}}}$$

input
```
integrate(f^(a+b/x^3)*x^3,x, algorithm="fricas")
```

output
```
-3/4*(-b*log(f))^(1/3)*b*f^a*gamma(2/3, -b*log(f)/x^3)*log(f) + 1/4*(x^4 +
 3*b*x*log(f))*f^((a*x^3 + b)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}} x^3 \, dx = \int f^{a + \frac{b}{x^{3}}} x^{3} \, dx$$

input
```
integrate(f**(a+b/x**3)*x**3,x)
```

output
```
Integral(f**(a + b/x**3)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x^3}}x^3\,dx = \frac{1}{3}\,f^a x^4 \left(-\frac{b\log{(f)}}{x^3}\right)^{\frac{4}{3}}\Gamma\left(-\frac{4}{3},-\frac{b\log{(f)}}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)*x^3,x, algorithm="maxima")
```

output
```
1/3*f^a*x^4*(-b*log(f)/x^3)^(4/3)*gamma(-4/3, -b*log(f)/x^3)
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x^3\,dx = \int f^{a+\frac{b}{x^3}}x^3\,dx$$

input
```
integrate(f^(a+b/x^3)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int f^{a+\frac{b}{x^3}}x^3\,dx = \frac{f^a\,f^{\frac{b}{x^3}}\,x^4}{4} - \frac{3\,f^a\,x^4\,\Gamma\left(\frac{2}{3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{4/3}}{4} + \frac{3\,f^a\,x^4\,\Gamma\left(\frac{2}{3},-\frac{b\,\ln(f)}{x^3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{4/3}}{4} + \frac{3\,b\,f^a\,f^{\frac{b}{x^3}}\,x\,\ln{(f)}}{4}$$

input
```
int(f^(a + b/x^3)*x^3,x)
```

output
```
(f^a*f^(b/x^3)*x^4)/4 - (3*f^a*x^4*gamma(2/3)*(-(b*log(f))/x^3)^(4/3))/4 +
 (3*f^a*x^4*igamma(2/3, -(b*log(f))/x^3)*(-(b*log(f))/x^3)^(4/3))/4 + (3*b
*f^a*f^(b/x^3)*x*log(f))/4
```

Reduce [F]

$$\int f^{a+\frac{b}{x^3}} x^3\,dx$$
$$= \frac{-9f^{\frac{a\,x^3+b}{x^3}}\log(f)^2 b^2 + 6f^{\frac{a\,x^3+b}{x^3}}\log(f)\,b\,x^3 + 2f^{\frac{a\,x^3+b}{x^3}}x^6 - 27\left(\int \frac{f^{\frac{a\,x^3+b}{x^3}}}{x^6}dx\right)\log(f)^3 b^3 x^2}{8x^2}$$

input
```
int(f^(a+b/x^3)*x^3,x)
```

output
```
( - 9*f**((a*x**3 + b)/x**3)*log(f)**2*b**2 + 6*f**((a*x**3 + b)/x**3)*log
(f)*b*x**3 + 2*f**((a*x**3 + b)/x**3)*x**6 - 27*int(f**((a*x**3 + b)/x**3)
/x**6,x)*log(f)**3*b**3*x**2)/(8*x**2)
```

3.102 $\int f^{a+\frac{b}{x^3}} x\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int f^{a+\frac{b}{x^3}} x\,dx = \frac{1}{3} f^a x^2 \Gamma\left(-\frac{2}{3}, -\frac{b\log(f)}{x^3}\right) \left(-\frac{b\log(f)}{x^3}\right)^{2/3}$$

output
```
1/3*f^a*x^2*GAMMA(-2/3,-b*ln(f)/x^3)*(-b*ln(f)/x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x\,dx = \frac{1}{3} f^a x^2 \Gamma\left(-\frac{2}{3}, -\frac{b\log(f)}{x^3}\right) \left(-\frac{b\log(f)}{x^3}\right)^{2/3}$$

input
```
Integrate[f^(a + b/x^3)*x,x]
```

output
```
(f^a*x^2*Gamma[-2/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(2/3))/3
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{1}{3} x^2 f^a \left(-\frac{b \log(f)}{x^3}\right)^{2/3} \Gamma\left(-\frac{2}{3}, -\frac{b \log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x,x]
```

output
```
(f^a*x^2*Gamma[-2/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(2/3))/3
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.09

method	result	size
meijerg	$-\frac{f^a(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}\left(\frac{\ln(f)^{\frac{1}{3}}b\pi\sqrt{3}}{x(-b)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)\left(-\frac{b\ln(f)}{x^3}\right)^{\frac{1}{3}}}-\frac{3x^2e^{\frac{b\ln(f)}{x^3}}}{2(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}}-\frac{3\ln(f)^{\frac{1}{3}}b\Gamma\left(\frac{1}{3},-\frac{b\ln(f)}{x^3}\right)}{2x(-b)^{\frac{2}{3}}\left(-\frac{b\ln(f)}{x^3}\right)^{\frac{1}{3}}}\right)}{3}$	105

input
```
int(f^(a+b/x^3)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*(-b)^(2/3)*ln(f)^(2/3)*(1/x/(-b)^(2/3)*ln(f)^(1/3)*b*Pi*3^(1/2)/G
AMMA(2/3)/(-b*ln(f)/x^3)^(1/3)-3/2*x^2/(-b)^(2/3)/ln(f)^(2/3)*exp(b*ln(f)/
x^3)-3/2/x/(-b)^(2/3)*ln(f)^(1/3)*b/(-b*ln(f)/x^3)^(1/3)*GAMMA(1/3,-b*ln(f
)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int f^{a+\frac{b}{x^3}}x\,dx=\frac{1}{2}f^{\frac{ax^3+b}{x^3}}x^2-\frac{1}{2}(-b\log(f))^{\frac{2}{3}}f^a\Gamma\left(\frac{1}{3},-\frac{b\log(f)}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)*x,x, algorithm="fricas")
```

output
```
1/2*f^((a*x^3 + b)/x^3)*x^2 - 1/2*(-b*log(f))^(2/3)*f^a*gamma(1/3, -b*log(
f)/x^3)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}x\,dx=\int f^{a+\frac{b}{x^3}}x\,dx$$

input
```
integrate(f**(a+b/x**3)*x,x)
```

output
```
Integral(f**(a + b/x**3)*x, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int f^{a+\frac{b}{x^3}}x\,dx = \frac{1}{3}\,f^a x^2 \left(-\frac{b\log(f)}{x^3}\right)^{\frac{2}{3}}\Gamma\left(-\frac{2}{3},-\frac{b\log(f)}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)*x,x, algorithm="maxima")
```

output
```
1/3*f^a*x^2*(-b*log(f)/x^3)^(2/3)*gamma(-2/3, -b*log(f)/x^3)
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x\,dx = \int f^{a+\frac{b}{x^3}}x\,dx$$

input
```
integrate(f^(a+b/x^3)*x,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.06

$$\int f^{a+\frac{b}{x^3}}x\,dx = \frac{f^a\,f^{\frac{b}{x^3}}\,x^2}{2} - \frac{f^a\,x^2\,\Gamma\left(\frac{1}{3},-\frac{b\,\ln(f)}{x^3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{2/3}}{2} + \frac{\pi\,\sqrt{3}\,f^a\,x^2\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{2/3}}{3\,\Gamma\left(\frac{2}{3}\right)}$$

input
```
int(f^(a + b/x^3)*x,x)
```

output
```
(f^a*f^(b/x^3)*x^2)/2 - (f^a*x^2*igamma(1/3, -(b*log(f))/x^3)*(-(b*log(f))
/x^3)^(2/3))/2 + (3^(1/2)*f^a*x^2*pi*(-(b*log(f))/x^3)^(2/3))/(3*gamma(2/3
))
```

Reduce [F]

$$\int f^{a+\frac{b}{x^3}}x\,dx = \frac{-3f^{\frac{a\,x^3+b}{x^3}}\log(f)\,b+f^{\frac{a\,x^3+b}{x^3}}x^3-9\left(\int \frac{f^{\frac{a\,x^3+b}{x^3}}}{x^5}dx\right)\log(f)^2\,b^2x}{2x}$$

input
```
int(f^(a+b/x^3)*x,x)
```

output
```
( - 3*f**((a*x**3 + b)/x**3)*log(f)*b + f**((a*x**3 + b)/x**3)*x**3 - 9*in
t(f**((a*x**3 + b)/x**3)/x**5,x)*log(f)**2*b**2*x)/(2*x)
```

3.103 $\int f^{a+\frac{b}{x^3}} dx$

Optimal result

Integrand size = 9, antiderivative size = 32

$$\int f^{a+\frac{b}{x^3}} dx = \frac{1}{3} f^a x \Gamma\left(-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right) \sqrt[3]{-\frac{b \log(f)}{x^3}}$$

output
```
1/3*f^a*x*GAMMA(-1/3,-b*ln(f)/x^3)*(-b*ln(f)/x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} dx = \frac{1}{3} f^a x \Gamma\left(-\frac{1}{3}, -\frac{b \log(f)}{x^3}\right) \sqrt[3]{-\frac{b \log(f)}{x^3}}$$

input
```
Integrate[f^(a + b/x^3),x]
```

output
```
(f^a*x*Gamma[-1/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(1/3))/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+\frac{b}{x^3}}\,dx$$

$$\Big\downarrow \ 2637$$

$$\frac{1}{3}xf^a\sqrt[3]{-\frac{b\log(f)}{x^3}}\Gamma\left(-\frac{1}{3},-\frac{b\log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3),x]
```

output
```
(f^a*x*Gamma[-1/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^(1/3))/3
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(26) = 52.

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.06

method	result	size
meijerg	$-\frac{f^{a}(-b)^{\frac{1}{3}}\ln(f)^{\frac{1}{3}}\left(\frac{3\ln(f)^{\frac{2}{3}}b\Gamma\left(\frac{2}{3}\right)}{x^{2}(-b)^{\frac{1}{3}}\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{2}{3}}}-\frac{3x\,\mathrm{e}^{\frac{b\ln(f)}{x^{3}}}}{(-b)^{\frac{1}{3}}\ln(f)^{\frac{1}{3}}}-\frac{3\ln(f)^{\frac{2}{3}}b\Gamma\left(\frac{2}{3},-\frac{b\ln(f)}{x^{3}}\right)}{x^{2}(-b)^{\frac{1}{3}}\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{2}{3}}}\right)}{3}$	98

input
```
int(f^(a+b/x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*(-b)^(1/3)*ln(f)^(1/3)*(3/x^2/(-b)^(1/3)*ln(f)^(2/3)*b*GAMMA(2/3)
/(-b*ln(f)/x^3)^(2/3)-3*x/(-b)^(1/3)/ln(f)^(1/3)*exp(b*ln(f)/x^3)-3/x^2/(-
b)^(1/3)*ln(f)^(2/3)*b/(-b*ln(f)/x^3)^(2/3)*GAMMA(2/3,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int f^{a+\frac{b}{x^3}}\,dx = -(-b\log(f))^{\frac{1}{3}}\,f^a\Gamma\left(\frac{2}{3}, -\frac{b\log(f)}{x^3}\right) + f^{\frac{ax^3+b}{x^3}}x$$

input
```
integrate(f^(a+b/x^3),x, algorithm="fricas")
```

output
```
-(-b*log(f))^(1/3)*f^a*gamma(2/3, -b*log(f)/x^3) + f^((a*x^3 + b)/x^3)*x
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}\,dx = \int f^{a+\frac{b}{x^3}}\,dx$$

input
```
integrate(f**(a+b/x**3),x)
```

output
```
Integral(f**(a + b/x**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int f^{a+\frac{b}{x^3}}\,dx = \frac{1}{3}\,f^a x\left(-\frac{b\log(f)}{x^3}\right)^{\frac{1}{3}}\Gamma\left(-\frac{1}{3},-\frac{b\log(f)}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3),x, algorithm="maxima")
```

output
```
1/3*f^a*x*(-b*log(f)/x^3)^(1/3)*gamma(-1/3, -b*log(f)/x^3)
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}\,dx = \int f^{a+\frac{b}{x^3}}\,dx$$

input
```
integrate(f^(a+b/x^3),x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

$$\int f^{a+\frac{b}{x^3}}\,dx = f^a\,x\left(f^{\frac{b}{x^3}}+\Gamma\left(\frac{2}{3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{1/3}-\Gamma\left(\frac{2}{3},-\frac{b\,\ln(f)}{x^3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{1/3}\right)$$

input
```
int(f^(a + b/x^3),x)
```

output
```
f^a*x*(f^(b/x^3) + gamma(2/3)*(-(b*log(f))/x^3)^(1/3) - igamma(2/3, -(b*lo
g(f))/x^3)*(-(b*log(f))/x^3)^(1/3))
```

Reduce [F]

$$\int f^{a+\frac{b}{x^3}}\,dx = \frac{-3f^{\frac{a\,x^3+b}{x^3}}\log(f)\,b + 2f^{\frac{a\,x^3+b}{x^3}}x^3 - 9\left(\int \frac{f^{\frac{a\,x^3+b}{x^3}}}{x^6}dx\right)\log(f)^2\,b^2x^2}{2x^2}$$

input
```
int(f^(a+b/x^3),x)
```

output
```
( - 3*f**((a*x**3 + b)/x**3)*log(f)*b + 2*f**((a*x**3 + b)/x**3)*x**3 - 9*
int(f**((a*x**3 + b)/x**3)/x**6,x)*log(f)**2*b**2*x**2)/(2*x**2)
```

3.104 $\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx = \frac{f^a\Gamma\left(\frac{1}{3},-\frac{b\log(f)}{x^3}\right)}{3x\sqrt[3]{-\frac{b\log(f)}{x^3}}}$$

output
```
1/3*f^a*GAMMA(1/3,-b*ln(f)/x^3)/x/(-b*ln(f)/x^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx = \frac{f^a\Gamma\left(\frac{1}{3},-\frac{b\log(f)}{x^3}\right)}{3x\sqrt[3]{-\frac{b\log(f)}{x^3}}}$$

input
```
Integrate[f^(a + b/x^3)/x^2,x]
```

output
```
(f^a*Gamma[1/3, -((b*Log[f])/x^3)])/(3*x*(-((b*Log[f])/x^3))^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{f^a \Gamma\left(\frac{1}{3},-\frac{b\log(f)}{x^3}\right)}{3x\sqrt[3]{-\frac{b\log(f)}{x^3}}}$$

input
```
Int[f^(a + b/x^3)/x^2,x]
```

output
```
(f^a*Gamma[1/3, -((b*Log[f])/x^3)])/(3*x*(-((b*Log[f])/x^3))^(1/3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(28) = 56.

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

method	result	size
meijerg	$-\frac{f^{a}\left(\frac{2(-b)^{\frac{1}{3}} \ln(f)^{\frac{1}{3}} \pi \sqrt{3}}{3x\Gamma\left(\frac{2}{3}\right)\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{1}{3}}}-\frac{(-b)^{\frac{1}{3}} \ln(f)^{\frac{1}{3}} \Gamma\left(\frac{1}{3},-\frac{b\ln(f)}{x^{3}}\right)}{x\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{1}{3}}}\right)}{3(-b)^{\frac{1}{3}} \ln(f)^{\frac{1}{3}}}$	82

input
```
int(f^(a+b/x^3)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a/(-b)^(1/3)/ln(f)^(1/3)*(2/3/x*(-b)^(1/3)*ln(f)^(1/3)*Pi*3^(1/2)/G
AMMA(2/3)/(-b*ln(f)/x^3)^(1/3)-1/x*(-b)^(1/3)*ln(f)^(1/3)/(-b*ln(f)/x^3)^(
1/3)*GAMMA(1/3,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2}\,dx = -\frac{\left(-b\log\left(f\right)\right)^{\frac{2}{3}} f^{a}\Gamma\left(\frac{1}{3},-\frac{b\log(f)}{x^{3}}\right)}{3\,b\log\left(f\right)}$$

input
```
integrate(f^(a+b/x^3)/x^2,x, algorithm="fricas")
```

output
```
-1/3*(-b*log(f))^(2/3)*f^a*gamma(1/3, -b*log(f)/x^3)/(b*log(f))
```

Sympy [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^2}\,dx$$

input
```
integrate(f**(a+b/x**3)/x**2,x)
```

output
```
Integral(f**(a + b/x**3)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2}\,dx = \frac{f^a\Gamma\left(\frac{1}{3}, -\frac{b\log(f)}{x^3}\right)}{3\,x\left(-\frac{b\log(f)}{x^3}\right)^{\frac{1}{3}}}$$

input
```
integrate(f^(a+b/x^3)/x^2,x, algorithm="maxima")
```

output
```
1/3*f^a*gamma(1/3, -b*log(f)/x^3)/(x*(-b*log(f)/x^3)^(1/3))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^2}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx = -\frac{2 \pi \sqrt{3} f^a - 3 f^a \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{1}{3}, -\frac{b \ln(f)}{x^3}\right)}{9 x \Gamma\left(\frac{2}{3}\right) \left(-\frac{b \ln(f)}{x^3}\right)^{1/3}}$$

input
```
int(f^(a + b/x^3)/x^2,x)
```

output
```
-(2*3^(1/2)*f^a*pi - 3*f^a*gamma(2/3)*igamma(1/3, -(b*log(f))/x^3))/(9*x*g
amma(2/3)*(-(b*log(f))/x^3)^(1/3))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^2} dx = \frac{-f^{\frac{a x^3+b}{x^3}} - 3 \left(\int \frac{f^{\frac{a x^3+b}{x^3}}}{x^5} dx\right) \log(f) b x}{x}$$

input
```
int(f^(a+b/x^3)/x^2,x)
```

output
```
( - f**((a*x**3 + b)/x**3) - 3*int(f**((a*x**3 + b)/x**3)/x**5,x)*log(f)*b
*x)/x
```

3.105 $\int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx = \frac{f^a \Gamma\left(\frac{2}{3}, -\frac{b\log(f)}{x^3}\right)}{3x^2\left(-\frac{b\log(f)}{x^3}\right)^{2/3}}$$

output
```
1/3*f^a*GAMMA(2/3,-b*ln(f)/x^3)/x^2/(-b*ln(f)/x^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3} dx = \frac{f^a \Gamma\left(\frac{2}{3}, -\frac{b\log(f)}{x^3}\right)}{3x^2\left(-\frac{b\log(f)}{x^3}\right)^{2/3}}$$

input
```
Integrate[f^(a + b/x^3)/x^3,x]
```

output
```
(f^a*Gamma[2/3, -((b*Log[f])/x^3)])/(3*x^2*(-((b*Log[f])/x^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3} \, dx$$

$$\Big\downarrow \ 2648$$

$$\frac{f^a \Gamma\left(\frac{2}{3},-\frac{b \log(f)}{x^3}\right)}{3 x^2 \left(-\frac{b \log(f)}{x^3}\right)^{2/3}}$$

input
```
Int[f^(a + b/x^3)/x^3,x]
```

output
```
(f^a*Gamma[2/3, -((b*Log[f])/x^3)])/(3*x^2*(-((b*Log[f])/x^3))^(2/3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(28) = 56$.

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

method	result	size
meijerg	$\frac{f^{a}(-b)^{\frac{1}{3}}\left(\frac{(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}{x^{2}\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{2}{3}}}-\frac{(-b)^{\frac{2}{3}}\ln(f)^{\frac{2}{3}}\Gamma\left(\frac{2}{3},-\frac{b\ln(f)}{x^{3}}\right)}{x^{2}\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{2}{3}}}\right)}{3b\ln(f)^{\frac{2}{3}}}$	78

input
```
int(f^(a+b/x^3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a/b/ln(f)^(2/3)*(-b)^(1/3)*(1/x^2*(-b)^(2/3)*ln(f)^(2/3)*GAMMA(2/3)/
(-b*ln(f)/x^3)^(2/3)-1/x^2*(-b)^(2/3)*ln(f)^(2/3)/(-b*ln(f)/x^3)^(2/3)*GAM
MA(2/3,-b*ln(f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx = -\frac{\left(-b\log\left(f\right)\right)^{\frac{1}{3}} f^{a}\Gamma\left(\frac{2}{3}, -\frac{b\log(f)}{x^3}\right)}{3\,b\log\left(f\right)}$$

input
```
integrate(f^(a+b/x^3)/x^3,x, algorithm="fricas")
```

output
```
-1/3*(-b*log(f))^(1/3)*f^a*gamma(2/3, -b*log(f)/x^3)/(b*log(f))
```

Sympy [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx$$

input
```
integrate(f**(a+b/x**3)/x**3,x)
```

output
```
Integral(f**(a + b/x**3)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx = \frac{f^a\Gamma\left(\frac{2}{3}, -\frac{b\log(f)}{x^3}\right)}{3\,x^2\left(-\frac{b\log(f)}{x^3}\right)^{\frac{2}{3}}}$$

input
```
integrate(f^(a+b/x^3)/x^3,x, algorithm="maxima")
```

output
```
1/3*f^a*gamma(2/3, -b*log(f)/x^3)/(x^2*(-b*log(f)/x^3)^(2/3))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx = -\frac{f^a\left(\Gamma\left(\frac{2}{3}\right)-\Gamma\left(\frac{2}{3},-\frac{b\ln(f)}{x^3}\right)\right)}{3\,x^2\left(-\frac{b\ln(f)}{x^3}\right)^{2/3}}$$

input
```
int(f^(a + b/x^3)/x^3,x)
```

output
```
-(f^a*(gamma(2/3) - igamma(2/3, -(b*log(f))/x^3)))/(3*x^2*(-(b*log(f))/x^3
)^(2/3))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^3}\,dx = \frac{-f^{\frac{a\,x^3+b}{x^3}}-3\left(\int \frac{f^{\frac{a\,x^3+b}{x^3}}}{x^6}dx\right)\log(f)\,b\,x^2}{2x^2}$$

input
```
int(f^(a+b/x^3)/x^3,x)
```

output
```
( - f**((a*x**3 + b)/x**3) - 3*int(f**((a*x**3 + b)/x**3)/x**6,x)*log(f)*b
*x**2)/(2*x**2)
```

3.106 $\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx=\frac{f^a\Gamma\left(\frac{4}{3},-\frac{b\log(f)}{x^3}\right)}{3x^4\left(-\frac{b\log(f)}{x^3}\right)^{4/3}}$$

output
```
1/3*f^a*GAMMA(4/3,-b*ln(f)/x^3)/x^4/(-b*ln(f)/x^3)^(4/3)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx=\frac{f^a\Gamma\left(\frac{4}{3},-\frac{b\log(f)}{x^3}\right)}{3x^4\left(-\frac{b\log(f)}{x^3}\right)^{4/3}}$$

input
```
Integrate[f^(a + b/x^3)/x^5,x]
```

output
```
(f^a*Gamma[4/3, -((b*Log[f])/x^3)])/(3*x^4*(-((b*Log[f])/x^3))^(4/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{f^a \Gamma\left(\frac{4}{3},-\frac{b \log(f)}{x^3}\right)}{3x^4 \left(-\frac{b \log(f)}{x^3}\right)^{4/3}}$$

input
```
Int[f^(a + b/x^3)/x^5,x]
```

output
```
(f^a*Gamma[4/3, -((b*Log[f])/x^3)])/(3*x^4*(-((b*Log[f])/x^3))^(4/3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(28) = 56.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.29

method	result	size
meijerg	$-\frac{f^{a}\left(-\frac{2(-b)^{\frac{4}{3}}\ln(f)^{\frac{1}{3}}\pi\sqrt{3}}{9xb\Gamma\left(\frac{2}{3}\right)\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{1}{3}}}+\frac{(-b)^{\frac{4}{3}}\ln(f)^{\frac{1}{3}}\mathrm{e}^{\frac{b\ln(f)}{x^{3}}}}{xb}+\frac{(-b)^{\frac{4}{3}}\ln(f)^{\frac{1}{3}}\Gamma\left(\frac{1}{3},-\frac{b\ln(f)}{x^{3}}\right)}{3xb\left(-\frac{b\ln(f)}{x^{3}}\right)^{\frac{1}{3}}}\right)}{3(-b)^{\frac{4}{3}}\ln(f)^{\frac{4}{3}}}$	112

input
```
int(f^(a+b/x^3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a/(-b)^(4/3)/ln(f)^(4/3)*(-2/9/x*(-b)^(4/3)*ln(f)^(1/3)/b*Pi*3^(1/2
)/GAMMA(2/3)/(-b*ln(f)/x^3)^(1/3)+1/x*(-b)^(4/3)*ln(f)^(1/3)/b*exp(b*ln(f)
/x^3)+1/3/x*(-b)^(4/3)*ln(f)^(1/3)/b/(-b*ln(f)/x^3)^(1/3)*GAMMA(1/3,-b*ln(
f)/x^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx = \frac{\left(-b\log\left(f\right)\right)^{\frac{2}{3}} f^{a} x\Gamma\left(\frac{1}{3}, -\frac{b\log(f)}{x^{3}}\right) - 3\,bf^{\frac{ax^{3}+b}{x^{3}}}\log\left(f\right)}{9\,b^{2}x\log\left(f\right)^{2}}$$

input
```
integrate(f^(a+b/x^3)/x^5,x, algorithm="fricas")
```

output
```
1/9*((-b*log(f))^(2/3)*f^a*x*gamma(1/3, -b*log(f)/x^3) - 3*b*f^((a*x^3 + b
)/x^3)*log(f))/(b^2*x*log(f)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate(f**(a+b/x**3)/x**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx = \frac{f^a\Gamma\left(\frac{4}{3}, -\frac{b\log(f)}{x^3}\right)}{3\,x^4\left(-\frac{b\log(f)}{x^3}\right)^{\frac{4}{3}}}$$

input
```
integrate(f^(a+b/x^3)/x^5,x, algorithm="maxima")
```

output
```
1/3*f^a*gamma(4/3, -b*log(f)/x^3)/(x^4*(-b*log(f)/x^3)^(4/3))
```

Giac [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx = \int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx$$

input
```
integrate(f^(a+b/x^3)/x^5,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)/x^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.26

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx = \frac{f^a\,\Gamma\left(\frac{1}{3},-\frac{b\,\ln(f)}{x^3}\right)}{9\,x^4\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{4/3}} - \frac{f^a\,f^{\frac{b}{x^3}}}{3\,b\,x\,\ln(f)} - \frac{2\,\pi\,\sqrt{3}\,f^a}{27\,x^4\,\Gamma\left(\frac{2}{3}\right)\,\left(-\frac{b\,\ln(f)}{x^3}\right)^{4/3}}$$

input
```
int(f^(a + b/x^3)/x^5,x)
```

output
```
(f^a*igamma(1/3, -(b*log(f))/x^3))/(9*x^4*(-(b*log(f))/x^3)^(4/3)) - (f^a*
f^(b/x^3))/(3*b*x*log(f)) - (2*3^(1/2)*f^a*pi)/(27*x^4*gamma(2/3)*(-(b*log
(f))/x^3)^(4/3))
```

Reduce [F]

$$\int \frac{f^{a+\frac{b}{x^3}}}{x^5}\,dx = \int \frac{f^{\frac{a\,x^3+b}{x^3}}}{x^5}dx$$

input
```
int(f^(a+b/x^3)/x^5,x)
```

output
```
int(f**((a*x**3 + b)/x**3)/x**5,x)
```

3.107 $\int f^{a+bx^3} x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int f^{a+bx^3} x^m \, dx = -\frac{1}{3} f^a x^{1+m} \Gamma\left(\frac{1+m}{3}, -bx^3 \log(f)\right) \left(-bx^3 \log(f)\right)^{\frac{1}{3}(-1-m)}$$

output
```
-1/3*f^a*x^(1+m)*GAMMA(1/3+1/3*m,-b*x^3*ln(f))*(-b*x^3*ln(f))^(-1/3-1/3*m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int f^{a+bx^3} x^m \, dx = -\frac{1}{3} f^a x^{1+m} \Gamma\left(\frac{1+m}{3}, -bx^3 \log(f)\right) \left(-bx^3 \log(f)\right)^{\frac{1}{3}(-1-m)}$$

input
```
Integrate[f^(a + b*x^3)*x^m,x]
```

output
```
-1/3*(f^a*x^(1 + m)*Gamma[(1 + m)/3, -(b*x^3*Log[f])]*(-(b*x^3*Log[f]))^((
-1 - m)/3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+bx^3} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{1}{3} f^a x^{m+1} \left(-bx^3 \log(f)\right)^{\frac{1}{3}(-m-1)} \Gamma\left(\frac{m+1}{3}, -bx^3 \log(f)\right)$$

input
```
Int[f^(a + b*x^3)*x^m,x]
```

output
```
-1/3*(f^a*x^(1 + m)*Gamma[(1 + m)/3, -(b*x^3*Log[f])]*(-(b*x^3*Log[f]))^((
-1 - m)/3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(38) = 76.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.04

method	result
meijerg	$\frac{f^{a}(-b)^{-\frac{1}{3}-\frac{m}{3}}\ln(f)^{-\frac{1}{3}-\frac{m}{3}}\left(\frac{3x^{1+m}(-b)^{\frac{1}{3}+\frac{m}{3}}\ln(f)^{\frac{1}{3}+\frac{m}{3}}\left(\frac{1}{3}+\frac{m}{3}\right)\left(-b\,x^{3}\ln(f)\right)^{-\frac{1}{3}-\frac{m}{3}}\Gamma\left(\frac{1}{3}+\frac{m}{3}\right)}{1+m}+\frac{3x^{1+m}(-b)^{\frac{1}{3}+\frac{m}{3}}\ln(f)^{\frac{1}{3}+\frac{m}{3}}\left(-\frac{1}{3}-\right.}{}\right.}{3}$

input
```
int(f^(b*x^3+a)*x^m,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^a*(-b)^(-1/3-1/3*m)*ln(f)^(-1/3-1/3*m)*(3/(1+m)*x^(1+m)*(-b)^(1/3+1/
3*m)*ln(f)^(1/3+1/3*m)*(1/3+1/3*m)*(-b*x^3*ln(f))^(-1/3-1/3*m)*GAMMA(1/3+1
/3*m)+3/(1+m)*x^(1+m)*(-b)^(1/3+1/3*m)*ln(f)^(1/3+1/3*m)*(-1/3-1/3*m)*(-b*
x^3*ln(f))^(-1/3-1/3*m)*GAMMA(1/3+1/3*m,-b*x^3*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int f^{a+bx^3}x^m\,dx=\frac{e^{\left(-\frac{1}{3}\,(m-2)\log(-b\log(f))+a\log(f)\right)}\Gamma\left(\frac{1}{3}\,m+\frac{1}{3},-bx^3\log\left(f\right)\right)}{3\,b\log\left(f\right)}$$

input
```
integrate(f^(b*x^3+a)*x^m,x, algorithm="fricas")
```

output
```
1/3*e^(-1/3*(m - 2)*log(-b*log(f)) + a*log(f))*gamma(1/3*m + 1/3, -b*x^3*l
og(f))/(b*log(f))
```

Sympy [F]

$$\int f^{a+bx^3}x^m\,dx=\int f^{a+bx^3}x^m\,dx$$

input
```
integrate(f**(b*x**3+a)*x**m,x)
```

output
```
Integral(f**(a + b*x**3)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int f^{a+bx^3}x^m\,dx = -\frac{1}{3}\left(-bx^3\log\left(f\right)\right)^{-\frac{1}{3}m-\frac{1}{3}}f^ax^{m+1}\Gamma\left(\frac{1}{3}m+\frac{1}{3},-bx^3\log\left(f\right)\right)$$

input
```
integrate(f^(b*x^3+a)*x^m,x, algorithm="maxima")
```

output
```
-1/3*(-b*x^3*log(f))^(-1/3*m - 1/3)*f^a*x^(m + 1)*gamma(1/3*m + 1/3, -b*x^
3*log(f))
```

Giac [F]

$$\int f^{a+bx^3}x^m\,dx = \int f^{bx^3+a}x^m\,dx$$

input
```
integrate(f^(b*x^3+a)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(b*x^3 + a)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.22

$$\int f^{a+bx^3}x^m\,dx = \frac{f^a\,x^{m+1}\,\mathrm{e}^{\frac{b\,x^3\,\ln(f)}{2}}\,\mathrm{M}_{\frac{1}{3}-\frac{m}{6},\frac{m}{6}+\frac{1}{6}}\left(b\,x^3\,\ln\left(f\right)\right)}{\left(m+1\right)\,{\left(b\,x^3\,\ln\left(f\right)\right)}^{\frac{m}{6}+\frac{2}{3}}}$$

input
```
int(f^(a + b*x^3)*x^m,x)
```

output
```
(f^a*x^(m + 1)*exp((b*x^3*log(f))/2)*whittakerM(1/3 - m/6, m/6 + 1/6, b*x^
3*log(f)))/((m + 1)*(b*x^3*log(f))^(m/6 + 2/3))
```

Reduce [F]

$$\int f^{a+bx^3} x^m \, dx = f^a \left(\int x^m f^{b\,x^3} dx \right)$$

input
```
int(f^(b*x^3+a)*x^m,x)
```

output
```
f**a*int(x**m*f**(b*x**3),x)
```

3.108 $\int f^{a+bx^2} x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int f^{a+bx^2} x^m \, dx = -\frac{1}{2} f^a x^{1+m} \Gamma\left(\frac{1+m}{2}, -bx^2 \log(f)\right) \left(-bx^2 \log(f)\right)^{\frac{1}{2}(-1-m)}$$

output
```
-1/2*f^a*x^(1+m)*GAMMA(1/2+1/2*m,-b*x^2*ln(f))*(-b*x^2*ln(f))^(-1/2-1/2*m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int f^{a+bx^2} x^m \, dx = -\frac{1}{2} f^a x^{1+m} \Gamma\left(\frac{1+m}{2}, -bx^2 \log(f)\right) \left(-bx^2 \log(f)\right)^{\frac{1}{2}(-1-m)}$$

input
```
Integrate[f^(a + b*x^2)*x^m,x]
```

output
```
-1/2*(f^a*x^(1 + m)*Gamma[(1 + m)/2, -(b*x^2*Log[f])]*(-(b*x^2*Log[f]))^((
-1 - m)/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+bx^2}\,dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{1}{2}f^a x^{m+1}\left(-bx^2\log(f)\right)^{\frac{1}{2}(-m-1)}\Gamma\left(\frac{m+1}{2},-bx^2\log(f)\right)$$

input
```
Int[f^(a + b*x^2)*x^m,x]
```

output
```
-1/2*(f^a*x^(1 + m)*Gamma[(1 + m)/2, -(b*x^2*Log[f])]*(-(b*x^2*Log[f]))^((
-1 - m)/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(38) = 76$.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.04

method	result
meijerg	$\frac{f^{a}(-b)^{-\frac{1}{2}-\frac{m}{2}}\ln(f)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}(-b)^{\frac{1}{2}+\frac{m}{2}}\ln(f)^{\frac{1}{2}+\frac{m}{2}}\left(\frac{1}{2}+\frac{m}{2}\right)\left(-b\,x^{2}\ln(f)\right)^{-\frac{1}{2}-\frac{m}{2}}\Gamma\left(\frac{1}{2}+\frac{m}{2}\right)}{1+m}+\frac{2x^{1+m}(-b)^{\frac{1}{2}+\frac{m}{2}}\ln(f)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{1}{2}-\right.}{}\right.}{2}$

input
```
int(f^(b*x^2+a)*x^m,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^a*(-b)^(-1/2-1/2*m)*ln(f)^(-1/2-1/2*m)*(2/(1+m)*x^(1+m)*(-b)^(1/2+1/
2*m)*ln(f)^(1/2+1/2*m)*(1/2+1/2*m)*(-b*x^2*ln(f))^(-1/2-1/2*m)*GAMMA(1/2+1
/2*m)+2/(1+m)*x^(1+m)*(-b)^(1/2+1/2*m)*ln(f)^(1/2+1/2*m)*(-1/2-1/2*m)*(-b*
x^2*ln(f))^(-1/2-1/2*m)*GAMMA(1/2+1/2*m,-b*x^2*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int f^{a+bx^2}x^m\,dx=\frac{e^{\left(-\frac{1}{2}\,(m-1)\log(-b\log(f))+a\log(f)\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-bx^2\log\left(f\right)\right)}{2\,b\log\left(f\right)}$$

input
```
integrate(f^(b*x^2+a)*x^m,x, algorithm="fricas")
```

output
```
1/2*e^(-1/2*(m - 1)*log(-b*log(f)) + a*log(f))*gamma(1/2*m + 1/2, -b*x^2*l
og(f))/(b*log(f))
```

Sympy [F]

$$\int f^{a+bx^2}x^m\,dx=\int f^{a+bx^2}x^m\,dx$$

input
```
integrate(f**(b*x**2+a)*x**m,x)
```

output
```
Integral(f**(a + b*x**2)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int f^{a+bx^2}x^m\,dx = -\frac{1}{2}\left(-bx^2\log(f)\right)^{-\frac{1}{2}m-\frac{1}{2}}f^a x^{m+1}\Gamma\left(\frac{1}{2}m+\frac{1}{2},-bx^2\log(f)\right)$$

input
```
integrate(f^(b*x^2+a)*x^m,x, algorithm="maxima")
```

output
```
-1/2*(-b*x^2*log(f))^(-1/2*m - 1/2)*f^a*x^(m + 1)*gamma(1/2*m + 1/2, -b*x^
2*log(f))
```

Giac [F]

$$\int f^{a+bx^2}x^m\,dx = \int f^{bx^2+a}x^m\,dx$$

input
```
integrate(f^(b*x^2+a)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(b*x^2 + a)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int f^{a+bx^2}x^m\,dx = \frac{f^a\,x^{m+1}\left(\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)-\Gamma\left(\frac{m}{2}+\frac{1}{2},-b\,x^2\,\ln(f)\right)\right)}{2\left(-b\,x^2\,\ln(f)\right)^{\frac{m}{2}+\frac{1}{2}}}$$

input
```
int(f^(a + b*x^2)*x^m,x)
```

output
```
(f^a*x^(m + 1)*(gamma(m/2 + 1/2) - igamma(m/2 + 1/2, -b*x^2*log(f))))/(2*(
-b*x^2*log(f))^(m/2 + 1/2))
```

Reduce [F]

$$\int f^{a+bx^2} x^m \, dx = f^a \left(\int x^m f^{b\,x^2} dx \right)$$

input
```
int(f^(b*x^2+a)*x^m,x)
```

output
```
f**a*int(x**m*f**(b*x**2),x)
```

3.109 $\int f^{a+bx}x^m\,dx$

Optimal result

Integrand size = 11, antiderivative size = 34

$$\int f^{a+bx}x^m\,dx = \frac{f^a x^m \Gamma(1+m,-bx\log(f))(-bx\log(f))^{-m}}{b\log(f)}$$

output
```
f^a*x^m*GAMMA(1+m,-b*x*ln(f))/b/ln(f)/((-b*x*ln(f))^m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx}x^m\,dx = \frac{f^a x^m \Gamma(1+m,-bx\log(f))(-bx\log(f))^{-m}}{b\log(f)}$$

input
```
Integrate[f^(a + b*x)*x^m,x]
```

output
```
(f^a*x^m*Gamma[1 + m, -(b*x*Log[f])])/(b*Log[f]*(-(b*x*Log[f]))^m)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+bx}\,dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{f^a x^m (-bx\log(f))^{-m}\Gamma(m+1,-bx\log(f))}{b\log(f)}$$

input
```
Int[f^(a + b*x)*x^m,x]
```

output
```
(f^a*x^m*Gamma[1 + m, -(b*x*Log[f])])/(b*Log[f]*(-(b*x*Log[f]))^m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.06

method	result
meijerg	$-\frac{f^{a}(-b)^{-m}\ln(f)^{-1-m}\left(x^{m}(-b)^{m}\ln(f)^{m}m\Gamma(m)(-bx\ln(f))^{-m}-x^{m}(-b)^{m}\ln(f)^{m}\mathrm{e}^{bx\ln(f)}-x^{m}(-b)^{m}\ln(f)^{m}m(-bx\ln(f))^{-m}\right.}{b}$

input
```
int(f^(b*x+a)*x^m,x,method=_RETURNVERBOSE)
```

output
```
-f^a*(-b)^(-m)*ln(f)^(-1-m)/b*(x^m*(-b)^m*ln(f)^m*m*GAMMA(m)*(-b*x*ln(f))^
(-m)-x^m*(-b)^m*ln(f)^m*exp(b*x*ln(f))-x^m*(-b)^m*ln(f)^m*m*(-b*x*ln(f))^(
-m)*GAMMA(m,-b*x*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int f^{a+bx}x^m\,dx = \frac{e^{(-m\log(-b\log(f))+a\log(f))}\Gamma(m+1,-bx\log(f))}{b\log(f)}$$

input
```
integrate(f^(b*x+a)*x^m,x, algorithm="fricas")
```

output
```
e^(-m*log(-b*log(f)) + a*log(f))*gamma(m + 1, -b*x*log(f))/(b*log(f))
```

Sympy [F]

$$\int f^{a+bx}x^m\,dx = \int f^{a+bx}x^m\,dx$$

input
```
integrate(f**(b*x+a)*x**m,x)
```

output
```
Integral(f**(a + b*x)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int f^{a+bx}x^m\,dx = -(-bx\log(f))^{-m-1}f^a x^{m+1}\Gamma(m+1,-bx\log(f))$$

input
```
integrate(f^(b*x+a)*x^m,x, algorithm="maxima")
```

output
```
-(-b*x*log(f))^(-m - 1)*f^a*x^(m + 1)*gamma(m + 1, -b*x*log(f))
```

Giac [F]

$$\int f^{a+bx}x^m\,dx = \int f^{bx+a}x^m\,dx$$

input
```
integrate(f^(b*x+a)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(b*x + a)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int f^{a+bx}x^m\,dx = \frac{f^a\,x^m\,\Gamma(m+1,-b\,x\,\ln(f))}{b\,\ln(f)\,(-b\,x\,\ln(f))^m}$$

input
```
int(f^(a + b*x)*x^m,x)
```

output
```
(f^a*x^m*igamma(m + 1, -b*x*log(f)))/(b*log(f)*(-b*x*log(f))^m)
```

Reduce [F]

$$\int f^{a+bx}x^m\,dx = \frac{f^a\left(x^m f^{bx} - \left(\int \frac{x^m f^{bx}}{x}dx\right)m\right)}{\log\left(f\right)b}$$

input
```
int(f^(b*x+a)*x^m,x)
```

output
```
(f**a*(x**m*f**(b*x) - int((x**m*f**(b*x))/x,x)*m))/(log(f)*b)
```

3.110 $\int f^{a+\frac{b}{x}} x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int f^{a+\frac{b}{x}} x^m \, dx = f^a x^{1+m} \Gamma\left(-1-m,-\frac{b\log(f)}{x}\right)\left(-\frac{b\log(f)}{x}\right)^{1+m}$$

output
```
f^a*x^(1+m)*GAMMA(-1-m,-b*ln(f)/x)*(-b*ln(f)/x)^(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}} x^m \, dx = f^a x^{1+m} \Gamma\left(-1-m,-\frac{b\log(f)}{x}\right)\left(-\frac{b\log(f)}{x}\right)^{1+m}$$

input
```
Integrate[f^(a + b/x)*x^m,x]
```

output
```
f^a*x^(1 + m)*Gamma[-1 - m, -((b*Log[f])/x)]*(-((b*Log[f])/x))^(1 + m)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+\frac{b}{x}} \, dx$$

$$\Big\downarrow \; 2648$$

$$f^a x^{m+1} \left(-\frac{b \log(f)}{x}\right)^{m+1} \Gamma\left(-m-1, -\frac{b \log(f)}{x}\right)$$

input
```
Int[f^(a + b/x)*x^m,x]
```

output
```
f^a*x^(1 + m)*Gamma[-1 - m, -((b*Log[f])/x)]*(-((b*Log[f])/x))^(1 + m)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(35) = 70.

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.89

method	result
meijerg	$f^a(-b)^m \ln(f)^{1+m} b\left(-\frac{x^m(-b)^{-m}\ln(f)^{-m}\Gamma(-m)\left(-\frac{b\ln(f)}{x}\right)^m}{1+m} + \frac{x^{1+m}\ln(f)^{-1-m}(-b)^{-m}e^{\frac{b\ln(f)}{x}}}{(1+m)b} + \frac{x^m(-b)^{-m}\ln\cdots}{\cdots}\right.$

input
```
int(f^(a+b/x)*x^m,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^m*ln(f)^(1+m)*b*(-1/(1+m)*x^m*(-b)^(-m)*ln(f)^(-m)*GAMMA(-m)*(-b*
ln(f)/x)^m+1/(1+m)*x^(1+m)/b*ln(f)^(-1-m)*(-b)^(-m)*exp(b*ln(f)/x)+1/(1+m)
*x^m*(-b)^(-m)*ln(f)^(-m)*(-b*ln(f)/x)^m*GAMMA(-m,-b*ln(f)/x))
```

Fricas [F]

$$\int f^{a+\frac{b}{x}} x^m \, dx = \int f^{a+\frac{b}{x}} x^m \, dx$$

input
```
integrate(f^(a+b/x)*x^m,x, algorithm="fricas")
```

output
```
integral(f^((a*x + b)/x)*x^m, x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x}} x^m \, dx = \int f^{a+\frac{b}{x}} x^m \, dx$$

input
```
integrate(f**(a+b/x)*x**m,x)
```

output
```
Integral(f**(a + b/x)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x}}x^m\,dx = f^a x^{m+1}\left(-\frac{b\log(f)}{x}\right)^{m+1}\Gamma\left(-m-1,-\frac{b\log(f)}{x}\right)$$

input
```
integrate(f^(a+b/x)*x^m,x, algorithm="maxima")
```

output
```
f^a*x^(m + 1)*(-b*log(f)/x)^(m + 1)*gamma(-m - 1, -b*log(f)/x)
```

Giac [F]

$$\int f^{a+\frac{b}{x}}x^m\,dx = \int f^{a+\frac{b}{x}}x^m\,dx$$

input
```
integrate(f^(a+b/x)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.49

$$\int f^{a+\frac{b}{x}}x^m\,dx = \frac{f^a\,x^{m+1}\,\mathrm{e}^{\frac{b\,\ln(f)}{2\,x}}\,\mathrm{M}_{\frac{m}{2}+1,-\frac{m}{2}-\frac{1}{2}}\left(\frac{b\,\ln(f)}{x}\right)\,\left(\frac{b\,\ln(f)}{x}\right)^{m/2}}{m+1}$$

input
```
int(f^(a + b/x)*x^m,x)
```

output
```
(f^a*x^(m + 1)*exp((b*log(f))/(2*x))*whittakerM(m/2 + 1, - m/2 - 1/2, (b*l
og(f))/x)*((b*log(f))/x)^(m/2))/(m + 1)
```

Reduce [F]

$$\int f^{a+\frac{b}{x}} x^m \, dx = \int x^m f^{\frac{ax+b}{x}} \, dx$$

input
```
int(f^(a+b/x)*x^m,x)
```

output
```
int(x**m*f**((a*x + b)/x),x)
```

3.111 $\int f^{a+\frac{b}{x^2}}x^m\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int f^{a+\frac{b}{x^2}}x^m\,dx=\frac{1}{2}f^ax^{1+m}\Gamma\left(\frac{1}{2}(-1-m),-\frac{b\log(f)}{x^2}\right)\left(-\frac{b\log(f)}{x^2}\right)^{\frac{1+m}{2}}$$

output
```
1/2*f^a*x^(1+m)*GAMMA(-1/2-1/2*m,-b*ln(f)/x^2)*(-b*ln(f)/x^2)^(1/2+1/2*m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^2}}x^m\,dx=\frac{1}{2}f^ax^{1+m}\Gamma\left(\frac{1}{2}(-1-m),-\frac{b\log(f)}{x^2}\right)\left(-\frac{b\log(f)}{x^2}\right)^{\frac{1+m}{2}}$$

input
```
Integrate[f^(a + b/x^2)*x^m,x]
```

output
```
(f^a*x^(1 + m)*Gamma[(-1 - m)/2, -((b*Log[f])/x^2)]*(-((b*Log[f])/x^2))^((
1 + m)/2))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+\frac{b}{x^2}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{1}{2}f^a x^{m+1}\left(-\frac{b\log(f)}{x^2}\right)^{\frac{m+1}{2}}\Gamma\left(\frac{1}{2}(-m-1),-\frac{b\log(f)}{x^2}\right)$$

input
```
Int[f^(a + b/x^2)*x^m,x]
```

output
```
(f^a*x^(1 + m)*Gamma[(-1 - m)/2, -((b*Log[f])/x^2)]*(-((b*Log[f])/x^2))^((
1 + m)/2))/2
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(38) = 76.

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.67

method	result
meijerg	$-\frac{f^{a}(-b)^{\frac{1}{2}+\frac{m}{2}}\ln(f)^{\frac{1}{2}+\frac{m}{2}}\left(\frac{2x^{-1+m}(-b)^{-\frac{1}{2}-\frac{m}{2}}\ln(f)^{\frac{1}{2}-\frac{m}{2}}b\left(-\frac{b\ln(f)}{x^{2}}\right)^{-\frac{1}{2}+\frac{m}{2}}\Gamma\left(\frac{1}{2}-\frac{m}{2}\right)}{1+m}-\frac{2x^{1+m}(-b)^{-\frac{1}{2}-\frac{m}{2}}\ln(f)^{-\frac{1}{2}-\frac{m}{2}}\mathrm{e}^{\frac{b\ln(f)}{x^{2}}}}{1+m}\right.}{2}$

input
```
int(f^(a+b/x^2)*x^m,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*(-b)^(1/2+1/2*m)*ln(f)^(1/2+1/2*m)*(2/(1+m)*x^(-1+m)*(-b)^(-1/2-1
/2*m)*ln(f)^(1/2-1/2*m)*b*(-b*ln(f)/x^2)^(-1/2+1/2*m)*GAMMA(1/2-1/2*m)-2/(
1+m)*x^(1+m)*(-b)^(-1/2-1/2*m)*ln(f)^(-1/2-1/2*m)*exp(b*ln(f)/x^2)-2/(1+m)
*x^(-1+m)*(-b)^(-1/2-1/2*m)*ln(f)^(1/2-1/2*m)*b*(-b*ln(f)/x^2)^(-1/2+1/2*m
)*GAMMA(1/2-1/2*m,-b*ln(f)/x^2))
```

Fricas [F]

$$\int f^{a+\frac{b}{x^2}}x^m\,dx=\int f^{a+\frac{b}{x^2}}x^m\,dx$$

input
```
integrate(f^(a+b/x^2)*x^m,x, algorithm="fricas")
```

output
```
integral(f^((a*x^2 + b)/x^2)*x^m, x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^2}}x^m\,dx=\int f^{a+\frac{b}{x^2}}x^m\,dx$$

input
```
integrate(f**(a+b/x**2)*x**m,x)
```

output
```
Integral(f**(a + b/x**2)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int f^{a+\frac{b}{x^2}} x^m \, dx = \frac{1}{2} f^a x^{m+1} \left(-\frac{b \log(f)}{x^2}\right)^{\frac{1}{2}m+\frac{1}{2}} \Gamma\left(-\frac{1}{2}m-\frac{1}{2}, -\frac{b \log(f)}{x^2}\right)$$

input
```
integrate(f^(a+b/x^2)*x^m,x, algorithm="maxima")
```

output
```
1/2*f^a*x^(m + 1)*(-b*log(f)/x^2)^(1/2*m + 1/2)*gamma(-1/2*m - 1/2, -b*log
(f)/x^2)
```

Giac [F]

$$\int f^{a+\frac{b}{x^2}} x^m \, dx = \int f^{a+\frac{b}{x^2}} x^m \, dx$$

input
```
integrate(f^(a+b/x^2)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^2)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int f^{a+\frac{b}{x^2}} x^m \, dx = \frac{f^a \, x^{m+1} \, e^{\frac{b \ln(f)}{2 x^2}} \, M_{\frac{m}{4}+\frac{3}{4}, -\frac{m}{4}-\frac{1}{4}}\left(\frac{b \ln(f)}{x^2}\right) \left(\frac{b \ln(f)}{x^2}\right)^{\frac{m}{4}-\frac{1}{4}}}{m+1}$$

input
```
int(f^(a + b/x^2)*x^m,x)
```

output
```
(f^a*x^(m + 1)*exp((b*log(f))/(2*x^2))*whittakerM(m/4 + 3/4, - m/4 - 1/4,
(b*log(f))/x^2)*((b*log(f))/x^2)^(m/4 - 1/4))/(m + 1)
```

Reduce [F]

$$\int f^{a+\frac{b}{x^2}}x^m\,dx = \frac{2x^m f^{\frac{a\,x^2+b}{x^2}}\log(f)\,b + x^m f^{\frac{a\,x^2+b}{x^2}}m\,x^2 - x^m f^{\frac{a\,x^2+b}{x^2}}x^2 + 4\left(\int \frac{x^m f^{\frac{a\,x^2+b}{x^2}}}{x^4}dx\right)\log(f)^2\,b^2x}{x\,(m^2-1)}$$

input
```
int(f^(a+b/x^2)*x^m,x)
```

output
```
(2*x**m*f**((a*x**2 + b)/x**2)*log(f)*b + x**m*f**((a*x**2 + b)/x**2)*m*x*
*2 - x**m*f**((a*x**2 + b)/x**2)*x**2 + 4*int((x**m*f**((a*x**2 + b)/x**2)
)/x**4,x)*log(f)**2*b**2*x)/(x*(m**2 - 1))
```

3.112 $\int f^{a+\frac{b}{x^3}} x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int f^{a+\frac{b}{x^3}} x^m \, dx = \frac{1}{3} f^a x^{1+m} \Gamma\left(\frac{1}{3}(-1-m), -\frac{b\log(f)}{x^3}\right)\left(-\frac{b\log(f)}{x^3}\right)^{\frac{1+m}{3}}$$

output
```
1/3*f^a*x^(1+m)*GAMMA(-1/3-1/3*m,-b*ln(f)/x^3)*(-b*ln(f)/x^3)^(1/3+1/3*m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int f^{a+\frac{b}{x^3}} x^m \, dx = \frac{1}{3} f^a x^{1+m} \Gamma\left(\frac{1}{3}(-1-m), -\frac{b\log(f)}{x^3}\right)\left(-\frac{b\log(f)}{x^3}\right)^{\frac{1+m}{3}}$$

input
```
Integrate[f^(a + b/x^3)*x^m,x]
```

output
```
(f^a*x^(1 + m)*Gamma[(-1 - m)/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^((
1 + m)/3))/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+\frac{b}{x^3}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{1}{3} f^a x^{m+1} \left(-\frac{b \log(f)}{x^3}\right)^{\frac{m+1}{3}} \Gamma\left(\frac{1}{3}(-m-1), -\frac{b \log(f)}{x^3}\right)$$

input
```
Int[f^(a + b/x^3)*x^m,x]
```

output
```
(f^a*x^(1 + m)*Gamma[(-1 - m)/3, -((b*Log[f])/x^3)]*(-((b*Log[f])/x^3))^((
1 + m)/3))/3
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.67

method	result
meijerg	$-\frac{f^{a}(-b)^{\frac{1}{3}+\frac{m}{3}}\ln(f)^{\frac{1}{3}+\frac{m}{3}}\left(\frac{3x^{-2+m}(-b)^{-\frac{1}{3}-\frac{m}{3}}\ln(f)^{\frac{2}{3}-\frac{m}{3}}b\left(-\frac{b\ln(f)}{x^{3}}\right)^{-\frac{2}{3}+\frac{m}{3}}\Gamma\left(\frac{2}{3}-\frac{m}{3}\right)}{1+m}-\frac{3x^{1+m}(-b)^{-\frac{1}{3}-\frac{m}{3}}\ln(f)^{-\frac{1}{3}-\frac{m}{3}}e^{\frac{b\ln(f)}{x^{3}}}}{1+m}\right.}{3}$

input
```
int(f^(a+b/x^3)*x^m,x,method=_RETURNVERBOSE)
```

output
```
-1/3*f^a*(-b)^(1/3+1/3*m)*ln(f)^(1/3+1/3*m)*(3/(1+m)*x^(-2+m)*(-b)^(-1/3-1
/3*m)*ln(f)^(2/3-1/3*m)*b*(-b*ln(f)/x^3)^(-2/3+1/3*m)*GAMMA(2/3-1/3*m)-3/(
1+m)*x^(1+m)*(-b)^(-1/3-1/3*m)*ln(f)^(-1/3-1/3*m)*exp(b*ln(f)/x^3)-3/(1+m)
*x^(-2+m)*(-b)^(-1/3-1/3*m)*ln(f)^(2/3-1/3*m)*b*(-b*ln(f)/x^3)^(-2/3+1/3*m
)*GAMMA(2/3-1/3*m,-b*ln(f)/x^3))
```

Fricas [F]

$$\int f^{a+\frac{b}{x^3}}x^m\,dx=\int f^{a+\frac{b}{x^3}}x^m\,dx$$

input
```
integrate(f^(a+b/x^3)*x^m,x, algorithm="fricas")
```

output
```
integral(f^((a*x^3 + b)/x^3)*x^m, x)
```

Sympy [F]

$$\int f^{a+\frac{b}{x^3}}x^m\,dx=\int f^{a+\frac{b}{x^3}}x^m\,dx$$

input
```
integrate(f**(a+b/x**3)*x**m,x)
```

output
```
Integral(f**(a + b/x**3)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int f^{a+\frac{b}{x^3}}x^m\,dx = \frac{1}{3}\,f^a x^{m+1}\left(-\frac{b\log{(f)}}{x^3}\right)^{\frac{1}{3}\,m+\frac{1}{3}}\Gamma\left(-\frac{1}{3}\,m-\frac{1}{3},-\frac{b\log{(f)}}{x^3}\right)$$

input
```
integrate(f^(a+b/x^3)*x^m,x, algorithm="maxima")
```

output
```
1/3*f^a*x^(m + 1)*(-b*log(f)/x^3)^(1/3*m + 1/3)*gamma(-1/3*m - 1/3, -b*log
(f)/x^3)
```

Giac [F]

$$\int f^{a+\frac{b}{x^3}}x^m\,dx = \int f^{a+\frac{b}{x^3}}x^m\,dx$$

input
```
integrate(f^(a+b/x^3)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(a + b/x^3)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17

$$\int f^{a+\frac{b}{x^3}}x^m\,dx = \frac{f^a\,x^{m+1}\,\mathrm{e}^{\frac{b\,\ln(f)}{2\,x^3}}\,\mathrm{M}_{\frac{m}{6}+\frac{2}{3},-\frac{m}{6}-\frac{1}{6}}\left(\frac{b\,\ln(f)}{x^3}\right)\,\left(\frac{b\,\ln(f)}{x^3}\right)^{\frac{m}{6}-\frac{1}{3}}}{m+1}$$

input
```
int(f^(a + b/x^3)*x^m,x)
```

output
```
(f^a*x^(m + 1)*exp((b*log(f))/(2*x^3))*whittakerM(m/6 + 2/3, - m/6 - 1/6,
(b*log(f))/x^3)*((b*log(f))/x^3)^(m/6 - 1/3))/(m + 1)
```

Reduce [F]

$$\int f^{a+\frac{b}{x^3}} x^m \, dx$$
$$= \frac{3x^m f^{\frac{a x^3+b}{x^3}} \log(f) \, b + x^m f^{\frac{a x^3+b}{x^3}} m \, x^3 - 2x^m f^{\frac{a x^3+b}{x^3}} x^3 + 9\left(\int \frac{x^m f^{\frac{a x^3+b}{x^3}}}{x^6} dx\right) \log(f)^2 \, b^2 x^2}{x^2 \left(m^2 - m - 2\right)}$$

input
```
int(f^(a+b/x^3)*x^m,x)
```

output
```
(3*x**m*f**((a*x**3 + b)/x**3)*log(f)*b + x**m*f**((a*x**3 + b)/x**3)*m*x*
*3 - 2*x**m*f**((a*x**3 + b)/x**3)*x**3 + 9*int((x**m*f**((a*x**3 + b)/x**
3))/x**6,x)*log(f)**2*b**2*x**2)/(x**2*(m**2 - m - 2))
```

3.113 $\int f^{a+bx^n}x^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int f^{a+bx^n}x^3\,dx=-\frac{f^a x^4\Gamma\left(\frac{4}{n},-bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{-4/n}}{n}$$

output
```
-f^a*x^4*GAMMA(4/n,-b*x^n*ln(f))/n/((-b*x^n*ln(f))^(4/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}x^3\,dx=-\frac{f^a x^4\Gamma\left(\frac{4}{n},-bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{-4/n}}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^3,x]
```

output
```
-((f^a*x^4*Gamma[4/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(4/n)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{a+bx^n}\,dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{x^4 f^a \left(-b\log(f)x^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-bx^n\log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n)*x^3,x]
```

output
```
-((f^a*x^4*Gamma[4/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(4/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.44

method	result
meijerg	$\frac{f^a(-b)^{-\frac{4}{n}}\ln(f)^{-\frac{4}{n}}\left(\frac{n\,x^4(-b)^{\frac{4}{n}}\ln(f)^{\frac{4}{n}}(x^n\ln(f)bn+n+4)\Gamma\left(1-\frac{4}{n}\right)\Gamma\left(\frac{n+4}{n}+1\right)\operatorname{LaguerreL}\left(-\frac{4}{n},\frac{n+4}{n},b\,x^n\ln(f)\right)}{4(n+4)\Gamma\left(-\frac{4}{n}+\frac{n+4}{n}+1\right)}-\frac{n^2x^{n+4}(-b)^{\frac{4}{n}}\ln(f)^{1+\frac{4}{n}}}{}\right.}{n}$

input
```
int(f^(a+b*x^n)*x^3,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(-4/n)*ln(f)^(-4/n)/n*(1/4*n*x^4*(-b)^(4/n)*ln(f)^(4/n)*(x^n*ln(f
)*b*n+n+4)/(n+4)/GAMMA(-4/n+(n+4)/n+1)*GAMMA(1-4/n)*GAMMA((n+4)/n+1)*Lague
rreL(-4/n,(n+4)/n,b*x^n*ln(f))-1/4*n^2*x^(n+4)*(-b)^(4/n)*ln(f)^(1+4/n)*b/
(n+4)*LaguerreL(-4/n,(n+4)/n+1,b*x^n*ln(f))*GAMMA(1-4/n)*GAMMA((n+4)/n+1)/
GAMMA(-4/n+(n+4)/n+1))
```

Fricas [F]

$$\int f^{a+bx^n}x^3\,dx=\int f^{bx^n+a}x^3\,dx$$

input
```
integrate(f^(a+b*x^n)*x^3,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)*x^3, x)
```

Sympy [F]

$$\int f^{a+bx^n}x^3\,dx=\int f^{a+bx^n}x^3\,dx$$

input
```
integrate(f**(a+b*x**n)*x**3,x)
```

output
```
Integral(f**(a + b*x**n)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int f^{a+bx^n}x^3\,dx=-\frac{f^ax^4\Gamma\left(\frac{4}{n},-bx^n\log(f)\right)}{(-bx^n\log(f))^{\frac{4}{n}}n}$$

input
```
integrate(f^(a+b*x^n)*x^3,x, algorithm="maxima")
```

output
```
-f^a*x^4*gamma(4/n, -b*x^n*log(f))/((-b*x^n*log(f))^(4/n)*n)
```

Giac [F]

$$\int f^{a+bx^n}x^3\,dx=\int f^{bx^n+a}x^3\,dx$$

input
```
integrate(f^(a+b*x^n)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int f^{a+bx^n}x^3\,dx=\frac{f^a\,x^4\,\mathrm{e}^{\frac{b\,x^n\,\ln(f)}{2}}\,\mathrm{M}_{\frac{1}{2}-\frac{2}{n},\frac{2}{n}}\left(b\,x^n\,\ln(f)\right)}{4\,(b\,x^n\,\ln(f))^{\frac{2}{n}+\frac{1}{2}}}$$

input
```
int(f^(a + b*x^n)*x^3,x)
```

output
```
(f^a*x^4*exp((b*x^n*log(f))/2)*whittakerM(1/2 - 2/n, 2/n, b*x^n*log(f)))/(
4*(b*x^n*log(f))^(2/n + 1/2))
```

Reduce [F]

$$\int f^{a+bx^n}x^3\,dx = f^a\left(\int f^{x^n b}x^3dx\right)$$

input
```
int(f^(a+b*x^n)*x^3,x)
```

output
```
f**a*int(f**(x**n*b)*x**3,x)
```

3.114 $\int f^{a+bx^n}x^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int f^{a+bx^n}x^2\,dx=-\frac{f^ax^3\Gamma\left(\frac{3}{n},-bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{-3/n}}{n}$$

output
```
-f^a*x^3*GAMMA(3/n,-b*x^n*ln(f))/n/((-b*x^n*ln(f))^(3/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}x^2\,dx=-\frac{f^ax^3\Gamma\left(\frac{3}{n},-bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{-3/n}}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^2,x]
```

output
```
-((f^a*x^3*Gamma[3/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(3/n)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+bx^n} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{x^3 f^a \left(-b \log(f) x^n\right)^{-3/n} \Gamma\left(\frac{3}{n}, -b x^n \log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n)*x^2,x]
```

output
```
-((f^a*x^3*Gamma[3/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(3/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.44

method	result
meijerg	$\frac{f^a (-b)^{-\frac{3}{n}} \ln(f)^{-\frac{3}{n}} \left(\frac{n x^3 (-b)^{\frac{3}{n}} \ln(f)^{\frac{3}{n}} (x^n \ln(f) b n+n+3) \Gamma\left(1-\frac{3}{n}\right) \Gamma\left(\frac{3+n}{n}+1\right) \text{LaguerreL}\left(-\frac{3}{n}, \frac{3+n}{n}, b x^n \ln(f)\right)}{3(3+n) \Gamma\left(-\frac{3}{n}+\frac{3+n}{n}+1\right)} - \frac{n^2 x^{3+n} (-b)^{\frac{3}{n}} \ln(f)^{1+\frac{3}{n}}}{}\right.}{n}$

input
```
int(f^(a+b*x^n)*x^2,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(-3/n)*ln(f)^(-3/n)/n*(1/3*n*x^3*(-b)^(3/n)*ln(f)^(3/n)*(x^n*ln(f
)*b*n+n+3)/(3+n)/GAMMA(-3/n+(3+n)/n+1)*GAMMA(1-3/n)*GAMMA((3+n)/n+1)*Lague
rreL(-3/n,(3+n)/n,b*x^n*ln(f))-1/3*n^2*x^(3+n)*(-b)^(3/n)*ln(f)^(1+3/n)*b/
(3+n)*LaguerreL(-3/n,(3+n)/n+1,b*x^n*ln(f))*GAMMA(1-3/n)*GAMMA((3+n)/n+1)/
GAMMA(-3/n+(3+n)/n+1))
```

Fricas [F]

$$\int f^{a+bx^n}x^2\,dx=\int f^{bx^n+a}x^2\,dx$$

input
```
integrate(f^(a+b*x^n)*x^2,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)*x^2, x)
```

Sympy [F]

$$\int f^{a+bx^n}x^2\,dx=\int f^{a+bx^n}x^2\,dx$$

input
```
integrate(f**(a+b*x**n)*x**2,x)
```

output
```
Integral(f**(a + b*x**n)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int f^{a+bx^n} x^2 \, dx = -\frac{f^a x^3 \Gamma\left(\frac{3}{n}, -bx^n \log(f)\right)}{(-bx^n \log(f))^{\frac{3}{n}} n}$$

input
```
integrate(f^(a+b*x^n)*x^2,x, algorithm="maxima")
```

output
```
-f^a*x^3*gamma(3/n, -b*x^n*log(f))/((-b*x^n*log(f))^(3/n)*n)
```

Giac [F]

$$\int f^{a+bx^n} x^2 \, dx = \int f^{bx^n+a} x^2 \, dx$$

input
```
integrate(f^(a+b*x^n)*x^2,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.38

$$\int f^{a+bx^n} x^2 \, dx = \frac{f^a x^3 e^{\frac{b x^n \ln(f)}{2}} M_{\frac{1}{2}-\frac{3}{2n},\frac{3}{2n}}(b x^n \ln(f))}{3 (b x^n \ln(f))^{\frac{3}{2n}+\frac{1}{2}}}$$

input
```
int(f^(a + b*x^n)*x^2,x)
```

output
```
(f^a*x^3*exp((b*x^n*log(f))/2)*whittakerM(1/2 - 3/(2*n), 3/(2*n), b*x^n*lo
g(f)))/(3*(b*x^n*log(f))^(3/(2*n) + 1/2))
```

Reduce [F]

$$\int f^{a+bx^n}x^2\,dx = f^a\left(\int f^{x^nb}x^2dx\right)$$

input
```
int(f^(a+b*x^n)*x^2,x)
```

output
```
f**a*int(f**(x**n*b)*x**2,x)
```

3.115 $\int f^{a+bx^n} x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int f^{a+bx^n} x \, dx = -\frac{f^a x^2 \Gamma\left(\frac{2}{n}, -bx^n \log(f)\right) \left(-bx^n \log(f)\right)^{-2/n}}{n}$$

output
```
-f^a*x^2*GAMMA(2/n,-b*x^n*ln(f))/n/((-b*x^n*ln(f))^(2/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} x \, dx = -\frac{f^a x^2 \Gamma\left(\frac{2}{n}, -bx^n \log(f)\right) \left(-bx^n \log(f)\right)^{-2/n}}{n}$$

input
```
Integrate[f^(a + b*x^n)*x,x]
```

output
```
-((f^a*x^2*Gamma[2/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(2/n)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{a+bx^n}\,dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{x^2 f^a (-b\log(f)x^n)^{-2/n}\Gamma\left(\frac{2}{n},-bx^n\log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n)*x,x]
```

output
```
-((f^a*x^2*Gamma[2/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(2/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.44

method	result
meijerg	$\frac{f^a(-b)^{-\frac{2}{n}}\ln(f)^{-\frac{2}{n}}\left(\frac{n\,x^2(-b)^{\frac{2}{n}}\ln(f)^{\frac{2}{n}}(x^n\ln(f)bn+n+2)\Gamma\left(1-\frac{2}{n}\right)\Gamma\left(\frac{2+n}{n}+1\right)\text{LaguerreL}\left(-\frac{2}{n},\frac{2+n}{n},b\,x^n\ln(f)\right)}{2(2+n)\Gamma\left(-\frac{2}{n}+\frac{2+n}{n}+1\right)}-\frac{n^2x^{2+n}(-b)^{\frac{2}{n}}\ln(f)^{1+\frac{2}{n}}}{}\right.}{n}$

input
```
int(f^(a+b*x^n)*x,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(-2/n)*ln(f)^(-2/n)/n*(1/2*n*x^2*(-b)^(2/n)*ln(f)^(2/n)*(x^n*ln(f
)*b*n+n+2)/(2+n)/GAMMA(-2/n+(2+n)/n+1)*GAMMA(1-2/n)*GAMMA((2+n)/n+1)*Lague
rreL(-2/n,(2+n)/n,b*x^n*ln(f))-1/2*n^2*x^(2+n)*(-b)^(2/n)*ln(f)^(1+2/n)*b/
(2+n)*LaguerreL(-2/n,(2+n)/n+1,b*x^n*ln(f))*GAMMA(1-2/n)*GAMMA((2+n)/n+1)/
GAMMA(-2/n+(2+n)/n+1))
```

Fricas [F]

$$\int f^{a+bx^n}x\,dx = \int f^{bx^n+a}x\,dx$$

input
```
integrate(f^(a+b*x^n)*x,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)*x, x)
```

Sympy [F]

$$\int f^{a+bx^n}x\,dx = \int f^{a+bx^n}x\,dx$$

input
```
integrate(f**(a+b*x**n)*x,x)
```

output
```
Integral(f**(a + b*x**n)*x, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int f^{a+bx^n} x\,dx = -\frac{f^a x^2 \Gamma\left(\frac{2}{n}, -bx^n \log(f)\right)}{(-bx^n \log(f))^{\frac{2}{n}} n}$$

input
```
integrate(f^(a+b*x^n)*x,x, algorithm="maxima")
```

output
```
-f^a*x^2*gamma(2/n, -b*x^n*log(f))/((-b*x^n*log(f))^(2/n)*n)
```

Giac [F]

$$\int f^{a+bx^n} x\,dx = \int f^{bx^n+a} x\,dx$$

input
```
integrate(f^(a+b*x^n)*x,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n} x\,dx = \int f^{a+b\,x^n}\,x\,dx$$

input
```
int(f^(a + b*x^n)*x,x)
```

output
```
int(f^(a + b*x^n)*x, x)
```

Reduce [F]

$$\int f^{a+bx^n} x \, dx = f^a \left(\int f^{x^n b} x dx \right)$$

input
```
int(f^(a+b*x^n)*x,x)
```

output
```
f**a*int(f**(x**n*b)*x,x)
```

3.116 $\int f^{a+bx^n}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int f^{a+bx^n}\,dx = -\frac{f^a x\Gamma\left(\frac{1}{n}, -bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{-1/n}}{n}$$

output
```
-f^a*x*GAMMA(1/n,-b*x^n*ln(f))/n/((-b*x^n*ln(f))^(1/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}\,dx = -\frac{f^a x\Gamma\left(\frac{1}{n}, -bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{-1/n}}{n}$$

input
```
Integrate[f^(a + b*x^n),x]
```

output
```
-((f^a*x*Gamma[n^(-1), -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^n^(-1)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx^n}\,dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{xf^a(-b\log(f)x^n)^{-1/n}\,\Gamma\left(\frac{1}{n},-bx^n\log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n),x]
```

output
```
-((f^a*x*Gamma[n^(-1), -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^n^(-1)))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 5.74

method	result
meijerg	$\frac{f^a(-b)^{-\frac{1}{n}}\ln(f)^{-\frac{1}{n}}\left(\frac{nx(-b)^{\frac{1}{n}}\ln(f)^{\frac{1}{n}}(x^n\ln(f)bn+n+1)\Gamma\left(1-\frac{1}{n}\right)\Gamma\left(\frac{1+n}{n}+1\right)\text{LaguerreL}\left(-\frac{1}{n},\frac{1+n}{n},b\,x^n\ln(f)\right)}{(1+n)\Gamma\left(-\frac{1}{n}+\frac{1+n}{n}+1\right)}-\frac{n^2x^{1+n}(-b)^{\frac{1}{n}}\ln(f)^{1+\frac{1}{n}}b}{}\right.}{n}$

input
```
int(f^(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
f^a/n*(-b)^(-1/n)*ln(f)^(-1/n)*(n*x*(-b)^(1/n)*ln(f)^(1/n)*(x^n*ln(f)*b*n+
n+1)/(1+n)/GAMMA(-1/n+(1+n)/n+1)*GAMMA(1-1/n)*GAMMA((1+n)/n+1)*LaguerreL(-
1/n,(1+n)/n,b*x^n*ln(f))-n^2*x^(1+n)*(-b)^(1/n)*ln(f)^(1+1/n)*b/(1+n)*Lagu
erreL(-1/n,(1+n)/n+1,b*x^n*ln(f))*GAMMA(1-1/n)*GAMMA((1+n)/n+1)/GAMMA(-1/n
+(1+n)/n+1))
```

Fricas [F]

$$\int f^{a+bx^n} \, dx = \int f^{bx^n+a} \, dx$$

input
```
integrate(f^(a+b*x^n),x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a), x)
```

Sympy [F]

$$\int f^{a+bx^n} \, dx = \int f^{a+bx^n} \, dx$$

input
```
integrate(f**(a+b*x**n),x)
```

output
```
Integral(f**(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}\,dx = -\frac{f^a x\Gamma\left(\frac{1}{n}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\left(\frac{1}{n}\right)} n}$$

input
```
integrate(f^(a+b*x^n),x, algorithm="maxima")
```

output
```
-f^a*x*gamma(1/n, -b*x^n*log(f))/((-b*x^n*log(f))^(1/n)*n)
```

Giac [F]

$$\int f^{a+bx^n}\,dx = \int f^{bx^n+a}\,dx$$

input
```
integrate(f^(a+b*x^n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n}\,dx = \int f^{a+b\,x^n}\,dx$$

input
```
int(f^(a + b*x^n),x)
```

output
```
int(f^(a + b*x^n), x)
```

Reduce [F]

$$\int f^{a+bx^n} dx = f^a \left(\int f^{x^n b} dx\right)$$

input
```
int(f^(a+b*x^n),x)
```

output
```
f**a*int(f**(x**n*b),x)
```

3.117 $\int \frac{f^{a+bx^n}}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{f^{a+bx^n}}{x} dx = \frac{f^a \text{ExpIntegralEi}(bx^n \log(f))}{n}$$

output
```
f^a*Ei(b*x^n*ln(f))/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x} dx = \frac{f^a \text{ExpIntegralEi}(bx^n \log(f))}{n}$$

input
```
Integrate[f^(a + b*x^n)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^n*Log[f]])/n
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^n}}{x} \, dx$$

$$\Big\downarrow \text{2639}$$

$$\frac{f^a \operatorname{ExpIntegralEi}\left(bx^n \log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^n*Log[f]])/n
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

method	result	size
risch	$-\frac{f^{a} \operatorname{expIntegral}_{1}\left(-b\,x^{n} \ln \left(f \right)\right)}{n}$	19
meijerg	$\frac{f^{a} \left(n \ln \left(x \right)+\ln \left(-b \right)+\ln \left(\ln \left(f \right)\right)-\ln \left(-b\,x^{n} \ln \left(f \right)\right)-\operatorname{expIntegral}_{1}\left(-b\,x^{n} \ln \left(f \right)\right)\right)}{n}$	43

input
```
int(f^(a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/n*f^a*Ei(1,-b*x^n*ln(f))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx=\frac{f^a\mathrm{Ei}(bx^n\log(f))}{n}$$

input
```
integrate(f^(a+b*x^n)/x,x, algorithm="fricas")
```

output
```
f^a*Ei(b*x^n*log(f))/n
```

Sympy [F]

$$\int \frac{f^{a+bx^n}}{x}\,dx=\int \frac{f^{a+bx^n}}{x}\,dx$$

input
```
integrate(f**(a+b*x**n)/x,x)
```

output
```
Integral(f**(a + b*x**n)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx=\frac{f^a\mathrm{Ei}(bx^n\log(f))}{n}$$

input
```
integrate(f^(a+b*x^n)/x,x, algorithm="maxima")
```

output
```
f^a*Ei(b*x^n*log(f))/n
```

Giac [F]

$$\int \frac{f^{a+bx^n}}{x}\,dx = \int \frac{f^{bx^n+a}}{x}\,dx$$

input
```
integrate(f^(a+b*x^n)/x,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{a+bx^n}}{x}\,dx = \int \frac{f^{a+b\,x^n}}{x}\,dx$$

input
```
int(f^(a + b*x^n)/x,x)
```

output
```
int(f^(a + b*x^n)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx = \frac{f^a\,ei(x^n\log(f)\,b)}{n}$$

input
```
int(f^(a+b*x^n)/x,x)
```

output
```
(f**a*ei(x**n*log(f)*b))/n
```

3.118 $\int \frac{f^{a+bx^n}}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = -\frac{f^a\Gamma\left(-\frac{1}{n}, -bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{\frac{1}{n}}}{nx}$$

output
```
-f^a*GAMMA(-1/n,-b*x^n*ln(f))*(-b*x^n*ln(f))^(1/n)/n/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = -\frac{f^a\Gamma\left(-\frac{1}{n}, -bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{\frac{1}{n}}}{nx}$$

input
```
Integrate[f^(a + b*x^n)/x^2,x]
```

output
```
-((f^a*Gamma[-n^(-1), -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^n^(-1))/(n*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^n}}{x^2} \, dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{f^a(-b\log(f)x^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-bx^n\log(f)\right)}{nx}$$

input
```
Int[f^(a + b*x^n)/x^2,x]
```

output
```
-((f^a*Gamma[-n^(-1), -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^n^(-1))/(n*x))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.07 (sec) , antiderivative size = 195, normalized size of antiderivative = 5.27

method	result
meijerg	$\frac{f^a(-b)^{\frac{1}{n}}\ln(f)^{\frac{1}{n}}\left(-\frac{n(-b)^{-\frac{1}{n}}\ln(f)^{-\frac{1}{n}}(x^n\ln(f)bn+n-1)\Gamma\left(1+\frac{1}{n}\right)\Gamma\left(\frac{-1+n}{n}+1\right)\text{LaguerreL}\left(\frac{1}{n},\frac{-1+n}{n},b\,x^n\ln(f)\right)}{x(-1+n)\Gamma\left(\frac{1}{n}+\frac{-1+n}{n}+1\right)}+\frac{n^2x^{-1+n}(-b)^{-\frac{1}{n}}\ln(f)}{}\right.}{n}$

input
```
int(f^(a+b*x^n)/x^2,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(1/n)*ln(f)^(1/n)/n*(-n/x*(-b)^(-1/n)*ln(f)^(-1/n)*(x^n*ln(f)*b*n
+n-1)/(-1+n)/GAMMA(1/n+(-1+n)/n+1)*GAMMA(1+1/n)*GAMMA((-1+n)/n+1)*Laguerre
L(1/n,(-1+n)/n,b*x^n*ln(f))+n^2*x^(-1+n)*(-b)^(-1/n)*ln(f)^(1-1/n)*b/(-1+n
)*LaguerreL(1/n,(-1+n)/n+1,b*x^n*ln(f))*GAMMA(1+1/n)*GAMMA((-1+n)/n+1)/GAM
MA(1/n+(-1+n)/n+1))
```

Fricas [F]

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = \int \frac{f^{bx^n+a}}{x^2}\,dx$$

input
```
integrate(f^(a+b*x^n)/x^2,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)/x^2, x)
```

Sympy [F]

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = \int \frac{f^{a+bx^n}}{x^2}\,dx$$

input
```
integrate(f**(a+b*x**n)/x**2,x)
```

output
```
Integral(f**(a + b*x**n)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = -\frac{\left(-bx^n\log\left(f\right)\right)^{\left(\frac{1}{n}\right)} f^a\Gamma\left(-\frac{1}{n}, -bx^n\log\left(f\right)\right)}{nx}$$

input
```
integrate(f^(a+b*x^n)/x^2,x, algorithm="maxima")
```

output
```
-(-b*x^n*log(f))^(1/n)*f^a*gamma(-1/n, -b*x^n*log(f))/(n*x)
```

Giac [F]

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = \int \frac{f^{bx^n+a}}{x^2}\,dx$$

input
```
integrate(f^(a+b*x^n)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{f^{a+bx^n}}{x^2}\,dx = -\frac{f^a\,\mathrm{e}^{\frac{b\,x^n\,\ln(f)}{2}}\,\mathrm{M}_{\frac{1}{2\,n}+\frac{1}{2},-\frac{1}{2\,n}}\left(b\,x^n\,\ln\left(f\right)\right)\,\left(b\,x^n\,\ln\left(f\right)\right)^{\frac{1}{2\,n}-\frac{1}{2}}}{x}$$

input
```
int(f^(a + b*x^n)/x^2,x)
```

output
```
-(f^a*exp((b*x^n*log(f))/2)*whittakerM(1/(2*n) + 1/2, -1/(2*n), b*x^n*log(
f))*(b*x^n*log(f))^(1/(2*n) - 1/2))/x
```

Reduce [F]

$$\int \frac{f^{a+bx^n}}{x^2} \, dx = f^a \left(\int \frac{f^{x^n b}}{x^2} dx \right)$$

input
```
int(f^(a+b*x^n)/x^2,x)
```

output
```
f**a*int(f**(x**n*b)/x**2,x)
```

3.119 $\int \frac{f^{a+bx^n}}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{f^{a+bx^n}}{x^3} dx = -\frac{f^a\Gamma\left(-\frac{2}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{2/n}}{nx^2}$$

output
```
-f^a*GAMMA(-2/n,-b*x^n*ln(f))*(-b*x^n*ln(f))^(2/n)/n/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x^3} dx = -\frac{f^a\Gamma\left(-\frac{2}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{2/n}}{nx^2}$$

input
```
Integrate[f^(a + b*x^n)/x^3,x]
```

output
```
-((f^a*Gamma[-2/n, -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^(2/n))/(n*x^2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^n}}{x^3} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{f^a(-b\log(f)x^n)^{2/n}\,\Gamma\left(-\frac{2}{n}, -bx^n\log(f)\right)}{nx^2}$$

input
```
Int[f^(a + b*x^n)/x^3,x]
```

output
```
-((f^a*Gamma[-2/n, -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^(2/n))/(n*x^2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.44

method	result
meijerg	$\frac{f^a(-b)^{\frac{2}{n}}\ln(f)^{\frac{2}{n}}\left(-\frac{n(-b)^{-\frac{2}{n}}\ln(f)^{-\frac{2}{n}}(x^n\ln(f)bn+n-2)\Gamma\left(1+\frac{2}{n}\right)\Gamma\left(\frac{-2+n}{n}+1\right)\text{LaguerreL}\left(\frac{2}{n},\frac{-2+n}{n},b\,x^n\ln(f)\right)}{2x^2(-2+n)\Gamma\left(\frac{2}{n}+\frac{-2+n}{n}+1\right)}+\frac{n^2x^{-2+n}(-b)^{-\frac{2}{n}}\ln(f)}{}\right.}{n}$

input
```
int(f^(a+b*x^n)/x^3,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(2/n)*ln(f)^(2/n)/n*(-1/2*n/x^2*(-b)^(-2/n)*ln(f)^(-2/n)*(x^n*ln(
f)*b*n+n-2)/(-2+n)/GAMMA(2/n+(-2+n)/n+1)*GAMMA(1+2/n)*GAMMA((-2+n)/n+1)*La
guerreL(2/n,(-2+n)/n,b*x^n*ln(f))+1/2*n^2*x^(-2+n)*(-b)^(-2/n)*ln(f)^(1-2/
n)*b/(-2+n)*LaguerreL(2/n,(-2+n)/n+1,b*x^n*ln(f))*GAMMA(1+2/n)*GAMMA((-2+n
)/n+1)/GAMMA(2/n+(-2+n)/n+1))
```

Fricas [F]

$$\int \frac{f^{a+bx^n}}{x^3}\,dx = \int \frac{f^{bx^n+a}}{x^3}\,dx$$

input
```
integrate(f^(a+b*x^n)/x^3,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)/x^3, x)
```

Sympy [F]

$$\int \frac{f^{a+bx^n}}{x^3}\,dx = \int \frac{f^{a+bx^n}}{x^3}\,dx$$

input
```
integrate(f**(a+b*x**n)/x**3,x)
```

output
```
Integral(f**(a + b*x**n)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x^3} dx = -\frac{(-bx^n \log(f))^{\frac{2}{n}} f^a \Gamma\left(-\frac{2}{n}, -bx^n \log(f)\right)}{nx^2}$$

input
```
integrate(f^(a+b*x^n)/x^3,x, algorithm="maxima")
```

output
```
-(-b*x^n*log(f))^(2/n)*f^a*gamma(-2/n, -b*x^n*log(f))/(n*x^2)
```

Giac [F]

$$\int \frac{f^{a+bx^n}}{x^3} dx = \int \frac{f^{bx^n+a}}{x^3} dx$$

input
```
integrate(f^(a+b*x^n)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \frac{f^{a+bx^n}}{x^3} dx = -\frac{f^a \, e^{\frac{b\,x^n\,\ln(f)}{2}} \, \mathrm{M}_{\frac{1}{n}+\frac{1}{2},-\frac{1}{n}}\left(b\,x^n\,\ln(f)\right) \, \left(b\,x^n\,\ln(f)\right)^{\frac{1}{n}-\frac{1}{2}}}{2\,x^2}$$

input
```
int(f^(a + b*x^n)/x^3,x)
```

output
```
-(f^a*exp((b*x^n*log(f))/2)*whittakerM(1/n + 1/2, -1/n, b*x^n*log(f))*(b*x
^n*log(f))^(1/n - 1/2))/(2*x^2)
```

Reduce [F]

$$\int \frac{f^{a+bx^n}}{x^3} \, dx = f^a \left(\int \frac{f^{x^n b}}{x^3} dx \right)$$

input
```
int(f^(a+b*x^n)/x^3,x)
```

output
```
f**a*int(f**(x**n*b)/x**3,x)
```

3.120 $\int \frac{f^{a+bx^n}}{x^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{f^{a+bx^n}}{x^4}\,dx = -\frac{f^a\Gamma\left(-\frac{3}{n},-bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{3/n}}{nx^3}$$

output
```
-f^a*GAMMA(-3/n,-b*x^n*ln(f))*(-b*x^n*ln(f))^(3/n)/n/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x^4}\,dx = -\frac{f^a\Gamma\left(-\frac{3}{n},-bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{3/n}}{nx^3}$$

input
```
Integrate[f^(a + b*x^n)/x^4,x]
```

output
```
-((f^a*Gamma[-3/n, -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^(3/n))/(n*x^3))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^n}}{x^4} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{f^a(-b\log(f)x^n)^{3/n}\,\Gamma\left(-\frac{3}{n},-bx^n\log(f)\right)}{nx^3}$$

input
```
Int[f^(a + b*x^n)/x^4,x]
```

output
```
-((f^a*Gamma[-3/n, -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^(3/n))/(n*x^3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 212, normalized size of antiderivative = 5.44

method	result
meijerg	$\frac{f^a(-b)^{\frac{3}{n}}\ln(f)^{\frac{3}{n}}\left(-\frac{n(-b)^{-\frac{3}{n}}\ln(f)^{-\frac{3}{n}}(x^n\ln(f)bn+n-3)\Gamma\left(1+\frac{3}{n}\right)\Gamma\left(\frac{n-3}{n}+1\right)\text{LaguerreL}\left(\frac{3}{n},\frac{n-3}{n},b\,x^n\ln(f)\right)}{3x^3(n-3)\Gamma\left(\frac{3}{n}+\frac{n-3}{n}+1\right)}+\frac{n^2x^{n-3}(-b)^{-\frac{3}{n}}\ln(f)^{1-\frac{3}{n}}}{}\right.}{n}$

input
```
int(f^(a+b*x^n)/x^4,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(3/n)*ln(f)^(3/n)/n*(-1/3*n/x^3*(-b)^(-3/n)*ln(f)^(-3/n)*(x^n*ln(
f)*b*n+n-3)/(n-3)/GAMMA(3/n+(n-3)/n+1)*GAMMA(1+3/n)*GAMMA((n-3)/n+1)*Lague
rreL(3/n,(n-3)/n,b*x^n*ln(f))+1/3*n^2*x^(n-3)*(-b)^(-3/n)*ln(f)^(1-3/n)*b/
(n-3)*LaguerreL(3/n,(n-3)/n+1,b*x^n*ln(f))*GAMMA(1+3/n)*GAMMA((n-3)/n+1)/G
AMMA(3/n+(n-3)/n+1))
```

Fricas [F]

$$\int \frac{f^{a+bx^n}}{x^4}\,dx=\int \frac{f^{bx^n+a}}{x^4}\,dx$$

input
```
integrate(f^(a+b*x^n)/x^4,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)/x^4, x)
```

Sympy [F]

$$\int \frac{f^{a+bx^n}}{x^4}\,dx=\int \frac{f^{a+bx^n}}{x^4}\,dx$$

input
```
integrate(f**(a+b*x**n)/x**4,x)
```

output
```
Integral(f**(a + b*x**n)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x^4} dx = -\frac{(-bx^n \log(f))^{\frac{3}{n}} f^a \Gamma\left(-\frac{3}{n}, -bx^n \log(f)\right)}{nx^3}$$

input
```
integrate(f^(a+b*x^n)/x^4,x, algorithm="maxima")
```

output
```
-(-b*x^n*log(f))^(3/n)*f^a*gamma(-3/n, -b*x^n*log(f))/(n*x^3)
```

Giac [F]

$$\int \frac{f^{a+bx^n}}{x^4} dx = \int \frac{f^{bx^n+a}}{x^4} dx$$

input
```
integrate(f^(a+b*x^n)/x^4,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

$$\int \frac{f^{a+bx^n}}{x^4} dx = -\frac{f^a \, e^{\frac{b\,x^n\,\ln(f)}{2}} \, M_{\frac{3}{2\,n}+\frac{1}{2},-\frac{3}{2\,n}}\left(b\,x^n\,\ln(f)\right)\,\left(b\,x^n\,\ln(f)\right)^{\frac{3}{2\,n}-\frac{1}{2}}}{3\,x^3}$$

input
```
int(f^(a + b*x^n)/x^4,x)
```

output
```
-(f^a*exp((b*x^n*log(f))/2)*whittakerM(3/(2*n) + 1/2, -3/(2*n), b*x^n*log(
f))*(b*x^n*log(f))^(3/(2*n) - 1/2))/(3*x^3)
```

Reduce [F]

$$\int \frac{f^{a+bx^n}}{x^4}\,dx = f^a \left(\int \frac{f^{x^n b}}{x^4}dx\right)$$

input
```
int(f^(a+b*x^n)/x^4,x)
```

output
```
f**a*int(f**(x**n*b)/x**4,x)
```

3.121 $\int e^{-x/10}x\,dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int e^{-x/10}x\,dx = -100e^{-x/10} - 10e^{-x/10}x$$

output
```
-100/exp(1/10*x)-10*x/exp(1/10*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int e^{-x/10}x\,dx = e^{-x/10}(-100 - 10x)$$

input
```
Integrate[x/E^(x/10),x]
```

output
```
(-100 - 10*x)/E^(x/10)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x/10}x\,dx$$

$$\Big\downarrow \text{2607}$$

$$10\int e^{-x/10}dx-10e^{-x/10}x$$

$$\Big\downarrow \text{2624}$$

$$-10e^{-x/10}x-100e^{-x/10}$$

input
```
Int[x/E^(x/10),x]
```

output
```
-100/E^(x/10) - (10*x)/E^(x/10)
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.55

method	result	size
risch	$(-100-10x)\,\mathrm{e}^{-\frac{x}{10}}$	11
gosper	$-10(x+10)\,\mathrm{e}^{-\frac{x}{10}}$	12
orering	$-10(x+10)\,\mathrm{e}^{-\frac{x}{10}}$	12
norman	$(-100-10x)\,\mathrm{e}^{-\frac{x}{10}}$	13
meijerg	$100-50\left(2+\frac{x}{5}\right)\mathrm{e}^{-\frac{x}{10}}$	14
parallelrisch	$-(100+10x)\,\mathrm{e}^{-\frac{x}{10}}$	14
derivativedivides	$-100\,\mathrm{e}^{-\frac{x}{10}}-10x\,\mathrm{e}^{-\frac{x}{10}}$	19
default	$-100\,\mathrm{e}^{-\frac{x}{10}}-10x\,\mathrm{e}^{-\frac{x}{10}}$	19

input
```
int(x/exp(1/10*x),x,method=_RETURNVERBOSE)
```

output
```
(-100-10*x)*exp(-1/10*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int e^{-x/10}x\,dx = -10\,(x+10)e^{\left(-\frac{1}{10}\,x\right)}$$

input
```
integrate(x/exp(1/10*x),x, algorithm="fricas")
```

output
```
-10*(x + 10)*e^(-1/10*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.50

$$\int e^{-x/10}x\,dx = (-10x - 100)\,e^{-\frac{x}{10}}$$

input
```
integrate(x/exp(1/10*x),x)
```

output
```
(-10*x - 100)*exp(-x/10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int e^{-x/10}x\,dx = -10\,(x + 10)e^{\left(-\frac{1}{10}\,x\right)}$$

input
```
integrate(x/exp(1/10*x),x, algorithm="maxima")
```

output
```
-10*(x + 10)*e^(-1/10*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int e^{-x/10}x\,dx = -10\,(x + 10)e^{\left(-\frac{1}{10}\,x\right)}$$

input
```
integrate(x/exp(1/10*x),x, algorithm="giac")
```

output
```
-10*(x + 10)*e^(-1/10*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int e^{-x/10}x\,dx=-10\,\mathrm{e}^{-\frac{x}{10}}\,(x+10)$$

input
```
int(x*exp(-x/10),x)
```

output
```
-10*exp(-x/10)*(x + 10)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int e^{-x/10}x\,dx=\frac{-10x-100}{e^{\frac{x}{10}}}$$

input
```
int(x/exp(1/10*x),x)
```

output
```
(10*( - x - 10))/e**(x/10)
```

3.122 $\int f^{a+bx^n} x^m \, dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int f^{a+bx^n} x^m \, dx = -\frac{f^a x^{1+m} \Gamma\left(\frac{1+m}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-\frac{1+m}{n}}}{n}$$

output
```
-f^a*x^(1+m)*GAMMA((1+m)/n,-b*x^n*ln(f))/n/((-b*x^n*ln(f))^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} x^m \, dx = -\frac{f^a x^{1+m} \Gamma\left(\frac{1+m}{n}, -bx^n \log(f)\right) (-bx^n \log(f))^{-\frac{1+m}{n}}}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^m,x]
```

output
```
-((f^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(
(1 + m)/n)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{a+bx^n}\,dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{f^a x^{m+1}(-b\log(f)x^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-bx^n\log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n)*x^m,x]
```

output
```
-((f^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(
(1 + m)/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 6.09

method	result
meijerg	$\frac{f^{a}(-b)^{-\frac{m}{n}-\frac{1}{n}} \ln(f)^{-\frac{m}{n}-\frac{1}{n}} \left(\frac{n\, x^{1+m} (-b)^{\frac{m}{n}+\frac{1}{n}} \ln(f)^{\frac{m}{n}+\frac{1}{n}} (x^{n} \ln(f) b n+m+n+1) \operatorname{LaguerreL}\left(-\frac{1+m}{n}, \frac{1+m+n}{n}, b\, x^{n} \ln(f)\right) \Gamma\left(-\frac{1+m}{n}+1\right) \Gamma}{(1+m)(1+m+n)\Gamma\left(-\frac{1+m}{n}+\frac{1+m+n}{n}+1\right)}\right.}{n}$

input
```
int(f^(a+b*x^n)*x^m,x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(-m/n-1/n)*ln(f)^(-m/n-1/n)/n*(n/(1+m)*x^(1+m)*(-b)^(m/n+1/n)*ln(
f)^(m/n+1/n)*(x^n*ln(f)*b*n+m+n+1)/(1+m+n)*LaguerreL(-(1+m)/n,(1+m+n)/n,b*
x^n*ln(f))*GAMMA(-(1+m)/n+1)*GAMMA((1+m+n)/n+1)/GAMMA(-(1+m)/n+(1+m+n)/n+1
)-n^2/(1+m)*x^(1+m+n)*(-b)^(m/n+1/n)*ln(f)^(1+m/n+1/n)*b/(1+m+n)*LaguerreL
(-(1+m)/n,(1+m+n)/n+1,b*x^n*ln(f))*GAMMA(-(1+m)/n+1)*GAMMA((1+m+n)/n+1)/GA
MMA(-(1+m)/n+(1+m+n)/n+1))
```

Fricas [F]

$$\int f^{a+bx^n} x^m\,dx = \int f^{bx^n+a} x^m\,dx$$

input
```
integrate(f^(a+b*x^n)*x^m,x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)*x^m, x)
```

Sympy [F]

$$\int f^{a+bx^n} x^m\,dx = \int f^{a+bx^n} x^m\,dx$$

input
```
integrate(f**(a+b*x**n)*x**m,x)
```

output
```
Integral(f**(a + b*x**n)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int f^{a+bx^n} x^m\,dx = -\frac{f^a x^{m+1}\Gamma\left(\frac{m+1}{n}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\frac{m+1}{n}} n}$$

input
```
integrate(f^(a+b*x^n)*x^m,x, algorithm="maxima")
```

output
```
-f^a*x^(m + 1)*gamma((m + 1)/n, -b*x^n*log(f))/((-b*x^n*log(f))^((m + 1)/n
)*n)
```

Giac [F]

$$\int f^{a+bx^n} x^m\,dx = \int f^{bx^n+a} x^m\,dx$$

input
```
integrate(f^(a+b*x^n)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int f^{a+bx^n} x^m\,dx = \frac{f^a\,f^{b\,x^n}\,x^{m+1}\,\mathrm{e}^{-\frac{b\,x^n\,\ln(f)}{2}}\,\mathrm{M}_{1-\frac{m+n+1}{2\,n},\frac{m+n+1}{2\,n}-\frac{1}{2}}\left(b\,x^n\,\ln(f)\right)}{(m+1)\,\left(b\,x^n\,\ln(f)\right)^{\frac{m+n+1}{2\,n}}}$$

input
```
int(f^(a + b*x^n)*x^m,x)
```

output
```
(f^a*f^(b*x^n)*x^(m + 1)*exp(-(b*x^n*log(f))/2)*whittakerM(1 - (m + n + 1)
/(2*n), (m + n + 1)/(2*n) - 1/2, b*x^n*log(f)))/((m + 1)*(b*x^n*log(f))^((
m + n + 1)/(2*n)))
```

Reduce [F]

$$\int f^{a+bx^n}x^m\,dx = f^a\left(\int x^m f^{x^n b}dx\right)$$

input
```
int(f^(a+b*x^n)*x^m,x)
```

output
```
f**a*int(x**m*f**(x**n*b),x)
```

3.123 $\int f^{a+bx^n} x^{-1+3n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int f^{a+bx^n} x^{-1+3n}\,dx = \frac{2f^{a+bx^n}}{b^3n\log^3(f)} - \frac{2f^{a+bx^n}x^n}{b^2n\log^2(f)} + \frac{f^{a+bx^n}x^{2n}}{bn\log(f)}$$

output
```
2*f^(a+b*x^n)/b^3/n/ln(f)^3-2*f^(a+b*x^n)*x^n/b^2/n/ln(f)^2+f^(a+b*x^n)*x^
(2*n)/b/n/ln(f)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.34

$$\int f^{a+bx^n} x^{-1+3n}\,dx = \frac{f^a\Gamma(3, -bx^n\log(f))}{b^3n\log^3(f)}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 + 3*n),x]
```

output
```
(f^a*Gamma[3, -(b*x^n*Log[f])])/(b^3*n*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2642, 2642, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3n-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \ 2642$$

$$\frac{x^{2n} f^{a+bx^n}}{bn \log(f)} - \frac{2 \int f^{bx^n+a} x^{2n-1} dx}{b \log(f)}$$

$$\Big\downarrow \ 2642$$

$$\frac{x^{2n} f^{a+bx^n}}{bn \log(f)} - \frac{2 \left(\frac{x^n f^{a+bx^n}}{bn \log(f)} - \frac{\int f^{bx^n+a} x^{n-1} dx}{b \log(f)} \right)}{b \log(f)}$$

$$\Big\downarrow \ 2638$$

$$\frac{x^{2n} f^{a+bx^n}}{bn \log(f)} - \frac{2 \left(\frac{x^n f^{a+bx^n}}{bn \log(f)} - \frac{f^{a+bx^n}}{b^2 n \log^2(f)} \right)}{b \log(f)}$$

input
```
Int[f^(a + b*x^n)*x^(-1 + 3*n),x]
```

output
```
(f^(a + b*x^n)*x^(2*n))/(b*n*Log[f]) - (2*(-(f^(a + b*x^n)/(b^2*n*Log[f]^2
)) + (f^(a + b*x^n)*x^n)/(b*n*Log[f])))/(b*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{\left(b^{2} x^{2 n} \ln \left(f \right)^{2}-2 b \,x^{n} \ln \left(f \right)+2\right) f^{a +b \,x^{n}}}{b^{3} \ln \left(f \right)^{3} n}$	44
meijerg	$-\frac{f^{a} \left(2-\frac{\left(3 b^{2} x^{2 n} \ln \left(f \right)^{2}-6 b \,x^{n} \ln \left(f \right)+6\right) {\mathrm e}^{b \,x^{n} \ln \left(f \right)}}{3}\right)}{\ln \left(f \right)^{3} b^{3} n}$	52

input
```
int(f^(a+b*x^n)*x^(-1+3*n),x,method=_RETURNVERBOSE)
```

output
```
(b^2*(x^n)^2*ln(f)^2-2*b*x^n*ln(f)+2)/b^3/ln(f)^3/n*f^(a+b*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int f^{a+bx^n} x^{-1+3n}\,dx = \frac{\left(b^2x^{2\,n}\log\left(f\right)^2 - 2\,bx^n\log\left(f\right) + 2\right)e^{\left(bx^n\log(f)+a\log(f)\right)}}{b^3n\log\left(f\right)^3}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+3*n),x, algorithm="fricas")
```

output
```
(b^2*x^(2*n)*log(f)^2 - 2*b*x^n*log(f) + 2)*e^(b*x^n*log(f) + a*log(f))/(b
^3*n*log(f)^3)
```

Sympy [A] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int f^{a+bx^n} x^{-1+3n}\,dx = \begin{cases} \log\left(x\right) & \text{for } b=0 \wedge f=1 \wedge n=0 \\ \frac{f^a x x^{3n-1}}{3n} & \text{for } b=0 \\ f^{a+b}\log\left(x\right) & \text{for } n=0 \\ \frac{x x^{3n-1}}{3n} & \text{for } f=1 \\ \frac{f^{a+bx^n}x^{2n}}{bn\log\left(f\right)} - \frac{2f^{a+bx^n}x^n}{b^2n\log\left(f\right)^2} + \frac{2f^{a+bx^n}}{b^3n\log\left(f\right)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(a+b*x**n)*x**(-1+3*n),x)
```

output
```
Piecewise((log(x), Eq(b, 0) & Eq(f, 1) & Eq(n, 0)), (f**a*x*x**(3*n - 1)/(
3*n), Eq(b, 0)), (f**(a + b)*log(x), Eq(n, 0)), (x*x**(3*n - 1)/(3*n), Eq(
f, 1)), (f**(a + b*x**n)*x**(2*n)/(b*n*log(f)) - 2*f**(a + b*x**n)*x**n/(b
**2*n*log(f)**2) + 2*f**(a + b*x**n)/(b**3*n*log(f)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int f^{a+bx^n}x^{-1+3n}\,dx = \frac{\left(b^2f^ax^{2\,n}\log\left(f\right)^2 - 2\,bf^ax^n\log\left(f\right) + 2\,f^a\right)f^{bx^n}}{b^3n\log\left(f\right)^3}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+3*n),x, algorithm="maxima")
```

output
```
(b^2*f^a*x^(2*n)*log(f)^2 - 2*b*f^a*x^n*log(f) + 2*f^a)*f^(b*x^n)/(b^3*n*l
og(f)^3)
```

Giac [F]

$$\int f^{a+bx^n}x^{-1+3n}\,dx = \int f^{bx^n+a}x^{3\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1+3*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n}x^{-1+3n}\,dx = \int f^{a+b\,x^n}\,x^{3\,n-1}\,dx$$

input
```
int(f^(a + b*x^n)*x^(3*n - 1),x)
```

output
```
int(f^(a + b*x^n)*x^(3*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int f^{a+bx^n} x^{-1+3n} \, dx = \frac{f^{x^n b+a} \left(x^{2n} \log(f)^2 b^2 - 2x^n \log(f) b + 2\right)}{\log(f)^3 b^3 n}$$

input
```
int(f^(a+b*x^n)*x^(-1+3*n),x)
```

output
```
(f**(x***n*b + a)*(x**(2*n)*log(f)**2*b**2 - 2*x***n*log(f)*b + 2))/(log(f)*
*3*b**3*n)
```

3.124 $\int f^{a+bx^n} x^{-1+2n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int f^{a+bx^n} x^{-1+2n}\,dx = -\frac{f^{a+bx^n}}{b^2 n \log^2(f)} + \frac{f^{a+bx^n} x^n}{bn \log(f)}$$

output
```
-f^(a+b*x^n)/b^2/n/ln(f)^2+f^(a+b*x^n)*x^n/b/n/ln(f)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int f^{a+bx^n} x^{-1+2n}\,dx = -\frac{f^a \Gamma(2, -bx^n \log(f))}{b^2 n \log^2(f)}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 + 2*n),x]
```

output
```
-((f^a*Gamma[2, -(b*x^n*Log[f])])/(b^2*n*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2642, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2n-1} f^{a+bx^n}\,dx$$

$$\Big\downarrow \; 2642$$

$$\frac{x^n f^{a+bx^n}}{bn\log(f)} - \frac{\int f^{bx^n+a}x^{n-1}dx}{b\log(f)}$$

$$\Big\downarrow \; 2638$$

$$\frac{x^n f^{a+bx^n}}{bn\log(f)} - \frac{f^{a+bx^n}}{b^2 n\log^2(f)}$$

input
```
Int[f^(a + b*x^n)*x^(-1 + 2*n),x]
```

output
```
-(f^(a + b*x^n)/(b^2*n*Log[f]^2)) + (f^(a + b*x^n)*x^n)/(b*n*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{(b\,x^{n}\ln(f)-1)f^{a+b\,x^{n}}}{\ln(f)^{2}b^{2}n}$	30
meijerg	$\frac{f^{a}\left(1-\frac{(-2b\,x^{n}\ln(f)+2)\mathrm{e}^{b\,x^{n}\ln(f)}}{2}\right)}{b^{2}\ln(f)^{2}n}$	37
norman	$\frac{\mathrm{e}^{n\ln(x)}\mathrm{e}^{\left(a+b\,\mathrm{e}^{n\ln(x)}\right)\ln(f)}}{\ln(f)bn}-\frac{\mathrm{e}^{\left(a+b\,\mathrm{e}^{n\ln(x)}\right)\ln(f)}}{\ln(f)^{2}b^{2}n}$	56

input
```
int(f^(a+b*x^n)*x^(2*n-1),x,method=_RETURNVERBOSE)
```

output
```
(b*x^n*ln(f)-1)/ln(f)^2/b^2/n*f^(a+b*x^n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int f^{a+bx^n}x^{-1+2n}\,dx=\frac{(bx^n\log(f)-1)e^{(bx^n\log(f)+a\log(f))}}{b^2n\log(f)^2}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+2*n),x, algorithm="fricas")
```

output
```
(b*x^n*log(f) - 1)*e^(b*x^n*log(f) + a*log(f))/(b^2*n*log(f)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.

Time = 1.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int f^{a+bx^n}x^{-1+2n}\,dx = \begin{cases} \log(x) & \text{for}\ b = 0 \wedge f = 1 \wedge n = 0 \\ \frac{f^a x x^{2n-1}}{2n} & \text{for}\ b = 0 \\ f^{a+b}\log(x) & \text{for}\ n = 0 \\ \frac{x x^{2n-1}}{2n} & \text{for}\ f = 1 \\ \frac{f^{a+bx^n}x^n}{bn\log(f)} - \frac{f^{a+bx^n}}{b^2 n\log(f)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(a+b*x**n)*x**(-1+2*n),x)
```

output
```
Piecewise((log(x), Eq(b, 0) & Eq(f, 1) & Eq(n, 0)), (f**a*x*x**(2*n - 1)/(
2*n), Eq(b, 0)), (f**(a + b)*log(x), Eq(n, 0)), (x*x**(2*n - 1)/(2*n), Eq(
f, 1)), (f**(a + b*x**n)*x**n/(b*n*log(f)) - f**(a + b*x**n)/(b**2*n*log(f
)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int f^{a+bx^n}x^{-1+2n}\,dx = \frac{(bf^a x^n\log(f) - f^a)f^{bx^n}}{b^2 n\log(f)^2}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+2*n),x, algorithm="maxima")
```

output
```
(b*f^a*x^n*log(f) - f^a)*f^(b*x^n)/(b^2*n*log(f)^2)
```

Giac [F]

$$\int f^{a+bx^n}x^{-1+2n}\,dx = \int f^{bx^n+a}x^{2\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1+2*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n}x^{-1+2n}\,dx = \int f^{a+b\,x^n}\,x^{2\,n-1}\,dx$$

input
```
int(f^(a + b*x^n)*x^(2*n - 1),x)
```

output
```
int(f^(a + b*x^n)*x^(2*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.64

$$\int f^{a+bx^n}x^{-1+2n}\,dx = \frac{f^{x^n b+a}(x^n \log(f)\,b-1)}{\log(f)^2\,b^2 n}$$

input
```
int(f^(a+b*x^n)*x^(-1+2*n),x)
```

output
```
(f**(x**n*b + a)*(x**n*log(f)*b - 1))/(log(f)**2*b**2*n)
```

3.125 $\int f^{a+bx^n} x^{-1+n} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int f^{a+bx^n} x^{-1+n} \, dx = \frac{f^{a+bx^n}}{bn \log(f)}$$

output
```
f^(a+b*x^n)/b/n/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} x^{-1+n} \, dx = \frac{f^{a+bx^n}}{bn \log(f)}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 + n),x]
```

output
```
f^(a + b*x^n)/(b*n*Log[f])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \; 2638$$

$$\frac{f^{a+bx^n}}{bn\log(f)}$$

input
```
Int[f^(a + b*x^n)*x^(-1 + n),x]
```

output
```
f^(a + b*x^n)/(b*n*Log[f])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{f^{a+b\,x^{n}}}{bn\ln\left(f\right)}$	21
norman	$\frac{\mathrm{e}^{\left(a+b\,\mathrm{e}^{n\ln\left(x\right)}\right)\ln\left(f\right)}}{\ln\left(f\right)bn}$	25
meijerg	$-\frac{f^{a}\left(-\frac{\left(-1\right)^{-\frac{1}{n}-\frac{-1+n}{n}}}{\Gamma\left(2-\frac{1}{n}-\frac{-1+n}{n}\right)}+\frac{\left(-1\right)^{-\frac{1}{n}-\frac{-1+n}{n}}\mathrm{e}^{b\,x^{n}\ln\left(f\right)}}{\Gamma\left(2-\frac{1}{n}-\frac{-1+n}{n}\right)}\right)}{\ln\left(f\right)bn}$	96

input
```
int(f^(a+b*x^n)*x^(-1+n),x,method=_RETURNVERBOSE)
```

output
```
f^(a+b*x^n)/b/n/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int f^{a+bx^n}x^{-1+n}\,dx=\frac{e^{(bx^n\log(f)+a\log(f))}}{bn\log(f)}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+n),x, algorithm="fricas")
```

output
```
e^(b*x^n*log(f) + a*log(f))/(b*n*log(f))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(14) = 28.

Time = 0.92 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int f^{a+bx^n}x^{-1+n}\,dx=\begin{cases}\log(x) & \text{for } b=0\wedge f=1\wedge n=0\\ \frac{f^axx^{n-1}}{n} & \text{for } b=0\\ f^{a+b}\log(x) & \text{for } n=0\\ \frac{xx^{n-1}}{n} & \text{for } f=1\\ \frac{f^{a+bx^n}}{bn\log(f)} & \text{otherwise}\end{cases}$$

input
```
integrate(f**(a+b*x**n)*x**(-1+n),x)
```

output
```
Piecewise((log(x), Eq(b, 0) & Eq(f, 1) & Eq(n, 0)), (f**a*x*x**(n - 1)/n,
Eq(b, 0)), (f**(a + b)*log(x), Eq(n, 0)), (x*x**(n - 1)/n, Eq(f, 1)), (f**
(a + b*x**n)/(b*n*log(f)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}x^{-1+n}\,dx=\frac{f^{bx^n+a}}{bn\log(f)}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+n),x, algorithm="maxima")
```

output
```
f^(b*x^n + a)/(b*n*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}x^{-1+n}\,dx=\frac{f^{bx^n+a}}{bn\log(f)}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+n),x, algorithm="giac")
```

output
```
f^(b*x^n + a)/(b*n*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} x^{-1+n} \, dx = \frac{f^{a+b\,x^n}}{b\,n\,\ln(f)}$$

input
```
int(f^(a + b*x^n)*x^(n - 1),x)
```

output
```
f^(a + b*x^n)/(b*n*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} x^{-1+n} \, dx = \frac{f^{x^n b+a}}{\log(f)\,bn}$$

input
```
int(f^(a+b*x^n)*x^(-1+n),x)
```

output
```
f**(x**n*b + a)/(log(f)*b*n)
```

3.126 $\int \frac{f^{a+bx^n}}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{f^{a+bx^n}}{x}\,dx=\frac{f^a\,\text{ExpIntegralEi}\left(bx^n\log(f)\right)}{n}$$

output
```
f^a*Ei(b*x^n*ln(f))/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx=\frac{f^a\,\text{ExpIntegralEi}\left(bx^n\log(f)\right)}{n}$$

input
```
Integrate[f^(a + b*x^n)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^n*Log[f]])/n
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx^n}}{x} \, dx$$

$$\Big\downarrow \; 2639$$

$$\frac{f^a \text{ExpIntegralEi}\left(bx^n \log(f)\right)}{n}$$

input
```
Int[f^(a + b*x^n)/x,x]
```

output
```
(f^a*ExpIntegralEi[b*x^n*Log[f]])/n
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

method	result	size
risch	$-\frac{f^{a} \operatorname{expIntegral}_{1}\left(-b \,x^{n} \ln \left(f \right)\right)}{n}$	19
meijerg	$\frac{f^{a} \left(n \ln \left(x \right)+\ln \left(-b \right)+\ln \left(\ln \left(f \right)\right)-\ln \left(-b \,x^{n} \ln \left(f \right)\right)-\operatorname{expIntegral}_{1}\left(-b \,x^{n} \ln \left(f \right)\right)\right)}{n}$	43

input
```
int(f^(a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/n*f^a*Ei(1,-b*x^n*ln(f))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx = \frac{f^a\text{Ei}(bx^n\log{(f)})}{n}$$

input
```
integrate(f^(a+b*x^n)/x,x, algorithm="fricas")
```

output
```
f^a*Ei(b*x^n*log(f))/n
```

Sympy [F]

$$\int \frac{f^{a+bx^n}}{x}\,dx = \int \frac{f^{a+bx^n}}{x}\,dx$$

input
```
integrate(f**(a+b*x**n)/x,x)
```

output
```
Integral(f**(a + b*x**n)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx = \frac{f^a\text{Ei}(bx^n\log{(f)})}{n}$$

input
```
integrate(f^(a+b*x^n)/x,x, algorithm="maxima")
```

output
```
f^a*Ei(b*x^n*log(f))/n
```

Giac [F]

$$\int \frac{f^{a+bx^n}}{x}\,dx = \int \frac{f^{bx^n+a}}{x}\,dx$$

input
```
integrate(f^(a+b*x^n)/x,x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{a+bx^n}}{x}\,dx = \int \frac{f^{a+b\,x^n}}{x}\,dx$$

input
```
int(f^(a + b*x^n)/x,x)
```

output
```
int(f^(a + b*x^n)/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx^n}}{x}\,dx = \frac{f^a\,ei(x^n\log(f)\,b)}{n}$$

input
```
int(f^(a+b*x^n)/x,x)
```

output
```
(f**a*ei(x**n*log(f)*b))/n
```

3.127 $\int f^{a+bx^n}x^{-1-n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int f^{a+bx^n}x^{-1-n}\,dx=-\frac{f^{a+bx^n}x^{-n}}{n}+\frac{bf^a\,\text{ExpIntegralEi}\left(bx^n\log(f)\right)\log(f)}{n}$$

output
```
-f^(a+b*x^n)/n/(x^n)+b*f^a*Ei(b*x^n*ln(f))*ln(f)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int f^{a+bx^n}x^{-1-n}\,dx=\frac{bf^a\Gamma(-1,-bx^n\log(f))\log(f)}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 - n),x]
```

output
```
(b*f^a*Gamma[-1, -(b*x^n*Log[f])]*Log[f])/n
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2644, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-n-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \text{2644}$$

$$b \log(f) \int \frac{f^{bx^n+a}}{x} dx - \frac{x^{-n} f^{a+bx^n}}{n}$$

$$\Big\downarrow \text{2639}$$

$$\frac{bf^a \log(f) \text{ExpIntegralEi}\left(bx^n \log(f)\right)}{n} - \frac{x^{-n} f^{a+bx^n}}{n}$$

input
```
Int[f^(a + b*x^n)*x^(-1 - n),x]
```

output
```
-(f^(a + b*x^n)/(n*x^n)) + (b*f^a*ExpIntegralEi[b*x^n*Log[f]]*Log[f])/n
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{f^{b\,x^{n}} f^{a} x^{-n}}{n}-\frac{\ln(f) b\, f^{a} \operatorname{expIntegral}_{1}(-b\,x^{n} \ln(f))}{n}$
meijerg	$-\frac{f^{a} b \ln(f) \left(-x^{\left(\frac{-n-1}{n}+\frac{1}{n}\right) n} (-b)^{\frac{-n-1}{n}+\frac{1}{n}} \ln(f)^{\frac{-n-1}{n}+\frac{1}{n}}+\frac{(-1)^{-\frac{-n-1}{n}-\frac{1}{n}} \left(-\Psi\left(1-\frac{-n-1}{n}-\frac{1}{n}\right)+n\ln(x)+\ln(-b)+\ln(\ln(f))\right)}{\Gamma\left(1-\frac{-n-1}{n}-\frac{1}{n}\right)}+\frac{(-1)^{}}{}\right.}{}$

input
```
int(f^(a+b*x^n)*x^(-n-1),x,method=_RETURNVERBOSE)
```

output
```
-1/n*f^(b*x^n)*f^a/(x^n)-1/n*ln(f)*b*f^a*Ei(1,-b*x^n*ln(f))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.13

$$\int f^{a+bx^n} x^{-1-n}\,dx = \frac{bf^a x^n \mathrm{Ei}(bx^n \log(f)) \log(f) - e^{(bx^n \log(f)+a\log(f))}}{nx^n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-n),x, algorithm="fricas")
```

output
```
(b*f^a*x^n*Ei(b*x^n*log(f))*log(f) - e^(b*x^n*log(f) + a*log(f)))/(n*x^n)
```

Sympy [F]

$$\int f^{a+bx^n} x^{-1-n}\,dx = \int f^{a+bx^n} x^{-n-1}\,dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1-n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(-n - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.53

$$\int f^{a+bx^n} x^{-1-n}\,dx = \frac{bf^a\Gamma(-1,-bx^n\log(f))\log(f)}{n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-n),x, algorithm="maxima")
```

output
```
b*f^a*gamma(-1, -b*x^n*log(f))*log(f)/n
```

Giac [F]

$$\int f^{a+bx^n} x^{-1-n}\,dx = \int f^{bx^n+a} x^{-n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1-n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(-n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n} x^{-1-n}\,dx = \int \frac{f^{a+b\,x^n}}{x^{n+1}}\,dx$$

input
```
int(f^(a + b*x^n)/x^(n + 1),x)
```

output
```
int(f^(a + b*x^n)/x^(n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int f^{a+bx^n} x^{-1-n}\, dx = \frac{f^a \left(x^n ei(x^n \log(f)\, b) \log(f)\, b - f^{x^n b}\right)}{x^n n}$$

input
```
int(f^(a+b*x^n)*x^(-1-n),x)
```

output
```
(f**a*(x**n*ei(x**n*log(f)*b)*log(f)*b - f**(x**n*b)))/(x**n*n)
```

3.128 $\int f^{a+bx^n}x^{-1-2n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int f^{a+bx^n}x^{-1-2n}\,dx=-\frac{f^{a+bx^n}x^{-2n}}{2n}-\frac{bf^{a+bx^n}x^{-n}\log(f)}{2n}+\frac{b^2f^a\,\text{ExpIntegralEi}\left(bx^n\log(f)\right)\log^2(f)}{2n}$$

output
```
-1/2*f^(a+b*x^n)/n/(x^(2*n))-1/2*b*f^(a+b*x^n)*ln(f)/n/(x^n)+1/2*b^2*f^a*E
i(b*x^n*ln(f))*ln(f)^2/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.35

$$\int f^{a+bx^n}x^{-1-2n}\,dx=-\frac{b^2f^a\Gamma(-2,-bx^n\log(f))\log^2(f)}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 - 2*n),x]
```

output
```
-((b^2*f^a*Gamma[-2, -(b*x^n*Log[f])]*Log[f]^2)/n)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2644, 2644, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2n-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \ 2644$$

$$\frac{1}{2} b \log(f) \int f^{bx^n+a} x^{-n-1} dx - \frac{x^{-2n} f^{a+bx^n}}{2n}$$

$$\Big\downarrow \ 2644$$

$$\frac{1}{2} b \log(f) \left(b \log(f) \int \frac{f^{bx^n+a}}{x} dx - \frac{x^{-n} f^{a+bx^n}}{n} \right) - \frac{x^{-2n} f^{a+bx^n}}{2n}$$

$$\Big\downarrow \ 2639$$

$$\frac{1}{2} b \log(f) \left(\frac{b f^a \log(f) \operatorname{ExpIntegralEi}(bx^n \log(f))}{n} - \frac{x^{-n} f^{a+bx^n}}{n} \right) - \frac{x^{-2n} f^{a+bx^n}}{2n}$$

input
```
Int[f^(a + b*x^n)*x^(-1 - 2*n),x]
```

output
```
-1/2*f^(a + b*x^n)/(n*x^(2*n)) + (b*Log[f]*(-(f^(a + b*x^n)/(n*x^n)) + (b*
f^a*ExpIntegralEi[b*x^n*Log[f]]*Log[f])/n))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] &&  !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{f^{b\,x^{n}} f^{a} x^{-2n}}{2n}-\frac{\ln(f) b\, f^{b\,x^{n}} f^{a} x^{-n}}{2n}-\frac{\ln(f)^{2} b^{2} f^{a}\,\text{expIntegral}_{1}(-b\,x^{n}\ln(f))}{2n}$
meijerg	$\frac{f^{a}\ln(f)^{2} b^{2}\left(-\frac{x^{\left(\frac{-1-2n}{n}+\frac{1}{n}\right)n}(-b)^{\frac{-1-2n}{n}+\frac{1}{n}}\ln(f)^{\frac{-1-2n}{n}+\frac{1}{n}}}{2}+x^{\left(1+\frac{-1-2n}{n}+\frac{1}{n}\right)n}(-b)^{1+\frac{-1-2n}{n}+\frac{1}{n}}\ln(f)^{1+\frac{-1-2n}{n}+\frac{1}{n}}+\frac{(-1)^{-\frac{-1}{}}}{}\right.}{}$

input
```
int(f^(a+b*x^n)*x^(-1-2*n),x,method=_RETURNVERBOSE)
```

output
```
-1/2/n*f^(b*x^n)*f^a/(x^n)^2-1/2/n*ln(f)*b*f^(b*x^n)*f^a/(x^n)-1/2/n*ln(f)
^2*b^2*f^a*Ei(1,-b*x^n*ln(f))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int f^{a+bx^n}x^{-1-2n}\,dx=\frac{b^2f^ax^{2\,n}\mathrm{Ei}(bx^n\log{(f)})\log{(f)}^2-(bx^n\log{(f)}+1)e^{(bx^n\log(f)+a\log(f))}}{2\,nx^{2\,n}}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-2*n),x, algorithm="fricas")
```

output
```
1/2*(b^2*f^a*x^(2*n)*Ei(b*x^n*log(f))*log(f)^2 - (b*x^n*log(f) + 1)*e^(b*x
^n*log(f) + a*log(f)))/(n*x^(2*n))
```

Sympy [F]

$$\int f^{a+bx^n}x^{-1-2n}\,dx=\int f^{a+bx^n}x^{-2n-1}\,dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1-2*n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(-2*n - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.35

$$\int f^{a+bx^n}x^{-1-2n}\,dx=-\frac{b^2f^a\Gamma(-2,-bx^n\log(f))\log(f)^2}{n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-2*n),x, algorithm="maxima")
```

output
```
-b^2*f^a*gamma(-2, -b*x^n*log(f))*log(f)^2/n
```

Giac [F]

$$\int f^{a+bx^n}x^{-1-2n}\,dx=\int f^{bx^n+a}x^{-2n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1-2*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(-2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n} x^{-1-2n} \, dx = \int \frac{f^{a+b\,x^n}}{x^{2\,n+1}} \, dx$$

input
```
int(f^(a + b*x^n)/x^(2*n + 1),x)
```

output
```
int(f^(a + b*x^n)/x^(2*n + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.86

$$\int f^{a+bx^n} x^{-1-2n} \, dx = \frac{f^a \left(x^{2n} ei(x^n \log(f)\, b) \log(f)^2\, b^2 - x^n f^{x^n b} \log(f)\, b - f^{x^n b}\right)}{2x^{2n} n}$$

input
```
int(f^(a+b*x^n)*x^(-1-2*n),x)
```

output
```
(f**a*(x**(2*n)*ei(x**n*log(f)*b)*log(f)**2*b**2 - x**n*f**(x**n*b)*log(f)
*b - f**(x**n*b)))/(2*x**(2*n)*n)
```

3.129 $\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 104

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx=\frac{3f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)}{4b^{5/2}n\log^{\frac{5}{2}}(f)}-\frac{3f^{a+bx^n}x^{n/2}}{2b^2n\log^2(f)}+\frac{f^{a+bx^n}x^{3n/2}}{bn\log(f)}$$

output
```
3/4*f^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2*n)*ln(f)^(1/2))/b^(5/2)/n/ln(f)^(5/2)
-3/2*f^(a+b*x^n)*x^(1/2*n)/b^2/n/ln(f)^2+f^(a+b*x^n)*x^(3/2*n)/b/n/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.38

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx=-\frac{f^ax^{5n/2}\Gamma\left(\frac{5}{2},-bx^n\log(f)\right)}{n\left(-bx^n\log(f)\right)^{5/2}}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 + (5*n)/2),x]
```

output
```
-((f^a*x^((5*n)/2)*Gamma[5/2, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(5/2)
))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2642, 2642, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{5n}{2}-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \text{2642}$$

$$\frac{x^{3n/2} f^{a+bx^n}}{bn\log(f)}-\frac{3\int f^{bx^n+a}x^{\frac{3n}{2}-1}dx}{2b\log(f)}$$

$$\Big\downarrow \text{2642}$$

$$\frac{x^{3n/2} f^{a+bx^n}}{bn\log(f)}-\frac{3\left(\frac{x^{n/2} f^{a+bx^n}}{bn\log(f)}-\frac{\int f^{bx^n+a}x^{\frac{n-2}{2}}dx}{2b\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{2640}$$

$$\frac{x^{3n/2} f^{a+bx^n}}{bn\log(f)}-\frac{3\left(\frac{x^{n/2} f^{a+bx^n}}{bn\log(f)}-\frac{\int f^{bx^n+a}dx^{n/2}}{bn\log(f)}\right)}{2b\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{x^{3n/2} f^{a+bx^n}}{bn\log(f)}-\frac{3\left(\frac{x^{n/2} f^{a+bx^n}}{bn\log(f)}-\frac{\sqrt{\pi}f^a\text{erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{2b^{3/2}n\log^{\frac{3}{2}}(f)}\right)}{2b\log(f)}$$

input
```
Int[f^(a + b*x^n)*x^(-1 + (5*n)/2),x]
```

output
```
(f^(a + b*x^n)*x^((3*n)/2))/(b*n*Log[f]) - (3*(-1/2*(f^a*Sqrt[Pi]*Erfi[Sqr
t[b]*x^(n/2)*Sqrt[Log[f]]])/(b^(3/2)*n*Log[f]^(3/2)) + (f^(a + b*x^n)*x^(n
/2))/(b*n*Log[f])))/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

method	result	size
meijerg	$\frac{f^{a} \left(-\frac{x^{\frac{n}{2}} \left(-b\right)^{\frac{5}{2}} \sqrt{\ln \left(f\right)}\, \left(-10 b \,x^{n} \ln \left(f\right)+15\right) {\mathrm e}^{b \,x^{n} \ln \left(f\right)}}{10 b^{2}}+\frac{3 \left(-b\right)^{\frac{5}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x^{\frac{n}{2}} \sqrt{\ln \left(f\right)}\right)}{4 b^{\frac{5}{2}}}\right)}{\left(-b\right)^{\frac{5}{2}} \ln \left(f\right)^{\frac{5}{2}} n}$	82
risch	$\frac{f^{a} f^{b \,x^{n}} x^{\frac{3 n}{2}}}{n b \ln \left(f\right)}-\frac{3 f^{a} x^{\frac{n}{2}} f^{b \,x^{n}}}{2 n \ln \left(f\right)^{2} b^{2}}+\frac{3 f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln \left(f\right)}\, x^{\frac{n}{2}}\right)}{4 n \ln \left(f\right)^{2} b^{2} \sqrt{-b \ln \left(f\right)}}$	96

input
```
int(f^(a+b*x^n)*x^(-1+5/2*n),x,method=_RETURNVERBOSE)
```

output
```
f^a/(-b)^(5/2)/ln(f)^(5/2)/n*(-1/10*x^(1/2*n)*(-b)^(5/2)*ln(f)^(1/2)*(-10*
b*x^n*ln(f)+15)/b^2*exp(b*x^n*ln(f))+3/4*(-b)^(5/2)/b^(5/2)*Pi^(1/2)*erfi(
b^(1/2)*x^(1/2*n)*ln(f)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.79

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx = -\frac{3\sqrt{\pi}\sqrt{-b\log(f)}f^a\operatorname{erf}\left(\sqrt{-b\log(f)}x^{\frac{1}{2}n}\right)-2\left(2b^2x^{\frac{3}{2}n}\log(f)^2-3bx^{\frac{1}{2}n}\log(f)\right)e^{\left(bx^n\log(f)+a\log(f)\right)}}{4b^3n\log(f)^3}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+5/2*n),x, algorithm="fricas")
```

output
```
-1/4*(3*sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x^(1/2*n)) - 2*(2
*b^2*x^(3/2*n)*log(f)^2 - 3*b*x^(1/2*n)*log(f))*e^(b*x^n*log(f) + a*log(f)
))/(b^3*n*log(f)^3)
```

Sympy [F]

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx = \int f^{a+bx^n}x^{\frac{5n}{2}-1}\,dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1+5/2*n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(5*n/2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.32

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx = -\frac{f^a x^{\frac{5}{2}n}\Gamma\left(\frac{5}{2},-bx^n\log(f)\right)}{\left(-bx^n\log(f)\right)^{\frac{5}{2}}n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+5/2*n),x, algorithm="maxima")
```

output
```
-f^a*x^(5/2*n)*gamma(5/2, -b*x^n*log(f))/((-b*x^n*log(f))^(5/2)*n)
```

Giac [F]

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx=\int f^{bx^n+a}x^{\frac{5}{2}\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1+5/2*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(5/2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx=\int f^{a+b\,x^n}\,x^{\frac{5\,n}{2}-1}\,dx$$

input
```
int(f^(a + b*x^n)*x^((5*n)/2 - 1),x)
```

output
```
int(f^(a + b*x^n)*x^((5*n)/2 - 1), x)
```

Reduce [F]

$$\int f^{a+bx^n}x^{-1+\frac{5n}{2}}\,dx=f^a\left(\int\frac{x^{\frac{5n}{2}}f^{x^nb}}{x}dx\right)$$

input
```
int(f^(a+b*x^n)*x^(-1+5/2*n),x)
```

output
```
f**a*int((x**((5*n)/2)*f**(x**n*b))/x,x)
```

3.130 $\int f^{a+bx^n} x^{-1+\frac{3n}{2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 74

$$\int f^{a+bx^n} x^{-1+\frac{3n}{2}}\,dx = -\frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)}{2b^{3/2}n\log^{\frac{3}{2}}(f)}+\frac{f^{a+bx^n}x^{n/2}}{bn\log(f)}$$

output
```
-1/2*f^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2*n)*ln(f)^(1/2))/b^(3/2)/n/ln(f)^(3/2
)+f^(a+b*x^n)*x^(1/2*n)/b/n/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int f^{a+bx^n} x^{-1+\frac{3n}{2}}\,dx = -\frac{f^a x^{3n/2}\Gamma\left(\frac{3}{2},-bx^n\log(f)\right)}{n\left(-bx^n\log(f)\right)^{3/2}}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 + (3*n)/2),x]
```

output
```
-((f^a*x^((3*n)/2)*Gamma[3/2, -(b*x^n*Log[f])])/(n*(-(b*x^n*Log[f]))^(3/2)
))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2642, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{3n}{2}-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \text{2642}$$

$$\frac{x^{n/2} f^{a+bx^n}}{bn \log(f)} - \frac{\int f^{bx^n+a} x^{\frac{n-2}{2}} dx}{2b \log(f)}$$

$$\Big\downarrow \text{2640}$$

$$\frac{x^{n/2} f^{a+bx^n}}{bn \log(f)} - \frac{\int f^{bx^n+a} dx^{n/2}}{bn \log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{x^{n/2} f^{a+bx^n}}{bn \log(f)} - \frac{\sqrt{\pi} f^a \text{erfi}\left(\sqrt{b} \sqrt{\log(f)} x^{n/2}\right)}{2b^{3/2} n \log^{\frac{3}{2}}(f)}$$

input
```
Int[f^(a + b*x^n)*x^(-1 + (3*n)/2),x]
```

output
```
-1/2*(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x^(n/2)*Sqrt[Log[f]]])/(b^(3/2)*n*Log[f]^(
3/2)) + (f^(a + b*x^n)*x^(n/2))/(b*n*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{f^{a} x^{\frac{n}{2}} f^{b \,x^{n}}}{n \ln \left(f \right) b}-\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln \left(f \right)}\, x^{\frac{n}{2}}\right)}{2 n \ln \left(f \right) b \sqrt{-b \ln \left(f \right)}}$	67
meijerg	$\frac{f^{a} \left(\frac{x^{\frac{n}{2}} \left(-b \right)^{\frac{3}{2}} \sqrt{\ln \left(f \right)}\, {\mathrm e}^{b \,x^{n} \ln \left(f \right)}}{b}-\frac{\left(-b \right)^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x^{\frac{n}{2}} \sqrt{\ln \left(f \right)}\right)}{2 b^{\frac{3}{2}}}\right)}{\left(-b \right)^{\frac{3}{2}} \ln \left(f \right)^{\frac{3}{2}} n}$	71

input
```
int(f^(a+b*x^n)*x^(-1+3/2*n),x,method=_RETURNVERBOSE)
```

output
```
1/n*f^a/ln(f)/b*x^(1/2*n)*f^(b*x^n)-1/2/n*f^a/ln(f)/b*Pi^(1/2)/(-b*ln(f))^
(1/2)*erf((-b*ln(f))^(1/2)*x^(1/2*n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int f^{a+bx^n} x^{-1+\frac{3n}{2}}\,dx = \frac{2\,bx^{\frac{1}{2}\,n} e^{\left(bx^n \log\left(f\right) + a\log\left(f\right)\right)} \log\left(f\right) + \sqrt{\pi}\sqrt{-b\log\left(f\right)} f^a \operatorname{erf}\left(\sqrt{-b\log\left(f\right)} x^{\frac{1}{2}\,n}\right)}{2\,b^2 n \log\left(f\right)^2}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+3/2*n),x, algorithm="fricas")
```

output
```
1/2*(2*b*x^(1/2*n)*e^(b*x^n*log(f) + a*log(f))*log(f) + sqrt(pi)*sqrt(-b*l
og(f))*f^a*erf(sqrt(-b*log(f))*x^(1/2*n)))/(b^2*n*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx^n} x^{-1+\frac{3n}{2}}\,dx = \int f^{a+bx^n} x^{\frac{3n}{2}-1}\,dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1+3/2*n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(3*n/2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.45

$$\int f^{a+bx^n} x^{-1+\frac{3n}{2}}\,dx = -\frac{f^a x^{\frac{3}{2}n}\Gamma\left(\frac{3}{2}, -bx^n \log(f)\right)}{\left(-bx^n \log(f)\right)^{\frac{3}{2}} n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+3/2*n),x, algorithm="maxima")
```

output
```
-f^a*x^(3/2*n)*gamma(3/2, -b*x^n*log(f))/((-b*x^n*log(f))^(3/2)*n)
```

Giac [F]

$$\int f^{a+bx^n}x^{-1+\frac{3n}{2}}\,dx = \int f^{bx^n+a}x^{\frac{3}{2}\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1+3/2*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(3/2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n}x^{-1+\frac{3n}{2}}\,dx = \int f^{a+b\,x^n}\,x^{\frac{3\,n}{2}-1}\,dx$$

input
```
int(f^(a + b*x^n)*x^((3*n)/2 - 1),x)
```

output
```
int(f^(a + b*x^n)*x^((3*n)/2 - 1), x)
```

Reduce [F]

$$\int f^{a+bx^n}x^{-1+\frac{3n}{2}}\,dx = f^a\left(\int \frac{x^{\frac{3n}{2}}f^{x^n b}}{x}dx\right)$$

input
```
int(f^(a+b*x^n)*x^(-1+3/2*n),x)
```

output
```
f**a*int((x**((3*n)/2)*f**(x**n*b))/x,x)
```

3.131 $\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 43

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)}{\sqrt{b}n\sqrt{\log(f)}}$$

output
```
f^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2*n)*ln(f)^(1/2))/b^(1/2)/n/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=\frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)}{\sqrt{b}n\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 + n/2),x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x^(n/2)*Sqrt[Log[f]]])/(Sqrt[b]*n*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{\frac{n}{2}-1} f^{a+bx^n} \, dx$$

$$\Big\downarrow \text{ 2640}$$

$$\frac{2 \int f^{bx^n+a} dx^{n/2}}{n}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\sqrt{\pi} f^a \text{erfi}\left(\sqrt{b}\sqrt{\log(f)} x^{n/2}\right)}{\sqrt{b} n \sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x^n)*x^(-1 + n/2),x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x^(n/2)*Sqrt[Log[f]]])/(Sqrt[b]*n*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.74

method	result	size
meijerg	$\frac{f^{a}\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{b}\,x^{\frac{n}{2}}\sqrt{\ln\left(f\right)}\right)}{\sqrt{b}\,n\sqrt{\ln\left(f\right)}}$	32
risch	$\frac{f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-b\ln\left(f\right)}\,x^{\frac{n}{2}}\right)}{n\sqrt{-b\ln\left(f\right)}}$	32

input
```
int(f^(a+b*x^n)*x^(-1+1/2*n),x,method=_RETURNVERBOSE)
```

output
```
f^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2*n)*ln(f)^(1/2))/b^(1/2)/n/ln(f)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=-\frac{\sqrt{\pi}\sqrt{-b\log\left(f\right)}f^{a}\operatorname{erf}\left(\sqrt{-b\log\left(f\right)}xx^{\frac{1}{2}n-1}\right)}{bn\log\left(f\right)}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+1/2*n),x, algorithm="fricas")
```

output
```
-sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))*x*x^(1/2*n - 1))/(b*n*lo
g(f))
```

Sympy [F]

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=\int f^{a+bx^n}x^{\frac{n}{2}-1}\,dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1+1/2*n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(n/2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}f^a x^{\frac{1}{2}n}\left(\operatorname{erf}\left(\sqrt{-bx^n\log(f)}\right)-1\right)}{\sqrt{-bx^n\log(f)}n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+1/2*n),x, algorithm="maxima")
```

output
```
sqrt(pi)*f^a*x^(1/2*n)*(erf(sqrt(-b*x^n*log(f))) - 1)/(sqrt(-b*x^n*log(f))
*n)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=-\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-b\log(f)}\sqrt{x^n}\right)}{\sqrt{-b\log(f)}n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1+1/2*n),x, algorithm="giac")
```

output
```
-sqrt(pi)*f^a*erf(-sqrt(-b*log(f))*sqrt(x^n))/(sqrt(-b*log(f))*n)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=\int f^{a+b\,x^n}\,x^{\frac{n}{2}-1}\,dx$$

input
```
int(f^(a + b*x^n)*x^(n/2 - 1),x)
```

output
```
int(f^(a + b*x^n)*x^(n/2 - 1), x)
```

Reduce [F]

$$\int f^{a+bx^n}x^{-1+\frac{n}{2}}\,dx=f^a\left(\int\frac{x^{\frac{n}{2}}f^{x^n b}}{x}dx\right)$$

input
```
int(f^(a+b*x^n)*x^(-1+1/2*n),x)
```

output
```
f**a*int((x**(n/2)*f**(x**n*b))/x,x)
```

3.132 $\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx=-\frac{2f^{a+bx^n}x^{-n/2}}{n}+\frac{2\sqrt{b}f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)\sqrt{\log(f)}}{n}$$

output
```
-2*f^(a+b*x^n)/n/(x^(1/2*n))+2*b^(1/2)*f^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2*n)
*ln(f)^(1/2))*ln(f)^(1/2)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx=-\frac{f^a x^{-n/2}\Gamma\left(-\frac{1}{2},-bx^n\log(f)\right)\sqrt{-bx^n\log(f)}}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 - n/2),x]
```

output
```
-((f^a*Gamma[-1/2, -(b*x^n*Log[f])]*Sqrt[-(b*x^n*Log[f])])/(n*x^(n/2)))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2644, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{n}{2}-1}f^{a+bx^n}\,dx$$

$$\Big\downarrow \text{2644}$$

$$2b\log(f)\int f^{bx^n+a}x^{\frac{n-2}{2}}dx-\frac{2x^{-n/2}f^{a+bx^n}}{n}$$

$$\Big\downarrow \text{2640}$$

$$\frac{4b\log(f)\int f^{bx^n+a}dx^{n/2}}{n}-\frac{2x^{-n/2}f^{a+bx^n}}{n}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2\sqrt{\pi}\sqrt{b}f^a\sqrt{\log(f)}\text{erfi}\left(\sqrt{b}\sqrt{\log(f)}x^{n/2}\right)}{n}-\frac{2x^{-n/2}f^{a+bx^n}}{n}$$

input
```
Int[f^(a + b*x^n)*x^(-1 - n/2),x]
```

output
```
(-2*f^(a + b*x^n))/(n*x^(n/2)) + (2*Sqrt[b]*f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x^(n
/2)*Sqrt[Log[f]]]*Sqrt[Log[f]])/n
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] &&  !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{2 f^{a} x^{-\frac{n}{2}} f^{b \,x^{n}}}{n}+\frac{2 f^{a} \ln \left(f \right) b \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln \left(f \right)}\, x^{\frac{n}{2}}\right)}{n \sqrt{-b \ln \left(f \right)}}$	59
meijerg	$\frac{f^{a} \sqrt{-b}\, \sqrt{\ln \left(f \right)}\, \left(-\frac{2 x^{-\frac{n}{2}} {\mathrm e}^{b \,x^{n} \ln \left(f \right)}}{\sqrt{-b}\, \sqrt{\ln \left(f \right)}}+\frac{2 \sqrt{b}\, \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x^{\frac{n}{2}} \sqrt{\ln \left(f \right)}\right)}{\sqrt{-b}}\right)}{n}$	69

input
```
int(f^(a+b*x^n)*x^(-1-1/2*n),x,method=_RETURNVERBOSE)
```

output
```
-2/n*f^a/(x^(1/2*n))*f^(b*x^n)+2/n*f^a*ln(f)*b*Pi^(1/2)/(-b*ln(f))^(1/2)*e
rf((-b*ln(f))^(1/2)*x^(1/2*n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx = -\frac{2\left(\sqrt{\pi}\sqrt{-b\log\left(f\right)}f^{a}\operatorname{erf}\left(\frac{\sqrt{-b\log(f)}}{x x^{-\frac{1}{2}\,n-1}}\right)+x x^{-\frac{1}{2}\,n-1}e^{\left(\frac{a x^2 x^{-n-2}\log(f)+b\log(f)}{x^2 x^{-n-2}}\right)}\right)}{n}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-1/2*n),x, algorithm="fricas")
```

output
```
-2*(sqrt(pi)*sqrt(-b*log(f))*f^a*erf(sqrt(-b*log(f))/(x*x^(-1/2*n - 1))) +
 x*x^(-1/2*n - 1)*e^((a*x^2*x^(-n - 2)*log(f) + b*log(f))/(x^2*x^(-n - 2))
))/n
```

Sympy [F]

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\, dx = \int f^{a+bx^n} x^{-\frac{n}{2}-1}\, dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1-1/2*n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(-n/2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\, dx = -\frac{\sqrt{-bx^n\log(f)}f^a\Gamma\left(-\frac{1}{2}, -bx^n\log(f)\right)}{nx^{\frac{1}{2}n}}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-1/2*n),x, algorithm="maxima")
```

output
```
-sqrt(-b*x^n*log(f))*f^a*gamma(-1/2, -b*x^n*log(f))/(n*x^(1/2*n))
```

Giac [F]

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx = \int f^{bx^n+a} x^{-\frac{1}{2}\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1-1/2*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(-1/2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx = \int \frac{f^{a+b\,x^n}}{x^{\frac{n}{2}+1}}\,dx$$

input
```
int(f^(a + b*x^n)/x^(n/2 + 1),x)
```

output
```
int(f^(a + b*x^n)/x^(n/2 + 1), x)
```

Reduce [F]

$$\int f^{a+bx^n} x^{-1-\frac{n}{2}}\,dx = f^a \left(\int \frac{f^{x^n b}}{x^{\frac{n}{2}} x}dx\right)$$

input
```
int(f^(a+b*x^n)*x^(-1-1/2*n),x)
```

output
```
f**a*int(f**(x**n*b)/(x**(n/2)*x),x)
```

3.133 $\int f^{a+bx^n} x^{-1-\frac{3n}{2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 96

$$\int f^{a+bx^n} x^{-1-\frac{3n}{2}} \, dx = -\frac{2f^{a+bx^n}x^{-3n/2}}{3n} - \frac{4bf^{a+bx^n}x^{-n/2}\log(f)}{3n} + \frac{4b^{3/2}f^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x^{n/2}\sqrt{\log(f)}\right)\log^{\frac{3}{2}}(f)}{3n}$$

output
```
-2/3*f^(a+b*x^n)/n/(x^(3/2*n))-4/3*b*f^(a+b*x^n)*ln(f)/n/(x^(1/2*n))+4/3*b
^(3/2)*f^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2*n)*ln(f)^(1/2))*ln(f)^(3/2)/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.41

$$\int f^{a+bx^n} x^{-1-\frac{3n}{2}} \, dx = -\frac{f^a x^{-3n/2}\Gamma\left(-\frac{3}{2}, -bx^n\log(f)\right)\left(-bx^n\log(f)\right)^{3/2}}{n}$$

input
```
Integrate[f^(a + b*x^n)*x^(-1 - (3*n)/2),x]
```

output
```
-((f^a*Gamma[-3/2, -(b*x^n*Log[f])]*(-(b*x^n*Log[f]))^(3/2))/(n*x^((3*n)/2
)))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2644, 2644, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{3n}{2}-1} f^{a+bx^n}\, dx$$

$$\Big\downarrow \text{ 2644}$$

$$\frac{2}{3} b \log(f) \int f^{bx^n+a} x^{-\frac{n}{2}-1} dx - \frac{2x^{-3n/2} f^{a+bx^n}}{3n}$$

$$\Big\downarrow \text{ 2644}$$

$$\frac{2}{3} b \log(f) \left(2b \log(f) \int f^{bx^n+a} x^{\frac{n-2}{2}}\, dx - \frac{2x^{-n/2} f^{a+bx^n}}{n}\right) - \frac{2x^{-3n/2} f^{a+bx^n}}{3n}$$

$$\Big\downarrow \text{ 2640}$$

$$\frac{2}{3} b \log(f) \left(\frac{4b \log(f) \int f^{bx^n+a} dx^{n/2}}{n} - \frac{2x^{-n/2} f^{a+bx^n}}{n}\right) - \frac{2x^{-3n/2} f^{a+bx^n}}{3n}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{2}{3} b \log(f) \left(\frac{2\sqrt{\pi}\sqrt{b} f^a \sqrt{\log(f)} \text{erfi}\left(\sqrt{b}\sqrt{\log(f)} x^{n/2}\right)}{n} - \frac{2x^{-n/2} f^{a+bx^n}}{n}\right) - \frac{2x^{-3n/2} f^{a+bx^n}}{3n}$$

input
```
Int[f^(a + b*x^n)*x^(-1 - (3*n)/2),x]
```

output
```
(-2*f^(a + b*x^n))/(3*n*x^((3*n)/2)) + (2*b*((-2*f^(a + b*x^n))/(n*x^(n/2)
) + (2*Sqrt[b]*f^a*Sqrt[Pi]*Erfi[Sqrt[b]*x^(n/2)*Sqrt[Log[f]]]*Sqrt[Log[f]
])/n)*Log[f])/3
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

method	result	size
meijerg	$\frac{f^{a} \left(-b\right)^{\frac{3}{2}} \ln \left(f\right)^{\frac{3}{2}} \left(-\frac{2 x^{-\frac{3 n}{2}} \left(2 b \,x^{n} \ln \left(f\right)+1\right) {\mathrm e}^{b \,x^{n} \ln \left(f\right)}}{3 \left(-b\right)^{\frac{3}{2}} \ln \left(f\right)^{\frac{3}{2}}}+\frac{4 b^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{b}\, x^{\frac{n}{2}} \sqrt{\ln \left(f\right)}\right)}{3 \left(-b\right)^{\frac{3}{2}}}\right)}{n}$	79
risch	$-\frac{2 f^{a} x^{-\frac{3 n}{2}} f^{b \,x^{n}}}{3 n}-\frac{4 f^{a} \ln \left(f\right) b \,x^{-\frac{n}{2}} f^{b \,x^{n}}}{3 n}+\frac{4 f^{a} \ln \left(f\right)^{2} b^{2} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-b \ln \left(f\right)}\, x^{\frac{n}{2}}\right)}{3 n \sqrt{-b \ln \left(f\right)}}$	88

input
```
int(f^(a+b*x^n)*x^(-1-3/2*n),x,method=_RETURNVERBOSE)
```

output
```
f^a*(-b)^(3/2)*ln(f)^(3/2)/n*(-2/3*x^(-3/2*n)/(-b)^(3/2)/ln(f)^(3/2)*(2*b*
x^n*ln(f)+1)*exp(b*x^n*ln(f))+4/3/(-b)^(3/2)*b^(3/2)*Pi^(1/2)*erfi(b^(1/2)
*x^(1/2*n)*ln(f)^(1/2)))
```

Fricas [F]

$$\int f^{a+bx^n}x^{-1-\frac{3n}{2}}\,dx=\int f^{bx^n+a}x^{-\frac{3}{2}\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1-3/2*n),x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)*x^(-3/2*n - 1), x)
```

Sympy [F]

$$\int f^{a+bx^n}x^{-1-\frac{3n}{2}}\,dx=\int f^{a+bx^n}x^{-\frac{3n}{2}-1}\,dx$$

input
```
integrate(f**(a+b*x**n)*x**(-1-3/2*n),x)
```

output
```
Integral(f**(a + b*x**n)*x**(-3*n/2 - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.36

$$\int f^{a+bx^n}x^{-1-\frac{3n}{2}}\,dx=-\frac{\left(-bx^n\log\left(f\right)\right)^{\frac{3}{2}}f^a\Gamma\left(-\frac{3}{2},-bx^n\log\left(f\right)\right)}{nx^{\frac{3}{2}\,n}}$$

input
```
integrate(f^(a+b*x^n)*x^(-1-3/2*n),x, algorithm="maxima")
```

output
```
-(-b*x^n*log(f))^(3/2)*f^a*gamma(-3/2, -b*x^n*log(f))/(n*x^(3/2*n))
```

Giac [F]

$$\int f^{a+bx^n} x^{-1-\frac{3n}{2}}\,dx = \int f^{bx^n+a} x^{-\frac{3}{2}\,n-1}\,dx$$

input
```
integrate(f^(a+b*x^n)*x^(-1-3/2*n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*x^(-3/2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n} x^{-1-\frac{3n}{2}}\,dx = \int \frac{f^{a+b\,x^n}}{x^{\frac{3\,n}{2}+1}}\,dx$$

input
```
int(f^(a + b*x^n)/x^((3*n)/2 + 1),x)
```

output
```
int(f^(a + b*x^n)/x^((3*n)/2 + 1), x)
```

Reduce [F]

$$\int f^{a+bx^n} x^{-1-\frac{3n}{2}}\,dx = f^a\left(\int \frac{f^{x^n b}}{x^{\frac{3n}{2}} x}dx\right)$$

input
```
int(f^(a+b*x^n)*x^(-1-3/2*n),x)
```

output
```
f**a*int(f**(x**n*b)/(x**((3*n)/2)*x),x)
```

3.134 $\int f^{c(a+bx)^2} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 203

$$\int f^{c(a+bx)^2} x^3 \, dx = -\frac{f^{c(a+bx)^2}}{2b^4c^2\log^2(f)} + \frac{3a\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{4b^4c^{3/2}\log^{\frac{3}{2}}(f)} + \frac{3a^2f^{c(a+bx)^2}}{2b^4c\log(f)} - \frac{3af^{c(a+bx)^2}(a+bx)}{2b^4c\log(f)} + \frac{f^{c(a+bx)^2}(a+bx)^2}{2b^4c\log(f)} - \frac{a^3\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{2b^4\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/2*f^(c*(b*x+a)^2)/b^4/c^2/ln(f)^2+3/4*a*Pi^(1/2)*erfi(c^(1/2)*(b*x+a)*l
n(f)^(1/2))/b^4/c^(3/2)/ln(f)^(3/2)+3/2*a^2*f^(c*(b*x+a)^2)/b^4/c/ln(f)-3/
2*a*f^(c*(b*x+a)^2)*(b*x+a)/b^4/c/ln(f)+1/2*f^(c*(b*x+a)^2)*(b*x+a)^2/b^4/
c/ln(f)-1/2*a^3*Pi^(1/2)*erfi(c^(1/2)*(b*x+a)*ln(f)^(1/2))/b^4/c^(1/2)/ln(
f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.47

$$\int f^{c(a+bx)^2}x^3\,dx$$
$$=\frac{a\sqrt{c}\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)\sqrt{\log(f)}(3-2a^2c\log(f))+2f^{c(a+bx)^2}(-1+c(a^2-abx+b^2x^2)\log(f}{4b^4c^2\log^2(f)}$$

input
```
Integrate[f^(c*(a + b*x)^2)*x^3,x]
```

output
```
(a*Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]]*Sqrt[Log[f]]*(3 -
 2*a^2*c*Log[f]) + 2*f^(c*(a + b*x)^2)*(-1 + c*(a^2 - a*b*x + b^2*x^2)*Log
[f]))/(4*b^4*c^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3f^{c(a+bx)^2}dx$$
$$\Big\downarrow \text{2656}$$
$$\int \left(-\frac{a^3f^{c(a+bx)^2}}{b^3}+\frac{3a^2(a+bx)f^{c(a+bx)^2}}{b^3}+\frac{(a+bx)^3f^{c(a+bx)^2}}{b^3}-\frac{3a(a+bx)^2f^{c(a+bx)^2}}{b^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\sqrt{\pi}a^3\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^4\sqrt{c}\sqrt{\log(f)}}+\frac{3a^2f^{c(a+bx)^2}}{2b^4c\log(f)}+\frac{3\sqrt{\pi}a\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{4b^4c^{3/2}\log^{\frac{3}{2}}(f)}-\frac{f^{c(a+bx)^2}}{2b^4c^2\log^2(f)}+\frac{(a+bx)^2f^{c(a+bx)^2}}{2b^4c\log(f)}-\frac{3a(a+bx)f^{c(a+bx)^2}}{2b^4c\log(f)}$$

input
```
Int[f^(c*(a + b*x)^2)*x^3,x]
```

output
```
-1/2*f^(c*(a + b*x)^2)/(b^4*c^2*Log[f]^2) + (3*a*Sqrt[Pi]*Erfi[Sqrt[c]*(a
+ b*x)*Sqrt[Log[f]]])/(4*b^4*c^(3/2)*Log[f]^(3/2)) + (3*a^2*f^(c*(a + b*x)
^2))/(2*b^4*c*Log[f]) - (3*a*f^(c*(a + b*x)^2)*(a + b*x))/(2*b^4*c*Log[f])
 + (f^(c*(a + b*x)^2)*(a + b*x)^2)/(2*b^4*c*Log[f]) - (a^3*Sqrt[Pi]*Erfi[S
qrt[c]*(a + b*x)*Sqrt[Log[f]]])/(2*b^4*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.23

method	result
risch	$\frac{x^{2} f^{b^{2} c \,x^{2}} f^{2 a b c x} f^{a^{2} c}}{2 c \,b^{2} \ln \left(f \right)}-\frac{a x \,f^{b^{2} c \,x^{2}} f^{2 a b c x} f^{a^{2} c}}{2 b^{3} c \ln \left(f \right)}+\frac{a^{2} f^{b^{2} c \,x^{2}} f^{2 a b c x} f^{a^{2} c}}{2 b^{4} c \ln \left(f \right)}+\frac{a^{3} \sqrt{\pi}\, \operatorname{erf}\left(-b \sqrt{-c \ln \left(f \right)}\, x +\frac{a c \ln \left(f \right)}{\sqrt{-c \ln \left(f \right)}}\right)}{2 b^{4} \sqrt{-c \ln \left(f \right)}}-\frac{3 a \sqrt{\pi}\, \mathrm{e}}{}$

input
```
int(f^(c*(b*x+a)^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/c/b^2/ln(f)*x^2*f^(b^2*c*x^2)*f^(2*a*b*c*x)*f^(a^2*c)-1/2*a/b^3/c/ln(f
)*x*f^(b^2*c*x^2)*f^(2*a*b*c*x)*f^(a^2*c)+1/2*a^2/b^4/c/ln(f)*f^(b^2*c*x^2
)*f^(2*a*b*c*x)*f^(a^2*c)+1/2*a^3/b^4*Pi^(1/2)/(-c*ln(f))^(1/2)*erf(-b*(-c
*ln(f))^(1/2)*x+a*c*ln(f)/(-c*ln(f))^(1/2))-3/4*a/b^4/c/ln(f)*Pi^(1/2)/(-c
*ln(f))^(1/2)*erf(-b*(-c*ln(f))^(1/2)*x+a*c*ln(f)/(-c*ln(f))^(1/2))-1/2/c^
2/b^4/ln(f)^2*f^(b^2*c*x^2)*f^(2*a*b*c*x)*f^(a^2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.56

$$\int f^{c(a+bx)^2}x^3\,dx$$
$$= \frac{\sqrt{\pi}(2\,a^3c\log{(f)} - 3\,a)\sqrt{-b^2c\log{(f)}}\,\mathrm{erf}\left(\frac{\sqrt{-b^2c\log(f)}(bx+a)}{b}\right) + 2\,((b^3cx^2 - ab^2cx + a^2bc)\log{(f)} - b)f^{b^2c}}{4\,b^5c^2\log{(f)}^2}$$

input
```
integrate(f^(c*(b*x+a)^2)*x^3,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(2*a^3*c*log(f) - 3*a)*sqrt(-b^2*c*log(f))*erf(sqrt(-b^2*c*l
og(f))*(b*x + a)/b) + 2*((b^3*c*x^2 - a*b^2*c*x + a^2*b*c)*log(f) - b)*f^(
b^2*c*x^2 + 2*a*b*c*x + a^2*c))/(b^5*c^2*log(f)^2)
```

Sympy [F]

$$\int f^{c(a+bx)^2}x^3\,dx = \int f^{c(a+bx)^2}x^3\,dx$$

input
```
integrate(f**(c*(b*x+a)**2)*x**3,x)
```

output
```
Integral(f**(c*(a + b*x)**2)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.30

$$\int f^{c(a+bx)^2}x^3\,dx =$$
$$-\frac{\frac{\sqrt{\pi}(b^2cx+abc)a^3c^3\left(\mathrm{erf}\left(\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}\right)-1\right)\log(f)^4}{(c\log(f))^{\frac{7}{2}}b^4\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}} - \frac{3\,a^2c^3f^{\frac{(b^2cx+abc)^2}{b^2c}}\log(f)^3}{(c\log(f))^{\frac{7}{2}}b^3} - \frac{3\,(b^2cx+abc)^3ac\Gamma\left(\frac{3}{2},-\frac{(b^2cx+abc)^2}{b^2c}\right.}{(c\log(f))^{\frac{7}{2}}b^6\left(-\frac{(b^2cx+abc)^2\,\mathrm{l}}{b^2c}\right.}}{2\sqrt{c\log{(f)}}b}$$

input
```
integrate(f^(c*(b*x+a)^2)*x^3,x, algorithm="maxima")
```

output
```
-1/2*(sqrt(pi)*(b^2*c*x + a*b*c)*a^3*c^3*(erf(sqrt(-(b^2*c*x + a*b*c)^2*lo
g(f)/(b^2*c))) - 1)*log(f)^4/((c*log(f))^(7/2)*b^4*sqrt(-(b^2*c*x + a*b*c)
^2*log(f)/(b^2*c))) - 3*a^2*c^3*f^((b^2*c*x + a*b*c)^2/(b^2*c))*log(f)^3/(
(c*log(f))^(7/2)*b^3) - 3*(b^2*c*x + a*b*c)^3*a*c*gamma(3/2, -(b^2*c*x + a
*b*c)^2*log(f)/(b^2*c))*log(f)^4/((c*log(f))^(7/2)*b^6*(-(b^2*c*x + a*b*c)
^2*log(f)/(b^2*c))^(3/2)) + c^2*gamma(2, -(b^2*c*x + a*b*c)^2*log(f)/(b^2*
c))*log(f)^2/((c*log(f))^(7/2)*b^3))/(sqrt(c*log(f))*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int f^{c(a+bx)^2}x^3\,dx = \frac{\frac{\sqrt{\pi}(2\,a^3c\log(f)-3\,a)\operatorname{erf}\left(-\sqrt{-c\log(f)}b\left(x+\frac{a}{b}\right)\right)}{\sqrt{-c\log(f)}bc\log(f)} + \frac{2\left(b^2c\left(x+\frac{a}{b}\right)^2\log(f)-3\,abc\left(x+\frac{a}{b}\right)\log(f)+3\,a^2c\log(f)-1\right)e^{(b^2cx^2\log(f)+2\,abcx\log(f)+a}}{bc^2\log(f)^2}}{4\,b^3}$$

input
```
integrate(f^(c*(b*x+a)^2)*x^3,x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*(2*a^3*c*log(f) - 3*a)*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sq
rt(-c*log(f))*b*c*log(f)) + 2*(b^2*c*(x + a/b)^2*log(f) - 3*a*b*c*(x + a/b
)*log(f) + 3*a^2*c*log(f) - 1)*e^(b^2*c*x^2*log(f) + 2*a*b*c*x*log(f) + a^
2*c*log(f))/(b*c^2*log(f)^2))/b^3
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.84

$$\int f^{c(a+bx)^2}x^3\,dx = \frac{f^{b^2\,c\,x^2}\,f^{a^2\,c}\,f^{2\,a\,b\,c\,x}\,x^2}{2\,b^2\,c\,\ln(f)} - \frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c\,\ln(f)}\,(a+b\,x)\right)\,\left(\frac{a^3}{b^4}-\frac{3\,a}{2\,b^4\,c\,\ln(f)}\right)}{2\,\sqrt{c\,\ln(f)}} + \frac{f^{b^2\,c\,x^2}\,f^{a^2\,c}\,f^{2\,a\,b\,c\,x}\,\left(\frac{a^2\,c\,\ln(f)}{2}-\frac{1}{2}\right)}{b^4\,c^2\,\ln(f)^2} - \frac{a\,f^{b^2\,c\,x^2}\,f^{a^2\,c}\,f^{2\,a\,b\,c\,x}\,x}{2\,b^3\,c\,\ln(f)}$$

input
```
int(f^(c*(a + b*x)^2)*x^3,x)
```

output
```
(f^(b^2*c*x^2)*f^(a^2*c)*f^(2*a*b*c*x)*x^2)/(2*b^2*c*log(f)) - (pi^(1/2)*e
rfi((c*log(f))^(1/2)*(a + b*x))*(a^3/b^4 - (3*a)/(2*b^4*c*log(f))))/(2*(c*
log(f))^(1/2)) + (f^(b^2*c*x^2)*f^(a^2*c)*f^(2*a*b*c*x)*((a^2*c*log(f))/2
- 1/2))/(b^4*c^2*log(f)^2) - (a*f^(b^2*c*x^2)*f^(a^2*c)*f^(2*a*b*c*x)*x)/(
2*b^3*c*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.15

$$\int f^{c(a+bx)^2}x^3\,dx = \frac{2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)aci+\log(f)bcix}{\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)^2\,a^3c^2i-3\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)aci+\log(f)bcix}{\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,aci+2f^{b^2cx^2+2abcx+a^2c}\sqrt{c}\,\cdots}{\cdots}$$

input
```
int(f^(c*(b*x+a)^2)*x^3,x)
```

output
```
(2*sqrt(pi)*erf((log(f)*a*c*i + log(f)*b*c*i*x)/(sqrt(c)*sqrt(log(f))))*lo
g(f)**2*a**3*c**2*i - 3*sqrt(pi)*erf((log(f)*a*c*i + log(f)*b*c*i*x)/(sqrt
(c)*sqrt(log(f))))*log(f)*a*c*i + 2*f**(a**2*c + 2*a*b*c*x + b**2*c*x**2)*
sqrt(c)*sqrt(log(f))*log(f)*a**2*c - 2*f**(a**2*c + 2*a*b*c*x + b**2*c*x**
2)*sqrt(c)*sqrt(log(f))*log(f)*a*b*c*x + 2*f**(a**2*c + 2*a*b*c*x + b**2*c
*x**2)*sqrt(c)*sqrt(log(f))*log(f)*b**2*c*x**2 - 2*f**(a**2*c + 2*a*b*c*x
+ b**2*c*x**2)*sqrt(c)*sqrt(log(f)))/(4*sqrt(c)*sqrt(log(f))*log(f)**2*b**
4*c**2)
```

3.135 $\int f^{c(a+bx)^2}x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int f^{c(a+bx)^2}x^2\,dx=-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{4b^3c^{3/2}\log^{\frac{3}{2}}(f)}-\frac{af^{c(a+bx)^2}}{b^3c\log(f)}+\frac{f^{c(a+bx)^2}(a+bx)}{2b^3c\log(f)}+\frac{a^2\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{2b^3\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/4*Pi^(1/2)*erfi(c^(1/2)*(b*x+a)*ln(f)^(1/2))/b^3/c^(3/2)/ln(f)^(3/2)-a*
f^(c*(b*x+a)^2)/b^3/c/ln(f)+1/2*f^(c*(b*x+a)^2)*(b*x+a)/b^3/c/ln(f)+1/2*a^
2*Pi^(1/2)*erfi(c^(1/2)*(b*x+a)*ln(f)^(1/2))/b^3/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int f^{c(a+bx)^2}x^2\,dx=\frac{-2\sqrt{c}f^{c(a+bx)^2}(a-bx)\sqrt{\log(f)}+\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)\left(-1+2a^2c\log(f)\right)}{4b^3c^{3/2}\log^{\frac{3}{2}}(f)}$$

input
```
Integrate[f^(c*(a + b*x)^2)*x^2,x]
```

output
```
(-2*Sqrt[c]*f^(c*(a + b*x)^2)*(a - b*x)*Sqrt[Log[f]] + Sqrt[Pi]*Erfi[Sqrt[
c]*(a + b*x)*Sqrt[Log[f]]]*(-1 + 2*a^2*c*Log[f]))/(4*b^3*c^(3/2)*Log[f]^(3
/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{c(a+bx)^2} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^2 f^{c(a+bx)^2}}{b^2}+\frac{(a+bx)^2 f^{c(a+bx)^2}}{b^2}-\frac{2a(a+bx) f^{c(a+bx)^2}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}a^2\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^3\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{4b^3c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{(a+bx)f^{c(a+bx)^2}}{2b^3c\log(f)}-\frac{af^{c(a+bx)^2}}{b^3c\log(f)}$$

input
```
Int[f^(c*(a + b*x)^2)*x^2,x]
```

output
```
-1/4*(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(b^3*c^(3/2)*Log[f]^(
3/2)) - (a*f^(c*(a + b*x)^2))/(b^3*c*Log[f]) + (f^(c*(a + b*x)^2)*(a + b*x
))/(2*b^3*c*Log[f]) + (a^2*Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/
(2*b^3*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.20

method	result
risch	$\frac{x f^{b^2 c x^2} f^{2abcx} f^{a^2 c}}{2c b^2 \ln(f)} - \frac{a f^{b^2 c x^2} f^{2abcx} f^{a^2 c}}{2b^3 c \ln(f)} - \frac{a^2\sqrt{\pi}\,\mathrm{erf}\left(-b\sqrt{-c\ln(f)}\,x+\frac{ac\ln(f)}{\sqrt{-c\ln(f)}}\right)}{2b^3\sqrt{-c\ln(f)}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(-b\sqrt{-c\ln(f)}\,x+\frac{ac\ln(f)}{\sqrt{-c\ln(f)}}\right)}{4c b^3\ln(f)\sqrt{-c\ln(f)}}$

input
```
int(f^(c*(b*x+a)^2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c/b^2/ln(f)*x*f^(b^2*c*x^2)*f^(2*a*b*c*x)*f^(a^2*c)-1/2*a/b^3/c/ln(f)*
f^(b^2*c*x^2)*f^(2*a*b*c*x)*f^(a^2*c)-1/2*a^2/b^3*Pi^(1/2)/(-c*ln(f))^(1/2
)*erf(-b*(-c*ln(f))^(1/2)*x+a*c*ln(f)/(-c*ln(f))^(1/2))+1/4/c/b^3/ln(f)*Pi
^(1/2)/(-c*ln(f))^(1/2)*erf(-b*(-c*ln(f))^(1/2)*x+a*c*ln(f)/(-c*ln(f))^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int f^{c(a+bx)^2}x^2\,dx = -\frac{\sqrt{\pi}(2\,a^2c\log(f)-1)\sqrt{-b^2c\log(f)}\,\mathrm{erf}\left(\frac{\sqrt{-b^2c\log(f)}(bx+a)}{b}\right)-2\,(b^2cx-abc)f^{b^2cx^2+2\,abcx+a^2c}\log(f)}{4\,b^4c^2\log(f)^2}$$

input
```
integrate(f^(c*(b*x+a)^2)*x^2,x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(2*a^2*c*log(f) - 1)*sqrt(-b^2*c*log(f))*erf(sqrt(-b^2*c*lo
g(f))*(b*x + a)/b) - 2*(b^2*c*x - a*b*c)*f^(b^2*c*x^2 + 2*a*b*c*x + a^2*c)
*log(f))/(b^4*c^2*log(f)^2)
```

Sympy [F]

$$\int f^{c(a+bx)^2}x^2\,dx=\int f^{c(a+bx)^2}x^2\,dx$$

input
```
integrate(f**(c*(b*x+a)**2)*x**2,x)
```

output
```
Integral(f**(c*(a + b*x)**2)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.56

$$\int f^{c(a+bx)^2}x^2\,dx = \frac{\frac{\sqrt{\pi}(b^2cx+abc)a^2c^2\left(\operatorname{erf}\left(\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}\right)-1\right)\log(f)^3}{(c\log(f))^{\frac{5}{2}}b^3\sqrt{-\frac{(b^2cx+abc)^2\log(f)}{b^2c}}}-\frac{2ac^2f^{\frac{(b^2cx+abc)^2}{b^2c}}\log(f)^2}{(c\log(f))^{\frac{5}{2}}b^2}-\frac{(b^2cx+abc)^3\Gamma\left(\frac{3}{2},-\frac{(b^2cx+abc)^2\log(f)}{b^2c}\right)\log(f)^3}{(c\log(f))^{\frac{5}{2}}b^5\left(-\frac{(b^2cx+abc)^2\log(f)}{b^2c}\right)^{\frac{3}{2}}}}{2\sqrt{c\log(f)}b}$$

input
```
integrate(f^(c*(b*x+a)^2)*x^2,x, algorithm="maxima")
```

output
```
1/2*(sqrt(pi)*(b^2*c*x + a*b*c)*a^2*c^2*(erf(sqrt(-(b^2*c*x + a*b*c)^2*log
(f)/(b^2*c))) - 1)*log(f)^3/((c*log(f))^(5/2)*b^3*sqrt(-(b^2*c*x + a*b*c)^
2*log(f)/(b^2*c))) - 2*a*c^2*f^((b^2*c*x + a*b*c)^2/(b^2*c))*log(f)^2/((c*
log(f))^(5/2)*b^2) - (b^2*c*x + a*b*c)^3*gamma(3/2, -(b^2*c*x + a*b*c)^2*l
og(f)/(b^2*c))*log(f)^3/((c*log(f))^(5/2)*b^5*(-(b^2*c*x + a*b*c)^2*log(f)
/(b^2*c))^(3/2)))/(sqrt(c*log(f))*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.76

$$\int f^{c(a+bx)^2}x^2\,dx = -\frac{\frac{\sqrt{\pi}(2a^2c\log(f)-1)\operatorname{erf}\left(-\sqrt{-c\log(f)}b\left(x+\frac{a}{b}\right)\right)}{\sqrt{-c\log(f)}bc\log(f)} - \frac{2\left(b\left(x+\frac{a}{b}\right)-2a\right)e^{\left(b^2cx^2\log(f)+2abcx\log(f)+a^2c\log(f)\right)}}{bc\log(f)}}{4b^2}$$

input
```
integrate(f^(c*(b*x+a)^2)*x^2,x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*(2*a^2*c*log(f) - 1)*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sqr
t(-c*log(f))*b*c*log(f)) - 2*(b*(x + a/b) - 2*a)*e^(b^2*c*x^2*log(f) + 2*a
*b*c*x*log(f) + a^2*c*log(f))/(b*c*log(f)))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int f^{c(a+bx)^2}x^2\,dx = \frac{\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c\ln(f)}\,(a+b\,x)\right)\,\left(\frac{a^2}{b^3}-\frac{1}{2\,b^3\,c\,\ln(f)}\right)}{2\,\sqrt{c\ln(f)}} - \frac{a\,f^{b^2\,c\,x^2}\,f^{a^2\,c}\,f^{2\,a\,b\,c\,x}}{2\,b^3\,c\,\ln(f)} + \frac{f^{b^2\,c\,x^2}\,f^{a^2\,c}\,f^{2\,a\,b\,c\,x}\,x}{2\,b^2\,c\,\ln(f)}$$

input
```
int(f^(c*(a + b*x)^2)*x^2,x)
```

output
```
(pi^(1/2)*erfi((c*log(f))^(1/2)*(a + b*x))*(a^2/b^3 - 1/(2*b^3*c*log(f))))
/(2*(c*log(f))^(1/2)) - (a*f^(b^2*c*x^2)*f^(a^2*c)*f^(2*a*b*c*x))/(2*b^3*c
*log(f)) + (f^(b^2*c*x^2)*f^(a^2*c)*f^(2*a*b*c*x)*x)/(2*b^2*c*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int f^{c(a+bx)^2}x^2\,dx$$
$$= \frac{-2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)aci+\log(f)bcix}{\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,a^2ci+\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)aci+\log(f)bcix}{\sqrt{c}\,\sqrt{\log(f)}}\right)i-2f^{b^2cx^2+2abcx+a^2c}\sqrt{c}\,\sqrt{\log(f)}\,a}{4\sqrt{c}\,\sqrt{\log(f)}\log(f)\,b^3c}$$

input
```
int(f^(c*(b*x+a)^2)*x^2,x)
```

output
```
( - 2*sqrt(pi)*erf((log(f)*a*c*i + log(f)*b*c*i*x)/(sqrt(c)*sqrt(log(f))))
*log(f)*a**2*c*i + sqrt(pi)*erf((log(f)*a*c*i + log(f)*b*c*i*x)/(sqrt(c)*s
qrt(log(f))))*i - 2*f**(a**2*c + 2*a*b*c*x + b**2*c*x**2)*sqrt(c)*sqrt(log
(f))*a + 2*f**(a**2*c + 2*a*b*c*x + b**2*c*x**2)*sqrt(c)*sqrt(log(f))*b*x)
/(4*sqrt(c)*sqrt(log(f))*log(f)*b**3*c)
```

3.136 $\int f^{c(a+bx)^2} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int f^{c(a+bx)^2} x \, dx = \frac{f^{c(a+bx)^2}}{2b^2 c \log(f)} - \frac{a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{2b^2\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/2*f^(c*(b*x+a)^2)/b^2/c/ln(f)-1/2*a*Pi^(1/2)*erfi(c^(1/2)*(b*x+a)*ln(f)^
(1/2))/b^2/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int f^{c(a+bx)^2} x \, dx = \frac{f^{c(a+bx)^2} - a\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)\sqrt{\log(f)}}{2b^2 c \log(f)}$$

input
```
Integrate[f^(c*(a + b*x)^2)*x,x]
```

output
```
(f^(c*(a + b*x)^2) - a*Sqrt[c]*Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]
]]*Sqrt[Log[f]])/(2*b^2*c*Log[f])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{c(a+bx)^2} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(a+bx) f^{c(a+bx)^2}}{b}-\frac{a f^{c(a+bx)^2}}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^{c(a+bx)^2}}{2b^2 c \log(f)}-\frac{\sqrt{\pi} a \text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b^2\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(c*(a + b*x)^2)*x,x]
```

output
```
f^(c*(a + b*x)^2)/(2*b^2*c*Log[f]) - (a*Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sq
rt[Log[f]]])/(2*b^2*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{f^{b^{2} c \,x^{2}} f^{2 a b c x} f^{a^{2} c}}{2 c \,b^{2} \ln \left(f \right)}+\frac{a \sqrt{\pi}\, \operatorname{erf}\left(-b \sqrt{-c \ln \left(f \right)}\, x +\frac{a c \ln \left(f \right)}{\sqrt{-c \ln \left(f \right)}}\right)}{2 b^{2} \sqrt{-c \ln \left(f \right)}}$	80

input
```
int(f^(c*(b*x+a)^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2/c/b^2/ln(f)*f^(b^2*c*x^2)*f^(2*a*b*c*x)*f^(a^2*c)+1/2*a/b^2*Pi^(1/2)/(
-c*ln(f))^(1/2)*erf(-b*(-c*ln(f))^(1/2)*x+a*c*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int f^{c(a+bx)^2} x \, dx = \frac{\sqrt{\pi} \sqrt{-b^2 c \log \left(f\right)} a \operatorname{erf}\left(\frac{\sqrt{-b^2 c \log \left(f\right)} (bx+a)}{b}\right) + b f^{b^2 c x^2 + 2\, a b c x + a^2 c}}{2\, b^3 c \log \left(f\right)}$$

input
```
integrate(f^(c*(b*x+a)^2)*x,x, algorithm="fricas")
```

output
```
1/2*(sqrt(pi)*sqrt(-b^2*c*log(f))*a*erf(sqrt(-b^2*c*log(f))*(b*x + a)/b) +
 b*f^(b^2*c*x^2 + 2*a*b*c*x + a^2*c))/(b^3*c*log(f))
```

Sympy [F]

$$\int f^{c(a+bx)^2} x \, dx = \int f^{c(a+bx)^2} x \, dx$$

input
```
integrate(f**(c*(b*x+a)**2)*x,x)
```

output
```
Integral(f**(c*(a + b*x)**2)*x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(54) = 108$.

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.93

$$\int f^{c(a+bx)^2}x\,dx = -\frac{\frac{\sqrt{\pi}(b^2cx+abc)ac\left(\operatorname{erf}\left(\sqrt{-\frac{\left(b^2cx+abc\right)^2\log(f)}{b^2c}}\right)-1\right)\log(f)^2}{(c\log(f))^{\frac{3}{2}}b^2\sqrt{-\frac{\left(b^2cx+abc\right)^2\log(f)}{b^2c}}}-\frac{cf^{\frac{\left(b^2cx+abc\right)^2}{b^2c}}\log(f)}{(c\log(f))^{\frac{3}{2}}b}}{2\sqrt{c\log(f)}b}$$

input
```
integrate(f^(c*(b*x+a)^2)*x,x, algorithm="maxima")
```

output
```
-1/2*(sqrt(pi)*(b^2*c*x + a*b*c)*a*c*(erf(sqrt(-(b^2*c*x + a*b*c)^2*log(f)
/(b^2*c))) - 1)*log(f)^2/((c*log(f))^(3/2)*b^2*sqrt(-(b^2*c*x + a*b*c)^2*l
og(f)/(b^2*c))) - c*f^((b^2*c*x + a*b*c)^2/(b^2*c))*log(f)/((c*log(f))^(3/
2)*b))/(sqrt(c*log(f))*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.13

$$\int f^{c(a+bx)^2}x\,dx = \frac{\frac{\sqrt{\pi}a\operatorname{erf}\left(-\sqrt{-c\log(f)}b\left(x+\frac{a}{b}\right)\right)}{\sqrt{-c\log(f)}b}+\frac{e^{\left(b^2cx^2\log(f)+2\,abcx\log(f)+a^2c\log(f)\right)}}{bc\log(f)}}{2\,b}$$

input
```
integrate(f^(c*(b*x+a)^2)*x,x, algorithm="giac")
```

output
```
1/2*(sqrt(pi)*a*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sqrt(-c*log(f))*b) + e^
(b^2*c*x^2*log(f) + 2*a*b*c*x*log(f) + a^2*c*log(f))/(b*c*log(f)))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int f^{c(a+bx)^2} x\, dx = \frac{f^{b^2 c x^2} f^{a^2 c} f^{2abcx}}{2 b^2 c \ln(f)} - \frac{a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c \ln(f)}\,(a+bx)\right)}{2 b^2 \sqrt{c \ln(f)}}$$

input
```
int(f^(c*(a + b*x)^2)*x,x)
```

output
```
(f^(b^2*c*x^2)*f^(a^2*c)*f^(2*a*b*c*x))/(2*b^2*c*log(f)) - (a*pi^(1/2)*erf
i((c*log(f))^(1/2)*(a + b*x)))/(2*b^2*(c*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int f^{c(a+bx)^2} x\, dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)aci+\log(f)bcix}{\sqrt{c}\sqrt{\log(f)}}\right)\log(f)\,aci + f^{b^2cx^2+2abcx+a^2c}\sqrt{c}\sqrt{\log(f)}}{2\sqrt{c}\sqrt{\log(f)}\log(f)\,b^2c}$$

input
```
int(f^(c*(b*x+a)^2)*x,x)
```

output
```
(sqrt(pi)*erf((log(f)*a*c*i + log(f)*b*c*i*x)/(sqrt(c)*sqrt(log(f))))*log(
f)*a*c*i + f**(a**2*c + 2*a*b*c*x + b**2*c*x**2)*sqrt(c)*sqrt(log(f)))/(2*
sqrt(c)*sqrt(log(f))*log(f)*b**2*c)
```

3.137 $\int f^{c(a+bx)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int f^{c(a+bx)^2} dx = \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/2*Pi^(1/2)*erfi(c^(1/2)*(b*x+a)*ln(f)^(1/2))/b/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int f^{c(a+bx)^2} dx = \frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)\sqrt{\log(f)}\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(c*(a + b*x)^2),x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(2*b*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{c(a+bx)^2}\,dx$$

$$\Big\downarrow \; 2633$$

$$\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)}{2b\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(c*(a + b*x)^2),x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)*Sqrt[Log[f]]])/(2*b*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{\sqrt{\pi}\,\operatorname{erf}\left(-b\sqrt{-c\ln(f)}\,x+\frac{ac\ln(f)}{\sqrt{-c\ln(f)}}\right)}{2b\sqrt{-c\ln(f)}}$	41

input
```
int(f^(c*(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)/b/(-c*ln(f))^(1/2)*erf(-b*(-c*ln(f))^(1/2)*x+a*c*ln(f)/(-c*l
n(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int f^{c(a+bx)^2}\,dx=-\frac{\sqrt{\pi}\sqrt{-b^2c\log{(f)}}\operatorname{erf}\left(\frac{\sqrt{-b^2c\log(f)}(bx+a)}{b}\right)}{2\,b^2c\log{(f)}}$$

input
```
integrate(f^(c*(b*x+a)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*c*log(f))*erf(sqrt(-b^2*c*log(f))*(b*x + a)/b)/(b^
2*c*log(f))
```

Sympy [F]

$$\int f^{c(a+bx)^2}\,dx=\int f^{c(a+bx)^2}\,dx$$

input
```
integrate(f**(c*(b*x+a)**2),x)
```

output
```
Integral(f**(c*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int f^{c(a+bx)^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)}bx - \frac{ac\log(f)}{\sqrt{-c\log(f)}}\right)}{2\,\sqrt{-c\log\left(f\right)}b}$$

input
```
integrate(f^(c*(b*x+a)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*erf(sqrt(-c*log(f))*b*x - a*c*log(f)/sqrt(-c*log(f)))/(sqrt(-
c*log(f))*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int f^{c(a+bx)^2}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)}b\left(x+\frac{a}{b}\right)\right)}{2\,\sqrt{-c\log\left(f\right)}b}$$

input
```
integrate(f^(c*(b*x+a)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-sqrt(-c*log(f))*b*(x + a/b))/(sqrt(-c*log(f))*b)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int f^{c(a+bx)^2}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{1\mathrm{i}\,c\,x\,\ln(f)\,b^2+1\mathrm{i}\,a\,c\,\ln(f)\,b}{\sqrt{b^2\,c\,\ln(f)}}\right)\,1\mathrm{i}}{2\,\sqrt{b^2\,c\,\ln\left(f\right)}}$$

input
```
int(f^(c*(a + b*x)^2),x)
```

output
```
-(pi^(1/2)*erf((a*b*c*log(f)*1i + b^2*c*x*log(f)*1i)/(b^2*c*log(f))^(1/2))
*1i)/(2*(b^2*c*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int f^{c(a+bx)^2}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)aci+\log(f)bcix}{\sqrt{c}\,\sqrt{\log(f)}}\right)i}{2\sqrt{c}\,\sqrt{\log(f)}\,b}$$

input
```
int(f^(c*(b*x+a)^2),x)
```

output
```
( - sqrt(pi)*erf((log(f)*a*c*i + log(f)*b*c*i*x)/(sqrt(c)*sqrt(log(f))))*i
)/(2*sqrt(c)*sqrt(log(f))*b)
```

3.138 $\int \frac{f^{c(a+bx)^2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{c(a+bx)^2}}{x} dx = \text{Int}\left(\frac{f^{c(a+bx)^2}}{x}, x\right)$$

output
```
Defer(Int)(f^(c*(b*x+a)^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x} dx = \int \frac{f^{c(a+bx)^2}}{x} dx$$

input
```
Integrate[f^(c*(a + b*x)^2)/x,x]
```

output
```
Integrate[f^(c*(a + b*x)^2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{c(a+bx)^2}}{x} \, dx$$

$$\Big\downarrow \text{2654}$$

$$\int \frac{f^{c(a+bx)^2}}{x}dx$$

input
```
Int[f^(c*(a + b*x)^2)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{c(bx+a)^2}}{x}dx$$

input
```
int(f^(c*(b*x+a)^2)/x,x)
```

output
```
int(f^(c*(b*x+a)^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{c(a+bx)^2}}{x} \, dx = \int \frac{f^{(bx+a)^2 c}}{x} \, dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
integral(f^(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{f^{c(a+bx)^2}}{x} \, dx = \int \frac{f^{c(a+bx)^2}}{x} \, dx$$

input
```
integrate(f**(c*(b*x+a)**2)/x,x)
```

output
```
Integral(f**(c*(a + b*x)**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x}\,dx = \int \frac{f^{(bx+a)^2c}}{x}\,dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^2*c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x}\,dx = \int \frac{f^{(bx+a)^2c}}{x}\,dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^2*c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x}\,dx = \int \frac{f^{c\,(a+b\,x)^2}}{x}\,dx$$

input
```
int(f^(c*(a + b*x)^2)/x,x)
```

output
```
int(f^(c*(a + b*x)^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{f^{c(a+bx)^2}}{x}\,dx = f^{a^2c}\left(\int \frac{f^{b^2c\,x^2+2abcx}}{x}dx\right)$$

input
```
int(f^(c*(b*x+a)^2)/x,x)
```

output
```
f**(a**2*c)*int(f**(2*a*b*c*x + b**2*c*x**2)/x,x)
```

3.139 $\int \frac{f^{c(a+bx)^2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{c(a+bx)^2}}{x^2} dx = \operatorname{Int}\left(\frac{f^{c(a+bx)^2}}{x^2}, x\right)$$

output
```
Defer(Int)(f^(c*(b*x+a)^2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^2} dx = \int \frac{f^{c(a+bx)^2}}{x^2} dx$$

input
```
Integrate[f^(c*(a + b*x)^2)/x^2,x]
```

output
```
Integrate[f^(c*(a + b*x)^2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2650, 2633, 2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{c(a+bx)^2}}{x^2} \, dx$$

$$\Big\downarrow \text{2650}$$

$$2b^2c\log(f)\int f^{c(a+bx)^2}dx + 2abc\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx - \frac{f^{c(a+bx)^2}}{x}$$

$$\Big\downarrow \text{2633}$$

$$2abc\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx + \sqrt{\pi}b\sqrt{c}\sqrt{\log(f)}\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right) - \frac{f^{c(a+bx)^2}}{x}$$

$$\Big\downarrow \text{2654}$$

$$2abc\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx + \sqrt{\pi}b\sqrt{c}\sqrt{\log(f)}\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right) - \frac{f^{c(a+bx)^2}}{x}$$

input
```
Int[f^(c*(a + b*x)^2)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2650
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)*((e_.) + (f_.)*(x_))^(m_),
x_Symbol] :> Simp[f*(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^2)/((m + 1)*f^2))
, x] + (-Simp[2*b*d^2*(Log[F]/(f^2*(m + 1)))   Int[(e + f*x)^(m + 2)*F^(a +
 b*(c + d*x)^2), x], x] + Simp[2*b*d*(d*e - c*f)*(Log[F]/(f^2*(m + 1)))   I
nt[(e + f*x)^(m + 1)*F^(a + b*(c + d*x)^2), x], x]) /; FreeQ[{F, a, b, c, d
, e, f}, x] && NeQ[d*e - c*f, 0] && LtQ[m, -1]
```

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{c(bx+a)^2}}{x^2}dx$$

input
```
int(f^(c*(b*x+a)^2)/x^2,x)
```

output
```
int(f^(c*(b*x+a)^2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{c(a+bx)^2}}{x^2}\,dx = \int \frac{f^{(bx+a)^2c}}{x^2}\,dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x^2,x, algorithm="fricas")
```

output
```
integral(f^(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{c(a+bx)^2}}{x^2} \, dx = \int \frac{f^{c(a+bx)^2}}{x^2} \, dx$$

input
```
integrate(f**(c*(b*x+a)**2)/x**2,x)
```

output
```
Integral(f**(c*(a + b*x)**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^2} \, dx = \int \frac{f^{(bx+a)^2 c}}{x^2} \, dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^2*c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^2} \, dx = \int \frac{f^{(bx+a)^2 c}}{x^2} \, dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x^2,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^2*c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^2}\, dx = \int \frac{f^{c\,(a+b\,x)^2}}{x^2}\, dx$$

input
```
int(f^(c*(a + b*x)^2)/x^2,x)
```

output
```
int(f^(c*(a + b*x)^2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{f^{c(a+bx)^2}}{x^2}\, dx = f^{a^2c}\left(\int \frac{f^{b^2c\,x^2+2abcx}}{x^2}dx\right)$$

input
```
int(f^(c*(b*x+a)^2)/x^2,x)
```

output
```
f**(a**2*c)*int(f**(2*a*b*c*x + b**2*c*x**2)/x**2,x)
```

3.140 $\int \frac{f^{c(a+bx)^2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{c(a+bx)^2}}{x^3} dx = \text{Int}\left(\frac{f^{c(a+bx)^2}}{x^3}, x\right)$$

output
```
Defer(Int)(f^(c*(b*x+a)^2)/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^3} dx = \int \frac{f^{c(a+bx)^2}}{x^3} dx$$

input
```
Integrate[f^(c*(a + b*x)^2)/x^3,x]
```

output
```
Integrate[f^(c*(a + b*x)^2)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2650, 2650, 2633, 2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{c(a+bx)^2}}{x^3}\,dx$$

$$\Big\downarrow \text{2650}$$

$$b^2c\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx+abc\log(f)\int \frac{f^{c(a+bx)^2}}{x^2}dx-\frac{f^{c(a+bx)^2}}{2x^2}$$

$$\Big\downarrow \text{2650}$$

$$b^2c\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx+ abc\log(f)\left(2b^2c\log(f)\int f^{c(a+bx)^2}dx+2abc\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx-\frac{f^{c(a+bx)^2}}{x}\right)-\frac{f^{c(a+bx)^2}}{2x^2}$$

$$\Big\downarrow \text{2633}$$

$$b^2c\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx+ abc\log(f)\left(2abc\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx+\sqrt{\pi}b\sqrt{c}\sqrt{\log(f)}\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)-\frac{f^{c(a+bx)^2}}{x}\right)- \frac{f^{c(a+bx)^2}}{2x^2}$$

$$\Big\downarrow \text{2654}$$

$$b^2c\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx+ abc\log(f)\left(2abc\log(f)\int \frac{f^{c(a+bx)^2}}{x}dx+\sqrt{\pi}b\sqrt{c}\sqrt{\log(f)}\text{erfi}\left(\sqrt{c}\sqrt{\log(f)}(a+bx)\right)-\frac{f^{c(a+bx)^2}}{x}\right)- \frac{f^{c(a+bx)^2}}{2x^2}$$

input
```
Int[f^(c*(a + b*x)^2)/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2650
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)*((e_.) + (f_.)*(x_))^(m_),
x_Symbol] :> Simp[f*(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^2)/((m + 1)*f^2))
, x] + (-Simp[2*b*d^2*(Log[F]/(f^2*(m + 1)))   Int[(e + f*x)^(m + 2)*F^(a +
 b*(c + d*x)^2), x], x] + Simp[2*b*d*(d*e - c*f)*(Log[F]/(f^2*(m + 1)))   I
nt[(e + f*x)^(m + 1)*F^(a + b*(c + d*x)^2), x], x]) /; FreeQ[{F, a, b, c, d
, e, f}, x] && NeQ[d*e - c*f, 0] && LtQ[m, -1]
```

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{c(bx+a)^2}}{x^3}dx$$

input
```
int(f^(c*(b*x+a)^2)/x^3,x)
```

output
```
int(f^(c*(b*x+a)^2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{c(a+bx)^2}}{x^3} \, dx = \int \frac{f^{(bx+a)^2 c}}{x^3} \, dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x^3,x, algorithm="fricas")
```

output
```
integral(f^(b^2*c*x^2 + 2*a*b*c*x + a^2*c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{c(a+bx)^2}}{x^3} \, dx = \int \frac{f^{c(a+bx)^2}}{x^3} \, dx$$

input
```
integrate(f**(c*(b*x+a)**2)/x**3,x)
```

output
```
Integral(f**(c*(a + b*x)**2)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^3} \, dx = \int \frac{f^{(bx+a)^2 c}}{x^3} \, dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^2*c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^3} \, dx = \int \frac{f^{(bx+a)^2 c}}{x^3} \, dx$$

input
```
integrate(f^(c*(b*x+a)^2)/x^3,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^2*c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^2}}{x^3} \, dx = \int \frac{f^{c\,(a+b\,x)^2}}{x^3} \, dx$$

input
```
int(f^(c*(a + b*x)^2)/x^3,x)
```

output
```
int(f^(c*(a + b*x)^2)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{f^{c(a+bx)^2}}{x^3}\,dx = f^{a^2c}\left(\int \frac{f^{b^2c\,x^2+2abcx}}{x^3}dx\right)$$

input
```
int(f^(c*(b*x+a)^2)/x^3,x)
```

output
```
f**(a**2*c)*int(f**(2*a*b*c*x + b**2*c*x**2)/x**3,x)
```

3.141 $\int f^{c(a+bx)^3} x^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int f^{c(a+bx)^3} x^2\,dx = \frac{f^{c(a+bx)^3}}{3b^3c\log(f)} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3},-c(a+bx)^3\log(f)\right)}{3b^3\left(-c(a+bx)^3\log(f)\right)^{2/3}} - \frac{a^2(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3b^3\sqrt[3]{-c(a+bx)^3\log(f)}}$$

output
```
1/3*f^(c*(b*x+a)^3)/b^3/c/ln(f)+2/3*a*(b*x+a)^2*GAMMA(2/3,-c*(b*x+a)^3*ln(
f))/b^3/(-c*(b*x+a)^3*ln(f))^(2/3)-1/3*a^2*(b*x+a)*GAMMA(1/3,-c*(b*x+a)^3*
ln(f))/b^3/(-c*(b*x+a)^3*ln(f))^(1/3)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int f^{c(a+bx)^3} x^2\,dx = \frac{\frac{f^{c(a+bx)^3}}{c\log(f)} + \frac{2a(a+bx)^2\Gamma\left(\frac{2}{3},-c(a+bx)^3\log(f)\right)}{\left(-c(a+bx)^3\log(f)\right)^{2/3}} - \frac{a^2(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{\sqrt[3]{-c(a+bx)^3\log(f)}}}{3b^3}$$

input
```
Integrate[f^(c*(a + b*x)^3)*x^2,x]
```

output
```
(f^(c*(a + b*x)^3)/(c*Log[f]) + (2*a*(a + b*x)^2*Gamma[2/3, -(c*(a + b*x)^
3*Log[f])])/(-(c*(a + b*x)^3*Log[f]))^(2/3) - (a^2*(a + b*x)*Gamma[1/3, -(
c*(a + b*x)^3*Log[f])])/(-(c*(a + b*x)^3*Log[f]))^(1/3))/(3*b^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{c(a+bx)^3} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^2 f^{c(a+bx)^3}}{b^2}+\frac{(a+bx)^2 f^{c(a+bx)^3}}{b^2}-\frac{2a(a+bx) f^{c(a+bx)^3}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3b^3\sqrt[3]{-c\log(f)(a+bx)^3}}+\frac{f^{c(a+bx)^3}}{3b^3c\log(f)}+\frac{2a(a+bx)^2\Gamma\left(\frac{2}{3},-c(a+bx)^3\log(f)\right)}{3b^3\left(-c\log(f)(a+bx)^3\right)^{2/3}}$$

input
```
Int[f^(c*(a + b*x)^3)*x^2,x]
```

output
```
f^(c*(a + b*x)^3)/(3*b^3*c*Log[f]) + (2*a*(a + b*x)^2*Gamma[2/3, -(c*(a +
b*x)^3*Log[f])])/(3*b^3*(-(c*(a + b*x)^3*Log[f]))^(2/3)) - (a^2*(a + b*x)*
Gamma[1/3, -(c*(a + b*x)^3*Log[f])])/(3*b^3*(-(c*(a + b*x)^3*Log[f]))^(1/3
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int f^{c(bx+a)^3}x^2dx$$

input
```
int(f^(c*(b*x+a)^3)*x^2,x)
```

output
```
int(f^(c*(b*x+a)^3)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.29

$$\int f^{c(a+bx)^3}x^2\,dx = \frac{(-b^3c\log(f))^{\frac{2}{3}}\,a^2\Gamma\left(\frac{1}{3}, -(b^3cx^3+3\,ab^2cx^2+3\,a^2bcx+a^3c)\log(f)\right) - 2\,(-b^3c\log(f))^{\frac{1}{3}}\,ab\Gamma\left(\frac{2}{3}, -(b^3cx^3\right.}{3\,b^5c\log(f)}$$

input
```
integrate(f^(c*(b*x+a)^3)*x^2,x, algorithm="fricas")
```

output
```
1/3*((-b^3*c*log(f))^(2/3)*a^2*gamma(1/3, -(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*
a^2*b*c*x + a^3*c)*log(f)) - 2*(-b^3*c*log(f))^(1/3)*a*b*gamma(2/3, -(b^3*
c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*log(f)) + b^2*f^(b^3*c*x^3 +
3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c))/(b^5*c*log(f))
```

Sympy [F]

$$\int f^{c(a+bx)^3}x^2\,dx = \int f^{c(a+bx)^3}x^2\,dx$$

input
```
integrate(f**(c*(b*x+a)**3)*x**2,x)
```

output
```
Integral(f**(c*(a + b*x)**3)*x**2, x)
```

Maxima [F]

$$\int f^{c(a+bx)^3}x^2\,dx = \int f^{(bx+a)^3c}x^2\,dx$$

input
```
integrate(f^(c*(b*x+a)^3)*x^2,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^3*c)*x^2, x)
```

Giac [F]

$$\int f^{c(a+bx)^3}x^2\,dx = \int f^{(bx+a)^3c}x^2\,dx$$

input
```
integrate(f^(c*(b*x+a)^3)*x^2,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^3*c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{c(a+bx)^3}x^2\,dx=\int f^{c\,(a+b\,x)^3}\,x^2\,dx$$

input
```
int(f^(c*(a + b*x)^3)*x^2,x)
```

output
```
int(f^(c*(a + b*x)^3)*x^2, x)
```

Reduce [F]

$$\int f^{c(a+bx)^3}x^2\,dx$$

$$=\frac{f^{a^3c}\left(f^{b^3cx^3+3a\,b^2cx^2+3a^2bcx}-3\left(\int f^{b^3cx^3+3a\,b^2cx^2+3a^2bcx}dx\right)\log(f)\,a^2bc-6\left(\int f^{b^3cx^3+3a\,b^2cx^2+3a^2bcx}xdx\right)\text{lo}\right.}{3\log\left(f\right)b^3c}$$

input
```
int(f^(c*(b*x+a)^3)*x^2,x)
```

output
```
(f**(a**3*c)*(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3) - 3*int(f*
*(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3),x)*log(f)*a**2*b*c - 6*int
(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3)*x,x)*log(f)*a*b**2*c))/
(3*log(f)*b**3*c)
```

3.142 $\int f^{c(a+bx)^3} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int f^{c(a+bx)^3} x\,dx = -\frac{(a+bx)^2\Gamma\left(\frac{2}{3},-c(a+bx)^3\log(f)\right)}{3b^2\left(-c(a+bx)^3\log(f)\right)^{2/3}}+\frac{a(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3b^2\sqrt[3]{-c(a+bx)^3\log(f)}}$$

output
```
-1/3*(b*x+a)^2*GAMMA(2/3,-c*(b*x+a)^3*ln(f))/b^2/(-c*(b*x+a)^3*ln(f))^(2/3
)+1/3*a*(b*x+a)*GAMMA(1/3,-c*(b*x+a)^3*ln(f))/b^2/(-c*(b*x+a)^3*ln(f))^(1/
3)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int f^{c(a+bx)^3} x\,dx = -\frac{(a+bx)\left((a+bx)\Gamma\left(\frac{2}{3},-c(a+bx)^3\log(f)\right)-a\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)\sqrt[3]{-c(a+bx)^3\log(f)}\right)}{3b^2\left(-c(a+bx)^3\log(f)\right)^{2/3}}$$

input
```
Integrate[f^(c*(a + b*x)^3)*x,x]
```

output
```
-1/3*((a + b*x)*((a + b*x)*Gamma[2/3, -(c*(a + b*x)^3*Log[f])] - a*Gamma[1
/3, -(c*(a + b*x)^3*Log[f])]*(-(c*(a + b*x)^3*Log[f]))^(1/3)))/(b^2*(-(c*(
a + b*x)^3*Log[f]))^(2/3))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{c(a+bx)^3} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(a+bx) f^{c(a+bx)^3}}{b} - \frac{a f^{c(a+bx)^3}}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3}, -c(a+bx)^3 \log(f)\right)}{3b^2 \sqrt[3]{-c \log(f)(a+bx)^3}} - \frac{(a+bx)^2 \Gamma\left(\frac{2}{3}, -c(a+bx)^3 \log(f)\right)}{3b^2 \left(-c \log(f)(a+bx)^3\right)^{2/3}}$$

input
```
Int[f^(c*(a + b*x)^3)*x,x]
```

output
```
-1/3*((a + b*x)^2*Gamma[2/3, -(c*(a + b*x)^3*Log[f])])/(b^2*(-(c*(a + b*x)
^3*Log[f]))^(2/3)) + (a*(a + b*x)*Gamma[1/3, -(c*(a + b*x)^3*Log[f])])/(3*
b^2*(-(c*(a + b*x)^3*Log[f]))^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int f^{c(bx+a)^3} x dx$$

input
```
int(f^(c*(b*x+a)^3)*x,x)
```

output
```
int(f^(c*(b*x+a)^3)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24

$$\int f^{c(a+bx)^3} x \, dx = -\frac{(-b^3 c \log(f))^{\frac{2}{3}} a\Gamma\left(\frac{1}{3}, -(b^3cx^3 + 3\,ab^2cx^2 + 3\,a^2bcx + a^3c)\log(f)\right) - (-b^3c\log(f))^{\frac{1}{3}} b\Gamma\left(\frac{2}{3}, -(b^3cx^3 + \right.}{3\,b^4c\log(f)}$$

input
```
integrate(f^(c*(b*x+a)^3)*x,x, algorithm="fricas")
```

output
```
-1/3*((-b^3*c*log(f))^(2/3)*a*gamma(1/3, -(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a
^2*b*c*x + a^3*c)*log(f)) - (-b^3*c*log(f))^(1/3)*b*gamma(2/3, -(b^3*c*x^3
 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*log(f)))/(b^4*c*log(f))
```

Sympy [F]

$$\int f^{c(a+bx)^3} x \, dx = \int f^{c(a+bx)^3} x \, dx$$

input
```
integrate(f**(c*(b*x+a)**3)*x,x)
```

output
```
Integral(f**(c*(a + b*x)**3)*x, x)
```

Maxima [F]

$$\int f^{c(a+bx)^3} x \, dx = \int f^{(bx+a)^3 c} x \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)*x,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^3*c)*x, x)
```

Giac [F]

$$\int f^{c(a+bx)^3} x \, dx = \int f^{(bx+a)^3 c} x \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)*x,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^3*c)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{c(a+bx)^3}x\,dx=\int f^{c\,(a+b\,x)^3}\,x\,dx$$

input
```
int(f^(c*(a + b*x)^3)*x,x)
```

output
```
int(f^(c*(a + b*x)^3)*x, x)
```

Reduce [F]

$$\int f^{c(a+bx)^3}x\,dx=f^{a^3c}\left(\int f^{b^3c\,x^3+3a\,b^2c\,x^2+3a^2bcx}xdx\right)$$

input
```
int(f^(c*(b*x+a)^3)*x,x)
```

output
```
f**(a**3*c)*int(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3)*x,x)
```

3.143 $\int f^{c(a+bx)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int f^{c(a+bx)^3}\,dx=-\frac{(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3b\sqrt[3]{-c(a+bx)^3\log(f)}}$$

output
```
-1/3*(b*x+a)*GAMMA(1/3,-c*(b*x+a)^3*ln(f))/b/(-c*(b*x+a)^3*ln(f))^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int f^{c(a+bx)^3}\,dx=-\frac{(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3b\sqrt[3]{-c(a+bx)^3\log(f)}}$$

input
```
Integrate[f^(c*(a + b*x)^3),x]
```

output
```
-1/3*((a + b*x)*Gamma[1/3, -(c*(a + b*x)^3*Log[f])])/(b*(-(c*(a + b*x)^3*L
og[f]))^(1/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{c(a+bx)^3}\,dx$$

$$\Big\downarrow \; 2637$$

$$-\frac{(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3b\sqrt[3]{-c\log(f)(a+bx)^3}}$$

input
```
Int[f^(c*(a + b*x)^3),x]
```

output
```
-1/3*((a + b*x)*Gamma[1/3, -(c*(a + b*x)^3*Log[f])])/(b*(-(c*(a + b*x)^3*L
og[f]))^(1/3))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Maple [F]

$$\int f^{c(bx+a)^3}dx$$

input
```
int(f^(c*(b*x+a)^3),x)
```

output
```
int(f^(c*(b*x+a)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int f^{c(a+bx)^3}\,dx = \frac{(-b^3c\log(f))^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3}, -(b^3cx^3 + 3\,ab^2cx^2 + 3\,a^2bcx + a^3c)\log(f)\right)}{3\,b^3c\log(f)}$$

input
```
integrate(f^(c*(b*x+a)^3),x, algorithm="fricas")
```

output
```
1/3*(-b^3*c*log(f))^(2/3)*gamma(1/3, -(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b
*c*x + a^3*c)*log(f))/(b^3*c*log(f))
```

Sympy [F]

$$\int f^{c(a+bx)^3}\,dx = \int f^{c(a+bx)^3}\,dx$$

input
```
integrate(f**(c*(b*x+a)**3),x)
```

output
```
Integral(f**(c*(a + b*x)**3), x)
```

Maxima [F]

$$\int f^{c(a+bx)^3}\,dx = \int f^{(bx+a)^3c}\,dx$$

input
```
integrate(f^(c*(b*x+a)^3),x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^3*c), x)
```

Giac [F]

$$\int f^{c(a+bx)^3}\,dx=\int f^{(bx+a)^3c}\,dx$$

input
```
integrate(f^(c*(b*x+a)^3),x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^3*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{c(a+bx)^3}\,dx=\int f^{c\,(a+b\,x)^3}\,dx$$

input
```
int(f^(c*(a + b*x)^3),x)
```

output
```
int(f^(c*(a + b*x)^3), x)
```

Reduce [F]

$$\int f^{c(a+bx)^3}\,dx=f^{a^3c}\left(\int f^{b^3c\,x^3+3a\,b^2c\,x^2+3a^2bcx}dx\right)$$

input
```
int(f^(c*(b*x+a)^3),x)
```

output
```
f**(a**3*c)*int(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3),x)
```

3.144 $\int \frac{f^{c(a+bx)^3}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{c(a+bx)^3}}{x} dx = \operatorname{Int}\left(\frac{f^{c(a+bx)^3}}{x}, x\right)$$

output
```
Defer(Int)(f^(c*(b*x+a)^3)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x} dx = \int \frac{f^{c(a+bx)^3}}{x} dx$$

input
```
Integrate[f^(c*(a + b*x)^3)/x,x]
```

output
```
Integrate[f^(c*(a + b*x)^3)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{c(a+bx)^3}}{x} \, dx$$
$$\Big\downarrow \text{ 2654}$$
$$\int \frac{f^{c(a+bx)^3}}{x}dx$$

input
```
Int[f^(c*(a + b*x)^3)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{c(bx+a)^3}}{x}dx$$

input
```
int(f^(c*(b*x+a)^3)/x,x)
```

output
```
int(f^(c*(b*x+a)^3)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

$$\int \frac{f^{c(a+bx)^3}}{x}\,dx = \int \frac{f^{(bx+a)^3c}}{x}\,dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x,x, algorithm="fricas")
```

output
```
integral(f^(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{f^{c(a+bx)^3}}{x}\,dx = \int \frac{f^{c(a+bx)^3}}{x}\,dx$$

input
```
integrate(f**(c*(b*x+a)**3)/x,x)
```

output
```
Integral(f**(c*(a + b*x)**3)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x}\,dx = \int \frac{f^{(bx+a)^3c}}{x}\,dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^3*c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x}\,dx = \int \frac{f^{(bx+a)^3c}}{x}\,dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^3*c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x}\,dx = \int \frac{f^{c\,(a+b\,x)^3}}{x}\,dx$$

input
```
int(f^(c*(a + b*x)^3)/x,x)
```

output
```
int(f^(c*(a + b*x)^3)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{f^{c(a+bx)^3}}{x} \, dx = f^{a^3c}\left(\int \frac{f^{b^3c\,x^3+3a\,b^2c\,x^2+3a^2bcx}}{x}dx\right)$$

input
```
int(f^(c*(b*x+a)^3)/x,x)
```

output
```
f**(a**3*c)*int(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3)/x,x)
```

3.145 $\int \frac{f^{c(a+bx)^3}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{c(a+bx)^3}}{x^2}\,dx=\text{Int}\left(\frac{f^{c(a+bx)^3}}{x^2},x\right)$$

output
```
Defer(Int)(f^(c*(b*x+a)^3)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^2}\,dx=\int \frac{f^{c(a+bx)^3}}{x^2}\,dx$$

input
```
Integrate[f^(c*(a + b*x)^3)/x^2,x]
```

output
```
Integrate[f^(c*(a + b*x)^3)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2651, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx$$

$$\Big\downarrow \text{2651}$$

$$3bc\log(f)\int \frac{f^{c(a+bx)^3}(a+bx)^2}{x}dx-\frac{f^{c(a+bx)^3}}{x}$$

$$\Big\downarrow \text{7293}$$

$$3bc\log(f)\int \left(abf^{c(a+bx)^3}+b(a+bx)f^{c(a+bx)^3}+\frac{a^2f^{c(a+bx)^3}}{x}\right)dx-\frac{f^{c(a+bx)^3}}{x}$$

$$\Big\downarrow \text{2009}$$

$$3bc\log(f)\left(a^2\int \frac{f^{c(a+bx)^3}}{x}dx-\frac{a(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{3\sqrt[3]{-c\log(f)(a+bx)^3}}-\frac{(a+bx)^2\Gamma\left(\frac{2}{3},-c(a+bx)^3\log(f)\right)}{3\left(-c\log(f)(a+bx)^3\right)^{2/3}}\right)-\frac{f^{c(a+bx)^3}}{x}$$

input
```
Int[f^(c*(a + b*x)^3)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2651
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(f*(m + 1))),
 x] - Simp[b*d*n*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(c + d*x)^(n
- 1)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && Ne
Q[d*e - c*f, 0] && IGtQ[n, 2] && LtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{c(bx+a)^{3}}}{x^{2}}dx$$

input
```
int(f^(c*(b*x+a)^3)/x^2,x)
```

output
```
int(f^(c*(b*x+a)^3)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx = \int \frac{f^{(bx+a)^{3}c}}{x^{2}} \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x^2,x, algorithm="fricas")
```

output
```
integral(f^(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx = \int \frac{f^{c(a+bx)^3}}{x^2} \, dx$$

input
```
integrate(f**(c*(b*x+a)**3)/x**2,x)
```

output
```
Integral(f**(c*(a + b*x)**3)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx = \int \frac{f^{(bx+a)^3c}}{x^2} \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x^2,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^3*c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx = \int \frac{f^{(bx+a)^3 c}}{x^2} \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x^2,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^3*c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx = \int \frac{f^{c\,(a+b\,x)^3}}{x^2} \, dx$$

input
```
int(f^(c*(a + b*x)^3)/x^2,x)
```

output
```
int(f^(c*(a + b*x)^3)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{f^{c(a+bx)^3}}{x^2} \, dx = f^{a^3 c} \left(\int \frac{f^{b^3 c\,x^3+3a\,b^2 c\,x^2+3a^2 bcx}}{x^2} dx \right)$$

input
```
int(f^(c*(b*x+a)^3)/x^2,x)
```

output
```
f**(a**3*c)*int(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3)/x**2,x)
```

3.146 $\int \frac{f^{c(a+bx)^3}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{c(a+bx)^3}}{x^3}\,dx = \text{Int}\left(\frac{f^{c(a+bx)^3}}{x^3},x\right)$$

output
```
Defer(Int)(f^(c*(b*x+a)^3)/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^3}\,dx = \int \frac{f^{c(a+bx)^3}}{x^3}\,dx$$

input
```
Integrate[f^(c*(a + b*x)^3)/x^3,x]
```

output
```
Integrate[f^(c*(a + b*x)^3)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2651, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{c(a+bx)^3}}{x^3}\,dx$$

$$\Big\downarrow \ 2651$$

$$\frac{3}{2}bc\log(f)\int \frac{f^{c(a+bx)^3}(a+bx)^2}{x^2}dx-\frac{f^{c(a+bx)^3}}{2x^2}$$

$$\Big\downarrow \ 7293$$

$$\frac{3}{2}bc\log(f)\int \left(b^2f^{c(a+bx)^3}+\frac{2abf^{c(a+bx)^3}}{x}+\frac{a^2f^{c(a+bx)^3}}{x^2}\right)dx-\frac{f^{c(a+bx)^3}}{2x^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{3}{2}bc\log(f)\left(3a^4bc\log(f)\int \frac{f^{c(a+bx)^3}}{x}dx+2ab\int \frac{f^{c(a+bx)^3}}{x}dx-\frac{a^3bc\log(f)(a+bx)\Gamma\left(\frac{1}{3},-c(a+bx)^3\log(f)\right)}{\sqrt[3]{-c\log(f)(a+bx)^3}}-\right.$$

$$\frac{f^{c(a+bx)^3}}{2x^2}$$

input
```
Int[f^(c*(a + b*x)^3)/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2651
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(f*(m + 1))),
 x] - Simp[b*d*n*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(c + d*x)^(n
- 1)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && Ne
Q[d*e - c*f, 0] && IGtQ[n, 2] && LtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{c(bx+a)^3}}{x^3}dx$$

input
```
int(f^(c*(b*x+a)^3)/x^3,x)
```

output
```
int(f^(c*(b*x+a)^3)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

$$\int \frac{f^{c(a+bx)^3}}{x^3}\,dx = \int \frac{f^{(bx+a)^3c}}{x^3}\,dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x^3,x, algorithm="fricas")
```

output
```
integral(f^(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{c(a+bx)^3}}{x^3} \, dx = \int \frac{f^{c(a+bx)^3}}{x^3} \, dx$$

input
```
integrate(f**(c*(b*x+a)**3)/x**3,x)
```

output
```
Integral(f**(c*(a + b*x)**3)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^3} \, dx = \int \frac{f^{(bx+a)^3 c}}{x^3} \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x^3,x, algorithm="maxima")
```

output
```
integrate(f^((b*x + a)^3*c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^3} \, dx = \int \frac{f^{(bx+a)^3c}}{x^3} \, dx$$

input
```
integrate(f^(c*(b*x+a)^3)/x^3,x, algorithm="giac")
```

output
```
integrate(f^((b*x + a)^3*c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{c(a+bx)^3}}{x^3} \, dx = \int \frac{f^{c\,(a+b\,x)^3}}{x^3} \, dx$$

input
```
int(f^(c*(a + b*x)^3)/x^3,x)
```

output
```
int(f^(c*(a + b*x)^3)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{f^{c(a+bx)^3}}{x^3} \, dx = f^{a^3c}\left(\int \frac{f^{b^3c\,x^3+3a\,b^2c\,x^2+3a^2bcx}}{x^3}dx\right)$$

input
```
int(f^(c*(b*x+a)^3)/x^3,x)
```

output
```
f**(a**3*c)*int(f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3)/x**3,x)
```

3.147 $\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx$

Optimal result

Integrand size = 33, antiderivative size = 33

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx=\text{Int}\left(e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m,x\right)$$

output
```
Defer(Int)(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx=\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx$$

input
```
Integrate[E^(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*x^m,x]
```

output
```
Integrate[E^(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*x^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{a^3+3a^2bx+3ab^2x^2+b^3x^3} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}dx$$

input
```
Int[E^(a^3 + 3*a^2*b*x + 3*a*b^2*x^2 + b^3*x^3)*x^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \mathrm{e}^{b^3x^3+3a\,b^2x^2+3a^2bx+a^3} x^m dx$$

input
```
int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x)
```

output
```
int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx=\int x^me^{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)}\,dx$$

input
```
integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x, algorithm="fricas"
)
```

output
```
integral(x^m*e^(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 100.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx=e^{a^3}\int x^me^{b^3x^3}e^{3ab^2x^2}e^{3a^2bx}\,dx$$

input
```
integrate(exp(b**3*x**3+3*a*b**2*x**2+3*a**2*b*x+a**3)*x**m,x)
```

output
```
exp(a**3)*Integral(x**m*exp(b**3*x**3)*exp(3*a*b**2*x**2)*exp(3*a**2*b*x),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx = \int x^m e^{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)}\,dx$$

input
```
integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x, algorithm="maxima"
)
```

output
```
integrate(x^m*e^(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx = \int x^m e^{\left(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3\right)}\,dx$$

input
```
integrate(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*e^(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx = \int x^m\,\mathrm{e}^{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}\,dx$$

input
```
int(x^m*exp(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x),x)
```

output
```
int(x^m*exp(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int e^{a^3+3a^2bx+3ab^2x^2+b^3x^3}x^m\,dx = e^{a^3}\left(\int x^m e^{b^3x^3+3a\,b^2x^2+3a^2bx}dx\right)$$

input
```
int(exp(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+a^3)*x^m,x)
```

output
```
e**(a**3)*int(x**m*e**(3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
```

3.148 $\int e^{\sqrt{5+3x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int e^{\sqrt{5+3x}}\,dx=-\frac{2}{3}e^{\sqrt{5+3x}}+\frac{2}{3}e^{\sqrt{5+3x}}\sqrt{5+3x}$$

output
```
-2/3*exp((5+3*x)^(1/2))+2/3*exp((5+3*x)^(1/2))*(5+3*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int e^{\sqrt{5+3x}}\,dx=\frac{2}{3}e^{\sqrt{5+3x}}\left(-1+\sqrt{5+3x}\right)$$

input
```
Integrate[E^Sqrt[5 + 3*x],x]
```

output
```
(2*E^Sqrt[5 + 3*x]*(-1 + Sqrt[5 + 3*x]))/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2636, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sqrt{3x+5}}dx$$

$$\Big\downarrow \text{2636}$$

$$\frac{2}{3}\int e^{\sqrt{3x+5}}\sqrt{3x+5}d\sqrt{3x+5}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2}{3}\left(e^{\sqrt{3x+5}}\sqrt{3x+5}-\int e^{\sqrt{3x+5}}d\sqrt{3x+5}\right)$$

$$\Big\downarrow \text{2624}$$

$$\frac{2}{3}\left(e^{\sqrt{3x+5}}\sqrt{3x+5}-e^{\sqrt{3x+5}}\right)$$

input
```
Int[E^Sqrt[5 + 3*x],x]
```

output
```
(2*(-E^Sqrt[5 + 3*x] + E^Sqrt[5 + 3*x]*Sqrt[5 + 3*x]))/3
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2636
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Simp[k/d   Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (
c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Int
egerQ[n]
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$-\frac{2\,\mathrm{e}^{\sqrt{3x+5}}}{3}+\frac{2\,\mathrm{e}^{\sqrt{3x+5}}\sqrt{3x+5}}{3}$	29
default	$-\frac{2\,\mathrm{e}^{\sqrt{3x+5}}}{3}+\frac{2\,\mathrm{e}^{\sqrt{3x+5}}\sqrt{3x+5}}{3}$	29

input
```
int(exp((3*x+5)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/3*exp((3*x+5)^(1/2))+2/3*exp((3*x+5)^(1/2))*(3*x+5)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int e^{\sqrt{5+3x}}\,dx = \frac{2}{3}\left(\sqrt{3\,x+5}-1\right)e^{\left(\sqrt{3\,x+5}\right)}$$

input
```
integrate(exp((5+3*x)^(1/2)),x, algorithm="fricas")
```

output
```
2/3*(sqrt(3*x + 5) - 1)*e^(sqrt(3*x + 5))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int e^{\sqrt{5+3x}}\,dx = \frac{2\sqrt{3x+5}e^{\sqrt{3x+5}}}{3} - \frac{2e^{\sqrt{3x+5}}}{3}$$

input
```
integrate(exp((5+3*x)**(1/2)),x)
```

output
```
2*sqrt(3*x + 5)*exp(sqrt(3*x + 5))/3 - 2*exp(sqrt(3*x + 5))/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int e^{\sqrt{5+3x}}\,dx = \frac{2}{3}\left(\sqrt{3\,x+5}-1\right)e^{\left(\sqrt{3\,x+5}\right)}$$

input
```
integrate(exp((5+3*x)^(1/2)),x, algorithm="maxima")
```

output
```
2/3*(sqrt(3*x + 5) - 1)*e^(sqrt(3*x + 5))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int e^{\sqrt{5+3x}}\,dx = \frac{2}{3}\left(\sqrt{3\,x+5}-1\right)e^{\left(\sqrt{3\,x+5}\right)}$$

input
```
integrate(exp((5+3*x)^(1/2)),x, algorithm="giac")
```

output
```
2/3*(sqrt(3*x + 5) - 1)*e^(sqrt(3*x + 5))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.48

$$\int e^{\sqrt{5+3x}}\,dx = \frac{2\,\mathrm{e}^{\sqrt{3\,x+5}}\,\left(\sqrt{3\,x+5}-1\right)}{3}$$

input
```
int(exp((3*x + 5)^(1/2)),x)
```

output
```
(2*exp((3*x + 5)^(1/2))*((3*x + 5)^(1/2) - 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.45

$$\int e^{\sqrt{5+3x}}\,dx = \frac{2e^{\sqrt{3x+5}}\left(\sqrt{3x+5}-1\right)}{3}$$

input
```
int(exp((5+3*x)^(1/2)),x)
```

output
```
(2*e**sqrt(3*x + 5)*(sqrt(3*x + 5) - 1))/3
```

3.149 $\int f^{\frac{c}{a+bx}} x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 291

$$\begin{aligned}\int f^{\frac{c}{a+bx}} x^4 \, dx = {} & \frac{a^4 f^{\frac{c}{a+bx}}(a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{a+bx}}(a+bx)^2}{b^5} + \frac{2a^2 f^{\frac{c}{a+bx}}(a+bx)^3}{b^5} \\ & - \frac{2a^3 c f^{\frac{c}{a+bx}}(a+bx)\log(f)}{b^5} + \frac{a^2 c f^{\frac{c}{a+bx}}(a+bx)^2\log(f)}{b^5} \\ & - \frac{a^4 c \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)}{b^5} + \frac{a^2 c^2 f^{\frac{c}{a+bx}}(a+bx)\log^2(f)}{b^5} \\ & + \frac{2a^3 c^2 \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^2(f)}{b^5} \\ & - \frac{a^2 c^3 \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^3(f)}{b^5} \\ & - \frac{4ac^4\Gamma\left(-4, -\frac{c\log(f)}{a+bx}\right)\log^4(f)}{b^5} - \frac{c^5\Gamma\left(-5, -\frac{c\log(f)}{a+bx}\right)\log^5(f)}{b^5}\end{aligned}$$

output
```
a^4*f^(c/(b*x+a))*(b*x+a)/b^5-2*a^3*f^(c/(b*x+a))*(b*x+a)^2/b^5+2*a^2*f^(c
/(b*x+a))*(b*x+a)^3/b^5-2*a^3*c*f^(c/(b*x+a))*(b*x+a)*ln(f)/b^5+a^2*c*f^(c
/(b*x+a))*(b*x+a)^2*ln(f)/b^5-a^4*c*Ei(c*ln(f)/(b*x+a))*ln(f)/b^5+a^2*c^2*
f^(c/(b*x+a))*(b*x+a)*ln(f)^2/b^5+2*a^3*c^2*Ei(c*ln(f)/(b*x+a))*ln(f)^2/b^
5-a^2*c^3*Ei(c*ln(f)/(b*x+a))*ln(f)^3/b^5-4*a*(b*x+a)^4*Ei(5,-c*ln(f)/(b*x
+a))/b^5+(b*x+a)^5*Ei(6,-c*ln(f)/(b*x+a))/b^5
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.83

$$\int f^{\frac{c}{a+bx}} x^4 \, dx = \frac{af^{\frac{c}{a+bx}}(24a^4 - 154a^3c\log(f) + 102a^2c^2\log^2(f) - 19ac^3\log^3(f) + c^4\log^4(f))}{120b^5} + \frac{-c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)\left(120a^4 - 240a^3c\log(f) + 120a^2c^2\log^2(f) - 20ac^3\log^3(f) + c^4\log^4\right.}{}$$

input
```
Integrate[f^(c/(a + b*x))*x^4,x]
```

output
```
(a*f^(c/(a + b*x))*(24*a^4 - 154*a^3*c*Log[f] + 102*a^2*c^2*Log[f]^2 - 19*
a*c^3*Log[f]^3 + c^4*Log[f]^4))/(120*b^5) + (-(c*ExpIntegralEi[(c*Log[f])/
(a + b*x)]*Log[f]*(120*a^4 - 240*a^3*c*Log[f] + 120*a^2*c^2*Log[f]^2 - 20*
a*c^3*Log[f]^3 + c^4*Log[f]^4)) + b*f^(c/(a + b*x))*x*(24*b^4*x^4 + 2*c*(-
48*a^3 + 18*a^2*b*x - 8*a*b^2*x^2 + 3*b^3*x^3)*Log[f] + 2*c^2*(43*a^2 - 7*
a*b*x + b^2*x^2)*Log[f]^2 + c^3*(-18*a + b*x)*Log[f]^3 + c^4*Log[f]^4))/(1
20*b^5)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{\frac{c}{a+bx}} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^4 f^{\frac{c}{a+bx}}}{b^4} - \frac{4a^3(a+bx)f^{\frac{c}{a+bx}}}{b^4} + \frac{6a^2(a+bx)^2 f^{\frac{c}{a+bx}}}{b^4} + \frac{(a+bx)^4 f^{\frac{c}{a+bx}}}{b^4} - \frac{4a(a+bx)^3 f^{\frac{c}{a+bx}}}{b^4}\right) dx$$

$$\downarrow \text{2009}$$

$$-\frac{a^4 c \log(f) \text{ExpIntegralEi}\left(\frac{c \log(f)}{a+bx}\right)}{b^5} + \frac{a^4 (a+bx) f^{\frac{c}{a+bx}}}{b^5} + \frac{2a^3 c^2 \log^2(f) \text{ExpIntegralEi}\left(\frac{c \log(f)}{a+bx}\right)}{b^5} - \frac{2a^3 (a+bx)^2 f^{\frac{c}{a+bx}}}{b^5} - \frac{2a^3 c \log(f)(a+bx) f^{\frac{c}{a+bx}}}{b^5} - \frac{a^2 c^3 \log^3(f) \text{ExpIntegralEi}\left(\frac{c \log(f)}{a+bx}\right)}{b^5} + \frac{a^2 c^2 \log^2(f)(a+bx) f^{\frac{c}{a+bx}}}{b^5} + \frac{2a^2 (a+bx)^3 f^{\frac{c}{a+bx}}}{b^5} + \frac{a^2 c \log(f)(a+bx)^2 f^{\frac{c}{a+bx}}}{b^5} - \frac{c^5 \log^5(f) \Gamma\left(-5, -\frac{c \log(f)}{a+bx}\right)}{b^5} - \frac{4ac^4 \log^4(f) \Gamma\left(-4, -\frac{c \log(f)}{a+bx}\right)}{b^5}$$

input
```
Int[f^(c/(a + b*x))*x^4,x]
```

output
```
(a^4*f^(c/(a + b*x))*(a + b*x))/b^5 - (2*a^3*f^(c/(a + b*x))*(a + b*x)^2)/
b^5 + (2*a^2*f^(c/(a + b*x))*(a + b*x)^3)/b^5 - (2*a^3*c*f^(c/(a + b*x))*(
a + b*x)*Log[f])/b^5 + (a^2*c*f^(c/(a + b*x))*(a + b*x)^2*Log[f])/b^5 - (a
^4*c*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f])/b^5 + (a^2*c^2*f^(c/(a +
b*x))*(a + b*x)*Log[f]^2)/b^5 + (2*a^3*c^2*ExpIntegralEi[(c*Log[f])/(a + b
*x)]*Log[f]^2)/b^5 - (a^2*c^3*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f]^3
)/b^5 - (4*a*c^4*Gamma[-4, -((c*Log[f])/(a + b*x))]*Log[f]^4)/b^5 - (c^5*G
amma[-5, -((c*Log[f])/(a + b*x))]*Log[f]^5)/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.78

method	result
risch	$\frac{\operatorname{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}\right)c^5\ln(f)^5}{120b^5}+\frac{\ln(f)^4 f^{\frac{c}{bx+a}}c^4x}{120b^4}+\frac{\ln(f)^3 f^{\frac{c}{bx+a}}c^3x^2}{120b^3}+\frac{\ln(f)^2 f^{\frac{c}{bx+a}}c^2x^3}{60b^2}+\frac{\ln(f) f^{\frac{c}{bx+a}}cx^4}{20b}+\frac{f^{\frac{c}{bx+}}}{5}$

input
```
int(f^(c/(b*x+a))*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/120/b^5*Ei(1,-c*ln(f)/(b*x+a))*c^5*ln(f)^5+1/120/b^4*ln(f)^4*f^(c/(b*x+a
))*c^4*x+1/120/b^3*ln(f)^3*f^(c/(b*x+a))*c^3*x^2+1/60/b^2*ln(f)^2*f^(c/(b*
x+a))*c^2*x^3+1/20/b*ln(f)*f^(c/(b*x+a))*c*x^4+1/5*f^(c/(b*x+a))*x^5+1/120
/b^5*ln(f)^4*f^(c/(b*x+a))*a*c^4-1/6/b^5*ln(f)^4*Ei(1,-c*ln(f)/(b*x+a))*a*
c^4-3/20/b^4*ln(f)^3*f^(c/(b*x+a))*a*c^3*x-7/60/b^3*ln(f)^2*f^(c/(b*x+a))*
a*c^2*x^2-2/15/b^2*ln(f)*f^(c/(b*x+a))*a*c*x^3-19/120/b^5*ln(f)^3*f^(c/(b*
x+a))*a^2*c^3+1/b^5*ln(f)^3*Ei(1,-c*ln(f)/(b*x+a))*a^2*c^3+43/60/b^4*ln(f)
^2*f^(c/(b*x+a))*a^2*c^2*x+3/10/b^3*ln(f)*f^(c/(b*x+a))*a^2*c*x^2+17/20/b^
5*ln(f)^2*f^(c/(b*x+a))*a^3*c^2-2/b^5*ln(f)^2*Ei(1,-c*ln(f)/(b*x+a))*a^3*c
^2-4/5/b^4*ln(f)*f^(c/(b*x+a))*a^3*c*x-77/60/b^5*ln(f)*f^(c/(b*x+a))*a^4*c
+1/b^5*ln(f)*Ei(1,-c*ln(f)/(b*x+a))*a^4*c+1/5/b^5*f^(c/(b*x+a))*a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.84

$$\int f^{\frac{c}{a+bx}}x^4\,dx$$
$$=\frac{\left(24\,b^5x^5+24\,a^5+(bc^4x+ac^4)\log\left(f\right)^4+(b^2c^3x^2-18\,abc^3x-19\,a^2c^3)\log\left(f\right)^3+2\,(b^3c^2x^3-7\,ab^2c^2x^2\right.}{}$$

input
```
integrate(f^(c/(b*x+a))*x^4,x, algorithm="fricas")
```

output
```
1/120*((24*b^5*x^5 + 24*a^5 + (b*c^4*x + a*c^4)*log(f)^4 + (b^2*c^3*x^2 -
18*a*b*c^3*x - 19*a^2*c^3)*log(f)^3 + 2*(b^3*c^2*x^3 - 7*a*b^2*c^2*x^2 + 4
3*a^2*b*c^2*x + 51*a^3*c^2)*log(f)^2 + 2*(3*b^4*c*x^4 - 8*a*b^3*c*x^3 + 18
*a^2*b^2*c*x^2 - 48*a^3*b*c*x - 77*a^4*c)*log(f))*f^(c/(b*x + a)) - (c^5*l
og(f)^5 - 20*a*c^4*log(f)^4 + 120*a^2*c^3*log(f)^3 - 240*a^3*c^2*log(f)^2
+ 120*a^4*c*log(f))*Ei(c*log(f)/(b*x + a)))/b^5
```

Sympy [F]

$$\int f^{\frac{c}{a+bx}} x^4 \, dx = \int f^{\frac{c}{a+bx}} x^4 \, dx$$

input
```
integrate(f**(c/(b*x+a))*x**4,x)
```

output
```
Integral(f**(c/(a + b*x))*x**4, x)
```

Maxima [F]

$$\int f^{\frac{c}{a+bx}} x^4 \, dx = \int f^{\frac{c}{bx+a}} x^4 \, dx$$

input
```
integrate(f^(c/(b*x+a))*x^4,x, algorithm="maxima")
```

output
```
1/120*(24*b^4*x^5 + 6*b^3*c*x^4*log(f) + 2*(b^2*c^2*log(f)^2 - 8*a*b^2*c*l
og(f))*x^3 + (b*c^3*log(f)^3 - 14*a*b*c^2*log(f)^2 + 36*a^2*b*c*log(f))*x^
2 + (c^4*log(f)^4 - 18*a*c^3*log(f)^3 + 86*a^2*c^2*log(f)^2 - 96*a^3*c*log
(f))*x)*f^(c/(b*x + a))/b^4 + integrate(-1/120*(a^2*c^4*log(f)^4 - 18*a^3*
c^3*log(f)^3 + 86*a^4*c^2*log(f)^2 - 96*a^5*c*log(f) - (b*c^5*log(f)^5 - 2
0*a*b*c^4*log(f)^4 + 120*a^2*b*c^3*log(f)^3 - 240*a^3*b*c^2*log(f)^2 + 120
*a^4*b*c*log(f))*x)*f^(c/(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), x)
```

Giac [F]

$$\int f^{\frac{c}{a+bx}}x^4\,dx = \int f^{\frac{c}{bx+a}}x^4\,dx$$

input
```
integrate(f^(c/(b*x+a))*x^4,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{a+bx}}x^4\,dx = \int f^{\frac{c}{a+b\,x}}\,x^4\,dx$$

input
```
int(f^(c/(a + b*x))*x^4,x)
```

output
```
int(f^(c/(a + b*x))*x^4, x)
```

Reduce [F]

$$\int f^{\frac{c}{a+bx}}x^4\,dx = \text{Too large to display}$$

input
```
int(f^(c/(b*x+a))*x^4,x)
```

output

```
(f**(c/(a + b*x))*log(f)**5*b**2*c**5*x**2 + 2*f**(c/(a + b*x))*log(f)**4*
a**2*b*c**4*x - 17*f**(c/(a + b*x))*log(f)**4*a*b**2*c**4*x**2 + f**(c/(a
+ b*x))*log(f)**4*b**3*c**4*x**3 + 2*f**(c/(a + b*x))*log(f)**3*a**4*c**3
- 32*f**(c/(a + b*x))*log(f)**3*a**3*b*c**3*x + 72*f**(c/(a + b*x))*log(f)
**3*a**2*b**2*c**3*x**2 - 12*f**(c/(a + b*x))*log(f)**3*a*b**3*c**3*x**3 +
 2*f**(c/(a + b*x))*log(f)**3*b**4*c**3*x**4 - 34*f**(c/(a + b*x))*log(f)*
*2*a**5*c**2 + 110*f**(c/(a + b*x))*log(f)**2*a**4*b*c**2*x - 60*f**(c/(a
+ b*x))*log(f)**2*a**3*b**2*c**2*x**2 + 20*f**(c/(a + b*x))*log(f)**2*a**2
*b**3*c**2*x**3 - 10*f**(c/(a + b*x))*log(f)**2*a*b**4*c**2*x**4 + 6*f**(c
/(a + b*x))*log(f)**2*b**5*c**2*x**5 + 144*f**(c/(a + b*x))*log(f)*a**6*c
+ 24*f**(c/(a + b*x))*log(f)*a**5*b*c*x + 24*f**(c/(a + b*x))*log(f)*a*b**
5*c*x**5 + 24*f**(c/(a + b*x))*log(f)*b**6*c*x**6 - 120*f**(c/(a + b*x))*a
**7 - 120*f**(c/(a + b*x))*a**6*b*x + int((f**(c/(a + b*x))*x**2)/(a**3 +
3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**6*a*b**3*c**6 + int((f*
*(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*lo
g(f)**6*b**4*c**6*x - 20*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3),x)*log(f)**5*a**2*b**3*c**5 - 20*int((f**(c/(a
+ b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**5
*a*b**4*c**5*x + 120*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3),x)*log(f)**4*a**3*b**3*c**4 + 120*int((f**(c/(a ...
```

3.150 $\int f^{\frac{c}{a+bx}} x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 269

$$\begin{aligned}\int f^{\frac{c}{a+bx}} x^3\,dx = & -\frac{a^3 f^{\frac{c}{a+bx}}(a+bx)}{b^4}+\frac{3a^2 f^{\frac{c}{a+bx}}(a+bx)^2}{2b^4}-\frac{a f^{\frac{c}{a+bx}}(a+bx)^3}{b^4} \\ & +\frac{3a^2 c f^{\frac{c}{a+bx}}(a+bx)\log(f)}{2b^4}-\frac{acf^{\frac{c}{a+bx}}(a+bx)^2\log(f)}{2b^4} \\ & +\frac{a^3 c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)}{b^4}-\frac{ac^2 f^{\frac{c}{a+bx}}(a+bx)\log^2(f)}{2b^4} \\ & -\frac{3a^2c^2\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^2(f)}{2b^4} \\ & +\frac{ac^3\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^3(f)}{2b^4}+\frac{c^4\Gamma\left(-4,-\frac{c\log(f)}{a+bx}\right)\log^4(f)}{b^4}\end{aligned}$$

output
```
-a^3*f^(c/(b*x+a))*(b*x+a)/b^4+3/2*a^2*f^(c/(b*x+a))*(b*x+a)^2/b^4-a*f^(c/
(b*x+a))*(b*x+a)^3/b^4+3/2*a^2*c*f^(c/(b*x+a))*(b*x+a)*ln(f)/b^4-1/2*a*c*f
^(c/(b*x+a))*(b*x+a)^2*ln(f)/b^4+a^3*c*Ei(c*ln(f)/(b*x+a))*ln(f)/b^4-1/2*a
*c^2*f^(c/(b*x+a))*(b*x+a)*ln(f)^2/b^4-3/2*a^2*c^2*Ei(c*ln(f)/(b*x+a))*ln(
f)^2/b^4+1/2*a*c^3*Ei(c*ln(f)/(b*x+a))*ln(f)^3/b^4+(b*x+a)^4*Ei(5,-c*ln(f)
/(b*x+a))/b^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.67

$$\int f^{\frac{c}{a+bx}}x^3\,dx = -\frac{af^{\frac{c}{a+bx}}\left(6a^3-26a^2c\log(f)+11ac^2\log^2(f)-c^3\log^3(f)\right)}{24b^4} + \frac{c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)\left(24a^3-36a^2c\log(f)+12ac^2\log^2(f)-c^3\log^3(f)\right)+bf^{\frac{c}{a+bx}}x\left(6b^3x^3\right.}{24b^4}$$

input
```
Integrate[f^(c/(a + b*x))*x^3,x]
```

output
```
-1/24*(a*f^(c/(a + b*x))*(6*a^3 - 26*a^2*c*Log[f] + 11*a*c^2*Log[f]^2 - c^
3*Log[f]^3))/b^4 + (c*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f]*(24*a^3 -
 36*a^2*c*Log[f] + 12*a*c^2*Log[f]^2 - c^3*Log[f]^3) + b*f^(c/(a + b*x))*x
*(6*b^3*x^3 + 2*c*(9*a^2 - 3*a*b*x + b^2*x^2)*Log[f] + c^2*(-10*a + b*x)*L
og[f]^2 + c^3*Log[f]^3))/(24*b^4)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{\frac{c}{a+bx}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(-\frac{a^3 f^{\frac{c}{a+bx}}}{b^3}+\frac{3a^2(a+bx)f^{\frac{c}{a+bx}}}{b^3}+\frac{(a+bx)^3 f^{\frac{c}{a+bx}}}{b^3}-\frac{3a(a+bx)^2 f^{\frac{c}{a+bx}}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3 c \log(f) \operatorname{ExpIntegralEi}\left(\frac{c \log(f)}{a+bx}\right)}{b^4} - \frac{a^3(a+bx)f^{\frac{c}{a+bx}}}{b^4} - \frac{3a^2c^2\log^2(f) \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)}{2b^4} + \frac{3a^2(a+bx)^2 f^{\frac{c}{a+bx}}}{2b^4} + \frac{3a^2 c \log(f)(a+bx) f^{\frac{c}{a+bx}}}{2b^4} + \frac{c^4\log^4(f)\Gamma\left(-4, -\frac{c\log(f)}{a+bx}\right)}{b^4} + \frac{ac^3\log^3(f) \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)}{2b^4} - \frac{ac^2\log^2(f)(a+bx)f^{\frac{c}{a+bx}}}{2b^4} - \frac{a(a+bx)^3 f^{\frac{c}{a+bx}}}{b^4} - \frac{ac\log(f)(a+bx)^2 f^{\frac{c}{a+bx}}}{2b^4}$$

input
```
Int[f^(c/(a + b*x))*x^3,x]
```

output
```
-((a^3*f^(c/(a + b*x))*(a + b*x))/b^4) + (3*a^2*f^(c/(a + b*x))*(a + b*x)^
2)/(2*b^4) - (a*f^(c/(a + b*x))*(a + b*x)^3)/b^4 + (3*a^2*c*f^(c/(a + b*x)
)*(a + b*x)*Log[f])/(2*b^4) - (a*c*f^(c/(a + b*x))*(a + b*x)^2*Log[f])/(2*
b^4) + (a^3*c*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f])/b^4 - (a*c^2*f^(
c/(a + b*x))*(a + b*x)*Log[f]^2)/(2*b^4) - (3*a^2*c^2*ExpIntegralEi[(c*Log
[f])/(a + b*x)]*Log[f]^2)/(2*b^4) + (a*c^3*ExpIntegralEi[(c*Log[f])/(a + b
*x)]*Log[f]^3)/(2*b^4) + (c^4*Gamma[-4, -((c*Log[f])/(a + b*x))]*Log[f]^4)
/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.33

method	result
risch	$\frac{\ln(f)^3 f^{\frac{c}{bx+a}} c^3 x}{24b^3}+\frac{\ln(f)^2 f^{\frac{c}{bx+a}} c^2 x^2}{24b^2}+\frac{\ln(f) f^{\frac{c}{bx+a}} c x^3}{12b}+\frac{f^{\frac{c}{bx+a}} x^4}{4}+\frac{\text{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}\right) c^4 \ln(f)^4}{24b^4}+\frac{\ln(f)^3 f^{\frac{c}{bx+a}}}{24b^4}$

input
```
int(f^(c/(b*x+a))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/b^3*ln(f)^3*f^(c/(b*x+a))*c^3*x+1/24/b^2*ln(f)^2*f^(c/(b*x+a))*c^2*x^
2+1/12/b*ln(f)*f^(c/(b*x+a))*c*x^3+1/4*f^(c/(b*x+a))*x^4+1/24/b^4*Ei(1,-c*
ln(f)/(b*x+a))*c^4*ln(f)^4+1/24/b^4*ln(f)^3*f^(c/(b*x+a))*a*c^3-5/12/b^3*l
n(f)^2*f^(c/(b*x+a))*a*c^2*x-1/4/b^2*ln(f)*f^(c/(b*x+a))*a*c*x^2-1/2/b^4*l
n(f)^3*Ei(1,-c*ln(f)/(b*x+a))*a*c^3-11/24/b^4*ln(f)^2*f^(c/(b*x+a))*a^2*c^
2+3/4/b^3*ln(f)*f^(c/(b*x+a))*a^2*c*x+3/2/b^4*ln(f)^2*Ei(1,-c*ln(f)/(b*x+a
))*a^2*c^2+13/12/b^4*ln(f)*f^(c/(b*x+a))*a^3*c-1/b^4*ln(f)*Ei(1,-c*ln(f)/(
b*x+a))*a^3*c-1/4/b^4*f^(c/(b*x+a))*a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.64

$$\int f^{\frac{c}{a+bx}} x^3 \, dx$$

$$= \frac{\left(6 b^4 x^4 - 6 a^4 + (bc^3 x + ac^3) \log(f)^3 + (b^2 c^2 x^2 - 10 abc^2 x - 11 a^2 c^2) \log(f)^2 + 2 (b^3 c x^3 - 3 ab^2 c x^2 + 9 \right.}{2}$$

input
```
integrate(f^(c/(b*x+a))*x^3,x, algorithm="fricas")
```

output
```
1/24*((6*b^4*x^4 - 6*a^4 + (b*c^3*x + a*c^3)*log(f)^3 + (b^2*c^2*x^2 - 10*
a*b*c^2*x - 11*a^2*c^2)*log(f)^2 + 2*(b^3*c*x^3 - 3*a*b^2*c*x^2 + 9*a^2*b*
c*x + 13*a^3*c)*log(f))*f^(c/(b*x + a)) - (c^4*log(f)^4 - 12*a*c^3*log(f)^
3 + 36*a^2*c^2*log(f)^2 - 24*a^3*c*log(f))*Ei(c*log(f)/(b*x + a)))/b^4
```

Sympy [F]

$$\int f^{\frac{c}{a+bx}} x^3 \, dx = \int f^{\frac{c}{a+bx}} x^3 \, dx$$

input
```
integrate(f**(c/(b*x+a))*x**3,x)
```

output
```
Integral(f**(c/(a + b*x))*x**3, x)
```

Maxima [F]

$$\int f^{\frac{c}{a+bx}} x^3 \, dx = \int f^{\frac{c}{bx+a}} x^3 \, dx$$

input
```
integrate(f^(c/(b*x+a))*x^3,x, algorithm="maxima")
```

output
```
1/24*(6*b^3*x^4 + 2*b^2*c*x^3*log(f) + (b*c^2*log(f)^2 - 6*a*b*c*log(f))*x
^2 + (c^3*log(f)^3 - 10*a*c^2*log(f)^2 + 18*a^2*c*log(f))*x)*f^(c/(b*x + a
))/b^3 - integrate(1/24*(a^2*c^3*log(f)^3 - 10*a^3*c^2*log(f)^2 + 18*a^4*c
*log(f) - (b*c^4*log(f)^4 - 12*a*b*c^3*log(f)^3 + 36*a^2*b*c^2*log(f)^2 -
24*a^3*b*c*log(f))*x)*f^(c/(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), x)
```

Giac [F]

$$\int f^{\frac{c}{a+bx}} x^3 \, dx = \int f^{\frac{c}{bx+a}} x^3 \, dx$$

input
```
integrate(f^(c/(b*x+a))*x^3,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{a+bx}}x^3\,dx=\int f^{\frac{c}{a+b\,x}}\,x^3\,dx$$

input
```
int(f^(c/(a + b*x))*x^3,x)
```

output
```
int(f^(c/(a + b*x))*x^3, x)
```

Reduce [F]

$$\int f^{\frac{c}{a+bx}}x^3\,dx=\text{Too large to display}$$

input
```
int(f^(c/(b*x+a))*x^3,x)
```

output
```
(f**(c/(a + b*x))*log(f)**4*b**2*c**4*x**2 + 2*f**(c/(a + b*x))*log(f)**3*
a**2*b*c**3*x - 9*f**(c/(a + b*x))*log(f)**3*a*b**2*c**3*x**2 + f**(c/(a +
 b*x))*log(f)**3*b**3*c**3*x**3 + 2*f**(c/(a + b*x))*log(f)**2*a**4*c**2 -
 16*f**(c/(a + b*x))*log(f)**2*a**3*b*c**2*x + 12*f**(c/(a + b*x))*log(f)*
*2*a**2*b**2*c**2*x**2 - 4*f**(c/(a + b*x))*log(f)**2*a*b**3*c**2*x**3 + 2
*f**(c/(a + b*x))*log(f)**2*b**4*c**2*x**4 - 18*f**(c/(a + b*x))*log(f)*a*
*5*c + 6*f**(c/(a + b*x))*log(f)*a**4*b*c*x + 6*f**(c/(a + b*x))*log(f)*a*
b**4*c*x**4 + 6*f**(c/(a + b*x))*log(f)*b**5*c*x**5 + 24*f**(c/(a + b*x))*
a**6 + 24*f**(c/(a + b*x))*a**5*b*x + int((f**(c/(a + b*x))*x**2)/(a**3 +
3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**5*a*b**3*c**5 + int((f*
*(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*lo
g(f)**5*b**4*c**5*x - 12*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3),x)*log(f)**4*a**2*b**3*c**4 - 12*int((f**(c/(a
+ b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**4
*a*b**4*c**4*x + 36*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b
**2*x**2 + b**3*x**3),x)*log(f)**3*a**3*b**3*c**3 + 36*int((f**(c/(a + b*x
))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**3*a**2
*b**4*c**3*x - 24*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3),x)*log(f)**2*a**4*b**3*c**2 - 24*int((f**(c/(a + b*x))
*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**2*a**...
```

3.151 $\int f^{\frac{c}{a+bx}} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int f^{\frac{c}{a+bx}} x^2 \, dx = {} & \frac{a^2 f^{\frac{c}{a+bx}}(a+bx)}{b^3} - \frac{a f^{\frac{c}{a+bx}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{a+bx}}(a+bx)^3}{3b^3} \\ & - \frac{acf^{\frac{c}{a+bx}}(a+bx)\log(f)}{b^3} + \frac{cf^{\frac{c}{a+bx}}(a+bx)^2\log(f)}{6b^3} \\ & - \frac{a^2 c \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)}{b^3} + \frac{c^2 f^{\frac{c}{a+bx}}(a+bx)\log^2(f)}{6b^3} \\ & + \frac{ac^2 \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^2(f)}{b^3} \\ & - \frac{c^3 \operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^3(f)}{6b^3}\end{aligned}$$

output
```
a^2*f^(c/(b*x+a))*(b*x+a)/b^3-a*f^(c/(b*x+a))*(b*x+a)^2/b^3+1/3*f^(c/(b*x+
a))*(b*x+a)^3/b^3-a*c*f^(c/(b*x+a))*(b*x+a)*ln(f)/b^3+1/6*c*f^(c/(b*x+a))*
(b*x+a)^2*ln(f)/b^3-a^2*c*Ei(c*ln(f)/(b*x+a))*ln(f)/b^3+1/6*c^2*f^(c/(b*x+
a))*(b*x+a)*ln(f)^2/b^3+a*c^2*Ei(c*ln(f)/(b*x+a))*ln(f)^2/b^3-1/6*c^3*Ei(c
*ln(f)/(b*x+a))*ln(f)^3/b^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.56

$$\int f^{\frac{c}{a+bx}}x^2\,dx = \frac{af^{\frac{c}{a+bx}}\left(2a^2-5ac\log(f)+c^2\log^2(f)\right)}{6b^3}$$
$$+\frac{-c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)\left(6a^2-6ac\log(f)+c^2\log^2(f)\right)+bf^{\frac{c}{a+bx}}x\left(2b^2x^2+(-4ac+bcx)\,\text{lo}\right.}{6b^3}$$

input
```
Integrate[f^(c/(a + b*x))*x^2,x]
```

output
```
(a*f^(c/(a + b*x))*(2*a^2 - 5*a*c*Log[f] + c^2*Log[f]^2))/(6*b^3) + (-(c*E
xpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f]*(6*a^2 - 6*a*c*Log[f] + c^2*Log[
f]^2)) + b*f^(c/(a + b*x))*x*(2*b^2*x^2 + (-4*a*c + b*c*x)*Log[f] + c^2*Lo
g[f]^2))/(6*b^3)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{\frac{c}{a+bx}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^2f^{\frac{c}{a+bx}}}{b^2}+\frac{(a+bx)^2f^{\frac{c}{a+bx}}}{b^2}-\frac{2a(a+bx)f^{\frac{c}{a+bx}}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2c\log(f)\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)}{b^3}+\frac{a^2(a+bx)f^{\frac{c}{a+bx}}}{b^3}-\frac{c^3\log^3(f)\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)}{6b^3}+\frac{ac^2\log^2(f)\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)}{b^3}+\frac{c^2\log^2(f)(a+bx)f^{\frac{c}{a+bx}}}{6b^3}+\frac{(a+bx)^3f^{\frac{c}{a+bx}}}{3b^3}-\frac{a(a+bx)^2f^{\frac{c}{a+bx}}}{b^3}+\frac{c\log(f)(a+bx)^2f^{\frac{c}{a+bx}}}{6b^3}-\frac{ac\log(f)(a+bx)f^{\frac{c}{a+bx}}}{b^3}$$

input
```
Int[f^(c/(a + b*x))*x^2,x]
```

output
```
(a^2*f^(c/(a + b*x))*(a + b*x))/b^3 - (a*f^(c/(a + b*x))*(a + b*x)^2)/b^3
+ (f^(c/(a + b*x))*(a + b*x)^3)/(3*b^3) - (a*c*f^(c/(a + b*x))*(a + b*x)*L
og[f])/b^3 + (c*f^(c/(a + b*x))*(a + b*x)^2*Log[f])/(6*b^3) - (a^2*c*ExpIn
tegralEi[(c*Log[f])/(a + b*x)]*Log[f])/b^3 + (c^2*f^(c/(a + b*x))*(a + b*x
)*Log[f]^2)/(6*b^3) + (a*c^2*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f]^2)
/b^3 - (c^3*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log[f]^3)/(6*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\ln(f)^2 f^{\frac{c}{bx+a}} c^2 x}{6b^2}+\frac{\ln(f) f^{\frac{c}{bx+a}} c\, x^2}{6b}+\frac{f^{\frac{c}{bx+a}} x^3}{3}+\frac{\text{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}\right) c^3 \ln(f)^3}{6b^3}+\frac{\ln(f)^2 f^{\frac{c}{bx+a}} a\, c^2}{6b^3}-\frac{2\ln(f) f^{\frac{c}{bx+a}} a}{3b^2}$

input
```
int(f^(c/(b*x+a))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6/b^2*ln(f)^2*f^(c/(b*x+a))*c^2*x+1/6/b*ln(f)*f^(c/(b*x+a))*c*x^2+1/3*f^
(c/(b*x+a))*x^3+1/6/b^3*Ei(1,-c*ln(f)/(b*x+a))*c^3*ln(f)^3+1/6/b^3*ln(f)^2
*f^(c/(b*x+a))*a*c^2-2/3/b^2*ln(f)*f^(c/(b*x+a))*a*c*x-1/b^3*ln(f)^2*Ei(1,
-c*ln(f)/(b*x+a))*a*c^2-5/6/b^3*ln(f)*f^(c/(b*x+a))*a^2*c+1/b^3*ln(f)*Ei(1
,-c*ln(f)/(b*x+a))*a^2*c+1/3/b^3*f^(c/(b*x+a))*a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.50

$$\int f^{\frac{c}{a+bx}}x^2\,dx$$
$$= \frac{\left(2\,b^3x^3 + 2\,a^3 + (bc^2x + ac^2)\log\left(f\right)^2 + (b^2cx^2 - 4\,abcx - 5\,a^2c)\log\left(f\right)\right)f^{\frac{c}{bx+a}} - \left(c^3\log\left(f\right)^3 - 6\,ac^2\log\left(\right.\right.}{6\,b^3}$$

input
```
integrate(f^(c/(b*x+a))*x^2,x, algorithm="fricas")
```

output
```
1/6*((2*b^3*x^3 + 2*a^3 + (b*c^2*x + a*c^2)*log(f)^2 + (b^2*c*x^2 - 4*a*b*
c*x - 5*a^2*c)*log(f))*f^(c/(b*x + a)) - (c^3*log(f)^3 - 6*a*c^2*log(f)^2
+ 6*a^2*c*log(f))*Ei(c*log(f)/(b*x + a)))/b^3
```

Sympy [F]

$$\int f^{\frac{c}{a+bx}}x^2\,dx = \int f^{\frac{c}{a+bx}}x^2\,dx$$

input
```
integrate(f**(c/(b*x+a))*x**2,x)
```

output
```
Integral(f**(c/(a + b*x))*x**2, x)
```

Maxima [F]

$$\int f^{\frac{c}{a+bx}} x^2 \, dx = \int f^{\frac{c}{bx+a}} x^2 \, dx$$

input
```
integrate(f^(c/(b*x+a))*x^2,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*x^3 + b*c*x^2*log(f) + (c^2*log(f)^2 - 4*a*c*log(f))*x)*f^(c/(b
*x + a))/b^2 + integrate(-1/6*(a^2*c^2*log(f)^2 - 4*a^3*c*log(f) - (b*c^3*
log(f)^3 - 6*a*b*c^2*log(f)^2 + 6*a^2*b*c*log(f))*x)*f^(c/(b*x + a))/(b^4*
x^2 + 2*a*b^3*x + a^2*b^2), x)
```

Giac [F]

$$\int f^{\frac{c}{a+bx}} x^2 \, dx = \int f^{\frac{c}{bx+a}} x^2 \, dx$$

input
```
integrate(f^(c/(b*x+a))*x^2,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.91

$$\int f^{\frac{c}{a+bx}} x^2 \, dx = \frac{\frac{b\, f^{\frac{c}{a+b\,x}}\, x^4}{3} + f^{\frac{c}{a+b\,x}}\, x^3 \left(\frac{a}{3} + \frac{c\, \ln(f)}{6}\right) + \frac{f^{\frac{c}{a+b\,x}}\, x \left(2\,a^3 - 9\,a^2\,c\,\ln(f) + 2\,a\,c^2\,\ln(f)^2\right)}{6\,b^2} + \frac{f^{\frac{c}{a+b\,x}}\, x^2 \left(c^2\,\ln(f)^2 - 3\,a\,c\,\ln(f)\right)}{6\,b} + \frac{a^2\, f^{\frac{c}{a}}\cdots}{\cdots}}{a + b\,x} - \frac{\mathrm{ei}\left(\frac{c\,\ln(f)}{a+b\,x}\right) \left(6\,a^2\,c\,\ln\left(f\right) - 6\,a\,c^2\,\ln\left(f\right)^2 + c^3\,\ln\left(f\right)^3\right)}{6\,b^3}$$

input
```
int(f^(c/(a + b*x))*x^2,x)
```

output
```
((b*f^(c/(a + b*x))*x^4)/3 + f^(c/(a + b*x))*x^3*(a/3 + (c*log(f))/6) + (f
^(c/(a + b*x))*x*(2*a^3 - 9*a^2*c*log(f) + 2*a*c^2*log(f)^2))/(6*b^2) + (f
^(c/(a + b*x))*x^2*(c^2*log(f)^2 - 3*a*c*log(f)))/(6*b) + (a^2*f^(c/(a + b
*x))*(c^2*log(f)^2 + 2*a^2 - 5*a*c*log(f)))/(6*b^3))/(a + b*x) - (ei((c*lo
g(f))/(a + b*x))*(c^3*log(f)^3 + 6*a^2*c*log(f) - 6*a*c^2*log(f)^2))/(6*b^
3)
```

Reduce [F]

$$\int f^{\frac{c}{a+bx}}x^2\,dx$$
$$= \frac{f^{\frac{c}{bx+a}}\log(f)^3 b^2c^3x^2 + 2f^{\frac{c}{bx+a}}\log(f)^2 a^2b\,c^2x - 3f^{\frac{c}{bx+a}}\log(f)^2 a\,b^2c^2x^2 + f^{\frac{c}{bx+a}}\log(f)^2 b^3c^2x^3 + 2f^{\frac{c}{bx+a}}\log}{}$$

input
```
int(f^(c/(b*x+a))*x^2,x)
```

output
```
(f**(c/(a + b*x))*log(f)**3*b**2*c**3*x**2 + 2*f**(c/(a + b*x))*log(f)**2*
a**2*b*c**2*x - 3*f**(c/(a + b*x))*log(f)**2*a*b**2*c**2*x**2 + f**(c/(a +
 b*x))*log(f)**2*b**3*c**2*x**3 + 2*f**(c/(a + b*x))*log(f)*a**4*c - 4*f**
(c/(a + b*x))*log(f)*a**3*b*c*x + 2*f**(c/(a + b*x))*log(f)*a*b**3*c*x**3
+ 2*f**(c/(a + b*x))*log(f)*b**4*c*x**4 - 6*f**(c/(a + b*x))*a**5 - 6*f**(
c/(a + b*x))*a**4*b*x + int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3
*a*b**2*x**2 + b**3*x**3),x)*log(f)**4*a*b**3*c**4 + int((f**(c/(a + b*x))
*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**4*b**4*c
**4*x - 6*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3),x)*log(f)**3*a**2*b**3*c**3 - 6*int((f**(c/(a + b*x))*x**2)/(a
**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**3*a*b**4*c**3*x +
 6*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3),x)*log(f)**2*a**3*b**3*c**2 + 6*int((f**(c/(a + b*x))*x**2)/(a**3 + 3
*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**2*a**2*b**4*c**2*x)/(6*l
og(f)*b**3*c*(a + b*x))
```

3.152 $\int f^{\frac{c}{a+bx}} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 120

$$\int f^{\frac{c}{a+bx}} x \, dx = -\frac{af^{\frac{c}{a+bx}}(a+bx)}{b^2} + \frac{f^{\frac{c}{a+bx}}(a+bx)^2}{2b^2} + \frac{cf^{\frac{c}{a+bx}}(a+bx)\log(f)}{2b^2} + \frac{ac\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)}{b^2} - \frac{c^2\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log^2(f)}{2b^2}$$

output
```
-a*f^(c/(b*x+a))*(b*x+a)/b^2+1/2*f^(c/(b*x+a))*(b*x+a)^2/b^2+1/2*c*f^(c/(b
*x+a))*(b*x+a)*ln(f)/b^2+a*c*Ei(c*ln(f)/(b*x+a))*ln(f)/b^2-1/2*c^2*Ei(c*ln
(f)/(b*x+a))*ln(f)^2/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int f^{\frac{c}{a+bx}} x \, dx = -\frac{af^{\frac{c}{a+bx}}(a-c\log(f))}{2b^2} + \frac{c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)(2a-c\log(f)) + bf^{\frac{c}{a+bx}}x(bx+c\log(f))}{2b^2}$$

input
```
Integrate[f^(c/(a + b*x))*x,x]
```

output
```
-1/2*(a*f^(c/(a + b*x))*(a - c*Log[f]))/b^2 + (c*ExpIntegralEi[(c*Log[f])/
(a + b*x)]*Log[f]*(2*a - c*Log[f]) + b*f^(c/(a + b*x))*x*(b*x + c*Log[f]))
/(2*b^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{\frac{c}{a+bx}} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(a+bx) f^{\frac{c}{a+bx}}}{b}-\frac{a f^{\frac{c}{a+bx}}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2 \log^2(f) \text{ExpIntegralEi}\left(\frac{c \log(f)}{a+bx}\right)}{2b^2}+\frac{ac \log(f) \text{ExpIntegralEi}\left(\frac{c \log(f)}{a+bx}\right)}{b^2}+\frac{(a+bx)^2 f^{\frac{c}{a+bx}}}{2b^2}-\frac{a(a+bx) f^{\frac{c}{a+bx}}}{b^2}+\frac{c \log(f)(a+bx) f^{\frac{c}{a+bx}}}{2b^2}$$

input
```
Int[f^(c/(a + b*x))*x,x]
```

output
```
-((a*f^(c/(a + b*x))*(a + b*x))/b^2) + (f^(c/(a + b*x))*(a + b*x)^2)/(2*b^
2) + (c*f^(c/(a + b*x))*(a + b*x)*Log[f])/(2*b^2) + (a*c*ExpIntegralEi[(c*
Log[f])/(a + b*x)]*Log[f])/b^2 - (c^2*ExpIntegralEi[(c*Log[f])/(a + b*x)]*
Log[f]^2)/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.05

method	result
risch	$\frac{f^{\frac{c}{bx+a}} x^{2}}{2}-\frac{f^{\frac{c}{bx+a}} a^{2}}{2 b^{2}}+\frac{\ln(f) f^{\frac{c}{bx+a}} c x}{2 b}+\frac{\ln(f) f^{\frac{c}{bx+a}} a c}{2 b^{2}}+\frac{c^{2} \ln(f)^{2} \operatorname{expIntegral}_{1}\left(-\frac{c \ln(f)}{bx+a}\right)}{2 b^{2}}-\frac{c \ln(f) a \operatorname{expIntegral}_{1}\left(-\frac{c}{\ldots}\right.}{b^{2}}$

input
```
int(f^(c/(b*x+a))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^(c/(b*x+a))*x^2-1/2/b^2*f^(c/(b*x+a))*a^2+1/2/b*ln(f)*f^(c/(b*x+a))*
c*x+1/2/b^2*ln(f)*f^(c/(b*x+a))*a*c+1/2*c^2*ln(f)^2/b^2*Ei(1,-c*ln(f)/(b*x
+a))-c*ln(f)/b^2*a*Ei(1,-c*ln(f)/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

$$\int f^{\frac{c}{a+bx}} x \, dx = \frac{\left(b^{2} x^{2}-a^{2}+(bcx+ac) \log(f)\right) f^{\frac{c}{bx+a}}-\left(c^{2} \log(f)^{2}-2 ac \log(f)\right) \operatorname{Ei}\left(\frac{c \log(f)}{bx+a}\right)}{2 b^{2}}$$

input
```
integrate(f^(c/(b*x+a))*x,x, algorithm="fricas")
```

output
```
1/2*((b^2*x^2 - a^2 + (b*c*x + a*c)*log(f))*f^(c/(b*x + a)) - (c^2*log(f)^
2 - 2*a*c*log(f))*Ei(c*log(f)/(b*x + a)))/b^2
```

Sympy [F]

$$\int f^{\frac{c}{a+bx}} x \, dx = \int f^{\frac{c}{a+bx}} x \, dx$$

input
```
integrate(f**(c/(b*x+a))*x,x)
```

output
```
Integral(f**(c/(a + b*x))*x, x)
```

Maxima [F]

$$\int f^{\frac{c}{a+bx}} x \, dx = \int f^{\frac{c}{bx+a}} x \, dx$$

input
```
integrate(f^(c/(b*x+a))*x,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + c*x*log(f))*f^(c/(b*x + a))/b - integrate(1/2*(a^2*c*log(f) -
 (b*c^2*log(f)^2 - 2*a*b*c*log(f))*x)*f^(c/(b*x + a))/(b^3*x^2 + 2*a*b^2*x
 + a^2*b), x)
```

Giac [F]

$$\int f^{\frac{c}{a+bx}} x \, dx = \int f^{\frac{c}{bx+a}} x \, dx$$

input
```
integrate(f^(c/(b*x+a))*x,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.13

$$\int f^{\frac{c}{a+bx}} x \, dx$$
$$= \frac{\frac{b\, f^{\frac{c}{a+b\,x}}\, x^3}{2} + f^{\frac{c}{a+b\,x}}\, x^2 \left(\frac{a}{2} + \frac{c\,\ln(f)}{2}\right) - \frac{a^2\, f^{\frac{c}{a+b\,x}}\,(a-c\,\ln(f))}{2\,b^2} - \frac{f^{\frac{c}{a+b\,x}}\, x\,(a^2-2\,a\,c\,\ln(f))}{2\,b}}{a+b\,x}$$
$$- \frac{\mathrm{ei}\left(\frac{c\,\ln(f)}{a+b\,x}\right)\,\left(c^2\ln\left(f\right)^2 - 2\,a\,c\,\ln\left(f\right)\right)}{2\,b^2}$$

input
```
int(f^(c/(a + b*x))*x,x)
```

output
```
((b*f^(c/(a + b*x))*x^3)/2 + f^(c/(a + b*x))*x^2*(a/2 + (c*log(f))/2) - (a
^2*f^(c/(a + b*x))*(a - c*log(f)))/(2*b^2) - (f^(c/(a + b*x))*x*(a^2 - 2*a
*c*log(f)))/(2*b))/(a + b*x) - (ei((c*log(f))/(a + b*x))*(c^2*log(f)^2 - 2
*a*c*log(f)))/(2*b^2)
```

Reduce [F]

$$\int f^{\frac{c}{a+bx}} x \, dx$$
$$= \frac{f^{\frac{c}{bx+a}} \log(f)^2 b^2c^2x^2 + 2f^{\frac{c}{bx+a}} \log(f)\, a^2bcx + f^{\frac{c}{bx+a}} \log(f)\, a\, b^2c\, x^2 + f^{\frac{c}{bx+a}} \log(f)\, b^3c\, x^3 + 2f^{\frac{c}{bx+a}} a^4 + 2f^{\frac{c}{bx+}}}{}$$

input
```
int(f^(c/(b*x+a))*x,x)
```

output

```
(f**(c/(a + b*x))*log(f)**2*b**2*c**2*x**2 + 2*f**(c/(a + b*x))*log(f)*a**
2*b*c*x + f**(c/(a + b*x))*log(f)*a*b**2*c*x**2 + f**(c/(a + b*x))*log(f)*
b**3*c*x**3 + 2*f**(c/(a + b*x))*a**4 + 2*f**(c/(a + b*x))*a**3*b*x + int(
(f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
*log(f)**3*a*b**3*c**3 + int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x +
3*a*b**2*x**2 + b**3*x**3),x)*log(f)**3*b**4*c**3*x - 2*int((f**(c/(a + b*
x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**2*a**
2*b**3*c**2 - 2*int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2*
x**2 + b**3*x**3),x)*log(f)**2*a*b**4*c**2*x)/(2*log(f)*b**2*c*(a + b*x))
```

3.153 $\int f^{\frac{c}{a+bx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 41

$$\int f^{\frac{c}{a+bx}}\,dx=\frac{f^{\frac{c}{a+bx}}(a+bx)}{b}-\frac{c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)}{b}$$

output
```
f^(c/(b*x+a))*(b*x+a)/b-c*Ei(c*ln(f)/(b*x+a))*ln(f)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int f^{\frac{c}{a+bx}}\,dx=\frac{f^{\frac{c}{a+bx}}(a+bx)}{b}-\frac{c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)\log(f)}{b}$$

input
```
Integrate[f^(c/(a + b*x)),x]
```

output
```
(f^(c/(a + b*x))*(a + b*x))/b - (c*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log
[f])/b
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{\frac{c}{a+bx}}\,dx$$

$$\Big\downarrow \text{2635}$$

$$c\log(f)\int \frac{f^{\frac{c}{a+bx}}}{a+bx}dx+\frac{(a+bx)f^{\frac{c}{a+bx}}}{b}$$

$$\Big\downarrow \text{2639}$$

$$\frac{(a+bx)f^{\frac{c}{a+bx}}}{b}-\frac{c\log(f)\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)}{b}$$

input
```
Int[f^(c/(a + b*x)),x]
```

output
```
(f^(c/(a + b*x))*(a + b*x))/b - (c*ExpIntegralEi[(c*Log[f])/(a + b*x)]*Log
[f])/b
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

method	result	size
risch	$f^{\frac{c}{bx+a}}x+\frac{f^{\frac{c}{bx+a}}a}{b}+\frac{c\ln\left(f\right)\mathrm{expIntegral}_{1}\left(-\frac{c\ln\left(f\right)}{bx+a}\right)}{b}$	52

input
```
int(f^(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
f^(c/(b*x+a))*x+1/b*f^(c/(b*x+a))*a+c/b*ln(f)*Ei(1,-c*ln(f)/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int f^{\frac{c}{a+bx}}\,dx=-\frac{c\mathrm{Ei}\left(\frac{c\log(f)}{bx+a}\right)\log\left(f\right)-(bx+a)f^{\frac{c}{bx+a}}}{b}$$

input
```
integrate(f^(c/(b*x+a)),x, algorithm="fricas")
```

output
```
-(c*Ei(c*log(f)/(b*x + a))*log(f) - (b*x + a)*f^(c/(b*x + a)))/b
```

Sympy [F]

$$\int f^{\frac{c}{a+bx}}\,dx=\int f^{\frac{c}{a+bx}}\,dx$$

input
```
integrate(f**(c/(b*x+a)),x)
```

output
```
Integral(f**(c/(a + b*x)), x)
```

Maxima [F]

$$\int f^{\frac{c}{a+bx}}\,dx = \int f^{\left(\frac{c}{bx+a}\right)}\,dx$$

input
```
integrate(f^(c/(b*x+a)),x, algorithm="maxima")
```

output
```
b*c*integrate(f^(c/(b*x + a))*x/(b^2*x^2 + 2*a*b*x + a^2), x)*log(f) + f^(
c/(b*x + a))*x
```

Giac [F]

$$\int f^{\frac{c}{a+bx}}\,dx = \int f^{\left(\frac{c}{bx+a}\right)}\,dx$$

input
```
integrate(f^(c/(b*x+a)),x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.22

$$\int f^{\frac{c}{a+bx}}\,dx = f^{\frac{c}{a+b\,x}}\,x + \frac{a\,f^{\frac{c}{a+b\,x}}}{b} - \frac{c\,\mathrm{ei}\left(\frac{c\,\ln(f)}{a+b\,x}\right)\,\ln(f)}{b}$$

input
```
int(f^(c/(a + b*x)),x)
```

output
```
f^(c/(a + b*x))*x + (a*f^(c/(a + b*x)))/b - (c*ei((c*log(f))/(a + b*x))*lo
g(f))/b
```

Reduce [F]

$$\int f^{\frac{c}{a+bx}}\,dx$$

$$= \frac{f^{\frac{c}{bx+a}}\log(f)\,b^2c\,x^2 - f^{\frac{c}{bx+a}}a^3 - f^{\frac{c}{bx+a}}a^2bx + \left(\int \frac{f^{\frac{c}{bx+a}}x^2}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)\log(f)^2\,a\,b^3c^2 + \left(\int \frac{f^{\frac{c}{bx+a}}}{b^3x^3+3a\,b^2x^2+}\right.}{\log\left(f\right)bc\left(bx+a\right)}$$

input
```
int(f^(c/(b*x+a)),x)
```

output
```
(f**(c/(a + b*x))*log(f)*b**2*c*x**2 - f**(c/(a + b*x))*a**3 - f**(c/(a +
b*x))*a**2*b*x + int((f**(c/(a + b*x))*x**2)/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3),x)*log(f)**2*a*b**3*c**2 + int((f**(c/(a + b*x))*x**2)/
(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*log(f)**2*b**4*c**2*x)/
(log(f)*b*c*(a + b*x))
```

3.154 $\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx = -\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right) + f^{\frac{c}{a}} \text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right)$$

output
```
-Ei(c*ln(f)/(b*x+a))+f^(c/a)*Ei(-b*c*x*ln(f)/a/(b*x+a))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx = -\text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right) + f^{\frac{c}{a}} \text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a^2+abx}\right)$$

input
```
Integrate[f^(c/(a + b*x))/x,x]
```

output
```
-ExpIntegralEi[(c*Log[f])/(a + b*x)] + f^(c/a)*ExpIntegralEi[-((b*c*x*Log[
f])/(a^2 + a*b*x))]
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2652, 2639, 2658, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx$$

$$\Big\downarrow \text{2652}$$

$$b \int \frac{f^{\frac{c}{a+bx}}}{a+bx}dx + a \int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)}dx$$

$$\Big\downarrow \text{2639}$$

$$a \int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)}dx - \text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)$$

$$\Big\downarrow \text{2658}$$

$$\int \frac{f^{\frac{c}{a}-\frac{bcx}{a(a+bx)}}(a+bx)}{x}d\frac{x}{a+bx} - \text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)$$

$$\Big\downarrow \text{2609}$$

$$f^{\frac{c}{a}} \text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right) - \text{ExpIntegralEi}\left(\frac{c\log(f)}{a+bx}\right)$$

input
```
Int[f^(c/(a + b*x))/x,x]
```

output
```
-ExpIntegralEi[(c*Log[f])/(a + b*x)] + f^(c/a)*ExpIntegralEi[-((b*c*x*Log[
f])/(a*(a + b*x)))]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2652
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/((e_.) + (f_.)*(x_)), x_Symbo
l] :> Simp[d/f   Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Simp[(d*e - c*
f)/f   Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a
, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

rule 2658
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.)
+ (h_.)*(x_))), x_Symbol] :> Simp[-d/(f*(d*g - c*h))   Subst[Int[F^(a - b*(
h/(d*g - c*h)) + d*b*(x/(d*g - c*h)))/x, x], x, (g + h*x)/(c + d*x)], x] /;
 FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.15

method	result	size
risch	$-f^{\frac{c}{a}} \operatorname{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}+\frac{c\ln(f)}{a}\right)+\operatorname{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}\right)$	47

input
```
int(f^(c/(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
-f^(1/a*c)*Ei(1,-c*ln(f)/(b*x+a)+c*ln(f)/a)+Ei(1,-c*ln(f)/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx = f^{\frac{c}{a}} \mathrm{Ei}\left(-\frac{bcx \log(f)}{abx + a^2}\right) - \mathrm{Ei}\left(\frac{c \log(f)}{bx + a}\right)$$

input
```
integrate(f^(c/(b*x+a))/x,x, algorithm="fricas")
```

output
```
f^(c/a)*Ei(-b*c*x*log(f)/(a*b*x + a^2)) - Ei(c*log(f)/(b*x + a))
```

Sympy [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx = \int \frac{f^{\frac{c}{a+bx}}}{x} \, dx$$

input
```
integrate(f**(c/(b*x+a))/x,x)
```

output
```
Integral(f**(c/(a + b*x))/x, x)
```

Maxima [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x} \, dx = \int \frac{f^{\frac{c}{bx+a}}}{x} \, dx$$

input
```
integrate(f^(c/(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a))/x, x)
```

Giac [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x}\,dx = \int \frac{f^{\frac{c}{bx+a}}}{x}\,dx$$

input
```
integrate(f^(c/(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{\frac{c}{a+bx}}}{x}\,dx = \int \frac{f^{\frac{c}{a+b\,x}}}{x}\,dx$$

input
```
int(f^(c/(a + b*x))/x,x)
```

output
```
int(f^(c/(a + b*x))/x, x)
```

Reduce [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x}\,dx = \int \frac{f^{\frac{c}{bx+a}}}{x}dx$$

input
```
int(f^(c/(b*x+a))/x,x)
```

output
```
int(f**(c/(a + b*x))/x,x)
```

3.155 $\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx=-\frac{bf^{\frac{c}{a+bx}}}{a}-\frac{f^{\frac{c}{a+bx}}}{x}-\frac{bcf^{\frac{c}{a}}\operatorname{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right)\log(f)}{a^2}$$

output
```
-b*f^(c/(b*x+a))/a-f^(c/(b*x+a))/x-b*c*f^(c/a)*Ei(-b*c*x*ln(f)/a/(b*x+a))*
ln(f)/a^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx=-\frac{bf^{\frac{c}{a+bx}}}{a}-\frac{f^{\frac{c}{a+bx}}}{x}-\frac{bcf^{\frac{c}{a}}\operatorname{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a^2+abx}\right)\log(f)}{a^2}$$

input
```
Integrate[f^(c/(a + b*x))/x^2,x]
```

output
```
-((b*f^(c/(a + b*x)))/a) - f^(c/(a + b*x))/x - (b*c*f^(c/a)*ExpIntegralEi[
-((b*c*x*Log[f])/(a^2 + a*b*x))]*Log[f])/a^2
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2} \, dx$$

$$\Big\downarrow \text{2653}$$

$$-bc\log(f)\int \frac{f^{\frac{c}{a+bx}}}{x(a+bx)^2}dx-\frac{f^{\frac{c}{a+bx}}}{x}$$

$$\Big\downarrow \text{7293}$$

$$-bc\log(f)\int \left(\frac{f^{\frac{c}{a+bx}}}{a^2x}-\frac{bf^{\frac{c}{a+bx}}}{a^2(a+bx)}-\frac{bf^{\frac{c}{a+bx}}}{a(a+bx)^2}\right)dx-\frac{f^{\frac{c}{a+bx}}}{x}$$

$$\Big\downarrow \text{2009}$$

$$-bc\log(f)\left(\frac{f^{\frac{c}{a}}\,\text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right)}{a^2}+\frac{f^{\frac{c}{a+bx}}}{ac\log(f)}\right)-\frac{f^{\frac{c}{a+bx}}}{x}$$

input
```
Int[f^(c/(a + b*x))/x^2,x]
```

output
```
-(f^(c/(a + b*x))/x) - b*c*((f^(c/a)*ExpIntegralEi[-((b*c*x*Log[f])/(a*(a
+ b*x)))])/a^2 + f^(c/(a + b*x))/(a*c*Log[f]))*Log[f]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{\ln(f) bc\, f^{\frac{c}{bx+a}}}{a^2\left(\frac{c\ln(f)}{bx+a}-\frac{c\ln(f)}{a}\right)}+\frac{\ln(f) bc\, f^{\frac{c}{a}}\, \mathrm{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}+\frac{c\ln(f)}{a}\right)}{a^2}$	80

input
```
int(f^(c/(b*x+a))/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2*ln(f)*b*c*f^(c/(b*x+a))/(c*ln(f)/(b*x+a)-c*ln(f)/a)+1/a^2*ln(f)*b*c*
f^(1/a*c)*Ei(1,-c*ln(f)/(b*x+a)+c*ln(f)/a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\, dx = -\frac{bcf^{\frac{c}{a}} x \mathrm{Ei}\left(-\frac{bcx\log(f)}{abx+a^2}\right)\log(f) + (abx+a^2)f^{\frac{c}{bx+a}}}{a^2 x}$$

input
```
integrate(f^(c/(b*x+a))/x^2,x, algorithm="fricas")
```

output
```
-(b*c*f^(c/a)*x*Ei(-b*c*x*log(f)/(a*b*x + a^2))*log(f) + (a*b*x + a^2)*f^(
c/(b*x + a)))/(a^2*x)
```

Sympy [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx = \int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx$$

input
```
integrate(f**(c/(b*x+a))/x**2,x)
```

output
```
Integral(f**(c/(a + b*x))/x**2, x)
```

Maxima [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx = \int \frac{f^{\frac{c}{bx+a}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a))/x^2,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a))/x^2, x)
```

Giac [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx = \int \frac{f^{\frac{c}{bx+a}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a))/x^2,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx = \int \frac{f^{\frac{c}{a+b\,x}}}{x^2}\,dx$$

input
```
int(f^(c/(a + b*x))/x^2,x)
```

output
```
int(f^(c/(a + b*x))/x^2, x)
```

Reduce [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^2}\,dx = \text{Too large to display}$$

input
```
int(f^(c/(b*x+a))/x^2,x)
```

output
```
( - 4*f**(c/(a + b*x))*log(f)**2*a**3*c**2 - 6*f**(c/(a + b*x))*log(f)**2*
a**2*b*c**2*x - 4*f**(c/(a + b*x))*log(f)**2*a*b**2*c**2*x**2 - f**(c/(a +
 b*x))*log(f)**2*b**3*c**2*x**3 - 4*f**(c/(a + b*x))*log(f)*a**3*b*c*x - 6
*f**(c/(a + b*x))*log(f)*a**2*b**2*c*x**2 - 2*f**(c/(a + b*x))*log(f)*a*b*
*3*c*x**3 + 2*f**(c/(a + b*x))*a**4*b*x + 4*f**(c/(a + b*x))*a**3*b**2*x**
2 + 2*f**(c/(a + b*x))*a**2*b**3*x**3 + int(f**(c/(a + b*x))/(log(f)*a**4*
c*x**2 + 4*log(f)*a**3*b*c*x**3 + 6*log(f)*a**2*b**2*c*x**4 + 4*log(f)*a*b
**3*c*x**5 + log(f)*b**4*c*x**6 + 4*a**5*x**2 + 16*a**4*b*x**3 + 24*a**3*b
**2*x**4 + 16*a**2*b**3*x**5 + 4*a*b**4*x**6),x)*log(f)**4*a**6*c**4*x + 2
*int(f**(c/(a + b*x))/(log(f)*a**4*c*x**2 + 4*log(f)*a**3*b*c*x**3 + 6*log
(f)*a**2*b**2*c*x**4 + 4*log(f)*a*b**3*c*x**5 + log(f)*b**4*c*x**6 + 4*a**
5*x**2 + 16*a**4*b*x**3 + 24*a**3*b**2*x**4 + 16*a**2*b**3*x**5 + 4*a*b**4
*x**6),x)*log(f)**4*a**5*b*c**4*x**2 + int(f**(c/(a + b*x))/(log(f)*a**4*c
*x**2 + 4*log(f)*a**3*b*c*x**3 + 6*log(f)*a**2*b**2*c*x**4 + 4*log(f)*a*b*
*3*c*x**5 + log(f)*b**4*c*x**6 + 4*a**5*x**2 + 16*a**4*b*x**3 + 24*a**3*b*
*2*x**4 + 16*a**2*b**3*x**5 + 4*a*b**4*x**6),x)*log(f)**4*a**4*b**2*c**4*x
**3 + 4*int(f**(c/(a + b*x))/(log(f)*a**4*c*x**2 + 4*log(f)*a**3*b*c*x**3
+ 6*log(f)*a**2*b**2*c*x**4 + 4*log(f)*a*b**3*c*x**5 + log(f)*b**4*c*x**6
+ 4*a**5*x**2 + 16*a**4*b*x**3 + 24*a**3*b**2*x**4 + 16*a**2*b**3*x**5 + 4
*a*b**4*x**6),x)*log(f)**3*a**7*c**3*x + 8*int(f**(c/(a + b*x))/(log(f)...
```

3.156 $\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 166

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \frac{b^2 f^{\frac{c}{a+bx}}}{2a^2} - \frac{f^{\frac{c}{a+bx}}}{2x^2} + \frac{b^2 c f^{\frac{c}{a+bx}} \log(f)}{2a^3} + \frac{bcf^{\frac{c}{a+bx}} \log(f)}{2a^2 x} + \frac{b^2 c f^{\frac{c}{a}} \text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right) \log(f)}{a^3} + \frac{b^2 c^2 f^{\frac{c}{a}} \text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right) \log^2(f)}{2a^4}$$

output
```
1/2*b^2*f^(c/(b*x+a))/a^2-1/2*f^(c/(b*x+a))/x^2+1/2*b^2*c*f^(c/(b*x+a))*ln
(f)/a^3+1/2*b*c*f^(c/(b*x+a))*ln(f)/a^2/x+b^2*c*f^(c/a)*Ei(-b*c*x*ln(f)/a/
(b*x+a))*ln(f)/a^3+1/2*b^2*c^2*f^(c/a)*Ei(-b*c*x*ln(f)/a/(b*x+a))*ln(f)^2/
a^4
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx$$
$$= \frac{b^2 f^{\frac{c}{a+bx}}(2a + c\log(f))}{2a^3}$$
$$+ \frac{b^2 c f^{\frac{c}{a}}\,\text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a^2+abx}\right)\log(f)(2a + c\log(f)) - \frac{a^2 f^{\frac{c}{a+bx}}(a^2+b^2x^2-bcx\log(f))}{x^2}}{2a^4}$$

input
```
Integrate[f^(c/(a + b*x))/x^3,x]
```

output
```
(b^2*f^(c/(a + b*x))*(2*a + c*Log[f]))/(2*a^3) + (b^2*c*f^(c/a)*ExpIntegra
lEi[-((b*c*x*Log[f])/(a^2 + a*b*x))]*Log[f]*(2*a + c*Log[f]) - (a^2*f^(c/(
a + b*x))*(a^2 + b^2*x^2 - b*c*x*Log[f]))/x^2)/(2*a^4)
```

Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx$$
$$\Big\downarrow \; 2653$$
$$-\frac{1}{2}bc\log(f)\int \frac{f^{\frac{c}{a+bx}}}{x^2(a+bx)^2}dx - \frac{f^{\frac{c}{a+bx}}}{2x^2}$$
$$\Big\downarrow \; 7293$$
$$-\frac{1}{2}bc\log(f)\int \left(-\frac{2bf^{\frac{c}{a+bx}}}{a^3x} + \frac{2b^2f^{\frac{c}{a+bx}}}{a^3(a+bx)} + \frac{f^{\frac{c}{a+bx}}}{a^2x^2} + \frac{b^2f^{\frac{c}{a+bx}}}{a^2(a+bx)^2}\right)dx - \frac{f^{\frac{c}{a+bx}}}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}bc\log(f)\left(-\frac{bc\log(f)f^{\frac{c}{a}}\,\text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right)}{a^4}-\frac{2bf^{\frac{c}{a}}\,\text{ExpIntegralEi}\left(-\frac{bcx\log(f)}{a(a+bx)}\right)}{a^3}-\frac{bf^{\frac{c}{a+bx}}}{a^3}-\frac{f^{\frac{c}{a+bx}}}{a^2x}\right.$$

$$\frac{f^{\frac{c}{a+bx}}}{2x^2}$$

input
```
Int[f^(c/(a + b*x))/x^3,x]
```

output
```
-1/2*f^(c/(a + b*x))/x^2 - (b*c*Log[f]*(-((b*f^(c/(a + b*x)))/a^3) - f^(c/
(a + b*x))/(a^2*x) - (2*b*f^(c/a)*ExpIntegralEi[-((b*c*x*Log[f])/(a*(a + b
*x)))])/a^3 - (b*f^(c/(a + b*x)))/(a^2*c*Log[f]) - (b*c*f^(c/a)*ExpIntegra
lEi[-((b*c*x*Log[f])/(a*(a + b*x)))]*Log[f])/a^4))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.05

method	result
risch	$\frac{-f^{\frac{c}{a}} \operatorname{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}+\frac{c\ln(f)}{a}\right)c^2b^2\ln(f)^2x^2-2\ln(f)f^{\frac{c}{a}}\operatorname{expIntegral}_1\left(-\frac{c\ln(f)}{bx+a}+\frac{c\ln(f)}{a}\right)a\,b^2c\,x^2+f^{\frac{c}{bx+a}}\ln(f)a\,b^2c\,x^2+f^{\frac{c}{b\ldots}}}{2a^4x^2}$

input
```
int(f^(c/(b*x+a))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-f^(1/a*c)*Ei(1,-c*ln(f)/(b*x+a)+c*ln(f)/a)*c^2*b^2*ln(f)^2*x^2-2*ln(
f)*f^(1/a*c)*Ei(1,-c*ln(f)/(b*x+a)+c*ln(f)/a)*a*b^2*c*x^2+f^(c/(b*x+a))*ln
(f)*a*b^2*c*x^2+f^(c/(b*x+a))*ln(f)*a^2*b*c*x+f^(c/(b*x+a))*a^2*b^2*x^2-f^
(c/(b*x+a))*a^4)/a^4/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.66

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \frac{\left(b^2c^2x^2\log(f)^2+2\,ab^2cx^2\log(f)\right)f^{\frac{c}{a}}\operatorname{Ei}\left(-\frac{bcx\log(f)}{abx+a^2}\right)+\left(a^2b^2x^2-a^4+\left(ab^2cx^2+a^2bcx\right)\log(f)\right)f^{\frac{c}{bx+a}}}{2\,a^4x^2}$$

input
```
integrate(f^(c/(b*x+a))/x^3,x, algorithm="fricas")
```

output
```
1/2*((b^2*c^2*x^2*log(f)^2 + 2*a*b^2*c*x^2*log(f))*f^(c/a)*Ei(-b*c*x*log(f
)/(a*b*x + a^2)) + (a^2*b^2*x^2 - a^4 + (a*b^2*c*x^2 + a^2*b*c*x)*log(f))*
f^(c/(b*x + a)))/(a^4*x^2)
```

Sympy [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx$$

input
```
integrate(f**(c/(b*x+a))/x**3,x)
```

output
```
Integral(f**(c/(a + b*x))/x**3, x)
```

Maxima [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \int \frac{f^{\frac{c}{bx+a}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a))/x^3,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a))/x^3, x)
```

Giac [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \int \frac{f^{\frac{c}{bx+a}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a))/x^3,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a))/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \int \frac{f^{\frac{c}{a+b\,x}}}{x^3}\,dx$$

input
```
int(f^(c/(a + b*x))/x^3,x)
```

output
```
int(f^(c/(a + b*x))/x^3, x)
```

Reduce [F]

$$\int \frac{f^{\frac{c}{a+bx}}}{x^3}\,dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a))/x^3,x)
```

output

```
( - 4*f**(c/(a + b*x))*log(f)**3*a**3*c**3 - 6*f**(c/(a + b*x))*log(f)**3*
a**2*b*c**3*x - 4*f**(c/(a + b*x))*log(f)**3*a*b**2*c**3*x**2 - f**(c/(a +
 b*x))*log(f)**3*b**3*c**3*x**3 - 10*f**(c/(a + b*x))*log(f)**2*a**4*c**2
- 20*f**(c/(a + b*x))*log(f)**2*a**3*b*c**2*x - 15*f**(c/(a + b*x))*log(f)
**2*a**2*b**2*c**2*x**2 - 2*f**(c/(a + b*x))*log(f)**2*a*b**3*c**2*x**3 +
f**(c/(a + b*x))*log(f)**2*b**4*c**2*x**4 + 8*f**(c/(a + b*x))*log(f)*a**3
*b**2*c*x**2 + 12*f**(c/(a + b*x))*log(f)*a**2*b**3*c*x**3 + 4*f**(c/(a +
b*x))*log(f)*a*b**4*c*x**4 - 4*f**(c/(a + b*x))*a**4*b**2*x**2 - 8*f**(c/(
a + b*x))*a**3*b**3*x**3 - 4*f**(c/(a + b*x))*a**2*b**4*x**4 + int(f**(c/(
a + b*x))/(log(f)**2*a**4*c**2*x**3 + 4*log(f)**2*a**3*b*c**2*x**4 + 6*log
(f)**2*a**2*b**2*c**2*x**5 + 4*log(f)**2*a*b**3*c**2*x**6 + log(f)**2*b**4
*c**2*x**7 + 10*log(f)*a**5*c*x**3 + 40*log(f)*a**4*b*c*x**4 + 60*log(f)*a
**3*b**2*c*x**5 + 40*log(f)*a**2*b**3*c*x**6 + 10*log(f)*a*b**4*c*x**7 + 2
0*a**6*x**3 + 80*a**5*b*x**4 + 120*a**4*b**2*x**5 + 80*a**3*b**3*x**6 + 20
*a**2*b**4*x**7),x)*log(f)**6*a**6*c**6*x**2 + 2*int(f**(c/(a + b*x))/(log
(f)**2*a**4*c**2*x**3 + 4*log(f)**2*a**3*b*c**2*x**4 + 6*log(f)**2*a**2*b*
*2*c**2*x**5 + 4*log(f)**2*a*b**3*c**2*x**6 + log(f)**2*b**4*c**2*x**7 + 1
0*log(f)*a**5*c*x**3 + 40*log(f)*a**4*b*c*x**4 + 60*log(f)*a**3*b**2*c*x**
5 + 40*log(f)*a**2*b**3*c*x**6 + 10*log(f)*a*b**4*c*x**7 + 20*a**6*x**3 +
80*a**5*b*x**4 + 120*a**4*b**2*x**5 + 80*a**3*b**3*x**6 + 20*a**2*b**4*...
```

3.157 $\int f^{\frac{c}{(a+bx)^2}} x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 415

$$\begin{aligned}\int f^{\frac{c}{(a+bx)^2}} x^4\,dx &= \frac{a^4 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^5} - \frac{2a^3 f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^5} \\ &+ \frac{2a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{b^5} - \frac{a f^{\frac{c}{(a+bx)^2}}(a+bx)^4}{b^5} \\ &+ \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^5}{5b^5} - \frac{a^4\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b^5} \\ &+ \frac{4a^2 c f^{\frac{c}{(a+bx)^2}}(a+bx)\log(f)}{b^5} - \frac{acf^{\frac{c}{(a+bx)^2}}(a+bx)^2\log(f)}{b^5} \\ &+ \frac{2cf^{\frac{c}{(a+bx)^2}}(a+bx)^3\log(f)}{15b^5} + \frac{2a^3c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{b^5} \\ &- \frac{4a^2c^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\log^{\frac{3}{2}}(f)}{b^5} + \frac{4c^2 f^{\frac{c}{(a+bx)^2}}(a+bx)\log^2(f)}{15b^5} \\ &+ \frac{ac^2\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log^2(f)}{b^5} \\ &- \frac{4c^{5/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\log^{\frac{5}{2}}(f)}{15b^5}\end{aligned}$$

output
```
a^4*f^(c/(b*x+a)^2)*(b*x+a)/b^5-2*a^3*f^(c/(b*x+a)^2)*(b*x+a)^2/b^5+2*a^2*
f^(c/(b*x+a)^2)*(b*x+a)^3/b^5-a*f^(c/(b*x+a)^2)*(b*x+a)^4/b^5+1/5*f^(c/(b*
x+a)^2)*(b*x+a)^5/b^5-a^4*c^(1/2)*Pi^(1/2)*erfi(c^(1/2)*ln(f)^(1/2)/(b*x+a
))*ln(f)^(1/2)/b^5+4*a^2*c*f^(c/(b*x+a)^2)*(b*x+a)*ln(f)/b^5-a*c*f^(c/(b*x
+a)^2)*(b*x+a)^2*ln(f)/b^5+2/15*c*f^(c/(b*x+a)^2)*(b*x+a)^3*ln(f)/b^5+2*a^
3*c*Ei(c*ln(f)/(b*x+a)^2)*ln(f)/b^5-4*a^2*c^(3/2)*Pi^(1/2)*erfi(c^(1/2)*ln
(f)^(1/2)/(b*x+a))*ln(f)^(3/2)/b^5+4/15*c^2*f^(c/(b*x+a)^2)*(b*x+a)*ln(f)^
2/b^5+a*c^2*Ei(c*ln(f)/(b*x+a)^2)*ln(f)^2/b^5-4/15*c^(5/2)*Pi^(1/2)*erfi(c
^(1/2)*ln(f)^(1/2)/(b*x+a))*ln(f)^(5/2)/b^5
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.47

$$\int f^{\frac{c}{(a+bx)^2}} x^4 \, dx = \frac{a f^{\frac{c}{(a+bx)^2}} \left(3a^4 + 47a^2 c \log(f) + 4c^2 \log^2(f)\right)}{15b^5} + \frac{15ac \operatorname{ExpIntegralEi}\left(\frac{c \log(f)}{(a+bx)^2}\right) \log(f) \left(2a^2 + c \log(f)\right) - \sqrt{c}\sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right) \sqrt{\log(f)} \left(15a^4 + 60a^2 c \log\right.}{15b^5}$$

input
```
Integrate[f^(c/(a + b*x)^2)*x^4,x]
```

output
```
(a*f^(c/(a + b*x)^2)*(3*a^4 + 47*a^2*c*Log[f] + 4*c^2*Log[f]^2))/(15*b^5)
+ (15*a*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f]*(2*a^2 + c*Log[f])
- Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]]*(15
*a^4 + 60*a^2*c*Log[f] + 4*c^2*Log[f]^2) + b*f^(c/(a + b*x)^2)*x*(3*b^4*x^
4 + c*(36*a^2 - 9*a*b*x + 2*b^2*x^2)*Log[f] + 4*c^2*Log[f]^2))/(15*b^5)
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{\frac{c}{(a+bx)^2}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^4 f^{\frac{c}{(a+bx)^2}}}{b^4} - \frac{4a^3(a+bx) f^{\frac{c}{(a+bx)^2}}}{b^4} + \frac{6a^2(a+bx)^2 f^{\frac{c}{(a+bx)^2}}}{b^4} + \frac{(a+bx)^4 f^{\frac{c}{(a+bx)^2}}}{b^4} - \frac{4a(a+bx)^3 f^{\frac{c}{(a+bx)^2}}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}a^4\sqrt{c}\sqrt{\log(f)}\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^5} + \frac{a^4(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^5} + \frac{2a^3c\log(f)\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^5} - \frac{2a^3(a+bx)^2 f^{\frac{c}{(a+bx)^2}}}{b^5} - \frac{4\sqrt{\pi}a^2c^{3/2}\log^{\frac{3}{2}}(f)\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^5} + \frac{2a^2(a+bx)^3 f^{\frac{c}{(a+bx)^2}}}{b^5} + \frac{4a^2c\log(f)(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^5} - \frac{4\sqrt{\pi}c^{5/2}\log^{\frac{5}{2}}(f)\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{15b^5} + \frac{ac^2\log^2(f)\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^5} + \frac{4c^2\log^2(f)(a+bx)f^{\frac{c}{(a+bx)^2}}}{15b^5} + \frac{(a+bx)^5 f^{\frac{c}{(a+bx)^2}}}{5b^5} - \frac{a(a+bx)^4 f^{\frac{c}{(a+bx)^2}}}{b^5} + \frac{2c\log(f)(a+bx)^3 f^{\frac{c}{(a+bx)^2}}}{15b^5} - \frac{ac\log(f)(a+bx)^2 f^{\frac{c}{(a+bx)^2}}}{b^5}$$

input
```
Int[f^(c/(a + b*x)^2)*x^4,x]
```

output
```
(a^4*f^(c/(a + b*x)^2)*(a + b*x))/b^5 - (2*a^3*f^(c/(a + b*x)^2)*(a + b*x)
^2)/b^5 + (2*a^2*f^(c/(a + b*x)^2)*(a + b*x)^3)/b^5 - (a*f^(c/(a + b*x)^2)
*(a + b*x)^4)/b^5 + (f^(c/(a + b*x)^2)*(a + b*x)^5)/(5*b^5) - (a^4*Sqrt[c]
*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]])/b^5 + (4*a^
2*c*f^(c/(a + b*x)^2)*(a + b*x)*Log[f])/b^5 - (a*c*f^(c/(a + b*x)^2)*(a +
b*x)^2*Log[f])/b^5 + (2*c*f^(c/(a + b*x)^2)*(a + b*x)^3*Log[f])/(15*b^5) +
 (2*a^3*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f])/b^5 - (4*a^2*c^(3/
2)*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Log[f]^(3/2))/b^5 + (4*
c^2*f^(c/(a + b*x)^2)*(a + b*x)*Log[f]^2)/(15*b^5) + (a*c^2*ExpIntegralEi[
(c*Log[f])/(a + b*x)^2]*Log[f]^2)/b^5 - (4*c^(5/2)*Sqrt[Pi]*Erfi[(Sqrt[c]*
Sqrt[Log[f]])/(a + b*x)]*Log[f]^(5/2))/(15*b^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.83

method	result
risch	$\frac{f^{\frac{c}{(bx+a)^2}} x^5}{5} + \frac{2 f^{\frac{c}{(bx+a)^2}} \ln(f) c x^3}{15 b^2} - \frac{3 f^{\frac{c}{(bx+a)^2}} \ln(f) a c x^2}{5 b^3} + \frac{4 f^{\frac{c}{(bx+a)^2}} \ln(f)^2 c^2 x}{15 b^4} + \frac{12 f^{\frac{c}{(bx+a)^2}} \ln(f) a^2 c x}{5 b^4} - \frac{4 \ln(f)^3 \sqrt{}}{1}$

input
```
int(f^(c/(b*x+a)^2)*x^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*f^(c/(b*x+a)^2)*x^5+2/15/b^2*f^(c/(b*x+a)^2)*ln(f)*c*x^3-3/5/b^3*f^(c/
(b*x+a)^2)*ln(f)*a*c*x^2+4/15/b^4*f^(c/(b*x+a)^2)*ln(f)^2*c^2*x+12/5/b^4*f
^(c/(b*x+a)^2)*ln(f)*a^2*c*x-4/15/b^5/(-c*ln(f))^(1/2)*ln(f)^3*Pi^(1/2)*er
f((-c*ln(f))^(1/2)/(b*x+a))*c^3-4/b^5/(-c*ln(f))^(1/2)*ln(f)^2*Pi^(1/2)*er
f((-c*ln(f))^(1/2)/(b*x+a))*a^2*c^2-1/b^5/(-c*ln(f))^(1/2)*ln(f)*Pi^(1/2)*
erf((-c*ln(f))^(1/2)/(b*x+a))*a^4*c+4/15/b^5*f^(c/(b*x+a)^2)*ln(f)^2*a*c^2
+47/15/b^5*f^(c/(b*x+a)^2)*ln(f)*a^3*c+1/5/b^5*f^(c/(b*x+a)^2)*a^5-1/b^5*l
n(f)^2*Ei(1,-c*ln(f)/(b*x+a)^2)*a*c^2-2/b^5*ln(f)*Ei(1,-c*ln(f)/(b*x+a)^2)
*a^3*c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.48

$$\int f^{\frac{c}{(a+bx)^2}}x^4\,dx$$

$$= \frac{\sqrt{\pi}\left(15\,a^4b + 60\,a^2bc\log\left(f\right) + 4\,bc^2\log\left(f\right)^2\right)\sqrt{-\frac{c\log(f)}{b^2}}\operatorname{erf}\left(\frac{b\sqrt{-\frac{c\log(f)}{b^2}}}{bx+a}\right) + \left(3\,b^5x^5 + 3\,a^5 + 4\left(bc^2x + ac^2\right.\right.}{}$$

input
```
integrate(f^(c/(b*x+a)^2)*x^4,x, algorithm="fricas")
```

output
```
1/15*(sqrt(pi)*(15*a^4*b + 60*a^2*b*c*log(f) + 4*b*c^2*log(f)^2)*sqrt(-c*l
og(f)/b^2)*erf(b*sqrt(-c*log(f)/b^2)/(b*x + a)) + (3*b^5*x^5 + 3*a^5 + 4*(
b*c^2*x + a*c^2)*log(f)^2 + (2*b^3*c*x^3 - 9*a*b^2*c*x^2 + 36*a^2*b*c*x +
47*a^3*c)*log(f))*f^(c/(b^2*x^2 + 2*a*b*x + a^2)) + 15*(2*a^3*c*log(f) + a
*c^2*log(f)^2)*Ei(c*log(f)/(b^2*x^2 + 2*a*b*x + a^2)))/b^5
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^4\,dx = \int f^{\frac{c}{(a+bx)^2}}x^4\,dx$$

input
```
integrate(f**(c/(b*x+a)**2)*x***4,x)
```

output
```
Integral(f**(c/(a + b*x)**2)*x***4, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^4\,dx=\int f^{\frac{c}{(bx+a)^2}}x^4\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x^4,x, algorithm="maxima")
```

output
```
1/15*(3*b^4*x^5 + 2*b^2*c*x^3*log(f) - 9*a*b*c*x^2*log(f) + 4*(9*a^2*c*log
(f) + c^2*log(f)^2)*x)*f^(c/(b^2*x^2 + 2*a*b*x + a^2))/b^4 - integrate(2/1
5*(18*a^5*c*log(f) + 2*a^3*c^2*log(f)^2 + 15*(2*a^3*b^2*c*log(f) + a*b^2*c
^2*log(f)^2)*x^2 + (45*a^4*b*c*log(f) - 30*a^2*b*c^2*log(f)^2 - 4*b*c^3*lo
g(f)^3)*x)*f^(c/(b^2*x^2 + 2*a*b*x + a^2))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*
b^5*x + a^3*b^4), x)
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^4\,dx=\int f^{\frac{c}{(bx+a)^2}}x^4\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x^4,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^2}}x^4\,dx=\int f^{\frac{c}{(a+b\,x)^2}}\,x^4\,dx$$

input
```
int(f^(c/(a + b*x)^2)*x^4,x)
```

output
```
int(f^(c/(a + b*x)^2)*x^4, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^2}} x^4 \, dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^2)*x^4,x)
```

output
```
(16*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**5*b*c**5*x + 8*f**(c/(a**2
 + 2*a*b*x + b**2*x**2))*log(f)**4*a**3*c**4 - 8*f**(c/(a**2 + 2*a*b*x + b
**2*x**2))*log(f)**4*a**2*b*c**4*x - 40*f**(c/(a**2 + 2*a*b*x + b**2*x**2)
)*log(f)**4*a*b**2*c**4*x**2 - 24*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(
f)**4*b**3*c**4*x**3 - 20*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a*
*5*c**3 - 368*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a**4*b*c**3*x
- 72*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a**3*b**2*c**3*x**2 - 8
*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a**2*b**3*c**3*x**3 + 28*f*
*(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a*b**4*c**3*x**4 + 12*f**(c/(a
**2 + 2*a*b*x + b**2*x**2))*log(f)**3*b**5*c**3*x**5 - 144*f**(c/(a**2 + 2
*a*b*x + b**2*x**2))*log(f)**2*a**7*c**2 + 432*f**(c/(a**2 + 2*a*b*x + b**
2*x**2))*log(f)**2*a**6*b*c**2*x + 876*f**(c/(a**2 + 2*a*b*x + b**2*x**2))
*log(f)**2*a**5*b**2*c**2*x**2 + 660*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*l
og(f)**2*a**4*b**3*c**2*x**3 + 165*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log
(f)**2*a**3*b**4*c**2*x**4 + 36*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)
**2*a**2*b**5*c**2*x**5 - 3*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*
a*b**6*c**2*x**6 + 6*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*b**7*c*
*2*x**7 + 504*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**9*c + 1236*f**
(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**8*b*c*x + 1074*f**(c/(a**2 + 2*
a*b*x + b**2*x**2))*log(f)*a**7*b**2*c*x**2 + 456*f**(c/(a**2 + 2*a*b*x...
```

3.158 $\int f^{\frac{c}{(a+bx)^2}} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 291

$$\begin{aligned}\int f^{\frac{c}{(a+bx)^2}} x^3\,dx = &-\frac{a^3 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^4}+\frac{3a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{2b^4}-\frac{a f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{b^4}\\&+\frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^4}{4b^4}+\frac{a^3\sqrt{c}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b^4}\\&-\frac{2acf^{\frac{c}{(a+bx)^2}}(a+bx)\log(f)}{b^4}+\frac{cf^{\frac{c}{(a+bx)^2}}(a+bx)^2\log(f)}{4b^4}\\&-\frac{3a^2c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{2b^4}\\&+\frac{2ac^{3/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\log^{\frac{3}{2}}(f)}{b^4}\\&-\frac{c^2\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log^2(f)}{4b^4}\end{aligned}$$

output
```
-a^3*f^(c/(b*x+a)^2)*(b*x+a)/b^4+3/2*a^2*f^(c/(b*x+a)^2)*(b*x+a)^2/b^4-a*f
^(c/(b*x+a)^2)*(b*x+a)^3/b^4+1/4*f^(c/(b*x+a)^2)*(b*x+a)^4/b^4+a^3*c^(1/2)
*Pi^(1/2)*erfi(c^(1/2)*ln(f)^(1/2)/(b*x+a))*ln(f)^(1/2)/b^4-2*a*c*f^(c/(b*
x+a)^2)*(b*x+a)*ln(f)/b^4+1/4*c*f^(c/(b*x+a)^2)*(b*x+a)^2*ln(f)/b^4-3/2*a^
2*c*Ei(c*ln(f)/(b*x+a)^2)*ln(f)/b^4+2*a*c^(3/2)*Pi^(1/2)*erfi(c^(1/2)*ln(f
)^(1/2)/(b*x+a))*ln(f)^(3/2)/b^4-1/4*c^2*Ei(c*ln(f)/(b*x+a)^2)*ln(f)^2/b^4
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.51

$$\int f^{\frac{c}{(a+bx)^2}}x^3\,dx = -\frac{a^2f^{\frac{c}{(a+bx)^2}}(a^2+7c\log(f))}{4b^4} + \frac{-c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)\left(6a^2+c\log(f)\right)+4a\sqrt{c}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}(a^2+2c\log(f)}{4b^4}$$

input
```
Integrate[f^(c/(a + b*x)^2)*x^3,x]
```

output
```
-1/4*(a^2*f^(c/(a + b*x)^2)*(a^2 + 7*c*Log[f]))/b^4 + (-(c*ExpIntegralEi[(
c*Log[f])/(a + b*x)^2]*Log[f]*(6*a^2 + c*Log[f])) + 4*a*Sqrt[c]*Sqrt[Pi]*E
rfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]]*(a^2 + 2*c*Log[f]) + b*
f^(c/(a + b*x)^2)*x*(b^3*x^3 - 6*a*c*Log[f] + b*c*x*Log[f]))/(4*b^4)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3f^{\frac{c}{(a+bx)^2}}\,dx$$

$$\Big\downarrow \text{ 2656}$$

$$\int \left(-\frac{a^3f^{\frac{c}{(a+bx)^2}}}{b^3}+\frac{3a^2(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^3}+\frac{(a+bx)^3f^{\frac{c}{(a+bx)^2}}}{b^3}-\frac{3a(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}a^3\sqrt{c}\sqrt{\log(f)}\mathrm{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^4}-\frac{a^3(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^4}-\frac{3a^2c\log(f)\,\mathrm{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{2b^4}+\frac{3a^2(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{2b^4}+\frac{2\sqrt{\pi}ac^{3/2}\log^{\frac{3}{2}}(f)\mathrm{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^4}-\frac{c^2\log^2(f)\,\mathrm{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{4b^4}+\frac{(a+bx)^4f^{\frac{c}{(a+bx)^2}}}{4b^4}-\frac{a(a+bx)^3f^{\frac{c}{(a+bx)^2}}}{b^4}+\frac{c\log(f)(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{4b^4}-\frac{2ac\log(f)(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^4}$$

input
```
Int[f^(c/(a + b*x)^2)*x^3,x]
```

output
```
-((a^3*f^(c/(a + b*x)^2)*(a + b*x))/b^4) + (3*a^2*f^(c/(a + b*x)^2)*(a + b
*x)^2)/(2*b^4) - (a*f^(c/(a + b*x)^2)*(a + b*x)^3)/b^4 + (f^(c/(a + b*x)^2
)*(a + b*x)^4)/(4*b^4) + (a^3*Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])
/(a + b*x)]*Sqrt[Log[f]])/b^4 - (2*a*c*f^(c/(a + b*x)^2)*(a + b*x)*Log[f])
/b^4 + (c*f^(c/(a + b*x)^2)*(a + b*x)^2*Log[f])/(4*b^4) - (3*a^2*c*ExpInte
gralEi[(c*Log[f])/(a + b*x)^2]*Log[f])/(2*b^4) + (2*a*c^(3/2)*Sqrt[Pi]*Erf
i[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Log[f]^(3/2))/b^4 - (c^2*ExpIntegralEi
[(c*Log[f])/(a + b*x)^2]*Log[f]^2)/(4*b^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.78

method	result
risch	$\frac{f^{\frac{c}{(bx+a)^2}} x^4}{4} + \frac{f^{\frac{c}{(bx+a)^2}} \ln(f) c x^2}{4b^2} - \frac{3 f^{\frac{c}{(bx+a)^2}} \ln(f) a c x}{2b^3} + \frac{2 \ln(f)^2 \sqrt{\pi}\, \mathrm{erf}\left(\frac{\sqrt{-c\ln(f)}}{bx+a}\right) a c^2}{b^4 \sqrt{-c\ln(f)}} + \frac{\ln(f)\sqrt{\pi}\, \mathrm{erf}\left(\frac{\sqrt{-c\ln(f)}}{bx+a}\right) a^3}{b^4\sqrt{-c\ln(f)}}$

input
```
int(f^(c/(b*x+a)^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*f^(c/(b*x+a)^2)*x^4+1/4/b^2*f^(c/(b*x+a)^2)*ln(f)*c*x^2-3/2/b^3*f^(c/(
b*x+a)^2)*ln(f)*a*c*x+2/b^4/(-c*ln(f))^(1/2)*ln(f)^2*Pi^(1/2)*erf((-c*ln(f
))^(1/2)/(b*x+a))*a*c^2+1/b^4/(-c*ln(f))^(1/2)*ln(f)*Pi^(1/2)*erf((-c*ln(f
))^(1/2)/(b*x+a))*a^3*c-7/4/b^4*f^(c/(b*x+a)^2)*ln(f)*a^2*c-1/4/b^4*f^(c/(
b*x+a)^2)*a^4+1/4/b^4*ln(f)^2*Ei(1,-c*ln(f)/(b*x+a)^2)*c^2+3/2/b^4*ln(f)*E
i(1,-c*ln(f)/(b*x+a)^2)*a^2*c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.54

$$\int f^{\frac{c}{(a+bx)^2}} x^3 \, dx = -\frac{4\sqrt{\pi}(a^3b + 2\,abc\log(f))\sqrt{-\frac{c\log(f)}{b^2}}\,\mathrm{erf}\left(\frac{b\sqrt{-\frac{c\log(f)}{b^2}}}{bx+a}\right) - (b^4x^4 - a^4 + (b^2cx^2 - 6\,abcx - 7\,a^2c)\log(f))f}{4\,b^4}$$

input
```
integrate(f^(c/(b*x+a)^2)*x^3,x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(pi)*(a^3*b + 2*a*b*c*log(f))*sqrt(-c*log(f)/b^2)*erf(b*sqrt(-
c*log(f)/b^2)/(b*x + a)) - (b^4*x^4 - a^4 + (b^2*c*x^2 - 6*a*b*c*x - 7*a^2
*c)*log(f))*f^(c/(b^2*x^2 + 2*a*b*x + a^2)) + (6*a^2*c*log(f) + c^2*log(f)
^2)*Ei(c*log(f)/(b^2*x^2 + 2*a*b*x + a^2)))/b^4
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^3\,dx = \int f^{\frac{c}{(a+bx)^2}}x^3\,dx$$

input
```
integrate(f**(c/(b*x+a)**2)*x**3,x)
```

output
```
Integral(f**(c/(a + b*x)**2)*x**3, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^3\,dx = \int f^{\frac{c}{(bx+a)^2}}x^3\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x^3,x, algorithm="maxima")
```

output
```
1/4*(b^3*x^4 + b*c*x^2*log(f) - 6*a*c*x*log(f))*f^(c/(b^2*x^2 + 2*a*b*x +
a^2))/b^3 + integrate(1/2*(3*a^4*c*log(f) + (6*a^2*b^2*c*log(f) + b^2*c^2*
log(f)^2)*x^2 + 2*(4*a^3*b*c*log(f) - 3*a*b*c^2*log(f)^2)*x)*f^(c/(b^2*x^2
 + 2*a*b*x + a^2))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3), x)
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^3\,dx = \int f^{\frac{c}{(bx+a)^2}}x^3\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^2}} x^3 \, dx = \int f^{\frac{c}{(a+b\,x)^2}} \, x^3 \, dx$$

input
```
int(f^(c/(a + b*x)^2)*x^3,x)
```

output
```
int(f^(c/(a + b*x)^2)*x^3, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^2}} x^3 \, dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^2)*x^3,x)
```

output
```
( - 16*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**4*a*b*c**4*x - 8*f**(c/
(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a**4*c**3 + 80*f**(c/(a**2 + 2*a*b
*x + b**2*x**2))*log(f)**3*a**3*b*c**3*x + 40*f**(c/(a**2 + 2*a*b*x + b**2
*x**2))*log(f)**3*a**2*b**2*c**3*x**2 + 24*f**(c/(a**2 + 2*a*b*x + b**2*x*
*2))*log(f)**3*a*b**3*c**3*x**3 + 56*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*l
og(f)**2*a**6*c**2 - 16*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**5
*b*c**2*x - 108*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**4*b**2*c*
*2*x**2 - 100*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**3*b**3*c**2
*x**3 - 25*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**2*b**4*c**2*x*
*4 - 6*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*b**5*c**2*x**5 + 3*
f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*b**6*c**2*x**6 - 120*f**(c/(
a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**8*c - 324*f**(c/(a**2 + 2*a*b*x + b
**2*x**2))*log(f)*a**7*b*c*x - 330*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log
(f)*a**6*b**2*c*x**2 - 168*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**5
*b**3*c*x**3 - 39*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**4*b**4*c*x
**4 + 12*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*b**5*c*x**5 + 18*
f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**2*b**6*c*x**6 + 12*f**(c/(a*
*2 + 2*a*b*x + b**2*x**2))*log(f)*a*b**7*c*x**7 + 3*f**(c/(a**2 + 2*a*b*x
+ b**2*x**2))*log(f)*b**8*c*x**8 + 78*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*
a**10 + 312*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**9*b*x + 468*f**(c/(a...
```

3.159 $\int f^{\frac{c}{(a+bx)^2}} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 206

$$\int f^{\frac{c}{(a+bx)^2}} x^2 \, dx = \frac{a^2 f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^3} - \frac{a f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{b^3} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^3}{3b^3} - \frac{a^2\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b^3} + \frac{2cf^{\frac{c}{(a+bx)^2}}(a+bx)\log(f)}{3b^3} + \frac{ac\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{b^3} - \frac{2c^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\log^{\frac{3}{2}}(f)}{3b^3}$$

output
```
a^2*f^(c/(b*x+a)^2)*(b*x+a)/b^3-a*f^(c/(b*x+a)^2)*(b*x+a)^2/b^3+1/3*f^(c/(
b*x+a)^2)*(b*x+a)^3/b^3-a^2*c^(1/2)*Pi^(1/2)*erfi(c^(1/2)*ln(f)^(1/2)/(b*x
+a))*ln(f)^(1/2)/b^3+2/3*c*f^(c/(b*x+a)^2)*(b*x+a)*ln(f)/b^3+a*c*Ei(c*ln(f
)/(b*x+a)^2)*ln(f)/b^3-2/3*c^(3/2)*Pi^(1/2)*erfi(c^(1/2)*ln(f)^(1/2)/(b*x+
a))*ln(f)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int f^{\frac{c}{(a+bx)^2}}x^2\,dx=\frac{af^{\frac{c}{(a+bx)^2}}\left(a^2+2c\log(f)\right)}{3b^3}+\frac{3ac\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)-\sqrt{c}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}(3a^2+2c\log(f))+bf^{\frac{c}{(a+bx)^2}}x(b^2}{3b^3}$$

input
```
Integrate[f^(c/(a + b*x)^2)*x^2,x]
```

output
```
(a*f^(c/(a + b*x)^2)*(a^2 + 2*c*Log[f]))/(3*b^3) + (3*a*c*ExpIntegralEi[(c
*Log[f])/(a + b*x)^2]*Log[f] - Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]]
)/(a + b*x)]*Sqrt[Log[f]]*(3*a^2 + 2*c*Log[f]) + b*f^(c/(a + b*x)^2)*x*(b^
2*x^2 + 2*c*Log[f]))/(3*b^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{\frac{c}{(a+bx)^2}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^2 f^{\frac{c}{(a+bx)^2}}}{b^2}+\frac{(a+bx)^2 f^{\frac{c}{(a+bx)^2}}}{b^2}-\frac{2a(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}a^2\sqrt{c}\sqrt{\log(f)}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^3}+\frac{a^2(a+bx)f^{\frac{c}{(a+bx)^2}}}{b^3}-\frac{2\sqrt{\pi}c^{3/2}\log^{\frac{3}{2}}(f)\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{3b^3}+\frac{ac\log(f)\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)}{b^3}+\frac{(a+bx)^3f^{\frac{c}{(a+bx)^2}}}{3b^3}-\frac{a(a+bx)^2f^{\frac{c}{(a+bx)^2}}}{b^3}+\frac{2c\log(f)(a+bx)f^{\frac{c}{(a+bx)^2}}}{3b^3}$$

input
```
Int[f^(c/(a + b*x)^2)*x^2,x]
```

output
```
(a^2*f^(c/(a + b*x)^2)*(a + b*x))/b^3 - (a*f^(c/(a + b*x)^2)*(a + b*x)^2)/
b^3 + (f^(c/(a + b*x)^2)*(a + b*x)^3)/(3*b^3) - (a^2*Sqrt[c]*Sqrt[Pi]*Erfi
[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]])/b^3 + (2*c*f^(c/(a + b*x)
^2)*(a + b*x)*Log[f])/(3*b^3) + (a*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]
*Log[f])/b^3 - (2*c^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*
Log[f]^(3/2))/(3*b^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

method	result
risch	$\frac{f^{\frac{c}{(bx+a)^2}}x^3}{3}+\frac{a^3f^{\frac{c}{(bx+a)^2}}}{3b^3}+\frac{2f^{\frac{c}{(bx+a)^2}}\ln(f)cx}{3b^2}+\frac{2f^{\frac{c}{(bx+a)^2}}\ln(f)ac}{3b^3}-\frac{2\ln(f)^2c^2\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-c\ln(f)}}{bx+a}\right)}{3b^3\sqrt{-c\ln(f)}}-\frac{a^2\ln(f)c\sqrt{\pi}\,\mathrm{e}}{b^3\sqrt{-}}$

input
```
int(f^(c/(b*x+a)^2)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*f^(c/(b*x+a)^2)*x^3+1/3/b^3*a^3*f^(c/(b*x+a)^2)+2/3/b^2*f^(c/(b*x+a)^2
)*ln(f)*c*x+2/3/b^3*f^(c/(b*x+a)^2)*ln(f)*a*c-2/3/b^3*ln(f)^2*c^2*Pi^(1/2)
/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)/(b*x+a))-1/b^3*a^2*ln(f)*c*Pi^(1/2)
/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)/(b*x+a))-1/b^3*a*ln(f)*c*Ei(1,-c*ln
(f)/(b*x+a)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62

$$\int f^{\frac{c}{(a+bx)^2}} x^2\,dx = \frac{3\,ac\mathrm{Ei}\left(\frac{c\log(f)}{b^2x^2+2\,abx+a^2}\right)\log\left(f\right)+\sqrt{\pi}(3\,a^2b+2\,bc\log\left(f\right))\sqrt{-\frac{c\log(f)}{b^2}}\,\mathrm{erf}\left(\frac{b\sqrt{-\frac{c\log(f)}{b^2}}}{bx+a}\right)+(b^3x^3+a^3+2\,(bcx}{3\,b^3}$$

input
```
integrate(f^(c/(b*x+a)^2)*x^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*c*Ei(c*log(f)/(b^2*x^2 + 2*a*b*x + a^2))*log(f) + sqrt(pi)*(3*a^2
*b + 2*b*c*log(f))*sqrt(-c*log(f)/b^2)*erf(b*sqrt(-c*log(f)/b^2)/(b*x + a)
) + (b^3*x^3 + a^3 + 2*(b*c*x + a*c)*log(f))*f^(c/(b^2*x^2 + 2*a*b*x + a^2
)))/b^3
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^2}} x^2\,dx = \int f^{\frac{c}{(a+bx)^2}} x^2\,dx$$

input
```
integrate(f**(c/(b*x+a)**2)*x**2,x)
```

output
```
Integral(f**(c/(a + b*x)**2)*x**2, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^2\,dx = \int f^{\frac{c}{(bx+a)^2}}x^2\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*x^3 + 2*c*x*log(f))*f^(c/(b^2*x^2 + 2*a*b*x + a^2))/b^2 - integra
te(2/3*(3*a*b^2*c*x^2*log(f) + a^3*c*log(f) + (3*a^2*b*c*log(f) - 2*b*c^2*
log(f)^2)*x)*f^(c/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^
2*b^3*x + a^3*b^2), x)
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^2}}x^2\,dx = \int f^{\frac{c}{(bx+a)^2}}x^2\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x^2,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^2}}x^2\,dx = \int f^{\frac{c}{(a+b\,x)^2}}\,x^2\,dx$$

input
```
int(f^(c/(a + b*x)^2)*x^2,x)
```

output
```
int(f^(c/(a + b*x)^2)*x^2, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^2}} x^2 \, dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^2)*x^2,x)
```

output
```
(8*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**4*b*c**4*x + 4*f**(c/(a**2
+ 2*a*b*x + b**2*x**2))*log(f)**3*a**3*c**3 - 40*f**(c/(a**2 + 2*a*b*x + b
**2*x**2))*log(f)**3*a**2*b*c**3*x - 20*f**(c/(a**2 + 2*a*b*x + b**2*x**2)
)*log(f)**3*a*b**2*c**3*x**2 - 12*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(
f)**3*b**3*c**3*x**3 - 28*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*
*5*c**2 + 8*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**4*b*c**2*x +
54*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**3*b**2*c**2*x**2 + 50*
f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**2*b**3*c**2*x**3 + 14*f**
(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*b**4*c**2*x**4 + 6*f**(c/(a**
2 + 2*a*b*x + b**2*x**2))*log(f)**2*b**5*c**2*x**5 + 60*f**(c/(a**2 + 2*a*
b*x + b**2*x**2))*log(f)*a**7*c + 162*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*
log(f)*a**6*b*c*x + 165*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**5*b*
*2*c*x**2 + 87*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**4*b**3*c*x**3
 + 33*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*b**4*c*x**4 + 18*f**
(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**2*b**5*c*x**5 + 12*f**(c/(a**2
+ 2*a*b*x + b**2*x**2))*log(f)*a*b**6*c*x**6 + 3*f**(c/(a**2 + 2*a*b*x + b
**2*x**2))*log(f)*b**7*c*x**7 - 39*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**
9 - 156*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**8*b*x - 234*f**(c/(a**2 + 2
*a*b*x + b**2*x**2))*a**7*b**2*x**2 - 156*f**(c/(a**2 + 2*a*b*x + b**2*x**
2))*a**6*b**3*x**3 - 39*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**5*b**4*x...
```

3.160 $\int f^{\frac{c}{(a+bx)^2}} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int f^{\frac{c}{(a+bx)^2}} x \, dx = -\frac{a f^{\frac{c}{(a+bx)^2}}(a+bx)}{b^2} + \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)^2}{2b^2} + \frac{a\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b^2} - \frac{c\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{2b^2}$$

output
```
-a*f^(c/(b*x+a)^2)*(b*x+a)/b^2+1/2*f^(c/(b*x+a)^2)*(b*x+a)^2/b^2+a*c^(1/2)
*Pi^(1/2)*erfi(c^(1/2)*ln(f)^(1/2)/(b*x+a))*ln(f)^(1/2)/b^2-1/2*c*Ei(c*ln(
f)/(b*x+a)^2)*ln(f)/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int f^{\frac{c}{(a+bx)^2}} x \, dx = \frac{f^{\frac{c}{(a+bx)^2}}(-a^2+b^2x^2) + 2a\sqrt{c}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)} - c\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^2}\right)\log(f)}{2b^2}$$

input
```
Integrate[f^(c/(a + b*x)^2)*x,x]
```

output
```
(f^(c/(a + b*x)^2)*(-a^2 + b^2*x^2) + 2*a*Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*S
qrt[Log[f]])/(a + b*x)]*Sqrt[Log[f]] - c*ExpIntegralEi[(c*Log[f])/(a + b*x
)^2]*Log[f])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{\frac{c}{(a+bx)^2}} \, dx$$

$$\Big\downarrow \text{ 2656}$$

$$\int \left(\frac{(a+bx) f^{\frac{c}{(a+bx)^2}}}{b} - \frac{a f^{\frac{c}{(a+bx)^2}}}{b}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi} a \sqrt{c} \sqrt{\log(f)} \text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b^2} - \frac{c \log(f) \text{ExpIntegralEi}\left(\frac{c \log(f)}{(a+bx)^2}\right)}{2b^2} + \frac{(a+bx)^2 f^{\frac{c}{(a+bx)^2}}}{2b^2} - \frac{a(a+bx) f^{\frac{c}{(a+bx)^2}}}{b^2}$$

input
```
Int[f^(c/(a + b*x)^2)*x,x]
```

output
```
-((a*f^(c/(a + b*x)^2)*(a + b*x))/b^2) + (f^(c/(a + b*x)^2)*(a + b*x)^2)/(
2*b^2) + (a*Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log[f]])/(a + b*x)]*Sqrt[L
og[f]])/b^2 - (c*ExpIntegralEi[(c*Log[f])/(a + b*x)^2]*Log[f])/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{f^{\frac{c}{(bx+a)^2}}x^2}{2}-\frac{f^{\frac{c}{(bx+a)^2}}a^2}{2b^2}+\frac{\ln(f)c\ \mathrm{expIntegral}_1\left(-\frac{c\ln(f)}{(bx+a)^2}\right)}{2b^2}+\frac{a\ln(f)c\sqrt{\pi}\ \mathrm{erf}\left(\frac{\sqrt{-c\ln(f)}}{bx+a}\right)}{b^2\sqrt{-c\ln(f)}}$	93

input
```
int(f^(c/(b*x+a)^2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^(c/(b*x+a)^2)*x^2-1/2/b^2*f^(c/(b*x+a)^2)*a^2+1/2/b^2*ln(f)*c*Ei(1,-
c*ln(f)/(b*x+a)^2)+1/b^2*a*ln(f)*c*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f)
)^(1/2)/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int f^{\frac{c}{(a+bx)^2}}x\,dx = -\frac{2\sqrt{\pi}ab\sqrt{-\frac{c\log(f)}{b^2}}\ \mathrm{erf}\left(\frac{b\sqrt{-\frac{c\log(f)}{b^2}}}{bx+a}\right)+c\mathrm{Ei}\left(\frac{c\log(f)}{b^2x^2+2\,abx+a^2}\right)\log(f)-(b^2x^2-a^2)f^{\frac{c}{b^2x^2+2\,abx+a^2}}}{2\,b^2}$$

input
```
integrate(f^(c/(b*x+a)^2)*x,x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(pi)*a*b*sqrt(-c*log(f)/b^2)*erf(b*sqrt(-c*log(f)/b^2)/(b*x +
a)) + c*Ei(c*log(f)/(b^2*x^2 + 2*a*b*x + a^2))*log(f) - (b^2*x^2 - a^2)*f^
(c/(b^2*x^2 + 2*a*b*x + a^2)))/b^2
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^2}}x\,dx = \int f^{\frac{c}{(a+bx)^2}}x\,dx$$

input
```
integrate(f**(c/(b*x+a)**2)*x,x)
```

output
```
Integral(f**(c/(a + b*x)**2)*x, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^2}}x\,dx = \int f^{\frac{c}{(bx+a)^2}}x\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x,x, algorithm="maxima")
```

output
```
b*c*integrate(f^(c/(b^2*x^2 + 2*a*b*x + a^2))*x^2/(b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3), x)*log(f) + 1/2*f^(c/(b^2*x^2 + 2*a*b*x + a^2))*x^2
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^2}}x\,dx = \int f^{\frac{c}{(bx+a)^2}}x\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)*x,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^2}}x\,dx=\int f^{\frac{c}{(a+b\,x)^2}}\,x\,dx$$

input
```
int(f^(c/(a + b*x)^2)*x,x)
```

output
```
int(f^(c/(a + b*x)^2)*x, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^2}}x\,dx=\text{too large to display}$$

input
```
int(f^(c/(b*x+a)^2)*x,x)
```

output
```
(8*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a*b*c**3*x + 4*f**(c/(a**
2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**4*c**2 - 12*f**(c/(a**2 + 2*a*b*x +
 b**2*x**2))*log(f)**2*a**3*b*c**2*x - 20*f**(c/(a**2 + 2*a*b*x + b**2*x**
2))*log(f)**2*a**2*b**2*c**2*x**2 - 12*f**(c/(a**2 + 2*a*b*x + b**2*x**2))
*log(f)**2*a*b**3*c**2*x**3 - 14*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f
)*a**6*c - 30*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**5*b*c*x - 16*f
**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**4*b**2*c*x**2 + 8*f**(c/(a**2
 + 2*a*b*x + b**2*x**2))*log(f)*a**3*b**3*c*x**3 + 17*f**(c/(a**2 + 2*a*b*
x + b**2*x**2))*log(f)*a**2*b**4*c*x**4 + 12*f**(c/(a**2 + 2*a*b*x + b**2*
x**2))*log(f)*a*b**5*c*x**5 + 3*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)
*b**6*c*x**6 + 13*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**8 + 52*f**(c/(a**
2 + 2*a*b*x + b**2*x**2))*a**7*b*x + 78*f**(c/(a**2 + 2*a*b*x + b**2*x**2)
)*a**6*b**2*x**2 + 52*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**5*b**3*x**3 +
 13*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b**4*x**4 + 6*int((f**(c/(a**
2 + 2*a*b*x + b**2*x**2))*x**4)/(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 1
0*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5),x)*log(f)**2*a**4*b**5*c**2
+ 24*int((f**(c/(a**2 + 2*a*b*x + b**2*x**2))*x**4)/(a**5 + 5*a**4*b*x + 1
0*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x**4 + b**5*x**5),x)*log(f
)**2*a**3*b**6*c**2*x + 36*int((f**(c/(a**2 + 2*a*b*x + b**2*x**2))*x**4)/
(a**5 + 5*a**4*b*x + 10*a**3*b**2*x**2 + 10*a**2*b**3*x**3 + 5*a*b**4*x...
```

3.161 $\int f^{\frac{c}{(a+bx)^2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 62

$$\int f^{\frac{c}{(a+bx)^2}} dx = \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)}{b} - \frac{\sqrt{c}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b}$$

output
```
f^(c/(b*x+a)^2)*(b*x+a)/b-c^(1/2)*Pi^(1/2)*erfi(c^(1/2)*ln(f)^(1/2)/(b*x+a
))*ln(f)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int f^{\frac{c}{(a+bx)^2}} dx = \frac{f^{\frac{c}{(a+bx)^2}}(a+bx)}{b} - \frac{\sqrt{c}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)\sqrt{\log(f)}}{b}$$

input
```
Integrate[f^(c/(a + b*x)^2),x]
```

output
```
(f^(c/(a + b*x)^2)*(a + b*x))/b - (Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log
[f]])/(a + b*x)]*Sqrt[Log[f]])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{\frac{c}{(a+bx)^2}}\,dx$$

$$\Big\downarrow \text{2635}$$

$$2c\log(f)\int \frac{f^{\frac{c}{(a+bx)^2}}}{(a+bx)^2}dx+\frac{(a+bx)f^{\frac{c}{(a+bx)^2}}}{b}$$

$$\Big\downarrow \text{2640}$$

$$\frac{(a+bx)f^{\frac{c}{(a+bx)^2}}}{b}-\frac{2c\log(f)\int f^{\frac{c}{(a+bx)^2}}d\frac{1}{a+bx}}{b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(a+bx)f^{\frac{c}{(a+bx)^2}}}{b}-\frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}\text{erfi}\left(\frac{\sqrt{c}\sqrt{\log(f)}}{a+bx}\right)}{b}$$

input
```
Int[f^(c/(a + b*x)^2),x]
```

output
```
(f^(c/(a + b*x)^2)*(a + b*x))/b - (Sqrt[c]*Sqrt[Pi]*Erfi[(Sqrt[c]*Sqrt[Log
[f]])/(a + b*x)]*Sqrt[Log[f]])/b
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

method	result	size
risch	$f^{\frac{c}{(bx+a)^2}}x+\frac{f^{\frac{c}{(bx+a)^2}}a}{b}-\frac{\ln(f)c\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-c\ln(f)}}{bx+a}\right)}{b\sqrt{-c\ln(f)}}$	65

input
```
int(f^(c/(b*x+a)^2),x,method=_RETURNVERBOSE)
```

output
```
f^(c/(b*x+a)^2)*x+1/b*f^(c/(b*x+a)^2)*a-1/b*ln(f)*c*Pi^(1/2)/(-c*ln(f))^(1
/2)*erf((-c*ln(f))^(1/2)/(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int f^{\frac{c}{(a+bx)^2}}\,dx=\frac{\sqrt{\pi}b\sqrt{-\frac{c\log(f)}{b^2}}\,\mathrm{erf}\left(\frac{b\sqrt{-\frac{c\log(f)}{b^2}}}{bx+a}\right)+(bx+a)f^{\frac{c}{b^2x^2+2\,abx+a^2}}}{b}$$

input
```
integrate(f^(c/(b*x+a)^2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*b*sqrt(-c*log(f)/b^2)*erf(b*sqrt(-c*log(f)/b^2)/(b*x + a)) + (b*
x + a)*f^(c/(b^2*x^2 + 2*a*b*x + a^2)))/b
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^2}}\,dx = \int f^{\frac{c}{(a+bx)^2}}\,dx$$

input
```
integrate(f**(c/(b*x+a)**2),x)
```

output
```
Integral(f**(c/(a + b*x)**2), x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^2}}\,dx = \int f^{\frac{c}{(bx+a)^2}}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2),x, algorithm="maxima")
```

output
```
2*b*c*integrate(f^(c/(b^2*x^2 + 2*a*b*x + a^2))*x/(b^3*x^3 + 3*a*b^2*x^2 +
 3*a^2*b*x + a^3), x)*log(f) + f^(c/(b^2*x^2 + 2*a*b*x + a^2))*x
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^2}}\,dx = \int f^{\frac{c}{(bx+a)^2}}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2),x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int f^{\frac{c}{(a+bx)^2}}\,dx=\frac{f^{\frac{c}{(a+b\,x)^2}}\,(a+b\,x)}{b}-\frac{c\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{c\,\ln(f)}}{a+b\,x}\right)\,\ln(f)}{b\,\sqrt{c\,\ln(f)}}$$

input
```
int(f^(c/(a + b*x)^2),x)
```

output
```
(f^(c/(a + b*x)^2)*(a + b*x))/b - (c*pi^(1/2)*erfi((c*log(f))^(1/2)/(a + b
*x))*log(f))/(b*(c*log(f))^(1/2))
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^2}}\,dx=\text{Too large to display}$$

input
```
int(f^(c/(b*x+a)^2),x)
```

output

```
(4*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*b*c**2*x + 2*f**(c/(a**2
+ 2*a*b*x + b**2*x**2))*log(f)*a**3*c - 2*f**(c/(a**2 + 2*a*b*x + b**2*x**
2))*log(f)*a**2*b*c*x - 10*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a*b*
*2*c*x**2 - 6*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*b**3*c*x**3 - 5*f
**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**5 - 17*f**(c/(a**2 + 2*a*b*x + b**2*
x**2))*a**4*b*x - 18*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**3*b**2*x**2 -
2*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**2*b**3*x**3 + 7*f**(c/(a**2 + 2*a
*b*x + b**2*x**2))*a*b**4*x**4 + 3*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*b**
5*x**5 + 8*int((f**(c/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a**7 + 7*a**6*b*x
+ 21*a**5*b**2*x**2 + 35*a**4*b**3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5
*x**5 + 7*a*b**6*x**6 + b**7*x**7),x)*log(f)**3*a**4*b**2*c**3 + 32*int((f
**(c/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a**7 + 7*a**6*b*x + 21*a**5*b**2*x*
*2 + 35*a**4*b**3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5*x**5 + 7*a*b**6*
x**6 + b**7*x**7),x)*log(f)**3*a**3*b**3*c**3*x + 48*int((f**(c/(a**2 + 2*
a*b*x + b**2*x**2))*x)/(a**7 + 7*a**6*b*x + 21*a**5*b**2*x**2 + 35*a**4*b*
*3*x**3 + 35*a**3*b**4*x**4 + 21*a**2*b**5*x**5 + 7*a*b**6*x**6 + b**7*x**
7),x)*log(f)**3*a**2*b**4*c**3*x**2 + 32*int((f**(c/(a**2 + 2*a*b*x + b**2
*x**2))*x)/(a**7 + 7*a**6*b*x + 21*a**5*b**2*x**2 + 35*a**4*b**3*x**3 + 35
*a**3*b**4*x**4 + 21*a**2*b**5*x**5 + 7*a*b**6*x**6 + b**7*x**7),x)*log(f)
**3*a*b**5*c**3*x**3 + 8*int((f**(c/(a**2 + 2*a*b*x + b**2*x**2))*x)/(a...
```

3.162 $\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx = \text{Int}\left(\frac{f^{\frac{c}{(a+bx)^2}}}{x}, x\right)$$

output
```
Defer(Int)(f^(c/(b*x+a)^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx = \int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx$$

input
```
Integrate[f^(c/(a + b*x)^2)/x,x]
```

output
```
Integrate[f^(c/(a + b*x)^2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx$$
$$\Big\downarrow \text{2654}$$
$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x}dx$$

input
```
Int[f^(c/(a + b*x)^2)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x}dx$$

input
```
int(f^(c/(b*x+a)^2)/x,x)
```

output
```
int(f^(c/(b*x+a)^2)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x}\,dx=\int \frac{f^{\frac{c}{(bx+a)^2}}}{x}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x,x, algorithm="fricas")
```

output
```
integral(f^(c/(b^2*x^2 + 2*a*b*x + a^2))/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x}\,dx=\int \frac{f^{\frac{c}{(a+bx)^2}}}{x}\,dx$$

input
```
integrate(f**(c/(b*x+a)**2)/x,x)
```

output
```
Integral(f**(c/(a + b*x)**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x} \, dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a)^2)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x} \, dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x} \, dx = \int \frac{f^{\frac{c}{(a+b\,x)^2}}}{x} \, dx$$

input
```
int(f^(c/(a + b*x)^2)/x,x)
```

output
```
int(f^(c/(a + b*x)^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x}\,dx = \int \frac{f^{\frac{c}{b^2x^2+2abx+a^2}}}{x}dx$$

input
```
int(f^(c/(b*x+a)^2)/x,x)
```

output
```
int(f**(c/(a**2 + 2*a*b*x + b**2*x**2))/x,x)
```

3.163 $\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} dx = \text{Int}\left(\frac{f^{\frac{c}{(a+bx)^2}}}{x^2}, x\right)$$

output
```
Defer(Int)(f^(c/(b*x+a)^2)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} dx = \int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} dx$$

input
```
Integrate[f^(c/(a + b*x)^2)/x^2,x]
```

output
```
Integrate[f^(c/(a + b*x)^2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} dx$$

input
```
Int[f^(c/(a + b*x)^2)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x^2} dx$$

input
```
int(f^(c/(b*x+a)^2)/x^2,x)
```

output
```
int(f^(c/(b*x+a)^2)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x^2,x, algorithm="fricas")
```

output
```
integral(f^(c/(b^2*x^2 + 2*a*b*x + a^2))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2}\,dx$$

input
```
integrate(f**(c/(b*x+a)**2)/x**2,x)
```

output
```
Integral(f**(c/(a + b*x)**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x^2,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a)^2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(a+b\,x)^2}}}{x^2}\,dx$$

input
```
int(f^(c/(a + b*x)^2)/x^2,x)
```

output
```
int(f^(c/(a + b*x)^2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 3627, normalized size of antiderivative = 241.80

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^2} \, dx = \text{Too large to display}$$

input
```
int(f^(c/(b*x+a)^2)/x^2,x)
```

output
```
(2*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a**3*c**3 - f**(c/(a**2 +
 2*a*b*x + b**2*x**2))*log(f)**3*b**3*c**3*x**3 - 3*f**(c/(a**2 + 2*a*b*x
+ b**2*x**2))*log(f)**2*a**5*c**2 - 11*f**(c/(a**2 + 2*a*b*x + b**2*x**2))
*log(f)**2*a**4*b*c**2*x - 6*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2
*a**3*b**2*c**2*x**2 - 2*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a**
2*b**3*c**2*x**3 + f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*b**4*c*
*2*x**4 + f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*b**5*c**2*x**5 + 4
*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**6*b*c*x - 16*f**(c/(a**2 +
2*a*b*x + b**2*x**2))*log(f)*a**4*b**3*c*x**3 - 16*f**(c/(a**2 + 2*a*b*x +
 b**2*x**2))*log(f)*a**3*b**4*c*x**4 - 4*f**(c/(a**2 + 2*a*b*x + b**2*x**2
))*log(f)*a**2*b**5*c*x**5 - 8*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**8*b*
x - 32*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**7*b**2*x**2 - 48*f**(c/(a**2
 + 2*a*b*x + b**2*x**2))*a**6*b**3*x**3 - 32*f**(c/(a**2 + 2*a*b*x + b**2*
x**2))*a**5*b**4*x**4 - 8*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b**5*x*
*5 + 2*int(f**(c/(a**2 + 2*a*b*x + b**2*x**2))/(a**7*x**2 + 7*a**6*b*x**3
+ 21*a**5*b**2*x**4 + 35*a**4*b**3*x**5 + 35*a**3*b**4*x**6 + 21*a**2*b**5
*x**7 + 7*a*b**6*x**8 + b**7*x**9),x)*log(f)**3*a**10*c**3*x + 8*int(f**(c
/(a**2 + 2*a*b*x + b**2*x**2))/(a**7*x**2 + 7*a**6*b*x**3 + 21*a**5*b**2*x
**4 + 35*a**4*b**3*x**5 + 35*a**3*b**4*x**6 + 21*a**2*b**5*x**7 + 7*a*b**6
*x**8 + b**7*x**9),x)*log(f)**3*a**9*b*c**3*x**2 + 12*int(f**(c/(a**2 +...
```

3.164 $\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} dx = \text{Int}\left(\frac{f^{\frac{c}{(a+bx)^2}}}{x^3}, x\right)$$

output
```
Defer(Int)(f^(c/(b*x+a)^2)/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} dx = \int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} dx$$

input
```
Integrate[f^(c/(a + b*x)^2)/x^3,x]
```

output
```
Integrate[f^(c/(a + b*x)^2)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3}dx$$

input
```
Int[f^(c/(a + b*x)^2)/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{(bx+a)^2}}}{x^3}dx$$

input
```
int(f^(c/(b*x+a)^2)/x^3,x)
```

output
```
int(f^(c/(b*x+a)^2)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} \, dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x^3} \, dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x^3,x, algorithm="fricas")
```

output
```
integral(f^(c/(b^2*x^2 + 2*a*b*x + a^2))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} \, dx = \int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} \, dx$$

input
```
integrate(f**(c/(b*x+a)**2)/x**3,x)
```

output
```
Integral(f**(c/(a + b*x)**2)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x^3,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a)^2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(bx+a)^2}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a)^2)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(a+b\,x)^2}}}{x^3}\,dx$$

input
```
int(f^(c/(a + b*x)^2)/x^3,x)
```

output
```
int(f^(c/(a + b*x)^2)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 3628, normalized size of antiderivative = 241.87

$$\int \frac{f^{\frac{c}{(a+bx)^2}}}{x^3} \, dx = \text{Too large to display}$$

input
```
int(f^(c/(b*x+a)^2)/x^3,x)
```

output
```
(12*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**3*a**2*c**3 + 28*f**(c/(a*
*2 + 2*a*b*x + b**2*x**2))*log(f)**3*a*b*c**3*x - 35*f**(c/(a**2 + 2*a*b*x
 + b**2*x**2))*log(f)**2*a**4*c**2 - 140*f**(c/(a**2 + 2*a*b*x + b**2*x**2
))*log(f)**2*a**3*b*c**2*x - 190*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f
)**2*a**2*b**2*c**2*x**2 - 70*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**
2*a*b**3*c**2*x**3 - 5*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**4*b**
2*c*x**2 - 80*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*b**3*c*x**3
- 180*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**2*b**4*c*x**4 - 140*f*
*(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a*b**5*c*x**5 - 35*f**(c/(a**2 +
2*a*b*x + b**2*x**2))*log(f)*b**6*c*x**6 - 30*f**(c/(a**2 + 2*a*b*x + b**2
*x**2))*a**6*b**2*x**2 - 120*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**5*b**3
*x**3 - 180*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*a**4*b**4*x**4 - 120*f**(c
/(a**2 + 2*a*b*x + b**2*x**2))*a**3*b**5*x**5 - 30*f**(c/(a**2 + 2*a*b*x +
 b**2*x**2))*a**2*b**6*x**6 + 24*int(f**(c/(a**2 + 2*a*b*x + b**2*x**2))/(
a**7*x**3 + 7*a**6*b*x**4 + 21*a**5*b**2*x**5 + 35*a**4*b**3*x**6 + 35*a**
3*b**4*x**7 + 21*a**2*b**5*x**8 + 7*a*b**6*x**9 + b**7*x**10),x)*log(f)**3
*a**9*c**3*x**2 + 96*int(f**(c/(a**2 + 2*a*b*x + b**2*x**2))/(a**7*x**3 +
7*a**6*b*x**4 + 21*a**5*b**2*x**5 + 35*a**4*b**3*x**6 + 35*a**3*b**4*x**7
+ 21*a**2*b**5*x**8 + 7*a*b**6*x**9 + b**7*x**10),x)*log(f)**3*a**8*b*c**3
*x**3 + 144*int(f**(c/(a**2 + 2*a*b*x + b**2*x**2))/(a**7*x**3 + 7*a**6...
```

3.165 $\int f^{\frac{c}{(a+bx)^3}} x^4 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 239

$$\int f^{\frac{c}{(a+bx)^3}} x^4 \, dx = \frac{2a^2 f^{\frac{c}{(a+bx)^3}}(a+bx)^3}{b^5} - \frac{2a^2 c \,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)\log(f)}{b^5} + \frac{a^4(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b^5} - \frac{4a^3(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}}{3b^5} - \frac{4a(a+bx)^4\Gamma\left(-\frac{4}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{4/3}}{3b^5} + \frac{(a+bx)^5\Gamma\left(-\frac{5}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{5/3}}{3b^5}$$

output

```
2*a^2*f^(c/(b*x+a)^3)*(b*x+a)^3/b^5-2*a^2*c*Ei(c*ln(f)/(b*x+a)^3)*ln(f)/b^
5+1/3*a^4*(b*x+a)*GAMMA(-1/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(1/3
)/b^5-4/3*a^3*(b*x+a)^2*GAMMA(-2/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3
)^(2/3)/b^5-4/3*a*(b*x+a)^4*GAMMA(-4/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+
a)^3)^(4/3)/b^5+1/3*(b*x+a)^5*GAMMA(-5/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*
x+a)^3)^(5/3)/b^5
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int f^{\frac{c}{(a+bx)^3}}x^4\,dx$$

$$= \frac{6a^2f^{\frac{c}{(a+bx)^3}}(a+bx)^3-6a^2c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)\log(f)+a^4(a+bx)\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{}$$

input
```
Integrate[f^(c/(a + b*x)^3)*x^4,x]
```

output
```
(6*a^2*f^(c/(a + b*x)^3)*(a + b*x)^3 - 6*a^2*c*ExpIntegralEi[(c*Log[f])/(a
 + b*x)^3]*Log[f] + a^4*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(
-((c*Log[f])/(a + b*x)^3))^(1/3) + 4*a*c*(a + b*x)*Gamma[-4/3, -((c*Log[f]
)/(a + b*x)^3)]*Log[f]*(-((c*Log[f])/(a + b*x)^3))^(1/3) - 4*a^3*(a + b*x)
^2*Gamma[-2/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3
) + (a + b*x)^5*Gamma[-5/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a +
b*x)^3))^(5/3))/(3*b^5)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 f^{\frac{c}{(a+bx)^3}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^4 f^{\frac{c}{(a+bx)^3}}}{b^4}-\frac{4a^3(a+bx)f^{\frac{c}{(a+bx)^3}}}{b^4}+\frac{6a^2(a+bx)^2f^{\frac{c}{(a+bx)^3}}}{b^4}+\frac{(a+bx)^4f^{\frac{c}{(a+bx)^3}}}{b^4}-\frac{4a(a+bx)^3f^{\frac{c}{(a+bx)^3}}}{b^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4(a+bx)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^5}-\frac{4a^3(a+bx)^2\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^5}-\frac{2a^2c\log(f)\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)}{b^5}+\frac{2a^2(a+bx)^3f^{\frac{c}{(a+bx)^3}}}{b^5}+\frac{(a+bx)^5\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{5/3}\Gamma\left(-\frac{5}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^5}-\frac{4a(a+bx)^4\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{4/3}\Gamma\left(-\frac{4}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^5}$$

input
```
Int[f^(c/(a + b*x)^3)*x^4,x]
```

output
```
(2*a^2*f^(c/(a + b*x)^3)*(a + b*x)^3)/b^5 - (2*a^2*c*ExpIntegralEi[(c*Log[
f])/(a + b*x)^3]*Log[f])/b^5 + (a^4*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a
+ b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(1/3))/(3*b^5) - (4*a^3*(a + b*x)^2
*Gamma[-2/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3))
/(3*b^5) - (4*a*(a + b*x)^4*Gamma[-4/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*L
og[f])/(a + b*x)^3))^(4/3))/(3*b^5) + ((a + b*x)^5*Gamma[-5/3, -((c*Log[f]
)/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(5/3))/(3*b^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int f^{\frac{c}{(bx+a)^3}}x^4dx$$

input
```
int(f^(c/(b*x+a)^3)*x^4,x)
```

output
```
int(f^(c/(b*x+a)^3)*x^4,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int f^{\frac{c}{(a+bx)^3}} x^4\, dx =$$

$$-\frac{20\, a^2 c \mathrm{Ei}\left(\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right)\log\left(f\right) - \left(20\, a^3b^2 - 3\, b^2c\log\left(f\right)\right)\left(-\frac{c\log(f)}{b^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+}\right.}{}$$

input
```
integrate(f^(c/(b*x+a)^3)*x^4,x, algorithm="fricas")
```

output
```
-1/10*(20*a^2*c*Ei(c*log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*log
(f) - (20*a^3*b^2 - 3*b^2*c*log(f))*(-c*log(f)/b^3)^(2/3)*gamma(1/3, -c*lo
g(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) + 10*(a^4*b - 3*a*b*c*log(
f))*(-c*log(f)/b^3)^(1/3)*gamma(2/3, -c*log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3)) - (2*b^5*x^5 + 2*a^5 + 3*(b^2*c*x^2 - 8*a*b*c*x - 9*a^2*c)
*log(f))*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)))/b^5
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^3}} x^4\, dx = \int f^{\frac{c}{(a+bx)^3}} x^4\, dx$$

input
```
integrate(f**(c/(b*x+a)**3)*x***4,x)
```

output
```
Integral(f**(c/(a + b*x)**3)*x***4, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^4\,dx=\int f^{\frac{c}{(bx+a)^3}}x^4\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x^4,x, algorithm="maxima")
```

output
```
1/10*(2*b^4*x^5 + 3*b*c*x^2*log(f) - 24*a*c*x*log(f))*f^(c/(b^3*x^3 + 3*a*
b^2*x^2 + 3*a^2*b*x + a^3))/b^4 + integrate(3/10*(20*a^2*b^3*c*x^3*log(f)
+ 8*a^5*c*log(f) + (40*a^3*b^2*c*log(f) + 3*b^2*c^2*log(f)^2)*x^2 + 6*(5*a
^4*b*c*log(f) - 4*a*b*c^2*log(f)^2)*x)*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x + a^3))/(b^8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^
4), x)
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^4\,dx=\int f^{\frac{c}{(bx+a)^3}}x^4\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x^4,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^3}}x^4\,dx=\int f^{\frac{c}{(a+b\,x)^3}}\,x^4\,dx$$

input
```
int(f^(c/(a + b*x)^3)*x^4,x)
```

output
```
int(f^(c/(a + b*x)^3)*x^4, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^4\,dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^3)*x^4,x)
```

output
```
( - 54*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**4*a*
b*c**4*x + 27*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f
)**4*b**2*c**4*x**2 - 18*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3))*log(f)**3*a**5*c**3 + 810*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3))*log(f)**3*a**4*b*c**3*x + 1134*f**(c/(a**3 + 3*a**2*b*x + 3*a
*b**2*x**2 + b**3*x**3))*log(f)**3*a**3*b**2*c**3*x**2 + 198*f**(c/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**3*a**2*b**3*c**3*x**3 -
36*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**3*b**5*c
**3*x**5 + 306*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(
f)**2*a**8*c**2 + 1836*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3))*log(f)**2*a**7*b*c**2*x - 1278*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**
2 + b**3*x**3))*log(f)**2*a**6*b**2*c**2*x**2 - 5616*f**(c/(a**3 + 3*a**2*
b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**5*b**3*c**2*x**3 - 5190*f**
(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**4*b**4*c*
*2*x**4 - 2076*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(
f)**2*a**3*b**5*c**2*x**5 - 342*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3))*log(f)**2*a**2*b**6*c**2*x**6 - 24*f**(c/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a*b**7*c**2*x**7 + 12*f**(c/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*b**8*c**2*x**8 - 3912*
f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**10*b*c...
```

3.166 $\int f^{\frac{c}{(a+bx)^3}} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 184

$$\int f^{\frac{c}{(a+bx)^3}} x^3 \, dx = -\frac{a f^{\frac{c}{(a+bx)^3}} (a+bx)^3}{b^4} + \frac{ac \operatorname{ExpIntegralEi}\left(\frac{c \log(f)}{(a+bx)^3}\right) \log(f)}{b^4} - \frac{a^3(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \sqrt[3]{-\frac{c \log(f)}{(a+bx)^3}}}{3b^4} + \frac{a^2(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{2/3}}{b^4} + \frac{(a+bx)^4\Gamma\left(-\frac{4}{3}, -\frac{c \log(f)}{(a+bx)^3}\right) \left(-\frac{c \log(f)}{(a+bx)^3}\right)^{4/3}}{3b^4}$$

output
```
-a*f^(c/(b*x+a)^3)*(b*x+a)^3/b^4+a*c*Ei(c*ln(f)/(b*x+a)^3)*ln(f)/b^4-1/3*a
^3*(b*x+a)*GAMMA(-1/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(1/3)/b^4+a
^2*(b*x+a)^2*GAMMA(-2/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(2/3)/b^4
+1/3*(b*x+a)^4*GAMMA(-4/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(4/3)/b
^4
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.91

$$\int f^{\frac{c}{(a+bx)^3}}x^3\,dx$$
$$=\frac{3ac\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)\log(f)-(a+bx)\left(a^3\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}+c\Gamma\left(-\frac{4}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\right.}{3b^4}$$

input
```
Integrate[f^(c/(a + b*x)^3)*x^3,x]
```

output
```
(3*a*c*ExpIntegralEi[(c*Log[f])/(a + b*x)^3]*Log[f] - (a + b*x)*(a^3*Gamma
[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(1/3) + c*Ga
mma[-4/3, -((c*Log[f])/(a + b*x)^3)]*Log[f]*(-((c*Log[f])/(a + b*x)^3))^(1
/3) + 3*a*(a + b*x)*(f^(c/(a + b*x)^3)*(a + b*x) - a*Gamma[-2/3, -((c*Log[
f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3))))/(3*b^4)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 f^{\frac{c}{(a+bx)^3}}\,dx$$

$$\Big\downarrow \text{ 2656}$$

$$\int \left(-\frac{a^3 f^{\frac{c}{(a+bx)^3}}}{b^3}+\frac{3a^2(a+bx)f^{\frac{c}{(a+bx)^3}}}{b^3}+\frac{(a+bx)^3 f^{\frac{c}{(a+bx)^3}}}{b^3}-\frac{3a(a+bx)^2 f^{\frac{c}{(a+bx)^3}}}{b^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3(a+bx)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^4}+\frac{a^2(a+bx)^2\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{b^4}+\frac{ac\log(f)\operatorname{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)}{b^4}-\frac{a(a+bx)^3f^{\frac{c}{(a+bx)^3}}}{b^4}+\frac{(a+bx)^4\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{4/3}\Gamma\left(-\frac{4}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^4}$$

input
```
Int[f^(c/(a + b*x)^3)*x^3,x]
```

output
```
-((a*f^(c/(a + b*x)^3)*(a + b*x)^3)/b^4) + (a*c*ExpIntegralEi[(c*Log[f])/(
a + b*x)^3]*Log[f])/b^4 - (a^3*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x
)^3)]*(-((c*Log[f])/(a + b*x)^3))^(1/3))/(3*b^4) + (a^2*(a + b*x)^2*Gamma[
-2/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3))/b^4 +
((a + b*x)^4*Gamma[-4/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x
)^3))^(4/3))/(3*b^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int f^{\frac{c}{(bx+a)^3}}x^3dx$$

input
```
int(f^(c/(b*x+a)^3)*x^3,x)
```

output
```
int(f^(c/(b*x+a)^3)*x^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.20

$$\int f^{\frac{c}{(a+bx)^3}} x^3 \, dx =$$

$$-\frac{6\,a^2b^2\left(-\frac{c\log(f)}{b^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},-\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right)-4\,ac\mathrm{Ei}\left(\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right)\log(f)-(4\,a^3b-3\,bc\,\mathrm{l}}{4}$$

input
```
integrate(f^(c/(b*x+a)^3)*x^3,x, algorithm="fricas")
```

output
```
-1/4*(6*a^2*b^2*(-c*log(f)/b^3)^(2/3)*gamma(1/3, -c*log(f)/(b^3*x^3 + 3*a*
b^2*x^2 + 3*a^2*b*x + a^3)) - 4*a*c*Ei(c*log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3
*a^2*b*x + a^3))*log(f) - (4*a^3*b - 3*b*c*log(f))*(-c*log(f)/b^3)^(1/3)*g
amma(2/3, -c*log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) - (b^4*x^4
- a^4 + 3*(b*c*x + a*c)*log(f))*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x +
a^3)))/b^4
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^3}} x^3 \, dx = \int f^{\frac{c}{(a+bx)^3}} x^3 \, dx$$

input
```
integrate(f**(c/(b*x+a)**3)*x**3,x)
```

output
```
Integral(f**(c/(a + b*x)**3)*x**3, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^3\,dx = \int f^{\frac{c}{(bx+a)^3}}x^3\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x^3,x, algorithm="maxima")
```

output
```
1/4*(b^3*x^4 + 3*c*x*log(f))*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3
))/b^3 - integrate(3/4*(4*a*b^3*c*x^3*log(f) + 6*a^2*b^2*c*x^2*log(f) + a^
4*c*log(f) + (4*a^3*b*c*log(f) - 3*b*c^2*log(f)^2)*x)*f^(c/(b^3*x^3 + 3*a*
b^2*x^2 + 3*a^2*b*x + a^3))/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3
*b^4*x + a^4*b^3), x)
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^3\,dx = \int f^{\frac{c}{(bx+a)^3}}x^3\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x^3,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^3}}x^3\,dx = \int f^{\frac{c}{(a+b\,x)^3}}\,x^3\,dx$$

input
```
int(f^(c/(a + b*x)^3)*x^3,x)
```

output
```
int(f^(c/(a + b*x)^3)*x^3, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^3}} x^3 \, dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^3)*x^3,x)
```

output
```
(27*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**4*b*c**
4*x + 9*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**3*a
**4*c**3 - 324*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(
f)**3*a**3*b*c**3*x - 513*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*
x**3))*log(f)**3*a**2*b**2*c**3*x**2 - 126*f**(c/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3))*log(f)**3*a*b**3*c**3*x**3 - 45*f**(c/(a**3 + 3*a*
*2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**3*b**4*c**3*x**4 - 126*f**(c/
(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**7*c**2 - 936
*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**6*b*c
**2*x + 324*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*
*2*a**5*b**2*c**2*x**2 + 2208*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b
**3*x**3))*log(f)**2*a**4*b**3*c**2*x**3 + 2100*f**(c/(a**3 + 3*a**2*b*x +
 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**3*b**4*c**2*x**4 + 846*f**(c/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**2*b**5*c**2*x**
5 + 156*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a
*b**6*c**2*x**6 + 30*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
)*log(f)**2*b**7*c**2*x**7 - 60*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
 b**3*x**3))*log(f)*a**10*c + 1596*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**
2 + b**3*x**3))*log(f)*a**9*b*c*x + 6924*f**(c/(a**3 + 3*a**2*b*x + 3*a*b*
*2*x**2 + b**3*x**3))*log(f)*a**8*b**2*c*x**2 + 11188*f**(c/(a**3 + 3*a...
```

3.167 $\int f^{\frac{c}{(a+bx)^3}} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 142

$$\int f^{\frac{c}{(a+bx)^3}} x^2 \, dx = \frac{f^{\frac{c}{(a+bx)^3}}(a+bx)^3}{3b^3} - \frac{c \, \text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)\log(f)}{3b^3} + \frac{a^2(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b^3} - \frac{2a(a+bx)^2\Gamma\left(-\frac{2}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}}{3b^3}$$

output
```
1/3*f^(c/(b*x+a)^3)*(b*x+a)^3/b^3-1/3*c*Ei(c*ln(f)/(b*x+a)^3)*ln(f)/b^3+1/
3*a^2*(b*x+a)*GAMMA(-1/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(1/3)/b^
3-2/3*a*(b*x+a)^2*GAMMA(-2/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(2/3
)/b^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int f^{\frac{c}{(a+bx)^3}} x^2\,dx$$
$$= \frac{f^{\frac{c}{(a+bx)^3}}(a+bx)^3 - c\,\text{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)\log(f) + a^2(a+bx)\Gamma\left(-\frac{1}{3}, -\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}} - 2a(}{3b^3}$$

input
```
Integrate[f^(c/(a + b*x)^3)*x^2,x]
```

output
```
(f^(c/(a + b*x)^3)*(a + b*x)^3 - c*ExpIntegralEi[(c*Log[f])/(a + b*x)^3]*L
og[f] + a^2*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])
/(a + b*x)^3))^(1/3) - 2*a*(a + b*x)^2*Gamma[-2/3, -((c*Log[f])/(a + b*x)^
3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3))/(3*b^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{\frac{c}{(a+bx)^3}}\,dx$$
$$\Big\downarrow \text{2656}$$
$$\int \left(\frac{a^2 f^{\frac{c}{(a+bx)^3}}}{b^2} + \frac{(a+bx)^2 f^{\frac{c}{(a+bx)^3}}}{b^2} - \frac{2a(a+bx) f^{\frac{c}{(a+bx)^3}}}{b^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2(a+bx)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^3}-\frac{c\log(f)\,\mathrm{ExpIntegralEi}\left(\frac{c\log(f)}{(a+bx)^3}\right)}{3b^3}+\frac{(a+bx)^3f^{\frac{c}{(a+bx)^3}}}{3b^3}-\frac{2a(a+bx)^2\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^3}$$

input
```
Int[f^(c/(a + b*x)^3)*x^2,x]
```

output
```
(f^(c/(a + b*x)^3)*(a + b*x)^3)/(3*b^3) - (c*ExpIntegralEi[(c*Log[f])/(a +
 b*x)^3]*Log[f])/(3*b^3) + (a^2*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*
x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(1/3))/(3*b^3) - (2*a*(a + b*x)^2*Gamma
[-2/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^3))^(2/3))/(3*b^
3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int f^{\frac{c}{(bx+a)^3}}x^2dx$$

input
```
int(f^(c/(b*x+a)^3)*x^2,x)
```

output
```
int(f^(c/(b*x+a)^3)*x^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.37

$$\int f^{\frac{c}{(a+bx)^3}} x^2 \, dx$$
$$= \frac{3\,ab^2\left(-\frac{c\log(f)}{b^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right) - 3\,a^2b\left(-\frac{c\log(f)}{b^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, -\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right) - c\mathrm{Ei}\left(\frac{}{b^3}\right.}{3\,b^3}$$

input
```
integrate(f^(c/(b*x+a)^3)*x^2,x, algorithm="fricas")
```

output
```
1/3*(3*a*b^2*(-c*log(f)/b^3)^(2/3)*gamma(1/3, -c*log(f)/(b^3*x^3 + 3*a*b^2
*x^2 + 3*a^2*b*x + a^3)) - 3*a^2*b*(-c*log(f)/b^3)^(1/3)*gamma(2/3, -c*log
(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) - c*Ei(c*log(f)/(b^3*x^3 +
3*a*b^2*x^2 + 3*a^2*b*x + a^3))*log(f) + (b^3*x^3 + a^3)*f^(c/(b^3*x^3 + 3
*a*b^2*x^2 + 3*a^2*b*x + a^3)))/b^3
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^3}} x^2 \, dx = \int f^{\frac{c}{(a+bx)^3}} x^2 \, dx$$

input
```
integrate(f**(c/(b*x+a)**3)*x**2,x)
```

output
```
Integral(f**(c/(a + b*x)**3)*x**2, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^2\,dx=\int f^{\frac{c}{(bx+a)^3}}x^2\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x^2,x, algorithm="maxima")
```

output
```
1/3*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*x^3 + b*c*integrate(f^
(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*x^3/(b^4*x^4 + 4*a*b^3*x^3 +
 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)*log(f)
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^3}}x^2\,dx=\int f^{\frac{c}{(bx+a)^3}}x^2\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x^2,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^3}}x^2\,dx=\int f^{\frac{c}{(a+b\,x)^3}}\,x^2\,dx$$

input
```
int(f^(c/(a + b*x)^3)*x^2,x)
```

output
```
int(f^(c/(a + b*x)^3)*x^2, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^3}} x^2 \, dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^3)*x^2,x)
```

output
```
(54*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**3*a**2*
b*c**3*x + 135*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(
f)**3*a*b**2*c**3*x**2 + 18*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3))*log(f)**2*a**6*c**2 + 432*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**
2 + b**3*x**3))*log(f)**2*a**5*b*c**2*x + 99*f**(c/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3))*log(f)**2*a**4*b**2*c**2*x**2 - 522*f**(c/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**3*b**3*c**2*x**3 -
 540*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**2
*b**4*c**2*x**4 - 180*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
))*log(f)**2*a*b**5*c**2*x**5 + 108*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x*
*2 + b**3*x**3))*log(f)*a**9*c - 330*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x
**2 + b**3*x**3))*log(f)*a**8*b*c*x - 2292*f**(c/(a**3 + 3*a**2*b*x + 3*a*
b**2*x**2 + b**3*x**3))*log(f)*a**7*b**2*c*x**2 - 4014*f**(c/(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**6*b**3*c*x**3 - 3150*f**(c/(
a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**5*b**4*c*x**4 -
1008*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**4*b*
*5*c*x**5 + 182*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log
(f)*a**3*b**6*c*x**6 + 300*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3))*log(f)*a**2*b**7*c*x**7 + 120*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*
x**2 + b**3*x**3))*log(f)*a*b**8*c*x**8 + 20*f**(c/(a**3 + 3*a**2*b*x +...
```

3.168 $\int f^{\frac{c}{(a+bx)^3}} x\,dx$

Optimal result

Integrand size = 13, antiderivative size = 92

$$\int f^{\frac{c}{(a+bx)^3}} x\,dx = -\frac{a(a+bx)\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b^2} + \frac{(a+bx)^2\Gamma\left(-\frac{2}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}}{3b^2}$$

output
```
-1/3*a*(b*x+a)*GAMMA(-1/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(1/3)/b
^2+1/3*(b*x+a)^2*GAMMA(-2/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(2/3)
/b^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int f^{\frac{c}{(a+bx)^3}} x\,dx = \frac{(a+bx)\left(-a\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}} + (a+bx)\Gamma\left(-\frac{2}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3}\right)}{3b^2}$$

input
```
Integrate[f^(c/(a + b*x)^3)*x,x]
```

output
```
((a + b*x)*(-(a*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a +
b*x)^3))^(1/3)) + (a + b*x)*Gamma[-2/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*L
og[f])/(a + b*x)^3))^(2/3)))/(3*b^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x f^{\frac{c}{(a+bx)^3}} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(a+bx) f^{\frac{c}{(a+bx)^3}}}{b} - \frac{a f^{\frac{c}{(a+bx)^3}}}{b}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+bx)^2 \left(-\frac{c\log(f)}{(a+bx)^3}\right)^{2/3} \Gamma\left(-\frac{2}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^2} - \frac{a(a+bx)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b^2}$$

input
```
Int[f^(c/(a + b*x)^3)*x,x]
```

output
```
-1/3*(a*(a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a
+ b*x)^3))^(1/3))/b^2 + ((a + b*x)^2*Gamma[-2/3, -((c*Log[f])/(a + b*x)^3)
]*(-((c*Log[f])/(a + b*x)^3))^(2/3))/(3*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int f^{\frac{c}{(bx+a)^3}} x dx$$

input
```
int(f^(c/(b*x+a)^3)*x,x)
```

output
```
int(f^(c/(b*x+a)^3)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.67

$$\int f^{\frac{c}{(a+bx)^3}} x\,dx = -\frac{b^2\left(-\frac{c\log(f)}{b^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right) - 2\,ab\left(-\frac{c\log(f)}{b^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, -\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right) - (b^2x^2 -}{2\,b^2}$$

input
```
integrate(f^(c/(b*x+a)^3)*x,x, algorithm="fricas")
```

output
```
-1/2*(b^2*(-c*log(f)/b^3)^(2/3)*gamma(1/3, -c*log(f)/(b^3*x^3 + 3*a*b^2*x^
2 + 3*a^2*b*x + a^3)) - 2*a*b*(-c*log(f)/b^3)^(1/3)*gamma(2/3, -c*log(f)/(
b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)) - (b^2*x^2 - a^2)*f^(c/(b^3*x^3
+ 3*a*b^2*x^2 + 3*a^2*b*x + a^3)))/b^2
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^3}} x \, dx = \int f^{\frac{c}{(a+bx)^3}} x \, dx$$

input
```
integrate(f**(c/(b*x+a)**3)*x,x)
```

output
```
Integral(f**(c/(a + b*x)**3)*x, x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^3}} x \, dx = \int f^{\frac{c}{(bx+a)^3}} x \, dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x,x, algorithm="maxima")
```

output
```
3*b*c*integrate(1/2*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*x^2/(b
^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)*log(f) + 1/2*f
^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*x^2
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^3}} x \, dx = \int f^{\frac{c}{(bx+a)^3}} x \, dx$$

input
```
integrate(f^(c/(b*x+a)^3)*x,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{\frac{c}{(a+bx)^3}}x\,dx=\int f^{\frac{c}{(a+b\,x)^3}}\,x\,dx$$

input
```
int(f^(c/(a + b*x)^3)*x,x)
```

output
```
int(f^(c/(a + b*x)^3)*x, x)
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^3}}x\,dx=\text{too large to display}$$

input
```
int(f^(c/(b*x+a)^3)*x,x)
```

output
```
(9*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a*c**2
 + 45*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*b*c
**2*x - 60*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a
**3*b*c*x - 180*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log
(f)*a**2*b**2*c*x**2 - 180*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3))*log(f)*a*b**3*c*x**3 - 60*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
 + b**3*x**3))*log(f)*b**4*c*x**4 - 20*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3))*a**7 - 100*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3))*a**6*b*x - 180*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3))*a**5*b**2*x**2 - 100*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3))*a**4*b**3*x**3 + 100*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3))*a**3*b**4*x**4 + 180*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3))*a**2*b**5*x**5 + 100*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**
2 + b**3*x**3))*a*b**6*x**6 + 20*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3))*b**7*x**7 + 27*int(f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3))/(a**9 + 9*a**8*b*x + 36*a**7*b**2*x**2 + 84*a**6*b**3*x**3 +
 126*a**5*b**4*x**4 + 126*a**4*b**5*x**5 + 84*a**3*b**6*x**6 + 36*a**2*b**
7*x**7 + 9*a*b**8*x**8 + b**9*x**9),x)*log(f)**3*a**6*b*c**3 + 135*int(f**
(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/(a**9 + 9*a**8*b*x + 3
6*a**7*b**2*x**2 + 84*a**6*b**3*x**3 + 126*a**5*b**4*x**4 + 126*a**4*b*...
```

3.169 $\int f^{\frac{c}{(a+bx)^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 44

$$\int f^{\frac{c}{(a+bx)^3}}\,dx=\frac{(a+bx)\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b}$$

output
```
1/3*(b*x+a)*GAMMA(-1/3,-c*ln(f)/(b*x+a)^3)*(-c*ln(f)/(b*x+a)^3)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int f^{\frac{c}{(a+bx)^3}}\,dx=\frac{(a+bx)\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}}{3b}$$

input
```
Integrate[f^(c/(a + b*x)^3),x]
```

output
```
((a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^
3))^(1/3))/(3*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{\frac{c}{(a+bx)^3}}dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{(a+bx)\sqrt[3]{-\frac{c\log(f)}{(a+bx)^3}}\Gamma\left(-\frac{1}{3},-\frac{c\log(f)}{(a+bx)^3}\right)}{3b}$$

input
```
Int[f^(c/(a + b*x)^3),x]
```

output
```
((a + b*x)*Gamma[-1/3, -((c*Log[f])/(a + b*x)^3)]*(-((c*Log[f])/(a + b*x)^
3))^(1/3))/(3*b)
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Maple [F]

$$\int f^{\frac{c}{(bx+a)^3}}dx$$

input
```
int(f^(c/(b*x+a)^3),x)
```

output
```
int(f^(c/(b*x+a)^3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(38) = 76.

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.14

$$\int f^{\frac{c}{(a+bx)^3}}\,dx = -\frac{b\left(-\frac{c\log(f)}{b^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, -\frac{c\log(f)}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}\right) - (bx+a)f^{\frac{c}{b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3}}}{b}$$

input
```
integrate(f^(c/(b*x+a)^3),x, algorithm="fricas")
```

output
```
-(b*(-c*log(f)/b^3)^(1/3)*gamma(2/3, -c*log(f)/(b^3*x^3 + 3*a*b^2*x^2 + 3*
a^2*b*x + a^3)) - (b*x + a)*f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)
))/b
```

Sympy [F]

$$\int f^{\frac{c}{(a+bx)^3}}\,dx = \int f^{\frac{c}{(a+bx)^3}}\,dx$$

input
```
integrate(f**(c/(b*x+a)**3),x)
```

output
```
Integral(f**(c/(a + b*x)**3), x)
```

Maxima [F]

$$\int f^{\frac{c}{(a+bx)^3}}\,dx = \int f^{\frac{c}{(bx+a)^3}}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3),x, algorithm="maxima")
```

output
```
3*b*c*integrate(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*x/(b^4*x^4
 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4), x)*log(f) + f^(c/(b^3*x
^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))*x
```

Giac [F]

$$\int f^{\frac{c}{(a+bx)^3}}\,dx = \int f^{\frac{c}{(bx+a)^3}}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3),x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int f^{\frac{c}{(a+bx)^3}}\,dx = \frac{(a+b\,x)\left(\Gamma\left(\frac{2}{3}\right)\left(-\frac{c\ln(f)}{(a+b\,x)^3}\right)^{1/3}-\Gamma\left(\frac{2}{3},-\frac{c\ln(f)}{(a+b\,x)^3}\right)\left(-\frac{c\ln(f)}{(a+b\,x)^3}\right)^{1/3}+f^{\frac{c}{(a+b\,x)^3}}\right)}{b}$$

input
```
int(f^(c/(a + b*x)^3),x)
```

output
```
((a + b*x)*(gamma(2/3)*(-(c*log(f))/(a + b*x)^3)^(1/3) - igamma(2/3, -(c*l
og(f))/(a + b*x)^3)*(-(c*log(f))/(a + b*x)^3)^(1/3) + f^(c/(a + b*x)^3)))/
b
```

Reduce [F]

$$\int f^{\frac{c}{(a+bx)^3}}\,dx = \text{too large to display}$$

input
```
int(f^(c/(b*x+a)^3),x)
```

output
```
(9*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*b*c**2
*x + 3*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**4*
c - 6*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**3*b
*c*x - 36*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a*
*2*b**2*c*x**2 - 42*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
*log(f)*a*b**3*c*x**3 - 15*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3))*log(f)*b**4*c*x**4 - 8*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3))*a**7 - 38*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
))*a**6*b*x - 60*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*a*
*5*b**2*x**2 - 10*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*a
**4*b**3*x**3 + 80*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*
a**3*b**4*x**4 + 102*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
)*a**2*b**5*x**5 + 52*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
))*a*b**6*x**6 + 10*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
*b**7*x**7 + 27*int((f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
)*x)/(a**10 + 10*a**9*b*x + 45*a**8*b**2*x**2 + 120*a**7*b**3*x**3 + 210*a
**6*b**4*x**4 + 252*a**5*b**5*x**5 + 210*a**4*b**6*x**6 + 120*a**3*b**7*x*
*7 + 45*a**2*b**8*x**8 + 10*a*b**9*x**9 + b**10*x**10),x)*log(f)**3*a**6*b
**2*c**3 + 162*int((f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
*x)/(a**10 + 10*a**9*b*x + 45*a**8*b**2*x**2 + 120*a**7*b**3*x**3 + 210...
```

3.170 $\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \text{Int}\left(\frac{f^{\frac{c}{(a+bx)^3}}}{x},x\right)$$

output
```
Defer(Int)(f^(c/(b*x+a)^3)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx$$

input
```
Integrate[f^(c/(a + b*x)^3)/x,x]
```

output
```
Integrate[f^(c/(a + b*x)^3)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x} \, dx$$
$$\Big\downarrow \text{ 2654}$$
$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}dx$$

input
```
Int[f^(c/(a + b*x)^3)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x}dx$$

input
```
int(f^(c/(b*x+a)^3)/x,x)
```

output
```
int(f^(c/(b*x+a)^3)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x,x, algorithm="fricas")
```

output
```
integral(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx$$

input
```
integrate(f**(c/(b*x+a)**3)/x,x)
```

output
```
Integral(f**(c/(a + b*x)**3)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a)^3)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x}\,dx = \int \frac{f^{\frac{c}{(a+b\,x)^3}}}{x}\,dx$$

input
```
int(f^(c/(a + b*x)^3)/x,x)
```

output
```
int(f^(c/(a + b*x)^3)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x} \, dx = \int \frac{f^{\frac{c}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}}}{x} dx$$

input
```
int(f^(c/(b*x+a)^3)/x,x)
```

output
```
int(f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))/x,x)
```

3.171 $\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} dx = \text{Int}\left(\frac{f^{\frac{c}{(a+bx)^3}}}{x^2}, x\right)$$

output
```
Defer(Int)(f^(c/(b*x+a)^3)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} dx = \int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} dx$$

input
```
Integrate[f^(c/(a + b*x)^3)/x^2,x]
```

output
```
Integrate[f^(c/(a + b*x)^3)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} dx$$

input
```
Int[f^(c/(a + b*x)^3)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x^2} dx$$

input
```
int(f^(c/(b*x+a)^3)/x^2,x)
```

output
```
int(f^(c/(b*x+a)^3)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x^2,x, algorithm="fricas")
```

output
```
integral(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2}\,dx$$

input
```
integrate(f**(c/(b*x+a)**3)/x**2,x)
```

output
```
Integral(f**(c/(a + b*x)**3)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x^2,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a)^3)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x^2}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2}\,dx = \int \frac{f^{\frac{c}{(a+b\,x)^3}}}{x^2}\,dx$$

input
```
int(f^(c/(a + b*x)^3)/x^2,x)
```

output
```
int(f^(c/(a + b*x)^3)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10349, normalized size of antiderivative = 689.93

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^2} \, dx = \text{Too large to display}$$

input
```
int(f^(c/(b*x+a)^3)/x^2,x)
```

output
```
(315*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**3*a**4
*c**3 + 594*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*
*3*a**2*b**2*c**3*x**2 - 110*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b*
*3*x**3))*log(f)**3*b**4*c**3*x**4 - 440*f**(c/(a**3 + 3*a**2*b*x + 3*a*b*
*2*x**2 + b**3*x**3))*log(f)**2*a**7*c**2 - 2322*f**(c/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**6*b*c**2*x - 2046*f**(c/(a**3 +
 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**5*b**2*c**2*x**2 -
3256*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**4
*b**3*c**2*x**3 - 3102*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3))*log(f)**2*a**3*b**4*c**2*x**4 - 990*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**
2*x**2 + b**3*x**3))*log(f)**2*a**2*b**5*c**2*x**5 + 220*f**(c/(a**3 + 3*a
**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a*b**6*c**2*x**6 + 110*f**
(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*b**7*c**2*x*
*7 + 882*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**
9*b*c*x - 1248*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(
f)*a**8*b**2*c*x**2 - 12930*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3))*log(f)*a**7*b**3*c*x**3 - 23780*f**(c/(a**3 + 3*a**2*b*x + 3*a*b*
*2*x**2 + b**3*x**3))*log(f)*a**6*b**4*c*x**4 - 19470*f**(c/(a**3 + 3*a**2
*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**5*b**5*c*x**5 - 7788*f**(c/(a
**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**4*b**6*c*x**6 ...
```

3.172 $\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} dx = \text{Int}\left(\frac{f^{\frac{c}{(a+bx)^3}}}{x^3}, x\right)$$

output
```
Defer(Int)(f^(c/(b*x+a)^3)/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} dx = \int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} dx$$

input
```
Integrate[f^(c/(a + b*x)^3)/x^3,x]
```

output
```
Integrate[f^(c/(a + b*x)^3)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{7299\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} dx$$

input
```
Int[f^(c/(a + b*x)^3)/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{f^{\frac{c}{(bx+a)^3}}}{x^3} dx$$

input
```
int(f^(c/(b*x+a)^3)/x^3,x)
```

output
```
int(f^(c/(b*x+a)^3)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x^3,x, algorithm="fricas")
```

output
```
integral(f^(c/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3}\,dx$$

input
```
integrate(f**(c/(b*x+a)**3)/x**3,x)
```

output
```
Integral(f**(c/(a + b*x)**3)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x^3,x, algorithm="maxima")
```

output
```
integrate(f^(c/(b*x + a)^3)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(bx+a)^3}}}{x^3}\,dx$$

input
```
integrate(f^(c/(b*x+a)^3)/x^3,x, algorithm="giac")
```

output
```
integrate(f^(c/(b*x + a)^3)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3}\,dx = \int \frac{f^{\frac{c}{(a+b\,x)^3}}}{x^3}\,dx$$

input
```
int(f^(c/(a + b*x)^3)/x^3,x)
```

output
```
int(f^(c/(a + b*x)^3)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12757, normalized size of antiderivative = 850.47

$$\int \frac{f^{\frac{c}{(a+bx)^3}}}{x^3} \, dx = \text{Too large to display}$$

input
```
int(f^(c/(b*x+a)^3)/x^3,x)
```

output
```
( - 9*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**4*b*c
**4*x + 66*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**
3*a**4*c**3 + 180*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*l
og(f)**3*a**3*b*c**3*x + 72*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3))*log(f)**3*a**2*b**2*c**3*x**2 + 66*f**(c/(a**3 + 3*a**2*b*x + 3*a
*b**2*x**2 + b**3*x**3))*log(f)**3*a*b**3*c**3*x**3 - 110*f**(c/(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**7*c**2 - 660*f**(c/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**6*b*c**2*x - 21
34*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**5*b
**2*c**2*x**2 - 1708*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
)*log(f)**2*a**4*b**3*c**2*x**3 - 1158*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3))*log(f)**2*a**3*b**4*c**2*x**4 - 496*f**(c/(a**3 + 3*a*
*2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a**2*b**5*c**2*x**5 - 110*f
**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)**2*a*b**6*c**
2*x**6 + 1132*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f
)*a**8*b**2*c*x**2 + 2904*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*
x**3))*log(f)*a**7*b**3*c*x**3 + 1428*f**(c/(a**3 + 3*a**2*b*x + 3*a*b**2*
x**2 + b**3*x**3))*log(f)*a**6*b**4*c*x**4 - 1984*f**(c/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**5*b**5*c*x**5 - 2460*f**(c/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))*log(f)*a**4*b**6*c*x**6 - 98...
```

3.173 $\int F^{c(a+bx)^3} x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int F^{c(a+bx)^3} x^m \, dx = \text{Int}\left(F^{c(a+bx)^3} x^m, x\right)$$

output
```
Defer(Int)(F^(c*(b*x+a)^3)*x^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^3} x^m \, dx = \int F^{c(a+bx)^3} x^m \, dx$$

input
```
Integrate[F^(c*(a + b*x)^3)*x^m,x]
```

output
```
Integrate[F^(c*(a + b*x)^3)*x^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{c(a+bx)^3} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int x^m F^{c(a+bx)^3} dx$$

input
```
Int[F^(c*(a + b*x)^3)*x^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{c(bx+a)^3} x^m dx$$

input
```
int(F^(c*(b*x+a)^3)*x^m,x)
```

output
```
int(F^(c*(b*x+a)^3)*x^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.67

$$\int F^{c(a+bx)^3}x^m\,dx = \int F^{(bx+a)^3c}x^m\,dx$$

input
```
integrate(F^(c*(b*x+a)^3)*x^m,x, algorithm="fricas")
```

output
```
integral(F^(b^3*c*x^3 + 3*a*b^2*c*x^2 + 3*a^2*b*c*x + a^3*c)*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)^3}x^m\,dx = \int F^{c(a+bx)^3}x^m\,dx$$

input
```
integrate(F**(c*(b*x+a)**3)*x**m,x)
```

output
```
Integral(F**(c*(a + b*x)**3)*x**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^3}x^m\,dx = \int F^{(bx+a)^3c}x^m\,dx$$

input
```
integrate(F^(c*(b*x+a)^3)*x^m,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^3*c)*x^m, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^3}x^m\,dx = \int F^{(bx+a)^3c}x^m\,dx$$

input
```
integrate(F^(c*(b*x+a)^3)*x^m,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^3*c)*x^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^3}x^m\,dx = \int F^{c\,(a+b\,x)^3}\,x^m\,dx$$

input
```
int(F^(c*(a + b*x)^3)*x^m,x)
```

output
```
int(F^(c*(a + b*x)^3)*x^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int F^{c(a+bx)^3}x^m\,dx = f^{a^3c}\left(\int x^m f^{b^3c\,x^3+3a\,b^2c\,x^2+3a^2bcx}dx\right)$$

input
```
int(F^(c*(b*x+a)^3)*x^m,x)
```

output
```
f**(a**3*c)*int(x**m*f**(3*a**2*b*c*x + 3*a*b**2*c*x**2 + b**3*c*x**3),x)
```

3.174 $\int F^{c(a+bx)^2} x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int F^{c(a+bx)^2} x^m \, dx = \text{Int}\left(F^{a^2c+2abcx+b^2cx^2} x^m, x\right)$$

output
```
Defer(Int)(F^(b^2*c*x^2+2*a*b*c*x+a^2*c)*x^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^2} x^m \, dx = \int F^{c(a+bx)^2} x^m \, dx$$

input
```
Integrate[F^(c*(a + b*x)^2)*x^m,x]
```

output
```
Integrate[F^(c*(a + b*x)^2)*x^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2674, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{c(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 2674}$$

$$\int x^m F^{a^2c+2abcx+b^2cx^2} dx$$

$$\Big\downarrow \text{ 2673}$$

$$\int x^m F^{a^2c+2abcx+b^2cx^2} dx$$

input
```
Int[F^(c*(a + b*x)^2)*x^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

rule 2674
```
Int[(F_)^(v_)*(u_)^(m_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*F^ExpandToSu
m[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !(
LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{c(bx+a)^2} x^m dx$$

input
```
int(F^(c*(b*x+a)^2)*x^m,x)
```

output
```
int(F^(c*(b*x+a)^2)*x^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int F^{c(a+bx)^2} x^m \, dx = \int F^{(bx+a)^2 c} x^m \, dx$$

input
```
integrate(F^(c*(b*x+a)^2)*x^m,x, algorithm="fricas")
```

output
```
integral(F^(b^2*c*x^2 + 2*a*b*c*x + a^2*c)*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)^2} x^m \, dx = \int F^{c(a+bx)^2} x^m \, dx$$

input
```
integrate(F**(c*(b*x+a)**2)*x**m,x)
```

output
```
Integral(F**(c*(a + b*x)**2)*x**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^2}x^m\,dx=\int F^{(bx+a)^2c}x^m\,dx$$

input
```
integrate(F^(c*(b*x+a)^2)*x^m,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^2*c)*x^m, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^2}x^m\,dx=\int F^{(bx+a)^2c}x^m\,dx$$

input
```
integrate(F^(c*(b*x+a)^2)*x^m,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^2*c)*x^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^2}x^m\,dx=\int F^{c\,(a+b\,x)^2}\,x^m\,dx$$

input
```
int(F^(c*(a + b*x)^2)*x^m,x)
```

output
```
int(F^(c*(a + b*x)^2)*x^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int F^{c(a+bx)^2}x^m\,dx = f^{a^2c}\left(\int x^m f^{b^2c\,x^2+2abcx}dx\right)$$

input
```
int(F^(c*(b*x+a)^2)*x^m,x)
```

output
```
f**(a**2*c)*int(x**m*f**(2*a*b*c*x + b**2*c*x**2),x)
```

3.175 $\int F^{c(a+bx)}x^m\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int F^{c(a+bx)}x^m\,dx=\frac{F^{ac}x^m\Gamma(1+m,-bcx\log(F))(-bcx\log(F))^{-m}}{bc\log(F)}$$

output
```
F^(a*c)*x^m*GAMMA(1+m,-b*c*x*ln(F))/b/c/ln(F)/((-b*c*x*ln(F))^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)}x^m\,dx=-F^{ac}x^{1+m}\Gamma(1+m,-bcx\log(F))(-bcx\log(F))^{-1-m}$$

input
```
Integrate[F^(c*(a + b*x))*x^m,x]
```

output
```
-(F^(a*c)*x^(1 + m)*Gamma[1 + m, -(b*c*x*Log[F])]*(-(b*c*x*Log[F]))^(-1 -
m))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{c(a+bx)} \, dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{x^m F^{ac} (-bcx \log(F))^{-m} \Gamma(m+1,-bcx \log(F))}{bc \log(F)}$$

input
```
Int[F^(c*(a + b*x))*x^m,x]
```

output
```
(F^(a*c)*x^m*Gamma[1 + m, -(b*c*x*Log[F])])/(b*c*Log[F]*(-(b*c*x*Log[F]))^
m)
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(41) = 82.

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.85

method	result
meijerg	$-\frac{F^{ac}(-bc)^{-m}\ln(F)^{-1-m}\left(x^m(-bc)^m\ln(F)^m m\Gamma(m)(-bcx\ln(F))^{-m}-x^m(-bc)^m\ln(F)^m \mathrm{e}^{bcx\ln(F)}-x^m(-bc)^m\ln(F)^m m(-bc\right.}{bc}$

input
```
int(F^((b*x+a)*c)*x^m,x,method=_RETURNVERBOSE)
```

output
```
-F^(a*c)*(-b*c)^(-m)*ln(F)^(-1-m)/b/c*(x^m*(-b*c)^m*ln(F)^m*m*GAMMA(m)*(-b
*c*x*ln(F))^(-m)-x^m*(-b*c)^m*ln(F)^m*exp(b*c*x*ln(F))-x^m*(-b*c)^m*ln(F)^
m*m*(-b*c*x*ln(F))^(-m)*GAMMA(m,-b*c*x*ln(F)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int F^{c(a+bx)}x^m\,dx = \frac{e^{(ac\log(F)-m\log(-bc\log(F)))}\Gamma(m+1,-bcx\log(F))}{bc\log(F)}$$

input
```
integrate(F^((b*x+a)*c)*x^m,x, algorithm="fricas")
```

output
```
e^(a*c*log(F) - m*log(-b*c*log(F)))*gamma(m + 1, -b*c*x*log(F))/(b*c*log(F
))
```

Sympy [F]

$$\int F^{c(a+bx)}x^m\,dx = \int F^{c(a+bx)}x^m\,dx$$

input
```
integrate(F**((b*x+a)*c)*x**m,x)
```

output
```
Integral(F**(c*(a + b*x))*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)}x^m\,dx = -(-bcx\log(F))^{-m-1}F^{ac}x^{m+1}\Gamma(m+1,-bcx\log(F))$$

input
```
integrate(F^((b*x+a)*c)*x^m,x, algorithm="maxima")
```

output
```
-(-b*c*x*log(F))^(-m - 1)*F^(a*c)*x^(m + 1)*gamma(m + 1, -b*c*x*log(F))
```

Giac [F]

$$\int F^{c(a+bx)}x^m\,dx = \int F^{(bx+a)c}x^m\,dx$$

input
```
integrate(F^((b*x+a)*c)*x^m,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}x^m\,dx=\frac{F^{a\,c}\,x^m\,\Gamma(m+1,-b\,c\,x\,\ln(F))}{b\,c\,\ln(F)\,\left(-b\,c\,x\,\ln(F)\right)^m}$$

input
```
int(F^(c*(a + b*x))*x^m,x)
```

output
```
(F^(a*c)*x^m*igamma(m + 1, -b*c*x*log(F)))/(b*c*log(F)*(-b*c*x*log(F))^m)
```

Reduce [F]

$$\int F^{c(a+bx)}x^m\,dx=\frac{f^{ac}\left(x^m f^{bcx}-\left(\int\frac{x^m f^{bcx}}{x}dx\right)m\right)}{\log(f)\,bc}$$

input
```
int(F^((b*x+a)*c)*x^m,x)
```

output
```
(f**(a*c)*(x**m*f**(b*c*x) - int((x**m*f**(b*c*x))/x,x)*m))/(log(f)*b*c)
```

3.176 $\int F^{\frac{c}{a+bx}} x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int F^{\frac{c}{a+bx}} x^m \, dx = \text{Int}\left(F^{\frac{c}{a+bx}} x^m, x\right)$$

output
```
Defer(Int)(F^(c/(b*x+a))*x^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{a+bx}} x^m \, dx = \int F^{\frac{c}{a+bx}} x^m \, dx$$

input
```
Integrate[F^(c/(a + b*x))*x^m,x]
```

output
```
Integrate[F^(c/(a + b*x))*x^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{\frac{c}{a+bx}} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int x^m F^{\frac{c}{a+bx}} dx$$

input
```
Int[F^(c/(a + b*x))*x^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{\frac{c}{bx+a}} x^m dx$$

input
```
int(F^(c/(b*x+a))*x^m,x)
```

output
```
int(F^(c/(b*x+a))*x^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{a+bx}}x^m\,dx = \int F^{\frac{c}{bx+a}}x^m\,dx$$

input
```
integrate(F^(c/(b*x+a))*x^m,x, algorithm="fricas")
```

output
```
integral(F^(c/(b*x + a))*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int F^{\frac{c}{a+bx}}x^m\,dx = \int F^{\frac{c}{a+bx}}x^m\,dx$$

input
```
integrate(F**(c/(b*x+a))*x**m,x)
```

output
```
Integral(F**(c/(a + b*x))*x**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{a+bx}}x^m\,dx=\int F^{\frac{c}{bx+a}}x^m\,dx$$

input
```
integrate(F^(c/(b*x+a))*x^m,x, algorithm="maxima")
```

output
```
integrate(F^(c/(b*x + a))*x^m, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{a+bx}}x^m\,dx=\int F^{\frac{c}{bx+a}}x^m\,dx$$

input
```
integrate(F^(c/(b*x+a))*x^m,x, algorithm="giac")
```

output
```
integrate(F^(c/(b*x + a))*x^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{a+bx}}x^m\,dx=\int F^{\frac{c}{a+b\,x}}\,x^m\,dx$$

input
```
int(F^(c/(a + b*x))*x^m,x)
```

output
```
int(F^(c/(a + b*x))*x^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 77721, normalized size of antiderivative = 5181.40

$$\int F^{\frac{c}{a+bx}} x^m \, dx = \text{Too large to display}$$

input
```
int(F^(c/(b*x+a))*x^m,x)
```

output
```
(x**m*f**(c/(a + b*x))*log(f)**4*c**4 - 4*x**m*f**(c/(a + b*x))*log(f)**3*
a*c**3*m + x**m*f**(c/(a + b*x))*log(f)**3*b*c**3*m*x - 2*x**m*f**(c/(a +
b*x))*log(f)**3*b*c**3*x + 5*x**m*f**(c/(a + b*x))*log(f)**2*a**2*c**2*m**
2 - 2*x**m*f**(c/(a + b*x))*log(f)**2*a**2*c**2 - 2*x**m*f**(c/(a + b*x))*
log(f)**2*a*b*c**2*m**2*x + 4*x**m*f**(c/(a + b*x))*log(f)**2*a*b*c**2*m*x
 + x**m*f**(c/(a + b*x))*log(f)**2*b**2*c**2*m**2*x**2 - 3*x**m*f**(c/(a +
 b*x))*log(f)**2*b**2*c**2*m*x**2 + 2*x**m*f**(c/(a + b*x))*log(f)**2*b**2
*c**2*x**2 - x**m*f**(c/(a + b*x))*log(f)*a**3*c*m**3 - 3*x**m*f**(c/(a +
b*x))*log(f)*a**3*c*m**2 + 2*x**m*f**(c/(a + b*x))*log(f)*a**3*c*m + 4*x**
m*f**(c/(a + b*x))*log(f)*a**3*c + x**m*f**(c/(a + b*x))*log(f)*a**2*b*c*m
**3*x + x**m*f**(c/(a + b*x))*log(f)*a**2*b*c*m**2*x - 10*x**m*f**(c/(a +
b*x))*log(f)*a**2*b*c*m*x + 8*x**m*f**(c/(a + b*x))*log(f)*a**2*b*c*x + 2*
x**m*f**(c/(a + b*x))*log(f)*a*b**2*c*m**2*x**2 - 6*x**m*f**(c/(a + b*x))*
log(f)*a*b**2*c*m*x**2 + 4*x**m*f**(c/(a + b*x))*log(f)*a*b**2*c*x**2 + x*
*m*f**(c/(a + b*x))*log(f)*b**3*c*m**3*x**3 - 3*x**m*f**(c/(a + b*x))*log(
f)*b**3*c*m**2*x**3 + 2*x**m*f**(c/(a + b*x))*log(f)*b**3*c*m*x**3 - 2*x**
m*f**(c/(a + b*x))*a**3*b*m**4*x + 8*x**m*f**(c/(a + b*x))*a**3*b*m**3*x -
 10*x**m*f**(c/(a + b*x))*a**3*b*m**2*x + 4*x**m*f**(c/(a + b*x))*a**3*b*m
*x - 4*x**m*f**(c/(a + b*x))*a**2*b**2*m**4*x**2 + 16*x**m*f**(c/(a + b*x)
)*a**2*b**2*m**3*x**2 - 20*x**m*f**(c/(a + b*x))*a**2*b**2*m**2*x**2 + ...
```

3.177 $\int F^{\frac{c}{(a+bx)^2}} x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int F^{\frac{c}{(a+bx)^2}} x^m \, dx = \text{Int}\left(F^{\frac{c}{(a+bx)^2}} x^m, x\right)$$

output
```
Defer(Int)(F^(c/(b*x+a)^2)*x^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{(a+bx)^2}} x^m \, dx = \int F^{\frac{c}{(a+bx)^2}} x^m \, dx$$

input
```
Integrate[F^(c/(a + b*x)^2)*x^m,x]
```

output
```
Integrate[F^(c/(a + b*x)^2)*x^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{\frac{c}{(a+bx)^2}} dx$$
$$\Big\downarrow \text{7299}$$
$$\int x^m F^{\frac{c}{(a+bx)^2}} dx$$

input
```
Int[F^(c/(a + b*x)^2)*x^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{\frac{c}{(bx+a)^2}} x^m dx$$

input
```
int(F^(c/(b*x+a)^2)*x^m,x)
```

output
```
int(F^(c/(b*x+a)^2)*x^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.87

$$\int F^{\frac{c}{(a+bx)^2}}x^m\,dx = \int F^{\frac{c}{(bx+a)^2}}x^m\,dx$$

input
```
integrate(F^(c/(b*x+a)^2)*x^m,x, algorithm="fricas")
```

output
```
integral(F^(c/(b^2*x^2 + 2*a*b*x + a^2))*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 80.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int F^{\frac{c}{(a+bx)^2}}x^m\,dx = \int F^{\frac{c}{(a+bx)^2}}x^m\,dx$$

input
```
integrate(F**(c/(b*x+a)**2)*x***m,x)
```

output
```
Integral(F**(c/(a + b*x)**2)*x***m, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{(a+bx)^2}} x^m \, dx = \int F^{\frac{c}{(bx+a)^2}} x^m \, dx$$

input
```
integrate(F^(c/(b*x+a)^2)*x^m,x, algorithm="maxima")
```

output
```
integrate(F^(c/(b*x + a)^2)*x^m, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{(a+bx)^2}} x^m \, dx = \int F^{\frac{c}{(bx+a)^2}} x^m \, dx$$

input
```
integrate(F^(c/(b*x+a)^2)*x^m,x, algorithm="giac")
```

output
```
integrate(F^(c/(b*x + a)^2)*x^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{\frac{c}{(a+bx)^2}} x^m \, dx = \int F^{\frac{c}{(a+b\,x)^2}} \, x^m \, dx$$

input
```
int(F^(c/(a + b*x)^2)*x^m,x)
```

output
```
int(F^(c/(a + b*x)^2)*x^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 441986, normalized size of antiderivative = 29465.73

$$\int F^{\frac{c}{(a+bx)^2}} x^m \, dx = \text{Too large to display}$$

input
```
int(F^(c/(b*x+a)^2)*x^m,x)
```

output
```
( - 24*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*c**2*m**3 + 10
0*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*c**2*m**2 - 36*x**m
*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*c**2*m - 160*x**m*f**(c/(
a**2 + 2*a*b*x + b**2*x**2))*log(f)**2*a*c**2 + 12*x**m*f**(c/(a**2 + 2*a*
b*x + b**2*x**2))*log(f)**2*b*c**2*m**3*x - 104*x**m*f**(c/(a**2 + 2*a*b*x
 + b**2*x**2))*log(f)**2*b*c**2*m**2*x + 288*x**m*f**(c/(a**2 + 2*a*b*x +
b**2*x**2))*log(f)**2*b*c**2*m*x - 256*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x
**2))*log(f)**2*b*c**2*x + 2*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(
f)*a**3*c*m**4 - 12*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*c
*m**3 - 62*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*c*m**2 + 1
44*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*c*m + 192*x**m*f**
(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a**3*c + 72*x**m*f**(c/(a**2 + 2*a
*b*x + b**2*x**2))*log(f)*a**2*b*c*m**2*x - 480*x**m*f**(c/(a**2 + 2*a*b*x
 + b**2*x**2))*log(f)*a**2*b*c*m*x + 768*x**m*f**(c/(a**2 + 2*a*b*x + b**2
*x**2))*log(f)*a**2*b*c*x + 6*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log
(f)*a*b**2*c*m**4*x**2 - 88*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f
)*a*b**2*c*m**3*x**2 + 474*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)
*a*b**2*c*m**2*x**2 - 1112*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)
*a*b**2*c*m*x**2 + 960*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*a*b
**2*c*x**2 + 6*x**m*f**(c/(a**2 + 2*a*b*x + b**2*x**2))*log(f)*b**3*c*m...
```

3.178 $\int F^{c(a+bx)^n} x^m \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int F^{c(a+bx)^n} x^m \, dx = \text{Int}\left(F^{c(a+bx)^n} x^m, x\right)$$

output
```
Defer(Int)(F^(c*(b*x+a)^n)*x^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^n} x^m \, dx = \int F^{c(a+bx)^n} x^m \, dx$$

input
```
Integrate[F^(c*(a + b*x)^n)*x^m,x]
```

output
```
Integrate[F^(c*(a + b*x)^n)*x^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{c(a+bx)^n} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m F^{c(a+bx)^n} dx$$

input
```
Int[F^(c*(a + b*x)^n)*x^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{c(bx+a)^n} x^m dx$$

input
```
int(F^(c*(b*x+a)^n)*x^m,x)
```

output
```
int(F^(c*(b*x+a)^n)*x^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^n} x^m\,dx = \int F^{(bx+a)^n c} x^m\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^m,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)*x^m, x)
```

Sympy [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int F^{c(a+bx)^n} x^m\,dx = \int F^{c(a+bx)^n} x^m\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)*x**m,x)
```

output
```
Integral(F**(c*(a + b*x)**n)*x**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^n} x^m \, dx = \int F^{(bx+a)^n c} x^m \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^m,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)*x^m, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^n} x^m \, dx = \int F^{(bx+a)^n c} x^m \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^m,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)*x^m, x)
```

Mupad [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^n} x^m \, dx = \int F^{c\,(a+b\,x)^n}\,x^m \, dx$$

input
```
int(F^(c*(a + b*x)^n)*x^m,x)
```

output
```
int(F^(c*(a + b*x)^n)*x^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int F^{c(a+bx)^n}x^m\,dx = \int x^m f^{(bx+a)^n c}dx$$

input
```
int(F^(c*(b*x+a)^n)*x^m,x)
```

output
```
int(x**m*f**((a + b*x)**n*c),x)
```

3.179 $\int F^{c(a+bx)^n} x^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 207

$$\int F^{c(a+bx)^n} x^3 \, dx = -\frac{(a+bx)^4 \Gamma\left(\frac{4}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-4/n}}{b^4 n} + \frac{3a(a+bx)^3 \Gamma\left(\frac{3}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-3/n}}{b^4 n} - \frac{3a^2(a+bx)^2 \Gamma\left(\frac{2}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-2/n}}{b^4 n} + \frac{a^3(a+bx) \Gamma\left(\frac{1}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-1/n}}{b^4 n}$$

output
```
-(b*x+a)^4*GAMMA(4/n,-c*(b*x+a)^n*ln(F))/b^4/n/((-c*(b*x+a)^n*ln(F))^(4/n)
)+3*a*(b*x+a)^3*GAMMA(3/n,-c*(b*x+a)^n*ln(F))/b^4/n/((-c*(b*x+a)^n*ln(F))^
(3/n))-3*a^2*(b*x+a)^2*GAMMA(2/n,-c*(b*x+a)^n*ln(F))/b^4/n/((-c*(b*x+a)^n*
ln(F))^(2/n))+a^3*(b*x+a)*GAMMA(1/n,-c*(b*x+a)^n*ln(F))/b^4/n/((-c*(b*x+a)
^n*ln(F))^(1/n))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)^n}x^3\,dx =$$

$$-\frac{(a+bx)\left(-c(a+bx)^n\log(F)\right)^{-4/n}\left((a+bx)^3\Gamma\left(\frac{4}{n},-c(a+bx)^n\log(F)\right)-a(-c(a+bx)^n\log(F))^{\frac{1}{n}}\left(3\right.\right.}{}$$

input
```
Integrate[F^(c*(a + b*x)^n)*x^3,x]
```

output
```
-(((a + b*x)*((a + b*x)^3*Gamma[4/n, -(c*(a + b*x)^n*Log[F])] - a*(-(c*(a
+ b*x)^n*Log[F]))^n^(-1)*(3*(a + b*x)^2*Gamma[3/n, -(c*(a + b*x)^n*Log[F])
] + a*(-(c*(a + b*x)^n*Log[F]))^n^(-1)*(-3*(a + b*x)*Gamma[2/n, -(c*(a + b
*x)^n*Log[F])] + a*Gamma[n^(-1), -(c*(a + b*x)^n*Log[F])]*(-(c*(a + b*x)^n
*Log[F]))^n^(-1)))))/(b^4*n*(-(c*(a + b*x)^n*Log[F]))^(4/n)))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3F^{c(a+bx)^n}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(-\frac{a^3F^{c(a+bx)^n}}{b^3}+\frac{3a^2(a+bx)F^{c(a+bx)^n}}{b^3}+\frac{(a+bx)^3F^{c(a+bx)^n}}{b^3}-\frac{3a(a+bx)^2F^{c(a+bx)^n}}{b^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(a+bx)\left(-c\log(F)(a+bx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-c(a+bx)^n\log(F)\right)}{b^4n}-\frac{3a^2(a+bx)^2\left(-c\log(F)(a+bx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-c(a+bx)^n\log(F)\right)}{b^4n}-\frac{(a+bx)^4\left(-c\log(F)(a+bx)^n\right)^{-4/n}\Gamma\left(\frac{4}{n},-c(a+bx)^n\log(F)\right)}{b^4n}+\frac{3a(a+bx)^3\left(-c\log(F)(a+bx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-c(a+bx)^n\log(F)\right)}{b^4n}$$

input
```
Int[F^(c*(a + b*x)^n)*x^3,x]
```

output
```
-(((a + b*x)^4*Gamma[4/n, -(c*(a + b*x)^n*Log[F])])/(b^4*n*(-(c*(a + b*x)^
n*Log[F]))^(4/n))) + (3*a*(a + b*x)^3*Gamma[3/n, -(c*(a + b*x)^n*Log[F])])
/(b^4*n*(-(c*(a + b*x)^n*Log[F]))^(3/n)) - (3*a^2*(a + b*x)^2*Gamma[2/n, -
(c*(a + b*x)^n*Log[F])])/(b^4*n*(-(c*(a + b*x)^n*Log[F]))^(2/n)) + (a^3*(a
 + b*x)*Gamma[n^(-1), -(c*(a + b*x)^n*Log[F])])/(b^4*n*(-(c*(a + b*x)^n*Lo
g[F]))^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int F^{c(bx+a)^n}x^3dx$$

input
```
int(F^(c*(b*x+a)^n)*x^3,x)
```

output
```
int(F^(c*(b*x+a)^n)*x^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)^n}x^3\,dx = \int F^{(bx+a)^n c}x^3\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^3,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)*x^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)^n}x^3\,dx = \int F^{c(a+bx)^n}x^3\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)*x**3,x)
```

output
```
Integral(F**(c*(a + b*x)**n)*x**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)^n}x^3\,dx = \int F^{(bx+a)^n c}x^3\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^3,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)*x^3, x)
```

Giac [F]

$$\int F^{c(a+bx)^n}x^3\,dx = \int F^{(bx+a)^n c}x^3\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)^n}x^3\,dx = \int F^{c\,(a+b\,x)^n}\,x^3\,dx$$

input
```
int(F^(c*(a + b*x)^n)*x^3,x)
```

output
```
int(F^(c*(a + b*x)^n)*x^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)^n}x^3\,dx = \int f^{(bx+a)^n c}x^3dx$$

input
```
int(F^(c*(b*x+a)^n)*x^3,x)
```

output
```
int(f**((a + b*x)**n*c)*x**3,x)
```

3.180 $\int F^{c(a+bx)^n} x^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int F^{c(a+bx)^n} x^2 \, dx = -\frac{(a+bx)^3 \Gamma\left(\frac{3}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-3/n}}{b^3 n} + \frac{2a(a+bx)^2 \Gamma\left(\frac{2}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-2/n}}{b^3 n} - \frac{a^2(a+bx) \Gamma\left(\frac{1}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-1/n}}{b^3 n}$$

output
```
-(b*x+a)^3*GAMMA(3/n,-c*(b*x+a)^n*ln(F))/b^3/n/((-c*(b*x+a)^n*ln(F))^(3/n)
)+2*a*(b*x+a)^2*GAMMA(2/n,-c*(b*x+a)^n*ln(F))/b^3/n/((-c*(b*x+a)^n*ln(F))^
(2/n))-a^2*(b*x+a)*GAMMA(1/n,-c*(b*x+a)^n*ln(F))/b^3/n/((-c*(b*x+a)^n*ln(F
))^(1/n))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)^n}x^2\,dx =$$
$$-\frac{(a+bx)\left(-c(a+bx)^n\log(F)\right)^{-3/n}\left((a+bx)^2\Gamma\left(\frac{3}{n},-c(a+bx)^n\log(F)\right)+a(-c(a+bx)^n\log(F))^{\frac{1}{n}}\left(-\right.\right.}{b^3n}$$

input
```
Integrate[F^(c*(a + b*x)^n)*x^2,x]
```

output
```
-(((a + b*x)*((a + b*x)^2*Gamma[3/n, -(c*(a + b*x)^n*Log[F])] + a*(-(c*(a
+ b*x)^n*Log[F]))^n^(-1)*(-2*(a + b*x)*Gamma[2/n, -(c*(a + b*x)^n*Log[F])]
 + a*Gamma[n^(-1), -(c*(a + b*x)^n*Log[F])]*(-(c*(a + b*x)^n*Log[F]))^n^(-
1))))/(b^3*n*(-(c*(a + b*x)^n*Log[F]))^(3/n)))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2F^{c(a+bx)^n}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{a^2F^{c(a+bx)^n}}{b^2}+\frac{(a+bx)^2F^{c(a+bx)^n}}{b^2}-\frac{2a(a+bx)F^{c(a+bx)^n}}{b^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2(a+bx)\left(-c\log(F)(a+bx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-c(a+bx)^n\log(F)\right)}{b^3n}-\frac{(a+bx)^3\left(-c\log(F)(a+bx)^n\right)^{-3/n}\Gamma\left(\frac{3}{n},-c(a+bx)^n\log(F)\right)}{b^3n}+\frac{2a(a+bx)^2\left(-c\log(F)(a+bx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-c(a+bx)^n\log(F)\right)}{b^3n}$$

input
```
Int[F^(c*(a + b*x)^n)*x^2,x]
```

output
```
-(((a + b*x)^3*Gamma[3/n, -(c*(a + b*x)^n*Log[F])])/(b^3*n*(-(c*(a + b*x)^
n*Log[F]))^(3/n))) + (2*a*(a + b*x)^2*Gamma[2/n, -(c*(a + b*x)^n*Log[F])])
/(b^3*n*(-(c*(a + b*x)^n*Log[F]))^(2/n)) - (a^2*(a + b*x)*Gamma[n^(-1), -(
c*(a + b*x)^n*Log[F])])/(b^3*n*(-(c*(a + b*x)^n*Log[F]))^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int F^{c(bx+a)^n}x^2dx$$

input
```
int(F^(c*(b*x+a)^n)*x^2,x)
```

output
```
int(F^(c*(b*x+a)^n)*x^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)^n}x^2\,dx = \int F^{(bx+a)^nc}x^2\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^2,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)*x^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)^n}x^2\,dx = \int F^{c(a+bx)^n}x^2\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)*x**2,x)
```

output
```
Integral(F**(c*(a + b*x)**n)*x**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)^n}x^2\,dx = \int F^{(bx+a)^nc}x^2\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^2,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)*x^2, x)
```

Giac [F]

$$\int F^{c(a+bx)^n}x^2\,dx = \int F^{(bx+a)^n c}x^2\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)^n}x^2\,dx = \int F^{c\,(a+b\,x)^n}\,x^2\,dx$$

input
```
int(F^(c*(a + b*x)^n)*x^2,x)
```

output
```
int(F^(c*(a + b*x)^n)*x^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)^n}x^2\,dx = \int f^{(bx+a)^n c}x^2dx$$

input
```
int(F^(c*(b*x+a)^n)*x^2,x)
```

output
```
int(f**((a + b*x)**n*c)*x**2,x)
```

3.181 $\int F^{c(a+bx)^n} x \, dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int F^{c(a+bx)^n} x \, dx = -\frac{(a+bx)^2 \Gamma\left(\frac{2}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-2/n}}{b^2 n} + \frac{a(a+bx) \Gamma\left(\frac{1}{n}, -c(a+bx)^n \log(F)\right) \left(-c(a+bx)^n \log(F)\right)^{-1/n}}{b^2 n}$$

output
```
-(b*x+a)^2*GAMMA(2/n,-c*(b*x+a)^n*ln(F))/b^2/n/((-c*(b*x+a)^n*ln(F))^(2/n)
)+a*(b*x+a)*GAMMA(1/n,-c*(b*x+a)^n*ln(F))/b^2/n/((-c*(b*x+a)^n*ln(F))^(1/n
))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int F^{c(a+bx)^n} x \, dx = -\frac{(a+bx) \left(-c(a+bx)^n \log(F)\right)^{-2/n} \left((a+bx) \Gamma\left(\frac{2}{n}, -c(a+bx)^n \log(F)\right) - a \Gamma\left(\frac{1}{n}, -c(a+bx)^n \log(F)\right) (\right.}{b^2 n}$$

input
```
Integrate[F^(c*(a + b*x)^n)*x,x]
```

output
```
-(((a + b*x)*((a + b*x)*Gamma[2/n, -(c*(a + b*x)^n*Log[F])] - a*Gamma[n^(-
1), -(c*(a + b*x)^n*Log[F])]*(-(c*(a + b*x)^n*Log[F]))^n^(-1)))/(b^2*n*(-(
c*(a + b*x)^n*Log[F]))^(2/n)))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xF^{c(a+bx)^n}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(a+bx)F^{c(a+bx)^n}}{b}-\frac{aF^{c(a+bx)^n}}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(a+bx)\left(-c\log(F)(a+bx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-c(a+bx)^n\log(F)\right)}{b^2n}-\frac{(a+bx)^2\left(-c\log(F)(a+bx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-c(a+bx)^n\log(F)\right)}{b^2n}$$

input
```
Int[F^(c*(a + b*x)^n)*x,x]
```

output
```
-(((a + b*x)^2*Gamma[2/n, -(c*(a + b*x)^n*Log[F])])/(b^2*n*(-(c*(a + b*x)^
n*Log[F]))^(2/n))) + (a*(a + b*x)*Gamma[n^(-1), -(c*(a + b*x)^n*Log[F])])/
(b^2*n*(-(c*(a + b*x)^n*Log[F]))^n^(-1))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int F^{c(bx+a)^n}xdx$$

input
```
int(F^(c*(b*x+a)^n)*x,x)
```

output
```
int(F^(c*(b*x+a)^n)*x,x)
```

Fricas [F]

$$\int F^{c(a+bx)^n}x\,dx=\int F^{(bx+a)^nc}x\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)*x, x)
```

Sympy [F]

$$\int F^{c(a+bx)^n} x \, dx = \int F^{c(a+bx)^n} x \, dx$$

input
```
integrate(F**(c*(b*x+a)**n)*x,x)
```

output
```
Integral(F**(c*(a + b*x)**n)*x, x)
```

Maxima [F]

$$\int F^{c(a+bx)^n} x \, dx = \int F^{(bx+a)^n c} x \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)*x, x)
```

Giac [F]

$$\int F^{c(a+bx)^n} x \, dx = \int F^{(bx+a)^n c} x \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)*x,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)^n}x\,dx=\int F^{c\,(a+b\,x)^n}\,x\,dx$$

input
```
int(F^(c*(a + b*x)^n)*x,x)
```

output
```
int(F^(c*(a + b*x)^n)*x, x)
```

Reduce [F]

$$\int F^{c(a+bx)^n}x\,dx=\int f^{(bx+a)^n c}xdx$$

input
```
int(F^(c*(b*x+a)^n)*x,x)
```

output
```
int(f**((a + b*x)**n*c)*x,x)
```

3.182 $\int F^{c(a+bx)^n}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 47

$$\int F^{c(a+bx)^n}\,dx=-\frac{(a+bx)\Gamma\left(\frac{1}{n},-c(a+bx)^n\log(F)\right)\left(-c(a+bx)^n\log(F)\right)^{-1/n}}{bn}$$

output
```
-(b*x+a)*GAMMA(1/n,-c*(b*x+a)^n*ln(F))/b/n/((-c*(b*x+a)^n*ln(F))^(1/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)^n}\,dx=-\frac{(a+bx)\Gamma\left(\frac{1}{n},-c(a+bx)^n\log(F)\right)\left(-c(a+bx)^n\log(F)\right)^{-1/n}}{bn}$$

input
```
Integrate[F^(c*(a + b*x)^n),x]
```

output
```
-(((a + b*x)*Gamma[n^(-1), -(c*(a + b*x)^n*Log[F])])/(b*n*(-(c*(a + b*x)^n
*Log[F]))^n^(-1)))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)^n} \, dx$$

$$\Big\downarrow \; 2637$$

$$-\frac{(a+bx)\left(-c\log(F)(a+bx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-c(a+bx)^n\log(F)\right)}{bn}$$

input
```
Int[F^(c*(a + b*x)^n),x]
```

output
```
-(((a + b*x)*Gamma[n^(-1), -(c*(a + b*x)^n*Log[F])])/(b*n*(-(c*(a + b*x)^n
*Log[F]))^n^(-1)))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Maple [F]

$$\int F^{c(bx+a)^n} dx$$

input
```
int(F^(c*(b*x+a)^n),x)
```

output
```
int(F^(c*(b*x+a)^n),x)
```

Fricas [F]

$$\int F^{c(a+bx)^n}\,dx=\int F^{(bx+a)^n c}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n),x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c), x)
```

Sympy [F]

$$\int F^{c(a+bx)^n}\,dx=\int F^{c(a+bx)^n}\,dx$$

input
```
integrate(F**(c*(b*x+a)**n),x)
```

output
```
Integral(F**(c*(a + b*x)**n), x)
```

Maxima [F]

$$\int F^{c(a+bx)^n}\,dx=\int F^{(bx+a)^n c}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n),x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c), x)
```

Giac [F]

$$\int F^{c(a+bx)^n} \, dx = \int F^{(bx+a)^n c} \, dx$$

input
```
integrate(F^(c*(b*x+a)^n),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)^n} \, dx = \int F^{c\,(a+b\,x)^n} \, dx$$

input
```
int(F^(c*(a + b*x)^n),x)
```

output
```
int(F^(c*(a + b*x)^n), x)
```

Reduce [F]

$$\int F^{c(a+bx)^n} \, dx = \int f^{(bx+a)^n c} dx$$

input
```
int(F^(c*(b*x+a)^n),x)
```

output
```
int(f**((a + b*x)**n*c),x)
```

3.183 $\int \frac{F^{c(a+bx)^n}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{F^{c(a+bx)^n}}{x} \, dx = \text{Int}\left(\frac{F^{c(a+bx)^n}}{x}, x\right)$$

output
```
Defer(Int)(F^(c*(b*x+a)^n)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x} \, dx = \int \frac{F^{c(a+bx)^n}}{x} \, dx$$

input
```
Integrate[F^(c*(a + b*x)^n)/x,x]
```

output
```
Integrate[F^(c*(a + b*x)^n)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)^n}}{x} \, dx$$

$$\Big\downarrow \text{2654}$$

$$\int \frac{F^{c(a+bx)^n}}{x}dx$$

input
```
Int[F^(c*(a + b*x)^n)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(bx+a)^n}}{x}dx$$

input
```
int(F^(c*(b*x+a)^n)/x,x)
```

output
```
int(F^(c*(b*x+a)^n)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x}\,dx = \int \frac{F^{(bx+a)^n c}}{x}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{F^{c(a+bx)^n}}{x}\,dx = \int \frac{F^{c(a+bx)^n}}{x}\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)/x,x)
```

output
```
Integral(F**(c*(a + b*x)**n)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x}\,dx = \int \frac{F^{(bx+a)^n c}}{x}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x}\,dx = \int \frac{F^{(bx+a)^n c}}{x}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x}\,dx = \int \frac{F^{c\,(a+b\,x)^n}}{x}\,dx$$

input
```
int(F^(c*(a + b*x)^n)/x,x)
```

output
```
int(F^(c*(a + b*x)^n)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x}\,dx = \int \frac{f^{(bx+a)^n c}}{x}dx$$

input
```
int(F^(c*(b*x+a)^n)/x,x)
```

output
```
int(f**((a + b*x)**n*c)/x,x)
```

3.184 $\int \frac{F^{c(a+bx)^n}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{F^{c(a+bx)^n}}{x^2} dx = \text{Int}\left(\frac{F^{c(a+bx)^n}}{x^2}, x\right)$$

output
```
Defer(Int)(F^(c*(b*x+a)^n)/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^2} dx = \int \frac{F^{c(a+bx)^n}}{x^2} dx$$

input
```
Integrate[F^(c*(a + b*x)^n)/x^2,x]
```

output
```
Integrate[F^(c*(a + b*x)^n)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)^n}}{x^2} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{F^{c(a+bx)^n}}{x^2}dx$$

input
```
Int[F^(c*(a + b*x)^n)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(bx+a)^n}}{x^2}dx$$

input
```
int(F^(c*(b*x+a)^n)/x^2,x)
```

output
```
int(F^(c*(b*x+a)^n)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^2}\,dx = \int \frac{F^{(bx+a)^n c}}{x^2}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x^2,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{F^{c(a+bx)^n}}{x^2}\,dx = \int \frac{F^{c(a+bx)^n}}{x^2}\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)/x**2,x)
```

output
```
Integral(F**(c*(a + b*x)**n)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^2} \, dx = \int \frac{F^{(bx+a)^n c}}{x^2} \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x^2,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^2} \, dx = \int \frac{F^{(bx+a)^n c}}{x^2} \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^2} \, dx = \int \frac{F^{c\,(a+b\,x)^n}}{x^2} \, dx$$

input
```
int(F^(c*(a + b*x)^n)/x^2,x)
```

output
```
int(F^(c*(a + b*x)^n)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^2}\,dx = \int \frac{f^{(bx+a)^n c}}{x^2}dx$$

input
```
int(F^(c*(b*x+a)^n)/x^2,x)
```

output
```
int(f**((a + b*x)**n*c)/x**2,x)
```

3.185 $\int \frac{F^{c(a+bx)^n}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{F^{c(a+bx)^n}}{x^3} dx = \text{Int}\left(\frac{F^{c(a+bx)^n}}{x^3}, x\right)$$

output
```
Defer(Int)(F^(c*(b*x+a)^n)/x^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^3} dx = \int \frac{F^{c(a+bx)^n}}{x^3} dx$$

input
```
Integrate[F^(c*(a + b*x)^n)/x^3,x]
```

output
```
Integrate[F^(c*(a + b*x)^n)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)^n}}{x^3} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{F^{c(a+bx)^n}}{x^3}dx$$

input
```
Int[F^(c*(a + b*x)^n)/x^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(bx+a)^n}}{x^3}dx$$

input
```
int(F^(c*(b*x+a)^n)/x^3,x)
```

output
```
int(F^(c*(b*x+a)^n)/x^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^3}\,dx = \int \frac{F^{(bx+a)^n c}}{x^3}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x^3,x, algorithm="fricas")
```

output
```
integral(F^((b*x + a)^n*c)/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{F^{c(a+bx)^n}}{x^3}\,dx = \int \frac{F^{c(a+bx)^n}}{x^3}\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)/x**3,x)
```

output
```
Integral(F**(c*(a + b*x)**n)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^3} \, dx = \int \frac{F^{(bx+a)^n c}}{x^3} \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x^3,x, algorithm="maxima")
```

output
```
integrate(F^((b*x + a)^n*c)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^3} \, dx = \int \frac{F^{(bx+a)^n c}}{x^3} \, dx$$

input
```
integrate(F^(c*(b*x+a)^n)/x^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)^n*c)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^3} \, dx = \int \frac{F^{c\,(a+b\,x)^n}}{x^3} \, dx$$

input
```
int(F^(c*(a + b*x)^n)/x^3,x)
```

output
```
int(F^(c*(a + b*x)^n)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{F^{c(a+bx)^n}}{x^3}\,dx = \int \frac{f^{(bx+a)^n c}}{x^3}dx$$

input
```
int(F^(c*(b*x+a)^n)/x^3,x)
```

output
```
int(f**((a + b*x)**n*c)/x**3,x)
```

3.186 $\int F^{d(c(a+bx)^n)^{\frac{1}{n}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int F^{d(c(a+bx)^n)^{\frac{1}{n}}}\,dx=\frac{F^{d(c(a+bx)^n)^{\frac{1}{n}}}(a+bx)\left(c(a+bx)^n\right)^{-1/n}}{bd\log(F)}$$

output
```
F^(d*(c*(b*x+a)^n)^(1/n))*(b*x+a)/b/d/((c*(b*x+a)^n)^(1/n))/ln(F)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int F^{d(c(a+bx)^n)^{\frac{1}{n}}}\,dx=\frac{F^{d(c(a+bx)^n)^{\frac{1}{n}}}(a+bx)\left(c(a+bx)^n\right)^{-1/n}}{bd\log(F)}$$

input
```
Integrate[F^(d*(c*(a + b*x)^n)^n^(-1)),x]
```

output
```
(F^(d*(c*(a + b*x)^n)^n^(-1))*(a + b*x))/(b*d*(c*(a + b*x)^n)^n^(-1)*Log[F
])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2718}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{d\left(c(a+bx)^n\right)^{\frac{1}{n}}}\,dx$$

$$\Big\downarrow \; 2718$$

$$\frac{(a+bx)\left(c(a+bx)^n\right)^{-1/n}F^{d\left(c(a+bx)^n\right)^{\frac{1}{n}}}}{bd\log(F)}$$

input
```
Int[F^(d*(c*(a + b*x)^n)^n^(-1)),x]
```

output
```
(F^(d*(c*(a + b*x)^n)^n^(-1))*(a + b*x))/(b*d*(c*(a + b*x)^n)^n^(-1)*Log[F
])
```

Defintions of rubi rules used

rule 2718
```
Int[(F_)^((d_.)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(m_)), x_Symbol] :> Simp[
(a + b*x)*(F^(d*(c*(a + b*x)^n)^m)/(b*d*(c*(a + b*x)^n)^m*Log[F])), x] /; F
reeQ[{F, a, b, c, d, m, n}, x] && EqQ[m*n, 1]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.17 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.94

method	result
risch	$\frac{F^{d\,c^{\frac{1}{n}}\left((bx+a)^{n}\right)^{\frac{1}{n}}\mathrm{e}^{-\frac{i\pi\ \mathrm{csgn}\left(ic(bx+a)^{n}\right)\left(\mathrm{csgn}\left(i(bx+a)^{n}\right)-\mathrm{csgn}\left(ic(bx+a)^{n}\right)\right)\left(\mathrm{csgn}\left(ic\right)-\mathrm{csgn}\left(ic(bx+a)^{n}\right)\right)}{2n}}}\left((bx+a)^{n}\right)^{-\frac{1}{n}}c^{-\frac{1}{n}}(bx+a)\mathrm{e}^{\frac{i\pi\ \mathrm{csg}}{}}}{db\ln(F)}$

input
```
int(F^(d*(c*(b*x+a)^n)^(1/n)),x,method=_RETURNVERBOSE)
```

output
```
F^(d*c^(1/n)*((b*x+a)^n)^(1/n)*exp(-1/2*I*Pi*csgn(I*c*(b*x+a)^n)*(csgn(I*(
b*x+a)^n)-csgn(I*c*(b*x+a)^n))*(csgn(I*c)-csgn(I*c*(b*x+a)^n))/n))/d/b/(((
b*x+a)^n)^(1/n))/(c^(1/n))*(b*x+a)*exp(1/2*I*Pi*csgn(I*c*(b*x+a)^n)*(csgn(
I*(b*x+a)^n)-csgn(I*c*(b*x+a)^n))*(csgn(I*c)-csgn(I*c*(b*x+a)^n))/n)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int F^{d\left(c(a+bx)^n\right)^{\frac{1}{n}}}\,dx=\frac{F^{(bdx+ad)c^{\left(\frac{1}{n}\right)}}}{bc^{\left(\frac{1}{n}\right)}d\log\left(F\right)}$$

input
```
integrate(F^(d*(c*(b*x+a)^n)^(1/n)),x, algorithm="fricas")
```

output
```
F^((b*d*x + a*d)*c^(1/n))/(b*c^(1/n)*d*log(F))
```

Sympy [F]

$$\int F^{d\left(c(a+bx)^n\right)^{\frac{1}{n}}}\,dx=\int F^{d\left(c(a+bx)^n\right)^{\frac{1}{n}}}\,dx$$

input
```
integrate(F**(d*(c*(b*x+a)**n)**(1/n)),x)
```

output
```
Integral(F**(d*(c*(a + b*x)**n)**(1/n)), x)
```

Maxima [F]

$$\int F^{d(c(a+bx)^n)^{\frac{1}{n}}}\,dx = \int F^{((bx+a)^nc)^{\left(\frac{1}{n}\right)}d}\,dx$$

input
```
integrate(F^(d*(c*(b*x+a)^n)^(1/n)),x, algorithm="maxima")
```

output
```
integrate(F^(((b*x + a)^n*c)^(1/n)*d), x)
```

Giac [A] (verification not implemented)

Time = 1.38 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int F^{d(c(a+bx)^n)^{\frac{1}{n}}}\,dx = \frac{e^{\left(bc^{\left(\frac{1}{n}\right)}dx\log(F)+ac^{\left(\frac{1}{n}\right)}d\log(F)\right)}}{bc^{\left(\frac{1}{n}\right)}d\log\left(F\right)}$$

input
```
integrate(F^(d*(c*(b*x+a)^n)^(1/n)),x, algorithm="giac")
```

output
```
e^(b*c^(1/n)*d*x*log(F) + a*c^(1/n)*d*log(F))/(b*c^(1/n)*d*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int F^{d(c(a+bx)^n)^{\frac{1}{n}}}\,dx = \int F^{d\,{\left(c\,{\left(a+b\,x\right)}^n\right)}^{1/n}}\,dx$$

input
```
int(F^(d*(c*(a + b*x)^n)^(1/n)),x)
```

output
```
int(F^(d*(c*(a + b*x)^n)^(1/n)), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{d\left(c(a+bx)^n\right)^{\frac{1}{n}}}\,dx=\frac{f^{c^{\frac{1}{n}}ad+c^{\frac{1}{n}}bdx}}{c^{\frac{1}{n}}\log\left(f\right)bd}$$

input
```
int(F^(d*(c*(b*x+a)^n)^(1/n)),x)
```

output
```
f**(c**(1/n)*a*d + c**(1/n)*b*d*x)/(c**(1/n)*log(f)*b*d)
```

3.187 $\int F^{d(c(a+bx)^n)^m}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int F^{d(c(a+bx)^n)^m}\,dx = -\frac{(a+bx)\Gamma\left(\frac{1}{mn},-d(c(a+bx)^n)^m\log(F)\right)\left(-d(c(a+bx)^n)^m\log(F)\right)^{-\frac{1}{mn}}}{bmn}$$

output
```
-(b*x+a)*GAMMA(1/m/n,-d*(c*(b*x+a)^n)^m*ln(F))/b/m/n/((-d*(c*(b*x+a)^n)^m*
ln(F))^(1/m/n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int F^{d(c(a+bx)^n)^m}\,dx = -\frac{(a+bx)\Gamma\left(\frac{1}{mn},-d(c(a+bx)^n)^m\log(F)\right)\left(-d(c(a+bx)^n)^m\log(F)\right)^{-\frac{1}{mn}}}{bmn}$$

input
```
Integrate[F^(d*(c*(a + b*x)^n)^m),x]
```

output
```
-(((a + b*x)*Gamma[1/(m*n), -(d*(c*(a + b*x)^n)^m*Log[F])])/(b*m*n*(-(d*(c
*(a + b*x)^n)^m*Log[F]))^(1/(m*n))))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2719\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{d(c(a+bx)^n)^m}\,dx$$

$$\Big\downarrow \text{2719}$$

$$-\frac{(a+bx)\left(-d\log(F)\left(c(a+bx)^n\right)^m\right)^{-\frac{1}{mn}}\Gamma\left(\frac{1}{mn},-d\left(c(a+bx)^n\right)^m\log(F)\right)}{bmn}$$

input
```
Int[F^(d*(c*(a + b*x)^n)^m),x]
```

output
```
-(((a + b*x)*Gamma[1/(m*n), -(d*(c*(a + b*x)^n)^m*Log[F])])/(b*m*n*(-(d*(c
*(a + b*x)^n)^m*Log[F]))^(1/(m*n))))
```


Defintions of rubi rules used

rule 2719
```
Int[(F_)^((d_.)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(m_)), x_Symbol] :> Simp[
(-(a + b*x))*(Gamma[1/(m*n), ((-d)*(c*(a + b*x)^n)^m)*Log[F]]/(b*m*n*(((-d)
*(c*(a + b*x)^n)^m)*Log[F])^(1/(m*n)))), x] /; FreeQ[{F, a, b, c, d, m, n},
 x]
```

Maple [F]

$$\int F^{d\left(c(bx+a)^n\right)^m}dx$$

input
```
int(F^(d*(c*(b*x+a)^n)^m),x)
```

output
```
int(F^(d*(c*(b*x+a)^n)^m),x)
```

Fricas [F]

$$\int F^{d\left(c(a+bx)^n\right)^m}\,dx=\int F^{\left((bx+a)^n c\right)^m d}\,dx$$

input
```
integrate(F^(d*(c*(b*x+a)^n)^m),x, algorithm="fricas")
```

output
```
integral(F^(((b*x + a)^n*c)^m*d), x)
```

Sympy [F]

$$\int F^{d\left(c(a+bx)^n\right)^m}\,dx=\int F^{d\left(c(a+bx)^n\right)^m}\,dx$$

input
```
integrate(F**(d*(c*(b*x+a)**n)**m),x)
```

output
```
Integral(F**(d*(c*(a + b*x)**n)**m), x)
```

Maxima [F]

$$\int F^{d(c(a+bx)^n)^m}\,dx = \int F^{((bx+a)^n c)^m d}\,dx$$

input
```
integrate(F^(d*(c*(b*x+a)^n)^m),x, algorithm="maxima")
```

output
```
integrate(F^(((b*x + a)^n*c)^m*d), x)
```

Giac [F]

$$\int F^{d(c(a+bx)^n)^m}\,dx = \int F^{((bx+a)^n c)^m d}\,dx$$

input
```
integrate(F^(d*(c*(b*x+a)^n)^m),x, algorithm="giac")
```

output
```
integrate(F^(((b*x + a)^n*c)^m*d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{d(c(a+bx)^n)^m}\,dx = \int F^{d\,(c\,(a+b\,x)^n)^m}\,dx$$

input
```
int(F^(d*(c*(a + b*x)^n)^m),x)
```

output
```
int(F^(d*(c*(a + b*x)^n)^m), x)
```

Reduce [F]

$$\int F^{d(c(a+bx)^n)^m} dx = \int f^{c^m (bx+a)^{mn} d} dx$$

input
```
int(F^(d*(c*(b*x+a)^n)^m),x)
```

output
```
int(f**(c**m*(a + b*x)**(m*n)*d),x)
```

3.188 $\int F^{a+b(c+dx)^2}(c+dx)^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{2},-b(c+dx)^2\log(F)\right)\left(-b(c+dx)^2\log(F)\right)^{\frac{1}{2}(-1-m)}}{2d}$$

output
```
-1/2*F^a*(d*x+c)^(1+m)*GAMMA(1/2+1/2*m,-b*(d*x+c)^2*ln(F))*(-b*(d*x+c)^2*l
n(F))^(-1/2-1/2*m)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{2},-b(c+dx)^2\log(F)\right)\left(-b(c+dx)^2\log(F)\right)^{\frac{1}{2}(-1-m)}}{2d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^m,x]
```

output
```
-1/2*(F^a*(c + d*x)^(1 + m)*Gamma[(1 + m)/2, -(b*(c + d*x)^2*Log[F])]*(-(b
*(c + d*x)^2*Log[F]))^((-1 - m)/2))/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m F^{a+b(c+dx)^2} \, dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{F^a(c+dx)^{m+1}\left(-b\log(F)(c+dx)^2\right)^{\frac{1}{2}(-m-1)}\Gamma\left(\frac{m+1}{2},-b(c+dx)^2\log(F)\right)}{2d}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^m,x]
```

output
```
-1/2*(F^a*(c + d*x)^(1 + m)*Gamma[(1 + m)/2, -(b*(c + d*x)^2*Log[F])]*(-(b
*(c + d*x)^2*Log[F]))^((-1 - m)/2))/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^2}(dx+c)^m\,dx$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x)
```

output
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx = \frac{e^{\left(-\frac{1}{2}\,(m-1)\log(-b\log(F))+a\log(F)\right)}\Gamma\left(\frac{1}{2}\,m+\frac{1}{2},-\left(bd^2x^2+2\,bcdx+bc^2\right)\log\left(F\right)\right)}{2\,bd\log\left(F\right)}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x, algorithm="fricas")
```

output
```
1/2*e^(-1/2*(m - 1)*log(-b*log(F)) + a*log(F))*gamma(1/2*m + 1/2, -(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*log(F))/(b*d*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx = \int F^{a+b(c+dx)^2}(c+dx)^m\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**m,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**m, x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx=\int (dx+c)^m F^{(dx+c)^2b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*F^((d*x + c)^2*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx=\int (dx+c)^m F^{(dx+c)^2b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*F^((d*x + c)^2*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx=\frac{F^a\,\mathrm{e}^{\frac{b\,\ln(F)\,(c+d\,x)^2}{2}}\,(c+d\,x)^{m+1}\,\mathrm{M}_{\frac{1}{4}-\frac{m}{4},\frac{m}{4}+\frac{1}{4}}\left(b\,\ln(F)\,(c+d\,x)^2\right)}{d\,(m+1)\,\left(b\,\ln(F)\,(c+d\,x)^2\right)^{\frac{m}{4}+\frac{3}{4}}}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^m,x)
```

output
```
(F^a*exp((b*log(F)*(c + d*x)^2)/2)*(c + d*x)^(m + 1)*whittakerM(1/4 - m/4,
 m/4 + 1/4, b*log(F)*(c + d*x)^2))/(d*(m + 1)*(b*log(F)*(c + d*x)^2)^(m/4
+ 3/4))
```

Reduce [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^m\,dx = f^{b\,c^2+a}\left(\int f^{b\,d^2x^2+2bcdx}(dx+c)^m\,dx\right)$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^m,x)
```

output
```
f**(a + b*c**2)*int(f**(2*b*c*d*x + b*d**2*x**2)*(c + d*x)**m,x)
```

3.189 $\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = -\frac{F^{a+b(c+dx)^2}\left(120-120b(c+dx)^2\log(F)+60b^2(c+dx)^4\log^2(F)-20b^3(c+dx)^6\log^3(F)+5b^4(c+d\ldots\right.}{2b^6d\log^6(F)}$$

output
```
-1/2*F^(a+b*(d*x+c)^2)*(120-120*b*(d*x+c)^2*ln(F)+60*b^2*(d*x+c)^4*ln(F)^2
-20*b^3*(d*x+c)^6*ln(F)^3+5*b^4*(d*x+c)^8*ln(F)^4-b^5*(d*x+c)^10*ln(F)^5)/
b^6/d/ln(F)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.32 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = -\frac{F^a\Gamma(6,-b(c+dx)^2\log(F))}{2b^6d\log^6(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^11,x]
```

output
```
-1/2*(F^a*Gamma[6, -(b*(c + d*x)^2*Log[F])])/(b^6*d*Log[F]^6)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2647\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{11}F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \; 2647$$

$$-\frac{F^{a+b(c+dx)^2}\left(-b^5\log^5(F)(c+dx)^{10}+5b^4\log^4(F)(c+dx)^8-20b^3\log^3(F)(c+dx)^6+60b^2\log^2(F)(c+dx)^4-\right.}{2b^6d\log^6(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^11,x]
```

output
```
-1/2*(F^(a + b*(c + d*x)^2)*(120 - 120*b*(c + d*x)^2*Log[F] + 60*b^2*(c +
d*x)^4*Log[F]^2 - 20*b^3*(c + d*x)^6*Log[F]^3 + 5*b^4*(c + d*x)^8*Log[F]^4
 - b^5*(c + d*x)^10*Log[F]^5))/(b^6*d*Log[F]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(103) = 206$.

Time = 1.03 (sec) , antiderivative size = 569, normalized size of antiderivative = 5.42

method	result
orering	$\frac{\left(-120+120\ln(F)b\,c^2+120\ln(F)b\,d^2x^2+d^{10}x^{10}b^5\ln(F)^5-5d^8x^8b^4\ln(F)^4+20d^6x^6b^3\ln(F)^3-60d^4x^4b^2\ln(F)^2-140\ln(F)^4\right.}{}$
gosper	$\frac{\left(-120+120\ln(F)b\,c^2+120\ln(F)b\,d^2x^2+d^{10}x^{10}b^5\ln(F)^5-5d^8x^8b^4\ln(F)^4+20d^6x^6b^3\ln(F)^3-60d^4x^4b^2\ln(F)^2-140\ln(F)^4\right.}{}$
risch	$\frac{\left(-120+120\ln(F)b\,c^2+120\ln(F)b\,d^2x^2+d^{10}x^{10}b^5\ln(F)^5-5d^8x^8b^4\ln(F)^4+20d^6x^6b^3\ln(F)^3-60d^4x^4b^2\ln(F)^2-140\ln(F)^4\right.}{}$
norman	$\frac{\left(\ln(F)^5b^5c^{10}-5\ln(F)^4b^4c^8+20\ln(F)^3b^3c^6-60\ln(F)^2b^2c^4+120\ln(F)b\,c^2-120\right)\mathrm{e}^{\left(a+b(dx+c)^2\right)\ln(F)}}{2b^6\ln(F)^6d}+\frac{d^9x^{10}\mathrm{e}^{\left(a+b(dx+c)\right.}}{2\ln(F)b}$
parallelrisch	Expression too large to display

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^11,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(-120+120*ln(F)*b*c^2+120*ln(F)*b*d^2*x^2+d^10*x^10*b^5*ln(F)^5-5*d^
8*x^8*b^4*ln(F)^4+20*d^6*x^6*b^3*ln(F)^3-60*d^4*x^4*b^2*ln(F)^2-140*ln(F)^
4*b^4*c^6*d^2*x^2-40*ln(F)^4*b^4*c^7*d*x+120*c*d^5*x^5*b^3*ln(F)^3+300*ln(
F)^3*b^3*c^2*d^4*x^4+400*ln(F)^3*b^3*c^3*d^3*x^3+300*ln(F)^3*b^3*c^4*d^2*x
^2+120*ln(F)^3*b^3*c^5*d*x-240*d^3*c*x^3*b^2*ln(F)^2-360*ln(F)^2*b^2*c^2*d
^2*x^2-240*ln(F)^2*b^2*c^3*d*x+10*d^9*c*x^9*b^5*ln(F)^5+45*ln(F)^5*b^5*c^2
*d^8*x^8+120*ln(F)^5*b^5*c^3*d^7*x^7+210*ln(F)^5*b^5*c^4*d^6*x^6+252*ln(F)
^5*b^5*c^5*d^5*x^5+210*ln(F)^5*b^5*c^6*d^4*x^4+120*ln(F)^5*b^5*c^7*d^3*x^3
-40*c*d^7*x^7*b^4*ln(F)^4-5*ln(F)^4*b^4*c^8+240*ln(F)*b*c*d*x+20*ln(F)^3*b
^3*c^6-60*ln(F)^2*b^2*c^4+ln(F)^5*b^5*c^10+45*ln(F)^5*b^5*c^8*d^2*x^2-140*
ln(F)^4*b^4*c^2*d^6*x^6+10*ln(F)^5*b^5*c^9*d*x-280*ln(F)^4*b^4*c^3*d^5*x^5
-350*ln(F)^4*b^4*c^4*d^4*x^4-280*ln(F)^4*b^4*c^5*d^3*x^3)/ln(F)^6/b^6*F^(a
+b*(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(102) = 204$.

Time = 0.09 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.46

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx$$
$$= \frac{\left(\left(b^5d^{10}x^{10} + 10\,b^5cd^9x^9 + 45\,b^5c^2d^8x^8 + 120\,b^5c^3d^7x^7 + 210\,b^5c^4d^6x^6 + 252\,b^5c^5d^5x^5 + 210\,b^5c^6d^4x^4 + 12\right.\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^11,x, algorithm="fricas")
```

output
```
1/2*((b^5*d^10*x^10 + 10*b^5*c*d^9*x^9 + 45*b^5*c^2*d^8*x^8 + 120*b^5*c^3*
d^7*x^7 + 210*b^5*c^4*d^6*x^6 + 252*b^5*c^5*d^5*x^5 + 210*b^5*c^6*d^4*x^4
+ 120*b^5*c^7*d^3*x^3 + 45*b^5*c^8*d^2*x^2 + 10*b^5*c^9*d*x + b^5*c^10)*lo
g(F)^5 - 5*(b^4*d^8*x^8 + 8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 + 56*b^4*c^
3*d^5*x^5 + 70*b^4*c^4*d^4*x^4 + 56*b^4*c^5*d^3*x^3 + 28*b^4*c^6*d^2*x^2 +
 8*b^4*c^7*d*x + b^4*c^8)*log(F)^4 + 20*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 1
5*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*
x + b^3*c^6)*log(F)^3 - 60*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*
x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)^2 + 120*(b*d^2*x^2 + 2*b*c*d*x + b*c
^2)*log(F) - 120)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b^6*d*log(F)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(105) = 210$.

Time = 0.24 (sec) , antiderivative size = 794, normalized size of antiderivative = 7.56

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = \text{Too large to display}$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**11,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**2)*(b**5*c**10*log(F)**5 + 10*b**5*c**9*d*
x*log(F)**5 + 45*b**5*c**8*d**2*x**2*log(F)**5 + 120*b**5*c**7*d**3*x**3*l
og(F)**5 + 210*b**5*c**6*d**4*x**4*log(F)**5 + 252*b**5*c**5*d**5*x**5*log
(F)**5 + 210*b**5*c**4*d**6*x**6*log(F)**5 + 120*b**5*c**3*d**7*x**7*log(F
)**5 + 45*b**5*c**2*d**8*x**8*log(F)**5 + 10*b**5*c*d**9*x**9*log(F)**5 +
b**5*d**10*x**10*log(F)**5 - 5*b**4*c**8*log(F)**4 - 40*b**4*c**7*d*x*log(
F)**4 - 140*b**4*c**6*d**2*x**2*log(F)**4 - 280*b**4*c**5*d**3*x**3*log(F)
**4 - 350*b**4*c**4*d**4*x**4*log(F)**4 - 280*b**4*c**3*d**5*x**5*log(F)**
4 - 140*b**4*c**2*d**6*x**6*log(F)**4 - 40*b**4*c*d**7*x**7*log(F)**4 - 5*
b**4*d**8*x**8*log(F)**4 + 20*b**3*c**6*log(F)**3 + 120*b**3*c**5*d*x*log(
F)**3 + 300*b**3*c**4*d**2*x**2*log(F)**3 + 400*b**3*c**3*d**3*x**3*log(F)
**3 + 300*b**3*c**2*d**4*x**4*log(F)**3 + 120*b**3*c*d**5*x**5*log(F)**3 +
 20*b**3*d**6*x**6*log(F)**3 - 60*b**2*c**4*log(F)**2 - 240*b**2*c**3*d*x*
log(F)**2 - 360*b**2*c**2*d**2*x**2*log(F)**2 - 240*b**2*c*d**3*x**3*log(F
)**2 - 60*b**2*d**4*x**4*log(F)**2 + 120*b*c**2*log(F) + 240*b*c*d*x*log(F
) + 120*b*d**2*x**2*log(F) - 120)/(2*b**6*d*log(F)**6), Ne(b**6*d*log(F)**
6, 0)), (c**11*x + 11*c**10*d*x**2/2 + 55*c**9*d**2*x**3/3 + 165*c**8*d**3
*x**4/4 + 66*c**7*d**4*x**5 + 77*c**6*d**5*x**6 + 66*c**5*d**6*x**7 + 165*
c**4*d**7*x**8/4 + 55*c**3*d**8*x**9/3 + 11*c**2*d**9*x**10/2 + c*d**10*x*
*11 + d**11*x**12/12, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.87 (sec) , antiderivative size = 5261, normalized size of antiderivative = 50.10

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^11,x, algorithm="maxima")
```

output
```
-11/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F
)/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*
log(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3
/2)*d))*F^a*c^10/sqrt(b*log(F)) + 55/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2
*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))
^(5/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*
c*d)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^
3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/
2)*d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^9*d/sqrt(b*log(
F)) - 165/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*
d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x +
 b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log
(F)^3/((b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^
2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x +
 b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F
)/(b*d^2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^8*d^2/sqrt(b*log(F)) + 1
65*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(
F)/(b*d^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2
*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b
*log(F))^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.38

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = \frac{\left(b^5d^{10}\left(x+\frac{c}{d}\right)^{10}\log\left(F\right)^5-5\,b^4d^8\left(x+\frac{c}{d}\right)^8\log\left(F\right)^4+20\,b^3d^6\left(x+\frac{c}{d}\right)^6\log\left(F\right)^3-60\,b^2d^4\left(x+\frac{c}{d}\right)^4\log\left(F\right)}{2\,b^6d\log\left(F\right)^6}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^11,x, algorithm="giac")
```

output
```
1/2*(b^5*d^10*(x + c/d)^10*log(F)^5 - 5*b^4*d^8*(x + c/d)^8*log(F)^4 + 20*
b^3*d^6*(x + c/d)^6*log(F)^3 - 60*b^2*d^4*(x + c/d)^4*log(F)^2 + 120*b*d^2
*(x + c/d)^2*log(F) - 120)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*
log(F) + a*log(F))/(b^6*d*log(F)^6)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 553, normalized size of antiderivative = 5.27

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,(c^{10}+10\,c^9\,d\,x+45\,c^8\,d^2\,x^2+120\,c^7\,d^3\,x^3+210\,c^6\,d^4\,x^4+252\,c^5\,d^5\,x^5+210\,c^4\,d}{2\,b\,d\,\ln(F)} - \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,(5\,c^8+40\,c^7\,d\,x+140\,c^6\,d^2\,x^2+280\,c^5\,d^3\,x^3+350\,c^4\,d^4\,x^4+280\,c^3\,d^5\,x^5+140}{2\,b^2\,d\ln(F)^2} - \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,(60\,c^4+240\,c^3\,d\,x+360\,c^2\,d^2\,x^2+240\,c\,d^3\,x^3+60\,d^4\,x^4)}{2\,b^4\,d\ln(F)^4} - \frac{60\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}}{b^6\,d\ln(F)^6} + \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,(20\,c^6+120\,c^5\,d\,x+300\,c^4\,d^2\,x^2+400\,c^3\,d^3\,x^3+300\,c^2\,d^4\,x^4+120\,c\,d^5\,x^5+20}{2\,b^3\,d\ln(F)^3} + \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,(120\,c^2+240\,c\,d\,x+120\,d^2\,x^2)}{2\,b^5\,d\ln(F)^5}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^11,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(c^10 + d^10*x^10 + 10*c*d^9*x^
9 + 45*c^8*d^2*x^2 + 120*c^7*d^3*x^3 + 210*c^6*d^4*x^4 + 252*c^5*d^5*x^5 +
 210*c^4*d^6*x^6 + 120*c^3*d^7*x^7 + 45*c^2*d^8*x^8 + 10*c^9*d*x))/(2*b*d*
log(F)) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(5*c^8 + 5*d^8*x^8 +
40*c*d^7*x^7 + 140*c^6*d^2*x^2 + 280*c^5*d^3*x^3 + 350*c^4*d^4*x^4 + 280*c
^3*d^5*x^5 + 140*c^2*d^6*x^6 + 40*c^7*d*x))/(2*b^2*d*log(F)^2) - (F^(b*d^2
*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(60*c^4 + 60*d^4*x^4 + 240*c*d^3*x^3 + 3
60*c^2*d^2*x^2 + 240*c^3*d*x))/(2*b^4*d*log(F)^4) - (60*F^(b*d^2*x^2)*F^a*
F^(b*c^2)*F^(2*b*c*d*x))/(b^6*d*log(F)^6) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F
^(2*b*c*d*x)*(20*c^6 + 20*d^6*x^6 + 120*c*d^5*x^5 + 300*c^4*d^2*x^2 + 400*
c^3*d^3*x^3 + 300*c^2*d^4*x^4 + 120*c^5*d*x))/(2*b^3*d*log(F)^3) + (F^(b*d
^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(120*c^2 + 120*d^2*x^2 + 240*c*d*x))/(
2*b^5*d*log(F)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 578, normalized size of antiderivative = 5.50

$$\int F^{a+b(c+dx)^2}(c+dx)^{11}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^11,x)
```

output
```
(f**(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*(log(f)**5*b**5*c**10 + 10*log(
f)**5*b**5*c**9*d*x + 45*log(f)**5*b**5*c**8*d**2*x**2 + 120*log(f)**5*b**
5*c**7*d**3*x**3 + 210*log(f)**5*b**5*c**6*d**4*x**4 + 252*log(f)**5*b**5*
c**5*d**5*x**5 + 210*log(f)**5*b**5*c**4*d**6*x**6 + 120*log(f)**5*b**5*c*
*3*d**7*x**7 + 45*log(f)**5*b**5*c**2*d**8*x**8 + 10*log(f)**5*b**5*c*d**9
*x**9 + log(f)**5*b**5*d**10*x**10 - 5*log(f)**4*b**4*c**8 - 40*log(f)**4*
b**4*c**7*d*x - 140*log(f)**4*b**4*c**6*d**2*x**2 - 280*log(f)**4*b**4*c**
5*d**3*x**3 - 350*log(f)**4*b**4*c**4*d**4*x**4 - 280*log(f)**4*b**4*c**3*
d**5*x**5 - 140*log(f)**4*b**4*c**2*d**6*x**6 - 40*log(f)**4*b**4*c*d**7*x
**7 - 5*log(f)**4*b**4*d**8*x**8 + 20*log(f)**3*b**3*c**6 + 120*log(f)**3*
b**3*c**5*d*x + 300*log(f)**3*b**3*c**4*d**2*x**2 + 400*log(f)**3*b**3*c**
3*d**3*x**3 + 300*log(f)**3*b**3*c**2*d**4*x**4 + 120*log(f)**3*b**3*c*d**
5*x**5 + 20*log(f)**3*b**3*d**6*x**6 - 60*log(f)**2*b**2*c**4 - 240*log(f)
**2*b**2*c**3*d*x - 360*log(f)**2*b**2*c**2*d**2*x**2 - 240*log(f)**2*b**2
*c*d**3*x**3 - 60*log(f)**2*b**2*d**4*x**4 + 120*log(f)*b*c**2 + 240*log(f
)*b*c*d*x + 120*log(f)*b*d**2*x**2 - 120))/(2*log(f)**6*b**6*d)
```

3.190 $\int F^{a+b(c+dx)^2}(c+dx)^9\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx = \frac{F^{a+b(c+dx)^2}\left(24-24b(c+dx)^2\log(F)+12b^2(c+dx)^4\log^2(F)-4b^3(c+dx)^6\log^3(F)+b^4(c+dx)^8\log^4\right.}{2b^5d\log^5(F)}$$

output
```
1/2*F^(a+b*(d*x+c)^2)*(24-24*b*(d*x+c)^2*ln(F)+12*b^2*(d*x+c)^4*ln(F)^2-4*
b^3*(d*x+c)^6*ln(F)^3+b^4*(d*x+c)^8*ln(F)^4)/b^5/d/ln(F)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.35

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx = \frac{F^a\Gamma(5,-b(c+dx)^2\log(F))}{2b^5d\log^5(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^9,x]
```

output
```
(F^a*Gamma[5, -(b*(c + d*x)^2*Log[F])])/(2*b^5*d*Log[F]^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^9 F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2647}$$

$$\frac{F^{a+b(c+dx)^2}\left(b^4\log^4(F)(c+dx)^8-4b^3\log^3(F)(c+dx)^6+12b^2\log^2(F)(c+dx)^4-24b\log(F)(c+dx)^2+24\right)}{2b^5d\log^5(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^9,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(24 - 24*b*(c + d*x)^2*Log[F] + 12*b^2*(c + d*x)^4*
Log[F]^2 - 4*b^3*(c + d*x)^6*Log[F]^3 + b^4*(c + d*x)^8*Log[F]^4))/(2*b^5*
d*Log[F]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(86) = 172$.

Time = 0.55 (sec) , antiderivative size = 386, normalized size of antiderivative = 4.39

method	result
orering	$\frac{\left(24-24\ln(F)b\,c^2-24\ln(F)b\,d^2x^2+d^8x^8b^4\ln(F)^4-4d^6x^6b^3\ln(F)^3+12d^4x^4b^2\ln(F)^2+28\ln(F)^4b^4c^6d^2x^2+8\ln(F)^4b^4c^7dx\cdots\right.}{}$
gosper	$\frac{\left(24-24\ln(F)b\,c^2-24\ln(F)b\,d^2x^2+d^8x^8b^4\ln(F)^4-4d^6x^6b^3\ln(F)^3+12d^4x^4b^2\ln(F)^2+28\ln(F)^4b^4c^6d^2x^2+8\ln(F)^4b^4c^7dx\cdots\right.}{}$
risch	$\frac{\left(24-24\ln(F)b\,c^2-24\ln(F)b\,d^2x^2+d^8x^8b^4\ln(F)^4-4d^6x^6b^3\ln(F)^3+12d^4x^4b^2\ln(F)^2+28\ln(F)^4b^4c^6d^2x^2+8\ln(F)^4b^4c^7dx\cdots\right.}{}$
norman	$\frac{d^3\left(35\ln(F)^2b^2c^4-30\ln(F)b\,c^2+6\right)x^4\mathrm{e}^{\left(a+b(dx+c)^2\right)\ln(F)}}{\ln(F)^3b^3}+\frac{\left(\ln(F)^4b^4c^8-4\ln(F)^3b^3c^6+12\ln(F)^2b^2c^4-24\ln(F)b\,c^2+24\cdots\right.}{2b^5\ln(F)^5d}$
parallelrisch	$\frac{24F^{a+b(dx+c)^2}+d^8F^{a+b(dx+c)^2}x^8b^4\ln(F)^4-4d^6F^{a+b(dx+c)^2}x^6b^3\ln(F)^3+12d^4F^{a+b(dx+c)^2}x^4b^2\ln(F)^2-24d^2F^{a+b(dx+c}\cdots}{}$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(24-24*ln(F)*b*c^2-24*ln(F)*b*d^2*x^2+d^8*x^8*b^4*ln(F)^4-4*d^6*x^6*
b^3*ln(F)^3+12*d^4*x^4*b^2*ln(F)^2+28*ln(F)^4*b^4*c^6*d^2*x^2+8*ln(F)^4*b^
4*c^7*d*x-24*c*d^5*x^5*b^3*ln(F)^3-60*ln(F)^3*b^3*c^2*d^4*x^4-80*ln(F)^3*b
^3*c^3*d^3*x^3-60*ln(F)^3*b^3*c^4*d^2*x^2-24*ln(F)^3*b^3*c^5*d*x+48*d^3*c*
x^3*b^2*ln(F)^2+72*ln(F)^2*b^2*c^2*d^2*x^2+48*ln(F)^2*b^2*c^3*d*x+8*c*d^7*
x^7*b^4*ln(F)^4+ln(F)^4*b^4*c^8-48*ln(F)*b*c*d*x-4*ln(F)^3*b^3*c^6+12*ln(F
)^2*b^2*c^4+28*ln(F)^4*b^4*c^2*d^6*x^6+56*ln(F)^4*b^4*c^3*d^5*x^5+70*ln(F)
^4*b^4*c^4*d^4*x^4+56*ln(F)^4*b^4*c^5*d^3*x^3)/ln(F)^5/b^5*F^(a+b*(d*x+c)^
2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(86) = 172$.

Time = 0.08 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.68

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx$$

$$= \frac{\left(\left(b^4d^8x^8+8\,b^4cd^7x^7+28\,b^4c^2d^6x^6+56\,b^4c^3d^5x^5+70\,b^4c^4d^4x^4+56\,b^4c^5d^3x^3+28\,b^4c^6d^2x^2+8\,b^4c^7dx+\cdots\right.\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x, algorithm="fricas")
```

output
```
1/2*((b^4*d^8*x^8 + 8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 + 56*b^4*c^3*d^5*
x^5 + 70*b^4*c^4*d^4*x^4 + 56*b^4*c^5*d^3*x^3 + 28*b^4*c^6*d^2*x^2 + 8*b^4
*c^7*d*x + b^4*c^8)*log(F)^4 - 4*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c
^2*d^4*x^4 + 20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3
*c^6)*log(F)^3 + 12*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4
*b^2*c^3*d*x + b^2*c^4)*log(F)^2 - 24*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(
F) + 24)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b^5*d*log(F)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 556 vs. $2(87) = 174$.

Time = 0.19 (sec) , antiderivative size = 556, normalized size of antiderivative = 6.32

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx$$

$$= \begin{cases} \frac{F^{a+b(c+dx)^2}\left(b^4c^8\log(F)^4+8b^4c^7dx\log(F)^4+28b^4c^6d^2x^2\log(F)^4+56b^4c^5d^3x^3\log(F)^4+70b^4c^4d^4x^4\log(F)^4+56b^4c^3d^5x^5\log(F)^4+28b^4\right.}{} \\ c^9x+\frac{9c^8dx^2}{2}+12c^7d^2x^3+21c^6d^3x^4+\frac{126c^5d^4x^5}{5}+21c^4d^5x^6+12c^3d^6x^7+\frac{9c^2d^7x^8}{2}+cd^8x^9+\frac{d^9x^{10}}{10} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**9,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**2)*(b**4*c**8*log(F)**4 + 8*b**4*c**7*d*x*
log(F)**4 + 28*b**4*c**6*d**2*x**2*log(F)**4 + 56*b**4*c**5*d**3*x**3*log(
F)**4 + 70*b**4*c**4*d**4*x**4*log(F)**4 + 56*b**4*c**3*d**5*x**5*log(F)**
4 + 28*b**4*c**2*d**6*x**6*log(F)**4 + 8*b**4*c*d**7*x**7*log(F)**4 + b**4
*d**8*x**8*log(F)**4 - 4*b**3*c**6*log(F)**3 - 24*b**3*c**5*d*x*log(F)**3
- 60*b**3*c**4*d**2*x**2*log(F)**3 - 80*b**3*c**3*d**3*x**3*log(F)**3 - 60
*b**3*c**2*d**4*x**4*log(F)**3 - 24*b**3*c*d**5*x**5*log(F)**3 - 4*b**3*d*
*6*x**6*log(F)**3 + 12*b**2*c**4*log(F)**2 + 48*b**2*c**3*d*x*log(F)**2 +
72*b**2*c**2*d**2*x**2*log(F)**2 + 48*b**2*c*d**3*x**3*log(F)**2 + 12*b**2
*d**4*x**4*log(F)**2 - 24*b*c**2*log(F) - 48*b*c*d*x*log(F) - 24*b*d**2*x*
*2*log(F) + 24)/(2*b**5*d*log(F)**5), Ne(b**5*d*log(F)**5, 0)), (c**9*x +
9*c**8*d*x**2/2 + 12*c**7*d**2*x**3 + 21*c**6*d**3*x**4 + 126*c**5*d**4*x*
*5/5 + 21*c**4*d**5*x**6 + 12*c**3*d**6*x**7 + 9*c**2*d**7*x**8/2 + c*d**8
*x**9 + d**9*x**10/10, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.38 (sec) , antiderivative size = 3727, normalized size of antiderivative = 42.35

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x, algorithm="maxima")
```

output
```
-9/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*c^8/sqrt(b*log(F)) + 18*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(er
f(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/
2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)
^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*ga
mma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d
^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^7*d/sqrt(b*log(F))
- 42*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*lo
g(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/(
(b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b
*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^
2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^6*d^2/sqrt(b*log(F)) + 63*(sqrt
(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^
2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/
(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log(F))
^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + b*c*...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx$$
$$= \frac{\left(b^4d^8\left(x+\frac{c}{d}\right)^8\log(F)^4-4\,b^3d^6\left(x+\frac{c}{d}\right)^6\log(F)^3+12\,b^2d^4\left(x+\frac{c}{d}\right)^4\log(F)^2-24\,bd^2\left(x+\frac{c}{d}\right)^2\log(F)+\right.}{2\,b^5d\log(F)^5}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x, algorithm="giac")
```

output
```
1/2*(b^4*d^8*(x + c/d)^8*log(F)^4 - 4*b^3*d^6*(x + c/d)^6*log(F)^3 + 12*b^
2*d^4*(x + c/d)^4*log(F)^2 - 24*b*d^2*(x + c/d)^2*log(F) + 24)*e^(b*d^2*x^
2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^5*d*log(F)^5)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 391, normalized size of antiderivative = 4.44

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx$$
$$= \frac{12\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x} - \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,b^3\ln(F)^3\,(4\,c^6+24\,c^5\,d\,x+60\,c^4\,d^2\,x^2+80\,c^3\,d^3\,x^3+60\,c^2\,d^4\,x^4+24\,c\,d^5\,x^5+4\,c}{2}}{}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^9,x)
```

output
```
(12*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x) - (F^(b*d^2*x^2)*F^a*F^(b*c^
2)*F^(2*b*c*d*x)*b^3*log(F)^3*(4*c^6 + 4*d^6*x^6 + 24*c*d^5*x^5 + 60*c^4*d
^2*x^2 + 80*c^3*d^3*x^3 + 60*c^2*d^4*x^4 + 24*c^5*d*x))/2 + (F^(b*d^2*x^2)
*F^a*F^(b*c^2)*F^(2*b*c*d*x)*b^4*log(F)^4*(c^8 + d^8*x^8 + 8*c*d^7*x^7 + 2
8*c^6*d^2*x^2 + 56*c^5*d^3*x^3 + 70*c^4*d^4*x^4 + 56*c^3*d^5*x^5 + 28*c^2*
d^6*x^6 + 8*c^7*d*x))/2 - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*b*log
(F)*(24*c^2 + 24*d^2*x^2 + 48*c*d*x))/2 + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(
2*b*c*d*x)*b^2*log(F)^2*(12*c^4 + 12*d^4*x^4 + 48*c*d^3*x^3 + 72*c^2*d^2*x
^2 + 48*c^3*d*x))/2)/(b^5*d*log(F)^5)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 395, normalized size of antiderivative = 4.49

$$\int F^{a+b(c+dx)^2}(c+dx)^9\,dx$$
$$= \frac{f^{b\,d^2x^2+2bcdx+b\,c^2+a}\left(24-48\log(f)\,bcdx+8\log(f)^4\,b^4c^7dx+28\log(f)^4\,b^4c^6d^2x^2+56\log(f)^4\,b^4c^5d^3x^3+70\right.}{}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^9,x)
```

output
```
(f**(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*(log(f)**4*b**4*c**8 + 8*log(f)
**4*b**4*c**7*d*x + 28*log(f)**4*b**4*c**6*d**2*x**2 + 56*log(f)**4*b**4*c
**5*d**3*x**3 + 70*log(f)**4*b**4*c**4*d**4*x**4 + 56*log(f)**4*b**4*c**3*
d**5*x**5 + 28*log(f)**4*b**4*c**2*d**6*x**6 + 8*log(f)**4*b**4*c*d**7*x**
7 + log(f)**4*b**4*d**8*x**8 - 4*log(f)**3*b**3*c**6 - 24*log(f)**3*b**3*c
**5*d*x - 60*log(f)**3*b**3*c**4*d**2*x**2 - 80*log(f)**3*b**3*c**3*d**3*x
**3 - 60*log(f)**3*b**3*c**2*d**4*x**4 - 24*log(f)**3*b**3*c*d**5*x**5 - 4
*log(f)**3*b**3*d**6*x**6 + 12*log(f)**2*b**2*c**4 + 48*log(f)**2*b**2*c**
3*d*x + 72*log(f)**2*b**2*c**2*d**2*x**2 + 48*log(f)**2*b**2*c*d**3*x**3 +
 12*log(f)**2*b**2*d**4*x**4 - 24*log(f)*b*c**2 - 48*log(f)*b*c*d*x - 24*l
og(f)*b*d**2*x**2 + 24))/(2*log(f)**5*b**5*d)
```

3.191 $\int F^{a+b(c+dx)^2}(c+dx)^7\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx=-\frac{3F^{a+b(c+dx)^2}}{b^4d\log^4(F)}+\frac{3F^{a+b(c+dx)^2}(c+dx)^2}{b^3d\log^3(F)}-\frac{3F^{a+b(c+dx)^2}(c+dx)^4}{2b^2d\log^2(F)}+\frac{F^{a+b(c+dx)^2}(c+dx)^6}{2bd\log(F)}$$

output
```
-3*F^(a+b*(d*x+c)^2)/b^4/d/ln(F)^4+3*F^(a+b*(d*x+c)^2)*(d*x+c)^2/b^3/d/ln(
F)^3-3/2*F^(a+b*(d*x+c)^2)*(d*x+c)^4/b^2/d/ln(F)^2+1/2*F^(a+b*(d*x+c)^2)*(
d*x+c)^6/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.57

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx=\frac{F^{a+b(c+dx)^2}\left(-6+6b(c+dx)^2\log(F)-3b^2(c+dx)^4\log^2(F)+b^3(c+dx)^6\log^3(F)\right)}{2b^4d\log^4(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^7,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(-6 + 6*b*(c + d*x)^2*Log[F] - 3*b^2*(c + d*x)^4*Lo
g[F]^2 + b^3*(c + d*x)^6*Log[F]^3))/(2*b^4*d*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^7 F^{a+b(c+dx)^2} \, dx$$

$$\Big\downarrow \; 2641$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{3\int F^{b(c+dx)^2+a}(c+dx)^5 dx}{b\log(F)}$$

$$\Big\downarrow \; 2641$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{3\left(\frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{2\int F^{b(c+dx)^2+a}(c+dx)^3 dx}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \; 2641$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{3\left(\frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{2\left(\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{\int F^{b(c+dx)^2+a}(c+dx)dx}{b\log(F)}\right)}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \; 2638$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{3\left(\frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{2\left(\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{2bd\log(F)} - \frac{F^{a+b(c+dx)^2}}{2b^2d\log^2(F)}\right)}{b\log(F)}\right)}{b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^7,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(c + d*x)^6)/(2*b*d*Log[F]) - (3*((F^(a + b*(c + d*
x)^2)*(c + d*x)^4)/(2*b*d*Log[F]) - (2*(-1/2*F^(a + b*(c + d*x)^2)/(b^2*d*
Log[F]^2) + (F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(2*b*d*Log[F])))/(b*Log[F]
)))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.90

method	result
orering	$\frac{\left(d^{6} x^{6} b^{3} \ln \left(F \right)^{3}+6 c \,d^{5} x^{5} b^{3} \ln \left(F \right)^{3}+15 \ln \left(F \right)^{3} b^{3} c^{2} d^{4} x^{4}+20 \ln \left(F \right)^{3} b^{3} c^{3} d^{3} x^{3}+15 \ln \left(F \right)^{3} b^{3} c^{4} d^{2} x^{2}+6 \ln \left(F \right)^{3} b^{3} c^{5} d x +\ln \left(F \right)^{3} b^{3}\right.}{}$
gosper	$\frac{\left(d^{6} x^{6} b^{3} \ln \left(F \right)^{3}+6 c \,d^{5} x^{5} b^{3} \ln \left(F \right)^{3}+15 \ln \left(F \right)^{3} b^{3} c^{2} d^{4} x^{4}+20 \ln \left(F \right)^{3} b^{3} c^{3} d^{3} x^{3}+15 \ln \left(F \right)^{3} b^{3} c^{4} d^{2} x^{2}+6 \ln \left(F \right)^{3} b^{3} c^{5} d x +\ln \left(F \right)^{3} b^{3}\right.}{}$
risch	$\frac{\left(d^{6} x^{6} b^{3} \ln \left(F \right)^{3}+6 c \,d^{5} x^{5} b^{3} \ln \left(F \right)^{3}+15 \ln \left(F \right)^{3} b^{3} c^{2} d^{4} x^{4}+20 \ln \left(F \right)^{3} b^{3} c^{3} d^{3} x^{3}+15 \ln \left(F \right)^{3} b^{3} c^{4} d^{2} x^{2}+6 \ln \left(F \right)^{3} b^{3} c^{5} d x +\ln \left(F \right)^{3} b^{3}\right.}{}$
norman	$\frac{\left(\ln \left(F \right)^{3} b^{3} c^{6}-3 \ln \left(F \right)^{2} b^{2} c^{4}+6 \ln \left(F \right) b \,c^{2}-6\right) {\mathrm e}^{\left(a +b \left(d x +c \right)^{2}\right) \ln \left(F \right)}}{2 \ln \left(F \right)^{4} b^{4} d}+\frac{d^{5} x^{6} {\mathrm e}^{\left(a +b \left(d x +c \right)^{2}\right) \ln \left(F \right)}}{2 \ln \left(F \right) b}+\frac{3 c \left(\ln \left(F \right)^{2} b^{2} c^{4}-2 \ln \left(F \right) b c\right.}{\ln(}$
parallelrisch	$\frac{d^{6} F^{a +b \left(d x +c \right)^{2}} x^{6} b^{3} \ln \left(F \right)^{3}+6 c \,d^{5} F^{a +b \left(d x +c \right)^{2}} x^{5} b^{3} \ln \left(F \right)^{3}+15 \ln \left(F \right)^{3} x^{4} F^{a +b \left(d x +c \right)^{2}} b^{3} c^{2} d^{4}+20 \ln \left(F \right)^{3} x^{3} F^{a +b \left(d x +c \right)^{2}} b^{3} c^{3}}{}$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(d^6*x^6*b^3*ln(F)^3+6*c*d^5*x^5*b^3*ln(F)^3+15*ln(F)^3*b^3*c^2*d^4*
x^4+20*ln(F)^3*b^3*c^3*d^3*x^3+15*ln(F)^3*b^3*c^4*d^2*x^2+6*ln(F)^3*b^3*c^
5*d*x+ln(F)^3*b^3*c^6-3*d^4*x^4*b^2*ln(F)^2-12*d^3*c*x^3*b^2*ln(F)^2-18*ln
(F)^2*b^2*c^2*d^2*x^2-12*ln(F)^2*b^2*c^3*d*x-3*ln(F)^2*b^2*c^4+6*ln(F)*b*d
^2*x^2+12*ln(F)*b*c*d*x+6*ln(F)*b*c^2-6)/b^4/ln(F)^4*F^(a+b*(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.65

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx$$
$$= \frac{\left(\left(b^3d^6x^6+6\,b^3cd^5x^5+15\,b^3c^2d^4x^4+20\,b^3c^3d^3x^3+15\,b^3c^4d^2x^2+6\,b^3c^5dx+b^3c^6\right)\log\left(F\right)^3-3\,\left(b^2d^4x^4+\right.\right.}{2\,b^4d}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/2*((b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3*
x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^3 - 3*(b^2*d^4*
x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F
)^2 + 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) - 6)*F^(b*d^2*x^2 + 2*b*c*d
*x + b*c^2 + a)/(b^4*d*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(112) = 224.

Time = 0.14 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.89

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx$$
$$= \begin{cases} \frac{F^{a+b(c+dx)^2}\left(b^3c^6\log\left(F\right)^3+6b^3c^5dx\log\left(F\right)^3+15b^3c^4d^2x^2\log\left(F\right)^3+20b^3c^3d^3x^3\log\left(F\right)^3+15b^3c^2d^4x^4\log\left(F\right)^3+6b^3cd^5x^5\log\left(F\right)^3+b^3d^6x^6\right.}{} \\ c^7x+\frac{7c^6dx^2}{2}+7c^5d^2x^3+\frac{35c^4d^3x^4}{4}+7c^3d^4x^5+\frac{7c^2d^5x^6}{2}+cd^6x^7+\frac{d^7x^8}{8} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**7,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**2)*(b**3*c**6*log(F)**3 + 6*b**3*c**5*d*x*
log(F)**3 + 15*b**3*c**4*d**2*x**2*log(F)**3 + 20*b**3*c**3*d**3*x**3*log(
F)**3 + 15*b**3*c**2*d**4*x**4*log(F)**3 + 6*b**3*c*d**5*x**5*log(F)**3 +
b**3*d**6*x**6*log(F)**3 - 3*b**2*c**4*log(F)**2 - 12*b**2*c**3*d*x*log(F)
**2 - 18*b**2*c**2*d**2*x**2*log(F)**2 - 12*b**2*c*d**3*x**3*log(F)**2 - 3
*b**2*d**4*x**4*log(F)**2 + 6*b*c**2*log(F) + 12*b*c*d*x*log(F) + 6*b*d**2
*x**2*log(F) - 6)/(2*b**4*d*log(F)**4), Ne(b**4*d*log(F)**4, 0)), (c**7*x
+ 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*d**4*x
**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.89 (sec) , antiderivative size = 2452, normalized size of antiderivative = 19.46

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^7,x, algorithm="maxima")
```

output
```
-7/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*c^6/sqrt(b*log(F)) + 21/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(
erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(
5/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*
d)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*
gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)
*d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^5*d/sqrt(b*log(F)
) - 35/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^
2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*
c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)
^3/((b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x
 + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*
c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^4*d^2/sqrt(b*log(F)) + 35/2
*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*l
og(F))^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x ...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx = \frac{\left(b^3d^6\left(x+\frac{c}{d}\right)^6\log(F)^3-3\,b^2d^4\left(x+\frac{c}{d}\right)^4\log(F)^2+6\,bd^2\left(x+\frac{c}{d}\right)^2\log(F)-6\right)e^{\left(bd^2x^2\log(F)+2\,bcdx\log(F)+bc^2\log(F)+a\log(F)\right)}}{2\,b^4d\log(F)^4}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^7,x, algorithm="giac")
```

output
```
1/2*(b^3*d^6*(x + c/d)^6*log(F)^3 - 3*b^2*d^4*(x + c/d)^4*log(F)^2 + 6*b*d
^2*(x + c/d)^2*log(F) - 6)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*
log(F) + a*log(F))/(b^4*d*log(F)^4)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.01

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx$$
$$= \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,\left(b^3\,c^6\,\ln\left(F\right)^3+6\,b^3\,c^5\,d\,x\,\ln\left(F\right)^3+15\,b^3\,c^4\,d^2\,x^2\,\ln\left(F\right)^3+20\,b^3\,c^3\,d^3\,x^3\,\ln\left(F\right)^3+\right.}{}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^7,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(6*b*c^2*log(F) - 3*b^2*c^4*log
(F)^2 + b^3*c^6*log(F)^3 + 6*b*d^2*x^2*log(F) - 3*b^2*d^4*x^4*log(F)^2 + b
^3*d^6*x^6*log(F)^3 - 12*b^2*c*d^3*x^3*log(F)^2 + 6*b^3*c*d^5*x^5*log(F)^3
 - 18*b^2*c^2*d^2*x^2*log(F)^2 + 15*b^3*c^4*d^2*x^2*log(F)^3 + 20*b^3*c^3*
d^3*x^3*log(F)^3 + 15*b^3*c^2*d^4*x^4*log(F)^3 + 12*b*c*d*x*log(F) - 12*b^
2*c^3*d*x*log(F)^2 + 6*b^3*c^5*d*x*log(F)^3 - 6))/(2*b^4*d*log(F)^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.97

$$\int F^{a+b(c+dx)^2}(c+dx)^7\,dx$$
$$= \frac{f^{b\,d^2x^2+2bcdx+b\,c^2+a}\left(\log(f)^3\,b^3c^6+6\log(f)^3\,b^3c^5dx+15\log(f)^3\,b^3c^4d^2x^2+20\log(f)^3\,b^3c^3d^3x^3+15\log(f)^3\right.}{}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^7,x)
```

output
```
(f**(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*(log(f)**3*b**3*c**6 + 6*log(f)
**3*b**3*c**5*d*x + 15*log(f)**3*b**3*c**4*d**2*x**2 + 20*log(f)**3*b**3*c
**3*d**3*x**3 + 15*log(f)**3*b**3*c**2*d**4*x**4 + 6*log(f)**3*b**3*c*d**5
*x**5 + log(f)**3*b**3*d**6*x**6 - 3*log(f)**2*b**2*c**4 - 12*log(f)**2*b*
*2*c**3*d*x - 18*log(f)**2*b**2*c**2*d**2*x**2 - 12*log(f)**2*b**2*c*d**3*
x**3 - 3*log(f)**2*b**2*d**4*x**4 + 6*log(f)*b*c**2 + 12*log(f)*b*c*d*x +
6*log(f)*b*d**2*x**2 - 6))/(2*log(f)**4*b**4*d)
```

3.192 $\int F^{a+b(c+dx)^2}(c+dx)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx=\frac{F^{a+b(c+dx)^2}}{b^3d\log^3(F)}-\frac{F^{a+b(c+dx)^2}(c+dx)^2}{b^2d\log^2(F)}+\frac{F^{a+b(c+dx)^2}(c+dx)^4}{2bd\log(F)}$$

output
```
F^(a+b*(d*x+c)^2)/b^3/d/ln(F)^3-F^(a+b*(d*x+c)^2)*(d*x+c)^2/b^2/d/ln(F)^2+
1/2*F^(a+b*(d*x+c)^2)*(d*x+c)^4/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx=\frac{F^{a+b(c+dx)^2}\left(2-2b(c+dx)^2\log(F)+b^2(c+dx)^4\log^2(F)\right)}{2b^3d\log^3(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^5,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(2 - 2*b*(c + d*x)^2*Log[F] + b^2*(c + d*x)^4*Log[F
]^2))/(2*b^3*d*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^5 F^{a+b(c+dx)^2} \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{2\int F^{b(c+dx)^2+a}(c+dx)^3dx}{b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{2\left(\frac{(c+dx)^2F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\int F^{b(c+dx)^2+a}(c+dx)dx}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{(c+dx)^4 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{2\left(\frac{(c+dx)^2F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{F^{a+b(c+dx)^2}}{2b^2d\log^2(F)}\right)}{b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^5,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(c + d*x)^4)/(2*b*d*Log[F]) - (2*(-1/2*F^(a + b*(c
+ d*x)^2)/(b^2*d*Log[F]^2) + (F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(2*b*d*Lo
g[F])))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.41

method	result
orering	$\frac{\left(d^4x^4b^2\ln(F)^2+4d^3c\,x^3b^2\ln(F)^2+6\ln(F)^2b^2c^2d^2x^2+4\ln(F)^2b^2c^3dx+\ln(F)^2b^2c^4-2\ln(F)b\,d^2x^2-4\ln(F)bcdx-2\ln(F)b\,c\right.}{2d\,b^3\ln(F)^3}$
gosper	$\frac{\left(d^4x^4b^2\ln(F)^2+4d^3c\,x^3b^2\ln(F)^2+6\ln(F)^2b^2c^2d^2x^2+4\ln(F)^2b^2c^3dx+\ln(F)^2b^2c^4-2\ln(F)b\,d^2x^2-4\ln(F)bcdx-2\ln(F)b\,c\right.}{2\ln(F)^3b^3d}$
risch	$\frac{\left(d^4x^4b^2\ln(F)^2+4d^3c\,x^3b^2\ln(F)^2+6\ln(F)^2b^2c^2d^2x^2+4\ln(F)^2b^2c^3dx+\ln(F)^2b^2c^4-2\ln(F)b\,d^2x^2-4\ln(F)bcdx-2\ln(F)b\,c\right.}{2\ln(F)^3b^3d}$
norman	$\frac{d(3\ln(F)b\,c^2-1)x^2\mathrm{e}^{\left(a+b(dx+c)^2\right)\ln(F)}}{\ln(F)^2b^2}+\frac{\left(\ln(F)^2b^2c^4-2\ln(F)b\,c^2+2\right)\mathrm{e}^{\left(a+b(dx+c)^2\right)\ln(F)}}{2\ln(F)^3b^3d}+\frac{d^3x^4\mathrm{e}^{\left(a+b(dx+c)^2\right)\ln(F)}}{2\ln(F)b}$
parallelrisch	$\frac{d^4F^{a+b(dx+c)^2}x^4b^2\ln(F)^2+4d^3c\,F^{a+b(dx+c)^2}x^3b^2\ln(F)^2+6\ln(F)^2x^2F^{a+b(dx+c)^2}b^2c^2d^2+4\ln(F)^2x\,F^{a+b(dx+c)^2}b^2c^3d+}{2\ln(F)}$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(d^4*x^4*b^2*ln(F)^2+4*d^3*c*x^3*b^2*ln(F)^2+6*ln(F)^2*b^2*c^2*d^2*x
^2+4*ln(F)^2*b^2*c^3*d*x+ln(F)^2*b^2*c^4-2*ln(F)*b*d^2*x^2-4*ln(F)*b*c*d*x
-2*ln(F)*b*c^2+2)/b^3/ln(F)^3*F^(a+b*(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.32

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx$$
$$=\frac{\left(\left(b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4\right)\log\left(F\right)^2-2\left(bd^2x^2+2\,bcdx+bc^2\right)\log\left(F\right)+2\right)F^{bd}}{2\,b^3d\log\left(F\right)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/2*((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
b^2*c^4)*log(F)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) + 2)*F^(b*d^2
*x^2 + 2*b*c*d*x + b*c^2 + a)/(b^3*d*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(76) = 152.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.33

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx$$
$$=\begin{cases}\frac{F^{a+b(c+dx)^2}\left(b^2c^4\log\left(F\right)^2+4b^2c^3dx\log\left(F\right)^2+6b^2c^2d^2x^2\log\left(F\right)^2+4b^2cd^3x^3\log\left(F\right)^2+b^2d^4x^4\log\left(F\right)^2-2bc^2\log\left(F\right)-4bcdx\log\left(F\right)-2bd^2x\right.}{2b^3d\log\left(F\right)^3} \\ c^5x+\frac{5c^4dx^2}{2}+\frac{10c^3d^2x^3}{3}+\frac{5c^2d^3x^4}{2}+cd^4x^5+\frac{d^5x^6}{6}\end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**5,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**2)*(b**2*c**4*log(F)**2 + 4*b**2*c**3*d*x*
log(F)**2 + 6*b**2*c**2*d**2*x**2*log(F)**2 + 4*b**2*c*d**3*x**3*log(F)**2
 + b**2*d**4*x**4*log(F)**2 - 2*b*c**2*log(F) - 4*b*c*d*x*log(F) - 2*b*d**
2*x**2*log(F) + 2)/(2*b**3*d*log(F)**3), Ne(b**3*d*log(F)**3, 0)), (c**5*x
 + 5*c**4*d*x**2/2 + 10*c**3*d**2*x**3/3 + 5*c**2*d**3*x**4/2 + c*d**4*x**
5 + d**5*x**6/6, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.57 (sec) , antiderivative size = 1438, normalized size of antiderivative = 15.80

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^5,x, algorithm="maxima")
```

output
```
-5/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*c^4/sqrt(b*log(F)) + 5*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(erf
(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/2
)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^
2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*gam
ma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d^
5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^3*d/sqrt(b*log(F)) -
 5*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(
F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)^2
*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/((b
*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b*c
*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)^2
*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)
)*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^2*d^2/sqrt(b*log(F)) + 5/2*(sqrt(
pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2
))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log(F))^
(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + b*c*d...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx = \frac{\left(b^2d^4\left(x+\frac{c}{d}\right)^4\log\left(F\right)^2-2\,bd^2\left(x+\frac{c}{d}\right)^2\log\left(F\right)+2\right)e^{\left(bd^2x^2\log(F)+2\,bcdx\log(F)+bc^2\log(F)+a\log(F)\right)}}{2\,b^3d\log\left(F\right)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^5,x, algorithm="giac")
```

output
```
1/2*(b^2*d^4*(x + c/d)^4*log(F)^2 - 2*b*d^2*(x + c/d)^2*log(F) + 2)*e^(b*d
^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^3*d*log(F)^
3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx = \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,\left(b^2\,c^4\ln\left(F\right)^2+4\,b^2\,c^3\,d\,x\ln\left(F\right)^2+6\,b^2\,c^2\,d^2\,x^2\ln\left(F\right)^2+4\,b^2\,c\,d^3\,x^3\ln\left(F\right)^2+b^2\,c\right.}{2\,b^3\,d\ln\left(F\right)^3}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^5,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(b^2*c^4*log(F)^2 - 2*b*c^2*log
(F) - 2*b*d^2*x^2*log(F) + b^2*d^4*x^4*log(F)^2 + 4*b^2*c*d^3*x^3*log(F)^2
 + 6*b^2*c^2*d^2*x^2*log(F)^2 - 4*b*c*d*x*log(F) + 4*b^2*c^3*d*x*log(F)^2
+ 2))/(2*b^3*d*log(F)^3)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.51

$$\int F^{a+b(c+dx)^2}(c+dx)^5\,dx$$
$$= \frac{f^{b\,d^2x^2+2bcdx+b\,c^2+a}\left(\log(f)^2\,b^2c^4+4\log(f)^2\,b^2c^3dx+6\log(f)^2\,b^2c^2d^2x^2+4\log(f)^2\,b^2c\,d^3x^3+\log(f)^2\,b^2d^4x\right.}{2\log\left(f\right)^3b^3d}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^5,x)
```

output
```
(f**(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*(log(f)**2*b**2*c**4 + 4*log(f)
**2*b**2*c**3*d*x + 6*log(f)**2*b**2*c**2*d**2*x**2 + 4*log(f)**2*b**2*c*d
**3*x**3 + log(f)**2*b**2*d**4*x**4 - 2*log(f)*b*c**2 - 4*log(f)*b*c*d*x -
 2*log(f)*b*d**2*x**2 + 2))/(2*log(f)**3*b**3*d)
```

3.193 $\int F^{a+b(c+dx)^2}(c+dx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx=-\frac{F^{a+b(c+dx)^2}}{2b^2d\log^2(F)}+\frac{F^{a+b(c+dx)^2}(c+dx)^2}{2bd\log(F)}$$

output
```
-1/2*F^(a+b*(d*x+c)^2)/b^2/d/ln(F)^2+1/2*F^(a+b*(d*x+c)^2)*(d*x+c)^2/b/d/l
n(F)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx=\frac{F^{a+b(c+dx)^2}(-1+b(c+dx)^2\log(F))}{2b^2d\log^2(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^3,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(-1 + b*(c + d*x)^2*Log[F]))/(2*b^2*d*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 F^{a+b(c+dx)^2} \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\int F^{b(c+dx)^2+a}(c+dx)dx}{b\log(F)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{(c+dx)^2 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{F^{a+b(c+dx)^2}}{2b^2d\log^2(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^3,x]
```

output
```
-1/2*F^(a + b*(c + d*x)^2)/(b^2*d*Log[F]^2) + (F^(a + b*(c + d*x)^2)*(c +
d*x)^2)/(2*b*d*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

method	result	size
orering	$\frac{\left(\ln \left(F \right) b \,d^{2} x^{2}+2 \ln \left(F \right) b c d x +\ln \left(F \right) b \,c^{2}-1\right) F^{a +b \left(d x +c \right)^{2}}}{2 d \,b^{2} \ln \left(F \right)^{2}}$	53
gosper	$\frac{\left(\ln \left(F \right) b \,d^{2} x^{2}+2 \ln \left(F \right) b c d x +\ln \left(F \right) b \,c^{2}-1\right) F^{b \,d^{2} x^{2}+2 b c d x +b \,c^{2}+a}}{2 d \,b^{2} \ln \left(F \right)^{2}}$	63
risch	$\frac{\left(\ln \left(F \right) b \,d^{2} x^{2}+2 \ln \left(F \right) b c d x +\ln \left(F \right) b \,c^{2}-1\right) F^{b \,d^{2} x^{2}+2 b c d x +b \,c^{2}+a}}{2 d \,b^{2} \ln \left(F \right)^{2}}$	63
norman	$\frac{c x \,{\mathrm e}^{\left(a +b \left(d x +c \right)^{2}\right) \ln \left(F \right)}}{\ln \left(F \right) b}+\frac{\left(\ln \left(F \right) b \,c^{2}-1\right) {\mathrm e}^{\left(a +b \left(d x +c \right)^{2}\right) \ln \left(F \right)}}{2 d \,b^{2} \ln \left(F \right)^{2}}+\frac{d \,x^{2} {\mathrm e}^{\left(a +b \left(d x +c \right)^{2}\right) \ln \left(F \right)}}{2 \ln \left(F \right) b}$	91
parallelrisch	$\frac{d^{2} F^{a +b \left(d x +c \right)^{2}} x^{2} b \ln \left(F \right)+2 c \,F^{a +b \left(d x +c \right)^{2}} x b \ln \left(F \right) d +\ln \left(F \right) F^{a +b \left(d x +c \right)^{2}} b \,c^{2}-F^{a +b \left(d x +c \right)^{2}}}{2 d \,b^{2} \ln \left(F \right)^{2}}$	93

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(ln(F)*b*d^2*x^2+2*ln(F)*b*c*d*x+ln(F)*b*c^2-1)/b^2/ln(F)^2*F^(a+b*(
d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.97

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx = \frac{((bd^2x^2+2\,bcdx+bc^2)\log(F)-1)F^{bd^2x^2+2\,bcdx+bc^2+a}}{2\,b^2d\log(F)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) - 1)*F^(b*d^2*x^2 + 2*b*c*d*x
+ b*c^2 + a)/(b^2*d*log(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx = \begin{cases} \frac{F^{a+b(c+dx)^2}(bc^2\log(F)+2bcdx\log(F)+bd^2x^2\log(F)-1)}{2b^2d\log(F)^2} & \text{for } b^2d\log(F)^2\neq 0 \\ c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**3,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**2)*(b*c**2*log(F) + 2*b*c*d*x*log(F) + b*d
**2*x**2*log(F) - 1)/(2*b**2*d*log(F)**2), Ne(b**2*d*log(F)**2, 0)), (c**3
*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.33 (sec) , antiderivative size = 683, normalized size of antiderivative = 11.02

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
-3/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*c^2/sqrt(b*log(F)) + 3/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(e
rf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5
/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d
)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*g
amma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*
d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c*d/sqrt(b*log(F)) -
 1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*lo
g(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/(
(b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b
*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^
2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*d^2/sqrt(b*log(F)) + 1/2*sqrt(pi)
*F^(b*c^2 + a)*c^3*erf(sqrt(-b*log(F))*d*x - b*c*log(F)/sqrt(-b*log(F)))/(
sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 1227, normalized size of antiderivative = 19.79

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^3,x, algorithm="giac")
```

output
```
1/2*(2*((pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))*(pi*b*c^2
*d*sgn(F) + pi*(d*x^2 + 2*c*x)*b*d^2*sgn(F) - pi*b*c^2*d - pi*(d*x^2 + 2*c
*x)*b*d^2)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)
^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))^2) + (pi^2
*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)*(b*c^2*d*log(abs
(F)) + (d*x^2 + 2*c*x)*b*d^2*log(abs(F)) - d)/((pi^2*b^2*d^2*sgn(F) - pi^2
*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) -
 pi*b^2*d^2*log(abs(F)))^2))*cos(-1/2*pi*b*d^2*x^2*sgn(F) + 1/2*pi*b*d^2*x
^2 - pi*b*c*d*x*sgn(F) + pi*b*c*d*x - 1/2*pi*b*c^2*sgn(F) + 1/2*pi*b*c^2 -
 1/2*pi*a*sgn(F) + 1/2*pi*a) + ((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^
2*d^2*log(abs(F))^2)*(pi*b*c^2*d*sgn(F) + pi*(d*x^2 + 2*c*x)*b*d^2*sgn(F)
- pi*b*c^2*d - pi*(d*x^2 + 2*c*x)*b*d^2)/((pi^2*b^2*d^2*sgn(F) - pi^2*b^2*
d^2 + 2*b^2*d^2*log(abs(F))^2)^2 + 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b
^2*d^2*log(abs(F)))^2) - 4*(pi*b^2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log
(abs(F)))*(b*c^2*d*log(abs(F)) + (d*x^2 + 2*c*x)*b*d^2*log(abs(F)) - d)/((
pi^2*b^2*d^2*sgn(F) - pi^2*b^2*d^2 + 2*b^2*d^2*log(abs(F))^2)^2 + 4*(pi*b^
2*d^2*log(abs(F))*sgn(F) - pi*b^2*d^2*log(abs(F)))^2))*sin(-1/2*pi*b*d^2*x
^2*sgn(F) + 1/2*pi*b*d^2*x^2 - pi*b*c*d*x*sgn(F) + pi*b*c*d*x - 1/2*pi*b*c
^2*sgn(F) + 1/2*pi*b*c^2 - 1/2*pi*a*sgn(F) + 1/2*pi*a))*e^(b*c^2*log(abs(F
)) + (d*x^2 + 2*c*x)*b*d*log(abs(F)) + a*log(abs(F))) - 1/4*I*((pi*b*c^...
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx = \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,\left(b\,\ln\left(F\right)\,c^2+2\,b\,\ln\left(F\right)\,c\,d\,x+b\,\ln\left(F\right)\,d^2\,x^2-1\right)}{2\,b^2\,d\,{\ln\left(F\right)}^2}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^3,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(b*c^2*log(F) + b*d^2*x^2*log(F
) + 2*b*c*d*x*log(F) - 1))/(2*b^2*d*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)^3\,dx = \frac{f^{b\,d^2x^2+2bcdx+b\,c^2+a}(\log(f)\,b\,c^2+2\log(f)\,bcdx+\log(f)\,b\,d^2x^2-1)}{2\log(f)^2\,b^2d}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^3,x)
```

output
```
(f**(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)*(log(f)*b*c**2 + 2*log(f)*b*c*d
*x + log(f)*b*d**2*x**2 - 1))/(2*log(f)**2*b**2*d)
```

3.194 $\int F^{a+b(c+dx)^2}(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx=\frac{F^{a+b(c+dx)^2}}{2bd\log(F)}$$

output
```
1/2*F^(a+b*(d*x+c)^2)/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx=\frac{F^{a+b(c+dx)^2}}{2bd\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x),x]
```

output
```
F^(a + b*(c + d*x)^2)/(2*b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{F^{a+b(c+dx)^2}}{2bd\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x),x]
```

output
```
F^(a + b*(c + d*x)^2)/(2*b*d*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{F^{a+b(dx+c)^2}}{2bd\ln(F)}$	26
default	$\frac{F^{a+b(dx+c)^2}}{2bd\ln(F)}$	26
parallelrisch	$\frac{F^{a+b(dx+c)^2}}{2bd\ln(F)}$	26
orering	$\frac{F^{a+b(dx+c)^2}}{2bd\ln(F)}$	26
norman	$\frac{e^{\left(a+b(dx+c)^2\right)\ln(F)}}{2\ln(F)bd}$	28
gosper	$\frac{F^{b\,d^2x^2+2bcdx+b\,c^2+a}}{2bd\ln(F)}$	36
risch	$\frac{F^{b\,d^2x^2+2bcdx+b\,c^2+a}}{2bd\ln(F)}$	36

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(a+b*(d*x+c)^2)/b/d/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx = \frac{F^{bd^2x^2+2\,bcdx+bc^2+a}}{2\,bd\log{(F)}}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c),x, algorithm="fricas")
```

output
```
1/2*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx = \begin{cases} \frac{F^{a+b(c+dx)^2}}{2bd\log{(F)}} & \text{for } bd\log{(F)} \neq 0 \\ cx + \frac{dx^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c),x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**2)/(2*b*d*log(F)), Ne(b*d*log(F), 0)), (c*
x + d*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx = \frac{F^{(dx+c)^2b+a}}{2\,bd\log{(F)}}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c),x, algorithm="maxima")
```

output
```
1/2*F^((d*x + c)^2*b + a)/(b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx = \frac{F^{bd^2x^2+2\,bcdx+bc^2+a}}{2\,bd\log{(F)}}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c),x, algorithm="giac")
```

output
```
1/2*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx = \frac{F^{a+b\,(c+d\,x)^2}}{2\,b\,d\,\ln(F)}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x),x)
```

output
```
F^(a + b*(c + d*x)^2)/(2*b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.30

$$\int F^{a+b(c+dx)^2}(c+dx)\,dx = \frac{f^{b\,d^2x^2+2bcdx+b\,c^2+a}}{2\log(f)\,bd}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c),x)
```

output
```
f**(a + b*c**2 + 2*b*c*d*x + b*d**2*x**2)/(2*log(f)*b*d)
```

3.195 $\int \frac{F^{a+b(c+dx)^2}}{c+dx} dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx} \, dx = \frac{F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^2 \log(F)\right)}{2d}$$

output
```
1/2*F^a*Ei(b*(d*x+c)^2*ln(F))/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx} \, dx = \frac{F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^2 \log(F)\right)}{2d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^2*Log[F]])/(2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx$$
$$\Big\downarrow \text{2639}$$
$$\frac{F^a \,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)}{2d}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^2*Log[F]])/(2*d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{F^{a} \operatorname{expIntegral}_{1}\left(-b(dx+c)^{2} \ln(F)\right)}{2d}$	23

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*F^a*Ei(1,-b*(d*x+c)^2*ln(F))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx = \frac{F^a\mathrm{Ei}\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\log\left(F\right)\right)}{2\,d}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*F^a*Ei((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))/d
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx = \int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c),x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx = \int \frac{F^{(dx+c)^2b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx = \int \frac{F^{(dx+c)^2b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx = \frac{F^a\,\mathrm{ei}\left(b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)}{2\,d}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x),x)
```

output
```
(F^a*ei(b*log(F)*(c + d*x)^2))/(2*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{c+dx}\,dx = f^{b\,c^2+a}\left(\int \frac{f^{b\,d^2x^2+2bcdx}}{dx+c}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c),x)
```

output
```
f**(a + b*c**2)*int(f**(2*b*c*d*x + b*d**2*x**2)/(c + d*x),x)
```

3.196 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx=-\frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}+\frac{bF^a\,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)\log(F)}{2d}$$

output
```
-1/2*F^(a+b*(d*x+c)^2)/d/(d*x+c)^2+1/2*b*F^a*Ei(b*(d*x+c)^2*ln(F))*ln(F)/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx=\frac{F^a\left(-\frac{F^{b(c+dx)^2}}{(c+dx)^2}+b\,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)\log(F)\right)}{2d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^3,x]
```

output
```
(F^a*(-(F^(b*(c + d*x)^2)/(c + d*x)^2) + b*ExpIntegralEi[b*(c + d*x)^2*Log
[F]]*Log[F]))/(2*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2643}$$

$$b\log(F)\int \frac{F^{b(c+dx)^2+a}}{c+dx}dx-\frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}$$

$$\Big\downarrow \text{2639}$$

$$\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)}{2d}-\frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^3,x]
```

output
```
-1/2*F^(a + b*(c + d*x)^2)/(d*(c + d*x)^2) + (b*F^a*ExpIntegralEi[b*(c + d
*x)^2*Log[F]]*Log[F])/(2*d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{F^{b(dx+c)^2} F^a}{2d(dx+c)^2}-\frac{b\ln(F)F^a \operatorname{expIntegral}_1\left(-b(dx+c)^2\ln(F)\right)}{2d}$	53

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(d*x+c)^2*F^(b*(d*x+c)^2)*F^a-1/2/d*b*ln(F)*F^a*Ei(1,-b*(d*x+c)^2*l
n(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.89

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx = \frac{(bd^2x^2+2\,bcdx+bc^2)F^a\mathrm{Ei}((bd^2x^2+2\,bcdx+bc^2)\log(F))\log(F)-F^{bd^2x^2+2\,bcdx+bc^2+a}}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*F^a*Ei((b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*log(F))*log(F) - F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(d^3*x^2 + 2*c*d
^2*x + c^2*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx = -\frac{F^a\left(F^{b\,(c+d\,x)^2}+b\,\ln(F)\,\mathrm{expint}\left(-b\,\ln(F)\,(c+d\,x)^2\right)\,(c+d\,x)^2\right)}{2\,d\,(c+d\,x)^2}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^3,x)
```

output
```
-(F^a*(F^(b*(c + d*x)^2) + b*log(F)*expint(-b*log(F)*(c + d*x)^2)*(c + d*x
)^2))/(2*d*(c + d*x)^2)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^3}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^3,x)
```

output

```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 4*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**5 + 3*log(f)*b*c**4*d*x + 3*log(f)*b*c**3*d**2*x
**2 + log(f)*b*c**2*d**3*x**3 - c**3 - 3*c**2*d*x - 3*c*d**2*x**2 - d**3*x
**3),x)*log(f)**2*b**2*c**6*d + 8*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)
*b*c**5 + 3*log(f)*b*c**4*d*x + 3*log(f)*b*c**3*d**2*x**2 + log(f)*b*c**2*
d**3*x**3 - c**3 - 3*c**2*d*x - 3*c*d**2*x**2 - d**3*x**3),x)*log(f)**2*b*
*2*c**5*d**2*x + 4*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**5 + 3*log
(f)*b*c**4*d*x + 3*log(f)*b*c**3*d**2*x**2 + log(f)*b*c**2*d**3*x**3 - c**
3 - 3*c**2*d*x - 3*c*d**2*x**2 - d**3*x**3),x)*log(f)**2*b**2*c**4*d**3*x*
*2 - 8*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**5 + 3*log(f)*b*c**4*d
*x + 3*log(f)*b*c**3*d**2*x**2 + log(f)*b*c**2*d**3*x**3 - c**3 - 3*c**2*d
*x - 3*c*d**2*x**2 - d**3*x**3),x)*log(f)*b*c**4*d - 16*int(f**(2*b*c*d*x
+ b*d**2*x**2)/(log(f)*b*c**5 + 3*log(f)*b*c**4*d*x + 3*log(f)*b*c**3*d**2
*x**2 + log(f)*b*c**2*d**3*x**3 - c**3 - 3*c**2*d*x - 3*c*d**2*x**2 - d**3
*x**3),x)*log(f)*b*c**3*d**2*x - 8*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f
)*b*c**5 + 3*log(f)*b*c**4*d*x + 3*log(f)*b*c**3*d**2*x**2 + log(f)*b*c**2
*d**3*x**3 - c**3 - 3*c**2*d*x - 3*c*d**2*x**2 - d**3*x**3),x)*log(f)*b*c*
*2*d**3*x**2 + 4*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**5 + 3*log(f
)*b*c**4*d*x + 3*log(f)*b*c**3*d**2*x**2 + log(f)*b*c**2*d**3*x**3 - c**3
- 3*c**2*d*x - 3*c*d**2*x**2 - d**3*x**3),x)*c**2*d + 8*int(f**(2*b*c*d...
```

3.197 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = -\frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4} - \frac{bF^{a+b(c+dx)^2}\log(F)}{4d(c+dx)^2} + \frac{b^2F^a\,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)\log^2(F)}{4d}$$

output
```
-1/4*F^(a+b*(d*x+c)^2)/d/(d*x+c)^4-1/4*b*F^(a+b*(d*x+c)^2)*ln(F)/d/(d*x+c)
^2+1/4*b^2*F^a*Ei(b*(d*x+c)^2*ln(F))*ln(F)^2/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = \frac{F^a\left(b^2\,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)\log^2(F) - \frac{F^{b(c+dx)^2}\left(1+b(c+dx)^2\log(F)\right)}{(c+dx)^4}\right)}{4d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^5,x]
```

output
```
(F^a*(b^2*ExpIntegralEi[b*(c + d*x)^2*Log[F]]*Log[F]^2 - (F^(b*(c + d*x)^2
)*(1 + b*(c + d*x)^2*Log[F]))/(c + d*x)^4))/(4*d)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5} dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2}b\log(F)\int \frac{F^{b(c+dx)^2+a}}{(c+dx)^3}dx - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2}b\log(F)\left(b\log(F)\int \frac{F^{b(c+dx)^2+a}}{c+dx}dx - \frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}\right) - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2}b\log(F)\left(\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)}{2d} - \frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}\right) - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^5,x]
```

output
```
-1/4*F^(a + b*(c + d*x)^2)/(d*(c + d*x)^4) + (b*Log[F]*(-1/2*F^(a + b*(c +
 d*x)^2)/(d*(c + d*x)^2) + (b*F^a*ExpIntegralEi[b*(c + d*x)^2*Log[F]]*Log[
F])/(2*d)))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{F^{b(dx+c)^2} F^a}{4d(dx+c)^4}-\frac{b\ln(F) F^{b(dx+c)^2} F^a}{4d(dx+c)^2}-\frac{b^2\ln(F)^2 F^a \operatorname{expIntegral}_1\left(-b(dx+c)^2\ln(F)\right)}{4d}$	86

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d/(d*x+c)^4*F^(b*(d*x+c)^2)*F^a-1/4/d*b*ln(F)/(d*x+c)^2*F^(b*(d*x+c)^
2)*F^a-1/4/d*b^2*ln(F)^2*F^a*Ei(1,-b*(d*x+c)^2*ln(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(81) = 162$.

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.10

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = \frac{(b^2d^4x^4+4\,b^2cd^3x^3+6\,b^2c^2d^2x^2+4\,b^2c^3dx+b^2c^4)F^a\mathrm{Ei}((bd^2x^2+2\,bcdx+bc^2)\log(F))\log(F)^2-((bd^2}{4\,(d^5x^4+4\,cd^4x^3+6\,c^2d^3x^2+4\,c^3d^2x+c^4d)}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/4*((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
b^2*c^4)*F^a*Ei((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))*log(F)^2 - ((b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*log(F) + 1)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
))/(d^5*x^4 + 4*c*d^4*x^3 + 6*c^2*d^3*x^2 + 4*c^3*d^2*x + c^4*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**5,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**5, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^5, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = -\frac{F^a\,b^2\ln(F)^2\left(\frac{\mathrm{expint}\left(-b\,\ln(F)\,(c+d\,x)^2\right)}{2}+F^{b\,(c+d\,x)^2}\left(\frac{1}{2\,b\,\ln(F)\,(c+d\,x)^2}+\frac{1}{2\,b^2\,\ln(F)^2\,(c+d\,x)^4}\right)\right)}{2\,d}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^5,x)
```

output
```
-(F^a*b^2*log(F)^2*(expint(-b*log(F)*(c + d*x)^2)/2 + F^(b*(c + d*x)^2)*(1
/(2*b*log(F)*(c + d*x)^2) + 1/(2*b^2*log(F)^2*(c + d*x)^4))))/(2*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^5}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^5,x)
```

output

```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 4*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**7 + 5*log(f)*b*c**6*d*x + 10*log(f)*b*c**5*d**2*
x**2 + 10*log(f)*b*c**4*d**3*x**3 + 5*log(f)*b*c**3*d**4*x**4 + log(f)*b*c
**2*d**5*x**5 - 2*c**5 - 10*c**4*d*x - 20*c**3*d**2*x**2 - 20*c**2*d**3*x*
*3 - 10*c*d**4*x**4 - 2*d**5*x**5),x)*log(f)**2*b**2*c**8*d + 16*int(f**(2
*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**7 + 5*log(f)*b*c**6*d*x + 10*log(f)*b
*c**5*d**2*x**2 + 10*log(f)*b*c**4*d**3*x**3 + 5*log(f)*b*c**3*d**4*x**4 +
 log(f)*b*c**2*d**5*x**5 - 2*c**5 - 10*c**4*d*x - 20*c**3*d**2*x**2 - 20*c
**2*d**3*x**3 - 10*c*d**4*x**4 - 2*d**5*x**5),x)*log(f)**2*b**2*c**7*d**2*
x + 24*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**7 + 5*log(f)*b*c**6*d
*x + 10*log(f)*b*c**5*d**2*x**2 + 10*log(f)*b*c**4*d**3*x**3 + 5*log(f)*b*
c**3*d**4*x**4 + log(f)*b*c**2*d**5*x**5 - 2*c**5 - 10*c**4*d*x - 20*c**3*
d**2*x**2 - 20*c**2*d**3*x**3 - 10*c*d**4*x**4 - 2*d**5*x**5),x)*log(f)**2
*b**2*c**6*d**3*x**2 + 16*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**7
+ 5*log(f)*b*c**6*d*x + 10*log(f)*b*c**5*d**2*x**2 + 10*log(f)*b*c**4*d**3
*x**3 + 5*log(f)*b*c**3*d**4*x**4 + log(f)*b*c**2*d**5*x**5 - 2*c**5 - 10*
c**4*d*x - 20*c**3*d**2*x**2 - 20*c**2*d**3*x**3 - 10*c*d**4*x**4 - 2*d**5
*x**5),x)*log(f)**2*b**2*c**5*d**4*x**3 + 4*int(f**(2*b*c*d*x + b*d**2*x**
2)/(log(f)*b*c**7 + 5*log(f)*b*c**6*d*x + 10*log(f)*b*c**5*d**2*x**2 + 10*
log(f)*b*c**4*d**3*x**3 + 5*log(f)*b*c**3*d**4*x**4 + log(f)*b*c**2*d**...
```

3.198 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7} dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7} dx = -\frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6} - \frac{bF^{a+b(c+dx)^2}\log(F)}{12d(c+dx)^4} - \frac{b^2F^{a+b(c+dx)^2}\log^2(F)}{12d(c+dx)^2} + \frac{b^3F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)\log^3(F)}{12d}$$

output
```
-1/6*F^(a+b*(d*x+c)^2)/d/(d*x+c)^6-1/12*b*F^(a+b*(d*x+c)^2)*ln(F)/d/(d*x+c
)^4-1/12*b^2*F^(a+b*(d*x+c)^2)*ln(F)^2/d/(d*x+c)^2+1/12*b^3*F^a*Ei(b*(d*x+
c)^2*ln(F))*ln(F)^3/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7} dx = \frac{F^a\left(b^3 \operatorname{ExpIntegralEi}\left(b(c+dx)^2\log(F)\right)\log^3(F) - \frac{F^{b(c+dx)^2}\left(2+b(c+dx)^2\log(F)+b^2(c+dx)^4\log^2(F)\right)}{(c+dx)^6}\right)}{12d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^7,x]
```

output
```
(F^a*(b^3*ExpIntegralEi[b*(c + d*x)^2*Log[F]]*Log[F]^3 - (F^(b*(c + d*x)^2
)*(2 + b*(c + d*x)^2*Log[F] + b^2*(c + d*x)^4*Log[F]^2))/(c + d*x)^6))/(12
*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3} b \log(F) \int \frac{F^{b(c+dx)^2+a}}{(c+dx)^5} dx - \frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \int \frac{F^{b(c+dx)^2+a}}{(c+dx)^3} dx - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4}\right) - \frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \left(b \log(F) \int \frac{F^{b(c+dx)^2+a}}{c+dx} dx - \frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}\right) - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4}\right) - \frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \left(\frac{b F^a \log(F) \operatorname{ExpIntegralEi}\left(b(c+dx)^2 \log(F)\right)}{2d} - \frac{F^{a+b(c+dx)^2}}{2d(c+dx)^2}\right) - \frac{F^{a+b(c+dx)^2}}{4d(c+dx)^4}\right) - \frac{F^{a+b(c+dx)^2}}{6d(c+dx)^6}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^7,x]
```

output
```
-1/6*F^(a + b*(c + d*x)^2)/(d*(c + d*x)^6) + (b*Log[F]*(-1/4*F^(a + b*(c +
 d*x)^2)/(d*(c + d*x)^4) + (b*Log[F]*(-1/2*F^(a + b*(c + d*x)^2)/(d*(c + d
*x)^2) + (b*F^a*ExpIntegralEi[b*(c + d*x)^2*Log[F]]*Log[F])/(2*d)))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{F^{b(dx+c)^2} F^a}{6d(dx+c)^6}-\frac{b\ln(F) F^{b(dx+c)^2} F^a}{12d(dx+c)^4}-\frac{b^2\ln(F)^2 F^{b(dx+c)^2} F^a}{12d(dx+c)^2}-\frac{b^3\ln(F)^3 F^a \operatorname{expIntegral}_1\left(-b(dx+c)^2\ln(F)\right)}{12d}$	119

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6/d/(d*x+c)^6*F^(b*(d*x+c)^2)*F^a-1/12/d*b*ln(F)/(d*x+c)^4*F^(b*(d*x+c)
^2)*F^a-1/12/d*b^2*ln(F)^2/(d*x+c)^2*F^(b*(d*x+c)^2)*F^a-1/12/d*b^3*ln(F)^
3*F^a*Ei(1,-b*(d*x+c)^2*ln(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(113) = 226$.

Time = 0.08 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.41

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx = \frac{(b^3d^6x^6 + 6\,b^3cd^5x^5 + 15\,b^3c^2d^4x^4 + 20\,b^3c^3d^3x^3 + 15\,b^3c^4d^2x^2 + 6\,b^3c^5dx + b^3c^6)F^a\mathrm{Ei}((bd^2x^2 + 2\,bcdx +}{12\,(d^7x^6 + 6\,cd^6x}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/12*((b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3
*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*F^a*Ei((b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*log(F))*log(F)^3 - ((b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b
^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)^2 + (b*d^2*x^2 + 2*b*c*d*
x + b*c^2)*log(F) + 2)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(d^7*x^6 + 6
*c*d^6*x^5 + 15*c^2*d^5*x^4 + 20*c^3*d^4*x^3 + 15*c^4*d^3*x^2 + 6*c^5*d^2*
x + c^6*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**7,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**7, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^7, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx = -\frac{F^a\,b^3\ln\left(F\right)^3\,\mathrm{expint}\left(-b\,\ln\left(F\right)\,{\left(c+d\,x\right)}^2\right)}{12\,d} - \frac{F^a\,F^{b\,{\left(c+d\,x\right)}^2}\,b^3\ln\left(F\right)^3\,\left(\frac{1}{6\,b\,\ln\left(F\right)\,{\left(c+d\,x\right)}^2}+\frac{1}{6\,b^2\ln\left(F\right)^2\,{\left(c+d\,x\right)}^4}+\frac{1}{3\,b^3\ln\left(F\right)^3\,{\left(c+d\,x\right)}^6}\right)}{2\,d}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^7,x)
```

output
```
- (F^a*b^3*log(F)^3*expint(-b*log(F)*(c + d*x)^2))/(12*d) - (F^a*F^(b*(c +
 d*x)^2)*b^3*log(F)^3*(1/(6*b*log(F)*(c + d*x)^2) + 1/(6*b^2*log(F)^2*(c +
 d*x)^4) + 1/(3*b^3*log(F)^3*(c + d*x)^6)))/(2*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^7}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^7,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 4*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**9 + 7*log(f)*b*c**8*d*x + 21*log(f)*b*c**7*d**2*
x**2 + 35*log(f)*b*c**6*d**3*x**3 + 35*log(f)*b*c**5*d**4*x**4 + 21*log(f)
*b*c**4*d**5*x**5 + 7*log(f)*b*c**3*d**6*x**6 + log(f)*b*c**2*d**7*x**7 -
3*c**7 - 21*c**6*d*x - 63*c**5*d**2*x**2 - 105*c**4*d**3*x**3 - 105*c**3*d
**4*x**4 - 63*c**2*d**5*x**5 - 21*c*d**6*x**6 - 3*d**7*x**7),x)*log(f)**2*
b**2*c**10*d + 24*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**9 + 7*log(
f)*b*c**8*d*x + 21*log(f)*b*c**7*d**2*x**2 + 35*log(f)*b*c**6*d**3*x**3 +
35*log(f)*b*c**5*d**4*x**4 + 21*log(f)*b*c**4*d**5*x**5 + 7*log(f)*b*c**3*
d**6*x**6 + log(f)*b*c**2*d**7*x**7 - 3*c**7 - 21*c**6*d*x - 63*c**5*d**2*
x**2 - 105*c**4*d**3*x**3 - 105*c**3*d**4*x**4 - 63*c**2*d**5*x**5 - 21*c*
d**6*x**6 - 3*d**7*x**7),x)*log(f)**2*b**2*c**9*d**2*x + 60*int(f**(2*b*c*
d*x + b*d**2*x**2)/(log(f)*b*c**9 + 7*log(f)*b*c**8*d*x + 21*log(f)*b*c**7
*d**2*x**2 + 35*log(f)*b*c**6*d**3*x**3 + 35*log(f)*b*c**5*d**4*x**4 + 21*
log(f)*b*c**4*d**5*x**5 + 7*log(f)*b*c**3*d**6*x**6 + log(f)*b*c**2*d**7*x
**7 - 3*c**7 - 21*c**6*d*x - 63*c**5*d**2*x**2 - 105*c**4*d**3*x**3 - 105*
c**3*d**4*x**4 - 63*c**2*d**5*x**5 - 21*c*d**6*x**6 - 3*d**7*x**7),x)*log(
f)**2*b**2*c**8*d**3*x**2 + 80*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*
c**9 + 7*log(f)*b*c**8*d*x + 21*log(f)*b*c**7*d**2*x**2 + 35*log(f)*b*c**6
*d**3*x**3 + 35*log(f)*b*c**5*d**4*x**4 + 21*log(f)*b*c**4*d**5*x**5 + ...
```

3.199 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx = -\frac{b^4 F^a \Gamma(-4, -b(c+dx)^2 \log(F)) \log^4(F)}{2d}$$

output
```
-1/2*F^a/(d*x+c)^8*Ei(5,-b*(d*x+c)^2*ln(F))/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx = -\frac{b^4 F^a \Gamma(-4, -b(c+dx)^2 \log(F)) \log^4(F)}{2d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^9,x]
```

output
```
-1/2*(b^4*F^a*Gamma[-4, -(b*(c + d*x)^2*Log[F])]*Log[F]^4)/d
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{b^4F^a \log^4(F)\Gamma\left(-4,-b(c+dx)^2\log(F)\right)}{2d}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^9,x]
```

output
```
-1/2*(b^4*F^a*Gamma[-4, -(b*(c + d*x)^2*Log[F])]*Log[F]^4)/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(29) = 58.

Time = 0.62 (sec) , antiderivative size = 152, normalized size of antiderivative = 4.90

method	result
risch	$-\frac{F^{b(dx+c)^2}F^a}{8d(dx+c)^8}-\frac{b\ln(F)F^{b(dx+c)^2}F^a}{24d(dx+c)^6}-\frac{b^2\ln(F)^2F^{b(dx+c)^2}F^a}{48d(dx+c)^4}-\frac{b^3\ln(F)^3F^{b(dx+c)^2}F^a}{48d(dx+c)^2}-\frac{b^4\ln(F)^4F^a\operatorname{expIntegral}_1\left(-\right.}{48d}$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8/d/(d*x+c)^8*F^(b*(d*x+c)^2)*F^a-1/24/d*b*ln(F)/(d*x+c)^6*F^(b*(d*x+c)
^2)*F^a-1/48/d*b^2*ln(F)^2/(d*x+c)^4*F^(b*(d*x+c)^2)*F^a-1/48/d*b^3*ln(F)^
3/(d*x+c)^2*F^(b*(d*x+c)^2)*F^a-1/48/d*b^4*ln(F)^4*F^a*Ei(1,-b*(d*x+c)^2*l
n(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 430, normalized size of antiderivative = 13.87

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx = \frac{(b^4d^8x^8 + 8\,b^4cd^7x^7 + 28\,b^4c^2d^6x^6 + 56\,b^4c^3d^5x^5 + 70\,b^4c^4d^4x^4 + 56\,b^4c^5d^3x^3 + 28\,b^4c^6d^2x^2 + 8\,b^4c^7dx +}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x, algorithm="fricas")
```

output
```
1/48*((b^4*d^8*x^8 + 8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 + 56*b^4*c^3*d^5
*x^5 + 70*b^4*c^4*d^4*x^4 + 56*b^4*c^5*d^3*x^3 + 28*b^4*c^6*d^2*x^2 + 8*b^
4*c^7*d*x + b^4*c^8)*F^a*Ei((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))*log(F)
^4 - ((b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3
*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^3 + (b^2*d^4*x
^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)
^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) + 6)*F^(b*d^2*x^2 + 2*b*c*d*
x + b*c^2 + a))/(d^9*x^8 + 8*c*d^8*x^7 + 28*c^2*d^7*x^6 + 56*c^3*d^6*x^5 +
 70*c^4*d^5*x^4 + 56*c^5*d^4*x^3 + 28*c^6*d^3*x^2 + 8*c^7*d^2*x + c^8*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**9,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**9, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^9} \, dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^9, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9} \, dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^9} \, dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^9, x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9}\,dx = -\frac{F^a\,b^4\ln(F)^4\operatorname{expint}\left(-b\,\ln(F)\,(c+d\,x)^2\right)}{48\,d} - \frac{F^a\,F^{b\,(c+d\,x)^2}\,b^4\ln(F)^4\left(\frac{1}{24\,b\,\ln(F)\,(c+d\,x)^2}+\frac{1}{24\,b^2\ln(F)^2\,(c+d\,x)^4}+\frac{1}{12\,b^3\ln(F)^3\,(c+d\,x)^6}+\frac{1}{4\,b^4\ln(F)^4\,(c+d\,x)^8}\right)}{2\,d}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^9,x)
```

output
```
- (F^a*b^4*log(F)^4*expint(-b*log(F)*(c + d*x)^2))/(48*d) - (F^a*F^(b*(c +
 d*x)^2)*b^4*log(F)^4*(1/(24*b*log(F)*(c + d*x)^2) + 1/(24*b^2*log(F)^2*(c
 + d*x)^4) + 1/(12*b^3*log(F)^3*(c + d*x)^6) + 1/(4*b^4*log(F)^4*(c + d*x)
^8)))/(2*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^9}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^9,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 4*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**11 + 9*log(f)*b*c**10*d*x + 36*log(f)*b*c**9*d**
2*x**2 + 84*log(f)*b*c**8*d**3*x**3 + 126*log(f)*b*c**7*d**4*x**4 + 126*lo
g(f)*b*c**6*d**5*x**5 + 84*log(f)*b*c**5*d**6*x**6 + 36*log(f)*b*c**4*d**7
*x**7 + 9*log(f)*b*c**3*d**8*x**8 + log(f)*b*c**2*d**9*x**9 - 4*c**9 - 36*
c**8*d*x - 144*c**7*d**2*x**2 - 336*c**6*d**3*x**3 - 504*c**5*d**4*x**4 -
504*c**4*d**5*x**5 - 336*c**3*d**6*x**6 - 144*c**2*d**7*x**7 - 36*c*d**8*x
**8 - 4*d**9*x**9),x)*log(f)**2*b**2*c**12*d + 32*int(f**(2*b*c*d*x + b*d*
*2*x**2)/(log(f)*b*c**11 + 9*log(f)*b*c**10*d*x + 36*log(f)*b*c**9*d**2*x*
*2 + 84*log(f)*b*c**8*d**3*x**3 + 126*log(f)*b*c**7*d**4*x**4 + 126*log(f)
*b*c**6*d**5*x**5 + 84*log(f)*b*c**5*d**6*x**6 + 36*log(f)*b*c**4*d**7*x**
7 + 9*log(f)*b*c**3*d**8*x**8 + log(f)*b*c**2*d**9*x**9 - 4*c**9 - 36*c**8
*d*x - 144*c**7*d**2*x**2 - 336*c**6*d**3*x**3 - 504*c**5*d**4*x**4 - 504*
c**4*d**5*x**5 - 336*c**3*d**6*x**6 - 144*c**2*d**7*x**7 - 36*c*d**8*x**8
- 4*d**9*x**9),x)*log(f)**2*b**2*c**11*d**2*x + 112*int(f**(2*b*c*d*x + b*
d**2*x**2)/(log(f)*b*c**11 + 9*log(f)*b*c**10*d*x + 36*log(f)*b*c**9*d**2*
x**2 + 84*log(f)*b*c**8*d**3*x**3 + 126*log(f)*b*c**7*d**4*x**4 + 126*log(
f)*b*c**6*d**5*x**5 + 84*log(f)*b*c**5*d**6*x**6 + 36*log(f)*b*c**4*d**7*x
**7 + 9*log(f)*b*c**3*d**8*x**8 + log(f)*b*c**2*d**9*x**9 - 4*c**9 - 36*c*
*8*d*x - 144*c**7*d**2*x**2 - 336*c**6*d**3*x**3 - 504*c**5*d**4*x**4 -...
```

3.200 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}} dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}} dx = \frac{b^5F^a\Gamma(-5,-b(c+dx)^2\log(F))\log^5(F)}{2d}$$

output
```
-1/2*F^a/(d*x+c)^10*Ei(6,-b*(d*x+c)^2*ln(F))/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}} dx = \frac{b^5F^a\Gamma(-5,-b(c+dx)^2\log(F))\log^5(F)}{2d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^11,x]
```

output
```
(b^5*F^a*Gamma[-5, -(b*(c + d*x)^2*Log[F])]*Log[F]^5)/(2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{b^5 F^a \log^5(F)\Gamma\left(-5,-b(c+dx)^2\log(F)\right)}{2d}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^11,x]
```

output
```
(b^5*F^a*Gamma[-5, -(b*(c + d*x)^2*Log[F])]*Log[F]^5)/(2*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(29) = 58.

Time = 1.00 (sec) , antiderivative size = 185, normalized size of antiderivative = 5.97

method	result
risch	$-\frac{F^{b(dx+c)^2} F^a}{10d(dx+c)^{10}} - \frac{b\ln(F) F^{b(dx+c)^2} F^a}{40d(dx+c)^8} - \frac{b^2\ln(F)^2 F^{b(dx+c)^2} F^a}{120d(dx+c)^6} - \frac{b^3\ln(F)^3 F^{b(dx+c)^2} F^a}{240d(dx+c)^4} - \frac{b^4\ln(F)^4 F^{b(dx+c)^2} F^a}{240d(dx+c)^2} - b^5$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10/d/(d*x+c)^10*F^(b*(d*x+c)^2)*F^a-1/40/d*b*ln(F)/(d*x+c)^8*F^(b*(d*x+
c)^2)*F^a-1/120/d*b^2*ln(F)^2/(d*x+c)^6*F^(b*(d*x+c)^2)*F^a-1/240/d*b^3*ln
(F)^3/(d*x+c)^4*F^(b*(d*x+c)^2)*F^a-1/240/d*b^4*ln(F)^4/(d*x+c)^2*F^(b*(d*
x+c)^2)*F^a-1/240/d*b^5*ln(F)^5*F^a*Ei(1,-b*(d*x+c)^2*ln(F))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. 2(29) = 58.

Time = 0.09 (sec) , antiderivative size = 596, normalized size of antiderivative = 19.23

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx = \frac{(b^5d^{10}x^{10} + 10\,b^5cd^9x^9 + 45\,b^5c^2d^8x^8 + 120\,b^5c^3d^7x^7 + 210\,b^5c^4d^6x^6 + 252\,b^5c^5d^5x^5 + 210\,b^5c^6d^4x^4 + 120}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x, algorithm="fricas")
```

output
```
1/240*((b^5*d^10*x^10 + 10*b^5*c*d^9*x^9 + 45*b^5*c^2*d^8*x^8 + 120*b^5*c^
3*d^7*x^7 + 210*b^5*c^4*d^6*x^6 + 252*b^5*c^5*d^5*x^5 + 210*b^5*c^6*d^4*x^
4 + 120*b^5*c^7*d^3*x^3 + 45*b^5*c^8*d^2*x^2 + 10*b^5*c^9*d*x + b^5*c^10)*
F^a*Ei((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))*log(F)^5 - ((b^4*d^8*x^8 +
8*b^4*c*d^7*x^7 + 28*b^4*c^2*d^6*x^6 + 56*b^4*c^3*d^5*x^5 + 70*b^4*c^4*d^4
*x^4 + 56*b^4*c^5*d^3*x^3 + 28*b^4*c^6*d^2*x^2 + 8*b^4*c^7*d*x + b^4*c^8)*
log(F)^4 + (b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^
3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^3 + 2*(b^
2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)
*log(F)^2 + 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F) + 24)*F^(b*d^2*x^2 +
2*b*c*d*x + b*c^2 + a))/(d^11*x^10 + 10*c*d^10*x^9 + 45*c^2*d^9*x^8 + 120*
c^3*d^8*x^7 + 210*c^4*d^7*x^6 + 252*c^5*d^6*x^5 + 210*c^6*d^5*x^4 + 120*c^
7*d^4*x^3 + 45*c^8*d^3*x^2 + 10*c^9*d^2*x + c^10*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**11,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**11, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^{11}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^11, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^{11}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^11, x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.39

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx = -\frac{F^a\,b^5\ln(F)^5\,\mathrm{expint}\left(-b\,\ln(F)\,(c+d\,x)^2\right)}{240\,d} - \frac{F^a\,F^{b\,(c+d\,x)^2}\,b^5\ln(F)^5\left(\frac{1}{120\,b\,\ln(F)\,(c+d\,x)^2}+\frac{1}{120\,b^2\ln(F)^2\,(c+d\,x)^4}+\frac{1}{60\,b^3\ln(F)^3\,(c+d\,x)^6}+\frac{1}{20\,b^4\ln(F)^4\,(c+d\,x)^8}+\frac{1}{5\,b^5\ln(F)^5\,(c+d\,x)^{10}}\right)}{2\,d}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^11,x)
```

output
```
- (F^a*b^5*log(F)^5*expint(-b*log(F)*(c + d*x)^2))/(240*d) - (F^a*F^(b*(c
+ d*x)^2)*b^5*log(F)^5*(1/(120*b*log(F)*(c + d*x)^2) + 1/(120*b^2*log(F)^2
*(c + d*x)^4) + 1/(60*b^3*log(F)^3*(c + d*x)^6) + 1/(20*b^4*log(F)^4*(c +
d*x)^8) + 1/(5*b^5*log(F)^5*(c + d*x)^10)))/(2*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{11}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^11,x)
```

output

```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 4*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**13 + 11*log(f)*b*c**12*d*x + 55*log(f)*b*c**11*d
**2*x**2 + 165*log(f)*b*c**10*d**3*x**3 + 330*log(f)*b*c**9*d**4*x**4 + 46
2*log(f)*b*c**8*d**5*x**5 + 462*log(f)*b*c**7*d**6*x**6 + 330*log(f)*b*c**
6*d**7*x**7 + 165*log(f)*b*c**5*d**8*x**8 + 55*log(f)*b*c**4*d**9*x**9 + 1
1*log(f)*b*c**3*d**10*x**10 + log(f)*b*c**2*d**11*x**11 - 5*c**11 - 55*c**
10*d*x - 275*c**9*d**2*x**2 - 825*c**8*d**3*x**3 - 1650*c**7*d**4*x**4 - 2
310*c**6*d**5*x**5 - 2310*c**5*d**6*x**6 - 1650*c**4*d**7*x**7 - 825*c**3*
d**8*x**8 - 275*c**2*d**9*x**9 - 55*c*d**10*x**10 - 5*d**11*x**11),x)*log(
f)**2*b**2*c**14*d + 40*int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**13 +
 11*log(f)*b*c**12*d*x + 55*log(f)*b*c**11*d**2*x**2 + 165*log(f)*b*c**10*
d**3*x**3 + 330*log(f)*b*c**9*d**4*x**4 + 462*log(f)*b*c**8*d**5*x**5 + 46
2*log(f)*b*c**7*d**6*x**6 + 330*log(f)*b*c**6*d**7*x**7 + 165*log(f)*b*c**
5*d**8*x**8 + 55*log(f)*b*c**4*d**9*x**9 + 11*log(f)*b*c**3*d**10*x**10 +
log(f)*b*c**2*d**11*x**11 - 5*c**11 - 55*c**10*d*x - 275*c**9*d**2*x**2 -
825*c**8*d**3*x**3 - 1650*c**7*d**4*x**4 - 2310*c**6*d**5*x**5 - 2310*c**5
*d**6*x**6 - 1650*c**4*d**7*x**7 - 825*c**3*d**8*x**8 - 275*c**2*d**9*x**9
 - 55*c*d**10*x**10 - 5*d**11*x**11),x)*log(f)**2*b**2*c**13*d**2*x + 180*
int(f**(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**13 + 11*log(f)*b*c**12*d*x +
 55*log(f)*b*c**11*d**2*x**2 + 165*log(f)*b*c**10*d**3*x**3 + 330*log(f...
```

3.201 $\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx=-\frac{F^a(c+dx)^{13}\Gamma\left(\frac{13}{2},-b(c+dx)^2\log(F)\right)}{2d\left(-b(c+dx)^2\log(F)\right)^{13/2}}$$

output
```
-1/2*F^a*(d*x+c)^13*(524288/5621533568633696205238621875*GAMMA(51/2,-b*(d*
x+c)^2*ln(F))-524288/5621533568633696205238621875*(-b*(d*x+c)^2*ln(F))^(49
/2)*exp(b*(d*x+c)^2*ln(F))-262144/114725174870075432759971875*(-b*(d*x+c)^
2*ln(F))^(47/2)*exp(b*(d*x+c)^2*ln(F))-131072/2440961167448413462978125*(-
b*(d*x+c)^2*ln(F))^(45/2)*exp(b*(d*x+c)^2*ln(F))-65536/5424358149885363251
0625*(-b*(d*x+c)^2*ln(F))^(43/2)*exp(b*(d*x+c)^2*ln(F))-32768/126147863950
8224011875*(-b*(d*x+c)^2*ln(F))^(41/2)*exp(b*(d*x+c)^2*ln(F))-16384/307677
71695322536875*(-b*(d*x+c)^2*ln(F))^(39/2)*exp(b*(d*x+c)^2*ln(F))-8192/788
917222956988125*(-b*(d*x+c)^2*ln(F))^(37/2)*exp(b*(d*x+c)^2*ln(F))-4096/21
322087106945625*(-b*(d*x+c)^2*ln(F))^(35/2)*exp(b*(d*x+c)^2*ln(F))-2048/60
9202488769875*(-b*(d*x+c)^2*ln(F))^(33/2)*exp(b*(d*x+c)^2*ln(F))-1024/1846
0681477875*(-b*(d*x+c)^2*ln(F))^(31/2)*exp(b*(d*x+c)^2*ln(F))-512/59550585
4125*(-b*(d*x+c)^2*ln(F))^(29/2)*exp(b*(d*x+c)^2*ln(F))-256/20534684625*(-
b*(d*x+c)^2*ln(F))^(27/2)*exp(b*(d*x+c)^2*ln(F))-128/760543875*(-b*(d*x+c)
^2*ln(F))^(25/2)*exp(b*(d*x+c)^2*ln(F))-64/30421755*(-b*(d*x+c)^2*ln(F))^(
23/2)*exp(b*(d*x+c)^2*ln(F))-32/1322685*(-b*(d*x+c)^2*ln(F))^(21/2)*exp(b*
(d*x+c)^2*ln(F))-16/62985*(-b*(d*x+c)^2*ln(F))^(19/2)*exp(b*(d*x+c)^2*ln(F
))-8/3315*(-b*(d*x+c)^2*ln(F))^(17/2)*exp(b*(d*x+c)^2*ln(F))-4/195*(-b*(d*
x+c)^2*ln(F))^(15/2)*exp(b*(d*x+c)^2*ln(F))-2/13*(-b*(d*x+c)^2*ln(F))^(13/
2)*exp(b*(d*x+c)^2*ln(F)))/d/(-b*(d*x+c)^2*ln(F))^(13/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx=-\frac{F^a(c+dx)^{13}\Gamma\left(\frac{13}{2},-b(c+dx)^2\log(F)\right)}{2d\left(-b(c+dx)^2\log(F)\right)^{13/2}}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^12,x]
```

output
```
-1/2*(F^a*(c + d*x)^13*Gamma[13/2, -(b*(c + d*x)^2*Log[F])])/(d*(-(b*(c +
d*x)^2*Log[F]))^(13/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{12}F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{F^a(c+dx)^{13}\Gamma\left(\frac{13}{2},-b(c+dx)^2\log(F)\right)}{2d\left(-b\log(F)(c+dx)^2\right)^{13/2}}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^12,x]
```

output
```
-1/2*(F^a*(c + d*x)^13*Gamma[13/2, -(b*(c + d*x)^2*Log[F])])/(d*(-(b*(c +
d*x)^2*Log[F]))^(13/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1910 vs. 2(578) = 1156.

Time = 2.98 (sec) , antiderivative size = 1911, normalized size of antiderivative = 39.00

method	result	size
risch	Expression too large to display	1911

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^12,x,method=_RETURNVERBOSE)
```

output
```
-3465/16*F^(b*c^2)*F^a*c^4/ln(F)^4/b^4*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+10395
/32*F^(b*c^2)*F^a*c^2/ln(F)^5/b^5*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+693/8*F^(b
*c^2)*F^a*c^6/ln(F)^3/b^3*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+11/2*F^(b*c^2)*F^a
*c^10/ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-99/4*F^(b*c^2)*F^a*c^8/ln(F)^2
/b^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3465/32*F^(b*c^2)*F^a/d*c^3/ln(F)^5/b^5
*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a/d*c^11/ln(F)/b*F^(b*d^2*x^2
)*F^(2*b*c*d*x)-11/4*F^(b*c^2)*F^a/d*c^9/ln(F)^2/b^2*F^(b*d^2*x^2)*F^(2*b*
c*d*x)+99/8*F^(b*c^2)*F^a/d*c^7/ln(F)^3/b^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)-69
3/16*F^(b*c^2)*F^a/d*c^5/ln(F)^4/b^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)-10395/64*
F^(b*c^2)*F^a/d*c/ln(F)^6/b^6*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^
a*d^10/ln(F)/b*x^11*F^(b*d^2*x^2)*F^(2*b*c*d*x)-11/4*F^(b*c^2)*F^a*d^8/ln(
F)^2/b^2*x^9*F^(b*d^2*x^2)*F^(2*b*c*d*x)-693/16*F^(b*c^2)*F^a*d^4/ln(F)^4/
b^4*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3465/32*F^(b*c^2)*F^a*d^2/ln(F)^5/b^5*
x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)+99/8*F^(b*c^2)*F^a*d^6/ln(F)^3/b^3*x^7*F^(
b*d^2*x^2)*F^(2*b*c*d*x)-693/2*F^(b*c^2)*F^a*d^4*c^4/ln(F)^2/b^2*x^5*F^(b*
d^2*x^2)*F^(2*b*c*d*x)+3465/8*F^(b*c^2)*F^a*d^2*c^4/ln(F)^3/b^3*x^3*F^(b*d
^2*x^2)*F^(2*b*c*d*x)-10395/64*F^(b*c^2)*F^a/ln(F)^6/b^6*x*F^(b*d^2*x^2)*F
^(2*b*c*d*x)+165/2*F^(b*c^2)*F^a*d^7*c^3/ln(F)/b*x^8*F^(b*d^2*x^2)*F^(2*b*
c*d*x)+2079/8*F^(b*c^2)*F^a*d^4*c^2/ln(F)^3/b^3*x^5*F^(b*d^2*x^2)*F^(2*b*c
*d*x)-3465/8*F^(b*c^2)*F^a*d^2*c^2/ln(F)^4/b^4*x^3*F^(b*d^2*x^2)*F^(2*b...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 617, normalized size of antiderivative = 12.59

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx =$$

$$-\frac{10395\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)-2\left(32\left(b^6d^{12}x^{11}+11\,b^6cd^{11}x^{10}+55\,b^6c^2d^{10}x^9+\right.\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^12,x, algorithm="fricas")
```

output
```
-1/128*(10395*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*
x + c)/d) - 2*(32*(b^6*d^12*x^11 + 11*b^6*c*d^11*x^10 + 55*b^6*c^2*d^10*x^
9 + 165*b^6*c^3*d^9*x^8 + 330*b^6*c^4*d^8*x^7 + 462*b^6*c^5*d^7*x^6 + 462*
b^6*c^6*d^6*x^5 + 330*b^6*c^7*d^5*x^4 + 165*b^6*c^8*d^4*x^3 + 55*b^6*c^9*d
^3*x^2 + 11*b^6*c^10*d^2*x + b^6*c^11*d)*log(F)^6 - 176*(b^5*d^10*x^9 + 9*
b^5*c*d^9*x^8 + 36*b^5*c^2*d^8*x^7 + 84*b^5*c^3*d^7*x^6 + 126*b^5*c^4*d^6*
x^5 + 126*b^5*c^5*d^5*x^4 + 84*b^5*c^6*d^4*x^3 + 36*b^5*c^7*d^3*x^2 + 9*b^
5*c^8*d^2*x + b^5*c^9*d)*log(F)^5 + 792*(b^4*d^8*x^7 + 7*b^4*c*d^7*x^6 + 2
1*b^4*c^2*d^6*x^5 + 35*b^4*c^3*d^5*x^4 + 35*b^4*c^4*d^4*x^3 + 21*b^4*c^5*d
^3*x^2 + 7*b^4*c^6*d^2*x + b^4*c^7*d)*log(F)^4 - 2772*(b^3*d^6*x^5 + 5*b^3
*c*d^5*x^4 + 10*b^3*c^2*d^4*x^3 + 10*b^3*c^3*d^3*x^2 + 5*b^3*c^4*d^2*x + b
^3*c^5*d)*log(F)^3 + 6930*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x
 + b^2*c^3*d)*log(F)^2 - 10395*(b*d^2*x + b*c*d)*log(F))*F^(b*d^2*x^2 + 2*
b*c*d*x + b*c^2 + a))/(b^7*d^2*log(F)^7)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx = \int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**12,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**12, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6135 vs. $2(559) = 1118$.

Time = 2.10 (sec) , antiderivative size = 6135, normalized size of antiderivative = 125.20

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^12,x, algorithm="maxima")
```

output
```
-6*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)
*d))*F^a*c^11/sqrt(b*log(F)) + 33*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(erf
(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/2
)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^
2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*gam
ma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d^
5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^10*d/sqrt(b*log(F))
- 110*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d
)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/
((b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x +
b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d
)^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d
^2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^9*d^2/sqrt(b*log(F)) + 495/2*(
sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log
(F))^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + ...
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.98

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx$$

$$= \frac{\left(32\,b^5d^{10}\left(x+\frac{c}{d}\right)^{11}\log(F)^5 - 176\,b^4d^8\left(x+\frac{c}{d}\right)^9\log(F)^4 + 792\,b^3d^6\left(x+\frac{c}{d}\right)^7\log(F)^3 - 2772\,b^2d^4\left(x+\frac{c}{d}\right)\cdots\right.}{64\,b^6\log(\cdots}$$

$$- \frac{10395\,\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}d\left(x+\frac{c}{d}\right)\right)}{128\,\sqrt{-b\log(F)}b^6d\log(F)^6}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^12,x, algorithm="giac")
```

output
```
1/64*(32*b^5*d^10*(x + c/d)^11*log(F)^5 - 176*b^4*d^8*(x + c/d)^9*log(F)^4
 + 792*b^3*d^6*(x + c/d)^7*log(F)^3 - 2772*b^2*d^4*(x + c/d)^5*log(F)^2 +
6930*b*d^2*(x + c/d)^3*log(F) - 10395*x - 10395*c/d)*e^(b*d^2*x^2*log(F) +
 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^6*log(F)^6) - 10395/128*sq
rt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b^6*d*log(F)
^6)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.27

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx$$
$$=\frac{\frac{F^a\left(\frac{10395\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(F)\,(c+d\,x)}{\sqrt{b\,\ln(F)}}\right)}{128}-\frac{10395\,F^{b\,(c+d\,x)^2}\,\sqrt{b\,\ln(F)}\,(c+d\,x)}{64}\right)}{\sqrt{b\,\ln(F)}}-\frac{693\,F^{a+b\,(c+d\,x)^2}\,b^2\,\ln(F)^2\,(c+d\,x)^5}{16}+\frac{99\,F^{a+b\,(c+d\,x)^2}\,b^3\,\ln(F)^{\cdots}}{8}}{b^6\,d\ln{(F)}^{\cdots}}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^12,x)
```

output
```
((F^a*((10395*pi^(1/2)*erfi((b*log(F)*(c + d*x))/(b*log(F))^(1/2)))/128 -
(10395*F^(b*(c + d*x)^2)*(b*log(F))^(1/2)*(c + d*x))/64))/(b*log(F))^(1/2)
 - (693*F^(a + b*(c + d*x)^2)*b^2*log(F)^2*(c + d*x)^5)/16 + (99*F^(a + b*
(c + d*x)^2)*b^3*log(F)^3*(c + d*x)^7)/8 - (11*F^(a + b*(c + d*x)^2)*b^4*l
og(F)^4*(c + d*x)^9)/4 + (F^(a + b*(c + d*x)^2)*b^5*log(F)^5*(c + d*x)^11)
/2 + (3465*F^(a + b*(c + d*x)^2)*b*log(F)*(c + d*x)^3)/32)/(b^6*d*log(F)^6
)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1825, normalized size of antiderivative = 37.24

$$\int F^{a+b(c+dx)^2}(c+dx)^{12}\,dx=\text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^12,x)
```

output

```
(f**a*( - 10395*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt
(log(f))))*i + 64*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f
))*log(f)**5*b**5*c**11 + 704*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b
)*sqrt(log(f))*log(f)**5*b**5*c**10*d*x + 3520*f**(b*c**2 + 2*b*c*d*x + b*
d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**5*b**5*c**9*d**2*x**2 + 10560*f**(
b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**5*b**5*c**8
*d**3*x**3 + 21120*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(
f))*log(f)**5*b**5*c**7*d**4*x**4 + 29568*f**(b*c**2 + 2*b*c*d*x + b*d**2*
x**2)*sqrt(b)*sqrt(log(f))*log(f)**5*b**5*c**6*d**5*x**5 + 29568*f**(b*c**
2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**5*b**5*c**5*d**6
*x**6 + 21120*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*l
og(f)**5*b**5*c**4*d**7*x**7 + 10560*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)
*sqrt(b)*sqrt(log(f))*log(f)**5*b**5*c**3*d**8*x**8 + 3520*f**(b*c**2 + 2*
b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**5*b**5*c**2*d**9*x**9
+ 704*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**5
*b**5*c*d**10*x**10 + 64*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqr
t(log(f))*log(f)**5*b**5*d**11*x**11 - 352*f**(b*c**2 + 2*b*c*d*x + b*d**2
*x**2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4*c**9 - 3168*f**(b*c**2 + 2*b*c*
d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4*c**8*d*x - 12672*f*
*(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4...
```

3.202 $\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx = -\frac{F^a(c+dx)^{11}\Gamma\left(\frac{11}{2},-b(c+dx)^2\log(F)\right)}{2d\left(-b(c+dx)^2\log(F)\right)^{11/2}}$$

output
```
-1/2*F^a*(d*x+c)^11*(1048576/618368692549706582576248406258*GAMMA(51/2,-b*(
d*x+c)^2*ln(F))-1048576/61836869254970658257624840625*(-b*(d*x+c)^2*ln(F))
^(49/2)*exp(b*(d*x+c)^2*ln(F))-524288/1261976923570829760359690625*(-b*(d*
x+c)^2*ln(F))^(47/2)*exp(b*(d*x+c)^2*ln(F))-262144/268505728419325480927593
375*(-b*(d*x+c)^2*ln(F))^(45/2)*exp(b*(d*x+c)^2*ln(F))-131072/596679396487
38957616875*(-b*(d*x+c)^2*ln(F))^(43/2)*exp(b*(d*x+c)^2*ln(F))-65536/1387
6265034590464130625*(-b*(d*x+c)^2*ln(F))^(41/2)*exp(b*(d*x+c)^2*ln(F))-327
68/338445488648547905625*(-b*(d*x+c)^2*ln(F))^(39/2)*exp(b*(d*x+c)^2*ln(F)
)-16384/8678089452526869375*(-b*(d*x+c)^2*ln(F))^(37/2)*exp(b*(d*x+c)^2*ln
(F))-8192/234542958176401875*(-b*(d*x+c)^2*ln(F))^(35/2)*exp(b*(d*x+c)^2*l
n(F))-4096/6701227376468625*(-b*(d*x+c)^2*ln(F))^(33/2)*exp(b*(d*x+c)^2*ln
(F))-2048/203067496256625*(-b*(d*x+c)^2*ln(F))^(31/2)*exp(b*(d*x+c)^2*ln(F
))-1024/6550564395375*(-b*(d*x+c)^2*ln(F))^(29/2)*exp(b*(d*x+c)^2*ln(F))-5
12/225881530875*(-b*(d*x+c)^2*ln(F))^(27/2)*exp(b*(d*x+c)^2*ln(F))-256/836
5982625*(-b*(d*x+c)^2*ln(F))^(25/2)*exp(b*(d*x+c)^2*ln(F))-128/334639305*(
-b*(d*x+c)^2*ln(F))^(23/2)*exp(b*(d*x+c)^2*ln(F))-64/14549535*(-b*(d*x+c)^
2*ln(F))^(21/2)*exp(b*(d*x+c)^2*ln(F))-32/692835*(-b*(d*x+c)^2*ln(F))^(19/
2)*exp(b*(d*x+c)^2*ln(F))-16/36465*(-b*(d*x+c)^2*ln(F))^(17/2)*exp(b*(d*x+
c)^2*ln(F))-8/2145*(-b*(d*x+c)^2*ln(F))^(15/2)*exp(b*(d*x+c)^2*ln(F))-4/14
3*(-b*(d*x+c)^2*ln(F))^(13/2)*exp(b*(d*x+c)^2*ln(F))-2/11*(-b*(d*x+c)^2...
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx=-\frac{F^a(c+dx)^{11}\Gamma\left(\frac{11}{2},-b(c+dx)^2\log(F)\right)}{2d\left(-b(c+dx)^2\log(F)\right)^{11/2}}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^10,x]
```

output
```
-1/2*(F^a*(c + d*x)^11*Gamma[11/2, -(b*(c + d*x)^2*Log[F])])/(d*(-(b*(c +
d*x)^2*Log[F]))^(11/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{10}F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{F^a(c+dx)^{11}\Gamma\left(\frac{11}{2},-b(c+dx)^2\log(F)\right)}{2d\left(-b\log(F)(c+dx)^2\right)^{11/2}}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^10,x]
```

output
```
-1/2*(F^a*(c + d*x)^11*Gamma[11/2, -(b*(c + d*x)^2*Log[F])])/(d*(-(b*(c +
d*x)^2*Log[F]))^(11/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. 2(606) = 1212.

Time = 1.56 (sec) , antiderivative size = 1374, normalized size of antiderivative = 28.04

method	result	size
risch	Expression too large to display	1374

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(b*c^2)*F^a*d^8/ln(F)/b*x^9*F^(b*d^2*x^2)*F^(2*b*c*d*x)+945/32*F^(b*
c^2)*F^a/d*c/ln(F)^5/b^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a/d*c
^9/ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)-9/4*F^(b*c^2)*F^a/d*c^7/ln(F)^2/b^2
*F^(b*d^2*x^2)*F^(2*b*c*d*x)+63/8*F^(b*c^2)*F^a/d*c^5/ln(F)^3/b^3*F^(b*d^2
*x^2)*F^(2*b*c*d*x)-315/16*F^(b*c^2)*F^a/d*c^3/ln(F)^4/b^4*F^(b*d^2*x^2)*F
^(2*b*c*d*x)-9/4*F^(b*c^2)*F^a*d^6/ln(F)^2/b^2*x^7*F^(b*d^2*x^2)*F^(2*b*c*
d*x)+63/8*F^(b*c^2)*F^a*d^4/ln(F)^3/b^3*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)-31
5/16*F^(b*c^2)*F^a*d^2/ln(F)^4/b^4*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)-945/16*
F^(b*c^2)*F^a*c^2/ln(F)^4/b^4*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+9/2*F^(b*c^2)*
F^a*c^8/ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-63/4*F^(b*c^2)*F^a*c^6/ln(F)
^2/b^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+315/8*F^(b*c^2)*F^a*c^4/ln(F)^3/b^3*x
*F^(b*d^2*x^2)*F^(2*b*c*d*x)+945/64*F^(b*c^2)*F^a/d/ln(F)^5/b^5*Pi^(1/2)*F
^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^
(1/2))+945/32*F^(b*c^2)*F^a/ln(F)^5/b^5*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+9/2*
F^(b*c^2)*F^a*d^7*c/ln(F)/b*x^8*F^(b*d^2*x^2)*F^(2*b*c*d*x)+42*F^(b*c^2)*F
^a*d^5*c^3/ln(F)/b*x^6*F^(b*d^2*x^2)*F^(2*b*c*d*x)+315/4*F^(b*c^2)*F^a*d*c
^3/ln(F)^3/b^3*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+63*F^(b*c^2)*F^a*d^4*c^4/ln
(F)/b*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)+18*F^(b*c^2)*F^a*d^6*c^2/ln(F)/b*x^7
*F^(b*d^2*x^2)*F^(2*b*c*d*x)-189/4*F^(b*c^2)*F^a*d^4*c^2/ln(F)^2/b^2*x^5*F
^(b*d^2*x^2)*F^(2*b*c*d*x)+315/4*F^(b*c^2)*F^a*d^2*c^2/ln(F)^3/b^3*x^3*...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 456, normalized size of antiderivative = 9.31

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx$$
$$= \frac{945\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)+2\left(16\left(b^5d^{10}x^9+9\,b^5cd^9x^8+36\,b^5c^2d^8x^7+84\,b^5c^3d^7x\right.\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^10,x, algorithm="fricas")
```

output
```
1/64*(945*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x +
c)/d) + 2*(16*(b^5*d^10*x^9 + 9*b^5*c*d^9*x^8 + 36*b^5*c^2*d^8*x^7 + 84*b^
5*c^3*d^7*x^6 + 126*b^5*c^4*d^6*x^5 + 126*b^5*c^5*d^5*x^4 + 84*b^5*c^6*d^4
*x^3 + 36*b^5*c^7*d^3*x^2 + 9*b^5*c^8*d^2*x + b^5*c^9*d)*log(F)^5 - 72*(b^
4*d^8*x^7 + 7*b^4*c*d^7*x^6 + 21*b^4*c^2*d^6*x^5 + 35*b^4*c^3*d^5*x^4 + 35
*b^4*c^4*d^4*x^3 + 21*b^4*c^5*d^3*x^2 + 7*b^4*c^6*d^2*x + b^4*c^7*d)*log(F
)^4 + 252*(b^3*d^6*x^5 + 5*b^3*c*d^5*x^4 + 10*b^3*c^2*d^4*x^3 + 10*b^3*c^3
*d^3*x^2 + 5*b^3*c^4*d^2*x + b^3*c^5*d)*log(F)^3 - 630*(b^2*d^4*x^3 + 3*b^
2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*log(F)^2 + 945*(b*d^2*x + b*c*d
)*log(F))*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^6*d^2*log(F)^6)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx = \int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**10,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4471 vs. $2(586) = 1172$.

Time = 1.53 (sec) , antiderivative size = 4471, normalized size of antiderivative = 91.24

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^10,x, algorithm="maxima")
```

output
```
-5*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)
*d))*F^a*c^9/sqrt(b*log(F)) + 45/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(er
f(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/
2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)
^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*ga
mma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d
^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^8*d/sqrt(b*log(F))
- 60*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*lo
g(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/(
(b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b
*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^
2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^7*d^2/sqrt(b*log(F)) + 105*(sqr
t(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d
^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log(F)
)^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + b*c...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.55

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx$$
$$= \frac{\left(16\,b^4d^8\left(x+\frac{c}{d}\right)^9\log\left(F\right)^4-72\,b^3d^6\left(x+\frac{c}{d}\right)^7\log\left(F\right)^3+252\,b^2d^4\left(x+\frac{c}{d}\right)^5\log\left(F\right)^2-630\,bd^2\left(x+\frac{c}{d}\right)^3\log}{32\,b^5\log\left(F\right)^5}$$
$$+\frac{945\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log\left(F\right)}d\left(x+\frac{c}{d}\right)\right)}{64\sqrt{-b\log\left(F\right)}b^5d\log\left(F\right)^5}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^10,x, algorithm="giac")
```

output
```
1/32*(16*b^4*d^8*(x + c/d)^9*log(F)^4 - 72*b^3*d^6*(x + c/d)^7*log(F)^3 +
252*b^2*d^4*(x + c/d)^5*log(F)^2 - 630*b*d^2*(x + c/d)^3*log(F) + 945*x +
945*c/d)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))
/(b^5*log(F)^5) + 945/64*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(s
qrt(-b*log(F))*b^5*d*log(F)^5)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 730, normalized size of antiderivative = 14.90

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx = \text{Too large to display}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^10,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*((945*c)/32 - (315*b*c^3*log(F)
)/16 + (63*b^2*c^5*log(F)^2)/8 - (9*b^3*c^7*log(F)^3)/4 + (b^4*c^9*log(F)^
4)/2))/(b^5*d*log(F)^5) - (945*F^a*pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*l
og(F))/(b*d^2*log(F))^(1/2)))/(64*b^5*log(F)^5*(b*d^2*log(F))^(1/2)) + (63
*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x^4*(5*c*d^3 + 8*b^2*c^5*d^3*lo
g(F)^2 - 10*b*c^3*d^3*log(F)))/(8*b^3*log(F)^3) + (F^(b*d^2*x^2)*F^a*F^(b*
c^2)*F^(2*b*c*d*x)*d^8*x^9)/(2*b*log(F)) - (9*F^(b*d^2*x^2)*F^a*F^(b*c^2)*
F^(2*b*c*d*x)*x^2*(105*c*d + 84*b^2*c^5*d*log(F)^2 - 32*b^3*c^7*d*log(F)^3
 - 140*b*c^3*d*log(F)))/(16*b^4*log(F)^4) + (63*F^(b*d^2*x^2)*F^a*F^(b*c^2
)*F^(2*b*c*d*x)*x^5*(d^4 + 8*b^2*c^4*d^4*log(F)^2 - 6*b*c^2*d^4*log(F)))/(
8*b^3*log(F)^3) - (21*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x^6*(3*c*d
^5 - 8*b*c^3*d^5*log(F)))/(4*b^2*log(F)^2) - (21*F^(b*d^2*x^2)*F^a*F^(b*c^
2)*F^(2*b*c*d*x)*x^3*(15*d^2 + 60*b^2*c^4*d^2*log(F)^2 - 32*b^3*c^6*d^2*lo
g(F)^3 - 60*b*c^2*d^2*log(F)))/(16*b^4*log(F)^4) + (9*F^(b*d^2*x^2)*F^a*F^
(b*c^2)*F^(2*b*c*d*x)*x*(140*b^2*c^4*log(F)^2 - 210*b*c^2*log(F) - 56*b^3*
c^6*log(F)^3 + 16*b^4*c^8*log(F)^4 + 105))/(32*b^5*log(F)^5) + (9*F^(b*d^2
*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*c*d^7*x^8)/(2*b*log(F)) + (9*F^(b*d^2*x^
2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*d^6*x^7*(8*b*c^2*log(F) - 1))/(4*b^2*log(F)
^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1300, normalized size of antiderivative = 26.53

$$\int F^{a+b(c+dx)^2}(c+dx)^{10}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^10,x)
```

output
```
(f**a*(945*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(
f))))*i + 32*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*lo
g(f)**4*b**4*c**9 + 288*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt
(log(f))*log(f)**4*b**4*c**8*d*x + 1152*f**(b*c**2 + 2*b*c*d*x + b*d**2*x*
*2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4*c**7*d**2*x**2 + 2688*f**(b*c**2 +
 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4*c**6*d**3*x*
*3 + 4032*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f
)**4*b**4*c**5*d**4*x**4 + 4032*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt
(b)*sqrt(log(f))*log(f)**4*b**4*c**4*d**5*x**5 + 2688*f**(b*c**2 + 2*b*c*d
*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4*c**3*d**6*x**6 + 115
2*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**
4*c**2*d**7*x**7 + 288*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(
log(f))*log(f)**4*b**4*c*d**8*x**8 + 32*f**(b*c**2 + 2*b*c*d*x + b*d**2*x*
*2)*sqrt(b)*sqrt(log(f))*log(f)**4*b**4*d**9*x**9 - 144*f**(b*c**2 + 2*b*c
*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*c**7 - 1008*f**(b*
c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*c**6*d
*x - 3024*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f
)**3*b**3*c**5*d**2*x**2 - 5040*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt
(b)*sqrt(log(f))*log(f)**3*b**3*c**4*d**3*x**3 - 5040*f**(b*c**2 + 2*b*c*d
*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*c**3*d**4*x**4 - ...
```

3.203 $\int F^{a+b(c+dx)^2}(c+dx)^8\,dx$

Optimal result

Integrand size = 21, antiderivative size = 179

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx = \frac{105F^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{32b^{9/2}d\log^{\frac{9}{2}}(F)} - \frac{105F^{a+b(c+dx)^2}(c+dx)}{16b^4d\log^4(F)} + \frac{35F^{a+b(c+dx)^2}(c+dx)^3}{8b^3d\log^3(F)} - \frac{7F^{a+b(c+dx)^2}(c+dx)^5}{4b^2d\log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^7}{2bd\log(F)}$$

output
```
105/32*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(9/2)/d/ln(F)^(9/2
)-105/16*F^(a+b*(d*x+c)^2)*(d*x+c)/b^4/d/ln(F)^4+35/8*F^(a+b*(d*x+c)^2)*(d
*x+c)^3/b^3/d/ln(F)^3-7/4*F^(a+b*(d*x+c)^2)*(d*x+c)^5/b^2/d/ln(F)^2+1/2*F^
(a+b*(d*x+c)^2)*(d*x+c)^7/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx = \frac{F^a\left(16F^{b(c+dx)^2}(c+dx)^7+\frac{105\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{b^{7/2}\log^{\frac{7}{2}}(F)}-\frac{210F^{b(c+dx)^2}(c+dx)}{b^3\log^3(F)}+\frac{140F^{b(c+dx)^2}(c+dx)^3}{b^2\log^2(F)}-\frac{56F^{b(c+dx)^2}(c+dx)^5}{b\log(F)}\right)}{32bd\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^8,x]
```

output
```
(F^a*(16*F^(b*(c + d*x)^2)*(c + d*x)^7 + (105*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d
*x)*Sqrt[Log[F]]])/(b^(7/2)*Log[F]^(7/2)) - (210*F^(b*(c + d*x)^2)*(c + d*
x))/(b^3*Log[F]^3) + (140*F^(b*(c + d*x)^2)*(c + d*x)^3)/(b^2*Log[F]^2) -
(56*F^(b*(c + d*x)^2)*(c + d*x)^5)/(b*Log[F])))/(32*b*d*Log[F])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2641, 2641, 2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^8F^{a+b(c+dx)^2}dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^7F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{7\int F^{b(c+dx)^2+a}(c+dx)^6dx}{2b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^7F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{7\left(\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\int F^{b(c+dx)^2+a}(c+dx)^4dx}{2b\log(F)}\right)}{2b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^7F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{7\left(\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\left(\frac{(c+dx)^3F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\int F^{b(c+dx)^2+a}(c+dx)^2dx}{2b\log(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^7F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{7\left(\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\left(\frac{(c+dx)^3F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\left(\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\int F^{b(c+dx)^2+a}dx}{2b\log(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)^7F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{7\left(\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\left(\frac{(c+dx)^3F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\left(\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\sqrt{\pi}F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^8,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(c + d*x)^7)/(2*b*d*Log[F]) - (7*((F^(a + b*(c + d*
x)^2)*(c + d*x)^5)/(2*b*d*Log[F]) - (5*((F^(a + b*(c + d*x)^2)*(c + d*x)^3
)/(2*b*d*Log[F]) - (3*(-1/4*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[
F]]])/(b^(3/2)*d*Log[F]^(3/2)) + (F^(a + b*(c + d*x)^2)*(c + d*x))/(2*b*d*
Log[F])))/(2*b*Log[F])))/(2*b*Log[F])))/(2*b*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(159) = 318$.

Time = 0.79 (sec) , antiderivative size = 929, normalized size of antiderivative = 5.19

method	result
risch	$-\frac{105F^{b\,c^2}F^a c\,F^{b\,d^2x^2}F^{2bcdx}}{16d\ln(F)^4b^4}+\frac{7F^{b\,c^2}F^a c^6x\,F^{b\,d^2x^2}F^{2bcdx}}{2\ln(F)b}-\frac{35F^{b\,c^2}F^a c^4x\,F^{b\,d^2x^2}F^{2bcdx}}{4\ln(F)^2b^2}+\frac{105F^{b\,c^2}F^a c^2x\,F^{b\,d^2x^2}F^{2b}}{8\ln(F)^3b^3}$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
-105/16*F^(b*c^2)*F^a/d*c/ln(F)^4/b^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)+7/2*F^(b
*c^2)*F^a*c^6/ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-35/4*F^(b*c^2)*F^a*c^4
/ln(F)^2/b^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+105/8*F^(b*c^2)*F^a*c^2/ln(F)^3
/b^3*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-7/4*F^(b*c^2)*F^a*d^4/ln(F)^2/b^2*x^5*F
^(b*d^2*x^2)*F^(2*b*c*d*x)+35/8*F^(b*c^2)*F^a*d^2/ln(F)^3/b^3*x^3*F^(b*d^2
*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a*d^6/ln(F)/b*x^7*F^(b*d^2*x^2)*F^(2*b
*c*d*x)+1/2*F^(b*c^2)*F^a/d*c^7/ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)-7/4*F^
(b*c^2)*F^a/d*c^5/ln(F)^2/b^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+35/8*F^(b*c^2)*F
^a/d*c^3/ln(F)^3/b^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)-105/32*F^(b*c^2)*F^a/d/ln
(F)^4/b^4*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b
*c*ln(F)/(-b*ln(F))^(1/2))+7/2*F^(b*c^2)*F^a*d^5*c/ln(F)/b*x^6*F^(b*d^2*x^
2)*F^(2*b*c*d*x)-35/4*F^(b*c^2)*F^a*d^3*c/ln(F)^2/b^2*x^4*F^(b*d^2*x^2)*F^
(2*b*c*d*x)+105/8*F^(b*c^2)*F^a*d*c/ln(F)^3/b^3*x^2*F^(b*d^2*x^2)*F^(2*b*c
*d*x)+21/2*F^(b*c^2)*F^a*d^4*c^2/ln(F)/b*x^5*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3
5/2*F^(b*c^2)*F^a*d^3*c^3/ln(F)/b*x^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)+35/2*F^(
b*c^2)*F^a*d^2*c^4/ln(F)/b*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)+21/2*F^(b*c^2)*
F^a*d*c^5/ln(F)/b*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)-35/2*F^(b*c^2)*F^a*d*c^3
/ln(F)^2/b^2*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)-35/2*F^(b*c^2)*F^a*d^2*c^2/ln
(F)^2/b^2*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)-105/16*F^(b*c^2)*F^a/ln(F)^4/b^4
*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(159) = 318$.

Time = 0.09 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.80

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx =$$

$$-\frac{105\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)-2\left(8\left(b^4d^8x^7+7b^4cd^7x^6+21b^4c^2d^6x^5+35b^4c^3d^5x\right.\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^8,x, algorithm="fricas")
```

output
```
-1/32*(105*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x +
 c)/d) - 2*(8*(b^4*d^8*x^7 + 7*b^4*c*d^7*x^6 + 21*b^4*c^2*d^6*x^5 + 35*b^4
*c^3*d^5*x^4 + 35*b^4*c^4*d^4*x^3 + 21*b^4*c^5*d^3*x^2 + 7*b^4*c^6*d^2*x +
 b^4*c^7*d)*log(F)^4 - 28*(b^3*d^6*x^5 + 5*b^3*c*d^5*x^4 + 10*b^3*c^2*d^4*
x^3 + 10*b^3*c^3*d^3*x^2 + 5*b^3*c^4*d^2*x + b^3*c^5*d)*log(F)^3 + 70*(b^2
*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*log(F)^2 - 105*(
b*d^2*x + b*c*d)*log(F))*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^5*d^2*l
og(F)^5)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx=\int F^{a+b(c+dx)^2}(c+dx)^8\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**8,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3066 vs. $2(159)=318$.

Time = 1.11 (sec) , antiderivative size = 3066, normalized size of antiderivative = 17.13

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx=\text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^8,x, algorithm="maxima")
```

output
```
-4*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)
*d))*F^a*c^7/sqrt(b*log(F)) + 14*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(erf(
sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/2)
*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^2
/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*gamm
a(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d^5
*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^6*d/sqrt(b*log(F)) -
28*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(
F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)^2
*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/((b
*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b*c
*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)^2
*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)
)*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^5*d^2/sqrt(b*log(F)) + 35*(sqrt(p
i)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)
)) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b
*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log(F))^(
9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + b*c*d)...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.85

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx$$

$$= \frac{\left(8\,b^3d^6\left(x+\frac{c}{d}\right)^7\log\left(F\right)^3 - 28\,b^2d^4\left(x+\frac{c}{d}\right)^5\log\left(F\right)^2 + 70\,bd^2\left(x+\frac{c}{d}\right)^3\log\left(F\right) - 105\,x - \frac{105\,c}{d}\right)e^{(bd^2x^2\log(F}}{16\,b^4\log\left(F\right)^4}$$

$$- \frac{105\,\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log\left(F\right)}d\left(x+\frac{c}{d}\right)\right)}{32\,\sqrt{-b\log\left(F\right)}b^4d\log\left(F\right)^4}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^8,x, algorithm="giac")
```

output
```
1/16*(8*b^3*d^6*(x + c/d)^7*log(F)^3 - 28*b^2*d^4*(x + c/d)^5*log(F)^2 + 7
0*b*d^2*(x + c/d)^3*log(F) - 105*x - 105*c/d)*e^(b*d^2*x^2*log(F) + 2*b*c*
d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^4*log(F)^4) - 105/32*sqrt(pi)*F^a
*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b^4*d*log(F)^4)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.98

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx$$
$$= \frac{105\,F^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)}{32\,b^4\ln\left(F\right)^4\sqrt{b\,d^2\,\ln\left(F\right)}}$$
$$+\frac{7\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x\,\left(8\,b^3\,c^6\ln\left(F\right)^3-20\,b^2\,c^4\ln\left(F\right)^2+30\,b\,c^2\,\ln\left(F\right)-15\right)}{16\,b^4\ln\left(F\right)^4}$$
$$-\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,\left(-\frac{b^3\,c^7\ln(F)^3}{2}+\frac{7\,b^2\,c^5\ln(F)^2}{4}-\frac{35\,b\,c^3\,\ln(F)}{8}+\frac{105\,c}{16}\right)}{b^4\,d\ln\left(F\right)^4}$$
$$+\frac{7\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x^2\,\left(12\,d\,b^2\,c^5\ln\left(F\right)^2-20\,d\,b\,c^3\,\ln\left(F\right)+15\,d\,c\right)}{8\,b^3\ln\left(F\right)^3}$$
$$+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,d^6\,x^7}{2\,b\,\ln\left(F\right)}$$
$$+\frac{35\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x^3\,\left(4\,b^2\,c^4\,d^2\ln\left(F\right)^2-4\,b\,c^2\,d^2\,\ln\left(F\right)+d^2\right)}{8\,b^3\ln\left(F\right)^3}$$
$$-\frac{35\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x^4\,\left(c\,d^3-2\,b\,c^3\,d^3\,\ln\left(F\right)\right)}{4\,b^2\ln\left(F\right)^2}$$
$$+\frac{7\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,c\,d^5\,x^6}{2\,b\,\ln\left(F\right)}+\frac{7\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,d^4\,x^5\,\left(6\,b\,c^2\,\ln\left(F\right)-1\right)}{4\,b^2\ln\left(F\right)^2}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^8,x)
```

output
```
(105*F^a*pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2
)))/(32*b^4*log(F)^4*(b*d^2*log(F))^(1/2)) + (7*F^(b*d^2*x^2)*F^a*F^(b*c^2
)*F^(2*b*c*d*x)*x*(30*b*c^2*log(F) - 20*b^2*c^4*log(F)^2 + 8*b^3*c^6*log(F
)^3 - 15))/(16*b^4*log(F)^4) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*
((105*c)/16 - (35*b*c^3*log(F))/8 + (7*b^2*c^5*log(F)^2)/4 - (b^3*c^7*log(
F)^3)/2))/(b^4*d*log(F)^4) + (7*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*
x^2*(15*c*d + 12*b^2*c^5*d*log(F)^2 - 20*b*c^3*d*log(F)))/(8*b^3*log(F)^3)
 + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*d^6*x^7)/(2*b*log(F)) + (35*
F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x^3*(d^2 + 4*b^2*c^4*d^2*log(F)^
2 - 4*b*c^2*d^2*log(F)))/(8*b^3*log(F)^3) - (35*F^(b*d^2*x^2)*F^a*F^(b*c^2
)*F^(2*b*c*d*x)*x^4*(c*d^3 - 2*b*c^3*d^3*log(F)))/(4*b^2*log(F)^2) + (7*F^
(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*c*d^5*x^6)/(2*b*log(F)) + (7*F^(b*
d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*d^4*x^5*(6*b*c^2*log(F) - 1))/(4*b^2*
log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 865, normalized size of antiderivative = 4.83

$$\int F^{a+b(c+dx)^2}(c+dx)^8\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^8,x)
```

output
```
(f**a*( - 105*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(l
og(f))))*i + 16*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))
*log(f)**3*b**3*c**7 + 112*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*s
qrt(log(f))*log(f)**3*b**3*c**6*d*x + 336*f**(b*c**2 + 2*b*c*d*x + b*d**2*
x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*c**5*d**2*x**2 + 560*f**(b*c**2
+ 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*c**4*d**3*x
**3 + 560*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f
)**3*b**3*c**3*d**4*x**4 + 336*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(
b)*sqrt(log(f))*log(f)**3*b**3*c**2*d**5*x**5 + 112*f**(b*c**2 + 2*b*c*d*x
 + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*c*d**6*x**6 + 16*f**(b
*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*d**7*
x**7 - 56*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f
)**2*b**2*c**5 - 280*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(lo
g(f))*log(f)**2*b**2*c**4*d*x - 560*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*
sqrt(b)*sqrt(log(f))*log(f)**2*b**2*c**3*d**2*x**2 - 560*f**(b*c**2 + 2*b*
c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2*c**2*d**3*x**3 -
280*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b
**2*c*d**4*x**4 - 56*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(lo
g(f))*log(f)**2*b**2*d**5*x**5 + 140*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)
*sqrt(b)*sqrt(log(f))*log(f)*b*c**3 + 420*f**(b*c**2 + 2*b*c*d*x + b*d*...
```

3.204 $\int F^{a+b(c+dx)^2}(c+dx)^6\,dx$

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx = -\frac{15F^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{16b^{7/2}d\log^{\frac{7}{2}}(F)} + \frac{15F^{a+b(c+dx)^2}(c+dx)}{8b^3d\log^3(F)} - \frac{5F^{a+b(c+dx)^2}(c+dx)^3}{4b^2d\log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^5}{2bd\log(F)}$$

output
```
-15/16*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(7/2)/d/ln(F)^(7/2
)+15/8*F^(a+b*(d*x+c)^2)*(d*x+c)/b^3/d/ln(F)^3-5/4*F^(a+b*(d*x+c)^2)*(d*x+
c)^3/b^2/d/ln(F)^2+1/2*F^(a+b*(d*x+c)^2)*(d*x+c)^5/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.87

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx$$
$$=\frac{F^a\left(8F^{b(c+dx)^2}(c+dx)^5-\frac{15\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{b^{5/2}\log^{\frac{5}{2}}(F)}+\frac{30F^{b(c+dx)^2}(c+dx)}{b^2\log^2(F)}-\frac{20F^{b(c+dx)^2}(c+dx)^3}{b\log(F)}\right)}{16bd\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^6,x]
```

output
```
(F^a*(8*F^(b*(c + d*x)^2)*(c + d*x)^5 - (15*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x
)*Sqrt[Log[F]]])/(b^(5/2)*Log[F]^(5/2)) + (30*F^(b*(c + d*x)^2)*(c + d*x))
/(b^2*Log[F]^2) - (20*F^(b*(c + d*x)^2)*(c + d*x)^3)/(b*Log[F])))/(16*b*d*
Log[F])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2641, 2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^6F^{a+b(c+dx)^2}dx$$
$$\Big\downarrow \text{2641}$$
$$\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\int F^{b(c+dx)^2+a}(c+dx)^4dx}{2b\log(F)}$$
$$\Big\downarrow \text{2641}$$
$$\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\left(\frac{(c+dx)^3F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\int F^{b(c+dx)^2+a}(c+dx)^2dx}{2b\log(F)}\right)}{2b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\left(\frac{(c+dx)^3F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\left(\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\int F^{b(c+dx)^2+a}dx}{2b\log(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)^5F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{5\left(\frac{(c+dx)^3F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\left(\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\sqrt{\pi}F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}\right)}{2b\log(F)}\right)}{2b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^6,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(c + d*x)^5)/(2*b*d*Log[F]) - (5*((F^(a + b*(c + d*
x)^2)*(c + d*x)^3)/(2*b*d*Log[F]) - (3*(-1/4*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c
 + d*x)*Sqrt[Log[F]]])/(b^(3/2)*d*Log[F]^(3/2)) + (F^(a + b*(c + d*x)^2)*(
c + d*x))/(2*b*d*Log[F])))/(2*b*Log[F])))/(2*b*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(127) = 254.

Time = 0.40 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.97

method	result
risch	$-\frac{5F^{b\,c^2}F^a c^3 F^{b\,d^2x^2}F^{2bcdx}}{4d\ln(F)^2b^2}-\frac{15F^{b\,c^2}F^a c^2 x F^{b\,d^2x^2}F^{2bcdx}}{4\ln(F)^2b^2}+\frac{15F^{b\,c^2}F^a c F^{b\,d^2x^2}F^{2bcdx}}{8d\ln(F)^3b^3}-\frac{5F^{b\,c^2}F^a d^2x^3 F^{b\,d^2x^2}F^{2bcdx}}{4\ln(F)^2b^2}$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-5/4*F^(b*c^2)*F^a/d*c^3/ln(F)^2/b^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)-15/4*F^(b
*c^2)*F^a*c^2/ln(F)^2/b^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+15/8*F^(b*c^2)*F^a
/d*c/ln(F)^3/b^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)-5/4*F^(b*c^2)*F^a*d^2/ln(F)^2
/b^2*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a*d^4/ln(F)/b*x^5*F^(
b*d^2*x^2)*F^(2*b*c*d*x)+5/2*F^(b*c^2)*F^a*c^4/ln(F)/b*x*F^(b*d^2*x^2)*F^(
2*b*c*d*x)+1/2*F^(b*c^2)*F^a/d*c^5/ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)+5/2
*F^(b*c^2)*F^a*d^3*c/ln(F)/b*x^4*F^(b*d^2*x^2)*F^(2*b*c*d*x)+5*F^(b*c^2)*F
^a*d^2*c^2/ln(F)/b*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)+5*F^(b*c^2)*F^a*d*c^3/l
n(F)/b*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)-15/4*F^(b*c^2)*F^a*d*c/ln(F)^2/b^2*
x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+15/16*F^(b*c^2)*F^a/d/ln(F)^3/b^3*Pi^(1/2)
*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F)
)^(1/2))+15/8*F^(b*c^2)*F^a/ln(F)^3/b^3*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.50

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx$$
$$=\frac{15\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)+2\left(4\left(b^3d^6x^5+5\,b^3cd^5x^4+10\,b^3c^2d^4x^3+10\,b^3c^3d^3x^2+\right.\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/16*(15*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c
)/d) + 2*(4*(b^3*d^6*x^5 + 5*b^3*c*d^5*x^4 + 10*b^3*c^2*d^4*x^3 + 10*b^3*c
^3*d^3*x^2 + 5*b^3*c^4*d^2*x + b^3*c^5*d)*log(F)^3 - 10*(b^2*d^4*x^3 + 3*b
^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*log(F)^2 + 15*(b*d^2*x + b*c*d
)*log(F))*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^4*d^2*log(F)^4)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx = \int F^{a+b(c+dx)^2}(c+dx)^6\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**6,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1922 vs. 2(127) = 254.

Time = 0.72 (sec) , antiderivative size = 1922, normalized size of antiderivative = 13.26

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^6,x, algorithm="maxima")
```

output
```
-3*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)
*d))*F^a*c^5/sqrt(b*log(F)) + 15/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(er
f(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/
2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)
^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*ga
mma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d
^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^4*d/sqrt(b*log(F))
- 10*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*lo
g(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/(
(b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b
*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^
2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c^3*d^2/sqrt(b*log(F)) + 15/2*(sq
rt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*
d^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log(F
)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log(F
))^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + b*...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.91

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx = \frac{\left(4\,b^2d^4\left(x+\frac{c}{d}\right)^5\log\left(F\right)^2-10\,bd^2\left(x+\frac{c}{d}\right)^3\log\left(F\right)+15\,x+\frac{15\,c}{d}\right)e^{\left(bd^2x^2\log(F)+2\,bcdx\log(F)+bc^2\log(F)+a\log(F)\right)}}{8\,b^3\log\left(F\right)^3} + \frac{15\,\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log\left(F\right)}d\left(x+\frac{c}{d}\right)\right)}{16\,\sqrt{-b\log\left(F\right)}b^3d\log\left(F\right)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^6,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^4*(x + c/d)^5*log(F)^2 - 10*b*d^2*(x + c/d)^3*log(F) + 15*x +
 15*c/d)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))
/(b^3*log(F)^3) + 15/16*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sq
rt(-b*log(F))*b^3*d*log(F)^3)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.61

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx$$
$$= F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\left(\frac{15\,c}{8\,b^3\,d\ln\left(F\right)^3}+\frac{c^5}{2\,b\,d\,\ln\left(F\right)}-\frac{5\,c^3}{4\,b^2\,d\ln\left(F\right)^2}\right)$$
$$-\frac{15\,F^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)}{16\,b^3\ln\left(F\right)^3\sqrt{b\,d^2\,\ln\left(F\right)}}$$
$$-\frac{5\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x^2\left(3\,c\,d-4\,b\,c^3\,d\,\ln\left(F\right)\right)}{4\,b^2\ln\left(F\right)^2}$$
$$+\frac{5\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x\left(4\,b^2\,c^4\ln\left(F\right)^2-6\,b\,c^2\,\ln\left(F\right)+3\right)}{8\,b^3\ln\left(F\right)^3}$$
$$+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,d^4\,x^5}{2\,b\,\ln\left(F\right)}+\frac{5\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,c\,d^3\,x^4}{2\,b\,\ln\left(F\right)}$$
$$+\frac{5\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,d^2\,x^3\left(4\,b\,c^2\,\ln\left(F\right)-1\right)}{4\,b^2\ln\left(F\right)^2}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^6,x)
```

output
```
F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*((15*c)/(8*b^3*d*log(F)^3) + c^5
/(2*b*d*log(F)) - (5*c^3)/(4*b^2*d*log(F)^2)) - (15*F^a*pi^(1/2)*erfi((b*c
*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2)))/(16*b^3*log(F)^3*(b*d^2
*log(F))^(1/2)) - (5*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x^2*(3*c*d
- 4*b*c^3*d*log(F)))/(4*b^2*log(F)^2) + (5*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(
2*b*c*d*x)*x*(4*b^2*c^4*log(F)^2 - 6*b*c^2*log(F) + 3))/(8*b^3*log(F)^3) +
 (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*d^4*x^5)/(2*b*log(F)) + (5*F^(
b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*c*d^3*x^4)/(2*b*log(F)) + (5*F^(b*d
^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*d^2*x^3*(4*b*c^2*log(F) - 1))/(4*b^2*l
og(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.59

$$\int F^{a+b(c+dx)^2}(c+dx)^6\,dx$$
$$= \frac{f^a\left(15\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)i+8f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{\log(f)}\log(f)^2\,b^2c^5+40f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{}\right.}{}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^6,x)
```

output
```
(f**a*(15*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f
))))*i + 8*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(
f)**2*b**2*c**5 + 40*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(lo
g(f))*log(f)**2*b**2*c**4*d*x + 80*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*s
qrt(b)*sqrt(log(f))*log(f)**2*b**2*c**3*d**2*x**2 + 80*f**(b*c**2 + 2*b*c*
d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2*c**2*d**3*x**3 + 40
*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2
*c*d**4*x**4 + 8*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f)
)*log(f)**2*b**2*d**5*x**5 - 20*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt
(b)*sqrt(log(f))*log(f)*b*c**3 - 60*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*
sqrt(b)*sqrt(log(f))*log(f)*b*c**2*d*x - 60*f**(b*c**2 + 2*b*c*d*x + b*d**
2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*c*d**2*x**2 - 20*f**(b*c**2 + 2*b*c*
d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*d**3*x**3 + 30*f**(b*c**2
 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*c + 30*f**(b*c**2 + 2*b*c
*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*d*x))/(16*sqrt(b)*sqrt(log(f))*lo
g(f)**3*b**3*d)
```

3.205 $\int F^{a+b(c+dx)^2}(c+dx)^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 111

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx = \frac{3F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{8b^{5/2}d\log^{\frac{5}{2}}(F)} - \frac{3F^{a+b(c+dx)^2}(c+dx)}{4b^2d\log^2(F)} + \frac{F^{a+b(c+dx)^2}(c+dx)^3}{2bd\log(F)}$$

output
```
3/8*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(5/2)/d/ln(F)^(5/2)-3
/4*F^(a+b*(d*x+c)^2)*(d*x+c)/b^2/d/ln(F)^2+1/2*F^(a+b*(d*x+c)^2)*(d*x+c)^3
/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.81

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx = \frac{F^a\left(3\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)+2\sqrt{b}F^{b(c+dx)^2}(c+dx)\sqrt{\log(F)}(-3+2b(c+dx)^2\log(F))\right)}{8b^{5/2}d\log^{\frac{5}{2}}(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^4,x]
```

output
```
(F^a*(3*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]] + 2*Sqrt[b]*F^(b*(c
+ d*x)^2)*(c + d*x)*Sqrt[Log[F]]*(-3 + 2*b*(c + d*x)^2*Log[F])))/(8*b^(5/2
)*d*Log[F]^(5/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2641, 2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \; 2641$$

$$\frac{(c+dx)^3 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\int F^{b(c+dx)^2+a}(c+dx)^2dx}{2b\log(F)}$$

$$\Big\downarrow \; 2641$$

$$\frac{(c+dx)^3 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\left(\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\int F^{b(c+dx)^2+a}dx}{2b\log(F)}\right)}{2b\log(F)}$$

$$\Big\downarrow \; 2633$$

$$\frac{(c+dx)^3 F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{3\left(\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\sqrt{\pi}F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}\right)}{2b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^4,x]
```

output
```
(F^(a + b*(c + d*x)^2)*(c + d*x)^3)/(2*b*d*Log[F]) - (3*(-1/4*(F^a*Sqrt[Pi
]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(b^(3/2)*d*Log[F]^(3/2)) + (F^(a +
 b*(c + d*x)^2)*(c + d*x))/(2*b*d*Log[F])))/(2*b*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(95) = 190$.

Time = 0.19 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.84

method	result
risch	$\frac{F^{b\,c^2} F^a d^2 x^3 F^{b\,d^2 x^2} F^{2bcdx}}{2\ln(F) b} + \frac{3F^{b\,c^2} F^a d c x^2 F^{b\,d^2 x^2} F^{2bcdx}}{2\ln(F) b} + \frac{3F^{b\,c^2} F^a c^2 x\, F^{b\,d^2 x^2} F^{2bcdx}}{2\ln(F) b} + \frac{F^{b\,c^2} F^a c^3 F^{b\,d^2 x^2} F^{2bcdx}}{2d\ln(F) b} -$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(b*c^2)*F^a*d^2/ln(F)/b*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3/2*F^(b*c^2
)*F^a*d*c/ln(F)/b*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3/2*F^(b*c^2)*F^a*c^2/ln
(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a/d*c^3/ln(F)/b*F^(b*d
^2*x^2)*F^(2*b*c*d*x)-3/4*F^(b*c^2)*F^a/d*c/ln(F)^2/b^2*F^(b*d^2*x^2)*F^(2
*b*c*d*x)-3/4*F^(b*c^2)*F^a/ln(F)^2/b^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-3/8*
F^(b*c^2)*F^a/d/ln(F)^2/b^2*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-
b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.27

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx =$$
$$-\frac{3\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)-2\left(2\left(b^2d^4x^3+3\,b^2cd^3x^2+3\,b^2c^2d^2x+b^2c^3d\right)\log(F)^2}{8\,b^3d^2\log(F)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/8*(3*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)
/d) - 2*(2*(b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*l
og(F)^2 - 3*(b*d^2*x + b*c*d)*log(F))*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a
))/(b^3*d^2*log(F)^3)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx = \int F^{a+b(c+dx)^2}(c+dx)^4\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**4,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. 2(95) = 190.

Time = 0.43 (sec) , antiderivative size = 1037, normalized size of antiderivative = 9.34

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^4,x, algorithm="maxima")
```

output
```
-2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)
*d))*F^a*c^3/sqrt(b*log(F)) + 3*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(erf(s
qrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/2)*
d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^2/
(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*gamma
(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d^5*
(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*c^2*d/sqrt(b*log(F)) - 2
*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*log(F)^3/((b*l
og(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d^2*x + b*c*d
)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*
log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*c*d^2/sqrt(b*log(F)) + 1/2*(sqrt(pi)*
(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2)))
- 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^
2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/((b*log(F))^(9/2
)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d^2*x + b*c*d)^2*...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx$$
$$= \frac{\left(2\,bd^2\left(x+\frac{c}{d}\right)^3\log\left(F\right)-3\,x-\frac{3\,c}{d}\right)e^{\left(bd^2x^2\log(F)+2\,bcdx\log(F)+bc^2\log(F)+a\log(F)\right)}}{4\,b^2\log\left(F\right)^2} - \frac{3\,\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log\left(F\right)}d\left(x+\frac{c}{d}\right)\right)}{8\,\sqrt{-b\log\left(F\right)}b^2d\log\left(F\right)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^4,x, algorithm="giac")
```

output
```
1/4*(2*b*d^2*(x + c/d)^3*log(F) - 3*x - 3*c/d)*e^(b*d^2*x^2*log(F) + 2*b*c
*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^2*log(F)^2) - 3/8*sqrt(pi)*F^a*e
rf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b^2*d*log(F)^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.19

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx = \frac{3\,F^a\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)}{8\,b^2\ln\left(F\right)^2\,\sqrt{b\,d^2\,\ln\left(F\right)}} - F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\left(\frac{3\,c}{4\,b^2\,d\ln\left(F\right)^2}-\frac{c^3}{2\,b\,d\,\ln\left(F\right)}\right) + \frac{3\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x\left(2\,b\,c^2\,\ln\left(F\right)-1\right)}{4\,b^2\ln\left(F\right)^2} + \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,d^2\,x^3}{2\,b\,\ln\left(F\right)} + \frac{3\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,c\,d\,x^2}{2\,b\,\ln\left(F\right)}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^4,x)
```

output
```
(3*F^a*pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))
)/(8*b^2*log(F)^2*(b*d^2*log(F))^(1/2)) - F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2
*b*c*d*x)*((3*c)/(4*b^2*d*log(F)^2) - c^3/(2*b*d*log(F))) + (3*F^(b*d^2*x^
2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x*(2*b*c^2*log(F) - 1))/(4*b^2*log(F)^2) +
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*d^2*x^3)/(2*b*log(F)) + (3*F^(b
*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*c*d*x^2)/(2*b*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.39

$$\int F^{a+b(c+dx)^2}(c+dx)^4\,dx$$
$$= \frac{f^a\left(-3\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)i+4f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{\log(f)}\log(f)\,b\,c^3+12f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{\mathrm{l}}}{}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^4,x)
```

output
```
(f**a*( - 3*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log
(f))))*i + 4*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*lo
g(f)*b*c**3 + 12*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f)
)*log(f)*b*c**2*d*x + 12*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqr
t(log(f))*log(f)*b*c*d**2*x**2 + 4*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*s
qrt(b)*sqrt(log(f))*log(f)*b*d**3*x**3 - 6*f**(b*c**2 + 2*b*c*d*x + b*d**2
*x**2)*sqrt(b)*sqrt(log(f))*c - 6*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sq
rt(b)*sqrt(log(f))*d*x))/(8*sqrt(b)*sqrt(log(f))*log(f)**2*b**2*d)
```

3.206 $\int F^{a+b(c+dx)^2}(c+dx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx=-\frac{F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}+\frac{F^{a+b(c+dx)^2}(c+dx)}{2bd\log(F)}$$

output
```
-1/4*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(3/2)/d/ln(F)^(3/2)+
1/2*F^(a+b*(d*x+c)^2)*(d*x+c)/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx=-\frac{F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}+\frac{F^{a+b(c+dx)^2}(c+dx)}{2bd\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(c + d*x)^2,x]
```

output
```
-1/4*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(b^(3/2)*d*Log[F]
^(3/2)) + (F^(a + b*(c + d*x)^2)*(c + d*x))/(2*b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2641, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 F^{a+b(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\int F^{b(c+dx)^2+a}dx}{2b\log(F)}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{(c+dx)F^{a+b(c+dx)^2}}{2bd\log(F)}-\frac{\sqrt{\pi}F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(c + d*x)^2,x]
```

output
```
-1/4*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(b^(3/2)*d*Log[F]
^(3/2)) + (F^(a + b*(c + d*x)^2)*(c + d*x))/(2*b*d*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(63) = 126$.

Time = 0.10 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.90

method	result	size
risch	$\frac{F^{b\,c^{2}} F^{a} x\,F^{b\,d^{2} x^{2}} F^{2 b c d x}}{2 \ln \left(F \right) b}+\frac{F^{b\,c^{2}} F^{a} c\,F^{b\,d^{2} x^{2}} F^{2 b c d x}}{2 d \ln \left(F \right) b}+\frac{F^{b\,c^{2}} F^{a} \sqrt{\pi}\,F^{-b\,c^{2}} \operatorname{erf}\left(-d \sqrt{-b \ln \left(F \right)}\,x +\frac{b c \ln \left(F \right)}{\sqrt{-b \ln \left(F \right)}}\right)}{4 d \ln \left(F \right) b \sqrt{-b \ln \left(F \right)}}$	146

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*F^(b*c^2)*F^a/ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a/
d*c/ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/4*F^(b*c^2)*F^a/d/ln(F)/b*Pi^(1/
2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(
F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx$$
$$=\frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)+2\left(bd^2x+bcd\right)F^{bd^2x^2+2\,bcdx+bc^2+a}\log(F)}{4\,b^2d^2\log(F)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d)
 + 2*(b*d^2*x + b*c*d)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*log(F))/(b^2*
d^2*log(F)^2)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx=\int F^{a+b(c+dx)^2}\left(c+dx\right)^2\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(d*x+c)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(63) = 126.

Time = 0.22 (sec) , antiderivative size = 413, normalized size of antiderivative = 5.36

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx$$

$$= -\frac{\left(\frac{\sqrt{\pi}(bd^2x+bcd)bc\left(\operatorname{erf}\left(\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}\right)-1\right)\log(F)^2}{(b\log(F))^{\frac{3}{2}}d^2\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}}-\frac{F^{\frac{(bd^2x+bcd)^2}{bd^2}}b\log(F)}{(b\log(F))^{\frac{3}{2}}d}\right)F^ac}{\sqrt{b\log(F)}}$$

$$+\frac{\left(\frac{\sqrt{\pi}(bd^2x+bcd)b^2c^2\left(\operatorname{erf}\left(\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}\right)-1\right)\log(F)^3}{(b\log(F))^{\frac{5}{2}}d^3\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}}-\frac{2F^{\frac{(bd^2x+bcd)^2}{bd^2}}b^2c\log(F)^2}{(b\log(F))^{\frac{5}{2}}d^2}-\frac{(bd^2x+bcd)^3\Gamma\left(\frac{3}{2},-\frac{(bd^2x+bcd)^2\,\mathrm{l}\cdots}{bd^2}\right.}{(b\log(F))^{\frac{5}{2}}d^5\left(-\frac{(bd^2x+bcd)\cdots}{bd^2}\right.}\cdots}{2\sqrt{b\log(F)}}$$

$$+\frac{\sqrt{\pi}F^{bc^2+a}c^2\operatorname{erf}\left(\sqrt{-b\log(F)}dx-\frac{bc\log(F)}{\sqrt{-b\log(F)}}\right)}{2\sqrt{-b\log(F)}F^{bc^2}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*
d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log(F
)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)*d
))*F^a*c/sqrt(b*log(F)) + 1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(erf(sqr
t(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5/2)*d^
3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d)^2/(b
*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*gamma(3
/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*d^5*(-
(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*d/sqrt(b*log(F)) + 1/2*sqr
t(pi)*F^(b*c^2 + a)*c^2*erf(sqrt(-b*log(F))*d*x - b*c*log(F)/sqrt(-b*log(F
)))/(sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx = \frac{\left(x+\frac{c}{d}\right)e^{\left(bd^2x^2\log(F)+2\,bcdx\log(F)+bc^2\log(F)+a\log(F)\right)}}{2\,b\log(F)} + \frac{\sqrt{\pi}F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}d\left(x+\frac{c}{d}\right)\right)}{4\sqrt{-b\log(F)}bd\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(x + c/d)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*lo
g(F))/(b*log(F)) + 1/4*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqr
t(-b*log(F))*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.69

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx = \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x}{2\,b\,\ln(F)} - \frac{F^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)}{4\,b\,\ln(F)\,\sqrt{b\,d^2\,\ln(F)}} + \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,c}{2\,b\,d\,\ln(F)}$$

input
```
int(F^(a + b*(c + d*x)^2)*(c + d*x)^2,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x)/(2*b*log(F)) - (F^a*pi^(1/2)
*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2)))/(4*b*log(F)*(
b*d^2*log(F))^(1/2)) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*c)/(2*b*
d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.49

$$\int F^{a+b(c+dx)^2}(c+dx)^2\,dx = \frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)i+2f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{\log(f)}\,c+2f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{\log(f)}\,dx\right)}{4\sqrt{b}\,\sqrt{\log(f)}\log(f)\,bd}$$

input
```
int(F^(a+b*(d*x+c)^2)*(d*x+c)^2,x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f)))
)*i + 2*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*c + 2*f
**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*d*x))/(4*sqrt(b)
*sqrt(log(f))*log(f)*b*d)
```

3.207 $\int F^{a+b(c+dx)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int F^{a+b(c+dx)^2} dx = \frac{F^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

output
```
1/2*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(1/2)/d/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2} dx = \frac{F^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

input
```
Integrate[F^(a + b*(c + d*x)^2),x]
```

output
```
(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d*Sqrt[Log[
F]])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

input
```
Int[F^(a + b*(c + d*x)^2),x]
```

output
```
(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d*Sqrt[Log[
F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{\sqrt{\pi}\, F^{b\,c^2+a} F^{-b\,c^2}\,\text{erf}\left(-d\sqrt{-b\ln(F)}\,x+\frac{b c\ln(F)}{\sqrt{-b\ln(F)}}\right)}{2d\sqrt{-b\ln(F)}}$	58

input
```
int(F^(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*F^(b*c^2+a)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^
(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)}{2\,bd^2\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d)
/(b*d^2*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}\,dx = \int F^{a+b(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2),x)
```

output
```
Integral(F**(a + b*(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int F^{a+b(c+dx)^2}\,dx = \frac{\sqrt{\pi}F^{bc^2+a}\,\mathrm{erf}\left(\sqrt{-b\log\left(F\right)}dx - \frac{bc\log(F)}{\sqrt{-b\log(F)}}\right)}{2\,\sqrt{-b\log\left(F\right)}F^{bc^2}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*F^(b*c^2 + a)*erf(sqrt(-b*log(F))*d*x - b*c*log(F)/sqrt(-b*lo
g(F)))/(sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{\sqrt{\pi}F^{a}\,\mathrm{erf}\left(-\sqrt{-b\log\left(F\right)}d\left(x+\frac{c}{d}\right)\right)}{2\,\sqrt{-b\log\left(F\right)}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{F^{a}\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{1\mathrm{i}\,b\,x\,\ln(F)\,d^2+1\mathrm{i}\,b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\,1\mathrm{i}}{2\,\sqrt{b\,d^2\,\ln\left(F\right)}}$$

input
```
int(F^(a + b*(c + d*x)^2),x)
```

output
```
-(F^a*pi^(1/2)*erf((b*c*d*log(F)*1i + b*d^2*x*log(F)*1i)/(b*d^2*log(F))^(1
/2))*1i)/(2*(b*d^2*log(F))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{\sqrt{\pi}\,f^a\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)i}{2\sqrt{b}\,\sqrt{\log(f)}\,d}$$

input
```
int(F^(a+b*(d*x+c)^2),x)
```

output
```
( - sqrt(pi)*f**a*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f)
)))*i)/(2*sqrt(b)*sqrt(log(f))*d)
```

3.208 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 67

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx = -\frac{F^{a+b(c+dx)^2}}{d(c+dx)} + \frac{\sqrt{b}F^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\sqrt{\log(F)}}{d}$$

output
```
-F^(a+b*(d*x+c)^2)/d/(d*x+c)+b^(1/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(
F)^(1/2))*ln(F)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} dx = \frac{F^a\left(-\frac{F^{b(c+dx)^2}}{c+dx} + \sqrt{b}\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\sqrt{\log(F)}\right)}{d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^2,x]
```

output
```
(F^a*(-(F^(b*(c + d*x)^2)/(c + d*x)) + Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[b]*(c +
d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]]))/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$2b\log(F)\int F^{b(c+dx)^2+a}dx - \frac{F^{a+b(c+dx)^2}}{d(c+dx)}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{d} - \frac{F^{a+b(c+dx)^2}}{d(c+dx)}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^2,x]
```

output
```
-(F^(a + b*(c + d*x)^2)/(d*(c + d*x))) + (Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b
]*(c + d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]])/d
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{F^{b(dx+c)^2} F^a}{d(dx+c)}+\frac{b\ln(F)\sqrt{\pi}\, F^a \operatorname{erf}\left(\sqrt{-b\ln(F)}\,(dx+c)\right)}{d\sqrt{-b\ln(F)}}$	62

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d/(d*x+c)*F^(b*(d*x+c)^2)*F^a+1/d*b*ln(F)*Pi^(1/2)*F^a/(-b*ln(F))^(1/2)
*erf((-b*ln(F))^(1/2)*(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx = -\frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}(dx+c)F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)+F^{bd^2x^2+2\,bcdx+bc^2+a}d}{d^3x+cd^2}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(sqrt(pi)*sqrt(-b*d^2*log(F))*(d*x + c)*F^a*erf(sqrt(-b*d^2*log(F))*(d*x
+ c)/d) + F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx = \frac{F^a\,b\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\,\ln(F)}{\sqrt{b\,d^2\,\ln(F)}} - \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}}{d\,(c+d\,x)}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^2,x)
```

output
```
(F^a*b*pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))
*log(F))/(b*d^2*log(F))^(1/2) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)
)/(d*(c + d*x))
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^2}\,dx = f^{b\,c^2+a}\left(\int \frac{f^{b\,d^2x^2+2bcdx}}{d^2x^2+2cdx+c^2}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^2,x)
```

output
```
f**(a + b*c**2)*int(f**(2*b*c*d*x + b*d**2*x**2)/(c**2 + 2*c*d*x + d**2*x*
*2),x)
```

3.209 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = -\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3} - \frac{2bF^{a+b(c+dx)^2}\log(F)}{3d(c+dx)} + \frac{2b^{3/2}F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\log^{\frac{3}{2}}(F)}{3d}$$

output
```
-1/3*F^(a+b*(d*x+c)^2)/d/(d*x+c)^3-2/3*b*F^(a+b*(d*x+c)^2)*ln(F)/d/(d*x+c)
+2/3*b^(3/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))*ln(F)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = \frac{F^a\left(2b^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\log^{\frac{3}{2}}(F) - \frac{F^{b(c+dx)^2}(1+2b(c+dx)^2\log(F))}{(c+dx)^3}\right)}{3d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^4,x]
```

output
```
(F^a*(2*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Log[F]^(3/2)
 - (F^(b*(c + d*x)^2)*(1 + 2*b*(c + d*x)^2*Log[F]))/(c + d*x)^3))/(3*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4} dx$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2}{3}b\log(F)\int \frac{F^{b(c+dx)^2+a}}{(c+dx)^2}dx - \frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2}{3}b\log(F)\left(2b\log(F)\int F^{b(c+dx)^2+a}dx - \frac{F^{a+b(c+dx)^2}}{d(c+dx)}\right) - \frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{2}{3}b\log(F)\left(\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{d} - \frac{F^{a+b(c+dx)^2}}{d(c+dx)}\right) - \frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^4,x]
```

output
```
-1/3*F^(a + b*(c + d*x)^2)/(d*(c + d*x)^3) + (2*b*(-(F^(a + b*(c + d*x)^2)
/(d*(c + d*x))) + (Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]
]]*Sqrt[Log[F]])/d)*Log[F])/3
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{F^{b(dx+c)^2} F^a}{3d(dx+c)^3}-\frac{2b\ln(F)F^{b(dx+c)^2}F^a}{3d(dx+c)}+\frac{2b^2\ln(F)^2\sqrt{\pi}\,F^a\,\mathrm{erf}\left(\sqrt{-b\ln(F)}\,(dx+c)\right)}{3d\sqrt{-b\ln(F)}}$	96

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3/d/(d*x+c)^3*F^(b*(d*x+c)^2)*F^a-2/3/d*b*ln(F)/(d*x+c)*F^(b*(d*x+c)^2)
*F^a+2/3/d*b^2*ln(F)^2*Pi^(1/2)*F^a/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)*
(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.60

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx =$$

$$-\frac{2\sqrt{\pi}(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3)\sqrt{-bd^2\log(F)}F^a\,\mathrm{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)\log(F)+(2\,(bd^3x^2+}{3\,(d^5x^3+3\,cd^4x^2+3\,c^2d^3x+c^3d^2)}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(2*sqrt(pi)*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sqrt(-b
*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d)*log(F) + (2*(b*d^3*x
^2 + 2*b*c*d^2*x + b*c^2*d)*log(F) + d)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 +
 a))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**4,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**4, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.97

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = \frac{2\,F^a\,b^2\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\,\ln\left(F\right)^2}{3\,\sqrt{b\,d^2\,\ln\left(F\right)}} - \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\left(\frac{1}{3\,d}+\frac{2\,b\,c^2\,\ln(F)}{3\,d}\right)+\frac{4\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,b\,c\,x\,\ln(F)}{3}+\frac{2\,F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,b\,d\,x^2\,\ln}{3}}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^4,x)
```

output
```
(2*F^a*b^2*pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1
/2))*log(F)^2)/(3*(b*d^2*log(F))^(1/2)) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(
2*b*c*d*x)*(1/(3*d) + (2*b*c^2*log(F))/(3*d)) + (4*F^(b*d^2*x^2)*F^a*F^(b*
c^2)*F^(2*b*c*d*x)*b*c*x*log(F))/3 + (2*F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b
*c*d*x)*b*d*x^2*log(F))/3)/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^4}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^4,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 8*int(f**(2*b*c*d*x +
b*d**2*x**2)/(2*log(f)*b*c**6 + 8*log(f)*b*c**5*d*x + 12*log(f)*b*c**4*d**
2*x**2 + 8*log(f)*b*c**3*d**3*x**3 + 2*log(f)*b*c**2*d**4*x**4 - 3*c**4 -
12*c**3*d*x - 18*c**2*d**2*x**2 - 12*c*d**3*x**3 - 3*d**4*x**4),x)*log(f)*
*2*b**2*c**7*d + 24*int(f**(2*b*c*d*x + b*d**2*x**2)/(2*log(f)*b*c**6 + 8*
log(f)*b*c**5*d*x + 12*log(f)*b*c**4*d**2*x**2 + 8*log(f)*b*c**3*d**3*x**3
 + 2*log(f)*b*c**2*d**4*x**4 - 3*c**4 - 12*c**3*d*x - 18*c**2*d**2*x**2 -
12*c*d**3*x**3 - 3*d**4*x**4),x)*log(f)**2*b**2*c**6*d**2*x + 24*int(f**(2
*b*c*d*x + b*d**2*x**2)/(2*log(f)*b*c**6 + 8*log(f)*b*c**5*d*x + 12*log(f)
*b*c**4*d**2*x**2 + 8*log(f)*b*c**3*d**3*x**3 + 2*log(f)*b*c**2*d**4*x**4
- 3*c**4 - 12*c**3*d*x - 18*c**2*d**2*x**2 - 12*c*d**3*x**3 - 3*d**4*x**4)
,x)*log(f)**2*b**2*c**5*d**3*x**2 + 8*int(f**(2*b*c*d*x + b*d**2*x**2)/(2*
log(f)*b*c**6 + 8*log(f)*b*c**5*d*x + 12*log(f)*b*c**4*d**2*x**2 + 8*log(f
)*b*c**3*d**3*x**3 + 2*log(f)*b*c**2*d**4*x**4 - 3*c**4 - 12*c**3*d*x - 18
*c**2*d**2*x**2 - 12*c*d**3*x**3 - 3*d**4*x**4),x)*log(f)**2*b**2*c**4*d**
4*x**3 - 24*int(f**(2*b*c*d*x + b*d**2*x**2)/(2*log(f)*b*c**6 + 8*log(f)*b
*c**5*d*x + 12*log(f)*b*c**4*d**2*x**2 + 8*log(f)*b*c**3*d**3*x**3 + 2*log
(f)*b*c**2*d**4*x**4 - 3*c**4 - 12*c**3*d*x - 18*c**2*d**2*x**2 - 12*c*d**
3*x**3 - 3*d**4*x**4),x)*log(f)*b*c**5*d - 72*int(f**(2*b*c*d*x + b*d**2*x
**2)/(2*log(f)*b*c**6 + 8*log(f)*b*c**5*d*x + 12*log(f)*b*c**4*d**2*x**...
```

3.210 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = -\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5} - \frac{2bF^{a+b(c+dx)^2}\log(F)}{15d(c+dx)^3} - \frac{4b^2F^{a+b(c+dx)^2}\log^2(F)}{15d(c+dx)} + \frac{4b^{5/2}F^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\log^{\frac{5}{2}}(F)}{15d}$$

output
```
-1/5*F^(a+b*(d*x+c)^2)/d/(d*x+c)^5-2/15*b*F^(a+b*(d*x+c)^2)*ln(F)/d/(d*x+c
)^3-4/15*b^2*F^(a+b*(d*x+c)^2)*ln(F)^2/d/(d*x+c)+4/15*b^(5/2)*F^a*Pi^(1/2)
*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))*ln(F)^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = \frac{F^a\left(4b^{5/2}\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\log^{\frac{5}{2}}(F) - \frac{F^{b(c+dx)^2}\left(3+2b(c+dx)^2\log(F)+4b^2(c+dx)^4\log^2(F)\right)}{(c+dx)^5}\right)}{15d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^6,x]
```

output
```
(F^a*(4*b^(5/2)*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Log[F]^(5/2)
 - (F^(b*(c + d*x)^2)*(3 + 2*b*(c + d*x)^2*Log[F] + 4*b^2*(c + d*x)^4*Log[
F]^2))/(c + d*x)^5))/(15*d)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2643, 2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(F)\int \frac{F^{b(c+dx)^2+a}}{(c+dx)^4}dx-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\int \frac{F^{b(c+dx)^2+a}}{(c+dx)^2}dx-\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}\right)-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(2b\log(F)\int F^{b(c+dx)^2+a}dx-\frac{F^{a+b(c+dx)^2}}{d(c+dx)}\right)-\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}\right)-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{d}-\frac{F^{a+b(c+dx)^2}}{d(c+dx)}\right)-\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}\right)-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^6,x]
```

output
```
-1/5*F^(a + b*(c + d*x)^2)/(d*(c + d*x)^5) + (2*b*Log[F]*(-1/3*F^(a + b*(c
 + d*x)^2)/(d*(c + d*x)^3) + (2*b*(-(F^(a + b*(c + d*x)^2)/(d*(c + d*x)))
+ (Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]])
/d)*Log[F])/3))/5
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{F^{b(dx+c)^2}F^a}{5d(dx+c)^5}-\frac{2b\ln(F)F^{b(dx+c)^2}F^a}{15d(dx+c)^3}-\frac{4b^2\ln(F)^2F^{b(dx+c)^2}F^a}{15d(dx+c)}+\frac{4b^3\ln(F)^3\sqrt{\pi}\,F^a\,\text{erf}\left(\sqrt{-b\ln(F)}\,(dx+c)\right)}{15d\sqrt{-b\ln(F)}}$	129

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5/d/(d*x+c)^5*F^(b*(d*x+c)^2)*F^a-2/15/d*b*ln(F)/(d*x+c)^3*F^(b*(d*x+c)
^2)*F^a-4/15/d*b^2*ln(F)^2/(d*x+c)*F^(b*(d*x+c)^2)*F^a+4/15/d*b^3*ln(F)^3*
Pi^(1/2)*F^a/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(118) = 236$.

Time = 0.08 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.12

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx =$$
$$-\frac{4\sqrt{\pi}(b^2d^5x^5 + 5\,b^2cd^4x^4 + 10\,b^2c^2d^3x^3 + 10\,b^2c^3d^2x^2 + 5\,b^2c^4dx + b^2c^5)\sqrt{-bd^2\log(F)}F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2\log \cdots}}{\cdots}\right.}{15\,(d^7x \cdots}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/15*(4*sqrt(pi)*(b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10
*b^2*c^3*d^2*x^2 + 5*b^2*c^4*d*x + b^2*c^5)*sqrt(-b*d^2*log(F))*F^a*erf(sq
rt(-b*d^2*log(F))*(d*x + c)/d)*log(F)^2 + (4*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^
3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x + b^2*c^4*d)*log(F)^2 + 2*(b*d^3*x
^2 + 2*b*c*d^2*x + b*c^2*d)*log(F) + 3*d)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2
 + a))/(d^7*x^5 + 5*c*d^6*x^4 + 10*c^2*d^5*x^3 + 10*c^3*d^4*x^2 + 5*c^4*d^
3*x + c^5*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**6,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**6, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^6}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^6,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^6, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^6}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^6,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = \frac{4\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2}\right)\,\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)^{5/2}}{15\,d\,\left(c+d\,x\right)^5} - \frac{4\,F^a\,\sqrt{\pi}\,\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)^{5/2}}{15\,d\,\left(c+d\,x\right)^5} - \frac{4\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^2\,\ln\left(F\right)^2}{15\,d\,\left(c+d\,x\right)} - \frac{2\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b\,\ln\left(F\right)}{15\,d\,\left(c+d\,x\right)^3} - \frac{F^a\,F^{b\,\left(c+d\,x\right)^2}}{5\,d\,\left(c+d\,x\right)^5}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^6,x)
```

output
```
(4*F^a*pi^(1/2)*erfc((-b*log(F)*(c + d*x)^2)^(1/2))*(-b*log(F)*(c + d*x)^2
)^(5/2))/(15*d*(c + d*x)^5) - (4*F^a*pi^(1/2)*(-b*log(F)*(c + d*x)^2)^(5/2
))/(15*d*(c + d*x)^5) - (4*F^a*F^(b*(c + d*x)^2)*b^2*log(F)^2)/(15*d*(c +
d*x)) - (2*F^a*F^(b*(c + d*x)^2)*b*log(F))/(15*d*(c + d*x)^3) - (F^a*F^(b*
(c + d*x)^2))/(5*d*(c + d*x)^5)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^6}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^6,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 8*int(f**(2*b*c*d*x +
b*d**2*x**2)/(2*log(f)*b*c**8 + 12*log(f)*b*c**7*d*x + 30*log(f)*b*c**6*d*
*2*x**2 + 40*log(f)*b*c**5*d**3*x**3 + 30*log(f)*b*c**4*d**4*x**4 + 12*log
(f)*b*c**3*d**5*x**5 + 2*log(f)*b*c**2*d**6*x**6 - 5*c**6 - 30*c**5*d*x -
75*c**4*d**2*x**2 - 100*c**3*d**3*x**3 - 75*c**2*d**4*x**4 - 30*c*d**5*x**
5 - 5*d**6*x**6),x)*log(f)**2*b**2*c**9*d + 40*int(f**(2*b*c*d*x + b*d**2*
x**2)/(2*log(f)*b*c**8 + 12*log(f)*b*c**7*d*x + 30*log(f)*b*c**6*d**2*x**2
 + 40*log(f)*b*c**5*d**3*x**3 + 30*log(f)*b*c**4*d**4*x**4 + 12*log(f)*b*c
**3*d**5*x**5 + 2*log(f)*b*c**2*d**6*x**6 - 5*c**6 - 30*c**5*d*x - 75*c**4
*d**2*x**2 - 100*c**3*d**3*x**3 - 75*c**2*d**4*x**4 - 30*c*d**5*x**5 - 5*d
**6*x**6),x)*log(f)**2*b**2*c**8*d**2*x + 80*int(f**(2*b*c*d*x + b*d**2*x*
*2)/(2*log(f)*b*c**8 + 12*log(f)*b*c**7*d*x + 30*log(f)*b*c**6*d**2*x**2 +
 40*log(f)*b*c**5*d**3*x**3 + 30*log(f)*b*c**4*d**4*x**4 + 12*log(f)*b*c**
3*d**5*x**5 + 2*log(f)*b*c**2*d**6*x**6 - 5*c**6 - 30*c**5*d*x - 75*c**4*d
**2*x**2 - 100*c**3*d**3*x**3 - 75*c**2*d**4*x**4 - 30*c*d**5*x**5 - 5*d**
6*x**6),x)*log(f)**2*b**2*c**7*d**3*x**2 + 80*int(f**(2*b*c*d*x + b*d**2*x
**2)/(2*log(f)*b*c**8 + 12*log(f)*b*c**7*d*x + 30*log(f)*b*c**6*d**2*x**2
+ 40*log(f)*b*c**5*d**3*x**3 + 30*log(f)*b*c**4*d**4*x**4 + 12*log(f)*b*c*
*3*d**5*x**5 + 2*log(f)*b*c**2*d**6*x**6 - 5*c**6 - 30*c**5*d*x - 75*c**4*
d**2*x**2 - 100*c**3*d**3*x**3 - 75*c**2*d**4*x**4 - 30*c*d**5*x**5 - 5...
```

3.211 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} \, dx = -\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7} - \frac{2bF^{a+b(c+dx)^2}\log(F)}{35d(c+dx)^5} - \frac{4b^2F^{a+b(c+dx)^2}\log^2(F)}{105d(c+dx)^3} - \frac{8b^3F^{a+b(c+dx)^2}\log^3(F)}{105d(c+dx)} + \frac{8b^{7/2}F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\log^{\frac{7}{2}}(F)}{105d}$$

output
```
-1/7*F^(a+b*(d*x+c)^2)/d/(d*x+c)^7-2/35*b*F^(a+b*(d*x+c)^2)*ln(F)/d/(d*x+c
)^5-4/105*b^2*F^(a+b*(d*x+c)^2)*ln(F)^2/d/(d*x+c)^3-8/105*b^3*F^(a+b*(d*x+
c)^2)*ln(F)^3/d/(d*x+c)+8/105*b^(7/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln
(F)^(1/2))*ln(F)^(7/2)/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.66

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8}\,dx = \frac{F^a\left(8b^{7/2}\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\log^{\frac{7}{2}}(F)+\frac{F^{b(c+dx)^2}(-15-6b(c+dx)^2\log(F)-4b^2(c+dx)^4\log^2(F)-8b^3(c+dx)^6\log^3(F))}{(c+dx)^7}\right)}{105d}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^8,x]
```

output
```
(F^a*(8*b^(7/2)*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Log[F]^(7/2)
 + (F^(b*(c + d*x)^2)*(-15 - 6*b*(c + d*x)^2*Log[F] - 4*b^2*(c + d*x)^4*Lo
g[F]^2 - 8*b^3*(c + d*x)^6*Log[F]^3))/(c + d*x)^7))/(105*d)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2643, 2643, 2643, 2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8}\,dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7}b\log(F)\int \frac{F^{b(c+dx)^2+a}}{(c+dx)^6}dx-\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\int \frac{F^{b(c+dx)^2+a}}{(c+dx)^4}dx-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}\right)-\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\int\frac{F^{b(c+dx)^2+a}}{(c+dx)^2}dx-\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}\right)-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}\right)-\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(2b\log(F)\int F^{b(c+dx)^2+a}dx-\frac{F^{a+b(c+dx)^2}}{d(c+dx)}\right)-\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}\right)-\frac{F^{a+b(c+dx)^2}}{5d(c+dx)^5}\right)-\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{d}-\frac{F^{a+b(c+dx)^2}}{d(c+dx)}\right)-\frac{F^{a+b(c+dx)^2}}{3d(c+dx)^3}\right)-\ldots\right)-\frac{F^{a+b(c+dx)^2}}{7d(c+dx)^7}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^8,x]
```

output
```
-1/7*F^(a + b*(c + d*x)^2)/(d*(c + d*x)^7) + (2*b*Log[F]*(-1/5*F^(a + b*(c
 + d*x)^2)/(d*(c + d*x)^5) + (2*b*Log[F]*(-1/3*F^(a + b*(c + d*x)^2)/(d*(c
 + d*x)^3) + (2*b*(-(F^(a + b*(c + d*x)^2)/(d*(c + d*x))) + (Sqrt[b]*F^a*S
qrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]])/d)*Log[F])/3))/
5))/7
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{F^{b(dx+c)^2} F^a}{7d(dx+c)^7}-\frac{2b\ln(F) F^{b(dx+c)^2} F^a}{35d(dx+c)^5}-\frac{4b^2\ln(F)^2 F^{b(dx+c)^2} F^a}{105d(dx+c)^3}-\frac{8b^3\ln(F)^3 F^{b(dx+c)^2} F^a}{105d(dx+c)}+\frac{8b^4\ln(F)^4\sqrt{\pi}\, F^a \operatorname{erf}\left(\sqrt{-}\right.}{105d\sqrt{-b\ln}}$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7/d/(d*x+c)^7*F^(b*(d*x+c)^2)*F^a-2/35/d*b*ln(F)/(d*x+c)^5*F^(b*(d*x+c)
^2)*F^a-4/105/d*b^2*ln(F)^2/(d*x+c)^3*F^(b*(d*x+c)^2)*F^a-8/105/d*b^3*ln(F
)^3/(d*x+c)*F^(b*(d*x+c)^2)*F^a+8/105/d*b^4*ln(F)^4*Pi^(1/2)*F^a/(-b*ln(F)
)^(1/2)*erf((-b*ln(F))^(1/2)*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. 2(150) = 300.

Time = 0.09 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.52

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} dx = -\frac{8\sqrt{\pi}(b^3d^7x^7 + 7b^3cd^6x^6 + 21b^3c^2d^5x^5 + 35b^3c^3d^4x^4 + 35b^3c^4d^3x^3 + 21b^3c^5d^2x^2 + 7b^3c^6dx + b^3c^7)\sqrt{}}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^8,x, algorithm="fricas")
```

output
```
-1/105*(8*sqrt(pi)*(b^3*d^7*x^7 + 7*b^3*c*d^6*x^6 + 21*b^3*c^2*d^5*x^5 + 3
5*b^3*c^3*d^4*x^4 + 35*b^3*c^4*d^3*x^3 + 21*b^3*c^5*d^2*x^2 + 7*b^3*c^6*d*
x + b^3*c^7)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d)*
log(F)^3 + (8*(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3
*c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d)*log(F)^3
+ 4*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x +
 b^2*c^4*d)*log(F)^2 + 6*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*log(F) + 15*d
)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d
^7*x^5 + 35*c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x +
c^7*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8} dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**8,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**8, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^8}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^8,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^8, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^8}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^8,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8}\,dx = \frac{8\,F^a\,\sqrt{\pi}\,\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)^{7/2}}{105\,d\,\left(c+d\,x\right)^7} - \frac{F^a\,F^{b\,\left(c+d\,x\right)^2}}{7\,d\,\left(c+d\,x\right)^7} - \frac{4\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^2\,\ln\left(F\right)^2}{105\,d\,\left(c+d\,x\right)^3} - \frac{8\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^3\,\ln\left(F\right)^3}{105\,d\,\left(c+d\,x\right)} - \frac{2\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b\,\ln\left(F\right)}{35\,d\,\left(c+d\,x\right)^5} - \frac{8\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2}\right)\,\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)^{7/2}}{105\,d\,\left(c+d\,x\right)^7}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^8,x)
```

output
```
(8*F^a*pi^(1/2)*(-b*log(F)*(c + d*x)^2)^(7/2))/(105*d*(c + d*x)^7) - (F^a*
F^(b*(c + d*x)^2))/(7*d*(c + d*x)^7) - (4*F^a*F^(b*(c + d*x)^2)*b^2*log(F)
^2)/(105*d*(c + d*x)^3) - (8*F^a*F^(b*(c + d*x)^2)*b^3*log(F)^3)/(105*d*(c
 + d*x)) - (2*F^a*F^(b*(c + d*x)^2)*b*log(F))/(35*d*(c + d*x)^5) - (8*F^a*
pi^(1/2)*erfc((-b*log(F)*(c + d*x)^2)^(1/2))*(-b*log(F)*(c + d*x)^2)^(7/2)
)/(105*d*(c + d*x)^7)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^8}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^8,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 8*int(f**(2*b*c*d*x +
b*d**2*x**2)/(2*log(f)*b*c**10 + 16*log(f)*b*c**9*d*x + 56*log(f)*b*c**8*d
**2*x**2 + 112*log(f)*b*c**7*d**3*x**3 + 140*log(f)*b*c**6*d**4*x**4 + 112
*log(f)*b*c**5*d**5*x**5 + 56*log(f)*b*c**4*d**6*x**6 + 16*log(f)*b*c**3*d
**7*x**7 + 2*log(f)*b*c**2*d**8*x**8 - 7*c**8 - 56*c**7*d*x - 196*c**6*d**
2*x**2 - 392*c**5*d**3*x**3 - 490*c**4*d**4*x**4 - 392*c**3*d**5*x**5 - 19
6*c**2*d**6*x**6 - 56*c*d**7*x**7 - 7*d**8*x**8),x)*log(f)**2*b**2*c**11*d
 + 56*int(f**(2*b*c*d*x + b*d**2*x**2)/(2*log(f)*b*c**10 + 16*log(f)*b*c**
9*d*x + 56*log(f)*b*c**8*d**2*x**2 + 112*log(f)*b*c**7*d**3*x**3 + 140*log
(f)*b*c**6*d**4*x**4 + 112*log(f)*b*c**5*d**5*x**5 + 56*log(f)*b*c**4*d**6
*x**6 + 16*log(f)*b*c**3*d**7*x**7 + 2*log(f)*b*c**2*d**8*x**8 - 7*c**8 -
56*c**7*d*x - 196*c**6*d**2*x**2 - 392*c**5*d**3*x**3 - 490*c**4*d**4*x**4
 - 392*c**3*d**5*x**5 - 196*c**2*d**6*x**6 - 56*c*d**7*x**7 - 7*d**8*x**8)
,x)*log(f)**2*b**2*c**10*d**2*x + 168*int(f**(2*b*c*d*x + b*d**2*x**2)/(2*
log(f)*b*c**10 + 16*log(f)*b*c**9*d*x + 56*log(f)*b*c**8*d**2*x**2 + 112*l
og(f)*b*c**7*d**3*x**3 + 140*log(f)*b*c**6*d**4*x**4 + 112*log(f)*b*c**5*d
**5*x**5 + 56*log(f)*b*c**4*d**6*x**6 + 16*log(f)*b*c**3*d**7*x**7 + 2*log
(f)*b*c**2*d**8*x**8 - 7*c**8 - 56*c**7*d*x - 196*c**6*d**2*x**2 - 392*c**
5*d**3*x**3 - 490*c**4*d**4*x**4 - 392*c**3*d**5*x**5 - 196*c**2*d**6*x**6
 - 56*c*d**7*x**7 - 7*d**8*x**8),x)*log(f)**2*b**2*c**9*d**3*x**2 + 280...
```

3.212 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} dx = -\frac{F^a\Gamma\left(-\frac{9}{2},-b(c+dx)^2\log(F)\right)\left(-b(c+dx)^2\log(F)\right)^{9/2}}{2d(c+dx)^9}$$

output
```
-1/2*F^a*(-32/945*Pi^(1/2)*erfc((-b*(d*x+c)^2*ln(F))^(1/2))+32/945/(-b*(d*
x+c)^2*ln(F))^(1/2)*exp(b*(d*x+c)^2*ln(F))-16/945/(-b*(d*x+c)^2*ln(F))^(3/
2)*exp(b*(d*x+c)^2*ln(F))+8/315/(-b*(d*x+c)^2*ln(F))^(5/2)*exp(b*(d*x+c)^2
*ln(F))-4/63/(-b*(d*x+c)^2*ln(F))^(7/2)*exp(b*(d*x+c)^2*ln(F))+2/9/(-b*(d*
x+c)^2*ln(F))^(9/2)*exp(b*(d*x+c)^2*ln(F)))*(-b*(d*x+c)^2*ln(F))^(9/2)/d/(
d*x+c)^9
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} dx = -\frac{F^a\Gamma\left(-\frac{9}{2},-b(c+dx)^2\log(F)\right)\left(-b(c+dx)^2\log(F)\right)^{9/2}}{2d(c+dx)^9}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^10,x]
```

output
```
-1/2*(F^a*Gamma[-9/2, -(b*(c + d*x)^2*Log[F])]*(-(b*(c + d*x)^2*Log[F]))^(
9/2))/(d*(c + d*x)^9)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{9/2} \Gamma\left(-\frac{9}{2}, -b(c+dx)^2 \log(F)\right)}{2d(c+dx)^9}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^10,x]
```

output
```
-1/2*(F^a*Gamma[-9/2, -(b*(c + d*x)^2*Log[F])]*(-(b*(c + d*x)^2*Log[F]))^(
9/2))/(d*(c + d*x)^9)
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.98

method	result
risch	$-\frac{F^{b(dx+c)^2} F^a}{9d(dx+c)^9}-\frac{2b\ln(F) F^{b(dx+c)^2} F^a}{63d(dx+c)^7}-\frac{4b^2\ln(F)^2 F^{b(dx+c)^2} F^a}{315d(dx+c)^5}-\frac{8b^3\ln(F)^3 F^{b(dx+c)^2} F^a}{945d(dx+c)^3}-\frac{16b^4\ln(F)^4 F^{b(dx+c)^2} F^a}{945d(dx+c)}$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9/d/(d*x+c)^9*F^(b*(d*x+c)^2)*F^a-2/63/d*b*ln(F)/(d*x+c)^7*F^(b*(d*x+c)
^2)*F^a-4/315/d*b^2*ln(F)^2/(d*x+c)^5*F^(b*(d*x+c)^2)*F^a-8/945/d*b^3*ln(F
)^3/(d*x+c)^3*F^(b*(d*x+c)^2)*F^a-16/945/d*b^4*ln(F)^4/(d*x+c)*F^(b*(d*x+c
)^2)*F^a+16/945/d*b^5*ln(F)^5*Pi^(1/2)*F^a/(-b*ln(F))^(1/2)*erf((-b*ln(F))
^(1/2)*(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(185) = 370$.

Time = 0.09 (sec) , antiderivative size = 598, normalized size of antiderivative = 12.20

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx =$$

$$-\frac{16\sqrt{\pi}(b^4d^9x^9+9\,b^4cd^8x^8+36\,b^4c^2d^7x^7+84\,b^4c^3d^6x^6+126\,b^4c^4d^5x^5+126\,b^4c^5d^4x^4+84\,b^4c^6d^3x^3+$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^10,x, algorithm="fricas")
```

output
```
-1/945*(16*sqrt(pi)*(b^4*d^9*x^9 + 9*b^4*c*d^8*x^8 + 36*b^4*c^2*d^7*x^7 +
84*b^4*c^3*d^6*x^6 + 126*b^4*c^4*d^5*x^5 + 126*b^4*c^5*d^4*x^4 + 84*b^4*c^
6*d^3*x^3 + 36*b^4*c^7*d^2*x^2 + 9*b^4*c^8*d*x + b^4*c^9)*sqrt(-b*d^2*log(
F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d)*log(F)^4 + (16*(b^4*d^9*x^8 +
 8*b^4*c*d^8*x^7 + 28*b^4*c^2*d^7*x^6 + 56*b^4*c^3*d^6*x^5 + 70*b^4*c^4*d^
5*x^4 + 56*b^4*c^5*d^4*x^3 + 28*b^4*c^6*d^3*x^2 + 8*b^4*c^7*d^2*x + b^4*c^
8*d)*log(F)^4 + 8*(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20
*b^3*c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d)*log(F
)^3 + 12*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^
2*x + b^2*c^4*d)*log(F)^2 + 30*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*log(F)
+ 105*d)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(d^11*x^9 + 9*c*d^10*x^8 +
 36*c^2*d^9*x^7 + 84*c^3*d^8*x^6 + 126*c^4*d^7*x^5 + 126*c^5*d^6*x^4 + 84*
c^6*d^5*x^3 + 36*c^7*d^4*x^2 + 9*c^8*d^3*x + c^9*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**10,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**10, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^10,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^10, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^10,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.78

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx = \frac{16\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,\ln(F)\,(c+d\,x)^2}\right)\,\left(-b\,\ln(F)\,(c+d\,x)^2\right)^{9/2}}{945\,d\,(c+d\,x)^9} - \frac{16\,F^a\,\sqrt{\pi}\,\left(-b\,\ln(F)\,(c+d\,x)^2\right)^{9/2}}{945\,d\,(c+d\,x)^9} - \frac{4\,F^a\,F^{b\,(c+d\,x)^2}\,b^2\ln(F)^2}{315\,d\,(c+d\,x)^5} - \frac{8\,F^a\,F^{b\,(c+d\,x)^2}\,b^3\ln(F)^3}{945\,d\,(c+d\,x)^3} - \frac{16\,F^a\,F^{b\,(c+d\,x)^2}\,b^4\ln(F)^4}{945\,d\,(c+d\,x)} - \frac{2\,F^a\,F^{b\,(c+d\,x)^2}\,b\,\ln(F)}{63\,d\,(c+d\,x)^7} - \frac{F^a\,F^{b\,(c+d\,x)^2}}{9\,d\,(c+d\,x)^9}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^10,x)
```

output
```
(16*F^a*pi^(1/2)*erfc((-b*log(F)*(c + d*x)^2)^(1/2))*(-b*log(F)*(c + d*x)^
2)^(9/2))/(945*d*(c + d*x)^9) - (16*F^a*pi^(1/2)*(-b*log(F)*(c + d*x)^2)^(
9/2))/(945*d*(c + d*x)^9) - (4*F^a*F^(b*(c + d*x)^2)*b^2*log(F)^2)/(315*d*
(c + d*x)^5) - (8*F^a*F^(b*(c + d*x)^2)*b^3*log(F)^3)/(945*d*(c + d*x)^3)
- (16*F^a*F^(b*(c + d*x)^2)*b^4*log(F)^4)/(945*d*(c + d*x)) - (2*F^a*F^(b*
(c + d*x)^2)*b*log(F))/(63*d*(c + d*x)^7) - (F^a*F^(b*(c + d*x)^2))/(9*d*(
c + d*x)^9)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{10}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^10,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 8*int(f**(2*b*c*d*x +
b*d**2*x**2)/(2*log(f)*b*c**12 + 20*log(f)*b*c**11*d*x + 90*log(f)*b*c**10
*d**2*x**2 + 240*log(f)*b*c**9*d**3*x**3 + 420*log(f)*b*c**8*d**4*x**4 + 5
04*log(f)*b*c**7*d**5*x**5 + 420*log(f)*b*c**6*d**6*x**6 + 240*log(f)*b*c*
*5*d**7*x**7 + 90*log(f)*b*c**4*d**8*x**8 + 20*log(f)*b*c**3*d**9*x**9 + 2
*log(f)*b*c**2*d**10*x**10 - 9*c**10 - 90*c**9*d*x - 405*c**8*d**2*x**2 -
1080*c**7*d**3*x**3 - 1890*c**6*d**4*x**4 - 2268*c**5*d**5*x**5 - 1890*c**
4*d**6*x**6 - 1080*c**3*d**7*x**7 - 405*c**2*d**8*x**8 - 90*c*d**9*x**9 -
9*d**10*x**10),x)*log(f)**2*b**2*c**13*d + 72*int(f**(2*b*c*d*x + b*d**2*x
**2)/(2*log(f)*b*c**12 + 20*log(f)*b*c**11*d*x + 90*log(f)*b*c**10*d**2*x*
*2 + 240*log(f)*b*c**9*d**3*x**3 + 420*log(f)*b*c**8*d**4*x**4 + 504*log(f
)*b*c**7*d**5*x**5 + 420*log(f)*b*c**6*d**6*x**6 + 240*log(f)*b*c**5*d**7*
x**7 + 90*log(f)*b*c**4*d**8*x**8 + 20*log(f)*b*c**3*d**9*x**9 + 2*log(f)*
b*c**2*d**10*x**10 - 9*c**10 - 90*c**9*d*x - 405*c**8*d**2*x**2 - 1080*c**
7*d**3*x**3 - 1890*c**6*d**4*x**4 - 2268*c**5*d**5*x**5 - 1890*c**4*d**6*x
**6 - 1080*c**3*d**7*x**7 - 405*c**2*d**8*x**8 - 90*c*d**9*x**9 - 9*d**10*
x**10),x)*log(f)**2*b**2*c**12*d**2*x + 288*int(f**(2*b*c*d*x + b*d**2*x**
2)/(2*log(f)*b*c**12 + 20*log(f)*b*c**11*d*x + 90*log(f)*b*c**10*d**2*x**2
 + 240*log(f)*b*c**9*d**3*x**3 + 420*log(f)*b*c**8*d**4*x**4 + 504*log(f)*
b*c**7*d**5*x**5 + 420*log(f)*b*c**6*d**6*x**6 + 240*log(f)*b*c**5*d**7...
```

3.213 $\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} dx = -\frac{F^a\Gamma\left(-\frac{11}{2}, -b(c+dx)^2\log(F)\right)\left(-b(c+dx)^2\log(F)\right)^{11/2}}{2d(c+dx)^{11}}$$

output
```
-1/2*F^a*(64/10395*Pi^(1/2)*erfc((-b*(d*x+c)^2*ln(F))^(1/2))-64/10395/(-b*
(d*x+c)^2*ln(F))^(1/2)*exp(b*(d*x+c)^2*ln(F))+32/10395/(-b*(d*x+c)^2*ln(F)
)^(3/2)*exp(b*(d*x+c)^2*ln(F))-16/3465/(-b*(d*x+c)^2*ln(F))^(5/2)*exp(b*(d
*x+c)^2*ln(F))+8/693/(-b*(d*x+c)^2*ln(F))^(7/2)*exp(b*(d*x+c)^2*ln(F))-4/9
9/(-b*(d*x+c)^2*ln(F))^(9/2)*exp(b*(d*x+c)^2*ln(F))+2/11/(-b*(d*x+c)^2*ln(
F))^(11/2)*exp(b*(d*x+c)^2*ln(F)))*(-b*(d*x+c)^2*ln(F))^(11/2)/d/(d*x+c)^1
1
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} dx = -\frac{F^a\Gamma\left(-\frac{11}{2}, -b(c+dx)^2\log(F)\right)\left(-b(c+dx)^2\log(F)\right)^{11/2}}{2d(c+dx)^{11}}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(c + d*x)^12,x]
```

output
```
-1/2*(F^a*Gamma[-11/2, -(b*(c + d*x)^2*Log[F])]*(-(b*(c + d*x)^2*Log[F]))^
(11/2))/(d*(c + d*x)^11)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a \left(-b \log(F)(c+dx)^2\right)^{11/2} \Gamma\left(-\frac{11}{2}, -b(c+dx)^2 \log(F)\right)}{2d(c+dx)^{11}}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(c + d*x)^12,x]
```

output
```
-1/2*(F^a*Gamma[-11/2, -(b*(c + d*x)^2*Log[F])]*(-(b*(c + d*x)^2*Log[F]))^
(11/2))/(d*(c + d*x)^11)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.65

method	result
risch	$-\frac{F^{b(dx+c)^2} F^a}{11d(dx+c)^{11}} - \frac{2b\ln(F) F^{b(dx+c)^2} F^a}{99d(dx+c)^9} - \frac{4b^2\ln(F)^2 F^{b(dx+c)^2} F^a}{693d(dx+c)^7} - \frac{8b^3\ln(F)^3 F^{b(dx+c)^2} F^a}{3465d(dx+c)^5} - \frac{16b^4\ln(F)^4 F^{b(dx+c)^2} F^a}{10395d(dx+c)^3}$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11/d/(d*x+c)^11*F^(b*(d*x+c)^2)*F^a-2/99/d*b*ln(F)/(d*x+c)^9*F^(b*(d*x+
c)^2)*F^a-4/693/d*b^2*ln(F)^2/(d*x+c)^7*F^(b*(d*x+c)^2)*F^a-8/3465/d*b^3*l
n(F)^3/(d*x+c)^5*F^(b*(d*x+c)^2)*F^a-16/10395/d*b^4*ln(F)^4/(d*x+c)^3*F^(b
*(d*x+c)^2)*F^a-32/10395/d*b^5*ln(F)^5/(d*x+c)*F^(b*(d*x+c)^2)*F^a+32/1039
5/d*b^6*ln(F)^6*Pi^(1/2)*F^a/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)*(d*x+c)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(212) = 424$.

Time = 0.09 (sec) , antiderivative size = 795, normalized size of antiderivative = 16.22

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^12,x, algorithm="fricas")
```

output
```
-1/10395*(32*sqrt(pi)*(b^5*d^11*x^11 + 11*b^5*c*d^10*x^10 + 55*b^5*c^2*d^9
*x^9 + 165*b^5*c^3*d^8*x^8 + 330*b^5*c^4*d^7*x^7 + 462*b^5*c^5*d^6*x^6 + 4
62*b^5*c^6*d^5*x^5 + 330*b^5*c^7*d^4*x^4 + 165*b^5*c^8*d^3*x^3 + 55*b^5*c^
9*d^2*x^2 + 11*b^5*c^10*d*x + b^5*c^11)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-
b*d^2*log(F))*(d*x + c)/d)*log(F)^5 + (32*(b^5*d^11*x^10 + 10*b^5*c*d^10*x
^9 + 45*b^5*c^2*d^9*x^8 + 120*b^5*c^3*d^8*x^7 + 210*b^5*c^4*d^7*x^6 + 252*
b^5*c^5*d^6*x^5 + 210*b^5*c^6*d^5*x^4 + 120*b^5*c^7*d^4*x^3 + 45*b^5*c^8*d
^3*x^2 + 10*b^5*c^9*d^2*x + b^5*c^10*d)*log(F)^5 + 16*(b^4*d^9*x^8 + 8*b^4
*c*d^8*x^7 + 28*b^4*c^2*d^7*x^6 + 56*b^4*c^3*d^6*x^5 + 70*b^4*c^4*d^5*x^4
+ 56*b^4*c^5*d^4*x^3 + 28*b^4*c^6*d^3*x^2 + 8*b^4*c^7*d^2*x + b^4*c^8*d)*l
og(F)^4 + 24*(b^3*d^7*x^6 + 6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3*
c^3*d^4*x^3 + 15*b^3*c^4*d^3*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d)*log(F)^3 +
 60*(b^2*d^5*x^4 + 4*b^2*c*d^4*x^3 + 6*b^2*c^2*d^3*x^2 + 4*b^2*c^3*d^2*x +
 b^2*c^4*d)*log(F)^2 + 210*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*log(F) + 94
5*d)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(d^13*x^11 + 11*c*d^12*x^10 +
55*c^2*d^11*x^9 + 165*c^3*d^10*x^8 + 330*c^4*d^9*x^7 + 462*c^5*d^8*x^6 + 4
62*c^6*d^7*x^5 + 330*c^7*d^6*x^4 + 165*c^8*d^5*x^3 + 55*c^9*d^4*x^2 + 11*c
^10*d^3*x + c^11*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(d*x+c)**12,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(c + d*x)**12, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx=\int \frac{F^{(dx+c)^2b+a}}{(dx+c)^{12}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^12,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^12, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx=\int \frac{F^{(dx+c)^2b+a}}{(dx+c)^{12}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(d*x+c)^12,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(d*x + c)^12, x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 267, normalized size of antiderivative = 5.45

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx=\frac{32\,F^a\,\sqrt{\pi}\,\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)^{11/2}}{10395\,d\,\left(c+d\,x\right)^{11}}-\frac{F^a\,F^{b\,\left(c+d\,x\right)^2}}{11\,d\,\left(c+d\,x\right)^{11}}-\frac{4\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^2\,\ln\left(F\right)^2}{693\,d\,\left(c+d\,x\right)^7}-\frac{8\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^3\,\ln\left(F\right)^3}{3465\,d\,\left(c+d\,x\right)^5}-\frac{16\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^4\,\ln\left(F\right)^4}{10395\,d\,\left(c+d\,x\right)^3}-\frac{32\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b^5\,\ln\left(F\right)^5}{10395\,d\,\left(c+d\,x\right)}-\frac{2\,F^a\,F^{b\,\left(c+d\,x\right)^2}\,b\,\ln\left(F\right)}{99\,d\,\left(c+d\,x\right)^9}-\frac{32\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2}\right)\,\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^2\right)^{11/2}}{10395\,d\,\left(c+d\,x\right)^{11}}$$

input
```
int(F^(a + b*(c + d*x)^2)/(c + d*x)^12,x)
```

output
```
(32*F^a*pi^(1/2)*(-b*log(F)*(c + d*x)^2)^(11/2))/(10395*d*(c + d*x)^11) -
(F^a*F^(b*(c + d*x)^2))/(11*d*(c + d*x)^11) - (4*F^a*F^(b*(c + d*x)^2)*b^2
*log(F)^2)/(693*d*(c + d*x)^7) - (8*F^a*F^(b*(c + d*x)^2)*b^3*log(F)^3)/(3
465*d*(c + d*x)^5) - (16*F^a*F^(b*(c + d*x)^2)*b^4*log(F)^4)/(10395*d*(c +
 d*x)^3) - (32*F^a*F^(b*(c + d*x)^2)*b^5*log(F)^5)/(10395*d*(c + d*x)) - (
2*F^a*F^(b*(c + d*x)^2)*b*log(F))/(99*d*(c + d*x)^9) - (32*F^a*pi^(1/2)*er
fc((-b*log(F)*(c + d*x)^2)^(1/2))*(-b*log(F)*(c + d*x)^2)^(11/2))/(10395*d
*(c + d*x)^11)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^2}}{(c+dx)^{12}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(d*x+c)^12,x)
```

output

```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 8*int(f**(2*b*c*d*x +
b*d**2*x**2)/(2*log(f)*b*c**14 + 24*log(f)*b*c**13*d*x + 132*log(f)*b*c**1
2*d**2*x**2 + 440*log(f)*b*c**11*d**3*x**3 + 990*log(f)*b*c**10*d**4*x**4
+ 1584*log(f)*b*c**9*d**5*x**5 + 1848*log(f)*b*c**8*d**6*x**6 + 1584*log(f
)*b*c**7*d**7*x**7 + 990*log(f)*b*c**6*d**8*x**8 + 440*log(f)*b*c**5*d**9*
x**9 + 132*log(f)*b*c**4*d**10*x**10 + 24*log(f)*b*c**3*d**11*x**11 + 2*lo
g(f)*b*c**2*d**12*x**12 - 11*c**12 - 132*c**11*d*x - 726*c**10*d**2*x**2 -
 2420*c**9*d**3*x**3 - 5445*c**8*d**4*x**4 - 8712*c**7*d**5*x**5 - 10164*c
**6*d**6*x**6 - 8712*c**5*d**7*x**7 - 5445*c**4*d**8*x**8 - 2420*c**3*d**9
*x**9 - 726*c**2*d**10*x**10 - 132*c*d**11*x**11 - 11*d**12*x**12),x)*log(
f)**2*b**2*c**15*d + 88*int(f**(2*b*c*d*x + b*d**2*x**2)/(2*log(f)*b*c**14
 + 24*log(f)*b*c**13*d*x + 132*log(f)*b*c**12*d**2*x**2 + 440*log(f)*b*c**
11*d**3*x**3 + 990*log(f)*b*c**10*d**4*x**4 + 1584*log(f)*b*c**9*d**5*x**5
 + 1848*log(f)*b*c**8*d**6*x**6 + 1584*log(f)*b*c**7*d**7*x**7 + 990*log(f
)*b*c**6*d**8*x**8 + 440*log(f)*b*c**5*d**9*x**9 + 132*log(f)*b*c**4*d**10
*x**10 + 24*log(f)*b*c**3*d**11*x**11 + 2*log(f)*b*c**2*d**12*x**12 - 11*c
**12 - 132*c**11*d*x - 726*c**10*d**2*x**2 - 2420*c**9*d**3*x**3 - 5445*c*
*8*d**4*x**4 - 8712*c**7*d**5*x**5 - 10164*c**6*d**6*x**6 - 8712*c**5*d**7
*x**7 - 5445*c**4*d**8*x**8 - 2420*c**3*d**9*x**9 - 726*c**2*d**10*x**10 -
 132*c*d**11*x**11 - 11*d**12*x**12),x)*log(f)**2*b**2*c**14*d**2*x + 4...
```

3.214 $\int F^{a+b(c+dx)^3}(c+dx)^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{3},-b(c+dx)^3\log(F)\right)\left(-b(c+dx)^3\log(F)\right)^{\frac{1}{3}(-1-m)}}{3d}$$

output
```
-1/3*F^a*(d*x+c)^(1+m)*GAMMA(1/3+1/3*m,-b*(d*x+c)^3*ln(F))*(-b*(d*x+c)^3*l
n(F))^(-1/3-1/3*m)/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{3},-b(c+dx)^3\log(F)\right)\left(-b(c+dx)^3\log(F)\right)^{\frac{1}{3}(-1-m)}}{3d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^m,x]
```

output
```
-1/3*(F^a*(c + d*x)^(1 + m)*Gamma[(1 + m)/3, -(b*(c + d*x)^3*Log[F])]*(-(b
*(c + d*x)^3*Log[F]))^((-1 - m)/3))/d
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m F^{a+b(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a(c+dx)^{m+1} \left(-b\log(F)(c+dx)^3\right)^{\frac{1}{3}(-m-1)} \Gamma\left(\frac{m+1}{3}, -b(c+dx)^3 \log(F)\right)}{3d}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^m,x]
```

output
```
-1/3*(F^a*(c + d*x)^(1 + m)*Gamma[(1 + m)/3, -(b*(c + d*x)^3*Log[F])]*(-(b
*(c + d*x)^3*Log[F]))^((-1 - m)/3))/d
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^3}(dx+c)^m\,dx$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x)
```

output
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = \frac{e^{\left(-\frac{1}{3}(m-2)\log(-b\log(F))+a\log(F)\right)}\Gamma\left(\frac{1}{3}m+\frac{1}{3},-(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3)\log(F)\right)}{3\,bd\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x, algorithm="fricas")
```

output
```
1/3*e^(-1/3*(m - 2)*log(-b*log(F)) + a*log(F))*gamma(1/3*m + 1/3, -(b*d^3*
x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))/(b*d*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = \int F^{a+b(c+dx)^3}(c+dx)^m\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**m,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)*(c + d*x)**m, x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = \int (dx+c)^m F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*F^((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = \int (dx+c)^m F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*F^((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = \frac{F^a\,\mathrm{e}^{\frac{b\,\ln(F)\,(c+d\,x)^3}{2}}\,(c+d\,x)^{m+1}\,\mathrm{M}_{\frac{1}{3}-\frac{m}{6},\frac{m}{6}+\frac{1}{6}}\left(b\,\ln(F)\,\left(c+d\,x\right)^3\right)}{d\,(m+1)\,\left(b\,\ln(F)\,\left(c+d\,x\right)^3\right)^{\frac{m}{6}+\frac{2}{3}}}$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^m,x)
```

output
```
(F^a*exp((b*log(F)*(c + d*x)^3)/2)*(c + d*x)^(m + 1)*whittakerM(1/3 - m/6,
 m/6 + 1/6, b*log(F)*(c + d*x)^3))/(d*(m + 1)*(b*log(F)*(c + d*x)^3)^(m/6
+ 2/3))
```

Reduce [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^m\,dx = f^{b\,c^3+a}\left(\int f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}(dx+c)^m\,dx\right)$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^m,x)
```

output
```
f**(a + b*c**3)*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*(c +
 d*x)**m,x)
```

3.215 $\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 105

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = -\frac{F^{a+b(c+dx)^3}\left(120-120b(c+dx)^3\log(F)+60b^2(c+dx)^6\log^2(F)-20b^3(c+dx)^9\log^3(F)+5b^4(c+d\right.}{3b^6d\log^6(F)}$$

output
```
-1/3*F^(a+b*(d*x+c)^3)*(120-120*b*(d*x+c)^3*ln(F)+60*b^2*(d*x+c)^6*ln(F)^2
-20*b^3*(d*x+c)^9*ln(F)^3+5*b^4*(d*x+c)^12*ln(F)^4-b^5*(d*x+c)^15*ln(F)^5)
/b^6/d/ln(F)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = -\frac{F^a\Gamma(6,-b(c+dx)^3\log(F))}{3b^6d\log^6(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^17,x]
```

output
```
-1/3*(F^a*Gamma[6, -(b*(c + d*x)^3*Log[F])])/(b^6*d*Log[F]^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{17} F^{a+b(c+dx)^3}\,dx$$

$$\Big\downarrow \; 2647$$

$$-\frac{F^{a+b(c+dx)^3}\left(-b^5\log^5(F)(c+dx)^{15}+5b^4\log^4(F)(c+dx)^{12}-20b^3\log^3(F)(c+dx)^9+60b^2\log^2(F)(c+dx)^6 -\right.}{3b^6d\log^6(F)}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^17,x]
```

output
```
-1/3*(F^(a + b*(c + d*x)^3)*(120 - 120*b*(c + d*x)^3*Log[F] + 60*b^2*(c +
d*x)^6*Log[F]^2 - 20*b^3*(c + d*x)^9*Log[F]^3 + 5*b^4*(c + d*x)^12*Log[F]^
4 - b^5*(c + d*x)^15*Log[F]^5))/(b^6*d*Log[F]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. 2(103) = 206.

Time = 5.60 (sec) , antiderivative size = 835, normalized size of antiderivative = 7.95

method	result
orering	$\frac{\left(-120+20\ln(F)^3b^3c^9-60\ln(F)^2b^2c^6+120\ln(F)b\,d^3x^3+120\ln(F)b\,c^3+d^{15}x^{15}\ln(F)^5b^5-5d^{12}x^{12}\ln(F)^4b^4+20d^9x^9\ln(F)^3\right.}{}$
gosper	$\frac{\left(-120+20\ln(F)^3b^3c^9-60\ln(F)^2b^2c^6+120\ln(F)b\,d^3x^3+120\ln(F)b\,c^3+d^{15}x^{15}\ln(F)^5b^5-5d^{12}x^{12}\ln(F)^4b^4+20d^9x^9\ln(F)^3\right.}{}$
risch	$\frac{\left(-120+20\ln(F)^3b^3c^9-60\ln(F)^2b^2c^6+120\ln(F)b\,d^3x^3+120\ln(F)b\,c^3+d^{15}x^{15}\ln(F)^5b^5-5d^{12}x^{12}\ln(F)^4b^4+20d^9x^9\ln(F)^3\right.}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^17,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-120+20*ln(F)^3*b^3*c^9-60*ln(F)^2*b^2*c^6+120*ln(F)*b*d^3*x^3+120*ln
(F)*b*c^3+d^15*x^15*ln(F)^5*b^5-5*d^12*x^12*ln(F)^4*b^4+20*d^9*x^9*ln(F)^3
*b^3-60*d^6*x^6*ln(F)^2*b^2+360*ln(F)*b*c*d^2*x^2+360*ln(F)*b*c^2*d*x+15*d
^14*c*x^14*ln(F)^5*b^5+105*d^13*c^2*x^13*ln(F)^5*b^5+455*ln(F)^5*b^5*c^3*d
^12*x^12+1365*ln(F)^5*b^5*c^4*d^11*x^11+3003*ln(F)^5*b^5*c^5*d^10*x^10+500
5*ln(F)^5*b^5*c^6*d^9*x^9+6435*ln(F)^5*b^5*c^7*d^8*x^8+6435*ln(F)^5*b^5*c^
8*d^7*x^7+5005*ln(F)^5*b^5*c^9*d^6*x^6-60*c*d^11*x^11*ln(F)^4*b^4+3003*ln(
F)^5*b^5*c^10*d^5*x^5-330*c^2*d^10*x^10*ln(F)^4*b^4+1365*ln(F)^5*b^5*c^11*
d^4*x^4-1100*ln(F)^4*b^4*c^3*d^9*x^9+455*ln(F)^5*b^5*c^12*d^3*x^3-2475*ln(
F)^4*b^4*c^4*d^8*x^8+105*ln(F)^5*b^5*c^13*d^2*x^2+ln(F)^5*b^5*c^15-5*ln(F)
^4*b^4*c^12-3960*ln(F)^4*b^4*c^5*d^7*x^7+15*ln(F)^5*b^5*c^14*d*x-4620*ln(F
)^4*b^4*c^6*d^6*x^6-3960*ln(F)^4*b^4*c^7*d^5*x^5-2475*ln(F)^4*b^4*c^8*d^4*
x^4-1100*ln(F)^4*b^4*c^9*d^3*x^3+180*c*d^8*x^8*ln(F)^3*b^3-330*ln(F)^4*b^4
*c^10*d^2*x^2+720*c^2*d^7*x^7*ln(F)^3*b^3-60*ln(F)^4*b^4*c^11*d*x+1680*ln(
F)^3*b^3*c^3*d^6*x^6+2520*ln(F)^3*b^3*c^4*d^5*x^5+2520*ln(F)^3*b^3*c^5*d^4
*x^4+1680*ln(F)^3*b^3*c^6*d^3*x^3+720*ln(F)^3*b^3*c^7*d^2*x^2+180*ln(F)^3*
b^3*c^8*d*x-360*c*d^5*x^5*ln(F)^2*b^2-900*c^2*d^4*x^4*ln(F)^2*b^2-1200*ln(
F)^2*b^2*c^3*d^3*x^3-900*ln(F)^2*b^2*c^4*d^2*x^2-360*ln(F)^2*b^2*c^5*d*x)/
b^6/ln(F)^6/d*F^(a+b*(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(102) = 204.

Time = 0.09 (sec) , antiderivative size = 688, normalized size of antiderivative = 6.55

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx$$
$$=\frac{((b^5d^{15}x^{15}+15\,b^5cd^{14}x^{14}+105\,b^5c^2d^{13}x^{13}+455\,b^5c^3d^{12}x^{12}+1365\,b^5c^4d^{11}x^{11}+3003\,b^5c^5d^{10}x^{10}+5005\,b}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^17,x, algorithm="fricas")
```

output
```
1/3*((b^5*d^15*x^15 + 15*b^5*c*d^14*x^14 + 105*b^5*c^2*d^13*x^13 + 455*b^5
*c^3*d^12*x^12 + 1365*b^5*c^4*d^11*x^11 + 3003*b^5*c^5*d^10*x^10 + 5005*b^
5*c^6*d^9*x^9 + 6435*b^5*c^7*d^8*x^8 + 6435*b^5*c^8*d^7*x^7 + 5005*b^5*c^9
*d^6*x^6 + 3003*b^5*c^10*d^5*x^5 + 1365*b^5*c^11*d^4*x^4 + 455*b^5*c^12*d^
3*x^3 + 105*b^5*c^13*d^2*x^2 + 15*b^5*c^14*d*x + b^5*c^15)*log(F)^5 - 5*(b
^4*d^12*x^12 + 12*b^4*c*d^11*x^11 + 66*b^4*c^2*d^10*x^10 + 220*b^4*c^3*d^9
*x^9 + 495*b^4*c^4*d^8*x^8 + 792*b^4*c^5*d^7*x^7 + 924*b^4*c^6*d^6*x^6 + 7
92*b^4*c^7*d^5*x^5 + 495*b^4*c^8*d^4*x^4 + 220*b^4*c^9*d^3*x^3 + 66*b^4*c^
10*d^2*x^2 + 12*b^4*c^11*d*x + b^4*c^12)*log(F)^4 + 20*(b^3*d^9*x^9 + 9*b^
3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6*x^6 + 126*b^3*c^4*d^5*x^
5 + 126*b^3*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36*b^3*c^7*d^2*x^2 + 9*b^3*
c^8*d*x + b^3*c^9)*log(F)^3 - 60*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c
^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2
*c^6)*log(F)^2 + 120*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log
(F) - 120)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b^6*d*
log(F)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. 2(105) = 210.

Time = 0.32 (sec) , antiderivative size = 1170, normalized size of antiderivative = 11.14

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = \text{Too large to display}$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**17,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**3)*(b**5*c**15*log(F)**5 + 15*b**5*c**14*d
*x*log(F)**5 + 105*b**5*c**13*d**2*x**2*log(F)**5 + 455*b**5*c**12*d**3*x*
*3*log(F)**5 + 1365*b**5*c**11*d**4*x**4*log(F)**5 + 3003*b**5*c**10*d**5*
x**5*log(F)**5 + 5005*b**5*c**9*d**6*x**6*log(F)**5 + 6435*b**5*c**8*d**7*
x**7*log(F)**5 + 6435*b**5*c**7*d**8*x**8*log(F)**5 + 5005*b**5*c**6*d**9*
x**9*log(F)**5 + 3003*b**5*c**5*d**10*x**10*log(F)**5 + 1365*b**5*c**4*d**
11*x**11*log(F)**5 + 455*b**5*c**3*d**12*x**12*log(F)**5 + 105*b**5*c**2*d
**13*x**13*log(F)**5 + 15*b**5*c*d**14*x**14*log(F)**5 + b**5*d**15*x**15*
log(F)**5 - 5*b**4*c**12*log(F)**4 - 60*b**4*c**11*d*x*log(F)**4 - 330*b**
4*c**10*d**2*x**2*log(F)**4 - 1100*b**4*c**9*d**3*x**3*log(F)**4 - 2475*b*
*4*c**8*d**4*x**4*log(F)**4 - 3960*b**4*c**7*d**5*x**5*log(F)**4 - 4620*b*
*4*c**6*d**6*x**6*log(F)**4 - 3960*b**4*c**5*d**7*x**7*log(F)**4 - 2475*b*
*4*c**4*d**8*x**8*log(F)**4 - 1100*b**4*c**3*d**9*x**9*log(F)**4 - 330*b**
4*c**2*d**10*x**10*log(F)**4 - 60*b**4*c*d**11*x**11*log(F)**4 - 5*b**4*d*
*12*x**12*log(F)**4 + 20*b**3*c**9*log(F)**3 + 180*b**3*c**8*d*x*log(F)**3
 + 720*b**3*c**7*d**2*x**2*log(F)**3 + 1680*b**3*c**6*d**3*x**3*log(F)**3
+ 2520*b**3*c**5*d**4*x**4*log(F)**3 + 2520*b**3*c**4*d**5*x**5*log(F)**3
+ 1680*b**3*c**3*d**6*x**6*log(F)**3 + 720*b**3*c**2*d**7*x**7*log(F)**3 +
 180*b**3*c*d**8*x**8*log(F)**3 + 20*b**3*d**9*x**9*log(F)**3 - 60*b**2*c*
*6*log(F)**2 - 360*b**2*c**5*d*x*log(F)**2 - 900*b**2*c**4*d**2*x**2*lo...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1268 vs. $2(102) = 204$.

Time = 0.25 (sec) , antiderivative size = 1268, normalized size of antiderivative = 12.08

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^17,x, algorithm="maxima")
```

output
```
1/3*(F^(b*c^3 + a)*b^5*d^15*x^15*log(F)^5 + 15*F^(b*c^3 + a)*b^5*c*d^14*x^
14*log(F)^5 + 105*F^(b*c^3 + a)*b^5*c^2*d^13*x^13*log(F)^5 + F^(b*c^3 + a)
*b^5*c^15*log(F)^5 - 5*F^(b*c^3 + a)*b^4*c^12*log(F)^4 + 20*F^(b*c^3 + a)*
b^3*c^9*log(F)^3 + 5*(91*F^(b*c^3 + a)*b^5*c^3*d^12*log(F)^5 - F^(b*c^3 +
a)*b^4*d^12*log(F)^4)*x^12 + 15*(91*F^(b*c^3 + a)*b^5*c^4*d^11*log(F)^5 -
4*F^(b*c^3 + a)*b^4*c*d^11*log(F)^4)*x^11 + 33*(91*F^(b*c^3 + a)*b^5*c^5*d
^10*log(F)^5 - 10*F^(b*c^3 + a)*b^4*c^2*d^10*log(F)^4)*x^10 - 60*F^(b*c^3
+ a)*b^2*c^6*log(F)^2 + 5*(1001*F^(b*c^3 + a)*b^5*c^6*d^9*log(F)^5 - 220*F
^(b*c^3 + a)*b^4*c^3*d^9*log(F)^4 + 4*F^(b*c^3 + a)*b^3*d^9*log(F)^3)*x^9
+ 45*(143*F^(b*c^3 + a)*b^5*c^7*d^8*log(F)^5 - 55*F^(b*c^3 + a)*b^4*c^4*d^
8*log(F)^4 + 4*F^(b*c^3 + a)*b^3*c*d^8*log(F)^3)*x^8 + 45*(143*F^(b*c^3 +
a)*b^5*c^8*d^7*log(F)^5 - 88*F^(b*c^3 + a)*b^4*c^5*d^7*log(F)^4 + 16*F^(b*
c^3 + a)*b^3*c^2*d^7*log(F)^3)*x^7 + 5*(1001*F^(b*c^3 + a)*b^5*c^9*d^6*log
(F)^5 - 924*F^(b*c^3 + a)*b^4*c^6*d^6*log(F)^4 + 336*F^(b*c^3 + a)*b^3*c^3
*d^6*log(F)^3 - 12*F^(b*c^3 + a)*b^2*d^6*log(F)^2)*x^6 + 3*(1001*F^(b*c^3
+ a)*b^5*c^10*d^5*log(F)^5 - 1320*F^(b*c^3 + a)*b^4*c^7*d^5*log(F)^4 + 840
*F^(b*c^3 + a)*b^3*c^4*d^5*log(F)^3 - 120*F^(b*c^3 + a)*b^2*c*d^5*log(F)^2
)*x^5 + 120*F^(b*c^3 + a)*b*c^3*log(F) + 15*(91*F^(b*c^3 + a)*b^5*c^11*d^4
*log(F)^5 - 165*F^(b*c^3 + a)*b^4*c^8*d^4*log(F)^4 + 168*F^(b*c^3 + a)*b^3
*c^5*d^4*log(F)^3 - 60*F^(b*c^3 + a)*b^2*c^2*d^4*log(F)^2)*x^4 + 5*(91*...
```

Giac [F(-2)]

Exception generated.

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^17,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error
: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 685, normalized size of antiderivative = 6.52

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = \text{Too large to display}$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^17,x)
```

output
```
F^(b*d^3*x^3)*F^(3*b*c^2*d*x)*F^a*F^(b*c^3)*F^(3*b*c*d^2*x^2)*((120*b*c^3*
log(F) - 60*b^2*c^6*log(F)^2 + 20*b^3*c^9*log(F)^3 - 5*b^4*c^12*log(F)^4 +
 b^5*c^15*log(F)^5 - 120)/(3*b^6*d*log(F)^6) + (d^14*x^15)/(3*b*log(F)) +
(5*c*d^13*x^14)/(b*log(F)) + (5*d^2*x^3*(336*b^2*c^6*log(F)^2 - 240*b*c^3*
log(F) - 220*b^3*c^9*log(F)^3 + 91*b^4*c^12*log(F)^4 + 24))/(3*b^5*log(F)^
5) + (5*d^5*x^6*(336*b*c^3*log(F) - 924*b^2*c^6*log(F)^2 + 1001*b^3*c^9*lo
g(F)^3 - 12))/(3*b^4*log(F)^4) + (5*d^8*x^9*(1001*b^2*c^6*log(F)^2 - 220*b
*c^3*log(F) + 4))/(3*b^3*log(F)^3) + (5*d^11*x^12*(91*b*c^3*log(F) - 1))/(
3*b^2*log(F)^2) + (35*c^2*d^12*x^13)/(b*log(F)) + (5*c^2*x*(12*b^2*c^6*log
(F)^2 - 24*b*c^3*log(F) - 4*b^3*c^9*log(F)^3 + b^4*c^12*log(F)^4 + 24))/(b
^5*log(F)^5) + (5*c^2*d^3*x^4*(168*b*c^3*log(F) - 165*b^2*c^6*log(F)^2 + 9
1*b^3*c^9*log(F)^3 - 60))/(b^4*log(F)^4) + (15*c^2*d^6*x^7*(143*b^2*c^6*lo
g(F)^2 - 88*b*c^3*log(F) + 16))/(b^3*log(F)^3) + (11*c^2*d^9*x^10*(91*b*c^
3*log(F) - 10))/(b^2*log(F)^2) + (5*c*d*x^2*(48*b^2*c^6*log(F)^2 - 60*b*c^
3*log(F) - 22*b^3*c^9*log(F)^3 + 7*b^4*c^12*log(F)^4 + 24))/(b^5*log(F)^5)
 + (c*d^4*x^5*(840*b*c^3*log(F) - 1320*b^2*c^6*log(F)^2 + 1001*b^3*c^9*log
(F)^3 - 120))/(b^4*log(F)^4) + (15*c*d^7*x^8*(143*b^2*c^6*log(F)^2 - 55*b*
c^3*log(F) + 4))/(b^3*log(F)^3) + (5*c*d^10*x^11*(91*b*c^3*log(F) - 4))/(b
^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 856, normalized size of antiderivative = 8.15

$$\int F^{a+b(c+dx)^3}(c+dx)^{17}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^17,x)
```

output
```
(f**(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*(log(f)**5
*b**5*c**15 + 15*log(f)**5*b**5*c**14*d*x + 105*log(f)**5*b**5*c**13*d**2*
x**2 + 455*log(f)**5*b**5*c**12*d**3*x**3 + 1365*log(f)**5*b**5*c**11*d**4
*x**4 + 3003*log(f)**5*b**5*c**10*d**5*x**5 + 5005*log(f)**5*b**5*c**9*d**
6*x**6 + 6435*log(f)**5*b**5*c**8*d**7*x**7 + 6435*log(f)**5*b**5*c**7*d**
8*x**8 + 5005*log(f)**5*b**5*c**6*d**9*x**9 + 3003*log(f)**5*b**5*c**5*d**
10*x**10 + 1365*log(f)**5*b**5*c**4*d**11*x**11 + 455*log(f)**5*b**5*c**3*
d**12*x**12 + 105*log(f)**5*b**5*c**2*d**13*x**13 + 15*log(f)**5*b**5*c*d*
*14*x**14 + log(f)**5*b**5*d**15*x**15 - 5*log(f)**4*b**4*c**12 - 60*log(f
)**4*b**4*c**11*d*x - 330*log(f)**4*b**4*c**10*d**2*x**2 - 1100*log(f)**4*
b**4*c**9*d**3*x**3 - 2475*log(f)**4*b**4*c**8*d**4*x**4 - 3960*log(f)**4*
b**4*c**7*d**5*x**5 - 4620*log(f)**4*b**4*c**6*d**6*x**6 - 3960*log(f)**4*
b**4*c**5*d**7*x**7 - 2475*log(f)**4*b**4*c**4*d**8*x**8 - 1100*log(f)**4*
b**4*c**3*d**9*x**9 - 330*log(f)**4*b**4*c**2*d**10*x**10 - 60*log(f)**4*b
**4*c*d**11*x**11 - 5*log(f)**4*b**4*d**12*x**12 + 20*log(f)**3*b**3*c**9
+ 180*log(f)**3*b**3*c**8*d*x + 720*log(f)**3*b**3*c**7*d**2*x**2 + 1680*l
og(f)**3*b**3*c**6*d**3*x**3 + 2520*log(f)**3*b**3*c**5*d**4*x**4 + 2520*l
og(f)**3*b**3*c**4*d**5*x**5 + 1680*log(f)**3*b**3*c**3*d**6*x**6 + 720*lo
g(f)**3*b**3*c**2*d**7*x**7 + 180*log(f)**3*b**3*c*d**8*x**8 + 20*log(f)**
3*b**3*d**9*x**9 - 60*log(f)**2*b**2*c**6 - 360*log(f)**2*b**2*c**5*d*x...
```

3.216 $\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 88

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx$$
$$= \frac{F^{a+b(c+dx)^3}\left(24-24b(c+dx)^3\log(F)+12b^2(c+dx)^6\log^2(F)-4b^3(c+dx)^9\log^3(F)+b^4(c+dx)^{12}\log\right.}{3b^5d\log^5(F)}$$

output
```
1/3*F^(a+b*(d*x+c)^3)*(24-24*b*(d*x+c)^3*ln(F)+12*b^2*(d*x+c)^6*ln(F)^2-4*
b^3*(d*x+c)^9*ln(F)^3+b^4*(d*x+c)^12*ln(F)^4)/b^5/d/ln(F)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.35

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx=\frac{F^a\Gamma(5,-b(c+dx)^3\log(F))}{3b^5d\log^5(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^14,x]
```

output
```
(F^a*Gamma[5, -(b*(c + d*x)^3*Log[F])])/(3*b^5*d*Log[F]^5)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{14} F^{a+b(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2647}$$

$$\frac{F^{a+b(c+dx)^3} \left(b^4 \log^4(F)(c+dx)^{12} - 4b^3 \log^3(F)(c+dx)^9 + 12b^2 \log^2(F)(c+dx)^6 - 24b \log(F)(c+dx)^3 + 24\right)}{3b^5 d \log^5(F)}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^14,x]
```

output
```
(F^(a + b*(c + d*x)^3)*(24 - 24*b*(c + d*x)^3*Log[F] + 12*b^2*(c + d*x)^6*
Log[F]^2 - 4*b^3*(c + d*x)^9*Log[F]^3 + b^4*(c + d*x)^12*Log[F]^4))/(3*b^5
*d*Log[F]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(86) = 172$.

Time = 2.86 (sec) , antiderivative size = 562, normalized size of antiderivative = 6.39

method	result
orering	$\frac{\left(24-4\ln(F)^3b^3c^9+12\ln(F)^2b^2c^6-24\ln(F)b\,d^3x^3-24\ln(F)b\,c^3+d^{12}x^{12}\ln(F)^4b^4-4d^9x^9\ln(F)^3b^3+12d^6x^6\ln(F)^2b^2-72\ln\right.}{}$
gosper	$\frac{\left(24-4\ln(F)^3b^3c^9+12\ln(F)^2b^2c^6-24\ln(F)b\,d^3x^3-24\ln(F)b\,c^3+d^{12}x^{12}\ln(F)^4b^4-4d^9x^9\ln(F)^3b^3+12d^6x^6\ln(F)^2b^2-72\ln\right.}{}$
risch	$\frac{\left(24-4\ln(F)^3b^3c^9+12\ln(F)^2b^2c^6-24\ln(F)b\,d^3x^3-24\ln(F)b\,c^3+d^{12}x^{12}\ln(F)^4b^4-4d^9x^9\ln(F)^3b^3+12d^6x^6\ln(F)^2b^2-72\ln\right.}{}$
norman	$\frac{\left(\ln(F)^4b^4c^{12}-4\ln(F)^3b^3c^9+12\ln(F)^2b^2c^6-24\ln(F)b\,c^3+24\right)e^{\left(a+b(dx+c)^3\right)\ln(F)}}{3d\ln(F)^5b^5}+\frac{d^{11}x^{12}e^{\left(a+b(dx+c)^3\right)\ln(F)}}{3\ln(F)b}+\frac{4c^2\left(\right.}{}$
parallelrisch	Expression too large to display

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x,method=_RETURNVERBOSE)
```

output
```
1/3*(24-4*ln(F)^3*b^3*c^9+12*ln(F)^2*b^2*c^6-24*ln(F)*b*d^3*x^3-24*ln(F)*b
*c^3+d^12*x^12*ln(F)^4*b^4-4*d^9*x^9*ln(F)^3*b^3+12*d^6*x^6*ln(F)^2*b^2-72
*ln(F)*b*c*d^2*x^2-72*ln(F)*b*c^2*d*x+12*c*d^11*x^11*ln(F)^4*b^4+66*c^2*d^
10*x^10*ln(F)^4*b^4+220*ln(F)^4*b^4*c^3*d^9*x^9+495*ln(F)^4*b^4*c^4*d^8*x^
8+ln(F)^4*b^4*c^12+792*ln(F)^4*b^4*c^5*d^7*x^7+924*ln(F)^4*b^4*c^6*d^6*x^6
+792*ln(F)^4*b^4*c^7*d^5*x^5+495*ln(F)^4*b^4*c^8*d^4*x^4+220*ln(F)^4*b^4*c
^9*d^3*x^3-36*c*d^8*x^8*ln(F)^3*b^3+66*ln(F)^4*b^4*c^10*d^2*x^2-144*c^2*d^
7*x^7*ln(F)^3*b^3+12*ln(F)^4*b^4*c^11*d*x-336*ln(F)^3*b^3*c^3*d^6*x^6-504*
ln(F)^3*b^3*c^4*d^5*x^5-504*ln(F)^3*b^3*c^5*d^4*x^4-336*ln(F)^3*b^3*c^6*d^
3*x^3-144*ln(F)^3*b^3*c^7*d^2*x^2-36*ln(F)^3*b^3*c^8*d*x+72*c*d^5*x^5*ln(F
)^2*b^2+180*c^2*d^4*x^4*ln(F)^2*b^2+240*ln(F)^2*b^2*c^3*d^3*x^3+180*ln(F)^
2*b^2*c^4*d^2*x^2+72*ln(F)^2*b^2*c^5*d*x)/d/ln(F)^5/b^5*F^(a+b*(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(86) = 172$.

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.39

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx$$
$$= \frac{((b^4d^{12}x^{12} + 12\,b^4cd^{11}x^{11} + 66\,b^4c^2d^{10}x^{10} + 220\,b^4c^3d^9x^9 + 495\,b^4c^4d^8x^8 + 792\,b^4c^5d^7x^7 + 924\,b^4c^6d^6x^6 +}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x, algorithm="fricas")
```

output
```
1/3*((b^4*d^12*x^12 + 12*b^4*c*d^11*x^11 + 66*b^4*c^2*d^10*x^10 + 220*b^4*
c^3*d^9*x^9 + 495*b^4*c^4*d^8*x^8 + 792*b^4*c^5*d^7*x^7 + 924*b^4*c^6*d^6*
x^6 + 792*b^4*c^7*d^5*x^5 + 495*b^4*c^8*d^4*x^4 + 220*b^4*c^9*d^3*x^3 + 66
*b^4*c^10*d^2*x^2 + 12*b^4*c^11*d*x + b^4*c^12)*log(F)^4 - 4*(b^3*d^9*x^9
+ 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6*x^6 + 126*b^3*c^4*
d^5*x^5 + 126*b^3*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36*b^3*c^7*d^2*x^2 +
9*b^3*c^8*d*x + b^3*c^9)*log(F)^3 + 12*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15
*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x
 + b^2*c^6)*log(F)^2 - 24*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
)*log(F) + 24)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b^
5*d*log(F)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(87) = 174$.

Time = 0.25 (sec) , antiderivative size = 821, normalized size of antiderivative = 9.33

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx = \text{Too large to display}$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**14,x)
```

output

```
Piecewise((F**(a + b*(c + d*x)**3)*(b**4*c**12*log(F)**4 + 12*b**4*c**11*d
*x*log(F)**4 + 66*b**4*c**10*d**2*x**2*log(F)**4 + 220*b**4*c**9*d**3*x**3
*log(F)**4 + 495*b**4*c**8*d**4*x**4*log(F)**4 + 792*b**4*c**7*d**5*x**5*l
og(F)**4 + 924*b**4*c**6*d**6*x**6*log(F)**4 + 792*b**4*c**5*d**7*x**7*log
(F)**4 + 495*b**4*c**4*d**8*x**8*log(F)**4 + 220*b**4*c**3*d**9*x**9*log(F
)**4 + 66*b**4*c**2*d**10*x**10*log(F)**4 + 12*b**4*c*d**11*x**11*log(F)**
4 + b**4*d**12*x**12*log(F)**4 - 4*b**3*c**9*log(F)**3 - 36*b**3*c**8*d*x*
log(F)**3 - 144*b**3*c**7*d**2*x**2*log(F)**3 - 336*b**3*c**6*d**3*x**3*lo
g(F)**3 - 504*b**3*c**5*d**4*x**4*log(F)**3 - 504*b**3*c**4*d**5*x**5*log(
F)**3 - 336*b**3*c**3*d**6*x**6*log(F)**3 - 144*b**3*c**2*d**7*x**7*log(F)
**3 - 36*b**3*c*d**8*x**8*log(F)**3 - 4*b**3*d**9*x**9*log(F)**3 + 12*b**2
*c**6*log(F)**2 + 72*b**2*c**5*d*x*log(F)**2 + 180*b**2*c**4*d**2*x**2*log
(F)**2 + 240*b**2*c**3*d**3*x**3*log(F)**2 + 180*b**2*c**2*d**4*x**4*log(F
)**2 + 72*b**2*c*d**5*x**5*log(F)**2 + 12*b**2*d**6*x**6*log(F)**2 - 24*b*
c**3*log(F) - 72*b*c**2*d*x*log(F) - 72*b*c*d**2*x**2*log(F) - 24*b*d**3*x
**3*log(F) + 24)/(3*b**5*d*log(F)**5), Ne(b**5*d*log(F)**5, 0)), (c**14*x
+ 7*c**13*d*x**2 + 91*c**12*d**2*x**3/3 + 91*c**11*d**3*x**4 + 1001*c**10*
d**4*x**5/5 + 1001*c**9*d**5*x**6/3 + 429*c**8*d**6*x**7 + 429*c**7*d**7*x
**8 + 1001*c**6*d**8*x**9/3 + 1001*c**5*d**9*x**10/5 + 91*c**4*d**10*x**11
 + 91*c**3*d**11*x**12/3 + 7*c**2*d**12*x**13 + c*d**13*x**14 + d**14*x...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 874 vs. $2(86) = 172$.

Time = 0.18 (sec) , antiderivative size = 874, normalized size of antiderivative = 9.93

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx = \text{Too large to display}$$

input

```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x, algorithm="maxima")
```

output
```
1/3*(F^(b*c^3 + a)*b^4*d^12*x^12*log(F)^4 + 12*F^(b*c^3 + a)*b^4*c*d^11*x^
11*log(F)^4 + 66*F^(b*c^3 + a)*b^4*c^2*d^10*x^10*log(F)^4 + F^(b*c^3 + a)*
b^4*c^12*log(F)^4 - 4*F^(b*c^3 + a)*b^3*c^9*log(F)^3 + 12*F^(b*c^3 + a)*b^
2*c^6*log(F)^2 + 4*(55*F^(b*c^3 + a)*b^4*c^3*d^9*log(F)^4 - F^(b*c^3 + a)*
b^3*d^9*log(F)^3)*x^9 + 9*(55*F^(b*c^3 + a)*b^4*c^4*d^8*log(F)^4 - 4*F^(b*
c^3 + a)*b^3*c*d^8*log(F)^3)*x^8 + 72*(11*F^(b*c^3 + a)*b^4*c^5*d^7*log(F)
^4 - 2*F^(b*c^3 + a)*b^3*c^2*d^7*log(F)^3)*x^7 + 12*(77*F^(b*c^3 + a)*b^4*
c^6*d^6*log(F)^4 - 28*F^(b*c^3 + a)*b^3*c^3*d^6*log(F)^3 + F^(b*c^3 + a)*b
^2*d^6*log(F)^2)*x^6 + 72*(11*F^(b*c^3 + a)*b^4*c^7*d^5*log(F)^4 - 7*F^(b*
c^3 + a)*b^3*c^4*d^5*log(F)^3 + F^(b*c^3 + a)*b^2*c*d^5*log(F)^2)*x^5 - 24
*F^(b*c^3 + a)*b*c^3*log(F) + 9*(55*F^(b*c^3 + a)*b^4*c^8*d^4*log(F)^4 - 5
6*F^(b*c^3 + a)*b^3*c^5*d^4*log(F)^3 + 20*F^(b*c^3 + a)*b^2*c^2*d^4*log(F)
^2)*x^4 + 4*(55*F^(b*c^3 + a)*b^4*c^9*d^3*log(F)^4 - 84*F^(b*c^3 + a)*b^3*
c^6*d^3*log(F)^3 + 60*F^(b*c^3 + a)*b^2*c^3*d^3*log(F)^2 - 6*F^(b*c^3 + a)
*b*d^3*log(F))*x^3 + 6*(11*F^(b*c^3 + a)*b^4*c^10*d^2*log(F)^4 - 24*F^(b*c
^3 + a)*b^3*c^7*d^2*log(F)^3 + 30*F^(b*c^3 + a)*b^2*c^4*d^2*log(F)^2 - 12*
F^(b*c^3 + a)*b*c*d^2*log(F))*x^2 + 12*(F^(b*c^3 + a)*b^4*c^11*d*log(F)^4
- 3*F^(b*c^3 + a)*b^3*c^8*d*log(F)^3 + 6*F^(b*c^3 + a)*b^2*c^5*d*log(F)^2
- 6*F^(b*c^3 + a)*b*c^2*d*log(F))*x + 24*F^(b*c^3 + a))*e^(b*d^3*x^3*log(F
) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F))/(b^5*d*log(F)^5)
```

Giac [F(-2)]

Exception generated.

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Polynomial exponent overflow. Error
: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 487, normalized size of antiderivative = 5.53

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx$$

$$= F^{b\,d^3\,x^3}\,F^{3\,b\,c^2\,d\,x}\,F^a\,F^{b\,c^3}\,F^{3\,b\,c\,d^2\,x^2}\left(\frac{b^4\,c^{12}\ln(F)^4-4\,b^3\,c^9\ln(F)^3+12\,b^2\,c^6\ln(F)^2-24\,b\,c^3\,\ln(F)+24}{3\,b^5\,d\ln(F)^5}+\frac{d^{11}\,x^{12}}{3\,b\,\ln(F)}+\frac{4\,c\,d^{10}\,x^{11}}{b\,\ln(F)}+\frac{4\,d^2\,x^3\,\left(55\,b^3\,c^9\ln(F)^3-84\,b^2\,c^6\ln(F)^2+60\,b\,c^3\,\ln(F)-6\right)}{3\,b^4\ln(F)^4}+\frac{4\,d^5\,x^6\,\left(77\,b^2\,c^6\ln(F)^2-28\,b\,c^3\,\ln(F)+1\right)}{b^3\ln(F)^3}+\frac{4\,d^8\,x^9\,(55\,b\,c^3\,\ln(F)-1)}{3\,b^2\ln(F)^2}+\frac{22\,c^2\,d^9\,x^{10}}{b\,\ln(F)}+\frac{4\,c^2\,x\,\left(b^3\,c^9\ln(F)^3-3\,b^2\,c^6\ln(F)^2+6\,b\,c^3\,\ln(F)-6\right)}{b^4\ln(F)^4}+\frac{3\,c^2\,d^3\,x^4\,\left(55\,b^2\,c^6\ln(F)^2-56\,b\,c^3\,\ln(F)+20\right)}{b^3\ln(F)^3}+\frac{24\,c^2\,d^6\,x^7\,(11\,b\,c^3\,\ln(F)-2)}{b^2\ln(F)^2}+\frac{2\,c\,d\,x^2\,\left(11\,b^3\,c^9\ln(F)^3-24\,b^2\,c^6\ln(F)^2+30\,b\,c^3\,\ln(F)-12\right)}{b^4\ln(F)^4}+\frac{24\,c\,d^4\,x^5\,\left(11\,b^2\,c^6\ln(F)^2-7\,b\,c^3\,\ln(F)+1\right)}{b^3\ln(F)^3}+\frac{3\,c\,d^7\,x^8\,(55\,b\,c^3\,\ln(F)-4)}{b^2\ln(F)^2}\right)$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^14,x)
```

output
```
F^(b*d^3*x^3)*F^(3*b*c^2*d*x)*F^a*F^(b*c^3)*F^(3*b*c*d^2*x^2)*((12*b^2*c^6
*log(F)^2 - 24*b*c^3*log(F) - 4*b^3*c^9*log(F)^3 + b^4*c^12*log(F)^4 + 24)
/(3*b^5*d*log(F)^5) + (d^11*x^12)/(3*b*log(F)) + (4*c*d^10*x^11)/(b*log(F)
) + (4*d^2*x^3*(60*b*c^3*log(F) - 84*b^2*c^6*log(F)^2 + 55*b^3*c^9*log(F)^
3 - 6))/(3*b^4*log(F)^4) + (4*d^5*x^6*(77*b^2*c^6*log(F)^2 - 28*b*c^3*log(
F) + 1))/(b^3*log(F)^3) + (4*d^8*x^9*(55*b*c^3*log(F) - 1))/(3*b^2*log(F)^
2) + (22*c^2*d^9*x^10)/(b*log(F)) + (4*c^2*x*(6*b*c^3*log(F) - 3*b^2*c^6*l
og(F)^2 + b^3*c^9*log(F)^3 - 6))/(b^4*log(F)^4) + (3*c^2*d^3*x^4*(55*b^2*c
^6*log(F)^2 - 56*b*c^3*log(F) + 20))/(b^3*log(F)^3) + (24*c^2*d^6*x^7*(11*
b*c^3*log(F) - 2))/(b^2*log(F)^2) + (2*c*d*x^2*(30*b*c^3*log(F) - 24*b^2*c
^6*log(F)^2 + 11*b^3*c^9*log(F)^3 - 12))/(b^4*log(F)^4) + (24*c*d^4*x^5*(1
1*b^2*c^6*log(F)^2 - 7*b*c^3*log(F) + 1))/(b^3*log(F)^3) + (3*c*d^7*x^8*(5
5*b*c^3*log(F) - 4))/(b^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 583, normalized size of antiderivative = 6.62

$$\int F^{a+b(c+dx)^3}(c+dx)^{14}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^14,x)
```

output
```
(f**(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*(log(f)**4
*b**4*c**12 + 12*log(f)**4*b**4*c**11*d*x + 66*log(f)**4*b**4*c**10*d**2*x
**2 + 220*log(f)**4*b**4*c**9*d**3*x**3 + 495*log(f)**4*b**4*c**8*d**4*x**
4 + 792*log(f)**4*b**4*c**7*d**5*x**5 + 924*log(f)**4*b**4*c**6*d**6*x**6
+ 792*log(f)**4*b**4*c**5*d**7*x**7 + 495*log(f)**4*b**4*c**4*d**8*x**8 +
220*log(f)**4*b**4*c**3*d**9*x**9 + 66*log(f)**4*b**4*c**2*d**10*x**10 + 1
2*log(f)**4*b**4*c*d**11*x**11 + log(f)**4*b**4*d**12*x**12 - 4*log(f)**3*
b**3*c**9 - 36*log(f)**3*b**3*c**8*d*x - 144*log(f)**3*b**3*c**7*d**2*x**2
 - 336*log(f)**3*b**3*c**6*d**3*x**3 - 504*log(f)**3*b**3*c**5*d**4*x**4 -
 504*log(f)**3*b**3*c**4*d**5*x**5 - 336*log(f)**3*b**3*c**3*d**6*x**6 - 1
44*log(f)**3*b**3*c**2*d**7*x**7 - 36*log(f)**3*b**3*c*d**8*x**8 - 4*log(f
)**3*b**3*d**9*x**9 + 12*log(f)**2*b**2*c**6 + 72*log(f)**2*b**2*c**5*d*x
+ 180*log(f)**2*b**2*c**4*d**2*x**2 + 240*log(f)**2*b**2*c**3*d**3*x**3 +
180*log(f)**2*b**2*c**2*d**4*x**4 + 72*log(f)**2*b**2*c*d**5*x**5 + 12*log
(f)**2*b**2*d**6*x**6 - 24*log(f)*b*c**3 - 72*log(f)*b*c**2*d*x - 72*log(f
)*b*c*d**2*x**2 - 24*log(f)*b*d**3*x**3 + 24))/(3*log(f)**5*b**5*d)
```

3.217 $\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 124

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx = -\frac{2F^{a+b(c+dx)^3}}{b^4d\log^4(F)}+\frac{2F^{a+b(c+dx)^3}(c+dx)^3}{b^3d\log^3(F)}-\frac{F^{a+b(c+dx)^3}(c+dx)^6}{b^2d\log^2(F)}+\frac{F^{a+b(c+dx)^3}(c+dx)^9}{3bd\log(F)}$$

output
```
-2*F^(a+b*(d*x+c)^3)/b^4/d/ln(F)^4+2*F^(a+b*(d*x+c)^3)*(d*x+c)^3/b^3/d/ln(
F)^3-F^(a+b*(d*x+c)^3)*(d*x+c)^6/b^2/d/ln(F)^2+1/3*F^(a+b*(d*x+c)^3)*(d*x+
c)^9/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx = \frac{F^{a+b(c+dx)^3}\left(-3b^2(c+dx)^6\log^2(F)+b^3(c+dx)^9\log^3(F)-6(1-b(c+dx)^3\log(F))\right)}{3b^4d\log^4(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^11,x]
```

output
```
(F^(a + b*(c + d*x)^3)*(-3*b^2*(c + d*x)^6*Log[F]^2 + b^3*(c + d*x)^9*Log[
F]^3 - 6*(1 - b*(c + d*x)^3*Log[F])))/(3*b^4*d*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{11}F^{a+b(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^9F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{3\int F^{b(c+dx)^3+a}(c+dx)^8dx}{b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^9F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{3\left(\frac{(c+dx)^6F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{2\int F^{b(c+dx)^3+a}(c+dx)^5dx}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^9F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{3\left(\frac{(c+dx)^6F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{2\left(\frac{(c+dx)^3F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{\int F^{b(c+dx)^3+a}(c+dx)^2dx}{b\log(F)}\right)}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{(c+dx)^9F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{3\left(\frac{(c+dx)^6F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{2\left(\frac{(c+dx)^3F^{a+b(c+dx)^3}}{3bd\log(F)}-\frac{F^{a+b(c+dx)^3}}{3b^2d\log^2(F)}\right)}{b\log(F)}\right)}{b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^11,x]
```

output

```
(F^(a + b*(c + d*x)^3)*(c + d*x)^9)/(3*b*d*Log[F]) - (3*((F^(a + b*(c + d*
x)^3)*(c + d*x)^6)/(3*b*d*Log[F]) - (2*(-1/3*F^(a + b*(c + d*x)^3)/(b^2*d*
Log[F]^2) + (F^(a + b*(c + d*x)^3)*(c + d*x)^3)/(3*b*d*Log[F])))/(b*Log[F]
)))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(122) = 244$.

Time = 1.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.77

method	result
orering	$\frac{\left(d^9x^9\ln(F)^3b^3+9c\,d^8x^8\ln(F)^3b^3+36c^2d^7x^7\ln(F)^3b^3+84\ln(F)^3b^3c^3d^6x^6+126\ln(F)^3b^3c^4d^5x^5+126\ln(F)^3b^3c^5d^4x^4+84\ln\right.}{}$
gosper	$\frac{\left(d^9x^9\ln(F)^3b^3+9c\,d^8x^8\ln(F)^3b^3+36c^2d^7x^7\ln(F)^3b^3+84\ln(F)^3b^3c^3d^6x^6+126\ln(F)^3b^3c^4d^5x^5+126\ln(F)^3b^3c^5d^4x^4+84\ln\right.}{}$
risch	$\frac{\left(d^9x^9\ln(F)^3b^3+9c\,d^8x^8\ln(F)^3b^3+36c^2d^7x^7\ln(F)^3b^3+84\ln(F)^3b^3c^3d^6x^6+126\ln(F)^3b^3c^4d^5x^5+126\ln(F)^3b^3c^5d^4x^4+84\ln\right.}{}$
norman	$\frac{d^5(28\ln(F)b\,c^3-1)x^6\mathrm{e}^{\left(a+b(dx+c)^3\right)\ln(F)}}{\ln(F)^2b^2}+\frac{\left(\ln(F)^3b^3c^9-3\ln(F)^2b^2c^6+6\ln(F)b\,c^3-6\right)\mathrm{e}^{\left(a+b(dx+c)^3\right)\ln(F)}}{3\ln(F)^4b^4d}+\frac{d^8x^9\mathrm{e}^{(a}}{3}$
parallelrisch	$\frac{d^9F^{a+b(dx+c)^3}x^9b^3\ln(F)^3+9c\,d^8F^{a+b(dx+c)^3}x^8b^3\ln(F)^3+36c^2d^7F^{a+b(dx+c)^3}x^7b^3\ln(F)^3+84\ln(F)^3x^6F^{a+b(dx+c)^3}b^3c^3}{}$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^11,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(d^9*x^9*ln(F)^3*b^3+9*c*d^8*x^8*ln(F)^3*b^3+36*c^2*d^7*x^7*ln(F)^3*
b^3+84*ln(F)^3*b^3*c^3*d^6*x^6+126*ln(F)^3*b^3*c^4*d^5*x^5+126*ln(F)^3*b^3
*c^5*d^4*x^4+84*ln(F)^3*b^3*c^6*d^3*x^3+36*ln(F)^3*b^3*c^7*d^2*x^2+9*ln(F)
^3*b^3*c^8*d*x-3*d^6*x^6*ln(F)^2*b^2+ln(F)^3*b^3*c^9-18*c*d^5*x^5*ln(F)^2*
b^2-45*c^2*d^4*x^4*ln(F)^2*b^2-60*ln(F)^2*b^2*c^3*d^3*x^3-45*ln(F)^2*b^2*c
^4*d^2*x^2-18*ln(F)^2*b^2*c^5*d*x-3*ln(F)^2*b^2*c^6+6*ln(F)*b*d^3*x^3+18*l
n(F)*b*c*d^2*x^2+18*ln(F)*b*c^2*d*x+6*ln(F)*b*c^3-6)/b^4/ln(F)^4*F^(a+b*(d
*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(122) = 244$.

Time = 0.08 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.44

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx$$
$$= \frac{((b^3d^9x^9 + 9\,b^3cd^8x^8 + 36\,b^3c^2d^7x^7 + 84\,b^3c^3d^6x^6 + 126\,b^3c^4d^5x^5 + 126\,b^3c^5d^4x^4 + 84\,b^3c^6d^3x^3 + 36\,b^3c^7 d}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^11,x, algorithm="fricas")
```

output
```
1/3*((b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6*
x^6 + 126*b^3*c^4*d^5*x^5 + 126*b^3*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36*
b^3*c^7*d^2*x^2 + 9*b^3*c^8*d*x + b^3*c^9)*log(F)^3 - 3*(b^2*d^6*x^6 + 6*b
^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^
2 + 6*b^2*c^5*d*x + b^2*c^6)*log(F)^2 + 6*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*log(F) - 6)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3 + a)/(b^4*d*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(109) = 218$.

Time = 0.20 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.32

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx$$
$$= \begin{cases} \frac{F^{a+b(c+dx)^3}\left(b^3c^9\log(F)^3+9b^3c^8dx\log(F)^3+36b^3c^7d^2x^2\log(F)^3+84b^3c^6d^3x^3\log(F)^3+126b^3c^5d^4x^4\log(F)^3+126b^3c^4d^5x^5\log(F)^3+84\right.}{} \\ c^{11}x+\frac{11c^{10}dx^2}{2}+\frac{55c^9d^2x^3}{3}+\frac{165c^8d^3x^4}{4}+66c^7d^4x^5+77c^6d^5x^6+66c^5d^6x^7+\frac{165c^4d^7x^8}{4}+\frac{55c^3d^8x^9}{3}+\frac{11c^2d^9x^1}{2} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**11,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**3)*(b**3*c**9*log(F)**3 + 9*b**3*c**8*d*x*
log(F)**3 + 36*b**3*c**7*d**2*x**2*log(F)**3 + 84*b**3*c**6*d**3*x**3*log(
F)**3 + 126*b**3*c**5*d**4*x**4*log(F)**3 + 126*b**3*c**4*d**5*x**5*log(F)
**3 + 84*b**3*c**3*d**6*x**6*log(F)**3 + 36*b**3*c**2*d**7*x**7*log(F)**3
+ 9*b**3*c*d**8*x**8*log(F)**3 + b**3*d**9*x**9*log(F)**3 - 3*b**2*c**6*lo
g(F)**2 - 18*b**2*c**5*d*x*log(F)**2 - 45*b**2*c**4*d**2*x**2*log(F)**2 -
60*b**2*c**3*d**3*x**3*log(F)**2 - 45*b**2*c**2*d**4*x**4*log(F)**2 - 18*b
**2*c*d**5*x**5*log(F)**2 - 3*b**2*d**6*x**6*log(F)**2 + 6*b*c**3*log(F) +
 18*b*c**2*d*x*log(F) + 18*b*c*d**2*x**2*log(F) + 6*b*d**3*x**3*log(F) - 6
)/(3*b**4*d*log(F)**4), Ne(b**4*d*log(F)**4, 0)), (c**11*x + 11*c**10*d*x*
*2/2 + 55*c**9*d**2*x**3/3 + 165*c**8*d**3*x**4/4 + 66*c**7*d**4*x**5 + 77
*c**6*d**5*x**6 + 66*c**5*d**6*x**7 + 165*c**4*d**7*x**8/4 + 55*c**3*d**8*
x**9/3 + 11*c**2*d**9*x**10/2 + c*d**10*x**11 + d**11*x**12/12, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(122) = 244$.

Time = 0.17 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.48

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx$$
$$= \frac{\left(F^{bc^3+a}b^3d^9x^9\log(F)^3+9\,F^{bc^3+a}b^3cd^8x^8\log(F)^3+36\,F^{bc^3+a}b^3c^2d^7x^7\log(F)^3+F^{bc^3+a}b^3c^9\log(F)^3-\right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^11,x, algorithm="maxima")
```

output
```
1/3*(F^(b*c^3 + a)*b^3*d^9*x^9*log(F)^3 + 9*F^(b*c^3 + a)*b^3*c*d^8*x^8*lo
g(F)^3 + 36*F^(b*c^3 + a)*b^3*c^2*d^7*x^7*log(F)^3 + F^(b*c^3 + a)*b^3*c^9
*log(F)^3 - 3*F^(b*c^3 + a)*b^2*c^6*log(F)^2 + 3*(28*F^(b*c^3 + a)*b^3*c^3
*d^6*log(F)^3 - F^(b*c^3 + a)*b^2*d^6*log(F)^2)*x^6 + 18*(7*F^(b*c^3 + a)*
b^3*c^4*d^5*log(F)^3 - F^(b*c^3 + a)*b^2*c*d^5*log(F)^2)*x^5 + 6*F^(b*c^3
+ a)*b*c^3*log(F) + 9*(14*F^(b*c^3 + a)*b^3*c^5*d^4*log(F)^3 - 5*F^(b*c^3
+ a)*b^2*c^2*d^4*log(F)^2)*x^4 + 6*(14*F^(b*c^3 + a)*b^3*c^6*d^3*log(F)^3
- 10*F^(b*c^3 + a)*b^2*c^3*d^3*log(F)^2 + F^(b*c^3 + a)*b*d^3*log(F))*x^3
+ 9*(4*F^(b*c^3 + a)*b^3*c^7*d^2*log(F)^3 - 5*F^(b*c^3 + a)*b^2*c^4*d^2*lo
g(F)^2 + 2*F^(b*c^3 + a)*b*c*d^2*log(F))*x^2 + 9*(F^(b*c^3 + a)*b^3*c^8*d*
log(F)^3 - 2*F^(b*c^3 + a)*b^2*c^5*d*log(F)^2 + 2*F^(b*c^3 + a)*b*c^2*d*lo
g(F))*x - 6*F^(b*c^3 + a))*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*
b*c^2*d*x*log(F))/(b^4*d*log(F)^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1320 vs. $2(122) = 244$.

Time = 0.60 (sec) , antiderivative size = 1320, normalized size of antiderivative = 10.65

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^11,x, algorithm="giac")
```

output
```
1/3*(b^3*d^9*x^9*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*
log(F) + b*c^3*log(F) + a*log(F))*log(F)^3 + 9*b^3*c*d^8*x^8*e^(b*d^3*x^3*
log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(
F))*log(F)^3 + 36*b^3*c^2*d^7*x^7*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(
F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F)^3 + 84*b^3*c^3*d
^6*x^6*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b
*c^3*log(F) + a*log(F))*log(F)^3 + 126*b^3*c^4*d^5*x^5*e^(b*d^3*x^3*log(F)
 + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*lo
g(F)^3 + 126*b^3*c^5*d^4*x^4*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) +
3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F)^3 + 84*b^3*c^6*d^3*x^
3*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*
log(F) + a*log(F))*log(F)^3 + 36*b^3*c^7*d^2*x^2*e^(b*d^3*x^3*log(F) + 3*b
*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F)^3
 - 3*b^2*d^6*x^6*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*
log(F) + b*c^3*log(F) + a*log(F))*log(F)^2 + 9*b^3*c^8*d*x*e^(b*d^3*x^3*lo
g(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F)
)*log(F)^3 - 18*b^2*c*d^5*x^5*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) +
 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F)^2 + b^3*c^9*e^(b*d^3
*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a
*log(F))*log(F)^3 - 45*b^2*c^2*d^4*x^4*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*...
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.60

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx = F^{b\,d^3\,x^3}\,F^{3\,b\,c^2\,d\,x}\,F^a\,F^{b\,c^3}\,F^{3\,b\,c\,d^2\,x^2}\left(\frac{b^3\,c^9\ln(F)^3-3\,b^2\,c^6\ln(F)^2+6\,b\,c^3\,\ln(F)-6}{3\,b^4\,d\ln(F)^4}+\frac{d^8\,x^9}{3\,b\,\ln(F)}+\frac{3\,c\,d^7\,x^8}{b\,\ln(F)}+\frac{2\,d^2\,x^3\left(14\,b^2\,c^6\ln(F)^2-10\,b\,c^3\,\ln(F)+1\right)}{b^3\ln(F)^3}+\frac{d^5\,x^6\,(28\,b\,c^3\,\ln(F)-1)}{b^2\ln(F)^2}+\frac{12\,c^2\,d^6\,x^7}{b\,\ln(F)}+\frac{3\,c^2\,x\left(b^2\,c^6\ln(F)^2-2\,b\,c^3\,\ln(F)+2\right)}{b^3\ln(F)^3}+\frac{3\,c^2\,d^3\,x^4\,(14\,b\,c^3\,\ln(F)-5)}{b^2\ln(F)^2}+\frac{3\,c\,d\,x^2\left(4\,b^2\,c^6\ln(F)^2-5\,b\,c^3\,\ln(F)+2\right)}{b^3\ln(F)^3}+\frac{6\,c\,d^4\,x^5\,(7\,b\,c^3\,\ln(F)-1)}{b^2\ln(F)^2}\right)$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^11,x)
```

output
```
F^(b*d^3*x^3)*F^(3*b*c^2*d*x)*F^a*F^(b*c^3)*F^(3*b*c*d^2*x^2)*((6*b*c^3*lo
g(F) - 3*b^2*c^6*log(F)^2 + b^3*c^9*log(F)^3 - 6)/(3*b^4*d*log(F)^4) + (d^
8*x^9)/(3*b*log(F)) + (3*c*d^7*x^8)/(b*log(F)) + (2*d^2*x^3*(14*b^2*c^6*lo
g(F)^2 - 10*b*c^3*log(F) + 1))/(b^3*log(F)^3) + (d^5*x^6*(28*b*c^3*log(F)
- 1))/(b^2*log(F)^2) + (12*c^2*d^6*x^7)/(b*log(F)) + (3*c^2*x*(b^2*c^6*log
(F)^2 - 2*b*c^3*log(F) + 2))/(b^3*log(F)^3) + (3*c^2*d^3*x^4*(14*b*c^3*log
(F) - 5))/(b^2*log(F)^2) + (3*c*d*x^2*(4*b^2*c^6*log(F)^2 - 5*b*c^3*log(F)
 + 2))/(b^3*log(F)^3) + (6*c*d^4*x^5*(7*b*c^3*log(F) - 1))/(b^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.94

$$\int F^{a+b(c+dx)^3}(c+dx)^{11}\,dx = \frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}\left(\log(f)^3\,b^3c^9+9\log(f)^3\,b^3c^8dx+36\log(f)^3\,b^3c^7d^2x^2+84\log(f)^3\,b^3c^6d^3x^3+1\right.}{}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^11,x)
```

output
```
(f**(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*(log(f)**3
*b**3*c**9 + 9*log(f)**3*b**3*c**8*d*x + 36*log(f)**3*b**3*c**7*d**2*x**2
+ 84*log(f)**3*b**3*c**6*d**3*x**3 + 126*log(f)**3*b**3*c**5*d**4*x**4 + 1
26*log(f)**3*b**3*c**4*d**5*x**5 + 84*log(f)**3*b**3*c**3*d**6*x**6 + 36*l
og(f)**3*b**3*c**2*d**7*x**7 + 9*log(f)**3*b**3*c*d**8*x**8 + log(f)**3*b*
*3*d**9*x**9 - 3*log(f)**2*b**2*c**6 - 18*log(f)**2*b**2*c**5*d*x - 45*log
(f)**2*b**2*c**4*d**2*x**2 - 60*log(f)**2*b**2*c**3*d**3*x**3 - 45*log(f)*
*2*b**2*c**2*d**4*x**4 - 18*log(f)**2*b**2*c*d**5*x**5 - 3*log(f)**2*b**2*
d**6*x**6 + 6*log(f)*b*c**3 + 18*log(f)*b*c**2*d*x + 18*log(f)*b*c*d**2*x*
*2 + 6*log(f)*b*d**3*x**3 - 6))/(3*log(f)**4*b**4*d)
```

3.218 $\int F^{a+b(c+dx)^3}(c+dx)^8\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx=\frac{2F^{a+b(c+dx)^3}}{3b^3d\log^3(F)}-\frac{2F^{a+b(c+dx)^3}(c+dx)^3}{3b^2d\log^2(F)}+\frac{F^{a+b(c+dx)^3}(c+dx)^6}{3bd\log(F)}$$

output
```
2/3*F^(a+b*(d*x+c)^3)/b^3/d/ln(F)^3-2/3*F^(a+b*(d*x+c)^3)*(d*x+c)^3/b^2/d/
ln(F)^2+1/3*F^(a+b*(d*x+c)^3)*(d*x+c)^6/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx=\frac{F^{a+b(c+dx)^3}\left(2-2b(c+dx)^3\log(F)+b^2(c+dx)^6\log^2(F)\right)}{3b^3d\log^3(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^8,x]
```

output
```
(F^(a + b*(c + d*x)^3)*(2 - 2*b*(c + d*x)^3*Log[F] + b^2*(c + d*x)^6*Log[F
]^2))/(3*b^3*d*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^8 F^{a+b(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{2\int F^{b(c+dx)^3+a}(c+dx)^5dx}{b\log(F)}$$

$$\Big\downarrow \text{2641}$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{2\left(\frac{(c+dx)^3 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{\int F^{b(c+dx)^3+a}(c+dx)^2dx}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{(c+dx)^6 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{2\left(\frac{(c+dx)^3 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{F^{a+b(c+dx)^3}}{3b^2d\log^2(F)}\right)}{b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^8,x]
```

output
```
(F^(a + b*(c + d*x)^3)*(c + d*x)^6)/(3*b*d*Log[F]) - (2*(-1/3*F^(a + b*(c
+ d*x)^3)/(b^2*d*Log[F]^2) + (F^(a + b*(c + d*x)^3)*(c + d*x)^3)/(3*b*d*Lo
g[F])))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.85

method	result
orering	$\frac{\left(d^{6}x^{6}\ln(F)^{2}b^{2}+6c\,d^{5}x^{5}\ln(F)^{2}b^{2}+15c^{2}d^{4}x^{4}\ln(F)^{2}b^{2}+20\ln(F)^{2}b^{2}c^{3}d^{3}x^{3}+15\ln(F)^{2}b^{2}c^{4}d^{2}x^{2}+6\ln(F)^{2}b^{2}c^{5}dx+\ln(F)^{2}b^{2}\right.}{3d\,b^{3}\ln(F)^{3}}$
gosper	$\frac{\left(d^{6}x^{6}\ln(F)^{2}b^{2}+6c\,d^{5}x^{5}\ln(F)^{2}b^{2}+15c^{2}d^{4}x^{4}\ln(F)^{2}b^{2}+20\ln(F)^{2}b^{2}c^{3}d^{3}x^{3}+15\ln(F)^{2}b^{2}c^{4}d^{2}x^{2}+6\ln(F)^{2}b^{2}c^{5}dx+\ln(F)^{2}b^{2}\right.}{3\ln(F)^{3}b^{3}d}$
risch	$\frac{\left(d^{6}x^{6}\ln(F)^{2}b^{2}+6c\,d^{5}x^{5}\ln(F)^{2}b^{2}+15c^{2}d^{4}x^{4}\ln(F)^{2}b^{2}+20\ln(F)^{2}b^{2}c^{3}d^{3}x^{3}+15\ln(F)^{2}b^{2}c^{4}d^{2}x^{2}+6\ln(F)^{2}b^{2}c^{5}dx+\ln(F)^{2}b^{2}\right.}{3\ln(F)^{3}b^{3}d}$
norman	$\frac{cd\left(5\ln(F)b\,c^{3}-2\right)x^{2}\mathrm{e}^{\left(a+b(dx+c)^{3}\right)\ln(F)}}{\ln(F)^{2}b^{2}}+\frac{\left(\ln(F)^{2}b^{2}c^{6}-2\ln(F)b\,c^{3}+2\right)\mathrm{e}^{\left(a+b(dx+c)^{3}\right)\ln(F)}}{3\ln(F)^{3}b^{3}d}+\frac{d^{5}x^{6}\mathrm{e}^{\left(a+b(dx+c)^{3}\right)\ln(F)}}{3\ln(F)b}$
parallelrisch	$d^{6}F^{a+b(dx+c)^{3}}x^{6}b^{2}\ln(F)^{2}+6c\,d^{5}F^{a+b(dx+c)^{3}}x^{5}b^{2}\ln(F)^{2}+15c^{2}d^{4}F^{a+b(dx+c)^{3}}x^{4}b^{2}\ln(F)^{2}+20\ln(F)^{2}x^{3}F^{a+b(dx+c)^{3}}b^{2}c^{3}$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(d^6*x^6*ln(F)^2*b^2+6*c*d^5*x^5*ln(F)^2*b^2+15*c^2*d^4*x^4*ln(F)^2*
b^2+20*ln(F)^2*b^2*c^3*d^3*x^3+15*ln(F)^2*b^2*c^4*d^2*x^2+6*ln(F)^2*b^2*c^
5*d*x+ln(F)^2*b^2*c^6-2*ln(F)*b*d^3*x^3-6*ln(F)*b*c*d^2*x^2-6*ln(F)*b*c^2*
d*x-2*ln(F)*b*c^3+2)/b^3/ln(F)^3*F^(a+b*(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.79

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx$$
$$=\frac{\left(\left(b^2d^6x^6+6\,b^2cd^5x^5+15\,b^2c^2d^4x^4+20\,b^2c^3d^3x^3+15\,b^2c^4d^2x^2+6\,b^2c^5dx+b^2c^6\right)\log\left(F\right)^2-2\left(bd^3x^3+\right.\right.}{3\,b^3d\log\left(F\right)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/3*((b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*
x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*log(F)^2 - 2*(b*d^3*x^
3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F) + 2)*F^(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b^3*d*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(83) = 166.

Time = 0.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.17

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx$$
$$=\begin{cases}\frac{F^{a+b(c+dx)^3}\left(b^2c^6\log\left(F\right)^2+6b^2c^5dx\log\left(F\right)^2+15b^2c^4d^2x^2\log\left(F\right)^2+20b^2c^3d^3x^3\log\left(F\right)^2+15b^2c^2d^4x^4\log\left(F\right)^2+6b^2cd^5x^5\log\left(F\right)^2+b^2d^6x^6\right.}{3b^3d\log\left(F\right)^3} \\ c^8x+4c^7dx^2+\frac{28c^6d^2x^3}{3}+14c^5d^3x^4+14c^4d^4x^5+\frac{28c^3d^5x^6}{3}+4c^2d^6x^7+cd^7x^8+\frac{d^8x^9}{9}\end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**8,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**3)*(b**2*c**6*log(F)**2 + 6*b**2*c**5*d*x*
log(F)**2 + 15*b**2*c**4*d**2*x**2*log(F)**2 + 20*b**2*c**3*d**3*x**3*log(
F)**2 + 15*b**2*c**2*d**4*x**4*log(F)**2 + 6*b**2*c*d**5*x**5*log(F)**2 +
b**2*d**6*x**6*log(F)**2 - 2*b*c**3*log(F) - 6*b*c**2*d*x*log(F) - 6*b*c*d
**2*x**2*log(F) - 2*b*d**3*x**3*log(F) + 2)/(3*b**3*d*log(F)**3), Ne(b**3*
d*log(F)**3, 0)), (c**8*x + 4*c**7*d*x**2 + 28*c**6*d**2*x**3/3 + 14*c**5*
d**3*x**4 + 14*c**4*d**4*x**5 + 28*c**3*d**5*x**6/3 + 4*c**2*d**6*x**7 + c
*d**7*x**8 + d**8*x**9/9, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(90) = 180$.

Time = 0.16 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.21

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx$$
$$= \frac{\left(F^{bc^3+a}b^2d^6x^6\log(F)^2 + 6\,F^{bc^3+a}b^2cd^5x^5\log(F)^2 + 15\,F^{bc^3+a}b^2c^2d^4x^4\log(F)^2 + F^{bc^3+a}b^2c^6\log(F)^2 - \right.}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/3*(F^(b*c^3 + a)*b^2*d^6*x^6*log(F)^2 + 6*F^(b*c^3 + a)*b^2*c*d^5*x^5*lo
g(F)^2 + 15*F^(b*c^3 + a)*b^2*c^2*d^4*x^4*log(F)^2 + F^(b*c^3 + a)*b^2*c^6
*log(F)^2 - 2*F^(b*c^3 + a)*b*c^3*log(F) + 2*(10*F^(b*c^3 + a)*b^2*c^3*d^3
*log(F)^2 - F^(b*c^3 + a)*b*d^3*log(F))*x^3 + 3*(5*F^(b*c^3 + a)*b^2*c^4*d
^2*log(F)^2 - 2*F^(b*c^3 + a)*b*c*d^2*log(F))*x^2 + 6*(F^(b*c^3 + a)*b^2*c
^5*d*log(F)^2 - F^(b*c^3 + a)*b*c^2*d*log(F))*x + 2*F^(b*c^3 + a))*e^(b*d^
3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F))/(b^3*d*log(F)^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 705 vs. $2(90) = 180$.

Time = 0.36 (sec) , antiderivative size = 705, normalized size of antiderivative = 7.34

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^8,x, algorithm="giac")
```

output
```
1/3*(b^2*d^6*x^6*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*
log(F) + b*c^3*log(F) + a*log(F))*log(F)^2 + 6*b^2*c*d^5*x^5*e^(b*d^3*x^3*
log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(
F))*log(F)^2 + 15*b^2*c^2*d^4*x^4*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(
F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F)^2 + 20*b^2*c^3*d
^3*x^3*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b
*c^3*log(F) + a*log(F))*log(F)^2 + 15*b^2*c^4*d^2*x^2*e^(b*d^3*x^3*log(F)
+ 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log
(F)^2 + 6*b^2*c^5*d*x*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2
*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F)^2 + b^2*c^6*e^(b*d^3*x^3*log
(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))
*log(F)^2 - 2*b*d^3*x^3*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c
^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F) - 6*b*c*d^2*x^2*e^(b*d^3*x
^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*l
og(F))*log(F) - 6*b*c^2*d*x*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3
*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log(F))*log(F) - 2*b*c^3*e^(b*d^3*x^3
*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F) + b*c^3*log(F) + a*log
(F))*log(F) + 2*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*l
og(F) + b*c^3*log(F) + a*log(F)))/(b^3*d*log(F)^3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.04

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx$$
$$= F^{b\,d^3\,x^3}\,F^{3\,b\,c^2\,d\,x}\,F^a\,F^{b\,c^3}\,F^{3\,b\,c\,d^2\,x^2}\left(\frac{b^2\,c^6\ln(F)^2-2\,b\,c^3\,\ln(F)+2}{3\,b^3\,d\ln(F)^3}+\frac{d^5\,x^6}{3\,b\,\ln(F)}+\frac{2\,c^2\,x\,(b\,c^3\,\ln(F)-1)}{b^2\ln(F)^2}+\frac{2\,c\,d^4\,x^5}{b\,\ln(F)}+\frac{2\,d^2\,x^3\,(10\,b\,c^3\,\ln(F)-1)}{3\,b^2\ln(F)^2}+\frac{5\,c^2\,d^3\,x^4}{b\,\ln(F)}+\frac{c\,d\,x^2\,(5\,b\,c^3\,\ln(F)-2)}{b^2\ln(F)^2}\right)$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^8,x)
```

output
```
F^(b*d^3*x^3)*F^(3*b*c^2*d*x)*F^a*F^(b*c^3)*F^(3*b*c*d^2*x^2)*((b^2*c^6*lo
g(F)^2 - 2*b*c^3*log(F) + 2)/(3*b^3*d*log(F)^3) + (d^5*x^6)/(3*b*log(F)) +
 (2*c^2*x*(b*c^3*log(F) - 1))/(b^2*log(F)^2) + (2*c*d^4*x^5)/(b*log(F)) +
(2*d^2*x^3*(10*b*c^3*log(F) - 1))/(3*b^2*log(F)^2) + (5*c^2*d^3*x^4)/(b*lo
g(F)) + (c*d*x^2*(5*b*c^3*log(F) - 2))/(b^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

$$\int F^{a+b(c+dx)^3}(c+dx)^8\,dx$$
$$= \frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}\left(\log(f)^2\,b^2c^6+6\log(f)^2\,b^2c^5dx+15\log(f)^2\,b^2c^4d^2x^2+20\log(f)^2\,b^2c^3d^3x^3+1\right.}{}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^8,x)
```

output
```
(f**(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*(log(f)**2
*b**2*c**6 + 6*log(f)**2*b**2*c**5*d*x + 15*log(f)**2*b**2*c**4*d**2*x**2
+ 20*log(f)**2*b**2*c**3*d**3*x**3 + 15*log(f)**2*b**2*c**2*d**4*x**4 + 6*
log(f)**2*b**2*c*d**5*x**5 + log(f)**2*b**2*d**6*x**6 - 2*log(f)*b*c**3 -
6*log(f)*b*c**2*d*x - 6*log(f)*b*c*d**2*x**2 - 2*log(f)*b*d**3*x**3 + 2))/
(3*log(f)**3*b**3*d)
```

3.219 $\int F^{a+b(c+dx)^3}(c+dx)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx=-\frac{F^{a+b(c+dx)^3}}{3b^2d\log^2(F)}+\frac{F^{a+b(c+dx)^3}(c+dx)^3}{3bd\log(F)}$$

output
```
-1/3*F^(a+b*(d*x+c)^3)/b^2/d/ln(F)^2+1/3*F^(a+b*(d*x+c)^3)*(d*x+c)^3/b/d/l
n(F)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx=\frac{F^{a+b(c+dx)^3}(-1+b(c+dx)^3\log(F))}{3b^2d\log^2(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^5,x]
```

output
```
(F^(a + b*(c + d*x)^3)*(-1 + b*(c + d*x)^3*Log[F]))/(3*b^2*d*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^5 F^{a+b(c+dx)^3} \, dx$$

$$\Big\downarrow \; 2641$$

$$\frac{(c+dx)^3 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{\int F^{b(c+dx)^3+a}(c+dx)^2dx}{b\log(F)}$$

$$\Big\downarrow \; 2638$$

$$\frac{(c+dx)^3 F^{a+b(c+dx)^3}}{3bd\log(F)} - \frac{F^{a+b(c+dx)^3}}{3b^2d\log^2(F)}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^5,x]
```

output
```
-1/3*F^(a + b*(c + d*x)^3)/(b^2*d*Log[F]^2) + (F^(a + b*(c + d*x)^3)*(c +
d*x)^3)/(3*b*d*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

method	result
orering	$\frac{\left(\ln \left(F\right) b \,d^{3} x^{3}+3 \ln \left(F\right) b c \,d^{2} x^{2}+3 \ln \left(F\right) b \,c^{2} d x +\ln \left(F\right) b \,c^{3}-1\right) F^{a +b \left(d x +c \right)^{3}}}{3 d \,b^{2} \ln \left(F\right)^{2}}$
gosper	$\frac{\left(\ln \left(F\right) b \,d^{3} x^{3}+3 \ln \left(F\right) b c \,d^{2} x^{2}+3 \ln \left(F\right) b \,c^{2} d x +\ln \left(F\right) b \,c^{3}-1\right) F^{b \,d^{3} x^{3}+3 b c \,d^{2} x^{2}+3 b \,c^{2} d x +b \,c^{3}+a}}{3 \ln \left(F\right)^{2} b^{2} d}$
risch	$\frac{\left(\ln \left(F\right) b \,d^{3} x^{3}+3 \ln \left(F\right) b c \,d^{2} x^{2}+3 \ln \left(F\right) b \,c^{2} d x +\ln \left(F\right) b \,c^{3}-1\right) F^{b \,d^{3} x^{3}+3 b c \,d^{2} x^{2}+3 b \,c^{2} d x +b \,c^{3}+a}}{3 \ln \left(F\right)^{2} b^{2} d}$
parallelrisch	$\frac{d^{3} F^{a +b \left(d x +c \right)^{3}} x^{3} b \ln \left(F\right)+3 c \,d^{2} F^{a +b \left(d x +c \right)^{3}} x^{2} b \ln \left(F\right)+3 c^{2} F^{a +b \left(d x +c \right)^{3}} x b \ln \left(F\right) d +\ln \left(F\right) F^{a +b \left(d x +c \right)^{3}} b \,c^{3}-F^{a +b \left(d x +c \right)^{3}}}{3 \ln \left(F\right)^{2} b^{2} d}$
norman	$\frac{c^{2} x \,{\mathrm e}^{\left(a +b \left(d x +c \right)^{3}\right) \ln \left(F\right)}}{\ln \left(F\right) b}+\frac{d c \,x^{2} {\mathrm e}^{\left(a +b \left(d x +c \right)^{3}\right) \ln \left(F\right)}}{\ln \left(F\right) b}+\frac{\left(\ln \left(F\right) b \,c^{3}-1\right) {\mathrm e}^{\left(a +b \left(d x +c \right)^{3}\right) \ln \left(F\right)}}{3 \ln \left(F\right)^{2} b^{2} d}+\frac{d^{2} x^{3} {\mathrm e}^{\left(a +b \left(d x +c \right)^{3}\right) \ln \left(F\right)}}{3 \ln \left(F\right) b}$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3/d*(ln(F)*b*d^3*x^3+3*ln(F)*b*c*d^2*x^2+3*ln(F)*b*c^2*d*x+ln(F)*b*c^3-1
)/b^2/ln(F)^2*F^(a+b*(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.35

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx = \frac{\left(\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(F\right)-1\right)F^{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}}{3\,b^2d\log\left(F\right)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F) - 1)*F^(b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b^2*d*log(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(49) = 98.

Time = 0.10 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.31

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx = \begin{cases} \frac{F^{a+b(c+dx)^3}\left(bc^3\log\left(F\right)+3bc^2dx\log\left(F\right)+3bcd^2x^2\log\left(F\right)+bd^3x^3\log\left(F\right)-1\right)}{3b^2d\log\left(F\right)^2} & \text{for}\ b^2d\log\left(F\right)^2\neq 0 \\ c^5x+\frac{5c^4dx^2}{2}+\frac{10c^3d^2x^3}{3}+\frac{5c^2d^3x^4}{2}+cd^4x^5+\frac{d^5x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**5,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**3)*(b*c**3*log(F) + 3*b*c**2*d*x*log(F) +
3*b*c*d**2*x**2*log(F) + b*d**3*x**3*log(F) - 1)/(3*b**2*d*log(F)**2), Ne(
b**2*d*log(F)**2, 0)), (c**5*x + 5*c**4*d*x**2/2 + 10*c**3*d**2*x**3/3 + 5
*c**2*d**3*x**4/2 + c*d**4*x**5 + d**5*x**6/6, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(58) = 116$.

Time = 0.16 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.15

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx = \frac{\left(F^{bc^3+a}bd^3x^3\log(F) + 3\,F^{bc^3+a}bcd^2x^2\log(F) + 3\,F^{bc^3+a}bc^2dx\log(F) + F^{bc^3+a}bc^3\log(F) - F^{bc^3+a}\right)e^{(b}}{3\,b^2d\log(F)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/3*(F^(b*c^3 + a)*b*d^3*x^3*log(F) + 3*F^(b*c^3 + a)*b*c*d^2*x^2*log(F) +
 3*F^(b*c^3 + a)*b*c^2*d*x*log(F) + F^(b*c^3 + a)*b*c^3*log(F) - F^(b*c^3
+ a))*e^(b*d^3*x^3*log(F) + 3*b*c*d^2*x^2*log(F) + 3*b*c^2*d*x*log(F))/(b^
2*d*log(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1014, normalized size of antiderivative = 16.35

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^5,x, algorithm="giac")
```

output
```
1/6*(2*(2*(((d*x + c)^3*b*log(abs(F)) - 1)*(pi^2*b^2*sgn(F) - pi^2*b^2 + 2
*b^2*log(abs(F))^2)/((pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)^2
+ 4*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F)))^2) + (pi*(d*x + c)^3*
b*sgn(F) - pi*(d*x + c)^3*b)*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F
)))/((pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)^2 + 4*(pi*b^2*log(
abs(F))*sgn(F) - pi*b^2*log(abs(F)))^2))*cos(-1/2*pi*b*d^3*x^3*sgn(F) + 1/
2*pi*b*d^3*x^3 - 3/2*pi*b*c*d^2*x^2*sgn(F) + 3/2*pi*b*c*d^2*x^2 - 3/2*pi*b
*c^2*d*x*sgn(F) + 3/2*pi*b*c^2*d*x - 1/2*pi*b*c^3*sgn(F) + 1/2*pi*b*c^3 -
1/2*pi*a*sgn(F) + 1/2*pi*a) + ((pi*(d*x + c)^3*b*sgn(F) - pi*(d*x + c)^3*b
)*(pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))^2)/((pi^2*b^2*sgn(F) - p
i^2*b^2 + 2*b^2*log(abs(F))^2)^2 + 4*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*l
og(abs(F)))^2) - 4*((d*x + c)^3*b*log(abs(F)) - 1)*(pi*b^2*log(abs(F))*sgn
(F) - pi*b^2*log(abs(F)))/((pi^2*b^2*sgn(F) - pi^2*b^2 + 2*b^2*log(abs(F))
^2)^2 + 4*(pi*b^2*log(abs(F))*sgn(F) - pi*b^2*log(abs(F)))^2))*sin(-1/2*pi
*b*d^3*x^3*sgn(F) + 1/2*pi*b*d^3*x^3 - 3/2*pi*b*c*d^2*x^2*sgn(F) + 3/2*pi*
b*c*d^2*x^2 - 3/2*pi*b*c^2*d*x*sgn(F) + 3/2*pi*b*c^2*d*x - 1/2*pi*b*c^3*sg
n(F) + 1/2*pi*b*c^3 - 1/2*pi*a*sgn(F) + 1/2*pi*a))*e^((d*x + c)^3*b*log(ab
s(F)) + a*log(abs(F))) - I*((pi*(d*x + c)^3*b*sgn(F) - pi*(d*x + c)^3*b -
2*I*(d*x + c)^3*b*log(abs(F)) + 2*I)*e^(1/2*I*pi*b*d^3*x^3*sgn(F) - 1/2*I*
pi*b*d^3*x^3 + 3/2*I*pi*b*c*d^2*x^2*sgn(F) - 3/2*I*pi*b*c*d^2*x^2 + 3/2...
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.53

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx = \frac{F^{b\,d^3\,x^3}\,F^{3\,b\,c^2\,d\,x}\,F^a\,F^{b\,c^3}\,F^{3\,b\,c\,d^2\,x^2}\,\left(b\,\ln\left(F\right)\,c^3+3\,b\,\ln\left(F\right)\,c^2\,d\,x+3\,b\,\ln\left(F\right)\,c\,d^2\,x^2+b\,\ln\left(F\right)\,d^3\,x^3-1\right)}{3\,b^2\,d\ln\left(F\right)^2}$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^5,x)
```

output
```
(F^(b*d^3*x^3)*F^(3*b*c^2*d*x)*F^a*F^(b*c^3)*F^(3*b*c*d^2*x^2)*(b*c^3*log(
F) + b*d^3*x^3*log(F) + 3*b*c^2*d*x*log(F) + 3*b*c*d^2*x^2*log(F) - 1))/(3
*b^2*d*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int F^{a+b(c+dx)^3}(c+dx)^5\,dx$$
$$=\frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}(\log(f)\,b\,c^3+3\log(f)\,b\,c^2dx+3\log(f)\,bc\,d^2x^2+\log(f)\,b\,d^3x^3-1)}{3\log{(f)}^2\,b^2d}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^5,x)
```

output
```
(f**(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*(log(f)*b*
c**3 + 3*log(f)*b*c**2*d*x + 3*log(f)*b*c*d**2*x**2 + log(f)*b*d**3*x**3 -
 1))/(3*log(f)**2*b**2*d)
```

3.220 $\int F^{a+b(c+dx)^3}(c+dx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx=\frac{F^{a+b(c+dx)^3}}{3bd\log(F)}$$

output
```
1/3*F^(a+b*(d*x+c)^3)/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx=\frac{F^{a+b(c+dx)^3}}{3bd\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^2,x]
```

output
```
F^(a + b*(c + d*x)^3)/(3*b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 F^{a+b(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{F^{a+b(c+dx)^3}}{3bd\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^2,x]
```

output
```
F^(a + b*(c + d*x)^3)/(3*b*d*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{F^{a+b(dx+c)^3}}{3bd\ln(F)}$	26
default	$\frac{F^{a+b(dx+c)^3}}{3bd\ln(F)}$	26
parallelrisch	$\frac{F^{a+b(dx+c)^3}}{3bd\ln(F)}$	26
orering	$\frac{F^{a+b(dx+c)^3}}{3bd\ln(F)}$	26
norman	$\frac{e^{\left(a+b(dx+c)^3\right)\ln(F)}}{3\ln(F)bd}$	28
gosper	$\frac{F^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}}{3bd\ln(F)}$	48
risch	$\frac{F^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}}{3bd\ln(F)}$	48

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*F^(a+b*(d*x+c)^3)/b/d/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx = \frac{F^{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}}{3\,bd\log\left(F\right)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(19) = 38$.

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx = \begin{cases} \frac{F^{a+b(c+dx)^3}}{3bd\log(F)} & \text{for } bd\log(F) \neq 0 \\ c^2x + cdx^2 + \frac{d^2x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**2,x)
```

output
```
Piecewise((F**(a + b*(c + d*x)**3)/(3*b*d*log(F)), Ne(b*d*log(F), 0)), (c*
*2*x + c*d*x**2 + d**2*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx = \frac{F^{(dx+c)^3b+a}}{3\,bd\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*F^((d*x + c)^3*b + a)/(b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx = \frac{F^{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}}{3\,bd\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x, algorithm="giac")
```

output
```
1/3*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx = \frac{F^{a+b\,(c+d\,x)^3}}{3\,b\,d\,\ln\left(F\right)}$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^2,x)
```

output
```
F^(a + b*(c + d*x)^3)/(3*b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int F^{a+b(c+dx)^3}(c+dx)^2\,dx = \frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx+b\,c^3+a}}{3\log\left(f\right)bd}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^2,x)
```

output
```
f**(a + b*c**3 + 3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*log(f)*b
*d)
```

3.221 $\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx=\frac{F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)}{3d}$$

output
```
1/3*F^a*Ei(b*(d*x+c)^3*ln(F))/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx=\frac{F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)}{3d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^3*Log[F]])/(3*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx} \, dx$$

$$\Big\downarrow \; 2639$$

$$\frac{F^a \text{ExpIntegralEi}\left(b(c+dx)^3 \log(F)\right)}{3d}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^3*Log[F]])/(3*d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{dx+c} dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c),x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx = \frac{F^a \operatorname{Ei}\left(\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3\right)\log\left(F\right)\right)}{3\,d}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c),x, algorithm="fricas")
```

output
```
1/3*F^a*Ei((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))/d
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx = \int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c),x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx = \int \frac{F^{(dx+c)^3b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx = \int \frac{F^{(dx+c)^3b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx = \frac{F^a\,\mathrm{ei}\left(b\,\ln\left(F\right)\,\left(c+d\,x\right)^3\right)}{3\,d}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x),x)
```

output
```
(F^a*ei(b*log(F)*(c + d*x)^3))/(3*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{c+dx}\,dx = f^{b\,c^3+a}\left(\int \frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}}{dx+c}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c),x)
```

output
```
f**(a + b*c**3)*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(c +
 d*x),x)
```

3.222 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx = -\frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3} + \frac{bF^a \text{ExpIntegralEi}\left(b(c+dx)^3 \log(F)\right) \log(F)}{3d}$$

output
```
-1/3*F^(a+b*(d*x+c)^3)/d/(d*x+c)^3+1/3*b*F^a*Ei(b*(d*x+c)^3*ln(F))*ln(F)/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} dx = \frac{F^a\left(-\frac{F^{b(c+dx)^3}}{(c+dx)^3} + b\,\text{ExpIntegralEi}\left(b(c+dx)^3 \log(F)\right) \log(F)\right)}{3d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^4,x]
```

output
```
(F^a*(-(F^(b*(c + d*x)^3)/(c + d*x)^3) + b*ExpIntegralEi[b*(c + d*x)^3*Log
[F]]*Log[F]))/(3*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$b\log(F)\int \frac{F^{b(c+dx)^3+a}}{c+dx}dx-\frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)}{3d}-\frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^4,x]
```

output
```
-1/3*F^(a + b*(c + d*x)^3)/(d*(c + d*x)^3) + (b*F^a*ExpIntegralEi[b*(c + d
*x)^3*Log[F]]*Log[F])/(3*d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^4}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.77

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4}\,dx = \frac{(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3)F^a\mathrm{Ei}((bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3)\log(F))\log(F) - F^{bd^3x^3+3\,bc}}{3\,(d^4x^3 + 3\,cd^3x^2 + 3\,c^2d^2x + c^3d)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*((b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*F^a*Ei((b*d^3*x^3 +
 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))*log(F) - F^(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x
 + c^3*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**4,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**4, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} \, dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^4} \, dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} \, dx = -\frac{F^a \left(F^{b\,(c+d\,x)^3} + b\,\ln\left(F\right)\,\mathrm{expint}\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^3\right)\,\left(c+d\,x\right)^3\right)}{3\,d\,\left(c+d\,x\right)^3}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^4,x)
```

output
```
-(F^a*(F^(b*(c + d*x)^3) + b*log(F)*expint(-b*log(F)*(c + d*x)^3)*(c + d*x
)^3))/(3*d*(c + d*x)^3)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^4} \, dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^4,x)
```

output
```
(f**(a + b*c**3)*( - 2*f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) +
 9*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**7 +
4*log(f)*b*c**6*d*x + 6*log(f)*b*c**5*d**2*x**2 + 4*log(f)*b*c**4*d**3*x**
3 + log(f)*b*c**3*d**4*x**4 - c**4 - 4*c**3*d*x - 6*c**2*d**2*x**2 - 4*c*d
**3*x**3 - d**4*x**4),x)*log(f)**2*b**2*c**9*d + 27*int(f**(3*b*c**2*d*x +
 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**7 + 4*log(f)*b*c**6*d*x + 6*l
og(f)*b*c**5*d**2*x**2 + 4*log(f)*b*c**4*d**3*x**3 + log(f)*b*c**3*d**4*x*
*4 - c**4 - 4*c**3*d*x - 6*c**2*d**2*x**2 - 4*c*d**3*x**3 - d**4*x**4),x)*
log(f)**2*b**2*c**8*d**2*x + 27*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b
*d**3*x**3)/(log(f)*b*c**7 + 4*log(f)*b*c**6*d*x + 6*log(f)*b*c**5*d**2*x*
*2 + 4*log(f)*b*c**4*d**3*x**3 + log(f)*b*c**3*d**4*x**4 - c**4 - 4*c**3*d
*x - 6*c**2*d**2*x**2 - 4*c*d**3*x**3 - d**4*x**4),x)*log(f)**2*b**2*c**7*
d**3*x**2 + 9*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f
)*b*c**7 + 4*log(f)*b*c**6*d*x + 6*log(f)*b*c**5*d**2*x**2 + 4*log(f)*b*c*
*4*d**3*x**3 + log(f)*b*c**3*d**4*x**4 - c**4 - 4*c**3*d*x - 6*c**2*d**2*x
**2 - 4*c*d**3*x**3 - d**4*x**4),x)*log(f)**2*b**2*c**6*d**4*x**3 - 18*int
(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**7 + 4*log(
f)*b*c**6*d*x + 6*log(f)*b*c**5*d**2*x**2 + 4*log(f)*b*c**4*d**3*x**3 + lo
g(f)*b*c**3*d**4*x**4 - c**4 - 4*c**3*d*x - 6*c**2*d**2*x**2 - 4*c*d**3*x*
*3 - d**4*x**4),x)*log(f)*b*c**6*d - 54*int(f**(3*b*c**2*d*x + 3*b*c*d*...
```

3.223 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} \, dx = -\frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6} - \frac{bF^{a+b(c+dx)^3}\log(F)}{6d(c+dx)^3} + \frac{b^2F^a \text{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)\log^2(F)}{6d}$$

output
```
-1/6*F^(a+b*(d*x+c)^3)/d/(d*x+c)^6-1/6*b*F^(a+b*(d*x+c)^3)*ln(F)/d/(d*x+c)
^3+1/6*b^2*F^a*Ei(b*(d*x+c)^3*ln(F))*ln(F)^2/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} \, dx = \frac{F^a\left(b^2 \text{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)\log^2(F) - \frac{F^{b(c+dx)^3}\left(1+b(c+dx)^3\log(F)\right)}{(c+dx)^6}\right)}{6d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^7,x]
```

output
```
(F^a*(b^2*ExpIntegralEi[b*(c + d*x)^3*Log[F]]*Log[F]^2 - (F^(b*(c + d*x)^3
)*(1 + b*(c + d*x)^3*Log[F]))/(c + d*x)^6))/(6*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(F) \int \frac{F^{b(c+dx)^3+a}}{(c+dx)^4} dx - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{2} b \log(F) \left(b \log(F) \int \frac{F^{b(c+dx)^3+a}}{c+dx} dx - \frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3} \right) - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{2} b \log(F) \left(\frac{bF^a \log(F) \operatorname{ExpIntegralEi}\left(b(c+dx)^3 \log(F)\right)}{3d} - \frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3} \right) - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^7,x]
```

output
```
-1/6*F^(a + b*(c + d*x)^3)/(d*(c + d*x)^6) + (b*Log[F]*(-1/3*F^(a + b*(c +
 d*x)^3)/(d*(c + d*x)^3) + (b*F^a*ExpIntegralEi[b*(c + d*x)^3*Log[F]]*Log[
F])/(3*d)))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^7}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^7,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^7,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(81) = 162.

Time = 0.08 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.09

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx = \frac{(b^2d^6x^6 + 6\,b^2cd^5x^5 + 15\,b^2c^2d^4x^4 + 20\,b^2c^3d^3x^3 + 15\,b^2c^4d^2x^2 + 6\,b^2c^5dx + b^2c^6)F^a\mathrm{Ei}((bd^3x^3 + 3\,bcd^2x^2}{6\,(d^7x^6 + 6\,cd^6x^5 + 15\,c^2d^5x^4 +}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/6*((b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*
x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*F^a*Ei((b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))*log(F)^2 - ((b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F) + 1)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3 + a))/(d^7*x^6 + 6*c*d^6*x^5 + 15*c^2*d^5*x^4 + 20*c^3
*d^4*x^3 + 15*c^4*d^3*x^2 + 6*c^5*d^2*x + c^6*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**7,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**7, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^7, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx = -\frac{F^a\,b^2\ln(F)^2\left(\frac{\mathrm{expint}\left(-b\,\ln(F)\,(c+d\,x)^3\right)}{2}+F^{b\,(c+d\,x)^3}\left(\frac{1}{2\,b\,\ln(F)\,(c+d\,x)^3}+\frac{1}{2\,b^2\,\ln(F)^2\,(c+d\,x)^6}\right)\right)}{3\,d}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^7,x)
```

output
```
-(F^a*b^2*log(F)^2*(expint(-b*log(F)*(c + d*x)^3)/2 + F^(b*(c + d*x)^3)*(1
/(2*b*log(F)*(c + d*x)^3) + 1/(2*b^2*log(F)^2*(c + d*x)^6))))/(3*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^7}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^7,x)
```

output
```
(f**(a + b*c**3)*( - 2*f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) +
 9*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**10 +
 7*log(f)*b*c**9*d*x + 21*log(f)*b*c**8*d**2*x**2 + 35*log(f)*b*c**7*d**3*
x**3 + 35*log(f)*b*c**6*d**4*x**4 + 21*log(f)*b*c**5*d**5*x**5 + 7*log(f)*
b*c**4*d**6*x**6 + log(f)*b*c**3*d**7*x**7 - 2*c**7 - 14*c**6*d*x - 42*c**
5*d**2*x**2 - 70*c**4*d**3*x**3 - 70*c**3*d**4*x**4 - 42*c**2*d**5*x**5 -
14*c*d**6*x**6 - 2*d**7*x**7),x)*log(f)**2*b**2*c**12*d + 54*int(f**(3*b*c
**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**10 + 7*log(f)*b*c**9
*d*x + 21*log(f)*b*c**8*d**2*x**2 + 35*log(f)*b*c**7*d**3*x**3 + 35*log(f)
*b*c**6*d**4*x**4 + 21*log(f)*b*c**5*d**5*x**5 + 7*log(f)*b*c**4*d**6*x**6
 + log(f)*b*c**3*d**7*x**7 - 2*c**7 - 14*c**6*d*x - 42*c**5*d**2*x**2 - 70
*c**4*d**3*x**3 - 70*c**3*d**4*x**4 - 42*c**2*d**5*x**5 - 14*c*d**6*x**6 -
 2*d**7*x**7),x)*log(f)**2*b**2*c**11*d**2*x + 135*int(f**(3*b*c**2*d*x +
3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**10 + 7*log(f)*b*c**9*d*x + 21*
log(f)*b*c**8*d**2*x**2 + 35*log(f)*b*c**7*d**3*x**3 + 35*log(f)*b*c**6*d*
*4*x**4 + 21*log(f)*b*c**5*d**5*x**5 + 7*log(f)*b*c**4*d**6*x**6 + log(f)*
b*c**3*d**7*x**7 - 2*c**7 - 14*c**6*d*x - 42*c**5*d**2*x**2 - 70*c**4*d**3
*x**3 - 70*c**3*d**4*x**4 - 42*c**2*d**5*x**5 - 14*c*d**6*x**6 - 2*d**7*x*
*7),x)*log(f)**2*b**2*c**10*d**3*x**2 + 180*int(f**(3*b*c**2*d*x + 3*b*c*d
**2*x**2 + b*d**3*x**3)/(log(f)*b*c**10 + 7*log(f)*b*c**9*d*x + 21*log(...
```

3.224 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx=-\frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9}-\frac{bF^{a+b(c+dx)^3}\log(F)}{18d(c+dx)^6}-\frac{b^2F^{a+b(c+dx)^3}\log^2(F)}{18d(c+dx)^3}+\frac{b^3F^a\,\text{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)\log^3(F)}{18d}$$

output
```
-1/9*F^(a+b*(d*x+c)^3)/d/(d*x+c)^9-1/18*b*F^(a+b*(d*x+c)^3)*ln(F)/d/(d*x+c
)^6-1/18*b^2*F^(a+b*(d*x+c)^3)*ln(F)^2/d/(d*x+c)^3+1/18*b^3*F^a*Ei(b*(d*x+
c)^3*ln(F))*ln(F)^3/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.66

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx=\frac{F^a\left(b^3\,\text{ExpIntegralEi}\left(b(c+dx)^3\log(F)\right)\log^3(F)+\frac{F^{b(c+dx)^3}\left(-2-b(c+dx)^3\log(F)-b^2(c+dx)^6\log^2(F)\right)}{(c+dx)^9}\right)}{18d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^10,x]
```

output
```
(F^a*(b^3*ExpIntegralEi[b*(c + d*x)^3*Log[F]]*Log[F]^3 + (F^(b*(c + d*x)^3
)*(-2 - b*(c + d*x)^3*Log[F] - b^2*(c + d*x)^6*Log[F]^2))/(c + d*x)^9))/(1
8*d)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(F) \int \frac{F^{b(c+dx)^3+a}}{(c+dx)^7} dx - \frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \int \frac{F^{b(c+dx)^3+a}}{(c+dx)^4} dx - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6}\right) - \frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \left(b \log(F) \int \frac{F^{b(c+dx)^3+a}}{c+dx} dx - \frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3}\right) - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6}\right) - \frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \left(\frac{bF^a \log(F) \text{ExpIntegralEi}\left(b(c+dx)^3 \log(F)\right)}{3d} - \frac{F^{a+b(c+dx)^3}}{3d(c+dx)^3}\right) - \frac{F^{a+b(c+dx)^3}}{6d(c+dx)^6}\right) - \frac{F^{a+b(c+dx)^3}}{9d(c+dx)^9}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^10,x]
```

output
```
-1/9*F^(a + b*(c + d*x)^3)/(d*(c + d*x)^9) + (b*Log[F]*(-1/6*F^(a + b*(c +
 d*x)^3)/(d*(c + d*x)^6) + (b*Log[F]*(-1/3*F^(a + b*(c + d*x)^3)/(d*(c + d
*x)^3) + (b*F^a*ExpIntegralEi[b*(c + d*x)^3*Log[F]]*Log[F])/(3*d)))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^{10}}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^10,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^10,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(113) = 226$.

Time = 0.08 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.56

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx = \frac{(b^3d^9x^9 + 9\,b^3cd^8x^8 + 36\,b^3c^2d^7x^7 + 84\,b^3c^3d^6x^6 + 126\,b^3c^4d^5x^5 + 126\,b^3c^5d^4x^4 + 84\,b^3c^6d^3x^3 + 36\,b^3c^7d^2}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^10,x, algorithm="fricas")
```

output
```
1/18*((b^3*d^9*x^9 + 9*b^3*c*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6
*x^6 + 126*b^3*c^4*d^5*x^5 + 126*b^3*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36
*b^3*c^7*d^2*x^2 + 9*b^3*c^8*d*x + b^3*c^9)*F^a*Ei((b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3)*log(F))*log(F)^3 - ((b^2*d^6*x^6 + 6*b^2*c*d^5*
x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2
*c^5*d*x + b^2*c^6)*log(F)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
b*c^3)*log(F) + 2)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)
)/(d^10*x^9 + 9*c*d^9*x^8 + 36*c^2*d^8*x^7 + 84*c^3*d^7*x^6 + 126*c^4*d^6*
x^5 + 126*c^5*d^5*x^4 + 84*c^6*d^4*x^3 + 36*c^7*d^3*x^2 + 9*c^8*d^2*x + c^
9*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**10,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**10, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^10,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^10, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^10,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.86

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx = -\frac{F^a\,b^3\ln(F)^3\,\mathrm{expint}\left(-b\,\ln(F)\,(c+d\,x)^3\right)}{18\,d} - \frac{F^a\,F^{b\,(c+d\,x)^3}\,b^3\ln(F)^3\left(\frac{1}{6\,b\,\ln(F)\,(c+d\,x)^3}+\frac{1}{6\,b^2\ln(F)^2\,(c+d\,x)^6}+\frac{1}{3\,b^3\ln(F)^3\,(c+d\,x)^9}\right)}{3\,d}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^10,x)
```

output
```
- (F^a*b^3*log(F)^3*expint(-b*log(F)*(c + d*x)^3))/(18*d) - (F^a*F^(b*(c +
 d*x)^3)*b^3*log(F)^3*(1/(6*b*log(F)*(c + d*x)^3) + 1/(6*b^2*log(F)^2*(c +
 d*x)^6) + 1/(3*b^3*log(F)^3*(c + d*x)^9)))/(3*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{10}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^10,x)
```

output
```
(f**(a + b*c**3)*( - 2*f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) +
 9*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**13 +
 10*log(f)*b*c**12*d*x + 45*log(f)*b*c**11*d**2*x**2 + 120*log(f)*b*c**10*
d**3*x**3 + 210*log(f)*b*c**9*d**4*x**4 + 252*log(f)*b*c**8*d**5*x**5 + 21
0*log(f)*b*c**7*d**6*x**6 + 120*log(f)*b*c**6*d**7*x**7 + 45*log(f)*b*c**5
*d**8*x**8 + 10*log(f)*b*c**4*d**9*x**9 + log(f)*b*c**3*d**10*x**10 - 3*c*
*10 - 30*c**9*d*x - 135*c**8*d**2*x**2 - 360*c**7*d**3*x**3 - 630*c**6*d**
4*x**4 - 756*c**5*d**5*x**5 - 630*c**4*d**6*x**6 - 360*c**3*d**7*x**7 - 13
5*c**2*d**8*x**8 - 30*c*d**9*x**9 - 3*d**10*x**10),x)*log(f)**2*b**2*c**15
*d + 81*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c*
*13 + 10*log(f)*b*c**12*d*x + 45*log(f)*b*c**11*d**2*x**2 + 120*log(f)*b*c
**10*d**3*x**3 + 210*log(f)*b*c**9*d**4*x**4 + 252*log(f)*b*c**8*d**5*x**5
 + 210*log(f)*b*c**7*d**6*x**6 + 120*log(f)*b*c**6*d**7*x**7 + 45*log(f)*b
*c**5*d**8*x**8 + 10*log(f)*b*c**4*d**9*x**9 + log(f)*b*c**3*d**10*x**10 -
 3*c**10 - 30*c**9*d*x - 135*c**8*d**2*x**2 - 360*c**7*d**3*x**3 - 630*c**
6*d**4*x**4 - 756*c**5*d**5*x**5 - 630*c**4*d**6*x**6 - 360*c**3*d**7*x**7
 - 135*c**2*d**8*x**8 - 30*c*d**9*x**9 - 3*d**10*x**10),x)*log(f)**2*b**2*
c**14*d**2*x + 324*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(
log(f)*b*c**13 + 10*log(f)*b*c**12*d*x + 45*log(f)*b*c**11*d**2*x**2 + 120
*log(f)*b*c**10*d**3*x**3 + 210*log(f)*b*c**9*d**4*x**4 + 252*log(f)*b*...
```

3.225 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = -\frac{b^4F^a\Gamma(-4,-b(c+dx)^3\log(F))\log^4(F)}{3d}$$

output
```
-1/3*F^a/(d*x+c)^12*Ei(5,-b*(d*x+c)^3*ln(F))/d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = -\frac{b^4F^a\Gamma(-4,-b(c+dx)^3\log(F))\log^4(F)}{3d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^13,x]
```

output
```
-1/3*(b^4*F^a*Gamma[-4, -(b*(c + d*x)^3*Log[F])]*Log[F]^4)/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{b^4 F^a \log^4(F) \Gamma\left(-4,-b(c+dx)^3 \log(F)\right)}{3d}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^13,x]
```

output
```
-1/3*(b^4*F^a*Gamma[-4, -(b*(c + d*x)^3*Log[F])]*Log[F]^4)/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^{13}}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 636 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 636, normalized size of antiderivative = 20.52

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = \frac{(b^4d^{12}x^{12} + 12\,b^4cd^{11}x^{11} + 66\,b^4c^2d^{10}x^{10} + 220\,b^4c^3d^9x^9 + 495\,b^4c^4d^8x^8 + 792\,b^4c^5d^7x^7 + 924\,b^4c^6d^6x^6 +}{}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x, algorithm="fricas")
```

output
```
1/72*((b^4*d^12*x^12 + 12*b^4*c*d^11*x^11 + 66*b^4*c^2*d^10*x^10 + 220*b^4
*c^3*d^9*x^9 + 495*b^4*c^4*d^8*x^8 + 792*b^4*c^5*d^7*x^7 + 924*b^4*c^6*d^6
*x^6 + 792*b^4*c^7*d^5*x^5 + 495*b^4*c^8*d^4*x^4 + 220*b^4*c^9*d^3*x^3 + 6
6*b^4*c^10*d^2*x^2 + 12*b^4*c^11*d*x + b^4*c^12)*F^a*Ei((b*d^3*x^3 + 3*b*c
*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))*log(F)^4 - ((b^3*d^9*x^9 + 9*b^3*c
*d^8*x^8 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6*x^6 + 126*b^3*c^4*d^5*x^5 +
 126*b^3*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36*b^3*c^7*d^2*x^2 + 9*b^3*c^8
*d*x + b^3*c^9)*log(F)^3 + (b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4
*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*
log(F)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F) + 6)
*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(d^13*x^12 + 12*
c*d^12*x^11 + 66*c^2*d^11*x^10 + 220*c^3*d^10*x^9 + 495*c^4*d^9*x^8 + 792*
c^5*d^8*x^7 + 924*c^6*d^7*x^6 + 792*c^7*d^6*x^5 + 495*c^8*d^5*x^4 + 220*c^
9*d^4*x^3 + 66*c^10*d^3*x^2 + 12*c^11*d^2*x + c^12*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**13,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**13, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{13}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^13, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{13}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^13, x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = -\frac{F^a\,b^4\ln(F)^4\,\mathrm{expint}\left(-b\,\ln(F)\,(c+d\,x)^3\right)}{72\,d} - \frac{F^a\,F^{b\,(c+d\,x)^3}\,b^4\ln(F)^4\left(\frac{1}{24\,b\,\ln(F)\,(c+d\,x)^3}+\frac{1}{24\,b^2\ln(F)^2\,(c+d\,x)^6}+\frac{1}{12\,b^3\ln(F)^3\,(c+d\,x)^9}+\frac{1}{4\,b^4\ln(F)^4\,(c+d\,x)^{12}}\right)}{3\,d}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^13,x)
```

output
```
- (F^a*b^4*log(F)^4*expint(-b*log(F)*(c + d*x)^3))/(72*d) - (F^a*F^(b*(c +
 d*x)^3)*b^4*log(F)^4*(1/(24*b*log(F)*(c + d*x)^3) + 1/(24*b^2*log(F)^2*(c
 + d*x)^6) + 1/(12*b^3*log(F)^3*(c + d*x)^9) + 1/(4*b^4*log(F)^4*(c + d*x)
^12)))/(3*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{13}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^13,x)
```

output
```
(f**(a + b*c**3)*( - 2*f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) +
 9*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**16 +
 13*log(f)*b*c**15*d*x + 78*log(f)*b*c**14*d**2*x**2 + 286*log(f)*b*c**13*
d**3*x**3 + 715*log(f)*b*c**12*d**4*x**4 + 1287*log(f)*b*c**11*d**5*x**5 +
 1716*log(f)*b*c**10*d**6*x**6 + 1716*log(f)*b*c**9*d**7*x**7 + 1287*log(f
)*b*c**8*d**8*x**8 + 715*log(f)*b*c**7*d**9*x**9 + 286*log(f)*b*c**6*d**10
*x**10 + 78*log(f)*b*c**5*d**11*x**11 + 13*log(f)*b*c**4*d**12*x**12 + log
(f)*b*c**3*d**13*x**13 - 4*c**13 - 52*c**12*d*x - 312*c**11*d**2*x**2 - 11
44*c**10*d**3*x**3 - 2860*c**9*d**4*x**4 - 5148*c**8*d**5*x**5 - 6864*c**7
*d**6*x**6 - 6864*c**6*d**7*x**7 - 5148*c**5*d**8*x**8 - 2860*c**4*d**9*x*
*9 - 1144*c**3*d**10*x**10 - 312*c**2*d**11*x**11 - 52*c*d**12*x**12 - 4*d
**13*x**13),x)*log(f)**2*b**2*c**18*d + 108*int(f**(3*b*c**2*d*x + 3*b*c*d
**2*x**2 + b*d**3*x**3)/(log(f)*b*c**16 + 13*log(f)*b*c**15*d*x + 78*log(f
)*b*c**14*d**2*x**2 + 286*log(f)*b*c**13*d**3*x**3 + 715*log(f)*b*c**12*d*
*4*x**4 + 1287*log(f)*b*c**11*d**5*x**5 + 1716*log(f)*b*c**10*d**6*x**6 +
1716*log(f)*b*c**9*d**7*x**7 + 1287*log(f)*b*c**8*d**8*x**8 + 715*log(f)*b
*c**7*d**9*x**9 + 286*log(f)*b*c**6*d**10*x**10 + 78*log(f)*b*c**5*d**11*x
**11 + 13*log(f)*b*c**4*d**12*x**12 + log(f)*b*c**3*d**13*x**13 - 4*c**13
- 52*c**12*d*x - 312*c**11*d**2*x**2 - 1144*c**10*d**3*x**3 - 2860*c**9*d*
*4*x**4 - 5148*c**8*d**5*x**5 - 6864*c**7*d**6*x**6 - 6864*c**6*d**7*x*...
```

3.226 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} \, dx = \frac{b^5 F^a \Gamma(-5, -b(c+dx)^3 \log(F)) \log^5(F)}{3d}$$

output
```
-1/3*F^a/(d*x+c)^15*Ei(6,-b*(d*x+c)^3*ln(F))/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} \, dx = \frac{b^5 F^a \Gamma(-5, -b(c+dx)^3 \log(F)) \log^5(F)}{3d}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^16,x]
```

output
```
(b^5*F^a*Gamma[-5, -(b*(c + d*x)^3*Log[F])]*Log[F]^5)/(3*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{b^5 F^a \log^5(F)\Gamma\left(-5,-b(c+dx)^3\log(F)\right)}{3d}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^16,x]
```

output
```
(b^5*F^a*Gamma[-5, -(b*(c + d*x)^3*Log[F])]*Log[F]^5)/(3*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^{16}}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 883, normalized size of antiderivative = 28.48

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x, algorithm="fricas")
```

output
```
1/360*((b^5*d^15*x^15 + 15*b^5*c*d^14*x^14 + 105*b^5*c^2*d^13*x^13 + 455*b
^5*c^3*d^12*x^12 + 1365*b^5*c^4*d^11*x^11 + 3003*b^5*c^5*d^10*x^10 + 5005*
b^5*c^6*d^9*x^9 + 6435*b^5*c^7*d^8*x^8 + 6435*b^5*c^8*d^7*x^7 + 5005*b^5*c
^9*d^6*x^6 + 3003*b^5*c^10*d^5*x^5 + 1365*b^5*c^11*d^4*x^4 + 455*b^5*c^12*
d^3*x^3 + 105*b^5*c^13*d^2*x^2 + 15*b^5*c^14*d*x + b^5*c^15)*F^a*Ei((b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))*log(F)^5 - ((b^4*d^12*
x^12 + 12*b^4*c*d^11*x^11 + 66*b^4*c^2*d^10*x^10 + 220*b^4*c^3*d^9*x^9 + 4
95*b^4*c^4*d^8*x^8 + 792*b^4*c^5*d^7*x^7 + 924*b^4*c^6*d^6*x^6 + 792*b^4*c
^7*d^5*x^5 + 495*b^4*c^8*d^4*x^4 + 220*b^4*c^9*d^3*x^3 + 66*b^4*c^10*d^2*x
^2 + 12*b^4*c^11*d*x + b^4*c^12)*log(F)^4 + (b^3*d^9*x^9 + 9*b^3*c*d^8*x^8
 + 36*b^3*c^2*d^7*x^7 + 84*b^3*c^3*d^6*x^6 + 126*b^3*c^4*d^5*x^5 + 126*b^3
*c^5*d^4*x^4 + 84*b^3*c^6*d^3*x^3 + 36*b^3*c^7*d^2*x^2 + 9*b^3*c^8*d*x + b
^3*c^9)*log(F)^3 + 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 +
 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*log(F)
^2 + 6*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F) + 24)*F^(b
*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a))/(d^16*x^15 + 15*c*d^1
5*x^14 + 105*c^2*d^14*x^13 + 455*c^3*d^13*x^12 + 1365*c^4*d^12*x^11 + 3003
*c^5*d^11*x^10 + 5005*c^6*d^10*x^9 + 6435*c^7*d^9*x^8 + 6435*c^8*d^8*x^7 +
 5005*c^9*d^7*x^6 + 3003*c^10*d^6*x^5 + 1365*c^11*d^5*x^4 + 455*c^12*d^4*x
^3 + 105*c^13*d^3*x^2 + 15*c^14*d^2*x + c^15*d)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**16,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**16, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{16}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^16, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^{16}}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^16, x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.39

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx = -\frac{F^a\,b^5\ln\left(F\right)^5\,\mathrm{expint}\left(-b\,\ln\left(F\right)\,\left(c+d\,x\right)^3\right)}{360\,d}$$
$$-\frac{F^a\,F^{b\,(c+d\,x)^3}\,b^5\ln\left(F\right)^5\left(\frac{1}{120\,b\,\ln(F)\,(c+d\,x)^3}+\frac{1}{120\,b^2\ln(F)^2\,(c+d\,x)^6}+\frac{1}{60\,b^3\ln(F)^3\,(c+d\,x)^9}+\frac{1}{20\,b^4\ln(F)^4\,(c+d\,x)^{12}}+\cdots\right.}{3\,d}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^16,x)
```

output
```
- (F^a*b^5*log(F)^5*expint(-b*log(F)*(c + d*x)^3))/(360*d) - (F^a*F^(b*(c
+ d*x)^3)*b^5*log(F)^5*(1/(120*b*log(F)*(c + d*x)^3) + 1/(120*b^2*log(F)^2
*(c + d*x)^6) + 1/(60*b^3*log(F)^3*(c + d*x)^9) + 1/(20*b^4*log(F)^4*(c +
d*x)^12) + 1/(5*b^5*log(F)^5*(c + d*x)^15)))/(3*d)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^{16}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^16,x)
```

output

```
(f**(a + b*c**3)*( - 2*f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) +
 9*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)*b*c**19 +
 16*log(f)*b*c**18*d*x + 120*log(f)*b*c**17*d**2*x**2 + 560*log(f)*b*c**16
*d**3*x**3 + 1820*log(f)*b*c**15*d**4*x**4 + 4368*log(f)*b*c**14*d**5*x**5
 + 8008*log(f)*b*c**13*d**6*x**6 + 11440*log(f)*b*c**12*d**7*x**7 + 12870*
log(f)*b*c**11*d**8*x**8 + 11440*log(f)*b*c**10*d**9*x**9 + 8008*log(f)*b*
c**9*d**10*x**10 + 4368*log(f)*b*c**8*d**11*x**11 + 1820*log(f)*b*c**7*d**
12*x**12 + 560*log(f)*b*c**6*d**13*x**13 + 120*log(f)*b*c**5*d**14*x**14 +
 16*log(f)*b*c**4*d**15*x**15 + log(f)*b*c**3*d**16*x**16 - 5*c**16 - 80*c
**15*d*x - 600*c**14*d**2*x**2 - 2800*c**13*d**3*x**3 - 9100*c**12*d**4*x*
*4 - 21840*c**11*d**5*x**5 - 40040*c**10*d**6*x**6 - 57200*c**9*d**7*x**7
- 64350*c**8*d**8*x**8 - 57200*c**7*d**9*x**9 - 40040*c**6*d**10*x**10 - 2
1840*c**5*d**11*x**11 - 9100*c**4*d**12*x**12 - 2800*c**3*d**13*x**13 - 60
0*c**2*d**14*x**14 - 80*c*d**15*x**15 - 5*d**16*x**16),x)*log(f)**2*b**2*c
**21*d + 135*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(log(f)
*b*c**19 + 16*log(f)*b*c**18*d*x + 120*log(f)*b*c**17*d**2*x**2 + 560*log(
f)*b*c**16*d**3*x**3 + 1820*log(f)*b*c**15*d**4*x**4 + 4368*log(f)*b*c**14
*d**5*x**5 + 8008*log(f)*b*c**13*d**6*x**6 + 11440*log(f)*b*c**12*d**7*x**
7 + 12870*log(f)*b*c**11*d**8*x**8 + 11440*log(f)*b*c**10*d**9*x**9 + 8008
*log(f)*b*c**9*d**10*x**10 + 4368*log(f)*b*c**8*d**11*x**11 + 1820*log(...
```

3.227 $\int F^{a+b(c+dx)^3}(c+dx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx=-\frac{F^a(c+dx)^4\Gamma\left(\frac{4}{3},-b(c+dx)^3\log(F)\right)}{3d\left(-b(c+dx)^3\log(F)\right)^{4/3}}$$

output
```
-1/3*F^a*(d*x+c)^4*GAMMA(4/3,-b*(d*x+c)^3*ln(F))/d/(-b*(d*x+c)^3*ln(F))^(4
/3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx=-\frac{F^a(c+dx)^4\Gamma\left(\frac{4}{3},-b(c+dx)^3\log(F)\right)}{3d\left(-b(c+dx)^3\log(F)\right)^{4/3}}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x)^3,x]
```

output
```
-1/3*(F^a*(c + d*x)^4*Gamma[4/3, -(b*(c + d*x)^3*Log[F])])/(d*(-(b*(c + d*
x)^3*Log[F]))^(4/3))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 F^{a+b(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a (c+dx)^4 \Gamma\left(\frac{4}{3},-b(c+dx)^3 \log(F)\right)}{3d \left(-b \log(F)(c+dx)^3\right)^{4/3}}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x)^3,x]
```

output
```
-1/3*(F^a*(c + d*x)^4*Gamma[4/3, -(b*(c + d*x)^3*Log[F])])/(d*(-(b*(c + d*
x)^3*Log[F]))^(4/3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^3} (dx+c)^3 \, dx$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x)
```

output
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(43) = 86$.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.41

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx$$
$$= \frac{3\,(bd^3x+bcd^2)F^{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3+a}\log\left(F\right)-\left(-bd^3\log\left(F\right)\right)^{\frac{2}{3}}F^a\Gamma\left(\frac{1}{3},-(bd^3x^3+3\,bcd^2x^2+3\,bc^2d}{9\,b^2d^3\log\left(F\right)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/9*(3*(b*d^3*x + b*c*d^2)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3 + a)*log(F) - (-b*d^3*log(F))^(2/3)*F^a*gamma(1/3, -(b*d^3*x^3 + 3*b*c
*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F)))/(b^2*d^3*log(F)^2)
```

Sympy [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx = \int F^{a+b(c+dx)^3}\left(c+dx\right)^3\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)*(c + d*x)**3, x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx=\int (dx+c)^3F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*F^((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx=\int (dx+c)^3F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*F^((d*x + c)^3*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx=\frac{F^a\,F^{b\,(c+d\,x)^3}\,(c+d\,x)}{3\,b\,d\,\ln(F)}-\frac{F^a\,\Gamma\left(\frac{1}{3},-b\,\ln(F)\,(c+d\,x)^3\right)\,(c+d\,x)^4}{9\,d\,\left(-b\,\ln(F)\,(c+d\,x)^3\right)^{4/3}}+\frac{2\,\pi\,\sqrt{3}\,F^a\,(c+d\,x)^4}{27\,d\,\Gamma\left(\frac{2}{3}\right)\,\left(-b\,\ln(F)\,(c+d\,x)^3\right)^{4/3}}$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x)^3,x)
```

output
```
(F^a*F^(b*(c + d*x)^3)*(c + d*x))/(3*b*d*log(F)) - (F^a*igamma(1/3, -b*log
(F)*(c + d*x)^3)*(c + d*x)^4)/(9*d*(-b*log(F)*(c + d*x)^3)^(4/3)) + (2*3^(
1/2)*F^a*pi*(c + d*x)^4)/(27*d*gamma(2/3)*(-b*log(F)*(c + d*x)^3)^(4/3))
```

Reduce [F]

$$\int F^{a+b(c+dx)^3}(c+dx)^3\,dx = \frac{f^{b\,c^3+a}\left(f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}c+f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}dx-\left(\int f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}dx\right)d\right)}{3\log\left(f\right)bd}$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c)^3,x)
```

output
```
(f**(a + b*c**3)*(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*c + f*
*(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*d*x - int(f**(3*b*c**2*d*x
 + 3*b*c*d**2*x**2 + b*d**3*x**3),x)*d))/(3*log(f)*b*d)
```

3.228 $\int F^{a+b(c+dx)^3}(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx=-\frac{F^a(c+dx)^2\Gamma\left(\frac{2}{3},-b(c+dx)^3\log(F)\right)}{3d\left(-b(c+dx)^3\log(F)\right)^{2/3}}$$

output
```
-1/3*F^a*(d*x+c)^2*GAMMA(2/3,-b*(d*x+c)^3*ln(F))/d/(-b*(d*x+c)^3*ln(F))^(2
/3)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx=-\frac{F^a(c+dx)^2\Gamma\left(\frac{2}{3},-b(c+dx)^3\log(F)\right)}{3d\left(-b(c+dx)^3\log(F)\right)^{2/3}}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)*(c + d*x),x]
```

output
```
-1/3*(F^a*(c + d*x)^2*Gamma[2/3, -(b*(c + d*x)^3*Log[F])])/(d*(-(b*(c + d*
x)^3*Log[F]))^(2/3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)F^{a+b(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{F^a(c+dx)^2\Gamma\left(\frac{2}{3},-b(c+dx)^3\log(F)\right)}{3d\left(-b\log(F)(c+dx)^3\right)^{2/3}}$$

input
```
Int[F^(a + b*(c + d*x)^3)*(c + d*x),x]
```

output
```
-1/3*(F^a*(c + d*x)^2*Gamma[2/3, -(b*(c + d*x)^3*Log[F])])/(d*(-(b*(c + d*
x)^3*Log[F]))^(2/3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^3}(dx+c)\,dx$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c),x)
```

output
```
int(F^(a+b*(d*x+c)^3)*(d*x+c),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx = \frac{\left(-bd^3\log(F)\right)^{\frac{1}{3}}F^a\Gamma\left(\frac{2}{3},-\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log(F)\right)}{3\,bd^2\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c),x, algorithm="fricas")
```

output
```
1/3*(-b*d^3*log(F))^(1/3)*F^a*gamma(2/3, -(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*log(F))/(b*d^2*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx = \int F^{a+b(c+dx)^3}(c+dx)\ dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)*(d*x+c),x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)*(c + d*x), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx = \int (dx+c)F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*F^((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx = \int (dx+c)F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*F^((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx = \int F^{a+b\,(c+d\,x)^3}\,(c+d\,x)\,dx$$

input
```
int(F^(a + b*(c + d*x)^3)*(c + d*x),x)
```

output
```
int(F^(a + b*(c + d*x)^3)*(c + d*x), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^3}(c+dx)\,dx = f^{b\,c^3+a}\left(\left(\int f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}dx\right)c + \left(\int f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}xdx\right)d\right)$$

input
```
int(F^(a+b*(d*x+c)^3)*(d*x+c),x)
```

output
```
f**(a + b*c**3)*(int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3),x)*
c + int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)*x,x)*d)
```

3.229 $\int F^{a+b(c+dx)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int F^{a+b(c+dx)^3}\,dx=-\frac{F^a(c+dx)\Gamma\left(\frac{1}{3},-b(c+dx)^3\log(F)\right)}{3d\sqrt[3]{-b(c+dx)^3\log(F)}}$$

output
```
-1/3*F^a*(d*x+c)*GAMMA(1/3,-b*(d*x+c)^3*ln(F))/d/(-b*(d*x+c)^3*ln(F))^(1/3
)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^3}\,dx=-\frac{F^a(c+dx)\Gamma\left(\frac{1}{3},-b(c+dx)^3\log(F)\right)}{3d\sqrt[3]{-b(c+dx)^3\log(F)}}$$

input
```
Integrate[F^(a + b*(c + d*x)^3),x]
```

output
```
-1/3*(F^a*(c + d*x)*Gamma[1/3, -(b*(c + d*x)^3*Log[F])])/(d*(-(b*(c + d*x)
^3*Log[F]))^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+b(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{F^a(c+dx)\Gamma\left(\frac{1}{3},-b(c+dx)^3\log(F)\right)}{3d\sqrt[3]{-b\log(F)(c+dx)^3}}$$

input
```
Int[F^(a + b*(c + d*x)^3),x]
```

output
```
-1/3*(F^a*(c + d*x)*Gamma[1/3, -(b*(c + d*x)^3*Log[F])])/(d*(-(b*(c + d*x)
^3*Log[F]))^(1/3))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Maple [F]

$$\int F^{a+b(dx+c)^3}dx$$

input
```
int(F^(a+b*(d*x+c)^3),x)
```

output
```
int(F^(a+b*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int F^{a+b(c+dx)^3}\,dx = \frac{\left(-bd^3\log\left(F\right)\right)^{\frac{2}{3}}F^a\Gamma\left(\frac{1}{3}, -\left(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3\right)\log\left(F\right)\right)}{3\,bd^3\log\left(F\right)}$$

input
```
integrate(F^(a+b*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*(-b*d^3*log(F))^(2/3)*F^a*gamma(1/3, -(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*log(F))/(b*d^3*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^3}\,dx = \int F^{a+b(c+dx)^3}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3),x)
```

output
```
Integral(F**(a + b*(c + d*x)**3), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^3}\,dx = \int F^{(dx+c)^3b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^3} \, dx = \int F^{(dx+c)^3 b+a} \, dx$$

input
```
integrate(F^(a+b*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^3} \, dx = \int F^{a+b\,(c+d\,x)^3} \, dx$$

input
```
int(F^(a + b*(c + d*x)^3),x)
```

output
```
int(F^(a + b*(c + d*x)^3), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^3} \, dx = f^{b\,c^3+a}\left(\int f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^3),x)
```

output
```
f**(a + b*c**3)*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3),x)
```

3.230 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx = -\frac{F^a\Gamma\left(-\frac{1}{3},-b(c+dx)^3\log(F)\right)\sqrt[3]{-b(c+dx)^3\log(F)}}{3d(c+dx)}$$

output
```
-1/3*F^a*GAMMA(-1/3,-b*(d*x+c)^3*ln(F))*(-b*(d*x+c)^3*ln(F))^(1/3)/d/(d*x+
c)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} dx = -\frac{F^a\Gamma\left(-\frac{1}{3},-b(c+dx)^3\log(F)\right)\sqrt[3]{-b(c+dx)^3\log(F)}}{3d(c+dx)}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^2,x]
```

output
```
-1/3*(F^a*Gamma[-1/3, -(b*(c + d*x)^3*Log[F])]*(-(b*(c + d*x)^3*Log[F]))^(
1/3))/(d*(c + d*x))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{F^a \sqrt[3]{-b\log(F)(c+dx)^3}\,\Gamma\left(-\frac{1}{3},-b(c+dx)^3\log(F)\right)}{3d(c+dx)}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^2,x]
```

output
```
-1/3*(F^a*Gamma[-1/3, -(b*(c + d*x)^3*Log[F])]*(-(b*(c + d*x)^3*Log[F]))^(
1/3))/(d*(c + d*x))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^2}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx = \frac{(-bd^3\log(F))^{\frac{1}{3}}(dx+c)F^a\Gamma\left(\frac{2}{3}, -(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3)\log(F)\right) - F^{bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3}}{d^3x+cd^2}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((-b*d^3*log(F))^(1/3)*(d*x + c)*F^a*gamma(2/3, -(b*d^3*x^3 + 3*b*c*d^2*x^
2 + 3*b*c^2*d*x + b*c^3)*log(F)) - F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + a)*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx = -\frac{F^a\left(F^{b\,(c+d\,x)^3}-\Gamma\left(\frac{2}{3},-b\,\ln\left(F\right)\,(c+d\,x)^3\right)\,\left(-b\,\ln\left(F\right)\,(c+d\,x)^3\right)^{1/3}+\Gamma\left(\frac{2}{3}\right)\,\left(-b\,\ln\left(F\right)\,(c+d\,x)^3\right)^{1/3}\right)}{d\,(c+d\,x)}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^2,x)
```

output
```
-(F^a*(F^(b*(c + d*x)^3) - igamma(2/3, -b*log(F)*(c + d*x)^3)*(-b*log(F)*(
c + d*x)^3)^(1/3) + gamma(2/3)*(-b*log(F)*(c + d*x)^3)^(1/3)))/(d*(c + d*x
))
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^2}\,dx = f^{b\,c^3+a}\left(\int \frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}}{d^2x^2+2cdx+c^2}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^2,x)
```

output
```
f**(a + b*c**3)*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(c**
2 + 2*c*d*x + d**2*x**2),x)
```

3.231 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx=-\frac{F^a\Gamma\left(-\frac{2}{3},-b(c+dx)^3\log(F)\right)\left(-b(c+dx)^3\log(F)\right)^{2/3}}{3d(c+dx)^2}$$

output
```
-1/3*F^a*GAMMA(-2/3,-b*(d*x+c)^3*ln(F))*(-b*(d*x+c)^3*ln(F))^(2/3)/d/(d*x+
c)^2
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx=-\frac{F^a\Gamma\left(-\frac{2}{3},-b(c+dx)^3\log(F)\right)\left(-b(c+dx)^3\log(F)\right)^{2/3}}{3d(c+dx)^2}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^3,x]
```

output
```
-1/3*(F^a*Gamma[-2/3, -(b*(c + d*x)^3*Log[F])]*(-(b*(c + d*x)^3*Log[F]))^(
2/3))/(d*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a\left(-b\log(F)(c+dx)^3\right)^{2/3}\Gamma\left(-\frac{2}{3},-b(c+dx)^3\log(F)\right)}{3d(c+dx)^2}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^3,x]
```

output
```
-1/3*(F^a*Gamma[-2/3, -(b*(c + d*x)^3*Log[F])]*(-(b*(c + d*x)^3*Log[F]))^(
2/3))/(d*(c + d*x)^2)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^3}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.76

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx = \frac{(-bd^3\log(F))^{\frac{2}{3}}(d^2x^2+2\,cdx+c^2)F^a\Gamma\left(\frac{1}{3}, -(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+bc^3)\log(F)\right) - F^{bd^3x^3+3\,bcd^2x}}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*((-b*d^3*log(F))^(2/3)*(d^2*x^2 + 2*c*d*x + c^2)*F^a*gamma(1/3, -(b*d^
3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F)) - F^(b*d^3*x^3 + 3*b*
c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + a)*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.78

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx =$$
$$-\frac{F^a\left(3\,F^{b\,(c+d\,x)^3}\,\Gamma\left(\frac{2}{3}\right)-3\,\Gamma\left(\frac{2}{3}\right)\,\Gamma\left(\frac{1}{3},-b\,\ln\left(F\right)\,(c+d\,x)^3\right)\,\left(-b\,\ln\left(F\right)\,(c+d\,x)^3\right)^{2/3}+2\,\pi\,\sqrt{3}\,\left(-b\,\ln\left(\right.\right.}{6\,d\,\Gamma\left(\frac{2}{3}\right)\,\left(c+d\,x\right)^2}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^3,x)
```

output
```
-(F^a*(3*F^(b*(c + d*x)^3)*gamma(2/3) - 3*gamma(2/3)*igamma(1/3, -b*log(F)
*(c + d*x)^3)*(-b*log(F)*(c + d*x)^3)^(2/3) + 2*3^(1/2)*pi*(-b*log(F)*(c +
 d*x)^3)^(2/3)))/(6*d*gamma(2/3)*(c + d*x)^2)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^3}\,dx = f^{b\,c^3+a}\left(\int \frac{f^{b\,d^3x^3+3bc\,d^2x^2+3b\,c^2dx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)$$

input

```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^3,x)
```

output

```
f**(a + b*c**3)*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
```

3.232 $\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5}\,dx=-\frac{F^a\Gamma\left(-\frac{4}{3},-b(c+dx)^3\log(F)\right)\left(-b(c+dx)^3\log(F)\right)^{4/3}}{3d(c+dx)^4}$$

output
```
-1/3*F^a*GAMMA(-4/3,-b*(d*x+c)^3*ln(F))*(-b*(d*x+c)^3*ln(F))^(4/3)/d/(d*x+
c)^4
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5}\,dx=-\frac{F^a\Gamma\left(-\frac{4}{3},-b(c+dx)^3\log(F)\right)\left(-b(c+dx)^3\log(F)\right)^{4/3}}{3d(c+dx)^4}$$

input
```
Integrate[F^(a + b*(c + d*x)^3)/(c + d*x)^5,x]
```

output
```
-1/3*(F^a*Gamma[-4/3, -(b*(c + d*x)^3*Log[F])]*(-(b*(c + d*x)^3*Log[F]))^(
4/3))/(d*(c + d*x)^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{F^a \left(-b \log(F)(c+dx)^3\right)^{4/3} \Gamma\left(-\frac{4}{3}, -b(c+dx)^3 \log(F)\right)}{3d(c+dx)^4}$$

input
```
Int[F^(a + b*(c + d*x)^3)/(c + d*x)^5,x]
```

output
```
-1/3*(F^a*Gamma[-4/3, -(b*(c + d*x)^3*Log[F])]*(-(b*(c + d*x)^3*Log[F]))^(
4/3))/(d*(c + d*x)^4)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^3}}{(dx+c)^5}dx$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x)
```

output
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 226, normalized size of antiderivative = 4.61

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5}\,dx = \frac{3\left(bd^4x^4+4\,bcd^3x^3+6\,bc^2d^2x^2+4\,bc^3dx+bc^4\right)(-bd^3\log\left(F\right))^{\frac{1}{3}}\,F^a\Gamma\left(\frac{2}{3}, -(bd^3x^3+3\,bcd^2x^2+3\,bc^2dx+\right.}{4\left(d^6x^4+4\,cd^5x^3+6\,c^2d\right.}$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/4*(3*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)
*(-b*d^3*log(F))^(1/3)*F^a*gamma(2/3, -(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^
2*d*x + b*c^3)*log(F))*log(F) - (3*(b*d^4*x^3 + 3*b*c*d^3*x^2 + 3*b*c^2*d^
2*x + b*c^3*d)*log(F) + d)*F^(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3 + a))/(d^6*x^4 + 4*c*d^5*x^3 + 6*c^2*d^4*x^2 + 4*c^3*d^3*x + c^4*d^2)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx = \int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx$$

input
```
integrate(F**(a+b*(d*x+c)**3)/(d*x+c)**5,x)
```

output
```
Integral(F**(a + b*(c + d*x)**3)/(c + d*x)**5, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^5} dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^5, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} dx = \int \frac{F^{(dx+c)^3b+a}}{(dx+c)^5} dx$$

input
```
integrate(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^3*b + a)/(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.65

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} \, dx = \frac{3 F^a \Gamma\left(\frac{2}{3}\right) \left(-b \ln(F) (c+dx)^3\right)^{4/3}}{4 d (c+dx)^4} - \frac{F^a F^{b (c+dx)^3}}{4 d (c+dx)^4} - \frac{3 F^a \Gamma\left(\frac{2}{3}, -b \ln(F) (c+dx)^3\right) \left(-b \ln(F) (c+dx)^3\right)^{4/3}}{4 d (c+dx)^4} - \frac{3 F^a F^{b (c+dx)^3} b \ln(F)}{4 d (c+dx)}$$

input
```
int(F^(a + b*(c + d*x)^3)/(c + d*x)^5,x)
```

output
```
(3*F^a*gamma(2/3)*(-b*log(F)*(c + d*x)^3)^(4/3))/(4*d*(c + d*x)^4) - (F^a*
F^(b*(c + d*x)^3))/(4*d*(c + d*x)^4) - (3*F^a*igamma(2/3, -b*log(F)*(c + d
*x)^3)*(-b*log(F)*(c + d*x)^3)^(4/3))/(4*d*(c + d*x)^4) - (3*F^a*F^(b*(c +
 d*x)^3)*b*log(F))/(4*d*(c + d*x))
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^3}}{(c+dx)^5} \, dx = \text{too large to display}$$

input
```
int(F^(a+b*(d*x+c)^3)/(d*x+c)^5,x)
```

output
```
(f**(a + b*c**3)*( - 2*f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3) +
 27*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*log(f)*b*c**8
 + 15*log(f)*b*c**7*d*x + 30*log(f)*b*c**6*d**2*x**2 + 30*log(f)*b*c**5*d*
*3*x**3 + 15*log(f)*b*c**4*d**4*x**4 + 3*log(f)*b*c**3*d**5*x**5 - 4*c**5
- 20*c**4*d*x - 40*c**3*d**2*x**2 - 40*c**2*d**3*x**3 - 20*c*d**4*x**4 - 4
*d**5*x**5),x)*log(f)**2*b**2*c**10*d + 108*int(f**(3*b*c**2*d*x + 3*b*c*d
**2*x**2 + b*d**3*x**3)/(3*log(f)*b*c**8 + 15*log(f)*b*c**7*d*x + 30*log(f
)*b*c**6*d**2*x**2 + 30*log(f)*b*c**5*d**3*x**3 + 15*log(f)*b*c**4*d**4*x*
*4 + 3*log(f)*b*c**3*d**5*x**5 - 4*c**5 - 20*c**4*d*x - 40*c**3*d**2*x**2
- 40*c**2*d**3*x**3 - 20*c*d**4*x**4 - 4*d**5*x**5),x)*log(f)**2*b**2*c**9
*d**2*x + 162*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*log
(f)*b*c**8 + 15*log(f)*b*c**7*d*x + 30*log(f)*b*c**6*d**2*x**2 + 30*log(f)
*b*c**5*d**3*x**3 + 15*log(f)*b*c**4*d**4*x**4 + 3*log(f)*b*c**3*d**5*x**5
 - 4*c**5 - 20*c**4*d*x - 40*c**3*d**2*x**2 - 40*c**2*d**3*x**3 - 20*c*d**
4*x**4 - 4*d**5*x**5),x)*log(f)**2*b**2*c**8*d**3*x**2 + 108*int(f**(3*b*c
**2*d*x + 3*b*c*d**2*x**2 + b*d**3*x**3)/(3*log(f)*b*c**8 + 15*log(f)*b*c*
*7*d*x + 30*log(f)*b*c**6*d**2*x**2 + 30*log(f)*b*c**5*d**3*x**3 + 15*log(
f)*b*c**4*d**4*x**4 + 3*log(f)*b*c**3*d**5*x**5 - 4*c**5 - 20*c**4*d*x - 4
0*c**3*d**2*x**2 - 40*c**2*d**3*x**3 - 20*c*d**4*x**4 - 4*d**5*x**5),x)*lo
g(f)**2*b**2*c**7*d**4*x**3 + 27*int(f**(3*b*c**2*d*x + 3*b*c*d**2*x**2...
```

3.233 $\int f^{a+b\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int f^{a+b\sqrt{c+dx}}\,dx=-\frac{2f^{a+b\sqrt{c+dx}}}{b^2d\log^2(f)}+\frac{2f^{a+b\sqrt{c+dx}}\sqrt{c+dx}}{bd\log(f)}$$

output
```
-2*f^(a+b*(d*x+c)^(1/2))/b^2/d/ln(f)^2+2*f^(a+b*(d*x+c)^(1/2))*(d*x+c)^(1/
2)/b/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int f^{a+b\sqrt{c+dx}}\,dx=\frac{2f^{a+b\sqrt{c+dx}}\left(-1+b\sqrt{c+dx}\log(f)\right)}{b^2d\log^2(f)}$$

input
```
Integrate[f^(a + b*Sqrt[c + d*x]),x]
```

output
```
(2*f^(a + b*Sqrt[c + d*x])*(-1 + b*Sqrt[c + d*x]*Log[f]))/(b^2*d*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2636, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+b\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2636}$$

$$\frac{2\int f^{a+b\sqrt{c+dx}}\sqrt{c+dx}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\sqrt{c+dx}f^{a+b\sqrt{c+dx}}}{b\log(f)}-\frac{\int f^{a+b\sqrt{c+dx}}d\sqrt{c+dx}}{b\log(f)}\right)}{d}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(\frac{\sqrt{c+dx}f^{a+b\sqrt{c+dx}}}{b\log(f)}-\frac{f^{a+b\sqrt{c+dx}}}{b^2\log^2(f)}\right)}{d}$$

input
```
Int[f^(a + b*Sqrt[c + d*x]),x]
```

output
```
(2*(-(f^(a + b*Sqrt[c + d*x])/(b^2*Log[f]^2)) + (f^(a + b*Sqrt[c + d*x])*S
qrt[c + d*x])/(b*Log[f])))/d
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2636
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Simp[k/d   Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (
c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Int
egerQ[n]
```

Maple [F]

$$\int f^{a+b\sqrt{dx+c}}dx$$

input
```
int(f^(a+b*(d*x+c)^(1/2)),x)
```

output
```
int(f^(a+b*(d*x+c)^(1/2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int f^{a+b\sqrt{c+dx}}\,dx = \frac{2\left(\sqrt{dx+c}b\log\left(f\right)-1\right)e^{\left(\sqrt{dx+c}b\log(f)+a\log(f)\right)}}{b^2d\log\left(f\right)^2}$$

input
```
integrate(f^(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

output
```
2*(sqrt(d*x + c)*b*log(f) - 1)*e^(sqrt(d*x + c)*b*log(f) + a*log(f))/(b^2*
d*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int f^{a+b\sqrt{c+dx}}\,dx = \begin{cases} x & \text{for } b = 0 \wedge d = 0 \wedge f = 1 \\ f^a x & \text{for } b = 0 \\ f^{a+b\sqrt{c}}x & \text{for } d = 0 \\ x & \text{for } f = 1 \\ \frac{2f^{a+b\sqrt{c+dx}}\sqrt{c+dx}}{bd\log(f)} - \frac{2f^{a+b\sqrt{c+dx}}}{b^2d\log(f)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(f**(a+b*(d*x+c)**(1/2)),x)
```

output
```
Piecewise((x, Eq(b, 0) & Eq(d, 0) & Eq(f, 1)), (f**a*x, Eq(b, 0)), (f**(a
+ b*sqrt(c))*x, Eq(d, 0)), (x, Eq(f, 1)), (2*f**(a + b*sqrt(c + d*x))*sqrt
(c + d*x)/(b*d*log(f)) - 2*f**(a + b*sqrt(c + d*x))/(b**2*d*log(f)**2), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int f^{a+b\sqrt{c+dx}}\,dx = \frac{2\left(\sqrt{dx+c}bf^a\log(f) - f^a\right)f^{\sqrt{dx+c}b}}{b^2d\log(f)^2}$$

input
```
integrate(f^(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")
```

output
```
2*(sqrt(d*x + c)*b*f^a*log(f) - f^a)*f^(sqrt(d*x + c)*b)/(b^2*d*log(f)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 781, normalized size of antiderivative = 12.20

$$\int f^{a+b\sqrt{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(f^(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

output
```
(2*(2*((pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))*(pi*sqrt(d*x + c)*
b*sgn(f) - pi*sqrt(d*x + c)*b)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(ab
s(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) + (pi^2
*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)*(sqrt(d*x + c)*b*log(abs(f))
 - 1)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*lo
g(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2))*cos(-1/2*pi*sqrt(d*x + c)*b*sgn
(f) - 1/2*pi*a*sgn(f) + 1/2*pi*sqrt(d*x + c)*b + 1/2*pi*a) + ((pi^2*b^2*sg
n(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)*(pi*sqrt(d*x + c)*b*sgn(f) - pi*sqr
t(d*x + c)*b)/((pi^2*b^2*sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(p
i*b^2*log(abs(f))*sgn(f) - pi*b^2*log(abs(f)))^2) - 4*(pi*b^2*log(abs(f))*
sgn(f) - pi*b^2*log(abs(f)))*(sqrt(d*x + c)*b*log(abs(f)) - 1)/((pi^2*b^2*
sgn(f) - pi^2*b^2 + 2*b^2*log(abs(f))^2)^2 + 4*(pi*b^2*log(abs(f))*sgn(f)
- pi*b^2*log(abs(f)))^2))*sin(-1/2*pi*sqrt(d*x + c)*b*sgn(f) - 1/2*pi*a*sg
n(f) + 1/2*pi*sqrt(d*x + c)*b + 1/2*pi*a))*e^(sqrt(d*x + c)*b*log(abs(f))
+ a*log(abs(f))) - I*((pi*sqrt(d*x + c)*b*sgn(f) - pi*sqrt(d*x + c)*b - 2*
I*sqrt(d*x + c)*b*log(abs(f)) + 2*I)*e^(1/2*I*pi*sqrt(d*x + c)*b*sgn(f) +
1/2*I*pi*a*sgn(f) - 1/2*I*pi*sqrt(d*x + c)*b - 1/2*I*pi*a)/(pi^2*b^2*sgn(f
) + 2*I*pi*b^2*log(abs(f))*sgn(f) - pi^2*b^2 - 2*I*pi*b^2*log(abs(f)) + 2*
b^2*log(abs(f))^2) + (pi*sqrt(d*x + c)*b*sgn(f) - pi*sqrt(d*x + c)*b + 2*I
*sqrt(d*x + c)*b*log(abs(f)) - 2*I)*e^(-1/2*I*pi*sqrt(d*x + c)*b*sgn(f)...
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

$$\int f^{a+b\sqrt{c+dx}}\,dx=\frac{f^{a+b\,\sqrt{c+d\,x}}\,\left(2\,b\,\ln\left(f\right)\,\sqrt{c+d\,x}-2\right)}{b^2\,d\,{\ln\left(f\right)}^2}$$

input
```
int(f^(a + b*(c + d*x)^(1/2)),x)
```

output
```
(f^(a + b*(c + d*x)^(1/2))*(2*b*log(f)*(c + d*x)^(1/2) - 2))/(b^2*d*log(f)
^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int f^{a+b\sqrt{c+dx}}\,dx=\frac{2f^{\sqrt{dx+c}\,b+a}\left(\sqrt{dx+c}\,\log(f)\,b-1\right)}{\log\left(f\right)^2 b^2 d}$$

input
```
int(f^(a+b*(d*x+c)^(1/2)),x)
```

output
```
(2*f**(sqrt(c + d*x)*b + a)*(sqrt(c + d*x)*log(f)*b - 1))/(log(f)**2*b**2*
d)
```

3.234 $\int f^{a+b\sqrt[3]{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx=\frac{6f^{a+b\sqrt[3]{c+dx}}}{b^3d\log^3(f)}-\frac{6f^{a+b\sqrt[3]{c+dx}}\sqrt[3]{c+dx}}{b^2d\log^2(f)}+\frac{3f^{a+b\sqrt[3]{c+dx}}(c+dx)^{2/3}}{bd\log(f)}$$

output
```
6*f^(a+b*(d*x+c)^(1/3))/b^3/d/ln(f)^3-6*f^(a+b*(d*x+c)^(1/3))*(d*x+c)^(1/3
)/b^2/d/ln(f)^2+3*f^(a+b*(d*x+c)^(1/3))*(d*x+c)^(2/3)/b/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.60

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx=\frac{3f^{a+b\sqrt[3]{c+dx}}\left(2-2b\sqrt[3]{c+dx}\log(f)+b^2(c+dx)^{2/3}\log^2(f)\right)}{b^3d\log^3(f)}$$

input
```
Integrate[f^(a + b*(c + d*x)^(1/3)),x]
```

output
```
(3*f^(a + b*(c + d*x)^(1/3))*(2 - 2*b*(c + d*x)^(1/3)*Log[f] + b^2*(c + d*
x)^(2/3)*Log[f]^2))/(b^3*d*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2636, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+b\sqrt[3]{c+dx}}dx$$

$$\Big\downarrow \text{2636}$$

$$\frac{3\int f^{a+b\sqrt[3]{c+dx}}(c+dx)^{2/3}d\sqrt[3]{c+dx}}{d}$$

$$\Big\downarrow \text{2607}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}f^{a+b\sqrt[3]{c+dx}}}{b\log(f)}-\frac{2\int f^{a+b\sqrt[3]{c+dx}}\sqrt[3]{c+dx}d\sqrt[3]{c+dx}}{b\log(f)}\right)}{d}$$

$$\Big\downarrow \text{2607}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}f^{a+b\sqrt[3]{c+dx}}}{b\log(f)}-\frac{2\left(\frac{\sqrt[3]{c+dx}f^{a+b\sqrt[3]{c+dx}}}{b\log(f)}-\frac{\int f^{a+b\sqrt[3]{c+dx}}d\sqrt[3]{c+dx}}{b\log(f)}\right)}{b\log(f)}\right)}{d}$$

$$\Big\downarrow \text{2624}$$

$$\frac{3\left(\frac{(c+dx)^{2/3}f^{a+b\sqrt[3]{c+dx}}}{b\log(f)}-\frac{2\left(\frac{\sqrt[3]{c+dx}f^{a+b\sqrt[3]{c+dx}}}{b\log(f)}-\frac{f^{a+b\sqrt[3]{c+dx}}}{b^2\log^2(f)}\right)}{b\log(f)}\right)}{d}$$

input
```
Int[f^(a + b*(c + d*x)^(1/3)),x]
```

output
```
(3*((f^(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/(b*Log[f]) - (2*(-(f^(a + 
b*(c + d*x)^(1/3))/(b^2*Log[f]^2)) + (f^(a + b*(c + d*x)^(1/3))*(c + d*x)^
(1/3))/(b*Log[f])))/(b*Log[f])))/d
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2636
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Simp[k/d   Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (
c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Int
egerQ[n]
```

Maple [F]

$$\int f^{a+b(dx+c)^{\frac{1}{3}}}dx$$

input
```
int(f^(a+b*(d*x+c)^(1/3)),x)
```

output
```
int(f^(a+b*(d*x+c)^(1/3)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx = \frac{3\left((dx+c)^{\frac{2}{3}}b^2\log(f)^2 - 2(dx+c)^{\frac{1}{3}}b\log(f) + 2\right)e^{\left((dx+c)^{\frac{1}{3}}b\log(f)+a\log(f)\right)}}{b^3d\log(f)^3}$$

input
```
integrate(f^(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")
```

output
```
3*((d*x + c)^(2/3)*b^2*log(f)^2 - 2*(d*x + c)^(1/3)*b*log(f) + 2)*e^((d*x
+ c)^(1/3)*b*log(f) + a*log(f))/(b^3*d*log(f)^3)
```

Sympy [F]

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx = \int f^{a+b\sqrt[3]{c+dx}}\,dx$$

input
```
integrate(f**(a+b*(d*x+c)**(1/3)),x)
```

output
```
Integral(f**(a + b*(c + d*x)**(1/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.62

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx = \frac{3\left((dx+c)^{\frac{2}{3}}b^2f^a\log(f)^2 - 2\,(dx+c)^{\frac{1}{3}}bf^a\log(f) + 2\,f^a\right)f^{(dx+c)^{\frac{1}{3}}b}}{b^3d\log(f)^3}$$

input
```
integrate(f^(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

output
```
3*((d*x + c)^(2/3)*b^2*f^a*log(f)^2 - 2*(d*x + c)^(1/3)*b*f^a*log(f) + 2*f
^a)*f^((d*x + c)^(1/3)*b)/(b^3*d*log(f)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1338, normalized size of antiderivative = 13.38

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx = \text{Too large to display}$$

input
```
integrate(f^(a+b*(d*x+c)^(1/3)),x, algorithm="giac")
```

output
```
3*((((3*pi^2*b^3*log(abs(f))*sgn(f) - 3*pi^2*b^3*log(abs(f)) + 2*b^3*log(a
bs(f))^3)*(pi^2*(d*x + c)^(2/3)*b^2*sgn(f) - pi^2*(d*x + c)^(2/3)*b^2 + 2*
(d*x + c)^(2/3)*b^2*log(abs(f))^2 - 4*(d*x + c)^(1/3)*b*log(abs(f)) + 4)/(
(pi^3*b^3*sgn(f) - 3*pi*b^3*log(abs(f))^2*sgn(f) - pi^3*b^3 + 3*pi*b^3*log
(abs(f))^2)^2 + (3*pi^2*b^3*log(abs(f))*sgn(f) - 3*pi^2*b^3*log(abs(f)) +
2*b^3*log(abs(f))^3)^2) - 2*(pi^3*b^3*sgn(f) - 3*pi*b^3*log(abs(f))^2*sgn(
f) - pi^3*b^3 + 3*pi*b^3*log(abs(f))^2)*(pi*(d*x + c)^(2/3)*b^2*log(abs(f)
)*sgn(f) - pi*(d*x + c)^(2/3)*b^2*log(abs(f)) - pi*(d*x + c)^(1/3)*b*sgn(f
) + pi*(d*x + c)^(1/3)*b)/((pi^3*b^3*sgn(f) - 3*pi*b^3*log(abs(f))^2*sgn(f
) - pi^3*b^3 + 3*pi*b^3*log(abs(f))^2)^2 + (3*pi^2*b^3*log(abs(f))*sgn(f)
- 3*pi^2*b^3*log(abs(f)) + 2*b^3*log(abs(f))^3)^2))*cos(-1/2*pi*(d*x + c)^
(1/3)*b*sgn(f) - 1/2*pi*a*sgn(f) + 1/2*pi*(d*x + c)^(1/3)*b + 1/2*pi*a) +
((pi^3*b^3*sgn(f) - 3*pi*b^3*log(abs(f))^2*sgn(f) - pi^3*b^3 + 3*pi*b^3*lo
g(abs(f))^2)*(pi^2*(d*x + c)^(2/3)*b^2*sgn(f) - pi^2*(d*x + c)^(2/3)*b^2 +
 2*(d*x + c)^(2/3)*b^2*log(abs(f))^2 - 4*(d*x + c)^(1/3)*b*log(abs(f)) + 4
)/((pi^3*b^3*sgn(f) - 3*pi*b^3*log(abs(f))^2*sgn(f) - pi^3*b^3 + 3*pi*b^3*
log(abs(f))^2)^2 + (3*pi^2*b^3*log(abs(f))*sgn(f) - 3*pi^2*b^3*log(abs(f))
 + 2*b^3*log(abs(f))^3)^2) + 2*(3*pi^2*b^3*log(abs(f))*sgn(f) - 3*pi^2*b^3
*log(abs(f)) + 2*b^3*log(abs(f))^3)*(pi*(d*x + c)^(2/3)*b^2*log(abs(f))*sg
n(f) - pi*(d*x + c)^(2/3)*b^2*log(abs(f)) - pi*(d*x + c)^(1/3)*b*sgn(f)...
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.54

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx=\frac{f^{a+b\,(c+d\,x)^{1/3}}\,\left(3\,b^2\ln\left(f\right)^2\,{\left(c+d\,x\right)}^{2/3}-6\,b\,\ln\left(f\right)\,{\left(c+d\,x\right)}^{1/3}+6\right)}{b^3\,d\ln\left(f\right)^3}$$

input
```
int(f^(a + b*(c + d*x)^(1/3)),x)
```

output
```
(f^(a + b*(c + d*x)^(1/3))*(3*b^2*log(f)^2*(c + d*x)^(2/3) - 6*b*log(f)*(c
 + d*x)^(1/3) + 6))/(b^3*d*log(f)^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.54

$$\int f^{a+b\sqrt[3]{c+dx}}\,dx=\frac{3f^{(dx+c)^{\frac{1}{3}}b+a}\left((dx+c)^{\frac{2}{3}}\log(f)^2\,b^2-2(dx+c)^{\frac{1}{3}}\log(f)\,b+2\right)}{\log\left(f\right)^3b^3d}$$

input
```
int(f^(a+b*(d*x+c)^(1/3)),x)
```

output
```
(3*f**((c + d*x)**(1/3)*b + a)*((c + d*x)**(2/3)*log(f)**2*b**2 - 2*(c + d
*x)**(1/3)*log(f)*b + 2))/(log(f)**3*b**3*d)
```

3.235 $\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 50

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx=\frac{F^a(c+dx)^{1+m}\Gamma\left(-1-m,-\frac{b\log(F)}{c+dx}\right)\left(-\frac{b\log(F)}{c+dx}\right)^{1+m}}{d}$$

output
```
F^a*(d*x+c)^(1+m)*GAMMA(-1-m,-b*ln(F)/(d*x+c))*(-b*ln(F)/(d*x+c))^(1+m)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx=\frac{F^a(c+dx)^{1+m}\Gamma\left(-1-m,-\frac{b\log(F)}{c+dx}\right)\left(-\frac{b\log(F)}{c+dx}\right)^{1+m}}{d}$$

input
```
Integrate[F^(a + b/(c + d*x))*(c + d*x)^m,x]
```

output
```
(F^a*(c + d*x)^(1 + m)*Gamma[-1 - m, -((b*Log[F])/(c + d*x))]*(-((b*Log[F]
)/(c + d*x)))^(1 + m))/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m F^{a+\frac{b}{c+dx}} \, dx$$

$$\Big\downarrow \; 2648$$

$$\frac{F^a (c+dx)^{m+1} \left(-\frac{b\log(F)}{c+dx}\right)^{m+1} \Gamma\left(-m-1,-\frac{b\log(F)}{c+dx}\right)}{d}$$

input
```
Int[F^(a + b/(c + d*x))*(c + d*x)^m,x]
```

output
```
(F^a*(c + d*x)^(1 + m)*Gamma[-1 - m, -((b*Log[F])/(c + d*x))]*(-((b*Log[F]
)/(c + d*x)))^(1 + m))/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{dx+c}}(dx+c)^m\,dx$$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^m,x)
```

output
```
int(F^(a+b/(d*x+c))*(d*x+c)^m,x)
```

Fricas [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx=\int (dx+c)^m F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*F^((a*d*x + a*c + b)/(d*x + c)), x)
```

Sympy [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx=\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx$$

input
```
integrate(F**(a+b/(d*x+c))*(d*x+c)**m,x)
```

output
```
Integral(F**(a + b/(c + d*x))*(c + d*x)**m, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx = \int (dx+c)^m F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*F^(a + b/(d*x + c)), x)
```

Giac [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx = \int (dx+c)^m F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*F^(a + b/(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.46

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx = \frac{F^a\,\mathrm{e}^{\frac{b\ln(F)}{2\,(c+d\,x)}}\,(c+d\,x)^{m+1}\,\mathrm{M}_{\frac{m}{2}+1,-\frac{m}{2}-\frac{1}{2}}\left(\frac{b\ln(F)}{c+d\,x}\right)\,\left(\frac{b\ln(F)}{c+d\,x}\right)^{m/2}}{d\,(m+1)}$$

input
```
int(F^(a + b/(c + d*x))*(c + d*x)^m,x)
```

output
```
(F^a*exp((b*log(F))/(2*(c + d*x)))*(c + d*x)^(m + 1)*whittakerM(m/2 + 1, -
 m/2 - 1/2, (b*log(F))/(c + d*x))*((b*log(F))/(c + d*x))^(m/2))/(d*(m + 1)
)
```

Reduce [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^m\,dx=\text{too large to display}$$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^m,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)**3*b**3 + f**((a*c +
 a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)**2*b**2*c*m - 2*f**((a*c + a*d*
x + b)/(c + d*x))*(c + d*x)**m*log(f)**2*b**2*c + f**((a*c + a*d*x + b)/(c
 + d*x))*(c + d*x)**m*log(f)**2*b**2*d*m*x - 2*f**((a*c + a*d*x + b)/(c +
d*x))*(c + d*x)**m*log(f)**2*b**2*d*x + f**((a*c + a*d*x + b)/(c + d*x))*(
c + d*x)**m*log(f)*b*c**2*m**2 - 3*f**((a*c + a*d*x + b)/(c + d*x))*(c + d
*x)**m*log(f)*b*c**2*m + 2*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*l
og(f)*b*c**2 + 2*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)*b*c*
d*m**2*x - 6*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)*b*c*d*m*
x + 4*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)*b*c*d*x + f**((
a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)*b*d**2*m**2*x**2 - 3*f**((
a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)*b*d**2*m*x**2 + 2*f**((a*c
 + a*d*x + b)/(c + d*x))*(c + d*x)**m*log(f)*b*d**2*x**2 + f**((a*c + a*d*
x + b)/(c + d*x))*(c + d*x)**m*c**3*m**3 - 3*f**((a*c + a*d*x + b)/(c + d*
x))*(c + d*x)**m*c**3*m**2 + 2*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)*
*m*c**3*m + 3*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*c**2*d*m**3*x
- 9*f**((a*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*c**2*d*m**2*x + 6*f**((a
*c + a*d*x + b)/(c + d*x))*(c + d*x)**m*c**2*d*m*x + 3*f**((a*c + a*d*x +
b)/(c + d*x))*(c + d*x)**m*c*d**2*m**3*x**2 - 9*f**((a*c + a*d*x + b)/(c +
 d*x))*(c + d*x)**m*c*d**2*m**2*x**2 + 6*f**((a*c + a*d*x + b)/(c + d*x...
```

3.236 $\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx=-\frac{b^5F^a\Gamma\left(-5,-\frac{b\log(F)}{c+dx}\right)\log^5(F)}{d}$$

output
```
F^a*(d*x+c)^5*Ei(6,-b*ln(F)/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx=-\frac{b^5F^a\Gamma\left(-5,-\frac{b\log(F)}{c+dx}\right)\log^5(F)}{d}$$

input
```
Integrate[F^(a + b/(c + d*x))*(c + d*x)^4,x]
```

output
```
-((b^5*F^a*Gamma[-5, -((b*Log[F])/(c + d*x))]*Log[F]^5)/d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 F^{a+\frac{b}{c+dx}}\,dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{b^5 F^a \log^5(F)\Gamma\left(-5,-\frac{b\log(F)}{c+dx}\right)}{d}$$

input
```
Int[F^(a + b/(c + d*x))*(c + d*x)^4,x]
```

output
```
-((b^5*F^a*Gamma[-5, -((b*Log[F])/(c + d*x))]*Log[F]^5)/d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(28) = 56.

Time = 0.43 (sec) , antiderivative size = 534, normalized size of antiderivative = 18.41

method	result
risch	$\frac{d^4 F^a F^{\frac{b}{dx+c}} x^5}{5} + d^3 F^a F^{\frac{b}{dx+c}} c\, x^4 + 2d^2 F^a F^{\frac{b}{dx+c}} c^2 x^3 + 2d\, F^a F^{\frac{b}{dx+c}} c^3 x^2 + F^a F^{\frac{b}{dx+c}} c^4 x + \frac{F^a F^{\frac{b}{dx+c}} c^5}{5d}$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*d^4*F^a*F^(b/(d*x+c))*x^5+d^3*F^a*F^(b/(d*x+c))*c*x^4+2*d^2*F^a*F^(b/(
d*x+c))*c^2*x^3+2*d*F^a*F^(b/(d*x+c))*c^3*x^2+F^a*F^(b/(d*x+c))*c^4*x+1/5/
d*F^a*F^(b/(d*x+c))*c^5+1/20*d^3*b*ln(F)*F^a*F^(b/(d*x+c))*x^4+1/5*d^2*b*l
n(F)*F^a*F^(b/(d*x+c))*c*x^3+3/10*d*b*ln(F)*F^a*F^(b/(d*x+c))*c^2*x^2+1/5*
b*ln(F)*F^a*F^(b/(d*x+c))*c^3*x+1/20/d*b*ln(F)*F^a*F^(b/(d*x+c))*c^4+1/60*
d^2*b^2*ln(F)^2*F^a*F^(b/(d*x+c))*x^3+1/20*d*b^2*ln(F)^2*F^a*F^(b/(d*x+c))
*c*x^2+1/20*b^2*ln(F)^2*F^a*F^(b/(d*x+c))*c^2*x+1/60/d*b^2*ln(F)^2*F^a*F^(
b/(d*x+c))*c^3+1/120*d*b^3*ln(F)^3*F^a*F^(b/(d*x+c))*x^2+1/60*b^3*ln(F)^3*
F^a*F^(b/(d*x+c))*c*x+1/120/d*b^3*ln(F)^3*F^a*F^(b/(d*x+c))*c^2+1/120*b^4*
ln(F)^4*F^a*F^(b/(d*x+c))*x+1/120/d*b^4*ln(F)^4*F^a*F^(b/(d*x+c))*c+1/120/
d*b^5*ln(F)^5*F^a*Ei(1,-b*ln(F)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 8.41

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx = -\frac{F^a b^5 \text{Ei}\left(\frac{b\log(F)}{dx+c}\right)\log(F)^5 - \left(24\,d^5x^5 + 120\,cd^4x^4 + 240\,c^2d^3x^3 + 240\,c^3d^2x^2 + 120\,c^4dx + 24\,c^5 + (b^4d\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/120*(F^a*b^5*Ei(b*log(F)/(d*x + c))*log(F)^5 - (24*d^5*x^5 + 120*c*d^4*
x^4 + 240*c^2*d^3*x^3 + 240*c^3*d^2*x^2 + 120*c^4*d*x + 24*c^5 + (b^4*d*x
+ b^4*c)*log(F)^4 + (b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*log(F)^3 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*log(F)^2 + 6*(b*d^4
*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*log(F))*F^((
a*d*x + a*c + b)/(d*x + c)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx = \int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx$$

input
```
integrate(F**(a+b/(d*x+c))*(d*x+c)**4,x)
```

output
```
Integral(F**(a + b/(c + d*x))*(c + d*x)**4, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx = \int (dx+c)^4 F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/120*(24*F^a*d^4*x^5 + 6*(F^a*b*d^3*log(F) + 20*F^a*c*d^3)*x^4 + 2*(F^a*b
^2*d^2*log(F)^2 + 12*F^a*b*c*d^2*log(F) + 120*F^a*c^2*d^2)*x^3 + (F^a*b^3*
d*log(F)^3 + 6*F^a*b^2*c*d*log(F)^2 + 36*F^a*b*c^2*d*log(F) + 240*F^a*c^3*
d)*x^2 + (F^a*b^4*log(F)^4 + 2*F^a*b^3*c*log(F)^3 + 6*F^a*b^2*c^2*log(F)^2
 + 24*F^a*b*c^3*log(F) + 120*F^a*c^4)*x)*F^(b/(d*x + c)) + integrate(1/120
*(F^a*b^5*d*x*log(F)^5 - F^a*b^4*c^2*log(F)^4 - 2*F^a*b^3*c^3*log(F)^3 - 6
*F^a*b^2*c^4*log(F)^2 - 24*F^a*b*c^5*log(F))*F^(b/(d*x + c))/(d^2*x^2 + 2*
c*d*x + c^2), x)
```

Giac [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx = \int (dx+c)^4 F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*F^(a + b/(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 6.24

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx=\frac{F^a\,F^{\frac{b}{c+d\,x}}\,(c+d\,x)^5}{5\,d}+\frac{F^a\,b^5\ln(F)^5\,\mathrm{expint}\left(-\frac{b\ln(F)}{c+d\,x}\right)}{120\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b^2\ln(F)^2\,(c+d\,x)^3}{60\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b^3\ln(F)^3\,(c+d\,x)^2}{120\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b\ln(F)\,(c+d\,x)^4}{20\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b^4\ln(F)^4\,(c+d\,x)}{120\,d}$$

input
```
int(F^(a + b/(c + d*x))*(c + d*x)^4,x)
```

output
```
(F^a*F^(b/(c + d*x))*(c + d*x)^5)/(5*d) + (F^a*b^5*log(F)^5*expint(-(b*log
(F))/(c + d*x)))/(120*d) + (F^a*F^(b/(c + d*x))*b^2*log(F)^2*(c + d*x)^3)/
(60*d) + (F^a*F^(b/(c + d*x))*b^3*log(F)^3*(c + d*x)^2)/(120*d) + (F^a*F^(
b/(c + d*x))*b*log(F)*(c + d*x)^4)/(20*d) + (F^a*F^(b/(c + d*x))*b^4*log(F
)^4*(c + d*x))/(120*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^4\,dx=\frac{f^{\frac{adx+ac+b}{dx+c}}\log(f)^4b^4d^2x^2+2f^{\frac{adx+ac+b}{dx+c}}\log(f)^3b^3c^2dx+3f^{\frac{adx+ac+b}{dx+c}}\log(f)^3b^3c\,d^2x^2+f^{\frac{adx+ac+b}{dx+c}}\log(f)^3b^3d^3x^3}{}$$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^4,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*log(f)**4*b**4*d**2*x**2 + 2*f**((a*c +
a*d*x + b)/(c + d*x))*log(f)**3*b**3*c**2*d*x + 3*f**((a*c + a*d*x + b)/(c
 + d*x))*log(f)**3*b**3*c*d**2*x**2 + f**((a*c + a*d*x + b)/(c + d*x))*log
(f)**3*b**3*d**3*x**3 + 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*
c**4 + 8*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**3*d*x + 12*f**
((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**2*d**2*x**2 + 8*f**((a*c +
 a*d*x + b)/(c + d*x))*log(f)**2*b**2*c*d**3*x**3 + 2*f**((a*c + a*d*x + b
)/(c + d*x))*log(f)**2*b**2*d**4*x**4 + 6*f**((a*c + a*d*x + b)/(c + d*x))
*log(f)*b*c**5 + 30*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c**4*d*x + 6
0*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c**3*d**2*x**2 + 60*f**((a*c +
 a*d*x + b)/(c + d*x))*log(f)*b*c**2*d**3*x**3 + 30*f**((a*c + a*d*x + b)/
(c + d*x))*log(f)*b*c*d**4*x**4 + 6*f**((a*c + a*d*x + b)/(c + d*x))*log(f
)*b*d**5*x**5 + 24*f**((a*c + a*d*x + b)/(c + d*x))*c**6 + 144*f**((a*c +
a*d*x + b)/(c + d*x))*c**5*d*x + 360*f**((a*c + a*d*x + b)/(c + d*x))*c**4
*d**2*x**2 + 480*f**((a*c + a*d*x + b)/(c + d*x))*c**3*d**3*x**3 + 360*f**
((a*c + a*d*x + b)/(c + d*x))*c**2*d**4*x**4 + 144*f**((a*c + a*d*x + b)/(
c + d*x))*c*d**5*x**5 + 24*f**((a*c + a*d*x + b)/(c + d*x))*d**6*x**6 + in
t((f**((a*c + a*d*x + b)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3),x)*log(f)**5*b**5*c*d**3 + int((f**((a*c + a*d*x + b)/(c +
 d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*log(f)*...
```

3.237 $\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx = \frac{b^4F^a\Gamma\left(-4,-\frac{b\log(F)}{c+dx}\right)\log^4(F)}{d}$$

output
```
F^a*(d*x+c)^4*Ei(5,-b*ln(F)/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx = \frac{b^4F^a\Gamma\left(-4,-\frac{b\log(F)}{c+dx}\right)\log^4(F)}{d}$$

input
```
Integrate[F^(a + b/(c + d*x))*(c + d*x)^3,x]
```

output
```
(b^4*F^a*Gamma[-4, -((b*Log[F])/(c + d*x))]*Log[F]^4)/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 F^{a+\frac{b}{c+dx}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{b^4 F^a \log^4(F) \Gamma\left(-4,-\frac{b\log(F)}{c+dx}\right)}{d}$$

input
```
Int[F^(a + b/(c + d*x))*(c + d*x)^3,x]
```

output
```
(b^4*F^a*Gamma[-4, -((b*Log[F])/(c + d*x))]*Log[F]^4)/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(28) = 56$.

Time = 0.30 (sec) , antiderivative size = 368, normalized size of antiderivative = 13.14

method	result
risch	$\frac{d^3 F^a F^{\frac{b}{dx+c}} x^4}{4} + d^2 F^a F^{\frac{b}{dx+c}} c x^3 + \frac{3d F^a F^{\frac{b}{dx+c}} c^2 x^2}{2} + F^a F^{\frac{b}{dx+c}} c^3 x + \frac{F^a F^{\frac{b}{dx+c}} c^4}{4d} + \frac{d^2 b \ln(F) F^a F^{\frac{b}{dx+c}} x^3}{12} +$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*F^a*F^(b/(d*x+c))*x^4+d^2*F^a*F^(b/(d*x+c))*c*x^3+3/2*d*F^a*F^(b/(
d*x+c))*c^2*x^2+F^a*F^(b/(d*x+c))*c^3*x+1/4/d*F^a*F^(b/(d*x+c))*c^4+1/12*d
^2*b*ln(F)*F^a*F^(b/(d*x+c))*x^3+1/4*d*b*ln(F)*F^a*F^(b/(d*x+c))*c*x^2+1/4
*b*ln(F)*F^a*F^(b/(d*x+c))*c^2*x+1/12/d*b*ln(F)*F^a*F^(b/(d*x+c))*c^3+1/24
*d*b^2*ln(F)^2*F^a*F^(b/(d*x+c))*x^2+1/12*b^2*ln(F)^2*F^a*F^(b/(d*x+c))*c*
x+1/24/d*b^2*ln(F)^2*F^a*F^(b/(d*x+c))*c^2+1/24*b^3*ln(F)^3*F^a*F^(b/(d*x+
c))*x+1/24/d*b^3*ln(F)^3*F^a*F^(b/(d*x+c))*c+1/24/d*b^4*ln(F)^4*F^a*Ei(1,-
b*ln(F)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 6.25

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx = -\frac{F^a b^4 \mathrm{Ei}\left(\frac{b\log(F)}{dx+c}\right)\log(F)^4 - \left(6\,d^4x^4 + 24\,cd^3x^3 + 36\,c^2d^2x^2 + 24\,c^3dx + 6\,c^4 + (b^3dx + b^3c)\log(F)^3 + \right.}{24\,d}$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/24*(F^a*b^4*Ei(b*log(F)/(d*x + c))*log(F)^4 - (6*d^4*x^4 + 24*c*d^3*x^3
 + 36*c^2*d^2*x^2 + 24*c^3*d*x + 6*c^4 + (b^3*d*x + b^3*c)*log(F)^3 + (b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(F)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2
+ 3*b*c^2*d*x + b*c^3)*log(F))*F^((a*d*x + a*c + b)/(d*x + c)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx=\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx$$

input
```
integrate(F**(a+b/(d*x+c))*(d*x+c)**3,x)
```

output
```
Integral(F**(a + b/(c + d*x))*(c + d*x)**3, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx=\int (dx+c)^3F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/24*(6*F^a*d^3*x^4 + 2*(F^a*b*d^2*log(F) + 12*F^a*c*d^2)*x^3 + (F^a*b^2*d
*log(F)^2 + 6*F^a*b*c*d*log(F) + 36*F^a*c^2*d)*x^2 + (F^a*b^3*log(F)^3 + 2
*F^a*b^2*c*log(F)^2 + 6*F^a*b*c^2*log(F) + 24*F^a*c^3)*x)*F^(b/(d*x + c))
+ integrate(1/24*(F^a*b^4*d*x*log(F)^4 - F^a*b^3*c^2*log(F)^3 - 2*F^a*b^2*
c^3*log(F)^2 - 6*F^a*b*c^4*log(F))*F^(b/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^
2), x)
```

Giac [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx=\int (dx+c)^3F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*F^(a + b/(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 5.29

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx = \frac{F^a\,F^{\frac{b}{c+d\,x}}\,{(c+d\,x)}^4}{4\,d}+\frac{F^a\,b^4\,{\ln\left(F\right)}^4\,\mathrm{expint}\left(-\frac{b\,\ln\left(F\right)}{c+d\,x}\right)}{24\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b^2\,{\ln\left(F\right)}^2\,{(c+d\,x)}^2}{24\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b\,\ln\left(F\right)\,{(c+d\,x)}^3}{12\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b^3\,{\ln\left(F\right)}^3\,(c+d\,x)}{24\,d}$$

input
```
int(F^(a + b/(c + d*x))*(c + d*x)^3,x)
```

output
```
(F^a*F^(b/(c + d*x))*(c + d*x)^4)/(4*d) + (F^a*b^4*log(F)^4*expint(-(b*log
(F))/(c + d*x)))/(24*d) + (F^a*F^(b/(c + d*x))*b^2*log(F)^2*(c + d*x)^2)/(
24*d) + (F^a*F^(b/(c + d*x))*b*log(F)*(c + d*x)^3)/(12*d) + (F^a*F^(b/(c +
 d*x))*b^3*log(F)^3*(c + d*x))/(24*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^3\,dx$$
$$= \frac{f^{\frac{adx+ac+b}{dx+c}}\log(f)^3 b^3 d^2 x^2 + 2f^{\frac{adx+ac+b}{dx+c}}\log(f)^2 b^2 c^2 dx + 3f^{\frac{adx+ac+b}{dx+c}}\log(f)^2 b^2 c\,d^2 x^2 + f^{\frac{adx+ac+b}{dx+c}}\log(f)^2 b^2 d^3 x^3}{}$$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^3,x)
```

output

```
(f**((a*c + a*d*x + b)/(c + d*x))*log(f)**3*b**3*d**2*x**2 + 2*f**((a*c +
a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**2*d*x + 3*f**((a*c + a*d*x + b)/(c
 + d*x))*log(f)**2*b**2*c*d**2*x**2 + f**((a*c + a*d*x + b)/(c + d*x))*log
(f)**2*b**2*d**3*x**3 + 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c**4 +
 8*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c**3*d*x + 12*f**((a*c + a*d*
x + b)/(c + d*x))*log(f)*b*c**2*d**2*x**2 + 8*f**((a*c + a*d*x + b)/(c + d
*x))*log(f)*b*c*d**3*x**3 + 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*d*
*4*x**4 + 6*f**((a*c + a*d*x + b)/(c + d*x))*c**5 + 30*f**((a*c + a*d*x +
b)/(c + d*x))*c**4*d*x + 60*f**((a*c + a*d*x + b)/(c + d*x))*c**3*d**2*x**
2 + 60*f**((a*c + a*d*x + b)/(c + d*x))*c**2*d**3*x**3 + 30*f**((a*c + a*d
*x + b)/(c + d*x))*c*d**4*x**4 + 6*f**((a*c + a*d*x + b)/(c + d*x))*d**5*x
**5 + int((f**((a*c + a*d*x + b)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3),x)*log(f)**4*b**4*c*d**3 + int((f**((a*c + a*d*x +
 b)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*lo
g(f)**4*b**4*d**4*x)/(24*d*(c + d*x))
```

3.238 $\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx=\frac{F^{a+\frac{b}{c+dx}}(c+dx)^3}{3d}+\frac{bF^{a+\frac{b}{c+dx}}(c+dx)^2\log(F)}{6d}+\frac{b^2F^{a+\frac{b}{c+dx}}(c+dx)\log^2(F)}{6d}-\frac{b^3F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)\log^3(F)}{6d}$$

output
```
1/3*F^(a+b/(d*x+c))*(d*x+c)^3/d+1/6*b*F^(a+b/(d*x+c))*(d*x+c)^2*ln(F)/d+1/
6*b^2*F^(a+b/(d*x+c))*(d*x+c)*ln(F)^2/d-1/6*b^3*F^a*Ei(b*ln(F)/(d*x+c))*ln
(F)^3/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx=\frac{F^a\left(-b^3\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)\log^3(F)+F^{\frac{b}{c+dx}}(c+dx)\left(2(c+dx)^2+b(c+dx)\log(F)+b^2\log^2(F)\right)\right)}{6d}$$

input
```
Integrate[F^(a + b/(c + d*x))*(c + d*x)^2,x]
```

output
```
(F^a*(-(b^3*ExpIntegralEi[(b*Log[F])/(c + d*x)]*Log[F]^3) + F^(b/(c + d*x)
)*(c + d*x)*(2*(c + d*x)^2 + b*(c + d*x)*Log[F] + b^2*Log[F]^2)))/(6*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2643, 2643, 2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 F^{a+\frac{b}{c+dx}} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3} b \log(F) \int F^{a+\frac{b}{c+dx}} (c+dx) dx + \frac{(c+dx)^3 F^{a+\frac{b}{c+dx}}}{3d}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \int F^{a+\frac{b}{c+dx}} dx + \frac{(c+dx)^2 F^{a+\frac{b}{c+dx}}}{2d}\right) + \frac{(c+dx)^3 F^{a+\frac{b}{c+dx}}}{3d}$$

$$\Big\downarrow \text{ 2635}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \left(b \log(F) \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} dx + \frac{(c+dx) F^{a+\frac{b}{c+dx}}}{d}\right) + \frac{(c+dx)^2 F^{a+\frac{b}{c+dx}}}{2d}\right) + \frac{(c+dx)^3 F^{a+\frac{b}{c+dx}}}{3d}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{3} b \log(F) \left(\frac{1}{2} b \log(F) \left(\frac{(c+dx) F^{a+\frac{b}{c+dx}}}{d} - \frac{b F^a \log(F) \text{ExpIntegralEi}\left(\frac{b \log(F)}{c+dx}\right)}{d}\right) + \frac{(c+dx)^2 F^{a+\frac{b}{c+dx}}}{2d}\right) + \frac{(c+dx)^3 F^{a+\frac{b}{c+dx}}}{3d}$$

input
```
Int[F^(a + b/(c + d*x))*(c + d*x)^2,x]
```

output
```
(F^(a + b/(c + d*x))*(c + d*x)^3)/(3*d) + (b*Log[F]*((F^(a + b/(c + d*x))*
(c + d*x)^2)/(2*d) + (b*Log[F]*((F^(a + b/(c + d*x))*(c + d*x))/d - (b*F^a
*ExpIntegralEi[(b*Log[F])/(c + d*x)]*Log[F])/d))/2))/3
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(111) = 222$.

Time = 0.22 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.97

method	result
risch	$\frac{d^2 F^a F^{\frac{b}{dx+c}} x^3}{3} + d F^a F^{\frac{b}{dx+c}} c x^2 + F^a F^{\frac{b}{dx+c}} c^2 x + \frac{F^a F^{\frac{b}{dx+c}} c^3}{3d} + \frac{db \ln(F) F^a F^{\frac{b}{dx+c}} x^2}{6} + \frac{b \ln(F) F^a F^{\frac{b}{dx+c}} c x}{3} +$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*F^a*F^(b/(d*x+c))*x^3+d*F^a*F^(b/(d*x+c))*c*x^2+F^a*F^(b/(d*x+c))*
c^2*x+1/3/d*F^a*F^(b/(d*x+c))*c^3+1/6*d*b*ln(F)*F^a*F^(b/(d*x+c))*x^2+1/3*
b*ln(F)*F^a*F^(b/(d*x+c))*c*x+1/6/d*b*ln(F)*F^a*F^(b/(d*x+c))*c^2+1/6*b^2*
ln(F)^2*F^a*F^(b/(d*x+c))*x+1/6/d*b^2*ln(F)^2*F^a*F^(b/(d*x+c))*c+1/6/d*b^
3*ln(F)^3*F^a*Ei(1,-b*ln(F)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx = -\frac{F^a b^3 \mathrm{Ei}\left(\frac{b\log(F)}{dx+c}\right)\log\left(F\right)^3 - \left(2\,d^3x^3 + 6\,cd^2x^2 + 6\,c^2dx + 2\,c^3 + \left(b^2dx + b^2c\right)\log\left(F\right)^2 + \left(bd^2x^2 + 2\,bcd\right.\right.}{6\,d}$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(F^a*b^3*Ei(b*log(F)/(d*x + c))*log(F)^3 - (2*d^3*x^3 + 6*c*d^2*x^2 +
 6*c^2*d*x + 2*c^3 + (b^2*d*x + b^2*c)*log(F)^2 + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*log(F))*F^((a*d*x + a*c + b)/(d*x + c)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx = \int F^{a+\frac{b}{c+dx}}(c+dx)^2\;dx$$

input
```
integrate(F**(a+b/(d*x+c))*(d*x+c)**2,x)
```

output
```
Integral(F**(a + b/(c + d*x))*(c + d*x)**2, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx = \int (dx+c)^2 F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/6*(2*F^a*d^2*x^3 + (F^a*b*d*log(F) + 6*F^a*c*d)*x^2 + (F^a*b^2*log(F)^2
+ 2*F^a*b*c*log(F) + 6*F^a*c^2)*x)*F^(b/(d*x + c)) + integrate(1/6*(F^a*b^
3*d*x*log(F)^3 - F^a*b^2*c^2*log(F)^2 - 2*F^a*b*c^3*log(F))*F^(b/(d*x + c)
)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Giac [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx = \int (dx+c)^2 F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*F^(a + b/(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx = \frac{F^a\,b^3\ln\left(F\right)^3\left(\frac{\mathrm{expint}\left(-\frac{b\,\ln(F)}{c+d\,x}\right)}{6}+F^{\frac{b}{c+d\,x}}\left(\frac{c+d\,x}{6\,b\,\ln(F)}+\frac{(c+d\,x)^2}{6\,b^2\ln(F)^2}+\frac{(c+d\,x)^3}{3\,b^3\ln(F)^3}\right)\right)}{d}$$

input
```
int(F^(a + b/(c + d*x))*(c + d*x)^2,x)
```

output
```
(F^a*b^3*log(F)^3*(expint(-(b*log(F))/(c + d*x))/6 + F^(b/(c + d*x))*((c +
 d*x)/(6*b*log(F)) + (c + d*x)^2/(6*b^2*log(F)^2) + (c + d*x)^3/(3*b^3*log
(F)^3))))/d
```

Reduce [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)^2\,dx$$
$$=\frac{f^{\frac{adx+ac+b}{dx+c}}\log(f)^2 b^2d^2x^2+2f^{\frac{adx+ac+b}{dx+c}}\log(f)\,b\,c^2dx+3f^{\frac{adx+ac+b}{dx+c}}\log(f)\,bc\,d^2x^2+f^{\frac{adx+ac+b}{dx+c}}\log(f)\,b\,d^3x^3+2}{}$$

input
```
int(F^(a+b/(d*x+c))*(d*x+c)^2,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*d**2*x**2 + 2*f**((a*c +
a*d*x + b)/(c + d*x))*log(f)*b*c**2*d*x + 3*f**((a*c + a*d*x + b)/(c + d*x
))*log(f)*b*c*d**2*x**2 + f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*d**3*x
**3 + 2*f**((a*c + a*d*x + b)/(c + d*x))*c**4 + 8*f**((a*c + a*d*x + b)/(c
 + d*x))*c**3*d*x + 12*f**((a*c + a*d*x + b)/(c + d*x))*c**2*d**2*x**2 + 8
*f**((a*c + a*d*x + b)/(c + d*x))*c*d**3*x**3 + 2*f**((a*c + a*d*x + b)/(c
 + d*x))*d**4*x**4 + int((f**((a*c + a*d*x + b)/(c + d*x))*x**2)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*log(f)**3*b**3*c*d**3 + int((f**
((a*c + a*d*x + b)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)*log(f)**3*b**3*d**4*x)/(6*d*(c + d*x))
```

3.239 $\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx = \frac{F^{a+\frac{b}{c+dx}}(c+dx)^2}{2d} + \frac{bF^{a+\frac{b}{c+dx}}(c+dx)\log(F)}{2d} - \frac{b^2F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)\log^2(F)}{2d}$$

output
```
1/2*F^(a+b/(d*x+c))*(d*x+c)^2/d+1/2*b*F^(a+b/(d*x+c))*(d*x+c)*ln(F)/d-1/2*
b^2*F^a*Ei(b*ln(F)/(d*x+c))*ln(F)^2/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx = \frac{F^a\left(-b^2\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)\log^2(F)+F^{\frac{b}{c+dx}}(c+dx)(c+dx+b\log(F))\right)}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x))*(c + d*x),x]
```

output
```
(F^a*(-(b^2*ExpIntegralEi[(b*Log[F])/(c + d*x)]*Log[F]^2) + F^(b/(c + d*x)
)*(c + d*x)*(c + d*x + b*Log[F])))/(2*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2643, 2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)F^{a+\frac{b}{c+dx}}\,dx$$

$$\Big\downarrow \; 2643$$

$$\frac{1}{2}b\log(F)\int F^{a+\frac{b}{c+dx}}dx+\frac{(c+dx)^2F^{a+\frac{b}{c+dx}}}{2d}$$

$$\Big\downarrow \; 2635$$

$$\frac{1}{2}b\log(F)\left(b\log(F)\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}dx+\frac{(c+dx)F^{a+\frac{b}{c+dx}}}{d}\right)+\frac{(c+dx)^2F^{a+\frac{b}{c+dx}}}{2d}$$

$$\Big\downarrow \; 2639$$

$$\frac{1}{2}b\log(F)\left(\frac{(c+dx)F^{a+\frac{b}{c+dx}}}{d}-\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{d}\right)+\frac{(c+dx)^2F^{a+\frac{b}{c+dx}}}{2d}$$

input
```
Int[F^(a + b/(c + d*x))*(c + d*x),x]
```

output
```
(F^(a + b/(c + d*x))*(c + d*x)^2)/(2*d) + (b*Log[F]*((F^(a + b/(c + d*x))*
(c + d*x))/d - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)]*Log[F])/d))/2
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

method	result
risch	$\frac{d\,F^{a} F^{\frac{b}{dx+c}} x^{2}}{2}+F^{a} F^{\frac{b}{dx+c}} c x+\frac{F^{a} F^{\frac{b}{dx+c}} c^{2}}{2 d}+\frac{b \ln\left(F\right) F^{a} F^{\frac{b}{dx+c}} x}{2}+\frac{b \ln\left(F\right) F^{a} F^{\frac{b}{dx+c}} c}{2 d}+\frac{b^{2} \ln\left(F\right)^{2} F^{a} \operatorname{expIntegral}_{1}\left(-\right.}{2 d}$

input
```
int(F^(a+b/(d*x+c))*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*F^a*F^(b/(d*x+c))*x^2+F^a*F^(b/(d*x+c))*c*x+1/2/d*F^a*F^(b/(d*x+c))*
c^2+1/2*b*ln(F)*F^a*F^(b/(d*x+c))*x+1/2/d*b*ln(F)*F^a*F^(b/(d*x+c))*c+1/2/
d*b^2*ln(F)^2*F^a*Ei(1,-b*ln(F)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx$$
$$= -\frac{F^a b^2 \text{Ei}\left(\frac{b\log(F)}{dx+c}\right)\log(F)^2 - (d^2x^2 + 2\,cdx + c^2 + (bdx + bc)\log(F))F^{\frac{adx+ac+b}{dx+c}}}{2\,d}$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(F^a*b^2*Ei(b*log(F)/(d*x + c))*log(F)^2 - (d^2*x^2 + 2*c*d*x + c^2 +
 (b*d*x + b*c)*log(F))*F^((a*d*x + a*c + b)/(d*x + c)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx = \int F^{a+\frac{b}{c+dx}}(c+dx)\,dx$$

input
```
integrate(F**(a+b/(d*x+c))*(d*x+c),x)
```

output
```
Integral(F**(a + b/(c + d*x))*(c + d*x), x)
```

Maxima [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx = \int (dx+c)F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(F^a*d*x^2 + (F^a*b*log(F) + 2*F^a*c)*x)*F^(b/(d*x + c)) + integrate(1
/2*(F^a*b^2*d*x*log(F)^2 - F^a*b*c^2*log(F))*F^(b/(d*x + c))/(d^2*x^2 + 2*
c*d*x + c^2), x)
```

Giac [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx=\int (dx+c)F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*F^(a + b/(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx=\frac{F^a\,F^{\frac{b}{c+d\,x}}\,{\left(c+d\,x\right)}^2}{2\,d}+\frac{F^a\,b^2\,{\ln\left(F\right)}^2\,\mathrm{expint}\left(-\frac{b\,\ln(F)}{c+d\,x}\right)}{2\,d}+\frac{F^a\,F^{\frac{b}{c+d\,x}}\,b\,\ln\left(F\right)\,\left(c+d\,x\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x))*(c + d*x),x)
```

output
```
(F^a*F^(b/(c + d*x))*(c + d*x)^2)/(2*d) + (F^a*b^2*log(F)^2*expint(-(b*log
(F))/(c + d*x)))/(2*d) + (F^a*F^(b/(c + d*x))*b*log(F)*(c + d*x))/(2*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{c+dx}}(c+dx)\,dx=\frac{f^{\frac{adx+ac+b}{dx+c}}\log(f)\,bd\,x^2+2f^{\frac{adx+ac+b}{dx+c}}c^2x+3f^{\frac{adx+ac+b}{dx+c}}cd\,x^2+f^{\frac{adx+ac+b}{dx+c}}d^2x^3+\left(\int\frac{f^{\frac{adx+ac+b}{dx+c}}x^2}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)\mathrm{lo}}{2dx+2c}$$

input
```
int(F^(a+b/(d*x+c))*(d*x+c),x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*d*x**2 + 2*f**((a*c + a*d*x + b
)/(c + d*x))*c**2*x + 3*f**((a*c + a*d*x + b)/(c + d*x))*c*d*x**2 + f**((a
*c + a*d*x + b)/(c + d*x))*d**2*x**3 + int((f**((a*c + a*d*x + b)/(c + d*x
))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*log(f)**2*b**2
*c*d**2 + int((f**((a*c + a*d*x + b)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3),x)*log(f)**2*b**2*d**3*x)/(2*(c + d*x))
```

3.240 $\int F^{a+\frac{b}{c+dx}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int F^{a+\frac{b}{c+dx}}\,dx=\frac{F^{a+\frac{b}{c+dx}}(c+dx)}{d}-\frac{bF^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)\log(F)}{d}$$

output
```
F^(a+b/(d*x+c))*(d*x+c)/d-b*F^a*Ei(b*ln(F)/(d*x+c))*ln(F)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int F^{a+\frac{b}{c+dx}}\,dx=\frac{F^a\left(F^{\frac{b}{c+dx}}(c+dx)-b\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)\log(F)\right)}{d}$$

input
```
Integrate[F^(a + b/(c + d*x)),x]
```

output
```
(F^a*(F^(b/(c + d*x))*(c + d*x) - b*ExpIntegralEi[(b*Log[F])/(c + d*x)]*Lo
g[F]))/d
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+\frac{b}{c+dx}}\,dx$$

$$\Big\downarrow \text{2635}$$

$$b\log(F)\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}dx+\frac{(c+dx)F^{a+\frac{b}{c+dx}}}{d}$$

$$\Big\downarrow \text{2639}$$

$$\frac{(c+dx)F^{a+\frac{b}{c+dx}}}{d}-\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{d}$$

input
```
Int[F^(a + b/(c + d*x)),x]
```

output
```
(F^(a + b/(c + d*x))*(c + d*x))/d - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d
*x)]*Log[F])/d
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

method	result	size
risch	$F^{a} F^{\frac{b}{d x +c}} x +\frac{F^{a} F^{\frac{b}{d x +c}} c}{d}+\frac{b \ln \left(F \right) F^{a} \operatorname{expIntegral}_{1}\left(-\frac{b \ln \left(F \right)}{d x +c}\right)}{d}$	61

input
```
int(F^(a+b/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
F^a*F^(b/(d*x+c))*x+1/d*F^a*F^(b/(d*x+c))*c+b/d*ln(F)*F^a*Ei(1,-b*ln(F)/(d
*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int F^{a+\frac{b}{c+dx}}\,dx=-\frac{F^{a} b {\rm Ei}\left(\frac{b \log\left(F\right)}{d x + c}\right) \log\left(F\right) - {\left(d x + c\right)} F^{\frac{a d x + a c + b}{d x + c}}}{d}$$

input
```
integrate(F^(a+b/(d*x+c)),x, algorithm="fricas")
```

output
```
-(F^a*b*Ei(b*log(F)/(d*x + c))*log(F) - (d*x + c)*F^((a*d*x + a*c + b)/(d*
x + c)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{c+dx}}\,dx=\int F^{a + \frac{b}{c + d x}}\, dx$$

input
```
integrate(F**(a+b/(d*x+c)),x)
```

output
```
Integral(F**(a + b/(c + d*x)), x)
```

Maxima [F]

$$\int F^{a+\frac{b}{c+dx}}\,dx = \int F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)),x, algorithm="maxima")
```

output
```
F^a*b*d*integrate(F^(b/(d*x + c))*x/(d^2*x^2 + 2*c*d*x + c^2), x)*log(F) +
 F^a*F^(b/(d*x + c))*x
```

Giac [F]

$$\int F^{a+\frac{b}{c+dx}}\,dx = \int F^{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int F^{a+\frac{b}{c+dx}}\,dx = \frac{F^a\,F^{\frac{b}{c+d\,x}}\,(c+d\,x)}{d} + \frac{F^a\,b\,\ln(F)\,\mathrm{expint}\left(-\frac{b\,\ln(F)}{c+d\,x}\right)}{d}$$

input
```
int(F^(a + b/(c + d*x)),x)
```

output
```
(F^a*F^(b/(c + d*x))*(c + d*x))/d + (F^a*b*log(F)*expint(-(b*log(F))/(c +
d*x)))/d
```

Reduce [F]

$$\int F^{a+\frac{b}{c+dx}}\,dx$$

$$= \frac{f^{\frac{adx+ac+b}{dx+c}}\log(f)\,b\,d^2x^2 - f^{\frac{adx+ac+b}{dx+c}}c^3 - f^{\frac{adx+ac+b}{dx+c}}c^2dx + \left(\int \frac{f^{\frac{adx+ac+b}{dx+c}}x^2}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)\log(f)^2\,b^2c\,d^3 + \left(\int \frac{}{d^3}\right.}{\log\left(f\right)bd\left(dx+c\right)}$$

input
```
int(F^(a+b/(d*x+c)),x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*d**2*x**2 - f**((a*c + a*d*x +
b)/(c + d*x))*c**3 - f**((a*c + a*d*x + b)/(c + d*x))*c**2*d*x + int((f**(
(a*c + a*d*x + b)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*log(f)**2*b**2*c*d**3 + int((f**((a*c + a*d*x + b)/(c + d*x))*
x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*log(f)**2*b**2*d*
*4*x)/(log(f)*b*d*(c + d*x))
```

3.241 $\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = -\frac{F^a \operatorname{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{d}$$

output
```
-F^a*Ei(b*ln(F)/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = -\frac{F^a \operatorname{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{d}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x),x]
```

output
```
-((F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)])/d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx} \, dx$$

$$\Big\downarrow \ 2639$$

$$-\frac{F^a \text{ExpIntegralEi}\left(\frac{b \log(F)}{c+dx}\right)}{d}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x),x]
```

output
```
-((F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)])/d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{F^a \operatorname{expIntegral}_1\left(-\frac{b \ln(F)}{dx+c}\right)}{d}$	22

input
```
int(F^(a+b/(d*x+c))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*F^a*Ei(1,-b*ln(F)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = -\frac{F^a \operatorname{Ei}\left(\frac{b\log(F)}{dx+c}\right)}{d}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c),x, algorithm="fricas")
```

output
```
-F^a*Ei(b*log(F)/(d*x + c))/d
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c),x)
```

output
```
Integral(F**(a + b/(c + d*x))/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{dx+c}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{dx+c}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = -\frac{F^a\,\mathrm{ei}\left(\frac{b\,\ln(F)}{c+d\,x}\right)}{d}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x),x)
```

output
```
-(F^a*ei((b*log(F))/(c + d*x)))/d
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}\,dx = \int \frac{f^{\frac{adx+ac+b}{dx+c}}}{dx+c}\,dx$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c),x)
```

output
```
int(f**((a*c + a*d*x + b)/(c + d*x))/(c + d*x),x)
```

3.242 $\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx = -\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)}$$

output
```
-F^(a+b/(d*x+c))/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2} dx = -\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x)^2,x]
```

output
```
-(F^(a + b/(c + d*x))/(b*d*Log[F]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 2638$$

$$-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x)^2,x]
```

output
```
-(F^(a + b/(c + d*x))/(b*d*Log[F]))
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{F^{a+\frac{b}{dx+c}}}{bd\ln(F)}$	26
default	$-\frac{F^{a+\frac{b}{dx+c}}}{bd\ln(F)}$	26
parallelrisch	$-\frac{F^{a+\frac{b}{dx+c}}}{bd\ln(F)}$	26
risch	$-\frac{F^{\frac{adx+ac+b}{dx+c}}}{bd\ln(F)}$	32
norman	$\frac{-\frac{x\,e^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)b}-\frac{c\,e^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)bd}}{dx+c}$	63

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-F^(a+b/(d*x+c))/b/d/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx = -\frac{F^{\frac{adx+ac+b}{dx+c}}}{bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-F^((a*d*x + a*c + b)/(d*x + c))/(b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx = \begin{cases} -\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)} & \text{for } bd\log(F)\neq 0 \\ -\frac{1}{cd+d^2x} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c)**2,x)
```

output
```
Piecewise((-F**(a + b/(c + d*x))/(b*d*log(F)), Ne(b*d*log(F), 0)), (-1/(c*
d + d**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx = -\frac{F^{a+\frac{b}{dx+c}}}{bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-F^(a + b/(d*x + c))/(b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx = -\frac{F^{\frac{adx+ac+b}{dx+c}}}{bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^2,x, algorithm="giac")
```

output
```
-F^((a*d*x + a*c + b)/(d*x + c))/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx = -\frac{F^{a+\frac{b}{c+d\,x}}}{b\,d\,\ln(F)}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x)^2,x)
```

output
```
-F^(a + b/(c + d*x))/(b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}\,dx = -\frac{f^{\frac{adx+ac+b}{dx+c}}}{\log(f)\,bd}$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^2,x)
```

output
```
( - f**((a*c + a*d*x + b)/(c + d*x)))/(log(f)*b*d)
```

3.243 $\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx = \frac{F^{a+\frac{b}{c+dx}}}{b^2 d \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)\log(F)}$$

output
```
F^(a+b/(d*x+c))/b^2/d/ln(F)^2-F^(a+b/(d*x+c))/b/d/(d*x+c)/ln(F)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} dx = \frac{F^{a+\frac{b}{c+dx}}(c+dx-b\log(F))}{b^2 d(c+dx)\log^2(F)}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x)^3,x]
```

output
```
(F^(a + b/(c + d*x))*(c + d*x - b*Log[F]))/(b^2*d*(c + d*x)*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}dx}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{F^{a+\frac{b}{c+dx}}}{b^2d\log^2(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x)^3,x]
```

output
```
F^(a + b/(c + d*x))/(b^2*d*Log[F]^2) - F^(a + b/(c + d*x))/(b*d*(c + d*x)*
Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{(b\ln(F)-dx-c)F^{\frac{adx+ac+b}{dx+c}}}{d\ln(F)^2b^2(dx+c)}$	51
parallelrisch	$\frac{-\ln(F)F^{a+\frac{b}{dx+c}}b\,d^4+x\,F^{a+\frac{b}{dx+c}}d^5+F^{a+\frac{b}{dx+c}}c\,d^4}{(dx+c)\ln(F)^2b^2d^5}$	77
norman	$\frac{\frac{d\,x^2\mathrm{e}^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)^2b^2}-\frac{(b\ln(F)-2c)x\,\mathrm{e}^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)^2b^2}-\frac{c(b\ln(F)-c)\mathrm{e}^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{d\ln(F)^2b^2}}{(dx+c)^2}$	106

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(F)-d*x-c)/d/ln(F)^2/b^2/(d*x+c)*F^((a*d*x+a*c+b)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}\,dx = \frac{(dx-b\log(F)+c)F^{\frac{adx+ac+b}{dx+c}}}{(b^2d^2x+b^2cd)\log(F)^2}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
(d*x - b*log(F) + c)*F^((a*d*x + a*c + b)/(d*x + c))/((b^2*d^2*x + b^2*c*d
)*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}\,dx = \frac{F^{a+\frac{b}{c+dx}}(-b\log(F)+c+dx)}{b^2cd\log(F)^2+b^2d^2x\log(F)^2}$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c)**3,x)
```

output
```
F**(a + b/(c + d*x))*(-b*log(F) + c + d*x)/(b**2*c*d*log(F)**2 + b**2*d**2
*x*log(F)**2)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}\,dx = \frac{F^{a+\frac{b}{c+d\,x}}\,(c+d\,x-b\,\ln(F))}{b^2\,d\ln(F)^2\,(c+d\,x)}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x)^3,x)
```

output
```
(F^(a + b/(c + d*x))*(c + d*x - b*log(F)))/(b^2*d*log(F)^2*(c + d*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}\,dx = \frac{f^{\frac{adx+ac+b}{dx+c}}(-\log(f)\,b+c+dx)}{\log(f)^2\,b^2d\,(dx+c)}$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^3,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*( - log(f)*b + c + d*x))/(log(f)**2*b**2
*d*(c + d*x))
```

3.244 $\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx = -\frac{2F^{a+\frac{b}{c+dx}}}{b^3 d\log^3(F)} + \frac{2F^{a+\frac{b}{c+dx}}}{b^2 d(c+dx)\log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^2\log(F)}$$

output
```
-2*F^(a+b/(d*x+c))/b^3/d/ln(F)^3+2*F^(a+b/(d*x+c))/b^2/d/(d*x+c)/ln(F)^2-F
^(a+b/(d*x+c))/b/d/(d*x+c)^2/ln(F)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx = -\frac{F^{a+\frac{b}{c+dx}}\left(2(c+dx)^2-2b(c+dx)\log(F)+b^2\log^2(F)\right)}{b^3 d(c+dx)^2\log^3(F)}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x)^4,x]
```

output
```
-((F^(a + b/(c + d*x))*(2*(c + d*x)^2 - 2*b*(c + d*x)*Log[F] + b^2*Log[F]^
2))/(b^3*d*(c + d*x)^2*Log[F]^3))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{2\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}dx}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^2}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{2\left(-\frac{\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}dx}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^2}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{2\left(\frac{F^{a+\frac{b}{c+dx}}}{b^2d\log^2(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^2}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x)^4,x]
```

output
```
-(F^(a + b/(c + d*x))/(b*d*(c + d*x)^2*Log[F])) - (2*(F^(a + b/(c + d*x))/
(b^2*d*Log[F]^2) - F^(a + b/(c + d*x))/(b*d*(c + d*x)*Log[F])))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{\left(\ln\left(F\right)^{2} b^{2}-2\ln\left(F\right) b d x+2x^{2} d^{2}-2 b c\ln\left(F\right)+4 c d x+2 c^{2}\right) F^{\frac{a d x+a c+b}{d x+c}}}{b^{3}\ln\left(F\right)^{3} d \left(d x+c\right)^{2}}$
parallelrisch	$\frac{-\ln\left(F\right)^{2} F^{a+\frac{b}{d x+c}} b^{2} d^{5}+2\ln\left(F\right) x\, F^{a+\frac{b}{d x+c}} b\, d^{6}-2x^{2} F^{a+\frac{b}{d x+c}} d^{7}+2\ln\left(F\right) F^{a+\frac{b}{d x+c}} b c\, d^{5}-4x\, F^{a+\frac{b}{d x+c}} c\, d^{6}-2F^{a+\frac{b}{d x+c}} c^{2} \cdots}{\left(d x+c\right)^{2} b^{3}\ln\left(F\right)^{3} d^{6}}$
norman	$\frac{-\frac{2d^{2} x^{3} {\mathrm e}^{\left(a+\frac{b}{d x+c}\right)\ln\left(F\right)}}{\ln\left(F\right)^{3} b^{3}}-\frac{\left(\ln\left(F\right)^{2} b^{2}-4 b c\ln\left(F\right)+6c^{2}\right) x\,{\mathrm e}^{\left(a+\frac{b}{d x+c}\right)\ln\left(F\right)}}{\ln\left(F\right)^{3} b^{3}}+\frac{2d\left(b\ln\left(F\right)-3c\right) x^{2} {\mathrm e}^{\left(a+\frac{b}{d x+c}\right)\ln\left(F\right)}}{\ln\left(F\right)^{3} b^{3}}-\left(\ln\left(F\right)^{2} b^{2}-2 b c\ln(\cdots\right.}{\left(d x+c\right)^{3}}$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-(ln(F)^2*b^2-2*ln(F)*b*d*x+2*x^2*d^2-2*b*c*ln(F)+4*c*d*x+2*c^2)/b^3/ln(F)
^3/d/(d*x+c)^2*F^((a*d*x+a*c+b)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.06

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx = -\frac{\left(2\,d^2x^2+b^2\log\left(F\right)^2+4\,cdx+2\,c^2-2\,(bdx+bc)\log\left(F\right)\right)F^{\frac{adx+ac+b}{dx+c}}}{\left(b^3d^3x^2+2\,b^3cd^2x+b^3c^2d\right)\log\left(F\right)^3}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-(2*d^2*x^2 + b^2*log(F)^2 + 4*c*d*x + 2*c^2 - 2*(b*d*x + b*c)*log(F))*F^(
(a*d*x + a*c + b)/(d*x + c))/((b^3*d^3*x^2 + 2*b^3*c*d^2*x + b^3*c^2*d)*lo
g(F)^3)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx = \frac{F^{a+\frac{b}{c+dx}}\left(-b^2\log\left(F\right)^2+2bc\log\left(F\right)+2bdx\log\left(F\right)-2c^2-4cdx-2d^2x^2\right)}{b^3c^2d\log\left(F\right)^3+2b^3cd^2x\log\left(F\right)^3+b^3d^3x^2\log\left(F\right)^3}$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c)**4,x)
```

output
```
F**(a + b/(c + d*x))*(-b**2*log(F)**2 + 2*b*c*log(F) + 2*b*d*x*log(F) - 2*
c**2 - 4*c*d*x - 2*d**2*x**2)/(b**3*c**2*d*log(F)**3 + 2*b**3*c*d**2*x*log
(F)**3 + b**3*d**3*x**2*log(F)**3)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx=\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx=\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx=-\frac{F^{a+\frac{b}{c+d\,x}}\left(\frac{b^2\ln(F)^2-2\,b\,c\,\ln(F)+2\,c^2}{b^3\,d^3\ln(F)^3}+\frac{2\,x^2}{b^3\,d\ln(F)^3}+\frac{2\,x\,(2\,c-b\,\ln(F))}{b^3\,d^2\ln(F)^3}\right)}{x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x)^4,x)
```

output
```
-(F^(a + b/(c + d*x))*((b^2*log(F)^2 + 2*c^2 - 2*b*c*log(F))/(b^3*d^3*log(
F)^3) + (2*x^2)/(b^3*d*log(F)^3) + (2*x*(2*c - b*log(F)))/(b^3*d^2*log(F)^
3)))/(x^2 + c^2/d^2 + (2*c*x)/d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}\,dx = \frac{f^{\frac{adx+ac+b}{dx+c}}\left(-\log(f)^2 b^2+2\log(f)\,bc+2\log(f)\,bdx-2c^2-4cdx-2d^2x^2\right)}{\log(f)^3 b^3 d\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^4,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*( - log(f)**2*b**2 + 2*log(f)*b*c + 2*lo
g(f)*b*d*x - 2*c**2 - 4*c*d*x - 2*d**2*x**2))/(log(f)**3*b**3*d*(c**2 + 2*
c*d*x + d**2*x**2))
```

3.245 $\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5} dx$

Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5} dx = \frac{6F^{a+\frac{b}{c+dx}}}{b^4 d \log^4(F)} - \frac{6F^{a+\frac{b}{c+dx}}}{b^3 d(c+dx) \log^3(F)} + \frac{3F^{a+\frac{b}{c+dx}}}{b^2 d(c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{c+dx}}}{bd(c+dx)^3 \log(F)}$$

output
```
6*F^(a+b/(d*x+c))/b^4/d/ln(F)^4-6*F^(a+b/(d*x+c))/b^3/d/(d*x+c)/ln(F)^3+3*
F^(a+b/(d*x+c))/b^2/d/(d*x+c)^2/ln(F)^2-F^(a+b/(d*x+c))/b/d/(d*x+c)^3/ln(F
)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.62

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5} dx = \frac{F^{a+\frac{b}{c+dx}} (6(c+dx)^3 - 6b(c+dx)^2 \log(F) + 3b^2(c+dx) \log^2(F) - b^3 \log^3(F))}{b^4 d(c+dx)^3 \log^4(F)}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x)^5,x]
```

output
```
(F^(a + b/(c + d*x))*(6*(c + d*x)^3 - 6*b*(c + d*x)^2*Log[F] + 3*b^2*(c +
d*x)*Log[F]^2 - b^3*Log[F]^3))/(b^4*d*(c + d*x)^3*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{3\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^4}dx}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{3\left(-\frac{2\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^3}dx}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2}dx}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{3\left(-\frac{2\left(\frac{F^{a+\frac{b}{c+dx}}}{b^2d\log^2(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{c+dx}}}{bd\log(F)(c+dx)^3}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x)^5,x]
```

output
```
-(F^(a + b/(c + d*x))/(b*d*(c + d*x)^3*Log[F])) - (3*(-(F^(a + b/(c + d*x)
)/(b*d*(c + d*x)^2*Log[F])) - (2*(F^(a + b/(c + d*x))/(b^2*d*Log[F]^2) - F
^(a + b/(c + d*x))/(b*d*(c + d*x)*Log[F])))/(b*Log[F])))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(\ln(F)^3 b^3-3\ln(F)^2 b^2 dx+6\ln(F) b\,d^2x^2-6d^3x^3-3\ln(F)^2 b^2 c+12\ln(F) bcdx-18c\,d^2x^2+6\ln(F) b\,c^2-18c^2dx-6c^3\right) F^{\frac{adx \cdots}{d \cdots}}}{b^4\ln(F)^4 d(dx+c)^3}$
norman	$\frac{-\frac{\left(\ln(F)^3b^3-6\ln(F)^2b^2c+18\ln(F)b\,c^2-24c^3\right)x\,e^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)^4b^4}+\frac{6d^3x^4e^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)^4b^4}+\frac{3d\left(\ln(F)^2b^2-6bc\ln(F)+12c^2\right)x^2e^{\left(a+\cdots\right.}}{\ln(F)^4b^4}\cdots}{(dx+c)^4}$
parallelrisch	$\frac{-\ln(F)^3F^{a+\frac{b}{dx+c}}b^3d^6+3\ln(F)^2x\,F^{a+\frac{b}{dx+c}}b^2d^7-6\ln(F)x^2F^{a+\frac{b}{dx+c}}b\,d^8+6x^3F^{a+\frac{b}{dx+c}}d^9+3\ln(F)^2F^{a+\frac{b}{dx+c}}b^2c\,d^6-12\cdots}{(dx+c)^3\ln(F)^4b^4d\cdots}$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-(ln(F)^3*b^3-3*ln(F)^2*b^2*d*x+6*ln(F)*b*d^2*x^2-6*d^3*x^3-3*ln(F)^2*b^2*
c+12*ln(F)*b*c*d*x-18*c*d^2*x^2+6*ln(F)*b*c^2-18*c^2*d*x-6*c^3)/b^4/ln(F)^
4/d/(d*x+c)^3*F^((a*d*x+a*c+b)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.23

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}\,dx$$
$$=\frac{\left(6\,d^3x^3-b^3\log\left(F\right)^3+18\,cd^2x^2+18\,c^2dx+6\,c^3+3\left(b^2dx+b^2c\right)\log\left(F\right)^2-6\left(bd^2x^2+2\,bcdx+bc^2\right)\log \cdots\right.}{\left(b^4d^4x^3+3\,b^4cd^3x^2+3\,b^4c^2d^2x+b^4c^3d\right)\log\left(F\right)^4}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^5,x, algorithm="fricas")
```

output
```
(6*d^3*x^3 - b^3*log(F)^3 + 18*c*d^2*x^2 + 18*c^2*d*x + 6*c^3 + 3*(b^2*d*x
 + b^2*c)*log(F)^2 - 6*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))*F^((a*d*x +
 a*c + b)/(d*x + c))/((b^4*d^4*x^3 + 3*b^4*c*d^3*x^2 + 3*b^4*c^2*d^2*x + b
^4*c^3*d)*log(F)^4)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.45

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}\,dx$$
$$= \frac{F^{a+\frac{b}{c+dx}}\left(-b^3\log{(F)}^3+3b^2c\log{(F)}^2+3b^2dx\log{(F)}^2-6bc^2\log{(F)}-12bcdx\log{(F)}-6bd^2x^2\log{(F)}\right.}{b^4c^3d\log{(F)}^4+3b^4c^2d^2x\log{(F)}^4+3b^4cd^3x^2\log{(F)}^4+b^4d^4x^3\log{(F}}$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c)**5,x)
```

output
```
F**(a + b/(c + d*x))*(-b**3*log(F)**3 + 3*b**2*c*log(F)**2 + 3*b**2*d*x*lo
g(F)**2 - 6*b*c**2*log(F) - 12*b*c*d*x*log(F) - 6*b*d**2*x**2*log(F) + 6*c
**3 + 18*c**2*d*x + 18*c*d**2*x**2 + 6*d**3*x**3)/(b**4*c**3*d*log(F)**4 +
 3*b**4*c**2*d**2*x*log(F)**4 + 3*b**4*c*d**3*x**2*log(F)**4 + b**4*d**4*x
**3*log(F)**4)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^5, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.32

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}\,dx = \frac{F^{a+\frac{b}{c+d\,x}}\left(\frac{6\,x^3}{b^4\,d\ln(F)^4}-\frac{b^3\ln(F)^3-3\,b^2\,c\ln(F)^2+6\,b\,c^2\,\ln(F)-6\,c^3}{b^4\,d^4\ln(F)^4}+\frac{x^2\,(18\,c-6\,b\,\ln(F))}{b^4\,d^2\ln(F)^4}+\frac{3\,x\,\left(b^2\ln(F)^2-4\,b\,c\,\ln(F)+6\,c^2\right)}{b^4\,d^3\ln(F)^4}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x)^5,x)
```

output
```
(F^(a + b/(c + d*x))*((6*x^3)/(b^4*d*log(F)^4) - (b^3*log(F)^3 - 6*c^3 + 6
*b*c^2*log(F) - 3*b^2*c*log(F)^2)/(b^4*d^4*log(F)^4) + (x^2*(18*c - 6*b*lo
g(F)))/(b^4*d^2*log(F)^4) + (3*x*(b^2*log(F)^2 + 6*c^2 - 4*b*c*log(F)))/(b
^4*d^3*log(F)^4)))/(x^3 + c^3/d^3 + (3*c*x^2)/d + (3*c^2*x)/d^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^5}\,dx = \frac{f^{\frac{adx+ac+b}{dx+c}}\left(-\log(f)^3 b^3+3\log(f)^2 b^2c+3\log(f)^2 b^2dx-6\log(f)\,b\,c^2-12\log(f)\,bcdx-6\log(f)\,b\,d^2x^2+\right.}{\log(f)^4 b^4d\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^5,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*( - log(f)**3*b**3 + 3*log(f)**2*b**2*c
+ 3*log(f)**2*b**2*d*x - 6*log(f)*b*c**2 - 12*log(f)*b*c*d*x - 6*log(f)*b*
d**2*x**2 + 6*c**3 + 18*c**2*d*x + 18*c*d**2*x**2 + 6*d**3*x**3))/(log(f)*
*4*b**4*d*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))
```

3.246 $\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx = -\frac{F^{a+\frac{b}{c+dx}}(24(c+dx)^4-24b(c+dx)^3\log(F)+12b^2(c+dx)^2\log^2(F)-4b^3(c+dx)\log^3(F)+b^4\log^4(}{b^5d(c+dx)^4\log^5(F)}$$

output
```
-F^(a+b/(d*x+c))*(24*(d*x+c)^4-24*b*(d*x+c)^3*ln(F)+12*b^2*(d*x+c)^2*ln(F)
^2-4*b^3*(d*x+c)*ln(F)^3+b^4*ln(F)^4)/b^5/d/(d*x+c)^4/ln(F)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx = -\frac{F^a\Gamma\left(5,-\frac{b\log(F)}{c+dx}\right)}{b^5d\log^5(F)}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x)^6,x]
```

output
```
-((F^a*Gamma[5, -((b*Log[F])/(c + d*x))])/(b^5*d*Log[F]^5))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6} \, dx$$

$$\Big\downarrow \text{2647}$$

$$-\frac{F^{a+\frac{b}{c+dx}} \left(b^4 \log^4(F) - 4b^3 \log^3(F)(c+dx) + 12b^2 \log^2(F)(c+dx)^2 - 24b \log(F)(c+dx)^3 + 24(c+dx)^4\right)}{b^5 d \log^5(F)(c+dx)^4}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x)^6,x]
```

output
```
-((F^(a + b/(c + d*x))*(24*(c + d*x)^4 - 24*b*(c + d*x)^3*Log[F] + 12*b^2*
(c + d*x)^2*Log[F]^2 - 4*b^3*(c + d*x)*Log[F]^3 + b^4*Log[F]^4))/(b^5*d*(c
 + d*x)^4*Log[F]^5))
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(92) = 184$.

Time = 0.46 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

method	result
risch	$-\frac{\left(b^4 \ln(F)^4-4 \ln(F)^3 b^3 dx+12 \ln(F)^2 b^2 d^2 x^2-24 \ln(F) b\, d^3 x^3+24 d^4 x^4-4 \ln(F)^3 b^3 c+24 \ln(F)^2 b^2 c d x-72 \ln(F) b c\, d^2 x^2+96 \cdots\right.}{b^5 \ln(F)^5 d (dx+c)^4}$
norman	$\frac{-\frac{24 d^4 x^5 \mathrm{e}^{\left(a+\frac{b}{dx+c}\right) \ln(F)}}{\ln(F)^5 b^5}-\frac{\left(b^4 \ln(F)^4-8 \ln(F)^3 b^3 c+36 \ln(F)^2 b^2 c^2-96 \ln(F) b\, c^3+120 c^4\right) x\, \mathrm{e}^{\left(a+\frac{b}{dx+c}\right) \ln(F)}}{\ln(F)^5 b^5}+\frac{4 d \left(\ln(F)^3 b^3-9 \ln(F) \cdots\right.}{\cdots}}{\cdots}$
parallelrisch	$\frac{-\ln(F)^4 F^{a+\frac{b}{dx+c}} b^4 d^7+4 \ln(F)^3 x\, F^{a+\frac{b}{dx+c}} b^3 d^8-12 \ln(F)^2 x^2 F^{a+\frac{b}{dx+c}} b^2 d^9+24 \ln(F) x^3 F^{a+\frac{b}{dx+c}} b\, d^{10}-24 d^{11} F^{a+\frac{b}{dx+c}} \cdots}{\cdots}$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-(b^4*ln(F)^4-4*ln(F)^3*b^3*d*x+12*ln(F)^2*b^2*d^2*x^2-24*ln(F)*b*d^3*x^3+
24*d^4*x^4-4*ln(F)^3*b^3*c+24*ln(F)^2*b^2*c*d*x-72*ln(F)*b*c*d^2*x^2+96*c*
d^3*x^3+12*ln(F)^2*b^2*c^2-72*ln(F)*b*c^2*d*x+144*c^2*d^2*x^2-24*ln(F)*b*c
^3+96*c^3*d*x+24*c^4)/b^5/ln(F)^5/d/(d*x+c)^4*F^((a*d*x+a*c+b)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(92) = 184$.

Time = 0.08 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.38

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\, dx = -\frac{\left(24\, d^4 x^4 + b^4 \log(F)^4 + 96\, c d^3 x^3 + 144\, c^2 d^2 x^2 + 96\, c^3 d x + 24\, c^4 - 4\, (b^3 d x + b^3 c) \log(F)^3 + 12\, (b^2 d^2 x \cdots\right.}{(b^5 d^5 x^4 + 4\, b^5 c d^4 x^3 + 6\, b^5 c^2 d^3 x^2 + 4\, b^5 \cdots}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^6,x, algorithm="fricas")
```

output
```
-(24*d^4*x^4 + b^4*log(F)^4 + 96*c*d^3*x^3 + 144*c^2*d^2*x^2 + 96*c^3*d*x
+ 24*c^4 - 4*(b^3*d*x + b^3*c)*log(F)^3 + 12*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*log(F)^2 - 24*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*l
og(F))*F^((a*d*x + a*c + b)/(d*x + c))/((b^5*d^5*x^4 + 4*b^5*c*d^4*x^3 + 6
*b^5*c^2*d^3*x^2 + 4*b^5*c^3*d^2*x + b^5*c^4*d)*log(F)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(90) = 180$.

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.96

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx = \frac{F^{a+\frac{b}{c+dx}}\left(-b^4\log(F)^4 + 4b^3c\log(F)^3 + 4b^3dx\log(F)^3 - 12b^2c^2\log(F)^2 - 24b^2cdx\log(F)^2 - 12b^2d^2x^2\right.}{b^5c^4d\log(F)^5 + 4b^5c^3d^2x\log(F)}$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c)**6,x)
```

output
```
F**(a + b/(c + d*x))*(-b**4*log(F)**4 + 4*b**3*c*log(F)**3 + 4*b**3*d*x*lo
g(F)**3 - 12*b**2*c**2*log(F)**2 - 24*b**2*c*d*x*log(F)**2 - 12*b**2*d**2*
x**2*log(F)**2 + 24*b*c**3*log(F) + 72*b*c**2*d*x*log(F) + 72*b*c*d**2*x**
2*log(F) + 24*b*d**3*x**3*log(F) - 24*c**4 - 96*c**3*d*x - 144*c**2*d**2*x
**2 - 96*c*d**3*x**3 - 24*d**4*x**4)/(b**5*c**4*d*log(F)**5 + 4*b**5*c**3*
d**2*x*log(F)**5 + 6*b**5*c**2*d**3*x**2*log(F)**5 + 4*b**5*c*d**4*x**3*lo
g(F)**5 + b**5*d**5*x**4*log(F)**5)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^6}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^6,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^6, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^6}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^6,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.51

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx = -\frac{F^{a+\frac{b}{c+d\,x}}\left(\frac{b^4\ln(F)^4-4\,b^3\,c\ln(F)^3+12\,b^2\,c^2\ln(F)^2-24\,b\,c^3\,\ln(F)+24\,c^4}{b^5\,d^5\ln(F)^5}+\frac{24\,x^4}{b^5\,d\ln(F)^5}+\frac{x^2\left(12\,b^2\ln(F)^2-72\,b\,c\,\ln(F)+144\,c^2\right)}{b^5\,d^3\ln(F)^5}+\right.}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x)^6,x)
```

output
```
-(F^(a + b/(c + d*x))*((b^4*log(F)^4 + 24*c^4 - 24*b*c^3*log(F) - 4*b^3*c*
log(F)^3 + 12*b^2*c^2*log(F)^2)/(b^5*d^5*log(F)^5) + (24*x^4)/(b^5*d*log(F
)^5) + (x^2*(12*b^2*log(F)^2 + 144*c^2 - 72*b*c*log(F)))/(b^5*d^3*log(F)^5
) + (x^3*(96*c - 24*b*log(F)))/(b^5*d^2*log(F)^5) - (4*x*(b^3*log(F)^3 - 2
4*c^3 + 18*b*c^2*log(F) - 6*b^2*c*log(F)^2))/(b^5*d^4*log(F)^5)))/(x^4 + c
^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d^3 + (6*c^2*x^2)/d^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.40

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^6}\,dx$$
$$= \frac{f^{\frac{adx+ac+b}{dx+c}}\left(-\log(f)^4 b^4 + 4\log(f)^3 b^3 c + 4\log(f)^3 b^3 dx - 12\log(f)^2 b^2 c^2 - 24\log(f)^2 b^2 c dx - 12\log(f)^2 b^2 c\right.}{\log(f)^5 b^5 d\,(d}$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^6,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*( - log(f)**4*b**4 + 4*log(f)**3*b**3*c
+ 4*log(f)**3*b**3*d*x - 12*log(f)**2*b**2*c**2 - 24*log(f)**2*b**2*c*d*x
- 12*log(f)**2*b**2*d**2*x**2 + 24*log(f)*b*c**3 + 72*log(f)*b*c**2*d*x +
72*log(f)*b*c*d**2*x**2 + 24*log(f)*b*d**3*x**3 - 24*c**4 - 96*c**3*d*x -
144*c**2*d**2*x**2 - 96*c*d**3*x**3 - 24*d**4*x**4))/(log(f)**5*b**5*d*(c*
*4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4))
```

3.247 $\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7} \, dx = \frac{F^{a+\frac{b}{c+dx}} \left(120(c+dx)^5 - 120b(c+dx)^4 \log(F) + 60b^2(c+dx)^3 \log^2(F) - 20b^3(c+dx)^2 \log^3(F) + 5b^4(c\right.}{b^6 d (c+dx)^5 \log^6(F)}$$

output
```
F^(a+b/(d*x+c))*(120*(d*x+c)^5-120*b*(d*x+c)^4*ln(F)+60*b^2*(d*x+c)^3*ln(F
)^2-20*b^3*(d*x+c)^2*ln(F)^3+5*b^4*(d*x+c)*ln(F)^4-b^5*ln(F)^5)/b^6/d/(d*x
+c)^5/ln(F)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.26

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7} \, dx = \frac{F^a \Gamma\left(6, -\frac{b \log(F)}{c+dx}\right)}{b^6 d \log^6(F)}$$

input
```
Integrate[F^(a + b/(c + d*x))/(c + d*x)^7,x]
```

output
```
(F^a*Gamma[6, -((b*Log[F])/(c + d*x))])/(b^6*d*Log[F]^6)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2647\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx$$

$$\Big\downarrow \; 2647$$

$$\frac{F^{a+\frac{b}{c+dx}}\left(-b^5\log^5(F)+5b^4\log^4(F)(c+dx)-20b^3\log^3(F)(c+dx)^2+60b^2\log^2(F)(c+dx)^3-120b\log(F)(c+\right.}{b^6d\log^6(F)(c+dx)^5}$$

input
```
Int[F^(a + b/(c + d*x))/(c + d*x)^7,x]
```

output
```
(F^(a + b/(c + d*x))*(120*(c + d*x)^5 - 120*b*(c + d*x)^4*Log[F] + 60*b^2*
(c + d*x)^3*Log[F]^2 - 20*b^3*(c + d*x)^2*Log[F]^3 + 5*b^4*(c + d*x)*Log[F
]^4 - b^5*Log[F]^5))/(b^6*d*(c + d*x)^5*Log[F]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(108) = 216$.

Time = 0.64 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.51

method	result
risch	$-\frac{\left(b^5\ln(F)^5-5\ln(F)^4b^4dx+20\ln(F)^3b^3d^2x^2-60\ln(F)^2b^2d^3x^3+120\ln(F)bd^4x^4-120d^5x^5-5\ln(F)^4b^4c+40\ln(F)^3b^3cdx\right.}{}$
norman	$\frac{120d^5x^6e^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)^6b^6}-\frac{\left(b^5\ln(F)^5-10\ln(F)^4b^4c+60\ln(F)^3b^3c^2-240\ln(F)^2b^2c^3+600\ln(F)bc^4-720c^5\right)xe^{\left(a+\frac{b}{dx+c}\right)\ln(F)}}{\ln(F)^6b^6}+\frac{5d(}{}$
parallelrisch	$120d^{13}F^{a+\frac{b}{dx+c}}x^5+120F^{a+\frac{b}{dx+c}}c^5d^8+5\ln(F)^4xF^{a+\frac{b}{dx+c}}b^4d^9-20\ln(F)^3x^2F^{a+\frac{b}{dx+c}}b^3d^{10}+60\ln(F)^2x^3F^{a+\frac{b}{dx+c}}b^2d^1$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-(b^5*ln(F)^5-5*ln(F)^4*b^4*d*x+20*ln(F)^3*b^3*d^2*x^2-60*ln(F)^2*b^2*d^3*
x^3+120*ln(F)*b*d^4*x^4-120*d^5*x^5-5*ln(F)^4*b^4*c+40*ln(F)^3*b^3*c*d*x-1
80*ln(F)^2*b^2*c*d^2*x^2+480*ln(F)*b*c*d^3*x^3-600*c*d^4*x^4+20*ln(F)^3*b^
3*c^2-180*ln(F)^2*b^2*c^2*d*x+720*ln(F)*b*c^2*d^2*x^2-1200*c^2*d^3*x^3-60*
ln(F)^2*b^2*c^3+480*ln(F)*b*c^3*d*x-1200*c^3*d^2*x^2+120*ln(F)*b*c^4-600*c
^4*d*x-120*c^5)/b^6/ln(F)^6/d/(d*x+c)^5*F^((a*d*x+a*c+b)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(108) = 216$.

Time = 0.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.80

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx = \frac{\left(120\,d^5x^5 - b^5\log\left(F\right)^5 + 600\,cd^4x^4 + 1200\,c^2d^3x^3 + 1200\,c^3d^2x^2 + 600\,c^4dx + 120\,c^5 + 5\left(b^4dx + b^4c\right)\log\right.}{\left(b^6d^6x\right.}$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^7,x, algorithm="fricas")
```

output
```
(120*d^5*x^5 - b^5*log(F)^5 + 600*c*d^4*x^4 + 1200*c^2*d^3*x^3 + 1200*c^3*
d^2*x^2 + 600*c^4*d*x + 120*c^5 + 5*(b^4*d*x + b^4*c)*log(F)^4 - 20*(b^3*d
^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*log(F)^3 + 60*(b^2*d^3*x^3 + 3*b^2*c*d^2*x
^2 + 3*b^2*c^2*d*x + b^2*c^3)*log(F)^2 - 120*(b*d^4*x^4 + 4*b*c*d^3*x^3 +
6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*log(F))*F^((a*d*x + a*c + b)/(d*x +
 c))/((b^6*d^6*x^5 + 5*b^6*c*d^5*x^4 + 10*b^6*c^2*d^4*x^3 + 10*b^6*c^3*d^3
*x^2 + 5*b^6*c^4*d^2*x + b^6*c^5*d)*log(F)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(105) = 210$.

Time = 0.17 (sec) , antiderivative size = 388, normalized size of antiderivative = 3.59

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx$$
$$= \frac{F^{a+\frac{b}{c+dx}}\left(-b^5\log(F)^5 + 5b^4c\log(F)^4 + 5b^4dx\log(F)^4 - 20b^3c^2\log(F)^3 - 40b^3cdx\log(F)^3 - 20b^3d^2x^2\right.}{}$$

input
```
integrate(F**(a+b/(d*x+c))/(d*x+c)**7,x)
```

output
```
F**(a + b/(c + d*x))*(-b**5*log(F)**5 + 5*b**4*c*log(F)**4 + 5*b**4*d*x*lo
g(F)**4 - 20*b**3*c**2*log(F)**3 - 40*b**3*c*d*x*log(F)**3 - 20*b**3*d**2*
x**2*log(F)**3 + 60*b**2*c**3*log(F)**2 + 180*b**2*c**2*d*x*log(F)**2 + 18
0*b**2*c*d**2*x**2*log(F)**2 + 60*b**2*d**3*x**3*log(F)**2 - 120*b*c**4*lo
g(F) - 480*b*c**3*d*x*log(F) - 720*b*c**2*d**2*x**2*log(F) - 480*b*c*d**3*
x**3*log(F) - 120*b*d**4*x**4*log(F) + 120*c**5 + 600*c**4*d*x + 1200*c**3
*d**2*x**2 + 1200*c**2*d**3*x**3 + 600*c*d**4*x**4 + 120*d**5*x**5)/(b**6*
c**5*d*log(F)**6 + 5*b**6*c**4*d**2*x*log(F)**6 + 10*b**6*c**3*d**3*x**2*l
og(F)**6 + 10*b**6*c**2*d**4*x**3*log(F)**6 + 5*b**6*c*d**5*x**4*log(F)**6
 + b**6*d**6*x**5*log(F)**6)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx=\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^7,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^7, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx=\int \frac{F^{a+\frac{b}{dx+c}}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(d*x + c)^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.92

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx = \frac{F^a\,F^{\frac{b}{c+d\,x}}\,\left(\frac{120\,x^5}{b^6\,d\,\ln(F)^6}-\frac{b^5\,\ln(F)^5-5\,b^4\,c\,\ln(F)^4+20\,b^3\,c^2\,\ln(F)^3-60\,b^2\,c^3\,\ln(F)^2+120\,b\,c^4\,\ln(F)-120\,c^5}{b^6\,d^6\,\ln(F)^6}-\frac{20\,x^2\,\left(b^3\,\ln(F)^3-9\,b^2\,c\,\ln\cdots\right.}{b^6\,d^4\cdots}\right.}{x^5+\frac{c^5}{d^5}+\cdots}$$

input
```
int(F^(a + b/(c + d*x))/(c + d*x)^7,x)
```

output
```
(F^a*F^(b/(c + d*x))*((120*x^5)/(b^6*d*log(F)^6) - (b^5*log(F)^5 - 120*c^5
 + 120*b*c^4*log(F) - 5*b^4*c*log(F)^4 - 60*b^2*c^3*log(F)^2 + 20*b^3*c^2*
log(F)^3)/(b^6*d^6*log(F)^6) - (20*x^2*(b^3*log(F)^3 - 60*c^3 + 36*b*c^2*l
og(F) - 9*b^2*c*log(F)^2))/(b^6*d^4*log(F)^6) + (60*x^3*(b^2*log(F)^2 + 20
*c^2 - 8*b*c*log(F)))/(b^6*d^3*log(F)^6) + (120*x^4*(5*c - b*log(F)))/(b^6
*d^2*log(F)^6) + (5*x*(b^4*log(F)^4 + 120*c^4 - 96*b*c^3*log(F) - 8*b^3*c*
log(F)^3 + 36*b^2*c^2*log(F)^2))/(b^6*d^5*log(F)^6)))/(x^5 + c^5/d^5 + (5*
c*x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.91

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^7}\,dx$$
$$= \frac{f^{\frac{adx+ac+b}{dx+c}}\left(-\log(f)^5 b^5+5\log(f)^4 b^4c+5\log(f)^4 b^4dx-20\log(f)^3 b^3c^2-40\log(f)^3 b^3cdx-20\log(f)^3 b^3c\right.}{}$$

input
```
int(F^(a+b/(d*x+c))/(d*x+c)^7,x)
```

output
```
(f**((a*c + a*d*x + b)/(c + d*x))*( - log(f)**5*b**5 + 5*log(f)**4*b**4*c
+ 5*log(f)**4*b**4*d*x - 20*log(f)**3*b**3*c**2 - 40*log(f)**3*b**3*c*d*x
- 20*log(f)**3*b**3*d**2*x**2 + 60*log(f)**2*b**2*c**3 + 180*log(f)**2*b**
2*c**2*d*x + 180*log(f)**2*b**2*c*d**2*x**2 + 60*log(f)**2*b**2*d**3*x**3
- 120*log(f)*b*c**4 - 480*log(f)*b*c**3*d*x - 720*log(f)*b*c**2*d**2*x**2
- 480*log(f)*b*c*d**3*x**3 - 120*log(f)*b*d**4*x**4 + 120*c**5 + 600*c**4*
d*x + 1200*c**3*d**2*x**2 + 1200*c**2*d**3*x**3 + 600*c*d**4*x**4 + 120*d*
*5*x**5))/(log(f)**6*b**6*d*(c**5 + 5*c**4*d*x + 10*c**3*d**2*x**2 + 10*c*
*2*d**3*x**3 + 5*c*d**4*x**4 + d**5*x**5))
```

3.248 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx=\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1}{2}(-1-m),-\frac{b\log(F)}{(c+dx)^2}\right)\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{\frac{1+m}{2}}}{2d}$$

output
```
1/2*F^a*(d*x+c)^(1+m)*GAMMA(-1/2-1/2*m,-b*ln(F)/(d*x+c)^2)*(-b*ln(F)/(d*x+
c)^2)^(1/2+1/2*m)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx=\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1}{2}(-1-m),-\frac{b\log(F)}{(c+dx)^2}\right)\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{\frac{1+m}{2}}}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^m,x]
```

output
```
(F^a*(c + d*x)^(1 + m)*Gamma[(-1 - m)/2, -((b*Log[F])/(c + d*x)^2)]*(-((b*
Log[F])/(c + d*x)^2))^((1 + m)/2))/(2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m F^{a+\frac{b}{(c+dx)^2}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{F^a (c+dx)^{m+1} \left(-\frac{b\log(F)}{(c+dx)^2}\right)^{\frac{m+1}{2}} \Gamma\left(\frac{1}{2}(-m-1),-\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^m,x]
```

output
```
(F^a*(c + d*x)^(1 + m)*Gamma[(-1 - m)/2, -((b*Log[F])/(c + d*x)^2)]*(-((b*
Log[F])/(c + d*x)^2))^((1 + m)/2))/(2*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^2}}(dx+c)^m\,dx$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x)
```

output
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x)
```

Fricas [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx = \int (dx+c)^m F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c
*d*x + c^2)), x)
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx = \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**m,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**m, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx=\int (dx+c)^m F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*F^(a + b/(d*x + c)^2), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx=\int (dx+c)^m F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx = \frac{F^a\,\mathrm{e}^{\frac{b\,\ln(F)}{2\,(c+d\,x)^2}}\,(c+d\,x)^{m+1}\,\mathrm{M}_{\frac{m}{4}+\frac{3}{4},-\frac{m}{4}-\frac{1}{4}}\left(\frac{b\,\ln(F)}{(c+d\,x)^2}\right)\,\left(\frac{b\,\ln(F)}{(c+d\,x)^2}\right)^{\frac{m}{4}-\frac{1}{4}}}{d\,(m+1)}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^m,x)
```

output
```
(F^a*exp((b*log(F))/(2*(c + d*x)^2))*(c + d*x)^(m + 1)*whittakerM(m/4 + 3/
4, - m/4 - 1/4, (b*log(F))/(c + d*x)^2)*((b*log(F))/(c + d*x)^2)^(m/4 - 1/
4))/(d*(m + 1))
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^m\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^m,x)
```

output
```
(4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*(c + d*x)**m*log(f)**2*b**2 + 2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**m*log(f)*b*c**2*m - 6*f**((a*c
**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)
**m*log(f)*b*c**2 + 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2
*c*d*x + d**2*x**2))*(c + d*x)**m*log(f)*b*c*d*m*x - 12*f**((a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**m*log(f)
*b*c*d*x + 2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
d**2*x**2))*(c + d*x)**m*log(f)*b*d**2*m*x**2 - 6*f**((a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**m*log(f)*b*d**
2*x**2 + f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2
*x**2))*(c + d*x)**m*c**4*m**2 - 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**m*c**4*m + 3*f**((a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**m*c**4
 + 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
2))*(c + d*x)**m*c**3*d*m**2*x - 16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**m*c**3*d*m*x + 12*f**((a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*(c + d*x)**
m*c**3*d*x + 6*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))*(c + d*x)**m*c**2*d**2*m**2*x**2 - 24*f**((a*c**2 + 2*a*c...
```

3.249 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx=-\frac{b^5F^a\Gamma\left(-5,-\frac{b\log(F)}{(c+dx)^2}\right)\log^5(F)}{2d}$$

output
```
1/2*F^a*(d*x+c)^10*Ei(6,-b*ln(F)/(d*x+c)^2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx=-\frac{b^5F^a\Gamma\left(-5,-\frac{b\log(F)}{(c+dx)^2}\right)\log^5(F)}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^9,x]
```

output
```
-1/2*(b^5*F^a*Gamma[-5, -((b*Log[F])/(c + d*x)^2)]*Log[F]^5)/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^9 F^{a+\frac{b}{(c+dx)^2}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{b^5 F^a \log^5(F) \Gamma\left(-5,-\frac{b \log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^9,x]
```

output
```
-1/2*(b^5*F^a*Gamma[-5, -((b*Log[F])/(c + d*x)^2)]*Log[F]^5)/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 960 vs. 2(29) = 58.

Time = 3.44 (sec) , antiderivative size = 961, normalized size of antiderivative = 31.00

method	result	size
risch	Expression too large to display	961

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
1/240*F^a/d*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^4+1/240*F^a/d*b^4*ln(F)^4*F^(b/(
d*x+c)^2)*c^2+1/120*F^a*d^5*b^2*ln(F)^2*F^(b/(d*x+c)^2)*x^6+1/240*F^a*d^3*
b^3*ln(F)^3*F^(b/(d*x+c)^2)*x^4+1/240*F^a*d*b^4*ln(F)^4*F^(b/(d*x+c)^2)*x^
2+1/40*F^a*d^7*b*ln(F)*F^(b/(d*x+c)^2)*x^8+1/10*F^a/d*F^(b/(d*x+c)^2)*c^10
+1/10*F^a*d^9*F^(b/(d*x+c)^2)*x^10+F^a*F^(b/(d*x+c)^2)*c^9*x+1/8*F^a*d^3*b
^2*ln(F)^2*F^(b/(d*x+c)^2)*c^2*x^4+1/6*F^a*d^2*b^2*ln(F)^2*F^(b/(d*x+c)^2)
*c^3*x^3+1/8*F^a*d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^4*x^2+1/60*F^a*d^2*b^3*ln
(F)^3*F^(b/(d*x+c)^2)*c*x^3+1/40*F^a*d*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^2*x^2
+1/5*F^a*d^6*b*ln(F)*F^(b/(d*x+c)^2)*c*x^7+7/10*F^a*d^5*b*ln(F)*F^(b/(d*x+
c)^2)*c^2*x^6+7/5*F^a*d^4*b*ln(F)*F^(b/(d*x+c)^2)*c^3*x^5+7/4*F^a*d^3*b*ln
(F)*F^(b/(d*x+c)^2)*c^4*x^4+7/5*F^a*d^2*b*ln(F)*F^(b/(d*x+c)^2)*c^5*x^3+7/
10*F^a*d*b*ln(F)*F^(b/(d*x+c)^2)*c^6*x^2+1/20*F^a*d^4*b^2*ln(F)^2*F^(b/(d*
x+c)^2)*c*x^5+126/5*F^a*d^4*F^(b/(d*x+c)^2)*c^5*x^5+21*F^a*d^3*F^(b/(d*x+c
)^2)*c^6*x^4+12*F^a*d^2*F^(b/(d*x+c)^2)*c^7*x^3+9/2*F^a*d*F^(b/(d*x+c)^2)*
c^8*x^2+1/240*F^a/d*b^5*ln(F)^5*Ei(1,-b*ln(F)/(d*x+c)^2)+F^a*d^8*F^(b/(d*x
+c)^2)*c*x^9+9/2*F^a*d^7*F^(b/(d*x+c)^2)*c^2*x^8+12*F^a*d^6*F^(b/(d*x+c)^2
)*c^3*x^7+21*F^a*d^5*F^(b/(d*x+c)^2)*c^4*x^6+1/20*F^a*b^2*ln(F)^2*F^(b/(d*
x+c)^2)*c^5*x+1/60*F^a*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^3*x+1/120*F^a*b^4*ln(
F)^4*F^(b/(d*x+c)^2)*c*x+1/5*F^a*b*ln(F)*F^(b/(d*x+c)^2)*c^7*x+1/40*F^a/d*
b*ln(F)*F^(b/(d*x+c)^2)*c^8+1/120*F^a/d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 465, normalized size of antiderivative = 15.00

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx =$$

$$-\frac{F^a b^5 \mathrm{Ei}\left(\frac{b\log(F)}{d^2x^2+2\,cdx+c^2}\right)\log\left(F\right)^5 - \left(24\,d^{10}x^{10}+240\,cd^9x^9+1080\,c^2d^8x^8+2880\,c^3d^7x^7+5040\,c^4d^6x^6+\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x, algorithm="fricas")
```

output
```
-1/240*(F^a*b^5*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))*log(F)^5 - (24*d^10
*x^10 + 240*c*d^9*x^9 + 1080*c^2*d^8*x^8 + 2880*c^3*d^7*x^7 + 5040*c^4*d^6
*x^6 + 6048*c^5*d^5*x^5 + 5040*c^6*d^4*x^4 + 2880*c^7*d^3*x^3 + 1080*c^8*d
^2*x^2 + 240*c^9*d*x + 24*c^10 + (b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2)*log
(F)^4 + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x
 + b^3*c^4)*log(F)^3 + 2*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x
^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*lo
g(F)^2 + 6*(b*d^8*x^8 + 8*b*c*d^7*x^7 + 28*b*c^2*d^6*x^6 + 56*b*c^3*d^5*x^
5 + 70*b*c^4*d^4*x^4 + 56*b*c^5*d^3*x^3 + 28*b*c^6*d^2*x^2 + 8*b*c^7*d*x +
 b*c^8)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
+ c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx=\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**9,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**9, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx=\int (dx+c)^9F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x, algorithm="maxima")
```

output
```
1/240*(24*F^a*d^9*x^10 + 240*F^a*c*d^8*x^9 + 6*(180*F^a*c^2*d^7 + F^a*b*d^
7*log(F))*x^8 + 48*(60*F^a*c^3*d^6 + F^a*b*c*d^6*log(F))*x^7 + 2*(2520*F^a
*c^4*d^5 + 84*F^a*b*c^2*d^5*log(F) + F^a*b^2*d^5*log(F)^2)*x^6 + 12*(504*F
^a*c^5*d^4 + 28*F^a*b*c^3*d^4*log(F) + F^a*b^2*c*d^4*log(F)^2)*x^5 + (5040
*F^a*c^6*d^3 + 420*F^a*b*c^4*d^3*log(F) + 30*F^a*b^2*c^2*d^3*log(F)^2 + F^
a*b^3*d^3*log(F)^3)*x^4 + 4*(720*F^a*c^7*d^2 + 84*F^a*b*c^5*d^2*log(F) + 1
0*F^a*b^2*c^3*d^2*log(F)^2 + F^a*b^3*c*d^2*log(F)^3)*x^3 + (1080*F^a*c^8*d
 + 168*F^a*b*c^6*d*log(F) + 30*F^a*b^2*c^4*d*log(F)^2 + 6*F^a*b^3*c^2*d*lo
g(F)^3 + F^a*b^4*d*log(F)^4)*x^2 + 2*(120*F^a*c^9 + 24*F^a*b*c^7*log(F) +
6*F^a*b^2*c^5*log(F)^2 + 2*F^a*b^3*c^3*log(F)^3 + F^a*b^4*c*log(F)^4)*x)*F
^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(1/120*(F^a*b^5*d^2*x^2*log(F)^5
 + 2*F^a*b^5*c*d*x*log(F)^5 - 24*F^a*b*c^10*log(F) - 6*F^a*b^2*c^8*log(F)^
2 - 2*F^a*b^3*c^6*log(F)^3 - F^a*b^4*c^4*log(F)^4)*F^(b/(d^2*x^2 + 2*c*d*x
 + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx=\int (dx+c)^9F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x, algorithm="giac")
```

output
```
integrate((d*x + c)^9*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.39

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx = \frac{F^a\,b^5\,\ln(F)^5\,\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)}{240\,d} + \frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^5\,\ln(F)^5\left(\frac{(c+d\,x)^2}{120\,b\,\ln(F)}+\frac{(c+d\,x)^4}{120\,b^2\,\ln(F)^2}+\frac{(c+d\,x)^6}{60\,b^3\,\ln(F)^3}+\frac{(c+d\,x)^8}{20\,b^4\,\ln(F)^4}+\frac{(c+d\,x)^{10}}{5\,b^5\,\ln(F)^5}\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^9,x)
```

output
```
(F^a*b^5*log(F)^5*expint(-(b*log(F))/(c + d*x)^2))/(240*d) + (F^a*F^(b/(c
+ d*x)^2)*b^5*log(F)^5*((c + d*x)^2/(120*b*log(F)) + (c + d*x)^4/(120*b^2*
log(F)^2) + (c + d*x)^6/(60*b^3*log(F)^3) + (c + d*x)^8/(20*b^4*log(F)^4)
+ (c + d*x)^10/(5*b^5*log(F)^5)))/(2*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^9\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^9,x)
```

output
```
(16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**6*b**6*c*d*x + 8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c**4 - 16*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c**3*d*x
 - 40*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**5*b**5*c**2*d**2*x**2 - 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c*d**3*x**3 - 24*f**
((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(
f)**4*b**4*c**6 - 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*
c*d*x + d**2*x**2))*log(f)**4*b**4*c**5*d*x - 52*f**((a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**4*d**2*x
**2 + 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*
x**2))*log(f)**4*b**4*c**3*d**3*x**3 + 31*f**((a*c**2 + 2*a*c*d*x + a*d**2
*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**2*d**4*x**4 + 1
8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))
*log(f)**4*b**4*c*d**5*x**5 + 3*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*d**6*x**6 + 16*f**((a*c**2 +
 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3
*c**8 + 76*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))*log(f)**3*b**3*c**7*d*x + 162*f**((a*c**2 + 2*a*c*d*x + a*d**...
```

3.250 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx = \frac{b^4F^a\Gamma\left(-4,-\frac{b\log(F)}{(c+dx)^2}\right)\log^4(F)}{2d}$$

output
```
1/2*F^a*(d*x+c)^8*Ei(5,-b*ln(F)/(d*x+c)^2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx = \frac{b^4F^a\Gamma\left(-4,-\frac{b\log(F)}{(c+dx)^2}\right)\log^4(F)}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^7,x]
```

output
```
(b^4*F^a*Gamma[-4, -((b*Log[F])/(c + d*x)^2)]*Log[F]^4)/(2*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^7 F^{a+\frac{b}{(c+dx)^2}}\, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{b^4 F^a \log^4(F)\Gamma\left(-4,-\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^7,x]
```

output
```
(b^4*F^a*Gamma[-4, -((b*Log[F])/(c + d*x)^2)]*Log[F]^4)/(2*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(29) = 58.

Time = 1.78 (sec) , antiderivative size = 646, normalized size of antiderivative = 20.84

method	result
risch	$\frac{F^a d^3 b^2 \ln(F)^2 F^{\frac{b}{(dx+c)^2}} x^4}{48} + \frac{F^a d b^3 \ln(F)^3 F^{\frac{b}{(dx+c)^2}} x^2}{48} + \frac{F^a b \ln(F) F^{\frac{b}{(dx+c)^2}} c^5 x}{4} + \frac{F^a b^2 \ln(F)^2 F^{\frac{b}{(dx+c)^2}} c^3 x}{12} + \frac{F^a b^3 \ln}{}$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
1/48*F^a*d^3*b^2*ln(F)^2*F^(b/(d*x+c)^2)*x^4+1/48*F^a*d*b^3*ln(F)^3*F^(b/(
d*x+c)^2)*x^2+1/4*F^a*b*ln(F)*F^(b/(d*x+c)^2)*c^5*x+1/12*F^a*b^2*ln(F)^2*F
^(b/(d*x+c)^2)*c^3*x+1/24*F^a*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c*x+1/48*F^a/d*b
^3*ln(F)^3*F^(b/(d*x+c)^2)*c^2+1/24*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c^6+1/48
*F^a/d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^4+1/24*F^a*d^5*b*ln(F)*F^(b/(d*x+c)^2
)*x^6+F^a*d^6*F^(b/(d*x+c)^2)*c*x^7+7/2*F^a*d^5*F^(b/(d*x+c)^2)*c^2*x^6+7*
F^a*d^4*F^(b/(d*x+c)^2)*c^3*x^5+35/4*F^a*d^3*F^(b/(d*x+c)^2)*c^4*x^4+7*F^a
*d^2*F^(b/(d*x+c)^2)*c^5*x^3+7/2*F^a*d*F^(b/(d*x+c)^2)*c^6*x^2+1/48*F^a/d*
b^4*ln(F)^4*Ei(1,-b*ln(F)/(d*x+c)^2)+5/8*F^a*d^3*b*ln(F)*F^(b/(d*x+c)^2)*c
^2*x^4+5/6*F^a*d^2*b*ln(F)*F^(b/(d*x+c)^2)*c^3*x^3+5/8*F^a*d*b*ln(F)*F^(b/
(d*x+c)^2)*c^4*x^2+1/12*F^a*d^2*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c*x^3+1/8*F^a*
d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^2*x^2+1/4*F^a*d^4*b*ln(F)*F^(b/(d*x+c)^2)*
c*x^5+F^a*F^(b/(d*x+c)^2)*c^7*x+1/8*F^a/d*F^(b/(d*x+c)^2)*c^8+1/8*F^a*d^7*
F^(b/(d*x+c)^2)*x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 331, normalized size of antiderivative = 10.68

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx = -\frac{F^a b^4 \mathrm{Ei}\left(\frac{b\log(F)}{d^2x^2+2\,cdx+c^2}\right)\log(F)^4 - \left(6\,d^8x^8 + 48\,cd^7x^7 + 168\,c^2d^6x^6 + 336\,c^3d^5x^5 + 420\,c^4d^4x^4 + 336\,c^5d\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x, algorithm="fricas")
```

output
```
-1/48*(F^a*b^4*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))*log(F)^4 - (6*d^8*x^
8 + 48*c*d^7*x^7 + 168*c^2*d^6*x^6 + 336*c^3*d^5*x^5 + 420*c^4*d^4*x^4 + 3
36*c^5*d^3*x^3 + 168*c^6*d^2*x^2 + 48*c^7*d*x + 6*c^8 + (b^3*d^2*x^2 + 2*b
^3*c*d*x + b^3*c^2)*log(F)^3 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*
d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)^2 + 2*(b*d^6*x^6 + 6*b*c*d^5*x^5
 + 15*b*c^2*d^4*x^4 + 20*b*c^3*d^3*x^3 + 15*b*c^4*d^2*x^2 + 6*b*c^5*d*x +
b*c^6)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx=\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**7,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**7, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx=\int (dx+c)^7F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/48*(6*F^a*d^7*x^8 + 48*F^a*c*d^6*x^7 + 2*(84*F^a*c^2*d^5 + F^a*b*d^5*log
(F))*x^6 + 12*(28*F^a*c^3*d^4 + F^a*b*c*d^4*log(F))*x^5 + (420*F^a*c^4*d^3
 + 30*F^a*b*c^2*d^3*log(F) + F^a*b^2*d^3*log(F)^2)*x^4 + 4*(84*F^a*c^5*d^2
 + 10*F^a*b*c^3*d^2*log(F) + F^a*b^2*c*d^2*log(F)^2)*x^3 + (168*F^a*c^6*d
+ 30*F^a*b*c^4*d*log(F) + 6*F^a*b^2*c^2*d*log(F)^2 + F^a*b^3*d*log(F)^3)*x
^2 + 2*(24*F^a*c^7 + 6*F^a*b*c^5*log(F) + 2*F^a*b^2*c^3*log(F)^2 + F^a*b^3
*c*log(F)^3)*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(1/24*(F^a*b^4*
d^2*x^2*log(F)^4 + 2*F^a*b^4*c*d*x*log(F)^4 - 6*F^a*b*c^8*log(F) - 2*F^a*b
^2*c^6*log(F)^2 - F^a*b^3*c^4*log(F)^3)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx=\int (dx+c)^7F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate((d*x + c)^7*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx=\frac{F^a\,b^4\,\ln\left(F\right)^4\,\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)}{48\,d}+\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^4\,\ln\left(F\right)^4\left(\frac{(c+d\,x)^2}{24\,b\,\ln(F)}+\frac{(c+d\,x)^4}{24\,b^2\,\ln(F)^2}+\frac{(c+d\,x)^6}{12\,b^3\,\ln(F)^3}+\frac{(c+d\,x)^8}{4\,b^4\,\ln(F)^4}\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^7,x)
```

output
```
(F^a*b^4*log(F)^4*expint(-(b*log(F))/(c + d*x)^2))/(48*d) + (F^a*F^(b/(c +
 d*x)^2)*b^4*log(F)^4*((c + d*x)^2/(24*b*log(F)) + (c + d*x)^4/(24*b^2*log
(F)^2) + (c + d*x)^6/(12*b^3*log(F)^3) + (c + d*x)^8/(4*b^4*log(F)^4)))/(2
*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^7,x)
```

output
```
(16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**5*b**5*c*d*x + 8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**4 - 16*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**3*d*x
 - 40*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**4*b**4*c**2*d**2*x**2 - 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c*d**3*x**3 - 24*f**
((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(
f)**3*b**3*c**6 - 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*
c*d*x + d**2*x**2))*log(f)**3*b**3*c**5*d*x - 52*f**((a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**4*d**2*x
**2 + 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*
x**2))*log(f)**3*b**3*c**3*d**3*x**3 + 31*f**((a*c**2 + 2*a*c*d*x + a*d**2
*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**2*d**4*x**4 + 1
8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))
*log(f)**3*b**3*c*d**5*x**5 + 3*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*d**6*x**6 + 16*f**((a*c**2 +
 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2
*c**8 + 76*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))*log(f)**2*b**2*c**7*d*x + 162*f**((a*c**2 + 2*a*c*d*x + a*d**...
```

3.251 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx=\frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6}{6d}+\frac{bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\log(F)}{12d}+\frac{b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\log^2(F)}{12d}-\frac{b^3F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)\log^3(F)}{12d}$$

output
```
1/6*F^(a+b/(d*x+c)^2)*(d*x+c)^6/d+1/12*b*F^(a+b/(d*x+c)^2)*(d*x+c)^4*ln(F)
/d+1/12*b^2*F^(a+b/(d*x+c)^2)*(d*x+c)^2*ln(F)^2/d-1/12*b^3*F^a*Ei(b*ln(F)/
(d*x+c)^2)*ln(F)^3/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx = \frac{F^a\left(2F^{\frac{b}{(c+dx)^2}}(c+dx)^6+b\log(F)\left(F^{\frac{b}{(c+dx)^2}}(c+dx)^4+b\log(F)\left(F^{\frac{b}{(c+dx)^2}}(c+dx)^2-b\,\text{ExpIntegralEi}\left(\right.\right.\right.\right.}{12d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^5,x]
```

output
```
(F^a*(2*F^(b/(c + d*x)^2)*(c + d*x)^6 + b*Log[F]*(F^(b/(c + d*x)^2)*(c + d
*x)^4 + b*Log[F]*(F^(b/(c + d*x)^2)*(c + d*x)^2 - b*ExpIntegralEi[(b*Log[F
])/(c + d*x)^2]*Log[F]))))/(12*d)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^5F^{a+\frac{b}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3dx+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^2}}}{6d}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)dx+\frac{(c+dx)^4F^{a+\frac{b}{(c+dx)^2}}}{4d}\right)+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^2}}}{6d}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\left(b\log(F)\int\frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}dx+\frac{(c+dx)^2F^{a+\frac{b}{(c+dx)^2}}}{2d}\right)+\frac{(c+dx)^4F^{a+\frac{b}{(c+dx)^2}}}{4d}\right)+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^2}}}{6d}$$

$\downarrow$ 2639

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\left(\frac{(c+dx)^2F^{a+\frac{b}{(c+dx)^2}}}{2d}-\frac{bF^a\log(F)\operatorname{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)}{2d}\right)+\frac{(c+dx)^4F^{a+\frac{b}{(c+dx)^2}}}{4d}\right)+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^2}}}{6d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^5,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x)^6)/(6*d) + (b*Log[F]*((F^(a + b/(c + d*x)
^2)*(c + d*x)^4)/(4*d) + (b*Log[F]*((F^(a + b/(c + d*x)^2)*(c + d*x)^2)/(2
*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^2]*Log[F])/(2*d)))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(113) = 226.

Time = 0.91 (sec) , antiderivative size = 395, normalized size of antiderivative = 3.26

method	result
risch	$\frac{F^{a} d^{5} F^{\frac{b}{(dx+c)^{2}}} x^{6}}{6}+F^{a} d^{4} F^{\frac{b}{(dx+c)^{2}}} c\,x^{5}+\frac{5 F^{a} d^{3} F^{\frac{b}{(dx+c)^{2}}} c^{2} x^{4}}{2}+\frac{10 F^{a} d^{2} F^{\frac{b}{(dx+c)^{2}}} c^{3} x^{3}}{3}+\frac{5 F^{a} d\,F^{\frac{b}{(dx+c)^{2}}} c^{4} x^{2}}{2}+F$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*F^a*d^5*F^(b/(d*x+c)^2)*x^6+F^a*d^4*F^(b/(d*x+c)^2)*c*x^5+5/2*F^a*d^3*
F^(b/(d*x+c)^2)*c^2*x^4+10/3*F^a*d^2*F^(b/(d*x+c)^2)*c^3*x^3+5/2*F^a*d*F^(
b/(d*x+c)^2)*c^4*x^2+F^a*F^(b/(d*x+c)^2)*c^5*x+1/6*F^a/d*F^(b/(d*x+c)^2)*c
^6+1/12*F^a*d^3*b*ln(F)*F^(b/(d*x+c)^2)*x^4+1/3*F^a*d^2*b*ln(F)*F^(b/(d*x+
c)^2)*c*x^3+1/2*F^a*d*b*ln(F)*F^(b/(d*x+c)^2)*c^2*x^2+1/3*F^a*b*ln(F)*F^(b
/(d*x+c)^2)*c^3*x+1/12*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c^4+1/12*F^a*d*b^2*ln
(F)^2*F^(b/(d*x+c)^2)*x^2+1/6*F^a*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c*x+1/12*F^a
/d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^2+1/12*F^a/d*b^3*ln(F)^3*Ei(1,-b*ln(F)/(d
*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.86

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx =$$

$$-\frac{F^{a}b^{3}\mathrm{Ei}\left(\frac{b\log(F)}{d^{2}x^{2}+2\,cdx+c^{2}}\right)\log\left(F\right)^{3}-\left(2\,d^{6}x^{6}+12\,cd^{5}x^{5}+30\,c^{2}d^{4}x^{4}+40\,c^{3}d^{3}x^{3}+30\,c^{4}d^{2}x^{2}+12\,c^{5}dx+\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/12*(F^a*b^3*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))*log(F)^3 - (2*d^6*x^
6 + 12*c*d^5*x^5 + 30*c^2*d^4*x^4 + 40*c^3*d^3*x^3 + 30*c^4*d^2*x^2 + 12*c
^5*d*x + 2*c^6 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(F)^2 + (b*d^4*x
^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x + b*c^4)*log(F))*F^((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx = \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**5,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**5, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx = \int (dx+c)^5F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/12*(2*F^a*d^5*x^6 + 12*F^a*c*d^4*x^5 + (30*F^a*c^2*d^3 + F^a*b*d^3*log(F
))*x^4 + 4*(10*F^a*c^3*d^2 + F^a*b*c*d^2*log(F))*x^3 + (30*F^a*c^4*d + 6*F
^a*b*c^2*d*log(F) + F^a*b^2*d*log(F)^2)*x^2 + 2*(6*F^a*c^5 + 2*F^a*b*c^3*l
og(F) + F^a*b^2*c*log(F)^2)*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate
(1/6*(F^a*b^3*d^2*x^2*log(F)^3 + 2*F^a*b^3*c*d*x*log(F)^3 - 2*F^a*b*c^6*lo
g(F) - F^a*b^2*c^4*log(F)^2)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*
c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx = \int (dx+c)^5 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((d*x + c)^5*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx = \frac{F^a\,b^3\ln(F)^3\left(\frac{\mathrm{expint}\left(-\frac{b\ln(F)}{(c+d\,x)^2}\right)}{6}+F^{\frac{b}{(c+d\,x)^2}}\left(\frac{(c+d\,x)^2}{6\,b\,\ln(F)}+\frac{(c+d\,x)^4}{6\,b^2\ln(F)^2}+\frac{(c+d\,x)^6}{3\,b^3\ln(F)^3}\right)\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^5,x)
```

output
```
(F^a*b^3*log(F)^3*(expint(-(b*log(F))/(c + d*x)^2)/6 + F^(b/(c + d*x)^2)*(
(c + d*x)^2/(6*b*log(F)) + (c + d*x)^4/(6*b^2*log(F)^2) + (c + d*x)^6/(3*b
^3*log(F)^3))))/(2*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^5,x)
```

output
```
(16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**4*b**4*c*d*x + 8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**4 - 16*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**3*d*x
 - 40*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**3*b**3*c**2*d**2*x**2 - 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c*d**3*x**3 - 24*f**
((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(
f)**2*b**2*c**6 - 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*
c*d*x + d**2*x**2))*log(f)**2*b**2*c**5*d*x - 52*f**((a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**4*d**2*x
**2 + 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*
x**2))*log(f)**2*b**2*c**3*d**3*x**3 + 31*f**((a*c**2 + 2*a*c*d*x + a*d**2
*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**2*d**4*x**4 + 1
8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))
*log(f)**2*b**2*c*d**5*x**5 + 3*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)
/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*d**6*x**6 + 16*f**((a*c**2 +
 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**8
+ 76*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
2))*log(f)*b*c**7*d*x + 162*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/...
```

3.252 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx=\frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4}{4d}+\frac{bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\log(F)}{4d}-\frac{b^2F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)\log^2(F)}{4d}$$

output
```
1/4*F^(a+b/(d*x+c)^2)*(d*x+c)^4/d+1/4*b*F^(a+b/(d*x+c)^2)*(d*x+c)^2*ln(F)/
d-1/4*b^2*F^a*Ei(b*ln(F)/(d*x+c)^2)*ln(F)^2/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx$$
$$=\frac{F^a\left(F^{\frac{b}{(c+dx)^2}}(c+dx)^4+b\log(F)\left(F^{\frac{b}{(c+dx)^2}}(c+dx)^2-b\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)\log(F)\right)\right)}{4d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^3,x]
```

output
```
(F^a*(F^(b/(c + d*x)^2)*(c + d*x)^4 + b*Log[F]*(F^(b/(c + d*x)^2)*(c + d*x
)^2 - b*ExpIntegralEi[(b*Log[F])/(c + d*x)^2]*Log[F])))/(4*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 F^{a+\frac{b}{(c+dx)^2}} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{2} b \log(F) \int F^{a+\frac{b}{(c+dx)^2}} (c+dx)dx + \frac{(c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{4d}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{2} b \log(F) \left(b \log(F) \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} dx + \frac{(c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{2d} \right) + \frac{(c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{4d}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{2} b \log(F) \left(\frac{(c+dx)^2 F^{a+\frac{b}{(c+dx)^2}}}{2d} - \frac{b F^a \log(F) \text{ExpIntegralEi}\left(\frac{b \log(F)}{(c+dx)^2}\right)}{2d} \right) + \frac{(c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}}{4d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^3,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x)^4)/(4*d) + (b*Log[F]*((F^(a + b/(c + d*x)
^2)*(c + d*x)^2)/(2*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^2]*Log[
F])/(2*d)))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(81) = 162$.

Time = 0.39 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.39

method	result
risch	$\frac{F^a d^3 F^{\frac{b}{(dx+c)^2}} x^4}{4} + F^a d^2 F^{\frac{b}{(dx+c)^2}} c x^3 + \frac{3F^a d F^{\frac{b}{(dx+c)^2}} c^2 x^2}{2} + F^a F^{\frac{b}{(dx+c)^2}} c^3 x + \frac{F^a F^{\frac{b}{(dx+c)^2}} c^4}{4d} + \frac{F^a d b \ln(F)}{}$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*F^a*d^3*F^(b/(d*x+c)^2)*x^4+F^a*d^2*F^(b/(d*x+c)^2)*c*x^3+3/2*F^a*d*F^
(b/(d*x+c)^2)*c^2*x^2+F^a*F^(b/(d*x+c)^2)*c^3*x+1/4*F^a/d*F^(b/(d*x+c)^2)*
c^4+1/4*F^a*d*b*ln(F)*F^(b/(d*x+c)^2)*x^2+1/2*F^a*b*ln(F)*F^(b/(d*x+c)^2)*
c*x+1/4*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c^2+1/4*F^a/d*b^2*ln(F)^2*Ei(1,-b*ln
(F)/(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.67

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx =$$
$$-\frac{F^a b^2 \mathrm{Ei}\left(\frac{b\log(F)}{d^2x^2+2\,cdx+c^2}\right)\log\left(F\right)^2-\left(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4+\left(bd^2x^2+2\,bcdx+bc^2\right)\mathrm{lo}\right.}{4\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(F^a*b^2*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))*log(F)^2 - (d^4*x^4 +
 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4 + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x +
 c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx = \int F^{a+\frac{b}{(c+dx)^2}}\left(c+dx\right)^3\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**3,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**3, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx = \int \left(dx+c\right)^3 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(F^a*d^3*x^4 + 4*F^a*c*d^2*x^3 + (6*F^a*c^2*d + F^a*b*d*log(F))*x^2 +
2*(2*F^a*c^3 + F^a*b*c*log(F))*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integr
ate(1/2*(F^a*b^2*d^2*x^2*log(F)^2 + 2*F^a*b^2*c*d*x*log(F)^2 - F^a*b*c^4*l
og(F))*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx=\int (dx+c)^3F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx=\frac{F^a\,b^2\ln(F)^2\left(\frac{\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)}{2}+F^{\frac{b}{(c+d\,x)^2}}\left(\frac{(c+d\,x)^2}{2\,b\,\ln(F)}+\frac{(c+d\,x)^4}{2\,b^2\ln(F)^2}\right)\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^3,x)
```

output
```
(F^a*b^2*log(F)^2*(expint(-(b*log(F))/(c + d*x)^2)/2 + F^(b/(c + d*x)^2)*(
(c + d*x)^2/(2*b*log(F)) + (c + d*x)^4/(2*b^2*log(F)^2))))/(2*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^3,x)
```

output
```
(16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**3*b**3*c*d*x + 8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**4 - 16*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**3*d*x
 - 40*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**2*b**2*c**2*d**2*x**2 - 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c*d**3*x**3 - 24*f**
((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(
f)*b*c**6 - 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))*log(f)*b*c**5*d*x - 52*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**4*d**2*x**2 + 4*f**((a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c*
*3*d**3*x**3 + 31*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d
*x + d**2*x**2))*log(f)*b*c**2*d**4*x**4 + 18*f**((a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c*d**5*x**5 + 3*f**(
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f
)*b*d**6*x**6 + 16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*
d*x + d**2*x**2))*c**8 + 76*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*c**7*d*x + 162*f**((a*c**2 + 2*a*c*d*x + a*d**2
*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**6*d**2*x**2 + 220*f**((a*c*...
```

3.253 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx=\frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2}{2d}-\frac{bF^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)\log(F)}{2d}$$

output
```
1/2*F^(a+b/(d*x+c)^2)*(d*x+c)^2/d-1/2*b*F^a*Ei(b*ln(F)/(d*x+c)^2)*ln(F)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx=\frac{F^a\left(F^{\frac{b}{(c+dx)^2}}(c+dx)^2-b\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)\log(F)\right)}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x),x]
```

output
```
(F^a*(F^(b/(c + d*x)^2)*(c + d*x)^2 - b*ExpIntegralEi[(b*Log[F])/(c + d*x)
^2]*Log[F]))/(2*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)F^{a+\frac{b}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \; 2643$$

$$b\log(F)\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}dx+\frac{(c+dx)^2F^{a+\frac{b}{(c+dx)^2}}}{2d}$$

$$\Big\downarrow \; 2639$$

$$\frac{(c+dx)^2F^{a+\frac{b}{(c+dx)^2}}}{2d}-\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x),x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x)^2)/(2*d) - (b*F^a*ExpIntegralEi[(b*Log[F]
)/(c + d*x)^2]*Log[F])/(2*d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.62

method	result	size
risch	$\frac{d\,F^{a}F^{\frac{b}{(dx+c)^{2}}}x^{2}}{2}+F^{a}F^{\frac{b}{(dx+c)^{2}}}cx+\frac{F^{a}F^{\frac{b}{(dx+c)^{2}}}c^{2}}{2d}+\frac{F^{a}b\ln(F)\operatorname{expIntegral}_{1}\left(-\frac{b\ln(F)}{(dx+c)^{2}}\right)}{2d}$	86

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*F^a*F^(b/(d*x+c)^2)*x^2+F^a*F^(b/(d*x+c)^2)*c*x+1/2/d*F^a*F^(b/(d*x+
c)^2)*c^2+1/2/d*F^a*b*ln(F)*Ei(1,-b*ln(F)/(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.81

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx = -\frac{F^{a}b\mathrm{Ei}\left(\frac{b\log(F)}{d^{2}x^{2}+2\,cdx+c^{2}}\right)\log\left(F\right)-\left(d^{2}x^{2}+2\,cdx+c^{2}\right)F^{\frac{ad^{2}x^{2}+2\,acdx+ac^{2}+b}{d^{2}x^{2}+2\,cdx+c^{2}}}}{2\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(F^a*b*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))*log(F) - (d^2*x^2 + 2*c
*d*x + c^2)*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^
2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx = \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\;dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c),x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x), x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx = \int (dx+c)F^{a+\frac{b}{(dx+c)^2}}\;dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(F^a*d*x^2 + 2*F^a*c*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate((F
^a*b*d^2*x^2*log(F) + 2*F^a*b*c*d*x*log(F))*F^(b/(d^2*x^2 + 2*c*d*x + c^2)
)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx = \int (dx+c)F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx = \frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,(c+d\,x)^2}{2\,d} + \frac{F^a\,b\,\ln(F)\,\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x),x)
```

output
```
(F^a*F^(b/(c + d*x)^2)*(c + d*x)^2)/(2*d) + (F^a*b*log(F)*expint(-(b*log(F
))/(c + d*x)^2))/(2*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c),x)
```

output

```
(16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**3*b**3*c*d*x + 8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**4 - 16*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**3*d*x
 - 40*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**2*b**2*c**2*d**2*x**2 - 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c*d**3*x**3 - 24*f**
((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(
f)*b*c**6 - 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))*log(f)*b*c**5*d*x - 52*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**4*d**2*x**2 + 4*f**((a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c*
*3*d**3*x**3 + 31*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d
*x + d**2*x**2))*log(f)*b*c**2*d**4*x**4 + 18*f**((a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c*d**5*x**5 + 3*f**(
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f
)*b*d**6*x**6 + 13*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*
d*x + d**2*x**2))*c**8 + 52*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*c**7*d*x + 78*f**((a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**6*d**2*x**2 + 52*f**((a*c**2...
```

3.254 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx=-\frac{F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

output
```
-1/2*F^a*Ei(b*ln(F)/(d*x+c)^2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx=-\frac{F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x),x]
```

output
```
-1/2*(F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^2])/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx} \, dx$$

$$\Big\downarrow \; 2639$$

$$-\frac{F^a \text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x),x]
```

output
```
-1/2*(F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^2])/d
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
risch	$\frac{F^a \operatorname{expIntegral}_1\left(-\frac{b\ln(F)}{(dx+c)^2}\right)}{2d}$	23

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*F^a*Ei(1,-b*ln(F)/(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx = -\frac{F^a \operatorname{Ei}\left(\frac{b\log(F)}{d^2x^2+2\,cdx+c^2}\right)}{2\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*F^a*Ei(b*log(F)/(d^2*x^2 + 2*c*d*x + c^2))/d
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c),x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{dx+c}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{dx+c}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx = -\frac{F^a \operatorname{ei}\left(\frac{b\ln(F)}{(c+d\,x)^2}\right)}{2\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x),x)
```

output
```
-(F^a*ei((b*log(F))/(c + d*x)^2))/(2*d)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{c+dx}\,dx = \int \frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}}{dx+c}dx$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c),x)
```

output
```
int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))/(c + d*x),x)
```

3.255 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)}$$

output
```
-1/2*F^(a+b/(d*x+c)^2)/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^3,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3} \, dx$$
$$\Big\downarrow \; 2638$$
$$-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^3,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{F^{a+\frac{b}{(dx+c)^2}}}{2bd\ln(F)}$	26
default	$-\frac{F^{a+\frac{b}{(dx+c)^2}}}{2bd\ln(F)}$	26
parallelrisch	$-\frac{F^{a+\frac{b}{(dx+c)^2}}}{2bd\ln(F)}$	26
risch	$-\frac{F^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{(dx+c)^2}}}{2bd\ln(F)}$	44
norman	$\frac{-\frac{c^2 e^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{2\ln(F)bd}-\frac{cx\,e^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{\ln(F)b}-\frac{d\,x^2 e^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{2\ln(F)b}}{(dx+c)^2}$	94

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^(a+b/(d*x+c)^2)/b/d/ln(F)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{F^{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{2\,bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(b*
d*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = \begin{cases} -\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)} & \text{for } bd\log(F) \neq 0 \\ -\frac{1}{2c^2d+4cd^2x+2d^3x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**3,x)
```

output
```
Piecewise((-F**(a + b/(c + d*x)**2)/(2*b*d*log(F)), Ne(b*d*log(F), 0)), (-
1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{F^{a+\frac{b}{(dx+c)^2}}}{2\,bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*F^(a + b/(d*x + c)^2)/(b*d*log(F))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{F^{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{2\,bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/(b*
d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{F^a\,F^{\frac{b}{c^2+2\,c\,d\,x+d^2\,x^2}}}{2\,b\,d\,\ln(F)}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^3,x)
```

output
```
-(F^a*F^(b/(c^2 + d^2*x^2 + 2*c*d*x)))/(2*b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}\,dx = -\frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}}{2\log(f)\,bd}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^3,x)
```

output
```
( - f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
)))/(2*log(f)*b*d)
```

3.256 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^2\log(F)}$$

output
```
1/2*F^(a+b/(d*x+c)^2)/b^2/d/ln(F)^2-1/2*F^(a+b/(d*x+c)^2)/b/d/(d*x+c)^2/ln
(F)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{F^{a+\frac{b}{(c+dx)^2}}\left((c+dx)^2 - b\log(F)\right)}{2b^2d(c+dx)^2\log^2(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^5,x]
```

output
```
(F^(a + b/(c + d*x)^2)*((c + d*x)^2 - b*Log[F]))/(2*b^2*d*(c + d*x)^2*Log[
F]^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^2}$$

$$\Big\downarrow \text{2638}$$

$$\frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^2}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^5,x]
```

output
```
F^(a + b/(c + d*x)^2)/(2*b^2*d*Log[F]^2) - F^(a + b/(c + d*x)^2)/(2*b*d*(c
 + d*x)^2*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{\left(-x^{2} d^{2}-2 c d x+b \ln \left(F\right)-c^{2}\right) F^{\frac{a\,d^{2} x^{2}+2 a c d x+a\,c^{2}+b}{\left(d x+c\right)^{2}}}}{2 d \ln \left(F\right)^{2} b^{2} \left(d x+c\right)^{2}}$
parallelrisch	$\frac{x^{2} F^{a+\frac{b}{\left(d x+c\right)^{2}}} d^{9}+2 x\,F^{a+\frac{b}{\left(d x+c\right)^{2}}} c\,d^{8}-\ln \left(F\right) F^{a+\frac{b}{\left(d x+c\right)^{2}}} b\,d^{7}+F^{a+\frac{b}{\left(d x+c\right)^{2}}} c^{2} d^{7}}{2 \left(d x+c\right)^{2} \ln \left(F\right)^{2} b^{2} d^{8}}$
norman	$\frac{\frac{d^{3} x^{4} {\mathrm e}^{\left(a+\frac{b}{\left(d x+c\right)^{2}}\right) \ln \left(F\right)}}{2 \ln \left(F\right)^{2} b^{2}}-\frac{c \left(b \ln \left(F\right)-2 c^{2}\right) x\,{\mathrm e}^{\left(a+\frac{b}{\left(d x+c\right)^{2}}\right) \ln \left(F\right)}}{\ln \left(F\right)^{2} b^{2}}-\frac{c^{2} \left(b \ln \left(F\right)-c^{2}\right) {\mathrm e}^{\left(a+\frac{b}{\left(d x+c\right)^{2}}\right) \ln \left(F\right)}}{2 d \ln \left(F\right)^{2} b^{2}}-\frac{d \left(b \ln \left(F\right)-6 c^{2}\right) x^{2} {\mathrm e}^{\left(a+ \right.}}{2 \ln \left(F\right)^{2} b}}{\left(d x+c\right)^{4}}$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-x^2*d^2-2*c*d*x+b*ln(F)-c^2)/d/ln(F)^2/b^2/(d*x+c)^2*F^((a*d^2*x^2+
2*a*c*d*x+a*c^2+b)/(d*x+c)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{(d^2x^2 + 2\,cdx + c^2 - b\log(F))F^{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}}{2\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d)\log(F)^2}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/2*(d^2*x^2 + 2*c*d*x + c^2 - b*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2
 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
)*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{F^{a+\frac{b}{(c+dx)^2}}(-b\log{(F)}+c^2+2cdx+d^2x^2)}{2b^2c^2d\log{(F)}^2+4b^2cd^2x\log{(F)}^2+2b^2d^3x^2\log{(F)}^2}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**5,x)
```

output
```
F**(a + b/(c + d*x)**2)*(-b*log(F) + c**2 + 2*c*d*x + d**2*x**2)/(2*b**2*c
**2*d*log(F)**2 + 4*b**2*c*d**2*x*log(F)**2 + 2*b**2*d**3*x**2*log(F)**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.63

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{(F^ad^2x^2+2\,F^acdx+F^ac^2-F^ab\log{(F)})F^{\frac{b}{d^2x^2+2\,cdx+c^2}}}{2\left(b^2d^3x^2\log{(F)}^2+2\,b^2cd^2x\log{(F)}^2+b^2c^2d\log{(F)}^2\right)}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/2*(F^a*d^2*x^2 + 2*F^a*c*d*x + F^a*c^2 - F^a*b*log(F))*F^(b/(d^2*x^2 + 2
*c*d*x + c^2))/(b^2*d^3*x^2*log(F)^2 + 2*b^2*c*d^2*x*log(F)^2 + b^2*c^2*d*
log(F)^2)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{F^a\,F^{\frac{b}{c^2+2\,c\,d\,x+d^2\,x^2}}\,\left(\frac{x^2}{2\,b^2\,d\ln(F)^2}-\frac{b\,\ln(F)-c^2}{2\,b^2\,d^3\,\ln(F)^2}+\frac{c\,x}{b^2\,d^2\,\ln(F)^2}\right)}{x^2+\frac{c^2}{d^2}+\frac{2\,c\,x}{d}}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^5,x)
```

output
```
(F^a*F^(b/(c^2 + d^2*x^2 + 2*c*d*x))*(x^2/(2*b^2*d*log(F)^2) - (b*log(F) -
 c^2)/(2*b^2*d^3*log(F)^2) + (c*x)/(b^2*d^2*log(F)^2)))/(x^2 + c^2/d^2 + (
2*c*x)/d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.50

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}\,dx = \frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}(-\log(f)\,b+c^2+2cdx+d^2x^2)}{2\log\left(f\right)^2b^2d\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^5,x)
```

output
```
(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*
( - log(f)*b + c**2 + 2*c*d*x + d**2*x**2))/(2*log(f)**2*b**2*d*(c**2 + 2*
c*d*x + d**2*x**2))
```

3.257 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx$

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx = -\frac{F^{a+\frac{b}{(c+dx)^2}}}{b^3 d \log^3(F)} + \frac{F^{a+\frac{b}{(c+dx)^2}}}{b^2 d (c+dx)^2 \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^4 \log(F)}$$

output
```
-F^(a+b/(d*x+c)^2)/b^3/d/ln(F)^3+F^(a+b/(d*x+c)^2)/b^2/d/(d*x+c)^2/ln(F)^2
-1/2*F^(a+b/(d*x+c)^2)/b/d/(d*x+c)^4/ln(F)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7} dx = -\frac{F^{a+\frac{b}{(c+dx)^2}} \left(2(c+dx)^4 - 2b(c+dx)^2 \log(F) + b^2 \log^2(F)\right)}{2b^3 d (c+dx)^4 \log^3(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^7,x]
```

output
```
-1/2*(F^(a + b/(c + d*x)^2)*(2*(c + d*x)^4 - 2*b*(c + d*x)^2*Log[F] + b^2*
Log[F]^2))/(b^3*d*(c + d*x)^4*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx$$

$$\Big\downarrow \ 2641$$

$$-\frac{2\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^4}$$

$$\Big\downarrow \ 2641$$

$$-\frac{2\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^4}$$

$$\Big\downarrow \ 2638$$

$$-\frac{2\left(\frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^4}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^7,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*(c + d*x)^4*Log[F]) - (2*(F^(a + b/(c + d*
x)^2)/(2*b^2*d*Log[F]^2) - F^(a + b/(c + d*x)^2)/(2*b*d*(c + d*x)^2*Log[F]
)))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.40

method	result
risch	$-\frac{\left(2d^{4}x^{4}+8c\,d^{3}x^{3}-2\ln\left(F\right)b\,d^{2}x^{2}+12c^{2}d^{2}x^{2}-4\ln\left(F\right)bcdx+8c^{3}dx+\ln\left(F\right)^{2}b^{2}-2\ln\left(F\right)b\,c^{2}+2c^{4}\right)F^{\frac{a\,d^{2}x^{2}+2acdx+a\,c^{2}+b}{(dx+c)^{2}}}}{2b^{3}\ln\left(F\right)^{3}d\left(dx+c\right)^{4}}$
parallelrisch	$\frac{-2x^{4}F^{a+\frac{b}{(dx+c)^{2}}}d^{13}-8x^{3}F^{a+\frac{b}{(dx+c)^{2}}}c\,d^{12}+2\ln\left(F\right)x^{2}F^{a+\frac{b}{(dx+c)^{2}}}b\,d^{11}-12x^{2}F^{a+\frac{b}{(dx+c)^{2}}}c^{2}d^{11}+4\ln\left(F\right)x\,F^{a+\frac{b}{(dx+c)^{2}}}b}{2\left(dx+c\right)^{4}\ln\left(F\right)^{3}b^{3}d^{10}}$
norman	$\frac{\frac{d^{3}\left(b\ln\left(F\right)-15c^{2}\right)x^{4}{\mathrm e}^{\left(a+\frac{b}{(dx+c)^{2}}\right)\ln\left(F\right)}}{\ln\left(F\right)^{3}b^{3}}-\frac{d^{5}x^{6}{\mathrm e}^{\left(a+\frac{b}{(dx+c)^{2}}\right)\ln\left(F\right)}}{\ln\left(F\right)^{3}b^{3}}-\frac{c\left(\ln\left(F\right)^{2}b^{2}-4\ln\left(F\right)b\,c^{2}+6c^{4}\right)x\,{\mathrm e}^{\left(a+\frac{b}{(dx+c)^{2}}\right)\ln\left(F\right)}}{b^{3}\ln\left(F\right)^{3}}-\frac{d\left(\ln\right.}{}}{}$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*d^4*x^4+8*c*d^3*x^3-2*ln(F)*b*d^2*x^2+12*c^2*d^2*x^2-4*ln(F)*b*c*d
*x+8*c^3*d*x+ln(F)^2*b^2-2*ln(F)*b*c^2+2*c^4)/b^3/ln(F)^3/d/(d*x+c)^4*F^((
a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(89) = 178$.

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx =$$
$$-\frac{\left(2\,d^4x^4 + 8\,cd^3x^3 + 12\,c^2d^2x^2 + 8\,c^3dx + 2\,c^4 + b^2\log\left(F\right)^2 - 2\left(bd^2x^2 + 2\,bcdx + bc^2\right)\log\left(F\right)\right)F^{\frac{ad^2x^2+\cdots}{d^2x^2\cdots}}}{2\left(b^3d^5x^4 + 4\,b^3cd^4x^3 + 6\,b^3c^2d^3x^2 + 4\,b^3c^3d^2x + b^3c^4d\right)\log\left(F\right)^3}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^7,x, algorithm="fricas")
```

output
```
-1/2*(2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 + 8*c^3*d*x + 2*c^4 + b^2*l
og(F)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))*F^((a*d^2*x^2 + 2*a*c*
d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((b^3*d^5*x^4 + 4*b^3*c*d^4*x^
3 + 6*b^3*c^2*d^3*x^2 + 4*b^3*c^3*d^2*x + b^3*c^4*d)*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(76) = 152$.

Time = 0.15 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.08

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx$$
$$= \frac{F^{a+\frac{b}{(c+dx)^2}}\left(-b^2\log\left(F\right)^2 + 2bc^2\log\left(F\right) + 4bcdx\log\left(F\right) + 2bd^2x^2\log\left(F\right) - 2c^4 - 8c^3dx - 12c^2d^2x^2 - 8c\cdots\right.}{2b^3c^4d\log\left(F\right)^3 + 8b^3c^3d^2x\log\left(F\right)^3 + 12b^3c^2d^3x^2\log\left(F\right)^3 + 8b^3cd^4x^3\log\left(F\right)^3 + 2b^3d^5x^4\log\left(\cdots\right.}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**7,x)
```

output
```
F**(a + b/(c + d*x)**2)*(-b**2*log(F)**2 + 2*b*c**2*log(F) + 4*b*c*d*x*log
(F) + 2*b*d**2*x**2*log(F) - 2*c**4 - 8*c**3*d*x - 12*c**2*d**2*x**2 - 8*c
*d**3*x**3 - 2*d**4*x**4)/(2*b**3*c**4*d*log(F)**3 + 8*b**3*c**3*d**2*x*lo
g(F)**3 + 12*b**3*c**2*d**3*x**2*log(F)**3 + 8*b**3*c*d**4*x**3*log(F)**3
+ 2*b**3*d**5*x**4*log(F)**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. 2(89) = 178.

Time = 0.04 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.29

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx = -\frac{\left(2\,F^a d^4 x^4 + 8\,F^a c d^3 x^3 + 2\,F^a c^4 - 2\,F^a b c^2 \log\left(F\right) + F^a b^2 \log\left(F\right)^2 + 2\left(6\,F^a c^2 d^2 - F^a b d^2 \log\left(F\right)\right)x^2 + \cdots}{2\left(b^3 d^5 x^4 \log\left(F\right)^3 + 4\,b^3 c d^4 x^3 \log\left(F\right)^3 + 6\,b^3 c^2 d^3 x^2 \log\left(F\right)^3 + 4\,b^3 c^3 d^2 x \log\left(F\right)^3 + \cdots\right)}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^7,x, algorithm="maxima")
```

output
```
-1/2*(2*F^a*d^4*x^4 + 8*F^a*c*d^3*x^3 + 2*F^a*c^4 - 2*F^a*b*c^2*log(F) + F
^a*b^2*log(F)^2 + 2*(6*F^a*c^2*d^2 - F^a*b*d^2*log(F))*x^2 + 4*(2*F^a*c^3*
d - F^a*b*c*d*log(F))*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*d^5*x^4*log(
F)^3 + 4*b^3*c*d^4*x^3*log(F)^3 + 6*b^3*c^2*d^3*x^2*log(F)^3 + 4*b^3*c^3*d
^2*x*log(F)^3 + b^3*c^4*d*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 1703, normalized size of antiderivative = 18.71

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^7,x, algorithm="giac")
```

output
```
-1/2*((2*(pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b
^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)*(pi*b*d^2*sgn(F)/(d*x + c)^2 - pi*b^2
*d^2*log(abs(F))*sgn(F)/(d*x + c)^4 - pi*b*d^2/(d*x + c)^2 + pi*b^2*d^2*lo
g(abs(F))/(d*x + c)^4)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*
sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*lo
g(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)^2
) + (3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^
3*d^3*log(abs(F))^3)*(pi^2*b^2*d^2*sgn(F)/(d*x + c)^4 - pi^2*b^2*d^2/(d*x
+ c)^4 + 4*d^2 - 4*b*d^2*log(abs(F))/(d*x + c)^2 + 2*b^2*d^2*log(abs(F))^2
/(d*x + c)^4)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) -
pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*log(abs(F))
*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)^2))*cos(-1
/2*pi*a*sgn(F) + 1/2*pi*a - 1/2*pi*b*sgn(F)/(d^2*x^2 + 2*c*d*x + c^2) + 1/
2*pi*b/(d^2*x^2 + 2*c*d*x + c^2)) - (2*(3*pi^2*b^3*d^3*log(abs(F))*sgn(F)
- 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)*(pi*b*d^2*sgn(F)/(
d*x + c)^2 - pi*b^2*d^2*log(abs(F))*sgn(F)/(d*x + c)^4 - pi*b*d^2/(d*x + c
)^2 + pi*b^2*d^2*log(abs(F))/(d*x + c)^4)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3
*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 +
 (3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d
^3*log(abs(F))^3)^2) - (pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^...
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.01

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx = -\frac{F^a\,F^{\frac{b}{c^2+2\,c\,d\,x+d^2\,x^2}}\,\left(\frac{x^4}{b^3\,d\ln(F)^3}+\frac{b^2\ln(F)^2-2\,b\,c^2\,\ln(F)+2\,c^4}{2\,b^3\,d^5\ln(F)^3}+\frac{4\,c\,x^3}{b^3\,d^2\ln(F)^3}-\frac{x^2\,\left(b\,\ln(F)-6\,c^2\right)}{b^3\,d^3\ln(F)^3}-\frac{2\,c\,x\,\left(b\,\ln(F)-2\,c^2\right)}{b^3\,d^4\ln(F)^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^7,x)
```

output
```
-(F^a*F^(b/(c^2 + d^2*x^2 + 2*c*d*x))*(x^4/(b^3*d*log(F)^3) + (b^2*log(F)^
2 + 2*c^4 - 2*b*c^2*log(F))/(2*b^3*d^5*log(F)^3) + (4*c*x^3)/(b^3*d^2*log(
F)^3) - (x^2*(b*log(F) - 6*c^2))/(b^3*d^3*log(F)^3) - (2*c*x*(b*log(F) - 2
*c^2))/(b^3*d^4*log(F)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d^3 +
 (6*c^2*x^2)/d^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}\,dx$$
$$= \frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}\left(-\log(f)^2\,b^2+2\log(f)\,b\,c^2+4\log(f)\,bcdx+2\log(f)\,b\,d^2x^2-2c^4-8c^3dx-12c^2d^2x^2\right.}{2\log\left(f\right)^3b^3d\left(d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4\right)}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^7,x)
```

output
```
(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*
( - log(f)**2*b**2 + 2*log(f)*b*c**2 + 4*log(f)*b*c*d*x + 2*log(f)*b*d**2*
x**2 - 2*c**4 - 8*c**3*d*x - 12*c**2*d**2*x**2 - 8*c*d**3*x**3 - 2*d**4*x*
*4))/(2*log(f)**3*b**3*d*(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*
x**3 + d**4*x**4))
```

3.258 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{3F^{a+\frac{b}{(c+dx)^2}}}{b^4d\log^4(F)} - \frac{3F^{a+\frac{b}{(c+dx)^2}}}{b^3d(c+dx)^2\log^3(F)} + \frac{3F^{a+\frac{b}{(c+dx)^2}}}{2b^2d(c+dx)^4\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^6\log(F)}$$

output
```
3*F^(a+b/(d*x+c)^2)/b^4/d/ln(F)^4-3*F^(a+b/(d*x+c)^2)/b^3/d/(d*x+c)^2/ln(F
)^3+3/2*F^(a+b/(d*x+c)^2)/b^2/d/(d*x+c)^4/ln(F)^2-1/2*F^(a+b/(d*x+c)^2)/b/
d/(d*x+c)^6/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{F^{a+\frac{b}{(c+dx)^2}}\left(6(c+dx)^6-6b(c+dx)^4\log(F)+3b^2(c+dx)^2\log^2(F)-b^3\log^3(F)\right)}{2b^4d(c+dx)^6\log^4(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^9,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(6*(c + d*x)^6 - 6*b*(c + d*x)^4*Log[F] + 3*b^2*(c
+ d*x)^2*Log[F]^2 - b^3*Log[F]^3))/(2*b^4*d*(c + d*x)^6*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^7}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^6}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\left(-\frac{2\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^5}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^4}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^6}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^3}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^4}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^6}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{3\left(-\frac{2\left(\frac{F^{a+\frac{b}{(c+dx)^2}}}{2b^2d\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^2}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^4}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^6}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^9,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*(c + d*x)^6*Log[F]) - (3*(-1/2*F^(a + b/(c
 + d*x)^2)/(b*d*(c + d*x)^4*Log[F]) - (2*(F^(a + b/(c + d*x)^2)/(2*b^2*d*L
og[F]^2) - F^(a + b/(c + d*x)^2)/(2*b*d*(c + d*x)^2*Log[F])))/(b*Log[F])))
/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\left(-6d^6x^6-36c\,d^5x^5+6\ln(F)b\,d^4x^4-90c^2d^4x^4+24\ln(F)bc\,d^3x^3-120c^3d^3x^3-3\ln(F)^2b^2d^2x^2+36\ln(F)b\,c^2d^2x^2-90c^4d^2\right.}{2b^4\ln(F)^4d(dx}$
parallelrisch	$\frac{6d^{17}F^{a+\frac{b}{(dx+c)^2}}x^6+36d^{16}c\,F^{a+\frac{b}{(dx+c)^2}}x^5-6\ln(F)x^4F^{a+\frac{b}{(dx+c)^2}}b\,d^{15}+90d^{15}c^2F^{a+\frac{b}{(dx+c)^2}}x^4-24\ln(F)x^3F^{a+\frac{b}{(dx+c)^2}}}{}$
norman	$\frac{\frac{3d^7x^8e^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{\ln(F)^4b^4}-\frac{c\left(\ln(F)^3b^3-6\ln(F)^2b^2c^2+18\ln(F)b\,c^4-24c^6\right)x\,e^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{b^4\ln(F)^4}-\frac{d\left(\ln(F)^3b^3-18\ln(F)^2b^2c^2+90\ln\right.}{2}}{}$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^9,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-6*d^6*x^6-36*c*d^5*x^5+6*ln(F)*b*d^4*x^4-90*c^2*d^4*x^4+24*ln(F)*b*
c*d^3*x^3-120*c^3*d^3*x^3-3*ln(F)^2*b^2*d^2*x^2+36*ln(F)*b*c^2*d^2*x^2-90*
c^4*d^2*x^2-6*ln(F)^2*b^2*c*d*x+24*ln(F)*b*c^3*d*x-36*c^5*d*x+ln(F)^3*b^3-
3*ln(F)^2*b^2*c^2+6*ln(F)*b*c^4-6*c^6)/b^4/ln(F)^4/d/(d*x+c)^6*F^((a*d^2*x
^2+2*a*c*d*x+a*c^2+b)/(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(122) = 244$.

Time = 0.09 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.28

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{\left(6\,d^6x^6+36\,cd^5x^5+90\,c^2d^4x^4+120\,c^3d^3x^3+90\,c^4d^2x^2+36\,c^5dx+6\,c^6-b^3\log\left(F\right)^3+3\,(b^2d^2x^2+2\,b^2\right.}{2\,(b^4d^7x^6+6\,b^4cd^6x^5+15\,b^4c^2d^5x^4+20\,b^4c^3d^4x^3}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^9,x, algorithm="fricas")
```

output
```
1/2*(6*d^6*x^6 + 36*c*d^5*x^5 + 90*c^2*d^4*x^4 + 120*c^3*d^3*x^3 + 90*c^4*
d^2*x^2 + 36*c^5*d*x + 6*c^6 - b^3*log(F)^3 + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*log(F)^2 - 6*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*
b*c^3*d*x + b*c^4)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
 + 2*c*d*x + c^2))/((b^4*d^7*x^6 + 6*b^4*c*d^6*x^5 + 15*b^4*c^2*d^5*x^4 +
20*b^4*c^3*d^4*x^3 + 15*b^4*c^4*d^3*x^2 + 6*b^4*c^5*d^2*x + b^4*c^6*d)*log
(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(112) = 224$.

Time = 0.17 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.64

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{F^{a+\frac{b}{(c+dx)^2}}\left(-b^3\log{(F)}^3+3b^2c^2\log{(F)}^2+6b^2cdx\log{(F)}^2+3b^2d^2x^2\log{(F)}^2-6bc^4\log{(F)}-24bc^3dx\,\mathrm{l}\right.}{2b^4c^6d\log{(F)}^4+12b^4c^5d^2x\log{(F)}^4+30b^4c^4d^3x^2\log{(}}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**9,x)
```

output
```
F**(a + b/(c + d*x)**2)*(-b**3*log(F)**3 + 3*b**2*c**2*log(F)**2 + 6*b**2*
c*d*x*log(F)**2 + 3*b**2*d**2*x**2*log(F)**2 - 6*b*c**4*log(F) - 24*b*c**3
*d*x*log(F) - 36*b*c**2*d**2*x**2*log(F) - 24*b*c*d**3*x**3*log(F) - 6*b*d
**4*x**4*log(F) + 6*c**6 + 36*c**5*d*x + 90*c**4*d**2*x**2 + 120*c**3*d**3
*x**3 + 90*c**2*d**4*x**4 + 36*c*d**5*x**5 + 6*d**6*x**6)/(2*b**4*c**6*d*l
og(F)**4 + 12*b**4*c**5*d**2*x*log(F)**4 + 30*b**4*c**4*d**3*x**2*log(F)**
4 + 40*b**4*c**3*d**4*x**3*log(F)**4 + 30*b**4*c**2*d**5*x**4*log(F)**4 +
12*b**4*c*d**6*x**5*log(F)**4 + 2*b**4*d**7*x**6*log(F)**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(122) = 244.

Time = 0.04 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.77

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{\left(6\,F^a d^6 x^6 + 36\,F^a c d^5 x^5 + 6\,F^a c^6 - 6\,F^a b c^4 \log(F) + 3\,F^a b^2 c^2 \log(F)^2 - F^a b^3 \log(F)^3 + 6\,(15\,F^a c^2 d^4\right.}{2\,\left(b^4 d^7 x^6 \log(F)^4 + 6\,b^4 c d^6 x^5 \log\right.}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^9,x, algorithm="maxima")
```

output
```
1/2*(6*F^a*d^6*x^6 + 36*F^a*c*d^5*x^5 + 6*F^a*c^6 - 6*F^a*b*c^4*log(F) + 3
*F^a*b^2*c^2*log(F)^2 - F^a*b^3*log(F)^3 + 6*(15*F^a*c^2*d^4 - F^a*b*d^4*l
og(F))*x^4 + 24*(5*F^a*c^3*d^3 - F^a*b*c*d^3*log(F))*x^3 + 3*(30*F^a*c^4*d
^2 - 12*F^a*b*c^2*d^2*log(F) + F^a*b^2*d^2*log(F)^2)*x^2 + 6*(6*F^a*c^5*d
- 4*F^a*b*c^3*d*log(F) + F^a*b^2*c*d*log(F)^2)*x)*F^(b/(d^2*x^2 + 2*c*d*x
+ c^2))/(b^4*d^7*x^6*log(F)^4 + 6*b^4*c*d^6*x^5*log(F)^4 + 15*b^4*c^2*d^5*
x^4*log(F)^4 + 20*b^4*c^3*d^4*x^3*log(F)^4 + 15*b^4*c^4*d^3*x^2*log(F)^4 +
 6*b^4*c^5*d^2*x*log(F)^4 + b^4*c^6*d*log(F)^4)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^9}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^9,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^9, x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.32

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{F^a\,F^{\frac{b}{c^2+2\,c\,d\,x+d^2\,x^2}}\left(\frac{3\,x^6}{b^4\,d\ln(F)^4}-\frac{b^3\ln(F)^3-3\,b^2\,c^2\ln(F)^2+6\,b\,c^4\ln(F)-6\,c^6}{2\,b^4\,d^7\ln(F)^4}+\frac{18\,c\,x^5}{b^4\,d^2\ln(F)^4}+\frac{3\,x^2\left(b^2\ln(F)^2-12\,b\,c^2\ln(F)+30\,c^4\right)}{2\,b^4\,d^5\ln(F)^4}\right.}{x^6+\frac{c^6}{d^6}+\frac{6\,c\,x^5}{d}+\frac{6\,c^5\,x}{d^5}+\frac{15\,c^2\,x^4}{d^2}+\frac{20\,c^3\,x^3}{d^3}}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^9,x)
```

output
```
(F^a*F^(b/(c^2 + d^2*x^2 + 2*c*d*x))*((3*x^6)/(b^4*d*log(F)^4) - (b^3*log(
F)^3 - 6*c^6 + 6*b*c^4*log(F) - 3*b^2*c^2*log(F)^2)/(2*b^4*d^7*log(F)^4) +
 (18*c*x^5)/(b^4*d^2*log(F)^4) + (3*x^2*(b^2*log(F)^2 + 30*c^4 - 12*b*c^2*
log(F)))/(2*b^4*d^5*log(F)^4) - (3*x^4*(b*log(F) - 15*c^2))/(b^4*d^3*log(F
)^4) - (12*c*x^3*(b*log(F) - 5*c^2))/(b^4*d^4*log(F)^4) + (3*c*x*(b^2*log(
F)^2 + 6*c^4 - 4*b*c^2*log(F)))/(b^4*d^6*log(F)^4)))/(x^6 + c^6/d^6 + (6*c
*x^5)/d + (6*c^5*x)/d^5 + (15*c^2*x^4)/d^2 + (20*c^3*x^3)/d^3 + (15*c^4*x^
2)/d^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.24

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^9}\,dx = \frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}\left(-\log(f)^3\,b^3+3\log(f)^2\,b^2c^2+6\log(f)^2\,b^2cdx+3\log(f)^2\,b^2d^2x^2-6\log(f)\,b\,c^4-24\log\right.}{2\log(f)^4\,b^4d\left(d^6x^6+\right.}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^9,x)
```

output
```
(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*
( - log(f)**3*b**3 + 3*log(f)**2*b**2*c**2 + 6*log(f)**2*b**2*c*d*x + 3*lo
g(f)**2*b**2*d**2*x**2 - 6*log(f)*b*c**4 - 24*log(f)*b*c**3*d*x - 36*log(f
)*b*c**2*d**2*x**2 - 24*log(f)*b*c*d**3*x**3 - 6*log(f)*b*d**4*x**4 + 6*c*
*6 + 36*c**5*d*x + 90*c**4*d**2*x**2 + 120*c**3*d**3*x**3 + 90*c**2*d**4*x
**4 + 36*c*d**5*x**5 + 6*d**6*x**6))/(2*log(f)**4*b**4*d*(c**6 + 6*c**5*d*
x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4*x**4 + 6*c*d**5*x
**5 + d**6*x**6))
```

3.259 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx = -\frac{F^{a+\frac{b}{(c+dx)^2}}\left(24(c+dx)^8-24b(c+dx)^6\log(F)+12b^2(c+dx)^4\log^2(F)-4b^3(c+dx)^2\log^3(F)+b^4\log\right.}{2b^5d(c+dx)^8\log^5(F)}$$

output
```
-1/2*F^(a+b/(d*x+c)^2)*(24*(d*x+c)^8-24*b*(d*x+c)^6*ln(F)+12*b^2*(d*x+c)^4
*ln(F)^2-4*b^3*(d*x+c)^2*ln(F)^3+b^4*ln(F)^4)/b^5/d/(d*x+c)^8/ln(F)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.32

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx = -\frac{F^a\Gamma\left(5,-\frac{b\log(F)}{(c+dx)^2}\right)}{2b^5d\log^5(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^11,x]
```

output
```
-1/2*(F^a*Gamma[5, -((b*Log[F])/(c + d*x)^2)])/(b^5*d*Log[F]^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx$$

$$\Big\downarrow \text{2647}$$

$$-\frac{F^{a+\frac{b}{(c+dx)^2}}\left(b^4\log^4(F)-4b^3\log^3(F)(c+dx)^2+12b^2\log^2(F)(c+dx)^4-24b\log(F)(c+dx)^6+24(c+dx)^8\right)}{2b^5d\log^5(F)(c+dx)^8}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^11,x]
```

output
```
-1/2*(F^(a + b/(c + d*x)^2)*(24*(c + d*x)^8 - 24*b*(c + d*x)^6*Log[F] + 12
*b^2*(c + d*x)^4*Log[F]^2 - 4*b^3*(c + d*x)^2*Log[F]^3 + b^4*Log[F]^4))/(b
^5*d*(c + d*x)^8*Log[F]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(94) = 188$.

Time = 2.92 (sec) , antiderivative size = 341, normalized size of antiderivative = 3.55

method	result
risch	$-\frac{\left(24c^8+24d^8x^8+12d^4x^4b^2\ln(F)^2+48d^3c\,x^3b^2\ln(F)^2+72\ln(F)^2b^2c^2d^2x^2+48\ln(F)^2b^2c^3dx+b^4\ln(F)^4+12\ln(F)^2b^2c^4-2\right.}{}$
norman	$\frac{-\frac{12d^9x^{10}\mathrm{e}^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{\ln(F)^5b^5}-\frac{c\left(b^4\ln(F)^4-8\ln(F)^3b^3c^2+36\ln(F)^2b^2c^4-96\ln(F)b\,c^6+120c^8\right)x\,\mathrm{e}^{\left(a+\frac{b}{(dx+c)^2}\right)\ln(F)}}{b^5\ln(F)^5}-\frac{d\left(b^4\ln(F)\right.}{}}{}$
parallelrisch	$\frac{8\ln(F)^3x\,F^{a+\frac{b}{(dx+c)^2}}b^3c\,d^{14}-48\ln(F)^2x\,F^{a+\frac{b}{(dx+c)^2}}b^2c^3d^{14}+144\ln(F)x\,F^{a+\frac{b}{(dx+c)^2}}b\,c^5d^{14}-24F^{a+\frac{b}{(dx+c)^2}}c^8d^{13}-24c}{}$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^11,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(24*c^8+24*d^8*x^8+12*d^4*x^4*b^2*ln(F)^2+48*d^3*c*x^3*b^2*ln(F)^2+72
*ln(F)^2*b^2*c^2*d^2*x^2+48*ln(F)^2*b^2*c^3*d*x+b^4*ln(F)^4+12*ln(F)^2*b^2
*c^4-24*ln(F)*b*c^6-4*ln(F)^3*b^3*c^2-8*ln(F)^3*b^3*c*d*x+192*c*d^7*x^7+67
2*c^2*d^6*x^6+1344*c^3*d^5*x^5+1680*c^4*d^4*x^4+1344*c^5*d^3*x^3+672*c^6*d
^2*x^2+192*c^7*d*x-24*ln(F)*b*d^6*x^6-4*ln(F)^3*b^3*d^2*x^2-144*ln(F)*b*c*
d^5*x^5-360*ln(F)*b*c^2*d^4*x^4-480*ln(F)*b*c^3*d^3*x^3-360*ln(F)*b*c^4*d^
2*x^2-144*ln(F)*b*c^5*d*x)/b^5/ln(F)^5/d/(d*x+c)^8*F^((a*d^2*x^2+2*a*c*d*x
+a*c^2+b)/(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(94) = 188$.

Time = 0.09 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.38

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx =$$

$$-\frac{\left(24\,d^8x^8+192\,cd^7x^7+672\,c^2d^6x^6+1344\,c^3d^5x^5+1680\,c^4d^4x^4+1344\,c^5d^3x^3+672\,c^6d^2x^2+192\,c^7dx\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^11,x, algorithm="fricas")
```

output
```
-1/2*(24*d^8*x^8 + 192*c*d^7*x^7 + 672*c^2*d^6*x^6 + 1344*c^3*d^5*x^5 + 16
80*c^4*d^4*x^4 + 1344*c^5*d^3*x^3 + 672*c^6*d^2*x^2 + 192*c^7*d*x + 24*c^8
 + b^4*log(F)^4 - 4*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*log(F)^3 + 12*(b
^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4
)*log(F)^2 - 24*(b*d^6*x^6 + 6*b*c*d^5*x^5 + 15*b*c^2*d^4*x^4 + 20*b*c^3*d
^3*x^3 + 15*b*c^4*d^2*x^2 + 6*b*c^5*d*x + b*c^6)*log(F))*F^((a*d^2*x^2 + 2
*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((b^5*d^9*x^8 + 8*b^5*c*d
^8*x^7 + 28*b^5*c^2*d^7*x^6 + 56*b^5*c^3*d^6*x^5 + 70*b^5*c^4*d^5*x^4 + 56
*b^5*c^5*d^4*x^3 + 28*b^5*c^6*d^3*x^2 + 8*b^5*c^7*d^2*x + b^5*c^8*d)*log(F
)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(95) = 190$.

Time = 0.30 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.40

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx$$
$$= \frac{F^{a+\frac{b}{(c+dx)^2}}\left(-b^4\log(F)^4+4b^3c^2\log(F)^3+8b^3cdx\log(F)^3+4b^3d^2x^2\log(F)^3-12b^2c^4\log(F)^2-48b^2c^3\right.}{}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**11,x)
```

output
```
F**(a + b/(c + d*x)**2)*(-b**4*log(F)**4 + 4*b**3*c**2*log(F)**3 + 8*b**3*
c*d*x*log(F)**3 + 4*b**3*d**2*x**2*log(F)**3 - 12*b**2*c**4*log(F)**2 - 48
*b**2*c**3*d*x*log(F)**2 - 72*b**2*c**2*d**2*x**2*log(F)**2 - 48*b**2*c*d*
*3*x**3*log(F)**2 - 12*b**2*d**4*x**4*log(F)**2 + 24*b*c**6*log(F) + 144*b
*c**5*d*x*log(F) + 360*b*c**4*d**2*x**2*log(F) + 480*b*c**3*d**3*x**3*log(
F) + 360*b*c**2*d**4*x**4*log(F) + 144*b*c*d**5*x**5*log(F) + 24*b*d**6*x*
*6*log(F) - 24*c**8 - 192*c**7*d*x - 672*c**6*d**2*x**2 - 1344*c**5*d**3*x
**3 - 1680*c**4*d**4*x**4 - 1344*c**3*d**5*x**5 - 672*c**2*d**6*x**6 - 192
*c*d**7*x**7 - 24*d**8*x**8)/(2*b**5*c**8*d*log(F)**5 + 16*b**5*c**7*d**2*
x*log(F)**5 + 56*b**5*c**6*d**3*x**2*log(F)**5 + 112*b**5*c**5*d**4*x**3*l
og(F)**5 + 140*b**5*c**4*d**5*x**4*log(F)**5 + 112*b**5*c**3*d**6*x**5*log
(F)**5 + 56*b**5*c**2*d**7*x**6*log(F)**5 + 16*b**5*c*d**8*x**7*log(F)**5
+ 2*b**5*d**9*x**8*log(F)**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(94) = 188$.

Time = 0.05 (sec) , antiderivative size = 526, normalized size of antiderivative = 5.48

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx = -\frac{\left(24\,F^a d^8 x^8 + 192\,F^a c d^7 x^7 + 24\,F^a c^8 - 24\,F^a b c^6 \log\left(F\right) + 12\,F^a b^2 c^4 \log\left(F\right)^2 - 4\,F^a b^3 c^2 \log\left(F\right)^3 + F\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^11,x, algorithm="maxima")
```

output
```
-1/2*(24*F^a*d^8*x^8 + 192*F^a*c*d^7*x^7 + 24*F^a*c^8 - 24*F^a*b*c^6*log(F
) + 12*F^a*b^2*c^4*log(F)^2 - 4*F^a*b^3*c^2*log(F)^3 + F^a*b^4*log(F)^4 +
24*(28*F^a*c^2*d^6 - F^a*b*d^6*log(F))*x^6 + 48*(28*F^a*c^3*d^5 - 3*F^a*b*
c*d^5*log(F))*x^5 + 12*(140*F^a*c^4*d^4 - 30*F^a*b*c^2*d^4*log(F) + F^a*b^
2*d^4*log(F)^2)*x^4 + 48*(28*F^a*c^5*d^3 - 10*F^a*b*c^3*d^3*log(F) + F^a*b
^2*c*d^3*log(F)^2)*x^3 + 4*(168*F^a*c^6*d^2 - 90*F^a*b*c^4*d^2*log(F) + 18
*F^a*b^2*c^2*d^2*log(F)^2 - F^a*b^3*d^2*log(F)^3)*x^2 + 8*(24*F^a*c^7*d -
18*F^a*b*c^5*d*log(F) + 6*F^a*b^2*c^3*d*log(F)^2 - F^a*b^3*c*d*log(F)^3)*x
)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(b^5*d^9*x^8*log(F)^5 + 8*b^5*c*d^8*x^7*
log(F)^5 + 28*b^5*c^2*d^7*x^6*log(F)^5 + 56*b^5*c^3*d^6*x^5*log(F)^5 + 70*
b^5*c^4*d^5*x^4*log(F)^5 + 56*b^5*c^5*d^4*x^3*log(F)^5 + 28*b^5*c^6*d^3*x^
2*log(F)^5 + 8*b^5*c^7*d^2*x*log(F)^5 + b^5*c^8*d*log(F)^5)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{11}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^11,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^11, x)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.45

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx = -\frac{F^a\,F^{\frac{b}{c^2+2\,c\,d\,x+d^2\,x^2}}\,\left(\frac{b^4\,\ln(F)^4-4\,b^3\,c^2\,\ln(F)^3+12\,b^2\,c^4\,\ln(F)^2-24\,b\,c^6\,\ln(F)+24\,c^8}{2\,b^5\,d^9\,\ln(F)^5}+\frac{12\,x^8}{b^5\,d\,\ln(F)^5}+\frac{96\,c\,x^7}{b^5\,d^2\,\ln(F)^5}-\frac{2\,x^2\,\left(b^3\,\ln(F\right.}{}\right.}{}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^11,x)
```

output
```
-(F^a*F^(b/(c^2 + d^2*x^2 + 2*c*d*x))*((b^4*log(F)^4 + 24*c^8 - 24*b*c^6*l
og(F) + 12*b^2*c^4*log(F)^2 - 4*b^3*c^2*log(F)^3)/(2*b^5*d^9*log(F)^5) + (
12*x^8)/(b^5*d*log(F)^5) + (96*c*x^7)/(b^5*d^2*log(F)^5) - (2*x^2*(b^3*log
(F)^3 - 168*c^6 + 90*b*c^4*log(F) - 18*b^2*c^2*log(F)^2))/(b^5*d^7*log(F)^
5) + (6*x^4*(b^2*log(F)^2 + 140*c^4 - 30*b*c^2*log(F)))/(b^5*d^5*log(F)^5)
 - (12*x^6*(b*log(F) - 28*c^2))/(b^5*d^3*log(F)^5) + (24*c*x^3*(b^2*log(F)
^2 + 28*c^4 - 10*b*c^2*log(F)))/(b^5*d^6*log(F)^5) - (24*c*x^5*(3*b*log(F)
 - 28*c^2))/(b^5*d^4*log(F)^5) - (4*c*x*(b^3*log(F)^3 - 24*c^6 + 18*b*c^4*
log(F) - 6*b^2*c^2*log(F)^2))/(b^5*d^8*log(F)^5)))/(x^8 + c^8/d^8 + (8*c*x
^7)/d + (8*c^7*x)/d^7 + (28*c^2*x^6)/d^2 + (56*c^3*x^5)/d^3 + (70*c^4*x^4)
/d^4 + (56*c^5*x^3)/d^5 + (28*c^6*x^2)/d^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 4.47

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{11}}\,dx = \frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}\left(24\log(f)\,b\,d^6x^6-48\log(f)^2\,b^2c^3dx-72\log(f)^2\,b^2c^2d^2x^2-48\log(f)^2\,b^2c\,d^3x^3-12\log\right.}{}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^11,x)
```

output
```
(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*
( - log(f)**4*b**4 + 4*log(f)**3*b**3*c**2 + 8*log(f)**3*b**3*c*d*x + 4*lo
g(f)**3*b**3*d**2*x**2 - 12*log(f)**2*b**2*c**4 - 48*log(f)**2*b**2*c**3*d
*x - 72*log(f)**2*b**2*c**2*d**2*x**2 - 48*log(f)**2*b**2*c*d**3*x**3 - 12
*log(f)**2*b**2*d**4*x**4 + 24*log(f)*b*c**6 + 144*log(f)*b*c**5*d*x + 360
*log(f)*b*c**4*d**2*x**2 + 480*log(f)*b*c**3*d**3*x**3 + 360*log(f)*b*c**2
*d**4*x**4 + 144*log(f)*b*c*d**5*x**5 + 24*log(f)*b*d**6*x**6 - 24*c**8 -
192*c**7*d*x - 672*c**6*d**2*x**2 - 1344*c**5*d**3*x**3 - 1680*c**4*d**4*x
**4 - 1344*c**3*d**5*x**5 - 672*c**2*d**6*x**6 - 192*c*d**7*x**7 - 24*d**8
*x**8))/(2*log(f)**5*b**5*d*(c**8 + 8*c**7*d*x + 28*c**6*d**2*x**2 + 56*c*
*5*d**3*x**3 + 70*c**4*d**4*x**4 + 56*c**3*d**5*x**5 + 28*c**2*d**6*x**6 +
 8*c*d**7*x**7 + d**8*x**8))
```

3.260 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}} \, dx = \frac{F^{a+\frac{b}{(c+dx)^2}} \left(120(c+dx)^{10} - 120b(c+dx)^8 \log(F) + 60b^2(c+dx)^6 \log^2(F) - 20b^3(c+dx)^4 \log^3(F) + 5b^4 \right.}{2b^6 d(c+dx)^{10} \log^6(F)}$$

output
```
1/2*F^(a+b/(d*x+c)^2)*(120*(d*x+c)^10-120*b*(d*x+c)^8*ln(F)+60*b^2*(d*x+c)
^6*ln(F)^2-20*b^3*(d*x+c)^4*ln(F)^3+5*b^4*(d*x+c)^2*ln(F)^4-b^5*ln(F)^5)/b
^6/d/(d*x+c)^10/ln(F)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.27

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}} \, dx = \frac{F^a \Gamma\left(6, -\frac{b \log(F)}{(c+dx)^2}\right)}{2b^6 d \log^6(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^13,x]
```

output
```
(F^a*Gamma[6, -((b*Log[F])/(c + d*x)^2)])/(2*b^6*d*Log[F]^6)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2647\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx$$

$$\Big\downarrow \; 2647$$

$$\frac{F^{a+\frac{b}{(c+dx)^2}}\left(-b^5\log^5(F)+5b^4\log^4(F)(c+dx)^2-20b^3\log^3(F)(c+dx)^4+60b^2\log^2(F)(c+dx)^6-120b\log(F)(\right.}{2b^6d\log^6(F)(c+dx)^{10}}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^13,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(120*(c + d*x)^10 - 120*b*(c + d*x)^8*Log[F] + 60*b
^2*(c + d*x)^6*Log[F]^2 - 20*b^3*(c + d*x)^4*Log[F]^3 + 5*b^4*(c + d*x)^2*
Log[F]^4 - b^5*Log[F]^5))/(2*b^6*d*(c + d*x)^10*Log[F]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(111) = 222$.

Time = 4.69 (sec) , antiderivative size = 502, normalized size of antiderivative = 4.44

method	result
risch	$-\frac{\left(-5\ln(F)^4 b^4 c^2+20\ln(F)^3 b^3 c^4-1200 c\, d^9 x^9-5400 c^2 d^8 x^8-14400 c^3 d^7 x^7-25200 c^4 d^6 x^6-30240 c^5 d^5 x^5-25200 c^6 d^4 x^4-14\right.}{}$
norman	Expression too large to display
parallelrisch	Expression too large to display

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^13,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-5*ln(F)^4*b^4*c^2+20*ln(F)^3*b^3*c^4-1200*c*d^9*x^9-5400*c^2*d^8*x^
8-14400*c^3*d^7*x^7-25200*c^4*d^6*x^6-30240*c^5*d^5*x^5-25200*c^6*d^4*x^4-
14400*c^7*d^3*x^3-5400*c^8*d^2*x^2-1200*c^9*d*x-60*ln(F)^2*b^2*c^6-120*c^1
0+120*ln(F)*b*c^8+b^5*ln(F)^5-60*d^6*x^6*ln(F)^2*b^2-120*d^10*x^10-360*c*d
^5*x^5*ln(F)^2*b^2-900*c^2*d^4*x^4*ln(F)^2*b^2-1200*ln(F)^2*b^2*c^3*d^3*x^
3-900*ln(F)^2*b^2*c^4*d^2*x^2-360*ln(F)^2*b^2*c^5*d*x+120*ln(F)*b*d^8*x^8+
20*ln(F)^3*b^3*d^4*x^4-5*ln(F)^4*b^4*d^2*x^2+960*ln(F)*b*c*d^7*x^7+3360*ln
(F)*b*c^2*d^6*x^6+6720*ln(F)*b*c^3*d^5*x^5+8400*ln(F)*b*c^4*d^4*x^4+80*ln(
F)^3*b^3*c*d^3*x^3+6720*ln(F)*b*c^5*d^3*x^3+120*ln(F)^3*b^3*c^2*d^2*x^2+33
60*ln(F)*b*c^6*d^2*x^2-10*ln(F)^4*b^4*c*d*x+80*ln(F)^3*b^3*c^3*d*x+960*ln(
F)*b*c^7*d*x)/b^6/ln(F)^6/d/(d*x+c)^10*F^((a*d^2*x^2+2*a*c*d*x+a*c^2+b)/(d
*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(111) = 222$.

Time = 0.11 (sec) , antiderivative size = 583, normalized size of antiderivative = 5.16

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx = \frac{(120\,d^{10}x^{10} + 1200\,cd^9x^9 + 5400\,c^2d^8x^8 + 14400\,c^3d^7x^7 + 25200\,c^4d^6x^6 + 30240\,c^5d^5x^5 + 25200\,c^6d^4x^4 +}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^13,x, algorithm="fricas")
```

output
```
1/2*(120*d^10*x^10 + 1200*c*d^9*x^9 + 5400*c^2*d^8*x^8 + 14400*c^3*d^7*x^7
 + 25200*c^4*d^6*x^6 + 30240*c^5*d^5*x^5 + 25200*c^6*d^4*x^4 + 14400*c^7*d
^3*x^3 + 5400*c^8*d^2*x^2 + 1200*c^9*d*x + 120*c^10 - b^5*log(F)^5 + 5*(b^
4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2)*log(F)^4 - 20*(b^3*d^4*x^4 + 4*b^3*c*d^
3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*log(F)^3 + 60*(b^2*d^
6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2
*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*log(F)^2 - 120*(b*d^8*x^8 + 8*b*c*
d^7*x^7 + 28*b*c^2*d^6*x^6 + 56*b*c^3*d^5*x^5 + 70*b*c^4*d^4*x^4 + 56*b*c^
5*d^3*x^3 + 28*b*c^6*d^2*x^2 + 8*b*c^7*d*x + b*c^8)*log(F))*F^((a*d^2*x^2
+ 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))/((b^6*d^11*x^10 + 10*b
^6*c*d^10*x^9 + 45*b^6*c^2*d^9*x^8 + 120*b^6*c^3*d^8*x^7 + 210*b^6*c^4*d^7
*x^6 + 252*b^6*c^5*d^6*x^5 + 210*b^6*c^6*d^5*x^4 + 120*b^6*c^7*d^4*x^3 + 4
5*b^6*c^8*d^3*x^2 + 10*b^6*c^9*d^2*x + b^6*c^10*d)*log(F)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(110) = 220$.

Time = 0.41 (sec) , antiderivative size = 745, normalized size of antiderivative = 6.59

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx = \text{Too large to display}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**13,x)
```

output
```
F**(a + b/(c + d*x)**2)*(-b**5*log(F)**5 + 5*b**4*c**2*log(F)**4 + 10*b**4
*c*d*x*log(F)**4 + 5*b**4*d**2*x**2*log(F)**4 - 20*b**3*c**4*log(F)**3 - 8
0*b**3*c**3*d*x*log(F)**3 - 120*b**3*c**2*d**2*x**2*log(F)**3 - 80*b**3*c*
d**3*x**3*log(F)**3 - 20*b**3*d**4*x**4*log(F)**3 + 60*b**2*c**6*log(F)**2
 + 360*b**2*c**5*d*x*log(F)**2 + 900*b**2*c**4*d**2*x**2*log(F)**2 + 1200*
b**2*c**3*d**3*x**3*log(F)**2 + 900*b**2*c**2*d**4*x**4*log(F)**2 + 360*b*
*2*c*d**5*x**5*log(F)**2 + 60*b**2*d**6*x**6*log(F)**2 - 120*b*c**8*log(F)
 - 960*b*c**7*d*x*log(F) - 3360*b*c**6*d**2*x**2*log(F) - 6720*b*c**5*d**3
*x**3*log(F) - 8400*b*c**4*d**4*x**4*log(F) - 6720*b*c**3*d**5*x**5*log(F)
 - 3360*b*c**2*d**6*x**6*log(F) - 960*b*c*d**7*x**7*log(F) - 120*b*d**8*x*
*8*log(F) + 120*c**10 + 1200*c**9*d*x + 5400*c**8*d**2*x**2 + 14400*c**7*d
**3*x**3 + 25200*c**6*d**4*x**4 + 30240*c**5*d**5*x**5 + 25200*c**4*d**6*x
**6 + 14400*c**3*d**7*x**7 + 5400*c**2*d**8*x**8 + 1200*c*d**9*x**9 + 120*
d**10*x**10)/(2*b**6*c**10*d*log(F)**6 + 20*b**6*c**9*d**2*x*log(F)**6 + 9
0*b**6*c**8*d**3*x**2*log(F)**6 + 240*b**6*c**7*d**4*x**3*log(F)**6 + 420*
b**6*c**6*d**5*x**4*log(F)**6 + 504*b**6*c**5*d**6*x**5*log(F)**6 + 420*b*
*6*c**4*d**7*x**6*log(F)**6 + 240*b**6*c**3*d**8*x**7*log(F)**6 + 90*b**6*
c**2*d**9*x**8*log(F)**6 + 20*b**6*c*d**10*x**9*log(F)**6 + 2*b**6*d**11*x
**10*log(F)**6)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(111) = 222$.

Time = 0.06 (sec) , antiderivative size = 740, normalized size of antiderivative = 6.55

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^13,x, algorithm="maxima")
```

output
```
1/2*(120*F^a*d^10*x^10 + 1200*F^a*c*d^9*x^9 + 120*F^a*c^10 - 120*F^a*b*c^8
*log(F) + 60*F^a*b^2*c^6*log(F)^2 - 20*F^a*b^3*c^4*log(F)^3 + 5*F^a*b^4*c^
2*log(F)^4 - F^a*b^5*log(F)^5 + 120*(45*F^a*c^2*d^8 - F^a*b*d^8*log(F))*x^
8 + 960*(15*F^a*c^3*d^7 - F^a*b*c*d^7*log(F))*x^7 + 60*(420*F^a*c^4*d^6 -
56*F^a*b*c^2*d^6*log(F) + F^a*b^2*d^6*log(F)^2)*x^6 + 120*(252*F^a*c^5*d^5
 - 56*F^a*b*c^3*d^5*log(F) + 3*F^a*b^2*c*d^5*log(F)^2)*x^5 + 20*(1260*F^a*
c^6*d^4 - 420*F^a*b*c^4*d^4*log(F) + 45*F^a*b^2*c^2*d^4*log(F)^2 - F^a*b^3
*d^4*log(F)^3)*x^4 + 80*(180*F^a*c^7*d^3 - 84*F^a*b*c^5*d^3*log(F) + 15*F^
a*b^2*c^3*d^3*log(F)^2 - F^a*b^3*c*d^3*log(F)^3)*x^3 + 5*(1080*F^a*c^8*d^2
 - 672*F^a*b*c^6*d^2*log(F) + 180*F^a*b^2*c^4*d^2*log(F)^2 - 24*F^a*b^3*c^
2*d^2*log(F)^3 + F^a*b^4*d^2*log(F)^4)*x^2 + 10*(120*F^a*c^9*d - 96*F^a*b*
c^7*d*log(F) + 36*F^a*b^2*c^5*d*log(F)^2 - 8*F^a*b^3*c^3*d*log(F)^3 + F^a*
b^4*c*d*log(F)^4)*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(b^6*d^11*x^10*log(F)
^6 + 10*b^6*c*d^10*x^9*log(F)^6 + 45*b^6*c^2*d^9*x^8*log(F)^6 + 120*b^6*c^
3*d^8*x^7*log(F)^6 + 210*b^6*c^4*d^7*x^6*log(F)^6 + 252*b^6*c^5*d^6*x^5*lo
g(F)^6 + 210*b^6*c^6*d^5*x^4*log(F)^6 + 120*b^6*c^7*d^4*x^3*log(F)^6 + 45*
b^6*c^8*d^3*x^2*log(F)^6 + 10*b^6*c^9*d^2*x*log(F)^6 + b^6*c^10*d*log(F)^6
)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{13}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^13,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^13, x)
```

Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 583, normalized size of antiderivative = 5.16

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx = \frac{F^a\,F^{\frac{b}{c^2+2\,c\,d\,x+d^2\,x^2}}\left(\frac{60\,x^{10}}{b^6\,d\ln(F)^6}-\frac{b^5\ln(F)^5-5\,b^4\,c^2\ln(F)^4+20\,b^3\,c^4\ln(F)^3-60\,b^2\,c^6\ln(F)^2+120\,b\,c^8\ln(F)-120\,c^{10}}{2\,b^6\,d^{11}\ln(F)^6}+\frac{600\,c\,x^9}{b^6\,d^2\ln(F)^6}\right.}{}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^13,x)
```

output
```
(F^a*F^(b/(c^2 + d^2*x^2 + 2*c*d*x))*((60*x^10)/(b^6*d*log(F)^6) - (b^5*lo
g(F)^5 - 120*c^10 + 120*b*c^8*log(F) - 60*b^2*c^6*log(F)^2 + 20*b^3*c^4*lo
g(F)^3 - 5*b^4*c^2*log(F)^4)/(2*b^6*d^11*log(F)^6) + (600*c*x^9)/(b^6*d^2*
log(F)^6) + (5*x^2*(b^4*log(F)^4 + 1080*c^8 - 672*b*c^6*log(F) + 180*b^2*c
^4*log(F)^2 - 24*b^3*c^2*log(F)^3))/(2*b^6*d^9*log(F)^6) - (10*x^4*(b^3*lo
g(F)^3 - 1260*c^6 + 420*b*c^4*log(F) - 45*b^2*c^2*log(F)^2))/(b^6*d^7*log(
F)^6) + (30*x^6*(b^2*log(F)^2 + 420*c^4 - 56*b*c^2*log(F)))/(b^6*d^5*log(F
)^6) - (60*x^8*(b*log(F) - 45*c^2))/(b^6*d^3*log(F)^6) - (40*c*x^3*(b^3*lo
g(F)^3 - 180*c^6 + 84*b*c^4*log(F) - 15*b^2*c^2*log(F)^2))/(b^6*d^8*log(F)
^6) + (60*c*x^5*(3*b^2*log(F)^2 + 252*c^4 - 56*b*c^2*log(F)))/(b^6*d^6*log
(F)^6) - (480*c*x^7*(b*log(F) - 15*c^2))/(b^6*d^4*log(F)^6) + (5*c*x*(b^4*
log(F)^4 + 120*c^8 - 96*b*c^6*log(F) + 36*b^2*c^4*log(F)^2 - 8*b^3*c^2*log
(F)^3))/(b^6*d^10*log(F)^6)))/(x^10 + c^10/d^10 + (10*c*x^9)/d + (10*c^9*x
)/d^9 + (45*c^2*x^8)/d^2 + (120*c^3*x^7)/d^3 + (210*c^4*x^6)/d^4 + (252*c^
5*x^5)/d^5 + (210*c^6*x^4)/d^6 + (120*c^7*x^3)/d^7 + (45*c^8*x^2)/d^8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 612, normalized size of antiderivative = 5.42

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{13}}\,dx = \frac{f^{\frac{a\,d^2x^2+2acdx+a\,c^2+b}{d^2x^2+2cdx+c^2}}\left(10\log(f)^4\,b^4cdx-80\log(f)^3\,b^3c^3dx-120\log(f)^3\,b^3c^2d^2x^2+5\log(f)^4\,b^4c^2-20\log(f)^3\right.}{}$$

input

```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^13,x)
```

output

```
(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*
( - log(f)**5*b**5 + 5*log(f)**4*b**4*c**2 + 10*log(f)**4*b**4*c*d*x + 5*l
og(f)**4*b**4*d**2*x**2 - 20*log(f)**3*b**3*c**4 - 80*log(f)**3*b**3*c**3*
d*x - 120*log(f)**3*b**3*c**2*d**2*x**2 - 80*log(f)**3*b**3*c*d**3*x**3 -
20*log(f)**3*b**3*d**4*x**4 + 60*log(f)**2*b**2*c**6 + 360*log(f)**2*b**2*
c**5*d*x + 900*log(f)**2*b**2*c**4*d**2*x**2 + 1200*log(f)**2*b**2*c**3*d*
*3*x**3 + 900*log(f)**2*b**2*c**2*d**4*x**4 + 360*log(f)**2*b**2*c*d**5*x*
*5 + 60*log(f)**2*b**2*d**6*x**6 - 120*log(f)*b*c**8 - 960*log(f)*b*c**7*d
*x - 3360*log(f)*b*c**6*d**2*x**2 - 6720*log(f)*b*c**5*d**3*x**3 - 8400*lo
g(f)*b*c**4*d**4*x**4 - 6720*log(f)*b*c**3*d**5*x**5 - 3360*log(f)*b*c**2*
d**6*x**6 - 960*log(f)*b*c*d**7*x**7 - 120*log(f)*b*d**8*x**8 + 120*c**10
+ 1200*c**9*d*x + 5400*c**8*d**2*x**2 + 14400*c**7*d**3*x**3 + 25200*c**6*
d**4*x**4 + 30240*c**5*d**5*x**5 + 25200*c**4*d**6*x**6 + 14400*c**3*d**7*
x**7 + 5400*c**2*d**8*x**8 + 1200*c*d**9*x**9 + 120*d**10*x**10))/(2*log(f
)**6*b**6*d*(c**10 + 10*c**9*d*x + 45*c**8*d**2*x**2 + 120*c**7*d**3*x**3
+ 210*c**6*d**4*x**4 + 252*c**5*d**5*x**5 + 210*c**4*d**6*x**6 + 120*c**3*
d**7*x**7 + 45*c**2*d**8*x**8 + 10*c*d**9*x**9 + d**10*x**10))
```

3.261 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx=\frac{F^a(c+dx)^{11}\Gamma\left(-\frac{11}{2},-\frac{b\log(F)}{(c+dx)^2}\right)\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{11/2}}{2d}$$

output
```
1/2*F^a*(d*x+c)^11*(64/10395*Pi^(1/2)*erfc((-b*ln(F)/(d*x+c)^2)^(1/2))-64/
10395/(-b*ln(F)/(d*x+c)^2)^(1/2)*exp(b*ln(F)/(d*x+c)^2)+32/10395/(-b*ln(F)
/(d*x+c)^2)^(3/2)*exp(b*ln(F)/(d*x+c)^2)-16/3465/(-b*ln(F)/(d*x+c)^2)^(5/2
)*exp(b*ln(F)/(d*x+c)^2)+8/693/(-b*ln(F)/(d*x+c)^2)^(7/2)*exp(b*ln(F)/(d*x
+c)^2)-4/99/(-b*ln(F)/(d*x+c)^2)^(9/2)*exp(b*ln(F)/(d*x+c)^2)+2/11/(-b*ln(
F)/(d*x+c)^2)^(11/2)*exp(b*ln(F)/(d*x+c)^2))*(-b*ln(F)/(d*x+c)^2)^(11/2)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx=\frac{F^a(c+dx)^{11}\Gamma\left(-\frac{11}{2},-\frac{b\log(F)}{(c+dx)^2}\right)\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{11/2}}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^10,x]
```

output
```
(F^a*(c + d*x)^11*Gamma[-11/2, -((b*Log[F])/(c + d*x)^2)]*(-((b*Log[F])/(c
 + d*x)^2))^(11/2))/(2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{10}F^{a+\frac{b}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \ 2648$$

$$\frac{F^a(c+dx)^{11}\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{11/2}\Gamma\left(-\frac{11}{2},-\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^10,x]
```

output

```
(F^a*(c + d*x)^11*Gamma[-11/2, -((b*Log[F])/(c + d*x)^2)]*(-((b*Log[F])/(c
 + d*x)^2))^(11/2))/(2*d)
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1172 vs. 2(218) = 436.

Time = 4.26 (sec) , antiderivative size = 1173, normalized size of antiderivative = 23.94

method	result	size
risch	Expression too large to display	1173

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^10,x,method=_RETURNVERBOSE)
```

output
```
1/11*F^a*d^10*F^(b/(d*x+c)^2)*x^11+1/11*F^a/d*F^(b/(d*x+c)^2)*c^11+F^a*F^(
b/(d*x+c)^2)*c^10*x+2/99*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c^9+4/693*F^a/d*b^2
*ln(F)^2*F^(b/(d*x+c)^2)*c^7+8/3465*F^a/d*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^5+
16/10395*F^a/d*b^4*ln(F)^4*F^(b/(d*x+c)^2)*c^3+32/10395*F^a/d*b^5*ln(F)^5*
F^(b/(d*x+c)^2)*c+4/693*F^a*d^6*b^2*ln(F)^2*F^(b/(d*x+c)^2)*x^7+8/3465*F^a
*d^4*b^3*ln(F)^3*F^(b/(d*x+c)^2)*x^5+16/10395*F^a*d^2*b^4*ln(F)^4*F^(b/(d*
x+c)^2)*x^3+2/99*F^a*d^8*b*ln(F)*F^(b/(d*x+c)^2)*x^9+4/99*F^a*b^2*ln(F)^2*
F^(b/(d*x+c)^2)*c^6*x+8/693*F^a*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^4*x+16/3465*
F^a*b^4*ln(F)^4*F^(b/(d*x+c)^2)*c^2*x+2/11*F^a*b*ln(F)*F^(b/(d*x+c)^2)*c^8
*x+4/99*F^a*d^5*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c*x^6+4/33*F^a*d^4*b^2*ln(F)^2
*F^(b/(d*x+c)^2)*c^2*x^5+20/99*F^a*d^3*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^3*x^4
+20/99*F^a*d^2*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^4*x^3+4/33*F^a*d*b^2*ln(F)^2*
F^(b/(d*x+c)^2)*c^5*x^2+8/693*F^a*d^3*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c*x^4+16
/693*F^a*d^2*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^2*x^3+16/693*F^a*d*b^3*ln(F)^3*
F^(b/(d*x+c)^2)*c^3*x^2+16/3465*F^a*d*b^4*ln(F)^4*F^(b/(d*x+c)^2)*c*x^2+2/
11*F^a*d^7*b*ln(F)*F^(b/(d*x+c)^2)*c*x^8+8/11*F^a*d^6*b*ln(F)*F^(b/(d*x+c)
^2)*c^2*x^7-32/10395*F^a/d*b^6*ln(F)^6*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*l
n(F))^(1/2)/(d*x+c))+56/33*F^a*d^5*b*ln(F)*F^(b/(d*x+c)^2)*c^3*x^6+28/11*F
^a*d^4*b*ln(F)*F^(b/(d*x+c)^2)*c^4*x^5+28/11*F^a*d^3*b*ln(F)*F^(b/(d*x+c)^
2)*c^5*x^4+56/33*F^a*d^2*b*ln(F)*F^(b/(d*x+c)^2)*c^6*x^3+8/11*F^a*d*b*l...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(212) = 424$.

Time = 0.09 (sec) , antiderivative size = 561, normalized size of antiderivative = 11.45

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx$$

$$= \frac{32\sqrt{\pi}F^a b^5 d\sqrt{-\frac{b\log(F)}{d^2}}\,\mathrm{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)\log(F)^5 + \left(945\,d^{11}x^{11} + 10395\,cd^{10}x^{10} + 51975\,c^2d^9x^9 + 15592\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^10,x, algorithm="fricas")
```

output
```
1/10395*(32*sqrt(pi)*F^a*b^5*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^
2)/(d*x + c))*log(F)^5 + (945*d^11*x^11 + 10395*c*d^10*x^10 + 51975*c^2*d^
9*x^9 + 155925*c^3*d^8*x^8 + 311850*c^4*d^7*x^7 + 436590*c^5*d^6*x^6 + 436
590*c^6*d^5*x^5 + 311850*c^7*d^4*x^4 + 155925*c^8*d^3*x^3 + 51975*c^9*d^2*
x^2 + 10395*c^10*d*x + 945*c^11 + 32*(b^5*d*x + b^5*c)*log(F)^5 + 16*(b^4*
d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*log(F)^4 + 24*(b^3*d^
5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d^2*x^2 + 5*b^3*
c^4*d*x + b^3*c^5)*log(F)^3 + 60*(b^2*d^7*x^7 + 7*b^2*c*d^6*x^6 + 21*b^2*c
^2*d^5*x^5 + 35*b^2*c^3*d^4*x^4 + 35*b^2*c^4*d^3*x^3 + 21*b^2*c^5*d^2*x^2
+ 7*b^2*c^6*d*x + b^2*c^7)*log(F)^2 + 210*(b*d^9*x^9 + 9*b*c*d^8*x^8 + 36*
b*c^2*d^7*x^7 + 84*b*c^3*d^6*x^6 + 126*b*c^4*d^5*x^5 + 126*b*c^5*d^4*x^4 +
 84*b*c^6*d^3*x^3 + 36*b*c^7*d^2*x^2 + 9*b*c^8*d*x + b*c^9)*log(F))*F^((a*
d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx = \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**10,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**10, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx = \int (dx+c)^{10}F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^10,x, algorithm="maxima")
```

output
```
1/10395*(945*F^a*d^10*x^11 + 10395*F^a*c*d^9*x^10 + 105*(495*F^a*c^2*d^8 +
 2*F^a*b*d^8*log(F))*x^9 + 945*(165*F^a*c^3*d^7 + 2*F^a*b*c*d^7*log(F))*x^
8 + 30*(10395*F^a*c^4*d^6 + 252*F^a*b*c^2*d^6*log(F) + 2*F^a*b^2*d^6*log(F
)^2)*x^7 + 210*(2079*F^a*c^5*d^5 + 84*F^a*b*c^3*d^5*log(F) + 2*F^a*b^2*c*d
^5*log(F)^2)*x^6 + 6*(72765*F^a*c^6*d^4 + 4410*F^a*b*c^4*d^4*log(F) + 210*
F^a*b^2*c^2*d^4*log(F)^2 + 4*F^a*b^3*d^4*log(F)^3)*x^5 + 30*(10395*F^a*c^7
*d^3 + 882*F^a*b*c^5*d^3*log(F) + 70*F^a*b^2*c^3*d^3*log(F)^2 + 4*F^a*b^3*
c*d^3*log(F)^3)*x^4 + (155925*F^a*c^8*d^2 + 17640*F^a*b*c^6*d^2*log(F) + 2
100*F^a*b^2*c^4*d^2*log(F)^2 + 240*F^a*b^3*c^2*d^2*log(F)^3 + 16*F^a*b^4*d
^2*log(F)^4)*x^3 + 3*(17325*F^a*c^9*d + 2520*F^a*b*c^7*d*log(F) + 420*F^a*
b^2*c^5*d*log(F)^2 + 80*F^a*b^3*c^3*d*log(F)^3 + 16*F^a*b^4*c*d*log(F)^4)*
x^2 + (10395*F^a*c^10 + 1890*F^a*b*c^8*log(F) + 420*F^a*b^2*c^6*log(F)^2 +
 120*F^a*b^3*c^4*log(F)^3 + 48*F^a*b^4*c^2*log(F)^4 + 32*F^a*b^5*log(F)^5)
*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(2/10395*(32*F^a*b^6*d*x*lo
g(F)^6 - 945*F^a*b*c^11*log(F) - 210*F^a*b^2*c^9*log(F)^2 - 60*F^a*b^3*c^7
*log(F)^3 - 24*F^a*b^4*c^5*log(F)^4 - 16*F^a*b^5*c^3*log(F)^5)*F^(b/(d^2*x
^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx = \int (dx+c)^{10}F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^10,x, algorithm="giac")
```

output
```
integrate((d*x + c)^10*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 265, normalized size of antiderivative = 5.41

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx = \frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,(c+d\,x)^{11}}{11\,d} - \frac{32\,F^a\,\sqrt{\pi}\,(c+d\,x)^{11}\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)^{11/2}}{10395\,d} + \frac{4\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^2\ln(F)^2\,(c+d\,x)^7}{693\,d} + \frac{8\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^3\ln(F)^3\,(c+d\,x)^5}{3465\,d} + \frac{16\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^4\ln(F)^4\,(c+d\,x)^3}{10395\,d} + \frac{2\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b\,\ln(F)\,(c+d\,x)^9}{99\,d} + \frac{32\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^5\ln(F)^5\,(c+d\,x)}{10395\,d} + \frac{32\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln(F)}{(c+d\,x)^2}}\right)\,(c+d\,x)^{11}\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)^{11/2}}{10395\,d}$$

input

```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^10,x)
```

output

```
(F^a*F^(b/(c + d*x)^2)*(c + d*x)^11)/(11*d) - (32*F^a*pi^(1/2)*(c + d*x)^1
1*(-(b*log(F))/(c + d*x)^2)^(11/2))/(10395*d) + (4*F^a*F^(b/(c + d*x)^2)*b
^2*log(F)^2*(c + d*x)^7)/(693*d) + (8*F^a*F^(b/(c + d*x)^2)*b^3*log(F)^3*(
c + d*x)^5)/(3465*d) + (16*F^a*F^(b/(c + d*x)^2)*b^4*log(F)^4*(c + d*x)^3)
/(10395*d) + (2*F^a*F^(b/(c + d*x)^2)*b*log(F)*(c + d*x)^9)/(99*d) + (32*F
^a*F^(b/(c + d*x)^2)*b^5*log(F)^5*(c + d*x))/(10395*d) + (32*F^a*pi^(1/2)*
erfc((-(b*log(F))/(c + d*x)^2)^(1/2))*(c + d*x)^11*(-(b*log(F))/(c + d*x)^
2)^(11/2))/(10395*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^{10}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^10,x)
```

output
```
(128*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
2))*log(f)**7*b**7*d*x + 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)**6*b**6*c**3 - 64*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**6*b**6*c**2*d*x
 - 320*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x
**2))*log(f)**6*b**6*c*d**2*x**2 - 192*f**((a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**6*b**6*d**3*x**3 - 160*f**((
a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)
**5*b**5*c**5 - 544*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c
*d*x + d**2*x**2))*log(f)**5*b**5*c**4*d*x - 576*f**((a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c**3*d**2*x
**2 - 64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2
*x**2))*log(f)**5*b**5*c**2*d**3*x**3 + 224*f**((a*c**2 + 2*a*c*d*x + a*d*
*2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c*d**4*x**4 + 96
*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*
log(f)**5*b**5*d**5*x**5 + 48*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(
c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**7 + 336*f**((a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**6*
d*x + 1008*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d*
*2*x**2))*log(f)**4*b**4*c**5*d**2*x**2 + 1680*f**((a*c**2 + 2*a*c*d*x ...
```

3.262 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx=\frac{F^a(c+dx)^9\Gamma\left(-\frac{9}{2},-\frac{b\log(F)}{(c+dx)^2}\right)\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{9/2}}{2d}$$

output
```
1/2*F^a*(d*x+c)^9*(-32/945*Pi^(1/2)*erfc((-b*ln(F)/(d*x+c)^2)^(1/2))+32/94
5/(-b*ln(F)/(d*x+c)^2)^(1/2)*exp(b*ln(F)/(d*x+c)^2)-16/945/(-b*ln(F)/(d*x+
c)^2)^(3/2)*exp(b*ln(F)/(d*x+c)^2)+8/315/(-b*ln(F)/(d*x+c)^2)^(5/2)*exp(b*
ln(F)/(d*x+c)^2)-4/63/(-b*ln(F)/(d*x+c)^2)^(7/2)*exp(b*ln(F)/(d*x+c)^2)+2/
9/(-b*ln(F)/(d*x+c)^2)^(9/2)*exp(b*ln(F)/(d*x+c)^2))*(-b*ln(F)/(d*x+c)^2)^
(9/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx=\frac{F^a(c+dx)^9\Gamma\left(-\frac{9}{2},-\frac{b\log(F)}{(c+dx)^2}\right)\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{9/2}}{2d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^8,x]
```

output
```
(F^a*(c + d*x)^9*Gamma[-9/2, -((b*Log[F])/(c + d*x)^2)]*(-((b*Log[F])/(c +
 d*x)^2))^(9/2))/(2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^8 F^{a+\frac{b}{(c+dx)^2}} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{F^a (c+dx)^9 \left(-\frac{b\log(F)}{(c+dx)^2}\right)^{9/2} \Gamma\left(-\frac{9}{2},-\frac{b\log(F)}{(c+dx)^2}\right)}{2d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^8,x]
```

output

```
(F^a*(c + d*x)^9*Gamma[-9/2, -((b*Log[F])/(c + d*x)^2)]*(-((b*Log[F])/(c +
 d*x)^2))^(9/2))/(2*d)
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. 2(190) = 380.

Time = 2.38 (sec) , antiderivative size = 826, normalized size of antiderivative = 16.86

method	result
risch	$\frac{2F^a d^6 b \ln(F) F^{\frac{b}{(dx+c)^2}} x^7}{63} + \frac{2F^a d b \ln(F) F^{\frac{b}{(dx+c)^2}} c^5 x^2}{3} + \frac{4F^a d^3 b^2 \ln(F)^2 F^{\frac{b}{(dx+c)^2}} c x^4}{63} + \frac{8F^a d^2 b^2 \ln(F)^2 F^{\frac{b}{(dx+c)^2}} c^2 x^3}{63}$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
2/63*F^a*d^6*b*ln(F)*F^(b/(d*x+c)^2)*x^7+2/3*F^a*d*b*ln(F)*F^(b/(d*x+c)^2)
*c^5*x^2+4/63*F^a*d^3*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c*x^4+8/63*F^a*d^2*b^2*l
n(F)^2*F^(b/(d*x+c)^2)*c^2*x^3+8/63*F^a*d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^3*
x^2+8/315*F^a*d*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c*x^2-16/945*F^a/d*b^5*ln(F)^5
*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))+2/9*F^a*d^5*b*ln(
F)*F^(b/(d*x+c)^2)*c*x^6+2/3*F^a*d^4*b*ln(F)*F^(b/(d*x+c)^2)*c^2*x^5+10/9*
F^a*d^3*b*ln(F)*F^(b/(d*x+c)^2)*c^3*x^4+10/9*F^a*d^2*b*ln(F)*F^(b/(d*x+c)^
2)*c^4*x^3+8/315*F^a*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^2*x+2/9*F^a*b*ln(F)*F^(
b/(d*x+c)^2)*c^6*x+4/63*F^a*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^4*x+2/63*F^a/d*b
*ln(F)*F^(b/(d*x+c)^2)*c^7+4/315*F^a/d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^5+8/9
45*F^a/d*b^3*ln(F)^3*F^(b/(d*x+c)^2)*c^3+16/945*F^a/d*b^4*ln(F)^4*F^(b/(d*
x+c)^2)*c+1/9*F^a/d*F^(b/(d*x+c)^2)*c^9+1/9*F^a*d^8*F^(b/(d*x+c)^2)*x^9+F^
a*F^(b/(d*x+c)^2)*c^8*x+16/945*F^a*b^4*ln(F)^4*F^(b/(d*x+c)^2)*x+F^a*d^7*F
^(b/(d*x+c)^2)*c*x^8+4*F^a*d^6*F^(b/(d*x+c)^2)*c^2*x^7+28/3*F^a*d^5*F^(b/(
d*x+c)^2)*c^3*x^6+14*F^a*d^4*F^(b/(d*x+c)^2)*c^4*x^5+14*F^a*d^3*F^(b/(d*x+
c)^2)*c^5*x^4+28/3*F^a*d^2*F^(b/(d*x+c)^2)*c^6*x^3+4*F^a*d*F^(b/(d*x+c)^2)
*c^7*x^2+4/315*F^a*d^4*b^2*ln(F)^2*F^(b/(d*x+c)^2)*x^5+8/945*F^a*d^2*b^3*l
n(F)^3*F^(b/(d*x+c)^2)*x^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(185) = 370$.

Time = 0.09 (sec) , antiderivative size = 413, normalized size of antiderivative = 8.43

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx$$
$$= \frac{16\sqrt{\pi}F^a b^4 d\sqrt{-\frac{b\log(F)}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)\log(F)^4 + \left(105\,d^9x^9 + 945\,cd^8x^8 + 3780\,c^2d^7x^7 + 8820\,c^3d^6x^6\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x, algorithm="fricas")
```

output
```
1/945*(16*sqrt(pi)*F^a*b^4*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)
/(d*x + c))*log(F)^4 + (105*d^9*x^9 + 945*c*d^8*x^8 + 3780*c^2*d^7*x^7 + 8
820*c^3*d^6*x^6 + 13230*c^4*d^5*x^5 + 13230*c^5*d^4*x^4 + 8820*c^6*d^3*x^3
 + 3780*c^7*d^2*x^2 + 945*c^8*d*x + 105*c^9 + 16*(b^4*d*x + b^4*c)*log(F)^
4 + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(F)^3 +
 12*(b^2*d^5*x^5 + 5*b^2*c*d^4*x^4 + 10*b^2*c^2*d^3*x^3 + 10*b^2*c^3*d^2*x
^2 + 5*b^2*c^4*d*x + b^2*c^5)*log(F)^2 + 30*(b*d^7*x^7 + 7*b*c*d^6*x^6 + 2
1*b*c^2*d^5*x^5 + 35*b*c^3*d^4*x^4 + 35*b*c^4*d^3*x^3 + 21*b*c^5*d^2*x^2 +
 7*b*c^6*d*x + b*c^7)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*
x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx = \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**8,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**8, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx = \int (dx+c)^8 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/945*(105*F^a*d^8*x^9 + 945*F^a*c*d^7*x^8 + 30*(126*F^a*c^2*d^6 + F^a*b*d
^6*log(F))*x^7 + 210*(42*F^a*c^3*d^5 + F^a*b*c*d^5*log(F))*x^6 + 6*(2205*F
^a*c^4*d^4 + 105*F^a*b*c^2*d^4*log(F) + 2*F^a*b^2*d^4*log(F)^2)*x^5 + 30*(
441*F^a*c^5*d^3 + 35*F^a*b*c^3*d^3*log(F) + 2*F^a*b^2*c*d^3*log(F)^2)*x^4
 + 2*(4410*F^a*c^6*d^2 + 525*F^a*b*c^4*d^2*log(F) + 60*F^a*b^2*c^2*d^2*log(
F)^2 + 4*F^a*b^3*d^2*log(F)^3)*x^3 + 6*(630*F^a*c^7*d + 105*F^a*b*c^5*d*lo
g(F) + 20*F^a*b^2*c^3*d*log(F)^2 + 4*F^a*b^3*c*d*log(F)^3)*x^2 + (945*F^a*
c^8 + 210*F^a*b*c^6*log(F) + 60*F^a*b^2*c^4*log(F)^2 + 24*F^a*b^3*c^2*log(
F)^3 + 16*F^a*b^4*log(F)^4)*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate
(2/945*(16*F^a*b^5*d*x*log(F)^5 - 105*F^a*b*c^9*log(F) - 30*F^a*b^2*c^7*lo
g(F)^2 - 12*F^a*b^3*c^5*log(F)^3 - 8*F^a*b^4*c^3*log(F)^4)*F^(b/(d^2*x^2 +
 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx = \int (dx+c)^8 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x, algorithm="giac")
```

output
```
integrate((d*x + c)^8*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.73

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx = \frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,(c+d\,x)^9}{9\,d} + \frac{16\,F^a\,\sqrt{\pi}\,(c+d\,x)^9\,\left(-\frac{b\ln(F)}{(c+d\,x)^2}\right)^{9/2}}{945\,d} + \frac{4\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^2\ln(F)^2\,(c+d\,x)^5}{315\,d} + \frac{8\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^3\ln(F)^3\,(c+d\,x)^3}{945\,d} + \frac{2\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b\,\ln(F)\,(c+d\,x)^7}{63\,d} + \frac{16\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^4\ln(F)^4\,(c+d\,x)}{945\,d} - \frac{16\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\ln(F)}{(c+d\,x)^2}}\right)\,(c+d\,x)^9\,\left(-\frac{b\ln(F)}{(c+d\,x)^2}\right)^{9/2}}{945\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^8,x)
```

output
```
(F^a*F^(b/(c + d*x)^2)*(c + d*x)^9)/(9*d) + (16*F^a*pi^(1/2)*(c + d*x)^9*(
-(b*log(F))/(c + d*x)^2)^(9/2))/(945*d) + (4*F^a*F^(b/(c + d*x)^2)*b^2*log
(F)^2*(c + d*x)^5)/(315*d) + (8*F^a*F^(b/(c + d*x)^2)*b^3*log(F)^3*(c + d*
x)^3)/(945*d) + (2*F^a*F^(b/(c + d*x)^2)*b*log(F)*(c + d*x)^7)/(63*d) + (1
6*F^a*F^(b/(c + d*x)^2)*b^4*log(F)^4*(c + d*x))/(945*d) - (16*F^a*pi^(1/2)
*erfc((-(b*log(F))/(c + d*x)^2)^(1/2))*(c + d*x)^9*(-(b*log(F))/(c + d*x)^
2)^(9/2))/(945*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^8\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^8,x)
```

output
```
(64*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**6*b**6*d*x + 32*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**
2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c**3 - 32*f**((a*c**2 + 2*a*c*d*x
 + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*c**2*d*x
- 160*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**5*b**5*c*d**2*x**2 - 96*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**5*b**5*d**3*x**3 - 80*f**((a*c
**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4
*b**4*c**5 - 272*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*
x + d**2*x**2))*log(f)**4*b**4*c**4*d*x - 288*f**((a*c**2 + 2*a*c*d*x + a*
d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**3*d**2*x**2
 - 32*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)**4*b**4*c**2*d**3*x**3 + 112*f**((a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c*d**4*x**4 + 48*f*
*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log
(f)**4*b**4*d**5*x**5 + 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**
2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**7 + 168*f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**6*d*x
 + 504*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x
**2))*log(f)**3*b**3*c**5*d**2*x**2 + 840*f**((a*c**2 + 2*a*c*d*x + a*d...
```

3.263 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx=\frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^7}{7d}+\frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^5\log(F)}{35d}+\frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\log^2(F)}{105d}+\frac{8b^3F^{a+\frac{b}{(c+dx)^2}}(c+dx)\log^3(F)}{105d}-\frac{8b^{7/2}F^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\log^{\frac{7}{2}}(F)}{105d}$$

output
```
1/7*F^(a+b/(d*x+c)^2)*(d*x+c)^7/d+2/35*b*F^(a+b/(d*x+c)^2)*(d*x+c)^5*ln(F)
/d+4/105*b^2*F^(a+b/(d*x+c)^2)*(d*x+c)^3*ln(F)^2/d+8/105*b^3*F^(a+b/(d*x+c
)^2)*(d*x+c)*ln(F)^3/d-8/105*b^(7/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)
/(d*x+c))*ln(F)^(7/2)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.66

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx = \frac{F^a\left(-8b^{7/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\log^{\frac{7}{2}}(F)+F^{\frac{b}{(c+dx)^2}}(c+dx)\left(15(c+dx)^6+6b(c+dx)^4\log(F)+4b^2(c+
$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^6,x]
```

output
```
(F^a*(-8*b^(7/2)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)]*Log[F]^(7
/2) + F^(b/(c + d*x)^2)*(c + d*x)*(15*(c + d*x)^6 + 6*b*(c + d*x)^4*Log[F]
 + 4*b^2*(c + d*x)^2*Log[F]^2 + 8*b^3*Log[F]^3)))/(105*d)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2643, 2643, 2643, 2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^6F^{a+\frac{b}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4dx+\frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2dx+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}\right)+\frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}dx+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right)+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}\right)+\frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

$$\Big\downarrow \text{2635}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(2b\log(F)\int\frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx+\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}\right)+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right)\right.$$
$$\frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

$$\Big\downarrow \text{2640}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}-\frac{2b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{d}\right)+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right.\right.$$
$$\frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{7}b\log(F)\left(\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}-\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{d}\right)+\frac{(c+dx)^3F^{a+}}{3d}\right.\right.$$
$$\frac{(c+dx)^7F^{a+\frac{b}{(c+dx)^2}}}{7d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^6,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x)^7)/(7*d) + (2*b*Log[F]*((F^(a + b/(c + d*
x)^2)*(c + d*x)^5)/(5*d) + (2*b*Log[F]*((F^(a + b/(c + d*x)^2)*(c + d*x)^3
)/(3*d) + (2*b*((F^(a + b/(c + d*x)^2)*(c + d*x))/d - (Sqrt[b]*F^a*Sqrt[Pi
]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)]*Sqrt[Log[F]])/d)*Log[F])/3))/5))/
7
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(150) = 300$.

Time = 1.21 (sec) , antiderivative size = 543, normalized size of antiderivative = 3.19

method	result
risch	$\frac{F^a d^6 F^{\frac{b}{(dx+c)^2}} x^7}{7} + F^a d^5 F^{\frac{b}{(dx+c)^2}} c x^6 + 3F^a d^4 F^{\frac{b}{(dx+c)^2}} c^2 x^5 + 5F^a d^3 F^{\frac{b}{(dx+c)^2}} c^3 x^4 + 5F^a d^2 F^{\frac{b}{(dx+c)^2}} c^4$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/7*F^a*d^6*F^(b/(d*x+c)^2)*x^7+F^a*d^5*F^(b/(d*x+c)^2)*c*x^6+3*F^a*d^4*F^
(b/(d*x+c)^2)*c^2*x^5+5*F^a*d^3*F^(b/(d*x+c)^2)*c^3*x^4+5*F^a*d^2*F^(b/(d*
x+c)^2)*c^4*x^3+3*F^a*d*F^(b/(d*x+c)^2)*c^5*x^2+F^a*F^(b/(d*x+c)^2)*c^6*x+
1/7*F^a/d*F^(b/(d*x+c)^2)*c^7+2/35*F^a*d^4*b*ln(F)*F^(b/(d*x+c)^2)*x^5+2/7
*F^a*d^3*b*ln(F)*F^(b/(d*x+c)^2)*c*x^4+4/7*F^a*d^2*b*ln(F)*F^(b/(d*x+c)^2)
*c^2*x^3+4/7*F^a*d*b*ln(F)*F^(b/(d*x+c)^2)*c^3*x^2+2/7*F^a*b*ln(F)*F^(b/(d
*x+c)^2)*c^4*x+2/35*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c^5+4/105*F^a*d^2*b^2*ln
(F)^2*F^(b/(d*x+c)^2)*x^3+4/35*F^a*d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c*x^2+4/3
5*F^a*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c^2*x+4/105*F^a/d*b^2*ln(F)^2*F^(b/(d*x+
c)^2)*c^3+8/105*F^a*b^3*ln(F)^3*F^(b/(d*x+c)^2)*x+8/105*F^a/d*b^3*ln(F)^3*
F^(b/(d*x+c)^2)*c-8/105*F^a/d*b^4*ln(F)^4*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-
b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.72

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx = \frac{8\sqrt{\pi}F^a b^3 d\sqrt{-\frac{b\log(F)}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)\log(F)^3 + \left(15\,d^7x^7 + 105\,cd^6x^6 + 315\,c^2d^5x^5 + 525\,c^3d^4x^4 + 525\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/105*(8*sqrt(pi)*F^a*b^3*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/
(d*x + c))*log(F)^3 + (15*d^7*x^7 + 105*c*d^6*x^6 + 315*c^2*d^5*x^5 + 525*
c^3*d^4*x^4 + 525*c^4*d^3*x^3 + 315*c^5*d^2*x^2 + 105*c^6*d*x + 15*c^7 + 8
*(b^3*d*x + b^3*c)*log(F)^3 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*log(F)^2 + 6*(b*d^5*x^5 + 5*b*c*d^4*x^4 + 10*b*c^2*d^3*x^3
 + 10*b*c^3*d^2*x^2 + 5*b*c^4*d*x + b*c^5)*log(F))*F^((a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx = \int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**6,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**6, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx = \int (dx+c)^6 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/105*(15*F^a*d^6*x^7 + 105*F^a*c*d^5*x^6 + 3*(105*F^a*c^2*d^4 + 2*F^a*b*d
^4*log(F))*x^5 + 15*(35*F^a*c^3*d^3 + 2*F^a*b*c*d^3*log(F))*x^4 + (525*F^a
*c^4*d^2 + 60*F^a*b*c^2*d^2*log(F) + 4*F^a*b^2*d^2*log(F)^2)*x^3 + 3*(105*
F^a*c^5*d + 20*F^a*b*c^3*d*log(F) + 4*F^a*b^2*c*d*log(F)^2)*x^2 + (105*F^a
*c^6 + 30*F^a*b*c^4*log(F) + 12*F^a*b^2*c^2*log(F)^2 + 8*F^a*b^3*log(F)^3)
*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + integrate(2/105*(8*F^a*b^4*d*x*log(F
)^4 - 15*F^a*b*c^7*log(F) - 6*F^a*b^2*c^5*log(F)^2 - 4*F^a*b^3*c^3*log(F)^
3)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^
3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx=\int (dx+c)^6F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x, algorithm="giac")
```

output
```
integrate((d*x + c)^6*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.17

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx=\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,(c+d\,x)^7}{7\,d}-\frac{8\,F^a\,\sqrt{\pi}\,(c+d\,x)^7\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)^{7/2}}{105\,d}+\frac{4\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^2\ln(F)^2\,(c+d\,x)^3}{105\,d}+\frac{2\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b\,\ln(F)\,(c+d\,x)^5}{35\,d}+\frac{8\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^3\ln(F)^3\,(c+d\,x)}{105\,d}+\frac{8\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln(F)}{(c+d\,x)^2}}\right)\,(c+d\,x)^7\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^2}\right)^{7/2}}{105\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^6,x)
```

output
```
(F^a*F^(b/(c + d*x)^2)*(c + d*x)^7)/(7*d) - (8*F^a*pi^(1/2)*(c + d*x)^7*(-
(b*log(F))/(c + d*x)^2)^(7/2))/(105*d) + (4*F^a*F^(b/(c + d*x)^2)*b^2*log(
F)^2*(c + d*x)^3)/(105*d) + (2*F^a*F^(b/(c + d*x)^2)*b*log(F)*(c + d*x)^5)
/(35*d) + (8*F^a*F^(b/(c + d*x)^2)*b^3*log(F)^3*(c + d*x))/(105*d) + (8*F^
a*pi^(1/2)*erfc((-(b*log(F))/(c + d*x)^2)^(1/2))*(c + d*x)^7*(-(b*log(F))/
(c + d*x)^2)^(7/2))/(105*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^6\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^6,x)
```

output
```
(32*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**5*b**5*d*x + 16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**
2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**3 - 16*f**((a*c**2 + 2*a*c*d*x
 + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*c**2*d*x
- 80*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
2))*log(f)**4*b**4*c*d**2*x**2 - 48*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**4*b**4*d**3*x**3 - 40*f**((a*c*
*2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*
b**3*c**5 - 136*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
 + d**2*x**2))*log(f)**3*b**3*c**4*d*x - 144*f**((a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**3*d**2*x**2
- 16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**
2))*log(f)**3*b**3*c**2*d**3*x**3 + 56*f**((a*c**2 + 2*a*c*d*x + a*d**2*x*
*2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c*d**4*x**4 + 24*f**(
(a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f
)**3*b**3*d**5*x**5 + 12*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
+ 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**7 + 84*f**((a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**6*d*x +
252*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**2*b**2*c**5*d**2*x**2 + 420*f**((a*c**2 + 2*a*c*d*x + a*d**2...
```

3.264 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 136

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx = \frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^5}{5d} + \frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)^3\log(F)}{15d} + \frac{4b^2F^{a+\frac{b}{(c+dx)^2}}(c+dx)\log^2(F)}{15d} - \frac{4b^{5/2}F^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\log^{\frac{5}{2}}(F)}{15d}$$

output
```
1/5*F^(a+b/(d*x+c)^2)*(d*x+c)^5/d+2/15*b*F^(a+b/(d*x+c)^2)*(d*x+c)^3*ln(F)
/d+4/15*b^2*F^(a+b/(d*x+c)^2)*(d*x+c)*ln(F)^2/d-4/15*b^(5/2)*F^a*Pi^(1/2)*
erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))*ln(F)^(5/2)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx = \frac{F^a\left(-4b^{5/2}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\log^{\frac{5}{2}}(F)+F^{\frac{b}{(c+dx)^2}}(c+dx)\left(3(c+dx)^4+2b(c+dx)^2\log(F)+4b^2\log^2(F\right.\right.}{15d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^4,x]
```

output
```
(F^a*(-4*b^(5/2)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)]*Log[F]^(5
/2) + F^(b/(c + d*x)^2)*(c + d*x)*(3*(c + d*x)^4 + 2*b*(c + d*x)^2*Log[F]
+ 4*b^2*Log[F]^2)))/(15*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {2643, 2643, 2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 F^{a+\frac{b}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2dx+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}dx+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right)+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}$$

$$\Big\downarrow \text{2635}$$

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(2b\log(F)\int\frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx+\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}\right)+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right)+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}$$

$\Big\downarrow$ 2640

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}-\frac{2b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{d}\right)+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right)+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}$$

$\Big\downarrow$ 2633

$$\frac{2}{5}b\log(F)\left(\frac{2}{3}b\log(F)\left(\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}-\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{d}\right)+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^2}}}{3d}\right)+\frac{(c+dx)^5F^{a+\frac{b}{(c+dx)^2}}}{5d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^4,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x)^5)/(5*d) + (2*b*Log[F]*((F^(a + b/(c + d*
x)^2)*(c + d*x)^3)/(3*d) + (2*b*((F^(a + b/(c + d*x)^2)*(c + d*x))/d - (Sq
rt[b]*F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)]*Sqrt[Log[F]])/d)
*Log[F])/3))/5
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(118) = 236$.

Time = 0.61 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.38

method	result
risch	$\frac{F^a d^4 F^{\frac{b}{(dx+c)^2}} x^5}{5} + F^a d^3 F^{\frac{b}{(dx+c)^2}} c x^4 + 2F^a d^2 F^{\frac{b}{(dx+c)^2}} c^2 x^3 + 2F^a d F^{\frac{b}{(dx+c)^2}} c^3 x^2 + F^a F^{\frac{b}{(dx+c)^2}} c^4 x +$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*F^a*d^4*F^(b/(d*x+c)^2)*x^5+F^a*d^3*F^(b/(d*x+c)^2)*c*x^4+2*F^a*d^2*F^
(b/(d*x+c)^2)*c^2*x^3+2*F^a*d*F^(b/(d*x+c)^2)*c^3*x^2+F^a*F^(b/(d*x+c)^2)*
c^4*x+1/5*F^a/d*F^(b/(d*x+c)^2)*c^5+2/15*F^a*d^2*b*ln(F)*F^(b/(d*x+c)^2)*x
^3+2/5*F^a*d*b*ln(F)*F^(b/(d*x+c)^2)*c*x^2+2/5*F^a*b*ln(F)*F^(b/(d*x+c)^2)
*c^2*x+2/15*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c^3+4/15*F^a*b^2*ln(F)^2*F^(b/(d
*x+c)^2)*x+4/15*F^a/d*b^2*ln(F)^2*F^(b/(d*x+c)^2)*c-4/15*F^a/d*b^3*ln(F)^3
*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.48

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx$$

$$= \frac{4\sqrt{\pi}F^a b^2 d\sqrt{-\frac{b\log(F)}{d^2}}\,\mathrm{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)\log(F)^2 + \left(3\,d^5x^5 + 15\,cd^4x^4 + 30\,c^2d^3x^3 + 30\,c^3d^2x^2 + 15\,c^4dx\right.}{15\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/15*(4*sqrt(pi)*F^a*b^2*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(
d*x + c))*log(F)^2 + (3*d^5*x^5 + 15*c*d^4*x^4 + 30*c^2*d^3*x^3 + 30*c^3*d
^2*x^2 + 15*c^4*d*x + 3*c^5 + 4*(b^2*d*x + b^2*c)*log(F)^2 + 2*(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x +
 a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx = \int F^{a+\frac{b}{(c+dx)^2}}\left(c+dx\right)^4\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**4,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**4, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx = \int (dx+c)^4 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/15*(3*F^a*d^4*x^5 + 15*F^a*c*d^3*x^4 + 2*(15*F^a*c^2*d^2 + F^a*b*d^2*log
(F))*x^3 + 6*(5*F^a*c^3*d + F^a*b*c*d*log(F))*x^2 + (15*F^a*c^4 + 6*F^a*b*
c^2*log(F) + 4*F^a*b^2*log(F)^2)*x)*F^(b/(d^2*x^2 + 2*c*d*x + c^2)) + inte
grate(2/15*(4*F^a*b^3*d*x*log(F)^3 - 3*F^a*b*c^5*log(F) - 2*F^a*b^2*c^3*lo
g(F)^2)*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx = \int (dx+c)^4 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.22

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx=\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,{\left(c+d\,x\right)}^5}{5\,d}+\frac{4\,F^a\,\sqrt{\pi}\,{\left(c+d\,x\right)}^5\,{\left(-\frac{b\,\ln\left(F\right)}{{\left(c+d\,x\right)}^2}\right)}^{5/2}}{15\,d}+\frac{2\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b\,\ln\left(F\right)\,{\left(c+d\,x\right)}^3}{15\,d}+\frac{4\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^2\,{\ln\left(F\right)}^2\,\left(c+d\,x\right)}{15\,d}-\frac{4\,F^a\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-\frac{b\,\ln\left(F\right)}{{\left(c+d\,x\right)}^2}}\right)\,{\left(c+d\,x\right)}^5\,{\left(-\frac{b\,\ln\left(F\right)}{{\left(c+d\,x\right)}^2}\right)}^{5/2}}{15\,d}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^4,x)
```

output
```
(F^a*F^(b/(c + d*x)^2)*(c + d*x)^5)/(5*d) + (4*F^a*pi^(1/2)*(c + d*x)^5*(-
(b*log(F))/(c + d*x)^2)^(5/2))/(15*d) + (2*F^a*F^(b/(c + d*x)^2)*b*log(F)*
(c + d*x)^3)/(15*d) + (4*F^a*F^(b/(c + d*x)^2)*b^2*log(F)^2*(c + d*x))/(15
*d) - (4*F^a*pi^(1/2)*erfc((-(b*log(F))/(c + d*x)^2)^(1/2))*(c + d*x)^5*(-
(b*log(F))/(c + d*x)^2)^(5/2))/(15*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^4\,dx=\text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^4,x)
```

output
```
(16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2
))*log(f)**4*b**4*d*x + 8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**3 - 8*f**((a*c**2 + 2*a*c*d*x +
 a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*c**2*d*x -
40*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)**3*b**3*c*d**2*x**2 - 24*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**3*b**3*d**3*x**3 - 20*f**((a*c**2
 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b*
*2*c**5 - 68*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
d**2*x**2))*log(f)**2*b**2*c**4*d*x - 72*f**((a*c**2 + 2*a*c*d*x + a*d**2*
x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**3*d**2*x**2 - 8*
f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*l
og(f)**2*b**2*c**2*d**3*x**3 + 28*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c*d**4*x**4 + 12*f**((a*c*
*2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*
b**2*d**5*x**5 + 6*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*
d*x + d**2*x**2))*log(f)*b*c**7 + 42*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**6*d*x + 126*f**((a*c**2 +
2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**5*d
**2*x**2 + 210*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d...
```

3.265 $\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx=\frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)^3}{3d}+\frac{2bF^{a+\frac{b}{(c+dx)^2}}(c+dx)\log(F)}{3d}-\frac{2b^{3/2}F^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\log^{\frac{3}{2}}(F)}{3d}$$

output
```
1/3*F^(a+b/(d*x+c)^2)*(d*x+c)^3/d+2/3*b*F^(a+b/(d*x+c)^2)*(d*x+c)*ln(F)/d-
2/3*b^(3/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))*ln(F)^(3/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.77

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx=\frac{F^a\left(-2b^{3/2}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\log^{\frac{3}{2}}(F)+F^{\frac{b}{(c+dx)^2}}(c+dx)\left((c+dx)^2+2b\log(F)\right)\right)}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)*(c + d*x)^2,x]
```

output
```
(F^a*(-2*b^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)]*Log[F]^(3
/2) + F^(b/(c + d*x)^2)*(c + d*x)*((c + d*x)^2 + 2*b*Log[F])))/(3*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2643, 2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 F^{a+\frac{b}{(c+dx)^2}} \, dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{2}{3} b \log(F) \int F^{a+\frac{b}{(c+dx)^2}} dx + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{3d}$$

$$\Big\downarrow \text{2635}$$

$$\frac{2}{3} b \log(F) \left(2b \log(F) \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx + \frac{(c+dx) F^{a+\frac{b}{(c+dx)^2}}}{d}\right) + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{3d}$$

$$\Big\downarrow \text{2640}$$

$$\frac{2}{3} b \log(F) \left(\frac{(c+dx) F^{a+\frac{b}{(c+dx)^2}}}{d} - \frac{2b \log(F) \int F^{a+\frac{b}{(c+dx)^2}} d\frac{1}{c+dx}}{d}\right) + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{3d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{3} b \log(F) \left(\frac{(c+dx) F^{a+\frac{b}{(c+dx)^2}}}{d} - \frac{\sqrt{\pi}\sqrt{b} F^a \sqrt{\log(F)} \text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{d}\right) + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^2}}}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^2)*(c + d*x)^2,x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x)^3)/(3*d) + (2*b*((F^(a + b/(c + d*x)^2)*(
c + d*x))/d - (Sqrt[b]*F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)]
*Sqrt[Log[F]])/d)*Log[F])/3
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.66

method	result
risch	$\frac{F^{a} d^{2} F^{\frac{b}{(dx+c)^{2}}} x^{3}}{3}+F^{a} d\, F^{\frac{b}{(dx+c)^{2}}} c\, x^{2}+F^{a} F^{\frac{b}{(dx+c)^{2}}} c^{2} x+\frac{F^{a} F^{\frac{b}{(dx+c)^{2}}} c^{3}}{3 d}+\frac{2 F^{a} b \ln(F) F^{\frac{b}{(dx+c)^{2}}} x}{3}+ 2F^{a} b\ln(F$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*F^a*d^2*F^(b/(d*x+c)^2)*x^3+F^a*d*F^(b/(d*x+c)^2)*c*x^2+F^a*F^(b/(d*x+
c)^2)*c^2*x+1/3*F^a/d*F^(b/(d*x+c)^2)*c^3+2/3*F^a*b*ln(F)*F^(b/(d*x+c)^2)*
x+2/3*F^a/d*b*ln(F)*F^(b/(d*x+c)^2)*c-2/3*F^a/d*b^2*ln(F)^2*Pi^(1/2)/(-b*l
n(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.27

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx$$
$$=\frac{2\sqrt{\pi}F^abd\sqrt{-\frac{b\log(F)}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)\log(F)+(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3+2\,(bdx+bc)\log(F))F^{\cdots}}{3\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(pi)*F^a*b*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x
 + c))*log(F) + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + 2*(b*d*x + b*c)
*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2))
)/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx=\int F^{a+\frac{b}{(c+dx)^2}}\left(c+dx\right)^2\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)*(d*x+c)**2,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)*(c + d*x)**2, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx = \int (dx+c)^2 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*(F^a*d^2*x^3 + 3*F^a*c*d*x^2 + (3*F^a*c^2 + 2*F^a*b*log(F))*x)*F^(b/(d
^2*x^2 + 2*c*d*x + c^2)) + integrate(2/3*(2*F^a*b^2*d*x*log(F)^2 - F^a*b*c
^3*log(F))*F^(b/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*
d*x + c^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx = \int (dx+c)^2 F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx = \frac{\left(\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}}{3}+\frac{2\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b\,\ln(F)}{3\,(c+d\,x)^2}\right)(c+d\,x)^3}{d} - \frac{2\,F^a\,b^2\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(F)}{\sqrt{b\,\ln(F)}\,(c+d\,x)}\right)\,\ln(F)^2}{3\,d\,\sqrt{b\,\ln(F)}}$$

input
```
int(F^(a + b/(c + d*x)^2)*(c + d*x)^2,x)
```

output
```
(((F^a*F^(b/(c + d*x)^2))/3 + (2*F^a*F^(b/(c + d*x)^2)*b*log(F))/(3*(c + d
*x)^2))*(c + d*x)^3)/d - (2*F^a*b^2*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(
1/2)*(c + d*x)))*log(F)^2)/(3*d*(b*log(F))^(1/2))
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}(c+dx)^2\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)*(d*x+c)^2,x)
```

output
```
(8*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)**3*b**3*d*x + 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
+ 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**3 - 4*f**((a*c**2 + 2*a*c*d*x +
a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*c**2*d*x - 2
0*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))
*log(f)**2*b**2*c*d**2*x**2 - 12*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)**2*b**2*d**3*x**3 - 10*f**((a*c**2
+ 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**5
 - 34*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x*
*2))*log(f)*b*c**4*d*x - 36*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c*
*2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**3*d**2*x**2 - 4*f**((a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**2*d**3*x
**3 + 14*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2
*x**2))*log(f)*b*c*d**4*x**4 + 6*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*d**5*x**5 + 3*f**((a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**7 + 21*f**((a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**6*d*x +
63*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*c**5*d**2*x**2 + 105*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 +
2*c*d*x + d**2*x**2))*c**4*d**3*x**3 + 105*f**((a*c**2 + 2*a*c*d*x + a*...
```

3.266 $\int F^{a+\frac{b}{(c+dx)^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx=\frac{F^{a+\frac{b}{(c+dx)^2}}(c+dx)}{d}-\frac{\sqrt{b}F^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\sqrt{\log(F)}}{d}$$

output
```
F^(a+b/(d*x+c)^2)*(d*x+c)/d-b^(1/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/
(d*x+c))*ln(F)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx=\frac{F^a\left(F^{\frac{b}{(c+dx)^2}}(c+dx)-\sqrt{b}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)\sqrt{\log(F)}\right)}{d}$$

input
```
Integrate[F^(a + b/(c + d*x)^2),x]
```

output
```
(F^a*(F^(b/(c + d*x)^2)*(c + d*x) - Sqrt[b]*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Lo
g[F]])/(c + d*x)]*Sqrt[Log[F]]))/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+\frac{b}{(c+dx)^2}}dx$$

$$\Big\downarrow \text{2635}$$

$$2b\log(F)\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx+\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}$$

$$\Big\downarrow \text{2640}$$

$$\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}-\frac{2b\log(F)\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)F^{a+\frac{b}{(c+dx)^2}}}{d}-\frac{\sqrt{\pi}\sqrt{b}F^a\sqrt{\log(F)}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{d}$$

input
```
Int[F^(a + b/(c + d*x)^2),x]
```

output
```
(F^(a + b/(c + d*x)^2)*(c + d*x))/d - (Sqrt[b]*F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*
Sqrt[Log[F]])/(c + d*x)]*Sqrt[Log[F]])/d
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

method	result	size
risch	$F^{a} F^{\frac{b}{\left(d x +c \right)^{2}}} x +\frac{F^{a} F^{\frac{b}{\left(d x +c \right)^{2}}} c}{d}-\frac{F^{a} b \ln \left(F \right) \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b \ln \left(F \right)}}{d x +c}\right)}{d \sqrt{-b \ln \left(F \right)}}$	74

input
```
int(F^(a+b/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
F^a*F^(b/(d*x+c)^2)*x+1/d*F^a*F^(b/(d*x+c)^2)*c-1/d*F^a*b*ln(F)*Pi^(1/2)/(
-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx=\frac{\sqrt{\pi}F^{a}d\sqrt{-\frac{b\log(F)}{d^{2}}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^{2}}}}{dx+c}\right)+(dx+c)F^{\frac{ad^{2}x^{2}+2\,acdx+ac^{2}+b}{d^{2}x^{2}+2\,cdx+c^{2}}}}{d}$$

input
```
integrate(F^(a+b/(d*x+c)^2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*F^a*d*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x + c)) +
 (d*x + c)*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2
)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx = \int F^{a+\frac{b}{(c+dx)^2}}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2),x)
```

output
```
Integral(F**(a + b/(c + d*x)**2), x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx = \int F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*F^a*b*d*integrate(F^(b/(d^2*x^2 + 2*c*d*x + c^2))*x/(d^3*x^3 + 3*c*d^2*x
^2 + 3*c^2*d*x + c^3), x)*log(F) + F^a*F^(b/(d^2*x^2 + 2*c*d*x + c^2))*x
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx = \int F^{a+\frac{b}{(dx+c)^2}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx=\frac{F^a\,F^{\frac{b}{{\left(c+d\,x\right)}^2}}\,\left(c+d\,x\right)}{d}-\frac{F^a\,b\,\sqrt{\pi }\,\mathrm{erfi}\left(\frac{b\,\ln\left(F\right)}{\sqrt{b\,\ln\left(F\right)}\,\left(c+d\,x\right)}\right)\,\ln\left(F\right)}{d\,\sqrt{b\,\ln\left(F\right)}}$$

input
```
int(F^(a + b/(c + d*x)^2),x)
```

output
```
(F^a*F^(b/(c + d*x)^2)*(c + d*x))/d - (F^a*b*pi^(1/2)*erfi((b*log(F))/((b*
log(F))^(1/2)*(c + d*x)))*log(F))/(d*(b*log(F))^(1/2))
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^2}}\,dx=\text{Too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2),x)
```

output
```
(4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)**2*b**2*d*x + 2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
+ 2*c*d*x + d**2*x**2))*log(f)*b*c**3 - 2*f**((a*c**2 + 2*a*c*d*x + a*d**2
*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**2*d*x - 10*f**((a*c**
2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c*
d**2*x**2 - 6*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))*log(f)*b*d**3*x**3 - 5*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)/(c**2 + 2*c*d*x + d**2*x**2))*c**5 - 17*f**((a*c**2 + 2*a*c*d*x + a*d
**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**4*d*x - 18*f**((a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**3*d**2*x**2 -
2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))
*c**2*d**3*x**3 + 7*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c
*d*x + d**2*x**2))*c*d**4*x**4 + 3*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)/(c**2 + 2*c*d*x + d**2*x**2))*d**5*x**5 + 8*int((f**((a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(c**7 + 7*c**6*d*x
+ 21*c**5*d**2*x**2 + 35*c**4*d**3*x**3 + 35*c**3*d**4*x**4 + 21*c**2*d**5
*x**5 + 7*c*d**6*x**6 + d**7*x**7),x)*log(f)**3*b**3*c**4*d**2 + 32*int((f
**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*x)
/(c**7 + 7*c**6*d*x + 21*c**5*d**2*x**2 + 35*c**4*d**3*x**3 + 35*c**3*d**4
*x**4 + 21*c**2*d**5*x**5 + 7*c*d**6*x**6 + d**7*x**7),x)*log(f)**3*b**...
```

3.267 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx = -\frac{F^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

output
```
-1/2*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))/b^(1/2)/d/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx = -\frac{F^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^2,x]
```

output
```
-1/2*(F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)])/(Sqrt[b]*d*Sqrt
[Log[F]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2} dx$$
$$\Big\downarrow \text{ 2640}$$
$$-\frac{\int F^{a+\frac{b}{(c+dx)^2}} d\frac{1}{c+dx}}{d}$$
$$\Big\downarrow \text{ 2633}$$
$$-\frac{\sqrt{\pi}F^a \text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^2,x]
```

output
```
-1/2*(F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)])/(Sqrt[b]*d*Sqrt
[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{F^{a}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b\ln(F)}}{dx+c}\right)}{2d\sqrt{-b\ln(F)}}$	35

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*F^a*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx = \frac{\sqrt{\pi}F^{a}\sqrt{-\frac{b\log(F)}{d^2}}\,\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)}{2\,b\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*F^a*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x + c))/
(b*log(F))
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx = \int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**2,x)
```

output
```
Integral(F**(a + b/(c + d*x)**2)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx = -\frac{F^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(F)}{\sqrt{b\,\ln(F)}\,(c+d\,x)}\right)}{2\,d\,\sqrt{b\,\ln\left(F\right)}}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^2,x)
```

output
```
-(F^a*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(1/2)*(c + d*x))))/(2*d*(b*log(
F))^(1/2))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^2,x)
```

output
```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)**2*b**2*d*x + f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 +
2*c*d*x + d**2*x**2))*log(f)*b*c**3 - f**((a*c**2 + 2*a*c*d*x + a*d**2*x**
2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c**2*d*x - 5*f**((a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*log(f)*b*c*d**2*
x**2 - 3*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2
*x**2))*log(f)*b*d**3*x**3 - 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/
(c**2 + 2*c*d*x + d**2*x**2))*c**5 - 16*f**((a*c**2 + 2*a*c*d*x + a*d**2*x
**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**4*d*x - 24*f**((a*c**2 + 2*a*c*d
*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**3*d**2*x**2 - 16*f*
*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*c**
2*d**3*x**3 - 4*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
 + d**2*x**2))*c*d**4*x**4 + 4*int((f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2
+ b)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(c**7 + 7*c**6*d*x + 21*c**5*d**2*x*
*2 + 35*c**4*d**3*x**3 + 35*c**3*d**4*x**4 + 21*c**2*d**5*x**5 + 7*c*d**6*
x**6 + d**7*x**7),x)*log(f)**3*b**3*c**4*d**2 + 16*int((f**((a*c**2 + 2*a*
c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))*x)/(c**7 + 7*c**6*d
*x + 21*c**5*d**2*x**2 + 35*c**4*d**3*x**3 + 35*c**3*d**4*x**4 + 21*c**2*d
**5*x**5 + 7*c*d**6*x**6 + d**7*x**7),x)*log(f)**3*b**3*c**3*d**3*x + 24*i
nt((f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x...
```

3.268 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \frac{F^a\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)\log(F)}$$

output
```
1/4*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))/b^(3/2)/d/ln(F)^(3/2)-1
/2*F^(a+b/(d*x+c)^2)/b/d/(d*x+c)/ln(F)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \frac{F^a\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)\log(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^4,x]
```

output
```
(F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)])/(4*b^(3/2)*d*Log[F]^
(3/2)) - F^(a + b/(c + d*x)^2)/(2*b*d*(c + d*x)*Log[F])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4} dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}$$

$$\Big\downarrow \text{2640}$$

$$\frac{\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{2bd\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^4,x]
```

output
```
(F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)])/(4*b^(3/2)*d*Log[F]^
(3/2)) - F^(a + b/(c + d*x)^2)/(2*b*d*(c + d*x)*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{F^{a} F^{\frac{b}{(dx+c)^{2}}}}{2d(dx+c)b\ln(F)}+\frac{F^{a}\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-b\ln(F)}}{dx+c}\right)}{4db\ln(F)\sqrt{-b\ln(F)}}$	76

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*F^a*F^(b/(d*x+c)^2)/(d*x+c)/b/ln(F)+1/4/d*F^a/b/ln(F)*Pi^(1/2)/(-b*
ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = -\frac{\sqrt{\pi}(d^2x+cd)F^a\sqrt{-\frac{b\log(F)}{d^2}}\,\mathrm{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)+2\,F^{\frac{ad^2x^2+2\,acdx+ac^2+b}{d^2x^2+2\,cdx+c^2}}b\log(F)}{4\,(b^2d^2x+b^2cd)\log(F)^2}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*(d^2*x + c*d)*F^a*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/
d^2)/(d*x + c)) + 2*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*
d*x + c^2))*b*log(F))/((b^2*d^2*x + b^2*c*d)*log(F)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \frac{F^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\ln(F)}{\sqrt{b\ln(F)}\,(c+d\,x)}\right)}{4\,b\,d\,\ln(F)\,\sqrt{b\ln(F)}} - \frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}}{2\,b\,d\,\ln(F)\,(c+d\,x)}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^4,x)
```

output
```
(F^a*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(1/2)*(c + d*x))))/(4*b*d*log(F)
*(b*log(F))^(1/2)) - (F^a*F^(b/(c + d*x)^2))/(2*b*d*log(F)*(c + d*x))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^4,x)
```

output
```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)*b - 5*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))*c**2 - 10*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 +
 2*c*d*x + d**2*x**2))*c*d*x - 5*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*d**2*x**2 + 4*int(f**((a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**8 + 8*c**7*d*x + 28*c
**6*d**2*x**2 + 56*c**5*d**3*x**3 + 70*c**4*d**4*x**4 + 56*c**3*d**5*x**5
+ 28*c**2*d**6*x**6 + 8*c*d**7*x**7 + d**8*x**8),x)*log(f)**2*b**2*c**5*d
+ 20*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2
*x**2))/(c**8 + 8*c**7*d*x + 28*c**6*d**2*x**2 + 56*c**5*d**3*x**3 + 70*c*
*4*d**4*x**4 + 56*c**3*d**5*x**5 + 28*c**2*d**6*x**6 + 8*c*d**7*x**7 + d**
8*x**8),x)*log(f)**2*b**2*c**4*d**2*x + 40*int(f**((a*c**2 + 2*a*c*d*x + a
*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**8 + 8*c**7*d*x + 28*c**6
*d**2*x**2 + 56*c**5*d**3*x**3 + 70*c**4*d**4*x**4 + 56*c**3*d**5*x**5 + 2
8*c**2*d**6*x**6 + 8*c*d**7*x**7 + d**8*x**8),x)*log(f)**2*b**2*c**3*d**3*
x**2 + 40*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x +
 d**2*x**2))/(c**8 + 8*c**7*d*x + 28*c**6*d**2*x**2 + 56*c**5*d**3*x**3 +
70*c**4*d**4*x**4 + 56*c**3*d**5*x**5 + 28*c**2*d**6*x**6 + 8*c*d**7*x**7
+ d**8*x**8),x)*log(f)**2*b**2*c**2*d**4*x**3 + 20*int(f**((a*c**2 + 2*a*c
*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**8 + 8*c**7*d*...
```

3.269 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6} dx$

Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6} dx = -\frac{3F^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{8b^{5/2}d\log^{\frac{5}{2}}(F)} + \frac{3F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^3\log(F)}$$

output
```
-3/8*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))/b^(5/2)/d/ln(F)^(5/2)+
3/4*F^(a+b/(d*x+c)^2)/b^2/d/(d*x+c)/ln(F)^2-1/2*F^(a+b/(d*x+c)^2)/b/d/(d*x
+c)^3/ln(F)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6} dx = \frac{F^a\left(-3\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right) - \frac{2\sqrt{b}F^{\frac{b}{(c+dx)^2}}\sqrt{\log(F)}(-3(c+dx)^2+2b\log(F))}{(c+dx)^3}\right)}{8b^{5/2}d\log^{\frac{5}{2}}(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^6,x]
```

output
```
(F^a*(-3*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)] - (2*Sqrt[b]*F^(b
/(c + d*x)^2)*Sqrt[Log[F]]*(-3*(c + d*x)^2 + 2*b*Log[F]))/(c + d*x)^3))/(8
*b^(5/2)*d*Log[F]^(5/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2641, 2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\,dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{3\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{2640}$$

$$-\frac{3\left(\frac{\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{2bd\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{3\left(\frac{\sqrt{\pi}F^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^6,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*(c + d*x)^3*Log[F]) - (3*((F^a*Sqrt[Pi]*Er
fi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)])/(4*b^(3/2)*d*Log[F]^(3/2)) - F^(a +
b/(c + d*x)^2)/(2*b*d*(c + d*x)*Log[F])))/(2*b*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{F^{a} F^{\frac{b}{(dx+c)^{2}}}}{2d(dx+c)^{3} b\ln(F)}+\frac{3F^{a} F^{\frac{b}{(dx+c)^{2}}}}{4d\, b^{2} \ln(F)^{2} (dx+c)}-\frac{3F^{a} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b\ln(F)}}{dx+c}\right)}{8d\, b^{2} \ln(F)^{2} \sqrt{-b\ln(F)}}$	109

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^a/d*F^(b/(d*x+c)^2)/(d*x+c)^3/b/ln(F)+3/4*F^a/d/b^2/ln(F)^2*F^(b/(d
*x+c)^2)/(d*x+c)-3/8*F^a/d/b^2/ln(F)^2*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*l
n(F))^(1/2)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(99) = 198$.

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.73

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\, dx$$
$$= \frac{3\sqrt{\pi}(d^4x^3 + 3\,cd^3x^2 + 3\,c^2d^2x + c^3d)F^a\sqrt{-\frac{b\log(F)}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right) - 2\left(2\,b^2\log\left(F\right)^2 - 3\left(bd^2x^2 + 2\,bc\right.\right.}{8\left(b^3d^4x^3 + 3\,b^3cd^3x^2 + 3\,b^3c^2d^2x + b^3c^3d\right)\log\left(F\right)^3}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(pi)*(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)*F^a*sqrt(-b*
log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x + c)) - 2*(2*b^2*log(F)^2 - 3*(
b*d^2*x^2 + 2*b*c*d*x + b*c^2)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + b^3*c^3*d)*log(F)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^6}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^6,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^6, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^6}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^6,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^6, x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\,dx = -\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}}{2\,b\,d\,\ln(F)\,(c+d\,x)^3} - \frac{F^a\left(3\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(F)}{\sqrt{b\,\ln(F)}\,(c+d\,x)}\right)-\frac{6\,F^{\frac{b}{(c+d\,x)^2}}\sqrt{b\,\ln(F)}}{c+d\,x}\right)}{8\,b^2\,d\ln(F)^2\,\sqrt{b\,\ln(F)}}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^6,x)
```

output
```
- (F^a*F^(b/(c + d*x)^2))/(2*b*d*log(F)*(c + d*x)^3) - (F^a*(3*pi^(1/2)*er
fi((b*log(F))/((b*log(F))^(1/2)*(c + d*x))) - (6*F^(b/(c + d*x)^2)*(b*log(
F))^(1/2))/(c + d*x)))/(8*b^2*d*log(F)^2*(b*log(F))^(1/2))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^6,x)
```

output

```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)*b - 7*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))*c**2 - 14*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 +
 2*c*d*x + d**2*x**2))*c*d*x - 7*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*d**2*x**2 + 4*int(f**((a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**10 + 10*c**9*d*x + 45
*c**8*d**2*x**2 + 120*c**7*d**3*x**3 + 210*c**6*d**4*x**4 + 252*c**5*d**5*
x**5 + 210*c**4*d**6*x**6 + 120*c**3*d**7*x**7 + 45*c**2*d**8*x**8 + 10*c*
d**9*x**9 + d**10*x**10),x)*log(f)**2*b**2*c**7*d + 28*int(f**((a*c**2 + 2
*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**10 + 10*c**9
*d*x + 45*c**8*d**2*x**2 + 120*c**7*d**3*x**3 + 210*c**6*d**4*x**4 + 252*c
**5*d**5*x**5 + 210*c**4*d**6*x**6 + 120*c**3*d**7*x**7 + 45*c**2*d**8*x**
8 + 10*c*d**9*x**9 + d**10*x**10),x)*log(f)**2*b**2*c**6*d**2*x + 84*int(f
**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c
**10 + 10*c**9*d*x + 45*c**8*d**2*x**2 + 120*c**7*d**3*x**3 + 210*c**6*d**
4*x**4 + 252*c**5*d**5*x**5 + 210*c**4*d**6*x**6 + 120*c**3*d**7*x**7 + 45
*c**2*d**8*x**8 + 10*c*d**9*x**9 + d**10*x**10),x)*log(f)**2*b**2*c**5*d**
3*x**2 + 140*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*
x + d**2*x**2))/(c**10 + 10*c**9*d*x + 45*c**8*d**2*x**2 + 120*c**7*d**3*x
**3 + 210*c**6*d**4*x**4 + 252*c**5*d**5*x**5 + 210*c**4*d**6*x**6 + 12...
```

3.270 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx=\frac{15F^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{16b^{7/2}d\log^{\frac{7}{2}}(F)}-\frac{15F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx)\log^3(F)}+\frac{5F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^3\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^5\log(F)}$$

output
```
15/16*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))/b^(7/2)/d/ln(F)^(7/2)
-15/8*F^(a+b/(d*x+c)^2)/b^3/d/(d*x+c)/ln(F)^3+5/4*F^(a+b/(d*x+c)^2)/b^2/d/
(d*x+c)^3/ln(F)^2-1/2*F^(a+b/(d*x+c)^2)/b/d/(d*x+c)^5/ln(F)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx=\frac{F^a\left(15\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)-\frac{2\sqrt{b}F^{\frac{b}{(c+dx)^2}}\sqrt{\log(F)}\left(15(c+dx)^4-10b(c+dx)^2\log(F)+4b^2\log^2(F)\right)}{(c+dx)^5}\right)}{16b^{7/2}d\log^{\frac{7}{2}}(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^8,x]
```

output
```
(F^a*(15*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)] - (2*Sqrt[b]*F^(b
/(c + d*x)^2)*Sqrt[Log[F]]*(15*(c + d*x)^4 - 10*b*(c + d*x)^2*Log[F] + 4*b
^2*Log[F]^2))/(c + d*x)^5))/(16*b^(7/2)*d*Log[F]^(7/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2641, 2641, 2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{5\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{5\left(-\frac{3\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{5\left(-\frac{3\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}$$

$$\Big\downarrow \text{2640}$$

$$-\frac{5\left(-\frac{3\left(\frac{\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{2bd\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{5\left(-\frac{3\left(\frac{\sqrt{\pi}F^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^8,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*(c + d*x)^5*Log[F]) - (5*(-1/2*F^(a + b/(c
 + d*x)^2)/(b*d*(c + d*x)^3*Log[F]) - (3*((F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt
[Log[F]])/(c + d*x)])/(4*b^(3/2)*d*Log[F]^(3/2)) - F^(a + b/(c + d*x)^2)/(
2*b*d*(c + d*x)*Log[F])))/(2*b*Log[F])))/(2*b*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 2.24 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

method	result	size
risch	$-\frac{F^{a} F^{\frac{b}{\left(dx+c\right)^{2}}}}{2d\left(dx+c\right)^{5} b \ln\left(F\right)}+\frac{5F^{a} F^{\frac{b}{\left(dx+c\right)^{2}}}}{4d\,b^{2} \ln\left(F\right)^{2} \left(dx+c\right)^{3}}-\frac{15F^{a} F^{\frac{b}{\left(dx+c\right)^{2}}}}{8d\,b^{3} \ln\left(F\right)^{3} \left(dx+c\right)}+\frac{15F^{a} \sqrt{\pi}\, \operatorname{erf}\left(\frac{\sqrt{-b\ln\left(F\right)}}{dx+c}\right)}{16d\,b^{3} \ln\left(F\right)^{3} \sqrt{-b\ln\left(F\right)}}$	142

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^8,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^a/d*F^(b/(d*x+c)^2)/(d*x+c)^5/b/ln(F)+5/4*F^a/d/b^2/ln(F)^2*F^(b/(d
*x+c)^2)/(d*x+c)^3-15/8*F^a/d/b^3/ln(F)^3*F^(b/(d*x+c)^2)/(d*x+c)+15/16*F^
a/d/b^3/ln(F)^3*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(131) = 262.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.05

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx =$$

$$-\frac{15\sqrt{\pi}(d^6x^5+5\,cd^5x^4+10\,c^2d^4x^3+10\,c^3d^3x^2+5\,c^4d^2x+c^5d)F^a\sqrt{-\frac{b\log(F)}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{b\log(F)}{d^2}}}{dx+c}\right)+2\,(4\,b}{16\,(b^4d^6x^5+5\,b^4cd^5x^4+10}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^8,x, algorithm="fricas")
```

output
```
-1/16*(15*sqrt(pi)*(d^6*x^5 + 5*c*d^5*x^4 + 10*c^2*d^4*x^3 + 10*c^3*d^3*x^
2 + 5*c^4*d^2*x + c^5*d)*F^a*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)
/(d*x + c)) + 2*(4*b^3*log(F)^3 - 10*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*log(F)^2 + 15*(b*d^4*x^4 + 4*b*c*d^3*x^3 + 6*b*c^2*d^2*x^2 + 4*b*c^3*d*x
+ b*c^4)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x
 + c^2)))/((b^4*d^6*x^5 + 5*b^4*c*d^5*x^4 + 10*b^4*c^2*d^4*x^3 + 10*b^4*c^
3*d^3*x^2 + 5*b^4*c^4*d^2*x + b^4*c^5*d)*log(F)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**8,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^8}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^8,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^8, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^8}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^8,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^8, x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx = \frac{5\,F^a\,F^{\frac{b}{(c+d\,x)^2}}}{4\,b^2\,d\ln(F)^2\,(c+d\,x)^3} - \frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}}{2\,b\,d\,\ln(F)\,(c+d\,x)^5} - \frac{15\,F^a\,F^{\frac{b}{(c+d\,x)^2}}}{8\,b^3\,d\ln(F)^3\,(c+d\,x)} + \frac{15\,F^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(F)}{\sqrt{b\,\ln(F)}\,(c+d\,x)}\right)}{16\,b^3\,d\ln(F)^3\,\sqrt{b\,\ln(F)}}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^8,x)
```

output
```
(5*F^a*F^(b/(c + d*x)^2))/(4*b^2*d*log(F)^2*(c + d*x)^3) - (F^a*F^(b/(c +
d*x)^2))/(2*b*d*log(F)*(c + d*x)^5) - (15*F^a*F^(b/(c + d*x)^2))/(8*b^3*d*
log(F)^3*(c + d*x)) + (15*F^a*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(1/2)*(
c + d*x))))/(16*b^3*d*log(F)^3*(b*log(F))^(1/2))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^8,x)
```

output
```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)*b - 9*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
+ d**2*x**2))*c**2 - 18*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 +
 2*c*d*x + d**2*x**2))*c*d*x - 9*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))*d**2*x**2 + 4*int(f**((a*c**2 + 2*a*c*d*x
+ a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**12 + 12*c**11*d*x + 6
6*c**10*d**2*x**2 + 220*c**9*d**3*x**3 + 495*c**8*d**4*x**4 + 792*c**7*d**
5*x**5 + 924*c**6*d**6*x**6 + 792*c**5*d**7*x**7 + 495*c**4*d**8*x**8 + 22
0*c**3*d**9*x**9 + 66*c**2*d**10*x**10 + 12*c*d**11*x**11 + d**12*x**12),x
)*log(f)**2*b**2*c**9*d + 36*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b
)/(c**2 + 2*c*d*x + d**2*x**2))/(c**12 + 12*c**11*d*x + 66*c**10*d**2*x**2
 + 220*c**9*d**3*x**3 + 495*c**8*d**4*x**4 + 792*c**7*d**5*x**5 + 924*c**6
*d**6*x**6 + 792*c**5*d**7*x**7 + 495*c**4*d**8*x**8 + 220*c**3*d**9*x**9
+ 66*c**2*d**10*x**10 + 12*c*d**11*x**11 + d**12*x**12),x)*log(f)**2*b**2*
c**8*d**2*x + 144*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2
*c*d*x + d**2*x**2))/(c**12 + 12*c**11*d*x + 66*c**10*d**2*x**2 + 220*c**9
*d**3*x**3 + 495*c**8*d**4*x**4 + 792*c**7*d**5*x**5 + 924*c**6*d**6*x**6
+ 792*c**5*d**7*x**7 + 495*c**4*d**8*x**8 + 220*c**3*d**9*x**9 + 66*c**2*d
**10*x**10 + 12*c*d**11*x**11 + d**12*x**12),x)*log(f)**2*b**2*c**7*d**3*x
**2 + 336*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*...
```

3.271 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 183

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = -\frac{105F^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{32b^{9/2}d\log^{\frac{9}{2}}(F)} + \frac{105F^{a+\frac{b}{(c+dx)^2}}}{16b^4d(c+dx)\log^4(F)} - \frac{35F^{a+\frac{b}{(c+dx)^2}}}{8b^3d(c+dx)^3\log^3(F)} + \frac{7F^{a+\frac{b}{(c+dx)^2}}}{4b^2d(c+dx)^5\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd(c+dx)^7\log(F)}$$

output
```
-105/32*F^a*Pi^(1/2)*erfi(b^(1/2)*ln(F)^(1/2)/(d*x+c))/b^(9/2)/d/ln(F)^(9/
2)+105/16*F^(a+b/(d*x+c)^2)/b^4/d/(d*x+c)/ln(F)^4-35/8*F^(a+b/(d*x+c)^2)/b
^3/d/(d*x+c)^3/ln(F)^3+7/4*F^(a+b/(d*x+c)^2)/b^2/d/(d*x+c)^5/ln(F)^2-1/2*F
^(a+b/(d*x+c)^2)/b/d/(d*x+c)^7/ln(F)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = \frac{F^a\left(-105\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)+\frac{2\sqrt{b}F^{\frac{b}{(c+dx)^2}}\sqrt{\log(F)}(105(c+dx)^6-70b(c+dx)^4\log(F)+28b^2(c+dx)^2\log^2(F)-8b^3\log^3(F))}{(c+dx)^7}\right)}{32b^{9/2}d\log^{\frac{9}{2}}(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^10,x]
```

output
```
(F^a*(-105*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d*x)] + (2*Sqrt[b]*F^
(b/(c + d*x)^2)*Sqrt[Log[F]]*(105*(c + d*x)^6 - 70*b*(c + d*x)^4*Log[F] +
28*b^2*(c + d*x)^2*Log[F]^2 - 8*b^3*Log[F]^3))/(c + d*x)^7))/(32*b^(9/2)*d
*Log[F]^(9/2))
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2641, 2641, 2641, 2641, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{7\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^8}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^7}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{7\left(-\frac{5\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^6}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^7}$$

$\big\downarrow$ 2641

$$-\frac{7\left(-\frac{5\left(-\frac{3\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^4}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^7}$$

$\big\downarrow$ 2641

$$-\frac{7\left(-\frac{5\left(-\frac{3\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^2}dx}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^7}$$

$\big\downarrow$ 2640

$$-\frac{7\left(-\frac{5\left(-\frac{3\left(\frac{\int F^{a+\frac{b}{(c+dx)^2}}d\frac{1}{c+dx}}{2bd\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^7}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{7\left(-\frac{5\left(-\frac{3\left(\frac{\sqrt{\pi}F^a\text{erfi}\left(\frac{\sqrt{b}\sqrt{\log(F)}}{c+dx}\right)}{4b^{3/2}d\log^{\frac{3}{2}}(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^3}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^5}\right)}{2b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^2}}}{2bd\log(F)(c+dx)^7}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^10,x]
```

output
```
-1/2*F^(a + b/(c + d*x)^2)/(b*d*(c + d*x)^7*Log[F]) - (7*(-1/2*F^(a + b/(c
 + d*x)^2)/(b*d*(c + d*x)^5*Log[F]) - (5*(-1/2*F^(a + b/(c + d*x)^2)/(b*d*
(c + d*x)^3*Log[F]) - (3*((F^a*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[Log[F]])/(c + d
*x)])/(4*b^(3/2)*d*Log[F]^(3/2)) - F^(a + b/(c + d*x)^2)/(2*b*d*(c + d*x)*
Log[F])))/(2*b*Log[F])))/(2*b*Log[F])))/(2*b*Log[F])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{F^{a} F^{\frac{b}{(dx+c)^{2}}}}{2d(dx+c)^{7} b\ln(F)}+\frac{7F^{a} F^{\frac{b}{(dx+c)^{2}}}}{4d\, b^{2}\ln(F)^{2}(dx+c)^{5}}-\frac{35F^{a} F^{\frac{b}{(dx+c)^{2}}}}{8d\, b^{3}\ln(F)^{3}(dx+c)^{3}}+\frac{105F^{a} F^{\frac{b}{(dx+c)^{2}}}}{16d\, b^{4}\ln(F)^{4}(dx+c)}-\frac{105F^{a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-b\ln(F)}}{dx+c}\right)}{32d\, b^{4}\ln(F)^{4}\sqrt{-b\ln(F)}}$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^a/d*F^(b/(d*x+c)^2)/(d*x+c)^7/b/ln(F)+7/4*F^a/d/b^2/ln(F)^2*F^(b/(d
*x+c)^2)/(d*x+c)^5-35/8*F^a/d/b^3/ln(F)^3*F^(b/(d*x+c)^2)/(d*x+c)^3+105/16
*F^a/d/b^4/ln(F)^4*F^(b/(d*x+c)^2)/(d*x+c)-105/32*F^a/d/b^4/ln(F)^4*Pi^(1/
2)/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(163) = 326$.

Time = 0.10 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.40

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = \frac{105\sqrt{\pi}(d^8x^7 + 7\,cd^7x^6 + 21\,c^2d^6x^5 + 35\,c^3d^5x^4 + 35\,c^4d^4x^3 + 21\,c^5d^3x^2 + 7\,c^6d^2x + c^7d)F^a\sqrt{-\frac{b\log(F)}{d^2}}\,\mathrm{er}}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^10,x, algorithm="fricas")
```

output
```
1/32*(105*sqrt(pi)*(d^8*x^7 + 7*c*d^7*x^6 + 21*c^2*d^6*x^5 + 35*c^3*d^5*x^
4 + 35*c^4*d^4*x^3 + 21*c^5*d^3*x^2 + 7*c^6*d^2*x + c^7*d)*F^a*sqrt(-b*log
(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x + c)) - 2*(8*b^4*log(F)^4 - 28*(b^
3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*log(F)^3 + 70*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*log(F)^2 - 105*(b*d^6
*x^6 + 6*b*c*d^5*x^5 + 15*b*c^2*d^4*x^4 + 20*b*c^3*d^3*x^3 + 15*b*c^4*d^2*
x^2 + 6*b*c^5*d*x + b*c^6)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/
(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*d^8*x^7 + 7*b^5*c*d^7*x^6 + 21*b^5*c^2*d
^6*x^5 + 35*b^5*c^3*d^5*x^4 + 35*b^5*c^4*d^4*x^3 + 21*b^5*c^5*d^3*x^2 + 7*
b^5*c^6*d^2*x + b^5*c^7*d)*log(F)^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**10,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^10,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^10, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^10,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = -\frac{\frac{F^a\left(105\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\ln(F)}{\sqrt{b\ln(F)}\,(c+d\,x)}\right)-\frac{210\,F^{\frac{b}{(c+d\,x)^2}}\sqrt{b\ln(F)}}{c+d\,x}\right)}{32\sqrt{b\ln(F)}}-\frac{7\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^2\ln(F)^2}{4\,(c+d\,x)^5}+\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^3\ln(F)^3}{2\,(c+d\,x)^7}+\frac{35\,F^a\,F^{\frac{b}{(c+d}}}{8\,(c+d}}{b^4\,d\ln\left(F\right)^4}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^10,x)
```

output
```
-((F^a*(105*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(1/2)*(c + d*x))) - (210*
F^(b/(c + d*x)^2)*(b*log(F))^(1/2))/(c + d*x)))/(32*(b*log(F))^(1/2)) - (7
*F^a*F^(b/(c + d*x)^2)*b^2*log(F)^2)/(4*(c + d*x)^5) + (F^a*F^(b/(c + d*x)
^2)*b^3*log(F)^3)/(2*(c + d*x)^7) + (35*F^a*F^(b/(c + d*x)^2)*b*log(F))/(8
*(c + d*x)^3))/(b^4*d*log(F)^4)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{10}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^10,x)
```

output
```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)*b - 11*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
 + d**2*x**2))*c**2 - 22*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
+ 2*c*d*x + d**2*x**2))*c*d*x - 11*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)/(c**2 + 2*c*d*x + d**2*x**2))*d**2*x**2 + 4*int(f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**14 + 14*c**13*d*x +
 91*c**12*d**2*x**2 + 364*c**11*d**3*x**3 + 1001*c**10*d**4*x**4 + 2002*c*
*9*d**5*x**5 + 3003*c**8*d**6*x**6 + 3432*c**7*d**7*x**7 + 3003*c**6*d**8*
x**8 + 2002*c**5*d**9*x**9 + 1001*c**4*d**10*x**10 + 364*c**3*d**11*x**11
+ 91*c**2*d**12*x**12 + 14*c*d**13*x**13 + d**14*x**14),x)*log(f)**2*b**2*
c**11*d + 44*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*
x + d**2*x**2))/(c**14 + 14*c**13*d*x + 91*c**12*d**2*x**2 + 364*c**11*d**
3*x**3 + 1001*c**10*d**4*x**4 + 2002*c**9*d**5*x**5 + 3003*c**8*d**6*x**6
+ 3432*c**7*d**7*x**7 + 3003*c**6*d**8*x**8 + 2002*c**5*d**9*x**9 + 1001*c
**4*d**10*x**10 + 364*c**3*d**11*x**11 + 91*c**2*d**12*x**12 + 14*c*d**13*
x**13 + d**14*x**14),x)*log(f)**2*b**2*c**10*d**2*x + 220*int(f**((a*c**2
+ 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**14 + 14*c
**13*d*x + 91*c**12*d**2*x**2 + 364*c**11*d**3*x**3 + 1001*c**10*d**4*x**4
 + 2002*c**9*d**5*x**5 + 3003*c**8*d**6*x**6 + 3432*c**7*d**7*x**7 + 3003*
c**6*d**8*x**8 + 2002*c**5*d**9*x**9 + 1001*c**4*d**10*x**10 + 364*c**3...
```

3.272 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx=\frac{F^a\Gamma\left(\frac{11}{2},-\frac{b\log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11}\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{11/2}}$$

output
```
1/2*F^a*(1048576/6183686925497065825762484062 5*GAMMA(51/2,-b*ln(F)/(d*x+c)
^2)-1048576/61836869254970658257624840625*(-b*ln(F)/(d*x+c)^2)^(49/2)*exp(
b*ln(F)/(d*x+c)^2)-524288/1261976923570829760359690625*(-b*ln(F)/(d*x+c)^2
)^(47/2)*exp(b*ln(F)/(d*x+c)^2)-262144/26850572841932548092759375*(-b*ln(F
)/(d*x+c)^2)^(45/2)*exp(b*ln(F)/(d*x+c)^2)-131072/596679396487389957616875
*(-b*ln(F)/(d*x+c)^2)^(43/2)*exp(b*ln(F)/(d*x+c)^2)-65536/1387626503459046
4130625*(-b*ln(F)/(d*x+c)^2)^(41/2)*exp(b*ln(F)/(d*x+c)^2)-32768/338445488
648547905625*(-b*ln(F)/(d*x+c)^2)^(39/2)*exp(b*ln(F)/(d*x+c)^2)-16384/8678
089452526869375*(-b*ln(F)/(d*x+c)^2)^(37/2)*exp(b*ln(F)/(d*x+c)^2)-8192/23
4542958176401875*(-b*ln(F)/(d*x+c)^2)^(35/2)*exp(b*ln(F)/(d*x+c)^2)-4096/6
701227376468625*(-b*ln(F)/(d*x+c)^2)^(33/2)*exp(b*ln(F)/(d*x+c)^2)-2048/20
3067496256625*(-b*ln(F)/(d*x+c)^2)^(31/2)*exp(b*ln(F)/(d*x+c)^2)-1024/6550
564395375*(-b*ln(F)/(d*x+c)^2)^(29/2)*exp(b*ln(F)/(d*x+c)^2)-512/225881530
875*(-b*ln(F)/(d*x+c)^2)^(27/2)*exp(b*ln(F)/(d*x+c)^2)-256/8365982625*(-b*
ln(F)/(d*x+c)^2)^(25/2)*exp(b*ln(F)/(d*x+c)^2)-128/334639305*(-b*ln(F)/(d*
x+c)^2)^(23/2)*exp(b*ln(F)/(d*x+c)^2)-64/14549535*(-b*ln(F)/(d*x+c)^2)^(21
/2)*exp(b*ln(F)/(d*x+c)^2)-32/692835*(-b*ln(F)/(d*x+c)^2)^(19/2)*exp(b*ln(
F)/(d*x+c)^2)-16/36465*(-b*ln(F)/(d*x+c)^2)^(17/2)*exp(b*ln(F)/(d*x+c)^2)-
8/2145*(-b*ln(F)/(d*x+c)^2)^(15/2)*exp(b*ln(F)/(d*x+c)^2)-4/143*(-b*ln(F)/
(d*x+c)^2)^(13/2)*exp(b*ln(F)/(d*x+c)^2)-2/11*(-b*ln(F)/(d*x+c)^2)^(11/...
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx=\frac{F^a\Gamma\left(\frac{11}{2},-\frac{b\log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11}\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{11/2}}$$

input
```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^12,x]
```

output
```
(F^a*Gamma[11/2, -((b*Log[F])/(c + d*x)^2)])/(2*d*(c + d*x)^11*(-((b*Log[F
])/(c + d*x)^2))^(11/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{F^a \Gamma\left(\frac{11}{2},-\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{11} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{11/2}}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^12,x]
```

output
```
(F^a*Gamma[11/2, -((b*Log[F])/(c + d*x)^2)])/(2*d*(c + d*x)^11*(-((b*Log[F
])/(c + d*x)^2))^(11/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 6.71 (sec) , antiderivative size = 208, normalized size of antiderivative = 4.24

method	result
risch	$-\frac{F^{a} F^{\frac{b}{(dx+c)^{2}}}}{2d(dx+c)^{9} b \ln(F)}+\frac{9F^{a} F^{\frac{b}{(dx+c)^{2}}}}{4d\, b^{2} \ln(F)^{2} (dx+c)^{7}}-\frac{63F^{a} F^{\frac{b}{(dx+c)^{2}}}}{8d\, b^{3} \ln(F)^{3} (dx+c)^{5}}+\frac{315F^{a} F^{\frac{b}{(dx+c)^{2}}}}{16d\, b^{4} \ln(F)^{4} (dx+c)^{3}}-\frac{945F^{a} F^{\frac{b}{(dx+c)^{2}}}}{32d\, b^{5} \ln(F)^{5} (dx+c)}+\frac{945}{64}$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^a/d*F^(b/(d*x+c)^2)/(d*x+c)^9/b/ln(F)+9/4*F^a/d/b^2/ln(F)^2*F^(b/(d
*x+c)^2)/(d*x+c)^7-63/8*F^a/d/b^3/ln(F)^3*F^(b/(d*x+c)^2)/(d*x+c)^5+315/16
*F^a/d/b^4/ln(F)^4*F^(b/(d*x+c)^2)/(d*x+c)^3-945/32*F^a/d/b^5/ln(F)^5*F^(b
/(d*x+c)^2)/(d*x+c)+945/64*F^a/d/b^5/ln(F)^5*Pi^(1/2)/(-b*ln(F))^(1/2)*erf
((-b*ln(F))^(1/2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 601, normalized size of antiderivative = 12.27

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx =$$

$$-\frac{945\sqrt{\pi}(d^{10}x^{9}+9\,cd^{9}x^{8}+36\,c^{2}d^{8}x^{7}+84\,c^{3}d^{7}x^{6}+126\,c^{4}d^{6}x^{5}+126\,c^{5}d^{5}x^{4}+84\,c^{6}d^{4}x^{3}+36\,c^{7}d^{3}x^{2}+}{}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x, algorithm="fricas")
```

output
```
-1/64*(945*sqrt(pi)*(d^10*x^9 + 9*c*d^9*x^8 + 36*c^2*d^8*x^7 + 84*c^3*d^7*
x^6 + 126*c^4*d^6*x^5 + 126*c^5*d^5*x^4 + 84*c^6*d^4*x^3 + 36*c^7*d^3*x^2
+ 9*c^8*d^2*x + c^9*d)*F^a*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(
d*x + c)) + 2*(16*b^5*log(F)^5 - 72*(b^4*d^2*x^2 + 2*b^4*c*d*x + b^4*c^2)*
log(F)^4 + 252*(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*
c^3*d*x + b^3*c^4)*log(F)^3 - 630*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*
c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^
2*c^6)*log(F)^2 + 945*(b*d^8*x^8 + 8*b*c*d^7*x^7 + 28*b*c^2*d^6*x^6 + 56*b
*c^3*d^5*x^5 + 70*b*c^4*d^4*x^4 + 56*b*c^5*d^3*x^3 + 28*b*c^6*d^2*x^2 + 8*
b*c^7*d*x + b*c^8)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2
 + 2*c*d*x + c^2)))/((b^6*d^10*x^9 + 9*b^6*c*d^9*x^8 + 36*b^6*c^2*d^8*x^7
+ 84*b^6*c^3*d^7*x^6 + 126*b^6*c^4*d^6*x^5 + 126*b^6*c^5*d^5*x^4 + 84*b^6*
c^6*d^4*x^3 + 36*b^6*c^7*d^3*x^2 + 9*b^6*c^8*d^2*x + b^6*c^9*d)*log(F)^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**12,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{12}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^12, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx=\int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{12}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^12, x)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 189, normalized size of antiderivative = 3.86

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx = \frac{\frac{F^a\left(945\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\ln(F)}{\sqrt{b\ln(F)}\,(c+d\,x)}\right)-\frac{1890\,F^{\frac{b}{(c+d\,x)^2}}\sqrt{b\ln(F)}}{c+d\,x}\right)}{64\sqrt{b\ln(F)}}-\frac{63\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^2\ln(F)^2}{8\,(c+d\,x)^5}+\frac{9\,F^a\,F^{\frac{b}{(c+d\,x)^2}}\,b^3\ln(F)^3}{4\,(c+d\,x)^7}-\frac{F^a\,F^{\frac{b}{(c+d\,x)^2}}}{2\,(c+d}}{b^5\,d\ln(F)^5}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^12,x)
```

output
```
((F^a*(945*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(1/2)*(c + d*x))) - (1890*
F^(b/(c + d*x)^2)*(b*log(F))^(1/2))/(c + d*x)))/(64*(b*log(F))^(1/2)) - (6
3*F^a*F^(b/(c + d*x)^2)*b^2*log(F)^2)/(8*(c + d*x)^5) + (9*F^a*F^(b/(c + d
*x)^2)*b^3*log(F)^3)/(4*(c + d*x)^7) - (F^a*F^(b/(c + d*x)^2)*b^4*log(F)^4
)/(2*(c + d*x)^9) + (315*F^a*F^(b/(c + d*x)^2)*b*log(F))/(16*(c + d*x)^3))
/(b^5*d*log(F)^5)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{12}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^12,x)
```

output
```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)*b - 13*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
 + d**2*x**2))*c**2 - 26*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
+ 2*c*d*x + d**2*x**2))*c*d*x - 13*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)/(c**2 + 2*c*d*x + d**2*x**2))*d**2*x**2 + 4*int(f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**16 + 16*c**15*d*x +
 120*c**14*d**2*x**2 + 560*c**13*d**3*x**3 + 1820*c**12*d**4*x**4 + 4368*c
**11*d**5*x**5 + 8008*c**10*d**6*x**6 + 11440*c**9*d**7*x**7 + 12870*c**8*
d**8*x**8 + 11440*c**7*d**9*x**9 + 8008*c**6*d**10*x**10 + 4368*c**5*d**11
*x**11 + 1820*c**4*d**12*x**12 + 560*c**3*d**13*x**13 + 120*c**2*d**14*x**
14 + 16*c*d**15*x**15 + d**16*x**16),x)*log(f)**2*b**2*c**13*d + 52*int(f*
*((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c*
*16 + 16*c**15*d*x + 120*c**14*d**2*x**2 + 560*c**13*d**3*x**3 + 1820*c**1
2*d**4*x**4 + 4368*c**11*d**5*x**5 + 8008*c**10*d**6*x**6 + 11440*c**9*d**
7*x**7 + 12870*c**8*d**8*x**8 + 11440*c**7*d**9*x**9 + 8008*c**6*d**10*x**
10 + 4368*c**5*d**11*x**11 + 1820*c**4*d**12*x**12 + 560*c**3*d**13*x**13
+ 120*c**2*d**14*x**14 + 16*c*d**15*x**15 + d**16*x**16),x)*log(f)**2*b**2
*c**12*d**2*x + 312*int(f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 +
 2*c*d*x + d**2*x**2))/(c**16 + 16*c**15*d*x + 120*c**14*d**2*x**2 + 560*c
**13*d**3*x**3 + 1820*c**12*d**4*x**4 + 4368*c**11*d**5*x**5 + 8008*c**...
```

3.273 $\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}} dx = \frac{F^a \Gamma\left(\frac{13}{2}, -\frac{b \log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13} \left(-\frac{b \log(F)}{(c+dx)^2}\right)^{13/2}}$$

output

```
1/2*F^a*(524288/5621533568633696205238621875*GAMMA(51/2,-b*ln(F)/(d*x+c)^2
)-524288/5621533568633696205238621875*(-b*ln(F)/(d*x+c)^2)^(49/2)*exp(b*ln
(F)/(d*x+c)^2)-262144/114725174870075432759971875*(-b*ln(F)/(d*x+c)^2)^(47
/2)*exp(b*ln(F)/(d*x+c)^2)-131072/2440961167448413462978125*(-b*ln(F)/(d*x
+c)^2)^(45/2)*exp(b*ln(F)/(d*x+c)^2)-65536/54243581498853632510625*(-b*ln(
F)/(d*x+c)^2)^(43/2)*exp(b*ln(F)/(d*x+c)^2)-32768/1261478639508224011875*(
-b*ln(F)/(d*x+c)^2)^(41/2)*exp(b*ln(F)/(d*x+c)^2)-16384/307677716953225368
75*(-b*ln(F)/(d*x+c)^2)^(39/2)*exp(b*ln(F)/(d*x+c)^2)-8192/788917222956988
125*(-b*ln(F)/(d*x+c)^2)^(37/2)*exp(b*ln(F)/(d*x+c)^2)-4096/21322087106945
625*(-b*ln(F)/(d*x+c)^2)^(35/2)*exp(b*ln(F)/(d*x+c)^2)-2048/60920248876987
5*(-b*ln(F)/(d*x+c)^2)^(33/2)*exp(b*ln(F)/(d*x+c)^2)-1024/18460681477875*(
-b*ln(F)/(d*x+c)^2)^(31/2)*exp(b*ln(F)/(d*x+c)^2)-512/595505854125*(-b*ln(
F)/(d*x+c)^2)^(29/2)*exp(b*ln(F)/(d*x+c)^2)-256/20534684625*(-b*ln(F)/(d*x
+c)^2)^(27/2)*exp(b*ln(F)/(d*x+c)^2)-128/760543875*(-b*ln(F)/(d*x+c)^2)^(2
5/2)*exp(b*ln(F)/(d*x+c)^2)-64/30421755*(-b*ln(F)/(d*x+c)^2)^(23/2)*exp(b*
ln(F)/(d*x+c)^2)-32/1322685*(-b*ln(F)/(d*x+c)^2)^(21/2)*exp(b*ln(F)/(d*x+c
)^2)-16/62985*(-b*ln(F)/(d*x+c)^2)^(19/2)*exp(b*ln(F)/(d*x+c)^2)-8/3315*(-
b*ln(F)/(d*x+c)^2)^(17/2)*exp(b*ln(F)/(d*x+c)^2)-4/195*(-b*ln(F)/(d*x+c)^2
)^(15/2)*exp(b*ln(F)/(d*x+c)^2)-2/13*(-b*ln(F)/(d*x+c)^2)^(13/2)*exp(b*ln(
F)/(d*x+c)^2))/d/(d*x+c)^13/(-b*ln(F)/(d*x+c)^2)^(13/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = \frac{F^a\Gamma\left(\frac{13}{2},-\frac{b\log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13}\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{13/2}}$$

input

```
Integrate[F^(a + b/(c + d*x)^2)/(c + d*x)^14,x]
```

output

```
(F^a*Gamma[13/2, -((b*Log[F])/(c + d*x)^2)])/(2*d*(c + d*x)^13*(-((b*Log[F
])/(c + d*x)^2))^(13/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{F^a\Gamma\left(\frac{13}{2},-\frac{b\log(F)}{(c+dx)^2}\right)}{2d(c+dx)^{13}\left(-\frac{b\log(F)}{(c+dx)^2}\right)^{13/2}}$$

input
```
Int[F^(a + b/(c + d*x)^2)/(c + d*x)^14,x]
```

output
```
(F^a*Gamma[13/2, -((b*Log[F])/(c + d*x)^2)])/(2*d*(c + d*x)^13*(-((b*Log[F
])/(c + d*x)^2))^(13/2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 10.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

method	result
risch	$-\frac{F^{a} F^{\frac{b}{(dx+c)^{2}}}}{2d(dx+c)^{11} b\ln(F)}+\frac{11F^{a} F^{\frac{b}{(dx+c)^{2}}}}{4d\,b^{2} \ln(F)^{2} (dx+c)^{9}}-\frac{99F^{a} F^{\frac{b}{(dx+c)^{2}}}}{8d\,b^{3} \ln(F)^{3} (dx+c)^{7}}+\frac{693F^{a} F^{\frac{b}{(dx+c)^{2}}}}{16d\,b^{4} \ln(F)^{4} (dx+c)^{5}}-\frac{3465F^{a} F^{\frac{b}{(dx+c)^{2}}}}{32d\,b^{5} \ln(F)^{5} (dx+c)^{3}}+$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^14,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^a/d*F^(b/(d*x+c)^2)/(d*x+c)^11/b/ln(F)+11/4*F^a/d/b^2/ln(F)^2*F^(b/
(d*x+c)^2)/(d*x+c)^9-99/8*F^a/d/b^3/ln(F)^3*F^(b/(d*x+c)^2)/(d*x+c)^7+693/
16*F^a/d/b^4/ln(F)^4*F^(b/(d*x+c)^2)/(d*x+c)^5-3465/32*F^a/d/b^5/ln(F)^5*F
^(b/(d*x+c)^2)/(d*x+c)^3+10395/64*F^a/d/b^6/ln(F)^6*F^(b/(d*x+c)^2)/(d*x+c
)-10395/128*F^a/d/b^6/ln(F)^6*Pi^(1/2)/(-b*ln(F))^(1/2)*erf((-b*ln(F))^(1/
2)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 791, normalized size of antiderivative = 16.14

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^14,x, algorithm="fricas")
```

output
```
1/128*(10395*sqrt(pi)*(d^12*x^11 + 11*c*d^11*x^10 + 55*c^2*d^10*x^9 + 165*
c^3*d^9*x^8 + 330*c^4*d^8*x^7 + 462*c^5*d^7*x^6 + 462*c^6*d^6*x^5 + 330*c^
7*d^5*x^4 + 165*c^8*d^4*x^3 + 55*c^9*d^3*x^2 + 11*c^10*d^2*x + c^11*d)*F^a
*sqrt(-b*log(F)/d^2)*erf(d*sqrt(-b*log(F)/d^2)/(d*x + c)) - 2*(32*b^6*log(
F)^6 - 176*(b^5*d^2*x^2 + 2*b^5*c*d*x + b^5*c^2)*log(F)^5 + 792*(b^4*d^4*x
^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(F)
^4 - 2772*(b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3
*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^3 + 6930*(
b^2*d^8*x^8 + 8*b^2*c*d^7*x^7 + 28*b^2*c^2*d^6*x^6 + 56*b^2*c^3*d^5*x^5 +
70*b^2*c^4*d^4*x^4 + 56*b^2*c^5*d^3*x^3 + 28*b^2*c^6*d^2*x^2 + 8*b^2*c^7*d
*x + b^2*c^8)*log(F)^2 - 10395*(b*d^10*x^10 + 10*b*c*d^9*x^9 + 45*b*c^2*d^
8*x^8 + 120*b*c^3*d^7*x^7 + 210*b*c^4*d^6*x^6 + 252*b*c^5*d^5*x^5 + 210*b*
c^6*d^4*x^4 + 120*b*c^7*d^3*x^3 + 45*b*c^8*d^2*x^2 + 10*b*c^9*d*x + b*c^10
)*log(F))*F^((a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b)/(d^2*x^2 + 2*c*d*x + c^2)
))/((b^7*d^12*x^11 + 11*b^7*c*d^11*x^10 + 55*b^7*c^2*d^10*x^9 + 165*b^7*c^
3*d^9*x^8 + 330*b^7*c^4*d^8*x^7 + 462*b^7*c^5*d^7*x^6 + 462*b^7*c^6*d^6*x^
5 + 330*b^7*c^7*d^5*x^4 + 165*b^7*c^8*d^4*x^3 + 55*b^7*c^9*d^3*x^2 + 11*b^
7*c^10*d^2*x + b^7*c^11*d)*log(F)^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**2)/(d*x+c)**14,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{14}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^14,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^14, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^2}}}{(dx+c)^{14}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^2)/(d*x+c)^14,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^2)/(d*x + c)^14, x)
```

Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 217, normalized size of antiderivative = 4.43

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = -\frac{\frac{F^a\left(\frac{10395\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,\ln(F)}{\sqrt{b\,\ln(F)}\,(c+d\,x)}\right)}{128}-\frac{10395\,F^{\frac{b}{(c+d\,x)^2}}\sqrt{b\,\ln(F)}}{64\,(c+d\,x)}\right)}{\sqrt{b\,\ln(F)}}-\frac{693\,F^{a+\frac{b}{(c+d\,x)^2}}\,b^2\ln(F)^2}{16\,(c+d\,x)^5}+\frac{99\,F^{a+\frac{b}{(c+d\,x)^2}}\,b^3\ln(F)^3}{8\,(c+d\,x)^7}-\frac{11\,F^{a+}}{}}{b^6\,d\ln\left(F\right)^6}$$

input
```
int(F^(a + b/(c + d*x)^2)/(c + d*x)^14,x)
```

output
```
-((F^a*((10395*pi^(1/2)*erfi((b*log(F))/((b*log(F))^(1/2)*(c + d*x))))/128
 - (10395*F^(b/(c + d*x)^2)*(b*log(F))^(1/2))/(64*(c + d*x))))/(b*log(F))^
(1/2) - (693*F^(a + b/(c + d*x)^2)*b^2*log(F)^2)/(16*(c + d*x)^5) + (99*F^
(a + b/(c + d*x)^2)*b^3*log(F)^3)/(8*(c + d*x)^7) - (11*F^(a + b/(c + d*x)
^2)*b^4*log(F)^4)/(4*(c + d*x)^9) + (F^(a + b/(c + d*x)^2)*b^5*log(F)^5)/(
2*(c + d*x)^11) + (3465*F^(a + b/(c + d*x)^2)*b*log(F))/(32*(c + d*x)^3))/
(b^6*d*log(F)^6)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^2}}}{(c+dx)^{14}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^2)/(d*x+c)^14,x)
```

output
```
(2*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2)
)*log(f)*b - 15*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x
 + d**2*x**2))*c**2 - 30*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 + b)/(c**2
+ 2*c*d*x + d**2*x**2))*c*d*x - 15*f**((a*c**2 + 2*a*c*d*x + a*d**2*x**2 +
 b)/(c**2 + 2*c*d*x + d**2*x**2))*d**2*x**2 + 4*int(f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**18 + 18*c**17*d*x +
 153*c**16*d**2*x**2 + 816*c**15*d**3*x**3 + 3060*c**14*d**4*x**4 + 8568*c
**13*d**5*x**5 + 18564*c**12*d**6*x**6 + 31824*c**11*d**7*x**7 + 43758*c**
10*d**8*x**8 + 48620*c**9*d**9*x**9 + 43758*c**8*d**10*x**10 + 31824*c**7*
d**11*x**11 + 18564*c**6*d**12*x**12 + 8568*c**5*d**13*x**13 + 3060*c**4*d
**14*x**14 + 816*c**3*d**15*x**15 + 153*c**2*d**16*x**16 + 18*c*d**17*x**1
7 + d**18*x**18),x)*log(f)**2*b**2*c**15*d + 60*int(f**((a*c**2 + 2*a*c*d*
x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**18 + 18*c**17*d*x +
 153*c**16*d**2*x**2 + 816*c**15*d**3*x**3 + 3060*c**14*d**4*x**4 + 8568*c
**13*d**5*x**5 + 18564*c**12*d**6*x**6 + 31824*c**11*d**7*x**7 + 43758*c**
10*d**8*x**8 + 48620*c**9*d**9*x**9 + 43758*c**8*d**10*x**10 + 31824*c**7*
d**11*x**11 + 18564*c**6*d**12*x**12 + 8568*c**5*d**13*x**13 + 3060*c**4*d
**14*x**14 + 816*c**3*d**15*x**15 + 153*c**2*d**16*x**16 + 18*c*d**17*x**1
7 + d**18*x**18),x)*log(f)**2*b**2*c**14*d**2*x + 420*int(f**((a*c**2 + 2*
a*c*d*x + a*d**2*x**2 + b)/(c**2 + 2*c*d*x + d**2*x**2))/(c**18 + 18*c*...
```

3.274 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx=\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1}{3}(-1-m),-\frac{b\log(F)}{(c+dx)^3}\right)\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{\frac{1+m}{3}}}{3d}$$

output
```
1/3*F^a*(d*x+c)^(1+m)*GAMMA(-1/3-1/3*m,-b*ln(F)/(d*x+c)^3)*(-b*ln(F)/(d*x+
c)^3)^(1/3+1/3*m)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx=\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1}{3}(-1-m),-\frac{b\log(F)}{(c+dx)^3}\right)\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{\frac{1+m}{3}}}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^m,x]
```

output
```
(F^a*(c + d*x)^(1 + m)*Gamma[(-1 - m)/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*
Log[F])/(c + d*x)^3))^((1 + m)/3))/(3*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m F^{a+\frac{b}{(c+dx)^3}} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{F^a (c+dx)^{m+1} \left(-\frac{b\log(F)}{(c+dx)^3}\right)^{\frac{m+1}{3}} \Gamma\left(\frac{1}{3}(-m-1),-\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^m,x]
```

output
```
(F^a*(c + d*x)^(1 + m)*Gamma[(-1 - m)/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*
Log[F])/(c + d*x)^3))^((1 + m)/3))/(3*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^m\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x)
```

Fricas [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx=\int (dx+c)^m F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 +
 b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)), x)
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx=\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**m,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**m, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx=\int (dx+c)^m F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*F^(a + b/(d*x + c)^3), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx=\int (dx+c)^m F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx = \frac{F^a\,\mathrm{e}^{\frac{b\,\ln(F)}{2\,(c+d\,x)^3}}\,(c+d\,x)^{m+1}\,\mathrm{M}_{\frac{m}{6}+\frac{2}{3},-\frac{m}{6}-\frac{1}{6}}\left(\frac{b\,\ln(F)}{(c+d\,x)^3}\right)\,\left(\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{\frac{m}{6}-\frac{1}{3}}}{d\,(m+1)}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^m,x)
```

output
```
(F^a*exp((b*log(F))/(2*(c + d*x)^3))*(c + d*x)^(m + 1)*whittakerM(m/6 + 2/
3, - m/6 - 1/6, (b*log(F))/(c + d*x)^3)*((b*log(F))/(c + d*x)^3)^(m/6 - 1/
3))/(d*(m + 1))
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^m\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^m,x)
```

output
```
(9*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)**2*b**2 + 3*
f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*c**3*m - 15*f
**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c
**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*c**3 + 9*f**((
a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*c**2*d*m*x - 45*f*
*((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*c**2*d*x + 9*f*
*((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*c*d**2*m*x**2 -
 45*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*c*d**2*x*
*2 + 3*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c*
*3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*d**3*m
*x**3 - 15*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)
/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*log(f)*b*d*
*3*x**3 + f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/
(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*(c + d*x)**m*c**6*m**2...
```

3.275 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx=-\frac{b^5F^a\Gamma\left(-5,-\frac{b\log(F)}{(c+dx)^3}\right)\log^5(F)}{3d}$$

output
```
1/3*F^a*(d*x+c)^15*Ei(6,-b*ln(F)/(d*x+c)^3)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx=-\frac{b^5F^a\Gamma\left(-5,-\frac{b\log(F)}{(c+dx)^3}\right)\log^5(F)}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^14,x]
```

output
```
-1/3*(b^5*F^a*Gamma[-5, -((b*Log[F])/(c + d*x)^3)]*Log[F]^5)/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{14}F^{a+\frac{b}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{b^5F^a\log^5(F)\Gamma\left(-5,-\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^14,x]
```

output
```
-1/3*(b^5*F^a*Gamma[-5, -((b*Log[F])/(c + d*x)^3)]*Log[F]^5)/d
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^{14}\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 686, normalized size of antiderivative = 22.13

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx =$$

$$-\frac{F^a b^5 \mathrm{Ei}\left(\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)\log(F)^5 - \left(24\,d^{15}x^{15} + 360\,cd^{14}x^{14} + 2520\,c^2d^{13}x^{13} + 10920\,c^3d^{12}x^{12} + \right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x, algorithm="fricas")
```

output
```
-1/360*(F^a*b^5*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log
(F)^5 - (24*d^15*x^15 + 360*c*d^14*x^14 + 2520*c^2*d^13*x^13 + 10920*c^3*d
^12*x^12 + 32760*c^4*d^11*x^11 + 72072*c^5*d^10*x^10 + 120120*c^6*d^9*x^9
+ 154440*c^7*d^8*x^8 + 154440*c^8*d^7*x^7 + 120120*c^9*d^6*x^6 + 72072*c^1
0*d^5*x^5 + 32760*c^11*d^4*x^4 + 10920*c^12*d^3*x^3 + 2520*c^13*d^2*x^2 +
360*c^14*d*x + 24*c^15 + (b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x +
b^4*c^3)*log(F)^4 + (b^3*d^6*x^6 + 6*b^3*c*d^5*x^5 + 15*b^3*c^2*d^4*x^4 +
20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*c^5*d*x + b^3*c^6)*log(F)^
3 + 2*(b^2*d^9*x^9 + 9*b^2*c*d^8*x^8 + 36*b^2*c^2*d^7*x^7 + 84*b^2*c^3*d^6
*x^6 + 126*b^2*c^4*d^5*x^5 + 126*b^2*c^5*d^4*x^4 + 84*b^2*c^6*d^3*x^3 + 36
*b^2*c^7*d^2*x^2 + 9*b^2*c^8*d*x + b^2*c^9)*log(F)^2 + 6*(b*d^12*x^12 + 12
*b*c*d^11*x^11 + 66*b*c^2*d^10*x^10 + 220*b*c^3*d^9*x^9 + 495*b*c^4*d^8*x^
8 + 792*b*c^5*d^7*x^7 + 924*b*c^6*d^6*x^6 + 792*b*c^7*d^5*x^5 + 495*b*c^8*
d^4*x^4 + 220*b*c^9*d^3*x^3 + 66*b*c^10*d^2*x^2 + 12*b*c^11*d*x + b*c^12)*
log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3
+ 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx = \int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**14,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**14, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx=\int (dx+c)^{14}F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x, algorithm="maxima")
```

output
```
1/360*(24*F^a*d^14*x^15 + 360*F^a*c*d^13*x^14 + 2520*F^a*c^2*d^12*x^13 + 6
*(1820*F^a*c^3*d^11 + F^a*b*d^11*log(F))*x^12 + 72*(455*F^a*c^4*d^10 + F^a
*b*c*d^10*log(F))*x^11 + 396*(182*F^a*c^5*d^9 + F^a*b*c^2*d^9*log(F))*x^10
 + 2*(60060*F^a*c^6*d^8 + 660*F^a*b*c^3*d^8*log(F) + F^a*b^2*d^8*log(F)^2)
*x^9 + 18*(8580*F^a*c^7*d^7 + 165*F^a*b*c^4*d^7*log(F) + F^a*b^2*c*d^7*log
(F)^2)*x^8 + 72*(2145*F^a*c^8*d^6 + 66*F^a*b*c^5*d^6*log(F) + F^a*b^2*c^2*
d^6*log(F)^2)*x^7 + (120120*F^a*c^9*d^5 + 5544*F^a*b*c^6*d^5*log(F) + 168*
F^a*b^2*c^3*d^5*log(F)^2 + F^a*b^3*d^5*log(F)^3)*x^6 + 6*(12012*F^a*c^10*d
^4 + 792*F^a*b*c^7*d^4*log(F) + 42*F^a*b^2*c^4*d^4*log(F)^2 + F^a*b^3*c*d^
4*log(F)^3)*x^5 + 3*(10920*F^a*c^11*d^3 + 990*F^a*b*c^8*d^3*log(F) + 84*F^
a*b^2*c^5*d^3*log(F)^2 + 5*F^a*b^3*c^2*d^3*log(F)^3)*x^4 + (10920*F^a*c^12
*d^2 + 1320*F^a*b*c^9*d^2*log(F) + 168*F^a*b^2*c^6*d^2*log(F)^2 + 20*F^a*b
^3*c^3*d^2*log(F)^3 + F^a*b^4*d^2*log(F)^4)*x^3 + 3*(840*F^a*c^13*d + 132*
F^a*b*c^10*d*log(F) + 24*F^a*b^2*c^7*d*log(F)^2 + 5*F^a*b^3*c^4*d*log(F)^3
 + F^a*b^4*c*d*log(F)^4)*x^2 + 3*(120*F^a*c^14 + 24*F^a*b*c^11*log(F) + 6*
F^a*b^2*c^8*log(F)^2 + 2*F^a*b^3*c^5*log(F)^3 + F^a*b^4*c^2*log(F)^4)*x)*F
^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(-1/120*(24*F^a*
b*c^15*log(F) + 6*F^a*b^2*c^12*log(F)^2 - F^a*b^5*d^3*x^3*log(F)^5 + 2*F^a
*b^3*c^9*log(F)^3 - 3*F^a*b^5*c*d^2*x^2*log(F)^5 + F^a*b^4*c^6*log(F)^4 -
3*F^a*b^5*c^2*d*x*log(F)^5)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c...
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx=\int (dx+c)^{14}F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x, algorithm="giac")
```

output
```
integrate((d*x + c)^14*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 4.39

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx$$
$$=\frac{F^a\,b^5\ln(F)^5\,\mathrm{expint}\left(-\frac{b\ln(F)}{(c+d\,x)^3}\right)}{360\,d}+\frac{F^a\,F^{\frac{b}{(c+d\,x)^3}}\,b^5\ln(F)^5\left(\frac{(c+d\,x)^3}{120\,b\,\ln(F)}+\frac{(c+d\,x)^6}{120\,b^2\ln(F)^2}+\frac{(c+d\,x)^9}{60\,b^3\ln(F)^3}+\frac{(c+d\,x)^{12}}{20\,b^4\ln(F)^4}+\frac{(c+d\,x)^{15}}{5\,b^5\ln(F)^5}\right)}{3\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^14,x)
```

output
```
(F^a*b^5*log(F)^5*expint(-(b*log(F))/(c + d*x)^3))/(360*d) + (F^a*F^(b/(c
+ d*x)^3)*b^5*log(F)^5*((c + d*x)^3/(120*b*log(F)) + (c + d*x)^6/(120*b^2*
log(F)^2) + (c + d*x)^9/(60*b^3*log(F)^3) + (c + d*x)^12/(20*b^4*log(F)^4)
 + (c + d*x)^15/(5*b^5*log(F)^5)))/(3*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{14}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^14,x)
```

output
```
(270*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**6*b**6*c**2*d*x + 270*
f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**6*b**6*c*d**2*x**2 + 90*f**
((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c**6 + 378*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c**5*d*x - 675*f**((a*c**3 + 3*a
*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d*
*2*x**2 + d**3*x**3))*log(f)**5*b**5*c**4*d**2*x**2 - 1800*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c**3*d**3*x**3 - 1350*f**((a*c**3 +
 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c**2*d**4*x**4 - 360*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c*d**5*x**5 - 54*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c**9 - 1242*f**((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*log(f)**4*b**4*c**8*d*x - 4302*f**((a*c**3 + 3*a*c**2*d...
```

3.276 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx=\frac{b^4F^a\Gamma\left(-4,-\frac{b\log(F)}{(c+dx)^3}\right)\log^4(F)}{3d}$$

output
```
1/3*F^a*(d*x+c)^12*Ei(5,-b*ln(F)/(d*x+c)^3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx=\frac{b^4F^a\Gamma\left(-4,-\frac{b\log(F)}{(c+dx)^3}\right)\log^4(F)}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^11,x]
```

output
```
(b^4*F^a*Gamma[-4, -((b*Log[F])/(c + d*x)^3)]*Log[F]^4)/(3*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{11}F^{a+\frac{b}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \ 2648$$

$$\frac{b^4F^a\log^4(F)\Gamma\left(-4,-\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^11,x]
```

output
```
(b^4*F^a*Gamma[-4, -((b*Log[F])/(c + d*x)^3)]*Log[F]^4)/(3*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^{11}\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 487, normalized size of antiderivative = 15.71

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx =$$

$$-\frac{F^a b^4 \text{Ei}\left(\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)\log(F)^4 - \left(6\,d^{12}x^{12}+72\,cd^{11}x^{11}+396\,c^2d^{10}x^{10}+1320\,c^3d^9x^9+2970\,c\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x, algorithm="fricas")
```

output
```
-1/72*(F^a*b^4*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log(
F)^4 - (6*d^12*x^12 + 72*c*d^11*x^11 + 396*c^2*d^10*x^10 + 1320*c^3*d^9*x^
9 + 2970*c^4*d^8*x^8 + 4752*c^5*d^7*x^7 + 5544*c^6*d^6*x^6 + 4752*c^7*d^5*
x^5 + 2970*c^8*d^4*x^4 + 1320*c^9*d^3*x^3 + 396*c^10*d^2*x^2 + 72*c^11*d*x
 + 6*c^12 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(
F)^3 + (b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^2*c^2*d^4*x^4 + 20*b^2*c^3*d^
3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x + b^2*c^6)*log(F)^2 + 2*(b*d^9*
x^9 + 9*b*c*d^8*x^8 + 36*b*c^2*d^7*x^7 + 84*b*c^3*d^6*x^6 + 126*b*c^4*d^5*
x^5 + 126*b*c^5*d^4*x^4 + 84*b*c^6*d^3*x^3 + 36*b*c^7*d^2*x^2 + 9*b*c^8*d*
x + b*c^9)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b
)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx = \int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**11,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**11, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx=\int (dx+c)^{11}F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x, algorithm="maxima")
```

output
```
1/72*(6*F^a*d^11*x^12 + 72*F^a*c*d^10*x^11 + 396*F^a*c^2*d^9*x^10 + 2*(660
*F^a*c^3*d^8 + F^a*b*d^8*log(F))*x^9 + 18*(165*F^a*c^4*d^7 + F^a*b*c*d^7*l
og(F))*x^8 + 72*(66*F^a*c^5*d^6 + F^a*b*c^2*d^6*log(F))*x^7 + (5544*F^a*c^
6*d^5 + 168*F^a*b*c^3*d^5*log(F) + F^a*b^2*d^5*log(F)^2)*x^6 + 6*(792*F^a*
c^7*d^4 + 42*F^a*b*c^4*d^4*log(F) + F^a*b^2*c*d^4*log(F)^2)*x^5 + 3*(990*F
^a*c^8*d^3 + 84*F^a*b*c^5*d^3*log(F) + 5*F^a*b^2*c^2*d^3*log(F)^2)*x^4 + (
1320*F^a*c^9*d^2 + 168*F^a*b*c^6*d^2*log(F) + 20*F^a*b^2*c^3*d^2*log(F)^2
+ F^a*b^3*d^2*log(F)^3)*x^3 + 3*(132*F^a*c^10*d + 24*F^a*b*c^7*d*log(F) +
5*F^a*b^2*c^4*d*log(F)^2 + F^a*b^3*c*d*log(F)^3)*x^2 + 3*(24*F^a*c^11 + 6*
F^a*b*c^8*log(F) + 2*F^a*b^2*c^5*log(F)^2 + F^a*b^3*c^2*log(F)^3)*x)*F^(b/
(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(-1/24*(6*F^a*b*c^12
*log(F) - F^a*b^4*d^3*x^3*log(F)^4 + 2*F^a*b^2*c^9*log(F)^2 - 3*F^a*b^4*c*
d^2*x^2*log(F)^4 + F^a*b^3*c^6*log(F)^3 - 3*F^a*b^4*c^2*d*x*log(F)^4)*F^(b
/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2
*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx=\int (dx+c)^{11}F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x, algorithm="giac")
```

output
```
integrate((d*x + c)^11*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.87

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx$$
$$=\frac{F^a\,b^4\ln{(F)}^4\,\mathrm{expint}\left(-\frac{b\ln(F)}{(c+d\,x)^3}\right)}{72\,d}$$
$$+\frac{F^a\,F^{\frac{b}{(c+d\,x)^3}}\,b^4\ln{(F)}^4\left(\frac{(c+d\,x)^3}{24\,b\,\ln(F)}+\frac{(c+d\,x)^6}{24\,b^2\ln(F)^2}+\frac{(c+d\,x)^9}{12\,b^3\ln(F)^3}+\frac{(c+d\,x)^{12}}{4\,b^4\ln(F)^4}\right)}{3\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^11,x)
```

output
```
(F^a*b^4*log(F)^4*expint(-(b*log(F))/(c + d*x)^3))/(72*d) + (F^a*F^(b/(c +
 d*x)^3)*b^4*log(F)^4*((c + d*x)^3/(24*b*log(F)) + (c + d*x)^6/(24*b^2*log
(F)^2) + (c + d*x)^9/(12*b^3*log(F)^3) + (c + d*x)^12/(4*b^4*log(F)^4)))/(
3*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^{11}\,dx=\text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^11,x)
```

output

```
(270*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c**2*d*x + 270*
f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**5*b**5*c*d**2*x**2 + 90*f**
((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c**6 + 378*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c**5*d*x - 675*f**((a*c**3 + 3*a
*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d*
*2*x**2 + d**3*x**3))*log(f)**4*b**4*c**4*d**2*x**2 - 1800*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c**3*d**3*x**3 - 1350*f**((a*c**3 +
 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c**2*d**4*x**4 - 360*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c*d**5*x**5 - 54*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**9 - 1242*f**((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*log(f)**3*b**3*c**8*d*x - 4302*f**((a*c**3 + 3*a*c**2*d...
```

3.277 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx$

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \frac{F^{a+\frac{b}{(c+dx)^3}}(c+dx)^9}{9d} + \frac{bF^{a+\frac{b}{(c+dx)^3}}(c+dx)^6\log(F)}{18d} + \frac{b^2F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\log^2(F)}{18d} - \frac{b^3F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)\log^3(F)}{18d}$$

output
```
1/9*F^(a+b/(d*x+c)^3)*(d*x+c)^9/d+1/18*b*F^(a+b/(d*x+c)^3)*(d*x+c)^6*ln(F)
/d+1/18*b^2*F^(a+b/(d*x+c)^3)*(d*x+c)^3*ln(F)^2/d-1/18*b^3*F^a*Ei(b*ln(F)/
(d*x+c)^3)*ln(F)^3/d
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.79

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \frac{F^a\left(2F^{\frac{b}{(c+dx)^3}}(c+dx)^9+b\log(F)\left(F^{\frac{b}{(c+dx)^3}}(c+dx)^6+b\log(F)\left(F^{\frac{b}{(c+dx)^3}}(c+dx)^3-b\,\text{ExpIntegralEi}\left(\right.\right.\right.\right.}{18d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^8,x]
```

output
```
(F^a*(2*F^(b/(c + d*x)^3)*(c + d*x)^9 + b*Log[F]*(F^(b/(c + d*x)^3)*(c + d
*x)^6 + b*Log[F]*(F^(b/(c + d*x)^3)*(c + d*x)^3 - b*ExpIntegralEi[(b*Log[F
])/(c + d*x)^3]*Log[F]))))/(18*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2643, 2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^8F^{a+\frac{b}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(F)\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5dx+\frac{(c+dx)^9F^{a+\frac{b}{(c+dx)^3}}}{9d}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2dx+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^3}}}{6d}\right)+\frac{(c+dx)^9F^{a+\frac{b}{(c+dx)^3}}}{9d}$$

$$\Big\downarrow \text{2643}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\left(b\log(F)\int\frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}dx+\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^3}}}{3d}\right)+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^3}}}{6d}\right)+\frac{(c+dx)^9F^{a+\frac{b}{(c+dx)^3}}}{9d}$$

$$\Big\downarrow \text{2639}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\left(\frac{(c+dx)^3F^{a+\frac{b}{(c+dx)^3}}}{3d}-\frac{bF^a\log(F)\operatorname{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)}{3d}\right)+\frac{(c+dx)^6F^{a+\frac{b}{(c+dx)^3}}}{6d}\right)+\frac{(c+dx)^9F^{a+\frac{b}{(c+dx)^3}}}{9d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^8,x]
```

output
```
(F^(a + b/(c + d*x)^3)*(c + d*x)^9)/(9*d) + (b*Log[F]*((F^(a + b/(c + d*x)
^3)*(c + d*x)^6)/(6*d) + (b*Log[F]*((F^(a + b/(c + d*x)^3)*(c + d*x)^3)/(3
*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^3]*Log[F])/(3*d)))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^8\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(113) = 226$.

Time = 0.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.73

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = -\frac{F^a b^3 \mathrm{Ei}\left(\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)\log(F)^3-\left(2\,d^9x^9+18\,cd^8x^8+72\,c^2d^7x^7+168\,c^3d^6x^6+252\,c^4d^5x^5+\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x, algorithm="fricas")
```

output
```
-1/18*(F^a*b^3*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log(
F)^3 - (2*d^9*x^9 + 18*c*d^8*x^8 + 72*c^2*d^7*x^7 + 168*c^3*d^6*x^6 + 252*
c^4*d^5*x^5 + 252*c^5*d^4*x^4 + 168*c^6*d^3*x^3 + 72*c^7*d^2*x^2 + 18*c^8*
d*x + 2*c^9 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*lo
g(F)^2 + (b*d^6*x^6 + 6*b*c*d^5*x^5 + 15*b*c^2*d^4*x^4 + 20*b*c^3*d^3*x^3
+ 15*b*c^4*d^2*x^2 + 6*b*c^5*d*x + b*c^6)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^
2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**8,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**8, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \int (dx+c)^8 F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x, algorithm="maxima")
```

output
```
1/18*(2*F^a*d^8*x^9 + 18*F^a*c*d^7*x^8 + 72*F^a*c^2*d^6*x^7 + (168*F^a*c^3
*d^5 + F^a*b*d^5*log(F))*x^6 + 6*(42*F^a*c^4*d^4 + F^a*b*c*d^4*log(F))*x^5
 + 3*(84*F^a*c^5*d^3 + 5*F^a*b*c^2*d^3*log(F))*x^4 + (168*F^a*c^6*d^2 + 20
*F^a*b*c^3*d^2*log(F) + F^a*b^2*d^2*log(F)^2)*x^3 + 3*(24*F^a*c^7*d + 5*F^
a*b*c^4*d*log(F) + F^a*b^2*c*d*log(F)^2)*x^2 + 3*(6*F^a*c^8 + 2*F^a*b*c^5*
log(F) + F^a*b^2*c^2*log(F)^2)*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3)) + integrate(1/6*(F^a*b^3*d^3*x^3*log(F)^3 - 2*F^a*b*c^9*log(F) + 3
*F^a*b^3*c*d^2*x^2*log(F)^3 - F^a*b^2*c^6*log(F)^2 + 3*F^a*b^3*c^2*d*x*log
(F)^3)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*
x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \int (dx+c)^8 F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x, algorithm="giac")
```

output
```
integrate((d*x + c)^8*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.76

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \frac{F^a\,b^3\ln(F)^3\left(\frac{\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)}{6}+F^{\frac{b}{(c+d\,x)^3}}\left(\frac{(c+d\,x)^3}{6\,b\,\ln(F)}+\frac{(c+d\,x)^6}{6\,b^2\ln(F)^2}+\frac{(c+d\,x)^9}{3\,b^3\ln(F)^3}\right)\right)}{3\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^8,x)
```

output
```
(F^a*b^3*log(F)^3*(expint(-(b*log(F))/(c + d*x)^3)/6 + F^(b/(c + d*x)^3)*(
(c + d*x)^3/(6*b*log(F)) + (c + d*x)^6/(6*b^2*log(F)^2) + (c + d*x)^9/(3*b
^3*log(F)^3))))/(3*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^8\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^8,x)
```

output
```
(270*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c**2*d*x + 270*
f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**4*b**4*c*d**2*x**2 + 90*f**
((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**6 + 378*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**5*d*x - 675*f**((a*c**3 + 3*a
*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d*
*2*x**2 + d**3*x**3))*log(f)**3*b**3*c**4*d**2*x**2 - 1800*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**3*d**3*x**3 - 1350*f**((a*c**3 +
 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**2*d**4*x**4 - 360*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c*d**5*x**5 - 54*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**9 - 1242*f**((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*log(f)**2*b**2*c**8*d*x - 4302*f**((a*c**3 + 3*a*c**2*d...
```

3.278 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 87

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\frac{F^{a+\frac{b}{(c+dx)^3}}(c+dx)^6}{6d}+\frac{bF^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\log(F)}{6d}-\frac{b^2F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)\log^2(F)}{6d}$$

output
```
1/6*F^(a+b/(d*x+c)^3)*(d*x+c)^6/d+1/6*b*F^(a+b/(d*x+c)^3)*(d*x+c)^3*ln(F)/
d-1/6*b^2*F^a*Ei(b*ln(F)/(d*x+c)^3)*ln(F)^2/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\frac{F^a\left(F^{\frac{b}{(c+dx)^3}}(c+dx)^6+b\log(F)\left(F^{\frac{b}{(c+dx)^3}}(c+dx)^3-b\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)\log(F)\right)\right)}{6d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^5,x]
```

output
```
(F^a*(F^(b/(c + d*x)^3)*(c + d*x)^6 + b*Log[F]*(F^(b/(c + d*x)^3)*(c + d*x
)^3 - b*ExpIntegralEi[(b*Log[F])/(c + d*x)^3]*Log[F])))/(6*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2643, 2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^5 F^{a+\frac{b}{(c+dx)^3}} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{2} b \log(F) \int F^{a+\frac{b}{(c+dx)^3}} (c+dx)^2 dx + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{6d}$$

$$\Big\downarrow \text{ 2643}$$

$$\frac{1}{2} b \log(F) \left(b \log(F) \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} dx + \frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{3d} \right) + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{6d}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{2} b \log(F) \left(\frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{3d} - \frac{b F^a \log(F) \text{ExpIntegralEi}\left(\frac{b \log(F)}{(c+dx)^3}\right)}{3d} \right) + \frac{(c+dx)^6 F^{a+\frac{b}{(c+dx)^3}}}{6d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^5,x]
```

output
```
(F^(a + b/(c + d*x)^3)*(c + d*x)^6)/(6*d) + (b*Log[F]*((F^(a + b/(c + d*x)
^3)*(c + d*x)^3)/(3*d) - (b*F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^3]*Log[
F])/(3*d)))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^5\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.45

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx =$$
$$-\frac{F^a b^2 \mathrm{Ei}\left(\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)\log(F)^2-(d^6x^6+6\,cd^5x^5+15\,c^2d^4x^4+20\,c^3d^3x^3+15\,c^4d^2x^2+6\,c^5dx}{6\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/6*(F^a*b^2*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log(F
)^2 - (d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^
2*x^2 + 6*c^5*d*x + c^6 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3
)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**5,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**5, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\int (dx+c)^5F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/6*(F^a*d^5*x^6 + 6*F^a*c*d^4*x^5 + 15*F^a*c^2*d^3*x^4 + (20*F^a*c^3*d^2
+ F^a*b*d^2*log(F))*x^3 + 3*(5*F^a*c^4*d + F^a*b*c*d*log(F))*x^2 + 3*(2*F^
a*c^5 + F^a*b*c^2*log(F))*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3
)) + integrate(1/2*(F^a*b^2*d^3*x^3*log(F)^2 + 3*F^a*b^2*c*d^2*x^2*log(F)^
2 - F^a*b*c^6*log(F) + 3*F^a*b^2*c^2*d*x*log(F)^2)*F^(b/(d^3*x^3 + 3*c*d^2
*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*
x + c^4), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\int (dx+c)^5F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate((d*x + c)^5*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\frac{F^a\,b^2\ln(F)^2\left(\frac{\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)}{2}+F^{\frac{b}{(c+d\,x)^3}}\left(\frac{(c+d\,x)^3}{2\,b\,\ln(F)}+\frac{(c+d\,x)^6}{2\,b^2\ln(F)^2}\right)\right)}{3\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^5,x)
```

output
```
(F^a*b^2*log(F)^2*(expint(-(b*log(F))/(c + d*x)^3)/2 + F^(b/(c + d*x)^3)*(
(c + d*x)^3/(2*b*log(F)) + (c + d*x)^6/(2*b^2*log(F)^2))))/(3*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^5\,dx=\text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^5,x)
```

output
```
(270*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**2*d*x + 270*
f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c*d**2*x**2 + 90*f**
((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**6 + 378*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**5*d*x - 675*f**((a*c**3 + 3*a
*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d*
*2*x**2 + d**3*x**3))*log(f)**2*b**2*c**4*d**2*x**2 - 1800*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**3*d**3*x**3 - 1350*f**((a*c**3 +
 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**2*d**4*x**4 - 360*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c*d**5*x**5 - 54*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)*b*c**9 - 1242*f**((a*c**3 + 3*a*c**2*d*x +
3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3))*log(f)*b*c**8*d*x - 4302*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d...
```

3.279 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx=\frac{F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3}{3d}-\frac{bF^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)\log(F)}{3d}$$

output
```
1/3*F^(a+b/(d*x+c)^3)*(d*x+c)^3/d-1/3*b*F^a*Ei(b*ln(F)/(d*x+c)^3)*ln(F)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx=\frac{F^a\left(F^{\frac{b}{(c+dx)^3}}(c+dx)^3-b\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)\log(F)\right)}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^2,x]
```

output
```
(F^a*(F^(b/(c + d*x)^3)*(c + d*x)^3 - b*ExpIntegralEi[(b*Log[F])/(c + d*x)
^3]*Log[F]))/(3*d)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2643, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 F^{a+\frac{b}{(c+dx)^3}} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$b\log(F)\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}dx+\frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{3d}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{(c+dx)^3 F^{a+\frac{b}{(c+dx)^3}}}{3d}-\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^2,x]
```

output
```
(F^(a + b/(c + d*x)^3)*(c + d*x)^3)/(3*d) - (b*F^a*ExpIntegralEi[(b*Log[F]
)/(c + d*x)^3]*Log[F])/(3*d)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^2\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^2,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(49) = 98$.

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.66

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx = -\frac{F^a b\,\mathrm{Ei}\left(\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)\log(F)-(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3)F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{3\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*(F^a*b*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*log(F)
- (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*F^((a*d^3*x^3 + 3*a*c*d^2*x^2
+ 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx=\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**2,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**2, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx=\int (dx+c)^2F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*(F^a*d^2*x^3 + 3*F^a*c*d*x^2 + 3*F^a*c^2*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^
2 + 3*c^2*d*x + c^3)) + integrate((F^a*b*d^3*x^3*log(F) + 3*F^a*b*c*d^2*x^
2*log(F) + 3*F^a*b*c^2*d*x*log(F))*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx = \int (dx+c)^2 F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx = \frac{F^a\,F^{\frac{b}{(c+d\,x)^3}}\,(c+d\,x)^3}{3\,d} + \frac{F^a\,b\,\ln\left(F\right)\,\mathrm{expint}\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)}{3\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^2,x)
```

output
```
(F^a*F^(b/(c + d*x)^3)*(c + d*x)^3)/(3*d) + (F^a*b*log(F)*expint(-(b*log(F
))/(c + d*x)^3))/(3*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^2\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^2,x)
```

output
```
(270*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c**2*d*x + 270*
f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*c*d**2*x**2 + 90*f**
((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**6 + 378*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**5*d*x - 675*f**((a*c**3 + 3*a
*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d*
*2*x**2 + d**3*x**3))*log(f)**2*b**2*c**4*d**2*x**2 - 1800*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**3*d**3*x**3 - 1350*f**((a*c**3 +
 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**2*d**4*x**4 - 360*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c*d**5*x**5 - 54*f**((a*c**3 + 3
*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*log(f)*b*c**9 - 1242*f**((a*c**3 + 3*a*c**2*d*x +
3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3))*log(f)*b*c**8*d*x - 4302*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d...
```

3.280 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx=-\frac{F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

output
```
-1/3*F^a*Ei(b*ln(F)/(d*x+c)^3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx=-\frac{F^a\,\text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x),x]
```

output
```
-1/3*(F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^3])/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx} \, dx$$

$$\Big\downarrow \ 2639$$

$$-\frac{F^a \text{ExpIntegralEi}\left(\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x),x]
```

output
```
-1/3*(F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)^3])/d
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{dx+c}dx$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c),x)
```

output
```
int(F^(a+b/(d*x+c)^3)/(d*x+c),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx = -\frac{F^a \operatorname{Ei}\left(\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{3\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c),x, algorithm="fricas")
```

output
```
-1/3*F^a*Ei(b*log(F)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/d
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c),x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{dx+c}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{dx+c}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx = -\frac{F^a\,\mathrm{ei}\left(\frac{b\,\ln(F)}{(c+d\,x)^3}\right)}{3\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x),x)
```

output
```
-(F^a*ei((b*log(F))/(c + d*x)^3))/(3*d)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{c+dx}\,dx = \int \frac{f^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}}{dx+c}dx$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c),x)
```

output
```
int(f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(c + d*x),x)
```

3.281 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx = -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)}$$

output
```
-1/3*F^(a+b/(d*x+c)^3)/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} dx = -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^4,x]
```

output
```
-1/3*F^(a + b/(c + d*x)^3)/(b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4} \, dx$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^4,x]
```

output
```
-1/3*F^(a + b/(c + d*x)^3)/(b*d*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{F^{a+\frac{b}{(dx+c)^3}}}{3bd\ln(F)}$	26
default	$-\frac{F^{a+\frac{b}{(dx+c)^3}}}{3bd\ln(F)}$	26
parallelrisch	$-\frac{F^{a+\frac{b}{(dx+c)^3}}}{3bd\ln(F)}$	26
risch	$-\frac{F^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{(dx+c)^3}}}{3bd\ln(F)}$	56
norman	$\frac{-\frac{c^3\mathrm{e}^{\left(a+\frac{b}{(dx+c)^3}\right)\ln(F)}}{3\ln(F)bd}-\frac{c^2x\,\mathrm{e}^{\left(a+\frac{b}{(dx+c)^3}\right)\ln(F)}}{\ln(F)b}-\frac{d^2x^3\mathrm{e}^{\left(a+\frac{b}{(dx+c)^3}\right)\ln(F)}}{3\ln(F)b}-\frac{dc\,x^2\mathrm{e}^{\left(a+\frac{b}{(dx+c)^3}\right)\ln(F)}}{\ln(F)b}}{(dx+c)^3}$	127

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*F^(a+b/(d*x+c)^3)/b/d/ln(F)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(25) = 50$.

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}\,dx = -\frac{F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{3\,bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3
*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b*d*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(20) = 40$.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}\,dx = \begin{cases} -\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)} & \text{for } bd\log(F)\neq 0 \\ -\frac{1}{3c^3d+9c^2d^2x+9cd^3x^2+3d^4x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**4,x)
```

output
```
Piecewise((-F**(a + b/(c + d*x)**3)/(3*b*d*log(F)), Ne(b*d*log(F), 0)), (-
1/(3*c**3*d + 9*c**2*d**2*x + 9*c*d**3*x**2 + 3*d**4*x**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}\,dx = -\frac{F^{a+\frac{b}{(dx+c)^3}}}{3\,bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/3*F^(a + b/(d*x + c)^3)/(b*d*log(F))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(25) = 50$.

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}\,dx = -\frac{F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{3\,bd\log(F)}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/3*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3
*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}\,dx = -\frac{F^a\,F^{\frac{b}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}}}{3\,b\,d\,\ln(F)}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^4,x)
```

output
```
-(F^a*F^(b/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x)))/(3*b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.85

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}\,dx = -\frac{f^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}}{3\log(f)\,bd}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^4,x)
```

output
```
( - f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)))/(3*log(f)*b*d)
```

3.282 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx$

Optimal result

Integrand size = 21, antiderivative size = 62

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx = \frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2 d \log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^3 \log(F)}$$

output
```
1/3*F^(a+b/(d*x+c)^3)/b^2/d/ln(F)^2-1/3*F^(a+b/(d*x+c)^3)/b/d/(d*x+c)^3/ln
(F)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} dx = \frac{F^{a+\frac{b}{(c+dx)^3}} ((c+dx)^3 - b \log(F))}{3b^2 d (c+dx)^3 \log^2(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^7,x]
```

output
```
(F^(a + b/(c + d*x)^3)*((c + d*x)^3 - b*Log[F]))/(3*b^2*d*(c + d*x)^3*Log[
F]^2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7} \, dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}dx}{b\log(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^3}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2d\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^3}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^7,x]
```

output
```
F^(a + b/(c + d*x)^3)/(3*b^2*d*Log[F]^2) - F^(a + b/(c + d*x)^3)/(3*b*d*(c
 + d*x)^3*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{\left(-d^{3} x^{3}-3 c\, d^{2} x^{2}-3 c^{2} d x -c^{3}+b \ln\left(F \right)\right) F^{\frac{a\, d^{3} x^{3}+3 a c\, d^{2} x^{2}+3 a\, c^{2} d x +a\, c^{3}+b}{\left(d x +c \right)^{3}}}}{3 d \ln\left(F \right)^{2} b^{2} \left(d x +c \right)^{3}}$
parallelrisch	$\frac{x^{3} F^{a +\frac{b}{\left(d x +c \right)^{3}}} d^{13}+3 x^{2} F^{a +\frac{b}{\left(d x +c \right)^{3}}} c\, d^{12}+3 x\, F^{a +\frac{b}{\left(d x +c \right)^{3}}} c^{2} d^{11}+F^{a +\frac{b}{\left(d x +c \right)^{3}}} c^{3} d^{10}-\ln\left(F \right) F^{a +\frac{b}{\left(d x +c \right)^{3}}} b\, d^{10}}{3 \left(d x +c \right)^{3} \ln\left(F \right)^{2} b^{2} d^{11}}$
norman	$\frac{d^{5} x^{6} {\mathrm e}^{\left(a +\frac{b}{\left(d x +c \right)^{3}}\right) \ln\left(F \right)}}{3 \ln\left(F \right)^{2} b^{2}}-\frac{c^{2} \left(-2 c^{3}+b \ln\left(F \right)\right) x\, {\mathrm e}^{\left(a +\frac{b}{\left(d x +c \right)^{3}}\right) \ln\left(F \right)}}{\ln\left(F \right)^{2} b^{2}}-\frac{c^{3} \left(-c^{3}+b \ln\left(F \right)\right) {\mathrm e}^{\left(a +\frac{b}{\left(d x +c \right)^{3}}\right) \ln\left(F \right)}}{3 d \ln\left(F \right)^{2} b^{2}}-\frac{d^{2} \left(-20 c^{3}+b \ln\left(F \right)\right)}{3 \mathrm{l}}$ ... $(dx-$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-d^3*x^3-3*c*d^2*x^2-3*c^2*d*x-c^3+b*ln(F))/d/ln(F)^2/b^2/(d*x+c)^3*
F^((a*d^3*x^3+3*a*c*d^2*x^2+3*a*c^2*d*x+a*c^3+b)/(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(58) = 116$.

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.39

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}\,dx = \frac{(d^3x^3 + 3\,cd^2x^2 + 3\,c^2dx + c^3 - b\log(F))F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{3\,(b^2d^4x^3 + 3\,b^2cd^3x^2 + 3\,b^2c^2d^2x + b^2c^3d)\log(F)^2}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^7,x, algorithm="fricas")
```

output
```
1/3*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 - b*log(F))*F^((a*d^3*x^3 + 3
*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3))/((b^2*d^4*x^3 + 3*b^2*c*d^3*x^2 + 3*b^2*c^2*d^2*x + b^2*c^3*d)*lo
g(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(49) = 98$.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.84

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}\,dx = \frac{F^{a+\frac{b}{(c+dx)^3}}\left(-b\log(F)+c^3+3c^2dx+3cd^2x^2+d^3x^3\right)}{3b^2c^3d\log(F)^2+9b^2c^2d^2x\log(F)^2+9b^2cd^3x^2\log(F)^2+3b^2d^4x^3\log(F)^2}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**7,x)
```

output
```
F**(a + b/(c + d*x)**3)*(-b*log(F) + c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3)/(3*b**2*c**3*d*log(F)**2 + 9*b**2*c**2*d**2*x*log(F)**2 + 9*b**2
*c*d**3*x**2*log(F)**2 + 3*b**2*d**4*x**3*log(F)**2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(58) = 116$.

Time = 0.04 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.32

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}\,dx = \frac{\left(F^a d^3x^3+3\,F^a cd^2x^2+3\,F^a c^2dx+F^a c^3-F^a b\log(F)\right)F^{\frac{b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{3\left(b^2d^4x^3\log(F)^2+3\,b^2cd^3x^2\log(F)^2+3\,b^2c^2d^2x\log(F)^2+b^2c^3d\log(F)^2\right)}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/3*(F^a*d^3*x^3 + 3*F^a*c*d^2*x^2 + 3*F^a*c^2*d*x + F^a*c^3 - F^a*b*log(F
))*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^2*d^4*x^3*log(F)^2 +
 3*b^2*c*d^3*x^2*log(F)^2 + 3*b^2*c^2*d^2*x*log(F)^2 + b^2*c^3*d*log(F)^2)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^7}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^7,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^7, x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.19

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}\,dx = \frac{F^a\,F^{\frac{b}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}}\left(\frac{x^3}{3\,b^2\,d\ln(F)^2}-\frac{b\ln(F)-c^3}{3\,b^2\,d^4\ln(F)^2}+\frac{c\,x^2}{b^2\,d^2\ln(F)^2}+\frac{c^2\,x}{b^2\,d^3\ln(F)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^7,x)
```

output
```
(F^a*F^(b/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x))*(x^3/(3*b^2*d*log(F)^
2) - (b*log(F) - c^3)/(3*b^2*d^4*log(F)^2) + (c*x^2)/(b^2*d^2*log(F)^2) +
(c^2*x)/(b^2*d^3*log(F)^2)))/(x^3 + c^3/d^3 + (3*c*x^2)/d + (3*c^2*x)/d^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}\,dx = \frac{f^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}\left(-\log(f)\,b+c^3+3c^2dx+3c\,d^2x^2+d^3x^3\right)}{3\log\left(f\right)^2b^2d\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^7,x)
```

output
```
(f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*( - log(f)*b + c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3))/(3*log(f)**2*b**2*d*(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))
```

3.283 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}} dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}} dx = -\frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^3d\log^3(F)} + \frac{2F^{a+\frac{b}{(c+dx)^3}}}{3b^2d(c+dx)^3\log^2(F)} - \frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^6\log(F)}$$

output
```
-2/3*F^(a+b/(d*x+c)^3)/b^3/d/ln(F)^3+2/3*F^(a+b/(d*x+c)^3)/b^2/d/(d*x+c)^3
/ln(F)^2-1/3*F^(a+b/(d*x+c)^3)/b/d/(d*x+c)^6/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}} dx = -\frac{F^{a+\frac{b}{(c+dx)^3}}\left(2(c+dx)^6 - 2b(c+dx)^3\log(F) + b^2\log^2(F)\right)}{3b^3d(c+dx)^6\log^3(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^10,x]
```

output
```
-1/3*(F^(a + b/(c + d*x)^3)*(2*(c + d*x)^6 - 2*b*(c + d*x)^3*Log[F] + b^2*
Log[F]^2))/(b^3*d*(c + d*x)^6*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}dx$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^6}$$

$$\Big\downarrow \text{2641}$$

$$-\frac{2\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^3}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^6}$$

$$\Big\downarrow \text{2638}$$

$$-\frac{2\left(\frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2d\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^3}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^6}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^10,x]
```

output
```
-1/3*F^(a + b/(c + d*x)^3)/(b*d*(c + d*x)^6*Log[F]) - (2*(F^(a + b/(c + d*
x)^3)/(3*b^2*d*Log[F]^2) - F^(a + b/(c + d*x)^3)/(3*b*d*(c + d*x)^3*Log[F]
)))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.82

method	result
risch	$-\frac{\left(2d^{6}x^{6}+12c\,d^{5}x^{5}+30c^{2}d^{4}x^{4}+40c^{3}d^{3}x^{3}-2\ln(F)b\,d^{3}x^{3}+30c^{4}d^{2}x^{2}-6\ln(F)bc\,d^{2}x^{2}+12c^{5}dx-6\ln(F)b\,c^{2}dx+2c^{6}-2\ln(F)}{3b^{3}\ln(F)^{3}d(dx+c)^{6}}$
parallelrisch	$\frac{-2x^{6}F^{a+\frac{b}{(dx+c)^{3}}}d^{19}-12x^{5}F^{a+\frac{b}{(dx+c)^{3}}}c\,d^{18}-30x^{4}F^{a+\frac{b}{(dx+c)^{3}}}c^{2}d^{17}-40x^{3}F^{a+\frac{b}{(dx+c)^{3}}}c^{3}d^{16}+2\ln(F)x^{3}F^{a+\frac{b}{(dx+c)^{3}}}b\,d^{1}}{}$
norman	$\frac{-\frac{2d^{8}x^{9}{\rm e}^{\left(a+\frac{b}{(dx+c)^{3}}\right)\ln(F)}}{3\ln(F)^{3}b^{3}}-\frac{c^{2}\left(6c^{6}-4\ln(F)b\,c^{3}+\ln(F)^{2}b^{2}\right)x\,{\rm e}^{\left(a+\frac{b}{(dx+c)^{3}}\right)\ln(F)}}{b^{3}\ln(F)^{3}}-\frac{d^{2}\left(168c^{6}-40\ln(F)b\,c^{3}+\ln(F)^{2}b^{2}\right)x^{3}{\rm e}^{\left(a+\frac{}{(d}\right.}}{3\ln(F)^{3}b^{3}}}{}$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*d^6*x^6+12*c*d^5*x^5+30*c^2*d^4*x^4+40*c^3*d^3*x^3-2*ln(F)*b*d^3*x
^3+30*c^4*d^2*x^2-6*ln(F)*b*c*d^2*x^2+12*c^5*d*x-6*ln(F)*b*c^2*d*x+2*c^6-2
*ln(F)*b*c^3+ln(F)^2*b^2)/b^3/ln(F)^3/d/(d*x+c)^6*F^((a*d^3*x^3+3*a*c*d^2*
x^2+3*a*c^2*d*x+a*c^3+b)/(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(90) = 180$.

Time = 0.09 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.76

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}\,dx = -\frac{\left(2\,d^6x^6+12\,cd^5x^5+30\,c^2d^4x^4+40\,c^3d^3x^3+30\,c^4d^2x^2+12\,c^5dx+2\,c^6+b^2\log\left(F\right)^2-2\left(bd^3x^3+3\,b\right.\right.}{3\left(b^3d^7x^6+6\,b^3cd^6x^5+15\,b^3c^2d^5x^4+20\,b^3c^3d^4x^3+15\,b^3c^4d^3x^2+6\,b^3\right.}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x, algorithm="fricas")
```

output
```
-1/3*(2*d^6*x^6 + 12*c*d^5*x^5 + 30*c^2*d^4*x^4 + 40*c^3*d^3*x^3 + 30*c^4*
d^2*x^2 + 12*c^5*d*x + 2*c^6 + b^2*log(F)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2
 + 3*b*c^2*d*x + b*c^3)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*
x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b^3*d^7*x^6 +
6*b^3*c*d^6*x^5 + 15*b^3*c^2*d^5*x^4 + 20*b^3*c^3*d^4*x^3 + 15*b^3*c^4*d^3
*x^2 + 6*b^3*c^5*d^2*x + b^3*c^6*d)*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(83) = 166$.

Time = 0.19 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.81

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}\,dx = \frac{F^{a+\frac{b}{(c+dx)^3}}\left(-b^2\log\left(F\right)^2+2bc^3\log\left(F\right)+6bc^2dx\log\left(F\right)+6bcd^2x^2\log\left(F\right)+2bd^3x^3\log\left(F\right)-2c^6-12c^5\right.}{3b^3c^6d\log\left(F\right)^3+18b^3c^5d^2x\log\left(F\right)^3+45b^3c^4d^3x^2\log\left(F\right)^3+60b^3c^3d^4x^3\log\left(F\right)^3+45b^3c^2d^5x^4}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**10,x)
```

output
```
F**(a + b/(c + d*x)**3)*(-b**2*log(F)**2 + 2*b*c**3*log(F) + 6*b*c**2*d*x*
log(F) + 6*b*c*d**2*x**2*log(F) + 2*b*d**3*x**3*log(F) - 2*c**6 - 12*c**5*
d*x - 30*c**4*d**2*x**2 - 40*c**3*d**3*x**3 - 30*c**2*d**4*x**4 - 12*c*d**
5*x**5 - 2*d**6*x**6)/(3*b**3*c**6*d*log(F)**3 + 18*b**3*c**5*d**2*x*log(F
)**3 + 45*b**3*c**4*d**3*x**2*log(F)**3 + 60*b**3*c**3*d**4*x**3*log(F)**3
 + 45*b**3*c**2*d**5*x**4*log(F)**3 + 18*b**3*c*d**6*x**5*log(F)**3 + 3*b*
*3*d**7*x**6*log(F)**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(90) = 180.

Time = 0.04 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.12

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}\,dx = -\frac{\left(2\,F^a d^6 x^6 + 12\,F^a c d^5 x^5 + 30\,F^a c^2 d^4 x^4 + 2\,F^a c^6 - 2\,F^a b c^3 \log\left(F\right) + F^a b^2 \log\left(F\right)^2 + 2\,(20\,F^a c^3 d^3 - \right.}{3\,\left(b^3 d^7 x^6 \log\left(F\right)^3 + 6\,b^3 c d^6 x^5 \log\left(F\right)^3 + 15\,b^3 c^2 d^5 x^4 \log\left(F\right)^3 + 20\,b^3 c^3\right.}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x, algorithm="maxima")
```

output
```
-1/3*(2*F^a*d^6*x^6 + 12*F^a*c*d^5*x^5 + 30*F^a*c^2*d^4*x^4 + 2*F^a*c^6 -
2*F^a*b*c^3*log(F) + F^a*b^2*log(F)^2 + 2*(20*F^a*c^3*d^3 - F^a*b*d^3*log(
F))*x^3 + 6*(5*F^a*c^4*d^2 - F^a*b*c*d^2*log(F))*x^2 + 6*(2*F^a*c^5*d - F^
a*b*c^2*d*log(F))*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^3*
d^7*x^6*log(F)^3 + 6*b^3*c*d^6*x^5*log(F)^3 + 15*b^3*c^2*d^5*x^4*log(F)^3
+ 20*b^3*c^3*d^4*x^3*log(F)^3 + 15*b^3*c^4*d^3*x^2*log(F)^3 + 6*b^3*c^5*d^
2*x*log(F)^3 + b^3*c^6*d*log(F)^3)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^{10}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^10, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.74

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}\,dx = -\frac{F^a\,F^{\frac{b}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}}\,\left(\frac{2\,x^6}{3\,b^3\,d\ln(F)^3}+\frac{b^2\ln(F)^2-2\,b\,c^3\,\ln(F)+2\,c^6}{3\,b^3\,d^7\,\ln(F)^3}+\frac{4\,c\,x^5}{b^3\,d^2\,\ln(F)^3}+\frac{10\,c^2\,x^4}{b^3\,d^3\,\ln(F)^3}-\frac{2\,x^3\,(b\,\ln(F)-20\,c^3}{3\,b^3\,d^4\,\ln(F)^3}}{x^6+\frac{c^6}{d^6}+\frac{6\,c\,x^5}{d}+\frac{6\,c^5\,x}{d^5}+\frac{15\,c^2\,x^4}{d^2}+\frac{20\,c^3\,x^3}{d^3}+\frac{15\,c^4\,x^2}{d^4}}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^10,x)
```

output
```
-(F^a*F^(b/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x))*((2*x^6)/(3*b^3*d*lo
g(F)^3) + (b^2*log(F)^2 + 2*c^6 - 2*b*c^3*log(F))/(3*b^3*d^7*log(F)^3) + (
4*c*x^5)/(b^3*d^2*log(F)^3) + (10*c^2*x^4)/(b^3*d^3*log(F)^3) - (2*x^3*(b*
log(F) - 20*c^3))/(3*b^3*d^4*log(F)^3) - (2*c^2*x*(b*log(F) - 2*c^3))/(b^3
*d^6*log(F)^3) - (2*c*x^2*(b*log(F) - 5*c^3))/(b^3*d^5*log(F)^3)))/(x^6 +
c^6/d^6 + (6*c*x^5)/d + (6*c^5*x)/d^5 + (15*c^2*x^4)/d^2 + (20*c^3*x^3)/d^
3 + (15*c^4*x^2)/d^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.62

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}\,dx$$
$$= \frac{f^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}\left(-\log(f)^2\,b^2+2\log(f)\,b\,c^3+6\log(f)\,b\,c^2dx+6\log(f)\,bc\,d^2x^2+2\log(f)\,b\,d^3x^3\right.}{3\log\left(f\right)^3b^3d\left(d^6x^6+6c\,d^5x^5+15c^2d^4x^4+20c^3d^3x^3+\right.}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^10,x)
```

output
```
(f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*( - log(f)**2*b**2 + 2*log(f)*b*c*
*3 + 6*log(f)*b*c**2*d*x + 6*log(f)*b*c*d**2*x**2 + 2*log(f)*b*d**3*x**3 -
 2*c**6 - 12*c**5*d*x - 30*c**4*d**2*x**2 - 40*c**3*d**3*x**3 - 30*c**2*d*
*4*x**4 - 12*c*d**5*x**5 - 2*d**6*x**6))/(3*log(f)**3*b**3*d*(c**6 + 6*c**
5*d*x + 15*c**4*d**2*x**2 + 20*c**3*d**3*x**3 + 15*c**2*d**4*x**4 + 6*c*d*
*5*x**5 + d**6*x**6))
```

3.284 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx=\frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^4d\log^4(F)}-\frac{2F^{a+\frac{b}{(c+dx)^3}}}{b^3d(c+dx)^3\log^3(F)}+\frac{F^{a+\frac{b}{(c+dx)^3}}}{b^2d(c+dx)^6\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd(c+dx)^9\log(F)}$$

output
```
2*F^(a+b/(d*x+c)^3)/b^4/d/ln(F)^4-2*F^(a+b/(d*x+c)^3)/b^3/d/(d*x+c)^3/ln(F
)^3+F^(a+b/(d*x+c)^3)/b^2/d/(d*x+c)^6/ln(F)^2-1/3*F^(a+b/(d*x+c)^3)/b/d/(d
*x+c)^9/ln(F)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx=\frac{F^{a+\frac{b}{(c+dx)^3}}\left(6-\frac{6b\log(F)}{(c+dx)^3}+\frac{3b^2\log^2(F)}{(c+dx)^6}-\frac{b^3\log^3(F)}{(c+dx)^9}\right)}{3b^4d\log^4(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^13,x]
```

output
```
(F^(a + b/(c + d*x)^3)*(6 - (6*b*Log[F])/(c + d*x)^3 + (3*b^2*Log[F]^2)/(c
 + d*x)^6 - (b^3*Log[F]^3)/(c + d*x)^9))/(3*b^4*d*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2641, 2641, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{3\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{10}}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^9}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{3\left(-\frac{2\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^7}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^6}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^9}$$

$$\Big\downarrow \text{ 2641}$$

$$-\frac{3\left(-\frac{2\left(-\frac{\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^4}dx}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^3}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^6}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^9}$$

$$\Big\downarrow \text{ 2638}$$

$$-\frac{3\left(-\frac{2\left(\frac{F^{a+\frac{b}{(c+dx)^3}}}{3b^2d\log^2(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^3}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^6}\right)}{b\log(F)}-\frac{F^{a+\frac{b}{(c+dx)^3}}}{3bd\log(F)(c+dx)^9}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^13,x]
```

output
```
-1/3*F^(a + b/(c + d*x)^3)/(b*d*(c + d*x)^9*Log[F]) - (3*(-1/3*F^(a + b/(c
 + d*x)^3)/(b*d*(c + d*x)^6*Log[F]) - (2*(F^(a + b/(c + d*x)^3)/(3*b^2*d*L
og[F]^2) - F^(a + b/(c + d*x)^3)/(3*b*d*(c + d*x)^3*Log[F])))/(b*Log[F])))
/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(121) = 242$.

Time = 5.80 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.50

method	result
risch	$-\frac{\left(-6d^9x^9-54c\,d^8x^8-216c^2d^7x^7-504c^3d^6x^6+6\ln(F)b\,d^6x^6-756c^4d^5x^5+36\ln(F)bc\,d^5x^5-756c^5d^4x^4+90\ln(F)b\,c^2d^4x^4}{}$
parallelrisch	$\frac{-6\ln(F)x^6F^{a+\frac{b}{(dx+c)^3}}b\,d^{22}+3\ln(F)^2x^3F^{a+\frac{b}{(dx+c)^3}}b^2d^{19}-6\ln(F)F^{a+\frac{b}{(dx+c)^3}}b\,c^6d^{16}+3\ln(F)^2F^{a+\frac{b}{(dx+c)^3}}b^2c^3d^{16}+6x}{}$
norman	$-\frac{\left(-6c^9+6\ln(F)b\,c^6-3\ln(F)^2b^2c^3+\ln(F)^3b^3\right)c^3\mathrm{e}^{\left(a+\frac{b}{(dx+c)^3}\right)\ln(F)}}{3b^4\ln(F)^4d}-\frac{c^2\left(-24c^9+18\ln(F)b\,c^6-6\ln(F)^2b^2c^3+\ln(F)^3b^3\right)x\,\mathrm{e}^{\left(a+\frac{b}{(dx+}\right.}}{b^4\ln(F)^4}$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^13,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-6*d^9*x^9-54*c*d^8*x^8-216*c^2*d^7*x^7-504*c^3*d^6*x^6+6*ln(F)*b*d^
6*x^6-756*c^4*d^5*x^5+36*ln(F)*b*c*d^5*x^5-756*c^5*d^4*x^4+90*ln(F)*b*c^2*
d^4*x^4-504*c^6*d^3*x^3+120*ln(F)*b*c^3*d^3*x^3-216*c^7*d^2*x^2-3*ln(F)^2*
b^2*d^3*x^3+90*ln(F)*b*c^4*d^2*x^2-54*c^8*d*x-9*ln(F)^2*b^2*c*d^2*x^2+36*l
n(F)*b*c^5*d*x-6*c^9-9*ln(F)^2*b^2*c^2*d*x+6*ln(F)*b*c^6-3*ln(F)^2*b^2*c^3
+ln(F)^3*b^3)/b^4/ln(F)^4/d/(d*x+c)^9*F^((a*d^3*x^3+3*a*c*d^2*x^2+3*a*c^2*
d*x+a*c^3+b)/(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(121) = 242$.

Time = 0.10 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.44

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx = \frac{\left(6\,d^9x^9+54\,cd^8x^8+216\,c^2d^7x^7+504\,c^3d^6x^6+756\,c^4d^5x^5+756\,c^5d^4x^4+504\,c^6d^3x^3+216\,c^7d^2x^2+54\right.}{3\left(b^4d^{10}x^9+9\,b^4cd^9x^8+36\right.}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^13,x, algorithm="fricas")
```

output
```
1/3*(6*d^9*x^9 + 54*c*d^8*x^8 + 216*c^2*d^7*x^7 + 504*c^3*d^6*x^6 + 756*c^
4*d^5*x^5 + 756*c^5*d^4*x^4 + 504*c^6*d^3*x^3 + 216*c^7*d^2*x^2 + 54*c^8*d
*x + 6*c^9 - b^3*log(F)^3 + 3*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*log(F)^2 - 6*(b*d^6*x^6 + 6*b*c*d^5*x^5 + 15*b*c^2*d^4*x^4 +
 20*b*c^3*d^3*x^3 + 15*b*c^4*d^2*x^2 + 6*b*c^5*d*x + b*c^6)*log(F))*F^((a*
d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2
+ 3*c^2*d*x + c^3))/((b^4*d^10*x^9 + 9*b^4*c*d^9*x^8 + 36*b^4*c^2*d^8*x^7
+ 84*b^4*c^3*d^7*x^6 + 126*b^4*c^4*d^6*x^5 + 126*b^4*c^5*d^5*x^4 + 84*b^4*
c^6*d^4*x^3 + 36*b^4*c^7*d^3*x^2 + 9*b^4*c^8*d^2*x + b^4*c^9*d)*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 484, normalized size of antiderivative = 3.93

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx$$
$$= \frac{F^{a+\frac{b}{(c+dx)^3}}\left(-b^3\log(F)^3 + 3b^2c^3\log(F)^2 + 9b^2c^2dx\log(F)^2 + 9b^2cd^2x^2\log(F)^2 + 3b^2d^3x^3\log(F)^2 - 6b\right.}{3b^4c^9d\log(F)^4 + 27b^4c^8d^2x\log(F)^4 + 10}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**13,x)
```

output
```
F**(a + b/(c + d*x)**3)*(-b**3*log(F)**3 + 3*b**2*c**3*log(F)**2 + 9*b**2*
c**2*d*x*log(F)**2 + 9*b**2*c*d**2*x**2*log(F)**2 + 3*b**2*d**3*x**3*log(F
)**2 - 6*b*c**6*log(F) - 36*b*c**5*d*x*log(F) - 90*b*c**4*d**2*x**2*log(F)
 - 120*b*c**3*d**3*x**3*log(F) - 90*b*c**2*d**4*x**4*log(F) - 36*b*c*d**5*
x**5*log(F) - 6*b*d**6*x**6*log(F) + 6*c**9 + 54*c**8*d*x + 216*c**7*d**2*
x**2 + 504*c**6*d**3*x**3 + 756*c**5*d**4*x**4 + 756*c**4*d**5*x**5 + 504*
c**3*d**6*x**6 + 216*c**2*d**7*x**7 + 54*c*d**8*x**8 + 6*d**9*x**9)/(3*b**
4*c**9*d*log(F)**4 + 27*b**4*c**8*d**2*x*log(F)**4 + 108*b**4*c**7*d**3*x*
*2*log(F)**4 + 252*b**4*c**6*d**4*x**3*log(F)**4 + 378*b**4*c**5*d**5*x**4
*log(F)**4 + 378*b**4*c**4*d**6*x**5*log(F)**4 + 252*b**4*c**3*d**7*x**6*l
og(F)**4 + 108*b**4*c**2*d**8*x**7*log(F)**4 + 27*b**4*c*d**9*x**8*log(F)*
*4 + 3*b**4*d**10*x**9*log(F)**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. $2(121) = 242$.

Time = 0.05 (sec) , antiderivative size = 507, normalized size of antiderivative = 4.12

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx = \frac{\left(6\,F^a d^9 x^9 + 54\,F^a c d^8 x^8 + 216\,F^a c^2 d^7 x^7 + 6\,F^a c^9 - 6\,F^a b c^6 \log(F) + 3\,F^a b^2 c^3 \log(F)^2 + 6\,(84\,F^a c^3 d^6\right.}{3\,(b^4 d^{}}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^13,x, algorithm="maxima")
```

output
```
1/3*(6*F^a*d^9*x^9 + 54*F^a*c*d^8*x^8 + 216*F^a*c^2*d^7*x^7 + 6*F^a*c^9 -
6*F^a*b*c^6*log(F) + 3*F^a*b^2*c^3*log(F)^2 + 6*(84*F^a*c^3*d^6 - F^a*b*d^
6*log(F))*x^6 - F^a*b^3*log(F)^3 + 36*(21*F^a*c^4*d^5 - F^a*b*c*d^5*log(F)
)*x^5 + 18*(42*F^a*c^5*d^4 - 5*F^a*b*c^2*d^4*log(F))*x^4 + 3*(168*F^a*c^6*
d^3 - 40*F^a*b*c^3*d^3*log(F) + F^a*b^2*d^3*log(F)^2)*x^3 + 9*(24*F^a*c^7*
d^2 - 10*F^a*b*c^4*d^2*log(F) + F^a*b^2*c*d^2*log(F)^2)*x^2 + 9*(6*F^a*c^8
*d - 4*F^a*b*c^5*d*log(F) + F^a*b^2*c^2*d*log(F)^2)*x)*F^(b/(d^3*x^3 + 3*c
*d^2*x^2 + 3*c^2*d*x + c^3))/(b^4*d^10*x^9*log(F)^4 + 9*b^4*c*d^9*x^8*log(
F)^4 + 36*b^4*c^2*d^8*x^7*log(F)^4 + 84*b^4*c^3*d^7*x^6*log(F)^4 + 126*b^4
*c^4*d^6*x^5*log(F)^4 + 126*b^4*c^5*d^5*x^4*log(F)^4 + 84*b^4*c^6*d^4*x^3*
log(F)^4 + 36*b^4*c^7*d^3*x^2*log(F)^4 + 9*b^4*c^8*d^2*x*log(F)^4 + b^4*c^
9*d*log(F)^4)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^{13}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^13,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^13, x)
```

Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.43

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx = \frac{F^a\,F^{\frac{b}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}}\left(\frac{2\,x^9}{b^4\,d\ln(F)^4}-\frac{b^3\ln(F)^3-3\,b^2\,c^3\ln(F)^2+6\,b\,c^6\,\ln(F)-6\,c^9}{3\,b^4\,d^{10}\ln(F)^4}+\frac{18\,c\,x^8}{b^4\,d^2\ln(F)^4}+\frac{72\,c^2\,x^7}{b^4\,d^3\ln(F)^4}+\frac{x^3\left(b^2\ln\right.}{}\right.}{x^9+\frac{c^9}{d^9}+\frac{9\,c\,x^8}{d}+}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^13,x)
```

output
```
(F^a*F^(b/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x))*((2*x^9)/(b^4*d*log(F
)^4) - (b^3*log(F)^3 - 6*c^9 + 6*b*c^6*log(F) - 3*b^2*c^3*log(F)^2)/(3*b^4
*d^10*log(F)^4) + (18*c*x^8)/(b^4*d^2*log(F)^4) + (72*c^2*x^7)/(b^4*d^3*lo
g(F)^4) + (x^3*(b^2*log(F)^2 + 168*c^6 - 40*b*c^3*log(F)))/(b^4*d^7*log(F)
^4) - (2*x^6*(b*log(F) - 84*c^3))/(b^4*d^4*log(F)^4) + (3*c^2*x*(b^2*log(F
)^2 + 6*c^6 - 4*b*c^3*log(F)))/(b^4*d^9*log(F)^4) + (3*c*x^2*(b^2*log(F)^2
 + 24*c^6 - 10*b*c^3*log(F)))/(b^4*d^8*log(F)^4) - (12*c*x^5*(b*log(F) - 2
1*c^3))/(b^4*d^5*log(F)^4) - (6*c^2*x^4*(5*b*log(F) - 42*c^3))/(b^4*d^6*lo
g(F)^4)))/(x^9 + c^9/d^9 + (9*c*x^8)/d + (9*c^8*x)/d^8 + (36*c^2*x^7)/d^2
+ (84*c^3*x^6)/d^3 + (126*c^4*x^5)/d^4 + (126*c^5*x^4)/d^5 + (84*c^6*x^3)/
d^6 + (36*c^7*x^2)/d^7)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.39

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{13}}\,dx = \frac{f^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}\left(-\log(f)^3\,b^3+3\log(f)^2\,b^2c^3+9\log(f)^2\,b^2c^2dx+9\log(f)^2\,b^2c\,d^2x^2+3\log(f)^2\,b\right.}{}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^13,x)
```

output
```
(f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*( - log(f)**3*b**3 + 3*log(f)**2*b
**2*c**3 + 9*log(f)**2*b**2*c**2*d*x + 9*log(f)**2*b**2*c*d**2*x**2 + 3*lo
g(f)**2*b**2*d**3*x**3 - 6*log(f)*b*c**6 - 36*log(f)*b*c**5*d*x - 90*log(f
)*b*c**4*d**2*x**2 - 120*log(f)*b*c**3*d**3*x**3 - 90*log(f)*b*c**2*d**4*x
**4 - 36*log(f)*b*c*d**5*x**5 - 6*log(f)*b*d**6*x**6 + 6*c**9 + 54*c**8*d*
x + 216*c**7*d**2*x**2 + 504*c**6*d**3*x**3 + 756*c**5*d**4*x**4 + 756*c**
4*d**5*x**5 + 504*c**3*d**6*x**6 + 216*c**2*d**7*x**7 + 54*c*d**8*x**8 + 6
*d**9*x**9))/(3*log(f)**4*b**4*d*(c**9 + 9*c**8*d*x + 36*c**7*d**2*x**2 +
84*c**6*d**3*x**3 + 126*c**5*d**4*x**4 + 126*c**4*d**5*x**5 + 84*c**3*d**6
*x**6 + 36*c**2*d**7*x**7 + 9*c*d**8*x**8 + d**9*x**9))
```

3.285 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = -\frac{F^{a+\frac{b}{(c+dx)^3}}\left(24(c+dx)^{12}-24b(c+dx)^9\log(F)+12b^2(c+dx)^6\log^2(F)-4b^3(c+dx)^3\log^3(F)+b^4\log^4(F)\right)}{3b^5d(c+dx)^{12}\log^5(F)}$$

output
```
-1/3*F^(a+b/(d*x+c)^3)*(24*(d*x+c)^12-24*b*(d*x+c)^9*ln(F)+12*b^2*(d*x+c)^
6*ln(F)^2-4*b^3*(d*x+c)^3*ln(F)^3+b^4*ln(F)^4)/b^5/d/(d*x+c)^12/ln(F)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.32

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = -\frac{F^a\Gamma\left(5,-\frac{b\log(F)}{(c+dx)^3}\right)}{3b^5d\log^5(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^16,x]
```

output
```
-1/3*(F^a*Gamma[5, -((b*Log[F])/(c + d*x)^3)])/(b^5*d*Log[F]^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx$$

$$\Big\downarrow \; 2647$$

$$-\frac{F^{a+\frac{b}{(c+dx)^3}}\left(b^4\log^4(F)-4b^3\log^3(F)(c+dx)^3+12b^2\log^2(F)(c+dx)^6-24b\log(F)(c+dx)^9+24(c+dx)^{12}\right)}{3b^5d\log^5(F)(c+dx)^{12}}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^16,x]
```

output
```
-1/3*(F^(a + b/(c + d*x)^3)*(24*(c + d*x)^12 - 24*b*(c + d*x)^9*Log[F] + 1
2*b^2*(c + d*x)^6*Log[F]^2 - 4*b^3*(c + d*x)^3*Log[F]^3 + b^4*Log[F]^4))/(
b^5*d*(c + d*x)^12*Log[F]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(94) = 188$.

Time = 10.55 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.14

method	result
risch	$-\frac{\left(12 \ln(F)^{2} b^{2} c^{6}+24 d^{12} x^{12}+24 c^{12}+b^{4} \ln(F)^{4}+12 d^{6} x^{6} \ln(F)^{2} b^{2}-3024 \ln(F) b c^{5} d^{4} x^{4}-2016 \ln(F) b c^{6} d^{3} x^{3}-864 \ln(F) b c^{7} d}{}$
norman	Expression too large to display
parallelrisch	$\frac{-288 x^{11} F^{a+\frac{b}{(dx+c)^{3}}} c d^{30}-1584 x^{10} F^{a+\frac{b}{(dx+c)^{3}}} c^{2} d^{29}-5280 x^{9} F^{a+\frac{b}{(dx+c)^{3}}} c^{3} d^{28}-11880 x^{8} F^{a+\frac{b}{(dx+c)^{3}}} c^{4} d^{27}-19008 x^{7} F^{a}}{}$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^16,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(12*ln(F)^2*b^2*c^6+24*d^12*x^12+24*c^12+b^4*ln(F)^4+12*d^6*x^6*ln(F)
^2*b^2-3024*ln(F)*b*c^5*d^4*x^4-2016*ln(F)*b*c^6*d^3*x^3-864*ln(F)*b*c^7*d
^2*x^2-216*ln(F)*b*c^8*d*x-12*ln(F)^3*b^3*c*d^2*x^2-12*ln(F)^3*b^3*c^2*d*x
+288*c*d^11*x^11+1584*c^2*d^10*x^10+5280*c^3*d^9*x^9+11880*c^4*d^8*x^8+190
08*c^5*d^7*x^7+22176*c^6*d^6*x^6+19008*c^7*d^5*x^5+11880*c^8*d^4*x^4+5280*
c^9*d^3*x^3+1584*c^10*d^2*x^2+288*c^11*d*x+72*c*d^5*x^5*ln(F)^2*b^2+180*c^
2*d^4*x^4*ln(F)^2*b^2+240*ln(F)^2*b^2*c^3*d^3*x^3+180*ln(F)^2*b^2*c^4*d^2*
x^2+72*ln(F)^2*b^2*c^5*d*x-24*ln(F)*b*c^9-4*ln(F)^3*b^3*c^3-216*ln(F)*b*c*
d^8*x^8-864*ln(F)*b*c^2*d^7*x^7-2016*ln(F)*b*c^3*d^6*x^6-3024*ln(F)*b*c^4*
d^5*x^5-24*ln(F)*b*d^9*x^9-4*ln(F)^3*b^3*d^3*x^3)/b^5/ln(F)^5/d/(d*x+c)^12
*F^((a*d^3*x^3+3*a*c*d^2*x^2+3*a*c^2*d*x+a*c^3+b)/(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(94) = 188$.

Time = 0.12 (sec) , antiderivative size = 621, normalized size of antiderivative = 6.47

$$\int \frac{F^{a+\frac{b}{(c+dx)^{3}}}}{(c+dx)^{16}}\,dx =$$

$$-\frac{\left(24\,d^{12}x^{12}+288\,cd^{11}x^{11}+1584\,c^{2}d^{10}x^{10}+5280\,c^{3}d^{9}x^{9}+11880\,c^{4}d^{8}x^{8}+19008\,c^{5}d^{7}x^{7}+22176\,c^{6}d^{6}x^{6}\right.}{}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^16,x, algorithm="fricas")
```

output
```
-1/3*(24*d^12*x^12 + 288*c*d^11*x^11 + 1584*c^2*d^10*x^10 + 5280*c^3*d^9*x
^9 + 11880*c^4*d^8*x^8 + 19008*c^5*d^7*x^7 + 22176*c^6*d^6*x^6 + 19008*c^7
*d^5*x^5 + 11880*c^8*d^4*x^4 + 5280*c^9*d^3*x^3 + 1584*c^10*d^2*x^2 + 288*
c^11*d*x + 24*c^12 + b^4*log(F)^4 - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*log(F)^3 + 12*(b^2*d^6*x^6 + 6*b^2*c*d^5*x^5 + 15*b^
2*c^2*d^4*x^4 + 20*b^2*c^3*d^3*x^3 + 15*b^2*c^4*d^2*x^2 + 6*b^2*c^5*d*x +
b^2*c^6)*log(F)^2 - 24*(b*d^9*x^9 + 9*b*c*d^8*x^8 + 36*b*c^2*d^7*x^7 + 84*
b*c^3*d^6*x^6 + 126*b*c^4*d^5*x^5 + 126*b*c^5*d^4*x^4 + 84*b*c^6*d^3*x^3 +
 36*b*c^7*d^2*x^2 + 9*b*c^8*d*x + b*c^9)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2
*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
/((b^5*d^13*x^12 + 12*b^5*c*d^12*x^11 + 66*b^5*c^2*d^11*x^10 + 220*b^5*c^3
*d^10*x^9 + 495*b^5*c^4*d^9*x^8 + 792*b^5*c^5*d^8*x^7 + 924*b^5*c^6*d^7*x^
6 + 792*b^5*c^7*d^6*x^5 + 495*b^5*c^8*d^5*x^4 + 220*b^5*c^9*d^4*x^3 + 66*b
^5*c^10*d^3*x^2 + 12*b^5*c^11*d^2*x + b^5*c^12*d)*log(F)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(95) = 190$.

Time = 0.57 (sec) , antiderivative size = 760, normalized size of antiderivative = 7.92

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = \text{Too large to display}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**16,x)
```

output
```
F**(a + b/(c + d*x)**3)*(-b**4*log(F)**4 + 4*b**3*c**3*log(F)**3 + 12*b**3
*c**2*d*x*log(F)**3 + 12*b**3*c*d**2*x**2*log(F)**3 + 4*b**3*d**3*x**3*log
(F)**3 - 12*b**2*c**6*log(F)**2 - 72*b**2*c**5*d*x*log(F)**2 - 180*b**2*c*
*4*d**2*x**2*log(F)**2 - 240*b**2*c**3*d**3*x**3*log(F)**2 - 180*b**2*c**2
*d**4*x**4*log(F)**2 - 72*b**2*c*d**5*x**5*log(F)**2 - 12*b**2*d**6*x**6*l
og(F)**2 + 24*b*c**9*log(F) + 216*b*c**8*d*x*log(F) + 864*b*c**7*d**2*x**2
*log(F) + 2016*b*c**6*d**3*x**3*log(F) + 3024*b*c**5*d**4*x**4*log(F) + 30
24*b*c**4*d**5*x**5*log(F) + 2016*b*c**3*d**6*x**6*log(F) + 864*b*c**2*d**
7*x**7*log(F) + 216*b*c*d**8*x**8*log(F) + 24*b*d**9*x**9*log(F) - 24*c**1
2 - 288*c**11*d*x - 1584*c**10*d**2*x**2 - 5280*c**9*d**3*x**3 - 11880*c**
8*d**4*x**4 - 19008*c**7*d**5*x**5 - 22176*c**6*d**6*x**6 - 19008*c**5*d**
7*x**7 - 11880*c**4*d**8*x**8 - 5280*c**3*d**9*x**9 - 1584*c**2*d**10*x**1
0 - 288*c*d**11*x**11 - 24*d**12*x**12)/(3*b**5*c**12*d*log(F)**5 + 36*b**
5*c**11*d**2*x*log(F)**5 + 198*b**5*c**10*d**3*x**2*log(F)**5 + 660*b**5*c
**9*d**4*x**3*log(F)**5 + 1485*b**5*c**8*d**5*x**4*log(F)**5 + 2376*b**5*c
**7*d**6*x**5*log(F)**5 + 2772*b**5*c**6*d**7*x**6*log(F)**5 + 2376*b**5*c
**5*d**8*x**7*log(F)**5 + 1485*b**5*c**4*d**9*x**8*log(F)**5 + 660*b**5*c*
*3*d**10*x**9*log(F)**5 + 198*b**5*c**2*d**11*x**10*log(F)**5 + 36*b**5*c*
d**12*x**11*log(F)**5 + 3*b**5*d**13*x**12*log(F)**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(94) = 188$.

Time = 0.06 (sec) , antiderivative size = 770, normalized size of antiderivative = 8.02

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^16,x, algorithm="maxima")
```

output
```
-1/3*(24*F^a*d^12*x^12 + 288*F^a*c*d^11*x^11 + 1584*F^a*c^2*d^10*x^10 + 24
*F^a*c^12 - 24*F^a*b*c^9*log(F) + 12*F^a*b^2*c^6*log(F)^2 + 24*(220*F^a*c^
3*d^9 - F^a*b*d^9*log(F))*x^9 - 4*F^a*b^3*c^3*log(F)^3 + 216*(55*F^a*c^4*d
^8 - F^a*b*c*d^8*log(F))*x^8 + F^a*b^4*log(F)^4 + 864*(22*F^a*c^5*d^7 - F^
a*b*c^2*d^7*log(F))*x^7 + 12*(1848*F^a*c^6*d^6 - 168*F^a*b*c^3*d^6*log(F)
+ F^a*b^2*d^6*log(F)^2)*x^6 + 72*(264*F^a*c^7*d^5 - 42*F^a*b*c^4*d^5*log(F
) + F^a*b^2*c*d^5*log(F)^2)*x^5 + 36*(330*F^a*c^8*d^4 - 84*F^a*b*c^5*d^4*l
og(F) + 5*F^a*b^2*c^2*d^4*log(F)^2)*x^4 + 4*(1320*F^a*c^9*d^3 - 504*F^a*b*
c^6*d^3*log(F) + 60*F^a*b^2*c^3*d^3*log(F)^2 - F^a*b^3*d^3*log(F)^3)*x^3 +
 12*(132*F^a*c^10*d^2 - 72*F^a*b*c^7*d^2*log(F) + 15*F^a*b^2*c^4*d^2*log(F
)^2 - F^a*b^3*c*d^2*log(F)^3)*x^2 + 12*(24*F^a*c^11*d - 18*F^a*b*c^8*d*log
(F) + 6*F^a*b^2*c^5*d*log(F)^2 - F^a*b^3*c^2*d*log(F)^3)*x)*F^(b/(d^3*x^3
+ 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^5*d^13*x^12*log(F)^5 + 12*b^5*c*d^12*
x^11*log(F)^5 + 66*b^5*c^2*d^11*x^10*log(F)^5 + 220*b^5*c^3*d^10*x^9*log(F
)^5 + 495*b^5*c^4*d^9*x^8*log(F)^5 + 792*b^5*c^5*d^8*x^7*log(F)^5 + 924*b^
5*c^6*d^7*x^6*log(F)^5 + 792*b^5*c^7*d^6*x^5*log(F)^5 + 495*b^5*c^8*d^5*x^
4*log(F)^5 + 220*b^5*c^9*d^4*x^3*log(F)^5 + 66*b^5*c^10*d^3*x^2*log(F)^5 +
 12*b^5*c^11*d^2*x*log(F)^5 + b^5*c^12*d*log(F)^5)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^{16}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^16,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^16, x)
```

Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 620, normalized size of antiderivative = 6.46

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = -\frac{F^a\,F^{\frac{b}{c^3+3\,c^2\,d\,x+3\,c\,d^2\,x^2+d^3\,x^3}}\left(\frac{b^4\ln(F)^4-4\,b^3\,c^3\ln(F)^3+12\,b^2\,c^6\ln(F)^2-24\,b\,c^9\,\ln(F)+24\,c^{12}}{3\,b^5\,d^{13}\ln(F)^5}+\frac{8\,x^{12}}{b^5\,d\ln(F)^5}+\frac{96\,c\,x^{11}}{b^5\,d^2\ln(F)^5}+\frac{5}{b^5}\right.}{}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^16,x)
```

output
```
-(F^a*F^(b/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x))*((b^4*log(F)^4 + 24*
c^12 - 24*b*c^9*log(F) - 4*b^3*c^3*log(F)^3 + 12*b^2*c^6*log(F)^2)/(3*b^5*
d^13*log(F)^5) + (8*x^12)/(b^5*d*log(F)^5) + (96*c*x^11)/(b^5*d^2*log(F)^5
) + (528*c^2*x^10)/(b^5*d^3*log(F)^5) - (4*x^3*(b^3*log(F)^3 - 1320*c^9 +
504*b*c^6*log(F) - 60*b^2*c^3*log(F)^2))/(3*b^5*d^10*log(F)^5) + (4*x^6*(b
^2*log(F)^2 + 1848*c^6 - 168*b*c^3*log(F)))/(b^5*d^7*log(F)^5) - (8*x^9*(b
*log(F) - 220*c^3))/(b^5*d^4*log(F)^5) - (4*c^2*x*(b^3*log(F)^3 - 24*c^9 +
 18*b*c^6*log(F) - 6*b^2*c^3*log(F)^2))/(b^5*d^12*log(F)^5) - (4*c*x^2*(b^
3*log(F)^3 - 132*c^9 + 72*b*c^6*log(F) - 15*b^2*c^3*log(F)^2))/(b^5*d^11*l
og(F)^5) + (24*c*x^5*(b^2*log(F)^2 + 264*c^6 - 42*b*c^3*log(F)))/(b^5*d^8*
log(F)^5) - (72*c*x^8*(b*log(F) - 55*c^3))/(b^5*d^5*log(F)^5) + (12*c^2*x^
4*(5*b^2*log(F)^2 + 330*c^6 - 84*b*c^3*log(F)))/(b^5*d^9*log(F)^5) - (288*
c^2*x^7*(b*log(F) - 22*c^3))/(b^5*d^6*log(F)^5)))/(x^12 + c^12/d^12 + (12*
c*x^11)/d + (12*c^11*x)/d^11 + (66*c^2*x^10)/d^2 + (220*c^3*x^9)/d^3 + (49
5*c^4*x^8)/d^4 + (792*c^5*x^7)/d^5 + (924*c^6*x^6)/d^6 + (792*c^7*x^5)/d^7
 + (495*c^8*x^4)/d^8 + (220*c^9*x^3)/d^9 + (66*c^10*x^2)/d^10)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 636, normalized size of antiderivative = 6.62

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{16}}\,dx = \frac{f^{\frac{a\,d^3x^3+3ac\,d^2x^2+3a\,c^2dx+a\,c^3+b}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}\left(-24d^{12}x^{12}+4\log(f)^3\,b^3d^3x^3+24\log(f)\,b\,d^9x^9-72\log(f)^2\,b^2c^5dx-180\log(f\right.}{}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^16,x)
```

output
```
(f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*( - log(f)**4*b**4 + 4*log(f)**3*b
**3*c**3 + 12*log(f)**3*b**3*c**2*d*x + 12*log(f)**3*b**3*c*d**2*x**2 + 4*
log(f)**3*b**3*d**3*x**3 - 12*log(f)**2*b**2*c**6 - 72*log(f)**2*b**2*c**5
*d*x - 180*log(f)**2*b**2*c**4*d**2*x**2 - 240*log(f)**2*b**2*c**3*d**3*x*
*3 - 180*log(f)**2*b**2*c**2*d**4*x**4 - 72*log(f)**2*b**2*c*d**5*x**5 - 1
2*log(f)**2*b**2*d**6*x**6 + 24*log(f)*b*c**9 + 216*log(f)*b*c**8*d*x + 86
4*log(f)*b*c**7*d**2*x**2 + 2016*log(f)*b*c**6*d**3*x**3 + 3024*log(f)*b*c
**5*d**4*x**4 + 3024*log(f)*b*c**4*d**5*x**5 + 2016*log(f)*b*c**3*d**6*x**
6 + 864*log(f)*b*c**2*d**7*x**7 + 216*log(f)*b*c*d**8*x**8 + 24*log(f)*b*d
**9*x**9 - 24*c**12 - 288*c**11*d*x - 1584*c**10*d**2*x**2 - 5280*c**9*d**
3*x**3 - 11880*c**8*d**4*x**4 - 19008*c**7*d**5*x**5 - 22176*c**6*d**6*x**
6 - 19008*c**5*d**7*x**7 - 11880*c**4*d**8*x**8 - 5280*c**3*d**9*x**9 - 15
84*c**2*d**10*x**10 - 288*c*d**11*x**11 - 24*d**12*x**12))/(3*log(f)**5*b*
*5*d*(c**12 + 12*c**11*d*x + 66*c**10*d**2*x**2 + 220*c**9*d**3*x**3 + 495
*c**8*d**4*x**4 + 792*c**7*d**5*x**5 + 924*c**6*d**6*x**6 + 792*c**5*d**7*
x**7 + 495*c**4*d**8*x**8 + 220*c**3*d**9*x**9 + 66*c**2*d**10*x**10 + 12*
c*d**11*x**11 + d**12*x**12))
```

3.286 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}} dx$

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}} dx = \frac{F^{a+\frac{b}{(c+dx)^3}}\left(120(c+dx)^{15}-120b(c+dx)^{12}\log(F)+60b^2(c+dx)^9\log^2(F)-20b^3(c+dx)^6\log^3(F)+5b\right.}{3b^6d(c+dx)^{15}\log^6(F)}$$

output
```
1/3*F^(a+b/(d*x+c)^3)*(120*(d*x+c)^15-120*b*(d*x+c)^12*ln(F)+60*b^2*(d*x+c
)^9*ln(F)^2-20*b^3*(d*x+c)^6*ln(F)^3+5*b^4*(d*x+c)^3*ln(F)^4-b^5*ln(F)^5)/
b^6/d/(d*x+c)^15/ln(F)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.27

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}} dx = \frac{F^a\Gamma\left(6,-\frac{b\log(F)}{(c+dx)^3}\right)}{3b^6d\log^6(F)}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^19,x]
```

output
```
(F^a*Gamma[6, -((b*Log[F])/(c + d*x)^3)])/(3*b^6*d*Log[F]^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx$$

$$\Big\downarrow \; 2647$$

$$\frac{F^{a+\frac{b}{(c+dx)^3}}\left(-b^5\log^5(F)+5b^4\log^4(F)(c+dx)^3-20b^3\log^3(F)(c+dx)^6+60b^2\log^2(F)(c+dx)^9-120b\log(F)(\right.}{3b^6d\log^6(F)(c+dx)^{15}}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^19,x]
```

output
```
(F^(a + b/(c + d*x)^3)*(120*(c + d*x)^15 - 120*b*(c + d*x)^12*Log[F] + 60*
b^2*(c + d*x)^9*Log[F]^2 - 20*b^3*(c + d*x)^6*Log[F]^3 + 5*b^4*(c + d*x)^3
*Log[F]^4 - b^5*Log[F]^5))/(3*b^6*d*(c + d*x)^15*Log[F]^6)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] &&  !TrueQ[$UseGamma]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(111) = 222$.

Time = 18.09 (sec) , antiderivative size = 733, normalized size of antiderivative = 6.49

method	result
risch	$-\frac{\left(-120c^{15}+20d^{6}x^{6}b^{3}\ln(F)^{3}+7920\ln(F)b\,c^{2}d^{10}x^{10}+26400\ln(F)b\,c^{3}d^{9}x^{9}+59400\ln(F)b\,c^{4}d^{8}x^{8}-540\ln(F)^{2}b^{2}c\,d^{8}x^{8}+95\right.}{}$
parallelrisch	Expression too large to display

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^19,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-120*c^15+20*d^6*x^6*b^3*ln(F)^3+7920*ln(F)*b*c^2*d^10*x^10+26400*ln
(F)*b*c^3*d^9*x^9+59400*ln(F)*b*c^4*d^8*x^8-540*ln(F)^2*b^2*c*d^8*x^8+9504
0*ln(F)*b*c^5*d^7*x^7-2160*ln(F)^2*b^2*c^2*d^7*x^7+120*c*d^5*x^5*b^3*ln(F)
^3+300*ln(F)^3*b^3*c^2*d^4*x^4+400*ln(F)^3*b^3*c^3*d^3*x^3+300*ln(F)^3*b^3
*c^4*d^2*x^2+120*ln(F)^3*b^3*c^5*d*x+1440*ln(F)*b*c*d^11*x^11+b^5*ln(F)^5-
1800*c*d^14*x^14-12600*c^2*d^13*x^13-54600*c^3*d^12*x^12-163800*c^4*d^11*x
^11-360360*c^5*d^10*x^10-600600*c^6*d^9*x^9-772200*c^7*d^8*x^8-772200*c^8*
d^7*x^7+110880*ln(F)*b*c^6*d^6*x^6-5040*ln(F)^2*b^2*c^3*d^6*x^6+95040*ln(F
)*b*c^7*d^5*x^5-7560*ln(F)^2*b^2*c^4*d^5*x^5+59400*ln(F)*b*c^8*d^4*x^4-756
0*ln(F)^2*b^2*c^5*d^4*x^4+26400*ln(F)*b*c^9*d^3*x^3+20*ln(F)^3*b^3*c^6-600
600*c^9*d^6*x^6-360360*c^10*d^5*x^5-163800*c^11*d^4*x^4-54600*c^12*d^3*x^3
-12600*c^13*d^2*x^2-1800*c^14*d*x-120*d^15*x^15-5040*ln(F)^2*b^2*c^6*d^3*x
^3+7920*ln(F)*b*c^10*d^2*x^2-2160*ln(F)^2*b^2*c^7*d^2*x^2+1440*ln(F)*b*c^1
1*d*x-540*ln(F)^2*b^2*c^8*d*x-15*ln(F)^4*b^4*c*d^2*x^2-15*ln(F)^4*b^4*c^2*
d*x+120*ln(F)*b*d^12*x^12-60*ln(F)^2*b^2*d^9*x^9-5*ln(F)^4*b^4*d^3*x^3+120
*ln(F)*b*c^12-60*ln(F)^2*b^2*c^9-5*ln(F)^4*b^4*c^3)/ln(F)^6/b^6/d/(d*x+c)^
15*F^((a*d^3*x^3+3*a*c*d^2*x^2+3*a*c^2*d*x+a*c^3+b)/(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. $2(111) = 222$.

Time = 0.17 (sec) , antiderivative size = 863, normalized size of antiderivative = 7.64

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^19,x, algorithm="fricas")
```

output
```
1/3*(120*d^15*x^15 + 1800*c*d^14*x^14 + 12600*c^2*d^13*x^13 + 54600*c^3*d^
12*x^12 + 163800*c^4*d^11*x^11 + 360360*c^5*d^10*x^10 + 600600*c^6*d^9*x^9
 + 772200*c^7*d^8*x^8 + 772200*c^8*d^7*x^7 + 600600*c^9*d^6*x^6 + 360360*c
^10*d^5*x^5 + 163800*c^11*d^4*x^4 + 54600*c^12*d^3*x^3 + 12600*c^13*d^2*x^
2 + 1800*c^14*d*x + 120*c^15 - b^5*log(F)^5 + 5*(b^4*d^3*x^3 + 3*b^4*c*d^2
*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*log(F)^4 - 20*(b^3*d^6*x^6 + 6*b^3*c*d^5*x
^5 + 15*b^3*c^2*d^4*x^4 + 20*b^3*c^3*d^3*x^3 + 15*b^3*c^4*d^2*x^2 + 6*b^3*
c^5*d*x + b^3*c^6)*log(F)^3 + 60*(b^2*d^9*x^9 + 9*b^2*c*d^8*x^8 + 36*b^2*c
^2*d^7*x^7 + 84*b^2*c^3*d^6*x^6 + 126*b^2*c^4*d^5*x^5 + 126*b^2*c^5*d^4*x^
4 + 84*b^2*c^6*d^3*x^3 + 36*b^2*c^7*d^2*x^2 + 9*b^2*c^8*d*x + b^2*c^9)*log
(F)^2 - 120*(b*d^12*x^12 + 12*b*c*d^11*x^11 + 66*b*c^2*d^10*x^10 + 220*b*c
^3*d^9*x^9 + 495*b*c^4*d^8*x^8 + 792*b*c^5*d^7*x^7 + 924*b*c^6*d^6*x^6 + 7
92*b*c^7*d^5*x^5 + 495*b*c^8*d^4*x^4 + 220*b*c^9*d^3*x^3 + 66*b*c^10*d^2*x
^2 + 12*b*c^11*d*x + b*c^12)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c
^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b^6*d^16*
x^15 + 15*b^6*c*d^15*x^14 + 105*b^6*c^2*d^14*x^13 + 455*b^6*c^3*d^13*x^12
+ 1365*b^6*c^4*d^12*x^11 + 3003*b^6*c^5*d^11*x^10 + 5005*b^6*c^6*d^10*x^9
+ 6435*b^6*c^7*d^9*x^8 + 6435*b^6*c^8*d^8*x^7 + 5005*b^6*c^9*d^7*x^6 + 300
3*b^6*c^10*d^6*x^5 + 1365*b^6*c^11*d^5*x^4 + 455*b^6*c^12*d^4*x^3 + 105*b^
6*c^13*d^3*x^2 + 15*b^6*c^14*d^2*x + b^6*c^15*d)*log(F)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(110) = 220$.

Time = 2.62 (sec) , antiderivative size = 1096, normalized size of antiderivative = 9.70

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx = \text{Too large to display}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**19,x)
```

output
```
F**(a + b/(c + d*x)**3)*(-b**5*log(F)**5 + 5*b**4*c**3*log(F)**4 + 15*b**4
*c**2*d*x*log(F)**4 + 15*b**4*c*d**2*x**2*log(F)**4 + 5*b**4*d**3*x**3*log
(F)**4 - 20*b**3*c**6*log(F)**3 - 120*b**3*c**5*d*x*log(F)**3 - 300*b**3*c
**4*d**2*x**2*log(F)**3 - 400*b**3*c**3*d**3*x**3*log(F)**3 - 300*b**3*c**
2*d**4*x**4*log(F)**3 - 120*b**3*c*d**5*x**5*log(F)**3 - 20*b**3*d**6*x**6
*log(F)**3 + 60*b**2*c**9*log(F)**2 + 540*b**2*c**8*d*x*log(F)**2 + 2160*b
**2*c**7*d**2*x**2*log(F)**2 + 5040*b**2*c**6*d**3*x**3*log(F)**2 + 7560*b
**2*c**5*d**4*x**4*log(F)**2 + 7560*b**2*c**4*d**5*x**5*log(F)**2 + 5040*b
**2*c**3*d**6*x**6*log(F)**2 + 2160*b**2*c**2*d**7*x**7*log(F)**2 + 540*b*
*2*c*d**8*x**8*log(F)**2 + 60*b**2*d**9*x**9*log(F)**2 - 120*b*c**12*log(F
) - 1440*b*c**11*d*x*log(F) - 7920*b*c**10*d**2*x**2*log(F) - 26400*b*c**9
*d**3*x**3*log(F) - 59400*b*c**8*d**4*x**4*log(F) - 95040*b*c**7*d**5*x**5
*log(F) - 110880*b*c**6*d**6*x**6*log(F) - 95040*b*c**5*d**7*x**7*log(F) -
 59400*b*c**4*d**8*x**8*log(F) - 26400*b*c**3*d**9*x**9*log(F) - 7920*b*c*
*2*d**10*x**10*log(F) - 1440*b*c*d**11*x**11*log(F) - 120*b*d**12*x**12*lo
g(F) + 120*c**15 + 1800*c**14*d*x + 12600*c**13*d**2*x**2 + 54600*c**12*d*
*3*x**3 + 163800*c**11*d**4*x**4 + 360360*c**10*d**5*x**5 + 600600*c**9*d*
*6*x**6 + 772200*c**8*d**7*x**7 + 772200*c**7*d**8*x**8 + 600600*c**6*d**9
*x**9 + 360360*c**5*d**10*x**10 + 163800*c**4*d**11*x**11 + 54600*c**3*d**
12*x**12 + 12600*c**2*d**13*x**13 + 1800*c*d**14*x**14 + 120*d**15*x**1...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(111) = 222$.

Time = 0.07 (sec) , antiderivative size = 1085, normalized size of antiderivative = 9.60

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^19,x, algorithm="maxima")
```

output
```
1/3*(120*F^a*d^15*x^15 + 1800*F^a*c*d^14*x^14 + 12600*F^a*c^2*d^13*x^13 +
120*F^a*c^15 - 120*F^a*b*c^12*log(F) + 60*F^a*b^2*c^9*log(F)^2 + 120*(455*
F^a*c^3*d^12 - F^a*b*d^12*log(F))*x^12 - 20*F^a*b^3*c^6*log(F)^3 + 360*(45
5*F^a*c^4*d^11 - 4*F^a*b*c*d^11*log(F))*x^11 + 5*F^a*b^4*c^3*log(F)^4 + 39
60*(91*F^a*c^5*d^10 - 2*F^a*b*c^2*d^10*log(F))*x^10 - F^a*b^5*log(F)^5 + 6
0*(10010*F^a*c^6*d^9 - 440*F^a*b*c^3*d^9*log(F) + F^a*b^2*d^9*log(F)^2)*x^
9 + 540*(1430*F^a*c^7*d^8 - 110*F^a*b*c^4*d^8*log(F) + F^a*b^2*c*d^8*log(F
)^2)*x^8 + 1080*(715*F^a*c^8*d^7 - 88*F^a*b*c^5*d^7*log(F) + 2*F^a*b^2*c^2
*d^7*log(F)^2)*x^7 + 20*(30030*F^a*c^9*d^6 - 5544*F^a*b*c^6*d^6*log(F) + 2
52*F^a*b^2*c^3*d^6*log(F)^2 - F^a*b^3*d^6*log(F)^3)*x^6 + 120*(3003*F^a*c^
10*d^5 - 792*F^a*b*c^7*d^5*log(F) + 63*F^a*b^2*c^4*d^5*log(F)^2 - F^a*b^3*
c*d^5*log(F)^3)*x^5 + 60*(2730*F^a*c^11*d^4 - 990*F^a*b*c^8*d^4*log(F) + 1
26*F^a*b^2*c^5*d^4*log(F)^2 - 5*F^a*b^3*c^2*d^4*log(F)^3)*x^4 + 5*(10920*F
^a*c^12*d^3 - 5280*F^a*b*c^9*d^3*log(F) + 1008*F^a*b^2*c^6*d^3*log(F)^2 -
80*F^a*b^3*c^3*d^3*log(F)^3 + F^a*b^4*d^3*log(F)^4)*x^3 + 15*(840*F^a*c^13
*d^2 - 528*F^a*b*c^10*d^2*log(F) + 144*F^a*b^2*c^7*d^2*log(F)^2 - 20*F^a*b
^3*c^4*d^2*log(F)^3 + F^a*b^4*c*d^2*log(F)^4)*x^2 + 15*(120*F^a*c^14*d - 9
6*F^a*b*c^11*d*log(F) + 36*F^a*b^2*c^8*d*log(F)^2 - 8*F^a*b^3*c^5*d*log(F)
^3 + F^a*b^4*c^2*d*log(F)^4)*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3))/(b^6*d^16*x^15*log(F)^6 + 15*b^6*c*d^15*x^14*log(F)^6 + 105*b^6*c...
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^{19}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^19,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^19, x)
```

Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 854, normalized size of antiderivative = 7.56

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx = \text{Too large to display}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^19,x)
```

output
```
(F^a*F^(b/(c^3 + d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x))*((40*x^15)/(b^6*d*log
(F)^6) - (b^5*log(F)^5 - 120*c^15 + 120*b*c^12*log(F) - 5*b^4*c^3*log(F)^4
 + 20*b^3*c^6*log(F)^3 - 60*b^2*c^9*log(F)^2)/(3*b^6*d^16*log(F)^6) + (600
*c*x^14)/(b^6*d^2*log(F)^6) + (4200*c^2*x^13)/(b^6*d^3*log(F)^6) + (5*x^3*
(b^4*log(F)^4 + 10920*c^12 - 5280*b*c^9*log(F) - 80*b^3*c^3*log(F)^3 + 100
8*b^2*c^6*log(F)^2))/(3*b^6*d^13*log(F)^6) - (20*x^6*(b^3*log(F)^3 - 30030
*c^9 + 5544*b*c^6*log(F) - 252*b^2*c^3*log(F)^2))/(3*b^6*d^10*log(F)^6) +
(20*x^9*(b^2*log(F)^2 + 10010*c^6 - 440*b*c^3*log(F)))/(b^6*d^7*log(F)^6)
- (40*x^12*(b*log(F) - 455*c^3))/(b^6*d^4*log(F)^6) + (5*c^2*x*(b^4*log(F)
^4 + 120*c^12 - 96*b*c^9*log(F) - 8*b^3*c^3*log(F)^3 + 36*b^2*c^6*log(F)^2
))/(b^6*d^15*log(F)^6) + (5*c*x^2*(b^4*log(F)^4 + 840*c^12 - 528*b*c^9*log
(F) - 20*b^3*c^3*log(F)^3 + 144*b^2*c^6*log(F)^2))/(b^6*d^14*log(F)^6) - (
40*c*x^5*(b^3*log(F)^3 - 3003*c^9 + 792*b*c^6*log(F) - 63*b^2*c^3*log(F)^2
))/(b^6*d^11*log(F)^6) + (180*c*x^8*(b^2*log(F)^2 + 1430*c^6 - 110*b*c^3*l
og(F)))/(b^6*d^8*log(F)^6) - (120*c*x^11*(4*b*log(F) - 455*c^3))/(b^6*d^5*
log(F)^6) - (20*c^2*x^4*(5*b^3*log(F)^3 - 2730*c^9 + 990*b*c^6*log(F) - 12
6*b^2*c^3*log(F)^2))/(b^6*d^12*log(F)^6) + (360*c^2*x^7*(2*b^2*log(F)^2 +
715*c^6 - 88*b*c^3*log(F)))/(b^6*d^9*log(F)^6) - (1320*c^2*x^10*(2*b*log(F
) - 91*c^3))/(b^6*d^6*log(F)^6)))/(x^15 + c^15/d^15 + (15*c*x^14)/d + (15*
c^14*x)/d^14 + (105*c^2*x^13)/d^2 + (455*c^3*x^12)/d^3 + (1365*c^4*x^11...
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 909, normalized size of antiderivative = 8.04

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^{19}}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^19,x)
```

output
```
(f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*( - log(f)**5*b**5 + 5*log(f)**4*b
**4*c**3 + 15*log(f)**4*b**4*c**2*d*x + 15*log(f)**4*b**4*c*d**2*x**2 + 5*
log(f)**4*b**4*d**3*x**3 - 20*log(f)**3*b**3*c**6 - 120*log(f)**3*b**3*c**
5*d*x - 300*log(f)**3*b**3*c**4*d**2*x**2 - 400*log(f)**3*b**3*c**3*d**3*x
**3 - 300*log(f)**3*b**3*c**2*d**4*x**4 - 120*log(f)**3*b**3*c*d**5*x**5 -
 20*log(f)**3*b**3*d**6*x**6 + 60*log(f)**2*b**2*c**9 + 540*log(f)**2*b**2
*c**8*d*x + 2160*log(f)**2*b**2*c**7*d**2*x**2 + 5040*log(f)**2*b**2*c**6*
d**3*x**3 + 7560*log(f)**2*b**2*c**5*d**4*x**4 + 7560*log(f)**2*b**2*c**4*
d**5*x**5 + 5040*log(f)**2*b**2*c**3*d**6*x**6 + 2160*log(f)**2*b**2*c**2*
d**7*x**7 + 540*log(f)**2*b**2*c*d**8*x**8 + 60*log(f)**2*b**2*d**9*x**9 -
 120*log(f)*b*c**12 - 1440*log(f)*b*c**11*d*x - 7920*log(f)*b*c**10*d**2*x
**2 - 26400*log(f)*b*c**9*d**3*x**3 - 59400*log(f)*b*c**8*d**4*x**4 - 9504
0*log(f)*b*c**7*d**5*x**5 - 110880*log(f)*b*c**6*d**6*x**6 - 95040*log(f)*
b*c**5*d**7*x**7 - 59400*log(f)*b*c**4*d**8*x**8 - 26400*log(f)*b*c**3*d**
9*x**9 - 7920*log(f)*b*c**2*d**10*x**10 - 1440*log(f)*b*c*d**11*x**11 - 12
0*log(f)*b*d**12*x**12 + 120*c**15 + 1800*c**14*d*x + 12600*c**13*d**2*x**
2 + 54600*c**12*d**3*x**3 + 163800*c**11*d**4*x**4 + 360360*c**10*d**5*x**
5 + 600600*c**9*d**6*x**6 + 772200*c**8*d**7*x**7 + 772200*c**7*d**8*x**8
+ 600600*c**6*d**9*x**9 + 360360*c**5*d**10*x**10 + 163800*c**4*d**11*x...
```

3.287 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx=\frac{F^a(c+dx)^4\Gamma\left(-\frac{4}{3},-\frac{b\log(F)}{(c+dx)^3}\right)\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{4/3}}{3d}$$

output
```
1/3*F^a*(d*x+c)^4*GAMMA(-4/3,-b*ln(F)/(d*x+c)^3)*(-b*ln(F)/(d*x+c)^3)^(4/3
)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx=\frac{F^a(c+dx)^4\Gamma\left(-\frac{4}{3},-\frac{b\log(F)}{(c+dx)^3}\right)\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{4/3}}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x)^3,x]
```

output
```
(F^a*(c + d*x)^4*Gamma[-4/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*Log[F])/(c +
 d*x)^3))^(4/3))/(3*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 F^{a+\frac{b}{(c+dx)^3}} \, dx$$

$$\Big\downarrow \; 2648$$

$$\frac{F^a (c+dx)^4 \left(-\frac{b \log(F)}{(c+dx)^3}\right)^{4/3} \Gamma\left(-\frac{4}{3}, -\frac{b \log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x)^3,x]
```

output
```
(F^a*(c + d*x)^4*Gamma[-4/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*Log[F])/(c +
 d*x)^3))^(4/3))/(3*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)^3\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^3,x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(43) = 86.

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.63

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx = -\frac{3\,F^a bd\left(-\frac{b\log(F)}{d^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},-\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)\log(F)-(d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4+}{4\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(3*F^a*b*d*(-b*log(F)/d^3)^(1/3)*gamma(2/3, -b*log(F)/(d^3*x^3 + 3*c*
d^2*x^2 + 3*c^2*d*x + c^3))*log(F) - (d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^
2 + 4*c^3*d*x + c^4 + 3*(b*d*x + b*c)*log(F))*F^((a*d^3*x^3 + 3*a*c*d^2*x^
2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx = \int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c)**3,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x)**3, x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx = \int (dx+c)^3 F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*(F^a*d^3*x^4 + 4*F^a*c*d^2*x^3 + 6*F^a*c^2*d*x^2 + (4*F^a*c^3 + 3*F^a*
b*log(F))*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + integrate(-
3/4*(F^a*b*c^4*log(F) - 3*F^a*b^2*d*x*log(F)^2)*F^(b/(d^3*x^3 + 3*c*d^2*x^
2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x +
 c^4), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx = \int (dx+c)^3 F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.61

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx = \frac{F^a\,F^{\frac{b}{(c+d\,x)^3}}\,(c+d\,x)^4}{4\,d} - \frac{3\,F^a\,\Gamma\left(\frac{2}{3}\right)\,(c+d\,x)^4\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{4/3}}{4\,d} + \frac{3\,F^a\,\Gamma\left(\frac{2}{3},-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)\,(c+d\,x)^4\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{4/3}}{4\,d} + \frac{3\,F^a\,F^{\frac{b}{(c+d\,x)^3}}\,b\,\ln(F)\,(c+d\,x)}{4\,d}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x)^3,x)
```

output
```
(F^a*F^(b/(c + d*x)^3)*(c + d*x)^4)/(4*d) - (3*F^a*gamma(2/3)*(c + d*x)^4*
(-(b*log(F))/(c + d*x)^3)^(4/3))/(4*d) + (3*F^a*igamma(2/3, -(b*log(F))/(c
 + d*x)^3)*(c + d*x)^4*(-(b*log(F))/(c + d*x)^3)^(4/3))/(4*d) + (3*F^a*F^(
b/(c + d*x)^3)*b*log(F)*(c + d*x))/(4*d)
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)^3\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c)^3,x)
```

output

```
(27*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**3*b**3*d*x + 9*f**((a*c
**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c**4 - 18*f**((a*c**3 + 3*a*
c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3))*log(f)**2*b**2*c**3*d*x - 108*f**((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*log(f)**2*b**2*c**2*d**2*x**2 - 126*f**((a*c**3 + 3*a*c**2
*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3))*log(f)**2*b**2*c*d**3*x**3 - 45*f**((a*c**3 + 3*a*c**2*d*
x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3))*log(f)**2*b**2*d**4*x**4 - 24*f**((a*c**3 + 3*a*c**2*d*x + 3
*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))*log(f)*b*c**7 - 114*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2
+ a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)
*b*c**6*d*x - 180*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x*
*3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*c**5*d**
2*x**2 - 30*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b
)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*c**4*d**3*x**3
 + 240*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/...
```

3.288 $\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx=\frac{F^a(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{b\log(F)}{(c+dx)^3}\right)\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{2/3}}{3d}$$

output
```
1/3*F^a*(d*x+c)^2*GAMMA(-2/3,-b*ln(F)/(d*x+c)^3)*(-b*ln(F)/(d*x+c)^3)^(2/3
)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx=\frac{F^a(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{b\log(F)}{(c+dx)^3}\right)\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{2/3}}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)*(c + d*x),x]
```

output
```
(F^a*(c + d*x)^2*Gamma[-2/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*Log[F])/(c +
 d*x)^3))^(2/3))/(3*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)F^{a+\frac{b}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{F^a(c+dx)^2\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3)*(c + d*x),x]
```

output
```
(F^a*(c + d*x)^2*Gamma[-2/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*Log[F])/(c +
 d*x)^3))^(2/3))/(3*d)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}(dx+c)\,dx$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c),x)
```

output
```
int(F^(a+b/(d*x+c)^3)*(d*x+c),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.90

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx = -\frac{F^a d^2\left(-\frac{b\log(F)}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right) - (d^2x^2+2\,cdx+c^2)F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{2\,d}$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(F^a*d^2*(-b*log(F)/d^3)^(2/3)*gamma(1/3, -b*log(F)/(d^3*x^3 + 3*c*d^
2*x^2 + 3*c^2*d*x + c^3)) - (d^2*x^2 + 2*c*d*x + c^2)*F^((a*d^3*x^3 + 3*a*
c*d^2*x^2 + 3*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx = \int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\;dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)*(d*x+c),x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)*(c + d*x), x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx = \int (dx+c)F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(F^a*d*x^2 + 2*F^a*c*x)*F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
) + integrate(3/2*(F^a*b*d^2*x^2*log(F) + 2*F^a*b*c*d*x*log(F))*F^(b/(d^3*
x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x
^2 + 4*c^3*d*x + c^4), x)
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx = \int (dx+c)F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.18

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx = \frac{F^a\,F^{\frac{b}{(c+d\,x)^3}}\,(c+d\,x)^2}{2\,d} - \frac{F^a\,\Gamma\left(\frac{1}{3},-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)\,(c+d\,x)^2\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{2/3}}{2\,d} + \frac{\pi\,\sqrt{3}\,F^a\,(c+d\,x)^2\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{2/3}}{3\,d\,\Gamma\left(\frac{2}{3}\right)}$$

input
```
int(F^(a + b/(c + d*x)^3)*(c + d*x),x)
```

output
```
(F^a*F^(b/(c + d*x)^3)*(c + d*x)^2)/(2*d) - (F^a*igamma(1/3, -(b*log(F))/(
c + d*x)^3)*(c + d*x)^2*(-(b*log(F))/(c + d*x)^3)^(2/3))/(2*d) + (3^(1/2)*
F^a*pi*(c + d*x)^2*(-(b*log(F))/(c + d*x)^3)^(2/3))/(3*d*gamma(2/3))
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}(c+dx)\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)*(d*x+c),x)
```

output
```
(18*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c*d*x + 9*f**((a
*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*d**2*x**2 + 6*f**((a*c**3
+ 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*log(f)*b*c**5 - 6*f**((a*c**3 + 3*a*c**2*d*x +
3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3))*log(f)*b*c**4*d*x - 66*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x
**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*lo
g(f)*b*c**3*d**2*x**2 - 102*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 +
 a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*
b*c**2*d**3*x**3 - 60*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**
3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*c*d*
*4*x**4 - 12*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 +
b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*d**5*x**5 - 1
4*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 +
3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**8 - 76*f**((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*c**7*d*x - 158*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**
2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c...
```

3.289 $\int F^{a+\frac{b}{(c+dx)^3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \frac{F^a(c+dx)\Gamma\left(-\frac{1}{3},-\frac{b\log(F)}{(c+dx)^3}\right)\sqrt[3]{-\frac{b\log(F)}{(c+dx)^3}}}{3d}$$

output
```
1/3*F^a*(d*x+c)*GAMMA(-1/3,-b*ln(F)/(d*x+c)^3)*(-b*ln(F)/(d*x+c)^3)^(1/3)/
d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \frac{F^a(c+dx)\Gamma\left(-\frac{1}{3},-\frac{b\log(F)}{(c+dx)^3}\right)\sqrt[3]{-\frac{b\log(F)}{(c+dx)^3}}}{3d}$$

input
```
Integrate[F^(a + b/(c + d*x)^3),x]
```

output
```
(F^a*(c + d*x)*Gamma[-1/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*Log[F])/(c + d
*x)^3))^(1/3))/(3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+\frac{b}{(c+dx)^3}} dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{F^a (c+dx) \sqrt[3]{-\frac{b\log(F)}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d}$$

input
```
Int[F^(a + b/(c + d*x)^3),x]
```

output
```
(F^a*(c + d*x)*Gamma[-1/3, -((b*Log[F])/(c + d*x)^3)]*(-((b*Log[F])/(c + d
*x)^3))^(1/3))/(3*d)
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Maple [F]

$$\int F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
int(F^(a+b/(d*x+c)^3),x)
```

output
```
int(F^(a+b/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(41) = 82$.

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.74

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = -\frac{F^a d\left(-\frac{b\log(F)}{d^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},-\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-(dx+c)F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}}{d}$$

input
```
integrate(F^(a+b/(d*x+c)^3),x, algorithm="fricas")
```

output
```
-(F^a*d*(-b*log(F)/d^3)^(1/3)*gamma(2/3, -b*log(F)/(d^3*x^3 + 3*c*d^2*x^2
+ 3*c^2*d*x + c^3)) - (d*x + c)*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3*a*c^2*d*
x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \int F^{a+\frac{b}{(c+dx)^3}}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3),x)
```

output
```
Integral(F**(a + b/(c + d*x)**3), x)
```

Maxima [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \int F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3),x, algorithm="maxima")
```

output
```
3*F^a*b*d*integrate(F^(b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*x/(d^4
*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)*log(F) + F^a*F^(
b/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*x
```

Giac [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \int F^{a+\frac{b}{(dx+c)^3}}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.51

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \frac{F^a\,(c+d\,x)\,\left(F^{\frac{b}{(c+d\,x)^3}}-\Gamma\left(\frac{2}{3},-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{1/3}+\Gamma\left(\frac{2}{3}\right)\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{1/3}\right)}{d}$$

input
```
int(F^(a + b/(c + d*x)^3),x)
```

output
```
(F^a*(c + d*x)*(F^(b/(c + d*x)^3) - igamma(2/3, -(b*log(F))/(c + d*x)^3)*(
-(b*log(F))/(c + d*x)^3)^(1/3) + gamma(2/3)*(-(b*log(F))/(c + d*x)^3)^(1/3
)))/d
```

Reduce [F]

$$\int F^{a+\frac{b}{(c+dx)^3}}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3),x)
```

output
```
(9*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*d*x + 3*f**((a*c*
*3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*c**4 - 6*f**((a*c**3 + 3*a*c**2*d*x
 + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3))*log(f)*b*c**3*d*x - 36*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**
2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))
*log(f)*b*c**2*d**2*x**2 - 42*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2
 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f
)*b*c*d**3*x**3 - 15*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3
*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*d**4*
x**4 - 8*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(
c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**7 - 38*f**((a*c**3 + 3*
a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d
**2*x**2 + d**3*x**3))*c**6*d*x - 60*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d*
*2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
)*c**5*d**2*x**2 - 10*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**
3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**4*d**3*x**
3 + 80*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c*
*3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**3*d**4*x**4 + 102*f**(...
```

3.290 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2} dx = \frac{F^a\Gamma\left(\frac{1}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)\sqrt[3]{-\frac{b\log(F)}{(c+dx)^3}}}$$

output
```
1/3*F^a*GAMMA(1/3,-b*ln(F)/(d*x+c)^3)/d/(d*x+c)/(-b*ln(F)/(d*x+c)^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2} dx = \frac{F^a\Gamma\left(\frac{1}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)\sqrt[3]{-\frac{b\log(F)}{(c+dx)^3}}}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^2,x]
```

output
```
(F^a*Gamma[1/3, -((b*Log[F])/(c + d*x)^3)])/(3*d*(c + d*x)*(-((b*Log[F])/(
c + d*x)^3))^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2648}$$

$$\frac{F^a\Gamma\left(\frac{1}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)\sqrt[3]{-\frac{b\log(F)}{(c+dx)^3}}}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^2,x]
```

output
```
(F^a*Gamma[1/3, -((b*Log[F])/(c + d*x)^3)])/(3*d*(c + d*x)*(-((b*Log[F])/(
c + d*x)^3))^(1/3))
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^2}dx$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x)
```

output
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx = -\frac{F^a d\left(-\frac{b\log(F)}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{3\,b\log\left(F\right)}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/3*F^a*d*(-b*log(F)/d^3)^(2/3)*gamma(1/3, -b*log(F)/(d^3*x^3 + 3*c*d^2*x
^2 + 3*c^2*d*x + c^3))/(b*log(F))
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx = \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**2,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx = \frac{F^a\left(3\,\Gamma\left(\frac{2}{3}\right)\,\Gamma\left(\frac{1}{3},-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)-2\,\pi\,\sqrt{3}\right)}{9\,d\,\Gamma\left(\frac{2}{3}\right)\,(c+d\,x)\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{1/3}}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^2,x)
```

output
```
(F^a*(3*gamma(2/3)*igamma(1/3, -(b*log(F))/(c + d*x)^3) - 2*3^(1/2)*pi))/(
9*d*gamma(2/3)*(c + d*x)*(-(b*log(F))/(c + d*x)^3)^(1/3))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^2}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^2,x)
```

output
```
(6*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*c*d*x + 3*f**((a*
c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*d**2*x**2 + 2*f**((a*c**3 +
 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3))*log(f)*b*c**5 - 2*f**((a*c**3 + 3*a*c**2*d*x + 3
*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))*log(f)*b*c**4*d*x - 22*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x*
*2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log
(f)*b*c**3*d**2*x**2 - 34*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a
*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*
c**2*d**3*x**3 - 20*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*
x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*c*d**4
*x**4 - 4*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/
(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*d**5*x**5 - 6*f*
*((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**8 - 36*f**((a*c**3 + 3*a*c**2*d*x
+ 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3))*c**7*d*x - 90*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a
*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**6*d...
```

3.291 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} dx = \frac{F^a\Gamma\left(\frac{2}{3}, -\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{2/3}}$$

output
```
1/3*F^a*GAMMA(2/3,-b*ln(F)/(d*x+c)^3)/d/(d*x+c)^2/(-b*ln(F)/(d*x+c)^3)^(2/
3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} dx = \frac{F^a\Gamma\left(\frac{2}{3}, -\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{2/3}}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^3,x]
```

output
```
(F^a*Gamma[2/3, -((b*Log[F])/(c + d*x)^3)])/(3*d*(c + d*x)^2*(-((b*Log[F])
/(c + d*x)^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{F^a\Gamma\left(\frac{2}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)^2\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{2/3}}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^3,x]
```

output

```
(F^a*Gamma[2/3, -((b*Log[F])/(c + d*x)^3)])/(3*d*(c + d*x)^2*(-((b*Log[F])
/(c + d*x)^3))^(2/3))
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^3}dx$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x)
```

output
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx = -\frac{F^a\left(-\frac{b\log(F)}{d^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3},-\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{3\,b\log\left(F\right)}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/3*F^a*(-b*log(F)/d^3)^(1/3)*gamma(2/3, -b*log(F)/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3))/(b*log(F))
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx = \int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**3,x)
```

output
```
Integral(F**(a + b/(c + d*x)**3)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx=\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx=\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx=-\frac{F^a\left(\Gamma\left(\frac{2}{3}\right)-\Gamma\left(\frac{2}{3},-\frac{b\ln(F)}{(c+d\,x)^3}\right)\right)}{3\,d\,(c+d\,x)^2\left(-\frac{b\ln(F)}{(c+d\,x)^3}\right)^{2/3}}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^3,x)
```

output
```
-(F^a*(gamma(2/3) - igamma(2/3, -(b*log(F))/(c + d*x)^3)))/(3*d*(c + d*x)^
2*(-(b*log(F))/(c + d*x)^3)^(2/3))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^3}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^3,x)
```

output
```
(3*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)**2*b**2*d*x + f**((a*c**3
 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3))*log(f)*b*c**4 - 2*f**((a*c**3 + 3*a*c**2*d*x +
 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3))*log(f)*b*c**3*d*x - 12*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*
x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*l
og(f)*b*c**2*d**2*x**2 - 14*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 +
 a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*
b*c*d**3*x**3 - 5*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x*
*3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b*d**4*x**
4 - 6*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**
3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**7 - 36*f**((a*c**3 + 3*a*c
**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3))*c**6*d*x - 90*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*
x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c
**5*d**2*x**2 - 120*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*
x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**4*d**3*x**3
- 90*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**3*d**4*x**4 - 36*f**((a*...
```

3.292 $\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx=\frac{F^a\Gamma\left(\frac{4}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{4/3}}$$

output
```
1/3*F^a*GAMMA(4/3,-b*ln(F)/(d*x+c)^3)/d/(d*x+c)^4/(-b*ln(F)/(d*x+c)^3)^(4/
3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx=\frac{F^a\Gamma\left(\frac{4}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{4/3}}$$

input
```
Integrate[F^(a + b/(c + d*x)^3)/(c + d*x)^5,x]
```

output
```
(F^a*Gamma[4/3, -((b*Log[F])/(c + d*x)^3)])/(3*d*(c + d*x)^4*(-((b*Log[F])
/(c + d*x)^3))^(4/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{F^a\Gamma\left(\frac{4}{3},-\frac{b\log(F)}{(c+dx)^3}\right)}{3d(c+dx)^4\left(-\frac{b\log(F)}{(c+dx)^3}\right)^{4/3}}$$

input
```
Int[F^(a + b/(c + d*x)^3)/(c + d*x)^5,x]
```

output

```
(F^a*Gamma[4/3, -((b*Log[F])/(c + d*x)^3)])/(3*d*(c + d*x)^4*(-((b*Log[F])
/(c + d*x)^3))^(4/3))
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^5}dx$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x)
```

output
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(43) = 86$.

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.16

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx = \frac{(d^3x+cd^2)F^a\left(-\frac{b\log(F)}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},-\frac{b\log(F)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-3\,F^{\frac{ad^3x^3+3\,acd^2x^2+3\,ac^2dx+ac^3+b}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}}b\log\left(F\right)}{9\left(b^2d^2x+b^2cd\right)\log\left(F\right)^2}$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/9*((d^3*x + c*d^2)*F^a*(-b*log(F)/d^3)^(2/3)*gamma(1/3, -b*log(F)/(d^3*x
^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 3*F^((a*d^3*x^3 + 3*a*c*d^2*x^2 + 3
*a*c^2*d*x + a*c^3 + b)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))*b*log(F
))/((b^2*d^2*x + b^2*c*d)*log(F)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx = \text{Timed out}$$

input
```
integrate(F**(a+b/(d*x+c)**3)/(d*x+c)**5,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^5, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx = \int \frac{F^{a+\frac{b}{(dx+c)^3}}}{(dx+c)^5}\,dx$$

input
```
integrate(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c)^3)/(d*x + c)^5, x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.33

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx = \frac{F^a\,\Gamma\left(\frac{1}{3},-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)}{9\,d\,(c+d\,x)^4\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{4/3}} - \frac{F^a\,F^{\frac{b}{(c+d\,x)^3}}}{3\,b\,d\,\ln(F)\,(c+d\,x)} - \frac{2\,\pi\,\sqrt{3}\,F^a}{27\,d\,\Gamma\left(\frac{2}{3}\right)\,(c+d\,x)^4\,\left(-\frac{b\,\ln(F)}{(c+d\,x)^3}\right)^{4/3}}$$

input
```
int(F^(a + b/(c + d*x)^3)/(c + d*x)^5,x)
```

output
```
(F^a*igamma(1/3, -(b*log(F))/(c + d*x)^3))/(9*d*(c + d*x)^4*(-(b*log(F))/(
c + d*x)^3)^(4/3)) - (F^a*F^(b/(c + d*x)^3))/(3*b*d*log(F)*(c + d*x)) - (2
*3^(1/2)*F^a*pi)/(27*d*gamma(2/3)*(c + d*x)^4*(-(b*log(F))/(c + d*x)^3)^(4
/3))
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{(c+dx)^3}}}{(c+dx)^5}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c)^3)/(d*x+c)^5,x)
```

output
```
(3*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 +
 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*log(f)*b - 7*f**((a*c**3 + 3*a*c
**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3))*c**3 - 21*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2
 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**2*
d*x - 21*f**((a*c**3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(
c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c*d**2*x**2 - 7*f**((a*c**
3 + 3*a*c**2*d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x +
 3*c*d**2*x**2 + d**3*x**3))*d**3*x**3 + 9*int(f**((a*c**3 + 3*a*c**2*d*x
+ 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3))/(c**11 + 11*c**10*d*x + 55*c**9*d**2*x**2 + 165*c**8*d**3*x**3
 + 330*c**7*d**4*x**4 + 462*c**6*d**5*x**5 + 462*c**5*d**6*x**6 + 330*c**4
*d**7*x**7 + 165*c**3*d**8*x**8 + 55*c**2*d**9*x**9 + 11*c*d**10*x**10 + d
**11*x**11),x)*log(f)**2*b**2*c**7*d + 63*int(f**((a*c**3 + 3*a*c**2*d*x +
 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3))/(c**11 + 11*c**10*d*x + 55*c**9*d**2*x**2 + 165*c**8*d**3*x**3
+ 330*c**7*d**4*x**4 + 462*c**6*d**5*x**5 + 462*c**5*d**6*x**6 + 330*c**4*
d**7*x**7 + 165*c**3*d**8*x**8 + 55*c**2*d**9*x**9 + 11*c*d**10*x**10 + d*
*11*x**11),x)*log(f)**2*b**2*c**6*d**2*x + 189*int(f**((a*c**3 + 3*a*c**2*
d*x + 3*a*c*d**2*x**2 + a*d**3*x**3 + b)/(c**3 + 3*c**2*d*x + 3*c*d**2*...
```

3.293 $\int F^{a+b(c+dx)^n}(c+dx)^m\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-\frac{1+m}{n}}}{dn}$$

output
```
-F^a*(d*x+c)^(1+m)*GAMMA((1+m)/n,-b*(d*x+c)^n*ln(F))/d/n/((-b*(d*x+c)^n*ln
(F))^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx = -\frac{F^a(c+dx)^{1+m}\Gamma\left(\frac{1+m}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-\frac{1+m}{n}}}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^m,x]
```

output
```
-((F^a*(c + d*x)^(1 + m)*Gamma[(1 + m)/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(
-(b*(c + d*x)^n*Log[F]))^((1 + m)/n)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m F^{a+b(c+dx)^n} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a(c+dx)^{m+1}\left(-b\log(F)(c+dx)^n\right)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-b(c+dx)^n\log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^m,x]
```

output
```
-((F^a*(c + d*x)^(1 + m)*Gamma[(1 + m)/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(
-(b*(c + d*x)^n*Log[F]))^((1 + m)/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^n}(dx+c)^m\,dx$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^m,x)
```

output
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^m,x)
```

Fricas [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx=\int (dx+c)^m F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*F^((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx=\int F^{a+b(c+dx)^n}(c+dx)^m\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**m,x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**m, x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx=\int (dx+c)^m F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx=\int (dx+c)^m F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*F^((d*x + c)^n*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.52

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx=\frac{F^{a+\frac{b\,(c+d\,x)^n}{2}}\,(c+d\,x)^{m+1}\,\mathrm{M}_{-\frac{\frac{m}{2}-\frac{n}{2}+\frac{1}{2}}{n},\frac{\frac{m}{2}+\frac{1}{2}}{n}}\left(b\,\ln(F)\,(c+d\,x)^n\right)}{d\,(m+1)\,\left(b\,\ln(F)\,(c+d\,x)^n\right)^{\frac{m}{2\,n}+\frac{1}{2\,n}+\frac{1}{2}}}$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^m,x)
```

output
```
(F^(a + (b*(c + d*x)^n)/2)*(c + d*x)^(m + 1)*whittakerM(-(m/2 - n/2 + 1/2)
/n, (m/2 + 1/2)/n, b*log(F)*(c + d*x)^n))/(d*(m + 1)*(b*log(F)*(c + d*x)^n
)^(m/(2*n) + 1/(2*n) + 1/2))
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^m\,dx = f^a\left(\int f^{(dx+c)^n b}(dx+c)^m\,dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^m,x)
```

output
```
f**a*int(f**((c + d*x)**n*b)*(c + d*x)**m,x)
```

3.294 $\int F^{a+b(c+dx)^n}(c+dx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx = -\frac{F^a(c+dx)^4\Gamma\left(\frac{4}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-4/n}}{dn}$$

output
```
-F^a*(d*x+c)^4*GAMMA(4/n,-b*(d*x+c)^n*ln(F))/d/n/((-b*(d*x+c)^n*ln(F))^(4/
n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx = -\frac{F^a(c+dx)^4\Gamma\left(\frac{4}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-4/n}}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^3,x]
```

output
```
-((F^a*(c + d*x)^4*Gamma[4/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x
)^n*Log[F]))^(4/n)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2648\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 F^{a+b(c+dx)^n} \, dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{F^a (c+dx)^4 \left(-b \log(F) (c+dx)^n\right)^{-4/n} \Gamma\left(\frac{4}{n}, -b(c+dx)^n \log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^3,x]
```

output
```
-((F^a*(c + d*x)^4*Gamma[4/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x
)^n*Log[F]))^(4/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^n}(dx+c)^3\,dx$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^3,x)
```

output
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^3,x)
```

Fricas [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx=\int (dx+c)^3F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*F^((d*x + c)^n*b + a),
x)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx=\int F^{a+b(c+dx)^n}(c+dx)^3\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**3, x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx=\int (dx+c)^3F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^3*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx=\int (dx+c)^3F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*F^((d*x + c)^n*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx=\frac{F^a\,\mathrm{e}^{\frac{b\,\ln(F)\,(c+d\,x)^n}{2}}\,(c+d\,x)^4\,\mathrm{M}_{\frac{1}{2}-\frac{2}{n},\frac{2}{n}}\left(b\,\ln\left(F\right)\,(c+d\,x)^n\right)}{4\,d\,\left(b\,\ln\left(F\right)\,(c+d\,x)^n\right)^{\frac{2}{n}+\frac{1}{2}}}$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^3,x)
```

output
```
(F^a*exp((b*log(F)*(c + d*x)^n)/2)*(c + d*x)^4*whittakerM(1/2 - 2/n, 2/n,
b*log(F)*(c + d*x)^n))/(4*d*(b*log(F)*(c + d*x)^n)^(2/n + 1/2))
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^3\,dx = f^a\left(\left(\int f^{(dx+c)^n b}dx\right)c^3+\left(\int f^{(dx+c)^n b}x^3dx\right)d^3 + 3\left(\int f^{(dx+c)^n b}x^2dx\right)c\,d^2+3\left(\int f^{(dx+c)^n b}xdx\right)c^2d\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^3,x)
```

output
```
f**a*(int(f**((c + d*x)**n*b),x)*c**3 + int(f**((c + d*x)**n*b)*x**3,x)*d*
*3 + 3*int(f**((c + d*x)**n*b)*x**2,x)*c*d**2 + 3*int(f**((c + d*x)**n*b)*
x,x)*c**2*d)
```

3.295 $\int F^{a+b(c+dx)^n}(c+dx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx = -\frac{F^a(c+dx)^3\Gamma\left(\frac{3}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-3/n}}{dn}$$

output
```
-F^a*(d*x+c)^3*GAMMA(3/n,-b*(d*x+c)^n*ln(F))/d/n/((-b*(d*x+c)^n*ln(F))^(3/
n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx = -\frac{F^a(c+dx)^3\Gamma\left(\frac{3}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-3/n}}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^2,x]
```

output
```
-((F^a*(c + d*x)^3*Gamma[3/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x
)^n*Log[F]))^(3/n)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 F^{a+b(c+dx)^n} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a (c+dx)^3 \left(-b \log(F)(c+dx)^n\right)^{-3/n} \Gamma\left(\frac{3}{n}, -b(c+dx)^n \log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^2,x]
```

output
```
-((F^a*(c + d*x)^3*Gamma[3/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x
)^n*Log[F]))^(3/n)))
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^n}(dx+c)^2\,dx$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^2,x)
```

output
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^2,x)
```

Fricas [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx = \int (dx+c)^2 F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((d^2*x^2 + 2*c*d*x + c^2)*F^((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx = \int F^{a+b(c+dx)^n}(c+dx)^2\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**2, x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx=\int (dx+c)^2F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx=\int (dx+c)^2F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*F^((d*x + c)^n*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx=\frac{F^a\,\mathrm{e}^{\frac{b\,\ln(F)\,(c+d\,x)^n}{2}}\,(c+d\,x)^3\,\mathrm{M}_{\frac{1}{2}-\frac{3}{2\,n},\frac{3}{2\,n}}\left(b\,\ln\left(F\right)\,(c+d\,x)^n\right)}{3\,d\,\left(b\,\ln\left(F\right)\,(c+d\,x)^n\right)^{\frac{3}{2\,n}+\frac{1}{2}}}$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^2,x)
```

output
```
(F^a*exp((b*log(F)*(c + d*x)^n)/2)*(c + d*x)^3*whittakerM(1/2 - 3/(2*n), 3
/(2*n), b*log(F)*(c + d*x)^n))/(3*d*(b*log(F)*(c + d*x)^n)^(3/(2*n) + 1/2)
)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^2\,dx = f^a\left(\left(\int f^{(dx+c)^n b}dx\right)c^2 + \left(\int f^{(dx+c)^n b}x^2dx\right)d^2 + 2\left(\int f^{(dx+c)^n b}xdx\right)cd\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^2,x)
```

output
```
f**a*(int(f**((c + d*x)**n*b),x)*c**2 + int(f**((c + d*x)**n*b)*x**2,x)*d*
*2 + 2*int(f**((c + d*x)**n*b)*x,x)*c*d)
```

3.296 $\int F^{a+b(c+dx)^n}(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 54

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx = -\frac{F^a(c+dx)^2\Gamma\left(\frac{2}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-2/n}}{dn}$$

output
```
-F^a*(d*x+c)^2*GAMMA(2/n,-b*(d*x+c)^n*ln(F))/d/n/((-b*(d*x+c)^n*ln(F))^(2/
n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx = -\frac{F^a(c+dx)^2\Gamma\left(\frac{2}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-2/n}}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x),x]
```

output
```
-((F^a*(c + d*x)^2*Gamma[2/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x
)^n*Log[F]))^(2/n)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{F^a(c+dx)^2\left(-b\log(F)(c+dx)^n\right)^{-2/n}\Gamma\left(\frac{2}{n},-b(c+dx)^n\log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x),x]
```

output
```
-((F^a*(c + d*x)^2*Gamma[2/n, -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*x
)^n*Log[F]))^(2/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int F^{a+b(dx+c)^n}(dx+c)\,dx$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c),x)
```

output
```
int(F^(a+b*(d*x+c)^n)*(d*x+c),x)
```

Fricas [F]

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx=\int (dx+c)F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c),x, algorithm="fricas")
```

output
```
integral((d*x + c)*F^((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx=\int F^{a+b(c+dx)^n}(c+dx)\;dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx = \int (dx+c)F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx = \int (dx+c)F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c),x, algorithm="giac")
```

output
```
integrate((d*x + c)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx = \int F^{a+b\,(c+d\,x)^n}\,(c+d\,x)\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x),x)
```

output
```
int(F^(a + b*(c + d*x)^n)*(c + d*x), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)\,dx = f^a\left(\left(\int f^{(dx+c)^n b}dx\right)c+\left(\int f^{(dx+c)^n b}xdx\right)d\right)$$

input

```
int(F^(a+b*(d*x+c)^n)*(d*x+c),x)
```

output

```
f**a*(int(f**((c + d*x)**n*b),x)*c + int(f**((c + d*x)**n*b)*x,x)*d)
```

3.297 $\int F^{a+b(c+dx)^n}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 50

$$\int F^{a+b(c+dx)^n}\,dx=-\frac{F^a(c+dx)\Gamma\left(\frac{1}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-1/n}}{dn}$$

output
```
-F^a*(d*x+c)*GAMMA(1/n,-b*(d*x+c)^n*ln(F))/d/n/((-b*(d*x+c)^n*ln(F))^(1/n)
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}\,dx=-\frac{F^a(c+dx)\Gamma\left(\frac{1}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{-1/n}}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n),x]
```

output
```
-((F^a*(c + d*x)*Gamma[n^(-1), -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*
x)^n*Log[F]))^n^(-1)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{F^a(c+dx)\left(-b\log(F)(c+dx)^n\right)^{-1/n}\Gamma\left(\frac{1}{n},-b(c+dx)^n\log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n),x]
```

output
```
-((F^a*(c + d*x)*Gamma[n^(-1), -(b*(c + d*x)^n*Log[F])])/(d*n*(-(b*(c + d*
x)^n*Log[F]))^n^(-1)))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] &&  !IntegerQ[2/n]
```

Maple [F]

$$\int F^{a+b(dx+c)^n}\,dx$$

input
```
int(F^(a+b*(d*x+c)^n),x)
```

output
```
int(F^(a+b*(d*x+c)^n),x)
```

Fricas [F]

$$\int F^{a+b(c+dx)^n}\,dx=\int F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n),x, algorithm="fricas")
```

output
```
integral(F^((d*x + c)^n*b + a), x)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}\,dx=\int F^{a+b(c+dx)^n}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}\,dx=\int F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}\,dx = \int F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}\,dx = \int F^{a+b\,(c+d\,x)^n}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n),x)
```

output
```
int(F^(a + b*(c + d*x)^n), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}\,dx = f^a\left(\int f^{(dx+c)^n b}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n),x)
```

output
```
f**a*int(f**((c + d*x)**n*b),x)
```

3.298 $\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx=\frac{F^a\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}$$

output
```
F^a*Ei(b*(d*x+c)^n*ln(F))/d/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx=\frac{F^a\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(d*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx} \, dx$$

$$\Big\downarrow \text{2639}$$

$$\frac{F^a \text{ExpIntegralEi}\left(b(c+dx)^n \log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(d*n)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{F^a \operatorname{expIntegral}_1(-b(dx+c)^n \ln(F))}{dn}$	26

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d/n*F^a*Ei(1,-b*(d*x+c)^n*ln(F))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \frac{F^a\mathrm{Ei}\left((dx+c)^n b\log(F)\right)}{dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="fricas")
```

output
```
F^a*Ei((d*x + c)^n*b*log(F))/(d*n)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)/(d*x+c),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^{(dx+c)^n b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^{(dx+c)^n b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^a\,F^{b\,(c+d\,x)^n}}{c+d\,x}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x),x)
```

output
```
int((F^a*F^(b*(c + d*x)^n))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{dx+c}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/(c + d*x),x)
```

3.299 $\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx$

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx = -\frac{F^a \Gamma\left(-\frac{1}{n}, -b(c+dx)^n \log(F)\right) \left(-b(c+dx)^n \log(F)\right)^{\frac{1}{n}}}{dn(c+dx)}$$

output
```
-F^a*GAMMA(-1/n,-b*(d*x+c)^n*ln(F))*(-b*(d*x+c)^n*ln(F))^(1/n)/d/n/(d*x+c)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx = -\frac{F^a \Gamma\left(-\frac{1}{n}, -b(c+dx)^n \log(F)\right) \left(-b(c+dx)^n \log(F)\right)^{\frac{1}{n}}}{dn(c+dx)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)/(c + d*x)^2,x]
```

output
```
-((F^a*Gamma[-n^(-1), -(b*(c + d*x)^n*Log[F])]*(-(b*(c + d*x)^n*Log[F]))^n
^(-1))/(d*n*(c + d*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2} dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a(-b\log(F)(c+dx)^n)^{\frac{1}{n}}\Gamma\left(-\frac{1}{n},-b(c+dx)^n\log(F)\right)}{dn(c+dx)}$$

input
```
Int[F^(a + b*(c + d*x)^n)/(c + d*x)^2,x]
```

output
```
-((F^a*Gamma[-n^(-1), -(b*(c + d*x)^n*Log[F])]*(-(b*(c + d*x)^n*Log[F]))^n
^(-1))/(d*n*(c + d*x)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^n}}{(dx+c)^2}dx$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x)
```

output
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x)
```

Fricas [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(F^((d*x + c)^n*b + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx = \int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)/(d*x+c)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c)^2, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx = -\frac{F^a\,e^{\frac{b\,\ln(F)\,(c+d\,x)^n}{2}}\,\left(b\,\ln(F)\,\left(c+d\,x\right)^n\right)^{\frac{1}{2\,n}-\frac{1}{2}}\,\mathrm{M}_{\frac{1}{2\,n}+\frac{1}{2},-\frac{1}{2\,n}}\left(b\,\ln(F)\,\left(c+d\,x\right)^n\right)}{d\,\left(c+d\,x\right)}$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^2,x)
```

output
```
-(F^a*exp((b*log(F)*(c + d*x)^n)/2)*(b*log(F)*(c + d*x)^n)^(1/(2*n) - 1/2)
*whittakerM(1/(2*n) + 1/2, -1/(2*n), b*log(F)*(c + d*x)^n))/(d*(c + d*x))
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^2}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{d^2x^2+2cdx+c^2}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^2,x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.300 $\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx=-\frac{F^a\Gamma\left(-\frac{2}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{2/n}}{dn(c+dx)^2}$$

output
```
-F^a*GAMMA(-2/n,-b*(d*x+c)^n*ln(F))*(-b*(d*x+c)^n*ln(F))^(2/n)/d/n/(d*x+c)
^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx=-\frac{F^a\Gamma\left(-\frac{2}{n},-b(c+dx)^n\log(F)\right)\left(-b(c+dx)^n\log(F)\right)^{2/n}}{dn(c+dx)^2}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)/(c + d*x)^3,x]
```

output
```
-((F^a*Gamma[-2/n, -(b*(c + d*x)^n*Log[F])]*(-(b*(c + d*x)^n*Log[F]))^(2/n
))/(d*n*(c + d*x)^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a(-b\log(F)(c+dx)^n)^{2/n}\,\Gamma\left(-\frac{2}{n},-b(c+dx)^n\log(F)\right)}{dn(c+dx)^2}$$

input
```
Int[F^(a + b*(c + d*x)^n)/(c + d*x)^3,x]
```

output
```
-((F^a*Gamma[-2/n, -(b*(c + d*x)^n*Log[F])]*(-(b*(c + d*x)^n*Log[F]))^(2/n
))/(d*n*(c + d*x)^2))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^n}}{(dx+c)^3}dx$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x)
```

output
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x)
```

Fricas [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(F^((d*x + c)^n*b + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3),
x)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx = \int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)/(d*x+c)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} \, dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^3} \, dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c)^3, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} \, dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^3} \, dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3} \, dx = -\frac{F^a \, \mathrm{e}^{\frac{b \, \ln(F) \, (c+d\,x)^n}{2}} \, \mathrm{M}_{\frac{1}{n}+\frac{1}{2},-\frac{1}{n}}\left(b \, \ln(F) \, (c+d\,x)^n\right) \, \left(b \, \ln(F) \, (c+d\,x)^n\right)^{\frac{1}{n}-\frac{1}{2}}}{2\,d\,(c+d\,x)^2}$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^3,x)
```

output
```
-(F^a*exp((b*log(F)*(c + d*x)^n)/2)*whittakerM(1/n + 1/2, -1/n, b*log(F)*(
c + d*x)^n)*(b*log(F)*(c + d*x)^n)^(1/n - 1/2))/(2*d*(c + d*x)^2)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^3}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^3,x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3),x)
```

3.301 $\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} dx = -\frac{F^a\Gamma\left(-\frac{3}{n}, -b(c+dx)^n \log(F)\right)\left(-b(c+dx)^n \log(F)\right)^{3/n}}{dn(c+dx)^3}$$

output
```
-F^a*GAMMA(-3/n,-b*(d*x+c)^n*ln(F))*(-b*(d*x+c)^n*ln(F))^(3/n)/d/n/(d*x+c)
^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} dx = -\frac{F^a\Gamma\left(-\frac{3}{n}, -b(c+dx)^n \log(F)\right)\left(-b(c+dx)^n \log(F)\right)^{3/n}}{dn(c+dx)^3}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)/(c + d*x)^4,x]
```

output
```
-((F^a*Gamma[-3/n, -(b*(c + d*x)^n*Log[F])]*(-(b*(c + d*x)^n*Log[F]))^(3/n
))/(d*n*(c + d*x)^3))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{F^a\left(-b\log(F)(c+dx)^n\right)^{3/n}\Gamma\left(-\frac{3}{n},-b(c+dx)^n\log(F)\right)}{dn(c+dx)^3}$$

input
```
Int[F^(a + b*(c + d*x)^n)/(c + d*x)^4,x]
```

output
```
-((F^a*Gamma[-3/n, -(b*(c + d*x)^n*Log[F])]*(-(b*(c + d*x)^n*Log[F]))^(3/n
))/(d*n*(c + d*x)^3))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \frac{F^{a+b(dx+c)^n}}{(dx+c)^4}dx$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x)
```

output
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x)
```

Fricas [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
integral(F^((d*x + c)^n*b + a)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*
c^3*d*x + c^4), x)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx = \int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)/(d*x+c)**4,x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)/(c + d*x)**4, x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c)^4, x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx = \int \frac{F^{(dx+c)^n b+a}}{(dx+c)^4}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c)^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4}\,dx = -\frac{F^a\, e^{\frac{b\,\ln(F)\,(c+d\,x)^n}{2}}\,\left(b\,\ln(F)\,(c+d\,x)^n\right)^{\frac{3}{2\,n}-\frac{1}{2}}\,\mathrm{M}_{\frac{3}{2\,n}+\frac{1}{2},-\frac{3}{2\,n}}\left(b\,\ln(F)\,(c+d\,x)^n\right)}{3\,d\,(c+d\,x)^3}$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^4,x)
```

output
```
-(F^a*exp((b*log(F)*(c + d*x)^n)/2)*(b*log(F)*(c + d*x)^n)^(3/(2*n) - 1/2)
*whittakerM(3/(2*n) + 1/2, -3/(2*n), b*log(F)*(c + d*x)^n))/(3*d*(c + d*x)
^3)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^n}}{(c+dx)^4} \, dx = f^a \left(\int \frac{f^{(dx+c)^n b}}{d^4x^4 + 4c\,d^3x^3 + 6c^2d^2x^2 + 4c^3dx + c^4} dx \right)$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c)^4,x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d
**3*x**3 + d**4*x**4),x)
```

3.302 $\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 114

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx = -\frac{F^{a+b(c+dx)^n}\left(120-120b(c+dx)^n\log(F)+60b^2(c+dx)^{2n}\log^2(F)-20b^3(c+dx)^{3n}\log^3(F)+5b^4(c+\right.}{b^6dn\log^6(F)}$$

output
```
-F^(a+b*(d*x+c)^n)*(120-120*b*(d*x+c)^n*ln(F)+60*b^2*(d*x+c)^(2*n)*ln(F)^2
-20*b^3*(d*x+c)^(3*n)*ln(F)^3+5*b^4*(d*x+c)^(4*n)*ln(F)^4-b^5*(d*x+c)^(5*n
)*ln(F)^5)/b^6/d/n/ln(F)^6
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.28

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx = -\frac{F^a\Gamma(6,-b(c+dx)^n\log(F))}{b^6dn\log^6(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 6*n),x]
```

output
```
-((F^a*Gamma[6, -(b*(c + d*x)^n*Log[F])])/(b^6*d*n*Log[F]^6))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{6n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \; 2647$$

$$-\frac{F^{a+b(c+dx)^n}\left(-b^5\log^5(F)(c+dx)^{5n}+5b^4\log^4(F)(c+dx)^{4n}-20b^3\log^3(F)(c+dx)^{3n}+60b^2\log^2(F)(c+dx)^{2}\right.}{b^6dn\log^6(F)}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 6*n),x]
```

output
```
-((F^(a + b*(c + d*x)^n)*(120 - 120*b*(c + d*x)^n*Log[F] + 60*b^2*(c + d*x
)^(2*n)*Log[F]^2 - 20*b^3*(c + d*x)^(3*n)*Log[F]^3 + 5*b^4*(c + d*x)^(4*n)
*Log[F]^4 - b^5*(c + d*x)^(5*n)*Log[F]^5))/(b^6*d*n*Log[F]^6))
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.99

method	result
risch	$\frac{\left(b^5 (dx+c)^{5n} \ln(F)^5 - 5b^4 (dx+c)^{4n} \ln(F)^4 + 20b^3 (dx+c)^{3n} \ln(F)^3 - 60b^2 (dx+c)^{2n} \ln(F)^2 + 120b (dx+c)^n \ln(F) - 120\right) F^{a+b(dx+c)^n}}{b^6 \ln(F)^6 nd}$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x,method=_RETURNVERBOSE)
```

output
```
(((d*x+c)^n)^5*b^5*ln(F)^5-5*((d*x+c)^n)^4*b^4*ln(F)^4+20*((d*x+c)^n)^3*b^
3*ln(F)^3-60*((d*x+c)^n)^2*b^2*ln(F)^2+120*b*(d*x+c)^n*ln(F)-120)/b^6/ln(F
)^6/n/d*F^(a+b*(d*x+c)^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int F^{a+b(c+dx)^n} (c+dx)^{-1+6n} \, dx = \frac{\left((dx+c)^{5n} b^5 \log(F)^5 - 5 (dx+c)^{4n} b^4 \log(F)^4 + 20 (dx+c)^{3n} b^3 \log(F)^3 - 60 (dx+c)^{2n} b^2 \log(F)^2 + \right.}{b^6 dn \log(F)^6}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x, algorithm="fricas")
```

output
```
((d*x + c)^(5*n)*b^5*log(F)^5 - 5*(d*x + c)^(4*n)*b^4*log(F)^4 + 20*(d*x +
 c)^(3*n)*b^3*log(F)^3 - 60*(d*x + c)^(2*n)*b^2*log(F)^2 + 120*(d*x + c)^n
*b*log(F) - 120)*e^((d*x + c)^n*b*log(F) + a*log(F))/(b^6*d*n*log(F)^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(112) = 224$.

Time = 41.76 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.52

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx$$

$$= \begin{cases} \frac{x}{c} \\ F^a\left(\frac{c(c+dx)^{6n-1}}{6dn}+\frac{x(c+dx)^{6n-1}}{6n}\right) \\ F^{a+bc^n}c^{6n-1}x \\ \frac{F^{a+b}\log\left(\frac{c}{d}+x\right)}{d} \\ \frac{c(c+dx)^{6n-1}}{6dn}+\frac{x(c+dx)^{6n-1}}{6n} \\ \frac{F^{a+b(c+dx)^n}(c+dx)^{5n}}{bdn\log(F)}-\frac{5F^{a+b(c+dx)^n}(c+dx)^{4n}}{b^2dn\log(F)^2}+\frac{20F^{a+b(c+dx)^n}(c+dx)^{3n}}{b^3dn\log(F)^3}-\frac{60F^{a+b(c+dx)^n}(c+dx)^{2n}}{b^4dn\log(F)^4}+\frac{120F^{a+b(c+dx)^n}(c+dx)}{b^5dn\log(F)^5} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+6*n),x)
```

output
```
Piecewise((x/c, Eq(F, 1) & Eq(b, 0) & Eq(d, 0) & Eq(n, 0)), (F**a*(c*(c +
d*x)**(6*n - 1)/(6*d*n) + x*(c + d*x)**(6*n - 1)/(6*n)), Eq(b, 0)), (F**(a
 + b*c**n)*c**(6*n - 1)*x, Eq(d, 0)), (F**(a + b)*log(c/d + x)/d, Eq(n, 0)
), (c*(c + d*x)**(6*n - 1)/(6*d*n) + x*(c + d*x)**(6*n - 1)/(6*n), Eq(F, 1
)), (F**(a + b*(c + d*x)**n)*(c + d*x)**(5*n)/(b*d*n*log(F)) - 5*F**(a + b
*(c + d*x)**n)*(c + d*x)**(4*n)/(b**2*d*n*log(F)**2) + 20*F**(a + b*(c + d
*x)**n)*(c + d*x)**(3*n)/(b**3*d*n*log(F)**3) - 60*F**(a + b*(c + d*x)**n)
*(c + d*x)**(2*n)/(b**4*d*n*log(F)**4) + 120*F**(a + b*(c + d*x)**n)*(c +
d*x)**n/(b**5*d*n*log(F)**5) - 120*F**(a + b*(c + d*x)**n)/(b**6*d*n*log(F
)**6), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.13

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx$$
$$=\frac{\left((dx+c)^{5n}F^ab^5\log(F)^5-5(dx+c)^{4n}F^ab^4\log(F)^4+20(dx+c)^{3n}F^ab^3\log(F)^3-60(dx+c)^{2n}F^ab^2\cdots\right.}{b^6dn\log(F)^6}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x, algorithm="maxima")
```

output
```
((d*x + c)^(5*n)*F^a*b^5*log(F)^5 - 5*(d*x + c)^(4*n)*F^a*b^4*log(F)^4 + 2
0*(d*x + c)^(3*n)*F^a*b^3*log(F)^3 - 60*(d*x + c)^(2*n)*F^a*b^2*log(F)^2 +
 120*(d*x + c)^n*F^a*b*log(F) - 120*F^a)*F^((d*x + c)^n*b)/(b^6*d*n*log(F)
^6)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx=\int(dx+c)^{6n-1}F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(6*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx=\int F^{a+b\,(c+d\,x)^n}\,(c+d\,x)^{6\,n-1}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(6*n - 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(6*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.98

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+6n}\,dx$$
$$= \frac{f^{(dx+c)^n b+a}\left((dx+c)^{5n}\log(f)^5 b^5 - 5(dx+c)^{4n}\log(f)^4 b^4 + 20(dx+c)^{3n}\log(f)^3 b^3 - 60(dx+c)^{2n}\log(f\right.}{\log(f)^6 b^6 dn}$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+6*n),x)
```

output
```
(f**((c + d*x)**n*b + a)*((c + d*x)**(5*n)*log(f)**5*b**5 - 5*(c + d*x)**(
4*n)*log(f)**4*b**4 + 20*(c + d*x)**(3*n)*log(f)**3*b**3 - 60*(c + d*x)**(
2*n)*log(f)**2*b**2 + 120*(c + d*x)**n*log(f)*b - 120))/(log(f)**6*b**6*d*
n)
```

3.303 $\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx = \frac{F^{a+b(c+dx)^n}\left(24-24b(c+dx)^n\log(F)+12b^2(c+dx)^{2n}\log^2(F)-4b^3(c+dx)^{3n}\log^3(F)+b^4(c+dx)^{4n}\right.}{b^5dn\log^5(F)}$$

output
```
F^(a+b*(d*x+c)^n)*(24-24*b*(d*x+c)^n*ln(F)+12*b^2*(d*x+c)^(2*n)*ln(F)^2-4*
b^3*(d*x+c)^(3*n)*ln(F)^3+b^4*(d*x+c)^(4*n)*ln(F)^4)/b^5/d/n/ln(F)^5
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.33

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx = \frac{F^a\Gamma(5,-b(c+dx)^n\log(F))}{b^5dn\log^5(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 5*n),x]
```

output
```
(F^a*Gamma[5, -(b*(c + d*x)^n*Log[F])])/(b^5*d*n*Log[F]^5)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2647}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5n-1} F^{a+b(c+dx)^n} \, dx$$

$$\Big\downarrow \text{2647}$$

$$\frac{F^{a+b(c+dx)^n} \left(b^4 \log^4(F)(c+dx)^{4n} - 4b^3 \log^3(F)(c+dx)^{3n} + 12b^2 \log^2(F)(c+dx)^{2n} - 24b \log(F)(c+dx)^n + 24\right.}{b^5 dn \log^5(F)}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 5*n),x]
```

output
```
(F^(a + b*(c + d*x)^n)*(24 - 24*b*(c + d*x)^n*Log[F] + 12*b^2*(c + d*x)^(2
*n)*Log[F]^2 - 4*b^3*(c + d*x)^(3*n)*Log[F]^3 + b^4*(c + d*x)^(4*n)*Log[F]
^4))/(b^5*d*n*Log[F]^5)
```

Defintions of rubi rules used

rule 2647
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> With[{p = Simplify[(m + 1)/n]}, Simp[(-F^a)*((f/d)^m/(d*n*
((-b)*Log[F])^p))*Simplify[FunctionExpand[Gamma[p, (-b)*(c + d*x)^n*Log[F]]
]], x] /; IGtQ[p, 0]] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e -
 c*f, 0] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

method	result	size
risch	$\frac{F^{a+b(dx+c)^{n}} \left(24-24 b (dx+c)^{n} \ln \left(F \right)+12 b^{2} (dx+c)^{2 n} \ln \left(F \right)^{2}-4 b^{3} (dx+c)^{3 n} \ln \left(F \right)^{3}+b^{4} (dx+c)^{4 n} \ln \left(F \right)^{4}\right)}{b^{5} d n \ln \left(F \right)^{5}}$	95

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x,method=_RETURNVERBOSE)
```

output
```
(((d*x+c)^n)^4*b^4*ln(F)^4-4*((d*x+c)^n)^3*b^3*ln(F)^3+12*((d*x+c)^n)^2*b^
2*ln(F)^2-24*b*(d*x+c)^n*ln(F)+24)/b^5/ln(F)^5/n/d*F^(a+b*(d*x+c)^n)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx$$
$$= \frac{\left((dx+c)^{4\,n}b^4\log\left(F\right)^4 - 4\,(dx+c)^{3\,n}b^3\log\left(F\right)^3 + 12\,(dx+c)^{2\,n}b^2\log\left(F\right)^2 - 24\,(dx+c)^n b\log\left(F\right) + 24}{b^5dn\log\left(F\right)^5}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x, algorithm="fricas")
```

output
```
((d*x + c)^(4*n)*b^4*log(F)^4 - 4*(d*x + c)^(3*n)*b^3*log(F)^3 + 12*(d*x +
 c)^(2*n)*b^2*log(F)^2 - 24*(d*x + c)^n*b*log(F) + 24)*e^((d*x + c)^n*b*lo
g(F) + a*log(F))/(b^5*d*n*log(F)^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(92) = 184$.

Time = 33.97 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.69

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx$$
$$= \begin{cases} \frac{x}{c} & \text{for } \ldots \\ F^a\left(\frac{c(c+dx)^{5n-1}}{5dn}+\frac{x(c+dx)^{5n-1}}{5n}\right) & \text{for } \ldots \\ F^{a+bc^n}c^{5n-1}x & \text{for } \ldots \\ \frac{F^{a+b}\log\left(\frac{c}{d}+x\right)}{d} & \text{for } \ldots \\ \frac{c(c+dx)^{5n-1}}{5dn}+\frac{x(c+dx)^{5n-1}}{5n} & \text{for } \ldots \\ \frac{F^{a+b(c+dx)^n}(c+dx)^{4n}}{bdn\log(F)}-\frac{4F^{a+b(c+dx)^n}(c+dx)^{3n}}{b^2dn\log(F)^2}+\frac{12F^{a+b(c+dx)^n}(c+dx)^{2n}}{b^3dn\log(F)^3}-\frac{24F^{a+b(c+dx)^n}(c+dx)^n}{b^4dn\log(F)^4}+\frac{24F^{a+b(c+dx)^n}}{b^5dn\log(F)^5} & \text{othe}\ldots \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+5*n),x)
```

output
```
Piecewise((x/c, Eq(F, 1) & Eq(b, 0) & Eq(d, 0) & Eq(n, 0)), (F**a*(c*(c +
d*x)**(5*n - 1)/(5*d*n) + x*(c + d*x)**(5*n - 1)/(5*n)), Eq(b, 0)), (F**(a
 + b*c**n)*c**(5*n - 1)*x, Eq(d, 0)), (F**(a + b)*log(c/d + x)/d, Eq(n, 0)
), (c*(c + d*x)**(5*n - 1)/(5*d*n) + x*(c + d*x)**(5*n - 1)/(5*n), Eq(F, 1
)), (F**(a + b*(c + d*x)**n)*(c + d*x)**(4*n)/(b*d*n*log(F)) - 4*F**(a + b
*(c + d*x)**n)*(c + d*x)**(3*n)/(b**2*d*n*log(F)**2) + 12*F**(a + b*(c + d
*x)**n)*(c + d*x)**(2*n)/(b**3*d*n*log(F)**3) - 24*F**(a + b*(c + d*x)**n)
*(c + d*x)**n/(b**4*d*n*log(F)**4) + 24*F**(a + b*(c + d*x)**n)/(b**5*d*n*
log(F)**5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx$$
$$= \frac{\left((dx+c)^{4n}F^ab^4\log(F)^4-4(dx+c)^{3n}F^ab^3\log(F)^3+12(dx+c)^{2n}F^ab^2\log(F)^2-24(dx+c)^nF^ab\,\mathrm{l}\ldots\right.}{b^5dn\log(F)^5}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x, algorithm="maxima")
```

output
```
((d*x + c)^(4*n)*F^a*b^4*log(F)^4 - 4*(d*x + c)^(3*n)*F^a*b^3*log(F)^3 + 1
2*(d*x + c)^(2*n)*F^a*b^2*log(F)^2 - 24*(d*x + c)^n*F^a*b*log(F) + 24*F^a)
*F^((d*x + c)^n*b)/(b^5*d*n*log(F)^5)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx = \int (dx+c)^{5\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx = \int F^{a+b\,(c+d\,x)^n}\,(c+d\,x)^{5\,n-1}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(5*n - 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(5*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+5n}\,dx$$
$$= \frac{f^{(dx+c)^n b+a}\left((dx+c)^{4n}\log(f)^4 b^4 - 4(dx+c)^{3n}\log(f)^3 b^3 + 12(dx+c)^{2n}\log(f)^2 b^2 - 24(dx+c)^n \log(f)\right.}{\log(f)^5 b^5 dn}$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+5*n),x)
```

output
```
(f**((c + d*x)**n*b + a)*((c + d*x)**(4*n)*log(f)**4*b**4 - 4*(c + d*x)**(
3*n)*log(f)**3*b**3 + 12*(c + d*x)**(2*n)*log(f)**2*b**2 - 24*(c + d*x)**n
*log(f)*b + 24))/(log(f)**5*b**5*d*n)
```

3.304 $\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 137

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = -\frac{6F^{a+b(c+dx)^n}}{b^4dn\log^4(F)}+\frac{6F^{a+b(c+dx)^n}(c+dx)^n}{b^3dn\log^3(F)} - \frac{3F^{a+b(c+dx)^n}(c+dx)^{2n}}{b^2dn\log^2(F)}+\frac{F^{a+b(c+dx)^n}(c+dx)^{3n}}{bdn\log(F)}$$

output
```
-6*F^(a+b*(d*x+c)^n)/b^4/d/n/ln(F)^4+6*F^(a+b*(d*x+c)^n)*(d*x+c)^n/b^3/d/n
/ln(F)^3-3*F^(a+b*(d*x+c)^n)*(d*x+c)^(2*n)/b^2/d/n/ln(F)^2+F^(a+b*(d*x+c)^
n)*(d*x+c)^(3*n)/b/d/n/ln(F)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.23

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = -\frac{F^a\Gamma(4,-b(c+dx)^n\log(F))}{b^4dn\log^4(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 4*n),x]
```

output
```
-((F^a*Gamma[4, -(b*(c + d*x)^n*Log[F])])/(b^4*d*n*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2642, 2642, 2642, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{4n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \text{ 2642}$$

$$\frac{(c+dx)^{3n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{3\int F^{b(c+dx)^n+a}(c+dx)^{3n-1}dx}{b\log(F)}$$

$$\Big\downarrow \text{ 2642}$$

$$\frac{(c+dx)^{3n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{3\left(\frac{(c+dx)^{2n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{2\int F^{b(c+dx)^n+a}(c+dx)^{2n-1}dx}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \text{ 2642}$$

$$\frac{(c+dx)^{3n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{3\left(\frac{(c+dx)^{2n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{2\left(\frac{(c+dx)^nF^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{\int F^{b(c+dx)^n+a}(c+dx)^{n-1}dx}{b\log(F)}\right)}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \text{ 2638}$$

$$\frac{(c+dx)^{3n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{3\left(\frac{(c+dx)^{2n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{2\left(\frac{(c+dx)^nF^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{F^{a+b(c+dx)^n}}{b^2dn\log^2(F)}\right)}{b\log(F)}\right)}{b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 4*n),x]
```

output
```
(F^(a + b*(c + d*x)^n)*(c + d*x)^(3*n))/(b*d*n*Log[F]) - (3*((F^(a + b*(c
+ d*x)^n)*(c + d*x)^(2*n))/(b*d*n*Log[F]) - (2*(-(F^(a + b*(c + d*x)^n)/(b
^2*d*n*Log[F]^2)) + (F^(a + b*(c + d*x)^n)*(c + d*x)^n)/(b*d*n*Log[F])))/(
b*Log[F])))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{\left(b^{3} (d x +c)^{3 n} \ln \left(F \right)^{3}-3 b^{2} (d x +c)^{2 n} \ln \left(F \right)^{2}+6 b (d x +c)^{n} \ln \left(F \right)-6\right) F^{a +b (d x +c)^{n}}}{b^{4} \ln \left(F \right)^{4} n d}$	77

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+4*n),x,method=_RETURNVERBOSE)
```

output
```
(((d*x+c)^n)^3*b^3*ln(F)^3-3*((d*x+c)^n)^2*b^2*ln(F)^2+6*b*(d*x+c)^n*ln(F)
-6)/b^4/ln(F)^4/n/d*F^(a+b*(d*x+c)^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = \frac{\left((dx+c)^{3\,n}b^3\log(F)^3-3\,(dx+c)^{2\,n}b^2\log(F)^2+6\,(dx+c)^n b\log(F)-6\right)e^{((dx+c)^n b\log(F)+a\log(F))}}{b^4dn\log(F)^4}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+4*n),x, algorithm="fricas")
```

output
```
((d*x + c)^(3*n)*b^3*log(F)^3 - 3*(d*x + c)^(2*n)*b^2*log(F)^2 + 6*(d*x +
c)^n*b*log(F) - 6)*e^((d*x + c)^n*b*log(F) + a*log(F))/(b^4*d*n*log(F)^4)
```

Sympy [A] (verification not implemented)

Time = 16.99 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.60

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = \begin{cases} \frac{x}{c} & \text{for}\: F=1 \wedge b=0 \wedge d=0 \wedge \\ F^a\left(\frac{c(c+dx)^{4n-1}}{4dn}+\frac{x(c+dx)^{4n-1}}{4n}\right) & \text{for}\: b=0 \\ F^{a+bc^n}c^{4n-1}x & \text{for}\: d=0 \\ \frac{F^{a+b}\log\left(\frac{c}{d}+x\right)}{d} & \text{for}\: n=0 \\ \frac{c(c+dx)^{4n-1}}{4dn}+\frac{x(c+dx)^{4n-1}}{4n} & \text{for}\: F=1 \\ \frac{F^{a+b(c+dx)^n}(c+dx)^{3n}}{bdn\log(F)}-\frac{3F^{a+b(c+dx)^n}(c+dx)^{2n}}{b^2dn\log(F)^2}+\frac{6F^{a+b(c+dx)^n}(c+dx)^n}{b^3dn\log(F)^3}-\frac{6F^{a+b(c+dx)^n}}{b^4dn\log(F)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+4*n),x)
```

output
```
Piecewise((x/c, Eq(F, 1) & Eq(b, 0) & Eq(d, 0) & Eq(n, 0)), (F**a*(c*(c +
d*x)**(4*n - 1)/(4*d*n) + x*(c + d*x)**(4*n - 1)/(4*n)), Eq(b, 0)), (F**(a
 + b*c**n)*c**(4*n - 1)*x, Eq(d, 0)), (F**(a + b)*log(c/d + x)/d, Eq(n, 0)
), (c*(c + d*x)**(4*n - 1)/(4*d*n) + x*(c + d*x)**(4*n - 1)/(4*n), Eq(F, 1
)), (F**(a + b*(c + d*x)**n)*(c + d*x)**(3*n)/(b*d*n*log(F)) - 3*F**(a + b
*(c + d*x)**n)*(c + d*x)**(2*n)/(b**2*d*n*log(F)**2) + 6*F**(a + b*(c + d*
x)**n)*(c + d*x)**n/(b**3*d*n*log(F)**3) - 6*F**(a + b*(c + d*x)**n)/(b**4
*d*n*log(F)**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = \frac{\left((dx+c)^{3\,n}F^ab^3\log\left(F\right)^3-3\,(dx+c)^{2\,n}F^ab^2\log\left(F\right)^2+6\,(dx+c)^nF^ab\log\left(F\right)-6\,F^a\right)F^{(dx+c)^nb}}{b^4dn\log\left(F\right)^4}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+4*n),x, algorithm="maxima")
```

output
```
((d*x + c)^(3*n)*F^a*b^3*log(F)^3 - 3*(d*x + c)^(2*n)*F^a*b^2*log(F)^2 + 6
*(d*x + c)^n*F^a*b*log(F) - 6*F^a)*F^((d*x + c)^n*b)/(b^4*d*n*log(F)^4)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = \int (dx+c)^{4\,n-1}F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+4*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(4*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx=\int F^{a+b\,(c+d\,x)^n}\,(c+d\,x)^{4\,n-1}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(4*n - 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(4*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.55

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+4n}\,dx = \frac{f^{(dx+c)^n b+a}\left((dx+c)^{3n}\log(f)^3 b^3-3(dx+c)^{2n}\log(f)^2 b^2+6(dx+c)^n\log(f)\,b-6\right)}{\log(f)^4 b^4 dn}$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+4*n),x)
```

output
```
(f**((c + d*x)**n*b + a)*((c + d*x)**(3*n)*log(f)**3*b**3 - 3*(c + d*x)**(
2*n)*log(f)**2*b**2 + 6*(c + d*x)**n*log(f)*b - 6))/(log(f)**4*b**4*d*n)
```

3.305 $\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx=\frac{2F^{a+b(c+dx)^n}}{b^3dn\log^3(F)}-\frac{2F^{a+b(c+dx)^n}(c+dx)^n}{b^2dn\log^2(F)}+\frac{F^{a+b(c+dx)^n}(c+dx)^{2n}}{bdn\log(F)}$$

output
```
2*F^(a+b*(d*x+c)^n)/b^3/d/n/ln(F)^3-2*F^(a+b*(d*x+c)^n)*(d*x+c)^n/b^2/d/n/
ln(F)^2+F^(a+b*(d*x+c)^n)*(d*x+c)^(2*n)/b/d/n/ln(F)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx=\frac{F^a\Gamma(3,-b(c+dx)^n\log(F))}{b^3dn\log^3(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 3*n),x]
```

output
```
(F^a*Gamma[3, -(b*(c + d*x)^n*Log[F])])/(b^3*d*n*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2642, 2642, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \; 2642$$

$$\frac{(c+dx)^{2n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{2\int F^{b(c+dx)^n+a}(c+dx)^{2n-1}dx}{b\log(F)}$$

$$\Big\downarrow \; 2642$$

$$\frac{(c+dx)^{2n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{2\left(\frac{(c+dx)^nF^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{\int F^{b(c+dx)^n+a}(c+dx)^{n-1}dx}{b\log(F)}\right)}{b\log(F)}$$

$$\Big\downarrow \; 2638$$

$$\frac{(c+dx)^{2n}F^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{2\left(\frac{(c+dx)^nF^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{F^{a+b(c+dx)^n}}{b^2dn\log^2(F)}\right)}{b\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 3*n),x]
```

output
```
(F^(a + b*(c + d*x)^n)*(c + d*x)^(2*n))/(b*d*n*Log[F]) - (2*(-(F^(a + b*(c
 + d*x)^n)/(b^2*d*n*Log[F]^2)) + (F^(a + b*(c + d*x)^n)*(c + d*x)^n)/(b*d*
n*Log[F])))/(b*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.59

method	result	size
risch	$\frac{\left(b^{2}(dx+c)^{2n}\ln(F)^{2}-2b(dx+c)^{n}\ln(F)+2\right)F^{a+b(dx+c)^{n}}}{b^{3}\ln(F)^{3}nd}$	59

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+3*n),x,method=_RETURNVERBOSE)
```

output
```
(((d*x+c)^n)^2*b^2*ln(F)^2-2*b*(d*x+c)^n*ln(F)+2)/b^3/ln(F)^3/n/d*F^(a+b*(
d*x+c)^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.62

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx = \frac{\left((dx+c)^{2n}b^2\log(F)^2-2\,(dx+c)^nb\log(F)+2\right)e^{\left((dx+c)^nb\log(F)+a\log(F)\right)}}{b^3dn\log(F)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+3*n),x, algorithm="fricas")
```

output
```
((d*x + c)^(2*n)*b^2*log(F)^2 - 2*(d*x + c)^n*b*log(F) + 2)*e^((d*x + c)^n
*b*log(F) + a*log(F))/(b^3*d*n*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(85) = 170.

Time = 11.72 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.85

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx$$
$$= \begin{cases} \frac{x}{c} & \text{for } F=1\wedge b=0\wedge d=0\wedge n=0 \\ F^a\left(\frac{c(c+dx)^{3n-1}}{3dn}+\frac{x(c+dx)^{3n-1}}{3n}\right) & \text{for } b=0 \\ F^{a+bc^n}c^{3n-1}x & \text{for } d=0 \\ \frac{F^{a+b}\log\left(\frac{c}{d}+x\right)}{d} & \text{for } n=0 \\ \frac{c(c+dx)^{3n-1}}{3dn}+\frac{x(c+dx)^{3n-1}}{3n} & \text{for } F=1 \\ \frac{F^{a+b(c+dx)^n}(c+dx)^{2n}}{bdn\log(F)}-\frac{2F^{a+b(c+dx)^n}(c+dx)^n}{b^2dn\log(F)^2}+\frac{2F^{a+b(c+dx)^n}}{b^3dn\log(F)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+3*n),x)
```

output
```
Piecewise((x/c, Eq(F, 1) & Eq(b, 0) & Eq(d, 0) & Eq(n, 0)), (F**a*(c*(c +
d*x)**(3*n - 1)/(3*d*n) + x*(c + d*x)**(3*n - 1)/(3*n)), Eq(b, 0)), (F**(a
 + b*c**n)*c**(3*n - 1)*x, Eq(d, 0)), (F**(a + b)*log(c/d + x)/d, Eq(n, 0)
), (c*(c + d*x)**(3*n - 1)/(3*d*n) + x*(c + d*x)**(3*n - 1)/(3*n), Eq(F, 1
)), (F**(a + b*(c + d*x)**n)*(c + d*x)**(2*n)/(b*d*n*log(F)) - 2*F**(a + b
*(c + d*x)**n)*(c + d*x)**n/(b**2*d*n*log(F)**2) + 2*F**(a + b*(c + d*x)**
n)/(b**3*d*n*log(F)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.66

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx = \frac{\left((dx+c)^{2\,n}F^ab^2\log\left(F\right)^2-2\,(dx+c)^nF^ab\log\left(F\right)+2\,F^a\right)F^{(dx+c)^nb}}{b^3dn\log\left(F\right)^3}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+3*n),x, algorithm="maxima")
```

output
```
((d*x + c)^(2*n)*F^a*b^2*log(F)^2 - 2*(d*x + c)^n*F^a*b*log(F) + 2*F^a)*F^
((d*x + c)^n*b)/(b^3*d*n*log(F)^3)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx = \int (dx+c)^{3\,n-1}F^{(dx+c)^nb+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+3*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx = \int F^{a+b\,{(c+d\,x)}^n}\,{(c+d\,x)}^{3\,n-1}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(3*n - 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(3*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+3n}\,dx$$
$$=\frac{f^{(dx+c)^n b+a}\left((dx+c)^{2n}\log(f)^2 b^2-2(dx+c)^n\log(f)\,b+2\right)}{\log(f)^3 b^3dn}$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+3*n),x)
```

output
```
(f**((c + d*x)**n*b + a)*((c + d*x)**(2*n)*log(f)**2*b**2 - 2*(c + d*x)**n
*log(f)*b + 2))/(log(f)**3*b**3*d*n)
```

3.306 $\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 63

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx=-\frac{F^{a+b(c+dx)^n}}{b^2dn\log^2(F)}+\frac{F^{a+b(c+dx)^n}(c+dx)^n}{bdn\log(F)}$$

output
```
-F^(a+b*(d*x+c)^n)/b^2/d/n/ln(F)^2+F^(a+b*(d*x+c)^n)*(d*x+c)^n/b/d/n/ln(F)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.51

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx=-\frac{F^a\Gamma(2,-b(c+dx)^n\log(F))}{b^2dn\log^2(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 2*n),x]
```

output
```
-((F^a*Gamma[2, -(b*(c + d*x)^n*Log[F])])/(b^2*d*n*Log[F]^2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2642, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{2n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \text{2642}$$

$$\frac{(c+dx)^nF^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{\int F^{b(c+dx)^n+a}(c+dx)^{n-1}dx}{b\log(F)}$$

$$\Big\downarrow \text{2638}$$

$$\frac{(c+dx)^nF^{a+b(c+dx)^n}}{bdn\log(F)}-\frac{F^{a+b(c+dx)^n}}{b^2dn\log^2(F)}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + 2*n),x]
```

output
```
-(F^(a + b*(c + d*x)^n)/(b^2*d*n*Log[F]^2)) + (F^(a + b*(c + d*x)^n)*(c +
d*x)^n)/(b*d*n*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2642
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^Simplify[m - n]
*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && Intege
rQ[2*Simplify[(m + 1)/n]] && LtQ[0, Simplify[(m + 1)/n], 5] &&  !RationalQ[
m] && SumSimplerQ[m, -n]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{\left(b \left(d x +c \right)^{n} \ln \left(F \right)-1\right) F^{a +b \left(d x +c \right)^{n}}}{b^{2} n d \ln \left(F \right)^{2}}$	41
norman	$\frac{{\mathrm e}^{n \ln \left(d x +c \right)} {\mathrm e}^{\left(a +b \,{\mathrm e}^{n \ln \left(d x +c \right)}\right) \ln \left(F \right)}}{d b n \ln \left(F \right)}-\frac{{\mathrm e}^{\left(a +b \,{\mathrm e}^{n \ln \left(d x +c \right)}\right) \ln \left(F \right)}}{b^{2} n d \ln \left(F \right)^{2}}$	74

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(2*n-1),x,method=_RETURNVERBOSE)
```

output
```
(b*(d*x+c)^n*ln(F)-1)/b^2/n/d/ln(F)^2*F^(a+b*(d*x+c)^n)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx=\frac{((dx+c)^nb\log(F)-1)e^{((dx+c)^nb\log(F)+a\log(F))}}{b^2dn\log(F)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+2*n),x, algorithm="fricas")
```

output
```
((d*x + c)^n*b*log(F) - 1)*e^((d*x + c)^n*b*log(F) + a*log(F))/(b^2*d*n*lo
g(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(49) = 98$.

Time = 4.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.38

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx = \begin{cases} \frac{x}{c} & \text{for}\ F=1 \wedge b=0 \wedge d=0 \wedge n=0 \\ F^a\left(\frac{c(c+dx)^{2n-1}}{2dn}+\frac{x(c+dx)^{2n-1}}{2n}\right) & \text{for}\ b=0 \\ F^{a+bc^n}c^{2n-1}x & \text{for}\ d=0 \\ \frac{F^{a+b}\log\left(\frac{c}{d}+x\right)}{d} & \text{for}\ n=0 \\ \frac{c(c+dx)^{2n-1}}{2dn}+\frac{x(c+dx)^{2n-1}}{2n} & \text{for}\ F=1 \\ \frac{F^{a+b(c+dx)^n}(c+dx)^n}{bdn\log(F)}-\frac{F^{a+b(c+dx)^n}}{b^2dn\log(F)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+2*n),x)
```

output
```
Piecewise((x/c, Eq(F, 1) & Eq(b, 0) & Eq(d, 0) & Eq(n, 0)), (F**a*(c*(c +
d*x)**(2*n - 1)/(2*d*n) + x*(c + d*x)**(2*n - 1)/(2*n)), Eq(b, 0)), (F**(a
 + b*c**n)*c**(2*n - 1)*x, Eq(d, 0)), (F**(a + b)*log(c/d + x)/d, Eq(n, 0)
), (c*(c + d*x)**(2*n - 1)/(2*d*n) + x*(c + d*x)**(2*n - 1)/(2*n), Eq(F, 1
)), (F**(a + b*(c + d*x)**n)*(c + d*x)**n/(b*d*n*log(F)) - F**(a + b*(c +
d*x)**n)/(b**2*d*n*log(F)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx = \frac{((dx+c)^n F^a b\log(F) - F^a)F^{(dx+c)^n b}}{b^2 dn\log(F)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+2*n),x, algorithm="maxima")
```

output
```
((d*x + c)^n*F^a*b*log(F) - F^a)*F^((d*x + c)^n*b)/(b^2*d*n*log(F)^2)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx = \int (dx+c)^{2n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+2*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(2*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx = \int F^{a+b\,(c+d\,x)^n}\,(c+d\,x)^{2\,n-1}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(2*n - 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(2*n - 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.63

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+2n}\,dx = \frac{f^{(dx+c)^n b+a}\left((dx+c)^n \log(f)\,b-1\right)}{\log\left(f\right)^2 b^2 dn}$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+2*n),x)
```

output
```
(f**((c + d*x)**n*b + a)*((c + d*x)**n*log(f)*b - 1))/(log(f)**2*b**2*d*n)
```

3.307 $\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 27

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx=\frac{F^{a+b(c+dx)^n}}{bdn\log(F)}$$

output
```
F^(a+b*(d*x+c)^n)/b/d/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx=\frac{F^{a+b(c+dx)^n}}{bdn\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + n),x]
```

output
```
F^(a + b*(c + d*x)^n)/(b*d*n*Log[F])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \; 2638$$

$$\frac{F^{a+b(c+dx)^n}}{bdn\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 + n),x]
```

output
```
F^(a + b*(c + d*x)^n)/(b*d*n*Log[F])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
risch	$\frac{F^{a+b(dx+c)^n}}{bdn\ln(F)}$	28
norman	$\frac{{\rm e}^{\left(a+b\,{\rm e}^{n\ln(dx+c)}\right)\ln(F)}}{dbn\ln(F)}$	32

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x,method=_RETURNVERBOSE)
```

output
```
F^(a+b*(d*x+c)^n)/b/d/n/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx=\frac{e^{\left((dx+c)^n b\log(F)+a\log(F)\right)}}{bdn\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x, algorithm="fricas")
```

output
```
e^((d*x + c)^n*b*log(F) + a*log(F))/(b*d*n*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(19) = 38$.

Time = 2.52 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.85

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx$$
$$=\begin{cases}\frac{x}{c} & \text{for}\: F=1\wedge b=0\wedge d=0\wedge n=0\\ F^{a}\left(\frac{c(c+dx)^{n-1}}{dn}+\frac{x(c+dx)^{n-1}}{n}\right) & \text{for}\: b=0\\ F^{a+bc^n}c^{n-1}x & \text{for}\: d=0\\ \frac{F^{a+b}\log\left(\frac{c}{d}+x\right)}{d} & \text{for}\: n=0\\ \frac{c(c+dx)^{n-1}}{dn}+\frac{x(c+dx)^{n-1}}{n} & \text{for}\: F=1\\ \frac{F^{a+b(c+dx)^n}}{bdn\log(F)} & \text{otherwise}\end{cases}$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1+n),x)
```

output
```
Piecewise((x/c, Eq(F, 1) & Eq(b, 0) & Eq(d, 0) & Eq(n, 0)), (F**a*(c*(c +
d*x)**(n - 1)/(d*n) + x*(c + d*x)**(n - 1)/n), Eq(b, 0)), (F**(a + b*c**n)
*c**(n - 1)*x, Eq(d, 0)), (F**(a + b)*log(c/d + x)/d, Eq(n, 0)), (c*(c + d
*x)**(n - 1)/(d*n) + x*(c + d*x)**(n - 1)/n, Eq(F, 1)), (F**(a + b*(c + d*
x)**n)/(b*d*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx = \frac{F^{(dx+c)^n b+a}}{bdn\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x, algorithm="maxima")
```

output
```
F^((d*x + c)^n*b + a)/(b*d*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx = \frac{F^{(dx+c)^n b+a}}{bdn\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x, algorithm="giac")
```

output
```
F^((d*x + c)^n*b + a)/(b*d*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx=\frac{F^{a+b\,(c+d\,x)^n}}{b\,d\,n\,\ln\left(F\right)}$$

input
```
int(F^(a + b*(c + d*x)^n)*(c + d*x)^(n - 1),x)
```

output
```
F^(a + b*(c + d*x)^n)/(b*d*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1+n}\,dx=\frac{f^{(dx+c)^n b+a}}{\log\left(f\right)bdn}$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1+n),x)
```

output
```
f**((c + d*x)**n*b + a)/(log(f)*b*d*n)
```

3.308 $\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 22

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx=\frac{F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}$$

output
```
F^a*Ei(b*(d*x+c)^n*ln(F))/d/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx=\frac{F^a \operatorname{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(d*n)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx$$

$$\Big\downarrow \; 2639$$

$$\frac{F^a \text{ExpIntegralEi}\left(b(c+dx)^n \log(F)\right)}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)/(c + d*x),x]
```

output
```
(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(d*n)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

method	result	size
risch	$-\frac{F^{a} \operatorname{expIntegral}_{1}\left(-b (d x +c)^{n} \ln \left(F \right)\right)}{d n}$	26

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d/n*F^a*Ei(1,-b*(d*x+c)^n*ln(F))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \frac{F^a\mathrm{Ei}\left((dx+c)^n b\log(F)\right)}{dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="fricas")
```

output
```
F^a*Ei((d*x + c)^n*b*log(F))/(d*n)
```

Sympy [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)/(d*x+c),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^{(dx+c)^n b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)
```

Giac [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^{(dx+c)^n b+a}}{dx+c}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^n*b + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = \int \frac{F^a\,F^{b\,(c+d\,x)^n}}{c+d\,x}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x),x)
```

output
```
int((F^a*F^(b*(c + d*x)^n))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{F^{a+b(c+dx)^n}}{c+dx}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{dx+c}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)/(d*x+c),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/(c + d*x),x)
```

3.309 $\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx=-\frac{F^{a+b(c+dx)^n}(c+dx)^{-n}}{dn}+\frac{bF^a\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)\log(F)}{dn}$$

output
```
-F^(a+b*(d*x+c)^n)/d/n/((d*x+c)^n)+b*F^a*Ei(b*(d*x+c)^n*ln(F))*ln(F)/d/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx=\frac{bF^a\Gamma(-1,-b(c+dx)^n\log(F))\log(F)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - n),x]
```

output
```
(b*F^a*Gamma[-1, -(b*(c + d*x)^n*Log[F])]*Log[F])/(d*n)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2644, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{-n-1}F^{a+b(c+dx)^n}dx$$
$$\Big\downarrow \text{2644}$$
$$b\log(F)\int \frac{F^{b(c+dx)^n+a}}{c+dx}dx-\frac{(c+dx)^{-n}F^{a+b(c+dx)^n}}{dn}$$
$$\Big\downarrow \text{2639}$$
$$\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}-\frac{(c+dx)^{-n}F^{a+b(c+dx)^n}}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - n),x]
```

output
```
-(F^(a + b*(c + d*x)^n)/(d*n*(c + d*x)^n)) + (b*F^a*ExpIntegralEi[b*(c + d
*x)^n*Log[F]]*Log[F])/(d*n)
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] &&  !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

method	result	size
risch	$-\frac{F^{a} F^{b (d x +c)^{n}} (d x +c)^{-n}}{n d}-\frac{\ln \left(F \right) b \,F^{a} \operatorname{expIntegral}_{1}\left(-b (d x +c)^{n} \ln \left(F \right)\right)}{n d}$	61

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-n-1),x,method=_RETURNVERBOSE)
```

output
```
-1/n/d*F^a*F^(b*(d*x+c)^n)/((d*x+c)^n)-1/n/d*ln(F)*b*F^a*Ei(1,-b*(d*x+c)^n
*ln(F))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx = \frac{(dx+c)^n F^a b\mathrm{Ei}\left((dx+c)^n b\log(F)\right)\log(F) - e^{\left((dx+c)^n b\log(F)+a\log(F)\right)}}{(dx+c)^n dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n),x, algorithm="fricas")
```

output
```
((d*x + c)^n*F^a*b*Ei((d*x + c)^n*b*log(F))*log(F) - e^((d*x + c)^n*b*log(
F) + a*log(F)))/((d*x + c)^n*d*n)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx = \int F^{a+b(c+dx)^n}(c+dx)^{-n-1}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-n),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**(-n - 1), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx=\int (dx+c)^{-n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(-n - 1)*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx=\int (dx+c)^{-n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(-n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx=\int \frac{F^{a+b\,(c+d\,x)^n}}{(c+d\,x)^{n+1}}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(n + 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(n + 1), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-n}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{(dx+c)^n c + (dx+c)^n dx}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-n),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/((c + d*x)**n*c + (c + d*x)**n*d*x),x)
```

3.310 $\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 100

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx = -\frac{F^{a+b(c+dx)^n}(c+dx)^{-2n}}{2dn} - \frac{bF^{a+b(c+dx)^n}(c+dx)^{-n}\log(F)}{2dn} + \frac{b^2F^a\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)\log^2(F)}{2dn}$$

output
```
-1/2*F^(a+b*(d*x+c)^n)/d/n/((d*x+c)^(2*n))-1/2*b*F^(a+b*(d*x+c)^n)*ln(F)/d
/n/((d*x+c)^n)+1/2*b^2*F^a*Ei(b*(d*x+c)^n*ln(F))*ln(F)^2/d/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.32

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx = -\frac{b^2F^a\Gamma(-2,-b(c+dx)^n\log(F))\log^2(F)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 2*n),x]
```

output
```
-((b^2*F^a*Gamma[-2, -(b*(c + d*x)^n*Log[F])]*Log[F]^2)/(d*n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2644, 2644, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{-2n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \text{ 2644}$$

$$\frac{1}{2}b\log(F)\int F^{b(c+dx)^n+a}(c+dx)^{-n-1}dx-\frac{(c+dx)^{-2n}F^{a+b(c+dx)^n}}{2dn}$$

$$\Big\downarrow \text{ 2644}$$

$$\frac{1}{2}b\log(F)\left(b\log(F)\int \frac{F^{b(c+dx)^n+a}}{c+dx}dx-\frac{(c+dx)^{-n}F^{a+b(c+dx)^n}}{dn}\right)-\frac{(c+dx)^{-2n}F^{a+b(c+dx)^n}}{2dn}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{2}b\log(F)\left(\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}-\frac{(c+dx)^{-n}F^{a+b(c+dx)^n}}{dn}\right)-\frac{(c+dx)^{-2n}F^{a+b(c+dx)^n}}{2dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 2*n),x]
```

output
```
-1/2*F^(a + b*(c + d*x)^n)/(d*n*(c + d*x)^(2*n)) + (b*Log[F]*(-(F^(a + b*(
c + d*x)^n)/(d*n*(c + d*x)^n)) + (b*F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]
]*Log[F])/(d*n)))/2
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] &&  !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

method	result	size
risch	$-\frac{F^{a} F^{b (d x+c)^{n}} (d x+c)^{-2 n}}{2 n d}-\frac{\ln \left(F \right) b \,F^{a} F^{b (d x+c)^{n}} (d x+c)^{-n}}{2 n d}-\frac{\ln \left(F \right)^{2} b^{2} F^{a} \operatorname{expIntegral}_{1}\left(-b (d x+c)^{n} \ln \left(F \right)\right)}{2 n d}$	99

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-2*n),x,method=_RETURNVERBOSE)
```

output
```
-1/2/n/d*F^a*F^(b*(d*x+c)^n)/((d*x+c)^n)^2-1/2/n/d*ln(F)*b*F^a*F^(b*(d*x+c
)^n)/((d*x+c)^n)-1/2/n/d*ln(F)^2*b^2*F^a*Ei(1,-b*(d*x+c)^n*ln(F))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx$$
$$=\frac{(dx+c)^{2\,n}F^ab^2\mathrm{Ei}((dx+c)^nb\log(F))\log(F)^2-((dx+c)^nb\log(F)+1)e^{((dx+c)^nb\log(F)+a\log(F))}}{2\,(dx+c)^{2\,n}dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-2*n),x, algorithm="fricas")
```

output
```
1/2*((d*x + c)^(2*n)*F^a*b^2*Ei((d*x + c)^n*b*log(F))*log(F)^2 - ((d*x + c
)^n*b*log(F) + 1)*e^((d*x + c)^n*b*log(F) + a*log(F)))/((d*x + c)^(2*n)*d*
n)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx=\int F^{a+b(c+dx)^n}(c+dx)^{-2n-1}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-2*n),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**(-2*n - 1), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx=\int (dx+c)^{-2n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-2*n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(-2*n - 1)*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx=\int (dx+c)^{-2n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-2*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(-2*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx=\int \frac{F^{a+b\,(c+d\,x)^n}}{(c+d\,x)^{2\,n+1}}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(2*n + 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(2*n + 1), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-2n}\,dx=f^a\left(\int \frac{f^{(dx+c)^n b}}{(dx+c)^{2n}\,c+(dx+c)^{2n}\,dx}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-2*n),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/((c + d*x)**(2*n)*c + (c + d*x)**(2*n)*d*x),x
)
```

3.311 $\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 139

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx = -\frac{F^{a+b(c+dx)^n}(c+dx)^{-3n}}{3dn} - \frac{bF^{a+b(c+dx)^n}(c+dx)^{-2n}\log(F)}{6dn} - \frac{b^2F^{a+b(c+dx)^n}(c+dx)^{-n}\log^2(F)}{6dn} + \frac{b^3F^a\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)\log^3(F)}{6dn}$$

output
```
-1/3*F^(a+b*(d*x+c)^n)/d/n/((d*x+c)^(3*n))-1/6*b*F^(a+b*(d*x+c)^n)*ln(F)/d
/n/((d*x+c)^(2*n))-1/6*b^2*F^(a+b*(d*x+c)^n)*ln(F)^2/d/n/((d*x+c)^n)+1/6*b
^3*F^a*Ei(b*(d*x+c)^n*ln(F))*ln(F)^3/d/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx=\frac{b^3F^a\Gamma(-3,-b(c+dx)^n\log(F))\log^3(F)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 3*n),x]
```

output
```
(b^3*F^a*Gamma[-3, -(b*(c + d*x)^n*Log[F])]*Log[F]^3)/(d*n)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2644, 2644, 2644, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{-3n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \text{2644}$$

$$\frac{1}{3}b\log(F)\int F^{b(c+dx)^n+a}(c+dx)^{-2n-1}dx-\frac{(c+dx)^{-3n}F^{a+b(c+dx)^n}}{3dn}$$

$$\Big\downarrow \text{2644}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\int F^{b(c+dx)^n+a}(c+dx)^{-n-1}dx-\frac{(c+dx)^{-2n}F^{a+b(c+dx)^n}}{2dn}\right)-\frac{(c+dx)^{-3n}F^{a+b(c+dx)^n}}{3dn}$$

$$\Big\downarrow \text{2644}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\left(b\log(F)\int\frac{F^{b(c+dx)^n+a}}{c+dx}dx-\frac{(c+dx)^{-n}F^{a+b(c+dx)^n}}{dn}\right)-\frac{(c+dx)^{-2n}F^{a+b(c+dx)^n}}{2dn}\right)-\frac{(c+dx)^{-3n}F^{a+b(c+dx)^n}}{3dn}$$

$$\Big\downarrow \text{ 2639}$$

$$\frac{1}{3}b\log(F)\left(\frac{1}{2}b\log(F)\left(\frac{bF^a\log(F)\,\text{ExpIntegralEi}\left(b(c+dx)^n\log(F)\right)}{dn}-\frac{(c+dx)^{-n}F^{a+b(c+dx)^n}}{dn}\right)-\frac{(c+dx)^{-2n}F^{a+b(c+dx)^n}}{2dn}\right)-\frac{(c+dx)^{-3n}F^{a+b(c+dx)^n}}{3dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 3*n),x]
```

output
```
-1/3*F^(a + b*(c + d*x)^n)/(d*n*(c + d*x)^(3*n)) + (b*Log[F]*(-1/2*F^(a +
b*(c + d*x)^n)/(d*n*(c + d*x)^(2*n)) + (b*Log[F]*(-(F^(a + b*(c + d*x)^n)/
(d*n*(c + d*x)^n)) + (b*F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]]*Log[F])/(d
*n)))/2))/3
```

Defintions of rubi rules used

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2644
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^Simplify[m + n]*F^(a + b*(
c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && IntegerQ[2*Simpli
fy[(m + 1)/n]] && LtQ[-4, Simplify[(m + 1)/n], 5] && !RationalQ[m] && SumS
implerQ[m, n]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{F^{a} F^{b (d x+c)^{n}} (d x+c)^{-3 n}}{3 n d}-\frac{\ln (F) b F^{a} F^{b (d x+c)^{n}} (d x+c)^{-2 n}}{6 n d}-\frac{\ln (F)^{2} b^{2} F^{a} F^{b (d x+c)^{n}} (d x+c)^{-n}}{6 n d}-\frac{\ln (F)^{3} b^{3} F^{a} \operatorname{expIntegra}}{6 n d}$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-3*n),x,method=_RETURNVERBOSE)
```

output
```
-1/3/n/d*F^a*F^(b*(d*x+c)^n)/((d*x+c)^n)^3-1/6/n/d*ln(F)*b*F^a*F^(b*(d*x+c
)^n)/((d*x+c)^n)^2-1/6/n/d*ln(F)^2*b^2*F^a*F^(b*(d*x+c)^n)/((d*x+c)^n)-1/6
/n/d*ln(F)^3*b^3*F^a*Ei(1,-b*(d*x+c)^n*ln(F))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx$$
$$=\frac{(dx+c)^{3n}F^a b^3\mathrm{Ei}((dx+c)^n b\log(F))\log(F)^3-\left((dx+c)^{2n}b^2\log(F)^2+(dx+c)^n b\log(F)+2\right)e^{((dx+c}}{6\,(dx+c)^{3n}dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-3*n),x, algorithm="fricas")
```

output
```
1/6*((d*x + c)^(3*n)*F^a*b^3*Ei((d*x + c)^n*b*log(F))*log(F)^3 - ((d*x + c
)^(2*n)*b^2*log(F)^2 + (d*x + c)^n*b*log(F) + 2)*e^((d*x + c)^n*b*log(F) +
 a*log(F)))/((d*x + c)^(3*n)*d*n)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx = \int F^{a+b(c+dx)^n}(c+dx)^{-3n-1}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-3*n),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**(-3*n - 1), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx = \int (dx+c)^{-3\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-3*n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(-3*n - 1)*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx = \int (dx+c)^{-3\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-3*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(-3*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx = \int \frac{F^{a+b\,(c+d\,x)^n}}{(c+d\,x)^{3\,n+1}}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(3*n + 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(3*n + 1), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-3n}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{(dx+c)^{3n}\,c+(dx+c)^{3n}\,dx}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-3*n),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/((c + d*x)**(3*n)*c + (c + d*x)**(3*n)*d*x),x
)
```

3.312 $\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 32

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx = -\frac{b^4F^a\Gamma(-4,-b(c+dx)^n\log(F))\log^4(F)}{dn}$$

output
```
-F^a/((d*x+c)^n)^4*Ei(5,-b*(d*x+c)^n*ln(F))/d/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx = -\frac{b^4F^a\Gamma(-4,-b(c+dx)^n\log(F))\log^4(F)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 4*n),x]
```

output
```
-((b^4*F^a*Gamma[-4, -(b*(c + d*x)^n*Log[F])]*Log[F]^4)/(d*n))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{-4n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \; 2648$$

$$-\frac{b^4F^a\log^4(F)\Gamma(-4,-b(c+dx)^n\log(F))}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 4*n),x]
```

output
```
-((b^4*F^a*Gamma[-4, -(b*(c + d*x)^n*Log[F])]*Log[F]^4)/(d*n))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(34) = 68.

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.53

method	result
risch	$-\frac{F^a\left(\text{expIntegral}_1(-b(dx+c)^n\ln(F))\ln(F)^4b^4(dx+c)^{4n}+F^{b(dx+c)^n}\ln(F)^3b^3(dx+c)^{3n}+F^{b(dx+c)^n}\ln(F)^2b^2(dx+c)^{2n}+2F^{b(dx\cdots}\right.}{24nd}$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x,method=_RETURNVERBOSE)
```

output
```
-1/24/n/d*F^a*(Ei(1,-b*(d*x+c)^n*ln(F))*ln(F)^4*b^4*((d*x+c)^n)^4+F^(b*(d*
x+c)^n)*ln(F)^3*b^3*((d*x+c)^n)^3+F^(b*(d*x+c)^n)*ln(F)^2*b^2*((d*x+c)^n)^
2+2*F^(b*(d*x+c)^n)*ln(F)*b*(d*x+c)^n+6*F^(b*(d*x+c)^n))/((d*x+c)^n)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(32) = 64$.

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.72

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx = \frac{(dx+c)^{4n}F^a b^4 \mathrm{Ei}((dx+c)^n b\log(F))\log(F)^4 - \left((dx+c)^{3n}b^3\log(F)^3 + (dx+c)^{2n}b^2\log(F)^2 + 2\,(dx}{24\,(dx+c)^{4n}dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x, algorithm="fricas")
```

output
```
1/24*((d*x + c)^(4*n)*F^a*b^4*Ei((d*x + c)^n*b*log(F))*log(F)^4 - ((d*x +
c)^(3*n)*b^3*log(F)^3 + (d*x + c)^(2*n)*b^2*log(F)^2 + 2*(d*x + c)^n*b*log
(F) + 6)*e^((d*x + c)^n*b*log(F) + a*log(F)))/((d*x + c)^(4*n)*d*n)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx = \int F^{a+b(c+dx)^n}(c+dx)^{-4n-1}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-4*n),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**(-4*n - 1), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx=\int (dx+c)^{-4\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(-4*n - 1)*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx=\int (dx+c)^{-4\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(-4*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx=\int \frac{F^{a+b\,(c+d\,x)^n}}{(c+d\,x)^{4\,n+1}}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(4*n + 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(4*n + 1), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-4n}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{(dx+c)^{4n}c+(dx+c)^{4n}dx}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-4*n),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/((c + d*x)**(4*n)*c + (c + d*x)**(4*n)*d*x),x
)
```

3.313 $\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 31

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx=\frac{b^5F^a\Gamma(-5,-b(c+dx)^n\log(F))\log^5(F)}{dn}$$

output
```
-F^a/((d*x+c)^n)^5*Ei(6,-b*(d*x+c)^n*ln(F))/d/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx=\frac{b^5F^a\Gamma(-5,-b(c+dx)^n\log(F))\log^5(F)}{dn}$$

input
```
Integrate[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 5*n),x]
```

output
```
(b^5*F^a*Gamma[-5, -(b*(c + d*x)^n*Log[F])]*Log[F]^5)/(d*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{-5n-1}F^{a+b(c+dx)^n}\,dx$$

$$\Big\downarrow \text{2648}$$

$$\frac{b^5F^a\log^5(F)\Gamma(-5,-b(c+dx)^n\log(F))}{dn}$$

input
```
Int[F^(a + b*(c + d*x)^n)*(c + d*x)^(-1 - 5*n),x]
```

output
```
(b^5*F^a*Gamma[-5, -(b*(c + d*x)^n*Log[F])]*Log[F]^5)/(d*n)
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(34) = 68.

Time = 0.36 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.61

method	result
risch	$-\frac{F^{a}\left(\mathrm{expIntegral}_{1}\left(-b\left(dx+c\right)^{n}\ln\left(F\right)\right)\ln\left(F\right)^{5}b^{5}\left(dx+c\right)^{5n}+F^{b\left(dx+c\right)^{n}}\ln\left(F\right)^{4}b^{4}\left(dx+c\right)^{4n}+F^{b\left(dx+c\right)^{n}}\ln\left(F\right)^{3}b^{3}\left(dx+c\right)^{3n}+2F^{b(dx}\right.}{120nd}$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x,method=_RETURNVERBOSE)
```

output
```
-1/120/n/d*F^a*(Ei(1,-b*(d*x+c)^n*ln(F))*ln(F)^5*b^5*((d*x+c)^n)^5+F^(b*(d
*x+c)^n)*ln(F)^4*b^4*((d*x+c)^n)^4+F^(b*(d*x+c)^n)*ln(F)^3*b^3*((d*x+c)^n)
^3+2*F^(b*(d*x+c)^n)*ln(F)^2*b^2*((d*x+c)^n)^2+6*F^(b*(d*x+c)^n)*ln(F)*b*(
d*x+c)^n+24*F^(b*(d*x+c)^n))/((d*x+c)^n)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(31) = 62.

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 4.42

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx = \frac{(dx+c)^{5n}F^ab^5\text{Ei}((dx+c)^nb\log(F))\log(F)^5 - \left((dx+c)^{4n}b^4\log(F)^4 + (dx+c)^{3n}b^3\log(F)^3 + 2\,(dx\right.}{120\,(dx+c)^{5n}dn}$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x, algorithm="fricas")
```

output
```
1/120*((d*x + c)^(5*n)*F^a*b^5*Ei((d*x + c)^n*b*log(F))*log(F)^5 - ((d*x +
 c)^(4*n)*b^4*log(F)^4 + (d*x + c)^(3*n)*b^3*log(F)^3 + 2*(d*x + c)^(2*n)*
b^2*log(F)^2 + 6*(d*x + c)^n*b*log(F) + 24)*e^((d*x + c)^n*b*log(F) + a*lo
g(F)))/((d*x + c)^(5*n)*d*n)
```

Sympy [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx = \int F^{a+b(c+dx)^n}(c+dx)^{-5n-1}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**n)*(d*x+c)**(-1-5*n),x)
```

output
```
Integral(F**(a + b*(c + d*x)**n)*(c + d*x)**(-5*n - 1), x)
```

Maxima [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx=\int (dx+c)^{-5\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^(-5*n - 1)*F^((d*x + c)^n*b + a), x)
```

Giac [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx=\int (dx+c)^{-5\,n-1}F^{(dx+c)^n b+a}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^(-5*n - 1)*F^((d*x + c)^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx=\int \frac{F^{a+b\,(c+d\,x)^n}}{(c+d\,x)^{5\,n+1}}\,dx$$

input
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(5*n + 1),x)
```

output
```
int(F^(a + b*(c + d*x)^n)/(c + d*x)^(5*n + 1), x)
```

Reduce [F]

$$\int F^{a+b(c+dx)^n}(c+dx)^{-1-5n}\,dx = f^a\left(\int \frac{f^{(dx+c)^n b}}{(dx+c)^{5n}c+(dx+c)^{5n}dx}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^n)*(d*x+c)^(-1-5*n),x)
```

output
```
f**a*int(f**((c + d*x)**n*b)/((c + d*x)**(5*n)*c + (c + d*x)**(5*n)*d*x),x
)
```

3.314 $\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)^{n/2}\sqrt{\log(F)}\right)}{b\sqrt{c}n\sqrt{\log(F)}}$$

output
```
Pi^(1/2)*erfi(c^(1/2)*(b*x+a)^(1/2*n)*ln(F)^(1/2))/b/c^(1/2)/n/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}(a+bx)^{n/2}\sqrt{\log(F)}\right)}{b\sqrt{c}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(c*(a + b*x)^n)*(a + b*x)^(-1 + n/2),x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)^(n/2)*Sqrt[Log[F]]])/(b*Sqrt[c]*n*Sqrt[Lo
g[F]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{\frac{n}{2}-1}F^{c(a+bx)^n}\,dx$$

$$\Big\downarrow \text{2640}$$

$$\frac{2\int F^{c(a+bx)^n}d(a+bx)^{n/2}}{bn}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{c}\sqrt{\log(F)}(a+bx)^{n/2}\right)}{b\sqrt{c}n\sqrt{\log(F)}}$$

input
```
Int[F^(c*(a + b*x)^n)*(a + b*x)^(-1 + n/2),x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[c]*(a + b*x)^(n/2)*Sqrt[Log[F]]])/(b*Sqrt[c]*n*Sqrt[Lo
g[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\sqrt{\pi}\, \mathrm{erf}\left(\sqrt{-\ln(F) c}\, (b x+a)^{\frac{n}{2}}\right)}{n b \sqrt{-\ln(F) c}}$	36

input
```
int(F^(c*(b*x+a)^n)*(b*x+a)^(-1+1/2*n),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*Pi^(1/2)/(-ln(F)*c)^(1/2)*erf((-ln(F)*c)^(1/2)*(b*x+a)^(1/2*n))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = -\frac{\sqrt{\pi}\sqrt{-c\log(F)}\,\mathrm{erf}\left((bx+a)\sqrt{-c\log(F)}(bx+a)^{\frac{1}{2}n-1}\right)}{bcn\log(F)}$$

input
```
integrate(F^(c*(b*x+a)^n)*(b*x+a)^(-1+1/2*n),x, algorithm="fricas")
```

output
```
-sqrt(pi)*sqrt(-c*log(F))*erf((b*x + a)*sqrt(-c*log(F))*(b*x + a)^(1/2*n -
 1))/(b*c*n*log(F))
```

Sympy [F]

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = \int F^{c(a+bx)^n}(a+bx)^{\frac{n}{2}-1}\,dx$$

input
```
integrate(F**(c*(b*x+a)**n)*(b*x+a)**(-1+1/2*n),x)
```

output
```
Integral(F**(c*(a + b*x)**n)*(a + b*x)**(n/2 - 1), x)
```

Maxima [F]

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = \int (bx+a)^{\frac{1}{2}n-1}F^{(bx+a)^n c}\,dx$$

input
```
integrate(F^(c*(b*x+a)^n)*(b*x+a)^(-1+1/2*n),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/2*n - 1)*F^((b*x + a)^n*c), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log(F)}\sqrt{(bx+a)^n}\right)}{\sqrt{-c\log(F)}bn}$$

input
```
integrate(F^(c*(b*x+a)^n)*(b*x+a)^(-1+1/2*n),x, algorithm="giac")
```

output
```
-sqrt(pi)*erf(-sqrt(-c*log(F))*sqrt((b*x + a)^n))/(sqrt(-c*log(F))*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}\,\sqrt{\ln(F)}\,(a+b\,x)^{n/2}\,1\mathrm{i}\right)\,1\mathrm{i}}{b\,\sqrt{c}\,n\,\sqrt{\ln(F)}}$$

input
```
int(F^(c*(a + b*x)^n)*(a + b*x)^(n/2 - 1),x)
```

output
```
-(pi^(1/2)*erf(c^(1/2)*log(F)^(1/2)*(a + b*x)^(n/2)*1i)*1i)/(b*c^(1/2)*n*l
og(F)^(1/2))
```

Reduce [F]

$$\int F^{c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = \int \frac{f^{(bx+a)^n c}(bx+a)^{\frac{n}{2}}}{bx+a}dx$$

input

```
int(F^(c*(b*x+a)^n)*(b*x+a)^(-1+1/2*n),x)
```

output

```
int((f**((a + b*x)**n*c)*(a + b*x)**(n/2))/(a + b*x),x)
```

3.315 $\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 47

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{c}(a+bx)^{n/2}\sqrt{\log(F)}\right)}{b\sqrt{c}n\sqrt{\log(F)}}$$

output
```
Pi^(1/2)*erf(c^(1/2)*(b*x+a)^(1/2*n)*ln(F)^(1/2))/b/c^(1/2)/n/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{c}(a+bx)^{n/2}\sqrt{\log(F)}\right)}{b\sqrt{c}n\sqrt{\log(F)}}$$

input
```
Integrate[(a + b*x)^(-1 + n/2)/F^(c*(a + b*x)^n),x]
```

output
```
(Sqrt[Pi]*Erf[Sqrt[c]*(a + b*x)^(n/2)*Sqrt[Log[F]]])/(b*Sqrt[c]*n*Sqrt[Log
[F]])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2640, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^{\frac{n}{2}-1} F^{-c(a+bx)^n} \, dx$$

$$\Big\downarrow \text{ 2640}$$

$$\frac{2\int F^{-c(a+bx)^n}d(a+bx)^{n/2}}{bn}$$

$$\Big\downarrow \text{ 2634}$$

$$\frac{\sqrt{\pi}\text{erf}\left(\sqrt{c}\sqrt{\log(F)}(a+bx)^{n/2}\right)}{b\sqrt{c}n\sqrt{\log(F)}}$$

input
```
Int[(a + b*x)^(-1 + n/2)/F^(c*(a + b*x)^n),x]
```

output
```
(Sqrt[Pi]*Erf[Sqrt[c]*(a + b*x)^(n/2)*Sqrt[Log[F]]])/(b*Sqrt[c]*n*Sqrt[Log
[F]])
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{\ln(F)c}\,(bx+a)^{\frac{n}{2}}\right)}{nb\sqrt{\ln(F)c}}$	34

input
```
int((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x,method=_RETURNVERBOSE)
```

output
```
1/n/b*Pi^(1/2)/(ln(F)*c)^(1/2)*erf((ln(F)*c)^(1/2)*(b*x+a)^(1/2*n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}\sqrt{c\log(F)}\,\mathrm{erf}\left((bx+a)\sqrt{c\log(F)}(bx+a)^{\frac{1}{2}n-1}\right)}{bcn\log(F)}$$

input
```
integrate((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x, algorithm="fricas")
```

output
```
sqrt(pi)*sqrt(c*log(F))*erf((b*x + a)*sqrt(c*log(F))*(b*x + a)^(1/2*n - 1)
)/(b*c*n*log(F))
```

Sympy [F]

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\int F^{-c(a+bx)^n}(a+bx)^{\frac{n}{2}-1}\,dx$$

input
```
integrate((b*x+a)**(-1+1/2*n)/(F**(c*(b*x+a)**n)),x)
```

output
```
Integral((a + b*x)**(n/2 - 1)/F**(c*(a + b*x)**n), x)
```

Maxima [F]

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\int\frac{(bx+a)^{\frac{1}{2}n-1}}{F^{(bx+a)^nc}}\,dx$$

input
```
integrate((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x, algorithm="maxima")
```

output
```
integrate((b*x + a)^(1/2*n - 1)/F^((b*x + a)^n*c), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{c\log(F)}\sqrt{(bx+a)^n}\right)}{\sqrt{c\log(F)}bn}$$

input
```
integrate((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x, algorithm="giac")
```

output
```
-sqrt(pi)*erf(-sqrt(c*log(F))*sqrt((b*x + a)^n))/(sqrt(c*log(F))*b*n)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}\,\sqrt{\ln(F)}\,(a+b\,x)^{n/2}\right)}{b\,\sqrt{c}\,n\,\sqrt{\ln(F)}}$$

input
```
int((a + b*x)^(n/2 - 1)/F^(c*(a + b*x)^n),x)
```

output
```
(pi^(1/2)*erf(c^(1/2)*log(F)^(1/2)*(a + b*x)^(n/2)))/(b*c^(1/2)*n*log(F)^(
1/2))
```

Reduce [F]

$$\int F^{-c(a+bx)^n}(a+bx)^{-1+\frac{n}{2}}\,dx = \int \frac{(bx+a)^{\frac{n}{2}}}{f^{(bx+a)^n c}a + f^{(bx+a)^n c}bx}dx$$

input
```
int((b*x+a)^(-1+1/2*n)/(F^(c*(b*x+a)^n)),x)
```

output
```
int((a + b*x)**(n/2)/(f**((a + b*x)**n*c)*a + f**((a + b*x)**n*c)*b*x),x)
```

3.316 $\int F^{a+b(c+dx)^2}(e+fx)^5\,dx$

Optimal result

Integrand size = 21, antiderivative size = 518

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx = \frac{f^5F^{a+b(c+dx)^2}}{b^3d^6\log^3(F)} + \frac{15f^4(de-cf)F^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{8b^{5/2}d^6\log^{\frac{5}{2}}(F)} - \frac{5f^3(de-cf)^2F^{a+b(c+dx)^2}}{b^2d^6\log^2(F)} - \frac{15f^4(de-cf)F^{a+b(c+dx)^2}(c+dx)}{4b^2d^6\log^2(F)} - \frac{f^5F^{a+b(c+dx)^2}(c+dx)^2}{b^2d^6\log^2(F)} - \frac{5f^2(de-cf)^3F^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2b^{3/2}d^6\log^{\frac{3}{2}}(F)} + \frac{5f(de-cf)^4F^{a+b(c+dx)^2}}{2bd^6\log(F)} + \frac{5f^2(de-cf)^3F^{a+b(c+dx)^2}(c+dx)}{bd^6\log(F)} + \frac{5f^3(de-cf)^2F^{a+b(c+dx)^2}(c+dx)^2}{bd^6\log(F)} + \frac{5f^4(de-cf)F^{a+b(c+dx)^2}(c+dx)^3}{2bd^6\log(F)} + \frac{f^5F^{a+b(c+dx)^2}(c+dx)^4}{2bd^6\log(F)} + \frac{(de-cf)^5F^a\sqrt{\pi}\,\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d^6\sqrt{\log(F)}}$$

output
```
f^5*F^(a+b*(d*x+c)^2)/b^3/d^6/ln(F)^3+15/8*f^4*(-c*f+d*e)*F^a*Pi^(1/2)*erf
i(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(5/2)/d^6/ln(F)^(5/2)-5*f^3*(-c*f+d*e)^2*
F^(a+b*(d*x+c)^2)/b^2/d^6/ln(F)^2-15/4*f^4*(-c*f+d*e)*F^(a+b*(d*x+c)^2)*(d
*x+c)/b^2/d^6/ln(F)^2-f^5*F^(a+b*(d*x+c)^2)*(d*x+c)^2/b^2/d^6/ln(F)^2-5/2*
f^2*(-c*f+d*e)^3*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(3/2)/d^
6/ln(F)^(3/2)+5/2*f*(-c*f+d*e)^4*F^(a+b*(d*x+c)^2)/b/d^6/ln(F)+5*f^2*(-c*f
+d*e)^3*F^(a+b*(d*x+c)^2)*(d*x+c)/b/d^6/ln(F)+5*f^3*(-c*f+d*e)^2*F^(a+b*(d
*x+c)^2)*(d*x+c)^2/b/d^6/ln(F)+5/2*f^4*(-c*f+d*e)*F^(a+b*(d*x+c)^2)*(d*x+c
)^3/b/d^6/ln(F)+1/2*f^5*F^(a+b*(d*x+c)^2)*(d*x+c)^4/b/d^6/ln(F)+1/2*(-c*f+
d*e)^5*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(1/2)/d^6/ln(F)^(1
/2)
```

Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.80

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx$$
$$= \frac{F^a\left(-40f^3(de-cf)^2F^{b(c+dx)^2}+\frac{15f^4(-de+cf)\left(-\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)+2\sqrt{b}F^{b(c+dx)^2}(c+dx)\sqrt{\log(F)}\right)}{\sqrt{b}\sqrt{\log(F)}}+20\sqrt{b}f^2\right.}{}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(e + f*x)^5,x]
```

output
```
(F^a*(-40*f^3*(d*e - c*f)^2*F^(b*(c + d*x)^2) + (15*f^4*(-(d*e) + c*f)*(-(
Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]) + 2*Sqrt[b]*F^(b*(c + d*x)^
2)*(c + d*x)*Sqrt[Log[F]]))/(Sqrt[b]*Sqrt[Log[F]]) + 20*Sqrt[b]*f^2*(-(d*e
) + c*f)^3*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]] + 20
*b*f*(d*e - c*f)^4*F^(b*(c + d*x)^2)*Log[F] + 40*b*f^2*(d*e - c*f)^3*F^(b*
(c + d*x)^2)*(c + d*x)*Log[F] + 40*b*f^3*(d*e - c*f)^2*F^(b*(c + d*x)^2)*(
c + d*x)^2*Log[F] + 20*b*f^4*(d*e - c*f)*F^(b*(c + d*x)^2)*(c + d*x)^3*Log
[F] + 4*b*f^5*F^(b*(c + d*x)^2)*(c + d*x)^4*Log[F] + 4*b^(3/2)*(d*e - c*f)
^5*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Log[F]^(3/2) + (8*f^5*F^(
b*(c + d*x)^2)*(1 - b*(c + d*x)^2*Log[F]))/(b*Log[F])))/(8*b^2*d^6*Log[F]^
2)
```

Rubi [A] (verified)

Time = 1.90 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^5 F^{a+b(c+dx)^2} \, dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{5f^4(c+dx)^4(de-cf)F^{a+b(c+dx)^2}}{d^5}+\frac{10f^3(c+dx)^3(de-cf)^2F^{a+b(c+dx)^2}}{d^5}+\frac{10f^2(c+dx)^2(de-cf)^3F^{a+b(}}{d^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{15\sqrt{\pi}f^4F^a(de-cf)\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{8b^{5/2}d^6\log^{\frac{5}{2}}(F)}-\frac{5\sqrt{\pi}f^2F^a(de-cf)^3\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2b^{3/2}d^6\log^{\frac{3}{2}}(F)}+\frac{f^5F^{a+b(c+dx)^2}}{b^3d^6\log^3(F)}-\frac{15f^4(c+dx)(de-cf)F^{a+b(c+dx)^2}}{4b^2d^6\log^2(F)}-\frac{5f^3(de-cf)^2F^{a+b(c+dx)^2}}{b^2d^6\log^2(F)}-\frac{f^5(c+dx)^2F^{a+b(c+dx)^2}}{b^2d^6\log^2(F)}+\frac{\sqrt{\pi}F^a(de-cf)^5\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^6\sqrt{\log(F)}}+\frac{5f^4(c+dx)^3(de-cf)F^{a+b(c+dx)^2}}{2bd^6\log(F)}+\frac{5f^3(c+dx)^2(de-cf)^2F^{a+b(c+dx)^2}}{bd^6\log(F)}+\frac{5f^2(c+dx)(de-cf)^3F^{a+b(c+dx)^2}}{bd^6\log(F)}+\frac{5f(de-cf)^4F^{a+b(c+dx)^2}}{2bd^6\log(F)}+\frac{f^5(c+dx)^4F^{a+b(c+dx)^2}}{2bd^6\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(e + f*x)^5,x]
```

output
```
(f^5*F^(a + b*(c + d*x)^2))/(b^3*d^6*Log[F]^3) + (15*f^4*(d*e - c*f)*F^a*S
qrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(8*b^(5/2)*d^6*Log[F]^(5/2))
 - (5*f^3*(d*e - c*f)^2*F^(a + b*(c + d*x)^2))/(b^2*d^6*Log[F]^2) - (15*f^
4*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x))/(4*b^2*d^6*Log[F]^2) - (f^5
*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b^2*d^6*Log[F]^2) - (5*f^2*(d*e - c*f
)^3*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*b^(3/2)*d^6*Log[
F]^(3/2)) + (5*f*(d*e - c*f)^4*F^(a + b*(c + d*x)^2))/(2*b*d^6*Log[F]) + (
5*f^2*(d*e - c*f)^3*F^(a + b*(c + d*x)^2)*(c + d*x))/(b*d^6*Log[F]) + (5*f
^3*(d*e - c*f)^2*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b*d^6*Log[F]) + (5*f^
4*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x)^3)/(2*b*d^6*Log[F]) + (f^5*F
^(a + b*(c + d*x)^2)*(c + d*x)^4)/(2*b*d^6*Log[F]) + ((d*e - c*f)^5*F^a*Sq
rt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^6*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1836 vs. $2(474) = 948$.

Time = 0.30 (sec) , antiderivative size = 1837, normalized size of antiderivative = 3.55

method	result	size
risch	Expression too large to display	1837

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^5,x,method=_RETURNVERBOSE)
```

output
```
5*f^3*e^2*F^a*F^(b*c^2)/ln(F)/b/d^2*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+5*f^3*
e^2*F^a*F^(b*c^2)*c^2/d^4/ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)+5*f^3*e^2*F^
a*F^(b*c^2)*c^3/d^4*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))
^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+5*f^2*e^3*F^a*F^(b*c^2)/ln(F)/b/d^2*x
*F^(b*d^2*x^2)*F^(2*b*c*d*x)-5*f^2*e^3*F^a*F^(b*c^2)*c/d^3/ln(F)/b*F^(b*d^
2*x^2)*F^(2*b*c*d*x)-5*f^2*e^3*F^a*F^(b*c^2)*c^2/d^3*Pi^(1/2)*F^(-b*c^2)/(
-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))-5/2*
f^5*F^a*F^(b*c^2)*c^3/d^6/ln(F)/b*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf
(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+15/8*f^5*F^a*F^(b*c^2)*
c/d^6/b^2/ln(F)^2*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(
1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))-5/2*f^4*e*F^a*F^(b*c^2)*c/d^3/ln(F)/b*x
^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+5/2*f^4*e*F^a*F^(b*c^2)*c^2/d^4/ln(F)/b*x*F
^(b*d^2*x^2)*F^(2*b*c*d*x)-15/8*f^4*e*F^a*F^(b*c^2)/d^5/b^2/ln(F)^2*Pi^(1/
2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(
F))^(1/2))-5*f^3*e^2*F^a*F^(b*c^2)*c/d^3/ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*
d*x)+5/2*f^2*e^3*F^a*F^(b*c^2)/ln(F)/b/d^3*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^
(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))-1/2*F^(b*c^2)*
F^a*e^5*Pi^(1/2)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b
*c*ln(F)/(-b*ln(F))^(1/2))+5/2*F^(b*c^2)*F^a*e^4*f*c/d^2*Pi^(1/2)*F^(-b*c^
2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.03

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx =$$

$$-\frac{\sqrt{\pi}\big(15\,def^4-15\,cf^5+4\,(b^2d^5e^5-5\,b^2cd^4e^4f+10\,b^2c^2d^3e^3f^2-10\,b^2c^3d^2e^2f^3+5\,b^2c^4def^4-b^2c^5f^5)}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^5,x, algorithm="fricas")
```

output
```
-1/8*(sqrt(pi)*(15*d*e*f^4 - 15*c*f^5 + 4*(b^2*d^5*e^5 - 5*b^2*c*d^4*e^4*f
 + 10*b^2*c^2*d^3*e^3*f^2 - 10*b^2*c^3*d^2*e^2*f^3 + 5*b^2*c^4*d*e*f^4 - b
^2*c^5*f^5)*log(F)^2 - 20*(b*d^3*e^3*f^2 - 3*b*c*d^2*e^2*f^3 + 3*b*c^2*d*e
*f^4 - b*c^3*f^5)*log(F))*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*
(d*x + c)/d) - 2*(4*d*f^5 + 2*(b^2*d^5*f^5*x^4 + 5*b^2*d^5*e^4*f - 10*b^2*
c*d^4*e^3*f^2 + 10*b^2*c^2*d^3*e^2*f^3 - 5*b^2*c^3*d^2*e*f^4 + b^2*c^4*d*f
^5 + (5*b^2*d^5*e*f^4 - b^2*c*d^4*f^5)*x^3 + (10*b^2*d^5*e^2*f^3 - 5*b^2*c
*d^4*e*f^4 + b^2*c^2*d^3*f^5)*x^2 + (10*b^2*d^5*e^3*f^2 - 10*b^2*c*d^4*e^2
*f^3 + 5*b^2*c^2*d^3*e*f^4 - b^2*c^3*d^2*f^5)*x)*log(F)^2 - (4*b*d^3*f^5*x
^2 + 20*b*d^3*e^2*f^3 - 25*b*c*d^2*e*f^4 + 9*b*c^2*d*f^5 + (15*b*d^3*e*f^4
 - 7*b*c*d^2*f^5)*x)*log(F))*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^3*d
^7*log(F)^3)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx=\int F^{a+b(c+dx)^2}(e+fx)^5\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(f*x+e)**5,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(e + f*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. $2(474) = 948$.

Time = 0.58 (sec) , antiderivative size = 1456, normalized size of antiderivative = 2.81

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx=\text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^5,x, algorithm="maxima")
```

output
```
-5/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*e^4*f/(sqrt(b*log(F))*d) + 5*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^
2*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F)
)^(5/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b
*c*d)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)
^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5
/2)*d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*e^3*f^2/(sqrt(b*
log(F))*d) - 5*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b
*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*
x + b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*
log(F)^3/((b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b
*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*
x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*lo
g(F)/(b*d^2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*e^2*f^3/(sqrt(b*log(F))
*d) + 5/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)
^2*log(F)/(b*d^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b
*c*d)^2*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F
)^4/((b*log(F))^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -...
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.36

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^5,x, algorithm="giac")
```

output

```
-1/8*(sqrt(pi)*(4*b^2*d^5*e^5*log(F)^2 - 20*b^2*c*d^4*e^4*f*log(F)^2 + 40*
b^2*c^2*d^3*e^3*f^2*log(F)^2 - 40*b^2*c^3*d^2*e^2*f^3*log(F)^2 + 20*b^2*c^
4*d*e*f^4*log(F)^2 - 4*b^2*c^5*f^5*log(F)^2 - 20*b*d^3*e^3*f^2*log(F) + 60
*b*c*d^2*e^2*f^3*log(F) - 60*b*c^2*d*e*f^4*log(F) + 20*b*c^3*f^5*log(F) +
15*d*e*f^4 - 15*c*f^5)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(
F))*b^2*d*log(F)^2) - 2*(2*b^2*d^4*f^5*(x + c/d)^4*log(F)^2 + 10*b^2*d^4*e
*f^4*(x + c/d)^3*log(F)^2 - 10*b^2*c*d^3*f^5*(x + c/d)^3*log(F)^2 + 20*b^2
*d^4*e^2*f^3*(x + c/d)^2*log(F)^2 - 40*b^2*c*d^3*e*f^4*(x + c/d)^2*log(F)^
2 + 20*b^2*c^2*d^2*f^5*(x + c/d)^2*log(F)^2 + 20*b^2*d^4*e^3*f^2*(x + c/d)
*log(F)^2 - 60*b^2*c*d^3*e^2*f^3*(x + c/d)*log(F)^2 + 60*b^2*c^2*d^2*e*f^4
*(x + c/d)*log(F)^2 - 20*b^2*c^3*d*f^5*(x + c/d)*log(F)^2 + 10*b^2*d^4*e^4
*f*log(F)^2 - 40*b^2*c*d^3*e^3*f^2*log(F)^2 + 60*b^2*c^2*d^2*e^2*f^3*log(F
)^2 - 40*b^2*c^3*d*e*f^4*log(F)^2 + 10*b^2*c^4*f^5*log(F)^2 - 4*b*d^2*f^5*
(x + c/d)^2*log(F) - 15*b*d^2*e*f^4*(x + c/d)*log(F) + 15*b*c*d*f^5*(x + c
/d)*log(F) - 20*b*d^2*e^2*f^3*log(F) + 40*b*c*d*e*f^4*log(F) - 20*b*c^2*f^
5*log(F) + 4*f^5)*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) +
a*log(F))/(b^3*d*log(F)^3))/d^5
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.38

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx$$

$$= \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\left(f^5+\frac{\ln(F)^2\,\left(2\,F^a\,b^2\,c^4\,f^5+10\,F^a\,b^2\,d^4\,e^4\,f+20\,F^a\,b^2\,c^2\,d^2\,e^2\,f^3-10\,F^a\,b^2\,c^3\,d\,e\,f^4-20\,F^a\,b^2\,c\,d^3\,e^3\,f^2\right)}{4\,F^a} - \cdots\right.}{b^3\,d^6\,\ln\left(F\right)^3}$$

$$-\operatorname{erfi}\left(\frac{b\,x\,\ln\left(F\right)\,d^2+b\,c\,\ln\left(F\right)\,d}{\sqrt{b\,d^2\,\ln\left(F\right)}}\right)\left(\frac{\frac{F^a\,\sqrt{\pi}\,\left(15\,c\,f^5-15\,d\,e\,f^4\right)}{8\,\sqrt{b\,d^2\,\ln(F)}}-\frac{F^a\,\sqrt{\pi}\,\ln(F)\,\left(20\,b\,c^3\,f^5-60\,b\,c^2\,d\,e\,f^4+60\,b\,c\,d^2\,e^2\,f^3-2\cdots\right.}{8\,\sqrt{b\,d^2\,\ln(F)}}}{b^2\,d^5\,\ln\left(F\right)^2}\right.$$

$$\left.+\frac{F^a\,\sqrt{\pi}\,\left(4\,b^2\,c^5\,f^5-20\,b^2\,c^4\,d\,e\,f^4+40\,b^2\,c^3\,d^2\,e^2\,f^3-40\,b^2\,c^2\,d^3\,e^3\,f^2+20\,b^2\,c\,d^4\,e^4\,f-4\,b^2\,d^5\,e^5\right)}{8\,b^2\,d^5\,\sqrt{b\,d^2\,\ln\left(F\right)}}\right)$$

$$-\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x\left(\ln\left(F\right)\,\left(\frac{b\,c^3\,f^5}{2}-\frac{5\,b\,c^2\,d\,e\,f^4}{2}+5\,b\,c\,d^2\,e^2\,f^3-5\,b\,d^3\,e^3\,f^2\right)-\frac{7\,c\,f^5}{4}+\frac{15\,d\,e\,f^4}{4}\right)}{b^2\,d^5\,\ln\left(F\right)^2}$$

$$+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f^5\,x^4}{2\,b\,d^2\,\ln\left(F\right)}$$

$$-\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x^2\left(f^5-\frac{b\,f^3\,\left(\ln(F)\,c^2\,f^2-5\,\ln(F)\,c\,d\,e\,f+10\,\ln(F)\,d^2\,e^2\right)}{2}\right)}{b^2\,d^4\,\ln\left(F\right)^2}$$

$$-\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f^4\,x^3\,\left(c\,f-5\,d\,e\right)}{2\,b\,d^3\,\ln\left(F\right)}$$

input
```
int(F^(a + b*(c + d*x)^2)*(e + f*x)^5,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(f^5 + (log(F)^2*(2*F^a*b^2*c^4
*f^5 + 10*F^a*b^2*d^4*e^4*f + 20*F^a*b^2*c^2*d^2*e^2*f^3 - 10*F^a*b^2*c^3*
d*e*f^4 - 20*F^a*b^2*c*d^3*e^3*f^2))/(4*F^a) - (log(F)*(9*F^a*b*c^2*f^5 +
20*F^a*b*d^2*e^2*f^3 - 25*F^a*b*c*d*e*f^4))/(4*F^a)))/(b^3*d^6*log(F)^3) -
 erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))*(((F^a*pi^(1/2
)*(15*c*f^5 - 15*d*e*f^4))/(8*(b*d^2*log(F))^(1/2)) - (F^a*pi^(1/2)*log(F)
*(20*b*c^3*f^5 - 20*b*d^3*e^3*f^2 - 60*b*c^2*d*e*f^4 + 60*b*c*d^2*e^2*f^3)
)/(8*(b*d^2*log(F))^(1/2)))/(b^2*d^5*log(F)^2) + (F^a*pi^(1/2)*(4*b^2*c^5*
f^5 - 4*b^2*d^5*e^5 - 40*b^2*c^2*d^3*e^3*f^2 + 40*b^2*c^3*d^2*e^2*f^3 + 20
*b^2*c*d^4*e^4*f - 20*b^2*c^4*d*e*f^4))/(8*b^2*d^5*(b*d^2*log(F))^(1/2)))
- (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x*(log(F)*((b*c^3*f^5)/2 - 5*
b*d^3*e^3*f^2 - (5*b*c^2*d*e*f^4)/2 + 5*b*c*d^2*e^2*f^3) - (7*c*f^5)/4 + (
15*d*e*f^4)/4))/(b^2*d^5*log(F)^2) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c
*d*x)*f^5*x^4)/(2*b*d^2*log(F)) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*
x)*x^2*(f^5 - (b*f^3*(c^2*f^2*log(F) + 10*d^2*e^2*log(F) - 5*c*d*e*f*log(F
)))/2))/(b^2*d^4*log(F)^2) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*f^
4*x^3*(c*f - 5*d*e))/(2*b*d^3*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1510, normalized size of antiderivative = 2.92

$$\int F^{a+b(c+dx)^2}(e+fx)^5\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^5,x)
```

output

```
(f**a*(4*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f)
)))*log(f)**3*b**3*c**5*f**5*i - 20*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*
d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**3*b**3*c**4*d*e*f**4*i + 40*sqrt(pi
)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**3*b*
*3*c**3*d**2*e**2*f**3*i - 40*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)
/(sqrt(b)*sqrt(log(f))))*log(f)**3*b**3*c**2*d**3*e**3*f**2*i + 20*sqrt(pi
)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**3*b*
*3*c*d**4*e**4*f*i - 4*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(
b)*sqrt(log(f))))*log(f)**3*b**3*d**5*e**5*i - 20*sqrt(pi)*erf((log(f)*b*c
*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2*c**3*f**5*i +
60*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*lo
g(f)**2*b**2*c**2*d*e*f**4*i - 60*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*
i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2*c*d**2*e**2*f**3*i + 20*sqrt(p
i)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b
**2*d**3*e**3*f**2*i + 15*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sq
rt(b)*sqrt(log(f))))*log(f)*b*c*f**5*i - 15*sqrt(pi)*erf((log(f)*b*c*i + l
og(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*d*e*f**4*i + 4*f**(b*c**2
+ 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2*c**4*f**5 -
 20*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)**2*b
**2*c**3*d*e*f**4 - 4*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqr...
```

3.317 $\int F^{a+b(c+dx)^2}(e+fx)^4\,dx$

Optimal result

Integrand size = 21, antiderivative size = 389

$$\begin{aligned}\int F^{a+b(c+dx)^2}(e+fx)^4\,dx &= \frac{3f^4F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{8b^{5/2}d^5\log^{\frac{5}{2}}(F)} \\ &\quad - \frac{2f^3(de-cf)F^{a+b(c+dx)^2}}{b^2d^5\log^2(F)} - \frac{3f^4F^{a+b(c+dx)^2}(c+dx)}{4b^2d^5\log^2(F)} \\ &\quad - \frac{3f^2(de-cf)^2F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2b^{3/2}d^5\log^{\frac{3}{2}}(F)} \\ &\quad + \frac{2f(de-cf)^3F^{a+b(c+dx)^2}}{bd^5\log(F)} \\ &\quad + \frac{3f^2(de-cf)^2F^{a+b(c+dx)^2}(c+dx)}{bd^5\log(F)} \\ &\quad + \frac{2f^3(de-cf)F^{a+b(c+dx)^2}(c+dx)^2}{bd^5\log(F)} \\ &\quad + \frac{f^4F^{a+b(c+dx)^2}(c+dx)^3}{2bd^5\log(F)} \\ &\quad + \frac{(de-cf)^4F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d^5\sqrt{\log(F)}}\end{aligned}$$

output
```
3/8*f^4*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(5/2)/d^5/ln(F)^(
5/2)-2*f^3*(-c*f+d*e)*F^(a+b*(d*x+c)^2)/b^2/d^5/ln(F)^2-3/4*f^4*F^(a+b*(d*
x+c)^2)*(d*x+c)/b^2/d^5/ln(F)^2-3/2*f^2*(-c*f+d*e)^2*F^a*Pi^(1/2)*erfi(b^(
1/2)*(d*x+c)*ln(F)^(1/2))/b^(3/2)/d^5/ln(F)^(3/2)+2*f*(-c*f+d*e)^3*F^(a+b*
(d*x+c)^2)/b/d^5/ln(F)+3*f^2*(-c*f+d*e)^2*F^(a+b*(d*x+c)^2)*(d*x+c)/b/d^5/
ln(F)+2*f^3*(-c*f+d*e)*F^(a+b*(d*x+c)^2)*(d*x+c)^2/b/d^5/ln(F)+1/2*f^4*F^(
a+b*(d*x+c)^2)*(d*x+c)^3/b/d^5/ln(F)+1/2*(-c*f+d*e)^4*F^a*Pi^(1/2)*erfi(b^
(1/2)*(d*x+c)*ln(F)^(1/2))/b^(1/2)/d^5/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.57

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx$$
$$= \frac{F^a\left(2\sqrt{b}fF^{b(c+dx)^2}\sqrt{\log(F)}(f^2(-8de+5cf-3dfx)+2b(-c^3f^3+c^2df^2(4e+fx)-cd^2f(6e^2+4efx\right.}{}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(e + f*x)^4,x]
```

output
```
(F^a*(2*Sqrt[b]*f*F^(b*(c + d*x)^2)*Sqrt[Log[F]]*(f^2*(-8*d*e + 5*c*f - 3*
d*f*x) + 2*b*(-(c^3*f^3) + c^2*d*f^2*(4*e + f*x) - c*d^2*f*(6*e^2 + 4*e*f*
x + f^2*x^2) + d^3*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))*Log[F]) +
Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*(3*f^4 - 12*b*f^2*(d*e - c*f
)^2*Log[F] + 4*b^2*(d*e - c*f)^4*Log[F]^2)))/(8*b^(5/2)*d^5*Log[F]^(5/2))
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^4 F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 2656}$$

$$\int \left(\frac{4f^3(c+dx)^3(de-cf)F^{a+b(c+dx)^2}}{d^4}+\frac{6f^2(c+dx)^2(de-cf)^2F^{a+b(c+dx)^2}}{d^4}+\frac{(de-cf)^4F^{a+b(c+dx)^2}}{d^4}+\frac{4f(c+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3\sqrt{\pi}f^2F^a(de-cf)^2\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2b^{3/2}d^5\log^{\frac{3}{2}}(F)}+\frac{3\sqrt{\pi}f^4F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{8b^{5/2}d^5\log^{\frac{5}{2}}(F)}-\frac{2f^3(de-cf)F^{a+b(c+dx)^2}}{b^2d^5\log^2(F)}-\frac{3f^4(c+dx)F^{a+b(c+dx)^2}}{4b^2d^5\log^2(F)}+\frac{\sqrt{\pi}F^a(de-cf)^4\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^5\sqrt{\log(F)}}+\frac{2f^3(c+dx)^2(de-cf)F^{a+b(c+dx)^2}}{bd^5\log(F)}+\frac{3f^2(c+dx)(de-cf)^2F^{a+b(c+dx)^2}}{bd^5\log(F)}+\frac{2f(de-cf)^3F^{a+b(c+dx)^2}}{bd^5\log(F)}+\frac{f^4(c+dx)^3F^{a+b(c+dx)^2}}{2bd^5\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(e + f*x)^4,x]
```

output
```
(3*f^4*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(8*b^(5/2)*d^5*L
og[F]^(5/2)) - (2*f^3*(d*e - c*f)*F^(a + b*(c + d*x)^2))/(b^2*d^5*Log[F]^2
) - (3*f^4*F^(a + b*(c + d*x)^2)*(c + d*x))/(4*b^2*d^5*Log[F]^2) - (3*f^2*
(d*e - c*f)^2*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*b^(3/2
)*d^5*Log[F]^(3/2)) + (2*f*(d*e - c*f)^3*F^(a + b*(c + d*x)^2))/(b*d^5*Log
[F]) + (3*f^2*(d*e - c*f)^2*F^(a + b*(c + d*x)^2)*(c + d*x))/(b*d^5*Log[F]
) + (2*f^3*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x)^2)/(b*d^5*Log[F]) +
 (f^4*F^(a + b*(c + d*x)^2)*(c + d*x)^3)/(2*b*d^5*Log[F]) + ((d*e - c*f)^4
*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^5*Sqrt[Lo
g[F]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1197 vs. $2(349) = 698$.

Time = 0.20 (sec) , antiderivative size = 1198, normalized size of antiderivative = 3.08

method	result	size
risch	Expression too large to display	1198

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^(b*c^2)*F^a*e^4*Pi^(1/2)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*l
n(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+1/2*f^4*F^a*F^(b*c^2)/ln(F)/b/d^
2*x^3*F^(b*d^2*x^2)*F^(2*b*c*d*x)-1/2*f^4*F^a*F^(b*c^2)*c/d^3/ln(F)/b*x^2*
F^(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*f^4*F^a*F^(b*c^2)*c^2/d^4/ln(F)/b*x*F^(b*d
^2*x^2)*F^(2*b*c*d*x)-1/2*f^4*F^a*F^(b*c^2)*c^3/d^5/ln(F)/b*F^(b*d^2*x^2)*
F^(2*b*c*d*x)-1/2*f^4*F^a*F^(b*c^2)*c^4/d^5*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))
^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+3/2*f^4*F^a*F
^(b*c^2)*c^2/d^5/ln(F)/b*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*l
n(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+5/4*f^4*F^a*F^(b*c^2)*c/d^5/b^2/
ln(F)^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)-3/4*f^4*F^a*F^(b*c^2)/d^4/b^2/ln(F)^2*
x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-3/8*f^4*F^a*F^(b*c^2)/d^5/b^2/ln(F)^2*Pi^(1/
2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(
F))^(1/2))+2*f^3*e*F^a*F^(b*c^2)/ln(F)/b/d^2*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*
x)-2*f^3*e*F^a*F^(b*c^2)*c/d^3/ln(F)/b*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)+2*f^3
*e*F^a*F^(b*c^2)*c^2/d^4/ln(F)/b*F^(b*d^2*x^2)*F^(2*b*c*d*x)+2*f^3*e*F^a*F
^(b*c^2)*c^3/d^4*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1
/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))-3*f^3*e*F^a*F^(b*c^2)*c/d^4/ln(F)/b*Pi^(
1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*l
n(F))^(1/2))-2*f^3*e*F^a*F^(b*c^2)/d^4/b^2/ln(F)^2*F^(b*d^2*x^2)*F^(2*b*c*
d*x)+3*f^2*e^2*F^a*F^(b*c^2)/ln(F)/b/d^2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.94

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx =$$

$$-\frac{\sqrt{\pi}\left(3\,f^4+4\,(b^2d^4e^4-4\,b^2cd^3e^3f+6\,b^2c^2d^2e^2f^2-4\,b^2c^3def^3+b^2c^4f^4)\log\left(F\right)^2-12\,(bd^2e^2f^2-2\,bc\cdots\right.}{\cdots}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^4,x, algorithm="fricas")
```

output
```
-1/8*(sqrt(pi)*(3*f^4 + 4*(b^2*d^4*e^4 - 4*b^2*c*d^3*e^3*f + 6*b^2*c^2*d^2
*e^2*f^2 - 4*b^2*c^3*d*e*f^3 + b^2*c^4*f^4)*log(F)^2 - 12*(b*d^2*e^2*f^2 -
 2*b*c*d*e*f^3 + b*c^2*f^4)*log(F))*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^
2*log(F))*(d*x + c)/d) - 2*(2*(b^2*d^4*f^4*x^3 + 4*b^2*d^4*e^3*f - 6*b^2*c
*d^3*e^2*f^2 + 4*b^2*c^2*d^2*e*f^3 - b^2*c^3*d*f^4 + (4*b^2*d^4*e*f^3 - b^
2*c*d^3*f^4)*x^2 + (6*b^2*d^4*e^2*f^2 - 4*b^2*c*d^3*e*f^3 + b^2*c^2*d^2*f^
4)*x)*log(F)^2 - (3*b*d^2*f^4*x + 8*b*d^2*e*f^3 - 5*b*c*d*f^4)*log(F))*F^(
b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a))/(b^3*d^6*log(F)^3)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx=\int F^{a+b(c+dx)^2}(e+fx)^4\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(f*x+e)**4,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(e + f*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(349) = 698$.

Time = 0.43 (sec) , antiderivative size = 1052, normalized size of antiderivative = 2.70

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^4,x, algorithm="maxima")
```

output
```
-2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(
b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log
(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)
*d))*F^a*e^3*f/(sqrt(b*log(F))*d) + 3*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*
(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^
(5/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c
*d)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3
*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2
)*d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*e^2*f^2/(sqrt(b*lo
g(F))*d) - 2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x + b*c
*d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^2*x
+ b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^2*lo
g(F)^3/((b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -(b*d
^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^2*x
+ b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*log(
F)/(b*d^2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*e*f^3/(sqrt(b*log(F))*d)
+ 1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^4*c^4*(erf(sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - 1)*log(F)^5/((b*log(F))^(9/2)*d^5*sqrt(-(b*d^2*x + b*c*d
)^2*log(F)/(b*d^2))) - 4*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^4*c^3*log(F)^4/
((b*log(F))^(9/2)*d^4) - 6*(b*d^2*x + b*c*d)^3*b^2*c^2*gamma(3/2, -(b*d...
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.07

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx =$$

$$-\frac{\frac{\sqrt{\pi}\left(4\,b^2d^4e^4\log(F)^2-16\,b^2cd^3e^3f\log(F)^2+24\,b^2c^2d^2e^2f^2\log(F)^2-16\,b^2c^3def^3\log(F)^2+4\,b^2c^4f^4\log(F)^2-12\,bd^2e^2f^2\log(F)+24\,bcdef^3\right.}{\sqrt{-b\log(F)}b^2d\log(F)^2}}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^4,x, algorithm="giac")
```

output
```
-1/8*(sqrt(pi)*(4*b^2*d^4*e^4*log(F)^2 - 16*b^2*c*d^3*e^3*f*log(F)^2 + 24*
b^2*c^2*d^2*e^2*f^2*log(F)^2 - 16*b^2*c^3*d*e*f^3*log(F)^2 + 4*b^2*c^4*f^4
*log(F)^2 - 12*b*d^2*e^2*f^2*log(F) + 24*b*c*d*e*f^3*log(F) - 12*b*c^2*f^4
*log(F) + 3*f^4)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b^
2*d*log(F)^2) - 2*(2*b*d^3*f^4*(x + c/d)^3*log(F) + 8*b*d^3*e*f^3*(x + c/d
)^2*log(F) - 8*b*c*d^2*f^4*(x + c/d)^2*log(F) + 12*b*d^3*e^2*f^2*(x + c/d)
*log(F) - 24*b*c*d^2*e*f^3*(x + c/d)*log(F) + 12*b*c^2*d*f^4*(x + c/d)*log
(F) + 8*b*d^3*e^3*f*log(F) - 24*b*c*d^2*e^2*f^2*log(F) + 24*b*c^2*d*e*f^3*
log(F) - 8*b*c^3*f^4*log(F) - 3*d*f^4*(x + c/d) - 8*d*e*f^3 + 8*c*f^4)*e^(
b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^2*d*log(
F)^2))/d^4
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.33

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx$$

$$= \operatorname{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\left(\frac{\frac{3\,F^a\,f^4\,\sqrt{\pi}}{8\,\sqrt{b\,d^2\,\ln(F)}}-\frac{F^a\,\sqrt{\pi}\,\ln(F)\,\left(12\,b\,c^2\,f^4-24\,b\,c\,d\,e\,f^3+12\,b\,d^2\,e^2\,f^2\right)}{8\,\sqrt{b\,d^2\,\ln(F)}}}{b^2\,d^4\,\ln(F)^2}\right.$$
$$\left.+\frac{F^a\,\sqrt{\pi}\,\left(4\,b^2\,c^4\,f^4-16\,b^2\,c^3\,d\,e\,f^3+24\,b^2\,c^2\,d^2\,e^2\,f^2-16\,b^2\,c\,d^3\,e^3\,f+4\,b^2\,d^4\,e^4\right)}{8\,b^2\,d^4\,\sqrt{b\,d^2\,\ln(F)}}\right)$$
$$+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,\left(\frac{5\,F^a\,c\,f^4-8\,F^a\,d\,e\,f^3}{4\,F^a}-\frac{b\,\left(2\,F^a\,c^3\,f^4\,\ln(F)-8\,F^a\,d^3\,e^3\,f\,\ln(F)-8\,F^a\,c^2\,d\,e\,f^3\,\ln(F)+12\,F^a\,c\,d^2\,e^2\,f^2\,\ln\right.}{4\,F^a}}{b^2\,d^5\,\ln(F)^2}$$
$$+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f^4\,x^3}{2\,b\,d^2\,\ln(F)}$$
$$+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x\,\left(b\,\left(\frac{\ln(F)\,c^2\,f^4}{2}-2\,\ln(F)\,c\,d\,e\,f^3+3\,\ln(F)\,d^2\,e^2\,f^2\right)-\frac{3\,f^4}{4}\right)}{b^2\,d^4\,\ln(F)^2}$$
$$-\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f^3\,x^2\,\left(c\,f-4\,d\,e\right)}{2\,b\,d^3\,\ln(F)}$$

input
```
int(F^(a + b*(c + d*x)^2)*(e + f*x)^4,x)
```

output
```
erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))*(((3*F^a*f^4*pi
^(1/2))/(8*(b*d^2*log(F))^(1/2)) - (F^a*pi^(1/2)*log(F)*(12*b*c^2*f^4 + 12
*b*d^2*e^2*f^2 - 24*b*c*d*e*f^3))/(8*(b*d^2*log(F))^(1/2)))/(b^2*d^4*log(F
)^2) + (F^a*pi^(1/2)*(4*b^2*c^4*f^4 + 4*b^2*d^4*e^4 + 24*b^2*c^2*d^2*e^2*f
^2 - 16*b^2*c*d^3*e^3*f - 16*b^2*c^3*d*e*f^3))/(8*b^2*d^4*(b*d^2*log(F))^(
1/2))) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*((5*F^a*c*f^4 - 8*F^a*
d*e*f^3)/(4*F^a) - (b*(2*F^a*c^3*f^4*log(F) - 8*F^a*d^3*e^3*f*log(F) - 8*F
^a*c^2*d*e*f^3*log(F) + 12*F^a*c*d^2*e^2*f^2*log(F)))/(4*F^a)))/(b^2*d^5*l
og(F)^2) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*f^4*x^3)/(2*b*d^2*lo
g(F)) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x*(b*((c^2*f^4*log(F))/
2 + 3*d^2*e^2*f^2*log(F) - 2*c*d*e*f^3*log(F)) - (3*f^4)/4))/(b^2*d^4*log(
F)^2) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*f^3*x^2*(c*f - 4*d*e))/
(2*b*d^3*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 911, normalized size of antiderivative = 2.34

$$\int F^{a+b(c+dx)^2}(e+fx)^4\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^4,x)
```

output
```
(f**a*( - 4*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log
(f))))*log(f)**2*b**2*c**4*f**4*i + 16*sqrt(pi)*erf((log(f)*b*c*i + log(f)
*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2*c**3*d*e*f**3*i - 24*sqrt
(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2
*b**2*c**2*d**2*e**2*f**2*i + 16*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i
*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2*c*d**3*e**3*f*i - 4*sqrt(pi)*er
f((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2*d
**4*e**4*i + 12*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt
(log(f))))*log(f)*b*c**2*f**4*i - 24*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b
*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*c*d*e*f**3*i + 12*sqrt(pi)*erf((l
og(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*d**2*e**2*f
**2*i - 3*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f
))))*f**4*i - 4*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))
*log(f)*b*c**3*f**4 + 16*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqr
t(log(f))*log(f)*b*c**2*d*e*f**3 + 4*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)
*sqrt(b)*sqrt(log(f))*log(f)*b*c**2*d*f**4*x - 24*f**(b*c**2 + 2*b*c*d*x +
 b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*c*d**2*e**2*f**2 - 16*f**(b*c*
*2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*c*d**2*e*f**3*
x - 4*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*
c*d**2*f**4*x**2 + 16*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqr...
```

3.318 $\int F^{a+b(c+dx)^2}(e+fx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 258

$$\begin{aligned}\int F^{a+b(c+dx)^2}(e+fx)^3\,dx &= -\frac{f^3F^{a+b(c+dx)^2}}{2b^2d^4\log^2(F)}\\&\quad-\frac{3f^2(de-cf)F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2}d^4\log^{\frac{3}{2}}(F)}\\&\quad+\frac{3f(de-cf)^2F^{a+b(c+dx)^2}}{2bd^4\log(F)}\\&\quad+\frac{3f^2(de-cf)F^{a+b(c+dx)^2}(c+dx)}{2bd^4\log(F)}\\&\quad+\frac{f^3F^{a+b(c+dx)^2}(c+dx)^2}{2bd^4\log(F)}\\&\quad+\frac{(de-cf)^3F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d^4\sqrt{\log(F)}}\end{aligned}$$

output
```
-1/2*f^3*F^(a+b*(d*x+c)^2)/b^2/d^4/ln(F)^2-3/4*f^2*(-c*f+d*e)*F^a*Pi^(1/2)
*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(3/2)/d^4/ln(F)^(3/2)+3/2*f*(-c*f+d*e
)^2*F^(a+b*(d*x+c)^2)/b/d^4/ln(F)+3/2*f^2*(-c*f+d*e)*F^(a+b*(d*x+c)^2)*(d*
x+c)/b/d^4/ln(F)+1/2*f^3*F^(a+b*(d*x+c)^2)*(d*x+c)^2/b/d^4/ln(F)+1/2*(-c*f
+d*e)^3*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(1/2)/d^4/ln(F)^(
1/2)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.57

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx$$
$$=\frac{F^a\left(\sqrt{b}(de-cf)\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\sqrt{\log(F)}(-3f^2+2b(de-cf)^2\log(F))+2fF^{b(c+dx)^2}(-\right.}{4b^2d^4\log^2(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(e + f*x)^3,x]
```

output
```
(F^a*(Sqrt[b]*(d*e - c*f)*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*Sq
rt[Log[F]]*(-3*f^2 + 2*b*(d*e - c*f)^2*Log[F]) + 2*f*F^(b*(c + d*x)^2)*(-f
^2 + b*(c^2*f^2 - c*d*f*(3*e + f*x) + d^2*(3*e^2 + 3*e*f*x + f^2*x^2))*Log
[F])))/(4*b^2*d^4*Log[F]^2)
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{3f^2(c+dx)^2(de-cf)F^{a+b(c+dx)^2}}{d^3}+\frac{(de-cf)^3F^{a+b(c+dx)^2}}{d^3}+\frac{3f(c+dx)(de-cf)^2F^{a+b(c+dx)^2}}{d^3}+\frac{f^3(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\pi}f^2F^a(de-cf)\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d^4\log^{\frac{3}{2}}(F)}-\frac{f^3F^{a+b(c+dx)^2}}{2b^2d^4\log^2(F)}+\frac{\sqrt{\pi}F^a(de-cf)^3\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^4\sqrt{\log(F)}}+\frac{3f^2(c+dx)(de-cf)F^{a+b(c+dx)^2}}{2bd^4\log(F)}+\frac{3f(de-cf)^2F^{a+b(c+dx)^2}}{2bd^4\log(F)}+\frac{f^3(c+dx)^2F^{a+b(c+dx)^2}}{2bd^4\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(e + f*x)^3,x]
```

output
```
-1/2*(f^3*F^(a + b*(c + d*x)^2))/(b^2*d^4*Log[F]^2) - (3*f^2*(d*e - c*f)*F
^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(4*b^(3/2)*d^4*Log[F]^(3
/2)) + (3*f*(d*e - c*f)^2*F^(a + b*(c + d*x)^2))/(2*b*d^4*Log[F]) + (3*f^2
*(d*e - c*f)*F^(a + b*(c + d*x)^2)*(c + d*x))/(2*b*d^4*Log[F]) + (f^3*F^(a
 + b*(c + d*x)^2)*(c + d*x)^2)/(2*b*d^4*Log[F]) + ((d*e - c*f)^3*F^a*Sqrt[
Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^4*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(226) = 452.

Time = 0.14 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.74

method	result
risch	$-\frac{F^{b\,c^2}F^a e^3\sqrt{\pi}\,F^{-b\,c^2}\,\text{erf}\left(-d\sqrt{-b\ln(F)}\,x+\frac{bc\ln(F)}{\sqrt{-b\ln(F)}}\right)}{2d\sqrt{-b\ln(F)}}+\frac{f^3F^aF^{b\,c^2}x^2F^{b\,d^2x^2}F^{2bcdx}}{2\ln(F)b\,d^2}-\frac{f^3F^aF^{b\,c^2}cx\,F^{b\,d^2x^2}F^{2bcdx}}{2d^3\ln(F)b}+$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^(b*c^2)*F^a*e^3*Pi^(1/2)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*l
n(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+1/2*f^3*F^a*F^(b*c^2)/ln(F)/b/d^
2*x^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)-1/2*f^3*F^a*F^(b*c^2)*c/d^3/ln(F)/b*x*F^
(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*f^3*F^a*F^(b*c^2)*c^2/d^4/ln(F)/b*F^(b*d^2*x
^2)*F^(2*b*c*d*x)+1/2*f^3*F^a*F^(b*c^2)*c^3/d^4*Pi^(1/2)*F^(-b*c^2)/(-b*ln
(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))-3/4*f^3*F
^a*F^(b*c^2)*c/d^4/ln(F)/b*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b
*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))-1/2*f^3*F^a*F^(b*c^2)/d^4/b^2/
ln(F)^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3/2*f^2*e*F^a*F^(b*c^2)/ln(F)/b/d^2*x*
F^(b*d^2*x^2)*F^(2*b*c*d*x)-3/2*f^2*e*F^a*F^(b*c^2)*c/d^3/ln(F)/b*F^(b*d^2
*x^2)*F^(2*b*c*d*x)-3/2*f^2*e*F^a*F^(b*c^2)*c^2/d^3*Pi^(1/2)*F^(-b*c^2)/(-
b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+3/4*f
^2*e*F^a*F^(b*c^2)/ln(F)/b/d^3*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d
*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+3/2*F^(b*c^2)*F^a*e^2*f/ln
(F)/b/d^2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+3/2*F^(b*c^2)*F^a*e^2*f*c/d^2*Pi^(1/
2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(
F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.81

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx = \frac{\sqrt{\pi}(3\,def^2 - 3\,cf^3 - 2\,(bd^3e^3 - 3\,bcd^2e^2f + 3\,bc^2def^2 - bc^3f^3)\log(F))\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(}}{d}\right.}{4\,b}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(3*d*e*f^2 - 3*c*f^3 - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*
c^2*d*e*f^2 - b*c^3*f^3)*log(F))*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*l
og(F))*(d*x + c)/d) - 2*(d*f^3 - (b*d^3*f^3*x^2 + 3*b*d^3*e^2*f - 3*b*c*d^
2*e*f^2 + b*c^2*d*f^3 + (3*b*d^3*e*f^2 - b*c*d^2*f^3)*x)*log(F))*F^(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + a))/(b^2*d^5*log(F)^2)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx=\int F^{a+b(c+dx)^2}(e+fx)^3\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(f*x+e)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(e + f*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. 2(226) = 452.

Time = 0.31 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.69

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx=\text{Too large to display}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^3,x, algorithm="maxima")
```

output
```
-3/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*e^2*f/(sqrt(b*log(F))*d) + 3/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*
c^2*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(
F))^(5/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x +
 b*c*d)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*
d)^3*gamma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^
(5/2)*d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*e*f^2/(sqrt(b*
log(F))*d) - 1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^3*c^3*(erf(sqrt(-(b*d^2*x +
 b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^4/((b*log(F))^(7/2)*d^4*sqrt(-(b*d^
2*x + b*c*d)^2*log(F)/(b*d^2))) - 3*F^((b*d^2*x + b*c*d)^2/(b*d^2))*b^3*c^
2*log(F)^3/((b*log(F))^(7/2)*d^3) - 3*(b*d^2*x + b*c*d)^3*b*c*gamma(3/2, -
(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^4/((b*log(F))^(7/2)*d^6*(-(b*d^
2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)) + b^2*gamma(2, -(b*d^2*x + b*c*d)^2*
log(F)/(b*d^2))*log(F)^2/((b*log(F))^(7/2)*d^3))*F^a*f^3/(sqrt(b*log(F))*d
) + 1/2*sqrt(pi)*F^(b*c^2 + a)*e^3*erf(sqrt(-b*log(F))*d*x - b*c*log(F)/sq
rt(-b*log(F)))/(sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx =$$

$$-\frac{\frac{\sqrt{\pi}\left(2\,bd^3e^3\log(F)-6\,bcd^2e^2f\log(F)+6\,bc^2def^2\log(F)-2\,bc^3f^3\log(F)-3\,def^2+3\,cf^3\right)F^a\,\mathrm{erf}\left(-\sqrt{-b\log(F)}d\left(x+\frac{c}{d}\right)\right)}{\sqrt{-b\log(F)}bd\log(F)}-\frac{2\left(bd^2f^3\left(x+\frac{c}{d}\right.\right.}{}}{}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^3,x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*(2*b*d^3*e^3*log(F) - 6*b*c*d^2*e^2*f*log(F) + 6*b*c^2*d*e*
f^2*log(F) - 2*b*c^3*f^3*log(F) - 3*d*e*f^2 + 3*c*f^3)*F^a*erf(-sqrt(-b*lo
g(F))*d*(x + c/d))/(sqrt(-b*log(F))*b*d*log(F)) - 2*(b*d^2*f^3*(x + c/d)^2
*log(F) + 3*b*d^2*e*f^2*(x + c/d)*log(F) - 3*b*c*d*f^3*(x + c/d)*log(F) +
3*b*d^2*e^2*f*log(F) - 6*b*c*d*e*f^2*log(F) + 3*b*c^2*f^3*log(F) - f^3)*e^
(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*log(F))/(b^2*d*log
(F)^2))/d^3
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.21

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx = \frac{F^a\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\,\left(-2\,b\,\ln\left(F\right)\,c^3\,f^3+6\,b\,\ln\left(F\right)\,c^2\,d\,e\,f^2-6\,b\,\ln\left(F\right)\,c\,d^2\,e^2\,f+3\,c\,f^3+2\right.}{4\,b\,d^3\,\ln\left(F\right)\,\sqrt{b\,d^2\,\ln\left(F\right)}} - F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,\left(\frac{f^3}{2\,b^2\,d^4\,\ln\left(F\right)^2}-\frac{3\,e^2\,f}{2\,b\,d^2\,\ln\left(F\right)}-\frac{c^2\,f^3}{2\,b\,d^4\,\ln\left(F\right)}+\frac{3\,c\,e\,f^2}{2\,b\,d^3\,\ln\left(F\right)}\right) - \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,x\,\left(c\,f^3-3\,d\,e\,f^2\right)}{2\,b\,d^3\,\ln\left(F\right)}+\frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f^3\,x^2}{2\,b\,d^2\,\ln\left(F\right)}$$

input
```
int(F^(a + b*(c + d*x)^2)*(e + f*x)^3,x)
```

output
```
(F^a*pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))*(
3*c*f^3 - 3*d*e*f^2 - 2*b*c^3*f^3*log(F) + 2*b*d^3*e^3*log(F) - 6*b*c*d^2*
e^2*f*log(F) + 6*b*c^2*d*e*f^2*log(F)))/(4*b*d^3*log(F)*(b*d^2*log(F))^(1/
2)) - F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*(f^3/(2*b^2*d^4*log(F)^2)
- (3*e^2*f)/(2*b*d^2*log(F)) - (c^2*f^3)/(2*b*d^4*log(F)) + (3*c*e*f^2)/(2
*b*d^3*log(F))) - (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*x*(c*f^3 - 3*
d*e*f^2))/(2*b*d^3*log(F)) + (F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*f^
3*x^2)/(2*b*d^2*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.10

$$\int F^{a+b(c+dx)^2}(e+fx)^3\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^3,x)
```

output

```
(f**a*(2*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f)
)))*log(f)**2*b**2*c**3*f**3*i - 6*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d
*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2*c**2*d*e*f**2*i + 6*sqrt(pi)*
erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)**2*b**2
*c*d**2*e**2*f*i - 2*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)
*sqrt(log(f))))*log(f)**2*b**2*d**3*e**3*i - 3*sqrt(pi)*erf((log(f)*b*c*i
+ log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*c*f**3*i + 3*sqrt(pi)*e
rf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*d*e*f*
*2*i + 2*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)
*b*c**2*f**3 - 6*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f)
)*log(f)*b*c*d*e*f**2 - 2*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sq
rt(log(f))*log(f)*b*c*d*f**3*x + 6*f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*s
qrt(b)*sqrt(log(f))*log(f)*b*d**2*e**2*f + 6*f**(b*c**2 + 2*b*c*d*x + b*d*
*2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*d**2*e*f**2*x + 2*f**(b*c**2 + 2*b*
c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*log(f)*b*d**2*f**3*x**2 - 2*f**(
b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*f**3))/(4*sqrt(b)*s
qrt(log(f))*log(f)**2*b**2*d**4)
```

3.319 $\int F^{a+b(c+dx)^2}(e+fx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 170

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx=-\frac{f^2F^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{4b^{3/2}d^3\log^{\frac{3}{2}}(F)}+\frac{f(de-cf)F^{a+b(c+dx)^2}}{bd^3\log(F)}+\frac{f^2F^{a+b(c+dx)^2}(c+dx)}{2bd^3\log(F)}+\frac{(de-cf)^2F^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d^3\sqrt{\log(F)}}$$

output
```
-1/4*f^2*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(3/2)/d^3/ln(F)^
(3/2)+f*(-c*f+d*e)*F^(a+b*(d*x+c)^2)/b/d^3/ln(F)+1/2*f^2*F^(a+b*(d*x+c)^2)
*(d*x+c)/b/d^3/ln(F)+1/2*(-c*f+d*e)^2*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln
(F)^(1/2))/b^(1/2)/d^3/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.62

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx$$
$$=\frac{F^a\left(2\sqrt{b}fF^{b(c+dx)^2}(2de-cf+dfx)\sqrt{\log(F)}+\sqrt{\pi}\text{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)(-f^2+2b(de-cf)^2\log(}{4b^{3/2}d^3\log^{\frac{3}{2}}(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(e + f*x)^2,x]
```

output
```
(F^a*(2*Sqrt[b]*f*F^(b*(c + d*x)^2)*(2*d*e - c*f + d*f*x)*Sqrt[Log[F]] + S
qrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]]*(-f^2 + 2*b*(d*e - c*f)^2*Log
[F])))/(4*b^(3/2)*d^3*Log[F]^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2F^{a+b(c+dx)^2}\,dx$$
$$\Big\downarrow \text{2656}$$
$$\int \left(\frac{(de-cf)^2F^{a+b(c+dx)^2}}{d^2}+\frac{2f(c+dx)(de-cf)F^{a+b(c+dx)^2}}{d^2}+\frac{f^2(c+dx)^2F^{a+b(c+dx)^2}}{d^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\sqrt{\pi}f^2F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{4b^{3/2}d^3\log^{\frac{3}{2}}(F)}+\frac{\sqrt{\pi}F^a(de-cf)^2\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^3\sqrt{\log(F)}}+\frac{f(de-cf)F^{a+b(c+dx)^2}}{bd^3\log(F)}+\frac{f^2(c+dx)F^{a+b(c+dx)^2}}{2bd^3\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(e + f*x)^2,x]
```

output
```
-1/4*(f^2*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(b^(3/2)*d^3*
Log[F]^(3/2)) + (f*(d*e - c*f)*F^(a + b*(c + d*x)^2))/(b*d^3*Log[F]) + (f^
2*F^(a + b*(c + d*x)^2)*(c + d*x))/(2*b*d^3*Log[F]) + ((d*e - c*f)^2*F^a*S
qrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^3*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(144) = 288$.

Time = 0.11 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.26

method	result
risch	$-\frac{F^{b\,c^2} F^a e^2 \sqrt{\pi}\, F^{-b\,c^2} \operatorname{erf}\left(-d\sqrt{-b\ln(F)}\, x+\frac{bc\ln(F)}{\sqrt{-b\ln(F)}}\right)}{2d\sqrt{-b\ln(F)}}+\frac{f^2 F^a F^{b\,c^2} x\, F^{b\,d^2 x^2} F^{2bcdx}}{2\ln(F) b\, d^2}-\frac{f^2 F^a F^{b\,c^2} c\, F^{b\,d^2 x^2} F^{2bcdx}}{2d^3\ln(F) b}-\frac{f^2}{}$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^(b*c^2)*F^a*e^2*Pi^(1/2)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*l
n(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+1/2*f^2*F^a*F^(b*c^2)/ln(F)/b/d^
2*x*F^(b*d^2*x^2)*F^(2*b*c*d*x)-1/2*f^2*F^a*F^(b*c^2)*c/d^3/ln(F)/b*F^(b*d
^2*x^2)*F^(2*b*c*d*x)-1/2*f^2*F^a*F^(b*c^2)*c^2/d^3*Pi^(1/2)*F^(-b*c^2)/(-
b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+1/4*f
^2*F^a*F^(b*c^2)/ln(F)/b/d^3*Pi^(1/2)*F^(-b*c^2)/(-b*ln(F))^(1/2)*erf(-d*(
-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+F^(b*c^2)*F^a*e*f/ln(F)/b/d^
2*F^(b*d^2*x^2)*F^(2*b*c*d*x)+F^(b*c^2)*F^a*e*f*c/d^2*Pi^(1/2)*F^(-b*c^2)/
(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx$$
$$= \frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}(f^2-2\left(bd^2e^2-2\,bcdef+bc^2f^2\right)\log(F))F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)+2\left(bd^2f^2x+2\,b\right.}{4\,b^2d^4\log(F)^2}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*sqrt(-b*d^2*log(F))*(f^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^
2*f^2)*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d) + 2*(b*d^2*f^2*x +
 2*b*d^2*e*f - b*c*d*f^2)*F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*log(F))/(b
^2*d^4*log(F)^2)
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx = \int F^{a+b(c+dx)^2}(e+fx)^2\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(f*x+e)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(e + f*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(144) = 288$.

Time = 0.22 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.48

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx$$
$$= -\frac{\left(\frac{\sqrt{\pi}(bd^2x+bcd)bc\left(\operatorname{erf}\left(\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}\right)-1\right)\log(F)^2}{(b\log(F))^{\frac{3}{2}}d^2\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}} - \frac{F^{\frac{(bd^2x+bcd)^2}{bd^2}}b\log(F)}{(b\log(F))^{\frac{3}{2}}d}\right)F^a ef}{\sqrt{b\log(F)}d}$$
$$+ \frac{\left(\frac{\sqrt{\pi}(bd^2x+bcd)b^2c^2\left(\operatorname{erf}\left(\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}\right)-1\right)\log(F)^3}{(b\log(F))^{\frac{5}{2}}d^3\sqrt{-\frac{(bd^2x+bcd)^2\log(F)}{bd^2}}} - \frac{2F^{\frac{(bd^2x+bcd)^2}{bd^2}}b^2c\log(F)^2}{(b\log(F))^{\frac{5}{2}}d^2} - \frac{(bd^2x+bcd)^3\Gamma\left(\frac{3}{2}, -\frac{(bd^2x+bcd)^2 l}{bd^2}\right.}{(b\log(F))^{\frac{5}{2}}d^5\left(-\frac{(bd^2x+bcd)}{bd^2}\right.}}{2\sqrt{b\log(F)}d}$$
$$+ \frac{\sqrt{\pi}F^{bc^2+a}e^2\operatorname{erf}\left(\sqrt{-b\log(F)}dx - \frac{bc\log(F)}{\sqrt{-b\log(F)}}\right)}{2\sqrt{-b\log(F)}F^{bc^2}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^2,x, algorithm="maxima")
```

output
```
-(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*
d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*log(F
)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/2)*d
))*F^a*e*f/(sqrt(b*log(F))*d) + 1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b^2*c^2*(e
rf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 1)*log(F)^3/((b*log(F))^(5
/2)*d^3*sqrt(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))) - 2*F^((b*d^2*x + b*c*d
)^2/(b*d^2))*b^2*c*log(F)^2/((b*log(F))^(5/2)*d^2) - (b*d^2*x + b*c*d)^3*g
amma(3/2, -(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))*log(F)^3/((b*log(F))^(5/2)*
d^5*(-(b*d^2*x + b*c*d)^2*log(F)/(b*d^2))^(3/2)))*F^a*f^2/(sqrt(b*log(F))*
d) + 1/2*sqrt(pi)*F^(b*c^2 + a)*e^2*erf(sqrt(-b*log(F))*d*x - b*c*log(F)/s
qrt(-b*log(F)))/(sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.89

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx = -\frac{\frac{\sqrt{\pi}\left(2\,bd^2e^2\log(F)-4\,bcdef\log(F)+2\,bc^2f^2\log(F)-f^2\right)F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}d\left(x+\frac{c}{d}\right)\right)}{\sqrt{-b\log(F)}bd\log(F)}-\frac{2\left(df^2\left(x+\frac{c}{d}\right)+2\,def-2\,cf^2\right)e^{\left(bd^2x^2\log(F)+2\,bc\right.}}{bd\log(F)}}{4\,d^2}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*(2*b*d^2*e^2*log(F) - 4*b*c*d*e*f*log(F) + 2*b*c^2*f^2*log(
F) - f^2)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*b*d*log(F
)) - 2*(d*f^2*(x + c/d) + 2*d*e*f - 2*c*f^2)*e^(b*d^2*x^2*log(F) + 2*b*c*d
*x*log(F) + b*c^2*log(F) + a*log(F))/(b*d*log(F)))/d^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.14

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx = \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f^2\,x}{2\,b\,d^2\,\ln(F)} - \frac{F^a\,\sqrt{\pi}\operatorname{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\left(-2\,b\,\ln(F)\,c^2\,f^2+4\,b\,\ln(F)\,c\,d\,e\,f-2\,b\,\ln(F)\,d^2\,e^2+f^2\right)}{4\,b\,d^2\,\ln(F)\,\sqrt{b\,d^2\,\ln(F)}} - F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\left(\frac{c\,f^2}{2\,b\,d^3\,\ln(F)}-\frac{e\,f}{b\,d^2\,\ln(F)}\right)$$

input
```
int(F^(a + b*(c + d*x)^2)*(e + f*x)^2,x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*f^2*x)/(2*b*d^2*log(F)) - (F^a*
pi^(1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))*(f^2 -
 2*b*c^2*f^2*log(F) - 2*b*d^2*e^2*log(F) + 4*b*c*d*e*f*log(F)))/(4*b*d^2*l
og(F)*(b*d^2*log(F))^(1/2)) - F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*((
c*f^2)/(2*b*d^3*log(F)) - (e*f)/(b*d^2*log(F)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.59

$$\int F^{a+b(c+dx)^2}(e+fx)^2\,dx$$
$$= \frac{f^a\left(-2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)\log(f)\,b\,c^2f^2i+4\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)\log(f)\,bcdefi-2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\mathrm{lo}}{}\right.}{}$$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e)^2,x)
```

output
```
(f**a*( - 2*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log
(f))))*log(f)*b*c**2*f**2*i + 4*sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*
x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*c*d*e*f*i - 2*sqrt(pi)*erf((log(f)*b*c
*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*log(f)*b*d**2*e**2*i + sqrt(p
i)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f))))*f**2*i - 2*
f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*c*f**2 + 4*f**(
b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*d*e*f + 2*f**(b*c**
2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(b)*sqrt(log(f))*d*f**2*x))/(4*sqrt(b)*sq
rt(log(f))*log(f)*b*d**3)
```

3.320 $\int F^{a+b(c+dx)^2}(e+fx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx=\frac{fF^{a+b(c+dx)^2}}{2bd^2\log(F)}+\frac{(de-cf)F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d^2\sqrt{\log(F)}}$$

output
```
1/2*f*F^(a+b*(d*x+c)^2)/b/d^2/ln(F)+1/2*(-c*f+d*e)*F^a*Pi^(1/2)*erfi(b^(1/
2)*(d*x+c)*ln(F)^(1/2))/b^(1/2)/d^2/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx =\frac{F^a\left(fF^{b(c+dx)^2}+\sqrt{b}(de-cf)\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)\sqrt{\log(F)}\right)}{2bd^2\log(F)}$$

input
```
Integrate[F^(a + b*(c + d*x)^2)*(e + f*x),x]
```

output
```
(F^a*(f*F^(b*(c + d*x)^2) + Sqrt[b]*(d*e - c*f)*Sqrt[Pi]*Erfi[Sqrt[b]*(c +
 d*x)*Sqrt[Log[F]]]*Sqrt[Log[F]]))/(2*b*d^2*Log[F])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(de-cf)F^{a+b(c+dx)^2}}{d}+\frac{f(c+dx)F^{a+b(c+dx)^2}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}F^a(de-cf)\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d^2\sqrt{\log(F)}}+\frac{fF^{a+b(c+dx)^2}}{2bd^2\log(F)}$$

input
```
Int[F^(a + b*(c + d*x)^2)*(e + f*x),x]
```

output
```
(f*F^(a + b*(c + d*x)^2))/(2*b*d^2*Log[F]) + ((d*e - c*f)*F^a*Sqrt[Pi]*Erf
i[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d^2*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(67) = 134.

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.00

method	result
risch	$-\frac{F^{b\,c^2} F^a e\sqrt{\pi}\, F^{-b\,c^2} \operatorname{erf}\left(-d\sqrt{-b\ln(F)}\,x+\frac{bc\ln(F)}{\sqrt{-b\ln(F)}}\right)}{2d\sqrt{-b\ln(F)}}+\frac{F^{b\,c^2} F^a f\, F^{b\,d^2 x^2} F^{2bcdx}}{2\ln(F) b\,d^2}+\frac{F^{b\,c^2} F^a f c\sqrt{\pi}\, F^{-b\,c^2} \operatorname{erf}\left(-d\sqrt{-b\ln(\right.}{2d^2\sqrt{-b\ln(F)}}$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/2*F^(b*c^2)*F^a*e*Pi^(1/2)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(
F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))+1/2*F^(b*c^2)*F^a*f/ln(F)/b/d^2*F^
(b*d^2*x^2)*F^(2*b*c*d*x)+1/2*F^(b*c^2)*F^a*f*c/d^2*Pi^(1/2)*F^(-b*c^2)/(-
b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx = -\frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}(de-cf)F^a \operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)-F^{bd^2x^2+2\,bcdx+bc^2+a}df}{2\,bd^3\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b*d^2*log(F))*(d*e - c*f)*F^a*erf(sqrt(-b*d^2*log(F))
*(d*x + c)/d) - F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)*d*f)/(b*d^3*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx=\int F^{a+b(c+dx)^2}(e+fx)\;dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)*(f*x+e),x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)*(e + f*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(67)=134$.

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.41

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx$$
$$=-\frac{\left(\frac{\sqrt{\pi}(bd^2x+bcd)bc\left(\operatorname{erf}\left(\sqrt{-\frac{\left(bd^2x+bcd\right)^2\log(F)}{bd^2}}\right)-1\right)\log(F)^2}{(b\log(F))^{\frac{3}{2}}d^2\sqrt{-\frac{\left(bd^2x+bcd\right)^2\log(F)}{bd^2}}}-\frac{F^{\frac{\left(bd^2x+bcd\right)^2}{bd^2}}b\log(F)}{(b\log(F))^{\frac{3}{2}}d}\right)F^af}{2\sqrt{b\log(F)}d}+\frac{\sqrt{\pi}F^{bc^2+a}e\operatorname{erf}\left(\sqrt{-b\log(F)}dx-\frac{bc\log(F)}{\sqrt{-b\log(F)}}\right)}{2\sqrt{-b\log(F)}F^{bc^2}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*(sqrt(pi)*(b*d^2*x + b*c*d)*b*c*(erf(sqrt(-(b*d^2*x + b*c*d)^2*log(F)
/(b*d^2))) - 1)*log(F)^2/((b*log(F))^(3/2)*d^2*sqrt(-(b*d^2*x + b*c*d)^2*l
og(F)/(b*d^2))) - F^((b*d^2*x + b*c*d)^2/(b*d^2))*b*log(F)/((b*log(F))^(3/
2)*d))*F^a*f/(sqrt(b*log(F))*d) + 1/2*sqrt(pi)*F^(b*c^2 + a)*e*erf(sqrt(-b
*log(F))*d*x - b*c*log(F)/sqrt(-b*log(F)))/(sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx = -\frac{\frac{\sqrt{\pi}(de-cf)F^a\operatorname{erf}\left(-\sqrt{-b\log(F)}d\left(x+\frac{c}{d}\right)\right)}{\sqrt{-b\log(F)}d} - \frac{fe^{\left(bd^2x^2\log(F)+2\,bcdx\log(F)+bc^2\log(F)+a\log(F)\right)}}{bd\log(F)}}{2\,d}$$

input
```
integrate(F^(a+b*(d*x+c)^2)*(f*x+e),x, algorithm="giac")
```

output
```
-1/2*(sqrt(pi)*(d*e - c*f)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*
log(F))*d) - f*e^(b*d^2*x^2*log(F) + 2*b*c*d*x*log(F) + b*c^2*log(F) + a*l
og(F))/(b*d*log(F)))/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx = \frac{F^{b\,d^2\,x^2}\,F^a\,F^{b\,c^2}\,F^{2\,b\,c\,d\,x}\,f}{2\,b\,d^2\,\ln(F)} - \frac{F^a\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{b\,x\,\ln(F)\,d^2+b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\,(c\,f-d\,e)}{2\,d\,\sqrt{b\,d^2\,\ln(F)}}$$

input
```
int(F^(a + b*(c + d*x)^2)*(e + f*x),x)
```

output
```
(F^(b*d^2*x^2)*F^a*F^(b*c^2)*F^(2*b*c*d*x)*f)/(2*b*d^2*log(F)) - (F^a*pi^(
1/2)*erfi((b*c*d*log(F) + b*d^2*x*log(F))/(b*d^2*log(F))^(1/2))*(c*f - d*e
))/(2*d*(b*d^2*log(F))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.52

$$\int F^{a+b(c+dx)^2}(e+fx)\,dx$$
$$= \frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)\log(f)\,bcfi-\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)\log(f)\,bdei+f^{b\,d^2x^2+2bcdx+b\,c^2}\sqrt{b}\,\sqrt{}\right.}{2\sqrt{b}\,\sqrt{\log\left(f\right)}\log\left(f\right)b\,d^2}$$

input
```
int(F^(a+b*(d*x+c)^2)*(f*x+e),x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f)))
)*log(f)*b*c*f*i - sqrt(pi)*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*s
qrt(log(f))))*log(f)*b*d*e*i + f**(b*c**2 + 2*b*c*d*x + b*d**2*x**2)*sqrt(
b)*sqrt(log(f))*f))/(2*sqrt(b)*sqrt(log(f))*log(f)*b*d**2)
```

3.321 $\int F^{a+b(c+dx)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int F^{a+b(c+dx)^2}\,dx=\frac{F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

output
```
1/2*F^a*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)*ln(F)^(1/2))/b^(1/2)/d/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)^2}\,dx=\frac{F^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{b}(c+dx)\sqrt{\log(F)}\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

input
```
Integrate[F^(a + b*(c + d*x)^2),x]
```

output
```
(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d*Sqrt[Log[
F]])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+b(c+dx)^2}\,dx$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^a\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{2\sqrt{b}d\sqrt{\log(F)}}$$

input
```
Int[F^(a + b*(c + d*x)^2),x]
```

output
```
(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*(c + d*x)*Sqrt[Log[F]]])/(2*Sqrt[b]*d*Sqrt[Log[
F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{\sqrt{\pi}\,F^{b\,c^{2}+a}\,F^{-b\,c^{2}}\,\operatorname{erf}\left(-d\sqrt{-b\ln\left(F\right)}\,x+\frac{b c\ln\left(F\right)}{\sqrt{-b\ln\left(F\right)}}\right)}{2 d\sqrt{-b\ln\left(F\right)}}$	58

input
```
int(F^(a+b*(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*F^(b*c^2+a)*F^(-b*c^2)/d/(-b*ln(F))^(1/2)*erf(-d*(-b*ln(F))^
(1/2)*x+b*c*ln(F)/(-b*ln(F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{\sqrt{\pi}\sqrt{-bd^2\log(F)}F^a\operatorname{erf}\left(\frac{\sqrt{-bd^2\log(F)}(dx+c)}{d}\right)}{2\,bd^2\log(F)}$$

input
```
integrate(F^(a+b*(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*d^2*log(F))*F^a*erf(sqrt(-b*d^2*log(F))*(d*x + c)/d)
/(b*d^2*log(F))
```

Sympy [F]

$$\int F^{a+b(c+dx)^2}\,dx = \int F^{a+b(c+dx)^2}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2),x)
```

output
```
Integral(F**(a + b*(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int F^{a+b(c+dx)^2}\,dx = \frac{\sqrt{\pi}F^{bc^2+a}\,\mathrm{erf}\left(\sqrt{-b\log\left(F\right)}dx - \frac{bc\log(F)}{\sqrt{-b\log(F)}}\right)}{2\sqrt{-b\log\left(F\right)}F^{bc^2}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*F^(b*c^2 + a)*erf(sqrt(-b*log(F))*d*x - b*c*log(F)/sqrt(-b*lo
g(F)))/(sqrt(-b*log(F))*F^(b*c^2)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{\sqrt{\pi}F^{a}\,\mathrm{erf}\left(-\sqrt{-b\log\left(F\right)}d\left(x+\frac{c}{d}\right)\right)}{2\sqrt{-b\log\left(F\right)}d}$$

input
```
integrate(F^(a+b*(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*d*(x + c/d))/(sqrt(-b*log(F))*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int F^{a+b(c+dx)^2}\,dx = -\frac{F^{a}\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{1\mathrm{i}\,b\,x\,\ln(F)\,d^2+1\mathrm{i}\,b\,c\,\ln(F)\,d}{\sqrt{b\,d^2\,\ln(F)}}\right)\,1\mathrm{i}}{2\sqrt{b\,d^2\,\ln\left(F\right)}}$$

input
```
int(F^(a + b*(c + d*x)^2),x)
```

output
```
-(F^a*pi^(1/2)*erf((b*c*d*log(F)*1i + b*d^2*x*log(F)*1i)/(b*d^2*log(F))^(1
/2))*1i)/(2*(b*d^2*log(F))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int F^{a+b(c+dx)^2}\,dx=-\frac{\sqrt{\pi}\,f^a\operatorname{erf}\left(\frac{\log(f)bci+\log(f)bdix}{\sqrt{b}\,\sqrt{\log(f)}}\right)i}{2\sqrt{b}\,\sqrt{\log(f)}\,d}$$

input
```
int(F^(a+b*(d*x+c)^2),x)
```

output
```
( - sqrt(pi)*f**a*erf((log(f)*b*c*i + log(f)*b*d*i*x)/(sqrt(b)*sqrt(log(f)
)))*i)/(2*sqrt(b)*sqrt(log(f))*d)
```

3.322 $\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \text{Int}\left(\frac{F^{a+b(c+dx)^2}}{e+fx},x\right)$$

output
```
Defer(Int)(F^(a+b*(d*x+c)^2)/(f*x+e),x)
```

Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(e + f*x),x]
```

output
```
Integrate[F^(a + b*(c + d*x)^2)/(e + f*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx} \, dx$$

$$\Big\downarrow \; 2654$$

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}dx$$

input
```
Int[F^(a + b*(c + d*x)^2)/(e + f*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(dx+c)^2}}{fx+e}dx$$

input
```
int(F^(a+b*(d*x+c)^2)/(f*x+e),x)
```

output
```
int(F^(a+b*(d*x+c)^2)/(f*x+e),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.57

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \int \frac{F^{(dx+c)^2b+a}}{fx+e}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="fricas")
```

output
```
integral(F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f*x + e), x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(f*x+e),x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(e + f*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \int \frac{F^{(dx+c)^2b+a}}{fx+e}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(f*x + e), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \int \frac{F^{(dx+c)^2b+a}}{fx+e}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e),x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(f*x + e), x)
```

Mupad [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = \int \frac{F^{a+b\,(c+d\,x)^2}}{e+f\,x}\,dx$$

input
```
int(F^(a + b*(c + d*x)^2)/(e + f*x),x)
```

output
```
int(F^(a + b*(c + d*x)^2)/(e + f*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{F^{a+b(c+dx)^2}}{e+fx}\,dx = f^{b\,c^2+a}\left(\int \frac{f^{b\,d^2x^2+2bcdx}}{fx+e}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^2)/(f*x+e),x)
```

output
```
f**(a + b*c**2)*int(f**(2*b*c*d*x + b*d**2*x**2)/(e + f*x),x)
```

3.323 $\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} \, dx = \text{Int}\left(\frac{F^{a+b(c+dx)^2}}{(e+fx)^2}, x\right)$$

output
```
Defer(Int)(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} \, dx = \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2} \, dx$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(e + f*x)^2,x]
```

output
```
Integrate[F^(a + b*(c + d*x)^2)/(e + f*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2650, 2633, 2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}dx$$

$$\Big\downarrow \text{2650}$$

$$\frac{2bd^2\log(F)\int F^{b(c+dx)^2+a}dx}{f^2}-\frac{2bd\log(F)(de-cf)\int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2}-\frac{F^{a+b(c+dx)^2}}{f(e+fx)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2bd\log(F)(de-cf)\int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2}-\frac{F^{a+b(c+dx)^2}}{f(e+fx)}+\frac{\sqrt{\pi}\sqrt{b}dF^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{f^2}$$

$$\Big\downarrow \text{2654}$$

$$-\frac{2bd\log(F)(de-cf)\int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2}-\frac{F^{a+b(c+dx)^2}}{f(e+fx)}+\frac{\sqrt{\pi}\sqrt{b}dF^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{f^2}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(e + f*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2650
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)*((e_.) + (f_.)*(x_))^(m_),
x_Symbol] :> Simp[f*(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^2)/((m + 1)*f^2))
, x] + (-Simp[2*b*d^2*(Log[F]/(f^2*(m + 1)))   Int[(e + f*x)^(m + 2)*F^(a +
 b*(c + d*x)^2), x], x] + Simp[2*b*d*(d*e - c*f)*(Log[F]/(f^2*(m + 1)))   I
nt[(e + f*x)^(m + 1)*F^(a + b*(c + d*x)^2), x], x]) /; FreeQ[{F, a, b, c, d
, e, f}, x] && NeQ[d*e - c*f, 0] && LtQ[m, -1]
```

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(dx+c)^2}}{(fx+e)^2}dx$$

input
```
int(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
int(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(fx+e)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^2*x^2 + 2*e*f*x + e^2),
x)
```

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(f*x+e)**2,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(e + f*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(fx+e)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(fx+e)^2}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(f*x + e)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx = \int \frac{F^{a+b\,(c+d\,x)^2}}{(e+f\,x)^2}\,dx$$

input
```
int(F^(a + b*(c + d*x)^2)/(e + f*x)^2,x)
```

output
```
int(F^(a + b*(c + d*x)^2)/(e + f*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^2}\,dx = f^{b\,c^2+a}\left(\int \frac{f^{b\,d^2x^2+2bcdx}}{f^2x^2+2efx+e^2}dx\right)$$

input
```
int(F^(a+b*(d*x+c)^2)/(f*x+e)^2,x)
```

output
```
f**(a + b*c**2)*int(f**(2*b*c*d*x + b*d**2*x**2)/(e**2 + 2*e*f*x + f**2*x*
*2),x)
```

3.324 $\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx = \text{Int}\left(\frac{F^{a+b(c+dx)^2}}{(e+fx)^3}, x\right)$$

output
```
Defer(Int)(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx = \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} dx$$

input
```
Integrate[F^(a + b*(c + d*x)^2)/(e + f*x)^3,x]
```

output
```
Integrate[F^(a + b*(c + d*x)^2)/(e + f*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2650, 2650, 2633, 2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3} \, dx$$

$$\Big\downarrow \text{ 2650}$$

$$\frac{bd^2 \log(F) \int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2} - \frac{bd\log(F)(de-cf) \int \frac{F^{b(c+dx)^2+a}}{(e+fx)^2}dx}{f^2} - \frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2}$$

$$\Big\downarrow \text{ 2650}$$

$$\frac{bd^2 \log(F) \int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2} - \frac{bd\log(F)(de-cf)\left(\frac{2bd^2\log(F)\int F^{b(c+dx)^2+a}dx}{f^2} - \frac{2bd\log(F)(de-cf)\int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2} - \frac{F^{a+b(c+dx)^2}}{f(e+fx)}\right)}{f^2} - \frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{bd^2 \log(F) \int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2} - \frac{bd\log(F)(de-cf)\left(-\frac{2bd\log(F)(de-cf)\int \frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2} - \frac{F^{a+b(c+dx)^2}}{f(e+fx)} + \frac{\sqrt{\pi}\sqrt{b}dF^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{f^2}\right)}{f^2} - \frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2}$$

$$\Big\downarrow \text{ 2654}$$

$$\frac{bd^2\log(F)\int\frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2}-\frac{bd\log(F)(de-cf)\left(-\frac{2bd\log(F)(de-cf)\int\frac{F^{b(c+dx)^2+a}}{e+fx}dx}{f^2}-\frac{F^{a+b(c+dx)^2}}{f(e+fx)}+\frac{\sqrt{\pi}\sqrt{b}dF^a\sqrt{\log(F)}\text{erfi}\left(\sqrt{b}\sqrt{\log(F)}(c+dx)\right)}{f^2}\right)}{f^2}-\frac{F^{a+b(c+dx)^2}}{2f(e+fx)^2}$$

input
```
Int[F^(a + b*(c + d*x)^2)/(e + f*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2650
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)*((e_.) + (f_.)*(x_))^(m_),
x_Symbol] :> Simp[f*(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^2)/((m + 1)*f^2))
, x] + (-Simp[2*b*d^2*(Log[F]/(f^2*(m + 1)))   Int[(e + f*x)^(m + 2)*F^(a +
 b*(c + d*x)^2), x], x] + Simp[2*b*d*(d*e - c*f)*(Log[F]/(f^2*(m + 1)))   I
nt[(e + f*x)^(m + 1)*F^(a + b*(c + d*x)^2), x], x]) /; FreeQ[{F, a, b, c, d
, e, f}, x] && NeQ[d*e - c*f, 0] && LtQ[m, -1]
```

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+b(dx+c)^2}}{(fx+e)^3}dx$$

input
```
int(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x)
```

output
```
int(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.62

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(fx+e)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + a)/(f^3*x^3 + 3*e*f^2*x^2 + 3*
e^2*f*x + e^3), x)
```

Sympy [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx = \int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx$$

input
```
integrate(F**(a+b*(d*x+c)**2)/(f*x+e)**3,x)
```

output
```
Integral(F**(a + b*(c + d*x)**2)/(e + f*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(fx+e)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(f*x + e)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx = \int \frac{F^{(dx+c)^2b+a}}{(fx+e)^3}\,dx$$

input
```
integrate(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(F^((d*x + c)^2*b + a)/(f*x + e)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx = \int \frac{F^{a+b\,(c+d\,x)^2}}{{(e+f\,x)}^3}\,dx$$

input
```
int(F^(a + b*(c + d*x)^2)/(e + f*x)^3,x)
```

output
```
int(F^(a + b*(c + d*x)^2)/(e + f*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 9784, normalized size of antiderivative = 465.90

$$\int \frac{F^{a+b(c+dx)^2}}{(e+fx)^3}\,dx = \text{Too large to display}$$

input
```
int(F^(a+b*(d*x+c)^2)/(f*x+e)^3,x)
```

output
```
(f**(a + b*c**2)*( - f**(2*b*c*d*x + b*d**2*x**2) + 2*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**2*d*e**4*f + 3*log(f)*b*c**2*d*e**3*f**2*x + 3*l
og(f)*b*c**2*d*e**2*f**3*x**2 + log(f)*b*c**2*d*e*f**4*x**3 + log(f)*b*c*d
**2*e**5 + 3*log(f)*b*c*d**2*e**4*f*x + 3*log(f)*b*c*d**2*e**3*f**2*x**2 +
 log(f)*b*c*d**2*e**2*f**3*x**3 - c*e**3*f**2 - 3*c*e**2*f**3*x - 3*c*e*f*
*4*x**2 - c*f**5*x**3 - d*e**4*f - 3*d*e**3*f**2*x - 3*d*e**2*f**3*x**2 -
d*e*f**4*x**3),x)*log(f)**2*b**2*c**3*d**2*e**4*f + 4*int(f**(2*b*c*d*x +
b*d**2*x**2)/(log(f)*b*c**2*d*e**4*f + 3*log(f)*b*c**2*d*e**3*f**2*x + 3*l
og(f)*b*c**2*d*e**2*f**3*x**2 + log(f)*b*c**2*d*e*f**4*x**3 + log(f)*b*c*d
**2*e**5 + 3*log(f)*b*c*d**2*e**4*f*x + 3*log(f)*b*c*d**2*e**3*f**2*x**2 +
 log(f)*b*c*d**2*e**2*f**3*x**3 - c*e**3*f**2 - 3*c*e**2*f**3*x - 3*c*e*f*
*4*x**2 - c*f**5*x**3 - d*e**4*f - 3*d*e**3*f**2*x - 3*d*e**2*f**3*x**2 -
d*e*f**4*x**3),x)*log(f)**2*b**2*c**3*d**2*e**3*f**2*x + 2*int(f**(2*b*c*d
*x + b*d**2*x**2)/(log(f)*b*c**2*d*e**4*f + 3*log(f)*b*c**2*d*e**3*f**2*x
+ 3*log(f)*b*c**2*d*e**2*f**3*x**2 + log(f)*b*c**2*d*e*f**4*x**3 + log(f)*
b*c*d**2*e**5 + 3*log(f)*b*c*d**2*e**4*f*x + 3*log(f)*b*c*d**2*e**3*f**2*x
**2 + log(f)*b*c*d**2*e**2*f**3*x**3 - c*e**3*f**2 - 3*c*e**2*f**3*x - 3*c
*e*f**4*x**2 - c*f**5*x**3 - d*e**4*f - 3*d*e**3*f**2*x - 3*d*e**2*f**3*x*
*2 - d*e*f**4*x**3),x)*log(f)**2*b**2*c**3*d**2*e**2*f**3*x**2 + 2*int(f**
(2*b*c*d*x + b*d**2*x**2)/(log(f)*b*c**2*d*e**4*f + 3*log(f)*b*c**2*d*e...
```

3.325 $\int e^{e(c+dx)^3}(a+bx)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 177

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx = -\frac{b^2(bc-ad)e^{e(c+dx)^3}}{d^4e} + \frac{(bc-ad)^3(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d^4\sqrt[3]{-e(c+dx)^3}} - \frac{b(bc-ad)^2(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{d^4\left(-e(c+dx)^3\right)^{2/3}} - \frac{b^3(c+dx)^4\Gamma\left(\frac{4}{3},-e(c+dx)^3\right)}{3d^4\left(-e(c+dx)^3\right)^{4/3}}$$

output
```
-b^2*(-a*d+b*c)*exp(e*(d*x+c)^3)/d^4/e+1/3*(-a*d+b*c)^3*(d*x+c)*GAMMA(1/3,
-e*(d*x+c)^3)/d^4/(-e*(d*x+c)^3)^(1/3)-b*(-a*d+b*c)^2*(d*x+c)^2*GAMMA(2/3,
-e*(d*x+c)^3)/d^4/(-e*(d*x+c)^3)^(2/3)-1/3*b^3*(d*x+c)^4*GAMMA(4/3,-e*(d*x
+c)^3)/d^4/(-e*(d*x+c)^3)^(4/3)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx = \frac{-\frac{3b^2(bc-ad)e^{e(c+dx)^3}}{e}+\frac{(bc-ad)^3(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{\sqrt[3]{-e(c+dx)^3}}-\frac{3b(bc-ad)^2(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{\left(-e(c+dx)^3\right)^{2/3}}+\frac{b^3(c+dx)\Gamma\left(\frac{4}{3},-e(c+dx)^3\right)}{e\sqrt[3]{-e(c+dx)^3}}}{3d^4}$$

input
```
Integrate[E^(e*(c + d*x)^3)*(a + b*x)^3,x]
```

output
```
((-3*b^2*(b*c - a*d)*E^(e*(c + d*x)^3))/e + ((b*c - a*d)^3*(c + d*x)*Gamma
[1/3, -(e*(c + d*x)^3)])/(-(e*(c + d*x)^3))^(1/3) - (3*b*(b*c - a*d)^2*(c
+ d*x)^2*Gamma[2/3, -(e*(c + d*x)^3)])/(-(e*(c + d*x)^3))^(2/3) + (b^3*(c
+ d*x)*Gamma[4/3, -(e*(c + d*x)^3)])/(e*(-(e*(c + d*x)^3))^(1/3)))/(3*d^4)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 e^{e(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(-\frac{3b^2(c+dx)^2(bc-ad)e^{e(c+dx)^3}}{d^3}+\frac{(ad-bc)^3e^{e(c+dx)^3}}{d^3}+\frac{3b(c+dx)(bc-ad)^2e^{e(c+dx)^3}}{d^3}+\frac{b^3(c+dx)^3e^{e(c+dx)^3}}{d^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2(bc-ad)e^{e(c+dx)^3}}{d^4e}-\frac{b(c+dx)^2(bc-ad)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{d^4\left(-e(c+dx)^3\right)^{2/3}}+\frac{(c+dx)(bc-ad)^3\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d^4\sqrt[3]{-e(c+dx)^3}}-\frac{b^3(c+dx)^4\Gamma\left(\frac{4}{3},-e(c+dx)^3\right)}{3d^4\left(-e(c+dx)^3\right)^{4/3}}$$

input
```
Int[E^(e*(c + d*x)^3)*(a + b*x)^3,x]
```

output
```
-((b^2*(b*c - a*d)*E^(e*(c + d*x)^3))/(d^4*e)) + ((b*c - a*d)^3*(c + d*x)*
Gamma[1/3, -(e*(c + d*x)^3)])/(3*d^4*(-(e*(c + d*x)^3))^(1/3)) - (b*(b*c -
 a*d)^2*(c + d*x)^2*Gamma[2/3, -(e*(c + d*x)^3)])/(d^4*(-(e*(c + d*x)^3))^
(2/3)) - (b^3*(c + d*x)^4*Gamma[4/3, -(e*(c + d*x)^3)])/(3*d^4*(-(e*(c + d
*x)^3))^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int e^{e(dx+c)^3}(bx+a)^3\,dx$$

input
```
int(exp(e*(d*x+c)^3)*(b*x+a)^3,x)
```

output
```
int(exp(e*(d*x+c)^3)*(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.35

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx$$
$$= \frac{9\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)(-d^3e)^{\frac{1}{3}}\,e\Gamma\left(\frac{2}{3},-d^3ex^3-3\,cd^2ex^2-3\,c^2dex-c^3e\right)-(-d^3e)^{\frac{2}{3}}\left(b^3+3\left(b^3c^3\right.\right.}{}$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/9*(9*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*(-d^3*e)^(1/3)*e*gamma(2/3,
 -d^3*e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*x - c^3*e) - (-d^3*e)^(2/3)*(b^3 +
 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e)*gamma(1/3, -d^3*
e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*x - c^3*e) + 3*(b^3*d^3*e*x - (2*b^3*c*d
^2 - 3*a*b^2*d^3)*e)*e^(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e))/
(d^6*e^2)
```

Sympy [F]

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx = \left(\int a^3e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx + \int b^3x^3e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx + \int 3ab^2x^2e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx + \int 3a^2bxe^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx\right)e^{c^3e}$$

input
```
integrate(exp(e*(d*x+c)**3)*(b*x+a)**3,x)
```

output
```
(Integral(a**3*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x)
 + Integral(b**3*x**3*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e
*x), x) + Integral(3*a*b**2*x**2*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp
(3*c**2*d*e*x), x) + Integral(3*a**2*b*x*exp(d**3*e*x**3)*exp(3*c*d**2*e*x
**2)*exp(3*c**2*d*e*x), x))*exp(c**3*e)
```

Maxima [F]

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx=\int (bx+a)^3e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^3*e^((d*x + c)^3*e), x)
```

Giac [F]

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx=\int (bx+a)^3e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((b*x + a)^3*e^((d*x + c)^3*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx=\int \mathrm{e}^{e\,(c+d\,x)^3}\,(a+b\,x)^3\,dx$$

input
```
int(exp(e*(c + d*x)^3)*(a + b*x)^3,x)
```

output
```
int(exp(e*(c + d*x)^3)*(a + b*x)^3, x)
```

Reduce [F]

$$\int e^{e(c+dx)^3}(a+bx)^3\,dx$$
$$= \frac{e^{c^3e}\left(3e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}a\,b^2d-2e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}b^3c+e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}b^3dx+3\left(\int e^{d^3e\,x^3+3c\,d^2\cdots}\right.\right.}{\cdots}$$

input
```
int(exp(e*(d*x+c)^3)*(b*x+a)^3,x)
```

output
```
(e**(c**3*e)*(3*e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*a*b**2*d
 - 2*e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*b**3*c + e**(3*c**2
*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*b**3*d*x + 3*int(e**(3*c**2*d*e*x
+ 3*c*d**2*e*x**2 + d**3*e*x**3),x)*a**3*d**4*e - 9*int(e**(3*c**2*d*e*x +
 3*c*d**2*e*x**2 + d**3*e*x**3),x)*a*b**2*c**2*d**2*e + 6*int(e**(3*c**2*d
*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3),x)*b**3*c**3*d*e - int(e**(3*c**2*d*
e*x + 3*c*d**2*e*x**2 + d**3*e*x**3),x)*b**3*d + 9*int(e**(3*c**2*d*e*x +
3*c*d**2*e*x**2 + d**3*e*x**3)*x,x)*a**2*b*d**4*e - 18*int(e**(3*c**2*d*e*
x + 3*c*d**2*e*x**2 + d**3*e*x**3)*x,x)*a*b**2*c*d**3*e + 9*int(e**(3*c**2
*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*x,x)*b**3*c**2*d**2*e))/(3*d**4*e)
```

3.326 $\int e^{e(c+dx)^3}(a+bx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx=\frac{b^2e^{e(c+dx)^3}}{3d^3e}-\frac{(bc-ad)^2(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d^3\sqrt[3]{-e(c+dx)^3}}+\frac{2b(bc-ad)(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{3d^3\left(-e(c+dx)^3\right)^{2/3}}$$

output
```
1/3*b^2*exp(e*(d*x+c)^3)/d^3/e-1/3*(-a*d+b*c)^2*(d*x+c)*GAMMA(1/3,-e*(d*x+
c)^3)/d^3/(-e*(d*x+c)^3)^(1/3)+2/3*b*(-a*d+b*c)*(d*x+c)^2*GAMMA(2/3,-e*(d*
x+c)^3)/d^3/(-e*(d*x+c)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx=\frac{\frac{b^2e^{e(c+dx)^3}}{e}-\frac{(bc-ad)^2(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{\sqrt[3]{-e(c+dx)^3}}+\frac{2b(bc-ad)(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{\left(-e(c+dx)^3\right)^{2/3}}}{3d^3}$$

input
```
Integrate[E^(e*(c + d*x)^3)*(a + b*x)^2,x]
```

output
```
((b^2*E^(e*(c + d*x)^3))/e - ((b*c - a*d)^2*(c + d*x)*Gamma[1/3, -(e*(c +
d*x)^3)])/(-(e*(c + d*x)^3))^(1/3) + (2*b*(b*c - a*d)*(c + d*x)^2*Gamma[2/
3, -(e*(c + d*x)^3)])/(-(e*(c + d*x)^3))^(2/3))/(3*d^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2 e^{e(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)^2 e^{e(c+dx)^3}}{d^2}-\frac{2b(c+dx)(bc-ad)e^{e(c+dx)^3}}{d^2}+\frac{b^2(c+dx)^2 e^{e(c+dx)^3}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b(c+dx)^2(bc-ad)\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{3d^3\left(-e(c+dx)^3\right)^{2/3}}-\frac{(c+dx)(bc-ad)^2\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d^3\sqrt[3]{-e(c+dx)^3}}+\frac{b^2 e^{e(c+dx)^3}}{3d^3 e}$$

input
```
Int[E^(e*(c + d*x)^3)*(a + b*x)^2,x]
```

output
```
(b^2*E^(e*(c + d*x)^3))/(3*d^3*e) - ((b*c - a*d)^2*(c + d*x)*Gamma[1/3, -(
e*(c + d*x)^3)])/(3*d^3*(-(e*(c + d*x)^3))^(1/3)) + (2*b*(b*c - a*d)*(c +
d*x)^2*Gamma[2/3, -(e*(c + d*x)^3)])/(3*d^3*(-(e*(c + d*x)^3))^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int e^{e(dx+c)^3}(bx+a)^2\,dx$$

input
```
int(exp(e*(d*x+c)^3)*(b*x+a)^2,x)
```

output
```
int(exp(e*(d*x+c)^3)*(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.39

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx = \frac{b^2d^2e^{\left(d^3ex^3+3\,cd^2ex^2+3\,c^2dex+c^3e\right)}+\left(b^2c^2-2\,abcd+a^2d^2\right)\left(-d^3e\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},-d^3ex^3-3\,cd^2ex^2-3\,c^2dex-c^3e\right)}{3\,d^5e}$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*d^2*e^(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e) + (b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*(-d^3*e)^(2/3)*gamma(1/3, -d^3*e*x^3 - 3*c*d^2*e*
x^2 - 3*c^2*d*e*x - c^3*e) - 2*(b^2*c*d - a*b*d^2)*(-d^3*e)^(1/3)*gamma(2/
3, -d^3*e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*x - c^3*e))/(d^5*e)
```

Sympy [F]

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx = \left(\int a^2e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx + \int b^2x^2e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx + \int 2abxe^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx\right)e^{c^3e}$$

input
```
integrate(exp(e*(d*x+c)**3)*(b*x+a)**2,x)
```

output
```
(Integral(a**2*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x)
 + Integral(b**2*x**2*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e
*x), x) + Integral(2*a*b*x*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**
2*d*e*x), x))*exp(c**3*e)
```

Maxima [F]

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx = \int (bx+a)^2e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^2*e^((d*x + c)^3*e), x)
```

Giac [F]

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx = \int (bx+a)^2e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*e^((d*x + c)^3*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx = \int e^{e\,(c+d\,x)^3}\,(a+b\,x)^2\,dx$$

input
```
int(exp(e*(c + d*x)^3)*(a + b*x)^2,x)
```

output
```
int(exp(e*(c + d*x)^3)*(a + b*x)^2, x)
```

Reduce [F]

$$\int e^{e(c+dx)^3}(a+bx)^2\,dx = \frac{e^{c^3e}\left(e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}b^2+3\left(\int e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}dx\right)a^2d^3e-3\left(\int e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}dx\right)b^2c^2de\right.}{3d^3e}$$

input
```
int(exp(e*(d*x+c)^3)*(b*x+a)^2,x)
```

output
```
(e**(c**3*e)*(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*b**2 + 3*i
nt(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3),x)*a**2*d**3*e - 3*in
t(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3),x)*b**2*c**2*d*e + 6*i
nt(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*x,x)*a*b*d**3*e - 6*i
nt(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*x,x)*b**2*c*d**2*e))/
(3*d**3*e)
```

3.327 $\int e^{e(c+dx)^3}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int e^{e(c+dx)^3}(a+bx)\,dx = \frac{(bc-ad)(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d^2\sqrt[3]{-e(c+dx)^3}} - \frac{b(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{3d^2\left(-e(c+dx)^3\right)^{2/3}}$$

output
```
1/3*(-a*d+b*c)*(d*x+c)*GAMMA(1/3,-e*(d*x+c)^3)/d^2/(-e*(d*x+c)^3)^(1/3)-1/
3*b*(d*x+c)^2*GAMMA(2/3,-e*(d*x+c)^3)/d^2/(-e*(d*x+c)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.93

$$\int e^{e(c+dx)^3}(a+bx)\,dx = -\frac{(c+dx)\left(-\left((bc-ad)\sqrt[3]{-e(c+dx)^3}\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)\right)+b(c+dx)\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)\right)}{3d^2\left(-e(c+dx)^3\right)^{2/3}}$$

input
```
Integrate[E^(e*(c + d*x)^3)*(a + b*x),x]
```

output
```
-1/3*((c + d*x)*(-((b*c - a*d)*(-(e*(c + d*x)^3))^(1/3)*Gamma[1/3, -(e*(c
+ d*x)^3)]) + b*(c + d*x)*Gamma[2/3, -(e*(c + d*x)^3)]))/(d^2*(-(e*(c + d*
x)^3))^(2/3))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)e^{e(c+dx)^3}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)e^{e(c+dx)^3}}{d}+\frac{b(c+dx)e^{e(c+dx)^3}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)(bc-ad)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d^2\sqrt[3]{-e(c+dx)^3}}-\frac{b(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{3d^2\left(-e(c+dx)^3\right)^{2/3}}$$

input
```
Int[E^(e*(c + d*x)^3)*(a + b*x),x]
```

output
```
((b*c - a*d)*(c + d*x)*Gamma[1/3, -(e*(c + d*x)^3)])/(3*d^2*(-(e*(c + d*x)
^3))^(1/3)) - (b*(c + d*x)^2*Gamma[2/3, -(e*(c + d*x)^3)])/(3*d^2*(-(e*(c
+ d*x)^3))^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int \mathrm{e}^{e(dx+c)^3}(bx+a)\,dx$$

input
```
int(exp(e*(d*x+c)^3)*(b*x+a),x)
```

output
```
int(exp(e*(d*x+c)^3)*(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int e^{e(c+dx)^3}(a+bx)\,dx = \frac{(-d^3e)^{\frac{1}{3}}\,bd\Gamma\left(\frac{2}{3},-d^3ex^3-3\,cd^2ex^2-3\,c^2dex-c^3e\right)-(-d^3e)^{\frac{2}{3}}\,(bc-ad)\Gamma\left(\frac{1}{3},-d^3ex^3-3\,cd^2ex^2-3\,c^2d\right.}{3\,d^4e}$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a),x, algorithm="fricas")
```

output
```
1/3*((-d^3*e)^(1/3)*b*d*gamma(2/3, -d^3*e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*
x - c^3*e) - (-d^3*e)^(2/3)*(b*c - a*d)*gamma(1/3, -d^3*e*x^3 - 3*c*d^2*e*
x^2 - 3*c^2*d*e*x - c^3*e))/(d^4*e)
```

Sympy [F]

$$\int e^{e(c+dx)^3}(a+bx)\,dx = \left(\int ae^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx + \int bxe^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx\right)e^{c^3e}$$

input
```
integrate(exp(e*(d*x+c)**3)*(b*x+a),x)
```

output
```
(Integral(a*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x) +
Integral(b*x*exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*x), x))*
exp(c**3*e)
```

Maxima [F]

$$\int e^{e(c+dx)^3}(a+bx)\,dx = \int (bx+a)e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a),x, algorithm="maxima")
```

output
```
integrate((b*x + a)*e^((d*x + c)^3*e), x)
```

Giac [F]

$$\int e^{e(c+dx)^3}(a+bx)\,dx = \int (bx+a)e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)*(b*x+a),x, algorithm="giac")
```

output
```
integrate((b*x + a)*e^((d*x + c)^3*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{e(c+dx)^3}(a+bx)\,dx=\int \mathrm{e}^{e\,(c+d\,x)^3}\,(a+b\,x)\,dx$$

input
```
int(exp(e*(c + d*x)^3)*(a + b*x),x)
```

output
```
int(exp(e*(c + d*x)^3)*(a + b*x), x)
```

Reduce [F]

$$\int e^{e(c+dx)^3}(a+bx)\,dx=e^{c^3e}\left(\left(\int e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}dx\right)a+\left(\int e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}xdx\right)b\right)$$

input
```
int(exp(e*(d*x+c)^3)*(b*x+a),x)
```

output
```
e**(c**3*e)*(int(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3),x)*a +
int(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)*x,x)*b)
```

3.328 $\int e^{e(c+dx)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int e^{e(c+dx)^3}\,dx = -\frac{(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

output
```
-1/3*(d*x+c)*GAMMA(1/3,-e*(d*x+c)^3)/d/(-e*(d*x+c)^3)^(1/3)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{e(c+dx)^3}\,dx = -\frac{(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

input
```
Integrate[E^(e*(c + d*x)^3),x]
```

output
```
-1/3*((c + d*x)*Gamma[1/3, -(e*(c + d*x)^3)])/(d*(-(e*(c + d*x)^3))^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{e(c+dx)^3} dx$$

$$\Big\downarrow \text{ 2637}$$

$$-\frac{(c+dx)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3d\sqrt[3]{-e(c+dx)^3}}$$

input
```
Int[E^(e*(c + d*x)^3),x]
```

output
```
-1/3*((c + d*x)*Gamma[1/3, -(e*(c + d*x)^3)])/(d*(-(e*(c + d*x)^3))^(1/3))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Maple [F]

$$\int e^{e(dx+c)^3}dx$$

input
```
int(exp(e*(d*x+c)^3),x)
```

output
```
int(exp(e*(d*x+c)^3),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int e^{e(c+dx)^3}\,dx=\frac{(-d^3e)^{\frac{2}{3}}\,\Gamma\left(\frac{1}{3},-d^3ex^3-3\,cd^2ex^2-3\,c^2dex-c^3e\right)}{3\,d^3e}$$

input
```
integrate(exp(e*(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*(-d^3*e)^(2/3)*gamma(1/3, -d^3*e*x^3 - 3*c*d^2*e*x^2 - 3*c^2*d*e*x - c
^3*e)/(d^3*e)
```

Sympy [F]

$$\int e^{e(c+dx)^3}\,dx=e^{c^3e}\int e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}\,dx$$

input
```
integrate(exp(e*(d*x+c)**3),x)
```

output
```
exp(c**3*e)*Integral(exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*
x), x)
```

Maxima [F]

$$\int e^{e(c+dx)^3}\,dx=\int e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3),x, algorithm="maxima")
```

output
```
integrate(e^((d*x + c)^3*e), x)
```

Giac [F]

$$\int e^{e(c+dx)^3}\,dx=\int e^{\left((dx+c)^3e\right)}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(e^((d*x + c)^3*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{e(c+dx)^3}\,dx=\int e^{e\,(c+d\,x)^3}\,dx$$

input
```
int(exp(e*(c + d*x)^3),x)
```

output
```
int(exp(e*(c + d*x)^3), x)
```

Reduce [F]

$$\int e^{e(c+dx)^3}\,dx=e^{c^3e}\left(\int e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}dx\right)$$

input
```
int(exp(e*(d*x+c)^3),x)
```

output
```
e**(c**3*e)*int(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3),x)
```

3.329 $\int \frac{e^{e(c+dx)^3}}{a+bx} dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{e(c+dx)^3}}{a+bx} dx = \text{Int}\left(\frac{e^{e(c+dx)^3}}{a+bx}, x\right)$$

output
```
Defer(Int)(exp(e*(d*x+c)^3)/(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{e(c+dx)^3}}{a+bx} dx = \int \frac{e^{e(c+dx)^3}}{a+bx} dx$$

input
```
Integrate[E^(e*(c + d*x)^3)/(a + b*x),x]
```

output
```
Integrate[E^(e*(c + d*x)^3)/(a + b*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{e(c+dx)^3}}{a+bx} \, dx$$
$$\Big\downarrow \; 2654$$
$$\int \frac{e^{e(c+dx)^3}}{a+bx} dx$$

input
```
Int[E^(e*(c + d*x)^3)/(a + b*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{e(dx+c)^3}}{bx+a} dx$$

input
```
int(exp(e*(d*x+c)^3)/(b*x+a),x)
```

output
```
int(exp(e*(d*x+c)^3)/(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{e^{e(c+dx)^3}}{a+bx}\,dx = \int \frac{e^{\left((dx+c)^3e\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(e^(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e)/(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 42.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.53

$$\int \frac{e^{e(c+dx)^3}}{a+bx}\,dx = e^{c^3e}\int \frac{e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}}{a+bx}\,dx$$

input
```
integrate(exp(e*(d*x+c)**3)/(b*x+a),x)
```

output
```
exp(c**3*e)*Integral(exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*
x)/(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{e(c+dx)^3}}{a+bx}\,dx = \int \frac{e^{\left((dx+c)^3e\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^((d*x + c)^3*e)/(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{e(c+dx)^3}}{a+bx}\,dx = \int \frac{e^{\left((dx+c)^3e\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^((d*x + c)^3*e)/(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{e(c+dx)^3}}{a+bx}\,dx = \int \frac{\mathrm{e}^{e\,{\left(c+d\,x\right)}^3}}{a+b\,x}\,dx$$

input
```
int(exp(e*(c + d*x)^3)/(a + b*x),x)
```

output
```
int(exp(e*(c + d*x)^3)/(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \frac{e^{e(c+dx)^3}}{a+bx}\,dx = e^{c^3e}\left(\int \frac{e^{d^3e\,x^3+3c\,d^2e\,x^2+3c^2dex}}{bx+a}dx\right)$$

input
```
int(exp(e*(d*x+c)^3)/(b*x+a),x)
```

output
```
e**(c**3*e)*int(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)/(a + b*x
),x)
```

3.330 $\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} \, dx = \text{Int}\left(\frac{e^{e(c+dx)^3}}{(a+bx)^2}, x\right)$$

output
```
Defer(Int)(exp(e*(d*x+c)^3)/(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} \, dx = \int \frac{e^{e(c+dx)^3}}{(a+bx)^2} \, dx$$

input
```
Integrate[E^(e*(c + d*x)^3)/(a + b*x)^2,x]
```

output
```
Integrate[E^(e*(c + d*x)^3)/(a + b*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2651, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{ 2651}$$

$$\frac{3de\int \frac{e^{e(c+dx)^3}(c+dx)^2}{a+bx}dx}{b} - \frac{e^{e(c+dx)^3}}{b(a+bx)}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{3de\int \left(\frac{e^{e(c+dx)^3}(bc-ad)^2}{b^2(a+bx)} + \frac{de^{e(c+dx)^3}(bc-ad)}{b^2} + \frac{de^{e(c+dx)^3}(c+dx)}{b}\right)dx}{b} - \frac{e^{e(c+dx)^3}}{b(a+bx)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3de\left(\frac{(bc-ad)^2\int \frac{e^{e(c+dx)^3}}{a+bx}dx}{b^2} - \frac{(c+dx)(bc-ad)\Gamma\left(\frac{1}{3},-e(c+dx)^3\right)}{3b^2\sqrt[3]{-e(c+dx)^3}} - \frac{(c+dx)^2\Gamma\left(\frac{2}{3},-e(c+dx)^3\right)}{3b(-e(c+dx)^3)^{2/3}}\right)}{b} - \frac{e^{e(c+dx)^3}}{b(a+bx)}$$

input
```
Int[E^(e*(c + d*x)^3)/(a + b*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2651
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(f*(m + 1))),
 x] - Simp[b*d*n*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(c + d*x)^(n
- 1)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && Ne
Q[d*e - c*f, 0] && IGtQ[n, 2] && LtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{e(dx+c)^3}}{(bx+a)^2}dx$$

input
```
int(exp(e*(d*x+c)^3)/(b*x+a)^2,x)
```

output
```
int(exp(e*(d*x+c)^3)/(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2}\,dx = \int \frac{e^{\left((dx+c)^3e\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(e^(d^3*e*x^3 + 3*c*d^2*e*x^2 + 3*c^2*d*e*x + c^3*e)/(b^2*x^2 + 2*
a*b*x + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 138.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.16

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2}\,dx = e^{c^3e}\int \frac{e^{d^3ex^3}e^{3cd^2ex^2}e^{3c^2dex}}{a^2+2abx+b^2x^2}\,dx$$

input
```
integrate(exp(e*(d*x+c)**3)/(b*x+a)**2,x)
```

output
```
exp(c**3*e)*Integral(exp(d**3*e*x**3)*exp(3*c*d**2*e*x**2)*exp(3*c**2*d*e*
x)/(a**2 + 2*a*b*x + b**2*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2}\,dx = \int \frac{e^{\left((dx+c)^3e\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(e^((d*x + c)^3*e)/(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.05

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2}\,dx = \int \frac{e^{\left(\left(dx+c\right)^3 e\right)}}{\left(bx+a\right)^2}\,dx$$

input
```
integrate(exp(e*(d*x+c)^3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2}\,dx = \int \frac{e^{e\,(c+d\,x)^3}}{\left(a+b\,x\right)^2}\,dx$$

input
```
int(exp(e*(c + d*x)^3)/(a + b*x)^2,x)
```

output
```
int(exp(e*(c + d*x)^3)/(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

$$\int \frac{e^{e(c+dx)^3}}{(a+bx)^2}\,dx = e^{c^3 e}\left(\int \frac{e^{d^3 e\,x^3+3c\,d^2 e\,x^2+3c^2 dex}}{b^2x^2+2abx+a^2}dx\right)$$

input
```
int(exp(e*(d*x+c)^3)/(b*x+a)^2,x)
```

output
```
e**(c**3*e)*int(e**(3*c**2*d*e*x + 3*c*d**2*e*x**2 + d**3*e*x**3)/(a**2 +
2*a*b*x + b**2*x**2),x)
```

3.331 $\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = -\frac{F^a \operatorname{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{f} + \frac{F^{a-\frac{bf}{de-cf}} \operatorname{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{f}$$

output
```
-F^a*Ei(b*ln(F)/(d*x+c))/f+F^(a-b*f/(-c*f+d*e))*Ei(b*d*(f*x+e)*ln(F)/(-c*f
+d*e)/(d*x+c))/f
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \frac{F^a\left(-\operatorname{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right) + F^{\frac{bf}{-de+cf}} \operatorname{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\right)}{f}$$

input
```
Integrate[F^(a + b/(c + d*x))/(e + f*x),x]
```

output
```
(F^a*(-ExpIntegralEi[(b*Log[F])/(c + d*x)] + F^((b*f)/(-(d*e) + c*f))*ExpI
ntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x))]))/f
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2652, 2639, 2658, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}dx$$

$$\Big\downarrow \text{ 2652}$$

$$\frac{d\int \frac{F^{a+\frac{b}{c+dx}}}{c+dx}dx}{f}-\frac{(de-cf)\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)(e+fx)}dx}{f}$$

$$\Big\downarrow \text{ 2639}$$

$$-\frac{(de-cf)\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)(e+fx)}dx}{f}-\frac{F^a \text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{f}$$

$$\Big\downarrow \text{ 2658}$$

$$\frac{\int \frac{F^{a+\frac{bd(e+fx)}{(de-cf)(c+dx)}-\frac{bf}{de-cf}}(c+dx)}{e+fx}d\frac{e+fx}{c+dx}}{f}-\frac{F^a \text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{f}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{F^{a-\frac{bf}{de-cf}} \text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{f}-\frac{F^a \text{ExpIntegralEi}\left(\frac{b\log(F)}{c+dx}\right)}{f}$$

input
```
Int[F^(a + b/(c + d*x))/(e + f*x),x]
```

output
```
-((F^a*ExpIntegralEi[(b*Log[F])/(c + d*x)])/f) + (F^(a - (b*f)/(d*e - c*f)
)*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x))])/f
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2652
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/((e_.) + (f_.)*(x_)), x_Symbo
l] :> Simp[d/f   Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Simp[(d*e - c*
f)/f   Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a
, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

rule 2658
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.)
+ (h_.)*(x_))), x_Symbol] :> Simp[-d/(f*(d*g - c*h))   Subst[Int[F^(a - b*(
h/(d*g - c*h)) + d*b*(x/(d*g - c*h)))/x, x], x, (g + h*x)/(c + d*x)], x] /;
 FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

method	result	size
risch	$-\frac{F^{\frac{acf-ade+bf}{cf-de}} \operatorname{expIntegral}_{1}\left(-\frac{b\ln(F)}{dx+c}-\ln(F)a-\frac{-\ln(F)acf+\ln(F)ade-\ln(F)bf}{cf-de}\right)}{f}+\frac{F^{a} \operatorname{expIntegral}_{1}\left(-\frac{b\ln(F)}{dx+c}\right)}{f}$	106

input
```
int(F^(a+b/(d*x+c))/(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/f*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln(F)/(d*x+c)-ln(F)*a-(-ln(F)
*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))+1/f*F^a*Ei(1,-b*ln(F)/(d*x+c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.25

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \frac{F^{\frac{ade-(ac+b)f}{de-cf}}\mathrm{Ei}\left(\frac{(bdfx+bde)\log(F)}{cde-c^2f+(d^2e-cdf)x}\right) - F^a\mathrm{Ei}\left(\frac{b\log(F)}{dx+c}\right)}{f}$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e),x, algorithm="fricas")
```

output
```
(F^((a*d*e - (a*c + b)*f)/(d*e - c*f))*Ei((b*d*f*x + b*d*e)*log(F)/(c*d*e
- c^2*f + (d^2*e - c*d*f)*x)) - F^a*Ei(b*log(F)/(d*x + c)))/f
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx$$

input
```
integrate(F**(a+b/(d*x+c))/(f*x+e),x)
```

output
```
Integral(F**(a + b/(c + d*x))/(e + f*x), x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{fx+e}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e), x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{fx+e}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e),x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \int \frac{F^{a+\frac{b}{c+d\,x}}}{e+f\,x}\,dx$$

input
```
int(F^(a + b/(c + d*x))/(e + f*x),x)
```

output
```
int(F^(a + b/(c + d*x))/(e + f*x), x)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{e+fx}\,dx = \int \frac{f^{\frac{adx+ac+b}{dx+c}}}{fx+e}dx$$

input
```
int(F^(a+b/(d*x+c))/(f*x+e),x)
```

output
```
int(f**((a*c + a*d*x + b)/(c + d*x))/(e + f*x),x)
```

3.332 $\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx=\frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)}-\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}-\frac{bdF^{a-\frac{bf}{de-cf}}\,\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\log(F)}{(de-cf)^2}$$

output
```
d*F^(a+b/(d*x+c))/f/(-c*f+d*e)-F^(a+b/(d*x+c))/f/(f*x+e)-b*d*F^(a-b*f/(-c*
f+d*e))*Ei(b*d*(f*x+e)*ln(F)/(-c*f+d*e)/(d*x+c))*ln(F)/(-c*f+d*e)^2
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx=\frac{dF^{a+\frac{b}{c+dx}}}{f(de-cf)}-\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}-\frac{bdF^{a+\frac{bf}{-de+cf}}\,\text{ExpIntegralEi}\left(-\frac{bf\log(F)}{-de+cf}+\frac{b\log(F)}{c+dx}\right)\log(F)}{(de-cf)^2}$$

input
```
Integrate[F^(a + b/(c + d*x))/(e + f*x)^2,x]
```

output
```
(d*F^(a + b/(c + d*x)))/(f*(d*e - c*f)) - F^(a + b/(c + d*x))/(f*(e + f*x)
) - (b*d*F^(a + (b*f)/(-(d*e) + c*f))*ExpIntegralEi[-((b*f*Log[F])/(-(d*e)
 + c*f)) + (b*Log[F])/(c + d*x)]*Log[F])/(d*e - c*f)^2
```

Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2} \, dx$$

$$\Big\downarrow \text{2653}$$

$$-\frac{bd\log(F)\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2(e+fx)}dx}{f}-\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{bd\log(F)\int \left(-\frac{dfF^{a+\frac{b}{c+dx}}}{(de-cf)^2(c+dx)}+\frac{f^2F^{a+\frac{b}{c+dx}}}{(de-cf)^2(e+fx)}+\frac{dF^{a+\frac{b}{c+dx}}}{(de-cf)(c+dx)^2}\right)dx}{f}-\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bd\log(F)\left(\frac{fF^{a-\frac{bf}{de-cf}}\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^2}-\frac{F^{a+\frac{b}{c+dx}}}{b\log(F)(de-cf)}\right)}{f}-\frac{F^{a+\frac{b}{c+dx}}}{f(e+fx)}$$

input
```
Int[F^(a + b/(c + d*x))/(e + f*x)^2,x]
```

output
```
-(F^(a + b/(c + d*x))/(f*(e + f*x))) - (b*d*((f*F^(a - (b*f)/(d*e - c*f))*
ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x))])/(d*e - c*f)
^2 - F^(a + b/(c + d*x))/(b*(d*e - c*f)*Log[F]))*Log[F])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.65

method	result
risch	$\frac{d\ln(F) b F^{a} F^{\frac{b}{dx+c}}}{(cf-de)^{2}\left(\frac{b\ln(F)}{dx+c}+\ln(F) a-\frac{\ln(F) a c f}{cf-de}+\frac{\ln(F) a d e}{cf-de}-\frac{\ln(F) b f}{cf-de}\right)}+\frac{d\ln(F) b F^{\frac{acf-ade+bf}{cf-de}} \operatorname{expIntegral}_{1}\left(-\frac{b\ln(F)}{dx+c}-\ln(F) a-\frac{-\ln(F) a c f}{}\right.}{(cf-de)^{2}}$

input
```
int(F^(a+b/(d*x+c))/(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
d*ln(F)*b/(c*f-d*e)^2*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F)*a-1/(c*f-d*
e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)+d*ln(F)*b/(c
*f-d*e)^2*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln(F)/(d*x+c)-ln(F)*a-(-
ln(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.54

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx = -\frac{(bdfx+bde)F^{\frac{ade-(ac+b)f}{de-cf}}\,\mathrm{Ei}\left(\frac{(bdfx+bde)\log(F)}{cde-c^2f+(d^2e-cdf)x}\right)\log(F)-(cde-c^2f+(d^2e-cdf)x)F^{\frac{adx+ac+b}{dx+c}}}{d^2e^3-2\,cde^2f+c^2ef^2+(d^2e^2f-2\,cdef^2+c^2f^3)x}$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^2,x, algorithm="fricas")
```

output
```
-((b*d*f*x + b*d*e)*F^((a*d*e - (a*c + b)*f)/(d*e - c*f))*Ei((b*d*f*x + b*
d*e)*log(F)/(c*d*e - c^2*f + (d^2*e - c*d*f)*x))*log(F) - (c*d*e - c^2*f +
 (d^2*e - c*d*f)*x)*F^((a*d*x + a*c + b)/(d*x + c)))/(d^2*e^3 - 2*c*d*e^2*
f + c^2*e*f^2 + (d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*x)
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx = \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx$$

input
```
integrate(F**(a+b/(d*x+c))/(f*x+e)**2,x)
```

output
```
Integral(F**(a + b/(c + d*x))/(e + f*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^2}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e)^2, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^2}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx = \int \frac{F^{a+\frac{b}{c+d\,x}}}{(e+f\,x)^2}\,dx$$

input
```
int(F^(a + b/(c + d*x))/(e + f*x)^2,x)
```

output
```
int(F^(a + b/(c + d*x))/(e + f*x)^2, x)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^2}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c))/(f*x+e)^2,x)
```

output
```
( - 4*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**3*f**3 + 6*f**((a
*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**2*d*e*f**2 - 6*f**((a*c + a*d
*x + b)/(c + d*x))*log(f)**2*b**2*c**2*d*f**3*x - 4*f**((a*c + a*d*x + b)/
(c + d*x))*log(f)**2*b**2*c*d**2*e**2*f + 4*f**((a*c + a*d*x + b)/(c + d*x
))*log(f)**2*b**2*c*d**2*e*f**2*x - 4*f**((a*c + a*d*x + b)/(c + d*x))*log
(f)**2*b**2*c*d**2*f**3*x**2 + f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*
b**2*d**3*e**3 - f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*d**3*e**2
*f*x + f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*d**3*e*f**2*x**2 -
f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*d**3*f**3*x**3 - 4*f**((a*
c + a*d*x + b)/(c + d*x))*log(f)*b*c**3*d*e*f**2 - 4*f**((a*c + a*d*x + b)
/(c + d*x))*log(f)*b*c**3*d*f**3*x + 6*f**((a*c + a*d*x + b)/(c + d*x))*lo
g(f)*b*c**2*d**2*e**2*f - 6*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c**2
*d**2*f**3*x**2 - 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c*d**3*e**3
+ 6*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*c*d**3*e**2*f*x + 6*f**((a*c
 + a*d*x + b)/(c + d*x))*log(f)*b*c*d**3*e*f**2*x**2 - 2*f**((a*c + a*d*x
+ b)/(c + d*x))*log(f)*b*c*d**3*f**3*x**3 - 2*f**((a*c + a*d*x + b)/(c + d
*x))*log(f)*b*d**4*e**3*x + 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)*b*d*
*4*e*f**2*x**3 + 2*f**((a*c + a*d*x + b)/(c + d*x))*c**4*d*e*f**2 + 2*f**(
(a*c + a*d*x + b)/(c + d*x))*c**4*d*f**3*x - 4*f**((a*c + a*d*x + b)/(c +
d*x))*c**3*d**2*e**2*f + 4*f**((a*c + a*d*x + b)/(c + d*x))*c**3*d**2*f...
```

3.333 $\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 267

$$\begin{aligned}\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx &= \frac{d^2F^{a+\frac{b}{c+dx}}}{2f(de-cf)^2}-\frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2} \\ &\quad -\frac{bd^2F^{a+\frac{b}{c+dx}}\log(F)}{2(de-cf)^3}+\frac{bdF^{a+\frac{b}{c+dx}}\log(F)}{2(de-cf)^2(e+fx)} \\ &\quad -\frac{bd^2F^{a-\frac{bf}{de-cf}}\operatorname{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\log(F)}{(de-cf)^3} \\ &\quad +\frac{b^2d^2fF^{a-\frac{bf}{de-cf}}\operatorname{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\log^2(F)}{2(de-cf)^4}\end{aligned}$$

output
```
1/2*d^2*F^(a+b/(d*x+c))/f/(-c*f+d*e)^2-1/2*F^(a+b/(d*x+c))/f/(f*x+e)^2-1/2
*b*d^2*F^(a+b/(d*x+c))*ln(F)/(-c*f+d*e)^3+1/2*b*d*F^(a+b/(d*x+c))*ln(F)/(-
c*f+d*e)^2/(f*x+e)-b*d^2*F^(a-b*f/(-c*f+d*e))*Ei(b*d*(f*x+e)*ln(F)/(-c*f+d
*e)/(d*x+c))*ln(F)/(-c*f+d*e)^3+1/2*b^2*d^2*f*F^(a-b*f/(-c*f+d*e))*Ei(b*d*
(f*x+e)*ln(F)/(-c*f+d*e)/(d*x+c))*ln(F)^2/(-c*f+d*e)^4
```

Mathematica [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} \, dx = \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} \, dx$$

input
```
Integrate[F^(a + b/(c + d*x))/(e + f*x)^3,x]
```

output
```
Integrate[F^(a + b/(c + d*x))/(e + f*x)^3, x]
```

Rubi [A] (verified)

Time = 2.91 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3} \, dx$$

$$\Big\downarrow \text{2653}$$

$$-\frac{bd\log(F)\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2(e+fx)^2}dx}{2f} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{bd\log(F)\int \left(-\frac{2d^2fF^{a+\frac{b}{c+dx}}}{(de-cf)^3(c+dx)} + \frac{2df^2F^{a+\frac{b}{c+dx}}}{(de-cf)^3(e+fx)} + \frac{d^2F^{a+\frac{b}{c+dx}}}{(de-cf)^2(c+dx)^2} + \frac{f^2F^{a+\frac{b}{c+dx}}}{(de-cf)^2(e+fx)^2}\right)dx}{2f} - \frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{bd\log(F)\left(-\frac{bdf^2\log(F)F^{a-\frac{bf}{de-cf}}\,\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^4}+\frac{2dfF^{a-\frac{bf}{de-cf}}\,\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^3}-\frac{fF^{a+\frac{b}{c+d}}}{(e+fx)(de-}\right.}{2f}$$

$$\frac{F^{a+\frac{b}{c+dx}}}{2f(e+fx)^2}$$

input
```
Int[F^(a + b/(c + d*x))/(e + f*x)^3,x]
```

output
```
-1/2*F^(a + b/(c + d*x))/(f*(e + f*x)^2) - (b*d*Log[F]*((d*f*F^(a + b/(c +
 d*x)))/(d*e - c*f)^3 - (f*F^(a + b/(c + d*x)))/((d*e - c*f)^2*(e + f*x))
+ (2*d*f*F^(a - (b*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((
d*e - c*f)*(c + d*x))])/(d*e - c*f)^3 - (d*F^(a + b/(c + d*x)))/(b*(d*e -
c*f)^2*Log[F]) - (b*d*f^2*F^(a - (b*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e
+ f*x)*Log[F])/((d*e - c*f)*(c + d*x))]*Log[F])/(d*e - c*f)^4))/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.90

method	result
risch	$-\frac{b^2 d^2 \ln(F)^2 f F^a F^{\frac{b}{dx+c}}}{2(cf-de)^4\left(\frac{b\ln(F)}{dx+c}+\ln(F)a-\frac{\ln(F)acf}{cf-de}+\frac{\ln(F)ade}{cf-de}-\frac{\ln(F)bf}{cf-de}\right)^2}-\frac{b^2 d^2 \ln(F)^2 f F^a F^{\frac{b}{dx+c}}}{2(cf-de)^4\left(\frac{b\ln(F)}{dx+c}+\ln(F)a-\frac{\ln(F)acf}{cf-de}+\frac{\ln(F)ade}{cf-de}-\frac{\ln(F)bf}{cf-de}\right)}-$

input
```
int(F^(a+b/(d*x+c))/(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^2*d^2*ln(F)^2*f/(c*f-d*e)^4*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F
)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)
^2-1/2*b^2*d^2*ln(F)^2*f/(c*f-d*e)^4*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln
(F)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*
f)-1/2*b^2*d^2*ln(F)^2*f/(c*f-d*e)^4*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,
-b*ln(F)/(d*x+c)-ln(F)*a-(-ln(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))-b
*d^2*ln(F)/(c*f-d*e)^3*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F)*a-1/(c*f-d
*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)-b*d^2*ln(F)
/(c*f-d*e)^3*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln(F)/(d*x+c)-ln(F)*a
-(-ln(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(257) = 514.

Time = 0.10 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.08

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx = \frac{((b^2d^2f^3x^2+2\,b^2d^2ef^2x+b^2d^2e^2f)\log(F)^2-2\,(bd^3e^3-bcd^2e^2f+(bd^3ef^2-bcd^2f^3)x^2+2\,(bd^3e^2f-}{2\,(d^4e^6-4\,cd}$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^3,x, algorithm="fricas")
```

output
```
1/2*(((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*log(F)^2 - 2*(
b*d^3*e^3 - b*c*d^2*e^2*f + (b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + 2*(b*d^3*e^2
*f - b*c*d^2*e*f^2)*x)*log(F))*F^((a*d*e - (a*c + b)*f)/(d*e - c*f))*Ei((b
*d*f*x + b*d*e)*log(F)/(c*d*e - c^2*f + (d^2*e - c*d*f)*x)) + (2*c*d^3*e^3
 - 5*c^2*d^2*e^2*f + 4*c^3*d*e*f^2 - c^4*f^3 + (d^4*e^2*f - 2*c*d^3*e*f^2
+ c^2*d^2*f^3)*x^2 + 2*(d^4*e^3 - 2*c*d^3*e^2*f + c^2*d^2*e*f^2)*x - (b*c*
d^2*e^2*f - b*c^2*d*e*f^2 + (b*d^3*e*f^2 - b*c*d^2*f^3)*x^2 + (b*d^3*e^2*f
 - b*c^2*d*f^3)*x)*log(F))*F^((a*d*x + a*c + b)/(d*x + c)))/(d^4*e^6 - 4*c
*d^3*e^5*f + 6*c^2*d^2*e^4*f^2 - 4*c^3*d*e^3*f^3 + c^4*e^2*f^4 + (d^4*e^4*
f^2 - 4*c*d^3*e^3*f^3 + 6*c^2*d^2*e^2*f^4 - 4*c^3*d*e*f^5 + c^4*f^6)*x^2 +
 2*(d^4*e^5*f - 4*c*d^3*e^4*f^2 + 6*c^2*d^2*e^3*f^3 - 4*c^3*d*e^2*f^4 + c^
4*e*f^5)*x)
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx = \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx$$

input
```
integrate(F**(a+b/(d*x+c))/(f*x+e)**3,x)
```

output
```
Integral(F**(a + b/(c + d*x))/(e + f*x)**3, x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^3}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e)^3, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^3}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx = \int \frac{F^{a+\frac{b}{c+d\,x}}}{(e+f\,x)^3}\,dx$$

input
```
int(F^(a + b/(c + d*x))/(e + f*x)^3,x)
```

output
```
int(F^(a + b/(c + d*x))/(e + f*x)^3, x)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^3}\,dx = \text{too large to display}$$

input
```
int(F^(a+b/(d*x+c))/(f*x+e)^3,x)
```

output
```
( - 4*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**3*b**3*c**3*f**4 + 6*f**((a
*c + a*d*x + b)/(c + d*x))*log(f)**3*b**3*c**2*d*e*f**3 - 6*f**((a*c + a*d
*x + b)/(c + d*x))*log(f)**3*b**3*c**2*d*f**4*x - 4*f**((a*c + a*d*x + b)/
(c + d*x))*log(f)**3*b**3*c*d**2*e**2*f**2 + 4*f**((a*c + a*d*x + b)/(c +
d*x))*log(f)**3*b**3*c*d**2*e*f**3*x - 4*f**((a*c + a*d*x + b)/(c + d*x))*
log(f)**3*b**3*c*d**2*f**4*x**2 + f**((a*c + a*d*x + b)/(c + d*x))*log(f)*
*3*b**3*d**3*e**3*f - f**((a*c + a*d*x + b)/(c + d*x))*log(f)**3*b**3*d**3
*e**2*f**2*x + f**((a*c + a*d*x + b)/(c + d*x))*log(f)**3*b**3*d**3*e*f**3
*x**2 - f**((a*c + a*d*x + b)/(c + d*x))*log(f)**3*b**3*d**3*f**4*x**3 - 1
0*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**4*f**4 + 20*f**((a*c
+ a*d*x + b)/(c + d*x))*log(f)**2*b**2*c**3*d*e*f**3 - 20*f**((a*c + a*d*x
 + b)/(c + d*x))*log(f)**2*b**2*c**3*d*f**4*x - 15*f**((a*c + a*d*x + b)/(
c + d*x))*log(f)**2*b**2*c**2*d**2*e**2*f**2 + 30*f**((a*c + a*d*x + b)/(c
 + d*x))*log(f)**2*b**2*c**2*d**2*e*f**3*x - 15*f**((a*c + a*d*x + b)/(c +
 d*x))*log(f)**2*b**2*c**2*d**2*f**4*x**2 + 8*f**((a*c + a*d*x + b)/(c + d
*x))*log(f)**2*b**2*c*d**3*e**3*f - 6*f**((a*c + a*d*x + b)/(c + d*x))*log
(f)**2*b**2*c*d**3*e**2*f**2*x + 24*f**((a*c + a*d*x + b)/(c + d*x))*log(f
)**2*b**2*c*d**3*e*f**3*x**2 - 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**
2*b**2*c*d**3*f**4*x**3 - 2*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**
2*d**4*e**4 - 3*f**((a*c + a*d*x + b)/(c + d*x))*log(f)**2*b**2*d**4*e*...
```

3.334 $\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 460

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \frac{d^3F^{a+\frac{b}{c+dx}}}{3f(de-cf)^3} - \frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3} - \frac{5bd^3F^{a+\frac{b}{c+dx}}\log(F)}{6(de-cf)^4} + \frac{bdF^{a+\frac{b}{c+dx}}\log(F)}{6(de-cf)^2(e+fx)^2} + \frac{2bd^2F^{a+\frac{b}{c+dx}}\log(F)}{3(de-cf)^3(e+fx)} - \frac{bd^3F^{a-\frac{bf}{de-cf}}\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\log(F)}{(de-cf)^4} + \frac{b^2d^3fF^{a+\frac{b}{c+dx}}\log^2(F)}{6(de-cf)^5} - \frac{b^2d^2fF^{a+\frac{b}{c+dx}}\log^2(F)}{6(de-cf)^4(e+fx)} + \frac{b^2d^3fF^{a-\frac{bf}{de-cf}}\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\log^2(F)}{(de-cf)^5} - \frac{b^3d^3f^2F^{a-\frac{bf}{de-cf}}\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)\log^3(F)}{6(de-cf)^6}$$

output
```
1/3*d^3*F^(a+b/(d*x+c))/f/(-c*f+d*e)^3-1/3*F^(a+b/(d*x+c))/f/(f*x+e)^3-5/6
*b*d^3*F^(a+b/(d*x+c))*ln(F)/(-c*f+d*e)^4+1/6*b*d*F^(a+b/(d*x+c))*ln(F)/(-
c*f+d*e)^2/(f*x+e)^2+2/3*b*d^2*F^(a+b/(d*x+c))*ln(F)/(-c*f+d*e)^3/(f*x+e)-
b*d^3*F^(a-b*f/(-c*f+d*e))*Ei(b*d*(f*x+e)*ln(F)/(-c*f+d*e)/(d*x+c))*ln(F)/
(-c*f+d*e)^4+1/6*b^2*d^3*f*F^(a+b/(d*x+c))*ln(F)^2/(-c*f+d*e)^5-1/6*b^2*d^
2*f*F^(a+b/(d*x+c))*ln(F)^2/(-c*f+d*e)^4/(f*x+e)+b^2*d^3*f*F^(a-b*f/(-c*f+
d*e))*Ei(b*d*(f*x+e)*ln(F)/(-c*f+d*e)/(d*x+c))*ln(F)^2/(-c*f+d*e)^5-1/6*b^
3*d^3*f^2*F^(a-b*f/(-c*f+d*e))*Ei(b*d*(f*x+e)*ln(F)/(-c*f+d*e)/(d*x+c))*ln
(F)^3/(-c*f+d*e)^6
```

Mathematica [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx=\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx$$

input
```
Integrate[F^(a + b/(c + d*x))/(e + f*x)^4,x]
```

output
```
Integrate[F^(a + b/(c + d*x))/(e + f*x)^4, x]
```

Rubi [A] (verified)

Time = 5.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx$$

$$\Big\downarrow \text{2653}$$

$$-\frac{bd\log(F)\int \frac{F^{a+\frac{b}{c+dx}}}{(c+dx)^2(e+fx)^3}dx}{3f}-\frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3}$$

$$\Big\downarrow \; 7293$$

$$-\frac{bd\log(F)\int\left(-\frac{3d^3fF^{a+\frac{b}{c+dx}}}{(de-cf)^4(c+dx)}+\frac{3d^2f^2F^{a+\frac{b}{c+dx}}}{(de-cf)^4(e+fx)}+\frac{d^3F^{a+\frac{b}{c+dx}}}{(de-cf)^3(c+dx)^2}+\frac{2df^2F^{a+\frac{b}{c+dx}}}{(de-cf)^3(e+fx)^2}+\frac{f^2F^{a+\frac{b}{c+dx}}}{(de-cf)^2(e+fx)^3}\right)dx}{3f}-\frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{bd\log(F)\left(\frac{b^2d^2f^3\log^2(F)F^{a-\frac{bf}{de-cf}}\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{2(de-cf)^6}-\frac{3bd^2f^2\log(F)F^{a-\frac{bf}{de-cf}}\text{ExpIntegralEi}\left(\frac{bd(e+fx)\log(F)}{(de-cf)(c+dx)}\right)}{(de-cf)^5}+\right.}{}$$

$$\frac{F^{a+\frac{b}{c+dx}}}{3f(e+fx)^3}$$

input
```
Int[F^(a + b/(c + d*x))/(e + f*x)^4,x]
```

output
```
-1/3*F^(a + b/(c + d*x))/(f*(e + f*x)^3) - (b*d*Log[F]*((5*d^2*f*F^(a + b/
(c + d*x)))/(2*(d*e - c*f)^4) - (f*F^(a + b/(c + d*x)))/(2*(d*e - c*f)^2*(
e + f*x)^2) - (2*d*f*F^(a + b/(c + d*x)))/((d*e - c*f)^3*(e + f*x)) + (3*d
^2*f*F^(a - (b*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e
- c*f)*(c + d*x))])/(d*e - c*f)^4 - (d^2*F^(a + b/(c + d*x)))/(b*(d*e - c*
f)^3*Log[F]) - (b*d^2*f^2*F^(a + b/(c + d*x))*Log[F])/(2*(d*e - c*f)^5) +
(b*d*f^2*F^(a + b/(c + d*x))*Log[F])/(2*(d*e - c*f)^4*(e + f*x)) - (3*b*d^
2*f^2*F^(a - (b*f)/(d*e - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e
 - c*f)*(c + d*x))]*Log[F])/(d*e - c*f)^5 + (b^2*d^2*f^3*F^(a - (b*f)/(d*e
 - c*f))*ExpIntegralEi[(b*d*(e + f*x)*Log[F])/((d*e - c*f)*(c + d*x))]*Log
[F]^2)/(2*(d*e - c*f)^6)))/(3*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 921 vs. $2(444) = 888$.

Time = 0.68 (sec) , antiderivative size = 922, normalized size of antiderivative = 2.00

method	result
risch	$\frac{b^3 d^3 \ln(F)^3 f^2 F^a F^{\frac{b}{dx+c}}}{3(cf-de)^6 \left(\frac{b\ln(F)}{dx+c}+\ln(F)a-\frac{\ln(F)acf}{cf-de}+\frac{\ln(F)ade}{cf-de}-\frac{\ln(F)bf}{cf-de}\right)^3}+\frac{b^3 d^3 \ln(F)^3 f^2 F^a F^{\frac{b}{dx+c}}}{6(cf-de)^6 \left(\frac{b\ln(F)}{dx+c}+\ln(F)a-\frac{\ln(F)acf}{cf-de}+\frac{\ln(F)ade}{cf-de}-\frac{\ln(F)bf}{cf-de}\right)^2}+$

input
```
int(F^(a+b/(d*x+c))/(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*b^3*d^3*ln(F)^3*f^2/(c*f-d*e)^6*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(
F)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f
)^3+1/6*b^3*d^3*ln(F)^3*f^2/(c*f-d*e)^6*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)
+ln(F)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)
*b*f)^2+1/6*b^3*d^3*ln(F)^3*f^2/(c*f-d*e)^6*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*
x+c)+ln(F)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*l
n(F)*b*f)+1/6*b^3*d^3*ln(F)^3*f^2/(c*f-d*e)^6*F^((a*c*f-a*d*e+b*f)/(c*f-d*
e))*Ei(1,-b*ln(F)/(d*x+c)-ln(F)*a-(-ln(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*
f-d*e))+b^2*d^3*ln(F)^2*f/(c*f-d*e)^5*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+l
n(F)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b
*f)^2+b^2*d^3*ln(F)^2*f/(c*f-d*e)^5*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(
F)*a-1/(c*f-d*e)*ln(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f
)+b^2*d^3*ln(F)^2*f/(c*f-d*e)^5*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln
(F)/(d*x+c)-ln(F)*a-(-ln(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))+b*d^3*
ln(F)/(c*f-d*e)^4*F^a*F^(b/(d*x+c))/(b*ln(F)/(d*x+c)+ln(F)*a-1/(c*f-d*e)*l
n(F)*a*c*f+1/(c*f-d*e)*ln(F)*a*d*e-1/(c*f-d*e)*ln(F)*b*f)+b*d^3*ln(F)/(c*f
-d*e)^4*F^((a*c*f-a*d*e+b*f)/(c*f-d*e))*Ei(1,-b*ln(F)/(d*x+c)-ln(F)*a-(-ln
(F)*a*c*f+ln(F)*a*d*e-ln(F)*b*f)/(c*f-d*e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. 2(444) = 888.

Time = 0.11 (sec) , antiderivative size = 1376, normalized size of antiderivative = 2.99

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^4,x, algorithm="fricas")
```

output
```
-1/6*(((b^3*d^3*f^5*x^3 + 3*b^3*d^3*e*f^4*x^2 + 3*b^3*d^3*e^2*f^3*x + b^3*
d^3*e^3*f^2)*log(F)^3 - 6*(b^2*d^4*e^4*f - b^2*c*d^3*e^3*f^2 + (b^2*d^4*e*
f^4 - b^2*c*d^3*f^5)*x^3 + 3*(b^2*d^4*e^2*f^3 - b^2*c*d^3*e*f^4)*x^2 + 3*(
b^2*d^4*e^3*f^2 - b^2*c*d^3*e^2*f^3)*x)*log(F)^2 + 6*(b*d^5*e^5 - 2*b*c*d^
4*e^4*f + b*c^2*d^3*e^3*f^2 + (b*d^5*e^2*f^3 - 2*b*c*d^4*e*f^4 + b*c^2*d^3
*f^5)*x^3 + 3*(b*d^5*e^3*f^2 - 2*b*c*d^4*e^2*f^3 + b*c^2*d^3*e*f^4)*x^2 +
3*(b*d^5*e^4*f - 2*b*c*d^4*e^3*f^2 + b*c^2*d^3*e^2*f^3)*x)*log(F))*F^((a*d
*e - (a*c + b)*f)/(d*e - c*f))*Ei((b*d*f*x + b*d*e)*log(F)/(c*d*e - c^2*f
+ (d^2*e - c*d*f)*x)) - (6*c*d^5*e^5 - 24*c^2*d^4*e^4*f + 38*c^3*d^3*e^3*f
^2 - 30*c^4*d^2*e^2*f^3 + 12*c^5*d*e*f^4 - 2*c^6*f^5 + 2*(d^6*e^3*f^2 - 3*
c*d^5*e^2*f^3 + 3*c^2*d^4*e*f^4 - c^3*d^3*f^5)*x^3 + 6*(d^6*e^4*f - 3*c*d^
5*e^3*f^2 + 3*c^2*d^4*e^2*f^3 - c^3*d^3*e*f^4)*x^2 + (b^2*c*d^3*e^3*f^2 -
b^2*c^2*d^2*e^2*f^3 + (b^2*d^4*e*f^4 - b^2*c*d^3*f^5)*x^3 + (2*b^2*d^4*e^2
*f^3 - b^2*c*d^3*e*f^4 - b^2*c^2*d^2*f^5)*x^2 + (b^2*d^4*e^3*f^2 + b^2*c*d
^3*e^2*f^3 - 2*b^2*c^2*d^2*e*f^4)*x)*log(F)^2 + 6*(d^6*e^5 - 3*c*d^5*e^4*f
 + 3*c^2*d^4*e^3*f^2 - c^3*d^3*e^2*f^3)*x - (6*b*c*d^4*e^4*f - 13*b*c^2*d^
3*e^3*f^2 + 8*b*c^3*d^2*e^2*f^3 - b*c^4*d*e*f^4 + 5*(b*d^5*e^2*f^3 - 2*b*c
*d^4*e*f^4 + b*c^2*d^3*f^5)*x^3 + (11*b*d^5*e^3*f^2 - 18*b*c*d^4*e^2*f^3 +
 3*b*c^2*d^3*e*f^4 + 4*b*c^3*d^2*f^5)*x^2 + (6*b*d^5*e^4*f - 2*b*c*d^4*e^3
*f^2 - 15*b*c^2*d^3*e^2*f^3 + 12*b*c^3*d^2*e*f^4 - b*c^4*d*f^5)*x)*log(...
```

Sympy [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx$$

input
```
integrate(F**(a+b/(d*x+c))/(f*x+e)**4,x)
```

output
```
Integral(F**(a + b/(c + d*x))/(e + f*x)**4, x)
```

Maxima [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^4}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e)^4, x)
```

Giac [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^4}\,dx$$

input
```
integrate(F^(a+b/(d*x+c))/(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate(F^(a + b/(d*x + c))/(f*x + e)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \int \frac{F^{a+\frac{b}{c+d\,x}}}{(e+f\,x)^4}\,dx$$

input
```
int(F^(a + b/(c + d*x))/(e + f*x)^4,x)
```

output
```
int(F^(a + b/(c + d*x))/(e + f*x)^4, x)
```

Reduce [F]

$$\int \frac{F^{a+\frac{b}{c+dx}}}{(e+fx)^4}\,dx = \int \frac{F^{a+\frac{b}{dx+c}}}{(fx+e)^4}dx$$

input
```
int(F^(a+b/(d*x+c))/(f*x+e)^4,x)
```

output
```
int(F^(a+b/(d*x+c))/(f*x+e)^4,x)
```

3.335 $\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx$

Optimal result

Integrand size = 19, antiderivative size = 346

$$\begin{aligned}\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx &= \frac{(bc-ad)^4 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e e^{\frac{e}{c+dx}}(c+dx)}{d^5}\\ &\quad + \frac{b^2(bc-ad)^2 e^2 e^{\frac{e}{c+dx}}(c+dx)}{d^5} - \frac{2b(bc-ad)^3 e^{\frac{e}{c+dx}}(c+dx)^2}{d^5}\\ &\quad + \frac{b^2(bc-ad)^2 e e^{\frac{e}{c+dx}}(c+dx)^2}{d^5} + \frac{2b^2(bc-ad)^2 e^{\frac{e}{c+dx}}(c+dx)^3}{d^5}\\ &\quad - \frac{(bc-ad)^4 e \operatorname{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^5}\\ &\quad + \frac{2b(bc-ad)^3 e^2 \operatorname{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^5}\\ &\quad - \frac{b^2(bc-ad)^2 e^3 \operatorname{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^5}\\ &\quad - \frac{b^4 e^5 \Gamma\left(-5,-\frac{e}{c+dx}\right)}{d^5} - \frac{4b^3(bc-ad)e^4\Gamma\left(-4,-\frac{e}{c+dx}\right)}{d^5}\end{aligned}$$

output
```
(-a*d+b*c)^4*exp(e/(d*x+c))*(d*x+c)/d^5-2*b*(-a*d+b*c)^3*e*exp(e/(d*x+c))*
(d*x+c)/d^5+b^2*(-a*d+b*c)^2*e^2*exp(e/(d*x+c))*(d*x+c)/d^5-2*b*(-a*d+b*c)
^3*exp(e/(d*x+c))*(d*x+c)^2/d^5+b^2*(-a*d+b*c)^2*e*exp(e/(d*x+c))*(d*x+c)^
2/d^5+2*b^2*(-a*d+b*c)^2*exp(e/(d*x+c))*(d*x+c)^3/d^5-(-a*d+b*c)^4*e*Ei(e/
(d*x+c))/d^5+2*b*(-a*d+b*c)^3*e^2*Ei(e/(d*x+c))/d^5-b^2*(-a*d+b*c)^2*e^3*E
i(e/(d*x+c))/d^5+b^4*(d*x+c)^5*Ei(6,-e/(d*x+c))/d^5-4*b^3*(-a*d+b*c)*(d*x+
c)^4*Ei(5,-e/(d*x+c))/d^5
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.35

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx$$

$$= \frac{c(120a^4d^4 - 240a^3bd^3(c-e) + 120a^2b^2d^2(2c^2-5ce+e^2) - 20ab^3d(6c^3-26c^2e+11ce^2-e^3) + b^4(24c^4}{120d^5}$$

$$+ \frac{de^{\frac{e}{c+dx}}x(120a^4d^4 + 240a^3bd^3(e+dx) + 120a^2b^2d^2(-4ce+e^2+dex+2d^2x^2) + 20ab^3d(18c^2e+e^3+c}{}$$

input
```
Integrate[E^(e/(c + d*x))*(a + b*x)^4,x]
```

output
```
(c*(120*a^4*d^4 - 240*a^3*b*d^3*(c - e) + 120*a^2*b^2*d^2*(2*c^2 - 5*c*e +
 e^2) - 20*a*b^3*d*(6*c^3 - 26*c^2*e + 11*c*e^2 - e^3) + b^4*(24*c^4 - 154
*c^3*e + 102*c^2*e^2 - 19*c*e^3 + e^4))*E^(e/(c + d*x)))/(120*d^5) + (d*E^
(e/(c + d*x))*x*(120*a^4*d^4 + 240*a^3*b*d^3*(e + d*x) + 120*a^2*b^2*d^2*(
-4*c*e + e^2 + d*e*x + 2*d^2*x^2) + 20*a*b^3*d*(18*c^2*e + e^3 + d*e^2*x +
 2*d^2*e*x^2 + 6*d^3*x^3 - 2*c*e*(5*e + 3*d*x)) + b^4*(-96*c^3*e + e^4 + d
*e^3*x + 2*d^2*e^2*x^2 + 6*d^3*e*x^3 + 24*d^4*x^4 + 2*c^2*e*(43*e + 18*d*x
) - 2*c*e*(9*e^2 + 7*d*e*x + 8*d^2*x^2))) - e*(120*a^4*d^4 - 240*a^3*b*d^3
*(2*c - e) + 120*a^2*b^2*d^2*(6*c^2 - 6*c*e + e^2) - 20*a*b^3*d*(24*c^3 -
36*c^2*e + 12*c*e^2 - e^3) + b^4*(120*c^4 - 240*c^3*e + 120*c^2*e^2 - 20*c
*e^3 + e^4))*ExpIntegralEi[e/(c + d*x)])/(120*d^5)
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^4 e^{\frac{e}{c+dx}}dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(-\frac{4b^3(c+dx)^3(bc-ad)e^{\frac{e}{c+dx}}}{d^4}+\frac{6b^2(c+dx)^2(bc-ad)^2e^{\frac{e}{c+dx}}}{d^4}+\frac{(ad-bc)^4e^{\frac{e}{c+dx}}}{d^4}-\frac{4b(c+dx)(bc-ad)^3e^{\frac{e}{c+d}}}{d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b^3e^4(bc-ad)\Gamma\left(-4,-\frac{e}{c+dx}\right)}{d^5}-\frac{b^2e^3(bc-ad)^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^5}+\frac{b^2e^2(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^5}+\frac{b^2e(c+dx)^2(bc-ad)^2e^{\frac{e}{c+dx}}}{d^5}+\frac{2b^2(c+dx)^3(bc-ad)^2e^{\frac{e}{c+dx}}}{d^5}+\frac{2be^2(bc-ad)^3\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^5}-\frac{e(bc-ad)^4\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^5}-\frac{2be(c+dx)(bc-ad)^3e^{\frac{e}{c+dx}}}{d^5}-\frac{2b(c+dx)^2(bc-ad)^3e^{\frac{e}{c+dx}}}{d^5}+\frac{(c+dx)(bc-ad)^4e^{\frac{e}{c+dx}}}{d^5}-\frac{b^4e^5\Gamma\left(-5,-\frac{e}{c+dx}\right)}{d^5}$$

input
```
Int[E^(e/(c + d*x))*(a + b*x)^4,x]
```

output
```
((b*c - a*d)^4*E^(e/(c + d*x))*(c + d*x))/d^5 - (2*b*(b*c - a*d)^3*e*E^(e/
(c + d*x))*(c + d*x))/d^5 + (b^2*(b*c - a*d)^2*e^2*E^(e/(c + d*x))*(c + d*
x))/d^5 - (2*b*(b*c - a*d)^3*E^(e/(c + d*x))*(c + d*x)^2)/d^5 + (b^2*(b*c
- a*d)^2*e*E^(e/(c + d*x))*(c + d*x)^2)/d^5 + (2*b^2*(b*c - a*d)^2*E^(e/(c
 + d*x))*(c + d*x)^3)/d^5 - ((b*c - a*d)^4*e*ExpIntegralEi[e/(c + d*x)])/d
^5 + (2*b*(b*c - a*d)^3*e^2*ExpIntegralEi[e/(c + d*x)])/d^5 - (b^2*(b*c -
a*d)^2*e^3*ExpIntegralEi[e/(c + d*x)])/d^5 - (b^4*e^5*Gamma[-5, -(e/(c + d
*x))])/d^5 - (4*b^3*(b*c - a*d)*e^4*Gamma[-4, -(e/(c + d*x))])/d^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1145 vs. $2(347) = 694$.

Time = 0.40 (sec) , antiderivative size = 1146, normalized size of antiderivative = 3.31

method	result	size
derivativedivides	Expression too large to display	1146
default	Expression too large to display	1146
risch	Expression too large to display	1273
parts	Expression too large to display	2204

input
```
int(exp(e/(d*x+c))*(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*(a^4*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+b^4/d^4*e^4*(-1
/5/e^5*(d*x+c)^5*exp(e/(d*x+c))-1/20/e^4*(d*x+c)^4*exp(e/(d*x+c))-1/60/e^3
*(d*x+c)^3*exp(e/(d*x+c))-1/120*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/120/e*(d*x+
c)*exp(e/(d*x+c))-1/120*Ei(1,-e/(d*x+c)))+b^4/d^4*c^4*(-1/e*(d*x+c)*exp(e/
(d*x+c))-Ei(1,-e/(d*x+c)))+4*b^3/d^3*e^3*a*(-1/4/e^4*(d*x+c)^4*exp(e/(d*x+
c))-1/12/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/24*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/
24/e*(d*x+c)*exp(e/(d*x+c))-1/24*Ei(1,-e/(d*x+c)))-4*b^4/d^4*e^3*c*(-1/4/e
^4*(d*x+c)^4*exp(e/(d*x+c))-1/12/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/24*exp(e/(
d*x+c))/e^2*(d*x+c)^2-1/24/e*(d*x+c)*exp(e/(d*x+c))-1/24*Ei(1,-e/(d*x+c)))
+6*b^2/d^2*e^2*a^2*(-1/3/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/6*exp(e/(d*x+c))/e
^2*(d*x+c)^2-1/6/e*(d*x+c)*exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+6*b^4/d^4*
e^2*c^2*(-1/3/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/6*exp(e/(d*x+c))/e^2*(d*x+c)^
2-1/6/e*(d*x+c)*exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+4*b/d*e*a^3*(-1/2*exp
(e/(d*x+c))/e^2*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)
))-4*b^4/d^4*e*c^3*(-1/2*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/
(d*x+c))-1/2*Ei(1,-e/(d*x+c)))-4*b/d*c*a^3*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei
(1,-e/(d*x+c)))+6*b^2/d^2*c^2*a^2*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*
x+c)))-4*b^3/d^3*c^3*a*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))-12*b
^3/d^3*e^2*c*a*(-1/3/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/6*exp(e/(d*x+c))/e^2*(
d*x+c)^2-1/6/e*(d*x+c)*exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))-12*b^2/d^2*...
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.84

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx =$$

$$-\frac{(b^4e^5 - 20\,(b^4c - ab^3d)e^4 + 120\,(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)e^3 - 240\,(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3}{}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/120*((b^4*e^5 - 20*(b^4*c - a*b^3*d)*e^4 + 120*(b^4*c^2 - 2*a*b^3*c*d +
 a^2*b^2*d^2)*e^3 - 240*(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b
*d^3)*e^2 + 120*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d
^3 + a^4*d^4)*e)*Ei(e/(d*x + c)) - (24*b^4*d^5*x^5 + 24*b^4*c^5 - 120*a*b^
3*c^4*d + 240*a^2*b^2*c^3*d^2 - 240*a^3*b*c^2*d^3 + 120*a^4*c*d^4 + b^4*c*
e^4 + 6*(20*a*b^3*d^5 + b^4*d^4*e)*x^4 - (19*b^4*c^2 - 20*a*b^3*c*d)*e^3 +
 2*(120*a^2*b^2*d^5 + b^4*d^3*e^2 - 4*(2*b^4*c*d^3 - 5*a*b^3*d^4)*e)*x^3 +
 2*(51*b^4*c^3 - 110*a*b^3*c^2*d + 60*a^2*b^2*c*d^2)*e^2 + (240*a^3*b*d^5
+ b^4*d^2*e^3 - 2*(7*b^4*c*d^2 - 10*a*b^3*d^3)*e^2 + 12*(3*b^4*c^2*d^2 - 1
0*a*b^3*c*d^3 + 10*a^2*b^2*d^4)*e)*x^2 - 2*(77*b^4*c^4 - 260*a*b^3*c^3*d +
 300*a^2*b^2*c^2*d^2 - 120*a^3*b*c*d^3)*e + (120*a^4*d^5 + b^4*d*e^4 - 2*(
9*b^4*c*d - 10*a*b^3*d^2)*e^3 + 2*(43*b^4*c^2*d - 100*a*b^3*c*d^2 + 60*a^2
*b^2*d^3)*e^2 - 24*(4*b^4*c^3*d - 15*a*b^3*c^2*d^2 + 20*a^2*b^2*c*d^3 - 10
*a^3*b*d^4)*e)*x)*e^(e/(d*x + c)))/d^5
```

Sympy [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx=\int (a+bx)^4\,e^{\frac{e}{c+dx}}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)**4,x)
```

output
```
Integral((a + b*x)**4*exp(e/(c + d*x)), x)
```

Maxima [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx=\int (bx+a)^4e^{\left(\frac{e}{dx+c}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/120*(24*b^4*d^4*x^5 + 6*(20*a*b^3*d^4 + b^4*d^3*e)*x^4 + 2*(120*a^2*b^2*
d^4 + 20*a*b^3*d^3*e - (8*c*d^2*e - d^2*e^2)*b^4)*x^3 + (240*a^3*b*d^4 + 1
20*a^2*b^2*d^3*e - 20*(6*c*d^2*e - d^2*e^2)*a*b^3 + (36*c^2*d*e - 14*c*d*e
^2 + d*e^3)*b^4)*x^2 + (120*a^4*d^4 + 240*a^3*b*d^3*e - 120*(4*c*d^2*e - d
^2*e^2)*a^2*b^2 + 20*(18*c^2*d*e - 10*c*d*e^2 + d*e^3)*a*b^3 - (96*c^3*e -
 86*c^2*e^2 + 18*c*e^3 - e^4)*b^4)*x)*e^(e/(d*x + c))/d^4 + integrate(-1/1
20*(240*a^3*b*c^2*d^3*e - 120*(4*c^3*d^2*e - c^2*d^2*e^2)*a^2*b^2 + 20*(18
*c^4*d*e - 10*c^3*d*e^2 + c^2*d*e^3)*a*b^3 - (96*c^5*e - 86*c^4*e^2 + 18*c
^3*e^3 - c^2*e^4)*b^4 - (120*a^4*d^5*e - 240*(2*c*d^4*e - d^4*e^2)*a^3*b +
 120*(6*c^2*d^3*e - 6*c*d^3*e^2 + d^3*e^3)*a^2*b^2 - 20*(24*c^3*d^2*e - 36
*c^2*d^2*e^2 + 12*c*d^2*e^3 - d^2*e^4)*a*b^3 + (120*c^4*d*e - 240*c^3*d*e^
2 + 120*c^2*d*e^3 - 20*c*d*e^4 + d*e^5)*b^4)*x)*e^(e/(d*x + c))/(d^6*x^2 +
 2*c*d^5*x + c^2*d^4), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1428 vs. $2(340) = 680$.

Time = 0.16 (sec) , antiderivative size = 1428, normalized size of antiderivative = 4.13

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx = \text{Too large to display}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/120*(120*b^4*c^4*e^7*Ei(e/(d*x + c))/(d*x + c)^5 - 480*a*b^3*c^3*d*e^7*
Ei(e/(d*x + c))/(d*x + c)^5 + 720*a^2*b^2*c^2*d^2*e^7*Ei(e/(d*x + c))/(d*x
 + c)^5 - 480*a^3*b*c*d^3*e^7*Ei(e/(d*x + c))/(d*x + c)^5 + 120*a^4*d^4*e^
7*Ei(e/(d*x + c))/(d*x + c)^5 - 240*b^4*c^3*e^8*Ei(e/(d*x + c))/(d*x + c)^
5 + 720*a*b^3*c^2*d*e^8*Ei(e/(d*x + c))/(d*x + c)^5 - 720*a^2*b^2*c*d^2*e^
8*Ei(e/(d*x + c))/(d*x + c)^5 + 240*a^3*b*d^3*e^8*Ei(e/(d*x + c))/(d*x + c
)^5 + 120*b^4*c^2*e^9*Ei(e/(d*x + c))/(d*x + c)^5 - 240*a*b^3*c*d*e^9*Ei(e
/(d*x + c))/(d*x + c)^5 + 120*a^2*b^2*d^2*e^9*Ei(e/(d*x + c))/(d*x + c)^5
- 20*b^4*c*e^10*Ei(e/(d*x + c))/(d*x + c)^5 + 20*a*b^3*d*e^10*Ei(e/(d*x +
c))/(d*x + c)^5 + b^4*e^11*Ei(e/(d*x + c))/(d*x + c)^5 - 24*b^4*e^6*e^(e/(
d*x + c)) + 120*b^4*c*e^6*e^(e/(d*x + c))/(d*x + c) - 240*b^4*c^2*e^6*e^(e
/(d*x + c))/(d*x + c)^2 + 240*b^4*c^3*e^6*e^(e/(d*x + c))/(d*x + c)^3 - 12
0*b^4*c^4*e^6*e^(e/(d*x + c))/(d*x + c)^4 - 120*a*b^3*d*e^6*e^(e/(d*x + c)
)/(d*x + c) + 480*a*b^3*c*d*e^6*e^(e/(d*x + c))/(d*x + c)^2 - 720*a*b^3*c^
2*d*e^6*e^(e/(d*x + c))/(d*x + c)^3 + 480*a*b^3*c^3*d*e^6*e^(e/(d*x + c))/
(d*x + c)^4 - 240*a^2*b^2*d^2*e^6*e^(e/(d*x + c))/(d*x + c)^2 + 720*a^2*b^
2*c*d^2*e^6*e^(e/(d*x + c))/(d*x + c)^3 - 720*a^2*b^2*c^2*d^2*e^6*e^(e/(d*
x + c))/(d*x + c)^4 - 240*a^3*b*d^3*e^6*e^(e/(d*x + c))/(d*x + c)^3 + 480*
a^3*b*c*d^3*e^6*e^(e/(d*x + c))/(d*x + c)^4 - 120*a^4*d^4*e^6*e^(e/(d*x +
c))/(d*x + c)^4 - 6*b^4*e^7*e^(e/(d*x + c))/(d*x + c) + 40*b^4*c*e^7*e^...
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx=\int \mathrm{e}^{\frac{e}{c+d\,x}}\,{\left(a+b\,x\right)}^4\,dx$$

input
```
int(exp(e/(c + d*x))*(a + b*x)^4,x)
```

output
```
int(exp(e/(c + d*x))*(a + b*x)^4, x)
```

Reduce [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^4\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c))*(b*x+a)^4,x)
```

output
```
( - 120*e**(e/(c + d*x))*a**4*c**3*d**4 - 120*e**(e/(c + d*x))*a**4*c**2*d
**5*x + 120*e**(e/(c + d*x))*a**4*d**6*e*x**2 + 480*e**(e/(c + d*x))*a**3*
b*c**4*d**3 + 480*e**(e/(c + d*x))*a**3*b*c**3*d**4*x + 480*e**(e/(c + d*x
))*a**3*b*c**2*d**4*e*x + 240*e**(e/(c + d*x))*a**3*b*c*d**5*e*x**2 + 240*
e**(e/(c + d*x))*a**3*b*d**6*e*x**3 + 240*e**(e/(c + d*x))*a**3*b*d**5*e**
2*x**2 - 720*e**(e/(c + d*x))*a**2*b**2*c**5*d**2 - 720*e**(e/(c + d*x))*a
**2*b**2*c**4*d**3*x + 240*e**(e/(c + d*x))*a**2*b**2*c**4*d**2*e - 480*e*
*(e/(c + d*x))*a**2*b**2*c**3*d**3*e*x + 240*e**(e/(c + d*x))*a**2*b**2*c*
*2*d**3*e**2*x + 240*e**(e/(c + d*x))*a**2*b**2*c*d**5*e*x**3 - 360*e**(e/
(c + d*x))*a**2*b**2*c*d**4*e**2*x**2 + 240*e**(e/(c + d*x))*a**2*b**2*d**
6*e*x**4 + 120*e**(e/(c + d*x))*a**2*b**2*d**5*e**2*x**3 + 120*e**(e/(c +
d*x))*a**2*b**2*d**4*e**3*x**2 + 480*e**(e/(c + d*x))*a*b**3*c**6*d + 480*
e**(e/(c + d*x))*a*b**3*c**5*d**2*x - 360*e**(e/(c + d*x))*a*b**3*c**5*d*e
 + 120*e**(e/(c + d*x))*a*b**3*c**4*d**2*e*x + 40*e**(e/(c + d*x))*a*b**3*
c**4*d*e**2 - 320*e**(e/(c + d*x))*a*b**3*c**3*d**2*e**2*x + 240*e**(e/(c
+ d*x))*a*b**3*c**2*d**3*e**2*x**2 + 40*e**(e/(c + d*x))*a*b**3*c**2*d**2*
e**3*x + 120*e**(e/(c + d*x))*a*b**3*c*d**5*e*x**4 - 80*e**(e/(c + d*x))*a
*b**3*c*d**4*e**2*x**3 - 180*e**(e/(c + d*x))*a*b**3*c*d**3*e**3*x**2 + 12
0*e**(e/(c + d*x))*a*b**3*d**6*e*x**5 + 40*e**(e/(c + d*x))*a*b**3*d**5*e*
*2*x**4 + 20*e**(e/(c + d*x))*a*b**3*d**4*e**3*x**3 + 20*e**(e/(c + d*x...
```

3.336 $\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 320

$$\begin{aligned}\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx =& -\frac{(bc-ad)^3e^{\frac{e}{c+dx}}(c+dx)}{d^4}+\frac{3b(bc-ad)^2ee^{\frac{e}{c+dx}}(c+dx)}{2d^4}\\&-\frac{b^2(bc-ad)e^2e^{\frac{e}{c+dx}}(c+dx)}{2d^4}+\frac{3b(bc-ad)^2e^{\frac{e}{c+dx}}(c+dx)^2}{2d^4}\\&-\frac{b^2(bc-ad)ee^{\frac{e}{c+dx}}(c+dx)^2}{2d^4}-\frac{b^2(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^3}{d^4}\\&+\frac{(bc-ad)^3e\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^4}\\&-\frac{3b(bc-ad)^2e^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^4}\\&+\frac{b^2(bc-ad)e^3\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^4}+\frac{b^3e^4\Gamma\left(-4,-\frac{e}{c+dx}\right)}{d^4}\end{aligned}$$

output

```
-(-a*d+b*c)^3*exp(e/(d*x+c))*(d*x+c)/d^4+3/2*b*(-a*d+b*c)^2*e*exp(e/(d*x+c
))*(d*x+c)/d^4-1/2*b^2*(-a*d+b*c)*e^2*exp(e/(d*x+c))*(d*x+c)/d^4+3/2*b*(-a
*d+b*c)^2*exp(e/(d*x+c))*(d*x+c)^2/d^4-1/2*b^2*(-a*d+b*c)*e*exp(e/(d*x+c))
*(d*x+c)^2/d^4-b^2*(-a*d+b*c)*exp(e/(d*x+c))*(d*x+c)^3/d^4+(-a*d+b*c)^3*e*
Ei(e/(d*x+c))/d^4-3/2*b*(-a*d+b*c)^2*e^2*Ei(e/(d*x+c))/d^4+1/2*b^2*(-a*d+b
*c)*e^3*Ei(e/(d*x+c))/d^4+b^3*(d*x+c)^4*Ei(5,-e/(d*x+c))/d^4
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.91

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx = -\frac{c(-24a^3d^3+36a^2bd^2(c-e)-12ab^2d(2c^2-5ce+e^2)+b^3(6c^3-26c^2e+11ce^2-e^3))\,e^{\frac{e}{c+dx}}}{24d^4} + \frac{de^{\frac{e}{c+dx}}x(24a^3d^3+36a^2bd^2(e+dx)+12ab^2d(-4ce+e^2+dex+2d^2x^2)+b^3(18c^2e+e^3+de^2x+2d^2\cdots}{}$$

input
```
Integrate[E^(e/(c + d*x))*(a + b*x)^3,x]
```

output
```
-1/24*(c*(-24*a^3*d^3 + 36*a^2*b*d^2*(c - e) - 12*a*b^2*d*(2*c^2 - 5*c*e +
 e^2) + b^3*(6*c^3 - 26*c^2*e + 11*c*e^2 - e^3))*E^(e/(c + d*x)))/d^4 + (d
*E^(e/(c + d*x))*x*(24*a^3*d^3 + 36*a^2*b*d^2*(e + d*x) + 12*a*b^2*d*(-4*c
*e + e^2 + d*e*x + 2*d^2*x^2) + b^3*(18*c^2*e + e^3 + d*e^2*x + 2*d^2*e*x^
2 + 6*d^3*x^3 - 2*c*e*(5*e + 3*d*x))) - e*(24*a^3*d^3 + 36*a^2*b*d^2*(-2*c
 + e) + 12*a*b^2*d*(6*c^2 - 6*c*e + e^2) + b^3*(-24*c^3 + 36*c^2*e - 12*c*
e^2 + e^3))*ExpIntegralEi[e/(c + d*x)])/(24*d^4)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 e^{\frac{e}{c+dx}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(-\frac{3b^2(c+dx)^2(bc-ad)e^{\frac{e}{c+dx}}}{d^3}+\frac{(ad-bc)^3e^{\frac{e}{c+dx}}}{d^3}+\frac{3b(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^3}+\frac{b^3(c+dx)^3e^{\frac{e}{c+dx}}}{d^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2e^3(bc-ad)\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^4}-\frac{b^2e^2(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{2d^4}-\frac{b^2e(c+dx)^2(bc-ad)e^{\frac{e}{c+dx}}}{2d^4}-\frac{b^2(c+dx)^3(bc-ad)e^{\frac{e}{c+dx}}}{d^4}-\frac{3be^2(bc-ad)^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^4}+\frac{e(bc-ad)^3\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^4}+\frac{3be(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{2d^4}+\frac{3b(c+dx)^2(bc-ad)^2e^{\frac{e}{c+dx}}}{2d^4}-\frac{(c+dx)(bc-ad)^3e^{\frac{e}{c+dx}}}{d^4}+\frac{b^3e^4\Gamma\left(-4,-\frac{e}{c+dx}\right)}{d^4}$$

input
```
Int[E^(e/(c + d*x))*(a + b*x)^3,x]
```

output
```
-(((b*c - a*d)^3*E^(e/(c + d*x))*(c + d*x))/d^4) + (3*b*(b*c - a*d)^2*e*E^
(e/(c + d*x))*(c + d*x))/(2*d^4) - (b^2*(b*c - a*d)*e^2*E^(e/(c + d*x))*(c
 + d*x))/(2*d^4) + (3*b*(b*c - a*d)^2*E^(e/(c + d*x))*(c + d*x)^2)/(2*d^4)
 - (b^2*(b*c - a*d)*e*E^(e/(c + d*x))*(c + d*x)^2)/(2*d^4) - (b^2*(b*c - a
*d)*E^(e/(c + d*x))*(c + d*x)^3)/d^4 + ((b*c - a*d)^3*e*ExpIntegralEi[e/(c
 + d*x)])/d^4 - (3*b*(b*c - a*d)^2*e^2*ExpIntegralEi[e/(c + d*x)])/(2*d^4)
 + (b^2*(b*c - a*d)*e^3*ExpIntegralEi[e/(c + d*x)])/(2*d^4) + (b^3*e^4*Gam
ma[-4, -(e/(c + d*x))])/d^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(306) = 612$.

Time = 0.29 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.13

method	result
derivativedivides	$-\frac{e\left(a^3\left(-\frac{(dx+c)\mathrm{e}^{\frac{e}{dx+c}}}{e}-\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)+\frac{b^3e^3\left(-\frac{(dx+c)^4\mathrm{e}^{\frac{e}{dx+c}}}{4e^4}-\frac{(dx+c)^3\mathrm{e}^{\frac{e}{dx+c}}}{12e^3}-\frac{\mathrm{e}^{\frac{e}{dx+c}}(dx+c)^2}{24e^2}-\frac{(dx+c)\mathrm{e}\cdots}{24\cdots}\cdots\right.}{d^3}\cdots}{\cdots}$
default	$-\frac{e\left(a^3\left(-\frac{(dx+c)\mathrm{e}^{\frac{e}{dx+c}}}{e}-\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)+\frac{b^3e^3\left(-\frac{(dx+c)^4\mathrm{e}^{\frac{e}{dx+c}}}{4e^4}-\frac{(dx+c)^3\mathrm{e}^{\frac{e}{dx+c}}}{12e^3}-\frac{\mathrm{e}^{\frac{e}{dx+c}}(dx+c)^2}{24e^2}-\frac{(dx+c)\mathrm{e}\cdots}{24\cdots}\cdots\right.}{d^3}\cdots}{\cdots}$
risch	$\frac{\mathrm{e}^{\frac{e}{dx+c}}b^3x^4}{4}+\mathrm{e}^{\frac{e}{dx+c}}a^3x+\frac{3\,\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)b^3c^2e^2}{2d^4}-\frac{\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)b^3c\,e^3}{2d^4}+\frac{\mathrm{e}^{\frac{e}{dx+c}}b^3e\,x^3}{12d}+\cdots$
parts	Expression too large to display

input
```
int(exp(e/(d*x+c))*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*(a^3*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+b^3/d^3*e^3*(-1
/4/e^4*(d*x+c)^4*exp(e/(d*x+c))-1/12/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/24*exp
(e/(d*x+c))/e^2*(d*x+c)^2-1/24/e*(d*x+c)*exp(e/(d*x+c))-1/24*Ei(1,-e/(d*x+
c)))-b^3/d^3*c^3*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+3*b^2/d^2*
e^2*a*(-1/3/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/6*exp(e/(d*x+c))/e^2*(d*x+c)^2-
1/6/e*(d*x+c)*exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))-3*b^3/d^3*e^2*c*(-1/3/e
^3*(d*x+c)^3*exp(e/(d*x+c))-1/6*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/6/e*(d*x+c)
*exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+3*b/d*e*a^2*(-1/2*exp(e/(d*x+c))/e^2
*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))+3*b^3/d^3*e*
c^2*(-1/2*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/(d*x+c))-1/2*Ei
(1,-e/(d*x+c)))-3*b/d*c*a^2*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))
+3*b^2/d^2*c^2*a*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))-6*b^2/d^2*
e*c*a*(-1/2*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/(d*x+c))-1/2*
Ei(1,-e/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.18

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx =$$

$$-\frac{(b^3e^4 - 12\,(b^3c - ab^2d)e^3 + 36\,(b^3c^2 - 2\,ab^2cd + a^2bd^2)e^2 - 24\,(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)e)\mathrm{Ei}}{}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/24*((b^3*e^4 - 12*(b^3*c - a*b^2*d)*e^3 + 36*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*e^2 - 24*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*e)*
Ei(e/(d*x + c)) - (6*b^3*d^4*x^4 - 6*b^3*c^4 + 24*a*b^2*c^3*d - 36*a^2*b*c
^2*d^2 + 24*a^3*c*d^3 + b^3*c*e^3 + 2*(12*a*b^2*d^4 + b^3*d^3*e)*x^3 - (11
*b^3*c^2 - 12*a*b^2*c*d)*e^2 + (36*a^2*b*d^4 + b^3*d^2*e^2 - 6*(b^3*c*d^2
- 2*a*b^2*d^3)*e)*x^2 + 2*(13*b^3*c^3 - 30*a*b^2*c^2*d + 18*a^2*b*c*d^2)*e
 + (24*a^3*d^4 + b^3*d*e^3 - 2*(5*b^3*c*d - 6*a*b^2*d^2)*e^2 + 6*(3*b^3*c^
2*d - 8*a*b^2*c*d^2 + 6*a^2*b*d^3)*e)*x)*e^(e/(d*x + c)))/d^4
```

Sympy [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx = \int (a+bx)^3\,e^{\frac{e}{c+dx}}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)**3,x)
```

output
```
Integral((a + b*x)**3*exp(e/(c + d*x)), x)
```

Maxima [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx=\int (bx+a)^3e^{\left(\frac{e}{dx+c}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*(6*b^3*d^3*x^4 + 2*(12*a*b^2*d^3 + b^3*d^2*e)*x^3 + (36*a^2*b*d^3 + 1
2*a*b^2*d^2*e - (6*c*d*e - d*e^2)*b^3)*x^2 + (24*a^3*d^3 + 36*a^2*b*d^2*e
- 12*(4*c*d*e - d*e^2)*a*b^2 + (18*c^2*e - 10*c*e^2 + e^3)*b^3)*x)*e^(e/(d
*x + c))/d^3 + integrate(-1/24*(36*a^2*b*c^2*d^2*e - 12*(4*c^3*d*e - c^2*d
*e^2)*a*b^2 + (18*c^4*e - 10*c^3*e^2 + c^2*e^3)*b^3 - (24*a^3*d^4*e - 36*(
2*c*d^3*e - d^3*e^2)*a^2*b + 12*(6*c^2*d^2*e - 6*c*d^2*e^2 + d^2*e^3)*a*b^
2 - (24*c^3*d*e - 36*c^2*d*e^2 + 12*c*d*e^3 - d*e^4)*b^3)*x)*e^(e/(d*x + c
))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(302) = 604.

Time = 0.16 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.60

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx=\text{Too large to display}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*(24*b^3*c^3*e^6*Ei(e/(d*x + c))/(d*x + c)^4 - 72*a*b^2*c^2*d*e^6*Ei(e
/(d*x + c))/(d*x + c)^4 + 72*a^2*b*c*d^2*e^6*Ei(e/(d*x + c))/(d*x + c)^4 -
 24*a^3*d^3*e^6*Ei(e/(d*x + c))/(d*x + c)^4 - 36*b^3*c^2*e^7*Ei(e/(d*x + c
))/(d*x + c)^4 + 72*a*b^2*c*d*e^7*Ei(e/(d*x + c))/(d*x + c)^4 - 36*a^2*b*d
^2*e^7*Ei(e/(d*x + c))/(d*x + c)^4 + 12*b^3*c*e^8*Ei(e/(d*x + c))/(d*x + c
)^4 - 12*a*b^2*d*e^8*Ei(e/(d*x + c))/(d*x + c)^4 - b^3*e^9*Ei(e/(d*x + c))
/(d*x + c)^4 + 6*b^3*e^5*e^(e/(d*x + c)) - 24*b^3*c*e^5*e^(e/(d*x + c))/(d
*x + c) + 36*b^3*c^2*e^5*e^(e/(d*x + c))/(d*x + c)^2 - 24*b^3*c^3*e^5*e^(e
/(d*x + c))/(d*x + c)^3 + 24*a*b^2*d*e^5*e^(e/(d*x + c))/(d*x + c) - 72*a*
b^2*c*d*e^5*e^(e/(d*x + c))/(d*x + c)^2 + 72*a*b^2*c^2*d*e^5*e^(e/(d*x + c
))/(d*x + c)^3 + 36*a^2*b*d^2*e^5*e^(e/(d*x + c))/(d*x + c)^2 - 72*a^2*b*c
*d^2*e^5*e^(e/(d*x + c))/(d*x + c)^3 + 24*a^3*d^3*e^5*e^(e/(d*x + c))/(d*x
 + c)^3 + 2*b^3*e^6*e^(e/(d*x + c))/(d*x + c) - 12*b^3*c*e^6*e^(e/(d*x + c
))/(d*x + c)^2 + 36*b^3*c^2*e^6*e^(e/(d*x + c))/(d*x + c)^3 + 12*a*b^2*d*e
^6*e^(e/(d*x + c))/(d*x + c)^2 - 72*a*b^2*c*d*e^6*e^(e/(d*x + c))/(d*x + c
)^3 + 36*a^2*b*d^2*e^6*e^(e/(d*x + c))/(d*x + c)^3 + b^3*e^7*e^(e/(d*x + c
))/(d*x + c)^2 - 12*b^3*c*e^7*e^(e/(d*x + c))/(d*x + c)^3 + 12*a*b^2*d*e^7
*e^(e/(d*x + c))/(d*x + c)^3 + b^3*e^8*e^(e/(d*x + c))/(d*x + c)^3)*(d*x +
 c)^4/(d^4*e^5)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx = \int \mathrm{e}^{\frac{e}{c+d\,x}}\,(a+b\,x)^3\,dx$$

input
```
int(exp(e/(c + d*x))*(a + b*x)^3,x)
```

output
```
int(exp(e/(c + d*x))*(a + b*x)^3, x)
```

Reduce [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^3\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c))*(b*x+a)^3,x)
```

output
```
( - 24*e**(e/(c + d*x))*a**3*c**3*d**3 - 24*e**(e/(c + d*x))*a**3*c**2*d**
4*x + 24*e**(e/(c + d*x))*a**3*d**5*e*x**2 + 72*e**(e/(c + d*x))*a**2*b*c*
*4*d**2 + 72*e**(e/(c + d*x))*a**2*b*c**3*d**3*x + 72*e**(e/(c + d*x))*a**
2*b*c**2*d**3*e*x + 36*e**(e/(c + d*x))*a**2*b*c*d**4*e*x**2 + 36*e**(e/(c
 + d*x))*a**2*b*d**5*e*x**3 + 36*e**(e/(c + d*x))*a**2*b*d**4*e**2*x**2 -
72*e**(e/(c + d*x))*a*b**2*c**5*d - 72*e**(e/(c + d*x))*a*b**2*c**4*d**2*x
 + 24*e**(e/(c + d*x))*a*b**2*c**4*d*e - 48*e**(e/(c + d*x))*a*b**2*c**3*d
**2*e*x + 24*e**(e/(c + d*x))*a*b**2*c**2*d**2*e**2*x + 24*e**(e/(c + d*x)
)*a*b**2*c*d**4*e*x**3 - 36*e**(e/(c + d*x))*a*b**2*c*d**3*e**2*x**2 + 24*
e**(e/(c + d*x))*a*b**2*d**5*e*x**4 + 12*e**(e/(c + d*x))*a*b**2*d**4*e**2
*x**3 + 12*e**(e/(c + d*x))*a*b**2*d**3*e**3*x**2 + 24*e**(e/(c + d*x))*b*
*3*c**6 + 24*e**(e/(c + d*x))*b**3*c**5*d*x - 18*e**(e/(c + d*x))*b**3*c**
5*e + 6*e**(e/(c + d*x))*b**3*c**4*d*e*x + 2*e**(e/(c + d*x))*b**3*c**4*e*
*2 - 16*e**(e/(c + d*x))*b**3*c**3*d*e**2*x + 12*e**(e/(c + d*x))*b**3*c**
2*d**2*e**2*x**2 + 2*e**(e/(c + d*x))*b**3*c**2*d*e**3*x + 6*e**(e/(c + d*
x))*b**3*c*d**4*e*x**4 - 4*e**(e/(c + d*x))*b**3*c*d**3*e**2*x**3 - 9*e**(
e/(c + d*x))*b**3*c*d**2*e**3*x**2 + 6*e**(e/(c + d*x))*b**3*d**5*e*x**5 +
 2*e**(e/(c + d*x))*b**3*d**4*e**2*x**4 + e**(e/(c + d*x))*b**3*d**3*e**3*
x**3 + e**(e/(c + d*x))*b**3*d**2*e**4*x**2 + 24*int((e**(e/(c + d*x))*x**
2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**3*c*d**6*e**2 ...
```

3.337 $\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 255

$$\begin{aligned}\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx =& \frac{(bc-ad)^2e^{\frac{e}{c+dx}}(c+dx)}{d^3} - \frac{b(bc-ad)ee^{\frac{e}{c+dx}}(c+dx)}{d^3} \\ &+ \frac{b^2e^2e^{\frac{e}{c+dx}}(c+dx)}{6d^3} - \frac{b(bc-ad)e^{\frac{e}{c+dx}}(c+dx)^2}{d^3} \\ &+ \frac{b^2ee^{\frac{e}{c+dx}}(c+dx)^2}{6d^3} + \frac{b^2e^{\frac{e}{c+dx}}(c+dx)^3}{3d^3} \\ &- \frac{(bc-ad)^2e\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^3} \\ &+ \frac{b(bc-ad)e^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^3} - \frac{b^2e^3\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{6d^3}\end{aligned}$$

output
```
(-a*d+b*c)^2*exp(e/(d*x+c))*(d*x+c)/d^3-b*(-a*d+b*c)*e*exp(e/(d*x+c))*(d*x
+c)/d^3+1/6*b^2*e^2*exp(e/(d*x+c))*(d*x+c)/d^3-b*(-a*d+b*c)*exp(e/(d*x+c))
*(d*x+c)^2/d^3+1/6*b^2*e*exp(e/(d*x+c))*(d*x+c)^2/d^3+1/3*b^2*exp(e/(d*x+c
))*(d*x+c)^3/d^3-(-a*d+b*c)^2*e*Ei(e/(d*x+c))/d^3+b*(-a*d+b*c)*e^2*Ei(e/(d
*x+c))/d^3-1/6*b^2*e^3*Ei(e/(d*x+c))/d^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.67

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx = \frac{c(6a^2d^2+6abd(-c+e)+b^2(2c^2-5ce+e^2))\,e^{\frac{e}{c+dx}}}{6d^3} + \frac{de^{\frac{e}{c+dx}}x(6a^2d^2+6abd(e+dx)+b^2(-4ce+e^2+dex+2d^2x^2))-e(6a^2d^2+6abd(-2c+e)+b^2(6c^2 -}{6d^3}$$

input
```
Integrate[E^(e/(c + d*x))*(a + b*x)^2,x]
```

output
```
(c*(6*a^2*d^2 + 6*a*b*d*(-c + e) + b^2*(2*c^2 - 5*c*e + e^2))*E^(e/(c + d*
x)))/(6*d^3) + (d*E^(e/(c + d*x))*x*(6*a^2*d^2 + 6*a*b*d*(e + d*x) + b^2*(
-4*c*e + e^2 + d*e*x + 2*d^2*x^2)) - e*(6*a^2*d^2 + 6*a*b*d*(-2*c + e) + b
^2*(6*c^2 - 6*c*e + e^2))*ExpIntegralEi[e/(c + d*x)])/(6*d^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2e^{\frac{e}{c+dx}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)^2e^{\frac{e}{c+dx}}}{d^2}-\frac{2b(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^2}+\frac{b^2(c+dx)^2e^{\frac{e}{c+dx}}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{be^2(bc-ad)\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^3}-\frac{e(bc-ad)^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^3}-\frac{be(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^3}-\frac{b(c+dx)^2(bc-ad)e^{\frac{e}{c+dx}}}{d^3}+\frac{(c+dx)(bc-ad)^2e^{\frac{e}{c+dx}}}{d^3}-\frac{b^2e^3\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{6d^3}+\frac{b^2e^2(c+dx)e^{\frac{e}{c+dx}}}{6d^3}+\frac{b^2e(c+dx)^2e^{\frac{e}{c+dx}}}{6d^3}+\frac{b^2(c+dx)^3e^{\frac{e}{c+dx}}}{3d^3}$$

input
```
Int[E^(e/(c + d*x))*(a + b*x)^2,x]
```

output
```
((b*c - a*d)^2*E^(e/(c + d*x))*(c + d*x))/d^3 - (b*(b*c - a*d)*e*E^(e/(c +
 d*x))*(c + d*x))/d^3 + (b^2*e^2*E^(e/(c + d*x))*(c + d*x))/(6*d^3) - (b*(
b*c - a*d)*E^(e/(c + d*x))*(c + d*x)^2)/d^3 + (b^2*e*E^(e/(c + d*x))*(c +
d*x)^2)/(6*d^3) + (b^2*E^(e/(c + d*x))*(c + d*x)^3)/(3*d^3) - ((b*c - a*d)
^2*e*ExpIntegralEi[e/(c + d*x)])/d^3 + (b*(b*c - a*d)*e^2*ExpIntegralEi[e/
(c + d*x)])/d^3 - (b^2*e^3*ExpIntegralEi[e/(c + d*x)])/(6*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.40

method	result
derivativedivides	$-\frac{e\left(a^2\left(-\frac{(dx+c)\mathrm{e}^{\frac{e}{dx+c}}}{e}-\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)+\frac{b^2e^2\left(-\frac{(dx+c)^3\mathrm{e}^{\frac{e}{dx+c}}}{3e^3}-\frac{\mathrm{e}^{\frac{e}{dx+c}}(dx+c)^2}{6e^2}-\frac{(dx+c)\mathrm{e}^{\frac{e}{dx+c}}}{6e}-\mathrm{expIntegr}\right.}{d^2}\right.}{}$
default	$-\frac{e\left(a^2\left(-\frac{(dx+c)\mathrm{e}^{\frac{e}{dx+c}}}{e}-\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)+\frac{b^2e^2\left(-\frac{(dx+c)^3\mathrm{e}^{\frac{e}{dx+c}}}{3e^3}-\frac{\mathrm{e}^{\frac{e}{dx+c}}(dx+c)^2}{6e^2}-\frac{(dx+c)\mathrm{e}^{\frac{e}{dx+c}}}{6e}-\mathrm{expIntegr}\right.}{d^2}\right.}{}$
risch	$-\frac{5e\,b^2\mathrm{e}^{\frac{e}{dx+c}}c^2}{6d^3}+ab\,\mathrm{e}^{\frac{e}{dx+c}}x^2+\frac{b^2\mathrm{e}^{\frac{e}{dx+c}}x^3}{3}+\frac{b^2\mathrm{e}^{\frac{e}{dx+c}}c^3}{3d^3}-\frac{2e\,b^2\mathrm{e}^{\frac{e}{dx+c}}cx}{3d^2}+\frac{e^2b^2\mathrm{e}^{\frac{e}{dx+c}}x}{6d^2}+\frac{e^2b^2\mathrm{e}^{\frac{e}{dx+c}}c}{6d^3}$
parts	$b^2\mathrm{e}^{\frac{e}{dx+c}}x^3+2ab\,\mathrm{e}^{\frac{e}{dx+c}}x^2+a^2\mathrm{e}^{\frac{e}{dx+c}}x+\frac{b^2\mathrm{e}^{\frac{e}{dx+c}}c\,x^2}{d}+\frac{2ab\,\mathrm{e}^{\frac{e}{dx+c}}cx}{d}+\frac{a^2\mathrm{e}^{\frac{e}{dx+c}}c}{d}+\frac{e\,\mathrm{expIntegral}_1(}{d}$

input
```
int(exp(e/(d*x+c))*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*(a^2*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+b^2/d^2*e^2*(-1
/3/e^3*(d*x+c)^3*exp(e/(d*x+c))-1/6*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/6/e*(d*
x+c)*exp(e/(d*x+c))-1/6*Ei(1,-e/(d*x+c)))+b^2/d^2*c^2*(-1/e*(d*x+c)*exp(e/
(d*x+c))-Ei(1,-e/(d*x+c)))+2*b/d*e*a*(-1/2*exp(e/(d*x+c))/e^2*(d*x+c)^2-1/
2/e*(d*x+c)*exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))-2*b^2/d^2*e*c*(-1/2*exp(e
/(d*x+c))/e^2*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))
-2*b/d*c*a*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.77

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx =$$

$$-\frac{(b^2e^3-6\,(b^2c-abd)e^2+6\,(b^2c^2-2\,abcd+a^2d^2)e)\mathrm{Ei}\left(\frac{e}{dx+c}\right)-(2\,b^2d^3x^3+2\,b^2c^3-6\,abc^2d+6\,a^2cd^2}{6\,d}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*((b^2*e^3 - 6*(b^2*c - a*b*d)*e^2 + 6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*e)*Ei(e/(d*x + c)) - (2*b^2*d^3*x^3 + 2*b^2*c^3 - 6*a*b*c^2*d + 6*a^2*c*d
^2 + b^2*c*e^2 + (6*a*b*d^3 + b^2*d^2*e)*x^2 - (5*b^2*c^2 - 6*a*b*c*d)*e +
 (6*a^2*d^3 + b^2*d*e^2 - 2*(2*b^2*c*d - 3*a*b*d^2)*e)*x)*e^(e/(d*x + c)))
/d^3
```

Sympy [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx = \int (a+bx)^2\,e^{\frac{e}{c+dx}}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)**2,x)
```

output
```
Integral((a + b*x)**2*exp(e/(c + d*x)), x)
```

Maxima [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx = \int (bx+a)^2 e^{\left(\frac{e}{dx+c}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*(2*b^2*d^2*x^3 + (6*a*b*d^2 + b^2*d*e)*x^2 + (6*a^2*d^2 + 6*a*b*d*e -
(4*c*e - e^2)*b^2)*x)*e^(e/(d*x + c))/d^2 + integrate(-1/6*(6*a*b*c^2*d*e
- (4*c^3*e - c^2*e^2)*b^2 - (6*a^2*d^3*e - 6*(2*c*d^2*e - d^2*e^2)*a*b + (
6*c^2*d*e - 6*c*d*e^2 + d*e^3)*b^2)*x)*e^(e/(d*x + c))/(d^4*x^2 + 2*c*d^3*
x + c^2*d^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.67

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx = -\frac{\left(\frac{6\,b^2c^2e^5\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^3} - \frac{12\,abcde^5\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^3} + \frac{6\,a^2d^2e^5\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^3} - \frac{6\,b^2ce^6\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^3} + \frac{6\,abde^6\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^3} + \frac{b^2e^7\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^3} - 2\right.}{}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/6*(6*b^2*c^2*e^5*Ei(e/(d*x + c))/(d*x + c)^3 - 12*a*b*c*d*e^5*Ei(e/(d*x
 + c))/(d*x + c)^3 + 6*a^2*d^2*e^5*Ei(e/(d*x + c))/(d*x + c)^3 - 6*b^2*c*e
^6*Ei(e/(d*x + c))/(d*x + c)^3 + 6*a*b*d*e^6*Ei(e/(d*x + c))/(d*x + c)^3 +
 b^2*e^7*Ei(e/(d*x + c))/(d*x + c)^3 - 2*b^2*e^4*e^(e/(d*x + c)) + 6*b^2*c
*e^4*e^(e/(d*x + c))/(d*x + c) - 6*b^2*c^2*e^4*e^(e/(d*x + c))/(d*x + c)^2
 - 6*a*b*d*e^4*e^(e/(d*x + c))/(d*x + c) + 12*a*b*c*d*e^4*e^(e/(d*x + c))/
(d*x + c)^2 - 6*a^2*d^2*e^4*e^(e/(d*x + c))/(d*x + c)^2 - b^2*e^5*e^(e/(d*
x + c))/(d*x + c) + 6*b^2*c*e^5*e^(e/(d*x + c))/(d*x + c)^2 - 6*a*b*d*e^5*
e^(e/(d*x + c))/(d*x + c)^2 - b^2*e^6*e^(e/(d*x + c))/(d*x + c)^2)*(d*x +
c)^3/(d^3*e^4)
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.20

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx = \frac{x\,\mathrm{e}^{\frac{e}{c+d\,x}}\left(2\,a^2\,c+\frac{\frac{b^2\,c^3}{3}-d\,(a\,b\,c^2-2\,a\,b\,c\,e)+\frac{b^2\,c\,e^2}{3}-\frac{3\,b^2\,c^2\,e}{2}}{d^2}\right)+\frac{\mathrm{e}^{\frac{e}{c+d\,x}}\left(\frac{b^2\,c^4}{3}-d\,(a\,b\,c^3-a\,b\,c^2\,e)-\frac{5\,b^2\,c^3\,e}{6}+a^2\,c^2\,d^2+\frac{b^2\,c^2\,e^2}{6}\right)}{d^3}+x}{c+d\,x} - \frac{\mathrm{ei}\left(\frac{e}{c+d\,x}\right)\left(\frac{b^2\,e^3}{6}+d\,(a\,b\,e^2-2\,a\,b\,c\,e)+a^2\,d^2\,e-b^2\,c\,e^2+b^2\,c^2\,e\right)}{d^3}$$

input
```
int(exp(e/(c + d*x))*(a + b*x)^2,x)
```

output
```
(x*exp(e/(c + d*x))*(2*a^2*c + ((b^2*c^3)/3 - d*(a*b*c^2 - 2*a*b*c*e) + (b
^2*c*e^2)/3 - (3*b^2*c^2*e)/2)/d^2) + (exp(e/(c + d*x))*((b^2*c^4)/3 - d*(
a*b*c^3 - a*b*c^2*e) - (5*b^2*c^3*e)/6 + a^2*c^2*d^2 + (b^2*c^2*e^2)/6))/d
^3 + x^2*exp(e/(c + d*x))*(((b^2*e^2)/6 - (b^2*c*e)/2)/d + a^2*d + a*b*c +
 a*b*e) + (b^2*d*x^4*exp(e/(c + d*x)))/3 + (b*x^3*exp(e/(c + d*x))*(6*a*d
+ 2*b*c + b*e))/6)/(c + d*x) - (ei(e/(c + d*x))*((b^2*e^3)/6 + d*(a*b*e^2
- 2*a*b*c*e) + a^2*d^2*e - b^2*c*e^2 + b^2*c^2*e))/d^3
```

Reduce [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)^2\,dx = \text{Too large to display}$$

input
```
int(exp(e/(d*x+c))*(b*x+a)^2,x)
```

output
```
( - 6*e**(e/(c + d*x))*a**2*c**3*d**2 - 6*e**(e/(c + d*x))*a**2*c**2*d**3*
x + 6*e**(e/(c + d*x))*a**2*d**4*e*x**2 + 12*e**(e/(c + d*x))*a*b*c**4*d +
 12*e**(e/(c + d*x))*a*b*c**3*d**2*x + 12*e**(e/(c + d*x))*a*b*c**2*d**2*e
*x + 6*e**(e/(c + d*x))*a*b*c*d**3*e*x**2 + 6*e**(e/(c + d*x))*a*b*d**4*e*
x**3 + 6*e**(e/(c + d*x))*a*b*d**3*e**2*x**2 - 6*e**(e/(c + d*x))*b**2*c**
5 - 6*e**(e/(c + d*x))*b**2*c**4*d*x + 2*e**(e/(c + d*x))*b**2*c**4*e - 4*
e**(e/(c + d*x))*b**2*c**3*d*e*x + 2*e**(e/(c + d*x))*b**2*c**2*d*e**2*x +
 2*e**(e/(c + d*x))*b**2*c*d**3*e*x**3 - 3*e**(e/(c + d*x))*b**2*c*d**2*e*
*2*x**2 + 2*e**(e/(c + d*x))*b**2*d**4*e*x**4 + e**(e/(c + d*x))*b**2*d**3
*e**2*x**3 + e**(e/(c + d*x))*b**2*d**2*e**3*x**2 + 6*int((e**(e/(c + d*x)
)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a**2*c*d**5*e**
2 + 6*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3),x)*a**2*d**6*e**2*x - 12*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*c**2*d**4*e**2 - 12*int((e**(e/(
c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*c*d
**5*e**2*x + 6*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*a*b*c*d**4*e**3 + 6*int((e**(e/(c + d*x))*x**2)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*d**5*e**3*x + 6*int((e**(
e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2
*c**3*d**3*e**2 + 6*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*...
```

3.338 $\int e^{\frac{e}{c+dx}}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 125

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = -\frac{(bc-ad)e^{\frac{e}{c+dx}}(c+dx)}{d^2}+\frac{bee^{\frac{e}{c+dx}}(c+dx)}{2d^2}+\frac{be^{\frac{e}{c+dx}}(c+dx)^2}{2d^2}+\frac{(bc-ad)e\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^2}-\frac{be^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^2}$$

output
```
-(-a*d+b*c)*exp(e/(d*x+c))*(d*x+c)/d^2+1/2*b*e*exp(e/(d*x+c))*(d*x+c)/d^2+
1/2*b*exp(e/(d*x+c))*(d*x+c)^2/d^2+(-a*d+b*c)*e*Ei(e/(d*x+c))/d^2-1/2*b*e^
2*Ei(e/(d*x+c))/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = \frac{c(2ad+b(-c+e))e^{\frac{e}{c+dx}}}{2d^2}+\frac{de^{\frac{e}{c+dx}}x(2ad+b(e+dx))-e(2ad+b(-2c+e))\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^2}$$

input
```
Integrate[E^(e/(c + d*x))*(a + b*x),x]
```

output
```
(c*(2*a*d + b*(-c + e))*E^(e/(c + d*x)))/(2*d^2) + (d*E^(e/(c + d*x))*x*(2
*a*d + b*(e + d*x)) - e*(2*a*d + b*(-2*c + e))*ExpIntegralEi[e/(c + d*x)])
/(2*d^2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)e^{\frac{e}{c+dx}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)e^{\frac{e}{c+dx}}}{d}+\frac{b(c+dx)e^{\frac{e}{c+dx}}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(bc-ad)\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d^2}-\frac{(c+dx)(bc-ad)e^{\frac{e}{c+dx}}}{d^2}-\frac{be^2\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{2d^2}+\frac{be(c+dx)e^{\frac{e}{c+dx}}}{2d^2}+\frac{b(c+dx)^2e^{\frac{e}{c+dx}}}{2d^2}$$

input
```
Int[E^(e/(c + d*x))*(a + b*x),x]
```

output
```
-(((b*c - a*d)*E^(e/(c + d*x))*(c + d*x))/d^2) + (b*e*E^(e/(c + d*x))*(c +
 d*x))/(2*d^2) + (b*E^(e/(c + d*x))*(c + d*x)^2)/(2*d^2) + ((b*c - a*d)*e*
ExpIntegralEi[e/(c + d*x)])/d^2 - (b*e^2*ExpIntegralEi[e/(c + d*x)])/(2*d^
2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.20

method	result
derivativedivides	$-\frac{e\left(a\left(-\frac{(dx+c)e^{\frac{e}{dx+c}}}{e}-\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)+\frac{be\left(-\frac{e^{\frac{e}{dx+c}}(dx+c)^2}{2e^2}-\frac{(dx+c)e^{\frac{e}{dx+c}}}{2e}-\frac{\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)}{2}\right)}{d}-\frac{bc(\cdots}{\cdots}\right.}{d}$
default	$-\frac{e\left(a\left(-\frac{(dx+c)e^{\frac{e}{dx+c}}}{e}-\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)+\frac{be\left(-\frac{e^{\frac{e}{dx+c}}(dx+c)^2}{2e^2}-\frac{(dx+c)e^{\frac{e}{dx+c}}}{2e}-\frac{\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)}{2}\right)}{d}-\frac{bc(\cdots}{\cdots}\right.}{d}$
risch	$a\,e^{\frac{e}{dx+c}}x+\frac{a\,e^{\frac{e}{dx+c}}c}{d}+\frac{ea\,\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)}{d}+\frac{b\,e^{\frac{e}{dx+c}}x^2}{2}-\frac{b\,e^{\frac{e}{dx+c}}c^2}{2d^2}+\frac{eb\,e^{\frac{e}{dx+c}}x}{2d}+\frac{eb\,e^{\frac{e}{dx+c}}c}{2d^2}\cdots$
parts	$b\,e^{\frac{e}{dx+c}}x^2+a\,e^{\frac{e}{dx+c}}x+\frac{b\,e^{\frac{e}{dx+c}}cx}{d}+\frac{a\,e^{\frac{e}{dx+c}}c}{d}+\frac{e\,\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)bx}{d}+\frac{ea\,\text{expIntegral}_1\left(-\frac{e}{dx+c}\right)}{d}\cdots$

input
```
int(exp(e/(d*x+c))*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*(a*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))+b/d*e*(-1/2*exp(e
/(d*x+c))/e^2*(d*x+c)^2-1/2/e*(d*x+c)*exp(e/(d*x+c))-1/2*Ei(1,-e/(d*x+c)))
-b/d*c*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.66

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = -\frac{(be^2-2(bc-ad)e)\mathrm{Ei}\left(\frac{e}{dx+c}\right)-(bd^2x^2-bc^2+2\,acd+bce+(2\,ad^2+bde)x)e^{\left(\frac{e}{dx+c}\right)}}{2\,d^2}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((b*e^2 - 2*(b*c - a*d)*e)*Ei(e/(d*x + c)) - (b*d^2*x^2 - b*c^2 + 2*a
*c*d + b*c*e + (2*a*d^2 + b*d*e)*x)*e^(e/(d*x + c)))/d^2
```

Sympy [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = \int (a+bx)\,e^{\frac{e}{c+dx}}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a),x)
```

output
```
Integral((a + b*x)*exp(e/(c + d*x)), x)
```

Maxima [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = \int (bx+a)e^{\left(\frac{e}{dx+c}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*d*x^2 + (2*a*d + b*e)*x)*e^(e/(d*x + c))/d + integrate(-1/2*(b*c^2*
e - (2*a*d^2*e - (2*c*d*e - d*e^2)*b)*x)*e^(e/(d*x + c))/(d^3*x^2 + 2*c*d^
2*x + c^2*d), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = \frac{\left(\frac{2\,bce^4\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^2} - \frac{2\,ade^4\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^2} - \frac{be^5\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{(dx+c)^2} + be^3e^{\left(\frac{e}{dx+c}\right)} - \frac{2\,bce^3e^{\left(\frac{e}{dx+c}\right)}}{dx+c} + \frac{2\,ade^3e^{\left(\frac{e}{dx+c}\right)}}{dx+c} + \frac{be^4e^{\left(\frac{e}{dx+c}\right)}}{dx+c}\right)(dx +}{2\,d^2e^3}$$

input
```
integrate(exp(e/(d*x+c))*(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b*c*e^4*Ei(e/(d*x + c))/(d*x + c)^2 - 2*a*d*e^4*Ei(e/(d*x + c))/(d*
x + c)^2 - b*e^5*Ei(e/(d*x + c))/(d*x + c)^2 + b*e^3*e^(e/(d*x + c)) - 2*b
*c*e^3*e^(e/(d*x + c))/(d*x + c) + 2*a*d*e^3*e^(e/(d*x + c))/(d*x + c) + b
*e^4*e^(e/(d*x + c))/(d*x + c))*(d*x + c)^2/(d^2*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx = \frac{\frac{e^{\frac{e}{c+d\,x}}\,\left(2\,a\,c^2\,d-b\,c^3+b\,c^2\,e\right)}{2\,d^2} + x\,e^{\frac{e}{c+d\,x}}\,\left(2\,a\,c-\frac{\frac{b\,c^2}{2}-b\,c\,e}{d}\right) + x^2\,e^{\frac{e}{c+d\,x}}\,\left(a\,d+\frac{b\,c}{2}+\frac{b\,e}{2}\right) + \frac{b\,d\,x^3\,e^{\frac{e}{c+d\,x}}}{2}}{c+d\,x} - \frac{\mathrm{ei}\left(\frac{e}{c+d\,x}\right)\,\left(b\,e^2+2\,a\,d\,e-2\,b\,c\,e\right)}{2\,d^2}$$

input
```
int(exp(e/(c + d*x))*(a + b*x),x)
```

output
```
((exp(e/(c + d*x))*(2*a*c^2*d - b*c^3 + b*c^2*e))/(2*d^2) + x*exp(e/(c + d
*x))*(2*a*c - ((b*c^2)/2 - b*c*e)/d) + x^2*exp(e/(c + d*x))*(a*d + (b*c)/2
 + (b*e)/2) + (b*d*x^3*exp(e/(c + d*x)))/2)/(c + d*x) - (ei(e/(c + d*x))*(
b*e^2 + 2*a*d*e - 2*b*c*e))/(2*d^2)
```

Reduce [F]

$$\int e^{\frac{e}{c+dx}}(a+bx)\,dx$$

$$= \frac{-2e^{\frac{e}{dx+c}}a\,c^3d - 2e^{\frac{e}{dx+c}}a\,c^2d^2x + 2e^{\frac{e}{dx+c}}a\,d^3e\,x^2 + 2e^{\frac{e}{dx+c}}b\,c^4 + 2e^{\frac{e}{dx+c}}b\,c^3dx + 2e^{\frac{e}{dx+c}}b\,c^2dex + e^{\frac{e}{dx+c}}bc\,d^2e}{}$$

input
```
int(exp(e/(d*x+c))*(b*x+a),x)
```

output
```
( - 2*e**(e/(c + d*x))*a*c**3*d - 2*e**(e/(c + d*x))*a*c**2*d**2*x + 2*e**
(e/(c + d*x))*a*d**3*e*x**2 + 2*e**(e/(c + d*x))*b*c**4 + 2*e**(e/(c + d*x
))*b*c**3*d*x + 2*e**(e/(c + d*x))*b*c**2*d*e*x + e**(e/(c + d*x))*b*c*d**
2*e*x**2 + e**(e/(c + d*x))*b*d**3*e*x**3 + e**(e/(c + d*x))*b*d**2*e**2*x
**2 + 2*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)*a*c*d**4*e**2 + 2*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*d**5*e**2*x - 2*int((e**(e/(c + d*x)
)*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b*c**2*d**3*e**
2 - 2*int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3),x)*b*c*d**4*e**2*x + int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3),x)*b*c*d**3*e**3 + int((e**(e/(c + d*x))*x*
*2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b*d**4*e**3*x)/(2*d
**2*e*(c + d*x))
```

3.339 $\int e^{\frac{e}{c+dx}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int e^{\frac{e}{c+dx}}\,dx=\frac{e^{\frac{e}{c+dx}}(c+dx)}{d}-\frac{e\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d}$$

output
```
exp(e/(d*x+c))*(d*x+c)/d-e*Ei(e/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{\frac{e}{c+dx}}\,dx=\frac{e^{\frac{e}{c+dx}}(c+dx)}{d}-\frac{e\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d}$$

input
```
Integrate[E^(e/(c + d*x)),x]
```

output
```
(E^(e/(c + d*x))*(c + d*x))/d - (e*ExpIntegralEi[e/(c + d*x)])/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2635, 2639}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{e}{c+dx}}\,dx$$

$$\Big\downarrow \text{2635}$$

$$e\int \frac{e^{\frac{e}{c+dx}}}{c+dx}dx+\frac{(c+dx)e^{\frac{e}{c+dx}}}{d}$$

$$\Big\downarrow \text{2639}$$

$$\frac{(c+dx)e^{\frac{e}{c+dx}}}{d}-\frac{e\,\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{d}$$

input
```
Int[E^(e/(c + d*x)),x]
```

output
```
(E^(e/(c + d*x))*(c + d*x))/d - (e*ExpIntegralEi[e/(c + d*x)])/d
```

Defintions of rubi rules used

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-\frac{e\left(-\frac{(dx+c)\,\mathrm{e}^{\frac{e}{dx+c}}}{e}-\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)}{d}$	42
default	$-\frac{e\left(-\frac{(dx+c)\,\mathrm{e}^{\frac{e}{dx+c}}}{e}-\mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)\right)}{d}$	42
risch	$\mathrm{e}^{\frac{e}{dx+c}}x+\frac{\mathrm{e}^{\frac{e}{dx+c}}c}{d}+\frac{e\ \mathrm{expIntegral}_1\left(-\frac{e}{dx+c}\right)}{d}$	46

input
```
int(exp(e/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*(-1/e*(d*x+c)*exp(e/(d*x+c))-Ei(1,-e/(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int e^{\frac{e}{c+dx}}\,dx=-\frac{e\mathrm{Ei}\left(\frac{e}{dx+c}\right)-(dx+c)e^{\left(\frac{e}{dx+c}\right)}}{d}$$

input
```
integrate(exp(e/(d*x+c)),x, algorithm="fricas")
```

output
```
-(e*Ei(e/(d*x + c)) - (d*x + c)*e^(e/(d*x + c)))/d
```

Sympy [F]

$$\int e^{\frac{e}{c+dx}}\,dx = \int e^{\frac{e}{c+dx}}\,dx$$

input
```
integrate(exp(e/(d*x+c)),x)
```

output
```
Integral(exp(e/(c + d*x)), x)
```

Maxima [F]

$$\int e^{\frac{e}{c+dx}}\,dx = \int e^{\left(\frac{e}{dx+c}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)),x, algorithm="maxima")
```

output
```
d*e*integrate(x*e^(e/(d*x + c))/(d^2*x^2 + 2*c*d*x + c^2), x) + x*e^(e/(d*
x + c))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int e^{\frac{e}{c+dx}}\,dx = -\frac{\left(\frac{e^3\mathrm{Ei}\left(\frac{e}{dx+c}\right)}{dx+c} - e^2e^{\left(\frac{e}{dx+c}\right)}\right)(dx+c)}{de^2}$$

input
```
integrate(exp(e/(d*x+c)),x, algorithm="giac")
```

output
```
-(e^3*Ei(e/(d*x + c))/(d*x + c) - e^2*e^(e/(d*x + c)))*(d*x + c)/(d*e^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int e^{\frac{e}{c+dx}}\,dx = x\,\mathrm{e}^{\frac{e}{c+d\,x}} - \frac{e\,\mathrm{ei}\left(\frac{e}{c+d\,x}\right) - c\,\mathrm{e}^{\frac{e}{c+d\,x}}}{d}$$

input
```
int(exp(e/(c + d*x)),x)
```

output
```
x*exp(e/(c + d*x)) - (e*ei(e/(c + d*x)) - c*exp(e/(c + d*x)))/d
```

Reduce [F]

$$\int e^{\frac{e}{c+dx}}\,dx$$
$$= \frac{-e^{\frac{e}{dx+c}}c^3 - e^{\frac{e}{dx+c}}c^2dx + e^{\frac{e}{dx+c}}d^2e\,x^2 + \left(\int \frac{e^{\frac{e}{dx+c}}x^2}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^3e^2 + \left(\int \frac{e^{\frac{e}{dx+c}}x^2}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)d^4}{de\,(dx+c)}$$

input
```
int(exp(e/(d*x+c)),x)
```

output
```
( - e**(e/(c + d*x))*c**3 - e**(e/(c + d*x))*c**2*d*x + e**(e/(c + d*x))*d
**2*e*x**2 + int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3),x)*c*d**3*e**2 + int((e**(e/(c + d*x))*x**2)/(c**3 + 3*c**2
*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*d**4*e**2*x)/(d*e*(c + d*x))
```

3.340 $\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 62

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx=-\frac{\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{b}+\frac{e^{\frac{be}{bc-ad}}\,\text{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{b}$$

output
```
-Ei(e/(d*x+c))/b+exp(b*e/(-a*d+b*c))*Ei(-d*e*(b*x+a)/(-a*d+b*c)/(d*x+c))/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx=\frac{-\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)+e^{\frac{be}{bc-ad}}\,\text{ExpIntegralEi}\left(e\left(\frac{b}{-bc+ad}+\frac{1}{c+dx}\right)\right)}{b}$$

input
```
Integrate[E^(e/(c + d*x))/(a + b*x),x]
```

output
```
(-ExpIntegralEi[e/(c + d*x)] + E^((b*e)/(b*c - a*d))*ExpIntegralEi[e*(b/(-
(b*c) + a*d) + (c + d*x)^(-1))])/b
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2652, 2639, 2658, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx} \, dx$$

$$\Big\downarrow \textbf{2652}$$

$$\frac{(bc-ad)\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)(c+dx)}dx}{b}+\frac{d\int \frac{e^{\frac{e}{c+dx}}}{c+dx}dx}{b}$$

$$\Big\downarrow \textbf{2639}$$

$$\frac{(bc-ad)\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)(c+dx)}dx}{b}-\frac{\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{b}$$

$$\Big\downarrow \textbf{2658}$$

$$\frac{\int \frac{\exp\left(\frac{be}{bc-ad}-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)(c+dx)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{b}$$

$$\Big\downarrow \textbf{2609}$$

$$\frac{e^{\frac{be}{bc-ad}}\text{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{b}-\frac{\text{ExpIntegralEi}\left(\frac{e}{c+dx}\right)}{b}$$

input
```
Int[E^(e/(c + d*x))/(a + b*x),x]
```

output
```
-(ExpIntegralEi[e/(c + d*x)]/b) + (E^((b*e)/(b*c - a*d))*ExpIntegralEi[-((
d*e*(a + b*x))/((b*c - a*d)*(c + d*x)))])/b
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2652
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/((e_.) + (f_.)*(x_)), x_Symbo
l] :> Simp[d/f   Int[F^(a + b/(c + d*x))/(c + d*x), x], x] - Simp[(d*e - c*
f)/f   Int[F^(a + b/(c + d*x))/((c + d*x)*(e + f*x)), x], x] /; FreeQ[{F, a
, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

rule 2658
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))/(((e_.) + (f_.)*(x_))*((g_.)
+ (h_.)*(x_))), x_Symbol] :> Simp[-d/(f*(d*g - c*h))   Subst[Int[F^(a - b*(
h/(d*g - c*h)) + d*b*(x/(d*g - c*h)))/x, x], x, (g + h*x)/(c + d*x)], x] /;
 FreeQ[{F, a, b, c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05

method	result	size
risch	$-\frac{\mathrm{e}^{-\frac{be}{ad-bc}} \operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{b}+\frac{\operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}\right)}{b}$	65
derivativedivides	$-\frac{e\left(-\frac{d \operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}\right)}{be}+\frac{d\,\mathrm{e}^{-\frac{be}{ad-bc}} \operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{be}\right)}{d}$	79
default	$-\frac{e\left(-\frac{d \operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}\right)}{be}+\frac{d\,\mathrm{e}^{-\frac{be}{ad-bc}} \operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{be}\right)}{d}$	79

input
```
int(exp(e/(d*x+c))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*exp(-b*e/(a*d-b*c))*Ei(1,-e/(d*x+c)-b*e/(a*d-b*c))+1/b*Ei(1,-e/(d*x+c
))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx} \, dx = \frac{\text{Ei}\left(-\frac{bdex+ade}{bc^2-acd+(bcd-ad^2)x}\right)e^{\left(\frac{be}{bc-ad}\right)} - \text{Ei}\left(\frac{e}{dx+c}\right)}{b}$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a),x, algorithm="fricas")
```

output
```
(Ei(-(b*d*e*x + a*d*e)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x))*e^(b*e/(b*c -
a*d)) - Ei(e/(d*x + c)))/b
```

Sympy [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx} \, dx = \int \frac{e^{\frac{e}{c+dx}}}{a+bx} \, dx$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a),x)
```

output
```
Integral(exp(e/(c + d*x))/(a + b*x), x)
```

Maxima [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx = \int \frac{e^{\left(\frac{e}{dx+c}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c))/(b*x + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(61) = 122$.

Time = 0.35 (sec) , antiderivative size = 483, normalized size of antiderivative = 7.79

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx = -\frac{\left(\frac{2\,b^2c^2e^3\mathrm{Ei}\left(-\frac{be-\frac{bce}{dx+c}+\frac{ade}{dx+c}}{bc-ad}\right)e^{\left(\frac{be}{bc-ad}\right)}}{(dx+c)^2} - \frac{4\,abcde^3\mathrm{Ei}\left(-\frac{be-\frac{bce}{dx+c}+\frac{ade}{dx+c}}{bc-ad}\right)e^{\left(\frac{be}{bc-ad}\right)}}{(dx+c)^2} + \frac{2\,a^2d^2e^3\mathrm{Ei}\left(-\frac{be-\frac{bce}{dx+c}+\frac{ade}{dx+c}}{bc-ad}\right)e^{\left(\frac{be}{bc-ad}\right)}}{(dx+c)^2}\right.}{}$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(2*b^2*c^2*e^3*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a
*d))*e^(b*e/(b*c - a*d))/(d*x + c)^2 - 4*a*b*c*d*e^3*Ei(-(b*e - b*c*e/(d*x
 + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c)^2 + 2*
a^2*d^2*e^3*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(
b*e/(b*c - a*d))/(d*x + c)^2 - 2*b^2*c^2*e^3*Ei(e/(d*x + c))/(d*x + c)^2 +
 4*a*b*c*d*e^3*Ei(e/(d*x + c))/(d*x + c)^2 - 2*a^2*d^2*e^3*Ei(e/(d*x + c))
/(d*x + c)^2 - 2*b^2*c*e^4*Ei(e/(d*x + c))/(d*x + c)^2 + 2*a*b*d*e^4*Ei(e/
(d*x + c))/(d*x + c)^2 - b^2*e^5*Ei(e/(d*x + c))/(d*x + c)^2 + b^2*e^3*e^(
e/(d*x + c)) + 2*b^2*c*e^3*e^(e/(d*x + c))/(d*x + c) - 2*a*b*d*e^3*e^(e/(d
*x + c))/(d*x + c) + b^2*e^4*e^(e/(d*x + c))/(d*x + c))*(d*x + c)^2/(b^3*d
*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx = \int \frac{\mathrm{e}^{\frac{e}{c+d\,x}}}{a+b\,x}\,dx$$

input
```
int(exp(e/(c + d*x))/(a + b*x),x)
```

output
```
int(exp(e/(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{a+bx}\,dx = \int \frac{e^{\frac{e}{dx+c}}}{bx+a}dx$$

input
```
int(exp(e/(d*x+c))/(b*x+a),x)
```

output
```
int(e**(e/(c + d*x))/(a + b*x),x)
```

3.341 $\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx=-\frac{de^{\frac{e}{c+dx}}}{b(bc-ad)}-\frac{e^{\frac{e}{c+dx}}}{b(a+bx)}-\frac{dee^{\frac{be}{bc-ad}}\operatorname{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^2}$$

output
```
-d*exp(e/(d*x+c))/b/(-a*d+b*c)-exp(e/(d*x+c))/b/(b*x+a)-d*e*exp(b*e/(-a*d+
b*c))*Ei(-d*e*(b*x+a)/(-a*d+b*c)/(d*x+c))/(-a*d+b*c)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx=-\frac{de^{\frac{e}{c+dx}}}{b(bc-ad)}-\frac{e^{\frac{e}{c+dx}}}{b(a+bx)}-\frac{dee^{\frac{be}{bc-ad}}\operatorname{ExpIntegralEi}\left(-\frac{be}{bc-ad}+\frac{e}{c+dx}\right)}{(-bc+ad)^2}$$

input
```
Integrate[E^(e/(c + d*x))/(a + b*x)^2,x]
```

output
```
-((d*E^(e/(c + d*x)))/(b*(b*c - a*d))) - E^(e/(c + d*x))/(b*(a + b*x)) - (
d*e*E^((b*e)/(b*c - a*d))*ExpIntegralEi[-((b*e)/(b*c - a*d)) + e/(c + d*x)
])/(-(b*c) + a*d)^2
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \text{2653}$$

$$-\frac{de\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)(c+dx)^2}dx}{b}-\frac{e^{\frac{e}{c+dx}}}{b(a+bx)}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{de\int \left(\frac{e^{\frac{e}{c+dx}}b^2}{(bc-ad)^2(a+bx)}-\frac{de^{\frac{e}{c+dx}}b}{(bc-ad)^2(c+dx)}-\frac{de^{\frac{e}{c+dx}}}{(bc-ad)(c+dx)^2}\right)dx}{b}-\frac{e^{\frac{e}{c+dx}}}{b(a+bx)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{de\left(\frac{be^{\frac{be}{bc-ad}}\text{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^2}+\frac{e^{\frac{e}{c+dx}}}{e(bc-ad)}\right)}{b}-\frac{e^{\frac{e}{c+dx}}}{b(a+bx)}$$

input
```
Int[E^(e/(c + d*x))/(a + b*x)^2,x]
```

output
```
-(E^(e/(c + d*x))/(b*(a + b*x))) - (d*e*(E^(e/(c + d*x))/((b*c - a*d)*e) +
 (b*E^((b*e)/(b*c - a*d))*ExpIntegralEi[-((d*e*(a + b*x))/((b*c - a*d)*(c
+ d*x)))])/(b*c - a*d)^2))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{de\left(-\frac{e^{\frac{e}{dx+c}}}{\frac{e}{dx+c}+\frac{be}{ad-bc}}-e^{-\frac{be}{ad-bc}}\operatorname{expIntegral}_1\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)\right)}{(ad-bc)^2}$	97
default	$-\frac{de\left(-\frac{e^{\frac{e}{dx+c}}}{\frac{e}{dx+c}+\frac{be}{ad-bc}}-e^{-\frac{be}{ad-bc}}\operatorname{expIntegral}_1\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)\right)}{(ad-bc)^2}$	97
risch	$\frac{de\,e^{\frac{e}{dx+c}}}{(ad-bc)^2\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)}+\frac{de\,e^{-\frac{be}{ad-bc}}\operatorname{expIntegral}_1\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{(ad-bc)^2}$	105

input
```
int(exp(e/(d*x+c))/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d*e/(a*d-b*c)^2*(-exp(e/(d*x+c))/(e/(d*x+c)+b*e/(a*d-b*c))-exp(-b*e/(a*d-
b*c))*Ei(1,-e/(d*x+c)-b*e/(a*d-b*c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.44

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx = -\frac{(bdex+ade)\operatorname{Ei}\left(-\frac{bdex+ade}{bc^2-acd+(bcd-ad^2)x}\right)e^{\left(\frac{be}{bc-ad}\right)}+(bc^2-acd+(bcd-ad^2)x)e^{\left(\frac{e}{dx+c}\right)}}{ab^2c^2-2\,a^2bcd+a^3d^2+(b^3c^2-2\,ab^2cd+a^2bd^2)x}$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-((b*d*e*x + a*d*e)*Ei(-(b*d*e*x + a*d*e)/(b*c^2 - a*c*d + (b*c*d - a*d^2)
*x))*e^(b*e/(b*c - a*d)) + (b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*e^(e/(d*x +
 c)))/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)*x)
```

Sympy [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx = \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)**2,x)
```

output
```
Integral(exp(e/(c + d*x))/(a + b*x)**2, x)
```

Maxima [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx = \int \frac{e^{\left(\frac{e}{dx+c}\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c))/(b*x + a)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(104) = 208.

Time = 0.14 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.08

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx = -\frac{\left(be^3\text{Ei}\left(-\frac{be-\frac{bce}{dx+c}+\frac{ade}{dx+c}}{bc-ad}\right)e^{\left(\frac{be}{bc-ad}\right)}-\frac{bce^3\text{Ei}\left(-\frac{be-\frac{bce}{dx+c}+\frac{ade}{dx+c}}{bc-ad}\right)e^{\left(\frac{be}{bc-ad}\right)}}{dx+c}+\frac{ade^3\text{Ei}\left(-\frac{be-\frac{bce}{dx+c}+\frac{ade}{dx+c}}{bc-ad}\right)e^{\left(\frac{be}{bc-ad}\right)}}{dx+c}+bc\cdots\right.}{\left(b^3c^2e-\frac{b^3c^3e}{dx+c}-2\,ab^2cde+\frac{3\,ab^2c^2de}{dx+c}+a^2bd^2e-\frac{3\,a^2bcd^2e}{dx+c}+\frac{a^3d^3e}{dx+c}\right)e\cdots}$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)^2,x, algorithm="giac")
```

output
```
-(b*e^3*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/
(b*c - a*d)) - b*c*e^3*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c
- a*d))*e^(b*e/(b*c - a*d))/(d*x + c) + a*d*e^3*Ei(-(b*e - b*c*e/(d*x + c)
 + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c) + b*c*e^2*e
^(e/(d*x + c)) - a*d*e^2*e^(e/(d*x + c)))*d/((b^3*c^2*e - b^3*c^3*e/(d*x +
 c) - 2*a*b^2*c*d*e + 3*a*b^2*c^2*d*e/(d*x + c) + a^2*b*d^2*e - 3*a^2*b*c*
d^2*e/(d*x + c) + a^3*d^3*e/(d*x + c))*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2}\,dx = \int \frac{\mathrm{e}^{\frac{e}{c+d\,x}}}{{\left(a+b\,x\right)}^2}\,dx$$

input
```
int(exp(e/(c + d*x))/(a + b*x)^2,x)
```

output
```
int(exp(e/(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2} \, dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c))/(b*x+a)^2,x)
```

output
```
( - 24*e**(e/(c + d*x))*a**5*c**4*d**5 - 96*e**(e/(c + d*x))*a**5*c**3*d**
6*x + 24*e**(e/(c + d*x))*a**5*c**3*d**5*e - 144*e**(e/(c + d*x))*a**5*c**
2*d**7*x**2 + 72*e**(e/(c + d*x))*a**5*c**2*d**6*e*x - 12*e**(e/(c + d*x))
*a**5*c**2*d**5*e**2 - 96*e**(e/(c + d*x))*a**5*c*d**8*x**3 + 72*e**(e/(c
+ d*x))*a**5*c*d**7*e*x**2 - 24*e**(e/(c + d*x))*a**5*c*d**6*e**2*x + 4*e*
*(e/(c + d*x))*a**5*c*d**5*e**3 - 24*e**(e/(c + d*x))*a**5*d**9*x**4 + 24*
e**(e/(c + d*x))*a**5*d**8*e*x**3 - 12*e**(e/(c + d*x))*a**5*d**7*e**2*x**
2 + 4*e**(e/(c + d*x))*a**5*d**6*e**3*x - e**(e/(c + d*x))*a**5*d**5*e**4
+ 96*e**(e/(c + d*x))*a**4*b*c**5*d**4 + 360*e**(e/(c + d*x))*a**4*b*c**4*
d**5*x - 108*e**(e/(c + d*x))*a**4*b*c**4*d**4*e + 480*e**(e/(c + d*x))*a*
*4*b*c**3*d**6*x**2 - 312*e**(e/(c + d*x))*a**4*b*c**3*d**5*e*x + 60*e**(e
/(c + d*x))*a**4*b*c**3*d**4*e**2 + 240*e**(e/(c + d*x))*a**4*b*c**2*d**7*
x**3 - 288*e**(e/(c + d*x))*a**4*b*c**2*d**6*e*x**2 + 120*e**(e/(c + d*x))
*a**4*b*c**2*d**5*e**2*x - 22*e**(e/(c + d*x))*a**4*b*c**2*d**4*e**3 - 72*
e**(e/(c + d*x))*a**4*b*c*d**7*e*x**3 + 60*e**(e/(c + d*x))*a**4*b*c*d**6*
e**2*x**2 - 24*e**(e/(c + d*x))*a**4*b*c*d**5*e**3*x + 6*e**(e/(c + d*x))*
a**4*b*c*d**4*e**4 - 24*e**(e/(c + d*x))*a**4*b*d**9*x**5 + 12*e**(e/(c +
d*x))*a**4*b*d**8*e*x**4 - 2*e**(e/(c + d*x))*a**4*b*d**6*e**3*x**2 + e**(
e/(c + d*x))*a**4*b*d**5*e**4*x - 144*e**(e/(c + d*x))*a**3*b**2*c**6*d**3
 - 480*e**(e/(c + d*x))*a**3*b**2*c**5*d**4*x + 180*e**(e/(c + d*x))*a*...
```

3.342 $\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 240

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \frac{d^2e^{\frac{e}{c+dx}}}{2b(bc-ad)^2} + \frac{d^2ee^{\frac{e}{c+dx}}}{2(bc-ad)^3} - \frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2} + \frac{dee^{\frac{e}{c+dx}}}{2(bc-ad)^2(a+bx)} + \frac{d^2ee^{\frac{be}{bc-ad}}\,\text{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3} + \frac{bd^2e^2e^{\frac{be}{bc-ad}}\,\text{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{2(bc-ad)^4}$$

output
```
1/2*d^2*exp(e/(d*x+c))/b/(-a*d+b*c)^2+1/2*d^2*e*exp(e/(d*x+c))/(-a*d+b*c)^
3-1/2*exp(e/(d*x+c))/b/(b*x+a)^2+1/2*d*e*exp(e/(d*x+c))/(-a*d+b*c)^2/(b*x+
a)+d^2*e*exp(b*e/(-a*d+b*c))*Ei(-d*e*(b*x+a)/(-a*d+b*c)/(d*x+c))/(-a*d+b*c
)^3+1/2*b*d^2*e^2*exp(b*e/(-a*d+b*c))*Ei(-d*e*(b*x+a)/(-a*d+b*c)/(d*x+c))/
(-a*d+b*c)^4
```

Mathematica [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx$$

input
```
Integrate[E^(e/(c + d*x))/(a + b*x)^3,x]
```

output
```
Integrate[E^(e/(c + d*x))/(a + b*x)^3, x]
```

Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2653, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{2653}$$

$$-\frac{de\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^2(c+dx)^2}dx}{2b}-\frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{de\int \left(-\frac{2de^{\frac{e}{c+dx}}b^2}{(bc-ad)^3(a+bx)}+\frac{e^{\frac{e}{c+dx}}b^2}{(bc-ad)^2(a+bx)^2}+\frac{2d^2e^{\frac{e}{c+dx}}b}{(bc-ad)^3(c+dx)}+\frac{d^2e^{\frac{e}{c+dx}}}{(bc-ad)^2(c+dx)^2}\right)dx}{2b}-\frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{de\left(-\frac{b^2dee^{\frac{be}{bc-ad}}\operatorname{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^4}-\frac{2bde^{\frac{be}{bc-ad}}\operatorname{ExpIntegralEi}\left(-\frac{de(a+bx)}{(bc-ad)(c+dx)}\right)}{(bc-ad)^3}-\frac{bde^{\frac{e}{c+dx}}}{(bc-ad)^3}-\frac{be^{\frac{e}{c+dx}}}{(a+bx)(bc-ad)^2}-\cdots\right.}{2b}$$

$$\frac{e^{\frac{e}{c+dx}}}{2b(a+bx)^2}$$

input
```
Int[E^(e/(c + d*x))/(a + b*x)^3,x]
```

output
```
-1/2*E^(e/(c + d*x))/(b*(a + b*x)^2) - (d*e*(-((b*d*E^(e/(c + d*x)))/(b*c
 - a*d)^3) - (d*E^(e/(c + d*x)))/((b*c - a*d)^2*e) - (b*E^(e/(c + d*x)))/((
b*c - a*d)^2*(a + b*x)) - (2*b*d*E^((b*e)/(b*c - a*d))*ExpIntegralEi[-((d*
e*(a + b*x))/((b*c - a*d)*(c + d*x)))])/(b*c - a*d)^3 - (b^2*d*e*E^((b*e)/
(b*c - a*d))*ExpIntegralEi[-((d*e*(a + b*x))/((b*c - a*d)*(c + d*x)))])/(b
*c - a*d)^4))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2653
```
Int[(F_)^((a_.) + (b_.)/((c_.) + (d_.)*(x_)))*((e_.) + (f_.)*(x_))^(m_), x_
Symbol] :> Simp[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(f*(m + 1))), x] + S
imp[b*d*(Log[F]/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(F^(a + b/(c + d*x))/(
c + d*x)^2), x], x] /; FreeQ[{F, a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0]
 && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00

method	result
derivativedivides	$-\frac{e\left(-\frac{d^{3}be\left(-\frac{e^{\frac{e}{dx+c}}}{2\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)^{2}}-\frac{e^{\frac{e}{dx+c}}}{2\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)}-\frac{e^{-\frac{be}{ad-bc}}\operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{2}\right)}{(ad-bc)^{4}}+\frac{d^{3}\left(-\frac{e^{\frac{e}{dx+c}}}{\frac{e}{dx+c}+\frac{be}{ad-bc}}-\right.}{}\right.}{d}$
default	$-\frac{e\left(-\frac{d^{3}be\left(-\frac{e^{\frac{e}{dx+c}}}{2\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)^{2}}-\frac{e^{\frac{e}{dx+c}}}{2\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)}-\frac{e^{-\frac{be}{ad-bc}}\operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{2}\right)}{(ad-bc)^{4}}+\frac{d^{3}\left(-\frac{e^{\frac{e}{dx+c}}}{\frac{e}{dx+c}+\frac{be}{ad-bc}}-\right.}{}\right.}{d}$
risch	$-\frac{e^{2}d^{2}b\,e^{\frac{e}{dx+c}}}{2(ad-bc)^{4}\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)^{2}}-\frac{e^{2}d^{2}b\,e^{\frac{e}{dx+c}}}{2(ad-bc)^{4}\left(\frac{e}{dx+c}+\frac{be}{ad-bc}\right)}-\frac{e^{2}d^{2}b\,e^{-\frac{be}{ad-bc}}\operatorname{expIntegral}_{1}\left(-\frac{e}{dx+c}-\frac{be}{ad-bc}\right)}{2(ad-bc)^{4}}+$

input
```
int(exp(e/(d*x+c))/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*e*(-d^3*b*e/(a*d-b*c)^4*(-1/2*exp(e/(d*x+c))/(e/(d*x+c)+b*e/(a*d-b*c)
)^2-1/2*exp(e/(d*x+c))/(e/(d*x+c)+b*e/(a*d-b*c))-1/2*exp(-b*e/(a*d-b*c))*E
i(1,-e/(d*x+c)-b*e/(a*d-b*c)))+d^3/(a*d-b*c)^3*(-exp(e/(d*x+c))/(e/(d*x+c)
+b*e/(a*d-b*c))-exp(-b*e/(a*d-b*c))*Ei(1,-e/(d*x+c)-b*e/(a*d-b*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(224) = 448.

Time = 0.09 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.15

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx$$
$$=\frac{(a^2bd^2e^2+(b^3d^2e^2+2\,(b^3cd^2-ab^2d^3)e)x^2+2\,(a^2bcd^2-a^3d^3)e+2\,(ab^2d^2e^2+2\,(ab^2cd^2-a^2bd^3)e)x)\mathrm{E}}{2\,(a^2b^4c^4-4\,a^3b^3c^3d+6\,a^4b^2c^2d}$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*((a^2*b*d^2*e^2 + (b^3*d^2*e^2 + 2*(b^3*c*d^2 - a*b^2*d^3)*e)*x^2 + 2*
(a^2*b*c*d^2 - a^3*d^3)*e + 2*(a*b^2*d^2*e^2 + 2*(a*b^2*c*d^2 - a^2*b*d^3)
*e)*x)*Ei(-(b*d*e*x + a*d*e)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x))*e^(b*e/(
b*c - a*d)) - (b^3*c^4 - 4*a*b^2*c^3*d + 5*a^2*b*c^2*d^2 - 2*a^3*c*d^3 - (
b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4 + (b^3*c*d^2 - a*b^2*d^3)*e)*x^2 -
 (a*b^2*c^2*d - a^2*b*c*d^2)*e - (2*a*b^2*c^2*d^2 - 4*a^2*b*c*d^3 + 2*a^3*
d^4 + (b^3*c^2*d - a^2*b*d^3)*e)*x)*e^(e/(d*x + c)))/(a^2*b^4*c^4 - 4*a^3*
b^3*c^3*d + 6*a^4*b^2*c^2*d^2 - 4*a^5*b*c*d^3 + a^6*d^4 + (b^6*c^4 - 4*a*b
^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^2 + 2*(a*b
^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4
)*x)
```

Sympy [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)**3,x)
```

output
```
Integral(exp(e/(c + d*x))/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \int \frac{e^{\left(\frac{e}{dx+c}\right)}}{(bx+a)^3}\,dx$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c))/(b*x + a)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1733 vs. $2(224) = 448$.

Time = 0.13 (sec) , antiderivative size = 1733, normalized size of antiderivative = 7.22

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(exp(e/(d*x+c))/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*(2*b^3*c*d*e^4*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*
d))*e^(b*e/(b*c - a*d)) - 4*b^3*c^2*d*e^4*Ei(-(b*e - b*c*e/(d*x + c) + a*d
*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c) + 2*b^3*c^3*d*e^4
*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c -
a*d))/(d*x + c)^2 - 2*a*b^2*d^2*e^4*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*
x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d)) + 8*a*b^2*c*d^2*e^4*Ei(-(b*e - b*
c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c
) - 6*a*b^2*c^2*d^2*e^4*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c
 - a*d))*e^(b*e/(b*c - a*d))/(d*x + c)^2 - 4*a^2*b*d^3*e^4*Ei(-(b*e - b*c*
e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c)
+ 6*a^2*b*c*d^3*e^4*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a
*d))*e^(b*e/(b*c - a*d))/(d*x + c)^2 - 2*a^3*d^4*e^4*Ei(-(b*e - b*c*e/(d*x
 + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c)^2 + b^
3*d*e^5*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/
(b*c - a*d)) - 2*b^3*c*d*e^5*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))
/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c) + b^3*c^2*d*e^5*Ei(-(b*e - b*c
*e/(d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c)
^2 + 2*a*b^2*d^2*e^5*Ei(-(b*e - b*c*e/(d*x + c) + a*d*e/(d*x + c))/(b*c -
a*d))*e^(b*e/(b*c - a*d))/(d*x + c) - 2*a*b^2*c*d^2*e^5*Ei(-(b*e - b*c*e/(
d*x + c) + a*d*e/(d*x + c))/(b*c - a*d))*e^(b*e/(b*c - a*d))/(d*x + c)^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \int \frac{\mathrm{e}^{\frac{e}{c+d\,x}}}{(a+b\,x)^3}\,dx$$

input
```
int(exp(e/(c + d*x))/(a + b*x)^3,x)
```

output
```
int(exp(e/(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{e^{\frac{e}{c+dx}}}{(a+bx)^3}\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c))/(b*x+a)^3,x)
```

output

```
( - 48*e**(e/(c + d*x))*a**6*c**4*d**6 - 192*e**(e/(c + d*x))*a**6*c**3*d*
*7*x + 48*e**(e/(c + d*x))*a**6*c**3*d**6*e - 288*e**(e/(c + d*x))*a**6*c*
*2*d**8*x**2 + 144*e**(e/(c + d*x))*a**6*c**2*d**7*e*x - 24*e**(e/(c + d*x
))*a**6*c**2*d**6*e**2 - 192*e**(e/(c + d*x))*a**6*c*d**9*x**3 + 144*e**(e
/(c + d*x))*a**6*c*d**8*e*x**2 - 48*e**(e/(c + d*x))*a**6*c*d**7*e**2*x +
8*e**(e/(c + d*x))*a**6*c*d**6*e**3 - 48*e**(e/(c + d*x))*a**6*d**10*x**4
+ 48*e**(e/(c + d*x))*a**6*d**9*e*x**3 - 24*e**(e/(c + d*x))*a**6*d**8*e**
2*x**2 + 8*e**(e/(c + d*x))*a**6*d**7*e**3*x - 2*e**(e/(c + d*x))*a**6*d**
6*e**4 + 192*e**(e/(c + d*x))*a**5*b*c**5*d**5 + 672*e**(e/(c + d*x))*a**5
*b*c**4*d**6*x - 204*e**(e/(c + d*x))*a**5*b*c**4*d**5*e + 768*e**(e/(c +
d*x))*a**5*b*c**3*d**7*x**2 - 528*e**(e/(c + d*x))*a**5*b*c**3*d**6*e*x +
108*e**(e/(c + d*x))*a**5*b*c**3*d**5*e**2 + 192*e**(e/(c + d*x))*a**5*b*c
**2*d**8*x**3 - 360*e**(e/(c + d*x))*a**5*b*c**2*d**7*e*x**2 + 180*e**(e/(
c + d*x))*a**5*b*c**2*d**6*e**2*x - 38*e**(e/(c + d*x))*a**5*b*c**2*d**5*e
**3 - 192*e**(e/(c + d*x))*a**5*b*c*d**9*x**4 + 48*e**(e/(c + d*x))*a**5*b
*c*d**8*e*x**3 + 36*e**(e/(c + d*x))*a**5*b*c*d**7*e**2*x**2 - 28*e**(e/(c
 + d*x))*a**5*b*c*d**6*e**3*x + 10*e**(e/(c + d*x))*a**5*b*c*d**5*e**4 - 9
6*e**(e/(c + d*x))*a**5*b*d**10*x**5 + 84*e**(e/(c + d*x))*a**5*b*d**9*e*x
**4 - 36*e**(e/(c + d*x))*a**5*b*d**8*e**2*x**3 + 10*e**(e/(c + d*x))*a**5
*b*d**7*e**3*x**2 - 2*e**(e/(c + d*x))*a**5*b*d**6*e**4*x + e**(e/(c + ...
```

3.343 $\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 322

$$\begin{aligned}\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx =& -\frac{(bc-ad)^3e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^4}-\frac{2b^2(bc-ad)ee^{\frac{e}{(c+dx)^2}}(c+dx)}{d^4}\\ &+\frac{3b(bc-ad)^2e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{2d^4}+\frac{b^3ee^{\frac{e}{(c+dx)^2}}(c+dx)^2}{4d^4}\\ &-\frac{b^2(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)^3}{d^4}+\frac{b^3e^{\frac{e}{(c+dx)^2}}(c+dx)^4}{4d^4}\\ &+\frac{(bc-ad)^3\sqrt{e}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4}+\frac{2b^2(bc-ad)e^{3/2}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4}\\ &-\frac{3b(bc-ad)^2e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{2d^4}\\ &-\frac{b^3e^2\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{4d^4}\end{aligned}$$

output
```
-(-a*d+b*c)^3*exp(e/(d*x+c)^2)*(d*x+c)/d^4-2*b^2*(-a*d+b*c)*e*exp(e/(d*x+c
)^2)*(d*x+c)/d^4+3/2*b*(-a*d+b*c)^2*exp(e/(d*x+c)^2)*(d*x+c)^2/d^4+1/4*b^3
*e*exp(e/(d*x+c)^2)*(d*x+c)^2/d^4-b^2*(-a*d+b*c)*exp(e/(d*x+c)^2)*(d*x+c)^
3/d^4+1/4*b^3*exp(e/(d*x+c)^2)*(d*x+c)^4/d^4+(-a*d+b*c)^3*e^(1/2)*Pi^(1/2)
*erfi(e^(1/2)/(d*x+c))/d^4+2*b^2*(-a*d+b*c)*e^(3/2)*Pi^(1/2)*erfi(e^(1/2)/
(d*x+c))/d^4-3/2*b*(-a*d+b*c)^2*e*Ei(e/(d*x+c)^2)/d^4-1/4*b^3*e^2*Ei(e/(d*
x+c)^2)/d^4
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.75

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx = -\frac{c(6a^2bcd^2-4a^3d^3-4ab^2d(c^2+2e)+b^3(c^3+7ce))\,e^{\frac{e}{(c+dx)^2}}}{4d^4} + \frac{de^{\frac{e}{(c+dx)^2}}x(4a^3d^3+6a^2bd^3x+4ab^2d(2e+d^2x^2)+b^3(-6ce+dex+d^3x^3))+4(bc-ad)\sqrt{e}(-2abcd+}{4d^4}$$

input
```
Integrate[E^(e/(c + d*x)^2)*(a + b*x)^3,x]
```

output
```
-1/4*(c*(6*a^2*b*c*d^2 - 4*a^3*d^3 - 4*a*b^2*d*(c^2 + 2*e) + b^3*(c^3 + 7*
c*e))*E^(e/(c + d*x)^2))/d^4 + (d*E^(e/(c + d*x)^2)*x*(4*a^3*d^3 + 6*a^2*b
*d^3*x + 4*a*b^2*d*(2*e + d^2*x^2) + b^3*(-6*c*e + d*e*x + d^3*x^3)) + 4*(
b*c - a*d)*Sqrt[e]*(-2*a*b*c*d + a^2*d^2 + b^2*(c^2 + 2*e))*Sqrt[Pi]*Erfi[
Sqrt[e]/(c + d*x)] - b*e*(-12*a*b*c*d + 6*a^2*d^2 + b^2*(6*c^2 + e))*ExpIn
tegralEi[e/(c + d*x)^2])/(4*d^4)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3 e^{\frac{e}{(c+dx)^2}} \, dx$$

$$\Big\downarrow \text{ 2656}$$

$$\int \left(-\frac{3b^2(c+dx)^2(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^3} + \frac{(ad-bc)^3 e^{\frac{e}{(c+dx)^2}}}{d^3} + \frac{3b(c+dx)(bc-ad)^2 e^{\frac{e}{(c+dx)^2}}}{d^3} + \frac{b^3(c+dx)^3 e^{\frac{e}{(c+dx)^2}}}{d^3} \right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\sqrt{\pi}b^2e^{3/2}(bc-ad)\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4} - \frac{b^2(c+dx)^3(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^4} - \frac{2b^2e(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^4} + \frac{\sqrt{\pi}\sqrt{e}(bc-ad)^3\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^4} - \frac{3be(bc-ad)^2 \text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{2d^4} + \frac{3b(c+dx)^2(bc-ad)^2e^{\frac{e}{(c+dx)^2}}}{2d^4} - \frac{(c+dx)(bc-ad)^3e^{\frac{e}{(c+dx)^2}}}{d^4} - \frac{b^3e^2 \text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{4d^4} + \frac{b^3(c+dx)^4e^{\frac{e}{(c+dx)^2}}}{4d^4} + \frac{b^3e(c+dx)^2e^{\frac{e}{(c+dx)^2}}}{4d^4}$$

input
```
Int[E^(e/(c + d*x)^2)*(a + b*x)^3,x]
```

output
```
-(((b*c - a*d)^3*E^(e/(c + d*x)^2)*(c + d*x))/d^4) - (2*b^2*(b*c - a*d)*e*
E^(e/(c + d*x)^2)*(c + d*x))/d^4 + (3*b*(b*c - a*d)^2*E^(e/(c + d*x)^2)*(c
 + d*x)^2)/(2*d^4) + (b^3*e*E^(e/(c + d*x)^2)*(c + d*x)^2)/(4*d^4) - (b^2*
(b*c - a*d)*E^(e/(c + d*x)^2)*(c + d*x)^3)/d^4 + (b^3*E^(e/(c + d*x)^2)*(c
 + d*x)^4)/(4*d^4) + ((b*c - a*d)^3*Sqrt[e]*Sqrt[Pi]*Erfi[Sqrt[e]/(c + d*x
)])/d^4 + (2*b^2*(b*c - a*d)*e^(3/2)*Sqrt[Pi]*Erfi[Sqrt[e]/(c + d*x)])/d^4
 - (3*b*(b*c - a*d)^2*e*ExpIntegralEi[e/(c + d*x)^2])/(2*d^4) - (b^3*e^2*E
xpIntegralEi[e/(c + d*x)^2])/(4*d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.72

method	result
risch	$\frac{e^{\frac{e}{(dx+c)^2}} b^3 x^4}{4} + e^{\frac{e}{(dx+c)^2}} a b^2 x^3 + \frac{3 e^{\frac{e}{(dx+c)^2}} a^2 b x^2}{2} + e^{\frac{e}{(dx+c)^2}} a^3 x + \frac{e^{\frac{e}{(dx+c)^2}} b^3 e x^2}{4d^2} + \frac{e^{\frac{e}{(dx+c)^2}} a^3 c}{d} - \ldots$
derivativedivides	$-\frac{a^3\left(-(dx+c)e^{\frac{e}{(dx+c)^2}} + \frac{e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right) + \frac{b^3\left(-\frac{(dx+c)^4 e^{\frac{e}{(dx+c)^2}}}{4} + \frac{e\left(-\frac{e^{\frac{e}{(dx+c)^2}}(dx+c)^2}{2} - \frac{e\,\mathrm{expIntegral}_1\left(-\frac{e}{(dx+c)^2}\right) \ldots}{2}\right)}{2} \ldots\right)}{d^3} \ldots}{\ldots}$
default	$-\frac{a^3\left(-(dx+c)e^{\frac{e}{(dx+c)^2}} + \frac{e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right) + \frac{b^3\left(-\frac{(dx+c)^4 e^{\frac{e}{(dx+c)^2}}}{4} + \frac{e\left(-\frac{e^{\frac{e}{(dx+c)^2}}(dx+c)^2}{2} - \frac{e\,\mathrm{expIntegral}_1\left(-\frac{e}{(dx+c)^2}\right) \ldots}{2}\right)}{2} \ldots\right)}{d^3} \ldots}{\ldots}$
parts	Expression too large to display

input
```
int(exp(e/(d*x+c)^2)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(e/(d*x+c)^2)*b^3*x^4+exp(e/(d*x+c)^2)*a*b^2*x^3+3/2*exp(e/(d*x+c)^
2)*a^2*b*x^2+exp(e/(d*x+c)^2)*a^3*x+1/4/d^2*exp(e/(d*x+c)^2)*b^3*e*x^2+1/d
*exp(e/(d*x+c)^2)*a^3*c-3/2/d^2*exp(e/(d*x+c)^2)*a^2*b*c^2+1/d^3*exp(e/(d*
x+c)^2)*a*b^2*c^3+2/d^2*exp(e/(d*x+c)^2)*a*b^2*e*x-1/4/d^4*exp(e/(d*x+c)^2
)*b^3*c^4-3/2/d^3*exp(e/(d*x+c)^2)*b^3*c*e*x-1/d/(-e)^(1/2)*Pi^(1/2)*erf((
-e)^(1/2)/(d*x+c))*a^3*e+3/d^2/(-e)^(1/2)*Pi^(1/2)*erf((-e)^(1/2)/(d*x+c))
*a^2*b*c*e-3/d^3/(-e)^(1/2)*Pi^(1/2)*erf((-e)^(1/2)/(d*x+c))*a*b^2*c^2*e+1
/d^4/(-e)^(1/2)*Pi^(1/2)*erf((-e)^(1/2)/(d*x+c))*b^3*c^3*e+2/d^3*exp(e/(d*
x+c)^2)*a*b^2*c*e-7/4/d^4*exp(e/(d*x+c)^2)*b^3*c^2*e+3/2/d^2*Ei(1,-e/(d*x+
c)^2)*a^2*b*e-3/d^3*Ei(1,-e/(d*x+c)^2)*a*b^2*c*e+3/2/d^4*Ei(1,-e/(d*x+c)^2
)*b^3*c^2*e-2/d^3/(-e)^(1/2)*Pi^(1/2)*erf((-e)^(1/2)/(d*x+c))*a*b^2*e^2+2/
d^4/(-e)^(1/2)*Pi^(1/2)*erf((-e)^(1/2)/(d*x+c))*b^3*c*e^2+1/4/d^4*Ei(1,-e/
(d*x+c)^2)*b^3*e^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.97

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx =$$

$$-\frac{4\sqrt{\pi}(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3-a^3d^4+2\,(b^3cd-ab^2d^2)e)\sqrt{-\frac{e}{d^2}}\,\mathrm{erf}\left(\frac{d\sqrt{-\frac{e}{d^2}}}{dx+c}\right)+(b^3e^2+6\,(b^3c^2-$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(pi)*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4 +
2*(b^3*c*d - a*b^2*d^2)*e)*sqrt(-e/d^2)*erf(d*sqrt(-e/d^2)/(d*x + c)) + (b
^3*e^2 + 6*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*e)*Ei(e/(d^2*x^2 + 2*c*d*x
+ c^2)) - (b^3*d^4*x^4 + 4*a*b^2*d^4*x^3 - b^3*c^4 + 4*a*b^2*c^3*d - 6*a^2
*b*c^2*d^2 + 4*a^3*c*d^3 + (6*a^2*b*d^4 + b^3*d^2*e)*x^2 - (7*b^3*c^2 - 8*
a*b^2*c*d)*e + 2*(2*a^3*d^4 - (3*b^3*c*d - 4*a*b^2*d^2)*e)*x)*e^(e/(d^2*x^
2 + 2*c*d*x + c^2)))/d^4
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx = \int (a+bx)^3\,e^{\frac{e}{c^2+2cdx+d^2x^2}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2)*(b*x+a)**3,x)
```

output
```
Integral((a + b*x)**3*exp(e/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx = \int (bx+a)^3 e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(b^3*d^3*x^4 + 4*a*b^2*d^3*x^3 + (6*a^2*b*d^3 + b^3*d*e)*x^2 + 2*(2*a^
3*d^3 - 3*b^3*c*e + 4*a*b^2*d*e)*x)*e^(e/(d^2*x^2 + 2*c*d*x + c^2))/d^3 +
integrate(1/2*(3*b^3*c^4*e - 4*a*b^2*c^3*d*e - (12*a*b^2*c*d^3*e - 6*a^2*b
*d^4*e - (6*c^2*d^2*e + d^2*e^2)*b^3)*x^2 + 2*(2*a^3*d^4*e - 2*(3*c^2*d^2*
e - 2*d^2*e^2)*a*b^2 + (4*c^3*d*e - 3*c*d*e^2)*b^3)*x)*e^(e/(d^2*x^2 + 2*c
*d*x + c^2))/(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3), x)
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx = \int (bx+a)^3 e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((b*x + a)^3*e^(e/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx = \int e^{\frac{e}{(c+d\,x)^2}}\,(a+b\,x)^3\,dx$$

input
```
int(exp(e/(c + d*x)^2)*(a + b*x)^3,x)
```

output
```
int(exp(e/(c + d*x)^2)*(a + b*x)^3, x)
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^3\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c)^2)*(b*x+a)^3,x)
```

output
```
( - 132*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c**7*d**3*e - 732*e**(e/(
c**2 + 2*c*d*x + d**2*x**2))*a**3*c**6*d**4*e*x - 1620*e**(e/(c**2 + 2*c*d
*x + d**2*x**2))*a**3*c**5*d**5*e*x**2 + 72*e**(e/(c**2 + 2*c*d*x + d**2*x
**2))*a**3*c**5*d**3*e**2 - 1740*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*
c**4*d**6*e*x**3 + 168*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c**4*d**4*
e**2*x - 780*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c**3*d**7*e*x**4 - 4
8*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c**3*d**5*e**2*x**2 - 16*e**(e/
(c**2 + 2*c*d*x + d**2*x**2))*a**3*c**3*d**3*e**3 + 108*e**(e/(c**2 + 2*c*
d*x + d**2*x**2))*a**3*c**2*d**8*e*x**5 - 432*e**(e/(c**2 + 2*c*d*x + d**2
*x**2))*a**3*c**2*d**6*e**2*x**3 + 48*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*
a**3*c**2*d**4*e**3*x + 228*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c*d**
9*e*x**6 - 408*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c*d**7*e**2*x**4 +
 144*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*c*d**5*e**3*x**2 + 60*e**(e/
(c**2 + 2*c*d*x + d**2*x**2))*a**3*d**10*e*x**7 - 120*e**(e/(c**2 + 2*c*d*
x + d**2*x**2))*a**3*d**8*e**2*x**5 + 80*e**(e/(c**2 + 2*c*d*x + d**2*x**2
))*a**3*d**6*e**3*x**3 - 32*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**3*d**4*
e**4*x + 486*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c**10*d**2 + 2916*
e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*b*c**9*d**3*x + 7290*e**(e/(c**2
+ 2*c*d*x + d**2*x**2))*a**2*b*c**8*d**4*x**2 - 612*e**(e/(c**2 + 2*c*d*x
+ d**2*x**2))*a**2*b*c**8*d**2*e + 9720*e**(e/(c**2 + 2*c*d*x + d**2*x*...
```

3.344 $\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 215

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx = \frac{(bc-ad)^2e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^3}+\frac{2b^2ee^{\frac{e}{(c+dx)^2}}(c+dx)}{3d^3} - \frac{b(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)^2}{d^3}+\frac{b^2e^{\frac{e}{(c+dx)^2}}(c+dx)^3}{3d^3} - \frac{(bc-ad)^2\sqrt{e}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^3}-\frac{2b^2e^{3/2}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{3d^3} + \frac{b(bc-ad)e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{d^3}$$

output
```
(-a*d+b*c)^2*exp(e/(d*x+c)^2)*(d*x+c)/d^3+2/3*b^2*e*exp(e/(d*x+c)^2)*(d*x+
c)/d^3-b*(-a*d+b*c)*exp(e/(d*x+c)^2)*(d*x+c)^2/d^3+1/3*b^2*exp(e/(d*x+c)^2
)*(d*x+c)^3/d^3-(-a*d+b*c)^2*e^(1/2)*Pi^(1/2)*erfi(e^(1/2)/(d*x+c))/d^3-2/
3*b^2*e^(3/2)*Pi^(1/2)*erfi(e^(1/2)/(d*x+c))/d^3+b*(-a*d+b*c)*e*Ei(e/(d*x+
c)^2)/d^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx = \frac{c(-3abcd+3a^2d^2+b^2(c^2+2e))\,e^{\frac{e}{(c+dx)^2}}}{3d^3} + \frac{de^{\frac{e}{(c+dx)^2}}x(3a^2d^2+3abd^2x+b^2(2e+d^2x^2)) - \sqrt{e}(-6abcd+3a^2d^2+b^2(3c^2+2e))\,\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right) + 3b}{3d^3}$$

input
```
Integrate[E^(e/(c + d*x)^2)*(a + b*x)^2,x]
```

output
```
(c*(-3*a*b*c*d + 3*a^2*d^2 + b^2*(c^2 + 2*e))*E^(e/(c + d*x)^2))/(3*d^3) +
 (d*E^(e/(c + d*x)^2)*x*(3*a^2*d^2 + 3*a*b*d^2*x + b^2*(2*e + d^2*x^2)) -
Sqrt[e]*(-6*a*b*c*d + 3*a^2*d^2 + b^2*(3*c^2 + 2*e))*Sqrt[Pi]*Erfi[Sqrt[e]
/(c + d*x)] + 3*b*(b*c - a*d)*e*ExpIntegralEi[e/(c + d*x)^2])/(3*d^3)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2e^{\frac{e}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)^2e^{\frac{e}{(c+dx)^2}}}{d^2} - \frac{2b(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^2} + \frac{b^2(c+dx)^2e^{\frac{e}{(c+dx)^2}}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}\sqrt{e}(bc-ad)^2\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^3}+\frac{be(bc-ad)\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{d^3}-\frac{b(c+dx)^2(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^3}+\frac{(c+dx)(bc-ad)^2e^{\frac{e}{(c+dx)^2}}}{d^3}-\frac{2\sqrt{\pi}b^2e^{3/2}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{3d^3}+\frac{b^2(c+dx)^3e^{\frac{e}{(c+dx)^2}}}{3d^3}+\frac{2b^2e(c+dx)e^{\frac{e}{(c+dx)^2}}}{3d^3}$$

input
```
Int[E^(e/(c + d*x)^2)*(a + b*x)^2,x]
```

output
```
((b*c - a*d)^2*E^(e/(c + d*x)^2)*(c + d*x))/d^3 + (2*b^2*e*E^(e/(c + d*x)^
2)*(c + d*x))/(3*d^3) - (b*(b*c - a*d)*E^(e/(c + d*x)^2)*(c + d*x)^2)/d^3
+ (b^2*E^(e/(c + d*x)^2)*(c + d*x)^3)/(3*d^3) - ((b*c - a*d)^2*Sqrt[e]*Sqr
t[Pi]*Erfi[Sqrt[e]/(c + d*x)])/d^3 - (2*b^2*e^(3/2)*Sqrt[Pi]*Erfi[Sqrt[e]/
(c + d*x)])/(3*d^3) + (b*(b*c - a*d)*e*ExpIntegralEi[e/(c + d*x)^2])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\frac{a^2\left(-(dx+c)e^{\frac{e}{(dx+c)^2}}+\frac{e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right)+\frac{b^2\left(-\frac{(dx+c)^3e^{\frac{e}{(dx+c)^2}}}{3}+\frac{2e\left(-(dx+c)e^{\frac{e}{(dx+c)^2}}+\frac{e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right)}{3}\right)}{d^2}+}{}$
default	$-\frac{a^2\left(-(dx+c)e^{\frac{e}{(dx+c)^2}}+\frac{e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right)+\frac{b^2\left(-\frac{(dx+c)^3e^{\frac{e}{(dx+c)^2}}}{3}+\frac{2e\left(-(dx+c)e^{\frac{e}{(dx+c)^2}}+\frac{e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right)}{3}\right)}{d^2}+}{}$
risch	$a^2e^{\frac{e}{(dx+c)^2}}x+\frac{a^2e^{\frac{e}{(dx+c)^2}}c}{d}-\frac{a^2e\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{d\sqrt{-e}}+\frac{b^2e^{\frac{e}{(dx+c)^2}}x^3}{3}+\frac{b^2e^{\frac{e}{(dx+c)^2}}c^3}{3d^3}+\frac{2b^2e\,e^{\frac{e}{(dx+c)^2}}x}{3d^2}+$
parts	$b^2e^{\frac{e}{(dx+c)^2}}x^3+2ab\,e^{\frac{e}{(dx+c)^2}}x^2+a^2e^{\frac{e}{(dx+c)^2}}x+\frac{b^2e^{\frac{e}{(dx+c)^2}}cx^2}{d}+\frac{2ab\,e^{\frac{e}{(dx+c)^2}}cx}{d}+\frac{a^2e^{\frac{e}{(dx+c)^2}}c}{d}-$

input
```
int(exp(e/(d*x+c)^2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(a^2*(-(d*x+c)*exp(e/(d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/
(d*x+c)))+b^2/d^2*(-1/3*(d*x+c)^3*exp(e/(d*x+c)^2)+2/3*e*(-(d*x+c)*exp(e/(
d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/(d*x+c))))+b^2/d^2*c^2*(-(d
*x+c)*exp(e/(d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/(d*x+c)))+2*b/
d*a*(-1/2*exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*Ei(1,-e/(d*x+c)^2))-2*b^2/d^2*c
*(-1/2*exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*Ei(1,-e/(d*x+c)^2))-2*b/d*c*a*(-(d
*x+c)*exp(e/(d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.91

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx$$
$$=\frac{3\,(b^2c-abd)e\mathrm{Ei}\left(\frac{e}{d^2x^2+2\,cdx+c^2}\right)+\sqrt{\pi}(3\,b^2c^2d-6\,abcd^2+3\,a^2d^3+2\,b^2de)\sqrt{-\frac{e}{d^2}}\,\mathrm{erf}\left(\frac{d\sqrt{-\frac{e}{d^2}}}{dx+c}\right)+(b^2d^3x^3}{3\,d^3}$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(b^2*c - a*b*d)*e*Ei(e/(d^2*x^2 + 2*c*d*x + c^2)) + sqrt(pi)*(3*b^2
*c^2*d - 6*a*b*c*d^2 + 3*a^2*d^3 + 2*b^2*d*e)*sqrt(-e/d^2)*erf(d*sqrt(-e/d
^2)/(d*x + c)) + (b^2*d^3*x^3 + 3*a*b*d^3*x^2 + b^2*c^3 - 3*a*b*c^2*d + 3*
a^2*c*d^2 + 2*b^2*c*e + (3*a^2*d^3 + 2*b^2*d*e)*x)*e^(e/(d^2*x^2 + 2*c*d*x
 + c^2)))/d^3
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx=\int(a+bx)^2\,e^{\frac{e}{c^2+2cdx+d^2x^2}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2)*(b*x+a)**2,x)
```

output
```
Integral((a + b*x)**2*exp(e/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx = \int (bx+a)^2 e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*d^2*x^3 + 3*a*b*d^2*x^2 + (3*a^2*d^2 + 2*b^2*e)*x)*e^(e/(d^2*x^2
+ 2*c*d*x + c^2))/d^2 + integrate(-2/3*(b^2*c^3*e + 3*(b^2*c*d^2*e - a*b*d
^3*e)*x^2 - (3*a^2*d^3*e - (3*c^2*d*e - 2*d*e^2)*b^2)*x)*e^(e/(d^2*x^2 + 2
*c*d*x + c^2))/(d^5*x^3 + 3*c*d^4*x^2 + 3*c^2*d^3*x + c^3*d^2), x)
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx = \int (bx+a)^2 e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*e^(e/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx = \int \mathrm{e}^{\frac{e}{{(c+d\,x)}^2}}\,{(a+b\,x)}^2\,dx$$

input
```
int(exp(e/(c + d*x)^2)*(a + b*x)^2,x)
```

output
```
int(exp(e/(c + d*x)^2)*(a + b*x)^2, x)
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)^2\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c)^2)*(b*x+a)^2,x)
```

output
```
( - 99*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c**7*d**2*e - 549*e**(e/(c
**2 + 2*c*d*x + d**2*x**2))*a**2*c**6*d**3*e*x - 1215*e**(e/(c**2 + 2*c*d*
x + d**2*x**2))*a**2*c**5*d**4*e*x**2 + 54*e**(e/(c**2 + 2*c*d*x + d**2*x*
*2))*a**2*c**5*d**2*e**2 - 1305*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c
**4*d**5*e*x**3 + 126*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c**4*d**3*e
**2*x - 585*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c**3*d**6*e*x**4 - 36
*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c**3*d**4*e**2*x**2 - 12*e**(e/(
c**2 + 2*c*d*x + d**2*x**2))*a**2*c**3*d**2*e**3 + 81*e**(e/(c**2 + 2*c*d*
x + d**2*x**2))*a**2*c**2*d**7*e*x**5 - 324*e**(e/(c**2 + 2*c*d*x + d**2*x
**2))*a**2*c**2*d**5*e**2*x**3 + 36*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*
*2*c**2*d**3*e**3*x + 171*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c*d**8*
e*x**6 - 306*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c*d**6*e**2*x**4 + 1
08*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*c*d**4*e**3*x**2 + 45*e**(e/(c
**2 + 2*c*d*x + d**2*x**2))*a**2*d**9*e*x**7 - 90*e**(e/(c**2 + 2*c*d*x +
d**2*x**2))*a**2*d**7*e**2*x**5 + 60*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a
**2*d**5*e**3*x**3 - 24*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**2*d**3*e**4
*x + 243*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c**10*d + 1458*e**(e/(c**
2 + 2*c*d*x + d**2*x**2))*a*b*c**9*d**2*x + 3645*e**(e/(c**2 + 2*c*d*x + d
**2*x**2))*a*b*c**8*d**3*x**2 - 306*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*
b*c**8*d*e + 4860*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*b*c**7*d**4*x**...
```

3.345 $\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 111

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx=-\frac{(bc-ad)e^{\frac{e}{(c+dx)^2}}(c+dx)}{d^2}+\frac{be^{\frac{e}{(c+dx)^2}}(c+dx)^2}{2d^2}+\frac{(bc-ad)\sqrt{e}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^2}-\frac{be\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{2d^2}$$

output
```
-(-a*d+b*c)*exp(e/(d*x+c)^2)*(d*x+c)/d^2+1/2*b*exp(e/(d*x+c)^2)*(d*x+c)^2/
d^2+(-a*d+b*c)*e^(1/2)*Pi^(1/2)*erfi(e^(1/2)/(d*x+c))/d^2-1/2*b*e*Ei(e/(d*
x+c)^2)/d^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.77

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx=-\frac{e^{\frac{e}{(c+dx)^2}}(c+dx)(bc-2ad-bdx)+2(-bc+ad)\sqrt{e}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)+be\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{2d^2}$$

input
```
Integrate[E^(e/(c + d*x)^2)*(a + b*x),x]
```

output
```
-1/2*(E^(e/(c + d*x)^2)*(c + d*x)*(b*c - 2*a*d - b*d*x) + 2*(-(b*c) + a*d)
*Sqrt[e]*Sqrt[Pi]*Erfi[Sqrt[e]/(c + d*x)] + b*e*ExpIntegralEi[e/(c + d*x)^
2])/d^2
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)e^{\frac{e}{(c+dx)^2}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)e^{\frac{e}{(c+dx)^2}}}{d}+\frac{b(c+dx)e^{\frac{e}{(c+dx)^2}}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}\sqrt{e}(bc-ad)\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d^2}-\frac{(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^2}}}{d^2}-\frac{be\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^2}\right)}{2d^2}+\frac{b(c+dx)^2e^{\frac{e}{(c+dx)^2}}}{2d^2}$$

input
```
Int[E^(e/(c + d*x)^2)*(a + b*x),x]
```

output
```
-(((b*c - a*d)*E^(e/(c + d*x)^2)*(c + d*x))/d^2) + (b*E^(e/(c + d*x)^2)*(c
 + d*x)^2)/(2*d^2) + ((b*c - a*d)*Sqrt[e]*Sqrt[Pi]*Erfi[Sqrt[e]/(c + d*x)]
)/d^2 - (b*e*ExpIntegralEi[e/(c + d*x)^2])/(2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26

method	result
derivativedivides	$-\frac{a\left(-(dx+c)\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}+\frac{e\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right)+\frac{b\left(-\frac{\mathrm{e}^{\frac{e}{(dx+c)^{2}}}(dx+c)^{2}}{2}-\frac{e\,\mathrm{expIntegral}_{1}\left(-\frac{e}{(dx+c)^{2}}\right)}{2}\right)}{d}-\frac{bc\left(-(dx+c)\,\mathrm{e}^{\frac{e}{(dx+}}\right.}{}}{d}$
default	$-\frac{a\left(-(dx+c)\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}+\frac{e\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}\right)+\frac{b\left(-\frac{\mathrm{e}^{\frac{e}{(dx+c)^{2}}}(dx+c)^{2}}{2}-\frac{e\,\mathrm{expIntegral}_{1}\left(-\frac{e}{(dx+c)^{2}}\right)}{2}\right)}{d}-\frac{bc\left(-(dx+c)\,\mathrm{e}^{\frac{e}{(dx+}}\right.}{}}{d}$
risch	$a\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}x+\frac{a\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}c}{d}-\frac{ae\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{d\sqrt{-e}}+\frac{b\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}x^{2}}{2}-\frac{b\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}c^{2}}{2d^{2}}+\frac{be\,\mathrm{expIntegral}_{1}\left(-\frac{e}{(dx+c}\right.}{2d^{2}}$
parts	$b\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}x^{2}+a\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}x+\frac{b\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}cx}{d}+\frac{a\,\mathrm{e}^{\frac{e}{(dx+c)^{2}}}c}{d}-\frac{e\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)bx}{d\sqrt{-e}}-\frac{ae\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{d\sqrt{-e}}+$

input
```
int(exp(e/(d*x+c)^2)*(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(a*(-(d*x+c)*exp(e/(d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/(d
*x+c)))+b/d*(-1/2*exp(e/(d*x+c)^2)*(d*x+c)^2-1/2*e*Ei(1,-e/(d*x+c)^2))-b/d
*c*(-(d*x+c)*exp(e/(d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/(d*x+c)
)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.10

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx =$$

$$-\frac{be\operatorname{Ei}\left(\frac{e}{d^2x^2+2\,cdx+c^2}\right)+2\,\sqrt{\pi}(bcd-ad^2)\sqrt{-\frac{e}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{e}{d^2}}}{dx+c}\right)-(bd^2x^2+2\,ad^2x-bc^2+2\,acd)e^{\left(\frac{e}{d^2x^2+2\,cd\cdots}\right.}}{2\,d^2}$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b*e*Ei(e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*sqrt(pi)*(b*c*d - a*d^2)*sqr
t(-e/d^2)*erf(d*sqrt(-e/d^2)/(d*x + c)) - (b*d^2*x^2 + 2*a*d^2*x - b*c^2 +
 2*a*c*d)*e^(e/(d^2*x^2 + 2*c*d*x + c^2)))/d^2
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx = \int (a+bx)\,e^{\frac{e}{c^2+2cdx+d^2x^2}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2)*(b*x+a),x)
```

output
```
Integral((a + b*x)*exp(e/(c**2 + 2*c*d*x + d**2*x**2)), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx = \int (bx+a)e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)*e^(e/(d^2*x^2 + 2*c*d*x + c^2)) + integrate((b*d*e*x^2
 + 2*a*d*e*x)*e^(e/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 + 3*c
^2*d*x + c^3), x)
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx=\int(bx+a)e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)*(b*x+a),x, algorithm="giac")
```

output
```
integrate((b*x + a)*e^(e/(d*x + c)^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx=\int \mathrm{e}^{\frac{e}{(c+d\,x)^2}}\,(a+b\,x)\,dx$$

input
```
int(exp(e/(c + d*x)^2)*(a + b*x),x)
```

output
```
int(exp(e/(c + d*x)^2)*(a + b*x), x)
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^2}}(a+bx)\,dx=\text{too large to display}$$

input
```
int(exp(e/(d*x+c)^2)*(b*x+a),x)
```

output
```
( - 66*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**7*d*e - 366*e**(e/(c**2 +
2*c*d*x + d**2*x**2))*a*c**6*d**2*e*x - 810*e**(e/(c**2 + 2*c*d*x + d**2*x
**2))*a*c**5*d**3*e*x**2 + 36*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**5*d
*e**2 - 870*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**4*d**4*e*x**3 + 84*e*
*(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**4*d**2*e**2*x - 390*e**(e/(c**2 + 2
*c*d*x + d**2*x**2))*a*c**3*d**5*e*x**4 - 24*e**(e/(c**2 + 2*c*d*x + d**2*
x**2))*a*c**3*d**3*e**2*x**2 - 8*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**
3*d*e**3 + 54*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**2*d**6*e*x**5 - 216
*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c**2*d**4*e**2*x**3 + 24*e**(e/(c**
2 + 2*c*d*x + d**2*x**2))*a*c**2*d**2*e**3*x + 114*e**(e/(c**2 + 2*c*d*x +
 d**2*x**2))*a*c*d**7*e*x**6 - 204*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c
*d**5*e**2*x**4 + 72*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*c*d**3*e**3*x**
2 + 30*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*d**8*e*x**7 - 60*e**(e/(c**2
+ 2*c*d*x + d**2*x**2))*a*d**6*e**2*x**5 + 40*e**(e/(c**2 + 2*c*d*x + d**2
*x**2))*a*d**4*e**3*x**3 - 16*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a*d**2*e
**4*x + 81*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*b*c**10 + 486*e**(e/(c**2 +
 2*c*d*x + d**2*x**2))*b*c**9*d*x + 1215*e**(e/(c**2 + 2*c*d*x + d**2*x**2
))*b*c**8*d**2*x**2 - 102*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*b*c**8*e + 1
620*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*b*c**7*d**3*x**3 - 450*e**(e/(c**2
 + 2*c*d*x + d**2*x**2))*b*c**7*d*e*x + 1215*e**(e/(c**2 + 2*c*d*x + d*...
```

3.346 $\int e^{\frac{e}{(c+dx)^2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 50

$$\int e^{\frac{e}{(c+dx)^2}} dx = \frac{e^{\frac{e}{(c+dx)^2}}(c+dx)}{d} - \frac{\sqrt{e}\sqrt{\pi}\mathrm{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d}$$

output
```
exp(e/(d*x+c)^2)*(d*x+c)/d-e^(1/2)*Pi^(1/2)*erfi(e^(1/2)/(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int e^{\frac{e}{(c+dx)^2}} dx = \frac{e^{\frac{e}{(c+dx)^2}}(c+dx)}{d} - \frac{\sqrt{e}\sqrt{\pi}\mathrm{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d}$$

input
```
Integrate[E^(e/(c + d*x)^2),x]
```

output
```
(E^(e/(c + d*x)^2)*(c + d*x))/d - (Sqrt[e]*Sqrt[Pi]*Erfi[Sqrt[e]/(c + d*x)
])/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2635, 2640, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{e}{(c+dx)^2}}dx$$

$$\Big\downarrow \text{2635}$$

$$2e\int \frac{e^{\frac{e}{(c+dx)^2}}}{(c+dx)^2}dx+\frac{(c+dx)e^{\frac{e}{(c+dx)^2}}}{d}$$

$$\Big\downarrow \text{2640}$$

$$\frac{(c+dx)e^{\frac{e}{(c+dx)^2}}}{d}-\frac{2e\int e^{\frac{e}{(c+dx)^2}}d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)e^{\frac{e}{(c+dx)^2}}}{d}-\frac{\sqrt{\pi}\sqrt{e}\text{erfi}\left(\frac{\sqrt{e}}{c+dx}\right)}{d}$$

input
```
Int[E^(e/(c + d*x)^2),x]
```

output
```
(E^(e/(c + d*x)^2)*(c + d*x))/d - (Sqrt[e]*Sqrt[Pi]*Erfi[Sqrt[e]/(c + d*x)
])/d
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2635
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(c +
d*x)*(F^(a + b*(c + d*x)^n)/d), x] - Simp[b*n*Log[F]   Int[(c + d*x)^n*F^(a
 + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&
ILtQ[n, 0]
```

rule 2640
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[1/(d*(m + 1))   Subst[Int[F^(a + b*x^2), x], x, (c +
d*x)^(m + 1)], x] /; FreeQ[{F, a, b, c, d, m, n}, x] && EqQ[n, 2*(m + 1)]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{-(dx+c)\,\mathrm{e}^{\frac{e}{(dx+c)^2}}+\frac{e\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}}{d}$	48
default	$-\frac{-(dx+c)\,\mathrm{e}^{\frac{e}{(dx+c)^2}}+\frac{e\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{\sqrt{-e}}}{d}$	48
risch	$\mathrm{e}^{\frac{e}{(dx+c)^2}}x+\frac{\mathrm{e}^{\frac{e}{(dx+c)^2}}c}{d}-\frac{e\sqrt{\pi}\,\operatorname{erf}\left(\frac{\sqrt{-e}}{dx+c}\right)}{d\sqrt{-e}}$	57

input
```
int(exp(e/(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-(d*x+c)*exp(e/(d*x+c)^2)+e*Pi^(1/2)/(-e)^(1/2)*erf((-e)^(1/2)/(d*x+
c)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int e^{\frac{e}{(c+dx)^2}}\,dx=\frac{\sqrt{\pi}d\sqrt{-\frac{e}{d^2}}\operatorname{erf}\left(\frac{d\sqrt{-\frac{e}{d^2}}}{dx+c}\right)+(dx+c)e^{\left(\frac{e}{d^2x^2+2\,cdx+c^2}\right)}}{d}$$

input
```
integrate(exp(e/(d*x+c)^2),x, algorithm="fricas")
```

output
```
(sqrt(pi)*d*sqrt(-e/d^2)*erf(d*sqrt(-e/d^2)/(d*x + c)) + (d*x + c)*e^(e/(d
^2*x^2 + 2*c*d*x + c^2)))/d
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^2}}\,dx = \int e^{\frac{e}{(c+dx)^2}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2),x)
```

output
```
Integral(exp(e/(c + d*x)**2), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^2}}\,dx = \int e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*d*e*integrate(x*e^(e/(d^2*x^2 + 2*c*d*x + c^2))/(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3), x) + x*e^(e/(d^2*x^2 + 2*c*d*x + c^2))
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^2}}\,dx = \int e^{\left(\frac{e}{(dx+c)^2}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(e^(e/(d*x + c)^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int e^{\frac{e}{(c+dx)^2}}\,dx = \frac{\mathrm{e}^{\frac{e}{(c+d\,x)^2}}\,(c+d\,x)}{d} - \frac{\sqrt{e}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{e}}{c+d\,x}\right)}{d}$$

input
```
int(exp(e/(c + d*x)^2),x)
```

output
```
(exp(e/(c + d*x)^2)*(c + d*x))/d - (e^(1/2)*pi^(1/2)*erfi(e^(1/2)/(c + d*x
)))/d
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^2}}\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c)^2),x)
```

output
```
( - 279*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**9 - 2127*e**(e/(c**2 + 2*c*
d*x + d**2*x**2))*c**8*d*x - 6972*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**7
*d**2*x**2 + 174*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**7*e - 12684*e**(e/
(c**2 + 2*c*d*x + d**2*x**2))*c**6*d**3*x**3 + 834*e**(e/(c**2 + 2*c*d*x +
 d**2*x**2))*c**6*d*e*x - 13650*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**5*d
**4*x**4 + 1350*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**5*d**2*e*x**2 - 52*
e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**5*e**2 - 8274*e**(e/(c**2 + 2*c*d*x
 + d**2*x**2))*c**4*d**5*x**5 + 330*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c*
*4*d**3*e*x**3 - 68*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**4*d*e**2*x - 19
32*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**3*d**6*x**6 - 1590*e**(e/(c**2 +
 2*c*d*x + d**2*x**2))*c**3*d**4*e*x**4 + 248*e**(e/(c**2 + 2*c*d*x + d**2
*x**2))*c**3*d**2*e**2*x**2 + 8*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**3*e
**3 + 708*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c**2*d**7*x**7 - 2106*e**(e/
(c**2 + 2*c*d*x + d**2*x**2))*c**2*d**5*e*x**5 + 632*e**(e/(c**2 + 2*c*d*x
 + d**2*x**2))*c**2*d**3*e**2*x**3 - 40*e**(e/(c**2 + 2*c*d*x + d**2*x**2)
)*c**2*d*e**3*x + 561*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c*d**8*x**8 - 10
86*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*c*d**6*e*x**6 + 508*e**(e/(c**2 + 2
*c*d*x + d**2*x**2))*c*d**4*e**2*x**4 - 104*e**(e/(c**2 + 2*c*d*x + d**2*x
**2))*c*d**2*e**3*x**2 + 105*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*d**9*x**9
 - 210*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*d**7*e*x**7 + 140*e**(e/(c**...
```

3.347 $\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} \, dx = \text{Int}\left(\frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}, x\right)$$

output
```
Defer(Int)(exp(e/(d*x+c)^2)/(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} \, dx = \int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} \, dx$$

input
```
Integrate[E^(e/(c + d*x)^2)/(a + b*x),x]
```

output
```
Integrate[E^(e/(c + d*x)^2)/(a + b*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx} \, dx$$

$$\Big\downarrow \; 2654$$

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}dx$$

input
```
Int[E^(e/(c + d*x)^2)/(a + b*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{\frac{e}{(dx+c)^2}}}{bx+a}dx$$

input
```
int(exp(e/(d*x+c)^2)/(b*x+a),x)
```

output
```
int(exp(e/(d*x+c)^2)/(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(e^(e/(d^2*x^2 + 2*c*d*x + c^2))/(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}\,dx = \int \frac{e^{\frac{e}{c^2+2cdx+d^2x^2}}}{a+bx}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2)/(b*x+a),x)
```

output
```
Integral(exp(e/(c**2 + 2*c*d*x + d**2*x**2))/(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}\,dx=\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c)^2)/(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}\,dx=\int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(e/(d*x + c)^2)/(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}\,dx=\int \frac{\mathrm{e}^{\frac{e}{{\left(c+d\,x\right)}^2}}}{a+b\,x}\,dx$$

input
```
int(exp(e/(c + d*x)^2)/(a + b*x),x)
```

output
```
int(exp(e/(c + d*x)^2)/(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{a+bx}\,dx = \int \frac{e^{\frac{e}{d^2x^2+2cdx+c^2}}}{bx+a}dx$$

input
```
int(exp(e/(d*x+c)^2)/(b*x+a),x)
```

output
```
int(e**(e/(c**2 + 2*c*d*x + d**2*x**2))/(a + b*x),x)
```

3.348 $\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx = \operatorname{Int}\left(\frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2},x\right)$$

output
```
Defer(Int)(exp(e/(d*x+c)^2)/(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx = \int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx$$

input
```
Integrate[E^(e/(c + d*x)^2)/(a + b*x)^2,x]
```

output
```
Integrate[E^(e/(c + d*x)^2)/(a + b*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}dx$$

input
```
Int[E^(e/(c + d*x)^2)/(a + b*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{\frac{e}{(dx+c)^2}}}{(bx+a)^2}dx$$

input
```
int(exp(e/(d*x+c)^2)/(b*x+a)^2,x)
```

output
```
int(exp(e/(d*x+c)^2)/(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(e^(e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*x^2 + 2*a*b*x + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx = \int \frac{e^{\frac{e}{c^2+2cdx+d^2x^2}}}{(a+bx)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2)/(b*x+a)**2,x)
```

output
```
Integral(exp(e/(c**2 + 2*c*d*x + d**2*x**2))/(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} \, dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^2} \, dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c)^2)/(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} \, dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^2} \, dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2} \, dx = \int \frac{e^{\frac{e}{(c+d\,x)^2}}}{(a+b\,x)^2} \, dx$$

input
```
int(exp(e/(c + d*x)^2)/(a + b*x)^2,x)
```

output
```
int(exp(e/(c + d*x)^2)/(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 318316, normalized size of antiderivative = 16753.47

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int(exp(e/(d*x+c)^2)/(b*x+a)^2,x)
```

output
```
(2520*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**8*d**8 + 20160*e**(e/(c*
*2 + 2*c*d*x + d**2*x**2))*a**8*c**7*d**9*x + 70560*e**(e/(c**2 + 2*c*d*x
+ d**2*x**2))*a**8*c**6*d**10*x**2 - 2520*e**(e/(c**2 + 2*c*d*x + d**2*x**
2))*a**8*c**6*d**8*e + 141120*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**
5*d**11*x**3 - 15120*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**5*d**9*e*
x + 176400*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**4*d**12*x**4 - 3780
0*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**4*d**10*e*x**2 + 1260*e**(e/
(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**4*d**8*e**2 + 141120*e**(e/(c**2 + 2
*c*d*x + d**2*x**2))*a**8*c**3*d**13*x**5 - 50400*e**(e/(c**2 + 2*c*d*x +
d**2*x**2))*a**8*c**3*d**11*e*x**3 + 5040*e**(e/(c**2 + 2*c*d*x + d**2*x**
2))*a**8*c**3*d**9*e**2*x + 70560*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8
*c**2*d**14*x**6 - 37800*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**2*d**
12*e*x**4 + 7560*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**2*d**10*e**2*
x**2 - 420*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**2*d**8*e**3 + 20160
*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c*d**15*x**7 - 15120*e**(e/(c**2
 + 2*c*d*x + d**2*x**2))*a**8*c*d**13*e*x**5 + 5040*e**(e/(c**2 + 2*c*d*x
+ d**2*x**2))*a**8*c*d**11*e**2*x**3 - 840*e**(e/(c**2 + 2*c*d*x + d**2*x*
*2))*a**8*c*d**9*e**3*x + 2520*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*d*
*16*x**8 - 2520*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*d**14*e*x**6 + 12
60*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*d**12*e**2*x**4 - 420*e**(e...
```

3.349 $\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx = \operatorname{Int}\left(\frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3}, x\right)$$

output
```
Defer(Int)(exp(e/(d*x+c)^2)/(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx = \int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} dx$$

input
```
Integrate[E^(e/(c + d*x)^2)/(a + b*x)^3,x]
```

output
```
Integrate[E^(e/(c + d*x)^2)/(a + b*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3}dx$$

input
```
Int[E^(e/(c + d*x)^2)/(a + b*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{\frac{e}{(dx+c)^2}}}{(bx+a)^3}dx$$

input
```
int(exp(e/(d*x+c)^2)/(b*x+a)^3,x)
```

output
```
int(exp(e/(d*x+c)^2)/(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^3}\,dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(e^(e/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*
x + a^3), x)
```

Sympy [N/A]

Not integrable

Time = 167.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3}\,dx = \int \frac{e^{\frac{e}{c^2+2cdx+d^2x^2}}}{(a+bx)^3}\,dx$$

input
```
integrate(exp(e/(d*x+c)**2)/(b*x+a)**3,x)
```

output
```
Integral(exp(e/(c**2 + 2*c*d*x + d**2*x**2))/(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} \, dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^3} \, dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c)^2)/(b*x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} \, dx = \int \frac{e^{\left(\frac{e}{(dx+c)^2}\right)}}{(bx+a)^3} \, dx$$

input
```
integrate(exp(e/(d*x+c)^2)/(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(e^(e/(d*x + c)^2)/(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3} \, dx = \int \frac{e^{\frac{e}{(c+d\,x)^2}}}{(a+b\,x)^3} \, dx$$

input
```
int(exp(e/(c + d*x)^2)/(a + b*x)^3,x)
```

output
```
int(exp(e/(c + d*x)^2)/(a + b*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 5.94 (sec) , antiderivative size = 771854, normalized size of antiderivative = 40623.89

$$\int \frac{e^{\frac{e}{(c+dx)^2}}}{(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int(exp(e/(d*x+c)^2)/(b*x+a)^3,x)
```

output
```
(2520*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**8*d**8 + 20160*e**(e/(c*
*2 + 2*c*d*x + d**2*x**2))*a**8*c**7*d**9*x + 70560*e**(e/(c**2 + 2*c*d*x
+ d**2*x**2))*a**8*c**6*d**10*x**2 - 2520*e**(e/(c**2 + 2*c*d*x + d**2*x**
2))*a**8*c**6*d**8*e + 141120*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**
5*d**11*x**3 - 15120*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**5*d**9*e*
x + 176400*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**4*d**12*x**4 - 3780
0*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**4*d**10*e*x**2 + 1260*e**(e/
(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**4*d**8*e**2 + 141120*e**(e/(c**2 + 2
*c*d*x + d**2*x**2))*a**8*c**3*d**13*x**5 - 50400*e**(e/(c**2 + 2*c*d*x +
d**2*x**2))*a**8*c**3*d**11*e*x**3 + 5040*e**(e/(c**2 + 2*c*d*x + d**2*x**
2))*a**8*c**3*d**9*e**2*x + 70560*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8
*c**2*d**14*x**6 - 37800*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**2*d**
12*e*x**4 + 7560*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**2*d**10*e**2*
x**2 - 420*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c**2*d**8*e**3 + 20160
*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*c*d**15*x**7 - 15120*e**(e/(c**2
 + 2*c*d*x + d**2*x**2))*a**8*c*d**13*e*x**5 + 5040*e**(e/(c**2 + 2*c*d*x
+ d**2*x**2))*a**8*c*d**11*e**2*x**3 - 840*e**(e/(c**2 + 2*c*d*x + d**2*x*
*2))*a**8*c*d**9*e**3*x + 2520*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*d*
*16*x**8 - 2520*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*d**14*e*x**6 + 12
60*e**(e/(c**2 + 2*c*d*x + d**2*x**2))*a**8*d**12*e**2*x**4 - 420*e**(e...
```

3.350 $\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx$

Optimal result

Integrand size = 19, antiderivative size = 206

$$\begin{aligned}\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx =& -\frac{b^2(bc-ad)e^{\frac{e}{(c+dx)^3}}(c+dx)^3}{d^4} \\ &+ \frac{b^2(bc-ad)e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^3}\right)}{d^4} \\ &+ \frac{b^3\left(-\frac{e}{(c+dx)^3}\right)^{4/3}(c+dx)^4\Gamma\left(-\frac{4}{3},-\frac{e}{(c+dx)^3}\right)}{3d^4} \\ &+ \frac{b(bc-ad)^2\left(-\frac{e}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)}{d^4} \\ &- \frac{(bc-ad)^3\sqrt[3]{-\frac{e}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d^4}\end{aligned}$$

output
```
-b^2*(-a*d+b*c)*exp(e/(d*x+c)^3)*(d*x+c)^3/d^4+b^2*(-a*d+b*c)*e*Ei(e/(d*x+
c)^3)/d^4+1/3*b^3*(-e/(d*x+c)^3)^(4/3)*(d*x+c)^4*GAMMA(-4/3,-e/(d*x+c)^3)/
d^4+b*(-a*d+b*c)^2*(-e/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-e/(d*x+c)^3)
/d^4-1/3*(-a*d+b*c)^3*(-e/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-e/(d*x+c)^3
)/d^4
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.95

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx$$
$$= \frac{-3b^2(bc-ad)e^{\frac{e}{(c+dx)^3}}(c+dx)^3+3b^2(bc-ad)e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^3}\right)+b^3\left(-\frac{e}{(c+dx)^3}\right)^{4/3}(c+dx)^4\Gamma\left(\cdot\right.}{}$$

input
```
Integrate[E^(e/(c + d*x)^3)*(a + b*x)^3,x]
```

output
```
(-3*b^2*(b*c - a*d)*E^(e/(c + d*x)^3)*(c + d*x)^3 + 3*b^2*(b*c - a*d)*e*Ex
pIntegralEi[e/(c + d*x)^3] + b^3*(-(e/(c + d*x)^3))^(4/3)*(c + d*x)^4*Gamm
a[-4/3, -(e/(c + d*x)^3)] + 3*b*(b*c - a*d)^2*(-(e/(c + d*x)^3))^(2/3)*(c
+ d*x)^2*Gamma[-2/3, -(e/(c + d*x)^3)] - (b*c - a*d)^3*(-(e/(c + d*x)^3))^
(1/3)*(c + d*x)*Gamma[-1/3, -(e/(c + d*x)^3)])/(3*d^4)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^3e^{\frac{e}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(-\frac{3b^2(c+dx)^2(bc-ad)e^{\frac{e}{(c+dx)^3}}}{d^3}+\frac{(ad-bc)^3e^{\frac{e}{(c+dx)^3}}}{d^3}+\frac{3b(c+dx)(bc-ad)^2e^{\frac{e}{(c+dx)^3}}}{d^3}+\frac{b^3(c+dx)^3e^{\frac{e}{(c+dx)^3}}}{d^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2e(bc-ad)\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^3}\right)}{d^4}-\frac{b^2(c+dx)^3(bc-ad)e^{\frac{e}{(c+dx)^3}}}{d^4}+\frac{b(c+dx)^2(bc-ad)^2\left(-\frac{e}{(c+dx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)}{d^4}-\frac{(c+dx)(bc-ad)^3\sqrt[3]{-\frac{e}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d^4}+\frac{b^3(c+dx)^4\left(-\frac{e}{(c+dx)^3}\right)^{4/3}\Gamma\left(-\frac{4}{3},-\frac{e}{(c+dx)^3}\right)}{3d^4}$$

input
```
Int[E^(e/(c + d*x)^3)*(a + b*x)^3,x]
```

output
```
-((b^2*(b*c - a*d)*E^(e/(c + d*x)^3)*(c + d*x)^3)/d^4) + (b^2*(b*c - a*d)*
e*ExpIntegralEi[e/(c + d*x)^3])/d^4 + (b^3*(-(e/(c + d*x)^3))^(4/3)*(c + d
*x)^4*Gamma[-4/3, -(e/(c + d*x)^3)])/(3*d^4) + (b*(b*c - a*d)^2*(-(e/(c +
d*x)^3))^(2/3)*(c + d*x)^2*Gamma[-2/3, -(e/(c + d*x)^3)])/d^4 - ((b*c - a*
d)^3*(-(e/(c + d*x)^3))^(1/3)*(c + d*x)*Gamma[-1/3, -(e/(c + d*x)^3)])/(3*
d^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int e^{\frac{e}{(dx+c)^3}} (bx+a)^3 \, dx$$

input
```
int(exp(e/(d*x+c)^3)*(b*x+a)^3,x)
```

output
```
int(exp(e/(d*x+c)^3)*(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.69

$$\int e^{\frac{e}{(c+dx)^3}} (a+bx)^3 \, dx$$
$$= \frac{4 (b^3c - ab^2d)e\mathrm{Ei}\left(\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right) - 6 \left(b^3c^2d^2 - 2\,ab^2cd^3 + a^2bd^4\right)\left(-\frac{e}{d^3}\right)^{\frac{2}{3}} \Gamma\left(\frac{1}{3}, -\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c}\right.}{}$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*(b^3*c - a*b^2*d)*e*Ei(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))
 - 6*(b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(-e/d^3)^(2/3)*gamma(1/3, -
e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + (4*b^3*c^3*d - 12*a*b^2*c^2
*d^2 + 12*a^2*b*c*d^3 - 4*a^3*d^4 - 3*b^3*d*e)*(-e/d^3)^(1/3)*gamma(2/3, -
e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + (b^3*d^4*x^4 + 4*a*b^2*d^4*
x^3 + 6*a^2*b*d^4*x^2 - b^3*c^4 + 4*a*b^2*c^3*d - 6*a^2*b*c^2*d^2 + 4*a^3*
c*d^3 + 3*b^3*c*e + (4*a^3*d^4 + 3*b^3*d*e)*x)*e^(e/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3)))/d^4
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx = \int (a+bx)^3\, e^{\frac{e}{c^3+3c^2dx+3cd^2x^2+d^3x^3}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**3)*(b*x+a)**3,x)
```

output
```
Integral((a + b*x)**3*exp(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
)), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx = \int (bx+a)^3 e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(b^3*d^3*x^4 + 4*a*b^2*d^3*x^3 + 6*a^2*b*d^3*x^2 + (4*a^3*d^3 + 3*b^3*
e)*x)*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/d^3 + integrate(-3/4
*(b^3*c^4*e + 4*(b^3*c*d^3*e - a*b^2*d^4*e)*x^3 + 6*(b^3*c^2*d^2*e - a^2*b
*d^4*e)*x^2 - (4*a^3*d^4*e - (4*c^3*d*e - 3*d*e^2)*b^3)*x)*e^(e/(d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^7*x^4 + 4*c*d^6*x^3 + 6*c^2*d^5*x^2 +
4*c^3*d^4*x + c^4*d^3), x)
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx = \int (bx+a)^3 e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((b*x + a)^3*e^(e/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx=\int \mathrm{e}^{\frac{e}{{(c+d\,x)}^3}}\,{(a+b\,x)}^3\,dx$$

input
```
int(exp(e/(c + d*x)^3)*(a + b*x)^3,x)
```

output
```
int(exp(e/(c + d*x)^3)*(a + b*x)^3, x)
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^3\,dx=\text{too large to display}$$

input
```
int(exp(e/(d*x+c)^3)*(b*x+a)^3,x)
```

output
```
( - 2296*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**10
*d**3*e - 18424*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**
3*c**9*d**4*e*x - 62496*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3))*a**3*c**8*d**5*e*x**2 - 112224*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3))*a**3*c**7*d**6*e*x**3 + 1176*e**(e/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*a**3*c**7*d**3*e**2 - 101136*e**(e/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**6*d**7*e*x**4 + 3696*e**(e/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**6*d**4*e**2*x - 705
6*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**5*d**8*e*
x**5 - 2520*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c*
*5*d**5*e**2*x**2 + 89376*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*
x**3))*a**3*c**4*d**9*e*x**6 - 26880*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3))*a**3*c**4*d**6*e**2*x**3 - 252*e**(e/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3))*a**3*c**4*d**3*e**3 + 105504*e**(e/(c**3 + 3
*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**3*d**10*e*x**7 - 49560*e**
(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**3*d**7*e**2*x*
*4 + 1260*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**3*c**3
*d**4*e**3*x + 59976*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
)*a**3*c**2*d**11*e*x**8 - 43344*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3))*a**3*c**2*d**8*e**2*x**5 + 5292*e**(e/(c**3 + 3*c**2*d*x ...
```

3.351 $\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx$

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx = \frac{b^2 e^{\frac{e}{(c+dx)^3}}(c+dx)^3}{3d^3} - \frac{b^2 e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^3}\right)}{3d^3} - \frac{2b(bc-ad)\left(-\frac{e}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)}{3d^3} + \frac{(bc-ad)^2\sqrt[3]{-\frac{e}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d^3}$$

output
```
1/3*b^2*exp(e/(d*x+c)^3)*(d*x+c)^3/d^3-1/3*b^2*e*Ei(e/(d*x+c)^3)/d^3-2/3*b
*(-a*d+b*c)*(-e/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-e/(d*x+c)^3)/d^3+1/
3*(-a*d+b*c)^2*(-e/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-e/(d*x+c)^3)/d^3
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.90

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx$$
$$=\frac{b^2e^{\frac{e}{(c+dx)^3}}(c+dx)^3-b^2e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^3}\right)-2b(bc-ad)\left(-\frac{e}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right.}{3d^3}$$

input
```
Integrate[E^(e/(c + d*x)^3)*(a + b*x)^2,x]
```

output
```
(b^2*E^(e/(c + d*x)^3)*(c + d*x)^3 - b^2*e*ExpIntegralEi[e/(c + d*x)^3] -
2*b*(b*c - a*d)*(-(e/(c + d*x)^3))^(2/3)*(c + d*x)^2*Gamma[-2/3, -(e/(c +
d*x)^3)] + (b*c - a*d)^2*(-(e/(c + d*x)^3))^(1/3)*(c + d*x)*Gamma[-1/3, -(
e/(c + d*x)^3)])/(3*d^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2e^{\frac{e}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)^2e^{\frac{e}{(c+dx)^3}}}{d^2}-\frac{2b(c+dx)(bc-ad)e^{\frac{e}{(c+dx)^3}}}{d^2}+\frac{b^2(c+dx)^2e^{\frac{e}{(c+dx)^3}}}{d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2b(c+dx)^2(bc-ad)\left(-\frac{e}{(c+dx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)}{3d^3}+\frac{(c+dx)(bc-ad)^2\sqrt[3]{-\frac{e}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d^3}-\frac{b^2e\,\text{ExpIntegralEi}\left(\frac{e}{(c+dx)^3}\right)}{3d^3}+\frac{b^2(c+dx)^3e^{\frac{e}{(c+dx)^3}}}{3d^3}$$

input
```
Int[E^(e/(c + d*x)^3)*(a + b*x)^2,x]
```

output
```
(b^2*E^(e/(c + d*x)^3)*(c + d*x)^3)/(3*d^3) - (b^2*e*ExpIntegralEi[e/(c +
d*x)^3])/(3*d^3) - (2*b*(b*c - a*d)*(-(e/(c + d*x)^3))^(2/3)*(c + d*x)^2*G
amma[-2/3, -(e/(c + d*x)^3)])/(3*d^3) + ((b*c - a*d)^2*(-(e/(c + d*x)^3))^
(1/3)*(c + d*x)*Gamma[-1/3, -(e/(c + d*x)^3)])/(3*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int e^{\frac{e}{(dx+c)^3}}(bx+a)^2\,dx$$

input
```
int(exp(e/(d*x+c)^3)*(b*x+a)^2,x)
```

output
```
int(exp(e/(d*x+c)^3)*(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx =$$

$$-\frac{b^2e\mathrm{Ei}\left(\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)-3\left(b^2cd^2-abd^3\right)\left(-\frac{e}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3},-\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)+3\left(b^2c^2d-2\,abcd\right.}{}$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3*(b^2*e*Ei(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - 3*(b^2*c*d^2
 - a*b*d^3)*(-e/d^3)^(2/3)*gamma(1/3, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3)) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(-e/d^3)^(1/3)*gamma(2/3
, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) - (b^2*d^3*x^3 + 3*a*b*d^3
*x^2 + 3*a^2*d^3*x + b^2*c^3 - 3*a*b*c^2*d + 3*a^2*c*d^2)*e^(e/(d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^3
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx = \int (a+bx)^2\,e^{\frac{e}{c^3+3c^2dx+3cd^2x^2+d^3x^3}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**3)*(b*x+a)**2,x)
```

output
```
Integral((a + b*x)**2*exp(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
)), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx=\int (bx+a)^2e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3)) + integrate((b^2*d*e*x^3 + 3*a*b*d*e*x^2 + 3*a^2*d*e*x)*e^(e/(d^
3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2
*x^2 + 4*c^3*d*x + c^4), x)
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx=\int (bx+a)^2e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((b*x + a)^2*e^(e/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx=\int \mathrm{e}^{\frac{e}{(c+d\,x)^3}}\,{(a+b\,x)}^2\,dx$$

input
```
int(exp(e/(c + d*x)^3)*(a + b*x)^2,x)
```

output
```
int(exp(e/(c + d*x)^3)*(a + b*x)^2, x)
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)^2\,dx = \text{too large to display}$$

input
```
int(exp(e/(d*x+c)^3)*(b*x+a)^2,x)
```

output
```
( - 1722*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**10
*d**2*e - 13818*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**
2*c**9*d**3*e*x - 46872*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3))*a**2*c**8*d**4*e*x**2 - 84168*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*a**2*c**7*d**5*e*x**3 + 882*e**(e/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3))*a**2*c**7*d**2*e**2 - 75852*e**(e/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**6*d**6*e*x**4 + 2772*e**(e/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**6*d**3*e**2*x - 5292*e
**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**5*d**7*e*x**
5 - 1890*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**5*
d**4*e**2*x**2 + 67032*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3))*a**2*c**4*d**8*e*x**6 - 20160*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3))*a**2*c**4*d**5*e**2*x**3 - 189*e**(e/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3))*a**2*c**4*d**2*e**3 + 79128*e**(e/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**3*d**9*e*x**7 - 37170*e**(e/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**3*d**6*e**2*x**4 +
945*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2*c**3*d**3*
e**3*x + 44982*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**2
*c**2*d**10*e*x**8 - 32508*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3))*a**2*c**2*d**7*e**2*x**5 + 3969*e**(e/(c**3 + 3*c**2*d*x + 3*c*...
```

3.352 $\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 92

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx=\frac{b\left(-\frac{e}{(c+dx)^3}\right)^{2/3}(c+dx)^2\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)}{3d^2}-\frac{(bc-ad)\sqrt[3]{-\frac{e}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d^2}$$

output
```
1/3*b*(-e/(d*x+c)^3)^(2/3)*(d*x+c)^2*GAMMA(-2/3,-e/(d*x+c)^3)/d^2-1/3*(-a*
d+b*c)*(-e/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-e/(d*x+c)^3)/d^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx=\frac{(c+dx)\left(b\left(-\frac{e}{(c+dx)^3}\right)^{2/3}(c+dx)\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)+(-bc+ad)\sqrt[3]{-\frac{e}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)\right)}{3d^2}$$

input
```
Integrate[E^(e/(c + d*x)^3)*(a + b*x),x]
```

output
```
((c + d*x)*(b*(-(e/(c + d*x)^3))^(2/3)*(c + d*x)*Gamma[-2/3, -(e/(c + d*x)
^3)] + (-(b*c) + a*d)*(-(e/(c + d*x)^3))^(1/3)*Gamma[-1/3, -(e/(c + d*x)^3
)]))/(3*d^2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)e^{\frac{e}{(c+dx)^3}}dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(\frac{(ad-bc)e^{\frac{e}{(c+dx)^3}}}{d}+\frac{b(c+dx)e^{\frac{e}{(c+dx)^3}}}{d}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b(c+dx)^2\left(-\frac{e}{(c+dx)^3}\right)^{2/3}\Gamma\left(-\frac{2}{3},-\frac{e}{(c+dx)^3}\right)}{3d^2}-\frac{(c+dx)(bc-ad)\sqrt[3]{-\frac{e}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d^2}$$

input
```
Int[E^(e/(c + d*x)^3)*(a + b*x),x]
```

output
```
(b*(-(e/(c + d*x)^3))^(2/3)*(c + d*x)^2*Gamma[-2/3, -(e/(c + d*x)^3)])/(3*
d^2) - ((b*c - a*d)*(-(e/(c + d*x)^3))^(1/3)*(c + d*x)*Gamma[-1/3, -(e/(c
+ d*x)^3)])/(3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int e^{\frac{e}{(dx+c)^3}}(bx+a)\,dx$$

input
```
int(exp(e/(d*x+c)^3)*(b*x+a),x)
```

output
```
int(exp(e/(d*x+c)^3)*(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(80) = 160$.

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.84

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx = -\frac{bd^2\left(-\frac{e}{d^3}\right)^{\frac{2}{3}}\Gamma\left(\frac{1}{3}, -\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right) - 2\left(bcd-ad^2\right)\left(-\frac{e}{d^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, -\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right) - \left(bd^2x^2 +\right.}{2\,d^2}$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b*d^2*(-e/d^3)^(2/3)*gamma(1/3, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3)) - 2*(b*c*d - a*d^2)*(-e/d^3)^(1/3)*gamma(2/3, -e/(d^3*x^3 + 3*c*
d^2*x^2 + 3*c^2*d*x + c^3)) - (b*d^2*x^2 + 2*a*d^2*x - b*c^2 + 2*a*c*d)*e^
(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d^2
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx = \int (a+bx)\,e^{\frac{e}{c^3+3c^2dx+3cd^2x^2+d^3x^3}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**3)*(b*x+a),x)
```

output
```
Integral((a + b*x)*exp(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)),
 x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx = \int (bx+a)e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + 2*a*x)*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)) + inte
grate(3/2*(b*d*e*x^2 + 2*a*d*e*x)*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3))/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x)
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx = \int (bx+a)e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)*(b*x+a),x, algorithm="giac")
```

output
```
integrate((b*x + a)*e^(e/(d*x + c)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx=\int \mathrm{e}^{\frac{e}{(c+d\,x)^3}}\,(a+b\,x)\,dx$$

input
```
int(exp(e/(c + d*x)^3)*(a + b*x),x)
```

output
```
int(exp(e/(c + d*x)^3)*(a + b*x), x)
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^3}}(a+bx)\,dx=\text{too large to display}$$

input
```
int(exp(e/(d*x+c)^3)*(b*x+a),x)
```

output

```
( - 7448*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**13*d
- 83216*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**12*d**
2*x - 417648*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**1
1*d**3*x**2 - 1232000*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
))*a*c**10*d**4*x**3 + 4368*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))*a*c**10*d*e - 2331560*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3))*a*c**9*d**5*x**4 + 30072*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3))*a*c**9*d**2*e*x - 2849616*e**(e/(c**3 + 3*c**2*d*x + 3*c
*d**2*x**2 + d**3*x**3))*a*c**8*d**6*x**5 + 74088*e**(e/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3))*a*c**8*d**3*e*x**2 - 2003232*e**(e/(c**3 +
3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**7*d**7*x**6 + 34272*e**(e/(c
**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**7*d**4*e*x**3 - 1260*e
**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**7*d*e**2 - 2069
76*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**6*d**8*x**7
 - 225792*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*c**6*d*
*5*e*x**4 - 2016*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a*
c**6*d**2*e**2*x + 1191960*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3))*a*c**5*d**9*x**8 - 613872*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
 + d**3*x**3))*a*c**5*d**6*e*x**5 + 14364*e**(e/(c**3 + 3*c**2*d*x + 3*c*d
**2*x**2 + d**3*x**3))*a*c**5*d**3*e**2*x**2 + 1410640*e**(e/(c**3 + 3*...
```

3.353 $\int e^{\frac{e}{(c+dx)^3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 40

$$\int e^{\frac{e}{(c+dx)^3}}\,dx=\frac{\sqrt[3]{-\frac{e}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d}$$

output
```
1/3*(-e/(d*x+c)^3)^(1/3)*(d*x+c)*GAMMA(-1/3,-e/(d*x+c)^3)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int e^{\frac{e}{(c+dx)^3}}\,dx=\frac{\sqrt[3]{-\frac{e}{(c+dx)^3}}(c+dx)\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d}$$

input
```
Integrate[E^(e/(c + d*x)^3),x]
```

output
```
((-(e/(c + d*x)^3))^(1/3)*(c + d*x)*Gamma[-1/3, -(e/(c + d*x)^3)])/(3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\frac{e}{(c+dx)^3}}\,dx$$

$$\Big\downarrow \text{2637}$$

$$\frac{(c+dx)\sqrt[3]{-\frac{e}{(c+dx)^3}}\Gamma\left(-\frac{1}{3},-\frac{e}{(c+dx)^3}\right)}{3d}$$

input
```
Int[E^(e/(c + d*x)^3),x]
```

output
```
((-(e/(c + d*x)^3))^(1/3)*(c + d*x)*Gamma[-1/3, -(e/(c + d*x)^3)])/(3*d)
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Maple [F]

$$\int \mathrm{e}^{\frac{e}{(dx+c)^3}}dx$$

input
```
int(exp(e/(d*x+c)^3),x)
```

output
```
int(exp(e/(d*x+c)^3),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.22

$$\int e^{\frac{e}{(c+dx)^3}}\,dx = -\frac{d\left(-\frac{e}{d^3}\right)^{\frac{1}{3}}\Gamma\left(\frac{2}{3}, -\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right) - (dx+c)e^{\left(\frac{e}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}}{d}$$

input
```
integrate(exp(e/(d*x+c)^3),x, algorithm="fricas")
```

output
```
-(d*(-e/d^3)^(1/3)*gamma(2/3, -e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
) - (d*x + c)*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)))/d
```

Sympy [F]

$$\int e^{\frac{e}{(c+dx)^3}}\,dx = \int e^{\frac{e}{(c+dx)^3}}\,dx$$

input
```
integrate(exp(e/(d*x+c)**3),x)
```

output
```
Integral(exp(e/(c + d*x)**3), x)
```

Maxima [F]

$$\int e^{\frac{e}{(c+dx)^3}}\,dx = \int e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3),x, algorithm="maxima")
```

output
```
3*d*e*integrate(x*e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(d^4*x^4
 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4), x) + x*e^(e/(d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3))
```

Giac [F]

$$\int e^{\frac{e}{(c+dx)^3}}\,dx=\int e^{\left(\frac{e}{(dx+c)^3}\right)}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3),x, algorithm="giac")
```

output
```
integrate(e^(e/(d*x + c)^3), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int e^{\frac{e}{(c+dx)^3}}\,dx=\frac{(c+d\,x)\,\left(\mathrm{e}^{\frac{e}{(c+d\,x)^3}}+\Gamma\left(\frac{2}{3}\right)\,{\left(-\frac{e}{(c+d\,x)^3}\right)}^{1/3}-{\left(-\frac{e}{(c+d\,x)^3}\right)}^{1/3}\,\Gamma\left(\frac{2}{3},-\frac{e}{(c+d\,x)^3}\right)\right)}{d}$$

input
```
int(exp(e/(c + d*x)^3),x)
```

output
```
((c + d*x)*(exp(e/(c + d*x)^3) + gamma(2/3)*(-e/(c + d*x)^3)^(1/3) - (-e/(
c + d*x)^3)^(1/3)*igamma(2/3, -e/(c + d*x)^3)))/d
```

Reduce [F]

$$\int e^{\frac{e}{(c+dx)^3}}\,dx=\text{too large to display}$$

input
```
int(exp(e/(d*x+c)^3),x)
```

output

```
( - 1064*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**13 - 11
888*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**12*d*x - 596
64*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**11*d**2*x**2
- 176000*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**10*d**3
*x**3 + 624*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**10*e
 - 333080*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**9*d**4
*x**4 + 4296*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**9*d
*e*x - 407088*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**8*
d**5*x**5 + 10584*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c
**8*d**2*e*x**2 - 286176*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3))*c**7*d**6*x**6 + 4896*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))*c**7*d**3*e*x**3 - 180*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3))*c**7*e**2 - 29568*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3))*c**6*d**7*x**7 - 32256*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**
2 + d**3*x**3))*c**6*d**4*e*x**4 - 288*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3))*c**6*d*e**2*x + 170280*e**(e/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3))*c**5*d**8*x**8 - 87696*e**(e/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3))*c**5*d**5*e*x**5 + 2052*e**(e/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3))*c**5*d**2*e**2*x**2 + 201520*e**(e/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*c**4*d**9*x**9 - 113904*e**(...
```

3.354 $\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} \, dx = \text{Int}\left(\frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}, x\right)$$

output
```
Defer(Int)(exp(e/(d*x+c)^3)/(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} \, dx = \int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} \, dx$$

input
```
Integrate[E^(e/(c + d*x)^3)/(a + b*x),x]
```

output
```
Integrate[E^(e/(c + d*x)^3)/(a + b*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2654}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx} \, dx$$

$$\Big\downarrow \text{ 2654}$$

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}dx$$

input
```
Int[E^(e/(c + d*x)^3)/(a + b*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2654
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Unintegrable[F^(a + b*(c + d*x)^n)/(e + f*x), x] /; FreeQ[{F, a,
 b, c, d, e, f, n}, x] && NeQ[d*e - c*f, 0]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{\frac{e}{(dx+c)^3}}}{bx+a}dx$$

input
```
int(exp(e/(d*x+c)^3)/(b*x+a),x)
```

output
```
int(exp(e/(d*x+c)^3)/(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)/(b*x+a),x, algorithm="fricas")
```

output
```
integral(e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}\,dx = \int \frac{e^{\frac{e}{c^3+3c^2dx+3cd^2x^2+d^3x^3}}}{a+bx}\,dx$$

input
```
integrate(exp(e/(d*x+c)**3)/(b*x+a),x)
```

output
```
Integral(exp(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(a + b*x),
 x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c)^3)/(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{bx+a}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)/(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(e/(d*x + c)^3)/(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}\,dx = \int \frac{\mathrm{e}^{\frac{e}{(c+d\,x)^3}}}{a+b\,x}\,dx$$

input
```
int(exp(e/(c + d*x)^3)/(a + b*x),x)
```

output
```
int(exp(e/(c + d*x)^3)/(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{a+bx}\,dx = \int \frac{e^{\frac{e}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}}}{bx+a}dx$$

input
```
int(exp(e/(d*x+c)^3)/(b*x+a),x)
```

output
```
int(e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(a + b*x),x)
```

3.355 $\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx = \text{Int}\left(\frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}, x\right)$$

output
```
Defer(Int)(exp(e/(d*x+c)^3)/(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx = \int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} dx$$

input
```
Integrate[E^(e/(c + d*x)^3)/(a + b*x)^2,x]
```

output
```
Integrate[E^(e/(c + d*x)^3)/(a + b*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}dx$$

input
```
Int[E^(e/(c + d*x)^3)/(a + b*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\mathrm{e}^{\frac{e}{(dx+c)^3}}}{(bx+a)^2}dx$$

input
```
int(exp(e/(d*x+c)^3)/(b*x+a)^2,x)
```

output
```
int(exp(e/(d*x+c)^3)/(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.79

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}\,dx=\int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(e^(e/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3))/(b^2*x^2 + 2*a*b*
x + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 9.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}\,dx=\int \frac{e^{\frac{e}{c^3+3c^2dx+3cd^2x^2+d^3x^3}}}{(a+bx)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c)**3)/(b*x+a)**2,x)
```

output
```
Integral(exp(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))/(a + b*x)*
*2, x)
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(e^(e/(d*x + c)^3)/(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}\,dx = \int \frac{e^{\left(\frac{e}{(dx+c)^3}\right)}}{(bx+a)^2}\,dx$$

input
```
integrate(exp(e/(d*x+c)^3)/(b*x+a)^2,x, algorithm="giac")
```

output
```
undef
```

Mupad [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}\,dx = \int \frac{\mathrm{e}^{\frac{e}{(c+d\,x)^3}}}{(a+b\,x)^2}\,dx$$

input
```
int(exp(e/(c + d*x)^3)/(a + b*x)^2,x)
```

output
```
int(exp(e/(c + d*x)^3)/(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 7.25 (sec) , antiderivative size = 1139170, normalized size of antiderivative = 59956.32

$$\int \frac{e^{\frac{e}{(c+dx)^3}}}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
int(exp(e/(d*x+c)^3)/(b*x+a)^2,x)
```

output
```
( - 221760*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c*
*13*d**11 - 2661120*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))
*a**11*c**12*d**12*x - 14636160*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
 d**3*x**3))*a**11*c**11*d**13*x**2 - 48787200*e**(e/(c**3 + 3*c**2*d*x +
3*c*d**2*x**2 + d**3*x**3))*a**11*c**10*d**14*x**3 + 221760*e**(e/(c**3 +
3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**10*d**11*e - 109771200*e
**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**9*d**15*x**
4 + 1995840*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c
**9*d**12*e*x - 175633920*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*
x**3))*a**11*c**8*d**16*x**5 + 7983360*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3))*a**11*c**8*d**13*e*x**2 - 204906240*e**(e/(c**3 + 3*c*
*2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**7*d**17*x**6 + 18627840*e**(
e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**7*d**14*e*x**3
 - 110880*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**
7*d**11*e**2 - 175633920*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3))*a**11*c**6*d**18*x**7 + 27941760*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3))*a**11*c**6*d**15*e*x**4 - 665280*e**(e/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**6*d**12*e**2*x - 109771200*e**(
e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*c**5*d**19*x**8 +
 27941760*e**(e/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3))*a**11*...
```

3.356 $\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 104

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = -\frac{F^{e+\frac{bf}{d}}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f\log(F)}{d(c+dx)}\right)}{h} + \frac{F^{e+\frac{f(bg-ah)}{dg-ch}}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{h}$$

output
```
-F^(e+b*f/d)*Ei(-(-a*d+b*c)*f*ln(F)/d/(d*x+c))/h+F^(e+f*(-a*h+b*g)/(-c*h+d
*g))*Ei(-(-a*d+b*c)*f*(h*x+g)*ln(F)/(-c*h+d*g)/(d*x+c))/h
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = \frac{F^{e+\frac{bf}{d}}\left(-\text{ExpIntegralEi}\left(\frac{(-bc+ad)f\log(F)}{d(c+dx)}\right) + F^{\frac{(bc-ad)fh}{d(dg-ch)}}\text{ExpIntegralEi}\left(\frac{(bc-ad)f(g+hx)\log(F)}{(-dg+ch)(c+dx)}\right)\right)}{h}$$

input
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x),x]
```

output
```
(F^(e + (b*f)/d)*(-ExpIntegralEi[((-(b*c) + a*d)*f*Log[F])/(d*(c + d*x))]
+ F^(((b*c - a*d)*f*h)/(d*(d*g - c*h)))*ExpIntegralEi[((b*c - a*d)*f*(g +
h*x)*Log[F])/((-(d*g) + c*h)*(c + d*x))]))/h
```

Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2661, 2660, 2639, 2663, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{f(a+bx)}{c+dx}+e}}{g+hx}dx$$

$$\Big\downarrow \text{2661}$$

$$\frac{d\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{c+dx}dx}{h}-\frac{(dg-ch)\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)(g+hx)}dx}{h}$$

$$\Big\downarrow \text{2660}$$

$$\frac{d\int \frac{F^{\frac{de+bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{c+dx}dx}{h}-\frac{(dg-ch)\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)(g+hx)}dx}{h}$$

$$\Big\downarrow \text{2639}$$

$$-\frac{(dg-ch)\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)(g+hx)}dx}{h}-\frac{F^{\frac{bf}{d}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f\log(F)}{d(c+dx)}\right)}{h}$$

$$\Big\downarrow \text{2663}$$

$$\frac{\int \frac{F^{e-\frac{(bc-ad)f(g+hx)}{(dg-ch)(c+dx)}+\frac{f(bg-ah)}{dg-ch}}(c+dx)}{g+hx}d\frac{g+hx}{c+dx}}{h}-\frac{F^{\frac{bf}{d}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f\log(F)}{d(c+dx)}\right)}{h}$$

$$\Big\downarrow \text{2609}$$

$$\frac{F^{\frac{f(bg-ah)}{dg-ch}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{h}-\frac{F^{\frac{bf}{d}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f\log(F)}{d(c+dx)}\right)}{h}$$

input
```
Int[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x),x]
```

output
```
-((F^(e + (b*f)/d)*ExpIntegralEi[-(((b*c - a*d)*f*Log[F])/(d*(c + d*x)))])
/h) + (F^(e + (f*(b*g - a*h))/(d*g - c*h))*ExpIntegralEi[-(((b*c - a*d)*f*
(g + h*x)*Log[F])/((d*g - c*h)*(c + d*x)))])/h
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2639
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[F^a*(ExpIntegralEi[b*(c + d*x)^n*Log[F]]/(f*n)), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

rule 2660
```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))*((g_.)
 + (h_.)*(x_))^(m_.), x_Symbol] :> Int[(g + h*x)^m*F^((d*e + b*f)/d - f*((b
*c - a*d)/(d*(c + d*x)))), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m}, x] &
& NeQ[b*c - a*d, 0] && EqQ[d*g - c*h, 0]
```

rule 2661
```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))/((g_.)
 + (h_.)*(x_)), x_Symbol] :> Simp[d/h   Int[F^(e + f*((a + b*x)/(c + d*x)))
/(c + d*x), x], x] - Simp[(d*g - c*h)/h   Int[F^(e + f*((a + b*x)/(c + d*x)
))/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] &
& NeQ[b*c - a*d, 0] && NeQ[d*g - c*h, 0]
```

rule 2663
```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))/(((g_.
) + (h_.)*(x_))*((i_.) + (j_.)*(x_))), x_Symbol] :> Simp[-d/(h*(d*i - c*j))
   Subst[Int[F^(e + f*((b*i - a*j)/(d*i - c*j)) - (b*c - a*d)*f*(x/(d*i - c
*j)))/x, x], x, (i + j*x)/(c + d*x)], x] /; FreeQ[{F, a, b, c, d, e, f, g,
h}, x] && EqQ[d*g - c*h, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(104) = 208$.

Time = 0.33 (sec) , antiderivative size = 436, normalized size of antiderivative = 4.19

method	result
risch	$\frac{F^{\frac{bf+de}{d}} \operatorname{expIntegral}_1\left(-\frac{(adf-bcf)\ln(F)}{d(dx+c)}-\frac{(bf+de)\ln(F)}{d}-\frac{-\ln(F)bf-\ln(F)de}{d}\right)ad}{(ad-bc)h} - \frac{F^{\frac{bf+de}{d}} \operatorname{expIntegral}_1\left(-\frac{(adf-bcf)\ln(F)}{d(dx+c)}-\frac{(bf+}{}\right.}{(ad-bc)h}$

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g),x,method=_RETURNVERBOSE)
```

output
```
1/(a*d-b*c)/h*F^((b*f+d*e)/d)*Ei(1,-(a*d*f-b*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e
)*ln(F)/d-(-ln(F)*b*f-ln(F)*d*e)/d)*a*d-1/(a*d-b*c)/h*F^((b*f+d*e)/d)*Ei(1
,-(a*d*f-b*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-ln(F)*b*f-ln(F)*d*e)/d
)*b*c-1/(a*d-b*c)/h*F^((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))*Ei(1,-(a*d*f-b
*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-ln(F)*a*f*h+ln(F)*b*f*g-ln(F)*c*
e*h+ln(F)*d*e*g)/(c*h-d*g))*a*d+1/(a*d-b*c)/h*F^((a*f*h-b*f*g+c*e*h-d*e*g)
/(c*h-d*g))*Ei(1,-(a*d*f-b*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-ln(F)*
a*f*h+ln(F)*b*f*g-ln(F)*c*e*h+ln(F)*d*e*g)/(c*h-d*g))*b*c
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.30

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = -\frac{F^{\frac{de+bf}{d}}\operatorname{Ei}\left(-\frac{(bc-ad)f\log(F)}{d^2x+cd}\right) - F^{\frac{(de+bf)g-(ce+af)h}{dg-ch}}\operatorname{Ei}\left(-\frac{((bc-ad)fhx+(bc-ad)fg)\log(F)}{cdg-c^2h+(d^2g-cdh)x}\right)}{h}$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g),x, algorithm="fricas")
```

output
```
-(F^((d*e + b*f)/d)*Ei(-(b*c - a*d)*f*log(F)/(d^2*x + c*d)) - F^(((d*e + b
*f)*g - (c*e + a*f)*h)/(d*g - c*h))*Ei(-((b*c - a*d)*f*h*x + (b*c - a*d)*f
*g)*log(F)/(c*d*g - c^2*h + (d^2*g - c*d*h)*x)))/h
```

Sympy [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx$$

input
```
integrate(F**(e+f*(b*x+a)/(d*x+c))/(h*x+g),x)
```

output
```
Integral(F**(e + f*(a + b*x)/(c + d*x))/(g + h*x), x)
```

Maxima [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = \int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{hx+g}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g),x, algorithm="maxima")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g), x)
```

Giac [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = \int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{hx+g}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g),x, algorithm="giac")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = \int \frac{F^{e+\frac{f\,(a+b\,x)}{c+d\,x}}}{g+h\,x}\,dx$$

input
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x),x)
```

output
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x), x)
```

Reduce [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{g+hx}\,dx = \int \frac{f^{\frac{bfx+dex+af+ce}{dx+c}}}{hx+g}dx$$

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g),x)
```

output
```
int(f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))/(g + h*x),x)
```

3.357 $\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 159

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx = \frac{dF^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{h(dg-ch)} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{h(g+hx)} + \frac{(bc-ad)fF^{e+\frac{f(bg-ah)}{dg-ch}}\,\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)\log(F)}{(dg-ch)^2}$$

output
```
d*F^(e+b*f/d-(-a*d+b*c)*f/d/(d*x+c))/h/(-c*h+d*g)-F^(e+f*(b*x+a)/(d*x+c))/
h/(h*x+g)+(-a*d+b*c)*f*F^(e+f*(-a*h+b*g)/(-c*h+d*g))*Ei(-(-a*d+b*c)*f*(h*x
+g)*ln(F)/(-c*h+d*g)/(d*x+c))*ln(F)/(-c*h+d*g)^2
```

Mathematica [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx = \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx$$

input
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^2,x]
```

output
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^2, x]
```

Rubi [A] (verified)

Time = 3.43 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2662, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{f(a+bx)}{c+dx}+e}}{(g+hx)^2}dx$$

$$\Big\downarrow \text{ 2662}$$

$$\frac{f\log(F)(bc-ad)\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)}dx}{h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{h(g+hx)}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{f\log(F)(bc-ad)\int \left(-\frac{dhF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(c+dx)}+\frac{h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)}+\frac{dF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)(c+dx)^2}\right)dx}{h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{h(g+hx)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f\log(F)(bc-ad)\left(\frac{hF^{\frac{f(bg-ah)}{dg-ch}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^2}+\frac{dF^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{f\log(F)(bc-ad)(dg-ch)}\right)}{h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{h(g+hx)}$$

input
```
Int[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^2,x]
```

output
```
-(F^(e + (f*(a + b*x))/(c + d*x))/(h*(g + h*x))) + ((b*c - a*d)*f*((F^(e +
 (f*(b*g - a*h))/(d*g - c*h))*h*ExpIntegralEi[-(((b*c - a*d)*f*(g + h*x)*L
og[F])/((d*g - c*h)*(c + d*x)))])/(d*g - c*h)^2 + (d*F^(e + (b*f)/d - ((b*
c - a*d)*f)/(d*(c + d*x))))/((b*c - a*d)*f*(d*g - c*h)*Log[F]))*Log[F])/h
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2662
```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))*((g_.)
 + (h_.)*(x_))^(m_), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(F^(e + f*((a + b*
x)/(c + d*x)))/(h*(m + 1))), x] - Simp[f*(b*c - a*d)*(Log[F]/(h*(m + 1)))
 Int[(g + h*x)^(m + 1)*(F^(e + f*((a + b*x)/(c + d*x)))/(c + d*x)^2), x], x
] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b*c - a*d, 0] && NeQ[d*g
- c*h, 0] && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(159) = 318$.

Time = 0.40 (sec) , antiderivative size = 582, normalized size of antiderivative = 3.66

method	result
risch	$\frac{\ln(F) F^{\frac{f(ad-bc)}{d(dx+c)}} F^{\frac{bf+de}{d}} adf}{(ch-dg)^2\left(\frac{f\ln(F)a}{dx+c}-\frac{bcf\ln(F)}{d(dx+c)}+\frac{\ln(F)bf}{d}+e\ln(F)-\frac{\ln(F)afh}{ch-dg}+\frac{\ln(F)bfg}{ch-dg}-\frac{\ln(F)ceh}{ch-dg}+\frac{\ln(F)deg}{ch-dg}\right)} - \frac{\ln}{(ch-dg)^2\left(\frac{f\ln(F)a}{dx+c}-\frac{bcf\ln(F)}{d(dx+c)}+\ln\right.}$

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(F)/(c*h-d*g)^2*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d*x+
c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1/(
c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln(F)*d*e*g)*a*d*
f-ln(F)/(c*h-d*g)^2*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d*
x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1
/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln(F)*d*e*g)*b*
c*f+ln(F)/(c*h-d*g)^2*F^((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))*Ei(1,-(a*d*f
-b*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-ln(F)*a*f*h+ln(F)*b*f*g-ln(F)*
c*e*h+ln(F)*d*e*g)/(c*h-d*g))*a*d*f-ln(F)/(c*h-d*g)^2*F^((a*f*h-b*f*g+c*e*
h-d*e*g)/(c*h-d*g))*Ei(1,-(a*d*f-b*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-
(-ln(F)*a*f*h+ln(F)*b*f*g-ln(F)*c*e*h+ln(F)*d*e*g)/(c*h-d*g))*b*c*f
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.38

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx$$
$$= \frac{((bc-ad)fhx+(bc-ad)fg)F^{\frac{(de+bf)g-(ce+af)h}{dg-ch}}\operatorname{Ei}\left(-\frac{((bc-ad)fhx+(bc-ad)fg)\log(F)}{cdg-c^2h+(d^2g-cdh)x}\right)\log(F)+(cdg-c^2h+(c}{d^2g^3-2\,cdg^2h+c^2gh^2+(d^2g^2h-2\,cdgh^2+c^2h^3)x}$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^2,x, algorithm="fricas")
```

output
```
(((b*c - a*d)*f*h*x + (b*c - a*d)*f*g)*F^(((d*e + b*f)*g - (c*e + a*f)*h)/
(d*g - c*h))*Ei(-((b*c - a*d)*f*h*x + (b*c - a*d)*f*g)*log(F)/(c*d*g - c^2
*h + (d^2*g - c*d*h)*x))*log(F) + (c*d*g - c^2*h + (d^2*g - c*d*h)*x)*F^((
c*e + a*f + (d*e + b*f)*x)/(d*x + c)))/(d^2*g^3 - 2*c*d*g^2*h + c^2*g*h^2
+ (d^2*g^2*h - 2*c*d*g*h^2 + c^2*h^3)*x)
```

Sympy [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx=\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx$$

input
```
integrate(F**(e+f*(b*x+a)/(d*x+c))/(h*x+g)**2,x)
```

output
```
Integral(F**(e + f*(a + b*x)/(c + d*x))/(g + h*x)**2, x)
```

Maxima [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx=\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^2, x)
```

Giac [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx=\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx = \int \frac{F^{e+\frac{f\,(a+b\,x)}{c+d\,x}}}{{(g+h\,x)}^2}\,dx$$

input
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^2,x)
```

output
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^2, x)
```

Reduce [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^2}\,dx = \text{too large to display}$$

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^2,x)
```

output
```
( - 4*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a**2*c**3*d**3*
f**2*h**3 + 6*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a**2*c*
*2*d**4*f**2*g*h**2 - 6*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)*
*2*a**2*c**2*d**4*f**2*h**3*x - 4*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x
))*log(f)**2*a**2*c*d**5*f**2*g**2*h + 4*f**((a*f + b*f*x + c*e + d*e*x)/(
c + d*x))*log(f)**2*a**2*c*d**5*f**2*g*h**2*x - 4*f**((a*f + b*f*x + c*e +
 d*e*x)/(c + d*x))*log(f)**2*a**2*c*d**5*f**2*h**3*x**2 + f**((a*f + b*f*x
 + c*e + d*e*x)/(c + d*x))*log(f)**2*a**2*d**6*f**2*g**3 - f**((a*f + b*f*
x + c*e + d*e*x)/(c + d*x))*log(f)**2*a**2*d**6*f**2*g**2*h*x + f**((a*f +
 b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a**2*d**6*f**2*g*h**2*x**2 - f*
*((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a**2*d**6*f**2*h**3*x**
3 + 8*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a*b*c**4*d**2*f
**2*h**3 - 12*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a*b*c**
3*d**3*f**2*g*h**2 + 12*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)*
*2*a*b*c**3*d**3*f**2*h**3*x + 8*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x)
)*log(f)**2*a*b*c**2*d**4*f**2*g**2*h - 8*f**((a*f + b*f*x + c*e + d*e*x)/
(c + d*x))*log(f)**2*a*b*c**2*d**4*f**2*g*h**2*x + 8*f**((a*f + b*f*x + c*
e + d*e*x)/(c + d*x))*log(f)**2*a*b*c**2*d**4*f**2*h**3*x**2 - 2*f**((a*f
+ b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a*b*c*d**5*f**2*g**3 + 2*f**((
a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**2*a*b*c*d**5*f**2*g**2*h*...
```

3.358 $\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 366

$$\begin{aligned}\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx &= \frac{d^2F^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{2h(dg-ch)^2}-\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{2h(g+hx)^2}\\
&\quad+\frac{d(bc-ad)fF^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}\log(F)}{2(dg-ch)^3}-\frac{(bc-ad)fF^{e+\frac{f(a+bx)}{c+dx}}\log(F)}{2(dg-ch)^2(g+hx)}\\
&\quad+\frac{d(bc-ad)fF^{e+\frac{f(bg-ah)}{dg-ch}}\operatorname{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)\log(F)}{(dg-ch)^3}\\
&\quad+\frac{(bc-ad)^2f^2F^{e+\frac{f(bg-ah)}{dg-ch}}h\operatorname{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)\log^2(F)}{2(dg-ch)^4}\end{aligned}$$

output
```
1/2*d^2*F^(e+b*f/d-(-a*d+b*c)*f/d/(d*x+c))/h/(-c*h+d*g)^2-1/2*F^(e+f*(b*x+
a)/(d*x+c))/h/(h*x+g)^2+1/2*d*(-a*d+b*c)*f*F^(e+b*f/d-(-a*d+b*c)*f/d/(d*x+
c))*ln(F)/(-c*h+d*g)^3-1/2*(-a*d+b*c)*f*F^(e+f*(b*x+a)/(d*x+c))*ln(F)/(-c*
h+d*g)^2/(h*x+g)+d*(-a*d+b*c)*f*F^(e+f*(-a*h+b*g)/(-c*h+d*g))*Ei(-(-a*d+b*
c)*f*(h*x+g)*ln(F)/(-c*h+d*g)/(d*x+c))*ln(F)/(-c*h+d*g)^3+1/2*(-a*d+b*c)^2
*f^2*F^(e+f*(-a*h+b*g)/(-c*h+d*g))*h*Ei(-(-a*d+b*c)*f*(h*x+g)*ln(F)/(-c*h+
d*g)/(d*x+c))*ln(F)^2/(-c*h+d*g)^4
```

Mathematica [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx = \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx$$

input
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^3,x]
```

output
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^3, x]
```

Rubi [A] (verified)

Time = 6.20 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2662, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{f(a+bx)}{c+dx}+e}}{(g+hx)^3}\,dx$$

$$\Big\downarrow \text{ 2662}$$

$$\frac{f\log(F)(bc-ad)\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)^2}dx}{2h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{2h(g+hx)^2}$$

$$\Big\downarrow \text{ 7293}$$

$$\frac{f\log(F)(bc-ad)\int \left(-\frac{2d^2hF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(c+dx)}+\frac{2dh^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(g+hx)}+\frac{d^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(c+dx)^2}+\frac{h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)^2}\right)dx}{2h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{2h(g+hx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f\log(F)(bc-ad)\left(\frac{d^2F^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{f\log(F)(bc-ad)(dg-ch)^2}+\frac{fh^2\log(F)(bc-ad)F^{\frac{f(bg-ah)}{dg-ch}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^4}+\frac{2dhF^{\frac{f(b}{dg}}}{}\right.}{2h}$$

$$\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{2h(g+hx)^2}$$

input
```
Int[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^3,x]
```

output
```
-1/2*F^(e + (f*(a + b*x))/(c + d*x))/(h*(g + h*x)^2) + ((b*c - a*d)*f*Log[
F]*((d*F^(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))*h)/(d*g - c*h)^3 -
(F^(e + (f*(a + b*x))/(c + d*x))*h)/((d*g - c*h)^2*(g + h*x)) + (2*d*F^(e
+ (f*(b*g - a*h))/(d*g - c*h))*h*ExpIntegralEi[-(((b*c - a*d)*f*(g + h*x)*
Log[F])/((d*g - c*h)*(c + d*x)))])/(d*g - c*h)^3 + (d^2*F^(e + (b*f)/d - (
(b*c - a*d)*f)/(d*(c + d*x))))/((b*c - a*d)*f*(d*g - c*h)^2*Log[F]) + ((b*
c - a*d)*f*F^(e + (f*(b*g - a*h))/(d*g - c*h))*h^2*ExpIntegralEi[-(((b*c -
 a*d)*f*(g + h*x)*Log[F])/((d*g - c*h)*(c + d*x)))]*Log[F])/(d*g - c*h)^4)
)/(2*h)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2662
```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))*((g_.)
 + (h_.)*(x_))^(m_), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(F^(e + f*((a + b*
x)/(c + d*x)))/(h*(m + 1))), x] - Simp[f*(b*c - a*d)*(Log[F]/(h*(m + 1)))
 Int[(g + h*x)^(m + 1)*(F^(e + f*((a + b*x)/(c + d*x)))/(c + d*x)^2), x], x
] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b*c - a*d, 0] && NeQ[d*g
- c*h, 0] && ILtQ[m, -1]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2018 vs. 2(356) = 712.

Time = 0.66 (sec) , antiderivative size = 2019, normalized size of antiderivative = 5.52

method	result	size
risch	Expression too large to display	2019

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(F)^2*d^2*h*f^2/(c*h-d*g)^4*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/
d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g
)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*
ln(F)*d*e*g)^2*a^2+ln(F)^2*d*h*f^2/(c*h-d*g)^4*F^(f*(a*d-b*c)/d/(d*x+c))*F
^((b*f+d*e)/d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F
)-1/(c*h-d*g)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+
1/(c*h-d*g)*ln(F)*d*e*g)^2*a*b*c-1/2*ln(F)^2*h*f^2/(c*h-d*g)^4*F^(f*(a*d-b
*c)/d/(d*x+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln
(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d
*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln(F)*d*e*g)^2*b^2*c^2-1/2*ln(F)^2*d^2*h*f^2/(
c*h-d*g)^4*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d*x+c)*a-b*
c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1/(c*h-d*g
)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln(F)*d*e*g)*a^2+ln(F)^2
*d*h*f^2/(c*h-d*g)^4*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d
*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+
1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln(F)*d*e*g)*a
*b*c-1/2*ln(F)^2*h*f^2/(c*h-d*g)^4*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/
d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g
)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*
ln(F)*d*e*g)*b^2*c^2-1/2*ln(F)^2*d^2*h*f^2/(c*h-d*g)^4*F^((a*f*h-b*f*g+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(356) = 712$.

Time = 0.10 (sec) , antiderivative size = 755, normalized size of antiderivative = 2.06

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx = \frac{(((b^2c^2-2\,abcd+a^2d^2)f^2h^3x^2+2\,(b^2c^2-2\,abcd+a^2d^2)f^2gh^2x+(b^2c^2-2\,abcd+a^2d^2)f^2g^2h)\log(F)}{}$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x, algorithm="fricas")
```

output
```
1/2*((((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f^2*h^3*x^2 + 2*(b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*f^2*g*h^2*x + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*f^2*g^2*h)*log(
F)^2 + 2*((b*c*d^2 - a*d^3)*f*g^3 - (b*c^2*d - a*c*d^2)*f*g^2*h + ((b*c*d^
2 - a*d^3)*f*g*h^2 - (b*c^2*d - a*c*d^2)*f*h^3)*x^2 + 2*((b*c*d^2 - a*d^3)
*f*g^2*h - (b*c^2*d - a*c*d^2)*f*g*h^2)*x)*log(F))*F^(((d*e + b*f)*g - (c*
e + a*f)*h)/(d*g - c*h))*Ei(-((b*c - a*d)*f*h*x + (b*c - a*d)*f*g)*log(F)/
(c*d*g - c^2*h + (d^2*g - c*d*h)*x)) + (2*c*d^3*g^3 - 5*c^2*d^2*g^2*h + 4*
c^3*d*g*h^2 - c^4*h^3 + (d^4*g^2*h - 2*c*d^3*g*h^2 + c^2*d^2*h^3)*x^2 + 2*
(d^4*g^3 - 2*c*d^3*g^2*h + c^2*d^2*g*h^2)*x + ((b*c^2*d - a*c*d^2)*f*g^2*h
 - (b*c^3 - a*c^2*d)*f*g*h^2 + ((b*c*d^2 - a*d^3)*f*g*h^2 - (b*c^2*d - a*c
*d^2)*f*h^3)*x^2 + ((b*c*d^2 - a*d^3)*f*g^2*h - (b*c^3 - a*c^2*d)*f*h^3)*x
)*log(F))*F^((c*e + a*f + (d*e + b*f)*x)/(d*x + c)))/(d^4*g^6 - 4*c*d^3*g^
5*h + 6*c^2*d^2*g^4*h^2 - 4*c^3*d*g^3*h^3 + c^4*g^2*h^4 + (d^4*g^4*h^2 - 4
*c*d^3*g^3*h^3 + 6*c^2*d^2*g^2*h^4 - 4*c^3*d*g*h^5 + c^4*h^6)*x^2 + 2*(d^4
*g^5*h - 4*c*d^3*g^4*h^2 + 6*c^2*d^2*g^3*h^3 - 4*c^3*d*g^2*h^4 + c^4*g*h^5
)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate(F**(e+f*(b*x+a)/(d*x+c))/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx=\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^3, x)
```

Giac [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx=\int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx=\int \frac{F^{e+\frac{f\,(a+b\,x)}{c+d\,x}}}{{(g+h\,x)}^3}\,dx$$

input
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^3,x)
```

output
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^3, x)
```

Reduce [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^3}\,dx = \text{too large to display}$$

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^3,x)
```

output
```
( - 4*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**3*c**3*d**4*
f**3*h**4 + 6*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**3*c*
*2*d**5*f**3*g*h**3 - 6*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)*
*3*a**3*c**2*d**5*f**3*h**4*x - 4*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x
))*log(f)**3*a**3*c*d**6*f**3*g**2*h**2 + 4*f**((a*f + b*f*x + c*e + d*e*x
)/(c + d*x))*log(f)**3*a**3*c*d**6*f**3*g*h**3*x - 4*f**((a*f + b*f*x + c*
e + d*e*x)/(c + d*x))*log(f)**3*a**3*c*d**6*f**3*h**4*x**2 + f**((a*f + b*
f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**3*d**7*f**3*g**3*h - f**((a*f +
 b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**3*d**7*f**3*g**2*h**2*x + f*
*((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**3*d**7*f**3*g*h**3*x
**2 - f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**3*d**7*f**3*
h**4*x**3 + 12*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**2*b
*c**4*d**3*f**3*h**4 - 18*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f
)**3*a**2*b*c**3*d**4*f**3*g*h**3 + 18*f**((a*f + b*f*x + c*e + d*e*x)/(c
+ d*x))*log(f)**3*a**2*b*c**3*d**4*f**3*h**4*x + 12*f**((a*f + b*f*x + c*e
 + d*e*x)/(c + d*x))*log(f)**3*a**2*b*c**2*d**5*f**3*g**2*h**2 - 12*f**((a
*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**2*b*c**2*d**5*f**3*g*h**
3*x + 12*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)**3*a**2*b*c**2*
d**5*f**3*h**4*x**2 - 3*f**((a*f + b*f*x + c*e + d*e*x)/(c + d*x))*log(f)*
*3*a**2*b*c*d**6*f**3*g**3*h + 3*f**((a*f + b*f*x + c*e + d*e*x)/(c + d...
```

3.359 $\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 634

$$\begin{aligned}
&\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx \\
&= \frac{d^3F^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}}{3h(dg-ch)^3} - \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{3h(g+hx)^3} + \frac{5d^2(bc-ad)fF^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}\log(F)}{6(dg-ch)^4} \\
&\quad - \frac{(bc-ad)fF^{e+\frac{f(a+bx)}{c+dx}}\log(F)}{6(dg-ch)^2(g+hx)^2} - \frac{2d(bc-ad)fF^{e+\frac{f(a+bx)}{c+dx}}\log(F)}{3(dg-ch)^3(g+hx)} \\
&\quad + \frac{d^2(bc-ad)fF^{e+\frac{f(bg-ah)}{dg-ch}}\,\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)\log(F)}{(dg-ch)^4} \\
&\quad + \frac{d(bc-ad)^2f^2F^{e+\frac{bf}{d}-\frac{(bc-ad)f}{d(c+dx)}}h\log^2(F)}{6(dg-ch)^5} - \frac{(bc-ad)^2f^2F^{e+\frac{f(a+bx)}{c+dx}}h\log^2(F)}{6(dg-ch)^4(g+hx)} \\
&\quad + \frac{d(bc-ad)^2f^2F^{e+\frac{f(bg-ah)}{dg-ch}}h\,\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)\log^2(F)}{(dg-ch)^5} \\
&\quad + \frac{(bc-ad)^3f^3F^{e+\frac{f(bg-ah)}{dg-ch}}h^2\,\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)\log^3(F)}{6(dg-ch)^6}
\end{aligned}$$

output
```
1/3*d^3*F^(e+b*f/d-(-a*d+b*c)*f/d/(d*x+c))/h/(-c*h+d*g)^3-1/3*F^(e+f*(b*x+
a)/(d*x+c))/h/(h*x+g)^3+5/6*d^2*(-a*d+b*c)*f*F^(e+b*f/d-(-a*d+b*c)*f/d/(d*
x+c))*ln(F)/(-c*h+d*g)^4-1/6*(-a*d+b*c)*f*F^(e+f*(b*x+a)/(d*x+c))*ln(F)/(-
c*h+d*g)^2/(h*x+g)^2-2/3*d*(-a*d+b*c)*f*F^(e+f*(b*x+a)/(d*x+c))*ln(F)/(-c*
h+d*g)^3/(h*x+g)+d^2*(-a*d+b*c)*f*F^(e+f*(-a*h+b*g)/(-c*h+d*g))*Ei(-(-a*d+
b*c)*f*(h*x+g)*ln(F)/(-c*h+d*g)/(d*x+c))*ln(F)/(-c*h+d*g)^4+1/6*d*(-a*d+b*
c)^2*f^2*F^(e+b*f/d-(-a*d+b*c)*f/d/(d*x+c))*h*ln(F)^2/(-c*h+d*g)^5-1/6*(-a
*d+b*c)^2*f^2*F^(e+f*(b*x+a)/(d*x+c))*h*ln(F)^2/(-c*h+d*g)^4/(h*x+g)+d*(-a
*d+b*c)^2*f^2*F^(e+f*(-a*h+b*g)/(-c*h+d*g))*h*Ei(-(-a*d+b*c)*f*(h*x+g)*ln(
F)/(-c*h+d*g)/(d*x+c))*ln(F)^2/(-c*h+d*g)^5+1/6*(-a*d+b*c)^3*f^3*F^(e+f*(-
a*h+b*g)/(-c*h+d*g))*h^2*Ei(-(-a*d+b*c)*f*(h*x+g)*ln(F)/(-c*h+d*g)/(d*x+c)
)*ln(F)^3/(-c*h+d*g)^6
```

Mathematica [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx$$

input
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^4,x]
```

output
```
Integrate[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^4, x]
```

Rubi [A] (verified)

Time = 34.21 (sec) , antiderivative size = 612, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2662, 7293, 7239, 7293, 7239, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{f(a+bx)}{c+dx}+e}}{(g+hx)^4}\,dx$$

$$\Big\downarrow \; 2662$$

$$\frac{f\log(F)(bc-ad)\int\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)^3}dx}{3h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

$\downarrow$ 7293

$$\frac{f\log(F)(bc-ad)\int\left(-\frac{3d^3hF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^4(c+dx)}+\frac{3d^2h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^4(g+hx)}+\frac{d^3F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(c+dx)^2}+\frac{2dh^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(g+hx)^2}+\frac{h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)^3}\right)dx}{3h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

$\downarrow$ 7239

$$\frac{f\log(F)(bc-ad)\int\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)^3}dx}{3h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

$\downarrow$ 7293

$$\frac{f\log(F)(bc-ad)\int\left(-\frac{3d^3hF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^4(c+dx)}+\frac{3d^2h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^4(g+hx)}+\frac{d^3F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(c+dx)^2}+\frac{2dh^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(g+hx)^2}+\frac{h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)^3}\right)dx}{3h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

$\downarrow$ 7239

$$\frac{f\log(F)(bc-ad)\int\frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(c+dx)^2(g+hx)^3}dx}{3h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

$\downarrow$ 7293

$$\frac{f\log(F)(bc-ad)\int\left(-\frac{3d^3hF^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^4(c+dx)}+\frac{3d^2h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^4(g+hx)}+\frac{d^3F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(c+dx)^2}+\frac{2dh^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^3(g+hx)^2}+\frac{h^2F^{e+\frac{f(a+bx)}{c+dx}}}{(dg-ch)^2(g+hx)^3}\right)dx}{3h}-\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

$\downarrow$ 2009

$$\frac{f\log(F)(bc-ad)\left(\frac{d^3F^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+e}}{f\log(F)(bc-ad)(dg-ch)^3}+\frac{3d^2hF^{\frac{f(bg-ah)}{dg-ch}+e}\text{ExpIntegralEi}\left(-\frac{(bc-ad)f(g+hx)\log(F)}{(dg-ch)(c+dx)}\right)}{(dg-ch)^4}+\frac{5d^2hF^{-\frac{f(bc-ad)}{d(c+dx)}+\frac{bf}{d}+}}{2(dg-ch)^4}\right.}{}$$

$$\frac{F^{\frac{f(a+bx)}{c+dx}+e}}{3h(g+hx)^3}$$

input
```
Int[F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^4,x]
```

output
```
-1/3*F^(e + (f*(a + b*x))/(c + d*x))/(h*(g + h*x)^3) + ((b*c - a*d)*f*Log[
F]*((5*d^2*F^(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))*h)/(2*(d*g - c*
h)^4) - (F^(e + (f*(a + b*x))/(c + d*x))*h)/(2*(d*g - c*h)^2*(g + h*x)^2)
- (2*d*F^(e + (f*(a + b*x))/(c + d*x))*h)/((d*g - c*h)^3*(g + h*x)) + (3*d
^2*F^(e + (f*(b*g - a*h))/(d*g - c*h))*h*ExpIntegralEi[-(((b*c - a*d)*f*(g
 + h*x)*Log[F])/((d*g - c*h)*(c + d*x)))])/(d*g - c*h)^4 + (d^3*F^(e + (b*
f)/d - ((b*c - a*d)*f)/(d*(c + d*x))))/((b*c - a*d)*f*(d*g - c*h)^3*Log[F]
) + (d*(b*c - a*d)*f*F^(e + (b*f)/d - ((b*c - a*d)*f)/(d*(c + d*x)))*h^2*L
og[F])/(2*(d*g - c*h)^5) - ((b*c - a*d)*f*F^(e + (f*(a + b*x))/(c + d*x))*
h^2*Log[F])/(2*(d*g - c*h)^4*(g + h*x)) + (3*d*(b*c - a*d)*f*F^(e + (f*(b*
g - a*h))/(d*g - c*h))*h^2*ExpIntegralEi[-(((b*c - a*d)*f*(g + h*x)*Log[F]
)/((d*g - c*h)*(c + d*x)))]*Log[F])/(d*g - c*h)^5 + ((b*c - a*d)^2*f^2*F^(
e + (f*(b*g - a*h))/(d*g - c*h))*h^3*ExpIntegralEi[-(((b*c - a*d)*f*(g + h
*x)*Log[F])/((d*g - c*h)*(c + d*x)))]*Log[F]^2)/(2*(d*g - c*h)^6)))/(3*h)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2662
```
Int[(F_)^((e_.) + ((f_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_)))*((g_.)
 + (h_.)*(x_))^(m_), x_Symbol] :> Simp[(g + h*x)^(m + 1)*(F^(e + f*((a + b*
x)/(c + d*x)))/(h*(m + 1))), x] - Simp[f*(b*c - a*d)*(Log[F]/(h*(m + 1)))
 Int[(g + h*x)^(m + 1)*(F^(e + f*((a + b*x)/(c + d*x)))/(c + d*x)^2), x], x
] /; FreeQ[{F, a, b, c, d, e, f, g, h}, x] && NeQ[b*c - a*d, 0] && NeQ[d*g
- c*h, 0] && ILtQ[m, -1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4679 vs. 2(618) = 1236.

Time = 1.08 (sec) , antiderivative size = 4680, normalized size of antiderivative = 7.38

method	result	size
risch	Expression too large to display	4680

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^4,x,method=_RETURNVERBOSE)
```

output
```
-2*ln(F)^2*d^2*f^2*h/(c*h-d*g)^5*F^(f*(a*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/d)
/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*
ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln
(F)*d*e*g)^2*a*b*c-2*ln(F)^2*d^2*f^2*h/(c*h-d*g)^5*F^(f*(a*d-b*c)/d/(d*x+c
))*F^((b*f+d*e)/d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+e*
ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(F)*c*
e*h+1/(c*h-d*g)*ln(F)*d*e*g)*a*b*c-1/6*ln(F)^3*f^3*h^2/(c*h-d*g)^6*F^(f*(a
*d-b*c)/d/(d*x+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c
)+ln(F)/d*b*f+e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c
*h-d*g)*ln(F)*c*e*h+1/(c*h-d*g)*ln(F)*d*e*g)^2*b^3*c^3+1/6*ln(F)^3*d^3*f^3
*h^2/(c*h-d*g)^6*F^((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))*Ei(1,-(a*d*f-b*c*
f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-ln(F)*a*f*h+ln(F)*b*f*g-ln(F)*c*e*h
+ln(F)*d*e*g)/(c*h-d*g))*a^3+ln(F)*d^3/(c*h-d*g)^4*F^((a*f*h-b*f*g+c*e*h-d
*e*g)/(c*h-d*g))*Ei(1,-(a*d*f-b*c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-l
n(F)*a*f*h+ln(F)*b*f*g-ln(F)*c*e*h+ln(F)*d*e*g)/(c*h-d*g))*a*f+ln(F)^2*d^3
*f^2*h/(c*h-d*g)^5*F^((a*f*h-b*f*g+c*e*h-d*e*g)/(c*h-d*g))*Ei(1,-(a*d*f-b*
c*f)*ln(F)/d/(d*x+c)-(b*f+d*e)*ln(F)/d-(-ln(F)*a*f*h+ln(F)*b*f*g-ln(F)*c*e
*h+ln(F)*d*e*g)/(c*h-d*g))*a^2+ln(F)*d^3/(c*h-d*g)^4*F^(f*(a*d-b*c)/d/(d*x
+c))*F^((b*f+d*e)/d)/(f*ln(F)/(d*x+c)*a-b*c*f*ln(F)/d/(d*x+c)+ln(F)/d*b*f+
e*ln(F)-1/(c*h-d*g)*ln(F)*a*f*h+1/(c*h-d*g)*ln(F)*b*f*g-1/(c*h-d*g)*ln(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2250 vs. $2(618) = 1236$.

Time = 0.13 (sec) , antiderivative size = 2250, normalized size of antiderivative = 3.55

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^4,x, algorithm="fricas")
```

output
```
1/6*((((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*f^3*h^5*x^3 + 3
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*f^3*g*h^4*x^2 + 3*(b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*f^3*g^2*h^3*x + (b^3*c^3
- 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*f^3*g^3*h^2)*log(F)^3 + 6*((b^2
*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*f^2*g^4*h - (b^2*c^3*d - 2*a*b*c^2*d^2 +
 a^2*c*d^3)*f^2*g^3*h^2 + ((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*f^2*g*h^4
 - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*f^2*h^5)*x^3 + 3*((b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*f^2*g^2*h^3 - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*
d^3)*f^2*g*h^4)*x^2 + 3*((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*f^2*g^3*h^2
 - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*f^2*g^2*h^3)*x)*log(F)^2 + 6*((
b*c*d^4 - a*d^5)*f*g^5 - 2*(b*c^2*d^3 - a*c*d^4)*f*g^4*h + (b*c^3*d^2 - a*
c^2*d^3)*f*g^3*h^2 + ((b*c*d^4 - a*d^5)*f*g^2*h^3 - 2*(b*c^2*d^3 - a*c*d^4
)*f*g*h^4 + (b*c^3*d^2 - a*c^2*d^3)*f*h^5)*x^3 + 3*((b*c*d^4 - a*d^5)*f*g^
3*h^2 - 2*(b*c^2*d^3 - a*c*d^4)*f*g^2*h^3 + (b*c^3*d^2 - a*c^2*d^3)*f*g*h^
4)*x^2 + 3*((b*c*d^4 - a*d^5)*f*g^4*h - 2*(b*c^2*d^3 - a*c*d^4)*f*g^3*h^2
+ (b*c^3*d^2 - a*c^2*d^3)*f*g^2*h^3)*x)*log(F))*F^(((d*e + b*f)*g - (c*e +
 a*f)*h)/(d*g - c*h))*Ei(-((b*c - a*d)*f*h*x + (b*c - a*d)*f*g)*log(F)/(c*
d*g - c^2*h + (d^2*g - c*d*h)*x)) + (6*c*d^5*g^5 - 24*c^2*d^4*g^4*h + 38*c
^3*d^3*g^3*h^2 - 30*c^4*d^2*g^2*h^3 + 12*c^5*d*g*h^4 - 2*c^6*h^5 + 2*(d^6*
g^3*h^2 - 3*c*d^5*g^2*h^3 + 3*c^2*d^4*g*h^4 - c^3*d^3*h^5)*x^3 + 6*(d^6...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \text{Timed out}$$

input
```
integrate(F**(e+f*(b*x+a)/(d*x+c))/(h*x+g)**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^4}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^4,x, algorithm="maxima")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^4, x)
```

Giac [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \int \frac{F^{e+\frac{(bx+a)f}{dx+c}}}{(hx+g)^4}\,dx$$

input
```
integrate(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^4,x, algorithm="giac")
```

output
```
integrate(F^(e + (b*x + a)*f/(d*x + c))/(h*x + g)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \int \frac{F^{e+\frac{f\,(a+b\,x)}{c+d\,x}}}{{(g+h\,x)}^4}\,dx$$

input
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^4,x)
```

output
```
int(F^(e + (f*(a + b*x))/(c + d*x))/(g + h*x)^4, x)
```

Reduce [F]

$$\int \frac{F^{e+\frac{f(a+bx)}{c+dx}}}{(g+hx)^4}\,dx = \int \frac{F^{e+\frac{f(bx+a)}{dx+c}}}{(hx+g)^4}dx$$

input
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^4,x)
```

output
```
int(F^(e+f*(b*x+a)/(d*x+c))/(h*x+g)^4,x)
```

3.360 $\int f^{a+bx+cx^2} x^3 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 217

$$\int f^{a+bx+cx^2} x^3 \, dx = -\frac{f^{a+bx+cx^2}}{2c^2 \log^2(f)} + \frac{3bf^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2}\log^{\frac{3}{2}}(f)} + \frac{b^2 f^{a+bx+cx^2}}{8c^3\log(f)} - \frac{bf^{a+bx+cx^2}x}{4c^2\log(f)} + \frac{f^{a+bx+cx^2}x^2}{2c\log(f)} - \frac{b^3 f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{16c^{7/2}\sqrt{\log(f)}}$$

output
```
-1/2*f^(c*x^2+b*x+a)/c^2/ln(f)^2+3/8*b*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(
2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(5/2)/ln(f)^(3/2)+1/8*b^2*f^(c*x^2+b*x+a)/
c^3/ln(f)-1/4*b*f^(c*x^2+b*x+a)*x/c^2/ln(f)+1/2*f^(c*x^2+b*x+a)*x^2/c/ln(f
)-1/16*b^3*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2)
)/c^(7/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.56

$$\int f^{a+bx+cx^2}x^3\,dx$$
$$=\frac{f^{a-\frac{b^2}{4c}}\left(b\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}(6c-b^2\log(f))+2\sqrt{c}f^{\frac{(b+2cx)^2}{4c}}(-4c+(b^2-2bcx+4c^2x^2)\log(f}{16c^{7/2}\log^2(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*x^3,x]
```

output
```
(f^(a - b^2/(4*c))*(b*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])
]*Sqrt[Log[f]]*(6*c - b^2*Log[f]) + 2*Sqrt[c]*f^((b + 2*c*x)^2/(4*c))*(-4*
c + (b^2 - 2*b*c*x + 4*c^2*x^2)*Log[f])))/(16*c^(7/2)*Log[f]^2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.36, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2671, 2670, 2664, 2633, 2671, 2664, 2633, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{2671}$$
$$-\frac{b\int f^{cx^2+bx+a}x^2dx}{2c}-\frac{\int f^{cx^2+bx+a}xdx}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$
$$\Big\downarrow \text{2670}$$
$$-\frac{b\int f^{cx^2+bx+a}x^2dx}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{b\int f^{cx^2+bx+a}dx}{2c}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$
$$\Big\downarrow \text{2664}$$

$$-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{bf^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}}{c\log(f)}-\frac{b\int f^{cx^2+bx+a}x^2dx}{2c}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{b\int f^{cx^2+bx+a}x^2dx}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{ 2671}$$

$$-\frac{b\left(-\frac{b\int f^{cx^2+bx+a}xdx}{2c}-\frac{\int f^{cx^2+bx+a}dx}{2c\log(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{ 2664}$$

$$-\frac{b\left(-\frac{f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c\log(f)}-\frac{b\int f^{cx^2+bx+a}xdx}{2c}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{b\left(-\frac{b\int f^{cx^2+bx+a}xdx}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{ 2670}$$

$$-\frac{b\left(-\frac{b\left(\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{b\int f^{cx^2+bx+a}dx}{2c}\right)}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2664

$$-\frac{b\left(-\frac{b\left(\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{bf^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}\right)}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2633

$$-\frac{b\left(-\frac{b\left(\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}\right)}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}}{c\log(f)}+\frac{x^2f^{a+bx+cx^2}}{2c\log(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*x^3,x]
```

output
```
-1/2*(b*(-1/2*(b*(f^(a + b*x + c*x^2)/(2*c*Log[f]) - (b*f^(a - b^2/(4*c))*
Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log
[f]])))/c - (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2
*Sqrt[c])])/(4*c^(3/2)*Log[f]^(3/2)) + (f^(a + b*x + c*x^2)*x)/(2*c*Log[f]
)))/c + (f^(a + b*x + c*x^2)*x^2)/(2*c*Log[f]) - (f^(a + b*x + c*x^2)/(2*c
*Log[f]) - (b*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(
2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f]]))/(c*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00

method	result
risch	$\frac{x^2 f^{c\,x^2} f^{bx} f^a}{2c\ln(f)} - \frac{bx\, f^{c\,x^2} f^{bx} f^a}{4c^2\ln(f)} + \frac{b^2 f^{c\,x^2} f^{bx} f^a}{8c^3\ln(f)} + \frac{b^3\sqrt{\pi}\, f^a f^{-\frac{b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x + \frac{b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{16c^3\sqrt{-c\ln(f)}} - \frac{3b\sqrt{\pi}\, f^a f^{-\frac{b^2}{4c}} \operatorname{erf}\left(-\right.}{8c^2\ln(}$

input
```
int(f^(c*x^2+b*x+a)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/c/ln(f)*x^2*f^(c*x^2)*f^(b*x)*f^a-1/4*b/c^2/ln(f)*x*f^(c*x^2)*f^(b*x)*
f^a+1/8*b^2/c^3/ln(f)*f^(c*x^2)*f^(b*x)*f^a+1/16*b^3/c^3*Pi^(1/2)*f^a*f^(-
1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))
^(1/2))-3/8*b/c^2/ln(f)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-
(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))-1/2/c^2/ln(f)^2*f^(c*x^2)
*f^(b*x)*f^a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.53

$$\int f^{a+bx+cx^2}x^3\,dx = -\frac{2\left(4c^2-\left(4c^3x^2-2bc^2x+b^2c\right)\log\left(f\right)\right)f^{cx^2+bx+a}-\frac{\sqrt{\pi}\left(b^3\log(f)-6bc\right)\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)}{f^{\frac{b^2-4ac}{4c}}}}{16c^4\log\left(f\right)^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*x^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(4*c^2 - (4*c^3*x^2 - 2*b*c^2*x + b^2*c)*log(f))*f^(c*x^2 + b*x +
 a) - sqrt(pi)*(b^3*log(f) - 6*b*c)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sq
rt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^4*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}x^3\,dx = \int f^{a+bx+cx^2}x^3\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*x**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.93

$$\int f^{a+bx+cx^2}x^3\,dx = -\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b^3\left(\operatorname{erf}\left(\frac{1}{2}\,\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^4}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{7}{2}}} - \frac{12\,(2\,cx+b)^3b\Gamma\left(\frac{3}{2},-\frac{(2\,cx+b)^2\log(f)}{4\,c}\right)\log(f)^4}{\left(-\frac{(2\,cx+b)^2\log(f)}{c}\right)^{\frac{3}{2}}(c\log(f))^{\frac{7}{2}}} - \frac{6\,b^2cf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)^3}{(c\log(f))^{\frac{7}{2}}}\right.}{16\,\sqrt{c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*x^3,x, algorithm="maxima")
```

output
```
-1/16*(sqrt(pi)*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) -
1)*log(f)^4/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(7/2)) - 12*(2*c*x +
 b)^3*b*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^4/((-(2*c*x + b)^2*
log(f)/c)^(3/2)*(c*log(f))^(7/2)) - 6*b^2*c*f^(1/4*(2*c*x + b)^2/c)*log(f)
^3/(c*log(f))^(7/2) + 8*c^2*gamma(2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^2
/(c*log(f))^(7/2))*f^(a - 1/4*b^2/c)/sqrt(c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.63

$$\int f^{a+bx+cx^2}x^3\,dx = \frac{\frac{\sqrt{\pi}(b^3\log(f)-6\,bc)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}\log(f)} + \frac{2\left(c^2\left(2\,x+\frac{b}{c}\right)^2\log(f)-3\,bc\left(2\,x+\frac{b}{c}\right)\log(f)+3\,b^2\log(f)-4\,c\right)}{\log(f)^2}}{16\,c^3}$$

input
```
integrate(f^(c*x^2+b*x+a)*x^3,x, algorithm="giac")
```

output
```
1/16*(sqrt(pi)*(b^3*log(f) - 6*b*c)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*
e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/(sqrt(-c*log(f))*log(f)) + 2*(c^2*(
2*x + b/c)^2*log(f) - 3*b*c*(2*x + b/c)*log(f) + 3*b^2*log(f) - 4*c)*e^(c*
x^2*log(f) + b*x*log(f) + a*log(f))/log(f)^2)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.71

$$\int f^{a+bx+cx^2}x^3\,dx = \frac{f^a\,f^{c\,x^2}\,f^{b\,x}\,x^2}{2\,c\,\ln(f)} - f^a\,f^{c\,x^2}\,f^{b\,x}\left(\frac{1}{2\,c^2\ln(f)^2} - \frac{b^2}{8\,c^3\,\ln(f)}\right) + \frac{f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\left(\frac{3\,b\,c}{8} - \frac{b^3\,\ln(f)}{16}\right)}{c^3\,\ln(f)\,\sqrt{c\,\ln(f)}} - \frac{b\,f^a\,f^{c\,x^2}\,f^{b\,x}\,x}{4\,c^2\,\ln(f)}$$

input
```
int(f^(a + b*x + c*x^2)*x^3,x)
```

output
```
(f^a*f^(c*x^2)*f^(b*x)*x^2)/(2*c*log(f)) - f^a*f^(c*x^2)*f^(b*x)*(1/(2*c^2
*log(f)^2) - b^2/(8*c^3*log(f))) + (f^(a - b^2/(4*c))*pi^(1/2)*erfi(((b*lo
g(f))/2 + c*x*log(f))/(c*log(f))^(1/2))*((3*b*c)/8 - (b^3*log(f))/16))/(c^
3*log(f)*(c*log(f))^(1/2)) - (b*f^a*f^(c*x^2)*f^(b*x)*x)/(4*c^2*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13

$$\int f^{a+bx+cx^2}x^3\,dx$$

$$= \frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)^2\,b^3i - 6\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,bci + 2f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{}\right.}{1}$$

input
```
int(f^(c*x^2+b*x+a)*x^3,x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f)))
)*log(f)**2*b**3*i - 6*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(
c)*sqrt(log(f))))*log(f)*b*c*i + 2*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*
c))*sqrt(c)*sqrt(log(f))*log(f)*b**2 - 4*f**(b*x + c*x**2)*e**((log(f)*b**
2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*b*c*x + 8*f**(b*x + c*x**2)*e**((log
(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*c**2*x**2 - 8*f**(b*x + c*x**
2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*c))/(16*e**((log(f)*b**2)
/(4*c))*sqrt(c)*sqrt(log(f))*log(f)**2*c**3)
```

3.361 $\int f^{a+bx+cx^2} x^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 164

$$\int f^{a+bx+cx^2} x^2 \, dx = -\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)} - \frac{bf^{a+bx+cx^2}}{4c^2\log(f)} + \frac{f^{a+bx+cx^2}x}{2c\log(f)} + \frac{b^2 f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2}\sqrt{\log(f)}}$$

output
```
-1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(3
/2)/ln(f)^(3/2)-1/4*b*f^(c*x^2+b*x+a)/c^2/ln(f)+1/2*f^(c*x^2+b*x+a)*x/c/ln
(f)+1/8*b^2*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2
))/c^(5/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int f^{a+bx+cx^2} x^2 \, dx = \frac{f^{a-\frac{b^2}{4c}}\left(-2\sqrt{c}f^{\frac{(b+2cx)^2}{4c}}(b-2cx)\sqrt{\log(f)} + \sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)(-2c+b^2\log(f))\right)}{8c^{5/2}\log^{\frac{3}{2}}(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*x^2,x]
```

output
```
(f^(a - b^2/(4*c))*(-2*Sqrt[c]*f^((b + 2*c*x)^2/(4*c))*(b - 2*c*x)*Sqrt[Lo
g[f]] + Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]*(-2*c + b^2*
Log[f])))/(8*c^(5/2)*Log[f]^(3/2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2671, 2664, 2633, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 f^{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{2671}$$

$$-\frac{b\int f^{cx^2+bx+a}xdx}{2c}-\frac{\int f^{cx^2+bx+a}dx}{2c\log(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c\log(f)}-\frac{b\int f^{cx^2+bx+a}xdx}{2c}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{b\int f^{cx^2+bx+a}xdx}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{b\left(\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{b\int f^{cx^2+bx+a}dx}{2c}\right)}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{b\left(\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{bf^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}\right)}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{b\left(\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}\right)}{2c}-\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{xf^{a+bx+cx^2}}{2c\log(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*x^2,x]
```

output
```
-1/2*(b*(f^(a + b*x + c*x^2)/(2*c*Log[f]) - (b*f^(a - b^2/(4*c))*Sqrt[Pi]*
Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f]])))/c
 - (f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])
])/(4*c^(3/2)*Log[f]^(3/2)) + (f^(a + b*x + c*x^2)*x)/(2*c*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.99

method	result
risch	$\frac{x f^{c x^2} f^{bx} f^a}{2c\ln(f)} - \frac{b f^{c x^2} f^{bx} f^a}{4c^2\ln(f)} - \frac{b^2\sqrt{\pi}\, f^a f^{-\frac{b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{8c^2\sqrt{-c\ln(f)}} + \frac{\sqrt{\pi}\, f^a f^{-\frac{b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{4c\ln(f)\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c/ln(f)*x*f^(c*x^2)*f^(b*x)*f^a-1/4*b/c^2/ln(f)*f^(c*x^2)*f^(b*x)*f^a-
1/8*b^2/c^2*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(
1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+1/4/c/ln(f)*Pi^(1/2)*f^a*f^(-1/4*b^2/
c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.58

$$\int f^{a+bx+cx^2} x^2\, dx = \frac{2\left(2\,c^2 x - bc\right) f^{cx^2+bx+a}\log\left(f\right) - \frac{\sqrt{\pi}\left(b^2\log(f) - 2\,c\right)\sqrt{-c\log(f)}\operatorname{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{8\,c^3\log\left(f\right)^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*x^2,x, algorithm="fricas")
```

output
```
1/8*(2*(2*c^2*x - b*c)*f^(c*x^2 + b*x + a)*log(f) - sqrt(pi)*(b^2*log(f) -
 2*c)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 -
 4*a*c)/c))/(c^3*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}x^2\,dx=\int f^{a+bx+cx^2}x^2\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*x**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01

$$\int f^{a+bx+cx^2}x^2\,dx$$
$$=\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^3}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{5}{2}}}-\frac{4\,(2\,cx+b)^3\Gamma\left(\frac{3}{2},-\frac{(2\,cx+b)^2\log(f)}{4\,c}\right)\log(f)^3}{\left(-\frac{(2\,cx+b)^2\log(f)}{c}\right)^{\frac{3}{2}}(c\log(f))^{\frac{5}{2}}}-\frac{4\,bcf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)^2}{(c\log(f))^{\frac{5}{2}}}\right)f^{a-\cdots}}{8\sqrt{c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*x^2,x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)
*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*x + b)
^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-(2*c*x + b)^2*log(f
)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^2/(c*l
og(f))^(5/2))*f^(a - 1/4*b^2/c)/sqrt(c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.66

$$\int f^{a+bx+cx^2} x^2 \, dx = -\frac{\frac{\sqrt{\pi}\left(b^2\log(f)-2\,c\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}\log(f)}-\frac{2\left(c\left(2\,x+\frac{b}{c}\right)-2\,b\right)e^{\left(cx^2\log(f)+bx\log(f)+a\log(f)\right)}}{\log(f)}}{8\,c^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*x^2,x, algorithm="giac")
```

output
```
-1/8*(sqrt(pi)*(b^2*log(f) - 2*c)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^
(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/(sqrt(-c*log(f))*log(f)) - 2*(c*(2*x
+ b/c) - 2*b)*e^(c*x^2*log(f) + b*x*log(f) + a*log(f))/log(f))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.68

$$\int f^{a+bx+cx^2} x^2 \, dx = \frac{f^a\,f^{c\,x^2}\,f^{b\,x}\,x}{2\,c\,\ln(f)} - \frac{b\,f^a\,f^{c\,x^2}\,f^{b\,x}}{4\,c^2\,\ln(f)} - \frac{f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\,\left(\frac{c}{4}-\frac{b^2\,\ln(f)}{8}\right)}{c^2\,\ln(f)\,\sqrt{c\,\ln(f)}}$$

input
```
int(f^(a + b*x + c*x^2)*x^2,x)
```

output
```
(f^a*f^(c*x^2)*f^(b*x)*x)/(2*c*log(f)) - (b*f^a*f^(c*x^2)*f^(b*x))/(4*c^2*
log(f)) - (f^(a - b^2/(4*c))*pi^(1/2)*erfi(((b*log(f))/2 + c*x*log(f))/(c*
log(f))^(1/2))*(c/4 - (b^2*log(f))/8))/(c^2*log(f)*(c*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01

$$\int f^{a+bx+cx^2}x^2\,dx$$

$$= \frac{f^a\left(-\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,b^2i+2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)ci-2f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}}{8e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}\log(f)\,c^2}$$

input
```
int(f^(c*x^2+b*x+a)*x^2,x)
```

output
```
(f**a*( - sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f
))))*log(f)*b**2*i + 2*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(
c)*sqrt(log(f))))*c*i - 2*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(
c)*sqrt(log(f))*b + 4*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*s
qrt(log(f))*c*x))/(8*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*
c**2)
```

3.362 $\int f^{a+bx+cx^2} x \, dx$

Optimal result

Integrand size = 14, antiderivative size = 81

$$\int f^{a+bx+cx^2} x \, dx = \frac{f^{a+bx+cx^2}}{2c\log(f)} - \frac{b f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}$$

output
```
1/2*f^(c*x^2+b*x+a)/c/ln(f)-1/4*b*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x
+b)*ln(f)^(1/2)/c^(1/2))/c^(3/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2} x \, dx = \frac{f^{a+bx+cx^2}}{2c\log(f)} - \frac{b f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*x,x]
```

output
```
f^(a + b*x + c*x^2)/(2*c*Log[f]) - (b*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xf^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2670}$$

$$\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{b\int f^{cx^2+bx+a}dx}{2c}$$

$$\Big\downarrow \text{2664}$$

$$\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{bf^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}$$

$$\Big\downarrow \text{2633}$$

$$\frac{f^{a+bx+cx^2}}{2c\log(f)}-\frac{\sqrt{\pi}bf^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*x,x]
```

output
```
f^(a + b*x + c*x^2)/(2*c*Log[f]) - (b*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b
 + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{f^{c\,x^{2}} f^{b x} f^{a}}{2 c \ln \left(f \right)}+\frac{b \sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{-c \ln \left(f \right)}\, x +\frac{b \ln \left(f \right)}{2 \sqrt{-c \ln \left(f \right)}}\right)}{4 c \sqrt{-c \ln \left(f \right)}}$	79

input
```
int(f^(c*x^2+b*x+a)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2/c/ln(f)*f^(c*x^2)*f^(b*x)*f^a+1/4*b/c*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*
ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}x\,dx=\frac{2\,cf^{cx^2+bx+a}+\frac{\sqrt{\pi}\sqrt{-c\log(f)}b\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{4\,c^2\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*x,x, algorithm="fricas")
```

output
```
1/4*(2*c*f^(c*x^2 + b*x + a) + sqrt(pi)*sqrt(-c*log(f))*b*erf(1/2*(2*c*x +
 b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}x\,dx=\int f^{a+bx+cx^2}x\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*x,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.32

$$\int f^{a+bx+cx^2}x\,dx=-\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b\left(\mathrm{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^2}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{3}{2}}}-\frac{2\,cf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)}{(c\log(f))^{\frac{3}{2}}}\right)f^{a-\frac{b^2}{4\,c}}}{4\sqrt{c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*x,x, algorithm="maxima")
```

output
```
-1/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*
log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*
c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*f^(a - 1/4*b^2/c)/sqrt(c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int f^{a+bx+cx^2}x\,dx = \frac{\frac{\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}}+\frac{2\,e^{\left(cx^2\log(f)+bx\log(f)+a\log(f)\right)}}{\log(f)}}{4\,c}$$

input
```
integrate(f^(c*x^2+b*x+a)*x,x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*b*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f)
- 4*a*c*log(f))/c)/sqrt(-c*log(f)) + 2*e^(c*x^2*log(f) + b*x*log(f) + a*lo
g(f))/log(f))/c
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}x\,dx = \frac{f^a\,f^{c\,x^2}\,f^{b\,x}}{2\,c\,\ln(f)} - \frac{b\,f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)}{4\,c\,\sqrt{c\,\ln(f)}}$$

input
```
int(f^(a + b*x + c*x^2)*x,x)
```

output
```
(f^a*f^(c*x^2)*f^(b*x))/(2*c*log(f)) - (b*f^(a - b^2/(4*c))*pi^(1/2)*erfi(
((b*log(f))/2 + c*x*log(f))/(c*log(f))^(1/2)))/(4*c*(c*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

$$\int f^{a+bx+cx^2}x\,dx$$
$$=\frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,bi+2f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log\left(f\right)}\right)}{4e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log\left(f\right)}\log\left(f\right)c}$$

input
```
int(f^(c*x^2+b*x+a)*x,x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f)))
)*log(f)*b*i + 2*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(l
og(f))))/(4*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*c)
```

3.363 $\int f^{a+bx+cx^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int f^{a+bx+cx^2} dx = \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/2*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1/
2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2} dx = \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2),x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(2*Sqrt[c]*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2664}$$

$$f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}\,dx$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{2\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2),x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(2*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{2\sqrt{-c\ln(f)}}$	50

input
```
int(f^(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*f^a*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+
1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int f^{a+bx+cx^2}\,dx=-\frac{\sqrt{\pi}\sqrt{-c\log{(f)}}\operatorname{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{2\,cf^{\frac{b^2-4\,ac}{4\,c}}\log{(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/(c*f^
(1/4*(b^2 - 4*a*c)/c)*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\,dx=\int f^{a+bx+cx^2}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a),x)
```

output
```
Integral(f**(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int f^{a+bx+cx^2}\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{2\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sq
rt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int f^{a+bx+cx^2}\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{2\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) -
4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\,dx = -\frac{f^a\sqrt{\pi}\,\mathrm{e}^{-\frac{b^2\ln(f)}{4c}}\operatorname{erf}\left(\frac{b\ln(f)\,1\mathrm{i}+c\,x\,\ln(f)\,2\mathrm{i}}{2\sqrt{c\ln(f)}}\right)1\mathrm{i}}{2\sqrt{c\ln(f)}}$$

input
```
int(f^(a + b*x + c*x^2),x)
```

output
```
-(f^a*pi^(1/2)*exp(-(b^2*log(f))/(4*c))*erf((b*log(f)*1i + c*x*log(f)*2i)/
(2*(c*log(f))^(1/2)))*1i)/(2*(c*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}\,dx = -\frac{\sqrt{\pi}\, f^a \mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right) i}{2e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}}$$

input
```
int(f^(c*x^2+b*x+a),x)
```

output
```
( - sqrt(pi)*f**a*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f)
)))*i)/(2*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f)))
```

3.364 $\int \frac{f^{a+bx+cx^2}}{x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{f^{a+bx+cx^2}}{x} \, dx = \operatorname{Int}\left(\frac{f^{a+bx+cx^2}}{x},x\right)$$

output
```
Defer(Int)(f^(c*x^2+b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x} \, dx = \int \frac{f^{a+bx+cx^2}}{x} \, dx$$

input
```
Integrate[f^(a + b*x + c*x^2)/x,x]
```

output
```
Integrate[f^(a + b*x + c*x^2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = $\{2673\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{x} \, dx$$

$$\Big\downarrow \text{2673}$$

$$\int \frac{f^{a+bx+cx^2}}{x}dx$$

input
```
Int[f^(a + b*x + c*x^2)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{f^{c\,x^2+bx+a}}{x}dx$$

input
```
int(f^(c*x^2+b*x+a)/x,x)
```

output
```
int(f^(c*x^2+b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x}\,dx = \int \frac{f^{cx^2+bx+a}}{x}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(f^(c*x^2 + b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{f^{a+bx+cx^2}}{x}\,dx = \int \frac{f^{a+bx+cx^2}}{x}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/x,x)
```

output
```
Integral(f**(a + b*x + c*x**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x} \, dx = \int \frac{f^{cx^2+bx+a}}{x} \, dx$$

input
```
integrate(f^(c*x^2+b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x} \, dx = \int \frac{f^{cx^2+bx+a}}{x} \, dx$$

input
```
integrate(f^(c*x^2+b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x} \, dx = \int \frac{f^{c\,x^2+b\,x+a}}{x} \, dx$$

input
```
int(f^(a + b*x + c*x^2)/x,x)
```

output
```
int(f^(a + b*x + c*x^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{f^{a+bx+cx^2}}{x}\,dx = f^a\left(\int \frac{f^{c\,x^2+bx}}{x}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)/x,x)
```

output
```
f**a*int(f**(b*x + c*x**2)/x,x)
```

3.365 $\int \frac{f^{a+bx+cx^2}}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{f^{a+bx+cx^2}}{x^2} \, dx = -\frac{f^{a+bx+cx^2}}{x} + \sqrt{c} f^{a-\frac{b^2}{4c}} \sqrt{\pi} \text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right) \sqrt{\log(f)} + b \log(f) \text{Int}\left(\frac{f^{a+bx+cx^2}}{x}, x\right)$$

output
```
-f^(c*x^2+b*x+a)/x+c^(1/2)*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(
f)^(1/2)/c^(1/2))*ln(f)^(1/2)+b*ln(f)*Defer(Int)(f^(c*x^2+b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x^2} \, dx = \int \frac{f^{a+bx+cx^2}}{x^2} \, dx$$

input
```
Integrate[f^(a + b*x + c*x^2)/x^2,x]
```

output
```
Integrate[f^(a + b*x + c*x^2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2672, 2664, 2633, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{x^2} \, dx$$

$$\Big\downarrow \text{2672}$$

$$2c\log(f)\int f^{cx^2+bx+a}dx + b\log(f)\int \frac{f^{cx^2+bx+a}}{x}dx - \frac{f^{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{2664}$$

$$2c\log(f)f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx + b\log(f)\int \frac{f^{cx^2+bx+a}}{x}dx - \frac{f^{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{2633}$$

$$b\log(f)\int \frac{f^{cx^2+bx+a}}{x}dx + \sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right) - \frac{f^{a+bx+cx^2}}{x}$$

$$\Big\downarrow \text{2673}$$

$$b\log(f)\int \frac{f^{cx^2+bx+a}}{x}dx + \sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right) - \frac{f^{a+bx+cx^2}}{x}$$

input
```
Int[f^(a + b*x + c*x^2)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2672
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] + (-
Simp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2)
, x], x] - Simp[(b*e - 2*c*d)*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 1
)*F^(a + b*x + c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e -
 2*c*d, 0] && LtQ[m, -1]
```

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{f^{c\,x^{2}+bx+a}}{x^{2}}dx$$

input
```
int(f^(c*x^2+b*x+a)/x^2,x)
```

output
```
int(f^(c*x^2+b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x^2} \, dx = \int \frac{f^{cx^2+bx+a}}{x^2} \, dx$$

input
```
integrate(f^(c*x^2+b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(f^(c*x^2 + b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{f^{a+bx+cx^2}}{x^2} \, dx = \int \frac{f^{a+bx+cx^2}}{x^2} \, dx$$

input
```
integrate(f**(c*x**2+b*x+a)/x**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x^2} \, dx = \int \frac{f^{cx^2+bx+a}}{x^2} \, dx$$

input
```
integrate(f^(c*x^2+b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x^2}\,dx = \int \frac{f^{cx^2+bx+a}}{x^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{f^{a+bx+cx^2}}{x^2}\,dx = \int \frac{f^{c\,x^2+b\,x+a}}{x^2}\,dx$$

input
```
int(f^(a + b*x + c*x^2)/x^2,x)
```

output
```
int(f^(a + b*x + c*x^2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \frac{f^{a+bx+cx^2}}{x^2}\,dx = f^a\left(\int \frac{f^{c\,x^2+bx}}{x^2}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)/x^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)/x**2,x)
```

3.366 $\int e^{a+bx-cx^2}x^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 181

$$\int e^{a+bx-cx^2}x^3\,dx = -\frac{b^2e^{a+bx-cx^2}}{8c^3}-\frac{e^{a+bx-cx^2}}{2c^2}-\frac{be^{a+bx-cx^2}x}{4c^2}-\frac{e^{a+bx-cx^2}x^2}{2c} -\frac{b^3e^{a+\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{16c^{7/2}}-\frac{3be^{a+\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}}$$

output
```
-1/8*b^2*exp(-c*x^2+b*x+a)/c^3-1/2*exp(-c*x^2+b*x+a)/c^2-1/4*b*exp(-c*x^2+
b*x+a)*x/c^2-1/2*exp(-c*x^2+b*x+a)*x^2/c-1/16*b^3*exp(a+1/4*b^2/c)*Pi^(1/2
)*erf(1/2*(-2*c*x+b)/c^(1/2))/c^(7/2)-3/8*b*exp(a+1/4*b^2/c)*Pi^(1/2)*erf(
1/2*(-2*c*x+b)/c^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.50

$$\int e^{a+bx-cx^2}x^3\,dx = -\frac{e^a\left(2\sqrt{c}e^{x(b-cx)}(b^2+2bcx+4c(1+cx^2))+b(b^2+6c)\,e^{\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)\right)}{16c^{7/2}}$$

input
```
Integrate[E^(a + b*x - c*x^2)*x^3,x]
```

output
```
-1/16*(E^a*(2*Sqrt[c]*E^(x*(b - c*x))*(b^2 + 2*b*c*x + 4*c*(1 + c*x^2)) +
b*(b^2 + 6*c)*E^(b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])]))/c^(7/2
)
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.34, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {2671, 2670, 2664, 2634, 2671, 2664, 2634, 2670, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3e^{a+bx-cx^2}\,dx$$

$$\Big\downarrow \text{2671}$$

$$\frac{\int e^{-cx^2+bx+a}xdx}{c}+\frac{b\int e^{-cx^2+bx+a}x^2dx}{2c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2670}$$

$$\frac{\frac{b\int e^{-cx^2+bx+a}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}}{c}+\frac{b\int e^{-cx^2+bx+a}x^2dx}{2c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2664}$$

$$\frac{\frac{be^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}}{c}+\frac{b\int e^{-cx^2+bx+a}x^2dx}{2c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\int e^{-cx^2+bx+a}x^2dx}{2c}+\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2671}$$

$$\frac{b\left(\frac{\int e^{-cx^2+bx+a}dx}{2c}+\frac{b\int e^{-cx^2+bx+a}xdx}{2c}-\frac{xe^{a+bx-cx^2}}{2c}\right)}{2c}+\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2664}$$

$$\frac{b\left(\frac{e^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx}{2c}+\frac{b\int e^{-cx^2+bx+a}xdx}{2c}-\frac{xe^{a+bx-cx^2}}{2c}\right)}{2c}+\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(\frac{b\int e^{-cx^2+bx+a}xdx}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}\right)}{2c}+\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2670}$$

$$\frac{b\left(\frac{b\left(\frac{b\int e^{-cx^2+bx+a}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}\right)}{2c}+\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2664}$$

$$\frac{b\left(\frac{b\left(\frac{be^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}\right)}{2c}+\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}}{c}+\frac{b\left(\frac{b\left(-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{x^2e^{a+bx-cx^2}}{2c}$$

input
```
Int[E^(a + b*x - c*x^2)*x^3,x]
```

output
```
-1/2*(E^(a + b*x - c*x^2)*x^2)/c + (-1/2*E^(a + b*x - c*x^2)/c - (b*E^(a +
 b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(4*c^(3/2)))/c + (b*(-1
/2*(E^(a + b*x - c*x^2)*x)/c - (E^(a + b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)
/(2*Sqrt[c])])/(4*c^(3/2)) + (b*(-1/2*E^(a + b*x - c*x^2)/c - (b*E^(a + b^
2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(4*c^(3/2))))/(2*c)))/(2*c
)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.85

method	result
risch	$-\frac{e^{-c\,x^2+bx+a}x^2}{2c}-\frac{b\,e^{-c\,x^2+bx+a}x}{4c^2}-\frac{b^2e^{-c\,x^2+bx+a}}{8c^3}-\frac{b^3\sqrt{\pi}\,e^{\frac{4ac+b^2}{4c}}\,\mathrm{erf}\left(-\sqrt{c}\,x+\frac{b}{2\sqrt{c}}\right)}{16c^{\frac{7}{2}}}-\frac{3b\sqrt{\pi}\,e^{\frac{4ac+b^2}{4c}}\,\mathrm{erf}\left(-\sqrt{c}\,x+\frac{b}{2\sqrt{c}}\right)}{8c^{\frac{5}{2}}}$
default	$-\frac{e^{-c\,x^2+bx+a}x^2}{2c}+\frac{b\left(-\frac{e^{-c\,x^2+bx+a}x}{2c}+\frac{b\left(-\frac{e^{-c\,x^2+bx+a}}{2c}-\frac{b\sqrt{\pi}\,e^{a+\frac{b^2}{4c}}\,\mathrm{erf}\left(-\sqrt{c}\,x+\frac{b}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)}{2c}-\frac{\sqrt{\pi}\,e^{a+\frac{b^2}{4c}}\,\mathrm{erf}\left(-\sqrt{c}\,x+\frac{b}{2\sqrt{c}}\right)}{4c^{\frac{3}{2}}}\right)}{2c}+$
parts	$-\frac{\sqrt{\pi}\,e^{a+\frac{b^2}{4c}}\,\mathrm{erf}\left(-\sqrt{c}\,x+\frac{b}{2\sqrt{c}}\right)x^3}{2\sqrt{c}}+\frac{e^{a+\frac{b^2}{4c}}\left(8\,\mathrm{erf}\left(\frac{-2cx+b}{2\sqrt{c}}\right)x^3c^{\frac{11}{2}}\sqrt{\pi}-\sqrt{\pi}\,c^{\frac{5}{2}}\,\mathrm{erf}\left(\frac{-2cx+b}{2\sqrt{c}}\right)b^3-8\,e^{-\frac{(-2cx+b)^2}{4c}}x^2c^5-6\sqrt{\pi}\,c^{\frac{7}{2}}\,\mathrm{er}\right.}{16c^6}$

input
```
int(exp(-c*x^2+b*x+a)*x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-c*x^2+b*x+a)*x^2/c-1/4*b*exp(-c*x^2+b*x+a)*x/c^2-1/8*b^2*exp(-c*
x^2+b*x+a)/c^3-1/16*b^3/c^(7/2)*Pi^(1/2)*exp(1/4*(4*a*c+b^2)/c)*erf(-c^(1/
2)*x+1/2*b/c^(1/2))-3/8*b/c^(5/2)*Pi^(1/2)*exp(1/4*(4*a*c+b^2)/c)*erf(-c^(
1/2)*x+1/2*b/c^(1/2))-1/2*exp(-c*x^2+b*x+a)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{a+bx-cx^2}x^3\,dx = \frac{\sqrt{\pi}(b^3+6\,bc)\sqrt{c}\operatorname{erf}\left(\frac{2\,cx-b}{2\,\sqrt{c}}\right)e^{\left(\frac{b^2+4\,ac}{4\,c}\right)}-2\,(4\,c^3x^2+2\,bc^2x+b^2c+4\,c^2)e^{(-cx^2+bx+a)}}{16\,c^4}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x^3,x, algorithm="fricas")
```

output
```
1/16*(sqrt(pi)*(b^3 + 6*b*c)*sqrt(c)*erf(1/2*(2*c*x - b)/sqrt(c))*e^(1/4*(
b^2 + 4*a*c)/c) - 2*(4*c^3*x^2 + 2*b*c^2*x + b^2*c + 4*c^2)*e^(-c*x^2 + b*
x + a))/c^4
```

Sympy [F]

$$\int e^{a+bx-cx^2}x^3\,dx = e^a\int x^3e^{bx}e^{-cx^2}\,dx$$

input
```
integrate(exp(-c*x**2+b*x+a)*x***3,x)
```

output
```
exp(a)*Integral(x**3*exp(b*x)*exp(-c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00

$$\int e^{a+bx-cx^2}x^3\,dx = \frac{\left(\frac{\sqrt{\pi}(2\,cx-b)b^3\left(\operatorname{erf}\left(\frac{1}{2}\,\sqrt{\frac{(2\,cx-b)^2}{c}}\right)-1\right)}{\sqrt{\frac{(2\,cx-b)^2}{c}}(-c)^{\frac{7}{2}}}-\frac{6\,b^2ce^{\left(-\frac{(2\,cx-b)^2}{4\,c}\right)}}{(-c)^{\frac{7}{2}}}-\frac{12\,(2\,cx-b)^3b\Gamma\left(\frac{3}{2},\frac{(2\,cx-b)^2}{4\,c}\right)}{\left(\frac{(2\,cx-b)^2}{c}\right)^{\frac{3}{2}}(-c)^{\frac{7}{2}}}-\frac{8\,c^2\Gamma\left(2,\frac{(2\,cx-b)^2}{4\,c}\right)}{(-c)^{\frac{7}{2}}}\right)e^{\left(a+\frac{b^2}{4\,c}\right)}}{16\,\sqrt{-c}}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x^3,x, algorithm="maxima")
```

output
```
1/16*(sqrt(pi)*(2*c*x - b)*b^3*(erf(1/2*sqrt((2*c*x - b)^2/c)) - 1)/(sqrt(
(2*c*x - b)^2/c)*(-c)^(7/2)) - 6*b^2*c*e^(-1/4*(2*c*x - b)^2/c)/(-c)^(7/2)
 - 12*(2*c*x - b)^3*b*gamma(3/2, 1/4*(2*c*x - b)^2/c)/((((2*c*x - b)^2/c)^(
3/2)*(-c)^(7/2)) - 8*c^2*gamma(2, 1/4*(2*c*x - b)^2/c)/(-c)^(7/2))*e^(a +
1/4*b^2/c)/sqrt(-c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.57

$$\int e^{a+bx-cx^2}x^3\,dx =$$

$$-\frac{\frac{\sqrt{\pi}(b^3+6\,bc)\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x-\frac{b}{c}\right)\right)e^{\left(\frac{b^2+4\,ac}{4\,c}\right)}}{\sqrt{c}}+2\left(c^2\left(2\,x-\frac{b}{c}\right)^2+3\,bc\left(2\,x-\frac{b}{c}\right)+3\,b^2+4\,c\right)e^{\left(-cx^2+bx+a\right)}}{16\,c^3}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x^3,x, algorithm="giac")
```

output
```
-1/16*(sqrt(pi)*(b^3 + 6*b*c)*erf(-1/2*sqrt(c)*(2*x - b/c))*e^(1/4*(b^2 +
4*a*c)/c)/sqrt(c) + 2*(c^2*(2*x - b/c)^2 + 3*b*c*(2*x - b/c) + 3*b^2 + 4*c
)*e^(-c*x^2 + b*x + a))/c^3
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.62

$$\int e^{a+bx-cx^2}x^3\,dx = -\mathrm{e}^{-c\,x^2+b\,x+a}\left(\frac{1}{2\,c^2}+\frac{b^2}{8\,c^3}\right)-\frac{x^2\,\mathrm{e}^{-c\,x^2+b\,x+a}}{2\,c} - \frac{b\,x\,\mathrm{e}^{-c\,x^2+b\,x+a}}{4\,c^2}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b}{2}-c\,x}{\sqrt{-c}}\right)\,\mathrm{e}^{a+\frac{b^2}{4\,c}}\,\left(b^3+6\,c\,b\right)}{16\,{\left(-c\right)}^{7/2}}$$

input
```
int(x^3*exp(a + b*x - c*x^2),x)
```

output
```
- exp(a + b*x - c*x^2)*(1/(2*c^2) + b^2/(8*c^3)) - (x^2*exp(a + b*x - c*x^
2))/(2*c) - (b*x*exp(a + b*x - c*x^2))/(4*c^2) - (pi^(1/2)*erfi((b/2 - c*x
)/(-c)^(1/2))*exp(a + b^2/(4*c))*(6*b*c + b^3))/(16*(-c)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.83

$$\int e^{a+bx-cx^2}x^3\,dx$$
$$= \frac{e^a\left(\sqrt{\pi}\,e^{\frac{4c^2x^2+b^2}{4c}}\operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)b^3+6\sqrt{\pi}\,e^{\frac{4c^2x^2+b^2}{4c}}\operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)bc-2e^{bx}\sqrt{c}\,b^2-4e^{bx}\sqrt{c}\,bcx-8e^{bx}\sqrt{c}\,c^2x^2-8}{16e^{c\,x^2}\sqrt{c}\,c^3}$$

input
```
int(exp(-c*x^2+b*x+a)*x^3,x)
```

output
```
(e**a*(sqrt(pi)*e**((b**2 + 4*c**2*x**2)/(4*c))*erf(( - b + 2*c*x)/(2*sqrt
(c)))*b**3 + 6*sqrt(pi)*e**((b**2 + 4*c**2*x**2)/(4*c))*erf(( - b + 2*c*x)
/(2*sqrt(c)))*b*c - 2*e**(b*x)*sqrt(c)*b**2 - 4*e**(b*x)*sqrt(c)*b*c*x - 8
*e**(b*x)*sqrt(c)*c**2*x**2 - 8*e**(b*x)*sqrt(c)*c))/(16*e**(c*x**2)*sqrt(
c)*c**3)
```

3.367 $\int e^{a+bx-cx^2} x^2 \, dx$

Optimal result

Integrand size = 17, antiderivative size = 134

$$\int e^{a+bx-cx^2} x^2 \, dx = -\frac{b e^{a+bx-cx^2}}{4c^2} - \frac{e^{a+bx-cx^2} x}{2c} - \frac{b^2 e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{8c^{5/2}} - \frac{e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}$$

output
```
-1/4*b*exp(-c*x^2+b*x+a)/c^2-1/2*exp(-c*x^2+b*x+a)*x/c-1/8*b^2*exp(a+1/4*b
^2/c)*Pi^(1/2)*erf(1/2*(-2*c*x+b)/c^(1/2))/c^(5/2)-1/4*exp(a+1/4*b^2/c)*Pi
^(1/2)*erf(1/2*(-2*c*x+b)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.59

$$\int e^{a+bx-cx^2} x^2 \, dx = \frac{e^a \left(-2\sqrt{c} e^{x(b-cx)} (b+2cx) + (b^2+2c) e^{\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{-b+2cx}{2\sqrt{c}}\right)\right)}{8c^{5/2}}$$

input
```
Integrate[E^(a + b*x - c*x^2)*x^2,x]
```

output
```
(E^a*(-2*Sqrt[c]*E^(x*(b - c*x))*(b + 2*c*x) + (b^2 + 2*c)*E^(b^2/(4*c))*S
qrt[Pi]*Erf[(-b + 2*c*x)/(2*Sqrt[c])]))/(8*c^(5/2))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2671, 2664, 2634, 2670, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{a+bx-cx^2}\,dx$$

$$\Big\downarrow \text{2671}$$

$$\frac{\int e^{-cx^2+bx+a}dx}{2c}+\frac{b\int e^{-cx^2+bx+a}xdx}{2c}-\frac{xe^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2664}$$

$$\frac{e^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx}{2c}+\frac{b\int e^{-cx^2+bx+a}xdx}{2c}-\frac{xe^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\int e^{-cx^2+bx+a}xdx}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2670}$$

$$\frac{b\left(\frac{b\int e^{-cx^2+bx+a}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2664}$$

$$\frac{b\left(\frac{be^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{2634}$$

$$\frac{b\left(-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}\right)}{2c}-\frac{\sqrt{\pi}e^{a+\frac{b^2}{4c}}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{xe^{a+bx-cx^2}}{2c}$$

input
```
Int[E^(a + b*x - c*x^2)*x^2,x]
```

output
```
-1/2*(E^(a + b*x - c*x^2)*x)/c - (E^(a + b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*
x)/(2*Sqrt[c])])/(4*c^(3/2)) + (b*(-1/2*E^(a + b*x - c*x^2)/c - (b*E^(a +
b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/(4*c^(3/2))))/(2*c)
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

method	result
default	$-\frac{{\rm e}^{-c\,x^{2}+b x+a} x}{2 c}+\frac{b \left(-\frac{{\rm e}^{-c\,x^{2}+b x+a}}{2 c}-\frac{b \sqrt{\pi}\, {\rm e}^{a+\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2 \sqrt{c}}\right)}{4 c^{\frac{3}{2}}}\right)}{2 c}-\frac{\sqrt{\pi}\, {\rm e}^{a+\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2 \sqrt{c}}\right)}{4 c^{\frac{3}{2}}}$
risch	$-\frac{{\rm e}^{-c\,x^{2}+b x+a} x}{2 c}-\frac{b\,{\rm e}^{-c\,x^{2}+b x+a}}{4 c^{2}}-\frac{b^{2} \sqrt{\pi}\, {\rm e}^{\frac{4 a c+b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2 \sqrt{c}}\right)}{8 c^{\frac{5}{2}}}-\frac{\sqrt{\pi}\, {\rm e}^{\frac{4 a c+b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2 \sqrt{c}}\right)}{4 c^{\frac{3}{2}}}$
parts	$-\frac{\sqrt{\pi}\, {\rm e}^{a+\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2 \sqrt{c}}\right) x^{2}}{2 \sqrt{c}}+\frac{{\rm e}^{a+\frac{b^{2}}{4 c}} \left(4 \operatorname{erf}\left(\frac{-2 c x+b}{2 \sqrt{c}}\right) x^{2} \sqrt{\pi}\, c^{\frac{7}{2}}-c^{\frac{3}{2}} \sqrt{\pi}\, \operatorname{erf}\left(\frac{-2 c x+b}{2 \sqrt{c}}\right) b^{2}-2 c^{\frac{5}{2}} \sqrt{\pi}\, \operatorname{erf}\left(\frac{-2 c x+b}{2 \sqrt{c}}\right)-4 x\,{\rm e}^{-(-}\right.}{8 c^{4}}$

input
```
int(exp(-c*x^2+b*x+a)*x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-c*x^2+b*x+a)*x/c+1/2*b/c*(-1/2*exp(-c*x^2+b*x+a)/c-1/4*b/c^(3/2)
*Pi^(1/2)*exp(a+1/4*b^2/c)*erf(-c^(1/2)*x+1/2*b/c^(1/2)))-1/4/c^(3/2)*Pi^(
1/2)*exp(a+1/4*b^2/c)*erf(-c^(1/2)*x+1/2*b/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.54

$$\int e^{a+bx-cx^2}x^2\,dx = \frac{\sqrt{\pi}(b^2+2\,c)\sqrt{c}\operatorname{erf}\left(\frac{2\,cx-b}{2\,\sqrt{c}}\right)e^{\left(\frac{b^2+4\,ac}{4\,c}\right)}-2\,(2\,c^2x+bc)e^{(-cx^2+bx+a)}}{8\,c^3}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x^2,x, algorithm="fricas")
```

output
```
1/8*(sqrt(pi)*(b^2 + 2*c)*sqrt(c)*erf(1/2*(2*c*x - b)/sqrt(c))*e^(1/4*(b^2
 + 4*a*c)/c) - 2*(2*c^2*x + b*c)*e^(-c*x^2 + b*x + a))/c^3
```

Sympy [F]

$$\int e^{a+bx-cx^2}x^2\,dx = e^a\int x^2e^{bx}e^{-cx^2}\,dx$$

input
```
integrate(exp(-c*x**2+b*x+a)*x**2,x)
```

output
```
exp(a)*Integral(x**2*exp(b*x)*exp(-c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.13

$$\int e^{a+bx-cx^2}x^2\,dx = -\frac{\left(\frac{\sqrt{\pi}(2\,cx-b)b^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{(2\,cx-b)^2}{c}}\right)-1\right)}{\sqrt{\frac{(2\,cx-b)^2}{c}}(-c)^{\frac{5}{2}}} - \frac{4\,bce^{\left(-\frac{(2\,cx-b)^2}{4\,c}\right)}}{(-c)^{\frac{5}{2}}} - \frac{4\,(2\,cx-b)^3\Gamma\left(\frac{3}{2},\frac{(2\,cx-b)^2}{4\,c}\right)}{\left(\frac{(2\,cx-b)^2}{c}\right)^{\frac{3}{2}}(-c)^{\frac{5}{2}}}\right)e^{\left(a+\frac{b^2}{4\,c}\right)}}{8\,\sqrt{-c}}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x^2,x, algorithm="maxima")
```

output
```
-1/8*(sqrt(pi)*(2*c*x - b)*b^2*(erf(1/2*sqrt((2*c*x - b)^2/c)) - 1)/(sqrt(
(2*c*x - b)^2/c)*(-c)^(5/2)) - 4*b*c*e^(-1/4*(2*c*x - b)^2/c)/(-c)^(5/2) -
 4*(2*c*x - b)^3*gamma(3/2, 1/4*(2*c*x - b)^2/c)/(((2*c*x - b)^2/c)^(3/2)*
(-c)^(5/2)))*e^(a + 1/4*b^2/c)/sqrt(-c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int e^{a+bx-cx^2}x^2\,dx$$
$$= -\frac{\frac{\sqrt{\pi}(b^2+2\,c)\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{c}\left(2\,x-\frac{b}{c}\right)\right)e^{\left(\frac{b^2+4\,ac}{4\,c}\right)}}{\sqrt{c}}+2\left(c\left(2\,x-\frac{b}{c}\right)+2\,b\right)e^{\left(-cx^2+bx+a\right)}}{8\,c^2}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x^2,x, algorithm="giac")
```

output
```
-1/8*(sqrt(pi)*(b^2 + 2*c)*erf(-1/2*sqrt(c)*(2*x - b/c))*e^(1/4*(b^2 + 4*a
*c)/c)/sqrt(c) + 2*(c*(2*x - b/c) + 2*b)*e^(-c*x^2 + b*x + a))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int e^{a+bx-cx^2}x^2\,dx = \frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b}{2}-c\,x}{\sqrt{-c}}\right)\,\mathrm{e}^{a+\frac{b^2}{4\,c}}\,\left(b^2+2\,c\right)}{8\,{\left(-c\right)}^{5/2}}-\frac{x\,\mathrm{e}^{-c\,x^2+b\,x+a}}{2\,c}-\frac{b\,\mathrm{e}^{-c\,x^2+b\,x+a}}{4\,c^2}$$

input
```
int(x^2*exp(a + b*x - c*x^2),x)
```

output
```
(pi^(1/2)*erfi((b/2 - c*x)/(-c)^(1/2))*exp(a + b^2/(4*c))*(2*c + b^2))/(8*
(-c)^(5/2)) - (x*exp(a + b*x - c*x^2))/(2*c) - (b*exp(a + b*x - c*x^2))/(4
*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.91

$$\int e^{a+bx-cx^2}x^2\,dx$$
$$= \frac{e^a\left(\sqrt{\pi}\,e^{\frac{4c^2x^2+b^2}{4c}}\operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)b^2+2\sqrt{\pi}\,e^{\frac{4c^2x^2+b^2}{4c}}\operatorname{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)c-2e^{bx}\sqrt{c}\,b-4e^{bx}\sqrt{c}\,cx\right)}{8e^{c\,x^2}\sqrt{c}\,c^2}$$

input
```
int(exp(-c*x^2+b*x+a)*x^2,x)
```

output
```
(e**a*(sqrt(pi)*e**((b**2 + 4*c**2*x**2)/(4*c))*erf(( - b + 2*c*x)/(2*sqrt
(c)))*b**2 + 2*sqrt(pi)*e**((b**2 + 4*c**2*x**2)/(4*c))*erf(( - b + 2*c*x)
/(2*sqrt(c)))*c - 2*e**(b*x)*sqrt(c)*b - 4*e**(b*x)*sqrt(c)*c*x))/(8*e**(c
*x**2)*sqrt(c)*c**2)
```

3.368 $\int e^{a+bx-cx^2} x \, dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int e^{a+bx-cx^2} x \, dx = -\frac{e^{a+bx-cx^2}}{2c} - \frac{b e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}$$

output
```
-1/2*exp(-c*x^2+b*x+a)/c-1/4*b*exp(a+1/4*b^2/c)*Pi^(1/2)*erf(1/2*(-2*c*x+b
)/c^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int e^{a+bx-cx^2} x \, dx = -\frac{e^{a+bx-cx^2}}{2c} + \frac{b e^{a+\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erf}\left(\frac{-b+2cx}{2\sqrt{c}}\right)}{4c^{3/2}}$$

input
```
Integrate[E^(a + b*x - c*x^2)*x,x]
```

output
```
-1/2*E^(a + b*x - c*x^2)/c + (b*E^(a + b^2/(4*c))*Sqrt[Pi]*Erf[(-b + 2*c*x
)/(2*Sqrt[c])])/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2670, 2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{a+bx-cx^2}\,dx$$

$$\Big\downarrow \text{ 2670}$$

$$\frac{b\int e^{-cx^2+bx+a}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{be^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx}{2c}-\frac{e^{a+bx-cx^2}}{2c}$$

$$\Big\downarrow \text{ 2634}$$

$$-\frac{\sqrt{\pi}be^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{e^{a+bx-cx^2}}{2c}$$

input
```
Int[E^(a + b*x - c*x^2)*x,x]
```

output
```
-1/2*E^(a + b*x - c*x^2)/c - (b*E^(a + b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)
/(2*Sqrt[c])])/(4*c^(3/2))
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{{\mathrm e}^{-c\,x^{2}+b x +a}}{2 c}-\frac{b \sqrt{\pi}\,{\mathrm e}^{a +\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\,x +\frac{b}{2 \sqrt{c}}\right)}{4 c^{\frac{3}{2}}}$	53
risch	$-\frac{{\mathrm e}^{-c\,x^{2}+b x +a}}{2 c}-\frac{b \sqrt{\pi}\,{\mathrm e}^{\frac{4 a c +b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\,x +\frac{b}{2 \sqrt{c}}\right)}{4 c^{\frac{3}{2}}}$	56
parts	$-\frac{\sqrt{\pi}\,{\mathrm e}^{a +\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{c}\,x +\frac{b}{2 \sqrt{c}}\right) x}{2 \sqrt{c}}+\frac{{\mathrm e}^{a +\frac{b^{2}}{4 c}} \left(2 \operatorname{erf}\left(\frac{-2 c x +b}{2 \sqrt{c}}\right) x \sqrt{\pi}\, c^{\frac{3}{2}}-b \operatorname{erf}\left(\frac{-2 c x +b}{2 \sqrt{c}}\right) \sqrt{\pi}\, \sqrt{c}-2 c \,{\mathrm e}^{-\frac{\left(-2 c x +b \right)^{2}}{4 c}}\right)}{4 c^{2}}$	112

input
```
int(exp(-c*x^2+b*x+a)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-c*x^2+b*x+a)/c-1/4*b/c^(3/2)*Pi^(1/2)*exp(a+1/4*b^2/c)*erf(-c^(1
/2)*x+1/2*b/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int e^{a+bx-cx^2} x\,dx = \frac{\sqrt{\pi} b \sqrt{c} \operatorname{erf}\left(\frac{2\,cx - b}{2\,\sqrt{c}}\right) e^{\left(\frac{b^{2} + 4\,ac}{4\,c}\right)} - 2\,c e^{\left(-cx^{2} + bx + a\right)}}{4\,c^{2}}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*b*sqrt(c)*erf(1/2*(2*c*x - b)/sqrt(c))*e^(1/4*(b^2 + 4*a*c)/
c) - 2*c*e^(-c*x^2 + b*x + a))/c^2
```

Sympy [F]

$$\int e^{a+bx-cx^2}x\,dx = e^a\int xe^{bx}e^{-cx^2}\,dx$$

input
```
integrate(exp(-c*x**2+b*x+a)*x,x)
```

output
```
exp(a)*Integral(x*exp(b*x)*exp(-c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48

$$\int e^{a+bx-cx^2}x\,dx = \frac{\left(\frac{\sqrt{\pi}(2\,cx-b)b\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{\frac{(2\,cx-b)^2}{c}}\right)-1\right)}{\sqrt{\frac{(2\,cx-b)^2}{c}}(-c)^{\frac{3}{2}}} - \frac{2\,ce^{\left(-\frac{(2\,cx-b)^2}{4\,c}\right)}}{(-c)^{\frac{3}{2}}}\right)e^{\left(a+\frac{b^2}{4\,c}\right)}}{4\sqrt{-c}}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x,x, algorithm="maxima")
```

output
```
1/4*(sqrt(pi)*(2*c*x - b)*b*(erf(1/2*sqrt((2*c*x - b)^2/c)) - 1)/(sqrt((2*
c*x - b)^2/c)*(-c)^(3/2)) - 2*c*e^(-1/4*(2*c*x - b)^2/c)/(-c)^(3/2))*e^(a
+ 1/4*b^2/c)/sqrt(-c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int e^{a+bx-cx^2}x\,dx = -\frac{\frac{\sqrt{\pi}b\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{b}{c}\right)\right)e^{\left(\frac{b^2+4ac}{4c}\right)}}{\sqrt{c}} + 2e^{\left(-cx^2+bx+a\right)}}{4c}$$

input
```
integrate(exp(-c*x^2+b*x+a)*x,x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*b*erf(-1/2*sqrt(c)*(2*x - b/c))*e^(1/4*(b^2 + 4*a*c)/c)/sqr
t(c) + 2*e^(-c*x^2 + b*x + a))/c
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int e^{a+bx-cx^2}x\,dx = -\frac{e^{bx}\,e^{a}\,e^{-cx^2}}{2c} - \frac{b\sqrt{\pi}\,e^{\frac{b^2}{4c}}\,e^{a}\,\mathrm{erfi}\left(\frac{b}{2\sqrt{-c}} + \sqrt{-c}\,x\right)}{4\,(-c)^{3/2}}$$

input
```
int(x*exp(a + b*x - c*x^2),x)
```

output
```
- (exp(b*x)*exp(a)*exp(-c*x^2))/(2*c) - (b*pi^(1/2)*exp(b^2/(4*c))*exp(a)*
erfi(b/(2*(-c)^(1/2)) + (-c)^(1/2)*x))/(4*(-c)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int e^{a+bx-cx^2}x\,dx = \frac{e^{a}\left(\sqrt{\pi}\,e^{\frac{4c^2x^2+b^2}{4c}}\,\mathrm{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)b - 2e^{bx}\sqrt{c}\right)}{4e^{cx^2}\sqrt{c}\,c}$$

input
```
int(exp(-c*x^2+b*x+a)*x,x)
```

output
```
(e**a*(sqrt(pi)*e**((b**2 + 4*c**2*x**2)/(4*c))*erf(( - b + 2*c*x)/(2*sqrt
(c)))*b - 2*e**(b*x)*sqrt(c)))/(4*e**(c*x**2)*sqrt(c)*c)
```

3.369 $\int e^{a+bx-cx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int e^{a+bx-cx^2}\,dx=-\frac{e^{a+\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

output
```
-1/2*exp(a+1/4*b^2/c)*Pi^(1/2)*erf(1/2*(-2*c*x+b)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int e^{a+bx-cx^2}\,dx=\frac{e^{a+\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{-b+2cx}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

input
```
Integrate[E^(a + b*x - c*x^2),x]
```

output
```
(E^(a + b^2/(4*c))*Sqrt[Pi]*Erf[(-b + 2*c*x)/(2*Sqrt[c])])/(2*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2664, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx-cx^2} \, dx$$

$$\Big\downarrow \text{2664}$$

$$e^{a+\frac{b^2}{4c}} \int e^{-\frac{(b-2cx)^2}{4c}} dx$$

$$\Big\downarrow \text{2634}$$

$$-\frac{\sqrt{\pi} e^{a+\frac{b^2}{4c}} \text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

input
```
Int[E^(a + b*x - c*x^2),x]
```

output
```
-1/2*(E^(a + b^2/(4*c))*Sqrt[Pi]*Erf[(b - 2*c*x)/(2*Sqrt[c])])/Sqrt[c]
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\sqrt{\pi}\, e^{a+\frac{b^2}{4c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2\sqrt{c}}\right)}{2\sqrt{c}}$	34
risch	$-\frac{\sqrt{\pi}\, e^{\frac{4ac+b^2}{4c}} \operatorname{erf}\left(-\sqrt{c}\, x+\frac{b}{2\sqrt{c}}\right)}{2\sqrt{c}}$	37

input
```
int(exp(-c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*exp(a+1/4*b^2/c)/c^(1/2)*erf(-c^(1/2)*x+1/2*b/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int e^{a+bx-cx^2}\,dx = \frac{\sqrt{\pi}\operatorname{erf}\left(\frac{2\,cx-b}{2\,\sqrt{c}}\right)e^{\left(\frac{b^2+4\,ac}{4\,c}\right)}}{2\,\sqrt{c}}$$

input
```
integrate(exp(-c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(pi)*erf(1/2*(2*c*x - b)/sqrt(c))*e^(1/4*(b^2 + 4*a*c)/c)/sqrt(c)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int e^{a+bx-cx^2}\,dx = \frac{\sqrt{\pi}\sqrt{-\frac{1}{c}}e^{a+\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{b-2cx}{2\sqrt{-c}}\right)}{2}$$

input
```
integrate(exp(-c*x**2+b*x+a),x)
```

output
```
sqrt(pi)*sqrt(-1/c)*exp(a + b**2/(4*c))*erfi((b - 2*c*x)/(2*sqrt(-c)))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int e^{a+bx-cx^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x - \frac{b}{2\sqrt{c}}\right)e^{\left(a+\frac{b^2}{4c}\right)}}{2\sqrt{c}}$$

input
```
integrate(exp(-c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*erf(sqrt(c)*x - 1/2*b/sqrt(c))*e^(a + 1/4*b^2/c)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int e^{a+bx-cx^2}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x - \frac{b}{c}\right)\right)e^{\left(\frac{b^2+4ac}{4c}\right)}}{2\sqrt{c}}$$

input
```
integrate(exp(-c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - b/c))*e^(1/4*(b^2 + 4*a*c)/c)/sqrt(c
)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

$$\int e^{a+bx-cx^2}\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\,1\mathrm{i}-c\,x\,2\mathrm{i}}{2\,\sqrt{-c}}\right)\,\mathrm{e}^{a+\frac{b^2}{4\,c}}\,1\mathrm{i}}{2\,\sqrt{-c}}$$

input
```
int(exp(a + b*x - c*x^2),x)
```

output
```
-(pi^(1/2)*erf((b*1i - c*x*2i)/(2*(-c)^(1/2)))*exp(a + b^2/(4*c))*1i)/(2*(
-c)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int e^{a+bx-cx^2}\,dx=\frac{\sqrt{\pi}\,e^{\frac{4ac+b^2}{4c}}\,\mathrm{erf}\left(\frac{2cx-b}{2\sqrt{c}}\right)}{2\sqrt{c}}$$

input
```
int(exp(-c*x^2+b*x+a),x)
```

output
```
(sqrt(pi)*e**((4*a*c + b**2)/(4*c))*erf(( - b + 2*c*x)/(2*sqrt(c))))/(2*sq
rt(c))
```

3.370 $\int \frac{e^{a+bx-cx^2}}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{a+bx-cx^2}}{x} \, dx = \text{Int}\left(\frac{e^{a+bx-cx^2}}{x}, x\right)$$

output
```
Defer(Int)(exp(-c*x^2+b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{a+bx-cx^2}}{x} \, dx = \int \frac{e^{a+bx-cx^2}}{x} \, dx$$

input
```
Integrate[E^(a + b*x - c*x^2)/x,x]
```

output
```
Integrate[E^(a + b*x - c*x^2)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{a+bx-cx^2}}{x} \, dx$$
$$\Big\downarrow \text{2673}$$
$$\int \frac{e^{a+bx-cx^2}}{x}dx$$

input
```
Int[E^(a + b*x - c*x^2)/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{-c\,x^{2}+b x +a}}{x}dx$$

input
```
int(exp(-c*x^2+b*x+a)/x,x)
```

output
```
int(exp(-c*x^2+b*x+a)/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x}\,dx = \int \frac{e^{\left(-cx^2+bx+a\right)}}{x}\,dx$$

input
```
integrate(exp(-c*x^2+b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(e^(-c*x^2 + b*x + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{a+bx-cx^2}}{x}\,dx = e^a \int \frac{e^{bx}e^{-cx^2}}{x}\,dx$$

input
```
integrate(exp(-c*x**2+b*x+a)/x,x)
```

output
```
exp(a)*Integral(exp(b*x)*exp(-c*x**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x}\,dx = \int \frac{e^{(-cx^2+bx+a)}}{x}\,dx$$

input
```
integrate(exp(-c*x^2+b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(e^(-c*x^2 + b*x + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x}\,dx = \int \frac{e^{(-cx^2+bx+a)}}{x}\,dx$$

input
```
integrate(exp(-c*x^2+b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(e^(-c*x^2 + b*x + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x}\,dx = \int \frac{\mathrm{e}^{-c\,x^2+b\,x+a}}{x}\,dx$$

input
```
int(exp(a + b*x - c*x^2)/x,x)
```

output
```
int(exp(a + b*x - c*x^2)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{a+bx-cx^2}}{x}\,dx = e^a\left(\int \frac{e^{bx}}{e^{c\,x^2}x}dx\right)$$

input
```
int(exp(-c*x^2+b*x+a)/x,x)
```

output
```
e**a*int(e**(b*x)/(e**(c*x**2)*x),x)
```

3.371 $\int \frac{e^{a+bx-cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{a+bx-cx^2}}{x^2}\,dx = -\frac{e^{a+bx-cx^2}}{x} + \sqrt{c}e^{a+\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right) + b\operatorname{Int}\left(\frac{e^{a+bx-cx^2}}{x}, x\right)$$

output
```
-exp(-c*x^2+b*x+a)/x+c^(1/2)*exp(a+1/4*b^2/c)*Pi^(1/2)*erf(1/2*(-2*c*x+b)/
c^(1/2))+b*Defer(Int)(exp(-c*x^2+b*x+a)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{a+bx-cx^2}}{x^2}\,dx = \int \frac{e^{a+bx-cx^2}}{x^2}\,dx$$

input
```
Integrate[E^(a + b*x - c*x^2)/x^2,x]
```

output
```
Integrate[E^(a + b*x - c*x^2)/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2672, 2664, 2634, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{a+bx-cx^2}}{x^2} \, dx$$

$$\Big\downarrow \text{2672}$$

$$-2c\int e^{-cx^2+bx+a}dx+b\int \frac{e^{-cx^2+bx+a}}{x}dx-\frac{e^{a+bx-cx^2}}{x}$$

$$\Big\downarrow \text{2664}$$

$$-2ce^{a+\frac{b^2}{4c}}\int e^{-\frac{(b-2cx)^2}{4c}}dx+b\int \frac{e^{-cx^2+bx+a}}{x}dx-\frac{e^{a+bx-cx^2}}{x}$$

$$\Big\downarrow \text{2634}$$

$$b\int \frac{e^{-cx^2+bx+a}}{x}dx+\sqrt{\pi}\sqrt{c}e^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)-\frac{e^{a+bx-cx^2}}{x}$$

$$\Big\downarrow \text{2673}$$

$$b\int \frac{e^{-cx^2+bx+a}}{x}dx+\sqrt{\pi}\sqrt{c}e^{a+\frac{b^2}{4c}}\text{erf}\left(\frac{b-2cx}{2\sqrt{c}}\right)-\frac{e^{a+bx-cx^2}}{x}$$

input
```
Int[E^(a + b*x - c*x^2)/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2672
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] + (-
Simp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2)
, x], x] - Simp[(b*e - 2*c*d)*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 1
)*F^(a + b*x + c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e -
 2*c*d, 0] && LtQ[m, -1]
```

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{-c\,x^{2}+b x+a}}{x^{2}}dx$$

input
```
int(exp(-c*x^2+b*x+a)/x^2,x)
```

output
```
int(exp(-c*x^2+b*x+a)/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x^2} dx = \int \frac{e^{\left(-cx^2+bx+a\right)}}{x^2} dx$$

input
```
integrate(exp(-c*x^2+b*x+a)/x^2,x, algorithm="fricas")
```

output
```
integral(e^(-c*x^2 + b*x + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{a+bx-cx^2}}{x^2} dx = e^a \int \frac{e^{bx}e^{-cx^2}}{x^2} dx$$

input
```
integrate(exp(-c*x**2+b*x+a)/x**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*exp(-c*x**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x^2} dx = \int \frac{e^{\left(-cx^2+bx+a\right)}}{x^2} dx$$

input
```
integrate(exp(-c*x^2+b*x+a)/x^2,x, algorithm="maxima")
```

output
```
integrate(e^(-c*x^2 + b*x + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x^2}\,dx = \int \frac{e^{(-cx^2+bx+a)}}{x^2}\,dx$$

input
```
integrate(exp(-c*x^2+b*x+a)/x^2,x, algorithm="giac")
```

output
```
integrate(e^(-c*x^2 + b*x + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx-cx^2}}{x^2}\,dx = \int \frac{\mathrm{e}^{-c\,x^2+b\,x+a}}{x^2}\,dx$$

input
```
int(exp(a + b*x - c*x^2)/x^2,x)
```

output
```
int(exp(a + b*x - c*x^2)/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{e^{a+bx-cx^2}}{x^2} dx = e^a \left(\int \frac{e^{bx}}{e^{c\,x^2} x^2} dx \right)$$

input
```
int(exp(-c*x^2+b*x+a)/x^2,x)
```

output
```
e**a*int(e**(b*x)/(e**(c*x**2)*x**2),x)
```

3.372 $\int e^{(a+bx)(c+dx)}x^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 297

$$\int e^{(a+bx)(c+dx)}x^3\,dx=-\frac{e^{ac+(bc+ad)x+bdx^2}}{2b^2d^2}+\frac{(bc+ad)^2e^{ac+(bc+ad)x+bdx^2}}{8b^3d^3}-\frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2}x}{4b^2d^2}+\frac{e^{ac+(bc+ad)x+bdx^2}x^2}{2bd}+\frac{3(bc+ad)e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{8b^{5/2}d^{5/2}}-\frac{(bc+ad)^3e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{16b^{7/2}d^{7/2}}$$

output
```
-1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b^2/d^2+1/8*(a*d+b*c)^2*exp(a*c+(a*d+b*c
)*x+b*d*x^2)/b^3/d^3-1/4*(a*d+b*c)*exp(a*c+(a*d+b*c)*x+b*d*x^2)*x/b^2/d^2+
1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)*x^2/b/d+3/8*(a*d+b*c)*Pi^(1/2)*erfi(1/2*(
2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2))/b^(5/2)/d^(5/2)/exp(1/4*(-a*d+b*c)^2/b/d
)-1/16*(a*d+b*c)^3*Pi^(1/2)*erfi(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2))/b^
(7/2)/d^(7/2)/exp(1/4*(-a*d+b*c)^2/b/d)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.64

$$\int e^{(a+bx)(c+dx)}x^3\,dx$$

$$= \frac{e^{-\frac{(bc-ad)^2}{4bd}}\left(2\sqrt{b}\sqrt{d}e^{\frac{(ad+b(c+2dx))^2}{4bd}}(a^2d^2-2bd(2-ac+adx)+b^2(c^2-2cdx+4d^2x^2))-(b^3c^3+3b^2c(-2+\right.}{16b^{7/2}d^{7/2}}$$

input
```
Integrate[E^((a + b*x)*(c + d*x))*x^3,x]
```

output
```
(2*Sqrt[b]*Sqrt[d]*E^((a*d + b*(c + 2*d*x))^2/(4*b*d))*(a^2*d^2 - 2*b*d*(2
 - a*c + a*d*x) + b^2*(c^2 - 2*c*d*x + 4*d^2*x^2)) - (b^3*c^3 + 3*b^2*c*(-
2 + a*c)*d + 3*a*b*(-2 + a*c)*d^2 + a^3*d^3)*Sqrt[Pi]*Erfi[(a*d + b*(c + 2
*d*x))/(2*Sqrt[b]*Sqrt[d])])/(16*b^(7/2)*d^(7/2)*E^((b*c - a*d)^2/(4*b*d))
)
```

Rubi [A] (verified)

Time = 2.67 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2674, 2671, 2670, 2664, 2633, 2671, 2664, 2633, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{2674}$$

$$\int x^3 e^{x(ad+bc)+ac+bdx^2}dx$$

$$\Big\downarrow \text{2671}$$

$$-\frac{\int e^{bdx^2+(bc+ad)x+ac}xdx}{bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}x^2dx}{2bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}dx}{2bd}}{bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}x^2dx}{2bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx}{2bd}}{bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}x^2dx}{2bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}x^2dx}{2bd}-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\operatorname{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2671}$$

$$-\frac{(ad+bc)\left(-\frac{\int e^{bdx^2+(bc+ad)x+ac}dx}{2bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}xdx}{2bd}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}\right)}{2bd}-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\operatorname{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{(ad+bc)\left(-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}xdx}{2bd}-\frac{e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx}{2bd}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}\right)}{2bd}-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\operatorname{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{(ad+bc)\left(-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}xdx}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\operatorname{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}\right)}{2bd}-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\operatorname{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$\Big\downarrow$ 2670

$$-\frac{(ad+bc)\left(-\frac{(ad+bc)\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}dx}{2bd}\right)}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+bc)+ac+b}}{2bd}\right.}{2bd}$$

$$\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$\Big\downarrow$ 2664

$$-\frac{(ad+bc)\left(-\frac{(ad+bc)\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx}{2bd}\right)}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+}}{}\right.}{2bd}$$

$$\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}+\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$\Big\downarrow$ 2633

$$-\frac{\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}}{bd}-$$

$$\frac{(ad+bc)\left(-\frac{(ad+bc)\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}\right)}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+}}{}\right.}{2bd}$$

$$\frac{x^2e^{x(ad+bc)+ac+bdx^2}}{2bd}$$

input

```
Int[E^((a + b*x)*(c + d*x))*x^3,x]
```

output
```
(E^(a*c + (b*c + a*d)*x + b*d*x^2)*x^2)/(2*b*d) - (E^(a*c + (b*c + a*d)*x
+ b*d*x^2)/(2*b*d) - ((b*c + a*d)*Sqrt[Pi]*Erfi[(b*c + a*d + 2*b*d*x)/(2*S
qrt[b]*Sqrt[d])])/(4*b^(3/2)*d^(3/2)*E^((b*c - a*d)^2/(4*b*d))))/(b*d) - (
(b*c + a*d)*((E^(a*c + (b*c + a*d)*x + b*d*x^2)*x)/(2*b*d) - (Sqrt[Pi]*Erf
i[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d])])/(4*b^(3/2)*d^(3/2)*E^((b*c -
 a*d)^2/(4*b*d))) - ((b*c + a*d)*(E^(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b*d
) - ((b*c + a*d)*Sqrt[Pi]*Erfi[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d])])
/(4*b^(3/2)*d^(3/2)*E^((b*c - a*d)^2/(4*b*d)))))/(2*b*d)))/(2*b*d)
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

rule 2674
```
Int[(F_)^(v_)*(u_)^(m_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*F^ExpandToSu
m[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(
LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.24

method	result
default	$\frac{e^{ac+(ad+bc)x+bd\,x^2}x^2}{2bd}-\frac{(ad+bc)\left(\frac{e^{ac+(ad+bc)x+bd\,x^2}x}{2bd}-\frac{(ad+bc)\left(\frac{e^{ac+(ad+bc)x+bd\,x^2}}{2bd}+\frac{(ad+bc)\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\operatorname{erf}\left(-\sqrt{-bd}\,x+\ldots\right.}{4bd\sqrt{-bd}}\right.}{2bd}\right.}{2bd}$
risch	$\frac{e^{(bx+a)(dx+c)}x^2}{2bd}-\frac{e^{(bx+a)(dx+c)}xa}{4b^2d}-\frac{e^{(bx+a)(dx+c)}xc}{4b\,d^2}+\frac{e^{(bx+a)(dx+c)}a^2}{8b^3d}+\frac{e^{(bx+a)(dx+c)}ac}{4b^2d^2}+\frac{e^{(bx+a)(dx+c)}c^2}{8b\,d^3}+\frac{\sqrt{\pi}\,e\ldots}{}$
parts	$-\frac{\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\operatorname{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)x^3}{2\sqrt{-bd}}-\frac{e^{ac-\frac{(ad+bc)^2}{4bd}}\left(8\operatorname{erf}\left(\frac{2bdx+ad+bc}{2\sqrt{-bd}}\right)x^3d^3b^3\sqrt{\pi}\sqrt{-bd}+\sqrt{\pi}\sqrt{-bd}\operatorname{erf}\left(\frac{2bdx+ad+\ldots}{2\sqrt{-bd}}\right.\right.}{}$

input
```
int(exp((b*x+a)*(d*x+c))*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)*x^2/b/d-1/2*(a*d+b*c)/b/d*(1/2*exp(a*c+(a
*d+b*c)*x+b*d*x^2)*x/b/d-1/2*(a*d+b*c)/b/d*(1/2*exp(a*c+(a*d+b*c)*x+b*d*x^
2)/b/d+1/4*(a*d+b*c)/b/d*Pi^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2
)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*d)^(1/2)))+1/4/b/d*Pi^(1/2)*exp(a*
c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*
d)^(1/2)))-1/b/d*(1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b/d+1/4*(a*d+b*c)/b/d*P
i^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*
(a*d+b*c)/(-b*d)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.71

$$\int e^{(a+bx)(c+dx)}x^3\,dx$$

$$=\frac{\sqrt{\pi}(b^3c^3+a^3d^3+3\,(a^2bc-2\,ab)d^2+3\,(ab^2c^2-2\,b^2c)d)\sqrt{-bd}\operatorname{erf}\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}}{2\,bd}\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}\ldots}{16\,b^4d^4}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x^3,x, algorithm="fricas")
```

output
```
1/16*(sqrt(pi)*(b^3*c^3 + a^3*d^3 + 3*(a^2*b*c - 2*a*b)*d^2 + 3*(a*b^2*c^2
 - 2*b^2*c)*d)*sqrt(-b*d)*erf(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b*d))*
e^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d)) + 2*(4*b^3*d^3*x^2 + b^3*c^
2*d + a^2*b*d^3 + 2*(a*b^2*c - 2*b^2)*d^2 - 2*(b^3*c*d^2 + a*b^2*d^3)*x)*e
^(b*d*x^2 + a*c + (b*c + a*d)*x))/(b^4*d^4)
```

Sympy [F]

$$\int e^{(a+bx)(c+dx)}x^3\,dx = e^{ac}\int x^3e^{adx}e^{bcx}e^{bdx^2}\,dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x**3,x)
```

output
```
exp(a*c)*Integral(x**3*exp(a*d*x)*exp(b*c*x)*exp(b*d*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.90

$$\int e^{(a+bx)(c+dx)}x^3\,dx = -\frac{\left(\frac{\sqrt{\pi}(2\,bdx+bc+ad)(bc+ad)^3\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,bdx+bc+ad)^2}{bd}}\right)-1\right)}{(bd)^{\frac{7}{2}}\sqrt{-\frac{(2\,bdx+bc+ad)^2}{bd}}}-\frac{6\,(bc+ad)^2bde^{\left(\frac{(2\,bdx+bc+ad)^2}{4\,bd}\right)}}{(bd)^{\frac{7}{2}}}+\frac{8\,b^2d^2\Gamma\left(2,-\frac{(2\,bdx+bc+ad)^2}{4\,bd}\right)}{(bd)^{\frac{7}{2}}}-\cdots}{16\,\sqrt{bd}}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x^3,x, algorithm="maxima")
```

output
```
-1/16*(sqrt(pi)*(2*b*d*x + b*c + a*d)*(b*c + a*d)^3*(erf(1/2*sqrt(-(2*b*d*
x + b*c + a*d)^2/(b*d))) - 1)/((b*d)^(7/2)*sqrt(-(2*b*d*x + b*c + a*d)^2/(
b*d))) - 6*(b*c + a*d)^2*b*d*e^(1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/(b*d)^(
7/2) + 8*b^2*d^2*gamma(2, -1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/(b*d)^(7/2)
 - 12*(2*b*d*x + b*c + a*d)^3*(b*c + a*d)*gamma(3/2, -1/4*(2*b*d*x + b*c +
a*d)^2/(b*d))/((b*d)^(7/2)*(-(2*b*d*x + b*c + a*d)^2/(b*d))^(3/2)))*e^(a*c
 - 1/4*(b*c + a*d)^2/(b*d))/sqrt(b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.84

$$\int e^{(a+bx)(c+dx)}x^3\,dx$$
$$= \frac{\frac{\sqrt{\pi}(b^3c^3+3\,ab^2c^2d+3\,a^2bcd^2+a^3d^3-6\,b^2cd-6\,abd^2)\,\text{erf}\left(-\frac{1}{2}\,\sqrt{-bd}\left(2\,x+\frac{bc+ad}{bd}\right)\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}}{\sqrt{-bd}}+2\left(b^2d^2\left(2\,x+\frac{bc+ad}{bd}\right)^2-\right.}{16\,b^3d^3}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x^3,x, algorithm="giac")
```

output
```
1/16*(sqrt(pi)*(b^3*c^3 + 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 - 6*b^2*
c*d - 6*a*b*d^2)*erf(-1/2*sqrt(-b*d)*(2*x + (b*c + a*d)/(b*d)))*e^(-1/4*(b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d))/sqrt(-b*d) + 2*(b^2*d^2*(2*x + (b*c +
 a*d)/(b*d))^2 - 3*b^2*c*d*(2*x + (b*c + a*d)/(b*d)) - 3*a*b*d^2*(2*x + (b
*c + a*d)/(b*d)) + 3*b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2 - 4*b*d)*e^(b*d*x^2 +
 b*c*x + a*d*x + a*c))/(b^3*d^3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.77

$$\int e^{(a+bx)(c+dx)}x^3\,dx = \frac{e^{a\,c+a\,d\,x+b\,c\,x+b\,d\,x^2}\left(\frac{a^2\,d^2}{8}-b\left(\frac{d}{2}-\frac{a\,c\,d}{4}\right)+\frac{b^2\,c^2}{8}\right)}{b^3\,d^3}$$
$$+\frac{x^2\,e^{a\,c+a\,d\,x+b\,c\,x+b\,d\,x^2}}{2\,b\,d}-\frac{x\,e^{a\,c+a\,d\,x+b\,c\,x+b\,d\,x^2}\,(a\,d+b\,c)}{4\,b^2\,d^2}$$
$$-\frac{\sqrt{\pi}\,e^{\frac{a\,c}{2}-\frac{a^2\,d}{4\,b}-\frac{b\,c^2}{4\,d}}\,\text{erfi}\left(\frac{\frac{a\,d}{2}+\frac{b\,c}{2}+b\,d\,x}{\sqrt{b\,d}}\right)\,(a^3\,d^3+3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-6\,a\,b\,d^2+b^3\,c^3-6\,b^2\,c\,d)}{16\,b^3\,d^3\,\sqrt{b\,d}}$$

input
```
int(x^3*exp((a + b*x)*(c + d*x)),x)
```

output
```
(exp(a*c + a*d*x + b*c*x + b*d*x^2)*((a^2*d^2)/8 - b*(d/2 - (a*c*d)/4) + (
b^2*c^2)/8))/(b^3*d^3) + (x^2*exp(a*c + a*d*x + b*c*x + b*d*x^2))/(2*b*d)
- (x*exp(a*c + a*d*x + b*c*x + b*d*x^2)*(a*d + b*c))/(4*b^2*d^2) - (pi^(1/
2)*exp((a*c)/2 - (a^2*d)/(4*b) - (b*c^2)/(4*d))*erfi(((a*d)/2 + (b*c)/2 +
b*d*x)/(b*d)^(1/2))*(a^3*d^3 + b^3*c^3 - 6*a*b*d^2 - 6*b^2*c*d + 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2))/(16*b^3*d^3*(b*d)^(1/2))
```

Reduce [F]

$$\int e^{(a+bx)(c+dx)}x^3\,dx = \frac{e^{ac}\left(e^{bd\,x^2+adx+bcx}a^2d^2+2e^{bd\,x^2+adx+bcx}abcd-2e^{bd\,x^2+adx+bcx}ab\,d^2x+e^{bd\,x^2+adx+bcx}b^2c^2-2e^{bd\,x^2+adx+bcx}b^2\right.}{}$$

input
```
int(exp((b*x+a)*(d*x+c))*x^3,x)
```

output
```
(e**(a*c)*(e**(a*d*x + b*c*x + b*d*x**2)*a**2*d**2 + 2*e**(a*d*x + b*c*x +
 b*d*x**2)*a*b*c*d - 2*e**(a*d*x + b*c*x + b*d*x**2)*a*b*d**2*x + e**(a*d*
x + b*c*x + b*d*x**2)*b**2*c**2 - 2*e**(a*d*x + b*c*x + b*d*x**2)*b**2*c*d
*x + 4*e**(a*d*x + b*c*x + b*d*x**2)*b**2*d**2*x**2 - 4*e**(a*d*x + b*c*x
+ b*d*x**2)*b*d - int(e**(a*d*x + b*c*x + b*d*x**2),x)*a**3*d**3 - 3*int(e
**(a*d*x + b*c*x + b*d*x**2),x)*a**2*b*c*d**2 - 3*int(e**(a*d*x + b*c*x +
b*d*x**2),x)*a*b**2*c**2*d + 6*int(e**(a*d*x + b*c*x + b*d*x**2),x)*a*b*d*
*2 - int(e**(a*d*x + b*c*x + b*d*x**2),x)*b**3*c**3 + 6*int(e**(a*d*x + b*
c*x + b*d*x**2),x)*b**2*c*d))/(8*b**3*d**3)
```

3.373 $\int e^{(a+bx)(c+dx)}x^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 216

$$\int e^{(a+bx)(c+dx)}x^2\,dx=-\frac{(bc+ad)e^{ac+(bc+ad)x+bdx^2}}{4b^2d^2}+\frac{e^{ac+(bc+ad)x+bdx^2}x}{2bd}-\frac{e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{(bc+ad)^2e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{8b^{5/2}d^{5/2}}$$

output
```
-1/4*(a*d+b*c)*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b^2/d^2+1/2*exp(a*c+(a*d+b*c)*
x+b*d*x^2)*x/b/d-1/4*Pi^(1/2)*erfi(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2))/
b^(3/2)/d^(3/2)/exp(1/4*(-a*d+b*c)^2/b/d)+1/8*(a*d+b*c)^2*Pi^(1/2)*erfi(1/
2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2))/b^(5/2)/d^(5/2)/exp(1/4*(-a*d+b*c)^2/
b/d)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.67

$$\int e^{(a+bx)(c+dx)}x^2\,dx = \frac{e^{-\frac{(bc-ad)^2}{4bd}}\left(-2\sqrt{b}\sqrt{d}e^{\frac{(ad+b(c+2dx))^2}{4bd}}(ad+b(c-2dx))+(b^2c^2+2b(-1+ac)d+a^2d^2)\sqrt{\pi}\text{erfi}\left(\frac{ad+b(c+2dx)}{2\sqrt{b}\sqrt{d}}\right)\right)}{8b^{5/2}d^{5/2}}$$

input
```
Integrate[E^((a + b*x)*(c + d*x))*x^2,x]
```

output
```
(-2*Sqrt[b]*Sqrt[d]*E^((a*d + b*(c + 2*d*x))^2/(4*b*d))*(a*d + b*(c - 2*d*
x)) + (b^2*c^2 + 2*b*(-1 + a*c)*d + a^2*d^2)*Sqrt[Pi]*Erfi[(a*d + b*(c + 2
*d*x))/(2*Sqrt[b]*Sqrt[d])])/(8*b^(5/2)*d^(5/2)*E^((b*c - a*d)^2/(4*b*d)))
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {2674, 2671, 2664, 2633, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2e^{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{2674}$$

$$\int x^2e^{x(ad+bc)+ac+bdx^2}dx$$

$$\Big\downarrow \text{2671}$$

$$-\frac{\int e^{bdx^2+(bc+ad)x+ac}dx}{2bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}xdx}{2bd}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}xdx}{2bd}-\frac{e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx}{2bd}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}xdx}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2670}$$

$$-\frac{(ad+bc)\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}dx}{2bd}\right)}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2664}$$

$$-\frac{(ad+bc)\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx}{2bd}\right)}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{(ad+bc)\left(\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}\right)}{2bd}-\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}+\frac{xe^{x(ad+bc)+ac+bdx^2}}{2bd}$$

input

```
Int[E^((a + b*x)*(c + d*x))*x^2,x]
```

output

```
(E^(a*c + (b*c + a*d)*x + b*d*x^2)*x)/(2*b*d) - (Sqrt[Pi]*Erfi[(b*c + a*d
+ 2*b*d*x)/(2*Sqrt[b]*Sqrt[d])])/(4*b^(3/2)*d^(3/2)*E^((b*c - a*d)^2/(4*b*
d))) - ((b*c + a*d)*(E^(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b*d) - ((b*c + a
*d)*Sqrt[Pi]*Erfi[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d])])/(4*b^(3/2)*d
^(3/2)*E^((b*c - a*d)^2/(4*b*d)))))/(2*b*d)
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

rule 2674
```
Int[(F_)^(v_)*(u_)^(m_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*F^ExpandToSu
m[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(
LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98

method	result
default	$\frac{e^{ac+(ad+bc)x+bd\,x^2}x}{2bd}-\frac{(ad+bc)\left(\frac{e^{ac+(ad+bc)x+bd\,x^2}}{2bd}+\frac{(ad+bc)\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\operatorname{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)}{4bd\sqrt{-bd}}\right)}{2bd}+\frac{\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\,e}{4b}$
risch	$\frac{e^{(bx+a)(dx+c)}x}{2bd}-\frac{e^{(bx+a)(dx+c)}a}{4b^2d}-\frac{e^{(bx+a)(dx+c)}c}{4b\,d^2}-\frac{\sqrt{\pi}\,e^{-\frac{(ad-bc)^2}{4bd}}\operatorname{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)a^2}{8b^2\sqrt{-bd}}-\frac{\sqrt{\pi}\,e^{-\frac{(ad-bc)^2}{4bd}}\operatorname{erf}\left(-\sqrt{}\right.}{4bd\sqrt{-}}$
parts	$-\frac{\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\operatorname{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)x^2}{2\sqrt{-bd}}-\frac{e^{ac-\frac{(ad+bc)^2}{4bd}}\left(4\operatorname{erf}\left(\frac{2bdx+ad+bc}{2\sqrt{-bd}}\right)x^2\sqrt{-bd}\,\sqrt{\pi}\,b^2d^2-\sqrt{\pi}\,\sqrt{-bd}\operatorname{erf}\left(\frac{2bdx+ad-}{2\sqrt{-bd}}\right.\right.}{}$

input
```
int(exp((b*x+a)*(d*x+c))*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)*x/b/d-1/2*(a*d+b*c)/b/d*(1/2*exp(a*c+(a*d
+b*c)*x+b*d*x^2)/b/d+1/4*(a*d+b*c)/b/d*Pi^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/
d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*d)^(1/2)))+1/4/b/d*P
i^(1/2)*exp(a*c-1/4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*
(a*d+b*c)/(-b*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.69

$$\int e^{(a+bx)(c+dx)}x^2\,dx =$$
$$-\frac{\sqrt{\pi}(b^2c^2+a^2d^2+2\,(abc-b)d)\sqrt{-bd}\operatorname{erf}\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}}{2\,bd}\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}-2\,(2\,b^2d^2x-b^2cd-a}{8\,b^3d^3}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x^2,x, algorithm="fricas")
```

output
```
-1/8*(sqrt(pi)*(b^2*c^2 + a^2*d^2 + 2*(a*b*c - b)*d)*sqrt(-b*d)*erf(1/2*(2
*b*d*x + b*c + a*d)*sqrt(-b*d)/(b*d))*e^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/(b*d)) - 2*(2*b^2*d^2*x - b^2*c*d - a*b*d^2)*e^(b*d*x^2 + a*c + (b*c +
 a*d)*x))/(b^3*d^3)
```

Sympy [F]

$$\int e^{(a+bx)(c+dx)}x^2\,dx = e^{ac}\int x^2e^{adx}e^{bcx}e^{bdx^2}\,dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x**2,x)
```

output
```
exp(a*c)*Integral(x**2*exp(a*d*x)*exp(b*c*x)*exp(b*d*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.02

$$\int e^{(a+bx)(c+dx)}x^2\,dx = \frac{\left(\frac{\sqrt{\pi}(2\,bdx+bc+ad)(bc+ad)^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,bdx+bc+ad)^2}{bd}}\right)-1\right)}{(bd)^{\frac{5}{2}}\sqrt{-\frac{(2\,bdx+bc+ad)^2}{bd}}} - \frac{4\,(bc+ad)bde^{\left(\frac{(2\,bdx+bc+ad)^2}{4\,bd}\right)}}{(bd)^{\frac{5}{2}}} - \frac{4\,(2\,bdx+bc+ad)^3\Gamma\left(\frac{3}{2},-\frac{(2\,bdx+bc+a}{4\,bd}\right.}{(bd)^{\frac{5}{2}}\left(-\frac{(2\,bdx+bc+ad)^2}{bd}\right)^{\frac{3}{2}}}}{8\,\sqrt{bd}}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x^2,x, algorithm="maxima")
```

output
```
1/8*(sqrt(pi)*(2*b*d*x + b*c + a*d)*(b*c + a*d)^2*(erf(1/2*sqrt(-(2*b*d*x
+ b*c + a*d)^2/(b*d))) - 1)/((b*d)^(5/2)*sqrt(-(2*b*d*x + b*c + a*d)^2/(b*
d))) - 4*(b*c + a*d)*b*d*e^(1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/(b*d)^(5/2)
 - 4*(2*b*d*x + b*c + a*d)^3*gamma(3/2, -1/4*(2*b*d*x + b*c + a*d)^2/(b*d)
)/((b*d)^(5/2)*(-(2*b*d*x + b*c + a*d)^2/(b*d))^(3/2)))*e^(a*c - 1/4*(b*c
+ a*d)^2/(b*d))/sqrt(b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.70

$$\int e^{(a+bx)(c+dx)}x^2\,dx =$$

$$-\frac{\frac{\sqrt{\pi}(b^2c^2+2\,abcd+a^2d^2-2\,bd)\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-bd}\left(2\,x+\frac{bc+ad}{bd}\right)\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}}{\sqrt{-bd}} - 2\left(bd\left(2\,x+\frac{bc+ad}{bd}\right)-2\,bc-2\,ad\right)e^{(bdx}}{8\,b^2d^2}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x^2,x, algorithm="giac")
```

output
```
-1/8*(sqrt(pi)*(b^2*c^2 + 2*a*b*c*d + a^2*d^2 - 2*b*d)*erf(-1/2*sqrt(-b*d)
*(2*x + (b*c + a*d)/(b*d)))*e^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d))
/sqrt(-b*d) - 2*(b*d*(2*x + (b*c + a*d)/(b*d)) - 2*b*c - 2*a*d)*e^(b*d*x^2
 + b*c*x + a*d*x + a*c))/(b^2*d^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.69

$$\int e^{(a+bx)(c+dx)}x^2\,dx = \frac{x\,\mathrm{e}^{a\,c+a\,d\,x+b\,c\,x+b\,d\,x^2}}{2\,b\,d} - \frac{\mathrm{e}^{a\,c+a\,d\,x+b\,c\,x+b\,d\,x^2}\,\left(\frac{a\,d}{4}+\frac{b\,c}{4}\right)}{b^2\,d^2} + \frac{\sqrt{\pi}\,\mathrm{e}^{\frac{a\,c}{2}-\frac{a^2\,d}{4\,b}-\frac{b\,c^2}{4\,d}}\,\mathrm{erfi}\left(\frac{\frac{a\,d}{2}+\frac{b\,c}{2}+b\,d\,x}{\sqrt{b\,d}}\right)\,\left(a^2\,d^2+2\,a\,b\,c\,d+b^2\,c^2-2\,b\,d\right)}{8\,b^2\,d^2\,\sqrt{b\,d}}$$

input
```
int(x^2*exp((a + b*x)*(c + d*x)),x)
```

output
```
(x*exp(a*c + a*d*x + b*c*x + b*d*x^2))/(2*b*d) - (exp(a*c + a*d*x + b*c*x
+ b*d*x^2)*((a*d)/4 + (b*c)/4))/(b^2*d^2) + (pi^(1/2)*exp((a*c)/2 - (a^2*d
)/(4*b) - (b*c^2)/(4*d))*erfi(((a*d)/2 + (b*c)/2 + b*d*x)/(b*d)^(1/2))*(a^
2*d^2 - 2*b*d + b^2*c^2 + 2*a*b*c*d))/(8*b^2*d^2*(b*d)^(1/2))
```

Reduce [F]

$$\int e^{(a+bx)(c+dx)}x^2\,dx$$
$$= \frac{e^{ac}\left(-e^{bd\,x^2+adx+bcx}ad-e^{bd\,x^2+adx+bcx}bc+2e^{bd\,x^2+adx+bcx}bdx+\left(\int e^{bd\,x^2+adx+bcx}dx\right)a^2d^2+2\left(\int e^{bd\,x^2+adx\right.\right.}{4b^2d^2}$$

input
```
int(exp((b*x+a)*(d*x+c))*x^2,x)
```

output
```
(e**(a*c)*( - e**(a*d*x + b*c*x + b*d*x**2)*a*d - e**(a*d*x + b*c*x + b*d*
x**2)*b*c + 2*e**(a*d*x + b*c*x + b*d*x**2)*b*d*x + int(e**(a*d*x + b*c*x
+ b*d*x**2),x)*a**2*d**2 + 2*int(e**(a*d*x + b*c*x + b*d*x**2),x)*a*b*c*d
+ int(e**(a*d*x + b*c*x + b*d*x**2),x)*b**2*c**2 - 2*int(e**(a*d*x + b*c*x
 + b*d*x**2),x)*b*d))/(4*b**2*d**2)
```

3.374 $\int e^{(a+bx)(c+dx)}x\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int e^{(a+bx)(c+dx)}x\,dx=\frac{e^{ac+(bc+ad)x+bdx^2}}{2bd}-\frac{(bc+ad)e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\operatorname{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}$$

output
```
1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b/d-1/4*(a*d+b*c)*Pi^(1/2)*erfi(1/2*(2*b*
d*x+a*d+b*c)/b^(1/2)/d^(1/2))/b^(3/2)/d^(3/2)/exp(1/4*(-a*d+b*c)^2/b/d)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int e^{(a+bx)(c+dx)}x\,dx=\frac{e^{-\frac{(bc-ad)^2}{4bd}}\left(2\sqrt{b}\sqrt{d}e^{\frac{(ad+b(c+2dx))^2}{4bd}}-(bc+ad)\sqrt{\pi}\operatorname{erfi}\left(\frac{ad+b(c+2dx)}{2\sqrt{b}\sqrt{d}}\right)\right)}{4b^{3/2}d^{3/2}}$$

input
```
Integrate[E^((a + b*x)*(c + d*x))*x,x]
```

output
```
(2*Sqrt[b]*Sqrt[d]*E^((a*d + b*(c + 2*d*x))^2/(4*b*d)) - (b*c + a*d)*Sqrt[
Pi]*Erfi[(a*d + b*(c + 2*d*x))/(2*Sqrt[b]*Sqrt[d])])/(4*b^(3/2)*d^(3/2)*E^
((b*c - a*d)^2/(4*b*d)))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2674, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{2674}$$

$$\int xe^{x(ad+bc)+ac+bdx^2}dx$$

$$\Big\downarrow \text{2670}$$

$$\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)\int e^{bdx^2+(bc+ad)x+ac}dx}{2bd}$$

$$\Big\downarrow \text{2664}$$

$$\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx}{2bd}$$

$$\Big\downarrow \text{2633}$$

$$\frac{e^{x(ad+bc)+ac+bdx^2}}{2bd}-\frac{\sqrt{\pi}(ad+bc)e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{4b^{3/2}d^{3/2}}$$

input
```
Int[E^((a + b*x)*(c + d*x))*x,x]
```

output
```
E^(a*c + (b*c + a*d)*x + b*d*x^2)/(2*b*d) - ((b*c + a*d)*Sqrt[Pi]*Erfi[(b*
c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d])])/(4*b^(3/2)*d^(3/2)*E^((b*c - a*d)
^2/(4*b*d)))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2674
```
Int[(F_)^(v_)*(u_)^(m_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*F^ExpandToSu
m[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(
LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

method	result
default	$\frac{e^{ac+(ad+bc)x+bd\,x^2}}{2bd}+\frac{(ad+bc)\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\,\mathrm{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)}{4bd\sqrt{-bd}}$
risch	$\frac{e^{(bx+a)(dx+c)}}{2bd}+\frac{\sqrt{\pi}\,e^{-\frac{(ad-bc)^2}{4bd}}\,\mathrm{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)a}{4b\sqrt{-bd}}+\frac{\sqrt{\pi}\,e^{-\frac{(ad-bc)^2}{4bd}}\,\mathrm{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)c}{4d\sqrt{-bd}}$
parts	$-\frac{\sqrt{\pi}\,e^{ac-\frac{(ad+bc)^2}{4bd}}\,\mathrm{erf}\left(-\sqrt{-bd}\,x+\frac{ad+bc}{2\sqrt{-bd}}\right)x}{2\sqrt{-bd}}+\frac{e^{ac-\frac{(ad+bc)^2}{4bd}}\left(2\,\mathrm{erf}\left(\frac{2bdx+ad+bc}{2\sqrt{-bd}}\right)x\sqrt{\pi}\,bd+\sqrt{\pi}\,\mathrm{erf}\left(\frac{2bdx+ad+bc}{2\sqrt{-bd}}\right)ad+\sqrt{\pi}\,\mathrm{erf}\left(\right.\right.}{4\sqrt{-bd}\,bd}$

input
```
int(exp((b*x+a)*(d*x+c))*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(a*c+(a*d+b*c)*x+b*d*x^2)/b/d+1/4*(a*d+b*c)/b/d*Pi^(1/2)*exp(a*c-1/
4*(a*d+b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1/2*(a*d+b*c)/(-b*d)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int e^{(a+bx)(c+dx)}x\,dx = \frac{\sqrt{\pi}(bc+ad)\sqrt{-bd}\,\mathrm{erf}\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}}{2\,bd}\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}+2\,bde^{(bdx^2+ac+(bc+ad)x)}}{4\,b^2d^2}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x,x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(b*c + a*d)*sqrt(-b*d)*erf(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b
*d)/(b*d))*e^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d)) + 2*b*d*e^(b*d*x
^2 + a*c + (b*c + a*d)*x))/(b^2*d^2)
```

Sympy [F]

$$\int e^{(a+bx)(c+dx)}x\,dx = e^{ac}\int xe^{adx}e^{bcx}e^{bdx^2}\,dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x,x)
```

output
```
exp(a*c)*Integral(x*exp(a*d*x)*exp(b*c*x)*exp(b*d*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.34

$$\int e^{(a+bx)(c+dx)}x\,dx$$
$$= -\frac{\left(\frac{\sqrt{\pi}(2\,bdx+bc+ad)(bc+ad)\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,bdx+bc+ad)^2}{bd}}\right)-1\right)}{(bd)^{\frac{3}{2}}\sqrt{-\frac{(2\,bdx+bc+ad)^2}{bd}}}-\frac{2\,bde^{\left(\frac{(2\,bdx+bc+ad)^2}{4\,bd}\right)}}{(bd)^{\frac{3}{2}}}\right)e^{\left(ac-\frac{(bc+ad)^2}{4\,bd}\right)}}{4\sqrt{bd}}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x,x, algorithm="maxima")
```

output
```
-1/4*(sqrt(pi)*(2*b*d*x + b*c + a*d)*(b*c + a*d)*(erf(1/2*sqrt(-(2*b*d*x +
 b*c + a*d)^2/(b*d))) - 1)/((b*d)^(3/2)*sqrt(-(2*b*d*x + b*c + a*d)^2/(b*d
))) - 2*b*d*e^(1/4*(2*b*d*x + b*c + a*d)^2/(b*d))/(b*d)^(3/2))*e^(a*c - 1/
4*(b*c + a*d)^2/(b*d))/sqrt(b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97

$$\int e^{(a+bx)(c+dx)}x\,dx$$
$$= \frac{\frac{\sqrt{\pi}(bc+ad)\operatorname{erf}\left(-\frac{1}{2}\sqrt{-bd}\left(2\,x+\frac{bc+ad}{bd}\right)\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}}{\sqrt{-bd}}+2\,e^{\left(bdx^2+bcx+adx+ac\right)}}{4\,bd}$$

input
```
integrate(exp((b*x+a)*(d*x+c))*x,x, algorithm="giac")
```

output
```
1/4*(sqrt(pi)*(b*c + a*d)*erf(-1/2*sqrt(-b*d)*(2*x + (b*c + a*d)/(b*d)))*e
^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d))/sqrt(-b*d) + 2*e^(b*d*x^2 +
b*c*x + a*d*x + a*c))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int e^{(a+bx)(c+dx)}x\,dx = \frac{e^{a\,c+a\,d\,x+b\,c\,x+b\,d\,x^2}}{2\,b\,d} - \frac{\sqrt{\pi}\,e^{\frac{a\,c}{2}-\frac{a^2\,d}{4\,b}-\frac{b\,c^2}{4\,d}}\,\mathrm{erfi}\left(\frac{\frac{a\,d}{2}+\frac{b\,c}{2}+b\,d\,x}{\sqrt{b\,d}}\right)\,(a\,d+b\,c)}{4\,b\,d\,\sqrt{b\,d}}$$

input
```
int(x*exp((a + b*x)*(c + d*x)),x)
```

output
```
exp(a*c + a*d*x + b*c*x + b*d*x^2)/(2*b*d) - (pi^(1/2)*exp((a*c)/2 - (a^2*
d)/(4*b) - (b*c^2)/(4*d))*erfi(((a*d)/2 + (b*c)/2 + b*d*x)/(b*d)^(1/2))*(a
*d + b*c))/(4*b*d*(b*d)^(1/2))
```

Reduce [F]

$$\int e^{(a+bx)(c+dx)}x\,dx = \frac{e^{ac}\left(e^{bd\,x^2+adx+bcx}-\left(\int e^{bd\,x^2+adx+bcx}dx\right)ad-\left(\int e^{bd\,x^2+adx+bcx}dx\right)bc\right)}{2bd}$$

input
```
int(exp((b*x+a)*(d*x+c))*x,x)
```

output
```
(e**(a*c)*(e**(a*d*x + b*c*x + b*d*x**2) - int(e**(a*d*x + b*c*x + b*d*x**
2),x)*a*d - int(e**(a*d*x + b*c*x + b*d*x**2),x)*b*c))/(2*b*d)
```

3.375 $\int e^{(a+bx)(c+dx)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int e^{(a+bx)(c+dx)}\,dx = \frac{e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
1/2*Pi^(1/2)*erfi(1/2*(2*b*d*x+a*d+b*c)/b^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)/e
xp(1/4*(-a*d+b*c)^2/b/d)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int e^{(a+bx)(c+dx)}\,dx = \frac{e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{ad+b(c+2dx)}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

input
```
Integrate[E^((a + b*x)*(c + d*x)),x]
```

output
```
(Sqrt[Pi]*Erfi[(a*d + b*(c + 2*d*x))/(2*Sqrt[b]*Sqrt[d])])/(2*Sqrt[b]*Sqrt
[d]*E^((b*c - a*d)^2/(4*b*d)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2665, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \ 2665$$

$$\int e^{x(ad+bc)+ac+bdx^2}dx$$

$$\Big\downarrow \ 2664$$

$$e^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx$$

$$\Big\downarrow \ 2633$$

$$\frac{\sqrt{\pi}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

input
```
Int[E^((a + b*x)*(c + d*x)),x]
```

output
```
(Sqrt[Pi]*Erfi[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d])])/(2*Sqrt[b]*Sqrt
[d]*E^((b*c - a*d)^2/(4*b*d)))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2665
```
Int[(F_)^(v_), x_Symbol] :> Int[F^ExpandToSum[v, x], x] /; FreeQ[F, x] && Q
uadraticQ[v, x] &&  !QuadraticMatchQ[v, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{\sqrt{\pi}\, \mathrm{e}^{-\frac{(ad-bc)^{2}}{4bd}} \operatorname{erf}\left(-\sqrt{-bd}\, x+\frac{ad+bc}{2\sqrt{-bd}}\right)}{2\sqrt{-bd}}$	57
default	$-\frac{\sqrt{\pi}\, \mathrm{e}^{ac-\frac{(ad+bc)^{2}}{4bd}} \operatorname{erf}\left(-\sqrt{-bd}\, x+\frac{ad+bc}{2\sqrt{-bd}}\right)}{2\sqrt{-bd}}$	60

input
```
int(exp((b*x+a)*(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Pi^(1/2)*exp(-1/4*(a*d-b*c)^2/b/d)/(-b*d)^(1/2)*erf(-(-b*d)^(1/2)*x+1
/2*(a*d+b*c)/(-b*d)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09

$$\int e^{(a+bx)(c+dx)}\,dx = -\frac{\sqrt{\pi}\sqrt{-bd}\operatorname{erf}\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}}{2\,bd}\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}}{2\,bd}$$

input
```
integrate(exp((b*x+a)*(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*d)*erf(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b*d))*e
^(-1/4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b*d))/(b*d)
```

Sympy [F]

$$\int e^{(a+bx)(c+dx)}\,dx = e^{ac}\int e^{adx}e^{bcx}e^{bdx^2}\,dx$$

input
```
integrate(exp((b*x+a)*(d*x+c)),x)
```

output
```
exp(a*c)*Integral(exp(a*d*x)*exp(b*c*x)*exp(b*d*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int e^{(a+bx)(c+dx)}\,dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-bd}x - \frac{bc+ad}{2\sqrt{-bd}}\right)e^{\left(ac-\frac{(bc+ad)^2}{4\,bd}\right)}}{2\sqrt{-bd}}$$

input
```
integrate(exp((b*x+a)*(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*erf(sqrt(-b*d)*x - 1/2*(b*c + a*d)/sqrt(-b*d))*e^(a*c - 1/4*(
b*c + a*d)^2/(b*d))/sqrt(-b*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int e^{(a+bx)(c+dx)}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-bd}\left(2\,x+\frac{bc+ad}{bd}\right)\right)e^{\left(-\frac{b^2c^2-2\,abcd+a^2d^2}{4\,bd}\right)}}{2\sqrt{-bd}}$$

input
```
integrate(exp((b*x+a)*(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-1/2*sqrt(-b*d)*(2*x + (b*c + a*d)/(b*d)))*e^(-1/4*(b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/(b*d))/sqrt(-b*d)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int e^{(a+bx)(c+dx)}\,dx = -\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{a\,c}{2}-\frac{a^2\,d}{4\,b}-\frac{b\,c^2}{4\,d}}\,\mathrm{erf}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{2\,\sqrt{b\,d}}\right)\,1\mathrm{i}}{2\,\sqrt{b\,d}}$$

input
```
int(exp((a + b*x)*(c + d*x)),x)
```

output
```
-(pi^(1/2)*exp((a*c)/2 - (a^2*d)/(4*b) - (b*c^2)/(4*d))*erf((a*d*1i + b*c*
1i + b*d*x*2i)/(2*(b*d)^(1/2)))*1i)/(2*(b*d)^(1/2))
```

Reduce [F]

$$\int e^{(a+bx)(c+dx)}\,dx = e^{ac}\left(\int e^{bd\,x^2+adx+bcx}dx\right)$$

input
```
int(exp((b*x+a)*(d*x+c)),x)
```

output
```
e**(a*c)*int(e**(a*d*x + b*c*x + b*d*x**2),x)
```

3.376 $\int \frac{e^{(a+bx)(c+dx)}}{x} dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{(a+bx)(c+dx)}}{x} dx = \text{Int}\left(\frac{e^{ac+(bc+ad)x+bdx^2}}{x}, x\right)$$

output
```
Defer(Int)(exp(a*c+(a*d+b*c)*x+b*d*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{(a+bx)(c+dx)}}{x} dx = \int \frac{e^{(a+bx)(c+dx)}}{x} dx$$

input
```
Integrate[E^((a + b*x)*(c + d*x))/x,x]
```

output
```
Integrate[E^((a + b*x)*(c + d*x))/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2674, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{(a+bx)(c+dx)}}{x} \, dx$$
$$\Big\downarrow \text{2674}$$
$$\int \frac{e^{x(ad+bc)+ac+bdx^2}}{x}dx$$
$$\Big\downarrow \text{2673}$$
$$\int \frac{e^{x(ad+bc)+ac+bdx^2}}{x}dx$$

input
```
Int[E^((a + b*x)*(c + d*x))/x,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

rule 2674
```
Int[(F_)^(v_)*(u_)^(m_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*F^ExpandToSu
m[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(
LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{(bx+a)(dx+c)}}{x}dx$$

input
```
int(exp((b*x+a)*(d*x+c))/x,x)
```

output
```
int(exp((b*x+a)*(d*x+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{e^{(a+bx)(c+dx)}}{x}\,dx = \int \frac{e^{((bx+a)(dx+c))}}{x}\,dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x,x, algorithm="fricas")
```

output
```
integral(e^(b*d*x^2 + a*c + (b*c + a*d)*x)/x, x)
```

Sympy [N/A]

Not integrable

Time = 7.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{e^{(a+bx)(c+dx)}}{x}\,dx = e^{ac} \int \frac{e^{adx}e^{bcx}e^{bdx^2}}{x}\,dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x,x)
```

output
```
exp(a*c)*Integral(exp(a*d*x)*exp(b*c*x)*exp(b*d*x**2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{(a+bx)(c+dx)}}{x} \, dx = \int \frac{e^{((bx+a)(dx+c))}}{x} \, dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x,x, algorithm="maxima")
```

output
```
integrate(e^((b*x + a)*(d*x + c))/x, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{(a+bx)(c+dx)}}{x} \, dx = \int \frac{e^{((bx+a)(dx+c))}}{x} \, dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x,x, algorithm="giac")
```

output
```
integrate(e^((b*x + a)*(d*x + c))/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{(a+bx)(c+dx)}}{x} \, dx = \int \frac{\mathrm{e}^{(a+b\,x)\,(c+d\,x)}}{x} \, dx$$

input
```
int(exp((a + b*x)*(c + d*x))/x,x)
```

output
```
int(exp((a + b*x)*(c + d*x))/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{e^{(a+bx)(c+dx)}}{x} \, dx = e^{ac}\left(\int \frac{e^{bd\,x^2+adx+bcx}}{x}dx\right)$$

input
```
int(exp((b*x+a)*(d*x+c))/x,x)
```

output
```
e**(a*c)*int(e**(a*d*x + b*c*x + b*d*x**2)/x,x)
```

3.377 $\int \frac{e^{(a+bx)(c+dx)}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2}\,dx = -\frac{e^{ac+(bc+ad)x+bdx^2}}{x} + \sqrt{b}\sqrt{d}e^{-\frac{(bc-ad)^2}{4bd}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}}\right) + (bc+ad)\mathrm{Int}\left(\frac{e^{ac+(bc+ad)x+bdx^2}}{x},x\right)$$

output
```
-exp(a*c+(a*d+b*c)*x+b*d*x^2)/x+b^(1/2)*d^(1/2)*Pi^(1/2)*erfi(1/2*(2*b*d*x
+a*d+b*c)/b^(1/2)/d^(1/2))/exp(1/4*(-a*d+b*c)^2/b/d)+(a*d+b*c)*Defer(Int)(
exp(a*c+(a*d+b*c)*x+b*d*x^2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2}\,dx = \int \frac{e^{(a+bx)(c+dx)}}{x^2}\,dx$$

input
```
Integrate[E^((a + b*x)*(c + d*x))/x^2,x]
```

output
```
Integrate[E^((a + b*x)*(c + d*x))/x^2, x]
```

Rubi [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2674, 2672, 2664, 2633, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} \, dx$$

$$\Big\downarrow \text{2674}$$

$$\int \frac{e^{x(ad+bc)+ac+bdx^2}}{x^2}dx$$

$$\Big\downarrow \text{2672}$$

$$2bd\int e^{bdx^2+(bc+ad)x+ac}dx+(ad+bc)\int \frac{e^{bdx^2+(bc+ad)x+ac}}{x}dx-\frac{e^{x(ad+bc)+ac+bdx^2}}{x}$$

$$\Big\downarrow \text{2664}$$

$$(ad+bc)\int \frac{e^{bdx^2+(bc+ad)x+ac}}{x}dx+2bde^{-\frac{(bc-ad)^2}{4bd}}\int e^{\frac{(bc+ad+2bdx)^2}{4bd}}dx-\frac{e^{x(ad+bc)+ac+bdx^2}}{x}$$

$$\Big\downarrow \text{2633}$$

$$(ad+bc)\int \frac{e^{bdx^2+(bc+ad)x+ac}}{x}dx+\sqrt{\pi}\sqrt{b}\sqrt{d}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)-\frac{e^{x(ad+bc)+ac+bdx^2}}{x}$$

$$\Big\downarrow \text{2673}$$

$$(ad+bc)\int \frac{e^{bdx^2+(bc+ad)x+ac}}{x}dx+\sqrt{\pi}\sqrt{b}\sqrt{d}e^{-\frac{(bc-ad)^2}{4bd}}\text{erfi}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}}\right)-\frac{e^{x(ad+bc)+ac+bdx^2}}{x}$$

input
```
Int[E^((a + b*x)*(c + d*x))/x^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2672
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] + (-
Simp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2)
, x], x] - Simp[(b*e - 2*c*d)*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 1
)*F^(a + b*x + c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e -
 2*c*d, 0] && LtQ[m, -1]
```

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

rule 2674
```
Int[(F_)^(v_)*(u_)^(m_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*F^ExpandToSu
m[v, x], x] /; FreeQ[{F, m}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !(
LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{e}^{(bx+a)(dx+c)}}{x^2} dx$$

input
```
int(exp((b*x+a)*(d*x+c))/x^2,x)
```

output
```
int(exp((b*x+a)*(d*x+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} dx = \int \frac{e^{((bx+a)(dx+c))}}{x^2} dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x^2,x, algorithm="fricas")
```

output
```
integral(e^(b*d*x^2 + a*c + (b*c + a*d)*x)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 9.36 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.82

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} dx = e^{ac} \int \frac{e^{adx} e^{bcx} e^{bdx^2}}{x^2} dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x**2,x)
```

output
```
exp(a*c)*Integral(exp(a*d*x)*exp(b*c*x)*exp(b*d*x**2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} \, dx = \int \frac{e^{((bx+a)(dx+c))}}{x^2} \, dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x^2,x, algorithm="maxima")
```

output
```
integrate(e^((b*x + a)*(d*x + c))/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2} \, dx = \int \frac{e^{((bx+a)(dx+c))}}{x^2} \, dx$$

input
```
integrate(exp((b*x+a)*(d*x+c))/x^2,x, algorithm="giac")
```

output
```
integrate(e^((b*x + a)*(d*x + c))/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2}\,dx = \int \frac{\mathrm{e}^{(a+b\,x)\,(c+d\,x)}}{x^2}\,dx$$

input
```
int(exp((a + b*x)*(c + d*x))/x^2,x)
```

output
```
int(exp((a + b*x)*(c + d*x))/x^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.71

$$\int \frac{e^{(a+bx)(c+dx)}}{x^2}\,dx = e^{ac}\left(\int \frac{e^{bd\,x^2+adx+bcx}}{x^2}dx\right)$$

input
```
int(exp((b*x+a)*(d*x+c))/x^2,x)
```

output
```
e**(a*c)*int(e**(a*d*x + b*c*x + b*d*x**2)/x**2,x)
```

3.378 $\int f^{a+bx+cx^2}(d+ex)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 266

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx = -\frac{e^3f^{a+bx+cx^2}}{2c^2\log^2(f)} - \frac{3e^2(2cd-be)f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2}\log^{\frac{3}{2}}(f)} + \frac{e(2cd-be)^2f^{a+bx+cx^2}}{8c^3\log(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}(d+ex)}{4c^2\log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)^2}{2c\log(f)} + \frac{(2cd-be)^3f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{16c^{7/2}\sqrt{\log(f)}}$$

output
```
-1/2*e^3*f^(c*x^2+b*x+a)/c^2/ln(f)^2-3/8*e^2*(-b*e+2*c*d)*f^(a-1/4*b^2/c)*
Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(5/2)/ln(f)^(3/2)+1/8*e
*(-b*e+2*c*d)^2*f^(c*x^2+b*x+a)/c^3/ln(f)+1/4*e*(-b*e+2*c*d)*f^(c*x^2+b*x+
a)*(e*x+d)/c^2/ln(f)+1/2*e*f^(c*x^2+b*x+a)*(e*x+d)^2/c/ln(f)+1/16*(-b*e+2*
c*d)^3*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^
(7/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.64

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx$$
$$=\frac{f^{a-\frac{b^2}{4c}}\left((2cd-be)\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}(-6ce^2+(-2cd+be)^2\log(f))+2\sqrt{c}ef^{\frac{(b+2cx)^2}{4c}}(-4ce^2\right.}{16c^{7/2}\log^2(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*(d + e*x)^3,x]
```

output
```
(f^(a - b^2/(4*c))*((2*c*d - b*e)*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])
/(2*Sqrt[c])]*Sqrt[Log[f]]*(-6*c*e^2 + (-2*c*d + b*e)^2*Log[f]) + 2*Sqrt[c
]*e*f^((b + 2*c*x)^2/(4*c))*(-4*c*e^2 + (b^2*e^2 - 2*b*c*e*(3*d + e*x) + 4
*c^2*(3*d^2 + 3*d*e*x + e^2*x^2))*Log[f])))/(16*c^(7/2)*Log[f]^2)
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.30, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2671, 2670, 2664, 2633, 2671, 2664, 2633, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^3f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{2671}$$
$$-\frac{e^2\int f^{cx^2+bx+a}(d+ex)dx}{c\log(f)}+\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)^2dx}{2c}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$
$$\Big\downarrow \text{2670}$$

$$-\frac{e^2\left(\frac{(2cd-be)\int f^{cx^2+bx+a}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)^2dx}{2c}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2664

$$-\frac{e^2\left(\frac{f^{a-\frac{b^2}{4c}}(2cd-be)\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)^2dx}{2c}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2633

$$\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)^2dx}{2c}-\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2671

$$\frac{(2cd-be)\left(\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)dx}{2c}-\frac{e^2\int f^{cx^2+bx+a}dx}{2c\log(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2664

$$\frac{(2cd-be)\left(-\frac{e^2f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c\log(f)}+\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)dx}{2c}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2633

$$\frac{(2cd-be)\left(\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)dx}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2670

$$\frac{(2cd-be)\left(\frac{(2cd-be)\left(\frac{(2cd-be)\int f^{cx^2+bx+a}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2664

$$\frac{(2cd-be)\left(\frac{(2cd-be)\left(\frac{f^{a-\frac{b^2}{4c}}(2cd-be)\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

$\downarrow$ 2633

$$\frac{(2cd-be)\Bigg(\frac{(2cd-be)\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx-}}{2c\log(f)}}{2c}$$

$$\frac{e^2\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{c\log(f)}+\frac{e(d+ex)^2f^{a+bx+cx^2}}{2c\log(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*(d + e*x)^3,x]
```

output
```
((2*c*d - b*e)*(((2*c*d - b*e)*((e*f^(a + b*x + c*x^2))/(2*c*Log[f]) + ((2
*c*d - b*e)*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*
Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f]])))/(2*c) - (e^2*f^(a - b^2/(4*c))*Sqrt[
Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Log[f]^(3/2))
 + (e*f^(a + b*x + c*x^2)*(d + e*x))/(2*c*Log[f])))/(2*c) + (e*f^(a + b*x
+ c*x^2)*(d + e*x)^2)/(2*c*Log[f]) - (e^2*((e*f^(a + b*x + c*x^2))/(2*c*Lo
g[f]) + ((2*c*d - b*e)*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[L
og[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f]])))/(c*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(226) = 452.

Time = 0.16 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{f^{a} d^{3} \sqrt{\pi}\, f^{-\frac{b^{2}}{4c}} \operatorname{erf}\left(-\sqrt{-c\ln(f)}\, x+\frac{b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{2\sqrt{-c\ln(f)}}+\frac{e^{3} f^{a} x^{2} f^{bx} f^{c\,x^{2}}}{2\ln(f) c}-\frac{e^{3} f^{a} b x\, f^{bx} f^{c\,x^{2}}}{4c^{2}\ln(f)}+\frac{e^{3} f^{a} b^{2} f^{bx} f^{c\,x^{2}}}{8c^{3}\ln(f)}+\frac{e^{3} f^{a} b^{3}\sqrt{\pi}\, f}{}$

input
```
int(f^(c*x^2+b*x+a)*(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*d^3*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2
)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+1/2*e^3*f^a/ln(f)/c*x^2*f^(b*x)*f^(c*x^2
)-1/4*e^3*f^a*b/c^2/ln(f)*x*f^(b*x)*f^(c*x^2)+1/8*e^3*f^a*b^2/c^3/ln(f)*f^
(b*x)*f^(c*x^2)+1/16*e^3*f^a*b^3/c^3*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1
/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))-3/8*e^3*f^a*b/c^
2/ln(f)*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1
/2*b*ln(f)/(-c*ln(f))^(1/2))-1/2*e^3*f^a/c^2/ln(f)^2*f^(b*x)*f^(c*x^2)+3/2
*e^2*d*f^a/ln(f)/c*x*f^(b*x)*f^(c*x^2)-3/4*e^2*d*f^a*b/c^2/ln(f)*f^(b*x)*f
^(c*x^2)-3/8*e^2*d*f^a*b^2/c^2*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*er
f(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+3/4*e^2*d*f^a/ln(f)/c*
Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(
f)/(-c*ln(f))^(1/2))+3/2*f^a*d^2*e/ln(f)/c*f^(b*x)*f^(c*x^2)+3/4*f^a*d^2*e
*b/c*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*
b*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.77

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx =$$

$$-\frac{2\left(4c^2e^3-\left(4c^3e^3x^2+12c^3d^2e-6bc^2de^2+b^2ce^3+2\left(6c^3de^2-bc^2e^3\right)x\right)\log(f)\right)f^{cx^2+bx+a}-\frac{\sqrt{\pi}\left(12c^2d\right.}{}}{16c^4\log(f)^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(4*c^2*e^3 - (4*c^3*e^3*x^2 + 12*c^3*d^2*e - 6*b*c^2*d*e^2 + b^2*
c*e^3 + 2*(6*c^3*d*e^2 - b*c^2*e^3)*x)*log(f))*f^(c*x^2 + b*x + a) - sqrt(
pi)*(12*c^2*d*e^2 - 6*b*c*e^3 - (8*c^3*d^3 - 12*b*c^2*d^2*e + 6*b^2*c*d*e^
2 - b^3*e^3)*log(f))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c
)/f^(1/4*(b^2 - 4*a*c)/c))/(c^4*log(f)^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx=\int f^{a+bx+cx^2}(d+ex)^3\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*(d + e*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(226) = 452.

Time = 0.24 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.03

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d)^3,x, algorithm="maxima")
```

output
```
-3/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*
log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*
c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*d^2*e*f^(a - 1/4*b^2/c)/sqrt(c*log(
f)) + 3/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)
) - 1)*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*
x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-(2*c*x + b)^2
*log(f)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^
2/(c*log(f))^(5/2))*d*e^2*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) - 1/16*(sqrt(pi
)*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(f)^4/(s
qrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(7/2)) - 12*(2*c*x + b)^3*b*gamma(
3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^4/((-(2*c*x + b)^2*log(f)/c)^(3/2
)*(c*log(f))^(7/2)) - 6*b^2*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^3/(c*log(f))^
(7/2) + 8*c^2*gamma(2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^2/(c*log(f))^(7
/2))*e^3*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) + 1/2*sqrt(pi)*d^3*f^a*erf(sqrt(
-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c
))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx =$$

$$-\frac{\frac{\sqrt{\pi}\left(8\,c^3d^3\log(f)-12\,bc^2d^2e\log(f)+6\,b^2cde^2\log(f)-b^3e^3\log(f)-12\,c^2de^2+6\,bce^3\right)\operatorname{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}\log(f)}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*(sqrt(pi)*(8*c^3*d^3*log(f) - 12*b*c^2*d^2*e*log(f) + 6*b^2*c*d*e^2*
log(f) - b^3*e^3*log(f) - 12*c^2*d*e^2 + 6*b*c*e^3)*erf(-1/2*sqrt(-c*log(f
))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/(sqrt(-c*log(f))*lo
g(f)) - 2*(c^2*e^3*(2*x + b/c)^2*log(f) + 6*c^2*d*e^2*(2*x + b/c)*log(f) -
 3*b*c*e^3*(2*x + b/c)*log(f) + 12*c^2*d^2*e*log(f) - 12*b*c*d*e^2*log(f)
+ 3*b^2*e^3*log(f) - 4*c*e^3)*e^(c*x^2*log(f) + b*x*log(f) + a*log(f))/log
(f)^2)/c^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.94

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx = \frac{e^3\,f^a\,f^{c\,x^2}\,f^{b\,x}\,x^2}{2\,c\,\ln(f)} - \frac{f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\left(\frac{\ln(f)\,b^3\,e^3}{16}-\frac{3\,\ln(f)\,b^2\,c\,d\,e^2}{8}+\frac{3\,\ln(f)\,b\,c^2\,d^2\,e}{4}-\frac{3\,b\,c\,e^3}{8}-\frac{\ln(f)\,c^3\,d^3}{2}+\frac{3\,c^2\,d\,e^2}{4}\right)}{c^3\,\ln(f)\,\sqrt{c\,\ln(f)}} - \frac{f^a\,f^{c\,x^2}\,f^{b\,x}\,x\,(b\,e^3-6\,c\,d\,e^2)}{4\,c^2\,\ln(f)} - f^a\,f^{c\,x^2}\,f^{b\,x}\left(\frac{e^3}{2\,c^2\,\ln(f)^2}-\frac{3\,d^2\,e}{2\,c\,\ln(f)}-\frac{b^2\,e^3}{8\,c^3\,\ln(f)}+\frac{3\,b\,d\,e^2}{4\,c^2\,\ln(f)}\right)$$

input
```
int(f^(a + b*x + c*x^2)*(d + e*x)^3,x)
```

output
```
(e^3*f^a*f^(c*x^2)*f^(b*x)*x^2)/(2*c*log(f)) - (f^(a - b^2/(4*c))*pi^(1/2)
*erfi(((b*log(f))/2 + c*x*log(f))/(c*log(f))^(1/2))*((3*c^2*d*e^2)/4 + (b^
3*e^3*log(f))/16 - (c^3*d^3*log(f))/2 - (3*b*c*e^3)/8 + (3*b*c^2*d^2*e*log
(f))/4 - (3*b^2*c*d*e^2*log(f))/8))/(c^3*log(f)*(c*log(f))^(1/2)) - (f^a*f
^(c*x^2)*f^(b*x)*x*(b*e^3 - 6*c*d*e^2))/(4*c^2*log(f)) - f^a*f^(c*x^2)*f^(
b*x)*(e^3/(2*c^2*log(f)^2) - (3*d^2*e)/(2*c*log(f)) - (b^2*e^3)/(8*c^3*log
(f)) + (3*b*d*e^2)/(4*c^2*log(f)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 543, normalized size of antiderivative = 2.04

$$\int f^{a+bx+cx^2}(d+ex)^3\,dx = \text{Too large to display}$$

input
```
int(f^(c*x^2+b*x+a)*(e*x+d)^3,x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f)))
)*log(f)**2*b**3*e**3*i - 6*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*
sqrt(c)*sqrt(log(f))))*log(f)**2*b**2*c*d*e**2*i + 12*sqrt(pi)*erf((log(f)
*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f))))*log(f)**2*b*c**2*d**2*e*i
 - 8*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f))))*
log(f)**2*c**3*d**3*i - 6*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sq
rt(c)*sqrt(log(f))))*log(f)*b*c*e**3*i + 12*sqrt(pi)*erf((log(f)*b*i + 2*l
og(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f))))*log(f)*c**2*d*e**2*i + 2*f**(b*x +
c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*b**2*e**3 - 1
2*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*b
*c*d*e**2 - 4*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(
f))*log(f)*b*c*e**3*x + 24*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt
(c)*sqrt(log(f))*log(f)*c**2*d**2*e + 24*f**(b*x + c*x**2)*e**((log(f)*b**
2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*c**2*d*e**2*x + 8*f**(b*x + c*x**2)*
e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*c**2*e**3*x**2 - 8*f*
*(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*c*e**3))/(16
*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)**2*c**3)
```

3.379 $\int f^{a+bx+cx^2}(d+ex)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 189

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx = -\frac{e^2 f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)} + \frac{e(2cd-be)f^{a+bx+cx^2}}{4c^2\log(f)} + \frac{ef^{a+bx+cx^2}(d+ex)}{2c\log(f)} + \frac{(2cd-be)^2 f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{8c^{5/2}\sqrt{\log(f)}}$$

output
```
-1/4*e^2*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/
c^(3/2)/ln(f)^(3/2)+1/4*e*(-b*e+2*c*d)*f^(c*x^2+b*x+a)/c^2/ln(f)+1/2*e*f^(
c*x^2+b*x+a)*(e*x+d)/c/ln(f)+1/8*(-b*e+2*c*d)^2*f^(a-1/4*b^2/c)*Pi^(1/2)*e
rfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(5/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx$$
$$=\frac{f^{a-\frac{b^2}{4c}}\left(2\sqrt{c}ef^{\frac{(b+2cx)^2}{4c}}(4cd-be+2cex)\sqrt{\log(f)}+\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)(-2ce^2+(-2cd+be)^2\log(f))\right)}{8c^{5/2}\log^{\frac{3}{2}}(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*(d + e*x)^2,x]
```

output
```
(f^(a - b^2/(4*c))*(2*Sqrt[c]*e*f^((b + 2*c*x)^2/(4*c))*(4*c*d - b*e + 2*c
*e*x)*Sqrt[Log[f]] + Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]
*(-2*c*e^2 + (-2*c*d + b*e)^2*Log[f])))/(8*c^(5/2)*Log[f]^(3/2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2671, 2664, 2633, 2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{2671}$$
$$\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)dx}{2c}-\frac{e^2\int f^{cx^2+bx+a}dx}{2c\log(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}$$
$$\Big\downarrow \text{2664}$$
$$-\frac{e^2f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c\log(f)}+\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)dx}{2c}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}$$
$$\Big\downarrow \text{2633}$$

$$\frac{(2cd-be)\int f^{cx^2+bx+a}(d+ex)dx}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2670}$$

$$\frac{(2cd-be)\left(\frac{(2cd-be)\int f^{cx^2+bx+a}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2664}$$

$$\frac{(2cd-be)\left(\frac{f^{a-\frac{b^2}{4c}}(2cd-be)\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(2cd-be)\left(\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}\right)}{2c}-\frac{\sqrt{\pi}e^2 f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\log^{\frac{3}{2}}(f)}+\frac{e(d+ex)f^{a+bx+cx^2}}{2c\log(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*(d + e*x)^2,x]
```

output
```
((2*c*d - b*e)*((e*f^(a + b*x + c*x^2))/(2*c*Log[f]) + ((2*c*d - b*e)*f^(a
 - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^
(3/2)*Sqrt[Log[f]])))/(2*c) - (e^2*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2
*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Log[f]^(3/2)) + (e*f^(a + b*x
 + c*x^2)*(d + e*x))/(2*c*Log[f])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

rule 2671
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] +
 (-Simp[(b*e - 2*c*d)/(2*c)   Int[(d + e*x)^(m - 1)*F^(a + b*x + c*x^2), x]
, x] - Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x +
 c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e - 2*c*d, 0] &&
GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{f^{a} d^{2} \sqrt{\pi}\, f^{-\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{-c \ln\left(f\right)}\, x+\frac{b \ln\left(f\right)}{2 \sqrt{-c \ln\left(f\right)}}\right)}{2 \sqrt{-c \ln\left(f\right)}}+\frac{e^{2} f^{a} x\, f^{b x} f^{c \,x^{2}}}{2 \ln\left(f\right) c}-\frac{e^{2} f^{a} b\, f^{b x} f^{c \,x^{2}}}{4 c^{2} \ln\left(f\right)}-\frac{e^{2} f^{a} b^{2} \sqrt{\pi}\, f^{-\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{-c \ln\left(f\right)}\right.}{8 c^{2} \sqrt{-c \ln\left(f\right)}}$

input
```
int(f^(c*x^2+b*x+a)*(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*d^2*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2
)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+1/2*e^2*f^a/ln(f)/c*x*f^(b*x)*f^(c*x^2)-
1/4*e^2*f^a*b/c^2/ln(f)*f^(b*x)*f^(c*x^2)-1/8*e^2*f^a*b^2/c^2*Pi^(1/2)*f^(
-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f)
)^(1/2))+1/4*e^2*f^a/ln(f)/c*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(
-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+f^a*d*e/ln(f)/c*f^(b*x)*
f^(c*x^2)+1/2*f^a*d*e*b/c*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-
c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.69

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx$$
$$=\frac{2\left(2\,c^2e^2x+4\,c^2de-bce^2\right)f^{cx^2+bx+a}\log\left(f\right)+\frac{\sqrt{\pi}\left(2\,ce^2-\left(4\,c^2d^2-4\,bcde+b^2e^2\right)\log(f)\right)\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{8\,c^3\log\left(f\right)^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/8*(2*(2*c^2*e^2*x + 4*c^2*d*e - b*c*e^2)*f^(c*x^2 + b*x + a)*log(f) + sq
rt(pi)*(2*c*e^2 - (4*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*log(f))*sqrt(-c*log(f)
)*erf(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^3*log
(f)^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx=\int f^{a+bx+cx^2}\left(d+ex\right)^2\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*(d + e*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(153) = 306$.

Time = 0.17 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.76

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx$$
$$= -\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^2}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{3}{2}}}-\frac{2\,cf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)}{(c\log(f))^{\frac{3}{2}}}\right)def^{a-\frac{b^2}{4\,c}}}{2\sqrt{c\log(f)}}$$
$$+\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^3}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{5}{2}}}-\frac{4\,(2\,cx+b)^3\Gamma\left(\frac{3}{2},-\frac{(2\,cx+b)^2\log(f)}{4\,c}\right)\log(f)^3}{\left(-\frac{(2\,cx+b)^2\log(f)}{c}\right)^{\frac{3}{2}}(c\log(f))^{\frac{5}{2}}}-\frac{4\,bcf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)^2}{(c\log(f))^{\frac{5}{2}}}\right)e^2}{8\sqrt{c\log(f)}}$$
$$+\frac{\sqrt{\pi}d^2f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{2\sqrt{-c\log(f)}f^{\frac{b^2}{4\,c}}}$$

input

```
integrate(f^(c*x^2+b*x+a)*(e*x+d)^2,x, algorithm="maxima")
```

output

```
-1/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*
log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*
c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*d*e*f^(a - 1/4*b^2/c)/sqrt(c*log(f)
) + 1/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c))
- 1)*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*x
+ b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-(2*c*x + b)^2*l
og(f)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^2/
(c*log(f))^(5/2))*e^2*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) + 1/2*sqrt(pi)*d^2*
f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))
*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx =$$

$$-\frac{\frac{\sqrt{\pi}\left(4\,c^2d^2\log(f)-4\,bcde\log(f)+b^2e^2\log(f)-2\,ce^2\right)\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}\log(f)}-\frac{2\left(ce^2\left(2\,x+\frac{b}{c}\right)+4\,cde-2\,b\right.}{}}{8\,c^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/8*(sqrt(pi)*(4*c^2*d^2*log(f) - 4*b*c*d*e*log(f) + b^2*e^2*log(f) - 2*c
*e^2)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*lo
g(f))/c)/(sqrt(-c*log(f))*log(f)) - 2*(c*e^2*(2*x + b/c) + 4*c*d*e - 2*b*e
^2)*e^(c*x^2*log(f) + b*x*log(f) + a*log(f))/log(f))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx$$
$$= f^a\,f^{c\,x^2}\,f^{b\,x}\left(\frac{d\,e}{c\,\ln(f)}-\frac{b\,e^2}{4\,c^2\,\ln(f)}\right)+\frac{e^2\,f^a\,f^{c\,x^2}\,f^{b\,x}\,x}{2\,c\,\ln(f)}$$
$$-\frac{f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\operatorname{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\left(-\frac{\ln(f)\,b^2\,e^2}{8}+\frac{\ln(f)\,b\,c\,d\,e}{2}-\frac{\ln(f)\,c^2\,d^2}{2}+\frac{c\,e^2}{4}\right)}{c^2\,\ln(f)\,\sqrt{c\,\ln(f)}}$$

input
```
int(f^(a + b*x + c*x^2)*(d + e*x)^2,x)
```

output
```
f^a*f^(c*x^2)*f^(b*x)*((d*e)/(c*log(f)) - (b*e^2)/(4*c^2*log(f))) + (e^2*f
^a*f^(c*x^2)*f^(b*x)*x)/(2*c*log(f)) - (f^(a - b^2/(4*c))*pi^(1/2)*erfi(((
b*log(f))/2 + c*x*log(f))/(c*log(f))^(1/2))*((c*e^2)/4 - (b^2*e^2*log(f))/
8 - (c^2*d^2*log(f))/2 + (b*c*d*e*log(f))/2))/(c^2*log(f)*(c*log(f))^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.50

$$\int f^{a+bx+cx^2}(d+ex)^2\,dx$$
$$= \frac{f^a\left(-\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,b^2e^2i+4\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,bcdei-4\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+}{2\sqrt{c}\,}\right.}{}$$

input
```
int(f^(c*x^2+b*x+a)*(e*x+d)^2,x)
```

output
```
(f**a*( - sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f
))))*log(f)*b**2*e**2*i + 4*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*
sqrt(c)*sqrt(log(f))))*log(f)*b*c*d*e*i - 4*sqrt(pi)*erf((log(f)*b*i + 2*l
og(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f))))*log(f)*c**2*d**2*i + 2*sqrt(pi)*erf
((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f))))*c*e**2*i - 2*f**(
b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*b*e**2 + 8*f**
(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*c*d*e + 4*f**
(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*c*e**2*x))/(8
*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f)*c**2)
```

3.380 $\int f^{a+bx+cx^2}(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 90

$$\int f^{a+bx+cx^2}(d+ex)\,dx=\frac{ef^{a+bx+cx^2}}{2c\log(f)}+\frac{(2cd-be)f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}$$

output
```
1/2*e*f^(c*x^2+b*x+a)/c/ln(f)+1/4*(-b*e+2*c*d)*f^(a-1/4*b^2/c)*Pi^(1/2)*er
fi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(3/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int f^{a+bx+cx^2}(d+ex)\,dx=\frac{f^{a-\frac{b^2}{4c}}\left(2\sqrt{c}ef^{\frac{(b+2cx)^2}{4c}}+(2cd-be)\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}\right)}{4c^{3/2}\log(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*(d + e*x),x]
```

output
```
(f^(a - b^2/(4*c))*(2*Sqrt[c]*e*f^((b + 2*c*x)^2/(4*c)) + (2*c*d - b*e)*Sq
rt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]*Sqrt[Log[f]]))/(4*c^(3
/2)*Log[f])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2670, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2670}$$

$$\frac{(2cd-be)\int f^{cx^2+bx+a}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2664}$$

$$\frac{f^{a-\frac{b^2}{4c}}(2cd-be)\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}(2cd-be)\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}\sqrt{\log(f)}}+\frac{ef^{a+bx+cx^2}}{2c\log(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*(d + e*x),x]
```

output
```
(e*f^(a + b*x + c*x^2))/(2*c*Log[f]) + ((2*c*d - b*e)*f^(a - b^2/(4*c))*Sq
rt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(4*c^(3/2)*Sqrt[Log[f
]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2670
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] - Simp[(b*e - 2*c*d)/(2*
c)   Int[F^(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ
[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

method	result	size
risch	$-\frac{f^{a} d \sqrt{\pi}\, f^{-\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{-c \ln \left(f \right)}\, x +\frac{b \ln \left(f \right)}{2 \sqrt{-c \ln \left(f \right)}}\right)}{2 \sqrt{-c \ln \left(f \right)}}+\frac{f^{a} e \,f^{b x} f^{c \,x^{2}}}{2 \ln \left(f \right) c}+\frac{f^{a} e b \sqrt{\pi}\, f^{-\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{-c \ln \left(f \right)}\, x +\frac{b \ln \left(f \right)}{2 \sqrt{-c \ln \left(f \right)}}\right)}{4 c \sqrt{-c \ln \left(f \right)}}$	131

input
```
int(f^(c*x^2+b*x+a)*(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*f^a*d*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*
x+1/2*b*ln(f)/(-c*ln(f))^(1/2))+1/2*f^a*e/ln(f)/c*f^(b*x)*f^(c*x^2)+1/4*f^
a*e*b/c*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1
/2*b*ln(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int f^{a+bx+cx^2}(d+ex)\,dx = \frac{2\,cef^{cx^2+bx+a} - \frac{\sqrt{\pi}(2\,cd-be)\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{4\,c^2\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d),x, algorithm="fricas")
```

output
```
1/4*(2*c*e*f^(c*x^2 + b*x + a) - sqrt(pi)*(2*c*d - b*e)*sqrt(-c*log(f))*er
f(1/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(c^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}(d+ex)\,dx = \int f^{a+bx+cx^2}(d+ex)\;dx$$

input
```
integrate(f**(c*x**2+b*x+a)*(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*(d + e*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(72) = 144.

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.78

$$\int f^{a+bx+cx^2}(d+ex)\,dx = -\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b\left(\mathrm{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^2}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\,(c\log(f))^{\frac{3}{2}}} - \frac{2\,cf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)}{(c\log(f))^{\frac{3}{2}}}\right)ef^{a-\frac{b^2}{4\,c}}}{4\sqrt{c\log(f)}} + \frac{\sqrt{\pi}df^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{2\sqrt{-c\log(f)}f^{\frac{b^2}{4\,c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*
log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*
c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*e*f^(a - 1/4*b^2/c)/sqrt(c*log(f))
+ 1/2*sqrt(pi)*d*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))
/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99

$$\int f^{a+bx+cx^2}(d+ex)\,dx = -\frac{\frac{\sqrt{\pi}(2\,cd-be)\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}} - \frac{2\,ee^{\left(cx^2\log(f)+bx\log(f)+a\log(f)\right)}}{\log(f)}}{4\,c}$$

input
```
integrate(f^(c*x^2+b*x+a)*(e*x+d),x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*(2*c*d - b*e)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4
*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f)) - 2*e*e^(c*x^2*log(f) + b*
x*log(f) + a*log(f))/log(f))/c
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int f^{a+bx+cx^2}(d+ex)\,dx = \frac{e\,f^a\,f^{c\,x^2}\,f^{b\,x}}{2\,c\,\ln(f)} - \frac{f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\,\left(\frac{b\,e}{4}-\frac{c\,d}{2}\right)}{c\,\sqrt{c\,\ln(f)}}$$

input
```
int(f^(a + b*x + c*x^2)*(d + e*x),x)
```

output
```
(e*f^a*f^(c*x^2)*f^(b*x))/(2*c*log(f)) - (f^(a - b^2/(4*c))*pi^(1/2)*erfi(
((b*log(f))/2 + c*x*log(f))/(c*log(f))^(1/2))*((b*e)/4 - (c*d)/2))/(c*(c*l
og(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.49

$$\int f^{a+bx+cx^2}(d+ex)\,dx$$
$$= \frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,bei-2\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)\log(f)\,cdi+2f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\mathrm{l}}\right.}{4e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}\log(f)\,c}$$

input
```
int(f^(c*x^2+b*x+a)*(e*x+d),x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f)))
)*log(f)*b*e*i - 2*sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*s
qrt(log(f))))*log(f)*c*d*i + 2*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*
sqrt(c)*sqrt(log(f))*e))/(4*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*
log(f)*c)
```

3.381 $\int \frac{f^{a+bx+cx^2}}{d+ex} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{f^{a+bx+cx^2}}{d+ex} dx = \text{Int}\left(\frac{f^{a+bx+cx^2}}{d+ex}, x\right)$$

output
```
Defer(Int)(f^(c*x^2+b*x+a)/(e*x+d),x)
```

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{d+ex} dx = \int \frac{f^{a+bx+cx^2}}{d+ex} dx$$

input
```
Integrate[f^(a + b*x + c*x^2)/(d + e*x),x]
```

output
```
Integrate[f^(a + b*x + c*x^2)/(d + e*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{d+ex} \, dx$$

$$\Big\downarrow \text{ 2673}$$

$$\int \frac{f^{a+bx+cx^2}}{d+ex}dx$$

input
```
Int[f^(a + b*x + c*x^2)/(d + e*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{f^{c\,x^2+bx+a}}{ex+d}dx$$

input
```
int(f^(c*x^2+b*x+a)/(e*x+d),x)
```

output
```
int(f^(c*x^2+b*x+a)/(e*x+d),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{d+ex}\,dx = \int \frac{f^{cx^2+bx+a}}{ex+d}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d),x, algorithm="fricas")
```

output
```
integral(f^(c*x^2 + b*x + a)/(e*x + d), x)
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{f^{a+bx+cx^2}}{d+ex}\,dx = \int \frac{f^{a+bx+cx^2}}{d+ex}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)/(d + e*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{d+ex}\,dx = \int \frac{f^{cx^2+bx+a}}{ex+d}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d),x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(e*x + d), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{d+ex}\,dx = \int \frac{f^{cx^2+bx+a}}{ex+d}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(e*x + d), x)
```

Mupad [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{d+ex}\,dx = \int \frac{f^{c\,x^2+b\,x+a}}{d+e\,x}\,dx$$

input
```
int(f^(a + b*x + c*x^2)/(d + e*x),x)
```

output
```
int(f^(a + b*x + c*x^2)/(d + e*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{f^{a+bx+cx^2}}{d+ex}\,dx = f^a\left(\int \frac{f^{c\,x^2+bx}}{ex+d}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)/(e*x+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)/(d + e*x),x)
```

3.382 $\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx = -\frac{f^{a+bx+cx^2}}{e(d+ex)} + \frac{\sqrt{c} f^{a-\frac{b^2}{4c}} \sqrt{\pi} \operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right) \sqrt{\log(f)}}{e^2} - \frac{(2cd-be)\log(f)\operatorname{Int}\left(\frac{f^{a+bx+cx^2}}{d+ex}, x\right)}{e^2}$$

output
```
-f^(c*x^2+b*x+a)/e/(e*x+d)+c^(1/2)*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*
x+b)*ln(f)^(1/2)/c^(1/2))*ln(f)^(1/2)/e^2-(-b*e+2*c*d)*ln(f)*Defer(Int)(f^
(c*x^2+b*x+a)/(e*x+d),x)/e^2
```

Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx = \int \frac{f^{a+bx+cx^2}}{(d+ex)^2} dx$$

input
```
Integrate[f^(a + b*x + c*x^2)/(d + e*x)^2,x]
```

output
```
Integrate[f^(a + b*x + c*x^2)/(d + e*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2672, 2664, 2633, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{ 2672}$$

$$-\frac{\log(f)(2cd-be)\int \frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}+\frac{2c\log(f)\int f^{cx^2+bx+a}dx}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{2c\log(f)f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{e^2}-\frac{\log(f)(2cd-be)\int \frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 2633}$$

$$-\frac{\log(f)(2cd-be)\int \frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}+\frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 2673}$$

$$-\frac{\log(f)(2cd-be)\int \frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}+\frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}$$

input
```
Int[f^(a + b*x + c*x^2)/(d + e*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2672
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] + (-
Simp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2)
, x], x] - Simp[(b*e - 2*c*d)*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 1
)*F^(a + b*x + c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e -
 2*c*d, 0] && LtQ[m, -1]
```

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{f^{c\,x^2+bx+a}}{(ex+d)^2}dx$$

input
```
int(f^(c*x^2+b*x+a)/(e*x+d)^2,x)
```

output
```
int(f^(c*x^2+b*x+a)/(e*x+d)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{f^{cx^2+bx+a}}{(ex+d)^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(f^(c*x^2 + b*x + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)/(d + e*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{f^{cx^2+bx+a}}{(ex+d)^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(e*x + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{f^{cx^2+bx+a}}{(ex+d)^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(e*x + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{f^{c\,x^2+b\,x+a}}{(d+e\,x)^2}\,dx$$

input
```
int(f^(a + b*x + c*x^2)/(d + e*x)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)/(d + e*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^2}\,dx = f^a\left(\int \frac{f^{c\,x^2+bx}}{e^2x^2+2dex+d^2}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)/(e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)/(d**2 + 2*d*e*x + e**2*x**2),x)
```

3.383 $\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = -\frac{f^{a+bx+cx^2}}{2e(d+ex)^2}+\frac{(2cd-be)f^{a+bx+cx^2}\log(f)}{2e^3(d+ex)} - \frac{\sqrt{c}(2cd-be)f^{a-\frac{b^2}{4c}}\sqrt{\pi}\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\log^{\frac{3}{2}}(f)}{2e^4} + \frac{c\log(f)\text{Int}\left(\frac{f^{a+bx+cx^2}}{d+ex},x\right)}{e^2}+\frac{(2cd-be)^2\log^2(f)\text{Int}\left(\frac{f^{a+bx+cx^2}}{d+ex},x\right)}{2e^4}$$

output
```
-1/2*f^(c*x^2+b*x+a)/e/(e*x+d)^2+1/2*(-b*e+2*c*d)*f^(c*x^2+b*x+a)*ln(f)/e^
3/(e*x+d)-1/2*c^(1/2)*(-b*e+2*c*d)*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*
x+b)*ln(f)^(1/2)/c^(1/2))*ln(f)^(3/2)/e^4+c*ln(f)*Defer(Int)(f^(c*x^2+b*x+
a)/(e*x+d),x)/e^2+1/2*(-b*e+2*c*d)^2*ln(f)^2*Defer(Int)(f^(c*x^2+b*x+a)/(e
*x+d),x)/e^4
```

Mathematica [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx$$

input
```
Integrate[f^(a + b*x + c*x^2)/(d + e*x)^3,x]
```

output
```
Integrate[f^(a + b*x + c*x^2)/(d + e*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2672, 2672, 2664, 2633, 2673}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx$$

$$\Big\downarrow \text{ 2672}$$

$$-\frac{\log(f)(2cd-be)\int \frac{f^{cx^2+bx+a}}{(d+ex)^2}dx}{2e^2}+\frac{c\log(f)\int \frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{f^{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{ 2672}$$

$$\frac{c\log(f)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{\log(f)(2cd-be)\left(-\frac{\log(f)(2cd-be)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}+\frac{2c\log(f)\int f^{cx^2+bx+a}dx}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}\right)}{2e^2}-\frac{f^{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2664$$

$$-\frac{\log(f)(2cd-be)\left(\frac{2c\log(f)f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{e^2}-\frac{\log(f)(2cd-be)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}\right)}{2e^2}+\frac{c\log(f)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{f^{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2633$$

$$-\frac{\log(f)(2cd-be)\left(-\frac{\log(f)(2cd-be)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}+\frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}\right)}{2e^2}+\frac{c\log(f)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{f^{a+bx+cx^2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 2673$$

$$-\frac{\log(f)(2cd-be)\left(-\frac{\log(f)(2cd-be)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}+\frac{\sqrt{\pi}\sqrt{c}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{e^2}-\frac{f^{a+bx+cx^2}}{e(d+ex)}\right)}{2e^2}+\frac{c\log(f)\int\frac{f^{cx^2+bx+a}}{d+ex}dx}{e^2}-\frac{f^{a+bx+cx^2}}{2e(d+ex)^2}$$

input
```
Int[f^(a + b*x + c*x^2)/(d + e*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2672
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] + (-
Simp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2)
, x], x] - Simp[(b*e - 2*c*d)*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 1
)*F^(a + b*x + c*x^2), x], x]) /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[b*e -
 2*c*d, 0] && LtQ[m, -1]
```

rule 2673
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_.), x_
Symbol] :> Unintegrable[F^(a + b*x + c*x^2)*(d + e*x)^m, x] /; FreeQ[{F, a,
 b, c, d, e, m}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{f^{c\,x^2+bx+a}}{\left(ex+d\right)^3}dx$$

input
```
int(f^(c*x^2+b*x+a)/(e*x+d)^3,x)
```

output
```
int(f^(c*x^2+b*x+a)/(e*x+d)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{f^{cx^2+bx+a}}{(ex+d)^3}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(f^(c*x^2 + b*x + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)/(d + e*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{f^{cx^2+bx+a}}{(ex+d)^3}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(e*x + d)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{f^{cx^2+bx+a}}{(ex+d)^3}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(e*x + d)^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{f^{c\,x^2+b\,x+a}}{(d+e\,x)^3}\,dx$$

input
```
int(f^(a + b*x + c*x^2)/(d + e*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)/(d + e*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 8588, normalized size of antiderivative = 429.40

$$\int \frac{f^{a+bx+cx^2}}{(d+ex)^3} \, dx = \text{Too large to display}$$

input
```
int(f^(c*x^2+b*x+a)/(e*x+d)^3,x)
```

output
```
(f**a*( - f**(b*x + c*x**2) + int(f**(b*x + c*x**2)/(log(f)*b**2*d**4*e +
3*log(f)*b**2*d**3*e**2*x + 3*log(f)*b**2*d**2*e**3*x**2 + log(f)*b**2*d*e
**4*x**3 + 2*log(f)*b*c*d**5 + 6*log(f)*b*c*d**4*e*x + 6*log(f)*b*c*d**3*e
**2*x**2 + 2*log(f)*b*c*d**2*e**3*x**3 - 2*b*d**3*e**2 - 6*b*d**2*e**3*x -
 6*b*d*e**4*x**2 - 2*b*e**5*x**3 - 4*c*d**4*e - 12*c*d**3*e**2*x - 12*c*d*
*2*e**3*x**2 - 4*c*d*e**4*x**3),x)*log(f)**2*b**3*d**4*e + 2*int(f**(b*x +
 c*x**2)/(log(f)*b**2*d**4*e + 3*log(f)*b**2*d**3*e**2*x + 3*log(f)*b**2*d
**2*e**3*x**2 + log(f)*b**2*d*e**4*x**3 + 2*log(f)*b*c*d**5 + 6*log(f)*b*c
*d**4*e*x + 6*log(f)*b*c*d**3*e**2*x**2 + 2*log(f)*b*c*d**2*e**3*x**3 - 2*
b*d**3*e**2 - 6*b*d**2*e**3*x - 6*b*d*e**4*x**2 - 2*b*e**5*x**3 - 4*c*d**4
*e - 12*c*d**3*e**2*x - 12*c*d**2*e**3*x**2 - 4*c*d*e**4*x**3),x)*log(f)**
2*b**3*d**3*e**2*x + int(f**(b*x + c*x**2)/(log(f)*b**2*d**4*e + 3*log(f)*
b**2*d**3*e**2*x + 3*log(f)*b**2*d**2*e**3*x**2 + log(f)*b**2*d*e**4*x**3
+ 2*log(f)*b*c*d**5 + 6*log(f)*b*c*d**4*e*x + 6*log(f)*b*c*d**3*e**2*x**2
+ 2*log(f)*b*c*d**2*e**3*x**3 - 2*b*d**3*e**2 - 6*b*d**2*e**3*x - 6*b*d*e*
*4*x**2 - 2*b*e**5*x**3 - 4*c*d**4*e - 12*c*d**3*e**2*x - 12*c*d**2*e**3*x
**2 - 4*c*d*e**4*x**3),x)*log(f)**2*b**3*d**2*e**3*x**2 + 2*int(f**(b*x +
c*x**2)/(log(f)*b**2*d**4*e + 3*log(f)*b**2*d**3*e**2*x + 3*log(f)*b**2*d*
*2*e**3*x**2 + log(f)*b**2*d*e**4*x**3 + 2*log(f)*b*c*d**5 + 6*log(f)*b*c*
d**4*e*x + 6*log(f)*b*c*d**3*e**2*x**2 + 2*log(f)*b*c*d**2*e**3*x**3 - ...
```

3.384 $\int f^{a+bx+cx^2}(b+2cx)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 45

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx=-\frac{4cf^{a+bx+cx^2}}{\log^2(f)}+\frac{f^{a+bx+cx^2}(b+2cx)^2}{\log(f)}$$

output
```
-4*c*f^(c*x^2+b*x+a)/ln(f)^2+f^(c*x^2+b*x+a)*(2*c*x+b)^2/ln(f)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx=\frac{f^{a+x(b+cx)}(-4c+(b+2cx)^2\log(f))}{\log^2(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*(b + 2*c*x)^3,x]
```

output
```
(f^(a + x*(b + c*x))*(-4*c + (b + 2*c*x)^2*Log[f]))/Log[f]^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 2666}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)^3 f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 2667$$

$$\frac{(b+2cx)^2 f^{a+bx+cx^2}}{\log(f)}-\frac{4c\int f^{cx^2+bx+a}(b+2cx)dx}{\log(f)}$$

$$\Big\downarrow \; 2666$$

$$\frac{(b+2cx)^2 f^{a+bx+cx^2}}{\log(f)}-\frac{4cf^{a+bx+cx^2}}{\log^2(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*(b + 2*c*x)^3,x]
```

output
```
(-4*c*f^(a + b*x + c*x^2))/Log[f]^2 + (f^(a + b*x + c*x^2)*(b + 2*c*x)^2)/
Log[f]
```

Defintions of rubi rules used

rule 2666
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

rule 2667
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] -
 Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x + c*x^2
), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && GtQ[m,
1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

method	result	size
gosper	$\frac{\left(4 \ln \left(f \right) c^{2} x^{2}+4 c b \ln \left(f \right) x +\ln \left(f \right) b^{2}-4 c \right) f^{c \,x^{2}+b x +a}}{\ln \left(f \right)^{2}}$	45
risch	$\frac{\left(4 \ln \left(f \right) c^{2} x^{2}+4 c b \ln \left(f \right) x +\ln \left(f \right) b^{2}-4 c \right) f^{c \,x^{2}+b x +a}}{\ln \left(f \right)^{2}}$	45
orering	$\frac{\left(4 \ln \left(f \right) c^{2} x^{2}+4 c b \ln \left(f \right) x +\ln \left(f \right) b^{2}-4 c \right) f^{c \,x^{2}+b x +a}}{\ln \left(f \right)^{2}}$	45
norman	$\frac{\left(\ln \left(f \right) b^{2}-4 c \right) {\mathrm e}^{\left(c \,x^{2}+b x +a \right) \ln \left(f \right)}}{\ln \left(f \right)^{2}}+\frac{4 c^{2} x^{2} {\mathrm e}^{\left(c \,x^{2}+b x +a \right) \ln \left(f \right)}}{\ln \left(f \right)}+\frac{4 c b x \,{\mathrm e}^{\left(c \,x^{2}+b x +a \right) \ln \left(f \right)}}{\ln \left(f \right)}$	80
parallelrisch	$\frac{4 c^{2} f^{c \,x^{2}+b x +a} x^{2} \ln \left(f \right)+4 c b x \,f^{c \,x^{2}+b x +a} \ln \left(f \right)+\ln \left(f \right) f^{c \,x^{2}+b x +a} b^{2}-4 c \,f^{c \,x^{2}+b x +a}}{\ln \left(f \right)^{2}}$	81

input
```
int(f^(c*x^2+b*x+a)*(2*c*x+b)^3,x,method=_RETURNVERBOSE)
```

output
```
(4*ln(f)*c^2*x^2+4*c*b*ln(f)*x+ln(f)*b^2-4*c)*f^(c*x^2+b*x+a)/ln(f)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx=\frac{\left((4\,c^2x^2+4\,bcx+b^2)\log\left(f\right)-4\,c\right)f^{cx^2+bx+a}}{\log\left(f\right)^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^3,x, algorithm="fricas")
```

output
```
((4*c^2*x^2 + 4*b*c*x + b^2)*log(f) - 4*c)*f^(c*x^2 + b*x + a)/log(f)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(42) = 84$.

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx = \begin{cases} \frac{f^{a+bx+cx^2}(b^2\log(f)+4bcx\log(f)+4c^2x^2\log(f)-4c)}{\log(f)^2} & \text{for } \log(f)^2 \neq 0 \\ b^3x+3b^2cx^2+4bc^2x^3+2c^3x^4 & \text{otherwise} \end{cases}$$

input
```
integrate(f**(c*x**2+b*x+a)*(2*c*x+b)**3,x)
```

output
```
Piecewise((f**(a + b*x + c*x**2)*(b**2*log(f) + 4*b*c*x*log(f) + 4*c**2*x*
*2*log(f) - 4*c)/log(f)**2, Ne(log(f)**2, 0)), (b**3*x + 3*b**2*c*x**2 + 4
*b*c**2*x**3 + 2*c**3*x**4, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 539, normalized size of antiderivative = 11.98

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^3,x, algorithm="maxima")
```

output
```
-3/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*
log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*
c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*b^2*c*f^(a - 1/4*b^2/c)/sqrt(c*log(
f)) + 3/2*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)
) - 1)*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*
x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-(2*c*x + b)^2
*log(f)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^
2/(c*log(f))^(5/2))*b*c^2*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) - 1/2*(sqrt(pi)
*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(f)^4/(sq
rt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(7/2)) - 12*(2*c*x + b)^3*b*gamma(3
/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^4/((-(2*c*x + b)^2*log(f)/c)^(3/2)
*(c*log(f))^(7/2)) - 6*b^2*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^3/(c*log(f))^(
7/2) + 8*c^2*gamma(2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^2/(c*log(f))^(7/
2))*c^3*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) + 1/2*sqrt(pi)*b^3*f^a*erf(sqrt(-
c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c)
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 798, normalized size of antiderivative = 17.73

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^3,x, algorithm="giac")
```

output
```
(2*((b^2*log(abs(f)) + 4*(c*x^2 + b*x)*c*log(abs(f)) - 4*c)*(pi^2*sgn(f) -
 pi^2 + 2*log(abs(f))^2)/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2 + 4*(pi
*log(abs(f))*sgn(f) - pi*log(abs(f)))^2) + (pi*b^2*sgn(f) + 4*pi*(c*x^2 +
b*x)*c*sgn(f) - pi*b^2 - 4*pi*(c*x^2 + b*x)*c)*(pi*log(abs(f))*sgn(f) - pi
*log(abs(f)))/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2 + 4*(pi*log(abs(f)
)*sgn(f) - pi*log(abs(f)))^2))*cos(-1/2*pi*c*x^2*sgn(f) + 1/2*pi*c*x^2 - 1
/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a) + ((pi*b^2*sgn
(f) + 4*pi*(c*x^2 + b*x)*c*sgn(f) - pi*b^2 - 4*pi*(c*x^2 + b*x)*c)*(pi^2*s
gn(f) - pi^2 + 2*log(abs(f))^2)/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2
+ 4*(pi*log(abs(f))*sgn(f) - pi*log(abs(f)))^2) - 4*(b^2*log(abs(f)) + 4*(
c*x^2 + b*x)*c*log(abs(f)) - 4*c)*(pi*log(abs(f))*sgn(f) - pi*log(abs(f)))
/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2 + 4*(pi*log(abs(f))*sgn(f) - pi
*log(abs(f)))^2))*sin(-1/2*pi*c*x^2*sgn(f) + 1/2*pi*c*x^2 - 1/2*pi*b*x*sgn
(f) + 1/2*pi*b*x - 1/2*pi*a*sgn(f) + 1/2*pi*a))*e^((c*x^2 + b*x)*log(abs(f
)) + a*log(abs(f))) - 1/2*I*((pi*b^2*sgn(f) + 4*pi*(c*x^2 + b*x)*c*sgn(f)
- pi*b^2 - 4*pi*(c*x^2 + b*x)*c - 2*I*b^2*log(abs(f)) + 8*(-I*c*x^2 - I*b*
x)*c*log(abs(f)) + 8*I*c)*e^(1/2*I*pi*c*x^2*sgn(f) - 1/2*I*pi*c*x^2 + 1/2*
I*pi*b*x*sgn(f) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(pi^2*sgn
(f) + 2*I*pi*log(abs(f))*sgn(f) - pi^2 - 2*I*pi*log(abs(f)) + 2*log(abs(f)
)^2) + (pi*b^2*sgn(f) + 4*pi*(c*x^2 + b*x)*c*sgn(f) - pi*b^2 - 4*pi*(c*...
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx=\frac{f^{c\,x^2+b\,x+a}\,\left(\ln\left(f\right)\,b^2+4\,\ln\left(f\right)\,b\,c\,x+4\,\ln\left(f\right)\,c^2\,x^2-4\,c\right)}{{\ln\left(f\right)}^2}$$

input
```
int(f^(a + b*x + c*x^2)*(b + 2*c*x)^3,x)
```

output
```
(f^(a + b*x + c*x^2)*(b^2*log(f) - 4*c + 4*c^2*x^2*log(f) + 4*b*c*x*log(f)
))/log(f)^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int f^{a+bx+cx^2}(b+2cx)^3\,dx = \frac{f^{c\,x^2+bx+a}(\log(f)\,b^2+4\log(f)\,bcx+4\log(f)\,c^2x^2-4c)}{\log\left(f\right)^2}$$

input
```
int(f^(c*x^2+b*x+a)*(2*c*x+b)^3,x)
```

output
```
(f**(a + b*x + c*x**2)*(log(f)*b**2 + 4*log(f)*b*c*x + 4*log(f)*c**2*x**2
- 4*c))/log(f)**2
```

3.385 $\int f^{a+bx+cx^2}(b+2cx)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx=-\frac{\sqrt{c}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}+\frac{f^{a+bx+cx^2}(b+2cx)}{\log(f)}$$

output
```
-c^(1/2)*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/
ln(f)^(3/2)+f^(c*x^2+b*x+a)*(2*c*x+b)/ln(f)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx=\frac{f^{a-\frac{b^2}{4c}}\left(-\sqrt{c}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+f^{\frac{(b+2cx)^2}{4c}}(b+2cx)\sqrt{\log(f)}\right)}{\log^{\frac{3}{2}}(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*(b + 2*c*x)^2,x]
```

output
```
(f^(a - b^2/(4*c))*(-(Sqrt[c]*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*
Sqrt[c])]) + f^((b + 2*c*x)^2/(4*c))*(b + 2*c*x)*Sqrt[Log[f]]))/Log[f]^(3/
2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2667, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)^2 f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 2667}$$

$$\frac{(b+2cx)f^{a+bx+cx^2}}{\log(f)}-\frac{2c\int f^{cx^2+bx+a}dx}{\log(f)}$$

$$\Big\downarrow \text{ 2664}$$

$$\frac{(b+2cx)f^{a+bx+cx^2}}{\log(f)}-\frac{2cf^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{\log(f)}$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{(b+2cx)f^{a+bx+cx^2}}{\log(f)}-\frac{\sqrt{\pi}\sqrt{c}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*(b + 2*c*x)^2,x]
```

output
```
-((Sqrt[c]*f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*S
qrt[c])])/Log[f]^(3/2)) + (f^(a + b*x + c*x^2)*(b + 2*c*x))/Log[f]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2667
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] -
 Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x + c*x^2
), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && GtQ[m,
1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27

method	result	size
risch	$\frac{2 f^{a} c x \,f^{b x} f^{c \,x^{2}}}{\ln \left(f \right)}+\frac{f^{a} b \,f^{b x} f^{c \,x^{2}}}{\ln \left(f \right)}+\frac{f^{a} c \sqrt{\pi}\, f^{-\frac{b^{2}}{4 c}} \operatorname{erf}\left(-\sqrt{-c \ln \left(f \right)}\, x +\frac{b \ln \left(f \right)}{2 \sqrt{-c \ln \left(f \right)}}\right)}{\ln \left(f \right) \sqrt{-c \ln \left(f \right)}}$	99

input
```
int(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
2*f^a*c/ln(f)*x*f^(b*x)*f^(c*x^2)+f^a*b/ln(f)*f^(b*x)*f^(c*x^2)+f^a*c/ln(f
)*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*l
n(f)/(-c*ln(f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx = \frac{(2\,cx+b)f^{cx^2+bx+a}\log{(f)} + \frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{\log{(f)}^2}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x, algorithm="fricas")
```

output
```
((2*c*x + b)*f^(c*x^2 + b*x + a)*log(f) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2
*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/log(f)^2
```

Sympy [F]

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx = \int f^{a+bx+cx^2}(b+2cx)^2\;dx$$

input
```
integrate(f**(c*x**2+b*x+a)*(2*c*x+b)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*(b + 2*c*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. 2(64) = 128.

Time = 0.17 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.26

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx$$
$$= -\frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^2}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{3}{2}}} - \frac{2\,cf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)}{(c\log(f))^{\frac{3}{2}}}\right)bcf^{a-\frac{b^2}{4\,c}}}{\sqrt{c\log(f)}}$$
$$+ \frac{\left(\frac{\sqrt{\pi}(2\,cx+b)b^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^3}{\sqrt{-\frac{(2\,cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{5}{2}}} - \frac{4\,(2\,cx+b)^3\Gamma\left(\frac{3}{2},-\frac{(2\,cx+b)^2\log(f)}{4\,c}\right)\log(f)^3}{\left(-\frac{(2\,cx+b)^2\log(f)}{c}\right)^{\frac{3}{2}}(c\log(f))^{\frac{5}{2}}} - \frac{4\,bcf^{\frac{(2\,cx+b)^2}{4\,c}}\log(f)^2}{(c\log(f))^{\frac{5}{2}}}\right)c^2}{2\sqrt{c\log(f)}}$$
$$+ \frac{\sqrt{\pi}b^2f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{2\sqrt{-c\log(f)}f^{\frac{b^2}{4\,c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x, algorithm="maxima")
```

output
```
-(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(
f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(3/2)) - 2*c*f^(1/4*(2*c*x
+ b)^2/c)*log(f)/(c*log(f))^(3/2))*b*c*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) +
1/2*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2*log(f)/c)) - 1)
*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(5/2)) - 4*(2*c*x + b)
^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^3/((-(2*c*x + b)^2*log(f
)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2*c*x + b)^2/c)*log(f)^2/(c*l
og(f))^(5/2))*c^2*f^(a - 1/4*b^2/c)/sqrt(c*log(f)) + 1/2*sqrt(pi)*b^2*f^a*
erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(
1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.13

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx = \frac{c\left(2\,x+\frac{b}{c}\right)e^{\left(cx^2\log(f)+bx\log(f)+a\log(f)\right)}}{\log(f)} + \frac{\sqrt{\pi}c\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{\sqrt{-c\log(f)}\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x, algorithm="giac")
```

output
```
c*(2*x + b/c)*e^(c*x^2*log(f) + b*x*log(f) + a*log(f))/log(f) + sqrt(pi)*c
*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))
/c)/(sqrt(-c*log(f))*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx=\frac{b\,f^a\,f^{c\,x^2}\,f^{b\,x}}{\ln\left(f\right)}-\frac{c\,f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)}{\ln\left(f\right)\,\sqrt{c\,\ln\left(f\right)}}+\frac{2\,c\,f^a\,f^{c\,x^2}\,f^{b\,x}\,x}{\ln\left(f\right)}$$

input
```
int(f^(a + b*x + c*x^2)*(b + 2*c*x)^2,x)
```

output
```
(b*f^a*f^(c*x^2)*f^(b*x))/log(f) - (c*f^(a - b^2/(4*c))*pi^(1/2)*erfi(((b*
log(f))/2 + c*x*log(f))/(c*log(f))^(1/2)))/(log(f)*(c*log(f))^(1/2)) + (2*
c*f^a*f^(c*x^2)*f^(b*x)*x)/log(f)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int f^{a+bx+cx^2}(b+2cx)^2\,dx=\frac{f^a\left(\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)ci+f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log\left(f\right)}\,b+2f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log\left(f\right)}\,cx\right)}{e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log\left(f\right)}\log\left(f\right)}$$

input
```
int(f^(c*x^2+b*x+a)*(2*c*x+b)^2,x)
```

output
```
(f**a*(sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f)))
)*c*i + f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*b
+ 2*f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*c*x))/
(e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f))
```

3.386 $\int f^{a+bx+cx^2}(b+2cx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 17

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{a+bx+cx^2}}{\log(f)}$$

output
```
f^(c*x^2+b*x+a)/ln(f)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{a+bx+cx^2}}{\log(f)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*(b + 2*c*x),x]
```

output
```
f^(a + b*x + c*x^2)/Log[f]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2666}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{2666}$$

$$\frac{f^{a+bx+cx^2}}{\log(f)}$$

input
```
Int[f^(a + b*x + c*x^2)*(b + 2*c*x),x]
```

output
```
f^(a + b*x + c*x^2)/Log[f]
```

Defintions of rubi rules used

rule 2666
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{f^{c\,x^2+bx+a}}{\ln(f)}$	18
derivativedivides	$\frac{f^{c\,x^2+bx+a}}{\ln(f)}$	18
default	$\frac{f^{c\,x^2+bx+a}}{\ln(f)}$	18
risch	$\frac{f^{c\,x^2+bx+a}}{\ln(f)}$	18
parallelrisch	$\frac{f^{c\,x^2+bx+a}}{\ln(f)}$	18
orering	$\frac{f^{c\,x^2+bx+a}}{\ln(f)}$	18
norman	$\frac{e^{\left(c\,x^2+bx+a\right)\ln(f)}}{\ln(f)}$	20

input
```
int(f^(c*x^2+b*x+a)*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
f^(c*x^2+b*x+a)/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{cx^2+bx+a}}{\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="fricas")
```

output
```
f^(c*x^2 + b*x + a)/log(f)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\begin{cases}\frac{f^{a+bx+cx^2}}{\log(f)} & \text{for } \log(f)\neq 0\\ bx+cx^2 & \text{otherwise}\end{cases}$$

input
```
integrate(f**(c*x**2+b*x+a)*(2*c*x+b),x)
```

output
```
Piecewise((f**(a + b*x + c*x**2)/log(f), Ne(log(f), 0)), (b*x + c*x**2, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{cx^2+bx+a}}{\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="maxima")
```

output
```
f^(c*x^2 + b*x + a)/log(f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{cx^2+bx+a}}{\log(f)}$$

input
```
integrate(f^(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="giac")
```

output
```
f^(c*x^2 + b*x + a)/log(f)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{c\,x^2+b\,x+a}}{\ln(f)}$$

input
```
int(f^(a + b*x + c*x^2)*(b + 2*c*x),x)
```

output
```
f^(a + b*x + c*x^2)/log(f)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int f^{a+bx+cx^2}(b+2cx)\,dx=\frac{f^{c\,x^2+bx+a}}{\log(f)}$$

input
```
int(f^(c*x^2+b*x+a)*(2*c*x+b),x)
```

output
```
f**(a + b*x + c*x**2)/log(f)
```

3.387 $\int \frac{f^{a+bx+cx^2}}{b+2cx} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{f^{a+bx+cx^2}}{b+2cx} \, dx = \frac{f^{a-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

output
```
1/4*f^(a-1/4*b^2/c)*Ei(1/4*(2*c*x+b)^2*ln(f)/c)/c
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx+cx^2}}{b+2cx} \, dx = \frac{f^{a-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

input
```
Integrate[f^(a + b*x + c*x^2)/(b + 2*c*x),x]
```

output
```
(f^(a - b^2/(4*c))*ExpIntegralEi[((b + 2*c*x)^2*Log[f])/(4*c)])/(4*c)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2668}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{b+2cx} \, dx$$

$$\Big\downarrow \text{ 2668}$$

$$\frac{f^{a-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

input
```
Int[f^(a + b*x + c*x^2)/(b + 2*c*x),x]
```

output
```
(f^(a - b^2/(4*c))*ExpIntegralEi[((b + 2*c*x)^2*Log[f])/(4*c)])/(4*c)
```

Defintions of rubi rules used

rule 2668
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(1/(2*e))*F^(a - b^2/(4*c))*ExpIntegralEi[(b + 2*c*x)^2*(Log[F]/(
4*c))], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{f^{\frac{4ac-b^2}{4c}} \operatorname{expIntegral}_1\left(-\frac{(2cx+b)^2 \ln(f)}{4c}\right)}{4c}$	40

input
```
int(f^(c*x^2+b*x+a)/(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c*f^(1/4*(4*a*c-b^2)/c)*Ei(1,-1/4*(2*c*x+b)^2*ln(f)/c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx = \frac{\mathrm{Ei}\left(\frac{(4\,c^2x^2+4\,bcx+b^2)\log(f)}{4\,c}\right)}{4\,cf^{\frac{b^2-4\,ac}{4\,c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b),x, algorithm="fricas")
```

output
```
1/4*Ei(1/4*(4*c^2*x^2 + 4*b*c*x + b^2)*log(f)/c)/(c*f^(1/4*(b^2 - 4*a*c)/c
))
```

Sympy [F]

$$\int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/(2*c*x+b),x)
```

output
```
Integral(f**(a + b*x + c*x**2)/(b + 2*c*x), x)
```

Maxima [F]

$$\int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{cx^2+bx+a}}{2\,cx+b}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b),x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(2*c*x + b), x)
```

Giac [F]

$$\int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{cx^2+bx+a}}{2\,cx+b}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(2*c*x + b), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{c\,x^2+b\,x+a}}{b+2\,c\,x}\,dx$$

input
```
int(f^(a + b*x + c*x^2)/(b + 2*c*x),x)
```

output
```
int(f^(a + b*x + c*x^2)/(b + 2*c*x), x)
```

Reduce [F]

$$\int \frac{f^{a+bx+cx^2}}{b+2cx}\,dx = f^a\left(\int \frac{f^{c\,x^2+bx}}{2cx+b}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)/(2*c*x+b),x)
```

output
```
f**a*int(f**(b*x + c*x**2)/(b + 2*c*x),x)
```

3.388 $\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} \, dx = -\frac{f^{a+bx+cx^2}}{2c(b+2cx)} + \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}}{4c^{3/2}}$$

output
```
-1/2*f^(c*x^2+b*x+a)/c/(2*c*x+b)+1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*
c*x+b)*ln(f)^(1/2)/c^(1/2))*ln(f)^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} \, dx = \frac{f^{a-\frac{b^2}{4c}}\left(-2\sqrt{c}f^{\frac{(b+2cx)^2}{4c}} + \sqrt{\pi}(b+2cx)\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}\right)}{4c^{3/2}(b+2cx)}$$

input
```
Integrate[f^(a + b*x + c*x^2)/(b + 2*c*x)^2,x]
```

output
```
(f^(a - b^2/(4*c))*(-2*Sqrt[c]*f^((b + 2*c*x)^2/(4*c)) + Sqrt[Pi]*(b + 2*c
*x)*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]*Sqrt[Log[f]]))/(4*c^(3/2)
*(b + 2*c*x))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2669, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2} \, dx$$

$$\Big\downarrow \text{2669}$$

$$\frac{\log(f)\int f^{cx^2+bx+a}dx}{2c}-\frac{f^{a+bx+cx^2}}{2c(b+2cx)}$$

$$\Big\downarrow \text{2664}$$

$$\frac{\log(f)f^{a-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}-\frac{f^{a+bx+cx^2}}{2c(b+2cx)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}\sqrt{\log(f)}f^{a-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{f^{a+bx+cx^2}}{2c(b+2cx)}$$

input
```
Int[f^(a + b*x + c*x^2)/(b + 2*c*x)^2,x]
```

output
```
-1/2*f^(a + b*x + c*x^2)/(c*(b + 2*c*x)) + (f^(a - b^2/(4*c))*Sqrt[Pi]*Erf
i[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]*Sqrt[Log[f]])/(4*c^(3/2))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2669
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] - Si
mp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2),
x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.20

method	result	size
risch	$-\frac{f^{\frac{(2cx+b)^{2}}{4c}} f^{\frac{4ac-b^{2}}{4c}}}{2c(2cx+b)}+\frac{\ln(f)\sqrt{\pi}\, f^{\frac{4ac-b^{2}}{4c}} \operatorname{erf}\left(\frac{\sqrt{-\frac{\ln(f)}{c}}\,(2cx+b)}{2}\right)}{4c^{2}\sqrt{-\frac{\ln(f)}{c}}}$	101

input
```
int(f^(c*x^2+b*x+a)/(2*c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/(2*c*x+b)*f^(1/4*(2*c*x+b)^2/c)*f^(1/4*(4*a*c-b^2)/c)+1/4/c^2*ln(f)
*Pi^(1/2)*f^(1/4*(4*a*c-b^2)/c)/(-ln(f)/c)^(1/2)*erf(1/2*(-ln(f)/c)^(1/2)*
(2*c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx = -\frac{2\,cf^{cx^2+bx+a} + \frac{\sqrt{\pi}(2\,cx+b)\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{4\,(2\,c^3x+bc^2)}$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*c*f^(c*x^2 + b*x + a) + sqrt(pi)*(2*c*x + b)*sqrt(-c*log(f))*erf(1
/2*(2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*(b^2 - 4*a*c)/c))/(2*c^3*x + b*c^
2)
```

Sympy [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx = \int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/(2*c*x+b)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)/(b + 2*c*x)**2, x)
```

Maxima [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx = \int \frac{f^{cx^2+bx+a}}{(2\,cx+b)^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^2,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(2*c*x + b)^2, x)
```

Giac [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx = \int \frac{f^{cx^2+bx+a}}{(2\,cx+b)^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(2*c*x + b)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx = \frac{f^{a-\frac{b^2}{4\,c}}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\,\ln(f)}{4\,c\,\sqrt{c\,\ln(f)}} - \frac{f^a\,f^{c\,x^2}\,f^{b\,x}}{2\,c\,(b+2\,c\,x)}$$

input
```
int(f^(a + b*x + c*x^2)/(b + 2*c*x)^2,x)
```

output
```
(f^(a - b^2/(4*c))*pi^(1/2)*erfi(((b*log(f))/2 + c*x*log(f))/(c*log(f))^(1
/2))*log(f))/(4*c*(c*log(f))^(1/2)) - (f^a*f^(c*x^2)*f^(b*x))/(2*c*(b + 2*
c*x))
```

Reduce [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^2}\,dx = f^a\left(\int \frac{f^{c\,x^2+bx}}{4c^2x^2+4bcx+b^2}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)/(2*c*x+b)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2),x)
```

3.389 $\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx=-\frac{f^{a+bx+cx^2}}{4c(b+2cx)^2}+\frac{f^{a-\frac{b^2}{4c}}\,\text{ExpIntegralEi}\left(\frac{(b+2cx)^2\log(f)}{4c}\right)\log(f)}{16c^2}$$

output
```
-1/4*f^(c*x^2+b*x+a)/c/(2*c*x+b)^2+1/16*f^(a-1/4*b^2/c)*Ei(1/4*(2*c*x+b)^2
*ln(f)/c)*ln(f)/c^2
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx=\frac{f^{a-\frac{b^2}{4c}}\left(-4cf^{\frac{(b+2cx)^2}{4c}}+(b+2cx)^2\,\text{ExpIntegralEi}\left(\frac{(b+2cx)^2\log(f)}{4c}\right)\log(f)\right)}{16c^2(b+2cx)^2}$$

input
```
Integrate[f^(a + b*x + c*x^2)/(b + 2*c*x)^3,x]
```

output
```
(f^(a - b^2/(4*c))*(-4*c*f^((b + 2*c*x)^2/(4*c)) + (b + 2*c*x)^2*ExpIntegr
alEi[((b + 2*c*x)^2*Log[f])/(4*c)]*Log[f]))/(16*c^2*(b + 2*c*x)^2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2669, 2668}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3} \, dx$$

$$\Big\downarrow \text{2669}$$

$$\frac{\log(f) \int \frac{f^{cx^2+bx+a}}{b+2cx}dx}{4c} - \frac{f^{a+bx+cx^2}}{4c(b+2cx)^2}$$

$$\Big\downarrow \text{2668}$$

$$\frac{\log(f) f^{a-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2\log(f)}{4c}\right)}{16c^2} - \frac{f^{a+bx+cx^2}}{4c(b+2cx)^2}$$

input
```
Int[f^(a + b*x + c*x^2)/(b + 2*c*x)^3,x]
```

output
```
-1/4*f^(a + b*x + c*x^2)/(c*(b + 2*c*x)^2) + (f^(a - b^2/(4*c))*ExpIntegra
lEi[((b + 2*c*x)^2*Log[f])/(4*c)]*Log[f])/(16*c^2)
```

Defintions of rubi rules used

rule 2668
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(1/(2*e))*F^(a - b^2/(4*c))*ExpIntegralEi[(b + 2*c*x)^2*(Log[F]/(
4*c))], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0]
```

rule 2669
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] - Si
mp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2),
x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.28

method	result	size
risch	$-\frac{f^{\frac{(2cx+b)^2}{4c}} f^{\frac{4ac-b^2}{4c}}}{4c(2cx+b)^2}-\frac{\ln(f) f^{\frac{4ac-b^2}{4c}} \operatorname{expIntegral}_1\left(-\frac{(2cx+b)^2 \ln(f)}{4c}\right)}{16c^2}$	88

input
```
int(f^(c*x^2+b*x+a)/(2*c*x+b)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/c/(2*c*x+b)^2*f^(1/4*(2*c*x+b)^2/c)*f^(1/4*(4*a*c-b^2)/c)-1/16/c^2*ln
(f)*f^(1/4*(4*a*c-b^2)/c)*Ei(1,-1/4*(2*c*x+b)^2*ln(f)/c)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx=-\frac{4\,cf^{cx^2+bx+a}-\frac{(4\,c^2x^2+4\,bcx+b^2)\operatorname{Ei}\left(\frac{(4\,c^2x^2+4\,bcx+b^2)\log(f)}{4\,c}\right)\log(f)}{f^{\frac{b^2-4\,ac}{4\,c}}}}{16\,(4\,c^4x^2+4\,bc^3x+b^2c^2)}$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^3,x, algorithm="fricas")
```

output
```
-1/16*(4*c*f^(c*x^2 + b*x + a) - (4*c^2*x^2 + 4*b*c*x + b^2)*Ei(1/4*(4*c^2
*x^2 + 4*b*c*x + b^2)*log(f)/c)*log(f)/f^(1/4*(b^2 - 4*a*c)/c))/(4*c^4*x^2
 + 4*b*c^3*x + b^2*c^2)
```

Sympy [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx = \int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)/(2*c*x+b)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)/(b + 2*c*x)**3, x)
```

Maxima [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx = \int \frac{f^{cx^2+bx+a}}{(2\,cx+b)^3}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^3,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(2*c*x + b)^3, x)
```

Giac [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx = \int \frac{f^{cx^2+bx+a}}{(2\,cx+b)^3}\,dx$$

input
```
integrate(f^(c*x^2+b*x+a)/(2*c*x+b)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x + a)/(2*c*x + b)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx = \int \frac{f^{c\,x^2+b\,x+a}}{{\left(b+2\,c\,x\right)}^3}\,dx$$

input
```
int(f^(a + b*x + c*x^2)/(b + 2*c*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)/(b + 2*c*x)^3, x)
```

Reduce [F]

$$\int \frac{f^{a+bx+cx^2}}{(b+2cx)^3}\,dx = \text{too large to display}$$

input
```
int(f^(c*x^2+b*x+a)/(2*c*x+b)^3,x)
```

output

```
(f**a*( - f**(b*x + c*x**2) + 2*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log
(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3
*c - 24*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f)**2*b**6 + 8
*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c
**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3
*x**2 - 32*c**4*x**3),x)*log(f)**2*b**5*c*x + 8*int(f**(b*x + c*x**2)/(log
(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c*
*3*x**3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*lo
g(f)**2*b**4*c**2*x**2 - 16*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*
b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c -
 24*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f)*b**4*c - 64*int
(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*
x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**
2 - 32*c**4*x**3),x)*log(f)*b**3*c**2*x - 64*int(f**(b*x + c*x**2)/(log(f)
*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*
x**3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f
)*b**2*c**3*x**2 + 32*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c
*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b*
*2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*b**2*c**2 + 128*int(f**(b*x
+ c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 +...
```

3.390 $\int f^{bx+cx^2}(b+2cx)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = -\frac{4cf^{bx+cx^2}}{\log^2(f)}+\frac{f^{bx+cx^2}(b+2cx)^2}{\log(f)}$$

output
```
-4*c*f^(c*x^2+b*x)/ln(f)^2+f^(c*x^2+b*x)*(2*c*x+b)^2/ln(f)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = \frac{f^{x(b+cx)}(-4c+(b+2cx)^2\log(f))}{\log^2(f)}$$

input
```
Integrate[f^(b*x + c*x^2)*(b + 2*c*x)^3,x]
```

output
```
(f^(x*(b + c*x))*(-4*c + (b + 2*c*x)^2*Log[f]))/Log[f]^2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2667, 2666}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)^3 f^{bx+cx^2} \, dx$$

$$\Big\downarrow \; 2667$$

$$\frac{(b+2cx)^2 f^{bx+cx^2}}{\log(f)}-\frac{4c \int f^{cx^2+bx}(b+2cx)dx}{\log(f)}$$

$$\Big\downarrow \; 2666$$

$$\frac{(b+2cx)^2 f^{bx+cx^2}}{\log(f)}-\frac{4cf^{bx+cx^2}}{\log^2(f)}$$

input
```
Int[f^(b*x + c*x^2)*(b + 2*c*x)^3,x]
```

output
```
(-4*c*f^(b*x + c*x^2))/Log[f]^2 + (f^(b*x + c*x^2)*(b + 2*c*x)^2)/Log[f]
```

Defintions of rubi rules used

rule 2666
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

rule 2667
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] -
 Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x + c*x^2
), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && GtQ[m,
1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{\left(4 \ln \left(f \right) c^{2} x^{2}+4 c b \ln \left(f \right) x +\ln \left(f \right) b^{2}-4 c \right) f^{x \left(c x +b \right)}}{\ln \left(f \right)^{2}}$	42
gosper	$\frac{\left(4 \ln \left(f \right) c^{2} x^{2}+4 c b \ln \left(f \right) x +\ln \left(f \right) b^{2}-4 c \right) f^{c \,x^{2}+b x}}{\ln \left(f \right)^{2}}$	44
orering	$\frac{\left(4 \ln \left(f \right) c^{2} x^{2}+4 c b \ln \left(f \right) x +\ln \left(f \right) b^{2}-4 c \right) f^{c \,x^{2}+b x}}{\ln \left(f \right)^{2}}$	44
norman	$\frac{\left(\ln \left(f \right) b^{2}-4 c \right) {\mathrm e}^{\left(c \,x^{2}+b x \right) \ln \left(f \right)}}{\ln \left(f \right)^{2}}+\frac{4 c^{2} x^{2} {\mathrm e}^{\left(c \,x^{2}+b x \right) \ln \left(f \right)}}{\ln \left(f \right)}+\frac{4 c b x \,{\mathrm e}^{\left(c \,x^{2}+b x \right) \ln \left(f \right)}}{\ln \left(f \right)}$	77
parallelrisch	$\frac{4 c^{2} f^{c \,x^{2}+b x} x^{2} \ln \left(f \right)+4 c b x \,f^{c \,x^{2}+b x} \ln \left(f \right)+\ln \left(f \right) f^{c \,x^{2}+b x} b^{2}-4 c \,f^{c \,x^{2}+b x}}{\ln \left(f \right)^{2}}$	77

input
```
int(f^(c*x^2+b*x)*(2*c*x+b)^3,x,method=_RETURNVERBOSE)
```

output
```
(4*ln(f)*c^2*x^2+4*c*b*ln(f)*x+ln(f)*b^2-4*c)/ln(f)^2*f^(x*(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int f^{bx+cx^2}(b+2cx)^3\,dx=\frac{((4\,c^2x^2+4\,bcx+b^2)\log(f)-4\,c)f^{cx^2+bx}}{\log(f)^2}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b)^3,x, algorithm="fricas")
```

output
```
((4*c^2*x^2 + 4*b*c*x + b^2)*log(f) - 4*c)*f^(c*x^2 + b*x)/log(f)^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(39) = 78$.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.93

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = \begin{cases} \frac{f^{bx+cx^2}(b^2\log(f)+4bcx\log(f)+4c^2x^2\log(f)-4c)}{\log(f)^2} & \text{for } \log(f)^2 \neq 0 \\ b^3x+3b^2cx^2+4bc^2x^3+2c^3x^4 & \text{otherwise} \end{cases}$$

input
```
integrate(f**(c*x**2+b*x)*(2*c*x+b)**3,x)
```

output
```
Piecewise((f**(b*x + c*x**2)*(b**2*log(f) + 4*b*c*x*log(f) + 4*c**2*x**2*l
og(f) - 4*c)/log(f)**2, Ne(log(f)**2, 0)), (b**3*x + 3*b**2*c*x**2 + 4*b*c
**2*x**3 + 2*c**3*x**4, True))
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.24 (sec) , antiderivative size = 536, normalized size of antiderivative = 12.47

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b)^3,x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*b^3*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sq
rt(-c*log(f))*f^(1/4*b^2/c)) - 3/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-
(2*c*x + b)^2*log(f)/c)) - 1)*log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*l
og(f))^(3/2)) - 2*c*f^(1/4*(2*c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*b^2*c
/(sqrt(c*log(f))*f^(1/4*b^2/c)) + 3/2*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*s
qrt(-(2*c*x + b)^2*log(f)/c)) - 1)*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)
*(c*log(f))^(5/2)) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/
c)*log(f)^3/((-(2*c*x + b)^2*log(f)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(
1/4*(2*c*x + b)^2/c)*log(f)^2/(c*log(f))^(5/2))*b*c^2/(sqrt(c*log(f))*f^(1
/4*b^2/c)) - 1/2*(sqrt(pi)*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2*lo
g(f)/c)) - 1)*log(f)^4/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f))^(7/2)) -
12*(2*c*x + b)^3*b*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log(f)^4/((-(2*
c*x + b)^2*log(f)/c)^(3/2)*(c*log(f))^(7/2)) - 6*b^2*c*f^(1/4*(2*c*x + b)^
2/c)*log(f)^3/(c*log(f))^(7/2) + 8*c^2*gamma(2, -1/4*(2*c*x + b)^2*log(f)/
c)*log(f)^2/(c*log(f))^(7/2))*c^3/(sqrt(c*log(f))*f^(1/4*b^2/c))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 742, normalized size of antiderivative = 17.26

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b)^3,x, algorithm="giac")
```

output
```
(2*((b^2*log(abs(f)) + 4*(c*x^2 + b*x)*c*log(abs(f)) - 4*c)*(pi^2*sgn(f) -
 pi^2 + 2*log(abs(f))^2)/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2 + 4*(pi
*log(abs(f))*sgn(f) - pi*log(abs(f)))^2) + (pi*b^2*sgn(f) + 4*pi*(c*x^2 +
b*x)*c*sgn(f) - pi*b^2 - 4*pi*(c*x^2 + b*x)*c)*(pi*log(abs(f))*sgn(f) - pi
*log(abs(f)))/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2 + 4*(pi*log(abs(f)
)*sgn(f) - pi*log(abs(f)))^2))*cos(-1/2*pi*c*x^2*sgn(f) + 1/2*pi*c*x^2 - 1
/2*pi*b*x*sgn(f) + 1/2*pi*b*x) + ((pi*b^2*sgn(f) + 4*pi*(c*x^2 + b*x)*c*sg
n(f) - pi*b^2 - 4*pi*(c*x^2 + b*x)*c)*(pi^2*sgn(f) - pi^2 + 2*log(abs(f))^
2)/((pi^2*sgn(f) - pi^2 + 2*log(abs(f))^2)^2 + 4*(pi*log(abs(f))*sgn(f) -
pi*log(abs(f)))^2) - 4*(b^2*log(abs(f)) + 4*(c*x^2 + b*x)*c*log(abs(f)) -
4*c)*(pi*log(abs(f))*sgn(f) - pi*log(abs(f)))/((pi^2*sgn(f) - pi^2 + 2*log
(abs(f))^2)^2 + 4*(pi*log(abs(f))*sgn(f) - pi*log(abs(f)))^2))*sin(-1/2*pi
*c*x^2*sgn(f) + 1/2*pi*c*x^2 - 1/2*pi*b*x*sgn(f) + 1/2*pi*b*x))*abs(f)^(c*
x^2 + b*x) - 1/2*I*abs(f)^(c*x^2 + b*x)*((pi*b^2*sgn(f) + 4*pi*(c*x^2 + b*
x)*c*sgn(f) - pi*b^2 - 4*pi*(c*x^2 + b*x)*c - 2*I*b^2*log(abs(f)) + 8*(-I*
c*x^2 - I*b*x)*c*log(abs(f)) + 8*I*c)*e^(1/2*I*pi*c*x^2*sgn(f) - 1/2*I*pi*
c*x^2 + 1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x)/(pi^2*sgn(f) + 2*I*pi*log(abs(
f))*sgn(f) - pi^2 - 2*I*pi*log(abs(f)) + 2*log(abs(f))^2) + (pi*b^2*sgn(f)
 + 4*pi*(c*x^2 + b*x)*c*sgn(f) - pi*b^2 - 4*pi*(c*x^2 + b*x)*c + 2*I*b^2*l
og(abs(f)) - 8*(-I*c*x^2 - I*b*x)*c*log(abs(f)) - 8*I*c)*e^(-1/2*I*pi*c...
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = \frac{f^{c\,x^2+b\,x}\,\left(\ln\left(f\right)\,b^2+4\,\ln\left(f\right)\,b\,c\,x+4\,\ln\left(f\right)\,c^2\,x^2-4\,c\right)}{{\ln\left(f\right)}^2}$$

input
```
int(f^(b*x + c*x^2)*(b + 2*c*x)^3,x)
```

output
```
(f^(b*x + c*x^2)*(b^2*log(f) - 4*c + 4*c^2*x^2*log(f) + 4*b*c*x*log(f)))/l
og(f)^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)^3\,dx = \frac{f^{c\,x^2+bx}(\log(f)\,b^2+4\log(f)\,bcx+4\log(f)\,c^2x^2-4c)}{\log(f)^2}$$

input
```
int(f^(c*x^2+b*x)*(2*c*x+b)^3,x)
```

output
```
(f**(b*x + c*x**2)*(log(f)*b**2 + 4*log(f)*b*c*x + 4*log(f)*c**2*x**2 - 4*
c))/log(f)**2
```

3.391 $\int f^{bx+cx^2}(b+2cx)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int f^{bx+cx^2}(b+2cx)^2\,dx = -\frac{\sqrt{c}f^{-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}+\frac{f^{bx+cx^2}(b+2cx)}{\log(f)}$$

output
```
-c^(1/2)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/(f^(1/4*b^2/c))/
ln(f)^(3/2)+f^(c*x^2+b*x)*(2*c*x+b)/ln(f)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int f^{bx+cx^2}(b+2cx)^2\,dx = \frac{f^{-\frac{b^2}{4c}}\left(-\sqrt{c}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+f^{\frac{(b+2cx)^2}{4c}}(b+2cx)\sqrt{\log(f)}\right)}{\log^{\frac{3}{2}}(f)}$$

input
```
Integrate[f^(b*x + c*x^2)*(b + 2*c*x)^2,x]
```

output
```
(-(Sqrt[c]*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])]) + f^((b
+ 2*c*x)^2/(4*c))*(b + 2*c*x)*Sqrt[Log[f]])/(f^(b^2/(4*c))*Log[f]^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2667, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)^2 f^{bx+cx^2} \, dx$$

$$\Big\downarrow \text{2667}$$

$$\frac{(b+2cx)f^{bx+cx^2}}{\log(f)}-\frac{2c\int f^{cx^2+bx}dx}{\log(f)}$$

$$\Big\downarrow \text{2664}$$

$$\frac{(b+2cx)f^{bx+cx^2}}{\log(f)}-\frac{2cf^{-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{\log(f)}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(b+2cx)f^{bx+cx^2}}{\log(f)}-\frac{\sqrt{\pi}\sqrt{c}f^{-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{\log^{\frac{3}{2}}(f)}$$

input
```
Int[f^(b*x + c*x^2)*(b + 2*c*x)^2,x]
```

output
```
-((Sqrt[c]*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(f^(b^2/
(4*c))*Log[f]^(3/2))) + (f^(b*x + c*x^2)*(b + 2*c*x))/Log[f]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2667
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] -
 Simp[(m - 1)*(e^2/(2*c*Log[F]))   Int[(d + e*x)^(m - 2)*F^(a + b*x + c*x^2
), x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && GtQ[m,
1]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

method	result	size
risch	$\frac{2cx\,f^{bx}f^{c\,x^2}}{\ln(f)}+\frac{b\,f^{bx}f^{c\,x^2}}{\ln(f)}+\frac{c\sqrt{\pi}\,f^{-\frac{b^2}{4c}}\,\mathrm{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)}{2\sqrt{-c\ln(f)}}\right)}{\ln(f)\sqrt{-c\ln(f)}}$	90

input
```
int(f^(c*x^2+b*x)*(2*c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
2*c/ln(f)*x*f^(b*x)*f^(c*x^2)+b/ln(f)*f^(b*x)*f^(c*x^2)+c/ln(f)*Pi^(1/2)*f
^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*b*ln(f)/(-c*ln(
f))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int f^{bx+cx^2}(b+2cx)^2\,dx = \frac{(2\,cx+b)f^{cx^2+bx}\log(f) + \frac{\sqrt{\pi}\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2}{4\,c}}}}{\log(f)^2}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b)^2,x, algorithm="fricas")
```

output
```
((2*c*x + b)*f^(c*x^2 + b*x)*log(f) + sqrt(pi)*sqrt(-c*log(f))*erf(1/2*(2*
c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*b^2/c))/log(f)^2
```

Sympy [F]

$$\int f^{bx+cx^2}(b+2cx)^2\,dx = \int f^{bx+cx^2}(b+2cx)^2\,dx$$

input
```
integrate(f**(c*x**2+b*x)*(2*c*x+b)**2,x)
```

output
```
Integral(f**(b*x + c*x**2)*(b + 2*c*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(63) = 126.

Time = 0.16 (sec) , antiderivative size = 329, normalized size of antiderivative = 4.39

$$\int f^{bx+cx^2}(b+2cx)^2\,dx = \frac{\sqrt{\pi}b^2\operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{2\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}} - \frac{\left(\frac{\sqrt{\pi}(2cx+b)b\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^2}{\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{3}{2}}} - \frac{2cf^{\frac{(2cx+b)^2}{4c}}\log(f)}{(c\log(f))^{\frac{3}{2}}}\right)bc}{\sqrt{c\log(f)}f^{\frac{b^2}{4c}}} + \frac{\left(\frac{\sqrt{\pi}(2cx+b)b^2\left(\operatorname{erf}\left(\frac{1}{2}\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}\right)-1\right)\log(f)^3}{\sqrt{-\frac{(2cx+b)^2\log(f)}{c}}(c\log(f))^{\frac{5}{2}}} - \frac{4(2cx+b)^3\Gamma\left(\frac{3}{2},-\frac{(2cx+b)^2\log(f)}{4c}\right)\log(f)^3}{\left(-\frac{(2cx+b)^2\log(f)}{c}\right)^{\frac{3}{2}}(c\log(f))^{\frac{5}{2}}} - \frac{4bcf^{\frac{(2cx+b)^2}{4c}}\log(f)^2}{(c\log(f))^{\frac{5}{2}}}\right)c^2}{2\sqrt{c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b)^2,x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*b^2*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sq
rt(-c*log(f))*f^(1/4*b^2/c)) - (sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c
*x + b)^2*log(f)/c)) - 1)*log(f)^2/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*log(f
))^(3/2)) - 2*c*f^(1/4*(2*c*x + b)^2/c)*log(f)/(c*log(f))^(3/2))*b*c/(sqrt
(c*log(f))*f^(1/4*b^2/c)) + 1/2*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(
2*c*x + b)^2*log(f)/c)) - 1)*log(f)^3/(sqrt(-(2*c*x + b)^2*log(f)/c)*(c*lo
g(f))^(5/2)) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2*log(f)/c)*log
(f)^3/((-(2*c*x + b)^2*log(f)/c)^(3/2)*(c*log(f))^(5/2)) - 4*b*c*f^(1/4*(2
*c*x + b)^2/c)*log(f)^2/(c*log(f))^(5/2))*c^2/(sqrt(c*log(f))*f^(1/4*b^2/c
))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int f^{bx+cx^2}(b+2cx)^2\,dx = \frac{c\left(2x+\frac{b}{c}\right)e^{\left(cx^2\log(f)+bx\log(f)\right)}}{\log(f)} + \frac{\sqrt{\pi}c\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)}{\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}\log(f)}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b)^2,x, algorithm="giac")
```

output
```
c*(2*x + b/c)*e^(c*x^2*log(f) + b*x*log(f))/log(f) + sqrt(pi)*c*erf(-1/2*s
qrt(-c*log(f))*(2*x + b/c))/(sqrt(-c*log(f))*f^(1/4*b^2/c)*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int f^{bx+cx^2}(b+2cx)^2\,dx=\frac{b\,f^{c\,x^2}\,f^{b\,x}}{\ln(f)}+\frac{2\,c\,f^{c\,x^2}\,f^{b\,x}\,x}{\ln(f)}-\frac{c\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)}{f^{\frac{b^2}{4\,c}}\,\ln(f)\,\sqrt{c\,\ln(f)}}$$

input
```
int(f^(b*x + c*x^2)*(b + 2*c*x)^2,x)
```

output
```
(b*f^(c*x^2)*f^(b*x))/log(f) + (2*c*f^(c*x^2)*f^(b*x)*x)/log(f) - (c*pi^(1
/2)*erfi(((b*log(f))/2 + c*x*log(f))/(c*log(f))^(1/2)))/(f^(b^2/(4*c))*log
(f)*(c*log(f))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.61

$$\int f^{bx+cx^2}(b+2cx)^2\,dx$$
$$=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\log(f)bi+2\log(f)cix}{2\sqrt{c}\,\sqrt{\log(f)}}\right)ci+f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}\,b+2f^{c\,x^2+bx}e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}\,cx}{e^{\frac{\log(f)b^2}{4c}}\sqrt{c}\,\sqrt{\log(f)}\log(f)}$$

input
```
int(f^(c*x^2+b*x)*(2*c*x+b)^2,x)
```

output
```
(sqrt(pi)*erf((log(f)*b*i + 2*log(f)*c*i*x)/(2*sqrt(c)*sqrt(log(f))))*c*i
+ f**(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*b + 2*f*
*(b*x + c*x**2)*e**((log(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*c*x)/(e**((l
og(f)*b**2)/(4*c))*sqrt(c)*sqrt(log(f))*log(f))
```

3.392 $\int f^{bx+cx^2}(b+2cx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 16

$$\int f^{bx+cx^2}(b+2cx)\,dx=\frac{f^{bx+cx^2}}{\log(f)}$$

output
```
f^(c*x^2+b*x)/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)\,dx=\frac{f^{bx+cx^2}}{\log(f)}$$

input
```
Integrate[f^(b*x + c*x^2)*(b + 2*c*x),x]
```

output
```
f^(b*x + c*x^2)/Log[f]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{2666\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)f^{bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 2666}$$

$$\frac{f^{bx+cx^2}}{\log(f)}$$

input
```
Int[f^(b*x + c*x^2)*(b + 2*c*x),x]
```

output
```
f^(b*x + c*x^2)/Log[f]
```

Defintions of rubi rules used

rule 2666
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{f^{x(cx+b)}}{\ln(f)}$	15
gosper	$\frac{f^{c\,x^2+bx}}{\ln(f)}$	17
derivativedivides	$\frac{f^{c\,x^2+bx}}{\ln(f)}$	17
default	$\frac{f^{c\,x^2+bx}}{\ln(f)}$	17
parallelrisch	$\frac{f^{c\,x^2+bx}}{\ln(f)}$	17
orering	$\frac{f^{c\,x^2+bx}}{\ln(f)}$	17
norman	$\frac{e^{\left(c\,x^2+bx\right)\ln(f)}}{\ln(f)}$	19

input
```
int(f^(c*x^2+b*x)*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
1/ln(f)*f^(x*(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)\,dx=\frac{f^{cx^2+bx}}{\log(f)}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b),x, algorithm="fricas")
```

output
```
f^(c*x^2 + b*x)/log(f)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int f^{bx+cx^2}(b+2cx)\,dx = \begin{cases} \frac{f^{bx+cx^2}}{\log(f)} & \text{for } \log(f) \neq 0 \\ bx+cx^2 & \text{otherwise} \end{cases}$$

input
```
integrate(f**(c*x**2+b*x)*(2*c*x+b),x)
```

output
```
Piecewise((f**(b*x + c*x**2)/log(f), Ne(log(f), 0)), (b*x + c*x**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)\,dx = \frac{f^{cx^2+bx}}{\log(f)}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b),x, algorithm="maxima")
```

output
```
f^(c*x^2 + b*x)/log(f)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)\,dx = \frac{f^{cx^2+bx}}{\log(f)}$$

input
```
integrate(f^(c*x^2+b*x)*(2*c*x+b),x, algorithm="giac")
```

output
```
f^(c*x^2 + b*x)/log(f)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)\,dx=\frac{f^{c\,x^2+b\,x}}{\ln{(f)}}$$

input
```
int(f^(b*x + c*x^2)*(b + 2*c*x),x)
```

output
```
f^(b*x + c*x^2)/log(f)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int f^{bx+cx^2}(b+2cx)\,dx=\frac{f^{c\,x^2+bx}}{\log{(f)}}$$

input
```
int(f^(c*x^2+b*x)*(2*c*x+b),x)
```

output
```
f**(b*x + c*x**2)/log(f)
```

3.393 $\int \frac{f^{bx+cx^2}}{b+2cx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{f^{bx+cx^2}}{b+2cx}\,dx = \frac{f^{-\frac{b^2}{4c}}\,\text{ExpIntegralEi}\left(\frac{(b+2cx)^2\log(f)}{4c}\right)}{4c}$$

output
```
1/4*Ei(1/4*(2*c*x+b)^2*ln(f)/c)/c/(f^(1/4*b^2/c))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{f^{bx+cx^2}}{b+2cx}\,dx = \frac{f^{-\frac{b^2}{4c}}\,\text{ExpIntegralEi}\left(\frac{(b+2cx)^2\log(f)}{4c}\right)}{4c}$$

input
```
Integrate[f^(b*x + c*x^2)/(b + 2*c*x),x]
```

output
```
ExpIntegralEi[((b + 2*c*x)^2*Log[f])/(4*c)]/(4*c*f^(b^2/(4*c)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2668}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{bx+cx^2}}{b+2cx} \, dx$$

$$\Big\downarrow \text{2668}$$

$$\frac{f^{-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{4c}$$

input
```
Int[f^(b*x + c*x^2)/(b + 2*c*x),x]
```

output
```
ExpIntegralEi[((b + 2*c*x)^2*Log[f])/(4*c)]/(4*c*f^(b^2/(4*c)))
```

Defintions of rubi rules used

rule 2668
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(1/(2*e))*F^(a - b^2/(4*c))*ExpIntegralEi[(b + 2*c*x)^2*(Log[F]/(
4*c))], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{f^{-\frac{b^2}{4c}} \text{expIntegral}_1\left(-\frac{(2cx+b)^2 \ln(f)}{4c}\right)}{4c}$	33

input
```
int(f^(c*x^2+b*x)/(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
-1/4/c*f^(-1/4*b^2/c)*Ei(1,-1/4*(2*c*x+b)^2*ln(f)/c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{f^{bx+cx^2}}{b+2cx} \, dx = \frac{\mathrm{Ei}\left(\frac{(4 \, c^2x^2+4 \, bcx+b^2) \log(f)}{4 \, c}\right)}{4 \, cf^{\frac{b^2}{4 \, c}}}$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b),x, algorithm="fricas")
```

output
```
1/4*Ei(1/4*(4*c^2*x^2 + 4*b*c*x + b^2)*log(f)/c)/(c*f^(1/4*b^2/c))
```

Sympy [F]

$$\int \frac{f^{bx+cx^2}}{b+2cx} \, dx = \int \frac{f^{bx+cx^2}}{b+2cx} \, dx$$

input
```
integrate(f**(c*x**2+b*x)/(2*c*x+b),x)
```

output
```
Integral(f**(b*x + c*x**2)/(b + 2*c*x), x)
```

Maxima [F]

$$\int \frac{f^{bx+cx^2}}{b+2cx} \, dx = \int \frac{f^{cx^2+bx}}{2 \, cx+b} \, dx$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b),x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x)/(2*c*x + b), x)
```

Giac [F]

$$\int \frac{f^{bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{cx^2+bx}}{2\,cx+b}\,dx$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b),x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x)/(2*c*x + b), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{c\,x^2+b\,x}}{b+2\,c\,x}\,dx$$

input
```
int(f^(b*x + c*x^2)/(b + 2*c*x),x)
```

output
```
int(f^(b*x + c*x^2)/(b + 2*c*x), x)
```

Reduce [F]

$$\int \frac{f^{bx+cx^2}}{b+2cx}\,dx = \int \frac{f^{c\,x^2+bx}}{2cx+b}dx$$

input
```
int(f^(c*x^2+b*x)/(2*c*x+b),x)
```

output
```
int(f**(b*x + c*x**2)/(b + 2*c*x),x)
```

3.394 $\int \frac{f^{bx+cx^2}}{(b+2cx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2}\,dx = -\frac{f^{bx+cx^2}}{2c(b+2cx)} + \frac{f^{-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}}{4c^{3/2}}$$

output
```
-1/2*f^(c*x^2+b*x)/c/(2*c*x+b)+1/4*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)
/c^(1/2))*ln(f)^(1/2)/c^(3/2)/(f^(1/4*b^2/c))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2}\,dx = \frac{f^{-\frac{b^2}{4c}}\left(-2\sqrt{c}f^{\frac{(b+2cx)^2}{4c}} + \sqrt{\pi}(b+2cx)\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)\sqrt{\log(f)}\right)}{4c^{3/2}(b+2cx)}$$

input
```
Integrate[f^(b*x + c*x^2)/(b + 2*c*x)^2,x]
```

output
```
(-2*Sqrt[c]*f^((b + 2*c*x)^2/(4*c)) + Sqrt[Pi]*(b + 2*c*x)*Erfi[((b + 2*c*
x)*Sqrt[Log[f]])/(2*Sqrt[c])]*Sqrt[Log[f]])/(4*c^(3/2)*f^(b^2/(4*c))*(b +
2*c*x))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2669, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2} \, dx$$

$$\Big\downarrow \ 2669$$

$$\frac{\log(f)\int f^{cx^2+bx}dx}{2c}-\frac{f^{bx+cx^2}}{2c(b+2cx)}$$

$$\Big\downarrow \ 2664$$

$$\frac{\log(f)f^{-\frac{b^2}{4c}}\int f^{\frac{(b+2cx)^2}{4c}}dx}{2c}-\frac{f^{bx+cx^2}}{2c(b+2cx)}$$

$$\Big\downarrow \ 2633$$

$$\frac{\sqrt{\pi}\sqrt{\log(f)}f^{-\frac{b^2}{4c}}\text{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4c^{3/2}}-\frac{f^{bx+cx^2}}{2c(b+2cx)}$$

input
```
Int[f^(b*x + c*x^2)/(b + 2*c*x)^2,x]
```

output
```
-1/2*f^(b*x + c*x^2)/(c*(b + 2*c*x)) + (Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Lo
g[f]])/(2*Sqrt[c])]*Sqrt[Log[f]])/(4*c^(3/2)*f^(b^2/(4*c)))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2669
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] - Si
mp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2),
x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

method	result	size
risch	$-\frac{f^{\frac{(2cx+b)^{2}}{4c}} f^{-\frac{b^{2}}{4c}}}{2c(2cx+b)}+\frac{\ln(f)\sqrt{\pi}\, f^{-\frac{b^{2}}{4c}} \operatorname{erf}\left(\frac{\sqrt{-\frac{\ln(f)}{c}}\,(2cx+b)}{2}\right)}{4c^{2}\sqrt{-\frac{\ln(f)}{c}}}$	87

input
```
int(f^(c*x^2+b*x)/(2*c*x+b)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c/(2*c*x+b)*f^(1/4*(2*c*x+b)^2/c)*f^(-1/4*b^2/c)+1/4/c^2*ln(f)*Pi^(1/
2)*f^(-1/4*b^2/c)/(-ln(f)/c)^(1/2)*erf(1/2*(-ln(f)/c)^(1/2)*(2*c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx = -\frac{2\,cf^{cx^2+bx} + \frac{\sqrt{\pi}(2\,cx+b)\sqrt{-c\log(f)}\,\mathrm{erf}\left(\frac{(2\,cx+b)\sqrt{-c\log(f)}}{2\,c}\right)}{f^{\frac{b^2}{4\,c}}}}{4\,(2\,c^3x+bc^2)}$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*c*f^(c*x^2 + b*x) + sqrt(pi)*(2*c*x + b)*sqrt(-c*log(f))*erf(1/2*(
2*c*x + b)*sqrt(-c*log(f))/c)/f^(1/4*b^2/c))/(2*c^3*x + b*c^2)
```

Sympy [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx = \int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx$$

input
```
integrate(f**(c*x**2+b*x)/(2*c*x+b)**2,x)
```

output
```
Integral(f**(b*x + c*x**2)/(b + 2*c*x)**2, x)
```

Maxima [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2} dx = \int \frac{f^{cx^2+bx}}{(2\,cx+b)^2} dx$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b)^2,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x)/(2*c*x + b)^2, x)
```

Giac [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2}\,dx = \int \frac{f^{cx^2+bx}}{(2\,cx+b)^2}\,dx$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b)^2,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x)/(2*c*x + b)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\frac{b\,\ln(f)}{2}+c\,x\,\ln(f)}{\sqrt{c\,\ln(f)}}\right)\,\ln(f)}{4\,c\,f^{\frac{b^2}{4\,c}}\,\sqrt{c\,\ln(f)}} - \frac{f^{c\,x^2}\,f^{b\,x}}{2\,c\,(b+2\,c\,x)}$$

input
```
int(f^(b*x + c*x^2)/(b + 2*c*x)^2,x)
```

output
```
(pi^(1/2)*erfi(((b*log(f))/2 + c*x*log(f))/(c*log(f))^(1/2))*log(f))/(4*c*
f^(b^2/(4*c))*(c*log(f))^(1/2)) - (f^(c*x^2)*f^(b*x))/(2*c*(b + 2*c*x))
```

Reduce [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^2}\,dx = \int \frac{f^{c\,x^2+bx}}{4c^2x^2+4bcx+b^2}dx$$

input
```
int(f^(c*x^2+b*x)/(2*c*x+b)^2,x)
```

output
```
int(f**(b*x + c*x**2)/(b**2 + 4*b*c*x + 4*c**2*x**2),x)
```

3.395 $\int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx = -\frac{f^{bx+cx^2}}{4c(b+2cx)^2} + \frac{f^{-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right) \log(f)}{16c^2}$$

output
```
-1/4*f^(c*x^2+b*x)/c/(2*c*x+b)^2+1/16*Ei(1/4*(2*c*x+b)^2*ln(f)/c)*ln(f)/c^
2/(f^(1/4*b^2/c))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} dx = \frac{f^{-\frac{b^2}{4c}}\left(-4cf^{\frac{(b+2cx)^2}{4c}} + (b+2cx)^2 \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right) \log(f)\right)}{16c^2(b+2cx)^2}$$

input
```
Integrate[f^(b*x + c*x^2)/(b + 2*c*x)^3,x]
```

output
```
(-4*c*f^((b + 2*c*x)^2/(4*c)) + (b + 2*c*x)^2*ExpIntegralEi[((b + 2*c*x)^2
*Log[f])/(4*c)]*Log[f])/(16*c^2*f^(b^2/(4*c))*(b + 2*c*x)^2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2669, 2668}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} \, dx$$

$$\Big\downarrow \text{2669}$$

$$\frac{\log(f) \int \frac{f^{cx^2+bx}}{b+2cx} dx}{4c} - \frac{f^{bx+cx^2}}{4c(b+2cx)^2}$$

$$\Big\downarrow \text{2668}$$

$$\frac{\log(f) f^{-\frac{b^2}{4c}} \text{ExpIntegralEi}\left(\frac{(b+2cx)^2 \log(f)}{4c}\right)}{16c^2} - \frac{f^{bx+cx^2}}{4c(b+2cx)^2}$$

input
```
Int[f^(b*x + c*x^2)/(b + 2*c*x)^3,x]
```

output
```
-1/4*f^(b*x + c*x^2)/(c*(b + 2*c*x)^2) + (ExpIntegralEi[((b + 2*c*x)^2*Log
[f])/(4*c)]*Log[f])/(16*c^2*f^(b^2/(4*c)))
```

Defintions of rubi rules used

rule 2668
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)/((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[(1/(2*e))*F^(a - b^2/(4*c))*ExpIntegralEi[(b + 2*c*x)^2*(Log[F]/(
4*c))], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0]
```

rule 2669
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_))^(m_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(F^(a + b*x + c*x^2)/(e*(m + 1))), x] - Si
mp[2*c*(Log[F]/(e^2*(m + 1)))   Int[(d + e*x)^(m + 2)*F^(a + b*x + c*x^2),
x], x] /; FreeQ[{F, a, b, c, d, e}, x] && EqQ[b*e - 2*c*d, 0] && LtQ[m, -1]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

method	result	size
risch	$-\frac{f^{\frac{(2cx+b)^2}{4c}} f^{-\frac{b^2}{4c}}}{4c(2cx+b)^2}-\frac{\ln(f) f^{-\frac{b^2}{4c}} \operatorname{expIntegral}_1\left(-\frac{(2cx+b)^2 \ln(f)}{4c}\right)}{16c^2}$	74

input
```
int(f^(c*x^2+b*x)/(2*c*x+b)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/c/(2*c*x+b)^2*f^(1/4*(2*c*x+b)^2/c)*f^(-1/4*b^2/c)-1/16/c^2*ln(f)*f^(
-1/4*b^2/c)*Ei(1,-1/4*(2*c*x+b)^2*ln(f)/c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.52

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3}\,dx=-\frac{4\,cf^{cx^2+bx}-\frac{(4\,c^2x^2+4\,bcx+b^2)\operatorname{Ei}\left(\frac{(4\,c^2x^2+4\,bcx+b^2)\log(f)}{4\,c}\right)\log(f)}{f^{\frac{b^2}{4\,c}}}}{16\,(4\,c^4x^2+4\,bc^3x+b^2c^2)}$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b)^3,x, algorithm="fricas")
```

output
```
-1/16*(4*c*f^(c*x^2 + b*x) - (4*c^2*x^2 + 4*b*c*x + b^2)*Ei(1/4*(4*c^2*x^2
 + 4*b*c*x + b^2)*log(f)/c)*log(f)/f^(1/4*b^2/c))/(4*c^4*x^2 + 4*b*c^3*x +
 b^2*c^2)
```

Sympy [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} \, dx = \int \frac{f^{bx+cx^2}}{(b+2cx)^3} \, dx$$

input
```
integrate(f**(c*x**2+b*x)/(2*c*x+b)**3,x)
```

output
```
Integral(f**(b*x + c*x**2)/(b + 2*c*x)**3, x)
```

Maxima [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} \, dx = \int \frac{f^{cx^2+bx}}{(2\,cx+b)^3} \, dx$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b)^3,x, algorithm="maxima")
```

output
```
integrate(f^(c*x^2 + b*x)/(2*c*x + b)^3, x)
```

Giac [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3} \, dx = \int \frac{f^{cx^2+bx}}{(2\,cx+b)^3} \, dx$$

input
```
integrate(f^(c*x^2+b*x)/(2*c*x+b)^3,x, algorithm="giac")
```

output
```
integrate(f^(c*x^2 + b*x)/(2*c*x + b)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3}\,dx = \int \frac{f^{c\,x^2+b\,x}}{\left(b+2\,c\,x\right)^3}\,dx$$

input
```
int(f^(b*x + c*x^2)/(b + 2*c*x)^3,x)
```

output
```
int(f^(b*x + c*x^2)/(b + 2*c*x)^3, x)
```

Reduce [F]

$$\int \frac{f^{bx+cx^2}}{(b+2cx)^3}\,dx = \text{too large to display}$$

input
```
int(f^(c*x^2+b*x)/(2*c*x+b)^3,x)
```

output

```
( - f**(b*x + c*x**2) + 2*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b*
*4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 2
4*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f)**2*b**6 + 8*int(f
**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x*
*2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**2
- 32*c**4*x**3),x)*log(f)**2*b**5*c*x + 8*int(f**(b*x + c*x**2)/(log(f)*b*
*5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**
3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f)**
2*b**4*c**2*x**2 - 16*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c
*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b*
*2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f)*b**4*c - 64*int(f**(b
*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 +
 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**2 - 32
*c**4*x**3),x)*log(f)*b**3*c**2*x - 64*int(f**(b*x + c*x**2)/(log(f)*b**5
+ 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 -
 4*b**3*c - 24*b**2*c**2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*log(f)*b**2
*c**3*x**2 + 32*int(f**(b*x + c*x**2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 1
2*log(f)*b**3*c**2*x**2 + 8*log(f)*b**2*c**3*x**3 - 4*b**3*c - 24*b**2*c**
2*x - 48*b*c**3*x**2 - 32*c**4*x**3),x)*b**2*c**2 + 128*int(f**(b*x + c*x*
*2)/(log(f)*b**5 + 6*log(f)*b**4*c*x + 12*log(f)*b**3*c**2*x**2 + 8*log...
```

3.396 $\int \frac{4^x}{a+2^x b} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{4^x}{a+2^x b} \, dx = \frac{2^x}{b \log(2)} - \frac{a \log(a+2^x b)}{b^2 \log(2)}$$

output
```
2^x/b/ln(2)-a*ln(a+2^x*b)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{4^x}{a+2^x b} \, dx = \frac{\frac{2^x}{b} - \frac{a \log(a+2^x b)}{b^2}}{\log(2)}$$

input
```
Integrate[4^x/(a + 2^x*b),x]
```

output
```
(2^x/b - (a*Log[a + 2^x*b])/b^2)/Log[2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{a+b2^x}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int \frac{2^x}{a+2^x b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{1}{b}-\frac{a}{b(a+2^x b)}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2^x}{b}-\frac{a\log(a+b2^x)}{b^2}}{\log(2)}$$

input
```
Int[4^x/(a + 2^x*b),x]
```

output
```
(2^x/b - (a*Log[a + 2^x*b])/b^2)/Log[2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

method	result	size
risch	$\frac{2^{x}}{b \ln \left(2\right)}-\frac{a \ln \left(2^{x}+\frac{a}{b}\right)}{b^{2} \ln \left(2\right)}$	33
norman	$\frac{{\mathrm e}^{x \ln \left(2\right)}}{b \ln \left(2\right)}-\frac{a \ln \left(a +{\mathrm e}^{x \ln \left(2\right)} b \right)}{b^{2} \ln \left(2\right)}$	35

input
```
int(4^x/(a+2^x*b),x,method=_RETURNVERBOSE)
```

output
```
2^x/b/ln(2)-1/b^2/ln(2)*a*ln(2^x+a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{4^x}{a+2^x b} \, dx=\frac{2^{x} b - a \log \left(2^{x} b + a\right)}{b^{2} \log \left(2\right)}$$

input
```
integrate(4^x/(a+2^x*b),x, algorithm="fricas")
```

output
```
(2^x*b - a*log(2^x*b + a))/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{4^x}{a+2^x b}\,dx = -\frac{a\log\left(\frac{a}{b}+e^{\frac{x\log(4)}{2}}\right)}{b^2\log(2)} + \begin{cases} \frac{e^{\frac{x\log(4)}{2}}}{b\log(2)} & \text{for } b\log(2)\neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(4**x/(a+2**x*b),x)
```

output
```
-a*log(a/b + exp(x*log(4)/2))/(b**2*log(2)) + Piecewise((exp(x*log(4)/2)/(
b*log(2)), Ne(b*log(2), 0)), (x/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{4^x}{a+2^x b}\,dx = \frac{2^x}{b\log(2)} - \frac{a\log(2^x b+a)}{b^2\log(2)}$$

input
```
integrate(4^x/(a+2^x*b),x, algorithm="maxima")
```

output
```
2^x/(b*log(2)) - a*log(2^x*b + a)/(b^2*log(2))
```

Giac [F]

$$\int \frac{4^x}{a+2^x b}\,dx = \int \frac{4^x}{2^x b+a}\,dx$$

input
```
integrate(4^x/(a+2^x*b),x, algorithm="giac")
```

output
```
integrate(4^x/(2^x*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{4^x}{a+2^x b}\,dx = -\frac{a\,\ln\left(a+2^x\,b\right)-2^x\,b}{b^2\,\ln\left(2\right)}$$

input
```
int(4^x/(a + 2^x*b),x)
```

output
```
-(a*log(a + 2^x*b) - 2^x*b)/(b^2*log(2))
```

Reduce [F]

$$\int \frac{4^x}{a+2^x b}\,dx = \int \frac{4^x}{2^x b+a}dx$$

input
```
int(4^x/(a+2^x*b),x)
```

output
```
int(4**x/(2**x*b + a),x)
```

3.397 $\int \frac{2^{2x}}{a+2^x b} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{2^{2x}}{a+2^x b} \, dx = \frac{2^x}{b \log(2)} - \frac{a \log(a+2^x b)}{b^2 \log(2)}$$

output
```
2^x/b/ln(2)-a*ln(a+2^x*b)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{2^{2x}}{a+2^x b} \, dx = \frac{\frac{2^x}{b} - \frac{a \log(a+2^x b)}{b^2}}{\log(2)}$$

input
```
Integrate[2^(2*x)/(a + 2^x*b),x]
```

output
```
(2^x/b - (a*Log[a + 2^x*b])/b^2)/Log[2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{a+b2^x}\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int \frac{2^x}{a+2^x b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{1}{b}-\frac{a}{b(a+2^x b)}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2^x}{b}-\frac{a\log(a+b2^x)}{b^2}}{\log(2)}$$

input

```
Int[2^(2*x)/(a + 2^x*b),x]
```

output

```
(2^x/b - (a*Log[a + 2^x*b])/b^2)/Log[2]
```

Defintions of rubi rules used

rule 49

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{2^{x}}{b}-\frac{a \ln \left(a+2^{x} b\right)}{b^{2}}}{\ln \left(2\right)}$	28
default	$\frac{\frac{2^{x}}{b}-\frac{a \ln \left(a+2^{x} b\right)}{b^{2}}}{\ln \left(2\right)}$	28
risch	$\frac{2^{x}}{b \ln \left(2\right)}-\frac{a \ln \left(2^{x}+\frac{a}{b}\right)}{b^{2} \ln \left(2\right)}$	33
norman	$\frac{{\mathrm e}^{x \ln \left(2\right)}}{b \ln \left(2\right)}-\frac{a \ln \left(a+{\mathrm e}^{x \ln \left(2\right)} b\right)}{b^{2} \ln \left(2\right)}$	35

input
```
int(2^(2*x)/(a+2^x*b),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(1/b*2^x-a/b^2*ln(a+2^x*b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{2^{2x}}{a+2^x b}\,dx=\frac{2^{x} b - a \log \left(2^{x} b + a\right)}{b^{2} \log \left(2\right)}$$

input
```
integrate(2^(2*x)/(a+2^x*b),x, algorithm="fricas")
```

output
```
(2^x*b - a*log(2^x*b + a))/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{2^{2x}}{a+2^x b}\,dx = -\frac{a\log\left(2^x+\frac{a}{b}\right)}{b^2\log(2)} + \begin{cases} \frac{2^x}{b\log(2)} & \text{for } b\log(2)\neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(2**(2*x)/(a+2**x*b),x)
```

output
```
-a*log(2**x + a/b)/(b**2*log(2)) + Piecewise((2**x/(b*log(2)), Ne(b*log(2)
, 0)), (x/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{2^{2x}}{a+2^x b}\,dx = \frac{2^x}{b\log(2)} - \frac{a\log(2^x b+a)}{b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a+2^x*b),x, algorithm="maxima")
```

output
```
2^x/(b*log(2)) - a*log(2^x*b + a)/(b^2*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{2^{2x}}{a+2^x b}\,dx = \frac{2^x}{b\log(2)} - \frac{a\log(|2^x b+a|)}{b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a+2^x*b),x, algorithm="giac")
```

output
```
2^x/(b*log(2)) - a*log(abs(2^x*b + a))/(b^2*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{2^{2x}}{a+2^x b}\,dx = -\frac{a\,\ln\left(a+2^x\,b\right)-2^x\,b}{b^2\,\ln\left(2\right)}$$

input
```
int(2^(2*x)/(a + 2^x*b),x)
```

output
```
-(a*log(a + 2^x*b) - 2^x*b)/(b^2*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{2^{2x}}{a+2^x b}\,dx = \frac{2^x b-\log(2^x b+a)\,a}{\log\left(2\right)b^2}$$

input
```
int(2^(2*x)/(a+2^x*b),x)
```

output
```
(2**x*b - log(2**x*b + a)*a)/(log(2)*b**2)
```

3.398 $\int \frac{4^x}{a-2^x b} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int \frac{4^x}{a-2^x b} \, dx = -\frac{2^x}{b \log(2)} - \frac{a \log(a-2^x b)}{b^2 \log(2)}$$

output
```
-2^x/b/ln(2)-a*ln(a-2^x*b)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{4^x}{a-2^x b} \, dx = -\frac{2^x b + a \log(a-2^x b)}{b^2 \log(2)}$$

input
```
Integrate[4^x/(a - 2^x*b),x]
```

output
```
-((2^x*b + a*Log[a - 2^x*b])/(b^2*Log[2]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{a-b2^x}dx$$

$$\Big\downarrow \text{ 2678}$$

$$\frac{\int \frac{2^x}{a-2^x b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(-\frac{a}{b(2^x b-a)}-\frac{1}{b}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{a\log(a-b2^x)}{b^2}-\frac{2^x}{b}}{\log(2)}$$

input
```
Int[4^x/(a - 2^x*b),x]
```

output
```
(-(2^x/b) - (a*Log[a - 2^x*b])/b^2)/Log[2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

method	result	size
risch	$-\frac{2^{x}}{b\ln(2)}-\frac{a\ln\left(2^{x}-\frac{a}{b}\right)}{\ln(2)b^{2}}$	35
norman	$-\frac{e^{x\ln(2)}}{b\ln(2)}-\frac{a\ln\left(a-e^{x\ln(2)}b\right)}{b^{2}\ln(2)}$	37

input
```
int(4^x/(a-2^x*b),x,method=_RETURNVERBOSE)
```

output
```
-2^x/b/ln(2)-a/ln(2)/b^2*ln(2^x-a/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{4^{x}}{a-2^{x}b}\,dx=-\frac{2^{x}b+a\log\left(2^{x}b-a\right)}{b^{2}\log\left(2\right)}$$

input
```
integrate(4^x/(a-2^x*b),x, algorithm="fricas")
```

output
```
-(2^x*b + a*log(2^x*b - a))/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{4^x}{a-2^x b}\,dx = -\frac{a\log\left(-\frac{a}{b}+e^{\frac{x\log(4)}{2}}\right)}{b^2\log(2)} + \begin{cases} -\frac{e^{\frac{x\log(4)}{2}}}{b\log(2)} & \text{for } b\log(2)\neq 0 \\ -\frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(4**x/(a-2**x*b),x)
```

output
```
-a*log(-a/b + exp(x*log(4)/2))/(b**2*log(2)) + Piecewise((-exp(x*log(4)/2)
/(b*log(2)), Ne(b*log(2), 0)), (-x/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{4^x}{a-2^x b}\,dx = -\frac{2^x}{b\log(2)} - \frac{a\log(2^x b-a)}{b^2\log(2)}$$

input
```
integrate(4^x/(a-2^x*b),x, algorithm="maxima")
```

output
```
-2^x/(b*log(2)) - a*log(2^x*b - a)/(b^2*log(2))
```

Giac [F]

$$\int \frac{4^x}{a-2^x b}\,dx = \int -\frac{4^x}{2^x b-a}\,dx$$

input
```
integrate(4^x/(a-2^x*b),x, algorithm="giac")
```

output
```
integrate(-4^x/(2^x*b - a), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{4^x}{a - 2^x b} \, dx = -\frac{2^x \, b + a \, \ln \left(2^x \, b - a\right)}{b^2 \, \ln \left(2\right)}$$

input
```
int(4^x/(a - 2^x*b),x)
```

output
```
-(2^x*b + a*log(2^x*b - a))/(b^2*log(2))
```

Reduce [F]

$$\int \frac{4^x}{a - 2^x b} \, dx = -\left(\int \frac{4^x}{2^x b - a} dx\right)$$

input
```
int(4^x/(a-2^x*b),x)
```

output
```
- int(4**x/(2**x*b - a),x)
```

3.399 $\int \frac{2^{2x}}{a-2^x b}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{2^{2x}}{a-2^x b}\,dx = -\frac{2^x}{b\log(2)} - \frac{a\log\left(a-2^x b\right)}{b^2\log(2)}$$

output
```
-2^x/b/ln(2)-a*ln(a-2^x*b)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{2^{2x}}{a-2^x b}\,dx = -\frac{2^x b + a\log\left(a-2^x b\right)}{b^2\log(2)}$$

input
```
Integrate[2^(2*x)/(a - 2^x*b),x]
```

output
```
-((2^x*b + a*Log[a - 2^x*b])/(b^2*Log[2]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{a-b2^x}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int \frac{2^x}{a-2^xb}d2^x}{\log(2)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(-\frac{a}{b(2^xb-a)}-\frac{1}{b}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a\log(a-b2^x)}{b^2}-\frac{2^x}{b}}{\log(2)}$$

input
```
Int[2^(2*x)/(a - 2^x*b),x]
```

output
```
(-(2^x/b) - (a*Log[a - 2^x*b])/b^2)/Log[2]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{-\frac{2^{x}}{b}-\frac{a\ln\left(a-2^{x}b\right)}{b^{2}}}{\ln\left(2\right)}$	30
default	$\frac{-\frac{2^{x}}{b}-\frac{a\ln\left(a-2^{x}b\right)}{b^{2}}}{\ln\left(2\right)}$	30
risch	$-\frac{2^{x}}{b\ln\left(2\right)}-\frac{a\ln\left(2^{x}-\frac{a}{b}\right)}{\ln\left(2\right)b^{2}}$	35
norman	$-\frac{{\mathrm e}^{x\ln\left(2\right)}}{b\ln\left(2\right)}-\frac{a\ln\left(a-{\mathrm e}^{x\ln\left(2\right)}b\right)}{b^{2}\ln\left(2\right)}$	37

input
```
int(2^(2*x)/(a-2^x*b),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(-1/b*2^x-a/b^2*ln(a-2^x*b))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{2^{2x}}{a-2^xb}\,dx=-\frac{2^xb+a\log\left(2^xb-a\right)}{b^2\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a-2^x*b),x, algorithm="fricas")
```

output
```
-(2^x*b + a*log(2^x*b - a))/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{2^{2x}}{a-2^x b}\,dx = -\frac{a\log\left(2^x-\frac{a}{b}\right)}{b^2\log\left(2\right)} + \begin{cases} -\frac{2^x}{b\log\left(2\right)} & \text{for } b\log\left(2\right)\neq 0 \\ -\frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(2**(2*x)/(a-2**x*b),x)
```

output
```
-a*log(2**x - a/b)/(b**2*log(2)) + Piecewise((-2**x/(b*log(2)), Ne(b*log(2
), 0)), (-x/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{2^{2x}}{a-2^x b}\,dx = -\frac{2^x}{b\log\left(2\right)} - \frac{a\log\left(2^x b-a\right)}{b^2\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a-2^x*b),x, algorithm="maxima")
```

output
```
-2^x/(b*log(2)) - a*log(2^x*b - a)/(b^2*log(2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{2^{2x}}{a-2^x b}\,dx = -\frac{2^x}{b\log\left(2\right)} - \frac{a\log\left(\left|2^x b-a\right|\right)}{b^2\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a-2^x*b),x, algorithm="giac")
```

output
```
-2^x/(b*log(2)) - a*log(abs(2^x*b - a))/(b^2*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{2^{2x}}{a - 2^x b}\,dx = -\frac{2^x\,b + a\,\ln\left(2^x\,b - a\right)}{b^2\,\ln\left(2\right)}$$

input
```
int(2^(2*x)/(a - 2^x*b),x)
```

output
```
-(2^x*b + a*log(2^x*b - a))/(b^2*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{2^{2x}}{a - 2^x b}\,dx = \frac{-2^x b - \log(2^x b - a)\,a}{\log\left(2\right) b^2}$$

input
```
int(2^(2*x)/(a-2^x*b),x)
```

output
```
( - (2**x*b + log(2**x*b - a)*a))/(log(2)*b**2)
```

3.400 $\int \frac{4^x}{a+2^{-x}b}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{4^x}{a+2^{-x}b}\,dx=\frac{b^2x}{a^3}+\frac{2^{-1+2x}}{a\log(2)}-\frac{2^xb}{a^2\log(2)}+\frac{b^2\log\left(a+2^{-x}b\right)}{a^3\log(2)}$$

output
```
b^2*x/a^3+2^(-1+2*x)/a/ln(2)-2^x*b/a^2/ln(2)+b^2*ln(a+b/(2^x))/a^3/ln(2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{4^x}{a+2^{-x}b}\,dx=\frac{2^xa(2^xa-2b)+2b^2\log\left(2^xa+b\right)}{a^3\log(4)}$$

input
```
Integrate[4^x/(a + b/2^x),x]
```

output
```
(2^x*a*(2^x*a - 2*b) + 2*b^2*Log[2^x*a + b])/(a^3*Log[4])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{a+b2^{-x}}dx$$

$$\Big\downarrow \text{ 2678}$$

$$-\frac{\int \frac{2^{3x}}{a+2^{-x}b}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{\int \left(-\frac{b^3}{a^3(a+2^{-x}b)}+\frac{2^xb^2}{a^3}-\frac{2^{2x}b}{a^2}+\frac{2^{3x}}{a}\right)d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{b^2\log(2^{-x})}{a^3}-\frac{b^2\log(a+b2^{-x})}{a^3}+\frac{b2^x}{a^2}-\frac{2^{2x-1}}{a}}{\log(2)}$$

input
```
Int[4^x/(a + b/2^x),x]
```

output
```
-((-(2^(-1 + 2*x)/a) + (2^x*b)/a^2 + (b^2*Log[2^(-x)])/a^3 - (b^2*Log[a +
b/2^x])/a^3)/Log[2])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{2^{2x}}{2\ln(2)a}-\frac{2^{x}b}{a^{2}\ln(2)}+\frac{b^{2}\ln\left(2^{x}+\frac{b}{a}\right)}{\ln(2)a^{3}}$	50
norman	$\frac{\mathrm{e}^{2x\ln(2)}}{2\ln(2)a}-\frac{b\,\mathrm{e}^{x\ln(2)}}{\ln(2)a^{2}}+\frac{b^{2}\ln\left(a\,\mathrm{e}^{x\ln(2)}+b\right)}{\ln(2)a^{3}}$	54

input
```
int(4^x/(a+b/(2^x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/ln(2)/a*(2^x)^2-2^x*b/a^2/ln(2)+1/ln(2)/a^3*b^2*ln(2^x+b/a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{4^x}{a+2^{-x}b}\,dx=\frac{2^{2\,x}a^2-2\cdot 2^xab+2\,b^2\log\left(2^xa+b\right)}{2\,a^3\log\left(2\right)}$$

input
```
integrate(4^x/(a+b/(2^x)),x, algorithm="fricas")
```

output
```
1/2*(2^(2*x)*a^2 - 2*2^x*a*b + 2*b^2*log(2^x*a + b))/(a^3*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{4^x}{a+2^{-x}b}\,dx = \begin{cases} \frac{2^{2x}a^2\log(2)-2\cdot 2^x ab\log(2)}{2a^3\log(2)^2} & \text{for } a^3\log(2)^2\neq 0 \\ x\left(-\frac{b^2}{a^3}+\frac{a^2-ab+b^2}{a^3}\right) & \text{otherwise} \end{cases} + \frac{b^2x}{a^3} + \frac{b^2\log\left(\frac{a}{b}+2^{-x}\right)}{a^3\log(2)}$$

input
```
integrate(4**x/(a+b/(2**x)),x)
```

output
```
Piecewise(((2**(2*x)*a**2*log(2) - 2*2**x*a*b*log(2))/(2*a**3*log(2)**2),
Ne(a**3*log(2)**2, 0)), (x*(-b**2/a**3 + (a**2 - a*b + b**2)/a**3), True))
 + b**2*x/a**3 + b**2*log(a/b + 2**(-x))/(a**3*log(2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{4^x}{a+2^{-x}b}\,dx = \frac{b^2x}{a^3} - \frac{(2^{-x+1}b-a)2^{2x-1}}{a^2\log(2)} + \frac{b^2\log\left(a+\frac{b}{2^x}\right)}{a^3\log(2)}$$

input
```
integrate(4^x/(a+b/(2^x)),x, algorithm="maxima")
```

output
```
b^2*x/a^3 - (2^(-x + 1)*b - a)*2^(2*x - 1)/(a^2*log(2)) + b^2*log(a + b/2^
x)/(a^3*log(2))
```

Giac [F]

$$\int \frac{4^x}{a+2^{-x}b}\,dx = \int \frac{4^x}{a+\frac{b}{2^x}}\,dx$$

input
```
integrate(4^x/(a+b/(2^x)),x, algorithm="giac")
```

output
```
integrate(4^x/(a + b/2^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4^x}{a+2^{-x}b}\,dx = \int \frac{4^x}{a+\frac{b}{2^x}}\,dx$$

input
```
int(4^x/(a + b/2^x),x)
```

output
```
int(4^x/(a + b/2^x), x)
```

Reduce [F]

$$\int \frac{4^x}{a+2^{-x}b}\,dx = \frac{4^x-\left(\int \frac{4^x}{2^x a+b}dx\right)\log(4)\,b}{\log(4)\,a}$$

input
```
int(4^x/(a+b/(2^x)),x)
```

output
```
(4**x - int(4**x/(2**x*a + b),x)*log(4)*b)/(log(4)*a)
```

3.401 $\int \frac{2^{2x}}{a+2^{-x}b} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 58

$$\int \frac{2^{2x}}{a+2^{-x}b} \, dx = \frac{b^2x}{a^3} + \frac{2^{-1+2x}}{a\log(2)} - \frac{2^xb}{a^2\log(2)} + \frac{b^2\log\left(a+2^{-x}b\right)}{a^3\log(2)}$$

output
```
b^2*x/a^3+2^(-1+2*x)/a/ln(2)-2^x*b/a^2/ln(2)+b^2*ln(a+b/(2^x))/a^3/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{2^{2x}}{a+2^{-x}b} \, dx = \frac{2^xa(2^xa-2b)+2b^2\log\left(2^xa+b\right)}{a^3\log(4)}$$

input
```
Integrate[2^(2*x)/(a + b/2^x),x]
```

output
```
(2^x*a*(2^x*a - 2*b) + 2*b^2*Log[2^x*a + b])/(a^3*Log[4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{a+b2^{-x}}dx$$

$$\Big\downarrow \text{ 2678}$$

$$-\frac{\int \frac{2^{3x}}{a+2^{-x}b}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{\int \left(-\frac{b^3}{a^3(a+2^{-x}b)}+\frac{2^xb^2}{a^3}-\frac{2^{2x}b}{a^2}+\frac{2^{3x}}{a}\right)d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{b^2\log(2^{-x})}{a^3}-\frac{b^2\log(a+b2^{-x})}{a^3}+\frac{b2^x}{a^2}-\frac{2^{2x-1}}{a}}{\log(2)}$$

input
```
Int[2^(2*x)/(a + b/2^x),x]
```

output
```
-((-(2^(-1 + 2*x)/a) + (2^x*b)/a^2 + (b^2*Log[2^(-x)])/a^3 - (b^2*Log[a +
b/2^x])/a^3)/Log[2])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{\frac{a 2^{2x}}{2}-2^{x} b}{a^{2}}+\frac{b^{2} \ln\left(a 2^{x}+b\right)}{a^{3}}}{\ln\left(2\right)}$	41
default	$\frac{\frac{\frac{a 2^{2x}}{2}-2^{x} b}{a^{2}}+\frac{b^{2} \ln\left(a 2^{x}+b\right)}{a^{3}}}{\ln\left(2\right)}$	41
risch	$\frac{2^{2x}}{2 \ln\left(2\right) a}-\frac{2^{x} b}{a^{2} \ln\left(2\right)}+\frac{b^{2} \ln\left(2^{x}+\frac{b}{a}\right)}{\ln\left(2\right) a^{3}}$	50
norman	$\frac{{\mathrm e}^{2 x \ln\left(2\right)}}{2 \ln\left(2\right) a}-\frac{b \,{\mathrm e}^{x \ln\left(2\right)}}{\ln\left(2\right) a^{2}}+\frac{b^{2} \ln\left(a \,{\mathrm e}^{x \ln\left(2\right)}+b\right)}{\ln\left(2\right) a^{3}}$	54

input
```
int(2^(2*x)/(a+b/(2^x)),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(1/a^2*(1/2*a*(2^x)^2-2^x*b)+b^2/a^3*ln(a*2^x+b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{2^{2x}}{a+2^{-x}b}\,dx = \frac{2^{2\,x}a^2 - 2\cdot 2^x ab + 2\,b^2\log\left(2^x a + b\right)}{2\,a^3\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a+b/(2^x)),x, algorithm="fricas")
```

output
```
1/2*(2^(2*x)*a^2 - 2*2^x*a*b + 2*b^2*log(2^x*a + b))/(a^3*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{2^{2x}}{a+2^{-x}b}\,dx = \begin{cases} \frac{2^{2x}a^2\log\left(2\right)-2\cdot 2^x ab\log\left(2\right)}{2a^3\log\left(2\right)^2} & \text{for } a^3\log\left(2\right)^2 \neq 0 \\ \frac{x(a-b)}{a^2} & \text{otherwise} \end{cases} + \frac{b^2\log\left(2^x+\frac{b}{a}\right)}{a^3\log\left(2\right)}$$

input
```
integrate(2**(2*x)/(a+b/(2**x)),x)
```

output
```
Piecewise(((2**(2*x)*a**2*log(2) - 2*2**x*a*b*log(2))/(2*a**3*log(2)**2),
Ne(a**3*log(2)**2, 0)), (x*(a - b)/a**2, True)) + b**2*log(2**x + b/a)/(a*
*3*log(2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

$$\int \frac{2^{2x}}{a+2^{-x}b}\,dx = \frac{b^2x}{a^3} - \frac{(2^{-x+1}b-a)2^{2\,x-1}}{a^2\log\left(2\right)} + \frac{b^2\log\left(a+\frac{b}{2^x}\right)}{a^3\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a+b/(2^x)),x, algorithm="maxima")
```

output
```
b^2*x/a^3 - (2^(-x + 1)*b - a)*2^(2*x - 1)/(a^2*log(2)) + b^2*log(a + b/2^
x)/(a^3*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{2^{2x}}{a+2^{-x}b}\,dx = \frac{b^2\log\left(|2^x a+b|\right)}{a^3\log(2)} + \frac{2^{2\,x}a\log(2) - 2\cdot 2^x b\log(2)}{2\,a^2\log(2)^2}$$

input
```
integrate(2^(2*x)/(a+b/(2^x)),x, algorithm="giac")
```

output
```
b^2*log(abs(2^x*a + b))/(a^3*log(2)) + 1/2*(2^(2*x)*a*log(2) - 2*2^x*b*log
(2))/(a^2*log(2)^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{2^{2x}}{a+2^{-x}b}\,dx = \frac{2^{2\,x}}{2\,a\,\ln(2)} - \frac{2^x\,b}{a^2\,\ln(2)} + \frac{b^2\,\ln\left(b+2^x\,a\right)}{a^3\,\ln(2)}$$

input
```
int(2^(2*x)/(a + b/2^x),x)
```

output
```
2^(2*x)/(2*a*log(2)) - (2^x*b)/(a^2*log(2)) + (b^2*log(b + 2^x*a))/(a^3*lo
g(2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.67

$$\int \frac{2^{2x}}{a+2^{-x}b}\,dx = \frac{2^{2x}a^2 - 2\,2^x ab + 2\log(2^x a+b)\,b^2}{2\log(2)\,a^3}$$

input
```
int(2^(2*x)/(a+b/(2^x)),x)
```

output
```
(2**(2*x)*a**2 - 2*2**x*a*b + 2*log(2**x*a + b)*b**2)/(2*log(2)*a**3)
```

3.402 $\int \frac{4^x}{a-2^{-x}b}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int \frac{4^x}{a-2^{-x}b}\,dx=\frac{b^2x}{a^3}+\frac{2^{-1+2x}}{a\log(2)}+\frac{2^xb}{a^2\log(2)}+\frac{b^2\log\left(a-2^{-x}b\right)}{a^3\log(2)}$$

output
```
b^2*x/a^3+2^(-1+2*x)/a/ln(2)+2^x*b/a^2/ln(2)+b^2*ln(a-b/(2^x))/a^3/ln(2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{4^x}{a-2^{-x}b}\,dx=\frac{2^xa(2^xa+2b)+2b^2\log\left(2^xa-b\right)}{a^3\log(4)}$$

input
```
Integrate[4^x/(a - b/2^x),x]
```

output
```
(2^x*a*(2^x*a + 2*b) + 2*b^2*Log[2^x*a - b])/(a^3*Log[4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{a-b2^{-x}}dx$$

$$\Big\downarrow \text{ 2678}$$

$$-\frac{\int \frac{2^{3x}}{a-2^{-x}b}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 54}$$

$$-\frac{\int \left(\frac{b^3}{a^3(a-2^{-x}b)}+\frac{2^xb^2}{a^3}+\frac{2^{2x}b}{a^2}+\frac{2^{3x}}{a}\right)d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{b^2\log(2^{-x})}{a^3}-\frac{b^2\log(a-b2^{-x})}{a^3}-\frac{b2^x}{a^2}-\frac{2^{2x-1}}{a}}{\log(2)}$$

input
```
Int[4^x/(a - b/2^x),x]
```

output
```
-((-(2^(-1 + 2*x)/a) - (2^x*b)/a^2 + (b^2*Log[2^(-x)])/a^3 - (b^2*Log[a -
b/2^x])/a^3)/Log[2])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{2^{2x}}{2\ln(2)a}+\frac{2^{x}b}{a^{2}\ln(2)}+\frac{b^{2}\ln\left(2^{x}-\frac{b}{a}\right)}{\ln(2)a^{3}}$	50
norman	$\frac{b\,{\mathrm e}^{x\ln(2)}}{\ln(2)a^{2}}+\frac{{\mathrm e}^{2x\ln(2)}}{2\ln(2)a}+\frac{b^{2}\ln\left(a\,{\mathrm e}^{x\ln(2)}-b\right)}{\ln(2)a^{3}}$	55

input
```
int(4^x/(a-b/(2^x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/ln(2)/a*(2^x)^2+2^x*b/a^2/ln(2)+1/ln(2)/a^3*b^2*ln(2^x-b/a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{4^x}{a-2^{-x}b}\,dx=\frac{2^{2\,x}a^2+2\cdot 2^x ab+2\,b^2\log\left(2^x a-b\right)}{2\,a^3\log\left(2\right)}$$

input
```
integrate(4^x/(a-b/(2^x)),x, algorithm="fricas")
```

output
```
1/2*(2^(2*x)*a^2 + 2*2^x*a*b + 2*b^2*log(2^x*a - b))/(a^3*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.55

$$\int \frac{4^x}{a-2^{-x}b}\,dx = \begin{cases} \frac{2^{2x}a^2\log(2)+2\cdot 2^x ab\log(2)}{2a^3\log(2)^2} & \text{for } a^3\log(2)^2\neq 0 \\ x\left(-\frac{b^2}{a^3}+\frac{a^2+ab+b^2}{a^3}\right) & \text{otherwise} \end{cases} + \frac{b^2x}{a^3} + \frac{b^2\log\left(-\frac{a}{b}+2^{-x}\right)}{a^3\log(2)}$$

input
```
integrate(4**x/(a-b/(2**x)),x)
```

output
```
Piecewise(((2**(2*x)*a**2*log(2) + 2*2**x*a*b*log(2))/(2*a**3*log(2)**2),
Ne(a**3*log(2)**2, 0)), (x*(-b**2/a**3 + (a**2 + a*b + b**2)/a**3), True))
 + b**2*x/a**3 + b**2*log(-a/b + 2**(-x))/(a**3*log(2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{4^x}{a-2^{-x}b}\,dx = \frac{b^2x}{a^3} + \frac{(2^{-x+1}b+a)2^{2x-1}}{a^2\log(2)} + \frac{b^2\log\left(-a+\frac{b}{2^x}\right)}{a^3\log(2)}$$

input
```
integrate(4^x/(a-b/(2^x)),x, algorithm="maxima")
```

output
```
b^2*x/a^3 + (2^(-x + 1)*b + a)*2^(2*x - 1)/(a^2*log(2)) + b^2*log(-a + b/2
^x)/(a^3*log(2))
```

Giac [F]

$$\int \frac{4^x}{a-2^{-x}b}\,dx = \int \frac{4^x}{a-\frac{b}{2^x}}\,dx$$

input
```
integrate(4^x/(a-b/(2^x)),x, algorithm="giac")
```

output
```
integrate(4^x/(a - b/2^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4^x}{a-2^{-x}b}\,dx=\int \frac{4^x}{a-\frac{b}{2^x}}\,dx$$

input
```
int(4^x/(a - b/2^x),x)
```

output
```
int(4^x/(a - b/2^x), x)
```

Reduce [F]

$$\int \frac{4^x}{a-2^{-x}b}\,dx=\frac{4^x+\left(\int \frac{4^x}{2^x a-b}dx\right)\log(4)\,b}{\log(4)\,a}$$

input
```
int(4^x/(a-b/(2^x)),x)
```

output
```
(4**x + int(4**x/(2**x*a - b),x)*log(4)*b)/(log(4)*a)
```

3.403 $\int \frac{2^{2x}}{a-2^{-x}b}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx=\frac{b^2x}{a^3}+\frac{2^{-1+2x}}{a\log(2)}+\frac{2^xb}{a^2\log(2)}+\frac{b^2\log\left(a-2^{-x}b\right)}{a^3\log(2)}$$

output
```
b^2*x/a^3+2^(-1+2*x)/a/ln(2)+2^x*b/a^2/ln(2)+b^2*ln(a-b/(2^x))/a^3/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx=\frac{2^xa(2^xa+2b)+2b^2\log\left(2^xa-b\right)}{a^3\log(4)}$$

input
```
Integrate[2^(2*x)/(a - b/2^x),x]
```

output
```
(2^x*a*(2^x*a + 2*b) + 2*b^2*Log[2^x*a - b])/(a^3*Log[4])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{a-b2^{-x}}dx$$

$$\Big\downarrow \text{2678}$$

$$-\frac{\int \frac{2^{3x}}{a-2^{-x}b}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{54}$$

$$-\frac{\int \left(\frac{b^3}{a^3(a-2^{-x}b)}+\frac{2^x b^2}{a^3}+\frac{2^{2x}b}{a^2}+\frac{2^{3x}}{a}\right)d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{b^2\log(2^{-x})}{a^3}-\frac{b^2\log(a-b2^{-x})}{a^3}-\frac{b2^x}{a^2}-\frac{2^{2x-1}}{a}}{\log(2)}$$

input
```
Int[2^(2*x)/(a - b/2^x),x]
```

output
```
-((-(2^(-1 + 2*x)/a) - (2^x*b)/a^2 + (b^2*Log[2^(-x)])/a^3 - (b^2*Log[a -
b/2^x])/a^3)/Log[2])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{\frac{\frac{a\,2^{2x}}{2}+2^{x} b}{a^{2}}+\frac{b^{2} \ln \left(a\,2^{x}-b\right)}{a^{3}}}{\ln \left(2\right)}$	42
default	$\frac{\frac{\frac{a\,2^{2x}}{2}+2^{x} b}{a^{2}}+\frac{b^{2} \ln \left(a\,2^{x}-b\right)}{a^{3}}}{\ln \left(2\right)}$	42
risch	$\frac{2^{2x}}{2 \ln \left(2\right) a}+\frac{2^{x} b}{a^{2} \ln \left(2\right)}+\frac{b^{2} \ln \left(2^{x}-\frac{b}{a}\right)}{\ln \left(2\right) a^{3}}$	50
norman	$\frac{b\,{\mathrm e}^{x \ln \left(2\right)}}{\ln \left(2\right) a^{2}}+\frac{{\mathrm e}^{2 x \ln \left(2\right)}}{2 \ln \left(2\right) a}+\frac{b^{2} \ln \left(a\,{\mathrm e}^{x \ln \left(2\right)}-b\right)}{\ln \left(2\right) a^{3}}$	55

input
```
int(2^(2*x)/(a-b/(2^x)),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(1/a^2*(1/2*a*(2^x)^2+2^x*b)+b^2/a^3*ln(a*2^x-b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx = \frac{2^{2x}a^2 + 2\cdot 2^x ab + 2\,b^2\log\left(2^x a - b\right)}{2\,a^3\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a-b/(2^x)),x, algorithm="fricas")
```

output
```
1/2*(2^(2*x)*a^2 + 2*2^x*a*b + 2*b^2*log(2^x*a - b))/(a^3*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx = \begin{cases} \frac{2^{2x}a^2\log\left(2\right)+2\cdot 2^x ab\log\left(2\right)}{2a^3\log\left(2\right)^2} & \text{for } a^3\log\left(2\right)^2 \neq 0 \\ \frac{x(a+b)}{a^2} & \text{otherwise} \end{cases} + \frac{b^2\log\left(2^x - \frac{b}{a}\right)}{a^3\log\left(2\right)}$$

input
```
integrate(2**(2*x)/(a-b/(2**x)),x)
```

output
```
Piecewise(((2**(2*x)*a**2*log(2) + 2*2**x*a*b*log(2))/(2*a**3*log(2)**2),
Ne(a**3*log(2)**2, 0)), (x*(a + b)/a**2, True)) + b**2*log(2**x - b/a)/(a*
*3*log(2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx = \frac{b^2x}{a^3} + \frac{\left(2^{-x+1}b+a\right)2^{2x-1}}{a^2\log\left(2\right)} + \frac{b^2\log\left(-a+\frac{b}{2^x}\right)}{a^3\log\left(2\right)}$$

input
```
integrate(2^(2*x)/(a-b/(2^x)),x, algorithm="maxima")
```

output
```
b^2*x/a^3 + (2^(-x + 1)*b + a)*2^(2*x - 1)/(a^2*log(2)) + b^2*log(-a + b/2
^x)/(a^3*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx = \frac{b^2 \log\left(|2^x a - b|\right)}{a^3 \log(2)} + \frac{2^{2x} a \log(2) + 2 \cdot 2^x b \log(2)}{2\, a^2 \log(2)^2}$$

input
```
integrate(2^(2*x)/(a-b/(2^x)),x, algorithm="giac")
```

output
```
b^2*log(abs(2^x*a - b))/(a^3*log(2)) + 1/2*(2^(2*x)*a*log(2) + 2*2^x*b*log
(2))/(a^2*log(2)^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx = \frac{2^{2x}}{2\,a\,\ln(2)} + \frac{2^x\,b}{a^2\,\ln(2)} + \frac{b^2\,\ln\left(b-2^x\,a\right)}{a^3\,\ln(2)}$$

input
```
int(2^(2*x)/(a - b/2^x),x)
```

output
```
2^(2*x)/(2*a*log(2)) + (2^x*b)/(a^2*log(2)) + (b^2*log(b - 2^x*a))/(a^3*lo
g(2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{2^{2x}}{a-2^{-x}b}\,dx = \frac{2^{2x}a^2+2\,2^xab+2\log(2^xa-b)\,b^2}{2\log(2)\,a^3}$$

input
```
int(2^(2*x)/(a-b/(2^x)),x)
```

output
```
(2**(2*x)*a**2 + 2*2**x*a*b + 2*log(2**x*a - b)*b**2)/(2*log(2)*a**3)
```

3.404 $\int \frac{2^x}{a+4^x b} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{2^x}{a+4^x b} \, dx = \frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

output
```
arctan(2^x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{a+4^x b} \, dx = \frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/(a + 4^x*b),x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2679, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a+b4^x}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{\int \frac{1}{a+2^{2x}b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Int[2^x/(a + 4^x*b),x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

method	result	size
risch	$-\frac{\ln\left(2^{x}-\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln(2)}+\frac{\ln\left(2^{x}+\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln(2)}$	53

input
```
int(2^x/(a+4^x*b),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a*b)^(1/2)/ln(2)*ln(2^x-1/(-a*b)^(1/2)*a)+1/2/(-a*b)^(1/2)/ln(2)*ln
(2^x+1/(-a*b)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.87

$$\int \frac{2^x}{a+4^xb}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{2^{2x}b-2\sqrt{-ab}2^x-a}{2^{2x}b+a}\right)}{2\,ab\log(2)}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{2^xb}\right)}{ab\log(2)}\right]$$

input
```
integrate(2^x/(a+4^x*b),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((2^(2*x)*b - 2*sqrt(-a*b)*2^x - a)/(2^(2*x)*b + a))/(
a*b*log(2)), -sqrt(a*b)*arctan(sqrt(a*b)/(2^x*b))/(a*b*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{2^x}{a+4^x b}\,dx = \frac{\operatorname{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(2ia+e^{\frac{x\log(4)}{2}}\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a+4**x*b),x)
```

output
```
RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2*_i*a + exp(x*log(4)/2))))/log
(2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{2^x}{a+4^x b}\,dx = \frac{\arctan\left(\frac{2^x b}{\sqrt{ab}}\right)}{\sqrt{ab}\log(2)}$$

input
```
integrate(2^x/(a+4^x*b),x, algorithm="maxima")
```

output
```
arctan(2^x*b/sqrt(a*b))/(sqrt(a*b)*log(2))
```

Giac [F]

$$\int \frac{2^x}{a+4^x b}\,dx = \int \frac{2^x}{4^x b+a}\,dx$$

input
```
integrate(2^x/(a+4^x*b),x, algorithm="giac")
```

output
```
integrate(2^x/(4^x*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{2^x}{a+4^x b} \, dx = \frac{\operatorname{atan}\left(\frac{2^x \sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\ln(2)}$$

input
```
int(2^x/(a + 4^x*b),x)
```

output
```
atan((2^x*b^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{a+4^x b} \, dx = \int \frac{2^x}{4^x b+a} dx$$

input
```
int(2^x/(a+4^x*b),x)
```

output
```
int(2**x/(4**x*b + a),x)
```

3.405 $\int \frac{2^x}{a+2^{2x}b}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{2^x}{a+2^{2x}b}\,dx=\frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

output
```
arctan(2^x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{a+2^{2x}b}\,dx=\frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/(a + 2^(2*x)*b),x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2679, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a+b2^{2x}}\,dx$$
$$\Big\downarrow \ 2679$$
$$\frac{\int \frac{1}{a+2^{2x}b}d2^x}{\log(2)}$$
$$\Big\downarrow \ 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Int[2^x/(a + 2^(2*x)*b),x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b2^x}{\sqrt{ab}}\right)}{\ln(2)\sqrt{ab}}$	22
default	$\frac{\arctan\left(\frac{b2^x}{\sqrt{ab}}\right)}{\ln(2)\sqrt{ab}}$	22
risch	$-\frac{\ln\left(2^x-\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln(2)}+\frac{\ln\left(2^x+\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln(2)}$	53

input
```
int(2^x/(a+2^(2*x)*b),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)/(a*b)^(1/2)*arctan(b*2^x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.87

$$\int \frac{2^x}{a+2^{2x}b}\,dx=\left[-\frac{\sqrt{-ab}\log\left(\frac{2^{2x}b-2\sqrt{-ab}2^x-a}{2^{2x}b+a}\right)}{2\,ab\log(2)},-\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{2^xb}\right)}{ab\log(2)}\right]$$

input
```
integrate(2^x/(a+2^(2*x)*b),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((2^(2*x)*b - 2*sqrt(-a*b)*2^x - a)/(2^(2*x)*b + a))/(
a*b*log(2)), -sqrt(a*b)*arctan(sqrt(a*b)/(2^x*b))/(a*b*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{2^x}{a+2^{2x}b}\,dx = \frac{\text{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(2^x+2ia\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a+2**(2*x)*b),x)
```

output
```
RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2**x + 2*_i*a)))/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{2^x}{a+2^{2x}b}\,dx = \frac{\arctan\left(\frac{2^x b}{\sqrt{ab}}\right)}{\sqrt{ab}\log(2)}$$

input
```
integrate(2^x/(a+2^(2*x)*b),x, algorithm="maxima")
```

output
```
arctan(2^x*b/sqrt(a*b))/(sqrt(a*b)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{2^x}{a+2^{2x}b}\,dx = \frac{\arctan\left(\frac{2^x b}{\sqrt{ab}}\right)}{\sqrt{ab}\log(2)}$$

input
```
integrate(2^x/(a+2^(2*x)*b),x, algorithm="giac")
```

output
```
arctan(2^x*b/sqrt(a*b))/(sqrt(a*b)*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{2^x}{a+2^{2x}b}\,dx = \frac{\operatorname{atan}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\ln(2)}$$

input
```
int(2^x/(a + 2^(2*x)*b),x)
```

output
```
atan((2^x*b^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{2^x}{a+2^{2x}b}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{2^x b}{\sqrt{b}\,\sqrt{a}}\right)}{\log(2)\,ab}$$

input
```
int(2^x/(a+2^(2*x)*b),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((2**x*b)/(sqrt(b)*sqrt(a))))/(log(2)*a*b)
```

3.406 $\int \frac{2^x}{a-4^x b} dx$

Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{2^x}{a-4^x b} dx = \frac{\operatorname{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

output
```
arctanh(2^x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{a-4^x b} dx = \frac{\operatorname{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/(a - 4^x*b),x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2679, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a-b4^x}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{\int \frac{1}{a-2^{2x}b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Int[2^x/(a - 4^x*b),x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(22) = 44$.

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.63

method	result	size
risch	$\frac{\ln\left(2^{x}+\frac{a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\ln(2)}-\frac{\ln\left(2^{x}-\frac{a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\ln(2)}$	49

input
```
int(2^x/(a-4^x*b),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*b)^(1/2)/ln(2)*ln(2^x+1/(a*b)^(1/2)*a)-1/2/(a*b)^(1/2)/ln(2)*ln(2^x
-1/(a*b)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.87

$$\int \frac{2^x}{a-4^xb}\,dx = \left[\frac{\sqrt{ab}\log\left(\frac{2^{2x}b+2\sqrt{ab}2^x+a}{2^{2x}b-a}\right)}{2\,ab\log(2)},-\frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{2^xb}\right)}{ab\log(2)}\right]$$

input
```
integrate(2^x/(a-4^x*b),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a*b)*log((2^(2*x)*b + 2*sqrt(a*b)*2^x + a)/(2^(2*x)*b - a))/(a*b
*log(2)), -sqrt(-a*b)*arctan(sqrt(-a*b)/(2^x*b))/(a*b*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{2^x}{a-4^x b}\,dx = \frac{\operatorname{RootSum}\left(4z^2ab-1,\left(i\mapsto i\log\left(2ia+e^{\frac{x\log(4)}{2}}\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a-4**x*b),x)
```

output
```
RootSum(4*_z**2*a*b - 1, Lambda(_i, _i*log(2*_i*a + exp(x*log(4)/2))))/log
(2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{2^x}{a-4^x b}\,dx = -\frac{\log\left(\frac{2^{x+1}b-2\sqrt{ab}}{2^{x+1}b+2\sqrt{ab}}\right)}{2\sqrt{ab}\log(2)}$$

input
```
integrate(2^x/(a-4^x*b),x, algorithm="maxima")
```

output
```
-1/2*log((2^(x + 1)*b - 2*sqrt(a*b))/(2^(x + 1)*b + 2*sqrt(a*b)))/(sqrt(a*
b)*log(2))
```

Giac [F]

$$\int \frac{2^x}{a-4^x b}\,dx = \int -\frac{2^x}{4^x b-a}\,dx$$

input
```
integrate(2^x/(a-4^x*b),x, algorithm="giac")
```

output
```
integrate(-2^x/(4^x*b - a), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{2^x}{a-4^x b} \, dx = \frac{\operatorname{atanh}\left(\frac{2^x \sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\ln(2)}$$

input
```
int(2^x/(a - 4^x*b),x)
```

output
```
atanh((2^x*b^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{a-4^x b} \, dx = -\left(\int \frac{2^x}{4^x b-a}dx\right)$$

input
```
int(2^x/(a-4^x*b),x)
```

output
```
- int(2**x/(4**x*b - a),x)
```

3.407 $\int \frac{2^x}{a-2^{2x}b} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 30

$$\int \frac{2^x}{a-2^{2x}b} \, dx = \frac{\operatorname{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

output
```
arctanh(2^x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{a-2^{2x}b} \, dx = \frac{\operatorname{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/(a - 2^(2*x)*b),x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2679, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a-b2^{2x}}dx$$
$$\Big\downarrow \text{2679}$$
$$\frac{\int \frac{1}{a-2^{2x}b}d2^x}{\log(2)}$$
$$\Big\downarrow \text{221}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}2^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(2)}$$

input
```
Int[2^x/(a - 2^(2*x)*b),x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{b2^x}{\sqrt{ab}}\right)}{\ln(2)\sqrt{ab}}$	22
default	$\frac{\operatorname{arctanh}\left(\frac{b2^x}{\sqrt{ab}}\right)}{\ln(2)\sqrt{ab}}$	22
risch	$\frac{\ln\left(2^x+\frac{a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\ln(2)}-\frac{\ln\left(2^x-\frac{a}{\sqrt{ab}}\right)}{2\sqrt{ab}\,\ln(2)}$	49

input
```
int(2^x/(a-2^(2*x)*b),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)/(a*b)^(1/2)*arctanh(b*2^x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.87

$$\int \frac{2^x}{a-2^{2x}b}\,dx=\left[\frac{\sqrt{ab}\log\left(\frac{2^{2x}b+2\sqrt{ab}2^x+a}{2^{2x}b-a}\right)}{2\,ab\log(2)},-\frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{2^xb}\right)}{ab\log(2)}\right]$$

input
```
integrate(2^x/(a-2^(2*x)*b),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a*b)*log((2^(2*x)*b + 2*sqrt(a*b)*2^x + a)/(2^(2*x)*b - a))/(a*b
*log(2)), -sqrt(-a*b)*arctan(sqrt(-a*b)/(2^x*b))/(a*b*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{2^x}{a-2^{2x}b}\,dx = \frac{\mathrm{RootSum}\left(4z^2ab-1,\left(i\mapsto i\log\left(2^x+2ia\right)\right)\right)}{\log\left(2\right)}$$

input
```
integrate(2**x/(a-2**(2*x)*b),x)
```

output
```
RootSum(4*_z**2*a*b - 1, Lambda(_i, _i*log(2**x + 2*_i*a)))/log(2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(22) = 44.

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{2^x}{a-2^{2x}b}\,dx = -\frac{\log\left(\frac{2^{x+1}b-2\sqrt{ab}}{2^{x+1}b+2\sqrt{ab}}\right)}{2\sqrt{ab}\log\left(2\right)}$$

input
```
integrate(2^x/(a-2^(2*x)*b),x, algorithm="maxima")
```

output
```
-1/2*log((2^(x + 1)*b - 2*sqrt(a*b))/(2^(x + 1)*b + 2*sqrt(a*b)))/(sqrt(a*
b)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{2^x}{a-2^{2x}b}\,dx = -\frac{\arctan\left(\frac{2^x b}{\sqrt{-ab}}\right)}{\sqrt{-ab}\log\left(2\right)}$$

input
```
integrate(2^x/(a-2^(2*x)*b),x, algorithm="giac")
```

output
```
-arctan(2^x*b/sqrt(-a*b))/(sqrt(-a*b)*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{2^x}{a-2^{2x}b}\,dx=\frac{\operatorname{atanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,\ln(2)}$$

input
```
int(2^x/(a - 2^(2*x)*b),x)
```

output
```
atanh((2^x*b^(1/2))/a^(1/2))/(a^(1/2)*b^(1/2)*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{2^x}{a-2^{2x}b}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,\left(-\log\left(-\sqrt{b}\,\sqrt{a}+2^x b\right)+\log\left(\sqrt{b}\,\sqrt{a}+2^x b\right)\right)}{2\log(2)\,ab}$$

input
```
int(2^x/(a-2^(2*x)*b),x)
```

output
```
(sqrt(b)*sqrt(a)*( - log( - sqrt(b)*sqrt(a) + 2**x*b) + log(sqrt(b)*sqrt(a
) + 2**x*b)))/(2*log(2)*a*b)
```

3.408 $\int \frac{2^x}{a+4^{-x}b}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \frac{2^x}{a\log(2)} - \frac{\sqrt{b}\arctan\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\log(2)}$$

output
```
2^x/a/ln(2)-b^(1/2)*arctan(2^x*a^(1/2)/b^(1/2))/a^(3/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \frac{\frac{2^x}{a} - \frac{\sqrt{b}\arctan\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Integrate[2^x/(a + b/4^x),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTan[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2679, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a+b4^{-x}}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{\int \frac{1}{a+2^{-2x}b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{772}$$

$$\frac{\int \frac{2^{2x}}{2^{2x}a+b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{2^x}{a}-\frac{b\int \frac{1}{2^{2x}a+b}d2^x}{a}}{\log(2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2^x}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Int[2^x/(a + b/4^x),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTan[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.72

method	result	size
risch	$\frac{2^{x}}{a \ln\left(2\right)}+\frac{\sqrt{-a b}\, \ln\left(2^{x}-\frac{\sqrt{-a b}}{a}\right)}{2 a^{2} \ln\left(2\right)}-\frac{\sqrt{-a b}\, \ln\left(2^{x}+\frac{\sqrt{-a b}}{a}\right)}{2 a^{2} \ln\left(2\right)}$	74

input
```
int(2^x/(a+b/(4^x)),x,method=_RETURNVERBOSE)
```

output
```
2^x/a/ln(2)+1/2/a^2*(-a*b)^(1/2)/ln(2)*ln(2^x-1/a*(-a*b)^(1/2))-1/2/a^2*(-
a*b)^(1/2)/ln(2)*ln(2^x+1/a*(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{2\cdot 2^x a\sqrt{-\frac{b}{a}}-2^{2\,x}a+b}{2^{2\,x}a+b}\right)+2\cdot 2^x}{2\,a\log(2)}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{2^x a\sqrt{\frac{b}{a}}}{b}\right)-2^x}{a\log(2)}\right]$$

input
```
integrate(2^x/(a+b/(4^x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(2*2^x*a*sqrt(-b/a) - 2^(2*x)*a + b)/(2^(2*x)*a + b)
) + 2*2^x)/(a*log(2)), -(sqrt(b/a)*arctan(2^x*a*sqrt(b/a)/b) - 2^x)/(a*log
(2))]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \begin{cases} \frac{e^{\frac{x\log(4)}{2}}}{a\log(2)} & \text{for } a\log(2)\neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{\operatorname{RootSum}\left(4z^2a^3+b, \left(i\mapsto i\log\left(\frac{2ia^2}{b}+e^{-\frac{x\log(4)}{2}}\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a+b/(4**x)),x)
```

output
```
Piecewise((exp(x*log(4)/2)/(a*log(2)), Ne(a*log(2), 0)), (x/a, True)) + Ro
otSum(4*_z**2*a**3 + b, Lambda(_i, _i*log(2*_i*a**2/b + exp(-x*log(4)/2)))
)/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \frac{b\arctan\left(\frac{b}{\sqrt{ab}2^x}\right)}{\sqrt{ab}a\log(2)} + \frac{4^{\frac{1}{2}x}a+\frac{b}{4^{\frac{1}{2}x}}}{a^2\log(2)} - \frac{b}{2^x a^2\log(2)}$$

input
```
integrate(2^x/(a+b/(4^x)),x, algorithm="maxima")
```

output
```
b*arctan(b/(sqrt(a*b)*2^x))/(sqrt(a*b)*a*log(2)) + (4^(1/2*x)*a + b/4^(1/2
*x))/(a^2*log(2)) - b/(2^x*a^2*log(2))
```

Giac [F]

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \int \frac{2^x}{a+\frac{b}{4^x}}\,dx$$

input
```
integrate(2^x/(a+b/(4^x)),x, algorithm="giac")
```

output
```
integrate(2^x/(a + b/4^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \frac{2^x}{a\ \ln(2)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\ \ln(2)}$$

input
```
int(2^x/(a + b/4^x),x)
```

output
```
2^x/(a*log(2)) - (b^(1/2)*atan((2^x*a^(1/2))/b^(1/2)))/(a^(3/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{a+4^{-x}b}\,dx = \frac{2^x - \left(\int \frac{2^x}{4^x a+b}dx\right)\log(2)\,b}{\log(2)\,a}$$

input
```
int(2^x/(a+b/(4^x)),x)
```

output
```
(2**x - int(2**x/(4**x*a + b),x)*log(2)*b)/(log(2)*a)
```

3.409 $\int \frac{2^x}{a+2^{-2x}b} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{2^x}{a+2^{-2x}b} \, dx = \frac{2^x}{a\log(2)} - \frac{\sqrt{b}\arctan\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\log(2)}$$

output
```
2^x/a/ln(2)-b^(1/2)*arctan(2^x*a^(1/2)/b^(1/2))/a^(3/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{2^x}{a+2^{-2x}b} \, dx = \frac{\frac{2^x}{a} - \frac{\sqrt{b}\arctan\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Integrate[2^x/(a + b/2^(2*x)),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTan[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2679, 772, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a+b2^{-2x}}\,dx$$

$$\Big\downarrow \ 2679$$

$$\frac{\int \frac{1}{a+2^{-2x}b}d2^x}{\log(2)}$$

$$\Big\downarrow \ 772$$

$$\frac{\int \frac{2^{2x}}{2^{2x}a+b}d2^x}{\log(2)}$$

$$\Big\downarrow \ 262$$

$$\frac{\frac{2^x}{a}-\frac{b\int \frac{1}{2^{2x}a+b}d2^x}{a}}{\log(2)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{2^x}{a}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Int[2^x/(a + b/2^(2*x)),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTan[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2^{x}}{a}-\frac{b \arctan\left(\frac{a 2^{x}}{\sqrt{a b}}\right)}{a \sqrt{a b}}}{\ln \left(2\right)}$	36
default	$\frac{\frac{2^{x}}{a}-\frac{b \arctan\left(\frac{a 2^{x}}{\sqrt{a b}}\right)}{a \sqrt{a b}}}{\ln \left(2\right)}$	36
risch	$\frac{2^{x}}{a \ln \left(2\right)}+\frac{\sqrt{-a b}\, \ln \left(2^{x}-\frac{\sqrt{-a b}}{a}\right)}{2 a^{2} \ln \left(2\right)}-\frac{\sqrt{-a b}\, \ln \left(2^{x}+\frac{\sqrt{-a b}}{a}\right)}{2 a^{2} \ln \left(2\right)}$	74

input
```
int(2^x/(a+b/(2^(2*x))),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(1/a*2^x-b/a/(a*b)^(1/2)*arctan(a*2^x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.37

$$\int \frac{2^x}{a+2^{-2x}b}\,dx = \left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{2\cdot 2^x a\sqrt{-\frac{b}{a}}-2^{2\,x}a+b}{2^{2\,x}a+b}\right)+2\cdot 2^x}{2\,a\log(2)}, -\frac{\sqrt{\frac{b}{a}}\arctan\left(\frac{2^x a\sqrt{\frac{b}{a}}}{b}\right)-2^x}{a\log(2)}\right]$$

input
```
integrate(2^x/(a+b/(2^(2*x))),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-b/a)*log(-(2*2^x*a*sqrt(-b/a) - 2^(2*x)*a + b)/(2^(2*x)*a + b)
) + 2*2^x)/(a*log(2)), -(sqrt(b/a)*arctan(2^x*a*sqrt(b/a)/b) - 2^x)/(a*log
(2))]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{2^x}{a+2^{-2x}b}\,dx = \begin{cases}\frac{2^x}{a\log(2)} & \text{for } a\log(2)\neq 0\\ \frac{x}{a} & \text{otherwise}\end{cases} + \frac{\operatorname{RootSum}\left(4z^2a^3+b,\left(i\mapsto i\log\left(\frac{2ia^2}{b}+2^{-x}\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a+b/(2**(2*x))),x)
```

output
```
Piecewise((2**x/(a*log(2)), Ne(a*log(2), 0)), (x/a, True)) + RootSum(4*_z*
*2*a**3 + b, Lambda(_i, _i*log(2*_i*a**2/b + 2**(-x))))/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{2^x}{a+2^{-2x}b}\,dx = \frac{b\arctan\left(\frac{b}{\sqrt{ab}2^x}\right)}{\sqrt{ab}a\log(2)} + \frac{2^x}{a\log(2)}$$

input
```
integrate(2^x/(a+b/(2^(2*x))),x, algorithm="maxima")
```

output
```
b*arctan(b/(sqrt(a*b)*2^x))/(sqrt(a*b)*a*log(2)) + 2^x/(a*log(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

$$\int \frac{2^x}{a+2^{-2x}b}\,dx = -\frac{b\arctan\left(\frac{2^xa}{\sqrt{ab}}\right)}{\sqrt{ab}a\log(2)} + \frac{2^x}{a\log(2)}$$

input
```
integrate(2^x/(a+b/(2^(2*x))),x, algorithm="giac")
```

output
```
-b*arctan(2^x*a/sqrt(a*b))/(sqrt(a*b)*a*log(2)) + 2^x/(a*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{2^x}{a+2^{-2x}b}\,dx = \frac{2^x}{a\,\ln(2)} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\,\ln(2)}$$

input
```
int(2^x/(a + b/2^(2*x)),x)
```

output
```
2^x/(a*log(2)) - (b^(1/2)*atan((2^x*a^(1/2))/b^(1/2)))/(a^(3/2)*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \frac{2^x}{a+2^{-2x}b}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{2^x a}{\sqrt{b}\,\sqrt{a}}\right)+2^x a}{\log(2)\,a^2}$$

input
```
int(2^x/(a+b/(2^(2*x))),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((2**x*a)/(sqrt(b)*sqrt(a))) + 2**x*a)/(log(2)*a**
2)
```

3.410 $\int \frac{2^x}{a-4^{-x}b}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{2^x}{a-4^{-x}b}\,dx=\frac{2^x}{a\log(2)}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\log(2)}$$

output
```
2^x/a/ln(2)-b^(1/2)*arctanh(2^x*a^(1/2)/b^(1/2))/a^(3/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{2^x}{a-4^{-x}b}\,dx=\frac{\frac{2^x}{a}-\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Integrate[2^x/(a - b/4^x),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTanh[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2679, 772, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a-b4^{-x}}dx$$

$$\Big\downarrow \text{ 2679}$$

$$\frac{\int \frac{1}{a-2^{-2x}b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 772}$$

$$\frac{\int \frac{2^{2x}}{2^{2x}a-b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{b\int \frac{1}{2^{2x}a-b}d2^x}{a}+\frac{2^x}{a}}{\log(2)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2^x}{a}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Int[2^x/(a - b/4^x),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTanh[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

method	result	size
risch	$\frac{2^{x}}{a \ln\left(2\right)}+\frac{\sqrt{a b}\, \ln\left(2^{x}-\frac{\sqrt{a b}}{a}\right)}{2 a^{2} \ln\left(2\right)}-\frac{\sqrt{a b}\, \ln\left(2^{x}+\frac{\sqrt{a b}}{a}\right)}{2 a^{2} \ln\left(2\right)}$	70

input
```
int(2^x/(a-b/(4^x)),x,method=_RETURNVERBOSE)
```

output
```
2^x/a/ln(2)+1/2/a^2*(a*b)^(1/2)/ln(2)*ln(2^x-1/a*(a*b)^(1/2))-1/2/a^2*(a*b
)^(1/2)/ln(2)*ln(2^x+1/a*(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \frac{2^x}{a-4^{-x}b}\,dx = \left[\frac{\sqrt{\frac{b}{a}}\log\left(-\frac{2\cdot 2^x a\sqrt{\frac{b}{a}}-2^{2\,x}a-b}{2^{2\,x}a-b}\right)+2\cdot 2^x}{2\,a\log(2)}, \frac{\sqrt{-\frac{b}{a}}\arctan\left(\frac{2^x a\sqrt{-\frac{b}{a}}}{b}\right)+2^x}{a\log(2)}\right]$$

input
```
integrate(2^x/(a-b/(4^x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/a)*log(-(2*2^x*a*sqrt(b/a) - 2^(2*x)*a - b)/(2^(2*x)*a - b))
+ 2*2^x)/(a*log(2)), (sqrt(-b/a)*arctan(2^x*a*sqrt(-b/a)/b) + 2^x)/(a*log(
2))]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{2^x}{a-4^{-x}b}\,dx = \begin{cases} \frac{e^{\frac{x\log(4)}{2}}}{a\log(2)} & \text{for } a\log(2)\neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{\operatorname{RootSum}\left(4z^2a^3-b, \left(i\mapsto i\log\left(-\frac{2ia^2}{b}+e^{-\frac{x\log(4)}{2}}\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a-b/(4**x)),x)
```

output
```
Piecewise((exp(x*log(4)/2)/(a*log(2)), Ne(a*log(2), 0)), (x/a, True)) + Ro
otSum(4*_z**2*a**3 - b, Lambda(_i, _i*log(-2*_i*a**2/b + exp(-x*log(4)/2))
))/log(2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

$$\int \frac{2^x}{a-4^{-x}b}\,dx = \frac{b\log\left(-\frac{\sqrt{ab}-\frac{b}{2^x}}{\sqrt{ab}+\frac{b}{2^x}}\right)}{2\sqrt{ab}a\log(2)} + \frac{4^{\frac{1}{2}x}a-\frac{b}{4^{\frac{1}{2}x}}}{a^2\log(2)} + \frac{b}{2^x a^2\log(2)}$$

input
```
integrate(2^x/(a-b/(4^x)),x, algorithm="maxima")
```

output
```
1/2*b*log(-(sqrt(a*b) - b/2^x)/(sqrt(a*b) + b/2^x))/(sqrt(a*b)*a*log(2)) +
 (4^(1/2*x)*a - b/4^(1/2*x))/(a^2*log(2)) + b/(2^x*a^2*log(2))
```

Giac [F]

$$\int \frac{2^x}{a-4^{-x}b}\,dx = \int \frac{2^x}{a-\frac{b}{4^x}}\,dx$$

input
```
integrate(2^x/(a-b/(4^x)),x, algorithm="giac")
```

output
```
integrate(2^x/(a - b/4^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{2^x}{a-4^{-x}b}\,dx = \frac{2^x}{a\,\ln(2)} - \frac{\sqrt{b}\,\mathrm{atanh}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\,\ln(2)}$$

input
```
int(2^x/(a - b/4^x),x)
```

output
```
2^x/(a*log(2)) - (b^(1/2)*atanh((2^x*a^(1/2))/b^(1/2)))/(a^(3/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{a-4^{-x}b}\,dx = \frac{2^x + \left(\int \frac{2^x}{4^x a-b}dx\right)\log(2)\,b}{\log\left(2\right)a}$$

input
```
int(2^x/(a-b/(4^x)),x)
```

output
```
(2**x + int(2**x/(4**x*a - b),x)*log(2)*b)/(log(2)*a)
```

3.411 $\int \frac{2^x}{a-2^{-2x}b}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 43

$$\int \frac{2^x}{a-2^{-2x}b}\,dx=\frac{2^x}{a\log(2)}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\log(2)}$$

output
```
2^x/a/ln(2)-b^(1/2)*arctanh(2^x*a^(1/2)/b^(1/2))/a^(3/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{2^x}{a-2^{-2x}b}\,dx=\frac{\frac{2^x}{a}-\frac{\sqrt{b}\,\text{arctanh}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Integrate[2^x/(a - b/2^(2*x)),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTanh[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2679, 772, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{a-b2^{-2x}}dx$$

$$\Big\downarrow \text{ 2679}$$

$$\frac{\int \frac{1}{a-2^{-2x}b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 772}$$

$$\frac{\int \frac{2^{2x}}{2^{2x}a-b}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{b\int \frac{1}{2^{2x}a-b}d2^x}{a}+\frac{2^x}{a}}{\log(2)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2^x}{a}-\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{a}2^x}{\sqrt{b}}\right)}{a^{3/2}}}{\log(2)}$$

input
```
Int[2^x/(a - b/2^(2*x)),x]
```

output
```
(2^x/a - (Sqrt[b]*ArcTanh[(2^x*Sqrt[a])/Sqrt[b]])/a^(3/2))/Log[2]
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{2^{x}}{a}-\frac{b\,\mathrm{arctanh}\left(\frac{a\,2^{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{\ln \left(2\right)}$	36
default	$\frac{\frac{2^{x}}{a}-\frac{b\,\mathrm{arctanh}\left(\frac{a\,2^{x}}{\sqrt{ab}}\right)}{a\sqrt{ab}}}{\ln \left(2\right)}$	36
risch	$\frac{2^{x}}{a\ln \left(2\right)}+\frac{\sqrt{ab}\,\ln \left(2^{x}-\frac{\sqrt{ab}}{a}\right)}{2a^{2}\ln \left(2\right)}-\frac{\sqrt{ab}\,\ln \left(2^{x}+\frac{\sqrt{ab}}{a}\right)}{2a^{2}\ln \left(2\right)}$	70

input
```
int(2^x/(a-b/(2^(2*x))),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(1/a*2^x-b/a/(a*b)^(1/2)*arctanh(a*2^x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \frac{2^x}{a-2^{-2x}b}\,dx = \left[\frac{\sqrt{\frac{b}{a}}\log\left(-\frac{2\cdot 2^x a\sqrt{\frac{b}{a}}-2^{2x}a-b}{2^{2x}a-b}\right)+2\cdot 2^x}{2\,a\log(2)}, \frac{\sqrt{-\frac{b}{a}}\arctan\left(\frac{2^x a\sqrt{-\frac{b}{a}}}{b}\right)+2^x}{a\log(2)}\right]$$

input
```
integrate(2^x/(a-b/(2^(2*x))),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/a)*log(-(2*2^x*a*sqrt(b/a) - 2^(2*x)*a - b)/(2^(2*x)*a - b))
+ 2*2^x)/(a*log(2)), (sqrt(-b/a)*arctan(2^x*a*sqrt(-b/a)/b) + 2^x)/(a*log(
2))]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{2^x}{a-2^{-2x}b}\,dx = \begin{cases}\frac{2^x}{a\log(2)} & \text{for } a\log(2)\neq 0\\ \frac{x}{a} & \text{otherwise}\end{cases} + \frac{\operatorname{RootSum}\left(4z^2a^3-b,\left(i\mapsto i\log\left(-\frac{2ia^2}{b}+2^{-x}\right)\right)\right)}{\log(2)}$$

input
```
integrate(2**x/(a-b/(2**(2*x))),x)
```

output
```
Piecewise((2**x/(a*log(2)), Ne(a*log(2), 0)), (x/a, True)) + RootSum(4*_z*
*2*a**3 - b, Lambda(_i, _i*log(-2*_i*a**2/b + 2**(-x))))/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \frac{2^x}{a - 2^{-2x}b}\,dx = \frac{b\log\left(\frac{2^{-x+1}b-2\sqrt{ab}}{2^{-x+1}b+2\sqrt{ab}}\right)}{2\sqrt{ab}a\log(2)} + \frac{2^x}{a\log(2)}$$

input
```
integrate(2^x/(a-b/(2^(2*x))),x, algorithm="maxima")
```

output
```
1/2*b*log((2^(-x + 1)*b - 2*sqrt(a*b))/(2^(-x + 1)*b + 2*sqrt(a*b)))/(sqrt
(a*b)*a*log(2)) + 2^x/(a*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{2^x}{a - 2^{-2x}b}\,dx = \frac{b\arctan\left(\frac{2^x a}{\sqrt{-ab}}\right)}{\sqrt{-ab}a\log(2)} + \frac{2^x}{a\log(2)}$$

input
```
integrate(2^x/(a-b/(2^(2*x))),x, algorithm="giac")
```

output
```
b*arctan(2^x*a/sqrt(-a*b))/(sqrt(-a*b)*a*log(2)) + 2^x/(a*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{2^x}{a - 2^{-2x}b}\,dx = \frac{2^x}{a\ \ln(2)} - \frac{\sqrt{b}\,\mathrm{atanh}\left(\frac{2^x\sqrt{a}}{\sqrt{b}}\right)}{a^{3/2}\ \ln(2)}$$

input
```
int(2^x/(a - b/2^(2*x)),x)
```

output
```
2^x/(a*log(2)) - (b^(1/2)*atanh((2^x*a^(1/2))/b^(1/2)))/(a^(3/2)*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{2^x}{a-2^{-2x}b}\,dx = \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}+2^x a\right)-\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}+2^x a\right)+2\,2^x a}{2\log(2)\,a^2}$$

input
```
int(2^x/(a-b/(2^(2*x))),x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) + 2**x*a) - sqrt(b)*sqrt(a)*log(sq
rt(b)*sqrt(a) + 2**x*a) + 2*2**x*a)/(2*log(2)*a**2)
```

3.412 $\int \frac{2^x}{\sqrt{a+4^x b}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx=\frac{\text{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a+4^x b}}\right)}{\sqrt{b}\log(2)}$$

output
```
arctanh(2^x*b^(1/2)/(a+4^x*b)^(1/2))/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx=\frac{\text{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a+2^{2x} b}}\right)}{\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a + 4^x*b],x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a + 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a+b4^x}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$\frac{\int \frac{1}{\sqrt{a+2^{2x}b}}d2^x}{\log(2)}$$
$$\Big\downarrow \text{ 224}$$
$$\frac{\int \frac{1}{1-2^{2x}b}d\frac{2^x}{\sqrt{a+2^{2x}b}}}{\log(2)}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}2^x}{\sqrt{a+b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

input
```
Int[2^x/Sqrt[a + 4^x*b],x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a + 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [F]

$$\int \frac{2^x}{\sqrt{a+4^x b}}dx$$

input
```
int(2^x/(a+4^x*b)^(1/2),x)
```

output
```
int(2^x/(a+4^x*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx = \left[\frac{\log\left(-2\sqrt{2^{2x}b+a}\,2^x\sqrt{b}-2\cdot 2^{2x}b-a\right)}{2\sqrt{b}\log(2)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{2^{2x}b+a}\sqrt{-b}}{2^x b}\right)}{b\log(2)}\right]$$

input
```
integrate(2^x/(a+4^x*b)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*sqrt(2^(2*x)*b + a)*2^x*sqrt(b) - 2*2^(2*x)*b - a)/(sqrt(b)*lo
g(2)), -sqrt(-b)*arctan(sqrt(2^(2*x)*b + a)*sqrt(-b)/(2^x*b))/(b*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx = \frac{\begin{cases} \frac{\log\left(2\cdot 2^x b+2\sqrt{b}\sqrt{2^{2x} b+a}\right)}{\sqrt{b}} & \text{for}\ a\neq 0\wedge b\neq 0 \\ \frac{2^x\log(2^x)}{\sqrt{2^{2x} b}} & \text{for}\ b\neq 0 \\ \frac{2^x}{\sqrt{a}} & \text{otherwise} \end{cases}}{\log(2)}$$

input
```
integrate(2**x/(a+4**x*b)**(1/2),x)
```

output
```
Piecewise((log(2*2**x*b + 2*sqrt(b)*sqrt(2**(2*x)*b + a))/sqrt(b), Ne(a, 0
) & Ne(b, 0)), (2**x*log(2**x)/sqrt(2**(2*x)*b), Ne(b, 0)), (2**x/sqrt(a),
 True))/log(2)
```

Maxima [F]

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx = \int \frac{2^x}{\sqrt{4^x b+a}}\,dx$$

input
```
integrate(2^x/(a+4^x*b)^(1/2),x, algorithm="maxima")
```

output
```
integrate(2^x/sqrt(4^x*b + a), x)
```

Giac [F]

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx = \int \frac{2^x}{\sqrt{4^x b+a}}\,dx$$

input
```
integrate(2^x/(a+4^x*b)^(1/2),x, algorithm="giac")
```

output
```
integrate(2^x/sqrt(4^x*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx = \frac{\ln\left(\sqrt{a+2^{2x}\,b}+2^x\,\sqrt{b}\right)}{\sqrt{b}\,\ln(2)}$$

input
```
int(2^x/(a + 4^x*b)^(1/2),x)
```

output
```
log((a + 2^(2*x)*b)^(1/2) + 2^x*b^(1/2))/(b^(1/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{\sqrt{a+4^x b}}\,dx = \int \frac{\sqrt{4^x b+a}\,2^x}{4^x b+a}dx$$

input
```
int(2^x/(a+4^x*b)^(1/2),x)
```

output
```
int((sqrt(4**x*b + a)*2**x)/(4**x*b + a),x)
```

3.413 $\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a+4^xb}}\right)}{\sqrt{b}\log(2)}$$

output
```
arctanh(2^x*b^(1/2)/(a+4^x*b)^(1/2))/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx = \frac{\operatorname{arctanh}\left(\frac{2^x\sqrt{b}}{\sqrt{a+2^{2x}b}}\right)}{\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a + 2^(2*x)*b],x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a + 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a+b2^{2x}}}dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{\int \frac{1}{\sqrt{a+2^{2x}b}}d2^x}{\log(2)}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-2^{2x}b}d\frac{2^x}{\sqrt{a+2^{2x}b}}}{\log(2)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}2^x}{\sqrt{a+b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

input
```
Int[2^x/Sqrt[a + 2^(2*x)*b],x]
```

output
```
ArcTanh[(2^x*Sqrt[b])/Sqrt[a + 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{\ln\left(\sqrt{b}\,2^{x}+\sqrt{a+2^{2x}b}\right)}{\ln(2)\sqrt{b}}$	29
default	$\frac{\ln\left(\sqrt{b}\,2^{x}+\sqrt{a+2^{2x}b}\right)}{\ln(2)\sqrt{b}}$	29

input
```
int(2^x/(a+2^(2*x)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*ln(b^(1/2)*2^x+(a+(2^x)^2*b)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx = \left[\frac{\log\left(-2\sqrt{2^{2x}b+a}2^x\sqrt{b}-2\cdot 2^{2x}b-a\right)}{2\sqrt{b}\log(2)}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{2^{2x}b+a}\sqrt{-b}}{2^x b}\right)}{b\log(2)}\right]$$

input
```
integrate(2^x/(a+2^(2*x)*b)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*sqrt(2^(2*x)*b + a)*2^x*sqrt(b) - 2*2^(2*x)*b - a)/(sqrt(b)*lo
g(2)), -sqrt(-b)*arctan(sqrt(2^(2*x)*b + a)*sqrt(-b)/(2^x*b))/(b*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.97

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx=\frac{\begin{cases}\frac{\log\left(2\cdot 2^x b+2\sqrt{b}\sqrt{2^{2x}b+a}\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{2^x\log\left(2^x\right)}{\sqrt{2^{2x}b}} & \text{for } b\neq 0\\ \frac{2^x}{\sqrt{a}} & \text{otherwise}\end{cases}}{\log\left(2\right)}$$

input
```
integrate(2**x/(a+2**(2*x)*b)**(1/2),x)
```

output
```
Piecewise((log(2*2**x*b + 2*sqrt(b)*sqrt(2**(2*x)*b + a))/sqrt(b), Ne(a, 0
) & Ne(b, 0)), (2**x*log(2**x)/sqrt(2**(2*x)*b), Ne(b, 0)), (2**x/sqrt(a),
 True))/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx=\frac{\operatorname{arsinh}\left(\frac{2^{x+1}b}{2\sqrt{ab}}\right)}{\sqrt{b}\log\left(2\right)}$$

input
```
integrate(2^x/(a+2^(2*x)*b)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/2*2^(x + 1)*b/sqrt(a*b))/(sqrt(b)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx = -\frac{\log\left(\left|-2^x\sqrt{b}+\sqrt{2^{2x}b+a}\right|\right)}{\sqrt{b}\log(2)}$$

input
```
integrate(2^x/(a+2^(2*x)*b)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-2^x*sqrt(b) + sqrt(2^(2*x)*b + a)))/(sqrt(b)*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx = \frac{\ln\left(\sqrt{a+2^{2x}b}+2^x\sqrt{b}\right)}{\sqrt{b}\,\ln(2)}$$

input
```
int(2^x/(a + 2^(2*x)*b)^(1/2),x)
```

output
```
log((a + 2^(2*x)*b)^(1/2) + 2^x*b^(1/2))/(b^(1/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{\sqrt{a+2^{2x}b}}\,dx = \int \frac{\sqrt{2^{2x}b+a}\,2^x}{2^{2x}b+a}dx$$

input
```
int(2^x/(a+2^(2*x)*b)^(1/2),x)
```

output
```
int((sqrt(2**(2*x)*b + a)*2**x)/(2**(2*x)*b + a),x)
```

3.414 $\int \frac{2^x}{\sqrt{a-4^x b}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 32

$$\int \frac{2^x}{\sqrt{a-4^x b}}\,dx=\frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a-4^x b}}\right)}{\sqrt{b}\log(2)}$$

output
```
arctan(2^x*b^(1/2)/(a-4^x*b)^(1/2))/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{2^x}{\sqrt{a-4^x b}}\,dx=\frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a-2^{2x} b}}\right)}{\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a - 4^x*b],x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a - 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2679, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a-b4^x}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$\frac{\int \frac{1}{\sqrt{a-2^{2x}b}}d2^x}{\log(2)}$$
$$\Big\downarrow \text{ 224}$$
$$\frac{\int \frac{1}{2^{2x}b+1}d\frac{2^x}{\sqrt{a-2^{2x}b}}}{\log(2)}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{\arctan\left(\frac{\sqrt{b}2^x}{\sqrt{a-b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

input
```
Int[2^x/Sqrt[a - 4^x*b],x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a - 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [F]

$$\int \frac{2^x}{\sqrt{a-4^x b}}dx$$

input
```
int(2^x/(a-4^x*b)^(1/2),x)
```

output
```
int(2^x/(a-4^x*b)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{2^x}{\sqrt{a-4^x b}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-2\sqrt{-2^{2x}b+a}\,2^x\sqrt{-b}+2\cdot 2^{2x}b-a\right)}{2\,b\log(2)},\; -\frac{\arctan\left(\frac{\sqrt{-2^{2x}b+a}}{2^x\sqrt{b}}\right)}{\sqrt{b}\log(2)}\right]$$

input
```
integrate(2^x/(a-4^x*b)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-2*sqrt(-2^(2*x)*b + a)*2^x*sqrt(-b) + 2*2^(2*x)*b - a)
/(b*log(2)), -arctan(sqrt(-2^(2*x)*b + a)/(2^x*sqrt(b)))/(sqrt(b)*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \frac{2^x}{\sqrt{a-4^xb}}\,dx = \frac{\begin{cases} \frac{\log\left(-2\cdot 2^x b+2\sqrt{-b}\sqrt{-2^{2x}b+a}\right)}{\sqrt{-b}} & \text{for } a\neq 0\wedge b\neq 0 \\ \frac{2^x\log(2^x)}{\sqrt{-2^{2x}b}} & \text{for } b\neq 0 \\ \frac{2^x}{\sqrt{a}} & \text{otherwise} \end{cases}}{\log(2)}$$

input
```
integrate(2**x/(a-4**x*b)**(1/2),x)
```

output
```
Piecewise((log(-2*2**x*b + 2*sqrt(-b)*sqrt(-2**(2*x)*b + a))/sqrt(-b), Ne(
a, 0) & Ne(b, 0)), (2**x*log(2**x)/sqrt(-2**(2*x)*b), Ne(b, 0)), (2**x/sqr
t(a), True))/log(2)
```

Maxima [F]

$$\int \frac{2^x}{\sqrt{a-4^xb}}\,dx = \int \frac{2^x}{\sqrt{-4^xb+a}}\,dx$$

input
```
integrate(2^x/(a-4^x*b)^(1/2),x, algorithm="maxima")
```

output
```
integrate(2^x/sqrt(-4^x*b + a), x)
```

Giac [F]

$$\int \frac{2^x}{\sqrt{a-4^xb}}\,dx = \int \frac{2^x}{\sqrt{-4^xb+a}}\,dx$$

input
```
integrate(2^x/(a-4^x*b)^(1/2),x, algorithm="giac")
```

output
```
integrate(2^x/sqrt(-4^x*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{2^x}{\sqrt{a-4^x b}}\,dx=\frac{\ln\left(\sqrt{a-2^{2\,x}\,b}+2^x\,\sqrt{-b}\right)}{\sqrt{-b}\,\ln\left(2\right)}$$

input
```
int(2^x/(a - 4^x*b)^(1/2),x)
```

output
```
log((a - 2^(2*x)*b)^(1/2) + 2^x*(-b)^(1/2))/((-b)^(1/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{\sqrt{a-4^x b}}\,dx=-\left(\int \frac{\sqrt{-4^x b+a}\,2^x}{4^x b-a}dx\right)$$

input
```
int(2^x/(a-4^x*b)^(1/2),x)
```

output
```
- int((sqrt( - 4**x*b + a)*2**x)/(4**x*b - a),x)
```

3.415 $\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = \frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a-4^xb}}\right)}{\sqrt{b}\log(2)}$$

output
```
arctan(2^x*b^(1/2)/(a-4^x*b)^(1/2))/b^(1/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = \frac{\arctan\left(\frac{2^x\sqrt{b}}{\sqrt{a-2^{2x}b}}\right)}{\sqrt{b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a - 2^(2*x)*b],x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a - 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2679, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a-b2^{2x}}}dx$$

$$\Big\downarrow \text{ 2679}$$

$$\frac{\int \frac{1}{\sqrt{a-2^{2x}b}}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\int \frac{1}{2^{2x}b+1}d\frac{2^x}{\sqrt{a-2^{2x}b}}}{\log(2)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{\sqrt{b}2^x}{\sqrt{a-b2^{2x}}}\right)}{\sqrt{b}\log(2)}$$

input
```
Int[2^x/Sqrt[a - 2^(2*x)*b],x]
```

output
```
ArcTan[(2^x*Sqrt[b])/Sqrt[a - 2^(2*x)*b]]/(Sqrt[b]*Log[2])
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sqrt{b}\,2^{x}}{\sqrt{a-2^{2x}b}}\right)}{\ln(2)\sqrt{b}}$	29
default	$\frac{\arctan\left(\frac{\sqrt{b}\,2^{x}}{\sqrt{a-2^{2x}b}}\right)}{\ln(2)\sqrt{b}}$	29

input
```
int(2^x/(a-2^(2*x)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)/b^(1/2)*arctan(b^(1/2)*2^x/(a-(2^x)^2*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = \left[-\frac{\sqrt{-b}\log\left(-2\sqrt{-2^{2x}b+a}\,2^x\sqrt{-b}+2\cdot 2^{2x}b-a\right)}{2\,b\log(2)},\ -\frac{\arctan\left(\frac{\sqrt{-2^{2x}b+a}}{2^x\sqrt{b}}\right)}{\sqrt{b}\log(2)}\right]$$

input
```
integrate(2^x/(a-2^(2*x)*b)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-b)*log(-2*sqrt(-2^(2*x)*b + a)*2^x*sqrt(-b) + 2*2^(2*x)*b - a)
/(b*log(2)), -arctan(sqrt(-2^(2*x)*b + a)/(2^x*sqrt(b)))/(sqrt(b)*log(2))]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.06

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = \frac{\begin{cases} \frac{\log\left(-2\cdot 2^x b+2\sqrt{-b}\sqrt{-2^{2x}b+a}\right)}{\sqrt{-b}} & \text{for } a\neq 0 \wedge b\neq 0 \\ \frac{2^x\log\left(2^x\right)}{\sqrt{-2^{2x}b}} & \text{for } b\neq 0 \\ \frac{2^x}{\sqrt{a}} & \text{otherwise} \end{cases}}{\log\left(2\right)}$$

input
```
integrate(2**x/(a-2**(2*x)*b)**(1/2),x)
```

output
```
Piecewise((log(-2*2**x*b + 2*sqrt(-b)*sqrt(-2**(2*x)*b + a))/sqrt(-b), Ne(
a, 0) & Ne(b, 0)), (2**x*log(2**x)/sqrt(-2**(2*x)*b), Ne(b, 0)), (2**x/sqr
t(a), True))/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = \frac{\arcsin\left(\frac{2^{x+1}b}{2\sqrt{ab}}\right)}{\sqrt{b}\log\left(2\right)}$$

input
```
integrate(2^x/(a-2^(2*x)*b)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(1/2*2^(x + 1)*b/sqrt(a*b))/(sqrt(b)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = -\frac{\log\left(\left|-2^x\sqrt{-b}+\sqrt{-2^{2x}b+a}\right|\right)}{\sqrt{-b}\log(2)}$$

input
```
integrate(2^x/(a-2^(2*x)*b)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(-2^x*sqrt(-b) + sqrt(-2^(2*x)*b + a)))/(sqrt(-b)*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = \frac{\ln\left(\sqrt{a-2^{2x}b}+2^x\sqrt{-b}\right)}{\sqrt{-b}\,\ln(2)}$$

input
```
int(2^x/(a - 2^(2*x)*b)^(1/2),x)
```

output
```
log((a - 2^(2*x)*b)^(1/2) + 2^x*(-b)^(1/2))/((-b)^(1/2)*log(2))
```

Reduce [F]

$$\int \frac{2^x}{\sqrt{a-2^{2x}b}}\,dx = -\left(\int \frac{\sqrt{-2^{2x}b+a}\,2^x}{2^{2x}b-a}dx\right)$$

input
```
int(2^x/(a-2^(2*x)*b)^(1/2),x)
```

output
```
- int((sqrt( - 2**(2*x)*b + a)*2**x)/(2**(2*x)*b - a),x)
```

3.416 $\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx=\frac{2^x\sqrt{a+2^{-2x}b}}{a\log(2)}$$

output
```
2^x*(a+b/(2^(2*x)))^(1/2)/a/ln(2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx=\frac{2^{-x}(2^{2x}a+b)}{a\sqrt{a+2^{-2x}b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a + b/4^x],x]
```

output
```
(2^(2*x)*a + b)/(2^x*a*Sqrt[a + b/2^(2*x)]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2679, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a+b4^{-x}}}dx$$
$$\Big\downarrow \; 2679$$
$$\frac{\int \frac{1}{\sqrt{a+2^{-2x}b}}d2^x}{\log(2)}$$
$$\Big\downarrow \; 746$$
$$\frac{2^x\sqrt{a+b2^{-2x}}}{a\log(2)}$$

input
```
Int[2^x/Sqrt[a + b/4^x],x]
```

output
```
(2^x*Sqrt[a + b/2^(2*x)])/(a*Log[2])
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result	size
risch	$\frac{(a2^{2x}+b)2^{-x}}{\sqrt{(a2^{2x}+b)2^{-2x}}\, a\ln(2)}$	40

input
```
int(2^x/(a+b/(4^x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*(2^x)^2+b)/(2^x)^2)^(1/2)*(a*(2^x)^2+b)/(2^x)/a/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx = \frac{2^x\sqrt{\frac{2^{2x}a+b}{2^{2x}}}}{a\log(2)}$$

input
```
integrate(2^x/(a+b/(4^x))^(1/2),x, algorithm="fricas")
```

output
```
2^x*sqrt((2^(2*x)*a + b)/2^(2*x))/(a*log(2))
```

Sympy [F]

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx = \int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx$$

input
```
integrate(2**x/(a+b/(4**x))**(1/2),x)
```

output
```
Integral(2**x/sqrt(a + b/4**x), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx = \frac{\sqrt{2^{2x}a+b}}{a\log(2)}$$

input
```
integrate(2^x/(a+b/(4^x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2^(2*x)*a + b)/(a*log(2))
```

Giac [F]

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx = \int \frac{2^x}{\sqrt{a+\frac{b}{4^x}}}\,dx$$

input
```
integrate(2^x/(a+b/(4^x))^(1/2),x, algorithm="giac")
```

output
```
integrate(2^x/sqrt(a + b/4^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx = \frac{2^x\sqrt{a+\frac{b}{2^{2x}}}}{a\ln(2)}$$

input
```
int(2^x/(a + b/4^x)^(1/2),x)
```

output
```
(2^x*(a + b/2^(2*x))^(1/2))/(a*log(2))
```

Reduce [F]

$$\int \frac{2^x}{\sqrt{a+4^{-x}b}}\,dx = \int \frac{\sqrt{4^x a+b}\,2^x 4^{\frac{x}{2}}}{4^x a+b}dx$$

input
```
int(2^x/(a+b/(4^x))^(1/2),x)
```

output
```
int((sqrt(4**x*a + b)*2**x*4**(x/2))/(4**x*a + b),x)
```

3.417 $\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx=\frac{2^x\sqrt{a+2^{-2x}b}}{a\log(2)}$$

output
```
2^x*(a+b/(2^(2*x)))^(1/2)/a/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx=\frac{2^{-x}(2^{2x}a+b)}{a\sqrt{a+2^{-2x}b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a + b/2^(2*x)],x]
```

output
```
(2^(2*x)*a + b)/(2^x*a*Sqrt[a + b/2^(2*x)]*Log[2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2679, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a+b2^{-2x}}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$\frac{\int \frac{1}{\sqrt{a+2^{-2x}b}}d2^x}{\log(2)}$$
$$\Big\downarrow \text{ 746}$$
$$\frac{2^x\sqrt{a+b2^{-2x}}}{a\log(2)}$$

input
```
Int[2^x/Sqrt[a + b/2^(2*x)],x]
```

output
```
(2^x*Sqrt[a + b/2^(2*x)])/(a*Log[2])
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

method	result	size
derivativedivides	$\frac{(a 2^{2x}+b)2^{-x}}{\sqrt{(a 2^{2x}+b)2^{-2x}}\, a \ln(2)}$	40
default	$\frac{(a 2^{2x}+b)2^{-x}}{\sqrt{(a 2^{2x}+b)2^{-2x}}\, a \ln(2)}$	40
risch	$\frac{(a 2^{2x}+b)2^{-x}}{\sqrt{(a 2^{2x}+b)2^{-2x}}\, a \ln(2)}$	40

input
```
int(2^x/(a+b/(2^(2*x)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*(2^x)^2+b)/(2^x)^2)^(1/2)*(a*(2^x)^2+b)/(2^x)/a/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx = \frac{2^x\sqrt{\frac{2^{2x}a+b}{2^{2x}}}}{a\log(2)}$$

input
```
integrate(2^x/(a+b/(2^(2*x)))^(1/2),x, algorithm="fricas")
```

output
```
2^x*sqrt((2^(2*x)*a + b)/2^(2*x))/(a*log(2))
```

Sympy [F]

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx = \int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx$$

input
```
integrate(2**x/(a+b/(2**(2*x)))**(1/2),x)
```

output
```
Integral(2**x/sqrt(a + b/2**(2*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx = \frac{2^x\sqrt{a+\frac{b}{2^{2\,x}}}}{a\log\left(2\right)}$$

input
```
integrate(2^x/(a+b/(2^(2*x)))^(1/2),x, algorithm="maxima")
```

output
```
2^x*sqrt(a + b/2^(2*x))/(a*log(2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx = \frac{\frac{\sqrt{2^{2\,x}a+b}}{a}-\frac{\sqrt{b}}{a}}{\log\left(2\right)}$$

input
```
integrate(2^x/(a+b/(2^(2*x)))^(1/2),x, algorithm="giac")
```

output
```
(sqrt(2^(2*x)*a + b)/a - sqrt(b)/a)/log(2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx = \frac{2^x\sqrt{a+\frac{b}{2^{2\,x}}}}{a\,\ln\left(2\right)}$$

input
```
int(2^x/(a + b/2^(2*x))^(1/2),x)
```

output
```
(2^x*(a + b/2^(2*x))^(1/2))/(a*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{2^x}{\sqrt{a+2^{-2x}b}}\,dx = \frac{\sqrt{2^{2x}a+b}}{\log(2)\,a}$$

input
```
int(2^x/(a+b/(2^(2*x)))^(1/2),x)
```

output
```
sqrt(2**(2*x)*a + b)/(log(2)*a)
```

3.418 $\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx=\frac{2^x\sqrt{a-2^{-2x}b}}{a\log(2)}$$

output
```
2^x*(a-b/(2^(2*x)))^(1/2)/a/ln(2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx=\frac{2^{-x}(2^{2x}a-b)}{a\sqrt{a-2^{-2x}b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a - b/4^x],x]
```

output
```
(2^(2*x)*a - b)/(2^x*a*Sqrt[a - b/2^(2*x)]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2679, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a-b4^{-x}}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$\frac{\int \frac{1}{\sqrt{a-2^{-2x}b}}d2^x}{\log(2)}$$
$$\Big\downarrow \text{ 746}$$
$$\frac{2^x\sqrt{a-b2^{-2x}}}{a\log(2)}$$

input
```
Int[2^x/Sqrt[a - b/4^x],x]
```

output
```
(2^x*Sqrt[a - b/2^(2*x)])/(a*Log[2])
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result	size
risch	$\frac{(a2^{2x}-b)2^{-x}}{\sqrt{(a2^{2x}-b)2^{-2x}}\,a\ln(2)}$	44

input
```
int(2^x/(a-b/(4^x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*(2^x)^2-b)/(2^x)^2)^(1/2)*(a*(2^x)^2-b)/(2^x)/a/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx = \frac{2^x\sqrt{\frac{2^{2x}a-b}{2^{2x}}}}{a\log(2)}$$

input
```
integrate(2^x/(a-b/(4^x))^(1/2),x, algorithm="fricas")
```

output
```
2^x*sqrt((2^(2*x)*a - b)/2^(2*x))/(a*log(2))
```

Sympy [F]

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx = \int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx$$

input
```
integrate(2**x/(a-b/(4**x))**(1/2),x)
```

output
```
Integral(2**x/sqrt(a - b/4**x), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx = \frac{\sqrt{2^{2\,x}a-b}}{a\log\left(2\right)}$$

input
```
integrate(2^x/(a-b/(4^x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2^(2*x)*a - b)/(a*log(2))
```

Giac [F]

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx = \int \frac{2^x}{\sqrt{a-\frac{b}{4^x}}}\,dx$$

input
```
integrate(2^x/(a-b/(4^x))^(1/2),x, algorithm="giac")
```

output
```
integrate(2^x/sqrt(a - b/4^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx = \frac{2^x\,\sqrt{a-\frac{b}{2^{2\,x}}}}{a\,\ln\left(2\right)}$$

input
```
int(2^x/(a - b/4^x)^(1/2),x)
```

output
```
(2^x*(a - b/2^(2*x))^(1/2))/(a*log(2))
```

Reduce [F]

$$\int \frac{2^x}{\sqrt{a-4^{-x}b}}\,dx = \int \frac{\sqrt{4^xa-b}\,2^x4^{\frac{x}{2}}}{4^xa-b}dx$$

input
```
int(2^x/(a-b/(4^x))^(1/2),x)
```

output
```
int((sqrt(4**x*a - b)*2**x*4**(x/2))/(4**x*a - b),x)
```

3.419 $\int \frac{2^x}{\sqrt{a-2^{-2x}b}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}} \, dx = \frac{2^x\sqrt{a-2^{-2x}b}}{a\log(2)}$$

output
```
2^x*(a-b/(2^(2*x)))^(1/2)/a/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}} \, dx = \frac{2^{-x}(2^{2x}a-b)}{a\sqrt{a-2^{-2x}b}\log(2)}$$

input
```
Integrate[2^x/Sqrt[a - b/2^(2*x)],x]
```

output
```
(2^(2*x)*a - b)/(2^x*a*Sqrt[a - b/2^(2*x)]*Log[2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2679, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^x}{\sqrt{a-b2^{-2x}}}dx$$
$$\Big\downarrow \text{2679}$$
$$\frac{\int \frac{1}{\sqrt{a-2^{-2x}b}}d2^x}{\log(2)}$$
$$\Big\downarrow \text{746}$$
$$\frac{2^x\sqrt{a-b2^{-2x}}}{a\log(2)}$$

input
```
Int[2^x/Sqrt[a - b/2^(2*x)],x]
```

output
```
(2^x*Sqrt[a - b/2^(2*x)])/(a*Log[2])
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result	size
derivativedivides	$\frac{(a2^{2x}-b)2^{-x}}{\sqrt{(a2^{2x}-b)2^{-2x}}\,a\ln(2)}$	44
default	$\frac{(a2^{2x}-b)2^{-x}}{\sqrt{(a2^{2x}-b)2^{-2x}}\,a\ln(2)}$	44
risch	$\frac{(a2^{2x}-b)2^{-x}}{\sqrt{(a2^{2x}-b)2^{-2x}}\,a\ln(2)}$	44

input
```
int(2^x/(a-b/(2^(2*x)))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((a*(2^x)^2-b)/(2^x)^2)^(1/2)*(a*(2^x)^2-b)/(2^x)/a/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx = \frac{2^x\sqrt{\frac{2^{2x}a-b}{2^{2x}}}}{a\log(2)}$$

input
```
integrate(2^x/(a-b/(2^(2*x)))^(1/2),x, algorithm="fricas")
```

output
```
2^x*sqrt((2^(2*x)*a - b)/2^(2*x))/(a*log(2))
```

Sympy [F]

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx = \int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx$$

input
```
integrate(2**x/(a-b/(2**(2*x)))**(1/2),x)
```

output
```
Integral(2**x/sqrt(a - b/2**(2*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx = \frac{2^x\sqrt{a-\frac{b}{2^{2x}}}}{a\log(2)}$$

input
```
integrate(2^x/(a-b/(2^(2*x)))^(1/2),x, algorithm="maxima")
```

output
```
2^x*sqrt(a - b/2^(2*x))/(a*log(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx = \frac{\frac{\sqrt{2^{2x}a-b}}{a}-\frac{\sqrt{-b}}{a}}{\log(2)}$$

input
```
integrate(2^x/(a-b/(2^(2*x)))^(1/2),x, algorithm="giac")
```

output
```
(sqrt(2^(2*x)*a - b)/a - sqrt(-b)/a)/log(2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx = \frac{2^x\sqrt{a-\frac{b}{2^{2x}}}}{a\ \ln(2)}$$

input
```
int(2^x/(a - b/2^(2*x))^(1/2),x)
```

output
```
(2^x*(a - b/2^(2*x))^(1/2))/(a*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{2^x}{\sqrt{a-2^{-2x}b}}\,dx=\frac{\sqrt{2^{2x}a-b}}{\log(2)\,a}$$

input
```
int(2^x/(a-b/(2^(2*x)))^(1/2),x)
```

output
```
sqrt(2**(2*x)*a - b)/(log(2)*a)
```

3.420 $\int \frac{4^x}{\sqrt{a+2^x b}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{4^x}{\sqrt{a+2^x b}} \, dx = -\frac{2a\sqrt{a+2^x b}}{b^2 \log(2)} + \frac{2(a+2^x b)^{3/2}}{3b^2 \log(2)}$$

output
```
-2*a*(a+2^x*b)^(1/2)/b^2/ln(2)+2/3*(a+2^x*b)^(3/2)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{4^x}{\sqrt{a+2^x b}} \, dx = \frac{2(-2a+2^x b)\sqrt{a+2^x b}}{b^2 \log(8)}$$

input
```
Integrate[4^x/Sqrt[a + 2^x*b],x]
```

output
```
(2*(-2*a + 2^x*b)*Sqrt[a + 2^x*b])/(b^2*Log[8])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{\sqrt{a+b2^x}}dx$$

$$\Big\downarrow \text{ 2678}$$

$$\frac{\int \frac{2^x}{\sqrt{a+2^xb}}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{\sqrt{a+2^xb}}{b}-\frac{a}{b\sqrt{a+2^xb}}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{2(a+b2^x)^{3/2}}{3b^2}-\frac{2a\sqrt{a+b2^x}}{b^2}}{\log(2)}$$

input
```
Int[4^x/Sqrt[a + 2^x*b],x]
```

output
```
((-2*a*Sqrt[a + 2^x*b])/b^2 + (2*(a + 2^x*b)^(3/2))/(3*b^2))/Log[2]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{2(-2^{x}b+2a)\sqrt{a+2^{x}b}}{3b^{2}\ln(2)}$	29

input
```
int(4^x/(a+2^x*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2^x*b+2*a)*(a+2^x*b)^(1/2)/b^2/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{4^x}{\sqrt{a+2^xb}}\,dx=\frac{2\,\sqrt{2^xb+a}(2^xb-2\,a)}{3\,b^2\log{(2)}}$$

input
```
integrate(4^x/(a+2^x*b)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(2^x*b + a)*(2^x*b - 2*a)/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{4^x}{\sqrt{a+2^x b}}\,dx = \begin{cases} \frac{2\cdot 2^x\sqrt{2^x b+a}}{3b\log(2)} - \frac{4a\sqrt{2^x b+a}}{3b^2\log(2)} & \text{for } b \neq 0 \\ \frac{4^x}{2\sqrt{a}\log(2)} & \text{otherwise} \end{cases}$$

input
```
integrate(4**x/(a+2**x*b)**(1/2),x)
```

output
```
Piecewise((2*2**x*sqrt(2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(2**x*b + a)/(3*
b**2*log(2)), Ne(b, 0)), (4**x/(2*sqrt(a)*log(2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{4^x}{\sqrt{a+2^x b}}\,dx = \frac{2^{2x+1}}{3\sqrt{2^x b+a}\log(2)} - \frac{2^{x+1}a}{3\sqrt{2^x b+a}b\log(2)} - \frac{4a^2}{3\sqrt{2^x b+a}b^2\log(2)}$$

input
```
integrate(4^x/(a+2^x*b)^(1/2),x, algorithm="maxima")
```

output
```
1/3*2^(2*x + 1)/(sqrt(2^x*b + a)*log(2)) - 1/3*2^(x + 1)*a/(sqrt(2^x*b + a
)*b*log(2)) - 4/3*a^2/(sqrt(2^x*b + a)*b^2*log(2))
```

Giac [F]

$$\int \frac{4^x}{\sqrt{a+2^x b}}\,dx = \int \frac{4^x}{\sqrt{2^x b+a}}\,dx$$

input
```
integrate(4^x/(a+2^x*b)^(1/2),x, algorithm="giac")
```

output
```
integrate(4^x/sqrt(2^x*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{4^x}{\sqrt{a+2^x b}}\,dx = -\frac{2\sqrt{a+2^x\,b}\,(2\,a-2^x\,b)}{3\,b^2\,\ln(2)}$$

input
```
int(4^x/(a + 2^x*b)^(1/2),x)
```

output
```
-(2*(a + 2^x*b)^(1/2)*(2*a - 2^x*b))/(3*b^2*log(2))
```

Reduce [F]

$$\int \frac{4^x}{\sqrt{a+2^x b}}\,dx = \int \frac{\sqrt{2^x b+a}\,4^x}{2^x b+a}dx$$

input
```
int(4^x/(a+2^x*b)^(1/2),x)
```

output
```
int((sqrt(2**x*b + a)*4**x)/(2**x*b + a),x)
```

3.421 $\int \frac{2^{2x}}{\sqrt{a+2^x b}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}} \, dx = -\frac{2a\sqrt{a+2^x b}}{b^2 \log(2)} + \frac{2(a+2^x b)^{3/2}}{3b^2 \log(2)}$$

output
```
-2*a*(a+2^x*b)^(1/2)/b^2/ln(2)+2/3*(a+2^x*b)^(3/2)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}} \, dx = \frac{2(-2a+2^x b)\sqrt{a+2^x b}}{b^2 \log(8)}$$

input
```
Integrate[2^(2*x)/Sqrt[a + 2^x*b],x]
```

output
```
(2*(-2*a + 2^x*b)*Sqrt[a + 2^x*b])/(b^2*Log[8])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{\sqrt{a+b2^x}}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int \frac{2^x}{\sqrt{a+2^x b}}d2^x}{\log(2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\sqrt{a+2^x b}}{b}-\frac{a}{b\sqrt{a+2^x b}}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(a+b2^x)^{3/2}}{3b^2}-\frac{2a\sqrt{a+b2^x}}{b^2}}{\log(2)}$$

input
```
Int[2^(2*x)/Sqrt[a + 2^x*b],x]
```

output
```
((-2*a*Sqrt[a + 2^x*b])/b^2 + (2*(a + 2^x*b)^(3/2))/(3*b^2))/Log[2]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{2(-2^{x} b+2 a)\sqrt{a+2^{x} b}}{3 b^{2} \ln(2)}$	29
derivativedivides	$\frac{\frac{2(a+2^{x} b)^{\frac{3}{2}}}{3}-2 a\sqrt{a+2^{x} b}}{b^{2} \ln(2)}$	34
default	$\frac{\frac{2(a+2^{x} b)^{\frac{3}{2}}}{3}-2 a\sqrt{a+2^{x} b}}{b^{2} \ln(2)}$	34

input
```
int(2^(2*x)/(a+2^x*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-2^x*b+2*a)*(a+2^x*b)^(1/2)/b^2/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}}\,dx = \frac{2\sqrt{2^x b+a}(2^x b-2\,a)}{3\,b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a+2^x*b)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(2^x*b + a)*(2^x*b - 2*a)/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}}\,dx = \begin{cases} \frac{2\cdot 2^x\sqrt{2^x b+a}}{3b\log(2)} - \frac{4a\sqrt{2^x b+a}}{3b^2\log(2)} & \text{for } b\neq 0 \\ \frac{2^{2x}}{2\sqrt{a}\log(2)} & \text{otherwise} \end{cases}$$

input
```
integrate(2**(2*x)/(a+2**x*b)**(1/2),x)
```

output
```
Piecewise((2*2**x*sqrt(2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(2**x*b + a)/(3*
b**2*log(2)), Ne(b, 0)), (2**(2*x)/(2*sqrt(a)*log(2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}}\,dx = \frac{2\,(2^x b+a)^{\frac{3}{2}}}{3\,b^2\log(2)} - \frac{2\,\sqrt{2^x b+a}a}{b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a+2^x*b)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(2^x*b + a)^(3/2)/(b^2*log(2)) - 2*sqrt(2^x*b + a)*a/(b^2*log(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}}\,dx = \frac{2\left((2^x b+a)^{\frac{3}{2}} - 3\sqrt{2^x b+a}\,a\right)}{3\,b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a+2^x*b)^(1/2),x, algorithm="giac")
```

output
```
2/3*((2^x*b + a)^(3/2) - 3*sqrt(2^x*b + a)*a)/(b^2*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}}\,dx = -\frac{2\sqrt{a+2^x\,b}\,(2\,a-2^x\,b)}{3\,b^2\,\ln(2)}$$

input
```
int(2^(2*x)/(a + 2^x*b)^(1/2),x)
```

output
```
-(2*(a + 2^x*b)^(1/2)*(2*a - 2^x*b))/(3*b^2*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{2^{2x}}{\sqrt{a+2^x b}}\,dx = \frac{2\sqrt{2^x b+a}\,(2^x b-2a)}{3\log(2)\,b^2}$$

input
```
int(2^(2*x)/(a+2^x*b)^(1/2),x)
```

output
```
(2*sqrt(2**x*b + a)*(2**x*b - 2*a))/(3*log(2)*b**2)
```

3.422 $\int \frac{4^x}{\sqrt{a-2^x b}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 46

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx = -\frac{2a\sqrt{a-2^x b}}{b^2\log(2)} + \frac{2(a-2^x b)^{3/2}}{3b^2\log(2)}$$

output
```
-2*a*(a-2^x*b)^(1/2)/b^2/ln(2)+2/3*(a-2^x*b)^(3/2)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx = -\frac{2\sqrt{a-2^x b}(2a+2^x b)}{b^2\log(8)}$$

input
```
Integrate[4^x/Sqrt[a - 2^x*b],x]
```

output
```
(-2*Sqrt[a - 2^x*b]*(2*a + 2^x*b))/(b^2*Log[8])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{\sqrt{a-b2^x}}dx$$
$$\Big\downarrow \text{2678}$$
$$\frac{\int \frac{2^x}{\sqrt{a-2^x b}}d2^x}{\log(2)}$$
$$\Big\downarrow \text{53}$$
$$\frac{\int \left(\frac{a}{b\sqrt{a-2^x b}}-\frac{\sqrt{a-2^x b}}{b}\right)d2^x}{\log(2)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{2(a-b2^x)^{3/2}}{3b^2}-\frac{2a\sqrt{a-b2^x}}{b^2}}{\log(2)}$$

input
```
Int[4^x/Sqrt[a - 2^x*b],x]
```

output
```
((-2*a*Sqrt[a - 2^x*b])/b^2 + (2*(a - 2^x*b)^(3/2))/(3*b^2))/Log[2]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
risch	$-\frac{2(2^x b+2a)\sqrt{a-2^x b}}{3b^2\ln(2)}$	29

input
```
int(4^x/(a-2^x*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2^x*b+2*a)/b^2*(a-2^x*b)^(1/2)/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx=-\frac{2\,(2^x b+2\,a)\sqrt{-2^x b+a}}{3\,b^2\log(2)}$$

input
```
integrate(4^x/(a-2^x*b)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2^x*b + 2*a)*sqrt(-2^x*b + a)/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx=\begin{cases} -\frac{2\cdot 2^x\sqrt{-2^x b+a}}{3b\log(2)}-\frac{4a\sqrt{-2^x b+a}}{3b^2\log(2)} & \text{for } b\neq 0\\ \frac{4^x}{2\sqrt{a}\log(2)} & \text{otherwise}\end{cases}$$

input
```
integrate(4**x/(a-2**x*b)**(1/2),x)
```

output
```
Piecewise((-2*2**x*sqrt(-2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(-2**x*b + a)/
(3*b**2*log(2)), Ne(b, 0)), (4**x/(2*sqrt(a)*log(2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx=\frac{2^{2\,x+1}}{3\,\sqrt{-2^x b+a}\log(2)}+\frac{2^{x+1}a}{3\,\sqrt{-2^x b+a}b\log(2)}-\frac{4\,a^2}{3\,\sqrt{-2^x b+a}b^2\log(2)}$$

input
```
integrate(4^x/(a-2^x*b)^(1/2),x, algorithm="maxima")
```

output
```
1/3*2^(2*x + 1)/(sqrt(-2^x*b + a)*log(2)) + 1/3*2^(x + 1)*a/(sqrt(-2^x*b +
 a)*b*log(2)) - 4/3*a^2/(sqrt(-2^x*b + a)*b^2*log(2))
```

Giac [F]

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx=\int \frac{4^x}{\sqrt{-2^x b+a}}\,dx$$

input
```
integrate(4^x/(a-2^x*b)^(1/2),x, algorithm="giac")
```

output
```
integrate(4^x/sqrt(-2^x*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx = -\frac{2\sqrt{a-2^x\,b}\,(2\,a+2^x\,b)}{3\,b^2\,\ln(2)}$$

input
```
int(4^x/(a - 2^x*b)^(1/2),x)
```

output
```
-(2*(a - 2^x*b)^(1/2)*(2*a + 2^x*b))/(3*b^2*log(2))
```

Reduce [F]

$$\int \frac{4^x}{\sqrt{a-2^x b}}\,dx = -\left(\int \frac{\sqrt{-2^x b+a}\,4^x}{2^x b-a}dx\right)$$

input
```
int(4^x/(a-2^x*b)^(1/2),x)
```

output
```
- int((sqrt( - 2**x*b + a)*4**x)/(2**x*b - a),x)
```

3.423 $\int \frac{2^{2x}}{\sqrt{a-2^x b}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 46

$$\int \frac{2^{2x}}{\sqrt{a-2^x b}}\,dx = -\frac{2a\sqrt{a-2^x b}}{b^2\log(2)} + \frac{2(a-2^x b)^{3/2}}{3b^2\log(2)}$$

output
```
-2*a*(a-2^x*b)^(1/2)/b^2/ln(2)+2/3*(a-2^x*b)^(3/2)/b^2/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{2^{2x}}{\sqrt{a-2^x b}}\,dx = -\frac{2\sqrt{a-2^x b}(2a+2^x b)}{b^2\log(8)}$$

input
```
Integrate[2^(2*x)/Sqrt[a - 2^x*b],x]
```

output
```
(-2*Sqrt[a - 2^x*b]*(2*a + 2^x*b))/(b^2*Log[8])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{\sqrt{a-b2^x}}\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int \frac{2^x}{\sqrt{a-2^xb}}d2^x}{\log(2)}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{a}{b\sqrt{a-2^xb}}-\frac{\sqrt{a-2^xb}}{b}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(a-b2^x)^{3/2}}{3b^2}-\frac{2a\sqrt{a-b2^x}}{b^2}}{\log(2)}$$

input
```
Int[2^(2*x)/Sqrt[a - 2^x*b],x]
```

output
```
((-2*a*Sqrt[a - 2^x*b])/b^2 + (2*(a - 2^x*b)^(3/2))/(3*b^2))/Log[2]
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.63

method	result	size
risch	$-\frac{2(2^{x}b+2a)\sqrt{a-2^{x}b}}{3b^{2}\ln(2)}$	29
derivativedivides	$\frac{\frac{2(a-2^{x}b)^{\frac{3}{2}}}{3}-2a\sqrt{a-2^{x}b}}{b^{2}\ln(2)}$	36
default	$\frac{\frac{2(a-2^{x}b)^{\frac{3}{2}}}{3}-2a\sqrt{a-2^{x}b}}{b^{2}\ln(2)}$	36

input
```
int(2^(2*x)/(a-2^x*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2^x*b+2*a)/b^2*(a-2^x*b)^(1/2)/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{2^{2x}}{\sqrt{a-2^{x}b}}\,dx = -\frac{2\,(2^{x}b+2\,a)\sqrt{-2^{x}b+a}}{3\,b^{2}\log{(2)}}$$

input
```
integrate(2^(2*x)/(a-2^x*b)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(2^x*b + 2*a)*sqrt(-2^x*b + a)/(b^2*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{2^{2x}}{\sqrt{a-2^x b}}\,dx = \begin{cases} -\frac{2\cdot 2^x\sqrt{-2^x b+a}}{3b\log(2)} - \frac{4a\sqrt{-2^x b+a}}{3b^2\log(2)} & \text{for } b \neq 0 \\ \frac{2^{2x}}{2\sqrt{a}\log(2)} & \text{otherwise} \end{cases}$$

input
```
integrate(2**(2*x)/(a-2**x*b)**(1/2),x)
```

output
```
Piecewise((-2*2**x*sqrt(-2**x*b + a)/(3*b*log(2)) - 4*a*sqrt(-2**x*b + a)/
(3*b**2*log(2)), Ne(b, 0)), (2**(2*x)/(2*sqrt(a)*log(2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{2^{2x}}{\sqrt{a-2^x b}}\,dx = \frac{2\left(-2^x b+a\right)^{\frac{3}{2}}}{3\,b^2\log(2)} - \frac{2\sqrt{-2^x b+a}a}{b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a-2^x*b)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(-2^x*b + a)^(3/2)/(b^2*log(2)) - 2*sqrt(-2^x*b + a)*a/(b^2*log(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \frac{2^{2x}}{\sqrt{a-2^xb}}\,dx = \frac{2\left((-2^xb+a)^{\frac{3}{2}}-3\sqrt{-2^xb+a}a\right)}{3\,b^2\log(2)}$$

input
```
integrate(2^(2*x)/(a-2^x*b)^(1/2),x, algorithm="giac")
```

output
```
2/3*((-2^x*b + a)^(3/2) - 3*sqrt(-2^x*b + a)*a)/(b^2*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{2^{2x}}{\sqrt{a-2^xb}}\,dx = -\frac{2\sqrt{a-2^x\,b}\,(2\,a+2^x\,b)}{3\,b^2\,\ln(2)}$$

input
```
int(2^(2*x)/(a - 2^x*b)^(1/2),x)
```

output
```
-(2*(a - 2^x*b)^(1/2)*(2*a + 2^x*b))/(3*b^2*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \frac{2^{2x}}{\sqrt{a-2^xb}}\,dx = \frac{2\sqrt{-2^xb+a}\,(-2^xb-2a)}{3\log(2)\,b^2}$$

input
```
int(2^(2*x)/(a-2^x*b)^(1/2),x)
```

output
```
(2*sqrt( - 2**x*b + a)*( - 2**x*b - 2*a))/(3*log(2)*b**2)
```

3.424 $\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx=\frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a\log(2)}-\frac{3\ 2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2\log(2)}+\frac{3b^2\operatorname{arctanh}\left(\frac{\sqrt{a+2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2}\log(2)}$$

output
```
2^(-1+2*x)*(a+b/(2^x))^(1/2)/a/ln(2)-3*2^(-2+x)*b*(a+b/(2^x))^(1/2)/a^2/ln
(2)+3/4*b^2*arctanh((a+b/(2^x))^(1/2)/a^(1/2))/a^(5/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx = \frac{2^{-2-\frac{x}{2}}\left(2^{x/2}\sqrt{a}(2^{1+2x}a^2-2^xab-3b^2)+3b^2\sqrt{2^xa+b}\operatorname{arctanh}\left(\frac{2^{x/2}\sqrt{a}}{\sqrt{2^xa+b}}\right)\right)}{a^{5/2}\sqrt{a+2^{-x}b}\log(2)}$$

input
```
Integrate[4^x/Sqrt[a + b/2^x],x]
```

output
```
(2^(-2 - x/2)*(2^(x/2)*Sqrt[a]*(2^(1 + 2*x)*a^2 - 2^x*a*b - 3*b^2) + 3*b^2
*Sqrt[2^x*a + b]*ArcTanh[(2^(x/2)*Sqrt[a])/Sqrt[2^x*a + b]]))/(a^(5/2)*Sqr
t[a + b/2^x]*Log[2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2678, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{\sqrt{a+b2^{-x}}}dx$$

$$\Big\downarrow \text{2678}$$

$$-\frac{\int \frac{2^{3x}}{\sqrt{a+2^{-x}b}}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}d2^{-x}}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\left(-\frac{b\int \frac{2^x}{\sqrt{a+2^{-x}b}}d2^{-x}}{2a}-\frac{2^x\sqrt{a+b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{3b\left(-\frac{\int \frac{1}{\frac{2^{-2x}}{b}-\frac{a}{b}}d\sqrt{a+2^{-x}b}}{a}-\frac{2^x\sqrt{a+b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+b2^{-x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2^x\sqrt{a+b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

input
```
Int[4^x/Sqrt[a + b/2^x],x]
```

output
```
-((-((2^(-1 + 2*x)*Sqrt[a + b/2^x])/a) - (3*b*(-((2^x*Sqrt[a + b/2^x])/a)
+ (b*ArcTanh[Sqrt[a + b/2^x]/Sqrt[a]])/a^(3/2)))/(4*a))/Log[2])
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [F]

$$\int \frac{4^x}{\sqrt{a+b2^{-x}}}\,dx$$

input
```
int(4^x/(a+b/(2^x))^(1/2),x)
```

output
```
int(4^x/(a+b/(2^x))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.88

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx = \left[\frac{3\sqrt{a}b^2\log\left(2\cdot 2^x a+2\cdot 2^x\sqrt{a}\sqrt{\frac{2^x a+b}{2^x}}+b\right)+2\left(2\cdot 2^{2x}a^2-3\cdot 2^x ab\right)\sqrt{\frac{2^x a+b}{2^x}}}{8\,a^3\log(2)},\right.$$
$$\left.-\frac{3\sqrt{-a}b^2\arctan\left(\frac{2^x\sqrt{-a}\sqrt{\frac{2^x a+b}{2^x}}}{2^x a+b}\right)-\left(2\cdot 2^{2x}a^2-3\cdot 2^x ab\right)\sqrt{\frac{2^x a+b}{2^x}}}{4\,a^3\log(2)}\right]$$

input
```
integrate(4^x/(a+b/(2^x))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*log(2*2^x*a + 2*2^x*sqrt(a)*sqrt((2^x*a + b)/2^x) + b)
 + 2*(2*2^(2*x)*a^2 - 3*2^x*a*b)*sqrt((2^x*a + b)/2^x))/(a^3*log(2)), -1/4
*(3*sqrt(-a)*b^2*arctan(2^x*sqrt(-a)*sqrt((2^x*a + b)/2^x)/(2^x*a + b)) -
(2*2^(2*x)*a^2 - 3*2^x*a*b)*sqrt((2^x*a + b)/2^x))/(a^3*log(2))]
```

Sympy [F]

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx=\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx$$

input
```
integrate(4**x/(a+b/(2**x))**(1/2),x)
```

output
```
Integral(4**x/sqrt(a + b/2**x), x)
```

Maxima [F]

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx=\int \frac{4^x}{\sqrt{a+\frac{b}{2^x}}}\,dx$$

input
```
integrate(4^x/(a+b/(2^x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(4^x/sqrt(a + b/2^x), x)
```

Giac [F]

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx=\int \frac{4^x}{\sqrt{a+\frac{b}{2^x}}}\,dx$$

input
```
integrate(4^x/(a+b/(2^x))^(1/2),x, algorithm="giac")
```

output
```
integrate(4^x/sqrt(a + b/2^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx = \int \frac{4^x}{\sqrt{a+\frac{b}{2^x}}}\,dx$$

input
```
int(4^x/(a + b/2^x)^(1/2),x)
```

output
```
int(4^x/(a + b/2^x)^(1/2), x)
```

Reduce [F]

$$\int \frac{4^x}{\sqrt{a+2^{-x}b}}\,dx = \int \frac{\sqrt{2^x a+b}\,2^{\frac{x}{2}}4^x}{2^x a+b}\,dx$$

input
```
int(4^x/(a+b/(2^x))^(1/2),x)
```

output
```
int((sqrt(2**x*a + b)*2**(x/2)*4**x)/(2**x*a + b),x)
```

3.425 $\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx=\frac{2^{-1+2x}\sqrt{a+2^{-x}b}}{a\log(2)}-\frac{3\,2^{-2+x}b\sqrt{a+2^{-x}b}}{a^2\log(2)}+\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2}\log(2)}$$

output
```
2^(-1+2*x)*(a+b/(2^x))^(1/2)/a/ln(2)-3*2^(-2+x)*b*(a+b/(2^x))^(1/2)/a^2/ln
(2)+3/4*b^2*arctanh((a+b/(2^x))^(1/2)/a^(1/2))/a^(5/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.19

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = \frac{2^{-2-\frac{x}{2}}\left(2^{x/2}\sqrt{a}(2^{1+2x}a^2-2^xab-3b^2)+3b^2\sqrt{2^xa+b}\,\text{arctanh}\left(\frac{2^{x/2}\sqrt{a}}{\sqrt{2^xa+b}}\right)\right)}{a^{5/2}\sqrt{a+2^{-x}b}\log(2)}$$

input
```
Integrate[2^(2*x)/Sqrt[a + b/2^x],x]
```

output
```
(2^(-2 - x/2)*(2^(x/2)*Sqrt[a]*(2^(1 + 2*x)*a^2 - 2^x*a*b - 3*b^2) + 3*b^2
*Sqrt[2^x*a + b]*ArcTanh[(2^(x/2)*Sqrt[a])/Sqrt[2^x*a + b]]))/(a^(5/2)*Sqr
t[a + b/2^x]*Log[2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2678, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{\sqrt{a+b2^{-x}}}dx$$

$$\Big\downarrow \text{2678}$$

$$-\frac{\int \frac{2^{3x}}{\sqrt{a+2^{-x}b}}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}d2^{-x}}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{-\frac{3b\left(-\frac{b\int \frac{2^{x}}{\sqrt{a+2^{-x}b}}d2^{-x}}{2a}-\frac{2^{x}\sqrt{a+b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{-\frac{3b\left(-\frac{\int \frac{1}{\frac{2^{-2x}}{b}-\frac{a}{b}}d\sqrt{a+2^{-x}b}}{a}-\frac{2^{x}\sqrt{a+b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+b2^{-x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2^x\sqrt{a+b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a+b2^{-x}}}{a}}{\log(2)}$$

input
```
Int[2^(2*x)/Sqrt[a + b/2^x],x]
```

output
```
-((-((2^(-1 + 2*x)*Sqrt[a + b/2^x])/a) - (3*b*(-((2^x*Sqrt[a + b/2^x])/a)
+ (b*ArcTanh[Sqrt[a + b/2^x]/Sqrt[a]])/a^(3/2)))/(4*a))/Log[2])
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(81) = 162$.

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.96

method	result
derivativedivides	$-\frac{\sqrt{(a2^x+b)2^{-x}}\,2^x\left(-4a^{\frac{5}{2}}\sqrt{a2^{2x}+2^x b}\,2^x+8\sqrt{(a2^x+b)2^x}\,a^{\frac{3}{2}}b-2a^{\frac{3}{2}}\sqrt{a2^{2x}+2^x b}\,b-4a\ln\left(\frac{2\sqrt{(a2^x+b)2^x}\,\sqrt{a}+2a2^x+b}{2\sqrt{a}}\right.}{8\ln(2)\sqrt{(a2^x+b)2^x}\,a^{\frac{7}{2}}}$
default	$-\frac{\sqrt{(a2^x+b)2^{-x}}\,2^x\left(-4a^{\frac{5}{2}}\sqrt{a2^{2x}+2^x b}\,2^x+8\sqrt{(a2^x+b)2^x}\,a^{\frac{3}{2}}b-2a^{\frac{3}{2}}\sqrt{a2^{2x}+2^x b}\,b-4a\ln\left(\frac{2\sqrt{(a2^x+b)2^x}\,\sqrt{a}+2a2^x+b}{2\sqrt{a}}\right.}{8\ln(2)\sqrt{(a2^x+b)2^x}\,a^{\frac{7}{2}}}$

input
```
int(2^(2*x)/(a+b/(2^x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/ln(2)*((a*2^x+b)/(2^x))^(1/2)*2^x*(-4*a^(5/2)*(a*(2^x)^2+2^x*b)^(1/2)
*2^x+8*((a*2^x+b)*2^x)^(1/2)*a^(3/2)*b-2*a^(3/2)*(a*(2^x)^2+2^x*b)^(1/2)*b
-4*a*ln(1/2*(2*((a*2^x+b)*2^x)^(1/2)*a^(1/2)+2*a*2^x+b)/a^(1/2))*b^2+b^2*l
n(1/2*(2*(a*(2^x)^2+2^x*b)^(1/2)*a^(1/2)+2*a*2^x+b)/a^(1/2))*a)/((a*2^x+b)
*2^x)^(1/2)/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.88

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = \left[\frac{3\sqrt{a}b^2\log\left(2\cdot 2^x a+2\cdot 2^x\sqrt{a}\sqrt{\frac{2^x a+b}{2^x}}+b\right)+2\left(2\cdot 2^{2x}a^2-3\cdot 2^x ab\right)\sqrt{\frac{2^x a+b}{2^x}}}{8\,a^3\log(2)}, -\frac{3\sqrt{-a}b^2\arctan\left(\frac{2^x\sqrt{-a}\sqrt{\frac{2^x a+b}{2^x}}}{2^x a+b}\right)-\left(2\cdot 2^{2x}a^2-3\cdot 2^x ab\right)\sqrt{\frac{2^x a+b}{2^x}}}{4\,a^3\log(2)}\right]$$

input
```
integrate(2^(2*x)/(a+b/(2^x))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*log(2*2^x*a + 2*2^x*sqrt(a)*sqrt((2^x*a + b)/2^x) + b)
 + 2*(2*2^(2*x)*a^2 - 3*2^x*a*b)*sqrt((2^x*a + b)/2^x))/(a^3*log(2)), -1/4
*(3*sqrt(-a)*b^2*arctan(2^x*sqrt(-a)*sqrt((2^x*a + b)/2^x)/(2^x*a + b)) -
(2*2^(2*x)*a^2 - 3*2^x*a*b)*sqrt((2^x*a + b)/2^x))/(a^3*log(2))]
```

Sympy [F]

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = \int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx$$

input
```
integrate(2**(2*x)/(a+b/(2**x))**(1/2),x)
```

output
```
Integral(2**(2*x)/sqrt(a + b/2**x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.33

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = -\frac{3\,b^2\log\left(\frac{\sqrt{a+\frac{b}{2^x}}-\sqrt{a}}{\sqrt{a+\frac{b}{2^x}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}\log(2)} - \frac{3\left(a+\frac{b}{2^x}\right)^{\frac{3}{2}}b^2 - 5\sqrt{a+\frac{b}{2^x}}ab^2}{4\left(\left(a+\frac{b}{2^x}\right)^2a^2 - 2\left(a+\frac{b}{2^x}\right)a^3 + a^4\right)\log(2)}$$

input
```
integrate(2^(2*x)/(a+b/(2^x))^(1/2),x, algorithm="maxima")
```

output
```
-3/8*b^2*log((sqrt(a + b/2^x) - sqrt(a))/(sqrt(a + b/2^x) + sqrt(a)))/(a^(
5/2)*log(2)) - 1/4*(3*(a + b/2^x)^(3/2)*b^2 - 5*sqrt(a + b/2^x)*a*b^2)/(((
a + b/2^x)^2*a^2 - 2*(a + b/2^x)*a^3 + a^4)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = \frac{2\sqrt{2^{2x}a+2^{x}b}\left(\frac{2\cdot 2^{x}}{a}-\frac{3b}{a^{2}}\right)-\frac{3b^{2}\log\left(\left|2\left(2^{x}\sqrt{a}-\sqrt{2^{2x}a+2^{x}b}\right)\sqrt{a}+b\right|\right)}{a^{\frac{5}{2}}}+\frac{3b^{2}\log(|b|)}{a^{\frac{5}{2}}}}{8\log(2)}$$

input
```
integrate(2^(2*x)/(a+b/(2^x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2^(2*x)*a + 2^x*b)*(2*2^x/a - 3*b/a^2) - 3*b^2*log(abs(2*(2^x*
sqrt(a) - sqrt(2^(2*x)*a + 2^x*b))*sqrt(a) + b))/a^(5/2) + 3*b^2*log(abs(b
))/a^(5/2))/log(2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = \int \frac{2^{2x}}{\sqrt{a+\frac{b}{2^{x}}}}\,dx$$

input
```
int(2^(2*x)/(a + b/2^x)^(1/2),x)
```

output
```
int(2^(2*x)/(a + b/2^x)^(1/2), x)
```

Reduce [F]

$$\int \frac{2^{2x}}{\sqrt{a+2^{-x}b}}\,dx = \frac{4\sqrt{2^{x}a+b}\,2^{x}2^{\frac{x}{2}}a-6\sqrt{2^{x}a+b}\,2^{\frac{x}{2}}b+3\left(\int \frac{\sqrt{2^{x}a+b}\,2^{\frac{x}{2}}}{2^{x}a+b}dx\right)\log(2)\,b^{2}}{8\log(2)\,a^{2}}$$

input
```
int(2^(2*x)/(a+b/(2^x))^(1/2),x)
```

output
```
(4*sqrt(2**x*a + b)*2**x*2**(x/2)*a - 6*sqrt(2**x*a + b)*2**(x/2)*b + 3*in
t((sqrt(2**x*a + b)*2**(x/2))/(2**x*a + b),x)*log(2)*b**2)/(8*log(2)*a**2)
```

3.426 $\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 96

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx=\frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a\log(2)}+\frac{3\,2^{-2+x}b\sqrt{a-2^{-x}b}}{a^2\log(2)}+\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a-2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2}\log(2)}$$

output
```
2^(-1+2*x)*(a-b/(2^x))^(1/2)/a/ln(2)+3*2^(-2+x)*b*(a-b/(2^x))^(1/2)/a^2/ln
(2)+3/4*b^2*arctanh((a-b/(2^x))^(1/2)/a^(1/2))/a^(5/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.20

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx =\frac{2^{-2-\frac{x}{2}}\left(2^{x/2}\sqrt{a}(2^{1+2x}a^2+2^xab-3b^2)+3\sqrt{2^xa-b}b^2\text{arctanh}\left(\frac{2^{x/2}\sqrt{a}}{\sqrt{2^xa-b}}\right)\right)}{a^{5/2}\sqrt{a-2^{-x}b}\log(2)}$$

input
```
Integrate[4^x/Sqrt[a - b/2^x],x]
```

output
```
(2^(-2 - x/2)*(2^(x/2)*Sqrt[a]*(2^(1 + 2*x)*a^2 + 2^x*a*b - 3*b^2) + 3*Sqr
t[2^x*a - b]*b^2*ArcTanh[(2^(x/2)*Sqrt[a])/Sqrt[2^x*a - b]]))/(a^(5/2)*Sqr
t[a - b/2^x]*Log[2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2678, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x}{\sqrt{a-b2^{-x}}}dx$$

$$\Big\downarrow \text{2678}$$

$$-\frac{\int \frac{2^{3x}}{\sqrt{a-2^{-x}b}}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\frac{3b\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}}d2^{-x}}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{52}$$

$$-\frac{\frac{3b\left(\frac{b\int \frac{2^{x}}{\sqrt{a-2^{-x}b}}d2^{-x}}{2a}-\frac{2^{x}\sqrt{a-b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\frac{3b\left(-\frac{\int \frac{1}{\frac{a}{b}-\frac{2^{-2x}}{b}}d\sqrt{a-2^{-x}b}}{a}-\frac{2^{x}\sqrt{a-b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3b\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a-b2^{-x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2^x\sqrt{a-b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

input
```
Int[4^x/Sqrt[a - b/2^x],x]
```

output
```
-((-((2^(-1 + 2*x)*Sqrt[a - b/2^x])/a) + (3*b*(-((2^x*Sqrt[a - b/2^x])/a)
- (b*ArcTanh[Sqrt[a - b/2^x]/Sqrt[a]])/a^(3/2)))/(4*a))/Log[2])
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [F]

$$\int \frac{4^x}{\sqrt{a - b2^{-x}}}dx$$

input
```
int(4^x/(a-b/(2^x))^(1/2),x)
```

output
```
int(4^x/(a-b/(2^x))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.93

$$\int \frac{4^x}{\sqrt{a - 2^{-x}b}}\,dx = \left[\frac{3\,\sqrt{a}b^2\log\left(-2\cdot 2^x a - 2\cdot 2^x\sqrt{a}\sqrt{\frac{2^x a - b}{2^x}} + b\right) + 2\left(2\cdot 2^{2\,x}a^2 + 3\cdot 2^x ab\right)\sqrt{\frac{2^x a - b}{2^x}}}{8\,a^3\log\left(2\right)},\; -\frac{3\,\sqrt{-a}b^2\arctan\left(\frac{2^x\sqrt{-a}\sqrt{\frac{2^x a-b}{2^x}}}{2^x a - b}\right) - \left(2\cdot 2^{2\,x}a^2 + 3\cdot 2^x ab\right)\sqrt{\frac{2^x a - b}{2^x}}}{4\,a^3\log\left(2\right)}\right]$$

input
```
integrate(4^x/(a-b/(2^x))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*log(-2*2^x*a - 2*2^x*sqrt(a)*sqrt((2^x*a - b)/2^x) + b
) + 2*(2*2^(2*x)*a^2 + 3*2^x*a*b)*sqrt((2^x*a - b)/2^x))/(a^3*log(2)), -1/
4*(3*sqrt(-a)*b^2*arctan(2^x*sqrt(-a)*sqrt((2^x*a - b)/2^x)/(2^x*a - b)) -
 (2*2^(2*x)*a^2 + 3*2^x*a*b)*sqrt((2^x*a - b)/2^x))/(a^3*log(2))]
```

Sympy [F]

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx = \int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx$$

input
```
integrate(4**x/(a-b/(2**x))**(1/2),x)
```

output
```
Integral(4**x/sqrt(a - b/2**x), x)
```

Maxima [F]

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx = \int \frac{4^x}{\sqrt{a-\frac{b}{2^x}}}\,dx$$

input
```
integrate(4^x/(a-b/(2^x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(4^x/sqrt(a - b/2^x), x)
```

Giac [F]

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx = \int \frac{4^x}{\sqrt{a-\frac{b}{2^x}}}\,dx$$

input
```
integrate(4^x/(a-b/(2^x))^(1/2),x, algorithm="giac")
```

output
```
integrate(4^x/sqrt(a - b/2^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx = \int \frac{4^x}{\sqrt{a-\frac{b}{2^x}}}\,dx$$

input
```
int(4^x/(a - b/2^x)^(1/2),x)
```

output
```
int(4^x/(a - b/2^x)^(1/2), x)
```

Reduce [F]

$$\int \frac{4^x}{\sqrt{a-2^{-x}b}}\,dx = \int \frac{\sqrt{2^x a-b}\,2^{\frac{x}{2}}4^x}{2^x a-b}dx$$

input
```
int(4^x/(a-b/(2^x))^(1/2),x)
```

output
```
int((sqrt(2**x*a - b)*2**(x/2)*4**x)/(2**x*a - b),x)
```

3.427 $\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}} \, dx = \frac{2^{-1+2x}\sqrt{a-2^{-x}b}}{a\log(2)} + \frac{3\,2^{-2+x}b\sqrt{a-2^{-x}b}}{a^2\log(2)} + \frac{3b^2\text{arctanh}\left(\frac{\sqrt{a-2^{-x}b}}{\sqrt{a}}\right)}{4a^{5/2}\log(2)}$$

output

```
2^(-1+2*x)*(a-b/(2^x))^(1/2)/a/ln(2)+3*2^(-2+x)*b*(a-b/(2^x))^(1/2)/a^2/ln
(2)+3/4*b^2*arctanh((a-b/(2^x))^(1/2)/a^(1/2))/a^(5/2)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.20

$$\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}} \, dx = \frac{2^{-2-\frac{x}{2}}\left(2^{x/2}\sqrt{a}(2^{1+2x}a^2+2^xab-3b^2)+3\sqrt{2^xa-b}\,b^2\text{arctanh}\left(\frac{2^{x/2}\sqrt{a}}{\sqrt{2^xa-b}}\right)\right)}{a^{5/2}\sqrt{a-2^{-x}b}\log(2)}$$

input

```
Integrate[2^(2*x)/Sqrt[a - b/2^x],x]
```

output
```
(2^(-2 - x/2)*(2^(x/2)*Sqrt[a]*(2^(1 + 2*x)*a^2 + 2^x*a*b - 3*b^2) + 3*Sqr
t[2^x*a - b]*b^2*ArcTanh[(2^(x/2)*Sqrt[a])/Sqrt[2^x*a - b]]))/(a^(5/2)*Sqr
t[a - b/2^x]*Log[2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2678, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{2x}}{\sqrt{a-b2^{-x}}}dx$$

$$\Big\downarrow \text{ 2678}$$

$$-\frac{\int \frac{2^{3x}}{\sqrt{a-2^{-x}b}}d2^{-x}}{\log(2)}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{\frac{3b\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}}d2^{-x}}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{\frac{3b\left(\frac{b\int \frac{2^{x}}{\sqrt{a-2^{-x}b}}d2^{-x}}{2a}-\frac{2^{x}\sqrt{a-b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{\frac{3b\left(-\frac{\int \frac{1}{\frac{a}{b}-\frac{2^{-2x}}{b}}d\sqrt{a-2^{-x}b}}{a}-\frac{2^{x}\sqrt{a-b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{3b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a-b2^{-x}}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2^x\sqrt{a-b2^{-x}}}{a}\right)}{4a}-\frac{2^{2x-1}\sqrt{a-b2^{-x}}}{a}}{\log(2)}$$

input
```
Int[2^(2*x)/Sqrt[a - b/2^x],x]
```

output
```
-((-((2^(-1 + 2*x)*Sqrt[a - b/2^x])/a) + (3*b*(-((2^x*Sqrt[a - b/2^x])/a)
- (b*ArcTanh[Sqrt[a - b/2^x]/Sqrt[a]])/a^(3/2)))/(4*a))/Log[2])
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(84) = 168$.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.06

method	result
derivativedivides	$\frac{\sqrt{2^{-x}(a2^x-b)}\,2^x\left(4a^{\frac{5}{2}}\sqrt{a2^{2x}-2^x b}\,2^x-2a^{\frac{3}{2}}\sqrt{a2^{2x}-2^x b}\,b+8a^{\frac{3}{2}}\sqrt{(a2^x-b)2^x}\,b+4a\ln\left(\frac{2\sqrt{(a2^x-b)2^x}\,\sqrt{a}+2a2^x-b}{2\sqrt{a}}\right)b^2-}{8\ln(2)\sqrt{(a2^x-b)2^x}\,a^{\frac{7}{2}}}$
default	$\frac{\sqrt{2^{-x}(a2^x-b)}\,2^x\left(4a^{\frac{5}{2}}\sqrt{a2^{2x}-2^x b}\,2^x-2a^{\frac{3}{2}}\sqrt{a2^{2x}-2^x b}\,b+8a^{\frac{3}{2}}\sqrt{(a2^x-b)2^x}\,b+4a\ln\left(\frac{2\sqrt{(a2^x-b)2^x}\,\sqrt{a}+2a2^x-b}{2\sqrt{a}}\right)b^2-}{8\ln(2)\sqrt{(a2^x-b)2^x}\,a^{\frac{7}{2}}}$

input
```
int(2^(2*x)/(a-b/(2^x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/ln(2)*(1/(2^x)*(a*2^x-b))^(1/2)*2^x*(4*a^(5/2)*(a*(2^x)^2-2^x*b)^(1/2)
*2^x-2*a^(3/2)*(a*(2^x)^2-2^x*b)^(1/2)*b+8*a^(3/2)*((a*2^x-b)*2^x)^(1/2)*b
+4*a*ln(1/2*(2*((a*2^x-b)*2^x)^(1/2)*a^(1/2)+2*a*2^x-b)/a^(1/2))*b^2-b^2*l
n(1/2*(2*(a*(2^x)^2-2^x*b)^(1/2)*a^(1/2)+2*a*2^x-b)/a^(1/2))*a)/((a*2^x-b)
*2^x)^(1/2)/a^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.93

$$\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}}\,dx = \left[\frac{3\sqrt{a}b^2\log\left(-2\cdot 2^x a-2\cdot 2^x\sqrt{a}\sqrt{\frac{2^x a-b}{2^x}}+b\right)+2\left(2\cdot 2^{2x}a^2+3\cdot 2^x ab\right)\sqrt{\frac{2^x a-b}{2^x}}}{8\,a^3\log(2)}, -\frac{3\sqrt{-a}b^2\arctan\left(\frac{2^x\sqrt{-a}\sqrt{\frac{2^x a-b}{2^x}}}{2^x a-b}\right)-\left(2\cdot 2^{2x}a^2+3\cdot 2^x ab\right)\sqrt{\frac{2^x a-b}{2^x}}}{4\,a^3\log(2)}\right]$$

input
```
integrate(2^(2*x)/(a-b/(2^x))^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(3*sqrt(a)*b^2*log(-2*2^x*a - 2*2^x*sqrt(a)*sqrt((2^x*a - b)/2^x) + b
) + 2*(2*2^(2*x)*a^2 + 3*2^x*a*b)*sqrt((2^x*a - b)/2^x))/(a^3*log(2)), -1/
4*(3*sqrt(-a)*b^2*arctan(2^x*sqrt(-a)*sqrt((2^x*a - b)/2^x)/(2^x*a - b)) -
 (2*2^(2*x)*a^2 + 3*2^x*a*b)*sqrt((2^x*a - b)/2^x))/(a^3*log(2))]
```

Sympy [F]

$$\int \frac{2^{2x}}{\sqrt{a - 2^{-x}b}}\,dx = \int \frac{2^{2x}}{\sqrt{a - 2^{-x}b}}\,dx$$

input
```
integrate(2**(2*x)/(a-b/(2**x))**(1/2),x)
```

output
```
Integral(2**(2*x)/sqrt(a - b/2**x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.35

$$\int \frac{2^{2x}}{\sqrt{a - 2^{-x}b}}\,dx = -\frac{3\,b^2\log\left(\frac{\sqrt{a-\frac{b}{2^x}}-\sqrt{a}}{\sqrt{a-\frac{b}{2^x}}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}\log(2)} - \frac{3\left(a-\frac{b}{2^x}\right)^{\frac{3}{2}}b^2 - 5\sqrt{a-\frac{b}{2^x}}\,ab^2}{4\left(\left(a-\frac{b}{2^x}\right)^2a^2 - 2\left(a-\frac{b}{2^x}\right)a^3 + a^4\right)\log(2)}$$

input
```
integrate(2^(2*x)/(a-b/(2^x))^(1/2),x, algorithm="maxima")
```

output
```
-3/8*b^2*log((sqrt(a - b/2^x) - sqrt(a))/(sqrt(a - b/2^x) + sqrt(a)))/(a^(
5/2)*log(2)) - 1/4*(3*(a - b/2^x)^(3/2)*b^2 - 5*sqrt(a - b/2^x)*a*b^2)/(((
a - b/2^x)^2*a^2 - 2*(a - b/2^x)*a^3 + a^4)*log(2))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00

$$\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}}\,dx = \frac{2\sqrt{2^{2x}a-2^{x}b}\left(\frac{2\cdot 2^{x}}{a}+\frac{3b}{a^{2}}\right)-\frac{3b^{2}\log\left(\left|2\left(2^{x}\sqrt{a}-\sqrt{2^{2x}a-2^{x}b}\right)\sqrt{a}-b\right|\right)}{a^{\frac{5}{2}}}+\frac{3b^{2}\log(|b|)}{a^{\frac{5}{2}}}}{8\log(2)}$$

input
```
integrate(2^(2*x)/(a-b/(2^x))^(1/2),x, algorithm="giac")
```

output
```
1/8*(2*sqrt(2^(2*x)*a - 2^x*b)*(2*2^x/a + 3*b/a^2) - 3*b^2*log(abs(2*(2^x*
sqrt(a) - sqrt(2^(2*x)*a - 2^x*b))*sqrt(a) - b))/a^(5/2) + 3*b^2*log(abs(b
))/a^(5/2))/log(2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}}\,dx = \int \frac{2^{2x}}{\sqrt{a-\frac{b}{2^{x}}}}\,dx$$

input
```
int(2^(2*x)/(a - b/2^x)^(1/2),x)
```

output
```
int(2^(2*x)/(a - b/2^x)^(1/2), x)
```

Reduce [F]

$$\int \frac{2^{2x}}{\sqrt{a-2^{-x}b}}\,dx = \frac{4\sqrt{2^{x}a-b}\,2^{x}2^{\frac{x}{2}}a+6\sqrt{2^{x}a-b}\,2^{\frac{x}{2}}b+3\left(\int \frac{\sqrt{2^{x}a-b}\,2^{\frac{x}{2}}}{2^{x}a-b}dx\right)\log(2)\,b^{2}}{8\log(2)\,a^{2}}$$

input
```
int(2^(2*x)/(a-b/(2^x))^(1/2),x)
```

output
```
(4*sqrt(2**x*a - b)*2**x*2**(x/2)*a + 6*sqrt(2**x*a - b)*2**(x/2)*b + 3*in
t((sqrt(2**x*a - b)*2**(x/2))/(2**x*a - b),x)*log(2)*b**2)/(8*log(2)*a**2)
```

3.428 $\int \frac{1}{1+2e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx=\frac{1}{1+e^x}+x-\log\left(1+e^x\right)$$

output
```
1/(1+exp(x))+x-ln(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx=\frac{1}{1+e^x}-2\text{arctanh}(1+2e^x)$$

input
```
Integrate[(1 + 2*E^x + E^(2*x))^(-1),x]
```

output
```
(1 + E^x)^(-1) - 2*ArcTanh[1 + 2*E^x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2e^x + e^{2x} + 1} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^{-x}}{(e^x + 1)^2} de^x$$

$$\Big\downarrow \text{ 54}$$

$$\int \left(e^{-x} - \frac{1}{(e^x + 1)^2} + \frac{1}{-e^x - 1} \right) de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{e^x + 1} + \log(e^x) - \log(e^x + 1)$$

input
```
Int[(1 + 2*E^x + E^(2*x))^(-1),x]
```

output
```
(1 + E^x)^(-1) + Log[E^x] - Log[1 + E^x]
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{1}{1+e^{x}}+x-\ln\left(1+e^{x}\right)$	16
default	$\ln\left(e^{x}\right)+\frac{1}{1+e^{x}}-\ln\left(1+e^{x}\right)$	18
norman	$\frac{x+e^{x}x+1}{1+e^{x}}-\ln\left(1+e^{x}\right)$	23

input
```
int(1/(1+2*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/(1+exp(x))+x-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx = \frac{xe^x-(e^x+1)\log(e^x+1)+x+1}{e^x+1}$$

input
```
integrate(1/(1+2*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
(x*e^x - (e^x + 1)*log(e^x + 1) + x + 1)/(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx = x - \log{(e^x+1)} + \frac{1}{e^x+1}$$

input
```
integrate(1/(1+2*exp(x)+exp(2*x)),x)
```

output
```
x - log(exp(x) + 1) + 1/(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx = x + \frac{1}{e^x+1} - \log{(e^x+1)}$$

input
```
integrate(1/(1+2*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
x + 1/(e^x + 1) - log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx = x + \frac{1}{e^x+1} - \log{(e^x+1)}$$

input
```
integrate(1/(1+2*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
x + 1/(e^x + 1) - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx = x-\ln\left(\mathrm{e}^x+1\right)+\frac{1}{\mathrm{e}^x+1}$$

input
```
int(1/(exp(2*x) + 2*exp(x) + 1),x)
```

output
```
x - log(exp(x) + 1) + 1/(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{1+2e^x+e^{2x}}\,dx = \frac{-e^x\log(e^x+1)+e^x x-e^x-\log(e^x+1)+x}{e^x+1}$$

input
```
int(1/(1+2*exp(x)+exp(2*x)),x)
```

output
```
( - e**x*log(e**x + 1) + e**x*x - e**x - log(e**x + 1) + x)/(e**x + 1)
```

3.429 $\int \frac{1}{2+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{x}{2} - \log(1+e^x) + \frac{1}{2}\log(2+e^x)$$

output
```
1/2*x-ln(1+exp(x))+1/2*ln(2+exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{\log(e^x)}{2} - \log(1+e^x) + \frac{1}{2}\log(2+e^x)$$

input
```
Integrate[(2 + 3*E^x + E^(2*x))^(-1),x]
```

output
```
Log[E^x]/2 - Log[1 + E^x] + Log[2 + E^x]/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3e^x + e^{2x} + 2} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^{-x}}{3e^x + e^{2x} + 2} de^x$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{e^{-x}}{2} + \frac{1}{2(e^x + 2)} + \frac{1}{-e^x - 1}\right) de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\log(e^x)}{2} - \log(e^x + 1) + \frac{1}{2}\log(e^x + 2)$$

input
```
Int[(2 + 3*E^x + E^(2*x))^(-1),x]
```

output
```
Log[E^x]/2 - Log[1 + E^x] + Log[2 + E^x]/2
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

method	result	size
norman	$\frac{x}{2}-\ln\left(1+{\mathrm e}^{x}\right)+\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	19
risch	$\frac{x}{2}-\ln\left(1+{\mathrm e}^{x}\right)+\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}$	19
default	$\frac{\ln\left({\mathrm e}^{x}\right)}{2}+\frac{\ln\left(2+{\mathrm e}^{x}\right)}{2}-\ln\left(1+{\mathrm e}^{x}\right)$	21

input
```
int(1/(2+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x-ln(1+exp(x))+1/2*ln(2+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx=\frac{1}{2}\,x+\frac{1}{2}\,\log\left(e^{x}+2\right)-\log\left(e^{x}+1\right)$$

input
```
integrate(1/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*x + 1/2*log(e^x + 2) - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{x}{2} - \log\left(e^x+1\right) + \frac{\log\left(e^x+2\right)}{2}$$

input
```
integrate(1/(2+3*exp(x)+exp(2*x)),x)
```

output
```
x/2 - log(exp(x) + 1) + log(exp(x) + 2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

input
```
integrate(1/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*log(e^x + 2) - log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

input
```
integrate(1/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
1/2*x + 1/2*log(e^x + 2) - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{x}{2} - \ln\left(\mathrm{e}^x+1\right) + \frac{\ln\left(\mathrm{e}^x+2\right)}{2}$$

input
```
int(1/(exp(2*x) + 3*exp(x) + 2),x)
```

output
```
x/2 - log(exp(x) + 1) + log(exp(x) + 2)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{1}{2+3e^x+e^{2x}}\,dx = \frac{\log(e^x+2)}{2} - \log(e^x+1) + \frac{x}{2}$$

input
```
int(1/(2+3*exp(x)+exp(2*x)),x)
```

output
```
(log(e**x + 2) - 2*log(e**x + 1) + x)/2
```

3.430 $\int \frac{1}{-1+e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 72

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx=-x+\frac{2\log\left(5-3\sqrt{5}+\left(5-\sqrt{5}\right)e^x\right)}{5-\sqrt{5}}+\frac{2\log\left(5+3\sqrt{5}+\left(5+\sqrt{5}\right)e^x\right)}{5+\sqrt{5}}$$

output
```
-x+2*ln(5-3*5^(1/2)+(5-5^(1/2))*exp(x))/(5-5^(1/2))+2*ln(5+3*5^(1/2)+(5+5^
(1/2))*exp(x))/(5+5^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx=\frac{1}{10}\left(-10\log\left(e^x\right)+\left(5+\sqrt{5}\right)\log\left(-1+\sqrt{5}-2e^x\right)-\left(-5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2e^x\right)\right)$$

input
```
Integrate[(-1 + E^x + E^(2*x))^(-1),x]
```

output
```
(-10*Log[E^x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*E^x] - (-5 + Sqrt[5])*L
og[1 + Sqrt[5] + 2*E^x])/10
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 25, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^x + e^{2x} - 1} \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{e^{-x}}{-e^x - e^{2x} + 1} de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{e^{-x}}{1 - e^x - e^{2x}} de^x$$

$$\Big\downarrow \text{1141}$$

$$\int \left(-e^{-x} + \frac{2}{\left(5+\sqrt{5}\right)e^x + 5 + 3\sqrt{5}} + \frac{2}{\left(5-\sqrt{5}\right)e^x + 5 - 3\sqrt{5}}\right) de^x$$

$$\Big\downarrow \text{2009}$$

$$-\log\left(e^x\right) + \frac{2\log\left(\left(5-\sqrt{5}\right)e^x + 5 - 3\sqrt{5}\right)}{5-\sqrt{5}} + \frac{2\log\left(\left(5+\sqrt{5}\right)e^x + 5 + 3\sqrt{5}\right)}{5+\sqrt{5}}$$

input
```
Int[(-1 + E^x + E^(2*x))^(-1),x]
```

output
```
-Log[E^x] + (2*Log[5 - 3*Sqrt[5] + (5 - Sqrt[5])*E^x])/(5 - Sqrt[5]) + (2*
Log[5 + 3*Sqrt[5] + (5 + Sqrt[5])*E^x])/(5 + Sqrt[5])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

method	result	size
default	$-\ln\left({\mathrm e}^{x}\right)+\frac{\ln\left(-1+{\mathrm e}^{x}+{\mathrm e}^{2 x}\right)}{2}-\frac{\sqrt{5}\, \operatorname{arctanh}\left(\frac{\left(1+2 \,{\mathrm e}^{x}\right) \sqrt{5}}{5}\right)}{5}$	35
risch	$-x+\frac{\ln\left({\mathrm e}^{x}+\frac{1}{2}-\frac{\sqrt{5}}{2}\right)}{2}+\frac{\ln\left({\mathrm e}^{x}+\frac{1}{2}-\frac{\sqrt{5}}{2}\right) \sqrt{5}}{10}+\frac{\ln\left({\mathrm e}^{x}+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)}{2}-\frac{\ln\left({\mathrm e}^{x}+\frac{1}{2}+\frac{\sqrt{5}}{2}\right) \sqrt{5}}{10}$	59

input
```
int(1/(-1+exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(exp(x))+1/2*ln(-1+exp(x)+exp(x)^2)-1/5*5^(1/2)*arctanh(1/5*(1+2*exp(x)
)*5^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx = \frac{1}{10}\sqrt{5}\log\left(-\frac{2\left(\sqrt{5}-1\right)e^x+\sqrt{5}-2\,e^{(2\,x)}-3}{e^{(2\,x)}+e^x-1}\right) - x + \frac{1}{2}\log\left(e^{(2\,x)}+e^x-1\right)$$

input
```
integrate(1/(-1+exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*log(-(2*(sqrt(5) - 1)*e^x + sqrt(5) - 2*e^(2*x) - 3)/(e^(2*x)
 + e^x - 1)) - x + 1/2*log(e^(2*x) + e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.31

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx = -x + \operatorname{RootSum}\left(5z^2-5z+1,\left(i\mapsto i\log\left(-5i+e^x+3\right)\right)\right)$$

input
```
integrate(1/(-1+exp(x)+exp(2*x)),x)
```

output
```
-x + RootSum(5*_z**2 - 5*_z + 1, Lambda(_i, _i*log(-5*_i + exp(x) + 3)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx = \frac{1}{10}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\,e^x-1}{\sqrt{5}+2\,e^x+1}\right) - x + \frac{1}{2}\log\left(e^{(2\,x)}+e^x-1\right)$$

input
```
integrate(1/(-1+exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/10*sqrt(5)*log(-(sqrt(5) - 2*e^x - 1)/(sqrt(5) + 2*e^x + 1)) - x + 1/2*l
og(e^(2*x) + e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx = \frac{1}{10}\,\sqrt{5}\log\left(\frac{\left|-\sqrt{5}+2\,e^x+1\right|}{\sqrt{5}+2\,e^x+1}\right) - x + \frac{1}{2}\,\log\left(\left|e^{(2\,x)}+e^x-1\right|\right)$$

input
```
integrate(1/(-1+exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
1/10*sqrt(5)*log(abs(-sqrt(5) + 2*e^x + 1)/(sqrt(5) + 2*e^x + 1)) - x + 1/
2*log(abs(e^(2*x) + e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.44

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^{2\,x}+\mathrm{e}^x-1\right)}{2} - x - \frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\left(2\,\mathrm{e}^x+1\right)}{5}\right)}{5}$$

input
```
int(1/(exp(2*x) + exp(x) - 1),x)
```

output
```
log(exp(2*x) + exp(x) - 1)/2 - x - (5^(1/2)*atanh((5^(1/2)*(2*exp(x) + 1))
/5))/5
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{1}{-1+e^x+e^{2x}}\,dx = \frac{\sqrt{5}\log\left(2e^x-\sqrt{5}+1\right)}{10} - \frac{\sqrt{5}\log\left(2e^x+\sqrt{5}+1\right)}{10} + \frac{\log\left(2e^x-\sqrt{5}+1\right)}{2} + \frac{\log\left(2e^x+\sqrt{5}+1\right)}{2} - x$$

input
```
int(1/(-1+exp(x)+exp(2*x)),x)
```

output
```
(sqrt(5)*log(2*e**x - sqrt(5) + 1) - sqrt(5)*log(2*e**x + sqrt(5) + 1) + 5
*log(2*e**x - sqrt(5) + 1) + 5*log(2*e**x + sqrt(5) + 1) - 10*x)/10
```

3.431 $\int \frac{1}{3+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx=\frac{x}{3}-\frac{\arctan\left(\frac{3+2e^x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(3+3e^x+e^{2x}\right)$$

output
```
1/3*x-1/3*arctan(1/3*(3+2*exp(x))*3^(1/2))*3^(1/2)-1/6*ln(3+3*exp(x)+exp(2
*x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx=\frac{1}{6}\left(-2\sqrt{3}\arctan\left(\frac{3+2e^x}{\sqrt{3}}\right)+2\log\left(e^x\right)-\log\left(3+3e^x+e^{2x}\right)\right)$$

input
```
Integrate[(3 + 3*E^x + E^(2*x))^(-1),x]
```

output
```
(-2*Sqrt[3]*ArcTan[(3 + 2*E^x)/Sqrt[3]] + 2*Log[E^x] - Log[3 + 3*E^x + E^(
2*x)])/6
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2720, 1144, 25, 1142, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3e^x+e^{2x}+3}dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{e^{-x}}{3e^x+e^{2x}+3}de^x$$

$$\Big\downarrow \text{1144}$$

$$\frac{1}{3}\int -\frac{3+e^x}{3+3e^x+e^{2x}}de^x+\frac{\log(e^x)}{3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\log(e^x)}{3}-\frac{1}{3}\int \frac{3+e^x}{3+3e^x+e^{2x}}de^x$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(-\frac{3}{2}\int \frac{1}{3+3e^x+e^{2x}}de^x-\frac{1}{2}\int \frac{3+2e^x}{3+3e^x+e^{2x}}de^x\right)+\frac{\log(e^x)}{3}$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(3\int \frac{1}{-3-e^{2x}}d(3+2e^x)-\frac{1}{2}\int \frac{3+2e^x}{3+3e^x+e^{2x}}de^x\right)+\frac{\log(e^x)}{3}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{3}\left(-\frac{1}{2}\int \frac{3+2e^x}{3+3e^x+e^{2x}}de^x-\sqrt{3}\arctan\left(\frac{2e^x+3}{\sqrt{3}}\right)\right)+\frac{\log(e^x)}{3}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{3}\left(-\sqrt{3}\arctan\left(\frac{2e^x+3}{\sqrt{3}}\right)-\frac{1}{2}\log\left(3e^x+e^{2x}+3\right)\right)+\frac{\log(e^x)}{3}$$

input
```
Int[(3 + 3*E^x + E^(2*x))^(-1),x]
```

output
```
Log[E^x]/3 + (-(Sqrt[3]*ArcTan[(3 + 2*E^x)/Sqrt[3]]) - Log[3 + 3*E^x + E^(
2*x)]/2)/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln(\mathrm{e}^{x})}{3}-\frac{\ln(3+3\,\mathrm{e}^{x}+\mathrm{e}^{2x})}{6}-\frac{\arctan\left(\frac{(3+2\,\mathrm{e}^{x})\sqrt{3}}{3}\right)\sqrt{3}}{3}$	37
risch	$\frac{x}{3}-\frac{\ln\left(\mathrm{e}^{x}+\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(\mathrm{e}^{x}+\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}-\frac{\ln\left(\mathrm{e}^{x}+\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(\mathrm{e}^{x}+\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	65

input
```
int(1/(3+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(exp(x))-1/6*ln(3+3*exp(x)+exp(x)^2)-1/3*arctan(1/3*(3+2*exp(x))*3^(
1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx=-\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{2}{3}\,\sqrt{3}e^{x}+\sqrt{3}\right)+\frac{1}{3}\,x-\frac{1}{6}\,\log\left(e^{(2\,x)}+3\,e^{x}+3\right)$$

input
```
integrate(1/(3+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(2/3*sqrt(3)*e^x + sqrt(3)) + 1/3*x - 1/6*log(e^(2*x) +
 3*e^x + 3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx = \frac{x}{3} + \text{RootSum}\left(9z^2+3z+1, (i \mapsto i\log(-3i+e^x+1))\right)$$

input
```
integrate(1/(3+3*exp(x)+exp(2*x)),x)
```

output
```
x/3 + RootSum(9*_z**2 + 3*_z + 1, Lambda(_i, _i*log(-3*_i + exp(x) + 1)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+3)\right) + \frac{1}{3}x - \frac{1}{6}\log\left(e^{(2x)}+3e^x+3\right)$$

input
```
integrate(1/(3+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 3)) + 1/3*x - 1/6*log(e^(2*x) + 3
*e^x + 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2e^x+3)\right) + \frac{1}{3}x - \frac{1}{6}\log\left(e^{(2x)}+3e^x+3\right)$$

input
```
integrate(1/(3+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
-1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^x + 3)) + 1/3*x - 1/6*log(e^(2*x) + 3
*e^x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx=\frac{x}{3}-\frac{\ln\left(e^{2x}+3\,e^x+3\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}+\frac{2\sqrt{3}\,e^x}{3}\right)}{3}$$

input
```
int(1/(exp(2*x) + 3*exp(x) + 3),x)
```

output
```
x/3 - log(exp(2*x) + 3*exp(x) + 3)/6 - (3^(1/2)*atan(3^(1/2) + (2*3^(1/2)*
exp(x))/3))/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{3+3e^x+e^{2x}}\,dx=-\frac{\sqrt{3}\,atan\left(\frac{2e^x+3}{\sqrt{3}}\right)}{3}-\frac{\log(e^{2x}+3e^x+3)}{6}+\frac{x}{3}$$

input
```
int(1/(3+3*exp(x)+exp(2*x)),x)
```

output
```
( - 2*sqrt(3)*atan((2*e**x + 3)/sqrt(3)) - log(e**(2*x) + 3*e**x + 3) + 2*
x)/6
```

3.432 $\int \frac{1}{a+be^x+ce^{2x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 67

$$\int \frac{1}{a+be^x+ce^{2x}} \, dx = \frac{x}{a}+\frac{b\text{arctanh}\left(\frac{b+2ce^x}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log \left(a+be^x+ce^{2x}\right)}{2a}$$

output
```
x/a+b*arctanh((b+2*c*exp(x))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)-1/2*
ln(a+b*exp(x)+c*exp(2*x))/a
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03

$$\int \frac{1}{a+be^x+ce^{2x}} \, dx = -\frac{\frac{2b\arctan\left(\frac{b+2ce^x}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log \left(e^x\right)+\log \left(a+e^x\left(b+ce^x\right)\right)}{2a}$$

input
```
Integrate[(a + b*E^x + c*E^(2*x))^(-1),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*E^x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] -
2*Log[E^x] + Log[a + E^x*(b + c*E^x)])/a
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2720, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+be^x+ce^{2x}}dx$$
$$\Big\downarrow \text{2720}$$
$$\int \frac{e^{-x}}{a+be^x+ce^{2x}}de^x$$
$$\Big\downarrow \text{1144}$$
$$\frac{\int -\frac{b+ce^x}{a+be^x+ce^{2x}}de^x}{a}+\frac{\log(e^x)}{a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\log(e^x)}{a}-\frac{\int \frac{b+ce^x}{a+be^x+ce^{2x}}de^x}{a}$$
$$\Big\downarrow \text{1142}$$
$$\frac{\log(e^x)}{a}-\frac{\frac{1}{2}b\int \frac{1}{a+be^x+ce^{2x}}de^x+\frac{1}{2}\int \frac{b+2ce^x}{a+be^x+ce^{2x}}de^x}{a}$$
$$\Big\downarrow \text{1083}$$
$$\frac{\log(e^x)}{a}-\frac{\frac{1}{2}\int \frac{b+2ce^x}{a+be^x+ce^{2x}}de^x-b\int \frac{1}{b^2-e^{2x}-4ac}d(b+2ce^x)}{a}$$
$$\Big\downarrow \text{219}$$
$$\frac{\log(e^x)}{a}-\frac{\frac{1}{2}\int \frac{b+2ce^x}{a+be^x+ce^{2x}}de^x-\frac{b\text{arctanh}\left(\frac{b+2ce^x}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$
$$\Big\downarrow \text{1103}$$
$$\frac{\log(e^x)}{a}-\frac{\frac{1}{2}\log\left(a+be^x+ce^{2x}\right)-\frac{b\text{arctanh}\left(\frac{b+2ce^x}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}$$

input
```
Int[(a + b*E^x + c*E^(2*x))^(-1),x]
```

output
```
Log[E^x]/a - (-((b*ArcTanh[(b + 2*c*E^x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*
a*c]) + Log[a + b*E^x + c*E^(2*x)]/2)/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

method	result
default	$\frac{\ln(e^{x})}{a}-\frac{\ln(a+b\,e^{x}+c\,e^{2x})}{2a}-\frac{b\arctan\left(\frac{b+2c\,e^{x}}{\sqrt{4ac-b^{2}}}\right)}{a\sqrt{4ac-b^{2}}}$
risch	$\frac{4xac}{4a^{2}c-a\,b^{2}}-\frac{x\,b^{2}}{4a^{2}c-a\,b^{2}}-\frac{2\ln\left(e^{x}-\frac{-b^{2}+\sqrt{-4b^{2}ac+b^{4}}}{2bc}\right)c}{4ac-b^{2}}+\frac{\ln\left(e^{x}-\frac{-b^{2}+\sqrt{-4b^{2}ac+b^{4}}}{2bc}\right)b^{2}}{2a(4ac-b^{2})}+\frac{\ln\left(e^{x}-\frac{-b^{2}+\sqrt{-4b^{2}ac+b^{4}}}{2bc}\right)\sqrt{-}}{2a(4ac-b^{2})}$

input
```
int(1/(a+b*exp(x)+c*exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(exp(x))-1/2/a*ln(a+b*exp(x)+c*exp(x)^2)-1/a*b/(4*a*c-b^2)^(1/2)*arc
tan((b+2*c*exp(x))/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.27

$$\int \frac{1}{a+be^x+ce^{2x}}\,dx$$
$$=\left[\frac{\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2e^{(2\,x)}+2\,bce^x+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,ce^x+b)}{ce^{(2\,x)}+be^x+a}\right)+2\,(b^2-4\,ac)x-(b^2-4\,ac)\log\left(ce^{(2\,x)}+be^x+\right.}{2\,(ab^2-4\,a^2c)}\right.$$

input
```
integrate(1/(a+b*exp(x)+c*exp(2*x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*e^(2*x) + 2*b*c*e^x + b^2 - 2*a*c + s
qrt(b^2 - 4*a*c)*(2*c*e^x + b))/(c*e^(2*x) + b*e^x + a)) + 2*(b^2 - 4*a*c)
*x - (b^2 - 4*a*c)*log(c*e^(2*x) + b*e^x + a))/(a*b^2 - 4*a^2*c), 1/2*(2*s
qrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*e^x + b)/(b^2 - 4*a*c)
) + 2*(b^2 - 4*a*c)*x - (b^2 - 4*a*c)*log(c*e^(2*x) + b*e^x + a))/(a*b^2 -
 4*a^2*c)]
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+be^x+ce^{2x}}\,dx = \operatorname{RootSum}\left(z^2\cdot\left(4a^2c-ab^2\right)+z\left(4ac-b^2\right)+c,\left(i\mapsto i\log\left(e^x+\frac{-4ia^2c+iab^2-2ac+b^2}{bc}\right)\right)\right) + \frac{x}{a}$$

input
```
integrate(1/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
RootSum(_z**2*(4*a**2*c - a*b**2) + _z*(4*a*c - b**2) + c, Lambda(_i, _i*l
og(exp(x) + (-4*_i*a**2*c + _i*a*b**2 - 2*a*c + b**2)/(b*c)))) + x/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+be^x+ce^{2x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*exp(x)+c*exp(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+be^x+ce^{2x}}\,dx = -\frac{b\arctan\left(\frac{2\,ce^x+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a} + \frac{x}{a} - \frac{\log\left(ce^{(2\,x)}+be^x+a\right)}{2\,a}$$

input
```
integrate(1/(a+b*exp(x)+c*exp(2*x)),x, algorithm="giac")
```

output
```
-b*arctan((2*c*e^x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) + x/a -
 1/2*log(c*e^(2*x) + b*e^x + a)/a
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{1}{a+be^x+ce^{2x}}\,dx = \frac{x}{a} - \frac{\ln\left(a+b\,e^x+c\,e^{2\,x}\right)}{2\,a} - \frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,e^x}{\sqrt{4\,a\,c-b^2}}\right)}{a\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(a + b*exp(x) + c*exp(2*x)),x)
```

output
```
x/a - log(a + b*exp(x) + c*exp(2*x))/(2*a) - (b*atan((b + 2*c*exp(x))/(4*a
*c - b^2)^(1/2)))/(a*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55

$$\int \frac{1}{a+be^x+ce^{2x}}\,dx = \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2e^x c+b}{\sqrt{4ac-b^2}}\right)b - 4\log(e^{2x}c+e^x b+a)\,ac + \log(e^{2x}c+e^x b+a)\,b^2 + 8acx - 2b^2x}{2a\left(4ac-b^2\right)}$$

input
```
int(1/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((2*e**x*c + b)/sqrt(4*a*c - b**2))*b - 4*log
(e**(2*x)*c + e**x*b + a)*a*c + log(e**(2*x)*c + e**x*b + a)*b**2 + 8*a*c*
x - 2*b**2*x)/(2*a*(4*a*c - b**2))
```

3.433 $\int \frac{x}{1+2e^x+e^{2x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 44

$$\int \frac{x}{1+2e^x+e^{2x}} \, dx = -x+\frac{x}{1+e^x}+\frac{x^2}{2}+\log(1+e^x)-x\log(1+e^x)-\text{PolyLog}(2,-e^x)$$

output
```
-x+x/(1+exp(x))+1/2*x^2+ln(1+exp(x))-x*ln(1+exp(x))-polylog(2,-exp(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x}{1+2e^x+e^{2x}} \, dx = \frac{1}{2}x\left(-2+\frac{2}{1+e^x}+x\right)-(-1+x)\log(1+e^x)-\text{PolyLog}(2,-e^x)$$

input
```
Integrate[x/(1 + 2*E^x + E^(2*x)),x]
```

output
```
(x*(-2 + 2/(1 + E^x) + x))/2 - (-1 + x)*Log[1 + E^x] - PolyLog[2, -E^x]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {7239, 2616, 2615, 2620, 2621, 2715, 2720, 47, 14, 16, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{2e^x + e^{2x} + 1} \, dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{x}{(e^x+1)^2}dx$$

$$\Big\downarrow \text{2616}$$

$$\int \frac{x}{1+e^x}dx - \int \frac{e^x x}{(1+e^x)^2}dx$$

$$\Big\downarrow \text{2615}$$

$$-\int \frac{e^x x}{(1+e^x)^2}dx - \int \frac{e^x x}{1+e^x}dx + \frac{x^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$-\int \frac{e^x x}{(1+e^x)^2}dx + \int \log(1+e^x)\, dx + \frac{x^2}{2} - x\log(e^x+1)$$

$$\Big\downarrow \text{2621}$$

$$-\int \frac{1}{1+e^x}dx + \int \log(1+e^x)\, dx + \frac{x^2}{2} + \frac{x}{e^x+1} - x\log(e^x+1)$$

$$\Big\downarrow \text{2715}$$

$$-\int \frac{1}{1+e^x}dx + \int e^{-x}\log(1+e^x)\, de^x + \frac{x^2}{2} + \frac{x}{e^x+1} - x\log(e^x+1)$$

$$\Big\downarrow \text{2720}$$

$$-\int \frac{e^{-x}}{1+e^x}de^x + \int e^{-x}\log(1+e^x)\, de^x + \frac{x^2}{2} + \frac{x}{e^x+1} - x\log(e^x+1)$$

$$\Big\downarrow \text{47}$$

$$-\int e^{-x}de^x+\int \frac{1}{1+e^x}de^x+\int e^{-x}\log(1+e^x)\,de^x+\frac{x^2}{2}+\frac{x}{e^x+1}-x\log(e^x+1)$$

$$\Big\downarrow \text{14}$$

$$\int \frac{1}{1+e^x}de^x+\int e^{-x}\log(1+e^x)\,de^x+\frac{x^2}{2}+\frac{x}{e^x+1}-x\log(e^x+1)-\log(e^x)$$

$$\Big\downarrow \text{16}$$

$$\int e^{-x}\log(1+e^x)\,de^x+\frac{x^2}{2}+\frac{x}{e^x+1}-x\log(e^x+1)-\log(e^x)+\log(e^x+1)$$

$$\Big\downarrow \text{2838}$$

$$-\operatorname{PolyLog}(2,-e^x)+\frac{x^2}{2}+\frac{x}{e^x+1}-x\log(e^x+1)-\log(e^x)+\log(e^x+1)$$

input
```
Int[x/(1 + 2*E^x + E^(2*x)),x]
```

output
```
x/(1 + E^x) + x^2/2 - Log[E^x] + Log[1 + E^x] - x*Log[1 + E^x] - PolyLog[2
, -E^x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{x^2}{2}+\ln\left(1+e^x\right)-\frac{x\,e^x}{1+e^x}-\operatorname{dilog}\left(1+e^x\right)-x\ln\left(1+e^x\right)$	38
risch	$\frac{x}{1+e^x}+\frac{x^2}{2}-x\ln\left(1+e^x\right)-\operatorname{polylog}\left(2,-e^x\right)-\ln\left(e^x\right)+\ln\left(1+e^x\right)$	41

input
```
int(x/(1+2*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+ln(1+exp(x))-x*exp(x)/(1+exp(x))-dilog(1+exp(x))-x*ln(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{x}{1+2e^x+e^{2x}}\,dx = \frac{x^2-2\left(e^x+1\right)\mathrm{Li}_2(-e^x)+(x^2-2\,x)e^x-2\left((x-1)e^x+x-1\right)\log\left(e^x+1\right)}{2\left(e^x+1\right)}$$

input
```
integrate(x/(1+2*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 2*(e^x + 1)*dilog(-e^x) + (x^2 - 2*x)*e^x - 2*((x - 1)*e^x + x
- 1)*log(e^x + 1))/(e^x + 1)
```

Sympy [F]

$$\int \frac{x}{1+2e^x+e^{2x}}\,dx = \frac{x}{e^x+1}+\int \frac{x-1}{e^x+1}\,dx$$

input
```
integrate(x/(1+2*exp(x)+exp(2*x)),x)
```

output
```
x/(exp(x) + 1) + Integral((x - 1)/(exp(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{x}{1+2e^x+e^{2x}}\,dx = \frac{1}{2}\,x^2 - x\log\left(e^x+1\right) - x + \frac{x}{e^x+1} - \mathrm{Li}_2(-e^x) + \log\left(e^x+1\right)$$

input
```
integrate(x/(1+2*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*x^2 - x*log(e^x + 1) - x + x/(e^x + 1) - dilog(-e^x) + log(e^x + 1)
```

Giac [F]

$$\int \frac{x}{1+2e^x+e^{2x}}\,dx = \int \frac{x}{e^{(2\,x)} + 2\,e^x + 1}\,dx$$

input
```
integrate(x/(1+2*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x/(e^(2*x) + 2*e^x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+2e^x+e^{2x}}\,dx = \int \frac{x}{\mathrm{e}^{2\,x} + 2\,\mathrm{e}^x + 1}\,dx$$

input
```
int(x/(exp(2*x) + 2*exp(x) + 1),x)
```

output
```
int(x/(exp(2*x) + 2*exp(x) + 1), x)
```

Reduce [F]

$$\int \frac{x}{1+2e^x+e^{2x}}\,dx=\int \frac{x}{e^{2x}+2e^x+1}dx$$

input
```
int(x/(1+2*exp(x)+exp(2*x)),x)
```

output
```
int(x/(e**(2*x) + 2*e**x + 1),x)
```

3.434 $\int \frac{x}{2+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 54

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx=\frac{x^2}{4}+\frac{1}{2}x\log\left(1+\frac{e^x}{2}\right)-x\log\left(1+e^x\right)-\operatorname{PolyLog}\left(2,-e^x\right)+\frac{1}{2}\operatorname{PolyLog}\left(2,-\frac{e^x}{2}\right)$$

output
```
1/4*x^2+1/2*x*ln(1+1/2*exp(x))-x*ln(1+exp(x))-polylog(2,-exp(x))+1/2*polyl
og(2,-1/2*exp(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx=-x\log\left(1+e^{-x}\right)+\frac{1}{2}x\log\left(1+2e^{-x}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,-2e^{-x}\right)+\operatorname{PolyLog}\left(2,-e^{-x}\right)$$

input
```
Integrate[x/(2 + 3*E^x + E^(2*x)),x]
```

output
```
-(x*Log[1 + E^(-x)]) + (x*Log[1 + 2/E^x])/2 - PolyLog[2, -2/E^x]/2 + PolyL
og[2, -E^(-x)]
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2693, 27, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{3e^x+e^{2x}+2}dx$$

$$\Big\downarrow \text{2693}$$

$$2\int \frac{x}{2(1+e^x)}dx-2\int \frac{x}{2(2+e^x)}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x}{1+e^x}dx-\int \frac{x}{2+e^x}dx$$

$$\Big\downarrow \text{2615}$$

$$-\int \frac{e^xx}{1+e^x}dx+\frac{1}{2}\int \frac{e^xx}{2+e^x}dx+\frac{x^2}{4}$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}\left(x\log\left(\frac{e^x}{2}+1\right)-\int \log\left(1+\frac{e^x}{2}\right)dx\right)+\int \log(1+e^x)dx+\frac{x^2}{4}-x\log(e^x+1)$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}\left(x\log\left(\frac{e^x}{2}+1\right)-\int e^{-x}\log\left(1+\frac{e^x}{2}\right)de^x\right)+\int e^{-x}\log(1+e^x)de^x+\frac{x^2}{4}-x\log(e^x+1)$$

$$\Big\downarrow \text{2838}$$

$$-\text{PolyLog}(2,-e^x)+\frac{1}{2}\left(\text{PolyLog}\left(2,-\frac{e^x}{2}\right)+x\log\left(\frac{e^x}{2}+1\right)\right)+\frac{x^2}{4}-x\log(e^x+1)$$

input
```
Int[x/(2 + 3*E^x + E^(2*x)),x]
```

output
```
x^2/4 - x*Log[1 + E^x] - PolyLog[2, -E^x] + (x*Log[1 + E^x/2] + PolyLog[2,
 -1/2*E^x])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{x^{2}}{4}+\frac{x \ln \left(1+\frac{{\mathrm e}^{x}}{2}\right)}{2}-x \ln \left(1+{\mathrm e}^{x}\right)-\operatorname{polylog}\left(2, -{\mathrm e}^{x}\right)+\frac{\operatorname{polylog}\left(2, -\frac{{\mathrm e}^{x}}{2}\right)}{2}$	41
risch	$\frac{x^{2}}{4}+\frac{x \ln \left(1+\frac{{\mathrm e}^{x}}{2}\right)}{2}-x \ln \left(1+{\mathrm e}^{x}\right)-\operatorname{polylog}\left(2, -{\mathrm e}^{x}\right)+\frac{\operatorname{polylog}\left(2, -\frac{{\mathrm e}^{x}}{2}\right)}{2}$	41

input
```
int(x/(2+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^2+1/2*x*ln(1+1/2*exp(x))-x*ln(1+exp(x))-polylog(2,-exp(x))+1/2*polyl
og(2,-1/2*exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx = \frac{1}{4}\,x^{2} - x\log\left(e^{x} + 1\right) + \frac{1}{2}\,x\log\left(\frac{1}{2}\,e^{x} + 1\right) + \frac{1}{2}\,{\rm Li}_2\left(-\frac{1}{2}\,e^{x}\right) - {\rm Li}_2\left(-e^{x}\right)$$

input
```
integrate(x/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/4*x^2 - x*log(e^x + 1) + 1/2*x*log(1/2*e^x + 1) + 1/2*dilog(-1/2*e^x) -
dilog(-e^x)
```

Sympy [F]

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx = \int \frac{x}{(e^x+1)(e^x+2)}\,dx$$

input
```
integrate(x/(2+3*exp(x)+exp(2*x)),x)
```

output
```
Integral(x/((exp(x) + 1)*(exp(x) + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx = \frac{1}{4}x^2 - x\log(e^x+1) + \frac{1}{2}x\log\left(\frac{1}{2}e^x+1\right) + \frac{1}{2}\operatorname{Li}_2\left(-\frac{1}{2}e^x\right) - \operatorname{Li}_2(-e^x)$$

input
```
integrate(x/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/4*x^2 - x*log(e^x + 1) + 1/2*x*log(1/2*e^x + 1) + 1/2*dilog(-1/2*e^x) -
dilog(-e^x)
```

Giac [F]

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx = \int \frac{x}{e^{(2x)}+3e^x+2}\,dx$$

input
```
integrate(x/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x/(e^(2*x) + 3*e^x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx=\int \frac{x}{\mathrm{e}^{2\,x}+3\,\mathrm{e}^x+2}\,dx$$

input
```
int(x/(exp(2*x) + 3*exp(x) + 2),x)
```

output
```
int(x/(exp(2*x) + 3*exp(x) + 2), x)
```

Reduce [F]

$$\int \frac{x}{2+3e^x+e^{2x}}\,dx=\int \frac{x}{e^{2x}+3e^x+2}dx$$

input
```
int(x/(2+3*exp(x)+exp(2*x)),x)
```

output
```
int(x/(e**(2*x) + 3*e**x + 2),x)
```

3.435 $\int \frac{x}{-1+e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 180

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx = \frac{x^2}{\sqrt{5}\,(1-\sqrt{5})} - \frac{x^2}{\sqrt{5}\,(1+\sqrt{5})} - \frac{2x\log\left(1+\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}\,(1-\sqrt{5})} + \frac{2x\log\left(1+\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}\,(1+\sqrt{5})} - \frac{2\,\mathrm{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}\,(1-\sqrt{5})} + \frac{2\,\mathrm{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}\,(1+\sqrt{5})}$$

output
```
1/5*x^2*5^(1/2)/(-5^(1/2)+1)-1/5*x^2*5^(1/2)/(5^(1/2)+1)-2/5*x*ln(1+2*exp(
x)/(-5^(1/2)+1))*5^(1/2)/(-5^(1/2)+1)+2/5*x*ln(1+2*exp(x)/(5^(1/2)+1))*5^(
1/2)/(5^(1/2)+1)-2/5*polylog(2,-2*exp(x)/(-5^(1/2)+1))*5^(1/2)/(-5^(1/2)+1
)+2/5*polylog(2,-2*exp(x)/(5^(1/2)+1))*5^(1/2)/(5^(1/2)+1)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx = \frac{\left(1+\sqrt{5}\right)x\log\left(1-\frac{1}{2}\left(-1+\sqrt{5}\right)e^{-x}\right)+\left(-1+\sqrt{5}\right)x\log\left(1+\frac{1}{2}\left(1+\sqrt{5}\right)e^{-x}\right)-\left(1+\sqrt{5}\right)\text{PolyLog}\left(2,\right.}{2\sqrt{5}}$$

input
```
Integrate[x/(-1 + E^x + E^(2*x)),x]
```

output
```
((1 + Sqrt[5])*x*Log[1 - (-1 + Sqrt[5])/(2*E^x)] + (-1 + Sqrt[5])*x*Log[1
+ (1 + Sqrt[5])/(2*E^x)] - (1 + Sqrt[5])*PolyLog[2, (-1 + Sqrt[5])/(2*E^x)
] - (-1 + Sqrt[5])*PolyLog[2, -1/2*(1 + Sqrt[5])/E^x])/(2*Sqrt[5])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2693, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{e^x+e^{2x}-1}\,dx$$

$$\Big\downarrow \text{2693}$$

$$\frac{2\int \frac{x}{1-\sqrt{5}+2e^x}dx}{\sqrt{5}}-\frac{2\int \frac{x}{1+\sqrt{5}+2e^x}dx}{\sqrt{5}}$$

$$\Big\downarrow \text{2615}$$

$$\frac{2\left(\frac{x^2}{2\left(1-\sqrt{5}\right)}-\frac{2\int \frac{e^x x}{1-\sqrt{5}+2e^x}dx}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^2}{2\left(1+\sqrt{5}\right)}-\frac{2\int \frac{e^x x}{1+\sqrt{5}+2e^x}dx}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2\left(\frac{x^2}{2\left(1-\sqrt{5}\right)}-\frac{2\left(\frac{1}{2}x\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)-\frac{1}{2}\int\log\left(1+\frac{2e^x}{1-\sqrt{5}}\right)dx\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^2}{2\left(1+\sqrt{5}\right)}-\frac{2\left(\frac{1}{2}x\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)-\frac{1}{2}\int\log\left(1+\frac{2e^x}{1+\sqrt{5}}\right)dx\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2\left(\frac{x^2}{2\left(1-\sqrt{5}\right)}-\frac{2\left(\frac{1}{2}x\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)-\frac{1}{2}\int e^{-x}\log\left(1+\frac{2e^x}{1-\sqrt{5}}\right)de^x\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^2}{2\left(1+\sqrt{5}\right)}-\frac{2\left(\frac{1}{2}x\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)-\frac{1}{2}\int e^{-x}\log\left(1+\frac{2e^x}{1+\sqrt{5}}\right)de^x\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2\left(\frac{x^2}{2\left(1-\sqrt{5}\right)}-\frac{2\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)+\frac{1}{2}x\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^2}{2\left(1+\sqrt{5}\right)}-\frac{2\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)+\frac{1}{2}x\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

input

```
Int[x/(-1 + E^x + E^(2*x)),x]
```

output

```
(2*(x^2/(2*(1 - Sqrt[5])) - (2*((x*Log[1 + (2*E^x)/(1 - Sqrt[5])])/2 + Pol
yLog[2, (-2*E^x)/(1 - Sqrt[5])]/2))/(1 - Sqrt[5])))/Sqrt[5] - (2*(x^2/(2*(
1 + Sqrt[5])) - (2*((x*Log[1 + (2*E^x)/(1 + Sqrt[5])])/2 + PolyLog[2, (-2*
E^x)/(1 + Sqrt[5])]/2))/(1 + Sqrt[5])))/Sqrt[5]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

method	result
default	$-\frac{x^2}{2}+\frac{\sqrt{5}\,x\ln\left(\frac{\sqrt{5}-1-2\,{\rm e}^{x}}{\sqrt{5}-1}\right)}{10}+\frac{x\ln\left(\frac{\sqrt{5}-1-2\,{\rm e}^{x}}{\sqrt{5}-1}\right)}{2}-\frac{\sqrt{5}\,x\ln\left(\frac{1+\sqrt{5}+2\,{\rm e}^{x}}{\sqrt{5}+1}\right)}{10}+\frac{x\ln\left(\frac{1+\sqrt{5}+2\,{\rm e}^{x}}{\sqrt{5}+1}\right)}{2}+\frac{\sqrt{5}\,\operatorname{dilog}\left(\frac{\sqrt{5}-1-2\,{\rm e}^{x}}{\sqrt{5}-1}\right)}{10}$
risch	$-\frac{x^2}{2}+\frac{\sqrt{5}\,x\ln\left(\frac{\sqrt{5}-1-2\,{\rm e}^{x}}{\sqrt{5}-1}\right)}{10}+\frac{x\ln\left(\frac{\sqrt{5}-1-2\,{\rm e}^{x}}{\sqrt{5}-1}\right)}{2}-\frac{\sqrt{5}\,x\ln\left(\frac{1+\sqrt{5}+2\,{\rm e}^{x}}{\sqrt{5}+1}\right)}{10}+\frac{x\ln\left(\frac{1+\sqrt{5}+2\,{\rm e}^{x}}{\sqrt{5}+1}\right)}{2}+\frac{\sqrt{5}\,\operatorname{dilog}\left(\frac{\sqrt{5}-1-2\,{\rm e}^{x}}{\sqrt{5}-1}\right)}{10}$

input
```
int(x/(-1+exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2+1/10*5^(1/2)*x*ln((5^(1/2)-1-2*exp(x))/(5^(1/2)-1))+1/2*x*ln((5^(
1/2)-1-2*exp(x))/(5^(1/2)-1))-1/10*5^(1/2)*x*ln((1+5^(1/2)+2*exp(x))/(5^(1
/2)+1))+1/2*x*ln((1+5^(1/2)+2*exp(x))/(5^(1/2)+1))+1/10*5^(1/2)*dilog((5^(
1/2)-1-2*exp(x))/(5^(1/2)-1))+1/2*dilog((5^(1/2)-1-2*exp(x))/(5^(1/2)-1))-
1/10*5^(1/2)*dilog((1+5^(1/2)+2*exp(x))/(5^(1/2)+1))+1/2*dilog((1+5^(1/2)+
2*exp(x))/(5^(1/2)+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.48

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx = -\frac{1}{2}\,x^2 + \frac{1}{10}\left(\sqrt{5}+5\right)\mathrm{Li}_2\left(\frac{1}{2}\left(\sqrt{5}+1\right)e^x\right) - \frac{1}{10}\left(\sqrt{5}-5\right)\mathrm{Li}_2\left(-\frac{1}{2}\left(\sqrt{5}-1\right)e^x\right) + \frac{1}{10}\left(\sqrt{5}x+5\,x\right)\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)e^x+1\right) - \frac{1}{10}\left(\sqrt{5}x-5\,x\right)\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)e^x+1\right)$$

input
```
integrate(x/(-1+exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/2*x^2 + 1/10*(sqrt(5) + 5)*dilog(1/2*(sqrt(5) + 1)*e^x) - 1/10*(sqrt(5)
 - 5)*dilog(-1/2*(sqrt(5) - 1)*e^x) + 1/10*(sqrt(5)*x + 5*x)*log(-1/2*(sqr
t(5) + 1)*e^x + 1) - 1/10*(sqrt(5)*x - 5*x)*log(1/2*(sqrt(5) - 1)*e^x + 1)
```

Sympy [F]

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx = \int \frac{x}{e^{2x}+e^x-1}\,dx$$

input
```
integrate(x/(-1+exp(x)+exp(2*x)),x)
```

output
```
Integral(x/(exp(2*x) + exp(x) - 1), x)
```

Maxima [F]

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx = \int \frac{x}{e^{(2x)}+e^x-1}\,dx$$

input
```
integrate(x/(-1+exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
integrate(x/(e^(2*x) + e^x - 1), x)
```

Giac [F]

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx = \int \frac{x}{e^{(2x)}+e^x-1}\,dx$$

input
```
integrate(x/(-1+exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x/(e^(2*x) + e^x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx=\int \frac{x}{\mathrm{e}^{2\,x}+\mathrm{e}^x-1}\,dx$$

input
```
int(x/(exp(2*x) + exp(x) - 1),x)
```

output
```
int(x/(exp(2*x) + exp(x) - 1), x)
```

Reduce [F]

$$\int \frac{x}{-1+e^x+e^{2x}}\,dx=\int \frac{x}{e^{2x}+e^x-1}dx$$

input
```
int(x/(-1+exp(x)+exp(2*x)),x)
```

output
```
int(x/(e**(2*x) + e**x - 1),x)
```

3.436 $\int \frac{x}{3+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 204

$$\begin{aligned}\int \frac{x}{3+3e^x+e^{2x}}\,dx =& -\frac{x^2}{\sqrt{3}\,(3i-\sqrt{3})}+\frac{x^2}{\sqrt{3}\,(3i+\sqrt{3})} \\ &-\frac{2x\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}\,(3i+\sqrt{3})}+\frac{2x\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}\,(3i-\sqrt{3})} \\ &-\frac{2\operatorname{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}\,(3i+\sqrt{3})}+\frac{2\operatorname{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}\,(3i-\sqrt{3})}\end{aligned}$$

output
```
-1/3*x^2*3^(1/2)/(3*I-3^(1/2))+1/3*x^2*3^(1/2)/(3*I+3^(1/2))-2/3*x*ln(1+2*
exp(x)/(3-I*3^(1/2)))*3^(1/2)/(3*I+3^(1/2))+2/3*x*ln(1+2*exp(x)/(3+I*3^(1/
2)))*3^(1/2)/(3*I-3^(1/2))-2/3*polylog(2,-2*exp(x)/(3-I*3^(1/2)))*3^(1/2)/
(3*I+3^(1/2))+2/3*polylog(2,-2*exp(x)/(3+I*3^(1/2)))*3^(1/2)/(3*I-3^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.71

$$\int \frac{x}{3+3e^x+e^{2x}}\,dx$$
$$= \frac{-x\left(\left(-3i+\sqrt{3}\right)\log\left(1+\frac{1}{2}\left(3-i\sqrt{3}\right)e^{-x}\right)+\left(3i+\sqrt{3}\right)\log\left(1+\frac{1}{2}\left(3+i\sqrt{3}\right)e^{-x}\right)\right)+\left(3i+\sqrt{3}\right)\text{PolyLog}}{6\sqrt{3}}$$

input
```
Integrate[x/(3 + 3*E^x + E^(2*x)),x]
```

output
```
(-(x*((-3*I + Sqrt[3])*Log[1 + (3 - I*Sqrt[3])/(2*E^x)] + (3*I + Sqrt[3])*
Log[1 + (3 + I*Sqrt[3])/(2*E^x)])) + (3*I + Sqrt[3])*PolyLog[2, ((-1/2*I)*
(-3*I + Sqrt[3]))/E^x] + (-3*I + Sqrt[3])*PolyLog[2, ((I/2)*(3*I + Sqrt[3]
))/E^x])/(6*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2693, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{3e^x+e^{2x}+3}dx$$
$$\Big\downarrow \text{2693}$$
$$\frac{2i\int \frac{x}{3+i\sqrt{3}+2e^x}dx}{\sqrt{3}}-\frac{2i\int \frac{x}{3-i\sqrt{3}+2e^x}dx}{\sqrt{3}}$$
$$\Big\downarrow \text{2615}$$
$$\frac{2i\left(\frac{x^2}{2\left(3+i\sqrt{3}\right)}-\frac{2\int \frac{e^x x}{3+i\sqrt{3}+2e^x}dx}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^2}{2\left(3-i\sqrt{3}\right)}-\frac{2\int \frac{e^x x}{3-i\sqrt{3}+2e^x}dx}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$
$$\Big\downarrow \text{2620}$$

$$\frac{2i\left(\frac{x^2}{2\left(3+i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}x\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)-\frac{1}{2}\int\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)dx\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^2}{2\left(3-i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}x\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)-\frac{1}{2}\int\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)dx\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2i\left(\frac{x^2}{2\left(3+i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}x\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)-\frac{1}{2}\int e^{-x}\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)de^x\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^2}{2\left(3-i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}x\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)-\frac{1}{2}\int e^{-x}\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)de^x\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2i\left(\frac{x^2}{2\left(3+i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)+\frac{1}{2}x\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^2}{2\left(3-i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}\text{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)+\frac{1}{2}x\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

input

```
Int[x/(3 + 3*E^x + E^(2*x)),x]
```

output

```
((-2*I)*(x^2/(2*(3 - I*Sqrt[3])) - (2*((x*Log[1 + (2*E^x)/(3 - I*Sqrt[3])]
)/2 + PolyLog[2, (-2*E^x)/(3 - I*Sqrt[3])]/2))/(3 - I*Sqrt[3])))/Sqrt[3] +
 ((2*I)*(x^2/(2*(3 + I*Sqrt[3])) - (2*((x*Log[1 + (2*E^x)/(3 + I*Sqrt[3])]
)/2 + PolyLog[2, (-2*E^x)/(3 + I*Sqrt[3])]/2))/(3 + I*Sqrt[3])))/Sqrt[3]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.15

method	result
default	$\frac{x^2}{6}+\frac{i\sqrt{3}\,x\ln\left(\frac{i\sqrt{3}-2\,\mathrm{e}^{x}-3}{i\sqrt{3}-3}\right)}{6}-\frac{x\ln\left(\frac{i\sqrt{3}-2\,\mathrm{e}^{x}-3}{i\sqrt{3}-3}\right)}{6}-\frac{i\sqrt{3}\,x\ln\left(\frac{i\sqrt{3}+2\,\mathrm{e}^{x}+3}{3+i\sqrt{3}}\right)}{6}-\frac{x\ln\left(\frac{i\sqrt{3}+2\,\mathrm{e}^{x}+3}{3+i\sqrt{3}}\right)}{6}+\frac{i\sqrt{3}\,\mathrm{dilog}\left(\frac{i\sqrt{3}-2\,\mathrm{e}^{x}-}{i\sqrt{3}-3}\right.}{6}$
risch	$\frac{x^2}{6}+\frac{i\sqrt{3}\,x\ln\left(\frac{i\sqrt{3}-2\,\mathrm{e}^{x}-3}{i\sqrt{3}-3}\right)}{6}-\frac{x\ln\left(\frac{i\sqrt{3}-2\,\mathrm{e}^{x}-3}{i\sqrt{3}-3}\right)}{6}-\frac{i\sqrt{3}\,x\ln\left(\frac{i\sqrt{3}+2\,\mathrm{e}^{x}+3}{3+i\sqrt{3}}\right)}{6}-\frac{x\ln\left(\frac{i\sqrt{3}+2\,\mathrm{e}^{x}+3}{3+i\sqrt{3}}\right)}{6}+\frac{i\sqrt{3}\,\mathrm{dilog}\left(\frac{i\sqrt{3}-2\,\mathrm{e}^{x}-}{i\sqrt{3}-3}\right.}{6}$

input
```
int(x/(3+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2+1/6*I*3^(1/2)*x*ln((I*3^(1/2)-2*exp(x)-3)/(I*3^(1/2)-3))-1/6*x*ln(
(I*3^(1/2)-2*exp(x)-3)/(I*3^(1/2)-3))-1/6*I*3^(1/2)*x*ln((I*3^(1/2)+2*exp(
x)+3)/(3+I*3^(1/2)))-1/6*x*ln((I*3^(1/2)+2*exp(x)+3)/(3+I*3^(1/2)))+1/6*I*
3^(1/2)*dilog((I*3^(1/2)-2*exp(x)-3)/(I*3^(1/2)-3))-1/6*dilog((I*3^(1/2)-2
*exp(x)-3)/(I*3^(1/2)-3))-1/6*I*3^(1/2)*dilog((I*3^(1/2)+2*exp(x)+3)/(3+I*
3^(1/2)))-1/6*dilog((I*3^(1/2)+2*exp(x)+3)/(3+I*3^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.44

$$\begin{aligned}\int \frac{x}{3+3e^x+e^{2x}}\,dx =& \frac{1}{6}\,x^2+\frac{1}{6}\left(3\sqrt{-\frac{1}{3}}-1\right)\mathrm{Li}_2\left(-\frac{1}{2}\left(\sqrt{-\frac{1}{3}}+1\right)e^x\right)\\&-\frac{1}{6}\left(3\sqrt{-\frac{1}{3}}+1\right)\mathrm{Li}_2\left(\frac{1}{2}\left(\sqrt{-\frac{1}{3}}-1\right)e^x\right)\\&+\frac{1}{6}\left(3\sqrt{-\frac{1}{3}}x-x\right)\log\left(\frac{1}{2}\left(\sqrt{-\frac{1}{3}}+1\right)e^x+1\right)\\&-\frac{1}{6}\left(3\sqrt{-\frac{1}{3}}x+x\right)\log\left(-\frac{1}{2}\left(\sqrt{-\frac{1}{3}}-1\right)e^x+1\right)\end{aligned}$$

input
```
integrate(x/(3+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/6*x^2 + 1/6*(3*sqrt(-1/3) - 1)*dilog(-1/2*(sqrt(-1/3) + 1)*e^x) - 1/6*(3
*sqrt(-1/3) + 1)*dilog(1/2*(sqrt(-1/3) - 1)*e^x) + 1/6*(3*sqrt(-1/3)*x - x
)*log(1/2*(sqrt(-1/3) + 1)*e^x + 1) - 1/6*(3*sqrt(-1/3)*x + x)*log(-1/2*(s
qrt(-1/3) - 1)*e^x + 1)
```

Sympy [F]

$$\int \frac{x}{3+3e^x+e^{2x}}\,dx=\int \frac{x}{e^{2x}+3e^x+3}\,dx$$

input
```
integrate(x/(3+3*exp(x)+exp(2*x)),x)
```

output
```
Integral(x/(exp(2*x) + 3*exp(x) + 3), x)
```

Maxima [F]

$$\int \frac{x}{3+3e^x+e^{2x}}\,dx=\int \frac{x}{e^{(2\,x)}+3\,e^x+3}\,dx$$

input
```
integrate(x/(3+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
integrate(x/(e^(2*x) + 3*e^x + 3), x)
```

Giac [F]

$$\int \frac{x}{3+3e^x+e^{2x}}\,dx=\int \frac{x}{e^{(2\,x)}+3\,e^x+3}\,dx$$

input
```
integrate(x/(3+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x/(e^(2*x) + 3*e^x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{3+3e^x+e^{2x}}\,dx=\int \frac{x}{\mathrm{e}^{2\,x}+3\,\mathrm{e}^x+3}\,dx$$

input
```
int(x/(exp(2*x) + 3*exp(x) + 3),x)
```

output
```
int(x/(exp(2*x) + 3*exp(x) + 3), x)
```

Reduce [F]

$$\int \frac{x}{3+3e^x+e^{2x}}\,dx=\int \frac{x}{e^{2x}+3e^x+3}dx$$

input
```
int(x/(3+3*exp(x)+exp(2*x)),x)
```

output
```
int(x/(e**(2*x) + 3*e**x + 3),x)
```

3.437 $\int \frac{x}{a+be^x+ce^{2x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 276

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx = -\frac{cx^2}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2}{b^2-4ac+b\sqrt{b^2-4ac}} + \frac{2cx\log\left(1+\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{2cx\log\left(1+\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}} + \frac{2c\,\mathrm{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{2c\,\mathrm{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output

```
-c*x^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2/(b*(-4*a*c+b^2)^(1/2)-4*a*c+
b^2)+2*c*x*ln(1+2*c*exp(x)/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^
2)^(1/2))+2*c*x*ln(1+2*c*exp(x)/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1
/2)-4*a*c+b^2)+2*c*polylog(2,-2*c*exp(x)/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a*
c-b*(-4*a*c+b^2)^(1/2))+2*c*polylog(2,-2*c*exp(x)/(b+(-4*a*c+b^2)^(1/2)))/
(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.74

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx =$$

$$-\frac{\left(b+\sqrt{b^2-4ac}\right)x\log\left(1+\frac{\left(b-\sqrt{b^2-4ac}\right)e^{-x}}{2c}\right)+\left(-b+\sqrt{b^2-4ac}\right)x\log\left(1+\frac{\left(b+\sqrt{b^2-4ac}\right)e^{-x}}{2c}\right)-(b+}{2a\sqrt{b^2-4}}$$

input
```
Integrate[x/(a + b*E^x + c*E^(2*x)),x]
```

output
```
-1/2*((b + Sqrt[b^2 - 4*a*c])*x*Log[1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*E^x)]
 + (-b + Sqrt[b^2 - 4*a*c])*x*Log[1 + (b + Sqrt[b^2 - 4*a*c])/(2*c*E^x)] -
 (b + Sqrt[b^2 - 4*a*c])*PolyLog[2, (-b + Sqrt[b^2 - 4*a*c])/(2*c*E^x)] +
(b - Sqrt[b^2 - 4*a*c])*PolyLog[2, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*E^x)])/
(a*Sqrt[b^2 - 4*a*c])
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {2693, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx$$

$$\Big\downarrow \; 2693$$

$$\frac{2c\int \frac{x}{b+2ce^x-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{b+2ce^x+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 2615$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int \frac{e^x x}{b+2ce^x-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int \frac{e^x x}{b+2ce^x+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}-\frac{\int\log\left(\frac{2e^xc}{b-\sqrt{b^2-4ac}}+1\right)dx}{2c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}-\frac{\int\log\left(\frac{2e^xc}{b+\sqrt{b^2-4ac}}+1\right)dx}{2c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}-\frac{\int e^{-x}\log\left(\frac{2e^xc}{b-\sqrt{b^2-4ac}}+1\right)de^x}{2c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}-\frac{\int e^{-x}\log\left(\frac{2e^xc}{b+\sqrt{b^2-4ac}}+1\right)de^x}{2c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{2c}+\frac{x\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{2c}+\frac{x\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x/(a + b*E^x + c*E^(2*x)),x]
```

output
```
(2*c*(x^2/(2*(b - Sqrt[b^2 - 4*a*c])) - (2*c*((x*Log[1 + (2*c*E^x)/(b - Sq
rt[b^2 - 4*a*c])])/(2*c) + PolyLog[2, (-2*c*E^x)/(b - Sqrt[b^2 - 4*a*c])]/
(2*c)))/(b - Sqrt[b^2 - 4*a*c])))/Sqrt[b^2 - 4*a*c] - (2*c*(x^2/(2*(b + Sq
rt[b^2 - 4*a*c])) - (2*c*((x*Log[1 + (2*c*E^x)/(b + Sqrt[b^2 - 4*a*c])])/(
2*c) + PolyLog[2, (-2*c*E^x)/(b + Sqrt[b^2 - 4*a*c])]/(2*c)))/(b + Sqrt[b^
2 - 4*a*c])))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.36

method	result
default	$\frac{x^2}{2a}+\frac{-\frac{x\left(\ln\left(\frac{-2c\,e^x+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)\sqrt{-4ac+b^2}+\ln\left(\frac{-2c\,e^x+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)b+\ln\left(\frac{2c\,e^x+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}\right)\sqrt{-4ac+b^2}-\ln\left(\frac{2c\,e^x+\sqrt{-4ac+}}{b+\sqrt{-4ac+}}\right.}{2\sqrt{-4ac+b^2}}}{}$
risch	$\frac{x^2}{2a}-\frac{\ln\left(\frac{-2c\,e^x+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)x}{2a}-\frac{\ln\left(\frac{2c\,e^x+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}\right)x}{2a}-\frac{x\ln\left(\frac{-2c\,e^x+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)b}{2a\sqrt{-4ac+b^2}}+\frac{x\ln\left(\frac{2c\,e^x+\sqrt{-4ac+b^2}+b}{b+\sqrt{-4ac+b^2}}\right.}{2a\sqrt{-4ac+b^2}}$

input
```
int(x/(a+b*exp(x)+c*exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a+(-1/2*x*(ln((-2*c*exp(x)+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^
(1/2)))*(-4*a*c+b^2)^(1/2)+ln((-2*c*exp(x)+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a
*c+b^2)^(1/2)))*b+ln((2*c*exp(x)+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/
2)))*(-4*a*c+b^2)^(1/2)-ln((2*c*exp(x)+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^
2)^(1/2)))*b)/(-4*a*c+b^2)^(1/2)-1/2*(dilog((-2*c*exp(x)+(-4*a*c+b^2)^(1/2
)-b)/(-b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)+dilog((-2*c*exp(x)+(-4*a*
c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b+dilog((2*c*exp(x)+(-4*a*c+b^2)^
(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*(-4*a*c+b^2)^(1/2)-dilog((2*c*exp(x)+(-4*
a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b)/(-4*a*c+b^2)^(1/2))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.01

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx = \frac{(b^2-4\,ac)x^2-\left(ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-4\,ac\right)\mathrm{Li}_2\left(-\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}e^x+be^x+2\,a}{2\,a}+1\right)+\left(ab\sqrt{\frac{b^2-4\,ac}{a^2}}-b^2+4\,ac\right)\mathrm{Li}_2}{}$$

input
```
integrate(x/(a+b*exp(x)+c*exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*((b^2 - 4*a*c)*x^2 - (a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 - 4*a*c)*dilog
(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x + 2*a)/a + 1) + (a*b*sqrt((b^
2 - 4*a*c)/a^2) - b^2 + 4*a*c)*dilog(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x -
b*e^x - 2*a)/a + 1) - (a*b*x*sqrt((b^2 - 4*a*c)/a^2) + (b^2 - 4*a*c)*x)*lo
g(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x + 2*a)/a) + (a*b*x*sqrt((b^2
- 4*a*c)/a^2) - (b^2 - 4*a*c)*x)*log(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x -
 b*e^x - 2*a)/a))/(a*b^2 - 4*a^2*c)
```

Sympy [F]

$$\int \frac{x}{a + be^x + ce^{2x}} \, dx = \int \frac{x}{a + be^x + ce^{2x}} \, dx$$

input
```
integrate(x/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
Integral(x/(a + b*exp(x) + c*exp(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + be^x + ce^{2x}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b*exp(x)+c*exp(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx = \int \frac{x}{ce^{(2x)}+be^x+a}\,dx$$

input
```
integrate(x/(a+b*exp(x)+c*exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x/(c*e^(2*x) + b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx = \int \frac{x}{a+b\,\mathrm{e}^x+c\,\mathrm{e}^{2\,x}}\,dx$$

input
```
int(x/(a + b*exp(x) + c*exp(2*x)),x)
```

output
```
int(x/(a + b*exp(x) + c*exp(2*x)), x)
```

Reduce [F]

$$\int \frac{x}{a+be^x+ce^{2x}}\,dx = \int \frac{x}{e^{2x}c+e^xb+a}dx$$

input
```
int(x/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
int(x/(e**(2*x)*c + e**x*b + a),x)
```

3.438 $\int \frac{x^2}{1+2e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx=-x^2+\frac{x^2}{1+e^x}+\frac{x^3}{3}+2x\log\left(1+e^x\right)-x^2\log\left(1+e^x\right)+2\,\text{PolyLog}\left(2,-e^x\right)-2x\,\text{PolyLog}\left(2,-e^x\right)+2\,\text{PolyLog}\left(3,-e^x\right)$$

output
```
-x^2+x^2/(1+exp(x))+1/3*x^3+2*x*ln(1+exp(x))-x^2*ln(1+exp(x))+2*polylog(2,
-exp(x))-2*x*polylog(2,-exp(x))+2*polylog(3,-exp(x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx=\frac{x^2\left(e^x(-3+x)+x\right)}{3\left(1+e^x\right)}-(-2+x)x\log\left(1+e^x\right)-2(-1+x)\,\text{PolyLog}\left(2,-e^x\right)+2\,\text{PolyLog}\left(3,-e^x\right)$$

input
```
Integrate[x^2/(1 + 2*E^x + E^(2*x)),x]
```

output
```
(x^2*(E^x*(-3 + x) + x))/(3*(1 + E^x)) - (-2 + x)*x*Log[1 + E^x] - 2*(-1 +
 x)*PolyLog[2, -E^x] + 2*PolyLog[3, -E^x]
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {7239, 2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2e^x+e^{2x}+1}dx$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{x^2}{(e^x+1)^2}dx$$

$$\Big\downarrow \text{ 2616}$$

$$\int \frac{x^2}{1+e^x}dx-\int \frac{e^x x^2}{(1+e^x)^2}dx$$

$$\Big\downarrow \text{ 2615}$$

$$-\int \frac{e^x x^2}{(1+e^x)^2}dx-\int \frac{e^x x^2}{1+e^x}dx+\frac{x^3}{3}$$

$$\Big\downarrow \text{ 2620}$$

$$-\int \frac{e^x x^2}{(1+e^x)^2}dx+2\int x\log(1+e^x)\,dx+\frac{x^3}{3}-x^2\log(e^x+1)$$

$$\Big\downarrow \text{ 2621}$$

$$-2\int \frac{x}{1+e^x}dx+2\int x\log(1+e^x)\,dx+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log(e^x+1)$$

$$\Big\downarrow \text{ 2615}$$

$$-2\left(\frac{x^2}{2}-\int \frac{e^x x}{1+e^x}dx\right)+2\int x\log(1+e^x)\,dx+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log(e^x+1)$$

$$\Big\downarrow \text{ 2620}$$

$$-2\left(\int \log(1+e^x)\,dx+\frac{x^2}{2}-x\log(e^x+1)\right)+2\int x\log(1+e^x)\,dx+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log(e^x+1)$$

$$\Big\downarrow \ 2715$$

$$-2\left(\int e^{-x}\log\left(1+e^x\right)de^x+\frac{x^2}{2}-x\log\left(e^x+1\right)\right)+2\int x\log\left(1+e^x\right)dx+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log\left(e^x+1\right)$$

$$\Big\downarrow \ 2838$$

$$2\int x\log\left(1+e^x\right)dx-2\left(-\text{PolyLog}\left(2,-e^x\right)+\frac{x^2}{2}-x\log\left(e^x+1\right)\right)+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log\left(e^x+1\right)$$

$$\Big\downarrow \ 3011$$

$$2\left(\int \text{PolyLog}\left(2,-e^x\right)dx-x\,\text{PolyLog}\left(2,-e^x\right)\right)-2\left(-\text{PolyLog}\left(2,-e^x\right)+\frac{x^2}{2}-x\log\left(e^x+1\right)\right)+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log\left(e^x+1\right)$$

$$\Big\downarrow \ 2720$$

$$2\left(\int e^{-x}\,\text{PolyLog}\left(2,-e^x\right)de^x-x\,\text{PolyLog}\left(2,-e^x\right)\right)-2\left(-\text{PolyLog}\left(2,-e^x\right)+\frac{x^2}{2}-x\log\left(e^x+1\right)\right)+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log\left(e^x+1\right)$$

$$\Big\downarrow \ 7143$$

$$-2\left(-\text{PolyLog}\left(2,-e^x\right)+\frac{x^2}{2}-x\log\left(e^x+1\right)\right)+2\left(\text{PolyLog}\left(3,-e^x\right)-x\,\text{PolyLog}\left(2,-e^x\right)\right)+\frac{x^3}{3}+\frac{x^2}{e^x+1}-x^2\log\left(e^x+1\right)$$

input
```
Int[x^2/(1 + 2*E^x + E^(2*x)),x]
```

output
```
x^2/(1 + E^x) + x^3/3 - x^2*Log[1 + E^x] - 2*(x^2/2 - x*Log[1 + E^x] - Pol
yLog[2, -E^x]) + 2*(-(x*PolyLog[2, -E^x]) + PolyLog[3, -E^x])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

method	result
risch	$-x^2 + \frac{x^2}{1+e^x} + \frac{x^3}{3} + 2x \ln(1+e^x) - x^2 \ln(1+e^x) + 2 \operatorname{polylog}(2, -e^x) - 2x \operatorname{polylog}(2, -e^x) +$

input
```
int(x^2/(1+2*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-x^2+x^2/(1+exp(x))+1/3*x^3+2*x*ln(1+exp(x))-x^2*ln(1+exp(x))+2*polylog(2,
-exp(x))-2*x*polylog(2,-exp(x))+2*polylog(3,-exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx = \frac{x^3 - 6\left((x-1)e^x + x - 1\right)\mathrm{Li}_2(-e^x) + (x^3 - 3x^2)e^x - 3\left(x^2 + (x^2 - 2x)e^x - 2x\right)\log\left(e^x + 1\right) + 6\left(e^x + \right.}{3\left(e^x + 1\right)}$$

input
```
integrate(x^2/(1+2*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/3*(x^3 - 6*((x - 1)*e^x + x - 1)*dilog(-e^x) + (x^3 - 3*x^2)*e^x - 3*(x^
2 + (x^2 - 2*x)*e^x - 2*x)*log(e^x + 1) + 6*(e^x + 1)*polylog(3, -e^x))/(e
^x + 1)
```

Sympy [F]

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx = \frac{x^2}{e^x+1} + \int \frac{x(x-2)}{e^x+1}\,dx$$

input
```
integrate(x**2/(1+2*exp(x)+exp(2*x)),x)
```

output
```
x**2/(exp(x) + 1) + Integral(x*(x - 2)/(exp(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx = \frac{1}{3}x^3 - x^2\log\left(e^x+1\right) - x^2 - 2x\mathrm{Li}_2(-e^x) + 2x\log\left(e^x+1\right) + \frac{x^2}{e^x+1} + 2\,\mathrm{Li}_2(-e^x) + 2\,\mathrm{Li}_3(-e^x)$$

input
```
integrate(x^2/(1+2*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/3*x^3 - x^2*log(e^x + 1) - x^2 - 2*x*dilog(-e^x) + 2*x*log(e^x + 1) + x^
2/(e^x + 1) + 2*dilog(-e^x) + 2*polylog(3, -e^x)
```

Giac [F]

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{(2\,x)}+2\,e^x+1}\,dx$$

input
```
integrate(x^2/(1+2*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(e^(2*x) + 2*e^x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx=\int \frac{x^2}{\mathrm{e}^{2\,x}+2\,\mathrm{e}^x+1}\,dx$$

input
```
int(x^2/(exp(2*x) + 2*exp(x) + 1),x)
```

output
```
int(x^2/(exp(2*x) + 2*exp(x) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1+2e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{2x}+2e^x+1}dx$$

input
```
int(x^2/(1+2*exp(x)+exp(2*x)),x)
```

output
```
int(x**2/(e**(2*x) + 2*e**x + 1),x)
```

3.439 $\int \frac{x^2}{2+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 77

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx=\frac{x^3}{6}+\frac{1}{2}x^2\log\left(1+\frac{e^x}{2}\right)-x^2\log\left(1+e^x\right)-2x\operatorname{PolyLog}\left(2,-e^x\right)+x\operatorname{PolyLog}\left(2,-\frac{e^x}{2}\right)+2\operatorname{PolyLog}\left(3,-e^x\right)-\operatorname{PolyLog}\left(3,-\frac{e^x}{2}\right)$$

output
```
1/6*x^3+1/2*x^2*ln(1+1/2*exp(x))-x^2*ln(1+exp(x))-2*x*polylog(2,-exp(x))+x
*polylog(2,-1/2*exp(x))+2*polylog(3,-exp(x))-polylog(3,-1/2*exp(x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx=-x^2\log\left(1+e^{-x}\right)+\frac{1}{2}x^2\log\left(1+2e^{-x}\right)-x\operatorname{PolyLog}\left(2,-2e^{-x}\right)+2x\operatorname{PolyLog}\left(2,-e^{-x}\right)-\operatorname{PolyLog}\left(3,-2e^{-x}\right)+2\operatorname{PolyLog}\left(3,-e^{-x}\right)$$

input
```
Integrate[x^2/(2 + 3*E^x + E^(2*x)),x]
```

output
```
-(x^2*Log[1 + E^(-x)]) + (x^2*Log[1 + 2/E^x])/2 - x*PolyLog[2, -2/E^x] + 2
*x*PolyLog[2, -E^(-x)] - PolyLog[3, -2/E^x] + 2*PolyLog[3, -E^(-x)]
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2693, 27, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{3e^x + e^{2x} + 2} \, dx$$

$$\Big\downarrow \text{2693}$$

$$2\int \frac{x^2}{2(1+e^x)}dx - 2\int \frac{x^2}{2(2+e^x)}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{x^2}{1+e^x}dx - \int \frac{x^2}{2+e^x}dx$$

$$\Big\downarrow \text{2615}$$

$$-\int \frac{e^x x^2}{1+e^x}dx + \frac{1}{2}\int \frac{e^x x^2}{2+e^x}dx + \frac{x^3}{6}$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}\left(x^2 \log\left(\frac{e^x}{2}+1\right) - 2\int x \log\left(1+\frac{e^x}{2}\right)dx\right) + 2\int x \log(1+e^x)\,dx + \frac{x^3}{6} - x^2 \log(e^x+1)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}\left(x^2 \log\left(\frac{e^x}{2}+1\right) - 2\left(\int \text{PolyLog}\left(2,-\frac{e^x}{2}\right)dx - x\,\text{PolyLog}\left(2,-\frac{e^x}{2}\right)\right)\right) + 2\left(\int \text{PolyLog}(2,-e^x)\,dx - x\,\text{PolyLog}(2,-e^x)\right) + \frac{x^3}{6} - x^2 \log(e^x+1)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(x^2\log\left(\frac{e^x}{2}+1\right)-2\left(\int e^{-x}\operatorname{PolyLog}\left(2,-\frac{e^x}{2}\right)de^x-x\operatorname{PolyLog}\left(2,-\frac{e^x}{2}\right)\right)\right)+2\left(\int e^{-x}\operatorname{PolyLog}(2,-e^x)\,de^x-x\operatorname{PolyLog}(2,-e^x)\right)+\frac{x^3}{6}-x^2\log(e^x+1)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}\left(x^2\log\left(\frac{e^x}{2}+1\right)-2\left(\operatorname{PolyLog}\left(3,-\frac{e^x}{2}\right)-x\operatorname{PolyLog}\left(2,-\frac{e^x}{2}\right)\right)\right)+2(\operatorname{PolyLog}(3,-e^x)-x\operatorname{PolyLog}(2,-e^x))+\frac{x^3}{6}-x^2\log(e^x+1)$$

input
```
Int[x^2/(2 + 3*E^x + E^(2*x)),x]
```

output
```
x^3/6 - x^2*Log[1 + E^x] + 2*(-(x*PolyLog[2, -E^x]) + PolyLog[3, -E^x]) +
(x^2*Log[1 + E^x/2] - 2*(-(x*PolyLog[2, -1/2*E^x]) + PolyLog[3, -1/2*E^x])
)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

method	result
default	$\frac{x^3}{6}+\frac{x^2\ln\left(1+\frac{e^x}{2}\right)}{2}-x^2\ln\left(1+e^x\right)-2x\,\mathrm{polylog}\left(2,-e^x\right)+x\,\mathrm{polylog}\left(2,-\frac{e^x}{2}\right)+2\,\mathrm{polylog}\left(3,-e^x\right)$
risch	$\frac{x^3}{6}+\frac{x^2\ln\left(1+\frac{e^x}{2}\right)}{2}-x^2\ln\left(1+e^x\right)-2x\,\mathrm{polylog}\left(2,-e^x\right)+x\,\mathrm{polylog}\left(2,-\frac{e^x}{2}\right)+2\,\mathrm{polylog}\left(3,-e^x\right)$

input
```
int(x^2/(2+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^3+1/2*x^2*ln(1+1/2*exp(x))-x^2*ln(1+exp(x))-2*x*polylog(2,-exp(x))+x
*polylog(2,-1/2*exp(x))+2*polylog(3,-exp(x))-polylog(3,-1/2*exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx = \frac{1}{6}\,x^3 - x^2\log\left(e^x+1\right) + \frac{1}{2}\,x^2\log\left(\frac{1}{2}\,e^x+1\right) + x\mathrm{Li}_2\left(-\frac{1}{2}\,e^x\right) - 2\,x\mathrm{Li}_2(-e^x) - \mathrm{polylog}\left(3,-\frac{1}{2}\,e^x\right) + 2\,\mathrm{polylog}(3,-e^x)$$

input
```
integrate(x^2/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/6*x^3 - x^2*log(e^x + 1) + 1/2*x^2*log(1/2*e^x + 1) + x*dilog(-1/2*e^x)
- 2*x*dilog(-e^x) - polylog(3, -1/2*e^x) + 2*polylog(3, -e^x)
```

Sympy [F]

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx = \int \frac{x^2}{(e^x+1)\,(e^x+2)}\,dx$$

input
```
integrate(x**2/(2+3*exp(x)+exp(2*x)),x)
```

output
```
Integral(x**2/((exp(x) + 1)*(exp(x) + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx = \frac{1}{6}\,x^3 - x^2\log\left(e^x+1\right) + \frac{1}{2}\,x^2\log\left(\frac{1}{2}\,e^x+1\right) + x\mathrm{Li}_2\left(-\frac{1}{2}\,e^x\right) - 2\,x\mathrm{Li}_2(-e^x) - \mathrm{Li}_3(-\frac{1}{2}\,e^x) + 2\,\mathrm{Li}_3(-e^x)$$

input
```
integrate(x^2/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/6*x^3 - x^2*log(e^x + 1) + 1/2*x^2*log(1/2*e^x + 1) + x*dilog(-1/2*e^x)
- 2*x*dilog(-e^x) - polylog(3, -1/2*e^x) + 2*polylog(3, -e^x)
```

Giac [F]

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{(2\,x)}+3\,e^x+2}\,dx$$

input
```
integrate(x^2/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(e^(2*x) + 3*e^x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx = \int \frac{x^2}{\mathrm{e}^{2\,x}+3\,\mathrm{e}^x+2}\,dx$$

input
```
int(x^2/(exp(2*x) + 3*exp(x) + 2),x)
```

output
```
int(x^2/(exp(2*x) + 3*exp(x) + 2), x)
```

Reduce [F]

$$\int \frac{x^2}{2+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{2x}+3e^x+2}dx$$

input
```
int(x^2/(2+3*exp(x)+exp(2*x)),x)
```

output
```
int(x**2/(e**(2*x) + 3*e**x + 2),x)
```

3.440 $\int \frac{x^2}{-1+e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 259

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx = \frac{2x^3}{3\sqrt{5}\,(1-\sqrt{5})} - \frac{2x^3}{3\sqrt{5}\,(1+\sqrt{5})} - \frac{2x^2\log\left(1+\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}\,(1-\sqrt{5})} + \frac{2x^2\log\left(1+\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}\,(1+\sqrt{5})} - \frac{4x\,\mathrm{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}\,(1-\sqrt{5})} + \frac{4x\,\mathrm{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}\,(1+\sqrt{5})} + \frac{4\,\mathrm{PolyLog}\left(3,-\frac{2e^x}{1-\sqrt{5}}\right)}{\sqrt{5}\,(1-\sqrt{5})} - \frac{4\,\mathrm{PolyLog}\left(3,-\frac{2e^x}{1+\sqrt{5}}\right)}{\sqrt{5}\,(1+\sqrt{5})}$$

output
```
2/15*x^3*5^(1/2)/(-5^(1/2)+1)-2/15*x^3*5^(1/2)/(5^(1/2)+1)-2/5*x^2*ln(1+2*
exp(x)/(-5^(1/2)+1))*5^(1/2)/(-5^(1/2)+1)+2/5*x^2*ln(1+2*exp(x)/(5^(1/2)+1
))*5^(1/2)/(5^(1/2)+1)-4/5*x*polylog(2,-2*exp(x)/(-5^(1/2)+1))*5^(1/2)/(-5
^(1/2)+1)+4/5*x*polylog(2,-2*exp(x)/(5^(1/2)+1))*5^(1/2)/(5^(1/2)+1)+4/5*p
olylog(3,-2*exp(x)/(-5^(1/2)+1))*5^(1/2)/(-5^(1/2)+1)-4/5*polylog(3,-2*exp
(x)/(5^(1/2)+1))*5^(1/2)/(5^(1/2)+1)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx$$
$$=\frac{2\left(\frac{x^2\log\left(1-\frac{1}{2}\left(-1+\sqrt{5}\right)e^{-x}\right)}{-1+\sqrt{5}}+\frac{x^2\log\left(1+\frac{1}{2}\left(1+\sqrt{5}\right)e^{-x}\right)}{1+\sqrt{5}}-\frac{2\left(x\,\mathrm{PolyLog}\left(2,\frac{1}{2}\left(-1+\sqrt{5}\right)e^{-x}\right)+\mathrm{PolyLog}\left(3,\frac{1}{2}\left(-1+\sqrt{5}\right)e^{-x}\right)\right)}{-1+\sqrt{5}}-\frac{2(\cdots}{\cdots}\right.}{\sqrt{5}}$$

input
```
Integrate[x^2/(-1 + E^x + E^(2*x)),x]
```

output
```
(2*((x^2*Log[1 - (-1 + Sqrt[5])/(2*E^x)])/(-1 + Sqrt[5]) + (x^2*Log[1 + (1
 + Sqrt[5])/(2*E^x)])/(1 + Sqrt[5]) - (2*(x*PolyLog[2, (-1 + Sqrt[5])/(2*E
^x)] + PolyLog[3, (-1 + Sqrt[5])/(2*E^x)]))/(-1 + Sqrt[5]) - (2*(x*PolyLog
[2, -1/2*(1 + Sqrt[5])/E^x] + PolyLog[3, -1/2*(1 + Sqrt[5])/E^x]))/(1 + Sq
rt[5])))/Sqrt[5]
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.78, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2693, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{e^x+e^{2x}-1}\,dx$$
$$\Big\downarrow \ 2693$$
$$\frac{2\int \frac{x^2}{1-\sqrt{5}+2e^x}dx}{\sqrt{5}}-\frac{2\int \frac{x^2}{1+\sqrt{5}+2e^x}dx}{\sqrt{5}}$$
$$\Big\downarrow \ 2615$$

$$\frac{2\left(\frac{x^3}{3(1-\sqrt{5})}-\frac{2\int \frac{e^x x^2}{1-\sqrt{5}+2e^x}dx}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^3}{3(1+\sqrt{5})}-\frac{2\int \frac{e^x x^2}{1+\sqrt{5}+2e^x}dx}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$\Big\downarrow$ 2620

$$\frac{2\left(\frac{x^3}{3(1-\sqrt{5})}-\frac{2\left(\frac{1}{2}x^2\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)-\int x\log\left(1+\frac{2e^x}{1-\sqrt{5}}\right)dx\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^3}{3(1+\sqrt{5})}-\frac{2\left(\frac{1}{2}x^2\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)-\int x\log\left(1+\frac{2e^x}{1+\sqrt{5}}\right)dx\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$\Big\downarrow$ 3011

$$\frac{2\left(\frac{x^3}{3(1-\sqrt{5})}-\frac{2\left(-\int \text{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)dx+x\,\text{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)+\frac{1}{2}x^2\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^3}{3(1+\sqrt{5})}-\frac{2\left(-\int \text{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)dx+x\,\text{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)+\frac{1}{2}x^2\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$\Big\downarrow$ 2720

$$\frac{2\left(\frac{x^3}{3(1-\sqrt{5})}-\frac{2\left(-\int e^{-x}\text{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)de^x+x\,\text{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)+\frac{1}{2}x^2\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^3}{3(1+\sqrt{5})}-\frac{2\left(-\int e^{-x}\text{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)de^x+x\,\text{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)+\frac{1}{2}x^2\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

$\Big\downarrow$ 7143

$$\frac{2\left(\frac{x^3}{3(1-\sqrt{5})}-\frac{2\left(x\,\text{PolyLog}\left(2,-\frac{2e^x}{1-\sqrt{5}}\right)-\text{PolyLog}\left(3,-\frac{2e^x}{1-\sqrt{5}}\right)+\frac{1}{2}x^2\log\left(\frac{2e^x}{1-\sqrt{5}}+1\right)\right)}{1-\sqrt{5}}\right)}{\sqrt{5}}-\frac{2\left(\frac{x^3}{3(1+\sqrt{5})}-\frac{2\left(x\,\text{PolyLog}\left(2,-\frac{2e^x}{1+\sqrt{5}}\right)-\text{PolyLog}\left(3,-\frac{2e^x}{1+\sqrt{5}}\right)+\frac{1}{2}x^2\log\left(\frac{2e^x}{1+\sqrt{5}}+1\right)\right)}{1+\sqrt{5}}\right)}{\sqrt{5}}$$

input
```
Int[x^2/(-1 + E^x + E^(2*x)),x]
```

output
```
(2*(x^3/(3*(1 - Sqrt[5])) - (2*((x^2*Log[1 + (2*E^x)/(1 - Sqrt[5])])/2 + x
*PolyLog[2, (-2*E^x)/(1 - Sqrt[5])] - PolyLog[3, (-2*E^x)/(1 - Sqrt[5])]))
/(1 - Sqrt[5])))/Sqrt[5] - (2*(x^3/(3*(1 + Sqrt[5])) - (2*((x^2*Log[1 + (2
*E^x)/(1 + Sqrt[5])])/2 + x*PolyLog[2, (-2*E^x)/(1 + Sqrt[5])] - PolyLog[3
, (-2*E^x)/(1 + Sqrt[5])]))/(1 + Sqrt[5])))/Sqrt[5]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{-1 + e^x + e^{2x}} dx$$

input
```
int(x^2/(-1+exp(x)+exp(2*x)),x)
```

output
```
int(x^2/(-1+exp(x)+exp(2*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.53

$$\begin{aligned}\int \frac{x^2}{-1+e^x+e^{2x}}\,dx = &-\frac{1}{3}\,x^3+\frac{1}{5}\left(\sqrt{5}x+5\,x\right)\mathrm{Li}_2\left(\frac{1}{2}\left(\sqrt{5}+1\right)e^x\right)\\ &-\frac{1}{5}\left(\sqrt{5}x-5\,x\right)\mathrm{Li}_2\left(-\frac{1}{2}\left(\sqrt{5}-1\right)e^x\right)\\ &+\frac{1}{10}\left(\sqrt{5}x^2+5\,x^2\right)\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)e^x+1\right)\\ &-\frac{1}{10}\left(\sqrt{5}x^2-5\,x^2\right)\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)e^x+1\right)\\ &-\frac{1}{5}\left(\sqrt{5}+5\right)\mathrm{polylog}\left(3,\frac{1}{2}\left(\sqrt{5}+1\right)e^x\right)\\ &+\frac{1}{5}\left(\sqrt{5}-5\right)\mathrm{polylog}\left(3,-\frac{1}{2}\left(\sqrt{5}-1\right)e^x\right)\end{aligned}$$

input
```
integrate(x^2/(-1+exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/3*x^3 + 1/5*(sqrt(5)*x + 5*x)*dilog(1/2*(sqrt(5) + 1)*e^x) - 1/5*(sqrt(
5)*x - 5*x)*dilog(-1/2*(sqrt(5) - 1)*e^x) + 1/10*(sqrt(5)*x^2 + 5*x^2)*log
(-1/2*(sqrt(5) + 1)*e^x + 1) - 1/10*(sqrt(5)*x^2 - 5*x^2)*log(1/2*(sqrt(5)
 - 1)*e^x + 1) - 1/5*(sqrt(5) + 5)*polylog(3, 1/2*(sqrt(5) + 1)*e^x) + 1/5
*(sqrt(5) - 5)*polylog(3, -1/2*(sqrt(5) - 1)*e^x)
```

Sympy [F]

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{2x}+e^x-1}\,dx$$

input
```
integrate(x**2/(-1+exp(x)+exp(2*x)),x)
```

output
```
Integral(x**2/(exp(2*x) + exp(x) - 1), x)
```

Maxima [F]

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{(2\,x)}+e^x-1}\,dx$$

input
```
integrate(x^2/(-1+exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(e^(2*x) + e^x - 1), x)
```

Giac [F]

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{(2x)}+e^x-1}\,dx$$

input
```
integrate(x^2/(-1+exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(e^(2*x) + e^x - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{2x}+e^x-1}\,dx$$

input
```
int(x^2/(exp(2*x) + exp(x) - 1),x)
```

output
```
int(x^2/(exp(2*x) + exp(x) - 1), x)
```

Reduce [F]

$$\int \frac{x^2}{-1+e^x+e^{2x}}\,dx=\int \frac{x^2}{e^{2x}+e^x-1}dx$$

input
```
int(x^2/(-1+exp(x)+exp(2*x)),x)
```

output
```
int(x**2/(e**(2*x) + e**x - 1),x)
```

3.441 $\int \frac{x^2}{3+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 293

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = -\frac{2x^3}{3\sqrt{3}\left(3i-\sqrt{3}\right)}+\frac{2x^3}{3\sqrt{3}\left(3i+\sqrt{3}\right)} -\frac{2x^2\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}\left(3i+\sqrt{3}\right)}+\frac{2x^2\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}\left(3i-\sqrt{3}\right)} -\frac{4x\,\mathrm{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}\left(3i+\sqrt{3}\right)}+\frac{4x\,\mathrm{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}\left(3i-\sqrt{3}\right)} +\frac{4\,\mathrm{PolyLog}\left(3,-\frac{2e^x}{3-i\sqrt{3}}\right)}{\sqrt{3}\left(3i+\sqrt{3}\right)}-\frac{4\,\mathrm{PolyLog}\left(3,-\frac{2e^x}{3+i\sqrt{3}}\right)}{\sqrt{3}\left(3i-\sqrt{3}\right)}$$

output
```
-2/9*x^3*3^(1/2)/(3*I-3^(1/2))+2/9*x^3*3^(1/2)/(3*I+3^(1/2))-2/3*x^2*ln(1+
2*exp(x)/(3-I*3^(1/2)))*3^(1/2)/(3*I+3^(1/2))+2/3*x^2*ln(1+2*exp(x)/(3+I*3
^(1/2)))*3^(1/2)/(3*I-3^(1/2))-4/3*x*polylog(2,-2*exp(x)/(3-I*3^(1/2)))*3^
(1/2)/(3*I+3^(1/2))+4/3*x*polylog(2,-2*exp(x)/(3+I*3^(1/2)))*3^(1/2)/(3*I-
3^(1/2))+4/3*polylog(3,-2*exp(x)/(3-I*3^(1/2)))*3^(1/2)/(3*I+3^(1/2))-4/3*
polylog(3,-2*exp(x)/(3+I*3^(1/2)))*3^(1/2)/(3*I-3^(1/2))
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \frac{2i\left(\frac{ix^2\log\left(1+\frac{1}{2}\left(3-i\sqrt{3}\right)e^{-x}\right)}{3i+\sqrt{3}}+\frac{ix^2\log\left(1+\frac{1}{2}\left(3+i\sqrt{3}\right)e^{-x}\right)}{-3i+\sqrt{3}}+\frac{2\left(x\,\text{PolyLog}\left(2,-\frac{1}{2}i\left(-3i+\sqrt{3}\right)e^{-x}\right)+\text{PolyLog}\left(3,-\frac{1}{2}i\left(-3i+\sqrt{3}\right)e^{-x}\right)\right.}{3+i\sqrt{3}}\right.}{\sqrt{3}}$$

input
```
Integrate[x^2/(3 + 3*E^x + E^(2*x)),x]
```

output
```
((2*I)*((I*x^2*Log[1 + (3 - I*Sqrt[3])/(2*E^x)])/(3*I + Sqrt[3]) + (I*x^2*
Log[1 + (3 + I*Sqrt[3])/(2*E^x)])/(-3*I + Sqrt[3]) + (2*(x*PolyLog[2, ((-1
/2*I)*(-3*I + Sqrt[3]))/E^x] + PolyLog[3, ((-1/2*I)*(-3*I + Sqrt[3]))/E^x]
))/(3 + I*Sqrt[3]) - ((2*I)*(x*PolyLog[2, ((I/2)*(3*I + Sqrt[3]))/E^x] + P
olyLog[3, ((I/2)*(3*I + Sqrt[3]))/E^x]))/(3*I + Sqrt[3])))/Sqrt[3]
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2693, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{3e^x+e^{2x}+3}\,dx$$

$$\Big\downarrow \text{2693}$$

$$\frac{2i\int \frac{x^2}{3+i\sqrt{3}+2e^x}dx}{\sqrt{3}}-\frac{2i\int \frac{x^2}{3-i\sqrt{3}+2e^x}dx}{\sqrt{3}}$$

$$\Big\downarrow \text{2615}$$

$$\frac{2i\left(\frac{x^3}{3\left(3+i\sqrt{3}\right)}-\frac{2\int \frac{e^x x^2}{3+i\sqrt{3}+2e^x}dx}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^3}{3\left(3-i\sqrt{3}\right)}-\frac{2\int \frac{e^x x^2}{3-i\sqrt{3}+2e^x}dx}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2i\left(\frac{x^3}{3\left(3+i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)-\int x\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)dx\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^3}{3\left(3-i\sqrt{3}\right)}-\frac{2\left(\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)-\int x\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)dx\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2i\left(\frac{x^3}{3\left(3+i\sqrt{3}\right)}-\frac{2\left(-\int \text{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)dx+x\,\text{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)+\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^3}{3\left(3-i\sqrt{3}\right)}-\frac{2\left(-\int \text{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)dx+x\,\text{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)+\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2i\left(\frac{x^3}{3\left(3+i\sqrt{3}\right)}-\frac{2\left(-\int e^{-x}\,\text{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)de^x+x\,\text{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)+\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^3}{3\left(3-i\sqrt{3}\right)}-\frac{2\left(-\int e^{-x}\,\text{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)de^x+x\,\text{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)+\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2i\left(\frac{x^3}{3\left(3+i\sqrt{3}\right)}-\frac{2\left(x\,\text{PolyLog}\left(2,-\frac{2e^x}{3+i\sqrt{3}}\right)-\text{PolyLog}\left(3,-\frac{2e^x}{3+i\sqrt{3}}\right)+\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3+i\sqrt{3}}\right)\right)}{3+i\sqrt{3}}\right)}{\sqrt{3}}-\frac{2i\left(\frac{x^3}{3\left(3-i\sqrt{3}\right)}-\frac{2\left(x\,\text{PolyLog}\left(2,-\frac{2e^x}{3-i\sqrt{3}}\right)-\text{PolyLog}\left(3,-\frac{2e^x}{3-i\sqrt{3}}\right)+\frac{1}{2}x^2\log\left(1+\frac{2e^x}{3-i\sqrt{3}}\right)\right)}{3-i\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[x^2/(3 + 3*E^x + E^(2*x)),x]
```

output
```
((-2*I)*(x^3/(3*(3 - I*Sqrt[3])) - (2*((x^2*Log[1 + (2*E^x)/(3 - I*Sqrt[3]
)])/2 + x*PolyLog[2, (-2*E^x)/(3 - I*Sqrt[3])] - PolyLog[3, (-2*E^x)/(3 -
I*Sqrt[3])]))/(3 - I*Sqrt[3])))/Sqrt[3] + ((2*I)*(x^3/(3*(3 + I*Sqrt[3]))
- (2*((x^2*Log[1 + (2*E^x)/(3 + I*Sqrt[3])])/2 + x*PolyLog[2, (-2*E^x)/(3
+ I*Sqrt[3])] - PolyLog[3, (-2*E^x)/(3 + I*Sqrt[3])]))/(3 + I*Sqrt[3])))/S
qrt[3]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{3+3\,e^x+e^{2x}}dx$$

input
```
int(x^2/(3+3*exp(x)+exp(2*x)),x)
```

output
```
int(x^2/(3+3*exp(x)+exp(2*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.48

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \frac{1}{9}x^3 + \frac{1}{3}\left(3\sqrt{-\frac{1}{3}}x - x\right)\mathrm{Li}_2\left(-\frac{1}{2}\left(\sqrt{-\frac{1}{3}}+1\right)e^x\right) - \frac{1}{3}\left(3\sqrt{-\frac{1}{3}}x + x\right)\mathrm{Li}_2\left(\frac{1}{2}\left(\sqrt{-\frac{1}{3}}-1\right)e^x\right) + \frac{1}{6}\left(3\sqrt{-\frac{1}{3}}x^2 - x^2\right)\log\left(\frac{1}{2}\left(\sqrt{-\frac{1}{3}}+1\right)e^x+1\right) - \frac{1}{6}\left(3\sqrt{-\frac{1}{3}}x^2 + x^2\right)\log\left(-\frac{1}{2}\left(\sqrt{-\frac{1}{3}}-1\right)e^x+1\right) - \frac{1}{3}\left(3\sqrt{-\frac{1}{3}}-1\right)\mathrm{polylog}\left(3,-\frac{1}{2}\left(\sqrt{-\frac{1}{3}}+1\right)e^x\right) + \frac{1}{3}\left(3\sqrt{-\frac{1}{3}}+1\right)\mathrm{polylog}\left(3,\frac{1}{2}\left(\sqrt{-\frac{1}{3}}-1\right)e^x\right)$$

input
```
integrate(x^2/(3+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/9*x^3 + 1/3*(3*sqrt(-1/3)*x - x)*dilog(-1/2*(sqrt(-1/3) + 1)*e^x) - 1/3*
(3*sqrt(-1/3)*x + x)*dilog(1/2*(sqrt(-1/3) - 1)*e^x) + 1/6*(3*sqrt(-1/3)*x
^2 - x^2)*log(1/2*(sqrt(-1/3) + 1)*e^x + 1) - 1/6*(3*sqrt(-1/3)*x^2 + x^2)
*log(-1/2*(sqrt(-1/3) - 1)*e^x + 1) - 1/3*(3*sqrt(-1/3) - 1)*polylog(3, -1
/2*(sqrt(-1/3) + 1)*e^x) + 1/3*(3*sqrt(-1/3) + 1)*polylog(3, 1/2*(sqrt(-1/
3) - 1)*e^x)
```

Sympy [F]

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{2x}+3e^x+3}\,dx$$

input
```
integrate(x**2/(3+3*exp(x)+exp(2*x)),x)
```

output
```
Integral(x**2/(exp(2*x) + 3*exp(x) + 3), x)
```

Maxima [F]

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{(2\,x)}+3\,e^x+3}\,dx$$

input
```
integrate(x^2/(3+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(e^(2*x) + 3*e^x + 3), x)
```

Giac [F]

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{(2\,x)}+3\,e^x+3}\,dx$$

input
```
integrate(x^2/(3+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(e^(2*x) + 3*e^x + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{2\,x}+3\,e^x+3}\,dx$$

input
```
int(x^2/(exp(2*x) + 3*exp(x) + 3),x)
```

output
```
int(x^2/(exp(2*x) + 3*exp(x) + 3), x)
```

Reduce [F]

$$\int \frac{x^2}{3+3e^x+e^{2x}}\,dx = \int \frac{x^2}{e^{2x}+3e^x+3}dx$$

input

```
int(x^2/(3+3*exp(x)+exp(2*x)),x)
```

output

```
int(x**2/(e**(2*x) + 3*e**x + 3),x)
```

3.442 $\int \frac{x^2}{a+be^x+ce^{2x}} dx$

Optimal result

Integrand size = 20, antiderivative size = 391

$$\int \frac{x^2}{a+be^x+ce^{2x}}\,dx = -\frac{2cx^3}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)} + \frac{2cx^2\log\left(1+\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{2cx^2\log\left(1+\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}} + \frac{4cx\,\mathrm{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}+\frac{4cx\,\mathrm{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}} - \frac{4c\,\mathrm{PolyLog}\left(3,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{4c\,\mathrm{PolyLog}\left(3,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{b^2-4ac+b\sqrt{b^2-4ac}}$$

output
```
-2*c*x^3/(3*b^2-12*a*c-3*b*(-4*a*c+b^2)^(1/2))-2*c*x^3/(3*b^2-12*a*c+3*b*(
-4*a*c+b^2)^(1/2))+2*c*x^2*ln(1+2*c*exp(x)/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*
a*c-b*(-4*a*c+b^2)^(1/2))+2*c*x^2*ln(1+2*c*exp(x)/(b+(-4*a*c+b^2)^(1/2)))/
(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)+4*c*x*polylog(2,-2*c*exp(x)/(b-(-4*a*c+b^
2)^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+4*c*x*polylog(2,-2*c*exp(x)/(b
+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)-4*c*polylog(3,-2*c*
exp(x)/(b-(-4*a*c+b^2)^(1/2)))/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-4*c*polylo
g(3,-2*c*exp(x)/(b+(-4*a*c+b^2)^(1/2)))/(b*(-4*a*c+b^2)^(1/2)-4*a*c+b^2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{a+be^x+ce^{2x}}\,dx = \frac{2c\left(\frac{x^2\log\left(1+\frac{\left(b-\sqrt{b^2-4ac}\right)e^{-x}}{2c}\right)}{-b+\sqrt{b^2-4ac}}+\frac{x^2\log\left(1+\frac{\left(b+\sqrt{b^2-4ac}\right)e^{-x}}{2c}\right)}{b+\sqrt{b^2-4ac}}+\frac{2\left(x\,\text{PolyLog}\left(2,\frac{\left(-b+\sqrt{b^2-4ac}\right)e^{-x}}{2c}\right)+\text{PolyLog}\left(3,\frac{\left(-b+\sqrt{b^2-4ac}\right)e^{-x}}{2c}\right.\right.}{b-\sqrt{b^2-4ac}}\right.}{\sqrt{b^2-4ac}}$$

input
```
Integrate[x^2/(a + b*E^x + c*E^(2*x)),x]
```

output
```
(2*c*((x^2*Log[1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*E^x)])/(-b + Sqrt[b^2 - 4*
a*c]) + (x^2*Log[1 + (b + Sqrt[b^2 - 4*a*c])/(2*c*E^x)])/(b + Sqrt[b^2 - 4
*a*c]) + (2*(x*PolyLog[2, (-b + Sqrt[b^2 - 4*a*c])/(2*c*E^x)] + PolyLog[3,
 (-b + Sqrt[b^2 - 4*a*c])/(2*c*E^x)]))/(b - Sqrt[b^2 - 4*a*c]) - (2*(x*Pol
yLog[2, -1/2*(b + Sqrt[b^2 - 4*a*c])/(c*E^x)] + PolyLog[3, -1/2*(b + Sqrt[
b^2 - 4*a*c])/(c*E^x)]))/(b + Sqrt[b^2 - 4*a*c])))/Sqrt[b^2 - 4*a*c]
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2693, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+be^x+ce^{2x}}\,dx$$

$$\Big\downarrow \text{2693}$$

$$\frac{2c\int \frac{x^2}{b+2ce^x-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2}{b+2ce^x+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2615}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int \frac{e^x x^2}{b+2ce^x-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int \frac{e^x x^2}{b+2ce^x+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}-\frac{\int x\log\left(\frac{2e^xc}{b-\sqrt{b^2-4ac}}+1\right)dx}{c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}-\frac{\int x\log\left(\frac{2e^xc}{b+\sqrt{b^2-4ac}}+1\right)dx}{c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}-\frac{\int \operatorname{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)dx-x\operatorname{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}-\frac{\int \operatorname{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)dx-x\operatorname{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}-\frac{\int e^{-x}\operatorname{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)de^x-x\operatorname{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}-\frac{\int e^{-x}\operatorname{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)de^x-x\operatorname{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{b-\sqrt{b^2-4ac}}+1\right)}{2c}-\frac{\text{PolyLog}\left(3,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{b-\sqrt{b^2-4ac}}\right)}{c}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{b^2-4ac}+b}+1\right)}{2c}-\frac{\text{PolyLog}\left(3,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{b+\sqrt{b^2-4ac}}\right)}{c}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a + b*E^x + c*E^(2*x)),x]
```

output
```
(2*c*(x^3/(3*(b - Sqrt[b^2 - 4*a*c])) - (2*c*((x^2*Log[1 + (2*c*E^x)/(b -
Sqrt[b^2 - 4*a*c])])/(2*c) - (-(x*PolyLog[2, (-2*c*E^x)/(b - Sqrt[b^2 - 4*
a*c])]) + PolyLog[3, (-2*c*E^x)/(b - Sqrt[b^2 - 4*a*c])])/c))/(b - Sqrt[b^
2 - 4*a*c])))/Sqrt[b^2 - 4*a*c] - (2*c*(x^3/(3*(b + Sqrt[b^2 - 4*a*c])) -
(2*c*((x^2*Log[1 + (2*c*E^x)/(b + Sqrt[b^2 - 4*a*c])])/(2*c) - (-(x*PolyLo
g[2, (-2*c*E^x)/(b + Sqrt[b^2 - 4*a*c])]) + PolyLog[3, (-2*c*E^x)/(b + Sqr
t[b^2 - 4*a*c])])/c))/(b + Sqrt[b^2 - 4*a*c])))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{a + b\,e^{x} + c\,e^{2x}}dx$$

input
```
int(x^2/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
int(x^2/(a+b*exp(x)+c*exp(2*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.06

$$\int \frac{x^2}{a+be^x+ce^{2x}}\,dx$$
$$= \frac{2\,(b^2-4\,ac)x^3-6\left(abx\sqrt{\frac{b^2-4\,ac}{a^2}}+(b^2-4\,ac)x\right)\mathrm{Li}_2\left(-\frac{a\sqrt{\frac{b^2-4\,ac}{a^2}}e^x+be^x+2\,a}{2\,a}+1\right)+6\left(abx\sqrt{\frac{b^2-4\,ac}{a^2}}-(b^2\right.}{}$$

input
```
integrate(x^2/(a+b*exp(x)+c*exp(2*x)),x, algorithm="fricas")
```

output
```
1/6*(2*(b^2 - 4*a*c)*x^3 - 6*(a*b*x*sqrt((b^2 - 4*a*c)/a^2) + (b^2 - 4*a*c
)*x)*dilog(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x + 2*a)/a + 1) + 6*(
a*b*x*sqrt((b^2 - 4*a*c)/a^2) - (b^2 - 4*a*c)*x)*dilog(1/2*(a*sqrt((b^2 -
4*a*c)/a^2)*e^x - b*e^x - 2*a)/a + 1) - 3*(a*b*x^2*sqrt((b^2 - 4*a*c)/a^2)
 + (b^2 - 4*a*c)*x^2)*log(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^x + 2*a
)/a) + 3*(a*b*x^2*sqrt((b^2 - 4*a*c)/a^2) - (b^2 - 4*a*c)*x^2)*log(-1/2*(a
*sqrt((b^2 - 4*a*c)/a^2)*e^x - b*e^x - 2*a)/a) + 6*(a*b*sqrt((b^2 - 4*a*c)
/a^2) + b^2 - 4*a*c)*polylog(3, -1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^x + b*e^
x)/a) - 6*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 4*a*c)*polylog(3, 1/2*(a*sq
rt((b^2 - 4*a*c)/a^2)*e^x - b*e^x)/a))/(a*b^2 - 4*a^2*c)
```

Sympy [F]

$$\int \frac{x^2}{a+be^x+ce^{2x}}\,dx = \int \frac{x^2}{a+be^x+ce^{2x}}\,dx$$

input
```
integrate(x**2/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
Integral(x**2/(a + b*exp(x) + c*exp(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + be^x + ce^{2x}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b*exp(x)+c*exp(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^2}{a + be^x + ce^{2x}} \, dx = \int \frac{x^2}{ce^{(2x)} + be^x + a} \, dx$$

input
```
integrate(x^2/(a+b*exp(x)+c*exp(2*x)),x, algorithm="giac")
```

output
```
integrate(x^2/(c*e^(2*x) + b*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + be^x + ce^{2x}} \, dx = \int \frac{x^2}{a + b\,e^x + c\,e^{2\,x}} \, dx$$

input
```
int(x^2/(a + b*exp(x) + c*exp(2*x)),x)
```

output
```
int(x^2/(a + b*exp(x) + c*exp(2*x)), x)
```

Reduce [F]

$$\int \frac{x^2}{a + be^x + ce^{2x}}\, dx = \int \frac{x^2}{e^{2x}c + e^x b + a} dx$$

input
```
int(x^2/(a+b*exp(x)+c*exp(2*x)),x)
```

output
```
int(x**2/(e**(2*x)*c + e**x*b + a),x)
```

3.443 $\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 40

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = x+\frac{1}{d\,(1+f^{c+dx})\log(f)}-\frac{\log\left(1+f^{c+dx}\right)}{d\log(f)}$$

output
```
x+1/d/(1+f^(d*x+c))/ln(f)-ln(1+f^(d*x+c))/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{\frac{1}{1+f^{c+dx}}+\log\left(f^{c+dx}\right)-\log\left(d\left(1+f^{c+dx}\right)\log(f)\right)}{d\log(f)}$$

input
```
Integrate[(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x))^(-1),x]
```

output
```
((1 + f^(c + d*x))^(-1) + Log[f^(c + d*x)] - Log[d*(1 + f^(c + d*x))*Log[f
]])/(d*Log[f])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2f^{c+dx}+f^{2c+2dx}+1}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{f^{-c-dx}}{\left(f^{c+dx}+1\right)^2}df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(f^{-c-dx}+\frac{1}{-f^{c+dx}-1}-\frac{1}{\left(f^{c+dx}+1\right)^2}\right)df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{f^{c+dx}+1}+\log\left(f^{c+dx}\right)-\log\left(f^{c+dx}+1\right)}{d\log(f)}$$

input
```
Int[(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x))^(-1),x]
```

output
```
((1 + f^(c + d*x))^(-1) + Log[f^(c + d*x)] - Log[1 + f^(c + d*x)])/(d*Log[
f])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result	size
risch	$x+\frac{c}{d}+\frac{1}{d\left(1+f^{dx+c}\right)\ln\left(f\right)}-\frac{\ln\left(1+f^{dx+c}\right)}{d\ln\left(f\right)}$	46
norman	$\frac{x+x\,\mathrm{e}^{\left(dx+c\right)\ln\left(f\right)}+\frac{1}{d\ln\left(f\right)}}{\mathrm{e}^{\left(dx+c\right)\ln\left(f\right)}+1}-\frac{\ln\left(\mathrm{e}^{\left(dx+c\right)\ln\left(f\right)}+1\right)}{d\ln\left(f\right)}$	58

input
```
int(1/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x,method=_RETURNVERBOSE)
```

output
```
x+c/d+1/d/(1+f^(d*x+c))/ln(f)-ln(1+f^(d*x+c))/d/ln(f)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{df^{dx+c}x\log(f)+dx\log(f)-\left(f^{dx+c}+1\right)\log\left(f^{dx+c}+1\right)+1}{df^{dx+c}\log(f)+d\log(f)}$$

input
```
integrate(1/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="fricas")
```

output
```
(d*f^(d*x + c)*x*log(f) + d*x*log(f) - (f^(d*x + c) + 1)*log(f^(d*x + c) +
 1) + 1)/(d*f^(d*x + c)*log(f) + d*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = x+\frac{1}{de^{\frac{(2c+2dx)\log(f)}{2}}\log(f)+d\log(f)}-\frac{\log\left(e^{\frac{(2c+2dx)\log(f)}{2}}+1\right)}{d\log(f)}$$

input
```
integrate(1/(1+2*f**(d*x+c)+f**(2*d*x+2*c)),x)
```

output
```
x + 1/(d*exp((2*c + 2*d*x)*log(f)/2)*log(f) + d*log(f)) - log(exp((2*c + 2
*d*x)*log(f)/2) + 1)/(d*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{dx+c}{d} - \frac{\log\left(f^{dx+c}+1\right)}{d\log\left(f\right)} + \frac{1}{d(f^{dx+c}+1)\log\left(f\right)}$$

input
```
integrate(1/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="maxima")
```

output
```
(d*x + c)/d - log(f^(d*x + c) + 1)/(d*log(f)) + 1/(d*(f^(d*x + c) + 1)*log
(f))
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{\frac{\log\left(|f|^{dx}|f|^{c}\right)}{\log(f)} - \frac{\log(|f^{dx}f^{c}+1|)}{\log(f)} + \frac{1}{(f^{dx}f^{c}+1)\log(f)}}{d}$$

input
```
integrate(1/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="giac")
```

output
```
(log(abs(f)^(d*x)*abs(f)^c)/log(f) - log(abs(f^(d*x)*f^c + 1))/log(f) + 1/
((f^(d*x)*f^c + 1)*log(f)))/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.25

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{1}{d\,\ln\left(f\right)\,\left(f^{d\,x}\,f^c+1\right)} - \frac{\ln\left(f^{d\,x}\,f^c+1\right)-d\,x\,\ln\left(f\right)}{d\,\ln\left(f\right)}$$

input
```
int(1/(f^(2*c + 2*d*x) + 2*f^(c + d*x) + 1),x)
```

output
```
1/(d*log(f)*(f^(d*x)*f^c + 1)) - (log(f^(d*x)*f^c + 1) - d*x*log(f))/(d*lo
g(f))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.92

$$\int \frac{1}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{-f^{dx+c}\log\left(f^{dx+c}+1\right)+f^{dx+c}\log(f)\,dx-f^{dx+c}-\log\left(f^{dx+c}+1\right)+\log(f)\,dx}{\log\left(f\right)d\left(f^{dx+c}+1\right)}$$

input
```
int(1/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x)
```

output
```
( - f**(c + d*x)*log(f**(c + d*x) + 1) + f**(c + d*x)*log(f)*d*x - f**(c +
 d*x) - log(f**(c + d*x) + 1) + log(f)*d*x)/(log(f)*d*(f**(c + d*x) + 1))
```

3.444 $\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx=\frac{x}{a}+\frac{b\,\text{arctanh}\left(\frac{b+2cf^{c+dx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}d\log(f)}-\frac{\log\left(a+bf^{c+dx}+cf^{2c+2dx}\right)}{2ad\log(f)}$$

output
```
x/a+b*arctanh((b+2*c*f^(d*x+c))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/d
/ln(f)-1/2*ln(a+b*f^(d*x+c)+c*f^(2*d*x+2*c))/a/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = -\frac{\frac{2b\arctan\left(\frac{b+2cf^{c+dx}}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log\left(f^{c+dx}\right)+\log\left(a+f^{c+dx}\left(b+cf^{c+dx}\right)\right)}{2ad\log(f)}$$

input
```
Integrate[(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x))^(-1),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*f^(c + d*x))/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] - 2*Log[f^(c + d*x)] + Log[a + f^(c + d*x)*(b + c*f^(c + d*x))])/(a*
d*Log[f])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2720, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{f^{-c-dx}}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{1144}$$

$$\frac{\frac{\int -\frac{cf^{c+dx}+b}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}}{a}+\frac{\log\left(f^{c+dx}\right)}{a}}{d\log(f)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\log\left(f^{c+dx}\right)}{a}-\frac{\int \frac{cf^{c+dx}+b}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}}{a}}{d\log(f)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\log\left(f^{c+dx}\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}+\frac{1}{2}\int \frac{2cf^{c+dx}+b}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}}{a}}{d\log(f)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\log\left(f^{c+dx}\right)}{a}-\frac{\frac{1}{2}\int \frac{2cf^{c+dx}+b}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}-b\int \frac{1}{-f^{2c+2dx}+b^2-4ac}d\left(2cf^{c+dx}+b\right)}{a}}{d\log(f)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\log(f^{c+dx})}{a}-\frac{\frac{1}{2}\int\frac{2cf^{c+dx}+b}{bf^{c+dx}+cf^{2c+2dx}+a}df^{c+dx}-\frac{b\text{arctanh}\left(\frac{b+2cf^{c+dx}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{d\log(f)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{\frac{\log(f^{c+dx})}{a}-\frac{\frac{1}{2}\log(a+bf^{c+dx}+cf^{2c+2dx})-\frac{b\text{arctanh}\left(\frac{b+2cf^{c+dx}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{d\log(f)}$$

input
```
Int[(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x))^(-1),x]
```

output
```
(Log[f^(c + d*x)]/a - (-((b*ArcTanh[(b + 2*c*f^(c + d*x))/Sqrt[b^2 - 4*a*c
]])/Sqrt[b^2 - 4*a*c]) + Log[a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)]/2)/a)/
(d*Log[f])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. $2(88) = 176$.

Time = 0.11 (sec) , antiderivative size = 547, normalized size of antiderivative = 5.82

method	result
risch	$\frac{4\ln(f)^2 a c\, d^2 x}{4\ln(f)^2 a^2 c\, d^2-\ln(f)^2 a\, b^2 d^2}-\frac{\ln(f)^2 b^2 d^2 x}{4\ln(f)^2 a^2 c\, d^2-\ln(f)^2 a\, b^2 d^2}+\frac{4\ln(f)^2 a\, c^2 d}{4\ln(f)^2 a^2 c\, d^2-\ln(f)^2 a\, b^2 d^2}-\frac{\ln(f)^2 b^2 c d}{4\ln(f)^2 a^2 c\, d^2-\ln(f)^2 a\, b^2 d^2}$

input
```
int(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x,method=_RETURNVERBOSE)
```

output
```
4/(4*ln(f)^2*a^2*c*d^2-ln(f)^2*a*b^2*d^2)*ln(f)^2*a*c*d^2*x-1/(4*ln(f)^2*a
^2*c*d^2-ln(f)^2*a*b^2*d^2)*ln(f)^2*b^2*d^2*x+4/(4*ln(f)^2*a^2*c*d^2-ln(f)
^2*a*b^2*d^2)*ln(f)^2*a*c^2*d-1/(4*ln(f)^2*a^2*c*d^2-ln(f)^2*a*b^2*d^2)*ln
(f)^2*b^2*c*d-2/(4*a*c-b^2)/d/ln(f)*ln(f^(d*x+c)-1/2*(-b^2+(-4*a*b^2*c+b^4
)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/d/ln(f)*ln(f^(d*x+c)-1/2*(-b^2+(-4*a*b^2
*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*c-b^2)/d/ln(f)*ln(f^(d*x+c)-1/2*(-b^2+(
-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)-2/(4*a*c-b^2)/d/ln(f)*l
n(f^(d*x+c)+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/d/ln
(f)*ln(f^(d*x+c)+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^
2)/d/ln(f)*ln(f^(d*x+c)+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+
b^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.29

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = \left[\frac{2\,(b^2-4\,ac)dx\log(f)+\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2f^{2\,dx+2\,c}+b^2-2\,ac+2\,\left(bc+\sqrt{b^2-4\,ac}c\right)f^{dx+c}+\sqrt{b^2-4\,ac}b}{cf^{2\,dx+2\,c}+bf^{dx+c}+a}\right)-(b^2-4\,ac}{2\,(ab^2-4\,a^2c)d\log(f)}\right.$$

input
```
integrate(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*d*x*log(f) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*f^(2*d*x
 + 2*c) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*f^(d*x + c) + sqrt(b
^2 - 4*a*c)*b)/(c*f^(2*d*x + 2*c) + b*f^(d*x + c) + a)) - (b^2 - 4*a*c)*lo
g(c*f^(2*d*x + 2*c) + b*f^(d*x + c) + a))/((a*b^2 - 4*a^2*c)*d*log(f)), 1/
2*(2*(b^2 - 4*a*c)*d*x*log(f) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*sqrt(-b^
2 + 4*a*c)*c*f^(d*x + c) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 - 4
*a*c)*log(c*f^(2*d*x + 2*c) + b*f^(d*x + c) + a))/((a*b^2 - 4*a^2*c)*d*log
(f))]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.19

$$\int \frac{1}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx$$
$$= \operatorname{RootSum}\left(z^2 \cdot \left(4a^2cd^2\log(f)^2 - ab^2d^2\log(f)^2\right) + z\left(4acd\log(f) - b^2d\log(f)\right) + c, \left(i \mapsto i\log\left(e^{\frac{(2c+2c}{}}\right.\right.\right.$$
$$+ \frac{x}{a}$$

input
```
integrate(1/(a+b*f**(d*x+c)+c*f**(2*d*x+2*c)),x)
```

output
```
RootSum(_z**2*(4*a**2*c*d**2*log(f)**2 - a*b**2*d**2*log(f)**2) + _z*(4*a*
c*d*log(f) - b**2*d*log(f)) + c, Lambda(_i, _i*log(exp((2*c + 2*d*x)*log(f
)/2) + (-4*_i*a**2*c*d*log(f) + _i*a*b**2*d*log(f) - 2*a*c + b**2)/(b*c)))
) + x/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = -\frac{\frac{2\,b\arctan\left(\frac{2\,cf^{dx}f^{c}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}a\log(f)}+\frac{\log\left(cf^{2\,dx}f^{2\,c}+bf^{dx}f^{c}+a\right)}{a\log(f)}-\frac{2\,\log\left(|f|^{dx}|f|^{c}\right)}{a\log(f)}}{2\,d}$$

input
```
integrate(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="giac")
```

output
```
-1/2*(2*b*arctan((2*c*f^(d*x)*f^c + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*
a*c)*a*log(f)) + log(c*f^(2*d*x)*f^(2*c) + b*f^(d*x)*f^c + a)/(a*log(f)) -
 2*log(abs(f)^(d*x)*abs(f)^c)/(a*log(f)))/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = \frac{x}{a}-\frac{\ln\left(a+c\,f^{2\,d\,x}\,f^{2\,c}+b\,f^{d\,x}\,f^{c}\right)}{2\,a\,d\,\ln\left(f\right)}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,f^{d\,x}\,f^{c}}{\sqrt{4\,a\,c-b^{2}}}\right)}{a\,d\,\ln\left(f\right)\,\sqrt{4\,a\,c-b^{2}}}$$

input
```
int(1/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x)
```

output
```
x/a - log(a + c*f^(2*d*x)*f^(2*c) + b*f^(d*x)*f^c)/(2*a*d*log(f)) - (b*ata
n((b + 2*c*f^(d*x)*f^c)/(4*a*c - b^2)^(1/2)))/(a*d*log(f)*(4*a*c - b^2)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.48

$$\int \frac{1}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2f^{dx+c}c+b}{\sqrt{4ac-b^2}}\right)b-4\log\left(f^{2dx+2c}c+f^{dx+c}b+a\right)ac+\log\left(f^{2dx+2c}c+f^{dx+c}b+a\right)b^2+8\,\mathrm{lo}}{2\log\left(f\right)ad\left(4ac-b^2\right)}$$

input
```
int(1/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((2*f**(c + d*x)*c + b)/sqrt(4*a*c - b**2))*b
 - 4*log(f**(2*c + 2*d*x)*c + f**(c + d*x)*b + a)*a*c + log(f**(2*c + 2*d*
x)*c + f**(c + d*x)*b + a)*b**2 + 8*log(f)*a*c*d*x - 2*log(f)*b**2*d*x)/(2
*log(f)*a*d*(4*a*c - b**2))
```

3.445 $\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx=\frac{x}{a}+\frac{b\operatorname{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}h\log(f)}-\frac{\log\left(a+bf^{g+hx}+cf^{2g+2hx}\right)}{2ah\log(f)}$$

output
```
x/a+b*arctanh((b+2*c*f^(h*x+g))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)/h
/ln(f)-1/2*ln(a+b*f^(h*x+g)+c*f^(2*h*x+2*g))/a/h/ln(f)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx=-\frac{\frac{2b\arctan\left(\frac{b+2cf^{g+hx}}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log\left(f^{g+hx}\right)+\log\left(a+f^{g+hx}\left(b+cf^{g+hx}\right)\right)}{2ah\log(f)}$$

input
```
Integrate[(a + b*f^(g + h*x) + c*f^(2*(g + h*x)))^(-1),x]
```

output
```
-1/2*((2*b*ArcTan[(b + 2*c*f^(g + h*x))/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] - 2*Log[f^(g + h*x)] + Log[a + f^(g + h*x)*(b + c*f^(g + h*x))])/(a*
h*Log[f])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {2720, 1144, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{f^{-g-hx}}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}}{h\log(f)}$$

$$\Big\downarrow \text{1144}$$

$$\frac{\frac{\int -\frac{cf^{g+hx}+b}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}}{a}+\frac{\log\left(f^{g+hx}\right)}{a}}{h\log(f)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\log\left(f^{g+hx}\right)}{a}-\frac{\int \frac{cf^{g+hx}+b}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}}{a}}{h\log(f)}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\frac{\log\left(f^{g+hx}\right)}{a}-\frac{\frac{1}{2}b\int \frac{1}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}+\frac{1}{2}\int \frac{2cf^{g+hx}+b}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}}{a}}{h\log(f)}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{\log\left(f^{g+hx}\right)}{a}-\frac{\frac{1}{2}\int \frac{2cf^{g+hx}+b}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}-b\int \frac{1}{-f^{2g+2hx}+b^2-4ac}d\left(2cf^{g+hx}+b\right)}{a}}{h\log(f)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\log\left(f^{g+hx}\right)}{a}-\frac{\frac{1}{2}\int\frac{2cf^{g+hx}+b}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}-\frac{b\operatorname{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{h\log(f)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log\left(f^{g+hx}\right)}{a}-\frac{\frac{1}{2}\log\left(a+bf^{g+hx}+cf^{2g+2hx}\right)-\frac{b\operatorname{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}}{a}}{h\log(f)}$$

input
```
Int[(a + b*f^(g + h*x) + c*f^(2*(g + h*x)))^(-1),x]
```

output
```
(Log[f^(g + h*x)]/a - (-((b*ArcTanh[(b + 2*c*f^(g + h*x))/Sqrt[b^2 - 4*a*c
]])/Sqrt[b^2 - 4*a*c]) + Log[a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)]/2)/a)/
(h*Log[f])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1144
```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Simp[e*(Log[RemoveContent[d + e*x, x]]/(c*d^2 - b*d*e + a*e^2)), x] + S
imp[1/(c*d^2 - b*d*e + a*e^2)   Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 545 vs. $2(88) = 176$.

Time = 0.12 (sec) , antiderivative size = 546, normalized size of antiderivative = 5.81

method	result
risch	$\frac{4\ln(f)^2 ac\,h^2 x}{4\ln(f)^2 a^2 c\,h^2-\ln(f)^2 a\,b^2 h^2}-\frac{\ln(f)^2 b^2 h^2 x}{4\ln(f)^2 a^2 c\,h^2-\ln(f)^2 a\,b^2 h^2}+\frac{4\ln(f)^2 acgh}{4\ln(f)^2 a^2 c\,h^2-\ln(f)^2 a\,b^2 h^2}-\frac{\ln(f)^2 b^2 gh}{4\ln(f)^2 a^2 c\,h^2-\ln(f)^2 a\,b^2 h^2}$

input
```
int(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x,method=_RETURNVERBOSE)
```

output
```
4/(4*ln(f)^2*a^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln(f)^2*a*c*h^2*x-1/(4*ln(f)^2*a
^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln(f)^2*b^2*h^2*x+4/(4*ln(f)^2*a^2*c*h^2-ln(f)
^2*a*b^2*h^2)*ln(f)^2*a*c*g*h-1/(4*ln(f)^2*a^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln
(f)^2*b^2*g*h-2/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-b^2+(-4*a*b^2*c+b^4
)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-b^2+(-4*a*b^2
*c+b^4)^(1/2))/b/c)*b^2+1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-b^2+(
-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+b^4)^(1/2)-2/(4*a*c-b^2)/h/ln(f)*l
n(f^(h*x+g)+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*c+1/2/a/(4*a*c-b^2)/h/ln
(f)*ln(f^(h*x+g)+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*b^2-1/2/a/(4*a*c-b^
2)/h/ln(f)*ln(f^(h*x+g)+1/2*(b^2+(-4*a*b^2*c+b^4)^(1/2))/b/c)*(-4*a*b^2*c+
b^4)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.29

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$
$$= \left[\frac{2\,(b^2-4\,ac)hx\log{(f)}+\sqrt{b^2-4\,ac}b\log\left(\frac{2\,c^2f^{2\,hx+2\,g}+b^2-2\,ac+2\left(bc+\sqrt{b^2-4\,ac}c\right)f^{hx+g}+\sqrt{b^2-4\,ac}b}{cf^{2\,hx+2\,g}+bf^{hx+g}+a}\right)-(b^2-4\,ac}{2\,(ab^2-4\,a^2c)h\log{(f)}}\right.$$

input
```
integrate(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="fricas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*h*x*log(f) + sqrt(b^2 - 4*a*c)*b*log((2*c^2*f^(2*h*x
 + 2*g) + b^2 - 2*a*c + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*f^(h*x + g) + sqrt(b
^2 - 4*a*c)*b)/(c*f^(2*h*x + 2*g) + b*f^(h*x + g) + a)) - (b^2 - 4*a*c)*lo
g(c*f^(2*h*x + 2*g) + b*f^(h*x + g) + a))/((a*b^2 - 4*a^2*c)*h*log(f)), 1/
2*(2*(b^2 - 4*a*c)*h*x*log(f) + 2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*sqrt(-b^
2 + 4*a*c)*c*f^(h*x + g) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4*a*c)) - (b^2 - 4
*a*c)*log(c*f^(2*h*x + 2*g) + b*f^(h*x + g) + a))/((a*b^2 - 4*a^2*c)*h*log
(f))]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$
$$=\text{RootSum}\left(z^2\cdot\left(4a^2ch^2\log\left(f\right)^2-ab^2h^2\log\left(f\right)^2\right)+z\left(4ach\log\left(f\right)-b^2h\log\left(f\right)\right)+c,\left(i\mapsto i\log\left(f^{g+hx}\right.\right.\right.$$
$$+\frac{x}{a}$$

input
```
integrate(1/(a+b*f**(h*x+g)+c*f**(2*h*x+2*g)),x)
```

output
```
RootSum(_z**2*(4*a**2*c*h**2*log(f)**2 - a*b**2*h**2*log(f)**2) + _z*(4*a*
c*h*log(f) - b**2*h*log(f)) + c, Lambda(_i, _i*log(f**(g + h*x) + (-4*_i*a
**2*c*h*log(f) + _i*a*b**2*h*log(f) - 2*a*c + b**2)/(b*c)))) + x/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.17

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx = -\frac{\frac{2\,b\arctan\left(\frac{2\,cf^{hx}f^{g}+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a\log(f)}+\frac{\log\left(cf^{2\,hx}f^{2\,g}+bf^{hx}f^{g}+a\right)}{a\log(f)}-\frac{2\,\log\left(|f|^{hx}|f|^{g}\right)}{a\log(f)}}{2\,h}$$

input
```
integrate(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="giac")
```

output
```
-1/2*(2*b*arctan((2*c*f^(h*x)*f^g + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*
a*c)*a*log(f)) + log(c*f^(2*h*x)*f^(2*g) + b*f^(h*x)*f^g + a)/(a*log(f)) -
 2*log(abs(f)^(h*x)*abs(f)^g)/(a*log(f)))/h
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx = \frac{x}{a}-\frac{\ln\left(a+c\,f^{2\,h\,x}\,f^{2\,g}+b\,f^{h\,x}\,f^{g}\right)}{2\,a\,h\,\ln\left(f\right)}-\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,f^{h\,x}\,f^{g}}{\sqrt{4\,a\,c-b^2}}\right)}{a\,h\,\ln\left(f\right)\,\sqrt{4\,a\,c-b^2}}$$

input
```
int(1/(a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)),x)
```

output
```
x/a - log(a + c*f^(2*h*x)*f^(2*g) + b*f^(h*x)*f^g)/(2*a*h*log(f)) - (b*ata
n((b + 2*c*f^(h*x)*f^g)/(4*a*c - b^2)^(1/2)))/(a*h*log(f)*(4*a*c - b^2)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.48

$$\int \frac{1}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$
$$= \frac{-2\sqrt{4ac-b^2}\,atan\left(\frac{2f^{hx+g}c+b}{\sqrt{4ac-b^2}}\right)b-4\log\left(f^{2hx+2g}c+f^{hx+g}b+a\right)ac+\log\left(f^{2hx+2g}c+f^{hx+g}b+a\right)b^2+8}{2\log\left(f\right)ah\left(4ac-b^2\right)}$$

input
```
int(1/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x)
```

output
```
( - 2*sqrt(4*a*c - b**2)*atan((2*f**(g + h*x)*c + b)/sqrt(4*a*c - b**2))*b
 - 4*log(f**(2*g + 2*h*x)*c + f**(g + h*x)*b + a)*a*c + log(f**(2*g + 2*h*
x)*c + f**(g + h*x)*b + a)*b**2 + 8*log(f)*a*c*h*x - 2*log(f)*b**2*h*x)/(2
*log(f)*a*h*(4*a*c - b**2))
```

3.446 $\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 96

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx=\frac{x^2}{2}-\frac{x}{d\log(f)}+\frac{x}{d\left(1+f^{c+dx}\right)\log(f)}+\frac{\log\left(1+f^{c+dx}\right)}{d^2\log^2(f)}-\frac{x\log\left(1+f^{c+dx}\right)}{d\log(f)}-\frac{\operatorname{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}$$

output
```
1/2*x^2-x/d/ln(f)+x/d/(1+f^(d*x+c))/ln(f)+ln(1+f^(d*x+c))/d^2/ln(f)^2-x*ln
(1+f^(d*x+c))/d/ln(f)-polylog(2,-f^(d*x+c))/d^2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx=\frac{1}{2}x\left(x+\frac{2}{d\log(f)+df^{c+dx}\log(f)}\right)+\frac{\log\left(1+f^{c+dx}\right)}{d^2\log^2(f)}-\frac{x\left(1+\log\left(1+f^{c+dx}\right)\right)}{d\log(f)}-\frac{\operatorname{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}$$

input
```
Integrate[x/(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x)),x]
```

output
```
(x*(x + 2/(d*Log[f] + d*f^(c + d*x)*Log[f])))/2 + Log[1 + f^(c + d*x)]/(d^
2*Log[f]^2) - (x*(1 + Log[1 + f^(c + d*x)]))/(d*Log[f]) - PolyLog[2, -f^(c
 + d*x)]/(d^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {7239, 2616, 2615, 2620, 2621, 2715, 2720, 47, 14, 16, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{2f^{c+dx}+f^{2c+2dx}+1}dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{x}{\left(f^{c+dx}+1\right)^2}dx$$

$$\Big\downarrow \text{2616}$$

$$\int \frac{x}{f^{c+dx}+1}dx-\int \frac{f^{c+dx}x}{\left(f^{c+dx}+1\right)^2}dx$$

$$\Big\downarrow \text{2615}$$

$$-\int \frac{f^{c+dx}x}{\left(f^{c+dx}+1\right)^2}dx-\int \frac{f^{c+dx}x}{f^{c+dx}+1}dx+\frac{x^2}{2}$$

$$\Big\downarrow \text{2620}$$

$$-\int \frac{f^{c+dx}x}{\left(f^{c+dx}+1\right)^2}dx+\frac{\int \log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{2621}$$

$$-\frac{\int \frac{1}{f^{c+dx}+1}dx}{d\log(f)}+\frac{\int \log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{\int \frac{1}{f^{c+dx}+1}dx}{d\log(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\int \frac{f^{-c-dx}}{f^{c+dx}+1}df^{c+dx}}{d^2\log^2(f)}+\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{47}$$

$$-\frac{\int f^{-c-dx}df^{c+dx}-\int \frac{1}{f^{c+dx}+1}df^{c+dx}}{d^2\log^2(f)}+\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{14}$$

$$-\frac{\log\left(f^{c+dx}\right)-\int \frac{1}{f^{c+dx}+1}df^{c+dx}}{d^2\log^2(f)}+\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{\log\left(f^{c+dx}\right)-\log\left(f^{c+dx}+1\right)}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\text{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}-\frac{\log\left(f^{c+dx}\right)-\log\left(f^{c+dx}+1\right)}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^2}{2}$$

input
```
Int[x/(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x)),x]
```

output
```
x^2/2 + x/(d*(1 + f^(c + d*x))*Log[f]) - (Log[f^(c + d*x)] - Log[1 + f^(c
 + d*x)])/(d^2*Log[f]^2) - (x*Log[1 + f^(c + d*x)])/(d*Log[f]) - PolyLog[2,
 -f^(c + d*x)]/(d^2*Log[f]^2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

method	result
risch	$\frac{x}{d\left(1+f^{dx+c}\right)\ln(f)}+\frac{x^{2}}{2}+\frac{cx}{d}+\frac{c^{2}}{2d^{2}}-\frac{\ln\left(1+f^{dx}f^{c}\right)x}{d\ln(f)}-\frac{\operatorname{polylog}\left(2,-f^{dx}f^{c}\right)}{d^{2}\ln(f)^{2}}+\frac{\ln\left(1+f^{dx}f^{c}\right)}{d^{2}\ln(f)^{2}}-\frac{\ln\left(f^{dx}f^{c}\right)}{d^{2}\ln(f)^{2}}-\frac{c\ln(f^{dx}}{d^{2}\ln(}$

input
```
int(x/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x,method=_RETURNVERBOSE)
```

output
```
x/d/(1+f^(d*x+c))/ln(f)+1/2*x^2+c*x/d+1/2/d^2*c^2-1/d/ln(f)*ln(1+f^(d*x)*f
^c)*x-1/d^2/ln(f)^2*polylog(2,-f^(d*x)*f^c)+1/d^2/ln(f)^2*ln(1+f^(d*x)*f^c
)-1/d^2/ln(f)^2*ln(f^(d*x)*f^c)-1/d^2/ln(f)*c*ln(f^(d*x)*f^c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx$$
$$= \frac{(d^2x^2-c^2)\log(f)^2+\left((d^2x^2-c^2)\log(f)^2-2\,(dx+c)\log(f)\right)f^{dx+c}-2\left(f^{dx+c}+1\right)\mathrm{Li}_2\left(-f^{dx+c}\right)-2\,(}{2\left(d^2f^{dx+c}\log(f)^2+d^2\log(f)^2\right)}$$

input
```
integrate(x/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="fricas")
```

output
```
1/2*((d^2*x^2 - c^2)*log(f)^2 + ((d^2*x^2 - c^2)*log(f)^2 - 2*(d*x + c)*lo
g(f))*f^(d*x + c) - 2*(f^(d*x + c) + 1)*dilog(-f^(d*x + c)) - 2*(d*x*log(f
) + (d*x*log(f) - 1)*f^(d*x + c) - 1)*log(f^(d*x + c) + 1) - 2*c*log(f))/(
d^2*f^(d*x + c)*log(f)^2 + d^2*log(f)^2)
```

Sympy [F]

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{x}{de^{\frac{(2c+2dx)\log(f)}{2}}\log(f)+d\log(f)} + \frac{\int \frac{dx\log(f)}{e^{c\log(f)}e^{dx\log(f)}+1}\,dx+\int\left(-\frac{1}{e^{c\log(f)}e^{dx\log(f)}+1}\right)dx}{d\log(f)}$$

input
```
integrate(x/(1+2*f**(d*x+c)+f**(2*d*x+2*c)),x)
```

output
```
x/(d*exp((2*c + 2*d*x)*log(f)/2)*log(f) + d*log(f)) + (Integral(d*x*log(f)
/(exp(c*log(f))*exp(d*x*log(f)) + 1), x) + Integral(-1/(exp(c*log(f))*exp(
d*x*log(f)) + 1), x))/(d*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{1}{2}x^2 + \frac{x}{df^{dx}f^c\log(f)+d\log(f)} - \frac{x}{d\log(f)} - \frac{dx\log\left(f^{dx}f^c+1\right)\log(f)+\mathrm{Li}_2\left(-f^{dx}f^c\right)}{d^2\log(f)^2} + \frac{\log\left(f^{dx}f^c+1\right)}{d^2\log(f)^2}$$

input
```
integrate(x/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="maxima")
```

output
```
1/2*x^2 + x/(d*f^(d*x)*f^c*log(f) + d*log(f)) - x/(d*log(f)) - (d*x*log(f^
(d*x)*f^c + 1)*log(f) + dilog(-f^(d*x)*f^c))/(d^2*log(f)^2) + log(f^(d*x)*
f^c + 1)/(d^2*log(f)^2)
```

Giac [F]

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \int \frac{x}{f^{2\,dx+2\,c}+2\,f^{dx+c}+1}\,dx$$

input
```
integrate(x/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="giac")
```

output
```
integrate(x/(f^(2*d*x + 2*c) + 2*f^(d*x + c) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \int \frac{x}{f^{2\,c+2\,d\,x}+2\,f^{c+d\,x}+1}\,dx$$

input
```
int(x/(f^(2*c + 2*d*x) + 2*f^(c + d*x) + 1),x)
```

output
```
int(x/(f^(2*c + 2*d*x) + 2*f^(c + d*x) + 1), x)
```

Reduce [F]

$$\int \frac{x}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \int \frac{x}{f^{2dx+2c}+2f^{dx+c}+1}dx$$

input
```
int(x/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x)
```

output
```
int(x/(f**(2*c + 2*d*x) + 2*f**(c + d*x) + 1),x)
```

3.447 $\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 338

$$\begin{aligned}\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = &-\frac{cx^2}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2}{b^2-4ac+b\sqrt{b^2-4ac}}\\
&-\frac{2cx\log\left(1+\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d\log(f)}\\
&+\frac{2cx\log\left(1+\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d\log(f)}\\
&-\frac{2c\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^2\log^2(f)}\\
&+\frac{2c\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^2\log^2(f)}\end{aligned}$$

output
```
-c*x^2/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2/(b*(-4*a*c+b^2)^(1/2)-4*a*c+
b^2)-2*c*x*ln(1+2*c*f^(d*x+c)/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(
b-(-4*a*c+b^2)^(1/2))/d/ln(f)+2*c*x*ln(1+2*c*f^(d*x+c)/(b+(-4*a*c+b^2)^(1/
2)))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d/ln(f)-2*c*polylog(2,-2*c*
f^(d*x+c)/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)/d^2/ln(f)^2+2*c*polylog(2,-2*c*f^(d*x+c)/(b+(-4*a*c+b^2)^(1/2)))/(-4*a*c
+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d^2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.70

$$\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = \frac{-dx\log(f)\left(\left(b+\sqrt{b^2-4ac}\right)\log\left(1+\frac{\left(b-\sqrt{b^2-4ac}\right)f^{-c-dx}}{2c}\right)+\left(-b+\sqrt{b^2-4ac}\right)\log\left(1+\frac{\left(b+\sqrt{b^2-4ac}\right)f^{-c}}{2c}\right.\right.}{2a\sqrt{b^2}}$$

input
```
Integrate[x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x]
```

output
```
(-(d*x*Log[f]*((b + Sqrt[b^2 - 4*a*c])*Log[1 + ((b - Sqrt[b^2 - 4*a*c])*f^
(-c - d*x))/(2*c)] + (-b + Sqrt[b^2 - 4*a*c])*Log[1 + ((b + Sqrt[b^2 - 4*a
*c])*f^(-c - d*x))/(2*c)])) + (b + Sqrt[b^2 - 4*a*c])*PolyLog[2, ((-b + Sq
rt[b^2 - 4*a*c])*f^(-c - d*x))/(2*c)] + (-b + Sqrt[b^2 - 4*a*c])*PolyLog[2
, -1/2*((b + Sqrt[b^2 - 4*a*c])*f^(-c - d*x))/c])/(2*a*Sqrt[b^2 - 4*a*c]*d
^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2693, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$$

$$\Big\downarrow \text{2693}$$

$$\frac{2c\int \frac{x}{2cf^{c+dx}+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x}{2cf^{c+dx}+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2615}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int\frac{f^{c+dx}x}{2cf^{c+dx}+b-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int\frac{f^{c+dx}x}{2cf^{c+dx}+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}-\frac{\int\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)dx}{2cd\log(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}-\frac{\int\log\left(\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}+1\right)dx}{2cd\log(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}-\frac{\int f^{-c-dx}\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)df^{c+dx}}{2cd^2\log^2(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}-\frac{\int f^{-c-dx}\log\left(\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}+1\right)df^{c+dx}}{2cd^2\log^2(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2c\left(\frac{x^2}{2\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{2cd^2\log^2(f)}+\frac{x\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^2}{2\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{2cd^2\log^2(f)}+\frac{x\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x]
```

output
```
(2*c*(x^2/(2*(b - Sqrt[b^2 - 4*a*c])) - (2*c*((x*Log[1 + (2*c*f^(c + d*x))
/(b - Sqrt[b^2 - 4*a*c])])/(2*c*d*Log[f]) + PolyLog[2, (-2*c*f^(c + d*x))/
(b - Sqrt[b^2 - 4*a*c])]/(2*c*d^2*Log[f]^2)))/(b - Sqrt[b^2 - 4*a*c])))/Sq
rt[b^2 - 4*a*c] - (2*c*(x^2/(2*(b + Sqrt[b^2 - 4*a*c])) - (2*c*((x*Log[1 +
 (2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c])])/(2*c*d*Log[f]) + PolyLog[2, (
-2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c])]/(2*c*d^2*Log[f]^2)))/(b + Sqrt[
b^2 - 4*a*c])))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. $2(310) = 620$.

Time = 0.11 (sec) , antiderivative size = 855, normalized size of antiderivative = 2.53

method	result
risch	$\frac{x^2}{2a}+\frac{cx}{da}+\frac{c^2}{2d^2a}-\frac{\ln\left(\frac{-2c\,f^{dx}f^{c}+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)x}{2d\ln(f)a}-\frac{\ln\left(\frac{-2c\,f^{dx}f^{c}+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)c}{2d^2\ln(f)a}-\frac{\ln\left(\frac{-2c\,f^{dx}f^{c}+\sqrt{-4ac+b^2}-b}{-b+\sqrt{-4ac+b^2}}\right)bx}{2d\ln(f)a\sqrt{-4ac+b^2}}$

input
```
int(x/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/a+1/d/a*c*x+1/2/d^2/a*c^2-1/2/d/ln(f)/a*ln((-2*c*f^(d*x)*f^c+(-4*a
*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*x-1/2/d^2/ln(f)/a*ln((-2*c*f^(d*
x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*c-1/2/d/ln(f)/a/(-4*
a*c+b^2)^(1/2)*ln((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)
^(1/2)))*b*x-1/2/d^2/ln(f)/a/(-4*a*c+b^2)^(1/2)*ln((-2*c*f^(d*x)*f^c+(-4*a
*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b*c-1/2/d/ln(f)/a*ln((2*c*f^(d*x
)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*x-1/2/d^2/ln(f)/a*ln((
2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*c+1/2/d/ln(f
)/a/(-4*a*c+b^2)^(1/2)*ln((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*
c+b^2)^(1/2)))*b*x+1/2/d^2/ln(f)/a/(-4*a*c+b^2)^(1/2)*ln((2*c*f^(d*x)*f^c+
(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))*b*c-1/2/d^2/ln(f)^2/a*dilog(
(2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+(-4*a*c+b^2)^(1/2)))+1/2/d^2/ln(
f)^2/a/(-4*a*c+b^2)^(1/2)*dilog((2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)+b)/(b+
(-4*a*c+b^2)^(1/2)))*b-1/2/d^2/ln(f)^2/a*dilog((-2*c*f^(d*x)*f^c+(-4*a*c+b
^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))-1/2/d^2/ln(f)^2/a/(-4*a*c+b^2)^(1/2)
*dilog((-2*c*f^(d*x)*f^c+(-4*a*c+b^2)^(1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))*b-
1/d^2/ln(f)*c/a*ln(f^(d*x)*f^c)+1/2/d^2/ln(f)*c/a*ln(a+b*f^(d*x)*f^c+c*(f^
(d*x))^2*(f^c)^2)+1/d^2/ln(f)*c/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*f^(d*x)*
f^c+b)/(4*a*c-b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.47

$$\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$$
$$= \frac{(b^2-4\,ac)d^2x^2\log(f)^2 - \left(ab\sqrt{\frac{b^2-4\,ac}{a^2}}+b^2-4\,ac\right)\mathrm{Li}_2\left(-\frac{\left(a\sqrt{\frac{b^2-4\,ac}{a^2}}+b\right)f^{dx+c}+2\,a}{2\,a}+1\right)+\left(ab\sqrt{\frac{b^2-4\,ac}{a^2}}-\right.}{}$$

input
```
integrate(x/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="fricas")
```

output
```
1/2*((b^2 - 4*a*c)*d^2*x^2*log(f)^2 - (a*b*sqrt((b^2 - 4*a*c)/a^2) + b^2 -
 4*a*c)*dilog(-1/2*((a*sqrt((b^2 - 4*a*c)/a^2) + b)*f^(d*x + c) + 2*a)/a +
 1) + (a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 4*a*c)*dilog(1/2*((a*sqrt((b^2
- 4*a*c)/a^2) - b)*f^(d*x + c) - 2*a)/a + 1) - (a*b*c*sqrt((b^2 - 4*a*c)/a
^2)*log(f) - (b^2*c - 4*a*c^2)*log(f))*log(2*c*f^(d*x + c) + a*sqrt((b^2 -
 4*a*c)/a^2) + b) + (a*b*c*sqrt((b^2 - 4*a*c)/a^2)*log(f) + (b^2*c - 4*a*c
^2)*log(f))*log(2*c*f^(d*x + c) - a*sqrt((b^2 - 4*a*c)/a^2) + b) - ((a*b*d
*x + a*b*c)*sqrt((b^2 - 4*a*c)/a^2)*log(f) + (b^2*c - 4*a*c^2 + (b^2 - 4*a
*c)*d*x)*log(f))*log(1/2*((a*sqrt((b^2 - 4*a*c)/a^2) + b)*f^(d*x + c) + 2*
a)/a) + ((a*b*d*x + a*b*c)*sqrt((b^2 - 4*a*c)/a^2)*log(f) - (b^2*c - 4*a*c
^2 + (b^2 - 4*a*c)*d*x)*log(f))*log(-1/2*((a*sqrt((b^2 - 4*a*c)/a^2) - b)*
f^(d*x + c) - 2*a)/a))/((a*b^2 - 4*a^2*c)*d^2*log(f)^2)
```

Sympy [F]

$$\int \frac{x}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \int \frac{x}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx$$

input
```
integrate(x/(a+b*f**(d*x+c)+c*f**(2*d*x+2*c)),x)
```

output
```
Integral(x/(a + b*f**(c + d*x) + c*f**(2*c + 2*d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx=\int \frac{x}{cf^{2\,dx+2\,c}+bf^{dx+c}+a}\,dx$$

input
```
integrate(x/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="giac")
```

output
```
integrate(x/(c*f^(2*d*x + 2*c) + b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx=\int \frac{x}{a+b\,f^{c+d\,x}+c\,f^{2\,c+2\,d\,x}}\,dx$$

input
```
int(x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x)
```

output
```
int(x/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)), x)
```

Reduce [F]

$$\int \frac{x}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx=\frac{2f^{2c}\left(\int \frac{f^{2dx}x}{f^{2dx+2c}c+f^{dx+c}b+a}dx\right)c+2f^{c}\left(\int \frac{f^{dx}x}{f^{2dx+2c}c+f^{dx+c}b+a}dx\right)b+4\left(\int \frac{x}{f^{2dx+2c}c+f^{dx+c}b+a}dx\right)a-x^{2}}{2a}$$

input
```
int(x/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x)
```

output
```
(2*f**(2*c)*int((f**(2*d*x)*x)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*b + a),x
)*c + 2*f**c*int((f**(d*x)*x)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*b + a),x)
*b + 4*int(x/(f**(2*c + 2*d*x)*c + f**(c + d*x)*b + a),x)*a - x**2)/(2*a)
```

3.448 $\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 145

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx=\frac{x^3}{3}-\frac{x^2}{d\log(f)}+\frac{x^2}{d\left(1+f^{c+dx}\right)\log(f)}+\frac{2x\log\left(1+f^{c+dx}\right)}{d^2\log^2(f)}-\frac{x^2\log\left(1+f^{c+dx}\right)}{d\log(f)}+\frac{2\,\mathrm{PolyLog}\left(2,-f^{c+dx}\right)}{d^3\log^3(f)}-\frac{2x\,\mathrm{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}+\frac{2\,\mathrm{PolyLog}\left(3,-f^{c+dx}\right)}{d^3\log^3(f)}$$

output
```
1/3*x^3-x^2/d/ln(f)+x^2/d/(1+f^(d*x+c))/ln(f)+2*x*ln(1+f^(d*x+c))/d^2/ln(f
)^2-x^2*ln(1+f^(d*x+c))/d/ln(f)+2*polylog(2,-f^(d*x+c))/d^3/ln(f)^3-2*x*po
lylog(2,-f^(d*x+c))/d^2/ln(f)^2+2*polylog(3,-f^(d*x+c))/d^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx$$
$$=\frac{d^3x^3\log^3(f)+6dx\log(f)\log\left(1+f^{c+dx}\right)-\frac{3d^2x^2\log^2(f)\left(f^{c+dx}+\left(1+f^{c+dx}\right)\log\left(1+f^{c+dx}\right)\right)}{1+f^{c+dx}}+(6-6dx\log(f))\,\mathrm{Po}}{3d^3\log^3(f)}$$

input
```
Integrate[x^2/(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x)),x]
```

output
```
(d^3*x^3*Log[f]^3 + 6*d*x*Log[f]*Log[1 + f^(c + d*x)] - (3*d^2*x^2*Log[f]^
2*(f^(c + d*x) + (1 + f^(c + d*x))*Log[1 + f^(c + d*x)]))/(1 + f^(c + d*x)
) + (6 - 6*d*x*Log[f])*PolyLog[2, -f^(c + d*x)] + 6*PolyLog[3, -f^(c + d*x
)])/(3*d^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {7239, 2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{2f^{c+dx}+f^{2c+2dx}+1}dx$$
$$\Big\downarrow \text{7239}$$
$$\int \frac{x^2}{\left(f^{c+dx}+1\right)^2}dx$$
$$\Big\downarrow \text{2616}$$
$$\int \frac{x^2}{f^{c+dx}+1}dx-\int \frac{f^{c+dx}x^2}{\left(f^{c+dx}+1\right)^2}dx$$
$$\Big\downarrow \text{2615}$$
$$-\int \frac{f^{c+dx}x^2}{\left(f^{c+dx}+1\right)^2}dx-\int \frac{f^{c+dx}x^2}{f^{c+dx}+1}dx+\frac{x^3}{3}$$
$$\Big\downarrow \text{2620}$$
$$-\int \frac{f^{c+dx}x^2}{\left(f^{c+dx}+1\right)^2}dx+\frac{2\int x\log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^3}{3}$$
$$\Big\downarrow \text{2621}$$
$$-\frac{2\int \frac{x}{f^{c+dx}+1}dx}{d\log(f)}+\frac{2\int x\log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 2615$$

$$-\frac{2\left(\frac{x^2}{2}-\int \frac{f^{c+dx}x}{f^{c+dx}+1}dx\right)}{d\log(f)}+\frac{2\int x\log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 2620$$

$$-\frac{2\left(\frac{\int \log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}+\frac{2\int x\log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 2715$$

$$-\frac{2\left(\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}+\frac{2\int x\log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 2838$$

$$\frac{2\int x\log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{2\left(-\frac{\text{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 3011$$

$$\frac{2\left(\frac{\int \text{PolyLog}\left(2,-f^{c+dx}\right)dx}{d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-f^{c+dx}\right)}{d\log(f)}\right)}{d\log(f)}-\frac{2\left(-\frac{\text{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 2720$$

$$\frac{2\left(\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,-f^{c+dx}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-f^{c+dx}\right)}{d\log(f)}\right)}{d\log(f)}-\frac{2\left(-\frac{\text{PolyLog}\left(2,-f^{c+dx}\right)}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

$$\Big\downarrow \ 7143$$

$$-\frac{2\left(-\frac{\text{PolyLog}(2,-f^{c+dx})}{d^2\log^2(f)}-\frac{x\log(f^{c+dx}+1)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}+\frac{2\left(\frac{\text{PolyLog}(3,-f^{c+dx})}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}(2,-f^{c+dx})}{d\log(f)}\right)}{d\log(f)}-\frac{x^2\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}+\frac{x^3}{3}$$

input
```
Int[x^2/(1 + 2*f^(c + d*x) + f^(2*c + 2*d*x)),x]
```

output
```
x^3/3 + x^2/(d*(1 + f^(c + d*x))*Log[f]) - (x^2*Log[1 + f^(c + d*x)])/(d*L
og[f]) - (2*(x^2/2 - (x*Log[1 + f^(c + d*x)])/(d*Log[f]) - PolyLog[2, -f^(
c + d*x)]/(d^2*Log[f]^2)))/(d*Log[f]) + (2*(-((x*PolyLog[2, -f^(c + d*x)])
/(d*Log[f])) + PolyLog[3, -f^(c + d*x)]/(d^2*Log[f]^2)))/(d*Log[f])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.60

method	result
risch	$\frac{x^2}{d(1+f^{dx+c})\ln(f)} - \frac{2c^3}{3d^3} - \frac{c^2}{d^3\ln(f)} - \frac{2cx}{d^2\ln(f)} + \frac{2\ln(1+f^{dx}f^c)x}{d^2\ln(f)^2} - \frac{c^2x}{d^2} - \frac{\ln(1+f^{dx}f^c)x^2}{d\ln(f)} - \frac{2\,\mathrm{polylog}(2,-f^{dx}f^c)x}{d^2\ln(f)^2}$

input
```
int(x^2/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x,method=_RETURNVERBOSE)
```

output
```
x^2/d/(1+f^(d*x+c))/ln(f)-2/3/d^3*c^3-1/d^3/ln(f)*c^2-2/d^2/ln(f)*c*x+2/d^
2/ln(f)^2*ln(1+f^(d*x)*f^c)*x-1/d^2*c^2*x-1/d/ln(f)*ln(1+f^(d*x)*f^c)*x^2-
2/d^2/ln(f)^2*polylog(2,-f^(d*x)*f^c)*x-x^2/d/ln(f)+2/d^3/ln(f)^3*polylog(
2,-f^(d*x)*f^c)+2/d^3/ln(f)^3*polylog(3,-f^(d*x)*f^c)+1/d^3/ln(f)*c^2*ln(f
^(d*x)*f^c)+2/d^3/ln(f)^2*c*ln(f^(d*x)*f^c)+1/3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.45

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{3\,c^2\log(f)^2 + (d^3x^3+c^3)\log(f)^3 + \left((d^3x^3+c^3)\log(f)^3 - 3\,(d^2x^2-c^2)\log(f)^2\right)f^{dx+c} - 6\,\left(dx\log(f)\right.}{}$$

input
```
integrate(x^2/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="fricas")
```

output
```
1/3*(3*c^2*log(f)^2 + (d^3*x^3 + c^3)*log(f)^3 + ((d^3*x^3 + c^3)*log(f)^3
 - 3*(d^2*x^2 - c^2)*log(f)^2)*f^(d*x + c) - 6*(d*x*log(f) + (d*x*log(f) -
 1)*f^(d*x + c) - 1)*dilog(-f^(d*x + c)) - 3*(d^2*x^2*log(f)^2 - 2*d*x*log
(f) + (d^2*x^2*log(f)^2 - 2*d*x*log(f))*f^(d*x + c))*log(f^(d*x + c) + 1)
+ 6*(f^(d*x + c) + 1)*polylog(3, -f^(d*x + c)))/(d^3*f^(d*x + c)*log(f)^3
+ d^3*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{x^2}{de^{\frac{(2c+2dx)\log(f)}{2}}\log(f)+d\log(f)} + \frac{\int\left(-\frac{2x}{e^{c\log(f)}e^{dx\log(f)}+1}\right)dx + \int\frac{dx^2\log(f)}{e^{c\log(f)}e^{dx\log(f)}+1}\,dx}{d\log(f)}$$

input
```
integrate(x**2/(1+2*f**(d*x+c)+f**(2*d*x+2*c)),x)
```

output
```
x**2/(d*exp((2*c + 2*d*x)*log(f)/2)*log(f) + d*log(f)) + (Integral(-2*x/(e
xp(c*log(f))*exp(d*x*log(f)) + 1), x) + Integral(d*x**2*log(f)/(exp(c*log(
f))*exp(d*x*log(f)) + 1), x))/(d*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \frac{x^2}{df^{dx}f^c\log(f)+d\log(f)} + \frac{d^3x^3\log(f)^3-3\,d^2x^2\log(f)^2}{3\,d^3\log(f)^3} - \frac{d^2x^2\log\left(f^{dx}f^c+1\right)\log(f)^2+2\,dx\mathrm{Li}_2\left(-f^{dx}f^c\right)\log(f)-2\,\mathrm{Li}_3(-f^{dx}f^c)}{d^3\log(f)^3} + \frac{2\left(dx\log\left(f^{dx}f^c+1\right)\log(f)+\mathrm{Li}_2\left(-f^{dx}f^c\right)\right)}{d^3\log(f)^3}$$

input
```
integrate(x^2/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="maxima")
```

output
```
x^2/(d*f^(d*x)*f^c*log(f) + d*log(f)) + 1/3*(d^3*x^3*log(f)^3 - 3*d^2*x^2*
log(f)^2)/(d^3*log(f)^3) - (d^2*x^2*log(f^(d*x)*f^c + 1)*log(f)^2 + 2*d*x*
dilog(-f^(d*x)*f^c)*log(f) - 2*polylog(3, -f^(d*x)*f^c))/(d^3*log(f)^3) +
2*(d*x*log(f^(d*x)*f^c + 1)*log(f) + dilog(-f^(d*x)*f^c))/(d^3*log(f)^3)
```

Giac [F]

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \int \frac{x^2}{f^{2\,dx+2\,c}+2\,f^{dx+c}+1}\,dx$$

input
```
integrate(x^2/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x, algorithm="giac")
```

output
```
integrate(x^2/(f^(2*d*x + 2*c) + 2*f^(d*x + c) + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \int \frac{x^2}{f^{2\,c+2\,d\,x}+2\,f^{c+d\,x}+1}\,dx$$

input
```
int(x^2/(f^(2*c + 2*d*x) + 2*f^(c + d*x) + 1),x)
```

output
```
int(x^2/(f^(2*c + 2*d*x) + 2*f^(c + d*x) + 1), x)
```

Reduce [F]

$$\int \frac{x^2}{1+2f^{c+dx}+f^{2c+2dx}}\,dx = \int \frac{x^2}{f^{2dx+2c}+2f^{dx+c}+1}dx$$

input
```
int(x^2/(1+2*f^(d*x+c)+f^(2*d*x+2*c)),x)
```

output
```
int(x**2/(f**(2*c + 2*d*x) + 2*f**(c + d*x) + 1),x)
```

3.449 $\int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 484

$$\begin{aligned}\int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx &= -\frac{2cx^3}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3}{3\left(b^2-4ac+b\sqrt{b^2-4ac}\right)}\\
&\quad-\frac{2cx^2\log\left(1+\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d\log(f)}\\
&\quad+\frac{2cx^2\log\left(1+\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d\log(f)}\\
&\quad-\frac{4cx\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^2\log^2(f)}\\
&\quad+\frac{4cx\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^2\log^2(f)}\\
&\quad+\frac{4c\,\text{PolyLog}\left(3,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)d^3\log^3(f)}\\
&\quad-\frac{4c\,\text{PolyLog}\left(3,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right)d^3\log^3(f)}\end{aligned}$$

output
```
-2*c*x^3/(3*b^2-12*a*c-3*b*(-4*a*c+b^2)^(1/2))-2*c*x^3/(3*b^2-12*a*c+3*b*(
-4*a*c+b^2)^(1/2))-2*c*x^2*ln(1+2*c*f^(d*x+c)/(b-(-4*a*c+b^2)^(1/2)))/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))/d/ln(f)+2*c*x^2*ln(1+2*c*f^(d*x+c)/(
b+(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d/ln(f)-4
*c*x*polylog(2,-2*c*f^(d*x+c)/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(
b-(-4*a*c+b^2)^(1/2))/d^2/ln(f)^2+4*c*x*polylog(2,-2*c*f^(d*x+c)/(b+(-4*a*
c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d^2/ln(f)^2+4*c*p
olylog(3,-2*c*f^(d*x+c)/(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)/(b-(-4*
a*c+b^2)^(1/2))/d^3/ln(f)^3-4*c*polylog(3,-2*c*f^(d*x+c)/(b+(-4*a*c+b^2)^(
1/2)))/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))/d^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = \frac{2c\left(\frac{x^2\log\left(1+\frac{\left(b-\sqrt{b^2-4ac}\right)f^{-c-dx}}{2c}\right)}{-b+\sqrt{b^2-4ac}}+\frac{x^2\log\left(1+\frac{\left(b+\sqrt{b^2-4ac}\right)f^{-c-dx}}{2c}\right)}{b+\sqrt{b^2-4ac}}-\frac{2\left(dx\log(f)\operatorname{PolyLog}\left(2,\frac{\left(-b+\sqrt{b^2-4ac}\right)f^{-c-dx}}{2c}\right)+\operatorname{PolyLo}\right.}{\left(-b+\sqrt{b^2-4ac}\right)d^2\log^2(f}\right.}{\sqrt{b^2-4ac}d\log}$$

input
```
Integrate[x^2/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x]
```

output
```
(2*c*((x^2*Log[1 + ((b - Sqrt[b^2 - 4*a*c])*f^(-c - d*x))/(2*c)])/(-b + Sq
rt[b^2 - 4*a*c]) + (x^2*Log[1 + ((b + Sqrt[b^2 - 4*a*c])*f^(-c - d*x))/(2*
c)])/(b + Sqrt[b^2 - 4*a*c]) - (2*(d*x*Log[f]*PolyLog[2, ((-b + Sqrt[b^2 -
 4*a*c])*f^(-c - d*x))/(2*c)] + PolyLog[3, ((-b + Sqrt[b^2 - 4*a*c])*f^(-c
 - d*x))/(2*c)]))/((-b + Sqrt[b^2 - 4*a*c])*d^2*Log[f]^2) - (2*(d*x*Log[f]
*PolyLog[2, -1/2*((b + Sqrt[b^2 - 4*a*c])*f^(-c - d*x))/c] + PolyLog[3, -1
/2*((b + Sqrt[b^2 - 4*a*c])*f^(-c - d*x))/c]))/((b + Sqrt[b^2 - 4*a*c])*d^
2*Log[f]^2)))/(Sqrt[b^2 - 4*a*c]*d*Log[f])
```

Rubi [A] (verified)

Time = 2.30 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {2693, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx$$

$$\Big\downarrow \text{ 2693}$$

$$\frac{2c\int \frac{x^2}{2cf^{c+dx}+b-\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}-\frac{2c\int \frac{x^2}{2cf^{c+dx}+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int \frac{f^{c+dx}x^2}{2cf^{c+dx}+b-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int \frac{f^{c+dx}x^2}{2cf^{c+dx}+b+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}-\frac{\int x\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)dx}{cd\log(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}-\frac{\int x\log\left(\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}+1\right)dx}{cd\log(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}-\frac{\frac{\int\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)dx}{d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d\log(f)}}{cd\log(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}-\frac{\frac{\int\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)dx}{d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$\big\downarrow$ 2720

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}-\frac{\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d\log(f)}}{cd\log(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}-\frac{\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

$\big\downarrow$ 7143

$$\frac{2c\left(\frac{x^3}{3\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}+1\right)}{2cd\log(f)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b-\sqrt{b^2-4ac}}\right)}{d\log(f)}}{cd\log(f)}\right)}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}-\frac{2c\left(\frac{x^3}{3\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{b^2-4ac}+b}+1\right)}{2cd\log(f)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{b+\sqrt{b^2-4ac}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{b^2-4ac}+b}\right)}{\sqrt{b^2-4ac}}$$

input
```
Int[x^2/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x]
```

output
```
(2*c*(x^3/(3*(b - Sqrt[b^2 - 4*a*c])) - (2*c*((x^2*Log[1 + (2*c*f^(c + d*x
))/(b - Sqrt[b^2 - 4*a*c])])/(2*c*d*Log[f]) - (-((x*PolyLog[2, (-2*c*f^(c
+ d*x))/(b - Sqrt[b^2 - 4*a*c])])/(d*Log[f])) + PolyLog[3, (-2*c*f^(c + d*
x))/(b - Sqrt[b^2 - 4*a*c])]/(d^2*Log[f]^2))/(c*d*Log[f])))/(b - Sqrt[b^2
- 4*a*c])))/Sqrt[b^2 - 4*a*c] - (2*c*(x^3/(3*(b + Sqrt[b^2 - 4*a*c])) - (2
*c*((x^2*Log[1 + (2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c])])/(2*c*d*Log[f]
) - (-((x*PolyLog[2, (-2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c])])/(d*Log[f
])) + PolyLog[3, (-2*c*f^(c + d*x))/(b + Sqrt[b^2 - 4*a*c])]/(d^2*Log[f]^2
))/(c*d*Log[f])))/(b + Sqrt[b^2 - 4*a*c])))/Sqrt[b^2 - 4*a*c]
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2693
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)),
 x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[(f + g*x)^m
/(b - q + 2*c*F^u), x], x] - Simp[2*(c/q)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{a + b\,f^{dx+c} + c\,f^{2dx+2c}}dx$$

input
```
int(x^2/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x)
```

output
```
int(x^2/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.43

$$\int \frac{x^2}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="fricas")
```

output
```
1/6*(2*(b^2 - 4*a*c)*d^3*x^3*log(f)^3 - 6*(a*b*d*x*sqrt((b^2 - 4*a*c)/a^2)
*log(f) + (b^2 - 4*a*c)*d*x*log(f))*dilog(-1/2*((a*sqrt((b^2 - 4*a*c)/a^2)
 + b)*f^(d*x + c) + 2*a)/a + 1) + 6*(a*b*d*x*sqrt((b^2 - 4*a*c)/a^2)*log(f
) - (b^2 - 4*a*c)*d*x*log(f))*dilog(1/2*((a*sqrt((b^2 - 4*a*c)/a^2) - b)*f
^(d*x + c) - 2*a)/a + 1) + 3*(a*b*c^2*sqrt((b^2 - 4*a*c)/a^2)*log(f)^2 - (
b^2*c^2 - 4*a*c^3)*log(f)^2)*log(2*c*f^(d*x + c) + a*sqrt((b^2 - 4*a*c)/a^
2) + b) - 3*(a*b*c^2*sqrt((b^2 - 4*a*c)/a^2)*log(f)^2 + (b^2*c^2 - 4*a*c^3
)*log(f)^2)*log(2*c*f^(d*x + c) - a*sqrt((b^2 - 4*a*c)/a^2) + b) - 3*((a*b
*d^2*x^2 - a*b*c^2)*sqrt((b^2 - 4*a*c)/a^2)*log(f)^2 + ((b^2 - 4*a*c)*d^2*
x^2 - b^2*c^2 + 4*a*c^3)*log(f)^2)*log(1/2*((a*sqrt((b^2 - 4*a*c)/a^2) + b
)*f^(d*x + c) + 2*a)/a) + 3*((a*b*d^2*x^2 - a*b*c^2)*sqrt((b^2 - 4*a*c)/a^
2)*log(f)^2 - ((b^2 - 4*a*c)*d^2*x^2 - b^2*c^2 + 4*a*c^3)*log(f)^2)*log(-1
/2*((a*sqrt((b^2 - 4*a*c)/a^2) - b)*f^(d*x + c) - 2*a)/a) + 6*(a*b*sqrt((b
^2 - 4*a*c)/a^2) + b^2 - 4*a*c)*polylog(3, -1/2*(a*sqrt((b^2 - 4*a*c)/a^2)
 + b)*f^(d*x + c)/a) - 6*(a*b*sqrt((b^2 - 4*a*c)/a^2) - b^2 + 4*a*c)*polyl
og(3, 1/2*(a*sqrt((b^2 - 4*a*c)/a^2) - b)*f^(d*x + c)/a))/((a*b^2 - 4*a^2*
c)*d^3*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \int \frac{x^2}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx$$

input
```
integrate(x**2/(a+b*f**(d*x+c)+c*f**(2*d*x+2*c)),x)
```

output
```
Integral(x**2/(a + b*f**(c + d*x) + c*f**(2*c + 2*d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^2}{a + bf^{c+dx} + cf^{2c+2dx}}\,dx = \int \frac{x^2}{cf^{2\,dx+2\,c} + bf^{dx+c} + a}\,dx$$

input
```
integrate(x^2/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x, algorithm="giac")
```

output
```
integrate(x^2/(c*f^(2*d*x + 2*c) + b*f^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx=\int \frac{x^2}{a+b\,f^{c+d\,x}+c\,f^{2\,c+2\,d\,x}}\,dx$$

input
```
int(x^2/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)),x)
```

output
```
int(x^2/(a + b*f^(c + d*x) + c*f^(2*c + 2*d*x)), x)
```

Reduce [F]

$$\int \frac{x^2}{a+bf^{c+dx}+cf^{2c+2dx}}\,dx = \frac{3f^{2c}\left(\int \frac{f^{2dx}x^2}{f^{2dx+2c}c+f^{dx+c}b+a}dx\right)c+3f^{c}\left(\int \frac{f^{dx}x^2}{f^{2dx+2c}c+f^{dx+c}b+a}dx\right)b+6\left(\int \frac{x^2}{f^{2dx+2c}c+f^{dx+c}b+a}dx\right)a-x^3}{3a}$$

input
```
int(x^2/(a+b*f^(d*x+c)+c*f^(2*d*x+2*c)),x)
```

output
```
(3*f**(2*c)*int((f**(2*d*x)*x**2)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*b + a
),x)*c + 3*f**c*int((f**(d*x)*x**2)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*b +
 a),x)*b + 6*int(x**2/(f**(2*c + 2*d*x)*c + f**(c + d*x)*b + a),x)*a - x**
3)/(3*a)
```

3.450 $\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 103

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx=\frac{dx}{a}+\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}h\log(f)}-\frac{d\log\left(a+bf^{g+hx}+cf^{2g+2hx}\right)}{2ah\log(f)}$$

output
```
d*x/a+(-2*a*e+b*d)*arctanh((b+2*c*f^(h*x+g))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c
+b^2)^(1/2)/h/ln(f)-1/2*d*ln(a+b*f^(h*x+g)+c*f^(2*h*x+2*g))/a/h/ln(f)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx=\frac{(-2bd+4ae)\arctan\left(\frac{b+2cf^{g+hx}}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}d\left(2\log\left(f^{g+hx}\right)-\log\left(a+f^{g+hx}\left(b+cf^{g+hx}\right)\right)\right)}{2a\sqrt{-b^2+4ac}h\log(f)}$$

input
```
Integrate[(d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)),x]
```

output
```
((-2*b*d + 4*a*e)*ArcTan[(b + 2*c*f^(g + h*x))/Sqrt[-b^2 + 4*a*c]] + Sqrt[
-b^2 + 4*a*c]*d*(2*Log[f^(g + h*x)] - Log[a + f^(g + h*x)*(b + c*f^(g + h*
x))]))/(2*a*Sqrt[-b^2 + 4*a*c]*h*Log[f])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2720, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{f^{-g-hx}(ef^{g+hx}+d)}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}}{h\log(f)}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{\int \left(\frac{df^{-g-hx}}{a}+\frac{-cdf^{g+hx}-bd+ae}{a\left(bf^{g+hx}+cf^{2g+2hx}+a\right)}\right)df^{g+hx}}{h\log(f)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bf^{g+hx}+cf^{2g+2hx}\right)}{2a}+\frac{d\log\left(f^{g+hx}\right)}{a}}{h\log(f)}$$

input
```
Int[(d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*f^(g + h*x))/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[
b^2 - 4*a*c]) + (d*Log[f^(g + h*x)])/a - (d*Log[a + b*f^(g + h*x) + c*f^(2
*g + 2*h*x)])/(2*a))/(h*Log[f])
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(97) = 194$.

Time = 0.18 (sec) , antiderivative size = 993, normalized size of antiderivative = 9.64

method	result
risch	$\frac{4\ln(f)^2 acd h^2 x}{4\ln(f)^2 a^2 c h^2-\ln(f)^2 a b^2 h^2}-\frac{\ln(f)^2 b^2 d h^2 x}{4\ln(f)^2 a^2 c h^2-\ln(f)^2 a b^2 h^2}+\frac{4\ln(f)^2 acdgh}{4\ln(f)^2 a^2 c h^2-\ln(f)^2 a b^2 h^2}-\frac{\ln(f)^2 b^2 dgh}{4\ln(f)^2 a^2 c h^2-\ln(f)^2 a b^2 h^2}$

input
```
int((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x,method=_RETURNVERBOS
E)
```

output
```
4/(4*ln(f)^2*a^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln(f)^2*a*c*d*h^2*x-1/(4*ln(f)^2
*a^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln(f)^2*b^2*d*h^2*x+4/(4*ln(f)^2*a^2*c*h^2-l
n(f)^2*a*b^2*h^2)*ln(f)^2*a*c*d*g*h-1/(4*ln(f)^2*a^2*c*h^2-ln(f)^2*a*b^2*h
^2)*ln(f)^2*b^2*d*g*h-2/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)+1/2*(2*a*b*e-b^2*
d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^
4*d^2)^(1/2))/c/(2*a*e-b*d))*c*d+1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)+1/
2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4
*a*b^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-b*d))*b^2*d+1/2/a/(4*a*c-b^2)/h/ln(f
)*ln(f^(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*
d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-b*d))*(-16*a^3*c*e^
2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2)-2/
(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*
b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-
b*d))*c*d+1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-2*a*b*e+b^2*d+(-16*
a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^
(1/2))/c/(2*a*e-b*d))*b^2*d-1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-2
*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b
^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-b*d))*(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^
2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.20

$$\int \frac{d + ef^{g+hx}}{a + bf^{g+hx} + cf^{2g+2hx}}\, dx$$
$$= \left[\frac{2\,(b^2 - 4\,ac)dhx\log{(f)} - (b^2 - 4\,ac)d\log{\left(cf^{2\,hx+2\,g} + bf^{hx+g} + a\right)} - \sqrt{b^2 - 4\,ac}(bd - 2\,ae)\log{\left(\frac{2\,c^2 f^2}{\ldots}\right.}}{2\,(ab^2 - 4\,a^2c)h\log{(f)}} \right.$$

input
```
integrate((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="fr
icas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*d*h*x*log(f) - (b^2 - 4*a*c)*d*log(c*f^(2*h*x + 2*g)
 + b*f^(h*x + g) + a) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*f^(2*h*
x + 2*g) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*f^(h*x + g) - sqrt(
b^2 - 4*a*c)*b)/(c*f^(2*h*x + 2*g) + b*f^(h*x + g) + a)))/((a*b^2 - 4*a^2*
c)*h*log(f)), 1/2*(2*(b^2 - 4*a*c)*d*h*x*log(f) - (b^2 - 4*a*c)*d*log(c*f^
(2*h*x + 2*g) + b*f^(h*x + g) + a) + 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*f^(h*x + g) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4
*a*c)))/((a*b^2 - 4*a^2*c)*h*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx$$
$$= \text{RootSum}\Big(z^2 \cdot \left(4a^2ch^2\log{(f)}^2 - ab^2h^2\log{(f)}^2\right) + z\left(4acdh\log{(f)} - b^2dh\log{(f)}\right) + ae^2 - bde + cd^2, \Big($$
$$+ \frac{dx}{a}$$

input
```
integrate((d+e*f**(h*x+g))/(a+b*f**(h*x+g)+c*f**(2*h*x+2*g)),x)
```

output
```
RootSum(_z**2*(4*a**2*c*h**2*log(f)**2 - a*b**2*h**2*log(f)**2) + _z*(4*a*
c*d*h*log(f) - b**2*d*h*log(f)) + a*e**2 - b*d*e + c*d**2, Lambda(_i, _i*l
og(f**(g + h*x) + (4*_i*a**2*c*h*log(f) - _i*a*b**2*h*log(f) + a*b*e + 2*a
*c*d - b**2*d)/(2*a*c*e - b*c*d)))) + d*x/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx = -\frac{\frac{d\log\left(cf^{2\,hx}f^{2\,g}+bf^{hx}f^{g}+a\right)}{a\log(f)}-\frac{2\,d\log\left(|f|^{hx}|f|^{g}\right)}{a\log(f)}+\frac{2\,(bd-2\,ae)\arctan\left(\frac{2\,cf^{hx}f^{g}+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a\log(f)}}{2\,h}$$

input
```
integrate((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="gi
ac")
```

output
```
-1/2*(d*log(c*f^(2*h*x)*f^(2*g) + b*f^(h*x)*f^g + a)/(a*log(f)) - 2*d*log(
abs(f)^(h*x)*abs(f)^g)/(a*log(f)) + 2*(b*d - 2*a*e)*arctan((2*c*f^(h*x)*f^
g + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a*log(f)))/h
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx = \frac{d\,x}{a}-\frac{d\,\ln\left(a+c\,f^{2\,h\,x}\,f^{2\,g}+b\,f^{h\,x}\,f^{g}\right)}{2\,a\,h\,\ln\left(f\right)}+\frac{\mathrm{atan}\left(\frac{b+2\,c\,f^{h\,x}\,f^{g}}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a\,e-b\,d\right)}{a\,h\,\ln\left(f\right)\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)),x)
```

output
```
(d*x)/a - (d*log(a + c*f^(2*h*x)*f^(2*g) + b*f^(h*x)*f^g))/(2*a*h*log(f))
+ (atan((b + 2*c*f^(h*x)*f^g)/(4*a*c - b^2)^(1/2))*(2*a*e - b*d))/(a*h*log
(f)*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2g+2hx}}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2f^{hx+g}c+b}{\sqrt{4ac-b^2}}\right)ae-2\sqrt{4ac-b^2}\,atan\left(\frac{2f^{hx+g}c+b}{\sqrt{4ac-b^2}}\right)bd-4\log\left(f^{2hx+2g}c+f^{hx+g}b+a\right)acd+}{2\log(f)\,ah\,(4ac-b^2)}$$

input
```
int((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((2*f**(g + h*x)*c + b)/sqrt(4*a*c - b**2))*a*e
- 2*sqrt(4*a*c - b**2)*atan((2*f**(g + h*x)*c + b)/sqrt(4*a*c - b**2))*b*d
 - 4*log(f**(2*g + 2*h*x)*c + f**(g + h*x)*b + a)*a*c*d + log(f**(2*g + 2*
h*x)*c + f**(g + h*x)*b + a)*b**2*d + 8*log(f)*a*c*d*h*x - 2*log(f)*b**2*d
*h*x)/(2*log(f)*a*h*(4*a*c - b**2))
```

3.451 $\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 103

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx=\frac{dx}{a}+\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}h\log(f)}-\frac{d\log\left(a+bf^{g+hx}+cf^{2g+2hx}\right)}{2ah\log(f)}$$

output
```
d*x/a+(-2*a*e+b*d)*arctanh((b+2*c*f^(h*x+g))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c
+b^2)^(1/2)/h/ln(f)-1/2*d*ln(a+b*f^(h*x+g)+c*f^(2*h*x+2*g))/a/h/ln(f)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.15

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx=\frac{(-2bd+4ae)\arctan\left(\frac{b+2cf^{g+hx}}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}d\left(2\log\left(f^{g+hx}\right)-\log\left(a+f^{g+hx}\left(b+cf^{g+hx}\right)\right)\right)}{2a\sqrt{-b^2+4ac}h\log(f)}$$

input
```
Integrate[(d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*(g + h*x))),x]
```

output
```
((-2*b*d + 4*a*e)*ArcTan[(b + 2*c*f^(g + h*x))/Sqrt[-b^2 + 4*a*c]] + Sqrt[
-b^2 + 4*a*c]*d*(2*Log[f^(g + h*x)] - Log[a + f^(g + h*x)*(b + c*f^(g + h*
x))]))/(2*a*Sqrt[-b^2 + 4*a*c]*h*Log[f])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2720, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{f^{-g-hx}(ef^{g+hx}+d)}{bf^{g+hx}+cf^{2g+2hx}+a}df^{g+hx}}{h\log(f)}$$
$$\Big\downarrow \text{1200}$$
$$\frac{\int \left(\frac{df^{-g-hx}}{a}+\frac{-cdf^{g+hx}-bd+ae}{a(bf^{g+hx}+cf^{2g+2hx}+a)}\right)df^{g+hx}}{h\log(f)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2cf^{g+hx}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bf^{g+hx}+cf^{2g+2hx}\right)}{2a}+\frac{d\log\left(f^{g+hx}\right)}{a}}{h\log(f)}$$

input
```
Int[(d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*(g + h*x))),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*f^(g + h*x))/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[
b^2 - 4*a*c]) + (d*Log[f^(g + h*x)])/a - (d*Log[a + b*f^(g + h*x) + c*f^(2
*g + 2*h*x)])/(2*a))/(h*Log[f])
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(97) = 194$.

Time = 0.00 (sec) , antiderivative size = 993, normalized size of antiderivative = 9.64

method	result
risch	$\frac{4\ln(f)^2 acd\,h^2x}{4\ln(f)^2a^2c\,h^2-\ln(f)^2a\,b^2h^2}-\frac{\ln(f)^2b^2d\,h^2x}{4\ln(f)^2a^2c\,h^2-\ln(f)^2a\,b^2h^2}+\frac{4\ln(f)^2acdgh}{4\ln(f)^2a^2c\,h^2-\ln(f)^2a\,b^2h^2}-\frac{\ln(f)^2b^2dgh}{4\ln(f)^2a^2c\,h^2-\ln(f)^2a\,b^2h^2}$

input
```
int((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x,method=_RETURNVERBOS
E)
```

output
```
4/(4*ln(f)^2*a^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln(f)^2*a*c*d*h^2*x-1/(4*ln(f)^2
*a^2*c*h^2-ln(f)^2*a*b^2*h^2)*ln(f)^2*b^2*d*h^2*x+4/(4*ln(f)^2*a^2*c*h^2-l
n(f)^2*a*b^2*h^2)*ln(f)^2*a*c*d*g*h-1/(4*ln(f)^2*a^2*c*h^2-ln(f)^2*a*b^2*h
^2)*ln(f)^2*b^2*d*g*h-2/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)+1/2*(2*a*b*e-b^2*
d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^
4*d^2)^(1/2))/c/(2*a*e-b*d))*c*d+1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)+1/
2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4
*a*b^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-b*d))*b^2*d+1/2/a/(4*a*c-b^2)/h/ln(f
)*ln(f^(h*x+g)+1/2*(2*a*b*e-b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*
d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-b*d))*(-16*a^3*c*e^
2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2)-2/
(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-2*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*
b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-
b*d))*c*d+1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-2*a*b*e+b^2*d+(-16*
a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^
(1/2))/c/(2*a*e-b*d))*b^2*d-1/2/a/(4*a*c-b^2)/h/ln(f)*ln(f^(h*x+g)-1/2*(-2
*a*b*e+b^2*d+(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^2*b*c*d*e-4*a*b^3*d*e-4*a*b
^2*c*d^2+b^4*d^2)^(1/2))/c/(2*a*e-b*d))*(-16*a^3*c*e^2+4*a^2*b^2*e^2+16*a^
2*b*c*d*e-4*a*b^3*d*e-4*a*b^2*c*d^2+b^4*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.20

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$
$$= \left[\frac{2\,(b^2-4\,ac)dhx\log{(f)}-(b^2-4\,ac)d\log{\left(cf^{2\,hx+2\,g}+bf^{hx+g}+a\right)}-\sqrt{b^2-4\,ac}(bd-2\,ae)\log{\left(\frac{2\,c^2f^2}{}\right.}}{2\,(ab^2-4\,a^2c)h\log{(f)}}\right.$$

input
```
integrate((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="fr
icas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*d*h*x*log(f) - (b^2 - 4*a*c)*d*log(c*f^(2*h*x + 2*g)
 + b*f^(h*x + g) + a) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*f^(2*h*
x + 2*g) + b^2 - 2*a*c + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*f^(h*x + g) - sqrt(
b^2 - 4*a*c)*b)/(c*f^(2*h*x + 2*g) + b*f^(h*x + g) + a)))/((a*b^2 - 4*a^2*
c)*h*log(f)), 1/2*(2*(b^2 - 4*a*c)*d*h*x*log(f) - (b^2 - 4*a*c)*d*log(c*f^
(2*h*x + 2*g) + b*f^(h*x + g) + a) + 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*ar
ctan(-(2*sqrt(-b^2 + 4*a*c)*c*f^(h*x + g) + sqrt(-b^2 + 4*a*c)*b)/(b^2 - 4
*a*c)))/((a*b^2 - 4*a^2*c)*h*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.35

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$
$$= \operatorname{RootSum}\left(z^2\cdot\left(4a^2ch^2\log{(f)}^2-ab^2h^2\log{(f)}^2\right)+z\left(4acdh\log{(f)}-b^2dh\log{(f)}\right)+ae^2-bde+cd^2,\ (\right.$$
$$+\frac{dx}{a}$$

input
```
integrate((d+e*f**(h*x+g))/(a+b*f**(h*x+g)+c*f**(2*h*x+2*g)),x)
```

output
```
RootSum(_z**2*(4*a**2*c*h**2*log(f)**2 - a*b**2*h**2*log(f)**2) + _z*(4*a*
c*d*h*log(f) - b**2*d*h*log(f)) + a*e**2 - b*d*e + c*d**2, Lambda(_i, _i*l
og(f**(g + h*x) + (4*_i*a**2*c*h*log(f) - _i*a*b**2*h*log(f) + a*b*e + 2*a
*c*d - b**2*d)/(2*a*c*e - b*c*d))) + d*x/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="ma
xima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.16

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx = -\frac{\frac{d\log\left(cf^{2\,hx}f^{2\,g}+bf^{hx}f^{g}+a\right)}{a\log(f)}-\frac{2\,d\log\left(|f|^{hx}|f|^{g}\right)}{a\log(f)}+\frac{2\,(bd-2\,ae)\arctan\left(\frac{2\,cf^{hx}f^{g}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\sqrt{-b^{2}+4\,ac}a\log(f)}}{2\,h}$$

input
```
integrate((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x, algorithm="gi
ac")
```

output
```
-1/2*(d*log(c*f^(2*h*x)*f^(2*g) + b*f^(h*x)*f^g + a)/(a*log(f)) - 2*d*log(
abs(f)^(h*x)*abs(f)^g)/(a*log(f)) + 2*(b*d - 2*a*e)*arctan((2*c*f^(h*x)*f^
g + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a*log(f)))/h
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx = \frac{d\,x}{a}-\frac{d\,\ln\left(a+c\,f^{2\,h\,x}\,f^{2\,g}+b\,f^{h\,x}\,f^{g}\right)}{2\,a\,h\,\ln\left(f\right)}+\frac{\mathrm{atan}\left(\frac{b+2\,c\,f^{h\,x}\,f^{g}}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a\,e-b\,d\right)}{a\,h\,\ln\left(f\right)\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e*f^(g + h*x))/(a + b*f^(g + h*x) + c*f^(2*g + 2*h*x)),x)
```

output
```
(d*x)/a - (d*log(a + c*f^(2*h*x)*f^(2*g) + b*f^(h*x)*f^g))/(2*a*h*log(f))
+ (atan((b + 2*c*f^(h*x)*f^g)/(4*a*c - b^2)^(1/2))*(2*a*e - b*d))/(a*h*log
(f)*(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.81

$$\int \frac{d+ef^{g+hx}}{a+bf^{g+hx}+cf^{2(g+hx)}}\,dx$$
$$= \frac{4\sqrt{4ac-b^2}\,atan\left(\frac{2f^{hx+g}c+b}{\sqrt{4ac-b^2}}\right)ae-2\sqrt{4ac-b^2}\,atan\left(\frac{2f^{hx+g}c+b}{\sqrt{4ac-b^2}}\right)bd-4\log\left(f^{2hx+2g}c+f^{hx+g}b+a\right)acd+}{2\log\left(f\right)ah\left(4ac-b^2\right)}$$

input
```
int((d+e*f^(h*x+g))/(a+b*f^(h*x+g)+c*f^(2*h*x+2*g)),x)
```

output
```
(4*sqrt(4*a*c - b**2)*atan((2*f**(g + h*x)*c + b)/sqrt(4*a*c - b**2))*a*e
- 2*sqrt(4*a*c - b**2)*atan((2*f**(g + h*x)*c + b)/sqrt(4*a*c - b**2))*b*d
 - 4*log(f**(2*g + 2*h*x)*c + f**(g + h*x)*b + a)*a*c*d + log(f**(2*g + 2*
h*x)*c + f**(g + h*x)*b + a)*b**2*d + 8*log(f)*a*c*d*h*x - 2*log(f)*b**2*d
*h*x)/(2*log(f)*a*h*(4*a*c - b**2))
```

3.452 $\int \frac{1}{2+e^{-x}+e^x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{1}{2+e^{-x}+e^x} \, dx = -\frac{1}{1+e^x}$$

output
```
-1/(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+e^{-x}+e^x} \, dx = -\frac{1}{1+e^x}$$

input
```
Integrate[(2 + E^(-x) + E^x)^(-1),x]
```

output
```
-(1 + E^x)^(-1)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^{-x}+e^x+2} \, dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \frac{1}{(e^x+1)^2}de^x$$
$$\Big\downarrow \text{ 17}$$
$$-\frac{1}{e^x+1}$$

input
```
Int[(2 + E^(-x) + E^x)^(-1),x]
```

output
```
-(1 + E^x)^(-1)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{1}{1+e^{x}}$	9
norman	$-\frac{1}{1+e^{x}}$	9
risch	$-\frac{1}{1+e^{x}}$	9

input
```
int(1/(2+exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+e^{-x}+e^{x}}\,dx=-\frac{1}{e^{x}+1}$$

input
```
integrate(1/(2+exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
-1/(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{1}{2+e^{-x}+e^{x}}\,dx=-\frac{1}{e^{x}+1}$$

input
```
integrate(1/(2+exp(-x)+exp(x)),x)
```

output
```
-1/(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+e^{-x}+e^{x}}\,dx = \frac{1}{e^{(-x)}+1}$$

input
```
integrate(1/(2+exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
1/(e^(-x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+e^{-x}+e^{x}}\,dx = -\frac{1}{e^{x}+1}$$

input
```
integrate(1/(2+exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
-1/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{1}{2+e^{-x}+e^{x}}\,dx = -\frac{1}{\mathrm{e}^{x}+1}$$

input
```
int(1/(exp(-x) + exp(x) + 2),x)
```

output
```
-1/(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{1}{2+e^{-x}+e^{x}}\,dx = \frac{e^{x}}{e^{x}+1}$$

input
```
int(1/(2+exp(-x)+exp(x)),x)
```

output
```
e**x/(e**x + 1)
```

3.453 $\int \frac{x}{2+e^{-x}+e^x} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{x}{2+e^{-x}+e^x} \, dx = x-\frac{x}{1+e^x}-\log \left(1+e^x\right)$$

output
```
x-x/(1+exp(x))-ln(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{2+e^{-x}+e^x} \, dx = x-\frac{x}{1+e^x}-\log \left(1+e^x\right)$$

input
```
Integrate[x/(2 + E^(-x) + E^x),x]
```

output
```
x - x/(1 + E^x) - Log[1 + E^x]
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2697, 7239, 2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{e^{-x}+e^x+2}dx$$
$$\Big\downarrow \text{2697}$$
$$\int \frac{e^x x}{2e^x+e^{2x}+1}dx$$
$$\Big\downarrow \text{7239}$$
$$\int \frac{e^x x}{(e^x+1)^2}dx$$
$$\Big\downarrow \text{2621}$$
$$\int \frac{1}{1+e^x}dx-\frac{x}{e^x+1}$$
$$\Big\downarrow \text{2720}$$
$$\int \frac{e^{-x}}{1+e^x}de^x-\frac{x}{e^x+1}$$
$$\Big\downarrow \text{47}$$
$$\int e^{-x}de^x-\int \frac{1}{1+e^x}de^x-\frac{x}{e^x+1}$$
$$\Big\downarrow \text{14}$$
$$-\int \frac{1}{1+e^x}de^x-\frac{x}{e^x+1}+\log \left(e^x\right)$$
$$\Big\downarrow \text{16}$$
$$-\frac{x}{e^x+1}+\log \left(e^x\right)-\log \left(e^x+1\right)$$

input
```
Int[x/(2 + E^(-x) + E^x),x]
```

output
```
-(x/(1 + E^x)) + Log[E^x] - Log[1 + E^x]
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$-\ln(1+e^x)+\frac{x\,e^x}{1+e^x}$	19
norman	$-\ln(1+e^x)+\frac{x\,e^x}{1+e^x}$	19
risch	$x-\frac{x}{1+e^x}-\ln(1+e^x)$	19

input
```
int(x/(2+exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+exp(x))+x*exp(x)/(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x}{2+e^{-x}+e^x}\,dx=\frac{xe^x-(e^x+1)\log(e^x+1)}{e^x+1}$$

input
```
integrate(x/(2+exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
(x*e^x - (e^x + 1)*log(e^x + 1))/(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{x}{2+e^{-x}+e^x}\,dx = x - \frac{x}{e^x+1} - \log\left(e^x+1\right)$$

input
```
integrate(x/(2+exp(-x)+exp(x)),x)
```

output
```
x - x/(exp(x) + 1) - log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{2+e^{-x}+e^x}\,dx = \frac{xe^x}{e^x+1} - \log\left(e^x+1\right)$$

input
```
integrate(x/(2+exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
x*e^x/(e^x + 1) - log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{x}{2+e^{-x}+e^x}\,dx = \frac{xe^x - e^x\log\left(e^x+1\right) - \log\left(e^x+1\right)}{e^x+1}$$

input
```
integrate(x/(2+exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
(x*e^x - e^x*log(e^x + 1) - log(e^x + 1))/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x}{2+e^{-x}+e^x}\,dx=\frac{x\,\mathrm{e}^x}{\mathrm{e}^x+1}-\ln\left(\mathrm{e}^x+1\right)$$

input
```
int(x/(exp(-x) + exp(x) + 2),x)
```

output
```
(x*exp(x))/(exp(x) + 1) - log(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{x}{2+e^{-x}+e^x}\,dx=\frac{-e^x\log(e^x+1)+e^x x-\log(e^x+1)}{e^x+1}$$

input
```
int(x/(2+exp(-x)+exp(x)),x)
```

output
```
( - e**x*log(e**x + 1) + e**x*x - log(e**x + 1))/(e**x + 1)
```

3.454 $\int \frac{x^2}{2+e^{-x}+e^x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{x^2}{2+e^{-x}+e^x}\,dx = x^2-\frac{x^2}{1+e^x}-2x\log\left(1+e^x\right)-2\,\text{PolyLog}\left(2,-e^x\right)$$

output
```
x^2-x^2/(1+exp(x))-2*x*ln(1+exp(x))-2*polylog(2,-exp(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{2+e^{-x}+e^x}\,dx = x\left(\frac{e^x x}{1+e^x}-2\log\left(1+e^x\right)\right)-2\,\text{PolyLog}\left(2,-e^x\right)$$

input
```
Integrate[x^2/(2 + E^(-x) + E^x),x]
```

output
```
x*((E^x*x)/(1 + E^x) - 2*Log[1 + E^x]) - 2*PolyLog[2, -E^x]
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {2697, 7239, 2621, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{e^{-x}+e^x+2} \, dx$$

$$\Big\downarrow \text{2697}$$

$$\int \frac{e^x x^2}{2e^x+e^{2x}+1}dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{e^x x^2}{(e^x+1)^2}dx$$

$$\Big\downarrow \text{2621}$$

$$2\int \frac{x}{1+e^x}dx-\frac{x^2}{e^x+1}$$

$$\Big\downarrow \text{2615}$$

$$2\left(\frac{x^2}{2}-\int \frac{e^x x}{1+e^x}dx\right)-\frac{x^2}{e^x+1}$$

$$\Big\downarrow \text{2620}$$

$$2\left(\int \log(1+e^x)\,dx+\frac{x^2}{2}-x\log(e^x+1)\right)-\frac{x^2}{e^x+1}$$

$$\Big\downarrow \text{2715}$$

$$2\left(\int e^{-x}\log(1+e^x)\,de^x+\frac{x^2}{2}-x\log(e^x+1)\right)-\frac{x^2}{e^x+1}$$

$$\Big\downarrow \text{2838}$$

$$2\left(-\text{PolyLog}(2,-e^x)+\frac{x^2}{2}-x\log(e^x+1)\right)-\frac{x^2}{e^x+1}$$

input
```
Int[x^2/(2 + E^(-x) + E^x),x]
```

output
```
-(x^2/(1 + E^x)) + 2*(x^2/2 - x*Log[1 + E^x] - PolyLog[2, -E^x])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
risch	$x^2 - \frac{x^2}{1+e^x} - 2x\ln\left(1+e^x\right) - 2\,\mathrm{polylog}\left(2, -e^x\right)$	32

input
```
int(x^2/(2+exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x^2-x^2/(1+exp(x))-2*x*ln(1+exp(x))-2*polylog(2,-exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int \frac{x^2}{2+e^{-x}+e^x}\,dx = \frac{x^2e^x - 2\left(e^x+1\right)\mathrm{Li}_2(-e^x) - 2\left(xe^x+x\right)\log\left(e^x+1\right)}{e^x+1}$$

input
```
integrate(x^2/(2+exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
(x^2*e^x - 2*(e^x + 1)*dilog(-e^x) - 2*(x*e^x + x)*log(e^x + 1))/(e^x + 1)
```

Sympy [F]

$$\int \frac{x^2}{2 + e^{-x} + e^x} \, dx = -\frac{x^2}{e^x + 1} + 2 \int \frac{x}{e^x + 1} \, dx$$

input
```
integrate(x**2/(2+exp(-x)+exp(x)),x)
```

output
```
-x**2/(exp(x) + 1) + 2*Integral(x/(exp(x) + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{2 + e^{-x} + e^x} \, dx = x^2 - 2\,x \log\left(e^x + 1\right) - \frac{x^2}{e^x + 1} - 2\,\mathrm{Li}_2(-e^x)$$

input
```
integrate(x^2/(2+exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
x^2 - 2*x*log(e^x + 1) - x^2/(e^x + 1) - 2*dilog(-e^x)
```

Giac [F]

$$\int \frac{x^2}{2 + e^{-x} + e^x} \, dx = \int \frac{x^2}{e^{(-x)} + e^x + 2} \, dx$$

input
```
integrate(x^2/(2+exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2/(e^(-x) + e^x + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{2+e^{-x}+e^x}\,dx=\int \frac{x^2}{\mathrm{e}^{-x}+\mathrm{e}^x+2}\,dx$$

input
```
int(x^2/(exp(-x) + exp(x) + 2),x)
```

output
```
int(x^2/(exp(-x) + exp(x) + 2), x)
```

Reduce [F]

$$\int \frac{x^2}{2+e^{-x}+e^x}\,dx =\frac{2e^x\left(\int \frac{x}{e^{2x}+2e^x+1}dx\right)-2e^x\log(e^x+1)+2e^x x+2\left(\int \frac{x}{e^{2x}+2e^x+1}dx\right)-2\log(e^x+1)-x^2}{e^x+1}$$

input
```
int(x^2/(2+exp(-x)+exp(x)),x)
```

output
```
(2*e**x*int(x/(e**(2*x) + 2*e**x + 1),x) - 2*e**x*log(e**x + 1) + 2*e**x*x
 + 2*int(x/(e**(2*x) + 2*e**x + 1),x) - 2*log(e**x + 1) - x**2)/(e**x + 1)
```

3.455 $\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx=-\frac{1}{d\left(1+f^{c+dx}\right)\log(f)}$$

output
```
-1/d/(1+f^(d*x+c))/ln(f)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx=-\frac{1}{d\left(1+f^{c+dx}\right)\log(f)}$$

input
```
Integrate[(2 + f^(-c - d*x) + f^(c + d*x))^(-1),x]
```

output
```
-(1/(d*(1 + f^(c + d*x))*Log[f]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2720, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{f^{-c-dx}+f^{c+dx}+2}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{1}{\left(f^{c+dx}+1\right)^2}df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{1}{d\log(f)\left(f^{c+dx}+1\right)}$$

input
```
Int[(2 + f^(-c - d*x) + f^(c + d*x))^(-1),x]
```

output
```
-(1/(d*(1 + f^(c + d*x))*Log[f]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
risch	$\frac{1}{d\ln(f)\left(f^{-dx-c}+1\right)}$	23
norman	$\frac{1}{d\ln(f)\left(\mathrm{e}^{(-dx-c)\ln(f)}+1\right)}$	25

input
```
int(1/(2+f^(-d*x-c)+f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)/(f^(-d*x-c)+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx = -\frac{1}{df^{dx+c}\log(f)+d\log(f)}$$

input
```
integrate(1/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="fricas")
```

output
```
-1/(d*f^(d*x + c)*log(f) + d*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx = -\frac{1}{df^{c+dx}\log(f)+d\log(f)}$$

input
```
integrate(1/(2+f**(-d*x-c)+f**(d*x+c)),x)
```

output
```
-1/(d*f**(c + d*x)*log(f) + d*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx = \frac{1}{d(f^{-dx-c}+1)\log(f)}$$

input
```
integrate(1/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="maxima")
```

output
```
1/(d*(f^(-d*x - c) + 1)*log(f))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx = -\frac{1}{(f^{dx}f^{c}+1)d\log(f)}$$

input
```
integrate(1/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="giac")
```

output
```
-1/((f^(d*x)*f^c + 1)*d*log(f))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx = -\frac{1}{d\,\ln(f)\,\left(f^{c+d\,x}+1\right)}$$

input
```
int(1/(1/f^(c + d*x) + f^(c + d*x) + 2),x)
```

output
```
-1/(d*log(f)*(f^(c + d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{2+f^{-c-dx}+f^{c+dx}}\,dx = \frac{f^{dx+c}}{\log(f)\,d\,(f^{dx+c}+1)}$$

input

```
int(1/(2+f^(-d*x-c)+f^(d*x+c)),x)
```

output

```
f**(c + d*x)/(log(f)*d*(f**(c + d*x) + 1))
```

3.456 $\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx=\frac{x}{d\log(f)}-\frac{x}{d\left(1+f^{c+dx}\right)\log(f)}-\frac{\log\left(1+f^{c+dx}\right)}{d^2\log^2(f)}$$

output
```
x/d/ln(f)-x/d/(1+f^(d*x+c))/ln(f)-ln(1+f^(d*x+c))/d^2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.88

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx=\frac{\frac{df^{c+dx}x\log(f)}{1+f^{c+dx}}-\log\left(1+f^{c+dx}\right)}{d^2\log^2(f)}$$

input
```
Integrate[x/(2 + f^(-c - d*x) + f^(c + d*x)),x]
```

output
```
((d*f^(c + d*x)*x*Log[f])/(1 + f^(c + d*x)) - Log[1 + f^(c + d*x)])/(d^2*L
og[f]^2)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {2697, 7239, 2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{f^{-c-dx}+f^{c+dx}+2}dx$$

$$\Big\downarrow \text{2697}$$

$$\int \frac{xf^{c+dx}}{2f^{c+dx}+f^{2(c+dx)}+1}dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{xf^{c+dx}}{\left(f^{c+dx}+1\right)^2}dx$$

$$\Big\downarrow \text{2621}$$

$$\frac{\int \frac{1}{f^{c+dx}+1}dx}{d\log(f)}-\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{f^{-c-dx}}{f^{c+dx}+1}df^{c+dx}}{d^2\log^2(f)}-\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{47}$$

$$\frac{\int f^{-c-dx}df^{c+dx}-\int \frac{1}{f^{c+dx}+1}df^{c+dx}}{d^2\log^2(f)}-\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log\left(f^{c+dx}\right)-\int \frac{1}{f^{c+dx}+1}df^{c+dx}}{d^2\log^2(f)}-\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log\left(f^{c+dx}\right)-\log\left(f^{c+dx}+1\right)}{d^2\log^2(f)}-\frac{x}{d\log(f)\left(f^{c+dx}+1\right)}$$

input
```
Int[x/(2 + f^(-c - d*x) + f^(c + d*x)),x]
```

output
```
-(x/(d*(1 + f^(c + d*x))*Log[f])) + (Log[f^(c + d*x)] - Log[1 + f^(c + d*x
)])/(d^2*Log[f]^2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

method	result	size
norman	$-\frac{x\,\mathrm{e}^{(-dx-c)\ln(f)}}{d\ln(f)\left(\mathrm{e}^{(-dx-c)\ln(f)}+1\right)}-\frac{\ln\left(\mathrm{e}^{(-dx-c)\ln(f)}+1\right)}{d^2\ln(f)^2}$	64
risch	$-\frac{x}{d\ln(f)}-\frac{c}{d^2\ln(f)}+\frac{x}{d\ln(f)\left(f^{-dx-c}+1\right)}-\frac{\ln\left(f^{-dx-c}+1\right)}{d^2\ln(f)^2}$	67

input
```
int(x/(2+f^(-d*x-c)+f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-x/d/ln(f)*exp((-d*x-c)*ln(f))/(exp((-d*x-c)*ln(f))+1)-1/d^2/ln(f)^2*ln(ex
p((-d*x-c)*ln(f))+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.22

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx=\frac{df^{dx+c}x\log(f)-\left(f^{dx+c}+1\right)\log\left(f^{dx+c}+1\right)}{d^2f^{dx+c}\log(f)^2+d^2\log(f)^2}$$

input
```
integrate(x/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="fricas")
```

output
```
(d*f^(d*x + c)*x*log(f) - (f^(d*x + c) + 1)*log(f^(d*x + c) + 1))/(d^2*f^(
d*x + c)*log(f)^2 + d^2*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx = -\frac{x}{df^{c+dx}\log(f)+d\log(f)}+\frac{x}{d\log(f)}-\frac{\log\left(f^{c+dx}+1\right)}{d^2\log(f)^2}$$

input
```
integrate(x/(2+f**(-d*x-c)+f**(d*x+c)),x)
```

output
```
-x/(d*f**(c + d*x)*log(f) + d*log(f)) + x/(d*log(f)) - log(f**(c + d*x) +
1)/(d**2*log(f)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx = \frac{f^{dx}f^{c}x}{df^{dx}f^{c}\log(f)+d\log(f)}-\frac{\log\left(\frac{f^{dx}f^{c}+1}{f^{c}}\right)}{d^2\log(f)^2}$$

input
```
integrate(x/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="maxima")
```

output
```
f^(d*x)*f^c*x/(d*f^(d*x)*f^c*log(f) + d*log(f)) - log((f^(d*x)*f^c + 1)/f^
c)/(d^2*log(f)^2)
```

Giac [F]

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx=\int \frac{x}{f^{dx+c}+f^{-dx-c}+2}\,dx$$

input
```
integrate(x/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x/(f^(d*x + c) + f^(-d*x - c) + 2), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx=\frac{f^{d\,x}\,f^c\,x}{d\,\ln\left(f\right)\,\left(f^{d\,x}\,f^c+1\right)}-\frac{\ln\left(f^{d\,x}\,f^c+1\right)}{d^2\ln\left(f\right)^2}$$

input
```
int(x/(1/f^(c + d*x) + f^(c + d*x) + 2),x)
```

output
```
(f^(d*x)*f^c*x)/(d*log(f)*(f^(d*x)*f^c + 1)) - log(f^(d*x)*f^c + 1)/(d^2*l
og(f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{x}{2+f^{-c-dx}+f^{c+dx}}\,dx=\frac{-f^{dx+c}\log\left(f^{dx+c}+1\right)+f^{dx+c}\log\left(f\right)dx-\log\left(f^{dx+c}+1\right)}{\log\left(f\right)^2d^2\left(f^{dx+c}+1\right)}$$

input
```
int(x/(2+f^(-d*x-c)+f^(d*x+c)),x)
```

output
```
( - f**(c + d*x)*log(f**(c + d*x) + 1) + f**(c + d*x)*log(f)*d*x - log(f**
(c + d*x) + 1))/(log(f)**2*d**2*(f**(c + d*x) + 1))
```

3.457 $\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 75

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx = \frac{x^2}{d\log(f)}-\frac{x^2}{d\left(1+f^{c+dx}\right)\log(f)} -\frac{2x\log\left(1+f^{c+dx}\right)}{d^2\log^2(f)}-\frac{2\,\text{PolyLog}\left(2,-f^{c+dx}\right)}{d^3\log^3(f)}$$

output
```
x^2/d/ln(f)-x^2/d/(1+f^(d*x+c))/ln(f)-2*x*ln(1+f^(d*x+c))/d^2/ln(f)^2-2*po
lylog(2,-f^(d*x+c))/d^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx = \frac{dx\log(f)\left(\frac{df^{c+dx}x\log(f)}{1+f^{c+dx}}-2\log\left(1+f^{c+dx}\right)\right)-2\,\text{PolyLog}\left(2,-f^{c+dx}\right)}{d^3\log^3(f)}$$

input
```
Integrate[x^2/(2 + f^(-c - d*x) + f^(c + d*x)),x]
```

output
```
(d*x*Log[f]*((d*f^(c + d*x)*x*Log[f])/(1 + f^(c + d*x)) - 2*Log[1 + f^(c +
 d*x)]) - 2*PolyLog[2, -f^(c + d*x)])/(d^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2697, 7239, 2621, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{f^{-c-dx}+f^{c+dx}+2}dx$$

$$\Big\downarrow \text{ 2697}$$

$$\int \frac{x^2f^{c+dx}}{2f^{c+dx}+f^{2(c+dx)}+1}dx$$

$$\Big\downarrow \text{ 7239}$$

$$\int \frac{x^2f^{c+dx}}{\left(f^{c+dx}+1\right)^2}dx$$

$$\Big\downarrow \text{ 2621}$$

$$\frac{2\int \frac{x}{f^{c+dx}+1}dx}{d\log(f)}-\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{2\left(\frac{x^2}{2}-\int \frac{f^{c+dx}x}{f^{c+dx}+1}dx\right)}{d\log(f)}-\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2\left(\frac{\int \log\left(f^{c+dx}+1\right)dx}{d\log(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}-\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{2\left(\frac{\int f^{-c-dx}\log\left(f^{c+dx}+1\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\log\left(f^{c+dx}+1\right)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}-\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2\left(-\frac{\text{PolyLog}(2,-f^{c+dx})}{d^2\log^2(f)}-\frac{x\log(f^{c+dx}+1)}{d\log(f)}+\frac{x^2}{2}\right)}{d\log(f)}-\frac{x^2}{d\log(f)\left(f^{c+dx}+1\right)}$$

input
```
Int[x^2/(2 + f^(-c - d*x) + f^(c + d*x)),x]
```

output
```
-(x^2/(d*(1 + f^(c + d*x))*Log[f])) + (2*(x^2/2 - (x*Log[1 + f^(c + d*x)])
/(d*Log[f]) - PolyLog[2, -f^(c + d*x)]/(d^2*Log[f]^2)))/(d*Log[f])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.79

method	result
risch	$\frac{x^2}{d\ln(f)\left(f^{-dx-c}+1\right)}-\frac{x^2}{d\ln(f)}-\frac{2cx}{d^2\ln(f)}-\frac{c^2}{d^3\ln(f)}-\frac{2\ln\left(f^{-dx}f^{-c}+1\right)x}{\ln(f)^2d^2}+\frac{2\,\mathrm{polylog}\left(2,-f^{-dx}f^{-c}\right)}{\ln(f)^3d^3}-\frac{2c\ln\left(f^{-dx}f^{-c}\right)}{\ln(f)^2d^3}$

input
```
int(x^2/(2+f^(-d*x-c)+f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)*x^2/(f^(-d*x-c)+1)-x^2/d/ln(f)-2/d^2/ln(f)*c*x-1/d^3/ln(f)*c^2-2
/ln(f)^2/d^2*ln(f^(-d*x)*f^(-c)+1)*x+2/ln(f)^3/d^3*polylog(2,-f^(-d*x)*f^(
-c))-2/ln(f)^2/d^3*c*ln(f^(-d*x)*f^(-c))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx = -\frac{c^2\log(f)^2-(d^2x^2-c^2)f^{dx+c}\log(f)^2+2\left(f^{dx+c}+1\right)\mathrm{Li}_2\left(-f^{dx+c}\right)+2\left(df^{dx+c}x\log(f)+dx\log(f)\right)}{d^3f^{dx+c}\log(f)^3+d^3\log(f)^3}$$

input
```
integrate(x^2/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="fricas")
```

output
```
-(c^2*log(f)^2 - (d^2*x^2 - c^2)*f^(d*x + c)*log(f)^2 + 2*(f^(d*x + c) + 1
)*dilog(-f^(d*x + c)) + 2*(d*f^(d*x + c)*x*log(f) + d*x*log(f))*log(f^(d*x
 + c) + 1))/(d^3*f^(d*x + c)*log(f)^3 + d^3*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx=-\frac{x^2}{df^{c+dx}\log(f)+d\log(f)}+\frac{2\int \frac{x}{e^{c\log(f)}e^{dx\log(f)}+1}\,dx}{d\log(f)}$$

input
```
integrate(x**2/(2+f**(-d*x-c)+f**(d*x+c)),x)
```

output
```
-x**2/(d*f**(c + d*x)*log(f) + d*log(f)) + 2*Integral(x/(exp(c*log(f))*exp
(d*x*log(f)) + 1), x)/(d*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx=-\frac{x^2}{df^{dx}f^c\log(f)+d\log(f)}+\frac{x^2}{d\log(f)}-\frac{2\left(dx\log\left(f^{dx}f^c+1\right)\log(f)+\mathrm{Li}_2\left(-f^{dx}f^c\right)\right)}{d^3\log(f)^3}$$

input
```
integrate(x^2/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="maxima")
```

output
```
-x^2/(d*f^(d*x)*f^c*log(f) + d*log(f)) + x^2/(d*log(f)) - 2*(d*x*log(f^(d*
x)*f^c + 1)*log(f) + dilog(-f^(d*x)*f^c))/(d^3*log(f)^3)
```

Giac [F]

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx=\int \frac{x^2}{f^{dx+c}+f^{-dx-c}+2}\,dx$$

input
```
integrate(x^2/(2+f^(-d*x-c)+f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2/(f^(d*x + c) + f^(-d*x - c) + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx=\int \frac{x^2}{\frac{1}{f^{c+d\,x}}+f^{c+d\,x}+2}\,dx$$

input
```
int(x^2/(1/f^(c + d*x) + f^(c + d*x) + 2),x)
```

output
```
int(x^2/(1/f^(c + d*x) + f^(c + d*x) + 2), x)
```

Reduce [F]

$$\int \frac{x^2}{2+f^{-c-dx}+f^{c+dx}}\,dx$$
$$=\frac{2f^{dx+c}\left(\int \frac{x}{f^{2dx+2c}+2f^{dx+c}+1}dx\right)\log(f)^2 d^2-2f^{dx+c}\log\left(f^{dx+c}+1\right)+2f^{dx+c}\log(f)\,dx+2\left(\int \frac{x}{f^{2dx+2c}+2f^{dx+c}}\right.}{\log\left(f\right)^3 d^3\left(f^{dx+c}+1\right)}$$

input
```
int(x^2/(2+f^(-d*x-c)+f^(d*x+c)),x)
```

output
```
(2*f**(c + d*x)*int(x/(f**(2*c + 2*d*x) + 2*f**(c + d*x) + 1),x)*log(f)**2
*d**2 - 2*f**(c + d*x)*log(f**(c + d*x) + 1) + 2*f**(c + d*x)*log(f)*d*x +
 2*int(x/(f**(2*c + 2*d*x) + 2*f**(c + d*x) + 1),x)*log(f)**2*d**2 - 2*log
(f**(c + d*x) + 1) - log(f)**2*d**2*x**2)/(log(f)**3*d**3*(f**(c + d*x) +
1))
```

3.458 $\int \frac{1}{2+3^{-x}+3^x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{1}{2+3^{-x}+3^x} \, dx = -\frac{1}{(1+3^x)\log(3)}$$

output
```
-1/(1+3^x)/ln(3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3^{-x}+3^x} \, dx = -\frac{1}{(1+3^x)\log(3)}$$

input
```
Integrate[(2 + 3^(-x) + 3^x)^(-1),x]
```

output
```
-(1/((1 + 3^x)*Log[3]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3^{-x}+3^x+2}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{(1+3^x)^2}d3^x}{\log(3)}$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{(3^x+1)\log(3)}$$

input
```
Int[(2 + 3^(-x) + 3^x)^(-1),x]
```

output
```
-(1/((1 + 3^x)*Log[3]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{1}{(1+3^{x})\ln(3)}$	14
default	$-\frac{1}{(1+3^{x})\ln(3)}$	14
risch	$-\frac{1}{(1+3^{x})\ln(3)}$	14
norman	$\frac{e^{x\ln(3)}}{\ln(3)(e^{x\ln(3)}+1)}$	20

input
```
int(1/(2+3^(-x)+3^x),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+3^x)/ln(3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3^{-x}+3^{x}}\,dx = -\frac{1}{3^{x}\log(3)+\log(3)}$$

input
```
integrate(1/(2+3^(-x)+3^x),x, algorithm="fricas")
```

output
```
-1/(3^x*log(3) + log(3))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{2+3^{-x}+3^{x}}\,dx = -\frac{1}{3^{x}\log(3)+\log(3)}$$

input
```
integrate(1/(2+3**(-x)+3**x),x)
```

output
```
-1/(3**x*log(3) + log(3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{2+3^{-x}+3^x}\,dx = \frac{1}{\left(\frac{1}{3^x}+1\right)\log(3)}$$

input
```
integrate(1/(2+3^(-x)+3^x),x, algorithm="maxima")
```

output
```
1/((1/3^x + 1)*log(3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3^{-x}+3^x}\,dx = -\frac{1}{(3^x+1)\log(3)}$$

input
```
integrate(1/(2+3^(-x)+3^x),x, algorithm="giac")
```

output
```
-1/((3^x + 1)*log(3))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2+3^{-x}+3^x}\,dx = -\frac{1}{\ln(3)\,(3^x+1)}$$

input
```
int(1/(1/3^x + 3^x + 2),x)
```

output
```
-1/(log(3)*(3^x + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{2+3^{-x}+3^x}\,dx=\frac{3^x}{\log(3)\,(3^x+1)}$$

input
```
int(1/(2+3^(-x)+3^x),x)
```

output
```
3**x/(log(3)*(3**x + 1))
```

3.459 $\int \frac{1}{1-e^{-x}+2e^x} dx$

Optimal result

Integrand size = 16, antiderivative size = 23

$$\int \frac{1}{1-e^{-x}+2e^x} dx = \frac{1}{3}\log(1-2e^x) - \frac{1}{3}\log(1+e^x)$$

output
```
1/3*ln(1-2*exp(x))-1/3*ln(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-e^{-x}+2e^x} dx = \frac{2}{3}\text{arctanh}\left(\frac{1}{3} - \frac{2e^{-x}}{3}\right)$$

input
```
Integrate[(1 - E^(-x) + 2*E^x)^(-1),x]
```

output
```
(2*ArcTanh[1/3 - 2/(3*E^x)])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2720, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-e^{-x}+2e^x+1}dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{e^x+2e^{2x}-1}de^x$$

$$\Big\downarrow \text{1081}$$

$$2\int \left(-\frac{1}{6(1+e^x)}-\frac{1}{3(1-2e^x)}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{6}\log(1-2e^x)-\frac{1}{6}\log(e^x+1)\right)$$

input
```
Int[(1 - E^(-x) + 2*E^x)^(-1),x]
```

output
```
2*(Log[1 - 2*E^x]/6 - Log[1 + E^x]/6)
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{\ln(1+e^{x})}{3}+\frac{\ln(e^{x}-\frac{1}{2})}{3}$	16
default	$\frac{\ln(2\,e^{x}-1)}{3}-\frac{\ln(1+e^{x})}{3}$	18
norman	$\frac{\ln(2\,e^{x}-1)}{3}-\frac{\ln(1+e^{x})}{3}$	18

input
```
int(1/(1-exp(-x)+2*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(1+exp(x))+1/3*ln(exp(x)-1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-e^{-x}+2e^{x}}\,dx=\frac{1}{3}\,\log\left(2\,e^{x}-1\right)-\frac{1}{3}\,\log\left(e^{x}+1\right)$$

input
```
integrate(1/(1-exp(-x)+2*exp(x)),x, algorithm="fricas")
```

output
```
1/3*log(2*e^x - 1) - 1/3*log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = \frac{\log\left(e^x-\frac{1}{2}\right)}{3}-\frac{\log\left(e^x+1\right)}{3}$$

input
```
integrate(1/(1-exp(-x)+2*exp(x)),x)
```

output
```
log(exp(x) - 1/2)/3 - log(exp(x) + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = -\frac{1}{3}\,\log\left(e^{(-x)}+1\right)+\frac{1}{3}\,\log\left(e^{(-x)}-2\right)$$

input
```
integrate(1/(1-exp(-x)+2*exp(x)),x, algorithm="maxima")
```

output
```
-1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = -\frac{1}{3}\,\log\left(e^x+1\right)+\frac{1}{3}\,\log\left(\left|2\,e^x-1\right|\right)$$

input
```
integrate(1/(1-exp(-x)+2*exp(x)),x, algorithm="giac")
```

output
```
-1/3*log(e^x + 1) + 1/3*log(abs(2*e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx=\frac{\ln\left(2\,\mathrm{e}^x-1\right)}{3}-\frac{\ln\left(\mathrm{e}^x+1\right)}{3}$$

input
```
int(1/(2*exp(x) - exp(-x) + 1),x)
```

output
```
log(2*exp(x) - 1)/3 - log(exp(x) + 1)/3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx=-\frac{\log(e^x+1)}{3}+\frac{\log(2e^x-1)}{3}$$

input
```
int(1/(1-exp(-x)+2*exp(x)),x)
```

output
```
( - log(e**x + 1) + log(2*e**x - 1))/3
```

3.460 $\int \frac{1}{a+be^{-x}+ce^x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{1}{a+be^{-x}+ce^x} \, dx = -\frac{2\text{arctanh}\left(\frac{a+2ce^x}{\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}$$

output
```
-2*arctanh((a+2*c*exp(x))/(a^2-4*b*c)^(1/2))/(a^2-4*b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int \frac{1}{a+be^{-x}+ce^x} \, dx = \frac{2\arctan\left(\frac{a+2ce^x}{\sqrt{-a^2+4bc}}\right)}{\sqrt{-a^2+4bc}}$$

input
```
Integrate[(a + b/E^x + c*E^x)^(-1),x]
```

output
```
(2*ArcTan[(a + 2*c*E^x)/Sqrt[-a^2 + 4*b*c]])/Sqrt[-a^2 + 4*b*c]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 1722, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+be^{-x}+ce^x}dx$$
$$\Big\downarrow \; 2720$$
$$\int \frac{e^{-x}}{a+be^{-x}+ce^x}de^x$$
$$\Big\downarrow \; 1722$$
$$\int \frac{1}{ae^x+b+ce^{2x}}de^x$$
$$\Big\downarrow \; 1083$$
$$-2\int \frac{1}{a^2-e^{2x}-4bc}d(a+2ce^x)$$
$$\Big\downarrow \; 219$$
$$-\frac{2\text{arctanh}\left(\frac{a+2ce^x}{\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}$$

input
```
Int[(a + b/E^x + c*E^x)^(-1),x]
```

output
```
(-2*ArcTanh[(a + 2*c*E^x)/Sqrt[a^2 - 4*b*c]])/Sqrt[a^2 - 4*b*c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{2 \arctan\left(\frac{a+2 c \,{\mathrm e}^{x}}{\sqrt{-a^{2}+4 b c}}\right)}{\sqrt{-a^{2}+4 b c}}$	36
default	$\frac{2 \arctan\left(\frac{a+2 c \,{\mathrm e}^{x}}{\sqrt{-a^{2}+4 b c}}\right)}{\sqrt{-a^{2}+4 b c}}$	36
risch	$\frac{\ln\left({\mathrm e}^{x}+\frac{a \sqrt{a^{2}-4 b c}-a^{2}+4 b c}{2 c \sqrt{a^{2}-4 b c}}\right)}{\sqrt{a^{2}-4 b c}}-\frac{\ln\left({\mathrm e}^{x}+\frac{a \sqrt{a^{2}-4 b c}+a^{2}-4 b c}{2 c \sqrt{a^{2}-4 b c}}\right)}{\sqrt{a^{2}-4 b c}}$	105

input
```
int(1/(a+b/exp(x)+c*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(-a^2+4*b*c)^(1/2)*arctan((a+2*c*exp(x))/(-a^2+4*b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.50

$$\int \frac{1}{a+be^{-x}+ce^x}\,dx = \left[\frac{\log\left(\frac{2\,c^2e^{(2\,x)}+2\,ace^x+a^2-2\,bc-\sqrt{a^2-4\,bc}(2\,ce^x+a)}{ce^{(2\,x)}+ae^x+b}\right)}{\sqrt{a^2-4\,bc}},\ -\frac{2\,\sqrt{-a^2+4\,bc}\arctan\left(-\frac{\sqrt{-a^2+4\,bc}(2\,ce^x+a)}{a^2-4\,bc}\right)}{a^2-4\,bc}\right]$$

input
```
integrate(1/(a+b/exp(x)+c*exp(x)),x, algorithm="fricas")
```

output
```
[log((2*c^2*e^(2*x) + 2*a*c*e^x + a^2 - 2*b*c - sqrt(a^2 - 4*b*c)*(2*c*e^x
 + a))/(c*e^(2*x) + a*e^x + b))/sqrt(a^2 - 4*b*c), -2*sqrt(-a^2 + 4*b*c)*a
rctan(-sqrt(-a^2 + 4*b*c)*(2*c*e^x + a)/(a^2 - 4*b*c))/(a^2 - 4*b*c)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+be^{-x}+ce^x}\,dx = \mathrm{RootSum}\left(z^2\left(a^2-4bc\right)-1,\left(i\mapsto i\log\left(e^x+\frac{-ia^2+4ibc+a}{2c}\right)\right)\right)$$

input
```
integrate(1/(a+b/exp(x)+c*exp(x)),x)
```

output
```
RootSum(_z**2*(a**2 - 4*b*c) - 1, Lambda(_i, _i*log(exp(x) + (-_i*a**2 + 4
*_i*b*c + a)/(2*c))))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+be^{-x}+ce^{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b/exp(x)+c*exp(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b*c-a^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+be^{-x}+ce^{x}}\,dx = \frac{2\arctan\left(\frac{2\,ce^{x}+a}{\sqrt{-a^{2}+4\,bc}}\right)}{\sqrt{-a^{2}+4\,bc}}$$

input
```
integrate(1/(a+b/exp(x)+c*exp(x)),x, algorithm="giac")
```

output
```
2*arctan((2*c*e^x + a)/sqrt(-a^2 + 4*b*c))/sqrt(-a^2 + 4*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{a+be^{-x}+ce^{x}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a+2\,c\,e^{x}}{\sqrt{4\,b\,c-a^{2}}}\right)}{\sqrt{4\,b\,c-a^{2}}}$$

input
```
int(1/(a + c*exp(x) + b*exp(-x)),x)
```

output
```
(2*atan((a + 2*c*exp(x))/(4*b*c - a^2)^(1/2)))/(4*b*c - a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+be^{-x}+ce^{x}}\,dx = -\frac{2\sqrt{-a^2+4bc}\,atan\left(\frac{2e^{x}c+a}{\sqrt{-a^2+4bc}}\right)}{a^2-4bc}$$

input
```
int(1/(a+b/exp(x)+c*exp(x)),x)
```

output
```
( - 2*sqrt( - a**2 + 4*b*c)*atan((2*e**x*c + a)/sqrt( - a**2 + 4*b*c)))/(a
**2 - 4*b*c)
```

3.461 $\int \frac{x}{a+be^{-x}+ce^x} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 159

$$\int \frac{x}{a+be^{-x}+ce^x} \, dx = \frac{x \log \left(1+\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{x \log \left(1+\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} + \frac{\text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{\text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}$$

output
```
x*ln(1+2*c*exp(x)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)-x*ln(1+2*c*exp(
x)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)+polylog(2,-2*c*exp(x)/(a-(a^2-
4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)-polylog(2,-2*c*exp(x)/(a+(a^2-4*b*c)^(1/2
)))/(a^2-4*b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.77

$$\int \frac{x}{a+be^{-x}+ce^x} \, dx = \frac{x\left(\log \left(1+\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right) - \log \left(1+\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)\right) + \text{PolyLog}\left(2,\frac{2ce^x}{-a+\sqrt{a^2-4bc}}\right) - \text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}$$

input
```
Integrate[x/(a + b/E^x + c*E^x),x]
```

output
```
(x*(Log[1 + (2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])] - Log[1 + (2*c*E^x)/(a + Sq
rt[a^2 - 4*b*c])]) + PolyLog[2, (2*c*E^x)/(-a + Sqrt[a^2 - 4*b*c])] - Poly
Log[2, (-2*c*E^x)/(a + Sqrt[a^2 - 4*b*c])])/Sqrt[a^2 - 4*b*c]
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2697, 2694, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+be^{-x}+ce^x}dx$$

$$\Big\downarrow \text{2697}$$

$$\int \frac{e^x x}{ae^x+b+ce^{2x}}dx$$

$$\Big\downarrow \text{2694}$$

$$\frac{2c\int \frac{e^x x}{a+2ce^x-\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}-\frac{2c\int \frac{e^x x}{a+2ce^x+\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}-\frac{\int \log\left(\frac{2e^xc}{a-\sqrt{a^2-4bc}}+1\right)dx}{2c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}-\frac{\int \log\left(\frac{2e^xc}{a+\sqrt{a^2-4bc}}+1\right)dx}{2c}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}-\frac{\int e^{-x}\log\left(\frac{2e^xc}{a-\sqrt{a^2-4bc}}+1\right)de^x}{2c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}-\frac{\int e^{-x}\log\left(\frac{2e^xc}{a+\sqrt{a^2-4bc}}+1\right)de^x}{2c}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \; 2838$$

$$\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{2c}+\frac{x\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{2c}+\frac{x\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}\right)}{\sqrt{a^2-4bc}}$$

input

```
Int[x/(a + b/E^x + c*E^x),x]
```

output

```
(2*c*((x*Log[1 + (2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])])/(2*c) + PolyLog[2, (-
2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])]/(2*c)))/Sqrt[a^2 - 4*b*c] - (2*c*((x*Log
[1 + (2*c*E^x)/(a + Sqrt[a^2 - 4*b*c])])/(2*c) + PolyLog[2, (-2*c*E^x)/(a
+ Sqrt[a^2 - 4*b*c])]/(2*c)))/Sqrt[a^2 - 4*b*c]
```

Defintions of rubi rules used

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))    Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{x\left(\ln\left(\frac{-2c\,e^{x}+\sqrt{a^{2}-4bc}-a}{-a+\sqrt{a^{2}-4bc}}\right)-\ln\left(\frac{2c\,e^{x}+\sqrt{a^{2}-4bc}+a}{a+\sqrt{a^{2}-4bc}}\right)\right)}{\sqrt{a^{2}-4bc}}+\frac{\operatorname{dilog}\left(\frac{-2c\,e^{x}+\sqrt{a^{2}-4bc}-a}{-a+\sqrt{a^{2}-4bc}}\right)-\operatorname{dilog}\left(\frac{2c\,e^{x}+\sqrt{a^{2}-4bc}+a}{a+\sqrt{a^{2}-4bc}}\right)}{\sqrt{a^{2}-4bc}}$	171
risch	$\frac{x\left(\ln\left(\frac{-2c\,e^{x}+\sqrt{a^{2}-4bc}-a}{-a+\sqrt{a^{2}-4bc}}\right)-\ln\left(\frac{2c\,e^{x}+\sqrt{a^{2}-4bc}+a}{a+\sqrt{a^{2}-4bc}}\right)\right)}{\sqrt{a^{2}-4bc}}+\frac{\operatorname{dilog}\left(\frac{-2c\,e^{x}+\sqrt{a^{2}-4bc}-a}{-a+\sqrt{a^{2}-4bc}}\right)}{\sqrt{a^{2}-4bc}}-\frac{\operatorname{dilog}\left(\frac{2c\,e^{x}+\sqrt{a^{2}-4bc}+a}{a+\sqrt{a^{2}-4bc}}\right)}{\sqrt{a^{2}-4bc}}$	180

input
```
int(x/(a+b/exp(x)+c*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x*(ln((-2*c*exp(x)+(a^2-4*b*c)^(1/2)-a)/(-a+(a^2-4*b*c)^(1/2)))-ln((2*c*ex
p(x)+(a^2-4*b*c)^(1/2)+a)/(a+(a^2-4*b*c)^(1/2))))/(a^2-4*b*c)^(1/2)+(dilog
((-2*c*exp(x)+(a^2-4*b*c)^(1/2)-a)/(-a+(a^2-4*b*c)^(1/2)))-dilog((2*c*exp(
x)+(a^2-4*b*c)^(1/2)+a)/(a+(a^2-4*b*c)^(1/2))))/(a^2-4*b*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.35

$$\int \frac{x}{a+be^{-x}+ce^{x}}\,dx$$
$$= \frac{bx\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(\frac{b\sqrt{\frac{a^2-4\,bc}{b^2}}e^{x}+ae^{x}+2\,b}{2\,b}\right)-bx\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(-\frac{b\sqrt{\frac{a^2-4\,bc}{b^2}}e^{x}-ae^{x}-2\,b}{2\,b}\right)+b\sqrt{\frac{a^2-4\,bc}{b^2}}\mathrm{Li}_2\left(-\frac{b\sqrt{\frac{a^2-4\,bc}{b^2}}}{}\right.}{a^2-4\,bc}$$

input
```
integrate(x/(a+b/exp(x)+c*exp(x)),x, algorithm="fricas")
```

output
```
(b*x*sqrt((a^2 - 4*b*c)/b^2)*log(1/2*(b*sqrt((a^2 - 4*b*c)/b^2)*e^x + a*e^
x + 2*b)/b) - b*x*sqrt((a^2 - 4*b*c)/b^2)*log(-1/2*(b*sqrt((a^2 - 4*b*c)/b
^2)*e^x - a*e^x - 2*b)/b) + b*sqrt((a^2 - 4*b*c)/b^2)*dilog(-1/2*(b*sqrt((
a^2 - 4*b*c)/b^2)*e^x + a*e^x + 2*b)/b + 1) - b*sqrt((a^2 - 4*b*c)/b^2)*di
log(1/2*(b*sqrt((a^2 - 4*b*c)/b^2)*e^x - a*e^x - 2*b)/b + 1))/(a^2 - 4*b*c
)
```

Sympy [F]

$$\int \frac{x}{a+be^{-x}+ce^{x}}\,dx=\int \frac{xe^{x}}{ae^{x}+b+ce^{2x}}\,dx$$

input
```
integrate(x/(a+b/exp(x)+c*exp(x)),x)
```

output
```
Integral(x*exp(x)/(a*exp(x) + b + c*exp(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a+be^{-x}+ce^{x}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b/exp(x)+c*exp(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a^2-4*b*c>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x}{a+be^{-x}+ce^{x}}\,dx = \int \frac{x}{be^{(-x)}+ce^{x}+a}\,dx$$

input
```
integrate(x/(a+b/exp(x)+c*exp(x)),x, algorithm="giac")
```

output
```
integrate(x/(b*e^(-x) + c*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+be^{-x}+ce^{x}}\,dx = \int \frac{x}{a+c\,\mathrm{e}^{x}+b\,\mathrm{e}^{-x}}\,dx$$

input
```
int(x/(a + c*exp(x) + b*exp(-x)),x)
```

output
```
int(x/(a + c*exp(x) + b*exp(-x)), x)
```

Reduce [F]

$$\int \frac{x}{a + be^{-x} + ce^{x}} \, dx = \int \frac{e^{x}x}{e^{2x}c + e^{x}a + b} dx$$

input

```
int(x/(a+b/exp(x)+c*exp(x)),x)
```

output

```
int((e**x*x)/(e**(2*x)*c + e**x*a + b),x)
```

3.462 $\int \frac{x^2}{a+be^{-x}+ce^x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 244

$$\begin{aligned}\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = {} & \frac{x^2\log\left(1+\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{x^2\log\left(1+\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} \\ & + \frac{2x\operatorname{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} - \frac{2x\operatorname{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} \\ & - \frac{2\operatorname{PolyLog}\left(3,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}} + \frac{2\operatorname{PolyLog}\left(3,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}}\end{aligned}$$

output
```
x^2*ln(1+2*c*exp(x)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)-x^2*ln(1+2*c*
exp(x)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)+2*x*polylog(2,-2*c*exp(x)/
(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)-2*x*polylog(2,-2*c*exp(x)/(a+(a^2
-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)-2*polylog(3,-2*c*exp(x)/(a-(a^2-4*b*c)^(
1/2)))/(a^2-4*b*c)^(1/2)+2*polylog(3,-2*c*exp(x)/(a+(a^2-4*b*c)^(1/2)))/(a
^2-4*b*c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx$$
$$= \frac{x^2\log\left(1+\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)-x^2\log\left(1+\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)+2x\,\text{PolyLog}\left(2,\frac{2ce^x}{-a+\sqrt{a^2-4bc}}\right)-2x\,\text{PolyLog}\left(2,-\frac{2c}{a+\sqrt{c}}\right.}{\sqrt{a^2-4bc}}$$

input
```
Integrate[x^2/(a + b/E^x + c*E^x),x]
```

output
```
(x^2*Log[1 + (2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])] - x^2*Log[1 + (2*c*E^x)/(a
 + Sqrt[a^2 - 4*b*c])] + 2*x*PolyLog[2, (2*c*E^x)/(-a + Sqrt[a^2 - 4*b*c])
] - 2*x*PolyLog[2, (-2*c*E^x)/(a + Sqrt[a^2 - 4*b*c])] - 2*PolyLog[3, (2*c
*E^x)/(-a + Sqrt[a^2 - 4*b*c])] + 2*PolyLog[3, (-2*c*E^x)/(a + Sqrt[a^2 -
4*b*c])])/Sqrt[a^2 - 4*b*c]
```

Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {2697, 2694, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx$$
$$\Big\downarrow \text{2697}$$
$$\int \frac{e^x x^2}{ae^x+b+ce^{2x}}dx$$
$$\Big\downarrow \text{2694}$$
$$\frac{2c\int \frac{e^x x^2}{a+2ce^x-\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}-\frac{2c\int \frac{e^x x^2}{a+2ce^x+\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}-\frac{\int x\log\left(\frac{2e^xc}{a-\sqrt{a^2-4bc}}+1\right)dx}{c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}-\frac{\int x\log\left(\frac{2e^xc}{a+\sqrt{a^2-4bc}}+1\right)dx}{c}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}-\frac{\int \text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)dx-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}-\frac{\int \text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)dx-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{c}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}-\frac{\int e^{-x}\,\text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)de^x-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}-\frac{\int e^{-x}\,\text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)de^x-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{c}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{a-\sqrt{a^2-4bc}}+1\right)}{2c}-\frac{\text{PolyLog}\left(3,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{a-\sqrt{a^2-4bc}}\right)}{c}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2ce^x}{\sqrt{a^2-4bc}+a}+1\right)}{2c}-\frac{\text{PolyLog}\left(3,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)-x\,\text{PolyLog}\left(2,-\frac{2ce^x}{a+\sqrt{a^2-4bc}}\right)}{c}\right)}{\sqrt{a^2-4bc}}$$

input
```
Int[x^2/(a + b/E^x + c*E^x),x]
```

output
```
(2*c*((x^2*Log[1 + (2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])])/(2*c) - (-(x*PolyLo
g[2, (-2*c*E^x)/(a - Sqrt[a^2 - 4*b*c])]) + PolyLog[3, (-2*c*E^x)/(a - Sqr
t[a^2 - 4*b*c])])/c))/Sqrt[a^2 - 4*b*c] - (2*c*((x^2*Log[1 + (2*c*E^x)/(a
+ Sqrt[a^2 - 4*b*c])])/(2*c) - (-(x*PolyLog[2, (-2*c*E^x)/(a + Sqrt[a^2 -
4*b*c])]) + PolyLog[3, (-2*c*E^x)/(a + Sqrt[a^2 - 4*b*c])])/c))/Sqrt[a^2 -
 4*b*c]
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{a+b\,\mathrm{e}^{-x}+c\,\mathrm{e}^{x}}dx$$

input
```
int(x^2/(a+b/exp(x)+c*exp(x)),x)
```

output
```
int(x^2/(a+b/exp(x)+c*exp(x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.30

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = \frac{bx^2\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(\frac{b\sqrt{\frac{a^2-4\,bc}{b^2}}e^x+ae^x+2\,b}{2\,b}\right)-bx^2\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(-\frac{b\sqrt{\frac{a^2-4\,bc}{b^2}}e^x-ae^x-2\,b}{2\,b}\right)+2\,bx\sqrt{\frac{a^2-4\,bc}{b^2}}\mathrm{Li}_2\left(-\frac{b\sqrt{\ldots}}{\ldots}\right.}{\ldots}$$

input
```
integrate(x^2/(a+b/exp(x)+c*exp(x)),x, algorithm="fricas")
```

output
```
(b*x^2*sqrt((a^2 - 4*b*c)/b^2)*log(1/2*(b*sqrt((a^2 - 4*b*c)/b^2)*e^x + a*
e^x + 2*b)/b) - b*x^2*sqrt((a^2 - 4*b*c)/b^2)*log(-1/2*(b*sqrt((a^2 - 4*b*
c)/b^2)*e^x - a*e^x - 2*b)/b) + 2*b*x*sqrt((a^2 - 4*b*c)/b^2)*dilog(-1/2*(
b*sqrt((a^2 - 4*b*c)/b^2)*e^x + a*e^x + 2*b)/b + 1) - 2*b*x*sqrt((a^2 - 4*
b*c)/b^2)*dilog(1/2*(b*sqrt((a^2 - 4*b*c)/b^2)*e^x - a*e^x - 2*b)/b + 1) -
 2*b*sqrt((a^2 - 4*b*c)/b^2)*polylog(3, -1/2*(b*sqrt((a^2 - 4*b*c)/b^2)*e^
x + a*e^x)/b) + 2*b*sqrt((a^2 - 4*b*c)/b^2)*polylog(3, 1/2*(b*sqrt((a^2 -
4*b*c)/b^2)*e^x - a*e^x)/b))/(a^2 - 4*b*c)
```

Sympy [F]

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = \int \frac{x^2e^x}{ae^x+b+ce^{2x}}\,dx$$

input
```
integrate(x**2/(a+b/exp(x)+c*exp(x)),x)
```

output
```
Integral(x**2*exp(x)/(a*exp(x) + b + c*exp(2*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b/exp(x)+c*exp(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a^2-4*b*c>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = \int \frac{x^2}{be^{(-x)}+ce^x+a}\,dx$$

input
```
integrate(x^2/(a+b/exp(x)+c*exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*e^(-x) + c*e^x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = \int \frac{x^2}{a+c\,\mathrm{e}^x+b\,\mathrm{e}^{-x}}\,dx$$

input
```
int(x^2/(a + c*exp(x) + b*exp(-x)),x)
```

output
```
int(x^2/(a + c*exp(x) + b*exp(-x)), x)
```

Reduce [F]

$$\int \frac{x^2}{a+be^{-x}+ce^x}\,dx = \int \frac{e^x x^2}{e^{2x}c+e^x a+b}dx$$

input
```
int(x^2/(a+b/exp(x)+c*exp(x)),x)
```

output
```
int((e**x*x**2)/(e**(2*x)*c + e**x*a + b),x)
```

3.463 $\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx=-\frac{2\text{arctanh}\left(\frac{a+2cf^{c+dx}}{\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d\log(f)}$$

output
```
-2*arctanh((a+2*c*f^(d*x+c))/(a^2-4*b*c)^(1/2))/(a^2-4*b*c)^(1/2)/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx=\frac{2\arctan\left(\frac{a+2cf^{c+dx}}{\sqrt{-a^2+4bc}}\right)}{\sqrt{-a^2+4bc}\,d\log(f)}$$

input
```
Integrate[(a + b*f^(-c - d*x) + c*f^(c + d*x))^(-1),x]
```

output
```
(2*ArcTan[(a + 2*c*f^(c + d*x))/Sqrt[-a^2 + 4*b*c]])/(Sqrt[-a^2 + 4*b*c]*d
*Log[f])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {2720, 1722, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{f^{-c-dx}}{bf^{-c-dx}+cf^{c+dx}+a}df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \ 1722$$

$$\frac{\int \frac{1}{af^{c+dx}+cf^{2c+2dx}+b}df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \ 1083$$

$$-\frac{2\int \frac{1}{-f^{2c+2dx}+a^2-4bc}d\left(2cf^{c+dx}+a\right)}{d\log(f)}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\text{arctanh}\left(\frac{a+2cf^{c+dx}}{\sqrt{a^2-4bc}}\right)}{d\log(f)\sqrt{a^2-4bc}}$$

input
```
Int[(a + b*f^(-c - d*x) + c*f^(c + d*x))^(-1),x]
```

output
```
(-2*ArcTanh[(a + 2*c*f^(c + d*x))/Sqrt[a^2 - 4*b*c]])/(Sqrt[a^2 - 4*b*c]*d
*Log[f])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1722
```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_.), x_Symbol
] :> Int[x^(m - n*p)*(b + a*x^n + c*x^(2*n))^p, x] /; FreeQ[{a, b, c, m, n}
, x] && EqQ[mn, -n] && IntegerQ[p] && PosQ[n]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(43) = 86$.

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.87

method	result	size
risch	$\frac{\ln\left(f^{-dx-c}+\frac{a\sqrt{a^2-4bc}+a^2-4bc}{2b\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d\ln(f)}-\frac{\ln\left(f^{-dx-c}+\frac{a\sqrt{a^2-4bc}-a^2+4bc}{2b\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d\ln(f)}$	135

input
```
int(1/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2-4*b*c)^(1/2)/d/ln(f)*ln(f^(-d*x-c)+1/2*(a*(a^2-4*b*c)^(1/2)+a^2-4*b
*c)/b/(a^2-4*b*c)^(1/2))-1/(a^2-4*b*c)^(1/2)/d/ln(f)*ln(f^(-d*x-c)+1/2*(a*
(a^2-4*b*c)^(1/2)-a^2+4*b*c)/b/(a^2-4*b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.02

$$\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx = \left[\frac{\log\left(\frac{2\,c^2f^{2\,dx+2\,c}+a^2-2\,bc+2\,\left(ac-\sqrt{a^2-4\,bc}c\right)f^{dx+c}-\sqrt{a^2-4\,bc}a}{cf^{2\,dx+2\,c}+af^{dx+c}+b}\right)}{\sqrt{a^2-4\,bc}d\log\left(f\right)}, -\frac{2\,\sqrt{-a^2+4\,bc}\arctan\left(-\frac{2\,\sqrt{-a^2+4\,bc}cf^{dx+c}+\sqrt{-a^2+4\,bc}a}{a^2-4\,bc}\right)}{(a^2-4\,bc)d\log\left(f\right)}\right]$$

input
```
integrate(1/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="fricas")
```

output
```
[log((2*c^2*f^(2*d*x + 2*c) + a^2 - 2*b*c + 2*(a*c - sqrt(a^2 - 4*b*c)*c)*
f^(d*x + c) - sqrt(a^2 - 4*b*c)*a)/(c*f^(2*d*x + 2*c) + a*f^(d*x + c) + b)
)/(sqrt(a^2 - 4*b*c)*d*log(f)), -2*sqrt(-a^2 + 4*b*c)*arctan(-(2*sqrt(-a^2
 + 4*b*c)*c*f^(d*x + c) + sqrt(-a^2 + 4*b*c)*a)/(a^2 - 4*b*c))/((a^2 - 4*b
*c)*d*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx$$
$$= \mathrm{RootSum}\left(z^2\left(a^2d^2\log\left(f\right)^2-4bcd^2\log\left(f\right)^2\right)-1, \left(i\mapsto i\log\left(f^{c+dx}+\frac{-ia^2d\log\left(f\right)+4ibcd\log\left(f\right)+a}{2c}\right.\right.\right.$$

input
```
integrate(1/(a+b*f**(-d*x-c)+c*f**(d*x+c)),x)
```

output
```
RootSum(_z**2*(a**2*d**2*log(f)**2 - 4*b*c*d**2*log(f)**2) - 1, Lambda(_i,
 _i*log(f**(c + d*x) + (-_i*a**2*d*log(f) + 4*_i*b*c*d*log(f) + a)/(2*c)))
)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + bf^{-c-dx} + cf^{c+dx}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b*c-a^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{a + bf^{-c-dx} + cf^{c+dx}}\, dx = \frac{2\,\arctan\left(\frac{2\,cf^{dx}f^{c}+a}{\sqrt{-a^{2}+4\,bc}}\right)}{\sqrt{-a^{2}+4\,bc}d\log\left(f\right)}$$

input
```
integrate(1/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="giac")
```

output
```
2*arctan((2*c*f^(d*x)*f^c + a)/sqrt(-a^2 + 4*b*c))/(sqrt(-a^2 + 4*b*c)*d*l
og(f))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{a + bf^{-c-dx} + cf^{c+dx}}\, dx = \frac{2\,\mathrm{atan}\left(\frac{a+2\,c\,f^{c+d\,x}}{\sqrt{4\,b\,c-a^2}}\right)}{d\,\ln\left(f\right)\,\sqrt{4\,b\,c-a^2}}$$

input
```
int(1/(a + c*f^(c + d*x) + b/f^(c + d*x)),x)
```

output
```
(2*atan((a + 2*c*f^(c + d*x))/(4*b*c - a^2)^(1/2)))/(d*log(f)*(4*b*c - a^2
)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{1}{a+bf^{-c-dx}+cf^{c+dx}}\,dx = -\frac{2\sqrt{-a^2+4bc}\,atan\left(\frac{2f^{dx+c}c+a}{\sqrt{-a^2+4bc}}\right)}{\log(f)\,d\,(a^2-4bc)}$$

input
```
int(1/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x)
```

output
```
( - 2*sqrt( - a**2 + 4*b*c)*atan((2*f**(c + d*x)*c + a)/sqrt( - a**2 + 4*b
*c)))/(log(f)*d*(a**2 - 4*b*c))
```

3.464 $\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 203

$$\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}} \, dx = \frac{x\log\left(1+\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}d\log(f)} - \frac{x\log\left(1+\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}d\log(f)} + \frac{\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}d^2\log^2(f)} - \frac{\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}d^2\log^2(f)}$$

output
```
x*ln(1+2*c*f^(d*x+c)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d/ln(f)-x*ln
(1+2*c*f^(d*x+c)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d/ln(f)+polylog(
2,-2*c*f^(d*x+c)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d^2/ln(f)^2-poly
log(2,-2*c*f^(d*x+c)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d^2/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.73

$$\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}} \, dx = \frac{dx\log(f)\left(\log\left(1+\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)-\log\left(1+\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)\right)+\text{PolyLog}\left(2,\frac{2cf^{c+dx}}{-a+\sqrt{a^2-4bc}}\right)-\text{PolyLog}\left(2,-\frac{2c}{a+\sqrt{}}\right.}{\sqrt{a^2-4bc}d^2\log^2(f)}$$

input
```
Integrate[x/(a + b*f^(-c - d*x) + c*f^(c + d*x)),x]
```

output
```
(d*x*Log[f]*(Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])] - Log[1 +
(2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])]) + PolyLog[2, (2*c*f^(c + d*x))
/(-a + Sqrt[a^2 - 4*b*c])] - PolyLog[2, (-2*c*f^(c + d*x))/(a + Sqrt[a^2 -
 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2697, 2694, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}}dx$$

$$\Big\downarrow \text{ 2697}$$

$$\int \frac{xf^{c+dx}}{af^{c+dx}+b+cf^{2(c+dx)}}dx$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2c\int \frac{f^{c+dx}x}{2cf^{c+dx}+a-\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}-\frac{2c\int \frac{f^{c+dx}x}{2cf^{c+dx}+a+\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}-\frac{\int \log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)dx}{2cd\log(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}-\frac{\int \log\left(\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}+1\right)dx}{2cd\log(f)}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{ 2715}$$

$$\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}-\frac{\int f^{-c-dx}\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)df^{c+dx}}{2cd^2\log^2(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}-\frac{\int f^{-c-dx}\log\left(\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}+1\right)df^{c+dx}}{2cd^2\log^2(f)}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{2cd^2\log^2(f)}+\frac{x\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{2cd^2\log^2(f)}+\frac{x\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}\right)}{\sqrt{a^2-4bc}}$$

input
```
Int[x/(a + b*f^(-c - d*x) + c*f^(c + d*x)),x]
```

output
```
(2*c*((x*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(2*c*d*Log[f]
) + PolyLog[2, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])]/(2*c*d^2*Log[f]
^2)))/Sqrt[a^2 - 4*b*c] - (2*c*((x*Log[1 + (2*c*f^(c + d*x))/(a + Sqrt[a^2
 - 4*b*c])])/(2*c*d*Log[f]) + PolyLog[2, (-2*c*f^(c + d*x))/(a + Sqrt[a^2
- 4*b*c])]/(2*c*d^2*Log[f]^2)))/Sqrt[a^2 - 4*b*c]
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(187) = 374$.

Time = 0.11 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.13

method	result
risch	$-\frac{\ln\left(\frac{-2b\,f^{-dx}f^{-c}+\sqrt{a^2-4bc}-a}{-a+\sqrt{a^2-4bc}}\right)x}{d\ln(f)\sqrt{a^2-4bc}}+\frac{\ln\left(\frac{2b\,f^{-dx}f^{-c}+\sqrt{a^2-4bc}+a}{a+\sqrt{a^2-4bc}}\right)x}{d\ln(f)\sqrt{a^2-4bc}}-\frac{\ln\left(\frac{-2b\,f^{-dx}f^{-c}+\sqrt{a^2-4bc}-a}{-a+\sqrt{a^2-4bc}}\right)c}{d^2\ln(f)\sqrt{a^2-4bc}}+\frac{\ln\left(\frac{2b\,f^{-dx}f^{-c}\ldots}{a+\sqrt{\ldots}}\right.}{d^2\ln(f)\ldots}$

input
```
int(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d/ln(f)/(a^2-4*b*c)^(1/2)*ln((-2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)-a)
/(-a+(a^2-4*b*c)^(1/2)))*x+1/d/ln(f)/(a^2-4*b*c)^(1/2)*ln((2*b*f^(-d*x)*f^
(-c)+(a^2-4*b*c)^(1/2)+a)/(a+(a^2-4*b*c)^(1/2)))*x-1/d^2/ln(f)/(a^2-4*b*c)
^(1/2)*ln((-2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)-a)/(-a+(a^2-4*b*c)^(1/2)
))*c+1/d^2/ln(f)/(a^2-4*b*c)^(1/2)*ln((2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/
2)+a)/(a+(a^2-4*b*c)^(1/2)))*c+1/d^2/ln(f)^2/(a^2-4*b*c)^(1/2)*dilog((-2*b
*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)-a)/(-a+(a^2-4*b*c)^(1/2)))-1/d^2/ln(f)^
2/(a^2-4*b*c)^(1/2)*dilog((2*b*f^(-d*x)*f^(-c)+(a^2-4*b*c)^(1/2)+a)/(a+(a^
2-4*b*c)^(1/2)))+2/d^2/ln(f)*c/(-a^2+4*b*c)^(1/2)*arctan((2*b*f^(-d*x)*f^(
-c)+a)/(-a^2+4*b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.74

$$\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}}\,dx$$
$$= \frac{bc\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(2\,cf^{dx+c}+b\sqrt{\frac{a^2-4\,bc}{b^2}}+a\right)\log(f)-bc\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(2\,cf^{dx+c}-b\sqrt{\frac{a^2-4\,bc}{b^2}}+a\right)\log(f)+\cdots}{\cdots}$$

input
```
integrate(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="fricas")
```

output
```
(b*c*sqrt((a^2 - 4*b*c)/b^2)*log(2*c*f^(d*x + c) + b*sqrt((a^2 - 4*b*c)/b^
2) + a)*log(f) - b*c*sqrt((a^2 - 4*b*c)/b^2)*log(2*c*f^(d*x + c) - b*sqrt(
(a^2 - 4*b*c)/b^2) + a)*log(f) + (b*d*x + b*c)*sqrt((a^2 - 4*b*c)/b^2)*log
(f)*log(1/2*((b*sqrt((a^2 - 4*b*c)/b^2) + a)*f^(d*x + c) + 2*b)/b) - (b*d*
x + b*c)*sqrt((a^2 - 4*b*c)/b^2)*log(f)*log(-1/2*((b*sqrt((a^2 - 4*b*c)/b^
2) - a)*f^(d*x + c) - 2*b)/b) + b*sqrt((a^2 - 4*b*c)/b^2)*dilog(-1/2*((b*s
qrt((a^2 - 4*b*c)/b^2) + a)*f^(d*x + c) + 2*b)/b + 1) - b*sqrt((a^2 - 4*b*
c)/b^2)*dilog(1/2*((b*sqrt((a^2 - 4*b*c)/b^2) - a)*f^(d*x + c) - 2*b)/b +
1))/((a^2 - 4*b*c)*d^2*log(f)^2)
```

Sympy [F]

$$\int \frac{x}{a + bf^{-c-dx} + cf^{c+dx}}\,dx = \int \frac{x}{a + bf^{-c-dx} + cf^{c+dx}}\,dx$$

input
```
integrate(x/(a+b*f**(-d*x-c)+c*f**(d*x+c)),x)
```

output
```
Integral(x/(a + b*f**(-c - d*x) + c*f**(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + bf^{-c-dx} + cf^{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a^2-4*b*c>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x}{a + bf^{-c-dx} + cf^{c+dx}}\,dx = \int \frac{x}{cf^{dx+c} + bf^{-dx-c} + a}\,dx$$

input
```
integrate(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x/(c*f^(d*x + c) + b*f^(-d*x - c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}}\,dx=\int \frac{x}{a+c\,f^{c+d\,x}+\frac{b}{f^{c+d\,x}}}\,dx$$

input
```
int(x/(a + c*f^(c + d*x) + b/f^(c + d*x)),x)
```

output
```
int(x/(a + c*f^(c + d*x) + b/f^(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x}{a+bf^{-c-dx}+cf^{c+dx}}\,dx = \frac{2f^{2c}\left(\int \frac{f^{2dx}x}{f^{2dx+2c}c+f^{dx+c}a+b}dx\right)c+4f^{c}\left(\int \frac{f^{dx}x}{f^{2dx+2c}c+f^{dx+c}a+b}dx\right)a+2\left(\int \frac{x}{f^{2dx+2c}c+f^{dx+c}a+b}dx\right)b-x^{2}}{2a}$$

input
```
int(x/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x)
```

output
```
(2*f**(2*c)*int((f**(2*d*x)*x)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*a + b),x
)*c + 4*f**c*int((f**(d*x)*x)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*a + b),x)
*a + 2*int(x/(f**(2*c + 2*d*x)*c + f**(c + d*x)*a + b),x)*b - x**2)/(2*a)
```

3.465 $\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 310

$$\begin{aligned}\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}\,dx &= \frac{x^2\log\left(1+\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d\log(f)}-\frac{x^2\log\left(1+\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d\log(f)}\\ &\quad+\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d^2\log^2(f)}\\ &\quad-\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d^2\log^2(f)}\\ &\quad-\frac{2\,\mathrm{PolyLog}\left(3,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d^3\log^3(f)}\\ &\quad+\frac{2\,\mathrm{PolyLog}\left(3,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{\sqrt{a^2-4bc}\,d^3\log^3(f)}\end{aligned}$$

output
```
x^2*ln(1+2*c*f^(d*x+c)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d/ln(f)-x^
2*ln(1+2*c*f^(d*x+c)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d/ln(f)+2*x*
polylog(2,-2*c*f^(d*x+c)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/d^2/ln(f
)^2-2*x*polylog(2,-2*c*f^(d*x+c)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^(1/2)/
d^2/ln(f)^2-2*polylog(3,-2*c*f^(d*x+c)/(a-(a^2-4*b*c)^(1/2)))/(a^2-4*b*c)^
(1/2)/d^3/ln(f)^3+2*polylog(3,-2*c*f^(d*x+c)/(a+(a^2-4*b*c)^(1/2)))/(a^2-4
*b*c)^(1/2)/d^3/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}\,dx$$
$$= \frac{d^2x^2\log^2(f)\log\left(1+\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)-d^2x^2\log^2(f)\log\left(1+\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)+2dx\log(f)\operatorname{PolyLog}\left(2,\frac{2cf^{c+dx}}{-a+\sqrt{a^2-4}}\right.}{\sqrt{a^2-4bcd}}$$

input
```
Integrate[x^2/(a + b*f^(-c - d*x) + c*f^(c + d*x)),x]
```

output
```
(d^2*x^2*Log[f]^2*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])] - d^2
*x^2*Log[f]^2*Log[1 + (2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])] + 2*d*x*L
og[f]*PolyLog[2, (2*c*f^(c + d*x))/(-a + Sqrt[a^2 - 4*b*c])] - 2*d*x*Log[f
]*PolyLog[2, (-2*c*f^(c + d*x))/(a + Sqrt[a^2 - 4*b*c])] - 2*PolyLog[3, (2
*c*f^(c + d*x))/(-a + Sqrt[a^2 - 4*b*c])] + 2*PolyLog[3, (-2*c*f^(c + d*x)
)/(a + Sqrt[a^2 - 4*b*c])])/(Sqrt[a^2 - 4*b*c]*d^3*Log[f]^3)
```

Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2697, 2694, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}dx$$

$$\Big\downarrow \text{2697}$$

$$\int \frac{x^2 f^{c+dx}}{af^{c+dx}+b+cf^{2(c+dx)}}dx$$

$$\Big\downarrow \text{2694}$$

$$\frac{2c\int \frac{f^{c+dx}x^2}{2cf^{c+dx}+a-\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}-\frac{2c\int \frac{f^{c+dx}x^2}{2cf^{c+dx}+a+\sqrt{a^2-4bc}}dx}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}-\frac{\int x\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)dx}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}-\frac{\int x\log\left(\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}+1\right)dx}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)dx}{d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)dx}{d\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}-\frac{\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}-\frac{\frac{\int f^{-c-dx}\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}+1\right)}{2cd\log(f)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a-\sqrt{a^2-4bc}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}-\frac{2c\left(\frac{x^2\log\left(\frac{2cf^{c+dx}}{\sqrt{a^2-4bc}+a}+1\right)}{2cd\log(f)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{d^2\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{2cf^{c+dx}}{a+\sqrt{a^2-4bc}}\right)}{d\log(f)}}{cd\log(f)}\right)}{\sqrt{a^2-4bc}}$$

input
```
Int[x^2/(a + b*f^(-c - d*x) + c*f^(c + d*x)),x]
```

output
```
(2*c*((x^2*Log[1 + (2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(2*c*d*Log[
f]) - (-((x*PolyLog[2, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])])/(d*Log
[f])) + PolyLog[3, (-2*c*f^(c + d*x))/(a - Sqrt[a^2 - 4*b*c])]/(d^2*Log[f]
^2))/(c*d*Log[f])))/Sqrt[a^2 - 4*b*c] - (2*c*((x^2*Log[1 + (2*c*f^(c + d*x
))/(a + Sqrt[a^2 - 4*b*c])])/(2*c*d*Log[f]) - (-((x*PolyLog[2, (-2*c*f^(c
+ d*x))/(a + Sqrt[a^2 - 4*b*c])])/(d*Log[f])) + PolyLog[3, (-2*c*f^(c + d*
x))/(a + Sqrt[a^2 - 4*b*c])]/(d^2*Log[f]^2))/(c*d*Log[f])))/Sqrt[a^2 - 4*b
*c]
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2697
```
Int[(u_)/((a_) + (b_.)*(F_)^(v_) + (c_.)*(F_)^(w_)), x_Symbol] :> Int[u*(F^
v/(c + a*F^v + b*F^(2*v))), x] /; FreeQ[{F, a, b, c}, x] && EqQ[w, -v] && L
inearQ[v, x] && If[RationalQ[D[v, x]], GtQ[D[v, x], 0], LtQ[LeafCount[v], L
eafCount[w]]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{a+b\,f^{-dx-c}+c\,f^{dx+c}}dx$$

input
```
int(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x)
```

output
```
int(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}\,dx = -\frac{bc^2\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(2\,cf^{dx+c}+b\sqrt{\frac{a^2-4\,bc}{b^2}}+a\right)\log\left(f\right)^2-bc^2\sqrt{\frac{a^2-4\,bc}{b^2}}\log\left(2\,cf^{dx+c}-b\sqrt{\frac{a^2-4\,bc}{b^2}}+a\right)\log}{}$$

input
```
integrate(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="fricas")
```

output
```
-(b*c^2*sqrt((a^2 - 4*b*c)/b^2)*log(2*c*f^(d*x + c) + b*sqrt((a^2 - 4*b*c)
/b^2) + a)*log(f)^2 - b*c^2*sqrt((a^2 - 4*b*c)/b^2)*log(2*c*f^(d*x + c) -
b*sqrt((a^2 - 4*b*c)/b^2) + a)*log(f)^2 - 2*b*d*x*sqrt((a^2 - 4*b*c)/b^2)*
dilog(-1/2*((b*sqrt((a^2 - 4*b*c)/b^2) + a)*f^(d*x + c) + 2*b)/b + 1)*log(
f) + 2*b*d*x*sqrt((a^2 - 4*b*c)/b^2)*dilog(1/2*((b*sqrt((a^2 - 4*b*c)/b^2)
 - a)*f^(d*x + c) - 2*b)/b + 1)*log(f) - (b*d^2*x^2 - b*c^2)*sqrt((a^2 - 4
*b*c)/b^2)*log(f)^2*log(1/2*((b*sqrt((a^2 - 4*b*c)/b^2) + a)*f^(d*x + c) +
 2*b)/b) + (b*d^2*x^2 - b*c^2)*sqrt((a^2 - 4*b*c)/b^2)*log(f)^2*log(-1/2*(
(b*sqrt((a^2 - 4*b*c)/b^2) - a)*f^(d*x + c) - 2*b)/b) + 2*b*sqrt((a^2 - 4*
b*c)/b^2)*polylog(3, -1/2*(b*sqrt((a^2 - 4*b*c)/b^2) + a)*f^(d*x + c)/b) -
 2*b*sqrt((a^2 - 4*b*c)/b^2)*polylog(3, 1/2*(b*sqrt((a^2 - 4*b*c)/b^2) - a
)*f^(d*x + c)/b))/((a^2 - 4*b*c)*d^3*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{a + bf^{-c-dx} + cf^{c+dx}}\,dx = \int \frac{x^2}{a + bf^{-c-dx} + cf^{c+dx}}\,dx$$

input
```
integrate(x**2/(a+b*f**(-d*x-c)+c*f**(d*x+c)),x)
```

output
```
Integral(x**2/(a + b*f**(-c - d*x) + c*f**(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a + bf^{-c-dx} + cf^{c+dx}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a^2-4*b*c>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{x^2}{a + bf^{-c-dx} + cf^{c+dx}}\,dx = \int \frac{x^2}{cf^{dx+c} + bf^{-dx-c} + a}\,dx$$

input
```
integrate(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2/(c*f^(d*x + c) + b*f^(-d*x - c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}\,dx=\int \frac{x^2}{a+c\,f^{c+d\,x}+\frac{b}{f^{c+d\,x}}}\,dx$$

input
```
int(x^2/(a + c*f^(c + d*x) + b/f^(c + d*x)),x)
```

output
```
int(x^2/(a + c*f^(c + d*x) + b/f^(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2}{a+bf^{-c-dx}+cf^{c+dx}}\,dx = \frac{3f^{2c}\left(\int \frac{f^{2dx}x^2}{f^{2dx+2c}c+f^{dx+c}a+b}dx\right)c+6f^{c}\left(\int \frac{f^{dx}x^2}{f^{2dx+2c}c+f^{dx+c}a+b}dx\right)a+3\left(\int \frac{x^2}{f^{2dx+2c}c+f^{dx+c}a+b}dx\right)b-x^3}{3a}$$

input
```
int(x^2/(a+b*f^(-d*x-c)+c*f^(d*x+c)),x)
```

output
```
(3*f**(2*c)*int((f**(2*d*x)*x**2)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*a + b
),x)*c + 6*f**c*int((f**(d*x)*x**2)/(f**(2*c + 2*d*x)*c + f**(c + d*x)*a +
 b),x)*a + 3*int(x**2/(f**(2*c + 2*d*x)*c + f**(c + d*x)*a + b),x)*b - x**
3)/(3*a)
```

3.466 $\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 50

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\text{Int}\left(\frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2},x\right)$$

output
```
Defer(Int)((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*
x^2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(d*f + (e*f + d*g)
*x + e*g*x^2),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(d*f + (e*f + d*g)
*x + e*g*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2730}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{x(dg+ef)+df+egx^2}dx$$

$$\Big\downarrow \text{2730}$$

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{x(dg+ef)+df+egx^2}dx$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(d*f + (e*f + d*g)*x + e
*g*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2730
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_)/((A_.) + (B_.)*(x_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrab
le[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(A + B*x + C*x^2), x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[B*e*g - C*(e*f + d*g), 0] && !IGtQ[n, 0]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}\right)^{n}}{df+\left(dg+ef\right)x+eg\,x^{2}}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [N/A]

Not integrable

Time = 74.71 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}b+a\right)^n}{(d+ex)\,(f+gx)}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))**n/(d*f+(d*g+e*f)*x+e
*g*x**2),x)
```

output
```
Integral((F**(c*sqrt(d + e*x)/sqrt(f + g*x))*b + a)**n/((d + e*x)*(f + g*x
)), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^n}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="maxima")
```

output
```
integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)^n/(e*g*x^2 + d*f + (e*
f + d*g)*x), x)
```

Giac [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^n}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="giac")
```

output
```
integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)^n/(e*g*x^2 + d*f + (e*
f + d*g)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(a+F^{\frac{c\,\sqrt{d+e\,x}}{\sqrt{f+g\,x}}}\,b\right)^n}{e\,g\,x^2+\left(d\,g+e\,f\right)\,x+d\,f}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^n/(d*f + x*(d*g + e*f)
 + e*g*x^2),x)
```

output
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^n/(d*f + x*(d*g + e*f)
 + e*g*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^n}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}b+a\right)^n}{eg\,x^2+dgx+efx+df}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^n/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int((f**((sqrt(d + e*x)*c)/sqrt(f + g*x))*b + a)**n/(d*f + d*g*x + e*f*x +
 e*g*x**2),x)
```

3.467 $\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 154

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\frac{6a^2b\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}+\frac{6ab^2\,\text{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}+\frac{2b^3\,\text{ExpIntegralEi}\left(\frac{3c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}+\frac{2a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg}$$

output
```
6*a^2*b*Ei(c*(e*x+d)^(1/2)*ln(F)/(g*x+f)^(1/2))/(-d*g+e*f)+6*a*b^2*Ei(2*c*
(e*x+d)^(1/2)*ln(F)/(g*x+f)^(1/2))/(-d*g+e*f)+2*b^3*Ei(3*c*(e*x+d)^(1/2)*l
n(F)/(g*x+f)^(1/2))/(-d*g+e*f)+2*a^3*ln((e*x+d)^(1/2)/(g*x+f)^(1/2))/(-d*g
+e*f)
```

Mathematica [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^3/(d*f + (e*f + d*g)
*x + e*g*x^2),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^3/(d*f + (e*f + d*g)
*x + e*g*x^2), x]
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.060, Rules used = {2728, 2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{x(dg+ef)+df+egx^2}\,dx$$

$$\Big\downarrow \text{2728}$$

$$\frac{2\int \frac{\left(bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}+a\right)^3\sqrt{f+gx}}{\sqrt{d+ex}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{ef-dg}$$

$$\Big\downarrow \text{2614}$$

$$\frac{2\int \left(\frac{3a^2b\sqrt{f+gx}F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{\sqrt{d+ex}}+\frac{3ab^2\sqrt{f+gx}F^{\frac{2c\sqrt{d+ex}}{\sqrt{f+gx}}}}{\sqrt{d+ex}}+\frac{b^3\sqrt{f+gx}F^{\frac{3c\sqrt{d+ex}}{\sqrt{f+gx}}}}{\sqrt{d+ex}}+\frac{a^3\sqrt{f+gx}}{\sqrt{d+ex}}\right)d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{ef-dg}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+3a^2b\,\mathrm{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)+3ab^2\,\mathrm{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)+b^3\,\mathrm{ExpIntegral}\right.}{ef-dg}$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^3/(d*f + (e*f + d*g)*x + e
*g*x^2),x]
```

output
```
(2*(3*a^2*b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]] + 3*a*b^
2*ExpIntegralEi[(2*c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]] + b^3*ExpIntegra
lEi[(3*c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]] + a^3*Log[Sqrt[d + e*x]/Sqrt
[f + g*x]]))/(e*f - d*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

rule 2728
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_.) + (B_.)*(x_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*
(g/(C*(e*f - d*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/S
qrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, C, F}, x] && EqQ[C*d
*f - A*e*g, 0] && EqQ[B*e*g - C*(e*f + d*g), 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}\right)^3}{df+(dg+ef)\,x+eg\,x^2}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

Fricas [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^3}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="fricas")
```

output
```
integral((3*F^(sqrt(e*x + d)*c/sqrt(g*x + f))*a^2*b + 3*F^(2*sqrt(e*x + d)
*c/sqrt(g*x + f))*a*b^2 + F^(3*sqrt(e*x + d)*c/sqrt(g*x + f))*b^3 + a^3)/(
e*g*x^2 + d*f + (e*f + d*g)*x), x)
```

Sympy [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}b+a\right)^3}{(d+ex)\,(f+gx)}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))**3/(d*f+(d*g+e*f)*x+e
*g*x**2),x)
```

output
```
Integral((F**(c*sqrt(d + e*x)/sqrt(f + g*x))*b + a)**3/((d + e*x)*(f + g*x
)), x)
```

Maxima [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^3}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="maxima")
```

output
```
a^3*(log(e*x + d)/(e*f - d*g) - log(g*x + f)/(e*f - d*g)) + b^3*integrate(
F^(3*sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f + d*g)*x), x) +
3*a*b^2*integrate(F^(2*sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*
f + d*g)*x), x) + 3*a^2*b*integrate(F^(sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g
*x^2 + d*f + (e*f + d*g)*x), x)
```

Giac [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^3}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="giac")
```

output
```
integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)^3/(e*g*x^2 + d*f + (e*
f + d*g)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(a+F^{\frac{c\sqrt{d+e\,x}}{\sqrt{f+g\,x}}}\,b\right)^3}{e\,g\,x^2+\left(d\,g+e\,f\right)\,x+d\,f}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^3/(d*f + x*(d*g + e*f)
 + e*g*x^2),x)
```

output
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^3/(d*f + x*(d*g + e*f)
 + e*g*x^2), x)
```

Reduce [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^3}{df+(ef+dg)x+egx^2}\,dx = \frac{3\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)a^2bdg-3\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)a^2bef+\left(\int \frac{f^{\frac{3\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)b^3dg-\left(\int \cdots\right.}{}$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^3/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
(3*int(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x + e*f*x + e*g*x**
2),x)*a**2*b*d*g - 3*int(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x
 + e*f*x + e*g*x**2),x)*a**2*b*e*f + int(f**((3*sqrt(d + e*x)*c)/sqrt(f +
g*x))/(d*f + d*g*x + e*f*x + e*g*x**2),x)*b**3*d*g - int(f**((3*sqrt(d + e
*x)*c)/sqrt(f + g*x))/(d*f + d*g*x + e*f*x + e*g*x**2),x)*b**3*e*f + 3*int
(f**((2*sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x + e*f*x + e*g*x**2),x
)*a*b**2*d*g - 3*int(f**((2*sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x +
 e*f*x + e*g*x**2),x)*a*b**2*e*f - log(d + e*x)*a**3 + log(f + g*x)*a**3)/
(d*g - e*f)
```

3.468 $\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 112

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\frac{4ab\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}+\frac{2b^2\,\text{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}+\frac{2a^2\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg}$$

output
```
4*a*b*Ei(c*(e*x+d)^(1/2)*ln(F)/(g*x+f)^(1/2))/(-d*g+e*f)+2*b^2*Ei(2*c*(e*x
+d)^(1/2)*ln(F)/(g*x+f)^(1/2))/(-d*g+e*f)+2*a^2*ln((e*x+d)^(1/2)/(g*x+f)^(
1/2))/(-d*g+e*f)
```

Mathematica [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^2/(d*f + (e*f + d*g)
*x + e*g*x^2),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^2/(d*f + (e*f + d*g)
*x + e*g*x^2), x]
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.060, Rules used = {2728, 2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{x(dg+ef)+df+egx^2}\,dx$$

$$\Big\downarrow \text{2728}$$

$$\frac{2\int \frac{\left(bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}+a\right)^2\sqrt{f+gx}}{\sqrt{d+ex}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{ef-dg}$$

$$\Big\downarrow \text{2614}$$

$$\frac{2\int \left(\frac{2ab\sqrt{f+gx}F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{\sqrt{d+ex}}+\frac{b^2\sqrt{f+gx}F^{\frac{2c\sqrt{d+ex}}{\sqrt{f+gx}}}}{\sqrt{d+ex}}+\frac{a^2\sqrt{f+gx}}{\sqrt{d+ex}}\right)d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{ef-dg}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(a^2\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+2ab\operatorname{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)+b^2\operatorname{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)\right)}{ef-dg}$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^2/(d*f + (e*f + d*g)*x + e
*g*x^2),x]
```

output
```
(2*(2*a*b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]] + b^2*ExpI
ntegralEi[(2*c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]] + a^2*Log[Sqrt[d + e*x
]/Sqrt[f + g*x]]))/(e*f - d*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

rule 2728
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_.) + (B_.)*(x_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*
(g/(C*(e*f - d*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/S
qrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, C, F}, x] && EqQ[C*d
*f - A*e*g, 0] && EqQ[B*e*g - C*(e*f + d*g), 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}\right)^2}{df+(dg+ef)\,x+eg\,x^2}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

Fricas [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^2}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="fricas")
```

output
```
integral((2*F^(sqrt(e*x + d)*c/sqrt(g*x + f))*a*b + F^(2*sqrt(e*x + d)*c/s
qrt(g*x + f))*b^2 + a^2)/(e*g*x^2 + d*f + (e*f + d*g)*x), x)
```

Sympy [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}b+a\right)^2}{(d+ex)\,(f+gx)}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))**2/(d*f+(d*g+e*f)*x+e
*g*x**2),x)
```

output
```
Integral((F**(c*sqrt(d + e*x)/sqrt(f + g*x))*b + a)**2/((d + e*x)*(f + g*x
)), x)
```

Maxima [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^2}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="maxima")
```

output
```
a^2*(log(e*x + d)/(e*f - d*g) - log(g*x + f)/(e*f - d*g)) + b^2*integrate(
F^(2*sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f + d*g)*x), x) +
2*a*b*integrate(F^(sqrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f +
d*g)*x), x)
```

Giac [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^2}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="giac")
```

output
```
integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)^2/(e*g*x^2 + d*f + (e*
f + d*g)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx=\int \frac{\left(a+F^{\frac{c\,\sqrt{d+e\,x}}{\sqrt{f+g\,x}}}\,b\right)^2}{e\,g\,x^2+\left(d\,g+e\,f\right)\,x+d\,f}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^2/(d*f + x*(d*g + e*f)
 + e*g*x^2),x)
```

output
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^2/(d*f + x*(d*g + e*f)
 + e*g*x^2), x)
```

Reduce [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}{df+(ef+dg)x+egx^2}\,dx$$
$$=\frac{2\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)abdg-2\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)abef+\left(\int \frac{f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)b^2dg-\left(\int \frac{\cdots}{eg\cdots}\right.\cdots}{dg-ef}$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
(2*int(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x + e*f*x + e*g*x**
2),x)*a*b*d*g - 2*int(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x +
e*f*x + e*g*x**2),x)*a*b*e*f + int(f**((2*sqrt(d + e*x)*c)/sqrt(f + g*x))/
(d*f + d*g*x + e*f*x + e*g*x**2),x)*b**2*d*g - int(f**((2*sqrt(d + e*x)*c)
/sqrt(f + g*x))/(d*f + d*g*x + e*f*x + e*g*x**2),x)*b**2*e*f - log(d + e*x
)*a**2 + log(f + g*x)*a**2)/(d*g - e*f)
```

3.469 $\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx$

Optimal result

Integrand size = 48, antiderivative size = 70

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx=\frac{2b\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)}{ef-dg}+\frac{2a\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)}{ef-dg}$$

output
```
2*b*Ei(c*(e*x+d)^(1/2)*ln(F)/(g*x+f)^(1/2))/(-d*g+e*f)+2*a*ln((e*x+d)^(1/2
)/(g*x+f)^(1/2))/(-d*g+e*f)
```

Mathematica [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx=\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))/(d*f + (e*f + d*g)*x
 + e*g*x^2),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))/(d*f + (e*f + d*g)*x
 + e*g*x^2), x]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2728, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{x(dg+ef)+df+egx^2}\,dx$$

$$\Big\downarrow \text{2728}$$

$$\frac{2\int \frac{\left(bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}+a\right)\sqrt{f+gx}}{\sqrt{d+ex}}d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{ef-dg}$$

$$\Big\downarrow \text{2010}$$

$$\frac{2\int \left(\frac{b\sqrt{f+gx}F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{\sqrt{d+ex}}+\frac{a\sqrt{f+gx}}{\sqrt{d+ex}}\right)d\frac{\sqrt{d+ex}}{\sqrt{f+gx}}}{ef-dg}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(a\log\left(\frac{\sqrt{d+ex}}{\sqrt{f+gx}}\right)+b\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{f+gx}}\right)\right)}{ef-dg}$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))/(d*f + (e*f + d*g)*x + e*g
*x^2),x]
```

output
```
(2*(b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[f + g*x]] + a*Log[Sqrt[d
 + e*x]/Sqrt[f + g*x]]))/(e*f - d*g)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2728
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_.) + (B_.)*(x_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*
(g/(C*(e*f - d*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/S
qrt[f + g*x]], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, C, F}, x] && EqQ[C*d
*f - A*e*g, 0] && EqQ[B*e*g - C*(e*f + d*g), 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}}{df+(dg+ef)\,x+eg\,x^{2}}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

Fricas [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^{2}}\,dx=\int \frac{F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a}{egx^{2}+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2
),x, algorithm="fricas")
```

output
```
integral((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)/(e*g*x^2 + d*f + (e*f +
 d*g)*x), x)
```

Sympy [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx=\int \frac{F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}b+a}{(d+ex)\,(f+gx)}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))/(d*f+(d*g+e*f)*x+e*g*
x**2),x)
```

output
```
Integral((F**(c*sqrt(d + e*x)/sqrt(f + g*x))*b + a)/((d + e*x)*(f + g*x)),
 x)
```

Maxima [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx=\int \frac{F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2
),x, algorithm="maxima")
```

output
```
a*(log(e*x + d)/(e*f - d*g) - log(g*x + f)/(e*f - d*g)) + b*integrate(F^(s
qrt(e*x + d)*c/sqrt(g*x + f))/(e*g*x^2 + d*f + (e*f + d*g)*x), x)
```

Giac [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx=\int \frac{F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a}{egx^2+df+(ef+dg)x}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2
),x, algorithm="giac")
```

output
```
integrate((F^(sqrt(e*x + d)*c/sqrt(g*x + f))*b + a)/(e*g*x^2 + d*f + (e*f
+ d*g)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx=\int \frac{a+F^{\frac{c\sqrt{d+e\,x}}{\sqrt{f+g\,x}}}\,b}{e\,g\,x^2+(d\,g+e\,f)\,x+d\,f}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)/(d*f + x*(d*g + e*f) +
 e*g*x^2),x)
```

output
```
int((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)/(d*f + x*(d*g + e*f) +
 e*g*x^2), x)
```

Reduce [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}}{df+(ef+dg)x+egx^2}\,dx$$
$$=\frac{\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)bdg-\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}}{eg\,x^2+dgx+efx+df}dx\right)bef-\log(ex+d)\,a+\log(gx+f)\,a}{dg-ef}$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
(int(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x + e*f*x + e*g*x**2)
,x)*b*d*g - int(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))/(d*f + d*g*x + e*f*x
+ e*g*x**2),x)*b*e*f - log(d + e*x)*a + log(f + g*x)*a)/(d*g - e*f)
```

3.470 $\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 50

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx = \operatorname{Int}\left(\frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)},x\right)$$

output
```
Defer(Int)(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*
x^2),x)
```

Mathematica [N/A]

Not integrable

Time = 3.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx$$

input
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))*(d*f + (e*f + d*g
)*x + e*g*x^2)),x]
```

output
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))*(d*f + (e*f + d*g
)*x + e*g*x^2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2730}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(dg+ef)+df+egx^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)}dx$$

$$\Big\downarrow \text{2730}$$

$$\int \frac{1}{(x(dg+ef)+df+egx^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)}dx$$

input
```
Int[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))*(d*f + (e*f + d*g)*x +
e*g*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2730
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_)/((A_.) + (B_.)*(x_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrab
le[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(A + B*x + C*x^2), x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[B*e*g - C*(e*f + d*g), 0] && !IGtQ[n, 0]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}\right)\left(df+\left(dg+ef\right)x+eg\,x^{2}\right)}dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{(egx^2+df+(ef+dg)x)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="fricas")
```

output
```
integral(1/(a*e*g*x^2 + a*d*f + (b*e*g*x^2 + b*d*f + (b*e*f + b*d*g)*x)*F^
(sqrt(e*x + d)*c/sqrt(g*x + f)) + (a*e*f + a*d*g)*x), x)
```

Sympy [N/A]

Not integrable

Time = 7.55 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{(d+ex)(f+gx)\left(F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))/(d*f+(d*g+e*f)*x+e*
g*x**2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)*(F**(c*sqrt(d + e*x)/sqrt(f + g*x))*b + a)
), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx$$
$$=\int \frac{1}{(egx^2+df+(ef+dg)x)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="maxima")
```

output
```
integrate(1/((e*g*x^2 + d*f + (e*f + d*g)*x)*(F^(sqrt(e*x + d)*c/sqrt(g*x
+ f))*b + a)), x)
```

Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx$$
$$=\int \frac{1}{(egx^2+df+(ef+dg)x)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x
^2),x, algorithm="giac")
```

output
```
integrate(1/((e*g*x^2 + d*f + (e*f + d*g)*x)*(F^(sqrt(e*x + d)*c/sqrt(g*x
+ f))*b + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{\left(a+F^{\frac{c\,\sqrt{d+e\,x}}{\sqrt{f+g\,x}}}\,b\right)\,\left(e\,g\,x^2+\left(d\,g+e\,f\right)\,x+d\,f\right)}\,dx$$

input
```
int(1/((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)*(d*f + x*(d*g + e*f
) + e*g*x^2)),x)
```

output
```
int(1/((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)*(d*f + x*(d*g + e*f
) + e*g*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)(df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}bdf+f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}bdgx+f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}befx+f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}beg\,x^2+adf+adgx+aefx+aeg\,x^2}\,dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))/(d*f+(d*g+e*f)*x+e*g*x^2),x)
```

output
```
int(1/(f**((sqrt(d + e*x)*c)/sqrt(f + g*x))*b*d*f + f**((sqrt(d + e*x)*c)/
sqrt(f + g*x))*b*d*g*x + f**((sqrt(d + e*x)*c)/sqrt(f + g*x))*b*e*f*x + f*
*((sqrt(d + e*x)*c)/sqrt(f + g*x))*b*e*g*x**2 + a*d*f + a*d*g*x + a*e*f*x
+ a*e*g*x**2),x)
```

3.471 $\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx$

Optimal result

Integrand size = 50, antiderivative size = 50

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx = \text{Int}\left(\frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)},x\right)$$

output
```
Defer(Int)(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*
g*x^2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx$$

input
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^2*(d*f + (e*f + d
*g)*x + e*g*x^2)),x]
```

output
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^2*(d*f + (e*f + d
*g)*x + e*g*x^2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2730}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x(dg+ef)+df+egx^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}dx$$

$$\Big\downarrow \text{2730}$$

$$\int \frac{1}{(x(dg+ef)+df+egx^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2}dx$$

input
```
Int[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^2*(d*f + (e*f + d*g)*x
+ e*g*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2730
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_)/((A_.) + (B_.)*(x_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrab
le[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(A + B*x + C*x^2), x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && E
qQ[B*e*g - C*(e*f + d*g), 0] &&  !IGtQ[n, 0]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{gx+f}}}\right)^{2}\left(df+\left(dg+ef\right)x+eg\,x^{2}\right)}dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^2),
x)
```

output
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^2),
x)
```

Fricas [N/A]

Not integrable

Time = 7.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.66

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx$$
$$=\int \frac{1}{(egx^2+df+(ef+dg)x)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g
*x^2),x, algorithm="fricas")
```

output
```
integral(1/(a^2*e*g*x^2 + a^2*d*f + (b^2*e*g*x^2 + b^2*d*f + (b^2*e*f + b^
2*d*g)*x)*F^(2*sqrt(e*x + d)*c/sqrt(g*x + f)) + 2*(a*b*e*g*x^2 + a*b*d*f +
 (a*b*e*f + a*b*d*g)*x)*F^(sqrt(e*x + d)*c/sqrt(g*x + f)) + (a^2*e*f + a^2
*d*g)*x), x)
```

Sympy [N/A]

Not integrable

Time = 66.55 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx$$
$$=\int \frac{1}{(d+ex)\,(f+gx)\left(F^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*F**(c*(e*x+d)**(1/2)/(g*x+f)**(1/2)))**2/(d*f+(d*g+e*f)*x
+e*g*x**2),x)
```

output
```
Integral(1/((d + e*x)*(f + g*x)*(F**(c*sqrt(d + e*x)/sqrt(f + g*x))*b + a)
**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 4.08

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{(egx^2+df+(ef+dg)x)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g
*x^2),x, algorithm="maxima")
```

output
```
2*sqrt(g*x + f)/((e*f - d*g)*sqrt(e*x + d)*F^(sqrt(e*x + d)*c/sqrt(g*x + f
))*a*b*c*log(F) + (e*f - d*g)*sqrt(e*x + d)*a^2*c*log(F)) + integrate((sqr
t(e*x + d)*c*log(F) + sqrt(g*x + f))/((a*b*c*e*g*x^2*log(F) + a*b*c*d*f*lo
g(F) + (e*f + d*g)*a*b*c*x*log(F))*sqrt(e*x + d)*F^(sqrt(e*x + d)*c/sqrt(g
*x + f)) + (a^2*c*e*g*x^2*log(F) + a^2*c*d*f*log(F) + (e*f + d*g)*a^2*c*x*
log(F))*sqrt(e*x + d)), x)
```

Giac [N/A]

Not integrable

Time = 5.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{(egx^2+df+(ef+dg)x)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{gx+f}}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g
*x^2),x, algorithm="giac")
```

output
```
integrate(1/((e*g*x^2 + d*f + (e*f + d*g)*x)*(F^(sqrt(e*x + d)*c/sqrt(g*x
+ f))*b + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{\left(a+F^{\frac{c\sqrt{d+e\,x}}{\sqrt{f+g\,x}}}\,b\right)^2\,\left(e\,g\,x^2+\left(d\,g+e\,f\right)\,x+d\,f\right)}\,dx$$

input
```
int(1/((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^2*(d*f + x*(d*g + e
*f) + e*g*x^2)),x)
```

output
```
int(1/((a + F^((c*(d + e*x)^(1/2))/(f + g*x)^(1/2))*b)^2*(d*f + x*(d*g + e
*f) + e*g*x^2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.80

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{f+gx}}}\right)^2 (df+(ef+dg)x+egx^2)}\,dx = \int \frac{1}{f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{gx+f}}}b^2df+f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{gx+f}}}b^2dgx+f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{gx+f}}}b^2efx+f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{gx+f}}}b^2eg\,x^2+2f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}abdf+2f^{\frac{\sqrt{ex+d}\,c}{\sqrt{gx+f}}}abdgx+}$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(g*x+f)^(1/2)))^2/(d*f+(d*g+e*f)*x+e*g*x^2),
x)
```

output

```
int(1/(f**((2*sqrt(d + e*x)*c)/sqrt(f + g*x))*b**2*d*f + f**((2*sqrt(d + e
*x)*c)/sqrt(f + g*x))*b**2*d*g*x + f**((2*sqrt(d + e*x)*c)/sqrt(f + g*x))*
b**2*e*f*x + f**((2*sqrt(d + e*x)*c)/sqrt(f + g*x))*b**2*e*g*x**2 + 2*f**(
(sqrt(d + e*x)*c)/sqrt(f + g*x))*a*b*d*f + 2*f**((sqrt(d + e*x)*c)/sqrt(f
+ g*x))*a*b*d*g*x + 2*f**((sqrt(d + e*x)*c)/sqrt(f + g*x))*a*b*e*f*x + 2*f
**((sqrt(d + e*x)*c)/sqrt(f + g*x))*a*b*e*g*x**2 + a**2*d*f + a**2*d*g*x +
 a**2*e*f*x + a**2*e*g*x**2),x)
```

3.472 $\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 47

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx=\text{Int}\left(\frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2},x\right)$$

output
```
Defer(Int)((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x
)
```

Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx=\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^n/(d^2 - e^2*x^2
),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^n/(d^2 - e^2*x^2
), x]
```

Rubi [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2731}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}dx$$

$$\Big\downarrow \; 2731$$

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}dx$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^n/(d^2 - e^2*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2731
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F^((
c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e,
 f, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !I
GtQ[n, 0]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\,F^{\frac{c\sqrt{e x +d}}{\sqrt{-e f x +d f}}}\right)^{n}}{-e^{2} x^{2}+d^{2}}d x$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x,
 algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   algl
ogextint: unimplemented
```

Sympy [N/A]

Not integrable

Time = 115.98 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx = -\int \frac{\left(F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}b+a\right)^n}{-d^2+e^2x^2}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))**n/(-e**2*x**2+d
**2),x)
```

output
```
-Integral((F**(c*sqrt(d + e*x)/sqrt(d*f - e*f*x))*b + a)**n/(-d**2 + e**2*
x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx = \int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^n}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x,
 algorithm="maxima")
```

output
```
-integrate((F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a)^n/(e^2*x^2 - d^2
), x)
```

Giac [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^n}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x,
 algorithm="giac")
```

output
```
integrate(-(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a)^n/(e^2*x^2 - d^2
), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx=\int \frac{{\left(a+b\,{\mathrm{e}}^{\frac{c\,\ln\left(F\right)\,\sqrt{d+e\,x}}{\sqrt{d\,f-e\,f\,x}}}\right)}^n}{d^2-e^2\,x^2}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2))*b)^n/(d^2 - e^2*x^2),
x)
```

output
```
int((a + b*exp((c*log(F)*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2)))^n/(d^2 - e
^2*x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^n}{d^2-e^2x^2}\,dx = \int \frac{\left(f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}b+a\right)^n}{-e^2x^2+d^2}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^n/(-e^2*x^2+d^2),x)
```

output
```
int((f**((sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))*b + a)**n/(d**2 - e**2
*x**2),x)
```

3.473 $\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 152

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=\frac{3a^2b\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}+\frac{3ab^2\,\text{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}+\frac{b^3\,\text{ExpIntegralEi}\left(\frac{3c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}+\frac{a^3\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de}$$

output
```
3*a^2*b*Ei(c*(e*x+d)^(1/2)*ln(F)/(-e*f*x+d*f)^(1/2))/d/e+3*a*b^2*Ei(2*c*(e
*x+d)^(1/2)*ln(F)/(-e*f*x+d*f)^(1/2))/d/e+b^3*Ei(3*c*(e*x+d)^(1/2)*ln(F)/(
-e*f*x+d*f)^(1/2))/d/e+a^3*ln((e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2))/d/e
```

Mathematica [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^3/(d^2 - e^2*x^2
),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^3/(d^2 - e^2*x^2
), x]
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}=0.064$, Rules used = {2729, 2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx$$

$$\Big\downarrow \text{ 2729}$$

$$\frac{\int \frac{\left(bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}+a\right)^3\sqrt{df-efx}}{\sqrt{d+ex}}d\frac{\sqrt{d+ex}}{\sqrt{df-efx}}}{de}$$

$$\Big\downarrow \text{ 2614}$$

$$\frac{\int \left(\frac{3a^2b\sqrt{df-efx}F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{\sqrt{d+ex}}+\frac{3ab^2\sqrt{df-efx}F^{\frac{2c\sqrt{d+ex}}{\sqrt{df-efx}}}}{\sqrt{d+ex}}+\frac{b^3\sqrt{df-efx}F^{\frac{3c\sqrt{d+ex}}{\sqrt{df-efx}}}}{\sqrt{d+ex}}+\frac{a^3\sqrt{df-efx}}{\sqrt{d+ex}}\right)d\frac{\sqrt{d+ex}}{\sqrt{df-efx}}}{de}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3 \log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right) + 3a^2 b \,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right) + 3ab^2 \,\text{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right) + b^3 \,\text{ExpIntegralE}}{de}$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^3/(d^2 - e^2*x^2),x]
```

output
```
(3*a^2*b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[d*f - e*f*x]] + 3*a*b
^2*ExpIntegralEi[(2*c*Sqrt[d + e*x]*Log[F])/Sqrt[d*f - e*f*x]] + b^3*ExpIn
tegralEi[(3*c*Sqrt[d + e*x]*Log[F])/Sqrt[d*f - e*f*x]] + a^3*Log[Sqrt[d +
e*x]/Sqrt[d*f - e*f*x]])/(d*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a + b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{-efx+df}}}\right)^3}{-e^2x^2 + d^2}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x)
```

Fricas [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^3}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x,
 algorithm="fricas")
```

output
```
integral(-(a^3 + 3*a^2*b/F^(sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*
f)) + 3*a*b^2/F^(2*sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)) + b^3
/F^(3*sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)))/(e^2*x^2 - d^2),
x)
```

Sympy [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=-\int \frac{a^3}{-d^2+e^2x^2}\,dx-\int \frac{F^{\frac{3c\sqrt{d+ex}}{\sqrt{df-efx}}}b^3}{-d^2+e^2x^2}\,dx-\int \frac{3F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}a^2b}{-d^2+e^2x^2}\,dx-\int \frac{3F^{\frac{2c\sqrt{d+ex}}{\sqrt{df-efx}}}ab^2}{-d^2+e^2x^2}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))**3/(-e**2*x**2+d
**2),x)
```

output
```
-Integral(a**3/(-d**2 + e**2*x**2), x) - Integral(F**(3*c*sqrt(d + e*x)/sq
rt(d*f - e*f*x))*b**3/(-d**2 + e**2*x**2), x) - Integral(3*F**(c*sqrt(d +
e*x)/sqrt(d*f - e*f*x))*a**2*b/(-d**2 + e**2*x**2), x) - Integral(3*F**(2*
c*sqrt(d + e*x)/sqrt(d*f - e*f*x))*a*b**2/(-d**2 + e**2*x**2), x)
```

Maxima [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^3}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x,
 algorithm="maxima")
```

output
```
1/2*a^3*(log(e*x + d)/(d*e) - log(e*x - d)/(d*e)) - b^3*integrate(F^(3*sqr
t(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x) - 3*a*b^2*integ
rate(F^(2*sqrt(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x) -
3*a^2*b*integrate(F^(sqrt(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 -
d^2), x)
```

Giac [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^3}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x,
 algorithm="giac")
```

output
```
integrate(-(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a)^3/(e^2*x^2 - d^2
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx=\int \frac{\left(a+b\,\mathrm{e}^{\frac{c\,\ln(F)\,\sqrt{d+e\,x}}{\sqrt{d\,f-e\,f\,x}}}\right)^3}{d^2-e^2\,x^2}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2))*b)^3/(d^2 - e^2*x^2),
x)
```

output
```
int((a + b*exp((c*log(F)*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2)))^3/(d^2 - e
^2*x^2), x)
```

Reduce [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^3}{d^2-e^2x^2}\,dx = \frac{6\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}}{-e^2x^2+d^2}dx\right)a^2bde+2\left(\int \frac{f^{\frac{3\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}}{-e^2x^2+d^2}dx\right)b^3de+6\left(\int \frac{f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}}{-e^2x^2+d^2}dx\right)a\,b^2de+\log(-ex-d)\,a^3-}{2de}$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^3/(-e^2*x^2+d^2),x)
```

output
```
(6*int(f**((sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))/(d**2 - e**2*x**2),x
)*a**2*b*d*e + 2*int(f**((3*sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))/(d**
2 - e**2*x**2),x)*b**3*d*e + 6*int(f**((2*sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d
 - e*x)))/(d**2 - e**2*x**2),x)*a*b**2*d*e + log( - d - e*x)*a**3 - log(d
- e*x)*a**3)/(2*d*e)
```

3.474 $\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 110

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx = \frac{2ab\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{b^2\,\text{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de} + \frac{a^2\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de}$$

output
```
2*a*b*Ei(c*(e*x+d)^(1/2)*ln(F)/(-e*f*x+d*f)^(1/2))/d/e+b^2*Ei(2*c*(e*x+d)^
(1/2)*ln(F)/(-e*f*x+d*f)^(1/2))/d/e+a^2*ln((e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2
))/d/e
```

Mathematica [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx=\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^2/(d^2 - e^2*x^2
),x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^2/(d^2 - e^2*x^2
), x]
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.064, Rules used = {2729, 2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx$$

$$\Big\downarrow \ 2729$$

$$\frac{\int \frac{\left(bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}+a\right)^2\sqrt{df-efx}}{\sqrt{d+ex}}d\frac{\sqrt{d+ex}}{\sqrt{df-efx}}}{de}$$

$$\Big\downarrow \ 2614$$

$$\frac{\int \left(\frac{2ab\sqrt{df-efx}F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{\sqrt{d+ex}}+\frac{b^2\sqrt{df-efx}F^{\frac{2c\sqrt{d+ex}}{\sqrt{df-efx}}}}{\sqrt{d+ex}}+\frac{a^2\sqrt{df-efx}}{\sqrt{d+ex}}\right)d\frac{\sqrt{d+ex}}{\sqrt{df-efx}}}{de}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)+2ab\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)+b^2\,\text{ExpIntegralEi}\left(\frac{2c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^2/(d^2 - e^2*x^2),x]
```

output
```
(2*a*b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[d*f - e*f*x]] + b^2*Exp
IntegralEi[(2*c*Sqrt[d + e*x]*Log[F])/Sqrt[d*f - e*f*x]] + a^2*Log[Sqrt[d
+ e*x]/Sqrt[d*f - e*f*x]])/(d*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{-efx+df}}}\right)^{2}}{-e^{2}x^{2}+d^{2}}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x)
```

Fricas [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^2}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x,
 algorithm="fricas")
```

output
```
integral(-(a^2 + 2*a*b/F^(sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)
) + b^2/F^(2*sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)))/(e^2*x^2 -
 d^2), x)
```

Sympy [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx=-\int \frac{a^2}{-d^2+e^2x^2}\,dx-\int \frac{F^{\frac{2c\sqrt{d+ex}}{\sqrt{df-efx}}}b^2}{-d^2+e^2x^2}\,dx-\int \frac{2F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}ab}{-d^2+e^2x^2}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))**2/(-e**2*x**2+d
**2),x)
```

output
```
-Integral(a**2/(-d**2 + e**2*x**2), x) - Integral(F**(2*c*sqrt(d + e*x)/sq
rt(d*f - e*f*x))*b**2/(-d**2 + e**2*x**2), x) - Integral(2*F**(c*sqrt(d +
e*x)/sqrt(d*f - e*f*x))*a*b/(-d**2 + e**2*x**2), x)
```

Maxima [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^2}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x,
 algorithm="maxima")
```

output
```
1/2*a^2*(log(e*x + d)/(d*e) - log(e*x - d)/(d*e)) - b^2*integrate(F^(2*sqr
t(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x) - 2*a*b*integra
te(F^(sqrt(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x)
```

Giac [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx=\int -\frac{\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^2}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x,
 algorithm="giac")
```

output
```
integrate(-(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a)^2/(e^2*x^2 - d^2
), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx=\int \frac{\left(a+b\,\mathrm{e}^{\frac{c\,\ln(F)\,\sqrt{d+e\,x}}{\sqrt{d\,f-e\,f\,x}}}\right)^2}{d^2-e^2\,x^2}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2))*b)^2/(d^2 - e^2*x^2),
x)
```

output
```
int((a + b*exp((c*log(F)*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2)))^2/(d^2 - e
^2*x^2), x)
```

Reduce [F]

$$\int \frac{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}{d^2-e^2x^2}\,dx$$
$$=\frac{4\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}}{-e^2x^2+d^2}dx\right)abde+2\left(\int \frac{f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}}{-e^2x^2+d^2}dx\right)b^2de+\log(-ex-d)\,a^2-\log(-ex+d)\,a^2}{2de}$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x)
```

output
```
(4*int(f**((sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))/(d**2 - e**2*x**2),x
)*a*b*d*e + 2*int(f**((2*sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))/(d**2 -
 e**2*x**2),x)*b**2*d*e + log( - d - e*x)*a**2 - log(d - e*x)*a**2)/(2*d*e
)
```

3.475 $\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx$

Optimal result

Integrand size = 45, antiderivative size = 68

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx=\frac{b\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}+\frac{a\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)}{de}$$

output
```
b*Ei(c*(e*x+d)^(1/2)*ln(F)/(-e*f*x+d*f)^(1/2))/d/e+a*ln((e*x+d)^(1/2)/(-e*
f*x+d*f)^(1/2))/d/e
```

Mathematica [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx=\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx$$

input
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))/(d^2 - e^2*x^2),
x]
```

output
```
Integrate[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))/(d^2 - e^2*x^2),
 x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2729, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx$$

$$\Big\downarrow \text{2729}$$

$$\frac{\int \frac{\left(bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}+a\right)\sqrt{df-efx}}{\sqrt{d+ex}}d\frac{\sqrt{d+ex}}{\sqrt{df-efx}}}{de}$$

$$\Big\downarrow \text{2010}$$

$$\frac{\int \left(\frac{b\sqrt{df-efx}F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{\sqrt{d+ex}}+\frac{a\sqrt{df-efx}}{\sqrt{d+ex}}\right)d\frac{\sqrt{d+ex}}{\sqrt{df-efx}}}{de}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log\left(\frac{\sqrt{d+ex}}{\sqrt{df-efx}}\right)+b\,\text{ExpIntegralEi}\left(\frac{c\sqrt{d+ex}\log(F)}{\sqrt{df-efx}}\right)}{de}$$

input
```
Int[(a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))/(d^2 - e^2*x^2),x]
```

output
```
(b*ExpIntegralEi[(c*Sqrt[d + e*x]*Log[F])/Sqrt[d*f - e*f*x]] + a*Log[Sqrt[
d + e*x]/Sqrt[d*f - e*f*x]])/(d*e)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{-efx+df}}}}{-e^{2}x^{2}+d^{2}}dx$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x)
```

output
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x)
```

Fricas [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx = \int -\frac{F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a}{e^{2}x^{2}-d^{2}}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x, a
lgorithm="fricas")
```

output
```
integral(-(a + b/F^(sqrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)))/(e^
2*x^2 - d^2), x)
```

Sympy [F]

$$\int \frac{a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2 - e^2x^2}\,dx = -\int \frac{a}{-d^2 + e^2x^2}\,dx - \int \frac{F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}b}{-d^2 + e^2x^2}\,dx$$

input
```
integrate((a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))/(-e**2*x**2+d**2
),x)
```

output
```
-Integral(a/(-d**2 + e**2*x**2), x) - Integral(F**(c*sqrt(d + e*x)/sqrt(d*
f - e*f*x))*b/(-d**2 + e**2*x**2), x)
```

Maxima [F]

$$\int \frac{a + bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2 - e^2x^2}\,dx = \int -\frac{F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b + a}{e^2x^2 - d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x, a
lgorithm="maxima")
```

output
```
1/2*a*(log(e*x + d)/(d*e) - log(e*x - d)/(d*e)) - b*integrate(F^(sqrt(e*x
+ d)*c/(sqrt(-e*x + d)*sqrt(f)))/(e^2*x^2 - d^2), x)
```

Giac [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx=\int -\frac{F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a}{e^2x^2-d^2}\,dx$$

input
```
integrate((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x, a
lgorithm="giac")
```

output
```
integrate(-(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b + a)/(e^2*x^2 - d^2),
 x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx=\int \frac{a+b\,\mathrm{e}^{\frac{c\,\ln(F)\,\sqrt{d+e\,x}}{\sqrt{d\,f-e\,f\,x}}}}{d^2-e^2\,x^2}\,dx$$

input
```
int((a + F^((c*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2))*b)/(d^2 - e^2*x^2),x)
```

output
```
int((a + b*exp((c*log(F)*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2)))/(d^2 - e^2
*x^2), x)
```

Reduce [F]

$$\int \frac{a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}}{d^2-e^2x^2}\,dx=\frac{2\left(\int \frac{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}}{-e^2x^2+d^2}dx\right)bde+\log(-ex-d)\,a-\log(-ex+d)\,a}{2de}$$

input
```
int((a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x)
```

output
```
(2*int(f**((sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))/(d**2 - e**2*x**2),x
)*b*d*e + log( - d - e*x)*a - log(d - e*x)*a)/(2*d*e)
```

3.476 $\int \frac{1}{d^2-e^2x^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{d^2 - e^2x^2} dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)}{de}$$

output
```
arctanh(e*x/d)/d/e
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{d^2 - e^2x^2} dx = \frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)}{de}$$

input
```
Integrate[(d^2 - e^2*x^2)^(-1),x]
```

output
```
ArcTanh[(e*x)/d]/(d*e)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{221\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{d^2-e^2x^2}\,dx$$
$$\Big\downarrow \text{ 221}$$
$$\frac{\operatorname{arctanh}\left(\frac{ex}{d}\right)}{de}$$

input
```
Int[(d^2 - e^2*x^2)^(-1),x]
```

output
```
ArcTanh[(e*x)/d]/(d*e)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

method	result	size
parallelrisch	$-\frac{\ln(ex-d)-\ln(ex+d)}{2de}$	26
default	$-\frac{\ln(-ex+d)}{2de}+\frac{\ln(ex+d)}{2de}$	31
norman	$-\frac{\ln(-ex+d)}{2de}+\frac{\ln(ex+d)}{2de}$	31
risch	$-\frac{\ln(-ex+d)}{2de}+\frac{\ln(ex+d)}{2de}$	31

input
```
int(1/(-e^2*x^2+d^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(ln(e*x-d)-ln(e*x+d))/d/e
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{d^2 - e^2x^2}\,dx = \frac{\log\left(ex + d\right) - \log\left(ex - d\right)}{2\,de}$$

input
```
integrate(1/(-e^2*x^2+d^2),x, algorithm="fricas")
```

output
```
1/2*(log(e*x + d) - log(e*x - d))/(d*e)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{d^2 - e^2x^2}\,dx = -\frac{\frac{\log\left(-\frac{d}{e}+x\right)}{2} - \frac{\log\left(\frac{d}{e}+x\right)}{2}}{de}$$

input
```
integrate(1/(-e**2*x**2+d**2),x)
```

output
```
-(log(-d/e + x)/2 - log(d/e + x)/2)/(d*e)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{1}{d^2-e^2x^2}\,dx = \frac{\log(ex+d)}{2\,de} - \frac{\log(ex-d)}{2\,de}$$

input
```
integrate(1/(-e^2*x^2+d^2),x, algorithm="maxima")
```

output
```
1/2*log(e*x + d)/(d*e) - 1/2*log(e*x - d)/(d*e)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{1}{d^2-e^2x^2}\,dx = \frac{\log(|ex+d|)}{2\,de} - \frac{\log(|ex-d|)}{2\,de}$$

input
```
integrate(1/(-e^2*x^2+d^2),x, algorithm="giac")
```

output
```
1/2*log(abs(e*x + d))/(d*e) - 1/2*log(abs(e*x - d))/(d*e)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{d^2-e^2x^2}\,dx = \frac{\mathrm{atanh}\left(\frac{e\,x}{d}\right)}{d\,e}$$

input
```
int(1/(d^2 - e^2*x^2),x)
```

output
```
atanh((e*x)/d)/(d*e)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{1}{d^2 - e^2x^2}\, dx = \frac{\log(-ex - d) - \log(-ex + d)}{2de}$$

input
```
int(1/(-e^2*x^2+d^2),x)
```

output
```
(log( - d - e*x) - log(d - e*x))/(2*d*e)
```

3.477 $\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 47

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx = \text{Int}\left(\frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)},x\right)$$

output
```
Defer(Int)(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x
)
```

Mathematica [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx = \int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx$$

input
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))*(d^2 - e^2*x^
2)),x]
```

output
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))*(d^2 - e^2*x^
2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2731}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d^2-e^2x^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)}dx$$

$$\Big\downarrow \text{2731}$$

$$\int \frac{1}{(d^2-e^2x^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)}dx$$

input
```
Int[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))*(d^2 - e^2*x^2)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2731
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F^((
c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e,
 f, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !I
GtQ[n, 0]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{-efx+df}}}\right)\left(-e^{2}x^{2}+d^{2}\right)}dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x)
```

output
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx=\int -\frac{1}{(e^2x^2-d^2)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x,
 algorithm="fricas")
```

output
```
integral(-1/(a*e^2*x^2 - a*d^2 + (b*e^2*x^2 - b*d^2)/F^(sqrt(-e*f*x + d*f)
*sqrt(e*x + d)*c/(e*f*x - d*f))), x)
```

Sympy [N/A]

Not integrable

Time = 11.74 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx = -\int \frac{1}{-F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}bd^2+F^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}be^2x^2-ad^2+ae^2x^2}\,dx$$

input
```
integrate(1/(a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))/(-e**2*x**2+d*
*2),x)
```

output
```
-Integral(1/(-F**(c*sqrt(d + e*x)/sqrt(d*f - e*f*x))*b*d**2 + F**(c*sqrt(d
 + e*x)/sqrt(d*f - e*f*x))*b*e**2*x**2 - a*d**2 + a*e**2*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx = \int -\frac{1}{(e^2x^2-d^2)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x,
 algorithm="maxima")
```

output
```
-integrate(1/((e^2*x^2 - d^2)*(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b +
a)), x)
```

Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx=\int -\frac{1}{(e^2x^2-d^2)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x,
 algorithm="giac")
```

output
```
integrate(-1/((e^2*x^2 - d^2)*(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b +
a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx=\int \frac{1}{(d^2-e^2\,x^2)\,\left(a+b\,\mathrm{e}^{\frac{c\,\ln(F)\,\sqrt{d+e\,x}}{\sqrt{d\,f-e\,f\,x}}}\right)}\,dx$$

input
```
int(1/((d^2 - e^2*x^2)*(a + F^((c*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2))*b)
),x)
```

output
```
int(1/((d^2 - e^2*x^2)*(a + b*exp((c*log(F)*(d + e*x)^(1/2))/(d*f - e*f*x)
^(1/2)))), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)(d^2-e^2x^2)}\,dx$$
$$=\int \frac{1}{f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}b\,d^2-f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}}b\,e^2x^2+a\,d^2-a\,e^2x^2}dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))/(-e^2*x^2+d^2),x)
```

output
```
int(1/(f**((sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))*b*d**2 - f**((sqrt(d
 + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))*b*e**2*x**2 + a*d**2 - a*e**2*x**2),x)
```

3.478 $\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 47

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx = \text{Int}\left(\frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)},x\right)$$

output
```
Defer(Int)(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2)
,x)
```

Mathematica [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx = \int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx$$

input
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^2*(d^2 - e^2*
x^2)),x]
```

output
```
Integrate[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^2*(d^2 - e^2*
x^2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2731}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d^2-e^2x^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}dx$$

$$\Big\downarrow \text{2731}$$

$$\int \frac{1}{(d^2-e^2x^2)\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2}dx$$

input
```
Int[1/((a + b*F^((c*Sqrt[d + e*x])/Sqrt[d*f - e*f*x]))^2*(d^2 - e^2*x^2)),
x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2731
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F^((
c*Sqrt[d + e*x])/Sqrt[f + g*x]))^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e,
 f, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !I
GtQ[n, 0]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(a+b\,F^{\frac{c\sqrt{ex+d}}{\sqrt{-efx+df}}}\right)^{2}\left(-e^{2}x^{2}+d^{2}\right)}dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x)
```

output
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x)
```

Fricas [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.85

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^{2}(d^{2}-e^{2}x^{2})}\,dx=\int -\frac{1}{(e^{2}x^{2}-d^{2})\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^{2}}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),
x, algorithm="fricas")
```

output
```
integral(-1/(a^2*e^2*x^2 - a^2*d^2 + 2*(a*b*e^2*x^2 - a*b*d^2)/F^(sqrt(-e*
f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f)) + (b^2*e^2*x^2 - b^2*d^2)/F^(2*s
qrt(-e*f*x + d*f)*sqrt(e*x + d)*c/(e*f*x - d*f))), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2(d^2-e^2x^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*F**(c*(e*x+d)**(1/2)/(-e*f*x+d*f)**(1/2)))**2/(-e**2*x**2
+d**2),x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.98

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2(d^2-e^2x^2)}\,dx = \int -\frac{1}{(e^2x^2-d^2)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),
x, algorithm="maxima")
```

output
```
sqrt(-e*x + d)*sqrt(f)/(sqrt(e*x + d)*F^(sqrt(e*x + d)*c/(sqrt(-e*x + d)*s
qrt(f)))*a*b*c*d*e*log(F) + sqrt(e*x + d)*a^2*c*d*e*log(F)) - integrate((s
qrt(e*x + d)*c*log(F) + sqrt(-e*x + d)*sqrt(f))/((a*b*c*e^2*x^2*log(F) - a
*b*c*d^2*log(F))*sqrt(e*x + d)*F^(sqrt(e*x + d)*c/(sqrt(-e*x + d)*sqrt(f))
) + (a^2*c*e^2*x^2*log(F) - a^2*c*d^2*log(F))*sqrt(e*x + d)), x)
```

Giac [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx = \int -\frac{1}{(e^2x^2-d^2)\left(F^{\frac{\sqrt{ex+d}c}{\sqrt{-efx+df}}}b+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),
x, algorithm="giac")
```

output
```
integrate(-1/((e^2*x^2 - d^2)*(F^(sqrt(e*x + d)*c/sqrt(-e*f*x + d*f))*b +
a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx = \int \frac{1}{(d^2-e^2\,x^2)\,\left(a+b\,\mathrm{e}^{\frac{c\,\ln(F)\,\sqrt{d+e\,x}}{\sqrt{d\,f-e\,f\,x}}}\right)^2}\,dx$$

input
```
int(1/((d^2 - e^2*x^2)*(a + F^((c*(d + e*x)^(1/2))/(d*f - e*f*x)^(1/2))*b)
^2),x)
```

output
```
int(1/((d^2 - e^2*x^2)*(a + b*exp((c*log(F)*(d + e*x)^(1/2))/(d*f - e*f*x)
^(1/2)))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.23

$$\int \frac{1}{\left(a+bF^{\frac{c\sqrt{d+ex}}{\sqrt{df-efx}}}\right)^2 (d^2-e^2x^2)}\,dx$$
$$= \int \frac{1}{f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}} b^2 d^2 - f^{\frac{2\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}} b^2 e^2 x^2 + 2 f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}} ab\, d^2 - 2 f^{\frac{\sqrt{ex+d}\,c}{\sqrt{f}\,\sqrt{-ex+d}}} ab\, e^2 x^2 + a^2 d^2 - a^2 e^2 x^2}dx$$

input
```
int(1/(a+b*F^(c*(e*x+d)^(1/2)/(-e*f*x+d*f)^(1/2)))^2/(-e^2*x^2+d^2),x)
```

output
```
int(1/(f**((2*sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))*b**2*d**2 - f**((2
*sqrt(d + e*x)*c)/(sqrt(f)*sqrt(d - e*x)))*b**2*e**2*x**2 + 2*f**((sqrt(d
+ e*x)*c)/(sqrt(f)*sqrt(d - e*x)))*a*b*d**2 - 2*f**((sqrt(d + e*x)*c)/(sqr
t(f)*sqrt(d - e*x)))*a*b*e**2*x**2 + a**2*d**2 - a**2*e**2*x**2),x)
```

3.479 $\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 77

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = -\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{1+ax}}}\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n \text{ExpIntegralEi}\left(\frac{n\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

output
```
-(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n*Ei(n*(-a*x+1)^(1/2)*ln(F)/(a*x+1)^(1
/2))/a/(F^(n*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))
```

Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = -\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{1+ax}}}\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n \text{ExpIntegralEi}\left(\frac{n\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[(F^(Sqrt[1 - a*x]/Sqrt[1 + a*x]))^n/(1 - a^2*x^2),x]
```

output
```
-(((F^(Sqrt[1 - a*x]/Sqrt[1 + a*x]))^n*ExpIntegralEi[(n*Sqrt[1 - a*x]*Log[
F])/Sqrt[1 + a*x]])/(a*F^((n*Sqrt[1 - a*x])/Sqrt[1 + a*x])))
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2717, 2729, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}\right)^n}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 2717}$$

$$F^{-\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}}\left(F^{\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}\right)^n\int \frac{F^{\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}}}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 2729}$$

$$-\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}}\left(F^{\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}\right)^n\int \frac{F^{\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}}\sqrt{ax+1}}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 2609}$$

$$-\frac{F^{-\frac{n\sqrt{1-ax}}{\sqrt{ax+1}}}\left(F^{\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}\right)^n\text{ExpIntegralEi}\left(\frac{n\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[(F^(Sqrt[1 - a*x]/Sqrt[1 + a*x]))^n/(1 - a^2*x^2),x]
```

output
```
-(((F^(Sqrt[1 - a*x]/Sqrt[1 + a*x]))^n*ExpIntegralEi[(n*Sqrt[1 - a*x]*Log[
F])/Sqrt[1 + a*x]])/(a*F^((n*Sqrt[1 - a*x])/Sqrt[1 + a*x])))
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\left(F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}\right)^{n}}{-a^{2} x^{2}+1}dx$$

input
```
int((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x)
```

output
```
int((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2} \, dx=\int -\frac{\left(F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}\right)^{n}}{a^{2} x^{2}-1}\, dx$$

input
```
integrate((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-(F^(sqrt(-a*x + 1)/sqrt(a*x + 1)))^n/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = -\int \frac{\left(F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}\right)^n}{a^2x^2-1}\,dx$$

input
```
integrate((F**((-a*x+1)**(1/2)/(a*x+1)**(1/2)))**n/(-a**2*x**2+1),x)
```

output
```
-Integral((F**(sqrt(-a*x + 1)/sqrt(a*x + 1)))**n/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = \int -\frac{\left(F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}\right)^n}{a^2x^2-1}\,dx$$

input
```
integrate((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
-integrate(F^(sqrt(-a*x + 1)*n/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = \int -\frac{\left(F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}\right)^n}{a^2x^2-1}\,dx$$

input
```
integrate((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-(F^(sqrt(-a*x + 1)/sqrt(a*x + 1)))^n/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = \int -\frac{\left(F^{\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}}\right)^n}{a^2\,x^2-1}\,dx$$

input
```
int(-(F^((1 - a*x)^(1/2)/(a*x + 1)^(1/2)))^n/(a^2*x^2 - 1),x)
```

output
```
int(-(F^((1 - a*x)^(1/2)/(a*x + 1)^(1/2)))^n/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\left(F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}\right)^n}{1-a^2x^2}\,dx = -\left(\int \frac{f^{\frac{\sqrt{-ax+1}\,n}{\sqrt{ax+1}}}}{a^2x^2-1}dx\right)$$

input
```
int((F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))^n/(-a^2*x^2+1),x)
```

output
```
- int(f**((sqrt( - a*x + 1)*n)/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.480 $\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 29

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\frac{\text{ExpIntegralEi}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Ei(3*(-a*x+1)^(1/2)*ln(F)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\frac{\text{ExpIntegralEi}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[F^((3*Sqrt[1 - a*x])/Sqrt[1 + a*x])/(1 - a^2*x^2),x]
```

output
```
-(ExpIntegralEi[(3*Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2729, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}}}{1-a^2x^2}dx$$

$$\Big\downarrow \text{2729}$$

$$-\frac{\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}}\sqrt{ax+1}}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\text{ExpIntegralEi}\left(\frac{3\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[F^((3*Sqrt[1 - a*x])/Sqrt[1 + a*x])/(1 - a^2*x^2),x]
```

output
```
-(ExpIntegralEi[(3*Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{-a^2x^2+1}dx$$

input
```
int(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-F^(3*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\int \frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F**(3*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(F**(3*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(F^(3*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-F^(3*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{3\sqrt{1-a\,x}}{\sqrt{a\,x+1}}}}{a^2\,x^2-1}\,dx$$

input
```
int(-F^((3*(1 - a*x)^(1/2))/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
int(-F^((3*(1 - a*x)^(1/2))/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{F^{\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\left(\int \frac{f^{\frac{3\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}dx\right)$$

input
```
int(F^(3*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
- int(f**((3*sqrt( - a*x + 1))/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.481 $\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 29

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=-\frac{\text{ExpIntegralEi}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Ei(2*(-a*x+1)^(1/2)*ln(F)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=-\frac{\text{ExpIntegralEi}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[F^((2*Sqrt[1 - a*x])/Sqrt[1 + a*x])/(1 - a^2*x^2),x]
```

output
```
-(ExpIntegralEi[(2*Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2729, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}}}{1-a^2x^2}dx$$

$$\Big\downarrow \text{2729}$$

$$-\frac{\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}}\sqrt{ax+1}}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\text{ExpIntegralEi}\left(\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[F^((2*Sqrt[1 - a*x])/Sqrt[1 + a*x])/(1 - a^2*x^2),x]
```

output
```
-(ExpIntegralEi[(2*Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{-a^2x^2+1}dx$$

input
```
int(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-F^(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\int \frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F**(2*(-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(F**(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(F^(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-F^(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x+1}}}}{a^2\,x^2-1}\,dx$$

input
```
int(-F^((2*(1 - a*x)^(1/2))/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
int(-F^((2*(1 - a*x)^(1/2))/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{F^{\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\left(\int \frac{f^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}dx\right)$$

input
```
int(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
- int(f**((2*sqrt( - a*x + 1))/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.482 $\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 28

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\frac{\text{ExpIntegralEi}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Ei((-a*x+1)^(1/2)*ln(F)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\frac{\text{ExpIntegralEi}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[F^(Sqrt[1 - a*x]/Sqrt[1 + a*x])/(1 - a^2*x^2),x]
```

output
```
-(ExpIntegralEi[(Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2729, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}}{1-a^2x^2}\,dx$$
$$\Big\downarrow \text{2729}$$
$$-\frac{\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}\sqrt{ax+1}}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{2609}$$
$$-\frac{\text{ExpIntegralEi}\left(\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[F^(Sqrt[1 - a*x]/Sqrt[1 + a*x])/(1 - a^2*x^2),x]
```

output
```
-(ExpIntegralEi[(Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{-a^2x^2+1}dx$$

input
```
int(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fric
as")
```

output
```
integral(-F^(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\int \frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F**((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(F**(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="maxi
ma")
```

output
```
-integrate(F^(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}\,dx$$

input
```
integrate(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="giac
")
```

output
```
integrate(-F^(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{F^{\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}}}{a^2\,x^2-1}\,dx$$

input
```
int(-F^((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
int(-F^((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{F^{\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\left(\int \frac{f^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{a^2x^2-1}dx\right)$$

input
```
int(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
- int(f**(sqrt( - a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.483 $\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx$

Optimal result

Integrand size = 36, antiderivative size = 29

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx = -\frac{\text{ExpIntegralEi}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Ei(-(-a*x+1)^(1/2)*ln(F)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2} dx = -\frac{\text{ExpIntegralEi}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[1/(F^(Sqrt[1 - a*x]/Sqrt[1 + a*x])*(1 - a^2*x^2)),x]
```

output
```
-(ExpIntegralEi[-((Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x])]/a)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2729, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 2729}$$

$$-\frac{\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}\sqrt{ax+1}}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 2609}$$

$$-\frac{\text{ExpIntegralEi}\left(-\frac{\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[1/(F^(Sqrt[1 - a*x]/Sqrt[1 + a*x])*(1 - a^2*x^2)),x]
```

output
```
-(ExpIntegralEi[-((Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x])]/a)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{F^{-\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}{-a^2x^2+1}dx$$

input
```
int(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x)
```

output
```
int(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*F^(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [F]

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\int \frac{1}{F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}a^2x^2 - F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F**((-a*x+1)**(1/2)/(a*x+1)**(1/2)))/(-a**2*x**2+1),x)
```

output
```
-Integral(1/(F**(sqrt(-a*x + 1)/sqrt(a*x + 1))*a**2*x**2 - F**(sqrt(-a*x +
 1)/sqrt(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=\int -\frac{1}{(a^2x^2-1)F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
-integrate(1/((a^2*x^2 - 1)*F^(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Giac [F]

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=\int -\frac{1}{(a^2x^2-1)F^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*F^(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=\int -\frac{1}{F^{\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}}\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(F^((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
int(-1/(F^((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{F^{-\frac{\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\left(\int \frac{1}{f^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}a^2x^2 - f^{\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}}}dx\right)$$

input
```
int(1/(F^((-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x)
```

output
```
 - int(1/(f**(sqrt( - a*x + 1)/sqrt(a*x + 1))*a**2*x**2 - f**(sqrt( - a*x
+ 1)/sqrt(a*x + 1))),x)
```

3.484 $\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 29

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=-\frac{\text{ExpIntegralEi}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Ei(-2*(-a*x+1)^(1/2)*ln(F)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx=-\frac{\text{ExpIntegralEi}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[1/(F^((2*Sqrt[1 - a*x])/Sqrt[1 + a*x])*(1 - a^2*x^2)),x]
```

output
```
-(ExpIntegralEi[(-2*Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2729, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}}}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{2729}$$

$$-\frac{\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}}\sqrt{ax+1}}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\text{ExpIntegralEi}\left(-\frac{2\sqrt{1-ax}\log(F)}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[1/(F^((2*Sqrt[1 - a*x])/Sqrt[1 + a*x])*(1 - a^2*x^2)),x]
```

output
```
-(ExpIntegralEi[(-2*Sqrt[1 - a*x]*Log[F])/Sqrt[1 + a*x]]/a)
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2729
```
Int[((a_.) + (b_.)*(F_)^(((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.
)*(x_)]))^(n_.)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F^(c*x))^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
 EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{F^{-\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}{-a^2x^2+1}dx$$

input
```
int(1/(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x)
```

output
```
int(1/(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm
="fricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*F^(2*sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Sympy [F]

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\int \frac{1}{F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}a^2x^2 - F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F**(2*(-a*x+1)**(1/2)/(a*x+1)**(1/2)))/(-a**2*x**2+1),x)
```

output
```
-Integral(1/(F**(2*sqrt(-a*x + 1)/sqrt(a*x + 1))*a**2*x**2 - F**(2*sqrt(-a
*x + 1)/sqrt(a*x + 1))), x)
```

Maxima [F]

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm
="maxima")
```

output
```
-integrate(1/((a^2*x^2 - 1)*F^(2*sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Giac [F]

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)F^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}\,dx$$

input
```
integrate(1/(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x, algorithm
="giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*F^(2*sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = \int -\frac{1}{F^{\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x+1}}}\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(F^((2*(1 - a*x)^(1/2))/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
int(-1/(F^((2*(1 - a*x)^(1/2))/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

Reduce [F]

$$\int \frac{F^{-\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}}}{1-a^2x^2}\,dx = -\left(\int \frac{1}{f^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}a^2x^2 - f^{\frac{2\sqrt{-ax+1}}{\sqrt{ax+1}}}}dx\right)$$

input
```
int(1/(F^(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2)))/(-a^2*x^2+1),x)
```

output
```
- int(1/(f**((2*sqrt( - a*x + 1))/sqrt(a*x + 1))*a**2*x**2 - f**((2*sqrt(
- a*x + 1))/sqrt(a*x + 1))),x)
```

3.485 $\int a^x b^x x^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 49

$$\int a^x b^x x^2 \, dx = \frac{2a^x b^x}{(\log(a)+\log(b))^3} - \frac{2a^x b^x x}{(\log(a)+\log(b))^2} + \frac{a^x b^x x^2}{\log(a)+\log(b)}$$

output
```
2*a^x*b^x/(ln(a)+ln(b))^3-2*a^x*b^x*x/(ln(a)+ln(b))^2+a^x*b^x*x^2/(ln(a)+l
n(b))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int a^x b^x x^2 \, dx = \frac{a^x b^x (2 - 2x(\log(a)+\log(b)) + x^2(\log(a)+\log(b))^2)}{(\log(a)+\log(b))^3}$$

input
```
Integrate[a^x*b^x*x^2,x]
```

output
```
(a^x*b^x*(2 - 2*x*(Log[a] + Log[b]) + x^2*(Log[a] + Log[b])^2))/(Log[a] +
Log[b])^3
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2725, 2607, 2725, 2607, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 a^x b^x \, dx$$

$$\Big\downarrow \text{ 2725}$$

$$\int x^2 e^{x(\log(a)+\log(b))}dx$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{x^2 a^x b^x}{\log(a)+\log(b)}-\frac{2\int a^x b^x x dx}{\log(a)+\log(b)}$$

$$\Big\downarrow \text{ 2725}$$

$$\frac{x^2 a^x b^x}{\log(a)+\log(b)}-\frac{2\int e^{x(\log(a)+\log(b))}x dx}{\log(a)+\log(b)}$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{x^2 a^x b^x}{\log(a)+\log(b)}-\frac{2\left(\frac{x a^x b^x}{\log(a)+\log(b)}-\frac{\int a^x b^x dx}{\log(a)+\log(b)}\right)}{\log(a)+\log(b)}$$

$$\Big\downarrow \text{ 2725}$$

$$\frac{x^2 a^x b^x}{\log(a)+\log(b)}-\frac{2\left(\frac{x a^x b^x}{\log(a)+\log(b)}-\frac{\int e^{x(\log(a)+\log(b))}dx}{\log(a)+\log(b)}\right)}{\log(a)+\log(b)}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{x^2 a^x b^x}{\log(a)+\log(b)}-\frac{2\left(\frac{x a^x b^x}{\log(a)+\log(b)}-\frac{a^x b^x}{(\log(a)+\log(b))^2}\right)}{\log(a)+\log(b)}$$

input
```
Int[a^x*b^x*x^2,x]
```

output
```
(a^x*b^x*x^2)/(Log[a] + Log[b]) - (2*(-((a^x*b^x)/(Log[a] + Log[b])^2) + (
a^x*b^x*x)/(Log[a] + Log[b])))/(Log[a] + Log[b])
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
risch	$\frac{\left(\ln(a)^2 x^2+2\ln(a)\ln(b)x^2+\ln(b)^2 x^2-2\ln(a)x-2\ln(b)x+2\right)a^x b^x}{(\ln(a)+\ln(b))^3}$	52
gosper	$\frac{\left(\ln(a)^2 x^2+2\ln(a)\ln(b)x^2+\ln(b)^2 x^2-2\ln(a)x-2\ln(b)x+2\right)a^x b^x}{(\ln(a)+\ln(b))\left(\ln(a)^2+2\ln(a)\ln(b)+\ln(b)^2\right)}$	69
orering	$\frac{\left(\ln(a)^2 x^2+2\ln(a)\ln(b)x^2+\ln(b)^2 x^2-2\ln(a)x-2\ln(b)x+2\right)a^x b^x}{(\ln(a)+\ln(b))\left(\ln(a)^2+2\ln(a)\ln(b)+\ln(b)^2\right)}$	69
meijerg	$-\frac{2-\frac{\left(3x^2\ln(b)^2\left(1+\frac{\ln(a)}{\ln(b)}\right)^2-6x\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)+6\right)e^{x\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)}}{3}}{\ln(b)^3\left(1+\frac{\ln(a)}{\ln(b)}\right)^3}$	72
norman	$\frac{x^2 e^{\ln(a)x}e^{\ln(b)x}}{\ln(a)+\ln(b)}-\frac{2x\,e^{\ln(a)x}e^{\ln(b)x}}{\ln(a)^2+2\ln(a)\ln(b)+\ln(b)^2}+\frac{2\,e^{\ln(a)x}e^{\ln(b)x}}{\left(\ln(a)^2+2\ln(a)\ln(b)+\ln(b)^2\right)(\ln(a)+\ln(b))}$	89
parallelrisch	$\frac{\ln(a)^2 x^2 a^x b^x+2\ln(a)\ln(b)x^2 a^x b^x+\ln(b)^2 x^2 a^x b^x-2b^x x\,a^x\ln(a)-2b^x x\,a^x\ln(b)+2a^x b^x}{(\ln(a)+\ln(b))\left(\ln(a)^2+2\ln(a)\ln(b)+\ln(b)^2\right)}$	100

input
```
int(a^x*b^x*x^2,x,method=_RETURNVERBOSE)
```

output
```
(ln(a)^2*x^2+2*ln(a)*ln(b)*x^2+ln(b)^2*x^2-2*ln(a)*x-2*ln(b)*x+2)*a^x/(ln(
a)+ln(b))^3*b^x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int a^x b^x x^2\,dx=\frac{\left(x^2\log(a)^2+x^2\log(b)^2-2\,x\log(a)+2\left(x^2\log(a)-x\right)\log(b)+2\right)a^x b^x}{\log(a)^3+3\,\log(a)^2\log(b)+3\,\log(a)\log(b)^2+\log(b)^3}$$

input
```
integrate(a^x*b^x*x^2,x, algorithm="fricas")
```

output
```
(x^2*log(a)^2 + x^2*log(b)^2 - 2*x*log(a) + 2*(x^2*log(a) - x)*log(b) + 2)
*a^x*b^x/(log(a)^3 + 3*log(a)^2*log(b) + 3*log(a)*log(b)^2 + log(b)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(51) = 102$.

Time = 0.69 (sec) , antiderivative size = 583, normalized size of antiderivative = 11.90

$$\int a^x b^x x^2\,dx = \text{Too large to display}$$

input
```
integrate(a**x*b**x*x**2,x)
```

output
```
Piecewise((a**x*b**x*x**2*log(a)**2/(log(a)**3 + 3*log(a)**2*log(b) + 3*lo
g(a)*log(b)**2 + log(b)**3) + 2*a**x*b**x*x**2*log(a)*log(b)/(log(a)**3 +
3*log(a)**2*log(b) + 3*log(a)*log(b)**2 + log(b)**3) + a**x*b**x*x**2*log(
b)**2/(log(a)**3 + 3*log(a)**2*log(b) + 3*log(a)*log(b)**2 + log(b)**3) -
2*a**x*b**x*x*log(a)/(log(a)**3 + 3*log(a)**2*log(b) + 3*log(a)*log(b)**2
+ log(b)**3) - 2*a**x*b**x*x*log(b)/(log(a)**3 + 3*log(a)**2*log(b) + 3*lo
g(a)*log(b)**2 + log(b)**3) + 2*a**x*b**x/(log(a)**3 + 3*log(a)**2*log(b)
+ 3*log(a)*log(b)**2 + log(b)**3), Ne(a, 1/b)), (b**x*x**2*(1/b)**x*log(1/
b)**2/(log(1/b)**3 + 3*log(1/b)**2*log(b) + 3*log(1/b)*log(b)**2 + log(b)*
*3) + 2*b**x*x**2*(1/b)**x*log(1/b)*log(b)/(log(1/b)**3 + 3*log(1/b)**2*lo
g(b) + 3*log(1/b)*log(b)**2 + log(b)**3) + b**x*x**2*(1/b)**x*log(b)**2/(l
og(1/b)**3 + 3*log(1/b)**2*log(b) + 3*log(1/b)*log(b)**2 + log(b)**3) - 2*
b**x*x*(1/b)**x*log(1/b)/(log(1/b)**3 + 3*log(1/b)**2*log(b) + 3*log(1/b)*
log(b)**2 + log(b)**3) - 2*b**x*x*(1/b)**x*log(b)/(log(1/b)**3 + 3*log(1/b
)**2*log(b) + 3*log(1/b)*log(b)**2 + log(b)**3) + 2*b**x*(1/b)**x/(log(1/b
)**3 + 3*log(1/b)**2*log(b) + 3*log(1/b)*log(b)**2 + log(b)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

$$\int a^x b^x x^2\,dx = \frac{\left(\left(\log(a)^2 + 2\log(a)\log(b) + \log(b)^2\right)x^2 - 2x(\log(a) + \log(b)) + 2\right)e^{(x\log(a) + x\log(b))}}{\log(a)^3 + 3\log(a)^2\log(b) + 3\log(a)\log(b)^2 + \log(b)^3}$$

input
```
integrate(a^x*b^x*x^2,x, algorithm="maxima")
```

output
```
((log(a)^2 + 2*log(a)*log(b) + log(b)^2)*x^2 - 2*x*(log(a) + log(b)) + 2)*
e^(x*log(a) + x*log(b))/(log(a)^3 + 3*log(a)^2*log(b) + 3*log(a)*log(b)^2
+ log(b)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 2631, normalized size of antiderivative = 53.69

$$\int a^x b^x x^2 \, dx = \text{Too large to display}$$

input
```
integrate(a^x*b^x*x^2,x, algorithm="giac")
```

output
```
((2*(pi*x^2*log(abs(a))*sgn(a) + pi*x^2*log(abs(b))*sgn(a) + pi*x^2*log(ab
s(a))*sgn(b) + pi*x^2*log(abs(b))*sgn(b) - 2*pi*x^2*log(abs(a)) - 2*pi*x^2
*log(abs(b)) - pi*x*sgn(a) - pi*x*sgn(b) + 2*pi*x)*(3*pi^3*sgn(a)*sgn(b) -
 4*pi^3*sgn(a) + 3*pi*log(abs(a))^2*sgn(a) + 6*pi*log(abs(a))*log(abs(b))*
sgn(a) + 3*pi*log(abs(b))^2*sgn(a) - 4*pi^3*sgn(b) + 3*pi*log(abs(a))^2*sg
n(b) + 6*pi*log(abs(a))*log(abs(b))*sgn(b) + 3*pi*log(abs(b))^2*sgn(b) + 5
*pi^3 - 6*pi*log(abs(a))^2 - 12*pi*log(abs(a))*log(abs(b)) - 6*pi*log(abs(
b))^2)/((3*pi^3*sgn(a)*sgn(b) - 4*pi^3*sgn(a) + 3*pi*log(abs(a))^2*sgn(a)
+ 6*pi*log(abs(a))*log(abs(b))*sgn(a) + 3*pi*log(abs(b))^2*sgn(a) - 4*pi^3
*sgn(b) + 3*pi*log(abs(a))^2*sgn(b) + 6*pi*log(abs(a))*log(abs(b))*sgn(b)
+ 3*pi*log(abs(b))^2*sgn(b) + 5*pi^3 - 6*pi*log(abs(a))^2 - 12*pi*log(abs(
a))*log(abs(b)) - 6*pi*log(abs(b))^2)^2 + (3*pi^2*log(abs(a))*sgn(a)*sgn(b
) + 3*pi^2*log(abs(b))*sgn(a)*sgn(b) - 6*pi^2*log(abs(a))*sgn(a) - 6*pi^2*
log(abs(b))*sgn(a) - 6*pi^2*log(abs(a))*sgn(b) - 6*pi^2*log(abs(b))*sgn(b)
 + 9*pi^2*log(abs(a)) - 2*log(abs(a))^3 + 9*pi^2*log(abs(b)) - 6*log(abs(a
))^2*log(abs(b)) - 6*log(abs(a))*log(abs(b))^2 - 2*log(abs(b))^3)^2) + (pi
^2*x^2*sgn(a)*sgn(b) - 2*pi^2*x^2*sgn(a) - 2*pi^2*x^2*sgn(b) + 3*pi^2*x^2
- 2*x^2*log(abs(a))^2 - 4*x^2*log(abs(a))*log(abs(b)) - 2*x^2*log(abs(b))^
2 + 4*x*log(abs(a)) + 4*x*log(abs(b)) - 4)*(3*pi^2*log(abs(a))*sgn(a)*sgn(
b) + 3*pi^2*log(abs(b))*sgn(a)*sgn(b) - 6*pi^2*log(abs(a))*sgn(a) - 6*p...
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int a^x b^x x^2 \, dx = \frac{a^x \, b^x \, \left(x^2 \, (\ln{(a)} + \ln{(b)})^2 - 2 \, x \, (\ln{(a)} + \ln{(b)}) + 2\right)}{(\ln{(a)} + \ln{(b)})^3}$$

input
```
int(a^x*b^x*x^2,x)
```

output
```
(a^x*b^x*(x^2*(log(a) + log(b))^2 - 2*x*(log(a) + log(b)) + 2))/(log(a) +
log(b))^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int a^x b^x x^2 \, dx = \frac{b^x a^x \left(\log(a)^2 \, x^2 + 2 \log(a) \log(b) \, x^2 - 2 \log(a) \, x + \log(b)^2 \, x^2 - 2 \log(b) \, x + 2\right)}{\log{(a)}^3 + 3\log{(a)}^2 \log{(b)} + 3 \log{(a)} \log{(b)}^2 + \log{(b)}^3}$$

input
```
int(a^x*b^x*x^2,x)
```

output
```
(b**x*a**x*(log(a)**2*x**2 + 2*log(a)*log(b)*x**2 - 2*log(a)*x + log(b)**2
*x**2 - 2*log(b)*x + 2))/(log(a)**3 + 3*log(a)**2*log(b) + 3*log(a)*log(b)
**2 + log(b)**3)
```

3.486 $\int a^x b^x x \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int a^x b^x x \, dx = -\frac{a^x b^x}{(\log(a)+\log(b))^2} + \frac{a^x b^x x}{\log(a)+\log(b)}$$

output
```
-a^x*b^x/(ln(a)+ln(b))^2+a^x*b^x*x/(ln(a)+ln(b))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int a^x b^x x \, dx = a^x b^x \left(-\frac{1}{(\log(a)+\log(b))^2} + \frac{x}{\log(a)+\log(b)}\right)$$

input
```
Integrate[a^x*b^x*x,x]
```

output
```
a^x*b^x*(-(Log[a] + Log[b])^(-2) + x/(Log[a] + Log[b]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2725, 2607, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xa^x b^x \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int xe^{x(\log(a)+\log(b))}dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{xa^x b^x}{\log(a)+\log(b)} - \frac{\int a^x b^x dx}{\log(a)+\log(b)}$$

$$\Big\downarrow \text{2725}$$

$$\frac{xa^x b^x}{\log(a)+\log(b)} - \frac{\int e^{x(\log(a)+\log(b))}dx}{\log(a)+\log(b)}$$

$$\Big\downarrow \text{2624}$$

$$\frac{xa^x b^x}{\log(a)+\log(b)} - \frac{a^x b^x}{(\log(a)+\log(b))^2}$$

input
```
Int[a^x*b^x*x,x]
```

output
```
-((a^x*b^x)/(Log[a] + Log[b])^2) + (a^x*b^x*x)/(Log[a] + Log[b])
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{(\ln(a)x+\ln(b)x-1)a^x b^x}{(\ln(a)+\ln(b))^2}$	25
risch	$\frac{(\ln(a)x+\ln(b)x-1)a^x b^x}{(\ln(a)+\ln(b))^2}$	25
orering	$\frac{(\ln(a)x+\ln(b)x-1)a^x b^x}{(\ln(a)+\ln(b))^2}$	25
parallelrisch	$\frac{b^x x\,a^x \ln(a)+b^x x\,a^x \ln(b)-a^x b^x}{(\ln(a)+\ln(b))^2}$	38
norman	$\frac{x\,e^{\ln(a)x}e^{\ln(b)x}}{\ln(a)+\ln(b)}-\frac{e^{\ln(a)x}e^{\ln(b)x}}{(\ln(a)+\ln(b))^2}$	40
meijerg	$\frac{1-\frac{\left(-2x\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)+2\right)e^{x\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)}}{2}}{\ln(b)^2\left(1+\frac{\ln(a)}{\ln(b)}\right)^2}$	51

input
```
int(a^x*b^x*x,x,method=_RETURNVERBOSE)
```

output
```
(ln(a)*x+ln(b)*x-1)*a^x*b^x/(ln(a)+ln(b))^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int a^x b^x x\,dx = \frac{(x\log(a) + x\log(b) - 1)a^x b^x}{\log(a)^2 + 2\log(a)\log(b) + \log(b)^2}$$

input
```
integrate(a^x*b^x*x,x, algorithm="fricas")
```

output
```
(x*log(a) + x*log(b) - 1)*a^x*b^x/(log(a)^2 + 2*log(a)*log(b) + log(b)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(29) = 58.

Time = 0.41 (sec) , antiderivative size = 190, normalized size of antiderivative = 6.13

$$\int a^x b^x x\,dx = \begin{cases} \frac{a^x b^x x\log(a)}{\log(a)^2+2\log(a)\log(b)+\log(b)^2} + \frac{a^x b^x x\log(b)}{\log(a)^2+2\log(a)\log(b)+\log(b)^2} - \frac{a^x b^x}{\log(a)^2+2\log(a)\log(b)+\log(b)^2} & \text{for } a \neq \frac{1}{b} \\ \frac{b^x x\left(\frac{1}{b}\right)^x\log\left(\frac{1}{b}\right)}{\log\left(\frac{1}{b}\right)^2+2\log\left(\frac{1}{b}\right)\log(b)+\log(b)^2} + \frac{b^x x\left(\frac{1}{b}\right)^x\log(b)}{\log\left(\frac{1}{b}\right)^2+2\log\left(\frac{1}{b}\right)\log(b)+\log(b)^2} - \frac{b^x\left(\frac{1}{b}\right)^x}{\log\left(\frac{1}{b}\right)^2+2\log\left(\frac{1}{b}\right)\log(b)+\log(b)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(a**x*b**x*x,x)
```

output
```
Piecewise((a**x*b**x*x*log(a)/(log(a)**2 + 2*log(a)*log(b) + log(b)**2) +
a**x*b**x*x*log(b)/(log(a)**2 + 2*log(a)*log(b) + log(b)**2) - a**x*b**x/(
log(a)**2 + 2*log(a)*log(b) + log(b)**2), Ne(a, 1/b)), (b**x*x*(1/b)**x*lo
g(1/b)/(log(1/b)**2 + 2*log(1/b)*log(b) + log(b)**2) + b**x*x*(1/b)**x*log
(b)/(log(1/b)**2 + 2*log(1/b)*log(b) + log(b)**2) - b**x*(1/b)**x/(log(1/b
)**2 + 2*log(1/b)*log(b) + log(b)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int a^x b^x x \, dx = \frac{(x(\log(a) + \log(b)) - 1)e^{(x \log(a) + x \log(b))}}{\log(a)^2 + 2 \log(a) \log(b) + \log(b)^2}$$

input
```
integrate(a^x*b^x*x,x, algorithm="maxima")
```

output
```
(x*(log(a) + log(b)) - 1)*e^(x*log(a) + x*log(b))/(log(a)^2 + 2*log(a)*log
(b) + log(b)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 994, normalized size of antiderivative = 32.06

$$\int a^x b^x x \, dx = \text{Too large to display}$$

input
```
integrate(a^x*b^x*x,x, algorithm="giac")
```

output
```
(2*((pi*x*sgn(a) + pi*x*sgn(b) - 2*pi*x)*(pi*log(abs(a))*sgn(a) + pi*log(a
bs(b))*sgn(a) + pi*log(abs(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(a
bs(a)) - 2*pi*log(abs(b)))/((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*s
gn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*log(abs(b
))^2)^2 + 4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) + pi*log(abs(a)
)*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log(abs(b)))^2)
 - (pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*log(ab
s(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*log(abs(b))^2)*(x*log(abs(a)) + x*
log(abs(b)) - 1)/((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn(a) - 2*pi^2*sgn(b) + 3*
pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b)) - 2*log(abs(b))^2)^2 +
4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))*sgn(a) + pi*log(abs(a))*sgn(b) +
 pi*log(abs(b))*sgn(b) - 2*pi*log(abs(a)) - 2*pi*log(abs(b)))^2))*cos(-1/2
*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi*x) - ((pi^2*sgn(a)*sgn(b) - 2*pi^2*sgn
(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs(a))*log(abs(b))
 - 2*log(abs(b))^2)*(pi*x*sgn(a) + pi*x*sgn(b) - 2*pi*x)/((pi^2*sgn(a)*sgn
(b) - 2*pi^2*sgn(a) - 2*pi^2*sgn(b) + 3*pi^2 - 2*log(abs(a))^2 - 4*log(abs
(a))*log(abs(b)) - 2*log(abs(b))^2)^2 + 4*(pi*log(abs(a))*sgn(a) + pi*log(
abs(b))*sgn(a) + pi*log(abs(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(
abs(a)) - 2*pi*log(abs(b)))^2) + 4*(pi*log(abs(a))*sgn(a) + pi*log(abs(b))
*sgn(a) + pi*log(abs(a))*sgn(b) + pi*log(abs(b))*sgn(b) - 2*pi*log(abs(...
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int a^x b^x x\,dx = \frac{a^x\,b^x\,(x\,(\ln{(a)}+\ln{(b)})-1)}{(\ln{(a)}+\ln{(b)})^2}$$

input
```
int(a^x*b^x*x,x)
```

output
```
(a^x*b^x*(x*(log(a) + log(b)) - 1))/(log(a) + log(b))^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int a^x b^x x\,dx = \frac{b^x a^x (\log(a)\,x + \log(b)\,x - 1)}{\log{(a)}^2 + 2\log{(a)}\log{(b)} + \log{(b)}^2}$$

input
```
int(a^x*b^x*x,x)
```

output
```
(b**x*a**x*(log(a)*x + log(b)*x - 1))/(log(a)**2 + 2*log(a)*log(b) + log(b
)**2)
```

3.487 $\int a^x b^x \, dx$

Optimal result

Integrand size = 7, antiderivative size = 14

$$\int a^x b^x \, dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

output
```
a^x*b^x/(ln(a)+ln(b))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{a^x b^x}{\log(a) + \log(b)}$$

input
```
Integrate[a^x*b^x,x]
```

output
```
(a^x*b^x)/(Log[a] + Log[b])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x b^x \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int e^{x(\log(a)+\log(b))}dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{a^x b^x}{\log(a)+\log(b)}$$

input
```
Int[a^x*b^x,x]
```

output
```
(a^x*b^x)/(Log[a] + Log[b])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{a^{x} b^{x}}{\ln\left(a\right)+\ln\left(b\right)}$	15
risch	$\frac{a^{x} b^{x}}{\ln\left(a\right)+\ln\left(b\right)}$	15
parallelrisch	$\frac{a^{x} b^{x}}{\ln\left(a\right)+\ln\left(b\right)}$	15
orering	$\frac{a^{x} b^{x}}{\ln\left(a\right)+\ln\left(b\right)}$	15
norman	$\frac{{\mathrm e}^{\ln\left(a\right) x} {\mathrm e}^{\ln\left(b\right) x}}{\ln\left(a\right)+\ln\left(b\right)}$	19
meijerg	$-\frac{1-{\mathrm e}^{x \ln\left(b\right) \left(1+\frac{\ln\left(a\right)}{\ln\left(b\right)}\right)}}{\ln\left(b\right) \left(1+\frac{\ln\left(a\right)}{\ln\left(b\right)}\right)}$	36

input
```
int(a^x*b^x,x,method=_RETURNVERBOSE)
```

output
```
a^x*b^x/(ln(a)+ln(b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x\,dx = \frac{a^x b^x}{\log\left(a\right) + \log\left(b\right)}$$

input
```
integrate(a^x*b^x,x, algorithm="fricas")
```

output
```
a^x*b^x/(log(a) + log(b))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int a^x b^x \, dx = \begin{cases} \frac{a^x b^x}{\log(a) + \log(b)} & \text{for } a \neq \frac{1}{b} \\ \frac{b^x \left(\frac{1}{b}\right)^x}{\log\left(\frac{1}{b}\right) + \log(b)} & \text{otherwise} \end{cases}$$

input
```
integrate(a**x*b**x,x)
```

output
```
Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (b**x*(1/b)**x/(log(1
/b) + log(b)), True))
```

Maxima [F(-2)]

Exception generated.

$$\int a^x b^x \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^x*b^x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(log(b)/log(a)>0)', see `assume?`
 for more
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 237, normalized size of antiderivative = 16.93

$$\int a^x b^x \, dx$$

$$= 2 \left(\frac{2 (\log (|a|) + \log (|b|)) \cos \left(-\frac{1}{2} \pi x \mathrm{sgn}(a) - \frac{1}{2} \pi x \mathrm{sgn}(b) + \pi x\right)}{(2 \pi - \pi \mathrm{sgn} (a) - \pi \mathrm{sgn} (b))^2 + 4 (\log (|a|) + \log (|b|))^2} + \frac{(2 \pi - \pi \mathrm{sgn}(a) - \pi \mathrm{sgn}(b)) \sin \left(-\frac{1}{2} \pi\right.}{(2 \pi - \pi \mathrm{sgn} (a) - \pi \mathrm{sgn} (b))^2 +} \right.$$
$$+ i \left(\frac{i e^{\left(\frac{1}{2} i \pi x \mathrm{sgn}(a) + \frac{1}{2} i \pi x \mathrm{sgn}(b) - i \pi x\right)}}{-2 i \pi + i \pi \mathrm{sgn} (a) + i \pi \mathrm{sgn} (b) + 2 \log (|a|) + 2 \log (|b|)} - \frac{i e^{\left(-\frac{1}{2} i \pi x \mathrm{sgn}(a) - \frac{1}{2} i \pi x \mathrm{sgn}(b) + i \pi\right.}}{2 i \pi - i \pi \mathrm{sgn} (a) - i \pi \mathrm{sgn} (b) + 2 \log (|a} \right.$$

input
```
integrate(a^x*b^x,x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) +
pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2)
 + (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) +
 pi*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2
))*e^(x*(log(abs(a)) + log(abs(b)))) + I*(I*e^(1/2*I*pi*x*sgn(a) + 1/2*I*p
i*x*sgn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a))
+ 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)/(
2*I*pi - I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))*e^(x*
(log(abs(a)) + log(abs(b))))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{a^x b^x}{\ln (a) + \ln (b)}$$

input
```
int(a^x*b^x,x)
```

output
```
(a^x*b^x)/(log(a) + log(b))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int a^x b^x \, dx = \frac{b^x a^x}{\log(a) + \log(b)}$$

input
```
int(a^x*b^x,x)
```

output
```
(b**x*a**x)/(log(a) + log(b))
```

3.488 $\int \frac{a^x b^x}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 8

$$\int \frac{a^x b^x}{x} \, dx = \text{ExpIntegralEi}(x(\log(a) + \log(b)))$$

output
```
Ei(x*(ln(a)+ln(b)))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{a^x b^x}{x} \, dx = \text{ExpIntegralEi}(x\log(a) + x\log(b))$$

input
```
Integrate[(a^x*b^x)/x,x]
```

output
```
ExpIntegralEi[x*Log[a] + x*Log[b]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2725, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^x b^x}{x} \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int \frac{e^{x(\log(a)+\log(b))}}{x}dx$$

$$\Big\downarrow \text{2609}$$

$$\text{ExpIntegralEi}(x(\log(a)+\log(b)))$$

input
```
Int[(a^x*b^x)/x,x]
```

output
```
ExpIntegralEi[x*(Log[a] + Log[b])]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 7.00

method	result
meijerg	$\ln(x) + i\pi + \ln(\ln(b)) + \ln\left(1 + \frac{\ln(a)}{\ln(b)}\right) - \ln\left(-x\ln(b)\left(1 + \frac{\ln(a)}{\ln(b)}\right)\right) - \text{expIntegral}_1\left(-x\ln(b)\right.$

input
```
int(a^x*b^x/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)+I*Pi+ln(ln(b))+ln(1+ln(a)/ln(b))-ln(-x*ln(b)*(1+ln(a)/ln(b)))-Ei(1,-
x*ln(b)*(1+ln(a)/ln(b)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{a^x b^x}{x}\,dx = \text{Ei}(x\log(a) + x\log(b))$$

input
```
integrate(a^x*b^x/x,x, algorithm="fricas")
```

output
```
Ei(x*log(a) + x*log(b))
```

Sympy [F]

$$\int \frac{a^x b^x}{x}\,dx = \int \frac{a^x b^x}{x}\,dx$$

input
```
integrate(a**x*b**x/x,x)
```

output
```
Integral(a**x*b**x/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a^x b^x}{x} \, dx = \text{Ei}(x(\log(a) + \log(b)))$$

input
```
integrate(a^x*b^x/x,x, algorithm="maxima")
```

output
```
Ei(x*(log(a) + log(b)))
```

Giac [F]

$$\int \frac{a^x b^x}{x} \, dx = \int \frac{a^x b^x}{x} \, dx$$

input
```
integrate(a^x*b^x/x,x, algorithm="giac")
```

output
```
integrate(a^x*b^x/x, x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{a^x b^x}{x} \, dx = \text{ei}(x(\ln(a) + \ln(b)))$$

input
```
int((a^x*b^x)/x,x)
```

output
```
ei(x*(log(a) + log(b)))
```

Reduce [F]

$$\int \frac{a^x b^x}{x} \, dx = \int \frac{b^x a^x}{x} dx$$

input
```
int(a^x*b^x/x,x)
```

output
```
int((b**x*a**x)/x,x)
```

3.489 $\int \frac{a^x b^x}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int \frac{a^x b^x}{x^2} \, dx = -\frac{a^x b^x}{x} + \text{ExpIntegralEi}(x(\log(a) + \log(b)))(\log(a) + \log(b))$$

output
```
-a^x*b^x/x+Ei(x*(ln(a)+ln(b)))*(ln(a)+ln(b))
```

Mathematica [F]

$$\int \frac{a^x b^x}{x^2} \, dx = \int \frac{a^x b^x}{x^2} \, dx$$

input
```
Integrate[(a^x*b^x)/x^2,x]
```

output
```
Integrate[(a^x*b^x)/x^2, x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2725, 2608, 2725, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^x b^x}{x^2} \, dx$$
$$\Big\downarrow \ 2725$$
$$\int \frac{e^{x(\log(a)+\log(b))}}{x^2}dx$$
$$\Big\downarrow \ 2608$$
$$(\log(a)+\log(b))\int \frac{a^x b^x}{x}dx-\frac{a^x b^x}{x}$$
$$\Big\downarrow \ 2725$$
$$(\log(a)+\log(b))\int \frac{e^{x(\log(a)+\log(b))}}{x}dx-\frac{a^x b^x}{x}$$
$$\Big\downarrow \ 2609$$
$$(\log(a)+\log(b))\,\text{ExpIntegralEi}(x(\log(a)+\log(b)))-\frac{a^x b^x}{x}$$

input
```
Int[(a^x*b^x)/x^2,x]
```

output
```
-((a^x*b^x)/x) + ExpIntegralEi[x*(Log[a] + Log[b])]*(Log[a] + Log[b])
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 6.15

method	result
meijerg	$-\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)\left(\frac{1}{x\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)}+1-\ln(x)-i\pi-\ln(\ln(b))-\ln\left(1+\frac{\ln(a)}{\ln(b)}\right)-\frac{2x\ln(b)\left(1+\frac{\ln}{\ln}\right.}{2x\ln(b)\left(1+\right.}\right.$ …

input
```
int(a^x*b^x/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(b)*(1+ln(a)/ln(b))*(1/x/ln(b)/(1+ln(a)/ln(b))+1-ln(x)-I*Pi-ln(ln(b))-l
n(1+ln(a)/ln(b))-1/2/x/ln(b)/(1+ln(a)/ln(b))*(2*x*ln(b)*(1+ln(a)/ln(b))+2)
+1/x/ln(b)/(1+ln(a)/ln(b))*exp(x*ln(b)*(1+ln(a)/ln(b)))+ln(-x*ln(b)*(1+ln(
a)/ln(b)))+Ei(1,-x*ln(b)*(1+ln(a)/ln(b))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{a^x b^x}{x^2} dx = -\frac{a^x b^x - (x \log(a) + x \log(b))\text{Ei}(x \log(a) + x \log(b))}{x}$$

input
```
integrate(a^x*b^x/x^2,x, algorithm="fricas")
```

output
```
-(a^x*b^x - (x*log(a) + x*log(b))*Ei(x*log(a) + x*log(b)))/x
```

Sympy [F]

$$\int \frac{a^x b^x}{x^2} dx = \int \frac{a^x b^x}{x^2} dx$$

input
```
integrate(a**x*b**x/x**2,x)
```

output
```
Integral(a**x*b**x/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{a^x b^x}{x^2} dx = (\log(a) + \log(b))\Gamma(-1, -x(\log(a) + \log(b)))$$

input
```
integrate(a^x*b^x/x^2,x, algorithm="maxima")
```

output
```
(log(a) + log(b))*gamma(-1, -x*(log(a) + log(b)))
```

Giac [F]

$$\int \frac{a^x b^x}{x^2} \, dx = \int \frac{a^x b^x}{x^2} \, dx$$

input
```
integrate(a^x*b^x/x^2,x, algorithm="giac")
```

output
```
integrate(a^x*b^x/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{a^x b^x}{x^2} \, dx = -\mathrm{expint}(-x\,(\ln(a)+\ln(b)))\,(\ln(a)+\ln(b)) - \frac{a^x\,b^x}{x}$$

input
```
int((a^x*b^x)/x^2,x)
```

output
```
- expint(-x*(log(a) + log(b)))*(log(a) + log(b)) - (a^x*b^x)/x
```

Reduce [F]

$$\int \frac{a^x b^x}{x^2} \, dx = \frac{-b^x a^x + \left(\int \frac{b^x a^x}{x} dx\right) \log(a)\, x + \left(\int \frac{b^x a^x}{x} dx\right) \log(b)\, x}{x}$$

input
```
int(a^x*b^x/x^2,x)
```

output
```
( - b**x*a**x + int((b**x*a**x)/x,x)*log(a)*x + int((b**x*a**x)/x,x)*log(b
)*x)/x
```

3.490 $\int \frac{a^x b^x}{x^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{a^x b^x}{x^3} \, dx = -\frac{a^x b^x}{2x^2} - \frac{a^x b^x (\log(a) + \log(b))}{2x} + \frac{1}{2} \text{ExpIntegralEi}(x(\log(a) + \log(b)))(\log(a) + \log(b))^2$$

output
```
-1/2*a^x*b^x/x^2-1/2*a^x*b^x*(ln(a)+ln(b))/x+1/2*Ei(x*(ln(a)+ln(b)))*(ln(a
)+ln(b))^2
```

Mathematica [F]

$$\int \frac{a^x b^x}{x^3} \, dx = \int \frac{a^x b^x}{x^3} \, dx$$

input
```
Integrate[(a^x*b^x)/x^3,x]
```

output
```
Integrate[(a^x*b^x)/x^3, x]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {2725, 2608, 2725, 2608, 2725, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^x b^x}{x^3} dx$$

$$\Big\downarrow \text{2725}$$

$$\int \frac{e^{x(\log(a)+\log(b))}}{x^3}dx$$

$$\Big\downarrow \text{2608}$$

$$\frac{1}{2}(\log(a)+\log(b))\int \frac{a^x b^x}{x^2}dx-\frac{a^x b^x}{2x^2}$$

$$\Big\downarrow \text{2725}$$

$$\frac{1}{2}(\log(a)+\log(b))\int \frac{e^{x(\log(a)+\log(b))}}{x^2}dx-\frac{a^x b^x}{2x^2}$$

$$\Big\downarrow \text{2608}$$

$$\frac{1}{2}(\log(a)+\log(b))\left((\log(a)+\log(b))\int \frac{a^x b^x}{x}dx-\frac{a^x b^x}{x}\right)-\frac{a^x b^x}{2x^2}$$

$$\Big\downarrow \text{2725}$$

$$\frac{1}{2}(\log(a)+\log(b))\left((\log(a)+\log(b))\int \frac{e^{x(\log(a)+\log(b))}}{x}dx-\frac{a^x b^x}{x}\right)-\frac{a^x b^x}{2x^2}$$

$$\Big\downarrow \text{2609}$$

$$\frac{1}{2}(\log(a)+\log(b))\left((\log(a)+\log(b))\text{ExpIntegralEi}(x(\log(a)+\log(b)))-\frac{a^x b^x}{x}\right)-\frac{a^x b^x}{2x^2}$$

input
```
Int[(a^x*b^x)/x^3,x]
```

output
```
-1/2*(a^x*b^x)/x^2 + ((Log[a] + Log[b])*(-((a^x*b^x)/x) + ExpIntegralEi[x*
(Log[a] + Log[b])]*(Log[a] + Log[b])))/2
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 4.41

method	result
meijerg	$\ln(b)^2\left(1+\frac{\ln(a)}{\ln(b)}\right)^2\left(-\frac{1}{2x^2\ln(b)^2\left(1+\frac{\ln(a)}{\ln(b)}\right)^2}-\frac{1}{x\ln(b)\left(1+\frac{\ln(a)}{\ln(b)}\right)}-\frac{3}{4}+\frac{\ln(x)}{2}+\frac{i\pi}{2}+\frac{\ln(\ln(b))}{2}+\frac{\ln\left(1+\frac{\ln(a)}{\ln(b)}\right)}{2}\right.$

input
```
int(a^x*b^x/x^3,x,method=_RETURNVERBOSE)
```

output
```
ln(b)^2*(1+ln(a)/ln(b))^2*(-1/2/x^2/ln(b)^2/(1+ln(a)/ln(b))^2-1/x/ln(b)/(1
+ln(a)/ln(b))-3/4+1/2*ln(x)+1/2*I*Pi+1/2*ln(ln(b))+1/2*ln(1+ln(a)/ln(b))+1
/12/x^2/ln(b)^2/(1+ln(a)/ln(b))^2*(9*x^2*ln(b)^2*(1+ln(a)/ln(b))^2+12*x*ln
(b)*(1+ln(a)/ln(b))+6)-1/6/x^2/ln(b)^2/(1+ln(a)/ln(b))^2*(3*x*ln(b)*(1+ln(
a)/ln(b))+3)*exp(x*ln(b)*(1+ln(a)/ln(b)))-1/2*ln(-x*ln(b)*(1+ln(a)/ln(b)))
-1/2*Ei(1,-x*ln(b)*(1+ln(a)/ln(b))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.20

$$\int \frac{a^x b^x}{x^3} \, dx = -\frac{(x \log(a) + x \log(b) + 1)a^x b^x - (x^2 \log(a)^2 + 2\,x^2 \log(a) \log(b) + x^2 \log(b)^2)\mathrm{Ei}(x \log(a) + x \log(b))}{2\,x^2}$$

input
```
integrate(a^x*b^x/x^3,x, algorithm="fricas")
```

output
```
-1/2*((x*log(a) + x*log(b) + 1)*a^x*b^x - (x^2*log(a)^2 + 2*x^2*log(a)*log
(b) + x^2*log(b)^2)*Ei(x*log(a) + x*log(b)))/x^2
```

Sympy [F]

$$\int \frac{a^x b^x}{x^3} \, dx = \int \frac{a^x b^x}{x^3} \, dx$$

input
```
integrate(a**x*b**x/x**3,x)
```

output
```
Integral(a**x*b**x/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.37

$$\int \frac{a^x b^x}{x^3} \, dx = -(\log(a) + \log(b))^2 \Gamma(-2, -x(\log(a) + \log(b)))$$

input
```
integrate(a^x*b^x/x^3,x, algorithm="maxima")
```

output
```
-(log(a) + log(b))^2*gamma(-2, -x*(log(a) + log(b)))
```

Giac [F]

$$\int \frac{a^x b^x}{x^3} \, dx = \int \frac{a^x b^x}{x^3} \, dx$$

input
```
integrate(a^x*b^x/x^3,x, algorithm="giac")
```

output
```
integrate(a^x*b^x/x^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int \frac{a^x b^x}{x^3} \, dx = -\frac{\operatorname{expint}(-x\,(\ln(a) + \ln(b)))\,(\ln(a) + \ln(b))^2}{2} - a^x\,b^x\left(\frac{1}{2\,x\,(\ln(a) + \ln(b))} + \frac{1}{2\,x^2\,(\ln(a) + \ln(b))^2}\right)(\ln(a) + \ln(b))^2$$

input
```
int((a^x*b^x)/x^3,x)
```

output
```
- (expint(-x*(log(a) + log(b)))*(log(a) + log(b))^2)/2 - a^x*b^x*(1/(2*x*(
log(a) + log(b))) + 1/(2*x^2*(log(a) + log(b))^2))*(log(a) + log(b))^2
```

Reduce [F]

$$\int \frac{a^x b^x}{x^3}\,dx$$
$$= \frac{-b^x a^x \log(a)\,x - b^x a^x \log(b)\,x - b^x a^x + \left(\int \frac{b^x a^x}{x}dx\right)\log(a)^2 x^2 + 2\left(\int \frac{b^x a^x}{x}dx\right)\log(a)\log(b)\,x^2 + \left(\int \frac{b^x a^x}{x}dx\right)}{2x^2}$$

input
```
int(a^x*b^x/x^3,x)
```

output
```
( - b**x*a**x*log(a)*x - b**x*a**x*log(b)*x - b**x*a**x + int((b**x*a**x)/
x,x)*log(a)**2*x**2 + 2*int((b**x*a**x)/x,x)*log(a)*log(b)*x**2 + int((b**
x*a**x)/x,x)*log(b)**2*x**2)/(2*x**2)
```

3.491 $\int a^x b^{-x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 18

$$\int a^x b^{-x} \, dx = \frac{a^x b^{-x}}{\log(a) - \log(b)}$$

output
```
a^x/(b^x)/(ln(a)-ln(b))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} \, dx = \frac{a^x b^{-x}}{\log(a) - \log(b)}$$

input
```
Integrate[a^x/b^x,x]
```

output
```
a^x/(b^x*(Log[a] - Log[b]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x b^{-x} \, dx$$

$$\Big\downarrow \text{ 2725}$$

$$\int e^{x(\log(a)-\log(b))}dx$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{a^x b^{-x}}{\log(a)-\log(b)}$$

input
```
Int[a^x/b^x,x]
```

output
```
a^x/(b^x*(Log[a] - Log[b]))
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{a^{x} b^{-x}}{\ln \left(a \right)-\ln \left(b \right)}$	19
risch	$\frac{a^{x} b^{-x}}{\ln \left(a \right)-\ln \left(b \right)}$	19
parallelrisch	$\frac{a^{x} b^{-x}}{\ln \left(a \right)-\ln \left(b \right)}$	19
orering	$\frac{a^{x} b^{-x}}{\ln \left(a \right)-\ln \left(b \right)}$	19
norman	$\frac{{\mathrm e}^{\ln \left(a \right) x} {\mathrm e}^{-\ln \left(b \right) x}}{\ln \left(a \right)-\ln \left(b \right)}$	23
meijerg	$-\frac{1-{\mathrm e}^{x \ln \left(a \right) \left(1-\frac{\ln \left(b \right)}{\ln \left(a \right)}\right)}}{\ln \left(a \right) \left(1-\frac{\ln \left(b \right)}{\ln \left(a \right)}\right)}$	38

input
```
int(a^x/(b^x),x,method=_RETURNVERBOSE)
```

output
```
a^x/(b^x)/(ln(a)-ln(b))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x} \, dx = \frac{a^x}{b^x(\log(a) - \log(b))}$$

input
```
integrate(a^x/(b^x),x, algorithm="fricas")
```

output
```
a^x/(b^x*(log(a) - log(b)))
```

Sympy [F(-2)]

Exception generated.

$$\int a^x b^{-x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(a**x/(b**x),x)
```

output
```
Exception raised: TypeError >> Invalid NaN comparison
```

Maxima [F(-2)]

Exception generated.

$$\int a^x b^{-x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^x/(b^x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-log(b)/log(a)>0)', see `assume?
` for more
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 216, normalized size of antiderivative = 12.00

$$\int a^x b^{-x}\,dx$$
$$= 2\left(\frac{2\left(\log\left(|a|\right)-\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)+\frac{1}{2}\pi x\mathrm{sgn}(b)\right)}{\left(\pi\mathrm{sgn}\left(a\right)-\pi\mathrm{sgn}\left(b\right)\right)^2+4\left(\log\left(|a|\right)-\log\left(|b|\right)\right)^2}-\frac{\left(\pi\mathrm{sgn}(a)-\pi\mathrm{sgn}(b)\right)\sin\left(-\frac{1}{2}\pi x\mathrm{sgn}(a)+\frac{1}{2}\right.}{\left(\pi\mathrm{sgn}\left(a\right)-\pi\mathrm{sgn}\left(b\right)\right)^2+4\left(\log\left(|a|\right)-\log\right.}\right.$$
$$\left.+i\left(\frac{i\,e^{\left(\frac{1}{2}i\,\pi x\mathrm{sgn}(a)-\frac{1}{2}i\,\pi x\mathrm{sgn}(b)\right)}}{i\,\pi\mathrm{sgn}\left(a\right)-i\,\pi\mathrm{sgn}\left(b\right)+2\,\log\left(|a|\right)-2\,\log\left(|b|\right)}-\frac{i\,e^{\left(-\frac{1}{2}i\,\pi x\mathrm{sgn}(a)+\frac{1}{2}i\,\pi x\mathrm{sgn}(b)\right)}}{-i\,\pi\mathrm{sgn}\left(a\right)+i\,\pi\mathrm{sgn}\left(b\right)+2\,\log\left(|a|\right)-2\,\log\left(|b\right.}\right.\right.$$

input
```
integrate(a^x/(b^x),x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/(
(pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a)
- pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(
b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))
) + I*(I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn
(b) + 2*log(abs(a)) - 2*log(abs(b))) - I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*
x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*e^
(x*(log(abs(a)) - log(abs(b))))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x}\,dx = \frac{a^x}{b^x\,\left(\ln\left(a\right)-\ln\left(b\right)\right)}$$

input
```
int(a^x/b^x,x)
```

output
```
a^x/(b^x*(log(a) - log(b)))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int a^x b^{-x}\,dx = \frac{a^x}{b^x(\log(a)-\log(b))}$$

input
```
int(a^x/(b^x),x)
```

output
```
a**x/(b**x*(log(a) - log(b)))
```

3.492 $\int a^x b^x c^x \, dx$

Optimal result

Integrand size = 10, antiderivative size = 19

$$\int a^x b^x c^x \, dx = \frac{a^x b^x c^x}{\log(a) + \log(b) + \log(c)}$$

output
```
a^x*b^x*c^x/(ln(a)+ln(b)+ln(c))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int a^x b^x c^x \, dx = \frac{e^{x(\log(a)+\log(b)+\log(c))}}{\log(a) + \log(b) + \log(c)}$$

input
```
Integrate[a^x*b^x*c^x,x]
```

output
```
E^(x*(Log[a] + Log[b] + Log[c]))/(Log[a] + Log[b] + Log[c])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {2725, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int a^x b^x c^x \, dx$$
$$\Big\downarrow \text{ 2725}$$
$$\int c^x e^{x(\log(a)+\log(b))} dx$$
$$\Big\downarrow \text{ 2725}$$
$$\int e^{x(\log(a)+\log(b)+\log(c))} dx$$
$$\Big\downarrow \text{ 2624}$$
$$\frac{a^x b^x c^x}{\log(a)+\log(b)+\log(c)}$$

input
```
Int[a^x*b^x*c^x,x]
```

output
```
(a^x*b^x*c^x)/(Log[a] + Log[b] + Log[c])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
gosper	$\frac{a^{x} b^{x} c^{x}}{\ln \left(a \right)+\ln \left(b \right)+\ln \left(c \right)}$	20
risch	$\frac{a^{x} b^{x} c^{x}}{\ln \left(a \right)+\ln \left(b \right)+\ln \left(c \right)}$	20
parallelrisch	$\frac{a^{x} b^{x} c^{x}}{\ln \left(a \right)+\ln \left(b \right)+\ln \left(c \right)}$	20
orering	$\frac{a^{x} b^{x} c^{x}}{\ln \left(a \right)+\ln \left(b \right)+\ln \left(c \right)}$	20
norman	$\frac{{\mathrm e}^{x \ln \left(c \right)} {\mathrm e}^{\ln \left(a \right) x} {\mathrm e}^{\ln \left(b \right) x}}{\ln \left(a \right)+\ln \left(b \right)+\ln \left(c \right)}$	26
meijerg	$-\frac{1-{\mathrm e}^{x \ln \left(b \right) \left(1+\frac{\ln \left(a \right)}{\ln \left(b \right)}\right) \left(1+\frac{\ln \left(c \right)}{\ln \left(b \right) \left(1+\frac{\ln \left(a \right)}{\ln \left(b \right)}\right)}\right)}}{\ln \left(b \right) \left(1+\frac{\ln \left(a \right)}{\ln \left(b \right)}\right) \left(1+\frac{\ln \left(c \right)}{\ln \left(b \right) \left(1+\frac{\ln \left(a \right)}{\ln \left(b \right)}\right)}\right)}$	78

input
```
int(a^x*b^x*c^x,x,method=_RETURNVERBOSE)
```

output
```
a^x*b^x*c^x/(ln(a)+ln(b)+ln(c))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int a^x b^x c^x \, dx = \frac{a^x b^x c^x}{\log (a) + \log (b) + \log (c)}$$

input
```
integrate(a^x*b^x*c^x,x, algorithm="fricas")
```

output
```
a^x*b^x*c^x/(log(a) + log(b) + log(c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(19) = 38$.

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.58

$$\int a^x b^x c^x \, dx = \begin{cases} \frac{a^x b^x c^x}{\log(a)+\log(b)+\log(c)} & \text{for } a \neq \frac{1}{bc} \\ \frac{b^x c^x \left(\frac{1}{bc}\right)^x}{\log(b)+\log(c)+\log\left(\frac{1}{bc}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(a**x*b**x*c**x,x)
```

output
```
Piecewise((a**x*b**x*c**x/(log(a) + log(b) + log(c)), Ne(a, 1/(b*c))), (b*
*x*c**x*(1/(b*c))**x/(log(b) + log(c) + log(1/(b*c))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int a^x b^x c^x \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(a^x*b^x*c^x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(log(c)/log(a)+log(b)/log(a)>0)',
 see `assu
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 313, normalized size of antiderivative = 16.47

$$\int a^x b^x c^x \, dx = \text{Too large to display}$$

input
```
integrate(a^x*b^x*c^x,x, algorithm="giac")
```

output
```
2*(2*(log(abs(a)) + log(abs(b)) + log(abs(c)))*cos(-1/2*pi*x*sgn(a) - 1/2*
pi*x*sgn(b) - 1/2*pi*x*sgn(c) + 3/2*pi*x)/((3*pi - pi*sgn(a) - pi*sgn(b) -
 pi*sgn(c))^2 + 4*(log(abs(a)) + log(abs(b)) + log(abs(c)))^2) + (3*pi - p
i*sgn(a) - pi*sgn(b) - pi*sgn(c))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) -
 1/2*pi*x*sgn(c) + 3/2*pi*x)/((3*pi - pi*sgn(a) - pi*sgn(b) - pi*sgn(c))^2
 + 4*(log(abs(a)) + log(abs(b)) + log(abs(c)))^2))*e^(x*(log(abs(a)) + log
(abs(b)) + log(abs(c)))) + I*(I*e^(1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b) +
 1/2*I*pi*x*sgn(c) - 3/2*I*pi*x)/(-3*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + I*
pi*sgn(c) + 2*log(abs(a)) + 2*log(abs(b)) + 2*log(abs(c))) - I*e^(-1/2*I*p
i*x*sgn(a) - 1/2*I*pi*x*sgn(b) - 1/2*I*pi*x*sgn(c) + 3/2*I*pi*x)/(3*I*pi -
 I*pi*sgn(a) - I*pi*sgn(b) - I*pi*sgn(c) + 2*log(abs(a)) + 2*log(abs(b)) +
 2*log(abs(c))))*e^(x*(log(abs(a)) + log(abs(b)) + log(abs(c))))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int a^x b^x c^x \, dx = \frac{a^x \, b^x \, c^x}{\ln(a) + \ln(b) + \ln(c)}$$

input
```
int(a^x*b^x*c^x,x)
```

output
```
(a^x*b^x*c^x)/(log(a) + log(b) + log(c))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int a^x b^x c^x \, dx = \frac{c^x b^x a^x}{\log(a) + \log(b) + \log(c)}$$

input
```
int(a^x*b^x*c^x,x)
```

output
```
(c**x*b**x*a**x)/(log(a) + log(b) + log(c))
```

3.493 $\int a^x b^{-x} x^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 61

$$\int a^x b^{-x} x^2 \, dx = \frac{2a^x b^{-x}}{(\log(a)-\log(b))^3} - \frac{2a^x b^{-x} x}{(\log(a)-\log(b))^2} + \frac{a^x b^{-x} x^2}{\log(a)-\log(b)}$$

output
```
2*a^x/(b^x)/(ln(a)-ln(b))^3-2*a^x*x/(b^x)/(ln(a)-ln(b))^2+a^x*x^2/(b^x)/(l
n(a)-ln(b))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int a^x b^{-x} x^2 \, dx = \frac{a^x b^{-x}(2-2x(\log(a)-\log(b))+x^2(\log(a)-\log(b))^2)}{(\log(a)-\log(b))^3}$$

input
```
Integrate[(a^x*x^2)/b^x,x]
```

output
```
(a^x*(2 - 2*x*(Log[a] - Log[b]) + x^2*(Log[a] - Log[b])^2))/(b^x*(Log[a] -
 Log[b])^3)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2725, 2607, 2725, 2607, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 a^x b^{-x} \, dx$$

$$\Big\downarrow \text{ 2725}$$

$$\int x^2 e^{x(\log(a)-\log(b))}dx$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{x^2 a^x b^{-x}}{\log(a)-\log(b)}-\frac{2\int a^x b^{-x}xdx}{\log(a)-\log(b)}$$

$$\Big\downarrow \text{ 2725}$$

$$\frac{x^2 a^x b^{-x}}{\log(a)-\log(b)}-\frac{2\int e^{x(\log(a)-\log(b))}xdx}{\log(a)-\log(b)}$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{x^2 a^x b^{-x}}{\log(a)-\log(b)}-\frac{2\left(\frac{xa^x b^{-x}}{\log(a)-\log(b)}-\frac{\int a^x b^{-x}dx}{\log(a)-\log(b)}\right)}{\log(a)-\log(b)}$$

$$\Big\downarrow \text{ 2725}$$

$$\frac{x^2 a^x b^{-x}}{\log(a)-\log(b)}-\frac{2\left(\frac{xa^x b^{-x}}{\log(a)-\log(b)}-\frac{\int e^{x(\log(a)-\log(b))}dx}{\log(a)-\log(b)}\right)}{\log(a)-\log(b)}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{x^2 a^x b^{-x}}{\log(a)-\log(b)}-\frac{2\left(\frac{xa^x b^{-x}}{\log(a)-\log(b)}-\frac{a^x b^{-x}}{(\log(a)-\log(b))^2}\right)}{\log(a)-\log(b)}$$

input
```
Int[(a^x*x^2)/b^x,x]
```

output
```
(a^x*x^2)/(b^x*(Log[a] - Log[b])) - (2*(-(a^x/(b^x*(Log[a] - Log[b])^2)) +
 (a^x*x)/(b^x*(Log[a] - Log[b]))))/(Log[a] - Log[b])
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\left(\ln(b)^2x^2-2\ln(a)\ln(b)x^2+\ln(a)^2x^2+2\ln(b)x-2\ln(a)x+2\right)a^xb^{-x}}{(-\ln(a)+\ln(b))^3}$	57
gosper	$-\frac{\left(\ln(b)^2x^2-2\ln(a)\ln(b)x^2+\ln(a)^2x^2+2\ln(b)x-2\ln(a)x+2\right)a^xb^{-x}}{(-\ln(a)+\ln(b))\left(\ln(b)^2-2\ln(a)\ln(b)+\ln(a)^2\right)}$	74
orering	$-\frac{\left(\ln(b)^2x^2-2\ln(a)\ln(b)x^2+\ln(a)^2x^2+2\ln(b)x-2\ln(a)x+2\right)a^xb^{-x}}{(-\ln(a)+\ln(b))\left(\ln(b)^2-2\ln(a)\ln(b)+\ln(a)^2\right)}$	74
meijerg	$-\frac{2-\frac{\left(3x^2\ln(a)^2\left(1-\frac{\ln(b)}{\ln(a)}\right)^2-6x\ln(a)\left(1-\frac{\ln(b)}{\ln(a)}\right)+6\right)e^{x\ln(a)\left(1-\frac{\ln(b)}{\ln(a)}\right)}}{3}}{\ln(a)^3\left(1-\frac{\ln(b)}{\ln(a)}\right)^3}$	76
norman	$\left(-\frac{x^2e^{\ln(a)x}}{-\ln(a)+\ln(b)}-\frac{2x\,e^{\ln(a)x}}{\ln(b)^2-2\ln(a)\ln(b)+\ln(a)^2}-\frac{2\,e^{\ln(a)x}}{\left(\ln(b)^2-2\ln(a)\ln(b)+\ln(a)^2\right)(-\ln(a)+\ln(b))}\right)e^{-\ln(b)x}$	87
parallelrisch	$-\frac{\left(\ln(b)^2x^2a^x-2\ln(b)\ln(a)x^2a^x+\ln(a)^2x^2a^x+2x\,a^x\ln(b)-2x\,a^x\ln(a)+2a^x\right)b^{-x}}{(-\ln(a)+\ln(b))\left(\ln(b)^2-2\ln(a)\ln(b)+\ln(a)^2\right)}$	90

input
```
int(a^x*x^2/(b^x),x,method=_RETURNVERBOSE)
```

output
```
-(ln(b)^2*x^2-2*ln(a)*ln(b)*x^2+ln(a)^2*x^2+2*ln(b)*x-2*ln(a)*x+2)*a^x/(-l
n(a)+ln(b))^3/(b^x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int a^xb^{-x}x^2\,dx=\frac{\left(x^2\log(a)^2+x^2\log(b)^2-2\,x\log(a)-2\left(x^2\log(a)-x\right)\log(b)+2\right)a^x}{\left(\log(a)^3-3\,\log(a)^2\log(b)+3\,\log(a)\log(b)^2-\log(b)^3\right)b^x}$$

input
```
integrate(a^x*x^2/(b^x),x, algorithm="fricas")
```

output
```
(x^2*log(a)^2 + x^2*log(b)^2 - 2*x*log(a) - 2*(x^2*log(a) - x)*log(b) + 2)
*a^x/((log(a)^3 - 3*log(a)^2*log(b) + 3*log(a)*log(b)^2 - log(b)^3)*b^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(51) = 102$.

Time = 0.46 (sec) , antiderivative size = 333, normalized size of antiderivative = 5.46

$$\int a^x b^{-x} x^2\,dx$$
$$= \begin{cases} \frac{a^x x^2 \log(a)^2}{b^x \log(a)^3 - 3b^x \log(a)^2 \log(b) + 3b^x \log(a) \log(b)^2 - b^x \log(b)^3} - \frac{2a^x x^2 \log(a) \log(b)}{b^x \log(a)^3 - 3b^x \log(a)^2 \log(b) + 3b^x \log(a) \log(b)^2 - b^x \log(b)^3} + \frac{}{b^x \log} \\ \frac{x^3}{3} \end{cases}$$

input
```
integrate(a**x*x**2/(b**x),x)
```

output
```
Piecewise((a**x*x**2*log(a)**2/(b**x*log(a)**3 - 3*b**x*log(a)**2*log(b) +
 3*b**x*log(a)*log(b)**2 - b**x*log(b)**3) - 2*a**x*x**2*log(a)*log(b)/(b*
*x*log(a)**3 - 3*b**x*log(a)**2*log(b) + 3*b**x*log(a)*log(b)**2 - b**x*lo
g(b)**3) + a**x*x**2*log(b)**2/(b**x*log(a)**3 - 3*b**x*log(a)**2*log(b) +
 3*b**x*log(a)*log(b)**2 - b**x*log(b)**3) - 2*a**x*x*log(a)/(b**x*log(a)*
*3 - 3*b**x*log(a)**2*log(b) + 3*b**x*log(a)*log(b)**2 - b**x*log(b)**3) +
 2*a**x*x*log(b)/(b**x*log(a)**3 - 3*b**x*log(a)**2*log(b) + 3*b**x*log(a)
*log(b)**2 - b**x*log(b)**3) + 2*a**x/(b**x*log(a)**3 - 3*b**x*log(a)**2*l
og(b) + 3*b**x*log(a)*log(b)**2 - b**x*log(b)**3), Ne(a, b)), (x**3/3, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int a^x b^{-x} x^2\,dx$$
$$= \frac{\left(\left(\log(a)^2 - 2\log(a)\log(b) + \log(b)^2\right)x^2 - 2x(\log(a) - \log(b)) + 2\right)e^{(x\log(a) - x\log(b))}}{\log(a)^3 - 3\log(a)^2\log(b) + 3\log(a)\log(b)^2 - \log(b)^3}$$

input
```
integrate(a^x*x^2/(b^x),x, algorithm="maxima")
```

output
```
((log(a)^2 - 2*log(a)*log(b) + log(b)^2)*x^2 - 2*x*(log(a) - log(b)) + 2)*
e^(x*log(a) - x*log(b))/(log(a)^3 - 3*log(a)^2*log(b) + 3*log(a)*log(b)^2
- log(b)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 1817, normalized size of antiderivative = 29.79

$$\int a^x b^{-x} x^2 \, dx = \text{Too large to display}$$

input
```
integrate(a^x*x^2/(b^x),x, algorithm="giac")
```

output
```
(((pi^2*x^2*sgn(a)*sgn(b) - pi^2*x^2 + 2*x^2*log(abs(a))^2 - 4*x^2*log(abs
(a))*log(abs(b)) + 2*x^2*log(abs(b))^2 - 4*x*log(abs(a)) + 4*x*log(abs(b))
 + 4)*(3*pi^2*log(abs(a))*sgn(a)*sgn(b) - 3*pi^2*log(abs(b))*sgn(a)*sgn(b)
 - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3 + 3*pi^2*log(abs(b)) - 6*log(abs(a
))^2*log(abs(b)) + 6*log(abs(a))*log(abs(b))^2 - 2*log(abs(b))^3)/((3*pi^2
*log(abs(a))*sgn(a)*sgn(b) - 3*pi^2*log(abs(b))*sgn(a)*sgn(b) - 3*pi^2*log
(abs(a)) + 2*log(abs(a))^3 + 3*pi^2*log(abs(b)) - 6*log(abs(a))^2*log(abs(
b)) + 6*log(abs(a))*log(abs(b))^2 - 2*log(abs(b))^3)^2 + (pi^3*sgn(a) - 3*
pi*log(abs(a))^2*sgn(a) + 6*pi*log(abs(a))*log(abs(b))*sgn(a) - 3*pi*log(a
bs(b))^2*sgn(a) - pi^3*sgn(b) + 3*pi*log(abs(a))^2*sgn(b) - 6*pi*log(abs(a
))*log(abs(b))*sgn(b) + 3*pi*log(abs(b))^2*sgn(b))^2) - 2*(pi*x^2*log(abs(
a))*sgn(a) - pi*x^2*log(abs(b))*sgn(a) - pi*x^2*log(abs(a))*sgn(b) + pi*x^
2*log(abs(b))*sgn(b) - pi*x*sgn(a) + pi*x*sgn(b))*(pi^3*sgn(a) - 3*pi*log(
abs(a))^2*sgn(a) + 6*pi*log(abs(a))*log(abs(b))*sgn(a) - 3*pi*log(abs(b))^
2*sgn(a) - pi^3*sgn(b) + 3*pi*log(abs(a))^2*sgn(b) - 6*pi*log(abs(a))*log(
abs(b))*sgn(b) + 3*pi*log(abs(b))^2*sgn(b))/((3*pi^2*log(abs(a))*sgn(a)*sg
n(b) - 3*pi^2*log(abs(b))*sgn(a)*sgn(b) - 3*pi^2*log(abs(a)) + 2*log(abs(a
))^3 + 3*pi^2*log(abs(b)) - 6*log(abs(a))^2*log(abs(b)) + 6*log(abs(a))*lo
g(abs(b))^2 - 2*log(abs(b))^3)^2 + (pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a
) + 6*pi*log(abs(a))*log(abs(b))*sgn(a) - 3*pi*log(abs(b))^2*sgn(a) - p...
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.70

$$\int a^x b^{-x} x^2 \, dx = \frac{a^x \left(x^2 \left(\ln(a) - \ln(b)\right)^2 - 2\,x\left(\ln(a) - \ln(b)\right) + 2\right)}{b^x \left(\ln(a) - \ln(b)\right)^3}$$

input
```
int((a^x*x^2)/b^x,x)
```

output
```
(a^x*(x^2*(log(a) - log(b))^2 - 2*x*(log(a) - log(b)) + 2))/(b^x*(log(a) -
 log(b))^3)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int a^x b^{-x} x^2 \, dx = \frac{a^x \left(\log(a)^2 x^2 - 2\log(a)\log(b)\,x^2 - 2\log(a)\,x + \log(b)^2 x^2 + 2\log(b)\,x + 2\right)}{b^x \left(\log(a)^3 - 3\log(a)^2 \log(b) + 3\log(a)\log(b)^2 - \log(b)^3\right)}$$

input
```
int(a^x*x^2/(b^x),x)
```

output
```
(a**x*(log(a)**2*x**2 - 2*log(a)*log(b)*x**2 - 2*log(a)*x + log(b)**2*x**2
 + 2*log(b)*x + 2))/(b**x*(log(a)**3 - 3*log(a)**2*log(b) + 3*log(a)*log(b
)**2 - log(b)**3))
```

3.494 $\int \frac{\left(d+ee^{h+ix}\right)(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 770

$$\int \frac{(d+ee^{h+ix})(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}}\,dx = \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^4}{4\left(b+\sqrt{b^2-4ac}\right)g} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^4}{4\left(b-\sqrt{b^2-4ac}\right)g}$$
$$-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^3\log\left(1+\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i}$$
$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^3\log\left(1+\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i}$$
$$-\frac{3\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g(f+gx)^2\,\mathrm{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i^2}$$
$$-\frac{3\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g(f+gx)^2\,\mathrm{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i^2}$$
$$+\frac{6\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g^2(f+gx)\,\mathrm{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i^3}$$
$$+\frac{6\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g^2(f+gx)\,\mathrm{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i^3}$$
$$-\frac{6\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g^3\,\mathrm{PolyLog}\left(4,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i^4}$$
$$-\frac{6\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g^3\,\mathrm{PolyLog}\left(4,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i^4}$$

output
```
1/4*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^4/(b+(-4*a*c+b^2)^(1/2))/g
+1/4*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^4/(b-(-4*a*c+b^2)^(1/2))/
g-(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^3*ln(1+2*c*exp(i*x+h)/(b-(-4
*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i-(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1
/2))*(g*x+f)^3*ln(1+2*c*exp(i*x+h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)
^(1/2))/i-3*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*g*(g*x+f)^2*polylog(2,-2*c
*exp(i*x+h)/(b-(-4*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i^2-3*(e-(-b*e+
2*c*d)/(-4*a*c+b^2)^(1/2))*g*(g*x+f)^2*polylog(2,-2*c*exp(i*x+h)/(b+(-4*a*
c+b^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2))/i^2+6*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(
1/2))*g^2*(g*x+f)*polylog(3,-2*c*exp(i*x+h)/(b-(-4*a*c+b^2)^(1/2)))/(b-(-4
*a*c+b^2)^(1/2))/i^3+6*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*g^2*(g*x+f)*pol
ylog(3,-2*c*exp(i*x+h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2))/i^3-
6*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*g^3*polylog(4,-2*c*exp(i*x+h)/(b-(-4
*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i^4-6*(e-(-b*e+2*c*d)/(-4*a*c+b^2
)^(1/2))*g^3*polylog(4,-2*c*exp(i*x+h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+
b^2)^(1/2))/i^4
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2448 vs. $2(770) = 1540$.

Time = 4.87 (sec) , antiderivative size = 2448, normalized size of antiderivative = 3.18

$$\int \frac{\left(d+ee^{h+ix}\right)(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d + e*E^(h + i*x))*(f + g*x)^3)/(a + b*E^(h + i*x) + c*E^(2*h
+ 2*i*x)),x]
```

output
```
-1/4*(-6*Sqrt[-(b^2 - 4*a*c)^2]*d*f^2*g*i^4*x^2 - 4*Sqrt[-(b^2 - 4*a*c)^2]
*d*f*g^2*i^4*x^3 - Sqrt[-(b^2 - 4*a*c)^2]*d*g^3*i^4*x^4 + 4*b*Sqrt[b^2 - 4
*a*c]*d*f^3*i^3*ArcTan[(b + 2*c*E^(h + i*x))/Sqrt[-b^2 + 4*a*c]] - 8*a*Sqr
t[b^2 - 4*a*c]*e*f^3*i^3*ArcTan[(b + 2*c*E^(h + i*x))/Sqrt[-b^2 + 4*a*c]]
- 4*Sqrt[-(b^2 - 4*a*c)^2]*d*f^3*i^3*Log[E^(h + i*x)] + 6*Sqrt[-(b^2 - 4*a
*c)^2]*d*f^2*g*i^3*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] +
6*b*Sqrt[-b^2 + 4*a*c]*d*f^2*g*i^3*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b
^2 - 4*a*c])] - 12*a*Sqrt[-b^2 + 4*a*c]*e*f^2*g*i^3*x*Log[1 + (2*c*E^(h +
i*x))/(b - Sqrt[b^2 - 4*a*c])] + 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g^2*i^3*x^2*
Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 6*b*Sqrt[-b^2 + 4*a*c
]*d*f*g^2*i^3*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] - 12*
a*Sqrt[-b^2 + 4*a*c]*e*f*g^2*i^3*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b
^2 - 4*a*c])] + 2*Sqrt[-(b^2 - 4*a*c)^2]*d*g^3*i^3*x^3*Log[1 + (2*c*E^(h +
 i*x))/(b - Sqrt[b^2 - 4*a*c])] + 2*b*Sqrt[-b^2 + 4*a*c]*d*g^3*i^3*x^3*Log
[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] - 4*a*Sqrt[-b^2 + 4*a*c]*e
*g^3*i^3*x^3*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 6*Sqrt[-
(b^2 - 4*a*c)^2]*d*f^2*g*i^3*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4
*a*c])] - 6*b*Sqrt[-b^2 + 4*a*c]*d*f^2*g*i^3*x*Log[1 + (2*c*E^(h + i*x))/(
b + Sqrt[b^2 - 4*a*c])] + 12*a*Sqrt[-b^2 + 4*a*c]*e*f^2*g*i^3*x*Log[1 + (2
*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f...
```

Rubi [A] (verified)

Time = 3.56 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.67, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.159$, Rules used = {2695, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3\left(d+ee^{h+ix}\right)}{a+be^{h+ix}+ce^{2h+2ix}}dx$$

$$\Big\downarrow \text{2695}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{(f+gx)^3}{b+2ce^{h+ix}-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(f+gx)^3}{b+2ce^{h+ix}+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{2615}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^4}{4g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int\frac{e^{h+ix}(f+gx)^3}{b+2ce^{h+ix}-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^4}{4g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int\frac{e^{h+ix}(f+gx)^3}{b+2ce^{h+ix}+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)$$

$\big\downarrow$ 2620

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^4}{4g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{3g\int(f+gx)^2\log\left(\frac{2e^{h+ix}c}{b-\sqrt{b^2-4ac}}+1\right)dx}{2ci}\right)}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^4}{4g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{3g\int(f+gx)^2\log\left(\frac{2e^{h+ix}c}{b+\sqrt{b^2-4ac}}+1\right)dx}{2ci}\right)}{\sqrt{b^2-4ac}+b}\right)$$

$\big\downarrow$ 3011

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^4}{4g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{3g\left(\frac{2g\int(f+gx)\operatorname{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)dx}{i}-\frac{(f}{}\right.}{2ci}\right.}{b-\sqrt{b^2-4ac}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^4}{4g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{3g\left(\frac{2g\int(f+gx)\operatorname{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)dx}{i}-\frac{(f}{}\right.}{2ci}\right.}{\sqrt{b^2-4ac}+b}\right.$$

$\downarrow$ 7163

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^4}{4g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i}-\frac{g\int \mathrm{P}\cdots}{\cdots}\right)}{i}\cdots}{\cdots}\right)}{b-\sqrt{b^2-4ac}\cdots}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^4}{4g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{i}-\frac{g\int \mathrm{P}\cdots}{\cdots}\right)}{i}\cdots}{\cdots}\right)}{\sqrt{b^2-4ac}+b\cdots}\right.$$

$\downarrow$ 2720

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^4}{4g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i}-\frac{g\int e\cdots}{\cdots}\right)}{i\cdots}\cdots\right)}{\cdots}\cdots\right)}{b-\sqrt{b^2-\cdots}}\cdots\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^4}{4g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{i}-\frac{g\int e\cdots}{\cdots}\right)}{i\cdots}\cdots\right)}{\cdots}\cdots\right)}{\sqrt{b^2-4a\cdots}}\cdots\right.$$

↓ 7143

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^4}{4g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i}-\frac{g\operatorname{Po}}{}\right.}{i}\right.}{}\right.}{b-\sqrt{b^2-4ac}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^4}{4g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^3\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{i}-\frac{g\operatorname{Po}}{}\right.}{i}\right.}{}\right.}{\sqrt{b^2-4ac}+b}\right.$$

input
```
Int[((d + e*E^(h + i*x))*(f + g*x)^3)/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*
x)),x]
```

output
```
(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*((f + g*x)^4/(4*(b - Sqrt[b^2 - 4*a*
c])*g) - (2*c*(((f + g*x)^3*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*
c])])/(2*c*i) - (3*g*(-(((f + g*x)^2*PolyLog[2, (-2*c*E^(h + i*x))/(b - Sq
rt[b^2 - 4*a*c])])/i) + (2*g*(((f + g*x)*PolyLog[3, (-2*c*E^(h + i*x))/(b
- Sqrt[b^2 - 4*a*c])])/i - (g*PolyLog[4, (-2*c*E^(h + i*x))/(b - Sqrt[b^2
- 4*a*c])])/i^2))/i))/(2*c*i)))/(b - Sqrt[b^2 - 4*a*c])) + (e - (2*c*d - b
*e)/Sqrt[b^2 - 4*a*c])*((f + g*x)^4/(4*(b + Sqrt[b^2 - 4*a*c])*g) - (2*c*(
((f + g*x)^3*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/(2*c*i) -
 (3*g*(-(((f + g*x)^2*PolyLog[2, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c]
)])/i) + (2*g*(((f + g*x)*PolyLog[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*
a*c])])/i - (g*PolyLog[4, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/i^2
))/i))/(2*c*i)))/(b + Sqrt[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2695
```
Int[(((i_.)*(F_)^(u_) + (h_))*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F
_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(Simplify[(2*c*h - b*i)/q] + i)   Int[(f + g*x)^m/(b - q + 2*c*F^u), x],
x] - Simp[(Simplify[(2*c*h - b*i)/q] - i)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g, h, i}, x] && EqQ[v, 2*u] && LinearQ
[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{\left(d+e\,\mathrm{e}^{ix+h}\right)\left(gx+f\right)^{3}}{a+b\,\mathrm{e}^{ix+h}+c\,\mathrm{e}^{2ix+2h}}dx$$

input
```
int((d+e*exp(i*x+h))*(g*x+f)^3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
int((d+e*exp(i*x+h))*(g*x+f)^3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1859 vs. $2(700) = 1400$.

Time = 0.15 (sec) , antiderivative size = 1859, normalized size of antiderivative = 2.41

$$\int \frac{(d + ee^{h+ix})(f + gx)^3}{a + be^{h+ix} + ce^{2h+2ix}} dx = \text{Too large to display}$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)^3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,
algorithm="fricas")
```

output
```
1/4*((b^2 - 4*a*c)*d*g^3*i^4*x^4 + 4*(b^2 - 4*a*c)*d*f*g^2*i^4*x^3 + 6*(b^
2 - 4*a*c)*d*f^2*g*i^4*x^2 + 4*(b^2 - 4*a*c)*d*f^3*i^4*x - 6*((b^2 - 4*a*c
)*d*g^3*i^2*x^2 + 2*(b^2 - 4*a*c)*d*f*g^2*i^2*x + (b^2 - 4*a*c)*d*f^2*g*i^
2 + ((a*b*d - 2*a^2*e)*g^3*i^2*x^2 + 2*(a*b*d - 2*a^2*e)*f*g^2*i^2*x + (a*
b*d - 2*a^2*e)*f^2*g*i^2)*sqrt((b^2 - 4*a*c)/a^2))*dilog(-1/2*(a*sqrt((b^2
 - 4*a*c)/a^2)*e^(i*x + h) + b*e^(i*x + h) + 2*a)/a + 1) - 6*((b^2 - 4*a*c
)*d*g^3*i^2*x^2 + 2*(b^2 - 4*a*c)*d*f*g^2*i^2*x + (b^2 - 4*a*c)*d*f^2*g*i^
2 - ((a*b*d - 2*a^2*e)*g^3*i^2*x^2 + 2*(a*b*d - 2*a^2*e)*f*g^2*i^2*x + (a*
b*d - 2*a^2*e)*f^2*g*i^2)*sqrt((b^2 - 4*a*c)/a^2))*dilog(1/2*(a*sqrt((b^2
- 4*a*c)/a^2)*e^(i*x + h) - b*e^(i*x + h) - 2*a)/a + 1) + 2*((b^2 - 4*a*c)
*d*g^3*h^3 - 3*(b^2 - 4*a*c)*d*f*g^2*h^2*i + 3*(b^2 - 4*a*c)*d*f^2*g*h*i^2
 - (b^2 - 4*a*c)*d*f^3*i^3 - ((a*b*d - 2*a^2*e)*g^3*h^3 - 3*(a*b*d - 2*a^2
*e)*f*g^2*h^2*i + 3*(a*b*d - 2*a^2*e)*f^2*g*h*i^2 - (a*b*d - 2*a^2*e)*f^3*
i^3)*sqrt((b^2 - 4*a*c)/a^2))*log(2*c*e^(i*x + h) + a*sqrt((b^2 - 4*a*c)/a
^2) + b) + 2*((b^2 - 4*a*c)*d*g^3*h^3 - 3*(b^2 - 4*a*c)*d*f*g^2*h^2*i + 3*
(b^2 - 4*a*c)*d*f^2*g*h*i^2 - (b^2 - 4*a*c)*d*f^3*i^3 + ((a*b*d - 2*a^2*e)
*g^3*h^3 - 3*(a*b*d - 2*a^2*e)*f*g^2*h^2*i + 3*(a*b*d - 2*a^2*e)*f^2*g*h*i
^2 - (a*b*d - 2*a^2*e)*f^3*i^3)*sqrt((b^2 - 4*a*c)/a^2))*log(2*c*e^(i*x +
h) - a*sqrt((b^2 - 4*a*c)/a^2) + b) - 2*((b^2 - 4*a*c)*d*g^3*i^3*x^3 + 3*(
b^2 - 4*a*c)*d*f*g^2*i^3*x^2 + 3*(b^2 - 4*a*c)*d*f^2*g*i^3*x + (b^2 - 4...
```

Sympy [F]

$$\int \frac{(d + e e^{h+ix}) (f + gx)^3}{a + b e^{h+ix} + c e^{2h+2ix}} dx = \int \frac{(d + e e^h e^{ix}) (f + gx)^3}{a + b e^h e^{ix} + c e^{2h} e^{2ix}} dx$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)**3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
Integral((d + e*exp(h)*exp(i*x))*(f + g*x)**3/(a + b*exp(h)*exp(i*x) + c*e
xp(2*h)*exp(2*i*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + e e^{h+ix}) (f + gx)^3}{a + b e^{h+ix} + c e^{2h+2ix}} dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)^3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,
algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(d + e e^{h+ix}) (f + gx)^3}{a + b e^{h+ix} + c e^{2h+2ix}} dx = \int \frac{(gx + f)^3 (e e^{(ix+h)} + d)}{c e^{(2\,ix+2\,h)} + b e^{(ix+h)} + a} dx$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)^3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,
algorithm="giac")
```

output
```
integrate((g*x + f)^3*(e*e^(i*x + h) + d)/(c*e^(2*i*x + 2*h) + b*e^(i*x +
h) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ee^{h+ix})(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}}\,dx=\int \frac{(f+g\,x)^3\,(d+e\,\mathrm{e}^{h+i\,x})}{a+b\,\mathrm{e}^{h+i\,x}+c\,\mathrm{e}^{2\,h+2\,i\,x}}\,dx$$

input
```
int(((f + g*x)^3*(d + e*exp(h + i*x)))/(a + b*exp(h + i*x) + c*exp(2*h + 2
*i*x)),x)
```

output
```
int(((f + g*x)^3*(d + e*exp(h + i*x)))/(a + b*exp(h + i*x) + c*exp(2*h + 2
*i*x)), x)
```

Reduce [F]

$$\int \frac{(d+ee^{h+ix})(f+gx)^3}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$$
$$=\frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right)ae\,f^3i+2\sqrt{4ac-b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right)bd\,f^3i+8e^h\left(\int\frac{e^{ix}x^3}{e^{2ix+2h}c+e^{ix+h}b+a}dx\right)a}{}$$

input
```
int((d+e*exp(i*x+h))*(g*x+f)^3/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c - b**2))*a
*e*f**3*i + 2*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c -
b**2))*b*d*f**3*i + 8*e**h*int((e**(i*x)*x**3)/(e**(2*h + 2*i*x)*c + e**(h
 + i*x)*b + a),x)*a**2*c*e*g**3 - 2*e**h*int((e**(i*x)*x**3)/(e**(2*h + 2*
i*x)*c + e**(h + i*x)*b + a),x)*a*b**2*e*g**3 + 24*e**h*int((e**(i*x)*x**2
)/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a**2*c*e*f*g**2 - 6*e**h*in
t((e**(i*x)*x**2)/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a*b**2*e*f*
g**2 + 24*e**h*int((e**(i*x)*x)/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),
x)*a**2*c*e*f**2*g - 6*e**h*int((e**(i*x)*x)/(e**(2*h + 2*i*x)*c + e**(h +
 i*x)*b + a),x)*a*b**2*e*f**2*g + 8*int(x**3/(e**(2*h + 2*i*x)*c + e**(h +
 i*x)*b + a),x)*a**2*c*d*g**3 - 2*int(x**3/(e**(2*h + 2*i*x)*c + e**(h + i
*x)*b + a),x)*a*b**2*d*g**3 + 24*int(x**2/(e**(2*h + 2*i*x)*c + e**(h + i*
x)*b + a),x)*a**2*c*d*f*g**2 - 6*int(x**2/(e**(2*h + 2*i*x)*c + e**(h + i*
x)*b + a),x)*a*b**2*d*f*g**2 + 24*int(x/(e**(2*h + 2*i*x)*c + e**(h + i*x)
*b + a),x)*a**2*c*d*f**2*g - 6*int(x/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b
+ a),x)*a*b**2*d*f**2*g + 4*log(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a)*a
*c*d*f**3*i - log(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a)*b**2*d*f**3*i +
 8*a*c*d*f**3*x - 2*b**2*d*f**3*x)/(2*a*(4*a*c - b**2))
```

3.495 $\int \frac{(d+ee^{h+ix})(f+gx)^2}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 599

$$\begin{aligned}\int \frac{(d+ee^{h+ix})(f+gx)^2}{a+be^{h+ix}+ce^{2h+2ix}}\,dx &= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^3}{3\left(b+\sqrt{b^2-4ac}\right)g}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^3}{3\left(b-\sqrt{b^2-4ac}\right)g} \\ &- \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^2\log\left(1+\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i} \\ &- \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^2\log\left(1+\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i} \\ &- \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g(f+gx)\operatorname{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i^2} \\ &- \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g(f+gx)\operatorname{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i^2} \\ &+ \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g^2\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i^3} \\ &+ \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g^2\operatorname{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i^3}\end{aligned}$$

output
```
1/3*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^3/(b+(-4*a*c+b^2)^(1/2))/g
+1/3*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^3/(b-(-4*a*c+b^2)^(1/2))/
g-(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^2*ln(1+2*c*exp(i*x+h)/(b-(-4
*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i-(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1
/2))*(g*x+f)^2*ln(1+2*c*exp(i*x+h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)
^(1/2))/i-2*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*g*(g*x+f)*polylog(2,-2*c*e
xp(i*x+h)/(b-(-4*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i^2-2*(e-(-b*e+2*
c*d)/(-4*a*c+b^2)^(1/2))*g*(g*x+f)*polylog(2,-2*c*exp(i*x+h)/(b+(-4*a*c+b^
2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2))/i^2+2*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2)
)*g^2*polylog(3,-2*c*exp(i*x+h)/(b-(-4*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1
/2))/i^3+2*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*g^2*polylog(3,-2*c*exp(i*x+
h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/2))/i^3
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1419 vs. $2(599) = 1198$.

Time = 2.73 (sec) , antiderivative size = 1419, normalized size of antiderivative = 2.37

$$\int \frac{(d + ee^{h+ix})(f + gx)^2}{a + be^{h+ix} + ce^{2h+2ix}} \, dx = \text{Too large to display}$$

input
```
Integrate[((d + e*E^(h + i*x))*(f + g*x)^2)/(a + b*E^(h + i*x) + c*E^(2*h
+ 2*i*x)),x]
```

output
```
-1/6*(-6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g*i^3*x^2 - 2*Sqrt[-(b^2 - 4*a*c)^2]*d
*g^2*i^3*x^3 + 6*b*Sqrt[b^2 - 4*a*c]*d*f^2*i^2*ArcTan[(b + 2*c*E^(h + i*x)
)/Sqrt[-b^2 + 4*a*c]] - 12*a*Sqrt[b^2 - 4*a*c]*e*f^2*i^2*ArcTan[(b + 2*c*E
^(h + i*x))/Sqrt[-b^2 + 4*a*c]] - 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f^2*i^2*Log[E
^(h + i*x)] + 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x
))/(b - Sqrt[b^2 - 4*a*c])] + 6*b*Sqrt[-b^2 + 4*a*c]*d*f*g*i^2*x*Log[1 + (
2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] - 12*a*Sqrt[-b^2 + 4*a*c]*e*f*g*
i^2*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + 3*Sqrt[-(b^2 -
4*a*c)^2]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])]
 + 3*b*Sqrt[-b^2 + 4*a*c]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqr
t[b^2 - 4*a*c])] - 6*a*Sqrt[-b^2 + 4*a*c]*e*g^2*i^2*x^2*Log[1 + (2*c*E^(h
+ i*x))/(b - Sqrt[b^2 - 4*a*c])] + 6*Sqrt[-(b^2 - 4*a*c)^2]*d*f*g*i^2*x*Lo
g[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - 6*b*Sqrt[-b^2 + 4*a*c]*
d*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 12*a*Sqrt
[-b^2 + 4*a*c]*e*f*g*i^2*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c
])] + 3*Sqrt[-(b^2 - 4*a*c)^2]*d*g^2*i^2*x^2*Log[1 + (2*c*E^(h + i*x))/(b
+ Sqrt[b^2 - 4*a*c])] - 3*b*Sqrt[-b^2 + 4*a*c]*d*g^2*i^2*x^2*Log[1 + (2*c*
E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 6*a*Sqrt[-b^2 + 4*a*c]*e*g^2*i^2*x
^2*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 3*Sqrt[-(b^2 - 4*a
*c)^2]*d*f^2*i^2*Log[a + E^(h + i*x)*(b + c*E^(h + i*x))] + 6*(Sqrt[-(b...
```

Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2695, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2 \left(d+ee^{h+ix}\right)}{a+be^{h+ix}+ce^{2h+2ix}} \, dx$$

$$\Big\downarrow \text{2695}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{(f+gx)^2}{b+2ce^{h+ix}-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(f+gx)^2}{b+2ce^{h+ix}+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{2615}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^3}{3g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int\frac{e^{h+ix}(f+gx)^2}{b+2ce^{h+ix}-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^3}{3g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int\frac{e^{h+ix}(f+gx)^2}{b+2ce^{h+ix}+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)$$

$$\downarrow 2620$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^3}{3g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{g\int(f+gx)\log\left(\frac{2e^{h+ix}c}{b-\sqrt{b^2-4ac}}+1\right)dx}{ci}\right)}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^3}{3g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{g\int(f+gx)\log\left(\frac{2e^{h+ix}c}{b+\sqrt{b^2-4ac}}+1\right)dx}{ci}\right)}{\sqrt{b^2-4ac}+b}\right)$$

$$\downarrow 3011$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^3}{3g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{g\left(\frac{g\int\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)dx}{i}-\frac{(f+gx)\,\text{Poly}}{}\right.}{ci}\right.}{b-\sqrt{b^2-4ac}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^3}{3g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{g\left(\frac{g\int\text{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)dx}{i}-\frac{(f+gx)\,\text{Poly}}{}\right.}{ci}\right.}{\sqrt{b^2-4ac}+b}\right.$$

$\downarrow$ 2720

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^3}{3g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{g\left(\frac{g\int e^{-h-ix}\operatorname{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)de^{h+ix}}{i^2}\right.}{ci}\right.}{b-\sqrt{b^2-4ac}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^3}{3g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{g\left(\frac{g\int e^{-h-ix}\operatorname{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)de^{h+ix}}{i^2}\right.}{ci}\right.}{\sqrt{b^2-4ac}+b}\right.$$

$\downarrow$ 7143

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^3}{3g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{g\left(\frac{g\,\mathrm{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{i^2}-\frac{(f+gx)\,\mathrm{PolyLog}}{}\right.}{ci}\right.}{b-\sqrt{b^2-4ac}}\right.$$

$$\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^3}{3g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)^2\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{g\left(\frac{g\,\mathrm{PolyLog}\left(3,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{i^2}-\frac{(f+gx)\,\mathrm{PolyLog}}{}\right.}{ci}\right.}{\sqrt{b^2-4ac}+b}\right.$$

input
```
Int[((d + e*E^(h + i*x))*(f + g*x)^2)/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*
x)),x]
```

output
```
(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*((f + g*x)^3/(3*(b - Sqrt[b^2 - 4*a*
c])*g) - (2*c*(((f + g*x)^2*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*
c])])/(2*c*i) - (g*(-(((f + g*x)*PolyLog[2, (-2*c*E^(h + i*x))/(b - Sqrt[b
^2 - 4*a*c])])/i) + (g*PolyLog[3, (-2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c
])])/i^2))/(c*i)))/(b - Sqrt[b^2 - 4*a*c])) + (e - (2*c*d - b*e)/Sqrt[b^2
- 4*a*c])*((f + g*x)^3/(3*(b + Sqrt[b^2 - 4*a*c])*g) - (2*c*(((f + g*x)^2*
Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/(2*c*i) - (g*(-(((f +
g*x)*PolyLog[2, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/i) + (g*PolyL
og[3, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/i^2))/(c*i)))/(b + Sqrt
[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2695
```
Int[(((i_.)*(F_)^(u_) + (h_))*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F
_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(Simplify[(2*c*h - b*i)/q] + i)   Int[(f + g*x)^m/(b - q + 2*c*F^u), x],
x] - Simp[(Simplify[(2*c*h - b*i)/q] - i)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g, h, i}, x] && EqQ[v, 2*u] && LinearQ
[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{\left(d+e\,\mathrm{e}^{ix+h}\right)\left(gx+f\right)^{2}}{a+b\,\mathrm{e}^{ix+h}+c\,\mathrm{e}^{2ix+2h}}dx$$

input
```
int((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
int((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(543) = 1086$.

Time = 0.12 (sec) , antiderivative size = 1193, normalized size of antiderivative = 1.99

$$\int \frac{\left(d+ee^{h+ix}\right)(f+gx)^2}{a+be^{h+ix}+ce^{2h+2ix}}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,
algorithm="fricas")
```

output
```
1/6*(2*(b^2 - 4*a*c)*d*g^2*i^3*x^3 + 6*(b^2 - 4*a*c)*d*f*g*i^3*x^2 + 6*(b^
2 - 4*a*c)*d*f^2*i^3*x - 6*((b^2 - 4*a*c)*d*g^2*i*x + (b^2 - 4*a*c)*d*f*g*
i + ((a*b*d - 2*a^2*e)*g^2*i*x + (a*b*d - 2*a^2*e)*f*g*i)*sqrt((b^2 - 4*a*
c)/a^2))*dilog(-1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^(i*x + h) + b*e^(i*x + h)
 + 2*a)/a + 1) - 6*((b^2 - 4*a*c)*d*g^2*i*x + (b^2 - 4*a*c)*d*f*g*i - ((a*
b*d - 2*a^2*e)*g^2*i*x + (a*b*d - 2*a^2*e)*f*g*i)*sqrt((b^2 - 4*a*c)/a^2))
*dilog(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^(i*x + h) - b*e^(i*x + h) - 2*a)/a
 + 1) - 3*((b^2 - 4*a*c)*d*g^2*h^2 - 2*(b^2 - 4*a*c)*d*f*g*h*i + (b^2 - 4*
a*c)*d*f^2*i^2 - ((a*b*d - 2*a^2*e)*g^2*h^2 - 2*(a*b*d - 2*a^2*e)*f*g*h*i
+ (a*b*d - 2*a^2*e)*f^2*i^2)*sqrt((b^2 - 4*a*c)/a^2))*log(2*c*e^(i*x + h)
+ a*sqrt((b^2 - 4*a*c)/a^2) + b) - 3*((b^2 - 4*a*c)*d*g^2*h^2 - 2*(b^2 - 4
*a*c)*d*f*g*h*i + (b^2 - 4*a*c)*d*f^2*i^2 + ((a*b*d - 2*a^2*e)*g^2*h^2 - 2
*(a*b*d - 2*a^2*e)*f*g*h*i + (a*b*d - 2*a^2*e)*f^2*i^2)*sqrt((b^2 - 4*a*c)
/a^2))*log(2*c*e^(i*x + h) - a*sqrt((b^2 - 4*a*c)/a^2) + b) - 3*((b^2 - 4*
a*c)*d*g^2*i^2*x^2 + 2*(b^2 - 4*a*c)*d*f*g*i^2*x - (b^2 - 4*a*c)*d*g^2*h^2
 + 2*(b^2 - 4*a*c)*d*f*g*h*i + ((a*b*d - 2*a^2*e)*g^2*i^2*x^2 + 2*(a*b*d -
 2*a^2*e)*f*g*i^2*x - (a*b*d - 2*a^2*e)*g^2*h^2 + 2*(a*b*d - 2*a^2*e)*f*g*
h*i)*sqrt((b^2 - 4*a*c)/a^2))*log(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^(i*x +
h) + b*e^(i*x + h) + 2*a)/a) - 3*((b^2 - 4*a*c)*d*g^2*i^2*x^2 + 2*(b^2 - 4
*a*c)*d*f*g*i^2*x - (b^2 - 4*a*c)*d*g^2*h^2 + 2*(b^2 - 4*a*c)*d*f*g*h*i...
```

Sympy [F]

$$\int \frac{\left(d + ee^{h+ix}\right)(f + gx)^2}{a + be^{h+ix} + ce^{2h+2ix}}\,dx = \int \frac{\left(d + ee^{h}e^{ix}\right)(f + gx)^2}{a + be^{h}e^{ix} + ce^{2h}e^{2ix}}\,dx$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)**2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
Integral((d + e*exp(h)*exp(i*x))*(f + g*x)**2/(a + b*exp(h)*exp(i*x) + c*e
xp(2*h)*exp(2*i*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d + ee^{h+ix})(f + gx)^2}{a + be^{h+ix} + ce^{2h+2ix}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,
algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(d + ee^{h+ix})(f + gx)^2}{a + be^{h+ix} + ce^{2h+2ix}}\,dx = \int \frac{(gx + f)^2 \left(ee^{(ix+h)} + d\right)}{ce^{(2\,ix+2\,h)} + be^{(ix+h)} + a}\,dx$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,
algorithm="giac")
```

output
```
integrate((g*x + f)^2*(e*e^(i*x + h) + d)/(c*e^(2*i*x + 2*h) + b*e^(i*x +
h) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ee^{h+ix})(f + gx)^2}{a + be^{h+ix} + ce^{2h+2ix}}\,dx = \int \frac{(f + g\,x)^2 \left(d + e\,e^{h+i\,x}\right)}{a + b\,e^{h+i\,x} + c\,e^{2\,h+2\,i\,x}}\,dx$$

input
```
int(((f + g*x)^2*(d + e*exp(h + i*x)))/(a + b*exp(h + i*x) + c*exp(2*h + 2
*i*x)),x)
```

output
```
int(((f + g*x)^2*(d + e*exp(h + i*x)))/(a + b*exp(h + i*x) + c*exp(2*h + 2
*i*x)), x)
```

Reduce [F]

$$\int \frac{(d + ee^{h+ix})(f + gx)^2}{a + be^{h+ix} + ce^{2h+2ix}}\,dx$$
$$= \frac{-4\sqrt{4ac - b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right) ae\,f^2 i + 2\sqrt{4ac - b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right) bd\,f^2 i + 8e^h\left(\int \frac{e^{ix}x^2}{e^{2ix+2h}c+e^{ix+h}b+a}dx\right) a}{}$$

input
```
int((d+e*exp(i*x+h))*(g*x+f)^2/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c - b**2))*a
*e*f**2*i + 2*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c -
b**2))*b*d*f**2*i + 8*e**h*int((e**(i*x)*x**2)/(e**(2*h + 2*i*x)*c + e**(h
 + i*x)*b + a),x)*a**2*c*e*g**2 - 2*e**h*int((e**(i*x)*x**2)/(e**(2*h + 2*
i*x)*c + e**(h + i*x)*b + a),x)*a*b**2*e*g**2 + 16*e**h*int((e**(i*x)*x)/(
e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a**2*c*e*f*g - 4*e**h*int((e**
(i*x)*x)/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a*b**2*e*f*g + 8*int
(x**2/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a**2*c*d*g**2 - 2*int(x
**2/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a*b**2*d*g**2 + 16*int(x/
(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a**2*c*d*f*g - 4*int(x/(e**(2
*h + 2*i*x)*c + e**(h + i*x)*b + a),x)*a*b**2*d*f*g + 4*log(e**(2*h + 2*i*
x)*c + e**(h + i*x)*b + a)*a*c*d*f**2*i - log(e**(2*h + 2*i*x)*c + e**(h +
 i*x)*b + a)*b**2*d*f**2*i + 8*a*c*d*f**2*x - 2*b**2*d*f**2*x)/(2*a*(4*a*c
 - b**2))
```

3.496 $\int \frac{(d+ee^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$

Optimal result

Integrand size = 42, antiderivative size = 428

$$\begin{aligned}\int \frac{(d+ee^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}}\,dx &= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^2}{2\left(b+\sqrt{b^2-4ac}\right)g}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)^2}{2\left(b-\sqrt{b^2-4ac}\right)g}\\
&\quad-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)\log\left(1+\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i}\\
&\quad-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(f+gx)\log\left(1+\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i}\\
&\quad-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g\,\mathrm{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)i^2}\\
&\quad-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)g\,\mathrm{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{\left(b+\sqrt{b^2-4ac}\right)i^2}\end{aligned}$$

output
```
1/2*(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^2/(b+(-4*a*c+b^2)^(1/2))/g
+1/2*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)^2/(b-(-4*a*c+b^2)^(1/2))/
g-(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*(g*x+f)*ln(1+2*c*exp(i*x+h)/(b-(-4*a
*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i-(e-(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2
))*(g*x+f)*ln(1+2*c*exp(i*x+h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^2)^(1/
2))/i-(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*g*polylog(2,-2*c*exp(i*x+h)/(b-(
-4*a*c+b^2)^(1/2)))/(b-(-4*a*c+b^2)^(1/2))/i^2-(e-(-b*e+2*c*d)/(-4*a*c+b^2
)^(1/2))*g*polylog(2,-2*c*exp(i*x+h)/(b+(-4*a*c+b^2)^(1/2)))/(b+(-4*a*c+b^
2)^(1/2))/i^2
```

Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.50

$$\int \frac{\left(d+ee^{h+ix}\right)(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}}\,dx =$$
$$-\frac{i\left(-\sqrt{-\left(b^2-4ac\right)^2}dgix^2+2\sqrt{b^2-4ac}(bd-2ae)f\arctan\left(\frac{b+2ce^{h+ix}}{\sqrt{-b^2+4ac}}\right)-2\sqrt{-\left(b^2-4ac\right)^2}df\log\left(e^{h+\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[((d + e*E^(h + i*x))*(f + g*x))/(a + b*E^(h + i*x) + c*E^(2*h +
2*i*x)),x]
```

output
```
-1/2*(i*(-(Sqrt[-(b^2 - 4*a*c)^2]*d*g*i*x^2) + 2*Sqrt[b^2 - 4*a*c]*(b*d -
2*a*e)*f*ArcTan[(b + 2*c*E^(h + i*x))/Sqrt[-b^2 + 4*a*c]] - 2*Sqrt[-(b^2 -
 4*a*c)^2]*d*f*Log[E^(h + i*x)] + Sqrt[-(b^2 - 4*a*c)^2]*d*g*x*Log[1 + (2*
c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + b*Sqrt[-b^2 + 4*a*c]*d*g*x*Log[1
 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] - 2*a*Sqrt[-b^2 + 4*a*c]*e*g
*x*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])] + Sqrt[-(b^2 - 4*a*c
)^2]*d*g*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] - b*Sqrt[-b^
2 + 4*a*c]*d*g*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])] + 2*a*
Sqrt[-b^2 + 4*a*c]*e*g*x*Log[1 + (2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])
] + Sqrt[-(b^2 - 4*a*c)^2]*d*f*Log[a + E^(h + i*x)*(b + c*E^(h + i*x))]) +
 (Sqrt[-(b^2 - 4*a*c)^2]*d + b*Sqrt[-b^2 + 4*a*c]*d - 2*a*Sqrt[-b^2 + 4*a*
c]*e)*g*PolyLog[2, (2*c*E^(h + i*x))/(-b + Sqrt[b^2 - 4*a*c])] + (Sqrt[-(b
^2 - 4*a*c)^2]*d - b*Sqrt[-b^2 + 4*a*c]*d + 2*a*Sqrt[-b^2 + 4*a*c]*e)*g*Po
lyLog[2, (-2*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/(a*Sqrt[-(b^2 - 4*a*
c)^2]*i^2)
```

Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {2695, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)\left(d+ee^{h+ix}\right)}{a+be^{h+ix}+ce^{2h+2ix}}dx$$

$$\Big\downarrow \textbf{2695}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{f+gx}{b+2ce^{h+ix}-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{f+gx}{b+2ce^{h+ix}+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \textbf{2615}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^2}{2g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\int\frac{e^{h+ix}(f+gx)}{b+2ce^{h+ix}-\sqrt{b^2-4ac}}dx}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^2}{2g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\int\frac{e^{h+ix}(f+gx)}{b+2ce^{h+ix}+\sqrt{b^2-4ac}}dx}{\sqrt{b^2-4ac}+b}\right)$$

$\downarrow$ 2620

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^2}{2g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{g\int\log\left(\frac{2e^{h+ix}c}{b-\sqrt{b^2-4ac}}+1\right)dx}{2ci}\right)}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^2}{2g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{g\int\log\left(\frac{2e^{h+ix}c}{b+\sqrt{b^2-4ac}}+1\right)dx}{2ci}\right)}{\sqrt{b^2-4ac}+b}\right)$$

$\downarrow$ 2715

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^2}{2g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}-\frac{g\int e^{-h-ix}\log\left(\frac{2e^{h+ix}c}{b-\sqrt{b^2-4ac}}+1\right)de^{h+ix}}{2ci^2}\right)}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^2}{2g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}-\frac{g\int e^{-h-ix}\log\left(\frac{2e^{h+ix}c}{b+\sqrt{b^2-4ac}}+1\right)de^{h+ix}}{2ci^2}\right)}{\sqrt{b^2-4ac}+b}\right)$$

$\downarrow$ 2838

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\left(\frac{(f+gx)^2}{2g\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c\left(\frac{(f+gx)\log\left(\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}+1\right)}{2ci}+\frac{g\,\mathrm{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b-\sqrt{b^2-4ac}}\right)}{2ci^2}\right)}{b-\sqrt{b^2-4ac}}\right)+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\left(\frac{(f+gx)^2}{2g\left(\sqrt{b^2-4ac}+b\right)}-\frac{2c\left(\frac{(f+gx)\log\left(\frac{2ce^{h+ix}}{\sqrt{b^2-4ac}+b}+1\right)}{2ci}+\frac{g\,\mathrm{PolyLog}\left(2,-\frac{2ce^{h+ix}}{b+\sqrt{b^2-4ac}}\right)}{2ci^2}\right)}{\sqrt{b^2-4ac}+b}\right)$$

input
```
Int[((d + e*E^(h + i*x))*(f + g*x))/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*x)
),x]
```

output
```
(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*((f + g*x)^2/(2*(b - Sqrt[b^2 - 4*a*
c])*g) - (2*c*(((f + g*x)*Log[1 + (2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c]
)])/(2*c*i) + (g*PolyLog[2, (-2*c*E^(h + i*x))/(b - Sqrt[b^2 - 4*a*c])])/(
2*c*i^2)))/(b - Sqrt[b^2 - 4*a*c])) + (e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c]
)*((f + g*x)^2/(2*(b + Sqrt[b^2 - 4*a*c])*g) - (2*c*(((f + g*x)*Log[1 + (2
*c*E^(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/(2*c*i) + (g*PolyLog[2, (-2*c*E^
(h + i*x))/(b + Sqrt[b^2 - 4*a*c])])/(2*c*i^2)))/(b + Sqrt[b^2 - 4*a*c]))
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2695
```
Int[(((i_.)*(F_)^(u_) + (h_))*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F
_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(Simplify[(2*c*h - b*i)/q] + i)   Int[(f + g*x)^m/(b - q + 2*c*F^u), x],
x] - Simp[(Simplify[(2*c*h - b*i)/q] - i)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g, h, i}, x] && EqQ[v, 2*u] && LinearQ
[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1181 vs. $2(388) = 776$.

Time = 0.16 (sec) , antiderivative size = 1182, normalized size of antiderivative = 2.76

method	result	size
default	Expression too large to display	1182
risch	Expression too large to display	1319

input
```
int((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,method=_R
ETURNVERBOSE)
```

output
```
d*f/i*(1/a*ln(exp(i*x))+1/a*(-1/2*ln(a+b*exp(i*x)*exp(h)+c*exp(i*x)^2*exp(
2*h))-exp(h)*b/(4*a*c*exp(2*h)-exp(h)^2*b^2)^(1/2)*arctan((exp(h)*b+2*exp(
2*h)*exp(i*x)*c)/(4*a*c*exp(2*h)-exp(h)^2*b^2)^(1/2))))+d*g/i^2*(1/2*i^2*x
^2/a+(-1/2*i*x*(exp(h)*ln((2*exp(2*h)*exp(i*x)*c+exp(h)*b-(-4*a*c*exp(2*h)
+exp(h)^2*b^2)^(1/2))/(exp(h)*b-(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)))*b-e
xp(h)*ln((2*exp(2*h)*exp(i*x)*c+exp(h)*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1
/2))/(exp(h)*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)))*b+ln((2*exp(2*h)*exp
(i*x)*c+exp(h)*b-(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))/(exp(h)*b-(-4*a*c*e
xp(2*h)+exp(h)^2*b^2)^(1/2)))*(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)+ln((2*e
xp(2*h)*exp(i*x)*c+exp(h)*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))/(exp(h)*
b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)))*(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1
/2))/(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)-1/2*(exp(h)*dilog((2*exp(2*h)*ex
p(i*x)*c+exp(h)*b-(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))/(exp(h)*b-(-4*a*c*
exp(2*h)+exp(h)^2*b^2)^(1/2)))*b-exp(h)*dilog((2*exp(2*h)*exp(i*x)*c+exp(h
)*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))/(exp(h)*b+(-4*a*c*exp(2*h)+exp(h
)^2*b^2)^(1/2)))*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)*dilog((2*exp(2*h)*
exp(i*x)*c+exp(h)*b-(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))/(exp(h)*b-(-4*a*
c*exp(2*h)+exp(h)^2*b^2)^(1/2)))+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2)*dilo
g((2*exp(2*h)*exp(i*x)*c+exp(h)*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))/(e
xp(h)*b+(-4*a*c*exp(2*h)+exp(h)^2*b^2)^(1/2))))/(-4*a*c*exp(2*h)+exp(h)...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 651, normalized size of antiderivative = 1.52

$$\int \frac{(d+ee^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, al
gorithm="fricas")
```

output
```
1/2*((b^2 - 4*a*c)*d*g*i^2*x^2 + 2*(b^2 - 4*a*c)*d*f*i^2*x - ((b^2 - 4*a*c
)*d*g + (a*b*d - 2*a^2*e)*g*sqrt((b^2 - 4*a*c)/a^2))*dilog(-1/2*(a*sqrt((b
^2 - 4*a*c)/a^2)*e^(i*x + h) + b*e^(i*x + h) + 2*a)/a + 1) - ((b^2 - 4*a*c
)*d*g - (a*b*d - 2*a^2*e)*g*sqrt((b^2 - 4*a*c)/a^2))*dilog(1/2*(a*sqrt((b^
2 - 4*a*c)/a^2)*e^(i*x + h) - b*e^(i*x + h) - 2*a)/a + 1) + ((b^2 - 4*a*c)
*d*g*h - (b^2 - 4*a*c)*d*f*i - ((a*b*d - 2*a^2*e)*g*h - (a*b*d - 2*a^2*e)*
f*i)*sqrt((b^2 - 4*a*c)/a^2))*log(2*c*e^(i*x + h) + a*sqrt((b^2 - 4*a*c)/a
^2) + b) + ((b^2 - 4*a*c)*d*g*h - (b^2 - 4*a*c)*d*f*i + ((a*b*d - 2*a^2*e)
*g*h - (a*b*d - 2*a^2*e)*f*i)*sqrt((b^2 - 4*a*c)/a^2))*log(2*c*e^(i*x + h)
 - a*sqrt((b^2 - 4*a*c)/a^2) + b) - ((b^2 - 4*a*c)*d*g*i*x + (b^2 - 4*a*c)
*d*g*h + ((a*b*d - 2*a^2*e)*g*i*x + (a*b*d - 2*a^2*e)*g*h)*sqrt((b^2 - 4*a
*c)/a^2))*log(1/2*(a*sqrt((b^2 - 4*a*c)/a^2)*e^(i*x + h) + b*e^(i*x + h) +
 2*a)/a) - ((b^2 - 4*a*c)*d*g*i*x + (b^2 - 4*a*c)*d*g*h - ((a*b*d - 2*a^2*
e)*g*i*x + (a*b*d - 2*a^2*e)*g*h)*sqrt((b^2 - 4*a*c)/a^2))*log(-1/2*(a*sqr
t((b^2 - 4*a*c)/a^2)*e^(i*x + h) - b*e^(i*x + h) - 2*a)/a))/((a*b^2 - 4*a^
2*c)*i^2)
```

Sympy [F]

$$\int \frac{(d+ee^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}}\,dx=\int \frac{(d+ee^{h}e^{ix})(f+gx)}{a+be^{h}e^{ix}+ce^{2h}e^{2ix}}\,dx$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
Integral((d + e*exp(h)*exp(i*x))*(f + g*x)/(a + b*exp(h)*exp(i*x) + c*exp(
2*h)*exp(2*i*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ee^{h+ix})(f+gx)}{a+be^{h+ix}+ce^{2h+2ix}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, al
gorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{(d + ee^{h+ix})(f + gx)}{a + be^{h+ix} + ce^{2h+2ix}} \, dx = \int \frac{(gx + f)(ee^{(ix+h)} + d)}{ce^{(2\,ix+2\,h)} + be^{(ix+h)} + a} \, dx$$

input
```
integrate((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, al
gorithm="giac")
```

output
```
integrate((g*x + f)*(e*e^(i*x + h) + d)/(c*e^(2*i*x + 2*h) + b*e^(i*x + h)
 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d + ee^{h+ix})(f + gx)}{a + be^{h+ix} + ce^{2h+2ix}} \, dx = \int \frac{(f + g\,x)\,\left(d + e\,\mathrm{e}^{h+i\,x}\right)}{a + b\,\mathrm{e}^{h+i\,x} + c\,\mathrm{e}^{2\,h+2\,i\,x}} \, dx$$

input
```
int(((f + g*x)*(d + e*exp(h + i*x)))/(a + b*exp(h + i*x) + c*exp(2*h + 2*i
*x)),x)
```

output
```
int(((f + g*x)*(d + e*exp(h + i*x)))/(a + b*exp(h + i*x) + c*exp(2*h + 2*i
*x)), x)
```

Reduce [F]

$$\int \frac{(d + ee^{h+ix})(f + gx)}{a + be^{h+ix} + ce^{2h+2ix}}\,dx$$
$$= \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right)aefi + 2\sqrt{4ac-b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right)bdfi + 8e^h\left(\int \frac{e^{ix}x}{e^{2ix+2h}c+e^{ix+h}b+a}dx\right)a^2ce}{}$$

input
```
int((d+e*exp(i*x+h))*(g*x+f)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c - b**2))*a
*e*f*i + 2*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c - b**
2))*b*d*f*i + 8*e**h*int((e**(i*x)*x)/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b
 + a),x)*a**2*c*e*g - 2*e**h*int((e**(i*x)*x)/(e**(2*h + 2*i*x)*c + e**(h
+ i*x)*b + a),x)*a*b**2*e*g + 8*int(x/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b
 + a),x)*a**2*c*d*g - 2*int(x/(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a),x)
*a*b**2*d*g + 4*log(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a)*a*c*d*f*i - l
og(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a)*b**2*d*f*i + 8*a*c*d*f*x - 2*b
**2*d*f*x)/(2*a*(4*a*c - b**2))
```

3.497 $\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$

Optimal result

Integrand size = 37, antiderivative size = 95

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx=\frac{dx}{a}+\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2ce^{h+ix}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}i}-\frac{d\log\left(a+be^{h+ix}+ce^{2h+2ix}\right)}{2ai}$$

output
```
d*x/a+(-2*a*e+b*d)*arctanh((b+2*c*exp(i*x+h))/(-4*a*c+b^2)^(1/2))/a/(-4*a*
c+b^2)^(1/2)/i-1/2*d*ln(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/a/i
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx=\frac{\frac{(-2bd+4ae)\arctan\left(\frac{b+2ce^{h+ix}}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+d\left(2\log\left(e^{h+ix}\right)-\log\left(a+e^{h+ix}\left(b+ce^{h+ix}\right)\right)\right)}{2ai}$$

input
```
Integrate[(d + e*E^(h + i*x))/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*x)),x]
```

output
```
(((-2*b*d + 4*a*e)*ArcTan[(b + 2*c*E^(h + i*x))/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] + d*(2*Log[E^(h + i*x)] - Log[a + E^(h + i*x)*(b + c*E^(h +
i*x))]))/(2*a*i)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2720, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{e^{-h-ix}(d+ee^{h+ix})}{a+be^{h+ix}+ce^{2h+2ix}}de^{h+ix}}{i}$$

$$\Big\downarrow \text{ 1200}$$

$$\frac{\int \left(\frac{e^{-h-ix}d}{a}+\frac{-ce^{h+ix}d-bd+ae}{a(a+be^{h+ix}+ce^{2h+2ix})}\right)de^{h+ix}}{i}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(bd-2ae)\text{arctanh}\left(\frac{b+2ce^{h+ix}}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{d\log(a+be^{h+ix}+ce^{2h+2ix})}{2a}+\frac{d\log(e^{h+ix})}{a}}{i}$$

input
```
Int[(d + e*E^(h + i*x))/(a + b*E^(h + i*x) + c*E^(2*h + 2*i*x)),x]
```

output
```
(((b*d - 2*a*e)*ArcTanh[(b + 2*c*E^(h + i*x))/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[
b^2 - 4*a*c]) + (d*Log[E^(h + i*x)])/a - (d*Log[a + b*E^(h + i*x) + c*E^(2
*h + 2*i*x)])/(2*a))/i
```

Defintions of rubi rules used

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(86) = 172$.

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.85

method	result
default	$\frac{d\left(\frac{\ln\left(e^{ix}\right)}{a}+\frac{-\frac{\ln\left(a+b\,e^{ix}e^{h}+c\,e^{2ix}e^{2h}\right)}{2}-\frac{e^{h}b\arctan\left(\frac{e^{h}b+2\,e^{2h}e^{ix}c}{\sqrt{4ac\,e^{2h}-e^{2h}b^{2}}}\right)}{\sqrt{4ac\,e^{2h}-e^{2h}b^{2}}}}{a}\right)}{i}+\frac{2e\,e^{h}\arctan\left(\frac{e^{h}b+2\,e^{2h}e^{ix}c}{\sqrt{4ac\,e^{2h}-e^{2h}b^{2}}}\right)}{i\sqrt{4ac\,e^{2h}-e^{2h}b^{2}}}$
risch	$\frac{4acd\,i^{2}x}{4a^{2}c\,i^{2}-a\,b^{2}i^{2}}-\frac{b^{2}d\,i^{2}x}{4a^{2}c\,i^{2}-a\,b^{2}i^{2}}+\frac{4acdhi}{4a^{2}c\,i^{2}-a\,b^{2}i^{2}}-\frac{b^{2}dhi}{4a^{2}c\,i^{2}-a\,b^{2}i^{2}}-\frac{2\ln\left(e^{ix+h}+\frac{2abe-b^{2}d+\sqrt{-16a^{3}c\,e^{2}+4a^{2}b^{2}e^{2}+16a^{2}bcd}}{2c(2ae-bd)}\right.}{(4ac-b^{2})i}$

input
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x,method=_RETURNVER
BOSE)
```

output
```
d/i*(1/a*ln(exp(i*x))+1/a*(-1/2*ln(a+b*exp(i*x)*exp(h)+c*exp(i*x)^2*exp(2*
h))-exp(h)*b/(4*a*c*exp(2*h)-exp(h)^2*b^2)^(1/2)*arctan((exp(h)*b+2*exp(2*
h)*exp(i*x)*c)/(4*a*c*exp(2*h)-exp(h)^2*b^2)^(1/2))))+2*e*exp(h)/i/(4*a*c*
exp(2*h)-exp(h)^2*b^2)^(1/2)*arctan((exp(h)*b+2*exp(2*h)*exp(i*x)*c)/(4*a*
c*exp(2*h)-exp(h)^2*b^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.06

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$$
$$= \left[\frac{2\,(b^2-4\,ac)dix-(b^2-4\,ac)d\log\left(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a\right)-\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2e^{(2\,ix+2\,h)}+\cdots}{\cdots}\right.}{2\,(ab^2-4\,a^2c)i}\right.$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm=
"fricas")
```

output
```
[1/2*(2*(b^2 - 4*a*c)*d*i*x - (b^2 - 4*a*c)*d*log(c*e^(2*i*x + 2*h) + b*e^
(i*x + h) + a) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*e^(2*i*x + 2*h
) + 2*b*c*e^(i*x + h) + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*e^(i*x + h) +
 b))/(c*e^(2*i*x + 2*h) + b*e^(i*x + h) + a)))/((a*b^2 - 4*a^2*c)*i), 1/2*
(2*(b^2 - 4*a*c)*d*i*x - (b^2 - 4*a*c)*d*log(c*e^(2*i*x + 2*h) + b*e^(i*x
+ h) + a) + 2*sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-sqrt(-b^2 + 4*a*c)*
(2*c*e^(i*x + h) + b)/(b^2 - 4*a*c)))/((a*b^2 - 4*a^2*c)*i)]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx$$
$$=\operatorname{RootSum}\left(z^2\cdot\left(4a^2ci^2-ab^2i^2\right)+z\left(4acdi-b^2di\right)+ae^2-bde+cd^2,\left(i\mapsto i\log\left(e^{h+ix}+\frac{4ia^2ci-iab^2\cdots}{2\cdots}\right.\right.\right.$$
$$+\frac{dx}{a}$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
RootSum(_z**2*(4*a**2*c*i**2 - a*b**2*i**2) + _z*(4*a*c*d*i - b**2*d*i) +
a*e**2 - b*d*e + c*d**2, Lambda(_i, _i*log(exp(h + i*x) + (4*_i*a**2*c*i -
 _i*a*b**2*i + a*b*e + 2*a*c*d - b**2*d)/(2*a*c*e - b*c*d)))) + d*x/a
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{d + ee^{h+ix}}{a + be^{h+ix} + ce^{2h+2ix}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm=
"maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{d + ee^{h+ix}}{a + be^{h+ix} + ce^{2h+2ix}}\,dx = \frac{\frac{2\,(ix+h)d}{a} - \frac{d\log\left(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a\right)}{a} - \frac{2\,(bd-2\,ae)\arctan\left(\frac{2\,ce^{(ix+h)}+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}a}}{2\,i}$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x, algorithm=
"giac")
```

output
```
1/2*(2*(i*x + h)*d/a - d*log(c*e^(2*i*x + 2*h) + b*e^(i*x + h) + a)/a - 2*
(b*d - 2*a*e)*arctan((2*c*e^(i*x + h) + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a))/i
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx = \frac{d\,x}{a}-\frac{d\,\ln\left(a+b\,e^{i\,x}\,e^{h}+c\,e^{2\,h}\,e^{2\,i\,x}\right)}{2\,a\,i}+\frac{\mathrm{atan}\left(\frac{b+2\,c\,e^{i\,x}\,e^{h}}{\sqrt{4\,a\,c-b^2}}\right)\,\left(2\,a\,e-b\,d\right)}{a\,i\,\sqrt{4\,a\,c-b^2}}$$

input
```
int((d + e*exp(h + i*x))/(a + b*exp(h + i*x) + c*exp(2*h + 2*i*x)),x)
```

output
```
(d*x)/a - (d*log(a + b*exp(i*x)*exp(h) + c*exp(2*h)*exp(2*i*x)))/(2*a*i) +
 (atan((b + 2*c*exp(i*x)*exp(h))/(4*a*c - b^2)^(1/2))*(2*a*e - b*d))/(a*i*
(4*a*c - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.87

$$\int \frac{d+ee^{h+ix}}{a+be^{h+ix}+ce^{2h+2ix}}\,dx = \frac{-4\sqrt{4ac-b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right)aei+2\sqrt{4ac-b^2}\,atan\left(\frac{2e^{ix+h}c+b}{\sqrt{4ac-b^2}}\right)bdi+4\log\left(e^{2ix+2h}c+e^{ix+h}b+a\right)acdi}{2a\left(4ac-b^2\right)}$$

input
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h)),x)
```

output
```
( - 4*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c - b**2))*a
*e*i + 2*sqrt(4*a*c - b**2)*atan((2*e**(h + i*x)*c + b)/sqrt(4*a*c - b**2)
)*b*d*i + 4*log(e**(2*h + 2*i*x)*c + e**(h + i*x)*b + a)*a*c*d*i - log(e**
(2*h + 2*i*x)*c + e**(h + i*x)*b + a)*b**2*d*i + 8*a*c*d*x - 2*b**2*d*x)/(
2*a*(4*a*c - b**2))
```

3.498 $\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 44

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx = d\text{Int}\left(\frac{1}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)},x\right) + e\text{Int}\left(\frac{e^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)},x\right)$$

output
```
d*Defer(Int)(1/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x)+e*Defer(Int)(e
xp(i*x+h)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x)
```

Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx = \int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx$$

input
```
Integrate[(d + e*E^(h + i*x))/((a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))*(f
+ g*x)),x]
```

output
```
Integrate[(d + e*E^(h + i*x))/((a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))*(f
+ g*x)), x]
```

Rubi [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ee^{h+ix}}{(f+gx)\left(a+be^{h+ix}+ce^{2h+2ix}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{d}{(f+gx)\left(a+be^{h+ix}+ce^{2h+2ix}\right)}+\frac{ee^{h+ix}}{(f+gx)\left(a+be^{h+ix}+ce^{2h+2ix}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{1}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)(f+gx)}dx+e\int \frac{e^{h+ix}}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)(f+gx)}dx$$

input
```
Int[(d + e*E^(h + i*x))/((a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))*(f + g*x)
),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{d+e\,\mathrm{e}^{ix+h}}{\left(a+b\,\mathrm{e}^{ix+h}+c\,\mathrm{e}^{2ix+2h}\right)(gx+f)}dx$$

input
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x)
```

output
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{d+ee^{h+ix}}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)(f+gx)}\,dx=\int \frac{ee^{(ix+h)}+d}{(gx+f)(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x, al
gorithm="fricas")
```

output
```
integral((e*e^(i*x + h) + d)/(a*g*x + a*f + (c*g*x + c*f)*e^(2*i*x + 2*h)
+ (b*g*x + b*f)*e^(i*x + h)), x)
```

Sympy [N/A]

Not integrable

Time = 54.71 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{d+ee^{h+ix}}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)\left(f+gx\right)}\,dx=\int \frac{d+ee^{h}e^{ix}}{\left(f+gx\right)\left(a+be^{h}e^{ix}+ce^{2h}e^{2ix}\right)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x)
```

output
```
Integral((d + e*exp(h)*exp(i*x))/((f + g*x)*(a + b*exp(h)*exp(i*x) + c*exp
(2*h)*exp(2*i*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{d+ee^{h+ix}}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)\left(f+gx\right)}\,dx=\int \frac{ee^{(ix+h)}+d}{(gx+f)(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x, al
gorithm="maxima")
```

output
```
integrate((e*e^(i*x + h) + d)/((g*x + f)*(c*e^(2*i*x + 2*h) + b*e^(i*x + h
) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx=\int \frac{ee^{(ix+h)}+d}{(gx+f)(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x, al
gorithm="giac")
```

output
```
integrate((e*e^(i*x + h) + d)/((g*x + f)*(c*e^(2*i*x + 2*h) + b*e^(i*x + h
) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx=\int \frac{d+e\,\mathrm{e}^{h+i\,x}}{(f+g\,x)\,\left(a+b\,\mathrm{e}^{h+i\,x}+c\,\mathrm{e}^{2\,h+2\,i\,x}\right)}\,dx$$

input
```
int((d + e*exp(h + i*x))/((f + g*x)*(a + b*exp(h + i*x) + c*exp(2*h + 2*i*
x))),x)
```

output
```
int((d + e*exp(h + i*x))/((f + g*x)*(a + b*exp(h + i*x) + c*exp(2*h + 2*i*
x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 3.05

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)}\,dx$$
$$= e^h\left(\int \frac{e^{ix}}{e^{2ix+2h}cf+e^{2ix+2h}cgx+e^{ix+h}bf+e^{ix+h}bgx+af+agx}dx\right)e$$
$$+\left(\int \frac{1}{e^{2ix+2h}cf+e^{2ix+2h}cgx+e^{ix+h}bf+e^{ix+h}bgx+af+agx}dx\right)d$$

input
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f),x)
```

output
```
e**h*int(e**(i*x)/(e**(2*h + 2*i*x)*c*f + e**(2*h + 2*i*x)*c*g*x + e**(h +
 i*x)*b*f + e**(h + i*x)*b*g*x + a*f + a*g*x),x)*e + int(1/(e**(2*h + 2*i*
x)*c*f + e**(2*h + 2*i*x)*c*g*x + e**(h + i*x)*b*f + e**(h + i*x)*b*g*x +
a*f + a*g*x),x)*d
```

3.499 $\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 44

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx = d\text{Int}\left(\frac{1}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2},x\right) + e\text{Int}\left(\frac{e^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2},x\right)$$

output
```
d*Defer(Int)(1/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x)+e*Defer(Int)
(exp(i*x+h)/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 12.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx = \int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx$$

input
```
Integrate[(d + e*E^(h + i*x))/((a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))*(f
+ g*x)^2),x]
```

output
```
Integrate[(d + e*E^(h + i*x))/((a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))*(f
+ g*x)^2), x]
```

Rubi [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ee^{h+ix}}{(f+gx)^2\left(a+be^{h+ix}+ce^{2h+2ix}\right)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{d}{(f+gx)^2\left(a+be^{h+ix}+ce^{2h+2ix}\right)}+\frac{ee^{h+ix}}{(f+gx)^2\left(a+be^{h+ix}+ce^{2h+2ix}\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$d\int \frac{1}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)(f+gx)^2}dx+e\int \frac{e^{h+ix}}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)(f+gx)^2}dx$$

input
```
Int[(d + e*E^(h + i*x))/((a + b*E^(h + i*x) + c*E^(2*h + 2*i*x))*(f + g*x)
^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \frac{d+e\,\mathrm{e}^{ix+h}}{\left(a+b\,\mathrm{e}^{ix+h}+c\,\mathrm{e}^{2ix+2h}\right)\left(gx+f\right)^{2}}dx$$

input
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x)
```

output
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.09

$$\int \frac{d+ee^{h+ix}}{\left(a+be^{h+ix}+ce^{2h+2ix}\right)(f+gx)^2}\,dx=\int \frac{ee^{(ix+h)}+d}{(gx+f)^2(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x,
algorithm="fricas")
```

output
```
integral((e*e^(i*x + h) + d)/(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (c*g^2*x^2 +
 2*c*f*g*x + c*f^2)*e^(2*i*x + 2*h) + (b*g^2*x^2 + 2*b*f*g*x + b*f^2)*e^(i
*x + h)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx = \int \frac{ee^{(ix+h)}+d}{(gx+f)^2(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x,
algorithm="maxima")
```

output
```
integrate((e*e^(i*x + h) + d)/((g*x + f)^2*(c*e^(2*i*x + 2*h) + b*e^(i*x +
 h) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx = \int \frac{ee^{(ix+h)}+d}{(gx+f)^2(ce^{(2\,ix+2\,h)}+be^{(ix+h)}+a)}\,dx$$

input
```
integrate((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x,
algorithm="giac")
```

output
```
integrate((e*e^(i*x + h) + d)/((g*x + f)^2*(c*e^(2*i*x + 2*h) + b*e^(i*x +
 h) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx=\int \frac{d+e\,\mathrm{e}^{h+i\,x}}{(f+g\,x)^2\,\left(a+b\,\mathrm{e}^{h+i\,x}+c\,\mathrm{e}^{2\,h+2\,i\,x}\right)}\,dx$$

input
```
int((d + e*exp(h + i*x))/((f + g*x)^2*(a + b*exp(h + i*x) + c*exp(2*h + 2*
i*x))),x)
```

output
```
int((d + e*exp(h + i*x))/((f + g*x)^2*(a + b*exp(h + i*x) + c*exp(2*h + 2*
i*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 5.45

$$\int \frac{d+ee^{h+ix}}{(a+be^{h+ix}+ce^{2h+2ix})(f+gx)^2}\,dx$$
$$=e^h\left(\int \frac{e^{ix}}{e^{2ix+2h}c\,f^2+2e^{2ix+2h}cfgx+e^{2ix+2h}c\,g^2x^2+e^{ix+h}b\,f^2+2e^{ix+h}bfgx+e^{ix+h}b\,g^2x^2+a\,f^2+2afg\right.$$
$$+\left(\int \frac{1}{e^{2ix+2h}c\,f^2+2e^{2ix+2h}cfgx+e^{2ix+2h}c\,g^2x^2+e^{ix+h}b\,f^2+2e^{ix+h}bfgx+e^{ix+h}b\,g^2x^2+a\,f^2+2afg\right.$$

input
```
int((d+e*exp(i*x+h))/(a+b*exp(i*x+h)+c*exp(2*i*x+2*h))/(g*x+f)^2,x)
```

output
```
e**h*int(e**(i*x)/(e**(2*h + 2*i*x)*c*f**2 + 2*e**(2*h + 2*i*x)*c*f*g*x +
e**(2*h + 2*i*x)*c*g**2*x**2 + e**(h + i*x)*b*f**2 + 2*e**(h + i*x)*b*f*g*
x + e**(h + i*x)*b*g**2*x**2 + a*f**2 + 2*a*f*g*x + a*g**2*x**2),x)*e + in
t(1/(e**(2*h + 2*i*x)*c*f**2 + 2*e**(2*h + 2*i*x)*c*f*g*x + e**(2*h + 2*i*
x)*c*g**2*x**2 + e**(h + i*x)*b*f**2 + 2*e**(h + i*x)*b*f*g*x + e**(h + i*
x)*b*g**2*x**2 + a*f**2 + 2*a*f*g*x + a*g**2*x**2),x)*d
```

3.500 $\int \frac{\left(be-aee^{c+dx}\right)x}{be-2aee^{c+dx}-bee^{2(c+dx)}}\,dx$

Optimal result

Integrand size = 47, antiderivative size = 150

$$\int \frac{\left(be-aee^{c+dx}\right)x}{be-2aee^{c+dx}-bee^{2(c+dx)}}\,dx=\frac{x^2}{2}-\frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d}-\frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2d}-\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2}-\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2d^2}$$

output
```
1/2*x^2-1/2*x*ln(1+b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)))/d-1/2*x*ln(1+b*exp(d*
x+c)/(a+(a^2+b^2)^(1/2)))/d-1/2*polylog(2,-b*exp(d*x+c)/(a-(a^2+b^2)^(1/2)
))/d^2-1/2*polylog(2,-b*exp(d*x+c)/(a+(a^2+b^2)^(1/2)))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 398 vs. $2(150) = 300$.

Time = 0.76 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.65

$$\int \frac{(be - aee^{c+dx})\,x}{be - 2aee^{c+dx} - bee^{2(c+dx)}}\,dx = \frac{-adx\log\left(1+\frac{\left(a-\sqrt{a^2+b^2}\right)e^{-c-dx}}{b}\right) - \sqrt{a^2+b^2}dx\log\left(1+\frac{\left(a-\sqrt{a^2+b^2}\right)e^{-c-dx}}{b}\right) + adx\log\left(1+\frac{\left(a+\sqrt{a^2+b^2}\right)e}{b}\right.}{}$$

input
```
Integrate[((b*e - a*e*E^(c + d*x))*x)/(b*e - 2*a*e*E^(c + d*x) - b*e*E^(2*
(c + d*x))),x]
```

output
```
(-(a*d*x*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b]) - Sqrt[a^2 + b^2
]*d*x*Log[1 + ((a - Sqrt[a^2 + b^2])*E^(-c - d*x))/b] + a*d*x*Log[1 + ((a
+ Sqrt[a^2 + b^2])*E^(-c - d*x))/b] - Sqrt[a^2 + b^2]*d*x*Log[1 + ((a + Sq
rt[a^2 + b^2])*E^(-c - d*x))/b] + a*d*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2])] - a*d*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + (a +
 Sqrt[a^2 + b^2])*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b] + (-
a + Sqrt[a^2 + b^2])*PolyLog[2, -(((a + Sqrt[a^2 + b^2])*E^(-c - d*x))/b)]
 + a*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - a*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*Sqrt[a^2 + b^2]*d^2)
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {2695, 27, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(be - aee^{c+dx})}{-2aee^{c+dx} - bee^{2(c+dx)} + be}\,dx$$

$$\Big\downarrow \; 2695$$

$$-\left(e\left(a-\sqrt{a^2+b^2}\right)\int -\frac{x}{2\left(be^{c+dx}e+\left(a-\sqrt{a^2+b^2}\right)e\right)}dx\right)-e\left(\sqrt{a^2+b^2}+a\right)\int -\frac{x}{2\left(be^{c+dx}e+\left(a+\sqrt{a^2+b^2}\right)e\right)}dx$$

$\Big\downarrow$ 27

$$\frac{1}{2}e\left(a-\sqrt{a^2+b^2}\right)\int \frac{x}{be^{c+dx}e+\left(a-\sqrt{a^2+b^2}\right)e}dx+\frac{1}{2}e\left(\sqrt{a^2+b^2}+a\right)\int \frac{x}{be^{c+dx}e+\left(a+\sqrt{a^2+b^2}\right)e}dx$$

$\Big\downarrow$ 2615

$$\frac{1}{2}e\left(a-\sqrt{a^2+b^2}\right)\left(\frac{x^2}{2e\left(a-\sqrt{a^2+b^2}\right)}-\frac{b\int \frac{e^{c+dx}x}{be^{c+dx}e+\left(a-\sqrt{a^2+b^2}\right)e}dx}{a-\sqrt{a^2+b^2}}\right)+\frac{1}{2}e\left(\sqrt{a^2+b^2}+a\right)\left(\frac{x^2}{2e\left(\sqrt{a^2+b^2}+a\right)}-\frac{b\int \frac{e^{c+dx}x}{be^{c+dx}e+\left(a+\sqrt{a^2+b^2}\right)e}dx}{\sqrt{a^2+b^2}+a}\right)$$

$\Big\downarrow$ 2620

$$\frac{1}{2}e\left(a-\sqrt{a^2+b^2}\right)\left(\frac{x^2}{2e\left(a-\sqrt{a^2+b^2}\right)}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bde}-\frac{\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)dx}{bde}\right)}{a-\sqrt{a^2+b^2}}\right)+\frac{1}{2}e\left(\sqrt{a^2+b^2}+a\right)\left(\frac{x^2}{2e\left(\sqrt{a^2+b^2}+a\right)}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bde}-\frac{\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)dx}{bde}\right)}{\sqrt{a^2+b^2}+a}\right)$$

$\Big\downarrow$ 2715

$$\frac{1}{2}e\left(a-\sqrt{a^2+b^2}\right)\left(\frac{x^2}{2e\left(a-\sqrt{a^2+b^2}\right)}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bde}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2e}\right)}{a-\sqrt{a^2+b^2}}\right)+\frac{1}{2}e\left(\sqrt{a^2+b^2}+a\right)\left(\frac{x^2}{2e\left(\sqrt{a^2+b^2}+a\right)}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bde}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)de^{c+dx}}{bd^2e}\right)}{\sqrt{a^2+b^2}+a}\right)$$

$$\Big\downarrow \; 2838$$

$$\frac{1}{2}e\left(a-\sqrt{a^2+b^2}\right)\left(\frac{x^2}{2e\left(a-\sqrt{a^2+b^2}\right)}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2e}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{bde}\right)}{a-\sqrt{a^2+b^2}}\right)+\frac{1}{2}e\left(\sqrt{a^2+b^2}+a\right)\left(\frac{x^2}{2e\left(\sqrt{a^2+b^2}+a\right)}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd^2e}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{bde}\right)}{\sqrt{a^2+b^2}+a}\right)$$

input
```
Int[((b*e - a*e*E^(c + d*x))*x)/(b*e - 2*a*e*E^(c + d*x) - b*e*E^(2*(c + d
*x))),x]
```

output
```
((a - Sqrt[a^2 + b^2])*e*(x^2/(2*(a - Sqrt[a^2 + b^2])*e) - (b*((x*Log[1 +
 (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*d*e) + PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))]/(b*d^2*e)))/(a - Sqrt[a^2 + b^2])))/2 + ((a
+ Sqrt[a^2 + b^2])*e*(x^2/(2*(a + Sqrt[a^2 + b^2])*e) - (b*((x*Log[1 + (b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*d*e) + PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))]/(b*d^2*e)))/(a + Sqrt[a^2 + b^2])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2695
```
Int[(((i_.)*(F_)^(u_) + (h_))*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F
_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(Simplify[(2*c*h - b*i)/q] + i)   Int[(f + g*x)^m/(b - q + 2*c*F^u), x],
x] - Simp[(Simplify[(2*c*h - b*i)/q] - i)   Int[(f + g*x)^m/(b + q + 2*c*F^
u), x], x]] /; FreeQ[{F, a, b, c, f, g, h, i}, x] && EqQ[v, 2*u] && LinearQ
[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(128) = 256.

Time = 0.15 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.12

method	result
risch	$\frac{c\ln\left(2\,\mathrm{e}^{dx+c}a+\mathrm{e}^{2dx+2c}b-b\right)}{2d^{2}}-\frac{c\ln\left(\mathrm{e}^{dx+c}\right)}{d^{2}}-\frac{\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}+b^{2}}-a}{-a+\sqrt{a^{2}+b^{2}}}\right)x}{2d}-\frac{\ln\left(\frac{-b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}+b^{2}}-a}{-a+\sqrt{a^{2}+b^{2}}}\right)c}{2d^{2}}-\frac{\ln\left(\frac{b\,\mathrm{e}^{dx+c}+\sqrt{a^{2}\cdots}}{a+\sqrt{a^{2}\cdots}}\right.}{2d}$
default	Expression too large to display

input
```
int((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c)),x,met
hod=_RETURNVERBOSE)
```

output
```
1/2/d^2*c*ln(2*exp(d*x+c)*a+exp(2*d*x+2*c)*b-b)-1/d^2*c*ln(exp(d*x+c))-1/2
/d*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/2/d^2*ln
((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/2/d*ln((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/2/d^2*ln((b*exp(d*x+c)
+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/2/d^2*dilog((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/2/d^2*dilog((b*exp(d*x+c)+(a^2+b^
2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/2*x^2+c*x/d+1/2/d^2*c^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.67

$$\int \frac{(be-aee^{c+dx})\,x}{be-2aee^{c+dx}-bee^{2(c+dx)}}\,dx$$
$$=\frac{d^2x^2+c\log\left(2\,be^{(dx+c)}+2\,b\sqrt{\frac{a^2+b^2}{b^2}}+2\,a\right)+c\log\left(2\,be^{(dx+c)}-2\,b\sqrt{\frac{a^2+b^2}{b^2}}+2\,a\right)-(dx+c)\log\left(-\frac{b\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c))
,x, algorithm="fricas")
```

output
```
1/2*(d^2*x^2 + c*log(2*b*e^(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
c*log(2*b*e^(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (d*x + c)*log(-
(b*sqrt((a^2 + b^2)/b^2)*e^(d*x + c) + a*e^(d*x + c) - b)/b) - (d*x + c)*l
og((b*sqrt((a^2 + b^2)/b^2)*e^(d*x + c) - a*e^(d*x + c) + b)/b) - dilog((b
*sqrt((a^2 + b^2)/b^2)*e^(d*x + c) + a*e^(d*x + c) - b)/b + 1) - dilog(-(b
*sqrt((a^2 + b^2)/b^2)*e^(d*x + c) - a*e^(d*x + c) + b)/b + 1))/d^2
```

Sympy [F]

$$\int \frac{(be - aee^{c+dx})\,x}{be - 2aee^{c+dx} - bee^{2(c+dx)}}\,dx = \int \frac{x(ae^ce^{dx} - b)}{2ae^ce^{dx} + be^{2c}e^{2dx} - b}\,dx$$

input
```
integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c))
,x)
```

output
```
Integral(x*(a*exp(c)*exp(d*x) - b)/(2*a*exp(c)*exp(d*x) + b*exp(2*c)*exp(2
*d*x) - b), x)
```

Maxima [F]

$$\int \frac{(be - aee^{c+dx})\,x}{be - 2aee^{c+dx} - bee^{2(c+dx)}}\,dx = \int \frac{(aee^{(dx+c)} - be)x}{bee^{(2\,dx+2\,c)} + 2\,aee^{(dx+c)} - be}\,dx$$

input
```
integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c))
,x, algorithm="maxima")
```

output
```
integrate((a*e*e^(d*x + c) - b*e)*x/(b*e*e^(2*d*x + 2*c) + 2*a*e*e^(d*x +
c) - b*e), x)
```

Giac [F]

$$\int \frac{(be - aee^{c+dx})\,x}{be - 2aee^{c+dx} - bee^{2(c+dx)}}\,dx = \int \frac{(aee^{(dx+c)} - be)\,x}{bee^{(2\,dx+2\,c)} + 2\,aee^{(dx+c)} - be}\,dx$$

input
```
integrate((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c))
,x, algorithm="giac")
```

output
```
integrate((a*e*e^(d*x + c) - b*e)*x/(b*e*e^(2*d*x + 2*c) + 2*a*e*e^(d*x +
c) - b*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(be - aee^{c+dx})\,x}{be - 2aee^{c+dx} - bee^{2(c+dx)}}\,dx = \int -\frac{x\,(b\,e - a\,e\,\mathrm{e}^{c+d\,x})}{2\,a\,e\,\mathrm{e}^{c+d\,x} - b\,e + b\,e\,\mathrm{e}^{2\,c+2\,d\,x}}\,dx$$

input
```
int(-(x*(b*e - a*e*exp(c + d*x)))/(2*a*e*exp(c + d*x) - b*e + b*e*exp(2*c
+ 2*d*x)),x)
```

output
```
int(-(x*(b*e - a*e*exp(c + d*x)))/(2*a*e*exp(c + d*x) - b*e + b*e*exp(2*c
+ 2*d*x)), x)
```

Reduce [F]

$$\int \frac{(be - aee^{c+dx})\,x}{be - 2aee^{c+dx} - bee^{2(c+dx)}}\,dx = e^{c}\left(\int \frac{e^{dx}x}{e^{2dx+2c}b + 2e^{dx+c}a - b}dx\right)a - \left(\int \frac{x}{e^{2dx+2c}b + 2e^{dx+c}a - b}dx\right)b$$

input
```
int((b*e-a*e*exp(d*x+c))*x/(b*e-2*a*e*exp(d*x+c)-b*e*exp(2*d*x+2*c)),x)
```

output
```
e**c*int((e**(d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*a - i
nt(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a - b),x)*b
```

3.501 $\int F^{a+b\log(c+dx^n)}x^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int F^{a+b\log(c+dx^n)}x^2\,dx=\frac{1}{3}F^ax^3(c+dx^n)^{b\log(F)}\left(1+\frac{dx^n}{c}\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(\frac{3}{n},-b\log(F),\frac{3+n}{n},-\frac{dx^n}{c}\right)$$

output
```
1/3*F^a*x^3*(c+d*x^n)^(b*ln(F))*hypergeom([3/n, -b*ln(F)],[(3+n)/n],-d*x^n
/c)/((1+d*x^n/c)^(b*ln(F)))
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int F^{a+b\log(c+dx^n)}x^2\,dx=-\frac{F^{a+b\log(c+dx^n)}x^3\left(-\frac{dx^n}{c}\right)^{-3/n}(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{-3+n}{n},1+b\log(F),2+b\log(F),1+\frac{dx^n}{c}\right)}{cn(1+b\log(F))}$$

input
```
Integrate[F^(a + b*Log[c + d*x^n])*x^2,x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*x^3*(c + d*x^n)*Hypergeometric2F1[(-3 + n)/n,
1 + b*Log[F], 2 + b*Log[F], 1 + (d*x^n)/c])/(c*n*(-((d*x^n)/c))^(3/n)*(1 +
 b*Log[F])))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2704, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 F^{a+b\log(c+dx^n)}\,dx$$

$$\Big\downarrow \text{2704}$$

$$\int x^2 F^a (c+dx^n)^{b\log(F)}\,dx$$

$$\Big\downarrow \text{27}$$

$$F^a \int x^2 (dx^n+c)^{b\log(F)}\,dx$$

$$\Big\downarrow \text{889}$$

$$F^a (c+dx^n)^{b\log(F)} \left(\frac{dx^n}{c}+1\right)^{-b\log(F)} \int x^2 \left(\frac{dx^n}{c}+1\right)^{b\log(F)}\,dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}x^3 F^a (c+dx^n)^{b\log(F)} \left(\frac{dx^n}{c}+1\right)^{-b\log(F)} \text{Hypergeometric2F1}\left(\frac{3}{n},-b\log(F),\frac{n+3}{n},-\frac{dx^n}{c}\right)$$

input
```
Int[F^(a + b*Log[c + d*x^n])*x^2,x]
```

output
```
(F^a*x^3*(c + d*x^n)^(b*Log[F])*Hypergeometric2F1[3/n, -(b*Log[F]), (3 + n
)/n, -((d*x^n)/c)])/(3*(1 + (d*x^n)/c)^(b*Log[F]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int F^{a+b\ln(c+d\,x^n)}x^2dx$$

input
```
int(F^(a+b*ln(c+d*x^n))*x^2,x)
```

output
```
int(F^(a+b*ln(c+d*x^n))*x^2,x)
```

Fricas [F]

$$\int F^{a+b\log(c+dx^n)}x^2\,dx = \int F^{b\log(dx^n+c)+a}x^2\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*x^2,x, algorithm="fricas")
```

output
```
integral(F^(b*log(d*x^n + c) + a)*x^2, x)
```

Sympy [F]

$$\int F^{a+b\log(c+dx^n)}x^2\,dx = \int F^{a+b\log{(c+dx^n)}}x^2\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n))*x**2,x)
```

output
```
Integral(F**(a + b*log(c + d*x**n))*x**2, x)
```

Maxima [F]

$$\int F^{a+b\log(c+dx^n)}x^2\,dx = \int F^{b\log(dx^n+c)+a}x^2\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*x^2,x, algorithm="maxima")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)*x^2, x)
```

Giac [F]

$$\int F^{a+b\log(c+dx^n)}x^2\,dx = \int F^{b\log(dx^n+c)+a}x^2\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*x^2,x, algorithm="giac")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b\log(c+dx^n)}x^2\,dx = \int F^{a+b\,\ln(c+d\,x^n)}\,x^2\,dx$$

input
```
int(F^(a + b*log(c + d*x^n))*x^2,x)
```

output
```
int(F^(a + b*log(c + d*x^n))*x^2, x)
```

Reduce [F]

$$\int F^{a+b\log(c+dx^n)}x^2\,dx$$
$$= \frac{f^a\left(f^{\log(x^n d+c)b}x^3 + \left(\int \frac{f^{\log(x^n d+c)b}x^2}{x^n\log(f)bdn+3x^n d+\log(f)bcn+3c}dx\right)\log(f)^2 b^2 c n^2 + 3\left(\int \frac{f^{\log(x^n d+c)b}x^2}{x^n\log(f)bdn+3x^n d+\log(f)bcn+3c}dx\right.}{\log(f)\,bn+3}$$

input
```
int(F^(a+b*log(c+d*x^n))*x^2,x)
```

output
```
(f**a*(f**(log(x**n*d + c)*b)*x**3 + int((f**(log(x**n*d + c)*b)*x**2)/(x*
*n*log(f)*b*d*n + 3*x**n*d + log(f)*b*c*n + 3*c),x)*log(f)**2*b**2*c*n**2
+ 3*int((f**(log(x**n*d + c)*b)*x**2)/(x**n*log(f)*b*d*n + 3*x**n*d + log(
f)*b*c*n + 3*c),x)*log(f)*b*c*n))/(log(f)*b*n + 3)
```

3.502 $\int F^{a+b\log(c+dx^n)}x\,dx$

Optimal result

Integrand size = 16, antiderivative size = 65

$$\int F^{a+b\log(c+dx^n)}x\,dx=\frac{1}{2}F^ax^2(c+dx^n)^{b\log(F)}\left(1+\frac{dx^n}{c}\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(\frac{2}{n},-b\log(F),\frac{2+n}{n},-\frac{dx^n}{c}\right)$$

output
```
1/2*F^a*x^2*(c+d*x^n)^(b*ln(F))*hypergeom([2/n, -b*ln(F)],[(2+n)/n],-d*x^n
/c)/((1+d*x^n/c)^(b*ln(F)))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.31

$$\int F^{a+b\log(c+dx^n)}x\,dx=-\frac{F^{a+b\log(c+dx^n)}x^2\left(-\frac{dx^n}{c}\right)^{-2/n}(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{-2+n}{n},1+b\log(F),2+b\log(F),1+\frac{dx^n}{c}\right)}{cn(1+b\log(F))}$$

input
```
Integrate[F^(a + b*Log[c + d*x^n])*x,x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*x^2*(c + d*x^n)*Hypergeometric2F1[(-2 + n)/n,
1 + b*Log[F], 2 + b*Log[F], 1 + (d*x^n)/c])/(c*n*(-((d*x^n)/c))^(2/n)*(1 +
 b*Log[F])))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2704, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xF^{a+b\log(c+dx^n)}\,dx$$

$$\Big\downarrow \text{2704}$$

$$\int xF^a(c+dx^n)^{b\log(F)}\,dx$$

$$\Big\downarrow \text{27}$$

$$F^a\int x(dx^n+c)^{b\log(F)}\,dx$$

$$\Big\downarrow \text{889}$$

$$F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\int x\left(\frac{dx^n}{c}+1\right)^{b\log(F)}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{2}x^2F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(\frac{2}{n},-b\log(F),\frac{n+2}{n},-\frac{dx^n}{c}\right)$$

input
```
Int[F^(a + b*Log[c + d*x^n])*x,x]
```

output
```
(F^a*x^2*(c + d*x^n)^(b*Log[F])*Hypergeometric2F1[2/n, -(b*Log[F]), (2 + n
)/n, -((d*x^n)/c)])/(2*(1 + (d*x^n)/c)^(b*Log[F]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int F^{a+b\ln(c+d\,x^n)}x dx$$

input
```
int(F^(a+b*ln(c+d*x^n))*x,x)
```

output
```
int(F^(a+b*ln(c+d*x^n))*x,x)
```

Fricas [F]

$$\int F^{a+b\log(c+dx^n)}x\,dx = \int F^{b\log(dx^n+c)+a}x\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*x,x, algorithm="fricas")
```

output
```
integral(F^(b*log(d*x^n + c) + a)*x, x)
```

Sympy [F]

$$\int F^{a+b\log(c+dx^n)}x\,dx = \int F^{a+b\log(c+dx^n)}x\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n))*x,x)
```

output
```
Integral(F**(a + b*log(c + d*x**n))*x, x)
```

Maxima [F]

$$\int F^{a+b\log(c+dx^n)}x\,dx = \int F^{b\log(dx^n+c)+a}x\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*x,x, algorithm="maxima")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)*x, x)
```

Giac [F]

$$\int F^{a+b\log(c+dx^n)}x\,dx = \int F^{b\log(dx^n+c)+a}x\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*x,x, algorithm="giac")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b\log(c+dx^n)}x\,dx = \int F^{a+b\,\ln(c+d\,x^n)}\,x\,dx$$

input
```
int(F^(a + b*log(c + d*x^n))*x,x)
```

output
```
int(F^(a + b*log(c + d*x^n))*x, x)
```

Reduce [F]

$$\int F^{a+b\log(c+dx^n)}x\,dx$$
$$= \frac{f^a\left(f^{\log(x^n d+c)b}x^2 + \left(\int \frac{f^{\log(x^n d+c)b}x}{x^n\log(f)bdn+2x^n d+\log(f)bcn+2c}dx\right)\log(f)^2 b^2cn^2 + 2\left(\int \frac{f^{\log(x^n d+c)b}x}{x^n\log(f)bdn+2x^n d+\log(f)bcn+2c}dx\right.}{\log(f)\,bn+2}$$

input
```
int(F^(a+b*log(c+d*x^n))*x,x)
```

output
```
(f**a*(f**(log(x**n*d + c)*b)*x**2 + int((f**(log(x**n*d + c)*b)*x)/(x**n*
log(f)*b*d*n + 2*x**n*d + log(f)*b*c*n + 2*c),x)*log(f)**2*b**2*c*n**2 + 2
*int((f**(log(x**n*d + c)*b)*x)/(x**n*log(f)*b*d*n + 2*x**n*d + log(f)*b*c
*n + 2*c),x)*log(f)*b*c*n))/(log(f)*b*n + 2)
```

3.503 $\int F^{a+b\log(c+dx^n)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 56

$$\int F^{a+b\log(c+dx^n)}\,dx = F^a x(c+dx^n)^{b\log(F)}\left(1+\frac{dx^n}{c}\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(\frac{1}{n},-b\log(F),1+\frac{1}{n},-\frac{dx^n}{c}\right)$$

output
```
F^a*x*(c+d*x^n)^(b*ln(F))*hypergeom([1/n, -b*ln(F)],[1+1/n],-d*x^n/c)/((1+
d*x^n/c)^(b*ln(F)))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.48

$$\int F^{a+b\log(c+dx^n)}\,dx = -\frac{F^{a+b\log(c+dx^n)}x\left(-\frac{dx^n}{c}\right)^{-1/n}(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{-1+n}{n},1+b\log(F),2+b\log(F),1+\frac{dx^n}{c}\right)}{cn(1+b\log(F))}$$

input
```
Integrate[F^(a + b*Log[c + d*x^n]),x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*x*(c + d*x^n)*Hypergeometric2F1[(-1 + n)/n, 1
+ b*Log[F], 2 + b*Log[F], 1 + (d*x^n)/c])/(c*n*(-((d*x^n)/c))^n^(-1)*(1 +
b*Log[F])))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2704, 27, 779, 778}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+b\log(c+dx^n)}\,dx$$

$$\Big\downarrow \text{2704}$$

$$\int F^a(c+dx^n)^{b\log(F)}dx$$

$$\Big\downarrow \text{27}$$

$$F^a\int (dx^n+c)^{b\log(F)}dx$$

$$\Big\downarrow \text{779}$$

$$F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\int \left(\frac{dx^n}{c}+1\right)^{b\log(F)}dx$$

$$\Big\downarrow \text{778}$$

$$xF^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(\frac{1}{n},-b\log(F),1+\frac{1}{n},-\frac{dx^n}{c}\right)$$

input
```
Int[F^(a + b*Log[c + d*x^n]),x]
```

output
```
(F^a*x*(c + d*x^n)^(b*Log[F])*Hypergeometric2F1[n^(-1), -(b*Log[F]), 1 + n
^(-1), -((d*x^n)/c)])/(1 + (d*x^n)/c)^(b*Log[F])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 778
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

rule 779
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(1 + b*(x^n/a))^p, x], x
] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Si
mplify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int F^{a+b\ln(c+d\,x^n)}dx$$

input
```
int(F^(a+b*ln(c+d*x^n)),x)
```

output
```
int(F^(a+b*ln(c+d*x^n)),x)
```

Fricas [F]

$$\int F^{a+b\log(c+dx^n)}\,dx = \int F^{b\log(dx^n+c)+a}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n)),x, algorithm="fricas")
```

output
```
integral(F^(b*log(d*x^n + c) + a), x)
```

Sympy [F]

$$\int F^{a+b\log(c+dx^n)}\,dx = \int F^{a+b\log{(c+dx^n)}}\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n)),x)
```

output
```
Integral(F**(a + b*log(c + d*x**n)), x)
```

Maxima [F]

$$\int F^{a+b\log(c+dx^n)}\,dx = \int F^{b\log(dx^n+c)+a}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n)),x, algorithm="maxima")
```

output
```
integrate(F^(b*log(d*x^n + c) + a), x)
```

Giac [F]

$$\int F^{a+b\log(c+dx^n)}\,dx=\int F^{b\log(dx^n+c)+a}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n)),x, algorithm="giac")
```

output
```
integrate(F^(b*log(d*x^n + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int F^{a+b\log(c+dx^n)}\,dx=\frac{F^a\,x\,(c+d\,x^n)^{b\,\ln(F)}\,{}_2\mathrm{F}_1\left(\frac{1}{n},-b\,\ln(F);\ \frac{1}{n}+1;\ -\frac{d\,x^n}{c}\right)}{\left(\frac{d\,x^n}{c}+1\right)^{b\,\ln(F)}}$$

input
```
int(F^(a + b*log(c + d*x^n)),x)
```

output
```
(F^a*x*(c + d*x^n)^(b*log(F))*hypergeom([1/n, -b*log(F)], 1/n + 1, -(d*x^n
)/c))/((d*x^n)/c + 1)^(b*log(F))
```

Reduce [F]

$$\int F^{a+b\log(c+dx^n)}\,dx$$
$$=\frac{f^a\left(f^{\log(x^nd+c)b}x+\left(\int\frac{f^{\log(x^nd+c)b}}{x^n\log(f)bdn+x^nd+\log(f)bcn+c}dx\right)\log(f)^2b^2cn^2+\left(\int\frac{f^{\log(x^nd+c)b}}{x^n\log(f)bdn+x^nd+\log(f)bcn+c}dx\right)\log(\right.}{\log(f)\,bn+1}$$

input
```
int(F^(a+b*log(c+d*x^n)),x)
```

output
```
(f**a*(f**(log(x**n*d + c)*b)*x + int(f**(log(x**n*d + c)*b)/(x**n*log(f)*
b*d*n + x**n*d + log(f)*b*c*n + c),x)*log(f)**2*b**2*c*n**2 + int(f**(log(
x**n*d + c)*b)/(x**n*log(f)*b*d*n + x**n*d + log(f)*b*c*n + c),x)*log(f)*b
*c*n))/(log(f)*b*n + 1)
```

3.504 $\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 58

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = -\frac{F^a(c+dx^n)^{1+b\log(F)}\,\text{Hypergeometric2F1}\left(1,1+b\log(F),2+b\log(F),\frac{c+dx^n}{c}\right)}{cn(1+b\log(F))}$$

output
```
-F^a*(c+d*x^n)^(1+b*ln(F))*hypergeom([1, 1+b*ln(F)],[2+b*ln(F)],(c+d*x^n)/
c)/c/n/(1+b*ln(F))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = -\frac{F^{a+b\log(c+dx^n)}\left(-1+\text{Hypergeometric2F1}\left(1,b\log(F),1+b\log(F),1+\frac{dx^n}{c}\right)\right)}{bn\log(F)}$$

input
```
Integrate[F^(a + b*Log[c + d*x^n])/x,x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*(-1 + Hypergeometric2F1[1, b*Log[F], 1 + b*Log
[F], 1 + (d*x^n)/c]))/(b*n*Log[F]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2704, 27, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}dx$$

$$\Big\downarrow \text{ 2704}$$

$$\int \frac{F^a(c+dx^n)^{b\log(F)}}{x}dx$$

$$\Big\downarrow \text{ 27}$$

$$F^a\int \frac{(dx^n+c)^{b\log(F)}}{x}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{F^a\int x^{-n}(dx^n+c)^{b\log(F)}dx^n}{n}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{F^a(c+dx^n)^{b\log(F)+1}\text{Hypergeometric2F1}\left(1,b\log(F)+1,b\log(F)+2,\frac{dx^n}{c}+1\right)}{cn(b\log(F)+1)}$$

input
```
Int[F^(a + b*Log[c + d*x^n])/x,x]
```

output
```
-((F^a*(c + d*x^n)^(1 + b*Log[F])*Hypergeometric2F1[1, 1 + b*Log[F], 2 + b
*Log[F], 1 + (d*x^n)/c])/(c*n*(1 + b*Log[F])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int \frac{F^{a+b\ln(c+d\,x^n)}}{x}dx$$

input
```
int(F^(a+b*ln(c+d*x^n))/x,x)
```

output
```
int(F^(a+b*ln(c+d*x^n))/x,x)
```

Fricas [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x,x, algorithm="fricas")
```

output
```
integral(F^(b*log(d*x^n + c) + a)/x, x)
```

Sympy [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = \int \frac{F^{a+b\log{(c+dx^n)}}}{x}\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n))/x,x)
```

output
```
Integral(F**(a + b*log(c + d*x**n))/x, x)
```

Maxima [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x,x, algorithm="maxima")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)/x, x)
```

Giac [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x,x, algorithm="giac")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = \int \frac{F^{a+b\,\ln(c+d\,x^n)}}{x}\,dx$$

input
```
int(F^(a + b*log(c + d*x^n))/x,x)
```

output
```
int(F^(a + b*log(c + d*x^n))/x, x)
```

Reduce [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x}\,dx = \frac{f^a\left(f^{\log(x^n d+c)b}+\left(\int \frac{f^{\log(x^n d+c)b}}{x^n dx+cx}dx\right)\log(f)\,bcn\right)}{\log(f)\,bn}$$

input
```
int(F^(a+b*log(c+d*x^n))/x,x)
```

output
```
(f**a*(f**(log(x**n*d + c)*b) + int(f**(log(x**n*d + c)*b)/(x**n*d*x + c*x
),x)*log(f)*b*c*n))/(log(f)*b*n)
```

3.505 $\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = -\frac{F^a(c+dx^n)^{b\log(F)}\left(1+\frac{dx^n}{c}\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(-\frac{1}{n},-b\log(F),-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{x}$$

output
```
-F^a*(c+d*x^n)^(b*ln(F))*hypergeom([-1/n, -b*ln(F)],[-(1-n)/n],-d*x^n/c)/x
/((1+d*x^n/c)^(b*ln(F)))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = -\frac{F^{a+b\log(c+dx^n)}\left(-\frac{dx^n}{c}\right)^{\frac{1}{n}}(c+dx^n)\,\text{Hypergeometric2F1}\left(1+\frac{1}{n},1+b\log(F),2+b\log(F),1+\frac{dx^n}{c}\right)}{cnx(1+b\log(F))}$$

input
```
Integrate[F^(a + b*Log[c + d*x^n])/x^2,x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*(-((d*x^n)/c))^n^(-1)*(c + d*x^n)*Hypergeometr
ic2F1[1 + n^(-1), 1 + b*Log[F], 2 + b*Log[F], 1 + (d*x^n)/c])/(c*n*x*(1 +
b*Log[F]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2704, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}dx$$

$$\Big\downarrow \text{ 2704}$$

$$\int \frac{F^a(c+dx^n)^{b\log(F)}}{x^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$F^a\int \frac{(dx^n+c)^{b\log(F)}}{x^2}dx$$

$$\Big\downarrow \text{ 889}$$

$$F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\int \frac{\left(\frac{dx^n}{c}+1\right)^{b\log(F)}}{x^2}dx$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(-\frac{1}{n},-b\log(F),-\frac{1-n}{n},-\frac{dx^n}{c}\right)}{x}$$

input
```
Int[F^(a + b*Log[c + d*x^n])/x^2,x]
```

output
```
-((F^a*(c + d*x^n)^(b*Log[F])*Hypergeometric2F1[-n^(-1), -(b*Log[F]), -((1
 - n)/n), -((d*x^n)/c)])/(x*(1 + (d*x^n)/c)^(b*Log[F])))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int \frac{F^{a+b\ln(c+d\,x^n)}}{x^2}dx$$

input
```
int(F^(a+b*ln(c+d*x^n))/x^2,x)
```

output
```
int(F^(a+b*ln(c+d*x^n))/x^2,x)
```

Fricas [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x^2}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x^2,x, algorithm="fricas")
```

output
```
integral(F^(b*log(d*x^n + c) + a)/x^2, x)
```

Sympy [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = \int \frac{F^{a+b\log{(c+dx^n)}}}{x^2}\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n))/x**2,x)
```

output
```
Integral(F**(a + b*log(c + d*x**n))/x**2, x)
```

Maxima [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x^2}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x^2,x, algorithm="maxima")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)/x^2, x)
```

Giac [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x^2}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = \int \frac{F^{a+b\,\ln(c+d\,x^n)}}{x^2}\,dx$$

input
```
int(F^(a + b*log(c + d*x^n))/x^2,x)
```

output
```
int(F^(a + b*log(c + d*x^n))/x^2, x)
```

Reduce [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^2}\,dx = \frac{f^a\left(f^{\log(x^n d+c)b} + \left(\int \frac{f^{\log(x^n d+c)b}}{x^n\log(f)bdn\,x^2-x^n d\,x^2+\log(f)bcn\,x^2-c\,x^2}dx\right)\log(f)^2\,b^2c\,n^2x - \left(\int \frac{f^{\log(x^n d+c)b}}{x^n\log(f)b\cdots}\right.\cdots\right.}{x\left(\log\left(f\right)bn-1\right)}$$

input
```
int(F^(a+b*log(c+d*x^n))/x^2,x)
```

output
```
(f**a*(f**(log(x**n*d + c)*b) + int(f**(log(x**n*d + c)*b)/(x**n*log(f)*b*
d*n*x**2 - x**n*d*x**2 + log(f)*b*c*n*x**2 - c*x**2),x)*log(f)**2*b**2*c*n
**2*x - int(f**(log(x**n*d + c)*b)/(x**n*log(f)*b*d*n*x**2 - x**n*d*x**2 +
 log(f)*b*c*n*x**2 - c*x**2),x)*log(f)*b*c*n*x))/(x*(log(f)*b*n - 1))
```

3.506 $\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 68

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = -\frac{F^a(c+dx^n)^{b\log(F)}\left(1+\frac{dx^n}{c}\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(-\frac{2}{n},-b\log(F),-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2x^2}$$

output
```
-1/2*F^a*(c+d*x^n)^(b*ln(F))*hypergeom([-2/n, -b*ln(F)],[-(2-n)/n],-d*x^n/
c)/x^2/((1+d*x^n/c)^(b*ln(F)))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = -\frac{F^{a+b\log(c+dx^n)}\left(-\frac{dx^n}{c}\right)^{2/n}(c+dx^n)\,\text{Hypergeometric2F1}\left(\frac{2+n}{n},1+b\log(F),2+b\log(F),1+\frac{dx^n}{c}\right)}{cnx^2(1+b\log(F))}$$

input
```
Integrate[F^(a + b*Log[c + d*x^n])/x^3,x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*(-((d*x^n)/c))^(2/n)*(c + d*x^n)*Hypergeometri
c2F1[(2 + n)/n, 1 + b*Log[F], 2 + b*Log[F], 1 + (d*x^n)/c])/(c*n*x^2*(1 +
b*Log[F])))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2704, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}dx$$
$$\Big\downarrow \text{2704}$$
$$\int \frac{F^a(c+dx^n)^{b\log(F)}}{x^3}dx$$
$$\Big\downarrow \text{27}$$
$$F^a\int \frac{(dx^n+c)^{b\log(F)}}{x^3}dx$$
$$\Big\downarrow \text{889}$$
$$F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\int \frac{\left(\frac{dx^n}{c}+1\right)^{b\log(F)}}{x^3}dx$$
$$\Big\downarrow \text{888}$$
$$-\frac{F^a(c+dx^n)^{b\log(F)}\left(\frac{dx^n}{c}+1\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(-\frac{2}{n},-b\log(F),-\frac{2-n}{n},-\frac{dx^n}{c}\right)}{2x^2}$$

input
```
Int[F^(a + b*Log[c + d*x^n])/x^3,x]
```

output
```
-1/2*(F^a*(c + d*x^n)^(b*Log[F])*Hypergeometric2F1[-2/n, -(b*Log[F]), -((2
 - n)/n), -((d*x^n)/c)])/(x^2*(1 + (d*x^n)/c)^(b*Log[F]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int \frac{F^{a+b\ln(c+d\,x^n)}}{x^3}dx$$

input
```
int(F^(a+b*ln(c+d*x^n))/x^3,x)
```

output
```
int(F^(a+b*ln(c+d*x^n))/x^3,x)
```

Fricas [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x^3}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x^3,x, algorithm="fricas")
```

output
```
integral(F^(b*log(d*x^n + c) + a)/x^3, x)
```

Sympy [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = \int \frac{F^{a+b\log{(c+dx^n)}}}{x^3}\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n))/x**3,x)
```

output
```
Integral(F**(a + b*log(c + d*x**n))/x**3, x)
```

Maxima [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x^3}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x^3,x, algorithm="maxima")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)/x^3, x)
```

Giac [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = \int \frac{F^{b\log(dx^n+c)+a}}{x^3}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate(F^(b*log(d*x^n + c) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx = \int \frac{F^{a+b\,\ln(c+d\,x^n)}}{x^3}\,dx$$

input
```
int(F^(a + b*log(c + d*x^n))/x^3,x)
```

output
```
int(F^(a + b*log(c + d*x^n))/x^3, x)
```

Reduce [F]

$$\int \frac{F^{a+b\log(c+dx^n)}}{x^3}\,dx$$
$$= \frac{f^a\left(f^{\log(x^n d+c)b} + \left(\int \frac{f^{\log(x^n d+c)b}}{x^n\log(f)bdn\,x^3-2x^n d\,x^3+\log(f)bcn\,x^3-2c\,x^3}dx\right)\log(f)^2 b^2 c\,n^2 x^2 - 2\left(\int \frac{f^{\log(x^n d+c}}{x^n\log(f)bdn\,x^3-2x^n d\,x^3+\mathrm{l}}\right.\right.}{x^2\left(\log\left(f\right)bn-2\right)}$$

input
```
int(F^(a+b*log(c+d*x^n))/x^3,x)
```

output
```
(f**a*(f**(log(x**n*d + c)*b) + int(f**(log(x**n*d + c)*b)/(x**n*log(f)*b*
d*n*x**3 - 2*x**n*d*x**3 + log(f)*b*c*n*x**3 - 2*c*x**3),x)*log(f)**2*b**2
*c*n**2*x**2 - 2*int(f**(log(x**n*d + c)*b)/(x**n*log(f)*b*d*n*x**3 - 2*x*
*n*d*x**3 + log(f)*b*c*n*x**3 - 2*c*x**3),x)*log(f)*b*c*n*x**2))/(x**2*(lo
g(f)*b*n - 2))
```

3.507 $\int F^{a+b\log(c+dx^n)}(dx)^m\,dx$

Optimal result

Integrand size = 20, antiderivative size = 77

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx = \frac{F^a(dx)^{1+m}(c+dx^n)^{b\log(F)}\left(1+\frac{dx^n}{c}\right)^{-b\log(F)}\text{Hypergeometric2F1}\left(\frac{1+m}{n},-b\log(F),\frac{1+m+n}{n},-\frac{dx^n}{c}\right)}{d(1+m)}$$

output

```
F^a*(d*x)^(1+m)*(c+d*x^n)^(b*ln(F))*hypergeom([(1+m)/n, -b*ln(F)],[(1+m+n)
/n],-d*x^n/c)/d/(1+m)/((1+d*x^n/c)^(b*ln(F)))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx = -\frac{F^{a+b\log(c+dx^n)}x(dx)^m\left(-\frac{dx^n}{c}\right)^{-\frac{1+m}{n}}(c+dx^n)\,\text{Hypergeometric2F1}\left(1-\frac{1+m}{n},1+b\log(F),2+b\log(F)\right.}{cn(1+b\log(F))}$$

input

```
Integrate[F^(a + b*Log[c + d*x^n])*(d*x)^m,x]
```

output
```
-((F^(a + b*Log[c + d*x^n])*x*(d*x)^m*(c + d*x^n)*Hypergeometric2F1[1 - (1
 + m)/n, 1 + b*Log[F], 2 + b*Log[F], 1 + (d*x^n)/c])/(c*n*(-((d*x^n)/c))^(
(1 + m)/n)*(1 + b*Log[F])))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2704, 27, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m F^{a+b\log(c+dx^n)} \, dx$$

$$\Big\downarrow \text{2704}$$

$$\int F^a (dx)^m (c+dx^n)^{b\log(F)} \, dx$$

$$\Big\downarrow \text{27}$$

$$F^a \int (dx)^m (dx^n+c)^{b\log(F)} \, dx$$

$$\Big\downarrow \text{889}$$

$$F^a (c+dx^n)^{b\log(F)} \left(\frac{dx^n}{c}+1\right)^{-b\log(F)} \int (dx)^m \left(\frac{dx^n}{c}+1\right)^{b\log(F)} dx$$

$$\Big\downarrow \text{888}$$

$$\frac{F^a (dx)^{m+1} (c+dx^n)^{b\log(F)} \left(\frac{dx^n}{c}+1\right)^{-b\log(F)} \text{Hypergeometric2F1}\left(\frac{m+1}{n},-b\log(F),\frac{m+n+1}{n},-\frac{dx^n}{c}\right)}{d(m+1)}$$

input
```
Int[F^(a + b*Log[c + d*x^n])*(d*x)^m,x]
```

output
```
(F^a*(d*x)^(1 + m)*(c + d*x^n)^(b*Log[F])*Hypergeometric2F1[(1 + m)/n, -(b
*Log[F]), (1 + m + n)/n, -((d*x^n)/c)])/(d*(1 + m)*(1 + (d*x^n)/c)^(b*Log[
F]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

Maple [F]

$$\int F^{a+b\ln(c+d\,x^{n})}(dx)^{m}\,dx$$

input
```
int(F^(a+b*ln(c+d*x^n))*(d*x)^m,x)
```

output
```
int(F^(a+b*ln(c+d*x^n))*(d*x)^m,x)
```

Fricas [F]

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx=\int (dx)^m\,F^{b\log(dx^n+c)+a}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*(d*x)^m,x, algorithm="fricas")
```

output
```
integral((d*x)^m*F^(b*log(d*x^n + c) + a), x)
```

Sympy [F]

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx=\int F^{a+b\log{(c+dx^n)}}(dx)^m\,dx$$

input
```
integrate(F**(a+b*ln(c+d*x**n))*(d*x)**m,x)
```

output
```
Integral(F**(a + b*log(c + d*x**n))*(d*x)**m, x)
```

Maxima [F]

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx=\int (dx)^m\,F^{b\log(dx^n+c)+a}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*(d*x)^m,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*F^(b*log(d*x^n + c) + a), x)
```

Giac [F]

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx = \int (dx)^m F^{b\log(dx^n+c)+a}\,dx$$

input
```
integrate(F^(a+b*log(c+d*x^n))*(d*x)^m,x, algorithm="giac")
```

output
```
integrate((d*x)^m*F^(b*log(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx = \int F^{a+b\ln(c+d\,x^n)}\,(d\,x)^m\,dx$$

input
```
int(F^(a + b*log(c + d*x^n))*(d*x)^m,x)
```

output
```
int(F^(a + b*log(c + d*x^n))*(d*x)^m, x)
```

Reduce [F]

$$\int F^{a+b\log(c+dx^n)}(dx)^m\,dx$$
$$= \frac{f^a d^m\left(x^m f^{\log(x^n d+c)b}x + \left(\int \frac{x^m f^{\log(x^n d+c)b}}{x^n\log(f)bdn+x^n dm+x^n d+\log(f)bcn+cm+c}dx\right)\log(f)^2 b^2 c n^2 + \left(\int \frac{x^m f^{\log(x}}{x^n\log(f)bdn+x^n dm+x^}\right.\right.}{\log(f)\,bn+m+}$$

input
```
int(F^(a+b*log(c+d*x^n))*(d*x)^m,x)
```

output
```
(f**a*d**m*(x**m*f**(log(x**n*d + c)*b)*x + int((x**m*f**(log(x**n*d + c)*
b))/(x**n*log(f)*b*d*n + x**n*d*m + x**n*d + log(f)*b*c*n + c*m + c),x)*lo
g(f)**2*b**2*c*n**2 + int((x**m*f**(log(x**n*d + c)*b))/(x**n*log(f)*b*d*n
 + x**n*d*m + x**n*d + log(f)*b*c*n + c*m + c),x)*log(f)*b*c*m*n + int((x*
*m*f**(log(x**n*d + c)*b))/(x**n*log(f)*b*d*n + x**n*d*m + x**n*d + log(f)
*b*c*n + c*m + c),x)*log(f)*b*c*n))/(log(f)*b*n + m + 1)
```

3.508 $\int x^{b\log(x)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 40

$$\int x^{b\log(x)}\,dx = \frac{e^{-\frac{1}{4}/b}\sqrt{\pi}\,\text{erfi}\left(\frac{1+2b\log(x)}{2\sqrt{b}}\right)}{2\sqrt{b}}$$

output
```
1/2*Pi^(1/2)*erfi(1/2*(1+2*b*ln(x))/b^(1/2))/b^(1/2)/exp(1/4/b)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int x^{b\log(x)}\,dx = \frac{e^{-\frac{1}{4}/b}\sqrt{\pi}\,\text{erfi}\left(\frac{1+2b\log(x)}{2\sqrt{b}}\right)}{2\sqrt{b}}$$

input
```
Integrate[x^(b*Log[x]),x]
```

output
```
(Sqrt[Pi]*Erfi[(1 + 2*b*Log[x])/(2*Sqrt[b])])/(2*Sqrt[b]*E^(1/(4*b)))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{b\log(x)}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^{b\log(x)}dx$$

input
```
Int[x^(b*Log[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [F]

$$\int x^{b\ln(x)}dx$$

input
```
int(x^(b*ln(x)),x)
```

output
```
int(x^(b*ln(x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int x^{b\log(x)}\,dx = -\frac{1}{2}\,\sqrt{\pi}\sqrt{-b}\operatorname{erf}\left(\frac{(2\,b\log(x)+1)\sqrt{-b}}{2\,b}\right)e^{\left(-\frac{1}{4\,b}\right)}$$

input
```
integrate(x^(b*log(x)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b)*erf(1/2*(2*b*log(x) + 1)*sqrt(-b)/b)*e^(-1/4/b)
```

Sympy [F]

$$\int x^{b\log(x)}\,dx = \int x^{b\log(x)}\,dx$$

input
```
integrate(x**(b*ln(x)),x)
```

output
```
Integral(x**(b*log(x)), x)
```

Maxima [F]

$$\int x^{b\log(x)}\,dx = \int x^{b\log(x)}\,dx$$

input
```
integrate(x^(b*log(x)),x, algorithm="maxima")
```

output
```
integrate(x^(b*log(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int x^{b\log(x)}\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-b}\log(x) - \frac{\sqrt{-b}}{2\,b}\right)e^{\left(-\frac{1}{4\,b}\right)}}{2\,\sqrt{-b}}$$

input
```
integrate(x^(b*log(x)),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-sqrt(-b)*log(x) - 1/2*sqrt(-b)/b)*e^(-1/4/b)/sqrt(-b)
```

Mupad [F(-1)]

Timed out.

$$\int x^{b\log(x)}\,dx = \int \mathrm{e}^{b\ln(x)^2}\,dx$$

input
```
int(x^(b*log(x)),x)
```

output
```
int(exp(b*log(x)^2), x)
```

Reduce [F]

$$\int x^{b\log(x)}\,dx = \int x^{\log(x)b}dx$$

input
```
int(x^(b*log(x)),x)
```

output
```
int(x**(log(x)*b),x)
```

3.509 $\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx$

Optimal result

Integrand size = 20, antiderivative size = 76

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = \frac{e^{-\frac{(1+m)^2}{4n^2}}\sqrt{\pi}(d+ex)^{1+m}\left((d+ex)^n\right)^{-\frac{1+m}{n}}\operatorname{erfi}\left(\frac{1+m+2n\log((d+ex)^n)}{2n}\right)}{2en}$$

output
```
1/2*Pi^(1/2)*(e*x+d)^(1+m)*erfi(1/2*(1+m+2*n*ln((e*x+d)^n))/n)/e/exp(1/4*(
1+m)^2/n^2)/n/(((e*x+d)^n)^((1+m)/n))
```

Mathematica [F]

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = \int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx$$

input
```
Integrate[E^Log[(d + e*x)^n]^2*(d + e*x)^m,x]
```

output
```
Integrate[E^Log[(d + e*x)^n]^2*(d + e*x)^m, x]
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m e^{\log^2((d+ex)^n)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{(d+ex)^{m+1}((d+ex)^n)^{-\frac{m+1}{n}}\int \exp\left(\log^2((d+ex)^n)+\frac{(m+1)\log((d+ex)^n)}{n}\right)d\log((d+ex)^n)}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{e^{-\frac{(m+1)^2}{4n^2}}(d+ex)^{m+1}((d+ex)^n)^{-\frac{m+1}{n}}\int e^{\frac{(m+2n\log((d+ex)^n)+1)^2}{4n^2}}d\log((d+ex)^n)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}e^{-\frac{(m+1)^2}{4n^2}}(d+ex)^{m+1}((d+ex)^n)^{-\frac{m+1}{n}}\text{erfi}\left(\frac{2n\log((d+ex)^n)+m+1}{2n}\right)}{2en}$$

input
```
Int[E^Log[(d + e*x)^n]^2*(d + e*x)^m,x]
```

output
```
(Sqrt[Pi]*(d + e*x)^(1 + m)*Erfi[(1 + m + 2*n*Log[(d + e*x)^n])/(2*n)])/(2
*e*E^((1 + m)^2/(4*n^2))*n*((d + e*x)^n)^((1 + m)/n))
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

Maple [F]

$$\int e^{\ln((ex+d)^n)^2}(ex+d)^m\,dx$$

input
```
int(exp(ln((e*x+d)^n)^2)*(e*x+d)^m,x)
```

output
```
int(exp(ln((e*x+d)^n)^2)*(e*x+d)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.78

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = -\frac{\sqrt{\pi}\sqrt{-n^2}\,\mathrm{erf}\left(\frac{(2\,n^2\log(ex+d)+m+1)\sqrt{-n^2}}{2\,n^2}\right)e^{\left(-\frac{m^2+2\,m+1}{4\,n^2}\right)}}{2\,en}$$

input
```
integrate(exp(log((e*x+d)^n)^2)*(e*x+d)^m,x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-n^2)*erf(1/2*(2*n^2*log(e*x + d) + m + 1)*sqrt(-n^2)/n
^2)*e^(-1/4*(m^2 + 2*m + 1)/n^2)/(e*n)
```

Sympy [F]

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = \int (d+ex)^m\,e^{\log((d+ex)^n)^2}\,dx$$

input
```
integrate(exp(ln((e*x+d)**n)**2)*(e*x+d)**m,x)
```

output
```
Integral((d + e*x)**m*exp(log((d + e*x)**n)**2), x)
```

Maxima [F]

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = \int (ex+d)^m e^{\left(\log((ex+d)^n)^2\right)}\,dx$$

input
```
integrate(exp(log((e*x+d)^n)^2)*(e*x+d)^m,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m*e^(log((e*x + d)^n)^2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.71

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(i\,n\log(ex+d)+\frac{i\,m}{2\,n}+\frac{i}{2\,n}\right)e^{\left(-\frac{m^2}{4\,n^2}-\frac{m}{2\,n^2}-\frac{1}{4\,n^2}\right)}}{2\,en}$$

input
```
integrate(exp(log((e*x+d)^n)^2)*(e*x+d)^m,x, algorithm="giac")
```

output
```
-1/2*I*sqrt(pi)*erf(I*n*log(e*x + d) + 1/2*I*m/n + 1/2*I/n)*e^(-1/4*m^2/n^
2 - 1/2*m/n^2 - 1/4/n^2)/(e*n)
```

Mupad [F(-1)]

Timed out.

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = \int \mathrm{e}^{\ln((d+e\,x)^n)^2}\,(d+e\,x)^m\,dx$$

input
```
int(exp(log((d + e*x)^n)^2)*(d + e*x)^m,x)
```

output
```
int(exp(log((d + e*x)^n)^2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int e^{\log^2((d+ex)^n)}(d+ex)^m\,dx = \int e^{\log((ex+d)^n)^2}(ex+d)^m\,dx$$

input
```
int(exp(log((e*x+d)^n)^2)*(e*x+d)^m,x)
```

output
```
int(e**(log((d + e*x)**n)**2)*(d + e*x)**m,x)
```

3.510 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx$

Optimal result

Integrand size = 31, antiderivative size = 137

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx = \frac{e^{-\frac{(1+m)^2}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}\left(c(d+ex)^n\right)^{-\frac{1+m}{n}}(dg+egx)^{1+m}\operatorname{erfi}\left(\frac{1+m+2bfn\log(F)\log\left(c(d+ex)^n\right)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}gn\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*Pi^(1/2)*(e*g*x+d*g)^(1+m)*erfi(1/2*(1+m+2*b*f*n*ln(F)*ln(c*(e
*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(1/4*(1+m)^2/b/f/n^2
/ln(F))/f^(1/2)/g/n/((c*(e*x+d)^n)^((1+m)/n))/ln(F)^(1/2)
```

Mathematica [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx = \int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x)^m,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x)^m, x]
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dg+egx)^m F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{(dg+egx)^{m+1}\left(c(d+ex)^n\right)^{-\frac{m+1}{n}}\int F^{bf\log^2\left(c(d+ex)^n\right)+af}\left(c(d+ex)^n\right)^{\frac{m+1}{n}}d\log\left(c(d+ex)^n\right)}{egn}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(dg+egx)^{m+1}\left(c(d+ex)^n\right)^{-\frac{m+1}{n}}\int \exp\left(bf\log(F)\log^2\left(c(d+ex)^n\right)+\frac{(m+1)\log(c(d+ex)^n)}{n}+af\log(F)\right)d\log\left(c(d}{egn}$$

$$\Big\downarrow \text{2664}$$

$$\frac{F^{af}(dg+egx)^{m+1}e^{-\frac{(m+1)^2}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-\frac{m+1}{n}}\int \exp\left(\frac{(m+2bfn\log(F)\log(c(d+ex)^n)+1)^2}{4bfn^2\log(F)}\right)d\log\left(c(d+ex)^n\right)}{egn}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^{af}(dg+egx)^{m+1}e^{-\frac{(m+1)^2}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-\frac{m+1}{n}}\text{erfi}\left(\frac{2bfn\log(F)\log(c(d+ex)^n)+m+1}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}gn\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x)^m,x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d*g + e*g*x)^(1 + m)*Erfi[(1 + m + 2*b*f*n*Log[F]*Log[c
*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^((1 + m
)^2/(4*b*f*n^2*Log[F]))*Sqrt[f]*g*n*(c*(d + e*x)^n)^((1 + m)/n)*Sqrt[Log[F
]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

$$\int F^{f\left(a+b\ln\left(c(ex+d)^n\right)^2\right)}(egx+dg)^m\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(e*g*x+d*g)^m,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(e*g*x+d*g)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)^m\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}\,\mathrm{erf}\left(\frac{(2\,bfn^2\log(ex+d)\log(F)+2\,bfn\log(F)\log(c)+m+1)\sqrt{-bfn^2\log(F)}}{2\,bfn^2\log(F)}\right)e^{\left(\frac{4\,abf^2n^2\log(F)^2+4\,bfmn^2\log}{}\right.}}{2\,en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^m,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F)
 + 2*b*f*n*log(F)*log(c) + m + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*
e^(1/4*(4*a*b*f^2*n^2*log(F)^2 + 4*b*f*m*n^2*log(F)*log(g) - 4*(b*f*m + b*
f)*n*log(F)*log(c) - m^2 - 2*m - 1)/(b*f*n^2*log(F)))/(e*n)
```

Sympy [F]

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)^m\,dx = \int F^{f\left(a+b\log\left(c(d+ex)^n\right)^2\right)}(g(d+ex))^m\,dx$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(e*g*x+d*g)**m,x)
```

output
```
Integral(F**(f*(a + b*log(c*(d + e*x)**n)**2))*(g*(d + e*x))**m, x)
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx=\int (egx+dg)^m F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^m,x, algorithm="maxim
a")
```

output
```
integrate((e*g*x + d*g)^m*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx=\int (egx+dg)^m F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^m,x, algorithm="giac"
)
```

output
```
integrate((e*g*x + d*g)^m*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx=\int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}\,\left(d\,g+e\,g\,x\right)^m\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(d*g + e*g*x)^m,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(d*g + e*g*x)^m, x)
```

Reduce [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^m\,dx = f^{af}\left(\int f^{\log\left((ex+d)^n c\right)^2 bf}(egx+dg)^m\,dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^m,x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)*(d*g + e*g*x)**m,x)
```

3.511 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 123

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^2\,dx = \frac{e^{-\frac{9}{4bfn^2\log(F)}}F^{af}g^2\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{3+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*g^2*Pi^(1/2)*(e*x+d)^3*erfi(1/2*(3+2*b*f*n*ln(F)*ln(c*(e*x+d)^
n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(9/4/b/f/n^2/ln(F))/f^(1/2
)/n/((c*(e*x+d)^n)^(3/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^2\,dx = \frac{e^{-\frac{9}{4bfn^2\log(F)}}F^{af}g^2\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{3+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x)^2,x]
```

output
```
(F^(a*f)*g^2*Sqrt[Pi]*(d + e*x)^3*Erfi[(3 + 2*b*f*n*Log[F]*Log[c*(d + e*x)
^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(9/(4*b*f*n^2*Log
[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dg+egx)^2F^{f\left(a+b\log^2(c(d+ex)^n)\right)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\int F^{bf\log^2(c(d+ex)^n)+af}\left(c(d+ex)^n\right)^{3/n}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\int \exp\left(bf\log(F)\log^2\left(c(d+ex)^n\right)+\frac{3\log(c(d+ex)^n)}{n}+af\log(F)\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{g^2F^{af}(d+ex)^3e^{-\frac{9}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-3/n}\int \exp\left(\frac{(2bfn\log(F)\log(c(d+ex)^n)+3)^2}{4bfn^2\log(F)}\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}g^2F^{af}(d+ex)^3e^{-\frac{9}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-3/n}\text{erfi}\left(\frac{2bfn\log(F)\log(c(d+ex)^n)+3}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x)^2,x]
```

output
```
(F^(a*f)*g^2*Sqrt[Pi]*(d + e*x)^3*Erfi[(3 + 2*b*f*n*Log[F]*Log[c*(d + e*x)
^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(9/(4*b*f*n^2*Log
[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

$$\int F^{f\left(a+b\ln\left(c(ex+d)^n\right)^2\right)}(egx+dg)^2\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(e*g*x+d*g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(e*g*x+d*g)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)^2\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}g^2\operatorname{erf}\left(\frac{(2\,bfn^2\log(ex+d)\log(F)+2\,bfn\log(F)\log(c)+3)\sqrt{-bfn^2\log(F)}}{2\,bfn^2\log(F)}\right)e^{\left(\frac{4\,abf^2n^2\log(F)^2-12\,bfn\log(F)}{4\,bfn^2\log(F)}\right.}}{2\,en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^2,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*g^2*erf(1/2*(2*b*f*n^2*log(e*x + d)*lo
g(F) + 2*b*f*n*log(F)*log(c) + 3)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*
e^(1/4*(4*a*b*f^2*n^2*log(F)^2 - 12*b*f*n*log(F)*log(c) - 9)/(b*f*n^2*log(
F)))/(e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. $2(110) = 220$.

Time = 104.10 (sec) , antiderivative size = 525, normalized size of antiderivative = 4.27

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)^2\,dx$$

$$= \begin{cases} -\frac{2F^{af+bf\log(c(d+ex)^n)^2}bd^3fg^2n^2\log(F)}{9e} - \frac{2F^{af+bf\log(c(d+ex)^n)^2}bd^3fg^2n\log(F)\log(c(d+ex)^n)}{9e} + \frac{2F^{af+bf\log(c(d+ex)^n)^2}bd^2fg^2}{9} \\ F^{f\left(a+b\log(cd^n)^2\right)}d^2g^2x \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(e*g*x+d*g)**2,x)
```

output
```
Piecewise((-2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**3*f*g**2*n**2*log
(F)/(9*e) - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**3*f*g**2*n*log(F)
*log(c*(d + e*x)**n)/(9*e) + 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**
2*f*g**2*n**2*x*log(F)/9 - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**2*
f*g**2*n*x*log(F)*log(c*(d + e*x)**n)/3 + 2*F**(a*f + b*f*log(c*(d + e*x)*
*n)**2)*b*d*e*f*g**2*n**2*x**2*log(F)/9 - 2*F**(a*f + b*f*log(c*(d + e*x)*
*n)**2)*b*d*e*f*g**2*n*x**2*log(F)*log(c*(d + e*x)**n)/3 + 2*F**(a*f + b*f
*log(c*(d + e*x)**n)**2)*b*e**2*f*g**2*n**2*x**3*log(F)/27 - 2*F**(a*f + b
*f*log(c*(d + e*x)**n)**2)*b*e**2*f*g**2*n*x**3*log(F)*log(c*(d + e*x)**n)
/9 + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*d**3*g**2/(3*e) + F**(a*f + b*f
*log(c*(d + e*x)**n)**2)*d**2*g**2*x + F**(a*f + b*f*log(c*(d + e*x)**n)**
2)*d*e*g**2*x**2 + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*e**2*g**2*x**3/3,
 Ne(e, 0)), (F**(f*(a + b*log(c*d**n)**2))*d**2*g**2*x, True))
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^2\,dx=\int (egx+dg)^2F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^2,x, algorithm="maxim
a")
```

output
```
integrate((e*g*x + d*g)^2*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)^2\,dx=\int (egx+dg)^2F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^2,x, algorithm="giac"
)
```

output
```
integrate((e*g*x + d*g)^2*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)^2\,dx=\int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}\,(d\,g+e\,g\,x)^2\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(d*g + e*g*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(d*g + e*g*x)^2, x)
```

Reduce [F]

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)^2\,dx=f^{af}g^2\left(\left(\int f^{\log((ex+d)^n c)^2bf}dx\right)d^2+\left(\int f^{\log((ex+d)^n c)^2bf}x^2dx\right)e^2+2\left(\int f^{\log((ex+d)^n c)^2bf}xdx\right)de\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g)^2,x)
```

output
```
f**(a*f)*g**2*(int(f**(log((d + e*x)**n*c)**2*b*f),x)*d**2 + int(f**(log((
d + e*x)**n*c)**2*b*f)*x**2,x)*e**2 + 2*int(f**(log((d + e*x)**n*c)**2*b*f
)*x,x)*d*e)
```

3.512 $\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 115

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)\,dx = \frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}g\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{1+bfn\log(F)\log(c(d+ex)^n)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*g*Pi^(1/2)*(e*x+d)^2*erfi((1+b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^(1
/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(1/b/f/n^2/ln(F))/f^(1/2)/n/((c*(e
*x+d)^n)^(2/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(dg+egx)\,dx = \frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}g\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{1+bfn\log(F)\log(c(d+ex)^n)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x),x]
```

output
```
(F^(a*f)*g*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*Log[c*(d + e*x)^n])
/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(b*f*n^2*Log[F]))*Sq
rt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dg+egx)F^{f\left(a+b\log^2(c(d+ex)^n)\right)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\int F^{bf\log^2(c(d+ex)^n)+af}\left(c(d+ex)^n\right)^{2/n}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{g(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\int \exp\left(bf\log(F)\log^2\left(c(d+ex)^n\right)+\frac{2\log(c(d+ex)^n)}{n}+af\log(F)\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{gF^{af}(d+ex)^2e^{-\frac{1}{bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-2/n}\int \exp\left(\frac{(bfn\log(F)\log(c(d+ex)^n)+1)^2}{bfn^2\log(F)}\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}gF^{af}(d+ex)^2e^{-\frac{1}{bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-2/n}\text{erfi}\left(\frac{bfn\log(F)\log(c(d+ex)^n)+1}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(d*g + e*g*x),x]
```

output
```
(F^(a*f)*g*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*Log[c*(d + e*x)^n])
/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(b*f*n^2*Log[F]))*Sq
rt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

$$\int F^{f\left(a+b\ln\left(c(ex+d)^n\right)^2\right)}(egx+dg)\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(e*g*x+d*g),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(e*g*x+d*g),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-bfn^2\log\left(F\right)}g\,\mathrm{erf}\left(\frac{\left(bfn^2\log(ex+d)\log(F)+bfn\log(F)\log(c)+1\right)\sqrt{-bfn^2\log(F)}}{bfn^2\log(F)}\right)e^{\left(\frac{abf^2n^2\log(F)^2-2\,bfn\log(F)\log(c)-}{bfn^2\log(F)}\right.}}{2\,en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g),x, algorithm="fricas"
)
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*g*erf((b*f*n^2*log(e*x + d)*log(F) + b
*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^((a*b*f^
2*n^2*log(F)^2 - 2*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F)))/(e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(105) = 210.

Time = 22.75 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.10

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)\,dx$$

$$= \begin{cases} -\frac{F^{af+bf\log\left(c(d+ex)^n\right)^2}bd^2fgn^2\log\left(F\right)}{2e} - \frac{F^{af+bf\log\left(c(d+ex)^n\right)^2}bd^2fgn\log\left(F\right)\log\left(c(d+ex)^n\right)}{2e} + \frac{F^{af+bf\log\left(c(d+ex)^n\right)^2}bdfgn^2x\log\left(\right.}{2} \\ F^{f\left(a+b\log\left(cd^n\right)^2\right)}dgx \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(e*g*x+d*g),x)
```

output
```
Piecewise((-F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**2*f*g*n**2*log(F)/(
2*e) - F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**2*f*g*n*log(F)*log(c*(d
+ e*x)**n)/(2*e) + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g*n**2*x*lo
g(F)/2 - F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g*n*x*log(F)*log(c*(d
 + e*x)**n) + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*e*f*g*n**2*x**2*log(
F)/4 - F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*e*f*g*n*x**2*log(F)*log(c*(
d + e*x)**n)/2 + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*d**2*g/(2*e) + F**(
a*f + b*f*log(c*(d + e*x)**n)**2)*d*g*x + F**(a*f + b*f*log(c*(d + e*x)**n
)**2)*e*g*x**2/2, Ne(e, 0)), (F**(f*(a + b*log(c*d**n)**2))*d*g*x, True))
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)\,dx=\int (egx+dg)F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g),x, algorithm="maxima"
)
```

output
```
integrate((e*g*x + d*g)*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)\,dx=\int (egx+dg)F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g),x, algorithm="giac")
```

output
```
integrate((e*g*x + d*g)*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)\,dx=\int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}\,(d\,g+e\,g\,x)\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(d*g + e*g*x),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(d*g + e*g*x), x)
```

Reduce [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(dg+egx)\,dx=f^{af}g\left(\left(\int f^{\log((ex+d)^n c)^2 bf}dx\right)d+\left(\int f^{\log((ex+d)^n c)^2 bf}xdx\right)e\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(e*g*x+d*g),x)
```

output
```
f**(a*f)*g*(int(f**(log((d + e*x)**n*c)**2*b*f),x)*d + int(f**(log((d + e*
x)**n*c)**2*b*f)*x,x)*e)
```

3.513 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx = \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log\left(c(d+ex)^n\right)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*Pi^(1/2)*(e*x+d)*erfi(1/2*(1+2*b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^
(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(1/4/b/f/n^2/ln(F))/f^(1/2)/n/((
c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx = \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log\left(c(d+ex)^n\right)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2)),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(
2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(4*b*f*n^2*Log[F]))*
Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2705, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}dx$$

$$\Big\downarrow \text{2705}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int F^{bf\log^2\left(c(d+ex)^n\right)+af}\left(c(d+ex)^n\right)^{\frac{1}{n}}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \exp\left(bf\log(F)\log^2\left(c(d+ex)^n\right)+\frac{\log(c(d+ex)^n)}{n}+af\log(F)\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{F^{af}(d+ex)e^{-\frac{1}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-1/n}\int \exp\left(\frac{(2bfn\log(F)\log(c(d+ex)^n)+1)^2}{4bfn^2\log(F)}\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^{af}(d+ex)e^{-\frac{1}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{2bfn\log(F)\log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2)),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(
2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(4*b*f*n^2*Log[F]))*
Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2705
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.)), x
_Symbol] :> Simp[(d + e*x)/(e*n*(c*(d + e*x)^n)^(1/n))   Subst[Int[E^(a*f*L
og[F] + x/n + b*f*Log[F]*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F,
a, b, c, d, e, f, n}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

$$\int F^{f\left(a+b\ln\left(c(ex+d)^n\right)^2\right)}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2)),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx = -\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}\operatorname{erf}\left(\frac{(2bfn^2\log(ex+d)\log(F)+2bfn\log(F)\log(c)+1)\sqrt{-bfn^2\log(F)}}{2bfn^2\log(F)}\right)e^{\left(\frac{4abf^2n^2\log(F)^2-4bfn\log(F)\log(}{4bfn^2\log(F)}\right.}}{2en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F)
 + 2*b*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^(1
/4*(4*a*b*f^2*n^2*log(F)^2 - 4*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F)))/
(e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(105) = 210.

Time = 5.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.85

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx$$

$$= \begin{cases} -\frac{2F^{af+bf\log\left(c(d+ex)^n\right)^2}bdfn^2\log(F)}{e} - \frac{2F^{af+bf\log\left(c(d+ex)^n\right)^2}bdfn\log(F)\log\left(c(d+ex)^n\right)}{e} + 2F^{af+bf\log\left(c(d+ex)^n\right)^2}bfn^2x\,\mathrm{lo} \\ F^{f\left(a+b\log\left(cd^n\right)^2\right)}x \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2)),x)
```

output
```
Piecewise((-2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*n**2*log(F)/e -
2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*n*log(F)*log(c*(d + e*x)**n)
/e + 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*n**2*x*log(F) - 2*F**(a*f
 + b*f*log(c*(d + e*x)**n)**2)*b*f*n*x*log(F)*log(c*(d + e*x)**n) + F**(a*
f + b*f*log(c*(d + e*x)**n)**2)*d/e + F**(a*f + b*f*log(c*(d + e*x)**n)**2
)*x, Ne(e, 0)), (F**(f*(a + b*log(c*d**n)**2))*x, True))
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx=\int F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx=-\frac{\sqrt{\pi}F^{af}\operatorname{erf}\left(-\sqrt{-bf\log\left(F\right)}n\log\left(ex+d\right)-\sqrt{-bf\log\left(F\right)}\log\left(c\right)-\frac{\sqrt{-bf\log(F)}}{2\,bfn\log(F)}\right)e^{\left(-\frac{1}{4\,bfn^2\log(F)}\right)}}{2\,\sqrt{-bf\log\left(F\right)}c^{\left(\frac{1}{n}\right)}en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*F^(a*f)*erf(-sqrt(-b*f*log(F))*n*log(e*x + d) - sqrt(-b*f*lo
g(F))*log(c) - 1/2*sqrt(-b*f*log(F))/(b*f*n*log(F)))*e^(-1/4/(b*f*n^2*log(
F)))/(sqrt(-b*f*log(F))*c^(1/n)*e*n)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx=\int F^{b\,f\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)^2}\,F^{a\,f}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2)),x)
```

output
```
int(F^(b*f*log(c*(d + e*x)^n)^2)*F^(a*f), x)
```

Reduce [F]

$$\int F^{f(a+b\log^2(c(d+ex)^n))}\,dx = f^{af}\left(\int f^{\log((ex+d)^n c)^2 bf}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f),x)
```

3.514 $\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{dg+egx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 67

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{dg+egx}\,dx=\frac{F^{af}\sqrt{\pi}\text{erfi}\left(\sqrt{b}\sqrt{f}\sqrt{\log(F)}\log\left(c(d+ex)^n\right)\right)}{2\sqrt{b}e\sqrt{f}gn\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*Pi^(1/2)*erfi(b^(1/2)*f^(1/2)*ln(F)^(1/2)*ln(c*(e*x+d)^n))/b^(
1/2)/e/f^(1/2)/g/n/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{dg+egx}\,dx=\frac{F^{af}\sqrt{\pi}\text{erfi}\left(\sqrt{b}\sqrt{f}\sqrt{\log(F)}\log\left(c(d+ex)^n\right)\right)}{2\sqrt{b}e\sqrt{f}gn\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(d*g + e*g*x),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[f]*Sqrt[Log[F]]*Log[c*(d + e*x)^n]])/(
2*Sqrt[b]*e*Sqrt[f]*g*n*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2706, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{dg+egx}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{\int F^{bf\log^2\left(c(d+ex)^n\right)+af}d\log\left(c(d+ex)^n\right)}{egn}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^{af}\operatorname{erfi}\left(\sqrt{b}\sqrt{f}\sqrt{\log(F)}\log\left(c(d+ex)^n\right)\right)}{2\sqrt{b}e\sqrt{f}gn\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(d*g + e*g*x),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[f]*Sqrt[Log[F]]*Log[c*(d + e*x)^n]])/(
2*Sqrt[b]*e*Sqrt[f]*g*n*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.00 (sec) , antiderivative size = 377, normalized size of antiderivative = 5.63

method	result
risch	$\sqrt{\pi}\, F^{f\left(-b\,\pi^2+b\ \mathrm{csgn}\left(ic(ex+d)^n\right)\pi^2\ \mathrm{csgn}(ic)+b\ \mathrm{csgn}\left(i(ex+d)^n\right)\ \mathrm{csgn}\left(ic(ex+d)^n\right)\pi^2-b\ \mathrm{csgn}\left(i(ex+d)^n\right)\pi^2\ \mathrm{csgn}(ic)+ib\ln(c)\ \mathrm{csgn}\left(i(ex+d)^n\right)\pi-i\right.}$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(e*g*x+d*g),x,method=_RETURNVERBOSE)
```

output
```
1/2/g/e/n*Pi^(1/2)*F^(f*(-b*Pi^2+b*csgn(I*c*(e*x+d)^n)*Pi^2*csgn(I*c)+b*cs
gn(I*(e*x+d)^n)*csgn(I*c*(e*x+d)^n)*Pi^2-b*csgn(I*(e*x+d)^n)*Pi^2*csgn(I*c
)+I*b*ln(c)*csgn(I*(e*x+d)^n)*Pi-I*b*ln(c)*csgn(I*(e*x+d)^n)*csgn(I*c*(e*x
+d)^n)*csgn(I*c)*Pi-I*b*ln(c)*csgn(I*c*(e*x+d)^n)*Pi+I*b*ln(c)*csgn(I*c)*P
i+b*ln(c)^2+a))*F^(1/4*f*b*(-csgn(I*(e*x+d)^n)*Pi+csgn(I*(e*x+d)^n)*csgn(I
*c*(e*x+d)^n)*csgn(I*c)*Pi+csgn(I*c*(e*x+d)^n)*Pi-csgn(I*c)*Pi+2*I*ln(c))^
2)/(-ln(F)*b*f)^(1/2)*erf((-ln(F)*b*f)^(1/2)*ln((e*x+d)^n)-1/2*f*b*(2*ln(c
)-I*Pi*csgn(I*c*(e*x+d)^n)*(-csgn(I*c*(e*x+d)^n)+csgn(I*c))*(-csgn(I*c*(e*
x+d)^n)+csgn(I*(e*x+d)^n)))*ln(F)/(-ln(F)*b*f)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{dg+egx}\,dx = -\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}F^{af}\operatorname{erf}\left(\frac{\sqrt{-bfn^2\log(F)}(n\log(ex+d)+\log(c))}{n}\right)}{2\,egn}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g),x, algorithm="fricas"
)
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*F^(a*f)*erf(sqrt(-b*f*n^2*log(F))*(n*l
og(e*x + d) + log(c))/n)/(e*g*n)
```

Sympy [F]

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{dg+egx}\,dx = \frac{\int \frac{F^{af+bf\log(c(d+ex)^n)^2}}{d+ex}\,dx}{g}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))/(e*g*x+d*g),x)
```

output
```
Integral(F**(a*f + b*f*log(c*(d + e*x)**n)**2)/(d + e*x), x)/g
```

Maxima [F]

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{dg+egx}\,dx = \int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{egx+dg}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g),x, algorithm="maxima"
)
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(e*g*x + d*g), x)
```

Giac [F]

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{dg+egx}\,dx = \int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{egx+dg}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g),x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(e*g*x + d*g), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.73

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{dg+egx}\,dx=\frac{F^{a\,f}\,\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\,f\,\ln(F)\,\ln\left(c\,(d+e\,x)^n\right)}{\sqrt{b\,f\,\ln(F)}}\right)}{2\,e\,g\,n\,\sqrt{b\,f\,\ln\left(F\right)}}$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))/(d*g + e*g*x),x)
```

output
```
(F^(a*f)*pi^(1/2)*erfi((b*f*log(F)*log(c*(d + e*x)^n))/(b*f*log(F))^(1/2))
)/(2*e*g*n*(b*f*log(F))^(1/2))
```

Reduce [F]

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{dg+egx}\,dx=\frac{f^{af}\left(\int \frac{f^{\log\left((ex+d)^n c\right)^2 bf}}{ex+d}dx\right)}{g}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g),x)
```

output
```
(f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)/(d + e*x),x))/g
```

3.515 $\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{(dg+egx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{(dg+egx)^2}\,dx = -\frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(c(d+ex)^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{1-2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}g^2n(d+ex)\sqrt{\log(F)}}$$

output
```
-1/2*F^(a*f)*Pi^(1/2)*(c*(e*x+d)^n)^(1/n)*erfi(1/2*(1-2*b*f*n*ln(F)*ln(c*(
e*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(1/4/b/f/n^2/ln(F))
/f^(1/2)/g^2/n/(e*x+d)/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{(dg+egx)^2}\,dx = \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(c(d+ex)^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{-1+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}g^2n(d+ex)\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(d*g + e*g*x)^2,x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)*Erfi[(-1 + 2*b*f*n*Log[F]*Log[c*(
d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(4*b*f
*n^2*Log[F]))*Sqrt[f]*g^2*n*(d + e*x)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{(dg+egx)^2}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{(c(d+ex)^n)^{\frac{1}{n}}\int F^{bf\log^2(c(d+ex)^n)+af}(c(d+ex)^n)^{-1/n}d\log(c(d+ex)^n)}{eg^2n(d+ex)}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(c(d+ex)^n)^{\frac{1}{n}}\int \exp\left(bf\log(F)\log^2(c(d+ex)^n)-\frac{\log(c(d+ex)^n)}{n}+af\log(F)\right)d\log(c(d+ex)^n)}{eg^2n(d+ex)}$$

$$\Big\downarrow \text{2664}$$

$$\frac{F^{af}e^{-\frac{1}{4bfn^2\log(F)}}(c(d+ex)^n)^{\frac{1}{n}}\int \exp\left(\frac{(1-2bfn\log(F)\log(c(d+ex)^n))^2}{4bfn^2\log(F)}\right)d\log(c(d+ex)^n)}{eg^2n(d+ex)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\sqrt{\pi}F^{af}e^{-\frac{1}{4bfn^2\log(F)}}(c(d+ex)^n)^{\frac{1}{n}}\text{erfi}\left(\frac{1-2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}g^2n\sqrt{\log(F)}(d+ex)}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(d*g + e*g*x)^2,x]
```

output
```
-1/2*(F^(a*f)*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)*Erfi[(1 - 2*b*f*n*Log[F]*Log
[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(Sqrt[b]*e*E^(1/(4*b
*f*n^2*Log[F]))*Sqrt[f]*g^2*n*(d + e*x)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

$$\int \frac{F^{f\left(a+b\ln\left(c(ex+d)^{n}\right)^{2}\right)}}{\left(egx+dg\right)^{2}}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(e*g*x+d*g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(e*g*x+d*g)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(dg+egx)^2}\,dx = -\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}\,\mathrm{erf}\left(\frac{(2\,bfn^2\log(ex+d)\log(F)+2\,bfn\log(F)\log(c)-1)\sqrt{-bfn^2\log(F)}}{2\,bfn^2\log(F)}\right)e^{\left(\frac{4\,abf^2n^2\log(F)^2+4\,bfn\log(F)\log(c)-1}{4\,bfn^2\log(F)}\right)}}{2\,eg^2n}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^2,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F)
 + 2*b*f*n*log(F)*log(c) - 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^(1
/4*(4*a*b*f^2*n^2*log(F)^2 + 4*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F)))/
(e*g^2*n)
```

Sympy [A] (verification not implemented)

Time = 71.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(dg+egx)^2}\,dx = \begin{cases} -\frac{2F^{af+bf\log(c(d+ex)^n)^2}bfn^2\log(F)}{deg^2+e^2g^2x} - \frac{2F^{af+bf\log(c(d+ex)^n)^2}bfn\log(F)\log(c(d+ex)^n)}{deg^2+e^2g^2x} - \frac{F^{af+bf\log(c(d+ex)^n)^2}}{deg^2+e^2g^2x} & \text{for } e\neq 0 \\ \frac{F^{f\left(a+b\log(cd^n)^2\right)}x}{d^2g^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))/(e*g*x+d*g)**2,x)
```

output
```
Piecewise((-2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*n**2*log(F)/(d*e*g
**2 + e**2*g**2*x) - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*n*log(F)*
log(c*(d + e*x)**n)/(d*e*g**2 + e**2*g**2*x) - F**(a*f + b*f*log(c*(d + e*
x)**n)**2)/(d*e*g**2 + e**2*g**2*x), Ne(e, 0)), (F**(f*(a + b*log(c*d**n)*
*2))*x/(d**2*g**2), True))
```

Maxima [F]

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^2}\,dx=\int \frac{F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}}{\left(egx+dg\right)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^2,x, algorithm="maxim
a")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(e*g*x + d*g)^2, x)
```

Giac [F]

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^2}\,dx=\int \frac{F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}}{\left(egx+dg\right)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^2,x, algorithm="giac"
)
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(e*g*x + d*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(dg+egx)^2}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,\left(b\ln(c\,(d+e\,x)^n)^2+a\right)}}{\left(d\,g+e\,g\,x\right)^2}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))/(d*g + e*g*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))/(d*g + e*g*x)^2, x)
```

Reduce [F]

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(dg+egx)^2}\,dx = \frac{f^{af}\left(\int \frac{f^{\log((ex+d)^n c)^2 bf}}{e^2x^2+2dex+d^2}dx\right)}{g^2}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^2,x)
```

output
```
(f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)/(d**2 + 2*d*e*x + e**2*x**2)
,x))/g**2
```

3.516 $\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = -\frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}\sqrt{\pi}\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{1-bfn\log(F)\log\left(c(d+ex)^n\right)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}g^3n(d+ex)^2\sqrt{\log(F)}}$$

output
```
-1/2*F^(a*f)*Pi^(1/2)*(c*(e*x+d)^n)^(2/n)*erfi((1-b*f*n*ln(F)*ln(c*(e*x+d)
^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(1/b/f/n^2/ln(F))/f^(1/2)
/g^3/n/(e*x+d)^2/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = \frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}\sqrt{\pi}\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{-1+bfn\log(F)\log\left(c(d+ex)^n\right)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}g^3n(d+ex)^2\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(d*g + e*g*x)^3,x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(c*(d + e*x)^n)^(2/n)*Erfi[(-1 + b*f*n*Log[F]*Log[c*(d +
 e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(b*f*n^2*Lo
g[F]))*Sqrt[f]*g^3*n*(d + e*x)^2*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{\left(c(d+ex)^n\right)^{2/n}\int F^{bf\log^2\left(c(d+ex)^n\right)+af}\left(c(d+ex)^n\right)^{-2/n}d\log\left(c(d+ex)^n\right)}{eg^3n(d+ex)^2}$$

$$\Big\downarrow \text{2725}$$

$$\frac{\left(c(d+ex)^n\right)^{2/n}\int \exp\left(bf\log(F)\log^2\left(c(d+ex)^n\right)-\frac{2\log\left(c(d+ex)^n\right)}{n}+af\log(F)\right)d\log\left(c(d+ex)^n\right)}{eg^3n(d+ex)^2}$$

$$\Big\downarrow \text{2664}$$

$$\frac{F^{af}e^{-\frac{1}{bfn^2\log(F)}}\left(c(d+ex)^n\right)^{2/n}\int \exp\left(\frac{\left(1-bfn\log(F)\log\left(c(d+ex)^n\right)\right)^2}{bfn^2\log(F)}\right)d\log\left(c(d+ex)^n\right)}{eg^3n(d+ex)^2}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\sqrt{\pi}F^{af}e^{-\frac{1}{bfn^2\log(F)}}\left(c(d+ex)^n\right)^{2/n}\text{erfi}\left(\frac{1-bfn\log(F)\log\left(c(d+ex)^n\right)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}g^3n\sqrt{\log(F)}(d+ex)^2}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(d*g + e*g*x)^3,x]
```

output
```
-1/2*(F^(a*f)*Sqrt[Pi]*(c*(d + e*x)^n)^(2/n)*Erfi[(1 - b*f*n*Log[F]*Log[c*
(d + e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(Sqrt[b]*e*E^(1/(b*f*n^2*
Log[F]))*Sqrt[f]*g^3*n*(d + e*x)^2*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

$$\int \frac{F^{f\left(a+b\ln\left(c(ex+d)^{n}\right)^{2}\right)}}{\left(egx+dg\right)^{3}}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(e*g*x+d*g)^3,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(e*g*x+d*g)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = -\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}\,\mathrm{erf}\left(\frac{\left(bfn^2\log(ex+d)\log(F)+bfn\log(F)\log(c)-1\right)\sqrt{-bfn^2\log(F)}}{bfn^2\log(F)}\right)e^{\left(\frac{abf^2n^2\log(F)^2+2\,bfn\log(F)\log(c)-1}{bfn^2\log(F)}\right.}}{2\,eg^3n}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^3,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*erf((b*f*n^2*log(e*x + d)*log(F) + b*f
*n*log(F)*log(c) - 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^((a*b*f^2*
n^2*log(F)^2 + 2*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F)))/(e*g^3*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))/(e*g*x+d*g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = \int \frac{F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}}{(egx+dg)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^3,x, algorithm="maxim
a")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(e*g*x + d*g)^3, x)
```

Giac [F]

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = \int \frac{F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}}{(egx+dg)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^3,x, algorithm="giac"
)
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(e*g*x + d*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}}{(d\,g+e\,g\,x)^3}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))/(d*g + e*g*x)^3,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))/(d*g + e*g*x)^3, x)
```

Reduce [F]

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(dg+egx)^3}\,dx=\frac{f^{af}\left(\int \frac{f^{\log\left((ex+d)^n c\right)^2 bf}}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)}{g^3}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(e*g*x+d*g)^3,x)
```

output
```
(f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)/(d**3 + 3*d**2*e*x + 3*d*e**
2*x**2 + e**3*x**3),x))/g**3
```

3.517 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\text{Int}\left(F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m,x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^m,x)
```

Mathematica [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^m,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2708}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^m F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx$$

$$\Big\downarrow \; 2708$$

$$\int (g+hx)^m F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2708
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n]^2))*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^m,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\int (hx+g)^m F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^m,x, algorithm="fricas")
```

output
```
integral((h*x + g)^m*F^(b*f*log((e*x + d)^n*c)^2 + a*f), x)
```

Sympy [F(-2)]

Exception generated.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(h*x+g)**m,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\int (hx+g)^m F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^m,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^m*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\int (hx+g)^m F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^m,x, algorithm="giac")
```

output
```
integrate((h*x + g)^m*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=\int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,{(d+e\,x)}^n\right)^2+a\right)}\,{(g+h\,x)}^m\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x)^m,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^m\,dx=f^{af}\left(\int f^{\log\left((ex+d)^n c\right)^2 bf}(hx+g)^m\,dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^m,x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)*(g + h*x)**m,x)
```

3.518 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 502

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx = \frac{3e^{-\frac{1}{bfn^2\log(F)}}F^{af}h(eg-dh)^2\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{1+bfn\log(F)\log(c(d+ex)^n)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^4\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{4}{bfn^2\log(F)}}F^{af}h^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{2+bfn\log(F)\log(c(d+ex)^n)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^4\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}(eg-dh)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^4\sqrt{f}n\sqrt{\log(F)}} + \frac{3e^{-\frac{9}{4bfn^2\log(F)}}F^{af}h^2(eg-dh)\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{3+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^4\sqrt{f}n\sqrt{\log(F)}}$$

output
```
3/2*F^(a*f)*h*(-d*h+e*g)^2*Pi^(1/2)*(e*x+d)^2*erfi((1+b*f*n*ln(F)*ln(c*(e*
x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e^4/exp(1/b/f/n^2/ln(F))/f
^(1/2)/n/((c*(e*x+d)^n)^(2/n))/ln(F)^(1/2)+1/2*F^(a*f)*h^3*Pi^(1/2)*(e*x+d
)^4*erfi((2+b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^
(1/2)/e^4/exp(4/b/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(4/n))/ln(F)^(1/2)
+1/2*F^(a*f)*(-d*h+e*g)^3*Pi^(1/2)*(e*x+d)*erfi(1/2*(1+2*b*f*n*ln(F)*ln(c*
(e*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e^4/exp(1/4/b/f/n^2/ln(
F))/f^(1/2)/n/((c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)+3/2*F^(a*f)*h^2*(-d*h+e*g)
*Pi^(1/2)*(e*x+d)^3*erfi(1/2*(3+2*b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^(1/2)/f^(
1/2)/n/ln(F)^(1/2))/b^(1/2)/e^4/exp(9/4/b/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+
d)^n)^(3/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 3.83 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.79

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx$$
$$= \frac{e^{-\frac{4}{bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-4/n}\left(3e^{\frac{3}{bfn^2\log(F)}}h(eg-dh)^2(d+ex)\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{1+bfn\log\cdots}{\sqrt{b}\cdots}\right.\right.}{\cdots}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^3,x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*(3*E^(3/(b*f*n^2*Log[F]))*h*(e*g - d*h)^2*(d +
 e*x)*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + b*f*n*Log[F]*Log[c*(d + e*x)^n])/(Sq
rt[b]*Sqrt[f]*n*Sqrt[Log[F]])] + h^3*(d + e*x)^3*Erfi[(2 + b*f*n*Log[F]*Lo
g[c*(d + e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])] + E^(7/(4*b*f*n^2*Log[
F]))*(e*g - d*h)*(c*(d + e*x)^n)^n^(-1)*(E^(2/(b*f*n^2*Log[F]))*(e*g - d*h
)^2*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*
Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])] + 3*h^2*(d + e*x)^2*Erfi[(3 + 2*b*f*n*Log
[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])))/(2*Sqrt[b]*
e^4*E^(4/(b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(4/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 494, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2707, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}dx$$

$$\Big\downarrow \text{2707}$$

$$\frac{\int \left((eg-dh)^3F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}+h^3(d+ex)^3F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}+3h^2(eg-dh)(d+ex)^2F^{f\left(b\log^2\left(c(d+ex)^n\right)\right.}\right.}{e^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}h^2F^{af}(d+ex)^3(eg-dh)e^{-\frac{9}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-3/n}\text{erfi}\left(\frac{2bfn\log(F)\log\left(c(d+ex)^n\right)+3}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}+\frac{3\sqrt{\pi}hF^{af}(d+ex)^2(eg-dh)^2e^{-\frac{1}{bfn^2\log(F)}}(c}{2\sqrt{b}\sqrt{f}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^3,x]
```

output
```
((3*F^(a*f)*h*(e*g - d*h)^2*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*Lo
g[c*(d + e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*E^(1/(b*f*
n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + (F^(a*f)*h^3*
Sqrt[Pi]*(d + e*x)^4*Erfi[(2 + b*f*n*Log[F]*Log[c*(d + e*x)^n])/(Sqrt[b]*S
qrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*E^(4/(b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d
 + e*x)^n)^(4/n)*Sqrt[Log[F]]) + (F^(a*f)*(e*g - d*h)^3*Sqrt[Pi]*(d + e*x)
*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Lo
g[F]])])/(2*Sqrt[b]*E^(1/(4*b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(
-1)*Sqrt[Log[F]]) + (3*F^(a*f)*h^2*(e*g - d*h)*Sqrt[Pi]*(d + e*x)^3*Erfi[(
3 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])]
)/(2*Sqrt[b]*E^(9/(4*b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt
[Log[F]]))/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2707
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/e^(m + 1)   Subst[Int[ExpandI
ntegrand[F^(f*(a + b*Log[c*x^n]^2)), (e*g - d*h + h*x)^m, x], x], x, d + e*
x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && IGtQ[m, 0]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^3,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.02

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-bfn^2\log\left(F\right)}h^3\operatorname{erf}\left(\frac{\left(bfn^2\log(ex+d)\log(F)+bfn\log(F)\log(c)+2\right)\sqrt{-bfn^2\log(F)}}{bfn^2\log(F)}\right)e^{\left(\frac{abf^2n^2\log(F)^2-4\,bfn\log(F)\log(c)}{bfn^2\log(F)}\right.}}{}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b*f*n^2*log(F))*h^3*erf((b*f*n^2*log(e*x + d)*log(F)
+ b*f*n*log(F)*log(c) + 2)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^((a*b
*f^2*n^2*log(F)^2 - 4*b*f*n*log(F)*log(c) - 4)/(b*f*n^2*log(F))) + sqrt(pi
)*(e^3*g^3 - 3*d*e^2*g^2*h + 3*d^2*e*g*h^2 - d^3*h^3)*sqrt(-b*f*n^2*log(F)
)*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F) + 2*b*f*n*log(F)*log(c) + 1)*sqrt
(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^(1/4*(4*a*b*f^2*n^2*log(F)^2 - 4*b*f
*n*log(F)*log(c) - 1)/(b*f*n^2*log(F))) + 3*sqrt(pi)*sqrt(-b*f*n^2*log(F))
*(e*g*h^2 - d*h^3)*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F) + 2*b*f*n*log(F)
*log(c) + 3)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^(1/4*(4*a*b*f^2*n^2
*log(F)^2 - 12*b*f*n*log(F)*log(c) - 9)/(b*f*n^2*log(F))) + 3*sqrt(pi)*(e^
2*g^2*h - 2*d*e*g*h^2 + d^2*h^3)*sqrt(-b*f*n^2*log(F))*erf((b*f*n^2*log(e*
x + d)*log(F) + b*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*lo
g(F)))*e^((a*b*f^2*n^2*log(F)^2 - 2*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(
F))))/(e^4*n)
```

Sympy [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx=\text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx=\int (hx+g)^3F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^3*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx=\int (hx+g)^3F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate((h*x + g)^3*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx=\int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}\,(g+h\,x)^3\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x)^3,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x)^3, x)
```

Reduce [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^3\,dx=f^{af}\left(\left(\int f^{\log\left((ex+d)^nc\right)^2bf}dx\right)g^3+\left(\int f^{\log\left((ex+d)^nc\right)^2bf}x^3dx\right)h^3+3\left(\int f^{\log\left((ex+d)^nc\right)^2bf}x^2dx\right)g\,h^2+3\left(\int f^{\log\left((ex+d)^nc\right)^2bf}xdx\right)g^2h\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^3,x)
```

output
```
f**(a*f)*(int(f**(log((d + e*x)**n*c)**2*b*f),x)*g**3 + int(f**(log((d + e
*x)**n*c)**2*b*f)*x**3,x)*h**3 + 3*int(f**(log((d + e*x)**n*c)**2*b*f)*x**
2,x)*g*h**2 + 3*int(f**(log((d + e*x)**n*c)**2*b*f)*x,x)*g**2*h)
```

3.519 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 372

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^2\,dx = \frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}h(eg-dh)\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{1+bfn\log(F)\log(c(d+ex)^n)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{\sqrt{b}e^3\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}(eg-dh)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^3\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{9}{4bfn^2\log(F)}}F^{af}h^2\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{3+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^3\sqrt{f}n\sqrt{\log(F)}}$$

output
```
F^(a*f)*h*(-d*h+e*g)*Pi^(1/2)*(e*x+d)^2*erfi((1+b*f*n*ln(F)*ln(c*(e*x+d)^n
))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e^3/exp(1/b/f/n^2/ln(F))/f^(1/2)
/n/((c*(e*x+d)^n)^(2/n))/ln(F)^(1/2)+1/2*F^(a*f)*(-d*h+e*g)^2*Pi^(1/2)*(e*
x+d)*erfi(1/2*(1+2*b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1
/2))/b^(1/2)/e^3/exp(1/4/b/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(1/n))/ln
(F)^(1/2)+1/2*F^(a*f)*h^2*Pi^(1/2)*(e*x+d)^3*erfi(1/2*(3+2*b*f*n*ln(F)*ln(
c*(e*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e^3/exp(9/4/b/f/n^2/l
n(F))/f^(1/2)/n/((c*(e*x+d)^n)^(3/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.81

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^2\,dx$$
$$= \frac{e^{-\frac{9}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(-2e^{\frac{5}{4bfn^2\log(F)}}h(-eg+dh)(d+ex)\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{1+b}{}\right.\right.}{}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^2,x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*(-2*E^(5/(4*b*f*n^2*Log[F]))*h*(-(e*g) + d*h)*
(d + e*x)*(c*(d + e*x)^n)^n^(-1)*Erfi[(1 + b*f*n*Log[F]*Log[c*(d + e*x)^n]
)/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])] + E^(2/(b*f*n^2*Log[F]))*(e*g - d*h)^2
*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqr
t[b]*Sqrt[f]*n*Sqrt[Log[F]])] + h^2*(d + e*x)^2*Erfi[(3 + 2*b*f*n*Log[F]*L
og[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])]))/(2*Sqrt[b]*e^3*E^
(9/(4*b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2707, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx$$

$$\Big\downarrow \text{2707}$$

$$\frac{\int \left((eg-dh)^2F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}+h^2(d+ex)^2F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}+2h(eg-dh)(d+ex)F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}\right.}{e^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{\pi}hF^{af}(d+ex)^2(eg-dh)e^{-\frac{1}{bfn^2\log(F)}}(c(d+ex)^n)^{-2/n}\text{erfi}\left(\frac{bfn\log(F)\log(c(d+ex)^n)+1}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}+\frac{\sqrt{\pi}F^{af}(d+ex)(eg-dh)^2e^{-\frac{1}{4bfn^2\log(F)}}(c(d+ex)}{2\sqrt{b}\sqrt{f}n\sqrt{l}}}{e^3}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x)^2,x]
```

output
```
((F^(a*f)*h*(e*g - d*h)*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*Log[c*
(d + e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(Sqrt[b]*E^(1/(b*f*n^2*Lo
g[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + (F^(a*f)*(e*g - d*h
)^2*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqr
t[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*E^(1/(4*b*f*n^2*Log[F]))*Sqrt[f]
*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]]) + (F^(a*f)*h^2*Sqrt[Pi]*(d + e*x)^
3*Erfi[(3 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[L
og[F]])])/(2*Sqrt[b]*E^(9/(4*b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3
/n)*Sqrt[Log[F]]))/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2707
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/e^(m + 1)   Subst[Int[ExpandI
ntegrand[F^(f*(a + b*Log[c*x^n]^2)), (e*g - d*h + h*x)^m, x], x], x, d + e*
x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && IGtQ[m, 0]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.99

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^2\,dx = -\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}h^2\,\mathrm{erf}\left(\frac{\left(2\,bfn^2\log(ex+d)\log(F)+2\,bfn\log(F)\log(c)+3\right)\sqrt{-bfn^2\log(F)}}{2\,bfn^2\log(F)}\right)e^{\left(\frac{4\,abf^2n^2\log(F)^2-12\,bfn\log(F}{4\,bfn^2\log(F)}\right.}}{}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b*f*n^2*log(F))*h^2*erf(1/2*(2*b*f*n^2*log(e*x + d)*l
og(F) + 2*b*f*n*log(F)*log(c) + 3)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))
*e^(1/4*(4*a*b*f^2*n^2*log(F)^2 - 12*b*f*n*log(F)*log(c) - 9)/(b*f*n^2*log
(F))) + sqrt(pi)*sqrt(-b*f*n^2*log(F))*(e^2*g^2 - 2*d*e*g*h + d^2*h^2)*erf
(1/2*(2*b*f*n^2*log(e*x + d)*log(F) + 2*b*f*n*log(F)*log(c) + 1)*sqrt(-b*f
*n^2*log(F))/(b*f*n^2*log(F)))*e^(1/4*(4*a*b*f^2*n^2*log(F)^2 - 4*b*f*n*lo
g(F)*log(c) - 1)/(b*f*n^2*log(F))) + 2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*(e*g
*h - d*h^2)*erf((b*f*n^2*log(e*x + d)*log(F) + b*f*n*log(F)*log(c) + 1)*sq
rt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^((a*b*f^2*n^2*log(F)^2 - 2*b*f*n*l
og(F)*log(c) - 1)/(b*f*n^2*log(F))))/(e^3*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(343) = 686.

Time = 105.67 (sec) , antiderivative size = 1027, normalized size of antiderivative = 2.76

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)^2\,dx = \text{Too large to display}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(h*x+g)**2,x)
```

output
```
Piecewise((-11*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**3*f*h**2*n**2*lo
g(F)/(9*e**3) - 11*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**3*f*h**2*n*l
og(F)*log(c*(d + e*x)**n)/(9*e**3) + 3*F**(a*f + b*f*log(c*(d + e*x)**n)**
2)*b*d**2*f*g*h*n**2*log(F)/e**2 + 3*F**(a*f + b*f*log(c*(d + e*x)**n)**2)
*b*d**2*f*g*h*n*log(F)*log(c*(d + e*x)**n)/e**2 + 11*F**(a*f + b*f*log(c*(
d + e*x)**n)**2)*b*d**2*f*h**2*n**2*x*log(F)/(9*e**2) - 2*F**(a*f + b*f*lo
g(c*(d + e*x)**n)**2)*b*d**2*f*h**2*n*x*log(F)*log(c*(d + e*x)**n)/(3*e**2
) - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g**2*n**2*log(F)/e - 2*F
**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g**2*n*log(F)*log(c*(d + e*x)**
n)/e - 3*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g*h*n**2*x*log(F)/e +
 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g*h*n*x*log(F)*log(c*(d + e
*x)**n)/e - 5*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*h**2*n**2*x**2*l
og(F)/(18*e) + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*h**2*n*x**2*log
(F)*log(c*(d + e*x)**n)/(3*e) + 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*
f*g**2*n**2*x*log(F) - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*g**2*n*
x*log(F)*log(c*(d + e*x)**n) + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*g
*h*n**2*x**2*log(F)/2 - F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*g*h*n*x*
*2*log(F)*log(c*(d + e*x)**n) + 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*
f*h**2*n**2*x**3*log(F)/27 - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*h
**2*n*x**3*log(F)*log(c*(d + e*x)**n)/9 + F**(a*f + b*f*log(c*(d + e*x)...
```

Maxima [F]

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(g+hx)^2\,dx=\int (hx+g)^2F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^2*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}(g+hx)^2\,dx = \int (hx+g)^2 F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate((h*x + g)^2*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}(g+hx)^2\,dx = \int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}\,(g+h\,x)^2\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x)^2, x)
```

Reduce [F]

$$\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}(g+hx)^2\,dx = f^{af}\left(\left(\int f^{\log\left((ex+d)^n c\right)^2 bf}dx\right)g^2 + \left(\int f^{\log\left((ex+d)^n c\right)^2 bf}x^2dx\right)h^2 + 2\left(\int f^{\log\left((ex+d)^n c\right)^2 bf}xdx\right)gh\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g)^2,x)
```

output
```
f**(a*f)*(int(f**(log((d + e*x)**n*c)**2*b*f),x)*g**2 + int(f**(log((d + e
*x)**n*c)**2*b*f)*x**2,x)*h**2 + 2*int(f**(log((d + e*x)**n*c)**2*b*f)*x,x
)*g*h)
```

3.520 $\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 242

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx = \frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}h\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{1+bfn\log(F)\log\left(c(d+ex)^n\right)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^2\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}(eg-dh)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log\left(c(d+ex)^n\right)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e^2\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*h*Pi^(1/2)*(e*x+d)^2*erfi((1+b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^(1
/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e^2/exp(1/b/f/n^2/ln(F))/f^(1/2)/n/((c*
(e*x+d)^n)^(2/n))/ln(F)^(1/2)+1/2*F^(a*f)*(-d*h+e*g)*Pi^(1/2)*(e*x+d)*erfi
(1/2*(1+2*b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1
/2)/e^2/exp(1/4/b/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.84

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx$$
$$=\frac{e^{-\frac{1}{bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(h(d+ex)\text{erfi}\left(\frac{1+bfn\log(F)\log(c(d+ex)^n)}{\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)+e^{\frac{3}{4bfn^2\log(F)}}(eg-dh$$
$$2\sqrt{b}e^2\sqrt{f}n\sqrt{\log(F)}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*(h*(d + e*x)*Erfi[(1 + b*f*n*Log[F]*Log[c*(d +
 e*x)^n])/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])] + E^(3/(4*b*f*n^2*Log[F]))*(e*
g - d*h)*(c*(d + e*x)^n)^n^(-1)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n
])/(2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])]))/(2*Sqrt[b]*e^2*E^(1/(b*f*n^2*Log[
F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2707, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx$$

$$\Big\downarrow \text{2707}$$

$$\frac{\int \left((eg-dh)F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}+h(d+ex)F^{f\left(b\log^2\left(c(d+ex)^n\right)+a\right)}\right)d(d+ex)}{e^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{\pi}F^{af}(d+ex)(eg-dh)e^{-\frac{1}{4bfn^2\log(F)}}(c(d+ex)^n)^{-1/n}\operatorname{erfi}\left(\frac{2bfn\log(F)\log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}+\frac{\sqrt{\pi}hF^{af}(d+ex)^2e^{-\frac{1}{bfn^2\log(F)}}(c(d+ex)^n)^{-2/n}e}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F}}}{e^2}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))*(g + h*x),x]
```

output
```
((F^(a*f)*h*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*Log[c*(d + e*x)^n]
)/(Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*E^(1/(b*f*n^2*Log[F]))*Sqr
t[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + (F^(a*f)*(e*g - d*h)*Sqrt[Pi]
*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*Sqrt[b]*Sqrt[f]
*n*Sqrt[Log[F]])])/(2*Sqrt[b]*E^(1/(4*b*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e
*x)^n)^n^(-1)*Sqrt[Log[F]]))/e^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2707
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/e^(m + 1)   Subst[Int[ExpandI
ntegrand[F^(f*(a + b*Log[c*x^n]^2)), (e*g - d*h + h*x)^m, x], x], x, d + e*
x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && IGtQ[m, 0]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))*(h*x+g),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(g+hx)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-bfn^2\log(F)}(eg-dh)\,\mathrm{erf}\left(\frac{(2\,bfn^2\log(ex+d)\log(F)+2\,bfn\log(F)\log(c)+1)\sqrt{-bfn^2\log(F)}}{2\,bfn^2\log(F)}\right)e^{\left(\frac{4\,abf^2n^2\log(F)^2-4}{4\,bfn^2}\right.}}{}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b*f*n^2*log(F))*(e*g - d*h)*erf(1/2*(2*b*f*n^2*log(e*
x + d)*log(F) + 2*b*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*
log(F)))*e^(1/4*(4*a*b*f^2*n^2*log(F)^2 - 4*b*f*n*log(F)*log(c) - 1)/(b*f*
n^2*log(F))) + sqrt(pi)*sqrt(-b*f*n^2*log(F))*h*erf((b*f*n^2*log(e*x + d)*
log(F) + b*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*
e^((a*b*f^2*n^2*log(F)^2 - 2*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F))))/(
e^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 558 vs. $2(223) = 446$.

Time = 23.42 (sec) , antiderivative size = 558, normalized size of antiderivative = 2.31

$$\int F^{f(a+b\log^2(c(d+ex)^n))}(g+hx)\,dx$$

$$=\begin{cases}\frac{3F^{af+bf\log(c(d+ex)^n)^2}bd^2fhn^2\log(F)}{2e^2}+\frac{3F^{af+bf\log(c(d+ex)^n)^2}bd^2fhn\log(F)\log(c(d+ex)^n)}{2e^2}-\frac{2F^{af+bf\log(c(d+ex)^n)^2}bdfgn^2\log(}{e} \\ F^{f\left(a+b\log(cd^n)^2\right)}\left(gx+\frac{hx^2}{2}\right)\end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))*(h*x+g),x)
```

output
```
Piecewise((3*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**2*f*h*n**2*log(F)/
(2*e**2) + 3*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d**2*f*h*n*log(F)*log
(c*(d + e*x)**n)/(2*e**2) - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*
g*n**2*log(F)/e - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*g*n*log(F)
*log(c*(d + e*x)**n)/e - 3*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*h*n
**2*x*log(F)/(2*e) + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*h*n*x*log
(F)*log(c*(d + e*x)**n)/e + 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*g*
n**2*x*log(F) - 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*g*n*x*log(F)*l
og(c*(d + e*x)**n) + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*h*n**2*x**2
*log(F)/4 - F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*h*n*x**2*log(F)*log(
c*(d + e*x)**n)/2 - F**(a*f + b*f*log(c*(d + e*x)**n)**2)*d**2*h/(2*e**2)
+ F**(a*f + b*f*log(c*(d + e*x)**n)**2)*d*g/e + F**(a*f + b*f*log(c*(d + e
*x)**n)**2)*g*x + F**(a*f + b*f*log(c*(d + e*x)**n)**2)*h*x**2/2, Ne(e, 0)
), (F**(f*(a + b*log(c*d**n)**2))*(g*x + h*x**2/2), True))
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx=\int (hx+g)F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((h*x + g)*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx=\int (hx+g)F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g),x, algorithm="giac")
```

output
```
integrate((h*x + g)*F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx=\int \mathrm{e}^{f\,\ln(F)\,\left(b\ln\left(c\,(d+e\,x)^n\right)^2+a\right)}\,(g+h\,x)\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))*(g + h*x), x)
```

Reduce [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}(g+hx)\,dx=f^{af}\left(\left(\int f^{\log((ex+d)^n c)^2 bf}dx\right)g+\left(\int f^{\log((ex+d)^n c)^2 bf}xdx\right)h\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))*(h*x+g),x)
```

output
```
f**(a*f)*(int(f**(log((d + e*x)**n*c)**2*b*f),x)*g + int(f**(log((d + e*x)
**n*c)**2*b*f)*x,x)*h)
```

3.521 $\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 118

$$\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}\,dx = \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*F^(a*f)*Pi^(1/2)*(e*x+d)*erfi(1/2*(1+2*b*f*n*ln(F)*ln(c*(e*x+d)^n))/b^
(1/2)/f^(1/2)/n/ln(F)^(1/2))/b^(1/2)/e/exp(1/4/b/f/n^2/ln(F))/f^(1/2)/n/((
c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}\,dx = \frac{e^{-\frac{1}{4bfn^2\log(F)}}F^{af}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)\log(c(d+ex)^n)}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2)),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(
2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(4*b*f*n^2*Log[F]))*
Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2705, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{f\left(a+b\log^2(c(d+ex)^n)\right)}dx$$

$$\Big\downarrow \text{2705}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int F^{bf\log^2(c(d+ex)^n)+af}\left(c(d+ex)^n\right)^{\frac{1}{n}}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \exp\left(bf\log(F)\log^2\left(c(d+ex)^n\right)+\frac{\log(c(d+ex)^n)}{n}+af\log(F)\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{F^{af}(d+ex)e^{-\frac{1}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-1/n}\int \exp\left(\frac{(2bfn\log(F)\log(c(d+ex)^n)+1)^2}{4bfn^2\log(F)}\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^{af}(d+ex)e^{-\frac{1}{4bfn^2\log(F)}}\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{2bfn\log(F)\log(c(d+ex)^n)+1}{2\sqrt{b}\sqrt{f}n\sqrt{\log(F)}}\right)}{2\sqrt{b}e\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2)),x]
```

output
```
(F^(a*f)*Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*Log[c*(d + e*x)^n])/(
2*Sqrt[b]*Sqrt[f]*n*Sqrt[Log[F]])])/(2*Sqrt[b]*e*E^(1/(4*b*f*n^2*Log[F]))*
Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2705
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.)), x
_Symbol] :> Simp[(d + e*x)/(e*n*(c*(d + e*x)^n)^(1/n))   Subst[Int[E^(a*f*L
og[F] + x/n + b*f*Log[F]*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F,
a, b, c, d, e, f, n}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2)),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx = -\frac{\sqrt{\pi}\sqrt{-bfn^2\log\left(F\right)}\operatorname{erf}\left(\frac{\left(2\,bfn^2\log(ex+d)\log(F)+2\,bfn\log(F)\log(c)+1\right)\sqrt{-bfn^2\log(F)}}{2\,bfn^2\log(F)}\right)e^{\left(\frac{4\,abf^2n^2\log(F)^2-4\,bfn\log(F)\log(c)-1}{4\,bfn^2\log(F)}\right)}}{2\,en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b*f*n^2*log(F))*erf(1/2*(2*b*f*n^2*log(e*x + d)*log(F)
 + 2*b*f*n*log(F)*log(c) + 1)*sqrt(-b*f*n^2*log(F))/(b*f*n^2*log(F)))*e^(1
/4*(4*a*b*f^2*n^2*log(F)^2 - 4*b*f*n*log(F)*log(c) - 1)/(b*f*n^2*log(F)))/
(e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(105) = 210.

Time = 4.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.85

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx = \begin{cases} -\frac{2F^{af+bf\log\left(c(d+ex)^n\right)^2}bdfn^2\log\left(F\right)}{e} - \frac{2F^{af+bf\log\left(c(d+ex)^n\right)^2}bdfn\log\left(F\right)\log\left(c(d+ex)^n\right)}{e} + 2F^{af+bf\log\left(c(d+ex)^n\right)^2}bfn^2x\log\left(F\right) - \cdots \\ F^{f\left(a+b\log\left(cd^n\right)^2\right)}x \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2)),x)
```

output
```
Piecewise((-2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*n**2*log(F)/e -
2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*d*f*n*log(F)*log(c*(d + e*x)**n)
/e + 2*F**(a*f + b*f*log(c*(d + e*x)**n)**2)*b*f*n**2*x*log(F) - 2*F**(a*f
 + b*f*log(c*(d + e*x)**n)**2)*b*f*n*x*log(F)*log(c*(d + e*x)**n) + F**(a*
f + b*f*log(c*(d + e*x)**n)**2)*d/e + F**(a*f + b*f*log(c*(d + e*x)**n)**2
)*x, Ne(e, 0)), (F**(f*(a + b*log(c*d**n)**2))*x, True))
```

Maxima [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx=\int F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx=-\frac{\sqrt{\pi}F^{af}\operatorname{erf}\left(-\sqrt{-bf\log\left(F\right)}n\log\left(ex+d\right)-\sqrt{-bf\log\left(F\right)}\log\left(c\right)-\frac{\sqrt{-bf\log(F)}}{2\,bfn\log(F)}\right)e^{\left(-\frac{1}{4\,bfn^2\log(F)}\right)}}{2\sqrt{-bf\log\left(F\right)}c^{\left(\frac{1}{n}\right)}en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*F^(a*f)*erf(-sqrt(-b*f*log(F))*n*log(e*x + d) - sqrt(-b*f*lo
g(F))*log(c) - 1/2*sqrt(-b*f*log(F))/(b*f*n*log(F)))*e^(-1/4/(b*f*n^2*log(
F)))/(sqrt(-b*f*log(F))*c^(1/n)*e*n)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx=\int F^{b\,f\,\ln\left(c\,{\left(d+e\,x\right)}^n\right)^2}\,F^{a\,f}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2)),x)
```

output
```
int(F^(b*f*log(c*(d + e*x)^n)^2)*F^(a*f), x)
```

Reduce [F]

$$\int F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}\,dx = f^{af}\left(\int f^{\log((ex+d)^n c)^2 bf} dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2)),x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f),x)
```

3.522 $\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{g+hx}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{g+hx}\,dx = \text{Int}\left(\frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{g+hx},x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g),x)
```

Mathematica [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{g+hx}\,dx = \int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{g+hx}\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x),x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2708}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{g+hx}\,dx$$
$$\Big\downarrow \text{ 2708}$$
$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{g+hx}dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2708
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n]^2))*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{g+hx}\,dx = \int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{hx+g}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g),x, algorithm="fricas")
```

output
```
integral(F^(b*f*log((e*x + d)^n*c)^2 + a*f)/(h*x + g), x)
```

Sympy [N/A]

Not integrable

Time = 9.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{g+hx}\,dx = \int \frac{F^{f\left(a+b\log{(c(d+ex)^n)}^2\right)}}{g+hx}\,dx$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))/(h*x+g),x)
```

output
```
Integral(F**(f*(a + b*log(c*(d + e*x)**n)**2))/(g + h*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{g+hx}\,dx = \int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{hx+g}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g),x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(h*x + g), x)
```

Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{g+hx}\,dx = \int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{hx+g}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g),x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(h*x + g), x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{g+hx}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,\left(b\ln(c\,(d+e\,x)^n)^2+a\right)}}{g+h\,x}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))/(g + h*x),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))/(g + h*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{g+hx}\,dx = f^{af}\left(\int \frac{f^{\log\left((ex+d)^n c\right)^2 bf}}{hx+g}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g),x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)/(g + h*x),x)
```

3.523 $\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2}\,dx=\text{Int}\left(\frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2},x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2}\,dx=\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2}\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x)^2,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2708}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2}dx$$
$$\Big\downarrow \text{ 2708}$$
$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^2}dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2708
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n]^2))*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.50

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^2}\,dx = \int \frac{F^{\left(b\log\left((ex+d)^n c\right)^2+a\right)f}}{\left(hx+g\right)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*f*log((e*x + d)^n*c)^2 + a*f)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^2}\,dx = \int \frac{F^{\left(b\log((ex+d)^nc)^2+a\right)f}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^2}\,dx = \int \frac{F^{\left(b\log((ex+d)^nc)^2+a\right)f}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^2}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,\left(b\ln(c\,(d+e\,x)^n)^2+a\right)}}{(g+h\,x)^2}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))/(g + h*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))/(g + h*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.57

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^2}\,dx = f^{af}\left(\int \frac{f^{\log((ex+d)^n c)^2 bf}}{h^2x^2+2ghx+g^2}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^2,x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)/(g**2 + 2*g*h*x + h**2*x**2),
x)
```

3.524 $\int \frac{F^{f\left(a+b\log^2(c(d+ex)^n)\right)}}{(g+hx)^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx=\text{Int}\left(\frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3},x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 5.62 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx=\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x)^3,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2708}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^3}dx$$
$$\Big\downarrow \text{ 2708}$$
$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^3}dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n]^2))/(g + h*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2708
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n]^2))*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g)^3,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n)^2))/(h*x+g)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx = \int \frac{F^{\left(b\log\left((ex+d)^nc\right)^2+a\right)f}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*f*log((e*x + d)^n*c)^2 + a*f)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2
*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n)**2))/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx=\int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx=\int \frac{F^{\left(b\log((ex+d)^n c)^2+a\right)f}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c)^2 + a)*f)/(h*x + g)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log^2(c(d+ex)^n))}}{(g+hx)^3}\,dx=\int \frac{{\mathrm{e}}^{f\,\ln(F)\,\left(b\,{\ln\left(c\,{\left(d+e\,x\right)}^n\right)}^2+a\right)}}{{\left(g+h\,x\right)}^3}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n)^2))/(g + h*x)^3,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n)^2))/(g + h*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{F^{f\left(a+b\log^2\left(c(d+ex)^n\right)\right)}}{(g+hx)^3}\,dx = f^{af}\left(\int \frac{f^{\log\left((ex+d)^n c\right)^2 bf}}{h^3x^3+3g\,h^2x^2+3g^2hx+g^3}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n)^2))/(h*x+g)^3,x)
```

output
```
f**(a*f)*int(f**(log((d + e*x)**n*c)**2*b*f)/(g**3 + 3*g**2*h*x + 3*g*h**2
*x**2 + h**3*x**3),x)
```

3.525 $\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx$

Optimal result

Integrand size = 31, antiderivative size = 153

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx = \frac{e^{-\frac{(1+m+2abfn\log(F))^2}{4b^2fn^2\log(F)}}F^{a^2f}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-\frac{1+m}{n}}(dg+egx)^m\operatorname{erfi}\left(\frac{1+m+2abfn\log(F)+2b^2fn\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*F^(a^2*f)*Pi^(1/2)*(e*x+d)*(e*g*x+d*g)^m*erfi(1/2*(1+m+2*a*b*f*n*ln(F)
+2*b^2*f*n*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/n/ln(F)^(1/2))/b/e/exp(1/4*(1+
m+2*a*b*f*n*ln(F))^2/b^2/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^((1+m)/n))/
ln(F)^(1/2)
```

Mathematica [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx = \int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^m,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^m, x]
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2712, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dg+egx)^m F^{f(a+b\log(c(d+ex)^n))^2}\,dx$$

$$\Big\downarrow \text{2712}$$

$$(dg+egx)^m \left(c(d+ex)^n\right)^{2abf\log(F)}(d+ex)^{-2abfn\log(F)-m}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{m+2abfn\log(F)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{(d+ex)(dg+egx)^m \left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2abfn\log(F)+m+1}{n}}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}(c(d+ex)^n)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(d+ex)(dg+egx)^m \left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2abfn\log(F)+m+1}{n}}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)+\right.}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{F^{a^2f}(d+ex)(dg+egx)^m \exp\left(-\frac{(2abfn\log(F)+m+1)^2}{4b^2fn^2\log(F)}\right)\left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2abfn\log(F)+m+1}{n}}\int \exp\left(\frac{(2fn\log(F)\log(}{}\right.}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}F^{a^2f}(d+ex)(dg+egx)^m \exp\left(-\frac{(2abfn\log(F)+m+1)^2}{4b^2fn^2\log(F)}\right)\left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2abfn\log(F)+m+1}{n}}\operatorname{erfi}\left(\frac{2abfn\log(F)+}{}\right.}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^m,x]
```

output
```
(F^(a^2*f)*Sqrt[Pi]*(d + e*x)*(c*(d + e*x)^n)^(2*a*b*f*Log[F] - (1 + m + 2
*a*b*f*n*Log[F])/n)*(d*g + e*g*x)^m*Erfi[(1 + m + 2*a*b*f*n*Log[F] + 2*b^2
*f*n*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*n*Sqrt[Log[F]])])/(2*b*e*E^((
1 + m + 2*a*b*f*n*Log[F])^2/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F(-1)]

Timed out.

hanged

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(e*g*x+d*g)^m,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(e*g*x+d*g)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\,\mathrm{erf}\left(\frac{(2\,b^2fn^2\log(ex+d)\log(F)+2\,b^2fn\log(F)\log(c)+2\,abfn\log(F)+m+1)\sqrt{-b^2fn^2\log(F)}}{2\,b^2fn^2\log(F)}\right)e^{\left(\frac{4\,b^2fmn}{\cdots}\right.}}{2\,ben}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^m,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*lo
g(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + m + 1)*sqrt(-b^2*f*n^2
*log(F))/(b^2*f*n^2*log(F)))*e^(1/4*(4*b^2*f*m*n^2*log(F)*log(g) - 4*(b^2*
f*m + b^2*f)*n*log(F)*log(c) - 4*(a*b*f*m + a*b*f)*n*log(F) - m^2 - 2*m -
1)/(b^2*f*n^2*log(F)))/(b*e*n)
```

Sympy [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx = \int F^{f(a+b\log(c(d+ex)^n))^2}(g(d+ex))^m\,dx$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(e*g*x+d*g)**m,x)
```

output
```
Integral(F**(f*(a + b*log(c*(d + e*x)**n))**2)*(g*(d + e*x))**m, x)
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx = \int (egx+dg)^m F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^m,x, algorithm="maxim
a")
```

output
```
integrate((e*g*x + d*g)^m*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx = \int (egx+dg)^m F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^m,x, algorithm="giac"
)
```

output
```
integrate((e*g*x + d*g)^m*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx=\int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(d\,g+e\,g\,x)^m\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(d*g + e*g*x)^m,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(d*g + e*g*x)^m, x)
```

Reduce [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^m\,dx=f^{a^2f}\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}(egx+dg)^m\,dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^m,x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f)*(d*g + e*g*x)**m,x)
```

3.526 $\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 133

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^2\,dx = \frac{e^{-\frac{3(3+4abfn\log(F))}{4b^2fn^2\log(F)}}g^2\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\frac{3}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*g^2*Pi^(1/2)*(e*x+d)^3*erfi(1/2*(3/n+2*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*
(e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e/exp(3/4*(3+4*a*b*f*n*ln(F))/b^2/f/n
^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(3/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.97

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^2\,dx = \frac{e^{-\frac{3(3+4bfn\log(F)(a+b(-n\log(d+ex)+\log(c(d+ex)^n))))}{4b^2fn^2\log(F)}}g^2\sqrt{\pi}\operatorname{erfi}\left(\frac{3+2bfn\log(F)(a+b\log(c(d+ex)^n))}{2b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^2,x]
```

output
```
(g^2*Sqrt[Pi]*Erfi[(3 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b*Sq
rt[f]*n*Sqrt[Log[F]])])/(2*b*e*E^((3*(3 + 4*b*f*n*Log[F]*(a + b*(-(n*Log[d
 + e*x]) + Log[c*(d + e*x)^n]))))/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*Sqrt[Log
[F]])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {2712, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dg+egx)^2 F^{f(a+b\log(c(d+ex)^n))^2}\,dx$$

$$\Big\downarrow \text{2712}$$

$$g^2(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)+2}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}\left(c(d+ex)^n\right)^{3/n}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)+\frac{(2abfn\log(F)+3)\log(c(d+ex)^n)}{n}\right)d\log}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\exp\left(-\frac{3(4abfn\log(F)+3)}{4b^2fn^2\log(F)}\right)\int \exp\left(\frac{\left(2f\log(F)\log(c(d+ex)^n)b^2+2af\log(F)b+\frac{3}{n}\right)^2}{4b^2f\log(F)}\right)d\log\left(c(d+\right.}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}g^2(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\exp\left(-\frac{3(4abfn\log(F)+3)}{4b^2fn^2\log(F)}\right)\operatorname{erfi}\left(\frac{2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x)^2,x]
```

output
```
(g^2*Sqrt[Pi]*(d + e*x)^3*Erfi[(3/n + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log[
c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*e*E^((3*(3 + 4*a*b*f*n*L
og[F]))/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]]
)
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^((((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int F^{f(a+b\ln(c(ex+d)^n))^2}(egx+dg)^2\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.01

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^2\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}g^2\operatorname{erf}\left(\frac{(2\,b^2fn^2\log(ex+d)\log(F)+2\,b^2fn\log(F)\log(c)+2\,abfn\log(F)+3)\sqrt{-b^2fn^2\log(F)}}{2\,b^2fn^2\log(F)}\right)e^{\left(-\frac{3\left(4\,b^2\right.}{}\right.}}{2\,ben}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*g^2*erf(1/2*(2*b^2*f*n^2*log(e*x + d
)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 3)*sqrt(-b^2*f*n^2
*log(F))/(b^2*f*n^2*log(F)))*e^(-3/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*
log(F) + 3)/(b^2*f*n^2*log(F)))/(b*e*n)
```

Sympy [F(-1)]

Timed out.

$$\int F^{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2}(dg+egx)^2\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(e*g*x+d*g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2}(dg+egx)^2\,dx = \int (egx+dg)^2 F^{\left(b\log\left((ex+d)^n c\right)+a\right)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x, algorithm="maxim
a")
```

output
```
integrate((e*g*x + d*g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [F]

$$\int F^{f\left(a+b\log\left(c(d+ex)^n\right)\right)^2}(dg+egx)^2\,dx = \int (egx+dg)^2 F^{\left(b\log\left((ex+d)^n c\right)+a\right)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x, algorithm="giac"
)
```

output
```
integrate((e*g*x + d*g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^2\,dx=\int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(d\,g+e\,g\,x)^2\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(d*g + e*g*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(d*g + e*g*x)^2, x)
```

Reduce [F]

$$\begin{aligned}&\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)^2\,dx\\&=f^{a^2f}g^2\Bigg(\left(\int f^{\log((ex+d)^n c)^2b^2f+2\log((ex+d)^n c)abf}dx\right)d^2\\&\quad+\left(\int f^{\log((ex+d)^n c)^2b^2f+2\log((ex+d)^n c)abf}x^2dx\right)e^2\\&\quad+2\left(\int f^{\log((ex+d)^n c)^2b^2f+2\log((ex+d)^n c)abf}xdx\right)de\Bigg)\end{aligned}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g)^2,x)
```

output
```
f**(a**2*f)*g**2*(int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)*
*n*c)*a*b*f),x)*d**2 + int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d +
e*x)**n*c)*a*b*f)*x**2,x)*e**2 + 2*int(f**(log((d + e*x)**n*c)**2*b**2*f +
 2*log((d + e*x)**n*c)*a*b*f)*x,x)*d*e)
```

3.527 $\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 122

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx = \frac{e^{-\frac{1+2abfn\log(F)}{b^2fn^2\log(F)}}g\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+abf\log(F)+b^2f\log(F)\log(c(d+ex)^n)}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*g*Pi^(1/2)*(e*x+d)^2*erfi((1/n+a*b*f*ln(F)+b^2*f*ln(F)*ln(c*(e*x+d)^n)
)/b/f^(1/2)/ln(F)^(1/2))/b/e/exp((1+2*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1
/2)/n/((c*(e*x+d)^n)^(2/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx = \frac{e^{-\frac{1+2abfn\log(F)}{b^2fn^2\log(F)}}g\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{1+bfn\log(F)(a+b\log(c(d+ex)^n))}{b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x),x]
```

output
```
(g*Sqrt[Pi]*(d + e*x)^2*Erfi[(1 + b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))
/(b*Sqrt[f]*n*Sqrt[Log[F]])])/(2*b*e*E^((1 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*
Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {2712, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dg+egx)F^{f(a+b\log(c(d+ex)^n))^2}dx$$

$$\Big\downarrow \text{2712}$$

$$g(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)+1}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{g(d+ex)^2\left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2(abfn\log(F)+1)}{n}}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}\left(c(d+ex)^n\right)^{2/n}d\log(c(\ldots}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{g(d+ex)^2\left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2(abfn\log(F)+1)}{n}}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)+2\left(abf\log(F)\ldots\right.\right.}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{g(d+ex)^2e^{-\frac{2abfn\log(F)+1}{b^2fn^2\log(F)}}\left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2(abfn\log(F)+1)}{n}}\int \exp\left(\frac{\left(f\log(F)\log(c(d+ex)^n)b^2+af\log(F)b+\frac{1}{n}\right)^2}{b^2f\log(F)}\right)d\log\ldots}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}g(d+ex)^2e^{-\frac{2abfn\log(F)+1}{b^2fn^2\log(F)}}\left(c(d+ex)^n\right)^{2abf\log(F)-\frac{2(abfn\log(F)+1)}{n}}\operatorname{erfi}\left(\frac{abf\log(F)+b^2f\log(F)\log(c(d+ex)^n)+\frac{1}{n}}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(d*g + e*g*x),x]
```

output
```
(g*Sqrt[Pi]*(d + e*x)^2*(c*(d + e*x)^n)^(2*a*b*f*Log[F] - (2*(1 + a*b*f*n*
Log[F]))/n)*Erfi[(n^(-1) + a*b*f*Log[F] + b^2*f*Log[F]*Log[c*(d + e*x)^n])
/(b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*e*E^((1 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Lo
g[F]))*Sqrt[f]*n*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int F^{f(a+b\ln(c(ex+d)^n))^2}(egx+dg)\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(e*g*x+d*g),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(e*g*x+d*g),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}g\,\mathrm{erf}\left(\frac{(b^2fn^2\log(ex+d)\log(F)+b^2fn\log(F)\log(c)+abfn\log(F)+1)\sqrt{-b^2fn^2\log(F)}}{b^2fn^2\log(F)}\right)e^{\left(-\frac{2b^2fn\log(F)\cdots}{b^2\cdots}\right)}}{2ben}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g),x, algorithm="fricas"
)
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*g*erf((b^2*f*n^2*log(e*x + d)*log(F)
 + b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log(F))/(b^
2*f*n^2*log(F)))*e^(-(2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)/(b^2
*f*n^2*log(F)))/(b*e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. 2(116) = 232.

Time = 68.68 (sec) , antiderivative size = 724, normalized size of antiderivative = 5.93

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx = \text{Too large to display}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(e*g*x+d*g),x)
```

output

```
Piecewise((F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*
x)**n)**2)*a*b*d**2*f*g*n*log(F)/e - F**(a**2*f + 2*a*b*f*log(c*(d + e*x)*
*n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*d*f*g*n*x*log(F) - F**(a**2*f + 2
*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*e*f*g*n*x*
*2*log(F)/2 - F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d +
 e*x)**n)**2)*b**2*d**2*f*g*n**2*log(F)/(2*e) - F**(a**2*f + 2*a*b*f*log(c
*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d**2*f*g*n*log(F)*log
(c*(d + e*x)**n)/(2*e) + F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f
*log(c*(d + e*x)**n)**2)*b**2*d*f*g*n**2*x*log(F)/2 - F**(a**2*f + 2*a*b*f
*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d*f*g*n*x*log(F
)*log(c*(d + e*x)**n) + F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*
log(c*(d + e*x)**n)**2)*b**2*e*f*g*n**2*x**2*log(F)/4 - F**(a**2*f + 2*a*b
*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*e*f*g*n*x**2*
log(F)*log(c*(d + e*x)**n)/2 + F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) +
b**2*f*log(c*(d + e*x)**n)**2)*d**2*g/(2*e) + F**(a**2*f + 2*a*b*f*log(c*(
d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*d*g*x + F**(a**2*f + 2*a*b*f
*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*e*g*x**2/2, Ne(e, 0)
), (F**(f*(a + b*log(c*d**n))**2)*d*g*x, True))
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx=\int (egx+dg)F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input

```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g),x, algorithm="maxima"
)
```

output

```
integrate((e*g*x + d*g)*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx=\int (egx+dg)F^{(b\log((ex+d)^nc)+a)^2f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g),x, algorithm="giac")
```

output
```
integrate((e*g*x + d*g)*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx=\int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(d\,g+e\,g\,x)\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(d*g + e*g*x),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(d*g + e*g*x), x)
```

Reduce [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(dg+egx)\,dx=f^{a^2f}g\left(\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}dx\right)d+\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}xdx\right)e\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(e*g*x+d*g),x)
```

output
```
f**(a**2*f)*g*(int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*
c)*a*b*f),x)*d + int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**
n*c)*a*b*f)*x,x)*e)
```

3.528 $\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*Pi^(1/2)*(e*x+d)*erfi(1/2*(1/n+2*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d
)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e/exp(1/4*(1+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(
F))/f^(1/2)/n/((c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{1+2bfn\log(F)(a+b\log(c(d+ex)^n))}{2b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(
2*b*Sqrt[f]*n*Sqrt[Log[F]])])/(2*b*e*E^((1 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^
2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2710, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}dx$$

$$\Big\downarrow \text{2710}$$

$$(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}\left(c(d+ex)^n\right)^{\frac{1}{n}}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)+\left(2abf\log(F)+\frac{1}{n}\right)\log\left(c(d+ex)^n\right.\right.}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}e^{-\frac{4abfn\log(F)+1}{4b^2fn^2\log(F)}}\int \exp\left(\frac{\left(2f\log(F)\log(c(d+ex)^n)b^2+2af\log(F)b+\frac{1}{n}\right)^2}{4b^2f\log(F)}\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}e^{-\frac{4abfn\log(F)+1}{4b^2fn^2\log(F)}}\text{erfi}\left(\frac{2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log[c*(
d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*e*E^((1 + 4*a*b*f*n*Log[F])
/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int F^{f(a+b\ln(c(ex+d)^n))^2}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\operatorname{erf}\left(\frac{(2\,b^2fn^2\log(ex+d)\log(F)+2\,b^2fn\log(F)\log(c)+2\,abfn\log(F)+1)\sqrt{-b^2fn^2\log(F)}}{2\,b^2fn^2\log(F)}\right)e^{\left(-\frac{4\,b^2fn\log(}{}\right.}}{2\,ben}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*lo
g(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log
(F))/(b^2*f*n^2*log(F)))*e^(-1/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(
F) + 1)/(b^2*f*n^2*log(F)))/(b*e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(114) = 228.

Time = 12.75 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.63

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx$$
$$= \begin{cases} \frac{2F^{a^2f+2abf\log(c(d+ex)^n)+b^2f\log(c(d+ex)^n)^2}abdfn\log(F)}{e} - 2F^{a^2f+2abf\log(c(d+ex)^n)+b^2f\log(c(d+ex)^n)^2}abfnx\log(F) - \\ F^{f(a+b\log(cd^n))^2}x \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2),x)
```

output
```
Piecewise((2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d +
e*x)**n)**2)*a*b*d*f*n*log(F)/e - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**
n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*f*n*x*log(F) - 2*F**(a**2*f + 2*a*
b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d*f*n**2*log
(F)/e - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x
)**n)**2)*b**2*d*f*n*log(F)*log(c*(d + e*x)**n)/e + 2*F**(a**2*f + 2*a*b*f
*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*f*n**2*x*log(F)
 - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)
**2)*b**2*f*n*x*log(F)*log(c*(d + e*x)**n) + F**(a**2*f + 2*a*b*f*log(c*(d
 + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*d/e + F**(a**2*f + 2*a*b*f*lo
g(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*x, Ne(e, 0)), (F**(f*(a
 + b*log(c*d**n))**2)*x, True))
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx=\int F^{(b\log((ex+d)^nc)+a)^2f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2),x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx=$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-f\log(F)}bn\log(ex+d)-\sqrt{-f\log(F)}b\log(c)-\sqrt{-f\log(F)}a-\frac{\sqrt{-f\log(F)}}{2\,bfn\log(F)}\right)e^{\left(-\frac{a}{bn}-\right.}}{2\sqrt{-f\log(F)}bc^{\left(\frac{1}{n}\right)}en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-sqrt(-f*log(F))*b*n*log(e*x + d) - sqrt(-f*log(F))*b*lo
g(c) - sqrt(-f*log(F))*a - 1/2*sqrt(-f*log(F))/(b*f*n*log(F)))*e^(-a/(b*n)
 - 1/4/(b^2*f*n^2*log(F)))/(sqrt(-f*log(F))*b*c^(1/n)*e*n)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = \int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2), x)
```

Reduce [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = f^{a^2f}\left(\int f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2),x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f),x)
```

3.529 $\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 70

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx=\frac{\sqrt{\pi}\text{erfi}\left(a\sqrt{f}\sqrt{\log(F)}+b\sqrt{f}\sqrt{\log(F)}\log\left(c(d+ex)^n\right)\right)}{2be\sqrt{f}gn\sqrt{\log(F)}}$$

output
```
1/2*Pi^(1/2)*erfi(a*f^(1/2)*ln(F)^(1/2)+b*f^(1/2)*ln(F)^(1/2)*ln(c*(e*x+d)
^n))/b/e/f^(1/2)/g/n/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx=\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{f}\sqrt{\log(F)}(a+b\log\left(c(d+ex)^n\right))\right)}{2be\sqrt{f}gn\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(d*g + e*g*x),x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[f]*Sqrt[Log[F]]*(a + b*Log[c*(d + e*x)^n])])/(2*b*e*Sq
rt[f]*g*n*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2712, 2706, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}dx$$

$$\Big\downarrow \text{2712}$$

$$\frac{(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)-1}dx}{g}$$

$$\Big\downarrow \text{2706}$$

$$\frac{\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}d\log\left(c(d+ex)^n\right)}{egn}$$

$$\Big\downarrow \text{2664}$$

$$\frac{\int F^{f(a+b\log(c(d+ex)^n))^2}d\log\left(c(d+ex)^n\right)}{egn}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}\text{erfi}\left(\sqrt{f}\sqrt{\log(F)}(a+b\log\left(c(d+ex)^n\right))\right)}{2be\sqrt{f}gn\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(d*g + e*g*x),x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[f]*Sqrt[Log[F]]*(a + b*Log[c*(d + e*x)^n])])/(2*b*e*Sq
rt[f]*g*n*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.94 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.79

method	result
risch	$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-b\sqrt{-f\ln(F)}\,\ln((ex+d)^{n})+\frac{f\left(a+b\left(\ln(c)-\frac{i\pi\ \mathrm{csgn}(ic(ex+d)^{n})(-\mathrm{csgn}(ic(ex+d)^{n})+\mathrm{csgn}(ic))(-\mathrm{csgn}(ic(ex+d)^{n})+\mathrm{csgn}(i(ex+}{2}}{\sqrt{-f\ln(F)}}}{2genb\sqrt{-f\ln(F)}}$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g),x,method=_RETURNVERBOSE)
```

output
```
-1/2/g/e/n*Pi^(1/2)/b/(-f*ln(F))^(1/2)*erf(-b*(-f*ln(F))^(1/2)*ln((e*x+d)^
n)+f*(a+b*(ln(c)-1/2*I*Pi*csgn(I*c*(e*x+d)^n)*(-csgn(I*c*(e*x+d)^n)+csgn(I
*c))*(-csgn(I*c*(e*x+d)^n)+csgn(I*(e*x+d)^n))))*ln(F)/(-f*ln(F))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx = -\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\operatorname{erf}\left(\frac{\sqrt{-b^2fn^2\log(F)}(bn\log(ex+d)+b\log(c)+a)}{bn}\right)}{2\,begn}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x, algorithm="fricas"
)
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf(sqrt(-b^2*f*n^2*log(F))*(b*n*log
(e*x + d) + b*log(c) + a)/(b*n))/(b*e*g*n)
```

Sympy [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx = \frac{\int \frac{F^{a^2f+2abf\log(c(d+ex)^n)+b^2f\log(c(d+ex)^n)^2}}{d+ex}\,dx}{g}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(e*g*x+d*g),x)
```

output
```
Integral(F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)
**n)**2)/(d + e*x), x)/g
```

Maxima [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx=\int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{egx+dg}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x, algorithm="maxima"
)
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g), x)
```

Giac [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx=\int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{egx+dg}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx=-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{1\mathrm{i}\,f\,\ln(F)\,\ln(c\,(d+e\,x)^n)\,b^2+1\mathrm{i}\,a\,f\,\ln(F)\,b}{\sqrt{b^2\,f\,\ln(F)}}\right)\,1\mathrm{i}}{2\,e\,g\,n\,\sqrt{b^2\,f\,\ln(F)}}$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)/(d*g + e*g*x),x)
```

output
```
-(pi^(1/2)*erf((b^2*f*log(F)*log(c*(d + e*x)^n)*1i + a*b*f*log(F)*1i)/(b^2
*f*log(F))^(1/2))*1i)/(2*e*g*n*(b^2*f*log(F))^(1/2))
```

Reduce [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{dg+egx}\,dx = \frac{f^{a^2f}\left(\int \frac{f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}}{ex+d}dx\right)}{g}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g),x)
```

output
```
(f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)
*a*b*f)/(d + e*x),x))/g
```

3.530 $\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 128

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = -\frac{e^{\frac{a}{bn}-\frac{1}{4b^2fn^2\log(F)}}\sqrt{\pi}\,(c(d+ex)^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{\frac{1}{n}-2abf\log(F)-2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n(d+ex)\sqrt{\log(F)}}$$

output
```
-1/2*exp(a/b/n-1/4/b^2/f/n^2/ln(F))*Pi^(1/2)*(c*(e*x+d)^n)^(1/n)*erfi(1/2*
(1/n-2*a*b*f*ln(F)-2*b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b
/e/f^(1/2)/g^2/n/(e*x+d)/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = \frac{e^{\frac{-1+4abfn\log(F)}{4b^2fn^2\log(F)}}\sqrt{\pi}\,(c(d+ex)^n)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{-1+2bfn\log(F)(a+b\log(c(d+ex)^n))}{2b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n(d+ex)\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(d*g + e*g*x)^2,x]
```

output
```
(E^((-1 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*Sqrt[Pi]*(c*(d + e*x)^n)
^n^(-1)*Erfi[(-1 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b*Sqrt[f]
*n*Sqrt[Log[F]])])/(2*b*e*Sqrt[f]*g^2*n*(d + e*x)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2712, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}dx$$

$$\Big\downarrow \text{2712}$$

$$\frac{(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)-2}dx}{g^2}$$

$$\Big\downarrow \text{2706}$$

$$\frac{\left(c(d+ex)^n\right)^{\frac{1}{n}}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}\left(c(d+ex)^n\right)^{-1/n}d\log\left(c(d+ex)^n\right)}{eg^2n(d+ex)}$$

$$\Big\downarrow \text{2725}$$

$$\frac{\left(c(d+ex)^n\right)^{\frac{1}{n}}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)-\frac{(1-2abfn\log(F))\log(c(d+ex)^n)}{n}\right)d\log\left(c(d+ex)^n\right)}{eg^2n(d+ex)}$$

$$\Big\downarrow \text{2664}$$

$$\frac{\left(c(d+ex)^n\right)^{\frac{1}{n}}e^{\frac{a}{bn}-\frac{1}{4b^2fn^2\log(F)}}\int \exp\left(\frac{\left(-2f\log(F)\log(c(d+ex)^n)b^2-2af\log(F)b+\frac{1}{n}\right)^2}{4b^2f\log(F)}\right)d\log\left(c(d+ex)^n\right)}{eg^2n(d+ex)}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\sqrt{\pi}(c(d+ex)^n)^{\frac{1}{n}}e^{\frac{a}{bn}-\frac{1}{4b^2fn^2\log(F)}}\operatorname{erfi}\left(\frac{-2abf\log(F)-2b^2f\log(F)\log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^2n\sqrt{\log(F)}(d+ex)}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(d*g + e*g*x)^2,x]
```

output
```
-1/2*(E^(a/(b*n) - 1/(4*b^2*f*n^2*Log[F]))*Sqrt[Pi]*(c*(d + e*x)^n)^n^(-1)
*Erfi[(n^(-1) - 2*a*b*f*Log[F] - 2*b^2*f*Log[F]*Log[c*(d + e*x)^n])/(2*b*S
qrt[f]*Sqrt[Log[F]])])/(b*e*Sqrt[f]*g^2*n*(d + e*x)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{(egx+dg)^2}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = -\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\,\mathrm{erf}\left(\frac{(2\,b^2fn^2\log(ex+d)\log(F)+2\,b^2fn\log(F)\log(c)+2\,abfn\log(F)-1)\sqrt{-b^2fn^2\log(F)}}{2\,b^2fn^2\log(F)}\right)e^{\left(\frac{4\,b^2fn\log(F}{4}\right.}}{2\,beg^2n}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*lo
g(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) - 1)*sqrt(-b^2*f*n^2*log
(F))/(b^2*f*n^2*log(F)))*e^(1/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(F
) - 1)/(b^2*f*n^2*log(F)))/(b*e*g^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(114) = 228.

Time = 156.04 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.31

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx$$
$$= \begin{cases} -\frac{2F^{a^2f+2abf\log(c(d+ex)^n)+b^2f\log(c(d+ex)^n)^2}abfn\log(F)}{deg^2+e^2g^2x} - \frac{2F^{a^2f+2abf\log(c(d+ex)^n)+b^2f\log(c(d+ex)^n)^2}b^2fn^2\log(F)}{deg^2+e^2g^2x} - \frac{2F^{a^2f+2a}}{} \\ \frac{F^{f(a+b\log(cd^n))^2}x}{d^2g^2} \end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(e*g*x+d*g)**2,x)
```

output
```
Piecewise((-2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d +
 e*x)**n)**2)*a*b*f*n*log(F)/(d*e*g**2 + e**2*g**2*x) - 2*F**(a**2*f + 2*a
*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*f*n**2*log(
F)/(d*e*g**2 + e**2*g**2*x) - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) +
 b**2*f*log(c*(d + e*x)**n)**2)*b**2*f*n*log(F)*log(c*(d + e*x)**n)/(d*e*g
**2 + e**2*g**2*x) - F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log
(c*(d + e*x)**n)**2)/(d*e*g**2 + e**2*g**2*x), Ne(e, 0)), (F**(f*(a + b*lo
g(c*d**n))**2)*x/(d**2*g**2), True))
```

Maxima [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = \int \frac{F^{(b\log((ex+d)^nc)+a)^2f}}{(egx+dg)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x, algorithm="maxim
a")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^2, x)
```

Giac [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(egx+dg)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x, algorithm="giac"
)
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = \int \frac{e^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}}{(d\,g+e\,g\,x)^2}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)/(d*g + e*g*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)/(d*g + e*g*x)^2, x)
```

Reduce [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^2}\,dx = \frac{f^{a^2 f}\left(\int \frac{f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}}{e^2x^2+2dex+d^2}dx\right)}{g^2}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^2,x)
```

output
```
(f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)
*a*b*f)/(d**2 + 2*d*e*x + e**2*x**2),x))/g**2
```

3.531 $\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 126

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = -\frac{e^{-\frac{1-2abfn\log(F)}{b^2fn^2\log(F)}}\sqrt{\pi}(c(d+ex)^n)^{2/n}\operatorname{erfi}\left(\frac{\frac{1}{n}-abf\log(F)-b^2f\log(F)\log(c(d+ex)^n)}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3n(d+ex)^2\sqrt{\log(F)}}$$

output
```
-1/2*Pi^(1/2)*(c*(e*x+d)^n)^(2/n)*erfi((1/n-a*b*f*ln(F)-b^2*f*ln(F)*ln(c*(
e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e/exp((1-2*a*b*f*n*ln(F))/b^2/f/n^2/ln
(F))/f^(1/2)/g^3/n/(e*x+d)^2/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = \frac{e^{\frac{-1+2abfn\log(F)}{b^2fn^2\log(F)}}\sqrt{\pi}(c(d+ex)^n)^{2/n}\operatorname{erfi}\left(\frac{-1+bfn\log(F)(a+b\log(c(d+ex)^n))}{b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3n(d+ex)^2\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(d*g + e*g*x)^3,x]
```

output
```
(E^((-1 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[Pi]*(c*(d + e*x)^n)^(
2/n)*Erfi[(-1 + b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(b*Sqrt[f]*n*Sqrt
[Log[F]])])/(2*b*e*Sqrt[f]*g^3*n*(d + e*x)^2*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {2712, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}dx$$

$$\Big\downarrow \text{2712}$$

$$\frac{(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)-3}dx}{g^3}$$

$$\Big\downarrow \text{2706}$$

$$\frac{\left(c(d+ex)^n\right)^{2\left(\frac{1}{n}-abf\log(F)\right)+2abf\log(F)}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}\left(c(d+ex)^n\right)^{-2/n}d\log\left(c(d+ex)^n\right)\cdots}{eg^3n(d+ex)^2}$$

$$\Big\downarrow \text{2725}$$

$$\frac{\left(c(d+ex)^n\right)^{2\left(\frac{1}{n}-abf\log(F)\right)+2abf\log(F)}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)-\frac{2(1-abfn\log(F))\log(c(d+e\cdots}{n}\cdots\right.}{eg^3n(d+ex)^2}$$

$$\Big\downarrow \text{2664}$$

$$\frac{e^{-\frac{1-2abfn\log(F)}{b^2fn^2\log(F)}}\left(c(d+ex)^n\right)^{2\left(\frac{1}{n}-abf\log(F)\right)+2abf\log(F)}\int \exp\left(\frac{\left(-f\log(F)\log(c(d+ex)^n)b^2-af\log(F)b+\frac{1}{n}\right)^2}{b^2f\log(F)}\right)d\log\left(c(d+e\cdots\right.}{eg^3n(d+ex)^2}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{\sqrt{\pi}e^{-\frac{1-2abfn\log(F)}{b^2fn^2\log(F)}}(c(d+ex)^n)^{2\left(\frac{1}{n}-abf\log(F)\right)+2abf\log(F)}\operatorname{erfi}\left(\frac{-abf\log(F)+b^2(-f)\log(F)\log(c(d+ex)^n)+\frac{1}{n}}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}g^3n\sqrt{\log(F)}(d+ex)^2}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(d*g + e*g*x)^3,x]
```

output
```
-1/2*(Sqrt[Pi]*(c*(d + e*x)^n)^(2*a*b*f*Log[F] + 2*(n^(-1) - a*b*f*Log[F])
)*Erfi[(n^(-1) - a*b*f*Log[F] - b^2*f*Log[F]*Log[c*(d + e*x)^n])/(b*Sqrt[f
]*Sqrt[Log[F]])])/(b*e*E^((1 - 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[
f]*g^3*n*(d + e*x)^2*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2712
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^m*((c*(d + e*x)^n)^(2
*a*b*f*Log[F])/(d + e*x)^(m + 2*a*b*f*n*Log[F]))*Int[(d + e*x)^(m + 2*a*b*f
*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^2), x], x] /; FreeQ[{F, a, b
, c, d, e, f, g, h, m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{(egx+dg)^3}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx =$$
$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\,\mathrm{erf}\left(\frac{(b^2fn^2\log(ex+d)\log(F)+b^2fn\log(F)\log(c)+abfn\log(F)-1)\sqrt{-b^2fn^2\log(F)}}{b^2fn^2\log(F)}\right)e^{\left(\frac{2b^2fn\log(F)\log(\ldots}{b^2fn^2\ldots}\right.}}{2\,beg^3n}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x, algorithm="frica
s")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf((b^2*f*n^2*log(e*x + d)*log(F) +
 b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) - 1)*sqrt(-b^2*f*n^2*log(F))/(b^2*
f*n^2*log(F)))*e^((2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) - 1)/(b^2*f*
n^2*log(F)))/(b*e*g^3*n)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(e*g*x+d*g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(egx+dg)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x, algorithm="maxim
a")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^3, x)
```

Giac [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(egx+dg)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x, algorithm="giac"
)
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(e*g*x + d*g)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}}{(d\,g+e\,g\,x)^3}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)/(d*g + e*g*x)^3,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)/(d*g + e*g*x)^3, x)
```

Reduce [F]

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(dg+egx)^3}\,dx = \frac{f^{a^2f}\left(\int \frac{f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}}{e^3x^3+3d\,e^2x^2+3d^2ex+d^3}dx\right)}{g^3}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(e*g*x+d*g)^3,x)
```

output
```
(f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)
*a*b*f)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x))/g**3
```

3.532 $\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx=\text{Int}\left(F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m,x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^m,x)
```

Mathematica [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx=\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^m,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^m F^{f(a+b\log(c(d+ex)^n))^2} \, dx$$

$$\Big\downarrow \text{ 2714}$$

$$\int (g+hx)^m F^{f(a+b\log(c(d+ex)^n))^2} dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2714
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n])^2)*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int F^{f(a+b\ln(c(ex+d)^n))^2} (hx+g)^m \, dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^m,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx = \int (hx+g)^m F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^m,x, algorithm="fricas")
```

output
```
integral((h*x + g)^m*F^(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)
^n*c) + a^2*f), x)
```

Sympy [F(-2)]

Exception generated.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(h*x+g)**m,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx=\int (hx+g)^m F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^m,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^m*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx=\int (hx+g)^m F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^m,x, algorithm="giac")
```

output
```
integrate((h*x + g)^m*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx=\int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(g+h\,x)^m\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x)^m,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^m\,dx = f^{a^2f}\left(\int f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}(hx+g)^m\,dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^m,x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f)*(g + h*x)**m,x)
```

3.533 $\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx$

Optimal result

Integrand size = 28, antiderivative size = 535

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx = \frac{3e^{-\frac{1+2abfn\log(F)}{b^2fn^2\log(F)}}h(eg-dh)^2\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+abf\log(F)+b^2f\log(F)\log(c(d+ex)^n)}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^4\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{4(1+abfn\log(F))}{b^2fn^2\log(F)}}h^3\sqrt{\pi}(d+ex)^4\left(c(d+ex)^n\right)^{-4/n}\operatorname{erfi}\left(\frac{\frac{2}{n}+abf\log(F)+b^2f\log(F)\log(c(d+ex)^n)}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^4\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}(eg-dh)^3\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^4\sqrt{f}n\sqrt{\log(F)}} + \frac{3e^{-\frac{3(3+4abfn\log(F))}{4b^2fn^2\log(F)}}h^2(eg-dh)\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\frac{3}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^4\sqrt{f}n\sqrt{\log(F)}}$$

output
```
3/2*h*(-d*h+e*g)^2*Pi^(1/2)*(e*x+d)^2*erfi((1/n+a*b*f*ln(F)+b^2*f*ln(F)*ln
(c*(e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e^4/exp((1+2*a*b*f*n*ln(F))/b^2/f/
n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(2/n))/ln(F)^(1/2)+1/2*h^3*Pi^(1/2)*(e
*x+d)^4*erfi((2/n+a*b*f*ln(F)+b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/ln(F)
^(1/2))/b/e^4/exp(4*(1+a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+
d)^n)^(4/n))/ln(F)^(1/2)+1/2*(-d*h+e*g)^3*Pi^(1/2)*(e*x+d)*erfi(1/2*(1/n+2
*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e^4/e
xp(1/4*(1+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(1/n)
)/ln(F)^(1/2)+3/2*h^2*(-d*h+e*g)*Pi^(1/2)*(e*x+d)^3*erfi(1/2*(3/n+2*a*b*f*
ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e^4/exp(3/4*
(3+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)^(3/n))/ln(F)
^(1/2)
```

Mathematica [A] (verified)

Time = 4.88 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.81

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx$$

$$= \frac{e^{-\frac{4(1+abfn\log(F))}{b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-4/n}\left(3e^{\frac{3+2abfn\log(F)}{b^2fn^2\log(F)}}h(eg-dh)^2(d+ex)\left(c(d+ex)^n\right)^{2/n}\operatorname{erfi}\left(\frac{1+bf}{\ldots}\right.\right.}{\ldots}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^3,x]
```

output
```
(Sqrt[Pi]*(d + e*x)*(3*E^((3 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*h*(e*
g - d*h)^2*(d + e*x)*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + b*f*n*Log[F]*(a + b*L
og[c*(d + e*x)^n]))/(b*Sqrt[f]*n*Sqrt[Log[F]])] + h^3*(d + e*x)^3*Erfi[(2
+ b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(b*Sqrt[f]*n*Sqrt[Log[F]])] + E
^((7 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*(e*g - d*h)*(c*(d + e*x)^n)
^n^(-1)*(E^((2 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*(e*g - d*h)^2*(c*(d
 + e*x)^n)^(2/n)*Erfi[(1 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b
*Sqrt[f]*n*Sqrt[Log[F]])] + 3*h^2*(d + e*x)^2*Erfi[(3 + 2*b*f*n*Log[F]*(a
+ b*Log[c*(d + e*x)^n]))/(2*b*Sqrt[f]*n*Sqrt[Log[F]])])))/(2*b*e^4*E^((4*(
1 + a*b*f*n*Log[F]))/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(4/n)*S
qrt[Log[F]])
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2713, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^3 F^{f(a+b\log(c(d+ex)^n))^2}\,dx$$

$$\Big\downarrow \; 2713$$

$$\frac{\int \Big((eg-dh)^3F^{f(a+b\log(c(d+ex)^n))^2}+h^3(d+ex)^3F^{f(a+b\log(c(d+ex)^n))^2}+3h^2(eg-dh)(d+ex)^2F^{f(a+b\log(c(d+ex)^n}}{e^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{3\sqrt{\pi}h^2(d+ex)^3(eg-dh)(c(d+ex)^n)^{-3/n}\exp\left(-\frac{3(4abfn\log(F)+3)}{4b^2fn^2\log(F)}\right)\text{erfi}\left(\frac{2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2b\sqrt{f}n\sqrt{\log(F)}}+\frac{3\sqrt{\pi}h(d+ex)^2(eg-dh)^2(}{}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^3,x]
```

output
```
((3*h*(e*g - d*h)^2*Sqrt[Pi]*(d + e*x)^2*Erfi[(n^(-1) + a*b*f*Log[F] + b^2
*f*Log[F]*Log[c*(d + e*x)^n])/(b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((1 + 2*a*
b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log
[F]]) + (h^3*Sqrt[Pi]*(d + e*x)^4*Erfi[(2/n + a*b*f*Log[F] + b^2*f*Log[F]*
Log[c*(d + e*x)^n])/(b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((4*(1 + a*b*f*n*Log
[F]))/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(4/n)*Sqrt[Log[F]]) +
((e*g - d*h)^3*Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*f*
Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((1 + 4*a*b
*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[L
og[F]]) + (3*h^2*(e*g - d*h)*Sqrt[Pi]*(d + e*x)^3*Erfi[(3/n + 2*a*b*f*Log[
F] + 2*b^2*f*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*
E^((3*(3 + 4*a*b*f*n*Log[F]))/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)
^n)^(3/n)*Sqrt[Log[F]]))/e^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2713
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/e^(m + 1)   Subst[Int[ExpandI
ntegrand[F^(f*(a + b*Log[c*x^n])^2), (e*g - d*h + h*x)^m, x], x], x, d + e*
x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int F^{f\left(a+b\ln\left(c(ex+d)^{n}\right)\right)^{2}}(hx+g)^{3}\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^3,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.05

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx = \text{Too large to display}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*h^3*erf((b^2*f*n^2*log(e*x + d)*log
(F) + b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 2)*sqrt(-b^2*f*n^2*log(F))/
(b^2*f*n^2*log(F)))*e^(-4*(b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 1)/(b^
2*f*n^2*log(F))) + 3*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*(e*g*h^2 - d*h^3)*er
f(1/2*(2*b^2*f*n^2*log(e*x + d)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f
*n*log(F) + 3)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-3/4*(4*b^2*
f*n*log(F)*log(c) + 4*a*b*f*n*log(F) + 3)/(b^2*f*n^2*log(F))) + sqrt(pi)*(
e^3*g^3 - 3*d*e^2*g^2*h + 3*d^2*e*g*h^2 - d^3*h^3)*sqrt(-b^2*f*n^2*log(F))
*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*
b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-1/4*(4*b
^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))) + 3*sqrt(
pi)*sqrt(-b^2*f*n^2*log(F))*(e^2*g^2*h - 2*d*e*g*h^2 + d^2*h^3)*erf((b^2*f
*n^2*log(e*x + d)*log(F) + b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 1)*sqr
t(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-(2*b^2*f*n*log(F)*log(c) + 2*
a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))))/(b*e^4*n)
```

Sympy [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx = \int (hx+g)^3 F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^3*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx=\int (hx+g)^3F^{(b\log((ex+d)^nc)+a)^2f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate((h*x + g)^3*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx=\int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(g+h\,x)^3\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x)^3,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x)^3, x)
```

Reduce [F]

$$\begin{aligned}\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^3\,dx=f^{a^2f}\Bigg(\left(\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}dx\right)g^3\right.\\+\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}x^3dx\right)h^3\\+3\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}x^2dx\right)g\,h^2\\+3\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}xdx\right)g^2h\Bigg)\end{aligned}$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^3,x)
```

output
```
f**(a**2*f)*(int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)
*a*b*f),x)*g**3 + int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)*
*n*c)*a*b*f)*x**3,x)*h**3 + 3*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*lo
g((d + e*x)**n*c)*a*b*f)*x**2,x)*g*h**2 + 3*int(f**(log((d + e*x)**n*c)**2
*b**2*f + 2*log((d + e*x)**n*c)*a*b*f)*x,x)*g**2*h)
```

3.534 $\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx$

Optimal result

Integrand size = 28, antiderivative size = 397

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx = \frac{e^{-\frac{1+2abfn\log(F)}{b^2fn^2\log(F)}}h(eg-dh)\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+abf\log(F)+b^2f\log(F)\log(c(d+ex)^n)}{b\sqrt{f}\sqrt{\log(F)}}\right)}{be^3\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}(eg-dh)^2\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^3\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{3(3+4abfn\log(F))}{4b^2fn^2\log(F)}}h^2\sqrt{\pi}(d+ex)^3\left(c(d+ex)^n\right)^{-3/n}\operatorname{erfi}\left(\frac{\frac{3}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^3\sqrt{f}n\sqrt{\log(F)}}$$

output
```
h*(-d*h+e*g)*Pi^(1/2)*(e*x+d)^2*erfi((1/n+a*b*f*ln(F)+b^2*f*ln(F)*ln(c*(e*
x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e^3/exp((1+2*a*b*f*n*ln(F))/b^2/f/n^2/ln
(F))/f^(1/2)/n/((c*(e*x+d)^n)^(2/n))/ln(F)^(1/2)+1/2*(-d*h+e*g)^2*Pi^(1/2)
*(e*x+d)*erfi(1/2*(1/n+2*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1
/2)/ln(F)^(1/2))/b/e^3/exp(1/4*(1+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1/2
)/n/((c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)+1/2*h^2*Pi^(1/2)*(e*x+d)^3*erfi(1/2*
(3/n+2*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/ln(F)^(1/2))/b
/e^3/exp(3/4*(3+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1/2)/n/((c*(e*x+d)^n)
^(3/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.83

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx$$
$$=\frac{e^{-\frac{3(3+4abfn\log(F))}{4b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-3/n}\left(-2e^{\frac{5+4abfn\log(F)}{4b^2fn^2\log(F)}}h(-eg+dh)(d+ex)\left(c(d+ex)^n\right)^{\frac{1}{n}}\operatorname{erfi}\left(\frac{1-}{}\right.\right.}{}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^2,x]
```

output
```
(Sqrt[Pi]*(d + e*x)*(-2*E^((5 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*h*
(-(e*g) + d*h)*(d + e*x)*(c*(d + e*x)^n)^n^(-1)*Erfi[(1 + b*f*n*Log[F]*(a
+ b*Log[c*(d + e*x)^n]))/(b*Sqrt[f]*n*Sqrt[Log[F]])] + E^((2 + 2*a*b*f*n*L
og[F])/(b^2*f*n^2*Log[F]))*(e*g - d*h)^2*(c*(d + e*x)^n)^(2/n)*Erfi[(1 + 2
*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b*Sqrt[f]*n*Sqrt[Log[F]])] +
h^2*(d + e*x)^2*Erfi[(3 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(2*b*
Sqrt[f]*n*Sqrt[Log[F]])]))/(2*b*e^3*E^((3*(3 + 4*a*b*f*n*Log[F]))/(4*b^2*f
*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2713, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)^2F^{f(a+b\log(c(d+ex)^n))^2}dx$$
$$\Big\downarrow \text{2713}$$
$$\frac{\int \left((eg-dh)^2F^{f(a+b\log(c(d+ex)^n))^2}+h^2(d+ex)^2F^{f(a+b\log(c(d+ex)^n))^2}+2h(eg-dh)(d+ex)F^{f(a+b\log(c(d+ex)^n))^2}\right.}{e^3}$$

$\downarrow$ 2009

$$\frac{\sqrt{\pi}h^2(d+ex)^3(c(d+ex)^n)^{-3/n}\exp\left(-\frac{3(4abfn\log(F)+3)}{4b^2fn^2\log(F)}\right)\text{erfi}\left(\frac{2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)+\frac{3}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2b\sqrt{f}n\sqrt{\log(F)}}+\frac{\sqrt{\pi}h(d+ex)^2(eg-dh)(c(d+ex)^n)^{-}}{}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x)^2,x]
```

output
```
((h*(e*g - d*h)*Sqrt[Pi]*(d + e*x)^2*Erfi[(n^(-1) + a*b*f*Log[F] + b^2*f*L
og[F]*Log[c*(d + e*x)^n])/(b*Sqrt[f]*Sqrt[Log[F]])])/(b*E^((1 + 2*a*b*f*n*
Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]])
 + ((e*g - d*h)^2*Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*
f*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((1 + 4*a
*b*f*n*Log[F])/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt
[Log[F]]) + (h^2*Sqrt[Pi]*(d + e*x)^3*Erfi[(3/n + 2*a*b*f*Log[F] + 2*b^2*f
*Log[F]*Log[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((3*(3 + 4
*a*b*f*n*Log[F]))/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(3/n)*Sq
rt[Log[F]]))/e^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2713
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/e^(m + 1)   Subst[Int[ExpandI
ntegrand[F^(f*(a + b*Log[c*x^n])^2), (e*g - d*h + h*x)^m, x], x], x, d + e*
x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int F^{f(a+b\ln(c(ex+d)^n))^2}(hx+g)^2\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.03

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}h^2\operatorname{erf}\left(\frac{(2b^2fn^2\log(ex+d)\log(F)+2b^2fn\log(F)\log(c)+2abfn\log(F)+3)\sqrt{-b^2fn^2\log(F)}}{2b^2fn^2\log(F)}\right)e^{\left(-\frac{3(4b^2}{}\right.}}{}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*h^2*erf(1/2*(2*b^2*f*n^2*log(e*x +
d)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 3)*sqrt(-b^2*f*n^
2*log(F))/(b^2*f*n^2*log(F)))*e^(-3/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n
*log(F) + 3)/(b^2*f*n^2*log(F))) + sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*(e^2*g
^2 - 2*d*e*g*h + d^2*h^2)*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*log(F) + 2*b^2
*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*
n^2*log(F)))*e^(-1/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(F) + 1)/(b^2
*f*n^2*log(F))) + 2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*(e*g*h - d*h^2)*erf((
b^2*f*n^2*log(e*x + d)*log(F) + b^2*f*n*log(F)*log(c) + a*b*f*n*log(F) + 1
)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-(2*b^2*f*n*log(F)*log(c)
 + 2*a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))))/(b*e^3*n)
```

Sympy [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx = \int (hx+g)^2F^{(b\log((ex+d)^nc)+a)^2f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate((h*x + g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx = \int (hx+g)^2F^{(b\log((ex+d)^nc)+a)^2f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate((h*x + g)^2*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx=\int e^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(g+h\,x)^2\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x)^2, x)
```

Reduce [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)^2\,dx=f^{a^2f}\left(\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}dx\right)g^2+\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}x^2dx\right)h^2+2\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}xdx\right)gh\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g)^2,x)
```

output
```
f**(a**2*f)*(int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)
*a*b*f),x)*g**2 + int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)*
*n*c)*a*b*f)*x**2,x)*h**2 + 2*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*lo
g((d + e*x)**n*c)*a*b*f)*x,x)*g*h)
```

3.535 $\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx = \frac{e^{-\frac{1+2abfn\log(F)}{b^2fn^2\log(F)}}h\sqrt{\pi}(d+ex)^2\left(c(d+ex)^n\right)^{-2/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+abf\log(F)+b^2f\log(F)\log(c(d+ex)^n)}{b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^2\sqrt{f}n\sqrt{\log(F)}} + \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}(eg-dh)\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\operatorname{erfi}\left(\frac{\frac{1}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be^2\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*h*Pi^(1/2)*(e*x+d)^2*erfi((1/n+a*b*f*ln(F)+b^2*f*ln(F)*ln(c*(e*x+d)^n)
)/b/f^(1/2)/ln(F)^(1/2))/b/e^2/exp((1+2*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^
(1/2)/n/((c*(e*x+d)^n)^(2/n))/ln(F)^(1/2)+1/2*(-d*h+e*g)*Pi^(1/2)*(e*x+d)*
erfi(1/2*(1/n+2*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d)^n))/b/f^(1/2)/ln(F)
^(1/2))/b/e^2/exp(1/4*(1+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(F))/f^(1/2)/n/((c*(
e*x+d)^n)^(1/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.86

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx$$
$$=\frac{e^{-\frac{1+2abfn\log(F)}{b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-2/n}\left(h(d+ex)\text{erfi}\left(\frac{1+bfn\log(F)(a+b\log(c(d+ex)^n))}{b\sqrt{f}n\sqrt{\log(F)}}\right)+e^{\frac{3+4abfn\log(F)}{4b^2fn^2\log(F)}}(eg\right.}{2be^2\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x),x]
```

output
```
(Sqrt[Pi]*(d + e*x)*(h*(d + e*x)*Erfi[(1 + b*f*n*Log[F]*(a + b*Log[c*(d +
e*x)^n]))/(b*Sqrt[f]*n*Sqrt[Log[F]])] + E^((3 + 4*a*b*f*n*Log[F])/(4*b^2*f
*n^2*Log[F]))*(e*g - d*h)*(c*(d + e*x)^n)^n^(-1)*Erfi[(1 + 2*b*f*n*Log[F]*
(a + b*Log[c*(d + e*x)^n]))/(2*b*Sqrt[f]*n*Sqrt[Log[F]])]))/(2*b*e^2*E^((1
 + 2*a*b*f*n*Log[F])/(b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*S
qrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2713, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g+hx)F^{f(a+b\log(c(d+ex)^n))^2}\,dx$$

$$\Big\downarrow \text{2713}$$

$$\frac{\int \left((eg-dh)F^{f(a+b\log(c(d+ex)^n))^2}+h(d+ex)F^{f(a+b\log(c(d+ex)^n))^2}\right)d(d+ex)}{e^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{\pi}(d+ex)(eg-dh)(c(d+ex)^n)^{-1/n}e^{-\frac{4abfn\log(F)+1}{4b^2fn^2\log(F)}}\operatorname{erfi}\left(\frac{2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2b\sqrt{f}n\sqrt{\log(F)}}+\frac{\sqrt{\pi}h(d+ex)^2(c(d+ex)^n)^{-2/n}e^{-\frac{2abfn}{b^2fn}}}{}}{e^2}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)*(g + h*x),x]
```

output
```
((h*Sqrt[Pi]*(d + e*x)^2*Erfi[(n^(-1) + a*b*f*Log[F] + b^2*f*Log[F]*Log[c*
(d + e*x)^n])/(b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((1 + 2*a*b*f*n*Log[F])/(b
^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^(2/n)*Sqrt[Log[F]]) + ((e*g -
d*h)*Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log
[c*(d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*E^((1 + 4*a*b*f*n*Log[F
])/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]]))/e
^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2713
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/e^(m + 1)   Subst[Int[ExpandI
ntegrand[F^(f*(a + b*Log[c*x^n])^2), (e*g - d*h + h*x)^m, x], x], x, d + e*
x], x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x] && IGtQ[m, 0]
```

Maple [F]

$$\int F^{f(a+b\ln(c(ex+d)^n))^2}(hx+g)\,dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)*(h*x+g),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.01

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}(eg-dh)\,\mathrm{erf}\left(\frac{(2\,b^2fn^2\log(ex+d)\log(F)+2\,b^2fn\log(F)\log(c)+2\,abfn\log(F)+1)\sqrt{-b^2fn^2\log(F)}}{2\,b^2fn^2\log(F)}\right)e$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*(e*g - d*h)*erf(1/2*(2*b^2*f*n^2*lo
g(e*x + d)*log(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)*sqrt(-
b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-1/4*(4*b^2*f*n*log(F)*log(c) + 4
*a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))) + sqrt(pi)*sqrt(-b^2*f*n^2*log(F)
)*h*erf((b^2*f*n^2*log(e*x + d)*log(F) + b^2*f*n*log(F)*log(c) + a*b*f*n*l
og(F) + 1)*sqrt(-b^2*f*n^2*log(F))/(b^2*f*n^2*log(F)))*e^(-(2*b^2*f*n*log(
F)*log(c) + 2*a*b*f*n*log(F) + 1)/(b^2*f*n^2*log(F))))/(b*e^2*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1149 vs. 2(243) = 486.

Time = 68.88 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.47

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx = \text{Too large to display}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)*(h*x+g),x)
```

output
```
Piecewise((-F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e
*x)**n)**2)*a*b*d**2*f*h*n*log(F)/e**2 + 2*F**(a**2*f + 2*a*b*f*log(c*(d +
 e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*d*f*g*n*log(F)/e + F**(a**2
*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*d*f*
h*n*x*log(F)/e - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c
*(d + e*x)**n)**2)*a*b*f*g*n*x*log(F) - F**(a**2*f + 2*a*b*f*log(c*(d + e*
x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*f*h*n*x**2*log(F)/2 + 3*F**(a*
*2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d
**2*f*h*n**2*log(F)/(2*e**2) + 3*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n)
+ b**2*f*log(c*(d + e*x)**n)**2)*b**2*d**2*f*h*n*log(F)*log(c*(d + e*x)**n
)/(2*e**2) - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d
+ e*x)**n)**2)*b**2*d*f*g*n**2*log(F)/e - 2*F**(a**2*f + 2*a*b*f*log(c*(d
+ e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d*f*g*n*log(F)*log(c*(d +
 e*x)**n)/e - 3*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d
 + e*x)**n)**2)*b**2*d*f*h*n**2*x*log(F)/(2*e) + F**(a**2*f + 2*a*b*f*log(
c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d*f*h*n*x*log(F)*log
(c*(d + e*x)**n)/e + 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*l
og(c*(d + e*x)**n)**2)*b**2*f*g*n**2*x*log(F) - 2*F**(a**2*f + 2*a*b*f*log
(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*f*g*n*x*log(F)*log(
c*(d + e*x)**n) + F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log...
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx = \int (hx+g)F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g),x, algorithm="maxima")
```

output
```
integrate((h*x + g)*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx = \int (hx+g)F^{(b\log((ex+d)^nc)+a)^2f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g),x, algorithm="giac")
```

output
```
integrate((h*x + g)*F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx = \int \mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}\,(g+h\,x)\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)*(g + h*x), x)
```

Reduce [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}(g+hx)\,dx = f^{a^2f}\left(\left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}dx\right)g + \left(\int f^{\log((ex+d)^nc)^2b^2f+2\log((ex+d)^nc)abf}xdx\right)h\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)*(h*x+g),x)
```

output
```
f**(a**2*f)*(int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)
*a*b*f),x)*g + int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*
c)*a*b*f)*x,x)*h)
```

3.536 $\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 126

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{\frac{1}{n}+2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

output
```
1/2*Pi^(1/2)*(e*x+d)*erfi(1/2*(1/n+2*a*b*f*ln(F)+2*b^2*f*ln(F)*ln(c*(e*x+d
)^n))/b/f^(1/2)/ln(F)^(1/2))/b/e/exp(1/4*(1+4*a*b*f*n*ln(F))/b^2/f/n^2/ln(
F))/f^(1/2)/n/((c*(e*x+d)^n)^(1/n))/ln(F)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.98

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = \frac{e^{-\frac{1+4abfn\log(F)}{4b^2fn^2\log(F)}}\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}\text{erfi}\left(\frac{1+2bfn\log(F)(a+b\log(c(d+ex)^n))}{2b\sqrt{f}n\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[(1 + 2*b*f*n*Log[F]*(a + b*Log[c*(d + e*x)^n]))/(
2*b*Sqrt[f]*n*Sqrt[Log[F]])])/(2*b*e*E^((1 + 4*a*b*f*n*Log[F])/(4*b^2*f*n^
2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2710, 2706, 2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{f(a+b\log(c(d+ex)^n))^2}dx$$

$$\Big\downarrow \text{2710}$$

$$(d+ex)^{-2abfn\log(F)}\left(c(d+ex)^n\right)^{2abf\log(F)}\int F^{fa^2+b^2f\log^2(c(d+ex)^n)}(d+ex)^{2abfn\log(F)}dx$$

$$\Big\downarrow \text{2706}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int F^{fa^2+2bf\log(c(d+ex)^n)a+b^2f\log^2(c(d+ex)^n)}\left(c(d+ex)^n\right)^{\frac{1}{n}}d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2725}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}\int \exp\left(f\log(F)a^2+b^2f\log(F)\log^2\left(c(d+ex)^n\right)+\left(2abf\log(F)+\frac{1}{n}\right)\log\left(c(d+ex)^n\right)\right.}{en}$$

$$\Big\downarrow \text{2664}$$

$$\frac{(d+ex)\left(c(d+ex)^n\right)^{-1/n}e^{-\frac{4abfn\log(F)+1}{4b^2fn^2\log(F)}}\int \exp\left(\frac{\left(2f\log(F)\log(c(d+ex)^n)b^2+2af\log(F)b+\frac{1}{n}\right)^2}{4b^2f\log(F)}\right)d\log\left(c(d+ex)^n\right)}{en}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{\pi}(d+ex)\left(c(d+ex)^n\right)^{-1/n}e^{-\frac{4abfn\log(F)+1}{4b^2fn^2\log(F)}}\text{erfi}\left(\frac{2abf\log(F)+2b^2f\log(F)\log(c(d+ex)^n)+\frac{1}{n}}{2b\sqrt{f}\sqrt{\log(F)}}\right)}{2be\sqrt{f}n\sqrt{\log(F)}}$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2),x]
```

output
```
(Sqrt[Pi]*(d + e*x)*Erfi[(n^(-1) + 2*a*b*f*Log[F] + 2*b^2*f*Log[F]*Log[c*(
d + e*x)^n])/(2*b*Sqrt[f]*Sqrt[Log[F]])])/(2*b*e*E^((1 + 4*a*b*f*n*Log[F])
/(4*b^2*f*n^2*Log[F]))*Sqrt[f]*n*(c*(d + e*x)^n)^n^(-1)*Sqrt[Log[F]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2706
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]^2*(b_.))*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[(g + h*x)^(m + 1)/(h*n*(c*(d +
e*x)^n)^((m + 1)/n))   Subst[Int[E^(a*f*Log[F] + ((m + 1)*x)/n + b*f*Log[F]
*x^2), x], x, Log[c*(d + e*x)^n]], x] /; FreeQ[{F, a, b, c, d, e, f, g, h,
m, n}, x] && EqQ[e*g - d*h, 0]
```

rule 2710
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.)), x
_Symbol] :> Simp[((c*(d + e*x)^n)^(2*a*b*f*Log[F])/(d + e*x)^(2*a*b*f*n*Log
[F]))*Int[(d + e*x)^(2*a*b*f*n*Log[F])*F^(a^2*f + b^2*f*Log[c*(d + e*x)^n]^
2), x], x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] &&  !IntegerQ[2*a*b*f*Log[
F]]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int F^{f(a+b\ln(c(ex+d)^n))^2}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-b^2fn^2\log(F)}\,\mathrm{erf}\left(\frac{(2\,b^2fn^2\log(ex+d)\log(F)+2\,b^2fn\log(F)\log(c)+2\,abfn\log(F)+1)\sqrt{-b^2fn^2\log(F)}}{2\,b^2fn^2\log(F)}\right)e^{\left(-\frac{4\,b^2fn\log(}{}\right.}}{2\,ben}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-b^2*f*n^2*log(F))*erf(1/2*(2*b^2*f*n^2*log(e*x + d)*lo
g(F) + 2*b^2*f*n*log(F)*log(c) + 2*a*b*f*n*log(F) + 1)*sqrt(-b^2*f*n^2*log
(F))/(b^2*f*n^2*log(F)))*e^(-1/4*(4*b^2*f*n*log(F)*log(c) + 4*a*b*f*n*log(
F) + 1)/(b^2*f*n^2*log(F)))/(b*e*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(114) = 228.

Time = 13.06 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.63

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx$$

$$=\begin{cases}\frac{2F^{a^2f+2abf\log\left(c(d+ex)^n\right)+b^2f\log\left(c(d+ex)^n\right)^2}abdfn\log\left(F\right)}{e}-2F^{a^2f+2abf\log\left(c(d+ex)^n\right)+b^2f\log\left(c(d+ex)^n\right)^2}abfnx\log\left(F\right)-\\ F^{f(a+b\log\left(cd^n\right))^2}x\end{cases}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2),x)
```

output
```
Piecewise((2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d +
e*x)**n)**2)*a*b*d*f*n*log(F)/e - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**
n) + b**2*f*log(c*(d + e*x)**n)**2)*a*b*f*n*x*log(F) - 2*F**(a**2*f + 2*a*
b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*d*f*n**2*log
(F)/e - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x
)**n)**2)*b**2*d*f*n*log(F)*log(c*(d + e*x)**n)/e + 2*F**(a**2*f + 2*a*b*f
*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*b**2*f*n**2*x*log(F)
 - 2*F**(a**2*f + 2*a*b*f*log(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)
**2)*b**2*f*n*x*log(F)*log(c*(d + e*x)**n) + F**(a**2*f + 2*a*b*f*log(c*(d
 + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*d/e + F**(a**2*f + 2*a*b*f*lo
g(c*(d + e*x)**n) + b**2*f*log(c*(d + e*x)**n)**2)*x, Ne(e, 0)), (F**(f*(a
 + b*log(c*d**n))**2)*x, True))
```

Maxima [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx = \int F^{(b\log((ex+d)^n c)+a)^2 f}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2),x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int F^{f(a+b\log(c(d+ex)^n))^2}\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-f\log(F)}bn\log(ex+d)-\sqrt{-f\log(F)}b\log(c)-\sqrt{-f\log(F)}a-\frac{\sqrt{-f\log(F)}}{2\,bfn\log(F)}\right)e^{\left(-\frac{a}{bn}-\right.}}{2\sqrt{-f\log(F)}bc^{\left(\frac{1}{n}\right)}en}$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-sqrt(-f*log(F))*b*n*log(e*x + d) - sqrt(-f*log(F))*b*lo
g(c) - sqrt(-f*log(F))*a - 1/2*sqrt(-f*log(F))/(b*f*n*log(F)))*e^(-a/(b*n)
 - 1/4/(b^2*f*n^2*log(F)))/(sqrt(-f*log(F))*b*c^(1/n)*e*n)
```

Mupad [F(-1)]

Timed out.

$$\int F^{f(a+b\log(c(d+ex)^n))^2} dx = \int \mathrm{e}^{f\ \ln(F)\ (a+b\ \ln(c\ (d+e\ x)^n))^2} dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2), x)
```

Reduce [F]

$$\int F^{f(a+b\log(c(d+ex)^n))^2} dx = f^{a^2f}\left(\int f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf} dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2),x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f),x)
```

3.537 $\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \text{Int}\left(\frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx},x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g),x)
```

Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x),x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx$$

$$\Big\downarrow \; 2714$$

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2714
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n])^2)*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{hx+g}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g),x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{hx+g}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g),x, algorithm="fricas")
```

output
```
integral(F^(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*
f)/(h*x + g), x)
```

Sympy [N/A]

Not integrable

Time = 15.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \int \frac{F^{f(a+b\log\left(c(d+ex)^n\right))^2}}{g+hx}\,dx$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(h*x+g),x)
```

output
```
Integral(F**(f*(a + b*log(c*(d + e*x)**n))**2)/(g + h*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{hx+g}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g),x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{hx+g}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g),x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g), x)
```

Mupad [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}}{g+h\,x}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)/(g + h*x),x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)/(g + h*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{g+hx}\,dx = f^{a^2 f}\left(\int \frac{f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}}{hx+g}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g),x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f)/(g + h*x),x)
```

3.538 $\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2} dx = \text{Int}\left(\frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}, x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 8.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2} dx = \int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2} dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x)^2,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2} \, dx$$
$$\Big\downarrow \text{2714}$$
$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2714
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n])^2)*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{(hx+g)^2}dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g)^2,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x, algorithm="fricas")
```

output
```
integral(F^(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*
f)/(h^2*x^2 + 2*g*h*x + g^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(h*x+g)**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(hx+g)^2}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}\,dx = \int \frac{e^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}}{(g+h\,x)^2}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)/(g + h*x)^2,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)/(g + h*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.29

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^2}\,dx = f^{a^2f}\left(\int \frac{f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}}{h^2x^2+2ghx+g^2}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^2,x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f)/(g**2 + 2*g*h*x + h**2*x**2),x)
```

3.539 $\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3} dx$

Optimal result

Integrand size = 28, antiderivative size = 28

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3} dx = \text{Int}\left(\frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}, x\right)$$

output
```
Defer(Int)(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 8.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3} dx = \int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3} dx$$

input
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x)^3,x]
```

output
```
Integrate[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2714}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3} \, dx$$
$$\Big\downarrow \text{2714}$$
$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3} dx$$

input
```
Int[F^(f*(a + b*Log[c*(d + e*x)^n])^2)/(g + h*x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2714
```
Int[(F_)^(((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^2*(f_.))*((
g_.) + (h_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[F^(f*(a + b*Log[c*(d +
e*x)^n])^2)*(g + h*x)^m, x] /; FreeQ[{F, a, b, c, d, e, f, g, h, m, n}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{f(a+b\ln(c(ex+d)^n))^2}}{(hx+g)^3} dx$$

input
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g)^3,x)
```

output
```
int(F^(f*(a+b*ln(c*(e*x+d)^n))^2)/(h*x+g)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}\,dx = \int \frac{F^{\left(b\log\left((ex+d)^n c\right)+a\right)^2 f}}{\left(hx+g\right)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x, algorithm="fricas")
```

output
```
integral(F^(b^2*f*log((e*x + d)^n*c)^2 + 2*a*b*f*log((e*x + d)^n*c) + a^2*
f)/(h^3*x^3 + 3*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}\,dx = \text{Timed out}$$

input
```
integrate(F**(f*(a+b*ln(c*(e*x+d)**n))**2)/(h*x+g)**3,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x, algorithm="maxima")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}\,dx = \int \frac{F^{(b\log((ex+d)^n c)+a)^2 f}}{(hx+g)^3}\,dx$$

input
```
integrate(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*log((e*x + d)^n*c) + a)^2*f)/(h*x + g)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}\,dx = \int \frac{\mathrm{e}^{f\,\ln(F)\,(a+b\,\ln(c\,(d+e\,x)^n))^2}}{(g+h\,x)^3}\,dx$$

input
```
int(F^(f*(a + b*log(c*(d + e*x)^n))^2)/(g + h*x)^3,x)
```

output
```
int(exp(f*log(F)*(a + b*log(c*(d + e*x)^n))^2)/(g + h*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{F^{f(a+b\log(c(d+ex)^n))^2}}{(g+hx)^3}\,dx = f^{a^2 f}\left(\int \frac{f^{\log((ex+d)^n c)^2 b^2 f+2\log((ex+d)^n c)abf}}{h^3x^3+3g\,h^2x^2+3g^2hx+g^3}dx\right)$$

input
```
int(F^(f*(a+b*log(c*(e*x+d)^n))^2)/(h*x+g)^3,x)
```

output
```
f**(a**2*f)*int(f**(log((d + e*x)**n*c)**2*b**2*f + 2*log((d + e*x)**n*c)*
a*b*f)/(g**3 + 3*g**2*h*x + 3*g*h**2*x**2 + h**3*x**3),x)
```

3.540 $\int F^{a+bx+cx^3}(b+3cx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 17

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{F^{a+bx+cx^3}}{\log(F)}$$

output
```
F^(c*x^3+b*x+a)/ln(F)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{F^{a+bx+cx^3}}{\log(F)}$$

input
```
Integrate[F^(a + b*x + c*x^3)*(b + 3*c*x^2),x]
```

output
```
F^(a + b*x + c*x^3)/Log[F]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {7257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b+3cx^2\right) F^{a+bx+cx^3} \, dx$$

$$\Big\downarrow \text{ 7257}$$

$$\frac{F^{a+bx+cx^3}}{\log(F)}$$

input
```
Int[F^(a + b*x + c*x^3)*(b + 3*c*x^2),x]
```

output
```
F^(a + b*x + c*x^3)/Log[F]
```

Defintions of rubi rules used

rule 7257
```
Int[(F_)^(v_)*(u_), x_Symbol] :> With[{q = DerivativeDivides[v, u, x]}, Sim
p[q*(F^v/Log[F]), x] /;  !FalseQ[q]] /; FreeQ[F, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{F^{c\,x^{3}+bx+a}}{\ln(F)}$	18
derivativedivides	$\frac{F^{c\,x^{3}+bx+a}}{\ln(F)}$	18
default	$\frac{F^{c\,x^{3}+bx+a}}{\ln(F)}$	18
risch	$\frac{F^{c\,x^{3}+bx+a}}{\ln(F)}$	18
parallelrisch	$\frac{F^{c\,x^{3}+bx+a}}{\ln(F)}$	18
orering	$\frac{F^{c\,x^{3}+bx+a}}{\ln(F)}$	18
norman	$\frac{{\mathrm e}^{\left(c\,x^{3}+bx+a\right)\ln(F)}}{\ln(F)}$	20

input
```
int(F^(c*x^3+b*x+a)*(3*c*x^2+b),x,method=_RETURNVERBOSE)
```

output
```
F^(c*x^3+b*x+a)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{F^{cx^3+bx+a}}{\log\left(F\right)}$$

input
```
integrate(F^(c*x^3+b*x+a)*(3*c*x^2+b),x, algorithm="fricas")
```

output
```
F^(c*x^3 + b*x + a)/log(F)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\begin{cases}\frac{F^{a+bx+cx^3}}{\log\left(F\right)} & \text{for } \log\left(F\right)\neq 0\\ bx+cx^3 & \text{otherwise}\end{cases}$$

input
```
integrate(F**(c*x**3+b*x+a)*(3*c*x**2+b),x)
```

output
```
Piecewise((F**(a + b*x + c*x**3)/log(F), Ne(log(F), 0)), (b*x + c*x**3, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{F^{cx^3+bx+a}}{\log\left(F\right)}$$

input
```
integrate(F^(c*x^3+b*x+a)*(3*c*x^2+b),x, algorithm="maxima")
```

output
```
F^(c*x^3 + b*x + a)/log(F)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{F^{cx^3+bx+a}}{\log\left(F\right)}$$

input
```
integrate(F^(c*x^3+b*x+a)*(3*c*x^2+b),x, algorithm="giac")
```

output
```
F^(c*x^3 + b*x + a)/log(F)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{F^{c\,x^3+b\,x+a}}{\ln\left(F\right)}$$

input
```
int(F^(a + b*x + c*x^3)*(b + 3*c*x^2),x)
```

output
```
F^(a + b*x + c*x^3)/log(F)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx+cx^3}\left(b+3cx^2\right)\,dx=\frac{f^{c\,x^3+bx+a}}{\log\left(f\right)}$$

input
```
int(F^(c*x^3+b*x+a)*(3*c*x^2+b),x)
```

output
```
f**(a + b*x + c*x**3)/log(f)
```

3.541 $\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 20

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{F^{\frac{1}{a+bx+cx^2}}}{\log(F)}$$

output
```
-F^(1/(c*x^2+b*x+a))/ln(F)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{F^{\frac{1}{a+x(b+cx)}}}{\log(F)}$$

input
```
Integrate[(F^(a + b*x + c*x^2)^(-1)*(b + 2*c*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-(F^(a + x*(b + c*x))^(-1)/Log[F])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {7257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)F^{\frac{1}{a+bx+cx^2}}}{(a+bx+cx^2)^2} \, dx$$
$$\Big\downarrow \; 7257$$
$$-\frac{F^{\frac{1}{a+bx+cx^2}}}{\log(F)}$$

input
```
Int[(F^(a + b*x + c*x^2)^(-1)*(b + 2*c*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-(F^(a + b*x + c*x^2)^(-1)/Log[F])
```

Defintions of rubi rules used

rule 7257
```
Int[(F_)^(v_)*(u_), x_Symbol] :> With[{q = DerivativeDivides[v, u, x]}, Sim
p[q*(F^v/Log[F]), x] /;  !FalseQ[q]] /; FreeQ[F, x]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{F^{\frac{1}{c x^{2}+b x+a}}}{\ln \left(F \right)}$	21
default	$-\frac{F^{\frac{1}{c x^{2}+b x+a}}}{\ln \left(F \right)}$	21
risch	$-\frac{F^{\frac{1}{c x^{2}+b x+a}}}{\ln \left(F \right)}$	21
parallelrisch	$-\frac{F^{\frac{1}{c x^{2}+b x+a}}}{\ln \left(F \right)}$	21
norman	$\frac{-\frac{a\,{\mathrm e}^{\frac{\ln \left(F \right)}{c x^{2}+b x+a}}}{\ln \left(F \right)}-\frac{b x\,{\mathrm e}^{\frac{\ln \left(F \right)}{c x^{2}+b x+a}}}{\ln \left(F \right)}-\frac{c\,x^{2} {\mathrm e}^{\frac{\ln \left(F \right)}{c x^{2}+b x+a}}}{\ln \left(F \right)}}{c x^{2}+b x+a}$	88

input
```
int(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-F^(1/(c*x^2+b*x+a))/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx=-\frac{F^{\left(\frac{1}{cx^2+bx+a}\right)}}{\log(F)}$$

input
```
integrate(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="fric
as")
```

output
```
-F^(1/(c*x^2 + b*x + a))/log(F)
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = \begin{cases} -\frac{F^{\frac{1}{a+bx+cx^2}}}{\log(F)} & \text{for } \log(F) \neq 0 \\ -\frac{1}{a+bx+cx^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(1/(c*x**2+b*x+a))*(2*c*x+b)/(c*x**2+b*x+a)**2,x)
```

output
```
Piecewise((-F**(1/(a + b*x + c*x**2))/log(F), Ne(log(F), 0)), (-1/(a + b*x
 + c*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{F^{\left(\frac{1}{cx^2+bx+a}\right)}}{\log(F)}$$

input
```
integrate(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="maxi
ma")
```

output
```
-F^(1/(c*x^2 + b*x + a))/log(F)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{F^{\left(\frac{1}{cx^2+bx+a}\right)}}{\log(F)}$$

input
```
integrate(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="giac
")
```

output
```
-F^(1/(c*x^2 + b*x + a))/log(F)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{F^{\frac{1}{c\,x^2+b\,x+a}}}{\ln(F)}$$

input
```
int((F^(1/(a + b*x + c*x^2))*(b + 2*c*x))/(a + b*x + c*x^2)^2,x)
```

output
```
-F^(1/(a + b*x + c*x^2))/log(F)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{F^{\frac{1}{a+bx+cx^2}}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{f^{\frac{1}{c\,x^2+bx+a}}}{\log(f)}$$

input
```
int(F^(1/(c*x^2+b*x+a))*(2*c*x+b)/(c*x^2+b*x+a)^2,x)
```

output
```
( - f**(1/(a + b*x + c*x**2)))/log(f)
```

3.542 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx$

Optimal result

Integrand size = 31, antiderivative size = 49

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx=\left(-a-bx-cx^2\right)^{-m}\left(a+bx+cx^2\right)^m\Gamma\left(1+m,-a-bx-cx^2\right)$$

output
```
(c*x^2+b*x+a)^m*GAMMA(1+m,-c*x^2-b*x-a)/((-c*x^2-b*x-a)^m)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx=(-a-x(b+cx))^{-m}(a+x(b+cx))^m\Gamma(1+m,-a-x(b+cx))$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^m,x]
```

output
```
((a + x*(b + c*x))^m*Gamma[1 + m, -a - x*(b + c*x)])/(-a - x*(b + c*x))^m
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {7258, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)^m\,dx$$

$$\Big\downarrow \text{ 7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)^m d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{ 2612}$$

$$\left(-a-bx-cx^2\right)^{-m}\left(a+bx+cx^2\right)^m\Gamma\left(m+1,-cx^2-bx-a\right)$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^m,x]
```

output
```
((a + b*x + c*x^2)^m*Gamma[1 + m, -a - b*x - c*x^2])/(-a - b*x - c*x^2)^m
```

Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [F]

$$\int e^{c\,x^{2}+bx+a}(2cx+b)\left(c\,x^{2}+bx+a\right)^{m}dx$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^m,x)
```

output
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^m,x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx = e^{(-i\,\pi m)}\Gamma\left(m+1,-cx^2-bx-a\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^m,x, algorithm="fricas"
)
```

output
```
e^(-I*pi*m)*gamma(m + 1, -c*x^2 - b*x - a)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx = \text{Timed out}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**m,x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx=\int (2\,cx+b)\big(cx^2+bx+a\big)^m e^{(cx^2+bx+a)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^m,x, algorithm="maxima"
)
```

output
```
integrate((2*c*x + b)*(c*x^2 + b*x + a)^m*e^(c*x^2 + b*x + a), x)
```

Giac [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx=\int (2\,cx+b)\big(cx^2+bx+a\big)^m e^{(cx^2+bx+a)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^m,x, algorithm="giac")
```

output
```
integrate((2*c*x + b)*(c*x^2 + b*x + a)^m*e^(c*x^2 + b*x + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx=\frac{\Gamma(m+1,-c\,x^2-b\,x-a)\,\left(c\,x^2+b\,x+a\right)^m}{\left(-c\,x^2-b\,x-a\right)^m}$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^m,x)
```

output
```
(igamma(m + 1, - a - b*x - c*x^2)*(a + b*x + c*x^2)^m)/(- a - b*x - c*x^2)
^m
```

Reduce [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^m\,dx = e^a\left(e^{c\,x^2+bx}\left(c\,x^2+bx+a\right)^m - 2\left(\int \frac{e^{c\,x^2+bx}\left(c\,x^2+bx+a\right)^m x}{c\,x^2+bx+a}dx\right)cm - \left(\int \frac{e^{c\,x^2+bx}\left(c\,x^2+bx+a\right)^m}{c\,x^2+bx+a}dx\right)bm\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^m,x)
```

output
```
e**a*(e**(b*x + c*x**2)*(a + b*x + c*x**2)**m - 2*int((e**(b*x + c*x**2)*(
a + b*x + c*x**2)**m*x)/(a + b*x + c*x**2),x)*c*m - int((e**(b*x + c*x**2)
*(a + b*x + c*x**2)**m)/(a + b*x + c*x**2),x)*b*m)
```

3.543 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 31, antiderivative size = 90

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx=-6e^{a+bx+cx^2}+6e^{a+bx+cx^2}\left(a+bx+cx^2\right)-3e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2+e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3$$

output
```
-6*exp(c*x^2+b*x+a)+6*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)-3*exp(c*x^2+b*x+a)*(c
*x^2+b*x+a)^2+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^3
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx=e^{a+x(b+cx)}\left(-6+6(a+x(b+cx))-3(a+x(b+cx))^2+(a+x(b+cx))^3\right)$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^3,x]
```

output
```
E^(a + x*(b + c*x))*(-6 + 6*(a + x*(b + c*x)) - 3*(a + x*(b + c*x))^2 + (a
 + x*(b + c*x))^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {7258, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3 d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3-3\int e^{cx^2+bx+a}\left(cx^2+bx+a\right)^2 d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3-3\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2-2\int e^{cx^2+bx+a}\left(cx^2+bx+a\right)d\left(cx^2+bx+a\right)\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3-3\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2-2\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)-\int e^{cx^2+bx+a}d\left(cx^2+bx+a\right)\right)\right)$$

$$\Big\downarrow \text{2624}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^3-3\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2-2\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)-e^{a+bx+cx^2}\right)\right)$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^3,x]
```

output
```
E^(a + b*x + c*x^2)*(a + b*x + c*x^2)^3 - 3*(E^(a + b*x + c*x^2)*(a + b*x
+ c*x^2)^2 - 2*(-E^(a + b*x + c*x^2) + E^(a + b*x + c*x^2)*(a + b*x + c*x^
2)))
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-6\,\mathrm{e}^{c\,x^2+bx+a}+6\,\mathrm{e}^{c\,x^2+bx+a}(c\,x^2+bx+a)-3\,\mathrm{e}^{c\,x^2+bx+a}(c\,x^2+bx+a)^2+\mathrm{e}^{c\,x^2+bx+a}(c$
default	$-6\,\mathrm{e}^{c\,x^2+bx+a}+6\,\mathrm{e}^{c\,x^2+bx+a}(c\,x^2+bx+a)-3\,\mathrm{e}^{c\,x^2+bx+a}(c\,x^2+bx+a)^2+\mathrm{e}^{c\,x^2+bx+a}(c$
gosper	$(c^3x^6+3b\,c^2x^5+3a\,c^2x^4+3b^2c\,x^4+6abc\,x^3+b^3x^3-3c^2x^4+3a^2c\,x^2+3a\,b^2x^2-6b$
risch	$(c^3x^6+3b\,c^2x^5+3a\,c^2x^4+3b^2c\,x^4+6abc\,x^3+b^3x^3-3c^2x^4+3a^2c\,x^2+3a\,b^2x^2-6b$
orering	$(c^3x^6+3b\,c^2x^5+3a\,c^2x^4+3b^2c\,x^4+6abc\,x^3+b^3x^3-3c^2x^4+3a^2c\,x^2+3a\,b^2x^2-6b$
norman	$(a^3-3a^2+6a-6)\,\mathrm{e}^{c\,x^2+bx+a}+c^3x^6\mathrm{e}^{c\,x^2+bx+a}+(3c^2a+3b^2c-3c^2)\,x^4\mathrm{e}^{c\,x^2+bx+a}+(3$
parallelrisch	$c^3x^6\mathrm{e}^{c\,x^2+bx+a}+3b\,c^2x^5\mathrm{e}^{c\,x^2+bx+a}+3x^4\mathrm{e}^{c\,x^2+bx+a}a\,c^2+3x^4\mathrm{e}^{c\,x^2+bx+a}b^2c-3x^4\mathrm{e}^{c\,x^2+bx+}$
parts	Expression too large to display

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-6*exp(c*x^2+b*x+a)+6*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)-3*exp(c*x^2+b*x+a)*(c
*x^2+b*x+a)^2+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.21

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx$$
$$=\left(c^3x^6+3\,bc^2x^5+3\,\left(b^2c+(a-1)c^2\right)x^4+\left(b^3+6\,(a-1)bc\right)x^3+a^3+3\,\left(a^2-2\,a+2\right)bx+3\,\left((a-1)b^2\right.\right.$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^3,x, algorithm="fricas"
)
```

output
```
(c^3*x^6 + 3*b*c^2*x^5 + 3*(b^2*c + (a - 1)*c^2)*x^4 + (b^3 + 6*(a - 1)*b*
c)*x^3 + a^3 + 3*(a^2 - 2*a + 2)*b*x + 3*((a - 1)*b^2 + (a^2 - 2*a + 2)*c)
*x^2 - 3*a^2 + 6*a - 6)*e^(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.78

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx=\left(a^3+3a^2bx+3a^2cx^2-3a^2+3ab^2x^2+6abcx^3-6abx+3ac^2x^4-6acx^2+6a+b^3x^3+3b^2cx^4-3b^2x^2+3bc^2x^5-6bcx^3+6bx+c^3x^6-3c^2x^4+6cx^2-6\right)e^{a+bx+cx^2}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**3,x)
```

output
```
(a**3 + 3*a**2*b*x + 3*a**2*c*x**2 - 3*a**2 + 3*a*b**2*x**2 + 6*a*b*c*x**3
 - 6*a*b*x + 3*a*c**2*x**4 - 6*a*c*x**2 + 6*a + b**3*x**3 + 3*b**2*c*x**4
- 3*b**2*x**2 + 3*b*c**2*x**5 - 6*b*c*x**3 + 6*b*x + c**3*x**6 - 3*c**2*x*
*4 + 6*c*x**2 - 6)*exp(a + b*x + c*x**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.78 (sec) , antiderivative size = 2381, normalized size of antiderivative = 26.46

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^3,x, algorithm="maxima"
)
```

output
```
1/2*sqrt(pi)*a^3*b*erf(sqrt(-c)*x - 1/2*b/sqrt(-c))*e^(a - 1/4*b^2/c)/sqrt
(-c) - 3/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(
sqrt(-(2*c*x + b)^2/c)*c^(3/2)) - 2*e^(1/4*(2*c*x + b)^2/c)/sqrt(c))*a^2*b
^2*e^(a - 1/4*b^2/c)/sqrt(c) + 3/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt
(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)*c^(5/2)) - 4*b*e^(1/4*(2*
c*x + b)^2/c)/c^(3/2) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2/c)/(
(-(2*c*x + b)^2/c)^(3/2)*c^(5/2)))*a*b^3*e^(a - 1/4*b^2/c)/sqrt(c) - 1/16*
(sqrt(pi)*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*
c*x + b)^2/c)*c^(7/2)) - 6*b^2*e^(1/4*(2*c*x + b)^2/c)/c^(5/2) - 12*(2*c*x
 + b)^3*b*gamma(3/2, -1/4*(2*c*x + b)^2/c)/((-(2*c*x + b)^2/c)^(3/2)*c^(7/
2)) + 8*gamma(2, -1/4*(2*c*x + b)^2/c)/c^(3/2))*b^4*e^(a - 1/4*b^2/c)/sqrt
(c) - 1/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(s
qrt(-(2*c*x + b)^2/c)*c^(3/2)) - 2*e^(1/4*(2*c*x + b)^2/c)/sqrt(c))*a^3*sq
rt(c)*e^(a - 1/4*b^2/c) + 9/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*
c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)*c^(5/2)) - 4*b*e^(1/4*(2*c*x +
 b)^2/c)/c^(3/2) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2/c)/((-(2*
c*x + b)^2/c)^(3/2)*c^(5/2)))*a^2*b*sqrt(c)*e^(a - 1/4*b^2/c) - 3/4*(sqrt(
pi)*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x +
b)^2/c)*c^(7/2)) - 6*b^2*e^(1/4*(2*c*x + b)^2/c)/c^(5/2) - 12*(2*c*x + b)^
3*b*gamma(3/2, -1/4*(2*c*x + b)^2/c)/((-(2*c*x + b)^2/c)^(3/2)*c^(7/2))...
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx$$
$$=\left(\left(cx^2+bx+a\right)^3+6\,cx^2-3\left(cx^2+bx+a\right)^2+6\,bx+6\,a-6\right)e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
((c*x^2 + b*x + a)^3 + 6*c*x^2 - 3*(c*x^2 + b*x + a)^2 + 6*b*x + 6*a - 6)*
e^(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.61

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx=\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}\,\left(a^3+3\,a^2\,b\,x+3\,a^2\,c\,x^2-3\,a^2+3\,a\,b^2\,x^2+6\,a\,b\,c\,x^3-6\,a\,b\,x+3\,a\,c^2\,x^4-6\,a\,c\,x^2+6\,a+b^3\,x^3+3\,b^2\,c\,x^4-3\,b^2\,x^2+3\,b\,c^2\,x^5-6\,b\,c\,x^3+6\,b\,x+c^3\,x^6-3\,c^2\,x^4+6\,c\,x^2-6\right)$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^3,x)
```

output
```
exp(b*x)*exp(a)*exp(c*x^2)*(6*a + 6*b*x + 6*c*x^2 - 3*a^2 + a^3 - 3*b^2*x^
2 + b^3*x^3 - 3*c^2*x^4 + c^3*x^6 + 3*a*b^2*x^2 + 3*a^2*c*x^2 + 3*a*c^2*x^
4 + 3*b^2*c*x^4 + 3*b*c^2*x^5 - 6*a*b*x + 3*a^2*b*x - 6*a*c*x^2 - 6*b*c*x^
3 + 6*a*b*c*x^3 - 6)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.61

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^3\,dx = e^{c\,x^2+bx+a}\big(c^3x^6+3b\,c^2x^5+3a\,c^2x^4+3b^2c\,x^4+6abc\,x^3+b^3x^3-3c^2x^4+3a^2c\,x^2+3a\,b^2x^2-6bc\,x^3+3a^2bx-6ac\,x^2-3b^2x^2+a^3-6abx+6c\,x^2-3a^2+6bx+6a-6\big)$$

input

```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^3,x)
```

output

```
e**(a + b*x + c*x**2)*(a**3 + 3*a**2*b*x + 3*a**2*c*x**2 - 3*a**2 + 3*a*b*
*2*x**2 + 6*a*b*c*x**3 - 6*a*b*x + 3*a*c**2*x**4 - 6*a*c*x**2 + 6*a + b**3
*x**3 + 3*b**2*c*x**4 - 3*b**2*x**2 + 3*b*c**2*x**5 - 6*b*c*x**3 + 6*b*x +
 c**3*x**6 - 3*c**2*x**4 + 6*c*x**2 - 6)
```

3.544 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 31, antiderivative size = 64

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx = 2e^{a+bx+cx^2}-2e^{a+bx+cx^2}\left(a+bx+cx^2\right)+e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2$$

output
```
2*exp(c*x^2+b*x+a)-2*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)+exp(c*x^2+b*x+a)*(c*x^
2+b*x+a)^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx = e^{a+x(b+cx)}\left(2-2(a+x(b+cx))+(a+x(b+cx))^2\right)$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^2,x]
```

output
```
E^(a + x*(b + c*x))*(2 - 2*(a + x*(b + c*x)) + (a + x*(b + c*x))^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {7258, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \text{7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2-2\int e^{cx^2+bx+a}\left(cx^2+bx+a\right)d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2-2\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)-\int e^{cx^2+bx+a}d\left(cx^2+bx+a\right)\right)$$

$$\Big\downarrow \text{2624}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^2-2\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)-e^{a+bx+cx^2}\right)$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^2,x]
```

output
```
E^(a + b*x + c*x^2)*(a + b*x + c*x^2)^2 - 2*(-E^(a + b*x + c*x^2) + E^(a +
 b*x + c*x^2)*(a + b*x + c*x^2))
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

method	result
derivativedivides	$2\,\mathrm{e}^{c\,x^{2}+bx+a}-2\,\mathrm{e}^{c\,x^{2}+bx+a}\left(c\,x^{2}+bx+a\right)+\mathrm{e}^{c\,x^{2}+bx+a}\left(c\,x^{2}+bx+a\right)^{2}$
default	$2\,\mathrm{e}^{c\,x^{2}+bx+a}-2\,\mathrm{e}^{c\,x^{2}+bx+a}\left(c\,x^{2}+bx+a\right)+\mathrm{e}^{c\,x^{2}+bx+a}\left(c\,x^{2}+bx+a\right)^{2}$
gosper	$\left(c^{2}x^{4}+2bc\,x^{3}+2ac\,x^{2}+b^{2}x^{2}+2abx-2c\,x^{2}+a^{2}-2bx-2a+2\right)\mathrm{e}^{c\,x^{2}+bx+a}$
risch	$\left(c^{2}x^{4}+2bc\,x^{3}+2ac\,x^{2}+b^{2}x^{2}+2abx-2c\,x^{2}+a^{2}-2bx-2a+2\right)\mathrm{e}^{c\,x^{2}+bx+a}$
orering	$\left(c^{2}x^{4}+2bc\,x^{3}+2ac\,x^{2}+b^{2}x^{2}+2abx-2c\,x^{2}+a^{2}-2bx-2a+2\right)\mathrm{e}^{c\,x^{2}+bx+a}$
norman	$\left(a^{2}-2a+2\right)\mathrm{e}^{c\,x^{2}+bx+a}+x^{4}\mathrm{e}^{c\,x^{2}+bx+a}c^{2}+\left(2ab-2b\right)x\,\mathrm{e}^{c\,x^{2}+bx+a}+\left(2ac+b^{2}-2c\right)x^{2}$
parallelrisch	$x^{4}\mathrm{e}^{c\,x^{2}+bx+a}c^{2}+2x^{3}\mathrm{e}^{c\,x^{2}+bx+a}bc+2x^{2}\mathrm{e}^{c\,x^{2}+bx+a}ac+x^{2}\mathrm{e}^{c\,x^{2}+bx+a}b^{2}-2x^{2}\mathrm{e}^{c\,x^{2}+bx+a}c+$
parts	$-\frac{\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^{2}}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-c}}\right)c^{3}x^{5}}{\sqrt{-c}}-\frac{5\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^{2}}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-c}}\right)b\,c^{2}x^{4}}{2\sqrt{-c}}-\frac{2\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^{2}}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{}{2}\right.}{\sqrt{-c}}$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*exp(c*x^2+b*x+a)-2*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)+exp(c*x^2+b*x+a)*(c*x^
2+b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx$$
$$= \left(c^2x^4+2\,bcx^3+2\,(a-1)bx+\left(b^2+2\,(a-1)c\right)x^2+a^2-2\,a+2\right)e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^2,x, algorithm="fricas"
)
```

output
```
(c^2*x^4 + 2*b*c*x^3 + 2*(a - 1)*b*x + (b^2 + 2*(a - 1)*c)*x^2 + a^2 - 2*a
 + 2)*e^(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx = \left(a^2+2abx+2acx^2-2a+b^2x^2+2bcx^3 - 2bx+c^2x^4-2cx^2+2\right)e^{a+bx+cx^2}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**2,x)
```

output
```
(a**2 + 2*a*b*x + 2*a*c*x**2 - 2*a + b**2*x**2 + 2*b*c*x**3 - 2*b*x + c**2
*x**4 - 2*c*x**2 + 2)*exp(a + b*x + c*x**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 1223, normalized size of antiderivative = 19.11

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^2,x, algorithm="maxima"
)
```

output
```
1/2*sqrt(pi)*a^2*b*erf(sqrt(-c)*x - 1/2*b/sqrt(-c))*e^(a - 1/4*b^2/c)/sqrt
(-c) - 1/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(
sqrt(-(2*c*x + b)^2/c)*c^(3/2)) - 2*e^(1/4*(2*c*x + b)^2/c)/sqrt(c))*a*b^2
*e^(a - 1/4*b^2/c)/sqrt(c) + 1/8*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-
(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)*c^(5/2)) - 4*b*e^(1/4*(2*c*
x + b)^2/c)/c^(3/2) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2/c)/((-
(2*c*x + b)^2/c)^(3/2)*c^(5/2)))*b^3*e^(a - 1/4*b^2/c)/sqrt(c) - 1/2*(sqrt
(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b
)^2/c)*c^(3/2)) - 2*e^(1/4*(2*c*x + b)^2/c)/sqrt(c))*a^2*sqrt(c)*e^(a - 1/
4*b^2/c) + 3/4*(sqrt(pi)*(2*c*x + b)*b^2*(erf(1/2*sqrt(-(2*c*x + b)^2/c))
- 1)/(sqrt(-(2*c*x + b)^2/c)*c^(5/2)) - 4*b*e^(1/4*(2*c*x + b)^2/c)/c^(3/2
) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x + b)^2/c)/((-(2*c*x + b)^2/c)^(
3/2)*c^(5/2)))*a*b*sqrt(c)*e^(a - 1/4*b^2/c) - 1/4*(sqrt(pi)*(2*c*x + b)*b
^3*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)*c^(7/2))
- 6*b^2*e^(1/4*(2*c*x + b)^2/c)/c^(5/2) - 12*(2*c*x + b)^3*b*gamma(3/2, -1
/4*(2*c*x + b)^2/c)/((-(2*c*x + b)^2/c)^(3/2)*c^(7/2)) + 8*gamma(2, -1/4*(
2*c*x + b)^2/c)/c^(3/2))*b^2*sqrt(c)*e^(a - 1/4*b^2/c) - 1/4*(sqrt(pi)*(2*
c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)
*c^(7/2)) - 6*b^2*e^(1/4*(2*c*x + b)^2/c)/c^(5/2) - 12*(2*c*x + b)^3*b*gam
ma(3/2, -1/4*(2*c*x + b)^2/c)/((-(2*c*x + b)^2/c)^(3/2)*c^(7/2)) + 8*ga...
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx = -\left(2\,cx^2-\left(cx^2+bx+a\right)^2+2\,bx+2\,a-2\right)e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
-(2*c*x^2 - (c*x^2 + b*x + a)^2 + 2*b*x + 2*a - 2)*e^(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx = \mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}\,\left(a^2+2\,a\,b\,x+2\,a\,c\,x^2-2\,a+b^2\,x^2+2\,b\,c\,x^3-2\,b\,x+c^2\,x^4-2\,c\,x^2+2\right)$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^2,x)
```

output
```
exp(b*x)*exp(a)*exp(c*x^2)*(a^2 - 2*b*x - 2*c*x^2 - 2*a + b^2*x^2 + c^2*x^
4 + 2*a*b*x + 2*a*c*x^2 + 2*b*c*x^3 + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^2\,dx = e^{c\,x^2+bx+a}\left(c^2x^4+2bc\,x^3+2ac\,x^2+b^2x^2+2abx-2c\,x^2+a^2-2bx-2a+2\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^2,x)
```

output
```
e**(a + b*x + c*x**2)*(a**2 + 2*a*b*x + 2*a*c*x**2 - 2*a + b**2*x**2 + 2*b
*c*x**3 - 2*b*x + c**2*x**4 - 2*c*x**2 + 2)
```

3.545 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx$

Optimal result

Integrand size = 29, antiderivative size = 38

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=-e^{a+bx+cx^2}+e^{a+bx+cx^2}\left(a+bx+cx^2\right)$$

output
```
-exp(c*x^2+b*x+a)+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=e^{a+x(b+cx)}\left(-1+a+bx+cx^2\right)$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2),x]
```

output
```
E^(a + x*(b + c*x))*(-1 + a + b*x + c*x^2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {7258, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{ 7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{ 2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)-\int e^{cx^2+bx+a}d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{ 2624}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)-e^{a+bx+cx^2}$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2),x]
```

output
```
-E^(a + b*x + c*x^2) + E^(a + b*x + c*x^2)*(a + b*x + c*x^2)
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result
gosper	$(c\,x^2+bx+a-1)\,\mathrm{e}^{c\,x^2+bx+a}$
risch	$(c\,x^2+bx+a-1)\,\mathrm{e}^{c\,x^2+bx+a}$
orering	$(c\,x^2+bx+a-1)\,\mathrm{e}^{c\,x^2+bx+a}$
derivativedivides	$-\mathrm{e}^{c\,x^2+bx+a}+\mathrm{e}^{c\,x^2+bx+a}(c\,x^2+bx+a)$
default	$-\mathrm{e}^{c\,x^2+bx+a}+\mathrm{e}^{c\,x^2+bx+a}(c\,x^2+bx+a)$
norman	$(a-1)\,\mathrm{e}^{c\,x^2+bx+a}+x\,\mathrm{e}^{c\,x^2+bx+a}b+x^2\mathrm{e}^{c\,x^2+bx+a}c$
parallelrisch	$x^2\mathrm{e}^{c\,x^2+bx+a}c+x\,\mathrm{e}^{c\,x^2+bx+a}b+\mathrm{e}^{c\,x^2+bx+a}a-\mathrm{e}^{c\,x^2+bx+a}$
parts	$-\frac{\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-c}}\right)c^2x^3}{\sqrt{-c}}-\frac{3\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-c}}\right)bc\,x^2}{2\sqrt{-c}}-\frac{\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4c}}\operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-}}\right.}{\sqrt{-c}}$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a-1)*exp(c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=\left(cx^2+bx+a-1\right)e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
(c*x^2 + b*x + a - 1)*e^(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.58

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=\left(a+bx+cx^2-1\right)e^{a+bx+cx^2}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a),x)
```

output
```
(a + b*x + c*x**2 - 1)*exp(a + b*x + c*x**2)
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.22 (sec) , antiderivative size = 501, normalized size of antiderivative = 13.18

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*a*b*erf(sqrt(-c)*x - 1/2*b/sqrt(-c))*e^(a - 1/4*b^2/c)/sqrt(-
c) - 1/4*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sq
rt(-(2*c*x + b)^2/c)*c^(3/2)) - 2*e^(1/4*(2*c*x + b)^2/c)/sqrt(c))*b^2*e^(
a - 1/4*b^2/c)/sqrt(c) - 1/2*(sqrt(pi)*(2*c*x + b)*b*(erf(1/2*sqrt(-(2*c*x
 + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)*c^(3/2)) - 2*e^(1/4*(2*c*x + b)^2
/c)/sqrt(c))*a*sqrt(c)*e^(a - 1/4*b^2/c) + 3/8*(sqrt(pi)*(2*c*x + b)*b^2*(
erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sqrt(-(2*c*x + b)^2/c)*c^(5/2)) - 4*
b*e^(1/4*(2*c*x + b)^2/c)/c^(3/2) - 4*(2*c*x + b)^3*gamma(3/2, -1/4*(2*c*x
 + b)^2/c)/((-(2*c*x + b)^2/c)^(3/2)*c^(5/2)))*b*sqrt(c)*e^(a - 1/4*b^2/c)
 - 1/8*(sqrt(pi)*(2*c*x + b)*b^3*(erf(1/2*sqrt(-(2*c*x + b)^2/c)) - 1)/(sq
rt(-(2*c*x + b)^2/c)*c^(7/2)) - 6*b^2*e^(1/4*(2*c*x + b)^2/c)/c^(5/2) - 12
*(2*c*x + b)^3*b*gamma(3/2, -1/4*(2*c*x + b)^2/c)/((-(2*c*x + b)^2/c)^(3/2
)*c^(7/2)) + 8*gamma(2, -1/4*(2*c*x + b)^2/c)/c^(3/2))*c^(3/2)*e^(a - 1/4*
b^2/c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=\left(cx^2+bx+a-1\right)e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
(c*x^2 + b*x + a - 1)*e^(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=\mathrm{e}^{c\,x^2+b\,x+a}\,\left(c\,x^2+b\,x+a-1\right)$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2),x)
```

output
```
exp(a + b*x + c*x^2)*(a + b*x + c*x^2 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)\,dx=e^{c\,x^2+bx+a}\left(c\,x^2+bx+a-1\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a),x)
```

output
```
e**(a + b*x + c*x**2)*(a + b*x + c*x**2 - 1)
```

3.546 $\int e^{a+bx+cx^2}(b+2cx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 12

$$\int e^{a+bx+cx^2}(b+2cx)\,dx=e^{a+bx+cx^2}$$

output
```
exp(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{a+bx+cx^2}(b+2cx)\,dx=e^{a+bx+cx^2}$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x),x]
```

output
```
E^(a + b*x + c*x^2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2666}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 2666}$$

$$e^{a+bx+cx^2}$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x),x]
```

output
```
E^(a + b*x + c*x^2)
```

Defintions of rubi rules used

rule 2666
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_.) + (e_.)*(x_)), x_Symbol
] :> Simp[e*(F^(a + b*x + c*x^2)/(2*c*Log[F])), x] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[b*e - 2*c*d, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result
gosper	$e^{c\,x^2+bx+a}$
derivativedivides	$e^{c\,x^2+bx+a}$
default	$e^{c\,x^2+bx+a}$
norman	$e^{c\,x^2+bx+a}$
risch	$e^{c\,x^2+bx+a}$
parallelrisch	$e^{c\,x^2+bx+a}$
orering	$e^{c\,x^2+bx+a}$
parts	$-\frac{\sqrt{\pi}\, e^{a-\frac{b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-c}}\right) c x}{\sqrt{-c}} - \frac{\sqrt{\pi}\, e^{a-\frac{b^2}{4c}} \operatorname{erf}\left(-\sqrt{-c}\,x+\frac{b}{2\sqrt{-c}}\right) b}{2\sqrt{-c}} + \frac{e^{a-\frac{b^2}{4c}}\left(2 \operatorname{erf}\left(\frac{2cx+b}{2\sqrt{-c}}\right) x\sqrt{\pi}\, c+b \operatorname{erf}\left(\right.\right.}{2\sqrt{-c}}$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b),x,method=_RETURNVERBOSE)
```

output
```
exp(c*x^2+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int e^{a+bx+cx^2}(b+2cx)\,dx = e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="fricas")
```

output
```
e^(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int e^{a+bx+cx^2}(b+2cx)\,dx=e^{a+bx+cx^2}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b),x)
```

output
```
exp(a + b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int e^{a+bx+cx^2}(b+2cx)\,dx=e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="maxima")
```

output
```
e^(c*x^2 + b*x + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int e^{a+bx+cx^2}(b+2cx)\,dx=e^{\left(cx^2+bx+a\right)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b),x, algorithm="giac")
```

output
```
e^(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int e^{a+bx+cx^2}(b+2cx)\,dx = \mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x),x)
```

output
```
exp(b*x)*exp(a)*exp(c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int e^{a+bx+cx^2}(b+2cx)\,dx = e^{c\,x^2+bx+a}$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b),x)
```

output
```
e**(a + b*x + c*x**2)
```

3.547 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 11

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \text{ExpIntegralEi}\left(a+bx+cx^2\right)$$

output
```
Ei(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \text{ExpIntegralEi}(a+x(b+cx))$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2),x]
```

output
```
ExpIntegralEi[a + x*(b + c*x)]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {7258, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{a+bx+cx^2}d(a+bx+cx^2)$$

$$\Big\downarrow \text{2609}$$

$$\text{ExpIntegralEi}\left(a+bx+cx^2\right)$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2),x]
```

output
```
ExpIntegralEi[a + b*x + c*x^2]
```

Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

method	result	size
derivativedivides	$-\operatorname{expIntegral}_1\left(-c\,x^2-bx-a\right)$	19
default	$-\operatorname{expIntegral}_1\left(-c\,x^2-bx-a\right)$	19
risch	$-\operatorname{expIntegral}_1\left(-c\,x^2-bx-a\right)$	19

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-Ei(1,-c*x^2-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \operatorname{Ei}\left(cx^2+bx+a\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
Ei(c*x^2 + b*x + a)
```

Sympy [A] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \operatorname{Ei}\left(a+bx+cx^2\right)$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a),x)
```

output
```
Ei(a + b*x + c*x**2)
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{cx^2+bx+a}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \operatorname{Ei}\left(cx^2+bx+a\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
Ei(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = \operatorname{ei}\left(c\,x^2+b\,x+a\right)$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2),x)
```

output
```
ei(a + b*x + c*x^2)
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{a+bx+cx^2}\,dx = e^a\left(\left(\int \frac{e^{c\,x^2+bx}}{c\,x^2+bx+a}dx\right)b+2\left(\int \frac{e^{c\,x^2+bx}x}{c\,x^2+bx+a}dx\right)c\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a),x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(a + b*x + c*x**2),x)*b + 2*int((e**(b*x + c*x
**2)*x)/(a + b*x + c*x**2),x)*c)
```

3.548 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 38

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx=-\frac{e^{a+bx+cx^2}}{a+bx+cx^2}+\text{ExpIntegralEi}\left(a+bx+cx^2\right)$$

output
```
-exp(c*x^2+b*x+a)/(c*x^2+b*x+a)+Ei(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx=-\frac{e^{a+x(b+cx)}}{a+x(b+cx)}+\text{ExpIntegralEi}(a+x(b+cx))$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-(E^(a + x*(b + c*x))/(a + x*(b + c*x))) + ExpIntegralEi[a + x*(b + c*x)]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {7258, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{(a+bx+cx^2)^2}dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{(a+bx+cx^2)^2}d(a+bx+cx^2)$$

$$\Big\downarrow \text{2608}$$

$$\int \frac{e^{cx^2+bx+a}}{cx^2+bx+a}d(cx^2+bx+a)-\frac{e^{a+bx+cx^2}}{a+bx+cx^2}$$

$$\Big\downarrow \text{2609}$$

$$\text{ExpIntegralEi}\left(cx^2+bx+a\right)-\frac{e^{a+bx+cx^2}}{a+bx+cx^2}$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^2,x]
```

output
```
-(E^(a + b*x + c*x^2)/(a + b*x + c*x^2)) + ExpIntegralEi[a + b*x + c*x^2]
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

method	result	size
derivativedivides	$-\frac{e^{c\,x^{2}+b x+a}}{c\,x^{2}+b x+a}-\operatorname{expIntegral}_{1}\left(-c\,x^{2}-b x-a\right)$	45
default	$-\frac{e^{c\,x^{2}+b x+a}}{c\,x^{2}+b x+a}-\operatorname{expIntegral}_{1}\left(-c\,x^{2}-b x-a\right)$	45
risch	$-\frac{e^{c\,x^{2}+b x+a}}{c\,x^{2}+b x+a}-\operatorname{expIntegral}_{1}\left(-c\,x^{2}-b x-a\right)$	45

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(c*x^2+b*x+a)/(c*x^2+b*x+a)-Ei(1,-c*x^2-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx=\frac{(cx^2+bx+a)\mathrm{Ei}(cx^2+bx+a)-e^{(cx^2+bx+a)}}{cx^2+bx+a}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="fricas"
)
```

output
```
((c*x^2 + b*x + a)*Ei(c*x^2 + b*x + a) - e^(c*x^2 + b*x + a))/(c*x^2 + b*x
 + a)
```

Sympy [A] (verification not implemented)

Time = 63.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\frac{\mathrm{E}_2\left(-a-bx-cx^2\right)}{a+bx+cx^2}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**2,x)
```

output
```
-expint(2, -a - b*x - c*x**2)/(a + b*x + c*x**2)
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="maxima"
)
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^2, x)
```

Giac [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^2}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx = -\mathrm{expint}\left(-c\,x^2-b\,x-a\right)-\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}}{c\,x^2+b\,x+a}$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^2,x)
```

output
```
- expint(- a - b*x - c*x^2) - (exp(b*x)*exp(a)*exp(c*x^2))/(a + b*x + c*x^
2)
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^2}\,dx = e^{a}\left(\left(\int \frac{e^{c\,x^2+bx}}{c^2x^4+2bc\,x^3+2ac\,x^2+b^2x^2+2abx+a^2}dx\right)b + 2\left(\int \frac{e^{c\,x^2+bx}x}{c^2x^4+2bc\,x^3+2ac\,x^2+b^2x^2+2abx+a^2}dx\right)c\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^2,x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(a**2 + 2*a*b*x + 2*a*c*x**2 + b**2*x**2 + 2*b
*c*x**3 + c**2*x**4),x)*b + 2*int((e**(b*x + c*x**2)*x)/(a**2 + 2*a*b*x +
2*a*c*x**2 + b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)*c)
```

3.549 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx = -\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2}-\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)}+\frac{1}{2}\text{ExpIntegralEi}\left(a+bx+cx^2\right)$$

output
```
-1/2*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^2-exp(c*x^2+b*x+a)/(2*c*x^2+2*b*x+2*a)
+1/2*Ei(c*x^2+b*x+a)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.69

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx = \frac{1}{2}\left(-\frac{e^{a+x(b+cx)}(1+a+bx+cx^2)}{(a+x(b+cx))^2}+\text{ExpIntegralEi}(a+x(b+cx))\right)$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^3,x]
```

output
```
(-((E^(a + x*(b + c*x))*(1 + a + b*x + c*x^2))/(a + x*(b + c*x))^2) + ExpI
ntegralEi[a + x*(b + c*x)])/2
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {7258, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{(a+bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 7258}$$

$$\int \frac{e^{a+bx+cx^2}}{(a+bx+cx^2)^3}d(a+bx+cx^2)$$

$$\Big\downarrow \text{ 2608}$$

$$\frac{1}{2}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^2}d(cx^2+bx+a)-\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 2608}$$

$$\frac{1}{2}\left(\int \frac{e^{cx^2+bx+a}}{cx^2+bx+a}d(cx^2+bx+a)-\frac{e^{a+bx+cx^2}}{a+bx+cx^2}\right)-\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{1}{2}\left(\text{ExpIntegralEi}\left(cx^2+bx+a\right)-\frac{e^{a+bx+cx^2}}{a+bx+cx^2}\right)-\frac{e^{a+bx+cx^2}}{2(a+bx+cx^2)^2}$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^3,x]
```

output
```
-1/2*E^(a + b*x + c*x^2)/(a + b*x + c*x^2)^2 + (-(E^(a + b*x + c*x^2)/(a +
 b*x + c*x^2)) + ExpIntegralEi[a + b*x + c*x^2])/2
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{e^{c\,x^{2}+bx+a}}{2(c\,x^{2}+bx+a)^{2}}-\frac{e^{c\,x^{2}+bx+a}}{2(c\,x^{2}+bx+a)}-\frac{\operatorname{expIntegral}_{1}(-c\,x^{2}-bx-a)}{2}$	70
default	$-\frac{e^{c\,x^{2}+bx+a}}{2(c\,x^{2}+bx+a)^{2}}-\frac{e^{c\,x^{2}+bx+a}}{2(c\,x^{2}+bx+a)}-\frac{\operatorname{expIntegral}_{1}(-c\,x^{2}-bx-a)}{2}$	70
risch	$-\frac{e^{c\,x^{2}+bx+a}}{2(c\,x^{2}+bx+a)^{2}}-\frac{e^{c\,x^{2}+bx+a}}{2(c\,x^{2}+bx+a)}-\frac{\operatorname{expIntegral}_{1}(-c\,x^{2}-bx-a)}{2}$	70

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^2-1/2*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)-1
/2*Ei(1,-c*x^2-b*x-a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.54

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx = \frac{(c^2x^4+2\,bcx^3+2\,abx+(b^2+2\,ac)x^2+a^2)\mathrm{Ei}(cx^2+bx+a)-(cx^2+bx+a+1)e^{(cx^2+bx+a)}}{2\,(c^2x^4+2\,bcx^3+2\,abx+(b^2+2\,ac)x^2+a^2)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^3,x, algorithm="fricas"
)
```

output
```
1/2*((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*Ei(c*x^2 +
b*x + a) - (c*x^2 + b*x + a + 1)*e^(c*x^2 + b*x + a))/(c^2*x^4 + 2*b*c*x^3
 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^3,x, algorithm="maxima"
)
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^3, x)
```

Giac [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx=\int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^3}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx=-\frac{\mathrm{expint}(-c\,x^2-b\,x-a)}{2}-\mathrm{e}^{c\,x^2+b\,x+a}\left(\frac{1}{2\,\left(c\,x^2+b\,x+a\right)}+\frac{1}{2\,{\left(c\,x^2+b\,x+a\right)}^2}\right)$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^3,x)
```

output
```
- expint(- a - b*x - c*x^2)/2 - exp(a + b*x + c*x^2)*(1/(2*(a + b*x + c*x^
2)) + 1/(2*(a + b*x + c*x^2)^2))
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^3}\,dx$$
$$= e^a\left(\left(\int \frac{e^{c\,x^2+bx}}{c^3x^6+3b\,c^2x^5+3a\,c^2x^4+3b^2c\,x^4+6abc\,x^3+b^3x^3+3a^2c\,x^2+3a\,b^2x^2+3a^2bx+a^3}dx\right)b\right.$$
$$\left.+2\left(\int \frac{e^{c\,x^2+bx}x}{c^3x^6+3b\,c^2x^5+3a\,c^2x^4+3b^2c\,x^4+6abc\,x^3+b^3x^3+3a^2c\,x^2+3a\,b^2x^2+3a^2bx+a^3}dx\right)c\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^3,x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(a**3 + 3*a**2*b*x + 3*a**2*c*x**2 + 3*a*b**2*
x**2 + 6*a*b*c*x**3 + 3*a*c**2*x**4 + b**3*x**3 + 3*b**2*c*x**4 + 3*b*c**2
*x**5 + c**3*x**6),x)*b + 2*int((e**(b*x + c*x**2)*x)/(a**3 + 3*a**2*b*x +
 3*a**2*c*x**2 + 3*a*b**2*x**2 + 6*a*b*c*x**3 + 3*a*c**2*x**4 + b**3*x**3
+ 3*b**2*c*x**4 + 3*b*c**2*x**5 + c**3*x**6),x)*c)
```

3.550 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 142

$$\begin{aligned}\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx &= -\frac{105}{8}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}\\ &+\frac{35}{4}e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{7}{2}e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}\\ &+e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}+\frac{105}{16}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{a+bx+cx^2}\right)\end{aligned}$$

output
```
-105/8*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)+35/4*exp(c*x^2+b*x+a)*(c*x^2+b
*x+a)^(3/2)-7/2*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(5/2)+exp(c*x^2+b*x+a)*(c*x
^2+b*x+a)^(7/2)+105/16*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 3.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.32

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx=\frac{\sqrt{-a-x(b+cx)}\,\Gamma\left(\frac{9}{2},-a-x(b+cx)\right)}{\sqrt{a+x(b+cx)}}$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(7/2),x]
```

output
```
(Sqrt[-a - x*(b + c*x)]*Gamma[9/2, -a - x*(b + c*x)])/Sqrt[a + x*(b + c*x)
]
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {7258, 2607, 2607, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}dx$$

$$\Big\downarrow \text{7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}-\frac{7}{2}\int e^{cx^2+bx+a}\left(cx^2+bx+a\right)^{5/2}d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}-\frac{7}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\int e^{cx^2+bx+a}\left(cx^2+bx+a\right)^{3/2}d\left(cx^2+bx+a\right)\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}-\frac{7}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\int e^{cx^2+bx+a}\sqrt{cx^2+bx+a}d\left(cx^2+bx+a\right)\right.\right.$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}-\frac{7}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\int\frac{e^{cx^2+b}}{\sqrt{cx^2+}}\right.\right.\right.$$

$$\Big\downarrow \text{2611}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}-\frac{7}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\int e^{cx^2+bx+a}d\right.\right.\right.$$

$$\Big\downarrow \text{2633}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{7/2}-\frac{7}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{a\right.\right.\right.\right.$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(7/2),x]
```

output
```
E^(a + b*x + c*x^2)*(a + b*x + c*x^2)^(7/2) - (7*(E^(a + b*x + c*x^2)*(a +
 b*x + c*x^2)^(5/2) - (5*(E^(a + b*x + c*x^2)*(a + b*x + c*x^2)^(3/2) - (3
*(E^(a + b*x + c*x^2)*Sqrt[a + b*x + c*x^2] - (Sqrt[Pi]*Erfi[Sqrt[a + b*x
+ c*x^2]])/2))/2))/2))/2
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{105\,{\mathrm e}^{c\,x^{2}+b x +a} \sqrt{c\,x^{2}+b x +a}}{8}+\frac{35\,{\mathrm e}^{c\,x^{2}+b x +a} \left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}{4}-\frac{7\,{\mathrm e}^{c\,x^{2}+b x +a} \left(c\,x^{2}+b x +a \right)^{\frac{5}{2}}}{2}+{\mathrm e}^{c\,x^{2}+b x +a} (c$
default	$-\frac{105\,{\mathrm e}^{c\,x^{2}+b x +a} \sqrt{c\,x^{2}+b x +a}}{8}+\frac{35\,{\mathrm e}^{c\,x^{2}+b x +a} \left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}{4}-\frac{7\,{\mathrm e}^{c\,x^{2}+b x +a} \left(c\,x^{2}+b x +a \right)^{\frac{5}{2}}}{2}+{\mathrm e}^{c\,x^{2}+b x +a} (c$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
-105/8*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)+35/4*exp(c*x^2+b*x+a)*(c*x^2+b
*x+a)^(3/2)-7/2*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(5/2)+exp(c*x^2+b*x+a)*(c*x
^2+b*x+a)^(7/2)+105/16*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Fricas [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx=\int\left(cx^2+bx+a\right)^{\frac{7}{2}}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(7/2),x, algorithm="fri
cas")
```

output
```
integral((2*c^4*x^7 + 7*b*c^3*x^6 + 3*(3*b^2*c^2 + 2*a*c^3)*x^5 + 5*(b^3*c
 + 3*a*b*c^2)*x^4 + a^3*b + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^3 + 3*(a*b^3
+ 3*a^2*b*c)*x^2 + (3*a^2*b^2 + 2*a^3*c)*x)*sqrt(c*x^2 + b*x + a)*e^(c*x^2
 + b*x + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx=\text{Timed out}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx=\int\left(cx^2+bx+a\right)^{\frac{7}{2}}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(7/2),x, algorithm="max
ima")
```

output
```
integrate((c*x^2 + b*x + a)^(7/2)*(2*c*x + b)*e^(c*x^2 + b*x + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.64

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx=\frac{1}{8}\left(8\left(cx^2+bx+a\right)^{\frac{7}{2}}-28\left(cx^2+bx+a\right)^{\frac{5}{2}}+70\left(cx^2+bx+a\right)^{\frac{3}{2}}-105\sqrt{cx^2+bx+a}\right)e^{(cx}+\frac{105}{16}i\sqrt{\pi}\,\mathrm{erf}\left(-i\sqrt{cx^2+bx+a}\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(7/2),x, algorithm="gia
c")
```

output
```
1/8*(8*(c*x^2 + b*x + a)^(7/2) - 28*(c*x^2 + b*x + a)^(5/2) + 70*(c*x^2 +
b*x + a)^(3/2) - 105*sqrt(c*x^2 + b*x + a))*e^(c*x^2 + b*x + a) + 105/16*I
*sqrt(pi)*erf(-I*sqrt(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.95

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx = \frac{\left(e^{c\,x^2+b\,x+a}\left(\frac{105\,\sqrt{-c\,x^2-b\,x-a}}{8}+\frac{35\,{\left(-c\,x^2-b\,x-a\right)}^{3/2}}{4}+\frac{7\,{\left(-c\,x^2-b\,x-a\right)}^{5/2}}{2}+{\left(-c\,x^2-b\,x-a\right)}^{7/2}\right)+\cdots\right.}{{\left(-c\,x^2-b\,x-a\right)}^{7/2}}$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(7/2),x)
```

output
```
((exp(a + b*x + c*x^2)*((105*(- a - b*x - c*x^2)^(1/2))/8 + (35*(- a - b*x
 - c*x^2)^(3/2))/4 + (7*(- a - b*x - c*x^2)^(5/2))/2 + (- a - b*x - c*x^2)
^(7/2)) + (105*pi^(1/2)*erfc((- a - b*x - c*x^2)^(1/2)))/16)*(a + b*x + c*
x^2)^(7/2))/(- a - b*x - c*x^2)^(7/2)
```

Reduce [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{7/2}\,dx = \int e^{c\,x^2+bx+a}(2cx+b)\left(c\,x^2+bx+a\right)^{\frac{7}{2}}dx$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(7/2),x)
```

output
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(7/2),x)
```

3.551 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 112

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{15}{4}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{5}{2}e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}+e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{15}{8}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
15/4*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)-5/2*exp(c*x^2+b*x+a)*(c*x^2+b*x+
a)^(3/2)+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(5/2)-15/8*Pi^(1/2)*erfi((c*x^2+b*
x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 2.46 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.41

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{\sqrt{a+x(b+cx)}\,\Gamma\left(\frac{7}{2},-a-x(b+cx)\right)}{\sqrt{-a-x(b+cx)}}$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[a + x*(b + c*x)]*Gamma[7/2, -a - x*(b + c*x)])/Sqrt[-a - x*(b + c*x)
]
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {7258, 2607, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\int e^{cx^2+bx+a}\left(cx^2+bx+a\right)^{3/2}d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\int e^{cx^2+bx+a}\sqrt{cx^2+bx+a}d\left(cx^2+bx+a\right)\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d\left(cx^2+bx+a\right)\right)\right)$$

$$\Big\downarrow \text{2611}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}\right)\right)$$

$$\Big\downarrow \text{2633}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{5/2}-\frac{5}{2}\left(e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)\right)\right)$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2),x]
```

output
```
E^(a + b*x + c*x^2)*(a + b*x + c*x^2)^(5/2) - (5*(E^(a + b*x + c*x^2)*(a +
 b*x + c*x^2)^(3/2) - (3*(E^(a + b*x + c*x^2)*Sqrt[a + b*x + c*x^2] - (Sqr
t[Pi]*Erfi[Sqrt[a + b*x + c*x^2]])/2))/2))/2
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{15\,\mathrm{e}^{c\,x^2+bx+a}\sqrt{c\,x^2+bx+a}}{4}-\frac{5\,\mathrm{e}^{c\,x^2+bx+a}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{2}+\mathrm{e}^{c\,x^2+bx+a}\left(c\,x^2+bx+a\right)^{\frac{5}{2}}-\frac{15\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\right.}{8}$
default	$\frac{15\,\mathrm{e}^{c\,x^2+bx+a}\sqrt{c\,x^2+bx+a}}{4}-\frac{5\,\mathrm{e}^{c\,x^2+bx+a}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{2}+\mathrm{e}^{c\,x^2+bx+a}\left(c\,x^2+bx+a\right)^{\frac{5}{2}}-\frac{15\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}\right.}{8}$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
15/4*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)-5/2*exp(c*x^2+b*x+a)*(c*x^2+b*x+
a)^(3/2)+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(5/2)-15/8*Pi^(1/2)*erfi((c*x^2+b*
x+a)^(1/2))
```

Fricas [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int\left(cx^2+bx+a\right)^{\frac{5}{2}}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2),x, algorithm="fri
cas")
```

output
```
integral((2*c^3*x^5 + 5*b*c^2*x^4 + 4*(b^2*c + a*c^2)*x^3 + a^2*b + (b^3 +
 6*a*b*c)*x^2 + 2*(a*b^2 + a^2*c)*x)*sqrt(c*x^2 + b*x + a)*e^(c*x^2 + b*x
 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{5}{2}}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2),x, algorithm="max
ima")
```

output
```
integrate((c*x^2 + b*x + a)^(5/2)*(2*c*x + b)*e^(c*x^2 + b*x + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{1}{4}\left(4\left(cx^2+bx+a\right)^{\frac{5}{2}}-10\left(cx^2+bx+a\right)^{\frac{3}{2}}+15\sqrt{cx^2+bx+a}\right)e^{\left(cx^2+bx+a\right)}-\frac{15}{8}i\sqrt{\pi}\,\text{erf}\left(-i\sqrt{cx^2+bx+a}\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2),x, algorithm="gia
c")
```

output
```
1/4*(4*(c*x^2 + b*x + a)^(5/2) - 10*(c*x^2 + b*x + a)^(3/2) + 15*sqrt(c*x^
2 + b*x + a))*e^(c*x^2 + b*x + a) - 15/8*I*sqrt(pi)*erf(-I*sqrt(c*x^2 + b*
x + a))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}\,dx=\frac{\left(e^{c\,x^2+b\,x+a}\left(\frac{15\,\sqrt{-c\,x^2-b\,x-a}}{4}+\frac{5\,{\left(-c\,x^2-b\,x-a\right)}^{3/2}}{2}+{\left(-c\,x^2-b\,x-a\right)}^{5/2}\right)+\frac{15\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-c\,x^2-b\,x-\cdots}\right.}{8}\right.}{{\left(-c\,x^2-b\,x-a\right)}^{5/2}}$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(5/2),x)
```

output
```
((exp(a + b*x + c*x^2)*((15*(- a - b*x - c*x^2)^(1/2))/4 + (5*(- a - b*x -
 c*x^2)^(3/2))/2 + (- a - b*x - c*x^2)^(5/2)) + (15*pi^(1/2)*erfc((- a - b
*x - c*x^2)^(1/2)))/8)*(a + b*x + c*x^2)^(5/2))/(- a - b*x - c*x^2)^(5/2)
```

Reduce [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{5/2}dx = e^a\left(2\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^5dx\right)c^3+5\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^4dx\right)b\,c^2+4\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^3dx\right)a\,c^2+4\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^3dx\right)b^2c+6\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^2dx\right)abc+\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^2dx\right)b^3+2\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,xdx\right)a^2c+2\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,xdx\right)a\,b^2+\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,dx\right)a^2b\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(5/2),x)
```

output
```
e**a*(2*int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x**5,x)*c**3 + 5*int(
e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x**4,x)*b*c**2 + 4*int(e**(b*x +
c*x**2)*sqrt(a + b*x + c*x**2)*x**3,x)*a*c**2 + 4*int(e**(b*x + c*x**2)*sq
rt(a + b*x + c*x**2)*x**3,x)*b**2*c + 6*int(e**(b*x + c*x**2)*sqrt(a + b*x
 + c*x**2)*x**2,x)*a*b*c + int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x*
*2,x)*b**3 + 2*int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x,x)*a**2*c +
2*int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x,x)*a*b**2 + int(e**(b*x +
 c*x**2)*sqrt(a + b*x + c*x**2),x)*a**2*b)
```

3.552 $\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 82

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx=-\frac{3}{2}e^{a+bx+cx^2}\sqrt{a+bx+cx^2}+e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}+\frac{3}{4}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
-3/2*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(
3/2)+3/4*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{\sqrt{-a-x(b+cx)}\Gamma\left(\frac{5}{2},-a-x(b+cx)\right)}{\sqrt{a+x(b+cx)}}$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[-a - x*(b + c*x)]*Gamma[5/2, -a - x*(b + c*x)])/Sqrt[a + x*(b + c*x)
]
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {7258, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{7258}$$

$$\int e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\int e^{cx^2+bx+a}\sqrt{cx^2+bx+a}d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d\left(cx^2+bx+a\right)\right)$$

$$\Big\downarrow \text{2611}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}\right)$$

$$\Big\downarrow \text{2633}$$

$$e^{a+bx+cx^2}\left(a+bx+cx^2\right)^{3/2}-\frac{3}{2}\left(e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)\right)$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2),x]
```

output
```
E^(a + b*x + c*x^2)*(a + b*x + c*x^2)^(3/2) - (3*(E^(a + b*x + c*x^2)*Sqrt
[a + b*x + c*x^2] - (Sqrt[Pi]*Erfi[Sqrt[a + b*x + c*x^2]])/2))/2
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{3\,{\mathrm e}^{c\,x^{2}+b x +a} \sqrt{c\,x^{2}+b x +a}}{2}+{\mathrm e}^{c\,x^{2}+b x +a} \left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}+\frac{3 \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{4}$	69
default	$-\frac{3\,{\mathrm e}^{c\,x^{2}+b x +a} \sqrt{c\,x^{2}+b x +a}}{2}+{\mathrm e}^{c\,x^{2}+b x +a} \left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}+\frac{3 \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{4}$	69

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-3/2*exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)+exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(
3/2)+3/4*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Fricas [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx+a\right)^{\frac{3}{2}}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2),x, algorithm="fri
cas")
```

output
```
integral((2*c^2*x^3 + 3*b*c*x^2 + a*b + (b^2 + 2*a*c)*x)*sqrt(c*x^2 + b*x
+ a)*e^(c*x^2 + b*x + a), x)
```

Sympy [A] (verification not implemented)

Time = 43.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.15

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{\left(-\sqrt{-a-bx-cx^2}\left(a+bx+cx^2-\frac{3}{2}\right)e^{a+bx+cx^2}+\frac{3\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-a-bx-cx^2}\right)}{4}\right)\left(a+bx+cx^2\right)^{\frac{3}{2}}}{\left(-a-bx-cx^2\right)^{\frac{3}{2}}}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**(3/2),x)
```

output
```
(-sqrt(-a - b*x - c*x**2)*(a + b*x + c*x**2 - 3/2)*exp(a + b*x + c*x**2) +
 3*sqrt(pi)*erfc(sqrt(-a - b*x - c*x**2))/4)*(a + b*x + c*x**2)**(3/2)/(-a
 - b*x - c*x**2)**(3/2)
```

Maxima [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx = \int \left(cx^2+bx+a\right)^{\frac{3}{2}}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((c*x^2 + b*x + a)^(3/2)*(2*c*x + b)*e^(c*x^2 + b*x + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx = \frac{1}{2}\left(2\left(cx^2+bx+a\right)^{\frac{3}{2}} - 3\sqrt{cx^2+bx+a}\right)e^{\left(cx^2+bx+a\right)} + \frac{3}{4}i\sqrt{\pi}\operatorname{erf}\left(-i\sqrt{cx^2+bx+a}\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2),x, algorithm="gia
c")
```

output
```
1/2*(2*(c*x^2 + b*x + a)^(3/2) - 3*sqrt(c*x^2 + b*x + a))*e^(c*x^2 + b*x +
 a) + 3/4*I*sqrt(pi)*erf(-I*sqrt(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.24

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx=\frac{3\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-c\,x^2-b\,x-a}\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{4\,\left(-c\,x^2-b\,x-a\right)^{3/2}}-\frac{3\,\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}\,\sqrt{c\,x^2+b\,x+a}}{2}+\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}\,\left(c\,x^2+b\,x+a\right)^{3/2}$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2),x)
```

output
```
(3*pi^(1/2)*erfc((- a - b*x - c*x^2)^(1/2))*(a + b*x + c*x^2)^(3/2))/(4*(-
 a - b*x - c*x^2)^(3/2)) - (3*exp(b*x)*exp(a)*exp(c*x^2)*(a + b*x + c*x^2)
^(1/2))/2 + exp(b*x)*exp(a)*exp(c*x^2)*(a + b*x + c*x^2)^(3/2)
```

Reduce [F]

$$\int e^{a+bx+cx^2}(b+2cx)\left(a+bx+cx^2\right)^{3/2}\,dx=e^{a}\left(2\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^3dx\right)c^2+3\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,x^2dx\right)bc+2\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,xdx\right)ac+\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,xdx\right)b^2+\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}dx\right)ab\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(3/2),x)
```

output
```
e**a*(2*int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x**3,x)*c**2 + 3*int(
e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x**2,x)*b*c + 2*int(e**(b*x + c*x
**2)*sqrt(a + b*x + c*x**2)*x,x)*a*c + int(e**(b*x + c*x**2)*sqrt(a + b*x
+ c*x**2)*x,x)*b**2 + int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2),x)*a*b)
```

3.553 $\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 52

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = e^{a+bx+cx^2}\sqrt{a+bx+cx^2} - \frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)-1/2*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2)
)
```

Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{a+x(b+cx)}\Gamma\left(\frac{3}{2},-a-x(b+cx)\right)}{\sqrt{-a-x(b+cx)}}$$

input
```
Integrate[E^(a + b*x + c*x^2)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[a + x*(b + c*x)]*Gamma[3/2, -a - x*(b + c*x)])/Sqrt[-a - x*(b + c*x)
]
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {7258, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b+2cx)e^{a+bx+cx^2}\sqrt{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{7258}$$

$$\int e^{a+bx+cx^2}\sqrt{a+bx+cx^2}d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2607}$$

$$e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d\left(cx^2+bx+a\right)$$

$$\Big\downarrow \text{2611}$$

$$e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}$$

$$\Big\downarrow \text{2633}$$

$$e^{a+bx+cx^2}\sqrt{a+bx+cx^2}-\frac{1}{2}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

input
```
Int[E^(a + b*x + c*x^2)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2],x]
```

output
```
E^(a + b*x + c*x^2)*Sqrt[a + b*x + c*x^2] - (Sqrt[Pi]*Erfi[Sqrt[a + b*x +
c*x^2]])/2
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\mathrm{e}^{c\,x^{2}+b x+a}\sqrt{c\,x^{2}+b x+a}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c\,x^{2}+b x+a}\right)}{2}$	44
default	$\mathrm{e}^{c\,x^{2}+b x+a}\sqrt{c\,x^{2}+b x+a}-\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c\,x^{2}+b x+a}\right)}{2}$	44

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
exp(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)-1/2*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2)
)
```

Fricas [F]

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(2*c*x + b)*e^(c*x^2 + b*x + a), x)
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.50

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = \frac{\left(\sqrt{-a-bx-cx^2}e^{a+bx+cx^2}+\frac{\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-a-bx-cx^2}\right)}{2}\right)\sqrt{a+bx+cx^2}}{\sqrt{-a-bx-cx^2}}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)*(c*x**2+b*x+a)**(1/2),x)
```

output
```
(sqrt(-a - b*x - c*x**2)*exp(a + b*x + c*x**2) + sqrt(pi)*erfc(sqrt(-a - b
*x - c*x**2))/2)*sqrt(a + b*x + c*x**2)/sqrt(-a - b*x - c*x**2)
```

Maxima [F]

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = \int \sqrt{cx^2+bx+a}(2\,cx+b)e^{\left(cx^2+bx+a\right)}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(2*c*x + b)*e^(c*x^2 + b*x + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = -\frac{1}{2}i\sqrt{\pi}\,\mathrm{erf}\left(-i\sqrt{cx^2+bx+a}\right) + \sqrt{cx^2+bx+a}e^{(cx^2+bx+a)}$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

output
```
-1/2*I*sqrt(pi)*erf(-I*sqrt(c*x^2 + b*x + a)) + sqrt(c*x^2 + b*x + a)*e^(c
*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-c\,x^2-b\,x-a}\right)\,\sqrt{c\,x^2+b\,x+a}}{2\,\sqrt{-c\,x^2-b\,x-a}} + \mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\mathrm{e}^{c\,x^2}\,\sqrt{c\,x^2+b\,x+a}$$

input
```
int(exp(a + b*x + c*x^2)*(b + 2*c*x)*(a + b*x + c*x^2)^(1/2),x)
```

output
```
(pi^(1/2)*erfc((- a - b*x - c*x^2)^(1/2))*(a + b*x + c*x^2)^(1/2))/(2*(- a
 - b*x - c*x^2)^(1/2)) + exp(b*x)*exp(a)*exp(c*x^2)*(a + b*x + c*x^2)^(1/2
)
```

Reduce [F]

$$\int e^{a+bx+cx^2}(b+2cx)\sqrt{a+bx+cx^2}\,dx = e^a\left(2\left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}\,xdx\right)c + \left(\int e^{c\,x^2+bx}\sqrt{c\,x^2+bx+a}dx\right)b\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2),x)
```

output
```
e**a*(2*int(e**(b*x + c*x**2)*sqrt(a + b*x + c*x**2)*x,x)*c + int(e**(b*x
+ c*x**2)*sqrt(a + b*x + c*x**2),x)*b)
```

3.554 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 21

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx=\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(21) = 42.

Time = 2.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx=\frac{\sqrt{-a-x(b+cx)}\Gamma\left(\frac{1}{2},-a-x(b+cx)\right)}{\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
(Sqrt[-a - x*(b + c*x)]*Gamma[1/2, -a - x*(b + c*x)])/Sqrt[a + x*(b + c*x)
]
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {7258, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}d(a+bx+cx^2)$$

$$\Big\downarrow \text{2611}$$

$$2\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}$$

$$\Big\downarrow \text{2633}$$

$$\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/Sqrt[a + b*x + c*x^2],x]
```

output
```
Sqrt[Pi]*Erfi[Sqrt[a + b*x + c*x^2]]
```

Defintions of rubi rules used

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{c\,x^{2}+bx+a}\right)$	18
default	$\sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{c\,x^{2}+bx+a}\right)$	18

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Fricas [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(1/2),x, algorithm="fri
cas")
```

output
```
integral((2*c*x + b)*e^(c*x^2 + b*x + a)/sqrt(c*x^2 + b*x + a), x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(19) = 38$.

Time = 1.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{\pi}\sqrt{-a-bx-cx^2}\,\operatorname{erfc}\left(\sqrt{-a-bx-cx^2}\right)}{\sqrt{a+bx+cx^2}}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
sqrt(pi)*sqrt(-a - b*x - c*x**2)*erfc(sqrt(-a - b*x - c*x**2))/sqrt(a + b*
x + c*x**2)
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{\sqrt{cx^2+bx+a}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/sqrt(c*x^2 + b*x + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx = i\sqrt{\pi}\,\operatorname{erf}\left(-i\sqrt{cx^2+bx+a}\right)$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(1/2),x, algorithm="gia
c")
```

output
```
I*sqrt(pi)*erf(-I*sqrt(c*x^2 + b*x + a))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx = \frac{\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-c\,x^2-b\,x-a}\right)\,\sqrt{-c\,x^2-b\,x-a}}{\sqrt{c\,x^2+b\,x+a}}$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(1/2),x)
```

output
```
(pi^(1/2)*erfc((- a - b*x - c*x^2)^(1/2))*(- a - b*x - c*x^2)^(1/2))/(a +
b*x + c*x^2)^(1/2)
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\sqrt{a+bx+cx^2}}\,dx = e^a\left(\left(\int \frac{e^{c\,x^2+bx}}{\sqrt{c\,x^2+bx+a}}dx\right)b+2\left(\int \frac{e^{c\,x^2+bx}x}{\sqrt{c\,x^2+bx+a}}dx\right)c\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(1/2),x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/sqrt(a + b*x + c*x**2),x)*b + 2*int((e**(b*x +
 c*x**2)*x)/sqrt(a + b*x + c*x**2),x)*c)
```

3.555 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 51

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}} + 2\sqrt{\pi}\,\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
-2*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(1/2)+2*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2
))
```

Mathematica [A] (verified)

Time = 2.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{-2e^{a+x(b+cx)} + 2\sqrt{-a-x(b+cx)}\,\Gamma\left(\frac{1}{2},-a-x(b+cx)\right)}{\sqrt{a+x(b+cx)}}$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*E^(a + x*(b + c*x)) + 2*Sqrt[-a - x*(b + c*x)]*Gamma[1/2, -a - x*(b +
c*x)])/Sqrt[a + x*(b + c*x)]
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {7258, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{(a+bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{(a+bx+cx^2)^{3/2}}d\left(a+bx+cx^2\right)$$

$$\Big\downarrow \text{2608}$$

$$2\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d\left(cx^2+bx+a\right)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{2611}$$

$$4\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}$$

$$\Big\downarrow \text{2633}$$

$$2\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(3/2),x]
```

output
```
(-2*E^(a + b*x + c*x^2))/Sqrt[a + b*x + c*x^2] + 2*Sqrt[Pi]*Erfi[Sqrt[a +
b*x + c*x^2]]
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{2\,\mathrm{e}^{c\,x^{2}+b x +a}}{\sqrt{c\,x^{2}+b x +a}}+2 \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)$	45
default	$-\frac{2\,\mathrm{e}^{c\,x^{2}+b x +a}}{\sqrt{c\,x^{2}+b x +a}}+2 \sqrt{\pi}\, \operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)$	45

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(3/2),x,method=_RETURNVERBOSE
)
```

output
```
-2*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(1/2)+2*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2
))
```

Fricas [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(3/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(2*c*x + b)*e^(c*x^2 + b*x + a)/(c^2*x^4 +
2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.57

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = \frac{\left(-2\sqrt{\pi}\operatorname{erfc}\left(\sqrt{-a-bx-cx^2}\right)+\frac{2e^{a+bx+cx^2}}{\sqrt{-a-bx-cx^2}}\right)(-a-bx-cx^2)^{\frac{3}{2}}}{(a+bx+cx^2)^{\frac{3}{2}}}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**(3/2),x)
```

output
```
(-2*sqrt(pi)*erfc(sqrt(-a - b*x - c*x**2)) + 2*exp(a + b*x + c*x**2)/sqrt(
-a - b*x - c*x**2))*(-a - b*x - c*x**2)**(3/2)/(a + b*x + c*x**2)**(3/2)
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(3/2),x, algorithm="max
ima")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(3/2), x)
```

Giac [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(3/2),x, algorithm="gia
c")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.55

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = -\frac{e^{c\,x^2+b\,x+a}\,(2\,c\,x^2+2\,b\,x+2\,a)+2\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-c\,x^2-b\,x-a}\right)\,(-c\,x^2-b\,x-a)^{3/2}}{(c\,x^2+b\,x+a)^{3/2}}$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(3/2),x)
```

output
```
-(exp(a + b*x + c*x^2)*(2*a + 2*b*x + 2*c*x^2) + 2*pi^(1/2)*erfc((- a - b*
x - c*x^2)^(1/2))*(- a - b*x - c*x^2)^(3/2))/(a + b*x + c*x^2)^(3/2)
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{3/2}}\,dx = e^{a}\left(\left(\int \frac{e^{c\,x^2+bx}}{\sqrt{c\,x^2+bx+a}\,a+\sqrt{c\,x^2+bx+a}\,bx+\sqrt{c\,x^2+bx+a}\,c\,x^2}dx\right)b + 2\left(\int \frac{e^{c\,x^2+bx}x}{\sqrt{c\,x^2+bx+a}\,a+\sqrt{c\,x^2+bx+a}\,bx+\sqrt{c\,x^2+bx+a}\,c\,x^2}dx\right)c\right)$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(3/2),x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(sqrt(a + b*x + c*x**2)*a + sqrt(a + b*x + c*x
**2)*b*x + sqrt(a + b*x + c*x**2)*c*x**2),x)*b + 2*int((e**(b*x + c*x**2)*
x)/(sqrt(a + b*x + c*x**2)*a + sqrt(a + b*x + c*x**2)*b*x + sqrt(a + b*x +
 c*x**2)*c*x**2),x)*c)
```

3.556 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 85

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2e^{a+bx+cx^2}}{3\,(a+bx+cx^2)^{3/2}} - \frac{4e^{a+bx+cx^2}}{3\sqrt{a+bx+cx^2}} + \frac{4}{3}\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
-2/3*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(3/2)-4/3*exp(c*x^2+b*x+a)/(c*x^2+b*x+
a)^(1/2)+4/3*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 4.93 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{2\left(e^{a+x(b+cx)}(1+2(a+x(b+cx)))+2(-a-x(b+cx))^{3/2}\Gamma\left(\frac{1}{2},-a-x(b+cx)\right)\right)}{3(a+x(b+cx))^{3/2}}$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(E^(a + x*(b + c*x))*(1 + 2*(a + x*(b + c*x))) + 2*(-a - x*(b + c*x))^
(3/2)*Gamma[1/2, -a - x*(b + c*x)]))/(3*(a + x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {7258, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{(a+bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{(a+bx+cx^2)^{5/2}}d(a+bx+cx^2)$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{3}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^{3/2}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{3}\left(2\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2}{3}\left(4\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{3}\left(2\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(5/2),x]
```

output
```
(-2*E^(a + b*x + c*x^2))/(3*(a + b*x + c*x^2)^(3/2)) + (2*((-2*E^(a + b*x
+ c*x^2))/Sqrt[a + b*x + c*x^2] + 2*Sqrt[Pi]*Erfi[Sqrt[a + b*x + c*x^2]]))
/3
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2\,{\mathrm e}^{c\,x^{2}+b x +a}}{3\left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}-\frac{4\,{\mathrm e}^{c\,x^{2}+b x +a}}{3\sqrt{c\,x^{2}+b x +a}}+\frac{4\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{3}$	70
default	$-\frac{2\,{\mathrm e}^{c\,x^{2}+b x +a}}{3\left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}-\frac{4\,{\mathrm e}^{c\,x^{2}+b x +a}}{3\sqrt{c\,x^{2}+b x +a}}+\frac{4\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{3}$	70

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(5/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/3*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(3/2)-4/3*exp(c*x^2+b*x+a)/(c*x^2+b*x+
a)^(1/2)+4/3*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Fricas [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(2\,cx+b)e^{\left(cx^2+bx+a\right)}}{(cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(5/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(2*c*x + b)*e^(c*x^2 + b*x + a)/(c^3*x^6 +
3*b*c^2*x^5 + 3*(b^2*c + a*c^2)*x^4 + 3*a^2*b*x + (b^3 + 6*a*b*c)*x^3 + a^
3 + 3*(a*b^2 + a^2*c)*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 17.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = \frac{\left(\frac{4\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-a-bx-cx^2}\right)}{3} - \frac{\left(-\frac{4a}{3}-\frac{4bx}{3}-\frac{4cx^2}{3}-\frac{2}{3}\right)e^{a+bx+cx^2}}{(-a-bx-cx^2)^{\frac{3}{2}}}\right)(-a-bx-cx^2)^{\frac{5}{2}}}{(a+bx+cx^2)^{\frac{5}{2}}}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**(5/2),x)
```

output
```
(4*sqrt(pi)*erfc(sqrt(-a - b*x - c*x**2))/3 - (-4*a/3 - 4*b*x/3 - 4*c*x**2
/3 - 2/3)*exp(a + b*x + c*x**2)/(-a - b*x - c*x**2)**(3/2))*(-a - b*x - c*
x**2)**(5/2)/(a + b*x + c*x**2)**(5/2)
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(5/2),x, algorithm="max
ima")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(5/2), x)
```

Giac [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(5/2),x, algorithm="gia
c")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = -\frac{\mathrm{e}^{c\,x^2+b\,x+a}\,(2\,c\,x^2+2\,b\,x+2\,a)+4\,\mathrm{e}^{c\,x^2+b\,x+a}\,{(c\,x^2+b\,x+a)}^2-4\,\sqrt{\pi}\,\mathrm{erfc}\left(\sqrt{-c\,x^2-b\,x-a}\right)\,(-c\,x^2-b}{3\,{(c\,x^2+b\,x+a)}^{5/2}}$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(5/2),x)
```

output
```
-(exp(a + b*x + c*x^2)*(2*a + 2*b*x + 2*c*x^2) + 4*exp(a + b*x + c*x^2)*(a
 + b*x + c*x^2)^2 - 4*pi^(1/2)*erfc((- a - b*x - c*x^2)^(1/2))*(- a - b*x
- c*x^2)^(5/2))/(3*(a + b*x + c*x^2)^(5/2))
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{5/2}}\,dx = e^a\left(\left(\left(\int \frac{e^{c\,x^2+b}}{\sqrt{c\,x^2+bx+a}\,a^2+2\sqrt{c\,x^2+bx+a}\,abx+2\sqrt{c\,x^2+bx+a}\,ac\,x^2+\sqrt{}}\right.\right.\right.$$

$$+2\left(\int \frac{e^{c\,x^2+bx}x}{\sqrt{c\,x^2+bx+a}\,a^2+2\sqrt{c\,x^2+bx+a}\,abx+2\sqrt{c\,x^2+bx+a}\,ac\,x^2+\sqrt{c\,x^2+bx+a}\,b^2x^2+2\sqrt{c\,x^2}}\right.$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(5/2),x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(sqrt(a + b*x + c*x**2)*a**2 + 2*sqrt(a + b*x
+ c*x**2)*a*b*x + 2*sqrt(a + b*x + c*x**2)*a*c*x**2 + sqrt(a + b*x + c*x**
2)*b**2*x**2 + 2*sqrt(a + b*x + c*x**2)*b*c*x**3 + sqrt(a + b*x + c*x**2)*
c**2*x**4),x)*b + 2*int((e**(b*x + c*x**2)*x)/(sqrt(a + b*x + c*x**2)*a**2
 + 2*sqrt(a + b*x + c*x**2)*a*b*x + 2*sqrt(a + b*x + c*x**2)*a*c*x**2 + sq
rt(a + b*x + c*x**2)*b**2*x**2 + 2*sqrt(a + b*x + c*x**2)*b*c*x**3 + sqrt(
a + b*x + c*x**2)*c**2*x**4),x)*c)
```

3.557 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{\left(a+bx+cx^2\right)^{7/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 115

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\left(a+bx+cx^2\right)^{7/2}}\,dx = -\frac{2e^{a+bx+cx^2}}{5\left(a+bx+cx^2\right)^{5/2}}-\frac{4e^{a+bx+cx^2}}{15\left(a+bx+cx^2\right)^{3/2}} -\frac{8e^{a+bx+cx^2}}{15\sqrt{a+bx+cx^2}}+\frac{8}{15}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
-2/5*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(5/2)-4/15*exp(c*x^2+b*x+a)/(c*x^2+b*x
+a)^(3/2)-8/15*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(1/2)+8/15*Pi^(1/2)*erfi((c*
x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 5.79 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{\left(a+bx+cx^2\right)^{7/2}}\,dx = \frac{-2e^{a+x(b+cx)}\left(3+2(a+x(b+cx))+4(a+x(b+cx))^2\right)+8(-a-x(b+cx))^{5/}}{15(a+x(b+cx))^{5/2}}$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(7/2),x]
```

output
```
(-2*E^(a + x*(b + c*x))*(3 + 2*(a + x*(b + c*x)) + 4*(a + x*(b + c*x))^2)
+ 8*(-a - x*(b + c*x))^(5/2)*Gamma[1/2, -a - x*(b + c*x)])/(15*(a + x*(b +
 c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7258, 2608, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{(a+bx+cx^2)^{7/2}}dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{(a+bx+cx^2)^{7/2}}d(a+bx+cx^2)$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{5}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^{5/2}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{5}\left(\frac{2}{3}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^{3/2}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{5}\left(\frac{2}{3}\left(2\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2}{5}\left(\frac{2}{3}\left(4\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3\left(a+bx+cx^2\right)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5\left(a+bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \ 2633$$

$$\frac{2}{5}\left(\frac{2}{3}\left(2\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3\left(a+bx+cx^2\right)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5\left(a+bx+cx^2\right)^{5/2}}$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(7/2),x]
```

output
```
(-2*E^(a + b*x + c*x^2))/(5*(a + b*x + c*x^2)^(5/2)) + (2*((-2*E^(a + b*x
+ c*x^2))/(3*(a + b*x + c*x^2)^(3/2)) + (2*((-2*E^(a + b*x + c*x^2))/Sqrt[
a + b*x + c*x^2] + 2*Sqrt[Pi]*Erfi[Sqrt[a + b*x + c*x^2]]))/3))/5
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$-\frac{2\,{\mathrm e}^{c\,x^{2}+b x +a}}{5\left(c\,x^{2}+b x +a \right)^{\frac{5}{2}}}-\frac{4\,{\mathrm e}^{c\,x^{2}+b x +a}}{15\left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}-\frac{8\,{\mathrm e}^{c\,x^{2}+b x +a}}{15\sqrt{c\,x^{2}+b x +a}}+\frac{8\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{15}$	95
default	$-\frac{2\,{\mathrm e}^{c\,x^{2}+b x +a}}{5\left(c\,x^{2}+b x +a \right)^{\frac{5}{2}}}-\frac{4\,{\mathrm e}^{c\,x^{2}+b x +a}}{15\left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}-\frac{8\,{\mathrm e}^{c\,x^{2}+b x +a}}{15\sqrt{c\,x^{2}+b x +a}}+\frac{8\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{15}$	95

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(7/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/5*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(5/2)-4/15*exp(c*x^2+b*x+a)/(c*x^2+b*x
+a)^(3/2)-8/15*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(1/2)+8/15*Pi^(1/2)*erfi((c*
x^2+b*x+a)^(1/2))
```

Fricas [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}}\,dx=\int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(7/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(2*c*x + b)*e^(c*x^2 + b*x + a)/(c^4*x^8 +
4*b*c^3*x^7 + 2*(3*b^2*c^2 + 2*a*c^3)*x^6 + 4*(b^3*c + 3*a*b*c^2)*x^5 + 4*
a^3*b*x + (b^4 + 12*a*b^2*c + 6*a^2*c^2)*x^4 + a^4 + 4*(a*b^3 + 3*a^2*b*c)
*x^3 + 2*(3*a^2*b^2 + 2*a^3*c)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(7/2),x, algorithm="max
ima")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(7/2), x)
```

Giac [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(7/2),x, algorithm="gia
c")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.12

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{\mathrm{e}^{c\,x^2+b\,x+a}\,(6\,c\,x^2+6\,b\,x+6\,a)+4\,\mathrm{e}^{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+a)^2+8\,\mathrm{e}^{c\,x^2+b\,x+a}\,(c\,x^2+b\,x+a)^3+8\,\sqrt{\pi}\,\mathrm{erfc}}{15\,(c\,x^2+b\,x+a)^{7/2}}$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(7/2),x)
```

output
```
-(exp(a + b*x + c*x^2)*(6*a + 6*b*x + 6*c*x^2) + 4*exp(a + b*x + c*x^2)*(a
 + b*x + c*x^2)^2 + 8*exp(a + b*x + c*x^2)*(a + b*x + c*x^2)^3 + 8*pi^(1/2
)*erfc((- a - b*x - c*x^2)^(1/2))*(- a - b*x - c*x^2)^(7/2))/(15*(a + b*x
+ c*x^2)^(7/2))
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{7/2}}\,dx = e^a\left(\left(\left(\int \frac{}{\sqrt{c\,x^2+bx+a}\,a^3+3\sqrt{c\,x^2+bx+a}\,a^2bx+3\sqrt{c\,x^2+bx+a}\,a^2c\,x^2+}\right.\right.\right.$$
$$+2\left(\int \frac{}{\sqrt{c\,x^2+bx+a}\,a^3+3\sqrt{c\,x^2+bx+a}\,a^2bx+3\sqrt{c\,x^2+bx+a}\,a^2c\,x^2+3\sqrt{c\,x^2+bx+a}\,a\,b^2x^2+6\sqrt{}}\right.$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(7/2),x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(sqrt(a + b*x + c*x**2)*a**3 + 3*sqrt(a + b*x
+ c*x**2)*a**2*b*x + 3*sqrt(a + b*x + c*x**2)*a**2*c*x**2 + 3*sqrt(a + b*x
 + c*x**2)*a*b**2*x**2 + 6*sqrt(a + b*x + c*x**2)*a*b*c*x**3 + 3*sqrt(a +
b*x + c*x**2)*a*c**2*x**4 + sqrt(a + b*x + c*x**2)*b**3*x**3 + 3*sqrt(a +
b*x + c*x**2)*b**2*c*x**4 + 3*sqrt(a + b*x + c*x**2)*b*c**2*x**5 + sqrt(a
+ b*x + c*x**2)*c**3*x**6),x)*b + 2*int((e**(b*x + c*x**2)*x)/(sqrt(a + b*
x + c*x**2)*a**3 + 3*sqrt(a + b*x + c*x**2)*a**2*b*x + 3*sqrt(a + b*x + c*
x**2)*a**2*c*x**2 + 3*sqrt(a + b*x + c*x**2)*a*b**2*x**2 + 6*sqrt(a + b*x
+ c*x**2)*a*b*c*x**3 + 3*sqrt(a + b*x + c*x**2)*a*c**2*x**4 + sqrt(a + b*x
 + c*x**2)*b**3*x**3 + 3*sqrt(a + b*x + c*x**2)*b**2*c*x**4 + 3*sqrt(a + b
*x + c*x**2)*b*c**2*x**5 + sqrt(a + b*x + c*x**2)*c**3*x**6),x)*c)
```

3.558 $\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 145

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx = -\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}} - \frac{4e^{a+bx+cx^2}}{35(a+bx+cx^2)^{5/2}} - \frac{8e^{a+bx+cx^2}}{105(a+bx+cx^2)^{3/2}} - \frac{16e^{a+bx+cx^2}}{105\sqrt{a+bx+cx^2}} + \frac{16}{105}\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{a+bx+cx^2}\right)$$

output
```
-2/7*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(7/2)-4/35*exp(c*x^2+b*x+a)/(c*x^2+b*x
+a)^(5/2)-8/105*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(3/2)-16/105*exp(c*x^2+b*x+
a)/(c*x^2+b*x+a)^(1/2)+16/105*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Mathematica [A] (verified)

Time = 7.36 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.71

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx =$$

$$-\frac{2\left(e^{a+x(b+cx)}(15+6(a+x(b+cx))+4(a+x(b+cx))^2+8(a+x(b+cx))^3)+8(-a-x(b+cx))^{7/2}\Gamma\left(\frac{1}{2}\right.\right.}{105(a+x(b+cx))^{7/2}}$$

input
```
Integrate[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(9/2),x]
```

output
```
(-2*(E^(a + x*(b + c*x))*(15 + 6*(a + x*(b + c*x)) + 4*(a + x*(b + c*x))^2
 + 8*(a + x*(b + c*x))^3) + 8*(-a - x*(b + c*x))^(7/2)*Gamma[1/2, -a - x*(
b + c*x)]))/(105*(a + x*(b + c*x))^(7/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {7258, 2608, 2608, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b+2cx)e^{a+bx+cx^2}}{(a+bx+cx^2)^{9/2}}dx$$

$$\Big\downarrow \text{7258}$$

$$\int \frac{e^{a+bx+cx^2}}{(a+bx+cx^2)^{9/2}}d(a+bx+cx^2)$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{7}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^{7/2}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{7}\left(\frac{2}{5}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^{5/2}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}\right)-\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{7}\left(\frac{2}{5}\left(\frac{2}{3}\int \frac{e^{cx^2+bx+a}}{(cx^2+bx+a)^{3/2}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{3(a+bx+cx^2)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5(a+bx+cx^2)^{5/2}}\right)-\frac{2e^{a+bx+cx^2}}{7(a+bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{2608}$$

$$\frac{2}{7}\left(\frac{2}{5}\left(\frac{2}{3}\left(2\int \frac{e^{cx^2+bx+a}}{\sqrt{cx^2+bx+a}}d(cx^2+bx+a)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3\left(a+bx+cx^2\right)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5\left(a+bx+cx^2\right)^{5/2}}\right)-\frac{2e^{a+bx+cx^2}}{7\left(a+bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{2}{7}\left(\frac{2}{5}\left(\frac{2}{3}\left(4\int e^{cx^2+bx+a}d\sqrt{cx^2+bx+a}-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3\left(a+bx+cx^2\right)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5\left(a+bx+cx^2\right)^{5/2}}\right)-\frac{2e^{a+bx+cx^2}}{7\left(a+bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2633}$$

$$\frac{2}{7}\left(\frac{2}{5}\left(\frac{2}{3}\left(2\sqrt{\pi}\text{erfi}\left(\sqrt{a+bx+cx^2}\right)-\frac{2e^{a+bx+cx^2}}{\sqrt{a+bx+cx^2}}\right)-\frac{2e^{a+bx+cx^2}}{3\left(a+bx+cx^2\right)^{3/2}}\right)-\frac{2e^{a+bx+cx^2}}{5\left(a+bx+cx^2\right)^{5/2}}\right)-\frac{2e^{a+bx+cx^2}}{7\left(a+bx+cx^2\right)^{7/2}}$$

input
```
Int[(E^(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(9/2),x]
```

output
```
(-2*E^(a + b*x + c*x^2))/(7*(a + b*x + c*x^2)^(7/2)) + (2*((-2*E^(a + b*x
+ c*x^2))/(5*(a + b*x + c*x^2)^(5/2)) + (2*((-2*E^(a + b*x + c*x^2))/(3*(a
 + b*x + c*x^2)^(3/2)) + (2*((-2*E^(a + b*x + c*x^2))/Sqrt[a + b*x + c*x^2
] + 2*Sqrt[Pi]*Erfi[Sqrt[a + b*x + c*x^2]]))/3))/5))/7
```

Defintions of rubi rules used

rule 2608
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1)))   Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
 f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] &&  !TrueQ[$UseGamma]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 7258
```
Int[(F_)^(v_)*(u_)*(w_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[v,
u, x]}, Simp[q   Subst[Int[x^m*F^x, x], x, v], x] /;  !FalseQ[q]] /; FreeQ[
{F, m}, x] && EqQ[w, v]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{2\,{\mathrm e}^{c\,x^{2}+b x +a}}{7\left(c\,x^{2}+b x +a \right)^{\frac{7}{2}}}-\frac{4\,{\mathrm e}^{c\,x^{2}+b x +a}}{35\left(c\,x^{2}+b x +a \right)^{\frac{5}{2}}}-\frac{8\,{\mathrm e}^{c\,x^{2}+b x +a}}{105\left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}-\frac{16\,{\mathrm e}^{c\,x^{2}+b x +a}}{105\sqrt{c\,x^{2}+b x +a}}+\frac{16\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{105}$
default	$-\frac{2\,{\mathrm e}^{c\,x^{2}+b x +a}}{7\left(c\,x^{2}+b x +a \right)^{\frac{7}{2}}}-\frac{4\,{\mathrm e}^{c\,x^{2}+b x +a}}{35\left(c\,x^{2}+b x +a \right)^{\frac{5}{2}}}-\frac{8\,{\mathrm e}^{c\,x^{2}+b x +a}}{105\left(c\,x^{2}+b x +a \right)^{\frac{3}{2}}}-\frac{16\,{\mathrm e}^{c\,x^{2}+b x +a}}{105\sqrt{c\,x^{2}+b x +a}}+\frac{16\sqrt{\pi}\,\operatorname{erfi}\left(\sqrt{c\,x^{2}+b x +a}\right)}{105}$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(9/2),x,method=_RETURNVERBOSE
)
```

output
```
-2/7*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(7/2)-4/35*exp(c*x^2+b*x+a)/(c*x^2+b*x
+a)^(5/2)-8/105*exp(c*x^2+b*x+a)/(c*x^2+b*x+a)^(3/2)-16/105*exp(c*x^2+b*x+
a)/(c*x^2+b*x+a)^(1/2)+16/105*Pi^(1/2)*erfi((c*x^2+b*x+a)^(1/2))
```

Fricas [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(9/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(c*x^2 + b*x + a)*(2*c*x + b)*e^(c*x^2 + b*x + a)/(c^5*x^10 +
 5*b*c^4*x^9 + 5*(2*b^2*c^3 + a*c^4)*x^8 + 10*(b^3*c^2 + 2*a*b*c^3)*x^7 +
5*(b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*x^6 + 5*a^4*b*x + (b^5 + 20*a*b^3*c +
30*a^2*b*c^2)*x^5 + a^5 + 5*(a*b^4 + 6*a^2*b^2*c + 2*a^3*c^2)*x^4 + 10*(a^
2*b^3 + 2*a^3*b*c)*x^3 + 5*(2*a^3*b^2 + a^4*c)*x^2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*x**2+b*x+a)*(2*c*x+b)/(c*x**2+b*x+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx = \int \frac{(2\,cx+b)e^{(cx^2+bx+a)}}{(cx^2+bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(9/2),x, algorithm="max
ima")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(9/2), x)
```

Giac [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx=\int \frac{(2\,cx+b)e^{\left(cx^2+bx+a\right)}}{(cx^2+bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(9/2),x, algorithm="gia
c")
```

output
```
integrate((2*c*x + b)*e^(c*x^2 + b*x + a)/(c*x^2 + b*x + a)^(9/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx=$$
$$-\frac{\mathrm{e}^{c\,x^2+b\,x+a}\,(30\,c\,x^2+30\,b\,x+30\,a)+12\,\mathrm{e}^{c\,x^2+b\,x+a}\,{(c\,x^2+b\,x+a)}^2+8\,\mathrm{e}^{c\,x^2+b\,x+a}\,{(c\,x^2+b\,x+a)}^3+16\,\mathrm{e}^{c}}{105\,{(c\,x^2+b\,x+a)}^{9/2}}$$

input
```
int((exp(a + b*x + c*x^2)*(b + 2*c*x))/(a + b*x + c*x^2)^(9/2),x)
```

output
```
-(exp(a + b*x + c*x^2)*(30*a + 30*b*x + 30*c*x^2) + 12*exp(a + b*x + c*x^2
)*(a + b*x + c*x^2)^2 + 8*exp(a + b*x + c*x^2)*(a + b*x + c*x^2)^3 + 16*ex
p(a + b*x + c*x^2)*(a + b*x + c*x^2)^4 - 16*pi^(1/2)*erfc((- a - b*x - c*x
^2)^(1/2))*(- a - b*x - c*x^2)^(9/2))/(105*(a + b*x + c*x^2)^(9/2))
```

Reduce [F]

$$\int \frac{e^{a+bx+cx^2}(b+2cx)}{(a+bx+cx^2)^{9/2}}\,dx = e^a\left(\left(\left(\int \frac{}{\sqrt{c\,x^2+bx+a}\,a^4+4\sqrt{c\,x^2+bx+a}\,a^3bx+4\sqrt{c\,x^2+bx+a}\,a^3c\,x^2+\right.\right.\right.$$

$$+2\left(\int \frac{}{\sqrt{c\,x^2+bx+a}\,a^4+4\sqrt{c\,x^2+bx+a}\,a^3bx+4\sqrt{c\,x^2+bx+a}\,a^3c\,x^2+6\sqrt{c\,x^2+bx+a}\,a^2b^2x^2+12\right.$$

input
```
int(exp(c*x^2+b*x+a)*(2*c*x+b)/(c*x^2+b*x+a)^(9/2),x)
```

output
```
e**a*(int(e**(b*x + c*x**2)/(sqrt(a + b*x + c*x**2)*a**4 + 4*sqrt(a + b*x
+ c*x**2)*a**3*b*x + 4*sqrt(a + b*x + c*x**2)*a**3*c*x**2 + 6*sqrt(a + b*x
 + c*x**2)*a**2*b**2*x**2 + 12*sqrt(a + b*x + c*x**2)*a**2*b*c*x**3 + 6*sq
rt(a + b*x + c*x**2)*a**2*c**2*x**4 + 4*sqrt(a + b*x + c*x**2)*a*b**3*x**3
 + 12*sqrt(a + b*x + c*x**2)*a*b**2*c*x**4 + 12*sqrt(a + b*x + c*x**2)*a*b
*c**2*x**5 + 4*sqrt(a + b*x + c*x**2)*a*c**3*x**6 + sqrt(a + b*x + c*x**2)
*b**4*x**4 + 4*sqrt(a + b*x + c*x**2)*b**3*c*x**5 + 6*sqrt(a + b*x + c*x**
2)*b**2*c**2*x**6 + 4*sqrt(a + b*x + c*x**2)*b*c**3*x**7 + sqrt(a + b*x +
c*x**2)*c**4*x**8),x)*b + 2*int((e**(b*x + c*x**2)*x)/(sqrt(a + b*x + c*x*
*2)*a**4 + 4*sqrt(a + b*x + c*x**2)*a**3*b*x + 4*sqrt(a + b*x + c*x**2)*a*
*3*c*x**2 + 6*sqrt(a + b*x + c*x**2)*a**2*b**2*x**2 + 12*sqrt(a + b*x + c*
x**2)*a**2*b*c*x**3 + 6*sqrt(a + b*x + c*x**2)*a**2*c**2*x**4 + 4*sqrt(a +
 b*x + c*x**2)*a*b**3*x**3 + 12*sqrt(a + b*x + c*x**2)*a*b**2*c*x**4 + 12*
sqrt(a + b*x + c*x**2)*a*b*c**2*x**5 + 4*sqrt(a + b*x + c*x**2)*a*c**3*x**
6 + sqrt(a + b*x + c*x**2)*b**4*x**4 + 4*sqrt(a + b*x + c*x**2)*b**3*c*x**
5 + 6*sqrt(a + b*x + c*x**2)*b**2*c**2*x**6 + 4*sqrt(a + b*x + c*x**2)*b*c
**3*x**7 + sqrt(a + b*x + c*x**2)*c**4*x**8),x)*c)
```

3.559 $\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 8

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} \, dx = -\arcsin\left(e^{-x}\right)$$

output
```
-arcsin(exp(-x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(8) = 16.

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 5.25

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} \, dx = \frac{e^{-x}\sqrt{-1+e^{2x}}\arctan\left(\sqrt{-1+e^{2x}}\right)}{\sqrt{1-e^{-2x}}}$$

input
```
Integrate[1/(E^x*Sqrt[1 - E^(-2*x)]),x]
```

output
```
(Sqrt[-1 + E^(2*x)]*ArcTan[Sqrt[-1 + E^(2*x)]])/(E^x*Sqrt[1 - E^(-2*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2679, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$$
$$\Big\downarrow \text{ 2679}$$
$$-\int \frac{1}{\sqrt{1-e^{-2x}}}de^{-x}$$
$$\Big\downarrow \text{ 223}$$
$$-\arcsin\left(e^{-x}\right)$$

input
```
Int[1/(E^x*Sqrt[1 - E^(-2*x)]),x]
```

output
```
-ArcSin[E^(-x)]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(7) = 14.

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 4.62

method	result	size
default	$-\frac{\mathrm{e}^{-x} \sqrt{-1+\mathrm{e}^{2 x}}\, \arctan\left(\frac{1}{\sqrt{-1+\mathrm{e}^{2 x}}}\right)}{\sqrt{\left(-1+\mathrm{e}^{2 x}\right) \mathrm{e}^{-2 x}}}$	37

input
```
int(1/exp(x)/(1-exp(-2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/((exp(x)^2-1)/exp(x)^2)^(1/2)/exp(x)*(exp(x)^2-1)^(1/2)*arctan(1/(exp(x
)^2-1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = 2\arctan\left(\left(\sqrt{-e^{(-2x)}+1}-1\right)e^{x}\right)$$

input
```
integrate(1/exp(x)/(1-exp(-2*x))^(1/2),x, algorithm="fricas")
```

output
```
2*arctan((sqrt(-e^(-2*x) + 1) - 1)*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = -\operatorname{asin}\left(e^{-x}\right)$$

input
```
integrate(1/exp(x)/(1-exp(-2*x))**(1/2),x)
```

output
```
-asin(exp(-x))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \arctan\left(\sqrt{-e^{(-2\,x)}+1}e^{x}\right)$$

input
```
integrate(1/exp(x)/(1-exp(-2*x))^(1/2),x, algorithm="maxima")
```

output
```
arctan(sqrt(-e^(-2*x) + 1)*e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

input
```
integrate(1/exp(x)/(1-exp(-2*x))^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(e^(2*x) - 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \int \frac{\mathrm{e}^{-x}}{\sqrt{1-\mathrm{e}^{-2\,x}}}\,dx$$

input
```
int(exp(-x)/(1 - exp(-2*x))^(1/2),x)
```

output
```
int(exp(-x)/(1 - exp(-2*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx = \int \frac{\sqrt{e^{2x}-1}}{e^{2x}-1}dx$$

input
```
int(1/exp(x)/(1-exp(-2*x))^(1/2),x)
```

output
```
int(sqrt(e**(2*x) - 1)/(e**(2*x) - 1),x)
```

3.560 $\int \frac{e^x}{4+e^{2x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^x}{4+e^{2x}} \, dx = \frac{1}{2} \arctan \left(\frac{e^x}{2}\right)$$

output
```
1/2*arctan(1/2*exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{4+e^{2x}} \, dx = \frac{1}{2} \arctan \left(\frac{e^x}{2}\right)$$

input
```
Integrate[E^x/(4 + E^(2*x)),x]
```

output
```
ArcTan[E^x/2]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}+4}dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{e^{2x}+4}de^x$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\arctan\left(\frac{e^x}{2}\right)$$

input
```
Int[E^x/(4 + E^(2*x)),x]
```

output
```
ArcTan[E^x/2]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{e^{x}}{2}\right)}{2}$	8
risch	$\frac{i\ln(e^{x}+2i)}{4}-\frac{i\ln(e^{x}-2i)}{4}$	20

input
```
int(exp(x)/(4+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x}{4+e^{2x}}\,dx=\frac{1}{2}\arctan\left(\frac{1}{2}e^x\right)$$

input
```
integrate(exp(x)/(4+exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*arctan(1/2*e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{4+e^{2x}}\,dx=\operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(8i+e^x\right)\right)\right)$$

input
```
integrate(exp(x)/(4+exp(2*x)),x)
```

output
```
RootSum(16*_z**2 + 1, Lambda(_i, _i*log(8*_i + exp(x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x}{4+e^{2x}}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}e^x\right)$$

input
```
integrate(exp(x)/(4+exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*arctan(1/2*e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x}{4+e^{2x}}\,dx = \frac{1}{2}\arctan\left(\frac{1}{2}e^x\right)$$

input
```
integrate(exp(x)/(4+exp(2*x)),x, algorithm="giac")
```

output
```
1/2*arctan(1/2*e^x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{e^x}{4+e^{2x}}\,dx = \frac{\mathrm{atan}\left(\frac{e^x}{2}\right)}{2}$$

input
```
int(exp(x)/(exp(2*x) + 4),x)
```

output
```
atan(exp(x)/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{4+e^{2x}}\,dx = \frac{atan\left(\frac{e^x}{2}\right)}{2}$$

input
```
int(exp(x)/(4+exp(2*x)),x)
```

output
```
atan(e**x/2)/2
```

3.561 $\int \frac{e^x}{1-e^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 4

$$\int \frac{e^x}{1-e^{2x}}\,dx = \operatorname{arctanh}(e^x)$$

output
```
arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1-e^{2x}}\,dx = \operatorname{arctanh}(e^x)$$

input
```
Integrate[E^x/(1 - E^(2*x)),x]
```

output
```
ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2679, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-e^{2x}} \, dx$$

$$\Big\downarrow \text{2679}$$

$$\int \frac{1}{1-e^{2x}} de^x$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}(e^x)$$

input
```
Int[E^x/(1 - E^(2*x)),x]
```

output
```
ArcTanh[E^x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\operatorname{arctanh}(\mathrm{e}^{x})$	4
norman	$-\frac{\ln(-1+\mathrm{e}^{x})}{2}+\frac{\ln(1+\mathrm{e}^{x})}{2}$	16
risch	$-\frac{\ln(-1+\mathrm{e}^{x})}{2}+\frac{\ln(1+\mathrm{e}^{x})}{2}$	16

input
```
int(exp(x)/(1-exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = \frac{1}{2}\log\left(e^x+1\right)-\frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(1-exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*log(e^x + 1) - 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = -\frac{\log\left(e^x-1\right)}{2}+\frac{\log\left(e^x+1\right)}{2}$$

input
```
integrate(exp(x)/(1-exp(2*x)),x)
```

output
```
-log(exp(x) - 1)/2 + log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1 - e^{2x}} \, dx = \frac{1}{2} \log \left(e^x + 1\right) - \frac{1}{2} \log \left(e^x - 1\right)$$

input
```
integrate(exp(x)/(1-exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*log(e^x + 1) - 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(3) = 6$.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 4.00

$$\int \frac{e^x}{1 - e^{2x}} \, dx = \frac{1}{2} \log \left(e^x + 1\right) - \frac{1}{2} \log \left(|e^x - 1|\right)$$

input
```
integrate(exp(x)/(1-exp(2*x)),x, algorithm="giac")
```

output
```
1/2*log(e^x + 1) - 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x+1\right)}{2}-\frac{\ln\left(\mathrm{e}^x-1\right)}{2}$$

input
```
int(-exp(x)/(exp(2*x) - 1),x)
```

output
```
log(exp(x) + 1)/2 - log(exp(x) - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 4.25

$$\int \frac{e^x}{1-e^{2x}}\,dx = -\frac{\log(e^x-1)}{2}+\frac{\log(e^x+1)}{2}$$

input
```
int(exp(x)/(1-exp(2*x)),x)
```

output
```
( - log(e**x - 1) + log(e**x + 1))/2
```

3.562 $\int \frac{e^x}{3-4e^{2x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{e^x}{3-4e^{2x}} \, dx = \frac{\operatorname{arctanh}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctanh(2/3*exp(x)*3^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{3-4e^{2x}} \, dx = \frac{\operatorname{arctanh}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Integrate[E^x/(3 - 4*E^(2*x)),x]
```

output
```
ArcTanh[(2*E^x)/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2679, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{3-4e^{2x}}\,dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{3-4e^{2x}}de^x$$
$$\Big\downarrow \text{219}$$
$$\frac{\text{arctanh}\left(\frac{2e^x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

input
```
Int[E^x/(3 - 4*E^(2*x)),x]
```

output
```
ArcTanh[(2*E^x)/Sqrt[3]]/(2*Sqrt[3])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{2\,\mathrm{e}^{x}\sqrt{3}}{3}\right)\sqrt{3}}{6}$	14
risch	$\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{x}+\frac{\sqrt{3}}{2}\right)}{12}-\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{x}-\frac{\sqrt{3}}{2}\right)}{12}$	30

input
```
int(exp(x)/(3-4*exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(2/3*exp(x)*3^(1/2))*3^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{e^x}{3-4e^{2x}}\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{4\sqrt{3}e^x+4\,e^{(2\,x)}+3}{4\,e^{(2\,x)}-3}\right)$$

input
```
integrate(exp(x)/(3-4*exp(2*x)),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((4*sqrt(3)*e^x + 4*e^(2*x) + 3)/(4*e^(2*x) - 3))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{3-4e^{2x}}\,dx = \operatorname{RootSum}\left(48z^2-1,\left(i\mapsto i\log\left(6i+e^x\right)\right)\right)$$

input
```
integrate(exp(x)/(3-4*exp(2*x)),x)
```

output
```
RootSum(48*_z**2 - 1, Lambda(_i, _i*log(6*_i + exp(x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{e^x}{3-4e^{2x}}\,dx = -\frac{1}{12}\,\sqrt{3}\log\left(-\frac{\sqrt{3}-2\,e^x}{\sqrt{3}+2\,e^x}\right)$$

input
```
integrate(exp(x)/(3-4*exp(2*x)),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(-(sqrt(3) - 2*e^x)/(sqrt(3) + 2*e^x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(13) = 26.

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{e^x}{3-4e^{2x}}\,dx = \frac{1}{12}\,\sqrt{3}\log\left(\frac{1}{2}\,\sqrt{3}+e^x\right)-\frac{1}{12}\,\sqrt{3}\log\left(\left|-\frac{1}{2}\,\sqrt{3}+e^x\right|\right)$$

input
```
integrate(exp(x)/(3-4*exp(2*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(1/2*sqrt(3) + e^x) - 1/12*sqrt(3)*log(abs(-1/2*sqrt(3) +
e^x))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \frac{e^x}{3-4e^{2x}}\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\,\mathrm{e}^x}{3}\right)}{6}$$

input
```
int(-exp(x)/(4*exp(2*x) - 3),x)
```

output
```
(3^(1/2)*atanh((2*3^(1/2)*exp(x))/3))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{e^x}{3-4e^{2x}}\,dx = \frac{\sqrt{3}\left(-\log\left(2e^x-\sqrt{3}\right)+\log\left(2e^x+\sqrt{3}\right)\right)}{12}$$

input
```
int(exp(x)/(3-4*exp(2*x)),x)
```

output
```
(sqrt(3)*( - log(2*e**x - sqrt(3)) + log(2*e**x + sqrt(3))))/12
```

3.563 $\int e^x\sqrt{3-4e^{2x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{3-4e^{2x}}+\frac{3}{4}\arcsin\left(\frac{2e^x}{\sqrt{3}}\right)$$

output
```
1/2*exp(x)*(3-4*exp(2*x))^(1/2)+3/4*arcsin(2/3*exp(x)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.47

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{3-4e^{2x}}+\frac{3}{2}\arctan\left(\frac{-\sqrt{3}+2e^x}{\sqrt{3-4e^{2x}}}\right)$$

input
```
Integrate[E^x*Sqrt[3 - 4*E^(2*x)],x]
```

output
```
(E^x*Sqrt[3 - 4*E^(2*x)])/2 + (3*ArcTan[(-Sqrt[3] + 2*E^x)/Sqrt[3 - 4*E^(2
*x)]])/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2679, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sqrt{3-4e^{2x}} \, dx$$

$$\Big\downarrow \text{ 2679}$$

$$\int \sqrt{3-4e^{2x}} de^x$$

$$\Big\downarrow \text{ 211}$$

$$\frac{3}{2}\int \frac{1}{\sqrt{3-4e^{2x}}}de^x + \frac{1}{2}e^x\sqrt{3-4e^{2x}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{3}{4}\arcsin\left(\frac{2e^x}{\sqrt{3}}\right) + \frac{1}{2}e^x\sqrt{3-4e^{2x}}$$

input
```
Int[E^x*Sqrt[3 - 4*E^(2*x)],x]
```

output
```
(E^x*Sqrt[3 - 4*E^(2*x)])/2 + (3*ArcSin[(2*E^x)/Sqrt[3]])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{e^{x}\sqrt{3-4\,e^{2x}}}{2}+\frac{3\arcsin\left(\frac{2\,e^{x}\sqrt{3}}{3}\right)}{4}$	26
risch	$-\frac{e^{x}\left(-3+4\,e^{2x}\right)}{2\sqrt{3-4\,e^{2x}}}+\frac{3\arcsin\left(\frac{2\,e^{x}\sqrt{3}}{3}\right)}{4}$	34

input
```
int(exp(x)*(3-4*exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(3-4*exp(x)^2)^(1/2)+3/4*arcsin(2/3*exp(x)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int e^{x}\sqrt{3-4e^{2x}}\,dx=\frac{1}{2}\,\sqrt{-4\,e^{(2\,x)}+3}e^{x}-\frac{3}{4}\,\arctan\left(\frac{1}{2}\,\sqrt{-4\,e^{(2\,x)}+3}e^{(-x)}\right)$$

input
```
integrate(exp(x)*(3-4*exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-4*e^(2*x) + 3)*e^x - 3/4*arctan(1/2*sqrt(-4*e^(2*x) + 3)*e^(-x))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{\sqrt{3-4e^{2x}}e^x}{2}+\frac{3\operatorname{asin}\left(\frac{2\sqrt{3}e^x}{3}\right)}{4}$$

input
```
integrate(exp(x)*(3-4*exp(2*x))**(1/2),x)
```

output
```
sqrt(3 - 4*exp(2*x))*exp(x)/2 + 3*asin(2*sqrt(3)*exp(x)/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{1}{2}\sqrt{-4\,e^{(2\,x)}+3}e^x+\frac{3}{4}\arcsin\left(\frac{2}{3}\sqrt{3}e^x\right)$$

input
```
integrate(exp(x)*(3-4*exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*e^(2*x) + 3)*e^x + 3/4*arcsin(2/3*sqrt(3)*e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{1}{2}\sqrt{-4\,e^{(2\,x)}+3}e^x+\frac{3}{4}\arcsin\left(\frac{2}{3}\sqrt{3}e^x\right)$$

input
```
integrate(exp(x)*(3-4*exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-4*e^(2*x) + 3)*e^x + 3/4*arcsin(2/3*sqrt(3)*e^x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{3\,\mathrm{asin}\left(\frac{2\,\sqrt{3}\,\mathrm{e}^x}{3}\right)}{4}+\mathrm{e}^x\,\sqrt{\frac{3}{4}-\mathrm{e}^{2\,x}}$$

input
```
int(exp(x)*(3 - 4*exp(2*x))^(1/2),x)
```

output
```
(3*asin((2*3^(1/2)*exp(x))/3))/4 + exp(x)*(3/4 - exp(2*x))^(1/2)
```

Reduce [F]

$$\int e^x\sqrt{3-4e^{2x}}\,dx=\frac{e^x\sqrt{-4e^{2x}+3}}{2}-\frac{3\left(\int\frac{e^x\sqrt{-4e^{2x}+3}}{4e^{2x}-3}dx\right)}{2}$$

input
```
int(exp(x)*(3-4*exp(2*x))^(1/2),x)
```

output
```
(e**x*sqrt( - 4*e**(2*x) + 3) - 3*int((e**x*sqrt( - 4*e**(2*x) + 3))/(4*e*
*(2*x) - 3),x))/2
```

3.564 $\int e^{x^2} x^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 22

$$\int e^{x^2} x^3 \, dx = -\frac{e^{x^2}}{2} + \frac{1}{2} e^{x^2} x^2$$

output
```
-1/2*exp(x^2)+1/2*exp(x^2)*x^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} e^{x^2} \left(-1 + x^2\right)$$

input
```
Integrate[E^x^2*x^3,x]
```

output
```
(E^x^2*(-1 + x^2))/2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}x^3\,dx$$

$$\Big\downarrow \text{2641}$$

$$\frac{1}{2}e^{x^2}x^2-\int e^{x^2}xdx$$

$$\Big\downarrow \text{2638}$$

$$\frac{1}{2}e^{x^2}x^2-\frac{e^{x^2}}{2}$$

input
```
Int[E^x^2*x^3,x]
```

output
```
-1/2*E^x^2 + (E^x^2*x^2)/2
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

method	result	size
gosper	$\frac{(x^2-1)e^{x^2}}{2}$	12
orering	$\frac{(x^2-1)e^{x^2}}{2}$	12
risch	$\left(\frac{x^2}{2}-\frac{1}{2}\right)e^{x^2}$	13
meijerg	$\frac{1}{2}-\frac{(-2x^2+2)e^{x^2}}{4}$	16
derivativedivides	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
default	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
norman	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
parallelrisch	$-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}$	17
parts	$\frac{\sqrt{\pi}\,\mathrm{erfi}(x)x^3}{2}-\frac{3\sqrt{\pi}\left(\frac{x^3\,\mathrm{erfi}(x)}{3}-\frac{2\left(-\frac{e^{x^2}}{2}+\frac{e^{x^2}x^2}{2}\right)}{3\sqrt{\pi}}\right)}{2}$	46

input
```
int(exp(x^2)*x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2-1)*exp(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2}x^3\,dx=\frac{1}{2}\left(x^2-1\right)e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.45

$$\int e^{x^2} x^3\,dx = \frac{(x^2-1)\,e^{x^2}}{2}$$

input
```
integrate(exp(x**2)*x**3,x)
```

output
```
(x**2 - 1)*exp(x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3\,dx = \frac{1}{2}\,(x^2-1)e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="maxima")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3\,dx = \frac{1}{2}\,(x^2-1)e^{(x^2)}$$

input
```
integrate(exp(x^2)*x^3,x, algorithm="giac")
```

output
```
1/2*(x^2 - 1)*e^(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int e^{x^2} x^3 \, dx = \frac{\mathrm{e}^{x^2} (x^2 - 1)}{2}$$

input
```
int(x^3*exp(x^2),x)
```

output
```
(exp(x^2)*(x^2 - 1))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int e^{x^2} x^3 \, dx = \frac{e^{x^2} (x^2 - 1)}{2}$$

input
```
int(exp(x^2)*x^3,x)
```

output
```
(e**(x**2)*(x**2 - 1))/2
```

3.565 $\int e^x\sqrt{1-e^{2x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 29

$$\int e^x\sqrt{1-e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{1-e^{2x}}+\frac{\arcsin(e^x)}{2}$$

output
```
1/2*exp(x)*(1-exp(2*x))^(1/2)+1/2*arcsin(exp(x))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int e^x\sqrt{1-e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{1-e^{2x}}-\arctan\left(\frac{\sqrt{1-e^{2x}}}{1+e^x}\right)$$

input
```
Integrate[E^x*Sqrt[1 - E^(2*x)],x]
```

output
```
(E^x*Sqrt[1 - E^(2*x)])/2 - ArcTan[Sqrt[1 - E^(2*x)]/(1 + E^x)]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2679, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x\sqrt{1-e^{2x}}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\int \sqrt{1-e^{2x}}de^x$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-e^{2x}}}de^x+\frac{1}{2}e^x\sqrt{1-e^{2x}}$$

$$\Big\downarrow \text{223}$$

$$\frac{\arcsin(e^x)}{2}+\frac{1}{2}e^x\sqrt{1-e^{2x}}$$

input
```
Int[E^x*Sqrt[1 - E^(2*x)],x]
```

output
```
(E^x*Sqrt[1 - E^(2*x)])/2 + ArcSin[E^x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{e^{x} \sqrt{1-e^{2 x}}}{2}+\frac{\arcsin\left(e^{x}\right)}{2}$	21
risch	$-\frac{e^{x} \left(-1+e^{2 x}\right)}{2 \sqrt{1-e^{2 x}}}+\frac{\arcsin\left(e^{x}\right)}{2}$	27

input
```
int(exp(x)*(1-exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(1-exp(x)^2)^(1/2)+1/2*arcsin(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int e^{x} \sqrt{1-e^{2x}}\, dx = \frac{1}{2} \sqrt{-e^{(2 x)} + 1} e^{x} - \arctan\left(\left(\sqrt{-e^{(2 x)} + 1} - 1\right) e^{(-x)}\right)$$

input
```
integrate(exp(x)*(1-exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-e^(2*x) + 1)*e^x - arctan((sqrt(-e^(2*x) + 1) - 1)*e^(-x))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int e^x\sqrt{1-e^{2x}}\,dx=\frac{\sqrt{1-e^{2x}}e^x}{2}+\frac{\operatorname{asin}\left(e^x\right)}{2}$$

input
```
integrate(exp(x)*(1-exp(2*x))**(1/2),x)
```

output
```
sqrt(1 - exp(2*x))*exp(x)/2 + asin(exp(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int e^x\sqrt{1-e^{2x}}\,dx=\frac{1}{2}\sqrt{-e^{(2x)}+1}e^x+\frac{1}{2}\arcsin\left(e^x\right)$$

input
```
integrate(exp(x)*(1-exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-e^(2*x) + 1)*e^x + 1/2*arcsin(e^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int e^x\sqrt{1-e^{2x}}\,dx=\frac{1}{2}\sqrt{-e^{(2x)}+1}e^x+\frac{1}{2}\arcsin\left(e^x\right)$$

input
```
integrate(exp(x)*(1-exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-e^(2*x) + 1)*e^x + 1/2*arcsin(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.69

$$\int e^x\sqrt{1-e^{2x}}\,dx = \frac{\operatorname{asin}(\mathrm{e}^x)}{2} + \frac{\mathrm{e}^x\,\sqrt{1-\mathrm{e}^{2\,x}}}{2}$$

input
```
int(exp(x)*(1 - exp(2*x))^(1/2),x)
```

output
```
asin(exp(x))/2 + (exp(x)*(1 - exp(2*x))^(1/2))/2
```

Reduce [F]

$$\int e^x\sqrt{1-e^{2x}}\,dx = \frac{e^x\sqrt{-e^{2x}+1}}{2} - \frac{\left(\int \frac{e^x\sqrt{-e^{2x}+1}}{e^{2x}-1}dx\right)}{2}$$

input
```
int(exp(x)*(1-exp(2*x))^(1/2),x)
```

output
```
(e**x*sqrt( - e**(2*x) + 1) - int((e**x*sqrt( - e**(2*x) + 1))/(e**(2*x) -
 1),x))/2
```

3.566 $\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} dx$

Optimal result

Integrand size = 18, antiderivative size = 14

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} \, dx = \operatorname{arcsinh}\left(\frac{1+2e^x}{\sqrt{3}}\right)$$

output
```
arcsinh(1/3*(1+2*exp(x))*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} \, dx = -\log\left(-1-2e^x+2\sqrt{1+e^x+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[1 + E^x + E^(2*x)],x]
```

output
```
-Log[-1 - 2*E^x + 2*Sqrt[1 + E^x + E^(2*x)]]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2720, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{e^x+e^{2x}+1}}dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \frac{1}{\sqrt{e^x+e^{2x}+1}}de^x$$
$$\Big\downarrow \text{ 1090}$$
$$\frac{\int \frac{1}{\sqrt{1+\frac{e^{2x}}{3}}}d(1+2e^x)}{\sqrt{3}}$$
$$\Big\downarrow \text{ 222}$$
$$\text{arcsinh}\left(\frac{2e^x+1}{\sqrt{3}}\right)$$

input
```
Int[E^x/Sqrt[1 + E^x + E^(2*x)],x]
```

output
```
ArcSinh[(1 + 2*E^x)/Sqrt[3]]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
default	$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\,(\mathrm{e}^{x}+\frac{1}{2})}{3}\right)$	11

input
```
int(exp(x)/(1+exp(x)+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(2/3*3^(1/2)*(exp(x)+1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx = -\log\left(2\sqrt{e^{(2x)}+e^x+1}-2e^x-1\right)$$

input
```
integrate(exp(x)/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(2*sqrt(e^(2*x) + e^x + 1) - 2*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx = \operatorname{asinh}\left(\frac{2\sqrt{3}\left(e^x+\frac{1}{2}\right)}{3}\right)$$

input
```
integrate(exp(x)/(1+exp(x)+exp(2*x))**(1/2),x)
```

output
```
asinh(2*sqrt(3)*(exp(x) + 1/2)/3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx = \operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2\,e^x+1)\right)$$

input
```
integrate(exp(x)/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(1/3*sqrt(3)*(2*e^x + 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx = -\log\left(2\sqrt{e^{(2x)}+e^x+1}-2\,e^x-1\right)$$

input
```
integrate(exp(x)/(1+exp(x)+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(2*sqrt(e^(2*x) + e^x + 1) - 2*e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx = \ln\left(\mathrm{e}^x+\sqrt{\mathrm{e}^{2\,x}+\mathrm{e}^x+1}+\frac{1}{2}\right)$$

input
```
int(exp(x)/(exp(2*x) + exp(x) + 1)^(1/2),x)
```

output
```
log(exp(x) + (exp(2*x) + exp(x) + 1)^(1/2) + 1/2)
```

Reduce [F]

$$\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx = \int \frac{e^x\sqrt{e^{2x}+e^x+1}}{e^{2x}+e^x+1}dx$$

input
```
int(exp(x)/(1+exp(x)+exp(2*x))^(1/2),x)
```

output
```
int((e**x*sqrt(e**(2*x) + e**x + 1))/(e**(2*x) + e**x + 1),x)
```

3.567 $\int \frac{e^x}{-4+e^{2x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^x}{-4+e^{2x}}\,dx=-\frac{1}{2}\text{arctanh}\left(\frac{e^x}{2}\right)$$

output
```
-1/2*arctanh(1/2*exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{-4+e^{2x}}\,dx=-\frac{1}{2}\text{arctanh}\left(\frac{e^x}{2}\right)$$

input
```
Integrate[E^x/(-4 + E^(2*x)),x]
```

output
```
-1/2*ArcTanh[E^x/2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2679, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}-4}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\int \frac{1}{e^{2x}-4}de^x$$

$$\Big\downarrow \text{220}$$

$$-\frac{1}{2}\text{arctanh}\left(\frac{e^x}{2}\right)$$

input
```
Int[E^x/(-4 + E^(2*x)),x]
```

output
```
-1/2*ArcTanh[E^x/2]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\ln(-2+{\mathrm e}^{x})}{4}-\frac{\ln(2+{\mathrm e}^{x})}{4}$	16
norman	$\frac{\ln(-2+{\mathrm e}^{x})}{4}-\frac{\ln(2+{\mathrm e}^{x})}{4}$	16
risch	$\frac{\ln(-2+{\mathrm e}^{x})}{4}-\frac{\ln(2+{\mathrm e}^{x})}{4}$	16

input
```
int(exp(x)/(-4+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(-2+exp(x))-1/4*ln(2+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{-4+e^{2x}}\,dx = -\frac{1}{4}\log\left(e^x+2\right)+\frac{1}{4}\log\left(e^x-2\right)$$

input
```
integrate(exp(x)/(-4+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/4*log(e^x + 2) + 1/4*log(e^x - 2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{-4+e^{2x}}\,dx = \frac{\log(e^x-2)}{4} - \frac{\log(e^x+2)}{4}$$

input
```
integrate(exp(x)/(-4+exp(2*x)),x)
```

output
```
log(exp(x) - 2)/4 - log(exp(x) + 2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{-4+e^{2x}}\,dx = -\frac{1}{4}\log(e^x+2) + \frac{1}{4}\log(e^x-2)$$

input
```
integrate(exp(x)/(-4+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/4*log(e^x + 2) + 1/4*log(e^x - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(7) = 14.

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{e^x}{-4+e^{2x}}\,dx = -\frac{1}{4}\log(e^x+2) + \frac{1}{4}\log(|e^x-2|)$$

input
```
integrate(exp(x)/(-4+exp(2*x)),x, algorithm="giac")
```

output
```
-1/4*log(e^x + 2) + 1/4*log(abs(e^x - 2))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{-4+e^{2x}}\,dx = \frac{\ln\left(e^x-2\right)}{4} - \frac{\ln\left(e^x+2\right)}{4}$$

input
```
int(exp(x)/(exp(2*x) - 4),x)
```

output
```
log(exp(x) - 2)/4 - log(exp(x) + 2)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{e^x}{-4+e^{2x}}\,dx = \frac{\log(e^x-2)}{4} - \frac{\log(e^x+2)}{4}$$

input
```
int(exp(x)/(-4+exp(2*x)),x)
```

output
```
(log(e**x - 2) - log(e**x + 2))/4
```

3.568 $\int e^{2-x^2} x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int e^{2-x^2} x \, dx = -\frac{1}{2} e^{2-x^2}$$

output
```
-1/2*exp(-x^2+2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{2-x^2} x \, dx = -\frac{1}{2} e^{2-x^2}$$

input
```
Integrate[E^(2 - x^2)*x,x]
```

output
```
-1/2*E^(2 - x^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{2-x^2} x \, dx$$

$$\Big\downarrow \text{2638}$$

$$-\frac{1}{2} e^{2-x^2}$$

input
```
Int[E^(2 - x^2)*x,x]
```

output
```
-1/2*E^(2 - x^2)
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{e^{-x^2+2}}{2}$	11
derivativedivides	$-\frac{e^{-x^2+2}}{2}$	11
default	$-\frac{e^{-x^2+2}}{2}$	11
norman	$-\frac{e^{-x^2+2}}{2}$	11
risch	$-\frac{e^{-x^2+2}}{2}$	11
parallelrisch	$-\frac{e^{-x^2+2}}{2}$	11
orering	$-\frac{e^{-x^2+2}}{2}$	11
meijerg	$\frac{e^2\left(1-e^{-x^2}\right)}{2}$	15
parts	$\frac{e^2\sqrt{\pi}\,\operatorname{erf}(x)x}{2}-\frac{e^2\left(\operatorname{erf}(x)x\sqrt{\pi}+e^{-x^2}\right)}{2}$	30

input
```
int(exp(-x^2+2)*x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-x^2+2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2-x^2}x\,dx=-\frac{1}{2}\,e^{(-x^2+2)}$$

input
```
integrate(exp(-x^2+2)*x,x, algorithm="fricas")
```

output
```
-1/2*e^(-x^2 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int e^{2-x^2}x\,dx = -\frac{e^{2-x^2}}{2}$$

input
```
integrate(exp(-x**2+2)*x,x)
```

output
```
-exp(2 - x**2)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2-x^2}x\,dx = -\frac{1}{2}\,e^{\left(-x^2+2\right)}$$

input
```
integrate(exp(-x^2+2)*x,x, algorithm="maxima")
```

output
```
-1/2*e^(-x^2 + 2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2-x^2}x\,dx = -\frac{1}{2}\,e^{\left(-x^2+2\right)}$$

input
```
integrate(exp(-x^2+2)*x,x, algorithm="giac")
```

output
```
-1/2*e^(-x^2 + 2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int e^{2-x^2}x\,dx = -\frac{\mathrm{e}^2\,\mathrm{e}^{-x^2}}{2}$$

input
```
int(x*exp(2 - x^2),x)
```

output
```
-(exp(2)*exp(-x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int e^{2-x^2}x\,dx = -\frac{e^2}{2e^{x^2}}$$

input
```
int(exp(-x^2+2)*x,x)
```

output
```
( - e**2)/(2*e**(x**2))
```

3.569 $\int (e^x - x^e) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 16

$$\int (e^x - x^e) \, dx = e^x - \frac{x^{1+e}}{1+e}$$

output
```
exp(x)-x^(1+exp(1))/(1+exp(1))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (e^x - x^e) \, dx = e^x - \frac{x^{1+e}}{1+e}$$

input
```
Integrate[E^x - x^E,x]
```

output
```
E^x - x^(1 + E)/(1 + E)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e^x - x^e) \, dx$$

$$\Big\downarrow \text{2009}$$

$$e^x - \frac{x^{1+e}}{1+e}$$

input
```
Int[E^x - x^E,x]
```

output
```
E^x - x^(1 + E)/(1 + E)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
risch	$-\frac{x\,x^{\mathrm{e}}}{1+\mathrm{e}}+\mathrm{e}^{x}$	17
default	$\mathrm{e}^{x}-\frac{x^{1+\mathrm{e}}}{1+\mathrm{e}}$	18
parts	$\mathrm{e}^{x}-\frac{x^{1+\mathrm{e}}}{1+\mathrm{e}}$	18
norman	$-\frac{x\,\mathrm{e}^{\mathrm{e}\ln(x)}}{1+\mathrm{e}}+\mathrm{e}^{x}$	19
parallelrisch	$\frac{\mathrm{e}^{x}\mathrm{e}-x\,x^{\mathrm{e}}+\mathrm{e}^{x}}{1+\mathrm{e}}$	23

input
```
int(exp(x)-x^exp(1),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+exp(1))*x*x^exp(1)+exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \left(e^{x}-x^{e}\right)\,dx=-\frac{xx^{e}-(e+1)e^{x}}{e+1}$$

input
```
integrate(exp(x)-x^exp(1),x, algorithm="fricas")
```

output
```
-(x*x^e - (e + 1)*e^x)/(e + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(e^{x}-x^{e}\right)\,dx=-\frac{x^{1+e}}{1+e}+e^{x}$$

input
```
integrate(exp(x)-x**exp(1),x)
```

output
```
-x**(1 + E)/(1 + E) + exp(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (e^x - x^e)\,dx = -\frac{x^{e+1}}{e+1} + e^x$$

input
```
integrate(exp(x)-x^exp(1),x, algorithm="maxima")
```

output
```
-x^(e + 1)/(e + 1) + e^x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (e^x - x^e)\,dx = -\frac{x^{e+1}}{e+1} + e^x$$

input
```
integrate(exp(x)-x^exp(1),x, algorithm="giac")
```

output
```
-x^(e + 1)/(e + 1) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (e^x - x^e)\,dx = \mathrm{e}^x - \frac{x\,x^{\mathrm{e}}}{\mathrm{e}+1}$$

input
```
int(exp(x) - x^exp(1),x)
```

output
```
exp(x) - (x*x^exp(1))/(exp(1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int (e^x - x^e) \, dx = \frac{e^x e + e^x - x^e x}{e + 1}$$

input
```
int(exp(x)-x^exp(1),x)
```

output
```
(e**x*e + e**x - x**e*x)/(e + 1)
```

3.570 $\int \frac{-1+e^{2x}}{3+e^{2x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{-1+e^{2x}}{3+e^{2x}} \, dx = -\frac{x}{3}+\frac{2}{3}\log\left(3+e^{2x}\right)$$

output
```
-1/3*x+2/3*ln(3+exp(2*x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{-1+e^{2x}}{3+e^{2x}} \, dx = -\frac{1}{3}\log\left(e^{x}\right)+\frac{2}{3}\log\left(3+e^{2x}\right)$$

input
```
Integrate[(-1 + E^(2*x))/(3 + E^(2*x)),x]
```

output
```
-1/3*Log[E^x] + (2*Log[3 + E^(2*x)])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 25, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}-1}{e^{2x}+3}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\int -\frac{e^{-2x}\left(1-e^{2x}\right)}{3+e^{2x}}de^{2x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{e^{-2x}\left(1-e^{2x}\right)}{3+e^{2x}}de^{2x}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{2}\int \left(\frac{e^{-2x}}{3}-\frac{4}{3\left(3+e^{2x}\right)}\right)de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{4}{3}\log\left(e^{2x}+3\right)-\frac{1}{3}\log\left(e^{2x}\right)\right)$$

input
```
Int[(-1 + E^(2*x))/(3 + E^(2*x)),x]
```

output
```
(-1/3*Log[E^(2*x)] + (4*Log[3 + E^(2*x)])/3)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
norman	$-\frac{x}{3}+\frac{2\ln\left(3+{\rm e}^{2x}\right)}{3}$	14
risch	$-\frac{x}{3}+\frac{2\ln\left(3+{\rm e}^{2x}\right)}{3}$	14
parallelrisch	$-\frac{x}{3}+\frac{2\ln\left(3+{\rm e}^{2x}\right)}{3}$	14
derivativedivides	$\frac{2\ln\left(3+{\rm e}^{2x}\right)}{3}-\frac{\ln\left({\rm e}^{2x}\right)}{6}$	18
default	$\frac{2\ln\left(3+{\rm e}^{2x}\right)}{3}-\frac{\ln\left({\rm e}^{2x}\right)}{6}$	18

input
```
int((-1+exp(2*x))/(3+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x+2/3*ln(3+exp(2*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{-1+e^{2x}}{3+e^{2x}}\,dx = -\frac{1}{3}\,x + \frac{2}{3}\,\log\left(e^{(2\,x)} + 3\right)$$

input
```
integrate((-1+exp(2*x))/(3+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/3*x + 2/3*log(e^(2*x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{-1+e^{2x}}{3+e^{2x}}\,dx = -\frac{x}{3} + \frac{2\log\left(e^{2x} + 3\right)}{3}$$

input
```
integrate((-1+exp(2*x))/(3+exp(2*x)),x)
```

output
```
-x/3 + 2*log(exp(2*x) + 3)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{-1+e^{2x}}{3+e^{2x}}\,dx = -\frac{1}{3}\,x + \frac{2}{3}\,\log\left(e^{(2\,x)} + 3\right)$$

input
```
integrate((-1+exp(2*x))/(3+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/3*x + 2/3*log(e^(2*x) + 3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{-1+e^{2x}}{3+e^{2x}}\,dx = -\frac{1}{3}\,x + \frac{2}{3}\,\log\left(e^{(2\,x)} + 3\right)$$

input
```
integrate((-1+exp(2*x))/(3+exp(2*x)),x, algorithm="giac")
```

output
```
-1/3*x + 2/3*log(e^(2*x) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{-1+e^{2x}}{3+e^{2x}}\,dx = \frac{2\,\ln\left(\mathrm{e}^{2\,x}+3\right)}{3} - \frac{x}{3}$$

input
```
int((exp(2*x) - 1)/(exp(2*x) + 3),x)
```

output
```
(2*log(exp(2*x) + 3))/3 - x/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{-1+e^{2x}}{3+e^{2x}}\,dx = \frac{2\log(e^{2x}+3)}{3} - \frac{x}{3}$$

input
```
int((-1+exp(2*x))/(3+exp(2*x)),x)
```

output
```
(2*log(e**(2*x) + 3) - x)/3
```

3.571 $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$

Optimal result

Integrand size = 17, antiderivative size = 4

$$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \arcsin(e^x)$$

output
```
arcsin(exp(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 24 vs. 2(4) = 8.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 6.00

$$\int \frac{e^x}{\sqrt{1-e^{2x}}} dx = -2\arctan\left(\frac{\sqrt{1-e^{2x}}}{1+e^x}\right)$$

input
```
Integrate[E^x/Sqrt[1 - E^(2*x)],x]
```

output
```
-2*ArcTan[Sqrt[1 - E^(2*x)]/(1 + E^x)]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2679, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{\sqrt{1-e^{2x}}}de^x$$
$$\Big\downarrow \text{223}$$
$$\arcsin\left(e^x\right)$$

input
```
Int[E^x/Sqrt[1 - E^(2*x)],x]
```

output
```
ArcSin[E^x]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\arcsin(e^x)$	4

input
```
int(exp(x)/(1-exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = -2\arctan\left(\left(\sqrt{-e^{(2x)}+1}-1\right)e^{(-x)}\right)$$

input
```
integrate(exp(x)/(1-exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan((sqrt(-e^(2*x) + 1) - 1)*e^(-x))
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \operatorname{asin}(e^x)$$

input
```
integrate(exp(x)/(1-exp(2*x))**(1/2),x)
```

output
```
asin(exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx = \arcsin\left(e^x\right)$$

input
```
integrate(exp(x)/(1-exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arcsin(e^x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx = \arcsin\left(e^x\right)$$

input
```
integrate(exp(x)/(1-exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
arcsin(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\, dx = \mathrm{asin}(\mathrm{e}^x)$$

input
```
int(exp(x)/(1 - exp(2*x))^(1/2),x)
```

output
```
asin(exp(x))
```

Reduce [F]

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = -\left(\int \frac{e^x\sqrt{-e^{2x}+1}}{e^{2x}-1}dx\right)$$

input
```
int(exp(x)/(1-exp(2*x))^(1/2),x)
```

output
```
- int((e**x*sqrt( - e**(2*x) + 1))/(e**(2*x) - 1),x)
```

3.572 $\int \frac{e^{2x}}{1+e^{4x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{e^{2x}}{1+e^{4x}} \, dx = \frac{1}{2}\arctan\left(e^{2x}\right)$$

output
```
1/2*arctan(exp(2*x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^{4x}} \, dx = \frac{1}{2}\arctan\left(e^{2x}\right)$$

input
```
Integrate[E^(2*x)/(1 + E^(4*x)),x]
```

output
```
ArcTan[E^(2*x)]/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{e^{4x}+1} \, dx$$
$$\Big\downarrow \text{ 2679}$$
$$\frac{1}{2} \int \frac{1}{1+e^{4x}} de^{2x}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{1}{2} \arctan \left(e^{2x}\right)$$

input
```
Int[E^(2*x)/(1 + E^(4*x)),x]
```

output
```
ArcTan[E^(2*x)]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(e^{2x})}{2}$	8
risch	$\frac{i\ln(e^{2x}+i)}{4}-\frac{i\ln(e^{2x}-i)}{4}$	24

input
```
int(exp(2*x)/(1+exp(4*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(exp(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \frac{1}{2}\arctan\left(e^{(2x)}\right)$$

input
```
integrate(exp(2*x)/(1+exp(4*x)),x, algorithm="fricas")
```

output
```
1/2*arctan(e^(2*x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(4i+e^{2x}\right)\right)\right)$$

input
```
integrate(exp(2*x)/(1+exp(4*x)),x)
```

output
```
RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(2*x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \frac{1}{2}\arctan\left(e^{(2x)}\right)$$

input
```
integrate(exp(2*x)/(1+exp(4*x)),x, algorithm="maxima")
```

output
```
1/2*arctan(e^(2*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \frac{1}{2}\arctan\left(e^{(2x)}\right)$$

input
```
integrate(exp(2*x)/(1+exp(4*x)),x, algorithm="giac")
```

output
```
1/2*arctan(e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \frac{\mathrm{atan}(\mathrm{e}^{2x})}{2}$$

input
```
int(exp(2*x)/(exp(4*x) + 1),x)
```

output
```
atan(exp(2*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 3.50

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx=\frac{atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)}{2}-\frac{atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right)}{2}$$

input
```
int(exp(2*x)/(1+exp(4*x)),x)
```

output
```
(atan((2*e**x - sqrt(2))/sqrt(2)) - atan((2*e**x + sqrt(2))/sqrt(2)))/2
```

3.573 $\int \frac{1}{-3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{-3e^x+e^{2x}}\,dx=\frac{e^{-x}}{3}-\frac{x}{9}+\frac{1}{9}\log\left(3-e^x\right)$$

output
```
1/3/exp(x)-1/9*x+1/9*ln(3-exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{-3e^x+e^{2x}}\,dx=\frac{1}{9}\left(3e^{-x}-\log\left(e^x\right)+\log\left(-3+e^x\right)\right)$$

input
```
Integrate[(-3*E^x + E^(2*x))^(-1),x]
```

output
```
(3/E^x - Log[E^x] + Log[-3 + E^x])/9
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2720, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^{2x}-3e^x}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{e^{-2x}}{3-e^x}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{e^{-2x}}{3-e^x}de^x$$

$$\Big\downarrow \text{54}$$

$$-\int \left(\frac{e^{-2x}}{3}+\frac{e^{-x}}{9}-\frac{1}{9(-3+e^x)}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{-x}}{3}-\frac{1}{9}\log\left(e^x\right)+\frac{1}{9}\log\left(3-e^x\right)$$

input
```
Int[(-3*E^x + E^(2*x))^(-1),x]
```

output
```
1/(3*E^x) - Log[E^x]/9 + Log[3 - E^x]/9
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{{\mathrm e}^{-x}}{3}-\frac{x}{9}+\frac{\ln \left({\mathrm e}^{x}-3\right)}{9}$	18
default	$\frac{{\mathrm e}^{-x}}{3}-\frac{\ln \left({\mathrm e}^{x}\right)}{9}+\frac{\ln \left({\mathrm e}^{x}-3\right)}{9}$	20
norman	$\left(\frac{1}{3}-\frac{{\mathrm e}^{x} x}{9}\right) {\mathrm e}^{-x}+\frac{\ln \left({\mathrm e}^{x}-3\right)}{9}$	21

input
```
int(1/(-3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(-x)-1/9*x+1/9*ln(exp(x)-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{1}{-3e^x + e^{2x}}\,dx = -\frac{1}{9}\left(xe^x - e^x\log\left(e^x - 3\right) - 3\right)e^{(-x)}$$

input
```
integrate(1/(-3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/9*(x*e^x - e^x*log(e^x - 3) - 3)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{-3e^x + e^{2x}}\,dx = -\frac{x}{9} + \frac{\log\left(e^x - 3\right)}{9} + \frac{e^{-x}}{3}$$

input
```
integrate(1/(-3*exp(x)+exp(2*x)),x)
```

output
```
-x/9 + log(exp(x) - 3)/9 + exp(-x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{-3e^x + e^{2x}}\,dx = -\frac{1}{9}\,x + \frac{1}{3}\,e^{(-x)} + \frac{1}{9}\,\log\left(e^x - 3\right)$$

input
```
integrate(1/(-3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/9*x + 1/3*e^(-x) + 1/9*log(e^x - 3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{1}{-3e^x + e^{2x}}\,dx = -\frac{1}{9}\,x + \frac{1}{3}\,e^{(-x)} + \frac{1}{9}\,\log\left(|e^x - 3|\right)$$

input
```
integrate(1/(-3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
-1/9*x + 1/3*e^(-x) + 1/9*log(abs(e^x - 3))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{1}{-3e^x + e^{2x}}\,dx = \frac{e^{-x}}{3} - \frac{x}{9} + \frac{\ln\left(e^x - 3\right)}{9}$$

input
```
int(1/(exp(2*x) - 3*exp(x)),x)
```

output
```
exp(-x)/3 - x/9 + log(exp(x) - 3)/9
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{-3e^x + e^{2x}}\,dx = \frac{e^x \log(e^x - 3) - e^x x + 3}{9e^x}$$

input
```
int(1/(-3*exp(x)+exp(2*x)),x)
```

output
```
(e**x*log(e**x - 3) - e**x*x + 3)/(9*e**x)
```

3.574 $\int \frac{e^x(-2+e^x)}{1+e^x} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{e^x(-2+e^x)}{1+e^x} \, dx = e^x - 3\log(1+e^x)$$

output
```
exp(x)-3*ln(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x(-2+e^x)}{1+e^x} \, dx = e^x - 3\log(1+e^x)$$

input
```
Integrate[(E^x*(-2 + E^x))/(1 + E^x),x]
```

output
```
E^x - 3*Log[1 + E^x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(e^x-2)}{e^x+1}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int -\frac{2-e^x}{e^x+1}de^x$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{2-e^x}{1+e^x}de^x$$

$$\Big\downarrow \text{ 49}$$

$$-\int \left(\frac{3}{1+e^x}-1\right)de^x$$

$$\Big\downarrow \text{ 2009}$$

$$e^x-3\log\left(e^x+1\right)$$

input
```
Int[(E^x*(-2 + E^x))/(1 + E^x),x]
```

output
```
E^x - 3*Log[1 + E^x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$e^{x}-3\ln\left(1+e^{x}\right)$	11
default	$e^{x}-3\ln\left(1+e^{x}\right)$	11
norman	$e^{x}-3\ln\left(1+e^{x}\right)$	11
risch	$e^{x}-3\ln\left(1+e^{x}\right)$	11
parallelrisch	$e^{x}-3\ln\left(1+e^{x}\right)$	11

input
```
int(exp(x)*(-2+exp(x))/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)-3*ln(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^x(-2+e^x)}{1+e^x}\,dx = e^x - 3\,\log\left(e^x+1\right)$$

input
```
integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x, algorithm="fricas")
```

output
```
e^x - 3*log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^x(-2+e^x)}{1+e^x}\,dx = e^x - 3\log\left(e^x+1\right)$$

input
```
integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x)
```

output
```
exp(x) - 3*log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^x(-2+e^x)}{1+e^x}\,dx = e^x - 3\,\log\left(e^x+1\right)$$

input
```
integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x, algorithm="maxima")
```

output
```
e^x - 3*log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^x(-2+e^x)}{1+e^x}\,dx = e^x - 3\log\left(e^x+1\right)$$

input
```
integrate(exp(x)*(-2+exp(x))/(1+exp(x)),x, algorithm="giac")
```

output
```
e^x - 3*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^x(-2+e^x)}{1+e^x}\,dx = \mathrm{e}^x - 3\,\ln\left(\mathrm{e}^x+1\right)$$

input
```
int((exp(x)*(exp(x) - 2))/(exp(x) + 1),x)
```

output
```
exp(x) - 3*log(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x(-2+e^x)}{1+e^x}\,dx = e^x - 3\log(e^x+1)$$

input
```
int(exp(x)*(-2+exp(x))/(1+exp(x)),x)
```

output
```
e**x - 3*log(e**x + 1)
```

3.575 $\int \frac{e^x}{-1+e^{2x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\text{arctanh}(e^x)$$

output
```
-arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{-1+e^{2x}} \, dx = -\text{arctanh}(e^x)$$

input
```
Integrate[E^x/(-1 + E^(2*x)),x]
```

output
```
-ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2679, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}-1} dx$$
$$\Big\downarrow \text{ 2679}$$
$$\int \frac{1}{e^{2x}-1} de^x$$
$$\Big\downarrow \text{ 220}$$
$$-\text{arctanh}(e^x)$$

input
```
Int[E^x/(-1 + E^(2*x)),x]
```

output
```
-ArcTanh[E^x]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
norman	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16
risch	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16

input
```
int(exp(x)/(-1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\log\left(e^x+1\right)+\frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\log\left(e^x-1\right)}{2}-\frac{\log\left(e^x+1\right)}{2}$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\,\log\left(e^x+1\right)+\frac{1}{2}\,\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{e^x}{-1+e^{2x}}\,dx = -\frac{1}{2}\,\log\left(e^x+1\right)+\frac{1}{2}\,\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2} - \frac{\ln\left(\mathrm{e}^x+1\right)}{2}$$

input
```
int(exp(x)/(exp(2*x) - 1),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{e^x}{-1+e^{2x}}\,dx = \frac{\log(e^x-1)}{2} - \frac{\log(e^x+1)}{2}$$

input
```
int(exp(x)/(-1+exp(2*x)),x)
```

output
```
(log(e**x - 1) - log(e**x + 1))/2
```

3.576 $\int \frac{e^x}{1+e^{2x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \frac{e^x}{1+e^{2x}} \, dx = \arctan \left(e^x\right)$$

output
```
arctan(exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^{2x}} \, dx = \arctan \left(e^x\right)$$

input
```
Integrate[E^x/(1 + E^(2*x)),x]
```

output
```
ArcTan[E^x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}+1}dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{e^{2x}+1}de^x$$
$$\Big\downarrow \text{216}$$
$$\arctan\left(e^x\right)$$

input
```
Int[E^x/(1 + E^(2*x)),x]
```

output
```
ArcTan[E^x]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan\left(\mathrm{e}^{x}\right)$	4
risch	$\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{2}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{2}$	20

input
```
int(exp(x)/(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
arctan(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx=\arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
```

output
```
arctan(e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1+e^{2x}}\,dx=\mathrm{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+e^x\right)\right)\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
```

output
```
arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
```

output
```
arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \mathrm{atan}(\mathrm{e}^x)$$

input
```
int(exp(x)/(exp(2*x) + 1),x)
```

output
```
atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^{2x}}\,dx = atan(e^x)$$

input
```
int(exp(x)/(1+exp(2*x)),x)
```

output
```
atan(e**x)
```

3.577 $\int \frac{e^{-x}+e^x}{-e^{-x}+e^x} dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{e^{-x}+e^x}{-e^{-x}+e^x} dx = \log \left(e^{-x}-e^x\right)$$

output
```
ln(exp(-x)-exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{e^{-x}+e^x}{-e^{-x}+e^x} dx = 2\text{arctanh}(1-2e^x)+\log \left(1+e^x\right)$$

input
```
Integrate[(E^(-x) + E^x)/(-E^(-x) + E^x),x]
```

output
```
2*ArcTanh[1 - 2*E^x] + Log[1 + E^x]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2720, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}+e^x}{e^x-e^{-x}}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int -\frac{e^{-x}\left(e^{2x}+1\right)}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{e^{-x}\left(1+e^{2x}\right)}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{ 354}$$

$$-\frac{1}{2}\int \frac{e^{-x}\left(1+e^{2x}\right)}{1-e^{2x}}de^{2x}$$

$$\Big\downarrow \text{ 86}$$

$$-\frac{1}{2}\int \left(e^{-x}-\frac{2}{-1+e^{2x}}\right)de^{2x}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(2\log\left(1-e^{2x}\right)-\log\left(e^{2x}\right)\right)$$

input
```
Int[(E^(-x) + E^x)/(-E^(-x) + E^x),x]
```

output
```
(-Log[E^(2*x)] + 2*Log[1 - E^(2*x)])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\ln\left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)$	11
default	$\ln\left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)$	11
parallelrisch	$\ln\left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)$	11
risch	$-x+\ln\left(-1+\mathrm{e}^{2x}\right)$	12
norman	$-x+\ln\left(-1+\mathrm{e}^{x}\right)+\ln\left(1+\mathrm{e}^{x}\right)$	15

input
```
int((exp(-x)+exp(x))/(-exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(-exp(-x)+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^{-x}+e^x}{-e^{-x}+e^x}\,dx=-x+\log\left(e^{(2\,x)}-1\right)$$

input
```
integrate((exp(-x)+exp(x))/(-exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
-x + log(e^(2*x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{-x}+e^x}{-e^{-x}+e^x}\,dx=-x+\log\left(e^{2x}-1\right)$$

input
```
integrate((exp(-x)+exp(x))/(-exp(-x)+exp(x)),x)
```

output
```
-x + log(exp(2*x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{-x}+e^{x}}{-e^{-x}+e^{x}}\,dx = \log\left(e^{(-x)}-e^{x}\right)$$

input
```
integrate((exp(-x)+exp(x))/(-exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
log(e^(-x) - e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}+e^{x}}{-e^{-x}+e^{x}}\,dx = -x+\log\left(\left|e^{(2x)}-1\right|\right)$$

input
```
integrate((exp(-x)+exp(x))/(-exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
-x + log(abs(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^{-x}+e^{x}}{-e^{-x}+e^{x}}\,dx = \ln\left(\mathrm{e}^{2x}-1\right)-x$$

input
```
int(-(exp(-x) + exp(x))/(exp(-x) - exp(x)),x)
```

output
```
log(exp(2*x) - 1) - x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{e^{-x}+e^{x}}{-e^{-x}+e^{x}}\,dx=\log(e^{x}-1)+\log(e^{x}+1)-x$$

input
```
int((exp(-x)+exp(x))/(-exp(-x)+exp(x)),x)
```

output
```
log(e**x - 1) + log(e**x + 1) - x
```

3.578 $\int \frac{-e^{-x}+e^x}{e^{-x}+e^x} dx$

Optimal result

Integrand size = 23, antiderivative size = 10

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x} dx = \log \left(e^{-x}+e^x\right)$$

output
```
ln(exp(-x)+exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x} dx = -\log \left(e^x\right)+\log \left(1+e^{2x}\right)$$

input
```
Integrate[(-E^(-x) + E^x)/(E^(-x) + E^x),x]
```

output
```
-Log[E^x] + Log[1 + E^(2*x)]
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2720, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x - e^{-x}}{e^{-x} + e^x} \, dx$$

$$\Big\downarrow \; 2720$$

$$\int -\frac{e^{-x}(1 - e^{2x})}{e^{2x} + 1} de^x$$

$$\Big\downarrow \; 25$$

$$-\int \frac{e^{-x}(1 - e^{2x})}{1 + e^{2x}} de^x$$

$$\Big\downarrow \; 354$$

$$-\frac{1}{2} \int \frac{e^{-x}(1 - e^{2x})}{1 + e^{2x}} de^{2x}$$

$$\Big\downarrow \; 86$$

$$-\frac{1}{2} \int \left(e^{-x} - \frac{2}{1 + e^{2x}}\right) de^{2x}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(2 \log \left(e^{2x} + 1\right) - \log \left(e^{2x}\right)\right)$$

input
```
Int[(-E^(-x) + E^x)/(E^(-x) + E^x),x]
```

output
```
(-Log[E^(2*x)] + 2*Log[1 + E^(2*x)])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\ln\left(\mathrm{e}^{-x}+\mathrm{e}^{x}\right)$	9
default	$\ln\left(\mathrm{e}^{-x}+\mathrm{e}^{x}\right)$	9
parallelrisch	$\ln\left(\mathrm{e}^{-x}+\mathrm{e}^{x}\right)$	9
norman	$-x+\ln\left(1+\mathrm{e}^{2x}\right)$	12
risch	$-x+\ln\left(1+\mathrm{e}^{2x}\right)$	12

input
```
int((-exp(-x)+exp(x))/(exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(-x)+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x}\,dx=-x+\log\left(e^{(2x)}+1\right)$$

input
```
integrate((-exp(-x)+exp(x))/(exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
-x + log(e^(2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x}\,dx=-x+\log\left(e^{2x}+1\right)$$

input
```
integrate((-exp(-x)+exp(x))/(exp(-x)+exp(x)),x)
```

output
```
-x + log(exp(2*x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x}\,dx = \log\left(e^{(-x)}+e^x\right)$$

input
```
integrate((-exp(-x)+exp(x))/(exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
log(e^(-x) + e^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x}\,dx = -x+\log\left(e^{(2x)}+1\right)$$

input
```
integrate((-exp(-x)+exp(x))/(exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
-x + log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{-e^{-x}+e^x}{e^{-x}+e^x}\,dx = \ln\left(\mathrm{e}^{2x}+1\right)-x$$

input
```
int(-(exp(-x) - exp(x))/(exp(-x) + exp(x)),x)
```

output
```
log(exp(2*x) + 1) - x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{-e^{-x}+e^{x}}{e^{-x}+e^{x}}\,dx=\log\left(e^{2x}+1\right)-x$$

input
```
int((-exp(-x)+exp(x))/(exp(-x)+exp(x)),x)
```

output
```
log(e**(2*x) + 1) - x
```

3.579 $\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 18

$$\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}}\,dx=-x+\frac{1}{2}\log\left(1-e^{4x}\right)$$

output
```
-x+1/2*ln(1-exp(4*x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(18) = 36.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}}\,dx=-\log\left(e^{x}\right)+\frac{1}{2}\log\left(-1+e^{x}\right)+\frac{1}{2}\log\left(1+e^{x}\right)+\frac{1}{2}\log\left(1+e^{2x}\right)$$

input
```
Integrate[(E^(-2*x) + E^(2*x))/(-E^(-2*x) + E^(2*x)),x]
```

output
```
-Log[E^x] + Log[-1 + E^x]/2 + Log[1 + E^x]/2 + Log[1 + E^(2*x)]/2
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2720, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-2x}+e^{2x}}{e^{2x}-e^{-2x}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\int -\frac{e^{-2x}\left(1+e^{4x}\right)}{1-e^{4x}}de^{2x}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{e^{-2x}\left(1+e^{4x}\right)}{1-e^{4x}}de^{2x}$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{4}\int \frac{e^{-2x}\left(1+e^{4x}\right)}{1-e^{4x}}de^{4x}$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{4}\int \left(e^{-2x}-\frac{2}{-1+e^{4x}}\right)de^{4x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(2\log\left(1-e^{4x}\right)-\log\left(e^{4x}\right)\right)$$

input
```
Int[(E^(-2*x) + E^(2*x))/(-E^(-2*x) + E^(2*x)),x]
```

output
```
(-Log[E^(4*x)] + 2*Log[1 - E^(4*x)])/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

method	result	size
risch	$x+\frac{\ln\left({\mathrm e}^{-4 x}-1\right)}{2}$	12
parallelrisch	$\frac{\ln\left({\mathrm e}^{-2 x}-{\mathrm e}^{2 x}\right)}{2}$	15
norman	$x+\frac{\ln\left(-1+{\mathrm e}^{-2 x}\right)}{2}+\frac{\ln\left({\mathrm e}^{-2 x}+1\right)}{2}$	21
default	$\frac{\ln\left(1+{\mathrm e}^{2 x}\right)}{2}-\ln\left({\mathrm e}^{x}\right)+\frac{\ln\left(-1+{\mathrm e}^{x}\right)}{2}+\frac{\ln\left(1+{\mathrm e}^{x}\right)}{2}$	30

input
```
int((exp(-2*x)+exp(2*x))/(-exp(-2*x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
x+1/2*ln(exp(-4*x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}}\,dx=-x+\frac{1}{2}\,\log\left(e^{(4\,x)}-1\right)$$

input
```
integrate((exp(-2*x)+exp(2*x))/(-exp(-2*x)+exp(2*x)),x, algorithm="fricas"
)
```

output
```
-x + 1/2*log(e^(4*x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}}\,dx=-x+\frac{\log\left(e^{4x}-1\right)}{2}$$

input
```
integrate((exp(-2*x)+exp(2*x))/(-exp(-2*x)+exp(2*x)),x)
```

output
```
-x + log(exp(4*x) - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2x} + e^{2x}}{-e^{-2x} + e^{2x}}\,dx = \frac{1}{2}\log\left(e^{(2\,x)} - e^{(-2\,x)}\right)$$

input
```
integrate((exp(-2*x)+exp(2*x))/(-exp(-2*x)+exp(2*x)),x, algorithm="maxima"
)
```

output
```
1/2*log(e^(2*x) - e^(-2*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-2x} + e^{2x}}{-e^{-2x} + e^{2x}}\,dx = -x + \frac{1}{2}\log\left(\left|e^{(4\,x)} - 1\right|\right)$$

input
```
integrate((exp(-2*x)+exp(2*x))/(-exp(-2*x)+exp(2*x)),x, algorithm="giac")
```

output
```
-x + 1/2*log(abs(e^(4*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{e^{-2x} + e^{2x}}{-e^{-2x} + e^{2x}}\,dx = \frac{\ln\left(e^{2\,x} - 1\right)}{2} - x + \frac{\ln\left(e^{2\,x} + 1\right)}{2}$$

input
```
int(-(exp(-2*x) + exp(2*x))/(exp(-2*x) - exp(2*x)),x)
```

output
```
log(exp(2*x) - 1)/2 - x + log(exp(2*x) + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{e^{-2x}+e^{2x}}{-e^{-2x}+e^{2x}}\,dx=\frac{\log(e^{2x}+1)}{2}+\frac{\log(e^{x}-1)}{2}+\frac{\log(e^{x}+1)}{2}-x$$

input
```
int((exp(-2*x)+exp(2*x))/(-exp(-2*x)+exp(2*x)),x)
```

output
```
(log(e**(2*x) + 1) + log(e**x - 1) + log(e**x + 1) - 2*x)/2
```

3.580 $\int \frac{e^x}{\sqrt{1+e^{2x}}} dx$

Optimal result

Integrand size = 15, antiderivative size = 4

$$\int \frac{e^x}{\sqrt{1+e^{2x}}} dx = \operatorname{arcsinh}(e^x)$$

output
```
arcsinh(exp(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(4) = 8.

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int \frac{e^x}{\sqrt{1+e^{2x}}} dx = -\log\left(-e^x+\sqrt{1+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[1 + E^(2*x)],x]
```

output
```
-Log[-E^x + Sqrt[1 + E^(2*x)]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2679, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{e^{2x}+1}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$\int \frac{1}{\sqrt{e^{2x}+1}}de^x$$
$$\Big\downarrow \text{ 222}$$
$$\text{arcsinh}(e^x)$$

input
```
Int[E^x/Sqrt[1 + E^(2*x)],x]
```

output
```
ArcSinh[E^x]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\operatorname{arcsinh}(e^x)$	4

input
```
int(exp(x)/(1+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 4.00

$$\int \frac{e^x}{\sqrt{1+e^{2x}}}\,dx = -\log\left(\sqrt{e^{(2x)}+1}-e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(e^(2*x) + 1) - e^x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1+e^{2x}}}\,dx = \operatorname{asinh}(e^x)$$

input
```
integrate(exp(x)/(1+exp(2*x))**(1/2),x)
```

output
```
asinh(exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1+e^{2x}}}\,dx = \operatorname{arsinh}\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(e^x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 4.00

$$\int \frac{e^x}{\sqrt{1+e^{2x}}}\,dx = -\log\left(\sqrt{e^{(2x)}+1}-e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(e^(2*x) + 1) - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{1+e^{2x}}}\,dx = \operatorname{asinh}(\mathrm{e}^x)$$

input
```
int(exp(x)/(exp(2*x) + 1)^(1/2),x)
```

output
```
asinh(exp(x))
```

Reduce [F]

$$\int \frac{e^x}{\sqrt{1+e^{2x}}}\,dx = \int \frac{e^x\sqrt{e^{2x}+1}}{e^{2x}+1}dx$$

input
```
int(exp(x)/(1+exp(2*x))^(1/2),x)
```

output
```
int((e**x*sqrt(e**(2*x) + 1))/(e**(2*x) + 1),x)
```

3.581 $\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2e^{\sqrt{4+x}}$$

output
```
2*exp((4+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2e^{\sqrt{4+x}}$$

input
```
Integrate[E^Sqrt[4 + x]/Sqrt[4 + x],x]
```

output
```
2*E^Sqrt[4 + x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sqrt{x+4}}}{\sqrt{x+4}} \, dx$$
$$\Big\downarrow \; 2638$$
$$2e^{\sqrt{x+4}}$$

input
```
Int[E^Sqrt[4 + x]/Sqrt[4 + x],x]
```

output
```
2*E^Sqrt[4 + x]
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\,{\mathrm e}^{\sqrt{4+x}}$	9
default	$2\,{\mathrm e}^{\sqrt{4+x}}$	9

input
```
int(exp((4+x)^(1/2))/(4+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*exp((4+x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2\,e^{\left(\sqrt{x+4}\right)}$$

input
```
integrate(exp((4+x)^(1/2))/(4+x)^(1/2),x, algorithm="fricas")
```

output
```
2*e^(sqrt(x + 4))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2e^{\sqrt{x+4}}$$

input
```
integrate(exp((4+x)**(1/2))/(4+x)**(1/2),x)
```

output
```
2*exp(sqrt(x + 4))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2\,e^{\left(\sqrt{x+4}\right)}$$

input
```
integrate(exp((4+x)^(1/2))/(4+x)^(1/2),x, algorithm="maxima")
```

output
```
2*e^(sqrt(x + 4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2\,e^{\left(\sqrt{x+4}\right)}$$

input
```
integrate(exp((4+x)^(1/2))/(4+x)^(1/2),x, algorithm="giac")
```

output
```
2*e^(sqrt(x + 4))
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2\,\mathrm{e}^{\sqrt{x+4}}$$

input
```
int(exp((x + 4)^(1/2))/(x + 4)^(1/2),x)
```

output
```
2*exp((x + 4)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^{\sqrt{4+x}}}{\sqrt{4+x}}\,dx = 2e^{\sqrt{x+4}}$$

input
```
int(exp((4+x)^(1/2))/(4+x)^(1/2),x)
```

output
```
2*e**sqrt(x + 4)
```

3.582 $\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{1}{2}\arctan\left(\sqrt{-1+e^{2x^2}}\right)$$

output
```
1/2*arctan((-1+exp(2*x^2))^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{1}{2}\arctan\left(\sqrt{-1+e^{2x^2}}\right)$$

input
```
Integrate[x/Sqrt[-1 + E^(2*x^2)],x]
```

output
```
ArcTan[Sqrt[-1 + E^(2*x^2)]]/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {7266, 2720, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{e^{2x^2}-1}}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-1+e^{2x^2}}}dx^2$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{-1+e^{2x^2}}x^2}de^{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt{-1+e^{2x^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(\sqrt{e^{2x^2}-1}\right)$$

input
```
Int[x/Sqrt[-1 + E^(2*x^2)],x]
```

output
```
ArcTan[Sqrt[-1 + E^(2*x^2)]]/2
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\arctan\left(\sqrt{-1+{\mathrm e}^{2 x^{2}}}\right)}{2}$	14
default	$\frac{\arctan\left(\sqrt{-1+{\mathrm e}^{2 x^{2}}}\right)}{2}$	14

input
```
int(x/(-1+exp(2*x^2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan((-1+exp(2*x^2))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{1}{2}\arctan\left(\sqrt{e^{(2\,x^2)}-1}\right)$$

input
```
integrate(x/(-1+exp(2*x^2))^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(sqrt(e^(2*x^2) - 1))
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{\operatorname{atan}\left(\sqrt{e^{2x^2}-1}\right)}{2}$$

input
```
integrate(x/(-1+exp(2*x**2))**(1/2),x)
```

output
```
atan(sqrt(exp(2*x**2) - 1))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{1}{2}\arctan\left(\sqrt{e^{(2\,x^2)}-1}\right)$$

input
```
integrate(x/(-1+exp(2*x^2))^(1/2),x, algorithm="maxima")
```

output
```
1/2*arctan(sqrt(e^(2*x^2) - 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{1}{2}\arctan\left(\sqrt{e^{(2x^2)}-1}\right)$$

input
```
integrate(x/(-1+exp(2*x^2))^(1/2),x, algorithm="giac")
```

output
```
1/2*arctan(sqrt(e^(2*x^2) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \frac{\mathrm{atan}\left(\sqrt{\mathrm{e}^{2x^2}-1}\right)}{2}$$

input
```
int(x/(exp(2*x^2) - 1)^(1/2),x)
```

output
```
atan((exp(2*x^2) - 1)^(1/2))/2
```

Reduce [F]

$$\int \frac{x}{\sqrt{-1+e^{2x^2}}}\,dx = \int \frac{\sqrt{e^{2x^2}-1}\,x}{e^{2x^2}-1}dx$$

input
```
int(x/(-1+exp(2*x^2))^(1/2),x)
```

output
```
int((sqrt(e**(2*x**2) - 1)*x)/(e**(2*x**2) - 1),x)
```

3.583 $\int e^x\sqrt{9+e^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{9+e^{2x}}+\frac{9}{2}\text{arcsinh}\left(\frac{e^x}{3}\right)$$

output
```
1/2*exp(x)*(9+exp(2*x))^(1/2)+9/2*arcsinh(1/3*exp(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{9+e^{2x}}-\frac{9}{2}\log\left(-e^x+\sqrt{9+e^{2x}}\right)$$

input
```
Integrate[E^x*Sqrt[9 + E^(2*x)],x]
```

output
```
(E^x*Sqrt[9 + E^(2*x)])/2 - (9*Log[-E^x + Sqrt[9 + E^(2*x)]])/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2679, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sqrt{e^{2x}+9}\,dx$$

$$\Big\downarrow \text{ 2679}$$

$$\int \sqrt{e^{2x}+9}de^x$$

$$\Big\downarrow \text{ 211}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{9+e^{2x}}}de^x+\frac{1}{2}e^x\sqrt{e^{2x}+9}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{9}{2}\text{arcsinh}\left(\frac{e^x}{3}\right)+\frac{1}{2}e^x\sqrt{e^{2x}+9}$$

input
```
Int[E^x*Sqrt[9 + E^(2*x)],x]
```

output
```
(E^x*Sqrt[9 + E^(2*x)])/2 + (9*ArcSinh[E^x/3])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\mathrm{e}^{x} \sqrt{9+\mathrm{e}^{2 x}}}{2}+\frac{9 \,\mathrm{arcsinh}\left(\frac{\mathrm{e}^{x}}{3}\right)}{2}$	21
risch	$\frac{\mathrm{e}^{x} \sqrt{9+\mathrm{e}^{2 x}}}{2}+\frac{9 \,\mathrm{arcsinh}\left(\frac{\mathrm{e}^{x}}{3}\right)}{2}$	21

input
```
int(exp(x)*(9+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(9+exp(x)^2)^(1/2)+9/2*arcsinh(1/3*exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int e^x \sqrt{9+e^{2x}}\, dx = \frac{1}{2} \, \sqrt{e^{(2\,x)} + 9} e^{x} - \frac{9}{2} \, \log\left(\sqrt{e^{(2\,x)} + 9} - e^{x}\right)$$

input
```
integrate(exp(x)*(9+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(e^(2*x) + 9)*e^x - 9/2*log(sqrt(e^(2*x) + 9) - e^x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{\sqrt{e^{2x}+9}e^x}{2}+\frac{9\operatorname{asinh}\left(\frac{e^x}{3}\right)}{2}$$

input
```
integrate(exp(x)*(9+exp(2*x))**(1/2),x)
```

output
```
sqrt(exp(2*x) + 9)*exp(x)/2 + 9*asinh(exp(x)/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{1}{2}\sqrt{e^{(2x)}+9}e^x+\frac{9}{2}\operatorname{arsinh}\left(\frac{1}{3}e^x\right)$$

input
```
integrate(exp(x)*(9+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(e^(2*x) + 9)*e^x + 9/2*arcsinh(1/3*e^x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{1}{2}\sqrt{e^{(2x)}+9}e^x-\frac{9}{2}\log\left(\sqrt{e^{(2x)}+9}-e^x\right)$$

input
```
integrate(exp(x)*(9+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(e^(2*x) + 9)*e^x - 9/2*log(sqrt(e^(2*x) + 9) - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{9\,\mathrm{asinh}\left(\frac{\mathrm{e}^x}{3}\right)}{2}+\frac{\mathrm{e}^x\,\sqrt{\mathrm{e}^{2\,x}+9}}{2}$$

input
```
int(exp(x)*(exp(2*x) + 9)^(1/2),x)
```

output
```
(9*asinh(exp(x)/3))/2 + (exp(x)*(exp(2*x) + 9)^(1/2))/2
```

Reduce [F]

$$\int e^x\sqrt{9+e^{2x}}\,dx=\frac{e^x\sqrt{e^{2x}+9}}{2}+\frac{9\left(\int\frac{e^x\sqrt{e^{2x}+9}}{e^{2x}+9}dx\right)}{2}$$

input
```
int(exp(x)*(9+exp(2*x))^(1/2),x)
```

output
```
(e**x*sqrt(e**(2*x) + 9) + 9*int((e**x*sqrt(e**(2*x) + 9))/(e**(2*x) + 9),
x))/2
```

3.584 $\int e^x\sqrt{1+e^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{1+e^{2x}}+\frac{\operatorname{arcsinh}(e^x)}{2}$$

output
```
1/2*exp(x)*(1+exp(2*x))^(1/2)+1/2*arcsinh(exp(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{1}{2}e^x\sqrt{1+e^{2x}}-\frac{1}{2}\log\left(-e^x+\sqrt{1+e^{2x}}\right)$$

input
```
Integrate[E^x*Sqrt[1 + E^(2*x)],x]
```

output
```
(E^x*Sqrt[1 + E^(2*x)])/2 - Log[-E^x + Sqrt[1 + E^(2*x)]]/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2679, 211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sqrt{e^{2x}+1}\,dx$$
$$\Big\downarrow \text{2679}$$
$$\int \sqrt{e^{2x}+1}de^x$$
$$\Big\downarrow \text{211}$$
$$\frac{1}{2}\int \frac{1}{\sqrt{1+e^{2x}}}de^x+\frac{1}{2}e^x\sqrt{e^{2x}+1}$$
$$\Big\downarrow \text{222}$$
$$\frac{\text{arcsinh}(e^x)}{2}+\frac{1}{2}e^x\sqrt{e^{2x}+1}$$

input
```
Int[E^x*Sqrt[1 + E^(2*x)],x]
```

output
```
(E^x*Sqrt[1 + E^(2*x)])/2 + ArcSinh[E^x]/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{e^{x}\sqrt{1+e^{2x}}}{2}+\frac{\operatorname{arcsinh}(e^{x})}{2}$	19
risch	$\frac{e^{x}\sqrt{1+e^{2x}}}{2}+\frac{\operatorname{arcsinh}(e^{x})}{2}$	19

input
```
int(exp(x)*(1+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(exp(x)^2+1)^(1/2)+1/2*arcsinh(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int e^{x}\sqrt{1+e^{2x}}\,dx=\frac{1}{2}\,\sqrt{e^{(2\,x)}+1}e^{x}-\frac{1}{2}\,\log\left(\sqrt{e^{(2\,x)}+1}-e^{x}\right)$$

input
```
integrate(exp(x)*(1+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(e^(2*x) + 1)*e^x - 1/2*log(sqrt(e^(2*x) + 1) - e^x)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{\sqrt{e^{2x}+1}e^x}{2}+\frac{\operatorname{asinh}(e^x)}{2}$$

input
```
integrate(exp(x)*(1+exp(2*x))**(1/2),x)
```

output
```
sqrt(exp(2*x) + 1)*exp(x)/2 + asinh(exp(x))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{1}{2}\sqrt{e^{(2x)}+1}e^x+\frac{1}{2}\operatorname{arsinh}(e^x)$$

input
```
integrate(exp(x)*(1+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(e^(2*x) + 1)*e^x + 1/2*arcsinh(e^x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{1}{2}\sqrt{e^{(2x)}+1}e^x-\frac{1}{2}\log\left(\sqrt{e^{(2x)}+1}-e^x\right)$$

input
```
integrate(exp(x)*(1+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(e^(2*x) + 1)*e^x - 1/2*log(sqrt(e^(2*x) + 1) - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{\operatorname{asinh}(e^x)}{2}+\frac{e^x\,\sqrt{e^{2\,x}+1}}{2}$$

input
```
int(exp(x)*(exp(2*x) + 1)^(1/2),x)
```

output
```
asinh(exp(x))/2 + (exp(x)*(exp(2*x) + 1)^(1/2))/2
```

Reduce [F]

$$\int e^x\sqrt{1+e^{2x}}\,dx=\frac{e^x\sqrt{e^{2x}+1}}{2}+\frac{\left(\int\frac{e^x\sqrt{e^{2x}+1}}{e^{2x}+1}dx\right)}{2}$$

input
```
int(exp(x)*(1+exp(2*x))^(1/2),x)
```

output
```
(e**x*sqrt(e**(2*x) + 1) + int((e**x*sqrt(e**(2*x) + 1))/(e**(2*x) + 1),x)
)/2
```

3.585 $\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx=\frac{1}{2}\arctan\left(e^{x^2}\right)$$

output
```
1/2*arctan(exp(x^2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx=\frac{1}{2}\arctan\left(e^{x^2}\right)$$

input
```
Integrate[(E^x^2*x)/(1 + E^(2*x^2)),x]
```

output
```
ArcTan[E^x^2]/2
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {7266, 2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x^2}x}{e^{2x^2}+1}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{e^{x^2}}{1+e^{2x^2}}dx^2$$

$$\Big\downarrow \text{2679}$$

$$\frac{1}{2}\int \frac{1}{x^4+1}de^{x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\arctan\left(e^{x^2}\right)$$

input
```
Int[(E^x^2*x)/(1 + E^(2*x^2)),x]
```

output
```
ArcTan[E^x^2]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\arctan\left({\mathrm e}^{x^{2}}\right)}{2}$	8
default	$\frac{\arctan\left({\mathrm e}^{x^{2}}\right)}{2}$	8
risch	$\frac{i \ln\left({\mathrm e}^{x^{2}}+i\right)}{4}-\frac{i \ln\left({\mathrm e}^{x^{2}}-i\right)}{4}$	24

input
```
int(exp(x^2)*x/(1+exp(2*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(exp(x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{x^2} x}{1+e^{2x^2}} \, dx = \frac{1}{2} \, \arctan\left(e^{\left(x^{2}\right)}\right)$$

input
```
integrate(exp(x^2)*x/(1+exp(2*x^2)),x, algorithm="fricas")
```

output
```
1/2*arctan(e^(x^2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx = \operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(4i+e^{x^2}\right)\right)\right)$$

input
```
integrate(exp(x**2)*x/(1+exp(2*x**2)),x)
```

output
```
RootSum(16*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x**2))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx = \frac{1}{2}\arctan\left(e^{\left(x^2\right)}\right)$$

input
```
integrate(exp(x^2)*x/(1+exp(2*x^2)),x, algorithm="maxima")
```

output
```
1/2*arctan(e^(x^2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx = \frac{1}{2}\arctan\left(e^{(x^2)}\right)$$

input
```
integrate(exp(x^2)*x/(1+exp(2*x^2)),x, algorithm="giac")
```

output
```
1/2*arctan(e^(x^2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx = \frac{\mathrm{atan}\left(\mathrm{e}^{x^2}\right)}{2}$$

input
```
int((x*exp(x^2))/(exp(2*x^2) + 1),x)
```

output
```
atan(exp(x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{x^2}x}{1+e^{2x^2}}\,dx = \frac{atan\left(e^{x^2}\right)}{2}$$

input
```
int(exp(x^2)*x/(1+exp(2*x^2)),x)
```

output
```
atan(e**(x**2))/2
```

3.586 $\int e^{x^{3/2}} x^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int e^{x^{3/2}} x^2 \, dx = -\frac{2}{3} e^{x^{3/2}} + \frac{2}{3} e^{x^{3/2}} x^{3/2}$$

output
```
-2/3*exp(x^(3/2))+2/3*exp(x^(3/2))*x^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.46

$$\int e^{x^{3/2}} x^2 \, dx = -\frac{2}{3} \Gamma\left(2, -x^{3/2}\right)$$

input
```
Integrate[E^x^(3/2)*x^2,x]
```

output
```
(-2*Gamma[2, -x^(3/2)])/3
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2645, 2641, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^{3/2}} x^2 \, dx$$

$$\Big\downarrow \text{2645}$$

$$2 \int e^{x^{3/2}} x^{5/2} d\sqrt{x}$$

$$\Big\downarrow \text{2641}$$

$$2 \left(\frac{1}{3} e^{x^{3/2}} x^{3/2} - \int e^{x^{3/2}} x d\sqrt{x}\right)$$

$$\Big\downarrow \text{2638}$$

$$2 \left(\frac{1}{3} e^{x^{3/2}} x^{3/2} - \frac{e^{x^{3/2}}}{3}\right)$$

input
```
Int[E^x^(3/2)*x^2,x]
```

output
```
2*(-1/3*E^x^(3/2) + (E^x^(3/2)*x^(3/2))/3)
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 2641
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m - n + 1)*(F^(a + b*(c + d*x)^n)/(b*d*n*L
og[F])), x] - Simp[(m - n + 1)/(b*n*Log[F])   Int[(c + d*x)^(m - n)*F^(a +
b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/
n)] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n
, 0])
```

rule 2645
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> With[{k = Denominator[n]}, Simp[k/d   Subst[Int[x^(k*(m +
1) - 1)*F^(a + b*x^(k*n)), x], x, (c + d*x)^(1/k)], x]] /; FreeQ[{F, a, b,
c, d, m, n}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[0, (m + 1)/n, 5] &&  !Int
egerQ[n]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

method	result	size
meijerg	$\frac{2}{3}-\frac{\left(2-2x^{\frac{3}{2}}\right) {\mathrm e}^{x^{\frac{3}{2}}}}{3}$	16
derivativedivides	$-\frac{2 \,{\mathrm e}^{x^{\frac{3}{2}}}}{3}+\frac{2 \,{\mathrm e}^{x^{\frac{3}{2}}} x^{\frac{3}{2}}}{3}$	17
default	$-\frac{2 \,{\mathrm e}^{x^{\frac{3}{2}}}}{3}+\frac{2 \,{\mathrm e}^{x^{\frac{3}{2}}} x^{\frac{3}{2}}}{3}$	17

input
```
int(exp(x^(3/2))*x^2,x,method=_RETURNVERBOSE)
```

output
```
2/3-1/3*(2-2*x^(3/2))*exp(x^(3/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.39

$$\int e^{x^{3/2}}x^2\,dx = \frac{2}{3}\left(x^{\frac{3}{2}} - 1\right)e^{\left(x^{\frac{3}{2}}\right)}$$

input
```
integrate(exp(x^(3/2))*x^2,x, algorithm="fricas")
```

output
```
2/3*(x^(3/2) - 1)*e^(x^(3/2))
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int e^{x^{3/2}}x^2\,dx = \frac{2x^{\frac{3}{2}}e^{x^{\frac{3}{2}}}}{3} - \frac{2e^{x^{\frac{3}{2}}}}{3}$$

input
```
integrate(exp(x**(3/2))*x**2,x)
```

output
```
2*x**(3/2)*exp(x**(3/2))/3 - 2*exp(x**(3/2))/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.39

$$\int e^{x^{3/2}}x^2\,dx = \frac{2}{3}\left(x^{\frac{3}{2}} - 1\right)e^{\left(x^{\frac{3}{2}}\right)}$$

input
```
integrate(exp(x^(3/2))*x^2,x, algorithm="maxima")
```

output
```
2/3*(x^(3/2) - 1)*e^(x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.39

$$\int e^{x^{3/2}} x^2 \, dx = \frac{2}{3} \left(x^{\frac{3}{2}} - 1\right) e^{\left(x^{\frac{3}{2}}\right)}$$

input
```
integrate(exp(x^(3/2))*x^2,x, algorithm="giac")
```

output
```
2/3*(x^(3/2) - 1)*e^(x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int e^{x^{3/2}} x^2 \, dx = \frac{2 \, x^{3/2} \, \mathrm{e}^{x^{3/2}}}{3} - \frac{2 \, \mathrm{e}^{x^{3/2}}}{3}$$

input
```
int(x^2*exp(x^(3/2)),x)
```

output
```
(2*x^(3/2)*exp(x^(3/2)))/3 - (2*exp(x^(3/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.50

$$\int e^{x^{3/2}} x^2 \, dx = \frac{2e^{\sqrt{x}\, x} \left(\sqrt{x}\, x - 1\right)}{3}$$

input
```
int(exp(x^(3/2))*x^2,x)
```

output
```
(2*e**(sqrt(x)*x)*(sqrt(x)*x - 1))/3
```

3.587 $\int \frac{e^x}{\sqrt{-3+e^{2x}}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}} \, dx = \operatorname{arctanh}\left(\frac{e^x}{\sqrt{-3+e^{2x}}}\right)$$

output
```
arctanh(exp(x)/(-3+exp(2*x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}} \, dx = -\log\left(-e^x + \sqrt{-3+e^{2x}}\right)$$

input
```
Integrate[E^x/Sqrt[-3 + E^(2*x)],x]
```

output
```
-Log[-E^x + Sqrt[-3 + E^(2*x)]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2679, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{\sqrt{e^{2x}-3}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$\int \frac{1}{\sqrt{e^{2x}-3}}de^x$$
$$\Big\downarrow \text{ 224}$$
$$\int \frac{1}{1-e^{2x}}d\frac{e^x}{\sqrt{e^{2x}-3}}$$
$$\Big\downarrow \text{ 219}$$
$$\text{arctanh}\left(\frac{e^x}{\sqrt{e^{2x}-3}}\right)$$

input
```
Int[E^x/Sqrt[-3 + E^(2*x)],x]
```

output
```
ArcTanh[E^x/Sqrt[-3 + E^(2*x)]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

method	result	size
default	$\ln\left(e^{x}+\sqrt{-3+e^{2x}}\right)$	13

input
```
int(exp(x)/(-3+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+(-3+exp(x)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}}\,dx=-\log\left(\sqrt{e^{(2x)}-3}-e^x\right)$$

input
```
integrate(exp(x)/(-3+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-log(sqrt(e^(2*x) - 3) - e^x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}}\,dx = \log\left(2\sqrt{e^{2x}-3}+2e^x\right)$$

input
```
integrate(exp(x)/(-3+exp(2*x))**(1/2),x)
```

output
```
log(2*sqrt(exp(2*x) - 3) + 2*exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}}\,dx = \log\left(2\,\sqrt{e^{(2\,x)}-3}+2\,e^x\right)$$

input
```
integrate(exp(x)/(-3+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(e^(2*x) - 3) + 2*e^x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}}\,dx = -\log\left(-\sqrt{e^{(2\,x)}-3}+e^x\right)$$

input
```
integrate(exp(x)/(-3+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
-log(-sqrt(e^(2*x) - 3) + e^x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}}\,dx = \ln\left(e^x + \sqrt{e^{2\,x}-3}\right)$$

input
```
int(exp(x)/(exp(2*x) - 3)^(1/2),x)
```

output
```
log(exp(x) + (exp(2*x) - 3)^(1/2))
```

Reduce [F]

$$\int \frac{e^x}{\sqrt{-3+e^{2x}}}\,dx = \int \frac{e^x\sqrt{e^{2x}-3}}{e^{2x}-3}dx$$

input
```
int(exp(x)/(-3+exp(2*x))^(1/2),x)
```

output
```
int((e**x*sqrt(e**(2*x) - 3))/(e**(2*x) - 3),x)
```

3.588 $\int \frac{e^x}{16-e^{2x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{e^x}{16-e^{2x}} dx = \frac{1}{4}\text{arctanh}\left(\frac{e^x}{4}\right)$$

output
```
1/4*arctanh(1/4*exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{16-e^{2x}} dx = \frac{1}{4}\text{arctanh}\left(\frac{e^x}{4}\right)$$

input
```
Integrate[E^x/(16 - E^(2*x)),x]
```

output
```
ArcTanh[E^x/4]/4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2679, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{16-e^{2x}}dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{16-e^{2x}}de^x$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{4}\text{arctanh}\left(\frac{e^x}{4}\right)$$

input
```
Int[E^x/(16 - E^(2*x)),x]
```

output
```
ArcTanh[E^x/4]/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

method	result	size
default	$-\frac{\ln(e^x-4)}{8}+\frac{\ln(e^x+4)}{8}$	16
norman	$-\frac{\ln(e^x-4)}{8}+\frac{\ln(e^x+4)}{8}$	16
risch	$-\frac{\ln(e^x-4)}{8}+\frac{\ln(e^x+4)}{8}$	16

input
```
int(exp(x)/(16-exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*ln(exp(x)-4)+1/8*ln(exp(x)+4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{16-e^{2x}}\,dx = \frac{1}{8}\,\log\left(e^x+4\right)-\frac{1}{8}\,\log\left(e^x-4\right)$$

input
```
integrate(exp(x)/(16-exp(2*x)),x, algorithm="fricas")
```

output
```
1/8*log(e^x + 4) - 1/8*log(e^x - 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{16-e^{2x}}\,dx = -\frac{\log\left(e^x-4\right)}{8} + \frac{\log\left(e^x+4\right)}{8}$$

input
```
integrate(exp(x)/(16-exp(2*x)),x)
```

output
```
-log(exp(x) - 4)/8 + log(exp(x) + 4)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{16-e^{2x}}\,dx = \frac{1}{8}\log\left(e^x+4\right) - \frac{1}{8}\log\left(e^x-4\right)$$

input
```
integrate(exp(x)/(16-exp(2*x)),x, algorithm="maxima")
```

output
```
1/8*log(e^x + 4) - 1/8*log(e^x - 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(7) = 14$.

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{e^x}{16-e^{2x}}\,dx = \frac{1}{8}\log\left(e^x+4\right) - \frac{1}{8}\log\left(|e^x-4|\right)$$

input
```
integrate(exp(x)/(16-exp(2*x)),x, algorithm="giac")
```

output
```
1/8*log(e^x + 4) - 1/8*log(abs(e^x - 4))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{e^x}{16-e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x+4\right)}{8} - \frac{\ln\left(\mathrm{e}^x-4\right)}{8}$$

input
```
int(-exp(x)/(exp(2*x) - 16),x)
```

output
```
log(exp(x) + 4)/8 - log(exp(x) - 4)/8
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{e^x}{16-e^{2x}}\,dx = -\frac{\log(e^x-4)}{8} + \frac{\log(e^x+4)}{8}$$

input
```
int(exp(x)/(16-exp(2*x)),x)
```

output
```
( - log(e**x - 4) + log(e**x + 4))/8
```

3.589 $\int \frac{e^{5x}}{1+e^{10x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \frac{1}{5}\arctan\left(e^{5x}\right)$$

output
```
1/5*arctan(exp(5*x))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \frac{1}{5}\arctan\left(e^{5x}\right)$$

input
```
Integrate[E^(5*x)/(1 + E^(10*x)),x]
```

output
```
ArcTan[E^(5*x)]/5
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{5x}}{e^{10x}+1} \, dx$$

$$\Big\downarrow \text{ 2679}$$

$$\frac{1}{5} \int \frac{1}{1+e^{10x}} de^{5x}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{5} \arctan \left(e^{5x}\right)$$

input
```
Int[E^(5*x)/(1 + E^(10*x)),x]
```

output
```
ArcTan[E^(5*x)]/5
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\arctan(\mathrm{e}^{5x})}{5}$	8
risch	$\frac{i\ln(\mathrm{e}^{5x}+i)}{10}-\frac{i\ln(\mathrm{e}^{5x}-i)}{10}$	24

input
```
int(exp(5*x)/(1+exp(10*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*arctan(exp(x)^5)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \frac{1}{5}\arctan\left(e^{(5\,x)}\right)$$

input
```
integrate(exp(5*x)/(1+exp(10*x)),x, algorithm="fricas")
```

output
```
1/5*arctan(e^(5*x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \operatorname{RootSum}\left(100z^2+1,\left(i\mapsto i\log\left(10i+e^{5x}\right)\right)\right)$$

input
```
integrate(exp(5*x)/(1+exp(10*x)),x)
```

output
```
RootSum(100*_z**2 + 1, Lambda(_i, _i*log(10*_i + exp(5*x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \frac{1}{5}\arctan\left(e^{(5x)}\right)$$

input
```
integrate(exp(5*x)/(1+exp(10*x)),x, algorithm="maxima")
```

output
```
1/5*arctan(e^(5*x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \frac{1}{5}\arctan\left(e^{(5x)}\right)$$

input
```
integrate(exp(5*x)/(1+exp(10*x)),x, algorithm="giac")
```

output
```
1/5*arctan(e^(5*x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \frac{\mathrm{atan}(\mathrm{e}^{5x})}{5}$$

input
```
int(exp(5*x)/(exp(10*x) + 1),x)
```

output
```
atan(exp(5*x))/5
```

Reduce [F]

$$\int \frac{e^{5x}}{1+e^{10x}}\,dx = \int \frac{e^{5x}}{e^{10x}+1}dx$$

input
```
int(exp(5*x)/(1+exp(10*x)),x)
```

output
```
int(e**(5*x)/(e**(10*x) + 1),x)
```

3.590 $\int \frac{e^{4x}}{\sqrt{16+e^{8x}}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}} \, dx = \frac{1}{4}\text{arcsinh}\left(\frac{e^{4x}}{4}\right)$$

output
```
1/4*arcsinh(1/4*exp(4*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}} \, dx = -\frac{1}{4}\log\left(-e^{4x}+\sqrt{16+e^{8x}}\right)$$

input
```
Integrate[E^(4*x)/Sqrt[16 + E^(8*x)],x]
```

output
```
-1/4*Log[-E^(4*x) + Sqrt[16 + E^(8*x)]]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2679, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{\sqrt{e^{8x}+16}}dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{16+e^{8x}}}de^{4x}$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{4}\text{arcsinh}\left(\frac{e^{4x}}{4}\right)$$

input
```
Int[E^(4*x)/Sqrt[16 + E^(8*x)],x]
```

output
```
ArcSinh[E^(4*x)/4]/4
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [F]

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}}dx$$

input
```
int(exp(4*x)/(16+exp(8*x))^(1/2),x)
```

output
```
int(exp(4*x)/(16+exp(8*x))^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}}\,dx = -\frac{1}{4}\,\log\left(\sqrt{e^{(8\,x)}+16}-e^{(4\,x)}\right)$$

input
```
integrate(exp(4*x)/(16+exp(8*x))^(1/2),x, algorithm="fricas")
```

output
```
-1/4*log(sqrt(e^(8*x) + 16) - e^(4*x))
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}}\,dx = \frac{\operatorname{asinh}\left(\frac{e^{4x}}{4}\right)}{4}$$

input
```
integrate(exp(4*x)/(16+exp(8*x))**(1/2),x)
```

output
```
asinh(exp(4*x)/4)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}}\,dx = \frac{1}{4}\,\operatorname{arsinh}\left(\frac{1}{4}\,e^{(4\,x)}\right)$$

input
```
integrate(exp(4*x)/(16+exp(8*x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*arcsinh(1/4*e^(4*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}}\,dx = -\frac{1}{4}\,\log\left(\sqrt{e^{(8\,x)}+16}-e^{(4\,x)}\right)$$

input
```
integrate(exp(4*x)/(16+exp(8*x))^(1/2),x, algorithm="giac")
```

output
```
-1/4*log(sqrt(e^(8*x) + 16) - e^(4*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{4x}}{\sqrt{16+e^{8x}}}\,dx = \int \frac{e^{4\,x}}{\sqrt{e^{8\,x}+16}}\,dx$$

input
```
int(exp(4*x)/(exp(8*x) + 16)^(1/2),x)
```

output
```
int(exp(4*x)/(exp(8*x) + 16)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{4x}}{\sqrt{16 + e^{8x}}}\, dx = \int \frac{e^{4x}\sqrt{e^{8x} + 16}}{e^{8x} + 16} dx$$

input
```
int(exp(4*x)/(16+exp(8*x))^(1/2),x)
```

output
```
int((e**(4*x)*sqrt(e**(8*x) + 16))/(e**(8*x) + 16),x)
```

3.591 $\int e^{4x^3} x^2 \cos(7x^3)\, dx$

Optimal result

Integrand size = 17, antiderivative size = 35

$$\int e^{4x^3} x^2 \cos(7x^3)\, dx = \frac{4}{195} e^{4x^3} \cos(7x^3) + \frac{7}{195} e^{4x^3} \sin(7x^3)$$

output
```
4/195*exp(4*x^3)*cos(7*x^3)+7/195*exp(4*x^3)*sin(7*x^3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int e^{4x^3} x^2 \cos(7x^3)\, dx = \frac{1}{195} e^{4x^3} (4\cos(7x^3) + 7\sin(7x^3))$$

input
```
Integrate[E^(4*x^3)*x^2*Cos[7*x^3],x]
```

output
```
(E^(4*x^3)*(4*Cos[7*x^3] + 7*Sin[7*x^3]))/195
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7266, 4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{4x^3} x^2 \cos\left(7x^3\right) \, dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{3} \int e^{4x^3} \cos\left(7x^3\right) dx^3$$

$$\Big\downarrow \; 4933$$

$$\frac{1}{3} \left(\frac{7}{65} e^{4x^3} \sin\left(7x^3\right) + \frac{4}{65} e^{4x^3} \cos\left(7x^3\right) \right)$$

input
```
Int[E^(4*x^3)*x^2*Cos[7*x^3],x]
```

output
```
((4*E^(4*x^3)*Cos[7*x^3])/65 + (7*E^(4*x^3)*Sin[7*x^3])/65)/3
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{4x^3}(4\cos(7x^3)+7\sin(7x^3))}{195}$	26
risch	$\frac{2\,e^{(4+7i)x^3}}{195}-\frac{7i e^{(4+7i)x^3}}{390}+\frac{2\,e^{(4-7i)x^3}}{195}+\frac{7i e^{(4-7i)x^3}}{390}$	44
norman	$\frac{\frac{14\,e^{4x^3}\tan\left(\frac{7x^3}{2}\right)}{195}-\frac{4\,e^{4x^3}\tan\left(\frac{7x^3}{2}\right)^2}{195}+\frac{4\,e^{4x^3}}{195}}{1+\tan\left(\frac{7x^3}{2}\right)^2}$	53
orering	$\frac{2(12x^3+1)e^{4x^3}\cos(7x^3)}{585x^3}-\frac{12\,e^{4x^3}x^4\cos(7x^3)+2\,e^{4x^3}x\cos(7x^3)-21\,e^{4x^3}x^4\sin(7x^3)}{585x^4}$	81

input
```
int(exp(4*x^3)*x^2*cos(7*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/195*exp(4*x^3)*(4*cos(7*x^3)+7*sin(7*x^3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{4x^3}x^2\cos\left(7x^3\right)\,dx=\frac{4}{195}\cos\left(7\,x^3\right)e^{\left(4\,x^3\right)}+\frac{7}{195}\,e^{\left(4\,x^3\right)}\sin\left(7\,x^3\right)$$

input
```
integrate(exp(4*x^3)*x^2*cos(7*x^3),x, algorithm="fricas")
```

output
```
4/195*cos(7*x^3)*e^(4*x^3) + 7/195*e^(4*x^3)*sin(7*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int e^{4x^3}x^2\cos\left(7x^3\right)\,dx=\frac{7e^{4x^3}\sin\left(7x^3\right)}{195}+\frac{4e^{4x^3}\cos\left(7x^3\right)}{195}$$

input
```
integrate(exp(4*x**3)*x**2*cos(7*x**3),x)
```

output
```
7*exp(4*x**3)*sin(7*x**3)/195 + 4*exp(4*x**3)*cos(7*x**3)/195
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{4x^3}x^2\cos\left(7x^3\right)\,dx=\frac{4}{195}\cos\left(7\,x^3\right)e^{\left(4\,x^3\right)}+\frac{7}{195}\,e^{\left(4\,x^3\right)}\sin\left(7\,x^3\right)$$

input
```
integrate(exp(4*x^3)*x^2*cos(7*x^3),x, algorithm="maxima")
```

output
```
4/195*cos(7*x^3)*e^(4*x^3) + 7/195*e^(4*x^3)*sin(7*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{4x^3}x^2\cos\left(7x^3\right)\,dx=\frac{1}{195}\left(4\cos\left(7\,x^3\right)+7\sin\left(7\,x^3\right)\right)e^{\left(4\,x^3\right)}$$

input
```
integrate(exp(4*x^3)*x^2*cos(7*x^3),x, algorithm="giac")
```

output
```
1/195*(4*cos(7*x^3) + 7*sin(7*x^3))*e^(4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{4x^3}x^2\cos\left(7x^3\right)\,dx=\frac{\mathrm{e}^{4\,x^3}\,\left(4\,\cos\left(7\,x^3\right)+7\,\sin\left(7\,x^3\right)\right)}{195}$$

input
```
int(x^2*exp(4*x^3)*cos(7*x^3),x)
```

output
```
(exp(4*x^3)*(4*cos(7*x^3) + 7*sin(7*x^3)))/195
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{4x^3}x^2\cos\left(7x^3\right)\,dx=\frac{e^{4x^3}(4\cos(7x^3)+7\sin(7x^3))}{195}$$

input
```
int(exp(4*x^3)*x^2*cos(7*x^3),x)
```

output
```
(e**(4*x**3)*(4*cos(7*x**3) + 7*sin(7*x**3)))/195
```

3.592 $\int e^{1+x^2} x\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int e^{1+x^2} x\,dx = \frac{e^{1+x^2}}{2}$$

output
```
1/2*exp(x^2+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{1+x^2} x\,dx = \frac{e^{1+x^2}}{2}$$

input
```
Integrate[E^(1 + x^2)*x,x]
```

output
```
E^(1 + x^2)/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2+1}x\,dx$$
$$\Big\downarrow \text{2638}$$
$$\frac{e^{x^2+1}}{2}$$

input
```
Int[E^(1 + x^2)*x,x]
```

output
```
E^(1 + x^2)/2
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{e^{x^2+1}}{2}$	9
derivativedivides	$\frac{e^{x^2+1}}{2}$	9
default	$\frac{e^{x^2+1}}{2}$	9
norman	$\frac{e^{x^2+1}}{2}$	9
risch	$\frac{e^{x^2+1}}{2}$	9
parallelrisch	$\frac{e^{x^2+1}}{2}$	9
orering	$\frac{e^{x^2+1}}{2}$	9
meijerg	$-\frac{e\left(1-e^{x^2}\right)}{2}$	13
parts	$\frac{e\sqrt{\pi}\,\operatorname{erfi}(x)x}{2}-\frac{e\left(\operatorname{erfi}(x)x\sqrt{\pi}-e^{x^2}\right)}{2}$	30

input
```
int(exp(x^2+1)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^2}x\,dx=\frac{1}{2}\,e^{\left(x^2+1\right)}$$

input
```
integrate(exp(x^2+1)*x,x, algorithm="fricas")
```

output
```
1/2*e^(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{1+x^2}x\,dx = \frac{e^{x^2+1}}{2}$$

input
```
integrate(exp(x**2+1)*x,x)
```

output
```
exp(x**2 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^2}x\,dx = \frac{1}{2}\,e^{(x^2+1)}$$

input
```
integrate(exp(x^2+1)*x,x, algorithm="maxima")
```

output
```
1/2*e^(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^2}x\,dx = \frac{1}{2}\,e^{(x^2+1)}$$

input
```
integrate(exp(x^2+1)*x,x, algorithm="giac")
```

output
```
1/2*e^(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^2} x\,dx = \frac{\mathrm{e}^{x^2}\,\mathrm{e}}{2}$$

input
```
int(x*exp(x^2 + 1),x)
```

output
```
(exp(x^2)*exp(1))/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^2} x\,dx = \frac{e^{x^2} e}{2}$$

input
```
int(exp(x^2+1)*x,x)
```

output
```
(e**(x**2)*e)/2
```

3.593 $\int e^{1+x^3} x^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int e^{1+x^3} x^2 \, dx = \frac{e^{1+x^3}}{3}$$

output
```
1/3*exp(x^3+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{1+x^3} x^2 \, dx = \frac{e^{1+x^3}}{3}$$

input
```
Integrate[E^(1 + x^3)*x^2,x]
```

output
```
E^(1 + x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^3+1}x^2\,dx$$

$$\Big\downarrow \; 2638$$

$$\frac{e^{x^3+1}}{3}$$

input
```
Int[E^(1 + x^3)*x^2,x]
```

output
```
E^(1 + x^3)/3
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{e^{x^3+1}}{3}$	9
derivativedivides	$\frac{e^{x^3+1}}{3}$	9
default	$\frac{e^{x^3+1}}{3}$	9
norman	$\frac{e^{x^3+1}}{3}$	9
parallelrisch	$\frac{e^{x^3+1}}{3}$	9
orering	$\frac{e^{x^3+1}}{3}$	9
meijerg	$-\frac{e\left(1-e^{x^3}\right)}{3}$	13
risch	$\frac{e^{(1+x)\left(x^2-x+1\right)}}{3}$	16

input
```
int(exp(x^3+1)*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*exp(x^3+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^3}x^2\,dx = \frac{1}{3}\,e^{\left(x^3+1\right)}$$

input
```
integrate(exp(x^3+1)*x^2,x, algorithm="fricas")
```

output
```
1/3*e^(x^3 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int e^{1+x^3}x^2\,dx = \frac{e^{x^3+1}}{3}$$

input
```
integrate(exp(x**3+1)*x**2,x)
```

output
```
exp(x**3 + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^3}x^2\,dx = \frac{1}{3}\,e^{(x^3+1)}$$

input
```
integrate(exp(x^3+1)*x^2,x, algorithm="maxima")
```

output
```
1/3*e^(x^3 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^3}x^2\,dx = \frac{1}{3}\,e^{(x^3+1)}$$

input
```
integrate(exp(x^3+1)*x^2,x, algorithm="giac")
```

output
```
1/3*e^(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^3}x^2\,dx = \frac{\mathrm{e}^{x^3}\,\mathrm{e}}{3}$$

input
```
int(x^2*exp(x^3 + 1),x)
```

output
```
(exp(x^3)*exp(1))/3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{1+x^3}x^2\,dx = \frac{e^{x^3}e}{3}$$

input
```
int(exp(x^3+1)*x^2,x)
```

output
```
(e**(x**3)*e)/3
```

3.594 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}}$$

output
```
2*exp(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}}$$

input
```
Integrate[E^Sqrt[x]/Sqrt[x],x]
```

output
```
2*E^Sqrt[x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$$
$$\Big\downarrow \; 2638$$
$$2e^{\sqrt{x}}$$

input
```
Int[E^Sqrt[x]/Sqrt[x],x]
```

output
```
2*E^Sqrt[x]
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$2\,{\mathrm e}^{\sqrt{x}}$	7
default	$2\,{\mathrm e}^{\sqrt{x}}$	7
meijerg	$-2+2\,{\mathrm e}^{\sqrt{x}}$	9

input
```
int(exp(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*exp(x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\,e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*e^sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}}$$

input
```
integrate(exp(x**(1/2))/x**(1/2),x)
```

output
```
2*exp(sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2\, e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*e^sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2\, e^{\sqrt{x}}$$

input
```
integrate(exp(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*e^sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx = 2\, \mathrm{e}^{\sqrt{x}}$$

input
```
int(exp(x^(1/2))/x^(1/2),x)
```

output
```
2*exp(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}}$$

input
```
int(exp(x^(1/2))/x^(1/2),x)
```

output
```
2*e**sqrt(x)
```

3.595 $\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 9

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} \, dx = 3e^{\sqrt[3]{x}}$$

output
```
3*exp(x^(1/3))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} \, dx = 3e^{\sqrt[3]{x}}$$

input
```
Integrate[E^x^(1/3)/x^(2/3),x]
```

output
```
3*E^x^(1/3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}} \, dx$$

$$\Big\downarrow \text{2638}$$

$$3e^{\sqrt[3]{x}}$$

input
```
Int[E^x^(1/3)/x^(2/3),x]
```

output
```
3*E^x^(1/3)
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$3\,{\mathrm e}^{x^{\frac{1}{3}}}$	7
default	$3\,{\mathrm e}^{x^{\frac{1}{3}}}$	7
meijerg	$-3+3\,{\mathrm e}^{x^{\frac{1}{3}}}$	9

input
```
int(exp(x^(1/3))/x^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3*exp(x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx = 3\,e^{\left(x^{\frac{1}{3}}\right)}$$

input
```
integrate(exp(x^(1/3))/x^(2/3),x, algorithm="fricas")
```

output
```
3*e^(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx = 3e^{\sqrt[3]{x}}$$

input
```
integrate(exp(x**(1/3))/x**(2/3),x)
```

output
```
3*exp(x**(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx = 3\,e^{\left(x^{\frac{1}{3}}\right)}$$

input
```
integrate(exp(x^(1/3))/x^(2/3),x, algorithm="maxima")
```

output
```
3*e^(x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx = 3\,e^{\left(x^{\frac{1}{3}}\right)}$$

input
```
integrate(exp(x^(1/3))/x^(2/3),x, algorithm="giac")
```

output
```
3*e^(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\,dx = 3\,\mathrm{e}^{x^{1/3}}$$

input
```
int(exp(x^(1/3))/x^(2/3),x)
```

output
```
3*exp(x^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{e^{\sqrt[3]{x}}}{x^{2/3}}\, dx = 3e^{x^{\frac{1}{3}}}$$

input
```
int(exp(x^(1/3))/x^(2/3),x)
```

output
```
3*e**(x**(1/3))
```

3.596 $\int e^{3x}(-8+2x^3+x^5)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=-\frac{724e^{3x}}{243}+\frac{76}{81}e^{3x}x-\frac{38}{27}e^{3x}x^2+\frac{38}{27}e^{3x}x^3-\frac{5}{9}e^{3x}x^4+\frac{1}{3}e^{3x}x^5$$

output
```
-724/243*exp(3*x)+76/81*exp(3*x)*x-38/27*exp(3*x)*x^2+38/27*exp(3*x)*x^3-5
/9*exp(3*x)*x^4+1/3*exp(3*x)*x^5
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{1}{243}e^{3x}\left(-724+228x-342x^2+342x^3-135x^4+81x^5\right)$$

input
```
Integrate[E^(3*x)*(-8 + 2*x^3 + x^5),x]
```

output
```
(E^(3*x)*(-724 + 228*x - 342*x^2 + 342*x^3 - 135*x^4 + 81*x^5))/243
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x}\left(x^5+2x^3-8\right)\,dx$$

$$\Big\downarrow \ 2626$$

$$\int \left(e^{3x}x^5+2e^{3x}x^3-8e^{3x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}e^{3x}x^5-\frac{5}{9}e^{3x}x^4+\frac{38}{27}e^{3x}x^3-\frac{38}{27}e^{3x}x^2+\frac{76}{81}e^{3x}x-\frac{724e^{3x}}{243}$$

input
```
Int[E^(3*x)*(-8 + 2*x^3 + x^5),x]
```

output
```
(-724*E^(3*x))/243 + (76*E^(3*x)*x)/81 - (38*E^(3*x)*x^2)/27 + (38*E^(3*x)
*x^3)/27 - (5*E^(3*x)*x^4)/9 + (E^(3*x)*x^5)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

method	result	size
risch	$\left(\frac{1}{3}x^5-\frac{5}{9}x^4+\frac{38}{27}x^3-\frac{38}{27}x^2+\frac{76}{81}x-\frac{724}{243}\right)\mathrm{e}^{3x}$	31
gosper	$\frac{\mathrm{e}^{3x}(81x^5-135x^4+342x^3-342x^2+228x-724)}{243}$	32
derivativedivides	$-\frac{724\,\mathrm{e}^{3x}}{243}+\frac{76\,\mathrm{e}^{3x}x}{81}-\frac{38\,\mathrm{e}^{3x}x^2}{27}+\frac{38\,\mathrm{e}^{3x}x^3}{27}-\frac{5\,\mathrm{e}^{3x}x^4}{9}+\frac{\mathrm{e}^{3x}x^5}{3}$	51
default	$-\frac{724\,\mathrm{e}^{3x}}{243}+\frac{76\,\mathrm{e}^{3x}x}{81}-\frac{38\,\mathrm{e}^{3x}x^2}{27}+\frac{38\,\mathrm{e}^{3x}x^3}{27}-\frac{5\,\mathrm{e}^{3x}x^4}{9}+\frac{\mathrm{e}^{3x}x^5}{3}$	51
norman	$-\frac{724\,\mathrm{e}^{3x}}{243}+\frac{76\,\mathrm{e}^{3x}x}{81}-\frac{38\,\mathrm{e}^{3x}x^2}{27}+\frac{38\,\mathrm{e}^{3x}x^3}{27}-\frac{5\,\mathrm{e}^{3x}x^4}{9}+\frac{\mathrm{e}^{3x}x^5}{3}$	51
parallelrisch	$-\frac{724\,\mathrm{e}^{3x}}{243}+\frac{76\,\mathrm{e}^{3x}x}{81}-\frac{38\,\mathrm{e}^{3x}x^2}{27}+\frac{38\,\mathrm{e}^{3x}x^3}{27}-\frac{5\,\mathrm{e}^{3x}x^4}{9}+\frac{\mathrm{e}^{3x}x^5}{3}$	51
parts	$-\frac{724\,\mathrm{e}^{3x}}{243}+\frac{76\,\mathrm{e}^{3x}x}{81}-\frac{38\,\mathrm{e}^{3x}x^2}{27}+\frac{38\,\mathrm{e}^{3x}x^3}{27}-\frac{5\,\mathrm{e}^{3x}x^4}{9}+\frac{\mathrm{e}^{3x}x^5}{3}$	51
meijerg	$\frac{724}{243}-\frac{(-1458x^5+2430x^4-3240x^3+3240x^2-2160x+720)\mathrm{e}^{3x}}{4374}-\frac{(-108x^3+108x^2-72x+24)\mathrm{e}^{3x}}{162}-\frac{8\,\mathrm{e}^{3x}}{3}$	61
orering	$\frac{(81x^5-135x^4+342x^3-342x^2+228x-724)\mathrm{e}^{3x}(x^5+2x^3-8)}{243(x^2+2x+2)(x^3-2x^2+4x-4)}$	67

input
```
int(exp(3*x)*(x^5+2*x^3-8),x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^5-5/9*x^4+38/27*x^3-38/27*x^2+76/81*x-724/243)*exp(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{1}{243}\left(81\,x^5-135\,x^4+342\,x^3-342\,x^2+228\,x-724\right)e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(x^5+2*x^3-8),x, algorithm="fricas")
```

output
```
1/243*(81*x^5 - 135*x^4 + 342*x^3 - 342*x^2 + 228*x - 724)*e^(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{\left(81x^5-135x^4+342x^3-342x^2+228x-724\right)e^{3x}}{243}$$

input
```
integrate(exp(3*x)*(x**5+2*x**3-8),x)
```

output
```
(81*x**5 - 135*x**4 + 342*x**3 - 342*x**2 + 228*x - 724)*exp(3*x)/243
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{1}{243}\left(81\,x^5-135\,x^4+180\,x^3-180\,x^2+120\,x-40\right)e^{(3\,x)}+\frac{2}{27}\left(9\,x^3-9\,x^2+6\,x-2\right)e^{(3\,x)}-\frac{8}{3}\,e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(x^5+2*x^3-8),x, algorithm="maxima")
```

output
```
1/243*(81*x^5 - 135*x^4 + 180*x^3 - 180*x^2 + 120*x - 40)*e^(3*x) + 2/27*(
9*x^3 - 9*x^2 + 6*x - 2)*e^(3*x) - 8/3*e^(3*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{1}{243}\left(81\,x^5-135\,x^4+342\,x^3-342\,x^2+228\,x-724\right)e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(x^5+2*x^3-8),x, algorithm="giac")
```

output
```
1/243*(81*x^5 - 135*x^4 + 342*x^3 - 342*x^2 + 228*x - 724)*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.46

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{\mathrm{e}^{3\,x}\,\left(81\,x^5-135\,x^4+342\,x^3-342\,x^2+228\,x-724\right)}{243}$$

input
```
int(exp(3*x)*(2*x^3 + x^5 - 8),x)
```

output
```
(exp(3*x)*(228*x - 342*x^2 + 342*x^3 - 135*x^4 + 81*x^5 - 724))/243
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.47

$$\int e^{3x}\left(-8+2x^3+x^5\right)\,dx=\frac{e^{3x}(81x^5-135x^4+342x^3-342x^2+228x-724)}{243}$$

input
```
int(exp(3*x)*(x^5+2*x^3-8),x)
```

output
```
(e**(3*x)*(81*x**5 - 135*x**4 + 342*x**3 - 342*x**2 + 228*x - 724))/243
```

3.597 $\int (e^x+x)^2 \, dx$

Optimal result

Integrand size = 7, antiderivative size = 28

$$\int (e^x+x)^2 \, dx = -2e^x+\frac{e^{2x}}{2}+2e^x x+\frac{x^3}{3}$$

output
```
-2*exp(x)+1/2*exp(2*x)+2*exp(x)*x+1/3*x^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int (e^x+x)^2 \, dx = \frac{e^{2x}}{2}+\frac{x^3}{3}+e^x(-2+2x)$$

input
```
Integrate[(E^x + x)^2,x]
```

output
```
E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+e^x)^2 \, dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(x^2+2e^x x+e^{2x}\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{x^3}{3}+2e^x x-2e^x+\frac{e^{2x}}{2}$$

input
```
Int[(E^x + x)^2,x]
```

output
```
-2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result
risch	$\frac{x^3}{3}+(-2+2x)\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}$
default	$-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^{x}x+\frac{x^3}{3}$
norman	$-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^{x}x+\frac{x^3}{3}$
parallelrisch	$-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^{x}x+\frac{x^3}{3}$
parts	$-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2x}}{2}+2\,\mathrm{e}^{x}x+\frac{x^3}{3}$
orering	$\frac{(2x^4-21x^2+39x-15)(\mathrm{e}^{x}+x)^2}{6(-1+x)^3}-\frac{(6x^4-25x^3+30x^2+6x-12)(\mathrm{e}^{x}+x)(1+\mathrm{e}^{x})}{6(-1+x)^3}+\frac{x(2x^2-9x+12)\left(2(1+\mathrm{e}^{x})^2+2(\mathrm{e}^{x}+x)\mathrm{e}^{x}\right)}{12(-1+x)^2}$

input
```
int((exp(x)+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3+(-2+2*x)*exp(x)+1/2*exp(2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x+x)^2\,dx = \frac{1}{3}\,x^3+2\,(x-1)e^x+\frac{1}{2}\,e^{(2\,x)}$$

input
```
integrate((exp(x)+x)^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int (e^x + x)^2 \, dx = \frac{x^3}{3} + \frac{(4x - 4)\, e^x}{2} + \frac{e^{2x}}{2}$$

input
```
integrate((exp(x)+x)**2,x)
```

output
```
x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3}\, x^3 + 2\,(x - 1)e^x + \frac{1}{2}\, e^{(2\,x)}$$

input
```
integrate((exp(x)+x)^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int (e^x + x)^2 \, dx = \frac{1}{3}\, x^3 + 2\,(x - 1)e^x + \frac{1}{2}\, e^{(2\,x)}$$

input
```
integrate((exp(x)+x)^2,x, algorithm="giac")
```

output
```
1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int (e^x + x)^2 \, dx = \frac{e^{2x}}{2} - 2e^x + 2x e^x + \frac{x^3}{3}$$

input
```
int((x + exp(x))^2,x)
```

output
```
exp(2*x)/2 - 2*exp(x) + 2*x*exp(x) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int (e^x + x)^2 \, dx = \frac{e^{2x}}{2} + 2e^x x - 2e^x + \frac{x^3}{3}$$

input
```
int((exp(x)+x)^2,x)
```

output
```
(3*e**(2*x) + 12*e**x*x - 12*e**x + 2*x**3)/6
```

3.598 $\int e^{-4x}(e^x + e^{2x} + e^{3x}) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int e^{-4x}\left(e^x + e^{2x} + e^{3x}\right) \, dx = -\frac{1}{3}e^{-3x} - \frac{e^{-2x}}{2} - e^{-x}$$

output
```
-1/3/exp(3*x)-1/2/exp(2*x)-exp(-x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{-4x}\left(e^x + e^{2x} + e^{3x}\right) \, dx = \frac{1}{6}e^{-3x}\left(-2 - 3e^x - 6e^{2x}\right)$$

input
```
Integrate[(E^x + E^(2*x) + E^(3*x))/E^(4*x),x]
```

output
```
(-2 - 3*E^x - 6*E^(2*x))/(6*E^(3*x))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {2720, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-4x}\left(e^x+e^{2x}+e^{3x}\right)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-4x}\left(e^x+e^{2x}+1\right)de^x$$

$$\Big\downarrow \text{1140}$$

$$\int \left(e^{-4x}+e^{-3x}+e^{-2x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}e^{-3x}-\frac{e^{-2x}}{2}-e^{-x}$$

input
```
Int[(E^x + E^(2*x) + E^(3*x))/E^(4*x),x]
```

output
```
-1/3*1/E^(3*x) - 1/(2*E^(2*x)) - E^(-x)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{e^{-3x}}{3}-\frac{e^{-2x}}{2}-e^{-x}$	20
risch	$-\frac{e^{-3x}}{3}-\frac{e^{-2x}}{2}-e^{-x}$	20
parts	$-\frac{e^{-3x}}{3}-\frac{e^{-2x}}{2}-e^{-x}$	20
meijerg	$\frac{11}{6}-\frac{e^{-3x}}{3}-\frac{e^{-2x}}{2}-e^{-x}$	21
norman	$\left(-\frac{e^{2x}}{2}-e^{3x}-\frac{e^{x}}{3}\right)e^{-4x}$	23
parallelrisch	$\frac{\left(-2\,e^{x}-3\,e^{2x}-6\,e^{3x}\right)e^{-4x}}{6}$	26
orering	$-\frac{\left(e^{x}+e^{2x}+e^{3x}\right)e^{-4x}}{2}+\frac{\left(e^{x}+2\,e^{2x}+3\,e^{3x}\right)e^{-4x}}{3}-\frac{\left(e^{x}+4\,e^{2x}+9\,e^{3x}\right)e^{-4x}}{6}$	67

input
```
int((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3/exp(x)^3-1/2/exp(x)^2-1/exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int e^{-4x}\left(e^{x}+e^{2x}+e^{3x}\right)\,dx=-\frac{1}{6}\left(6\,e^{(2\,x)}+3\,e^{x}+2\right)e^{(-3\,x)}$$

input
```
integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x, algorithm="fricas")
```

output
```
-1/6*(6*e^(2*x) + 3*e^x + 2)*e^(-3*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int e^{-4x}\left(e^x+e^{2x}+e^{3x}\right)\,dx=-e^{-x}-\frac{e^{-2x}}{2}-\frac{e^{-3x}}{3}$$

input
```
integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x)
```

output
```
-exp(-x) - exp(-2*x)/2 - exp(-3*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int e^{-4x}\left(e^x+e^{2x}+e^{3x}\right)\,dx=-e^{(-x)}-\frac{1}{2}\,e^{(-2\,x)}-\frac{1}{3}\,e^{(-3\,x)}$$

input
```
integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x, algorithm="maxima")
```

output
```
-e^(-x) - 1/2*e^(-2*x) - 1/3*e^(-3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int e^{-4x}\left(e^x+e^{2x}+e^{3x}\right)\,dx=-\frac{1}{6}\,\left(6\,e^{(2\,x)}+3\,e^{x}+2\right)e^{(-3\,x)}$$

input
```
integrate((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x, algorithm="giac")
```

output
```
-1/6*(6*e^(2*x) + 3*e^x + 2)*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int e^{-4x}\left(e^x+e^{2x}+e^{3x}\right)\,dx=-\frac{\mathrm{e}^{-3\,x}\,\left(6\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^x+2\right)}{6}$$

input
```
int(exp(-4*x)*(exp(2*x) + exp(3*x) + exp(x)),x)
```

output
```
-(exp(-3*x)*(6*exp(2*x) + 3*exp(x) + 2))/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int e^{-4x}\left(e^x+e^{2x}+e^{3x}\right)\,dx=\frac{-6e^{2x}-3e^x-2}{6e^{3x}}$$

input
```
int((exp(x)+exp(2*x)+exp(3*x))/exp(4*x),x)
```

output
```
( - 6*e**(2*x) - 3*e**x - 2)/(6*e**(3*x))
```

3.599 $\int \frac{e^x}{1+2e^x+e^{2x}} dx$

Optimal result

Integrand size = 18, antiderivative size = 9

$$\int \frac{e^x}{1+2e^x+e^{2x}} dx = -\frac{1}{1+e^x}$$

output
```
-1/(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+2e^x+e^{2x}} dx = -\frac{1}{1+e^x}$$

input
```
Integrate[E^x/(1 + 2*E^x + E^(2*x)),x]
```

output
```
-(1 + E^x)^(-1)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2720, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{2e^x + e^{2x} + 1} \, dx$$
$$\Big\downarrow \; 2720$$
$$\int \frac{1}{(e^x + 1)^2} de^x$$
$$\Big\downarrow \; 17$$
$$-\frac{1}{e^x + 1}$$

input
```
Int[E^x/(1 + 2*E^x + E^(2*x)),x]
```

output
```
-(1 + E^x)^(-1)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{1}{1+e^{x}}$	9
norman	$-\frac{1}{1+e^{x}}$	9
risch	$-\frac{1}{1+e^{x}}$	9

input
```
int(exp(x)/(1+2*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x}{1+2e^x+e^{2x}}\,dx=-\frac{1}{e^x+1}$$

input
```
integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
-1/(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{e^x}{1+2e^x+e^{2x}}\,dx=-\frac{1}{e^x+1}$$

input
```
integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x)
```

output
```
-1/(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x}{1+2e^x+e^{2x}}\,dx = -\frac{1}{e^x+1}$$

input
```
integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
-1/(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x}{1+2e^x+e^{2x}}\,dx = -\frac{1}{e^x+1}$$

input
```
integrate(exp(x)/(1+2*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
-1/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{e^x}{1+2e^x+e^{2x}}\,dx = -\frac{1}{\mathrm{e}^x+1}$$

input
```
int(exp(x)/(exp(2*x) + 2*exp(x) + 1),x)
```

output
```
-1/(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{e^x}{1+2e^x+e^{2x}}\,dx=\frac{e^x}{e^x+1}$$

input
```
int(exp(x)/(1+2*exp(x)+exp(2*x)),x)
```

output
```
e**x/(e**x + 1)
```

3.600 $\int e^{-x}\cos(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-x}\cos(3x)\,dx = -\frac{1}{10}e^{-x}\cos(3x) + \frac{3}{10}e^{-x}\sin(3x)$$

output
```
-1/10*cos(3*x)/exp(x)+3/10*sin(3*x)/exp(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{-x}\cos(3x)\,dx = -\frac{1}{10}e^{-x}(\cos(3x) - 3\sin(3x))$$

input
```
Integrate[Cos[3*x]/E^x,x]
```

output
```
-1/10*(Cos[3*x] - 3*Sin[3*x])/E^x
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\cos(3x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{3}{10}e^{-x}\sin(3x)-\frac{1}{10}e^{-x}\cos(3x)$$

input
```
Int[Cos[3*x]/E^x,x]
```

output
```
-1/10*Cos[3*x]/E^x + (3*Sin[3*x])/(10*E^x)
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{e^{-x}(\cos(3x)-3\sin(3x))}{10}$	18
default	$-\frac{e^{-x}\cos(3x)}{10}+\frac{3\,e^{-x}\sin(3x)}{10}$	22
orering	$-\frac{e^{-x}\cos(3x)}{10}+\frac{3\,e^{-x}\sin(3x)}{10}$	22
norman	$\frac{\left(-\frac{1}{10}+\frac{\tan\left(\frac{3x}{2}\right)^{2}}{10}+\frac{3\tan\left(\frac{3x}{2}\right)}{5}\right)e^{-x}}{1+\tan\left(\frac{3x}{2}\right)^{2}}$	32
risch	$-\frac{e^{(-1+3i)x}}{20}-\frac{3i e^{(-1+3i)x}}{20}-\frac{e^{(-1-3i)x}}{20}+\frac{3i e^{(-1-3i)x}}{20}$	36

input
```
int(cos(3*x)/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-1/10*exp(-x)*(cos(3*x)-3*sin(3*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-x}\cos(3x)\,dx = -\frac{1}{10}\cos(3x)\,e^{(-x)}+\frac{3}{10}\,e^{(-x)}\sin(3x)$$

input
```
integrate(cos(3*x)/exp(x),x, algorithm="fricas")
```

output
```
-1/10*cos(3*x)*e^(-x) + 3/10*e^(-x)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{-x}\cos(3x)\,dx = \frac{3e^{-x}\sin(3x)}{10}-\frac{e^{-x}\cos(3x)}{10}$$

input
```
integrate(cos(3*x)/exp(x),x)
```

output
```
3*exp(-x)*sin(3*x)/10 - exp(-x)*cos(3*x)/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{-x}\cos(3x)\,dx = -\frac{1}{10}\left(\cos\left(3\,x\right) - 3\,\sin\left(3\,x\right)\right)e^{(-x)}$$

input
```
integrate(cos(3*x)/exp(x),x, algorithm="maxima")
```

output
```
-1/10*(cos(3*x) - 3*sin(3*x))*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{-x}\cos(3x)\,dx = -\frac{1}{10}\left(\cos\left(3\,x\right) - 3\,\sin\left(3\,x\right)\right)e^{(-x)}$$

input
```
integrate(cos(3*x)/exp(x),x, algorithm="giac")
```

output
```
-1/10*(cos(3*x) - 3*sin(3*x))*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{-x}\cos(3x)\,dx = -\frac{\mathrm{e}^{-x}\left(\cos\left(3\,x\right) - 3\,\sin\left(3\,x\right)\right)}{10}$$

input
```
int(cos(3*x)*exp(-x),x)
```

output
```
-(exp(-x)*(cos(3*x) - 3*sin(3*x)))/10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{-x}\cos(3x)\,dx = \frac{-\cos(3x)+3\sin(3x)}{10e^{x}}$$

input
```
int(cos(3*x)/exp(x),x)
```

output
```
( - cos(3*x) + 3*sin(3*x))/(10*e**x)
```

3.601 $\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = -\log(1+e^x)+2\log(2+e^x)$$

output
```
-ln(1+exp(x))+2*ln(2+exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = -\log(1+e^x)+2\log(2+e^x)$$

input
```
Integrate[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

output
```
-Log[1 + E^x] + 2*Log[2 + E^x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2720, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{3e^x+e^{2x}+2}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^x}{3e^x+e^{2x}+2}de^x$$

$$\Big\downarrow \text{ 1141}$$

$$\int \left(\frac{2}{e^x+2}+\frac{1}{-e^x-1}\right)de^x$$

$$\Big\downarrow \text{ 2009}$$

$$2\log\left(e^x+2\right)-\log\left(e^x+1\right)$$

input
```
Int[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

output
```
-Log[1 + E^x] + 2*Log[2 + E^x]
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
default	$-\ln \left(1+{\mathrm e}^{x}\right)+2 \ln \left(2+{\mathrm e}^{x}\right)$	16
norman	$-\ln \left(1+{\mathrm e}^{x}\right)+2 \ln \left(2+{\mathrm e}^{x}\right)$	16
risch	$-\ln \left(1+{\mathrm e}^{x}\right)+2 \ln \left(2+{\mathrm e}^{x}\right)$	16

input
```
int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+exp(x))+2*ln(2+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\log\left(e^{x}+2\right)-\log\left(e^{x}+1\right)$$

input
```
integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
2*log(e^x + 2) - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = -\log\left(e^x+1\right)+2\log\left(e^x+2\right)$$

input
```
integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

output
```
-log(exp(x) + 1) + 2*log(exp(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\log\left(e^x+2\right)-\log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
2*log(e^x + 2) - log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\log\left(e^x+2\right)-\log\left(e^x+1\right)$$

input
```
integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
2*log(e^x + 2) - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\,\ln\left(\mathrm{e}^x+2\right)-\ln\left(\mathrm{e}^x+1\right)$$

input
```
int(exp(2*x)/(exp(2*x) + 3*exp(x) + 2),x)
```

output
```
2*log(exp(x) + 2) - log(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx = 2\log(e^x+2)-\log(e^x+1)$$

input
```
int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

output
```
2*log(e**x + 2) - log(e**x + 1)
```

3.602 $\int \frac{e^{2x}}{1+e^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^{2x}}{1+e^x} \, dx = e^x - \log(1+e^x)$$

output
```
exp(x)-ln(1+exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x} \, dx = e^x - \log(1+e^x)$$

input
```
Integrate[E^(2*x)/(1 + E^x),x]
```

output
```
E^x - Log[1 + E^x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{e^x+1} dx$$
$$\Big\downarrow \text{2678}$$
$$\int \frac{e^x}{e^x+1} de^x$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{1}{-e^x-1}+1\right) de^x$$
$$\Big\downarrow \text{2009}$$
$$e^x-\log \left(e^x+1\right)$$

input
```
Int[E^(2*x)/(1 + E^x),x]
```

output
```
E^x - Log[1 + E^x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
default	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11
norman	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11
risch	$\mathrm{e}^{x}-\ln\left(1+\mathrm{e}^{x}\right)$	11

input
```
int(exp(2*x)/(1+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)-ln(1+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

output
```
e^x - log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x)
```

output
```
exp(x) - log(exp(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

output
```
e^x - log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x}\,dx = e^x - \log\left(e^x + 1\right)$$

input
```
integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

output
```
e^x - log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^{2x}}{1+e^x} \, dx = e^x - \ln\left(e^x + 1\right)$$

input
```
int(exp(2*x)/(exp(x) + 1),x)
```

output
```
exp(x) - log(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{1+e^x} \, dx = e^x - \log(e^x + 1)$$

input
```
int(exp(2*x)/(1+exp(x)),x)
```

output
```
e**x - log(e**x + 1)
```

3.603 $\int e^{3x}\cos(5x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{3x}\cos(5x)\,dx=\frac{3}{34}e^{3x}\cos(5x)+\frac{5}{34}e^{3x}\sin(5x)$$

output
```
3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{3x}\cos(5x)\,dx=\frac{1}{34}e^{3x}(3\cos(5x)+5\sin(5x))$$

input
```
Integrate[E^(3*x)*Cos[5*x],x]
```

output
```
(E^(3*x)*(3*Cos[5*x] + 5*Sin[5*x]))/34
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x}\cos(5x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

input
```
Int[E^(3*x)*Cos[5*x],x]
```

output
```
(3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{3x}(3\cos(5x)+5\sin(5x))}{34}$	20
default	$\frac{3\,e^{3x}\cos(5x)}{34}+\frac{5\,e^{3x}\sin(5x)}{34}$	22
orering	$\frac{3\,e^{3x}\cos(5x)}{34}+\frac{5\,e^{3x}\sin(5x)}{34}$	22
risch	$\frac{3\,e^{(3+5i)x}}{68}-\frac{5ie^{(3+5i)x}}{68}+\frac{3\,e^{(3-5i)x}}{68}+\frac{5ie^{(3-5i)x}}{68}$	36
norman	$\frac{\frac{5\,e^{3x}\tan\left(\frac{5x}{2}\right)}{17}-\frac{3\,e^{3x}\tan\left(\frac{5x}{2}\right)^{2}}{34}+\frac{3\,e^{3x}}{34}}{1+\tan\left(\frac{5x}{2}\right)^{2}}$	41

input
```
int(exp(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/34*exp(3*x)*(3*cos(5*x)+5*sin(5*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{3x}\cos(5x)\,dx = \frac{3}{34}\cos\left(5\,x\right)e^{(3\,x)}+\frac{5}{34}\,e^{(3\,x)}\sin\left(5\,x\right)$$

input
```
integrate(exp(3*x)*cos(5*x),x, algorithm="fricas")
```

output
```
3/34*cos(5*x)*e^(3*x) + 5/34*e^(3*x)*sin(5*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{3x}\cos(5x)\,dx = \frac{5e^{3x}\sin\left(5x\right)}{34}+\frac{3e^{3x}\cos\left(5x\right)}{34}$$

input
```
integrate(exp(3*x)*cos(5*x),x)
```

output
```
5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}\cos(5x)\,dx = \frac{1}{34}\left(3\cos\left(5x\right)+5\sin\left(5x\right)\right)e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(5*x),x, algorithm="maxima")
```

output
```
1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}\cos(5x)\,dx = \frac{1}{34}\left(3\cos\left(5x\right)+5\sin\left(5x\right)\right)e^{(3x)}$$

input
```
integrate(exp(3*x)*cos(5*x),x, algorithm="giac")
```

output
```
1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{3x}\cos(5x)\,dx = \frac{e^{3x}\left(3\cos\left(5x\right)+5\sin\left(5x\right)\right)}{34}$$

input
```
int(cos(5*x)*exp(3*x),x)
```

output
```
(exp(3*x)*(3*cos(5*x) + 5*sin(5*x)))/34
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{3x}\cos(5x)\,dx = \frac{e^{3x}(3\cos(5x) + 5\sin(5x))}{34}$$

input
```
int(exp(3*x)*cos(5*x),x)
```

output
```
(e**(3*x)*(3*cos(5*x) + 5*sin(5*x)))/34
```

3.604 $\int e^x \text{sech}(e^x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 5

$$\int e^x \text{sech}(e^x)\,dx = \arctan(\sinh(e^x))$$

output
```
arctan(sinh(exp(x)))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int e^x \text{sech}(e^x)\,dx = -\cot^{-1}(\sinh(e^x))$$

input
```
Integrate[E^x*Sech[E^x],x]
```

output
```
-ArcCot[Sinh[E^x]]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}(e^x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \text{sech}(e^x) de^x$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\frac{\pi}{2}+ie^x\right) de^x$$

$$\Big\downarrow \text{4257}$$

$$\arctan(\sinh(e^x))$$

input
```
Int[E^x*Sech[E^x],x]
```

output
```
ArcTan[Sinh[E^x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\arctan(\sinh(e^{x}))$	5
default	$\arctan(\sinh(e^{x}))$	5
risch	$i\ln\left(e^{e^{x}}+i\right)-i\ln\left(e^{e^{x}}-i\right)$	22
parallelrisch	$-i\left(\ln\left(\tanh\left(\frac{e^{x}}{2}\right)-i\right)-\ln\left(\tanh\left(\frac{e^{x}}{2}\right)+i\right)\right)$	25

input
```
int(exp(x)*sech(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
arctan(sinh(exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(4) = 8$.

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 3.20

$$\int e^{x}\text{sech}(e^{x})\,dx = 2\arctan\left(\cosh\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+\sinh\left(\cosh\left(x\right)+\sinh\left(x\right)\right)\right)$$

input
```
integrate(exp(x)*sech(exp(x)),x, algorithm="fricas")
```

output
```
2*arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int e^x \text{sech}(e^x)\, dx = 2\,\text{atan}\left(\tanh\left(\frac{e^x}{2}\right)\right)$$

input
```
integrate(exp(x)*sech(exp(x)),x)
```

output
```
2*atan(tanh(exp(x)/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.80

$$\int e^x \text{sech}(e^x)\, dx = \arctan\left(\sinh\left(e^x\right)\right)$$

input
```
integrate(exp(x)*sech(exp(x)),x, algorithm="maxima")
```

output
```
arctan(sinh(e^x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int e^x \text{sech}(e^x)\, dx = 2\,\arctan\left(e^{\left(e^x\right)}\right)$$

input
```
integrate(exp(x)*sech(exp(x)),x, algorithm="giac")
```

output
```
2*arctan(e^(e^x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int e^x \operatorname{sech}(e^x)\,dx = 2\,\operatorname{atan}\left(\mathrm{e}^{\mathrm{e}^x}\right)$$

input
```
int(exp(x)/cosh(exp(x)),x)
```

output
```
2*atan(exp(exp(x)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int e^x \operatorname{sech}(e^x)\,dx = 2atan\left(e^{e^x}\right)$$

input
```
int(exp(x)*sech(exp(x)),x)
```

output
```
2*atan(e**(e**x))
```

3.605 $\int \frac{e^{-x}}{1+2e^x} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{e^{-x}}{1+2e^x} dx = -e^{-x} - 2x + 2\log(1+2e^x)$$

output
```
-exp(-x)-2*x+2*ln(1+2*exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{e^{-x}}{1+2e^x} dx = -e^{-x} - 2\log(e^x) + 2\log(1+2e^x)$$

input
```
Integrate[1/(E^x*(1 + 2*E^x)),x]
```

output
```
-E^(-x) - 2*Log[E^x] + 2*Log[1 + 2*E^x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}}{2e^x+1}dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^{-2x}}{2e^x+1}de^x$$

$$\Big\downarrow \text{54}$$

$$\int \left(e^{-2x}-2e^{-x}+\frac{4}{2e^x+1}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$-e^{-x}-2\log\left(e^x\right)+2\log\left(2e^x+1\right)$$

input
```
Int[1/(E^x*(1 + 2*E^x)),x]
```

output
```
-E^(-x) - 2*Log[E^x] + 2*Log[1 + 2*E^x]
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
risch	$-{\mathrm e}^{-x}-2 x +2 \ln \left({\mathrm e}^{x}+\frac{1}{2}\right)$	18
derivativedivides	$-{\mathrm e}^{-x}-2 \ln \left({\mathrm e}^{x}\right)+2 \ln \left(1+2 \,{\mathrm e}^{x}\right)$	22
default	$-{\mathrm e}^{-x}-2 \ln \left({\mathrm e}^{x}\right)+2 \ln \left(1+2 \,{\mathrm e}^{x}\right)$	22
parallelrisch	$\left(-1+2 \ln \left({\mathrm e}^{x}+\frac{1}{2}\right) {\mathrm e}^{x}-2 \,{\mathrm e}^{x} x \right) {\mathrm e}^{-x}$	22
norman	$\left(-1-2 \,{\mathrm e}^{x} x \right) {\mathrm e}^{-x}+2 \ln \left(1+2 \,{\mathrm e}^{x}\right)$	23

input
```
int(1/exp(x)/(1+2*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-exp(-x)-2*x+2*ln(exp(x)+1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{e^{-x}}{1+2e^x}\, dx=-{\left(2 \, x e^{x} - 2 \, e^{x} \log \left(2 \, e^{x} + 1\right) + 1\right)} e^{\left(-x\right)}$$

input
```
integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="fricas")
```

output
```
-(2*x*e^x - 2*e^x*log(2*e^x + 1) + 1)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -2x + 2\log\left(e^x + \frac{1}{2}\right) - e^{-x}$$

input
```
integrate(1/exp(x)/(1+2*exp(x)),x)
```

output
```
-2*x + 2*log(exp(x) + 1/2) - exp(-x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -e^{(-x)} + 2\,\log\left(e^{(-x)} + 2\right)$$

input
```
integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="maxima")
```

output
```
-e^(-x) + 2*log(e^(-x) + 2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}}{1+2e^x}\,dx = -2\,x - e^{(-x)} + 2\,\log\left(2\,e^x + 1\right)$$

input
```
integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="giac")
```

output
```
-2*x - e^(-x) + 2*log(2*e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{e^{-x}}{1+2e^x}\,dx = 2\,\ln\left(2\,\mathrm{e}^x+1\right)-2\,x-\mathrm{e}^{-x}$$

input
```
int(exp(-x)/(2*exp(x) + 1),x)
```

output
```
2*log(2*exp(x) + 1) - 2*x - exp(-x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

$$\int \frac{e^{-x}}{1+2e^x}\,dx = \frac{2e^x \log(2e^x+1)-2e^x x-1}{e^x}$$

input
```
int(1/exp(x)/(1+2*exp(x)),x)
```

output
```
(2*e**x*log(2*e**x + 1) - 2*e**x*x - 1)/e**x
```

3.606 $\int e^x \cos(4+3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}e^x \cos(4+3x) + \frac{3}{10}e^x \sin(4+3x)$$

output
```
1/10*exp(x)*cos(4+3*x)+3/10*exp(x)*sin(4+3*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}e^x(\cos(4+3x) + 3\sin(4+3x))$$

input
```
Integrate[E^x*Cos[4 + 3*x],x]
```

output
```
(E^x*(Cos[4 + 3*x] + 3*Sin[4 + 3*x]))/10
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cos(3x+4)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{3}{10}e^x \sin(3x+4)+\frac{1}{10}e^x \cos(3x+4)$$

input
```
Int[E^x*Cos[4 + 3*x],x]
```

output
```
(E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{e^{x}\left(\cos \left(4+3 x \right)+3 \sin \left(4+3 x \right)\right)}{10}$	20
default	$\frac{{\mathrm e}^{x} \cos \left(4+3 x \right)}{10}+\frac{3 \,{\mathrm e}^{x} \sin \left(4+3 x \right)}{10}$	22
orering	$\frac{{\mathrm e}^{x} \cos \left(4+3 x \right)}{10}+\frac{3 \,{\mathrm e}^{x} \sin \left(4+3 x \right)}{10}$	22
risch	$\left(\frac{1}{20}-\frac{3 i}{20}\right) {\mathrm e}^{x} {\mathrm e}^{3 i x} {\mathrm e}^{4 i}+\left(\frac{1}{20}+\frac{3 i}{20}\right) {\mathrm e}^{x} {\mathrm e}^{-3 i x} {\mathrm e}^{-4 i}$	30
norman	$\frac{\frac{3 \,{\mathrm e}^{x} \tan \left(2+\frac{3 x}{2}\right)}{5}-\frac{{\mathrm e}^{x} \tan \left(2+\frac{3 x}{2}\right)^{2}}{10}+\frac{{\mathrm e}^{x}}{10}}{1+\tan \left(2+\frac{3 x}{2}\right)^{2}}$	41

input
```
int(exp(x)*cos(4+3*x),x,method=_RETURNVERBOSE)
```

output
```
1/10*exp(x)*(cos(4+3*x)+3*sin(4+3*x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}\,\cos\left(3\,x+4\right)e^{x} + \frac{3}{10}\,e^{x}\sin\left(3\,x+4\right)$$

input
```
integrate(exp(x)*cos(4+3*x),x, algorithm="fricas")
```

output
```
1/10*cos(3*x + 4)*e^x + 3/10*e^x*sin(3*x + 4)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int e^x \cos(4+3x)\,dx = \frac{3e^{x}\sin\left(3x+4\right)}{10} + \frac{e^{x}\cos\left(3x+4\right)}{10}$$

input
```
integrate(exp(x)*cos(4+3*x),x)
```

output
```
3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}\left(\cos\left(3\,x+4\right)+3\,\sin\left(3\,x+4\right)\right)e^x$$

input
```
integrate(exp(x)*cos(4+3*x),x, algorithm="maxima")
```

output
```
1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^x \cos(4+3x)\,dx = \frac{1}{10}\left(\cos\left(3\,x+4\right)+3\,\sin\left(3\,x+4\right)\right)e^x$$

input
```
integrate(exp(x)*cos(4+3*x),x, algorithm="giac")
```

output
```
1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^x \cos(4+3x)\,dx = \frac{\mathrm{e}^x\,\left(\cos\left(3\,x+4\right)+3\,\sin\left(3\,x+4\right)\right)}{10}$$

input
```
int(exp(x)*cos(3*x + 4),x)
```

output
```
(exp(x)*(cos(3*x + 4) + 3*sin(3*x + 4)))/10
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^x \cos(4+3x)\,dx = \frac{e^x(\cos(3x+4)+3\sin(3x+4))}{10}$$

input
```
int(exp(x)*cos(4+3*x),x)
```

output
```
(e**x*(cos(3*x + 4) + 3*sin(3*x + 4)))/10
```

3.607 $\int e^x \sec^3(1-e^x)\, dx$

Optimal result

Integrand size = 14, antiderivative size = 34

$$\int e^x \sec^3(1-e^x)\, dx = -\frac{1}{2}\text{arctanh}(\sin(1-e^x)) - \frac{1}{2}\sec(1-e^x)\tan(1-e^x)$$

output
```
1/2*arctanh(sin(-1+exp(x)))+1/2*sec(-1+exp(x))*tan(-1+exp(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int e^x \sec^3(1-e^x)\, dx = -\frac{1}{2}\text{arctanh}(\sin(1-e^x)) - \frac{1}{2}\sec(1-e^x)\tan(1-e^x)$$

input
```
Integrate[E^x*Sec[1 - E^x]^3,x]
```

output
```
-1/2*ArcTanh[Sin[1 - E^x]] - (Sec[1 - E^x]*Tan[1 - E^x])/2
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2720, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sec^3\left(1-e^x\right) \, dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \sec^3\left(1-e^x\right)de^x$$
$$\Big\downarrow \text{ 3042}$$
$$\int \csc\left(-e^x+1+\frac{\pi}{2}\right)^3de^x$$
$$\Big\downarrow \text{ 4255}$$
$$\frac{1}{2}\int \sec\left(1-e^x\right)de^x-\frac{1}{2}\tan\left(1-e^x\right)\sec\left(1-e^x\right)$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{2}\int \csc\left(1-e^x+\frac{\pi}{2}\right)de^x-\frac{1}{2}\tan\left(1-e^x\right)\sec\left(1-e^x\right)$$
$$\Big\downarrow \text{ 4257}$$
$$-\frac{1}{2}\text{arctanh}(\sin\left(1-e^x\right))-\frac{1}{2}\tan\left(1-e^x\right)\sec\left(1-e^x\right)$$

input
```
Int[E^x*Sec[1 - E^x]^3,x]
```

output
```
-1/2*ArcTanh[Sin[1 - E^x]] - (Sec[1 - E^x]*Tan[1 - E^x])/2
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\sec(-1+e^{x})\tan(-1+e^{x})}{2}+\frac{\ln(\sec(-1+e^{x})+\tan(-1+e^{x}))}{2}$	28
default	$\frac{\sec(-1+e^{x})\tan(-1+e^{x})}{2}+\frac{\ln(\sec(-1+e^{x})+\tan(-1+e^{x}))}{2}$	28
norman	$\frac{\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)^{3}+\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)}{\left(\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)^{2}-1\right)^{2}}-\frac{\ln\left(\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)+1\right)}{2}$	57
risch	$-\frac{i\left(e^{3i(-1+e^{x})}-e^{i(-1+e^{x})}\right)}{\left(e^{2i(-1+e^{x})}+1\right)^{2}}+\frac{\ln\left(e^{i(-1+e^{x})}+i\right)}{2}-\frac{\ln\left(e^{i(-1+e^{x})}-i\right)}{2}$	64
parallelrisch	$\frac{(-\cos(-2+2e^{x})-1)\ln\left(\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)-1\right)+(\cos(-2+2e^{x})+1)\ln\left(\tan\left(-\frac{1}{2}+\frac{e^{x}}{2}\right)+1\right)+2\sin(-1+e^{x})}{2+2\cos(-2+2e^{x})}$	65

input
```
int(exp(x)*sec(-1+exp(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*sec(-1+exp(x))*tan(-1+exp(x))+1/2*ln(sec(-1+exp(x))+tan(-1+exp(x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.53

$$\int e^x \sec^3\left(1-e^x\right)\,dx = \frac{\cos\left(e^x-1\right)^2 \log\left(\sin\left(e^x-1\right)+1\right) - \cos\left(e^x-1\right)^2 \log\left(-\sin\left(e^x-1\right)+1\right) + 2\,\sin\left(e^x-1\right)}{4\,\cos\left(e^x-1\right)^2}$$

input
```
integrate(exp(x)*sec(-1+exp(x))^3,x, algorithm="fricas")
```

output
```
1/4*(cos(e^x - 1)^2*log(sin(e^x - 1) + 1) - cos(e^x - 1)^2*log(-sin(e^x -
1) + 1) + 2*sin(e^x - 1))/cos(e^x - 1)^2
```

Sympy [F]

$$\int e^x \sec^3\left(1-e^x\right)\,dx = \int e^x \sec^3\left(e^x-1\right)dx$$

input
```
integrate(exp(x)*sec(-1+exp(x))**3,x)
```

output
```
Integral(exp(x)*sec(exp(x) - 1)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int e^x \sec^3(1-e^x)\,dx = -\frac{\sin(e^x-1)}{2\left(\sin(e^x-1)^2-1\right)} + \frac{1}{4}\log(\sin(e^x-1)+1) - \frac{1}{4}\log(\sin(e^x-1)-1)$$

input
```
integrate(exp(x)*sec(-1+exp(x))^3,x, algorithm="maxima")
```

output
```
-1/2*sin(e^x - 1)/(sin(e^x - 1)^2 - 1) + 1/4*log(sin(e^x - 1) + 1) - 1/4*l
og(sin(e^x - 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int e^x \sec^3(1-e^x)\,dx = -\frac{\sin(e^x-1)}{2\left(\sin(e^x-1)^2-1\right)} + \frac{1}{4}\log(\sin(e^x-1)+1) - \frac{1}{4}\log(-\sin(e^x-1)+1)$$

input
```
integrate(exp(x)*sec(-1+exp(x))^3,x, algorithm="giac")
```

output
```
-1/2*sin(e^x - 1)/(sin(e^x - 1)^2 - 1) + 1/4*log(sin(e^x - 1) + 1) - 1/4*l
og(-sin(e^x - 1) + 1)
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.29

$$\int e^x \sec^3(1-e^x)\,dx = -\mathrm{atan}\left(\mathrm{e}^{-\mathrm{i}}\,\mathrm{e}^{\mathrm{e}^x\,1\mathrm{i}}\right)\,1\mathrm{i} - \frac{\mathrm{e}^{-\mathrm{i}}\,\mathrm{e}^{\mathrm{e}^x\,1\mathrm{i}}\,1\mathrm{i}}{\mathrm{e}^{-2\mathrm{i}}\,\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}}+1} + \frac{\mathrm{e}^{-\mathrm{i}}\,\mathrm{e}^{\mathrm{e}^x\,1\mathrm{i}}\,2\mathrm{i}}{2\,\mathrm{e}^{-2\mathrm{i}}\,\mathrm{e}^{\mathrm{e}^x\,2\mathrm{i}}+\mathrm{e}^{-4\mathrm{i}}\,\mathrm{e}^{\mathrm{e}^x\,4\mathrm{i}}+1}$$

input
```
int(exp(x)/cos(exp(x) - 1)^3,x)
```

output
```
(exp(-1i)*exp(exp(x)*1i)*2i)/(2*exp(-2i)*exp(exp(x)*2i) + exp(-4i)*exp(exp
(x)*4i) + 1) - (exp(-1i)*exp(exp(x)*1i)*1i)/(exp(-2i)*exp(exp(x)*2i) + 1)
- atan(exp(-1i)*exp(exp(x)*1i))*1i
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.62

$$\int e^x \sec^3(1-e^x)\,dx$$
$$= \frac{-\log\left(\tan\left(\frac{e^x}{2}-\frac{1}{2}\right)-1\right)\sin(e^x-1)^2 + \log\left(\tan\left(\frac{e^x}{2}-\frac{1}{2}\right)-1\right) + \log\left(\tan\left(\frac{e^x}{2}-\frac{1}{2}\right)+1\right)\sin(e^x-1)^2 - \mathrm{lo}}{2\sin(e^x-1)^2-2}$$

input
```
int(exp(x)*sec(-1+exp(x))^3,x)
```

output
```
( - log(tan((e**x - 1)/2) - 1)*sin(e**x - 1)**2 + log(tan((e**x - 1)/2) -
1) + log(tan((e**x - 1)/2) + 1)*sin(e**x - 1)**2 - log(tan((e**x - 1)/2) +
 1) - sin(e**x - 1))/(2*(sin(e**x - 1)**2 - 1))
```

3.608 $\int (e^{-x}+e^x)\, x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 26

$$\int (e^{-x}+e^x)\, x \, dx = -e^{-x} - e^x - e^{-x}x + e^x x$$

output
```
-exp(-x)-exp(x)-x/exp(x)+exp(x)*x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int (e^{-x}+e^x)\, x \, dx = e^{-x}\left(-1+e^{2x}(-1+x)-x\right)$$

input
```
Integrate[(E^(-x) + E^x)*x,x]
```

output
```
(-1 + E^(2*x)*(-1 + x) - x)/E^x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{-x}+e^x\right) x \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(e^{-x} x+e^x x\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-e^{-x} x+e^x x-e^{-x}-e^x$$

input
```
Int[(E^(-x) + E^x)*x,x]
```

output
```
-E^(-x) - E^x - x/E^x + E^x*x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

method	result	size
risch	$(-1+x)\,\mathrm{e}^{x}+(-1-x)\,\mathrm{e}^{-x}$	18
default	$-x\,\mathrm{e}^{-x}+\mathrm{e}^{x}x-\mathrm{e}^{-x}-\mathrm{e}^{x}$	23
norman	$\left(-1+x\,\mathrm{e}^{2x}-x-\mathrm{e}^{2x}\right)\mathrm{e}^{-x}$	23
meijerg	$2-\frac{(2+2x)\mathrm{e}^{-x}}{2}-\frac{(2-2x)\mathrm{e}^{x}}{2}$	23
parallelrisch	$-x\,\mathrm{e}^{-x}+\mathrm{e}^{x}x-\mathrm{e}^{-x}-\mathrm{e}^{x}$	23
parts	$-x\,\mathrm{e}^{-x}+\mathrm{e}^{x}x-\mathrm{e}^{-x}-\mathrm{e}^{x}$	23
orering	$-\mathrm{e}^{-x}-\mathrm{e}^{x}+\left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)x$	23

input
```
int((exp(-x)+exp(x))*x,x,method=_RETURNVERBOSE)
```

output
```
(-1+x)*exp(x)+(-1-x)*exp(-x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.69

$$\int \left(e^{-x}+e^{x}\right)x\,dx=\left((x-1)e^{(2\,x)}-x-1\right)e^{(-x)}$$

input
```
integrate((exp(-x)+exp(x))*x,x, algorithm="fricas")
```

output
```
((x - 1)*e^(2*x) - x - 1)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.54

$$\int \left(e^{-x}+e^{x}\right)x\,dx=(-x-1)\,e^{-x}+(x-1)\,e^{x}$$

input
```
integrate((exp(-x)+exp(x))*x,x)
```

output
```
(-x - 1)*exp(-x) + (x - 1)*exp(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \left(e^{-x}+e^{x}\right)x\,dx=-(x+1)e^{(-x)}+(x-1)e^{x}$$

input
```
integrate((exp(-x)+exp(x))*x,x, algorithm="maxima")
```

output
```
-(x + 1)*e^(-x) + (x - 1)*e^x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \left(e^{-x}+e^{x}\right)x\,dx=-(x+1)e^{(-x)}+(x-1)e^{x}$$

input
```
integrate((exp(-x)+exp(x))*x,x, algorithm="giac")
```

output
```
-(x + 1)*e^(-x) + (x - 1)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.38

$$\int \left(e^{-x}+e^x\right)x\,dx=2\,x\sinh(x)-2\cosh(x)$$

input
```
int(x*(exp(-x) + exp(x)),x)
```

output
```
2*x*sinh(x) - 2*cosh(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \left(e^{-x}+e^x\right)x\,dx=\frac{e^{2x}x-e^{2x}-x-1}{e^x}$$

input
```
int((exp(-x)+exp(x))*x,x)
```

output
```
(e**(2*x)*x - e**(2*x) - x - 1)/e**x
```

3.609 $\int \frac{e^x}{2+3e^x+e^{2x}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 15

$$\int \frac{e^x}{2+3e^x+e^{2x}} \, dx = \log(1+e^x) - \log(2+e^x)$$

output
```
ln(1+exp(x))-ln(2+exp(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{2+3e^x+e^{2x}} \, dx = -2\text{arctanh}(3+2e^x)$$

input
```
Integrate[E^x/(2 + 3*E^x + E^(2*x)),x]
```

output
```
-2*ArcTanh[3 + 2*E^x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{3e^x+e^{2x}+2}dx$$
$$\Big\downarrow \text{2720}$$
$$\int \frac{1}{3e^x+e^{2x}+2}de^x$$
$$\Big\downarrow \text{1081}$$
$$\int \left(\frac{1}{e^x+1}+\frac{1}{-e^x-2}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$\log \left(e^x+1\right)-\log \left(e^x+2\right)$$

input
```
Int[E^x/(2 + 3*E^x + E^(2*x)),x]
```

output
```
Log[1 + E^x] - Log[2 + E^x]
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln(1+e^x)-\ln(2+e^x)$	14
norman	$\ln(1+e^x)-\ln(2+e^x)$	14
risch	$\ln(1+e^x)-\ln(2+e^x)$	14

input
```
int(exp(x)/(2+3*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+exp(x))-ln(2+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{e^x}{2+3e^x+e^{2x}}\,dx = -\log\left(e^x+2\right)+\log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
-log(e^x + 2) + log(e^x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{e^x}{2+3e^x+e^{2x}}\,dx = \log\left(e^x+1\right) - \log\left(e^x+2\right)$$

input
```
integrate(exp(x)/(2+3*exp(x)+exp(2*x)),x)
```

output
```
log(exp(x) + 1) - log(exp(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{e^x}{2+3e^x+e^{2x}}\,dx = -\log\left(e^x+2\right) + \log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
-log(e^x + 2) + log(e^x + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{e^x}{2+3e^x+e^{2x}}\,dx = -\log\left(e^x+2\right) + \log\left(e^x+1\right)$$

input
```
integrate(exp(x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
-log(e^x + 2) + log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{e^x}{2+3e^x+e^{2x}}\,dx = \ln\left(\mathrm{e}^x+1\right)-\ln\left(\mathrm{e}^x+2\right)$$

input
```
int(exp(x)/(exp(2*x) + 3*exp(x) + 2),x)
```

output
```
log(exp(x) + 1) - log(exp(x) + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{2+3e^x+e^{2x}}\,dx = -\log(e^x+2)+\log(e^x+1)$$

input
```
int(exp(x)/(2+3*exp(x)+exp(2*x)),x)
```

output
```
- log(e**x + 2) + log(e**x + 1)
```

3.610 $\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = -\frac{3}{2}(1+e^x)^{2/3} + \frac{3}{5}(1+e^x)^{5/3}$$

output
```
-3/2*(1+exp(x))^(2/3)+3/5*(1+exp(x))^(5/3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \frac{3}{10}(1+e^x)^{2/3}\,(-5+2(1+e^x))$$

input
```
Integrate[E^(2*x)/(1 + E^x)^(1/3),x]
```

output
```
(3*(1 + E^x)^(2/3)*(-5 + 2*(1 + E^x)))/10
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{\sqrt[3]{e^x+1}}dx$$
$$\Big\downarrow \text{2678}$$
$$\int \frac{e^x}{\sqrt[3]{e^x+1}}de^x$$
$$\Big\downarrow \text{53}$$
$$\int \left((e^x+1)^{2/3}-\frac{1}{\sqrt[3]{e^x+1}}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$\frac{3}{5}(e^x+1)^{5/3}-\frac{3}{2}(e^x+1)^{2/3}$$

input
```
Int[E^(2*x)/(1 + E^x)^(1/3),x]
```

output
```
(-3*(1 + E^x)^(2/3))/2 + (3*(1 + E^x)^(5/3))/5
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{3(2\,e^{x}-3)(1+e^{x})^{\frac{2}{3}}}{10}$	15
default	$-\frac{3(1+e^{x})^{\frac{2}{3}}}{2}+\frac{3(1+e^{x})^{\frac{5}{3}}}{5}$	18

input
```
int(exp(2*x)/(1+exp(x))^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/10*(2*exp(x)-3)*(1+exp(x))^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \frac{3}{10}\,(2\,e^x-3)(e^x+1)^{\frac{2}{3}}$$

input
```
integrate(exp(2*x)/(1+exp(x))^(1/3),x, algorithm="fricas")
```

output
```
3/10*(2*e^x - 3)*(e^x + 1)^(2/3)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \frac{3(e^x+1)^{\frac{5}{3}}}{5} - \frac{3(e^x+1)^{\frac{2}{3}}}{2}$$

input
```
integrate(exp(2*x)/(1+exp(x))**(1/3),x)
```

output
```
3*(exp(x) + 1)**(5/3)/5 - 3*(exp(x) + 1)**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \frac{3}{5}\,(e^x+1)^{\frac{5}{3}} - \frac{3}{2}\,(e^x+1)^{\frac{2}{3}}$$

input
```
integrate(exp(2*x)/(1+exp(x))^(1/3),x, algorithm="maxima")
```

output
```
3/5*(e^x + 1)^(5/3) - 3/2*(e^x + 1)^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \frac{3}{5}\,(e^x+1)^{\frac{5}{3}} - \frac{3}{2}\,(e^x+1)^{\frac{2}{3}}$$

input
```
integrate(exp(2*x)/(1+exp(x))^(1/3),x, algorithm="giac")
```

output
```
3/5*(e^x + 1)^(5/3) - 3/2*(e^x + 1)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \frac{3\,{\left({\mathrm{e}}^x+1\right)}^{2/3}\,\left(2\,{\mathrm{e}}^x-3\right)}{10}$$

input
```
int(exp(2*x)/(exp(x) + 1)^(1/3),x)
```

output
```
(3*(exp(x) + 1)^(2/3)*(2*exp(x) - 3))/10
```

Reduce [F]

$$\int \frac{e^{2x}}{\sqrt[3]{1+e^x}}\,dx = \int \frac{e^{2x}}{(e^x+1)^{\frac{1}{3}}}dx$$

input
```
int(exp(2*x)/(1+exp(x))^(1/3),x)
```

output
```
int(e**(2*x)/(e**x + 1)**(1/3),x)
```

3.611 $\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx=-\frac{4}{3}(1+e^x)^{3/4}+\frac{4}{7}(1+e^x)^{7/4}$$

output
```
-4/3*(1+exp(x))^(3/4)+4/7*(1+exp(x))^(7/4)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx=\frac{4}{21}(1+e^x)^{3/4}(-7+3(1+e^x))$$

input
```
Integrate[E^(2*x)/(1 + E^x)^(1/4),x]
```

output
```
(4*(1 + E^x)^(3/4)*(-7 + 3*(1 + E^x)))/21
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{\sqrt[4]{e^x+1}}dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^x}{\sqrt[4]{e^x+1}}de^x$$

$$\Big\downarrow \text{53}$$

$$\int \left((e^x+1)^{3/4}-\frac{1}{\sqrt[4]{e^x+1}}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{7}(e^x+1)^{7/4}-\frac{4}{3}(e^x+1)^{3/4}$$

input
```
Int[E^(2*x)/(1 + E^x)^(1/4),x]
```

output
```
(-4*(1 + E^x)^(3/4))/3 + (4*(1 + E^x)^(7/4))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.56

method	result	size
risch	$\frac{4(-4+3\,e^{x})(1+e^{x})^{\frac{3}{4}}}{21}$	15
default	$-\frac{4(1+e^{x})^{\frac{3}{4}}}{3}+\frac{4(1+e^{x})^{\frac{7}{4}}}{7}$	18

input
```
int(exp(2*x)/(1+exp(x))^(1/4),x,method=_RETURNVERBOSE)
```

output
```
4/21*(-4+3*exp(x))*(1+exp(x))^(3/4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx=\frac{4}{21}\,(3\,e^{x}-4)(e^{x}+1)^{\frac{3}{4}}$$

input
```
integrate(exp(2*x)/(1+exp(x))^(1/4),x, algorithm="fricas")
```

output
```
4/21*(3*e^x - 4)*(e^x + 1)^(3/4)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx = \frac{4(e^x+1)^{\frac{7}{4}}}{7} - \frac{4(e^x+1)^{\frac{3}{4}}}{3}$$

input
```
integrate(exp(2*x)/(1+exp(x))**(1/4),x)
```

output
```
4*(exp(x) + 1)**(7/4)/7 - 4*(exp(x) + 1)**(3/4)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx = \frac{4}{7}(e^x+1)^{\frac{7}{4}} - \frac{4}{3}(e^x+1)^{\frac{3}{4}}$$

input
```
integrate(exp(2*x)/(1+exp(x))^(1/4),x, algorithm="maxima")
```

output
```
4/7*(e^x + 1)^(7/4) - 4/3*(e^x + 1)^(3/4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx = \frac{4}{7}(e^x+1)^{\frac{7}{4}} - \frac{4}{3}(e^x+1)^{\frac{3}{4}}$$

input
```
integrate(exp(2*x)/(1+exp(x))^(1/4),x, algorithm="giac")
```

output
```
4/7*(e^x + 1)^(7/4) - 4/3*(e^x + 1)^(3/4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.52

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx = \frac{4\,{\left(\mathrm{e}^x+1\right)}^{3/4}\,\left(3\,\mathrm{e}^x-4\right)}{21}$$

input
```
int(exp(2*x)/(exp(x) + 1)^(1/4),x)
```

output
```
(4*(exp(x) + 1)^(3/4)*(3*exp(x) - 4))/21
```

Reduce [F]

$$\int \frac{e^{2x}}{\sqrt[4]{1+e^x}}\,dx = \int \frac{e^{2x}}{(e^x+1)^{\frac{1}{4}}}dx$$

input
```
int(exp(2*x)/(1+exp(x))^(1/4),x)
```

output
```
int(e**(2*x)/(e**x + 1)**(1/4),x)
```

3.612 $\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 62

$$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx=\frac{2}{3}\sqrt{-1-6e^x+3e^{2x}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{3}(1-e^x)}{\sqrt{-1-6e^x+3e^{2x}}}\right)}{\sqrt{3}}$$

output
```
2/3*(-1-6*exp(x)+3*exp(2*x))^(1/2)-1/3*arctanh(3^(1/2)*(1-exp(x))/(-1-6*ex
p(x)+3*exp(2*x))^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx=\frac{2}{3}\sqrt{-1-6e^x+3e^{2x}}-\frac{\log\left(3-3e^x+\sqrt{-3-18e^x+9e^{2x}}\right)}{\sqrt{3}}$$

input
```
Integrate[(-E^x + 2*E^(2*x))/Sqrt[-1 - 6*E^x + 3*E^(2*x)],x]
```

output
```
(2*Sqrt[-1 - 6*E^x + 3*E^(2*x)])/3 - Log[3 - 3*E^x + Sqrt[-3 - 18*E^x + 9*
E^(2*x)]]/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2720, 25, 1160, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2e^{2x}-e^x}{\sqrt{-6e^x+3e^{2x}-1}}dx$$

$$\Big\downarrow \text{2720}$$

$$\int -\frac{1-2e^x}{\sqrt{-6e^x+3e^{2x}-1}}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1-2e^x}{\sqrt{-1-6e^x+3e^{2x}}}de^x$$

$$\Big\downarrow \text{1160}$$

$$\int \frac{1}{\sqrt{-1-6e^x+3e^{2x}}}de^x+\frac{2}{3}\sqrt{-6e^x+3e^{2x}-1}$$

$$\Big\downarrow \text{1092}$$

$$2\int \frac{1}{12-e^{2x}}d\left(-\frac{6(1-e^x)}{\sqrt{-1-6e^x+3e^{2x}}}\right)+\frac{2}{3}\sqrt{-6e^x+3e^{2x}-1}$$

$$\Big\downarrow \text{219}$$

$$\frac{2}{3}\sqrt{-6e^x+3e^{2x}-1}-\frac{\text{arctanh}\left(\frac{\sqrt{3}(1-e^x)}{\sqrt{-6e^x+3e^{2x}-1}}\right)}{\sqrt{3}}$$

input
```
Int[(-E^x + 2*E^(2*x))/Sqrt[-1 - 6*E^x + 3*E^(2*x)],x]
```

output
```
(2*Sqrt[-1 - 6*E^x + 3*E^(2*x)])/3 - ArcTanh[(Sqrt[3]*(1 - E^x))/Sqrt[-1 -
 6*E^x + 3*E^(2*x)]]/Sqrt[3]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln\left(\frac{\left(-3+3\,\mathrm{e}^{x}\right)\sqrt{3}}{3}+\sqrt{-1-6\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}}\right)\sqrt{3}}{3}+\frac{2\sqrt{-1-6\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}}}{3}$	50
risch	$\frac{\ln\left(\frac{\left(-3+3\,\mathrm{e}^{x}\right)\sqrt{3}}{3}+\sqrt{-1-6\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}}\right)\sqrt{3}}{3}+\frac{2\sqrt{-1-6\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}}}{3}$	50
parts	$\frac{\ln\left(\frac{\left(-3+3\,\mathrm{e}^{x}\right)\sqrt{3}}{3}+\sqrt{-1-6\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}}\right)\sqrt{3}}{3}+\frac{2\sqrt{-1-6\,\mathrm{e}^{x}+3\,\mathrm{e}^{2x}}}{3}$	50

input
```
int((-exp(x)+2*exp(2*x))/(-1-6*exp(x)+3*exp(2*x))^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/3*ln(1/3*(-3+3*exp(x))*3^(1/2)+(-1-6*exp(x)+3*exp(x)^2)^(1/2))*3^(1/2)+2
/3*(-1-6*exp(x)+3*exp(x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx = \frac{1}{6}\sqrt{3}\log\left(\left(\sqrt{3}e^x-\sqrt{3}\right)\sqrt{3\,e^{(2\,x)}-6\,e^x-1}+3\,e^{(2\,x)}-6\,e^x+1\right)+\frac{2}{3}\sqrt{3\,e^{(2\,x)}-6\,e^x-1}$$

input
```
integrate((-exp(x)+2*exp(2*x))/(-1-6*exp(x)+3*exp(2*x))^(1/2),x, algorithm
="fricas")
```

output
```
1/6*sqrt(3)*log((sqrt(3)*e^x - sqrt(3))*sqrt(3*e^(2*x) - 6*e^x - 1) + 3*e^
(2*x) - 6*e^x + 1) + 2/3*sqrt(3*e^(2*x) - 6*e^x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx = \frac{2\sqrt{3e^{2x}-6e^x-1}}{3}-\frac{\sqrt{3}\log\left(2\sqrt{3}\sqrt{3e^{2x}-6e^x-1}-6e^x+6\right)}{3}$$

input
```
integrate((-exp(x)+2*exp(2*x))/(-1-6*exp(x)+3*exp(2*x))**(1/2),x)
```

output
```
2*sqrt(3*exp(2*x) - 6*exp(x) - 1)/3 - sqrt(3)*log(2*sqrt(3)*sqrt(3*exp(2*x
) - 6*exp(x) - 1) - 6*exp(x) + 6)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.77

$$\int \frac{-e^x + 2e^{2x}}{\sqrt{-1 - 6e^x + 3e^{2x}}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3\,e^{(2\,x)} - 6\,e^x - 1} + 6\,e^x - 6\right) + \frac{2}{3}\sqrt{3\,e^{(2\,x)} - 6\,e^x - 1}$$

input
```
integrate((-exp(x)+2*exp(2*x))/(-1-6*exp(x)+3*exp(2*x))^(1/2),x, algorithm
="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*e^(2*x) - 6*e^x - 1) + 6*e^x - 6) + 2/3*s
qrt(3*e^(2*x) - 6*e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{-e^x + 2e^{2x}}{\sqrt{-1 - 6e^x + 3e^{2x}}}\,dx = -\frac{1}{3}\sqrt{3}\log\left(\left|-\sqrt{3}e^x + \sqrt{3} + \sqrt{3\,e^{(2\,x)} - 6\,e^x - 1}\right|\right) + \frac{2}{3}\sqrt{3\,e^{(2\,x)} - 6\,e^x - 1}$$

input
```
integrate((-exp(x)+2*exp(2*x))/(-1-6*exp(x)+3*exp(2*x))^(1/2),x, algorithm
="giac")
```

output
```
-1/3*sqrt(3)*log(abs(-sqrt(3)*e^x + sqrt(3) + sqrt(3*e^(2*x) - 6*e^x - 1))
) + 2/3*sqrt(3*e^(2*x) - 6*e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3\,e^{2\,x}-6\,e^x-1}-\sqrt{3}+\sqrt{3}\,e^x\right)}{3}+\frac{2\,\sqrt{3\,e^{2\,x}-6\,e^x-1}}{3}$$

input
```
int((2*exp(2*x) - exp(x))/(3*exp(2*x) - 6*exp(x) - 1)^(1/2),x)
```

output
```
(3^(1/2)*log((3*exp(2*x) - 6*exp(x) - 1)^(1/2) - 3^(1/2) + 3^(1/2)*exp(x))
)/3 + (2*(3*exp(2*x) - 6*exp(x) - 1)^(1/2))/3
```

Reduce [F]

$$\int \frac{-e^x+2e^{2x}}{\sqrt{-1-6e^x+3e^{2x}}}\,dx = \frac{\sqrt{3e^{2x}-6e^x-1}}{3}+\int \frac{e^{2x}\sqrt{3e^{2x}-6e^x-1}}{3e^{2x}-6e^x-1}dx$$

input
```
int((-exp(x)+2*exp(2*x))/(-1-6*exp(x)+3*exp(2*x))^(1/2),x)
```

output
```
(sqrt(3*e**(2*x) - 6*e**x - 1) + 3*int((e**(2*x)*sqrt(3*e**(2*x) - 6*e**x
- 1))/(3*e**(2*x) - 6*e**x - 1),x))/3
```

3.613 $\int e^x(-5x+x^2)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int e^x\left(-5x+x^2\right)\,dx=7e^x-7e^xx+e^xx^2$$

output
```
7*exp(x)-7*exp(x)*x+exp(x)*x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x\left(-5x+x^2\right)\,dx=e^x\left(7-7x+x^2\right)$$

input
```
Integrate[E^x*(-5*x + x^2),x]
```

output
```
E^x*(7 - 7*x + x^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2027, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \left(x^2-5x\right) \, dx$$

$$\Big\downarrow \text{2027}$$

$$\int e^x (x-5)x dx$$

$$\Big\downarrow \text{2626}$$

$$\int \left(e^x x^2-5e^x x\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$e^x x^2-7e^x x+7e^x$$

input
```
Int[E^x*(-5*x + x^2),x]
```

output
```
7*E^x - 7*E^x*x + E^x*x^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
gosper	$\mathrm{e}^{x}\left(x^{2}-7x+7\right)$	12
risch	$\mathrm{e}^{x}\left(x^{2}-7x+7\right)$	12
default	$7\,\mathrm{e}^{x}-7\,\mathrm{e}^{x}x+\mathrm{e}^{x}x^{2}$	17
norman	$7\,\mathrm{e}^{x}-7\,\mathrm{e}^{x}x+\mathrm{e}^{x}x^{2}$	17
parallelrisch	$7\,\mathrm{e}^{x}-7\,\mathrm{e}^{x}x+\mathrm{e}^{x}x^{2}$	17
parts	$7\,\mathrm{e}^{x}-7\,\mathrm{e}^{x}x+\mathrm{e}^{x}x^{2}$	17
meijerg	$-7+\frac{(3x^{2}-6x+6)\mathrm{e}^{x}}{3}+\frac{5(2-2x)\mathrm{e}^{x}}{2}$	26
orering	$\frac{(x^{2}-7x+7)\mathrm{e}^{x}(x^{2}-5x)}{x(x-5)}$	27

input
```
int(exp(x)*(x^2-5*x),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*(x^2-7*x+7)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^{x}\left(-5x+x^{2}\right)\,dx=\left(x^{2}-7\,x+7\right)e^{x}$$

input
```
integrate(exp(x)*(x^2-5*x),x, algorithm="fricas")
```

output
```
(x^2 - 7*x + 7)*e^x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int e^x\left(-5x+x^2\right)\,dx=\left(x^2-7x+7\right)e^x$$

input
```
integrate(exp(x)*(x**2-5*x),x)
```

output
```
(x**2 - 7*x + 7)*exp(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^x\left(-5x+x^2\right)\,dx=\left(x^2-2\,x+2\right)e^x-5\,(x-1)e^x$$

input
```
integrate(exp(x)*(x^2-5*x),x, algorithm="maxima")
```

output
```
(x^2 - 2*x + 2)*e^x - 5*(x - 1)*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x\left(-5x+x^2\right)\,dx=\left(x^2-7\,x+7\right)e^x$$

input
```
integrate(exp(x)*(x^2-5*x),x, algorithm="giac")
```

output
```
(x^2 - 7*x + 7)*e^x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int e^x(-5x+x^2)\ dx = \mathrm{e}^x\left(x^2-7\,x+7\right)$$

input
```
int(-exp(x)*(5*x - x^2),x)
```

output
```
exp(x)*(x^2 - 7*x + 7)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x(-5x+x^2)\ dx = e^x(x^2-7x+7)$$

input
```
int(exp(x)*(x^2-5*x),x)
```

output
```
e**x*(x**2 - 7*x + 7)
```

3.614 $\int e^{3x}(-x+x^2)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{5e^{3x}}{27}-\frac{5}{9}e^{3x}x+\frac{1}{3}e^{3x}x^2$$

output
```
5/27*exp(3*x)-5/9*exp(3*x)*x+1/3*exp(3*x)*x^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{1}{27}e^{3x}\left(5-15x+9x^2\right)$$

input
```
Integrate[E^(3*x)*(-x + x^2),x]
```

output
```
(E^(3*x)*(5 - 15*x + 9*x^2))/27
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2027, 2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{3x}(x^2-x)\,dx$$

$$\Big\downarrow \; 2027$$

$$\int e^{3x}(x-1)xdx$$

$$\Big\downarrow \; 2626$$

$$\int \left(e^{3x}x^2-e^{3x}x\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}e^{3x}x^2-\frac{5}{9}e^{3x}x+\frac{5e^{3x}}{27}$$

input
```
Int[E^(3*x)*(-x + x^2),x]
```

output
```
(5*E^(3*x))/27 - (5*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] && !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

method	result	size
risch	$\left(\frac{1}{3}x^{2}-\frac{5}{9}x+\frac{5}{27}\right) {\mathrm e}^{3 x}$	16
gosper	$\frac{{\mathrm e}^{3 x} \left(9 x^{2}-15 x +5\right)}{27}$	17
derivativedivides	$\frac{5 \,{\mathrm e}^{3 x}}{27}-\frac{5 \,{\mathrm e}^{3 x} x}{9}+\frac{{\mathrm e}^{3 x} x^{2}}{3}$	24
default	$\frac{5 \,{\mathrm e}^{3 x}}{27}-\frac{5 \,{\mathrm e}^{3 x} x}{9}+\frac{{\mathrm e}^{3 x} x^{2}}{3}$	24
norman	$\frac{5 \,{\mathrm e}^{3 x}}{27}-\frac{5 \,{\mathrm e}^{3 x} x}{9}+\frac{{\mathrm e}^{3 x} x^{2}}{3}$	24
parallelrisch	$\frac{5 \,{\mathrm e}^{3 x}}{27}-\frac{5 \,{\mathrm e}^{3 x} x}{9}+\frac{{\mathrm e}^{3 x} x^{2}}{3}$	24
parts	$\frac{5 \,{\mathrm e}^{3 x}}{27}-\frac{5 \,{\mathrm e}^{3 x} x}{9}+\frac{{\mathrm e}^{3 x} x^{2}}{3}$	24
meijerg	$-\frac{5}{27}+\frac{\left(27 x^{2}-18 x +6\right) {\mathrm e}^{3 x}}{81}+\frac{\left(-6 x +2\right) {\mathrm e}^{3 x}}{18}$	30
orering	$\frac{\left(9 x^{2}-15 x +5\right) {\mathrm e}^{3 x} \left(x^{2}-x \right)}{27 x \left(-1+x \right)}$	32

input
```
int(exp(3*x)*(x^2-x),x,method=_RETURNVERBOSE)
```

output
```
(1/3*x^2-5/9*x+5/27)*exp(3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{1}{27}\,{\left(9\,x^{2}-15\,x+5\right)}e^{\left(3\,x\right)}$$

input
```
integrate(exp(3*x)*(x^2-x),x, algorithm="fricas")
```

output
```
1/27*(9*x^2 - 15*x + 5)*e^(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{(9x^2-15x+5)\,e^{3x}}{27}$$

input
```
integrate(exp(3*x)*(x**2-x),x)
```

output
```
(9*x**2 - 15*x + 5)*exp(3*x)/27
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{1}{27}\,(9\,x^2-6\,x+2)e^{(3\,x)}-\frac{1}{9}\,(3\,x-1)e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(x^2-x),x, algorithm="maxima")
```

output
```
1/27*(9*x^2 - 6*x + 2)*e^(3*x) - 1/9*(3*x - 1)*e^(3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{1}{27}\,(9\,x^2-15\,x+5)e^{(3\,x)}$$

input
```
integrate(exp(3*x)*(x^2-x),x, algorithm="giac")
```

output
```
1/27*(9*x^2 - 15*x + 5)*e^(3*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{\mathrm{e}^{3\,x}\,\left(9\,x^2-15\,x+5\right)}{27}$$

input
```
int(-exp(3*x)*(x - x^2),x)
```

output
```
(exp(3*x)*(9*x^2 - 15*x + 5))/27
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.53

$$\int e^{3x}\left(-x+x^2\right)\,dx=\frac{e^{3x}(9x^2-15x+5)}{27}$$

input
```
int(exp(3*x)*(x^2-x),x)
```

output
```
(e**(3*x)*(9*x**2 - 15*x + 5))/27
```

3.615 $\int e^{x^x} x^{2x}(1+\log(x))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = e^{x^x}(-1+x^x)$$

output
```
exp(x^x)*(-1+x^x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = e^{x^x}(-1+x^x)$$

input
```
Integrate[E^x^x*x^(2*x)*(1 + Log[x]),x]
```

output
```
E^x^x*(-1 + x^x)
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^x} x^{2x} (\log(x)+1)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(e^{x^x} x^{2x} + e^{x^x} x^{2x}\log(x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\int e^{x^x} x^{2x} dx - \int \frac{\int e^{x^x} x^{2x} dx}{x} dx + \log(x) \int e^{x^x} x^{2x} dx$$

input
```
Int[E^x^x*x^(2*x)*(1 + Log[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
risch	$e^{x^x}(-1+x^x)$	11
norman	$e^{\ln(x)x} e^{e^{\ln(x)x}} - e^{e^{\ln(x)x}}$	22

input
```
int(exp(x^x)*x^(2*x)*(1+ln(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x^x)*(-1+x^x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = (x^x-1)e^{(x^x)}$$

input
```
integrate(exp(x^x)*x^(2*x)*(1+log(x)),x, algorithm="fricas")
```

output
```
(x^x - 1)*e^(x^x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = (x^x-1)\,e^{x^x}$$

input
```
integrate(exp(x**x)*x**(2*x)*(1+ln(x)),x)
```

output
```
(x**x - 1)*exp(x**x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = (x^x-1)e^{(x^x)}$$

input
```
integrate(exp(x^x)*x^(2*x)*(1+log(x)),x, algorithm="maxima")
```

output
```
(x^x - 1)*e^(x^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = (x^x-1)e^{(x^x)}$$

input
```
integrate(exp(x^x)*x^(2*x)*(1+log(x)),x, algorithm="giac")
```

output
```
(x^x - 1)*e^(x^x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = e^{x^x}\,(x^x-1)$$

input
```
int(x^(2*x)*exp(x^x)*(log(x) + 1),x)
```

output
```
exp(x^x)*(x^x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{x^x} x^{2x}(1+\log(x))\,dx = e^{x^x}(x^x-1)$$

input
```
int(exp(x^x)*x^(2*x)*(1+log(x)),x)
```

output
```
e**(x**x)*(x**x - 1)
```

3.616 $\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 9

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x} \, dx = \frac{e^{6x}}{6}$$

output
```
1/6*exp(6*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x} \, dx = \frac{e^{6x}}{6}$$

input
```
Integrate[(E^(5*x) + E^(7*x))/(E^(-x) + E^x),x]
```

output
```
E^(6*x)/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2720, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x}dx$$

$$\Big\downarrow \ 2720$$

$$\int e^{5x}de^x$$

$$\Big\downarrow \ 15$$

$$\frac{e^{6x}}{6}$$

input
```
Int[(E^(5*x) + E^(7*x))/(E^(-x) + E^x),x]
```

output
```
E^(6*x)/6
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{e^{6x}}{6}$	7
norman	$\frac{e^{6x}}{6}$	7
risch	$\frac{e^{6x}}{6}$	7
orering	$\frac{e^{5x}+e^{7x}}{6\,e^{-x}+6\,e^{x}}$	21

input
```
int((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(x)^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^{x}}\,dx=\frac{1}{6}\,e^{(6\,x)}$$

input
```
integrate((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
1/6*e^(6*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.56

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^{x}}\,dx=\frac{e^{6x}}{6}$$

input
```
integrate((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x)
```

output
```
exp(6*x)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x}\,dx=\frac{1}{6}\,e^{(6\,x)}$$

input
```
integrate((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
1/6*e^(6*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x}\,dx=\frac{1}{6}\,e^{(6\,x)}$$

input
```
integrate((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
1/6*e^(6*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^x}\,dx=\frac{\mathrm{e}^{6\,x}}{6}$$

input
```
int((exp(5*x) + exp(7*x))/(exp(-x) + exp(x)),x)
```

output
```
exp(6*x)/6
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{e^{5x}+e^{7x}}{e^{-x}+e^{x}}\,dx=\frac{e^{6x}}{6}$$

input
```
int((exp(5*x)+exp(7*x))/(exp(-x)+exp(x)),x)
```

output
```
e**(6*x)/6
```

3.617 $\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 9

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx=-x^{-1/x}$$

output
```
-x^(-1/x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx=-x^{-1/x}$$

input
```
Integrate[x^(-2 - x^(-1))*(1 - Log[x]),x]
```

output
```
-x^(-x^(-1))
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-\frac{1}{x}-2}(1-\log(x))\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(x^{-\frac{1}{x}-2}-x^{-\frac{1}{x}-2}\log(x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\int x^{-2-\frac{1}{x}}dx+\int \frac{\int x^{-2-\frac{1}{x}}dx}{x}dx-\log(x)\int x^{-2-\frac{1}{x}}dx$$

input
```
Int[x^(-2 - x^(-1))*(1 - Log[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.67

method	result	size
parallelrisch	$-x^{-2-\frac{1}{x}}x^{2}$	15
risch	$-x^{2}x^{-\frac{2x+1}{x}}$	18

input
```
int(x^(-2-1/x)*(1-ln(x)),x,method=_RETURNVERBOSE)
```

output
```
-x^(-2-1/x)*x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx=-\frac{x^2}{x^{\frac{2x+1}{x}}}$$

input
```
integrate(x^(-2-1/x)*(1-log(x)),x, algorithm="fricas")
```

output
```
-x^2/x^((2*x + 1)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx=-x^2x^{-2-\frac{1}{x}}$$

input
```
integrate(x**(-2-1/x)*(1-ln(x)),x)
```

output
```
-x**2*x**(-2 - 1/x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx = -\frac{1}{x^{\left(\frac{1}{x}\right)}}$$

input
```
integrate(x^(-2-1/x)*(1-log(x)),x, algorithm="maxima")
```

output
```
-1/x^(1/x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx = -xe^{\left(-\frac{x\log(x)+\log(x)}{x}\right)}$$

input
```
integrate(x^(-2-1/x)*(1-log(x)),x, algorithm="giac")
```

output
```
-x*e^(-(x*log(x) + log(x))/x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx = -\frac{1}{x^{1/x}}$$

input
```
int(-(log(x) - 1)/x^(1/x + 2),x)
```

output
```
-1/x^(1/x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x^{-2-\frac{1}{x}}(1-\log(x))\,dx=-\frac{1}{x^{\frac{1}{x}}}$$

input
```
int(x^(-2-1/x)*(1-log(x)),x)
```

output
```
( - 1)/x**(1/x)
```

3.618 $\int (a+be^x)^2 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+be^x)^2 \, dx = 2abe^x + \frac{1}{2}b^2e^{2x} + a^2x$$

output
```
2*a*b*exp(x)+1/2*b^2*exp(2*x)+a^2*x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int (a+be^x)^2 \, dx = \frac{1}{2}be^x(4a+be^x) + a^2\log(e^x)$$

input
```
Integrate[(a + b*E^x)^2,x]
```

output
```
(b*E^x*(4*a + b*E^x))/2 + a^2*Log[E^x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^x)^2\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}(a+be^x)^2\,de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^2e^{-x}+2ab+b^2e^x\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$a^2\log\left(e^x\right)+2abe^x+\frac{1}{2}b^2e^{2x}$$

input
```
Int[(a + b*E^x)^2,x]
```

output
```
2*a*b*E^x + (b^2*E^(2*x))/2 + a^2*Log[E^x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
norman	$2ab\,\mathrm{e}^{x}+\frac{b^{2}\mathrm{e}^{2x}}{2}+a^{2}x$	22
risch	$2ab\,\mathrm{e}^{x}+\frac{b^{2}\mathrm{e}^{2x}}{2}+a^{2}x$	22
parallelrisch	$2ab\,\mathrm{e}^{x}+\frac{b^{2}\mathrm{e}^{2x}}{2}+a^{2}x$	22
parts	$2ab\,\mathrm{e}^{x}+\frac{b^{2}\mathrm{e}^{2x}}{2}+a^{2}x$	22
derivativedivides	$\frac{b^{2}\mathrm{e}^{2x}}{2}+2ab\,\mathrm{e}^{x}+a^{2}\ln\left(\mathrm{e}^{x}\right)$	24
default	$\frac{b^{2}\mathrm{e}^{2x}}{2}+2ab\,\mathrm{e}^{x}+a^{2}\ln\left(\mathrm{e}^{x}\right)$	24
orering	$\left(x+\frac{3}{2}\right)\left(a+b\,\mathrm{e}^{x}\right)^{2}+2\left(-\frac{3x}{2}-\frac{1}{2}\right)\left(a+b\,\mathrm{e}^{x}\right)b\,\mathrm{e}^{x}+\frac{x\left(2b^{2}\mathrm{e}^{2x}+2\left(a+b\,\mathrm{e}^{x}\right)b\,\mathrm{e}^{x}\right)}{2}$	54

input
```
int((a+b*exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+1/2*b^2*exp(x)^2+2*a*b*exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+be^x)^2\,dx=a^2x+\frac{1}{2}\,b^2e^{(2\,x)}+2\,abe^x$$

input
```
integrate((a+b*exp(x))^2,x, algorithm="fricas")
```

output
```
a^2*x + 1/2*b^2*e^(2*x) + 2*a*b*e^x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (a + be^x)^2 \, dx = a^2x + 2abe^x + \frac{b^2e^{2x}}{2}$$

input
```
integrate((a+b*exp(x))**2,x)
```

output
```
a**2*x + 2*a*b*exp(x) + b**2*exp(2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a + be^x)^2 \, dx = a^2x + \frac{1}{2}\, b^2e^{(2\,x)} + 2\,abe^x$$

input
```
integrate((a+b*exp(x))^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/2*b^2*e^(2*x) + 2*a*b*e^x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a + be^x)^2 \, dx = a^2x + \frac{1}{2}\, b^2e^{(2\,x)} + 2\,abe^x$$

input
```
integrate((a+b*exp(x))^2,x, algorithm="giac")
```

output
```
a^2*x + 1/2*b^2*e^(2*x) + 2*a*b*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+be^x)^2 \, dx = x\,a^2 + 2\,\mathrm{e}^x\,a\,b + \frac{\mathrm{e}^{2\,x}\,b^2}{2}$$

input
```
int((a + b*exp(x))^2,x)
```

output
```
(b^2*exp(2*x))/2 + a^2*x + 2*a*b*exp(x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int (a+be^x)^2 \, dx = \frac{e^{2x}b^2}{2} + 2e^x ab + a^2 x$$

input
```
int((a+b*exp(x))^2,x)
```

output
```
(e**(2*x)*b**2 + 4*e**x*a*b + 2*a**2*x)/2
```

3.619 $\int (a+be^x)^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 40

$$\int (a+be^x)^3 \, dx = 3a^2be^x + \frac{3}{2}ab^2e^{2x} + \frac{1}{3}b^3e^{3x} + a^3x$$

output
```
3*a^2*b*exp(x)+3/2*a*b^2*exp(2*x)+1/3*b^3*exp(3*x)+a^3*x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int (a+be^x)^3 \, dx = \frac{1}{6}be^x\left(18a^2+9abe^x+2b^2e^{2x}\right) + a^3\log\left(e^x\right)$$

input
```
Integrate[(a + b*E^x)^3,x]
```

output
```
(b*E^x*(18*a^2 + 9*a*b*E^x + 2*b^2*E^(2*x)))/6 + a^3*Log[E^x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^x)^3\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}(a+be^x)^3\,de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^3e^{-x}+3a^2b+3ab^2e^x+b^3e^{2x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$a^3\log\left(e^x\right)+3a^2be^x+\frac{3}{2}ab^2e^{2x}+\frac{1}{3}b^3e^{3x}$$

input
```
Int[(a + b*E^x)^3,x]
```

output
```
3*a^2*b*E^x + (3*a*b^2*E^(2*x))/2 + (b^3*E^(3*x))/3 + a^3*Log[E^x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result
norman	$3a^2b\,e^x+\frac{3a\,b^2e^{2x}}{2}+\frac{b^3e^{3x}}{3}+a^3x$
risch	$3a^2b\,e^x+\frac{3a\,b^2e^{2x}}{2}+\frac{b^3e^{3x}}{3}+a^3x$
parallelrisch	$3a^2b\,e^x+\frac{3a\,b^2e^{2x}}{2}+\frac{b^3e^{3x}}{3}+a^3x$
parts	$3a^2b\,e^x+\frac{3a\,b^2e^{2x}}{2}+\frac{b^3e^{3x}}{3}+a^3x$
derivativedivides	$\frac{b^3e^{3x}}{3}+\frac{3a\,b^2e^{2x}}{2}+3a^2b\,e^x+a^3\ln\left(e^x\right)$
default	$\frac{b^3e^{3x}}{3}+\frac{3a\,b^2e^{2x}}{2}+3a^2b\,e^x+a^3\ln\left(e^x\right)$
orering	$\left(x+\frac{11}{6}\right)\left(a+b\,e^x\right)^3+3\left(-\frac{11x}{6}-1\right)\left(a+b\,e^x\right)^2b\,e^x+\left(x+\frac{1}{6}\right)\left(6(a+b\,e^x)\,b^2e^{2x}+3(a$

input
```
int((a+b*exp(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+1/3*b^3*exp(x)^3+3/2*a*b^2*exp(x)^2+3*a^2*b*exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int (a+be^x)^3\,dx=a^3x+\frac{1}{3}\,b^3e^{(3\,x)}+\frac{3}{2}\,ab^2e^{(2\,x)}+3\,a^2be^x$$

input
```
integrate((a+b*exp(x))^3,x, algorithm="fricas")
```

output
```
a^3*x + 1/3*b^3*e^(3*x) + 3/2*a*b^2*e^(2*x) + 3*a^2*b*e^x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int (a+be^x)^3\,dx=a^3x+3a^2be^x+\frac{3ab^2e^{2x}}{2}+\frac{b^3e^{3x}}{3}$$

input
```
integrate((a+b*exp(x))**3,x)
```

output
```
a**3*x + 3*a**2*b*exp(x) + 3*a*b**2*exp(2*x)/2 + b**3*exp(3*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int (a+be^x)^3\,dx=a^3x+\frac{1}{3}\,b^3e^{(3\,x)}+\frac{3}{2}\,ab^2e^{(2\,x)}+3\,a^2be^x$$

input
```
integrate((a+b*exp(x))^3,x, algorithm="maxima")
```

output
```
a^3*x + 1/3*b^3*e^(3*x) + 3/2*a*b^2*e^(2*x) + 3*a^2*b*e^x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int (a+be^x)^3\,dx=a^3x+\frac{1}{3}\,b^3e^{(3\,x)}+\frac{3}{2}\,ab^2e^{(2\,x)}+3\,a^2be^x$$

input
```
integrate((a+b*exp(x))^3,x, algorithm="giac")
```

output
```
a^3*x + 1/3*b^3*e^(3*x) + 3/2*a*b^2*e^(2*x) + 3*a^2*b*e^x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int (a+be^x)^3\,dx = x\,a^3+3\,\mathrm{e}^x\,a^2\,b+\frac{3\,\mathrm{e}^{2\,x}\,a\,b^2}{2}+\frac{\mathrm{e}^{3\,x}\,b^3}{3}$$

input
```
int((a + b*exp(x))^3,x)
```

output
```
(b^3*exp(3*x))/3 + a^3*x + 3*a^2*b*exp(x) + (3*a*b^2*exp(2*x))/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int (a+be^x)^3\,dx = \frac{e^{3x}b^3}{3}+\frac{3e^{2x}a\,b^2}{2}+3e^x a^2 b+a^3 x$$

input
```
int((a+b*exp(x))^3,x)
```

output
```
(2*e**(3*x)*b**3 + 9*e**(2*x)*a*b**2 + 18*e**x*a**2*b + 6*a**3*x)/6
```

3.620 $\int (a+be^x)^4 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 53

$$\int (a+be^x)^4 \, dx = 4a^3be^x + 3a^2b^2e^{2x} + \frac{4}{3}ab^3e^{3x} + \frac{1}{4}b^4e^{4x} + a^4x$$

output
```
4*a^3*b*exp(x)+3*a^2*b^2*exp(2*x)+4/3*a*b^3*exp(3*x)+1/4*b^4*exp(4*x)+a^4*
x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int (a+be^x)^4 \, dx = \frac{1}{12}be^x\left(48a^3 + 36a^2be^x + 16ab^2e^{2x} + 3b^3e^{3x}\right) + a^4\log\left(e^x\right)$$

input
```
Integrate[(a + b*E^x)^4,x]
```

output
```
(b*E^x*(48*a^3 + 36*a^2*b*E^x + 16*a*b^2*E^(2*x) + 3*b^3*E^(3*x)))/12 + a^
4*Log[E^x]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^x)^4\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x}(a+be^x)^4\,de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(a^4e^{-x}+4a^3b+6a^2b^2e^x+4ab^3e^{2x}+b^4e^{3x}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$a^4\log\left(e^x\right)+4a^3be^x+3a^2b^2e^{2x}+\frac{4}{3}ab^3e^{3x}+\frac{1}{4}b^4e^{4x}$$

input
```
Int[(a + b*E^x)^4,x]
```

output
```
4*a^3*b*E^x + 3*a^2*b^2*E^(2*x) + (4*a*b^3*E^(3*x))/3 + (b^4*E^(4*x))/4 +
a^4*Log[E^x]
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result
norman	$4a^3b\,\mathrm{e}^x+3a^2b^2\mathrm{e}^{2x}+\frac{4a\,b^3\mathrm{e}^{3x}}{3}+\frac{b^4\mathrm{e}^{4x}}{4}+a^4x$
risch	$4a^3b\,\mathrm{e}^x+3a^2b^2\mathrm{e}^{2x}+\frac{4a\,b^3\mathrm{e}^{3x}}{3}+\frac{b^4\mathrm{e}^{4x}}{4}+a^4x$
parallelrisch	$4a^3b\,\mathrm{e}^x+3a^2b^2\mathrm{e}^{2x}+\frac{4a\,b^3\mathrm{e}^{3x}}{3}+\frac{b^4\mathrm{e}^{4x}}{4}+a^4x$
parts	$4a^3b\,\mathrm{e}^x+3a^2b^2\mathrm{e}^{2x}+\frac{4a\,b^3\mathrm{e}^{3x}}{3}+\frac{b^4\mathrm{e}^{4x}}{4}+a^4x$
derivativedivides	$\frac{b^4\mathrm{e}^{4x}}{4}+\frac{4a\,b^3\mathrm{e}^{3x}}{3}+3a^2b^2\mathrm{e}^{2x}+4a^3b\,\mathrm{e}^x+a^4\ln\left(\mathrm{e}^x\right)$
default	$\frac{b^4\mathrm{e}^{4x}}{4}+\frac{4a\,b^3\mathrm{e}^{3x}}{3}+3a^2b^2\mathrm{e}^{2x}+4a^3b\,\mathrm{e}^x+a^4\ln\left(\mathrm{e}^x\right)$
orering	$\left(x+\frac{25}{12}\right)\left(a+b\,\mathrm{e}^x\right)^4+4\left(-\frac{25x}{12}-\frac{35}{24}\right)\left(a+b\,\mathrm{e}^x\right)^3b\,\mathrm{e}^x+\left(\frac{35x}{24}+\frac{5}{12}\right)\left(12\left(a+b\,\mathrm{e}^x\right)^2b^2\mathrm{e}^{2x}\right.$

input
```
int((a+b*exp(x))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*x+1/4*b^4*exp(x)^4+4/3*a*b^3*exp(x)^3+3*a^2*b^2*exp(x)^2+4*a^3*b*exp(x
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int (a+be^x)^4\,dx=a^4x+\frac{1}{4}\,b^4e^{(4\,x)}+\frac{4}{3}\,ab^3e^{(3\,x)}+3\,a^2b^2e^{(2\,x)}+4\,a^3be^x$$

input
```
integrate((a+b*exp(x))^4,x, algorithm="fricas")
```

output
```
a^4*x + 1/4*b^4*e^(4*x) + 4/3*a*b^3*e^(3*x) + 3*a^2*b^2*e^(2*x) + 4*a^3*b*
e^x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int (a+be^x)^4 \, dx = a^4x + 4a^3be^x + 3a^2b^2e^{2x} + \frac{4ab^3e^{3x}}{3} + \frac{b^4e^{4x}}{4}$$

input
```
integrate((a+b*exp(x))**4,x)
```

output
```
a**4*x + 4*a**3*b*exp(x) + 3*a**2*b**2*exp(2*x) + 4*a*b**3*exp(3*x)/3 + b*
*4*exp(4*x)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int (a+be^x)^4 \, dx = a^4x + \frac{1}{4}\,b^4e^{(4\,x)} + \frac{4}{3}\,ab^3e^{(3\,x)} + 3\,a^2b^2e^{(2\,x)} + 4\,a^3be^x$$

input
```
integrate((a+b*exp(x))^4,x, algorithm="maxima")
```

output
```
a^4*x + 1/4*b^4*e^(4*x) + 4/3*a*b^3*e^(3*x) + 3*a^2*b^2*e^(2*x) + 4*a^3*b*
e^x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int (a+be^x)^4\,dx = a^4x + \frac{1}{4}\,b^4e^{(4\,x)} + \frac{4}{3}\,ab^3e^{(3\,x)} + 3\,a^2b^2e^{(2\,x)} + 4\,a^3be^x$$

input
```
integrate((a+b*exp(x))^4,x, algorithm="giac")
```

output
```
a^4*x + 1/4*b^4*e^(4*x) + 4/3*a*b^3*e^(3*x) + 3*a^2*b^2*e^(2*x) + 4*a^3*b*
e^x
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int (a+be^x)^4\,dx = x\,a^4 + 4\,\mathrm{e}^x\,a^3\,b + 3\,\mathrm{e}^{2\,x}\,a^2\,b^2 + \frac{4\,\mathrm{e}^{3\,x}\,a\,b^3}{3} + \frac{\mathrm{e}^{4\,x}\,b^4}{4}$$

input
```
int((a + b*exp(x))^4,x)
```

output
```
(b^4*exp(4*x))/4 + a^4*x + 3*a^2*b^2*exp(2*x) + 4*a^3*b*exp(x) + (4*a*b^3*
exp(3*x))/3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int (a+be^x)^4\,dx = \frac{e^{4x}b^4}{4} + \frac{4e^{3x}a\,b^3}{3} + 3e^{2x}a^2b^2 + 4e^xa^3b + a^4x$$

input
```
int((a+b*exp(x))^4,x)
```

output
```
(3*e**(4*x)*b**4 + 16*e**(3*x)*a*b**3 + 36*e**(2*x)*a**2*b**2 + 48*e**x*a*
*3*b + 12*a**4*x)/12
```

3.621 $\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-2*arctanh((a+b*exp(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx=-\frac{2\text{arctanh}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[1/Sqrt[a + b*E^(c + d*x)],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*E^(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2720, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{-c-dx}}{\sqrt{a+be^{c+dx}}}de^{c+dx}}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{e^{2c+2dx}}{b}-\frac{a}{b}}d\sqrt{a+be^{c+dx}}}{bd}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + b*E^(c + d*x)],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*E^(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{d x+c}}}{\sqrt{a}}\right)}{\sqrt{a}\, d}$	26
default	$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{d x+c}}}{\sqrt{a}}\right)}{\sqrt{a}\, d}$	26

input
```
int(1/(a+b*exp(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((a+b*exp(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx = \left[\frac{\log\left(\left(be^{(dx+c)}-2\sqrt{be^{(dx+c)}+a}\sqrt{a}+2\,a\right)e^{(-dx-c)}\right)}{\sqrt{a}d}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{be^{(dx+c)}+a}}\right)}{ad}\right]$$

input
```
integrate(1/(a+b*exp(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[log((b*e^(d*x + c) - 2*sqrt(b*e^(d*x + c) + a)*sqrt(a) + 2*a)*e^(-d*x - c
))/(sqrt(a)*d), 2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*e^(d*x + c) + a))/(a*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.59 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.88

$$\int \frac{1}{\sqrt{a + be^{c+dx}}}\,dx = \begin{cases} \frac{\begin{cases} \frac{2\operatorname{atan}\left(\frac{\sqrt{a+be^c e^{dx}}}{\sqrt{-a}}\right)}{\sqrt{-a}} & \text{for}\ be^c \neq 0 \\ \frac{\log\left(e^{dx}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}}{d} & \text{for}\ d \neq 0 \\ \frac{x}{\sqrt{a+be^c}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*exp(d*x+c))**(1/2),x)
```

output
```
Piecewise((Piecewise((2*atan(sqrt(a + b*exp(c)*exp(d*x))/sqrt(-a))/sqrt(-a
), Ne(b*exp(c), 0)), (log(exp(d*x))/sqrt(a), True))/d, Ne(d, 0)), (x/sqrt(
a + b*exp(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{a + be^{c+dx}}}\,dx = \frac{\log\left(\frac{\sqrt{be^{(dx+c)}+a}-\sqrt{a}}{\sqrt{be^{(dx+c)}+a}+\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(b*e^(d*x + c) + a) - sqrt(a))/(sqrt(b*e^(d*x + c) + a) + sqrt(a)
))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx=\frac{2\arctan\left(\frac{\sqrt{be^{(dx+c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*e^(d*x + c) + a)/sqrt(-a))/(sqrt(-a)*d)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx=-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,e^{d\,x}\,e^c}}{\sqrt{a}}\right)}{\sqrt{a}\,d}$$

input
```
int(1/(a + b*exp(c + d*x))^(1/2),x)
```

output
```
-(2*atanh((a + b*exp(d*x)*exp(c))^(1/2)/a^(1/2)))/(a^(1/2)*d)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+be^{c+dx}}}\,dx=\int \frac{1}{\sqrt{e^{dx+c}b+a}}dx$$

input
```
int(1/(a+b*exp(d*x+c))^(1/2),x)
```

output
```
int(1/sqrt(e**(c + d*x)*b + a),x)
```

3.622 $\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 34

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
2*arctan((-a+b*exp(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx=\frac{2\arctan\left(\frac{\sqrt{-a+be^{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[1/Sqrt[-a + b*E^(c + d*x)],x]
```

output
```
(2*ArcTan[Sqrt[-a + b*E^(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2720, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{be^{c+dx}-a}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{e^{-c-dx}}{\sqrt{be^{c+dx}-a}}de^{c+dx}}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\int \frac{1}{\frac{a}{b}+\frac{e^{2c+2dx}}{b}}d\sqrt{be^{c+dx}-a}}{bd}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\arctan\left(\frac{\sqrt{be^{c+dx}-a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[-a + b*E^(c + d*x)],x]
```

output
```
(2*ArcTan[Sqrt[-a + b*E^(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2 \arctan\left(\frac{\sqrt{-a+b\,{\rm e}^{d x+c}}}{\sqrt{a}}\right)}{\sqrt{a}\, d}$	28
default	$\frac{2 \arctan\left(\frac{\sqrt{-a+b\,{\rm e}^{d x+c}}}{\sqrt{a}}\right)}{\sqrt{a}\, d}$	28

input
```
int(1/(-a+b*exp(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((-a+b*exp(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.50

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\left(be^{(dx+c)} - 2\sqrt{be^{(dx+c)}-a}\sqrt{-a} - 2\,a\right)e^{(-dx-c)}\right)}{ad}, -\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{be^{(dx+c)}-a}}\right)}{\sqrt{a}d}\right]$$

input
```
integrate(1/(-a+b*exp(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log((b*e^(d*x + c) - 2*sqrt(b*e^(d*x + c) - a)*sqrt(-a) - 2*a)*
e^(-d*x - c))/(a*d), -2*arctan(sqrt(a)/sqrt(b*e^(d*x + c) - a))/(sqrt(a)*d
)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx = \begin{cases} \frac{\begin{cases} \frac{2\,\operatorname{atan}\left(\frac{\sqrt{-a+be^{c}e^{dx}}}{\sqrt{a}}\right)}{\sqrt{a}} & \text{for}\ be^{c} \neq 0 \\ \frac{\log\left(e^{dx}\right)}{\sqrt{-a}} & \text{otherwise} \end{cases}}{d} & \text{for}\ d \neq 0 \\ \frac{x}{\sqrt{-a+be^{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-a+b*exp(d*x+c))**(1/2),x)
```

output
```
Piecewise((Piecewise((2*atan(sqrt(-a + b*exp(c)*exp(d*x))/sqrt(a))/sqrt(a)
, Ne(b*exp(c), 0)), (log(exp(d*x))/sqrt(-a), True))/d, Ne(d, 0)), (x/sqrt(
-a + b*exp(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx = \frac{2\,\arctan\left(\frac{\sqrt{be^{(dx+c)}-a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
integrate(1/(-a+b*exp(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(b*e^(d*x + c) - a)/sqrt(a))/(sqrt(a)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx = \frac{2\arctan\left(\frac{\sqrt{be^{(dx+c)}-a}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
integrate(1/(-a+b*exp(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(b*e^(d*x + c) - a)/sqrt(a))/(sqrt(a)*d)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,d}$$

input
```
int(1/(b*exp(c + d*x) - a)^(1/2),x)
```

output
```
-(2*atanh((b*exp(d*x)*exp(c) - a)^(1/2)/(-a)^(1/2)))/((-a)^(1/2)*d)
```

Reduce [F]

$$\int \frac{1}{\sqrt{-a+be^{c+dx}}}\,dx = \int \frac{1}{\sqrt{e^{dx+c}b-a}}dx$$

input
```
int(1/(-a+b*exp(d*x+c))^(1/2),x)
```

output
```
int(1/sqrt(e**(c + d*x)*b - a),x)
```

3.623 $\int \sqrt{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \sqrt{a+be^{c+dx}}\,dx=\frac{2\sqrt{a+be^{c+dx}}}{d}-\frac{2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{d}$$

output
```
2*(a+b*exp(d*x+c))^(1/2)/d-2*a^(1/2)*arctanh((a+b*exp(d*x+c))^(1/2)/a^(1/2
))/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \sqrt{a+be^{c+dx}}\,dx=\frac{2\left(\sqrt{a+be^{c+dx}}-\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)\right)}{d}$$

input
```
Integrate[Sqrt[a + b*E^(c + d*x)],x]
```

output
```
(2*(Sqrt[a + b*E^(c + d*x)] - Sqrt[a]*ArcTanh[Sqrt[a + b*E^(c + d*x)]/Sqrt
[a]]))/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2720, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+be^{c+dx}}dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int e^{-c-dx}\sqrt{a+be^{c+dx}}de^{c+dx}}{d}$$

$$\Big\downarrow \ 60$$

$$\frac{a\int \frac{e^{-c-dx}}{\sqrt{a+be^{c+dx}}}de^{c+dx}+2\sqrt{a+be^{c+dx}}}{d}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{2a\int \frac{1}{\frac{e^{2c+2dx}}{b}-\frac{a}{b}}d\sqrt{a+be^{c+dx}}}{b}+2\sqrt{a+be^{c+dx}}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{2\sqrt{a+be^{c+dx}}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+be^{c+dx}}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + b*E^(c + d*x)],x]
```

output
```
(2*Sqrt[a + b*E^(c + d*x)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*E^(c + d*x)]/Sqr
t[a]])/d
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2\sqrt{a+b\,{\mathrm e}^{d x +c}}-2\sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{d x +c}}}{\sqrt{a}}\right)}{d}$	42
default	$\frac{2\sqrt{a+b\,{\mathrm e}^{d x +c}}-2\sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{d x +c}}}{\sqrt{a}}\right)}{d}$	42
risch	$\frac{2\sqrt{a+b\,{\mathrm e}^{d x +c}}}{d}-\frac{2\sqrt{a}\, \operatorname{arctanh}\left(\frac{\sqrt{a+b\,{\mathrm e}^{d x +c}}}{\sqrt{a}}\right)}{d}$	44

input
```
int((a+b*exp(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(a+b*exp(d*x+c))^(1/2)-2*a^(1/2)*arctanh((a+b*exp(d*x+c))^(1/2)/a^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \sqrt{a+be^{c+dx}}\,dx = \left[\frac{\sqrt{a}\log\left(\left(be^{(dx+c)}-2\sqrt{be^{(dx+c)}+a}\sqrt{a}+2a\right)e^{(-dx-c)}\right)+2\sqrt{be^{(dx+c)}+a}}{d}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{be^{(dx+c)}+a}}\right.\right.}{d}\right.$$

input
```
integrate((a+b*exp(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(a)*log((b*e^(d*x + c) - 2*sqrt(b*e^(d*x + c) + a)*sqrt(a) + 2*a)*e^
(-d*x - c)) + 2*sqrt(b*e^(d*x + c) + a))/d, 2*(sqrt(-a)*arctan(sqrt(-a)/sq
rt(b*e^(d*x + c) + a)) + sqrt(b*e^(d*x + c) + a))/d]
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.47

$$\int \sqrt{a+be^{c+dx}}\,dx = \begin{cases} \frac{\begin{cases} \frac{2a\,\mathrm{atan}\left(\frac{\sqrt{a+be^{c}e^{dx}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{a+be^{c}e^{dx}} & \text{for}\ be^{c}\neq 0 \\ \sqrt{a}\log\left(e^{dx}\right) & \text{otherwise} \end{cases}}{d} & \text{for}\ d\neq 0 \\ x\sqrt{a+be^{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*exp(d*x+c))**(1/2),x)
```

output
```
Piecewise((Piecewise((2*a*atan(sqrt(a + b*exp(c)*exp(d*x))/sqrt(-a))/sqrt(
-a) + 2*sqrt(a + b*exp(c)*exp(d*x)), Ne(b*exp(c), 0)), (sqrt(a)*log(exp(d*
x)), True))/d, Ne(d, 0)), (x*sqrt(a + b*exp(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \sqrt{a+be^{c+dx}}\,dx = \frac{\sqrt{a}\log\left(\frac{\sqrt{be^{(dx+c)}+a}-\sqrt{a}}{\sqrt{be^{(dx+c)}+a}+\sqrt{a}}\right)}{d} + \frac{2\sqrt{be^{(dx+c)}+a}}{d}$$

input
```
integrate((a+b*exp(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(a)*log((sqrt(b*e^(d*x + c) + a) - sqrt(a))/(sqrt(b*e^(d*x + c) + a) +
 sqrt(a)))/d + 2*sqrt(b*e^(d*x + c) + a)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \sqrt{a+be^{c+dx}}\,dx = \frac{2\left(\frac{a\arctan\left(\frac{\sqrt{be^{(dx+c)}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{be^{(dx+c)}+a}\right)}{d}$$

input
```
integrate((a+b*exp(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*(a*arctan(sqrt(b*e^(d*x + c) + a)/sqrt(-a))/sqrt(-a) + sqrt(b*e^(d*x + c
) + a))/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \sqrt{a+be^{c+dx}}\,dx = \frac{2\sqrt{a+b\,\mathrm{e}^{c+d\,x}}}{d} - \frac{2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}}{\sqrt{a}}\right)}{d}$$

input
```
int((a + b*exp(c + d*x))^(1/2),x)
```

output
```
(2*(a + b*exp(c + d*x))^(1/2))/d - (2*a^(1/2)*atanh((a + b*exp(d*x)*exp(c)
)^(1/2)/a^(1/2)))/d
```

Reduce [F]

$$\int \sqrt{a+be^{c+dx}}\,dx = \int \sqrt{e^{dx+c}b+a}dx$$

input
```
int((a+b*exp(d*x+c))^(1/2),x)
```

output
```
int(sqrt(e**(c + d*x)*b + a),x)
```

3.624 $\int \sqrt{-a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \sqrt{-a+be^{c+dx}}\,dx=\frac{2\sqrt{-a+be^{c+dx}}}{d}-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{-a+be^{c+dx}}}{\sqrt{a}}\right)}{d}$$

output
```
2*(-a+b*exp(d*x+c))^(1/2)/d-2*a^(1/2)*arctan((-a+b*exp(d*x+c))^(1/2)/a^(1/
2))/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \sqrt{-a+be^{c+dx}}\,dx=\frac{2\left(\sqrt{-a+be^{c+dx}}-\sqrt{a}\arctan\left(\frac{\sqrt{-a+be^{c+dx}}}{\sqrt{a}}\right)\right)}{d}$$

input
```
Integrate[Sqrt[-a + b*E^(c + d*x)],x]
```

output
```
(2*(Sqrt[-a + b*E^(c + d*x)] - Sqrt[a]*ArcTan[Sqrt[-a + b*E^(c + d*x)]/Sqr
t[a]]))/d
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{be^{c+dx}-a}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-c-dx}\sqrt{be^{c+dx}-a}de^{c+dx}}{d}$$

$$\Big\downarrow \text{60}$$

$$\frac{2\sqrt{be^{c+dx}-a}-a\int \frac{e^{-c-dx}}{\sqrt{be^{c+dx}-a}}de^{c+dx}}{d}$$

$$\Big\downarrow \text{73}$$

$$\frac{2\sqrt{be^{c+dx}-a}-\frac{2a\int \frac{1}{\frac{a}{b}+\frac{e^{2c+2dx}}{b}}d\sqrt{be^{c+dx}-a}}{b}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{be^{c+dx}-a}-2\sqrt{a}\arctan\left(\frac{\sqrt{be^{c+dx}-a}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[-a + b*E^(c + d*x)],x]
```

output
```
(2*Sqrt[-a + b*E^(c + d*x)] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*E^(c + d*x)]/Sq
rt[a]])/d
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{2\sqrt{-a+b\,\mathrm{e}^{dx+c}}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{dx+c}}}{\sqrt{a}}\right)}{d}$	46
default	$\frac{2\sqrt{-a+b\,\mathrm{e}^{dx+c}}-2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{dx+c}}}{\sqrt{a}}\right)}{d}$	46
risch	$-\frac{2\left(a-b\,\mathrm{e}^{dx+c}\right)}{d\sqrt{-a+b\,\mathrm{e}^{dx+c}}}-\frac{2\sqrt{a}\,\arctan\left(\frac{\sqrt{-a+b\,\mathrm{e}^{dx+c}}}{\sqrt{a}}\right)}{d}$	59

input
```
int((-a+b*exp(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-a+b*exp(d*x+c))^(1/2)-2*a^(1/2)*arctan((-a+b*exp(d*x+c))^(1/2)/a^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \sqrt{-a+be^{c+dx}}\,dx = \left[\frac{\sqrt{-a}\log\left(\left(be^{(dx+c)}-2\sqrt{be^{(dx+c)}-a}\sqrt{-a}-2a\right)e^{(-dx-c)}\right)+2\sqrt{be^{(dx+c)}-a}}{d}, \frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{a}}{\sqrt{be^{(dx+c)}}}\right.\right.}{d}\right.$$

input
```
integrate((-a+b*exp(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(-a)*log((b*e^(d*x + c) - 2*sqrt(b*e^(d*x + c) - a)*sqrt(-a) - 2*a)*
e^(-d*x - c)) + 2*sqrt(b*e^(d*x + c) - a))/d, 2*(sqrt(a)*arctan(sqrt(a)/sq
rt(b*e^(d*x + c) - a)) + sqrt(b*e^(d*x + c) - a))/d]
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \sqrt{-a+be^{c+dx}}\,dx = \begin{cases} \frac{\begin{cases} -2\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{-a+be^{c}e^{dx}}}{\sqrt{a}}\right)+2\sqrt{-a+be^{c}e^{dx}} & \text{for}\: be^{c}\neq 0 \\ \sqrt{-a}\log\left(e^{dx}\right) & \text{otherwise} \end{cases}}{d} & \text{for}\: d\neq 0 \\ x\sqrt{-a+be^{c}} & \text{otherwise} \end{cases}$$

input
```
integrate((-a+b*exp(d*x+c))**(1/2),x)
```

output
```
Piecewise((Piecewise((-2*sqrt(a)*atan(sqrt(-a + b*exp(c)*exp(d*x))/sqrt(a)
) + 2*sqrt(-a + b*exp(c)*exp(d*x)), Ne(b*exp(c), 0)), (sqrt(-a)*log(exp(d*
x)), True))/d, Ne(d, 0)), (x*sqrt(-a + b*exp(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \sqrt{-a+be^{c+dx}}\,dx=-\frac{2\sqrt{a}\arctan\left(\frac{\sqrt{be^{(dx+c)}-a}}{\sqrt{a}}\right)}{d}+\frac{2\sqrt{be^{(dx+c)}-a}}{d}$$

input
```
integrate((-a+b*exp(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(a)*arctan(sqrt(b*e^(d*x + c) - a)/sqrt(a))/d + 2*sqrt(b*e^(d*x + c
) - a)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \sqrt{-a+be^{c+dx}}\,dx=-\frac{2\left(\sqrt{a}\arctan\left(\frac{\sqrt{be^{(dx+c)}-a}}{\sqrt{a}}\right)-\sqrt{be^{(dx+c)}-a}\right)}{d}$$

input
```
integrate((-a+b*exp(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*(sqrt(a)*arctan(sqrt(b*e^(d*x + c) - a)/sqrt(a)) - sqrt(b*e^(d*x + c) -
 a))/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \sqrt{-a+be^{c+dx}}\,dx=\frac{2\sqrt{b\,\mathrm{e}^{c+d\,x}-a}}{d}-\frac{2\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a}}{\sqrt{a}}\right)}{d}$$

input
```
int((b*exp(c + d*x) - a)^(1/2),x)
```

output
```
(2*(b*exp(c + d*x) - a)^(1/2))/d - (2*a^(1/2)*atan((b*exp(d*x)*exp(c) - a)
^(1/2)/a^(1/2)))/d
```

Reduce [F]

$$\int \sqrt{-a+be^{c+dx}}\,dx=\int \sqrt{e^{dx+c}b-a}dx$$

input
```
int((-a+b*exp(d*x+c))^(1/2),x)
```

output
```
int(sqrt(e**(c + d*x)*b - a),x)
```

3.625 $\int e^{6x}\sin(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{6x}\sin(3x)\,dx = -\frac{1}{15}e^{6x}\cos(3x) + \frac{2}{15}e^{6x}\sin(3x)$$

output
```
-1/15*exp(6*x)*cos(3*x)+2/15*exp(6*x)*sin(3*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{6x}\sin(3x)\,dx = -\frac{1}{15}e^{6x}(\cos(3x) - 2\sin(3x))$$

input
```
Integrate[E^(6*x)*Sin[3*x],x]
```

output
```
-1/15*(E^(6*x)*(Cos[3*x] - 2*Sin[3*x]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{6x}\sin(3x)\,dx$$

$$\Big\downarrow \text{ 4932}$$

$$\frac{2}{15}e^{6x}\sin(3x)-\frac{1}{15}e^{6x}\cos(3x)$$

input
```
Int[E^(6*x)*Sin[3*x],x]
```

output
```
-1/15*(E^(6*x)*Cos[3*x]) + (2*E^(6*x)*Sin[3*x])/15
```

Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$-\frac{e^{6x}(\cos(3x)-2\sin(3x))}{15}$	18
default	$-\frac{e^{6x}\cos(3x)}{15}+\frac{2\,e^{6x}\sin(3x)}{15}$	22
orering	$-\frac{e^{6x}\cos(3x)}{15}+\frac{2\,e^{6x}\sin(3x)}{15}$	22
risch	$-\frac{e^{(6+3i)x}}{30}-\frac{i e^{(6+3i)x}}{15}-\frac{e^{(6-3i)x}}{30}+\frac{i e^{(6-3i)x}}{15}$	36
norman	$\frac{\frac{4\,e^{6x}\tan\left(\frac{3x}{2}\right)}{15}+\frac{e^{6x}\tan\left(\frac{3x}{2}\right)^{2}}{15}-\frac{e^{6x}}{15}}{1+\tan\left(\frac{3x}{2}\right)^{2}}$	41

input
```
int(exp(6*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/15*exp(6*x)*(cos(3*x)-2*sin(3*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{6x}\sin(3x)\,dx=-\frac{1}{15}\cos(3x)e^{(6x)}+\frac{2}{15}e^{(6x)}\sin(3x)$$

input
```
integrate(exp(6*x)*sin(3*x),x, algorithm="fricas")
```

output
```
-1/15*cos(3*x)*e^(6*x) + 2/15*e^(6*x)*sin(3*x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int e^{6x}\sin(3x)\,dx=\frac{2e^{6x}\sin(3x)}{15}-\frac{e^{6x}\cos(3x)}{15}$$

input
```
integrate(exp(6*x)*sin(3*x),x)
```

output
```
2*exp(6*x)*sin(3*x)/15 - exp(6*x)*cos(3*x)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{6x}\sin(3x)\,dx = -\frac{1}{15}\left(\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right)\right)e^{\left(6\,x\right)}$$

input
```
integrate(exp(6*x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/15*(cos(3*x) - 2*sin(3*x))*e^(6*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int e^{6x}\sin(3x)\,dx = -\frac{1}{15}\left(\cos\left(3\,x\right) - 2\,\sin\left(3\,x\right)\right)e^{\left(6\,x\right)}$$

input
```
integrate(exp(6*x)*sin(3*x),x, algorithm="giac")
```

output
```
-1/15*(cos(3*x) - 2*sin(3*x))*e^(6*x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{6x}\sin(3x)\,dx = -\frac{\mathrm{e}^{6\,x}\,\left(3\,\cos\left(3\,x\right)-6\,\sin\left(3\,x\right)\right)}{45}$$

input
```
int(sin(3*x)*exp(6*x),x)
```

output
```
-(exp(6*x)*(3*cos(3*x) - 6*sin(3*x)))/45
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int e^{6x}\sin(3x)\,dx = \frac{e^{6x}(-\cos(3x)+2\sin(3x))}{15}$$

input
```
int(exp(6*x)*sin(3*x),x)
```

output
```
(e**(6*x)*( - cos(3*x) + 2*sin(3*x)))/15
```

3.626 $\int \frac{e^{3x}}{1+e^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = e^x - \arctan(e^x)$$

output
```
exp(x)-arctan(exp(x))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = e^x - \arctan(e^x)$$

input
```
Integrate[E^(3*x)/(1 + E^(2*x)),x]
```

output
```
E^x - ArcTan[E^x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2678, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3x}}{e^{2x}+1} \, dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^{2x}}{e^{2x}+1}de^x$$

$$\Big\downarrow \text{262}$$

$$e^x-\int \frac{1}{1+e^{2x}}de^x$$

$$\Big\downarrow \text{216}$$

$$e^x-\arctan \left(e^x\right)$$

input
```
Int[E^(3*x)/(1 + E^(2*x)),x]
```

output
```
E^x - ArcTan[E^x]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$e^{x}-\arctan\left(e^{x}\right)$	9
risch	$e^{x}+\frac{i\ln\left(e^{x}-i\right)}{2}-\frac{i\ln\left(e^{x}+i\right)}{2}$	22

input
```
int(exp(3*x)/(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)-arctan(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx=-\arctan\left(e^{x}\right)+e^{x}$$

input
```
integrate(exp(3*x)/(1+exp(2*x)),x, algorithm="fricas")
```

output
```
-arctan(e^x) + e^x
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = e^{x} + \operatorname{RootSum}\left(4z^{2} + 1, \left(i \mapsto i \log\left(-2i + e^{x}\right)\right)\right)$$

input
```
integrate(exp(3*x)/(1+exp(2*x)),x)
```

output
```
exp(x) + RootSum(4*_z**2 + 1, Lambda(_i, _i*log(-2*_i + exp(x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = -\arctan\left(e^{x}\right) + e^{x}$$

input
```
integrate(exp(3*x)/(1+exp(2*x)),x, algorithm="maxima")
```

output
```
-arctan(e^x) + e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = -\arctan\left(e^{x}\right) + e^{x}$$

input
```
integrate(exp(3*x)/(1+exp(2*x)),x, algorithm="giac")
```

output
```
-arctan(e^x) + e^x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = \mathrm{e}^x - \mathrm{atan}(\mathrm{e}^x)$$

input
```
int(exp(3*x)/(exp(2*x) + 1),x)
```

output
```
exp(x) - atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{3x}}{1+e^{2x}}\,dx = -atan(e^x) + e^x$$

input
```
int(exp(3*x)/(1+exp(2*x)),x)
```

output
```
 - atan(e**x) + e**x
```

3.627 $\int \frac{e^{3x}}{-1+e^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 10

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^x - \operatorname{arctanh}(e^x)$$

output
```
exp(x)-arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^x - \operatorname{arctanh}(e^x)$$

input
```
Integrate[E^(3*x)/(-1 + E^(2*x)),x]
```

output
```
E^x - ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2678, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3x}}{e^{2x}-1}dx$$

$$\Big\downarrow \text{2678}$$

$$\int -\frac{e^{2x}}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{e^{2x}}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{262}$$

$$e^x-\int \frac{1}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{219}$$

$$e^x-\text{arctanh}(e^x)$$

input
```
Int[E^(3*x)/(-1 + E^(2*x)),x]
```

output
```
E^x - ArcTanh[E^x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

method	result	size
default	$e^{x}+\frac{\ln(-1+e^{x})}{2}-\frac{\ln(1+e^{x})}{2}$	18
norman	$e^{x}+\frac{\ln(-1+e^{x})}{2}-\frac{\ln(1+e^{x})}{2}$	18
risch	$e^{x}+\frac{\ln(-1+e^{x})}{2}-\frac{\ln(1+e^{x})}{2}$	18

input
```
int(exp(3*x)/(-1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)+1/2*ln(-1+exp(x))-1/2*ln(1+exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(8) = 16.

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^{x} - \frac{1}{2}\,\log\left(e^{x}+1\right) + \frac{1}{2}\,\log\left(e^{x}-1\right)$$

input
```
integrate(exp(3*x)/(-1+exp(2*x)),x, algorithm="fricas")
```

output
```
e^x - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^{x} + \frac{\log\left(e^{x}-1\right)}{2} - \frac{\log\left(e^{x}+1\right)}{2}$$

input
```
integrate(exp(3*x)/(-1+exp(2*x)),x)
```

output
```
exp(x) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(8) = 16$.

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^x - \frac{1}{2}\,\log\left(e^x+1\right) + \frac{1}{2}\,\log\left(e^x-1\right)$$

input
```
integrate(exp(3*x)/(-1+exp(2*x)),x, algorithm="maxima")
```

output
```
e^x - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^x - \frac{1}{2}\,\log\left(e^x+1\right) + \frac{1}{2}\,\log\left(|e^x-1|\right)$$

input
```
integrate(exp(3*x)/(-1+exp(2*x)),x, algorithm="giac")
```

output
```
e^x - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x-1\right)}{2} - \frac{\ln\left(\mathrm{e}^x+1\right)}{2} + \mathrm{e}^x$$

input
```
int(exp(3*x)/(exp(2*x) - 1),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{e^{3x}}{-1+e^{2x}}\,dx = e^x + \frac{\log(e^x-1)}{2} - \frac{\log(e^x+1)}{2}$$

input
```
int(exp(3*x)/(-1+exp(2*x)),x)
```

output
```
(2*e**x + log(e**x - 1) - log(e**x + 1))/2
```

3.628 $\int \frac{e^{-x}}{\sqrt{1+e^{2x}}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 18

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}} \, dx = -e^{-x}\sqrt{1+e^{2x}}$$

output
```
-(1+exp(2*x))^(1/2)/exp(x)
```

Mathematica [A] (verified)

Time = 3.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}} \, dx = -e^{-x}\sqrt{1+e^{2x}}$$

input
```
Integrate[1/(E^x*Sqrt[1 + E^(2*x)]),x]
```

output
```
-(Sqrt[1 + E^(2*x)]/E^x)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2679, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-x}}{\sqrt{e^{2x}+1}}dx$$
$$\Big\downarrow \text{ 2679}$$
$$-\int \frac{1}{\sqrt{1+e^{2x}}}de^{-x}$$
$$\Big\downarrow \text{ 746}$$
$$-e^{-x}\sqrt{e^{2x}+1}$$

input
```
Int[1/(E^x*Sqrt[1 + E^(2*x)]),x]
```

output
```
-(Sqrt[1 + E^(2*x)]/E^x)
```

Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$-\mathrm{e}^{-x}\sqrt{1+\mathrm{e}^{2x}}$	15

input
```
int(1/exp(x)/(1+exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/exp(x)*(exp(x)^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}}\,dx = -\sqrt{e^{(-2\,x)}+1}$$

input
```
integrate(1/exp(x)/(1+exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(e^(-2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}}\,dx = -\sqrt{e^{2x}+1}e^{-x}$$

input
```
integrate(1/exp(x)/(1+exp(2*x))**(1/2),x)
```

output
```
-sqrt(exp(2*x) + 1)*exp(-x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}}\,dx = -\sqrt{e^{(2\,x)}+1}e^{(-x)}$$

input
```
integrate(1/exp(x)/(1+exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(e^(2*x) + 1)*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}}\,dx = \frac{2}{\left(\sqrt{e^{(2\,x)}+1}-e^{x}\right)^{2}-1}$$

input
```
integrate(1/exp(x)/(1+exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
2/((sqrt(e^(2*x) + 1) - e^x)^2 - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}}\,dx = \int \frac{\mathrm{e}^{-x}}{\sqrt{\mathrm{e}^{2\,x}+1}}\,dx$$

input
```
int(exp(-x)/(exp(2*x) + 1)^(1/2),x)
```

output
```
int(exp(-x)/(exp(2*x) + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{e^{-x}}{\sqrt{1+e^{2x}}}\,dx = -\frac{\sqrt{e^{2x}+1}}{e^{x}}$$

input
```
int(1/exp(x)/(1+exp(2*x))^(1/2),x)
```

output
```
( - sqrt(e**(2*x) + 1))/e**x
```

3.629 $\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 47

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = -\frac{\log\left(4-\sqrt{17}-e^x\right)}{2\sqrt{17}}+\frac{\log\left(4+\sqrt{17}-e^x\right)}{2\sqrt{17}}$$

output
```
-1/34*ln(4-17^(1/2)-exp(x))*17^(1/2)+1/34*ln(4+17^(1/2)-exp(x))*17^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.40

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{-4+e^x}{\sqrt{17}}\right)}{\sqrt{17}}$$

input
```
Integrate[E^x/(-1 - 8*E^x + E^(2*x)),x]
```

output
```
-(ArcTanh[(-4 + E^x)/Sqrt[17]]/Sqrt[17])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2720, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{-8e^x+e^{2x}-1}dx$$
$$\Big\downarrow \text{2720}$$
$$\int \frac{1}{-8e^x+e^{2x}-1}de^x$$
$$\Big\downarrow \text{1081}$$
$$\int \left(\frac{1}{2\sqrt{17}\left(-e^x+4-\sqrt{17}\right)}-\frac{1}{2\sqrt{17}\left(-e^x+4+\sqrt{17}\right)}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$\frac{\log\left(-e^x+4+\sqrt{17}\right)}{2\sqrt{17}}-\frac{\log\left(-e^x+4-\sqrt{17}\right)}{2\sqrt{17}}$$

input
```
Int[E^x/(-1 - 8*E^x + E^(2*x)),x]
```

output
```
-1/2*Log[4 - Sqrt[17] - E^x]/Sqrt[17] + Log[4 + Sqrt[17] - E^x]/(2*Sqrt[17
])
```

Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

method	result	size
default	$-\frac{\sqrt{17}\, \operatorname{arctanh}\left(\frac{(2\,e^{x}-8)\sqrt{17}}{34}\right)}{17}$	18
risch	$\frac{\sqrt{17}\, \ln\left(e^{x}-4-\sqrt{17}\right)}{34}-\frac{\sqrt{17}\, \ln\left(e^{x}-4+\sqrt{17}\right)}{34}$	30

input
```
int(exp(x)/(-1-8*exp(x)+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/17*17^(1/2)*arctanh(1/34*(2*exp(x)-8)*17^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = \frac{1}{34}\,\sqrt{17}\log\left(-\frac{2\,\left(\sqrt{17}+4\right)e^x - 8\,\sqrt{17} - e^{(2\,x)} - 33}{e^{(2\,x)} - 8\,e^x - 1}\right)$$

input
```
integrate(exp(x)/(-1-8*exp(x)+exp(2*x)),x, algorithm="fricas")
```

output
```
1/34*sqrt(17)*log(-(2*(sqrt(17) + 4)*e^x - 8*sqrt(17) - e^(2*x) - 33)/(e^(
2*x) - 8*e^x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.36

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = \text{RootSum}\left(68z^2-1,\left(i\mapsto i\log\left(-34i+e^x-4\right)\right)\right)$$

input
```
integrate(exp(x)/(-1-8*exp(x)+exp(2*x)),x)
```

output
```
RootSum(68*_z**2 - 1, Lambda(_i, _i*log(-34*_i + exp(x) - 4)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = \frac{1}{34}\sqrt{17}\log\left(-\frac{\sqrt{17}-e^x+4}{\sqrt{17}+e^x-4}\right)$$

input
```
integrate(exp(x)/(-1-8*exp(x)+exp(2*x)),x, algorithm="maxima")
```

output
```
1/34*sqrt(17)*log(-(sqrt(17) - e^x + 4)/(sqrt(17) + e^x - 4))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = \frac{1}{34}\sqrt{17}\log\left(\frac{\left|-2\sqrt{17}+2e^x-8\right|}{\left|2\sqrt{17}+2e^x-8\right|}\right)$$

input
```
integrate(exp(x)/(-1-8*exp(x)+exp(2*x)),x, algorithm="giac")
```

output
```
1/34*sqrt(17)*log(abs(-2*sqrt(17) + 2*e^x - 8)/abs(2*sqrt(17) + 2*e^x - 8)
)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.36

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = -\frac{\sqrt{17}\,\mathrm{atanh}\left(\frac{\sqrt{17}\,(2\,\mathrm{e}^x-8)}{34}\right)}{17}$$

input
```
int(-exp(x)/(8*exp(x) - exp(2*x) + 1),x)
```

output
```
-(17^(1/2)*atanh((17^(1/2)*(2*exp(x) - 8))/34))/17
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

$$\int \frac{e^x}{-1-8e^x+e^{2x}}\,dx = \frac{\sqrt{17}\,\left(\log\left(e^x-\sqrt{17}-4\right)-\log\left(e^x+\sqrt{17}-4\right)\right)}{34}$$

input
```
int(exp(x)/(-1-8*exp(x)+exp(2*x)),x)
```

output
```
(sqrt(17)*(log(e**x - sqrt(17) - 4) - log(e**x + sqrt(17) - 4)))/34
```

3.630 $\int e^{7x}x^3\,dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int e^{7x}x^3\,dx = -\frac{6e^{7x}}{2401}+\frac{6}{343}e^{7x}x-\frac{3}{49}e^{7x}x^2+\frac{1}{7}e^{7x}x^3$$

output
```
-6/2401*exp(7*x)+6/343*exp(7*x)*x-3/49*exp(7*x)*x^2+1/7*exp(7*x)*x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int e^{7x}x^3\,dx = \frac{e^{7x}(-6+42x-147x^2+343x^3)}{2401}$$

input
```
Integrate[E^(7*x)*x^3,x]
```

output
```
(E^(7*x)*(-6 + 42*x - 147*x^2 + 343*x^3))/2401
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{7x}x^3\,dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{1}{7}e^{7x}x^3-\frac{3}{7}\int e^{7x}x^2dx$$

$$\Big\downarrow \text{2607}$$

$$\frac{1}{7}e^{7x}x^3-\frac{3}{7}\left(\frac{1}{7}e^{7x}x^2-\frac{2}{7}\int e^{7x}xdx\right)$$

$$\Big\downarrow \text{2607}$$

$$\frac{1}{7}e^{7x}x^3-\frac{3}{7}\left(\frac{1}{7}e^{7x}x^2-\frac{2}{7}\left(\frac{1}{7}e^{7x}x-\frac{\int e^{7x}dx}{7}\right)\right)$$

$$\Big\downarrow \text{2624}$$

$$\frac{1}{7}e^{7x}x^3-\frac{3}{7}\left(\frac{1}{7}e^{7x}x^2-\frac{2}{7}\left(\frac{1}{7}e^{7x}x-\frac{e^{7x}}{49}\right)\right)$$

input
```
Int[E^(7*x)*x^3,x]
```

output
```
(E^(7*x)*x^3)/7 - (3*((E^(7*x)*x^2)/7 - (2*(-1/49*E^(7*x) + (E^(7*x)*x)/7)
)/7))/7
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

method	result	size
risch	$\left(\frac{1}{7}x^3-\frac{3}{49}x^2+\frac{6}{343}x-\frac{6}{2401}\right)e^{7x}$	21
gosper	$\frac{(343x^3-147x^2+42x-6)e^{7x}}{2401}$	22
orering	$\frac{(343x^3-147x^2+42x-6)e^{7x}}{2401}$	22
meijerg	$\frac{6}{2401}-\frac{(-1372x^3+588x^2-168x+24)e^{7x}}{9604}$	24
derivativedivides	$-\frac{6\,e^{7x}}{2401}+\frac{6\,e^{7x}x}{343}-\frac{3\,e^{7x}x^2}{49}+\frac{e^{7x}x^3}{7}$	33
default	$-\frac{6\,e^{7x}}{2401}+\frac{6\,e^{7x}x}{343}-\frac{3\,e^{7x}x^2}{49}+\frac{e^{7x}x^3}{7}$	33
norman	$-\frac{6\,e^{7x}}{2401}+\frac{6\,e^{7x}x}{343}-\frac{3\,e^{7x}x^2}{49}+\frac{e^{7x}x^3}{7}$	33
parallelrisch	$-\frac{6\,e^{7x}}{2401}+\frac{6\,e^{7x}x}{343}-\frac{3\,e^{7x}x^2}{49}+\frac{e^{7x}x^3}{7}$	33
parts	$-\frac{6\,e^{7x}}{2401}+\frac{6\,e^{7x}x}{343}-\frac{3\,e^{7x}x^2}{49}+\frac{e^{7x}x^3}{7}$	33

input
```
int(exp(7*x)*x^3,x,method=_RETURNVERBOSE)
```

output
```
(1/7*x^3-3/49*x^2+6/343*x-6/2401)*exp(7*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{7x} x^3 \, dx = \frac{1}{2401} \left(343 \, x^3 - 147 \, x^2 + 42 \, x - 6\right) e^{(7 \, x)}$$

input
```
integrate(exp(7*x)*x^3,x, algorithm="fricas")
```

output
```
1/2401*(343*x^3 - 147*x^2 + 42*x - 6)*e^(7*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int e^{7x} x^3 \, dx = \frac{(343x^3 - 147x^2 + 42x - 6) \, e^{7x}}{2401}$$

input
```
integrate(exp(7*x)*x**3,x)
```

output
```
(343*x**3 - 147*x**2 + 42*x - 6)*exp(7*x)/2401
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{7x} x^3 \, dx = \frac{1}{2401} \left(343 \, x^3 - 147 \, x^2 + 42 \, x - 6\right) e^{(7 \, x)}$$

input
```
integrate(exp(7*x)*x^3,x, algorithm="maxima")
```

output
```
1/2401*(343*x^3 - 147*x^2 + 42*x - 6)*e^(7*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{7x}x^3\,dx = \frac{1}{2401}\left(343\,x^3 - 147\,x^2 + 42\,x - 6\right)e^{(7\,x)}$$

input
```
integrate(exp(7*x)*x^3,x, algorithm="giac")
```

output
```
1/2401*(343*x^3 - 147*x^2 + 42*x - 6)*e^(7*x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int e^{7x}x^3\,dx = \frac{e^{7\,x}\,(343\,x^3 - 147\,x^2 + 42\,x - 6)}{2401}$$

input
```
int(x^3*exp(7*x),x)
```

output
```
(exp(7*x)*(42*x - 147*x^2 + 343*x^3 - 6))/2401
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int e^{7x}x^3\,dx = \frac{e^{7x}(343x^3 - 147x^2 + 42x - 6)}{2401}$$

input
```
int(exp(7*x)*x^3,x)
```

output
```
(e**(7*x)*(343*x**3 - 147*x**2 + 42*x - 6))/2401
```

3.631 $\int e^{8-2x} x^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int e^{8-2x} x^3 \, dx = -\frac{3}{8} e^{8-2x} - \frac{3}{4} e^{8-2x} x - \frac{3}{4} e^{8-2x} x^2 - \frac{1}{2} e^{8-2x} x^3$$

output
```
-3/8*exp(8-2*x)-3/4*exp(8-2*x)*x-3/4*exp(8-2*x)*x^2-1/2*exp(8-2*x)*x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.50

$$\int e^{8-2x} x^3 \, dx = -\frac{1}{8} e^{8-2x} \left(3 + 6x + 6x^2 + 4x^3\right)$$

input
```
Integrate[E^(8 - 2*x)*x^3,x]
```

output
```
-1/8*(E^(8 - 2*x)*(3 + 6*x + 6*x^2 + 4*x^3))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{8-2x}x^3\,dx$$

$$\Big\downarrow \ 2607$$

$$\frac{3}{2}\int e^{8-2x}x^2dx-\frac{1}{2}e^{8-2x}x^3$$

$$\Big\downarrow \ 2607$$

$$\frac{3}{2}\left(\int e^{8-2x}xdx-\frac{1}{2}e^{8-2x}x^2\right)-\frac{1}{2}e^{8-2x}x^3$$

$$\Big\downarrow \ 2607$$

$$\frac{3}{2}\left(\frac{1}{2}\int e^{8-2x}dx-\frac{1}{2}e^{8-2x}x^2-\frac{1}{2}e^{8-2x}x\right)-\frac{1}{2}e^{8-2x}x^3$$

$$\Big\downarrow \ 2624$$

$$\frac{3}{2}\left(-\frac{1}{2}e^{8-2x}x^2-\frac{1}{2}e^{8-2x}x-\frac{1}{4}e^{8-2x}\right)-\frac{1}{2}e^{8-2x}x^3$$

input
```
Int[E^(8 - 2*x)*x^3,x]
```

output
```
-1/2*(E^(8 - 2*x)*x^3) + (3*(-1/4*E^(8 - 2*x) - (E^(8 - 2*x)*x)/2 - (E^(8
- 2*x)*x^2)/2))/2
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

method	result	size
risch	$\left(-\frac{1}{2}x^{3}-\frac{3}{4}x^{2}-\frac{3}{4}x-\frac{3}{8}\right)\mathrm{e}^{8-2x}$	23
gosper	$-\frac{\left(4x^{3}+6x^{2}+6x+3\right)\mathrm{e}^{8-2x}}{8}$	24
orering	$-\frac{\left(4x^{3}+6x^{2}+6x+3\right)\mathrm{e}^{8-2x}}{8}$	24
meijerg	$\frac{\mathrm{e}^{8}\left(6-\frac{\left(32x^{3}+48x^{2}+48x+24\right)\mathrm{e}^{-2x}}{4}\right)}{16}$	28
norman	$-\frac{3\,\mathrm{e}^{8-2x}}{8}-\frac{3\,\mathrm{e}^{8-2x}x}{4}-\frac{3\,\mathrm{e}^{8-2x}x^{2}}{4}-\frac{\mathrm{e}^{8-2x}x^{3}}{2}$	41
parallelrisch	$-\frac{3\,\mathrm{e}^{8-2x}}{8}-\frac{3\,\mathrm{e}^{8-2x}x}{4}-\frac{3\,\mathrm{e}^{8-2x}x^{2}}{4}-\frac{\mathrm{e}^{8-2x}x^{3}}{2}$	41
parts	$-\frac{\mathrm{e}^{8-2x}x^{3}}{2}-\frac{3\,\mathrm{e}^{8-2x}\left(8-2x\right)^{2}}{16}+\frac{27\,\mathrm{e}^{8-2x}\left(8-2x\right)}{8}-\frac{123\,\mathrm{e}^{8-2x}}{8}$	49
derivativedivides	$\frac{123\,\mathrm{e}^{8-2x}\left(8-2x\right)}{8}-\frac{379\,\mathrm{e}^{8-2x}}{8}-\frac{27\,\mathrm{e}^{8-2x}\left(8-2x\right)^{2}}{16}+\frac{\mathrm{e}^{8-2x}\left(8-2x\right)^{3}}{16}$	53
default	$\frac{123\,\mathrm{e}^{8-2x}\left(8-2x\right)}{8}-\frac{379\,\mathrm{e}^{8-2x}}{8}-\frac{27\,\mathrm{e}^{8-2x}\left(8-2x\right)^{2}}{16}+\frac{\mathrm{e}^{8-2x}\left(8-2x\right)^{3}}{16}$	53

input
```
int(exp(8-2*x)*x^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^3-3/4*x^2-3/4*x-3/8)*exp(8-2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

$$\int e^{8-2x}x^3\,dx = -\frac{1}{8}\left(4\,x^3+6\,x^2+6\,x+3\right)e^{(-2\,x+8)}$$

input
```
integrate(exp(8-2*x)*x^3,x, algorithm="fricas")
```

output
```
-1/8*(4*x^3 + 6*x^2 + 6*x + 3)*e^(-2*x + 8)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.46

$$\int e^{8-2x}x^3\,dx = \frac{(-4x^3-6x^2-6x-3)\,e^{8-2x}}{8}$$

input
```
integrate(exp(8-2*x)*x**3,x)
```

output
```
(-4*x**3 - 6*x**2 - 6*x - 3)*exp(8 - 2*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int e^{8-2x}x^3\,dx = -\frac{1}{8}\left(4\,x^3e^8+6\,x^2e^8+6\,xe^8+3\,e^8\right)e^{(-2\,x)}$$

input
```
integrate(exp(8-2*x)*x^3,x, algorithm="maxima")
```

output
```
-1/8*(4*x^3*e^8 + 6*x^2*e^8 + 6*x*e^8 + 3*e^8)*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

$$\int e^{8-2x}x^3\,dx = -\frac{1}{8}\left(4\,x^3+6\,x^2+6\,x+3\right)e^{(-2\,x+8)}$$

input
```
integrate(exp(8-2*x)*x^3,x, algorithm="giac")
```

output
```
-1/8*(4*x^3 + 6*x^2 + 6*x + 3)*e^(-2*x + 8)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.44

$$\int e^{8-2x}x^3\,dx = -\mathrm{e}^{8-2\,x}\left(\frac{x^3}{2}+\frac{3\,x^2}{4}+\frac{3\,x}{4}+\frac{3}{8}\right)$$

input
```
int(x^3*exp(8 - 2*x),x)
```

output
```
-exp(8 - 2*x)*((3*x)/4 + (3*x^2)/4 + x^3/2 + 3/8)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

$$\int e^{8-2x}x^3\,dx = \frac{e^8(-4x^3-6x^2-6x-3)}{8e^{2x}}$$

input
```
int(exp(8-2*x)*x^3,x)
```

output
```
(e**8*( - 4*x**3 - 6*x**2 - 6*x - 3))/(8*e**(2*x))
```

3.632 $\int e^x \sqrt{9-e^{2x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 33

$$\int e^x \sqrt{9-e^{2x}}\,dx = \frac{1}{2}e^x\sqrt{9-e^{2x}} + \frac{9}{2}\arcsin\left(\frac{e^x}{3}\right)$$

output
```
1/2*exp(x)*(9-exp(2*x))^(1/2)+9/2*arcsin(1/3*exp(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int e^x \sqrt{9-e^{2x}}\,dx = \frac{1}{2}e^x\sqrt{9-e^{2x}} - 9\arctan\left(\frac{\sqrt{9-e^{2x}}}{3+e^x}\right)$$

input
```
Integrate[E^x*Sqrt[9 - E^(2*x)],x]
```

output
```
(E^x*Sqrt[9 - E^(2*x)])/2 - 9*ArcTan[Sqrt[9 - E^(2*x)]/(3 + E^x)]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2679, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \sqrt{9-e^{2x}}\,dx$$

$$\Big\downarrow \text{ 2679}$$

$$\int \sqrt{9-e^{2x}}de^x$$

$$\Big\downarrow \text{ 211}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{9-e^{2x}}}de^x+\frac{1}{2}e^x\sqrt{9-e^{2x}}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{9}{2}\arcsin\left(\frac{e^x}{3}\right)+\frac{1}{2}e^x\sqrt{9-e^{2x}}$$

input
```
Int[E^x*Sqrt[9 - E^(2*x)],x]
```

output
```
(E^x*Sqrt[9 - E^(2*x)])/2 + (9*ArcSin[E^x/3])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))    Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{e^{x}\sqrt{9-e^{2x}}}{2}+\frac{9\arcsin\left(\frac{e^{x}}{3}\right)}{2}$	23
risch	$-\frac{e^{x}\left(-9+e^{2x}\right)}{2\sqrt{9-e^{2x}}}+\frac{9\arcsin\left(\frac{e^{x}}{3}\right)}{2}$	29

input
```
int(exp(x)*(9-exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)*(9-exp(x)^2)^(1/2)+9/2*arcsin(1/3*exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int e^{x}\sqrt{9-e^{2x}}\,dx=\frac{1}{2}\,\sqrt{-e^{(2\,x)}+9}e^{x}-9\,\arctan\left(\left(\sqrt{-e^{(2\,x)}+9}-3\right)e^{(-x)}\right)$$

input
```
integrate(exp(x)*(9-exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-e^(2*x) + 9)*e^x - 9*arctan((sqrt(-e^(2*x) + 9) - 3)*e^(-x))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

$$\int e^x\sqrt{9-e^{2x}}\,dx = \frac{\sqrt{9-e^{2x}}e^x}{2} + \frac{9\,\mathrm{asin}\left(\frac{e^x}{3}\right)}{2}$$

input
```
integrate(exp(x)*(9-exp(2*x))**(1/2),x)
```

output
```
sqrt(9 - exp(2*x))*exp(x)/2 + 9*asin(exp(x)/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int e^x\sqrt{9-e^{2x}}\,dx = \frac{1}{2}\sqrt{-e^{(2x)}+9}e^x + \frac{9}{2}\arcsin\left(\frac{1}{3}e^x\right)$$

input
```
integrate(exp(x)*(9-exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-e^(2*x) + 9)*e^x + 9/2*arcsin(1/3*e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int e^x\sqrt{9-e^{2x}}\,dx = \frac{1}{2}\sqrt{-e^{(2x)}+9}e^x + \frac{9}{2}\arcsin\left(\frac{1}{3}e^x\right)$$

input
```
integrate(exp(x)*(9-exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-e^(2*x) + 9)*e^x + 9/2*arcsin(1/3*e^x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

$$\int e^x \sqrt{9-e^{2x}}\,dx = \frac{9\,\mathrm{asin}\left(\frac{\mathrm{e}^x}{3}\right)}{2} + \frac{\mathrm{e}^x\,\sqrt{9-\mathrm{e}^{2\,x}}}{2}$$

input
```
int(exp(x)*(9 - exp(2*x))^(1/2),x)
```

output
```
(9*asin(exp(x)/3))/2 + (exp(x)*(9 - exp(2*x))^(1/2))/2
```

Reduce [F]

$$\int e^x \sqrt{9-e^{2x}}\,dx = \frac{e^x\sqrt{-e^{2x}+9}}{2} - \frac{9\left(\int \frac{e^x\sqrt{-e^{2x}+9}}{e^{2x}-9}dx\right)}{2}$$

input
```
int(exp(x)*(9-exp(2*x))^(1/2),x)
```

output
```
(e**x*sqrt( - e**(2*x) + 9) - 9*int((e**x*sqrt( - e**(2*x) + 9))/(e**(2*x)
 - 9),x))/2
```

3.633 $\int e^{6x}\sqrt{9-e^{2x}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx=-27\left(9-e^{2x}\right)^{3/2}+\frac{18}{5}\left(9-e^{2x}\right)^{5/2}-\frac{1}{7}\left(9-e^{2x}\right)^{7/2}$$

output
```
-27*(9-exp(2*x))^(3/2)+18/5*(9-exp(2*x))^(5/2)-1/7*(9-exp(2*x))^(7/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx=-\frac{1}{35}\left(9-e^{2x}\right)^{3/2}\left(216+36e^{2x}+5e^{4x}\right)$$

input
```
Integrate[E^(6*x)*Sqrt[9 - E^(2*x)],x]
```

output
```
-1/35*((9 - E^(2*x))^(3/2)*(216 + 36*E^(2*x) + 5*E^(4*x)))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{1}{2}\int e^{4x}\sqrt{9-e^{2x}}de^{2x}$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\left(9-e^{2x}\right)^{5/2}-18\left(9-e^{2x}\right)^{3/2}+81\sqrt{9-e^{2x}}\right)de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2}{7}\left(9-e^{2x}\right)^{7/2}+\frac{36}{5}\left(9-e^{2x}\right)^{5/2}-54\left(9-e^{2x}\right)^{3/2}\right)$$

input
```
Int[E^(6*x)*Sqrt[9 - E^(2*x)],x]
```

output
```
(-54*(9 - E^(2*x))^(3/2) + (36*(9 - E^(2*x))^(5/2))/5 - (2*(9 - E^(2*x))^(
7/2))/7)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{\left(5\,{\mathrm e}^{6x}-9\,{\mathrm e}^{4x}-108\,{\mathrm e}^{2x}-1944\right)\left(-9+{\mathrm e}^{2x}\right)}{35\sqrt{9-{\mathrm e}^{2x}}}$	39
default	$-\frac{{\mathrm e}^{4x}\left(9-{\mathrm e}^{2x}\right)^{\frac{3}{2}}}{7}-\frac{36\,{\mathrm e}^{2x}\left(9-{\mathrm e}^{2x}\right)^{\frac{3}{2}}}{35}-\frac{216\left(9-{\mathrm e}^{2x}\right)^{\frac{3}{2}}}{35}$	46

input
```
int(exp(6*x)*(9-exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(5*exp(6*x)-9*exp(4*x)-108*exp(2*x)-1944)*(-9+exp(2*x))/(9-exp(2*x))
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx = \frac{1}{35}\left(5\,e^{(6\,x)}-9\,e^{(4\,x)}-108\,e^{(2\,x)}-1944\right)\sqrt{-e^{(2\,x)}+9}$$

input
```
integrate(exp(6*x)*(9-exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/35*(5*e^(6*x) - 9*e^(4*x) - 108*e^(2*x) - 1944)*sqrt(-e^(2*x) + 9)
```

Sympy [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx = -\frac{(9-e^{2x})^{\frac{7}{2}}}{7} + \frac{18(9-e^{2x})^{\frac{5}{2}}}{5} - 27(9-e^{2x})^{\frac{3}{2}}$$

input
```
integrate(exp(6*x)*(9-exp(2*x))**(1/2),x)
```

output
```
-(9 - exp(2*x))**(7/2)/7 + 18*(9 - exp(2*x))**(5/2)/5 - 27*(9 - exp(2*x))*
*(3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx = -\frac{1}{7}\left(-e^{(2x)}+9\right)^{\frac{7}{2}} + \frac{18}{5}\left(-e^{(2x)}+9\right)^{\frac{5}{2}} - 27\left(-e^{(2x)}+9\right)^{\frac{3}{2}}$$

input
```
integrate(exp(6*x)*(9-exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
-1/7*(-e^(2*x) + 9)^(7/2) + 18/5*(-e^(2*x) + 9)^(5/2) - 27*(-e^(2*x) + 9)^
(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx = \frac{1}{7}\left(e^{(2x)}-9\right)^{3}\sqrt{-e^{(2x)}+9} + \frac{18}{5}\left(e^{(2x)}-9\right)^{2}\sqrt{-e^{(2x)}+9} - 27\left(-e^{(2x)}+9\right)^{\frac{3}{2}}$$

input
```
integrate(exp(6*x)*(9-exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/7*(e^(2*x) - 9)^3*sqrt(-e^(2*x) + 9) + 18/5*(e^(2*x) - 9)^2*sqrt(-e^(2*x
) + 9) - 27*(-e^(2*x) + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx = -\sqrt{9-\mathrm{e}^{2\,x}}\,\left(\frac{108\,\mathrm{e}^{2\,x}}{35}+\frac{9\,\mathrm{e}^{4\,x}}{35}-\frac{\mathrm{e}^{6\,x}}{7}+\frac{1944}{35}\right)$$

input
```
int(exp(6*x)*(9 - exp(2*x))^(1/2),x)
```

output
```
-(9 - exp(2*x))^(1/2)*((108*exp(2*x))/35 + (9*exp(4*x))/35 - exp(6*x)/7 +
1944/35)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int e^{6x}\sqrt{9-e^{2x}}\,dx = \frac{\sqrt{-e^{2x}+9}\,(5e^{6x}-9e^{4x}-108e^{2x}-1944)}{35}$$

input
```
int(exp(6*x)*(9-exp(2*x))^(1/2),x)
```

output
```
(sqrt( - e**(2*x) + 9)*(5*e**(6*x) - 9*e**(4*x) - 108*e**(2*x) - 1944))/35
```

3.634 $\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = \frac{39366}{(9-e^x)^{3/2}} - \frac{65610}{\sqrt{9-e^x}} - 14580\sqrt{9-e^x} + 540(9-e^x)^{3/2} - 18(9-e^x)^{5/2} + \frac{2}{7}(9-e^x)^{7/2}$$

output
```
39366/(9-exp(x))^(3/2)-65610/(9-exp(x))^(1/2)-14580*(9-exp(x))^(1/2)+540*(
9-exp(x))^(3/2)-18*(9-exp(x))^(5/2)+2/7*(9-exp(x))^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = -\frac{2(5038848 - 839808e^x + 23328e^{2x} + 432e^{3x} + 18e^{4x} + e^{5x})}{7\,(9-e^x)^{3/2}}$$

input
```
Integrate[E^(6*x)/(9 - E^x)^(5/2),x]
```

output
```
(-2*(5038848 - 839808*E^x + 23328*E^(2*x) + 432*E^(3*x) + 18*E^(4*x) + E^(
5*x)))/(7*(9 - E^x)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^{5x}}{(9-e^x)^{5/2}}de^x$$

$$\Big\downarrow \text{53}$$

$$\int \left(-(9-e^x)^{5/2}+45(9-e^x)^{3/2}-810\sqrt{9-e^x}+\frac{7290}{\sqrt{9-e^x}}-\frac{32805}{(9-e^x)^{3/2}}+\frac{59049}{(9-e^x)^{5/2}}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}(9-e^x)^{7/2}-18(9-e^x)^{5/2}+540(9-e^x)^{3/2}-14580\sqrt{9-e^x}-\frac{65610}{\sqrt{9-e^x}}+\frac{39366}{(9-e^x)^{3/2}}$$

input
```
Int[E^(6*x)/(9 - E^x)^(5/2),x]
```

output
```
39366/(9 - E^x)^(3/2) - 65610/Sqrt[9 - E^x] - 14580*Sqrt[9 - E^x] + 540*(9
 - E^x)^(3/2) - 18*(9 - E^x)^(5/2) + (2*(9 - E^x)^(7/2))/7
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{39366}{(9-e^{x})^{\frac{3}{2}}}-\frac{65610}{\sqrt{9-e^{x}}}-14580\sqrt{9-e^{x}}+540(9-e^{x})^{\frac{3}{2}}-18(9-e^{x})^{\frac{5}{2}}+\frac{2(9-e^{x})^{\frac{7}{2}}}{7}$	62

input
```
int(exp(6*x)/(9-exp(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
39366/(9-exp(x))^(3/2)-65610/(9-exp(x))^(1/2)-14580*(9-exp(x))^(1/2)+540*(
9-exp(x))^(3/2)-18*(9-exp(x))^(5/2)+2/7*(9-exp(x))^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = -\frac{2\left(e^{(5x)}+18\,e^{(4x)}+432\,e^{(3x)}+23328\,e^{(2x)}-839808\,e^{x}+5038848\right)\sqrt{-e^{x}+9}}{7\left(e^{(2x)}-18\,e^{x}+81\right)}$$

input
```
integrate(exp(6*x)/(9-exp(x))^(5/2),x, algorithm="fricas")
```

output
```
-2/7*(e^(5*x) + 18*e^(4*x) + 432*e^(3*x) + 23328*e^(2*x) - 839808*e^x + 50
38848)*sqrt(-e^x + 9)/(e^(2*x) - 18*e^x + 81)
```

Sympy [A] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = \frac{2(9-e^x)^{\frac{7}{2}}}{7} - 18(9-e^x)^{\frac{5}{2}} + 540(9-e^x)^{\frac{3}{2}} - 14580\sqrt{9-e^x} - \frac{65610}{\sqrt{9-e^x}} + \frac{39366}{(9-e^x)^{\frac{3}{2}}}$$

input
```
integrate(exp(6*x)/(9-exp(x))**(5/2),x)
```

output
```
2*(9 - exp(x))**(7/2)/7 - 18*(9 - exp(x))**(5/2) + 540*(9 - exp(x))**(3/2)
 - 14580*sqrt(9 - exp(x)) - 65610/sqrt(9 - exp(x)) + 39366/(9 - exp(x))**(
3/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = \frac{2}{7}\left(-e^x+9\right)^{\frac{7}{2}} - 18\left(-e^x+9\right)^{\frac{5}{2}} + 540\left(-e^x+9\right)^{\frac{3}{2}} - 14580\sqrt{-e^x+9} - \frac{65610}{\sqrt{-e^x+9}} + \frac{39366}{\left(-e^x+9\right)^{\frac{3}{2}}}$$

input
```
integrate(exp(6*x)/(9-exp(x))^(5/2),x, algorithm="maxima")
```

output
```
2/7*(-e^x + 9)^(7/2) - 18*(-e^x + 9)^(5/2) + 540*(-e^x + 9)^(3/2) - 14580*
sqrt(-e^x + 9) - 65610/sqrt(-e^x + 9) + 39366/(-e^x + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = -\frac{2}{7}\left(e^x-9\right)^3\sqrt{-e^x+9} - 18\left(e^x-9\right)^2\sqrt{-e^x+9} + 540\left(-e^x+9\right)^{\frac{3}{2}} - 14580\sqrt{-e^x+9} - \frac{13122\left(5e^x-42\right)}{\left(e^x-9\right)\sqrt{-e^x+9}}$$

input
```
integrate(exp(6*x)/(9-exp(x))^(5/2),x, algorithm="giac")
```

output
```
-2/7*(e^x - 9)^3*sqrt(-e^x + 9) - 18*(e^x - 9)^2*sqrt(-e^x + 9) + 540*(-e^
x + 9)^(3/2) - 14580*sqrt(-e^x + 9) - 13122*(5*e^x - 42)/((e^x - 9)*sqrt(-
e^x + 9))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.47

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = -\frac{2\left(23328\,e^{2x} + 432\,e^{3x} + 18\,e^{4x} + e^{5x} - 839808\,e^x + 5038848\right)}{7\left(9-e^x\right)^{3/2}}$$

input
```
int(exp(6*x)/(9 - exp(x))^(5/2),x)
```

output
```
-(2*(23328*exp(2*x) + 432*exp(3*x) + 18*exp(4*x) + exp(5*x) - 839808*exp(x
) + 5038848))/(7*(9 - exp(x))^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{e^{6x}}{(9-e^x)^{5/2}}\,dx = \frac{2\sqrt{-e^x+9}\left(-e^{5x} - 18e^{4x} - 432e^{3x} - 23328e^{2x} + 839808e^x - 5038848\right)}{7e^{2x} - 126e^x + 567}$$

input
```
int(exp(6*x)/(9-exp(x))^(5/2),x)
```

output
```
(2*sqrt( - e**x + 9)*( - e**(5*x) - 18*e**(4*x) - 432*e**(3*x) - 23328*e**
(2*x) + 839808*e**x - 5038848))/(7*(e**(2*x) - 18*e**x + 81))
```

3.635 $\int \left(2-7e^{x^4}\right)^5 x^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx=-140e^{x^4}+490e^{2x^4}-\frac{3430e^{3x^4}}{3}+\frac{12005e^{4x^4}}{8}-\frac{16807e^{5x^4}}{20}+8x^4$$

output
```
-140*exp(x^4)+490*exp(2*x^4)-3430/3*exp(3*x^4)+12005/8*exp(4*x^4)-16807/20
*exp(5*x^4)+8*x^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx=-\frac{7}{120}e^{x^4}\left(2400-8400e^{x^4}+19600e^{2x^4}-25725e^{3x^4}+14406e^{4x^4}\right)+8\log\left(e^{x^4}\right)$$

input
```
Integrate[(2 - 7*E^x^4)^5*x^3,x]
```

output
```
(-7*E^x^4*(2400 - 8400*E^x^4 + 19600*E^(2*x^4) - 25725*E^(3*x^4) + 14406*E
^(4*x^4)))/120 + 8*Log[E^x^4]
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {7266, 2720, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2-7 e^{x^4}\right)^5 x^3 \, dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{4} \int \left(2-7 e^{x^4}\right)^5 dx^4$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{4} \int \frac{\left(2-7 e^{x^4}\right)^5}{x^4}de^{x^4}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4} \int \left(-16807 x^{16}+24010 x^{12}-13720 x^8+3920 e^{x^4}-560+\frac{32}{x^4}\right)de^{x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{16807 x^{20}}{5}+\frac{12005 x^{16}}{2}-\frac{13720 x^{12}}{3}+1960 x^8-560 e^{x^4}+32 \log \left(e^{x^4}\right)\right)$$

input
```
Int[(2 - 7*E^x^4)^5*x^3,x]
```

output
```
(-560*E^x^4 + 1960*x^8 - (13720*x^12)/3 + (12005*x^16)/2 - (16807*x^20)/5
+ 32*Log[E^x^4])/4
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
norman	$-140\,\mathrm{e}^{x^4}+490\,\mathrm{e}^{2x^4}-\frac{3430\,\mathrm{e}^{3x^4}}{3}+\frac{12005\,\mathrm{e}^{4x^4}}{8}-\frac{16807\,\mathrm{e}^{5x^4}}{20}+8x^4$	45
risch	$-140\,\mathrm{e}^{x^4}+490\,\mathrm{e}^{2x^4}-\frac{3430\,\mathrm{e}^{3x^4}}{3}+\frac{12005\,\mathrm{e}^{4x^4}}{8}-\frac{16807\,\mathrm{e}^{5x^4}}{20}+8x^4$	45
parallelrisch	$-140\,\mathrm{e}^{x^4}+490\,\mathrm{e}^{2x^4}-\frac{3430\,\mathrm{e}^{3x^4}}{3}+\frac{12005\,\mathrm{e}^{4x^4}}{8}-\frac{16807\,\mathrm{e}^{5x^4}}{20}+8x^4$	45
parts	$-140\,\mathrm{e}^{x^4}+490\,\mathrm{e}^{2x^4}-\frac{3430\,\mathrm{e}^{3x^4}}{3}+\frac{12005\,\mathrm{e}^{4x^4}}{8}-\frac{16807\,\mathrm{e}^{5x^4}}{20}+8x^4$	45
derivativedivides	$-\frac{16807\,\mathrm{e}^{5x^4}}{20}+\frac{12005\,\mathrm{e}^{4x^4}}{8}-\frac{3430\,\mathrm{e}^{3x^4}}{3}+490\,\mathrm{e}^{2x^4}-140\,\mathrm{e}^{x^4}+8\ln\left(\mathrm{e}^{x^4}\right)$	47
default	$-\frac{16807\,\mathrm{e}^{5x^4}}{20}+\frac{12005\,\mathrm{e}^{4x^4}}{8}-\frac{3430\,\mathrm{e}^{3x^4}}{3}+490\,\mathrm{e}^{2x^4}-140\,\mathrm{e}^{x^4}+8\ln\left(\mathrm{e}^{x^4}\right)$	47
orering	Expression too large to display	861

input
```
int((2-7*exp(x^4))^5*x^3,x,method=_RETURNVERBOSE)
```

output
```
8*x^4+490*exp(x^4)^2-3430/3*exp(x^4)^3+12005/8*exp(x^4)^4-16807/20*exp(x^4
)^5-140*exp(x^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx = 8\,x^4 - \frac{16807}{20}\,e^{\left(5\,x^4\right)} + \frac{12005}{8}\,e^{\left(4\,x^4\right)} - \frac{3430}{3}\,e^{\left(3\,x^4\right)} + 490\,e^{\left(2\,x^4\right)} - 140\,e^{\left(x^4\right)}$$

input
```
integrate((2-7*exp(x^4))^5*x^3,x, algorithm="fricas")
```

output
```
8*x^4 - 16807/20*e^(5*x^4) + 12005/8*e^(4*x^4) - 3430/3*e^(3*x^4) + 490*e^
(2*x^4) - 140*e^(x^4)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx = 8x^4 - \frac{16807e^{5x^4}}{20} + \frac{12005e^{4x^4}}{8} - \frac{3430e^{3x^4}}{3} + 490e^{2x^4} - 140e^{x^4}$$

input
```
integrate((2-7*exp(x**4))**5*x**3,x)
```

output
```
8*x**4 - 16807*exp(5*x**4)/20 + 12005*exp(4*x**4)/8 - 3430*exp(3*x**4)/3 +
 490*exp(2*x**4) - 140*exp(x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx = 8\,x^4 - \frac{16807}{20}\,e^{\left(5\,x^4\right)} + \frac{12005}{8}\,e^{\left(4\,x^4\right)} - \frac{3430}{3}\,e^{\left(3\,x^4\right)} + 490\,e^{\left(2\,x^4\right)} - 140\,e^{\left(x^4\right)}$$

input
```
integrate((2-7*exp(x^4))^5*x^3,x, algorithm="maxima")
```

output
```
8*x^4 - 16807/20*e^(5*x^4) + 12005/8*e^(4*x^4) - 3430/3*e^(3*x^4) + 490*e^
(2*x^4) - 140*e^(x^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx = 8\,x^4 - \frac{16807}{20}\,e^{\left(5\,x^4\right)} + \frac{12005}{8}\,e^{\left(4\,x^4\right)} - \frac{3430}{3}\,e^{\left(3\,x^4\right)} + 490\,e^{\left(2\,x^4\right)} - 140\,e^{\left(x^4\right)}$$

input
```
integrate((2-7*exp(x^4))^5*x^3,x, algorithm="giac")
```

output
```
8*x^4 - 16807/20*e^(5*x^4) + 12005/8*e^(4*x^4) - 3430/3*e^(3*x^4) + 490*e^
(2*x^4) - 140*e^(x^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx = 490\,\mathrm{e}^{2\,x^4} - 140\,\mathrm{e}^{x^4} - \frac{3430\,\mathrm{e}^{3\,x^4}}{3} + \frac{12005\,\mathrm{e}^{4\,x^4}}{8} - \frac{16807\,\mathrm{e}^{5\,x^4}}{20} + 8\,x^4$$

input
```
int(-x^3*(7*exp(x^4) - 2)^5,x)
```

output
```
490*exp(2*x^4) - 140*exp(x^4) - (3430*exp(3*x^4))/3 + (12005*exp(4*x^4))/8
 - (16807*exp(5*x^4))/20 + 8*x^4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \left(2-7e^{x^4}\right)^5 x^3\,dx = -\frac{16807e^{5x^4}}{20}+\frac{12005e^{4x^4}}{8}-\frac{3430e^{3x^4}}{3}+490e^{2x^4}-140e^{x^4}+8x^4$$

input
```
int((2-7*exp(x^4))^5*x^3,x)
```

output
```
( - 100842*e**(5*x**4) + 180075*e**(4*x**4) - 137200*e**(3*x**4) + 58800*e
**(2*x**4) - 16800*e**(x**4) + 960*x**4)/120
```

3.636 $\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx=\frac{1}{4}e^{x^2}\sqrt{1-e^{2x^2}}+\frac{1}{4}\arcsin\left(e^{x^2}\right)$$

output
```
1/4*exp(x^2)*(1-exp(2*x^2))^(1/2)+1/4*arcsin(exp(x^2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx=\frac{1}{4}e^{x^2}\sqrt{1-e^{2x^2}}-\frac{1}{2}\arctan\left(\frac{\sqrt{1-e^{2x^2}}}{1+e^{x^2}}\right)$$

input
```
Integrate[E^x^2*Sqrt[1 - E^(2*x^2)]*x,x]
```

output
```
(E^x^2*Sqrt[1 - E^(2*x^2)])/4 - ArcTan[Sqrt[1 - E^(2*x^2)]/(1 + E^x^2)]/2
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {7266, 2679, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int e^{x^2}\sqrt{1-e^{2x^2}}dx^2$$

$$\Big\downarrow \text{2679}$$

$$\frac{1}{2}\int \sqrt{1-x^4}de^{x^2}$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x^4}}de^{x^2}+\frac{1}{2}e^{x^2}\sqrt{1-x^4}\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(\frac{1}{2}\arcsin\left(e^{x^2}\right)+\frac{1}{2}e^{x^2}\sqrt{1-x^4}\right)$$

input
```
Int[E^x^2*Sqrt[1 - E^(2*x^2)]*x,x]
```

output
```
((E^x^2*Sqrt[1 - x^4])/2 + ArcSin[E^x^2]/2)/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{e^{x^2}\sqrt{1-e^{2x^2}}}{4}+\frac{\arcsin\left(e^{x^2}\right)}{4}$	27
default	$\frac{e^{x^2}\sqrt{1-e^{2x^2}}}{4}+\frac{\arcsin\left(e^{x^2}\right)}{4}$	27
risch	$-\frac{e^{x^2}\left(-1+e^{2x^2}\right)}{4\sqrt{1-e^{2x^2}}}+\frac{\arcsin\left(e^{x^2}\right)}{4}$	35

input
```
int(exp(x^2)*(1-exp(2*x^2))^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(x^2)*(1-exp(x^2)^2)^(1/2)+1/4*arcsin(exp(x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx = \frac{1}{4}\sqrt{-e^{(2x^2)}+1}e^{(x^2)} - \frac{1}{2}\arctan\left(\left(\sqrt{-e^{(2x^2)}+1}-1\right)e^{(-x^2)}\right)$$

input
```
integrate(exp(x^2)*(1-exp(2*x^2))^(1/2)*x,x, algorithm="fricas")
```

output
```
1/4*sqrt(-e^(2*x^2) + 1)*e^(x^2) - 1/2*arctan((sqrt(-e^(2*x^2) + 1) - 1)*e
^(-x^2))
```

Sympy [A] (verification not implemented)

Time = 15.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx = \frac{\sqrt{1-e^{2x^2}}e^{x^2}}{4} + \frac{\operatorname{asin}\left(e^{x^2}\right)}{4}$$

input
```
integrate(exp(x**2)*(1-exp(2*x**2))**(1/2)*x,x)
```

output
```
sqrt(1 - exp(2*x**2))*exp(x**2)/4 + asin(exp(x**2))/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx = \frac{1}{4}\sqrt{-e^{(2x^2)}+1}e^{(x^2)} + \frac{1}{4}\arcsin\left(e^{(x^2)}\right)$$

input
```
integrate(exp(x^2)*(1-exp(2*x^2))^(1/2)*x,x, algorithm="maxima")
```

output
```
1/4*sqrt(-e^(2*x^2) + 1)*e^(x^2) + 1/4*arcsin(e^(x^2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx = \frac{1}{4}\sqrt{-e^{(2x^2)}+1}e^{(x^2)} + \frac{1}{4}\arcsin\left(e^{(x^2)}\right)$$

input
```
integrate(exp(x^2)*(1-exp(2*x^2))^(1/2)*x,x, algorithm="giac")
```

output
```
1/4*sqrt(-e^(2*x^2) + 1)*e^(x^2) + 1/4*arcsin(e^(x^2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx = \frac{\mathrm{asin}\left(\mathrm{e}^{x^2}\right)}{4} + \frac{\mathrm{e}^{x^2}\sqrt{1-\mathrm{e}^{2x^2}}}{4}$$

input
```
int(x*exp(x^2)*(1 - exp(2*x^2))^(1/2),x)
```

output
```
asin(exp(x^2))/4 + (exp(x^2)*(1 - exp(2*x^2))^(1/2))/4
```

Reduce [F]

$$\int e^{x^2}\sqrt{1-e^{2x^2}}x\,dx = \frac{e^{x^2}\sqrt{-e^{2x^2}+1}}{4} - \frac{\left(\int \frac{e^{x^2}\sqrt{-e^{2x^2}+1}\,x}{e^{2x^2}-1}dx\right)}{2}$$

input
```
int(exp(x^2)*(1-exp(2*x^2))^(1/2)*x,x)
```

output
```
(e**(x**2)*sqrt( - e**(2*x**2) + 1) - 2*int((e**(x**2)*sqrt( - e**(2*x**2)
 + 1)*x)/(e**(2*x**2) - 1),x))/4
```

3.637 $\int e^{x^3} \left(1-e^{4x^3}\right)^2 x^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 32

$$\int e^{x^3} \left(1-e^{4x^3}\right)^2 x^2 \, dx = \frac{e^{x^3}}{3} - \frac{2e^{5x^3}}{15} + \frac{e^{9x^3}}{27}$$

output
```
1/3*exp(x^3)-2/15*exp(5*x^3)+1/27*exp(9*x^3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{x^3} \left(1-e^{4x^3}\right)^2 x^2 \, dx = \frac{1}{135} e^{x^3} \left(45-18e^{4x^3}+5e^{8x^3}\right)$$

input
```
Integrate[E^x^3*(1 - E^(4*x^3))^2*x^2,x]
```

output
```
(E^x^3*(45 - 18*E^(4*x^3) + 5*E^(8*x^3)))/135
```

Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {7266, 2679, 747, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2 x^2\,dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{3}\int e^{x^3}\left(1-e^{4x^3}\right)^2 dx^3$$

$$\Big\downarrow \text{2679}$$

$$\frac{1}{3}\int \left(1-x^{12}\right)^2 de^{x^3}$$

$$\Big\downarrow \text{747}$$

$$\frac{1}{3}\int \left(x^{24}-2x^{12}+1\right) de^{x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{x^{27}}{9}-\frac{2x^{15}}{5}+e^{x^3}\right)$$

input
```
Int[E^x^3*(1 - E^(4*x^3))^2*x^2,x]
```

output
```
(E^x^3 - (2*x^15)/5 + x^27/9)/3
```

Defintions of rubi rules used

rule 747
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{e^{x^3}}{3}-\frac{2\,e^{5x^3}}{15}+\frac{e^{9x^3}}{27}$
default	$\frac{e^{x^3}}{3}-\frac{2\,e^{5x^3}}{15}+\frac{e^{9x^3}}{27}$
risch	$\frac{e^{x^3}}{3}-\frac{2\,e^{5x^3}}{15}+\frac{e^{9x^3}}{27}$
meijerg	$-\frac{32}{135}+\frac{e^{x^3}}{3}-\frac{2\,e^{5x^3}}{15}+\frac{e^{9x^3}}{27}$
parallelrisch	$\frac{e^{8x^3}e^{x^3}}{27}-\frac{2\,e^{x^3}e^{4x^3}}{15}+\frac{e^{x^3}}{3}$
orering	$\frac{(531x^6+90x^3+10)e^{x^3}\left(1-e^{4x^3}\right)^2}{1215x^6}-\frac{(15x^3+2)\left(3x^4e^{x^3}\left(1-e^{4x^3}\right)^2-24\,e^{x^3}\left(1-e^{4x^3}\right)x^4e^{4x^3}+2\,e^{x^3}\left(1-e^{4x^3}\right)^2x\right)}{405x^7}$

input
```
int(exp(x^3)*(1-exp(4*x^3))^2*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/27*exp(x^3)^9-2/15*exp(x^3)^5+1/3*exp(x^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2 x^2\,dx = \frac{1}{27}\,e^{\left(9\,x^3\right)} - \frac{2}{15}\,e^{\left(5\,x^3\right)} + \frac{1}{3}\,e^{\left(x^3\right)}$$

input
```
integrate(exp(x^3)*(1-exp(4*x^3))^2*x^2,x, algorithm="fricas")
```

output
```
1/27*e^(9*x^3) - 2/15*e^(5*x^3) + 1/3*e^(x^3)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2 x^2\,dx = \frac{e^{9x^3}}{27} - \frac{2e^{5x^3}}{15} + \frac{e^{x^3}}{3}$$

input
```
integrate(exp(x**3)*(1-exp(4*x**3))**2*x**2,x)
```

output
```
exp(9*x**3)/27 - 2*exp(5*x**3)/15 + exp(x**3)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2 x^2\,dx = \frac{1}{27}\,e^{\left(9\,x^3\right)} - \frac{2}{15}\,e^{\left(5\,x^3\right)} + \frac{1}{3}\,e^{\left(x^3\right)}$$

input
```
integrate(exp(x^3)*(1-exp(4*x^3))^2*x^2,x, algorithm="maxima")
```

output
```
1/27*e^(9*x^3) - 2/15*e^(5*x^3) + 1/3*e^(x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2x^2\,dx=\frac{1}{27}\,e^{\left(9\,x^3\right)}-\frac{2}{15}\,e^{\left(5\,x^3\right)}+\frac{1}{3}\,e^{\left(x^3\right)}$$

input
```
integrate(exp(x^3)*(1-exp(4*x^3))^2*x^2,x, algorithm="giac")
```

output
```
1/27*e^(9*x^3) - 2/15*e^(5*x^3) + 1/3*e^(x^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2x^2\,dx=\frac{\mathrm{e}^{x^3}\,\left(5\,\mathrm{e}^{8\,x^3}-18\,\mathrm{e}^{4\,x^3}+45\right)}{135}$$

input
```
int(x^2*exp(x^3)*(exp(4*x^3) - 1)^2,x)
```

output
```
(exp(x^3)*(5*exp(8*x^3) - 18*exp(4*x^3) + 45))/135
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int e^{x^3}\left(1-e^{4x^3}\right)^2x^2\,dx=\frac{e^{x^3}\left(5e^{8x^3}-18e^{4x^3}+45\right)}{135}$$

input
```
int(exp(x^3)*(1-exp(4*x^3))^2*x^2,x)
```

output
```
(e**(x**3)*(5*e**(8*x**3) - 18*e**(4*x**3) + 45))/135
```

3.638 $\int e^{e^x+x} \, dx$

Optimal result

Integrand size = 7, antiderivative size = 5

$$\int e^{e^x+x} \, dx = e^{e^x}$$

output
```
exp(exp(1)^x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int e^{e^x+x} \, dx = e^{e^x}$$

input
```
Integrate[E^(E^x + x),x]
```

output
```
E^E^x
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2720, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x+e^x} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^{e^x} de^x$$

$$\Big\downarrow \text{ 2624}$$

$$e^{e^x}$$

input
```
Int[E^(E^x + x),x]
```

output
```
E^E^x
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.80

method	result	size
default	e^{e^x}	4
risch	e^{e^x}	4

input
```
int(exp(exp(1)^x+x),x,method=_RETURNVERBOSE)
```

output
```
exp(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{(e^x)}$$

input
```
integrate(exp(exp(1)^x+x),x, algorithm="fricas")
```

output
```
e^(e^x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{e^x}$$

input
```
integrate(exp(exp(1)**x+x),x)
```

output
```
exp(exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{(e^x)}$$

input
```
integrate(exp(exp(1)^x+x),x, algorithm="maxima")
```

output
```
e^(e^x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = e^{(e^x)}$$

input
```
integrate(exp(exp(1)^x+x),x, algorithm="giac")
```

output
```
e^(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int e^{e^x+x}\,dx = \mathrm{e}^{\mathrm{e}^x}$$

input
```
int(exp(x + exp(x)),x)
```

output
```
exp(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int e^{e^x+x}\,dx=e^{e^x}$$

input
```
int(exp(exp(1)^x+x),x)
```

output
```
e**(e**x)
```

3.639 $\int e^{e^{e^x}+e^x+x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 7

$$\int e^{e^{e^x}+e^x+x} \, dx = e^{e^{e^x}}$$

output
```
exp(exp(1)^(exp(1)^x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^{e^{e^x}+e^x+x} \, dx = e^{e^{e^x}}$$

input
```
Integrate[E^(E^E^x + E^x + x),x]
```

output
```
E^E^E^x
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 2720, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x+e^{e^x}+e^x} \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{e^{e^x}+e^x} de^x$$

$$\Big\downarrow \text{2720}$$

$$\int e^{e^{e^x}} de^{e^x}$$

$$\Big\downarrow \text{2624}$$

$$e^{e^{e^x}}$$

input
```
Int[E^(E^E^x + E^x + x),x]
```

output
```
E^E^E^x
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

method	result	size
default	$e^{e^{e^x}}$	5
risch	$e^{e^{e^x}}$	5

input
```
int(exp(exp(1)^(exp(1)^x)+exp(1)^x+x),x,method=_RETURNVERBOSE)
```

output
```
exp(exp(exp(x)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.57

$$\int e^{e^{e^x}+e^x+x}\,dx = e^{\left(e^{\left(e^x\right)}\right)}$$

input
```
integrate(exp(exp(1)^(exp(1)^x)+exp(1)^x+x),x, algorithm="fricas")
```

output
```
e^(e^(e^x))
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int e^{e^{e^x}+e^x+x}\,dx = e^{e^{e^x}}$$

input
```
integrate(exp(exp(1)**(exp(1)**x)+exp(1)**x+x),x)
```

output
```
exp(exp(exp(x)))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.57

$$\int e^{e^{e^x}+e^x+x}\,dx = e^{\left(e^{\left(e^x\right)}\right)}$$

input
```
integrate(exp(exp(1)^(exp(1)^x)+exp(1)^x+x),x, algorithm="maxima")
```

output
```
e^(e^(e^x))
```

Giac [F]

$$\int e^{e^{e^x}+e^x+x}\,dx = \int e^{\left(x+e^x+e^{\left(e^x\right)}\right)}\,dx$$

input
```
integrate(exp(exp(1)^(exp(1)^x)+exp(1)^x+x),x, algorithm="giac")
```

output
```
integrate(e^(x + e^x + e^(e^x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.57

$$\int e^{e^{e^x}+e^x+x}\,dx = e^{e^{e^x}}$$

input
```
int(exp(x + exp(exp(x)) + exp(x)),x)
```

output
```
exp(exp(exp(x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int e^{e^{e^x}+e^x+x}\,dx = e^{e^{e^x}}$$

input
```
int(exp(exp(1)^(exp(1)^x)+exp(1)^x+x),x)
```

output
```
e**(e**(e**x))
```

3.640 $\int \left(e^{-x}+e^x\right)^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \left(e^{-x}+e^x\right)^2 \, dx=-\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}+2x$$

output
```
-1/2/exp(2*x)+1/2*exp(2*x)+2*x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \left(e^{-x}+e^x\right)^2 \, dx=\frac{1}{2}e^{-2x}\left(-1+e^{4x}\right)+\log\left(e^{2x}\right)$$

input
```
Integrate[(E^(-x) + E^x)^2,x]
```

output
```
(-1 + E^(4*x))/(2*E^(2*x)) + Log[E^(2*x)]
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2720, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e^{-x}+e^x\right)^2 dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-3x}\left(e^{2x}+1\right)^2 de^x$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int e^{-2x}\left(1+e^{2x}\right)^2 de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(1+e^{-2x}+2e^{-x}\right) de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-e^{-x}+e^{2x}+2\log\left(e^{2x}\right)\right)$$

input
```
Int[(E^(-x) + E^x)^2,x]
```

output
```
(-E^(-x) + E^(2*x) + 2*Log[E^(2*x)])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$2x-\frac{{\mathrm e}^{-2x}}{2}+\frac{{\mathrm e}^{2x}}{2}$	17
risch	$2x-\frac{{\mathrm e}^{-2x}}{2}+\frac{{\mathrm e}^{2x}}{2}$	17
parts	$2x-\frac{{\mathrm e}^{-2x}}{2}+\frac{{\mathrm e}^{2x}}{2}$	19
norman	$\left(-\frac{1}{2}+\frac{{\mathrm e}^{4x}}{2}+2x\,{\mathrm e}^{2x}\right){\mathrm e}^{-2x}$	21
parallelrisch	$2\,{\mathrm e}^{-x}{\mathrm e}^{x}x+\frac{{\mathrm e}^{2x}}{2}-\frac{{\mathrm e}^{-2x}}{2}$	25
orering	$x({\mathrm e}^{-x}+{\mathrm e}^{x})^{2}+\frac{({\mathrm e}^{-x}+{\mathrm e}^{x})(-{\mathrm e}^{-x}+{\mathrm e}^{x})}{2}-\frac{x\left(2(-{\mathrm e}^{-x}+{\mathrm e}^{x})^{2}+2({\mathrm e}^{-x}+{\mathrm e}^{x})^{2}\right)}{4}$	59

input
```
int((exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*x-1/2/exp(x)^2+1/2*exp(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \left(e^{-x}+e^{x}\right)^{2}\,dx=\frac{1}{2}\left(4\,xe^{(2\,x)}+e^{(4\,x)}-1\right)e^{(-2\,x)}$$

input
```
integrate((exp(-x)+exp(x))^2,x, algorithm="fricas")
```

output
```
1/2*(4*x*e^(2*x) + e^(4*x) - 1)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \left(e^{-x}+e^{x}\right)^{2}\,dx=2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$$

input
```
integrate((exp(-x)+exp(x))**2,x)
```

output
```
2*x + exp(2*x)/2 - exp(-2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \left(e^{-x}+e^{x}\right)^{2}\,dx=2\,x+\frac{1}{2}\,e^{(2\,x)}-\frac{1}{2}\,e^{(-2\,x)}$$

input
```
integrate((exp(-x)+exp(x))^2,x, algorithm="maxima")
```

output
```
2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(e^{-x}+e^{x}\right)^2 \, dx = -\frac{1}{2} \left(2 \, e^{(2 \, x)}+1\right) e^{(-2 \, x)}+2 \, x+\frac{1}{2} \, e^{(2 \, x)}$$

input
```
integrate((exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
-1/2*(2*e^(2*x) + 1)*e^(-2*x) + 2*x + 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.36

$$\int \left(e^{-x}+e^{x}\right)^2 \, dx = 2 \, x+\sinh(2 \, x)$$

input
```
int((exp(-x) + exp(x))^2,x)
```

output
```
2*x + sinh(2*x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \left(e^{-x}+e^{x}\right)^2 \, dx = \frac{e^{4x}+4e^{2x}x-1}{2e^{2x}}$$

input
```
int((exp(-x)+exp(x))^2,x)
```

output
```
(e**(4*x) + 4*e**(2*x)*x - 1)/(2*e**(2*x))
```

3.641 $\int \frac{1}{e^{-x}+e^x} dx$

Optimal result

Integrand size = 11, antiderivative size = 4

$$\int \frac{1}{e^{-x}+e^x} dx = \arctan(e^x)$$

output
```
arctan(exp(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{e^{-x}+e^x} dx = \arctan(e^x)$$

input
```
Integrate[(E^(-x) + E^x)^(-1),x]
```

output
```
ArcTan[E^x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2720, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^{-x}+e^x} \, dx$$
$$\Big\downarrow \text{2720}$$
$$\int \frac{1}{e^{2x}+1}de^x$$
$$\Big\downarrow \text{216}$$
$$\arctan\left(e^x\right)$$

input
```
Int[(E^(-x) + E^x)^(-1),x]
```

output
```
ArcTan[E^x]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan(\mathrm{e}^{x})$	4
risch	$\frac{i \ln(\mathrm{e}^{x}+i)}{2}-\frac{i \ln(\mathrm{e}^{x}-i)}{2}$	20

input
```
int(1/(exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
arctan(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \arctan\left(e^{x}\right)$$

input
```
integrate(1/(exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
arctan(e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

input
```
integrate(1/(exp(-x)+exp(x)),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7 vs. 2(3) = 6.

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.75

$$\int \frac{1}{e^{-x} + e^{x}}\, dx = -\arctan\left(e^{(-x)}\right)$$

input
```
integrate(1/(exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
-arctan(e^(-x))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{e^{-x} + e^{x}}\, dx = \arctan\left(e^{x}\right)$$

input
```
integrate(1/(exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{1}{e^{-x} + e^{x}}\, dx = \mathrm{atan}(\mathrm{e}^{x})$$

input
```
int(1/(exp(-x) + exp(x)),x)
```

output
```
atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{e^{-x} + e^x} \, dx = atan(e^x)$$

input
```
int(1/(exp(-x)+exp(x)),x)
```

output
```
atan(e**x)
```

3.642 $\int \frac{1}{(e^{-x}+e^x)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{1}{(e^{-x}+e^x)^2}\,dx = -\frac{1}{2(1+e^{2x})}$$

output
```
-1/2/(1+exp(2*x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{(e^{-x}+e^x)^2}\,dx = -\frac{1}{2+2e^{2x}}$$

input
```
Integrate[(E^(-x) + E^x)^(-2),x]
```

output
```
-(2 + 2*E^(2*x))^(-1)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2720, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e^{-x}+e^x)^2} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{e^x}{(e^{2x}+1)^2} de^x$$

$$\Big\downarrow \text{ 241}$$

$$-\frac{1}{2\left(e^{2x}+1\right)}$$

input
```
Int[(E^(-x) + E^x)^(-2),x]
```

output
```
-1/2*1/(1 + E^(2*x))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{1}{2(1+e^{2x})}$	11
norman	$-\frac{1}{2(1+e^{2x})}$	11
risch	$-\frac{1}{2(1+e^{2x})}$	11

input
```
int(1/(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(exp(x)^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(e^{-x}+e^{x}\right)^{2}}\,dx=-\frac{1}{2\left(e^{(2x)}+1\right)}$$

input
```
integrate(1/(exp(-x)+exp(x))^2,x, algorithm="fricas")
```

output
```
-1/2/(e^(2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{\left(e^{-x}+e^{x}\right)^{2}}\,dx=-\frac{1}{2e^{2x}+2}$$

input
```
integrate(1/(exp(-x)+exp(x))**2,x)
```

output
```
-1/(2*exp(2*x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(e^{-x}+e^x)^2}\,dx = \frac{1}{2\,(e^{(-2\,x)}+1)}$$

input
```
integrate(1/(exp(-x)+exp(x))^2,x, algorithm="maxima")
```

output
```
1/2/(e^(-2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{(e^{-x}+e^x)^2}\,dx = -\frac{1}{2\,(e^{(2\,x)}+1)}$$

input
```
integrate(1/(exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
-1/2/(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{(e^{-x}+e^x)^2}\,dx = -\frac{1}{2\,\left(\mathrm{e}^{2\,x}+1\right)}$$

input
```
int(1/(exp(-x) + exp(x))^2,x)
```

output
```
-1/(2*(exp(2*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{1}{(e^{-x}+e^{x})^{2}}\,dx=\frac{e^{2x}}{2e^{2x}+2}$$

input
```
int(1/(exp(-x)+exp(x))^2,x)
```

output
```
e**(2*x)/(2*(e**(2*x) + 1))
```

3.643 $\int \frac{1}{-e^{-x}+e^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{1}{-e^{-x}+e^x} \, dx = -\text{arctanh}(e^x)$$

output
```
-arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1}{-e^{-x}+e^x} \, dx = -\text{arctanh}(e^x)$$

input
```
Integrate[(-E^(-x) + E^x)^(-1),x]
```

output
```
-ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2720, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^x - e^{-x}} \, dx$$
$$\Big\downarrow \text{ 2720}$$
$$\int \frac{1}{e^{2x} - 1} de^x$$
$$\Big\downarrow \text{ 220}$$
$$-\text{arctanh}(e^x)$$

input
```
Int[(-E^(-x) + E^x)^(-1),x]
```

output
```
-ArcTanh[E^x]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
default	$-\operatorname{arctanh}\left(\mathrm{e}^{x}\right)$	6
norman	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16
risch	$\frac{\ln\left(-1+\mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{x}\right)}{2}$	16

input
```
int(1/(-exp(-x)+exp(x)),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{-e^{-x}+e^{x}}\,dx=-\frac{1}{2}\,\log\left(e^{x}+1\right)+\frac{1}{2}\,\log\left(e^{x}-1\right)$$

input
```
integrate(1/(-exp(-x)+exp(x)),x, algorithm="fricas")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{-e^{-x}+e^{x}}\,dx=\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

input
```
integrate(1/(-exp(-x)+exp(x)),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 3.17

$$\int \frac{1}{-e^{-x}+e^{x}}\,dx = -\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

input
```
integrate(1/(-exp(-x)+exp(x)),x, algorithm="maxima")
```

output
```
-1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{1}{-e^{-x}+e^{x}}\,dx = -\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

input
```
integrate(1/(-exp(-x)+exp(x)),x, algorithm="giac")
```

output
```
-1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.50

$$\int \frac{1}{-e^{-x}+e^x}\,dx = \frac{\ln\left(e^x-1\right)}{2}-\frac{\ln\left(e^x+1\right)}{2}$$

input
```
int(-1/(exp(-x) - exp(x)),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.83

$$\int \frac{1}{-e^{-x}+e^x}\,dx = \frac{\log(e^x-1)}{2}-\frac{\log(e^x+1)}{2}$$

input
```
int(1/(-exp(-x)+exp(x)),x)
```

output
```
(log(e**x - 1) - log(e**x + 1))/2
```

3.644 $\int \frac{1}{(-e^{-x}+e^x)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = \frac{1}{2(1-e^{2x})}$$

output
```
1/(2-2*exp(2*x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = \frac{1}{2-2e^{2x}}$$

input
```
Integrate[(-E^(-x) + E^x)^(-2),x]
```

output
```
(2 - 2*E^(2*x))^(-1)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2720, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(e^x - e^{-x})^2} \, dx$$

$$\Big\downarrow \; 2720$$

$$\int \frac{e^x}{(1-e^{2x})^2} de^x$$

$$\Big\downarrow \; 241$$

$$\frac{1}{2(1-e^{2x})}$$

input
```
Int[(-E^(-x) + E^x)^(-2),x]
```

output
```
1/(2*(1 - E^(2*x)))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

method	result	size
default	$-\frac{1}{2(-1+e^{2x})}$	11
norman	$-\frac{1}{2(-1+e^{2x})}$	11
risch	$-\frac{1}{2(-1+e^{2x})}$	11

input
```
int(1/(-exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(exp(x)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = -\frac{1}{2\left(e^{(2x)}-1\right)}$$

input
```
integrate(1/(-exp(-x)+exp(x))^2,x, algorithm="fricas")
```

output
```
-1/2/(e^(2*x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = -\frac{1}{2e^{2x}-2}$$

input
```
integrate(1/(-exp(-x)+exp(x))**2,x)
```

output
```
-1/(2*exp(2*x) - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = \frac{1}{2\,(e^{(-2\,x)}-1)}$$

input
```
integrate(1/(-exp(-x)+exp(x))^2,x, algorithm="maxima")
```

output
```
1/2/(e^(-2*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = -\frac{1}{2\,(e^{(2\,x)}-1)}$$

input
```
integrate(1/(-exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
-1/2/(e^(2*x) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = -\frac{1}{2\,(\mathrm{e}^{2\,x}-1)}$$

input
```
int(1/(exp(-x) - exp(x))^2,x)
```

output
```
-1/(2*(exp(2*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{(-e^{-x}+e^x)^2}\,dx = -\frac{e^{2x}}{2e^{2x}-2}$$

input
```
int(1/(-exp(-x)+exp(x))^2,x)
```

output
```
( - e**(2*x))/(2*(e**(2*x) - 1))
```

3.645 $\int e^x(-e^{-x}+e^x)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 22

$$\int e^x\left(-e^{-x}+e^x\right)^2\,dx=-e^{-x}-2e^x+\frac{e^{3x}}{3}$$

output
```
-exp(-x)-2*exp(x)+1/3*exp(3*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int e^x\left(-e^{-x}+e^x\right)^2\,dx=\frac{1}{3}e^{-x}\left(-3-6e^{2x}+e^{4x}\right)$$

input
```
Integrate[E^x*(-E^(-x) + E^x)^2,x]
```

output
```
(-3 - 6*E^(2*x) + E^(4*x))/(3*E^x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2720, 11, 1380, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x (e^x - e^{-x})^2 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int e^{-x} (e^{-x} - 2e^x + e^{3x}) \, de^x$$

$$\Big\downarrow \text{11}$$

$$\int e^{-2x} (-2e^{2x} + e^{4x} + 1) \, de^x$$

$$\Big\downarrow \text{1380}$$

$$\int e^{-2x} (1 - e^{2x})^2 \, de^x$$

$$\Big\downarrow \text{244}$$

$$\int (e^{-2x} + e^{2x} - 2) \, de^x$$

$$\Big\downarrow \text{2009}$$

$$-e^{-x} - 2e^x + \frac{e^{3x}}{3}$$

input
```
Int[E^x*(-E^(-x) + E^x)^2,x]
```

output
```
-E^(-x) - 2*E^x + E^(3*x)/3
```

Defintions of rubi rules used

rule 11
```
Int[(u_.)*((e_.)*(x_))^(m_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(
x_)^(t_.))^(p_.), x_Symbol] :> Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*(a +
b*x^(s - r) + c*x^(t - r))^p, x], x] /; FreeQ[{a, b, c, e, m, r, s, t}, x]
&& IntegerQ[p] && (IntegerQ[p*r] || GtQ[e, 0]) && PosQ[s - r] && PosQ[t - r
]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 1380
```
Int[(u_)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> S
imp[1/c^p   Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n, p}, x]
&& EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
default	$-\mathrm{e}^{-x}-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{3x}}{3}$	18
risch	$-\mathrm{e}^{-x}-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{3x}}{3}$	18
parts	$-\mathrm{e}^{-x}-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{3x}}{3}$	18
meijerg	$\frac{8}{3}-\mathrm{e}^{-x}-2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{3x}}{3}$	19
norman	$\left(-2\,\mathrm{e}^{3x}+\frac{\mathrm{e}^{5x}}{3}-\mathrm{e}^{x}\right)\mathrm{e}^{-2x}$	23
parallelrisch	$-2\,\mathrm{e}^{-x}\mathrm{e}^{2x}-\mathrm{e}^{-2x}\mathrm{e}^{x}+\frac{\mathrm{e}^{3x}}{3}$	28
orering	$\frac{\mathrm{e}^{x}\left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)^{2}}{3}+\frac{2\,\mathrm{e}^{x}\left(-\mathrm{e}^{-x}+\mathrm{e}^{x}\right)\left(\mathrm{e}^{-x}+\mathrm{e}^{x}\right)}{3}-\frac{2\,\mathrm{e}^{x}\left(\mathrm{e}^{-x}+\mathrm{e}^{x}\right)^{2}}{3}$	50

input
```
int(exp(x)*(-exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/exp(x)+1/3*exp(x)^3-2*exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int e^{x}\left(-e^{-x}+e^{x}\right)^{2}\,dx=\frac{1}{3}\,\left(e^{(4\,x)}-6\,e^{(2\,x)}-3\right)e^{(-x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))^2,x, algorithm="fricas")
```

output
```
1/3*(e^(4*x) - 6*e^(2*x) - 3)*e^(-x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int e^x(-e^{-x}+e^x)^2\,dx=\frac{e^{3x}}{3}-2e^x-e^{-x}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))**2,x)
```

output
```
exp(3*x)/3 - 2*exp(x) - exp(-x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int e^x(-e^{-x}+e^x)^2\,dx=-\frac{1}{3}\left(6\,e^{(-2\,x)}-1\right)e^{(3\,x)}-e^{(-x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))^2,x, algorithm="maxima")
```

output
```
-1/3*(6*e^(-2*x) - 1)*e^(3*x) - e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int e^x(-e^{-x}+e^x)^2\,dx=\frac{1}{3}\,e^{(3\,x)}-e^{(-x)}-2\,e^x$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
1/3*e^(3*x) - e^(-x) - 2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int e^x \left(-e^{-x}+e^x\right)^2 \, dx = \frac{\mathrm{e}^{3\,x}}{3} - \mathrm{e}^{-x} - 2\,\mathrm{e}^{x}$$

input
```
int(exp(x)*(exp(-x) - exp(x))^2,x)
```

output
```
exp(3*x)/3 - exp(-x) - 2*exp(x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int e^x \left(-e^{-x}+e^x\right)^2 \, dx = \frac{e^{4x} - 6e^{2x} - 3}{3e^x}$$

input
```
int(exp(x)*(-exp(-x)+exp(x))^2,x)
```

output
```
(e**(4*x) - 6*e**(2*x) - 3)/(3*e**x)
```

3.646 $\int e^x(-e^{-x}+e^x)^3\,dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int e^x\left(-e^{-x}+e^x\right)^3\,dx=\frac{e^{-2x}}{2}-\frac{3e^{2x}}{2}+\frac{e^{4x}}{4}+3x$$

output
```
1/2/exp(2*x)-3/2*exp(2*x)+1/4*exp(4*x)+3*x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int e^x\left(-e^{-x}+e^x\right)^3\,dx=\frac{1}{4}e^{-2x}\left(2-6e^{4x}+e^{6x}\right)+3\log\left(e^x\right)$$

input
```
Integrate[E^x*(-E^(-x) + E^x)^3,x]
```

output
```
(2 - 6*E^(4*x) + E^(6*x))/(4*E^(2*x)) + 3*Log[E^x]
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2720, 25, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \left(e^x-e^{-x}\right)^3 \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int -e^{-3x} \left(1-e^{2x}\right)^3 \, de^x$$

$$\Big\downarrow \text{25}$$

$$-\int e^{-3x} \left(1-e^{2x}\right)^3 \, de^x$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2} \int e^{-2x} \left(1-e^{2x}\right)^3 \, de^{2x}$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2} \int \left(3+e^{-2x}-3e^{-x}-e^{2x}\right) de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(e^{-x}-\frac{5e^{2x}}{2}+3\log\left(e^{2x}\right)\right)$$

input
```
Int[E^x*(-E^(-x) + E^x)^3,x]
```

output
```
(E^(-x) - (5*E^(2*x))/2 + 3*Log[E^(2*x)])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result
default	$3x+\frac{{\mathrm e}^{4x}}{4}+\frac{{\mathrm e}^{-2x}}{2}-\frac{3\,{\mathrm e}^{2x}}{2}$
risch	$3x+\frac{{\mathrm e}^{4x}}{4}+\frac{{\mathrm e}^{-2x}}{2}-\frac{3\,{\mathrm e}^{2x}}{2}$
parts	$3x+\frac{{\mathrm e}^{4x}}{4}+\frac{{\mathrm e}^{-2x}}{2}-\frac{3\,{\mathrm e}^{2x}}{2}$
norman	$\left(-\frac{3\,{\mathrm e}^{5x}}{2}+\frac{{\mathrm e}^{7x}}{4}+3\,{\mathrm e}^{3x}x+\frac{{\mathrm e}^{x}}{2}\right){\mathrm e}^{-3x}$
parallelrisch	$-\frac{3\,{\mathrm e}^{-x}{\mathrm e}^{3x}}{2}+\frac{{\mathrm e}^{-3x}{\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{4x}}{4}+3\,{\mathrm e}^{-2x}{\mathrm e}^{2x}x$
orering	$\left(x+\frac{1}{4}\right){\mathrm e}^{x}\left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{3}+\left(-\frac{x}{4}+\frac{1}{4}\right)\left({\mathrm e}^{x}\left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{3}+3\,{\mathrm e}^{x}\left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{2}\left({\mathrm e}^{-x}+{\mathrm e}^{x}\right)\right)+(-$

input
```
int(exp(x)*(-exp(-x)+exp(x))^3,x,method=_RETURNVERBOSE)
```

output
```
3*x+1/4*exp(x)^4+1/2/exp(x)^2-3/2*exp(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int e^x\left(-e^{-x}+e^x\right)^3\,dx=\frac{1}{4}\left(12\,xe^{(2\,x)}+e^{(6\,x)}-6\,e^{(4\,x)}+2\right)e^{(-2\,x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))^3,x, algorithm="fricas")
```

output
```
1/4*(12*x*e^(2*x) + e^(6*x) - 6*e^(4*x) + 2)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int e^x\left(-e^{-x}+e^x\right)^3\,dx=3x+\frac{e^{4x}}{4}-\frac{3e^{2x}}{2}+\frac{e^{-2x}}{2}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))**3,x)
```

output
```
3*x + exp(4*x)/4 - 3*exp(2*x)/2 + exp(-2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int e^x \left(-e^{-x}+e^x\right)^3 \, dx = -\frac{1}{4} \left(6 \, e^{(-2 \, x)} - 1\right) e^{(4 \, x)} + 3 \, x + \frac{1}{2} \, e^{(-2 \, x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))^3,x, algorithm="maxima")
```

output
```
-1/4*(6*e^(-2*x) - 1)*e^(4*x) + 3*x + 1/2*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int e^x \left(-e^{-x}+e^x\right)^3 \, dx = -\frac{1}{2} \left(3 \, e^{(2 \, x)} - 1\right) e^{(-2 \, x)} + 3 \, x + \frac{1}{4} \, e^{(4 \, x)} - \frac{3}{2} \, e^{(2 \, x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))^3,x, algorithm="giac")
```

output
```
-1/2*(3*e^(2*x) - 1)*e^(-2*x) + 3*x + 1/4*e^(4*x) - 3/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int e^x \left(-e^{-x}+e^x\right)^3 \, dx = 3\,x + \frac{\mathrm{e}^{-2\,x}}{2} - \frac{3\,\mathrm{e}^{2\,x}}{2} + \frac{\mathrm{e}^{4\,x}}{4}$$

input
```
int(-exp(x)*(exp(-x) - exp(x))^3,x)
```

output
```
3*x + exp(-2*x)/2 - (3*exp(2*x))/2 + exp(4*x)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^x\left(-e^{-x}+e^x\right)^3\,dx=\frac{e^{6x}-6e^{4x}+12e^{2x}x+2}{4e^{2x}}$$

input
```
int(exp(x)*(-exp(-x)+exp(x))^3,x)
```

output
```
(e**(6*x) - 6*e**(4*x) + 12*e**(2*x)*x + 2)/(4*e**(2*x))
```

3.647 $\int \frac{1+4^x}{1+2^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1+4^x}{1+2^x} \, dx = x + \frac{2^x}{\log(2)} - \frac{2 \log(1+2^x)}{\log(2)}$$

output
```
x+2^x/ln(2)-2*ln(1+2^x)/ln(2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1+4^x}{1+2^x} \, dx = \frac{2^x + x\log(2) - 2 \log(1+2^x)}{\log(2)}$$

input
```
Integrate[(1 + 4^x)/(1 + 2^x),x]
```

output
```
(2^x + x*Log[2] - 2*Log[1 + 2^x])/Log[2]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2720, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x+1}{2^x+1} \, dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{2^{-x}\left(1+2^{2x}\right)}{1+2^x}d2^x}{\log(2)}$$
$$\Big\downarrow \text{522}$$
$$\frac{\int \left(2^{-x}+1-\frac{2}{1+2^x}\right)d2^x}{\log(2)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{2^x+\log\left(2^x\right)-2\log\left(2^x+1\right)}{\log(2)}$$

input
```
Int[(1 + 4^x)/(1 + 2^x),x]
```

output
```
(2^x + Log[2^x] - 2*Log[1 + 2^x])/Log[2]
```

Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
risch	$x+\frac{2^{x}}{\ln(2)}-\frac{2\ln(1+2^{x})}{\ln(2)}$	23
norman	$x+\frac{{\mathrm e}^{x\ln(2)}}{\ln(2)}-\frac{2\ln\left(1+{\mathrm e}^{x\ln(2)}\right)}{\ln(2)}$	27

input
```
int((1+4^x)/(1+2^x),x,method=_RETURNVERBOSE)
```

output
```
x+2^x/ln(2)-2*ln(1+2^x)/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1+4^x}{1+2^x}\,dx=\frac{x\log{(2)}+2^{x}-2\,\log{(2^{x}+1)}}{\log{(2)}}$$

input
```
integrate((1+4^x)/(1+2^x),x, algorithm="fricas")
```

output
```
(x*log(2) + 2^x - 2*log(2^x + 1))/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{1+4^x}{1+2^x}\,dx = x + \frac{e^{\frac{x\log(4)}{2}}}{\log(2)} - \frac{2\log\left(e^{\frac{x\log(4)}{2}}+1\right)}{\log(2)}$$

input
```
integrate((1+4**x)/(1+2**x),x)
```

output
```
x + exp(x*log(4)/2)/log(2) - 2*log(exp(x*log(4)/2) + 1)/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1+4^x}{1+2^x}\,dx = x + \frac{2^x}{\log(2)} - \frac{2\,\log(2^x+1)}{\log(2)}$$

input
```
integrate((1+4^x)/(1+2^x),x, algorithm="maxima")
```

output
```
x + 2^x/log(2) - 2*log(2^x + 1)/log(2)
```

Giac [F]

$$\int \frac{1+4^x}{1+2^x}\,dx = \int \frac{4^x+1}{2^x+1}\,dx$$

input
```
integrate((1+4^x)/(1+2^x),x, algorithm="giac")
```

output
```
integrate((4^x + 1)/(2^x + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1+4^x}{1+2^x}\,dx = \frac{x\,\ln(2) - 2\,\ln(2^x+1) + 2^x}{\ln(2)}$$

input
```
int((4^x + 1)/(2^x + 1),x)
```

output
```
(x*log(2) - 2*log(2^x + 1) + 2^x)/log(2)
```

Reduce [F]

$$\int \frac{1+4^x}{1+2^x}\,dx = \frac{\left(\int \frac{4^x}{2^x+1}dx\right)\log(2) - \log(2^x+1) + \log(2)\,x}{\log(2)}$$

input
```
int((1+4^x)/(1+2^x),x)
```

output
```
(int(4**x/(2**x + 1),x)*log(2) - log(2**x + 1) + log(2)*x)/log(2)
```

3.648 $\int \frac{1+4^x}{1+2^{-x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{1+4^x}{1+2^{-x}} \, dx = -\frac{2^x}{\log(2)} + \frac{2^{-1+2x}}{\log(2)} + \frac{2\log(1+2^x)}{\log(2)}$$

output
```
-2^x/ln(2)+2^(-1+2*x)/ln(2)+2*ln(1+2^x)/ln(2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{1+4^x}{1+2^{-x}} \, dx = \frac{2^x(-2+2^x)+4\log(1+2^x)}{\log(4)}$$

input
```
Integrate[(1 + 4^x)/(1 + 2^(-x)),x]
```

output
```
(2^x*(-2 + 2^x) + 4*Log[1 + 2^x])/Log[4]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2720, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4^x+1}{2^{-x}+1} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{1+2^{2x}}{1+2^x}d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 476}$$

$$\frac{\int \left(2^x-1+\frac{2}{1+2^x}\right)d2^x}{\log(2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-2^x+2^{2x-1}+2\log\left(2^x+1\right)}{\log(2)}$$

input
```
Int[(1 + 4^x)/(1 + 2^(-x)),x]
```

output
```
(-2^x + 2^(-1 + 2*x)) + 2*Log[1 + 2^x])/Log[2]
```

Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result	size
risch	$-\frac{2^{x}}{\ln(2)}+\frac{2^{2x}}{2\ln(2)}+2x+\frac{2\ln\left(1+\left(\frac{1}{2}\right)^{x}\right)}{\ln(2)}$	39
norman	$\left(\frac{1}{2\ln(2)}+2x\,\mathrm{e}^{-2x\ln(2)}-\frac{\mathrm{e}^{-x\ln(2)}}{\ln(2)}\right)\mathrm{e}^{2x\ln(2)}+\frac{2\ln\left(1+\mathrm{e}^{-x\ln(2)}\right)}{\ln(2)}$	56

input
```
int((1+4^x)/(1+2^(-x)),x,method=_RETURNVERBOSE)
```

output
```
-1/ln(2)/((1/2)^x)+1/2/ln(2)/((1/2)^x)^2+2*x+2/ln(2)*ln(1+(1/2)^x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{1+4^x}{1+2^{-x}}\,dx=\frac{2^{2\,x}-2\cdot 2^{x}+4\,\log\left(2^{x}+1\right)}{2\,\log\left(2\right)}$$

input
```
integrate((1+4^x)/(1+2^(-x)),x, algorithm="fricas")
```

output
```
1/2*(2^(2*x) - 2*2^x + 4*log(2^x + 1))/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{1+4^x}{1+2^{-x}}\,dx = 2x + \frac{2^{2x}\log(2) - 2\cdot 2^x\log(2)}{2\log(2)^2} + \frac{2\log(1+2^{-x})}{\log(2)}$$

input
```
integrate((1+4**x)/(1+2**(-x)),x)
```

output
```
2*x + (2**(2*x)*log(2) - 2*2**x*log(2))/(2*log(2)**2) + 2*log(1 + 2**(-x))
/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.18

$$\int \frac{1+4^x}{1+2^{-x}}\,dx = 2\,x - \frac{2^{2\,x-1}(2^{-x+1}-1)}{\log(2)} + \frac{2\,\log\left(\frac{1}{2^x}+1\right)}{\log(2)}$$

input
```
integrate((1+4^x)/(1+2^(-x)),x, algorithm="maxima")
```

output
```
2*x - 2^(2*x - 1)*(2^(-x + 1) - 1)/log(2) + 2*log(1/2^x + 1)/log(2)
```

Giac [F]

$$\int \frac{1+4^x}{1+2^{-x}}\,dx = \int \frac{4^x+1}{\frac{1}{2^x}+1}\,dx$$

input
```
integrate((1+4^x)/(1+2^(-x)),x, algorithm="giac")
```

output
```
integrate((4^x + 1)/(1/2^x + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+4^x}{1+2^{-x}}\,dx = \int \frac{4^x+1}{\frac{1}{2^x}+1}\,dx$$

input
```
int((4^x + 1)/(1/2^x + 1),x)
```

output
```
int((4^x + 1)/(1/2^x + 1), x)
```

Reduce [F]

$$\int \frac{1+4^x}{1+2^{-x}}\,dx = \frac{4^x\log(2)-\left(\int \frac{4^x}{2^x+1}dx\right)\log(4)\log(2)+\log(2^x+1)\log(4)}{\log(4)\log(2)}$$

input
```
int((1+4^x)/(1+2^(-x)),x)
```

output
```
(4**x*log(2) - int(4**x/(2**x + 1),x)*log(4)*log(2) + log(2**x + 1)*log(4)
)/(log(4)*log(2))
```

3.649 $\int \left(\frac{e^{(a+x)^2}}{x^2} - \frac{2ae^{(a+x)^2}}{x}\right) dx$

Optimal result

Integrand size = 25, antiderivative size = 23

$$\int \left(\frac{e^{(a+x)^2}}{x^2} - \frac{2ae^{(a+x)^2}}{x}\right) dx = -\frac{e^{(a+x)^2}}{x} + \sqrt{\pi}\,\mathrm{erfi}(a+x)$$

output
```
-exp((a+x)^2)/x+Pi^(1/2)*erfi(a+x)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \left(\frac{e^{(a+x)^2}}{x^2} - \frac{2ae^{(a+x)^2}}{x}\right) dx = -\frac{e^{(a+x)^2}}{x} + \sqrt{\pi}\,\mathrm{erfi}(a+x)$$

input
```
Integrate[E^(a + x)^2/x^2 - (2*a*E^(a + x)^2)/x,x]
```

output
```
-(E^(a + x)^2/x) + Sqrt[Pi]*Erfi[a + x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\sqrt{\pi}\text{erfi}(a+x)-\frac{e^{(a+x)^2}}{x}$$

input
```
Int[E^(a + x)^2/x^2 - (2*a*E^(a + x)^2)/x,x]
```

output
```
-(E^(a + x)^2/x) + Sqrt[Pi]*Erfi[a + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{\mathrm{e}^{(a+x)^2}}{x^2}-\frac{2a\,\mathrm{e}^{(a+x)^2}}{x}\right) dx$$

input
```
int(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x)
```

output
```
int(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right)\,dx=\frac{\sqrt{\pi}x\,\mathrm{erfi}\,(a+x)-e^{\left(a^2+2\,ax+x^2\right)}}{x}$$

input
```
integrate(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x, algorithm="fricas")
```

output
```
(sqrt(pi)*x*erfi(a + x) - e^(a^2 + 2*a*x + x^2))/x
```

Sympy [F]

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right)\,dx=-\left(\int \left(-\frac{e^{x^2}e^{2ax}}{x^2}\right)\,dx+\int \frac{2ae^{x^2}e^{2ax}}{x}\,dx\right)e^{a^2}$$

input
```
integrate(exp((a+x)**2)/x**2-2*a*exp((a+x)**2)/x,x)
```

output
```
-(Integral(-exp(x**2)*exp(2*a*x)/x**2, x) + Integral(2*a*exp(x**2)*exp(2*a
**x)/x, x))*exp(a**2)
```

Maxima [F]

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right)\,dx=\int -\frac{2\,ae^{\left((a+x)^2\right)}}{x}+\frac{e^{\left((a+x)^2\right)}}{x^2}\,dx$$

input
```
integrate(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x, algorithm="maxima")
```

output
```
integrate(-2*a*e^((a + x)^2)/x + e^((a + x)^2)/x^2, x)
```

Giac [F]

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right)dx=\int -\frac{2\,ae^{\left((a+x)^2\right)}}{x}+\frac{e^{\left((a+x)^2\right)}}{x^2}\,dx$$

input
```
integrate(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x, algorithm="giac")
```

output
```
integrate(-2*a*e^((a + x)^2)/x + e^((a + x)^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right)dx=\sqrt{\pi}\,\mathrm{erfi}(a+x)-\frac{\mathrm{e}^{a^2}\,\mathrm{e}^{x^2}\,\mathrm{e}^{2\,a\,x}}{x}$$

input
```
int(exp((a + x)^2)/x^2 - (2*a*exp((a + x)^2))/x,x)
```

output
```
pi^(1/2)*erfi(a + x) - (exp(a^2)*exp(x^2)*exp(2*a*x))/x
```

Reduce [F]

$$\int \left(\frac{e^{(a+x)^2}}{x^2}-\frac{2ae^{(a+x)^2}}{x}\right)dx=e^{a^2}\left(\int \frac{e^{2ax+x^2}}{x^2}dx-2\left(\int \frac{e^{2ax+x^2}}{x}dx\right)a\right)$$

input
```
int(exp((a+x)^2)/x^2-2*a*exp((a+x)^2)/x,x)
```

output
```
e**(a**2)*(int(e**(2*a*x + x**2)/x**2,x) - 2*int(e**(2*a*x + x**2)/x,x)*a)
```

3.650 $\int e^{-x^2}(x^4+x^6+x^8)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 66

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=-\frac{147}{16}e^{-x^2}x-\frac{49}{8}e^{-x^2}x^3-\frac{9}{4}e^{-x^2}x^5-\frac{1}{2}e^{-x^2}x^7+\frac{147}{32}\sqrt{\pi}\operatorname{erf}(x)$$

output
```
-147/16*x/exp(x^2)-49/8*x^3/exp(x^2)-9/4*x^5/exp(x^2)-1/2*x^7/exp(x^2)+147
/32*Pi^(1/2)*erf(x)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=\frac{1}{32}\left(-2e^{-x^2}x\left(147+98x^2+36x^4+8x^6\right)+147\sqrt{\pi}\operatorname{erf}(x)\right)$$

input
```
Integrate[(x^4 + x^6 + x^8)/E^x^2,x]
```

output
```
((-2*x*(147 + 98*x^2 + 36*x^4 + 8*x^6))/E^x^2 + 147*Sqrt[Pi]*Erf[x])/32
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2028, 2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x^2} \left(x^8+x^6+x^4\right) \, dx$$

$$\Big\downarrow \text{2028}$$

$$\int e^{-x^2} x^4 \left(x^4+x^2+1\right) dx$$

$$\Big\downarrow \text{2656}$$

$$\int \left(e^{-x^2} x^8+e^{-x^2} x^6+e^{-x^2} x^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{147}{32} \sqrt{\pi } \text{erf}(x)-\frac{147}{16} e^{-x^2} x-\frac{1}{2} e^{-x^2} x^7-\frac{9}{4} e^{-x^2} x^5-\frac{49}{8} e^{-x^2} x^3$$

input
```
Int[(x^4 + x^6 + x^8)/E^x^2,x]
```

output
```
(-147*x)/(16*E^x^2) - (49*x^3)/(8*E^x^2) - (9*x^5)/(4*E^x^2) - x^7/(2*E^x^
2) + (147*Sqrt[Pi]*Erf[x])/32
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2028
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*(x_)^(t_.))^(p_.),
x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r))^p*Fx, x] /; FreeQ[
{a, b, c, r, s, t}, x] && IntegerQ[p] && PosQ[s - r] && PosQ[t - r] && !(E
qQ[p, 1] && EqQ[u, 1])
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{147x\,\mathrm{e}^{-x^2}}{16}-\frac{49x^3\mathrm{e}^{-x^2}}{8}-\frac{9x^5\mathrm{e}^{-x^2}}{4}-\frac{x^7\mathrm{e}^{-x^2}}{2}+\frac{147\sqrt{\pi}\ \mathrm{erf}(x)}{32}$	51
risch	$-\frac{147x\,\mathrm{e}^{-x^2}}{16}-\frac{49x^3\mathrm{e}^{-x^2}}{8}-\frac{9x^5\mathrm{e}^{-x^2}}{4}-\frac{x^7\mathrm{e}^{-x^2}}{2}+\frac{147\sqrt{\pi}\ \mathrm{erf}(x)}{32}$	51
meijerg	$-\frac{x(72x^6+252x^4+630x^2+945)\mathrm{e}^{-x^2}}{144}+\frac{147\sqrt{\pi}\ \mathrm{erf}(x)}{32}-\frac{x(28x^4+70x^2+105)\mathrm{e}^{-x^2}}{56}-\frac{x(10x^2+15)\mathrm{e}^{-x^2}}{20}$	72

input
```
int((x^8+x^6+x^4)/exp(x^2),x,method=_RETURNVERBOSE)
```

output
```
-147/16*x/exp(x^2)-49/8*x^3/exp(x^2)-9/4*x^5/exp(x^2)-1/2*x^7/exp(x^2)+147
/32*Pi^(1/2)*erf(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=-\frac{1}{16}\left(8\,x^7+36\,x^5+98\,x^3+147\,x\right)e^{\left(-x^2\right)}+\frac{147}{32}\,\sqrt{\pi}\,\mathrm{erf}\left(x\right)$$

input
```
integrate((x^8+x^6+x^4)/exp(x^2),x, algorithm="fricas")
```

output
```
-1/16*(8*x^7 + 36*x^5 + 98*x^3 + 147*x)*e^(-x^2) + 147/32*sqrt(pi)*erf(x)
```

Sympy [A] (verification not implemented)

Time = 4.74 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=-\frac{x^7e^{-x^2}}{2}-\frac{9x^5e^{-x^2}}{4}-\frac{49x^3e^{-x^2}}{8}-\frac{147xe^{-x^2}}{16}+\frac{147\sqrt{\pi}\,\mathrm{erf}\,(x)}{32}$$

input
```
integrate((x**8+x**6+x**4)/exp(x**2),x)
```

output
```
-x**7*exp(-x**2)/2 - 9*x**5*exp(-x**2)/4 - 49*x**3*exp(-x**2)/8 - 147*x*ex
p(-x**2)/16 + 147*sqrt(pi)*erf(x)/32
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\begin{aligned}\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=&-\frac{1}{16}\left(8\,x^7+28\,x^5+70\,x^3+105\,x\right)e^{\left(-x^2\right)}\\&-\frac{1}{8}\left(4\,x^5+10\,x^3+15\,x\right)e^{\left(-x^2\right)}\\&-\frac{1}{4}\left(2\,x^3+3\,x\right)e^{\left(-x^2\right)}+\frac{147}{32}\,\sqrt{\pi}\,\mathrm{erf}\,(x)\end{aligned}$$

input
```
integrate((x^8+x^6+x^4)/exp(x^2),x, algorithm="maxima")
```

output
```
-1/16*(8*x^7 + 28*x^5 + 70*x^3 + 105*x)*e^(-x^2) - 1/8*(4*x^5 + 10*x^3 + 1
5*x)*e^(-x^2) - 1/4*(2*x^3 + 3*x)*e^(-x^2) + 147/32*sqrt(pi)*erf(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.53

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=-\frac{1}{16}\left(8\,x^7+36\,x^5+98\,x^3+147\,x\right)e^{\left(-x^2\right)}+\frac{147}{32}\,\sqrt{\pi}\,\mathrm{erf}\left(x\right)$$

input
```
integrate((x^8+x^6+x^4)/exp(x^2),x, algorithm="giac")
```

output
```
-1/16*(8*x^7 + 36*x^5 + 98*x^3 + 147*x)*e^(-x^2) + 147/32*sqrt(pi)*erf(x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=\frac{147\,\sqrt{\pi}\,\mathrm{erf}(x)}{32}-\frac{49\,x^3\,\mathrm{e}^{-x^2}}{8}-\frac{9\,x^5\,\mathrm{e}^{-x^2}}{4}-\frac{x^7\,\mathrm{e}^{-x^2}}{2}-\frac{147\,x\,\mathrm{e}^{-x^2}}{16}$$

input
```
int(exp(-x^2)*(x^4 + x^6 + x^8),x)
```

output
```
(147*pi^(1/2)*erf(x))/32 - (49*x^3*exp(-x^2))/8 - (9*x^5*exp(-x^2))/4 - (x
^7*exp(-x^2))/2 - (147*x*exp(-x^2))/16
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int e^{-x^2}\left(x^4+x^6+x^8\right)\,dx=\frac{147\sqrt{\pi}\,e^{x^2}\mathrm{erf}(x)-16x^7-72x^5-196x^3-294x}{32e^{x^2}}$$

input
```
int((x^8+x^6+x^4)/exp(x^2),x)
```

output
```
(147*sqrt(pi)*e**(x**2)*erf(x) - 16*x**7 - 72*x**5 - 196*x**3 - 294*x)/(32
*e**(x**2))
```

3.651 $\int \frac{1}{-e^x+e^{3x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{-e^x+e^{3x}} dx = e^{-x} - \operatorname{arctanh}(e^x)$$

output
```
exp(-x)-arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{-e^x+e^{3x}} dx = e^{-x} - \operatorname{arctanh}(e^x)$$

input
```
Integrate[(-E^x + E^(3*x))^(-1),x]
```

output
```
E^(-x) - ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2720, 25, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e^{3x}-e^x} \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int -\frac{e^{-2x}}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{e^{-2x}}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{ 264}$$

$$e^{-x}-\int \frac{1}{1-e^{2x}}de^x$$

$$\Big\downarrow \text{ 219}$$

$$e^{-x}-\text{arctanh}(e^x)$$

input
```
Int[(-E^x + E^(3*x))^(-1),x]
```

output
```
E^(-x) - ArcTanh[E^x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

method	result	size
default	$-\frac{\ln(1+{\mathrm e}^{x})}{2}+\frac{\ln(-1+{\mathrm e}^{x})}{2}+{\mathrm e}^{-x}$	20
norman	$-\frac{\ln(1+{\mathrm e}^{x})}{2}+\frac{\ln(-1+{\mathrm e}^{x})}{2}+{\mathrm e}^{-x}$	20
risch	$-\frac{\ln(1+{\mathrm e}^{x})}{2}+\frac{\ln(-1+{\mathrm e}^{x})}{2}+{\mathrm e}^{-x}$	20

input
```
int(1/(-exp(x)+exp(3*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(1+exp(x))+1/2*ln(-1+exp(x))+1/exp(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(10) = 20$.

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \frac{1}{-e^x + e^{3x}}\,dx = -\frac{1}{2}\left(e^x \log\left(e^x + 1\right) - e^x \log\left(e^x - 1\right) - 2\right)e^{(-x)}$$

input
```
integrate(1/(-exp(x)+exp(3*x)),x, algorithm="fricas")
```

output
```
-1/2*(e^x*log(e^x + 1) - e^x*log(e^x - 1) - 2)*e^(-x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{-e^x + e^{3x}}\,dx = \frac{\log\left(e^x - 1\right)}{2} - \frac{\log\left(e^x + 1\right)}{2} + e^{-x}$$

input
```
integrate(1/(-exp(x)+exp(3*x)),x)
```

output
```
log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(-x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{-e^x + e^{3x}}\,dx = e^{(-x)} - \frac{1}{2}\log\left(e^x + 1\right) + \frac{1}{2}\log\left(e^x - 1\right)$$

input
```
integrate(1/(-exp(x)+exp(3*x)),x, algorithm="maxima")
```

output
```
e^(-x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{1}{-e^x + e^{3x}} \, dx = e^{(-x)} - \frac{1}{2} \log\left(e^x + 1\right) + \frac{1}{2} \log\left(|e^x - 1|\right)$$

input
```
integrate(1/(-exp(x)+exp(3*x)),x, algorithm="giac")
```

output
```
e^(-x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{1}{-e^x + e^{3x}} \, dx = \mathrm{e}^{-x} + \frac{\ln\left(\mathrm{e}^x - 1\right)}{2} - \frac{\ln\left(\mathrm{e}^x + 1\right)}{2}$$

input
```
int(1/(exp(3*x) - exp(x)),x)
```

output
```
exp(-x) + log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int \frac{1}{-e^x + e^{3x}} \, dx = \frac{e^x \log(e^x - 1) - e^x \log(e^x + 1) + 2}{2e^x}$$

input
```
int(1/(-exp(x)+exp(3*x)),x)
```

output
```
(e**x*log(e**x - 1) - e**x*log(e**x + 1) + 2)/(2*e**x)
```

3.652 $\int \frac{e^x(-5+x+x^2)}{(-1+x)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2} dx = e^x - \frac{3e^x}{1-x}$$

output
```
exp(x)-3*exp(x)/(1-x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2} dx = e^x\left(1+\frac{3}{-1+x}\right)$$

input
```
Integrate[(E^x*(-5 + x + x^2))/(-1 + x)^2,x]
```

output
```
E^x*(1 + 3/(-1 + x))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x (x^2+x-5)}{(x-1)^2} \, dx$$

$$\Big\downarrow \text{ 2629}$$

$$\int \left(e^x+\frac{3e^x}{x-1}-\frac{3e^x}{(x-1)^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$e^x-\frac{3e^x}{1-x}$$

input
```
Int[(E^x*(-5 + x + x^2))/(-1 + x)^2,x]
```

output
```
E^x - (3*E^x)/(1 - x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629
```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] &&  !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

method	result	size
gosper	$\frac{(x+2)\,\mathrm{e}^{x}}{-1+x}$	12
risch	$\frac{(x+2)\,\mathrm{e}^{x}}{-1+x}$	12
orering	$\frac{(x+2)\,\mathrm{e}^{x}}{-1+x}$	12
default	$\frac{3\,\mathrm{e}^{x}}{-1+x}+\mathrm{e}^{x}$	13
norman	$\frac{\mathrm{e}^{x}x+2\,\mathrm{e}^{x}}{-1+x}$	16
parallelrisch	$\frac{\mathrm{e}^{x}x+2\,\mathrm{e}^{x}}{-1+x}$	16

input
```
int(exp(x)*(x^2+x-5)/(-1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/(-1+x)*(x+2)*exp(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2}\,dx = \frac{(x+2)e^x}{x-1}$$

input
```
integrate(exp(x)*(x^2+x-5)/(x-1)^2,x, algorithm="fricas")
```

output
```
(x + 2)*e^x/(x - 1)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2}\,dx = \frac{(x+2)\,e^x}{x-1}$$

input
```
integrate(exp(x)*(x**2+x-5)/(x-1)**2,x)
```

output
```
(x + 2)*exp(x)/(x - 1)
```

Maxima [F]

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2}\,dx = \int \frac{(x^2+x-5)e^x}{(x-1)^2}\,dx$$

input
```
integrate(exp(x)*(x^2+x-5)/(x-1)^2,x, algorithm="maxima")
```

output
```
(x^2 + x)*e^x/(x^2 - 2*x + 1) + 5*e*exp_integral_e(2, -x + 1)/(x - 1) + in
tegrate((3*x + 1)*e^x/(x^3 - 3*x^2 + 3*x - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(12) = 24.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.38

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2}\,dx = \frac{(x-1)\left(\frac{1}{x-1}+1\right)e^{\left((x-1)\left(\frac{1}{x-1}+1\right)\right)}+2\,e^{\left((x-1)\left(\frac{1}{x-1}+1\right)\right)}}{(x-1)\left(\frac{1}{x-1}+1\right)-1}$$

input
```
integrate(exp(x)*(x^2+x-5)/(x-1)^2,x, algorithm="giac")
```

output
```
((x - 1)*(1/(x - 1) + 1)*e^((x - 1)*(1/(x - 1) + 1)) + 2*e^((x - 1)*(1/(x
- 1) + 1)))/((x - 1)*(1/(x - 1) + 1) - 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2}\,dx = \frac{\mathrm{e}^x\,(x+2)}{x-1}$$

input
```
int((exp(x)*(x + x^2 - 5))/(x - 1)^2,x)
```

output
```
(exp(x)*(x + 2))/(x - 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{e^x(-5+x+x^2)}{(-1+x)^2}\,dx = \frac{e^x(x+2)}{x-1}$$

input
```
int(exp(x)*(x^2+x-5)/(x-1)^2,x)
```

output
```
(e**x*(x + 2))/(x - 1)
```

3.653 $\int \frac{e^{x^2} x^3}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{e^{x^2} x^3}{(1+x^2)^2} \, dx = \frac{e^{x^2}}{2(1+x^2)}$$

output
```
exp(x^2)/(2*x^2+2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{e^{x^2} x^3}{(1+x^2)^2} \, dx = \frac{e^{x^2}}{2(1+x^2)}$$

input
```
Integrate[(E^x^2*x^3)/(1 + x^2)^2,x]
```

output
```
E^x^2/(2*(1 + x^2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2727}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x^2} x^3}{(x^2+1)^2} \, dx$$
$$\Big\downarrow \text{ 2727}$$
$$\frac{e^{x^2}}{2(x^2+1)}$$

input
```
Int[(E^x^2*x^3)/(1 + x^2)^2,x]
```

output
```
E^x^2/(2*(1 + x^2))
```

Defintions of rubi rules used

rule 2727
```
Int[(F_)^(u_)*(v_)^(n_.)*(w_), x_Symbol] :> With[{z = Log[F]*v*D[u, x] + (n
 + 1)*D[v, x]}, Simp[(Coefficient[w, x, Exponent[w, x]]/Coefficient[z, x, E
xponent[z, x]])*F^u*v^(n + 1), x] /; EqQ[Exponent[w, x], Exponent[z, x]] &&
 EqQ[w*Coefficient[z, x, Exponent[z, x]], z*Coefficient[w, x, Exponent[w, x
]]]] /; FreeQ[{F, n}, x] && PolynomialQ[u, x] && PolynomialQ[v, x] && Polyn
omialQ[w, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
gosper	$\frac{e^{x^2}}{2x^2+2}$	14
derivativedivides	$\frac{e^{x^2}}{2x^2+2}$	14
default	$\frac{e^{x^2}}{2x^2+2}$	14
norman	$\frac{e^{x^2}}{2x^2+2}$	14
risch	$\frac{e^{x^2}}{2x^2+2}$	14
parallelrisch	$\frac{e^{x^2}}{2x^2+2}$	14
orering	$\frac{e^{x^2}}{2x^2+2}$	14

input
```
int(exp(x^2)*x^3/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(x^2+1)*exp(x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{x^2}x^3}{(1+x^2)^2}\,dx = \frac{e^{(x^2)}}{2\,(x^2+1)}$$

input
```
integrate(exp(x^2)*x^3/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*e^(x^2)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \frac{e^{x^2}x^3}{(1+x^2)^2}\,dx = \frac{e^{x^2}}{2x^2+2}$$

input
```
integrate(exp(x**2)*x**3/(x**2+1)**2,x)
```

output
```
exp(x**2)/(2*x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{x^2}x^3}{(1+x^2)^2}\,dx = \frac{e^{(x^2)}}{2\,(x^2+1)}$$

input
```
integrate(exp(x^2)*x^3/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2*e^(x^2)/(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{e^{x^2}x^3}{(1+x^2)^2}\,dx = \frac{e^{(x^2)}}{2\,(x^2+1)}$$

input
```
integrate(exp(x^2)*x^3/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*e^(x^2)/(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{e^{x^2}x^3}{(1+x^2)^2}\,dx = \frac{e^{x^2}}{2\,(x^2+1)}$$

input
```
int((x^3*exp(x^2))/(x^2 + 1)^2,x)
```

output
```
exp(x^2)/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{e^{x^2}x^3}{(1+x^2)^2}\,dx = \frac{e^{x^2}}{2x^2+2}$$

input
```
int(exp(x^2)*x^3/(x^2+1)^2,x)
```

output
```
e**(x**2)/(2*(x**2 + 1))
```

3.654 $\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}} dx$

Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}} dx = \frac{1}{32}e^x\sqrt{25+16e^{2x}} - \frac{25}{128}\text{arcsinh}\left(\frac{4e^x}{5}\right)$$

output
```
1/32*exp(x)*(25+16*exp(2*x))^(1/2)-25/128*arcsinh(4/5*exp(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}} dx = \frac{1}{32}e^x\sqrt{25+16e^{2x}} - \frac{25}{128}\text{arcsinh}\left(\frac{4e^x}{5}\right)$$

input
```
Integrate[E^(3*x)/Sqrt[25 + 16*E^(2*x)],x]
```

output
```
(E^x*Sqrt[25 + 16*E^(2*x)])/32 - (25*ArcSinh[(4*E^x)/5])/128
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2678, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{3x}}{\sqrt{16e^{2x}+25}}dx$$
$$\Big\downarrow \text{2678}$$
$$\int \frac{e^{2x}}{\sqrt{16e^{2x}+25}}de^x$$
$$\Big\downarrow \text{262}$$
$$\frac{1}{32}e^x\sqrt{16e^{2x}+25}-\frac{25}{32}\int \frac{1}{\sqrt{25+16e^{2x}}}de^x$$
$$\Big\downarrow \text{222}$$
$$\frac{1}{32}e^x\sqrt{16e^{2x}+25}-\frac{25}{128}\text{arcsinh}\left(\frac{4e^x}{5}\right)$$

input
```
Int[E^(3*x)/Sqrt[25 + 16*E^(2*x)],x]
```

output
```
(E^x*Sqrt[25 + 16*E^(2*x)])/32 - (25*ArcSinh[(4*E^x)/5])/128
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{e^{x} \sqrt{25+16 e^{2 x}}}{32}-\frac{25 \,\operatorname{arcsinh}\left(\frac{4 e^{x}}{5}\right)}{128}$	23
risch	$\frac{e^{x} \sqrt{25+16 e^{2 x}}}{32}-\frac{25 \,\operatorname{arcsinh}\left(\frac{4 e^{x}}{5}\right)}{128}$	23

input
```
int(exp(3*x)/(25+16*exp(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*exp(x)*(25+16*exp(x)^2)^(1/2)-25/128*arcsinh(4/5*exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx=\frac{1}{32}\,\sqrt{16\,e^{(2\,x)}+25}e^{x}+\frac{25}{128}\,\log\left(\sqrt{16\,e^{(2\,x)}+25}-4\,e^{x}\right)$$

input
```
integrate(exp(3*x)/(25+16*exp(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/32*sqrt(16*e^(2*x) + 25)*e^x + 25/128*log(sqrt(16*e^(2*x) + 25) - 4*e^x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx = \frac{\sqrt{16e^{2x}+25}e^{x}}{32} - \frac{25\operatorname{asinh}\left(\frac{4e^{x}}{5}\right)}{128}$$

input
```
integrate(exp(3*x)/(25+16*exp(2*x))**(1/2),x)
```

output
```
sqrt(16*exp(2*x) + 25)*exp(x)/32 - 25*asinh(4*exp(x)/5)/128
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(22) = 44$.

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx = & \frac{25\sqrt{16\,e^{(2\,x)}+25}e^{(-x)}}{32\left((16\,e^{(2\,x)}+25)e^{(-2\,x)}-16\right)} \\ & - \frac{25}{256}\log\left(\sqrt{16\,e^{(2\,x)}+25}e^{(-x)}+4\right) \\ & + \frac{25}{256}\log\left(\sqrt{16\,e^{(2\,x)}+25}e^{(-x)}-4\right)\end{aligned}$$

input
```
integrate(exp(3*x)/(25+16*exp(2*x))^(1/2),x, algorithm="maxima")
```

output
```
25/32*sqrt(16*e^(2*x) + 25)*e^(-x)/((16*e^(2*x) + 25)*e^(-2*x) - 16) - 25/
256*log(sqrt(16*e^(2*x) + 25)*e^(-x) + 4) + 25/256*log(sqrt(16*e^(2*x) + 2
5)*e^(-x) - 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx = \frac{1}{32}\,\sqrt{16\,e^{(2\,x)}+25}e^{x} + \frac{25}{128}\,\log\left(\sqrt{16\,e^{(2\,x)}+25}-4\,e^{x}\right)$$

input
```
integrate(exp(3*x)/(25+16*exp(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/32*sqrt(16*e^(2*x) + 25)*e^x + 25/128*log(sqrt(16*e^(2*x) + 25) - 4*e^x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx = \int \frac{e^{3\,x}}{\sqrt{16\,e^{2\,x}+25}}\,dx$$

input
```
int(exp(3*x)/(16*exp(2*x) + 25)^(1/2),x)
```

output
```
int(exp(3*x)/(16*exp(2*x) + 25)^(1/2), x)
```

Reduce [F]

$$\int \frac{e^{3x}}{\sqrt{25+16e^{2x}}}\,dx = \frac{e^{x}\sqrt{16e^{2x}+25}}{32} - \frac{25\left(\int \frac{e^{x}\sqrt{16e^{2x}+25}}{16e^{2x}+25}dx\right)}{32}$$

input
```
int(exp(3*x)/(25+16*exp(2*x))^(1/2),x)
```

output
```
(e**x*sqrt(16*e**(2*x) + 25) - 25*int((e**x*sqrt(16*e**(2*x) + 25))/(16*e*
*(2*x) + 25),x))/32
```

3.655 $\int \frac{1+e^x}{\sqrt{e^x+x}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1+e^x}{\sqrt{e^x+x}} \, dx = 2\sqrt{e^x+x}$$

output
```
2*(exp(x)+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1+e^x}{\sqrt{e^x+x}} \, dx = 2\sqrt{e^x+x}$$

input
```
Integrate[(1 + E^x)/Sqrt[E^x + x],x]
```

output
```
2*Sqrt[E^x + x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x+1}{\sqrt{x+e^x}}\,dx$$

$$\Big\downarrow \text{ 7237}$$

$$2\sqrt{x+e^x}$$

input
```
Int[(1 + E^x)/Sqrt[E^x + x],x]
```

output
```
2*Sqrt[E^x + x]
```

Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\sqrt{{e}^{x}+x}$	9
default	$2\sqrt{{e}^{x}+x}$	9
risch	$2\sqrt{{e}^{x}+x}$	9

input
```
int((1+exp(x))/(exp(x)+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(exp(x)+x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1+e^x}{\sqrt{e^x+x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+exp(x))/(exp(x)+x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+e^x}{\sqrt{e^x+x}}\,dx = 2\sqrt{x+e^x}$$

input
```
integrate((1+exp(x))/(exp(x)+x)**(1/2),x)
```

output
```
2*sqrt(x + exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+e^x}{\sqrt{e^x+x}}\,dx = 2\,\sqrt{x+e^x}$$

input
```
integrate((1+exp(x))/(exp(x)+x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x + e^x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+e^x}{\sqrt{e^x+x}}\,dx = 2\sqrt{x+e^x}$$

input
```
integrate((1+exp(x))/(exp(x)+x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x + e^x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+e^x}{\sqrt{e^x+x}}\,dx = 2\sqrt{x+\mathrm{e}^x}$$

input
```
int((exp(x) + 1)/(x + exp(x))^(1/2),x)
```

output
```
2*(x + exp(x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{1+e^x}{\sqrt{e^x+x}}\,dx = 2\sqrt{e^x+x}$$

input
```
int((1+exp(x))/(exp(x)+x)^(1/2),x)
```

output
```
2*sqrt(e**x + x)
```

3.656 $\int \frac{1+e^x}{e^x+x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{1+e^x}{e^x+x} \, dx = \log \left(e^x+x\right)$$

output
```
ln(exp(x)+x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1+e^x}{e^x+x} \, dx = \log \left(e^x+x\right)$$

input
```
Integrate[(1 + E^x)/(E^x + x),x]
```

output
```
Log[E^x + x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {7235}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x+1}{x+e^x} \, dx$$

$$\Big\downarrow \text{7235}$$

$$\log \left(x+e^x\right)$$

input
```
Int[(1 + E^x)/(E^x + x),x]
```

output
```
Log[E^x + x]
```

Defintions of rubi rules used

rule 7235
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\ln \left({\mathrm e}^{x}+x \right)$	6
default	$\ln \left({\mathrm e}^{x}+x \right)$	6
norman	$\ln \left({\mathrm e}^{x}+x \right)$	6
risch	$\ln \left({\mathrm e}^{x}+x \right)$	6
parallelrisch	$\ln \left({\mathrm e}^{x}+x \right)$	6

input
```
int((1+exp(x))/(exp(x)+x),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(x)+x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{e^x+x}\,dx = \log\left(x+e^x\right)$$

input
```
integrate((1+exp(x))/(exp(x)+x),x, algorithm="fricas")
```

output
```
log(x + e^x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{e^x+x}\,dx = \log\left(x+e^x\right)$$

input
```
integrate((1+exp(x))/(exp(x)+x),x)
```

output
```
log(x + exp(x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{e^x+x}\,dx = \log\left(x+e^x\right)$$

input
```
integrate((1+exp(x))/(exp(x)+x),x, algorithm="maxima")
```

output
```
log(x + e^x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{e^x+x}\,dx = \log\left(x+e^x\right)$$

input
```
integrate((1+exp(x))/(exp(x)+x),x, algorithm="giac")
```

output
```
log(x + e^x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{1+e^x}{e^x+x}\,dx = \ln\left(x+\mathrm{e}^x\right)$$

input
```
int((exp(x) + 1)/(x + exp(x)),x)
```

output
```
log(x + exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{1+e^x}{e^x+x}\,dx = \log(e^x+x)$$

input
```
int((1+exp(x))/(exp(x)+x),x)
```

output
```
log(e**x + x)
```

3.657 $\int \frac{e^{x^2}}{x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 19

$$\int \frac{e^{x^2}}{x^2}\,dx = -\frac{e^{x^2}}{x} + \sqrt{\pi}\,\mathrm{erfi}(x)$$

output
```
-exp(x^2)/x+Pi^(1/2)*erfi(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{x^2}}{x^2}\,dx = -\frac{e^{x^2}}{x} + \sqrt{\pi}\,\mathrm{erfi}(x)$$

input
```
Integrate[E^x^2/x^2,x]
```

output
```
-(E^x^2/x) + Sqrt[Pi]*Erfi[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2643, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x^2}}{x^2} \, dx$$

$$\Big\downarrow \text{ 2643}$$

$$2 \int e^{x^2} dx - \frac{e^{x^2}}{x}$$

$$\Big\downarrow \text{ 2633}$$

$$\sqrt{\pi}\text{erfi}(x) - \frac{e^{x^2}}{x}$$

input
```
Int[E^x^2/x^2,x]
```

output
```
-(E^x^2/x) + Sqrt[Pi]*Erfi[x]
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2643
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(c + d*x)^(m + 1)*(F^(a + b*(c + d*x)^n)/(d*(m + 1)))
, x] - Simp[b*n*(Log[F]/(m + 1))   Int[(c + d*x)^(m + n)*F^(a + b*(c + d*x)
^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2*((m + 1)/n)] && LtQ[
-4, (m + 1)/n, 5] && IntegerQ[n] && ((GtQ[n, 0] && LtQ[m, -1]) || (GtQ[-n,
0] && LeQ[-n, m + 1]))
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{e^{x^2}}{x}+\sqrt{\pi}\,\operatorname{erfi}(x)$	17
risch	$-\frac{e^{x^2}}{x}+\sqrt{\pi}\,\operatorname{erfi}(x)$	17
meijerg	$\frac{i\left(\frac{2ie^{x^2}}{x}-2i\operatorname{erfi}(x)\sqrt{\pi}\right)}{2}$	23

input
```
int(exp(x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(x^2)/x+Pi^(1/2)*erfi(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{x^2}}{x^2}\,dx=\frac{\sqrt{\pi}x\operatorname{erfi}(x)-e^{(x^2)}}{x}$$

input
```
integrate(exp(x^2)/x^2,x, algorithm="fricas")
```

output
```
(sqrt(pi)*x*erfi(x) - e^(x^2))/x
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{e^{x^2}}{x^2}\,dx=\sqrt{\pi}\operatorname{erfi}(x)-\frac{e^{x^2}}{x}$$

input
```
integrate(exp(x**2)/x**2,x)
```

output
```
sqrt(pi)*erfi(x) - exp(x**2)/x
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{x^2}}{x^2}\,dx = -\frac{\sqrt{-x^2}\,\Gamma\left(-\frac{1}{2},-x^2\right)}{2\,x}$$

input
```
integrate(exp(x^2)/x^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2)*gamma(-1/2, -x^2)/x
```

Giac [F]

$$\int \frac{e^{x^2}}{x^2}\,dx = \int \frac{e^{(x^2)}}{x^2}\,dx$$

input
```
integrate(exp(x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(e^(x^2)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{e^{x^2}}{x^2}\,dx = -\frac{e^{x^2}}{x} + \sqrt{\pi}\,\mathrm{erfc}(x\,1\mathrm{i})\,1\mathrm{i}$$

input
```
int(exp(x^2)/x^2,x)
```

output
```
pi^(1/2)*erfc(x*1i)*1i - exp(x^2)/x
```

Reduce [F]

$$\int \frac{e^{x^2}}{x^2} dx = \int \frac{e^{x^2}}{x^2} dx$$

input
```
int(exp(x^2)/x^2,x)
```

output
```
int(e**(x**2)/x**2,x)
```

3.658 $\int \frac{e^{x^2}(1+4x^4)}{x^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{e^{x^2}(1+4x^4)}{x^2} \, dx = -\frac{e^{x^2}}{x} + 2e^{x^2}x$$

output
```
-exp(x^2)/x+2*exp(x^2)*x
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{e^{x^2}(1+4x^4)}{x^2} \, dx = \frac{e^{x^2}(-1+2x^2)}{x}$$

input
```
Integrate[(E^x^2*(1 + 4*x^4))/x^2,x]
```

output
```
(E^x^2*(-1 + 2*x^2))/x
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{x^2}(4x^4+1)}{x^2}\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(4e^{x^2}x^2+\frac{e^{x^2}}{x^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$2e^{x^2}x-\frac{e^{x^2}}{x}$$

input
```
Int[(E^x^2*(1 + 4*x^4))/x^2,x]
```

output
```
-(E^x^2/x) + 2*E^x^2*x
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{e^{x^2}(2x^2-1)}{x}$	16
risch	$\frac{e^{x^2}(2x^2-1)}{x}$	16
default	$-\frac{e^{x^2}}{x}+2\,e^{x^2}x$	18
norman	$\frac{2\,e^{x^2}x^2-e^{x^2}}{x}$	21
parallelrisch	$\frac{2\,e^{x^2}x^2-e^{x^2}}{x}$	21
meijerg	$2i\left(-ix\,e^{x^2}+\frac{i\,\mathrm{erfi}(x)\sqrt{\pi}}{2}\right)+\frac{i\left(\frac{2i\,e^{x^2}}{x}-2i\,\mathrm{erfi}(x)\sqrt{\pi}\right)}{2}$	44
orering	$\frac{(2x^2-1)e^{x^2}(4x^4+1)}{x(2x^2-2x+1)(2x^2+2x+1)}$	47

input
```
int(exp(x^2)*(4*x^4+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
exp(x^2)*(2*x^2-1)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{e^{x^2}(1+4x^4)}{x^2}\,dx=\frac{(2\,x^2-1)e^{(x^2)}}{x}$$

input
```
integrate(exp(x^2)*(4*x^4+1)/x^2,x, algorithm="fricas")
```

output
```
(2*x^2 - 1)*e^(x^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{e^{x^2}(1+4x^4)}{x^2} \, dx = \frac{(2x^2-1)\, e^{x^2}}{x}$$

input
```
integrate(exp(x**2)*(4*x**4+1)/x**2,x)
```

output
```
(2*x**2 - 1)*exp(x**2)/x
```

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{e^{x^2}(1+4x^4)}{x^2} \, dx = 2\, xe^{(x^2)} + i\,\sqrt{\pi}\, \mathrm{erf}\,(i\,x) - \frac{\sqrt{-x^2}\Gamma\left(-\frac{1}{2}, -x^2\right)}{2\,x}$$

input
```
integrate(exp(x^2)*(4*x^4+1)/x^2,x, algorithm="maxima")
```

output
```
2*x*e^(x^2) + I*sqrt(pi)*erf(I*x) - 1/2*sqrt(-x^2)*gamma(-1/2, -x^2)/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{e^{x^2}(1+4x^4)}{x^2} \, dx = \frac{2\, x^2 e^{(x^2)} - e^{(x^2)}}{x}$$

input
```
integrate(exp(x^2)*(4*x^4+1)/x^2,x, algorithm="giac")
```

output
```
(2*x^2*e^(x^2) - e^(x^2))/x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{e^{x^2}(1+4x^4)}{x^2}\,dx = \frac{e^{x^2}\,(2\,x^2-1)}{x}$$

input
```
int((exp(x^2)*(4*x^4 + 1))/x^2,x)
```

output
```
(exp(x^2)*(2*x^2 - 1))/x
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{e^{x^2}(1+4x^4)}{x^2}\,dx = \frac{e^{x^2}(2x^2-1)}{x}$$

input
```
int(exp(x^2)*(4*x^4+1)/x^2,x)
```

output
```
(e**(x**2)*(2*x**2 - 1))/x
```

3.659 $\int \sqrt{f^x}(a+bx)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \sqrt{f^x}(a+bx)^2\,dx=\frac{16b^2\sqrt{f^x}}{\log^3(f)}-\frac{8b\sqrt{f^x}(a+bx)}{\log^2(f)}+\frac{2\sqrt{f^x}(a+bx)^2}{\log(f)}$$

output
```
16*b^2*(f^x)^(1/2)/ln(f)^3-8*b*(f^x)^(1/2)*(b*x+a)/ln(f)^2+2*(f^x)^(1/2)*(
b*x+a)^2/ln(f)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \sqrt{f^x}(a+bx)^2\,dx=\frac{2\sqrt{f^x}(8b^2-4b(a+bx)\log(f)+(a+bx)^2\log^2(f))}{\log^3(f)}$$

input
```
Integrate[Sqrt[f^x]*(a + b*x)^2,x]
```

output
```
(2*Sqrt[f^x]*(8*b^2 - 4*b*(a + b*x)*Log[f] + (a + b*x)^2*Log[f]^2))/Log[f]
^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{f^x}(a+bx)^2 \, dx$$

$$\Big\downarrow \; 2607$$

$$\frac{2\sqrt{f^x}(a+bx)^2}{\log(f)} - \frac{4b\int \sqrt{f^x}(a+bx)dx}{\log(f)}$$

$$\Big\downarrow \; 2607$$

$$\frac{2\sqrt{f^x}(a+bx)^2}{\log(f)} - \frac{4b\left(\frac{2\sqrt{f^x}(a+bx)}{\log(f)} - \frac{2b\int \sqrt{f^x}dx}{\log(f)}\right)}{\log(f)}$$

$$\Big\downarrow \; 2624$$

$$\frac{2\sqrt{f^x}(a+bx)^2}{\log(f)} - \frac{4b\left(\frac{2\sqrt{f^x}(a+bx)}{\log(f)} - \frac{4b\sqrt{f^x}}{\log^2(f)}\right)}{\log(f)}$$

input
```
Int[Sqrt[f^x]*(a + b*x)^2,x]
```

output
```
(2*Sqrt[f^x]*(a + b*x)^2)/Log[f] - (4*b*((-4*b*Sqrt[f^x])/Log[f]^2 + (2*Sq
rt[f^x]*(a + b*x))/Log[f]))/Log[f]
```

Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

method	result
gosper	$\frac{2\left(b^{2} x^{2} \ln \left(f\right)^{2}+2 \ln \left(f\right)^{2} a b x+\ln \left(f\right)^{2} a^{2}-4 \ln \left(f\right) b^{2} x-4 \ln \left(f\right) b a+8 b^{2}\right) \sqrt{f^{x}}}{\ln \left(f\right)^{3}}$
risch	$\frac{2\left(b^{2} x^{2} \ln \left(f\right)^{2}+2 \ln \left(f\right)^{2} a b x+\ln \left(f\right)^{2} a^{2}-4 \ln \left(f\right) b^{2} x-4 \ln \left(f\right) b a+8 b^{2}\right) \sqrt{f^{x}}}{\ln \left(f\right)^{3}}$
orering	$\frac{2\left(b^{2} x^{2} \ln \left(f\right)^{2}+2 \ln \left(f\right)^{2} a b x+\ln \left(f\right)^{2} a^{2}-4 \ln \left(f\right) b^{2} x-4 \ln \left(f\right) b a+8 b^{2}\right) \sqrt{f^{x}}}{\ln \left(f\right)^{3}}$
parallelrisch	$\frac{2 x^{2} \sqrt{f^{x}}\, b^{2} \ln \left(f\right)^{2}+4 \ln \left(f\right)^{2} x \sqrt{f^{x}}\, a b+2 \ln \left(f\right)^{2} \sqrt{f^{x}}\, a^{2}-8 \ln \left(f\right) x \sqrt{f^{x}}\, b^{2}-8 \ln \left(f\right) \sqrt{f^{x}}\, a b+16 \sqrt{f^{x}}\, b^{2}}{\ln \left(f\right)^{3}}$
meijerg	$-\frac{8 \sqrt{f^{x}}\, f^{-\frac{x}{2}} b^{2} \left(2-\frac{\left(\frac{3 x^{2} \ln \left(f\right)^{2}}{4}-3 x \ln \left(f\right)+6\right) {\mathrm e}^{\frac{x \ln \left(f\right)}{2}}}{3}\right)}{\ln \left(f\right)^{3}}+\frac{8 \sqrt{f^{x}}\, f^{-\frac{x}{2}} b a \left(1-\frac{\left(-x \ln \left(f\right)+2\right) {\mathrm e}^{\frac{x \ln \left(f\right)}{2}}}{2}\right)}{\ln \left(f\right)^{2}}-\frac{2 \sqrt{f^{x}}\, f^{-\frac{x}{2}} a^{2} \left(1-\right.}{\ln \left(f\right)}$

input
```
int((f^x)^(1/2)*(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*(b^2*x^2*ln(f)^2+2*ln(f)^2*a*b*x+ln(f)^2*a^2-4*ln(f)*b^2*x-4*ln(f)*b*a+8
*b^2)*(f^x)^(1/2)/ln(f)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{f^x}(a+bx)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((f^x)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.73

$$\int \sqrt{f^x}(a+bx)^2\,dx = \begin{cases} \frac{\left(2a^2\log{(f)}^2+4abx\log{(f)}^2-8ab\log{(f)}+2b^2x^2\log{(f)}^2-8b^2x\log{(f)}+16b^2\right)\sqrt{e^{x\log{(f)}}}}{\log{(f)}^3} & \text{for}\ \log{(f)}^3\neq 0 \\ a^2x+abx^2+\frac{b^2x^3}{3} & \text{otherwise} \end{cases}$$

input
```
integrate((f**x)**(1/2)*(b*x+a)**2,x)
```

output
```
Piecewise(((2*a**2*log(f)**2 + 4*a*b*x*log(f)**2 - 8*a*b*log(f) + 2*b**2*x
**2*log(f)**2 - 8*b**2*x*log(f) + 16*b**2)*sqrt(exp(x*log(f)))/log(f)**3,
Ne(log(f)**3, 0)), (a**2*x + a*b*x**2 + b**2*x**3/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \sqrt{f^x}(a+bx)^2\,dx = \frac{4\left(x\log{(f)}-2\right)abf^{\frac{1}{2}x}}{\log{(f)}^2}+\frac{2a^2f^{\frac{1}{2}x}}{\log{(f)}}+\frac{2\left(x^2\log{(f)}^2-4x\log{(f)}+8\right)b^2f^{\frac{1}{2}x}}{\log{(f)}^3}$$

input
```
integrate((f^x)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

output
```
4*(x*log(f) - 2)*a*b*f^(1/2*x)/log(f)^2 + 2*a^2*f^(1/2*x)/log(f) + 2*(x^2*
log(f)^2 - 4*x*log(f) + 8)*b^2*f^(1/2*x)/log(f)^3
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 1392, normalized size of antiderivative = 24.86

$$\int \sqrt{f^x}(a+bx)^2 \, dx = \text{Too large to display}$$

input
```
integrate((f^x)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*((2*(pi*b^2*x^2*log(abs(f))*sgn(f) - pi*b^2*x^2*log(abs(f)) + 2*pi*a*b*
x*log(abs(f))*sgn(f) - 2*pi*a*b*x*log(abs(f)) - 2*pi*b^2*x*sgn(f) + pi*a^2
*log(abs(f))*sgn(f) + 2*pi*b^2*x - pi*a^2*log(abs(f)) - 2*pi*a*b*sgn(f) +
2*pi*a*b)*(pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(f
))^2)/((pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(f))^
2)^2 + (3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^3)^
2) - (pi^2*b^2*x^2*sgn(f) - pi^2*b^2*x^2 + 2*b^2*x^2*log(abs(f))^2 + 2*pi^
2*a*b*x*sgn(f) - 2*pi^2*a*b*x + 4*a*b*x*log(abs(f))^2 + pi^2*a^2*sgn(f) -
pi^2*a^2 - 8*b^2*x*log(abs(f)) + 2*a^2*log(abs(f))^2 - 8*a*b*log(abs(f)) +
 16*b^2)*(3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^3
)/((pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn(f) - pi^3 + 3*pi*log(abs(f))^2)^2
 + (3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*log(abs(f)) + 2*log(abs(f))^3)^2))*
cos(-1/4*pi*x*sgn(f) + 1/4*pi*x) - ((pi^2*b^2*x^2*sgn(f) - pi^2*b^2*x^2 +
2*b^2*x^2*log(abs(f))^2 + 2*pi^2*a*b*x*sgn(f) - 2*pi^2*a*b*x + 4*a*b*x*log
(abs(f))^2 + pi^2*a^2*sgn(f) - pi^2*a^2 - 8*b^2*x*log(abs(f)) + 2*a^2*log(
abs(f))^2 - 8*a*b*log(abs(f)) + 16*b^2)*(pi^3*sgn(f) - 3*pi*log(abs(f))^2*
sgn(f) - pi^3 + 3*pi*log(abs(f))^2)/((pi^3*sgn(f) - 3*pi*log(abs(f))^2*sgn
(f) - pi^3 + 3*pi*log(abs(f))^2)^2 + (3*pi^2*log(abs(f))*sgn(f) - 3*pi^2*l
og(abs(f)) + 2*log(abs(f))^3)^2) + 2*(pi*b^2*x^2*log(abs(f))*sgn(f) - pi*b
^2*x^2*log(abs(f)) + 2*pi*a*b*x*log(abs(f))*sgn(f) - 2*pi*a*b*x*log(abs...
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \sqrt{f^x}(a+bx)^2\,dx = \sqrt{f^x}\left(\frac{2\,a^2\ln(f)^2-8\,a\,b\,\ln(f)+16\,b^2}{\ln(f)^3}+\frac{2\,b^2\,x^2}{\ln(f)}-\frac{4\,b\,x\,(2\,b-a\,\ln(f))}{\ln(f)^2}\right)$$

input
```
int((f^x)^(1/2)*(a + b*x)^2,x)
```

output
```
(f^x)^(1/2)*((2*a^2*log(f)^2 + 16*b^2 - 8*a*b*log(f))/log(f)^3 + (2*b^2*x^
2)/log(f) - (4*b*x*(2*b - a*log(f)))/log(f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

$$\int \sqrt{f^x}(a+bx)^2\,dx = \frac{2f^{\frac{x}{2}}\left(\log(f)^2 a^2+2\log(f)^2 abx+\log(f)^2 b^2x^2-4\log(f)\,ab-4\log(f)\,b^2x+8b^2\right)}{\log(f)^3}$$

input
```
int((f^x)^(1/2)*(b*x+a)^2,x)
```

output
```
(2*f**(x/2)*(log(f)**2*a**2 + 2*log(f)**2*a*b*x + log(f)**2*b**2*x**2 - 4*
log(f)*a*b - 4*log(f)*b**2*x + 8*b**2))/log(f)**3
```

3.660 $\int 3^{1+x^2} x \, dx$

Optimal result

Integrand size = 9, antiderivative size = 15

$$\int 3^{1+x^2} x \, dx = \frac{3^{1+x^2}}{2 \log(3)}$$

output
```
1/2*3^(x^2+1)/ln(3)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int 3^{1+x^2} x \, dx = \frac{3^{1+x^2}}{\log(9)}$$

input
```
Integrate[3^(1 + x^2)*x,x]
```

output
```
3^(1 + x^2)/Log[9]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 3^{x^2+1} x \, dx$$
$$\Big\downarrow \text{ 2638}$$
$$\frac{3^{x^2+1}}{2\log(3)}$$

input
```
Int[3^(1 + x^2)*x,x]
```

output
```
3^(1 + x^2)/(2*Log[3])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{3\,3^{x^2}}{2\ln(3)}$	12
default	$\frac{3\,3^{x^2}}{2\ln(3)}$	12
gosper	$\frac{3^{x^2+1}}{2\ln(3)}$	14
risch	$\frac{3^{x^2+1}}{2\ln(3)}$	14
parallelrisch	$\frac{3^{x^2+1}}{2\ln(3)}$	14
orering	$\frac{3^{x^2+1}}{2\ln(3)}$	14
norman	$\frac{\mathrm{e}^{\left(x^2+1\right)\ln(3)}}{2\ln(3)}$	16
meijerg	$-\frac{3\left(1-\mathrm{e}^{\ln(3)x^2}\right)}{2\ln(3)}$	18

input
```
int(3^(x^2+1)*x,x,method=_RETURNVERBOSE)
```

output
```
3/2*3^(x^2)/ln(3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int 3^{1+x^2}x\,dx = \frac{3^{x^2+1}}{2\,\log{(3)}}$$

input
```
integrate(3^(x^2+1)*x,x, algorithm="fricas")
```

output
```
1/2*3^(x^2 + 1)/log(3)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int 3^{1+x^2}x\,dx = \frac{3^{x^2+1}}{2\log(3)}$$

input
```
integrate(3**(x**2+1)*x,x)
```

output
```
3**(x**2 + 1)/(2*log(3))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int 3^{1+x^2}x\,dx = \frac{3^{x^2+1}}{2\log(3)}$$

input
```
integrate(3^(x^2+1)*x,x, algorithm="maxima")
```

output
```
1/2*3^(x^2 + 1)/log(3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int 3^{1+x^2}x\,dx = \frac{3^{x^2+1}}{2\log(3)}$$

input
```
integrate(3^(x^2+1)*x,x, algorithm="giac")
```

output
```
1/2*3^(x^2 + 1)/log(3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int 3^{1+x^2} x \, dx = \frac{3\, 3^{x^2}}{2 \ln(3)}$$

input
```
int(3^(x^2 + 1)*x,x)
```

output
```
(3*3^(x^2))/(2*log(3))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int 3^{1+x^2} x \, dx = \frac{3\, 3^{x^2}}{2 \log(3)}$$

input
```
int(3^(x^2+1)*x,x)
```

output
```
(3*3**(x**2))/(2*log(3))
```

3.661 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx=\frac{2^{1+\sqrt{x}}}{\log(2)}$$

output
```
2^(1+x^(1/2))/ln(2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx=\frac{2^{1+\sqrt{x}}}{\log(2)}$$

input
```
Integrate[2^Sqrt[x]/Sqrt[x],x]
```

output
```
2^(1 + Sqrt[x])/Log[2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

input
```
Int[2^Sqrt[x]/Sqrt[x],x]
```

output
```
2^(1 + Sqrt[x])/Log[2]
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
default	$\frac{2\,2^{\sqrt{x}}}{\ln(2)}$	12
meijerg	$-\frac{2\left(1-\mathrm{e}^{\sqrt{x}\,\ln(2)}\right)}{\ln(2)}$	18

input
```
int(2^(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(x^(1/2))/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\cdot 2^{(\sqrt{x})}}{\log(2)}$$

input
```
integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*2^sqrt(x)/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\cdot 2^{\sqrt{x}}}{\log(2)}$$

input
```
integrate(2**(x**(1/2))/x**(1/2),x)
```

output
```
2*2**(sqrt(x))/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{\sqrt{x}+1}}{\log(2)}$$

input
```
integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2^(sqrt(x) + 1)/log(2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2 \cdot 2^{(\sqrt{x})}}{\log(2)}$$

input
```
integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*2^sqrt(x)/log(2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\,2^{\sqrt{x}}}{\ln(2)}$$

input
```
int(2^(x^(1/2))/x^(1/2),x)
```

output
```
(2*2^(x^(1/2)))/log(2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\,2^{\sqrt{x}}}{\log(2)}$$

input
```
int(2^(x^(1/2))/x^(1/2),x)
```

output
```
(2*2**sqrt(x))/log(2)
```

3.662 $\int \frac{2^{\frac{1}{x}}}{x^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 11

$$\int \frac{2^{\frac{1}{x}}}{x^2}\,dx = -\frac{2^{\frac{1}{x}}}{\log(2)}$$

output
```
-2^(1/x)/ln(2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2^{\frac{1}{x}}}{x^2}\,dx = -\frac{2^{\frac{1}{x}}}{\log(2)}$$

input
```
Integrate[2^x^(-1)/x^2,x]
```

output
```
-(2^x^(-1)/Log[2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2^{\frac{1}{x}}}{x^2} \, dx$$

$$\Big\downarrow \text{2638}$$

$$-\frac{2^{\frac{1}{x}}}{\log(2)}$$

input
```
Int[2^x^(-1)/x^2,x]
```

output
```
-(2^x^(-1)/Log[2])
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
gosper	$-\frac{2^{\frac{1}{x}}}{\ln(2)}$	12
derivativedivides	$-\frac{2^{\frac{1}{x}}}{\ln(2)}$	12
default	$-\frac{2^{\frac{1}{x}}}{\ln(2)}$	12
risch	$-\frac{2^{\frac{1}{x}}}{\ln(2)}$	12
parallelrisch	$-\frac{2^{\frac{1}{x}}}{\ln(2)}$	12
norman	$-\frac{e^{\frac{\ln(2)}{x}}}{\ln(2)}$	14
meijerg	$\frac{1-e^{\frac{\ln(2)}{x}}}{\ln(2)}$	17

input
```
int(2^(1/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2^(1/x)/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2^{\frac{1}{x}}}{x^2}\,dx = -\frac{2^{\left(\frac{1}{x}\right)}}{\log(2)}$$

input
```
integrate(2^(1/x)/x^2,x, algorithm="fricas")
```

output
```
-2^(1/x)/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{2^{\frac{1}{x}}}{x^2} \, dx = -\frac{2^{\frac{1}{x}}}{\log (2)}$$

input
```
integrate(2**(1/x)/x**2,x)
```

output
```
-2**(1/x)/log(2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2^{\frac{1}{x}}}{x^2} \, dx = -\frac{2^{\left(\frac{1}{x}\right)}}{\log (2)}$$

input
```
integrate(2^(1/x)/x^2,x, algorithm="maxima")
```

output
```
-2^(1/x)/log(2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2^{\frac{1}{x}}}{x^2} \, dx = -\frac{2^{\left(\frac{1}{x}\right)}}{\log (2)}$$

input
```
integrate(2^(1/x)/x^2,x, algorithm="giac")
```

output
```
-2^(1/x)/log(2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2^{\frac{1}{x}}}{x^2} \, dx = -\frac{2^{1/x}}{\ln(2)}$$

input
```
int(2^(1/x)/x^2,x)
```

output
```
-2^(1/x)/log(2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{2^{\frac{1}{x}}}{x^2} \, dx = -\frac{2^{\frac{1}{x}}}{\log(2)}$$

input
```
int(2^(1/x)/x^2,x)
```

output
```
( - 2**(1/x))/log(2)
```

3.663 $\int (2^{-x} + 2^x) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 20

$$\int (2^{-x} + 2^x) \, dx = -\frac{2^{-x}}{\log(2)} + \frac{2^x}{\log(2)}$$

output
```
-1/(2^x)/ln(2)+2^x/ln(2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (2^{-x} + 2^x) \, dx = -\frac{2^{-x}}{\log(2)} + \frac{2^x}{\log(2)}$$

input
```
Integrate[2^(-x) + 2^x,x]
```

output
```
-(1/(2^x*Log[2])) + 2^x/Log[2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2^{-x}+2^x\right) \, dx$$

$$\Big\downarrow \ \color{blue}{2009}$$

$$\frac{2^x}{\log(2)}-\frac{2^{-x}}{\log(2)}$$

input
```
Int[2^(-x) + 2^x,x]
```

output
```
-(1/(2^x*Log[2])) + 2^x/Log[2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2^{x}-2^{-x}}{\ln(2)}$	17
parallelrisch	$\frac{2^{x}-2^{-x}}{\ln(2)}$	17
risch	$-\frac{\left(\frac{1}{2}\right)^{x}}{\ln(2)}+\frac{2^{x}}{\ln(2)}$	19
default	$-\frac{2^{-x}}{\ln(2)}+\frac{2^{x}}{\ln(2)}$	21
parts	$-\frac{2^{-x}}{\ln(2)}+\frac{2^{x}}{\ln(2)}$	21
orering	$\frac{-2^{-x}\ln(2)+2^{x}\ln(2)}{\ln(2)^{2}}$	22
norman	$\left(\frac{e^{2x\ln(2)}}{\ln(2)}-\frac{1}{\ln(2)}\right)e^{-x\ln(2)}$	28
meijerg	$\frac{1-e^{-x\ln(2)}}{\ln(2)}-\frac{1-e^{x\ln(2)}}{\ln(2)}$	32

input
```
int(2^(-x)+2^x,x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(2^x-1/(2^x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(2^{-x}+2^{x}\right)\,dx=\frac{2^{2\,x}-1}{2^{x}\log\left(2\right)}$$

input
```
integrate(2^(-x)+2^x,x, algorithm="fricas")
```

output
```
(2^(2*x) - 1)/(2^x*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(2^{-x}+2^x\right) \, dx=\frac{2^x \log (2)-2^{-x} \log (2)}{\log (2)^2}$$

input
```
integrate(2**(-x)+2**x,x)
```

output
```
(2**x*log(2) - log(2)/2**x)/log(2)**2
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(2^{-x}+2^x\right) \, dx=\frac{2^x}{\log (2)}-\frac{1}{2^x \log (2)}$$

input
```
integrate(2^(-x)+2^x,x, algorithm="maxima")
```

output
```
2^x/log(2) - 1/(2^x*log(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(2^{-x}+2^x\right) \, dx=\frac{2^x}{\log (2)}-\frac{1}{2^x \log (2)}$$

input
```
integrate(2^(-x)+2^x,x, algorithm="giac")
```

output
```
2^x/log(2) - 1/(2^x*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(2^{-x}+2^x\right)\,dx=\frac{2^{2\,x}-1}{2^x\,\ln\left(2\right)}$$

input
```
int(1/2^x + 2^x,x)
```

output
```
(2^(2*x) - 1)/(2^x*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \left(2^{-x}+2^x\right)\,dx=\frac{2^{2x}-1}{2^x\log\left(2\right)}$$

input
```
int(2^(-x)+2^x,x)
```

output
```
(2**(2*x) - 1)/(2**x*log(2))
```

3.664 $\int e^{-4x}(2-3x+x^2)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 32

$$\int e^{-4x}\left(2-3x+x^2\right)\,dx=-\frac{11}{32}e^{-4x}+\frac{5}{8}e^{-4x}x-\frac{1}{4}e^{-4x}x^2$$

output
```
-11/32/exp(4*x)+5/8*x/exp(4*x)-1/4*x^2/exp(4*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int e^{-4x}\left(2-3x+x^2\right)\,dx=-\frac{1}{32}e^{-4x}\left(11-20x+8x^2\right)$$

input
```
Integrate[(2 - 3*x + x^2)/E^(4*x),x]
```

output
```
-1/32*(11 - 20*x + 8*x^2)/E^(4*x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-4x}\left(x^2-3x+2\right)\,dx$$

$$\Big\downarrow \text{2626}$$

$$\int \left(e^{-4x}x^2-3e^{-4x}x+2e^{-4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{4}e^{-4x}x^2+\frac{5}{8}e^{-4x}x-\frac{11e^{-4x}}{32}$$

input
```
Int[(2 - 3*x + x^2)/E^(4*x),x]
```

output
```
-11/(32*E^(4*x)) + (5*x)/(8*E^(4*x)) - x^2/(4*E^(4*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2626
```
Int[(F_)^(v_)*(Px_), x_Symbol] :> Int[ExpandIntegrand[F^v, Px, x], x] /; Fr
eeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

method	result	size
risch	$\left(-\frac{1}{4}x^2+\frac{5}{8}x-\frac{11}{32}\right)\mathrm{e}^{-4x}$	16
norman	$\left(-\frac{1}{4}x^2+\frac{5}{8}x-\frac{11}{32}\right)\mathrm{e}^{-4x}$	18
gosper	$-\frac{(8x^2-20x+11)\mathrm{e}^{-4x}}{32}$	19
parallelrisch	$\frac{(-8x^2+20x-11)\mathrm{e}^{-4x}}{32}$	19
derivativedivides	$-\frac{11\,\mathrm{e}^{-4x}}{32}+\frac{5\,\mathrm{e}^{-4x}x}{8}-\frac{\mathrm{e}^{-4x}x^2}{4}$	30
default	$-\frac{11\,\mathrm{e}^{-4x}}{32}+\frac{5\,\mathrm{e}^{-4x}x}{8}-\frac{\mathrm{e}^{-4x}x^2}{4}$	30
meijerg	$\frac{11}{32}-\frac{(48x^2+24x+6)\mathrm{e}^{-4x}}{192}+\frac{3(2+8x)\mathrm{e}^{-4x}}{32}-\frac{\mathrm{e}^{-4x}}{2}$	36
orering	$-\frac{(8x^2-20x+11)(x^2-3x+2)\mathrm{e}^{-4x}}{32(-1+x)(-2+x)}$	37

input
```
int((x^2-3*x+2)/exp(4*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/4*x^2+5/8*x-11/32)*exp(-4*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-4x}(2-3x+x^2)\,dx=-\frac{1}{32}\left(8x^2-20x+11\right)e^{(-4x)}$$

input
```
integrate((x^2-3*x+2)/exp(4*x),x, algorithm="fricas")
```

output
```
-1/32*(8*x^2 - 20*x + 11)*e^(-4*x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.47

$$\int e^{-4x}(2-3x+x^2)\ dx=\frac{(-8x^2+20x-11)\,e^{-4x}}{32}$$

input
```
integrate((x**2-3*x+2)/exp(4*x),x)
```

output
```
(-8*x**2 + 20*x - 11)*exp(-4*x)/32
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int e^{-4x}(2-3x+x^2)\ dx=-\frac{1}{32}\,(8\,x^2+4\,x+1)e^{(-4\,x)}+\frac{3}{16}\,(4\,x+1)e^{(-4\,x)}-\frac{1}{2}\,e^{(-4\,x)}$$

input
```
integrate((x^2-3*x+2)/exp(4*x),x, algorithm="maxima")
```

output
```
-1/32*(8*x^2 + 4*x + 1)*e^(-4*x) + 3/16*(4*x + 1)*e^(-4*x) - 1/2*e^(-4*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-4x}(2-3x+x^2)\ dx=-\frac{1}{32}\,(8\,x^2-20\,x+11)e^{(-4\,x)}$$

input
```
integrate((x^2-3*x+2)/exp(4*x),x, algorithm="giac")
```

output
```
-1/32*(8*x^2 - 20*x + 11)*e^(-4*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.50

$$\int e^{-4x}(2-3x+x^2)\,dx = -\frac{\mathrm{e}^{-4\,x}\,(8\,x^2-20\,x+11)}{32}$$

input
```
int(exp(-4*x)*(x^2 - 3*x + 2),x)
```

output
```
-(exp(-4*x)*(8*x^2 - 20*x + 11))/32
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.59

$$\int e^{-4x}(2-3x+x^2)\,dx = \frac{-8x^2+20x-11}{32e^{4x}}$$

input
```
int((x^2-3*x+2)/exp(4*x),x)
```

output
```
( - 8*x**2 + 20*x - 11)/(32*e**(4*x))
```

3.665 $\int \left(k^{x/2}+x^{\sqrt{k}}\right) dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right) dx=\frac{x^{1+\sqrt{k}}}{1+\sqrt{k}}+\frac{2k^{x/2}}{\log(k)}$$

output
```
x^(1+k^(1/2))/(1+k^(1/2))+2*k^(1/2*x)/ln(k)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right) dx=\frac{x^{1+\sqrt{k}}}{1+\sqrt{k}}+\frac{2k^{x/2}}{\log(k)}$$

input
```
Integrate[k^(x/2) + x^Sqrt[k],x]
```

output
```
x^(1 + Sqrt[k])/(1 + Sqrt[k]) + (2*k^(x/2))/Log[k]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2k^{x/2}}{\log(k)}+\frac{x^{\sqrt{k}+1}}{\sqrt{k}+1}$$

input
```
Int[k^(x/2) + x^Sqrt[k],x]
```

output
```
x^(1 + Sqrt[k])/(1 + Sqrt[k]) + (2*k^(x/2))/Log[k]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{x^{1+\sqrt{k}}}{1+\sqrt{k}}+\frac{2k^{\frac{x}{2}}}{\ln(k)}$	28
parts	$\frac{x^{1+\sqrt{k}}}{1+\sqrt{k}}+\frac{2k^{\frac{x}{2}}}{\ln(k)}$	28
norman	$\frac{x\,\mathrm{e}^{\sqrt{k}\,\ln(x)}}{1+\sqrt{k}}+\frac{2\,\mathrm{e}^{\frac{x\ln(k)}{2}}}{\ln(k)}$	30
risch	$\frac{2k^{\frac{x}{2}}}{\ln(k)}+\frac{\left(\sqrt{k}-1\right)x\,x^{\sqrt{k}}}{k-1}$	30
parallelrisch	$\frac{x\,x^{\sqrt{k}}\ln(k)+2k^{\frac{x}{2}}\sqrt{k}+2k^{\frac{x}{2}}}{\left(1+\sqrt{k}\right)\ln(k)}$	40

input
```
int(k^(1/2*x)+x^(k^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x^(1+k^(1/2))/(1+k^(1/2))+2*k^(1/2*x)/ln(k)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int\left(k^{x/2}+x^{\sqrt{k}}\right)\,dx=\frac{2\,(k-1)k^{\frac{1}{2}\,x}+\left(\sqrt{k}x\log\left(k\right)-x\log\left(k\right)\right)x^{\left(\sqrt{k}\right)}}{(k-1)\log\left(k\right)}$$

input
```
integrate(k^(1/2*x)+x^(k^(1/2)),x, algorithm="fricas")
```

output
```
(2*(k - 1)*k^(1/2*x) + (sqrt(k)*x*log(k) - x*log(k))*x^sqrt(k))/((k - 1)*l
og(k))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right)\,dx=\begin{cases}\frac{2k^{\frac{x}{2}}}{\log\left(k\right)} & \text{for}\ \log\left(k\right)\neq 0\\ x & \text{otherwise}\end{cases}+\begin{cases}\frac{x^{\sqrt{k}+1}}{\sqrt{k}+1} & \text{for}\ \sqrt{k}\neq -1\\ \log\left(x\right) & \text{otherwise}\end{cases}$$

input
```
integrate(k**(1/2*x)+x**(k**(1/2)),x)
```

output
```
Piecewise((2*k**(x/2)/log(k), Ne(log(k), 0)), (x, True)) + Piecewise((x**(
sqrt(k) + 1)/(sqrt(k) + 1), Ne(sqrt(k), -1)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right)\,dx=\frac{x^{\sqrt{k}+1}}{\sqrt{k}+1}+\frac{2\,k^{\frac{1}{2}\,x}}{\log\left(k\right)}$$

input
```
integrate(k^(1/2*x)+x^(k^(1/2)),x, algorithm="maxima")
```

output
```
x^(sqrt(k) + 1)/(sqrt(k) + 1) + 2*k^(1/2*x)/log(k)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right)\,dx=\frac{x^{\sqrt{k}+1}}{\sqrt{k}+1}+\frac{2\,\sqrt{k^{x}}}{\log\left(k\right)}$$

input
```
integrate(k^(1/2*x)+x^(k^(1/2)),x, algorithm="giac")
```

output
```
x^(sqrt(k) + 1)/(sqrt(k) + 1) + 2*sqrt(k^x)/log(k)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right)\,dx=\frac{2\,k^{x/2}}{\ln\left(k\right)}+\frac{x\,x^{\sqrt{k}}}{\sqrt{k}+1}$$

input
```
int(k^(x/2) + x^(k^(1/2)),x)
```

output
```
(2*k^(x/2))/log(k) + (x*x^(k^(1/2)))/(k^(1/2) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \left(k^{x/2}+x^{\sqrt{k}}\right)\,dx=\frac{2k^{\frac{x}{2}}k-2k^{\frac{x}{2}}+x^{\sqrt{k}}\sqrt{k}\log(k)\,x-x^{\sqrt{k}}\log(k)\,x}{\log\left(k\right)\left(k-1\right)}$$

input
```
int(k^(1/2*x)+x^(k^(1/2)),x)
```

output
```
(2*k**(x/2)*k - 2*k**(x/2) + x**sqrt(k)*sqrt(k)*log(k)*x - x**sqrt(k)*log(
k)*x)/(log(k)*(k - 1))
```

3.666 $\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{1+\sqrt{x}}5^{\sqrt{x}}}{\log(10)}$$

output
```
2^(1+x^(1/2))*5^(x^(1/2))/ln(10)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2^{1+\sqrt{x}}5^{\sqrt{x}}}{\log(10)}$$

input
```
Integrate[10^Sqrt[x]/Sqrt[x],x]
```

output
```
(2^(1 + Sqrt[x])*5^Sqrt[x])/Log[10]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{2638}$$

$$\frac{2^{\sqrt{x}+1} 5^{\sqrt{x}}}{\log(10)}$$

input
```
Int[10^Sqrt[x]/Sqrt[x],x]
```

output
```
(2^(1 + Sqrt[x])*5^Sqrt[x])/Log[10]
```

Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
derivativedivides	$\frac{2\,10^{\sqrt{x}}}{\ln(10)}$	12
default	$\frac{2\,10^{\sqrt{x}}}{\ln(10)}$	12
meijerg	$-\frac{2\left(1-\mathrm{e}^{\sqrt{x}\,\ln(10)}\right)}{\ln(10)}$	18

input
```
int(10^(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*10^(x^(1/2))/ln(10)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\cdot 10^{\left(\sqrt{x}\right)}}{\log\left(10\right)}$$

input
```
integrate(10^(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*10^sqrt(x)/log(10)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.48

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\cdot 10^{\sqrt{x}}}{\log\left(10\right)}$$

input
```
integrate(10**(x**(1/2))/x**(1/2),x)
```

output
```
2*10**(sqrt(x))/log(10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2 \cdot 10^{\left(\sqrt{x}\right)}}{\log\left(10\right)}$$

input
```
integrate(10^(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*10^sqrt(x)/log(10)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2 \cdot 10^{\left(\sqrt{x}\right)}}{\log\left(10\right)}$$

input
```
integrate(10^(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*10^sqrt(x)/log(10)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\,10^{\sqrt{x}}}{\ln\left(10\right)}$$

input
```
int(10^(x^(1/2))/x^(1/2),x)
```

output
```
(2*10^(x^(1/2)))/log(10)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.48

$$\int \frac{10^{\sqrt{x}}}{\sqrt{x}}\,dx = \frac{2\,10^{\sqrt{x}}}{\log(10)}$$

input

```
int(10^(x^(1/2))/x^(1/2),x)
```

output

```
(2*10**sqrt(x))/log(10)
```

3.667 $\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right) dx$

Optimal result

Integrand size = 23, antiderivative size = 11

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right) dx = 2\sqrt{e^x+x}$$

output
```
2*(exp(x)+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right) dx = 2\sqrt{e^x+x}$$

input
```
Integrate[1/Sqrt[E^x + x] + E^x/Sqrt[E^x + x],x]
```

output
```
2*Sqrt[E^x + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e^x}{\sqrt{x+e^x}}+\frac{1}{\sqrt{x+e^x}}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$2\sqrt{x+e^x}$$

input
```
Int[1/Sqrt[E^x + x] + E^x/Sqrt[E^x + x],x]
```

output
```
2*Sqrt[E^x + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
risch	$2\sqrt{e^x+x}$	9

input
```
int(1/(exp(x)+x)^(1/2)+exp(x)/(exp(x)+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(exp(x)+x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(exp(x)+x)^(1/2)+exp(x)/(exp(x)+x)^(1/2),x, algorithm="fricas"
)
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right)\,dx=\int \frac{e^x+1}{\sqrt{x+e^x}}\,dx$$

input
```
integrate(1/(exp(x)+x)**(1/2)+exp(x)/(exp(x)+x)**(1/2),x)
```

output
```
Integral((exp(x) + 1)/sqrt(x + exp(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right)\,dx=2\sqrt{x+e^x}$$

input
```
integrate(1/(exp(x)+x)^(1/2)+exp(x)/(exp(x)+x)^(1/2),x, algorithm="maxima"
)
```

output
```
2*sqrt(x + e^x)
```

Giac [F]

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right) dx = \int \frac{e^x}{\sqrt{x+e^x}}+\frac{1}{\sqrt{x+e^x}}\, dx$$

input
```
integrate(1/(exp(x)+x)^(1/2)+exp(x)/(exp(x)+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(e^x/sqrt(x + e^x) + 1/sqrt(x + e^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right) dx = 2\sqrt{x+\mathrm{e}^x}$$

input
```
int(1/(x + exp(x))^(1/2) + exp(x)/(x + exp(x))^(1/2),x)
```

output
```
2*(x + exp(x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \left(\frac{1}{\sqrt{e^x+x}}+\frac{e^x}{\sqrt{e^x+x}}\right) dx = 2\sqrt{e^x+x}$$

input
```
int(1/(exp(x)+x)^(1/2)+exp(x)/(exp(x)+x)^(1/2),x)
```

output
```
2*sqrt(e**x + x)
```

3.668 $\int \left(\frac{(1+e^x)x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx$

Optimal result

Integrand size = 28, antiderivative size = 12

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2x\sqrt{e^x+x}$$

output
```
2*x*(exp(x)+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2x\sqrt{e^x+x}$$

input
```
Integrate[((1 + E^x)*x)/Sqrt[E^x + x] + 2*Sqrt[E^x + x],x]
```

output
```
2*x*Sqrt[E^x + x]
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{(e^x+1)x}{\sqrt{x+e^x}}+2\sqrt{x+e^x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$2x\sqrt{x+e^x}$$

input
```
Int[((1 + E^x)*x)/Sqrt[E^x + x] + 2*Sqrt[E^x + x],x]
```

output
```
2*x*Sqrt[E^x + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
risch	$2x\sqrt{{\rm e}^{x}+x}$	10

input
```
int((1+exp(x))*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
2*x*(exp(x)+x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x, algorithm="f
ricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right)\,dx=\int \frac{xe^x+3x+2e^x}{\sqrt{x+e^x}}\,dx$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)**(1/2)+2*(exp(x)+x)**(1/2),x)
```

output
```
Integral((x*exp(x) + 3*x + 2*exp(x))/sqrt(x + exp(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right)\,dx=\frac{2\,(x^2+xe^x)}{\sqrt{x+e^x}}$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x, algorithm="m
axima")
```

output
```
2*(x^2 + x*e^x)/sqrt(x + e^x)
```

Giac [F]

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right)\,dx=\int \frac{x(e^x+1)}{\sqrt{x+e^x}}+2\,\sqrt{x+e^x}\,dx$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x, algorithm="g
iac")
```

output
```
integrate(x*(e^x + 1)/sqrt(x + e^x) + 2*sqrt(x + e^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right)\,dx=2\,x\,\sqrt{x+\mathrm{e}^x}$$

input
```
int(2*(x + exp(x))^(1/2) + (x*(exp(x) + 1))/(x + exp(x))^(1/2),x)
```

output
```
2*x*(x + exp(x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{(1+e^x)\,x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right)\,dx=2\sqrt{e^x+x}\,x$$

input
```
int((1+exp(x))*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x)
```

output
```
2*sqrt(e**x + x)*x
```

3.669 $\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx$

Optimal result

Integrand size = 37, antiderivative size = 12

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2x\sqrt{e^x+x}$$

output
```
2*x*(exp(x)+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2x\sqrt{e^x+x}$$

input
```
Integrate[x/Sqrt[E^x + x] + (E^x*x)/Sqrt[E^x + x] + 2*Sqrt[E^x + x],x]
```

output
```
2*x*Sqrt[E^x + x]
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e^x x}{\sqrt{x+e^x}}+\frac{x}{\sqrt{x+e^x}}+2\sqrt{x+e^x}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$2x\sqrt{x+e^x}$$

input
```
Int[x/Sqrt[E^x + x] + (E^x*x)/Sqrt[E^x + x] + 2*Sqrt[E^x + x],x]
```

output
```
2*x*Sqrt[E^x + x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
risch	$2x\sqrt{\mathrm{e}^{x}+x}$	10

input
```
int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
2*x*(exp(x)+x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),
x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = \int \frac{xe^x+3x+2e^x}{\sqrt{x+e^x}}\, dx$$

input
```
integrate(x/(exp(x)+x)**(1/2)+exp(x)*x/(exp(x)+x)**(1/2)+2*(exp(x)+x)**(1/
2),x)
```

output
```
Integral((x*exp(x) + 3*x + 2*exp(x))/sqrt(x + exp(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2\sqrt{x+e^x}x$$

input
```
integrate(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),
x, algorithm="maxima")
```

output
```
2*sqrt(x + e^x)*x
```

Giac [F]

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = \int \frac{xe^x}{\sqrt{x+e^x}}+2\sqrt{x+e^x}+\frac{x}{\sqrt{x+e^x}}\,dx$$

input
```
integrate(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),
x, algorithm="giac")
```

output
```
integrate(x*e^x/sqrt(x + e^x) + 2*sqrt(x + e^x) + x/sqrt(x + e^x), x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2\,x\,\sqrt{x+\mathrm{e}^x}$$

input
```
int(2*(x + exp(x))^(1/2) + x/(x + exp(x))^(1/2) + (x*exp(x))/(x + exp(x))^
(1/2),x)
```

output
```
2*x*(x + exp(x))^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}+2\sqrt{e^x+x}\right) dx = 2\sqrt{e^x+x}\,x$$

input
```
int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2)+2*(exp(x)+x)^(1/2),x)
```

output
```
2*sqrt(e**x + x)*x
```

3.670 $\int \frac{(1+e^x)x}{\sqrt{e^x+x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(1+e^x)x}{\sqrt{e^x+x}} \, dx = 2x\sqrt{e^x+x} - 2\text{Int}\left(\sqrt{e^x+x}, x\right)$$

output
```
2*x*(exp(x)+x)^(1/2)-2*Defer(Int)((exp(x)+x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(1+e^x)x}{\sqrt{e^x+x}} \, dx = \int \frac{(1+e^x)x}{\sqrt{e^x+x}} \, dx$$

input
```
Integrate[((1 + E^x)*x)/Sqrt[E^x + x],x]
```

output
```
Integrate[((1 + E^x)*x)/Sqrt[E^x + x], x]
```

Rubi [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e^x+1)\,x}{\sqrt{x+e^x}}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^x x}{\sqrt{x+e^x}}+\frac{x}{\sqrt{x+e^x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2x\sqrt{x+e^x}-2\int \sqrt{x+e^x}dx$$

input
```
Int[((1 + E^x)*x)/Sqrt[E^x + x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{(1+\mathrm{e}^{x})\,x}{\sqrt{\mathrm{e}^{x}+x}}dx$$

input
```
int((1+exp(x))*x/(exp(x)+x)^(1/2),x)
```

output
```
int((1+exp(x))*x/(exp(x)+x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(1+e^{x})\,x}{\sqrt{e^{x}+x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(1+e^{x})\,x}{\sqrt{e^{x}+x}}\,dx = \int \frac{x(e^{x}+1)}{\sqrt{x+e^{x}}}\,dx$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)**(1/2),x)
```

output
```
Integral(x*(exp(x) + 1)/sqrt(x + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(1+e^x)\,x}{\sqrt{e^x+x}}\,dx = \int \frac{x(e^x+1)}{\sqrt{x+e^x}}\,dx$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*(e^x + 1)/sqrt(x + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(1+e^x)\,x}{\sqrt{e^x+x}}\,dx = \int \frac{x(e^x+1)}{\sqrt{x+e^x}}\,dx$$

input
```
integrate((1+exp(x))*x/(exp(x)+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*(e^x + 1)/sqrt(x + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(1+e^x)\,x}{\sqrt{e^x+x}}\,dx = \int \frac{x\,(\mathrm{e}^x+1)}{\sqrt{x+\mathrm{e}^x}}\,dx$$

input
```
int((x*(exp(x) + 1))/(x + exp(x))^(1/2),x)
```

output
```
int((x*(exp(x) + 1))/(x + exp(x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

$$\int \frac{(1+e^x)\,x}{\sqrt{e^x+x}}\,dx = 2\sqrt{e^x+x}\,x - 4\sqrt{e^x+x} + 2\left(\int \frac{\sqrt{e^x+x}}{e^x+x}dx\right) - 2\left(\int \frac{\sqrt{e^x+x}\,x}{e^x+x}dx\right)$$

input
```
int((1+exp(x))*x/(exp(x)+x)^(1/2),x)
```

output
```
2*(sqrt(e**x + x)*x - 2*sqrt(e**x + x) + int(sqrt(e**x + x)/(e**x + x),x)
- int((sqrt(e**x + x)*x)/(e**x + x),x))
```

3.671 $\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx$

Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = 2x\sqrt{e^x+x}-2\text{Int}\left(\sqrt{e^x+x},x\right)$$

output
```
2*x*(exp(x)+x)^(1/2)-2*Defer(Int)((exp(x)+x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = \int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx$$

input
```
Integrate[x/Sqrt[E^x + x] + (E^x*x)/Sqrt[E^x + x],x]
```

output
```
Integrate[x/Sqrt[E^x + x] + (E^x*x)/Sqrt[E^x + x], x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e^x x}{\sqrt{x+e^x}}+\frac{x}{\sqrt{x+e^x}}\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$2x\sqrt{x+e^x}-2\int \sqrt{x+e^x}dx$$

input
```
Int[x/Sqrt[E^x + x] + (E^x*x)/Sqrt[E^x + x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \left(\frac{x}{\sqrt{\mathrm{e}^x+x}}+\frac{\mathrm{e}^x x}{\sqrt{\mathrm{e}^x+x}}\right)dx$$

input
```
int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x)
```

output
```
int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x, algorithm="frica
s")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = \int \frac{x(e^x+1)}{\sqrt{x+e^x}}\, dx$$

input
```
integrate(x/(exp(x)+x)**(1/2)+exp(x)*x/(exp(x)+x)**(1/2),x)
```

output
```
Integral(x*(exp(x) + 1)/sqrt(x + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = \int \frac{xe^x}{\sqrt{x+e^x}}+\frac{x}{\sqrt{x+e^x}}\, dx$$

input
```
integrate(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x, algorithm="maxim
a")
```

output
```
integrate(x*e^x/sqrt(x + e^x) + x/sqrt(x + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = \int \frac{xe^x}{\sqrt{x+e^x}}+\frac{x}{\sqrt{x+e^x}}\, dx$$

input
```
integrate(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x, algorithm="giac"
)
```

output
```
integrate(x*e^x/sqrt(x + e^x) + x/sqrt(x + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = \int \frac{x}{\sqrt{x+\mathrm{e}^x}}+\frac{x\,\mathrm{e}^x}{\sqrt{x+\mathrm{e}^x}}\, dx$$

input
```
int(x/(x + exp(x))^(1/2) + (x*exp(x))/(x + exp(x))^(1/2),x)
```

output
```
int(x/(x + exp(x))^(1/2) + (x*exp(x))/(x + exp(x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \left(\frac{x}{\sqrt{e^x+x}}+\frac{e^x x}{\sqrt{e^x+x}}\right) dx = 2\sqrt{e^x+x}\,x-4\sqrt{e^x+x} +2\left(\int \frac{\sqrt{e^x+x}}{e^x+x}dx\right)-2\left(\int \frac{\sqrt{e^x+x}\,x}{e^x+x}dx\right)$$

input
```
int(x/(exp(x)+x)^(1/2)+exp(x)*x/(exp(x)+x)^(1/2),x)
```

output
```
2*(sqrt(e**x + x)*x - 2*sqrt(e**x + x) + int(sqrt(e**x + x)/(e**x + x),x)
- int((sqrt(e**x + x)*x)/(e**x + x),x))
```

3.672 $\int \frac{e^x x}{\sqrt{e^x+x}} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{e^x x}{\sqrt{e^x+x}} dx = 2\sqrt{e^x+x} + 2x\sqrt{e^x+x} - \text{Int}\left(\frac{1}{\sqrt{e^x+x}}, x\right) - 3\text{Int}\left(\sqrt{e^x+x}, x\right)$$

output
```
2*(exp(x)+x)^(1/2)+2*x*(exp(x)+x)^(1/2)-Defer(Int)(1/(exp(x)+x)^(1/2),x)-3
*Defer(Int)((exp(x)+x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{e^x x}{\sqrt{e^x+x}} dx = \int \frac{e^x x}{\sqrt{e^x+x}} dx$$

input
```
Integrate[(E^x*x)/Sqrt[E^x + x],x]
```

output
```
Integrate[(E^x*x)/Sqrt[E^x + x], x]
```

Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2692, 2703, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x x}{\sqrt{x+e^x}} \, dx$$

$$\Big\downarrow \text{2692}$$

$$-\int \frac{x}{\sqrt{x+e^x}}dx-2\int \sqrt{x+e^x}dx+2\sqrt{x+e^x}x$$

$$\Big\downarrow \text{2703}$$

$$-\int \frac{1}{\sqrt{x+e^x}}dx-3\int \sqrt{x+e^x}dx+2\sqrt{x+e^x}x+2\sqrt{x+e^x}$$

$$\Big\downarrow \text{7299}$$

$$-\int \frac{1}{\sqrt{x+e^x}}dx-3\int \sqrt{x+e^x}dx+2\sqrt{x+e^x}x+2\sqrt{x+e^x}$$

input
```
Int[(E^x*x)/Sqrt[E^x + x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2692
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*(x_)^(m_.)*((b_.)*(F_)^((e_.)*((c_.)
+ (d_.)*(x_))) + (a_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[x^m*((a*x^n + b
*F^(e*(c + d*x)))^(p + 1)/(b*d*e*(p + 1)*Log[F])), x] + (-Simp[m/(b*d*e*(p
+ 1)*Log[F])   Int[x^(m - 1)*(a*x^n + b*F^(e*(c + d*x)))^(p + 1), x], x] -
Simp[a*(n/(b*d*e*Log[F]))   Int[x^(m + n - 1)*(a*x^n + b*F^(e*(c + d*x)))^p
, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]
```

rule 2703
```
Int[(x_)^(m_.)*(E^(x_) + (x_)^(m_.))^(n_), x_Symbol] :> Simp[-(E^x + x^m)^(
n + 1)/(n + 1), x] + (Int[(E^x + x^m)^(n + 1), x] + Simp[m   Int[x^(m - 1)*
(E^x + x^m)^n, x], x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[
n, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\mathrm{e}^{x} x}{\sqrt{\mathrm{e}^{x}+x}}dx$$

input
```
int(exp(x)*x/(exp(x)+x)^(1/2),x)
```

output
```
int(exp(x)*x/(exp(x)+x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{e^{x} x}{\sqrt{e^{x}+x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(x)*x/(exp(x)+x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{e^x x}{\sqrt{e^x + x}} \, dx = \int \frac{x e^x}{\sqrt{x + e^x}} \, dx$$

input
```
integrate(exp(x)*x/(exp(x)+x)**(1/2),x)
```

output
```
Integral(x*exp(x)/sqrt(x + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^x x}{\sqrt{e^x + x}} \, dx = \int \frac{x e^x}{\sqrt{x + e^x}} \, dx$$

input
```
integrate(exp(x)*x/(exp(x)+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*e^x/sqrt(x + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^x x}{\sqrt{e^x + x}} \, dx = \int \frac{x e^x}{\sqrt{x + e^x}} \, dx$$

input
```
integrate(exp(x)*x/(exp(x)+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*e^x/sqrt(x + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{e^x x}{\sqrt{e^x+x}} \, dx = \int \frac{x\,\mathrm{e}^x}{\sqrt{x+\mathrm{e}^x}} \,dx$$

input
```
int((x*exp(x))/(x + exp(x))^(1/2),x)
```

output
```
int((x*exp(x))/(x + exp(x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.93

$$\int \frac{e^x x}{\sqrt{e^x+x}} \, dx = 2\sqrt{e^x+x}\,x-4\sqrt{e^x+x}+2\left(\int \frac{\sqrt{e^x+x}}{e^x+x}dx\right)-3\left(\int \frac{\sqrt{e^x+x}\,x}{e^x+x}dx\right)$$

input
```
int(exp(x)*x/(exp(x)+x)^(1/2),x)
```

output
```
2*sqrt(e**x + x)*x - 4*sqrt(e**x + x) + 2*int(sqrt(e**x + x)/(e**x + x),x)
 - 3*int((sqrt(e**x + x)*x)/(e**x + x),x)
```

3.673 $\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx$

Optimal result

Integrand size = 50, antiderivative size = 20

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx=\frac{2}{5}x^2\sqrt{5e^x+x^3}$$

output
```
2/5*x^2*(5*exp(x)+x^3)^(1/2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx=\frac{2}{5}x^2\sqrt{5e^x+x^3}$$

input
```
Integrate[(x^2*(5*E^x + 3*x^2))/(5*Sqrt[5*E^x + x^3]) + (4*x*Sqrt[5*E^x +
x^3])/5,x]
```

output
```
(2*x^2*Sqrt[5*E^x + x^3])/5
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.020, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{4}{5}\sqrt{x^3+5e^x}x+\frac{(3x^2+5e^x)\,x^2}{5\sqrt{x^3+5e^x}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{5}x^2\sqrt{x^3+5e^x}$$

input
```
Int[(x^2*(5*E^x + 3*x^2))/(5*Sqrt[5*E^x + x^3]) + (4*x*Sqrt[5*E^x + x^3])/
5,x]
```

output
```
(2*x^2*Sqrt[5*E^x + x^3])/5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{2x^2\sqrt{5\,\mathrm{e}^x+x^3}}{5}$	16

input
```
int(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^3)^(1/
2),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^2*(5*exp(x)+x^3)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^
3)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx = \frac{\int \frac{7x^4}{\sqrt{x^3+5e^x}}\,dx+\int \frac{20xe^x}{\sqrt{x^3+5e^x}}\,dx+\int \frac{5x^2e^x}{\sqrt{x^3+5e^x}}\,dx}{5}$$

input
```
integrate(1/5*x**2*(5*exp(x)+3*x**2)/(5*exp(x)+x**3)**(1/2)+4/5*x*(5*exp(x
)+x**3)**(1/2),x)
```

output
```
(Integral(7*x**4/sqrt(x**3 + 5*exp(x)), x) + Integral(20*x*exp(x)/sqrt(x**
3 + 5*exp(x)), x) + Integral(5*x**2*exp(x)/sqrt(x**3 + 5*exp(x)), x))/5
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx=\frac{2\,(x^5+5\,x^2e^x)}{5\,\sqrt{x^3+5\,e^x}}$$

input
```
integrate(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^
3)^(1/2),x, algorithm="maxima")
```

output
```
2/5*(x^5 + 5*x^2*e^x)/sqrt(x^3 + 5*e^x)
```

Giac [F]

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx=\int \frac{(3\,x^2+5\,e^x)x^2}{5\,\sqrt{x^3+5\,e^x}}+\frac{4}{5}\,\sqrt{x^3+5\,e^x}x\,dx$$

input
```
integrate(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^
3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/5*(3*x^2 + 5*e^x)*x^2/sqrt(x^3 + 5*e^x) + 4/5*sqrt(x^3 + 5*e^x
)*x, x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx=\frac{2\,x^2\,\sqrt{5\,\mathrm{e}^x+x^3}}{5}$$

input
```
int((4*x*(5*exp(x) + x^3)^(1/2))/5 + (x^2*(5*exp(x) + 3*x^2))/(5*(5*exp(x)
 + x^3)^(1/2)),x)
```

output
```
(2*x^2*(5*exp(x) + x^3)^(1/2))/5
```

Reduce [F]

$$\int \left(\frac{x^2(5e^x+3x^2)}{5\sqrt{5e^x+x^3}}+\frac{4}{5}x\sqrt{5e^x+x^3}\right)\,dx=\frac{7\left(\int\frac{x^4}{\sqrt{5e^x+x^3}}dx\right)}{5}+\int\frac{e^x x^2}{\sqrt{5e^x+x^3}}dx+4\left(\int\frac{e^x x}{\sqrt{5e^x+x^3}}dx\right)$$

input
```
int(1/5*x^2*(5*exp(x)+3*x^2)/(5*exp(x)+x^3)^(1/2)+4/5*x*(5*exp(x)+x^3)^(1/
2),x)
```

output
```
(7*int(x**4/sqrt(5*e**x + x**3),x) + 5*int((e**x*x**2)/sqrt(5*e**x + x**3)
,x) + 20*int((e**x*x)/sqrt(5*e**x + x**3),x))/5
```

3.674 $\int \frac{e^x x^2}{\sqrt{5e^x+x^3}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{e^x x^2}{\sqrt{5e^x+x^3}}\,dx=\frac{2}{5}x^2\sqrt{5e^x+x^3}-\frac{3}{5}\text{Int}\left(\frac{x^4}{\sqrt{5e^x+x^3}},x\right)-\frac{4}{5}\text{Int}\left(x\sqrt{5e^x+x^3},x\right)$$

output
```
2/5*x^2*(5*exp(x)+x^3)^(1/2)-3/5*Defer(Int)(x^4/(5*exp(x)+x^3)^(1/2),x)-4/
5*Defer(Int)(x*(5*exp(x)+x^3)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{e^x x^2}{\sqrt{5e^x+x^3}}\,dx=\int \frac{e^x x^2}{\sqrt{5e^x+x^3}}\,dx$$

input
```
Integrate[(E^x*x^2)/Sqrt[5*E^x + x^3],x]
```

output
```
Integrate[(E^x*x^2)/Sqrt[5*E^x + x^3], x]
```

Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2692, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x x^2}{\sqrt{x^3+5e^x}}dx$$
$$\Big\downarrow \text{2692}$$
$$-\frac{4}{5}\int x\sqrt{x^3+5e^x}dx-\frac{3}{5}\int \frac{x^4}{\sqrt{x^3+5e^x}}dx+\frac{2}{5}\sqrt{x^3+5e^x}x^2$$
$$\Big\downarrow \text{7299}$$
$$-\frac{4}{5}\int x\sqrt{x^3+5e^x}dx-\frac{3}{5}\int \frac{x^4}{\sqrt{x^3+5e^x}}dx+\frac{2}{5}\sqrt{x^3+5e^x}x^2$$

input
```
Int[(E^x*x^2)/Sqrt[5*E^x + x^3],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2692
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*(x_)^(m_.)*((b_.)*(F_)^((e_.)*((c_.)
+ (d_.)*(x_))) + (a_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[x^m*((a*x^n + b
*F^(e*(c + d*x)))^(p + 1)/(b*d*e*(p + 1)*Log[F])), x] + (-Simp[m/(b*d*e*(p
+ 1)*Log[F])   Int[x^(m - 1)*(a*x^n + b*F^(e*(c + d*x)))^(p + 1), x], x] -
Simp[a*(n/(b*d*e*Log[F]))   Int[x^(m + n - 1)*(a*x^n + b*F^(e*(c + d*x)))^p
, x], x]) /; FreeQ[{F, a, b, c, d, e, m, n, p}, x] && NeQ[p, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\mathrm{e}^{x} x^{2}}{\sqrt{5\,\mathrm{e}^{x}+x^{3}}}dx$$

input
```
int(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x)
```

output
```
int(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{e^{x} x^{2}}{\sqrt{5e^{x}+x^{3}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{e^{x} x^{2}}{\sqrt{5e^{x}+x^{3}}}\,dx = \int \frac{x^{2} e^{x}}{\sqrt{x^{3}+5e^{x}}}\,dx$$

input
```
integrate(exp(x)*x**2/(5*exp(x)+x**3)**(1/2),x)
```

output
```
Integral(x**2*exp(x)/sqrt(x**3 + 5*exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^x x^2}{\sqrt{5e^x + x^3}}\, dx = \int \frac{x^2 e^x}{\sqrt{x^3 + 5\, e^x}}\, dx$$

input
```
integrate(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*e^x/sqrt(x^3 + 5*e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^x x^2}{\sqrt{5e^x + x^3}}\, dx = \int \frac{x^2 e^x}{\sqrt{x^3 + 5\, e^x}}\, dx$$

input
```
integrate(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*e^x/sqrt(x^3 + 5*e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{e^x x^2}{\sqrt{5e^x + x^3}}\, dx = \int \frac{x^2\, \mathrm{e}^x}{\sqrt{5\, \mathrm{e}^x + x^3}}\, dx$$

input
```
int((x^2*exp(x))/(5*exp(x) + x^3)^(1/2),x)
```

output
```
int((x^2*exp(x))/(5*exp(x) + x^3)^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{e^x x^2}{\sqrt{5e^x+x^3}}\,dx=\int \frac{e^x x^2}{\sqrt{5e^x+x^3}}dx$$

input
```
int(exp(x)*x^2/(5*exp(x)+x^3)^(1/2),x)
```

output
```
int((e**x*x**2)/sqrt(5*e**x + x**3),x)
```

3.675 $\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 13

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx = -\frac{3}{2}(e^x+x)^{2/3}$$

output
```
-3/2*(exp(x)+x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx = -\frac{3}{2}(e^x+x)^{2/3}$$

input
```
Integrate[-((1 + E^x)/(E^x + x)^(1/3)),x]
```

output
```
(-3*(E^x + x)^(2/3))/2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {25, 7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{e^x+1}{\sqrt[3]{x+e^x}}dx$$
$$\Big\downarrow \text{ 25}$$
$$-\int \frac{1+e^x}{\sqrt[3]{x+e^x}}dx$$
$$\Big\downarrow \text{ 7237}$$
$$-\frac{3}{2}(x+e^x)^{2/3}$$

input
```
Int[-((1 + E^x)/(E^x + x)^(1/3)),x]
```

output
```
(-3*(E^x + x)^(2/3))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{3(e^{x}+x)^{\frac{2}{3}}}{2}$	9
default	$-\frac{3(e^{x}+x)^{\frac{2}{3}}}{2}$	9
risch	$-\frac{3(e^{x}+x)^{\frac{2}{3}}}{2}$	9

input
```
int(-(1+exp(x))/(exp(x)+x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*(exp(x)+x)^(2/3)
```

Fricas [F(-2)]

Exception generated.

$$\int -\frac{1+e^{x}}{\sqrt[3]{e^{x}+x}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(-(1+exp(x))/(exp(x)+x)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int -\frac{1+e^{x}}{\sqrt[3]{e^{x}+x}}\,dx = -\frac{3(x+e^{x})^{\frac{2}{3}}}{2}$$

input
```
integrate(-(1+exp(x))/(exp(x)+x)**(1/3),x)
```

output
```
-3*(x + exp(x))**(2/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx = -\frac{3}{2}\,(x+e^x)^{\frac{2}{3}}$$

input
```
integrate(-(1+exp(x))/(exp(x)+x)^(1/3),x, algorithm="maxima")
```

output
```
-3/2*(x + e^x)^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx = -\frac{3}{2}\,(x+e^x)^{\frac{2}{3}}$$

input
```
integrate(-(1+exp(x))/(exp(x)+x)^(1/3),x, algorithm="giac")
```

output
```
-3/2*(x + e^x)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx = -\frac{3\,(x+e^x)^{2/3}}{2}$$

input
```
int(-(exp(x) + 1)/(x + exp(x))^(1/3),x)
```

output
```
-(3*(x + exp(x))^(2/3))/2
```

Reduce [F]

$$\int -\frac{1+e^x}{\sqrt[3]{e^x+x}}\,dx = -\left(\int \frac{e^x}{(e^x+x)^{\frac{1}{3}}}dx\right) - \left(\int \frac{1}{(e^x+x)^{\frac{1}{3}}}dx\right)$$

input
```
int(-(1+exp(x))/(exp(x)+x)^(1/3),x)
```

output
```
- (int(e**x/(e**x + x)**(1/3),x) + int(1/(e**x + x)**(1/3),x))
```

3.676 $\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right)\,dx$

Optimal result

Integrand size = 34, antiderivative size = 13

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right)\,dx=-\frac{3}{2}(e^x+x)^{2/3}$$

output
```
-3/2*(exp(x)+x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right)\,dx=-\frac{3}{2}(e^x+x)^{2/3}$$

input
```
Integrate[-(E^x + x)^(-1/3) + x/(E^x + x)^(1/3) - (E^x + x)^(2/3),x]
```

output
```
(-3*(E^x + x)^(2/3))/2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.029, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\sqrt[3]{x+e^x}}-(x+e^x)^{2/3}-\frac{1}{\sqrt[3]{x+e^x}}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{2}(x+e^x)^{2/3}$$

input
```
Int[-(E^x + x)^(-1/3) + x/(E^x + x)^(1/3) - (E^x + x)^(2/3),x]
```

output
```
(-3*(E^x + x)^(2/3))/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{3(\mathrm{e}^{x}+x)^{\frac{2}{3}}}{2}$	9

input
```
int(-1/(exp(x)+x)^(1/3)+x/(exp(x)+x)^(1/3)-(exp(x)+x)^(2/3),x,method=_RETU
RNVERBOSE)
```

output
```
-3/2*(exp(x)+x)^(2/3)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(-1/(exp(x)+x)^(1/3)+x/(exp(x)+x)^(1/3)-(exp(x)+x)^(2/3),x, algor
ithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right)\,dx=-\int \frac{e^x}{\sqrt[3]{x+e^x}}\,dx-\int \frac{1}{\sqrt[3]{x+e^x}}\,dx$$

input
```
integrate(-1/(exp(x)+x)**(1/3)+x/(exp(x)+x)**(1/3)-(exp(x)+x)**(2/3),x)
```

output
```
-Integral(exp(x)/(x + exp(x))**(1/3), x) - Integral((x + exp(x))**(-1/3),
x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right) dx = -\frac{3}{2}\left(x+e^x\right)^{\frac{2}{3}}$$

input
```
integrate(-1/(exp(x)+x)^(1/3)+x/(exp(x)+x)^(1/3)-(exp(x)+x)^(2/3),x, algor
ithm="maxima")
```

output
```
-3/2*(x + e^x)^(2/3)
```

Giac [F]

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right) dx = \int -\left(x+e^x\right)^{\frac{2}{3}}+\frac{x}{\left(x+e^x\right)^{\frac{1}{3}}}-\frac{1}{\left(x+e^x\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(-1/(exp(x)+x)^(1/3)+x/(exp(x)+x)^(1/3)-(exp(x)+x)^(2/3),x, algor
ithm="giac")
```

output
```
integrate(-(x + e^x)^(2/3) + x/(x + e^x)^(1/3) - 1/(x + e^x)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right) dx = -\frac{3\left(x+\mathrm{e}^x\right)^{2/3}}{2}$$

input
```
int(x/(x + exp(x))^(1/3) - (x + exp(x))^(2/3) - 1/(x + exp(x))^(1/3),x)
```

output
```
-(3*(x + exp(x))^(2/3))/2
```

Reduce [F]

$$\int \left(-\frac{1}{\sqrt[3]{e^x+x}}+\frac{x}{\sqrt[3]{e^x+x}}-(e^x+x)^{2/3}\right) dx = -\left(\int \frac{e^x}{(e^x+x)^{\frac{1}{3}}}dx\right)-\left(\int \frac{1}{(e^x+x)^{\frac{1}{3}}}dx\right)$$

input
```
int(-1/(exp(x)+x)^(1/3)+x/(exp(x)+x)^(1/3)-(exp(x)+x)^(2/3),x)
```

output
```
- (int(e**x/(e**x + x)**(1/3),x) + int(1/(e**x + x)**(1/3),x))
```

3.677 $\int \frac{x}{\sqrt[3]{e^x+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{x}{\sqrt[3]{e^x+x}}\,dx=-\frac{3}{2}(e^x+x)^{2/3}+\text{Int}\left(\frac{1}{\sqrt[3]{e^x+x}},x\right)+\text{Int}\left((e^x+x)^{2/3},x\right)$$

output
```
-3/2*(exp(x)+x)^(2/3)+Defer(Int)(1/(exp(x)+x)^(1/3),x)+Defer(Int)((exp(x)+
x)^(2/3),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{x}{\sqrt[3]{e^x+x}}\,dx=\int \frac{x}{\sqrt[3]{e^x+x}}\,dx$$

input
```
Integrate[x/(E^x + x)^(1/3),x]
```

output
```
Integrate[x/(E^x + x)^(1/3), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2703, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{x+e^x}}\,dx$$

$$\Big\downarrow \text{2703}$$

$$\int \frac{1}{\sqrt[3]{x+e^x}}dx+\int (x+e^x)^{2/3}\,dx-\frac{3}{2}(x+e^x)^{2/3}$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{1}{\sqrt[3]{x+e^x}}dx+\int (x+e^x)^{2/3}\,dx-\frac{3}{2}(x+e^x)^{2/3}$$

input
```
Int[x/(E^x + x)^(1/3),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2703
```
Int[(x_)^(m_.)*(E^(x_) + (x_)^(m_.))^(n_), x_Symbol] :> Simp[-(E^x + x^m)^(
n + 1)/(n + 1), x] + (Int[(E^x + x^m)^(n + 1), x] + Simp[m   Int[x^(m - 1)*
(E^x + x^m)^n, x], x]) /; RationalQ[m, n] && GtQ[m, 0] && LtQ[n, 0] && NeQ[
n, -1]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{x}{(\mathrm{e}^{x} + x)^{\frac{1}{3}}} dx$$

input
```
int(x/(exp(x)+x)^(1/3),x)
```

output
```
int(x/(exp(x)+x)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt[3]{e^{x} + x}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(exp(x)+x)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt[3]{e^{x} + x}} \, dx = \int \frac{x}{\sqrt[3]{x + e^{x}}} \, dx$$

input
```
integrate(x/(exp(x)+x)**(1/3),x)
```

output
```
Integral(x/(x + exp(x))**(1/3), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt[3]{e^x + x}} \, dx = \int \frac{x}{(x + e^x)^{\frac{1}{3}}} \, dx$$

input
```
integrate(x/(exp(x)+x)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x/(x + e^x)^(1/3), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt[3]{e^x + x}} \, dx = \int \frac{x}{(x + e^x)^{\frac{1}{3}}} \, dx$$

input
```
integrate(x/(exp(x)+x)^(1/3),x, algorithm="giac")
```

output
```
integrate(x/(x + e^x)^(1/3), x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt[3]{e^x + x}} \, dx = \int \frac{x}{(x + \mathrm{e}^x)^{1/3}} \, dx$$

input
```
int(x/(x + exp(x))^(1/3),x)
```

output
```
int(x/(x + exp(x))^(1/3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt[3]{e^x+x}}\,dx=\int \frac{x}{(e^x+x)^{\frac{1}{3}}}dx$$

input
```
int(x/(exp(x)+x)^(1/3),x)
```

output
```
int(x/(e**x + x)**(1/3),x)
```

3.678 $\int \frac{5x+e^x(3+2x)}{\sqrt[3]{e^x+x}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{5x+e^x(3+2x)}{\sqrt[3]{e^x+x}}\,dx = 3x(e^x+x)^{2/3}$$

output
```
3*x*(exp(x)+x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{5x+e^x(3+2x)}{\sqrt[3]{e^x+x}}\,dx = 3x(e^x+x)^{2/3}$$

input
```
Integrate[(5*x + E^x*(3 + 2*x))/(E^x + x)^(1/3),x]
```

output
```
3*x*(E^x + x)^(2/3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x+e^x(2x+3)}{\sqrt[3]{x+e^x}}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{5x}{\sqrt[3]{x+e^x}}+\frac{e^x(2x+3)}{\sqrt[3]{x+e^x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3x(x+e^x)^{2/3}$$

input
```
Int[(5*x + E^x*(3 + 2*x))/(E^x + x)^(1/3),x]
```

output
```
3*x*(E^x + x)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
risch	$3x(e^{x}+x)^{\frac{2}{3}}$	10

input
```
int((5*x+exp(x)*(3+2*x))/(exp(x)+x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3*x*(exp(x)+x)^(2/3)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{5x+e^{x}(3+2x)}{\sqrt[3]{e^{x}+x}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((5*x+exp(x)*(3+2*x))/(exp(x)+x)^(1/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{5x+e^{x}(3+2x)}{\sqrt[3]{e^{x}+x}}\,dx=\int \frac{2xe^{x}+5x+3e^{x}}{\sqrt[3]{x+e^{x}}}\,dx$$

input
```
integrate((5*x+exp(x)*(3+2*x))/(exp(x)+x)**(1/3),x)
```

output
```
Integral((2*x*exp(x) + 5*x + 3*exp(x))/(x + exp(x))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{5x + e^x(3 + 2x)}{\sqrt[3]{e^x + x}}\, dx = \frac{3\left(x^2 + xe^x\right)}{(x + e^x)^{\frac{1}{3}}}$$

input
```
integrate((5*x+exp(x)*(3+2*x))/(exp(x)+x)^(1/3),x, algorithm="maxima")
```

output
```
3*(x^2 + x*e^x)/(x + e^x)^(1/3)
```

Giac [F]

$$\int \frac{5x + e^x(3 + 2x)}{\sqrt[3]{e^x + x}}\, dx = \int \frac{(2\,x + 3)e^x + 5\,x}{(x + e^x)^{\frac{1}{3}}}\, dx$$

input
```
integrate((5*x+exp(x)*(3+2*x))/(exp(x)+x)^(1/3),x, algorithm="giac")
```

output
```
integrate(((2*x + 3)*e^x + 5*x)/(x + e^x)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{5x + e^x(3 + 2x)}{\sqrt[3]{e^x + x}}\, dx = 3\,x\,\left(x + \mathrm{e}^x\right)^{2/3}$$

input
```
int((5*x + exp(x)*(2*x + 3))/(x + exp(x))^(1/3),x)
```

output
```
3*x*(x + exp(x))^(2/3)
```

Reduce [F]

$$\int \frac{5x+e^x(3+2x)}{\sqrt[3]{e^x+x}}\,dx$$
$$= 3\left(\int \frac{e^x}{(e^x+x)^{\frac{1}{3}}}dx\right)+2\left(\int \frac{e^x x}{(e^x+x)^{\frac{1}{3}}}dx\right)+5\left(\int \frac{x}{(e^x+x)^{\frac{1}{3}}}dx\right)$$

input
```
int((5*x+exp(x)*(3+2*x))/(exp(x)+x)^(1/3),x)
```

output
```
3*int(e**x/(e**x + x)**(1/3),x) + 2*int((e**x*x)/(e**x + x)**(1/3),x) + 5*
int(x/(e**x + x)**(1/3),x)
```

3.679 $\int \left(\frac{2x}{\sqrt[3]{e^x + x}} + \frac{2e^x x}{\sqrt[3]{e^x + x}} + 3(e^x + x)^{2/3} \right) dx$

Optimal result

Integrand size = 39, antiderivative size = 12

$$\int \left(\frac{2x}{\sqrt[3]{e^x + x}} + \frac{2e^x x}{\sqrt[3]{e^x + x}} + 3(e^x + x)^{2/3} \right) dx = 3x(e^x + x)^{2/3}$$

output
```
3*x*(exp(x)+x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \left(\frac{2x}{\sqrt[3]{e^x + x}} + \frac{2e^x x}{\sqrt[3]{e^x + x}} + 3(e^x + x)^{2/3} \right) dx = 3x(e^x + x)^{2/3}$$

input
```
Integrate[(2*x)/(E^x + x)^(1/3) + (2*E^x*x)/(E^x + x)^(1/3) + 3*(E^x + x)^
(2/3),x]
```

output
```
3*x*(E^x + x)^(2/3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{2e^x x}{\sqrt[3]{x+e^x}}+\frac{2x}{\sqrt[3]{x+e^x}}+3(x+e^x)^{2/3}\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$3x(x+e^x)^{2/3}$$

input
```
Int[(2*x)/(E^x + x)^(1/3) + (2*E^x*x)/(E^x + x)^(1/3) + 3*(E^x + x)^(2/3),
x]
```

output
```
3*x*(E^x + x)^(2/3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

method	result	size
risch	$3x(\mathrm{e}^{x}+x)^{\frac{2}{3}}$	10

input
```
int(2*x/(exp(x)+x)^(1/3)+2*exp(x)*x/(exp(x)+x)^(1/3)+3*(exp(x)+x)^(2/3),x,
method=_RETURNVERBOSE)
```

output
```
3*x*(exp(x)+x)^(2/3)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{2x}{\sqrt[3]{e^x+x}}+\frac{2e^xx}{\sqrt[3]{e^x+x}}+3(e^x+x)^{2/3}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(2*x/(exp(x)+x)^(1/3)+2*exp(x)*x/(exp(x)+x)^(1/3)+3*(exp(x)+x)^(2
/3),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{2x}{\sqrt[3]{e^x+x}}+\frac{2e^xx}{\sqrt[3]{e^x+x}}+3(e^x+x)^{2/3}\right)dx = \int \frac{2xe^x+5x+3e^x}{\sqrt[3]{x+e^x}}\,dx$$

input
```
integrate(2*x/(exp(x)+x)**(1/3)+2*exp(x)*x/(exp(x)+x)**(1/3)+3*(exp(x)+x)*
*(2/3),x)
```

output
```
Integral((2*x*exp(x) + 5*x + 3*exp(x))/(x + exp(x))**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \left(\frac{2x}{\sqrt[3]{e^x+x}}+\frac{2e^x x}{\sqrt[3]{e^x+x}}+3(e^x+x)^{2/3}\right)\,dx=\frac{3\,(x^2+xe^x)}{(x+e^x)^{\frac{1}{3}}}$$

input
```
integrate(2*x/(exp(x)+x)^(1/3)+2*exp(x)*x/(exp(x)+x)^(1/3)+3*(exp(x)+x)^(2
/3),x, algorithm="maxima")
```

output
```
3*(x^2 + x*e^x)/(x + e^x)^(1/3)
```

Giac [F]

$$\int \left(\frac{2x}{\sqrt[3]{e^x+x}}+\frac{2e^x x}{\sqrt[3]{e^x+x}}+3(e^x+x)^{2/3}\right)\,dx=\int \frac{2\,xe^x}{(x+e^x)^{\frac{1}{3}}}+3\,(x+e^x)^{\frac{2}{3}}+\frac{2\,x}{(x+e^x)^{\frac{1}{3}}}\,dx$$

input
```
integrate(2*x/(exp(x)+x)^(1/3)+2*exp(x)*x/(exp(x)+x)^(1/3)+3*(exp(x)+x)^(2
/3),x, algorithm="giac")
```

output
```
integrate(2*x*e^x/(x + e^x)^(1/3) + 3*(x + e^x)^(2/3) + 2*x/(x + e^x)^(1/3
), x)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \left(\frac{2x}{\sqrt[3]{e^x+x}}+\frac{2e^x x}{\sqrt[3]{e^x+x}}+3(e^x+x)^{2/3}\right)\,dx=3\,x\,(x+\mathrm{e}^x)^{2/3}$$

input
```
int(3*(x + exp(x))^(2/3) + (2*x)/(x + exp(x))^(1/3) + (2*x*exp(x))/(x + ex
p(x))^(1/3),x)
```

output
```
3*x*(x + exp(x))^(2/3)
```

Reduce [F]

$$\int \left(\frac{2x}{\sqrt[3]{e^x+x}}+\frac{2e^x x}{\sqrt[3]{e^x+x}}+3(e^x+x)^{2/3}\right)\,dx=3\left(\int \frac{e^x}{(e^x+x)^{\frac{1}{3}}}dx\right)+2\left(\int \frac{e^x x}{(e^x+x)^{\frac{1}{3}}}dx\right)+5\left(\int \frac{x}{(e^x+x)^{\frac{1}{3}}}dx\right)$$

input
```
int(2*x/(exp(x)+x)^(1/3)+2*exp(x)*x/(exp(x)+x)^(1/3)+3*(exp(x)+x)^(2/3),x)
```

output
```
3*int(e**x/(e**x + x)**(1/3),x) + 2*int((e**x*x)/(e**x + x)**(1/3),x) + 5*
int(x/(e**x + x)**(1/3),x)
```

3.680 $\int e^x(-e^{-x}+e^x)\left(e^{-x}+e^x\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 31

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2\,dx=\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}+\frac{e^{4x}}{4}-x$$

output
```
1/2/exp(2*x)+1/2*exp(2*x)+1/4*exp(4*x)-x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2\,dx=\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}+\frac{e^{4x}}{4}-x$$

input
```
Integrate[E^x*(-E^(-x) + E^x)*(E^(-x) + E^x)^2,x]
```

output
```
1/(2*E^(2*x)) + E^(2*x)/2 + E^(4*x)/4 - x
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2720, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \left(e^x-e^{-x}\right) \left(e^{-x}+e^x\right)^2 \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int -e^{-x} \left(e^{-2x}-e^{2x}-e^{4x}+1\right)de^x$$

$$\Big\downarrow \text{ 25}$$

$$-\int e^{-x} \left(1+e^{-2x}-e^{2x}-e^{4x}\right)de^x$$

$$\Big\downarrow \text{ 2010}$$

$$-\int \left(e^{-3x}+e^{-x}-e^x-e^{3x}\right)de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}+\frac{e^{4x}}{4}-\log \left(e^x\right)$$

input
```
Int[E^x*(-E^(-x) + E^x)*(E^(-x) + E^x)^2,x]
```

output
```
1/(2*E^(2*x)) + E^(2*x)/2 + E^(4*x)/4 - Log[E^x]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

method	result
default	$-x+\frac{{\mathrm e}^{2 x}}{2}+\frac{{\mathrm e}^{4 x}}{4}+\frac{{\mathrm e}^{-2 x}}{2}$
risch	$-x+\frac{{\mathrm e}^{2 x}}{2}+\frac{{\mathrm e}^{4 x}}{4}+\frac{{\mathrm e}^{-2 x}}{2}$
parts	$-x+\frac{{\mathrm e}^{2 x}}{2}+\frac{{\mathrm e}^{4 x}}{4}+\frac{{\mathrm e}^{-2 x}}{2}$
norman	$\left(\frac{{\mathrm e}^{5 x}}{2}+\frac{{\mathrm e}^{7 x}}{4}-{\mathrm e}^{3 x} x+\frac{{\mathrm e}^{x}}{2}\right) {\mathrm e}^{-3 x}$
parallelrisch	$\frac{{\mathrm e}^{-x} {\mathrm e}^{3 x}}{2}+\frac{{\mathrm e}^{-3 x} {\mathrm e}^{x}}{2}+\frac{{\mathrm e}^{4 x}}{4}-{\mathrm e}^{-2 x} {\mathrm e}^{2 x} x$
orering	$\left(x+\frac{1}{4}\right) {\mathrm e}^{x} \left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right) \left({\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{2}+\left(-\frac{x}{4}+\frac{1}{4}\right) \left({\mathrm e}^{x} \left(-{\mathrm e}^{-x}+{\mathrm e}^{x}\right) \left({\mathrm e}^{-x}+{\mathrm e}^{x}\right)^{2}+{\mathrm e}^{x} \left({\mathrm e}^{-x}+{\mathrm e}^{x}\right)$

input
```
int(exp(x)*(-exp(-x)+exp(x))*(exp(-x)+exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*exp(x)^2+1/4*exp(x)^4+1/2/exp(x)^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2 dx = -\frac{1}{4}\left(4\,xe^{(2\,x)}-e^{(6\,x)}-2\,e^{(4\,x)}-2\right)e^{(-2\,x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))*(exp(-x)+exp(x))^2,x, algorithm="fricas
")
```

output
```
-1/4*(4*x*e^(2*x) - e^(6*x) - 2*e^(4*x) - 2)*e^(-2*x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2 dx = -x+\frac{e^{4x}}{4}+\frac{e^{2x}}{2}+\frac{e^{-2x}}{2}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))*(exp(-x)+exp(x))**2,x)
```

output
```
-x + exp(4*x)/4 + exp(2*x)/2 + exp(-2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2 dx = \frac{1}{4}\left(2\,e^{(-2\,x)}+1\right)e^{(4\,x)}-x+\frac{1}{2}\,e^{(-2\,x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))*(exp(-x)+exp(x))^2,x, algorithm="maxima
")
```

output
```
1/4*(2*e^(-2*x) + 1)*e^(4*x) - x + 1/2*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2\,dx=\frac{1}{2}\left(e^{(2x)}+1\right)e^{(-2x)}-x+\frac{1}{4}\,e^{(4x)}+\frac{1}{2}\,e^{(2x)}$$

input
```
integrate(exp(x)*(-exp(-x)+exp(x))*(exp(-x)+exp(x))^2,x, algorithm="giac")
```

output
```
1/2*(e^(2*x) + 1)*e^(-2*x) - x + 1/4*e^(4*x) + 1/2*e^(2*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2\,dx=\frac{e^{-2x}}{2}-x+\frac{e^{2x}}{2}+\frac{e^{4x}}{4}$$

input
```
int(-exp(x)*(exp(-x) + exp(x))^2*(exp(-x) - exp(x)),x)
```

output
```
exp(-2*x)/2 - x + exp(2*x)/2 + exp(4*x)/4
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int e^x\left(-e^{-x}+e^x\right)\left(e^{-x}+e^x\right)^2\,dx=\frac{e^{6x}+2e^{4x}-4e^{2x}x+2}{4e^{2x}}$$

input
```
int(exp(x)*(-exp(-x)+exp(x))*(exp(-x)+exp(x))^2,x)
```

output
```
(e**(6*x) + 2*e**(4*x) - 4*e**(2*x)*x + 2)/(4*e**(2*x))
```

3.681 $\int \frac{x}{e^x+x} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{x}{e^x+x} \, dx=\text{Int}\left(\frac{x}{e^x+x},x\right)$$

output
```
Defer(Int)(x/(exp(x)+x),x)
```

Mathematica [N/A]

Not integrable

Time = 5.72 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{x}{e^x+x} \, dx=\int \frac{x}{e^x+x} \, dx$$

input
```
Integrate[x/(E^x + x),x]
```

output
```
Integrate[x/(E^x + x), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x+e^x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{x}{x+e^x}dx$$

input
```
Int[x/(E^x + x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{x}{e^x + x}dx$$

input
```
int(x/(exp(x)+x),x)
```

output
```
int(x/(exp(x)+x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{x}{e^x + x} \, dx = \int \frac{x}{x + e^x} \, dx$$

input
```
integrate(x/(exp(x)+x),x, algorithm="fricas")
```

output
```
integral(x/(x + e^x), x)
```

Sympy [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{x}{e^x + x} \, dx = \int \frac{x}{x + e^x} \, dx$$

input
```
integrate(x/(exp(x)+x),x)
```

output
```
Integral(x/(x + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{x}{e^x + x} \, dx = \int \frac{x}{x + e^x} \, dx$$

input
```
integrate(x/(exp(x)+x),x, algorithm="maxima")
```

output
```
integrate(x/(x + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{x}{e^x + x} \, dx = \int \frac{x}{x + e^x} \, dx$$

input
```
integrate(x/(exp(x)+x),x, algorithm="giac")
```

output
```
integrate(x/(x + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{x}{e^x + x} \, dx = \int \frac{x}{x + e^x} \, dx$$

input
```
int(x/(x + exp(x)),x)
```

output
```
int(x/(x + exp(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{x}{e^x + x} \, dx = \int \frac{1}{e^x + x} dx - \log(e^x + x) + x$$

input
```
int(x/(exp(x)+x),x)
```

output
```
int(1/(e**x + x),x) - log(e**x + x) + x
```

3.682 $\int \frac{x^2}{\sqrt{e^x+x}} dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{x^2}{\sqrt{e^x + x}} dx = \text{Int}\left(\frac{x^2}{\sqrt{e^x + x}}, x\right)$$

output
```
Defer(Int)(x^2/(exp(x)+x)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{\sqrt{e^x + x}} dx = \int \frac{x^2}{\sqrt{e^x + x}} dx$$

input
```
Integrate[x^2/Sqrt[E^x + x],x]
```

output
```
Integrate[x^2/Sqrt[E^x + x], x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{x+e^x}} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{x^2}{\sqrt{x+e^x}}dx$$

input
```
Int[x^2/Sqrt[E^x + x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{\sqrt{\mathrm{e}^x+x}}dx$$

input
```
int(x^2/(exp(x)+x)^(1/2),x)
```

output
```
int(x^2/(exp(x)+x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{e^x + x}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(exp(x)+x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\sqrt{e^x + x}}\, dx = \int \frac{x^2}{\sqrt{x + e^x}}\, dx$$

input
```
integrate(x**2/(exp(x)+x)**(1/2),x)
```

output
```
Integral(x**2/sqrt(x + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\sqrt{e^x+x}}\,dx = \int \frac{x^2}{\sqrt{x+e^x}}\,dx$$

input
```
integrate(x^2/(exp(x)+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt(x + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\sqrt{e^x+x}}\,dx = \int \frac{x^2}{\sqrt{x+e^x}}\,dx$$

input
```
integrate(x^2/(exp(x)+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(x + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\sqrt{e^x+x}}\,dx = \int \frac{x^2}{\sqrt{x+\mathrm{e}^x}}\,dx$$

input
```
int(x^2/(x + exp(x))^(1/2),x)
```

output
```
int(x^2/(x + exp(x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \frac{x^2}{\sqrt{e^x + x}}\,dx = \int \frac{\sqrt{e^x + x}\,x^2}{e^x + x}dx$$

input
```
int(x^2/(exp(x)+x)^(1/2),x)
```

output
```
int((sqrt(e**x + x)*x**2)/(e**x + x),x)
```

3.683 $\int \frac{e^x}{e^x+x} dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{e^x}{e^x+x} dx = \text{Int}\left(\frac{e^x}{e^x+x}, x\right)$$

output
```
Defer(Int)(exp(x)/(exp(x)+x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{e^x}{e^x+x} dx = \int \frac{e^x}{e^x+x} dx$$

input
```
Integrate[E^x/(E^x + x),x]
```

output
```
Integrate[E^x/(E^x + x), x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{x+e^x} \, dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{e^x}{x+e^x}dx$$

input
```
Int[E^x/(E^x + x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\mathrm{e}^{x}}{\mathrm{e}^{x}+x}dx$$

input
```
int(exp(x)/(exp(x)+x),x)
```

output
```
int(exp(x)/(exp(x)+x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{e^x + x}\,dx = \int \frac{e^x}{x + e^x}\,dx$$

input
```
integrate(exp(x)/(exp(x)+x),x, algorithm="fricas")
```

output
```
integral(e^x/(x + e^x), x)
```

Sympy [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{e^x}{e^x + x}\,dx = \int \frac{e^x}{x + e^x}\,dx$$

input
```
integrate(exp(x)/(exp(x)+x),x)
```

output
```
Integral(exp(x)/(x + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{e^x}{e^x + x} \, dx = \int \frac{e^x}{x + e^x} \, dx$$

input
```
integrate(exp(x)/(exp(x)+x),x, algorithm="maxima")
```

output
```
x - integrate(x/(x + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{e^x + x} \, dx = \int \frac{e^x}{x + e^x} \, dx$$

input
```
integrate(exp(x)/(exp(x)+x),x, algorithm="giac")
```

output
```
integrate(e^x/(x + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{e^x + x} \, dx = \int \frac{\mathrm{e}^x}{x + \mathrm{e}^x} \, dx$$

input
```
int(exp(x)/(x + exp(x)),x)
```

output
```
int(exp(x)/(x + exp(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{e^x}{e^x + x}\,dx = -\left(\int \frac{1}{e^x + x}dx\right) + \log(e^x + x)$$

input
```
int(exp(x)/(exp(x)+x),x)
```

output
```
- int(1/(e**x + x),x) + log(e**x + x)
```

3.684 $\int \frac{e^x}{e^x+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{e^x}{e^x+x^2} \, dx = \text{Int}\left(\frac{e^x}{e^x+x^2},x\right)$$

output
```
Defer(Int)(exp(x)/(exp(x)+x^2),x)
```

Mathematica [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{e^x}{e^x+x^2} \, dx = \int \frac{e^x}{e^x+x^2} \, dx$$

input
```
Integrate[E^x/(E^x + x^2),x]
```

output
```
Integrate[E^x/(E^x + x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{x^2+e^x} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{e^x}{x^2+e^x}dx$$

input
```
Int[E^x/(E^x + x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{{\mathrm e}^{x}}{{\mathrm e}^{x}+x^{2}}dx$$

input
```
int(exp(x)/(exp(x)+x^2),x)
```

output
```
int(exp(x)/(exp(x)+x^2),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{e^x + x^2} \, dx = \int \frac{e^x}{x^2 + e^x} \, dx$$

input
```
integrate(exp(x)/(exp(x)+x^2),x, algorithm="fricas")
```

output
```
integral(e^x/(x^2 + e^x), x)
```

Sympy [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{e^x}{e^x + x^2} \, dx = \int \frac{e^x}{x^2 + e^x} \, dx$$

input
```
integrate(exp(x)/(exp(x)+x**2),x)
```

output
```
Integral(exp(x)/(x**2 + exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{e^x}{e^x + x^2} \, dx = \int \frac{e^x}{x^2 + e^x} \, dx$$

input
```
integrate(exp(x)/(exp(x)+x^2),x, algorithm="maxima")
```

output
```
x - integrate(x^2/(x^2 + e^x), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{e^x + x^2} \, dx = \int \frac{e^x}{x^2 + e^x} \, dx$$

input
```
integrate(exp(x)/(exp(x)+x^2),x, algorithm="giac")
```

output
```
integrate(e^x/(x^2 + e^x), x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{e^x + x^2} \, dx = \int \frac{\mathrm{e}^x}{\mathrm{e}^x + x^2} \, dx$$

input
```
int(exp(x)/(exp(x) + x^2),x)
```

output
```
int(exp(x)/(exp(x) + x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{e^x}{e^x + x^2} \, dx = -2\left(\int \frac{x}{e^x + x^2} dx\right) + \log\left(e^x + x^2\right)$$

input
```
int(exp(x)/(exp(x)+x^2),x)
```

output
```
 - 2*int(x/(e**x + x**2),x) + log(e**x + x**2)
```

3.685 $\int \frac{\mathbf{F0}(x)}{x+\mathbf{F0}(x)}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx = x-\mathrm{Int}\left(\frac{x}{x+\mathrm{F0}(x)},x\right)$$

output
```
x-Defer(Int)(x/(x+F0(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx = \int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx$$

input
```
Integrate[F0[x]/(x + F0[x]),x]
```

output
```
Integrate[F0[x]/(x + F0[x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{F0}(x)}{\text{F0}(x)+x}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(1-\frac{x}{\text{F0}(x)+x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-\int \frac{x}{x+\text{F0}(x)}dx$$

input
```
Int[F0[x]/(x + F0[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{F0}\left(x\right)}{x+\mathrm{F0}\left(x\right)}dx$$

input
```
int(F0(x)/(x+F0(x)),x)
```

output
```
int(F0(x)/(x+F0(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx = \int \frac{F_0(x)}{x+F_0\left(x\right)}\,dx$$

input
```
integrate(F0(x)/(x+F0(x)),x, algorithm="fricas")
```

output
```
integral(F0(x)/(x + F0(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx = \int \frac{F_0(x)}{x+F_0(x)}\,dx$$

input
```
integrate(F0(x)/(x+F0(x)),x)
```

output
```
Integral(F0(x)/(x + F0(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx=\int \frac{F_0(x)}{x+F_0\left(x\right)}\,dx$$

input
```
integrate(F0(x)/(x+F0(x)),x, algorithm="maxima")
```

output
```
integrate(F0(x)/(x + F0(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx=\int \frac{F_0(x)}{x+F_0\left(x\right)}\,dx$$

input
```
integrate(F0(x)/(x+F0(x)),x, algorithm="giac")
```

output
```
integrate(F0(x)/(x + F0(x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx=\int \frac{F_0(x)}{x+F_0\left(x\right)}\,dx$$

input
```
int(F0(x)/(x + F0(x)),x)
```

output
```
int(F0(x)/(x + F0(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{x+\mathrm{F0}(x)}\,dx=\int \frac{\mathit{F0}(x)}{x+\mathit{F0}\left(x\right)}dx$$

input
```
int(F0(x)/(x+F0(x)),x)
```

output
```
int(F0(x)/(x+F0(x)),x)
```

3.686 $\int \frac{\mathbf{F0}(x)}{x^2+\mathbf{F0}(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{F0}(x)}{x^2+\text{F0}(x)}\,dx = x-\text{Int}\left(\frac{x^2}{x^2+\text{F0}(x)},x\right)$$

output
```
x-Defer(Int)(x^2/(x^2+F0(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{F0}(x)}{x^2+\text{F0}(x)}\,dx=\int \frac{\text{F0}(x)}{x^2+\text{F0}(x)}\,dx$$

input
```
Integrate[F0[x]/(x^2 + F0[x]),x]
```

output
```
Integrate[F0[x]/(x^2 + F0[x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{F0}(x)}{\text{F0}(x)+x^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(1-\frac{x^2}{\text{F0}(x)+x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x-\int \frac{x^2}{x^2+\text{F0}(x)}dx$$

input
```
Int[F0[x]/(x^2 + F0[x]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{F0}\,(x)}{x^2+\mathrm{F0}\,(x)}dx$$

input
```
int(F0(x)/(x^2+F0(x)),x)
```

output
```
int(F0(x)/(x^2+F0(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{x^2+\mathrm{F0}(x)}\,dx = \int \frac{F_0(x)}{x^2+F_0\,(x)}\,dx$$

input
```
integrate(F0(x)/(x^2+F0(x)),x, algorithm="fricas")
```

output
```
integral(F0(x)/(x^2 + F0(x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\mathrm{F0}(x)}{x^2+\mathrm{F0}(x)}\,dx = \int \frac{F_0(x)}{x^2+F_0(x)}\,dx$$

input
```
integrate(F0(x)/(x**2+F0(x)),x)
```

output
```
Integral(F0(x)/(x**2 + F0(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{x^2 + \mathrm{F0}(x)}\,dx = \int \frac{F_0(x)}{x^2 + F_0\left(x\right)}\,dx$$

input
```
integrate(F0(x)/(x^2+F0(x)),x, algorithm="maxima")
```

output
```
integrate(F0(x)/(x^2 + F0(x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{x^2 + \mathrm{F0}(x)}\,dx = \int \frac{F_0(x)}{x^2 + F_0\left(x\right)}\,dx$$

input
```
integrate(F0(x)/(x^2+F0(x)),x, algorithm="giac")
```

output
```
integrate(F0(x)/(x^2 + F0(x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{x^2+\mathrm{F0}(x)}\,dx=\int \frac{F_0(x)}{F_0\left(x\right)+x^2}\,dx$$

input
```
int(F0(x)/(F0(x) + x^2),x)
```

output
```
int(F0(x)/(F0(x) + x^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{x^2+\mathrm{F0}(x)}\,dx=\int \frac{\mathit{F0}(x)}{x^2+\mathit{F0}\left(x\right)}dx$$

input
```
int(F0(x)/(x^2+F0(x)),x)
```

output
```
int(F0(x)/(x^2+F0(x)),x)
```

3.687 $\int \frac{\mathbf{F0}(x)}{(x+\mathbf{F0}(x))^2}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 9

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx = -\mathrm{Int}\left(\frac{x}{(x+\mathrm{F0}(x))^2},x\right)+\mathrm{Int}\left(\frac{1}{x+\mathrm{F0}(x)},x\right)$$

output
```
-Defer(Int)(x/(x+F0(x))^2,x)+Defer(Int)(1/(x+F0(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx = \int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx$$

input
```
Integrate[F0[x]/(x + F0[x])^2,x]
```

output
```
Integrate[F0[x]/(x + F0[x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{F0}(x)}{(\text{F0}(x)+x)^2} \, dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{1}{\text{F0}(x)+x}-\frac{x}{(\text{F0}(x)+x)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\int \frac{1}{x+\text{F0}(x)}dx-\int \frac{x}{(x+\text{F0}(x))^2}dx$$

input
```
Int[F0[x]/(x + F0[x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{F0}\,(x)}{\left(x+\mathrm{F0}\,(x)\right)^{2}}dx$$

input
```
int(F0(x)/(x+F0(x))^2,x)
```

output
```
int(F0(x)/(x+F0(x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx=\int \frac{F_0(x)}{\left(x+F_0\,(x)\right)^{2}}\,dx$$

input
```
integrate(F0(x)/(x+F0(x))^2,x, algorithm="fricas")
```

output
```
integral(F0(x)/(x^2 + 2*x*F0(x) + F0(x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx=\int \frac{F_0(x)}{\left(x+F_0(x)\right)^{2}}\,dx$$

input
```
integrate(F0(x)/(x+F0(x))**2,x)
```

output
```
Integral(F0(x)/(x + F0(x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx = \int \frac{F_0(x)}{\left(x+F_0\left(x\right)\right)^2}\,dx$$

input
```
integrate(F0(x)/(x+F0(x))^2,x, algorithm="maxima")
```

output
```
integrate(F0(x)/(x + F0(x))^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx = \int \frac{F_0(x)}{\left(x+F_0\left(x\right)\right)^2}\,dx$$

input
```
integrate(F0(x)/(x+F0(x))^2,x, algorithm="giac")
```

output
```
integrate(F0(x)/(x + F0(x))^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx=\int \frac{F_0(x)}{\left(x+F_0\left(x\right)\right)^2}\,dx$$

input
```
int(F0(x)/(x + F0(x))^2,x)
```

output
```
int(F0(x)/(x + F0(x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\mathrm{F0}(x)}{(x+\mathrm{F0}(x))^2}\,dx=\int \frac{\mathit{F0}(x)}{\left(x+\mathit{F0}\left(x\right)\right)^2}dx$$

input
```
int(F0(x)/(x+F0(x))^2,x)
```

output
```
int(F0(x)/(x+F0(x))^2,x)
```

3.688 $\int \frac{\mathbf{F0}(x)}{\left(x^2+\mathbf{F0}(x)\right)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\text{F0}(x)}{\left(x^2+\text{F0}(x)\right)^2}\,dx = -\text{Int}\left(\frac{x^2}{\left(x^2+\text{F0}(x)\right)^2},x\right)+\text{Int}\left(\frac{1}{x^2+\text{F0}(x)},x\right)$$

output
```
-Defer(Int)(x^2/(x^2+F0(x))^2,x)+Defer(Int)(1/(x^2+F0(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\text{F0}(x)}{\left(x^2+\text{F0}(x)\right)^2}\,dx = \int \frac{\text{F0}(x)}{\left(x^2+\text{F0}(x)\right)^2}\,dx$$

input
```
Integrate[F0[x]/(x^2 + F0[x])^2,x]
```

output
```
Integrate[F0[x]/(x^2 + F0[x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{F0}(x)}{(\text{F0}(x)+x^2)^2} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{1}{\text{F0}(x)+x^2}-\frac{x^2}{(\text{F0}(x)+x^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{1}{x^2+\text{F0}(x)}dx-\int \frac{x^2}{(x^2+\text{F0}(x))^2}dx$$

input
```
Int[F0[x]/(x^2 + F0[x])^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\mathrm{F0}\left(x\right)}{\left(x^{2}+\mathrm{F0}\left(x\right)\right)^{2}}dx$$

input
```
int(F0(x)/(x^2+F0(x))^2,x)
```

output
```
int(F0(x)/(x^2+F0(x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\mathrm{F0}(x)}{\left(x^{2}+\mathrm{F0}(x)\right)^{2}}\, dx=\int \frac{F_{0}(x)}{\left(x^{2}+F_{0}\left(x\right)\right)^{2}}\, dx$$

input
```
integrate(F0(x)/(x^2+F0(x))^2,x, algorithm="fricas")
```

output
```
integral(F0(x)/(x^4 + 2*x^2*F0(x) + F0(x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\mathrm{F0}(x)}{\left(x^{2}+\mathrm{F0}(x)\right)^{2}}\, dx=\int \frac{F_{0}(x)}{\left(x^{2}+F_{0}(x)\right)^{2}}\, dx$$

input
```
integrate(F0(x)/(x**2+F0(x))**2,x)
```

output
```
Integral(F0(x)/(x**2 + F0(x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{(x^2+\mathrm{F0}(x))^2}\,dx = \int \frac{F_0(x)}{(x^2+F_0\,(x))^2}\,dx$$

input
```
integrate(F0(x)/(x^2+F0(x))^2,x, algorithm="maxima")
```

output
```
integrate(F0(x)/(x^2 + F0(x))^2, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{(x^2+\mathrm{F0}(x))^2}\,dx = \int \frac{F_0(x)}{(x^2+F_0\,(x))^2}\,dx$$

input
```
integrate(F0(x)/(x^2+F0(x))^2,x, algorithm="giac")
```

output
```
integrate(F0(x)/(x^2 + F0(x))^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{(x^2+\mathrm{F0}(x))^2}\,dx = \int \frac{F_0(x)}{(F_0(x)+x^2)^2}\,dx$$

input
```
int(F0(x)/(F0(x) + x^2)^2,x)
```

output
```
int(F0(x)/(F0(x) + x^2)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\mathrm{F0}(x)}{(x^2+\mathrm{F0}(x))^2}\,dx = \int \frac{F0(x)}{(x^2+F0(x))^2}dx$$

input
```
int(F0(x)/(x^2+F0(x))^2,x)
```

output
```
int(F0(x)/(x^2+F0(x))^2,x)
```

3.689 $\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \frac{\left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n}{(dm+fn)\log(F)}$$

output
```
(a*F^(d*x+c))^m*(b*F^(f*x+e))^n/(d*m+f*n)/ln(F)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \frac{\left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n}{dm\log(F) + fn\log(F)}$$

input
```
Integrate[(a*F^(c + d*x))^m*(b*F^(e + f*x))^n,x]
```

output
```
((a*F^(c + d*x))^m*(b*F^(e + f*x))^n)/(d*m*Log[F] + f*n*Log[F])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2717, 2717, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx$$
$$\Big\downarrow \text{ 2717}$$
$$F^{-m(c+dx)}\left(aF^{c+dx}\right)^m \int F^{m(c+dx)}\left(bF^{e+fx}\right)^n dx$$
$$\Big\downarrow \text{ 2717}$$
$$\left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n F^{-m(c+dx)-n(e+fx)} \int F^{m(c+dx)+n(e+fx)}dx$$
$$\Big\downarrow \text{ 2624}$$
$$\frac{\left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n}{\log(F)(dm+fn)}$$

input
```
Int[(a*F^(c + d*x))^m*(b*F^(e + f*x))^n,x]
```

output
```
((a*F^(c + d*x))^m*(b*F^(e + f*x))^n)/((d*m + f*n)*Log[F])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

method	result
gosper	$\frac{(a\,F^{dx+c})^{m}(b\,F^{fx+e})^{n}}{(dm+fn)\ln(F)}$
parallelrisch	$\frac{(a\,F^{dx+c})^{m}(b\,F^{fx+e})^{n}}{(dm+fn)\ln(F)}$
orering	$\frac{(a\,F^{dx+c})^{m}(b\,F^{fx+e})^{n}}{(dm+fn)\ln(F)}$
risch	$\frac{b^{n}(F^{fx+e})^{n}a^{m}(F^{dx+c})^{m}\mathrm{e}^{-\frac{i\pi\left(-\operatorname{csgn}\left(iF^{fx+e}\right)\operatorname{csgn}\left(ib\,F^{fx+e}\right)^{2}n+\operatorname{csgn}\left(iF^{fx+e}\right)\operatorname{csgn}\left(ib\,F^{fx+e}\right)\operatorname{csgn}(ib)n+\operatorname{csgn}\left(ia\,F^{dx+c}\right)^{3}n}{}}}{}$

input
```
int((a*F^(d*x+c))^m*(b*F^(f*x+e))^n,x,method=_RETURNVERBOSE)
```

output
```
(a*F^(d*x+c))^m*(b*F^(f*x+e))^n/(d*m+f*n)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \frac{e^{((dmx+cm)\log(F)+(fnx+en)\log(F)+m\log(a)+n\log(b))}}{(dm+fn)\log(F)}$$

input
```
integrate((a*F^(d*x+c))^m*(b*F^(f*x+e))^n,x, algorithm="fricas")
```

output
```
e^((d*m*x + c*m)*log(F) + (f*n*x + e*n)*log(F) + m*log(a) + n*log(b))/((d*
m + f*n)*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.17

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \begin{cases} a^m b^n x & \text{for } F = 1 \wedge \left(F = 1 \vee d = -\frac{fn}{m}\right) \\ x\left(F^{c-\frac{fnx}{m}}a\right)^m \left(F^{e+fx}b\right)^n & \text{for } d = -\frac{fn}{m} \\ \frac{\left(F^{c+dx}a\right)^m \left(F^{e+fx}b\right)^n}{dm\log(F)+fn\log(F)} & \text{otherwise} \end{cases}$$

input
```
integrate((a*F**(d*x+c))**m*(b*F**(f*x+e))**n,x)
```

output
```
Piecewise((a**m*b**n*x, Eq(F, 1) & (Eq(F, 1) | Eq(d, -f*n/m))), (x*(F**(c
- f*n*x/m)*a)**m*(F**(e + f*x)*b)**n, Eq(d, -f*n/m)), ((F**(c + d*x)*a)**m
*(F**(e + f*x)*b)**n/(d*m*log(F) + f*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.61

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \frac{F^{en-\frac{cfn}{d}} a^m b^n e^{\left((dx+c)m\log(F)+\frac{(dx+c)fn\log(F)}{d}\right)}}{(dm+fn)\log(F)}$$

input
```
integrate((a*F^(d*x+c))^m*(b*F^(f*x+e))^n,x, algorithm="maxima")
```

output
```
F^(e*n - c*f*n/d)*a^m*b^n*e^((d*x + c)*m*log(F) + (d*x + c)*f*n*log(F)/d)/
((d*m + f*n)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \frac{e^{(dmx\log(F)+fnx\log(F)+cm\log(F)+en\log(F)+m\log(a)+n\log(b))}}{dm\log(F)+fn\log(F)}$$

input
```
integrate((a*F^(d*x+c))^m*(b*F^(f*x+e))^n,x, algorithm="giac")
```

output
```
e^(d*m*x*log(F) + f*n*x*log(F) + c*m*log(F) + e*n*log(F) + m*log(a) + n*lo
g(b))/(d*m*log(F) + f*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \int \left(F^{c+d\,x}\,a\right)^m \left(F^{e+f\,x}\,b\right)^n \,dx$$

input
```
int((F^(c + d*x)*a)^m*(F^(e + f*x)*b)^n,x)
```

output
```
int((F^(c + d*x)*a)^m*(F^(e + f*x)*b)^n, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \left(aF^{c+dx}\right)^m \left(bF^{e+fx}\right)^n \, dx = \frac{f^{dmx+fnx+cm+en}b^n a^m}{\log(f)(dm+fn)}$$

input
```
int((a*F^(d*x+c))^m*(b*F^(f*x+e))^n,x)
```

output
```
(f**(c*m + d*m*x + e*n + f*n*x)*b**n*a**m)/(log(f)*(d*m + f*n))
```

3.690 $\int e^{a+c+bx^n+dx^n}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int e^{a+c+bx^n+dx^n}\,dx=-\frac{e^{a+c}x(-((b+d)x^n))^{-1/n}\,\Gamma\left(\frac{1}{n},-((b+d)x^n)\right)}{n}$$

output
```
-exp(a+c)*x*GAMMA(1/n,-(b+d)*x^n)/n/((-(b+d)*x^n)^(1/n))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{a+c+bx^n+dx^n}\,dx=-\frac{e^{a+c}x(-((b+d)x^n))^{-1/n}\,\Gamma\left(\frac{1}{n},-((b+d)x^n)\right)}{n}$$

input
```
Integrate[E^(a + c + b*x^n + d*x^n),x]
```

output
```
-((E^(a + c)*x*Gamma[n^(-1), -((b + d)*x^n)])/(n*(-((b + d)*x^n))^n^(-1)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7292, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx^n+c+dx^n}\,dx$$

$$\Big\downarrow \text{7292}$$

$$\int e^{a+(b+d)x^n+c}dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{xe^{a+c}(-((b+d)x^n))^{-1/n}\,\Gamma\left(\frac{1}{n},-((b+d)x^n)\right)}{n}$$

input
```
Int[E^(a + c + b*x^n + d*x^n),x]
```

output
```
-((E^(a + c)*x*Gamma[n^(-1), -((b + d)*x^n)])/(n*(-((b + d)*x^n))^n^(-1)))
```

Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.06 (sec) , antiderivative size = 241, normalized size of antiderivative = 6.51

method	result
meijerg	$\frac{\mathrm{e}^{a+c}(-b-d)^{-\frac{1}{n}}\left(\frac{n^{2}x^{1-n}(-b-d)^{\frac{1}{n}-1}(n\,x^{n}(-b-d)+n+1)(x^{n}(-b-d))^{-\frac{n+1}{2n}}\mathrm{e}^{-\frac{x^{n}(-b-d)}{2}}\operatorname{WhittakerM}\left(\frac{1}{n}-\frac{n+1}{2n},\frac{n+1}{2n}+\frac{1}{2},x^{n}(-b-d)\right)}{(n+1)(2n+1)}+\frac{n}{}\right.}{n}$

input
```
int(exp(a+c+b*x^n+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
exp(a+c)/n*(-b-d)^(-1/n)*(n^2*x^(1-n)*(-b-d)^(1/n-1)*(n*x^n*(-b-d)+n+1)/(n
+1)/(2*n+1)*(x^n*(-b-d))^(-1/2*(n+1)/n)*exp(-1/2*x^n*(-b-d))*WhittakerM(1/
n-1/2*(n+1)/n,1/2*(n+1)/n+1/2,x^n*(-b-d))+n*x^(1-n)*(-b-d)^(1/n-1)*(n+1)/(
2*n+1)*(x^n*(-b-d))^(-1/2*(n+1)/n)*exp(-1/2*x^n*(-b-d))*WhittakerM(1/n-1/2
*(n+1)/n+1,1/2*(n+1)/n+1/2,x^n*(-b-d)))
```

Fricas [F]

$$\int e^{a+c+bx^n+dx^n}\,dx=\int e^{(bx^n+dx^n+a+c)}\,dx$$

input
```
integrate(exp(a+c+b*x^n+d*x^n),x, algorithm="fricas")
```

output
```
integral(e^((b + d)*x^n + a + c), x)
```

Sympy [F]

$$\int e^{a+c+bx^n+dx^n}\,dx = e^a e^c \int e^{bx^n} e^{dx^n}\,dx$$

input
```
integrate(exp(a+c+b*x**n+d*x**n),x)
```

output
```
exp(a)*exp(c)*Integral(exp(b*x**n)*exp(d*x**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{a+c+bx^n+dx^n}\,dx = -\frac{xe^{(a+c)}\Gamma\left(\frac{1}{n}, -(b+d)x^n\right)}{\left(-(b+d)x^n\right)^{\left(\frac{1}{n}\right)} n}$$

input
```
integrate(exp(a+c+b*x^n+d*x^n),x, algorithm="maxima")
```

output
```
-x*e^(a + c)*gamma(1/n, -(b + d)*x^n)/((-(b + d)*x^n)^(1/n)*n)
```

Giac [F]

$$\int e^{a+c+bx^n+dx^n}\,dx = \int e^{(bx^n+dx^n+a+c)}\,dx$$

input
```
integrate(exp(a+c+b*x^n+d*x^n),x, algorithm="giac")
```

output
```
integrate(e^(b*x^n + d*x^n + a + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{a+c+bx^n+dx^n}\,dx = \int \mathrm{e}^{a+c+b\,x^n+d\,x^n}\,dx$$

input
```
int(exp(a + c + b*x^n + d*x^n),x)
```

output
```
int(exp(a + c + b*x^n + d*x^n), x)
```

Reduce [F]

$$\int e^{a+c+bx^n+dx^n}\,dx = e^{a+c}\left(\int e^{x^n b+x^n d}dx\right)$$

input
```
int(exp(a+c+b*x^n+d*x^n),x)
```

output
```
e**(a + c)*int(e**(x**n*b + x**n*d),x)
```

3.691 $\int f^{a+bx^n} g^{c+dx^n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 50

$$\int f^{a+bx^n} g^{c+dx^n} \, dx = -\frac{f^a g^c x \Gamma\left(\frac{1}{n}, -x^n(b\log(f)+d\log(g))\right) \left(-x^n(b\log(f)+d\log(g))\right)^{-1/n}}{n}$$

output
```
-f^a*g^c*x*GAMMA(1/n,-x^n*(b*ln(f)+d*ln(g)))/n/((-x^n*(b*ln(f)+d*ln(g)))^(
1/n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} g^{c+dx^n} \, dx = -\frac{f^a g^c x \Gamma\left(\frac{1}{n}, -x^n(b\log(f)+d\log(g))\right) \left(-x^n(b\log(f)+d\log(g))\right)^{-1/n}}{n}$$

input
```
Integrate[f^(a + b*x^n)*g^(c + d*x^n),x]
```

output
```
-((f^a*g^c*x*Gamma[n^(-1), -(x^n*(b*Log[f] + d*Log[g]))])/(n*(-(x^n*(b*Log
[f] + d*Log[g])))^n^(-1)))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2725, 2637}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx^n} g^{c+dx^n} \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int \exp \left(a \log(f) + x^n (b \log(f) + d \log(g)) + c \log(g)\right) dx$$

$$\Big\downarrow \text{2637}$$

$$-\frac{x f^a g^c (-x^n (b \log(f) + d \log(g)))^{-1/n} \Gamma\left(\frac{1}{n}, -x^n (b \log(f) + d \log(g))\right)}{n}$$

input
```
Int[f^(a + b*x^n)*g^(c + d*x^n),x]
```

output

```
-((f^a*g^c*x*Gamma[n^(-1), -(x^n*(b*Log[f] + d*Log[g]))])/(n*(-(x^n*(b*Log
[f] + d*Log[g])))^n^(-1)))
```


Defintions of rubi rules used

rule 2637
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F])^(1/n))), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 298, normalized size of antiderivative = 5.96

method	result
meijerg	$\frac{f^{a} g^{c} (-d)^{-\frac{1}{n}} \ln(g)^{-\frac{1}{n}} \left(1+\frac{b\ln(f)}{d\ln(g)}\right)^{-\frac{1}{n}} \left(\frac{n x (-d)^{\frac{1}{n}} \ln(g)^{\frac{1}{n}} \left(1+\frac{b \ln(f)}{d \ln(g)}\right)^{\frac{1}{n}} \left(n x^{n} d \ln(g) \left(1+\frac{b\ln(f)}{d\ln(g)}\right)+n+1\right) \Gamma\left(1-\frac{1}{n}\right) \Gamma\left(\frac{n+1}{n}+1\right) \mathrm{LaguerreL}(}{(n+1)\Gamma\left(-\frac{1}{n}+\frac{n+1}{n}+1\right)}\right.}{}$

input
```
int(f^(a+b*x^n)*g^(c+d*x^n),x,method=_RETURNVERBOSE)
```

output
```
f^a*g^c/n*(-d)^(-1/n)*ln(g)^(-1/n)*(1+b*ln(f)/d/ln(g))^(-1/n)*(n*x*(-d)^(1
/n)*ln(g)^(1/n)*(1+b*ln(f)/d/ln(g))^(1/n)*(n*x^n*d*ln(g)*(1+b*ln(f)/d/ln(g
))+n+1)/(n+1)/GAMMA(-1/n+(n+1)/n+1)*GAMMA(1-1/n)*GAMMA((n+1)/n+1)*Laguerre
L(-1/n,(n+1)/n,x^n*d*ln(g)*(1+b*ln(f)/d/ln(g)))-n^2*x^(n+1)*(-d)^(1/n)*ln(
g)^(1+1/n)*(1+b*ln(f)/d/ln(g))^(1+1/n)*d/(n+1)*LaguerreL(-1/n,(n+1)/n+1,x^
n*d*ln(g)*(1+b*ln(f)/d/ln(g)))*GAMMA(1-1/n)*GAMMA((n+1)/n+1)/GAMMA(-1/n+(n
+1)/n+1))
```

Fricas [F]

$$\int f^{a+bx^n} g^{c+dx^n}\,dx = \int f^{bx^n+a} g^{dx^n+c}\,dx$$

input
```
integrate(f^(a+b*x^n)*g^(c+d*x^n),x, algorithm="fricas")
```

output
```
integral(f^(b*x^n + a)*g^(d*x^n + c), x)
```

Sympy [F]

$$\int f^{a+bx^n} g^{c+dx^n}\,dx = \int f^{a+bx^n} g^{c+dx^n}\,dx$$

input
```
integrate(f**(a+b*x**n)*g**(c+d*x**n),x)
```

output
```
Integral(f**(a + b*x**n)*g**(c + d*x**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int f^{a+bx^n} g^{c+dx^n}\,dx = -\frac{f^a g^c x \Gamma\left(\frac{1}{n}, -(b\log(f) + d\log(g))x^n\right)}{\left(-(b\log(f) + d\log(g))x^n\right)^{\left(\frac{1}{n}\right)} n}$$

input
```
integrate(f^(a+b*x^n)*g^(c+d*x^n),x, algorithm="maxima")
```

output
```
-f^a*g^c*x*gamma(1/n, -(b*log(f) + d*log(g))*x^n)/((-(b*log(f) + d*log(g))
*x^n)^(1/n)*n)
```

Giac [F]

$$\int f^{a+bx^n} g^{c+dx^n}\,dx = \int f^{bx^n+a} g^{dx^n+c}\,dx$$

input
```
integrate(f^(a+b*x^n)*g^(c+d*x^n),x, algorithm="giac")
```

output
```
integrate(f^(b*x^n + a)*g^(d*x^n + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx^n} g^{c+dx^n} \, dx = \int f^{a+b\,x^n} \, g^{c+d\,x^n} \, dx$$

input
```
int(f^(a + b*x^n)*g^(c + d*x^n),x)
```

output
```
int(f^(a + b*x^n)*g^(c + d*x^n), x)
```

Reduce [F]

$$\int f^{a+bx^n} g^{c+dx^n} \, dx = g^c f^a \left(\int g^{x^n d} f^{x^n b} dx\right)$$

input
```
int(f^(a+b*x^n)*g^(c+d*x^n),x)
```

output
```
g**c*f**a*int(g**(x**n*d)*f**(x**n*b),x)
```

3.692 $\int e^{x^n} x^m \, dx$

Optimal result

Integrand size = 9, antiderivative size = 37

$$\int e^{x^n} x^m \, dx = -\frac{x^{1+m}(-x^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -x^n\right)}{n}$$

output
```
-x^(1+m)*GAMMA((1+m)/n,-x^n)/n/((-x^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int e^{x^n} x^m \, dx = -\frac{x^{1+m}(-x^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -x^n\right)}{n}$$

input
```
Integrate[E^x^n*x^m,x]
```

output
```
-((x^(1 + m)*Gamma[(1 + m)/n, -x^n])/(n*(-x^n)^((1 + m)/n)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{x^n} \, dx$$

$$\Big\downarrow \ 2648$$

$$-\frac{x^{m+1}(-x^n)^{-\frac{m+1}{n}}\Gamma\left(\frac{m+1}{n},-x^n\right)}{n}$$

input
```
Int[E^x^n*x^m,x]
```

output
```
-((x^(1 + m)*Gamma[(1 + m)/n, -x^n])/(n*(-x^n)^((1 + m)/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 219, normalized size of antiderivative = 5.92

method	result
meijerg	$\frac{(-1)^{-\frac{m}{n}-\frac{1}{n}}\left(\frac{n\,x^{1+m}(-1)^{\frac{m}{n}+\frac{1}{n}}(x^n n+m+n+1)\,\mathrm{LaguerreL}\left(-\frac{1+m}{n},\frac{1+m+n}{n},x^n\right)\Gamma\left(-\frac{1+m}{n}+1\right)\Gamma\left(\frac{1+m+n}{n}+1\right)}{(1+m)(1+m+n)\Gamma\left(-\frac{1+m}{n}+\frac{1+m+n}{n}+1\right)}-\frac{(-1)^{\frac{m}{n}+\frac{1}{n}}n^2x^{1+m+n}\,\mathrm{La}}{(1}\right.}{n}$

input
```
int(exp(x^n)*x^m,x,method=_RETURNVERBOSE)
```

output
```
(-1)^(-m/n-1/n)/n*(n/(1+m)*x^(1+m)*(-1)^(m/n+1/n)*(x^n*n+m+n+1)/(1+m+n)*La
guerreL(-(1+m)/n,(1+m+n)/n,x^n)*GAMMA(-(1+m)/n+1)*GAMMA((1+m+n)/n+1)/GAMMA
(-(1+m)/n+(1+m+n)/n+1)-(-1)^(m/n+1/n)*n^2/(1+m)*x^(1+m+n)/(1+m+n)*Laguerre
L(-(1+m)/n,(1+m+n)/n+1,x^n)*GAMMA(-(1+m)/n+1)*GAMMA((1+m+n)/n+1)/GAMMA(-(1
+m)/n+(1+m+n)/n+1))
```

Fricas [F]

$$\int e^{x^n} x^m \, dx = \int x^m e^{(x^n)} \, dx$$

input
```
integrate(exp(x^n)*x^m,x, algorithm="fricas")
```

output
```
integral(x^m*e^(x^n), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.84

$$\int e^{x^n} x^m \, dx = \frac{me^{-\frac{i\pi}{n}} e^{-\frac{i\pi m}{n}} \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) \gamma\left(\frac{m}{n} + \frac{1}{n}, x^n e^{i\pi}\right)}{n^2 \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)} + \frac{e^{-\frac{i\pi}{n}} e^{-\frac{i\pi m}{n}} \Gamma\left(\frac{m}{n} + \frac{1}{n}\right) \gamma\left(\frac{m}{n} + \frac{1}{n}, x^n e^{i\pi}\right)}{n^2 \Gamma\left(\frac{m}{n} + 1 + \frac{1}{n}\right)}$$

input
```
integrate(exp(x**n)*x**m,x)
```

output
```
m*exp(-I*pi/n)*exp(-I*pi*m/n)*gamma(m/n + 1/n)*lowergamma(m/n + 1/n, x**n*
exp_polar(I*pi))/(n**2*gamma(m/n + 1 + 1/n)) + exp(-I*pi/n)*exp(-I*pi*m/n)
*gamma(m/n + 1/n)*lowergamma(m/n + 1/n, x**n*exp_polar(I*pi))/(n**2*gamma(
m/n + 1 + 1/n))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int e^{x^n} x^m \, dx = -\frac{x^{m+1}\Gamma\left(\frac{m+1}{n}, -x^n\right)}{n\left(-x^n\right)^{\frac{m+1}{n}}}$$

input
```
integrate(exp(x^n)*x^m,x, algorithm="maxima")
```

output
```
-x^(m + 1)*gamma((m + 1)/n, -x^n)/(n*(-x^n)^((m + 1)/n))
```

Giac [F]

$$\int e^{x^n} x^m \, dx = \int x^m e^{(x^n)} \, dx$$

input
```
integrate(exp(x^n)*x^m,x, algorithm="giac")
```

output
```
integrate(x^m*e^(x^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int e^{x^n}x^m\,dx=\frac{x^{m+1}\,\mathrm{e}^{\frac{x^n}{2}}\,\mathrm{M}_{1-\frac{m+n+1}{2\,n},\frac{m+n+1}{2\,n}-\frac{1}{2}}(x^n)}{(x^n)^{\frac{m+n+1}{2\,n}}\,(m+1)}$$

input
```
int(x^m*exp(x^n),x)
```

output
```
(x^(m + 1)*exp(x^n/2)*whittakerM(1 - (m + n + 1)/(2*n), (m + n + 1)/(2*n)
- 1/2, x^n))/((x^n)^((m + n + 1)/(2*n))*(m + 1))
```

Reduce [F]

$$\int e^{x^n}x^m\,dx=\int x^me^{x^n}\,dx$$

input
```
int(exp(x^n)*x^m,x)
```

output
```
int(x**m*e**(x**n),x)
```

3.693 $\int f^{x^n} x^m \, dx$

Optimal result

Integrand size = 9, antiderivative size = 41

$$\int f^{x^n} x^m \, dx = -\frac{x^{1+m} \Gamma\left(\frac{1+m}{n}, -x^n \log(f)\right) \left(-x^n \log(f)\right)^{-\frac{1+m}{n}}}{n}$$

output
```
-x^(1+m)*GAMMA((1+m)/n,-x^n*ln(f))/n/((-x^n*ln(f))^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int f^{x^n} x^m \, dx = -\frac{x^{1+m} \Gamma\left(\frac{1+m}{n}, -x^n \log(f)\right) \left(-x^n \log(f)\right)^{-\frac{1+m}{n}}}{n}$$

input
```
Integrate[f^x^n*x^m,x]
```

output
```
-((x^(1 + m)*Gamma[(1 + m)/n, -(x^n*Log[f])])/(n*(-(x^n*Log[f]))^((1 + m)/
n)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m f^{x^n} \, dx$$

$$\Big\downarrow \text{2648}$$

$$-\frac{x^{m+1}(\log(f)\,(-x^n))^{-\frac{m+1}{n}}\,\Gamma\left(\frac{m+1}{n},-x^n\log(f)\right)}{n}$$

input
```
Int[f^x^n*x^m,x]
```

output
```
-((x^(1 + m)*Gamma[(1 + m)/n, -(x^n*Log[f])])/(n*(-(x^n*Log[f]))^((1 + m)/
n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 267, normalized size of antiderivative = 6.51

method	result
meijerg	$\frac{(-1)^{-\frac{m}{n}-\frac{1}{n}} \ln(f)^{-\frac{m}{n}-\frac{1}{n}} \left(\frac{n\, x^{1+m} (-1)^{\frac{m}{n}+\frac{1}{n}} \ln(f)^{\frac{m}{n}+\frac{1}{n}} (x^n \ln(f) n+m+n+1) \operatorname{LaguerreL}\left(-\frac{1+m}{n}, \frac{1+m+n}{n}, x^n \ln(f)\right) \Gamma\left(-\frac{1+m}{n}+1\right) \Gamma\left(\frac{1+m}{}\right.}{(1+m)(1+m+n) \Gamma\left(-\frac{1+m}{n}+\frac{1+m+n}{n}+1\right)} \right.}{n}$

input
```
int(f^(x^n)*x^m,x,method=_RETURNVERBOSE)
```

output
```
(-1)^(-m/n-1/n)*ln(f)^(-m/n-1/n)/n*(n/(1+m)*x^(1+m)*(-1)^(m/n+1/n)*ln(f)^(
m/n+1/n)*(x^n*ln(f)*n+m+n+1)/(1+m+n)*LaguerreL(-(1+m)/n,(1+m+n)/n,x^n*ln(f
))*GAMMA(-(1+m)/n+1)*GAMMA((1+m+n)/n+1)/GAMMA(-(1+m)/n+(1+m+n)/n+1)-(-1)^(
m/n+1/n)*n^2/(1+m)*x^(1+m+n)*ln(f)^(1+m/n+1/n)/(1+m+n)*LaguerreL(-(1+m)/n,
(1+m+n)/n+1,x^n*ln(f))*GAMMA(-(1+m)/n+1)*GAMMA((1+m+n)/n+1)/GAMMA(-(1+m)/n
+(1+m+n)/n+1))
```

Fricas [F]

$$\int f^{x^n} x^m \, dx = \int f^{(x^n)} x^m \, dx$$

input
```
integrate(f^(x^n)*x^m,x, algorithm="fricas")
```

output
```
integral(f^(x^n)*x^m, x)
```

Sympy [F]

$$\int f^{x^n} x^m \, dx = \int f^{x^n} x^m \, dx$$

input
```
integrate(f**(x**n)*x**m,x)
```

output
```
Integral(f**(x**n)*x**m, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int f^{x^n} x^m \, dx = -\frac{x^{m+1}\Gamma\left(\frac{m+1}{n}, -x^n \log(f)\right)}{(-x^n \log(f))^{\frac{m+1}{n}} n}$$

input
```
integrate(f^(x^n)*x^m,x, algorithm="maxima")
```

output
```
-x^(m + 1)*gamma((m + 1)/n, -x^n*log(f))/((-x^n*log(f))^((m + 1)/n)*n)
```

Giac [F]

$$\int f^{x^n} x^m \, dx = \int f^{(x^n)} x^m \, dx$$

input
```
integrate(f^(x^n)*x^m,x, algorithm="giac")
```

output
```
integrate(f^(x^n)*x^m, x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.73

$$\int f^{x^n} x^m \, dx = \frac{f^{x^n} x^{m+1} e^{-\frac{x^n \ln(f)}{2}} M_{1-\frac{m+n+1}{2n}, \frac{m+n+1}{2n}-\frac{1}{2}}(x^n \ln(f))}{(x^n \ln(f))^{\frac{m+n+1}{2n}} (m+1)}$$

input
```
int(f^(x^n)*x^m,x)
```

output
```
(f^(x^n)*x^(m + 1)*exp(-(x^n*log(f))/2)*whittakerM(1 - (m + n + 1)/(2*n),
(m + n + 1)/(2*n) - 1/2, x^n*log(f)))/((x^n*log(f))^((m + n + 1)/(2*n))*(m
 + 1))
```

Reduce [F]

$$\int f^{x^n} x^m \, dx = \int x^m f^{x^n} \, dx$$

input
```
int(f^(x^n)*x^m,x)
```

output
```
int(x**m*f**(x**n),x)
```

3.694 $\int e^{(a+bx)^n}(a+bx)^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=-\frac{(a+bx)^{1+m}\left(-(a+bx)^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-(a+bx)^n\right)}{bn}$$

output
```
-(b*x+a)^(1+m)*GAMMA((1+m)/n,-(b*x+a)^n)/b/n/((-(b*x+a)^n)^((1+m)/n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=-\frac{(a+bx)^{1+m}\left(-(a+bx)^n\right)^{-\frac{1+m}{n}}\Gamma\left(\frac{1+m}{n},-(a+bx)^n\right)}{bn}$$

input
```
Integrate[E^(a + b*x)^n*(a + b*x)^m,x]
```

output
```
-(((a + b*x)^(1 + m)*Gamma[(1 + m)/n, -(a + b*x)^n])/(b*n*(-(a + b*x)^n)^(
(1 + m)/n)))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^m e^{(a+bx)^n} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{(a+bx)^{m+1} \left(-(a+bx)^n\right)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n},-(a+bx)^n\right)}{bn}$$

input
```
Int[E^(a + b*x)^n*(a + b*x)^m,x]
```

output
```
-(((a + b*x)^(1 + m)*Gamma[(1 + m)/n, -(a + b*x)^n])/(b*n*(-(a + b*x)^n)^(
(1 + m)/n)))
```

Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int \mathrm{e}^{(bx+a)^n} (bx+a)^m \, dx$$

input
```
int(exp((b*x+a)^n)*(b*x+a)^m,x)
```

output
```
int(exp((b*x+a)^n)*(b*x+a)^m,x)
```

Fricas [F]

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=\int (bx+a)^m e^{\left((bx+a)^n\right)}\,dx$$

input
```
integrate(exp((b*x+a)^n)*(b*x+a)^m,x, algorithm="fricas")
```

output
```
integral((b*x + a)^m*e^((b*x + a)^n), x)
```

Sympy [F]

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=\int (a+bx)^m\,e^{(a+bx)^n}\,dx$$

input
```
integrate(exp((b*x+a)**n)*(b*x+a)**m,x)
```

output
```
Integral((a + b*x)**m*exp((a + b*x)**n), x)
```

Maxima [F]

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=\int (bx+a)^m e^{\left((bx+a)^n\right)}\,dx$$

input
```
integrate(exp((b*x+a)^n)*(b*x+a)^m,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m*e^((b*x + a)^n), x)
```

Giac [F]

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=\int (bx+a)^m e^{\left((bx+a)^n\right)}\,dx$$

input
```
integrate(exp((b*x+a)^n)*(b*x+a)^m,x, algorithm="giac")
```

output
```
integrate((b*x + a)^m*e^((b*x + a)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.48

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=\frac{\mathrm{e}^{\frac{(a+b\,x)^n}{2}}\,(a+b\,x)^{m+1}\,\mathrm{M}_{1-\frac{m+n+1}{2\,n},\frac{m+n+1}{2\,n}-\frac{1}{2}}\left((a+b\,x)^n\right)}{b\,\left((a+b\,x)^n\right)^{\frac{m+n+1}{2\,n}}\,(m+1)}$$

input
```
int(exp((a + b*x)^n)*(a + b*x)^m,x)
```

output
```
(exp((a + b*x)^n/2)*(a + b*x)^(m + 1)*whittakerM(1 - (m + n + 1)/(2*n), (m
 + n + 1)/(2*n) - 1/2, (a + b*x)^n))/(b*((a + b*x)^n)^((m + n + 1)/(2*n))*
(m + 1))
```

Reduce [F]

$$\int e^{(a+bx)^n}(a+bx)^m\,dx=\int e^{(bx+a)^n}(bx+a)^m\,dx$$

input
```
int(exp((b*x+a)^n)*(b*x+a)^m,x)
```

output
```
int(e**((a + b*x)**n)*(a + b*x)**m,x)
```

3.695 $\int f^{(a+bx)^n}(a+bx)^m\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int f^{(a+bx)^n}(a+bx)^m\,dx = -\frac{(a+bx)^{1+m}\Gamma\left(\frac{1+m}{n},-(a+bx)^n\log(f)\right)\left(-(a+bx)^n\log(f)\right)^{-\frac{1+m}{n}}}{bn}$$

output
```
-(b*x+a)^(1+m)*GAMMA((1+m)/n,-(b*x+a)^n*ln(f))/b/n/((-(b*x+a)^n*ln(f))^((1
+m)/n))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int f^{(a+bx)^n}(a+bx)^m\,dx = -\frac{(a+bx)^{1+m}\Gamma\left(\frac{1+m}{n},-(a+bx)^n\log(f)\right)\left(-(a+bx)^n\log(f)\right)^{-\frac{1+m}{n}}}{bn}$$

input
```
Integrate[f^(a + b*x)^n*(a + b*x)^m,x]
```

output
```
-(((a + b*x)^(1 + m)*Gamma[(1 + m)/n, -((a + b*x)^n*Log[f])])/(b*n*(-((a +
 b*x)^n*Log[f]))^((1 + m)/n)))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2648}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^m f^{(a+bx)^n} \, dx$$

$$\Big\downarrow \text{ 2648}$$

$$-\frac{(a+bx)^{m+1} (\log(f) (-(a+bx)^n))^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n},-(a+bx)^n \log(f)\right)}{bn}$$

input
```
Int[f^(a + b*x)^n*(a + b*x)^m,x]
```

output

```
-(((a + b*x)^(1 + m)*Gamma[(1 + m)/n, -((a + b*x)^n*Log[f])])/(b*n*(-((a +
 b*x)^n*Log[f]))^((1 + m)/n)))
```


Defintions of rubi rules used

rule 2648
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[
F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F
, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Maple [F]

$$\int f^{(bx+a)^n}(bx+a)^m\,dx$$

input
```
int(f^((b*x+a)^n)*(b*x+a)^m,x)
```

output
```
int(f^((b*x+a)^n)*(b*x+a)^m,x)
```

Fricas [F]

$$\int f^{(a+bx)^n}(a+bx)^m\,dx = \int (bx+a)^m f^{\left((bx+a)^n\right)}\,dx$$

input
```
integrate(f^((b*x+a)^n)*(b*x+a)^m,x, algorithm="fricas")
```

output
```
integral((b*x + a)^m*f^((b*x + a)^n), x)
```

Sympy [F]

$$\int f^{(a+bx)^n}(a+bx)^m\,dx = \int f^{(a+bx)^n}(a+bx)^m\,dx$$

input
```
integrate(f**((b*x+a)**n)*(b*x+a)**m,x)
```

output
```
Integral(f**((a + b*x)**n)*(a + b*x)**m, x)
```

Maxima [F]

$$\int f^{(a+bx)^n}(a+bx)^m\,dx=\int (bx+a)^m f^{\left((bx+a)^n\right)}\,dx$$

input
```
integrate(f^((b*x+a)^n)*(b*x+a)^m,x, algorithm="maxima")
```

output
```
integrate((b*x + a)^m*f^((b*x + a)^n), x)
```

Giac [F]

$$\int f^{(a+bx)^n}(a+bx)^m\,dx=\int (bx+a)^m f^{\left((bx+a)^n\right)}\,dx$$

input
```
integrate(f^((b*x+a)^n)*(b*x+a)^m,x, algorithm="giac")
```

output
```
integrate((b*x + a)^m*f^((b*x + a)^n), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.68

$$\int f^{(a+bx)^n}(a+bx)^m\,dx = \frac{f^{(a+b\,x)^n}\,\mathrm{e}^{-\frac{\ln(f)\,(a+b\,x)^n}{2}}\,(a+b\,x)^{m+1}\,\mathrm{M}_{1-\frac{m+n+1}{2\,n},\frac{m+n+1}{2\,n}-\frac{1}{2}}\left(\ln(f)\,(a+b\,x)^n\right)}{b\,(m+1)\,\left(\ln(f)\,(a+b\,x)^n\right)^{\frac{m+n+1}{2\,n}}}$$

input
```
int(f^((a + b*x)^n)*(a + b*x)^m,x)
```

output
```
(f^((a + b*x)^n)*exp(-(log(f)*(a + b*x)^n)/2)*(a + b*x)^(m + 1)*whittakerM
(1 - (m + n + 1)/(2*n), (m + n + 1)/(2*n) - 1/2, log(f)*(a + b*x)^n))/(b*(
m + 1)*(log(f)*(a + b*x)^n)^((m + n + 1)/(2*n)))
```

Reduce [F]

$$\int f^{(a+bx)^n}(a+bx)^m\,dx = \int f^{(bx+a)^n}(bx+a)^m\,dx$$

input
```
int(f^((b*x+a)^n)*(b*x+a)^m,x)
```

output
```
int(f**((a + b*x)**n)*(a + b*x)**m,x)
```

3.696 $\int e^{(a+bx)^3} x \, dx$

Optimal result

Integrand size = 11, antiderivative size = 80

$$\int e^{(a+bx)^3} x \, dx = \frac{a(a+bx)\Gamma\left(\frac{1}{3}, -(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}} - \frac{(a+bx)^2\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)}{3b^2\left(-(a+bx)^3\right)^{2/3}}$$

output
```
1/3*a*(b*x+a)*GAMMA(1/3,-(b*x+a)^3)/b^2/(-(b*x+a)^3)^(1/3)-1/3*(b*x+a)^2*G
AMMA(2/3,-(b*x+a)^3)/b^2/(-(b*x+a)^3)^(2/3)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int e^{(a+bx)^3} x \, dx = \frac{(a+bx)\left(a\sqrt[3]{-(a+bx)^3}\Gamma\left(\frac{1}{3}, -(a+bx)^3\right) - (a+bx)\Gamma\left(\frac{2}{3}, -(a+bx)^3\right)\right)}{3b^2\left(-(a+bx)^3\right)^{2/3}}$$

input
```
Integrate[E^(a + b*x)^3*x,x]
```

output
```
((a + b*x)*(a*(-(a + b*x)^3)^(1/3)*Gamma[1/3, -(a + b*x)^3] - (a + b*x)*Ga
mma[2/3, -(a + b*x)^3]))/(3*b^2*(-(a + b*x)^3)^(2/3))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2656, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{(a+bx)^3}\,dx$$

$$\Big\downarrow \text{ 2656}$$

$$\int \left(\frac{e^{(a+bx)^3}(a+bx)}{b}-\frac{ae^{(a+bx)^3}}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(a+bx)\Gamma\left(\frac{1}{3},-(a+bx)^3\right)}{3b^2\sqrt[3]{-(a+bx)^3}}-\frac{(a+bx)^2\Gamma\left(\frac{2}{3},-(a+bx)^3\right)}{3b^2\left(-(a+bx)^3\right)^{2/3}}$$

input
```
Int[E^(a + b*x)^3*x,x]
```

output
```
(a*(a + b*x)*Gamma[1/3, -(a + b*x)^3])/(3*b^2*(-(a + b*x)^3)^(1/3)) - ((a
+ b*x)^2*Gamma[2/3, -(a + b*x)^3])/(3*b^2*(-(a + b*x)^3)^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2656
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(Px_), x_Symbol] :> Int[
ExpandLinearProduct[F^(a + b*(c + d*x)^n), Px, c, d, x], x] /; FreeQ[{F, a,
 b, c, d, n}, x] && PolynomialQ[Px, x]
```

Maple [F]

$$\int e^{(bx+a)^3} x dx$$

input
```
int(exp((b*x+a)^3)*x,x)
```

output
```
int(exp((b*x+a)^3)*x,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.11

$$\int e^{(a+bx)^3} x \, dx = -\frac{(-b^3)^{\frac{2}{3}} a\Gamma\left(\frac{1}{3}, -b^3x^3 - 3\, ab^2x^2 - 3\, a^2bx - a^3\right) - (-b^3)^{\frac{1}{3}} b\Gamma\left(\frac{2}{3}, -b^3x^3 - 3\, ab^2x^2 - 3\, a^2bx - a^3\right)}{3\, b^4}$$

input
```
integrate(exp((b*x+a)^3)*x,x, algorithm="fricas")
```

output
```
-1/3*((-b^3)^(2/3)*a*gamma(1/3, -b^3*x^3 - 3*a*b^2*x^2 - 3*a^2*b*x - a^3)
- (-b^3)^(1/3)*b*gamma(2/3, -b^3*x^3 - 3*a*b^2*x^2 - 3*a^2*b*x - a^3))/b^4
```

Sympy [F]

$$\int e^{(a+bx)^3}x\,dx=e^{a^3}\int xe^{b^3x^3}e^{3ab^2x^2}e^{3a^2bx}\,dx$$

input
```
integrate(exp((b*x+a)**3)*x,x)
```

output
```
exp(a**3)*Integral(x*exp(b**3*x**3)*exp(3*a*b**2*x**2)*exp(3*a**2*b*x), x)
```

Maxima [F]

$$\int e^{(a+bx)^3}x\,dx=\int xe^{\left((bx+a)^3\right)}\,dx$$

input
```
integrate(exp((b*x+a)^3)*x,x, algorithm="maxima")
```

output
```
integrate(x*e^((b*x + a)^3), x)
```

Giac [F]

$$\int e^{(a+bx)^3}x\,dx=\int xe^{\left((bx+a)^3\right)}\,dx$$

input
```
integrate(exp((b*x+a)^3)*x,x, algorithm="giac")
```

output
```
integrate(x*e^((b*x + a)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int e^{(a+bx)^3} x\,dx = \int x\,\mathrm{e}^{(a+b\,x)^3}\,dx$$

input
```
int(x*exp((a + b*x)^3),x)
```

output
```
int(x*exp((a + b*x)^3), x)
```

Reduce [F]

$$\int e^{(a+bx)^3} x\,dx = e^{a^3}\left(\int e^{b^3x^3+3a\,b^2x^2+3a^2bx}xdx\right)$$

input
```
int(exp((b*x+a)^3)*x,x)
```

output
```
e**(a**3)*int(e**(3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)*x,x)
```

3.697 $\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}}\,dx$

Optimal result

Integrand size = 43, antiderivative size = 17

$$\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}}\,dx = 3x(e^x+x)^{2/3}+3\log(x)$$

output
```
3*x*(exp(x)+x)^(2/3)+3*ln(x)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}}\,dx = 3x(e^x+x)^{2/3}+3\log(x)$$

input
```
Integrate[(5*x^2 + 3*(E^x + x)^(1/3) + E^x*(3*x + 2*x^2))/(x*(E^x + x)^(1/
3)),x]
```

output
```
3*x*(E^x + x)^(2/3) + 3*Log[x]
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{5x^2+e^x(2x^2+3x)+3\sqrt[3]{x+e^x}}{x\sqrt[3]{x+e^x}}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{(2e^x+5)x}{\sqrt[3]{x+e^x}}+\frac{3e^x}{\sqrt[3]{x+e^x}}+\frac{3}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$3(x+e^x)^{2/3}x+3\log(x)$$

input
```
Int[(5*x^2 + 3*(E^x + x)^(1/3) + E^x*(3*x + 2*x^2))/(x*(E^x + x)^(1/3)),x]
```

output
```
3*x*(E^x + x)^(2/3) + 3*Log[x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \frac{5x^2 + 3(\mathrm{e}^x + x)^{\frac{1}{3}} + \mathrm{e}^x (2x^2 + 3x)}{x (\mathrm{e}^x + x)^{\frac{1}{3}}} dx$$

input
```
int((5*x^2+3*(exp(x)+x)^(1/3)+exp(x)*(2*x^2+3*x))/x/(exp(x)+x)^(1/3),x)
```

output
```
int((5*x^2+3*(exp(x)+x)^(1/3)+exp(x)*(2*x^2+3*x))/x/(exp(x)+x)^(1/3),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{5x^2 + 3\sqrt[3]{e^x + x} + e^x(3x + 2x^2)}{x\sqrt[3]{e^x + x}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((5*x^2+3*(exp(x)+x)^(1/3)+exp(x)*(2*x^2+3*x))/x/(exp(x)+x)^(1/3)
,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{5x^2 + 3\sqrt[3]{e^x + x} + e^x(3x + 2x^2)}{x\sqrt[3]{e^x + x}} dx = \int \frac{2x^2e^x + 5x^2 + 3xe^x + 3\sqrt[3]{x + e^x}}{x\sqrt[3]{x + e^x}} dx$$

input
```
integrate((5*x**2+3*(exp(x)+x)**(1/3)+exp(x)*(2*x**2+3*x))/x/(exp(x)+x)**(
1/3),x)
```

output
```
Integral((2*x**2*exp(x) + 5*x**2 + 3*x*exp(x) + 3*(x + exp(x))**(1/3))/(x*
(x + exp(x))**(1/3)), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}}\,dx=\frac{3\left(x^2+xe^x\right)}{\left(x+e^x\right)^{\frac{1}{3}}}+3\,\log\left(x\right)$$

input
```
integrate((5*x^2+3*(exp(x)+x)^(1/3)+exp(x)*(2*x^2+3*x))/x/(exp(x)+x)^(1/3)
,x, algorithm="maxima")
```

output
```
3*(x^2 + x*e^x)/(x + e^x)^(1/3) + 3*log(x)
```

Giac [F]

$$\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}}\,dx=\int \frac{5\,x^2+\left(2\,x^2+3\,x\right)e^x+3\left(x+e^x\right)^{\frac{1}{3}}}{\left(x+e^x\right)^{\frac{1}{3}}x}\,dx$$

input
```
integrate((5*x^2+3*(exp(x)+x)^(1/3)+exp(x)*(2*x^2+3*x))/x/(exp(x)+x)^(1/3)
,x, algorithm="giac")
```

output
```
integrate((5*x^2 + (2*x^2 + 3*x)*e^x + 3*(x + e^x)^(1/3))/((x + e^x)^(1/3)
*x), x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{5x^2+3\sqrt[3]{e^x+x}+e^x(3x+2x^2)}{x\sqrt[3]{e^x+x}}\,dx=3\,\ln\left(x\right)+3\,x\,{\left(x+\mathrm{e}^x\right)}^{2/3}$$

input
```
int((3*(x + exp(x))^(1/3) + exp(x)*(3*x + 2*x^2) + 5*x^2)/(x*(x + exp(x))^
(1/3)),x)
```

output
```
3*log(x) + 3*x*(x + exp(x))^(2/3)
```

Reduce [F]

$$\int \frac{5x^2 + 3\sqrt[3]{e^x + x} + e^x(3x + 2x^2)}{x\sqrt[3]{e^x + x}}\,dx = 3\left(\int \frac{e^x}{(e^x + x)^{\frac{1}{3}}}dx\right) + 2\left(\int \frac{e^x x}{(e^x + x)^{\frac{1}{3}}}dx\right) + 5\left(\int \frac{x}{(e^x + x)^{\frac{1}{3}}}dx\right) + 3\log(x)$$

input
```
int((5*x^2+3*(exp(x)+x)^(1/3)+exp(x)*(2*x^2+3*x))/x/(exp(x)+x)^(1/3),x)
```

output
```
3*int(e**x/(e**x + x)**(1/3),x) + 2*int((e**x*x)/(e**x + x)**(1/3),x) + 5*
int(x/(e**x + x)**(1/3),x) + 3*log(x)
```

3.698 $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 18

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx=\frac{e^x(1+x)}{\sqrt{1-x^2}}$$

output
```
exp(x)*(1+x)/(-x^2+1)^(1/2)
```

Mathematica [F]

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx=\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx$$

input
```
Integrate[(E^x*(2 - x^2))/((1 - x)*Sqrt[1 - x^2]),x]
```

output
```
Integrate[(E^x*(2 - x^2))/((1 - x)*Sqrt[1 - x^2]), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{e^x x}{\sqrt{1-x^2}}+\frac{e^x}{\sqrt{1-x^2}}+\frac{e^x}{(1-x)\sqrt{1-x^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\int \frac{e^x}{\sqrt{1-x^2}}dx+\int \frac{e^x}{(1-x)\sqrt{1-x^2}}dx+\int \frac{e^x x}{\sqrt{1-x^2}}dx$$

input
```
Int[(E^x*(2 - x^2))/((1 - x)*Sqrt[1 - x^2]),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
gosper	$\frac{e^{x}(1+x)}{\sqrt{-x^{2}+1}}$	16
orering	$\frac{(-1+x)(1+x)e^{x}(-x^{2}+2)}{(x^{2}-2)(1-x)\sqrt{-x^{2}+1}}$	40

input
```
int(exp(x)*(-x^2+2)/(1-x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
exp(x)*(1+x)/(-x^2+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx = -\frac{\sqrt{-x^2+1}e^x}{x-1}$$

input
```
integrate(exp(x)*(-x^2+2)/(1-x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-x^2 + 1)*e^x/(x - 1)
```

Sympy [F]

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx = \int \frac{(x^2-2)\,e^x}{\sqrt{-(x-1)\,(x+1)}\,(x-1)}\,dx$$

input
```
integrate(exp(x)*(-x**2+2)/(1-x)/(-x**2+1)**(1/2),x)
```

output
```
Integral((x**2 - 2)*exp(x)/(sqrt(-(x - 1)*(x + 1))*(x - 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx = -\frac{\sqrt{x+1}\sqrt{-x+1}e^x}{x-1}$$

input
```
integrate(exp(x)*(-x^2+2)/(1-x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x + 1)*sqrt(-x + 1)*e^x/(x - 1)
```

Giac [F]

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx = \int \frac{(x^2-2)e^x}{\sqrt{-x^2+1}(x-1)}\,dx$$

input
```
integrate(exp(x)*(-x^2+2)/(1-x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate((x^2 - 2)*e^x/(sqrt(-x^2 + 1)*(x - 1)), x)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx = -\frac{e^x\sqrt{1-x^2}}{x-1}$$

input
```
int((exp(x)*(x^2 - 2))/((1 - x^2)^(1/2)*(x - 1)),x)
```

output
```
-(exp(x)*(1 - x^2)^(1/2))/(x - 1)
```

Reduce [F]

$$\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}}\,dx = -2\left(\int \frac{e^x}{\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}}dx\right) + \int \frac{e^x x^2}{\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}}dx$$

input
```
int(exp(x)*(-x^2+2)/(1-x)/(-x^2+1)^(1/2),x)
```

output
```
- 2*int(e**x/(sqrt( - x**2 + 1)*x - sqrt( - x**2 + 1)),x) + int((e**x*x**
2)/(sqrt( - x**2 + 1)*x - sqrt( - x**2 + 1)),x)
```

3.699 $\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 11

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx=e^aF^bG^cx$$

output
```
exp(a)*F^b*G^c*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx=e^aF^bG^cx$$

input
```
Integrate[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x),x]
```

output
```
E^a*F^b*G^c*x
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2725, 2704, 27, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{b+x}G^{c+x}e^{a-x(\log(F)+\log(G))}\,dx$$

$$\Big\downarrow \text{ 2725}$$

$$\int G^{c+x}e^{a+b\log(F)-x\log(G)}dx$$

$$\Big\downarrow \text{ 2704}$$

$$\int F^bG^{c+x}e^{a-x\log(G)}dx$$

$$\Big\downarrow \text{ 27}$$

$$F^b\int e^aG^cdx$$

$$\Big\downarrow \text{ 24}$$

$$e^axF^bG^c$$

input
```
Int[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x),x]
```

output
```
E^a*F^b*G^c*x
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

method	result	size
default	$\mathrm{e}^{a} F^{b} G^{c} x$	11
meijerg	$\mathrm{e}^{a} F^{b} G^{c} x$	11
parallelrisch	$\mathrm{e}^{a-x(\ln(F)+\ln(G))} G^{c+x} F^{b+x} x$	24
orering	$\mathrm{e}^{a-x(\ln(F)+\ln(G))} G^{c+x} F^{b+x} x$	24
norman	$x\,\mathrm{e}^{(b+x)\ln(F)} \mathrm{e}^{(c+x)\ln(G)} \mathrm{e}^{a-x(\ln(F)+\ln(G))}$	28

input
```
int(exp(a-x*(ln(F)+ln(G)))*F^(b+x)*G^(c+x),x,method=_RETURNVERBOSE)
```

output
```
exp(a)*F^b*G^c*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx = xe^{(b\log(F)+c\log(G)+a)}$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x),x, algorithm="fricas")
```

output
```
x*e^(b*log(F) + c*log(G) + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(10) = 20.

Time = 2.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx = F^{b+x}G^{c+x}xe^{a}e^{-x\log{(F)}}e^{-x\log{(G)}}$$

input
```
integrate(exp(a-x*(ln(F)+ln(G)))*F**(b+x)*G**(c+x),x)
```

output
```
F**(b + x)*G**(c + x)*x*exp(a)*exp(-x*log(F))*exp(-x*log(G))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx = F^{b}G^{c}xe^{a}$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x),x, algorithm="maxima")
```

output
```
F^b*G^c*x*e^a
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 6.91

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx = x\cos\left(-\frac{1}{2}\,\pi b\mathrm{sgn}(F) - \frac{1}{2}\,\pi c\mathrm{sgn}(G) + \frac{1}{2}\,\pi b + \frac{1}{2}\,\pi c\right)e^{(b\log(|F|)+c\log(|G|)+a)} + i\,xe^{(b\log(|F|)+c\log(|G|)+a)}\sin\left(-\frac{1}{2}\,\pi b\mathrm{sgn}(F) - \frac{1}{2}\,\pi c\mathrm{sgn}(G) + \frac{1}{2}\,\pi b + \frac{1}{2}\,\pi c\right)$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x),x, algorithm="giac")
```

output
```
x*cos(-1/2*pi*b*sgn(F) - 1/2*pi*c*sgn(G) + 1/2*pi*b + 1/2*pi*c)*e^(b*log(a
bs(F)) + c*log(abs(G)) + a) + I*x*e^(b*log(abs(F)) + c*log(abs(G)) + a)*si
n(-1/2*pi*b*sgn(F) - 1/2*pi*c*sgn(G) + 1/2*pi*b + 1/2*pi*c)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx = F^b\,G^c\,x\,\mathrm{e}^a$$

input
```
int(F^(b + x)*G^(c + x)*exp(a - x*(log(F) + log(G))),x)
```

output
```
F^b*G^c*x*exp(a)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}\,dx = g^c f^b e^a x$$

input
```
int(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x),x)
```

output
```
g**c*f**b*e**a*x
```

3.700 $\int e^{a-x(\log(F)+\log(G))} F^{b+x} G^{c+x} x^m \, dx$

Optimal result

Integrand size = 26, antiderivative size = 20

$$\int e^{a-x(\log(F)+\log(G))} F^{b+x} G^{c+x} x^m \, dx = \frac{e^a F^b G^c x^{1+m}}{1+m}$$

output
```
exp(a)*F^b*G^c*x^(1+m)/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{a-x(\log(F)+\log(G))} F^{b+x} G^{c+x} x^m \, dx = \frac{e^a F^b G^c x^{1+m}}{1+m}$$

input
```
Integrate[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x)*x^m,x]
```

output
```
(E^a*F^b*G^c*x^(1 + m))/(1 + m)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2725, 2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m F^{b+x} G^{c+x} e^{a-x(\log(F)+\log(G))} \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int x^m G^{c+x} e^{a+b\log(F)-x\log(G)} dx$$

$$\Big\downarrow \text{2021}$$

$$\frac{e^a F^b G^c x^{m+1}}{m+1}$$

input
```
Int[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x)*x^m,x]
```

output
```
(E^a*F^b*G^c*x^(1 + m))/(1 + m)
```

Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\mathrm{e}^{a} F^{b} G^{c} x^{1+m}}{1+m}$	20
meijerg	$\frac{\mathrm{e}^{a} F^{b} G^{c} x^{1+m}}{1+m}$	20
parallelrisch	$\frac{\mathrm{e}^{a-x(\ln(F)+\ln(G))} G^{c+x} F^{b+x} x^{m} x}{1+m}$	32
orering	$\frac{\mathrm{e}^{a-x(\ln(F)+\ln(G))} G^{c+x} F^{b+x} x^{m} x}{1+m}$	32
risch	$\frac{x F^{b+x} G^{c+x} F^{-x} G^{-x} \mathrm{e}^{a} x^{m}}{1+m}$	33
gosper	$\frac{x^{1+m} F^{b+x} G^{c+x} \mathrm{e}^{-x\ln(F)-x\ln(G)+a}}{1+m}$	35

input
```
int(exp(a-x*(ln(F)+ln(G)))*F^(b+x)*G^(c+x)*x^m,x,method=_RETURNVERBOSE)
```

output
```
exp(a)*F^b*G^c*x^(1+m)/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int e^{a-x(\log(F)+\log(G))} F^{b+x} G^{c+x} x^m \, dx = \frac{x x^m e^{(b\log(F)+c\log(G)+a)}}{m+1}$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*x^m,x, algorithm="frica
s")
```

output
```
x*x^m*e^(b*log(F) + c*log(G) + a)/(m + 1)
```

Sympy [F]

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}x^m\,dx = e^a\int F^{b+x}G^{c+x}x^m e^{-x\log(F)}e^{-x\log(G)}\,dx$$

input
```
integrate(exp(a-x*(ln(F)+ln(G)))*F**(b+x)*G**(c+x)*x**m,x)
```

output
```
exp(a)*Integral(F**(b + x)*G**(c + x)*x**m*exp(-x*log(F))*exp(-x*log(G)),
x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}x^m\,dx = \frac{F^bG^cxe^{(m\log(x)+a)}}{m+1}$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*x^m,x, algorithm="maxim
a")
```

output
```
F^b*G^c*x*e^(m*log(x) + a)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}x^m\,dx = \frac{xx^me^{(b\log(F)+c\log(G)+a)}}{m+1}$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*x^m,x, algorithm="giac"
)
```

output
```
x*x^m*e^(b*log(F) + c*log(G) + a)/(m + 1)
```

Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}x^m\,dx = \frac{F^b\,G^c\,x^{m+1}\,\mathrm{e}^a}{m+1}$$

input
```
int(F^(b + x)*G^(c + x)*x^m*exp(a - x*(log(F) + log(G))),x)
```

output
```
(F^b*G^c*x^(m + 1)*exp(a))/(m + 1)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}x^m\,dx = \frac{x^m g^c f^b e^a x}{m+1}$$

input
```
int(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*x^m,x)
```

output
```
(x**m*g**c*f**b*e**a*x)/(m + 1)
```

3.701 $\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx=e^aF^bG^c\text{Int}(F(x),x)$$

output
```
exp(a)*F^b*G^c*Defer(Int)(F(x),x)
```

Mathematica [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx=\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx$$

input
```
Integrate[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x)*F[x],x]
```

output
```
Integrate[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x)*F[x], x]
```

Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2725, 2704, 27, 27, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F(x)F^{b+x}G^{c+x}e^{a-x(\log(F)+\log(G))}\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int F(x)G^{c+x}e^{a+b\log(F)-x\log(G)}dx$$

$$\Big\downarrow \text{2704}$$

$$\int F^bF(x)G^{c+x}e^{a-x\log(G)}dx$$

$$\Big\downarrow \text{27}$$

$$F^b\int e^aG^cF(x)dx$$

$$\Big\downarrow \text{27}$$

$$e^aF^bG^c\int F(x)dx$$

$$\Big\downarrow \text{7299}$$

$$e^aF^bG^c\int F(x)dx$$

input
```
Int[E^(a - x*(Log[F] + Log[G]))*F^(b + x)*G^(c + x)*F[x],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \mathrm{e}^{a-x(\ln(F)+\ln(G))} F^{b+x} G^{c+x} F(x) \, dx$$

input
```
int(exp(a-x*(ln(F)+ln(G)))*F^(b+x)*G^(c+x)*F(x),x)
```

output
```
int(exp(a-x*(ln(F)+ln(G)))*F^(b+x)*G^(c+x)*F(x),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx = \int F^{b+x}G^{c+x}F(x)\,e^{(-x(\log(F)+\log(G))+a)}\,dx$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*F(x),x, algorithm="fric
as")
```

output
```
integral(F^(b + x)*G^(c + x)*F(x)*e^(-x*log(F) - x*log(G) + a), x)
```

Sympy [N/A]

Not integrable

Time = 16.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx = e^{a}\int F^{b+x}G^{c+x}F(x)e^{-x\log{(F)}}e^{-x\log{(G)}}\,dx$$

input
```
integrate(exp(a-x*(ln(F)+ln(G)))*F**(b+x)*G**(c+x)*F(x),x)
```

output
```
exp(a)*Integral(F**(b + x)*G**(c + x)*F(x)*exp(-x*log(F))*exp(-x*log(G)),
x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx = \int F^{b+x}G^{c+x}F(x)\,e^{(-x(\log(F)+\log(G))+a)}\,dx$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*F(x),x, algorithm="maxi
ma")
```

output
```
integrate(F^(b + x)*G^(c + x)*F(x)*e^(-x*(log(F) + log(G)) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx = \int F^{b+x}G^{c+x}F(x)\,e^{(-x(\log(F)+\log(G))+a)}\,dx$$

input
```
integrate(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*F(x),x, algorithm="giac
")
```

output
```
integrate(F^(b + x)*G^(c + x)*F(x)*e^(-x*(log(F) + log(G)) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx = \int F^{b+x}\,G^{c+x}\,\mathrm{e}^{a-x\,(\ln(F)+\ln(G))}\,F(x)\,dx$$

input
```
int(F^(b + x)*G^(c + x)*exp(a - x*(log(F) + log(G)))*F(x),x)
```

output
```
int(F^(b + x)*G^(c + x)*exp(a - x*(log(F) + log(G)))*F(x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int e^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx = \int \mathrm{e}^{a-x(\log(F)+\log(G))}F^{b+x}G^{c+x}F(x)\,dx$$

input
```
int(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*F(x),x)
```

output
```
int(exp(a-x*(log(F)+log(G)))*F^(b+x)*G^(c+x)*F(x),x)
```

3.702 $\int F^{a+bx} dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \log(F)}$$

output
```
F^(b*x+a)/b/ln(F)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \log(F)}$$

input
```
Integrate[F^(a + b*x),x]
```

output
```
F^(a + b*x)/(b*Log[F])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx}\,dx$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{F^{a+bx}}{b\log(F)}$$

input
```
Int[F^(a + b*x),x]
```

output
```
F^(a + b*x)/(b*Log[F])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{F^{bx+a}}{b\ln(F)}$	16
derivativedivides	$\frac{F^{bx+a}}{b\ln(F)}$	16
default	$\frac{F^{bx+a}}{b\ln(F)}$	16
risch	$\frac{F^{bx+a}}{b\ln(F)}$	16
parallelrisch	$\frac{F^{bx+a}}{b\ln(F)}$	16
orering	$\frac{F^{bx+a}}{b\ln(F)}$	16
norman	$\frac{e^{(bx+a)\ln(F)}}{b\ln(F)}$	18
meijerg	$-\frac{F^{a}\left(1-e^{x\ln(F)b}\right)}{b\ln(F)}$	23

input
```
int(F^(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
F^(b*x+a)/b/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx}\,dx=\frac{F^{bx+a}}{b\log(F)}$$

input
```
integrate(F^(b*x+a),x, algorithm="fricas")
```

output
```
F^(b*x + a)/(b*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx}\,dx = \begin{cases} \frac{F^{a+bx}}{b\log{(F)}} & \text{for}\ b\log{(F)} \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(F**(b*x+a),x)
```

output
```
Piecewise((F**(a + b*x)/(b*log(F)), Ne(b*log(F), 0)), (x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx}\,dx = \frac{F^{bx+a}}{b\log{(F)}}$$

input
```
integrate(F^(b*x+a),x, algorithm="maxima")
```

output
```
F^(b*x + a)/(b*log(F))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx}\,dx = \frac{F^{bx+a}}{b\log{(F)}}$$

input
```
integrate(F^(b*x+a),x, algorithm="giac")
```

output
```
F^(b*x + a)/(b*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx}\,dx = \frac{F^{a+b\,x}}{b\,\ln\left(F\right)}$$

input
```
int(F^(a + b*x),x)
```

output
```
F^(a + b*x)/(b*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx}\,dx = \frac{f^{bx+a}}{\log\left(f\right)b}$$

input
```
int(F^(b*x+a),x)
```

output
```
f**(a + b*x)/(log(f)*b)
```

3.703 $\int 2^{a+bx} dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int 2^{a+bx} dx = \frac{2^{a+bx}}{b\log(2)}$$

output
```
2^(b*x+a)/b/ln(2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int 2^{a+bx} dx = \frac{2^{a+bx}}{b\log(2)}$$

input
```
Integrate[2^(a + b*x),x]
```

output
```
2^(a + b*x)/(b*Log[2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{a+bx}\,dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{2^{a+bx}}{b\log(2)}$$

input
```
Int[2^(a + b*x),x]
```

output
```
2^(a + b*x)/(b*Log[2])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
gosper	$\frac{2^{bx+a}}{b\ln(2)}$	16
risch	$\frac{2^{bx+a}}{b\ln(2)}$	16
parallelrisch	$\frac{2^{bx+a}}{b\ln(2)}$	16
orering	$\frac{2^{bx+a}}{b\ln(2)}$	16
derivativedivides	$\frac{2^{bx}2^{a}}{b\ln(2)}$	17
default	$\frac{2^{bx}2^{a}}{b\ln(2)}$	17
norman	$\frac{\mathrm{e}^{(bx+a)\ln(2)}}{b\ln(2)}$	18
meijerg	$-\frac{2^{a}\left(1-\mathrm{e}^{xb\ln(2)}\right)}{b\ln(2)}$	23

input
```
int(2^(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2^(b*x+a)/b/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}\,dx = \frac{2^{bx+a}}{b\log(2)}$$

input
```
integrate(2^(b*x+a),x, algorithm="fricas")
```

output
```
2^(b*x + a)/(b*log(2))
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}\,dx = \begin{cases} \frac{2^{a+bx}}{b\log(2)} & \text{for } b\log(2) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input
```
integrate(2**(b*x+a),x)
```

output
```
Piecewise((2**(a + b*x)/(b*log(2)), Ne(b*log(2), 0)), (x, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}\,dx = \frac{2^{bx+a}}{b\log(2)}$$

input
```
integrate(2^(b*x+a),x, algorithm="maxima")
```

output
```
2^(b*x + a)/(b*log(2))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}\,dx = \frac{2^{bx+a}}{b\log(2)}$$

input
```
integrate(2^(b*x+a),x, algorithm="giac")
```

output
```
2^(b*x + a)/(b*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}\,dx=\frac{2^{a+b\,x}}{b\,\ln{(2)}}$$

input
```
int(2^(a + b*x),x)
```

output
```
2^(a + b*x)/(b*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int 2^{a+bx}\,dx=\frac{2^a 2^{bx}}{\log{(2)}\,b}$$

input
```
int(2^(b*x+a),x)
```

output
```
(2**a*2**(b*x))/(log(2)*b)
```

3.704 $\int 2^{2+3x} dx$

Optimal result

Integrand size = 7, antiderivative size = 15

$$\int 2^{2+3x} dx = \frac{2^{2+3x}}{3\log(2)}$$

output
```
1/3*2^(2+3*x)/ln(2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int 2^{2+3x} dx = \frac{2^{2+3x}}{\log(8)}$$

input
```
Integrate[2^(2 + 3*x),x]
```

output
```
2^(2 + 3*x)/Log[8]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{2624\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{3x+2} \, dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{2^{3x+2}}{3 \log(2)}$$

input
```
Int[2^(2 + 3*x),x]
```

output
```
2^(2 + 3*x)/(3*Log[2])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{4\,2^{3x}}{3\ln(2)}$	12
default	$\frac{4\,2^{3x}}{3\ln(2)}$	12
gosper	$\frac{2^{2+3x}}{3\ln(2)}$	14
risch	$\frac{2^{2+3x}}{3\ln(2)}$	14
parallelrisch	$\frac{2^{2+3x}}{3\ln(2)}$	14
orering	$\frac{2^{2+3x}}{3\ln(2)}$	14
norman	$\frac{\mathrm{e}^{(2+3x)\ln(2)}}{3\ln(2)}$	16
meijerg	$-\frac{4\left(1-\mathrm{e}^{3x\ln(2)}\right)}{3\ln(2)}$	17

input
```
int(2^(2+3*x),x,method=_RETURNVERBOSE)
```

output
```
4/3*(2^x)^3/ln(2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int 2^{2+3x}\,dx = \frac{2^{3\,x+2}}{3\,\log\left(2\right)}$$

input
```
integrate(2^(2+3*x),x, algorithm="fricas")
```

output
```
1/3*2^(3*x + 2)/log(2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int 2^{2+3x}\,dx = \frac{2^{3x+2}}{3\log(2)}$$

input
```
integrate(2**(2+3*x),x)
```

output
```
2**(3*x + 2)/(3*log(2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int 2^{2+3x}\,dx = \frac{2^{3\,x+2}}{3\,\log(2)}$$

input
```
integrate(2^(2+3*x),x, algorithm="maxima")
```

output
```
1/3*2^(3*x + 2)/log(2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int 2^{2+3x}\,dx = \frac{2^{3\,x+2}}{3\,\log(2)}$$

input
```
integrate(2^(2+3*x),x, algorithm="giac")
```

output
```
1/3*2^(3*x + 2)/log(2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int 2^{2+3x}\,dx = \frac{4\,2^{3\,x}}{3\,\ln(2)}$$

input
```
int(2^(3*x + 2),x)
```

output
```
(4*2^(3*x))/(3*log(2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int 2^{2+3x}\,dx = \frac{4\,2^{3x}}{3\log(2)}$$

input
```
int(2^(2+3*x),x)
```

output
```
(4*2**(3*x))/(3*log(2))
```

3.705 $\int F^{a+bx} G^{c+dx}\, dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int F^{a+bx} G^{c+dx}\, dx = \frac{F^{a+bx} G^{c+dx}}{b\log(F) + d\log(G)}$$

output
```
F^(b*x+a)*G^(d*x+c)/(b*ln(F)+d*ln(G))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int F^{a+bx} G^{c+dx}\, dx = \frac{F^{a+bx} G^{c+dx}}{b\log(F) + d\log(G)}$$

input
```
Integrate[F^(a + b*x)*G^(c + d*x),x]
```

output
```
(F^(a + b*x)*G^(c + d*x))/(b*Log[F] + d*Log[G])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx}G^{c+dx}\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int \exp(a\log(F)+x(b\log(F)+d\log(G))+c\log(G))dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{F^{a+bx}G^{c+dx}}{b\log(F)+d\log(G)}$$

input
```
Int[F^(a + b*x)*G^(c + d*x),x]
```

output
```
(F^(a + b*x)*G^(c + d*x))/(b*Log[F] + d*Log[G])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{F^{bx+a}G^{dx+c}}{b\ln(F)+d\ln(G)}$	27
risch	$\frac{F^{bx+a}G^{dx+c}}{b\ln(F)+d\ln(G)}$	27
parallelrisch	$\frac{F^{bx+a}G^{dx+c}}{b\ln(F)+d\ln(G)}$	27
orering	$\frac{F^{bx+a}G^{dx+c}}{b\ln(F)+d\ln(G)}$	27
norman	$\frac{\mathrm{e}^{(bx+a)\ln(F)}\mathrm{e}^{(dx+c)\ln(G)}}{b\ln(F)+d\ln(G)}$	31
meijerg	$-\frac{F^{a}G^{c}\left(1-\mathrm{e}^{xd\ln(G)\left(1+\frac{b\ln(F)}{d\ln(G)}\right)}\right)}{d\ln(G)\left(1+\frac{b\ln(F)}{d\ln(G)}\right)}$	54

input
```
int(F^(b*x+a)*G^(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
F^(b*x+a)*G^(d*x+c)/(b*ln(F)+d*ln(G))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int F^{a+bx}G^{c+dx}\,dx=\frac{F^{bx+a}G^{dx+c}}{b\log\left(F\right)+d\log\left(G\right)}$$

input
```
integrate(F^(b*x+a)*G^(d*x+c),x, algorithm="fricas")
```

output
```
F^(b*x + a)*G^(d*x + c)/(b*log(F) + d*log(G))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int F^{a+bx}G^{c+dx}\,dx = \begin{cases} \frac{F^{a+bx}G^{c+dx}}{b\log(F)+d\log(G)} & \text{for } b \neq -\frac{d\log(G)}{\log(F)} \\ F^{a-\frac{dx\log(G)}{\log(F)}}G^{c+dx}x & \text{otherwise} \end{cases}$$

input
```
integrate(F**(b*x+a)*G**(d*x+c),x)
```

output
```
Piecewise((F**(a + b*x)*G**(c + d*x)/(b*log(F) + d*log(G)), Ne(b, -d*log(G
)/log(F))), (F**(a - d*x*log(G)/log(F))*G**(c + d*x)*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int F^{a+bx}G^{c+dx}\,dx = \frac{F^aG^ce^{(bx\log(F)+dx\log(G))}}{b\log(F)+d\log(G)}$$

input
```
integrate(F^(b*x+a)*G^(d*x+c),x, algorithm="maxima")
```

output
```
F^a*G^c*e^(b*x*log(F) + d*x*log(G))/(b*log(F) + d*log(G))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 427, normalized size of antiderivative = 16.42

$$\int F^{a+bx}G^{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(F^(b*x+a)*G^(d*x+c),x, algorithm="giac")
```

output
```
2*(2*(b*log(abs(F)) + d*log(abs(G)))*cos(-1/2*pi*b*x*sgn(F) - 1/2*pi*d*x*s
gn(G) + 1/2*pi*b*x + 1/2*pi*d*x - 1/2*pi*a*sgn(F) - 1/2*pi*c*sgn(G) + 1/2*
pi*a + 1/2*pi*c)/((pi*b*sgn(F) + pi*d*sgn(G) - pi*b - pi*d)^2 + 4*(b*log(a
bs(F)) + d*log(abs(G)))^2) - (pi*b*sgn(F) + pi*d*sgn(G) - pi*b - pi*d)*sin
(-1/2*pi*b*x*sgn(F) - 1/2*pi*d*x*sgn(G) + 1/2*pi*b*x + 1/2*pi*d*x - 1/2*pi
*a*sgn(F) - 1/2*pi*c*sgn(G) + 1/2*pi*a + 1/2*pi*c)/((pi*b*sgn(F) + pi*d*sg
n(G) - pi*b - pi*d)^2 + 4*(b*log(abs(F)) + d*log(abs(G)))^2))*e^((b*log(ab
s(F)) + d*log(abs(G)))*x + a*log(abs(F)) + c*log(abs(G))) + I*(I*e^(1/2*I*
pi*b*x*sgn(F) + 1/2*I*pi*d*x*sgn(G) - 1/2*I*pi*b*x - 1/2*I*pi*d*x + 1/2*I*
pi*a*sgn(F) + 1/2*I*pi*c*sgn(G) - 1/2*I*pi*a - 1/2*I*pi*c)/(I*pi*b*sgn(F)
+ I*pi*d*sgn(G) - I*pi*b - I*pi*d + 2*b*log(abs(F)) + 2*d*log(abs(G))) - I
*e^(-1/2*I*pi*b*x*sgn(F) - 1/2*I*pi*d*x*sgn(G) + 1/2*I*pi*b*x + 1/2*I*pi*d
*x - 1/2*I*pi*a*sgn(F) - 1/2*I*pi*c*sgn(G) + 1/2*I*pi*a + 1/2*I*pi*c)/(-I*
pi*b*sgn(F) - I*pi*d*sgn(G) + I*pi*b + I*pi*d + 2*b*log(abs(F)) + 2*d*log(
abs(G))))*e^((b*log(abs(F)) + d*log(abs(G)))*x + a*log(abs(F)) + c*log(abs
(G)))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int F^{a+bx}G^{c+dx}\,dx = \frac{F^{a+b\,x}\,G^{c+d\,x}}{b\,\ln(F)+d\,\ln(G)}$$

input
```
int(F^(a + b*x)*G^(c + d*x),x)
```

output
```
(F^(a + b*x)*G^(c + d*x))/(b*log(F) + d*log(G))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int F^{a+bx}G^{c+dx}\,dx = \frac{g^{dx+c}f^{bx+a}}{\log(f)\,b+\log(g)\,d}$$

input
```
int(F^(b*x+a)*G^(d*x+c),x)
```

output
```
(g**(c + d*x)*f**(a + b*x))/(log(f)*b + log(g)*d)
```

3.706 $\int 2^{a+bx} 3^{c+dx} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 26

$$\int 2^{a+bx} 3^{c+dx} \, dx = \frac{2^{a+bx} 3^{c+dx}}{b \log(2) + d \log(3)}$$

output
```
2^(b*x+a)*3^(d*x+c)/(b*ln(2)+d*ln(3))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int 2^{a+bx} 3^{c+dx} \, dx = \frac{2^{a+bx} 3^{c+dx}}{b \log(2) + d \log(3)}$$

input
```
Integrate[2^(a + b*x)*3^(c + d*x),x]
```

output
```
(2^(a + b*x)*3^(c + d*x))/(b*Log[2] + d*Log[3])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{a+bx}3^{c+dx}\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int \exp(a\log(2)+x(b\log(2)+d\log(3))+c\log(3))dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{2^{a+bx}3^{c+dx}}{b\log(2)+d\log(3)}$$

input
```
Int[2^(a + b*x)*3^(c + d*x),x]
```

output
```
(2^(a + b*x)*3^(c + d*x))/(b*Log[2] + d*Log[3])
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
gosper	$\frac{2^{bx+a}3^{dx+c}}{b\ln(2)+d\ln(3)}$	27
risch	$\frac{2^{bx+a}3^{dx+c}}{b\ln(2)+d\ln(3)}$	27
parallelrisch	$\frac{2^{bx+a}3^{dx+c}}{b\ln(2)+d\ln(3)}$	27
orering	$\frac{2^{bx+a}3^{dx+c}}{b\ln(2)+d\ln(3)}$	27
norman	$\frac{e^{(bx+a)\ln(2)}e^{(dx+c)\ln(3)}}{b\ln(2)+d\ln(3)}$	31
meijerg	$-\frac{2^{a}3^{c}\left(1-e^{xd\ln(3)\left(1+\frac{b\ln(2)}{d\ln(3)}\right)}\right)}{d\ln(3)\left(1+\frac{b\ln(2)}{d\ln(3)}\right)}$	54

input
```
int(2^(b*x+a)*3^(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2^(b*x+a)*3^(d*x+c)/(b*ln(2)+d*ln(3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}3^{c+dx}\,dx=\frac{3^{dx+c}2^{bx+a}}{d\log(3)+b\log(2)}$$

input
```
integrate(2^(b*x+a)*3^(d*x+c),x, algorithm="fricas")
```

output
```
3^(d*x + c)*2^(b*x + a)/(d*log(3) + b*log(2))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(22) = 44$.

Time = 0.34 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.15

$$\int 2^{a+bx}3^{c+dx}\,dx = \begin{cases} \frac{2^a 2^{bx} 3^c 3^{dx}}{b\log(2)+d\log(3)} & \text{for } b \neq -\frac{d\log(3)}{\log(2)} \\ 2^a 3^c 3^{dx} x e^{-dx\log(3)} + \frac{2^a 3^c 3^{dx} e^{-dx\log(3)}}{d\log(3)} & \text{otherwise} \end{cases}$$

input
```
integrate(2**(b*x+a)*3**(d*x+c),x)
```

output
```
Piecewise((2**a*2**(b*x)*3**c*3**(d*x)/(b*log(2) + d*log(3)), Ne(b, -d*log
(3)/log(2))), (2**a*3**c*3**(d*x)*x*exp(-d*x*log(3)) + 2**a*3**c*3**(d*x)*
exp(-d*x*log(3))/(d*log(3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int 2^{a+bx}3^{c+dx}\,dx = \frac{3^c 2^a e^{(dx\log(3)+bx\log(2))}}{d\log(3)+b\log(2)}$$

input
```
integrate(2^(b*x+a)*3^(d*x+c),x, algorithm="maxima")
```

output
```
3^c*2^a*e^(d*x*log(3) + b*x*log(2))/(d*log(3) + b*log(2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int 2^{a+bx}3^{c+dx}\,dx = \frac{e^{(dx\log(3)+bx\log(2)+c\log(3)+a\log(2))}}{d\log(3)+b\log(2)}$$

input
```
integrate(2^(b*x+a)*3^(d*x+c),x, algorithm="giac")
```

output
```
e^(d*x*log(3) + b*x*log(2) + c*log(3) + a*log(2))/(d*log(3) + b*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}3^{c+dx}\,dx=\frac{2^{a+b\,x}\,3^{c+d\,x}}{b\,\ln\left(2\right)+d\,\ln\left(3\right)}$$

input
```
int(2^(a + b*x)*3^(c + d*x),x)
```

output
```
(2^(a + b*x)*3^(c + d*x))/(b*log(2) + d*log(3))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int 2^{a+bx}3^{c+dx}\,dx=\frac{2^{a}2^{bx}3^{c}3^{dx}}{\log\left(3\right)d+\log\left(2\right)b}$$

input
```
int(2^(b*x+a)*3^(d*x+c),x)
```

output
```
(2**a*2**(b*x)*3**c*3**(d*x))/(log(3)*d + log(2)*b)
```

3.707 $\int 2^{2+3x}3^{5+7x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int 2^{2+3x}3^{5+7x}\,dx=\frac{2^{2+3x}3^{5+7x}}{\log(17496)}$$

output
```
2^(2+3*x)*3^(5+7*x)/ln(17496)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int 2^{2+3x}3^{5+7x}\,dx=\frac{2^{2+3x}3^{5+7x}}{\log(17496)}$$

input
```
Integrate[2^(2 + 3*x)*3^(5 + 7*x),x]
```

output
```
(2^(2 + 3*x)*3^(5 + 7*x))/Log[17496]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{3x+2}3^{7x+5}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int 2^{3x+2}3^{7x+5}dx$$

input
```
Int[2^(2 + 3*x)*3^(5 + 7*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

method	result	size
gosper	$\frac{2^{2+3x}3^{5+7x}}{3\ln(2)+7\ln(3)}$	27
risch	$\frac{2^{2+3x}3^{5+7x}}{3\ln(2)+7\ln(3)}$	27
parallelrisch	$\frac{2^{2+3x}3^{5+7x}}{3\ln(2)+7\ln(3)}$	27
orering	$\frac{2^{2+3x}3^{5+7x}}{3\ln(2)+7\ln(3)}$	27
norman	$\frac{{\mathrm e}^{(2+3x)\ln(2)}{\mathrm e}^{(5+7x)\ln(3)}}{3\ln(2)+7\ln(3)}$	31
meijerg	$-\frac{972\left(1-{\mathrm e}^{x\ln(3)\left(7+\frac{3\ln(2)}{\ln(3)}\right)}\right)}{\ln(3)\left(7+\frac{3\ln(2)}{\ln(3)}\right)}$	38

input
```
int(2^(2+3*x)*3^(5+7*x),x,method=_RETURNVERBOSE)
```

output
```
1/(3*ln(2)+7*ln(3))*2^(2+3*x)*3^(5+7*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int 2^{2+3x}3^{5+7x}\,dx = \frac{3^{7\,x+5}2^{3\,x+2}}{7\,\log\left(3\right)+3\,\log\left(2\right)}$$

input
```
integrate(2^(2+3*x)*3^(5+7*x),x, algorithm="fricas")
```

output
```
3^(7*x + 5)*2^(3*x + 2)/(7*log(3) + 3*log(2))
```

Sympy [F(-2)]

Exception generated.

$$\int 2^{2+3x}3^{5+7x}\,dx = \text{Exception raised: IndexError}$$

input
```
integrate(2**(2+3*x)*3**(5+7*x),x)
```

output
```
Exception raised: IndexError >> Index out of range: a[1]
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int 2^{2+3x}3^{5+7x}\,dx = \frac{972\,e^{(7\,x\log(3)+3\,x\log(2))}}{7\,\log(3)+3\,\log(2)}$$

input
```
integrate(2^(2+3*x)*3^(5+7*x),x, algorithm="maxima")
```

output
```
972*e^(7*x*log(3) + 3*x*log(2))/(7*log(3) + 3*log(2))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int 2^{2+3x}3^{5+7x}\,dx = \frac{e^{(7\,x\log(3)+3\,x\log(2)+5\,\log(3)+2\,\log(2))}}{7\,\log(3)+3\,\log(2)}$$

input
```
integrate(2^(2+3*x)*3^(5+7*x),x, algorithm="giac")
```

output
```
e^(7*x*log(3) + 3*x*log(2) + 5*log(3) + 2*log(2))/(7*log(3) + 3*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int 2^{2+3x}3^{5+7x}\,dx = \frac{972\,2^{3\,x}\,3^{7\,x}}{\ln(17496)}$$

input
```
int(2^(3*x + 2)*3^(7*x + 5),x)
```

output
```
(972*2^(3*x)*3^(7*x))/log(17496)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int 2^{2+3x}3^{5+7x}\,dx = \frac{972\,2^{3x}3^{7x}}{7\log(3)+3\log(2)}$$

input
```
int(2^(2+3*x)*3^(5+7*x),x)
```

output
```
(972*2**(3*x)*3**(7*x))/(7*log(3) + 3*log(2))
```

3.708 $\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx=\frac{F^{a+bx}G^{c+dx}H^{e+fx}}{b\log(F)+d\log(G)+f\log(H)}$$

output
```
F^(b*x+a)*G^(d*x+c)*H^(f*x+e)/(b*ln(F)+d*ln(G)+f*ln(H))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx=\frac{F^{a+bx}G^{c+dx}H^{e+fx}}{b\log(F)+d\log(G)+f\log(H)}$$

input
```
Integrate[F^(a + b*x)*G^(c + d*x)*H^(e + f*x),x]
```

output
```
(F^(a + b*x)*G^(c + d*x)*H^(e + f*x))/(b*Log[F] + d*Log[G] + f*Log[H])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {2725, 2704, 27, 2725, 2704, 27, 2725, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx} G^{c+dx} H^{e+fx} \, dx$$

$$\Big\downarrow \text{2725}$$

$$\int H^{e+fx} \exp(a \log(F) + x(b \log(F) + d \log(G)) + c \log(G)) dx$$

$$\Big\downarrow \text{2704}$$

$$\int F^a H^{e+fx} e^{x(b \log(F)+d \log(G))+c \log(G)} dx$$

$$\Big\downarrow \text{27}$$

$$F^a \int F^{bx} G^{c+dx} H^{e+fx} dx$$

$$\Big\downarrow \text{2725}$$

$$F^a \int e^{c \log(G)+x(b \log(F)+d \log(G))} H^{e+fx} dx$$

$$\Big\downarrow \text{2704}$$

$$F^a \int e^{x(b \log(F)+d \log(G))} G^c H^{e+fx} dx$$

$$\Big\downarrow \text{27}$$

$$F^a G^c \int F^{bx} G^{dx} H^{e+fx} dx$$

$$\Big\downarrow \text{2725}$$

$$F^a G^c \int e^{x(b \log(F)+d \log(G))} H^{e+fx} dx$$

$$\Big\downarrow \text{2725}$$

$$F^a G^c \int \exp(e \log(H) + x(b \log(F) + d \log(G) + f \log(H))) dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{F^{a+bx}G^{c+dx}H^{e+fx}}{b\log(F)+d\log(G)+f\log(H)}$$

input
```
Int[F^(a + b*x)*G^(c + d*x)*H^(e + f*x),x]
```

output
```
(F^(a + b*x)*G^(c + d*x)*H^(e + f*x))/(b*Log[F] + d*Log[G] + f*Log[H])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{F^{bx+a} G^{dx+c} H^{fx+e}}{b \ln(F)+d \ln(G)+f \ln(H)}$	38
risch	$\frac{F^{bx+a} G^{dx+c} H^{fx+e}}{b \ln(F)+d \ln(G)+f \ln(H)}$	38
parallelrisch	$\frac{F^{bx+a} G^{dx+c} H^{fx+e}}{b \ln(F)+d \ln(G)+f \ln(H)}$	38
orering	$\frac{F^{bx+a} G^{dx+c} H^{fx+e}}{b \ln(F)+d \ln(G)+f \ln(H)}$	38
norman	$\frac{{\mathrm e}^{(bx+a) \ln(F)} {\mathrm e}^{(dx+c) \ln(G)} {\mathrm e}^{(fx+e) \ln(H)}}{b \ln(F)+d \ln(G)+f \ln(H)}$	44
meijerg	$-\frac{F^{a} G^{c} H^{e} \left(1-{\mathrm e}^{x d \ln(G) \left(1+\frac{b \ln(F)}{d \ln(G)}\right) \left(1+\frac{f \ln(H)}{d \ln(G) \left(1+\frac{b \ln(F)}{d \ln(G)}\right)}\right)}\right)}{d \ln(G) \left(1+\frac{b \ln(F)}{d \ln(G)}\right) \left(1+\frac{f \ln(H)}{d \ln(G) \left(1+\frac{b \ln(F)}{d \ln(G)}\right)}\right)}$	115

input
```
int(F^(b*x+a)*G^(d*x+c)*H^(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
F^(b*x+a)*G^(d*x+c)*H^(f*x+e)/(b*ln(F)+d*ln(G)+f*ln(H))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int F^{a+bx} G^{c+dx} H^{e+fx} \, dx = \frac{F^{bx+a} G^{dx+c} H^{fx+e}}{b \log{(F)} + d \log{(G)} + f \log{(H)}}$$

input
```
integrate(F^(b*x+a)*G^(d*x+c)*H^(f*x+e),x, algorithm="fricas")
```

output
```
F^(b*x + a)*G^(d*x + c)*H^(f*x + e)/(b*log(F) + d*log(G) + f*log(H))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(34) = 68$.

Time = 1.16 (sec) , antiderivative size = 204, normalized size of antiderivative = 5.51

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx = \begin{cases} \frac{F^{a+bx}G^{c+dx}H^{e+fx}}{b\log(F)+d\log(G)+f\log(H)} \\ \frac{F^{a-\frac{dx\log(G)}{\log(F)}-\frac{fx\log(H)}{\log(F)}}G^{c+dx}H^{e+fx}dx\log(G)}{d\log(G)+f\log(H)} + \frac{F^{a-\frac{dx\log(G)}{\log(F)}-\frac{fx\log(H)}{\log(F)}}G^{c+dx}H^{e+fx}fx\log(H)}{d\log(G)+f\log(H)} + \frac{F^{a-\frac{dx\log(G)}{\log(F)}-\frac{fx\log(H)}{\log(F)}}G^{c+dx}}{d\log(G)+f\log(H)} \end{cases}$$

input
```
integrate(F**(b*x+a)*G**(d*x+c)*H**(f*x+e),x)
```

output
```
Piecewise((F**(a + b*x)*G**(c + d*x)*H**(e + f*x)/(b*log(F) + d*log(G) + f
*log(H)), Ne(b, (-d*log(G) - f*log(H))/log(F))), (F**(a - d*x*log(G)/log(F
) - f*x*log(H)/log(F))*G**(c + d*x)*H**(e + f*x)*d*x*log(G)/(d*log(G) + f*
log(H)) + F**(a - d*x*log(G)/log(F) - f*x*log(H)/log(F))*G**(c + d*x)*H**(
e + f*x)*f*x*log(H)/(d*log(G) + f*log(H)) + F**(a - d*x*log(G)/log(F) - f*
x*log(H)/log(F))*G**(c + d*x)*H**(e + f*x)/(d*log(G) + f*log(H)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx = \frac{F^aG^cH^e e^{(bx\log(F)+dx\log(G)+fx\log(H))}}{b\log(F)+d\log(G)+f\log(H)}$$

input
```
integrate(F^(b*x+a)*G^(d*x+c)*H^(f*x+e),x, algorithm="maxima")
```

output
```
F^a*G^c*H^e*e^(b*x*log(F) + d*x*log(G) + f*x*log(H))/(b*log(F) + d*log(G)
+ f*log(H))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 609, normalized size of antiderivative = 16.46

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx = \text{Too large to display}$$

input
```
integrate(F^(b*x+a)*G^(d*x+c)*H^(f*x+e),x, algorithm="giac")
```

output
```
2*(2*(b*log(abs(F)) + d*log(abs(G)) + f*log(abs(H)))*cos(-1/2*pi*b*x*sgn(F
) - 1/2*pi*d*x*sgn(G) - 1/2*pi*f*x*sgn(H) + 1/2*pi*b*x + 1/2*pi*d*x + 1/2*
pi*f*x - 1/2*pi*a*sgn(F) - 1/2*pi*c*sgn(G) - 1/2*pi*e*sgn(H) + 1/2*pi*a +
1/2*pi*c + 1/2*pi*e)/((pi*b*sgn(F) + pi*d*sgn(G) + pi*f*sgn(H) - pi*b - pi
*d - pi*f)^2 + 4*(b*log(abs(F)) + d*log(abs(G)) + f*log(abs(H)))^2) - (pi*
b*sgn(F) + pi*d*sgn(G) + pi*f*sgn(H) - pi*b - pi*d - pi*f)*sin(-1/2*pi*b*x
*sgn(F) - 1/2*pi*d*x*sgn(G) - 1/2*pi*f*x*sgn(H) + 1/2*pi*b*x + 1/2*pi*d*x
+ 1/2*pi*f*x - 1/2*pi*a*sgn(F) - 1/2*pi*c*sgn(G) - 1/2*pi*e*sgn(H) + 1/2*p
i*a + 1/2*pi*c + 1/2*pi*e)/((pi*b*sgn(F) + pi*d*sgn(G) + pi*f*sgn(H) - pi*
b - pi*d - pi*f)^2 + 4*(b*log(abs(F)) + d*log(abs(G)) + f*log(abs(H)))^2))
*e^((b*log(abs(F)) + d*log(abs(G)) + f*log(abs(H)))*x + a*log(abs(F)) + c*
log(abs(G)) + e*log(abs(H))) + I*(I*e^(1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*d*x*
sgn(G) + 1/2*I*pi*f*x*sgn(H) - 1/2*I*pi*b*x - 1/2*I*pi*d*x - 1/2*I*pi*f*x
+ 1/2*I*pi*a*sgn(F) + 1/2*I*pi*c*sgn(G) + 1/2*I*pi*e*sgn(H) - 1/2*I*pi*a -
 1/2*I*pi*c - 1/2*I*pi*e)/(I*pi*b*sgn(F) + I*pi*d*sgn(G) + I*pi*f*sgn(H) -
 I*pi*b - I*pi*d - I*pi*f + 2*b*log(abs(F)) + 2*d*log(abs(G)) + 2*f*log(ab
s(H))) - I*e^(-1/2*I*pi*b*x*sgn(F) - 1/2*I*pi*d*x*sgn(G) - 1/2*I*pi*f*x*sg
n(H) + 1/2*I*pi*b*x + 1/2*I*pi*d*x + 1/2*I*pi*f*x - 1/2*I*pi*a*sgn(F) - 1/
2*I*pi*c*sgn(G) - 1/2*I*pi*e*sgn(H) + 1/2*I*pi*a + 1/2*I*pi*c + 1/2*I*pi*e
)/(-I*pi*b*sgn(F) - I*pi*d*sgn(G) - I*pi*f*sgn(H) + I*pi*b + I*pi*d + I...
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx=\frac{F^{a+b\,x}\,G^{c+d\,x}\,H^{e+f\,x}}{b\,\ln\left(F\right)+d\,\ln\left(G\right)+f\,\ln\left(H\right)}$$

input
```
int(F^(a + b*x)*G^(c + d*x)*H^(e + f*x),x)
```

output
```
(F^(a + b*x)*G^(c + d*x)*H^(e + f*x))/(b*log(F) + d*log(G) + f*log(H))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int F^{a+bx}G^{c+dx}H^{e+fx}\,dx=\frac{h^{fx+e}g^{dx+c}f^{bx+a}}{\log\left(f\right)b+\log\left(g\right)d+\log\left(h\right)f}$$

input
```
int(F^(b*x+a)*G^(d*x+c)*H^(f*x+e),x)
```

output
```
(h**(e + f*x)*g**(c + d*x)*f**(a + b*x))/(log(f)*b + log(g)*d + log(h)*f)
```

3.709 $\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 37

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx=\frac{2^{a+bx}3^{c+dx}5^{e+fx}}{b\log(2)+d\log(3)+f\log(5)}$$

output
```
2^(b*x+a)*3^(d*x+c)*5^(f*x+e)/(b*ln(2)+d*ln(3)+f*ln(5))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx=\frac{2^{a+bx}3^{c+dx}5^{e+fx}}{b\log(2)+d\log(3)+f\log(5)}$$

input
```
Integrate[2^(a + b*x)*3^(c + d*x)*5^(e + f*x),x]
```

output
```
(2^(a + b*x)*3^(c + d*x)*5^(e + f*x))/(b*Log[2] + d*Log[3] + f*Log[5])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {2725, 2704, 27, 2725, 2704, 27, 2725, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx$$

$$\Big\downarrow \text{2725}$$

$$\int 5^{e+fx}\exp(a\log(2)+x(b\log(2)+d\log(3))+c\log(3))dx$$

$$\Big\downarrow \text{2704}$$

$$\int 2^a5^{e+fx}e^{x(b\log(2)+d\log(3))+c\log(3)}dx$$

$$\Big\downarrow \text{27}$$

$$2^a\int 2^{bx}3^{c+dx}5^{e+fx}dx$$

$$\Big\downarrow \text{2725}$$

$$2^a\int 5^{e+fx}e^{\log(3)c+x(\log(2)b+d\log(3))}dx$$

$$\Big\downarrow \text{2704}$$

$$2^a\int 3^c5^{e+fx}e^{x(\log(2)b+d\log(3))}dx$$

$$\Big\downarrow \text{27}$$

$$2^a3^c\int 2^{bx}3^{dx}5^{e+fx}dx$$

$$\Big\downarrow \text{2725}$$

$$2^a3^c\int 5^{e+fx}e^{x(\log(2)b+d\log(3))}dx$$

$$\Big\downarrow \text{2725}$$

$$2^a3^c\int \exp(\log(5)e+x(\log(2)b+f\log(5)+d\log(3)))dx$$

$$\Big\downarrow \text{2624}$$

$$\frac{2^{a+bx}3^{c+dx}5^{e+fx}}{b\log(2)+d\log(3)+f\log(5)}$$

input
```
Int[2^(a + b*x)*3^(c + d*x)*5^(e + f*x),x]
```

output
```
(2^(a + b*x)*3^(c + d*x)*5^(e + f*x))/(b*Log[2] + d*Log[3] + f*Log[5])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2704
```
Int[(u_.)*(F_)^((a_.)*(Log[z_]*(b_.) + (v_.))), x_Symbol] :> Int[u*F^(a*v)*
z^(a*b*Log[F]), x] /; FreeQ[{F, a, b}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
gosper	$\frac{2^{bx+a}3^{dx+c}5^{fx+e}}{b\ln(2)+d\ln(3)+f\ln(5)}$	38
risch	$\frac{2^{bx+a}3^{dx+c}5^{fx+e}}{b\ln(2)+d\ln(3)+f\ln(5)}$	38
parallelrisch	$\frac{2^{bx+a}3^{dx+c}5^{fx+e}}{b\ln(2)+d\ln(3)+f\ln(5)}$	38
orering	$\frac{2^{bx+a}3^{dx+c}5^{fx+e}}{b\ln(2)+d\ln(3)+f\ln(5)}$	38
norman	$\frac{e^{(bx+a)\ln(2)}e^{(dx+c)\ln(3)}e^{(fx+e)\ln(5)}}{b\ln(2)+d\ln(3)+f\ln(5)}$	44
meijerg	$-\frac{2^{a}3^{c}5^{e}\left(1-e^{xd\ln(3)\left(1+\frac{b\ln(2)}{d\ln(3)}\right)\left(1+\frac{f\ln(5)}{d\ln(3)\left(1+\frac{b\ln(2)}{d\ln(3)}\right)}\right)}\right)}{d\ln(3)\left(1+\frac{b\ln(2)}{d\ln(3)}\right)\left(1+\frac{f\ln(5)}{d\ln(3)\left(1+\frac{b\ln(2)}{d\ln(3)}\right)}\right)}$	115

input
```
int(2^(b*x+a)*3^(d*x+c)*5^(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
2^(b*x+a)*3^(d*x+c)*5^(f*x+e)/(b*ln(2)+d*ln(3)+f*ln(5))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx=\frac{5^{fx+e}3^{dx+c}2^{bx+a}}{f\log{(5)}+d\log{(3)}+b\log{(2)}}$$

input
```
integrate(2^(b*x+a)*3^(d*x+c)*5^(f*x+e),x, algorithm="fricas")
```

output
```
5^(f*x + e)*3^(d*x + c)*2^(b*x + a)/(f*log(5) + d*log(3) + b*log(2))
```

Sympy [A] (verification not implemented)

Time = 24.93 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx = \begin{cases} \frac{2^a2^{bx}3^c3^{dx}5^e5^{fx}}{b\log(2)+d\log(3)+f\log(5)} & \text{for } b \neq -\frac{d\log(3)}{\log(2)} - \frac{f\log(5)}{\log(2)} \\ 2^a3^c5^ex & \text{otherwise} \end{cases}$$

input
```
integrate(2**(b*x+a)*3**(d*x+c)*5**(f*x+e),x)
```

output
```
Piecewise((2**a*2**(b*x)*3**c*3**(d*x)*5**e*5**(f*x)/(b*log(2) + d*log(3)
+ f*log(5)), Ne(b, -d*log(3)/log(2) - f*log(5)/log(2))), (2**a*3**c*5**e*x
, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx = \frac{5^e3^c2^ae^{(fx\log(5)+dx\log(3)+bx\log(2))}}{f\log(5)+d\log(3)+b\log(2)}$$

input
```
integrate(2^(b*x+a)*3^(d*x+c)*5^(f*x+e),x, algorithm="maxima")
```

output
```
5^e*3^c*2^a*e^(f*x*log(5) + d*x*log(3) + b*x*log(2))/(f*log(5) + d*log(3)
+ b*log(2))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx = \frac{e^{(fx\log(5)+dx\log(3)+bx\log(2)+e\log(5)+c\log(3)+a\log(2))}}{f\log(5)+d\log(3)+b\log(2)}$$

input
```
integrate(2^(b*x+a)*3^(d*x+c)*5^(f*x+e),x, algorithm="giac")
```

output
```
e^(f*x*log(5) + d*x*log(3) + b*x*log(2) + e*log(5) + c*log(3) + a*log(2))/
(f*log(5) + d*log(3) + b*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx=\frac{2^{a+b\,x}\,3^{c+d\,x}\,5^{e+f\,x}}{b\,\ln\left(2\right)+d\,\ln\left(3\right)+f\,\ln\left(5\right)}$$

input
```
int(2^(a + b*x)*3^(c + d*x)*5^(e + f*x),x)
```

output
```
(2^(a + b*x)*3^(c + d*x)*5^(e + f*x))/(b*log(2) + d*log(3) + f*log(5))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int 2^{a+bx}3^{c+dx}5^{e+fx}\,dx=\frac{2^{a}2^{bx}3^{c}3^{dx}5^{e}5^{fx}}{\log\left(5\right)f+\log\left(3\right)d+\log\left(2\right)b}$$

input
```
int(2^(b*x+a)*3^(d*x+c)*5^(f*x+e),x)
```

output
```
(2**a*2**(b*x)*3**c*3**(d*x)*5**e*5**(f*x))/(log(5)*f + log(3)*d + log(2)*
b)
```

3.710 $\int 2^{2+3x}3^{5+7x}5^{11+13x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 35

$$\int 2^{2+3x}3^{5+7x}5^{11+13x}\,dx=\frac{2^{2+3x}3^{5+7x}5^{11+13x}}{7\log(3)+13\log(5)+\log(8)}$$

output
```
2^(2+3*x)*3^(5+7*x)*5^(11+13*x)/(7*ln(3)+13*ln(5)+3*ln(2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int 2^{2+3x}3^{5+7x}5^{11+13x}\,dx=\frac{47460937500e^{x(3\log(2)+7\log(3)+13\log(5))}}{3\log(2)+7\log(3)+13\log(5)}$$

input
```
Integrate[2^(2 + 3*x)*3^(5 + 7*x)*5^(11 + 13*x),x]
```

output
```
(47460937500*E^(x*(3*Log[2] + 7*Log[3] + 13*Log[5])))/(3*Log[2] + 7*Log[3]
 + 13*Log[5])
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{3x+2}3^{7x+5}5^{13x+11} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int 2^{3x+2}3^{7x+5}5^{13x+11}dx$$

input
```
Int[2^(2 + 3*x)*3^(5 + 7*x)*5^(11 + 13*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
gosper	$\frac{2^{2+3x}3^{5+7x}5^{11+13x}}{7\ln(3)+13\ln(5)+3\ln(2)}$	38
risch	$\frac{2^{2+3x}3^{5+7x}5^{11+13x}}{7\ln(3)+13\ln(5)+3\ln(2)}$	38
parallelrisch	$\frac{2^{2+3x}3^{5+7x}5^{11+13x}}{7\ln(3)+13\ln(5)+3\ln(2)}$	38
orering	$\frac{2^{2+3x}3^{5+7x}5^{11+13x}}{7\ln(3)+13\ln(5)+3\ln(2)}$	38
norman	$\frac{e^{(2+3x)\ln(2)}e^{(5+7x)\ln(3)}e^{(11+13x)\ln(5)}}{7\ln(3)+13\ln(5)+3\ln(2)}$	44
meijerg	$-\frac{47460937500\left(1-e^{x\ln(3)\left(7+\frac{3\ln(2)}{\ln(3)}\right)\left(1+\frac{13\ln(5)}{\ln(3)\left(7+\frac{3\ln(2)}{\ln(3)}\right)}\right)}\right)}{\ln(3)\left(7+\frac{3\ln(2)}{\ln(3)}\right)\left(1+\frac{13\ln(5)}{\ln(3)\left(7+\frac{3\ln(2)}{\ln(3)}\right)}\right)}$	84

input
```
int(2^(2+3*x)*3^(5+7*x)*5^(11+13*x),x,method=_RETURNVERBOSE)
```

output
```
2^(2+3*x)*3^(5+7*x)*5^(11+13*x)/(7*ln(3)+13*ln(5)+3*ln(2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int 2^{2+3x}3^{5+7x}5^{11+13x}\,dx=\frac{5^{13\,x+11}3^{7\,x+5}2^{3\,x+2}}{13\,\log\left(5\right)+7\,\log\left(3\right)+3\,\log\left(2\right)}$$

input
```
integrate(2^(2+3*x)*3^(5+7*x)*5^(11+13*x),x, algorithm="fricas")
```

output
```
5^(13*x + 11)*3^(7*x + 5)*2^(3*x + 2)/(13*log(5) + 7*log(3) + 3*log(2))
```

Sympy [F(-2)]

Exception generated.

$$\int 2^{2+3x} 3^{5+7x} 5^{11+13x} \, dx = \text{Exception raised: IndexError}$$

input
```
integrate(2**(2+3*x)*3**(5+7*x)*5**(11+13*x),x)
```

output
```
Exception raised: IndexError >> Index out of range: a[1]
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int 2^{2+3x} 3^{5+7x} 5^{11+13x} \, dx = \frac{47460937500 \, e^{(13 \, x \log(5) + 7 \, x \log(3) + 3 \, x \log(2))}}{13 \, \log(5) + 7 \, \log(3) + 3 \, \log(2)}$$

input
```
integrate(2^(2+3*x)*3^(5+7*x)*5^(11+13*x),x, algorithm="maxima")
```

output
```
47460937500*e^(13*x*log(5) + 7*x*log(3) + 3*x*log(2))/(13*log(5) + 7*log(3
) + 3*log(2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int 2^{2+3x} 3^{5+7x} 5^{11+13x} \, dx = \frac{e^{(13 \, x \log(5) + 7 \, x \log(3) + 3 \, x \log(2) + 11 \, \log(5) + 5 \, \log(3) + 2 \, \log(2))}}{13 \, \log(5) + 7 \, \log(3) + 3 \, \log(2)}$$

input
```
integrate(2^(2+3*x)*3^(5+7*x)*5^(11+13*x),x, algorithm="giac")
```

output
```
e^(13*x*log(5) + 7*x*log(3) + 3*x*log(2) + 11*log(5) + 5*log(3) + 2*log(2)
)/(13*log(5) + 7*log(3) + 3*log(2))
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.60

$$\int 2^{2+3x}3^{5+7x}5^{11+13x}\,dx = \frac{47460937500\,2^{3\,x}\,3^{7\,x}\,5^{13\,x}}{\ln\left(21357421875000\right)}$$

input
```
int(2^(3*x + 2)*3^(7*x + 5)*5^(13*x + 11),x)
```

output
```
(47460937500*2^(3*x)*3^(7*x)*5^(13*x))/log(21357421875000)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int 2^{2+3x}3^{5+7x}5^{11+13x}\,dx = \frac{47460937500\,2^{3x}3^{7x}5^{13x}}{13\log\left(5\right)+7\log\left(3\right)+3\log\left(2\right)}$$

input
```
int(2^(2+3*x)*3^(5+7*x)*5^(11+13*x),x)
```

output
```
(47460937500*2**(3*x)*3**(7*x)*5**(13*x))/(13*log(5) + 7*log(3) + 3*log(2)
)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file