

# Computer Algebra Independent Integration Tests

Summer 2024

2-Exponentials/156-2.1

Nasser M. Abbasi

May 17, 2024

Compiled on May 17, 2024 at 9:07pm

# Contents

<b>1</b>	<b>Introduction</b>	<b>6</b>
1.1	Listing of CAS systems tested . . . . .	7
1.2	Results . . . . .	8
1.3	Time and leaf size Performance . . . . .	12
1.4	Performance based on number of rules Rubi used . . . . .	14
1.5	Performance based on number of steps Rubi used . . . . .	15
1.6	Solved integrals histogram based on leaf size of result . . . . .	16
1.7	Solved integrals histogram based on CPU time used . . . . .	17
1.8	Leaf size vs. CPU time used . . . . .	18
1.9	list of integrals with no known antiderivative . . . . .	19
1.10	List of integrals solved by CAS but has no known antiderivative . . . . .	19
1.11	list of integrals solved by CAS but failed verification . . . . .	19
1.12	Timing . . . . .	20
1.13	Verification . . . . .	20
1.14	Important notes about some of the results . . . . .	21
1.15	Current tree layout of integration tests . . . . .	24
1.16	Design of the test system . . . . .	25
<b>2</b>	<b>detailed summary tables of results</b>	<b>26</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	27
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	32
2.3	Detailed conclusion table specific for Rubi results . . . . .	67
<b>3</b>	<b>Listing of integrals</b>	<b>72</b>
3.1	$\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx$ . . . . .	77
3.2	$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx$ . . . . .	82
3.3	$\int F^{c(a+bx)}(d + bcdx \log(F)) dx$ . . . . .	87
3.4	$\int F^{c\left(a+\frac{b}{x}\right)}\left(d - \frac{bcd \log(F)}{x}\right) dx$ . . . . .	93
3.5	$\int F^{c\left(a+\frac{b}{x^2}\right)}\left(d - \frac{2bcd \log(F)}{x^2}\right) dx$ . . . . .	98

3.6	$\int F^{c(a+\frac{b}{x^3})} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx$	103
3.7	$\int F^{c(a+bx^n)} (d + bcdnx^n \log(F)) dx$	108
3.8	$\int F^{c(a+bx^3)} (d + ex^2 + 3bcdx^3 \log(F)) dx$	113
3.9	$\int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx$	118
3.10	$\int F^{c(a+bx)} (d + e + bcdx \log(F)) dx$	123
3.11	$\int F^{c(a+\frac{b}{x})} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx$	129
3.12	$\int F^{c(a+\frac{b}{x^2})} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx$	134
3.13	$\int F^{c(a+\frac{b}{x^3})} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx$	139
3.14	$\int F^{c(a+bx^n)} (d + ex^{-1+n} + bcdnx^n \log(F)) dx$	144
3.15	$\int F^{c(a+bx)} (d - ex^2)^2 dx$	150
3.16	$\int F^{c(a+bx)} (d - ex^2) dx$	157
3.17	$\int \frac{F^{c(a+bx)}}{d-ex^2} dx$	163
3.18	$\int \frac{F^{c(a+bx)}}{(d-ex^2)^2} dx$	169
3.19	$\int F^{c(a+bx)} (d - ex^3)^2 dx$	176
3.20	$\int F^{c(a+bx)} (d - ex^3) dx$	184
3.21	$\int \frac{F^{c(a+bx)}}{d-ex^3} dx$	190
3.22	$\int \frac{F^{c(a+bx)}}{(d-ex^3)^2} dx$	196
3.23	$\int F^{c(a+bx)} (d + ex + fx^2)^2 dx$	205
3.24	$\int F^{c(a+bx)} (d + ex + fx^2) dx$	213
3.25	$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx$	219
3.26	$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$	225
3.27	$\int \frac{e^{a+bx} x^2}{c+dx^2} dx$	233
3.28	$\int \frac{e^{a+bx} x}{c+dx^2} dx$	238
3.29	$\int \frac{e^{a+bx}}{c+dx^2} dx$	243
3.30	$\int \frac{e^{a+bx}}{x(c+dx^2)} dx$	248
3.31	$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$	253
3.32	$\int \frac{e^{d+ex} x^3}{a+bx+cx^2} dx$	259
3.33	$\int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx$	265
3.34	$\int \frac{e^{d+ex} x}{a+bx+cx^2} dx$	271
3.35	$\int \frac{e^{d+ex}}{a+bx+cx^2} dx$	277
3.36	$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$	283
3.37	$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$	289
3.38	$\int F^{c(a+bx)} (d + ex)^4 dx$	295
3.39	$\int F^{c(a+bx)} (d + ex)^3 dx$	303

3.40	$\int F^{c(a+bx)}(d+ex)^2 dx$	310
3.41	$\int F^{c(a+bx)}(d+ex) dx$	317
3.42	$\int F^{c(a+bx)} dx$	323
3.43	$\int \frac{F^{c(a+bx)}}{d+ex} dx$	328
3.44	$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$	332
3.45	$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$	337
3.46	$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$	342
3.47	$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$	348
3.48	$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$	355
3.49	$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$	364
3.50	$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx$	372
3.51	$\int \frac{F^{c(a+bx)}}{d^2+2dex+e^2x^2} dx$	379
3.52	$\int \frac{F^{c(a+bx)}}{d^3+3d^2ex+3de^2x^2+e^3x^3} dx$	384
3.53	$\int \frac{F^{c(a+bx)}}{d^4+4d^3ex+6d^2e^2x^2+4de^3x^3+e^4x^4} dx$	390
3.54	$\int \frac{F^{c(a+bx)}}{d^5+5d^4ex+10d^3e^2x^2+10d^2e^3x^3+5de^4x^4+e^5x^5} dx$	396
3.55	$\int F^{a+bx}(c + bcx \log(F)) dx$	403
3.56	$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx$	409
3.57	$\int F^{2+5x} dx$	415
3.58	$\int F^{a+bx} dx$	420
3.59	$\int 10^{2+5x} dx$	425
3.60	$\int F^{a+bx} x^{7/2} dx$	430
3.61	$\int F^{a+bx} x^{5/2} dx$	436
3.62	$\int F^{a+bx} x^{3/2} dx$	442
3.63	$\int F^{a+bx} \sqrt{x} dx$	448
3.64	$\int \frac{F^{a+bx}}{\sqrt{x}} dx$	453
3.65	$\int \frac{F^{a+bx}}{x^{3/2}} dx$	458
3.66	$\int \frac{F^{a+bx}}{x^{5/2}} dx$	463
3.67	$\int \frac{F^{a+bx}}{x^{7/2}} dx$	468
3.68	$\int \frac{F^{a+bx}}{x^{9/2}} dx$	473
3.69	$\int F^{c(a+bx)}(d+ex)^{7/2} dx$	479
3.70	$\int F^{c(a+bx)}(d+ex)^{5/2} dx$	487
3.71	$\int F^{c(a+bx)}(d+ex)^{3/2} dx$	493
3.72	$\int F^{c(a+bx)} \sqrt{d+ex} dx$	499
3.73	$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$	504
3.74	$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$	509
3.75	$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$	514

3.76	$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$	520
3.77	$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$	526
3.78	$\int e^{-bx} x^{13/2} dx$	533
3.79	$\int F^{c(a+bx)}(d+ex)^{4/3} dx$	547
3.80	$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx$	552
3.81	$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$	557
3.82	$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$	563
3.83	$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$	568
3.84	$\int F^{c(a+bx)}(d+ex)^m dx$	573
3.85	$\int F^{c(a+bx)}(d+ex)^{-m} dx$	578
3.86	$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx$	583
3.87	$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$	588
3.88	$\int F^{c(a+bx)}(d+ex)^m dx$	593
3.89	$\int F^{c(a+bx)}((d+ex)^n)^m dx$	598
3.90	$\int F^{c(a+bx)}(d+ex) dx$	603
3.91	$\int F^{c(a+bx)}(d+ex+fx^2) dx$	609
3.92	$\int F^{c(a+bx)}(d+ex+fx^2+gx^3) dx$	615
3.93	$\int F^{c(a+bx)}(d+ex+fx^2+gx^3+hx^4) dx$	622
3.94	$\int e^{-a-bx} x^m (a+bx)^3 dx$	630
3.95	$\int e^{-a-bx} x^3 (a+bx)^3 dx$	636
3.96	$\int e^{-a-bx} x^2 (a+bx)^3 dx$	644
3.97	$\int e^{-a-bx} x (a+bx)^3 dx$	651
3.98	$\int e^{-a-bx} (a+bx)^3 dx$	657
3.99	$\int \frac{e^{-a-bx} (a+bx)^3}{x} dx$	663
3.100	$\int \frac{e^{-a-bx} (a+bx)^3}{x^2} dx$	668
3.101	$\int \frac{e^{-a-bx} (a+bx)^3}{x^3} dx$	674
3.102	$\int \frac{e^{-a-bx} (a+bx)^3}{x^4} dx$	680
3.103	$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx$	686
3.104	$\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx$	692
3.105	$\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx$	700
3.106	$\int F^{a+b(c+dx)} x (e+fx)^2 dx$	708
3.107	$\int F^{a+b(c+dx)} (e+fx)^2 dx$	715
3.108	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x} dx$	722
3.109	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^2} dx$	728
3.110	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^3} dx$	734
3.111	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^4} dx$	740
3.112	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^5} dx$	747

3.113	$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx$	754
3.114	$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$	767
3.115	$\int e^{-a-bx}(a+bx)^4(c+dx) dx$	779
3.116	$\int e^{-a-bx}(a+bx)^4 dx$	788
3.117	$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$	795
3.118	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$	802
3.119	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$	810
3.120	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$	818
3.121	$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$	828
3.122	$\int F^{c(a+bx)} x^m \log^n(dx)(e+en+e(1+m+bcx \log(F)) \log(dx)) dx$	838
3.123	$\int F^{c(a+bx)} x^2 \log^n(dx)(e+en+e(3+bcx \log(F)) \log(dx)) dx$	843
3.124	$\int F^{c(a+bx)} x \log^n(dx)(e+en+e(2+bcx \log(F)) \log(dx)) dx$	848
3.125	$\int F^{c(a+bx)} \log^n(dx)(e+en+e(1+bcx \log(F)) \log(dx)) dx$	853
3.126	$\int \frac{F^{c(a+bx)} \log^n(dx)(e+en+bcex \log(F) \log(dx))}{x} dx$	858
3.127	$\int \frac{F^{c(a+bx)} \log^n(dx)(e+en+e(-1+bcx \log(F)) \log(dx))}{x^2} dx$	863
3.128	$\int \frac{F^{c(a+bx)} \log^n(dx)(e+en+e(-2+bcx \log(F)) \log(dx))}{x^3} dx$	869
3.129	$\int \sqrt{e^{a+bx}} x^4 dx$	875
3.130	$\int \sqrt{e^{a+bx}} x^3 dx$	881
3.131	$\int \sqrt{e^{a+bx}} x^2 dx$	886
3.132	$\int \sqrt{e^{a+bx}} x dx$	891
3.133	$\int \sqrt{e^{a+bx}} dx$	896
3.134	$\int \frac{\sqrt{e^{a+bx}}}{x} dx$	901
3.135	$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$	906
3.136	$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx$	911
3.137	$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx$	916
<b>4</b>	<b>Appendix</b>	<b>921</b>
4.1	Listing of Grading functions	921
4.2	Links to plain text integration problems used in this report for each CAS939	

# CHAPTER 1

## INTRODUCTION

1.1	Listing of CAS systems tested . . . . .	7
1.2	Results . . . . .	8
1.3	Time and leaf size Performance . . . . .	12
1.4	Performance based on number of rules Rubi used . . . . .	14
1.5	Performance based on number of steps Rubi used . . . . .	15
1.6	Solved integrals histogram based on leaf size of result . . . . .	16
1.7	Solved integrals histogram based on CPU time used . . . . .	17
1.8	Leaf size vs. CPU time used . . . . .	18
1.9	list of integrals with no known antiderivative . . . . .	19
1.10	List of integrals solved by CAS but has no known antiderivative . . . . .	19
1.11	list of integrals solved by CAS but failed verification . . . . .	19
1.12	Timing . . . . .	20
1.13	Verification . . . . .	20
1.14	Important notes about some of the results . . . . .	21
1.15	Current tree layout of integration tests . . . . .	24
1.16	Design of the test system . . . . .	25

This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 137 ]. This is test number [ 156 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.



## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 ( 137 )	0.00 ( 0 )
Mathematica	100.00 ( 137 )	0.00 ( 0 )
Fricas	96.35 ( 132 )	3.65 ( 5 )
Maple	83.94 ( 115 )	16.06 ( 22 )
Maxima	62.77 ( 86 )	37.23 ( 51 )
Mupad	57.66 ( 79 )	42.34 ( 58 )
Reduce	54.74 ( 75 )	45.26 ( 62 )
Giac	52.55 ( 72 )	47.45 ( 65 )
Sympy	43.80 ( 60 )	56.20 ( 77 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

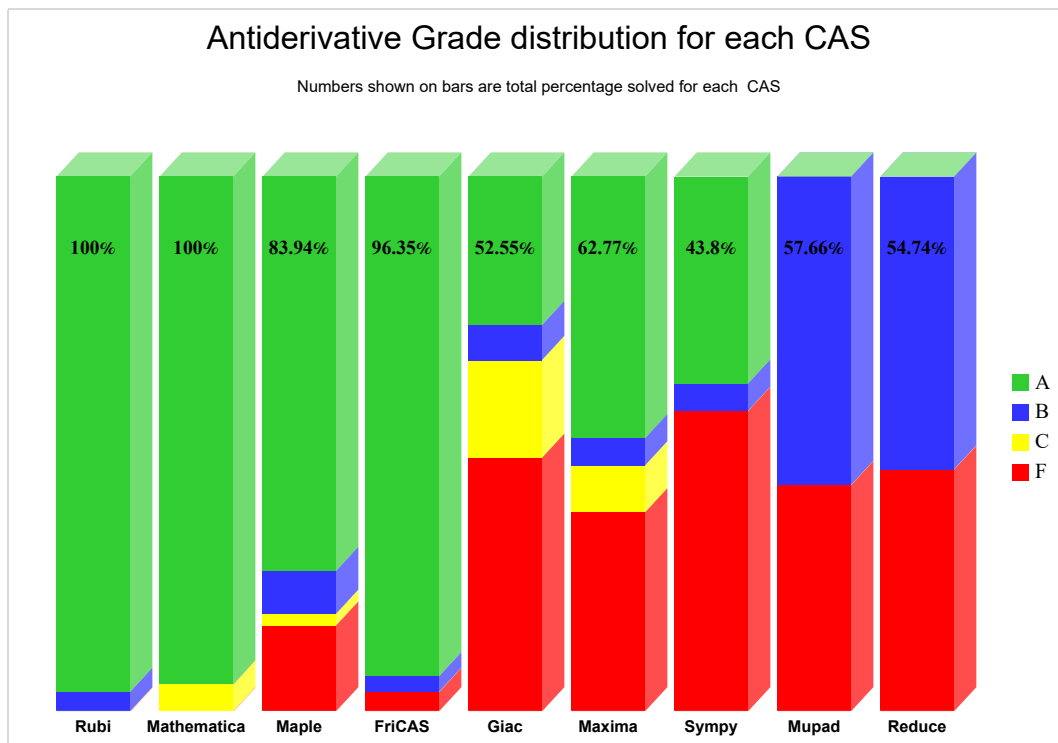
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

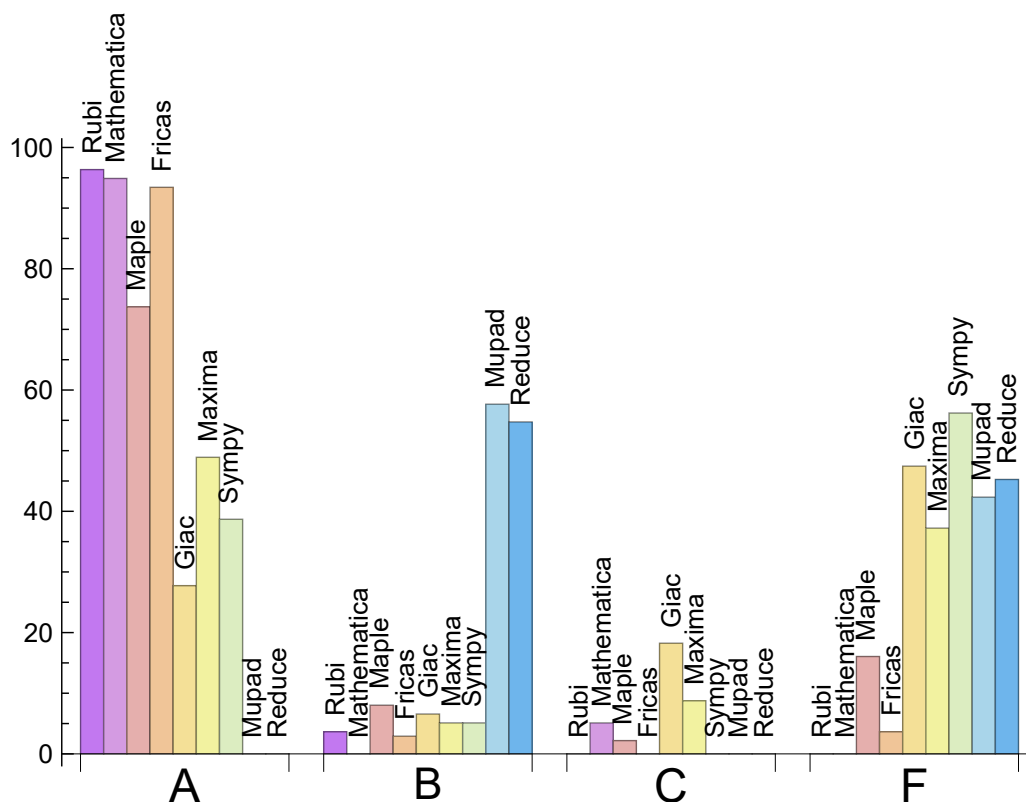
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.350	3.650	0.000	0.000
Mathematica	94.891	0.000	5.109	0.000
Fricas	93.431	2.920	0.000	3.650
Maple	73.723	8.029	2.190	16.058
Maxima	48.905	5.109	8.759	37.226
Sympy	38.686	5.109	0.000	56.204
Giac	27.737	6.569	18.248	47.445
Mupad	0.000	57.664	0.000	42.336
Reduce	0.000	54.745	0.000	45.255

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	5	100.00	0.00	0.00
Maple	22	100.00	0.00	0.00
Maxima	51	100.00	0.00	0.00
Mupad	58	0.00	100.00	0.00
Reduce	62	100.00	0.00	0.00
Giac	65	92.31	0.00	7.69
Sympy	77	93.51	5.19	1.30

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Fricas	0.08
Giac	0.15
Reduce	0.18
Mathematica	0.42
Rubi	0.65
Maple	1.99
Sympy	2.49
Mupad	20.67

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	86.88	0.79	67.00	0.77
Mupad	98.76	0.96	69.00	1.00
Reduce	104.84	1.01	61.00	1.03
Maxima	112.33	1.61	71.50	1.25
Fricas	133.31	1.04	82.50	1.00
Rubi	138.07	1.18	102.00	1.00
Sympy	152.42	1.31	92.00	1.27
Maple	158.97	1.31	99.00	1.10
Giac	1886.17	14.64	266.50	1.38

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the  $y$  axis is the percentage solved which Rubi itself needed the number of rules given the  $x$  axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

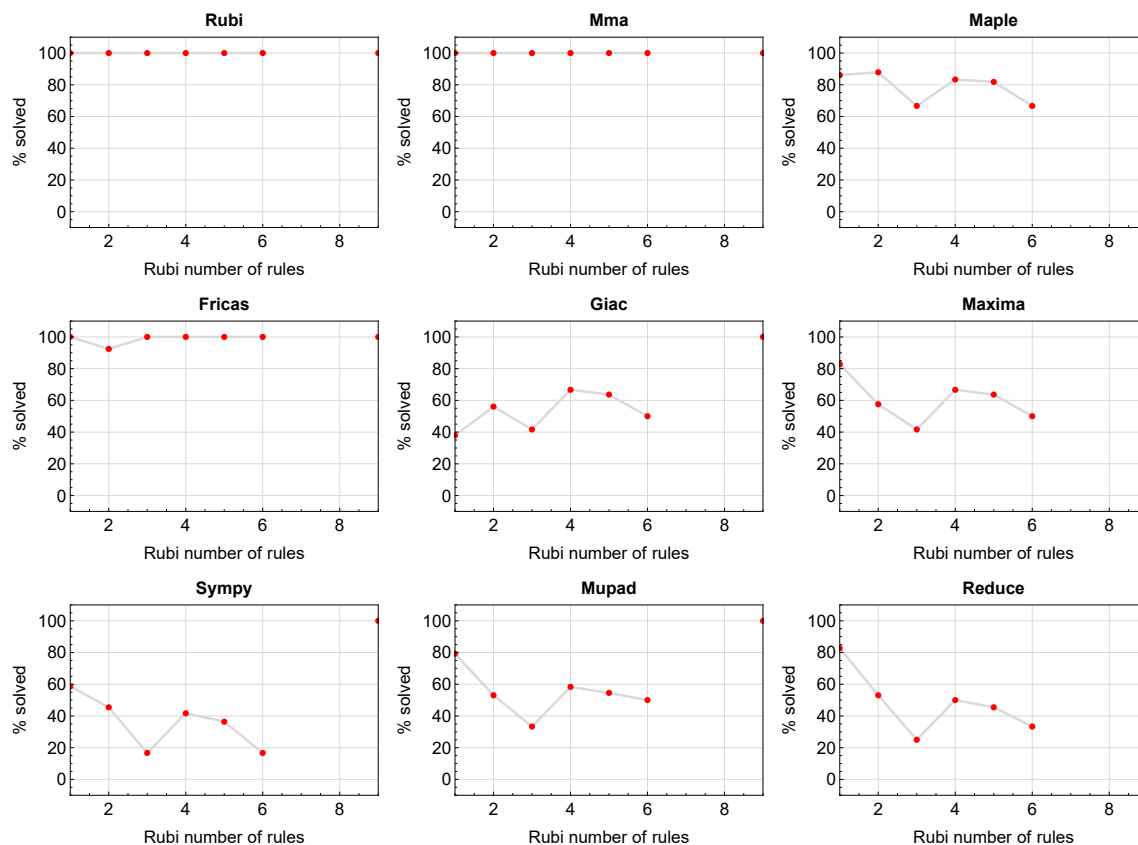


Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

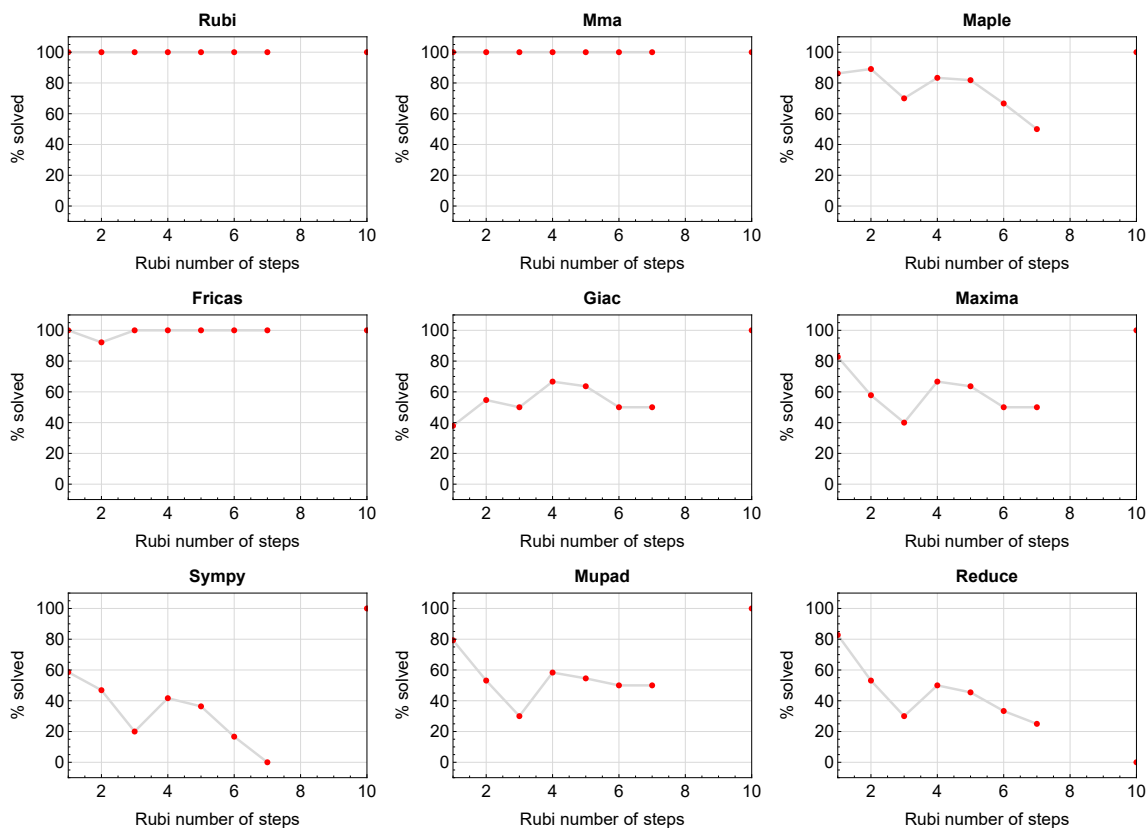


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.



## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

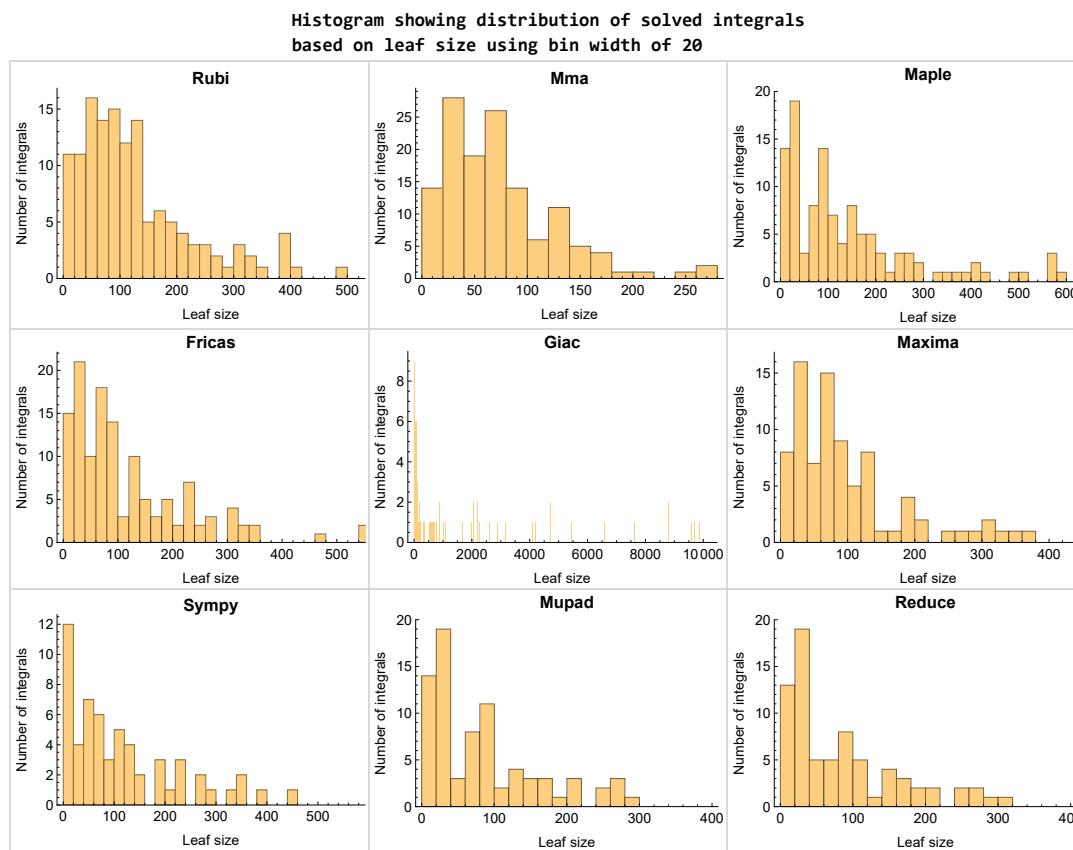


Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

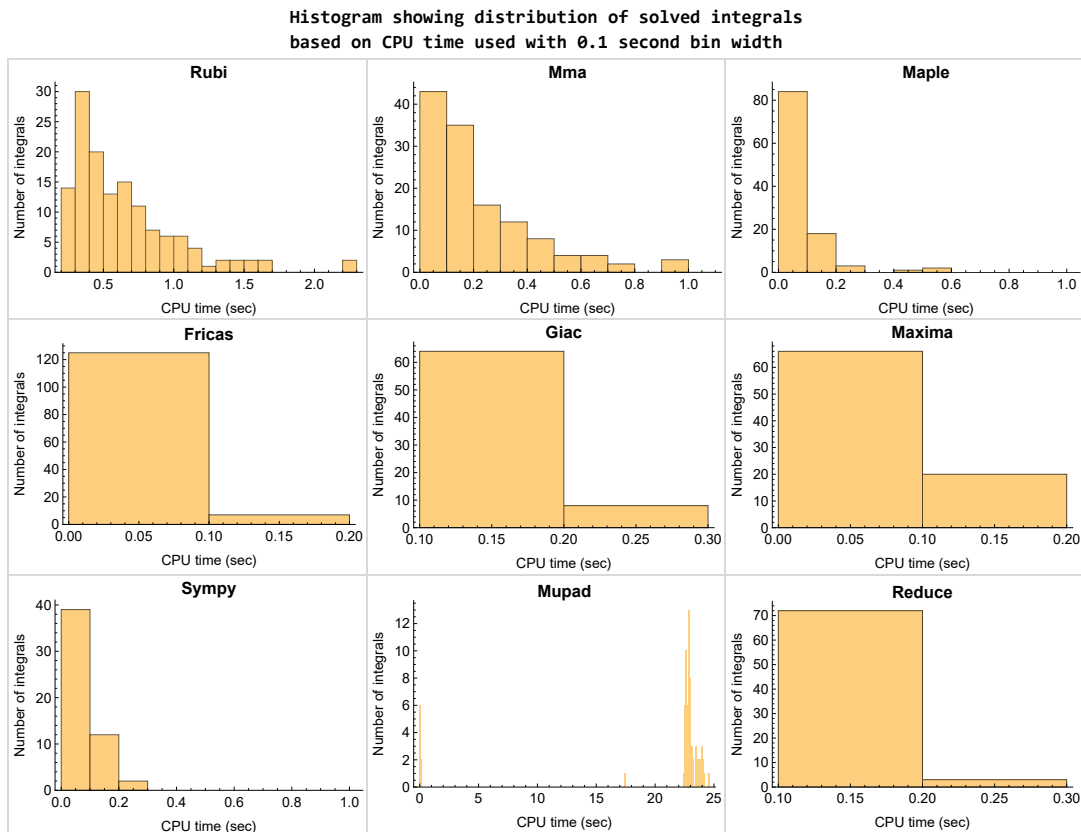


Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

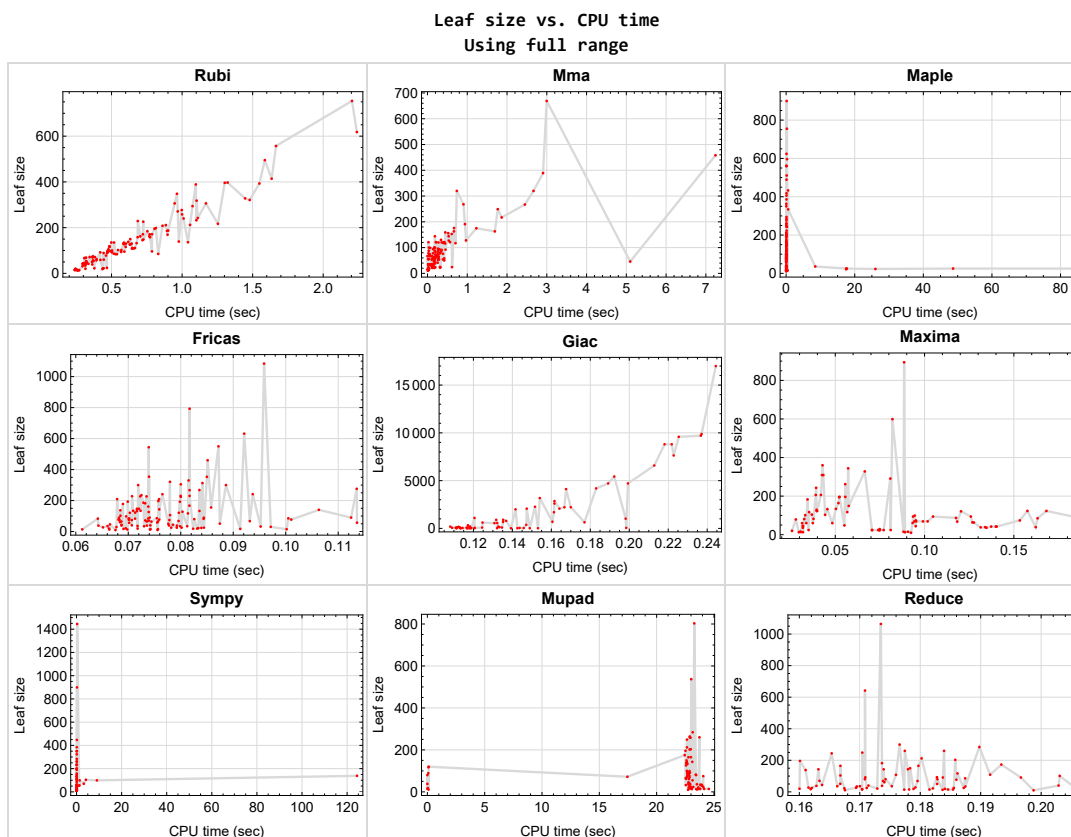


Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{}

## 1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {1, 2, 3, 4, 5, 6, 7, 55, 122}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for **Rubi** and **Mathematica**.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.14 Important notes about some of the results

### Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima `integrate` was run using SageMath with the following settings set by default

```
'besselexpand : true'  
'display2d : false'  
'domain : complex'  
'keepfloat : true'  
'load(to_poly_solve)'  
'load(simplify_sum)'  
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib  
maxima_lib.set('extra_definite_integration_methods', '[]')  
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:  
    # 1.7 is a fudge factor since it is low side from actual leaf count  
    leafCount = round(1.7*count_ops(anti))  
  
except Exception as ee:  
    leafCount = 1
```

### Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')  
the_variable = evalin(symengine, 'x')  
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$



# 1.15 Current tree layout of integration tests

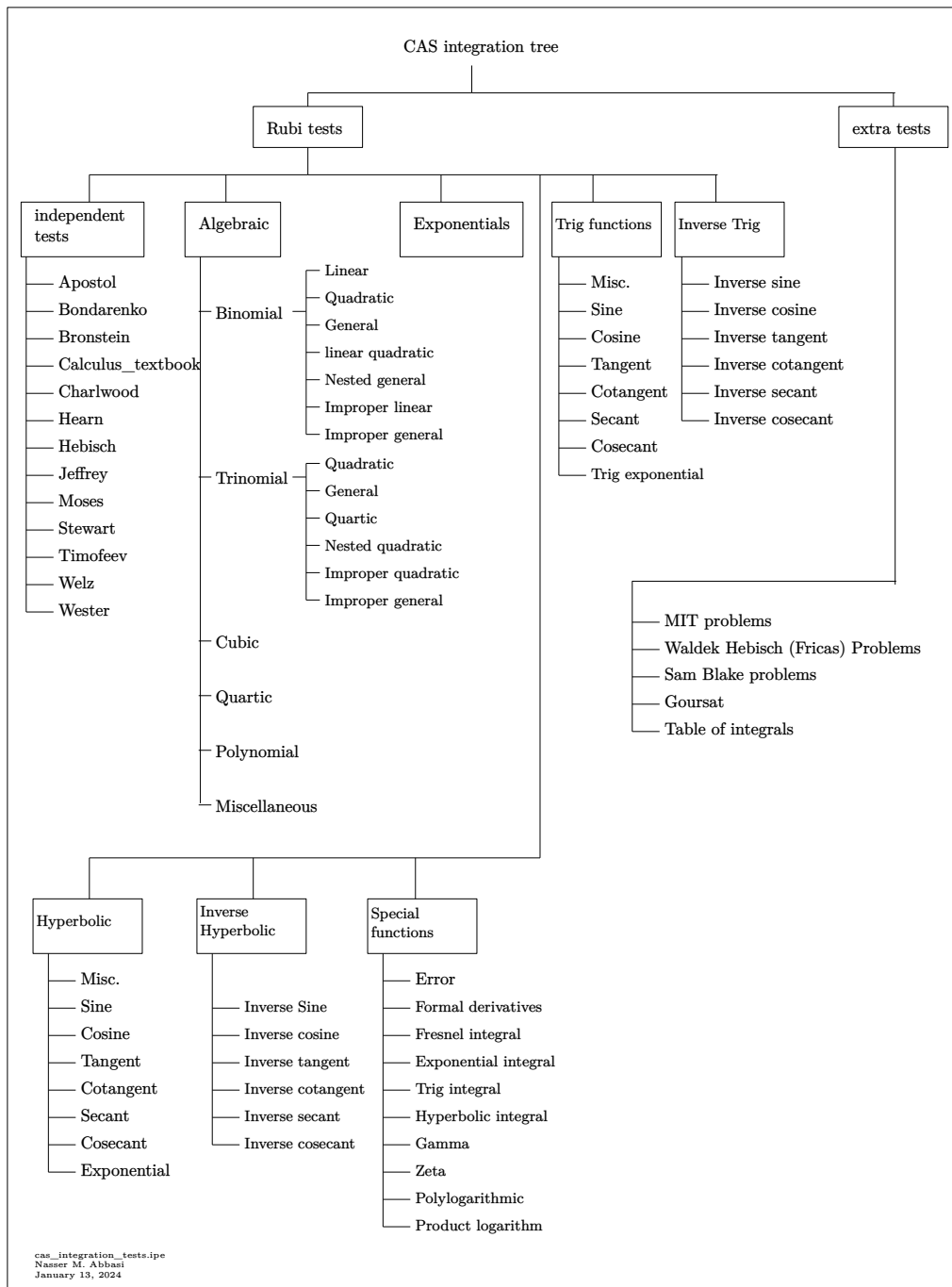
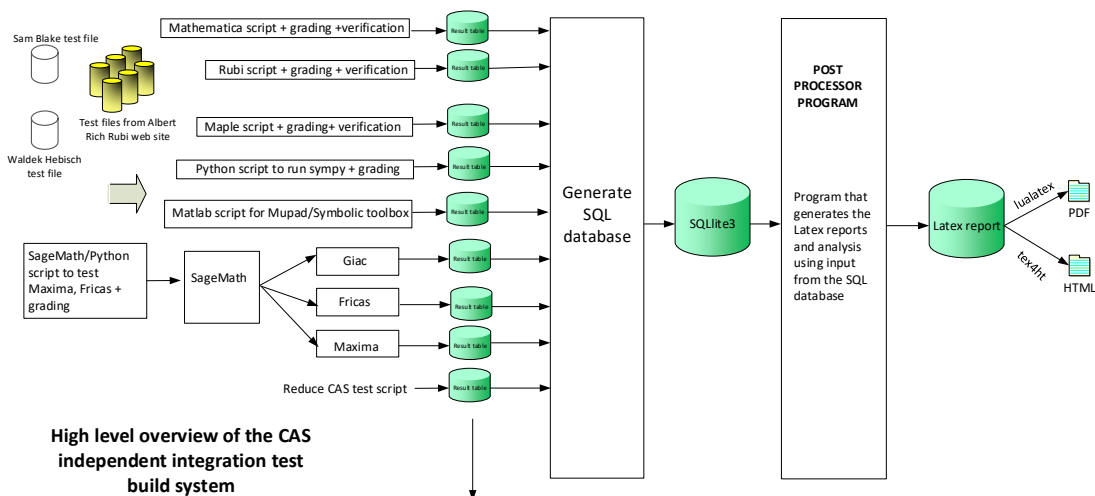


Figure 1.6: CAS integration tests tree

# 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



**High level overview of the CAS independent integration test build system**

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "E"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi  
January 13, 2024  
Design note

# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

2.1	List of integrals sorted by grade for each CAS . . . . .	27
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	32
2.3	Detailed conclusion table specific for Rubi results . . . . .	67

## 2.1 List of integrals sorted by grade for each CAS

Rubi . . . . .	27
Mma . . . . .	27
Maple . . . . .	28
Fricas . . . . .	28
Maxima . . . . .	29
Giac . . . . .	29
Mupad . . . . .	30
Sympy . . . . .	30
Reduce . . . . .	31

### Rubi

**A grade** { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137 }

**B grade** { 3, 23, 55, 56, 93 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137 }

**B grade** { }

**C grade** { 21, 22, 27, 28, 29, 30, 31 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 23, 24, 25, 27, 28, 29, 30, 31, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 78, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133 }

**B grade** { 26, 32, 33, 34, 36, 37, 103, 134, 135, 136, 137 }

**C grade** { 21, 94, 122 }

**F normal fail** { 18, 22, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137 }

**B grade** { 25, 26, 120, 121 }

**C grade** { }

**F normal fail** { 81, 82, 83, 86, 87 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Maxima

**A grade** { 15, 16, 19, 20, 24, 39, 40, 41, 42, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 78, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137 }

**B grade** { 3, 10, 23, 38, 48, 55, 56 }

**C grade** { 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14 }

**F normal fail** { 17, 18, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 51, 52, 53, 54, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 117, 118, 119, 120, 121 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Giac

**A grade** { 1, 2, 7, 8, 9, 14, 42, 57, 58, 59, 60, 61, 62, 63, 64, 73, 78, 95, 96, 97, 98, 99, 100, 101, 102, 113, 114, 115, 116, 129, 130, 131, 132, 133, 134, 135, 136, 137 }

**B grade** { 69, 70, 71, 72, 117, 118, 119, 120, 121 }

**C grade** { 3, 10, 15, 16, 19, 20, 23, 24, 38, 39, 40, 41, 48, 49, 50, 55, 56, 90, 91, 92, 93, 104, 105, 106, 107 }

**F normal fail** { 4, 5, 6, 11, 12, 13, 17, 18, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 51, 52, 53, 54, 65, 66, 67, 68, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 103, 108, 109, 110, 111, 112, 122, 128 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 123, 124, 125, 126, 127 }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 19, 20, 23, 24, 38, 39, 40, 41, 42, 48, 49, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 78, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 14, 17, 18, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 51, 52, 53, 54, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 103, 117, 118, 119, 120, 121, 134, 135, 136, 137 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 19, 20, 24, 40, 41, 42, 50, 55, 57, 58, 59, 78, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 113, 114, 115, 116, 129, 130, 131, 132, 133 }

**B grade** { 14, 23, 38, 39, 48, 49, 56 }

**C grade** { }

**F normal fail** { 17, 18, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 51, 52, 53, 54, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 79, 81, 82, 83, 84, 86, 87, 88, 89, 103, 108, 109, 110, 111, 112, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 134, 135, 136, 137 }

**F(-1) timedout fail** { 69, 77, 80, 122 }

**F(-2) exception fail** { 85 }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 23, 24, 38, 39, 40, 41, 42, 48, 49, 50, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133 }

**C grade** { }

**F normal fail** { 17, 18, 21, 22, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 51, 52, 53, 54, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 94, 103, 117, 118, 119, 120, 121, 134, 135, 136, 137 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }



## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$ . To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	69	15	12	16	15	14
N.S.	1	1.00	1.00	1.07	4.93	1.07	0.86	1.14	1.07	1.00
time (sec)	N/A	0.271	0.018	0.121	0.095	0.100	0.057	0.149	0.185	23.623

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	104	15	12	18	15	14
N.S.	1	1.00	1.00	1.07	7.43	1.07	0.86	1.29	1.07	1.00
time (sec)	N/A	0.256	0.018	0.056	0.037	0.074	0.085	0.143	0.170	23.748

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	12	52	13	13	60	13	10	701	13	12
N.S.	1	4.33	1.08	1.08	5.00	1.08	0.83	58.42	1.08	1.00
time (sec)	N/A	0.319	0.016	0.043	0.037	0.070	0.079	0.161	0.184	23.541

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	<b>F</b>	B	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	44	17	10	0	17	14
N.S.	1	1.00	1.00	1.07	3.14	1.21	0.71	0.00	1.21	1.00
time (sec)	N/A	0.259	0.021	0.079	0.095	0.069	0.078	0.000	0.162	24.549

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	<b>F</b>	B	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	74	19	12	0	19	14
N.S.	1	1.00	1.00	1.07	5.29	1.36	0.86	0.00	1.36	1.00
time (sec)	N/A	0.257	0.021	0.072	0.095	0.083	0.080	0.000	0.184	24.138

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	<b>F</b>	B	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	69	19	12	0	19	14
N.S.	1	1.00	1.00	1.07	4.93	1.36	0.86	0.00	1.36	1.00
time (sec)	N/A	0.260	0.021	0.093	0.102	0.072	0.085	0.000	0.179	23.802

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	94	18	14	16	15	15
N.S.	1	1.00	1.00	1.07	6.71	1.29	1.00	1.14	1.07	1.07
time (sec)	N/A	0.268	0.026	0.460	0.126	0.082	0.229	0.153	0.182	22.983

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	35	34	94	34	61	49	34	34
N.S.	1	1.26	1.00	0.97	2.69	0.97	1.74	1.40	0.97	0.97
time (sec)	N/A	0.309	0.024	0.057	0.105	0.081	0.060	0.144	0.187	22.833

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	42	35	34	129	34	61	37	34	34
N.S.	1	1.20	1.00	0.97	3.69	0.97	1.74	1.06	0.97	0.97
time (sec)	N/A	0.295	0.022	0.048	0.040	0.080	0.079	0.146	0.170	23.923

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	53	30	30	82	30	49	787	30	30
N.S.	1	1.83	1.03	1.03	2.83	1.03	1.69	27.14	1.03	1.03
time (sec)	N/A	0.322	0.021	0.034	0.038	0.097	0.080	0.137	0.205	23.976

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	42	33	35	69	36	49	0	36	34
N.S.	1	1.27	1.00	1.06	2.09	1.09	1.48	0.00	1.09	1.03
time (sec)	N/A	0.296	0.026	0.058	0.099	0.072	0.093	0.000	0.175	23.803

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	35	39	99	40	58	0	40	36
N.S.	1	1.26	1.00	1.11	2.83	1.14	1.66	0.00	1.14	1.03
time (sec)	N/A	0.303	0.028	0.072	0.094	0.074	0.113	0.000	0.171	23.512

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	44	35	39	94	40	60	0	40	36
N.S.	1	1.26	1.00	1.11	2.69	1.14	1.71	0.00	1.14	1.03
time (sec)	N/A	0.302	0.029	0.092	0.094	0.071	0.107	0.000	0.203	22.927

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	50	35	36	121	39	104	51	36	0
N.S.	1	1.43	1.00	1.03	3.46	1.11	2.97	1.46	1.03	0.00
time (sec)	N/A	0.361	0.068	8.490	0.120	0.078	4.176	0.198	0.166	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	226	100	150	183	143	202	5426	150	150
N.S.	1	1.60	0.71	1.06	1.30	1.01	1.43	38.48	1.06	1.06
time (sec)	N/A	0.725	0.173	0.065	0.034	0.076	0.089	0.192	0.178	22.609

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	92	53	61	81	60	85	1690	61	62
N.S.	1	1.24	0.72	0.82	1.09	0.81	1.15	22.84	0.82	0.84
time (sec)	N/A	0.420	0.107	0.030	0.033	0.068	0.086	0.161	0.174	22.578

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	91	130	0	155	0	0	25	0
N.S.	1	1.00	0.78	1.12	0.00	1.34	0.00	0.00	0.22	0.00
time (sec)	N/A	0.486	0.264	0.098	0.000	0.086	0.000	0.000	0.167	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	306	176	0	0	276	0	0	35	0
N.S.	1	1.01	0.58	0.00	0.00	0.91	0.00	0.00	0.12	0.00
time (sec)	N/A	1.169	0.668	0.000	0.000	0.114	0.000	0.000	0.183	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	306	144	203	243	193	262	9878	203	203
N.S.	1	1.51	0.71	1.00	1.20	0.95	1.29	48.66	1.00	1.00
time (sec)	N/A	0.947	0.186	0.082	0.039	0.070	0.104	0.237	0.186	22.975

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	117	67	77	101	76	104	2601	77	77
N.S.	1	1.18	0.68	0.78	1.02	0.77	1.05	26.27	0.78	0.78
time (sec)	N/A	0.483	0.115	0.033	0.033	0.068	0.068	0.162	0.186	22.857

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	46	120	0	268	0	0	25	0
N.S.	1	1.00	0.22	0.57	0.00	1.27	0.00	0.00	0.12	0.00
time (sec)	N/A	0.885	5.098	0.086	0.000	0.084	0.000	0.000	0.180	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	<b>F</b>	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	617	618	121	0	0	460	0	0	35	0
N.S.	1	1.00	0.20	0.00	0.00	0.75	0.00	0.00	0.06	0.00
time (sec)	N/A	2.239	0.214	0.000	0.000	0.085	0.000	0.000	0.176	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	389	120	284	360	241	384	9704	284	284
N.S.	1	2.20	0.68	1.60	2.03	1.36	2.17	54.82	1.60	1.60
time (sec)	N/A	1.097	0.275	0.116	0.043	0.077	0.115	0.237	0.190	23.144

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	135	56	80	117	74	116	2068	80	80
N.S.	1	1.69	0.70	1.00	1.46	0.92	1.45	25.85	1.00	1.00
time (sec)	N/A	0.518	0.156	0.036	0.035	0.071	0.073	0.164	0.171	23.439

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	128	223	0	305	0	0	27	0
N.S.	1	1.00	0.86	1.51	0.00	2.06	0.00	0.00	0.18	0.00
time (sec)	N/A	0.726	0.970	0.139	0.000	0.080	0.000	0.000	0.194	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	<b>F</b>	B	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	393	249	624	0	632	0	0	0	0
N.S.	1	1.29	0.82	2.05	0.00	2.08	0.00	0.00	0.00	0.00
time (sec)	N/A	1.548	1.767	0.118	0.000	0.092	0.000	0.000	0.215	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	120	127	0	106	0	0	37	0
N.S.	1	1.00	0.91	0.96	0.00	0.80	0.00	0.00	0.28	0.00
time (sec)	N/A	0.677	0.359	0.079	0.000	0.078	0.000	0.000	0.181	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	83	100	0	72	0	0	22	0
N.S.	1	1.00	0.83	1.00	0.00	0.72	0.00	0.00	0.22	0.00
time (sec)	N/A	0.503	0.198	0.038	0.000	0.073	0.000	0.000	0.178	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	94	102	0	98	0	0	21	0
N.S.	1	1.00	0.80	0.86	0.00	0.83	0.00	0.00	0.18	0.00
time (sec)	N/A	0.486	0.142	0.033	0.000	0.070	0.000	0.000	0.167	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	93	112	0	80	0	0	23	0
N.S.	1	1.00	0.84	1.01	0.00	0.72	0.00	0.00	0.21	0.00
time (sec)	N/A	0.666	0.304	0.046	0.000	0.083	0.000	0.000	0.189	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	121	142	0	128	0	0	25	0
N.S.	1	1.00	0.83	0.98	0.00	0.88	0.00	0.00	0.17	0.00
time (sec)	N/A	0.720	0.567	0.056	0.000	0.080	0.000	0.000	0.183	0.000



Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	268	755	0	330	0	0	119	0
N.S.	1	1.00	1.16	3.25	0.00	1.42	0.00	0.00	0.51	0.00
time (sec)	N/A	1.101	0.905	0.226	0.000	0.081	0.000	0.000	0.182	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	217	561	0	267	0	0	65	0
N.S.	1	1.00	1.17	3.02	0.00	1.44	0.00	0.00	0.35	0.00
time (sec)	N/A	0.893	1.865	0.073	0.000	0.082	0.000	0.000	0.185	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	153	350	0	224	0	0	25	0
N.S.	1	1.00	0.97	2.22	0.00	1.42	0.00	0.00	0.16	0.00
time (sec)	N/A	0.685	0.610	0.059	0.000	0.080	0.000	0.000	0.174	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	127	169	0	192	0	0	24	0
N.S.	1	1.00	0.92	1.22	0.00	1.39	0.00	0.00	0.17	0.00
time (sec)	N/A	0.619	0.536	0.048	0.000	0.080	0.000	0.000	0.184	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	163	363	0	241	0	0	28	0
N.S.	1	1.00	0.96	2.15	0.00	1.43	0.00	0.00	0.17	0.00
time (sec)	N/A	0.897	0.665	0.065	0.000	0.094	0.000	0.000	0.169	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	320	561	0	313	0	0	30	0
N.S.	1	1.00	1.51	2.65	0.00	1.48	0.00	0.00	0.14	0.00
time (sec)	N/A	1.056	0.738	0.082	0.000	0.084	0.000	0.000	0.194	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	171	100	260	309	227	350	8802	260	260
N.S.	1	1.21	0.71	1.84	2.19	1.61	2.48	62.43	1.84	1.84
time (sec)	N/A	0.770	0.176	0.077	0.043	0.073	0.116	0.218	0.184	23.736

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	130	78	165	206	147	231	4706	165	165
N.S.	1	1.18	0.71	1.50	1.87	1.34	2.10	42.78	1.50	1.50
time (sec)	N/A	0.584	0.139	0.058	0.039	0.072	0.090	0.199	0.167	22.994

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	89	56	91	123	84	133	2214	91	91
N.S.	1	1.13	0.71	1.15	1.56	1.06	1.68	28.03	1.15	1.15
time (sec)	N/A	0.444	0.125	0.040	0.040	0.070	0.070	0.167	0.184	22.808

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	38	60	38	60	898	38	38
N.S.	1	1.00	0.71	0.79	1.25	0.79	1.25	18.71	0.79	0.79
time (sec)	N/A	0.324	0.091	0.028	0.034	0.074	0.057	0.132	0.163	22.774

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	21	21	20	21	20	21	21	21
N.S.	1	1.00	1.05	1.05	1.00	1.05	1.00	1.05	1.05	1.05
time (sec)	N/A	0.243	0.027	0.016	0.026	0.070	0.035	0.132	0.173	22.796

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	56	0	39	0	0	22	0
N.S.	1	1.00	1.00	1.81	0.00	1.26	0.00	0.00	0.71	0.00
time (sec)	N/A	0.294	0.098	0.052	0.000	0.069	0.000	0.000	0.193	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	99	0	77	0	0	33	0
N.S.	1	1.00	0.96	1.74	0.00	1.35	0.00	0.00	0.58	0.00
time (sec)	N/A	0.369	0.177	0.063	0.000	0.068	0.000	0.000	0.164	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	92	88	155	0	134	0	0	44	0
N.S.	1	0.97	0.93	1.63	0.00	1.41	0.00	0.00	0.46	0.00
time (sec)	N/A	0.472	0.179	0.083	0.000	0.071	0.000	0.000	0.207	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	127	99	199	0	209	0	0	55	0
N.S.	1	0.99	0.77	1.55	0.00	1.63	0.00	0.00	0.43	0.00
time (sec)	N/A	0.590	0.190	0.115	0.000	0.068	0.000	0.000	0.176	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	121	243	0	300	0	0	66	0
N.S.	1	1.01	0.75	1.51	0.00	1.86	0.00	0.00	0.41	0.00
time (sec)	N/A	0.704	0.213	0.157	0.000	0.072	0.000	0.000	0.182	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	171	100	260	309	227	350	8802	260	260
N.S.	1	1.21	0.71	1.84	2.19	1.61	2.48	62.43	1.84	1.84
time (sec)	N/A	0.787	0.035	0.073	0.043	0.072	0.105	0.222	0.177	22.891

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	130	78	165	206	147	231	4706	165	165
N.S.	1	1.18	0.71	1.50	1.87	1.34	2.10	42.78	1.50	1.50
time (sec)	N/A	0.602	0.027	0.050	0.042	0.072	0.089	0.189	0.180	22.774

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	89	56	91	123	84	133	2214	91	91
N.S.	1	1.13	0.71	1.15	1.56	1.06	1.68	28.03	1.15	1.15
time (sec)	N/A	0.468	0.025	0.037	0.040	0.074	0.066	0.170	0.197	22.902

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	99	0	77	0	0	33	0
N.S.	1	1.00	0.96	1.74	0.00	1.35	0.00	0.00	0.58	0.00
time (sec)	N/A	0.393	0.016	0.080	0.000	0.076	0.000	0.000	0.178	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	92	88	155	0	134	0	0	44	0
N.S.	1	0.97	0.93	1.63	0.00	1.41	0.00	0.00	0.46	0.00
time (sec)	N/A	0.503	0.033	0.103	0.000	0.072	0.000	0.000	0.183	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	127	99	199	0	209	0	0	55	0
N.S.	1	0.99	0.77	1.55	0.00	1.63	0.00	0.00	0.43	0.00
time (sec)	N/A	0.635	0.069	0.151	0.000	0.076	0.000	0.000	0.179	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	162	121	243	0	300	0	0	66	0
N.S.	1	1.01	0.75	1.51	0.00	1.86	0.00	0.00	0.41	0.00
time (sec)	N/A	0.768	0.024	0.221	0.000	0.089	0.000	0.000	0.174	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD	TBD
size	10	41	10	11	45	10	8	569	10	10
N.S.	1	4.10	1.00	1.10	4.50	1.00	0.80	56.90	1.00	1.00
time (sec)	N/A	0.321	0.011	0.042	0.032	0.076	0.049	0.132	0.199	22.746

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	42	17	18	45	17	32	609	17	17
N.S.	1	2.47	1.00	1.06	2.65	1.00	1.88	35.82	1.00	1.00
time (sec)	N/A	0.336	0.015	0.030	0.032	0.091	0.052	0.125	0.170	22.842

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	14	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	0.93	0.87
time (sec)	N/A	0.239	0.023	0.021	0.030	0.066	0.037	0.118	0.177	23.070

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	15	15	15	15	15
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.242	0.001	0.017	0.030	0.076	0.036	0.139	0.178	0.002

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	13	13	10	13	11	11
N.S.	1	1.00	1.00	0.63	0.68	0.68	0.53	0.68	0.58	0.58
time (sec)	N/A	0.247	0.025	0.051	0.032	0.074	0.035	0.117	0.168	0.096

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	158	36	99	24	89	0	94	117	82
N.S.	1	1.21	0.27	0.76	0.18	0.68	0.00	0.72	0.89	0.63
time (sec)	N/A	0.773	0.114	0.034	0.074	0.084	0.000	0.117	0.186	23.417

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	126	36	87	24	77	0	82	93	72
N.S.	1	1.17	0.33	0.81	0.22	0.71	0.00	0.76	0.86	0.67
time (sec)	N/A	0.631	0.112	0.026	0.071	0.101	0.000	0.116	0.183	17.450

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	94	36	75	24	65	0	70	69	75
N.S.	1	1.11	0.42	0.88	0.28	0.76	0.00	0.82	0.81	0.88
time (sec)	N/A	0.500	0.147	0.022	0.075	0.076	0.000	0.117	0.174	24.068

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	30	66	24	51	0	58	50	55
N.S.	1	1.00	0.48	1.06	0.39	0.82	0.00	0.94	0.81	0.89
time (sec)	N/A	0.394	0.097	0.022	0.075	0.087	0.000	0.121	0.183	23.131



Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	30	27	29	34	0	28	27	32
N.S.	1	1.00	0.79	0.71	0.76	0.89	0.00	0.74	0.71	0.84
time (sec)	N/A	0.302	0.102	0.022	0.075	0.073	0.000	0.121	0.162	22.547

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	38	64	24	44	0	0	19	42
N.S.	1	1.00	0.70	1.19	0.44	0.81	0.00	0.00	0.35	0.78
time (sec)	N/A	0.372	0.185	0.020	0.074	0.079	0.000	0.000	0.185	22.855

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	78	49	72	24	58	0	0	19	61
N.S.	1	1.01	0.64	0.94	0.31	0.75	0.00	0.00	0.25	0.79
time (sec)	N/A	0.461	0.164	0.023	0.077	0.076	0.000	0.000	0.180	22.817

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	102	61	84	24	74	0	0	19	80
N.S.	1	1.02	0.61	0.84	0.24	0.74	0.00	0.00	0.19	0.80
time (sec)	N/A	0.546	0.187	0.023	0.081	0.078	0.000	0.000	0.188	22.807

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	126	73	96	24	86	0	0	19	99
N.S.	1	1.02	0.59	0.78	0.20	0.70	0.00	0.00	0.15	0.80
time (sec)	N/A	0.631	0.205	0.023	0.071	0.075	0.000	0.000	0.190	22.869

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F(-1)</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	208	240	72	0	0	230	0	1023	94	0
N.S.	1	1.15	0.35	0.00	0.00	1.11	0.00	4.92	0.45	0.00
time (sec)	N/A	1.010	0.243	0.000	0.000	0.080	0.000	0.198	0.178	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	173	195	72	0	0	167	0	643	68	0
N.S.	1	1.13	0.42	0.00	0.00	0.97	0.00	3.72	0.39	0.00
time (sec)	N/A	0.804	0.200	0.000	0.000	0.082	0.000	0.177	0.206	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	150	63	0	0	121	0	372	42	0
N.S.	1	1.09	0.46	0.00	0.00	0.88	0.00	2.70	0.30	0.00
time (sec)	N/A	0.633	0.240	0.000	0.000	0.080	0.000	0.147	0.182	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	63	0	0	90	0	189	21	0
N.S.	1	1.00	0.60	0.00	0.00	0.86	0.00	1.80	0.20	0.00
time (sec)	N/A	0.478	0.194	0.000	0.000	0.112	0.000	0.133	0.182	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	A	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	64	0	56	23	0
N.S.	1	1.00	0.88	0.00	0.00	0.89	0.00	0.78	0.32	0.00
time (sec)	N/A	0.363	0.155	0.000	0.000	0.074	0.000	0.119	0.180	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	0	0	90	0	0	35	0
N.S.	1	1.00	0.77	0.00	0.00	0.93	0.00	0.00	0.36	0.00
time (sec)	N/A	0.470	0.272	0.000	0.000	0.074	0.000	0.000	0.158	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	92	0	0	140	0	0	52	0
N.S.	1	1.03	0.71	0.00	0.00	1.08	0.00	0.00	0.40	0.00
time (sec)	N/A	0.587	0.453	0.000	0.000	0.106	0.000	0.000	0.199	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	171	118	0	0	230	0	0	69	0
N.S.	1	1.04	0.72	0.00	0.00	1.39	0.00	0.00	0.42	0.00
time (sec)	N/A	0.722	0.402	0.000	0.000	0.082	0.000	0.000	0.164	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F(-1)</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	208	144	0	0	321	0	0	86	0
N.S.	1	1.04	0.72	0.00	0.00	1.60	0.00	0.00	0.43	0.00
time (sec)	N/A	0.861	0.532	0.000	0.000	0.078	0.000	0.000	0.173	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	<b>F</b>	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	187	24	78	79	82	138	80	95	89
N.S.	1	1.24	0.16	0.52	0.52	0.54	0.91	0.53	0.63	0.59
time (sec)	N/A	0.901	0.617	0.105	0.028	0.084	124.397	0.111	0.176	0.083

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	0	0	117	0	0	44	0
N.S.	1	1.00	0.89	0.00	0.00	1.65	0.00	0.00	0.62	0.00
time (sec)	N/A	0.345	0.284	0.000	0.000	0.081	0.000	0.000	0.176	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	A	<b>F(-1)</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	78	0	0	133	0	0	47	0
N.S.	1	1.00	0.80	0.00	0.00	1.36	0.00	0.00	0.48	0.00
time (sec)	N/A	0.487	0.318	0.000	0.000	0.084	0.000	0.000	0.190	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	124	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.75	0.00
time (sec)	N/A	0.382	0.061	0.000	0.000	0.000	0.000	0.000	0.171	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	102	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.44	0.00
time (sec)	N/A	0.367	0.054	0.000	0.000	0.000	0.000	0.000	0.174	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	0	0	0	80	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.13	0.00
time (sec)	N/A	0.365	0.054	0.000	0.000	0.000	0.000	0.000	0.175	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	65	0	0	58	0
N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	0.87	0.00
time (sec)	N/A	0.313	0.066	0.000	0.000	0.069	0.000	0.000	0.162	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	67	0	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	0.35	0.00
time (sec)	N/A	0.320	0.054	0.000	0.000	0.093	0.000	0.000	0.180	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	35	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.48	0.00
time (sec)	N/A	0.365	0.054	0.000	0.000	0.000	0.000	0.000	0.176	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	0	0	0	0	0	46	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.63	0.00
time (sec)	N/A	0.371	0.055	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	65	0	0	58	0
N.S.	1	1.00	1.00	0.00	0.00	0.97	0.00	0.00	0.87	0.00
time (sec)	N/A	0.307	0.001	0.000	0.000	0.074	0.000	0.000	0.199	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	68	0	0	63	0
N.S.	1	1.00	1.00	0.00	0.00	0.94	0.00	0.00	0.88	0.00
time (sec)	N/A	0.368	0.050	0.000	0.000	0.080	0.000	0.000	0.170	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	34	38	60	38	60	898	38	38
N.S.	1	1.00	0.71	0.79	1.25	0.79	1.25	18.71	0.79	0.79
time (sec)	N/A	0.320	0.019	0.000	0.048	0.079	0.057	0.135	0.174	0.002

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	135	56	80	117	74	116	2068	80	80
N.S.	1	1.69	0.70	1.00	1.46	0.92	1.45	25.85	1.00	1.00
time (sec)	N/A	0.498	0.039	0.000	0.057	0.068	0.070	0.147	0.183	0.002

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	229	84	138	194	122	190	4188	138	138
N.S.	1	1.94	0.71	1.17	1.64	1.03	1.61	35.49	1.17	1.17
time (sec)	N/A	0.686	0.260	0.046	0.052	0.071	0.088	0.183	0.161	22.575

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	348	117	212	291	182	284	7630	212	212
N.S.	1	2.15	0.72	1.31	1.80	1.12	1.75	47.10	1.31	1.31
time (sec)	N/A	0.964	0.704	0.053	0.081	0.073	0.110	0.223	0.180	22.645

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	61	334	123	126	99	0	448	0
N.S.	1	1.00	0.53	2.88	1.06	1.09	0.85	0.00	3.86	0.00
time (sec)	N/A	0.600	0.131	0.588	0.158	0.074	9.028	0.000	0.172	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	397	121	178	196	121	236	202	182	175
N.S.	1	1.75	0.53	0.78	0.86	0.53	1.04	0.89	0.80	0.77
time (sec)	N/A	1.323	0.402	0.079	0.056	0.070	0.106	0.118	0.174	22.452



Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	318	130	143	164	102	196	163	143	126
N.S.	1	1.70	0.70	0.76	0.88	0.55	1.05	0.87	0.76	0.67
time (sec)	N/A	1.103	0.339	0.054	0.052	0.069	0.094	0.112	0.174	22.563

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	184	96	102	132	78	148	123	104	117
N.S.	1	1.34	0.70	0.74	0.96	0.57	1.08	0.90	0.76	0.85
time (sec)	N/A	0.754	0.269	0.049	0.046	0.068	0.080	0.112	0.167	0.107

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	86	41	68	103	57	104	87	69	66
N.S.	1	1.08	0.51	0.85	1.29	0.71	1.30	1.09	0.86	0.82
time (sec)	N/A	0.517	0.255	0.037	0.044	0.114	0.077	0.124	0.180	22.535

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	52	96	69	50	70	95	51	69
N.S.	1	1.00	0.51	0.94	0.68	0.49	0.69	0.93	0.50	0.68
time (sec)	N/A	0.570	0.267	0.046	0.094	0.078	3.218	0.118	0.167	22.872

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	54	88	61	56	99	92	70	72
N.S.	1	1.00	0.57	0.94	0.65	0.60	1.05	0.98	0.74	0.77
time (sec)	N/A	0.568	0.287	0.055	0.093	0.070	1.372	0.135	0.163	22.661

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	68	112	64	70	56	125	101	100
N.S.	1	1.00	0.52	0.86	0.49	0.54	0.43	0.96	0.78	0.77
time (sec)	N/A	0.643	0.273	0.066	0.127	0.074	1.262	0.113	0.203	22.639

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	81	167	63	83	53	183	143	142
N.S.	1	1.00	0.41	0.84	0.32	0.42	0.27	0.92	0.72	0.72
time (sec)	N/A	0.814	0.322	0.075	0.128	0.064	1.146	0.108	0.163	22.643

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	<b>F</b>	<b>F</b>	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	86	433	123	161	0	0	359	0
N.S.	1	1.00	0.62	3.12	0.88	1.16	0.00	0.00	2.58	0.00
time (sec)	N/A	0.977	0.344	0.563	0.168	0.074	0.000	0.000	0.193	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	414	159	250	328	228	323	9584	249	249
N.S.	1	1.89	0.73	1.14	1.50	1.04	1.47	43.76	1.14	1.14
time (sec)	N/A	1.635	0.447	0.079	0.067	0.071	0.112	0.225	0.170	22.647

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	328	121	197	262	178	260	6582	196	196
N.S.	1	1.93	0.71	1.16	1.54	1.05	1.53	38.72	1.15	1.15
time (sec)	N/A	1.447	0.425	0.066	0.056	0.069	0.102	0.213	0.160	22.517

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	242	91	144	196	132	199	4114	143	143
N.S.	1	1.88	0.71	1.12	1.52	1.02	1.54	31.89	1.11	1.11
time (sec)	N/A	1.111	0.356	0.055	0.052	0.072	0.083	0.168	0.178	23.112

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	58	93	134	85	134	2264	92	92
N.S.	1	1.12	0.68	1.09	1.58	1.00	1.58	26.64	1.08	1.08
time (sec)	N/A	0.591	0.269	0.045	0.050	0.100	0.073	0.152	0.171	22.677

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	54	118	87	75	0	0	80	80
N.S.	1	1.00	0.56	1.23	0.91	0.78	0.00	0.00	0.83	0.83
time (sec)	N/A	0.787	0.389	0.066	0.118	0.070	0.000	0.000	0.174	22.676

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	58	153	68	83	0	0	86	89
N.S.	1	1.00	0.68	1.80	0.80	0.98	0.00	0.00	1.01	1.05
time (sec)	N/A	0.831	0.420	0.086	0.118	0.071	0.000	0.000	0.187	22.690

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	76	208	74	89	0	0	108	133
N.S.	1	1.00	0.56	1.53	0.54	0.65	0.00	0.00	0.79	0.98
time (sec)	N/A	1.042	0.389	0.100	0.153	0.068	0.000	0.000	0.176	22.689

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	116	294	85	137	0	0	173	202
N.S.	1	1.00	0.53	1.35	0.39	0.63	0.00	0.00	0.80	0.93
time (sec)	N/A	1.254	0.444	0.112	0.163	0.068	0.000	0.000	0.193	22.879

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	156	386	93	186	0	0	244	258
N.S.	1	1.00	0.49	1.20	0.29	0.58	0.00	0.00	0.76	0.80
time (sec)	N/A	1.479	0.521	0.115	0.184	0.073	0.000	0.000	0.165	22.958

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	805	754	458	900	894	544	1445	1096	1064	803
N.S.	1	0.94	0.57	1.12	1.11	0.68	1.80	1.36	1.32	1.00
time (sec)	N/A	2.204	7.242	0.152	0.089	0.074	0.273	0.121	0.173	23.286

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	495	320	560	599	354	899	674	642	537
N.S.	1	0.99	0.64	1.12	1.20	0.71	1.81	1.35	1.29	1.08
time (sec)	N/A	1.586	2.666	0.111	0.082	0.074	0.196	0.135	0.171	23.020

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	271	191	276	344	197	447	331	300	264
N.S.	1	0.99	0.69	1.00	1.25	0.72	1.63	1.20	1.09	0.96
time (sec)	N/A	0.969	0.942	0.078	0.057	0.076	0.129	0.117	0.177	22.890

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	111	50	99	149	83	158	132	109	120
N.S.	1	1.09	0.49	0.97	1.46	0.81	1.55	1.29	1.07	1.18
time (sec)	N/A	0.647	0.420	0.049	0.058	0.070	0.083	0.133	0.192	0.115

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	175	489	0	235	0	546	366	0
N.S.	1	1.00	0.63	1.77	0.00	0.85	0.00	1.97	1.32	0.00
time (sec)	N/A	0.998	1.229	0.113	0.000	0.073	0.000	0.131	0.167	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	163	406	0	353	0	2861	0	0
N.S.	1	1.00	0.63	1.57	0.00	1.37	0.00	11.09	0.00	0.00
time (sec)	N/A	1.000	1.693	0.115	0.000	0.085	0.000	0.162	0.180	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	A	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	267	418	0	550	0	1995	0	0
N.S.	1	1.00	0.91	1.42	0.00	1.87	0.00	6.79	0.00	0.00
time (sec)	N/A	1.074	2.448	0.118	0.000	0.087	0.000	0.142	0.226	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	B	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	389	511	0	793	0	3178	0	0
N.S.	1	1.00	0.98	1.29	0.00	2.00	0.00	8.03	0.00	0.00
time (sec)	N/A	1.302	2.906	0.164	0.000	0.082	0.000	0.154	0.213	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	<b>F</b>	B	<b>F</b>	B	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	669	596	0	1084	0	16988	0	0
N.S.	1	1.00	1.20	1.07	0.00	1.95	0.00	30.50	0.00	0.00
time (sec)	N/A	1.666	2.998	0.261	0.000	0.096	0.000	0.244	0.236	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	<b>F(-1)</b>	<b>F</b>	B	B
verified	N/A	Yes	Yes	<b>No</b>	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	25	192	42	32	0	0	26	25
N.S.	1	1.00	1.04	8.00	1.75	1.33	0.00	0.00	1.08	1.04
time (sec)	N/A	0.467	0.315	0.027	0.137	0.095	0.000	0.000	0.169	22.966

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F(-2)</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	25	42	25	0	0	25	23
N.S.	1	1.00	1.05	1.14	1.91	1.14	0.00	0.00	1.14	1.05
time (sec)	N/A	0.440	0.196	83.951	0.140	0.078	0.000	0.000	0.187	23.204

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F(-2)</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	25	42	25	0	0	25	23
N.S.	1	1.00	1.05	1.14	1.91	1.14	0.00	0.00	1.14	1.05
time (sec)	N/A	0.378	0.176	48.674	0.140	0.080	0.000	0.000	0.162	24.071

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F(-2)</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	21	23	38	23	0	0	23	21
N.S.	1	1.00	1.05	1.15	1.90	1.15	0.00	0.00	1.15	1.05
time (sec)	N/A	0.315	0.167	26.037	0.131	0.084	0.000	0.000	0.167	23.635

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F(-2)</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	36	22	0	0	22	20
N.S.	1	1.00	1.00	1.16	1.89	1.16	0.00	0.00	1.16	1.05
time (sec)	N/A	0.436	0.040	17.519	0.135	0.083	0.000	0.000	0.182	23.026

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	<b>F</b>	<b>F(-2)</b>	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	25	39	25	0	0	25	23
N.S.	1	1.00	1.05	1.14	1.77	1.14	0.00	0.00	1.14	1.05
time (sec)	N/A	0.441	0.322	17.518	0.134	0.079	0.000	0.000	0.171	22.888



Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	25	39	25	0	0	25	23
N.S.	1	1.00	1.05	1.14	1.77	1.14	0.00	0.00	1.14	1.05
time (sec)	N/A	0.442	0.315	17.685	0.162	0.084	0.000	0.000	0.186	22.948

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	109	45	43	60	43	51	43	44	45
N.S.	1	1.20	0.49	0.47	0.66	0.47	0.56	0.47	0.48	0.49
time (sec)	N/A	0.658	0.191	0.067	0.031	0.069	0.057	0.114	0.164	22.773

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	84	37	35	48	35	42	35	36	37
N.S.	1	1.17	0.51	0.49	0.67	0.49	0.58	0.49	0.50	0.51
time (sec)	N/A	0.528	0.161	0.042	0.055	0.066	0.054	0.113	0.170	22.785

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	59	29	27	36	27	34	27	28	29
N.S.	1	1.11	0.55	0.51	0.68	0.51	0.64	0.51	0.53	0.55
time (sec)	N/A	0.421	0.150	0.034	0.032	0.065	0.050	0.114	0.179	23.933

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	21	19	24	19	26	19	20	18
N.S.	1	1.00	0.62	0.56	0.71	0.56	0.76	0.56	0.59	0.53
time (sec)	N/A	0.313	0.118	0.028	0.035	0.066	0.044	0.119	0.160	0.046

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	14	14	14	14	14	15	13
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.88	0.88	0.94	0.81
time (sec)	N/A	0.240	0.036	0.026	0.031	0.061	0.037	0.111	0.185	23.450

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	<b>F</b>	A	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	39	27	57	10	10	0	10	16	0
N.S.	1	1.44	1.00	2.11	0.37	0.37	0.00	0.37	0.59	0.00
time (sec)	N/A	0.364	0.137	0.030	0.092	0.068	0.000	0.117	0.171	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	<b>F</b>	A	<b>F</b>	<b>F(-1)</b>
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	60	47	116	13	29	0	29	37	0
N.S.	1	1.25	0.98	2.42	0.27	0.60	0.00	0.60	0.77	0.00
time (sec)	N/A	0.445	0.164	0.036	0.089	0.068	0.000	0.113	0.165	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	56	155	15	38	0	46	55	0
N.S.	1	1.18	0.79	2.18	0.21	0.54	0.00	0.65	0.77	0.00
time (sec)	N/A	0.538	0.190	0.049	0.091	0.064	0.000	0.111	0.177	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	108	64	189	15	46	0	63	73	0
N.S.	1	1.17	0.70	2.05	0.16	0.50	0.00	0.68	0.79	0.00
time (sec)	N/A	0.645	0.200	0.063	0.088	0.067	0.000	0.109	0.162	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [78] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	24	0.042
2	A	1	1	1.00	24	0.042
3	B	2	2	4.33	19	0.105
4	A	1	1	1.00	24	0.042
5	A	1	1	1.00	24	0.042
6	A	1	1	1.00	24	0.042
7	A	1	1	1.00	24	0.042
8	A	1	1	1.26	29	0.034
9	A	1	1	1.20	27	0.037
10	A	2	2	1.83	20	0.100
11	A	1	1	1.27	29	0.034
12	A	1	1	1.26	29	0.034
13	A	1	1	1.26	29	0.034
14	A	1	1	1.43	31	0.032
15	A	2	2	1.60	20	0.100
16	A	2	2	1.24	18	0.111
17	A	2	2	1.00	20	0.100
18	A	3	3	1.01	20	0.150
19	A	2	2	1.51	20	0.100
20	A	2	2	1.18	18	0.111
21	A	2	2	1.00	20	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	20	0.150
23	B	2	2	2.20	22	0.091
24	A	2	2	1.69	20	0.100
25	A	2	2	1.00	22	0.091
26	A	3	3	1.29	22	0.136
27	A	2	2	1.00	20	0.100
28	A	2	2	1.00	18	0.111
29	A	2	2	1.00	17	0.118
30	A	2	2	1.00	20	0.100
31	A	2	2	1.00	20	0.100
32	A	2	2	1.00	23	0.087
33	A	2	2	1.00	23	0.087
34	A	2	2	1.00	21	0.095
35	A	2	2	1.00	20	0.100
36	A	2	2	1.00	23	0.087
37	A	2	2	1.00	23	0.087
38	A	5	5	1.21	17	0.294
39	A	4	4	1.18	17	0.235
40	A	3	3	1.13	17	0.176
41	A	2	2	1.00	15	0.133
42	A	1	1	1.00	9	0.111
43	A	1	1	1.00	17	0.059
44	A	2	2	1.00	17	0.118
45	A	3	3	0.97	17	0.176
46	A	4	4	0.99	17	0.235
47	A	5	5	1.01	17	0.294
48	A	6	6	1.21	48	0.125
49	A	5	5	1.18	37	0.135
50	A	4	4	1.13	26	0.154
51	A	3	3	1.00	28	0.107
52	A	4	4	0.97	39	0.103
53	A	5	5	0.99	50	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	6	1.01	61	0.098
55	B	2	2	4.10	16	0.125
56	B	2	2	2.47	21	0.095
57	A	1	1	1.00	7	0.143
58	A	1	1	1.00	7	0.143
59	A	1	1	1.00	7	0.143
60	A	7	6	1.21	13	0.462
61	A	6	5	1.17	13	0.385
62	A	5	4	1.11	13	0.308
63	A	4	3	1.00	13	0.231
64	A	3	2	1.00	13	0.154
65	A	4	3	1.00	13	0.231
66	A	5	4	1.01	13	0.308
67	A	6	5	1.02	13	0.385
68	A	7	6	1.02	13	0.462
69	A	7	6	1.15	19	0.316
70	A	6	5	1.13	19	0.263
71	A	5	4	1.09	19	0.211
72	A	4	3	1.00	19	0.158
73	A	3	2	1.00	19	0.105
74	A	4	3	1.00	19	0.158
75	A	5	4	1.03	19	0.211
76	A	6	5	1.04	19	0.263
77	A	7	6	1.04	19	0.316
78	A	10	9	1.24	12	0.750
79	A	1	1	1.00	19	0.053
80	A	2	2	1.00	21	0.095
81	A	2	2	1.00	50	0.040
82	A	2	2	1.00	39	0.051
83	A	2	2	1.00	28	0.071
84	A	1	1	1.00	17	0.059
85	A	1	1	1.00	19	0.053

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	30	0.067
87	A	2	2	1.00	41	0.049
88	A	1	1	1.00	17	0.059
89	A	2	2	1.00	19	0.105
90	A	2	2	1.00	15	0.133
91	A	2	2	1.69	20	0.100
92	A	2	2	1.94	25	0.080
93	B	2	2	2.15	30	0.067
94	A	2	2	1.00	21	0.095
95	A	2	2	1.75	21	0.095
96	A	2	2	1.70	21	0.095
97	A	2	2	1.34	19	0.105
98	A	4	4	1.08	18	0.222
99	A	2	2	1.00	21	0.095
100	A	2	2	1.00	21	0.095
101	A	2	2	1.00	21	0.095
102	A	2	2	1.00	21	0.095
103	A	2	2	1.00	22	0.091
104	A	2	2	1.89	22	0.091
105	A	2	2	1.93	22	0.091
106	A	2	2	1.88	20	0.100
107	A	4	4	1.12	19	0.211
108	A	2	2	1.00	22	0.091
109	A	2	2	1.00	22	0.091
110	A	2	2	1.00	22	0.091
111	A	2	2	1.00	22	0.091
112	A	2	2	1.00	22	0.091
113	A	2	2	0.94	25	0.080
114	A	2	2	0.99	25	0.080
115	A	2	2	0.99	23	0.087
116	A	5	5	1.09	18	0.278
117	A	2	2	1.00	25	0.080

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	25	0.080
119	A	2	2	1.00	25	0.080
120	A	2	2	1.00	25	0.080
121	A	2	2	1.00	25	0.080
122	A	1	1	1.00	39	0.026
123	A	1	1	1.00	38	0.026
124	A	1	1	1.00	36	0.028
125	A	1	1	1.00	35	0.029
126	A	1	1	1.00	35	0.029
127	A	1	1	1.00	38	0.026
128	A	1	1	1.00	38	0.026
129	A	5	5	1.20	15	0.333
130	A	4	4	1.17	15	0.267
131	A	3	3	1.11	15	0.200
132	A	2	2	1.00	13	0.154
133	A	1	1	1.00	11	0.091
134	A	2	2	1.44	15	0.133
135	A	3	3	1.25	15	0.200
136	A	4	4	1.18	15	0.267
137	A	5	5	1.17	15	0.333



# CHAPTER 3

## LISTING OF INTEGRALS

3.1	$\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx \dots\dots\dots$	77
3.2	$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx \dots\dots\dots$	82
3.3	$\int F^{c(a+bx)}(d + bcdx \log(F)) dx \dots\dots\dots$	87
3.4	$\int F^{c(a+\frac{b}{x})}\left(d - \frac{bcd \log(F)}{x}\right) dx \dots\dots\dots$	93
3.5	$\int F^{c(a+\frac{b}{x^2})}\left(d - \frac{2bcd \log(F)}{x^2}\right) dx \dots\dots\dots$	98
3.6	$\int F^{c(a+\frac{b}{x^3})}\left(d - \frac{3bcd \log(F)}{x^3}\right) dx \dots\dots\dots$	103
3.7	$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx \dots\dots\dots$	108
3.8	$\int F^{c(a+bx^3)}(d + ex^2 + 3bcdx^3 \log(F)) dx \dots\dots\dots$	113
3.9	$\int F^{c(a+bx^2)}(d + ex + 2bcdx^2 \log(F)) dx \dots\dots\dots$	118
3.10	$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx \dots\dots\dots$	123
3.11	$\int F^{c(a+\frac{b}{x})}\left(d + \frac{e}{x^2} - \frac{bcd \log(F)}{x}\right) dx \dots\dots\dots$	129
3.12	$\int F^{c(a+\frac{b}{x^2})}\left(d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2}\right) dx \dots\dots\dots$	134
3.13	$\int F^{c(a+\frac{b}{x^3})}\left(d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3}\right) dx \dots\dots\dots$	139
3.14	$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx \dots\dots\dots$	144
3.15	$\int F^{c(a+bx)}(d - ex^2)^2 dx \dots\dots\dots$	150
3.16	$\int F^{c(a+bx)}(d - ex^2) dx \dots\dots\dots$	157
3.17	$\int \frac{F^{c(a+bx)}}{d - ex^2} dx \dots\dots\dots$	163
3.18	$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx \dots\dots\dots$	169
3.19	$\int F^{c(a+bx)}(d - ex^3)^2 dx \dots\dots\dots$	176
3.20	$\int F^{c(a+bx)}(d - ex^3) dx \dots\dots\dots$	184
3.21	$\int \frac{F^{c(a+bx)}}{d - ex^3} dx \dots\dots\dots$	190
3.22	$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx \dots\dots\dots$	196
3.23	$\int F^{c(a+bx)}(d + ex + fx^2)^2 dx \dots\dots\dots$	205

3.24	$\int F^{c(a+bx)}(d+ex+fx^2) dx$	213
3.25	$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx$	219
3.26	$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$	225
3.27	$\int \frac{e^{a+bx}x^2}{c+dx^2} dx$	233
3.28	$\int \frac{e^{a+bx}x}{c+dx^2} dx$	238
3.29	$\int \frac{e^{a+bx}}{c+dx^2} dx$	243
3.30	$\int \frac{e^{a+bx}}{x(c+dx^2)} dx$	248
3.31	$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$	253
3.32	$\int \frac{e^{d+ex}x^3}{a+bx+cx^2} dx$	259
3.33	$\int \frac{e^{d+ex}x^2}{a+bx+cx^2} dx$	265
3.34	$\int \frac{e^{d+ex}x}{a+bx+cx^2} dx$	271
3.35	$\int \frac{e^{d+ex}}{a+bx+cx^2} dx$	277
3.36	$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$	283
3.37	$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$	289
3.38	$\int F^{c(a+bx)}(d+ex)^4 dx$	295
3.39	$\int F^{c(a+bx)}(d+ex)^3 dx$	303
3.40	$\int F^{c(a+bx)}(d+ex)^2 dx$	310
3.41	$\int F^{c(a+bx)}(d+ex) dx$	317
3.42	$\int F^{c(a+bx)} dx$	323
3.43	$\int \frac{F^{c(a+bx)}}{d+ex} dx$	328
3.44	$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$	332
3.45	$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$	337
3.46	$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$	342
3.47	$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$	348
3.48	$\int F^{c(a+bx)}(d^4+4d^3ex+6d^2e^2x^2+4de^3x^3+e^4x^4) dx$	355
3.49	$\int F^{c(a+bx)}(d^3+3d^2ex+3de^2x^2+e^3x^3) dx$	364
3.50	$\int F^{c(a+bx)}(d^2+2dex+e^2x^2) dx$	372
3.51	$\int \frac{F^{c(a+bx)}}{d^2+2dex+e^2x^2} dx$	379
3.52	$\int \frac{F^{c(a+bx)}}{d^3+3d^2ex+3de^2x^2+e^3x^3} dx$	384
3.53	$\int \frac{F^{c(a+bx)}}{d^4+4d^3ex+6d^2e^2x^2+4de^3x^3+e^4x^4} dx$	390
3.54	$\int \frac{F^{c(a+bx)}}{d^5+5d^4ex+10d^3e^2x^2+10d^2e^3x^3+5de^4x^4+e^5x^5} dx$	396
3.55	$\int F^{a+bx}(c+bcx \log(F)) dx$	403
3.56	$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx$	409
3.57	$\int F^{2+5x} dx$	415
3.58	$\int F^{a+bx} dx$	420

3.59	$\int 10^{2+5x} dx$	425
3.60	$\int F^{a+bx} x^{7/2} dx$	430
3.61	$\int F^{a+bx} x^{5/2} dx$	436
3.62	$\int F^{a+bx} x^{3/2} dx$	442
3.63	$\int F^{a+bx} \sqrt{x} dx$	448
3.64	$\int \frac{F^{a+bx}}{\sqrt{x}} dx$	453
3.65	$\int \frac{F^{a+bx}}{x^{3/2}} dx$	458
3.66	$\int \frac{F^{a+bx}}{x^{5/2}} dx$	463
3.67	$\int \frac{F^{a+bx}}{x^{7/2}} dx$	468
3.68	$\int \frac{F^{a+bx}}{x^{9/2}} dx$	473
3.69	$\int F^{c(a+bx)} (d+ex)^{7/2} dx$	479
3.70	$\int F^{c(a+bx)} (d+ex)^{5/2} dx$	487
3.71	$\int F^{c(a+bx)} (d+ex)^{3/2} dx$	493
3.72	$\int F^{c(a+bx)} \sqrt{d+ex} dx$	499
3.73	$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$	504
3.74	$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$	509
3.75	$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$	514
3.76	$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$	520
3.77	$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$	526
3.78	$\int e^{-bx} x^{13/2} dx$	533
3.79	$\int F^{c(a+bx)} (d+ex)^{4/3} dx$	547
3.80	$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx$	552
3.81	$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$	557
3.82	$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$	563
3.83	$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^m dx$	568
3.84	$\int F^{c(a+bx)} (d+ex)^m dx$	573
3.85	$\int F^{c(a+bx)} (d+ex)^{-m} dx$	578
3.86	$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^{-m} dx$	583
3.87	$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$	588
3.88	$\int F^{c(a+bx)} (d+ex)^m dx$	593
3.89	$\int F^{c(a+bx)} ((d+ex)^n)^m dx$	598
3.90	$\int F^{c(a+bx)} (d+ex) dx$	603
3.91	$\int F^{c(a+bx)} (d+ex+fx^2) dx$	609
3.92	$\int F^{c(a+bx)} (d+ex+fx^2+gx^3) dx$	615
3.93	$\int F^{c(a+bx)} (d+ex+fx^2+gx^3+hx^4) dx$	622
3.94	$\int e^{-a-bx} x^m (a+bx)^3 dx$	630
3.95	$\int e^{-a-bx} x^3 (a+bx)^3 dx$	636

3.96	$\int e^{-a-bx} x^2 (a+bx)^3 dx$	644
3.97	$\int e^{-a-bx} x (a+bx)^3 dx$	651
3.98	$\int e^{-a-bx} (a+bx)^3 dx$	657
3.99	$\int \frac{e^{-a-bx} (a+bx)^3}{x} dx$	663
3.100	$\int \frac{e^{-a-bx} (a+bx)^3}{x^2} dx$	668
3.101	$\int \frac{e^{-a-bx} (a+bx)^3}{x^3} dx$	674
3.102	$\int \frac{e^{-a-bx} (a+bx)^3}{x^4} dx$	680
3.103	$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx$	686
3.104	$\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx$	692
3.105	$\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx$	700
3.106	$\int F^{a+b(c+dx)} x (e+fx)^2 dx$	708
3.107	$\int F^{a+b(c+dx)} (e+fx)^2 dx$	715
3.108	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x} dx$	722
3.109	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^2} dx$	728
3.110	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^3} dx$	734
3.111	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^4} dx$	740
3.112	$\int \frac{F^{a+b(c+dx)} (e+fx)^2}{x^5} dx$	747
3.113	$\int e^{-a-bx} (a+bx)^4 (c+dx)^3 dx$	754
3.114	$\int e^{-a-bx} (a+bx)^4 (c+dx)^2 dx$	767
3.115	$\int e^{-a-bx} (a+bx)^4 (c+dx) dx$	779
3.116	$\int e^{-a-bx} (a+bx)^4 dx$	788
3.117	$\int \frac{e^{-a-bx} (a+bx)^4}{c+dx} dx$	795
3.118	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^2} dx$	802
3.119	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^3} dx$	810
3.120	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^4} dx$	818
3.121	$\int \frac{e^{-a-bx} (a+bx)^4}{(c+dx)^5} dx$	828
3.122	$\int F^{c(a+bx)} x^m \log^n(dx) (e+en+e(1+m+bcx \log(F)) \log(dx)) dx$	838
3.123	$\int F^{c(a+bx)} x^2 \log^n(dx) (e+en+e(3+bcx \log(F)) \log(dx)) dx$	843
3.124	$\int F^{c(a+bx)} x \log^n(dx) (e+en+e(2+bcx \log(F)) \log(dx)) dx$	848
3.125	$\int F^{c(a+bx)} \log^n(dx) (e+en+e(1+bcx \log(F)) \log(dx)) dx$	853
3.126	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+bce x \log(F) \log(dx))}{x} dx$	858
3.127	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+e(-1+bcx \log(F)) \log(dx))}{x^2} dx$	863
3.128	$\int \frac{F^{c(a+bx)} \log^n(dx) (e+en+e(-2+bcx \log(F)) \log(dx))}{x^3} dx$	869
3.129	$\int \sqrt{e^{a+bx}} x^4 dx$	875
3.130	$\int \sqrt{e^{a+bx}} x^3 dx$	881
3.131	$\int \sqrt{e^{a+bx}} x^2 dx$	886

---

3.132	$\int \sqrt{e^{a+bx}} x dx$	891
3.133	$\int \sqrt{e^{a+bx}} dx$	896
3.134	$\int \frac{\sqrt{e^{a+bx}}}{x} dx$	901
3.135	$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$	906
3.136	$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx$	911
3.137	$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx$	916

### 3.1 $\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx$

Optimal result	77
Mathematica [A] (verified)	77
Rubi [A] (verified)	78
Maple [A] (warning: unable to verify)	79
Fricas [A] (verification not implemented)	79
Sympy [A] (verification not implemented)	80
Maxima [C] (verification not implemented)	80
Giac [A] (verification not implemented)	80
Mupad [B] (verification not implemented)	81
Reduce [B] (verification not implemented)	81

#### Optimal result

Integrand size = 24, antiderivative size = 14

$$\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx = dF^{c(a+bx^3)}x$$

output

```
d*F^(c*(b*x^3+a))*x
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx = dF^{c(a+bx^3)}x$$

input

```
Integrate[F^(c*(a + b*x^3))*(d + 3*b*c*d*x^3*Log[F]),x]
```

output

```
d*F^(c*(a + b*x^3))*x
```

**Rubi [A] (verified)**

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx^3)} (3bcdx^3 \log(F) + d) dx$$

$$\downarrow 2726$$

$$dx F^{c(a+bx^3)}$$

input `Int[F^(c*(a + b*x^3))*(d + 3*b*c*d*x^3*Log[F]),x]`

output `d*F^(c*(a + b*x^3))*x`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (warning: unable to verify)**

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
gospers	$d F^{c(b x^3+a)} x$
risch	$d F^{c(b x^3+a)} x$
parallelrisch	$d F^{c(b x^3+a)} x$
norman	$dx e^{c(b x^3+a) \ln(F)}$
meijerg	$\frac{F^{ac} d \left( \frac{2x(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}} \pi \sqrt{3}}{3\Gamma\left(\frac{2}{3}\right) (-\ln(F)bcx^3)^{\frac{1}{3}}} - \frac{x(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}} \Gamma\left(\frac{1}{3}, -\ln(F)bcx^3\right)}{(-\ln(F)bcx^3)^{\frac{1}{3}}} \right)}{3(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}}} - \frac{F^{ac} d \left( -\frac{2x(-bc)^{\frac{4}{3}} \ln(F)^{\frac{1}{3}} \pi \sqrt{3}}{9bc\Gamma\left(\frac{2}{3}\right) (-\ln(F)bcx^3)^{\frac{1}{3}}} + \frac{x(-bc)^{\frac{4}{3}} \ln(F)^{\frac{1}{3}} e^{\frac{1}{3} \ln(F)bcx^3}}{bc} \right)}{(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}}}$

input

```
int(F^(c*(b*x^3+a))*(d+3*b*c*d*x^3*ln(F)),x,method=_RETURNVERBOSE)
```

output

```
d*F^(c*(b*x^3+a))*x
```

**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int F^{c(a+bx^3)} (d + 3bcdx^3 \log(F)) dx = F^{bcx^3+ac} dx$$

input

```
integrate(F^(c*(b*x^3+a))*(d+3*b*c*d*x^3*log(F)),x, algorithm="fricas")
```

output

```
F^(b*c*x^3 + a*c)*d*x
```



**Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx^3)} (d + 3bcdx^3 \log(F)) dx = F^{c(a+bx^3)} dx$$

input `integrate(F**(c*(b*x**3+a))*(d+3*b*c*d*x**3*ln(F)),x)`

output `F**(c*(a + b*x**3))*d*x`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.93

$$\int F^{c(a+bx^3)} (d + 3bcdx^3 \log(F)) dx = -\frac{F^{ac}bcdx^4\Gamma(\frac{4}{3}, -bcx^3 \log(F)) \log(F)}{(-bcx^3 \log(F))^{\frac{4}{3}}} - \frac{F^{ac}dx\Gamma(\frac{1}{3}, -bcx^3 \log(F))}{3(-bcx^3 \log(F))^{\frac{1}{3}}}$$

input `integrate(F^(c*(b*x^3+a))*(d+3*b*c*d*x^3*log(F)),x, algorithm="maxima")`

output `-F^(a*c)*b*c*d*x^4*gamma(4/3, -b*c*x^3*log(F))*log(F)/(-b*c*x^3*log(F))^(4/3) - 1/3*F^(a*c)*d*x*gamma(1/3, -b*c*x^3*log(F))/(-b*c*x^3*log(F))^(1/3)`

**Giac [A] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx^3)} (d + 3bcdx^3 \log(F)) dx = F^{bcx^3} F^{ac} dx$$

input `integrate(F^(c*(b*x^3+a))*(d+3*b*c*d*x^3*log(F)),x, algorithm="giac")`

output  $F^{(b*c*x^3)*F^{(a*c)}*d*x$

### Mupad [B] (verification not implemented)

Time = 23.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx = F^{c(bx^3+a)} dx$$

input  $\text{int}(F^{(c*(a + b*x^3))}*(d + 3*b*c*d*x^3*\log(F)),x)$

output  $F^{(c*(a + b*x^3))*d*x$

### Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int F^{c(a+bx^3)}(d + 3bcdx^3 \log(F)) dx = f^{bcx^3+ac} dx$$

input  $\text{int}(F^{(c*(b*x^3+a))}*(d+3*b*c*d*x^3*\log(F)),x)$

output  $f^{**}(a*c + b*c*x**3)*d*x$

## 3.2 $\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx$

Optimal result	82
Mathematica [A] (verified)	82
Rubi [A] (verified)	83
Maple [A] (warning: unable to verify)	84
Fricas [A] (verification not implemented)	84
Sympy [A] (verification not implemented)	85
Maxima [C] (verification not implemented)	85
Giac [A] (verification not implemented)	86
Mupad [B] (verification not implemented)	86
Reduce [B] (verification not implemented)	86

### Optimal result

Integrand size = 24, antiderivative size = 14

$$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx = dF^{c(a+bx^2)}x$$

output

```
d*F^((b*x^2+a)*c)*x
```

### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx = dF^{c(a+bx^2)}x$$

input

```
Integrate[F^(c*(a + b*x^2))*(d + 2*b*c*d*x^2*Log[F]),x]
```

output

```
d*F^(c*(a + b*x^2))*x
```

**Rubi [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx^2)} (2bcdx^2 \log(F) + d) dx$$

$$\downarrow 2726$$

$$dx F^{c(a+bx^2)}$$

input `Int[F^(c*(a + b*x^2))*(d + 2*b*c*d*x^2*Log[F]),x]`

output `d*F^(c*(a + b*x^2))*x`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (warning: unable to verify)**

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
gospers	$d F^{c(bx^2+a)} x$	15
risch	$d F^{c(bx^2+a)} x$	15
parallelrisch	$d F^{c(bx^2+a)} x$	15
norman	$dx e^{c(bx^2+a) \ln(F)}$	16
meijerg	$\frac{F^{ac} d \sqrt{\pi} \operatorname{erfi}\left(\sqrt{c} \sqrt{b} x \sqrt{\ln(F)}\right)}{2 \sqrt{\ln(F)} \sqrt{c} \sqrt{b}} - \frac{F^{ac} d \left( \frac{x(-bc)^{\frac{3}{2}} \sqrt{\ln(F)} e^{cbx^2 \ln(F)}}{cb} - \frac{(-bc)^{\frac{3}{2}} \sqrt{\pi} \operatorname{erfi}\left(\sqrt{c} \sqrt{b} x \sqrt{\ln(F)}\right)}{2c^{\frac{3}{2}} b^{\frac{3}{2}}} \right)}{\sqrt{-bc} \sqrt{\ln(F)}}$	112

input `int(F^(c*(b*x^2+a))*(d+2*b*c*d*x^2*ln(F)),x,method=_RETURNVERBOSE)`

output `d*F^(c*(b*x^2+a))*x`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int F^{c(a+bx^2)} (d + 2bcdx^2 \log(F)) dx = F^{bcx^2+ac} dx$$

input `integrate(F^((b*x^2+a)*c)*(d+2*b*c*d*x^2*log(F)),x, algorithm="fricas")`

output `F^(b*c*x^2 + a*c)*d*x`

**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx^2)} (d + 2bcdx^2 \log(F)) dx = F^{c(a+bx^2)} dx$$

input `integrate(F**((b*x**2+a)*c)*(d+2*b*c*d*x**2*ln(F)),x)`

output `F**(c*(a + b*x**2))*d*x`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 7.43

$$\begin{aligned} & \int F^{c(a+bx^2)} (d + 2bcdx^2 \log(F)) dx \\ &= \frac{1}{2} bcd \left( \frac{2 F^{bcx^2} F^{ac} x}{bc \log(F)} - \frac{\sqrt{\pi} F^{ac} \operatorname{erf}(\sqrt{-bc \log(F)} x)}{\sqrt{-bc \log(F)} bc \log(F)} \right) \log(F) \\ &+ \frac{\sqrt{\pi} F^{ac} d \operatorname{erf}(\sqrt{-bc \log(F)} x)}{2 \sqrt{-bc \log(F)}} \end{aligned}$$

input `integrate(F^((b*x^2+a)*c)*(d+2*b*c*d*x^2*log(F)),x, algorithm="maxima")`

output `1/2*b*c*d*(2*F^(b*c*x^2)*F^(a*c)*x/(b*c*log(F)) - sqrt(pi)*F^(a*c)*erf(sqrt(-b*c*log(F))*x)/(sqrt(-b*c*log(F))*b*c*log(F))*log(F) + 1/2*sqrt(pi)*F^(a*c)*d*erf(sqrt(-b*c*log(F))*x)/sqrt(-b*c*log(F))`

**Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx = dx e^{(bcx^2 \log(F) + ac \log(F))}$$

input `integrate(F^((b*x^2+a)*c)*(d+2*b*c*d*x^2*log(F)),x, algorithm="giac")`

output `d*x*e^(b*c*x^2*log(F) + a*c*log(F))`

**Mupad [B] (verification not implemented)**

Time = 23.75 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx = F^{c(bx^2+a)} dx$$

input `int(F^(c*(a + b*x^2))*(d + 2*b*c*d*x^2*log(F)),x)`

output `F^(c*(a + b*x^2))*d*x`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int F^{c(a+bx^2)}(d + 2bcdx^2 \log(F)) dx = f^{bcx^2+ac} dx$$

input `int(F^((b*x^2+a)*c)*(d+2*b*c*d*x^2*log(F)),x)`

output `f**(a*c + b*c*x**2)*d*x`

### 3.3 $\int F^{c(a+bx)}(d + bcdx \log(F)) dx$

Optimal result . . . . .	87
Mathematica [A] (verified) . . . . .	87
Rubi [B] (verified) . . . . .	88
Maple [A] (warning: unable to verify) . . . . .	89
Fricas [A] (verification not implemented) . . . . .	89
Sympy [A] (verification not implemented) . . . . .	90
Maxima [B] (verification not implemented) . . . . .	90
Giac [C] (verification not implemented) . . . . .	90
Mupad [B] (verification not implemented) . . . . .	91
Reduce [B] (verification not implemented) . . . . .	92

#### Optimal result

Integrand size = 19, antiderivative size = 12

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = dF^{c(a+bx)}x$$

output `d*F^(c*(b*x+a))*x`

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = dF^{ac+bcx}x$$

input `Integrate[F^(c*(a + b*x))*(d + b*c*d*x*Log[F]),x]`

output `d*F^(a*c + b*c*x)*x`



**Rubi [B] (verified)**

Leaf count is larger than twice the leaf count of optimal. 52 vs.  $2(12) = 24$ .

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 4.33, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(bcdx \log(F) + d) dx$$

$$\downarrow 2607$$

$$\frac{dF^{c(a+bx)}(bcx \log(F) + 1)}{bc \log(F)} - d \int F^{c(a+bx)} dx$$

$$\downarrow 2624$$

$$\frac{dF^{c(a+bx)}(bcx \log(F) + 1)}{bc \log(F)} - \frac{dF^{c(a+bx)}}{bc \log(F)}$$

input `Int [F^(c*(a + b*x))*(d + b*c*d*x*Log[F]), x]`

output `-((d*F^(c*(a + b*x)))/(b*c*Log[F])) + (d*F^(c*(a + b*x))*(1 + b*c*x*Log[F]))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (warning: unable to verify)**

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
gospers	$d F^{c(bx+a)} x$	13
risch	$d F^{c(bx+a)} x$	13
parallelrisc	$d F^{c(bx+a)} x$	13
norman	$dx e^{c(bx+a) \ln(F)}$	14
meijerg	$-\frac{F^{ac} d(1-e^{bcx \ln(F)})}{bc \ln(F)} + \frac{F^{ac} d \left( 1 - \frac{(-2bcx \ln(F)+2)e^{bcx \ln(F)}}{2} \right)}{bc \ln(F)}$	68

input `int(F^(c*(b*x+a))*(d+b*c*d*x*ln(F)),x,method=_RETURNVERBOSE)`

output `d*F^(c*(b*x+a))*x`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)} (d + bcdx \log(F)) dx = F^{bcx+ac} dx$$

input `integrate(F^((b*x+a)*c)*(d+b*c*d*x*log(F)),x, algorithm="fricas")`

output `F^(b*c*x + a*c)*d*x`

**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = F^{c(a+bx)} dx$$

input `integrate(F**((b*x+a)*c)*(d+b*c*d*x*ln(F)), x)`

output `F**(c*(a + b*x))*d*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 5.00

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d}{bc \log(F)} + \frac{F^{bcx+ac}d}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c)*(d+b*c*d*x*log(F)), x, algorithm="maxima")`

output `(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d/(b*c*log(F)) + F^(b*c*x + a*c)*d/(b*c*log(F))`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 701, normalized size of antiderivative = 58.42

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(d+b*c*d*x*log(F)), x, algorithm="giac")`

output

```

-1/2*I*((pi*b^2*c^2*d*x*log(F)*sgn(F) - pi*b^2*c^2*d*x*log(F) - 2*I*b^2*c^
2*d*x*log(F)*log(abs(F)) + pi*b*c*d*sgn(F) - pi*b*c*d + 2*I*b*c*d*log(F) -
2*I*b*c*d*log(abs(F)))*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*
pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(ab
s(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(a
bs(F))^2) + (pi*b^2*c^2*d*x*log(F)*sgn(F) - pi*b^2*c^2*d*x*log(F) + 2*I*b^
2*c^2*d*x*log(F)*log(abs(F)) + pi*b*c*d*sgn(F) - pi*b*c*d - 2*I*b*c*d*log(
F) + 2*I*b*c*d*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1
/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*l
og(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*
log(abs(F))^2)*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*((-I*pi*b^2*
c^2*d*x*log(F)*sgn(F) + I*pi*b^2*c^2*d*x*log(F) - 2*b^2*c^2*d*x*log(F)*log
(abs(F)) - I*pi*b*c*d*sgn(F) + I*pi*b*c*d + 2*b*c*d*log(F) - 2*b*c*d*log(a
bs(F)))*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) -
1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - p
i^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2) + (I*p
i*b^2*c^2*d*x*log(F)*sgn(F) - I*pi*b^2*c^2*d*x*log(F) - 2*b^2*c^2*d*x*log(
F)*log(abs(F)) + I*pi*b*c*d*sgn(F) - I*pi*b*c*d + 2*b*c*d*log(F) - 2*b*c*d
*log(abs(F)))*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sg
n(F) + 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*...

```

### Mupad [B] (verification not implemented)

Time = 23.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = F^{c(a+bx)} dx$$

input

```
int(F^(c*(a + b*x))*(d + b*c*d*x*log(F)),x)
```

output

```
F^(c*(a + b*x))*d*x
```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)}(d + bcdx \log(F)) dx = f^{bcx+ac} dx$$

input `int(F^((b*x+a)*c)*(d+b*c*d*x*log(F)),x)`

output `f**(a*c + b*c*x)*d*x`

$$3.4 \quad \int F^{c\left(a+\frac{b}{x}\right)} \left( d - \frac{bcd \log(F)}{x} \right) dx$$

Optimal result	93
Mathematica [A] (verified)	93
Rubi [A] (verified)	94
Maple [A] (warning: unable to verify)	94
Fricas [A] (verification not implemented)	95
Sympy [A] (verification not implemented)	95
Maxima [C] (verification not implemented)	96
Giac [F]	96
Mupad [B] (verification not implemented)	96
Reduce [B] (verification not implemented)	97

### Optimal result

Integrand size = 24, antiderivative size = 14

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d - \frac{bcd \log(F)}{x} \right) dx = dF^{c\left(a+\frac{b}{x}\right)} x$$

output

```
d*F^(c*(a+b/x))*x
```

### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d - \frac{bcd \log(F)}{x} \right) dx = dF^{c\left(a+\frac{b}{x}\right)} x$$

input

```
Integrate[F^(c*(a + b/x))*(d - (b*c*d*Log[F])/x),x]
```

output

```
d*F^(c*(a + b/x))*x
```

### Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d - \frac{bcd \log(F)}{x} \right) dx$$

↓ 2726

$$dx F^{c\left(a+\frac{b}{x}\right)}$$

input

```
Int [F^(c*(a + b/x))*(d - (b*c*d*Log[F])/x) , x]
```

output

```
d*F^(c*(a + b/x))*x
```

#### Defintions of rubi rules used

rule 2726

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

### Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
parallelrisch	$d F^{c\left(a+\frac{b}{x}\right)} x$
norman	$dx e^{c\left(a+\frac{b}{x}\right) \ln(F)}$
risch	$dx F^{\frac{c(ax+b)}{x}}$
meijerg	$F^{ac} d \ln(F) bc \left( \frac{x}{bc \ln(F)} + 1 + \ln(x) - \ln(-bc) - \ln(\ln(F)) - \frac{x \left( \frac{2bc \ln(F)}{x} + 2 \right)}{2bc \ln(F)} + \frac{x e^{\frac{bc \ln(F)}{x}}}{bc \ln(F)} + \ln \right)$

input `int(F^(c*(a+b/x))*(d-b*c*d*ln(F)/x),x,method=_RETURNVERBOSE)`

output `d*F^(c*(a+b/x))*x`

### **Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int F^{c\left(a+\frac{b}{x}\right)}\left(d-\frac{bcd\log(F)}{x}\right)dx = F^{\frac{acx+bc}{x}}dx$$

input `integrate(F^(c*(a+b/x))*(d-b*c*d*log(F)/x),x, algorithm="fricas")`

output `F^((a*c*x + b*c)/x)*d*x`

### **Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int F^{c\left(a+\frac{b}{x}\right)}\left(d-\frac{bcd\log(F)}{x}\right)dx = F^{c\left(a+\frac{b}{x}\right)}dx$$

input `integrate(F**(c*(a+b/x))*(d-b*c*d*ln(F)/x),x)`

output `F**(c*(a + b/x))*d*x`



**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 3.14

$$\int F^{c\left(a+\frac{b}{x}\right)}\left(d-\frac{bcd\log(F)}{x}\right)dx = F^{ac}bcd\text{Ei}\left(\frac{bc\log(F)}{x}\right)\log(F) - F^{ac}bcd\Gamma\left(-1,-\frac{bc\log(F)}{x}\right)\log(F)$$

input `integrate(F^(c*(a+b/x))*(d-b*c*d*log(F)/x),x, algorithm="maxima")`

output `F^(a*c)*b*c*d*Ei(b*c*log(F)/x)*log(F) - F^(a*c)*b*c*d*gamma(-1, -b*c*log(F)/x)*log(F)`

**Giac [F]**

$$\int F^{c\left(a+\frac{b}{x}\right)}\left(d-\frac{bcd\log(F)}{x}\right)dx = \int -\left(\frac{bcd\log(F)}{x}-d\right)F^{\left(a+\frac{b}{x}\right)c}dx$$

input `integrate(F^(c*(a+b/x))*(d-b*c*d*log(F)/x),x, algorithm="giac")`

output `integrate(-(b*c*d*log(F)/x - d)*F^((a + b/x)*c), x)`

**Mupad [B] (verification not implemented)**

Time = 24.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x}\right)}\left(d-\frac{bcd\log(F)}{x}\right)dx = F^{c\left(a+\frac{b}{x}\right)}dx$$

input `int(F^(c*(a + b/x))*(d - (b*c*d*log(F))/x),x)`

output `F^(c*(a + b/x))*d*x`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int F^{c(a+\frac{b}{x})} \left( d - \frac{bcd \log(F)}{x} \right) dx = f^{\frac{acx+bc}{x}} dx$$

input `int(F^(c*(a+b/x))*(d-b*c*d*log(F)/x),x)`

output `f**((a*c*x + b*c)/x)*d*x`

$$3.5 \quad \int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx$$

Optimal result	98
Mathematica [A] (verified)	98
Rubi [A] (verified)	99
Maple [A] (warning: unable to verify)	100
Fricas [A] (verification not implemented)	100
Sympy [A] (verification not implemented)	101
Maxima [C] (verification not implemented)	101
Giac [F]	102
Mupad [B] (verification not implemented)	102
Reduce [B] (verification not implemented)	102

### Optimal result

Integrand size = 24, antiderivative size = 14

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = dF^{c\left(a+\frac{b}{x^2}\right)} x$$

output `d*F^(c*(a+b/x^2))*x`

### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = dF^{c\left(a+\frac{b}{x^2}\right)} x$$

input `Integrate[F^(c*(a + b/x^2))*(d - (2*b*c*d*Log[F])/x^2),x]`

output `d*F^(c*(a + b/x^2))*x`

**Rubi [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx$$

↓ 2726

$$dx F^{c\left(a+\frac{b}{x^2}\right)}$$

input `Int [F^(c*(a + b/x^2))*(d - (2*b*c*d*Log[F])/x^2),x]`

output `d*F^(c*(a + b/x^2))*x`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (warning: unable to verify)**

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
parallelrisch	$d F^{c\left(a+\frac{b}{x^2}\right)} x$
norman	$dx e^{c\left(a+\frac{b}{x^2}\right) \ln(F)}$
risch	$dx F^{\frac{c\left(a x^2+b\right)}{x^2}}$
meijerg	$-\frac{F^{ac} d \sqrt{-bc} \sqrt{\ln(F)} \left( -\frac{2x e^{-\frac{bc \ln(F)}{x^2}}}{\sqrt{-bc} \sqrt{\ln(F)}} + \frac{2\sqrt{b} \sqrt{c} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\ln(F)}}{x}\right)}{\sqrt{-bc}} \right)}{2} + F^{ac} d \sqrt{b} \sqrt{c} \sqrt{\ln(F)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\ln(F)}}{x}\right)$

input `int(F^(c*(a+b/x^2))*(d-2*b*c*d*ln(F)/x^2),x,method=_RETURNVERBOSE)`output `d*F^(c*(a+b/x^2))*x`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = F^{\frac{acx^2+bc}{x^2}} dx$$

input `integrate(F^(c*(a+b/x^2))*(d-2*b*c*d*log(F)/x^2),x, algorithm="fricas")`output `F^((a*c*x^2 + b*c)/x^2)*d*x`

**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F^{c(a+\frac{b}{x^2})} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = F^{c(a+\frac{b}{x^2})} dx$$

input `integrate(F**(c*(a+b/x**2))*(d-2*b*c*d*ln(F)/x**2),x)`

output `F**(c*(a + b/x**2))*d*x`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 5.29

$$\int F^{c(a+\frac{b}{x^2})} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = \frac{\sqrt{\pi} F^{ac} bcd \left( \operatorname{erf} \left( \sqrt{-\frac{bc \log(F)}{x^2}} \right) - 1 \right) \log(F)}{x \sqrt{-\frac{bc \log(F)}{x^2}}} + \frac{1}{2} F^{ac} dx \sqrt{-\frac{bc \log(F)}{x^2}} \Gamma \left( -\frac{1}{2}, -\frac{bc \log(F)}{x^2} \right)$$

input `integrate(F^(c*(a+b/x^2))*(d-2*b*c*d*log(F)/x^2),x, algorithm="maxima")`

output `sqrt(pi)*F^(a*c)*b*c*d*(erf(sqrt(-b*c*log(F)/x^2)) - 1)*log(F)/(x*sqrt(-b*c*log(F)/x^2)) + 1/2*F^(a*c)*d*x*sqrt(-b*c*log(F)/x^2)*gamma(-1/2, -b*c*log(F)/x^2)`

**Giac [F]**

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = \int - \left( \frac{2bcd \log(F)}{x^2} - d \right) F^{c\left(a+\frac{b}{x^2}\right)} dx$$

input `integrate(F^(c*(a+b/x^2))*(d-2*b*c*d*log(F)/x^2),x, algorithm="giac")`

output `integrate(-(2*b*c*d*log(F)/x^2 - d)*F^((a + b/x^2)*c), x)`

**Mupad [B] (verification not implemented)**

Time = 24.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = F^{c\left(a+\frac{b}{x^2}\right)} dx$$

input `int(F^(c*(a + b/x^2))*(d - (2*b*c*d*log(F))/x^2),x)`

output `F^(c*(a + b/x^2))*d*x`

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d - \frac{2bcd \log(F)}{x^2} \right) dx = f^{\frac{acx^2+bc}{x^2}} dx$$

input `int(F^(c*(a+b/x^2))*(d-2*b*c*d*log(F)/x^2),x)`

output `f**((a*c*x**2 + b*c)/x**2)*d*x`

$$3.6 \quad \int F^{c\left(a+\frac{b}{x^3}\right)} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx$$

Optimal result	103
Mathematica [A] (verified)	103
Rubi [A] (verified)	104
Maple [A] (warning: unable to verify)	105
Fricas [A] (verification not implemented)	105
Sympy [A] (verification not implemented)	106
Maxima [C] (verification not implemented)	106
Giac [F]	107
Mupad [B] (verification not implemented)	107
Reduce [B] (verification not implemented)	107

### Optimal result

Integrand size = 24, antiderivative size = 14

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = dF^{c\left(a+\frac{b}{x^3}\right)} x$$

output

```
d*F^(c*(a+b/x^3))*x
```

### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = dF^{c\left(a+\frac{b}{x^3}\right)} x$$

input

```
Integrate[F^(c*(a + b/x^3))*(d - (3*b*c*d*Log[F])/x^3),x]
```

output

```
d*F^(c*(a + b/x^3))*x
```



**Rubi [A] (verified)**

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx$$

↓ 2726

$$dx F^{c\left(a+\frac{b}{x^3}\right)}$$

input `Int[F^(c*(a + b/x^3))*(d - (3*b*c*d*Log[F])/x^3),x]`

output `d*F^(c*(a + b/x^3))*x`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (warning: unable to verify)**

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
parallelrisch	$d F^{c\left(a+\frac{b}{x^3}\right)} x$
norman	$dx e^{c\left(a+\frac{b}{x^3}\right) \ln(F)}$
risch	$dx F^{\frac{c\left(ax^3+b\right)}{x^3}}$
meijerg	$- \frac{F^{ac} d(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}} \left( \frac{3 \ln(F)^{\frac{2}{3}} cb \Gamma\left(\frac{2}{3}\right)}{x^2 (-bc)^{\frac{1}{3}} \left(-\frac{cb \ln(F)}{x^3}\right)^{\frac{2}{3}}} - \frac{3 x e^{\frac{cb \ln(F)}{x^3}}}{(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}}} - \frac{3 \ln(F)^{\frac{2}{3}} cb \Gamma\left(\frac{2}{3}, -\frac{cb \ln(F)}{x^3}\right)}{x^2 (-bc)^{\frac{1}{3}} \left(-\frac{cb \ln(F)}{x^3}\right)^{\frac{2}{3}}} \right)}{3} - F^{ac} d(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}}$

input `int(F^(c*(a+b/x^3))*(d-3*b*c*d*ln(F)/x^3),x,method=_RETURNVERBOSE)`output `d*F^(c*(a+b/x^3))*x`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = F^{\frac{acx^3+bc}{x^3}} dx$$

input `integrate(F^(c*(a+b/x^3))*(d-3*b*c*d*log(F)/x^3),x, algorithm="fricas")`output `F^((a*c*x^3 + b*c)/x^3)*d*x`

**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int F^{c(a+\frac{b}{x^3})} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = F^{c(a+\frac{b}{x^3})} dx$$

input `integrate(F**(c*(a+b/x**3))*(d-3*b*c*d*ln(F)/x**3),x)`

output `F**(c*(a + b/x**3))*d*x`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.93

$$\int F^{c(a+\frac{b}{x^3})} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = \frac{1}{3} F^{ac} dx \left( -\frac{bc \log(F)}{x^3} \right)^{\frac{1}{3}} \Gamma \left( -\frac{1}{3}, -\frac{bc \log(F)}{x^3} \right) - \frac{F^{ac} bcd \Gamma \left( \frac{2}{3}, -\frac{bc \log(F)}{x^3} \right) \log(F)}{x^2 \left( -\frac{bc \log(F)}{x^3} \right)^{\frac{2}{3}}}$$

input `integrate(F^(c*(a+b/x^3))*(d-3*b*c*d*log(F)/x^3),x, algorithm="maxima")`

output `1/3*F^(a*c)*d*x*(-b*c*log(F)/x^3)^(1/3)*gamma(-1/3, -b*c*log(F)/x^3) - F^(a*c)*b*c*d*gamma(2/3, -b*c*log(F)/x^3)*log(F)/(x^2*(-b*c*log(F)/x^3)^(2/3))`

**Giac [F]**

$$\int F^{c(a+\frac{b}{x^3})} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = \int F^{(a+\frac{b}{x^3})c} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx$$

input `integrate(F^(c*(a+b/x^3))*(d-3*b*c*d*log(F)/x^3),x, algorithm="giac")`

output `integrate(F^((a + b/x^3)*c)*(d - 3*b*c*d*log(F)/x^3), x)`

**Mupad [B] (verification not implemented)**

Time = 23.80 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+\frac{b}{x^3})} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = F^{c(a+\frac{b}{x^3})} dx$$

input `int(F^(c*(a + b/x^3))*(d - (3*b*c*d*log(F))/x^3),x)`

output `F^(c*(a + b/x^3))*d*x`

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int F^{c(a+\frac{b}{x^3})} \left( d - \frac{3bcd \log(F)}{x^3} \right) dx = f^{\frac{acx^3+bc}{x^3}} dx$$

input `int(F^(c*(a+b/x^3))*(d-3*b*c*d*log(F)/x^3),x)`

output `f**((a*c*x**3 + b*c)/x**3)*d*x`

### 3.7 $\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx$

Optimal result	108
Mathematica [A] (verified)	108
Rubi [A] (verified)	109
Maple [A] (warning: unable to verify)	109
Fricas [A] (verification not implemented)	110
Sympy [A] (verification not implemented)	110
Maxima [C] (verification not implemented)	111
Giac [A] (verification not implemented)	111
Mupad [B] (verification not implemented)	112
Reduce [B] (verification not implemented)	112

#### Optimal result

Integrand size = 24, antiderivative size = 14

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = dF^{c(a+bx^n)}x$$

output

`d*F^(c*(a+b*x^n))*x`

#### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = dF^{c(a+bx^n)}x$$

input

`Integrate[F^(c*(a + b*x^n))*(d + b*c*d*n*x^n*Log[F]),x]`

output

`d*F^(c*(a + b*x^n))*x`

### Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx^n)}(bcdn \log(F)x^n + d) dx$$

$\downarrow$  2726  
 $dx F^{c(a+bx^n)}$

input `Int[F^(c*(a + b*x^n))*(d + b*c*d*n*x^n*Log[F]),x]`

output `d*F^(c*(a + b*x^n))*x`

#### Defintions of rubi rules used

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

### Maple [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
risch	$d F^{c(a+bx^n)} x$
parallelrisc	$d F^{c(a+bx^n)} x$
norman	$dx e^{c(a+be^n \ln(x)) \ln(F)}$
meijerg	$F^{ac} d(-bc)^{-\frac{1}{n}} \ln(F)^{-\frac{1}{n}} \left( \frac{nx(-bc)^{\frac{1}{n}} \ln(F)^{\frac{1}{n}} (x^n \ln(F)bcn+n+1)\Gamma(1-\frac{1}{n})\Gamma(\frac{1+n}{n}+1) \text{LaguerreL}(-\frac{1}{n}, \frac{1+n}{n}, x^n bc \ln(F))}{(1+n)\Gamma(-\frac{1}{n} + \frac{1+n}{n} + 1)} \right) - n^2 x^{1+n} (-$
	$n$

input `int(F^(c*(a+b*x^n))*(d+b*c*d*n*x^n*ln(F)),x,method=_RETURNVERBOSE)`

output `d*F^(c*(a+b*x^n))*x`

### **Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = dx e^{(bcx^n \log(F) + ac \log(F))}$$

input `integrate(F^(c*(a+b*x^n))*(d+b*c*d*n*x^n*log(F)),x, algorithm="fricas")`

output `d*x*e^(b*c*x^n*log(F) + a*c*log(F))`

### **Sympy [A] (verification not implemented)**

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = F^{ac+bcx^n} dx$$

input `integrate(F**(c*(a+b*x**n))*(d+b*c*d*n*x**n*ln(F)),x)`

output `F**(a*c + b*c*x**n)*d*x`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 6.71

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = -\frac{F^{ac}bcdx^{n+1}\Gamma\left(\frac{n+1}{n}, -bcx^n \log(F)\right) \log(F)}{(-bcx^n \log(F))^{\frac{n+1}{n}}} - \frac{F^{ac}dx\Gamma\left(\frac{1}{n}, -bcx^n \log(F)\right)}{(-bcx^n \log(F))^{\frac{1}{n}} n}$$

input `integrate(F^(c*(a+b*x^n))*(d+b*c*d*n*x^n*log(F)),x, algorithm="maxima")`

output `-F^(a*c)*b*c*d*x^(n + 1)*gamma((n + 1)/n, -b*c*x^n*log(F))*log(F)/(-b*c*x^n*log(F))^(n + 1)/n - F^(a*c)*d*x*gamma(1/n, -b*c*x^n*log(F))/((-b*c*x^n*log(F))^(1/n)*n)`

**Giac [A] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = F^{bcx^n} F^{ac} dx$$

input `integrate(F^(c*(a+b*x^n))*(d+b*c*d*n*x^n*log(F)),x, algorithm="giac")`

output `F^(b*c*x^n)*F^(a*c)*d*x`



**Mupad [B] (verification not implemented)**

Time = 22.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = F^{ac+bcx^n} dx$$

input `int(F^(c*(a + b*x^n))*(d + b*c*d*n*x^n*log(F)),x)`output `F^(a*c + b*c*x^n)*d*x`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int F^{c(a+bx^n)}(d + bcdnx^n \log(F)) dx = f^{x^n bc+ac} dx$$

input `int(F^(c*(a+b*x^n))*(d+b*c*d*n*x^n*log(F)),x)`output `f**(x**n*b*c + a*c)*d*x`

### 3.8 $\int F^{c(a+bx^3)}(d + ex^2 + 3bcdx^3 \log(F)) dx$

Optimal result . . . . .	113
Mathematica [A] (verified) . . . . .	113
Rubi [A] (verified) . . . . .	114
Maple [A] (verified) . . . . .	115
Fricas [A] (verification not implemented) . . . . .	115
Sympy [A] (verification not implemented) . . . . .	116
Maxima [C] (verification not implemented) . . . . .	116
Giac [A] (verification not implemented) . . . . .	117
Mupad [B] (verification not implemented) . . . . .	117
Reduce [B] (verification not implemented) . . . . .	117

#### Optimal result

Integrand size = 29, antiderivative size = 35

$$\int F^{c(a+bx^3)}(d + ex^2 + 3bcdx^3 \log(F)) dx = \frac{F^{c(a+bx^3)}(e + 3bcdx \log(F))}{3bc \log(F)}$$

output `1/3*F^(c*(b*x^3+a))*(e+3*b*c*d*x*ln(F))/b/c/ln(F)`

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^3)}(d + ex^2 + 3bcdx^3 \log(F)) dx = \frac{F^{c(a+bx^3)}(e + 3bcdx \log(F))}{3bc \log(F)}$$

input `Integrate[F^(c*(a + b*x^3))*(d + e*x^2 + 3*b*c*d*x^3*Log[F]),x]`

output `(F^(c*(a + b*x^3))*(e + 3*b*c*d*x*Log[F]))/(3*b*c*Log[F])`

**Rubi [A] (verified)**

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx^3)} (3bcdx^3 \log(F) + d + ex^2) dx$$

$$\downarrow 2726$$

$$\frac{F^{c(a+bx^3)} (3bcdx^3 \log(F) + ex^2)}{3bcx^2 \log(F)}$$

input `Int[F^(c*(a + b*x^3))*(d + e*x^2 + 3*b*c*d*x^3*Log[F]),x]`

output `(F^(c*(a + b*x^3))*(e*x^2 + 3*b*c*d*x^3*Log[F]))/(3*b*c*x^2*Log[F])`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result
gospers	$\frac{F^{c(bx^3+a)}(e+3bcdx \ln(F))}{3bc \ln(F)}$
risch	$\frac{F^{c(bx^3+a)}(e+3bcdx \ln(F))}{3bc \ln(F)}$
norman	$dx e^{c(bx^3+a) \ln(F)} + \frac{e e^{c(bx^3+a) \ln(F)}}{3 \ln(F) bc}$
paralelrisch	$\frac{3d F^{c(bx^3+a)} x \ln(F) bc + e F^{c(bx^3+a)}}{3 \ln(F) bc}$
meijerg	$\frac{F^{ac} d \left( \frac{2x(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}} \pi \sqrt{3}}{3\Gamma(\frac{2}{3})(-\ln(F)bcx^3)^{\frac{1}{3}}} - \frac{x(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}} \Gamma(\frac{1}{3}, -\ln(F)bcx^3)}{(-\ln(F)bcx^3)^{\frac{1}{3}}} \right)}{3(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}}} - \frac{F^{ac} e (1 - e^{\ln(F)bcx^3})}{3bc \ln(F)} - \frac{F^{ac} d \left( -\frac{2x(-bc)^{\frac{4}{3}} \ln(F)^{\frac{1}{3}}}{9bc\Gamma(\frac{2}{3})(-\ln(F)bcx^3)^{\frac{1}{3}}} \right)}{9bc\Gamma(\frac{2}{3})(-\ln(F)bcx^3)^{\frac{1}{3}}}$

input `int(F^(c*(b*x^3+a))*(d+e*x^2+3*b*c*d*x^3*ln(F)),x,method=_RETURNVERBOSE)`

output `1/3*F^(c*(b*x^3+a))*(e+3*b*c*d*x*ln(F))/b/c/ln(F)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx^3)}(d + ex^2 + 3bcdx^3 \log(F)) dx = \frac{(3bcdx \log(F) + e)F^{bcx^3+ac}}{3bc \log(F)}$$

input `integrate(F^(c*(b*x^3+a))*(d+e*x^2+3*b*c*d*x^3*log(F)),x, algorithm="fricas")`

output `1/3*(3*b*c*d*x*log(F) + e)*F^(b*c*x^3 + a*c)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int F^{c(a+bx^3)} (d + ex^2 + 3bcdx^3 \log(F)) dx = \begin{cases} \frac{F^{c(a+bx^3)} (3bcdx \log(F) + e)}{3bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ \frac{3bcdx^4 \log(F)}{4} + dx + \frac{ex^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(F**(c*(b*x**3+a))*(d+e*x**2+3*b*c*d*x**3*ln(F)),x)`

output `Piecewise((F**(c*(a + b*x**3))*(3*b*c*d*x*log(F) + e)/(3*b*c*log(F)), Ne(b*c*log(F), 0)), (3*b*c*d*x**4*log(F)/4 + d*x + e*x**3/3, True))`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.69

$$\int F^{c(a+bx^3)} (d + ex^2 + 3bcdx^3 \log(F)) dx = -\frac{F^{ac}bcdx^4\Gamma\left(\frac{4}{3}, -bcx^3 \log(F)\right) \log(F)}{(-bcx^3 \log(F))^{\frac{4}{3}}} - \frac{F^{ac}dx\Gamma\left(\frac{1}{3}, -bcx^3 \log(F)\right)}{3(-bcx^3 \log(F))^{\frac{1}{3}}} + \frac{F^{bcx^3+ac}e}{3bc \log(F)}$$

input `integrate(F^(c*(b*x^3+a))*(d+e*x^2+3*b*c*d*x^3*log(F)),x, algorithm="maxima")`

output `-F^(a*c)*b*c*d*x^4*gamma(4/3, -b*c*x^3*log(F))*log(F)/(-b*c*x^3*log(F))^(4/3) - 1/3*F^(a*c)*d*x*gamma(1/3, -b*c*x^3*log(F))/(-b*c*x^3*log(F))^(1/3) + 1/3*F^(b*c*x^3 + a*c)*e/(b*c*log(F))`

**Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.40

$$\int F^{c(a+bx^3)} (d + ex^2 + 3bcdx^3 \log(F)) dx = \frac{3 F^{bcx^3} F^{ac} bcdx \log(F) + F^{bcx^3} F^{ac} e}{3 bc \log(F)}$$

input `integrate(F^(c*(b*x^3+a))*(d+e*x^2+3*b*c*d*x^3*log(F)),x, algorithm="giac")`

output `1/3*(3*F^(b*c*x^3)*F^(a*c)*b*c*d*x*log(F) + F^(b*c*x^3)*F^(a*c)*e)/(b*c*log(F))`

**Mupad [B] (verification not implemented)**

Time = 22.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx^3)} (d + ex^2 + 3bcdx^3 \log(F)) dx = \frac{F^{bcx^3+ac} \left( \frac{e}{3} + bcdx \ln(F) \right)}{bc \ln(F)}$$

input `int(F^(c*(a + b*x^3))*(d + e*x^2 + 3*b*c*d*x^3*log(F)),x)`

output `(F^(a*c + b*c*x^3)*(e/3 + b*c*d*x*log(F)))/(b*c*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx^3)} (d + ex^2 + 3bcdx^3 \log(F)) dx = \frac{f^{bcx^3+ac}(3 \log(f) bcdx + e)}{3 \log(f) bc}$$

input `int(F^(c*(b*x^3+a))*(d+e*x^2+3*b*c*d*x^3*log(F)),x)`

output `(f**(a*c + b*c*x**3)*(3*log(f)*b*c*d*x + e))/(3*log(f)*b*c)`

### 3.9 $\int F^{c(a+bx^2)}(d+ex+2bcdx^2 \log(F)) dx$

Optimal result	118
Mathematica [A] (verified)	118
Rubi [A] (verified)	119
Maple [A] (verified)	120
Fricas [A] (verification not implemented)	120
Sympy [A] (verification not implemented)	121
Maxima [C] (verification not implemented)	121
Giac [A] (verification not implemented)	122
Mupad [B] (verification not implemented)	122
Reduce [B] (verification not implemented)	122

#### Optimal result

Integrand size = 27, antiderivative size = 35

$$\int F^{c(a+bx^2)}(d+ex+2bcdx^2 \log(F)) dx = \frac{F^{c(a+bx^2)}(e+2bcdx \log(F))}{2bc \log(F)}$$

output `1/2*F^((b*x^2+a)*c)*(e+2*b*c*d*x*ln(F))/b/c/ln(F)`

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^2)}(d+ex+2bcdx^2 \log(F)) dx = \frac{F^{c(a+bx^2)}(e+2bcdx \log(F))}{2bc \log(F)}$$

input `Integrate[F^(c*(a + b*x^2))*(d + e*x + 2*b*c*d*x^2*Log[F]), x]`

output `(F^(c*(a + b*x^2))*(e + 2*b*c*d*x*Log[F]))/(2*b*c*Log[F])`

**Rubi [A] (verified)**

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx^2)} (2bcdx^2 \log(F) + d + ex) dx$$

$$\downarrow 2726$$

$$\frac{F^{c(a+bx^2)} (2bcdx^2 \log(F) + ex)}{2bcx \log(F)}$$

input `Int[F^(c*(a + b*x^2))*(d + e*x + 2*b*c*d*x^2*Log[F]),x]`

output `(F^(c*(a + b*x^2))*(e*x + 2*b*c*d*x^2*Log[F]))/(2*b*c*x*Log[F])`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`



**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

method	result
gospers	$\frac{F^{c(bx^2+a)}(e+2bcdx \ln(F))}{2bc \ln(F)}$
risch	$\frac{F^{c(bx^2+a)}(e+2bcdx \ln(F))}{2bc \ln(F)}$
norman	$dx e^{c(bx^2+a) \ln(F)} + \frac{e e^{c(bx^2+a) \ln(F)}}{2 \ln(F) bc}$
paralelrisch	$\frac{2F^{c(bx^2+a)} bcdx \ln(F) + e F^{c(bx^2+a)}}{2 \ln(F) bc}$
meijerg	$\frac{F^{ac} d \sqrt{\pi} \operatorname{erfi}(\sqrt{c} \sqrt{b} x \sqrt{\ln(F)})}{2 \sqrt{\ln(F)} \sqrt{c} \sqrt{b}} - \frac{F^{ac} e (1 - e^{cbx^2 \ln(F)})}{2bc \ln(F)} - \frac{F^{ac} d \left( \frac{x(-bc)^{\frac{3}{2}} \sqrt{\ln(F)} e^{cbx^2 \ln(F)}}{cb} - \frac{(-bc)^{\frac{3}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{c} \sqrt{b} x \sqrt{\ln(F)})}{2c^{\frac{3}{2}} b^{\frac{3}{2}}} \right)}{\sqrt{-bc} \sqrt{\ln(F)}}$

input `int(F^(c*(b*x^2+a))*(d+e*x+2*b*c*d*x^2*ln(F)),x,method=_RETURNVERBOSE)`

output `1/2*F^(c*(b*x^2+a))*(e+2*b*c*d*x*ln(F))/b/c/ln(F)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx = \frac{(2bcdx \log(F) + e) F^{bcx^2+ac}}{2bc \log(F)}$$

input `integrate(F^((b*x^2+a)*c)*(d+e*x+2*b*c*d*x^2*log(F)),x, algorithm="fricas")`

output `1/2*(2*b*c*d*x*log(F) + e)*F^(b*c*x^2 + a*c)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

$$\int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx = \begin{cases} \frac{F^{c(a+bx^2)} (2bcdx \log(F) + e)}{2bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ \frac{2bcdx^3 \log(F)}{3} + dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(F**(b*x**2+a)*c)*(d+e*x+2*b*c*d*x**2*ln(F)),x`

output `Piecewise((F**(c*(a + b*x**2))*(2*b*c*d*x*log(F) + e)/(2*b*c*log(F)), Ne(b*c*log(F), 0)), (2*b*c*d*x**3*log(F)/3 + d*x + e*x**2/2, True))`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.69

$$\begin{aligned} & \int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx \\ &= \frac{1}{2} bcd \left( \frac{2 F^{bcx^2} F^{ac} x}{bc \log(F)} - \frac{\sqrt{\pi} F^{ac} \operatorname{erf}(\sqrt{-bc \log(F)} x)}{\sqrt{-bc \log(F)} bc \log(F)} \right) \log(F) \\ &+ \frac{\sqrt{\pi} F^{ac} d \operatorname{erf}(\sqrt{-bc \log(F)} x)}{2 \sqrt{-bc \log(F)}} + \frac{F^{bcx^2+ac} e}{2 bc \log(F)} \end{aligned}$$

input `integrate(F^(b*x^2+a)*c)*(d+e*x+2*b*c*d*x^2*log(F)),x, algorithm="maxima"`

output `1/2*b*c*d*(2*F^(b*c*x^2)*F^(a*c)*x/(b*c*log(F)) - sqrt(pi)*F^(a*c)*erf(sqrt(-b*c*log(F))*x)/(sqrt(-b*c*log(F))*b*c*log(F)))*log(F) + 1/2*sqrt(pi)*F^(a*c)*d*erf(sqrt(-b*c*log(F))*x)/sqrt(-b*c*log(F)) + 1/2*F^(b*c*x^2 + a*c)*e/(b*c*log(F))`

**Giac [A] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx = \frac{(2bcdx \log(F) + e)e^{(bcx^2 \log(F) + ac \log(F))}}{2bc \log(F)}$$

input `integrate(F^((b*x^2+a)*c)*(d+e*x+2*b*c*d*x^2*log(F)),x, algorithm="giac")`

output `1/2*(2*b*c*d*x*log(F) + e)*e^(b*c*x^2*log(F) + a*c*log(F))/(b*c*log(F))`

**Mupad [B] (verification not implemented)**

Time = 23.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx = \frac{F^{bcx^2+ac} \left( \frac{e}{2} + bcdx \ln(F) \right)}{bc \ln(F)}$$

input `int(F^(c*(a + b*x^2))*(d + e*x + 2*b*c*d*x^2*log(F)),x)`

output `(F^(a*c + b*c*x^2)*(e/2 + b*c*d*x*log(F)))/(b*c*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx^2)} (d + ex + 2bcdx^2 \log(F)) dx = \frac{f^{bcx^2+ac} (2 \log(f) bcdx + e)}{2 \log(f) bc}$$

input `int(F^((b*x^2+a)*c)*(d+e*x+2*b*c*d*x^2*log(F)),x)`

output `(f**(a*c + b*c*x**2)*(2*log(f)*b*c*d*x + e))/(2*log(f)*b*c)`

### 3.10 $\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx$

Optimal result . . . . .	123
Mathematica [A] (verified) . . . . .	123
Rubi [A] (verified) . . . . .	124
Maple [A] (verified) . . . . .	125
Fricas [A] (verification not implemented) . . . . .	125
Sympy [A] (verification not implemented) . . . . .	126
Maxima [B] (verification not implemented) . . . . .	126
Giac [C] (verification not implemented) . . . . .	127
Mupad [B] (verification not implemented) . . . . .	128
Reduce [B] (verification not implemented) . . . . .	128

#### Optimal result

Integrand size = 20, antiderivative size = 29

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \frac{F^{c(a+bx)}(e + bcdx \log(F))}{bc \log(F)}$$

output

```
F^(c*(b*x+a))*(e+b*c*d*x*ln(F))/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \frac{F^{ac+bcx}(e + bcdx \log(F))}{bc \log(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e + b*c*d*x*Log[F]),x]
```

output

```
(F^(a*c + b*c*x)*(e + b*c*d*x*Log[F]))/(b*c*Log[F])
```

**Rubi [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(bcdx \log(F) + d + e) dx$$

$$\downarrow 2607$$

$$\frac{F^{c(a+bx)}(bcdx \log(F) + d + e)}{bc \log(F)} - d \int F^{c(a+bx)} dx$$

$$\downarrow 2624$$

$$\frac{F^{c(a+bx)}(bcdx \log(F) + d + e)}{bc \log(F)} - \frac{dF^{c(a+bx)}}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e + b*c*d*x*Log[F]),x]`

output `-((d*F^(c*(a + b*x)))/(b*c*Log[F])) + (F^(c*(a + b*x))*(d + e + b*c*d*x*Log[F]))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
gospers	$\frac{F^{c(bx+a)}(e+bc dx \ln(F))}{bc \ln(F)}$	30
risch	$\frac{F^{c(bx+a)}(e+bc dx \ln(F))}{bc \ln(F)}$	30
norman	$dx e^{c(bx+a) \ln(F)} + \frac{e e^{c(bx+a) \ln(F)}}{\ln(F)bc}$	37
parallelrisch	$\frac{d F^{c(bx+a) \ln(F)} bc x + F^{c(bx+a)} e}{cb \ln(F)}$	40
meijerg	$-\frac{F^{ac} d(1-e^{bcx \ln(F)})}{bc \ln(F)} - \frac{F^{ac} e(1-e^{bcx \ln(F)})}{bc \ln(F)} + \frac{F^{ac} d \left(1 - \frac{(-2bcx \ln(F)+2)e^{bcx \ln(F)}}{2}\right)}{bc \ln(F)}$	97

input `int(F^(c*(b*x+a))*(d+e+b*c*d*x*ln(F)),x,method=_RETURNVERBOSE)`

output `F^(c*(b*x+a))*(e+b*c*d*x*ln(F))/b/c/ln(F)`

**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(d+e+bc dx \log(F)) dx = \frac{(bc dx \log(F) + e)F^{bcx+ac}}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c)*(d+e+b*c*d*x*log(F)),x, algorithm="fricas")`

output `(b*c*d*x*log(F) + e)*F^(b*c*x + a*c)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.69

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \begin{cases} \frac{F^{c(a+bx)}(bcdx \log(F)+e)}{bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ \frac{bcdx^2 \log(F)}{2} + x(d + e) & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(d+e+b*c*d*x*ln(F)),x)`

output `Piecewise((F**(c*(a + b*x))*(b*c*d*x*log(F) + e)/(b*c*log(F)), Ne(b*c*log(F), 0)), (b*c*d*x**2*log(F)/2 + x*(d + e), True))`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(29) = 58.

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.83

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d}{bc \log(F)} + \frac{F^{bcx+ac}d}{bc \log(F)} + \frac{F^{bcx+ac}e}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c)*(d+e+b*c*d*x*log(F)),x, algorithm="maxima")`

output `(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d/(b*c*log(F)) + F^(b*c*x + a*c)*d/(b*c*log(F)) + F^(b*c*x + a*c)*e/(b*c*log(F))`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 787, normalized size of antiderivative = 27.14

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(d+e+b*c*d*x*log(F)),x, algorithm="giac")`

output

```
-1/2*I*((pi*b^2*c^2*d*x*log(F)*sgn(F) - pi*b^2*c^2*d*x*log(F) - 2*I*b^2*c^2*d*x*log(F)*log(abs(F)) + pi*b*c*d*sgn(F) + pi*b*c*e*sgn(F) - pi*b*c*d - pi*b*c*e + 2*I*b*c*d*log(F) - 2*I*b*c*d*log(abs(F)) - 2*I*b*c*e*log(abs(F))) * e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2) + (pi*b^2*c^2*d*x*log(F)*sgn(F) - pi*b^2*c^2*d*x*log(F) + 2*I*b^2*c^2*d*x*log(F)*log(abs(F)) + pi*b*c*d*sgn(F) + pi*b*c*e*sgn(F) - pi*b*c*d - pi*b*c*e - 2*I*b*c*d*log(F) + 2*I*b*c*d*log(abs(F)) + 2*I*b*c*e*log(abs(F))) * e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) - 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 + 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2)) * e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*((-I*pi*b^2*c^2*d*x*log(F)*sgn(F) + I*pi*b^2*c^2*d*x*log(F) - 2*b^2*c^2*d*x*log(F)*log(abs(F)) - I*pi*b*c*d*sgn(F) - I*pi*b*c*e*sgn(F) + I*pi*b*c*d + I*pi*b*c*e + 2*b*c*d*log(F) - 2*b*c*d*log(abs(F)) - 2*b*c*e*log(abs(F))) * e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2*log(abs(F)) + 2*b^2*c^2*log(abs(F))^2) + (I*pi*b^2*c^2*d*x*log(F)*sgn(F) - I*pi*b^2*c^2*d*x*log(F) - 2*b^2*c^2*d*x*log(F)*log(abs(F)) + I*pi*b*c*d*sgn(F) + I*pi*b*c*e*sgn(F) - I...
```



**Mupad [B] (verification not implemented)**

Time = 23.98 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \frac{F^{ac+bcx} (e + bcdx \ln(F))}{bc \ln(F)}$$

input `int(F^(c*(a + b*x))*(d + e + b*c*d*x*log(F)),x)`output `(F^(a*c + b*c*x)*(e + b*c*d*x*log(F)))/(b*c*log(F))`**Reduce [B] (verification not implemented)**

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(d + e + bcdx \log(F)) dx = \frac{f^{bcx+ac}(\log(f) bcdx + e)}{\log(f) bc}$$

input `int(F^((b*x+a)*c)*(d+e+b*c*d*x*log(F)),x)`output `(f**(a*c + b*c*x)*(log(f)*b*c*d*x + e))/(log(f)*b*c)`

$$3.11 \quad \int F^{c\left(a+\frac{b}{x}\right)} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx$$

Optimal result	129
Mathematica [A] (verified)	129
Rubi [A] (verified)	130
Maple [A] (verified)	131
Fricas [A] (verification not implemented)	131
Sympy [A] (verification not implemented)	132
Maxima [C] (verification not implemented)	132
Giac [F]	133
Mupad [B] (verification not implemented)	133
Reduce [B] (verification not implemented)	133

### Optimal result

Integrand size = 29, antiderivative size = 33

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = -\frac{F^{c\left(a+\frac{b}{x}\right)}(e - bcdx \log(F))}{bc \log(F)}$$

output `-F^(c*(a+b/x))*(e-b*c*d*x*ln(F))/b/c/ln(F)`

### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = -\frac{F^{c\left(a+\frac{b}{x}\right)}(e - bcdx \log(F))}{bc \log(F)}$$

input `Integrate[F^(c*(a + b/x))*(d + e/x^2 - (b*c*d*Log[F])/x),x]`

output `-((F^(c*(a + b/x))*(e - b*c*d*x*Log[F]))/(b*c*Log[F]))`

**Rubi [A] (verified)**

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( -\frac{bcd \log(F)}{x} + d + \frac{e}{x^2} \right) dx$$

↓ 2726

$$-\frac{x^2 F^{c\left(a+\frac{b}{x}\right)} \left( \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right)}{bc \log(F)}$$

input `Int[F^(c*(a + b/x))*(d + e/x^2 - (b*c*d*Log[F])/x),x]`

output `-((F^(c*(a + b/x))*x^2*(e/x^2 - (b*c*d*Log[F])/x))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] := With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

method	result
risch	$\frac{(bcdx \ln(F) - e)F^{\frac{c(ax+b)}{x}}}{bc \ln(F)}$
parallelrisc	$\frac{\ln(F)x F^{c(a+\frac{b}{x})}bcd - F^{c(a+\frac{b}{x})}e}{bc \ln(F)}$
norman	$\frac{dx^2 e^{c(a+\frac{b}{x}) \ln(F)} - \frac{e x e^{c(a+\frac{b}{x}) \ln(F)}}{\ln(F)bc}}{x}$
meijerg	$F^{ac}d \ln(F)bc \left( \frac{x}{bc \ln(F)} + 1 + \ln(x) - \ln(-bc) - \ln(\ln(F)) - \frac{x \left( \frac{2bc \ln(F)}{x} + 2 \right)}{2bc \ln(F)} + \frac{x e^{\frac{bc \ln(F)}{x}}}{bc \ln(F)} + \ln \right)$

input `int(F^(c*(a+b/x))*(d+e/x^2-b*c*d*ln(F)/x),x,method=_RETURNVERBOSE)`

output `(b*c*d*x*ln(F)-e)/b/c/ln(F)*F^(c*(a*x+b)/x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int F^{c(a+\frac{b}{x})} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = \frac{(bcdx \log(F) - e)F^{\frac{acx+bc}{x}}}{bc \log(F)}$$

input `integrate(F^(c*(a+b/x))*(d+e/x^2-b*c*d*log(F)/x),x, algorithm="fricas")`

output `(b*c*d*x*log(F) - e)*F^((a*c*x + b*c)/x)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int F^{c(a+\frac{b}{x})} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx$$

$$= \begin{cases} \frac{F^{c(a+\frac{b}{x})} (bcdx \log(F) - e)}{bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ -bcd \log(F) \log(x) + dx - \frac{e}{x} & \text{otherwise} \end{cases}$$

input `integrate(F**(c*(a+b/x))*(d+e/x**2-b*c*d*ln(F)/x),x)`

output `Piecewise((F**(c*(a + b/x))*(b*c*d*x*log(F) - e)/(b*c*log(F)), Ne(b*c*log(F), 0)), (-b*c*d*log(F)*log(x) + d*x - e/x, True))`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int F^{c(a+\frac{b}{x})} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = F^{ac} bcd \operatorname{Ei} \left( \frac{bc \log(F)}{x} \right) \log(F)$$

$$- F^{ac} bcd \Gamma \left( -1, -\frac{bc \log(F)}{x} \right) \log(F)$$

$$- \frac{F^{ac+\frac{bc}{x}} e}{bc \log(F)}$$

input `integrate(F^(c*(a+b/x))*(d+e/x^2-b*c*d*log(F)/x),x, algorithm="maxima")`

output `F^(a*c)*b*c*d*Ei(b*c*log(F)/x)*log(F) - F^(a*c)*b*c*d*gamma(-1, -b*c*log(F)/x)*log(F) - F^(a*c + b*c/x)*e/(b*c*log(F))`

**Giac [F]**

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = \int - \left( \frac{bcd \log(F)}{x} - d - \frac{e}{x^2} \right) F^{\left(a+\frac{b}{x}\right)c} dx$$

input `integrate(F^(c*(a+b/x))*(d+e/x^2-b*c*d*log(F)/x),x, algorithm="giac")`

output `integrate(-(b*c*d*log(F)/x - d - e/x^2)*F^((a + b/x)*c), x)`

**Mupad [B] (verification not implemented)**

Time = 23.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = -\frac{F^{a c + \frac{b c}{x}} (e - b c d x \ln(F))}{b c \ln(F)}$$

input `int(F^(c*(a + b/x))*(d + e/x^2 - (b*c*d*log(F))/x),x)`

output `-(F^(a*c + (b*c)/x)*(e - b*c*d*x*log(F)))/(b*c*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int F^{c\left(a+\frac{b}{x}\right)} \left( d + \frac{e}{x^2} - \frac{bcd \log(F)}{x} \right) dx = \frac{f^{\frac{acx+bc}{x}} (\log(f) bcdx - e)}{\log(f) bc}$$

input `int(F^(c*(a+b/x))*(d+e/x^2-b*c*d*log(F)/x),x)`

output `(f**((a*c*x + b*c)/x)*(log(f)*b*c*d*x - e))/(log(f)*b*c)`

$$3.12 \quad \int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx$$

Optimal result	134
Mathematica [A] (verified)	134
Rubi [A] (verified)	135
Maple [A] (verified)	136
Fricas [A] (verification not implemented)	136
Sympy [A] (verification not implemented)	137
Maxima [C] (verification not implemented)	137
Giac [F]	138
Mupad [B] (verification not implemented)	138
Reduce [B] (verification not implemented)	138

### Optimal result

Integrand size = 29, antiderivative size = 35

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = -\frac{F^{c\left(a+\frac{b}{x^2}\right)}(e - 2bcdx \log(F))}{2bc \log(F)}$$

output `-1/2*F^(c*(a+b/x^2))*(e-2*b*c*d*x*ln(F))/b/c/ln(F)`

### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = -\frac{F^{c\left(a+\frac{b}{x^2}\right)}(e - 2bcdx \log(F))}{2bc \log(F)}$$

input `Integrate[F^(c*(a + b/x^2))*(d + e/x^3 - (2*b*c*d*Log[F])/x^2),x]`

output `-1/2*(F^(c*(a + b/x^2))*(e - 2*b*c*d*x*Log[F]))/(b*c*Log[F])`

**Rubi [A] (verified)**

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( -\frac{2bcd \log(F)}{x^2} + d + \frac{e}{x^3} \right) dx$$

↓ 2726

$$-\frac{x^3 F^{c\left(a+\frac{b}{x^2}\right)} \left( \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right)}{2bc \log(F)}$$

input `Int[F^(c*(a + b/x^2))*(d + e/x^3 - (2*b*c*d*Log[F])/x^2),x]`

output `-1/2*(F^(c*(a + b/x^2))*x^3*(e/x^3 - (2*b*c*d*Log[F])/x^2))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`



**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(2bcdx \ln(F) - e)F^{\frac{c(ax^2+b)}{x^2}}}{2cb \ln(F)}$
parallelrisc	$\frac{2 \ln(F) x F^{c\left(a+\frac{b}{x^2}\right)} bcd - F^{c\left(a+\frac{b}{x^2}\right)} e}{2cb \ln(F)}$
norman	$\frac{d x^3 e^{c\left(a+\frac{b}{x^2}\right) \ln(F)} - e x^2 e^{c\left(a+\frac{b}{x^2}\right) \ln(F)}}{x^2 \cdot 2 \ln(F) bc}$
meijerg	$-\frac{F^{ac} d \sqrt{-bc} \sqrt{\ln(F)} \left( -\frac{2xe}{\sqrt{-bc} \sqrt{\ln(F)}} + \frac{2\sqrt{b} \sqrt{c} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\ln(F)}}{x}\right)}{\sqrt{-bc}} \right)}{2} + \frac{F^{ac} e \left( 1 - e^{\frac{bc \ln(F)}{x^2}} \right)}{2bc \ln(F)} + F^{ac} d \sqrt{b} \sqrt{c} \sqrt{\ln(F)}$

input `int(F^(c*(a+b/x^2))*(d+e/x^3-2*b*c*d*ln(F)/x^2),x,method=_RETURNVERBOSE)`

output `1/2*(2*b*c*d*x*ln(F)-e)/c/b/ln(F)*F^(c*(a*x^2+b)/x^2)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = \frac{(2bcdx \log(F) - e)F^{\frac{acx^2+bc}{x^2}}}{2bc \log(F)}$$

input `integrate(F^(c*(a+b/x^2))*(d+e/x^3-2*b*c*d*log(F)/x^2),x, algorithm="fricas")`

output `1/2*(2*b*c*d*x*log(F) - e)*F^((a*c*x^2 + b*c)/x^2)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = \begin{cases} \frac{F^{c\left(a+\frac{b}{x^2}\right)} (2bcdx \log(F) - e)}{2bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ dx + \frac{4bcdx \log(F) - e}{2x^2} & \text{otherwise} \end{cases}$$

input `integrate(F**(c*(a+b/x**2))*(d+e/x**3-2*b*c*d*ln(F)/x**2), x)`

output `Piecewise((F**(c*(a + b/x**2))*(2*b*c*d*x*log(F) - e)/(2*b*c*log(F)), Ne(b*c*log(F), 0)), (d*x + (4*b*c*d*x*log(F) - e)/(2*x**2), True))`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.83

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = \frac{\sqrt{\pi} F^{ac} bcd \left( \operatorname{erf} \left( \sqrt{-\frac{bc \log(F)}{x^2}} \right) - 1 \right) \log(F)}{x \sqrt{-\frac{bc \log(F)}{x^2}}} + \frac{1}{2} F^{ac} dx \sqrt{-\frac{bc \log(F)}{x^2}} \Gamma \left( -\frac{1}{2}, -\frac{bc \log(F)}{x^2} \right) - \frac{F^{ac+\frac{bc}{x^2}} e}{2bc \log(F)}$$

input `integrate(F^(c*(a+b/x^2))*(d+e/x^3-2*b*c*d*log(F)/x^2), x, algorithm="maxima")`

output `sqrt(pi)*F^(a*c)*b*c*d*(erf(sqrt(-b*c*log(F)/x^2)) - 1)*log(F)/(x*sqrt(-b*c*log(F)/x^2)) + 1/2*F^(a*c)*d*x*sqrt(-b*c*log(F)/x^2)*gamma(-1/2, -b*c*log(F)/x^2) - 1/2*F^(a*c + b*c/x^2)*e/(b*c*log(F))`

**Giac [F]**

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = \int - \left( \frac{2bcd \log(F)}{x^2} - d - \frac{e}{x^3} \right) F^{\left(a+\frac{b}{x^2}\right)c} dx$$

input `integrate(F^(c*(a+b/x^2))*(d+e/x^3-2*b*c*d*log(F)/x^2),x, algorithm="giac")`

output `integrate(-(2*b*c*d*log(F)/x^2 - d - e/x^3)*F^((a + b/x^2)*c), x)`

**Mupad [B] (verification not implemented)**

Time = 23.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = -\frac{F^{a+c+\frac{bc}{x^2}} \left( \frac{e}{2} - bcdx \ln(F) \right)}{bc \ln(F)}$$

input `int(F^(c*(a + b/x^2))*(d + e/x^3 - (2*b*c*d*log(F))/x^2),x)`

output `-(F^(a*c + (b*c)/x^2)*(e/2 - b*c*d*x*log(F)))/(b*c*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int F^{c\left(a+\frac{b}{x^2}\right)} \left( d + \frac{e}{x^3} - \frac{2bcd \log(F)}{x^2} \right) dx = \frac{f^{\frac{acx^2+bc}{x^2}} (2 \log(f) bcdx - e)}{2 \log(f) bc}$$

input `int(F^(c*(a+b/x^2))*(d+e/x^3-2*b*c*d*log(F)/x^2),x)`

output `(f**((a*c*x**2 + b*c)/x**2)*(2*log(f)*b*c*d*x - e))/(2*log(f)*b*c)`

$$3.13 \quad \int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx$$

Optimal result . . . . .	139
Mathematica [A] (verified) . . . . .	139
Rubi [A] (verified) . . . . .	140
Maple [A] (verified) . . . . .	141
Fricas [A] (verification not implemented) . . . . .	141
Sympy [A] (verification not implemented) . . . . .	142
Maxima [C] (verification not implemented) . . . . .	142
Giac [F] . . . . .	143
Mupad [B] (verification not implemented) . . . . .	143
Reduce [B] (verification not implemented) . . . . .	143

### Optimal result

Integrand size = 29, antiderivative size = 35

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = -\frac{F^{c\left(a+\frac{b}{x^3}\right)}(e - 3bcdx \log(F))}{3bc \log(F)}$$

output `-1/3*F^(c*(a+b/x^3))*(e-3*b*c*d*x*ln(F))/b/c/ln(F)`

### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = -\frac{F^{c\left(a+\frac{b}{x^3}\right)}(e - 3bcdx \log(F))}{3bc \log(F)}$$

input `Integrate[F^(c*(a + b/x^3))*(d + e/x^4 - (3*b*c*d*Log[F])/x^3),x]`

output `-1/3*(F^(c*(a + b/x^3))*(e - 3*b*c*d*x*Log[F]))/(b*c*Log[F])`

**Rubi [A] (verified)**

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( -\frac{3bcd \log(F)}{x^3} + d + \frac{e}{x^4} \right) dx$$

↓ 2726

$$-\frac{x^4 F^{c\left(a+\frac{b}{x^3}\right)} \left( \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right)}{3bc \log(F)}$$

input `Int[F^(c*(a + b/x^3))*(d + e/x^4 - (3*b*c*d*Log[F])/x^3),x]`

output `-1/3*(F^(c*(a + b/x^3))*x^4*(e/x^4 - (3*b*c*d*Log[F])/x^3))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(3bcdx \ln(F) - e)F^{\frac{c(ax^3+b)}{x^3}}}{3cb \ln(F)}$
parallelrisch	$\frac{3 \ln(F)x F^{c\left(a+\frac{b}{x^3}\right)} bcd - F^{c\left(a+\frac{b}{x^3}\right)} e}{3cb \ln(F)}$
norman	$\frac{dx^4 e^{c\left(a+\frac{b}{x^3}\right) \ln(F)} - e x^3 e^{c\left(a+\frac{b}{x^3}\right) \ln(F)}}{x^3 3 \ln(F) bc}$
meijerg	$-\frac{F^{ac} d (-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}} \left( \frac{3 \ln(F)^{\frac{2}{3}} cb \Gamma\left(\frac{2}{3}\right)}{x^2 (-bc)^{\frac{1}{3}} \left(-\frac{cb \ln(F)}{x^3}\right)^{\frac{2}{3}}} - \frac{3xe^{\frac{cb \ln(F)}{x^3}}}{(-bc)^{\frac{1}{3}} \ln(F)^{\frac{1}{3}}} - \frac{3 \ln(F)^{\frac{2}{3}} cb \Gamma\left(\frac{2}{3}, -\frac{cb \ln(F)}{x^3}\right)}{x^2 (-bc)^{\frac{1}{3}} \left(-\frac{cb \ln(F)}{x^3}\right)^{\frac{2}{3}}} \right)}{3} + \frac{F^{ac} e \left(1 - e^{-\frac{cb \ln(F)}{x^3}}\right)}{3bc \ln(F)}$

input `int(F^(c*(a+b/x^3))*(d+e/x^4-3*b*c*d*ln(F)/x^3),x,method=_RETURNVERBOSE)`

output `1/3*(3*b*c*d*x*ln(F)-e)/c/b/ln(F)*F^(c*(a*x^3+b)/x^3)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = \frac{(3bcdx \log(F) - e)F^{\frac{acx^3+bc}{x^3}}}{3bc \log(F)}$$

input `integrate(F^(c*(a+b/x^3))*(d+e/x^4-3*b*c*d*log(F)/x^3),x, algorithm="fricas")`

output `1/3*(3*b*c*d*x*log(F) - e)*F^((a*c*x^3 + b*c)/x^3)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.71

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = \begin{cases} \frac{F^{c\left(a+\frac{b}{x^3}\right)} (3bcdx \log(F) - e)}{3bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ dx + \frac{9bcdx \log(F) - 2e}{6x^3} & \text{otherwise} \end{cases}$$

input `integrate(F**(c*(a+b/x**3))*(d+e/x**4-3*b*c*d*ln(F)/x**3), x)`

output `Piecewise((F**(c*(a + b/x**3))*(3*b*c*d*x*log(F) - e)/(3*b*c*log(F)), Ne(b*c*log(F), 0)), (d*x + (9*b*c*d*x*log(F) - 2*e)/(6*x**3), True))`

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.69

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = \frac{1}{3} F^{ac} dx \left( -\frac{bc \log(F)}{x^3} \right)^{\frac{1}{3}} \Gamma \left( -\frac{1}{3}, -\frac{bc \log(F)}{x^3} \right) - \frac{F^{ac} bcd \Gamma \left( \frac{2}{3}, -\frac{bc \log(F)}{x^3} \right) \log(F)}{x^2 \left( -\frac{bc \log(F)}{x^3} \right)^{\frac{2}{3}}} - \frac{F^{ac+\frac{bc}{x^3}} e}{3bc \log(F)}$$

input `integrate(F^(c*(a+b/x^3))*(d+e/x^4-3*b*c*d*log(F)/x^3), x, algorithm="maxima")`

output `1/3*F^(a*c)*d*x*(-b*c*log(F)/x^3)^(1/3)*gamma(-1/3, -b*c*log(F)/x^3) - F^(a*c)*b*c*d*gamma(2/3, -b*c*log(F)/x^3)*log(F)/(x^2*(-b*c*log(F)/x^3)^(2/3)) - 1/3*F^(a*c + b*c/x^3)*e/(b*c*log(F))`

**Giac [F]**

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = \int F^{\left(a+\frac{b}{x^3}\right)c} \left( d - \frac{3bcd \log(F)}{x^3} + \frac{e}{x^4} \right) dx$$

input `integrate(F^(c*(a+b/x^3))*(d+e/x^4-3*b*c*d*log(F)/x^3),x, algorithm="giac")`

output `integrate(F^((a + b/x^3)*c)*(d - 3*b*c*d*log(F)/x^3 + e/x^4), x)`

**Mupad [B] (verification not implemented)**

Time = 22.93 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = -\frac{F^{a+\frac{bc}{x^3}} \left( \frac{e}{3} - bcdx \ln(F) \right)}{bc \ln(F)}$$

input `int(F^(c*(a + b/x^3))*(d + e/x^4 - (3*b*c*d*log(F))/x^3),x)`

output `-(F^(a*c + (b*c)/x^3)*(e/3 - b*c*d*x*log(F)))/(b*c*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.14

$$\int F^{c\left(a+\frac{b}{x^3}\right)} \left( d + \frac{e}{x^4} - \frac{3bcd \log(F)}{x^3} \right) dx = \frac{f^{\frac{acx^3+bc}{x^3}} (3 \log(f) bcdx - e)}{3 \log(f) bc}$$

input `int(F^(c*(a+b/x^3))*(d+e/x^4-3*b*c*d*log(F)/x^3),x)`

output `(f**((a*c*x**3 + b*c)/x**3)*(3*log(f)*b*c*d*x - e))/(3*log(f)*b*c)`



### 3.14 $\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx$

Optimal result . . . . .	144
Mathematica [A] (verified) . . . . .	144
Rubi [A] (verified) . . . . .	145
Maple [A] (verified) . . . . .	146
Fricas [A] (verification not implemented) . . . . .	146
Sympy [B] (verification not implemented) . . . . .	147
Maxima [C] (verification not implemented) . . . . .	147
Giac [A] (verification not implemented) . . . . .	148
Mupad [F(-1)] . . . . .	148
Reduce [B] (verification not implemented) . . . . .	149

#### Optimal result

Integrand size = 31, antiderivative size = 35

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx = \frac{F^{c(a+bx^n)}(e + bcdnx \log(F))}{bcn \log(F)}$$

output `F^(c*(a+b*x^n))*(e+b*c*d*n*x*ln(F))/b/c/n/ln(F)`

#### Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx = \frac{F^{c(a+bx^n)}(e + bcdnx \log(F))}{bcn \log(F)}$$

input `Integrate[F^(c*(a + b*x^n))*(d + e*x^(-1 + n) + b*c*d*n*x^n*Log[F]),x]`

output `(F^(c*(a + b*x^n))*(e + b*c*d*n*x*Log[F]))/(b*c*n*Log[F])`

**Rubi [A] (verified)**

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$ , Rules used = {2726}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx^n)} (bcdn \log(F)x^n + d + ex^{n-1}) dx$$

$$\downarrow 2726$$

$$\frac{x^{1-n} F^{c(a+bx^n)} (bcdn \log(F)x^n + ex^{n-1})}{bcn \log(F)}$$

input `Int[F^(c*(a + b*x^n))*(d + e*x^(-1 + n) + b*c*d*n*x^n*Log[F]),x]`

output `(F^(c*(a + b*x^n))*x^(1 - n)*(e*x^(-1 + n) + b*c*d*n*x^n*Log[F]))/(b*c*n*Log[F])`

**Defintions of rubi rules used**

rule 2726 `Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = v*(y/(Log[F]*D[u, x]))}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]`

**Maple [A] (verified)**

Time = 8.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

method	result
risch	$\frac{F^{c(a+bx^n)}(e+bcn \ln(F))}{bcn \ln(F)}$
norman	$dx e^{c(a+be^n \ln(x)) \ln(F)} + \frac{e e^{c(a+be^n \ln(x)) \ln(F)}}{\ln(F)bcn}$
meijerg	$F^{ac} d(-bc)^{-\frac{1}{n}} \ln(F)^{-\frac{1}{n}} \left( \frac{nx(-bc)^{\frac{1}{n}} \ln(F)^{\frac{1}{n}} (x^n \ln(F)bcn+n+1) \Gamma(1-\frac{1}{n}) \Gamma(\frac{1+n}{n}+1) \text{LaguerreL}(-\frac{1}{n}, \frac{1+n}{n}, x^n bc \ln(F))}{(1+n) \Gamma(-\frac{1}{n} + \frac{1+n}{n} + 1)} \right) - \frac{n^2 x^{1+n} (-bc)^{\frac{1}{n}} \ln(F)^{\frac{1}{n}}}{n}$

input

```
int(F^(c*(a+b*x^n))*(d+e*x^(-1+n)+b*c*d*n*x^n*ln(F)),x,method=_RETURNVERBOSE)
```

output

$$F^{c(a+bx^n)}(e+bcn \ln(F))/bcn \ln(F)$$
**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx = \frac{(bcdnx \log(F) + e)e^{(bcx^n \log(F) + ac \log(F))}}{bcn \log(F)}$$

input

```
integrate(F^(c*(a+b*x^n))*(d+e*x^(-1+n)+b*c*d*n*x^n*log(F)),x, algorithm="fricas")
```

output

$$(b*c*d*n*x*log(F) + e)*e^{(b*c*x^n*log(F) + a*c*log(F))}/(b*c*n*log(F))$$

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 104 vs.  $2(31) = 62$ .

Time = 4.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.97

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx$$

$$= \begin{cases} dx + e \log(x) & \text{for } F = 1 \wedge b = 0 \wedge c = 0 \wedge n = 0 \\ F^{ac} \left( dx + \frac{exx^{n-1}}{n} \right) & \text{for } b = 0 \\ dx + \frac{exx^{n-1}}{n} & \text{for } c = 0 \\ F^{c(a+b)}(dx + e \log(x)) & \text{for } n = 0 \\ dx + \frac{exx^{n-1}}{n} & \text{for } F = 1 \\ F^{ac+bcx^n} dx + \frac{F^{ac+bcx^n} e}{bcn \log(F)} & \text{otherwise} \end{cases}$$

input

```
integrate(F**(c*(a+b*x**n))*(d+e*x**(-1+n)+b*c*d*n*x**n*ln(F)),x)
```

output

```
Piecewise((d*x + e*log(x), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (F*
*(a*c)*(d*x + e*x*x**(n - 1)/n), Eq(b, 0)), (d*x + e*x*x**(n - 1)/n, Eq(c,
0)), (F**(c*(a + b))*(d*x + e*log(x)), Eq(n, 0)), (d*x + e*x*x**(n - 1)/n
, Eq(F, 1)), (F**(a*c + b*c*x**n)*d*x + F**(a*c + b*c*x**n)*e/(b*c*n*log(F
)), True))
```

**Maxima [C] (verification not implemented)**

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.46

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx$$

$$= -\frac{F^{ac}bcdx^{n+1}\Gamma\left(\frac{n+1}{n}, -bcx^n \log(F)\right) \log(F)}{(-bcx^n \log(F))^{\frac{n+1}{n}}}$$

$$- \frac{F^{ac}dx\Gamma\left(\frac{1}{n}, -bcx^n \log(F)\right)}{(-bcx^n \log(F))^{\left(\frac{1}{n}\right)} n} + \frac{F^{bcx^n+ac}e}{bcn \log(F)}$$

input `integrate(F^(c*(a+b*x^n))*(d+e*x^(-1+n)+b*c*d*n*x^n*log(F)),x, algorithm="maxima")`

output 
$$-F^{(a*c)}*b*c*d*x^{(n+1)}*\text{gamma}((n+1)/n, -b*c*x^n*\log(F))*\log(F)/(-b*c*x^n*\log(F))^{(n+1)/n} - F^{(a*c)}*d*x*\text{gamma}(1/n, -b*c*x^n*\log(F))/((-b*c*x^n*\log(F))^{(1/n)*n}) + F^{(b*c*x^n+a*c)}*e/(b*c*n*\log(F))$$

### Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.46

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx = \frac{F^{bcx^n} F^{ac} bcdnx \log(F) + F^{bcx^n} F^{ac} e}{bcn \log(F)}$$

input `integrate(F^(c*(a+b*x^n))*(d+e*x^(-1+n)+b*c*d*n*x^n*log(F)),x, algorithm="giac")`

output 
$$(F^{(b*c*x^n)}*F^{(a*c)}*b*c*d*n*x*\log(F) + F^{(b*c*x^n)}*F^{(a*c)}*e)/(b*c*n*\log(F))$$

### Mupad [F(-1)]

Timed out.

$$\begin{aligned} & \int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx \\ &= \int F^{c(a+bx^n)}(d + ex^{n-1} + bcdnx^n \ln(F)) dx \end{aligned}$$

input `int(F^(c*(a + b*x^n))*(d + e*x^(n - 1) + b*c*d*n*x^n*log(F)),x)`

output `int(F^(c*(a + b*x^n))*(d + e*x^(n - 1) + b*c*d*n*x^n*log(F)), x)`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx^n)}(d + ex^{-1+n} + bcdnx^n \log(F)) dx = \frac{f^{x^{nbc+ac}}(\log(f) bcdnx + e)}{\log(f) bcn}$$

input `int(F^(c*(a+b*x^n))*(d+e*x^(-1+n)+b*c*d*n*x^n*log(F)),x)`

output `(f**(x**n*b*c + a*c)*(log(f)*b*c*d*n*x + e))/(log(f)*b*c*n)`

### 3.15 $\int F^{c(a+bx)}(d - ex^2)^2 dx$

Optimal result . . . . .	150
Mathematica [A] (verified) . . . . .	150
Rubi [A] (verified) . . . . .	151
Maple [A] (verified) . . . . .	152
Fricas [A] (verification not implemented) . . . . .	153
Sympy [A] (verification not implemented) . . . . .	153
Maxima [A] (verification not implemented) . . . . .	154
Giac [C] (verification not implemented) . . . . .	154
Mupad [B] (verification not implemented) . . . . .	155
Reduce [B] (verification not implemented) . . . . .	156

#### Optimal result

Integrand size = 20, antiderivative size = 141

$$\int F^{c(a+bx)}(d - ex^2)^2 dx = \frac{24e^2 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{24e^2 F^{c(a+bx)} x}{b^4 c^4 \log^4(F)} - \frac{4e F^{c(a+bx)}(d - 3ex^2)}{b^3 c^3 \log^3(F)} + \frac{4e F^{c(a+bx)} x(d - ex^2)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d - ex^2)^2}{bc \log(F)}$$

output

```
24*e^2*F^(c*(b*x+a))/b^5/c^5/ln(F)^5-24*e^2*F^(c*(b*x+a))*x/b^4/c^4/ln(F)^4-4*e*F^(c*(b*x+a))*(-3*e*x^2+d)/b^3/c^3/ln(F)^3+4*e*F^(c*(b*x+a))*x*(-e*x^2+d)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(-e*x^2+d)^2/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d - ex^2)^2 dx = \frac{F^{c(a+bx)} \left( 24e^2 - 24bce^2 x \log(F) - 4b^2 c^2 e(d - 3ex^2) \log^2(F) + 4b^3 c^3 ex(d - ex^2) \log^3(F) + b^4 c^4 (d - ex^2) \log^4(F) \right)}{b^5 c^5 \log^5(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d - e*x^2)^2,x]
```

output

$$\frac{(F^{c(a+bx)}) \cdot (24e^2 - 24b^2c^2e^2x \operatorname{Log}[F] - 4b^2c^2e(d - 3e^2x^2) \operatorname{Log}[F]^2 + 4b^3c^3e^2x(d - e^2x^2) \operatorname{Log}[F]^3 + b^4c^4(d - e^2x^2)^2 \operatorname{Log}[F]^4)}{b^5c^5 \operatorname{Log}[F]^5}$$

**Rubi [A] (verified)**

Time = 0.72 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.60, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - ex^2)^2 F^{c(a+bx)} dx$$

↓ 2626

$$\int (d^2 F^{c(a+bx)} - 2dex^2 F^{c(a+bx)} + e^2 x^4 F^{c(a+bx)}) dx$$

↓ 2009

$$\frac{24e^2 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{24e^2 x F^{c(a+bx)}}{b^4 c^4 \log^4(F)} - \frac{4de F^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \frac{12e^2 x^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \frac{4dex F^{c(a+bx)}}{b^2 c^2 \log^2(F)} - \frac{4e^2 x^3 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{d^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2dex^2 F^{c(a+bx)}}{bc \log(F)} + \frac{e^2 x^4 F^{c(a+bx)}}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a+bx)} \cdot (d - e^2 x^2)^2, x]$$

output

$$\frac{(24e^2 F^{c(a+bx)})}{b^5 c^5 \operatorname{Log}[F]^5} - \frac{(24e^2 F^{c(a+bx)}) \cdot x}{b^4 c^4 \operatorname{Log}[F]^4} - \frac{(4d e F^{c(a+bx)})}{b^3 c^3 \operatorname{Log}[F]^3} + \frac{(12e^2 F^{c(a+bx)}) \cdot x^2}{b^3 c^3 \operatorname{Log}[F]^3} + \frac{(4d e F^{c(a+bx)}) \cdot x}{b^2 c^2 \operatorname{Log}[F]^2} - \frac{(4e^2 F^{c(a+bx)}) \cdot x^3}{b^2 c^2 \operatorname{Log}[F]^2} + \frac{(d^2 F^{c(a+bx)})}{(b c \operatorname{Log}[F])} - \frac{(2d e F^{c(a+bx)}) \cdot x^2}{(b c \operatorname{Log}[F])} + \frac{(e^2 F^{c(a+bx)}) \cdot x^4}{(b c \operatorname{Log}[F])}$$



Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

method	result
gospers	$\frac{(e^2 x^4 \ln(F)^4 b^4 c^4 - 2 \ln(F)^4 b^4 c^4 d e x^2 + \ln(F)^4 b^4 c^4 d^2 - 4 e^2 x^3 \ln(F)^3 b^3 c^3 + 4 \ln(F)^3 b^3 c^3 d e x + 12 \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d e \ln(F)^2 b^2 c^2 e^2 x - 4 d^2 \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d e \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d^2 \ln(F)^2 b^2 c^2 e^2 x^2) \ln(F)^5 b^5 c^5}{\ln(F)^5 b^5 c^5}$
risch	$\frac{(e^2 x^4 \ln(F)^4 b^4 c^4 - 2 \ln(F)^4 b^4 c^4 d e x^2 + \ln(F)^4 b^4 c^4 d^2 - 4 e^2 x^3 \ln(F)^3 b^3 c^3 + 4 \ln(F)^3 b^3 c^3 d e x + 12 \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d e \ln(F)^2 b^2 c^2 e^2 x - 4 d^2 \ln(F)^2 b^2 c^2 e^2 x^2) \ln(F)^5 b^5 c^5}{\ln(F)^5 b^5 c^5}$
orering	$\frac{(e^2 x^4 \ln(F)^4 b^4 c^4 - 2 \ln(F)^4 b^4 c^4 d e x^2 + \ln(F)^4 b^4 c^4 d^2 - 4 e^2 x^3 \ln(F)^3 b^3 c^3 + 4 \ln(F)^3 b^3 c^3 d e x + 12 \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d e \ln(F)^2 b^2 c^2 e^2 x - 4 d^2 \ln(F)^2 b^2 c^2 e^2 x^2) \ln(F)^5 b^5 c^5}{\ln(F)^5 b^5 c^5}$
meijerg	$-\frac{F^{ac} e^2 \left( 24 - \frac{(5 b^4 c^4 x^4 \ln(F)^4 - 20 b^3 c^3 x^3 \ln(F)^3 + 60 b^2 c^2 x^2 \ln(F)^2 - 120 b c x \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{b^5 c^5 \ln(F)^5} + \frac{2 F^{ac} e d \left( 2 - \frac{(3 b^2 c^2 x^2 \ln(F)^2 - 2 b c x \ln(F) + 2) e^{bcx \ln(F)}}{b^3 c^3} \right)}{b^3 c^3}$
norman	$\frac{(\ln(F)^4 b^4 c^4 d^2 - 4 d e \ln(F)^2 b^2 c^2 + 24 e^2) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5} + \frac{e^2 x^4 e^{c(bx+a) \ln(F)}}{\ln(F) b c} - \frac{4 e^2 x^3 e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} - \frac{2 e (\ln(F)^2 b^2 c^2 d - 2 b c x \ln(F) + 2) e^{c(bx+a) \ln(F)}}{c^3 b^3}$
parallelrisch	$\frac{x^4 F^{c(bx+a)} e^2 \ln(F)^4 b^4 c^4 - 2 \ln(F)^4 x^2 F^{c(bx+a)} b^4 c^4 d e + \ln(F)^4 F^{c(bx+a)} b^4 c^4 d^2 - 4 e^2 F^{c(bx+a)} x^3 \ln(F)^3 b^3 c^3 + 4 \ln(F)^3 x F^{c(bx+a)} b^3 c^3 d e x + 12 \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d e \ln(F)^2 b^2 c^2 e^2 x - 4 d^2 \ln(F)^2 b^2 c^2 e^2 x^2) \ln(F)^5 b^5 c^5}{\ln(F)^5 b^5 c^5}$

input `int(F^(c*(b*x+a))*(-e*x^2+d)^2,x,method=_RETURNVERBOSE)`

output  $(e^2 x^4 \ln(F)^4 b^4 c^4 - 2 \ln(F)^4 b^4 c^4 d e x^2 + \ln(F)^4 b^4 c^4 d^2 - 4 e^2 x^3 \ln(F)^3 b^3 c^3 + 4 \ln(F)^3 b^3 c^3 d e x + 12 \ln(F)^2 b^2 c^2 e^2 x^2 - 4 d e \ln(F)^2 b^2 c^2 e^2 x - 4 d^2 \ln(F)^2 b^2 c^2 e^2 x^2) \ln(F)^5 b^5 c^5 / c^5$

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int F^{c(a+bx)} (d - ex^2)^2 dx = \frac{(24 bce^2 x \log(F) - (b^4 c^4 e^2 x^4 - 2 b^4 c^4 dex^2 + b^4 c^4 d^2) \log(F)^4 + 4 (b^3 c^3 e^2 x^3 - b^3 c^3 dex) \log(F)^3 - 4 (3 b^2 c^2 e^2 x^2 - b^2 c^2 d e) \log(F)^2 - 24 e^2) F^{(b c x + a c)}}{b^5 c^5 \log(F)^5}$$

input `integrate(F^((b*x+a)*c)*(-e*x^2+d)^2,x, algorithm="fricas")`

output `-(24*b*c*e^2*x*log(F) - (b^4*c^4*e^2*x^4 - 2*b^4*c^4*d*e*x^2 + b^4*c^4*d^2)*log(F)^4 + 4*(b^3*c^3*e^2*x^3 - b^3*c^3*d*e*x)*log(F)^3 - 4*(3*b^2*c^2*e^2*x^2 - b^2*c^2*d*e)*log(F)^2 - 24*e^2)*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5)`

**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.43

$$\int F^{c(a+bx)} (d - ex^2)^2 dx = \begin{cases} \frac{F^{c(a+bx)} (b^4 c^4 d^2 \log(F)^4 - 2 b^4 c^4 dex^2 \log(F)^4 + b^4 c^4 e^2 x^4 \log(F)^4 + 4 b^3 c^3 dex \log(F)^3 - 4 b^3 c^3 e^2 x^3 \log(F)^3 - 4 b^2 c^2 de \log(F)^2 + 12 b^2 c^2 e^2 x^2 \log(F)^2 - 24 e^2) F^{(b c x + a c)}}{b^5 c^5 \log(F)^5} \\ d^2 x - \frac{2 dex^3}{3} + \frac{e^2 x^5}{5} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(-e*x**2+d)**2,x)`

output `Piecewise((F**(c*(a + b*x))*(b**4*c**4*d**2*log(F)**4 - 2*b**4*c**4*d*e*x**2*log(F)**4 + b**4*c**4*e**2*x**4*log(F)**4 + 4*b**3*c**3*d*e*x*log(F)**3 - 4*b**3*c**3*e**2*x**3*log(F)**3 - 4*b**2*c**2*d*e*log(F)**2 + 12*b**2*c**2*e**2*x**2*log(F)**2 - 24*b*c*e**2*x*log(F) + 24*e**2)/(b**5*c**5*log(F)**5), Ne(b**5*c**5*log(F)**5, 0)), (d**2*x - 2*d*e*x**3/3 + e**2*x**5/5, True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.30

$$\int F^{c(a+bx)} (d - ex^2)^2 dx$$

$$= \frac{F^{bcx+ac} d^2}{bc \log(F)} - \frac{2(F^{ac} b^2 c^2 x^2 \log(F)^2 - 2 F^{ac} bcx \log(F) + 2 F^{ac}) F^{bcx} de}{b^3 c^3 \log(F)^3}$$

$$+ \frac{(F^{ac} b^4 c^4 x^4 \log(F)^4 - 4 F^{ac} b^3 c^3 x^3 \log(F)^3 + 12 F^{ac} b^2 c^2 x^2 \log(F)^2 - 24 F^{ac} bcx \log(F) + 24 F^{ac}) F^{bcx}}{b^5 c^5 \log(F)^5}$$

input `integrate(F^((b*x+a)*c)*(-e*x^2+d)^2,x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d^2/(b*c*log(F)) - 2*(F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*d*e/(b^3*c^3*log(F)^3) + (F^(a*c)*b^4*c^4*x^4*log(F)^4 - 4*F^(a*c)*b^3*c^3*x^3*log(F)^3 + 12*F^(a*c)*b^2*c^2*x^2*log(F)^2 - 24*F^(a*c)*b*c*x*log(F) + 24*F^(a*c))*F^(b*c*x)*e^2/(b^5*c^5*log(F)^5)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 5426, normalized size of antiderivative = 38.48

$$\int F^{c(a+bx)} (d - ex^2)^2 dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(-e*x^2+d)^2,x, algorithm="giac")`

output

```

-((4*(pi^3*b^4*c^4*e^2*x^4*log(abs(F))*sgn(F) - pi*b^4*c^4*e^2*x^4*log(abs
(F))^3*sgn(F) - pi^3*b^4*c^4*e^2*x^4*log(abs(F)) + pi*b^4*c^4*e^2*x^4*log(
abs(F))^3 - 2*pi^3*b^4*c^4*d*e*x^2*log(abs(F))*sgn(F) + 2*pi*b^4*c^4*d*e*x
^2*log(abs(F))^3*sgn(F) + 2*pi^3*b^4*c^4*d*e*x^2*log(abs(F)) - 2*pi*b^4*c^
4*d*e*x^2*log(abs(F))^3 - pi^3*b^3*c^3*e^2*x^3*sgn(F) + pi^3*b^4*c^4*d^2*l
og(abs(F))*sgn(F) + 3*pi*b^3*c^3*e^2*x^3*log(abs(F))^2*sgn(F) - pi*b^4*c^4
*d^2*log(abs(F))^3*sgn(F) + pi^3*b^3*c^3*e^2*x^3 - pi^3*b^4*c^4*d^2*log(ab
s(F)) - 3*pi*b^3*c^3*e^2*x^3*log(abs(F))^2 + pi*b^4*c^4*d^2*log(abs(F))^3
+ pi^3*b^3*c^3*d*e*x*sgn(F) - 3*pi*b^3*c^3*d*e*x*log(abs(F))^2*sgn(F) - pi
^3*b^3*c^3*d*e*x + 3*pi*b^3*c^3*d*e*x*log(abs(F))^2 - 6*pi*b^2*c^2*e^2*x^2
*log(abs(F))*sgn(F) + 6*pi*b^2*c^2*e^2*x^2*log(abs(F)) + 2*pi*b^2*c^2*d*e*
log(abs(F))*sgn(F) - 2*pi*b^2*c^2*d*e*log(abs(F)) + 6*pi*b*c*e^2*x*sgn(F)
- 6*pi*b*c*e^2*x)*(pi^5*b^5*c^5*sgn(F) - 10*pi^3*b^5*c^5*log(abs(F))^2*sgn
(F) + 5*pi*b^5*c^5*log(abs(F))^4*sgn(F) - pi^5*b^5*c^5 + 10*pi^3*b^5*c^5*l
og(abs(F))^2 - 5*pi*b^5*c^5*log(abs(F))^4)/(pi^5*b^5*c^5*sgn(F) - 10*pi^3
*b^5*c^5*log(abs(F))^2*sgn(F) + 5*pi*b^5*c^5*log(abs(F))^4*sgn(F) - pi^5*b
^5*c^5 + 10*pi^3*b^5*c^5*log(abs(F))^2 - 5*pi*b^5*c^5*log(abs(F))^4)^2 + (
5*pi^4*b^5*c^5*log(abs(F))*sgn(F) - 10*pi^2*b^5*c^5*log(abs(F))^3*sgn(F) -
5*pi^4*b^5*c^5*log(abs(F)) + 10*pi^2*b^5*c^5*log(abs(F))^3 - 2*b^5*c^5*lo
g(abs(F))^5)^2) - (pi^4*b^4*c^4*e^2*x^4*sgn(F) - 6*pi^2*b^4*c^4*e^2*x^4...

```

### Mupad [B] (verification not implemented)

Time = 22.61 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(d - ex^2)^2 dx$$

$$= \frac{F^{ac+bcx} (b^4 c^4 d^2 \ln(F)^4 - 2b^4 c^4 d e x^2 \ln(F)^4 + b^4 c^4 e^2 x^4 \ln(F)^4 + 4b^3 c^3 d e x \ln(F)^3 - 4b^3 c^3 e^2 x^3 \ln(F)^3 - 4b^3 c^3 e^2 x^3 \ln(F)^3 + 4b^3 c^3 e^2 x^3 \ln(F)^3 - 4b^3 c^3 e^2 x^3 \ln(F)^3)}{b^5 c^5 \ln(F)^5}$$

input

```
int(F^(c*(a + b*x))*(d - e*x^2)^2,x)
```

output

```

(F^(a*c + b*c*x)*(24*e^2 + b^4*c^4*d^2*log(F)^4 - 24*b*c*e^2*x*log(F) + 12
*b^2*c^2*e^2*x^2*log(F)^2 - 4*b^3*c^3*e^2*x^3*log(F)^3 + b^4*c^4*e^2*x^4*
log(F)^4 - 4*b^2*c^2*d*e*log(F)^2 + 4*b^3*c^3*d*e*x*log(F)^3 - 2*b^4*c^4*d*
e*x^2*log(F)^4))/(b^5*c^5*log(F)^5)

```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)} (d - ex^2)^2 dx$$

$$= \frac{f^{bcx+ac} (\log(f)^4 b^4 c^4 d^2 - 2\log(f)^4 b^4 c^4 d e x^2 + \log(f)^4 b^4 c^4 e^2 x^4 + 4\log(f)^3 b^3 c^3 d e x - 4\log(f)^3 b^3 c^3 e^2 x^3 - 2\log(f)^3 b^3 c^3 d^2 x + 2\log(f)^3 b^3 c^3 e x^2 - 2\log(f)^3 b^3 c^3 d e x^2 + \log(f)^3 b^3 c^3 e^2 x^3 - 2\log(f)^2 b^2 c^2 d^2 e x + 2\log(f)^2 b^2 c^2 d e^2 x^2 - 2\log(f)^2 b^2 c^2 d e x^3 + \log(f)^2 b^2 c^2 e^2 x^4 - 2\log(f)^2 b^2 c^2 d e x^2 + \log(f)^2 b^2 c^2 e^2 x^3 - 2\log(f)^2 b^2 c^2 d^2 e x^2 + 2\log(f)^2 b^2 c^2 d e^2 x^3 - 2\log(f)^2 b^2 c^2 d e x^4 + \log(f)^2 b^2 c^2 e^2 x^5 - 2\log(f) b c d^2 e x + 2\log(f) b c d e^2 x^2 - 2\log(f) b c d e x^3 + \log(f) b c e^2 x^4 - 2\log(f) b c d^2 e x^2 + 2\log(f) b c d e^2 x^3 - 2\log(f) b c d e x^4 + \log(f) b c e^2 x^5 - 2d^2 e x + 2d e^2 x^2 - 2d e x^3 + e^2 x^4 - 2d^2 e x^2 + 2d e^2 x^3 - 2d e x^4 + e^2 x^5)}{\log(f)^5 b^5 c^5}$$

input `int(F^((b*x+a)*c)*(-e*x^2+d)^2,x)`output `(f**(a*c + b*c*x)*(log(f)**4*b**4*c**4*d**2 - 2*log(f)**4*b**4*c**4*d*e*x**2 + log(f)**4*b**4*c**4*e**2*x**4 + 4*log(f)**3*b**3*c**3*d*e*x - 4*log(f)**3*b**3*c**3*e**2*x**3 - 4*log(f)**2*b**2*c**2*d*e + 12*log(f)**2*b**2*c**2*e**2*x**2 - 24*log(f)*b*c*e**2*x + 24*e**2))/(log(f)**5*b**5*c**5)`

### 3.16 $\int F^{c(a+bx)}(d - ex^2) dx$

Optimal result	157
Mathematica [A] (verified)	157
Rubi [A] (verified)	158
Maple [A] (verified)	159
Fricas [A] (verification not implemented)	159
Sympy [A] (verification not implemented)	160
Maxima [A] (verification not implemented)	160
Giac [C] (verification not implemented)	161
Mupad [B] (verification not implemented)	162
Reduce [B] (verification not implemented)	162

#### Optimal result

Integrand size = 18, antiderivative size = 74

$$\int F^{c(a+bx)}(d - ex^2) dx = -\frac{2eF^{c(a+bx)}}{b^3c^3 \log^3(F)} + \frac{2eF^{c(a+bx)}x}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d - ex^2)}{bc \log(F)}$$

output

```
-2*e*F^(c*(b*x+a))/b^3/c^3/ln(F)^3+2*e*F^(c*(b*x+a))*x/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(-e*x^2+d)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int F^{c(a+bx)}(d - ex^2) dx = -\frac{F^{c(a+bx)}(2e - 2bcex \log(F) + b^2c^2(-d + ex^2) \log^2(F))}{b^3c^3 \log^3(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d - e*x^2), x]
```

output

```
-((F^(c*(a + b*x))*(2*e - 2*b*c*e*x*Log[F] + b^2*c^2*(-d + e*x^2)*Log[F]^2))/b^3*c^3*Log[F]^3)
```

**Rubi [A] (verified)**

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.24, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - ex^2) F^{c(a+bx)} dx$$

$$\downarrow 2626$$

$$\int (dF^{c(a+bx)} - ex^2 F^{c(a+bx)}) dx$$

$$\downarrow 2009$$

$$-\frac{2eF^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \frac{2exF^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} - \frac{ex^2 F^{c(a+bx)}}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d - e*x^2),x]`

output `(-2*e*F^(c*(a + b*x)))/(b^3*c^3*Log[F]^3) + (2*e*F^(c*(a + b*x))*x)/(b^2*c^2*Log[F]^2) + (d*F^(c*(a + b*x)))/(b*c*Log[F]) - (e*F^(c*(a + b*x))*x^2)/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

method	result	size
gospers	$\frac{(-e x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 d + 2e x \ln(F) b c - 2e) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$	61
risch	$\frac{(-e x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 d + 2e x \ln(F) b c - 2e) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$	61
orering	$\frac{(-e x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 d + 2e x \ln(F) b c - 2e) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$	61
meijerg	$\frac{F^{ac} e \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} - \frac{F^{ac} d (1 - e^{bcx \ln(F)})}{bc \ln(F)}$	83
parallelrisch	$\frac{-x^2 F^{c(bx+a)} e \ln(F)^2 b^2 c^2 + \ln(F)^2 F^{c(bx+a)} b^2 c^2 d + 2e F^{c(bx+a)} x \ln(F) b c - 2F^{c(bx+a)} e}{\ln(F)^3 b^3 c^3}$	88
norman	$\frac{(\ln(F)^2 b^2 c^2 d - 2e) e^{c(bx+a) \ln(F)}}{c^3 b^3 \ln(F)^3} + \frac{2e x e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} - \frac{e x^2 e^{c(bx+a) \ln(F)}}{\ln(F) b c}$	89

input `int(F^(c*(b*x+a))*(-e*x^2+d),x,method=_RETURNVERBOSE)`

output `(-e*x^2*ln(F)^2*b^2*c^2+ln(F)^2*b^2*c^2*d+2*e*x*ln(F)*b*c-2*e)*F^(c*(b*x+a))/ln(F)^3/b^3/c^3`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int F^{c(a+bx)} (d - ex^2) dx = \frac{(2bcex \log(F) - (b^2c^2ex^2 - b^2c^2d) \log(F)^2 - 2e) F^{bcx+ac}}{b^3c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c))*(-e*x^2+d),x, algorithm="fricas")`

output `(2*b*c*e*x*log(F) - (b^2*c^2*e*x^2 - b^2*c^2*d)*log(F)^2 - 2*e)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3)`



**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(d - ex^2) dx = \begin{cases} \frac{F^{c(a+bx)}(b^2c^2d \log(F)^2 - b^2c^2ex^2 \log(F)^2 + 2bcex \log(F) - 2e)}{b^3c^3 \log(F)^3} & \text{for } b^3c^3 \log(F)^3 \neq 0 \\ dx - \frac{ex^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(-e*x**2+d), x)`output `Piecewise((F**(c*(a + b*x))*(b**2*c**2*d*log(F)**2 - b**2*c**2*e*x**2*log(F)**2 + 2*b*c*e*x*log(F) - 2*e)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d*x - e*x**3/3, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int F^{c(a+bx)}(d - ex^2) dx = \frac{F^{bcx+ac}d}{bc \log(F)} - \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}e}{b^3c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(-e*x^2+d), x, algorithm="maxima")`output `F^(b*c*x + a*c)*d/(b*c*log(F)) - (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*e/(b^3*c^3*log(F)^3)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 1690, normalized size of antiderivative = 22.84

$$\int F^{c(a+bx)}(d - ex^2) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(-e*x^2+d),x, algorithm="giac")`

output

```
-(((pi^2*b^2*c^2*e*x^2*sgn(F) - pi^2*b^2*c^2*e*x^2 + 2*b^2*c^2*e*x^2*log(
abs(F))^2 - pi^2*b^2*c^2*d*sgn(F) + pi^2*b^2*c^2*d - 2*b^2*c^2*d*log(abs(F)
)^2 - 4*b*c*e*x*log(abs(F)) + 4*e)*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*
pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)/((pi^3*b^3*c^3*sgn(F)
- 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(
F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F))
+ 2*b^3*c^3*log(abs(F))^3)^2) - 2*(pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log
(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi*b^2*c^2
*e*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*e*x^2*log(abs(F)) - pi*b^2*c^2*d*lo
g(abs(F))*sgn(F) + pi*b^2*c^2*d*log(abs(F)) - pi*b*c*e*x*sgn(F) + pi*b*c*e
*x)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c
^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) -
3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*
x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^3*b^3*c^3
*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3
*log(abs(F))^2)*(pi^2*b^2*c^2*e*x^2*sgn(F) - pi^2*b^2*c^2*e*x^2 + 2*b^2*c^2
*e*x^2*log(abs(F))^2 - pi^2*b^2*c^2*d*sgn(F) + pi^2*b^2*c^2*d - 2*b^2*c^2*
d*log(abs(F))^2 - 4*b*c*e*x*log(abs(F)) + 4*e)/((pi^3*b^3*c^3*sgn(F) - 3*p
i*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2
)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + ...
```

**Mupad [B] (verification not implemented)**

Time = 22.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}(d - ex^2) dx$$

$$= -\frac{F^{ac+bcx} (eb^2c^2x^2 \ln(F)^2 - db^2c^2 \ln(F)^2 - 2ebcx \ln(F) + 2e)}{b^3c^3 \ln(F)^3}$$

input `int(F^(c*(a + b*x))*(d - e*x^2),x)`output `-(F^(a*c + b*c*x)*(2*e - b^2*c^2*d*log(F)^2 + b^2*c^2*e*x^2*log(F)^2 - 2*b*c*e*x*log(F)))/(b^3*c^3*log(F)^3)`**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int F^{c(a+bx)}(d - ex^2) dx = \frac{f^{bcx+ac}(\log(f)^2 b^2 c^2 d - \log(f)^2 b^2 c^2 e x^2 + 2 \log(f) bcex - 2e)}{\log(f)^3 b^3 c^3}$$

input `int(F^((b*x+a)*c)*(-e*x^2+d),x)`output `(f**(a*c + b*c*x)*(log(f)**2*b**2*c**2*d - log(f)**2*b**2*c**2*e*x**2 + 2*log(f)*b*c*e*x - 2*e))/(log(f)**3*b**3*c**3)`

### 3.17 $\int \frac{F^{c(a+bx)}}{d-ex^2} dx$

Optimal result	163
Mathematica [A] (verified)	163
Rubi [A] (verified)	164
Maple [A] (verified)	165
Fricas [A] (verification not implemented)	166
Sympy [F]	166
Maxima [F]	167
Giac [F]	167
Mupad [F(-1)]	167
Reduce [F]	168

#### Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{F^{c(a+bx)}}{d-ex^2} dx = -\frac{F^{c(a+\frac{b\sqrt{d}}{\sqrt{e}})} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{d}-\sqrt{e}x) \log(F)}{\sqrt{e}}\right)}{2\sqrt{d}\sqrt{e}} + \frac{F^{c(a-\frac{b\sqrt{d}}{\sqrt{e}})} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{d}+\sqrt{e}x) \log(F)}{\sqrt{e}}\right)}{2\sqrt{d}\sqrt{e}}$$

output

```
-1/2*F^(c*(a+b*d^(1/2)/e^(1/2)))*Ei(-b*c*(d^(1/2)-e^(1/2)*x)*ln(F)/e^(1/2)
)/d^(1/2)/e^(1/2)+1/2*F^(c*(a-b*d^(1/2)/e^(1/2)))*Ei(b*c*(d^(1/2)+e^(1/2)*
x)*ln(F)/e^(1/2))/d^(1/2)/e^(1/2)
```

#### Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{F^{c(a+bx)}}{d-ex^2} dx = \frac{F^{c(a-\frac{b\sqrt{d}}{\sqrt{e}})} \left( -F^{\frac{2bc\sqrt{d}}{\sqrt{e}}} \text{ExpIntegralEi}\left(bc\left(-\frac{\sqrt{d}}{\sqrt{e}}+x\right) \log(F)\right) + \text{ExpIntegralEi}\left(bc\left(\frac{\sqrt{d}}{\sqrt{e}}+x\right) \log(F)\right) \right)}{2\sqrt{d}\sqrt{e}}$$

input `Integrate[F^(c*(a + b*x))/(d - e*x^2),x]`

output `(F^(c*(a - (b*Sqrt[d])/Sqrt[e]))*(-(F^((2*b*c*Sqrt[d])/Sqrt[e])*ExpIntegralEi[b*c*(-(Sqrt[d]/Sqrt[e]) + x)*Log[F]]) + ExpIntegralEi[b*c*(Sqrt[d]/Sqrt[e] + x)*Log[F])))/(2*Sqrt[d]*Sqrt[e])`

### Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx$$

↓ 2699

$$\int \left( \frac{F^{c(a+bx)}}{2\sqrt{d}(\sqrt{d} - \sqrt{ex})} + \frac{F^{c(a+bx)}}{2\sqrt{d}(\sqrt{d} + \sqrt{ex})} \right) dx$$

↓ 2009

$$\frac{F^{c\left(a - \frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{ex} + \sqrt{d}) \log(F)}{\sqrt{e}}\right)}{2\sqrt{d}\sqrt{e}} - \frac{F^{c\left(a + \frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt{d} - \sqrt{ex}) \log(F)}{\sqrt{e}}\right)}{2\sqrt{d}\sqrt{e}}$$

input `Int[F^(c*(a + b*x))/(d - e*x^2),x]`

output

$$-1/2*(F^{(c*(a + (b*\sqrt{d}))/\sqrt{e}))}*\text{ExpIntegralEi}[-((b*c*(\sqrt{d} - \sqrt{e})*x)*\text{Log}[F])/\sqrt{e}]]/(\sqrt{d}*\sqrt{e}) + (F^{(c*(a - (b*\sqrt{d}))/\sqrt{e}))}*\text{ExpIntegralEi}[(b*c*(\sqrt{d} + \sqrt{e})*x)*\text{Log}[F])/\sqrt{e}])/(2*\sqrt{d}*\sqrt{e})$$

### Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x\_Symbol] \text{ :> Simp}[\text{IntSum}[u, x], x] \text{ /; SumQ}[u]$$

rule 2699

$$\text{Int}[(F_)^{((g_.)*((d_.) + (e_.)*(x_))^{(n_.)})}/((a_) + (c_.)*(x_)^2), x\_Symbol] \text{ :> Int}[\text{ExpandIntegrand}[F^{(g*(d + e*x)^n)}, 1/(a + c*x^2), x], x] \text{ /; FreeQ}[\{F, a, c, d, e, g, n\}, x]$$

### Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

method	result
risch	$\frac{F^{\frac{(b\sqrt{de}+ea)c}{e}} \text{expIntegral}_1\left(\frac{\sqrt{de} \ln(F)bc+ea \ln(F)c-e(bc \ln(F)+ac \ln(F))}{e}\right)}{2\sqrt{de}} - \frac{F^{\frac{(-b\sqrt{de}+ea)c}{e}} \text{expIntegral}_1\left(-\frac{\sqrt{de} \ln(F)bc-ea \ln(F)}{e}\right)}{2\sqrt{de}}$

input

$$\text{int}(F^{(c*(b*x+a))}/(-e*x^2+d), x, \text{method}=\_RETURNVERBOSE)$$

output

$$1/2/(d*e)^{(1/2)}*F^{((b*(d*e)^{(1/2)}+e*a)/e*c)*\text{Ei}(1, ((d*e)^{(1/2)}*\ln(F)*b*c+e*a*\ln(F)*c-e*(b*c*x*\ln(F)+a*c*\ln(F)))/e)} - 1/2/(d*e)^{(1/2)}*F^{((-b*(d*e)^{(1/2)}+e*a)/e*c)*\text{Ei}(1, -((d*e)^{(1/2)}*\ln(F)*b*c-e*a*\ln(F)*c+e*(b*c*x*\ln(F)+a*c*\ln(F)))/e)}$$

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.34

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx = \frac{\sqrt{\frac{b^2c^2d\log(F)^2}{e}} \operatorname{Ei}\left(bcx \log(F) - \sqrt{\frac{b^2c^2d\log(F)^2}{e}}\right) e^{\left(ac\log(F) + \sqrt{\frac{b^2c^2d\log(F)^2}{e}}\right)} - \sqrt{\frac{b^2c^2d\log(F)^2}{e}} \operatorname{Ei}\left(bcx \log(F) + \sqrt{\frac{b^2c^2d\log(F)^2}{e}}\right) e^{\left(ac\log(F) - \sqrt{\frac{b^2c^2d\log(F)^2}{e}}\right)}}{2bcd \log(F)}$$

input `integrate(F^((b*x+a)*c)/(-e*x^2+d),x, algorithm="fricas")`

output `-1/2*(sqrt(b^2*c^2*d*log(F)^2/e)*Ei(b*c*x*log(F) - sqrt(b^2*c^2*d*log(F)^2/e))*e^(a*c*log(F) + sqrt(b^2*c^2*d*log(F)^2/e)) - sqrt(b^2*c^2*d*log(F)^2/e)*Ei(b*c*x*log(F) + sqrt(b^2*c^2*d*log(F)^2/e))*e^(a*c*log(F) - sqrt(b^2*c^2*d*log(F)^2/e)))/(b*c*d*log(F))`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx = - \int \frac{F^{ac+bcx}}{-d + ex^2} dx$$

input `integrate(F**((b*x+a)*c)/(-e*x**2+d),x)`

output `-Integral(F**(a*c + b*c*x)/(-d + e*x**2), x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx = \int -\frac{F^{(bx+a)c}}{ex^2 - d} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^2+d),x, algorithm="maxima")`

output `-integrate(F^((b*x + a)*c)/(e*x^2 - d), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx = \int -\frac{F^{(bx+a)c}}{ex^2 - d} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^2+d),x, algorithm="giac")`

output `integrate(-F^((b*x + a)*c)/(e*x^2 - d), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx = \int \frac{F^{c(a+bx)}}{d - ex^2} dx$$

input `int(F^(c*(a + b*x))/(d - e*x^2),x)`

output `int(F^(c*(a + b*x))/(d - e*x^2), x)`



**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^2} dx = f^{ac} \left( \int \frac{f^{bcx}}{-ex^2 + d} dx \right)$$

input `int(F^((b*x+a)*c)/(-e*x^2+d),x)`

output `f**(a*c)*int(f**(b*c*x)/(d - e*x**2),x)`

### 3.18 $\int \frac{F^{c(a+bx)}}{(d-ex^2)^2} dx$

Optimal result	169
Mathematica [A] (verified)	170
Rubi [A] (verified)	170
Maple [F]	172
Fricas [A] (verification not implemented)	173
Sympy [F]	173
Maxima [F]	174
Giac [F]	174
Mupad [F(-1)]	174
Reduce [F]	175

#### Optimal result

Integrand size = 20, antiderivative size = 304

$$\int \frac{F^{c(a+bx)}}{(d-ex^2)^2} dx = \frac{F^{c(a+bx)}}{4d\sqrt{e}(\sqrt{d}-\sqrt{ex})} - \frac{F^{c(a+bx)}}{4d\sqrt{e}(\sqrt{d}+\sqrt{ex})}$$

$$- \frac{F^{c\left(a+\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt{d}-\sqrt{ex})\log(F)}{\sqrt{e}}\right)}{4d^{3/2}\sqrt{e}}$$

$$+ \frac{F^{c\left(a-\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{d}+\sqrt{ex})\log(F)}{\sqrt{e}}\right)}{4d^{3/2}\sqrt{e}}$$

$$+ \frac{bcF^{c\left(a+\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt{d}-\sqrt{ex})\log(F)}{\sqrt{e}}\right) \log(F)}{4de}$$

$$+ \frac{bcF^{c\left(a-\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{d}+\sqrt{ex})\log(F)}{\sqrt{e}}\right) \log(F)}{4de}$$

output

```
1/4*F^(c*(b*x+a))/d/e^(1/2)/(d^(1/2)-e^(1/2)*x)-1/4*F^(c*(b*x+a))/d/e^(1/2)
)/(d^(1/2)+e^(1/2)*x)-1/4*F^(c*(a+b*d^(1/2)/e^(1/2)))*Ei(-b*c*(d^(1/2)-e^(
1/2)*x)*ln(F)/e^(1/2))/d^(3/2)/e^(1/2)+1/4*F^(c*(a-b*d^(1/2)/e^(1/2)))*Ei(
b*c*(d^(1/2)+e^(1/2)*x)*ln(F)/e^(1/2))/d^(3/2)/e^(1/2)+1/4*b*c*F^(c*(a+b*d
^(1/2)/e^(1/2)))*Ei(-b*c*(d^(1/2)-e^(1/2)*x)*ln(F)/e^(1/2))*ln(F)/d/e+1/4*
b*c*F^(c*(a-b*d^(1/2)/e^(1/2)))*Ei(b*c*(d^(1/2)+e^(1/2)*x)*ln(F)/e^(1/2))*
ln(F)/d/e
```

**Mathematica [A] (verified)**

Time = 0.67 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.58

$$\int \frac{F^{c(a+bx)}}{(d-ex^2)^2} dx$$

$$= \frac{F^{c\left(a-\frac{b\sqrt{d}}{\sqrt{e}}\right)} \left( 2\sqrt{de} F^{bc\left(\frac{\sqrt{d}}{\sqrt{e}}+x\right)} x + F^{\frac{2bc\sqrt{d}}{\sqrt{e}}}(d-ex^2) \text{ExpIntegralEi}\left(bc\left(-\frac{\sqrt{d}}{\sqrt{e}}+x\right)\log(F)\right) \left(-\sqrt{e}+bc\sqrt{d}\right) \right)}{4d^{3/2}e(d-ex^2)}$$

input

```
Integrate[F^(c*(a + b*x))/(d - e*x^2)^2,x]
```

output

```
(F^(c*(a - (b*Sqrt[d])/Sqrt[e]))*(2*Sqrt[d]*e*F^(b*c*(Sqrt[d]/Sqrt[e] + x)
)*x + F^((2*b*c*Sqrt[d])/Sqrt[e])*(d - e*x^2)*ExpIntegralEi[b*c*(-(Sqrt[d]
/Sqrt[e]) + x)*Log[F]]*(-Sqrt[e] + b*c*Sqrt[d]*Log[F]) + (d - e*x^2)*ExpIn
tegralEi[b*c*(Sqrt[d]/Sqrt[e] + x)*Log[F]]*(Sqrt[e] + b*c*Sqrt[d]*Log[F]))
)/(4*d^(3/2)*e*(d - e*x^2))
```

**Rubi [A] (verified)**Time = 1.17 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
& \int \frac{F^{c(a+bx)}}{(d-ex^2)^2} dx \\
& \quad \downarrow \text{7292} \\
& \int \frac{F^{ac+bcx}}{(d-ex^2)^2} dx \\
& \quad \downarrow \text{7293} \\
& \int \left( \frac{eF^{ac+bcx}}{2d(de-e^2x^2)} + \frac{eF^{ac+bcx}}{4d(\sqrt{d}\sqrt{e}-ex)^2} + \frac{eF^{ac+bcx}}{4d(\sqrt{d}\sqrt{e}+ex)^2} \right) dx \\
& \quad \downarrow \text{2009} \\
& \frac{F^{c\left(a-\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{ex}+\sqrt{d})\log(F)}{\sqrt{e}}\right)}{4d^{3/2}\sqrt{e}} - \\
& \frac{F^{c\left(a+\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt{d}-\sqrt{ex})\log(F)}{\sqrt{e}}\right)}{4d^{3/2}\sqrt{e}} + \\
& \frac{bc\log(F)F^{c\left(a-\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(\frac{bc(\sqrt{ex}+\sqrt{d})\log(F)}{\sqrt{e}}\right)}{4de} + \\
& \frac{bc\log(F)F^{c\left(a+\frac{b\sqrt{d}}{\sqrt{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt{d}-\sqrt{ex})\log(F)}{\sqrt{e}}\right)}{4de} + \frac{F^{ac+bcx}}{4d\sqrt{e}(\sqrt{d}-\sqrt{ex})} - \\
& \frac{F^{ac+bcx}}{4d\sqrt{e}(\sqrt{d}+\sqrt{ex})}
\end{aligned}$$

input

```
Int[F^(c*(a + b*x))/(d - e*x^2)^2,x]
```

output

```
F^(a*c + b*c*x)/(4*d*Sqrt[e]*(Sqrt[d] - Sqrt[e]*x)) - F^(a*c + b*c*x)/(4*d
*Sqrt[e]*(Sqrt[d] + Sqrt[e]*x)) - (F^(c*(a + (b*Sqrt[d])/Sqrt[e]))*ExpInte
gralEi[-((b*c*(Sqrt[d] - Sqrt[e]*x)*Log[F])/Sqrt[e])])/(4*d^(3/2)*Sqrt[e])
+ (F^(c*(a - (b*Sqrt[d])/Sqrt[e]))*ExpIntegralEi[(b*c*(Sqrt[d] + Sqrt[e]*
x)*Log[F])/Sqrt[e])]/(4*d^(3/2)*Sqrt[e]) + (b*c*F^(c*(a + (b*Sqrt[d])/Sqrt
[e]))*ExpIntegralEi[-((b*c*(Sqrt[d] - Sqrt[e]*x)*Log[F])/Sqrt[e])]*Log[F])
/(4*d*e) + (b*c*F^(c*(a - (b*Sqrt[d])/Sqrt[e]))*ExpIntegralEi[(b*c*(Sqrt[d]
] + Sqrt[e]*x)*Log[F])/Sqrt[e]*Log[F])/(4*d*e)
```

### Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

rule 7293

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Maple [F]

$$\int \frac{F^{c(bx+a)}}{(-ex^2+d)^2} dx$$

input

```
int(F^(c*(b*x+a))/(-e*x^2+d)^2,x)
```

output

```
int(F^(c*(b*x+a))/(-e*x^2+d)^2,x)
```

**Fricas [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.91

$$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx =$$

$$\frac{2 F^{bcx+ac} bc dex \log(F) - \left( (b^2 c^2 dex^2 - b^2 c^2 d^2) \log(F)^2 - \sqrt{\frac{b^2 c^2 d \log(F)^2}{e}} (e^2 x^2 - de) \right) \text{Ei} \left( bcx \log(F) - \right)}{\dots}$$

input `integrate(F^((b*x+a)*c)/(-e*x^2+d)^2,x, algorithm="fricas")`

output `-1/4*(2*F^(b*c*x + a*c)*b*c*d*e*x*log(F) - ((b^2*c^2*d*e*x^2 - b^2*c^2*d^2)*log(F)^2 - sqrt(b^2*c^2*d*log(F)^2/e)*(e^2*x^2 - d*e))*Ei(b*c*x*log(F) - sqrt(b^2*c^2*d*log(F)^2/e))*e^(a*c*log(F) + sqrt(b^2*c^2*d*log(F)^2/e)) - ((b^2*c^2*d*e*x^2 - b^2*c^2*d^2)*log(F)^2 + sqrt(b^2*c^2*d*log(F)^2/e)*(e^2*x^2 - d*e))*Ei(b*c*x*log(F) + sqrt(b^2*c^2*d*log(F)^2/e))*e^(a*c*log(F) - sqrt(b^2*c^2*d*log(F)^2/e)))/((b*c*d^2*e^2*x^2 - b*c*d^3*e)*log(F))`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx = \int \frac{F^{c(a+bx)}}{(-d + ex^2)^2} dx$$

input `integrate(F**((b*x+a)*c)/(-e*x**2+d)**2,x)`

output `Integral(F**(c*(a + b*x))/(-d + e*x**2)**2, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx = \int \frac{F^{(bx+a)c}}{(ex^2 - d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^2+d)^2,x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x^2 - d)^2, x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx = \int \frac{F^{(bx+a)c}}{(ex^2 - d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^2+d)^2,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x^2 - d)^2, x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx = \int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx$$

input `int(F^(c*(a + b*x))/(d - e*x^2)^2,x)`

output `int(F^(c*(a + b*x))/(d - e*x^2)^2, x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d - ex^2)^2} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^2 x^4 - 2dex^2 + d^2} dx \right)$$

input `int(F^((b*x+a)*c)/(-e*x^2+d)^2,x)`

output `f**(a*c)*int(f**(b*c*x)/(d**2 - 2*d*e*x**2 + e**2*x**4),x)`



### 3.19 $\int F^{c(a+bx)}(d - ex^3)^2 dx$

Optimal result . . . . .	176
Mathematica [A] (verified) . . . . .	177
Rubi [A] (verified) . . . . .	177
Maple [A] (verified) . . . . .	179
Fricas [A] (verification not implemented) . . . . .	179
Sympy [A] (verification not implemented) . . . . .	180
Maxima [A] (verification not implemented) . . . . .	180
Giac [C] (verification not implemented) . . . . .	181
Mupad [B] (verification not implemented) . . . . .	182
Reduce [B] (verification not implemented) . . . . .	183

#### Optimal result

Integrand size = 20, antiderivative size = 203

$$\int F^{c(a+bx)}(d - ex^3)^2 dx = \frac{720e^2 F^{c(a+bx)}}{b^7 c^7 \log^7(F)} - \frac{720e^2 F^{c(a+bx)} x}{b^6 c^6 \log^6(F)} + \frac{360e^2 F^{c(a+bx)} x^2}{b^5 c^5 \log^5(F)} + \frac{12e F^{c(a+bx)}(d - 10ex^3)}{b^4 c^4 \log^4(F)} - \frac{6e F^{c(a+bx)} x(2d - 5ex^3)}{b^3 c^3 \log^3(F)} + \frac{6e F^{c(a+bx)} x^2(d - ex^3)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d - ex^3)^2}{bc \log(F)}$$

output

```
720*e^2*F^(c*(b*x+a))/b^7/c^7/ln(F)^7-720*e^2*F^(c*(b*x+a))*x/b^6/c^6/ln(F)^6+360*e^2*F^(c*(b*x+a))*x^2/b^5/c^5/ln(F)^5+12*e*F^(c*(b*x+a))*(-10*e*x^3+d)/b^4/c^4/ln(F)^4-6*e*F^(c*(b*x+a))*x*(-5*e*x^3+2*d)/b^3/c^3/ln(F)^3+6*e*F^(c*(b*x+a))*x^2*(-e*x^3+d)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(-e*x^3+d)^2/b/c/ln(F)
```

**Mathematica [A] (verified)**

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)} (d - ex^3)^2 dx$$

$$= \frac{F^{c(a+bx)} \left( 720e^2 - 720bce^2x \log(F) + 360b^2c^2e^2x^2 \log^2(F) + 12b^3c^3e(d - 10ex^3) \log^3(F) + 6b^4c^4ex(-2d - 10ex^3) \log^4(F) + 6b^5c^5e^2x^2(d - ex^3) \log^5(F) + b^6c^6(d - ex^3)^2 \log^6(F) \right)}{b^7c^7 \log^7(F)}$$

input `Integrate[F^(c*(a + b*x))*(d - e*x^3)^2,x]`

output  $(F^{c(a+bx)}(720e^2 - 720bce^2x \log[F] + 360b^2c^2e^2x^2 \log[F]^2 + 12b^3c^3e(d - 10ex^3) \log[F]^3 + 6b^4c^4e^2x^2(-2d - 10ex^3) \log[F]^4 + 6b^5c^5e^2x^2(d - ex^3) \log[F]^5 + b^6c^6(d - ex^3)^2 \log[F]^6)) / (b^7c^7 \log[F]^7)$

**Rubi [A] (verified)**

Time = 0.95 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - ex^3)^2 F^{c(a+bx)} dx$$

$$\downarrow 2626$$

$$\int \left( d^2 F^{c(a+bx)} - 2dex^3 F^{c(a+bx)} + e^2 x^6 F^{c(a+bx)} \right) dx$$

$$\downarrow 2009$$

$$\frac{720e^2 F^{c(a+bx)}}{b^7 c^7 \log^7(F)} - \frac{720e^2 x F^{c(a+bx)}}{b^6 c^6 \log^6(F)} + \frac{360e^2 x^2 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} + \frac{12de F^{c(a+bx)}}{b^4 c^4 \log^4(F)} - \frac{120e^2 x^3 F^{c(a+bx)}}{b^4 c^4 \log^4(F)} - \frac{12dex F^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \frac{30e^2 x^4 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \frac{6dex^2 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} - \frac{6e^2 x^5 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \frac{d^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2dex^3 F^{c(a+bx)}}{bc \log(F)} + \frac{e^2 x^6 F^{c(a+bx)}}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d - e*x^3)^2,x]`

output `(720*e^2*F^(c*(a + b*x)))/(b^7*c^7*Log[F]^7) - (720*e^2*F^(c*(a + b*x))*x)/(b^6*c^6*Log[F]^6) + (360*e^2*F^(c*(a + b*x))*x^2)/(b^5*c^5*Log[F]^5) + (12*d*e*F^(c*(a + b*x)))/(b^4*c^4*Log[F]^4) - (120*e^2*F^(c*(a + b*x))*x^3)/(b^4*c^4*Log[F]^4) - (12*d*e*F^(c*(a + b*x))*x)/(b^3*c^3*Log[F]^3) + (30*e^2*F^(c*(a + b*x))*x^4)/(b^3*c^3*Log[F]^3) + (6*d*e*F^(c*(a + b*x))*x^2)/(b^2*c^2*Log[F]^2) - (6*e^2*F^(c*(a + b*x))*x^5)/(b^2*c^2*Log[F]^2) + (d^2*F^(c*(a + b*x)))/(b*c*Log[F]) - (2*d*e*F^(c*(a + b*x))*x^3)/(b*c*Log[F]) + (e^2*F^(c*(a + b*x))*x^6)/(b*c*Log[F])`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(P_x_), x_Symbol] := Int[ExpandIntegrand[F^v, P_x, x], x] /; FreeQ[F, x] && PolynomialQ[P_x, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`



output

```
(360*b^2*c^2*e^2*x^2*log(F)^2 + (b^6*c^6*e^2*x^6 - 2*b^6*c^6*d*e*x^3 + b^6*c^6*d^2)*log(F)^6 - 720*b*c*e^2*x*log(F) - 6*(b^5*c^5*e^2*x^5 - b^5*c^5*d*e*x^2)*log(F)^5 + 6*(5*b^4*c^4*e^2*x^4 - 2*b^4*c^4*d*e*x)*log(F)^4 - 12*(10*b^3*c^3*e^2*x^3 - b^3*c^3*d*e)*log(F)^3 + 720*e^2)*F^(b*c*x + a*c)/(b^7*c^7*log(F)^7)
```

**Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.29

$$\int F^{c(a+bx)} (d - ex^3)^2 dx$$

$$= \left\{ \frac{F^{c(a+bx)} (b^6 c^6 d^2 \log(F)^6 - 2b^6 c^6 dex^3 \log(F)^6 + b^6 c^6 e^2 x^6 \log(F)^6 + 6b^5 c^5 dex^2 \log(F)^5 - 6b^5 c^5 e^2 x^5 \log(F)^5 - 12b^4 c^4 dex \log(F)^4 + 30b^4 c^4 e^2 x^4 \log(F)^4 - 12b^3 c^3 dex^3 \log(F)^3 + 720e^2)}{b^7 c^7 \log(F)^7} \right.$$

$$\left. d^2 x - \frac{dex^4}{2} + \frac{e^2 x^7}{7} \right.$$

input

```
integrate(F**(b*x+a)*c*(-e*x**3+d)**2,x)
```

output

```
Piecewise((F**(c*(a + b*x))*(b**6*c**6*d**2*log(F)**6 - 2*b**6*c**6*d*e*x**3*log(F)**6 + b**6*c**6*e**2*x**6*log(F)**6 + 6*b**5*c**5*d*e*x**2*log(F)**5 - 6*b**5*c**5*e**2*x**5*log(F)**5 - 12*b**4*c**4*d*e*x*log(F)**4 + 30*b**4*c**4*e**2*x**4*log(F)**4 + 12*b**3*c**3*d*e*log(F)**3 - 120*b**3*c**3*e**2*x**3*log(F)**3 + 360*b**2*c**2*e**2*x**2*log(F)**2 - 720*b*c*e**2*x*log(F) + 720*e**2)/(b**7*c**7*log(F)**7), Ne(b**7*c**7*log(F)**7, 0)), (d**2*x - d*e*x**4/2 + e**2*x**7/7, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.20

$$\int F^{c(a+bx)} (d - ex^3)^2 dx = \frac{F^{bcx+ac} d^2}{bc \log(F)}$$

$$- \frac{2(F^{ac} b^3 c^3 x^3 \log(F)^3 - 3F^{ac} b^2 c^2 x^2 \log(F)^2 + 6F^{ac} b c x \log(F) - 6F^{ac}) F^{bcx} de}{b^4 c^4 \log(F)^4}$$

$$+ \frac{(F^{ac} b^6 c^6 x^6 \log(F)^6 - 6F^{ac} b^5 c^5 x^5 \log(F)^5 + 30F^{ac} b^4 c^4 x^4 \log(F)^4 - 120F^{ac} b^3 c^3 x^3 \log(F)^3 + 360F^{ac} b^2 c^2 x^2 \log(F)^2 - 720F^{ac} b c x \log(F) + 720e^2)}{b^7 c^7 \log(F)^7}$$

input `integrate(F^((b*x+a)*c)*(-e*x^3+d)^2,x, algorithm="maxima")`

output 
$$F^{(b*c*x + a*c)}*d^2/(b*c*\log(F)) - 2*(F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 - 3*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 + 6*F^{(a*c)}*b*c*x*\log(F) - 6*F^{(a*c)})*F^{(b*c*x)}*d*e/(b^4*c^4*\log(F)^4) + (F^{(a*c)}*b^6*c^6*x^6*\log(F)^6 - 6*F^{(a*c)}*b^5*c^5*x^5*\log(F)^5 + 30*F^{(a*c)}*b^4*c^4*x^4*\log(F)^4 - 120*F^{(a*c)}*b^3*c^3*x^3*\log(F)^3 + 360*F^{(a*c)}*b^2*c^2*x^2*\log(F)^2 - 720*F^{(a*c)}*b*c*x*\log(F) + 720*F^{(a*c)})*F^{(b*c*x)}*e^2/(b^7*c^7*\log(F)^7)$$

### Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 9878, normalized size of antiderivative = 48.66

$$\int F^{c(a+bx)}(d - ex^3)^2 dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(-e*x^3+d)^2,x, algorithm="giac")`

output

```

-((2*(3*pi^5*b^6*c^6*e^2*x^6*log(abs(F))*sgn(F) - 10*pi^3*b^6*c^6*e^2*x^6*
log(abs(F))^3*sgn(F) + 3*pi*b^6*c^6*e^2*x^6*log(abs(F))^5*sgn(F) - 3*pi^5*
b^6*c^6*e^2*x^6*log(abs(F)) + 10*pi^3*b^6*c^6*e^2*x^6*log(abs(F))^3 - 3*pi
*b^6*c^6*e^2*x^6*log(abs(F))^5 - 6*pi^5*b^6*c^6*d*e*x^3*log(abs(F))*sgn(F)
+ 20*pi^3*b^6*c^6*d*e*x^3*log(abs(F))^3*sgn(F) - 6*pi*b^6*c^6*d*e*x^3*log
(abs(F))^5*sgn(F) + 6*pi^5*b^6*c^6*d*e*x^3*log(abs(F)) - 20*pi^3*b^6*c^6*d
*e*x^3*log(abs(F))^3 + 6*pi*b^6*c^6*d*e*x^3*log(abs(F))^5 - 3*pi^5*b^5*c^5
*e^2*x^5*sgn(F) + 30*pi^3*b^5*c^5*e^2*x^5*log(abs(F))^2*sgn(F) - 15*pi*b^5
*c^5*e^2*x^5*log(abs(F))^4*sgn(F) + 3*pi^5*b^5*c^5*e^2*x^5 - 30*pi^3*b^5*c
^5*e^2*x^5*log(abs(F))^2 + 15*pi*b^5*c^5*e^2*x^5*log(abs(F))^4 + 3*pi^5*b^
6*c^6*d^2*log(abs(F))*sgn(F) - 10*pi^3*b^6*c^6*d^2*log(abs(F))^3*sgn(F) +
3*pi*b^6*c^6*d^2*log(abs(F))^5*sgn(F) - 3*pi^5*b^6*c^6*d^2*log(abs(F)) + 1
0*pi^3*b^6*c^6*d^2*log(abs(F))^3 - 3*pi*b^6*c^6*d^2*log(abs(F))^5 + 3*pi^5
*b^5*c^5*d*e*x^2*sgn(F) - 30*pi^3*b^5*c^5*d*e*x^2*log(abs(F))^2*sgn(F) + 1
5*pi*b^5*c^5*d*e*x^2*log(abs(F))^4*sgn(F) - 3*pi^5*b^5*c^5*d*e*x^2 + 30*pi
^3*b^5*c^5*d*e*x^2*log(abs(F))^2 - 15*pi*b^5*c^5*d*e*x^2*log(abs(F))^4 - 6
0*pi^3*b^4*c^4*e^2*x^4*log(abs(F))*sgn(F) + 60*pi*b^4*c^4*e^2*x^4*log(abs(
F))^3*sgn(F) + 60*pi^3*b^4*c^4*e^2*x^4*log(abs(F)) - 60*pi*b^4*c^4*e^2*x^4
*log(abs(F))^3 + 24*pi^3*b^4*c^4*d*e*x*log(abs(F))*sgn(F) - 24*pi*b^4*c^4*
d*e*x*log(abs(F))^3*sgn(F) - 24*pi^3*b^4*c^4*d*e*x*log(abs(F)) + 24*pi*...

```

### Mupad [B] (verification not implemented)

Time = 22.98 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} (d - ex^3)^2 dx$$

$$= \frac{F^{a+c b x} (b^6 c^6 d^2 \ln(F)^6 - 2 b^6 c^6 d e x^3 \ln(F)^6 + b^6 c^6 e^2 x^6 \ln(F)^6 + 6 b^5 c^5 d e x^2 \ln(F)^5 - 6 b^5 c^5 e^2 x^5 \ln(F)^5 - 6 b^4 c^4 d^2 \ln(F)^4 + 2 b^4 c^4 d e x^3 \ln(F)^4 - 2 b^4 c^4 e^2 x^6 \ln(F)^4 + 6 b^3 c^3 d^2 \ln(F)^3 - 6 b^3 c^3 d e x^3 \ln(F)^3 + 6 b^3 c^3 e^2 x^6 \ln(F)^3 + 6 b^2 c^2 d^3 \ln(F)^2 - 6 b^2 c^2 d^2 e x^3 \ln(F)^2 + 6 b^2 c^2 d e^2 x^6 \ln(F)^2 + 6 b c d^4 \ln(F) - 6 b c d^3 e x^3 \ln(F) + 6 b c d^2 e^2 x^6 \ln(F) + 6 b^2 c^3 d^2 \ln(F) - 6 b^2 c^3 d e x^3 \ln(F) + 6 b^2 c^3 e^2 x^6 \ln(F) + 6 b c^4 d^2 \ln(F) - 6 b c^4 d e x^3 \ln(F) + 6 b c^4 e^2 x^6 \ln(F) + 6 c^5 d^2 \ln(F) - 6 c^5 d e x^3 \ln(F) + 6 c^5 e^2 x^6 \ln(F) + 6 b^6 c^6 d^2 \ln(F) - 2 b^6 c^6 d e x^3 \ln(F) + b^6 c^6 e^2 x^6 \ln(F) + 6 b^5 c^5 d e x^2 \ln(F) - 6 b^5 c^5 e^2 x^5 \ln(F) + 6 b^4 c^4 d^2 \ln(F) - 6 b^4 c^4 d e x^3 \ln(F) + 6 b^4 c^4 e^2 x^6 \ln(F) + 6 b^3 c^3 d^2 \ln(F) - 6 b^3 c^3 d e x^3 \ln(F) + 6 b^3 c^3 e^2 x^6 \ln(F) + 6 b^2 c^2 d^3 \ln(F) - 6 b^2 c^2 d^2 e x^3 \ln(F) + 6 b^2 c^2 d e^2 x^6 \ln(F) + 6 b c d^4 \ln(F) - 6 b c d^3 e x^3 \ln(F) + 6 b c d^2 e^2 x^6 \ln(F) + 6 c^5 d^2 \ln(F) - 6 c^5 d e x^3 \ln(F) + 6 c^5 e^2 x^6 \ln(F)}{b^7 c^7 \log(F)^7}$$

input

```
int(F^(c*(a + b*x))*(d - e*x^3)^2,x)
```

output

```

(F^(a*c + b*c*x)*(720*e^2 + b^6*c^6*d^2*log(F)^6 - 720*b*c*e^2*x*log(F) +
360*b^2*c^2*e^2*x^2*log(F)^2 - 120*b^3*c^3*e^2*x^3*log(F)^3 + 30*b^4*c^4*e
^2*x^4*log(F)^4 - 6*b^5*c^5*e^2*x^5*log(F)^5 + b^6*c^6*e^2*x^6*log(F)^6 +
12*b^3*c^3*d*e*log(F)^3 - 12*b^4*c^4*d*e*x*log(F)^4 + 6*b^5*c^5*d*e*x^2*lo
g(F)^5 - 2*b^6*c^6*d*e*x^3*log(F)^6))/(b^7*c^7*log(F)^7)

```

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d - ex^3)^2 dx$$

$$= \frac{f^{bcx+ac}(\log(f)^6 b^6 c^6 d^2 - 2\log(f)^6 b^6 c^6 d e x^3 + \log(f)^6 b^6 c^6 e^2 x^6 + 6\log(f)^5 b^5 c^5 d e x^2 - 6\log(f)^5 b^5 c^5 e^2 x^5 - \dots)}{\dots}$$

input `int(F^((b*x+a)*c)*(-e*x^3+d)^2,x)`output `(f**(a*c + b*c*x)*(log(f)**6*b**6*c**6*d**2 - 2*log(f)**6*b**6*c**6*d*e*x**3 + log(f)**6*b**6*c**6*e**2*x**6 + 6*log(f)**5*b**5*c**5*d*e*x**2 - 6*log(f)**5*b**5*c**5*e**2*x**5 - 12*log(f)**4*b**4*c**4*d*e*x + 30*log(f)**4*b**4*c**4*e**2*x**4 + 12*log(f)**3*b**3*c**3*d*e - 120*log(f)**3*b**3*c**3*e**2*x**3 + 360*log(f)**2*b**2*c**2*e**2*x**2 - 720*log(f)*b*c*e**2*x + 720*e**2))/(log(f)**7*b**7*c**7)`



### 3.20 $\int F^{c(a+bx)}(d - ex^3) dx$

Optimal result	184
Mathematica [A] (verified)	184
Rubi [A] (verified)	185
Maple [A] (verified)	186
Fricas [A] (verification not implemented)	186
Sympy [A] (verification not implemented)	187
Maxima [A] (verification not implemented)	187
Giac [C] (verification not implemented)	188
Mupad [B] (verification not implemented)	189
Reduce [B] (verification not implemented)	189

#### Optimal result

Integrand size = 18, antiderivative size = 99

$$\int F^{c(a+bx)}(d - ex^3) dx = \frac{6eF^{c(a+bx)}}{b^4c^4 \log^4(F)} - \frac{6eF^{c(a+bx)}x}{b^3c^3 \log^3(F)} + \frac{3eF^{c(a+bx)}x^2}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d - ex^3)}{bc \log(F)}$$

output

$6eF^{c(bx+a)}/b^4/c^4/\ln(F)^4 - 6eF^{c(bx+a)}x/b^3/c^3/\ln(F)^3 + 3eF^{c(bx+a)}x^2/b^2/c^2/\ln(F)^2 + F^{c(bx+a)}(-ex^3+d)/b/c/\ln(F)$

#### Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}(d - ex^3) dx = \frac{F^{c(a+bx)}(6e - 6bcex \log(F) + 3b^2c^2ex^2 \log^2(F) + b^3c^3(d - ex^3) \log^3(F))}{b^4c^4 \log^4(F)}$$

input

`Integrate[F^(c*(a + b*x))*(d - e*x^3),x]`

output

$(F^{c(a + bx)})(6e - 6b*c*e*x*\text{Log}[F] + 3*b^2*c^2*e*x^2*\text{Log}[F]^2 + b^3*c^3*(d - e*x^3)*\text{Log}[F]^3)/(b^4*c^4*\text{Log}[F]^4)$

**Rubi [A] (verified)**

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d - ex^3) F^{c(a+bx)} dx$$

$$\downarrow 2626$$

$$\int (dF^{c(a+bx)} - ex^3 F^{c(a+bx)}) dx$$

$$\downarrow 2009$$

$$\frac{6eF^{c(a+bx)}}{b^4c^4 \log^4(F)} - \frac{6exF^{c(a+bx)}}{b^3c^3 \log^3(F)} + \frac{3ex^2F^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{dF^{c(a+bx)}}{bc \log(F)} - \frac{ex^3F^{c(a+bx)}}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d - e*x^3),x]`

output `(6*e*F^(c*(a + b*x)))/(b^4*c^4*Log[F]^4) - (6*e*F^(c*(a + b*x))*x)/(b^3*c^3*Log[F]^3) + (3*e*F^(c*(a + b*x))*x^2)/(b^2*c^2*Log[F]^2) + (d*F^(c*(a + b*x)))/(b*c*Log[F]) - (e*F^(c*(a + b*x))*x^3)/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

method	result	size
gospers	$\frac{(-e x^3 \ln(F)^3 b^3 c^3 + \ln(F)^3 b^3 c^3 d + 3e x^2 \ln(F)^2 b^2 c^2 - 6ex \ln(F)bc + 6e) F^{c(bx+a)}}{\ln(F)^4 b^4 c^4}$	77
risch	$\frac{(-e x^3 \ln(F)^3 b^3 c^3 + \ln(F)^3 b^3 c^3 d + 3e x^2 \ln(F)^2 b^2 c^2 - 6ex \ln(F)bc + 6e) F^{c(bx+a)}}{\ln(F)^4 b^4 c^4}$	77
orering	$\frac{(-e x^3 \ln(F)^3 b^3 c^3 + \ln(F)^3 b^3 c^3 d + 3e x^2 \ln(F)^2 b^2 c^2 - 6ex \ln(F)bc + 6e) F^{c(bx+a)}}{\ln(F)^4 b^4 c^4}$	77
meijerg	$-\frac{F^{ac} e \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^2 x^2 \ln(F)^2 - 24bcx \ln(F) + 24) e^{bcx \ln(F)}}{4} \right)}{\ln(F)^4 b^4 c^4} - \frac{F^{ac} d (1 - e^{bcx \ln(F)})}{bc \ln(F)}$	99
parallelrisc	$\frac{-x^3 F^{c(bx+a)} e \ln(F)^3 b^3 c^3 + \ln(F)^3 F^{c(bx+a)} b^3 c^3 d + 3x^2 F^{c(bx+a)} e \ln(F)^2 b^2 c^2 - 6e F^{c(bx+a)} x \ln(F)bc + 6F^{c(bx+a)} e}{\ln(F)^4 b^4 c^4}$	113
norman	$\frac{(\ln(F)^3 b^3 c^3 d + 6e) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4} - \frac{6ex e^{c(bx+a) \ln(F)}}{\ln(F)^3 b^3 c^3} + \frac{3e x^2 e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} - \frac{e x^3 e^{c(bx+a) \ln(F)}}{\ln(F)bc}$	115

input `int(F^(c*(b*x+a))*(-e*x^3+d),x,method=_RETURNVERBOSE)`

output `(-e*x^3*ln(F)^3*b^3*c^3+ln(F)^3*b^3*c^3*d+3*e*x^2*ln(F)^2*b^2*c^2-6*e*x*ln(F)*b*c+6*e)*F^(c*(b*x+a))/ln(F)^4/b^4/c^4`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}(d - ex^3) dx$$

$$= \frac{(3b^2c^2ex^2 \log(F)^2 - 6bcex \log(F) - (b^3c^3ex^3 - b^3c^3d) \log(F)^3 + 6e) F^{bcx+ac}}{b^4c^4 \log(F)^4}$$

input `integrate(F^((b*x+a)*c))*(-e*x^3+d),x, algorithm="fricas")`

output `(3*b^2*c^2*e*x^2*log(F)^2 - 6*b*c*e*x*log(F) - (b^3*c^3*e*x^3 - b^3*c^3*d)*log(F)^3 + 6*e)*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4)`

**Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)}(d - ex^3) dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^3c^3d \log(F)^3 - b^3c^3ex^3 \log(F)^3 + 3b^2c^2ex^2 \log(F)^2 - 6bcex \log(F) + 6e)}{b^4c^4 \log(F)^4} & \text{for } b^4c^4 \log(F)^4 \neq 0 \\ dx - \frac{ex^4}{4} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(-e*x**3+d), x)`output `Piecewise((F**(c*(a + b*x))*(b**3*c**3*d*log(F)**3 - b**3*c**3*e*x**3*log(F)**3 + 3*b**2*c**2*e*x**2*log(F)**2 - 6*b*c*e*x*log(F) + 6*e)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d*x - e*x**4/4, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int F^{c(a+bx)}(d - ex^3) dx$$

$$= \frac{F^{bcx+ac}d}{bc \log(F)} - \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}e}{b^4c^4 \log(F)^4}$$

input `integrate(F^((b*x+a)*c)*(-e*x^3+d), x, algorithm="maxima")`output `F^(b*c*x + a*c)*d/(b*c*log(F)) - (F^(a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))*F^(b*c*x)*e/(b^4*c^4*log(F)^4)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 2601, normalized size of antiderivative = 26.27

$$\int F^{c(a+bx)}(d - ex^3) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(-e*x^3+d),x, algorithm="giac")`

output

```
((3*pi^2*b^3*c^3*e*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*e*x^3*log(abs(F)) + 2*b^3*c^3*e*x^3*log(abs(F))^3 - 3*pi^2*b^3*c^3*d*log(abs(F))*sgn(F) + 3*pi^2*b^3*c^3*d*log(abs(F))^3 - 2*b^2*c^2*e*x^2*sgn(F) + 3*pi^2*b^2*c^2*e*x^2 - 6*b^2*c^2*e*x^2*log(abs(F))^2 + 12*b*c*e*x*log(abs(F)) - 12*e)*(pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)/(pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2 - 4*(pi^3*b^3*c^3*e*x^3*sgn(F) - 3*pi*b^3*c^3*e*x^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3*e*x^3 + 3*pi*b^3*c^3*e*x^3*log(abs(F))^2 - pi^3*b^3*c^3*d*sgn(F) + 3*pi*b^3*c^3*d*log(abs(F))^2*sgn(F) + pi^3*b^3*c^3*d - 3*pi*b^3*c^3*d*log(abs(F))^2 + 6*pi*b^2*c^2*e*x^2*log(abs(F))*sgn(F) - 6*pi*b^2*c^2*e*x^2*log(abs(F)) - 6*pi*b*c*e*x*sgn(F) + 6*pi*b*c*e*x*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)/(pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2)*cos(-1/2*pi*b*c*x...
```

**Mupad [B] (verification not implemented)**

Time = 22.86 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}(d - ex^3) dx$$

$$= \frac{F^{ac+bcx} (-e b^3 c^3 x^3 \ln(F)^3 + d b^3 c^3 \ln(F)^3 + 3 e b^2 c^2 x^2 \ln(F)^2 - 6 e b c x \ln(F) + 6 e)}{b^4 c^4 \ln(F)^4}$$

input `int(F^(c*(a + b*x))*(d - e*x^3),x)`output `(F^(a*c + b*c*x)*(6*e + b^3*c^3*d*log(F)^3 + 3*b^2*c^2*e*x^2*log(F)^2 - b^3*c^3*e*x^3*log(F)^3 - 6*b*c*e*x*log(F)))/(b^4*c^4*log(F)^4)`**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int F^{c(a+bx)}(d - ex^3) dx$$

$$= \frac{f^{bcx+ac}(\log(f)^3 b^3 c^3 d - \log(f)^3 b^3 c^3 e x^3 + 3 \log(f)^2 b^2 c^2 e x^2 - 6 \log(f) b c e x + 6 e)}{\log(f)^4 b^4 c^4}$$

input `int(F^((b*x+a)*c)*(-e*x^3+d),x)`output `(f**(a*c + b*c*x)*(log(f)**3*b**3*c**3*d - log(f)**3*b**3*c**3*e*x**3 + 3*log(f)**2*b**2*c**2*e*x**2 - 6*log(f)*b*c*e*x + 6*e))/(log(f)**4*b**4*c**4)`

### 3.21 $\int \frac{F^{c(a+bx)}}{d-ex^3} dx$

Optimal result	190
Mathematica [C] (verified)	191
Rubi [A] (verified)	191
Maple [C] (verified)	193
Fricas [A] (verification not implemented)	193
Sympy [F]	194
Maxima [F]	194
Giac [F]	194
Mupad [F(-1)]	195
Reduce [F]	195

#### Optimal result

Integrand size = 20, antiderivative size = 211

$$\int \frac{F^{c(a+bx)}}{d-ex^3} dx$$

$$= \frac{F^{c\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc\left(\sqrt[3]{d}-\sqrt[3]{ex}\right) \log(F)}{\sqrt[3]{e}}\right)}{3d^{2/3}\sqrt[3]{e}} - \frac{(-1)^{2/3} F^{c\left(a+\frac{(-1)^{2/3}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc\left((-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{ex}\right) \log(F)}{\sqrt[3]{e}}\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\sqrt[3]{-1} F^{c\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(\frac{\sqrt[3]{-1}bc\left(\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{ex}\right) \log(F)}{\sqrt[3]{e}}\right)}{3d^{2/3}\sqrt[3]{e}}$$

output

```
-1/3*F^(c*(a+b*d^(1/3)/e^(1/3)))*Ei(-b*c*(d^(1/3)-e^(1/3)*x)*ln(F)/e^(1/3)
)/d^(2/3)/e^(1/3)-1/3*(-1)^(2/3)*F^(c*(a+(-1)^(2/3)*b*d^(1/3)/e^(1/3)))*Ei
(-b*c*((-1)^(2/3)*d^(1/3)-e^(1/3)*x)*ln(F)/e^(1/3))/d^(2/3)/e^(1/3)+1/3*(-
1)^(1/3)*F^(c*(a-(-1)^(1/3)*b*d^(1/3)/e^(1/3)))*Ei((-1)^(1/3)*b*c*(d^(1/3)
-(-1)^(2/3)*e^(1/3)*x)*ln(F)/e^(1/3))/d^(2/3)/e^(1/3)
```

**Mathematica [C] (verified)**

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = - \frac{\text{RootSum}\left[d - e\#1^3 \&, \frac{F^{c(a+b\#1)} \text{ExpIntegralEi}(bc \log(F)(x - \#1)) \&}{\#1^2}\right]}{3e}$$

input `Integrate[F^(c*(a + b*x))/(d - e*x^3),x]`

output `-1/3*RootSum[d - e*#1^3 & , (F^(c*(a + b*#1))*ExpIntegralEi[b*c*Log[F]*(x - #1)])/#1^2 & ]/e`

**Rubi [A] (verified)**

Time = 0.89 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx$$

↓ 7276

$$\int \left( \frac{F^{ac+bcx}}{3d^{2/3} (\sqrt[3]{d} - \sqrt[3]{ex})} + \frac{F^{ac+bcx}}{3d^{2/3} (\sqrt[3]{d} + \sqrt[3]{-1}\sqrt[3]{ex})} + \frac{F^{ac+bcx}}{3d^{2/3} (\sqrt[3]{d} - (-1)^{2/3}\sqrt[3]{ex})} \right) dx$$

↓ 2009



$$\frac{F^{c\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \operatorname{ExpIntegralEi}\left(-\frac{bc\left(\sqrt[3]{d}-\sqrt[3]{ex}\right)\log(F)}{\sqrt[3]{e}}\right)}{3d^{2/3}\sqrt[3]{e}} + \frac{\sqrt[3]{-1}F^{c\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \operatorname{ExpIntegralEi}\left(\frac{\sqrt[3]{-1}bc\left(\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{ex}\right)\log(F)}{\sqrt[3]{e}}\right)}{3d^{2/3}\sqrt[3]{e}} - \frac{(-1)^{2/3}F^{c\left(a+\frac{(-1)^{2/3}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \operatorname{ExpIntegralEi}\left(-\frac{bc\left((-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{ex}\right)\log(F)}{\sqrt[3]{e}}\right)}{3d^{2/3}\sqrt[3]{e}}$$

input `Int[F^(c*(a + b*x))/(d - e*x^3),x]`

output `-1/3*(F^(c*(a + (b*d^(1/3))/e^(1/3)))*ExpIntegralEi[-((b*c*(d^(1/3) - e^(1/3)*x)*Log[F])/e^(1/3))]/(d^(2/3)*e^(1/3)) - ((-1)^(2/3)*F^(c*(a + ((-1)^(2/3)*b*d^(1/3))/e^(1/3)))*ExpIntegralEi[-((b*c*((-1)^(2/3)*d^(1/3) - e^(1/3)*x)*Log[F])/e^(1/3))]/(3*d^(2/3)*e^(1/3)) + ((-1)^(1/3)*F^(c*(a - ((-1)^(1/3)*b*d^(1/3))/e^(1/3)))*ExpIntegralEi[((-1)^(1/3)*b*c*(d^(1/3) - (-1)^(2/3)*e^(1/3)*x)*Log[F])/e^(1/3)]/(3*d^(2/3)*e^(1/3))`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7276 `Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]`

### Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.57

method	result
risch	$\frac{\ln(F)^2 b^2 c^2 \left( \sum_{R1=\text{RootOf}(-\ln(F)^3 a^3 c^3 e - \ln(F)^3 b^3 c^3 d + 3 \ln(F)^2 a^2 c^2 e - Z - 3 \ln(F) a c e - Z^2 + e - Z^3)} \frac{e^{-R1} \exp(\text{Integral}_1(-bcx \ln(F) - ac \ln(F) - R1 ac + \dots))}{\ln(F)^2 a^2 c^2 - 2 \ln(F) - R1 ac + \dots} \right)}{3e}$

```
input int(F^(c*(b*x+a))/(-e*x^3+d),x,method=_RETURNVERBOSE)
```

```
output 1/3*ln(F)^2*b^2*c^2/e*sum(1/(ln(F)^2*a^2*c^2-2*ln(F)*_R1*a*c+_R1^2)*exp(_R1)*Ei(1,-b*c*x*ln(F)-a*c*ln(F)+_R1),_R1=RootOf(-ln(F)^3*a^3*c^3*e-ln(F)^3*b^3*c^3*d+3*ln(F)^2*a^2*c^2*e*_Z-3*ln(F)*a*c*e*_Z^2+e*_Z^3))
```

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.27

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = \frac{\left(-\frac{b^3 c^3 d \log(F)^3}{e}\right)^{\frac{1}{3}} (\sqrt{-3} + 1) \text{Ei}\left(bc x \log(F) - \frac{1}{2} \left(-\frac{b^3 c^3 d \log(F)^3}{e}\right)^{\frac{1}{3}} (\sqrt{-3} + 1)\right) e^{\left(ac \log(F) + \frac{1}{2} \left(-\frac{b^3 c^3 d \log(F)^3}{e}\right)^{\frac{1}{3}} (\sqrt{-3} + 1)\right)}}{\dots}$$

```
input integrate(F^((b*x+a)*c)/(-e*x^3+d),x, algorithm="fricas")
```

```
output -1/6*((-b^3*c^3*d*log(F)^3/e)^(1/3)*(sqrt(-3) + 1)*Ei(b*c*x*log(F) - 1/2*(-b^3*c^3*d*log(F)^3/e)^(1/3)*(sqrt(-3) + 1)))*e^(a*c*log(F) + 1/2*(-b^3*c^3*d*log(F)^3/e)^(1/3)*(sqrt(-3) + 1)) - (-b^3*c^3*d*log(F)^3/e)^(1/3)*(sqrt(-3) - 1)*Ei(b*c*x*log(F) + 1/2*(-b^3*c^3*d*log(F)^3/e)^(1/3)*(sqrt(-3) - 1))*e^(a*c*log(F) - 1/2*(-b^3*c^3*d*log(F)^3/e)^(1/3)*(sqrt(-3) - 1)) - 2*(-b^3*c^3*d*log(F)^3/e)^(1/3)*Ei(b*c*x*log(F) + (-b^3*c^3*d*log(F)^3/e)^(1/3))*e^(a*c*log(F) - (-b^3*c^3*d*log(F)^3/e)^(1/3))/(b*c*d*log(F))
```

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = - \int \frac{F^{ac+bcx}}{-d + ex^3} dx$$

input `integrate(F**((b*x+a)*c)/(-e*x**3+d), x)`

output `-Integral(F**(a*c + b*c*x)/(-d + e*x**3), x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = \int -\frac{F^{(bx+a)c}}{ex^3 - d} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^3+d), x, algorithm="maxima")`

output `-integrate(F^((b*x + a)*c)/(e*x^3 - d), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = \int -\frac{F^{(bx+a)c}}{ex^3 - d} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^3+d), x, algorithm="giac")`

output `integrate(-F^((b*x + a)*c)/(e*x^3 - d), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = \int \frac{F^{c(a+bx)}}{d - ex^3} dx$$

input `int(F^(c*(a + b*x))/(d - e*x^3), x)`output `int(F^(c*(a + b*x))/(d - e*x^3), x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d - ex^3} dx = f^{ac} \left( \int \frac{f^{bcx}}{-ex^3 + d} dx \right)$$

input `int(F^((b*x+a)*c)/(-e*x^3+d), x)`output `f**(a*c)*int(f**(b*c*x)/(d - e*x**3), x)`

$$3.22 \quad \int \frac{F^{c(a+bx)}}{(d-ex^3)^2} dx$$

Optimal result . . . . .	197
Mathematica [C] (verified) . . . . .	198
Rubi [A] (verified) . . . . .	199
Maple [F] . . . . .	201
Fricas [A] (verification not implemented) . . . . .	202
Sympy [F] . . . . .	202
Maxima [F] . . . . .	203
Giac [F] . . . . .	203
Mupad [F(-1)] . . . . .	203
Reduce [F] . . . . .	204

**Optimal result**

Integrand size = 20, antiderivative size = 617

$$\begin{aligned}
& \int \frac{F^{c(a+bx)}}{(d-ex^3)^2} dx \\
&= \frac{F^{c(a+bx)}}{9d^{4/3}\sqrt[3]{e}(\sqrt[3]{d}-\sqrt[3]{ex})} - \frac{(-1)^{2/3}F^{c(a+bx)}}{9d^{4/3}\sqrt[3]{e}(\sqrt[3]{-1}\sqrt[3]{d}+\sqrt[3]{ex})} \\
&\quad - \frac{\sqrt[3]{-1}F^{c(a+bx)}}{(1+\sqrt[3]{-1})^4 d^{4/3}\sqrt[3]{e}(\sqrt[3]{d}+\sqrt[3]{-1}\sqrt[3]{ex})} \\
&\quad - \frac{2F^{c\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt[3]{d}-\sqrt[3]{ex})\log(F)}{\sqrt[3]{e}}\right)}{9d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{2i\sqrt{3}F^{c\left(a+\frac{(-1)^{2/3}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc((-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{ex})\log(F)}{\sqrt[3]{e}}\right)}{(1+\sqrt[3]{-1})^5 d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{4F^{c\left(a-\frac{2ib\sqrt[3]{d}}{(i+\sqrt{3})\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(\frac{bc(2\sqrt[3]{d}+(1-i\sqrt{3})\sqrt[3]{ex})\log(F)}{(1-i\sqrt{3})\sqrt[3]{e}}\right)}{9(1-i\sqrt{3})d^{5/3}\sqrt[3]{e}} \\
&\quad + \frac{bcF^{c\left(a+\frac{b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc(\sqrt[3]{d}-\sqrt[3]{ex})\log(F)}{\sqrt[3]{e}}\right) \log(F)}{9d^{4/3}e^{2/3}} \\
&\quad + \frac{bcF^{c\left(a+\frac{(-1)^{2/3}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(-\frac{bc((-1)^{2/3}\sqrt[3]{d}-\sqrt[3]{ex})\log(F)}{\sqrt[3]{e}}\right) \log(F)}{(1+\sqrt[3]{-1})^4 d^{4/3}e^{2/3}} \\
&\quad + \frac{(-1)^{2/3}bcF^{c\left(a-\frac{\sqrt[3]{-1}b\sqrt[3]{d}}{\sqrt[3]{e}}\right)} \text{ExpIntegralEi}\left(\frac{\sqrt[3]{-1}bc(\sqrt[3]{d}-(-1)^{2/3}\sqrt[3]{ex})\log(F)}{\sqrt[3]{e}}\right) \log(F)}{9d^{4/3}e^{2/3}}
\end{aligned}$$

output

```
1/9*F^(c*(b*x+a))/d^(4/3)/e^(1/3)/(d^(1/3)-e^(1/3)*x)-1/9*(-1)^(2/3)*F^(c*(
(b*x+a))/d^(4/3)/e^(1/3)/((-1)^(1/3)*d^(1/3)+e^(1/3)*x)-(-1)^(1/3)*F^(c*(b
*x+a))/(1+(-1)^(1/3))^4/d^(4/3)/e^(1/3)/(d^(1/3)+(-1)^(1/3)*e^(1/3)*x)-2/9
*F^(c*(a+b*d^(1/3)/e^(1/3)))*Ei(-b*c*(d^(1/3)-e^(1/3)*x)*ln(F)/e^(1/3))/d^
(5/3)/e^(1/3)+2*I*3^(1/2)*F^(c*(a+(-1)^(2/3)*b*d^(1/3)/e^(1/3)))*Ei(-b*c*(
(-1)^(2/3)*d^(1/3)-e^(1/3)*x)*ln(F)/e^(1/3))/(1+(-1)^(1/3))^5/d^(5/3)/e^(1
/3)+4/9*F^(c*(a-2*I*b*d^(1/3)/(3^(1/2)+I)/e^(1/3)))*Ei(b*c*(2*d^(1/3)+(1-I
*3^(1/2))*e^(1/3)*x)*ln(F)/(1-I*3^(1/2))/e^(1/3))/(1-I*3^(1/2))/d^(5/3)/e^
(1/3)+1/9*b*c*F^(c*(a+b*d^(1/3)/e^(1/3)))*Ei(-b*c*(d^(1/3)-e^(1/3)*x)*ln(F
)/e^(1/3))*ln(F)/d^(4/3)/e^(2/3)+b*c*F^(c*(a+(-1)^(2/3)*b*d^(1/3)/e^(1/3)
))*Ei(-b*c*((-1)^(2/3)*d^(1/3)-e^(1/3)*x)*ln(F)/e^(1/3))*ln(F)/(1+(-1)^(1/3
))^4/d^(4/3)/e^(2/3)+1/9*(-1)^(2/3)*b*c*F^(c*(a-(-1)^(1/3)*b*d^(1/3)/e^(1/
3)))*Ei((-1)^(1/3)*b*c*(d^(1/3)-(-1)^(2/3)*e^(1/3)*x)*ln(F)/e^(1/3))*ln(F)
/d^(4/3)/e^(2/3)
```

### Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.20

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx$$

$$= \frac{\frac{3F^{c(a+bx)}x}{d-ex^3} - \frac{{}_2\text{RootSum}\left[d-e\#1^3 \&, \frac{F^{c(a+b\#1)} \text{ExpIntegralEi}(bc \log(F)(x-\#1))}{\#1^2} \& \right]}{e}}{9d} + \frac{bc \log(F) \text{RootSum}\left[d-e\#1^3 \&, \frac{F^{c(a+b\#1)}}{e} \right]}{e}$$

input

```
Integrate[F^(c*(a + b*x))/(d - e*x^3)^2,x]
```

output

```
((3*F^(c*(a + b*x))*x)/(d - e*x^3) - (2*RootSum[d - e*#1^3 & , (F^(c*(a +
b*#1))*ExpIntegralEi[b*c*Log[F]*(x - #1)])/#1^2 & ])/e + (b*c*Log[F]*RootS
um[d - e*#1^3 & , (F^(c*(a + b*#1))*ExpIntegralEi[b*c*Log[F]*(x - #1)])/#1
& ])/e)/(9*d)
```

**Rubi [A] (verified)**

Time = 2.24 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx$$

↓ 7292

$$\int \frac{F^{ac+bcx}}{(d - ex^3)^2} dx$$

↓ 7293

$$\int \left( \frac{2F^{ac+bcx}}{9d^{5/3} (\sqrt[3]{d} - \sqrt[3]{ex})} + \frac{2(-1)^{5/6} \sqrt{3} F^{ac+bcx}}{(1 + \sqrt[3]{-1})^5 d^{5/3} (\sqrt[3]{d} + \sqrt[3]{-1} \sqrt[3]{ex})} + \frac{4i F^{ac+bcx}}{9d^{5/3} (2i \sqrt[3]{d} + (\sqrt{3} + i) \sqrt[3]{ex})} + \frac{F^{ac+bcx}}{9d^{4/3} (\sqrt[3]{d} + \sqrt[3]{ex})} \right)$$

↓ 2009



$$\begin{aligned}
& \frac{bc \log(F) F^{c \left( a + \frac{b \sqrt[3]{d}}{\sqrt[3]{e}} \right)} \operatorname{ExpIntegralEi} \left( -\frac{bc \left( \sqrt[3]{d} - \sqrt[3]{ex} \right) \log(F)}{\sqrt[3]{e}} \right)}{9d^{4/3} e^{2/3}} + \\
& \frac{(-1)^{2/3} bc \log(F) F^{c \left( a - \frac{\sqrt[3]{-1} b \sqrt[3]{d}}{\sqrt[3]{e}} \right)} \operatorname{ExpIntegralEi} \left( \frac{\sqrt[3]{-1} bc \left( \sqrt[3]{d} - (-1)^{2/3} \sqrt[3]{ex} \right) \log(F)}{\sqrt[3]{e}} \right)}{9d^{4/3} e^{2/3}} + \\
& \frac{bc \log(F) F^{c \left( a + \frac{(-1)^{2/3} b \sqrt[3]{d}}{\sqrt[3]{e}} \right)} \operatorname{ExpIntegralEi} \left( -\frac{bc \left( (-1)^{2/3} \sqrt[3]{d} - \sqrt[3]{ex} \right) \log(F)}{\sqrt[3]{e}} \right)}{(1 + \sqrt[3]{-1})^4 d^{4/3} e^{2/3}} - \\
& \frac{2F^{c \left( a + \frac{b \sqrt[3]{d}}{\sqrt[3]{e}} \right)} \operatorname{ExpIntegralEi} \left( -\frac{bc \left( \sqrt[3]{d} - \sqrt[3]{ex} \right) \log(F)}{\sqrt[3]{e}} \right)}{9d^{5/3} \sqrt[3]{e}} + \\
& \frac{2i\sqrt{3} F^{c \left( a + \frac{(-1)^{2/3} b \sqrt[3]{d}}{\sqrt[3]{e}} \right)} \operatorname{ExpIntegralEi} \left( -\frac{bc \left( (-1)^{2/3} \sqrt[3]{d} - \sqrt[3]{ex} \right) \log(F)}{\sqrt[3]{e}} \right)}{(1 + \sqrt[3]{-1})^5 d^{5/3} \sqrt[3]{e}} + \\
& \frac{4F^{c \left( a - \frac{2ib \sqrt[3]{d}}{(\sqrt{3}+i) \sqrt[3]{e}} \right)} \operatorname{ExpIntegralEi} \left( \frac{bc \left( (i+\sqrt{3}) \sqrt[3]{ex} + 2i \sqrt[3]{d} \right) \log(F)}{(i+\sqrt{3}) \sqrt[3]{e}} \right)}{9(1-i\sqrt{3}) d^{5/3} \sqrt[3]{e}} + \frac{F^{ac+bcx}}{9d^{4/3} \sqrt[3]{e} \left( \sqrt[3]{d} - \sqrt[3]{ex} \right)} - \\
& \frac{(-1)^{2/3} F^{ac+bcx}}{9d^{4/3} \sqrt[3]{e} \left( \sqrt[3]{-1} \sqrt[3]{d} + \sqrt[3]{ex} \right)} - \frac{\sqrt[3]{-1} F^{ac+bcx}}{(1 + \sqrt[3]{-1})^4 d^{4/3} \sqrt[3]{e} \left( \sqrt[3]{d} + \sqrt[3]{-1} \sqrt[3]{ex} \right)}
\end{aligned}$$

input `Int [F^(c*(a + b*x))/(d - e*x^3)^2,x]`

output

```

F^(a*c + b*c*x)/(9*d^(4/3)*e^(1/3)*(d^(1/3) - e^(1/3)*x)) - ((-1)^(2/3)*F^(
(a*c + b*c*x))/(9*d^(4/3)*e^(1/3)*((-1)^(1/3)*d^(1/3) + e^(1/3)*x)) - ((-1)
)^(1/3)*F^(a*c + b*c*x)/((1 + (-1)^(1/3))^4*d^(4/3)*e^(1/3)*(d^(1/3) + (-
1)^(1/3)*e^(1/3)*x)) - (2*F^(c*(a + (b*d^(1/3))/e^(1/3)))*ExpIntegralEi[-(
(b*c*(d^(1/3) - e^(1/3)*x)*Log[F])/e^(1/3))]/(9*d^(5/3)*e^(1/3)) + ((2*I)
)*Sqrt[3]*F^(c*(a + ((-1)^(2/3)*b*d^(1/3))/e^(1/3)))*ExpIntegralEi[-((b*c*(
(-1)^(2/3)*d^(1/3) - e^(1/3)*x)*Log[F])/e^(1/3))]/((1 + (-1)^(1/3))^5*d^(
5/3)*e^(1/3)) + (4*F^(c*(a - ((2*I)*b*d^(1/3))/(I + Sqrt[3])*e^(1/3)))*E
xpIntegralEi[(b*c*((2*I)*d^(1/3) + (I + Sqrt[3])*e^(1/3)*x)*Log[F])/((I +
Sqrt[3])*e^(1/3))]/(9*(1 - I*Sqrt[3])*d^(5/3)*e^(1/3)) + (b*c*F^(c*(a + (
b*d^(1/3))/e^(1/3)))*ExpIntegralEi[-((b*c*(d^(1/3) - e^(1/3)*x)*Log[F])/e^(
1/3))]*Log[F])/((9*d^(4/3)*e^(2/3)) + (b*c*F^(c*(a + ((-1)^(2/3)*b*d^(1/3)
)/e^(1/3)))*ExpIntegralEi[-((b*c*((-1)^(2/3)*d^(1/3) - e^(1/3)*x)*Log[F])/
e^(1/3))]*Log[F])/((1 + (-1)^(1/3))^4*d^(4/3)*e^(2/3)) + ((-1)^(2/3)*b*c*F
^(c*(a - ((-1)^(1/3)*b*d^(1/3))/e^(1/3)))*ExpIntegralEi[((-1)^(1/3)*b*c*(d
^(1/3) - (-1)^(2/3)*e^(1/3)*x)*Log[F])/e^(1/3)]*Log[F])/((9*d^(4/3)*e^(2/3)
)

```

### Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7292

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

rule 7293

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Maple [F]

$$\int \frac{F^{c(bx+a)}}{(-ex^3+d)^2} dx$$

input

```
int(F^(c*(b*x+a))/(-e*x^3+d)^2,x)
```

output `int(F^(c*(b*x+a))/(-e*x^3+d)^2,x)`

### Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.75

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx =$$

$$\frac{6 F^{bcx+ac} bcdx \log(F) + \left( \left( -\frac{b^3 c^3 d \log(F)^3}{e} \right)^{\frac{2}{3}} (ex^3 - \sqrt{-3}(ex^3 - d) - d) + 2 \left( -\frac{b^3 c^3 d \log(F)^3}{e} \right)^{\frac{1}{3}} (ex^3 + \sqrt{-3}(ex^3 - d) - d) \right)}{(d - ex^3)^2}$$

input `integrate(F^((b*x+a)*c)/(-e*x^3+d)^2,x, algorithm="fricas")`

output

$$\begin{aligned} & -1/18*(6*F^{(b*c*x + a*c)}*b*c*d*x*\log(F) + ((-b^3*c^3*d*\log(F)^3/e)^{(2/3)}*( \\ & e*x^3 - \sqrt{-3}*(e*x^3 - d) - d) + 2*(-b^3*c^3*d*\log(F)^3/e)^{(1/3)}*(e*x^3 \\ & + \sqrt{-3}*(e*x^3 - d) - d))*Ei(b*c*x*\log(F) - 1/2*(-b^3*c^3*d*\log(F)^3/e \\ & )^{(1/3)}*(\sqrt{-3} + 1))*e^{(a*c*\log(F) + 1/2*(-b^3*c^3*d*\log(F)^3/e)^{(1/3)}* \\ & (\sqrt{-3} + 1))} + ((-b^3*c^3*d*\log(F)^3/e)^{(2/3)}*(e*x^3 + \sqrt{-3}*(e*x^3 \\ & - d) - d) + 2*(-b^3*c^3*d*\log(F)^3/e)^{(1/3)}*(e*x^3 - \sqrt{-3}*(e*x^3 - d) \\ & - d))*Ei(b*c*x*\log(F) + 1/2*(-b^3*c^3*d*\log(F)^3/e)^{(1/3)}*(\sqrt{-3} - 1))* \\ & e^{(a*c*\log(F) - 1/2*(-b^3*c^3*d*\log(F)^3/e)^{(1/3)}*(\sqrt{-3} - 1))} - 2*((-b \\ & ^3*c^3*d*\log(F)^3/e)^{(2/3)}*(e*x^3 - d) + 2*(-b^3*c^3*d*\log(F)^3/e)^{(1/3)}*( \\ & e*x^3 - d))*Ei(b*c*x*\log(F) + (-b^3*c^3*d*\log(F)^3/e)^{(1/3))*e^{(a*c*\log(F) \\ & - (-b^3*c^3*d*\log(F)^3/e)^{(1/3))}}/(b*c*d^2*e*x^3 - b*c*d^3)*\log(F)) \end{aligned}$$

### Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx = \int \frac{F^{c(a+bx)}}{(-d + ex^3)^2} dx$$

input `integrate(F**((b*x+a)*c)/(-e*x**3+d)**2,x)`

output `Integral(F**(c*(a + b*x))/(-d + e*x**3)**2, x)`

### Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx = \int \frac{F^{(bx+a)c}}{(ex^3 - d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^3+d)^2,x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x^3 - d)^2, x)`

### Giac [F]

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx = \int \frac{F^{(bx+a)c}}{(ex^3 - d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(-e*x^3+d)^2,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x^3 - d)^2, x)`

### Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx = \int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx$$

input `int(F^((c*(a + b*x)))/(d - e*x^3)^2,x)`

output `int(F^((c*(a + b*x)))/(d - e*x^3)^2, x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d - ex^3)^2} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^2 x^6 - 2de x^3 + d^2} dx \right)$$

input `int(F^((b*x+a)*c)/(-e*x^3+d)^2,x)`

output `f**(a*c)*int(f**(b*c*x)/(d**2 - 2*d*e*x**3 + e**2*x**6),x)`

### 3.23 $\int F^{c(a+bx)}(d+ex+fx^2)^2 dx$

Optimal result . . . . .	205
Mathematica [A] (verified) . . . . .	206
Rubi [B] (verified) . . . . .	206
Maple [A] (verified) . . . . .	207
Fricas [A] (verification not implemented) . . . . .	208
Sympy [B] (verification not implemented) . . . . .	209
Maxima [B] (verification not implemented) . . . . .	210
Giac [C] (verification not implemented) . . . . .	210
Mupad [B] (verification not implemented) . . . . .	211
Reduce [B] (verification not implemented) . . . . .	212

#### Optimal result

Integrand size = 22, antiderivative size = 177

$$\int F^{c(a+bx)}(d+ex+fx^2)^2 dx = \frac{24f^2F^{c(a+bx)}}{b^5c^5\log^5(F)} - \frac{12fF^{c(a+bx)}(e+2fx)}{b^4c^4\log^4(F)} + \frac{2F^{c(a+bx)}(e^2+2df+6efx+6f^2x^2)}{b^3c^3\log^3(F)} - \frac{2F^{c(a+bx)}(de+(e^2+2df)x+3efx^2+2f^2x^3)}{b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex+fx^2)^2}{bc\log(F)}$$

output

```
24*f^2*F^(c*(b*x+a))/b^5/c^5/ln(F)^5-12*f*F^(c*(b*x+a))*(2*f*x+e)/b^4/c^4/ln(F)^4+2*F^(c*(b*x+a))*(6*f^2*x^2+6*e*f*x+2*d*f+e^2)/b^3/c^3/ln(F)^3-2*F^(c*(b*x+a))*(d*e+(2*d*f+e^2)*x+3*e*f*x^2+2*f^2*x^3)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(f*x^2+e*x+d)^2/b/c/ln(F)
```

**Mathematica [A] (verified)**

Time = 0.27 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}(d+ex+fx^2)^2 dx$$

$$= \frac{F^{c(a+bx)}(24f^2 - 12bcf(e+2fx)\log(F) + 2b^2c^2(e^2 + 6efx + 2f(d+3fx^2))\log^2(F) - 2b^3c^3(e+2fx)(d+fx)\log(F) + b^4c^4(d+x(e+fx))^2\log^2(F) + b^5c^5\log^3(F))}{b^5c^5\log^5(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x + f*x^2)^2,x]
```

output

```
(F^(c*(a + b*x))*(24*f^2 - 12*b*c*f*(e + 2*f*x)*Log[F] + 2*b^2*c^2*(e^2 + 6*e*f*x + 2*f*(d + 3*f*x^2))*Log[F]^2 - 2*b^3*c^3*(e + 2*f*x)*(d + x*(e + f*x))*Log[F]^3 + b^4*c^4*(d + x*(e + f*x))^2*Log[F]^4)/(b^5*c^5*Log[F]^5)
```

**Rubi [B] (verified)**

Leaf count is larger than twice the leaf count of optimal. 389 vs.  $2(177) = 354$ .

Time = 1.10 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.20, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex+fx^2)^2 F^{c(a+bx)} dx$$

$$\downarrow 2626$$

$$\int \left( d^2 F^{c(a+bx)} + e^2 x^2 \left( \frac{2df}{e^2} + 1 \right) F^{c(a+bx)} + 2dex F^{c(a+bx)} + 2efx^3 F^{c(a+bx)} + f^2 x^4 F^{c(a+bx)} \right) dx$$

$$\downarrow 2009$$

$$\begin{aligned} & \frac{24f^2 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{12ef F^{c(a+bx)}}{b^4 c^4 \log^4(F)} - \frac{24f^2 x F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{2(2df + e^2) F^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \frac{12efx F^{c(a+bx)}}{b^3 c^3 \log^3(F)} + \\ & \frac{12f^2 x^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2de F^{c(a+bx)}}{2x(2df + e^2) F^{c(a+bx)}} - \frac{6efx^2 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} - \frac{4f^2 x^3 F^{c(a+bx)}}{b^2 c^2 \log^2(F)} + \\ & \frac{d^2 F^{c(a+bx)}}{bc \log(F)} + \frac{x^2(2df + e^2) F^{c(a+bx)}}{bc \log(F)} + \frac{2dex F^{c(a+bx)}}{bc \log(F)} + \frac{2efx^3 F^{c(a+bx)}}{bc \log(F)} + \frac{f^2 x^4 F^{c(a+bx)}}{bc \log(F)} \end{aligned}$$

input `Int[F^(c*(a + b*x))*(d + e*x + f*x^2)^2,x]`

output `(24*f^2*F^(c*(a + b*x)))/(b^5*c^5*Log[F]^5) - (12*e*f*F^(c*(a + b*x)))/(b^4*c^4*Log[F]^4) - (24*f^2*F^(c*(a + b*x))*x)/(b^4*c^4*Log[F]^4) + (2*(e^2 + 2*d*f)*F^(c*(a + b*x)))/(b^3*c^3*Log[F]^3) + (12*e*f*F^(c*(a + b*x))*x)/(b^3*c^3*Log[F]^3) + (12*f^2*F^(c*(a + b*x))*x^2)/(b^3*c^3*Log[F]^3) - (2*d*e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2) - (2*(e^2 + 2*d*f)*F^(c*(a + b*x))*x)/(b^2*c^2*Log[F]^2) - (6*e*f*F^(c*(a + b*x))*x^2)/(b^2*c^2*Log[F]^2) - (4*f^2*F^(c*(a + b*x))*x^3)/(b^2*c^2*Log[F]^2) + (d^2*F^(c*(a + b*x)))/(b*c*Log[F]) + (2*d*e*F^(c*(a + b*x))*x)/(b*c*Log[F]) + ((e^2 + 2*d*f)*F^(c*(a + b*x))*x^2)/(b*c*Log[F]) + (2*e*f*F^(c*(a + b*x))*x^3)/(b*c*Log[F]) + (f^2*F^(c*(a + b*x))*x^4)/(b*c*Log[F])`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.60



method	result
gospers	$(f^2 x^4 \ln(F)^4 b^4 c^4 + 2 \ln(F)^4 b^4 c^4 e f x^3 + 2 \ln(F)^4 b^4 c^4 d f x^2 + \ln(F)^4 b^4 c^4 e^2 x^2 + 2 \ln(F)^4 b^4 c^4 d e x + \ln(F)^4 b^4 c^4 d^2 - 4 \ln(F)^3 b^3 c^3 f^2) F^{c(bx+a)}$
risch	$(f^2 x^4 \ln(F)^4 b^4 c^4 + 2 \ln(F)^4 b^4 c^4 e f x^3 + 2 \ln(F)^4 b^4 c^4 d f x^2 + \ln(F)^4 b^4 c^4 e^2 x^2 + 2 \ln(F)^4 b^4 c^4 d e x + \ln(F)^4 b^4 c^4 d^2 - 4 \ln(F)^3 b^3 c^3 f^2) F^{c(bx+a)}$
orering	$(f^2 x^4 \ln(F)^4 b^4 c^4 + 2 \ln(F)^4 b^4 c^4 e f x^3 + 2 \ln(F)^4 b^4 c^4 d f x^2 + \ln(F)^4 b^4 c^4 e^2 x^2 + 2 \ln(F)^4 b^4 c^4 d e x + \ln(F)^4 b^4 c^4 d^2 - 4 \ln(F)^3 b^3 c^3 f^2) F^{c(bx+a)}$
norman	$\frac{(\ln(F)^4 b^4 c^4 d^2 - 2 \ln(F)^3 b^3 c^3 d e + 4 \ln(F)^2 b^2 c^2 d f + 2 \ln(F)^2 b^2 c^2 e^2 - 12 \ln(F) b c e f + 24 f^2) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5} + \frac{(2 \ln(F)^2 b^2 c^2 d f - 4 \ln(F) b c e f + 24 f^2) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5}$
parallelrisc	$x^4 F^{c(bx+a)} f^2 \ln(F)^4 b^4 c^4 + 2 \ln(F)^4 x^3 F^{c(bx+a)} b^4 c^4 e f + 2 \ln(F)^4 x^2 F^{c(bx+a)} b^4 c^4 d f + \ln(F)^4 x^2 F^{c(bx+a)} b^4 c^4 e^2 + 2 \ln(F)^4 x F^{c(bx+a)} b^4 c^4 d e + \ln(F)^4 x F^{c(bx+a)} b^4 c^4 d^2 - 4 \ln(F)^3 b^3 c^3 f^2 F^{c(bx+a)}$
meijerg	$-\frac{F^{ac} f^2 \left( 24 - \frac{(5b^4 c^4 x^4 \ln(F)^4 - 20b^3 c^3 x^3 \ln(F)^3 + 60b^2 c^2 x^2 \ln(F)^2 - 120bcx \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5} + \frac{F^{ac} f \left( e + \sqrt{-4df + e^2} \right) \left( 6 \ln(F)^4 b^4 c^4 d^2 - 4 \ln(F)^3 b^3 c^3 d e + 4 \ln(F)^2 b^2 c^2 d f + 2 \ln(F)^2 b^2 c^2 e^2 - 12 \ln(F) b c e f + 24 f^2 \right)}{\ln(F)^5 b^5 c^5}$

input

```
int(F^(c*(b*x+a))*(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output

```
(f^2*x^4*ln(F)^4*b^4*c^4+2*ln(F)^4*b^4*c^4*e*f*x^3+2*ln(F)^4*b^4*c^4*d*f*x^2+ln(F)^4*b^4*c^4*e^2*x^2+2*ln(F)^4*b^4*c^4*d*e*x+ln(F)^4*b^4*c^4*d^2-4*ln(F)^3*b^3*c^3*f^2*x^3-6*ln(F)^3*b^3*c^3*e*f*x^2-4*ln(F)^3*b^3*c^3*d*f*x-2*ln(F)^3*b^3*c^3*e^2*x-2*ln(F)^3*b^3*c^3*d*e+12*ln(F)^2*b^2*c^2*f^2*x^2+12*ln(F)^2*b^2*c^2*e*f*x+4*ln(F)^2*b^2*c^2*d*f+2*ln(F)^2*b^2*c^2*e^2-24*ln(F)*b*c*f^2*x-12*ln(F)*b*c*e*f+24*f^2)*F^(c*(b*x+a))/ln(F)^5/b^5/c^5
```

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.36

$$\int F^{c(a+bx)} (d + ex + fx^2)^2 dx$$

$$= \frac{((b^4 c^4 f^2 x^4 + 2 b^4 c^4 e f x^3 + 2 b^4 c^4 d e x + b^4 c^4 d^2 + (b^4 c^4 e^2 + 2 b^4 c^4 d f) x^2) \log(F)^4 - 2(2 b^3 c^3 f^2 x^3 + 3 b^3 c^3 e f x^2 + 2 b^3 c^3 d f x + b^3 c^3 d e + b^3 c^3 d^2) \log(F)^3 - 4(2 b^2 c^2 f^2 x^2 + 2 b^2 c^2 e f x + b^2 c^2 d f) \log(F)^2 - 4(2 b^2 c^2 e^2 x + 2 b^2 c^2 d e) \log(F) + 24 f^2) F^{c(a+bx)}}{\ln(F)^5 b^5 c^5}$$

input

```
integrate(F^((b*x+a)*c)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output

```
((b^4*c^4*f^2*x^4 + 2*b^4*c^4*e*f*x^3 + 2*b^4*c^4*d*e*x + b^4*c^4*d^2 + (b^4*c^4*e^2 + 2*b^4*c^4*d*f)*x^2)*log(F)^4 - 2*(2*b^3*c^3*f^2*x^3 + 3*b^3*c^3*e*f*x^2 + b^3*c^3*d*e + (b^3*c^3*e^2 + 2*b^3*c^3*d*f)*x)*log(F)^3 + 2*(6*b^2*c^2*f^2*x^2 + 6*b^2*c^2*e*f*x + b^2*c^2*e^2 + 2*b^2*c^2*d*f)*log(F)^2 + 24*f^2 - 12*(2*b*c*f^2*x + b*c*e*f)*log(F))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5)
```

### Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs.  $2(180) = 360$ .

Time = 0.12 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.17

$$\int F^{c(a+bx)}(d+ex+fx^2)^2 dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^4c^4d^2 \log(F)^4 + 2b^4c^4dex \log(F)^4 + 2b^4c^4dfx^2 \log(F)^4 + b^4c^4e^2x^2 \log(F)^4 + 2b^4c^4efx^3 \log(F)^4 + b^4c^4f^2x^4 \log(F)^4 - 2b^3c^3de \log(F)^4}{d^2x + dex^2 + \frac{efx^4}{2} + \frac{f^2x^5}{5} + x^3 \cdot \left(\frac{2df}{3} + \frac{e^2}{3}\right)} \end{cases}$$

input

```
integrate(F**((b*x+a)*c)*(f*x**2+e*x+d)**2,x)
```

output

```
Piecewise((F**(c*(a + b*x))*(b**4*c**4*d**2*log(F)**4 + 2*b**4*c**4*d*e*x*log(F)**4 + 2*b**4*c**4*d*f*x**2*log(F)**4 + b**4*c**4*e**2*x**2*log(F)**4 + 2*b**4*c**4*e*f*x**3*log(F)**4 + b**4*c**4*f**2*x**4*log(F)**4 - 2*b**3*c**3*d*e*log(F)**3 - 4*b**3*c**3*d*f*x*log(F)**3 - 2*b**3*c**3*e**2*x*log(F)**3 - 6*b**3*c**3*e*f*x**2*log(F)**3 - 4*b**3*c**3*f**2*x**3*log(F)**3 + 4*b**2*c**2*d*f*log(F)**2 + 2*b**2*c**2*e**2*log(F)**2 + 12*b**2*c**2*e*f*x*log(F)**2 + 12*b**2*c**2*f**2*x**2*log(F)**2 - 12*b*c*e*f*log(F) - 24*b*c*f**2*x*log(F) + 24*f**2)/(b**5*c**5*log(F)**5), Ne(b**5*c**5*log(F)**5, 0)), (d**2*x + d*e*x**2 + e*f*x**4/2 + f**2*x**5/5 + x**3*(2*d*f/3 + e**2/3), True))
```

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 360 vs.  $2(177) = 354$ .

Time = 0.04 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.03

$$\int F^{c(a+bx)}(d+ex+fx^2)^2 dx = \frac{F^{bcx+ac}d^2}{bc \log(F)} + \frac{2(F^{ac}bcx \log(F) - F^{ac})F^{bcx}de}{b^2c^2 \log(F)^2}$$

$$+ \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}e^2}{b^3c^3 \log(F)^3}$$

$$+ \frac{2(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}df}{b^3c^3 \log(F)^3}$$

$$+ \frac{2(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}ef}{b^4c^4 \log(F)^4}$$

$$+ \frac{(F^{ac}b^4c^4x^4 \log(F)^4 - 4F^{ac}b^3c^3x^3 \log(F)^3 + 12F^{ac}b^2c^2x^2 \log(F)^2 - 24F^{ac}bcx \log(F) + 24F^{ac})F^{bcx}}{b^5c^5 \log(F)^5}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d)^2,x, algorithm="maxima")`

output

```
F^(b*c*x + a*c)*d^2/(b*c*log(F)) + 2*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*e^2/(b^3*c^3*log(F)^3) + 2*(F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*d*f/(b^3*c^3*log(F)^3) + 2*(F^(a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))*F^(b*c*x)*e*f/(b^4*c^4*log(F)^4) + (F^(a*c)*b^4*c^4*x^4*log(F)^4 - 4*F^(a*c)*b^3*c^3*x^3*log(F)^3 + 12*F^(a*c)*b^2*c^2*x^2*log(F)^2 - 24*F^(a*c)*b*c*x*log(F) + 24*F^(a*c))*F^(b*c*x)*f^2/(b^5*c^5*log(F)^5)
```

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 9704, normalized size of antiderivative = 54.82

$$\int F^{c(a+bx)}(d+ex+fx^2)^2 dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d)^2,x, algorithm="giac")`

output

$$\begin{aligned}
& -((2*(2*\pi^3*b^4*c^4*f^2*x^4*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 2*\pi*b^4*c^4*f^2*x^4*\log(\operatorname{abs}(F)))^3*\operatorname{sgn}(F) - 2*\pi^3*b^4*c^4*f^2*x^4*\log(\operatorname{abs}(F)) + 2*\pi*b^4*c^4*f^2*x^4*\log(\operatorname{abs}(F))^3 + 4*\pi^3*b^4*c^4*e*f*x^3*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 4*\pi*b^4*c^4*e*f*x^3*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 4*\pi^3*b^4*c^4*e*f*x^3*\log(\operatorname{abs}(F)) + 4*\pi*b^4*c^4*e*f*x^3*\log(\operatorname{abs}(F))^3 + 2*\pi^3*b^4*c^4*e^2*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) + 4*\pi^3*b^4*c^4*d*f*x^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 2*\pi*b^4*c^4*e^2*x^2*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 4*\pi*b^4*c^4*d*f*x^2*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 2*\pi^3*b^4*c^4*e^2*x^2*\log(\operatorname{abs}(F)) - 4*\pi^3*b^4*c^4*d*f*x^2*\log(\operatorname{abs}(F)) + 2*\pi*b^4*c^4*e^2*x^2*\log(\operatorname{abs}(F))^3 + 4*\pi*b^4*c^4*d*f*x^2*\log(\operatorname{abs}(F))^3 + 4*\pi^3*b^4*c^4*d*e*x*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) - 4*\pi*b^4*c^4*d*e*x*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) - 4*\pi^3*b^4*c^4*d*e*x*\log(\operatorname{abs}(F)) + 4*\pi*b^4*c^4*d*e*x*\log(\operatorname{abs}(F))^3 - 2*\pi^3*b^3*c^3*f^2*x^3*\operatorname{sgn}(F) + 2*\pi^3*b^4*c^4*d^2*\log(\operatorname{abs}(F))*\operatorname{sgn}(F) + 6*\pi*b^3*c^3*f^2*x^3*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) - 2*\pi*b^4*c^4*d^2*\log(\operatorname{abs}(F))^3*\operatorname{sgn}(F) + 2*\pi^3*b^3*c^3*f^2*x^3 - 2*\pi^3*b^4*c^4*d^2*\log(\operatorname{abs}(F)) - 6*\pi*b^3*c^3*f^2*x^3*\log(\operatorname{abs}(F))^2 + 2*\pi*b^4*c^4*d^2*\log(\operatorname{abs}(F))^3 - 3*\pi^3*b^3*c^3*e*f*x^2*\operatorname{sgn}(F) + 9*\pi*b^3*c^3*e*f*x^2*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 3*\pi^3*b^3*c^3*e*f*x^2 - 9*\pi*b^3*c^3*e*f*x^2*\log(\operatorname{abs}(F))^2 - \pi^3*b^3*c^3*e^2*x*\operatorname{sgn}(F) - 2*\pi^3*b^3*c^3*d*f*x*\operatorname{sgn}(F) + 3*\pi*b^3*c^3*e^2*x*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + 6*\pi*b^3*c^3*d*f*x*\log(\operatorname{abs}(F))^2*\operatorname{sgn}(F) + \pi^3*b^3*c^3*e^2*x + 2*\pi^3*b^3*c^3*d*f*x - 3*\pi*b^3*c^3*e^2*x*\log(\operatorname{abs}(F))^2 - 6*\pi*b^3*c^3*d*f*x*\log(\operatorname{abs}(\dots)
\end{aligned}$$

### Mupad [B] (verification not implemented)

Time = 23.14 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.60

$$\begin{aligned}
& \int F^{c(a+bx)}(d+ex+fx^2)^2 dx \\
& = \frac{F^{ac+bcx} (b^4 c^4 d^2 \ln(F)^4 + 2b^4 c^4 d e x \ln(F)^4 + 2b^4 c^4 d f x^2 \ln(F)^4 + b^4 c^4 e^2 x^2 \ln(F)^4 + 2b^4 c^4 e f x^3 \ln(F)^4 + \dots}{\dots}
\end{aligned}$$

input `int(F^(c*(a + b*x))*(d + e*x + f*x^2)^2,x)`

output

```
(F^(a*c + b*c*x)*(24*f^2 + b^4*c^4*d^2*log(F)^4 + 2*b^2*c^2*e^2*log(F)^2 -
24*b*c*f^2*x*log(F) - 2*b^3*c^3*e^2*x*log(F)^3 + b^4*c^4*e^2*x^2*log(F)^4
+ 12*b^2*c^2*f^2*x^2*log(F)^2 - 4*b^3*c^3*f^2*x^3*log(F)^3 + b^4*c^4*f^2*
x^4*log(F)^4 - 12*b*c*e*f*log(F) - 2*b^3*c^3*d*e*log(F)^3 + 4*b^2*c^2*d*f*
log(F)^2 + 2*b^4*c^4*d*e*x*log(F)^4 - 4*b^3*c^3*d*f*x*log(F)^3 + 12*b^2*c^
2*e*f*x*log(F)^2 + 2*b^4*c^4*d*f*x^2*log(F)^4 - 6*b^3*c^3*e*f*x^2*log(F)^3
+ 2*b^4*c^4*e*f*x^3*log(F)^4))/(b^5*c^5*log(F)^5)
```

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.60

$$\int F^{c(a+bx)}(d+ex+fx^2)^2 dx$$

$$= \frac{f^{bcx+ac}(\log(f)^4 b^4 c^4 d^2 + 2\log(f)^4 b^4 c^4 dex + 2\log(f)^4 b^4 c^4 df x^2 + \log(f)^4 b^4 c^4 e^2 x^2 + 2\log(f)^4 b^4 c^4 ef x^3 +$$

input

```
int(F^((b*x+a)*c)*(f*x^2+e*x+d)^2,x)
```

output

```
(f**(a*c + b*c*x)*(log(f)**4*b**4*c**4*d**2 + 2*log(f)**4*b**4*c**4*d*e*x
+ 2*log(f)**4*b**4*c**4*d*f*x**2 + log(f)**4*b**4*c**4*e**2*x**2 + 2*log(f)
)**4*b**4*c**4*e*f*x**3 + log(f)**4*b**4*c**4*f**2*x**4 - 2*log(f)**3*b**3
*c**3*d*e - 4*log(f)**3*b**3*c**3*d*f*x - 2*log(f)**3*b**3*c**3*e**2*x - 6
*log(f)**3*b**3*c**3*e*f*x**2 - 4*log(f)**3*b**3*c**3*f**2*x**3 + 4*log(f)
**2*b**2*c**2*d*f + 2*log(f)**2*b**2*c**2*e**2 + 12*log(f)**2*b**2*c**2*e*
f*x + 12*log(f)**2*b**2*c**2*f**2*x**2 - 12*log(f)*b*c*e*f - 24*log(f)*b*c
*f**2*x + 24*f**2))/(log(f)**5*b**5*c**5)
```

### 3.24 $\int F^{c(a+bx)}(d + ex + fx^2) dx$

Optimal result	213
Mathematica [A] (verified)	213
Rubi [A] (verified)	214
Maple [A] (verified)	215
Fricas [A] (verification not implemented)	215
Sympy [A] (verification not implemented)	216
Maxima [A] (verification not implemented)	216
Giac [C] (verification not implemented)	217
Mupad [B] (verification not implemented)	218
Reduce [B] (verification not implemented)	218

#### Optimal result

Integrand size = 20, antiderivative size = 80

$$\int F^{c(a+bx)}(d+ex+fx^2) dx = \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{F^{c(a+bx)}(e+2fx)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex+fx^2)}{bc \log(F)}$$

output

$2*f*F^{(c*(b*x+a))/b^3/c^3/\ln(F)^3-F^{(c*(b*x+a))*(2*f*x+e)/b^2/c^2/\ln(F)^2+F^{(c*(b*x+a))*(f*x^2+e*x+d)/b/c/\ln(F)}$

#### Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int F^{c(a+bx)}(d + ex + fx^2) dx = \frac{F^{c(a+bx)}(2f - bc(e + 2fx) \log(F) + b^2c^2(d + x(e + fx)) \log^2(F))}{b^3c^3 \log^3(F)}$$

input

`Integrate[F^{(c*(a + b*x))*(d + e*x + f*x^2)}, x]`

output

$(F^{(c*(a + b*x))*(2*f - b*c*(e + 2*f*x)*\text{Log}[F] + b^2*c^2*(d + x*(e + f*x))*\text{Log}[F]^2))/(b^3*c^3*\text{Log}[F]^3)$

**Rubi [A] (verified)**

Time = 0.52 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex + fx^2) F^{c(a+bx)} dx$$

$$\downarrow 2626$$

$$\int \left( dF^{c(a+bx)} + exF^{c(a+bx)} + fx^2F^{c(a+bx)} \right) dx$$

$$\downarrow 2009$$

$$\frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{exF^{c(a+bx)}}{bc\log(F)} + \frac{fx^2F^{c(a+bx)}}{bc\log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x + f*x^2),x]`

output 
$$\frac{(2*f*F^{c*(a + b*x)})}{(b^3*c^3*\text{Log}[F]^3)} - \frac{(e*F^{c*(a + b*x)})}{(b^2*c^2*\text{Log}[F]^2)} - \frac{(2*f*F^{c*(a + b*x)}*x)}{(b^2*c^2*\text{Log}[F]^2)} + \frac{(d*F^{c*(a + b*x)})}{(b*c*\text{Log}[F])} + \frac{(e*F^{c*(a + b*x)}*x)}{(b*c*\text{Log}[F])} + \frac{(f*F^{c*(a + b*x)}*x^2)}{(b*c*\text{Log}[F])}$$

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result
gospers	$\frac{(f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 e x + \ln(F)^2 b^2 c^2 d - 2 \ln(F) b c f x - \ln(F) b c e + 2 f) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
risch	$\frac{(f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 e x + \ln(F)^2 b^2 c^2 d - 2 \ln(F) b c f x - \ln(F) b c e + 2 f) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
orering	$\frac{(f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 e x + \ln(F)^2 b^2 c^2 d - 2 \ln(F) b c f x - \ln(F) b c e + 2 f) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
norman	$\frac{(\ln(F)^2 b^2 c^2 d - \ln(F) b c e + 2 f) e^{c(bx+a) \ln(F)}}{c^3 b^3 \ln(F)^3} + \frac{(\ln(F) b c e - 2 f) x e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} + \frac{f x^2 e^{c(bx+a) \ln(F)}}{\ln(F) b c}$
meijerg	$- \frac{F^{ac} f \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} + \frac{F^{ac} e \left( 1 - \frac{(-2bcx \ln(F) + 2) e^{bcx \ln(F)}}{2} \right)}{\ln(F)^2 b^2 c^2} - \frac{F^{ac} d (1 - e^{bcx \ln(F)})}{bc \ln(F)}$
parallelrisch	$\frac{x^2 F^{c(bx+a)} f \ln(F)^2 b^2 c^2 + \ln(F)^2 x F^{c(bx+a)} b^2 c^2 e + \ln(F)^2 F^{c(bx+a)} b^2 c^2 d - 2 \ln(F) x F^{c(bx+a)} b c f - \ln(F) F^{c(bx+a)} b c e + 2 F^{c(bx+a)} f}{\ln(F)^3 b^3 c^3}$

input `int(F^(c*(b*x+a))*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)`

output `(f*x^2*ln(F)^2*b^2*c^2+ln(F)^2*b^2*c^2*e*x+ln(F)^2*b^2*c^2*d-2*ln(F)*b*c*f*x-ln(F)*b*c*e+2*f)*F^(c*(b*x+a))/ln(F)^3/b^3/c^3`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int F^{c(a+bx)} (d + ex + fx^2) dx$$

$$= \frac{((b^2 c^2 f x^2 + b^2 c^2 e x + b^2 c^2 d) \log(F)^2 - (2 b c f x + b c e) \log(F) + 2 f) F^{bcx+ac}}{b^3 c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d),x, algorithm="fricas")`

output `((b^2*c^2*f*x^2 + b^2*c^2*e*x + b^2*c^2*d)*log(F)^2 - (2*b*c*f*x + b*c*e)*log(F) + 2*f)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3)`



**Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^2c^2d\log(F)^2+b^2c^2ex\log(F)^2+b^2c^2fx^2\log(F)^2-bce\log(F)-2bcfx\log(F)+2f)}{b^3c^3\log(F)^3} & \text{for } b^3c^3\log(F)^3 \neq 0 \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(f*x**2+e*x+d),x)`output `Piecewise((F**(c*(a + b*x))*(b**2*c**2*d*log(F)**2 + b**2*c**2*e*x*log(F)**2 + b**2*c**2*f*x**2*log(F)**2 - b*c*e*log(F) - 2*b*c*f*x*log(F) + 2*f)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d*x + e*x**2/2 + f*x**3/3, True))`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int F^{c(a+bx)}(d+ex+fx^2) dx = \frac{F^{bcx+ac}d}{bc\log(F)} + \frac{(F^{ac}bcx\log(F) - F^{ac})F^{bcx}e}{b^2c^2\log(F)^2} + \frac{(F^{ac}b^2c^2x^2\log(F)^2 - 2F^{ac}bcx\log(F) + 2F^{ac})F^{bcx}f}{b^3c^3\log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d),x, algorithm="maxima")`output `F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*f/(b^3*c^3*log(F)^3)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 2068, normalized size of antiderivative = 25.85

$$\int F^{c(a+bx)}(d+ex+fx^2) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d),x, algorithm="giac")`

output

```
((pi^2*b^2*c^2*f*x^2*sgn(F) - pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*log(abs(F))^2 + pi^2*b^2*c^2*e*x*sgn(F) - pi^2*b^2*c^2*e*x + 2*b^2*c^2*e*x*log(abs(F))^2 + pi^2*b^2*c^2*d*sgn(F) - pi^2*b^2*c^2*d + 2*b^2*c^2*d*log(abs(F))^2 - 4*b*c*f*x*log(abs(F)) - 2*b*c*e*log(abs(F)) + 4*f)*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) - (pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(2*pi*b^2*c^2*f*x^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*f*x^2*log(abs(F)) + 2*pi*b^2*c^2*e*x*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*e*x*log(abs(F)) + 2*pi*b^2*c^2*d*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*d*log(abs(F)) - 2*pi*b*c*f*x*sgn(F) + 2*pi*b*c*f*x - pi*b*c*e*sgn(F) + pi*b*c*e)/(pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi^2*b^2*c^2*f*x^2*sgn(F) - pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*log(abs(F))^2 + pi^2*b^2*c^2*e*x*sgn(F) - pi^2*b^2*c^2*e*x + 2*b^2*c^2*e*x*log(abs...
```

**Mupad [B] (verification not implemented)**

Time = 23.44 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \frac{F^{ac+bcx} (fb^2c^2x^2 \ln(F)^2 + eb^2c^2x \ln(F)^2 + db^2c^2 \ln(F)^2 - 2fbcx \ln(F) - ebc \ln(F) + 2f)}{b^3c^3 \ln(F)^3}$$

input `int(F^(c*(a + b*x))*(d + e*x + f*x^2),x)`output `(F^(a*c + b*c*x)*(2*f - b*c*e*log(F) + b^2*c^2*d*log(F)^2 + b^2*c^2*f*x^2*log(F)^2 - 2*b*c*f*x*log(F) + b^2*c^2*e*x*log(F)^2))/(b^3*c^3*log(F)^3)`**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \frac{f^{bcx+ac}(\log(f)^2 b^2 c^2 d + \log(f)^2 b^2 c^2 ex + \log(f)^2 b^2 c^2 f x^2 - \log(f) bce - 2 \log(f) bcf x + 2f)}{\log(f)^3 b^3 c^3}$$

input `int(F^((b*x+a)*c)*(f*x^2+e*x+d),x)`output `(f**(a*c + b*c*x)*(log(f)**2*b**2*c**2*d + log(f)**2*b**2*c**2*e*x + log(f)**2*b**2*c**2*f*x**2 - log(f)*b*c*e - 2*log(f)*b*c*f*x + 2*f))/(log(f)**3*b**3*c**3)`

### 3.25 $\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx$

Optimal result	219
Mathematica [A] (verified)	220
Rubi [A] (verified)	220
Maple [A] (verified)	221
Fricas [B] (verification not implemented)	222
Sympy [F]	223
Maxima [F]	223
Giac [F]	223
Mupad [F(-1)]	224
Reduce [F]	224

#### Optimal result

Integrand size = 22, antiderivative size = 148

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx = \frac{F^c \left( a - \frac{b(e - \sqrt{e^2 - 4df})}{2f} \right) \text{ExpIntegralEi} \left( \frac{bc(e - \sqrt{e^2 - 4df} + 2fx) \log(F)}{2f} \right)}{\sqrt{e^2 - 4df}} - \frac{F^c \left( a - \frac{b(e + \sqrt{e^2 - 4df})}{2f} \right) \text{ExpIntegralEi} \left( \frac{bc(e + \sqrt{e^2 - 4df} + 2fx) \log(F)}{2f} \right)}{\sqrt{e^2 - 4df}}$$

output

```
F^(c*(a-1/2*b*(e-(-4*d*f+e^2)^(1/2))/f))*Ei(1/2*b*c*(e-(-4*d*f+e^2)^(1/2)+2*f*x)*ln(F)/f)/(-4*d*f+e^2)^(1/2)-F^(c*(a-1/2*b*(e+(-4*d*f+e^2)^(1/2))/f))*Ei(1/2*b*c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)*ln(F)/f)/(-4*d*f+e^2)^(1/2)
```

**Mathematica [A] (verified)**

Time = 0.97 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx$$

$$= \frac{F^{ac - \frac{bc(e + \sqrt{e^2 - 4df})}{2f}} \left( F^{\frac{bc\sqrt{e^2 - 4df}}{f}} \text{ExpIntegralEi} \left( \frac{bc(e - \sqrt{e^2 - 4df} + 2fx) \log(F)}{2f} \right) - \text{ExpIntegralEi} \left( \frac{bc(e + \sqrt{e^2 - 4df} + 2fx) \log(F)}{2f} \right) \right)}{\sqrt{e^2 - 4df}}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x + f*x^2), x]`

output `(F^(a*c - (b*c*(e + Sqrt[e^2 - 4*d*f]))/(2*f))*F^((b*c*Sqrt[e^2 - 4*d*f])/f)*ExpIntegralEi[(b*c*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F])/(2*f)] - ExpIntegralEi[(b*c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F])/(2*f))]/Sqrt[e^2 - 4*d*f]`

**Rubi [A] (verified)**

Time = 0.73 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2698, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx$$

$$\downarrow \text{2698}$$

$$\int \left( \frac{2fF^{c(a+bx)}}{\sqrt{e^2 - 4df}(-\sqrt{e^2 - 4df} + e + 2fx)} - \frac{2fF^{c(a+bx)}}{\sqrt{e^2 - 4df}(\sqrt{e^2 - 4df} + e + 2fx)} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{F^{c\left(a-\frac{b(e-\sqrt{e^2-4df})}{2f}\right)} \operatorname{ExpIntegralEi}\left(\frac{bc(e+2fx-\sqrt{e^2-4df})\log(F)}{2f}\right)}{\sqrt{e^2-4df}} - \frac{F^{c\left(a-\frac{b(\sqrt{e^2-4df}+e)}{2f}\right)} \operatorname{ExpIntegralEi}\left(\frac{bc(e+2fx+\sqrt{e^2-4df})\log(F)}{2f}\right)}{\sqrt{e^2-4df}}$$

input `Int[F^(c*(a + b*x))/(d + e*x + f*x^2),x]`

output `(F^(c*(a - (b*(e - Sqrt[e^2 - 4*d*f]))/(2*f)))*ExpIntegralEi[(b*c*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F])/(2*f)]/Sqrt[e^2 - 4*d*f] - (F^(c*(a - (b*(e + Sqrt[e^2 - 4*d*f]))/(2*f)))*ExpIntegralEi[(b*c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F])/(2*f)]/Sqrt[e^2 - 4*d*f]`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2698 `Int[(F_)^((g_)*((d_) + (e_)*(x_)^(n_)))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]`

**Maple [A] (verified)**

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{b F^{\frac{(2fa-be+\sqrt{-b^2(4df-e^2)})c}{2f}} \operatorname{expIntegral}_1\left(\frac{2fa \ln(F)c-\ln(F)bce+\sqrt{-4b^2df+b^2e^2} \ln(F)c-2f(bcx \ln(F)+ac \ln(F))}{2f}\right)}{\sqrt{-4b^2df+b^2e^2}} + \frac{(-2f)}{b F^{\dots}}$

input `int(F^(c*(b*x+a))/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)`

output

```
-b/(-4*b^2*d*f+b^2*e^2)^(1/2)*F^(1/2/f*(2*f*a-b*e+(-b^2*(4*d*f-e^2))^(1/2)
)*c)*Ei(1,1/2*(2*f*a*ln(F)*c-ln(F)*b*c*e+(-4*b^2*d*f+b^2*e^2)^(1/2)*ln(F)*
c-2*f*(b*c*x*ln(F)+a*c*ln(F)))/f)+b/(-4*b^2*d*f+b^2*e^2)^(1/2)*F^(-1/2*(-
2*f*a+b*e+(-b^2*(4*d*f-e^2))^(1/2))*c/f)*Ei(1,-1/2*(-2*f*a*ln(F)*c+ln(F)*b*
c*e+(-4*b^2*d*f+b^2*e^2)^(1/2)*ln(F)*c+2*f*(b*c*x*ln(F)+a*c*ln(F)))/f)
```

### Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs.  $2(134) = 268$ .

Time = 0.08 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.06

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx =$$

$$f \sqrt{\frac{(b^2c^2e^2-4b^2c^2df)\log(F)^2}{f^2}} \operatorname{Ei}\left(\frac{(2bcfx+bce)\log(F)+f\sqrt{\frac{(b^2c^2e^2-4b^2c^2df)\log(F)^2}{f^2}}}{2f}\right) e^{\left(-\frac{(bce-2acf)\log(F)+f\sqrt{\frac{(b^2c^2e^2-4b^2c^2df)\log(F)^2}{f^2}}}{2f}\right)}$$

input

```
integrate(F^((b*x+a)*c)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output

```
-(f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^2/f^2)*Ei(1/2*((2*b*c*f*x +
b*c*e)*log(F) + f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^2/f^2))/f)*e^(
-1/2*((b*c*e - 2*a*c*f)*log(F) + f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(
F)^2/f^2))/f) - f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^2/f^2)*Ei(1/2*
((2*b*c*f*x + b*c*e)*log(F) - f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^
2/f^2))/f)*e^(-1/2*((b*c*e - 2*a*c*f)*log(F) - f*sqrt((b^2*c^2*e^2 - 4*b^2
*c^2*d*f)*log(F)^2/f^2))/f))/((b*c*e^2 - 4*b*c*d*f)*log(F))
```

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx = \int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx$$

input `integrate(F**((b*x+a)*c)/(f*x**2+e*x+d), x)`

output `Integral(F**(c*(a + b*x))/(d + e*x + f*x**2), x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx = \int \frac{F^{(bx+a)c}}{fx^2+ex+d} dx$$

input `integrate(F^((b*x+a)*c)/(f*x^2+e*x+d), x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(f*x^2 + e*x + d), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx = \int \frac{F^{(bx+a)c}}{fx^2+ex+d} dx$$

input `integrate(F^((b*x+a)*c)/(f*x^2+e*x+d), x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(f*x^2 + e*x + d), x)`



**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx = \int \frac{F^{c(a+bx)}}{fx^2+ex+d} dx$$

input `int(F^(c*(a + b*x))/(d + e*x + f*x^2), x)`output `int(F^(c*(a + b*x))/(d + e*x + f*x^2), x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d+ex+fx^2} dx = f^{ac} \left( \int \frac{f^{bcx}}{fx^2+ex+d} dx \right)$$

input `int(F^((b*x+a)*c)/(f*x^2+e*x+d), x)`output `f**(a*c)*int(f**(b*c*x)/(d + e*x + f*x**2), x)`

**3.26** 
$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$$

Optimal result	225
Mathematica [A] (verified)	226
Rubi [A] (verified)	226
Maple [B] (verified)	228
Fricas [B] (verification not implemented)	229
Sympy [F]	230
Maxima [F]	230
Giac [F]	231
Mupad [F(-1)]	231
Reduce [F]	231

**Optimal result**

Integrand size = 22, antiderivative size = 304

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx = -\frac{F^{c\left(a-\frac{be}{f}\right)+\frac{bc\left(e-\sqrt{e^2-4df}\right)}{2f}+\frac{bc\left(e+\sqrt{e^2-4df}+2fx\right)}{2f}}{(e^2-4df)(d+x(e+fx))} (e+2fx)$$

$$-\frac{F^{c\left(a-\frac{be}{f}\right)+\frac{bc\left(e+\sqrt{e^2-4df}\right)}{2f}} \operatorname{ExpIntegralEi}\left(\frac{bc\left(e-\sqrt{e^2-4df}+2fx\right)\log(F)}{2f}\right) (2f-bc\sqrt{e^2-4df}\log(F))}{(e^2-4df)^{3/2}}$$

$$+\frac{F^{c\left(a-\frac{be}{f}\right)+\frac{bc\left(e-\sqrt{e^2-4df}\right)}{2f}} \operatorname{ExpIntegralEi}\left(\frac{bc\left(e+\sqrt{e^2-4df}+2fx\right)\log(F)}{2f}\right) (2f+bc\sqrt{e^2-4df}\log(F))}{(e^2-4df)^{3/2}}$$

output

```
-F^(c*(a-b*e/f)+1/2*b*c*(e-(-4*d*f+e^2)^(1/2))/f+1/2*b*c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)/f)*(2*f*x+e)/(-4*d*f+e^2)/(d+x*(f*x+e))-F^(c*(a-b*e/f)+1/2*b*c*(e+(-4*d*f+e^2)^(1/2))/f)*Ei(1/2*b*c*(e-(-4*d*f+e^2)^(1/2)+2*f*x)*ln(F)/f)*(2*f-b*c*(-4*d*f+e^2)^(1/2)*ln(F))/(-4*d*f+e^2)^(3/2)+F^(c*(a-b*e/f)+1/2*b*c*(e-(-4*d*f+e^2)^(1/2))/f)*Ei(1/2*b*c*(2*f*x+(-4*d*f+e^2)^(1/2)+e)*ln(F)/f)*(2*f+b*c*(-4*d*f+e^2)^(1/2)*ln(F))/(-4*d*f+e^2)^(3/2)
```

**Mathematica [A] (verified)**

Time = 1.77 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.82

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$$

$$= \frac{F^{ac - \frac{bc(e+\sqrt{e^2-4df})}{2f}} \left( -\sqrt{e^2-4df} F^{\frac{bc(e+\sqrt{e^2-4df}+2fx)}{2f}} (e+2fx) - F^{\frac{bc\sqrt{e^2-4df}}{f}} (d+x(e+fx)) \text{ExpIntegralEi} \left( \frac{bc(e+\sqrt{e^2-4df})}{2f} \log \left( \frac{d+ex+fx^2}{(d+x(e+fx))^2} \right) \right) \right)}{(e^2-4df)^{3/2} (d+x(e+fx))}$$

input

```
Integrate[F^(c*(a + b*x))/(d + e*x + f*x^2)^2,x]
```

output

```
(F^(a*c - (b*c*(e + Sqrt[e^2 - 4*d*f]))/(2*f))*(-(Sqrt[e^2 - 4*d*f]*F^((b*c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*f))*(e + 2*f*x)) - F^((b*c*Sqrt[e^2 - 4*d*f])/f)*(d + x*(e + f*x))*ExpIntegralEi[(b*c*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F])/(2*f)]*(2*f - b*c*Sqrt[e^2 - 4*d*f]*Log[F]) + (d + x*(e + f*x))*ExpIntegralEi[(b*c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F])/(2*f)]*(2*f + b*c*Sqrt[e^2 - 4*d*f]*Log[F]))) / ((e^2 - 4*d*f)^(3/2)*(d + x*(e + f*x)))
```

**Rubi [A] (verified)**

Time = 1.55 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {7292, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$$

$$\downarrow 7292$$

$$\int \frac{F^{ac+bcx}}{(d+ex+fx^2)^2} dx$$

$$\downarrow 7293$$

$$\int \left( \frac{4f^2 F^{ac+bcx}}{(e^2 - 4df)^{3/2} (\sqrt{e^2 - 4df} - e - 2fx)} + \frac{4f^2 F^{ac+bcx}}{(e^2 - 4df)^{3/2} (\sqrt{e^2 - 4df} + e + 2fx)} + \frac{4f^2 F^{ac+bcx}}{(e^2 - 4df) (\sqrt{e^2 - 4df} - e - 2fx)} \right)$$

↓ 2009

$$\frac{2f F^{ac - \frac{bc(e - \sqrt{e^2 - 4df})}{2f}} \text{ExpIntegralEi} \left( \frac{bc(e + 2fx - \sqrt{e^2 - 4df}) \log(F)}{2f} \right)}{(e^2 - 4df)^{3/2}} +$$

$$\frac{bc \log(F) F^{ac - \frac{bc(e - \sqrt{e^2 - 4df})}{2f}} \text{ExpIntegralEi} \left( \frac{bc(e + 2fx - \sqrt{e^2 - 4df}) \log(F)}{2f} \right)}{e^2 - 4df} +$$

$$\frac{2f F^{ac - \frac{bc(\sqrt{e^2 - 4df} + e)}{2f}} \text{ExpIntegralEi} \left( \frac{bc(e + 2fx + \sqrt{e^2 - 4df}) \log(F)}{2f} \right)}{(e^2 - 4df)^{3/2}} +$$

$$\frac{bc \log(F) F^{ac - \frac{bc(\sqrt{e^2 - 4df} + e)}{2f}} \text{ExpIntegralEi} \left( \frac{bc(e + 2fx + \sqrt{e^2 - 4df}) \log(F)}{2f} \right)}{e^2 - 4df} -$$

$$\frac{2f F^{ac+bcx}}{(e^2 - 4df) (-\sqrt{e^2 - 4df} + e + 2fx)} - \frac{2f F^{ac+bcx}}{(e^2 - 4df) (\sqrt{e^2 - 4df} + e + 2fx)}$$

input

```
Int [F^(c*(a + b*x))/(d + e*x + f*x^2)^2,x]
```

output

```
(-2*f*F^(a*c + b*c*x))/((e^2 - 4*d*f)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)) - (2*f*F^(a*c + b*c*x))/((e^2 - 4*d*f)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) - (2*f*F^(a*c - (b*c*(e - Sqrt[e^2 - 4*d*f]))/(2*f))*ExpIntegralEi[(b*c*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F]]/(2*f)))/(e^2 - 4*d*f)^(3/2) + (2*f*F^(a*c - (b*c*(e + Sqrt[e^2 - 4*d*f]))/(2*f))*ExpIntegralEi[(b*c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F]]/(2*f)))/(e^2 - 4*d*f)^(3/2) + (b*c*F^(a*c - (b*c*(e - Sqrt[e^2 - 4*d*f]))/(2*f))*ExpIntegralEi[(b*c*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F]]/(2*f))*Log[F])/(e^2 - 4*d*f) + (b*c*F^(a*c - (b*c*(e + Sqrt[e^2 - 4*d*f]))/(2*f))*ExpIntegralEi[(b*c*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)*Log[F]]/(2*f))*Log[F])/(e^2 - 4*d*f)
```

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 7292 `Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]`

rule 7293 `Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]`  
`]`

### Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs.  $2(272) = 544$ .

Time = 0.12 (sec) , antiderivative size = 624, normalized size of antiderivative = 2.05

method	result
risch	$\frac{2 \ln(F)^2 b^2 c^2 F^{bcx} F^{ac} x f}{(4df - e^2) (f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 ex + \ln(F)^2 b^2 c^2 d)} + \frac{\ln(F)^2 b^2 c^2 F^{bcx} F^{ac} e}{(4df - e^2) (f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 ex + \ln(F)^2 b^2 c^2 d)} + \frac{\ln(F)}{\dots}$

input `int(F^(c*(b*x+a))/(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)`

output

```

2*ln(F)^2*b^2*c^2*F^(b*c*x)*F^(a*c)/(4*d*f-e^2)/(f*x^2*ln(F)^2*b^2*c^2+ln(
F)^2*b^2*c^2*e*x+ln(F)^2*b^2*c^2*d)*x*f+ln(F)^2*b^2*c^2*F^(b*c*x)*F^(a*c)/
(4*d*f-e^2)/(f*x^2*ln(F)^2*b^2*c^2+ln(F)^2*b^2*c^2*e*x+ln(F)^2*b^2*c^2*d)*
e+ln(F)*b*c/(4*d*f-e^2)*F^(1/2/f*(2*f*a-b*e+(-b^2*(4*d*f-e^2))^(1/2))*c)*E
i(1,1/2*(2*f*a*ln(F)*c-ln(F)*b*c*e+(-4*b^2*d*f+b^2*e^2)^(1/2)*ln(F)*c-2*f*
(b*c*x*ln(F)+a*c*ln(F)))/f)+ln(F)*b*c/(4*d*f-e^2)*F^(-1/2*(-2*f*a+b*e+(-b^
2*(4*d*f-e^2))^(1/2))*c/f)*Ei(1,-1/2*(-2*f*a*ln(F)*c+ln(F)*b*c*e+(-4*b^2*d
*f+b^2*e^2)^(1/2)*ln(F)*c+2*f*(b*c*x*ln(F)+a*c*ln(F)))/f)-2*b/(4*d*f-e^2)/
(-4*b^2*d*f+b^2*e^2)^(1/2)*F^(1/2/f*(2*f*a-b*e+(-b^2*(4*d*f-e^2))^(1/2))*c
)*Ei(1,1/2*(2*f*a*ln(F)*c-ln(F)*b*c*e+(-4*b^2*d*f+b^2*e^2)^(1/2)*ln(F)*c-2
*f*(b*c*x*ln(F)+a*c*ln(F)))/f)*f+2*b/(4*d*f-e^2)/(-4*b^2*d*f+b^2*e^2)^(1/2
)*F^(-1/2*(-2*f*a+b*e+(-b^2*(4*d*f-e^2))^(1/2))*c/f)*Ei(1,-1/2*(-2*f*a*ln(
F)*c+ln(F)*b*c*e+(-4*b^2*d*f+b^2*e^2)^(1/2)*ln(F)*c+2*f*(b*c*x*ln(F)+a*c*ln
(F)))/f)*f

```

### Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 632 vs.  $2(270) = 540$ .

Time = 0.09 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.08

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$$

$$\left( (b^2c^2de^2 - 4b^2c^2d^2f + (b^2c^2e^2f - 4b^2c^2df^2)x^2 + (b^2c^2e^3 - 4b^2c^2def)x \right) \log(F)^2 + 2(f^3x^2 + ef^2x + d$$

=

input

```
integrate(F^((b*x+a)*c)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output

```
((b^2*c^2*d*e^2 - 4*b^2*c^2*d^2*f + (b^2*c^2*e^2*f - 4*b^2*c^2*d*f^2)*x^2
+ (b^2*c^2*e^3 - 4*b^2*c^2*d*e*f)*x)*log(F)^2 + 2*(f^3*x^2 + e*f^2*x + d*
f^2)*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^2/f^2))*Ei(1/2*((2*b*c*f*x
+ b*c*e)*log(F) + f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^2/f^2))/f)*e
^(-1/2*((b*c*e - 2*a*c*f)*log(F) + f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*lo
g(F)^2/f^2))/f) + ((b^2*c^2*d*e^2 - 4*b^2*c^2*d^2*f + (b^2*c^2*e^2*f - 4*b
^2*c^2*d*f^2)*x^2 + (b^2*c^2*e^3 - 4*b^2*c^2*d*e*f)*x)*log(F)^2 - 2*(f^3*x
^2 + e*f^2*x + d*f^2)*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*log(F)^2/f^2))*Ei
(1/2*((2*b*c*f*x + b*c*e)*log(F) - f*sqrt((b^2*c^2*e^2 - 4*b^2*c^2*d*f)*lo
g(F)^2/f^2))/f)*e^(-1/2*((b*c*e - 2*a*c*f)*log(F) - f*sqrt((b^2*c^2*e^2 -
4*b^2*c^2*d*f)*log(F)^2/f^2))/f) - (b*c*e^3 - 4*b*c*d*e*f + 2*(b*c*e^2*f -
4*b*c*d*f^2)*x)*F^(b*c*x + a*c)*log(F))/((b*c*d*e^4 - 8*b*c*d^2*e^2*f + 1
6*b*c*d^3*f^2 + (b*c*e^4*f - 8*b*c*d*e^2*f^2 + 16*b*c*d^2*f^3)*x^2 + (b*c*
e^5 - 8*b*c*d*e^3*f + 16*b*c*d^2*e*f^2)*x)*log(F))
```

## Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx = \int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx$$

input

```
integrate(F**((b*x+a)*c)/(f*x**2+e*x+d)**2,x)
```

output

```
Integral(F**(c*(a + b*x))/(d + e*x + f*x**2)**2, x)
```

## Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx = \int \frac{F^{(bx+a)c}}{(fx^2+ex+d)^2} dx$$

input

```
integrate(F^((b*x+a)*c)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output

```
integrate(F^((b*x + a)*c)/(f*x^2 + e*x + d)^2, x)
```

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx = \int \frac{F^{(bx+a)c}}{(fx^2+ex+d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(f*x^2+e*x+d)^2,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(f*x^2 + e*x + d)^2, x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx = \int \frac{F^{c(a+bx)}}{(fx^2+ex+d)^2} dx$$

input `int(F^(c*(a + b*x))/(d + e*x + f*x^2)^2,x)`

output `int(F^(c*(a + b*x))/(d + e*x + f*x^2)^2, x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex+fx^2)^2} dx = \text{too large to display}$$

input `int(F^((b*x+a)*c)/(f*x^2+e*x+d)^2,x)`



output

```
(f**(a*c)*(- f**(b*c*x) + int(f**(b*c*x)/(log(f)**2*b**2*c**2*d**3*e + 2*
log(f)**2*b**2*c**2*d**2*e**2*x + 2*log(f)**2*b**2*c**2*d**2*e*f*x**2 + lo
g(f)**2*b**2*c**2*d*e**3*x**2 + 2*log(f)**2*b**2*c**2*d*e**2*f*x**3 + log(
f)**2*b**2*c**2*d*e*f**2*x**4 - 2*log(f)*b*c*d**3*f - log(f)*b*c*d**2*e**2
- 4*log(f)*b*c*d**2*e*f*x - 4*log(f)*b*c*d**2*f**2*x**2 - 2*log(f)*b*c*d*
e**3*x - 4*log(f)*b*c*d*e**2*f*x**2 - 4*log(f)*b*c*d*e*f**2*x**3 - 2*log(f)
)*b*c*d*f**3*x**4 - log(f)*b*c*e**4*x**2 - 2*log(f)*b*c*e**3*f*x**3 - log(
f)*b*c*e**2*f**2*x**4 + 2*d**2*e*f + 4*d*e**2*f*x + 4*d*e*f**2*x**2 + 2*e*
**3*f*x**2 + 4*e**2*f**2*x**3 + 2*e*f**3*x**4),x)*log(f)**3*b**3*c**3*d**3*
e + int(f**(b*c*x)/(log(f)**2*b**2*c**2*d**3*e + 2*log(f)**2*b**2*c**2*d**
2*e**2*x + 2*log(f)**2*b**2*c**2*d**2*e*f*x**2 + log(f)**2*b**2*c**2*d*e**
3*x**2 + 2*log(f)**2*b**2*c**2*d*e**2*f*x**3 + log(f)**2*b**2*c**2*d*e*f**
2*x**4 - 2*log(f)*b*c*d**3*f - log(f)*b*c*d**2*e**2 - 4*log(f)*b*c*d**2*e*
f*x - 4*log(f)*b*c*d**2*f**2*x**2 - 2*log(f)*b*c*d*e**3*x - 4*log(f)*b*c*d
*e**2*f*x**2 - 4*log(f)*b*c*d*e*f**2*x**3 - 2*log(f)*b*c*d*f**3*x**4 - log
(f)*b*c*e**4*x**2 - 2*log(f)*b*c*e**3*f*x**3 - log(f)*b*c*e**2*f**2*x**4 +
2*d**2*e*f + 4*d*e**2*f*x + 4*d*e*f**2*x**2 + 2*e**3*f*x**2 + 4*e**2*f**2
*x**3 + 2*e*f**3*x**4),x)*log(f)**3*b**3*c**3*d**2*e**2*x + int(f**(b*c*x)
/(log(f)**2*b**2*c**2*d**3*e + 2*log(f)**2*b**2*c**2*d**2*e**2*x + 2*log(f)
)**2*b**2*c**2*d**2*e*f*x**2 + log(f)**2*b**2*c**2*d*e**3*x**2 + 2*log(...
```

### 3.27 $\int \frac{e^{a+bx} x^2}{c+dx^2} dx$

Optimal result	233
Mathematica [C] (verified)	233
Rubi [A] (verified)	234
Maple [A] (verified)	235
Fricas [A] (verification not implemented)	236
Sympy [F]	236
Maxima [F]	236
Giac [F]	237
Mupad [F(-1)]	237
Reduce [F]	237

#### Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{e^{a+bx} x^2}{c+dx^2} dx = \frac{e^{a+bx}}{bd} + \frac{\sqrt{-c} e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{\sqrt{-c} e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2d^{3/2}}$$

output `exp(b*x+a)/b/d+1/2*(-c)^(1/2)*exp(a+b*(-c)^(1/2)/d^(1/2))*Ei(-b*((-c)^(1/2)-d^(1/2)*x)/d^(1/2))/d^(3/2)-1/2*(-c)^(1/2)*exp(a-b*(-c)^(1/2)/d^(1/2))*Ei(b*((-c)^(1/2)+d^(1/2)*x)/d^(1/2))/d^(3/2)`

#### Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.91

$$\int \frac{e^{a+bx} x^2}{c+dx^2} dx = \frac{e^a \left( 2\sqrt{d} e^{bx} + ib\sqrt{c} \frac{ib\sqrt{c}}{\sqrt{d}} \text{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right) - ib\sqrt{c} e^{-\frac{ib\sqrt{c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right) \right)}{2bd^{3/2}}$$

input `Integrate[(E^(a + b*x)*x^2)/(c + d*x^2),x]`

output `(E^a*(2*Sqrt[d]*E^(b*x) + I*b*Sqrt[c]*E^((I*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[b*((-I)*Sqrt[c])/Sqrt[d] + x]) - (I*b*Sqrt[c]*ExpIntegralEi[b*(I*Sqrt[c])/Sqrt[d] + x])/E^((I*b*Sqrt[c])/Sqrt[d]))/(2*b*d^(3/2))`

### Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2701, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{a+bx}}{c+dx^2} dx$$

$$\downarrow \text{2701}$$

$$\int \left( \frac{e^{a+bx}}{d} - \frac{ce^{a+bx}}{d(c+dx^2)} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{-c} e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d^{3/2}} - \frac{\sqrt{-c} e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{e^{a+bx}}{bd}$$

input `Int[(E^(a + b*x)*x^2)/(c + d*x^2),x]`

output `E^(a + b*x)/(b*d) + (Sqrt[-c]*E^(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d])])/(2*d^(3/2)) - (Sqrt[-c]*E^(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d]])/(2*d^(3/2))`

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2701 Int[((F_)^((g_)*((d_) + (e_)*(x_))^(n_))* (u_)^(m_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + c*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

method	result
risch	$\frac{e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}_1\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right)c}{2d\sqrt{-cd}} - \frac{e^{-\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}_1\left(\frac{-b\sqrt{-cd}-da+d(bx+a)}{d}\right)c}{2d\sqrt{-cd}} + \frac{e^{bx+a}}{bd}$
derivativedivides	$-\frac{a^2 b \left( e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}_1\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right) - e^{-\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}_1\left(\frac{-b\sqrt{-cd}-da+d(bx+a)}{d}\right) \right)}{2\sqrt{-cd}} + \frac{b^2 e^{bx+a}}{d}$
default	$-\frac{a^2 b \left( e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}_1\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right) - e^{-\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}_1\left(\frac{-b\sqrt{-cd}-da+d(bx+a)}{d}\right) \right)}{2\sqrt{-cd}} + \frac{b^2 e^{bx+a}}{d}$

```
input int(exp(b*x+a)*x^2/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
output 1/2/d/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+d*a-d*(b*x+a))/d)*c-1/2/d/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(1/2)-d*a+d*(b*x+a))/d)*c+exp(b*x+a)/b/d
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \frac{e^{a+bx} x^2}{c + dx^2} dx$$

$$= \frac{\sqrt{-\frac{b^2c}{d}} \operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} - \sqrt{-\frac{b^2c}{d}} \operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)} + 2e^{(bx+a)}}{2bd}$$

input `integrate(exp(b*x+a)*x^2/(d*x^2+c),x, algorithm="fricas")`

output `1/2*(sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d))*e^(a + sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b^2*c/d))*e^(a - sqrt(-b^2*c/d)) + 2*e^(b*x + a))/(b*d)`

**Sympy [F]**

$$\int \frac{e^{a+bx} x^2}{c + dx^2} dx = e^a \int \frac{x^2 e^{bx}}{c + dx^2} dx$$

input `integrate(exp(b*x+a)*x**2/(d*x**2+c),x)`

output `exp(a)*Integral(x**2*exp(b*x)/(c + d*x**2), x)`

**Maxima [F]**

$$\int \frac{e^{a+bx} x^2}{c + dx^2} dx = \int \frac{x^2 e^{(bx+a)}}{dx^2 + c} dx$$

input `integrate(exp(b*x+a)*x^2/(d*x^2+c),x, algorithm="maxima")`

output  $x^2 e^{(bx+a)/(bdx^2+bc)} - 2c \int x e^{(bx+a)/(bdx^2+bc)} / (bd^2x^4 + 2b^2cdx^2 + b^2c^2) dx$

### Giac [F]

$$\int \frac{e^{a+bx} x^2}{c+dx^2} dx = \int \frac{x^2 e^{(bx+a)}}{dx^2+c} dx$$

input `integrate(exp(b*x+a)*x^2/(d*x^2+c),x, algorithm="giac")`

output `integrate(x^2*e^(b*x+a)/(d*x^2+c),x)`

### Mupad [F(-1)]

Timed out.

$$\int \frac{e^{a+bx} x^2}{c+dx^2} dx = \int \frac{x^2 e^{a+bx}}{dx^2+c} dx$$

input `int((x^2*exp(a+b*x))/(c+d*x^2),x)`

output `int((x^2*exp(a+b*x))/(c+d*x^2),x)`

### Reduce [F]

$$\int \frac{e^{a+bx} x^2}{c+dx^2} dx = \frac{e^a \left( e^{bx} - \left( \int \frac{e^{bx}}{dx^2+c} dx \right) bc \right)}{bd}$$

input `int(exp(b*x+a)*x^2/(d*x^2+c),x)`

output `(e**a*(e**(b*x) - int(e**(b*x)/(c+d*x**2),x)*b*c))/(b*d)`

### 3.28 $\int \frac{e^{a+bx} x}{c+dx^2} dx$

Optimal result	238
Mathematica [C] (verified)	238
Rubi [A] (verified)	239
Maple [A] (verified)	240
Fricas [A] (verification not implemented)	240
Sympy [F]	241
Maxima [F]	241
Giac [F]	241
Mupad [F(-1)]	242
Reduce [F]	242

#### Optimal result

Integrand size = 18, antiderivative size = 100

$$\int \frac{e^{a+bx} x}{c + dx^2} dx = \frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d} + \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2d}$$

output

$1/2*\exp(a+b*(-c)^{(1/2)}/d^{(1/2)})*Ei(-b*((-c)^{(1/2)}-d^{(1/2)}*x)/d^{(1/2)})/d+1/2*\exp(a-b*(-c)^{(1/2)}/d^{(1/2)})*Ei(b*((-c)^{(1/2)}+d^{(1/2)}*x)/d^{(1/2)})/d$

#### Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int \frac{e^{a+bx} x}{c + dx^2} dx = \frac{e^{a-\frac{ib\sqrt{c}}{\sqrt{d}}} \left( e^{\frac{2ib\sqrt{c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right) + \text{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right) \right)}{2d}$$

input `Integrate[(E^(a + b*x)*x)/(c + d*x^2),x]`

output `(E^(a - (I*b*Sqrt[c])/Sqrt[d])*(E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[b*(((I)*Sqrt[c])/Sqrt[d] + x)] + ExpIntegralEi[b*((I*Sqrt[c])/Sqrt[d] + x)]))/((2*d)`

### Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2701, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x e^{a+bx}}{c + dx^2} dx$$

↓ 2701

$$\int \left( \frac{e^{a+bx}}{2\sqrt{d}(\sqrt{-c} + \sqrt{dx})} - \frac{e^{a+bx}}{2\sqrt{d}(\sqrt{-c} - \sqrt{dx})} \right) dx$$

↓ 2009

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2d} + \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2d}$$

input `Int[(E^(a + b*x)*x)/(c + d*x^2),x]`

output `(E^(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d]))]/(2*d) + (E^(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d]])/(2*d)`



**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2701 Int[((F_)^((g_)*((d_) + (e_)*(x_))^(n_))* (u_)^(m_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + c*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]
```

**Maple [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right)}{2d} - \frac{e^{-\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}\left(-\frac{b\sqrt{-cd+da}+d(bx+a)}{d}\right)}{2d}$
derivativedivides	$-\frac{b\left(e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right)\sqrt{-cd} + e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}\left(\frac{b\sqrt{-cd+da}+d(bx+a)}{d}\right)ad + e^{-\frac{b\sqrt{-cd+da}}{d}}\right)}{2d\sqrt{-cd}}$
default	$-\frac{b\left(e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right)\sqrt{-cd} + e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegralEi}\left(\frac{b\sqrt{-cd+da}+d(bx+a)}{d}\right)ad + e^{-\frac{b\sqrt{-cd+da}}{d}}\right)}{2d\sqrt{-cd}}$

```
input int(exp(b*x+a)*x/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
output -1/2/d*exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+d*a-d*(b*x+a))/d)-1/2/d*exp((-b*(-c*d)^(1/2)+d*a)/d)*Ei(1,-(b*(-c*d)^(1/2)-d*a+d*(b*x+a))/d)
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int \frac{e^{a+bx} x}{c + dx^2} dx = \frac{\operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} + \operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)}}{2d}$$

```
input integrate(exp(b*x+a)*x/(d*x^2+c),x, algorithm="fricas")
```

output  $\frac{1}{2} * (\text{Ei}(b*x - \sqrt{-b^2*c/d}) * e^{(a + \sqrt{-b^2*c/d})} + \text{Ei}(b*x + \sqrt{-b^2*c/d}) * e^{(a - \sqrt{-b^2*c/d})}) / d$

### Sympy [F]

$$\int \frac{e^{a+bx} x}{c + dx^2} dx = e^a \int \frac{x e^{bx}}{c + dx^2} dx$$

input `integrate(exp(b*x+a)*x/(d*x**2+c), x)`

output `exp(a)*Integral(x*exp(b*x)/(c + d*x**2), x)`

### Maxima [F]

$$\int \frac{e^{a+bx} x}{c + dx^2} dx = \int \frac{x e^{(bx+a)}}{dx^2 + c} dx$$

input `integrate(exp(b*x+a)*x/(d*x^2+c), x, algorithm="maxima")`

output `x*e^(b*x + a)/(b*d*x^2 + b*c) + integrate((d*x^2*e^a - c*e^a)*e^(b*x)/(b*d^2*x^4 + 2*b*c*d*x^2 + b*c^2), x)`

### Giac [F]

$$\int \frac{e^{a+bx} x}{c + dx^2} dx = \int \frac{x e^{(bx+a)}}{dx^2 + c} dx$$

input `integrate(exp(b*x+a)*x/(d*x^2+c), x, algorithm="giac")`

output `integrate(x*e^(b*x + a)/(d*x^2 + c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{a+bx}x}{c+dx^2} dx = \int \frac{x e^{a+bx}}{dx^2+c} dx$$

input `int((x*exp(a + b*x))/(c + d*x^2),x)`output `int((x*exp(a + b*x))/(c + d*x^2), x)`**Reduce [F]**

$$\int \frac{e^{a+bx}x}{c+dx^2} dx = e^a \left( \int \frac{e^{bx}x}{dx^2+c} dx \right)$$

input `int(exp(b*x+a)*x/(d*x^2+c),x)`output `e**a*int((e**(b*x)*x)/(c + d*x**2),x)`

### 3.29 $\int \frac{e^{a+bx}}{c+dx^2} dx$

Optimal result	243
Mathematica [C] (verified)	243
Rubi [A] (verified)	244
Maple [A] (verified)	245
Fricas [A] (verification not implemented)	245
Sympy [F]	246
Maxima [F]	246
Giac [F]	247
Mupad [F(-1)]	247
Reduce [F]	247

#### Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{e^{a+bx}}{c+dx^2} dx = \frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

output

```
1/2*exp(a+b*(-c)^(1/2)/d^(1/2))*Ei(-b*((-c)^(1/2)-d^(1/2)*x)/d^(1/2))/(-c)^(1/2)/d^(1/2)-1/2*exp(a-b*(-c)^(1/2)/d^(1/2))*Ei(b*((-c)^(1/2)+d^(1/2)*x)/d^(1/2))/(-c)^(1/2)/d^(1/2)
```

#### Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

$$\int \frac{e^{a+bx}}{c+dx^2} dx = \frac{ie^{a-\frac{ib\sqrt{c}}{\sqrt{d}}} \left( e^{\frac{2ib\sqrt{c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right) - \text{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right) \right)}{2\sqrt{c}\sqrt{d}}$$

input `Integrate[E^(a + b*x)/(c + d*x^2),x]`

output `((-1/2*I)*E^(a - (I*b*Sqrt[c])/Sqrt[d])*(E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[b*((-I)*Sqrt[c])/Sqrt[d] + x] - ExpIntegralEi[b*(I*Sqrt[c])/Sqrt[d] + x]))/(Sqrt[c]*Sqrt[d])`

### Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2699, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{a+bx}}{c+dx^2} dx$$

↓ 2699

$$\int \left( \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}-\sqrt{dx})} + \frac{\sqrt{-c}e^{a+bx}}{2c(\sqrt{-c}+\sqrt{dx})} \right) dx$$

↓ 2009

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

input `Int[E^(a + b*x)/(c + d*x^2),x]`

output `(E^(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d]))/(2*Sqrt[-c]*Sqrt[d]) - (E^(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]))`

**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2699 Int[(F_)^((g_)*((d_) + (e_)*(x_)^n_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + c*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{e^{\frac{b\sqrt{-cd}+da}{d}} \operatorname{ExpIntegral}_1\left(\frac{b\sqrt{-cd}+da-d(bx+a)}{d}\right) - e^{-\frac{b\sqrt{-cd}-da}{d}} \operatorname{ExpIntegral}_1\left(-\frac{b\sqrt{-cd}-da+d(bx+a)}{d}\right)}{2\sqrt{-cd}}$	102
default	$-\frac{e^{\frac{b\sqrt{-cd}+da}{d}} \operatorname{ExpIntegral}_1\left(\frac{b\sqrt{-cd}+da-d(bx+a)}{d}\right) - e^{-\frac{b\sqrt{-cd}-da}{d}} \operatorname{ExpIntegral}_1\left(-\frac{b\sqrt{-cd}-da+d(bx+a)}{d}\right)}{2\sqrt{-cd}}$	102
risch	$-\frac{e^{\frac{b\sqrt{-cd}+da}{d}} \operatorname{ExpIntegral}_1\left(\frac{b\sqrt{-cd}+da-d(bx+a)}{d}\right)}{2\sqrt{-cd}} + \frac{e^{-\frac{b\sqrt{-cd}-da}{d}} \operatorname{ExpIntegral}_1\left(-\frac{b\sqrt{-cd}-da+d(bx+a)}{d}\right)}{2\sqrt{-cd}}$	106

```
input int(exp(b*x+a)/(d*x^2+c), x, method=_RETURNVERBOSE)
```

```
output -1/2*(exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1, (b*(-c*d)^(1/2)+d*a-d*(b*x+a))/d) - exp(-(b*(-c*d)^(1/2)-d*a)/d)*Ei(1, -(b*(-c*d)^(1/2)-d*a+d*(b*x+a))/d))/(-c*d)^(1/2)
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{e^{a+bx}}{c + dx^2} dx = -\frac{\sqrt{-\frac{b^2c}{d}} \operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} - \sqrt{-\frac{b^2c}{d}} \operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)}}{2bc}$$

input `integrate(exp(b*x+a)/(d*x^2+c),x, algorithm="fricas")`

output `-1/2*(sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d))*e^(a + sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(b*x + sqrt(-b^2*c/d))*e^(a - sqrt(-b^2*c/d)))/(b*c)`

### Sympy [F]

$$\int \frac{e^{a+bx}}{c+dx^2} dx = e^a \int \frac{e^{bx}}{c+dx^2} dx$$

input `integrate(exp(b*x+a)/(d*x**2+c),x)`

output `exp(a)*Integral(exp(b*x)/(c + d*x**2), x)`

### Maxima [F]

$$\int \frac{e^{a+bx}}{c+dx^2} dx = \int \frac{e^{(bx+a)}}{dx^2+c} dx$$

input `integrate(exp(b*x+a)/(d*x^2+c),x, algorithm="maxima")`

output `integrate(e^(b*x + a)/(d*x^2 + c), x)`

**Giac [F]**

$$\int \frac{e^{a+bx}}{c+dx^2} dx = \int \frac{e^{(bx+a)}}{dx^2+c} dx$$

input `integrate(exp(b*x+a)/(d*x^2+c),x, algorithm="giac")`

output `integrate(e^(b*x + a)/(d*x^2 + c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{a+bx}}{c+dx^2} dx = \int \frac{e^{a+bx}}{dx^2+c} dx$$

input `int(exp(a + b*x)/(c + d*x^2),x)`

output `int(exp(a + b*x)/(c + d*x^2), x)`

**Reduce [F]**

$$\int \frac{e^{a+bx}}{c+dx^2} dx = e^a \left( \int \frac{e^{bx}}{dx^2+c} dx \right)$$

input `int(exp(b*x+a)/(d*x^2+c),x)`

output `e**a*int(e**(b*x)/(c + d*x**2),x)`



### 3.30 $\int \frac{e^{a+bx}}{x(c+dx^2)} dx$

Optimal result	248
Mathematica [C] (verified)	248
Rubi [A] (verified)	249
Maple [A] (verified)	250
Fricas [A] (verification not implemented)	250
Sympy [F]	251
Maxima [F]	251
Giac [F]	252
Mupad [F(-1)]	252
Reduce [F]	252

#### Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = \frac{e^a \operatorname{ExpIntegralEi}(bx)}{c} - \frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2c} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{ExpIntegralEi}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2c}$$

output

```
exp(a)*Ei(b*x)/c-1/2*exp(a+b*(-c)^(1/2)/d^(1/2))*Ei(-b*((-c)^(1/2)-d^(1/2))
*x)/d^(1/2))/c-1/2*exp(a-b*(-c)^(1/2)/d^(1/2))*Ei(b*((-c)^(1/2)+d^(1/2)*x)
/d^(1/2))/c
```

#### Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = \frac{e^a \left( 2 \operatorname{ExpIntegralEi}(bx) - e^{-\frac{ib\sqrt{c}}{\sqrt{d}}} \left( e^{\frac{2ib\sqrt{c}}{\sqrt{d}}} \operatorname{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right) + \operatorname{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}} + x\right)\right) \right) \right)}{2c}$$

input `Integrate[E^(a + b*x)/(x*(c + d*x^2)),x]`

output `(E^a*(2*ExpIntegralEi[b*x] - (E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[b*((-I)*Sqrt[c])/Sqrt[d] + x]) + ExpIntegralEi[b*((I*Sqrt[c])/Sqrt[d] + x)]) / E^(((I*b*Sqrt[c])/Sqrt[d])) / (2*c)`

### Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2701, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx$$

$$\downarrow 2701$$

$$\int \left( \frac{e^{a+bx}}{cx} - \frac{dxe^{a+bx}}{c(c+dx^2)} \right) dx$$

$$\downarrow 2009$$

$$\frac{e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2c} - \frac{e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \text{ExpIntegralEi}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2c} + \frac{e^a \text{ExpIntegralEi}(bx)}{c}$$

input `Int[E^(a + b*x)/(x*(c + d*x^2)),x]`

output `(E^a*ExpIntegralEi[b*x])/c - (E^(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d])]) / (2*c) - (E^(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d]]) / (2*c)`

**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2701 Int[((F_)^((g_)*((d_) + (e_)*(x_))^(n_))* (u_)^(m_))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + c*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]
```

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegral}_1\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right) + e^{-\frac{b\sqrt{-cd}-da}{d}} \operatorname{ExpIntegral}_1\left(-\frac{b\sqrt{-cd}-da+d(bx+a)}{d}\right)}{2c} - \frac{e^a \operatorname{ExpIntegral}_1\left(-\frac{bx+a}{c}\right)}{c}$
default	$\frac{e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegral}_1\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right) + e^{-\frac{b\sqrt{-cd}-da}{d}} \operatorname{ExpIntegral}_1\left(-\frac{b\sqrt{-cd}-da+d(bx+a)}{d}\right)}{2c} - \frac{e^a \operatorname{ExpIntegral}_1\left(-\frac{bx+a}{c}\right)}{c}$
risch	$-\frac{e^a \operatorname{ExpIntegral}_1(-bx)}{c} + \frac{e^{\frac{b\sqrt{-cd+da}}{d}} \operatorname{ExpIntegral}_1\left(\frac{b\sqrt{-cd+da}-d(bx+a)}{d}\right)}{2c} + \frac{e^{-\frac{b\sqrt{-cd}-da}{d}} \operatorname{ExpIntegral}_1\left(-\frac{b\sqrt{-cd}-da+d(bx+a)}{d}\right)}{2c}$

```
input int(exp(b*x+a)/x/(d*x^2+c),x,method=_RETURNVERBOSE)
```

```
output 1/2*(exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+d*a-d*(b*x+a))/d)+exp(-b*(-c*d)^(1/2)-d*a/d)*Ei(1,-(b*(-c*d)^(1/2)-d*a+d*(b*x+a))/d))/c-1/c*exp(a)*Ei(1,-b*x)
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = -\frac{\operatorname{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} + \operatorname{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right) e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)} - 2 \operatorname{Ei}(bx) e^a}{2c}$$

input `integrate(exp(b*x+a)/x/(d*x^2+c),x, algorithm="fricas")`

output `-1/2*(Ei(b*x - sqrt(-b^2*c/d))*e^(a + sqrt(-b^2*c/d)) + Ei(b*x + sqrt(-b^2*c/d))*e^(a - sqrt(-b^2*c/d)) - 2*Ei(b*x)*e^a)/c`

## Sympy [F]

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = e^a \int \frac{e^{bx}}{cx+dx^3} dx$$

input `integrate(exp(b*x+a)/x/(d*x**2+c),x)`

output `exp(a)*Integral(exp(b*x)/(c*x + d*x**3), x)`

## Maxima [F]

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = \int \frac{e^{(bx+a)}}{(dx^2+c)x} dx$$

input `integrate(exp(b*x+a)/x/(d*x^2+c),x, algorithm="maxima")`

output `integrate(e^(b*x + a)/((d*x^2 + c)*x), x)`

**Giac [F]**

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = \int \frac{e^{(bx+a)}}{(dx^2+c)x} dx$$

input `integrate(exp(b*x+a)/x/(d*x^2+c),x, algorithm="giac")`

output `integrate(e^(b*x + a)/((d*x^2 + c)*x), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = \int \frac{e^{a+bx}}{x(dx^2+c)} dx$$

input `int(exp(a + b*x)/(x*(c + d*x^2)),x)`

output `int(exp(a + b*x)/(x*(c + d*x^2)), x)`

**Reduce [F]**

$$\int \frac{e^{a+bx}}{x(c+dx^2)} dx = e^a \left( \int \frac{e^{bx}}{dx^3+cx} dx \right)$$

input `int(exp(b*x+a)/x/(d*x^2+c),x)`

output `e**a*int(e**(b*x)/(c*x + d*x**3),x)`

### 3.31 $\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$

Optimal result	253
Mathematica [C] (verified)	254
Rubi [A] (verified)	254
Maple [A] (verified)	255
Fricas [A] (verification not implemented)	256
Sympy [F]	256
Maxima [F]	257
Giac [F]	257
Mupad [F(-1)]	257
Reduce [F]	258

#### Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx = -\frac{e^{a+bx}}{cx} + \frac{be^a \operatorname{ExpIntegralEi}(bx)}{c} + \frac{\sqrt{d}e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}} \operatorname{ExpIntegralEi}\left(\frac{b(\sqrt{-c}+\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}}$$

output

```
-exp(b*x+a)/c/x+b*exp(a)*Ei(b*x)/c+1/2*d^(1/2)*exp(a+b*(-c)^(1/2)/d^(1/2))
 *Ei(-b*((-c)^(1/2)-d^(1/2)*x)/d^(1/2))/(-c)^(3/2)-1/2*d^(1/2)*exp(a-b*(-c)
 ^ (1/2)/d^(1/2))*Ei(b*((-c)^(1/2)+d^(1/2)*x)/d^(1/2))/(-c)^(3/2)
```

**Mathematica [C] (verified)**

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$$

$$= \frac{-\frac{e^{a+bx}}{x} + be^a \operatorname{ExpIntegralEi}(bx)}{c}$$

$$+ \frac{i\sqrt{d}e^{a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\operatorname{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right) - \operatorname{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)\right)}{2c^{3/2}}$$

input `Integrate[E^(a + b*x)/(x^2*(c + d*x^2)),x]`

output `(-(E^(a + b*x)/x) + b*E^a*ExpIntegralEi[b*x])/c + ((I/2)*Sqrt[d]*E^(a - (I*b*Sqrt[c])/Sqrt[d])*(E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[b*((-I)*Sqrt[c])/Sqrt[d] + x]) - ExpIntegralEi[b*((I*Sqrt[c])/Sqrt[d] + x)]))/c^(3/2)`

**Rubi [A] (verified)**

Time = 0.72 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2701, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$$

$$\downarrow \text{2701}$$

$$\int \left( \frac{e^{a+bx}}{cx^2} - \frac{de^{a+bx}}{c(c+dx^2)} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{d}e^{a+\frac{b\sqrt{-c}}{\sqrt{d}}}\text{ExpIntegralEi}\left(-\frac{b(\sqrt{-c}-\sqrt{dx})}{\sqrt{d}}\right)}{2(-c)^{3/2}} - \frac{\sqrt{d}e^{a-\frac{b\sqrt{-c}}{\sqrt{d}}}\text{ExpIntegralEi}\left(\frac{b(\sqrt{dx}+\sqrt{-c})}{\sqrt{d}}\right)}{2(-c)^{3/2}} + \frac{e^ab\text{ExpIntegralEi}(bx)}{c} - \frac{e^{a+bx}}{cx}$$

input `Int[E^(a + b*x)/(x^2*(c + d*x^2)),x]`

output `-(E^(a + b*x)/(c*x)) + (b*E^a*ExpIntegralEi[b*x])/c + (Sqrt[d]*E^(a + (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[-((b*(Sqrt[-c] - Sqrt[d]*x))/Sqrt[d])])/(2*(-c)^(3/2)) - (Sqrt[d]*E^(a - (b*Sqrt[-c])/Sqrt[d])*ExpIntegralEi[(b*(Sqrt[-c] + Sqrt[d]*x))/Sqrt[d])]/(2*(-c)^(3/2))`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2701 `Int[((F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))* (u_)^(m_.))/((a_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + c*x^2), x], x] /; FreeQ[{F, a, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.98

method	result
derivativedivides	$b \left( -\frac{e^{bx+a}}{cbx} + \frac{d \left( e^{\frac{b\sqrt{-cd}+da}{d}} \text{expIntegral}_1 \left( \frac{b\sqrt{-cd}+da-d(bx+a)}{d} \right) - e^{-\frac{b\sqrt{-cd}-da}{d}} \text{expIntegral}_1 \left( -\frac{b\sqrt{-cd}-da+d(bx+a)}{d} \right) \right)}{2cb\sqrt{-cd}} \right)$
default	$b \left( -\frac{e^{bx+a}}{cbx} + \frac{d \left( e^{\frac{b\sqrt{-cd}+da}{d}} \text{expIntegral}_1 \left( \frac{b\sqrt{-cd}+da-d(bx+a)}{d} \right) - e^{-\frac{b\sqrt{-cd}-da}{d}} \text{expIntegral}_1 \left( -\frac{b\sqrt{-cd}-da+d(bx+a)}{d} \right) \right)}{2cb\sqrt{-cd}} \right)$
risch	$-\frac{e^{bx+a}}{cx} + \frac{de^{\frac{b\sqrt{-cd}+da}{d}} \text{expIntegral}_1 \left( \frac{b\sqrt{-cd}+da-d(bx+a)}{d} \right)}{2c\sqrt{-cd}} - \frac{de^{-\frac{b\sqrt{-cd}-da}{d}} \text{expIntegral}_1 \left( -\frac{b\sqrt{-cd}-da+d(bx+a)}{d} \right)}{2c\sqrt{-cd}}$



input `int(exp(b*x+a)/x^2/(d*x^2+c),x,method=_RETURNVERBOSE)`

output `b*(-exp(b*x+a)/c/b/x+1/2*d*(exp((b*(-c*d)^(1/2)+d*a)/d)*Ei(1,(b*(-c*d)^(1/2)+d*a-d*(b*x+a))/d)-exp(-(b*(-c*d)^(1/2)-d*a)/d)*Ei(1,-(b*(-c*d)^(1/2)-d*a+d*(b*x+a))/d))/c/b/(-c*d)^(1/2)-1/c*exp(a)*Ei(1,-b*x)`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx$$

$$= \frac{2b^2cx\text{Ei}(bx)e^a + \sqrt{-\frac{b^2c}{d}}dx\text{Ei}\left(bx - \sqrt{-\frac{b^2c}{d}}\right)e^{\left(a + \sqrt{-\frac{b^2c}{d}}\right)} - \sqrt{-\frac{b^2c}{d}}dx\text{Ei}\left(bx + \sqrt{-\frac{b^2c}{d}}\right)e^{\left(a - \sqrt{-\frac{b^2c}{d}}\right)} - 2bc^2x}{2bc^2x}$$

input `integrate(exp(b*x+a)/x^2/(d*x^2+c),x, algorithm="fricas")`

output `1/2*(2*b^2*c*x*Ei(b*x)*e^a + sqrt(-b^2*c/d)*d*x*Ei(b*x - sqrt(-b^2*c/d))*e^(a + sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*d*x*Ei(b*x + sqrt(-b^2*c/d))*e^(a - sqrt(-b^2*c/d)) - 2*b*c*e^(b*x + a))/(b*c^2*x)`

### Sympy [F]

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx = e^a \int \frac{e^{bx}}{cx^2 + dx^4} dx$$

input `integrate(exp(b*x+a)/x**2/(d*x**2+c),x)`

output `exp(a)*Integral(exp(b*x)/(c*x**2 + d*x**4), x)`

**Maxima [F]**

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx = \int \frac{e^{(bx+a)}}{(dx^2+c)x^2} dx$$

input `integrate(exp(b*x+a)/x^2/(d*x^2+c),x, algorithm="maxima")`

output `integrate(e^(b*x + a)/((d*x^2 + c)*x^2), x)`

**Giac [F]**

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx = \int \frac{e^{(bx+a)}}{(dx^2+c)x^2} dx$$

input `integrate(exp(b*x+a)/x^2/(d*x^2+c),x, algorithm="giac")`

output `integrate(e^(b*x + a)/((d*x^2 + c)*x^2), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx = \int \frac{e^{a+bx}}{x^2(dx^2+c)} dx$$

input `int(exp(a + b*x)/(x^2*(c + d*x^2)),x)`

output `int(exp(a + b*x)/(x^2*(c + d*x^2)), x)`

**Reduce [F]**

$$\int \frac{e^{a+bx}}{x^2(c+dx^2)} dx = e^a \left( \int \frac{e^{bx}}{dx^4 + cx^2} dx \right)$$

input `int(exp(b*x+a)/x^2/(d*x^2+c),x)`

output `e**a*int(e**(b*x)/(c*x**2 + d*x**4),x)`

### 3.32 $\int \frac{e^{d+ex} x^3}{a+bx+cx^2} dx$

Optimal result . . . . .	259
Mathematica [A] (verified) . . . . .	260
Rubi [A] (verified) . . . . .	260
Maple [B] (verified) . . . . .	261
Fricas [A] (verification not implemented) . . . . .	262
Sympy [F] . . . . .	263
Maxima [F] . . . . .	263
Giac [F] . . . . .	264
Mupad [F(-1)] . . . . .	264
Reduce [F] . . . . .	264

#### Optimal result

Integrand size = 23, antiderivative size = 232

$$\int \frac{e^{d+ex} x^3}{a+bx+cx^2} dx = -\frac{e^{d+ex}}{ce^2} - \frac{be^{d+ex}}{c^2e} + \frac{e^{d+ex} x}{ce} + \frac{\left(b^2 - ac - \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c^3} + \frac{\left(b^2 - ac + \frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c^3}$$

output

```
-exp(e*x+d)/c/e^2-b*exp(e*x+d)/c^2/e+exp(e*x+d)*x/c/e+1/2*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b-(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c)/c^3+1/2*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b+(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c)/c^3
```

**Mathematica [A] (verified)**

Time = 0.90 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.16

$$\int \frac{e^{d+ex} x^3}{a+bx+cx^2} dx$$

$$= \frac{e^{d-\frac{be}{c}} \left( -2c\sqrt{b^2-4ac} e^{\left(\frac{b}{c}+x\right)} (c+be-cex) + (-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}) e^2 e^{\frac{(b+\sqrt{b^2-4ac})}{2c}} \right)}{c^2}$$

input

```
Integrate[(E^(d + e*x)*x^3)/(a + b*x + c*x^2), x]
```

output

```
(E^(d - (b*e)/c)*(-2*c*Sqrt[b^2 - 4*a*c]*E^(e*(b/c + x))*(c + b*e - c*e*x)
+ (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*e^2*E^
(((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c]
] + 2*c*x))/(2*c)] + (b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2
- 4*a*c])*e^2*E^(((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c])]/(2*c^3*Sqrt[b^2 - 4*a*c]*e^2)
```

**Rubi [A] (verified)**

Time = 1.10 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2700, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 e^{d+ex}}{a+bx+cx^2} dx$$

$$\downarrow \text{2700}$$

$$\int \left( \frac{e^{d+ex} (x(b^2 - ac) + ab)}{c^2 (a + bx + cx^2)} - \frac{be^{d+ex}}{c^2} + \frac{xe^{d+ex}}{c} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \operatorname{ExpIntegralEi}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2c^3} +$$

$$\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}} - ac + b^2\right) e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \operatorname{ExpIntegralEi}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2c^3} - \frac{be^{d+ex}}{c^2e} - \frac{e^{d+ex}}{ce^2} +$$

$$\frac{xe^{d+ex}}{ce}$$

input `Int[(E^(d + e*x)*x^3)/(a + b*x + c*x^2),x]`

output `-(E^(d + e*x)/(c*e^2)) - (b*E^(d + e*x))/(c^2*e) + (E^(d + e*x)*x)/(c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c)))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/(2*c^3) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c)))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/(2*c^3)`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2700 `Int[((F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))* (u_)^(m_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]`

### Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(203) = 406.

Time = 0.23 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.25

method	result
risch	$\frac{3e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralEi}\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right)ab}{2c^2\sqrt{-4ace^2+b^2e^2}} + \frac{e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}}}{2c^2\sqrt{-4ace^2+b^2e^2}}$
derivativdivides	Expression too large to display
default	Expression too large to display

```
input int(exp(e*x+d)*x^3/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
output -3/2*e/c^2/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a*b+1/2*e/c^3/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b^3+3/2*e/c^2/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a*b-1/2*e/c^3/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b^3+1/2/c^2*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a-1/2/c^3*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b^2+1/2/c^2*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a-1/2/c^3*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b^2+exp(e*x+d)*x/c/e-b*exp(e*x+d)/c^2/e-exp(e*x+d)/c/e^2
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.42

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx$$

$$= \frac{\left( (b^4 - 5ab^2c + 4a^2c^2)e^2 - (b^3c - 3abc^2)e\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \right) \operatorname{Ei}\left(\frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\left(\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)}}{2c^2\sqrt{-4ace^2+b^2e^2}}$$

input `integrate(exp(e*x+d)*x^3/(c*x^2+b*x+a),x, algorithm="fricas")`

output 
$$\frac{1}{2} \left( (b^4 - 5ab^2c + 4a^2c^2)e^2 - (b^3c - 3ab^2c^2)e \sqrt{(b^2 - 4ac)e^2/c^2} \right) \operatorname{Ei} \left( \frac{1}{2} (2cex + be - c\sqrt{(b^2 - 4ac)e^2/c^2})/c \right) / c + \left( (b^4 - 5ab^2c + 4a^2c^2)e^2 + (b^3c - 3ab^2c^2)e \sqrt{(b^2 - 4ac)e^2/c^2} \right) \operatorname{Ei} \left( \frac{1}{2} (2cex + be + c\sqrt{(b^2 - 4ac)e^2/c^2})/c \right) / c - 2(b^2c^2 - 4ac^3 - (b^2c^2 - 4ac^3)ex + (b^3c - 4ab^2c^2)e)e^{(ex+d)} / ((b^2c^3 - 4ac^4)e^2)$$

### Sympy [F]

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx = e^d \int \frac{x^3 e^{ex}}{a + bx + cx^2} dx$$

input `integrate(exp(e*x+d)*x**3/(c*x**2+b*x+a),x)`

output `exp(d)*Integral(x**3*exp(e*x)/(a + b*x + c*x**2), x)`

### Maxima [F]

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx = \int \frac{x^3 e^{(ex+d)}}{cx^2 + bx + a} dx$$

input `integrate(exp(e*x+d)*x^3/(c*x^2+b*x+a),x, algorithm="maxima")`

output 
$$(cex^3e^d - cx^2e^d - bxe^d)e^{(ex)} / (c^2e^2x^2 + bce^2x + ace^2) - \int \frac{-(b^2e^d + 2c^2e^d)ax + (b^2e^d - 2ac^2e^d)x^2 + abe^d}{(c^3e^2x^4 + 2b^2c^2e^2x^3 + 2abce^2x + a^2c^2e^2 + (b^2c^2e^2 + 2ac^2e^2)x^2)} dx$$



**Giac [F]**

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx = \int \frac{x^3 e^{(ex+d)}}{cx^2 + bx + a} dx$$

input `integrate(exp(e*x+d)*x^3/(c*x^2+b*x+a),x, algorithm="giac")`

output `integrate(x^3*e^(e*x + d)/(c*x^2 + b*x + a), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx = \int \frac{x^3 e^{d+ex}}{cx^2 + bx + a} dx$$

input `int((x^3*exp(d + e*x))/(a + b*x + c*x^2),x)`

output `int((x^3*exp(d + e*x))/(a + b*x + c*x^2), x)`

**Reduce [F]**

$$\int \frac{e^{d+ex} x^3}{a + bx + cx^2} dx = \frac{e^d (-e^{ex} b e + e^{ex} c e x - e^{ex} c + (\int \frac{e^{ex}}{cx^2+bx+a} dx) a b e^2 - (\int \frac{e^{ex} x}{cx^2+bx+a} dx) a c e^2 + (\int \frac{e^{ex} x}{cx^2+bx+a} dx) b^2 e^2)}{c^2 e^2}$$

input `int(exp(e*x+d)*x^3/(c*x^2+b*x+a),x)`

output `(e**d*( - e**(e*x)*b*e + e**(e*x)*c*e*x - e**(e*x)*c + int(e**(e*x)/(a + b*x + c*x**2),x)*a*b*e**2 - int((e**(e*x)*x)/(a + b*x + c*x**2),x)*a*c*e**2 + int((e**(e*x)*x)/(a + b*x + c*x**2),x)*b**2*e**2))/(c**2*e**2)`

### 3.33 $\int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx$

Optimal result	265
Mathematica [A] (verified)	266
Rubi [A] (verified)	266
Maple [B] (verified)	267
Fricas [A] (verification not implemented)	268
Sympy [F]	269
Maxima [F]	269
Giac [F]	269
Mupad [F(-1)]	270
Reduce [F]	270

#### Optimal result

Integrand size = 23, antiderivative size = 186

$$\int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx = \frac{e^{d+ex}}{ce} - \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c^2} - \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c^2}$$

output

```
exp(e*x+d)/c/e-1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b-(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c)/c^2-1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b+(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c)/c^2
```

**Mathematica [A] (verified)**

Time = 1.87 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{e^{d+ex} x^2}{a + bx + cx^2} dx =$$

$$e^{d - \frac{(b + \sqrt{b^2 - 4ac})e}{2c}} \left( -2c\sqrt{b^2 - 4ac} e^{\frac{e(b + \sqrt{b^2 - 4ac} + 2cx)}{2c}} + (-b^2 + 2ac + b\sqrt{b^2 - 4ac}) e e^{\frac{\sqrt{b^2 - 4ac}e}{c}} \text{ExpIntegralEi} \right)$$


---


$$2c^2\sqrt{b^2 - 4ac}$$

input

```
Integrate[(E^(d + e*x)*x^2)/(a + b*x + c*x^2),x]
```

output

```
-1/2*(E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*(-2*c*Sqrt[b^2 - 4*a*c]*E^
((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)) + (-b^2 + 2*a*c + b*Sqrt[b^2 -
4*a*c])*e*E^((Sqrt[b^2 - 4*a*c]*e)/c)*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*
a*c] + 2*c*x))/(2*c]) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*e*ExpIntegralE
i[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]))/(c^2*Sqrt[b^2 - 4*a*c]*e)
```

**Rubi [A] (verified)**

Time = 0.89 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2700, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 e^{d+ex}}{a + bx + cx^2} dx$$

$$\downarrow \text{2700}$$

$$\int \left( \frac{e^{d+ex}}{c} - \frac{(a + bx)e^{d+ex}}{c(a + bx + cx^2)} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{\left(b - \frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}}\right) e^{d - \frac{e(b - \sqrt{b^2 - 4ac})}{2c}} \text{ExpIntegralEi}\left(\frac{e(b + 2cx - \sqrt{b^2 - 4ac})}{2c}\right) - \left(\frac{b^2 - 2ac}{\sqrt{b^2 - 4ac}} + b\right) e^{d - \frac{e(\sqrt{b^2 - 4ac} + b)}{2c}} \text{ExpIntegralEi}\left(\frac{e(b + 2cx + \sqrt{b^2 - 4ac})}{2c}\right)}{2c^2} + \frac{e^{d+ex}}{ce}$$

input `Int[(E^(d + e*x)*x^2)/(a + b*x + c*x^2),x]`

output `E^(d + e*x)/(c*e) - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c))]/(2*c^2) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c))]/(2*c^2)`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2700 `Int[((F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))*u_)^(m_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]`

**Maple [B] (verified)**

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. 2(159) = 318.

Time = 0.07 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.02

method	result
risch	$\frac{e e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \text{expIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) a}{c\sqrt{-4ace^2+b^2e^2}} - \frac{e e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \text{expIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) a}{c\sqrt{-4ace^2+b^2e^2}}$
derivativedivides	Expression too large to display
default	Expression too large to display

input `int(exp(e*x+d)*x^2/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)`

output 
$$\begin{aligned} & e/c/(-4*a*c*e^2+b^2*e^2)^{(1/2)}*\exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^{(1/2)})) * Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c)*a- \\ & 1/2*e/c^2/(-4*a*c*e^2+b^2*e^2)^{(1/2)}*\exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^{(1/2)})) * Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * b^2-e/c/(-4*a*c*e^2+b^2*e^2)^{(1/2)}*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * a+1/2*e/c^2/(-4*a*c*e^2+b^2*e^2)^{(1/2)}*\exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * b^2+1/2/c^2*\exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^{(1/2)})) * Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * b+1/2/c^2 * \exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^{(1/2)})/c) * b+\exp(e*x+d)/c/e \end{aligned}$$

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.44

$$\int \frac{e^{d+ex} x^2}{a+bx+cx^2} dx =$$

$$\frac{\left( (b^3 - 4abc)e - (b^2c - 2ac^2) \sqrt{\frac{(b^2-4ac)e^2}{c^2}} \right) Ei\left( \frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c} \right) e^{\left( \frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c} \right)} + \left( b^3 - \dots \right)}{2(b^2 \dots)}$$

input `integrate(exp(e*x+d)*x^2/(c*x^2+b*x+a),x, algorithm="fricas")`

output 
$$\begin{aligned} & -1/2*((b^3 - 4*a*b*c)*e - (b^2*c - 2*a*c^2)*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))* \\ & Ei(1/2*(2*c*e*x + b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c)*e^{(1/2*(2*c*d - \\ & b*e + c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c)} + ((b^3 - 4*a*b*c)*e + (b^2*c - 2* \\ & a*c^2)*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))*Ei(1/2*(2*c*e*x + b*e + c*\text{sqrt}((b^2 - \\ & 4*a*c)*e^2/c^2))/c)*e^{(1/2*(2*c*d - b*e - c*\text{sqrt}((b^2 - 4*a*c)*e^2/c^2))/c)} \\ & ) - 2*(b^2*c - 4*a*c^2)*e^{(e*x + d)}/((b^2*c^2 - 4*a*c^3)*e) \end{aligned}$$

**Sympy [F]**

$$\int \frac{e^{d+ex} x^2}{a + bx + cx^2} dx = e^d \int \frac{x^2 e^{ex}}{a + bx + cx^2} dx$$

input `integrate(exp(e*x+d)*x**2/(c*x**2+b*x+a), x)`

output `exp(d)*Integral(x**2*exp(e*x)/(a + b*x + c*x**2), x)`

**Maxima [F]**

$$\int \frac{e^{d+ex} x^2}{a + bx + cx^2} dx = \int \frac{x^2 e^{(ex+d)}}{cx^2 + bx + a} dx$$

input `integrate(exp(e*x+d)*x^2/(c*x^2+b*x+a), x, algorithm="maxima")`

output `x^2*e^(e*x + d)/(c*e*x^2 + b*e*x + a*e) - integrate((b*x^2*e^d + 2*a*x*e^d)*e^(e*x)/(c^2*e*x^4 + 2*b*c*e*x^3 + 2*a*b*e*x + a^2*e + (b^2*e + 2*a*c*e)*x^2), x)`

**Giac [F]**

$$\int \frac{e^{d+ex} x^2}{a + bx + cx^2} dx = \int \frac{x^2 e^{(ex+d)}}{cx^2 + bx + a} dx$$

input `integrate(exp(e*x+d)*x^2/(c*x^2+b*x+a), x, algorithm="giac")`

output `integrate(x^2*e^(e*x + d)/(c*x^2 + b*x + a), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{d+ex} x^2}{a + bx + cx^2} dx = \int \frac{x^2 e^{d+ex}}{cx^2 + bx + a} dx$$

input `int((x^2*exp(d + e*x))/(a + b*x + c*x^2), x)`output `int((x^2*exp(d + e*x))/(a + b*x + c*x^2), x)`**Reduce [F]**

$$\int \frac{e^{d+ex} x^2}{a + bx + cx^2} dx = \frac{e^d (e^{ex} - (\int \frac{e^{ex}}{cx^2+bx+a} dx) ae - (\int \frac{e^{ex} x}{cx^2+bx+a} dx) be)}{ce}$$

input `int(exp(e*x+d)*x^2/(c*x^2+b*x+a), x)`output `(e**d*(e**(e*x) - int(e**(e*x)/(a + b*x + c*x**2), x)*a*e - int((e**(e*x)*x)/(a + b*x + c*x**2), x)*b*e))/(c*e)`

### 3.34 $\int \frac{e^{d+ex} x}{a+bx+cx^2} dx$

Optimal result	271
Mathematica [A] (verified)	271
Rubi [A] (verified)	272
Maple [B] (verified)	273
Fricas [A] (verification not implemented)	274
Sympy [F]	275
Maxima [F]	275
Giac [F]	275
Mupad [F(-1)]	276
Reduce [F]	276

#### Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{e^{d+ex} x}{a+bx+cx^2} dx = \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c} + \frac{\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2c}$$

output

```
1/2*(1-b/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b-(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*
e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c)/c+1/2*(1+b/(-4*a*c+b^2)^(1/2))*exp(d-1/2
*(b+(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c)/c
```

#### Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{e^{d+ex} x}{a+bx+cx^2} dx = \frac{e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \left( (-b + \sqrt{b^2-4ac}) e^{\frac{\sqrt{b^2-4ac}e}{c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right) + (b + \sqrt{b^2-4ac}) \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right) \right)}{2c\sqrt{b^2-4ac}}$$



input `Integrate[(E^(d + e*x)*x)/(a + b*x + c*x^2), x]`

output `(E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*((-b + Sqrt[b^2 - 4*a*c])*E^(Sqrt[b^2 - 4*a*c]*e)/c)*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)] + (b + Sqrt[b^2 - 4*a*c])*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c]))/(2*c*Sqrt[b^2 - 4*a*c])`

### Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2700, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x e^{d+ex}}{a + bx + cx^2} dx$$

↓ 2700

$$\int \left( \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d+ex}}{-\sqrt{b^2-4ac} + b + 2cx} + \frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d+ex}}{\sqrt{b^2-4ac} + b + 2cx} \right) dx$$

↓ 2009

$$\frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(b - \sqrt{b^2-4ac})}{2c}} \text{ExpIntegralEi}\left(\frac{e(b + 2cx - \sqrt{b^2-4ac})}{2c}\right)}{2c} +$$

$$\frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d - \frac{e(\sqrt{b^2-4ac} + b)}{2c}} \text{ExpIntegralEi}\left(\frac{e(b + 2cx + \sqrt{b^2-4ac})}{2c}\right)}{2c}$$

input `Int[(E^(d + e*x)*x)/(a + b*x + c*x^2), x]`

```
output ((1 - b/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/(2*c) + ((1 + b/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/(2*c)
```

**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2700 Int[((F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))*(u_)^(m_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]
```

**Maple [B] (verified)**

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(132) = 264.

Time = 0.06 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.22

method	result
risch	$\frac{e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be}{2c\sqrt{-4ace^2+b^2e^2}} - \frac{e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be}{2c\sqrt{-4ace^2+b^2e^2}}$
derivativedivides	$e^2 \left( -e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be + 2e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be \right)$
default	$e^2 \left( -e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be + 2e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be \right)$

```
input int(exp(e*x+d)*x/(c*x^2+b*x+a), x, method=_RETURNVERBOSE)
```

output

```

1/2/c/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*
b*e-1/2/c/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2)))/c)*
b*e-1/2/c*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)-1/2/c*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)

```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.42

$$\int \frac{e^{d+ex} x}{a + bx + cx^2} dx =$$

$$\frac{\left( bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} - (b^2 - 4ac)e \right) \operatorname{Ei} \left( \frac{2cex + be - c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c} \right) e^{\left( \frac{2cd - be + c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c} \right)} - \left( bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} + \right)}{2(b^2c - 4ac^2)e}$$

input

```
integrate(exp(e*x+d)*x/(c*x^2+b*x+a),x, algorithm="fricas")
```

output

```

-1/2*((b*c*sqrt((b^2 - 4*a*c)*e^2/c^2) - (b^2 - 4*a*c)*e)*Ei(1/2*(2*c*e*x + b*e - c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c)*e^(1/2*(2*c*d - b*e + c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c) - (b*c*sqrt((b^2 - 4*a*c)*e^2/c^2) + (b^2 - 4*a*c)*e)*Ei(1/2*(2*c*e*x + b*e + c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c)*e^(1/2*(2*c*d - b*e - c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c))/((b^2*c - 4*a*c^2)*e)

```

**Sympy [F]**

$$\int \frac{e^{d+ex}x}{a+bx+cx^2} dx = e^d \int \frac{xe^{ex}}{a+bx+cx^2} dx$$

input `integrate(exp(e*x+d)*x/(c*x**2+b*x+a), x)`

output `exp(d)*Integral(x*exp(e*x)/(a + b*x + c*x**2), x)`

**Maxima [F]**

$$\int \frac{e^{d+ex}x}{a+bx+cx^2} dx = \int \frac{xe^{(ex+d)}}{cx^2+bx+a} dx$$

input `integrate(exp(e*x+d)*x/(c*x^2+b*x+a), x, algorithm="maxima")`

output `x*e^(e*x + d)/(c*e*x^2 + b*e*x + a*e) + integrate((c*x^2*e^d - a*e^d)*e^(e*x)/(c^2*e*x^4 + 2*b*c*e*x^3 + 2*a*b*e*x + a^2*e + (b^2*e + 2*a*c*e)*x^2), x)`

**Giac [F]**

$$\int \frac{e^{d+ex}x}{a+bx+cx^2} dx = \int \frac{xe^{(ex+d)}}{cx^2+bx+a} dx$$

input `integrate(exp(e*x+d)*x/(c*x^2+b*x+a), x, algorithm="giac")`

output `integrate(x*e^(e*x + d)/(c*x^2 + b*x + a), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{d+ex}x}{a+bx+cx^2} dx = \int \frac{x e^{d+ex}}{cx^2+bx+a} dx$$

input `int((x*exp(d + e*x))/(a + b*x + c*x^2), x)`output `int((x*exp(d + e*x))/(a + b*x + c*x^2), x)`**Reduce [F]**

$$\int \frac{e^{d+ex}x}{a+bx+cx^2} dx = e^d \left( \int \frac{e^{ex}x}{cx^2+bx+a} dx \right)$$

input `int(exp(e*x+d)*x/(c*x^2+b*x+a), x)`output `e**d*int((e**(e*x)*x)/(a + b*x + c*x**2), x)`

### 3.35 $\int \frac{e^{d+ex}}{a+bx+cx^2} dx$

Optimal result	277
Mathematica [A] (verified)	277
Rubi [A] (verified)	278
Maple [A] (verified)	279
Fricas [A] (verification not implemented)	280
Sympy [F]	280
Maxima [F]	281
Giac [F]	281
Mupad [F(-1)]	281
Reduce [F]	282

#### Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = \frac{e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{\sqrt{b^2-4ac}} - \frac{e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{\sqrt{b^2-4ac}}$$

output

```
exp(d-1/2*(b-(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c)/(-4*a*c+b^2)^(1/2)-exp(d-1/2*(b+(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c)/(-4*a*c+b^2)^(1/2)
```

#### Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = \frac{e^{d+\frac{(-b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right) - e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{\sqrt{b^2-4ac}}$$

input `Integrate[E^(d + e*x)/(a + b*x + c*x^2),x]`

output `(E^(d + ((-b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)] - E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c]))/Sqrt[b^2 - 4*a*c]`

### Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2698, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx$$

↓ 2698

$$\int \left( \frac{2ce^{d+ex}}{\sqrt{b^2-4ac}(-\sqrt{b^2-4ac}+b+2cx)} - \frac{2ce^{d+ex}}{\sqrt{b^2-4ac}(\sqrt{b^2-4ac}+b+2cx)} \right) dx$$

↓ 2009

$$\frac{e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}} - \frac{e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{\sqrt{b^2-4ac}}$$

input `Int[E^(d + e*x)/(a + b*x + c*x^2),x]`

```
output (E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/Sqrt[b^2 - 4*a*c] - (E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/Sqrt[b^2 - 4*a*c]
```

**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2698 Int[(F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]
```

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{e \left( e^{-\frac{-be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1 \left( \frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c} \right) - e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1 \left( \frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c} \right) \right)}{\sqrt{-4ace^2+b^2e^2}}$
default	$\frac{e \left( e^{-\frac{-be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1 \left( \frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c} \right) - e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1 \left( \frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c} \right) \right)}{\sqrt{-4ace^2+b^2e^2}}$
risch	$- \frac{e e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1 \left( \frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c} \right)}{\sqrt{-4ace^2+b^2e^2}} + \frac{e e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1 \left( \frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c} \right)}{\sqrt{-4ace^2+b^2e^2}}$

```
input int(exp(e*x+d)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
output -e*(exp(1/2*c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2)))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)-exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c))/(-4*a*c*e^2+b^2*e^2)^(1/2)
```



**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.39

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx$$

$$= \frac{c\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \operatorname{Ei}\left(\frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\left(\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)} - c\sqrt{\frac{(b^2-4ac)e^2}{c^2}} \operatorname{Ei}\left(\frac{2cex+be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right) e^{\left(\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)}}{(b^2-4ac)e}$$

input `integrate(exp(e*x+d)/(c*x^2+b*x+a),x, algorithm="fricas")`output `(c*sqrt((b^2 - 4*a*c)*e^2/c^2)*Ei(1/2*(2*c*e*x + b*e - c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c)*e^(1/2*(2*c*d - b*e + c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c) - c*sqrt((b^2 - 4*a*c)*e^2/c^2)*Ei(1/2*(2*c*e*x + b*e + c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c)*e^(1/2*(2*c*d - b*e - c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c))/((b^2 - 4*a*c)*e)`**Sympy [F]**

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = e^d \int \frac{e^{ex}}{a+bx+cx^2} dx$$

input `integrate(exp(e*x+d)/(c*x**2+b*x+a), x)`output `exp(d)*Integral(exp(e*x)/(a + b*x + c*x**2), x)`

**Maxima [F]**

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = \int \frac{e^{(ex+d)}}{cx^2+bx+a} dx$$

input `integrate(exp(e*x+d)/(c*x^2+b*x+a),x, algorithm="maxima")`

output `integrate(e^(e*x + d)/(c*x^2 + b*x + a), x)`

**Giac [F]**

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = \int \frac{e^{(ex+d)}}{cx^2+bx+a} dx$$

input `integrate(exp(e*x+d)/(c*x^2+b*x+a),x, algorithm="giac")`

output `integrate(e^(e*x + d)/(c*x^2 + b*x + a), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = \int \frac{e^{d+ex}}{cx^2+bx+a} dx$$

input `int(exp(d + e*x)/(a + b*x + c*x^2),x)`

output `int(exp(d + e*x)/(a + b*x + c*x^2), x)`

**Reduce [F]**

$$\int \frac{e^{d+ex}}{a+bx+cx^2} dx = e^d \left( \int \frac{e^{ex}}{cx^2+bx+a} dx \right)$$

input `int(exp(e*x+d)/(c*x^2+b*x+a),x)`

output `e**d*int(e**(e*x)/(a + b*x + c*x**2),x)`

### 3.36 $\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$

Optimal result	283
Mathematica [A] (verified)	284
Rubi [A] (verified)	284
Maple [B] (verified)	286
Fricas [A] (verification not implemented)	286
Sympy [F]	287
Maxima [F]	287
Giac [F]	288
Mupad [F(-1)]	288
Reduce [F]	288

#### Optimal result

Integrand size = 23, antiderivative size = 169

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx = \frac{e^d \text{ExpIntegralEi}(ex)}{a} - \frac{\left(1 + \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2a} - \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2a}$$

output

```
exp(d)*Ei(e*x)/a-1/2*(1+b/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b-(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c)/a-1/2*(1-b/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b+(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c)/a
```

**Mathematica [A] (verified)**

Time = 0.66 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$$

$$= \frac{e^d \left( 2 \operatorname{ExpIntegralEi}(ex) + \frac{e^{-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \left( -\left( (b+\sqrt{b^2-4ac}) e^{\frac{\sqrt{b^2-4ac}e}{c}} \operatorname{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right) \right) + (b-\sqrt{b^2-4ac}) \operatorname{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right) \right)}{\sqrt{b^2-4ac}} \right)}{2a}$$

input

```
Integrate[E^(d + e*x)/(x*(a + b*x + c*x^2)), x]
```

output

```
(E^d*(2*ExpIntegralEi[e*x] + (-((b + Sqrt[b^2 - 4*a*c])*E^((Sqrt[b^2 - 4*a*c]*e)/c)*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]) + (b - Sqrt[b^2 - 4*a*c])*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]))/(Sqrt[b^2 - 4*a*c]*E^(((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))))/(2*a)
```

**Rubi [A] (verified)**Time = 0.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2700, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$$

$$\downarrow \text{2700}$$

$$\int \left( \frac{(-b-cx)e^{d+ex}}{a(a+bx+cx^2)} + \frac{e^{d+ex}}{ax} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) e^{d - \frac{e(b - \sqrt{b^2-4ac})}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+2cx - \sqrt{b^2-4ac})}{2c}\right)}{2a} - \frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) e^{d - \frac{e(\sqrt{b^2-4ac} + b)}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+2cx + \sqrt{b^2-4ac})}{2c}\right)}{2a} + \frac{e^d \text{ExpIntegralEi}(ex)}{a}$$

input `Int[E^(d + e*x)/(x*(a + b*x + c*x^2)),x]`

output `(E^d*ExpIntegralEi[e*x])/a - ((1 + b/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c)))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/(2*a) - ((1 - b/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c)))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)]/(2*a)`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2700 `Int[((F_)^((g_)*((d_) + (e_)*(x_))^(n_))* (u_)^(m_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]`

### Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(142) = 284.

Time = 0.06 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.15

method	result
risch	$\frac{e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be - e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right)}{2a\sqrt{-4ace^2+b^2e^2}}$
derivativdivides	$\frac{e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be - e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right)}{2a\sqrt{-4ace^2+b^2e^2}}$
default	$\frac{e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) be - e^{-\frac{be-2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegral}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right)}{2a\sqrt{-4ace^2+b^2e^2}}$

```
input int (exp(e*x+d)/x/(c*x^2+b*x+a) , x, method=_RETURNVERBOSE)
```

```
output 1/2/a/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))
*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*
b*e-1/2/a/(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)
*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*
b*e+1/2/a*exp(-1/2/c*(b*e-2*c*d-(-4*a*c*e^2+b^2*e^2)^(1/2)))
*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)+1/2/a*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)
*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)-1/a*exp(d)*Ei(1,-e*x)
```

### Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.43

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx$$

$$= \frac{2(b^2-4ac)e\operatorname{Ei}(ex)e^d - \left(bc\sqrt{\frac{(b^2-4ac)e^2}{c^2}} + (b^2-4ac)e\right)\operatorname{Ei}\left(\frac{2cex+be-c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)e^{\left(\frac{2cd-be+c\sqrt{\frac{(b^2-4ac)e^2}{c^2}}}{2c}\right)}}{2(ab^2-4a^2c)e}$$

input `integrate(exp(e*x+d)/x/(c*x^2+b*x+a),x, algorithm="fricas")`

output 
$$\frac{1/2*(2*(b^2 - 4*a*c)*e*Ei(e*x)*e^d - (b*c*\sqrt{(b^2 - 4*a*c)*e^2/c^2} + (b^2 - 4*a*c)*e)*Ei(1/2*(2*c*e*x + b*e - c*\sqrt{(b^2 - 4*a*c)*e^2/c^2})/c)*e^{1/2*(2*c*d - b*e + c*\sqrt{(b^2 - 4*a*c)*e^2/c^2})/c} + (b*c*\sqrt{(b^2 - 4*a*c)*e^2/c^2} - (b^2 - 4*a*c)*e)*Ei(1/2*(2*c*e*x + b*e + c*\sqrt{(b^2 - 4*a*c)*e^2/c^2})/c)*e^{1/2*(2*c*d - b*e - c*\sqrt{(b^2 - 4*a*c)*e^2/c^2})/c}}{(a*b^2 - 4*a^2*c)*e}$$

### Sympy [F]

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx = e^d \int \frac{e^{ex}}{ax+bx^2+cx^3} dx$$

input `integrate(exp(e*x+d)/x/(c*x**2+b*x+a),x)`

output `exp(d)*Integral(exp(e*x)/(a*x + b*x**2 + c*x**3), x)`

### Maxima [F]

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx = \int \frac{e^{(ex+d)}}{(cx^2+bx+a)x} dx$$

input `integrate(exp(e*x+d)/x/(c*x^2+b*x+a),x, algorithm="maxima")`

output `integrate(e^(e*x + d)/((c*x^2 + b*x + a)*x), x)`



**Giac [F]**

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx = \int \frac{e^{(ex+d)}}{(cx^2+bx+a)x} dx$$

input `integrate(exp(e*x+d)/x/(c*x^2+b*x+a),x, algorithm="giac")`

output `integrate(e^(e*x + d)/((c*x^2 + b*x + a)*x), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx = \int \frac{e^{d+ex}}{x(cx^2+bx+a)} dx$$

input `int(exp(d + e*x)/(x*(a + b*x + c*x^2)),x)`

output `int(exp(d + e*x)/(x*(a + b*x + c*x^2)), x)`

**Reduce [F]**

$$\int \frac{e^{d+ex}}{x(a+bx+cx^2)} dx = e^d \left( \int \frac{e^{ex}}{cx^3+bx^2+ax} dx \right)$$

input `int(exp(e*x+d)/x/(c*x^2+b*x+a),x)`

output `e**d*int(e**(e*x)/(a*x + b*x**2 + c*x**3),x)`

### 3.37 $\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$

Optimal result	289
Mathematica [A] (verified)	290
Rubi [A] (verified)	290
Maple [B] (verified)	291
Fricas [A] (verification not implemented)	292
Sympy [F]	293
Maxima [F]	293
Giac [F]	294
Mupad [F(-1)]	294
Reduce [F]	294

#### Optimal result

Integrand size = 23, antiderivative size = 212

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx = -\frac{e^{d+ex}}{ax} - \frac{be^d \text{ExpIntegralEi}(ex)}{a^2} + \frac{ee^d \text{ExpIntegralEi}(ex)}{a} + \frac{\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b-\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b-\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2a^2} + \frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+\sqrt{b^2-4ac}+2cx)}{2c}\right)}{2a^2}$$

output

```
-exp(e*x+d)/a/x-b*exp(d)*Ei(e*x)/a^2+e*exp(d)*Ei(e*x)/a+1/2*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b-(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b-(-4*a*c+b^2)^(1/2)+2*c*x)/c)/a^2+1/2*(b-(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))*exp(d-1/2*(b+(-4*a*c+b^2)^(1/2))*e/c)*Ei(1/2*e*(b+(-4*a*c+b^2)^(1/2)+2*c*x)/c)/a^2
```

**Mathematica [A] (verified)**

Time = 0.74 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.51

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$$

$$= e^{d-\frac{(b+\sqrt{b^2-4ac})e}{2c}} \left( -2a\sqrt{b^2-4ac} e^{\frac{(b+\sqrt{b^2-4ac}+2cx)e}{2c}} - 2\sqrt{b^2-4ac}(b-ae) e^{\frac{(b+\sqrt{b^2-4ac})e}{2c}} x \operatorname{ExpIntegralEi}(ex) + \right.$$

input

```
Integrate[E^(d + e*x)/(x^2*(a + b*x + c*x^2)),x]
```

output

```
(E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*(-2*a*Sqrt[b^2 - 4*a*c]*E^((e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c)) - 2*Sqrt[b^2 - 4*a*c]*(b - a*e)*E^(((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*x*ExpIntegralEi[e*x] + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*E^((Sqrt[b^2 - 4*a*c]*e)/c)*x*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c]) - b^2*x*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c]) + 2*a*c*x*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c]) + b*Sqrt[b^2 - 4*a*c]*x*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c])]))/(2*a^2*Sqrt[b^2 - 4*a*c]*x)
```

**Rubi [A] (verified)**

Time = 1.06 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2700, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$$

$$\downarrow 2700$$

$$\int \left( \frac{e^{d+ex}(-ac+b^2+bcx)}{a^2(a+bx+cx^2)} - \frac{be^{d+ex}}{a^2x} + \frac{e^{d+ex}}{ax^2} \right) dx$$

$$\downarrow 2009$$

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) e^{d-\frac{e(b-\sqrt{b^2-4ac})}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+2cx-\sqrt{b^2-4ac})}{2c}\right)}{2a^2} +$$

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) e^{d-\frac{e(\sqrt{b^2-4ac}+b)}{2c}} \text{ExpIntegralEi}\left(\frac{e(b+2cx+\sqrt{b^2-4ac})}{2c}\right)}{2a^2} - \frac{be^d \text{ExpIntegralEi}(ex)}{a^2} +$$

$$\frac{2a^2}{e^d e \text{ExpIntegralEi}(ex)} - \frac{e^{d+ex}}{ax}$$

input `Int[E^(d + e*x)/(x^2*(a + b*x + c*x^2)),x]`

output `-(E^(d + e*x)/(a*x)) - (b*E^d*ExpIntegralEi[e*x])/a^2 + (e*E^d*ExpIntegralEi[e*x])/a + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b - Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b - Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c] ])/(2*a^2) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*E^(d - ((b + Sqrt[b^2 - 4*a*c])*e)/(2*c))*ExpIntegralEi[(e*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(2*c] ])/(2*a^2)`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2700 `Int[((F_)^((g_)*((d_) + (e_)*(x_))^(n_))* (u_)^(m_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[F^(g*(d + e*x)^n), u^m/(a + b*x + c*x^2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x] && PolynomialQ[u, x] && IntegerQ[m]`

### Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs.  $2(183) = 366$ .

Time = 0.08 (sec) , antiderivative size = 561, normalized size of antiderivative = 2.65

method	result
derivativedivides	$e \left( -\frac{e^{ex+d}}{ax} - \frac{-2e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) ace + e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}}}{a\sqrt{-4ace^2+b^2e^2}} \right)$
default	$e \left( -\frac{e^{ex+d}}{ax} - \frac{-2e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) ace + e^{-\frac{be+2cd+\sqrt{-4ace^2+b^2e^2}}{2c}}}{a\sqrt{-4ace^2+b^2e^2}} \right)$
risch	$-\frac{e^{ex+d}}{ax} + \frac{e e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}} \operatorname{ExpIntegralE}_1\left(\frac{-be+2cd-2c(ex+d)+\sqrt{-4ace^2+b^2e^2}}{2c}\right) c}{a\sqrt{-4ace^2+b^2e^2}} - \frac{e e^{-\frac{be-2cd-\sqrt{-4ace^2+b^2e^2}}{2c}}}{a\sqrt{-4ace^2+b^2e^2}}$

```
input int(exp(e*x+d)/x^2/(c*x^2+b*x+a), x, method=_RETURNVERBOSE)
```

```
output e*(-exp(e*x+d)/a/e/x-1/2*(-2*exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a*c*e+exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b^2*e+2*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*a*c*e-exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b^2*e+(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(1/2/c*(-b*e+2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))))*Ei(1,1/2*(-b*e+2*c*d-2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b+(-4*a*c*e^2+b^2*e^2)^(1/2)*exp(-1/2*(b*e-2*c*d+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*Ei(1,-1/2*(b*e-2*c*d+2*c*(e*x+d)+(-4*a*c*e^2+b^2*e^2)^(1/2))/c)*b)/a^2/e/(-4*a*c*e^2+b^2*e^2)^(1/2)-1/a^2/e*(a*e-b)*exp(d)*Ei(1,-e*x)
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.48

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx$$

$$= \frac{2((ab^2 - 4a^2c)e^2 - (b^3 - 4abc)e)x\operatorname{Ei}(ex)e^d - 2(ab^2 - 4a^2c)ee^{(ex+d)} + ((b^3 - 4abc)ex + (b^2c - 2ac^2))}{a^2e}$$

input `integrate(exp(e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="fricas")`

output `1/2*(2*((a*b^2 - 4*a^2*c)*e^2 - (b^3 - 4*a*b*c)*e)*x*Ei(e*x)*e^d - 2*(a*b^2 - 4*a^2*c)*e*e^(e*x + d) + ((b^3 - 4*a*b*c)*e*x + (b^2*c - 2*a*c^2)*sqrt((b^2 - 4*a*c)*e^2/c^2)*x)*Ei(1/2*(2*c*e*x + b*e - c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c) + ((b^3 - 4*a*b*c)*e*x - (b^2*c - 2*a*c^2)*sqrt((b^2 - 4*a*c)*e^2/c^2)*x)*Ei(1/2*(2*c*e*x + b*e + c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c)*e^(1/2*(2*c*d - b*e - c*sqrt((b^2 - 4*a*c)*e^2/c^2))/c))/((a^2*b^2 - 4*a^3*c)*e*x)`

### Sympy [F]

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx = e^d \int \frac{e^{ex}}{ax^2+bx^3+cx^4} dx$$

input `integrate(exp(e*x+d)/x**2/(c*x**2+b*x+a),x)`

output `exp(d)*Integral(exp(e*x)/(a*x**2 + b*x**3 + c*x**4), x)`

### Maxima [F]

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx = \int \frac{e^{(ex+d)}}{(cx^2+bx+a)x^2} dx$$

input `integrate(exp(e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="maxima")`

output `integrate(e^(e*x + d)/((c*x^2 + b*x + a)*x^2), x)`

**Giac [F]**

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx = \int \frac{e^{(ex+d)}}{(cx^2+bx+a)x^2} dx$$

input `integrate(exp(e*x+d)/x^2/(c*x^2+b*x+a),x, algorithm="giac")`

output `integrate(e^(e*x + d)/((c*x^2 + b*x + a)*x^2), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx = \int \frac{e^{d+ex}}{x^2(cx^2+bx+a)} dx$$

input `int(exp(d + e*x)/(x^2*(a + b*x + c*x^2)),x)`

output `int(exp(d + e*x)/(x^2*(a + b*x + c*x^2)), x)`

**Reduce [F]**

$$\int \frac{e^{d+ex}}{x^2(a+bx+cx^2)} dx = e^d \left( \int \frac{e^{ex}}{cx^4+bx^3+ax^2} dx \right)$$

input `int(exp(e*x+d)/x^2/(c*x^2+b*x+a),x)`

output `e**d*int(e**(e*x)/(a*x**2 + b*x**3 + c*x**4),x)`

### 3.38 $\int F^{c(a+bx)}(d+ex)^4 dx$

Optimal result	295
Mathematica [A] (verified)	295
Rubi [A] (verified)	296
Maple [A] (verified)	298
Fricas [A] (verification not implemented)	298
Sympy [B] (verification not implemented)	299
Maxima [B] (verification not implemented)	300
Giac [C] (verification not implemented)	300
Mupad [B] (verification not implemented)	301
Reduce [B] (verification not implemented)	302

#### Optimal result

Integrand size = 17, antiderivative size = 141

$$\int F^{c(a+bx)}(d+ex)^4 dx = \frac{24e^4 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{24e^3 F^{c(a+bx)}(d+ex)}{b^4 c^4 \log^4(F)} + \frac{12e^2 F^{c(a+bx)}(d+ex)^2}{b^3 c^3 \log^3(F)} - \frac{4e F^{c(a+bx)}(d+ex)^3}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)}$$

output

$24*e^4*F^{(c*(b*x+a))}/b^5/c^5/\ln(F)^5-24*e^3*F^{(c*(b*x+a))*(e*x+d)}/b^4/c^4/\ln(F)^4+12*e^2*F^{(c*(b*x+a))*(e*x+d)^2}/b^3/c^3/\ln(F)^3-4*e*F^{(c*(b*x+a))*(e*x+d)^3}/b^2/c^2/\ln(F)^2+F^{(c*(b*x+a))*(e*x+d)^4}/b/c/\ln(F)$

#### Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d+ex)^4 dx = \frac{F^{c(a+bx)}(24e^4 - 24bce^3(d+ex)\log(F) + 12b^2c^2e^2(d+ex)^2\log^2(F) - 4b^3c^3e(d+ex)^3\log^3(F) + b^4c^4(d+ex)^4)}{b^5c^5\log^5(F)}$$

input

`Integrate[F^(c*(a + b*x))*(d + e*x)^4,x]`



output

$$\frac{(F^{c(a+bx)})*(24e^4 - 24*b*c*e^3*(d+e*x)*\text{Log}[F] + 12*b^2*c^2*e^2*(d+e*x)^2*\text{Log}[F]^2 - 4*b^3*c^3*e*(d+e*x)^3*\text{Log}[F]^3 + b^4*c^4*(d+e*x)^4*\text{Log}[F]^4)}{(b^5*c^5*\text{Log}[F]^5)}$$

**Rubi [A] (verified)**

Time = 0.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2607, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^4 F^{c(a+bx)} dx$$

$$\downarrow 2607$$

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \int F^{c(a+bx)} (d+ex)^3 dx}{bc \log(F)}$$

$$\downarrow 2607$$

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} (d+ex)^2 dx}{bc \log(F)} \right)}{bc \log(F)}$$

$$\downarrow 2607$$

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \int F^{c(a+bx)} (d+ex) dx}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

$$\downarrow 2607$$

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

$$\downarrow 2624$$

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e F^{c(a+bx)}}{b^2 c^2 \log^2(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x)^4,x]`

output 
$$\frac{(F^{c(a+bx)}(d+ex)^4)/(bc \log(F)) - (4e((F^{c(a+bx)}(d+ex)^3)/(bc \log(F)) - (3e((F^{c(a+bx)}(d+ex)^2)/(bc \log(F)) - (2e(-((eF^{c(a+bx)})/(b^2 c^2 \log^2(F))) + (F^{c(a+bx)}(d+ex))/(bc \log(F)))))/(bc \log(F)))))/(bc \log(F)))/(bc \log(F)))/(bc \log(F))}{bc \log(F)}$$

### Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((bF^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(bF^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

### Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.84

method	result
gospers	$(e^4 x^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 e^4 x^2 - 12 \ln(F)^3 b^3 c^3 e^4 x - 12 \ln(F)^3 b^3 c^3 e^4) e^{c(bx+a)}$
risch	$(e^4 x^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 e^4 x^2 - 12 \ln(F)^3 b^3 c^3 e^4 x - 12 \ln(F)^3 b^3 c^3 e^4) e^{c(bx+a)}$
orering	$(e^4 x^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 e^4 x^2 - 12 \ln(F)^3 b^3 c^3 e^4 x - 12 \ln(F)^3 b^3 c^3 e^4) e^{c(bx+a)}$
norman	$\frac{(\ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 d^3 e + 12 \ln(F)^2 b^2 c^2 d^2 e^2 - 24 d e^3 \ln(F) b c + 24 e^4) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{\ln(F) b c} + \frac{4 e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2}$
meijerg	$- \frac{F^{ac} e^4 \left( 24 - \frac{(5 b^4 c^4 x^4 \ln(F)^4 - 20 b^3 c^3 x^3 \ln(F)^3 + 60 b^2 c^2 x^2 \ln(F)^2 - 120 b c x \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5} + \frac{4 F^{ac} e^3 d \left( 6 - \frac{(-4 b^3 c^3 x^3 \ln(F)^3 + 12 b^2 c^2 x^2 \ln(F)^2 - 12 b c x \ln(F) + 12) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5}$
parallelrisch	$x^4 F^{c(bx+a)} e^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 x^3 F^{c(bx+a)} b^4 c^4 d e^3 + 6 \ln(F)^4 x^2 F^{c(bx+a)} b^4 c^4 d^2 e^2 + 4 \ln(F)^4 x F^{c(bx+a)} b^4 c^4 d^3 e + \ln(F)^4 F^{c(bx+a)} b^4 c^4 d^4$

```
input int(F^(c*(b*x+a))*(e*x+d)^4,x,method=_RETURNVERBOSE)
```

```
output (e^4*x^4*ln(F)^4*b^4*c^4+4*ln(F)^4*b^4*c^4*d*e^3*x^3+6*ln(F)^4*b^4*c^4*d^2
*e^2*x^2+4*ln(F)^4*b^4*c^4*d^3*e*x+ln(F)^4*b^4*c^4*d^4-4*ln(F)^3*b^3*c^3*e
^4*x^3-12*ln(F)^3*b^3*c^3*d*e^3*x^2-12*ln(F)^3*b^3*c^3*d^2*e^2*x-4*ln(F)^3
*b^3*c^3*d^3*e+12*ln(F)^2*b^2*c^2*e^4*x^2+24*ln(F)^2*b^2*c^2*d*e^3*x+12*ln
(F)^2*b^2*c^2*d^2*e^2-24*ln(F)*b*c*e^4*x-24*d*e^3*ln(F)*b*c+24*e^4)*F^(c*(
b*x+a))/ln(F)^5/b^5/c^5
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.61

$$\int F^{c(a+bx)}(d+ex)^4 dx = \frac{((b^4 c^4 e^4 x^4 + 4 b^4 c^4 d e^3 x^3 + 6 b^4 c^4 d^2 e^2 x^2 + 4 b^4 c^4 d^3 e x + b^4 c^4 d^4) \log(F)^4 + 24 e^4 - 4 (b^3 c^3 e^4 x^3 + 3 b^3 c^3 d e^3 x^2 + 3 b^3 c^3 d^2 e^2 x + b^3 c^3 d^3 e) \log(F)^3 + 12 (b^2 c^2 e^4 x^2 + 2 b^2 c^2 d e^3 x + b^2 c^2 d^2 e^2) \log(F)^2 - 24 b c e^4 x - 24 d e^3 \log(F) + 24 e^4) F^{c(a+bx)}}{c^5}$$

```
input integrate(F^((b*x+a)*c)*(e*x+d)^4,x, algorithm="fricas")
```

output

$$\left( (b^4 c^4 e^{4x^4} + 4b^4 c^4 d e^{3x^3} + 6b^4 c^4 d^2 e^{2x^2} + 4b^4 c^4 d^3 e^x + b^4 c^4 d^4) \log(F)^4 + 24e^4 - 4(b^3 c^3 e^{4x^3} + 3b^3 c^3 d e^{3x^2} + 3b^3 c^3 d^2 e^{2x} + b^3 c^3 d^3 e) \log(F)^3 + 12(b^2 c^2 e^{4x^2} + 2b^2 c^2 d e^{3x} + b^2 c^2 d^2 e^2) \log(F)^2 - 24(b c e^{4x} + b c d e^3) \log(F) \right) F^{(b c x + a c)} / (b^5 c^5 \log(F)^5)$$

### Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs.  $2(139) = 278$ .

Time = 0.12 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.48

$$\int F^{c(a+bx)} (d+ex)^4 dx$$

$$= \left\{ \frac{F^{c(a+bx)} (b^4 c^4 d^4 \log(F)^4 + 4b^4 c^4 d^3 e x \log(F)^4 + 6b^4 c^4 d^2 e^2 x^2 \log(F)^4 + 4b^4 c^4 d e^3 x^3 \log(F)^4 + b^4 c^4 e^4 x^4 \log(F)^4 - 4b^3 c^3 d^3 e \log(F)^3 - 12b^3 c^3 d^2 e^2 \log(F)^2 - 24b^2 c^2 d e^3 \log(F) - 24b c e^4) F^{c(a+bx)}}{d^4 x + 2d^3 e x^2 + 2d^2 e^2 x^3 + d e^3 x^4 + \frac{e^4 x^5}{5}} \right.$$

input

```
integrate(F**((b*x+a)*c)*(e*x+d)**4,x)
```

output

```
Piecewise((F**(c*(a + b*x))*(b**4*c**4*d**4*log(F)**4 + 4*b**4*c**4*d**3*e*x*log(F)**4 + 6*b**4*c**4*d**2*e**2*x**2*log(F)**4 + 4*b**4*c**4*d*e**3*x**3*log(F)**4 + b**4*c**4*e**4*x**4*log(F)**4 - 4*b**3*c**3*d**3*e*log(F)**3 - 12*b**3*c**3*d**2*e**2*x*log(F)**3 - 12*b**3*c**3*d*e**3*x**2*log(F)**3 - 4*b**3*c**3*e**4*x**3*log(F)**3 + 12*b**2*c**2*d**2*e**2*log(F)**2 + 24*b**2*c**2*d*e**3*x*log(F)**2 + 12*b**2*c**2*e**4*x**2*log(F)**2 - 24*b*c*d*e**3*log(F) - 24*b*c*e**4*x*log(F) + 24*e**4)/(b**5*c**5*log(F)**5), Ne(b**5*c**5*log(F)**5, 0)), (d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d*e**3*x**4 + e**4*x**5/5, True))
```

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 309 vs.  $2(141) = 282$ .

Time = 0.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.19

$$\int F^{c(a+bx)}(d+ex)^4 dx = \frac{F^{bcx+ac}d^4}{bc \log(F)} + \frac{4(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d^3e}{b^2c^2 \log(F)^2} + \frac{6(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}d^2e^2}{b^3c^3 \log(F)^3} + \frac{4(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}de^3}{b^4c^4 \log(F)^4} + \frac{(F^{ac}b^4c^4x^4 \log(F)^4 - 4F^{ac}b^3c^3x^3 \log(F)^3 + 12F^{ac}b^2c^2x^2 \log(F)^2 - 24F^{ac}bcx \log(F) + 24F^{ac})F^{bcx}}{b^5c^5 \log(F)^5}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^4,x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d^4/(b*c*log(F)) + 4*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d^3*e/(b^2*c^2*log(F)^2) + 6*(F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*d^2*e^2/(b^3*c^3*log(F)^3) + 4*(F^(a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))*F^(b*c*x)*d*e^3/(b^4*c^4*log(F)^4) + (F^(a*c)*b^4*c^4*x^4*log(F)^4 - 4*F^(a*c)*b^3*c^3*x^3*log(F)^3 + 12*F^(a*c)*b^2*c^2*x^2*log(F)^2 - 24*F^(a*c)*b*c*x*log(F) + 24*F^(a*c))*F^(b*c*x)*e^4/(b^5*c^5*log(F)^5)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 8802, normalized size of antiderivative = 62.43

$$\int F^{c(a+bx)}(d+ex)^4 dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^4,x, algorithm="giac")`

output

```

-((4*(pi^3*b^4*c^4*e^4*x^4*log(abs(F))*sgn(F) - pi*b^4*c^4*e^4*x^4*log(abs
(F))^3*sgn(F) - pi^3*b^4*c^4*e^4*x^4*log(abs(F)) + pi*b^4*c^4*e^4*x^4*log(
abs(F))^3 + 4*pi^3*b^4*c^4*d*e^3*x^3*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*d*e
^3*x^3*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*d*e^3*x^3*log(abs(F)) + 4*pi*
b^4*c^4*d*e^3*x^3*log(abs(F))^3 + 6*pi^3*b^4*c^4*d^2*e^2*x^2*log(abs(F))*s
gn(F) - 6*pi*b^4*c^4*d^2*e^2*x^2*log(abs(F))^3*sgn(F) - 6*pi^3*b^4*c^4*d^2
*e^2*x^2*log(abs(F)) + 6*pi*b^4*c^4*d^2*e^2*x^2*log(abs(F))^3 + 4*pi^3*b^4
*c^4*d^3*e*x*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*d^3*e*x*log(abs(F))^3*sgn(F
) - 4*pi^3*b^4*c^4*d^3*e*x*log(abs(F)) + 4*pi*b^4*c^4*d^3*e*x*log(abs(F))^
3 - pi^3*b^3*c^3*e^4*x^3*sgn(F) + pi^3*b^4*c^4*d^4*log(abs(F))*sgn(F) + 3*
pi*b^3*c^3*e^4*x^3*log(abs(F))^2*sgn(F) - pi*b^4*c^4*d^4*log(abs(F))^3*sgn
(F) + pi^3*b^3*c^3*e^4*x^3 - pi^3*b^4*c^4*d^4*log(abs(F)) - 3*pi*b^3*c^3*e
^4*x^3*log(abs(F))^2 + pi*b^4*c^4*d^4*log(abs(F))^3 - 3*pi^3*b^3*c^3*d*e^3
*x^2*sgn(F) + 9*pi*b^3*c^3*d*e^3*x^2*log(abs(F))^2*sgn(F) + 3*pi^3*b^3*c^3
*d*e^3*x^2 - 9*pi*b^3*c^3*d*e^3*x^2*log(abs(F))^2 - 3*pi^3*b^3*c^3*d^2*e^2
*x*sgn(F) + 9*pi*b^3*c^3*d^2*e^2*x*log(abs(F))^2*sgn(F) + 3*pi^3*b^3*c^3*d
^2*e^2*x - 9*pi*b^3*c^3*d^2*e^2*x*log(abs(F))^2 - pi^3*b^3*c^3*d^3*e*sgn(F
) + 3*pi*b^3*c^3*d^3*e*log(abs(F))^2*sgn(F) + pi^3*b^3*c^3*d^3*e - 3*pi*b^
3*c^3*d^3*e*log(abs(F))^2 - 6*pi*b^2*c^2*e^4*x^2*log(abs(F))*sgn(F) + 6*pi
*b^2*c^2*e^4*x^2*log(abs(F)) - 12*pi*b^2*c^2*d*e^3*x*log(abs(F))*sgn(F)...

```

### Mupad [B] (verification not implemented)

Time = 23.74 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.84

$$\int F^{c(a+bx)}(d+ex)^4 dx$$

$$= \frac{F^{a+bcx} (b^4 c^4 d^4 \ln(F)^4 + 4b^4 c^4 d^3 e x \ln(F)^4 + 6b^4 c^4 d^2 e^2 x^2 \ln(F)^4 + 4b^4 c^4 d e^3 x^3 \ln(F)^4 + b^4 c^4 e^4 x^4 \ln(F)^4 + b^4 c^4 e^4 x^4)}{b^5 c^5 \log(F)^5}$$

input

```
int(F^(c*(a + b*x))*(d + e*x)^4,x)
```

output

```

(F^(a*c + b*c*x)*(24*e^4 + b^4*c^4*d^4*log(F)^4 - 24*b*c*e^4*x*log(F) - 4*
b^3*c^3*d^3*e*log(F)^3 + 12*b^2*c^2*d^2*e^2*log(F)^2 + 12*b^2*c^2*e^4*x^2*
log(F)^2 - 4*b^3*c^3*e^4*x^3*log(F)^3 + b^4*c^4*e^4*x^4*log(F)^4 - 24*b*c*
d*e^3*log(F) + 6*b^4*c^4*d^2*e^2*x^2*log(F)^4 + 24*b^2*c^2*d*e^3*x*log(F)^
2 + 4*b^4*c^4*d^3*e*x*log(F)^4 - 12*b^3*c^3*d^2*e^2*x*log(F)^3 - 12*b^3*c^
3*d*e^3*x^2*log(F)^3 + 4*b^4*c^4*d*e^3*x^3*log(F)^4))/(b^5*c^5*log(F)^5)

```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.84

$$\int F^{c(a+bx)}(d+ex)^4 dx$$

$$= \frac{f^{bcx+ac}(\log(f)^4 b^4 c^4 d^4 + 4\log(f)^4 b^4 c^4 d^3 ex + 6\log(f)^4 b^4 c^4 d^2 e^2 x^2 + 4\log(f)^4 b^4 c^4 d e^3 x^3 + \log(f)^4 b^4 c^4 e^4 x^4)}{(\log(f)^5 b^5 c^5)}$$

input `int(F^((b*x+a)*c)*(e*x+d)^4,x)`output `(f**(a*c + b*c*x)*(log(f)**4*b**4*c**4*d**4 + 4*log(f)**4*b**4*c**4*d**3*e*x + 6*log(f)**4*b**4*c**4*d**2*e**2*x**2 + 4*log(f)**4*b**4*c**4*d*e**3*x**3 + log(f)**4*b**4*c**4*e**4*x**4 - 4*log(f)**3*b**3*c**3*d**3*e - 12*log(f)**3*b**3*c**3*d**2*e**2*x - 12*log(f)**3*b**3*c**3*d*e**3*x**2 - 4*log(f)**3*b**3*c**3*e**4*x**3 + 12*log(f)**2*b**2*c**2*d**2*e**2 + 24*log(f)**2*b**2*c**2*d*e**3*x + 12*log(f)**2*b**2*c**2*e**4*x**2 - 24*log(f)*b*c*d*e**3 - 24*log(f)*b*c*e**4*x + 24*e**4))/(log(f)**5*b**5*c**5)`

### 3.39 $\int F^{c(a+bx)}(d+ex)^3 dx$

Optimal result . . . . .	303
Mathematica [A] (verified) . . . . .	303
Rubi [A] (verified) . . . . .	304
Maple [A] (verified) . . . . .	305
Fricas [A] (verification not implemented) . . . . .	306
Sympy [B] (verification not implemented) . . . . .	306
Maxima [A] (verification not implemented) . . . . .	307
Giac [C] (verification not implemented) . . . . .	308
Mupad [B] (verification not implemented) . . . . .	309
Reduce [B] (verification not implemented) . . . . .	309

#### Optimal result

Integrand size = 17, antiderivative size = 110

$$\int F^{c(a+bx)}(d+ex)^3 dx = -\frac{6e^3 F^{c(a+bx)}}{b^4 c^4 \log^4(F)} + \frac{6e^2 F^{c(a+bx)}(d+ex)}{b^3 c^3 \log^3(F)} - \frac{3e F^{c(a+bx)}(d+ex)^2}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc \log(F)}$$

output

```
-6*e^3*F^(c*(b*x+a))/b^4/c^4/ln(F)^4+6*e^2*F^(c*(b*x+a))*(e*x+d)/b^3/c^3/ln(F)^3-3*e*F^(c*(b*x+a))*(e*x+d)^2/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)^3/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d+ex)^3 dx = \frac{F^{c(a+bx)}(-6e^3 + 6bce^2(d+ex)\log(F) - 3b^2c^2e(d+ex)^2\log^2(F) + b^3c^3(d+ex)^3\log^3(F))}{b^4c^4\log^4(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^3,x]
```



output

$$\frac{(F^{c(a+bx)})^3(-6e^3 + 6b^2c^2e^2(d+ex)\text{Log}[F] - 3b^2c^2e^2(d+ex)^2\text{Log}[F]^2 + b^3c^3(d+ex)^3\text{Log}[F]^3)}{(b^4c^4\text{Log}[F]^4)}$$

**Rubi [A] (verified)**

Time = 0.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (d+ex)^3 F^{c(a+bx)} dx \\ & \quad \downarrow 2607 \\ & \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} (d+ex)^2 dx}{bc \log(F)} \\ & \quad \downarrow 2607 \\ & \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \int F^{c(a+bx)} (d+ex) dx}{bc \log(F)} \right)}{bc \log(F)} \\ & \quad \downarrow 2607 \\ & \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \\ & \quad \downarrow 2624 \\ & \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e F^{c(a+bx)}}{b^2 c^2 \log^2(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \end{aligned}$$

input

$$\text{Int}[F^{c(a+bx)}(d+ex)^3, x]$$

output

```
(F^(c*(a + b*x))*(d + e*x)^3)/(b*c*Log[F]) - (3*e*((F^(c*(a + b*x))*(d + e
*x)^2)/(b*c*Log[F]) - (2*e*(-((e*F^(c*(a + b*x))))/(b^2*c^2*Log[F]^2)) + (F
^(c*(a + b*x))*(d + e*x))/(b*c*Log[F]))) / (b*c*Log[F])
```

**Defintions of rubi rules used**

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

rule 2624

```
Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

method	result
gosp	$\frac{(e^3 x^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 b^3 c^3 d e^2 x^2 + 3 \ln(F)^3 b^3 c^3 d^2 e x + \ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 e^3 x^2 - 6 \ln(F)^2 b^2 c^2 d e^2 x - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
risch	$\frac{(e^3 x^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 b^3 c^3 d e^2 x^2 + 3 \ln(F)^3 b^3 c^3 d^2 e x + \ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 e^3 x^2 - 6 \ln(F)^2 b^2 c^2 d e^2 x - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
oring	$\frac{(e^3 x^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 b^3 c^3 d e^2 x^2 + 3 \ln(F)^3 b^3 c^3 d^2 e x + \ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 e^3 x^2 - 6 \ln(F)^2 b^2 c^2 d e^2 x - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
norman	$\frac{(\ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4} + \frac{e^3 x^3 e^{c(bx+a) \ln(F)}}{\ln(F) b c} + \frac{3e(\ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c d e^2 + e^3)}{\ln(F)^3}$
meijerg	$\frac{F^{ac} e^3 \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^2 x^2 \ln(F)^2 - 24bcx \ln(F) + 24) e^{bcx \ln(F)}}{4} \right)}{\ln(F)^4 b^4 c^4} - \frac{3F^{ac} e^2 d \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{b^3 c^3 \ln(F)^3}$
parallelrisch	$\frac{x^3 F^{c(bx+a)} e^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 x^2 F^{c(bx+a)} b^3 c^3 d e^2 + 3 \ln(F)^3 x F^{c(bx+a)} b^3 c^3 d^2 e + \ln(F)^3 F^{c(bx+a)} b^3 c^3 d^3 - 3 \ln(F)^2 x^2 F^{c(bx+a)} e^3}{\ln(F)^4 b^4 c^4}$

input

```
int(F^(c*(b*x+a))*(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output

```
(e^3*x^3*ln(F)^3*b^3*c^3+3*ln(F)^3*b^3*c^3*d*e^2*x^2+3*ln(F)^3*b^3*c^3*d^2
*e*x+ln(F)^3*b^3*c^3*d^3-3*ln(F)^2*b^2*c^2*e^3*x^2-6*ln(F)^2*b^2*c^2*d*e^2
*x-3*ln(F)^2*b^2*c^2*d^2*e+6*ln(F)*b*c*e^3*x+6*ln(F)*b*c*d*e^2-6*e^3)*F^(c
*(b*x+a))/ln(F)^4/b^4/c^4
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

$$\int F^{c(a+bx)}(d+ex)^3 dx$$

$$= \frac{((b^3c^3e^3x^3 + 3b^3c^3de^2x^2 + 3b^3c^3d^2ex + b^3c^3d^3) \log(F)^3 - 6e^3 - 3(b^2c^2e^3x^2 + 2b^2c^2de^2x + b^2c^2d^2e) \log(F)^2 + 6(b*c*e^3*x + b*c*d*e^2) \log(F)) * F^{(b*c*x + a*c)}}{b^4c^4 \log(F)^4}$$

input

```
integrate(F^((b*x+a)*c)*(e*x+d)^3,x, algorithm="fricas")
```

output

```
((b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)
*log(F)^3 - 6*e^3 - 3*(b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)
)*log(F)^2 + 6*(b*c*e^3*x + b*c*d*e^2)*log(F))*F^(b*c*x + a*c)/(b^4*c^4*lo
g(F)^4)
```

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(107) = 214.

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.10

$$\int F^{c(a+bx)}(d+ex)^3 dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^3c^3d^3 \log(F)^3 + 3b^3c^3d^2ex \log(F)^3 + 3b^3c^3de^2x^2 \log(F)^3 + b^3c^3e^3x^3 \log(F)^3 - 3b^2c^2d^2e \log(F)^2 - 6b^2c^2de^2x \log(F)^2 - 3b^2c^2e^3x^2 \log(F)^2 + 6(b*c*e^3*x + b*c*d*e^2) \log(F)) * F^{(b*c*x + a*c)}}{b^4c^4 \log(F)^4} \\ d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4} \end{cases}$$

input

```
integrate(F**((b*x+a)*c)*(e*x+d)**3,x)
```

output

```
Piecewise((F**(c*(a + b*x))*(b**3*c**3*d**3*log(F)**3 + 3*b**3*c**3*d**2*e
*x*log(F)**3 + 3*b**3*c**3*d*e**2*x**2*log(F)**3 + b**3*c**3*e**3*x**3*log
(F)**3 - 3*b**2*c**2*d**2*e*log(F)**2 - 6*b**2*c**2*d*e**2*x*log(F)**2 - 3
*b**2*c**2*e**3*x**2*log(F)**2 + 6*b*c*d*e**2*log(F) + 6*b*c*e**3*x*log(F)
- 6*e**3)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d**3*x + 3
*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4, True))
```

### Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int F^{c(a+bx)}(d+ex)^3 dx$$

$$= \frac{F^{bcx+ac}d^3}{bc \log(F)} + \frac{3(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d^2e}{b^2c^2 \log(F)^2}$$

$$+ \frac{3(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}de^2}{b^3c^3 \log(F)^3}$$

$$+ \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}e^3}{b^4c^4 \log(F)^4}$$

input

```
integrate(F^((b*x+a)*c)*(e*x+d)^3,x, algorithm="maxima")
```

output

```
F^(b*c*x + a*c)*d^3/(b*c*log(F)) + 3*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b
*c*x)*d^2*e/(b^2*c^2*log(F)^2) + 3*(F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*
c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*d*e^2/(b^3*c^3*log(F)^3) + (F^(a*c)
*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log
(F) - 6*F^(a*c))*F^(b*c*x)*e^3/(b^4*c^4*log(F)^4)
```

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 4706, normalized size of antiderivative = 42.78

$$\int F^{c(a+bx)}(d+ex)^3 dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^3,x, algorithm="giac")`

output

```
-(((3*pi^2*b^3*c^3*e^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*e^3*x^3*log
(abs(F)) + 2*b^3*c^3*e^3*x^3*log(abs(F))^3 + 9*pi^2*b^3*c^3*d*e^2*x^2*log(
abs(F))*sgn(F) - 9*pi^2*b^3*c^3*d*e^2*x^2*log(abs(F)) + 6*b^3*c^3*d*e^2*x^
2*log(abs(F))^3 + 9*pi^2*b^3*c^3*d^2*e*x*log(abs(F))*sgn(F) - 9*pi^2*b^3*c
^3*d^2*e*x*log(abs(F)) + 6*b^3*c^3*d^2*e*x*log(abs(F))^3 + 3*pi^2*b^3*c^3*
d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*d^3*log(abs(F)) + 2*b^3*c^3*d^3*lo
g(abs(F))^3 - 3*pi^2*b^2*c^2*e^3*x^2*sgn(F) + 3*pi^2*b^2*c^2*e^3*x^2 - 6*b
^2*c^2*e^3*x^2*log(abs(F))^2 - 6*pi^2*b^2*c^2*d*e^2*x*sgn(F) + 6*pi^2*b^2*
c^2*d*e^2*x - 12*b^2*c^2*d*e^2*x*log(abs(F))^2 - 3*pi^2*b^2*c^2*d^2*e*sgn(
F) + 3*pi^2*b^2*c^2*d^2*e - 6*b^2*c^2*d^2*e*log(abs(F))^2 + 12*b*c*e^3*x*1
og(abs(F)) + 12*b*c*d*e^2*log(abs(F)) - 12*e^3)*(pi^4*b^4*c^4*sgn(F) - 6*p
i^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F
))^2 - 2*b^4*c^4*log(abs(F))^4)/((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log
(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^
4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(
abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2)
- 4*(pi^3*b^3*c^3*e^3*x^3*sgn(F) - 3*pi*b^3*c^3*e^3*x^3*log(abs(F))^2*sgn
(F) - pi^3*b^3*c^3*e^3*x^3 + 3*pi*b^3*c^3*e^3*x^3*log(abs(F))^2 + 3*pi^3*b
^3*c^3*d*e^2*x^2*sgn(F) - 9*pi*b^3*c^3*d*e^2*x^2*log(abs(F))^2*sgn(F) - 3*
pi^3*b^3*c^3*d*e^2*x^2 + 9*pi*b^3*c^3*d*e^2*x^2*log(abs(F))^2 + 3*pi^3*...
```



### 3.40 $\int F^{c(a+bx)}(d+ex)^2 dx$

Optimal result	310
Mathematica [A] (verified)	310
Rubi [A] (verified)	311
Maple [A] (verified)	312
Fricas [A] (verification not implemented)	313
Sympy [A] (verification not implemented)	313
Maxima [A] (verification not implemented)	314
Giac [C] (verification not implemented)	314
Mupad [B] (verification not implemented)	315
Reduce [B] (verification not implemented)	316

#### Optimal result

Integrand size = 17, antiderivative size = 79

$$\int F^{c(a+bx)}(d+ex)^2 dx = \frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e F^{c(a+bx)}(d+ex)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^2}{bc \log(F)}$$

output

```
2*e^2*F^(c*(b*x+a))/b^3/c^3/ln(F)^3-2*e*F^(c*(b*x+a))*(e*x+d)/b^2/c^2/ln(F)
)^2+F^(c*(b*x+a))*(e*x+d)^2/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d+ex)^2 dx = \frac{F^{c(a+bx)}(2e^2 - 2bce(d+ex)\log(F) + b^2c^2(d+ex)^2\log^2(F))}{b^3c^3\log^3(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^2,x]
```

output

```
(F^(c*(a + b*x))*(2*e^2 - 2*b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(b^3*c^3*Log[F]^3)
```

**Rubi [A] (verified)**

Time = 0.44 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d + ex)^2 F^{c(a+bx)} dx \\
 & \quad \downarrow 2607 \\
 & \frac{(d + ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \int F^{c(a+bx)} (d + ex) dx}{bc \log(F)} \\
 & \quad \downarrow 2607 \\
 & \frac{(d + ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \right)}{bc \log(F)} \\
 & \quad \downarrow 2624 \\
 & \frac{(d + ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{e F^{c(a+bx)}}{b^2 c^2 \log^2(F)} \right)}{bc \log(F)}
 \end{aligned}$$

input `Int [F^(c*(a + b*x))*(d + e*x)^2,x]`

output `(F^(c*(a + b*x))*(d + e*x)^2)/(b*c*Log[F]) - (2*e*(-((e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2)) + (F^(c*(a + b*x))*(d + e*x))/(b*c*Log[F]))/(b*c*Log[F])`



Defintions of rubi rules used

```
rule 2607 Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

```
rule 2624 Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

method	result
gospers	$\frac{(\ln(F)^2 b^2 c^2 e^2 x^2 + 2 \ln(F)^2 b^2 c^2 d e x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e^2 x - 2 \ln(F) b c e d + 2 e^2) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
risch	$\frac{(\ln(F)^2 b^2 c^2 e^2 x^2 + 2 \ln(F)^2 b^2 c^2 d e x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e^2 x - 2 \ln(F) b c e d + 2 e^2) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
orering	$\frac{(\ln(F)^2 b^2 c^2 e^2 x^2 + 2 \ln(F)^2 b^2 c^2 d e x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e^2 x - 2 \ln(F) b c e d + 2 e^2) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
norman	$\frac{(\ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e d + 2 e^2) e^{c(bx+a) \ln(F)}}{c^3 b^3 \ln(F)^3} + \frac{e^2 x^2 e^{c(bx+a) \ln(F)}}{\ln(F) b c} + \frac{2 e (b c d \ln(F) - e) x e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2}$
meijerg	$- \frac{F^{a c} e^2 \left( 2 - \frac{(3 b^2 c^2 x^2 \ln(F)^2 - 6 b c x \ln(F) + 6) e^{b c x \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} + \frac{2 F^{a c} e d \left( 1 - \frac{(-2 b c x \ln(F) + 2) e^{b c x \ln(F)}}{2} \right)}{\ln(F)^2 b^2 c^2} - \frac{F^{a c} d^2 (1 - e^{b c x \ln(F)})}{\ln(F) b c}$
parallelsch	$\frac{\ln(F)^2 x^2 F^{c(bx+a)} b^2 c^2 e^2 + 2 \ln(F)^2 x F^{c(bx+a)} b^2 c^2 d e + \ln(F)^2 F^{c(bx+a)} b^2 c^2 d^2 - 2 \ln(F) x F^{c(bx+a)} b c e^2 - 2 F^{c(bx+a)} \ln(F) b c d}{\ln(F)^3 b^3 c^3}$

```
input int(F^(c*(b*x+a))*(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
output (ln(F)^2*b^2*c^2*e^2*x^2+2*ln(F)^2*b^2*c^2*d*e*x+ln(F)^2*b^2*c^2*d^2-2*ln(F)*
b*c*e^2*x-2*ln(F)*b*c*e*d+2*e^2)*F^(c*(b*x+a))/ln(F)^3/b^3/c^3
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)}(d+ex)^2 dx$$

$$= \frac{((b^2c^2e^2x^2 + 2b^2c^2dex + b^2c^2d^2) \log(F)^2 + 2e^2 - 2(bce^2x + bcde) \log(F)) F^{bcx+ac}}{b^3c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^2,x, algorithm="fricas")`

output `((b^2*c^2*e^2*x^2 + 2*b^2*c^2*d*e*x + b^2*c^2*d^2)*log(F)^2 + 2*e^2 - 2*(b*c*e^2*x + b*c*d*e)*log(F))*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3)`

**Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

$$\int F^{c(a+bx)}(d+ex)^2 dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^2c^2d^2 \log(F)^2 + 2b^2c^2dex \log(F)^2 + b^2c^2e^2x^2 \log(F)^2 - 2bcde \log(F) - 2bce^2x \log(F) + 2e^2)}{b^3c^3 \log(F)^3} & \text{for } b^3c^3 \log(F)^3 \neq 0 \\ d^2x + dex^2 + \frac{e^2x^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(e*x+d)**2,x)`

output `Piecewise((F**(c*(a + b*x))*(b**2*c**2*d**2*log(F)**2 + 2*b**2*c**2*d*e*x*log(F)**2 + b**2*c**2*e**2*x**2*log(F)**2 - 2*b*c*d*e*log(F) - 2*b*c*e**2*x*log(F) + 2*e**2)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d**2*x + d*e*x**2 + e**2*x**3/3, True))`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int F^{c(a+bx)}(d+ex)^2 dx = \frac{F^{bcx+ac}d^2}{bc \log(F)} + \frac{2(F^{ac}bcx \log(F) - F^{ac})F^{bcx}de}{b^2c^2 \log(F)^2} + \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}e^2}{b^3c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^2,x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d^2/(b*c*log(F)) + 2*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*e^2/(b^3*c^3*log(F)^3)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 2214, normalized size of antiderivative = 28.03

$$\int F^{c(a+bx)}(d+ex)^2 dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^2,x, algorithm="giac")`

output

```

-((2*(pi*b^2*c^2*e^2*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*e^2*x^2*log(abs(F)
)) + 2*pi*b^2*c^2*d*e*x*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*d*e*x*log(abs(F)
) + pi*b^2*c^2*d^2*log(abs(F))*sgn(F) - pi*b^2*c^2*d^2*log(abs(F)) - pi*b*
c*e^2*x*sgn(F) + pi*b*c*e^2*x - pi*b*c*d*e*sgn(F) + pi*b*c*d*e)*(pi^3*b^3*
c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c
^3*log(abs(F))^2)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F)
) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs
(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2 - (
pi^2*b^2*c^2*e^2*x^2*sgn(F) - pi^2*b^2*c^2*e^2*x^2 + 2*b^2*c^2*e^2*x^2*log
(abs(F))^2 + 2*pi^2*b^2*c^2*d*e*x*sgn(F) - 2*pi^2*b^2*c^2*d*e*x + 4*b^2*c^
2*d*e*x*log(abs(F))^2 + pi^2*b^2*c^2*d^2*sgn(F) - pi^2*b^2*c^2*d^2 + 2*b^2
*c^2*d^2*log(abs(F))^2 - 4*b*c*e^2*x*log(abs(F)) - 4*b*c*d*e*log(abs(F)) +
4*e^2)*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) +
2*b^3*c^3*log(abs(F))^3)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^
2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*
log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)
^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*
a*c) - ((pi^2*b^2*c^2*e^2*x^2*sgn(F) - pi^2*b^2*c^2*e^2*x^2 + 2*b^2*c^2*e^
2*x^2*log(abs(F))^2 + 2*pi^2*b^2*c^2*d*e*x*sgn(F) - 2*pi^2*b^2*c^2*d*e*x +
4*b^2*c^2*d*e*x*log(abs(F))^2 + pi^2*b^2*c^2*d^2*sgn(F) - pi^2*b^2*c^2...

```

### Mupad [B] (verification not implemented)

Time = 22.81 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(d+ex)^2 dx = \frac{F^{ac+bcx} (b^2 c^2 d^2 \ln(F)^2 + 2b^2 c^2 d e x \ln(F)^2 + b^2 c^2 e^2 x^2 \ln(F)^2 - 2bcde \ln(F) - 2bce^2 x \ln(F) + 2bcde^2 x^2)}{b^3 c^3 \ln(F)^3}$$

input

```
int(F^(c*(a + b*x))*(d + e*x)^2,x)
```

output

```

(F^(a*c + b*c*x)*(2*e^2 + b^2*c^2*d^2*log(F)^2 - 2*b*c*e^2*x*log(F) + b^2*
c^2*e^2*x^2*log(F)^2 - 2*b*c*d*e*log(F) + 2*b^2*c^2*d*e*x*log(F)^2))/(b^3*
c^3*log(F)^3)

```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(d+ex)^2 dx$$

$$= \frac{f^{bcx+ac}(\log(f)^2 b^2 c^2 d^2 + 2\log(f)^2 b^2 c^2 dex + \log(f)^2 b^2 c^2 e^2 x^2 - 2\log(f) bcde - 2\log(f) bc e^2 x + 2e^2)}{\log(f)^3 b^3 c^3}$$

input `int(F^((b*x+a)*c)*(e*x+d)^2,x)`output `(f**(a*c + b*c*x)*(log(f)**2*b**2*c**2*d**2 + 2*log(f)**2*b**2*c**2*d*e*x + log(f)**2*b**2*c**2*e**2*x**2 - 2*log(f)*b*c*d*e - 2*log(f)*b*c*e**2*x + 2*e**2))/(log(f)**3*b**3*c**3)`

### 3.41 $\int F^{c(a+bx)}(d+ex) dx$

Optimal result	317
Mathematica [A] (verified)	317
Rubi [A] (verified)	318
Maple [A] (verified)	319
Fricas [A] (verification not implemented)	319
Sympy [A] (verification not implemented)	320
Maxima [A] (verification not implemented)	320
Giac [C] (verification not implemented)	320
Mupad [B] (verification not implemented)	321
Reduce [B] (verification not implemented)	322

#### Optimal result

Integrand size = 15, antiderivative size = 48

$$\int F^{c(a+bx)}(d+ex) dx = -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$$

output

```
-e*F^(c*(b*x+a))/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{c(a+bx)}(-e+bc(d+ex)\log(F))}{b^2c^2 \log^2(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x), x]
```

output

```
(F^(c*(a + b*x))*(-e + b*c*(d + e*x)*Log[F]))/(b^2*c^2*Log[F]^2)
```

**Rubi [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)F^{c(a+bx)} dx$$

$$\downarrow 2607$$

$$\frac{(d + ex)F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)}$$

$$\downarrow 2624$$

$$\frac{(d + ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x),x]`

output `-((e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2)) + (F^(c*(a + b*x))*(d + e*x))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
gospers	$\frac{(ex \ln(F)bc + bcd \ln(F) - e)F^{c(bx+a)}}{\ln(F)^2 b^2 c^2}$	38
risch	$\frac{(ex \ln(F)bc + bcd \ln(F) - e)F^{c(bx+a)}}{\ln(F)^2 b^2 c^2}$	38
orering	$\frac{(ex \ln(F)bc + bcd \ln(F) - e)F^{c(bx+a)}}{\ln(F)^2 b^2 c^2}$	38
norman	$\frac{(bcd \ln(F) - e)e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} + \frac{ex e^{c(bx+a) \ln(F)}}{\ln(F)bc}$	56
parallelrisch	$\frac{e F^{c(bx+a)} x \ln(F) bc + \ln(F) F^{c(bx+a)} bcd - F^{c(bx+a)} e}{\ln(F)^2 b^2 c^2}$	56
meijerg	$\frac{F^{ac} e \left( 1 - \frac{(-2bcx \ln(F) + 2)e^{bcx \ln(F)}}{2} \right)}{\ln(F)^2 b^2 c^2} - \frac{F^{ac} d (1 - e^{bcx \ln(F)})}{bc \ln(F)}$	68

input `int(F^(c*(b*x+a))*(e*x+d),x,method=_RETURNVERBOSE)`

output `(e*x*ln(F)*b*c+b*c*d*ln(F)-e)*F^(c*(b*x+a))/ln(F)^2/b^2/c^2`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}(d+ex) dx = \frac{((bcex + bcd) \log(F) - e)F^{bcx+ac}}{b^2 c^2 \log(F)^2}$$

input `integrate(F^((b*x+a)*c)*(e*x+d),x, algorithm="fricas")`

output `((b*c*e*x + b*c*d)*log(F) - e)*F^(b*c*x + a*c)/(b^2*c^2*log(F)^2)`



**Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int F^{c(a+bx)}(d+ex) dx = \begin{cases} \frac{F^{c(a+bx)}(bcd \log(F) + bce x \log(F) - e)}{b^2 c^2 \log(F)^2} & \text{for } b^2 c^2 \log(F)^2 \neq 0 \\ dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(e*x+d), x)`

output `Piecewise((F**(c*(a + b*x))*(b*c*d*log(F) + b*c*e*x*log(F) - e)/(b**2*c**2*log(F)**2), Ne(b**2*c**2*log(F)**2, 0)), (d*x + e*x**2/2, True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2 c^2 \log(F)^2}$$

input `integrate(F^((b*x+a)*c)*(e*x+d), x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*e/(b^2*c^2*log(F)^2)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 898, normalized size of antiderivative = 18.71

$$\int F^{c(a+bx)}(d+ex) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d), x, algorithm="giac")`

output

```
(2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*e*x*sgn(F) - pi*b*c*e*x + pi*b*c*d*sgn(F) - pi*b*c*d)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2*(b*c*e*x*log(abs(F)) + b*c*d*log(abs(F)) - e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2*(pi*b*c*e*x*sgn(F) - pi*b*c*e*x + pi*b*c*d*sgn(F) - pi*b*c*d)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*e*x*log(abs(F)) + b*c*d*log(abs(F)) - e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*((pi*b*c*e*x*sgn(F) - pi*b*c*e*x - 2*I*b*c*e*x*log(abs(F)) + pi*b*c*d*sgn(F) - pi*b*c*d - 2*I*b*c*d*log(abs(F)) + 2*I*e))*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2...
```

**Mupad [B] (verification not implemented)**

Time = 22.77 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{a+bcx} (bcd \ln(F) - e + bce x \ln(F))}{b^2 c^2 \ln(F)^2}$$

input

```
int(F^(c*(a + b*x))*(d + e*x),x)
```

output

```
(F^(a*c + b*c*x)*(b*c*d*log(F) - e + b*c*e*x*log(F)))/(b^2*c^2*log(F)^2)
```

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}(d+ex) dx = \frac{f^{bcx+ac}(\log(f) bcd + \log(f) bce x - e)}{\log(f)^2 b^2 c^2}$$

input `int(F^((b*x+a)*c)*(e*x+d),x)`

output `(f**(a*c + b*c*x)*(log(f)*b*c*d + log(f)*b*c*e*x - e))/(log(f)**2*b**2*c**2)`

### 3.42 $\int F^{c(a+bx)} dx$

Optimal result	323
Mathematica [A] (verified)	323
Rubi [A] (verified)	324
Maple [A] (verified)	325
Fricas [A] (verification not implemented)	325
Sympy [A] (verification not implemented)	326
Maxima [A] (verification not implemented)	326
Giac [A] (verification not implemented)	326
Mupad [B] (verification not implemented)	327
Reduce [B] (verification not implemented)	327

#### Optimal result

Integrand size = 9, antiderivative size = 20

$$\int F^{c(a+bx)} dx = \frac{F^{c(a+bx)}}{bc \log(F)}$$

output

```
F^(c*(b*x+a))/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} dx = \frac{F^{ac+bcx}}{bc \log(F)}$$

input

```
Integrate[F^(c*(a + b*x)),x]
```

output

```
F^(a*c + b*c*x)/(b*c*Log[F])
```

**Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)} dx$$

$$\downarrow 2624$$

$$\frac{F^{c(a+bx)}}{bc \log(F)}$$

input `Int[F^(c*(a + b*x)), x]`

output `F^(c*(a + b*x))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
gospers	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
derivativedivides	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
default	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
risch	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
parallelrisch	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
orering	$\frac{F^{c(bx+a)}}{bc \ln(F)}$	21
norman	$\frac{e^{c(bx+a) \ln(F)}}{\ln(F)bc}$	22
meijerg	$-\frac{F^{ac} (1 - e^{bcx \ln(F)})}{\ln(F)bc}$	29

input `int(F^(c*(b*x+a)),x,method=_RETURNVERBOSE)`output `F^(c*(b*x+a))/b/c/ln(F)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} dx = \frac{F^{bcx+ac}}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c),x, algorithm="fricas")`output `F^(b*c*x + a*c)/(b*c*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} dx = \begin{cases} \frac{F^{c(a+bx)}}{bc \log(F)} & \text{for } bc \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c),x)`output `Piecewise((F**(c*(a + b*x))/(b*c*log(F)), Ne(b*c*log(F), 0)), (x, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)} dx = \frac{F^{(bx+a)c}}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c),x, algorithm="maxima")`output `F^((b*x + a)*c)/(b*c*log(F))`**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} dx = \frac{F^{bcx+ac}}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c),x, algorithm="giac")`output `F^(b*c*x + a*c)/(b*c*log(F))`

**Mupad [B] (verification not implemented)**

Time = 22.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} dx = \frac{F^{a+bcx}}{bc \ln(F)}$$

input `int(F^(c*(a + b*x)),x)`

output `F^(a*c + b*c*x)/(b*c*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} dx = \frac{f^{bcx+ac}}{\log(f) bc}$$

input `int(F^((b*x+a)*c),x)`

output `f**(a*c + b*c*x)/(log(f)*b*c)`



### 3.43 $\int \frac{F^{c(a+bx)}}{d+ex} dx$

Optimal result . . . . .	328
Mathematica [A] (verified) . . . . .	328
Rubi [A] (verified) . . . . .	329
Maple [A] (verified) . . . . .	329
Fricas [A] (verification not implemented) . . . . .	330
Sympy [F] . . . . .	330
Maxima [F] . . . . .	330
Giac [F] . . . . .	331
Mupad [F(-1)] . . . . .	331
Reduce [F] . . . . .	331

#### Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \frac{F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

output `F^(c*(a-b*d/e))*Ei(b*c*(e*x+d)*ln(F)/e)/e`

#### Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \frac{F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x), x]`

output `(F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e])/e`

**Rubi [A] (verified)**

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{d+ex} dx$$

↓ 2609

$$\frac{F^{c\left(a-\frac{bd}{e}\right)} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e}$$

input `Int[F^(c*(a + b*x))/(d + e*x),x]`

output `(F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e])/e`

**Defintions of rubi rules used**

rule 2609

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] :> Simp[
(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[
{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

method	result	size
risch	$\frac{F^{\frac{c(ea-bd)}{e}} \text{expIntegral}_1\left(-bcx \ln(F) - ac \ln(F) - \frac{-ea \ln(F)c + bcd \ln(F)}{e}\right)}{e}$	56

input `int(F^(c*(b*x+a))/(e*x+d),x,method=_RETURNVERBOSE)`

output  $-1/e * F^{(c*(a*e-b*d)/e)} * Ei(1, -b*c*x*\ln(F) - a*c*\ln(F) - (-e*a*\ln(F)*c + b*c*d*\ln(F))/e)$

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \frac{Ei\left(\frac{(bcex+bcd)\log(F)}{e}\right)}{F^{\frac{bcd-ace}{e}} e}$$

input `integrate(F^((b*x+a)*c)/(e*x+d),x, algorithm="fricas")`

output `Ei((b*c*e*x + b*c*d)*log(F)/e)/(F^((b*c*d - a*c*e)/e)*e)`

### Sympy [F]

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \int \frac{F^{c(a+bx)}}{d+ex} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d),x)`

output `Integral(F**(c*(a + b*x))/(d + e*x), x)`

### Maxima [F]

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \int \frac{F^{(bx+a)c}}{ex+d} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \int \frac{F^{(bx+a)c}}{ex+d} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = \int \frac{F^{c(a+bx)}}{d+ex} dx$$

input `int(F^(c*(a + b*x))/(d + e*x),x)`

output `int(F^(c*(a + b*x))/(d + e*x), x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d+ex} dx = f^{ac} \left( \int \frac{f^{bcx}}{ex+d} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d),x)`

output `f**(a*c)*int(f**(b*c*x)/(d + e*x),x)`

### 3.44 $\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$

Optimal result	332
Mathematica [A] (verified)	332
Rubi [A] (verified)	333
Maple [A] (verified)	334
Fricas [A] (verification not implemented)	334
Sympy [F]	335
Maxima [F]	335
Giac [F]	335
Mupad [F(-1)]	336
Reduce [F]	336

#### Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2}$$

output 
$$-F^{c(bx+a)}/e/(ex+d)+bcF^{c(a-bd/e)}*Ei(bc*(ex+d)*ln(F)/e)*ln(F)/e^2$$

#### Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = \frac{F^{ac} \left( -\frac{eF^{bcx}}{d+ex} + bcF^{-\frac{bcd}{e}} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F) \right)}{e^2}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x)^2,x]`

output 
$$(F^{a*c})*(-((eF^{b*c*x})/(d + e*x)) + (b*c*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/F^{(b*c*d)/e}))/e^2$$

**Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

↓ 2608

$$\frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

↓ 2609

$$\frac{bc \log(F) F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}$$

input `Int[F^(c*(a + b*x))/(d + e*x)^2,x]`

output `-(F^(c*(a + b*x))/(e*(d + e*x))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/e^2`

**Defintions of rubi rules used**

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_)), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2609

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

method	result	size
risch	$-\frac{\ln(F)bc F^{bcx} F^{ac}}{e^2 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)} - \frac{\ln(F)bc F^{\frac{c(ea-bd)}{e}} \expIntegral_1 \left( -bcx \ln(F) - ac \ln(F) - \frac{-ea \ln(F)c + bcd \ln(F)}{e} \right)}{e^2}$	99

input

```
int(F^(c*(b*x+a))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output

```
-ln(F)*b*c/e^2*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)-ln(F)*b*c/e^2*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*ln(F)-a*c*ln(F)-(-e*a*ln(F)*c+b*c*d*ln(F))/e)
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = -\frac{F^{bcx+ac} e^{-\frac{(bcx+bcd) \operatorname{Ei}\left(\frac{bcx+bcd}{e} \log(F)\right) \log(F)}{F^{\frac{bcd-ace}{e}}}}}{e^3 x + de^2}$$

input

```
integrate(F^((b*x+a)*c)/(e*x+d)^2,x, algorithm="fricas")
```

output

```
-(F^(b*c*x + a*c)*e - (b*c*e*x + b*c*d)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)/F^((b*c*d - a*c*e)/e))/(e^3*x + d*e^2)
```

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**2,x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**2, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^2,x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^2, x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^2} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^2,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^2, x)`



**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^2,x)`output `int(F^(c*(a + b*x))/(d + e*x)^2, x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^2} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^2x^2 + 2dex + d^2} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^2,x)`output `f**(a*c)*int(f**(b*c*x)/(d**2 + 2*d*e*x + e**2*x**2),x)`

### 3.45 $\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$

Optimal result	337
Mathematica [A] (verified)	337
Rubi [A] (verified)	338
Maple [A] (verified)	339
Fricas [A] (verification not implemented)	339
Sympy [F]	340
Maxima [F]	340
Giac [F]	341
Mupad [F(-1)]	341
Reduce [F]	341

#### Optimal result

Integrand size = 17, antiderivative size = 95

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)} + \frac{b^2c^2F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F)}{2e^3}$$

output

```
-1/2*F^(c*(b*x+a))/e/(e*x+d)^2-1/2*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)+1/2
*b^2*c^2*F^(c*(a-b*d/e))*Ei(b*c*(e*x+d)*ln(F)/e)*ln(F)^2/e^3
```

#### Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = \frac{F^{c(a-\frac{bd}{e})} \left( b^2c^2(d+ex)^2 \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F) - eF^{\frac{bc(d+ex)}{e}} (e+bc(d+ex)\log(F)) \right)}{2e^3(d+ex)^2}$$

input

```
Integrate[F^(c*(a + b*x))/(d + e*x)^3,x]
```

output

$$(F^{c(a - (b*d)/e)}) * (b^2 * c^2 * (d + e*x)^2 * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e] * \text{Log}[F]^2 - e * F^{c((b*c*(d + e*x))/e)} * (e + b*c*(d + e*x)*\text{Log}[F])) / (2 * e^3 * (d + e*x)^2)$$
**Rubi [A] (verified)**

Time = 0.47 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

↓ 2608

$$\frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

↓ 2608

$$\frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

↓ 2609

$$\frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a - \frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}$$

input

$$\text{Int}[F^{c(a + b*x)} / (d + e*x)^3, x]$$

output

$$-1/2 * F^{c(a + b*x)} / (e*(d + e*x)^2) + (b*c*\text{Log}[F] * (-F^{c(a + b*x)} / (e*(d + e*x))) + (b*c * F^{c(a - (b*d)/e)} * \text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e] * \text{Log}[F]) / e^2) / (2*e)$$

## Definitions of rubi rules used

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{\ln(F)^2 b^2 c^2 F^{bcx} F^{ac}}{2e^3 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^2} - \frac{\ln(F)^2 b^2 c^2 F^{bcx} F^{ac}}{2e^3 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)} - \frac{\ln(F)^2 b^2 c^2 F^{\frac{c(ea-bd)}{e}} \operatorname{expIntegral}_1 \left( -bcx \ln(F) - ac \ln(F) - \frac{ea}{e} \right)}{2e^3}$

input

```
int(F^(c*(b*x+a))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*ln(F)^2*b^2*c^2/e^3*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^2-1
/2*ln(F)^2*b^2*c^2/e^3*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)-1/2*ln
n(F)^2*b^2*c^2/e^3*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*ln(F)-a*c*ln(F)-(-e*a*ln(
F)*c+b*c*d*ln(F))/e)
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

$$= \frac{(b^2 c^2 e^2 x^2 + 2 b^2 c^2 d e x + b^2 c^2 d^2) \operatorname{Ei} \left( \frac{(bcex + bcd) \log(F)}{e} \right) \log(F)^2}{F^{\frac{bcd-ace}{e}}} - (e^2 + (bce^2 x + bcde) \log(F)) F^{bcx+ac}$$

$$2(e^5 x^2 + 2de^4 x + d^2 e^3)$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^3,x, algorithm="fricas")`

output `1/2*((b^2*c^2*e^2*x^2 + 2*b^2*c^2*d*e*x + b^2*c^2*d^2)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)^2/F^((b*c*d - a*c*e)/e) - (e^2 + (b*c*e^2*x + b*c*d*e)*log(F))*F^(b*c*x + a*c))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)`

## Sympy [F]

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**3,x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**3, x)`

## Maxima [F]

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^3} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^3,x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^3, x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^3} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^3,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^3, x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^3,x)`

output `int(F^(c*(a + b*x))/(d + e*x)^3, x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^3} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^3,x)`

output `f**(a*c)*int(f**(b*c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)`

### 3.46 $\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$

Optimal result	342
Mathematica [A] (verified)	342
Rubi [A] (verified)	343
Maple [A] (verified)	345
Fricas [A] (verification not implemented)	345
Sympy [F]	346
Maxima [F]	346
Giac [F]	346
Mupad [F(-1)]	347
Reduce [F]	347

#### Optimal result

Integrand size = 17, antiderivative size = 128

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)} + \frac{b^3c^3F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F)}{6e^4}$$

output

```
-1/3*F^(c*(b*x+a))/e/(e*x+d)^3-1/6*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^2-1/6*b^2*c^2*F^(c*(b*x+a))*ln(F)^2/e^3/(e*x+d)+1/6*b^3*c^3*F^(c*(a-b*d/e))*Ei(b*c*(e*x+d)*ln(F)/e)*ln(F)^3/e^4
```

#### Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = \frac{F^{ac} \left( b^3c^3F^{-\frac{bcd}{e}} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F) - \frac{eF^{bcx}(2e^2+bce(d+ex)\log(F)+b^2c^2(d+ex)^2\log^2(F))}{(d+ex)^3} \right)}{6e^4}$$

input

```
Integrate[F^(c*(a + b*x))/(d + e*x)^4,x]
```

output

$$\frac{(F^{(a*c)}*((b^3*c^3*\text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e]*\text{Log}[F]^3)/F^{(b*c*d)/e} - (e*F^{(b*c*x)}*(2*e^2 + b*c*e*(d + e*x)*\text{Log}[F] + b^2*c^2*(d + e*x)^2*\text{Log}[F]^2)))/(d + e*x)^3)/(6*e^4)}$$
**Rubi [A] (verified)**

Time = 0.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {2608, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

$$\downarrow 2608$$

$$\frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

$$\downarrow 2608$$

$$\frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

$$\downarrow 2608$$

$$\frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}$$

$$\downarrow 2609$$



$$bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a - \frac{bd}{e})} \operatorname{ExpIntegralEi} \left( \frac{bc(d+ex) \log(F)}{e} \right) - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{e^2} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)$$


---


$$\frac{3e F^{c(a+bx)}}{3e(d+ex)^3}$$

input `Int[F^(c*(a + b*x))/(d + e*x)^4,x]`

output `-1/3*F^(c*(a + b*x))/(e*(d + e*x)^3) + (b*c*Log[F]*(-1/2*F^(c*(a + b*x))/(e*(d + e*x)^2) + (b*c*Log[F]*(-(F^(c*(a + b*x))/(e*(d + e*x)))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e*Log[F]/e^2]))/(2*e)))/(3*e)`

### Defintions of rubi rules used

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2609 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{\ln(F)^3 b^3 c^3 F^{bcx} F^{ac}}{3e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^3} - \frac{\ln(F)^3 b^3 c^3 F^{bcx} F^{ac}}{6e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^2} - \frac{\ln(F)^3 b^3 c^3 F^{bcx} F^{ac}}{6e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)} - \frac{\ln(F)^3 b^3 c^3 F^{\frac{c(ea-bd)}{e}} \expIntegr$

input `int(F^(c*(b*x+a))/(e*x+d)^4,x,method=_RETURNVERBOSE)`

output

$$-1/3*\ln(F)^3*b^3*c^3/e^4*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)^3-1/6*\ln(F)^3*b^3*c^3/e^4*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)^2-1/6*\ln(F)^3*b^3*c^3/e^4*F^(b*c*x)*F^(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)-1/6*\ln(F)^3*b^3*c^3/e^4*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-e*a*\ln(F)*c+b*c*d*\ln(F))/e)$$
**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.63

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

$$= \frac{(b^3 c^3 e^3 x^3 + 3 b^3 c^3 d e^2 x^2 + 3 b^3 c^3 d^2 e x + b^3 c^3 d^3) \operatorname{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right) \log(F)^3}{F^{\frac{bcd-ace}{e}}} - (2e^3 + (b^2 c^2 e^3 x^2 + 2 b^2 c^2 d e^2 x + b^2 c^2 d^2 e) \log(F)) \log(F)$$

$$6(e^7 x^3 + 3 d e^6 x^2 + 3 d^2 e^5 x + d^3 e^4)$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^4,x, algorithm="fricas")`

output

$$1/6*((b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)^3/F^((b*c*d - a*c*e)/e) - (2*e^3 + (b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)*log(F)^2 + (b*c*e^3*x + b*c*d*e^2)*log(F))*F^(b*c*x + a*c))/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)$$

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**4,x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**4, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^4} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^4,x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^4, x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^4} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^4,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^4, x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^4,x)`output `int(F^(c*(a + b*x))/(d + e*x)^4, x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^4} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^4 x^4 + 4d e^3 x^3 + 6d^2 e^2 x^2 + 4d^3 e x + d^4} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^4,x)`output `f**(a*c)*int(f**(b*c*x)/(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4),x)`

### 3.47 $\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$

Optimal result	348
Mathematica [A] (verified)	349
Rubi [A] (verified)	349
Maple [A] (verified)	351
Fricas [A] (verification not implemented)	352
Sympy [F]	352
Maxima [F]	353
Giac [F]	353
Mupad [F(-1)]	353
Reduce [F]	354

#### Optimal result

Integrand size = 17, antiderivative size = 161

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx = -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2} - \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{b^4c^4F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^4(F)}{24e^5}$$

output

```
-1/4*F^(c*(b*x+a))/e/(e*x+d)^4-1/12*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^3-1/24*b^2*c^2*F^(c*(b*x+a))*ln(F)^2/e^3/(e*x+d)^2-1/24*b^3*c^3*F^(c*(b*x+a))*ln(F)^3/e^4/(e*x+d)+1/24*b^4*c^4*F^(c*(a-b*d/e))*Ei(b*c*(e*x+d)*ln(F)/e)*ln(F)^4/e^5
```

**Mathematica [A] (verified)**

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

$$= \frac{F^{ac} \left( b^4 c^4 F^{-\frac{bcd}{e}} \text{ExpIntegralEi} \left( \frac{bc(d+ex) \log(F)}{e} \right) \log^4(F) - \frac{eF^{bcx} (6e^3 + 2bce^2(d+ex) \log(F) + b^2 c^2 e(d+ex)^2 \log^2(F) + b^3 c^3 (d+ex)^3 \log^3(F))}{(d+ex)^4} \right)}{24e^5}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x)^5,x]`

output

```
(F^(a*c))*((b^4*c^4*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^4)/F^((b*c*d)/e) - (e*F^(b*c*x)*(6*e^3 + 2*b*c*e^2*(d + e*x)*Log[F] + b^2*c^2*e*(d + e*x)^2*Log[F]^2 + b^3*c^3*(d + e*x)^3*Log[F]^3))/(d + e*x)^4)/(24*e^5)
```

**Rubi [A] (verified)**

Time = 0.70 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {2608, 2608, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

$$\downarrow 2608$$

$$\frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

$$\downarrow 2608$$

$$\frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \right)}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}$$

$$\downarrow 2608$$

$$\begin{aligned}
 & \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \\
 & \frac{\hspace{10em}}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4} \\
 & \quad \downarrow \text{2608} \\
 & bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \\
 & \frac{\hspace{10em}}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4} \\
 & \quad \downarrow \text{2609} \\
 & bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex) \log(F)}{e}\right) - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \\
 & \frac{\hspace{10em}}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}
 \end{aligned}$$

input

```
Int [F^(c*(a + b*x))/(d + e*x)^5,x]
```

output

```
-1/4*F^(c*(a + b*x))/(e*(d + e*x)^4) + (b*c*Log[F]*(-1/3*F^(c*(a + b*x)))/(e*(d + e*x)^3) + (b*c*Log[F]*(-1/2*F^(c*(a + b*x)))/(e*(d + e*x)^2) + (b*c*Log[F]*(-F^(c*(a + b*x)))/(e*(d + e*x))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/e^2)/(2*e))/(3*e))/(4*e)
```

## Definitions of rubi rules used

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^(n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{4e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^4} - \frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{12e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^3} - \frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{24e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^2} - \frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{24e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)}$

input

```
int(F^(c*(b*x+a))/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output

```
-1/4*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^4-1
/12*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^3-1/
24*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^2-1/2
4*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)-1/24*l
n(F)^4*b^4*c^4/e^5*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*ln(F)-a*c*ln(F)-(-e*a*ln(
F)*c+b*c*d*ln(F))/e)
```



**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.86

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

$$= \frac{(b^4c^4e^4x^4 + 4b^4c^4de^3x^3 + 6b^4c^4d^2e^2x^2 + 4b^4c^4d^3ex + b^4c^4d^4) \operatorname{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right) \log(F)^4}{F^{\frac{bcd-ace}{e}}} - \frac{(6e^4 + (b^3c^3e^4x^3 + 3b^3c^3de^3x^2 + 3b^3c^3d^2e^2x + b^3c^3d^3e) \log(F)^3 + (b^2c^2e^4x^2 + 2b^2c^2d^2e^2) \log(F)^2 + 2(b^2c^2e^4x + b^2c^2de^3) \log(F)) F^{(b^2c^2e^4x^2 + 2b^2c^2d^2e^2) \log(F)^2 + 2(b^2c^2e^4x + b^2c^2de^3) \log(F)}}{24(e^9x^4 + 4de^8x^3 + 6d^2e^7x^2 + 4d^3e^6x + d^4e^5)}$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^5,x, algorithm="fricas")`

output `1/24*((b^4*c^4*e^4*x^4 + 4*b^4*c^4*d*e^3*x^3 + 6*b^4*c^4*d^2*e^2*x^2 + 4*b^4*c^4*d^3*e*x + b^4*c^4*d^4)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)^4/F^((b*c*d - a*c*e)/e) - (6*e^4 + (b^3*c^3*e^4*x^3 + 3*b^3*c^3*d*e^3*x^2 + 3*b^3*c^3*d^2*e^2*x + b^3*c^3*d^3*e)*log(F)^3 + (b^2*c^2*e^4*x^2 + 2*b^2*c^2*d^2*e^2)*log(F)^2 + 2*(b^2*c^2*e^4*x + b^2*c^2*d^2*e^2)*log(F))*F^((b^2*c^2*e^4*x^2 + 2*b^2*c^2*d^2*e^2) *log(F)^2 + 2*(b^2*c^2*e^4*x + b^2*c^2*d^2*e^2)*log(F))/ (e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**5,x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**5, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^5} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^5,x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^5, x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^5} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^5,x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^5, x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^5} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^5,x)`

output `int(F^(c*(a + b*x))/(d + e*x)^5, x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^5} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^5 x^5 + 5d e^4 x^4 + 10d^2 e^3 x^3 + 10d^3 e^2 x^2 + 5d^4 e x + d^5} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^5,x)`

output `f**(a*c)*int(f**(b*c*x)/(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5),x)`

### 3.48 $\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$

Optimal result	355
Mathematica [A] (verified)	355
Rubi [A] (verified)	356
Maple [A] (verified)	358
Fricas [A] (verification not implemented)	359
Sympy [B] (verification not implemented)	359
Maxima [B] (verification not implemented)	360
Giac [C] (verification not implemented)	361
Mupad [B] (verification not implemented)	362
Reduce [B] (verification not implemented)	363

#### Optimal result

Integrand size = 48, antiderivative size = 141

$$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \frac{24e^4 F^{c(a+bx)}}{b^5 c^5 \log^5(F)} - \frac{24e^3 F^{c(a+bx)}(d+ex)}{b^4 c^4 \log^4(F)} + \frac{12e^2 F^{c(a+bx)}(d+ex)^2}{b^3 c^3 \log^3(F)}$$

$$- \frac{4e F^{c(a+bx)}(d+ex)^3}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^4}{bc \log(F)}$$

output

```
24*e^4*F^(c*(b*x+a))/b^5/c^5/ln(F)^5-24*e^3*F^(c*(b*x+a))*(e*x+d)/b^4/c^4/ln(F)^4+12*e^2*F^(c*(b*x+a))*(e*x+d)^2/b^3/c^3/ln(F)^3-4*e*F^(c*(b*x+a))*(e*x+d)^3/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)^4/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \frac{F^{c(a+bx)}(24e^4 - 24bce^3(d+ex) \log(F) + 12b^2c^2e^2(d+ex)^2 \log^2(F) - 4b^3c^3e(d+ex)^3 \log^3(F) + b^4c^4(d+ex)^4)}{b^5c^5 \log^5(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4), x]
```

output

```
(F^(c*(a + b*x))*(24*e^4 - 24*b*c*e^3*(d + e*x)*Log[F] + 12*b^2*c^2*e^2*(d + e*x)^2*Log[F]^2 - 4*b^3*c^3*e*(d + e*x)^3*Log[F]^3 + b^4*c^4*(d + e*x)^4*Log[F]^4))/(b^5*c^5*Log[F]^5)
```

### Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2006, 2607, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) F^{c(a+bx)} dx \\
 & \quad \downarrow 2006 \\
 & \int (d + ex)^4 F^{c(a+bx)} dx \\
 & \quad \downarrow 2607 \\
 & \frac{(d + ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \int F^{c(a+bx)} (d + ex)^3 dx}{bc \log(F)} \\
 & \quad \downarrow 2607 \\
 & \frac{(d + ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} (d+ex)^2 dx}{bc \log(F)} \right)}{bc \log(F)} \\
 & \quad \downarrow 2607 \\
 & \frac{(d + ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \int F^{c(a+bx)} (d+ex) dx}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \\
 & \quad \downarrow 2607
 \end{aligned}$$

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

↓ 2624

$$\frac{(d+ex)^4 F^{c(a+bx)}}{bc \log(F)} - \frac{4e \left( \frac{(d+ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e F^{c(a+bx)}}{b^2 c^2 \log^2(F)} \right)}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4),x]`

output `(F^(c*(a + b*x))*(d + e*x)^4)/(b*c*Log[F]) - (4*e*((F^(c*(a + b*x))*(d + e*x)^3)/(b*c*Log[F]) - (3*e*((F^(c*(a + b*x))*(d + e*x)^2)/(b*c*Log[F]) - (2*e*(-((e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2)) + (F^(c*(a + b*x))*(d + e*x))/(b*c*Log[F])))/(b*c*Log[F])))/(b*c*Log[F]))/(b*c*Log[F]))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2006 `Int[(u_.)*(Px_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_.)*(v_)^Expon[Px, x]] /; FreeQ[a, x] && LinearQ[v, x]]`

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624

```
Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

### Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.84

method	result
gospers	$\frac{(e^4 x^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 - 12 \ln(F)^3 b^3 c^3 d^2 e^2 x - 12 \ln(F)^3 b^3 c^3 d^3 e}{\ln(F)^5 b^5 c^5}$
risch	$\frac{(e^4 x^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 - 12 \ln(F)^3 b^3 c^3 d^2 e^2 x - 12 \ln(F)^3 b^3 c^3 d^3 e}{\ln(F)^5 b^5 c^5}$
norman	$\frac{(\ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 d^3 e + 12 \ln(F)^2 b^2 c^2 d^2 e^2 - 24 d e^3 \ln(F) b c + 24 e^4) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5} + \frac{e^4 x^4 e^{c(bx+a) \ln(F)}}{\ln(F) b c} + \frac{4 e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2}$
meijerg	$- \frac{F^{ac} e^4 \left( 24 - \frac{(5 b^4 c^4 x^4 \ln(F)^4 - 20 b^3 c^3 x^3 \ln(F)^3 + 60 b^2 c^2 x^2 \ln(F)^2 - 120 b c x \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5} + \frac{4 F^{ac} e^3 d \left( 6 - \frac{(-4 b^3 c^3 x^3 \ln(F)^3 + 12 b^2 c^2 x^2 \ln(F)^2 - 12 b c x \ln(F) + 12) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5}$
orering	$\frac{(e^4 x^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 b^4 c^4 d e^3 x^3 + 6 \ln(F)^4 b^4 c^4 d^2 e^2 x^2 + 4 \ln(F)^4 b^4 c^4 d^3 e x + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 - 12 \ln(F)^3 b^3 c^3 d^2 e^2 x - 12 \ln(F)^3 b^3 c^3 d^3 e)}{\ln(F)^5 b^5 c^5}$
parallelrisch	$\frac{x^4 F^{c(bx+a)} e^4 \ln(F)^4 b^4 c^4 + 4 \ln(F)^4 x^3 F^{c(bx+a)} b^4 c^4 d e^3 + 6 \ln(F)^4 x^2 F^{c(bx+a)} b^4 c^4 d^2 e^2 + 4 \ln(F)^4 x F^{c(bx+a)} b^4 c^4 d^3 e + \ln(F)^4 b^4 c^4 d^4 - 4 \ln(F)^3 b^3 c^3 e^4 x^3 - 12 \ln(F)^3 b^3 c^3 d e^3 x^2 - 12 \ln(F)^3 b^3 c^3 d^2 e^2 x - 12 \ln(F)^3 b^3 c^3 d^3 e}{\ln(F)^5 b^5 c^5}$

input

```
int(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4), x, method=
_RETURNVERBOSE)
```

output

```
(e^4*x^4*ln(F)^4*b^4*c^4+4*ln(F)^4*b^4*c^4*d*e^3*x^3+6*ln(F)^4*b^4*c^4*d^2
*e^2*x^2+4*ln(F)^4*b^4*c^4*d^3*e*x+ln(F)^4*b^4*c^4*d^4-4*ln(F)^3*b^3*c^3*e
^4*x^3-12*ln(F)^3*b^3*c^3*d*e^3*x^2-12*ln(F)^3*b^3*c^3*d^2*e^2*x-4*ln(F)^3
*b^3*c^3*d^3*e+12*ln(F)^2*b^2*c^2*e^4*x^2+24*ln(F)^2*b^2*c^2*d*e^3*x+12*ln
(F)^2*b^2*c^2*d^2*e^2-24*ln(F)*b*c*e^4*x-24*d*e^3*ln(F)*b*c+24*e^4)*F^(c*(
b*x+a))/ln(F)^5/b^5/c^5
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.61

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \frac{((b^4c^4e^4x^4 + 4b^4c^4de^3x^3 + 6b^4c^4d^2e^2x^2 + 4b^4c^4d^3ex + b^4c^4d^4) \log(F)^4 + 24e^4 - 4(b^3c^3e^4x^3 + 3b^3c^3de^3x^2 + 3b^3c^3d^2e^2x + b^3c^3d^3e) \log(F)^3 + 12(b^2c^2e^4x^2 + 2b^2c^2d^2e^3x + b^2c^2d^2e^2) \log(F)^2 - 24(b^2c^2e^4x + b^2c^2d^2e^3) \log(F) + b^2c^2d^2e^2) F^{(b^2c^2x + a^2c^2)}}{(b^5c^5 \log(F)^5)}$$

input

```
integrate(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),
x, algorithm="fricas")
```

output

```
((b^4*c^4*e^4*x^4 + 4*b^4*c^4*d*e^3*x^3 + 6*b^4*c^4*d^2*e^2*x^2 + 4*b^4*c^4*d^3*e*x + b^4*c^4*d^4)*log(F)^4 + 24*e^4 - 4*(b^3*c^3*e^4*x^3 + 3*b^3*c^3*d*e^3*x^2 + 3*b^3*c^3*d^2*e^2*x + b^3*c^3*d^3*e)*log(F)^3 + 12*(b^2*c^2*e^4*x^2 + 2*b^2*c^2*d^2*e^3*x + b^2*c^2*d^2*e^2)*log(F)^2 - 24*(b*c*e^4*x + b*c*d^2*e^3)*log(F))*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5)
```

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(139) = 278.

Time = 0.11 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.48

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \left\{ \frac{F^{c(a+bx)} (b^4c^4d^4 \log(F)^4 + 4b^4c^4d^3ex \log(F)^4 + 6b^4c^4d^2e^2x^2 \log(F)^4 + 4b^4c^4de^3x^3 \log(F)^4 + b^4c^4e^4x^4 \log(F)^4 - 4b^3c^3d^3e \log(F)^3 - 12b^3c^3de^2 \log(F)^2 - 24b^2c^2d^2e^2 \log(F) + b^2c^2d^2e^2)}{d^4x + 2d^3ex^2 + 2d^2e^2x^3 + de^3x^4 + \frac{e^4x^5}{5}} \right.$$

input

```
integrate(F**((b*x+a)*c)*(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*
e*x+d**4), x)
```



output

```
Piecewise((F**(c*(a + b*x))*(b**4*c**4*d**4*log(F)**4 + 4*b**4*c**4*d**3*
*x*log(F)**4 + 6*b**4*c**4*d**2*e**2*x**2*log(F)**4 + 4*b**4*c**4*d*e**3*x
**3*log(F)**4 + b**4*c**4*e**4*x**4*log(F)**4 - 4*b**3*c**3*d**3*e*log(F)*
*3 - 12*b**3*c**3*d**2*e**2*x*log(F)**3 - 12*b**3*c**3*d*e**3*x**2*log(F)*
*3 - 4*b**3*c**3*e**4*x**3*log(F)**3 + 12*b**2*c**2*d**2*e**2*log(F)**2 +
24*b**2*c**2*d*e**3*x*log(F)**2 + 12*b**2*c**2*e**4*x**2*log(F)**2 - 24*b*
c*d*e**3*log(F) - 24*b*c*e**4*x*log(F) + 24*e**4)/(b**5*c**5*log(F)**5), N
e(b**5*c**5*log(F)**5, 0)), (d**4*x + 2*d**3*e*x**2 + 2*d**2*e**2*x**3 + d
*e**3*x**4 + e**4*x**5/5, True))
```

### Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs.  $2(141) = 282$ .

Time = 0.04 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.19

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \frac{F^{bcx+ac}d^4}{bc \log(F)} + \frac{4(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d^3e}{b^2c^2 \log(F)^2}$$

$$+ \frac{6(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}d^2e^2}{b^3c^3 \log(F)^3}$$

$$+ \frac{4(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}de^3}{b^4c^4 \log(F)^4}$$

$$+ \frac{(F^{ac}b^4c^4x^4 \log(F)^4 - 4F^{ac}b^3c^3x^3 \log(F)^3 + 12F^{ac}b^2c^2x^2 \log(F)^2 - 24F^{ac}bcx \log(F) + 24F^{ac})F^{bcx}}{b^5c^5 \log(F)^5}$$

input

```
integrate(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),
x, algorithm="maxima")
```

output

```
F^(b*c*x + a*c)*d^4/(b*c*log(F)) + 4*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b
*c*x)*d^3*e/(b^2*c^2*log(F)^2) + 6*(F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*
c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*d^2*e^2/(b^3*c^3*log(F)^3) + 4*(F^(
a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c
*x*log(F) - 6*F^(a*c))*F^(b*c*x)*d*e^3/(b^4*c^4*log(F)^4) + (F^(a*c)*b^4*c
^4*x^4*log(F)^4 - 4*F^(a*c)*b^3*c^3*x^3*log(F)^3 + 12*F^(a*c)*b^2*c^2*x^2*
log(F)^2 - 24*F^(a*c)*b*c*x*log(F) + 24*F^(a*c))*F^(b*c*x)*e^4/(b^5*c^5*lo
g(F)^5)
```

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 8802, normalized size of antiderivative = 62.43

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx = \text{Too large to display}$$

input

```
integrate(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),
x, algorithm="giac")
```

output

```

-((4*(pi^3*b^4*c^4*e^4*x^4*log(abs(F))*sgn(F) - pi*b^4*c^4*e^4*x^4*log(abs
(F))^3*sgn(F) - pi^3*b^4*c^4*d^4*x^4*log(abs(F)) + pi*b^4*c^4*e^4*x^4*log(
abs(F))^3 + 4*pi^3*b^4*c^4*d^3*e^3*x^3*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*d^3
e^3*x^3*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*d^3*e^3*x^3*log(abs(F)) + 4*pi*
b^4*c^4*d^3*e^3*x^3*log(abs(F))^3 + 6*pi^3*b^4*c^4*d^2*e^2*x^2*log(abs(F))*s
gn(F) - 6*pi*b^4*c^4*d^2*e^2*x^2*log(abs(F))^3*sgn(F) - 6*pi^3*b^4*c^4*d^2
e^2*x^2*log(abs(F)) + 6*pi*b^4*c^4*d^2*e^2*x^2*log(abs(F))^3 + 4*pi^3*b^4
*c^4*d^3*e*x*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*d^3*e*x*log(abs(F))^3*sgn(F)
) - 4*pi^3*b^4*c^4*d^3*e*x*log(abs(F)) + 4*pi*b^4*c^4*d^3*e*x*log(abs(F))^
3 - pi^3*b^3*c^3*e^4*x^3*sgn(F) + pi^3*b^4*c^4*d^4*log(abs(F))*sgn(F) + 3*
pi*b^3*c^3*e^4*x^3*log(abs(F))^2*sgn(F) - pi*b^4*c^4*d^4*log(abs(F))^3*sgn
(F) + pi^3*b^3*c^3*e^4*x^3 - pi^3*b^4*c^4*d^4*log(abs(F)) - 3*pi*b^3*c^3*e
^4*x^3*log(abs(F))^2 + pi*b^4*c^4*d^4*log(abs(F))^3 - 3*pi^3*b^3*c^3*d^3
e^3*x^2*sgn(F) + 9*pi*b^3*c^3*d^3*e^3*x^2*log(abs(F))^2*sgn(F) + 3*pi^3*b^3
c^3*d^3*e^3*x^2 - 9*pi*b^3*c^3*d^3*e^3*x^2*log(abs(F))^2 - 3*pi^3*b^3*c^3*d^2
e^2*x*sgn(F) + 9*pi*b^3*c^3*d^2*e^2*x*log(abs(F))^2*sgn(F) + 3*pi^3*b^3*c^3
d^2*e^2*x - 9*pi*b^3*c^3*d^2*e^2*x*log(abs(F))^2 - pi^3*b^3*c^3*d^3*e*sgn(F)
) + 3*pi*b^3*c^3*d^3*e*log(abs(F))^2*sgn(F) + pi^3*b^3*c^3*d^3*e - 3*pi*b^
3*c^3*d^3*e*log(abs(F))^2 - 6*pi*b^2*c^2*e^4*x^2*log(abs(F))*sgn(F) + 6*pi
*b^2*c^2*e^4*x^2*log(abs(F)) - 12*pi*b^2*c^2*d^3*e^3*x*log(abs(F))*sgn(F)...

```

### Mupad [B] (verification not implemented)

Time = 22.89 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.84

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \frac{F^{ac+bcx} (b^4 c^4 d^4 \ln(F)^4 + 4 b^4 c^4 d^3 e x \ln(F)^4 + 6 b^4 c^4 d^2 e^2 x^2 \ln(F)^4 + 4 b^4 c^4 d e^3 x^3 \ln(F)^4 + b^4 c^4 e^4 x^4 \ln(F)^4)}{b^4 c^4 d^4}$$

input

```

int(F^(c*(a + b*x))*(d^4 + e^4*x^4 + 4*d^3*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e
*x), x)

```

output

```
(F^(a*c + b*c*x)*(24*e^4 + b^4*c^4*d^4*log(F)^4 - 24*b*c*e^4*x*log(F) - 4*
b^3*c^3*d^3*e*log(F)^3 + 12*b^2*c^2*d^2*e^2*log(F)^2 + 12*b^2*c^2*e^4*x^2*
log(F)^2 - 4*b^3*c^3*e^4*x^3*log(F)^3 + b^4*c^4*e^4*x^4*log(F)^4 - 24*b*c*
d*e^3*log(F) + 6*b^4*c^4*d^2*e^2*x^2*log(F)^4 + 24*b^2*c^2*d*e^3*x*log(F)^
2 + 4*b^4*c^4*d^3*e*x*log(F)^4 - 12*b^3*c^3*d^2*e^2*x*log(F)^3 - 12*b^3*c^
3*d*e^3*x^2*log(F)^3 + 4*b^4*c^4*d*e^3*x^3*log(F)^4))/(b^5*c^5*log(F)^5)
```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.84

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4) dx$$

$$= \frac{f^{bcx+ac} (\log(f)^4 b^4 c^4 d^4 + 4\log(f)^4 b^4 c^4 d^3 ex + 6\log(f)^4 b^4 c^4 d^2 e^2 x^2 + 4\log(f)^4 b^4 c^4 d e^3 x^3 + \log(f)^4 b^4 c^4 e^4 x^4)}{b^5 c^5 \log(f)^5}$$

input

```
int(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x)
```

output

```
(f**(a*c + b*c*x)*(log(f)**4*b**4*c**4*d**4 + 4*log(f)**4*b**4*c**4*d**3*e
*x + 6*log(f)**4*b**4*c**4*d**2*e**2*x**2 + 4*log(f)**4*b**4*c**4*d*e**3*x
**3 + log(f)**4*b**4*c**4*e**4*x**4 - 4*log(f)**3*b**3*c**3*d**3*e - 12*lo
g(f)**3*b**3*c**3*d**2*e**2*x - 12*log(f)**3*b**3*c**3*d*e**3*x**2 - 4*log
(f)**3*b**3*c**3*e**4*x**3 + 12*log(f)**2*b**2*c**2*d**2*e**2 + 24*log(f)*
*2*b**2*c**2*d*e**3*x + 12*log(f)**2*b**2*c**2*e**4*x**2 - 24*log(f)*b*c*d
*e**3 - 24*log(f)*b*c*e**4*x + 24*e**4))/(log(f)**5*b**5*c**5)
```

### 3.49 $\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$

Optimal result . . . . .	364
Mathematica [A] (verified) . . . . .	364
Rubi [A] (verified) . . . . .	365
Maple [A] (verified) . . . . .	367
Fricas [A] (verification not implemented) . . . . .	367
Sympy [B] (verification not implemented) . . . . .	368
Maxima [A] (verification not implemented) . . . . .	369
Giac [C] (verification not implemented) . . . . .	369
Mupad [B] (verification not implemented) . . . . .	370
Reduce [B] (verification not implemented) . . . . .	371

#### Optimal result

Integrand size = 37, antiderivative size = 110

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$$

$$= -\frac{6e^3F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{6e^2F^{c(a+bx)}(d+ex)}{b^3c^3\log^3(F)} - \frac{3eF^{c(a+bx)}(d+ex)^2}{b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^3}{bc\log(F)}$$

output

```
-6*e^3*F^(c*(b*x+a))/b^4/c^4/ln(F)^4+6*e^2*F^(c*(b*x+a))*(e*x+d)/b^3/c^3/ln(F)^3-3*e*F^(c*(b*x+a))*(e*x+d)^2/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)^3/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$$

$$= \frac{F^{c(a+bx)}(-6e^3 + 6bce^2(d+ex)\log(F) - 3b^2c^2e(d+ex)^2\log^2(F) + b^3c^3(d+ex)^3\log^3(F))}{b^4c^4\log^4(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3),x]
```

output

$$(F^{c(a+bx)}) * (-6e^3 + 6b^2c^2e^2(d+ex) * \text{Log}[F] - 3b^2c^2e^2(d+ex)^2 * \text{Log}[F]^2 + b^3c^3(d+ex)^3 * \text{Log}[F]^3) / (b^4c^4 * \text{Log}[F]^4)$$

### Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$ , Rules used = {2006, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) F^{c(a+bx)} dx$$

↓ 2006

$$\int (d + ex)^3 F^{c(a+bx)} dx$$

↓ 2607

$$\frac{(d + ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} (d + ex)^2 dx}{bc \log(F)}$$

↓ 2607

$$\frac{(d + ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \int F^{c(a+bx)} (d+ex) dx}{bc \log(F)} \right)}{bc \log(F)}$$

↓ 2607

$$\frac{(d + ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

↓ 2624

$$\frac{(d + ex)^3 F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{(d+ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex) F^{c(a+bx)}}{bc \log(F)} - \frac{e F^{c(a+bx)}}{b^2 c^2 \log^2(F)} \right)}{bc \log(F)} \right)}{bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3),x]`

output `(F^(c*(a + b*x))*(d + e*x)^3)/(b*c*Log[F]) - (3*e*((F^(c*(a + b*x))*(d + e*x)^2)/(b*c*Log[F]) - (2*e*(-((e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2)) + (F^(c*(a + b*x))*(d + e*x))/(b*c*Log[F])))/(b*c*Log[F])))/(b*c*Log[F])`

### Defintions of rubi rules used

rule 2006 `Int[(u_)*(Px_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_)*(v_)^Expon[Px, x]] /; FreeQ[a, x] && LinearQ[v, x]]`

rule 2607 `Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

### Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

method	result
gospers	$\frac{(e^3 x^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 b^3 c^3 d e^2 x^2 + 3 \ln(F)^3 b^3 c^3 d^2 e x + \ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 e^3 x^2 - 6 \ln(F)^2 b^2 c^2 d e^2 x - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
risch	$\frac{(e^3 x^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 b^3 c^3 d e^2 x^2 + 3 \ln(F)^3 b^3 c^3 d^2 e x + \ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 e^3 x^2 - 6 \ln(F)^2 b^2 c^2 d e^2 x - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
norman	$\frac{(\ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4} + \frac{e^3 x^3 e^{c(bx+a) \ln(F)}}{\ln(F) b c} + \frac{3e(\ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c d e^2 + 6 \ln(F) b c d e - 6 e^3)}{\ln(F)^3 b^3 c^3}$
meijerg	$\frac{F^{ac} e^3 \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^2 x^2 \ln(F)^2 - 24bcx \ln(F) + 24) e^{bcx \ln(F)}}{4} \right)}{\ln(F)^4 b^4 c^4} - \frac{3F^{ac} e^2 d \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{b^3 c^3 \ln(F)^3}$
oring	$\frac{(e^3 x^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 b^3 c^3 d e^2 x^2 + 3 \ln(F)^3 b^3 c^3 d^2 e x + \ln(F)^3 b^3 c^3 d^3 - 3 \ln(F)^2 b^2 c^2 e^3 x^2 - 6 \ln(F)^2 b^2 c^2 d e^2 x - 3 \ln(F)^2 b^2 c^2 d^2 e + 6 \ln(F) b c d e^2 - 6 e^3) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4 (ex+d)^3}$
parallelrisc	$\frac{x^3 F^{c(bx+a)} e^3 \ln(F)^3 b^3 c^3 + 3 \ln(F)^3 x^2 F^{c(bx+a)} b^3 c^3 d e^2 + 3 \ln(F)^3 x F^{c(bx+a)} b^3 c^3 d^2 e + \ln(F)^3 F^{c(bx+a)} b^3 c^3 d^3 - 3 \ln(F)^2 x^2 F^{c(bx+a)} b^2 c^2 e^3 + 6 \ln(F)^2 x F^{c(bx+a)} b^2 c^2 d e^2 + 6 \ln(F)^2 F^{c(bx+a)} b^2 c^2 d^2 e - 6 \ln(F) x F^{c(bx+a)} b c d e^2 + 6 F^{c(bx+a)} b c d e - 6 e^3}{\ln(F)^4 b^4 c^4}$

```
input int(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x,method=_RETURNVERBOSE)
```

```
output (e^3*x^3*ln(F)^3*b^3*c^3+3*ln(F)^3*b^3*c^3*d*e^2*x^2+3*ln(F)^3*b^3*c^3*d^2*e*x+ln(F)^3*b^3*c^3*d^3-3*ln(F)^2*b^2*c^2*e^3*x^2-6*ln(F)^2*b^2*c^2*d*e^2*x-3*ln(F)^2*b^2*c^2*d^2*e+6*ln(F)*b*c*e^3*x+6*ln(F)*b*c*d*e^2-6*e^3)*F^(c*(b*x+a))/ln(F)^4/b^4/c^4
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

$$\int F^{c(a+bx)} (d^3 + 3d^2 e x + 3d e^2 x^2 + e^3 x^3) dx$$

$$= \frac{((b^3 c^3 e^3 x^3 + 3 b^3 c^3 d e^2 x^2 + 3 b^3 c^3 d^2 e x + b^3 c^3 d^3) \log(F)^3 - 6 e^3 - 3 (b^2 c^2 e^3 x^2 + 2 b^2 c^2 d e^2 x + b^2 c^2 d^2 e) \log(F)^2 + 6 b c d e^2 \log(F) - 6 e^3)}{b^4 c^4 \log(F)^4}$$

```
input integrate(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="fricas")
```



output

```
((b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)
*log(F)^3 - 6*e^3 - 3*(b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)
)*log(F)^2 + 6*(b*c*e^3*x + b*c*d*e^2)*log(F))*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4)
```

### Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs.  $2(107) = 214$ .

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.10

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$$

$$= \left\{ \frac{F^{c(a+bx)} (b^3c^3d^3 \log(F)^3 + 3b^3c^3d^2ex \log(F)^3 + 3b^3c^3de^2x^2 \log(F)^3 + b^3c^3e^3x^3 \log(F)^3 - 3b^2c^2d^2e \log(F)^2 - 6b^2c^2de^2x \log(F)^2 - 3b^2c^2e^3x^2 \log(F)^2 + 6b^2c^2d^2e \log(F)^2 + 6b^2c^2de^2x \log(F)^2 + 6b^2c^2e^3x^2 \log(F)^2 - 6e^3 \log(F)^2 + 6b^2c^2d^2e \log(F)^2 + 6b^2c^2de^2x \log(F)^2 + 6b^2c^2e^3x^2 \log(F)^2 - 6e^3 \log(F)^2)}{b^4c^4 \log(F)^4} \right.$$

$$\left. d^3x + \frac{3d^2ex^2}{2} + de^2x^3 + \frac{e^3x^4}{4} \right.$$

input

```
integrate(F**((b*x+a)*c)*(e**3*x**3+3*d*e**2*x**2+3*d**2*e*x+d**3),x)
```

output

```
Piecewise((F**(c*(a + b*x))*(b**3*c**3*d**3*log(F)**3 + 3*b**3*c**3*d**2*e
*x*log(F)**3 + 3*b**3*c**3*d*e**2*x**2*log(F)**3 + b**3*c**3*e**3*x**3*log
(F)**3 - 3*b**2*c**2*d**2*e*log(F)**2 - 6*b**2*c**2*d*e**2*x*log(F)**2 - 3
*b**2*c**2*e**3*x**2*log(F)**2 + 6*b*c*d*e**2*log(F) + 6*b*c*e**3*x*log(F)
- 6*e**3)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d**3*x + 3
*d**2*e*x**2/2 + d*e**2*x**3 + e**3*x**4/4, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$$

$$= \frac{F^{bcx+ac}d^3}{bc \log(F)} + \frac{3(F^{ac}bcx \log(F) - F^{ac})F^{bcx}d^2e}{b^2c^2 \log(F)^2}$$

$$+ \frac{3(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}de^2}{b^3c^3 \log(F)^3}$$

$$+ \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}e^3}{b^4c^4 \log(F)^4}$$

input `integrate(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d^3/(b*c*log(F)) + 3*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d^2*e/(b^2*c^2*log(F)^2) + 3*(F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*d*e^2/(b^3*c^3*log(F)^3) + (F^(a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))*F^(b*c*x)*e^3/(b^4*c^4*log(F)^4)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 4706, normalized size of antiderivative = 42.78

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="giac")`

output

```
-(((3*pi^2*b^3*c^3*e^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*e^3*x^3*log
(abs(F)) + 2*b^3*c^3*e^3*x^3*log(abs(F))^3 + 9*pi^2*b^3*c^3*d*e^2*x^2*log(
abs(F))*sgn(F) - 9*pi^2*b^3*c^3*d*e^2*x^2*log(abs(F)) + 6*b^3*c^3*d*e^2*x^
2*log(abs(F))^3 + 9*pi^2*b^3*c^3*d^2*e*x*log(abs(F))*sgn(F) - 9*pi^2*b^3*c
^3*d^2*e*x*log(abs(F)) + 6*b^3*c^3*d^2*e*x*log(abs(F))^3 + 3*pi^2*b^3*c^3*
d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*d^3*log(abs(F)) + 2*b^3*c^3*d^3*lo
g(abs(F))^3 - 3*pi^2*b^2*c^2*e^3*x^2*sgn(F) + 3*pi^2*b^2*c^2*e^3*x^2 - 6*b
^2*c^2*e^3*x^2*log(abs(F))^2 - 6*pi^2*b^2*c^2*d*e^2*x*sgn(F) + 6*pi^2*b^2*
c^2*d*e^2*x - 12*b^2*c^2*d*e^2*x*log(abs(F))^2 - 3*pi^2*b^2*c^2*d^2*e*sgn(
F) + 3*pi^2*b^2*c^2*d^2*e - 6*b^2*c^2*d^2*e*log(abs(F))^2 + 12*b*c*e^3*x*1
og(abs(F)) + 12*b*c*d*e^2*log(abs(F)) - 12*e^3)*(pi^4*b^4*c^4*sgn(F) - 6*p
i^2*b^4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F
))^2 - 2*b^4*c^4*log(abs(F))^4)/((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log
(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^
4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(
abs(F))^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2)
- 4*(pi^3*b^3*c^3*e^3*x^3*sgn(F) - 3*pi*b^3*c^3*e^3*x^3*log(abs(F))^2*sgn
(F) - pi^3*b^3*c^3*e^3*x^3 + 3*pi*b^3*c^3*e^3*x^3*log(abs(F))^2 + 3*pi^3*b
^3*c^3*d*e^2*x^2*sgn(F) - 9*pi*b^3*c^3*d*e^2*x^2*log(abs(F))^2*sgn(F) - 3*
pi^3*b^3*c^3*d*e^2*x^2 + 9*pi*b^3*c^3*d*e^2*x^2*log(abs(F))^2 + 3*pi^3*...
```

### Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$$

$$= \frac{F^{ac+bcx} (b^3 c^3 d^3 \ln(F)^3 + 3b^3 c^3 d^2 e x \ln(F)^3 + 3b^3 c^3 d e^2 x^2 \ln(F)^3 + b^3 c^3 e^3 x^3 \ln(F)^3 - 3b^2 c^2 d^2 e \ln(F)^4 + b^4 c^4 \ln(F)^5)}{b^4 c^4 \ln(F)^4}$$

input

```
int(F^(c*(a + b*x))*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x),x)
```

output

```
(F^(a*c + b*c*x)*(b^3*c^3*d^3*log(F)^3 - 6*e^3 + 6*b*c*e^3*x*log(F) - 3*b^
2*c^2*d^2*e*log(F)^2 - 3*b^2*c^2*e^3*x^2*log(F)^2 + b^3*c^3*e^3*x^3*log(F)
^3 + 6*b*c*d*e^2*log(F) - 6*b^2*c^2*d*e^2*x*log(F)^2 + 3*b^3*c^3*d^2*e*x*1
og(F)^3 + 3*b^3*c^3*d*e^2*x^2*log(F)^3))/(b^4*c^4*log(F)^4)
```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.50

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3) dx$$

$$= \frac{f^{bcx+ac} (\log(f)^3 b^3 c^3 d^3 + 3\log(f)^3 b^3 c^3 d^2 e x + 3\log(f)^3 b^3 c^3 d e^2 x^2 + \log(f)^3 b^3 c^3 e^3 x^3 - 3\log(f)^2 b^2 c^2 d^2 e - 6\log(f)^2 b^2 c^2 d e^2 x - 3\log(f)^2 b^2 c^2 e^3 x^2 + 6\log(f) b^2 c^2 d e^2 x + 6\log(f) b^2 c^2 e^3 x^2 - 6e^3 x^3)}{\log(f)^4 b^4 c^4}$$

input `int(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x)`

output `(f**(a*c + b*c*x)*(log(f)**3*b**3*c**3*d**3 + 3*log(f)**3*b**3*c**3*d**2*e*x + 3*log(f)**3*b**3*c**3*d*e**2*x**2 + log(f)**3*b**3*c**3*e**3*x**3 - 3*log(f)**2*b**2*c**2*d**2*e - 6*log(f)**2*b**2*c**2*d*e**2*x - 3*log(f)**2*b**2*c**2*e**3*x**2 + 6*log(f)*b*c*d*e**2 + 6*log(f)*b*c*e**3*x - 6*e**3)/(log(f)**4*b**4*c**4)`

### 3.50 $\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx$

Optimal result . . . . .	372
Mathematica [A] (verified) . . . . .	372
Rubi [A] (verified) . . . . .	373
Maple [A] (verified) . . . . .	374
Fricas [A] (verification not implemented) . . . . .	375
Sympy [A] (verification not implemented) . . . . .	375
Maxima [A] (verification not implemented) . . . . .	376
Giac [C] (verification not implemented) . . . . .	376
Mupad [B] (verification not implemented) . . . . .	377
Reduce [B] (verification not implemented) . . . . .	378

#### Optimal result

Integrand size = 26, antiderivative size = 79

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx = \frac{2e^2 F^{c(a+bx)}}{b^3 c^3 \log^3(F)} - \frac{2e F^{c(a+bx)}(d + ex)}{b^2 c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d + ex)^2}{bc \log(F)}$$

output

```
2*e^2*F^(c*(b*x+a))/b^3/c^3/ln(F)^3-2*e*F^(c*(b*x+a))*(e*x+d)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)^2/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx = \frac{F^{c(a+bx)}(2e^2 - 2bce(d + ex) \log(F) + b^2c^2(d + ex)^2 \log^2(F))}{b^3c^3 \log^3(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d^2 + 2*d*e*x + e^2*x^2),x]
```

output

```
(F^(c*(a + b*x))*(2*e^2 - 2*b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(b^3*c^3*Log[F]^3)
```

**Rubi [A] (verified)**

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2006, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (d^2 + 2dex + e^2x^2) F^{c(a+bx)} dx \\
 & \quad \downarrow \text{2006} \\
 & \int (d + ex)^2 F^{c(a+bx)} dx \\
 & \quad \downarrow \text{2607} \\
 & \frac{(d + ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \int F^{c(a+bx)} (d + ex) dx}{bc \log(F)} \\
 & \quad \downarrow \text{2607} \\
 & \frac{(d + ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)} \right)}{bc \log(F)} \\
 & \quad \downarrow \text{2624} \\
 & \frac{(d + ex)^2 F^{c(a+bx)}}{bc \log(F)} - \frac{2e \left( \frac{(d+ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} \right)}{bc \log(F)}
 \end{aligned}$$

input

```
Int[F^(c*(a + b*x))*(d^2 + 2*d*e*x + e^2*x^2),x]
```

output

```
(F^(c*(a + b*x))*(d + e*x)^2)/(b*c*Log[F]) - (2*e*(-((e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2)) + (F^(c*(a + b*x))*(d + e*x))/(b*c*Log[F]))/(b*c*Log[F])
```

Defintions of rubi rules used

```
rule 2006 Int[(u_.)*(Px_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]],
b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^Expon[Px
, x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; PolyQ[Px, x] && GtQ[Expon[P
x, x], 1] && NeQ[Coeff[Px, x, 0], 0] && !MatchQ[Px, (a_.)*(v_)^Expon[Px, x
] /; FreeQ[a, x] && LinearQ[v, x]]
```

```
rule 2607 Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_)^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

```
rule 2624 Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

method	result
gospers	$\frac{(\ln(F)^2 b^2 c^2 e^2 x^2 + 2 \ln(F)^2 b^2 c^2 d e x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e^2 x - 2 \ln(F) b c e d + 2 e^2) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
risch	$\frac{(\ln(F)^2 b^2 c^2 e^2 x^2 + 2 \ln(F)^2 b^2 c^2 d e x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e^2 x - 2 \ln(F) b c e d + 2 e^2) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
norman	$\frac{(\ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e d + 2 e^2) e^{c(bx+a) \ln(F)}}{c^3 b^3 \ln(F)^3} + \frac{e^2 x^2 e^{c(bx+a) \ln(F)}}{\ln(F) b c} + \frac{2 e (b c d \ln(F) - e) x e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2}$
oring	$\frac{(\ln(F)^2 b^2 c^2 e^2 x^2 + 2 \ln(F)^2 b^2 c^2 d e x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b c e^2 x - 2 \ln(F) b c e d + 2 e^2) F^{c(bx+a)} (e^2 x^2 + 2 e x d + d^2)}{\ln(F)^3 b^3 c^3 (e x + d)^2}$
meijerg	$-\frac{F^{a c} e^2 \left( 2 - \frac{(3 b^2 c^2 x^2 \ln(F)^2 - 6 b c x \ln(F) + 6) e^{b c x \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} + \frac{2 F^{a c} e d \left( 1 - \frac{(-2 b c x \ln(F) + 2) e^{b c x \ln(F)}}{2} \right)}{\ln(F)^2 b^2 c^2} - \frac{F^{a c} d^2 (1 - e^{b c x \ln(F)})}{\ln(F) b c}$
parallelrisc	$\frac{\ln(F)^2 x^2 F^{c(bx+a)} b^2 c^2 e^2 + 2 \ln(F)^2 x F^{c(bx+a)} b^2 c^2 d e + \ln(F)^2 F^{c(bx+a)} b^2 c^2 d^2 - 2 \ln(F) x F^{c(bx+a)} b c e^2 - 2 F^{c(bx+a)} \ln(F) b c d}{\ln(F)^3 b^3 c^3}$

```
input int(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2), x, method=_RETURNVERBOSE)
```

output  $(\ln(F)^2 b^2 c^2 e^{2x^2} + 2 \ln(F)^2 b^2 c^2 d e^x + \ln(F)^2 b^2 c^2 d^2 - 2 \ln(F) b^2 c^2 e^{2x} - 2 \ln(F) b^2 c^2 d e^x + 2 e^{2x}) F^{c(bx+a)} / \ln(F)^3 / b^3 / c^3$

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.06

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2 x^2) dx$$

$$= \frac{((b^2 c^2 e^2 x^2 + 2 b^2 c^2 dex + b^2 c^2 d^2) \log(F)^2 + 2 e^2 - 2 (bce^2 x + bcde) \log(F)) F^{bcx+ac}}{b^3 c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2),x, algorithm="fricas")`

output  $((b^2 c^2 e^{2x^2} + 2 b^2 c^2 d e^x + b^2 c^2 d^2) \log(F)^2 + 2 e^2 - 2 (b^2 c^2 e^{2x} + b^2 c^2 d e^x) \log(F)) F^{(b^2 c^2 x + a^2 c)} / (b^3 c^3 \log(F)^3)$

### Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.68

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2 x^2) dx$$

$$= \begin{cases} \frac{F^{c(a+bx)} (b^2 c^2 d^2 \log(F)^2 + 2 b^2 c^2 dex \log(F)^2 + b^2 c^2 e^2 x^2 \log(F)^2 - 2 bcde \log(F) - 2 bce^2 x \log(F) + 2 e^2)}{b^3 c^3 \log(F)^3} & \text{for } b^3 c^3 \log(F)^3 \neq 0 \\ d^2 x + dex^2 + \frac{e^2 x^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(e**2*x**2+2*d*e*x+d**2),x)`

output `Piecewise((F**(c*(a + b*x))*(b**2*c**2*d**2*log(F)**2 + 2*b**2*c**2*d*e*x*log(F)**2 + b**2*c**2*e**2*x**2*log(F)**2 - 2*b*c*d*e*log(F) - 2*b*c*e**2*x*log(F) + 2*e**2)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d**2*x + d*e*x**2 + e**2*x**3/3, True))`



**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.56

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx$$

$$= \frac{F^{bcx+ac}d^2}{bc \log(F)} + \frac{2(F^{ac}bcx \log(F) - F^{ac})F^{bcx}de}{b^2c^2 \log(F)^2}$$

$$+ \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}e^2}{b^3c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2),x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d^2/(b*c*log(F)) + 2*(F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*d*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*e^2/(b^3*c^3*log(F)^3)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 2214, normalized size of antiderivative = 28.03

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2),x, algorithm="giac")`

output

```

-((2*(pi*b^2*c^2*e^2*x^2*log(abs(F))*sgn(F) - pi*b^2*c^2*e^2*x^2*log(abs(F)
)) + 2*pi*b^2*c^2*d*e*x*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*d*e*x*log(abs(F)
) + pi*b^2*c^2*d^2*log(abs(F))*sgn(F) - pi*b^2*c^2*d^2*log(abs(F)) - pi*b*
c*e^2*x*sgn(F) + pi*b*c*e^2*x - pi*b*c*d*e*sgn(F) + pi*b*c*d*e)*(pi^3*b^3*
c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c
^3*log(abs(F))^2)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F)
) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs
(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) - (
pi^2*b^2*c^2*e^2*x^2*sgn(F) - pi^2*b^2*c^2*e^2*x^2 + 2*b^2*c^2*e^2*x^2*log
(abs(F))^2 + 2*pi^2*b^2*c^2*d*e*x*sgn(F) - 2*pi^2*b^2*c^2*d*e*x + 4*b^2*c^
2*d*e*x*log(abs(F))^2 + pi^2*b^2*c^2*d^2*sgn(F) - pi^2*b^2*c^2*d^2 + 2*b^2
*c^2*d^2*log(abs(F))^2 - 4*b*c*e^2*x*log(abs(F)) - 4*b*c*d*e*log(abs(F)) +
4*e^2)*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) +
2*b^3*c^3*log(abs(F))^3)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^
2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*
log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)
^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*
a*c) - ((pi^2*b^2*c^2*e^2*x^2*sgn(F) - pi^2*b^2*c^2*e^2*x^2 + 2*b^2*c^2*e^
2*x^2*log(abs(F))^2 + 2*pi^2*b^2*c^2*d*e*x*sgn(F) - 2*pi^2*b^2*c^2*d*e*x +
4*b^2*c^2*d*e*x*log(abs(F))^2 + pi^2*b^2*c^2*d^2*sgn(F) - pi^2*b^2*c^2...

```

### Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx$$

$$= \frac{F^{ac+bcx} (b^2 c^2 d^2 \ln(F)^2 + 2b^2 c^2 dex \ln(F)^2 + b^2 c^2 e^2 x^2 \ln(F)^2 - 2bcde \ln(F) - 2bce^2 x \ln(F) + 2bcde^2 x^2)}{b^3 c^3 \ln(F)^3}$$

input

```
int(F^(c*(a + b*x))*(d^2 + e^2*x^2 + 2*d*e*x),x)
```

output

```

(F^(a*c + b*c*x)*(2*e^2 + b^2*c^2*d^2*log(F)^2 - 2*b*c*e^2*x*log(F) + b^2*
c^2*e^2*x^2*log(F)^2 - 2*b*c*d*e*log(F) + 2*b^2*c^2*d*e*x*log(F)^2))/(b^3*
c^3*log(F)^3)

```

**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2) dx$$

$$= \frac{f^{bcx+ac}(\log(f)^2 b^2 c^2 d^2 + 2\log(f)^2 b^2 c^2 dex + \log(f)^2 b^2 c^2 e^2 x^2 - 2\log(f) bcde - 2\log(f) bc e^2 x + 2e^2)}{\log(f)^3 b^3 c^3}$$

input `int(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2),x)`output `(f**(a*c + b*c*x)*(log(f)**2*b**2*c**2*d**2 + 2*log(f)**2*b**2*c**2*d*e*x + log(f)**2*b**2*c**2*e**2*x**2 - 2*log(f)*b*c*d*e - 2*log(f)*b*c*e**2*x + 2*e**2))/(log(f)**3*b**3*c**3)`

### 3.51 $\int \frac{F^{c(a+bx)}}{d^2+2dex+e^2x^2} dx$

Optimal result	379
Mathematica [A] (verified)	379
Rubi [A] (verified)	380
Maple [A] (verified)	381
Fricas [A] (verification not implemented)	382
Sympy [F]	382
Maxima [F]	382
Giac [F]	383
Mupad [F(-1)]	383
Reduce [F]	383

#### Optimal result

Integrand size = 28, antiderivative size = 57

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = -\frac{F^{c(a+bx)}}{e(d+ex)} + \frac{bcF^{c\left(a-\frac{bd}{e}\right)} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)}{e^2}$$

output

$$-F^{c*(b*x+a)}/e/(e*x+d)+b*c*F^{c*(a-b*d/e)}*Ei(b*c*(e*x+d)*ln(F)/e)*ln(F)/e^2$$

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = \frac{F^{ac}\left(-\frac{eF^{bcx}}{d+ex} + bcF^{-\frac{bcd}{e}} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log(F)\right)}{e^2}$$

input

`Integrate[F^(c*(a + b*x))/(d^2 + 2*d*e*x + e^2*x^2), x]`

output

$$(F^{a*c})*(-((eF^{b*c*x})/(d + e*x)) + (b*c*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/F^{((b*c*d)/e)}))/e^2$$

**Rubi [A] (verified)**

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {2007, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx \\
 & \quad \downarrow \text{2007} \\
 & \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \\
 & \quad \downarrow \text{2609} \\
 & \frac{bc \log(F) F^{c\left(a-\frac{bd}{e}\right)} \text{ExpIntegralEi}\left(\frac{bc(d+ex) \log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)}
 \end{aligned}$$

input `Int [F^(c*(a + b*x))/(d^2 + 2*d*e*x + e^2*x^2), x]`

output `-(F^(c*(a + b*x))/(e*(d + e*x))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/e^2`

## Definitions of rubi rules used

rule 2007 `Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Expon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]`

rule 2608 `Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n/(d*(m + 1)), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2609 `Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

method	result	size
risch	$-\frac{\ln(F)bc F^{bcx} F^{ac}}{e^2 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)} - \frac{\ln(F)bc F^{\frac{c(ea-bd)}{e}} \expIntegral_1 \left( -bcx \ln(F) - ac \ln(F) - \frac{-ea \ln(F)c + bcd \ln(F)}{e} \right)}{e^2}$	99

input `int(F^(c*(b*x+a))/(e^2*x^2+2*d*e*x+d^2),x,method=_RETURNVERBOSE)`

output `-ln(F)*b*c/e^2*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)-ln(F)*b*c/e^2*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*ln(F)-a*c*ln(F)-(-e*a*ln(F)*c+b*c*d*ln(F))/e)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = -\frac{F^{bcx+ac} e^{-\frac{(bcex+bcd)\text{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right)}{F^{\frac{bcd-ace}{e}}}}}{e^3x + de^2}$$

input `integrate(F^((b*x+a)*c)/(e^2*x^2+2*d*e*x+d^2),x, algorithm="fricas")`

output `-(F^(b*c*x + a*c)*e - (b*c*e*x + b*c*d)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)/F^((b*c*d - a*c*e)/e))/(e^3*x + d*e^2)`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = \int \frac{F^{c(a+bx)}}{(d + ex)^2} dx$$

input `integrate(F**((b*x+a)*c)/(e**2*x**2+2*d*e*x+d**2),x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**2, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = \int \frac{F^{(bx+a)c}}{e^2x^2 + 2dex + d^2} dx$$

input `integrate(F^((b*x+a)*c)/(e^2*x^2+2*d*e*x+d^2),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e^2*x^2 + 2*d*e*x + d^2), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = \int \frac{F^{(bx+a)c}}{e^2x^2 + 2dex + d^2} dx$$

input `integrate(F^((b*x+a)*c)/(e^2*x^2+2*d*e*x+d^2),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e^2*x^2 + 2*d*e*x + d^2), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = \int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx$$

input `int(F^(c*(a + b*x))/(d^2 + e^2*x^2 + 2*d*e*x),x)`

output `int(F^(c*(a + b*x))/(d^2 + e^2*x^2 + 2*d*e*x), x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d^2 + 2dex + e^2x^2} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^2x^2 + 2dex + d^2} dx \right)$$

input `int(F^((b*x+a)*c)/(e^2*x^2+2*d*e*x+d^2),x)`

output `f**(a*c)*int(f**(b*c*x)/(d**2 + 2*d*e*x + e**2*x**2),x)`



**3.52**  $\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$

Optimal result . . . . .	384
Mathematica [A] (verified) . . . . .	384
Rubi [A] (verified) . . . . .	385
Maple [A] (verified) . . . . .	386
Fricas [A] (verification not implemented) . . . . .	387
Sympy [F] . . . . .	387
Maxima [F] . . . . .	388
Giac [F] . . . . .	388
Mupad [F(-1)] . . . . .	388
Reduce [F] . . . . .	389

**Optimal result**

Integrand size = 39, antiderivative size = 95

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

$$= -\frac{F^{c(a+bx)}}{2e(d+ex)^2} - \frac{bcF^{c(a+bx)} \log(F)}{2e^2(d+ex)}$$

$$+ \frac{b^2c^2F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F)}{2e^3}$$

output -1/2\*F^(c\*(b\*x+a))/e/(e\*x+d)^2-1/2\*b\*c\*F^(c\*(b\*x+a))\*ln(F)/e^2/(e\*x+d)+1/2\*b^2\*c^2\*F^(c\*(a-b\*d/e))\*Ei(b\*c\*(e\*x+d)\*ln(F)/e)\*ln(F)^2/e^3

**Mathematica [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

$$= \frac{F^{c(a-\frac{bd}{e})} \left( b^2c^2(d+ex)^2 \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^2(F) - eF^{\frac{bc(d+ex)}{e}}(e+bc(d+ex)\log(F)) \right)}{2e^3(d+ex)^2}$$

input `Integrate[F^(c*(a + b*x))/(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3),x]`

output  $(F^{c*(a - (b*d)/e)}*(b^2*c^2*(d + e*x)^2*\text{ExpIntegralEi}[(b*c*(d + e*x)*\text{Log}[F])/e]*\text{Log}[F]^2 - e*F^{((b*c*(d + e*x))/e)*(e + b*c*(d + e*x)*\text{Log}[F])})/(2*e^3*(d + e*x)^2)$

### Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2007, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx \\
 & \quad \downarrow \text{2007} \\
 & \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \\
 & \quad \downarrow \text{2609} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right)}{e^2} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2}
 \end{aligned}$$

input `Int[F^(c*(a + b*x))/(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3),x]`

output `-1/2*F^(c*(a + b*x))/(e*(d + e*x)^2) + (b*c*Log[F]*(-(F^(c*(a + b*x))/(e*(d + e*x)))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/e^2)/(2*e)`

### Defintions of rubi rules used

rule 2007 `Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Expon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]`

rule 2608 `Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_)), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2609 `Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{\ln(F)^2 b^2 c^2 F^{bcx} F^{ac}}{2e^3 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^2} - \frac{\ln(F)^2 b^2 c^2 F^{bcx} F^{ac}}{2e^3 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)} - \frac{\ln(F)^2 b^2 c^2 F^{\frac{c(ea-bd)}{e}} \expIntegral_1 \left( -bcx \ln(F) - ac \ln(F) - \frac{-ea}{e} \right)}{2e^3}$

input `int(F^(c*(b*x+a))/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x,method=_RETURNVERBOSE)`

output

$$-1/2*\ln(F)^2*b^2*c^2/e^3*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)^2-1/2*\ln(F)^2*b^2*c^2/e^3*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)-1/2*\ln(F)^2*b^2*c^2/e^3*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-e*a*\ln(F)*c+b*c*d*\ln(F))/e)}$$
**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.41

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

$$= \frac{\frac{(b^2c^2e^2x^2+2b^2c^2dex+b^2c^2d^2)Ei\left(\frac{(bcex+bcd)\log(F)}{e}\right)\log(F)^2}{F^{\frac{bcd-ace}{e}}} - (e^2 + (bce^2x + bcde)\log(F))F^{bcx+ac}}{2(e^5x^2 + 2de^4x + d^2e^3)}$$

input

```
integrate(F^((b*x+a)*c)/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="fricas")
```

output

$$1/2*((b^2*c^2*e^2*x^2 + 2*b^2*c^2*d*e*x + b^2*c^2*d^2)*Ei((b*c*e*x + b*c*d)*\log(F)/e)*\log(F)^2/F^{(b*c*d - a*c*e)/e} - (e^2 + (b*c*e^2*x + b*c*d*e)*\log(F))*F^{(b*c*x + a*c)})/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)$$
**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx = \int \frac{F^{c(a+bx)}}{(d + ex)^3} dx$$

input

```
integrate(F**(b*x+a)*c)/(e**3*x**3+3*d*e**2*x**2+3*d**2*e*x+d**3),x)
```

output

```
Integral(F**(c*(a + b*x))/(d + e*x)**3, x)
```

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx = \int \frac{F^{(bx+a)c}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3} dx$$

input `integrate(F^((b*x+a)*c)/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx = \int \frac{F^{(bx+a)c}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3} dx$$

input `integrate(F^((b*x+a)*c)/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx = \int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx$$

input `int(F^((c*(a + b*x)))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x), x)`

output `int(F^((c*(a + b*x)))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x), x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d^3 + 3d^2ex + 3de^2x^2 + e^3x^3} dx = f^{ac} \left( \int \frac{f^{bcx}}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3} dx \right)$$

input `int(F^((b*x+a)*c)/(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3),x)`

output `f**(a*c)*int(f**(b*c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3),x)`

**3.53** 
$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

Optimal result	390
Mathematica [A] (verified)	390
Rubi [A] (verified)	391
Maple [A] (verified)	393
Fricas [A] (verification not implemented)	393
Sympy [F]	394
Maxima [F]	394
Giac [F]	394
Mupad [F(-1)]	395
Reduce [F]	395

**Optimal result**

Integrand size = 50, antiderivative size = 128

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= -\frac{F^{c(a+bx)}}{3e(d+ex)^3} - \frac{bcF^{c(a+bx)} \log(F)}{6e^2(d+ex)^2} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{6e^3(d+ex)}$$

$$+ \frac{b^3c^3F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F)}{6e^4}$$

output

```
-1/3*F^(c*(b*x+a))/e/(e*x+d)^3-1/6*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^2-1/6*b^2*c^2*F^(c*(b*x+a))*ln(F)^2/e^3/(e*x+d)+1/6*b^3*c^3*F^(c*(a-b*d/e))*Ei(b*c*(e*x+d)*ln(F)/e)*ln(F)^3/e^4
```

**Mathematica [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= \frac{F^{ac} \left( b^3c^3F^{-\frac{bcd}{e}} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^3(F) - \frac{eF^{bcx}(2e^2 + bce(d+ex)\log(F) + b^2c^2(d+ex)^2\log^2(F))}{(d+ex)^3} \right)}{6e^4}$$

input

```
Integrate[F^(c*(a + b*x))/(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4), x]
```

output

```
(F^(a*c)*((b^3*c^3*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^3)/F^((b*c*d)/e) - (e*F^(b*c*x)*(2*e^2 + b*c*e*(d + e*x)*Log[F] + b^2*c^2*(d + e*x)^2*Log[F]^2))/(d + e*x)^3))/(6*e^4)
```

### Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2007, 2608, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx \\
 & \quad \downarrow \text{2007} \\
 & \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3}
 \end{aligned}$$



$$\begin{array}{c} \downarrow 2609 \\ bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a - \frac{bd}{e})} \text{ExpIntegralEi} \left( \frac{bc(d+ex) \log(F)}{e} \right) - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{e^2} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{2e} \right) \\ \hline \frac{3e F^{c(a+bx)}}{3e(d+ex)^3} \end{array}$$

input

```
Int [F^(c*(a + b*x))/(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4), x]
```

output

```
-1/3*F^(c*(a + b*x))/(e*(d + e*x)^3) + (b*c*Log[F]*(-1/2*F^(c*(a + b*x)))/(e*(d + e*x)^2) + (b*c*Log[F]*(-(F^(c*(a + b*x)))/(e*(d + e*x)))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]/e^2))/(2*e))/ (3*e)
```

### Defintions of rubi rules used

rule 2007

```
Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Expon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2608

```
Int[((b_)*(F_)^((g_)*((e_)+(f_)*(x_))))^(n_)*((c_)+(d_)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2609

```
Int[(F_)^((g_)*((e_)+(f_)*(x_)))/((c_)+(d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

**Maple [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{\ln(F)^3 b^3 c^3 F^{bcx} F^{ac}}{3e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^3} - \frac{\ln(F)^3 b^3 c^3 F^{bcx} F^{ac}}{6e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^2} - \frac{\ln(F)^3 b^3 c^3 F^{bcx} F^{ac}}{6e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)} - \frac{\ln(F)^3 b^3 c^3 F^{\frac{c(ea-bd)}{e}} \operatorname{ExpIntegralEi}\left(1, -bcx \ln(F) - a \ln(F) - \frac{e^4}{e} \right)}{6e^4 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)}$

input `int(F^(c*(b*x+a))/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,method=_RETURNVERBOSE)`

output 
$$-1/3*\ln(F)^3*b^3*c^3/e^4*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)}^3-1/6*\ln(F)^3*b^3*c^3/e^4*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)}^2-1/6*\ln(F)^3*b^3*c^3/e^4*F^{(b*c*x)*F^{(a*c)/(b*c*x*\ln(F)+b*c*\ln(F)/e*d)}-1/6*\ln(F)^3*b^3*c^3/e^4*F^{(c*(a*e-b*d)/e)*Ei(1,-b*c*x*\ln(F)-a*c*\ln(F)-(-e*a*\ln(F)*c+b*c*d*\ln(F))/e)}$$

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.63

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= \frac{(b^3c^3e^3x^3 + 3b^3c^3de^2x^2 + 3b^3c^3d^2ex + b^3c^3d^3) \operatorname{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right) \log(F)^3}{F^{\frac{bcd-ace}{e}}} - \frac{(2e^3 + (b^2c^2e^3x^2 + 2b^2c^2de^2x + b^2c^2d^2e) \log(F))}{6(e^7x^3 + 3de^6x^2 + 3d^2e^5x + d^3e^4)}$$

input `integrate(F^((b*x+a)*c)/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4),x,algorithm="fricas")`

output 
$$1/6*((b^3*c^3*e^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)*Ei((b*c*e*x + b*c*d)*\log(F)/e)*\log(F)^3/F^{((b*c*d - a*c*e)/e)} - (2*e^3 + (b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)*\log(F)^2 + (b*c*e^3*x + b*c*d*e^2)*\log(F))*F^{(b*c*x + a*c)}/(e^7*x^3 + 3*d*e^6*x^2 + 3*d^2*e^5*x + d^3*e^4)$$

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx = \int \frac{F^{c(a+bx)}}{(d + ex)^4} dx$$

input `integrate(F**(b*x+a)*c)/(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e*x+d**4), x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**4, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= \int \frac{F^{(bx+a)c}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4} dx$$

input `integrate(F^((b*x+a)*c)/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4), x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= \int \frac{F^{(bx+a)c}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4} dx$$

input `integrate(F^((b*x+a)*c)/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4), x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)`

### Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= \int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

input `int(F^(c*(a + b*x))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x), x)`

output `int(F^(c*(a + b*x))/(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x), x)`

### Reduce [F]

$$\int \frac{F^{c(a+bx)}}{d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4} dx$$

$$= f^{ac} \left( \int \frac{f^{bcx}}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4} dx \right)$$

input `int(F^((b*x+a)*c)/(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4), x)`

output `f**(a*c)*int(f**(b*c*x)/(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4), x)`

**3.54** 
$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

Optimal result . . . . .	396
Mathematica [A] (verified) . . . . .	397
Rubi [A] (verified) . . . . .	397
Maple [A] (verified) . . . . .	399
Fricas [A] (verification not implemented) . . . . .	400
Sympy [F] . . . . .	401
Maxima [F] . . . . .	401
Giac [F] . . . . .	401
Mupad [F(-1)] . . . . .	402
Reduce [F] . . . . .	402

**Optimal result**

Integrand size = 61, antiderivative size = 161

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= -\frac{F^{c(a+bx)}}{4e(d+ex)^4} - \frac{bcF^{c(a+bx)} \log(F)}{12e^2(d+ex)^3} - \frac{b^2c^2F^{c(a+bx)} \log^2(F)}{24e^3(d+ex)^2}$$

$$- \frac{b^3c^3F^{c(a+bx)} \log^3(F)}{24e^4(d+ex)} + \frac{b^4c^4F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi}\left(\frac{bc(d+ex)\log(F)}{e}\right) \log^4(F)}{24e^5}$$

output

```
-1/4*F^(c*(b*x+a))/e/(e*x+d)^4-1/12*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^3-
1/24*b^2*c^2*F^(c*(b*x+a))*ln(F)^2/e^3/(e*x+d)^2-1/24*b^3*c^3*F^(c*(b*x+a)
)*ln(F)^3/e^4/(e*x+d)+1/24*b^4*c^4*F^(c*(a-b*d/e))*Ei(b*c*(e*x+d)*ln(F)/e)
*ln(F)^4/e^5
```

### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= \frac{F^{ac} \left( b^4 c^4 F^{-\frac{bcd}{e}} \text{ExpIntegralEi} \left( \frac{bc(d+ex) \log(F)}{e} \right) \log^4(F) - \frac{eF^{bcx} (6e^3 + 2bce^2(d+ex) \log(F) + b^2c^2e(d+ex)^2 \log^2(F) + b^3c^3(d+ex)^3 \log^3(F))}{(d+ex)^4} \right)}{24e^5}$$

input

```
Integrate[F^(c*(a + b*x))/(d^5 + 5*d^4*e*x + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d*e^4*x^4 + e^5*x^5),x]
```

output

```
(F^(a*c)*((b^4*c^4*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F]^4)/F^((b*c*d)/e) - (e*F^(b*c*x)*(6*e^3 + 2*b*c*e^2*(d + e*x)*Log[F] + b^2*c^2*e*(d + e*x)^2*Log[F]^2 + b^3*c^3*(d + e*x)^3*Log[F]^3))/(d + e*x)^4))/(24*e^5)
```

### Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$ , Rules used = {2007, 2608, 2608, 2608, 2608, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

↓ 2007

$$\int \frac{F^{c(a+bx)}}{(d + ex)^5} dx$$

↓ 2608

$$\frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^4} dx}{4e} - \frac{F^{c(a+bx)}}{4e(d + ex)^4}$$

↓ 2608

$$\begin{aligned}
 & \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^3} dx}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \right)}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4} \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^2} dx}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \right)}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4} \\
 & \quad \downarrow \text{2608} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) \int \frac{F^{c(a+bx)}}{d+ex} dx}{e} - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \right)}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4} \\
 & \quad \downarrow \text{2609} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi} \left( \frac{bc(d+ex) \log(F)}{e} \right) - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \right)}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4} \\
 & \quad \downarrow \text{2609} \\
 & \frac{bc \log(F) \left( \frac{bc \log(F) \left( \frac{bc \log(F) F^{c(a-\frac{bd}{e})} \text{ExpIntegralEi} \left( \frac{bc(d+ex) \log(F)}{e} \right) - \frac{F^{c(a+bx)}}{e(d+ex)} \right)}{2e} - \frac{F^{c(a+bx)}}{2e(d+ex)^2} \right)}{3e} - \frac{F^{c(a+bx)}}{3e(d+ex)^3} \right)}{4e} - \frac{F^{c(a+bx)}}{4e(d+ex)^4}
 \end{aligned}$$

input `Int[F^(c*(a + b*x))/(d^5 + 5*d^4*e*x + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d*e^4*x^4 + e^5*x^5),x]`

output `-1/4*F^(c*(a + b*x))/(e*(d + e*x)^4) + (b*c*Log[F]*(-1/3*F^(c*(a + b*x))/(e*(d + e*x)^3) + (b*c*Log[F]*(-1/2*F^(c*(a + b*x))/(e*(d + e*x)^2) + (b*c*Log[F]*(-F^(c*(a + b*x))/(e*(d + e*x))) + (b*c*F^(c*(a - (b*d)/e))*ExpIntegralEi[(b*c*(d + e*x)*Log[F])/e]*Log[F])/e^2))/(2*e))/(3*e))/(4*e)`

### Defintions of rubi rules used

rule 2007 `Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Int[u*(a + b*x)^(Expon[Px, x]*p), x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]`

rule 2608 `Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_)))^(n_)*((c_) + (d_)*(x_))^(m_)), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2609 `Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{4e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^4} - \frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{12e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^3} - \frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{24e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)^2} - \frac{\ln(F)^4 b^4 c^4 F^{bcx} F^{ac}}{24e^5 \left( bcx \ln(F) + \frac{bc \ln(F)d}{e} \right)}$



input `int(F^(c*(b*x+a))/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x,method=_RETURNVERBOSE)`

output `-1/4*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^4-1/12*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^3-1/24*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)^2-1/24*ln(F)^4*b^4*c^4/e^5*F^(b*c*x)*F^(a*c)/(b*c*x*ln(F)+b*c*ln(F)/e*d)-1/24*ln(F)^4*b^4*c^4/e^5*F^(c*(a*e-b*d)/e)*Ei(1,-b*c*x*ln(F)-a*c*ln(F)-(-e*a*ln(F)*c+b*c*d*ln(F))/e)`

### Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.86

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= \frac{(b^4c^4e^4x^4 + 4b^4c^4de^3x^3 + 6b^4c^4d^2e^2x^2 + 4b^4c^4d^3ex + b^4c^4d^4) \operatorname{Ei}\left(\frac{(bcex+bcd)\log(F)}{e}\right) \log(F)^4}{F^{\frac{bcd-ace}{e}}} - (6e^4 + (b^3c^3e^4x^3 + 3b^3c^3de^3x^2 + 3b^3c^3d^2e^2x + b^3c^3d^3e) \log(F)^3 + (b^2c^2e^4x^2 + 2b^2c^2d^2e^2) \log(F)^2 + 2(b*c*e^4*x + b*c*d*e^3) \log(F)) * F^(b*c*x + a*c) / (e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)$$

input `integrate(F^((b*x+a)*c)/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5),x, algorithm="fricas")`

output `1/24*((b^4*c^4*e^4*x^4 + 4*b^4*c^4*d*e^3*x^3 + 6*b^4*c^4*d^2*e^2*x^2 + 4*b^4*c^4*d^3*e*x + b^4*c^4*d^4)*Ei((b*c*e*x + b*c*d)*log(F)/e)*log(F)^4/F^((b*c*d - a*c*e)/e) - (6*e^4 + (b^3*c^3*e^4*x^3 + 3*b^3*c^3*d*e^3*x^2 + 3*b^3*c^3*d^2*e^2*x + b^3*c^3*d^3*e)*log(F)^3 + (b^2*c^2*e^4*x^2 + 2*b^2*c^2*d^2*e^2)*log(F)^2 + 2*(b*c*e^4*x + b*c*d*e^3)*log(F))*F^(b*c*x + a*c)/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx = \int \frac{F^{c(a+bx)}}{(d + ex)^5} dx$$

input `integrate(F**((b*x+a)*c)/(e**5*x**5+5*d*e**4*x**4+10*d**2*e**3*x**3+10*d**3*e**2*x**2+5*d**4*e*x+d**5), x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**5, x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= \int \frac{F^{(bx+a)c}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5} dx$$

input `integrate(F^((b*x+a)*c)/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5), x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3*e^2*x^2 + 5*d^4*e*x + d^5), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= \int \frac{F^{(bx+a)c}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5} dx$$

input `integrate(F^((b*x+a)*c)/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4*e*x+d^5), x, algorithm="giac")`

output

```
integrate(F^((b*x + a)*c)/(e^5*x^5 + 5*d*e^4*x^4 + 10*d^2*e^3*x^3 + 10*d^3
*e^2*x^2 + 5*d^4*e*x + d^5), x)
```

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= \int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

input

```
int(F^(c*(a + b*x))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2
*e^3*x^3 + 5*d^4*e*x), x)
```

output

```
int(F^(c*(a + b*x))/(d^5 + e^5*x^5 + 5*d*e^4*x^4 + 10*d^3*e^2*x^2 + 10*d^2
*e^3*x^3 + 5*d^4*e*x), x)
```

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{d^5 + 5d^4ex + 10d^3e^2x^2 + 10d^2e^3x^3 + 5de^4x^4 + e^5x^5} dx$$

$$= f^{ac} \left( \int \frac{f^{bcx}}{e^5x^5 + 5de^4x^4 + 10d^2e^3x^3 + 10d^3e^2x^2 + 5d^4ex + d^5} dx \right)$$

input

```
int(F^((b*x+a)*c)/(e^5*x^5+5*d*e^4*x^4+10*d^2*e^3*x^3+10*d^3*e^2*x^2+5*d^4
*e*x+d^5), x)
```

output

```
f**(a*c)*int(f**(b*c*x)/(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*
e**3*x**3 + 5*d*e**4*x**4 + e**5*x**5), x)
```

### 3.55 $\int F^{a+bx}(c + bcx \log(F)) dx$

Optimal result	403
Mathematica [A] (verified)	403
Rubi [B] (verified)	404
Maple [A] (warning: unable to verify)	405
Fricas [A] (verification not implemented)	405
Sympy [A] (verification not implemented)	406
Maxima [B] (verification not implemented)	406
Giac [C] (verification not implemented)	406
Mupad [B] (verification not implemented)	407
Reduce [B] (verification not implemented)	408

#### Optimal result

Integrand size = 16, antiderivative size = 10

$$\int F^{a+bx}(c + bcx \log(F)) dx = cF^{a+bx}x$$

output `c*F^(b*x+a)*x`

#### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F^{a+bx}(c + bcx \log(F)) dx = cF^{a+bx}x$$

input `Integrate[F^(a + b*x)*(c + b*c*x*Log[F]),x]`

output `c*F^(a + b*x)*x`

**Rubi [B] (verified)**

Leaf count is larger than twice the leaf count of optimal. 41 vs.  $2(10) = 20$ .

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 4.10, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx}(bcx \log(F) + c) dx$$

$$\downarrow 2607$$

$$\frac{cF^{a+bx}(bx \log(F) + 1)}{b \log(F)} - c \int F^{a+bx} dx$$

$$\downarrow 2624$$

$$\frac{cF^{a+bx}(bx \log(F) + 1)}{b \log(F)} - \frac{cF^{a+bx}}{b \log(F)}$$

input `Int[F^(a + b*x)*(c + b*c*x*Log[F]),x]`

output `-((c*F^(a + b*x))/(b*Log[F])) + (c*F^(a + b*x)*(1 + b*x*Log[F]))/(b*Log[F])`

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (warning: unable to verify)**

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
gospers	$c F^{bx+a} x$	11
risch	$c F^{bx+a} x$	11
parallelrisch	$c F^{bx+a} x$	11
norman	$cx e^{(bx+a) \ln(F)}$	13
meijerg	$-\frac{F^a c (1 - e^{\ln(F) bx})}{b \ln(F)} + \frac{F^a c \left(1 - \frac{(2 - 2 \ln(F) bx) e^{\ln(F) bx}}{2}\right)}{b \ln(F)}$	55

input `int(F^(b*x+a)*(c+b*c*x*ln(F)),x,method=_RETURNVERBOSE)`

output `c*F^(b*x+a)*x`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F^{a+bx} (c + bcx \log(F)) dx = F^{bx+a} cx$$

input `integrate(F^(b*x+a)*(c+b*c*x*log(F)),x, algorithm="fricas")`

output `F^(b*x + a)*c*x`

**Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int F^{a+bx}(c + bcx \log(F)) dx = F^{a+bx} cx$$

input `integrate(F**(b*x+a)*(c+b*c*x*ln(F)),x)`

output `F**(a + b*x)*c*x`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.50

$$\int F^{a+bx}(c + bcx \log(F)) dx = \frac{(F^a bx \log(F) - F^a) F^{bx} c}{b \log(F)} + \frac{F^{bx+a} c}{b \log(F)}$$

input `integrate(F^(b*x+a)*(c+b*c*x*log(F)),x, algorithm="maxima")`

output `(F^a*b*x*log(F) - F^a)*F^(b*x)*c/(b*log(F)) + F^(b*x + a)*c/(b*log(F))`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 569, normalized size of antiderivative = 56.90

$$\int F^{a+bx}(c + bcx \log(F)) dx = \text{Too large to display}$$

input `integrate(F^(b*x+a)*(c+b*c*x*log(F)),x, algorithm="giac")`

output

```

-1/2*I*((pi*b^2*c*x*log(F)*sgn(F) - pi*b^2*c*x*log(F) - 2*I*b^2*c*x*log(F)
*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c + 2*I*b*c*log(F) - 2*I*b*c*log(abs(F)
)))*e^(1/2*I*pi*b*x*sgn(F) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a
)/(pi^2*b^2*sgn(F) + 2*I*pi*b^2*log(abs(F))*sgn(F) - pi^2*b^2 - 2*I*pi*b^2
*log(abs(F)) + 2*b^2*log(abs(F))^2) + (pi*b^2*c*x*log(F)*sgn(F) - pi*b^2*c
*x*log(F) + 2*I*b^2*c*x*log(F)*log(abs(F)) + pi*b*c*sgn(F) - pi*b*c - 2*I*
b*c*log(F) + 2*I*b*c*log(abs(F)))*e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x -
1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(pi^2*b^2*sgn(F) - 2*I*pi*b^2*log(abs(F))
*sgn(F) - pi^2*b^2 + 2*I*pi*b^2*log(abs(F)) + 2*b^2*log(abs(F))^2)*e^(b*x
*log(abs(F)) + a*log(abs(F))) - 1/2*((-I*pi*b^2*c*x*log(F)*sgn(F) + I*pi*b
^2*c*x*log(F) - 2*b^2*c*x*log(F)*log(abs(F)) - I*pi*b*c*sgn(F) + I*pi*b*c
+ 2*b*c*log(F) - 2*b*c*log(abs(F)))*e^(1/2*I*pi*b*x*sgn(F) - 1/2*I*pi*b*x
+ 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(pi^2*b^2*sgn(F) + 2*I*pi*b^2*log(abs(F)
)*sgn(F) - pi^2*b^2 - 2*I*pi*b^2*log(abs(F)) + 2*b^2*log(abs(F))^2) + (I*
pi*b^2*c*x*log(F)*sgn(F) - I*pi*b^2*c*x*log(F) - 2*b^2*c*x*log(F)*log(abs(F)
)) + I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(F) - 2*b*c*log(abs(F)))*e^(-1/
2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(pi^2*b
^2*sgn(F) - 2*I*pi*b^2*log(abs(F))*sgn(F) - pi^2*b^2 + 2*I*pi*b^2*log(abs(
F)) + 2*b^2*log(abs(F))^2)*e^(b*x*log(abs(F)) + a*log(abs(F)))

```

### Mupad [B] (verification not implemented)

Time = 22.75 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F^{a+bx}(c + bcx \log(F)) dx = F^{a+bx} cx$$

input

```
int(F^(a + b*x)*(c + b*c*x*log(F)),x)
```

output

```
F^(a + b*x)*c*x
```



**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int F^{a+bx}(c + bcx \log(F)) dx = f^{bx+a}cx$$

input `int(F^(b*x+a)*(c+b*c*x*log(F)),x)`

output `f**(a + b*x)*c*x`

$$3.56 \quad \int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx$$

Optimal result	409
Mathematica [A] (verified)	409
Rubi [B] (verified)	410
Maple [A] (verified)	411
Fricas [A] (verification not implemented)	411
Sympy [B] (verification not implemented)	412
Maxima [B] (verification not implemented)	412
Giac [C] (verification not implemented)	413
Mupad [B] (verification not implemented)	414
Reduce [B] (verification not implemented)	414

### Optimal result

Integrand size = 21, antiderivative size = 17

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \frac{dF^{a+bx}x}{b \log(F)}$$

output `d*F^(b*x+a)*x/b/ln(F)`

### Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \frac{dF^{a+bx}x}{b \log(F)}$$

input `Integrate[F^(a + b*x)*(d*x + d/(b*Log[F])),x]`

output `(d*F^(a + b*x)*x)/(b*Log[F])`

**Rubi [B] (verified)**

Leaf count is larger than twice the leaf count of optimal. 42 vs.  $2(17) = 34$ .

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx} \left( \frac{d}{b \log(F)} + dx \right) dx$$

$$\downarrow \text{2607}$$

$$\frac{dF^{a+bx} \left( bx + \frac{1}{\log(F)} \right)}{b^2 \log(F)} - \frac{d \int F^{a+bx} dx}{b \log(F)}$$

$$\downarrow \text{2624}$$

$$\frac{dF^{a+bx} \left( bx + \frac{1}{\log(F)} \right)}{b^2 \log(F)} - \frac{dF^{a+bx}}{b^2 \log^2(F)}$$

input `Int[F^(a + b*x)*(d*x + d/(b*Log[F])),x]`

output `-((d*F^(a + b*x))/(b^2*Log[F]^2)) + (d*F^(a + b*x)*(b*x + Log[F]^(-1)))/(b^2*Log[F])`

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624

```
Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
gospers	$\frac{d F^{bx+a} x}{b \ln(F)}$	18
risch	$\frac{d F^{bx+a} x}{b \ln(F)}$	18
parallelrisch	$\frac{d F^{bx+a} x}{b \ln(F)}$	18
norman	$\frac{dx e^{(bx+a) \ln(F)}}{b \ln(F)}$	20
meijerg	$\frac{F^a d \left( 1 - \frac{(2-2 \ln(F) bx) e^{\ln(F) bx}}{2} \right)}{b^2 \ln(F)^2} - \frac{F^a d (1 - e^{\ln(F) bx})}{b^2 \ln(F)^2}$	55

input

```
int(F^(b*x+a)*(d*x+d/b/ln(F)),x,method=_RETURNVERBOSE)
```

output

```
d*F^(b*x+a)*x/b/ln(F)
```

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \frac{F^{bx+a} dx}{b \log(F)}$$

input

```
integrate(F^(b*x+a)*(d*x+d/b/log(F)),x, algorithm="fricas")
```

output

```
F^(b*x + a)*d*x/(b*log(F))
```

**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 32 vs.  $2(14) = 28$ .

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.88

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \begin{cases} \frac{F^{a+bx} dx}{b \log(F)} & \text{for } b \log(F) \neq 0 \\ \frac{dx^2}{2} + \frac{dx}{b \log(F)} & \text{otherwise} \end{cases}$$

input `integrate(F**(b*x+a)*(d*x+d/b/ln(F)),x)`

output `Piecewise((F**(a + b*x)*d*x/(b*log(F)), Ne(b*log(F), 0)), (d*x**2/2 + d*x/(b*log(F)), True))`

**Maxima [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 45 vs.  $2(17) = 34$ .

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.65

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \frac{(F^a b x \log(F) - F^a) F^{bx} d}{b^2 \log(F)^2} + \frac{F^{bx+a} d}{b^2 \log(F)^2}$$

input `integrate(F^(b*x+a)*(d*x+d/b/log(F)),x, algorithm="maxima")`

output `(F^a*b*x*log(F) - F^a)*F^(b*x)*d/(b^2*log(F)^2) + F^(b*x + a)*d/(b^2*log(F)^2)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 609, normalized size of antiderivative = 35.82

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \text{Too large to display}$$

input `integrate(F^(b*x+a)*(d*x+d/b/log(F)),x, algorithm="giac")`

output

```
-1/2*I*((pi*b^2*d*x*log(F)*sgn(F) - pi*b^2*d*x*log(F) - 2*I*b^2*d*x*log(F)
*log(abs(F)) + pi*b*d*sgn(F) - pi*b*d + 2*I*b*d*log(F) - 2*I*b*d*log(abs(F)
)))e^(1/2*I*pi*b*x*sgn(F) - 1/2*I*pi*b*x + 1/2*I*pi*a*sgn(F) - 1/2*I*pi*a
)/(pi^2*b^3*log(F)*sgn(F) + 2*I*pi*b^3*log(F)*log(abs(F))*sgn(F) - pi^2*b^
3*log(F) - 2*I*pi*b^3*log(F)*log(abs(F)) + 2*b^3*log(F)*log(abs(F))^2) + (
pi*b^2*d*x*log(F)*sgn(F) - pi*b^2*d*x*log(F) + 2*I*b^2*d*x*log(F)*log(abs(
F)) + pi*b*d*sgn(F) - pi*b*d - 2*I*b*d*log(F) + 2*I*b*d*log(abs(F)))e^(-1
/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(F) + 1/2*I*pi*a)/(pi^2*
b^3*log(F)*sgn(F) - 2*I*pi*b^3*log(F)*log(abs(F))*sgn(F) - pi^2*b^3*log(F)
+ 2*I*pi*b^3*log(F)*log(abs(F)) + 2*b^3*log(F)*log(abs(F))^2))e^(b*x*log
(abs(F)) + a*log(abs(F))) - 1/2*((-I*pi*b^2*d*x*log(F)*sgn(F) + I*pi*b^2*d
*x*log(F) - 2*b^2*d*x*log(F)*log(abs(F)) - I*pi*b*d*sgn(F) + I*pi*b*d + 2*
b*d*log(F) - 2*b*d*log(abs(F)))e^(1/2*I*pi*b*x*sgn(F) - 1/2*I*pi*b*x + 1/
2*I*pi*a*sgn(F) - 1/2*I*pi*a)/(pi^2*b^3*log(F)*sgn(F) + 2*I*pi*b^3*log(F)*
log(abs(F))*sgn(F) - pi^2*b^3*log(F) - 2*I*pi*b^3*log(F)*log(abs(F)) + 2*b
^3*log(F)*log(abs(F))^2) + (I*pi*b^2*d*x*log(F)*sgn(F) - I*pi*b^2*d*x*log(
F) - 2*b^2*d*x*log(F)*log(abs(F)) + I*pi*b*d*sgn(F) - I*pi*b*d + 2*b*d*log
(F) - 2*b*d*log(abs(F)))e^(-1/2*I*pi*b*x*sgn(F) + 1/2*I*pi*b*x - 1/2*I*pi
*a*sgn(F) + 1/2*I*pi*a)/(pi^2*b^3*log(F)*sgn(F) - 2*I*pi*b^3*log(F)*log(ab
s(F))*sgn(F) - pi^2*b^3*log(F) + 2*I*pi*b^3*log(F)*log(abs(F)) + 2*b^3*...
```

**Mupad [B] (verification not implemented)**

Time = 22.84 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \frac{F^{a+bx} dx}{b \ln(F)}$$

input `int(F^(a + b*x)*(d*x + d/(b*log(F))),x)`output `(F^(a + b*x)*d*x)/(b*log(F))`**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int F^{a+bx} \left( dx + \frac{d}{b \log(F)} \right) dx = \frac{f^{bx+a} dx}{\log(f) b}$$

input `int(F^(b*x+a)*(d*x+d/b/log(F)),x)`output `(f**(a + b*x)*d*x)/(log(f)*b)`

### 3.57 $\int F^{2+5x} dx$

Optimal result	415
Mathematica [A] (verified)	415
Rubi [A] (verified)	416
Maple [A] (verified)	417
Fricas [A] (verification not implemented)	417
Sympy [A] (verification not implemented)	418
Maxima [A] (verification not implemented)	418
Giac [A] (verification not implemented)	418
Mupad [B] (verification not implemented)	419
Reduce [B] (verification not implemented)	419

#### Optimal result

Integrand size = 7, antiderivative size = 15

$$\int F^{2+5x} dx = \frac{F^{2+5x}}{5 \log(F)}$$

output

```
1/5*F^(2+5*x)/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{2+5x} dx = \frac{F^{2+5x}}{5 \log(F)}$$

input

```
Integrate[F^(2 + 5*x), x]
```

output

```
F^(2 + 5*x)/(5*Log[F])
```



**Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{5x+2} dx$$

$$\downarrow 2624$$

$$\frac{F^{5x+2}}{5 \log(F)}$$

input `Int [F^(2 + 5*x), x]`

output `F^(2 + 5*x)/(5*Log[F])`

**Defintions of rubi rules used**

rule 2624

```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gospers	$\frac{F^{2+5x}}{5 \ln(F)}$	14
derivativdivides	$\frac{F^{2+5x}}{5 \ln(F)}$	14
default	$\frac{F^{2+5x}}{5 \ln(F)}$	14
risch	$\frac{F^{2+5x}}{5 \ln(F)}$	14
parallelrisch	$\frac{F^{2+5x}}{5 \ln(F)}$	14
orering	$\frac{F^{2+5x}}{5 \ln(F)}$	14
norman	$\frac{e^{(2+5x) \ln(F)}}{5 \ln(F)}$	16
meijerg	$-\frac{F^2 (1 - e^{5x \ln(F)})}{5 \ln(F)}$	20

input `int(F^(2+5*x),x,method=_RETURNVERBOSE)`output `1/5*F^(2+5*x)/ln(F)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int F^{2+5x} dx = \frac{F^{5x+2}}{5 \log(F)}$$

input `integrate(F^(2+5*x),x, algorithm="fricas")`output `1/5*F^(5*x + 2)/log(F)`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int F^{2+5x} dx = \begin{cases} \frac{F^{5x+2}}{5 \log(F)} & \text{for } \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(F**(2+5*x), x)`output `Piecewise((F**(5*x + 2)/(5*log(F)), Ne(log(F), 0)), (x, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int F^{2+5x} dx = \frac{F^{5x+2}}{5 \log(F)}$$

input `integrate(F^(2+5*x), x, algorithm="maxima")`output `1/5*F^(5*x + 2)/log(F)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int F^{2+5x} dx = \frac{F^{5x+2}}{5 \log(F)}$$

input `integrate(F^(2+5*x), x, algorithm="giac")`output `1/5*F^(5*x + 2)/log(F)`

**Mupad [B] (verification not implemented)**

Time = 23.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int F^{2+5x} dx = \frac{F^{5x+2}}{5 \ln(F)}$$

input `int(F^(5*x + 2),x)`

output `F^(5*x + 2)/(5*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int F^{2+5x} dx = \frac{f^{5x} f^2}{5 \log(f)}$$

input `int(F^(2+5*x),x)`

output `(f**(5*x)*f**2)/(5*log(f))`

### 3.58 $\int F^{a+bx} dx$

Optimal result . . . . .	420
Mathematica [A] (verified) . . . . .	420
Rubi [A] (verified) . . . . .	421
Maple [A] (verified) . . . . .	422
Fricas [A] (verification not implemented) . . . . .	422
Sympy [A] (verification not implemented) . . . . .	423
Maxima [A] (verification not implemented) . . . . .	423
Giac [A] (verification not implemented) . . . . .	423
Mupad [B] (verification not implemented) . . . . .	424
Reduce [B] (verification not implemented) . . . . .	424

#### Optimal result

Integrand size = 7, antiderivative size = 15

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \log(F)}$$

output

$F^{(b*x+a)}/b/\ln(F)$

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \log(F)}$$

input

`Integrate[F^(a + b*x), x]`

output

$F^{(a + b*x)}/(b*\text{Log}[F])$

**Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{a+bx} dx$$

$$\downarrow 2624$$

$$\frac{F^{a+bx}}{b \log(F)}$$

input `Int[F^(a + b*x), x]`

output `F^(a + b*x)/(b*Log[F])`

**Defintions of rubi rules used**

rule 2624

```
Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
gospers	$\frac{F^{bx+a}}{b \ln(F)}$	16
derivativdivides	$\frac{F^{bx+a}}{b \ln(F)}$	16
default	$\frac{F^{bx+a}}{b \ln(F)}$	16
risch	$\frac{F^{bx+a}}{b \ln(F)}$	16
parallelrisc	$\frac{F^{bx+a}}{b \ln(F)}$	16
orering	$\frac{F^{bx+a}}{b \ln(F)}$	16
norman	$\frac{e^{(bx+a) \ln(F)}}{b \ln(F)}$	18
meijerg	$-\frac{F^a (1 - e^{\ln(F)bx})}{b \ln(F)}$	23

input `int(F^(b*x+a),x,method=_RETURNVERBOSE)`output `F^(b*x+a)/b/ln(F)`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{F^{bx+a}}{b \log(F)}$$

input `integrate(F^(b*x+a),x, algorithm="fricas")`output `F^(b*x + a)/(b*log(F))`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \begin{cases} \frac{F^{a+bx}}{b \log(F)} & \text{for } b \log(F) \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(F**(b*x+a), x)`output `Piecewise((F**(a + b*x)/(b*log(F)), Ne(b*log(F), 0)), (x, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{F^{bx+a}}{b \log(F)}$$

input `integrate(F^(b*x+a), x, algorithm="maxima")`output `F^(b*x + a)/(b*log(F))`**Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{F^{bx+a}}{b \log(F)}$$

input `integrate(F^(b*x+a), x, algorithm="giac")`output `F^(b*x + a)/(b*log(F))`



**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{F^{a+bx}}{b \ln(F)}$$

input `int(F^(a + b*x), x)`

output `F^(a + b*x)/(b*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int F^{a+bx} dx = \frac{f^{bx+a}}{\log(f) b}$$

input `int(F^(b*x+a), x)`

output `f**(a + b*x)/(log(f)*b)`

### 3.59 $\int 10^{2+5x} dx$

Optimal result	425
Mathematica [A] (verified)	425
Rubi [A] (verified)	426
Maple [A] (verified)	427
Fricas [A] (verification not implemented)	427
Sympy [A] (verification not implemented)	428
Maxima [A] (verification not implemented)	428
Giac [A] (verification not implemented)	428
Mupad [B] (verification not implemented)	429
Reduce [B] (verification not implemented)	429

#### Optimal result

Integrand size = 7, antiderivative size = 19

$$\int 10^{2+5x} dx = \frac{2^{2+5x} 5^{1+5x}}{\log(10)}$$

output

```
2^(2+5*x)*5^(1+5*x)/ln(10)
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int 10^{2+5x} dx = \frac{2^{2+5x} 5^{1+5x}}{\log(10)}$$

input

```
Integrate[10^(2 + 5*x), x]
```

output

```
(2^(2 + 5*x)*5^(1 + 5*x))/Log[10]
```

**Rubi [A] (verified)**

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 10^{5x+2} dx$$

$$\downarrow 2624$$

$$\frac{2^{5x+2}5^{5x+1}}{\log(10)}$$

input `Int[10^(2 + 5*x), x]`

output `(2^(2 + 5*x)*5^(1 + 5*x))/Log[10]`

**Defintions of rubi rules used**

rule 2624

```
Int[((F_)^(v_))^(n_), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{20 \cdot 10^{5x}}{\ln(10)}$	12
default	$\frac{20 \cdot 10^{5x}}{\ln(10)}$	12
gosper	$\frac{10^{2+5x}}{5 \ln(10)}$	14
parallelrisch	$\frac{10^{2+5x}}{5 \ln(10)}$	14
norman	$\frac{e^{(2+5x) \ln(10)}}{5 \ln(10)}$	16
risch	$\frac{20 \cdot 3125^x \cdot 32^x}{\ln(2) + \ln(5)}$	16
meijerg	$-\frac{20(1 - e^{5x \ln(10)})}{\ln(10)}$	17
orering	$\frac{10^{2+5x}}{5 \ln(2) + 5 \ln(5)}$	17

input `int(10^(2+5*x), x, method=_RETURNVERBOSE)`output `20*(10^x)^5/ln(10)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int 10^{2+5x} dx = \frac{10^{5x+2}}{5 \log(10)}$$

input `integrate(10^(2+5*x), x, algorithm="fricas")`output `1/5*10^(5*x + 2)/log(10)`

**Sympy [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

$$\int 10^{2+5x} dx = \frac{10^{5x+2}}{5 \log(10)}$$

input `integrate(10**(2+5*x),x)`

output `10**(5*x + 2)/(5*log(10))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int 10^{2+5x} dx = \frac{10^{5x+2}}{5 \log(10)}$$

input `integrate(10^(2+5*x),x, algorithm="maxima")`

output `1/5*10^(5*x + 2)/log(10)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int 10^{2+5x} dx = \frac{10^{5x+2}}{5 \log(10)}$$

input `integrate(10^(2+5*x),x, algorithm="giac")`

output `1/5*10^(5*x + 2)/log(10)`

**Mupad [B] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int 10^{2+5x} dx = \frac{20 \cdot 10^{5x}}{\ln(10)}$$

input `int(10^(5*x + 2), x)`

output `(20*10^(5*x))/log(10)`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int 10^{2+5x} dx = \frac{20 \cdot 10^{5x}}{\log(10)}$$

input `int(10^(2+5*x), x)`

output `(20*10**(5*x))/log(10)`

### 3.60 $\int F^{a+bx} x^{7/2} dx$

Optimal result	430
Mathematica [A] (verified)	430
Rubi [A] (verified)	431
Maple [A] (verified)	433
Fricas [A] (verification not implemented)	433
Sympy [F]	434
Maxima [A] (verification not implemented)	434
Giac [A] (verification not implemented)	434
Mupad [B] (verification not implemented)	435
Reduce [B] (verification not implemented)	435

#### Optimal result

Integrand size = 13, antiderivative size = 131

$$\int F^{a+bx} x^{7/2} dx = \frac{105F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{16b^{9/2} \log^{9/2}(F)} - \frac{105F^{a+bx} \sqrt{x}}{8b^4 \log^4(F)} + \frac{35F^{a+bx} x^{3/2}}{4b^3 \log^3(F)} - \frac{7F^{a+bx} x^{5/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{7/2}}{b \log(F)}$$

output

```
105/16*F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))/b^(9/2)/ln(F)^(9/2)-
105/8*F^(b*x+a)*x^(1/2)/b^4/ln(F)^4+35/4*F^(b*x+a)*x^(3/2)/b^3/ln(F)^3-7/2
*F^(b*x+a)*x^(5/2)/b^2/ln(F)^2+F^(b*x+a)*x^(7/2)/b/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.27

$$\int F^{a+bx} x^{7/2} dx = \frac{F^a \Gamma\left(\frac{9}{2}, -bx \log(F)\right) \sqrt{-bx \log(F)}}{b^5 \sqrt{x} \log^5(F)}$$

input

```
Integrate[F^(a + b*x)*x^(7/2), x]
```

```
output (F^a*Gamma[9/2, -(b*x*Log[F])]*Sqrt[-(b*x*Log[F])])/(b^5*Sqrt[x]*Log[F]^5)
```

**Rubi [A] (verified)**

Time = 0.77 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$ , Rules used = {2607, 2607, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^{7/2} F^{a+bx} dx \\
 & \quad \downarrow \text{2607} \\
 & \frac{x^{7/2} F^{a+bx}}{b \log(F)} - \frac{7 \int F^{a+bx} x^{5/2} dx}{2b \log(F)} \\
 & \quad \downarrow \text{2607} \\
 & \frac{x^{7/2} F^{a+bx}}{b \log(F)} - \frac{7 \left( \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \int F^{a+bx} x^{3/2} dx}{2b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow \text{2607} \\
 & \frac{x^{7/2} F^{a+bx}}{b \log(F)} - \frac{7 \left( \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \int F^{a+bx} \sqrt{x} dx}{2b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow \text{2607} \\
 & \frac{x^{7/2} F^{a+bx}}{b \log(F)} - \frac{7 \left( \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int F^{a+bx} dx}{2b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)}
 \end{aligned}$$



$$\begin{aligned}
 & \downarrow \text{2611} \\
 & \frac{x^{7/2} F^{a+bx}}{b \log(F)} - \frac{7 \left( \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int F^{a+bx} d\sqrt{x}}{b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \\
 & \downarrow \text{2633} \\
 & \frac{x^{7/2} F^{a+bx}}{b \log(F)} - \frac{7 \left( \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi} F^a \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\log(F)})}{2b^{3/2} \log^3(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)}
 \end{aligned}$$

input

```
Int[F^(a + b*x)*x^(7/2),x]
```

output

```
(F^(a + b*x)*x^(7/2))/(b*Log[F]) - (7*((F^(a + b*x)*x^(5/2))/(b*Log[F]) - (5*((F^(a + b*x)*x^(3/2))/(b*Log[F]) - (3*(-1/2*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]])]/(b^(3/2)*Log[F]^(3/2)) + (F^(a + b*x)*Sqrt[x])/(b*Log[F])))/(2*b*Log[F])))/(2*b*Log[F]))/(2*b*Log[F])
```

**Defintions of rubi rules used**

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76

method	result	size
meijerg	$F^a \left( -\frac{\sqrt{x}(-b)^{\frac{9}{2}}\sqrt{\ln(F)}(-72b^3x^3\ln(F)^3+252b^2x^2\ln(F)^2-630\ln(F)bx+945)e^{\ln(F)bx}+105(-b)^{\frac{9}{2}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{x}\sqrt{\ln(F)}}{16b^{\frac{9}{2}}}\right)}{(-b)^{\frac{7}{2}}\ln(F)^{\frac{9}{2}}b} \right)$	99

input

```
int(F^(b*x+a)*x^(7/2),x,method=_RETURNVERBOSE)
```

output

```
-F^a/(-b)^(7/2)/ln(F)^(9/2)/b*(-1/72*x^(1/2)*(-b)^(9/2)*ln(F)^(1/2)*(-72*b
^3*x^3*ln(F)^3+252*b^2*x^2*ln(F)^2-630*ln(F)*b*x+945)/b^4*exp(ln(F)*b*x)+
105/16*(-b)^(9/2)/b^(9/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int F^{a+bx} x^{7/2} dx =$$

$$\frac{105\sqrt{\pi}\sqrt{-b\log(F)}F^a\operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right)-2(8b^4x^3\log(F)^4-28b^3x^2\log(F)^3+70b^2x\log(F)^2-105b\log(F))F^{(b*x+a)}\sqrt{x}}{16b^5\log(F)^5}$$

input

```
integrate(F^(b*x+a)*x^(7/2),x, algorithm="fricas")
```

output

```
-1/16*(105*sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x))-2*(
8*b^4*x^3*log(F)^4-28*b^3*x^2*log(F)^3+70*b^2*x*log(F)^2-105*b*log(F)
))*F^(b*x+a)*sqrt(x)/(b^5*log(F)^5)
```

**Sympy [F]**

$$\int F^{a+bx} x^{7/2} dx = \int F^{a+bx} x^{\frac{7}{2}} dx$$

input `integrate(F**(b*x+a)*x**(7/2),x)`

output `Integral(F**(a + b*x)*x**(7/2), x)`

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.18

$$\int F^{a+bx} x^{7/2} dx = -\frac{F^a x^{\frac{9}{2}} \Gamma\left(\frac{9}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{9}{2}}}$$

input `integrate(F^(b*x+a)*x^(7/2),x, algorithm="maxima")`

output `-F^a*x^(9/2)*gamma(9/2, -b*x*log(F))/(-b*x*log(F))^(9/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

$$\int F^{a+bx} x^{7/2} dx = -\frac{105 \sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{16 \sqrt{-b \log(F)} b^4 \log(F)^4} + \frac{\left(8 b^3 x^{\frac{7}{2}} \log(F)^3 - 28 b^2 x^{\frac{5}{2}} \log(F)^2 + 70 b x^{\frac{3}{2}} \log(F) - 105 \sqrt{x}\right) e^{(bx \log(F)+a \log(F))}}{8 b^4 \log(F)^4}$$

input `integrate(F^(b*x+a)*x^(7/2),x, algorithm="giac")`

output

```
-105/16*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/(sqrt(-b*log(F))*b^4*log(F)^4) + 1/8*(8*b^3*x^(7/2)*log(F)^3 - 28*b^2*x^(5/2)*log(F)^2 + 70*b*x^(3/2)*log(F) - 105*sqrt(x))*e^(b*x*log(F) + a*log(F))/(b^4*log(F)^4)
```

**Mupad [B] (verification not implemented)**

Time = 23.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.63

$$\int F^{a+bx} x^{7/2} dx = \frac{F^a x^{7/2} \left( \frac{105\sqrt{\pi} \operatorname{erfc}(\sqrt{-bx \ln(F)})}{16} + F^{bx} \left( \frac{105\sqrt{-bx \ln(F)}}{8} + \frac{35(-bx \ln(F))^{3/2}}{4} + \frac{7(-bx \ln(F))^{5/2}}{2} + \dots \right) \right)}{b \ln(F) (-bx \ln(F))^{7/2}}$$

input

```
int(F^(a + b*x)*x^(7/2), x)
```

output

```
(F^a*x^(7/2)*((105*pi^(1/2)*erfc((-b*x*log(F))^(1/2)))/16 + F^(b*x)*((105*(-b*x*log(F))^(1/2))/8 + (35*(-b*x*log(F))^(3/2))/4 + (7*(-b*x*log(F))^(5/2))/2 + (-b*x*log(F))^(7/2))))/(b*log(F)*(-b*x*log(F))^(7/2))
```

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int F^{a+bx} x^{7/2} dx = \frac{f^a \left( -105\sqrt{\pi} \operatorname{erf}(\sqrt{x} \sqrt{b} \sqrt{\log(f)}) i + 16\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \log(f)^3 b^3 x^3 - 56\sqrt{x} f^{bx} \dots \right)}{16\sqrt{b} \sqrt{\log(f)}}$$

input

```
int(F^(b*x+a)*x^(7/2), x)
```

output

```
(f**a*( - 105*sqrt(pi)*erf(sqrt(x)*sqrt(b)*sqrt(log(f))*i)*i + 16*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f))*log(f)**3*b**3*x**3 - 56*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f))*log(f)**2*b**2*x**2 + 140*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f))*log(f)*b*x - 210*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f))))/(16*sqrt(b)*sqrt(log(f))*log(f)**4*b**4)
```

### 3.61 $\int F^{a+bx} x^{5/2} dx$

Optimal result	436
Mathematica [A] (verified)	436
Rubi [A] (verified)	437
Maple [A] (verified)	439
Fricas [A] (verification not implemented)	439
Sympy [F]	440
Maxima [A] (verification not implemented)	440
Giac [A] (verification not implemented)	440
Mupad [B] (verification not implemented)	441
Reduce [B] (verification not implemented)	441

#### Optimal result

Integrand size = 13, antiderivative size = 108

$$\int F^{a+bx} x^{5/2} dx = -\frac{15F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{8b^{7/2} \log^{7/2}(F)} + \frac{15F^{a+bx} \sqrt{x}}{4b^3 \log^3(F)} - \frac{5F^{a+bx} x^{3/2}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{5/2}}{b \log(F)}$$

output

```
-15/8*F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))/b^(7/2)/ln(F)^(7/2)+15/4*F^(b*x+a)*x^(1/2)/b^3/ln(F)^3-5/2*F^(b*x+a)*x^(3/2)/b^2/ln(F)^2+F^(b*x+a)*x^(5/2)/b/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.33

$$\int F^{a+bx} x^{5/2} dx = \frac{F^a \sqrt{x} \Gamma\left(\frac{7}{2}, -bx \log(F)\right)}{b^3 \log^3(F) \sqrt{-bx \log(F)}}$$

input

```
Integrate[F^(a + b*x)*x^(5/2), x]
```

output  $(F^a \sqrt{x} \Gamma[7/2, -(b*x*\text{Log}[F])]) / (b^3 * \text{Log}[F]^3 * \sqrt{-(b*x*\text{Log}[F])})$

### Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2607, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^{5/2} F^{a+bx} dx \\
 & \quad \downarrow 2607 \\
 & \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \int F^{a+bx} x^{3/2} dx}{2b \log(F)} \\
 & \quad \downarrow 2607 \\
 & \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \int F^{a+bx} \sqrt{x} dx}{2b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow 2607 \\
 & \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int \frac{F^{a+bx}}{\sqrt{x}} dx}{2b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow 2611 \\
 & \frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int F^{a+bx} d\sqrt{x}}{b \log(F)} \right)}{2b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow 2633
 \end{aligned}$$

$$\frac{x^{5/2} F^{a+bx}}{b \log(F)} - \frac{5 \left( \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi} F^a \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\log(F)})}{2b^{3/2} \log^2(F)} \right)}{2b \log(F)} \right)}{2b \log(F)}$$

input `Int[F^(a + b*x)*x^(5/2),x]`

output `(F^(a + b*x)*x^(5/2))/(b*Log[F]) - (5*((F^(a + b*x)*x^(3/2))/(b*Log[F]) - (3*(-1/2*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]])])/(b^(3/2)*Log[F]^(3/2)) + (F^(a + b*x)*Sqrt[x])/(b*Log[F])))/(2*b*Log[F]))/(2*b*Log[F])`

### Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

method	result	size
meijerg	$F^a \left( \frac{\sqrt{x} (-b)^{\frac{7}{2}} \sqrt{\ln(F)} (28b^2 x^2 \ln(F)^2 - 70 \ln(F) b x + 105) e^{\ln(F) b x}}{28b^3} - \frac{15(-b)^{\frac{7}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{8b^{\frac{7}{2}}} \right)$ $\frac{\quad}{(-b)^{\frac{5}{2}} \ln(F)^{\frac{7}{2}} b}$	87

input `int(F^(b*x+a)*x^(5/2),x,method=_RETURNVERBOSE)`

output `-F^a/(-b)^(5/2)/ln(F)^(7/2)/b*(1/28*x^(1/2)*(-b)^(7/2)*ln(F)^(1/2)*(28*b^2*x^2*ln(F)^2-70*ln(F)*b*x+105)/b^3*exp(ln(F)*b*x)-15/8*(-b)^(7/2)/b^(7/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`

**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int F^{a+bx} x^{5/2} dx = \frac{15 \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}(\sqrt{-b \log(F)} \sqrt{x}) + 2(4b^3 x^2 \log(F)^3 - 10b^2 x \log(F)^2 + 15b \log(F)) F^{b x + a} \sqrt{x}}{8b^4 \log(F)^4}$$

input `integrate(F^(b*x+a)*x^(5/2),x, algorithm="fricas")`

output `1/8*(15*sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x)) + 2*(4*b^3*x^2*log(F)^3 - 10*b^2*x*log(F)^2 + 15*b*log(F))*F^(b*x + a)*sqrt(x))/(b^4*log(F)^4)`



**Sympy [F]**

$$\int F^{a+bx} x^{5/2} dx = \int F^{a+bx} x^{\frac{5}{2}} dx$$

input `integrate(F**(b*x+a)*x**(5/2),x)`

output `Integral(F**(a + b*x)*x**(5/2), x)`

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.22

$$\int F^{a+bx} x^{5/2} dx = -\frac{F^a x^{\frac{7}{2}} \Gamma\left(\frac{7}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{7}{2}}}$$

input `integrate(F^(b*x+a)*x^(5/2),x, algorithm="maxima")`

output `-F^a*x^(7/2)*gamma(7/2, -b*x*log(F))/(-b*x*log(F))^(7/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int F^{a+bx} x^{5/2} dx = \frac{15 \sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{8 \sqrt{-b \log(F)} b^3 \log(F)^3} + \frac{\left(4 b^2 x^{\frac{5}{2}} \log(F)^2 - 10 b x^{\frac{3}{2}} \log(F) + 15 \sqrt{x}\right) e^{(bx \log(F) + a \log(F))}}{4 b^3 \log(F)^3}$$

input `integrate(F^(b*x+a)*x^(5/2),x, algorithm="giac")`

output 
$$\frac{15\sqrt{8}\sqrt{\pi}F^a \operatorname{erf}(-\sqrt{-b\log(F)}\sqrt{x})/(\sqrt{-b\log(F)}b^3\log(F)^3) + 1/4(4b^2x^{5/2}\log(F)^2 - 10b^2x^{3/2}\log(F) + 15\sqrt{x})e^{(b^2x\log(F) + a\log(F))/b^3\log(F)^3}}{b^3\log(F)^3}$$

### Mupad [B] (verification not implemented)

Time = 17.45 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.67

$$\int F^{a+bx} x^{5/2} dx = \frac{F^a x^{5/2} \left( F^{bx} \left( \frac{15\sqrt{-bx \ln(F)}}{4} + \frac{5(-bx \ln(F))^{3/2}}{2} + (-bx \ln(F))^{5/2} \right) + \frac{15\sqrt{\pi} \operatorname{erfc}\left(\frac{\sqrt{-bx \ln(F)}}{8}\right)}{8} \right)}{b \ln(F) (-bx \ln(F))^{5/2}}$$

input `int(F^(a + b*x)*x^(5/2),x)`

output 
$$\frac{(F^a x^{5/2} (F^{bx} ((15\sqrt{-bx \log(F)})^{1/2})/4 + (5(-bx \log(F))^{3/2})/2 + (-bx \log(F))^{5/2})) + (15\pi^{1/2} \operatorname{erfc}((-bx \log(F))^{1/2}))/8)}{b \log(F) (-bx \log(F))^{5/2}}$$

### Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int F^{a+bx} x^{5/2} dx = \frac{f^a \left( 15\sqrt{\pi} \operatorname{erf}\left(\sqrt{x} \sqrt{b} \sqrt{\log(f)}\right) i + 8\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \log(f)^2 b^2 x^2 - 20\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \log(f) + 15\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \right)}{8\sqrt{b} \sqrt{\log(f)} \log(f)^3 b^3}$$

input `int(F^(b*x+a)*x^(5/2),x)`

output 
$$\frac{(f^{a+bx} (15\sqrt{\pi} \operatorname{erf}(\sqrt{x} \sqrt{b} \sqrt{\log(f)}) i) i + 8\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \log(f)^2 b^2 x^2 - 20\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \log(f) + 15\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)})}{8\sqrt{b} \sqrt{\log(f)} \log(f)^3 b^3}$$

### 3.62 $\int F^{a+bx} x^{3/2} dx$

Optimal result	442
Mathematica [A] (verified)	442
Rubi [A] (verified)	443
Maple [A] (verified)	444
Fricas [A] (verification not implemented)	445
Sympy [F]	445
Maxima [A] (verification not implemented)	445
Giac [A] (verification not implemented)	446
Mupad [B] (verification not implemented)	446
Reduce [B] (verification not implemented)	447

#### Optimal result

Integrand size = 13, antiderivative size = 85

$$\int F^{a+bx} x^{3/2} dx = \frac{3F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{4b^{5/2} \log^{5/2}(F)} - \frac{3F^{a+bx} \sqrt{x}}{2b^2 \log^2(F)} + \frac{F^{a+bx} x^{3/2}}{b \log(F)}$$

output

```
3/4*F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))/b^(5/2)/ln(F)^(5/2)-3/2
*F^(b*x+a)*x^(1/2)/b^2/ln(F)^2+F^(b*x+a)*x^(3/2)/b/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

$$\int F^{a+bx} x^{3/2} dx = \frac{F^a \Gamma\left(\frac{5}{2}, -bx \log(F)\right) \sqrt{-bx \log(F)}}{b^3 \sqrt{x} \log^3(F)}$$

input

```
Integrate[F^(a + b*x)*x^(3/2),x]
```

output

```
(F^a*Gamma[5/2, -(b*x*Log[F])]*Sqrt[-(b*x*Log[F])])/(b^3*Sqrt[x]*Log[F]^3)
```

**Rubi [A] (verified)**

Time = 0.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^{3/2} F^{a+bx} dx \\
 & \quad \downarrow 2607 \\
 & \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \int F^{a+bx} \sqrt{x} dx}{2b \log(F)} \\
 & \quad \downarrow 2607 \\
 & \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int \frac{F^{a+bx}}{\sqrt{x}} dx}{2b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow 2611 \\
 & \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int F^{a+bx} d\sqrt{x}}{b \log(F)} \right)}{2b \log(F)} \\
 & \quad \downarrow 2633 \\
 & \frac{x^{3/2} F^{a+bx}}{b \log(F)} - \frac{3 \left( \frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi} F^a \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\log(F)})}{2b^{3/2} \log^{3/2}(F)} \right)}{2b \log(F)}
 \end{aligned}$$

input `Int[F^(a + b*x)*x^(3/2),x]`

output `(F^(a + b*x)*x^(3/2))/(b*Log[F]) - (3*(-1/2*(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]])]/(b^(3/2)*Log[F]^(3/2)) + (F^(a + b*x)*Sqrt[x])/(b*Log[F])))/(2*b*Log[F])`

## Definitions of rubi rules used

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

method	result	size
meijerg	$F^a \left( \frac{-\sqrt{x} (-b)^{\frac{5}{2}} \sqrt{\ln(F)} (-10 \ln(F) b x + 15) e^{\ln(F) b x} + \frac{3(-b)^{\frac{5}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{4b^{\frac{5}{2}}}}{(-b)^{\frac{3}{2}} \ln(F)^{\frac{5}{2}} b} \right)$	75

input

```
int(F^(b*x+a)*x^(3/2), x, method=_RETURNVERBOSE)
```

output

```
-F^a/(-b)^(3/2)/ln(F)^(5/2)/b*(-1/10*x^(1/2)*(-b)^(5/2)*ln(F)^(1/2)*(-10*1
n(F)*b*x+15)/b^2*exp(ln(F)*b*x)+3/4*(-b)^(5/2)/b^(5/2)*Pi^(1/2)*erfi(b^(1/
2)*x^(1/2)*ln(F)^(1/2))
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int F^{a+bx} x^{3/2} dx = \frac{3\sqrt{\pi}\sqrt{-b\log(F)}F^a \operatorname{erf}\left(\sqrt{-b\log(F)}\sqrt{x}\right) - 2(2b^2x\log(F)^2 - 3b\log(F))F^{bx+a}\sqrt{x}}{4b^3\log(F)^3}$$

input `integrate(F^(b*x+a)*x^(3/2),x, algorithm="fricas")`

output `-1/4*(3*sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x)) - 2*(2*b^2*x*log(F)^2 - 3*b*log(F))*F^(b*x + a)*sqrt(x))/(b^3*log(F)^3)`

**Sympy [F]**

$$\int F^{a+bx} x^{3/2} dx = \int F^{a+bx} x^{\frac{3}{2}} dx$$

input `integrate(F**(b*x+a)*x**(3/2),x)`

output `Integral(F**(a + b*x)*x**(3/2), x)`

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.28

$$\int F^{a+bx} x^{3/2} dx = -\frac{F^a x^{\frac{5}{2}} \Gamma\left(\frac{5}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{5}{2}}}$$

input `integrate(F^(b*x+a)*x^(3/2),x, algorithm="maxima")`

output `-F^a*x^(5/2)*gamma(5/2, -b*x*log(F))/(-b*x*log(F))^(5/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.82

$$\int F^{a+bx} x^{3/2} dx = -\frac{3\sqrt{\pi}F^a \operatorname{erf}\left(-\sqrt{-b\log(F)}\sqrt{x}\right)}{4\sqrt{-b\log(F)}b^2\log(F)^2} + \frac{\left(2bx^{\frac{3}{2}}\log(F) - 3\sqrt{x}\right)e^{(bx\log(F)+a\log(F))}}{2b^2\log(F)^2}$$

input `integrate(F^(b*x+a)*x^(3/2),x, algorithm="giac")`output `-3/4*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/(sqrt(-b*log(F))*b^2*log(F)^2) + 1/2*(2*b*x^(3/2)*log(F) - 3*sqrt(x))*e^(b*x*log(F) + a*log(F))/(b^2*log(F)^2)`**Mupad [B] (verification not implemented)**

Time = 24.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int F^{a+bx} x^{3/2} dx = \frac{F^a F^{bx} x^{3/2}}{b \ln(F)} - \frac{3 F^a F^{bx} \sqrt{x}}{2 b^2 \ln(F)^2} + \frac{3 F^a x^{3/2} \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-b x \ln(F)}\right)}{4 b \ln(F) (-b x \ln(F))^{3/2}}$$

input `int(F^(a + b*x)*x^(3/2),x)`output `(F^a*F^(b*x)*x^(3/2))/(b*log(F)) - (3*F^a*F^(b*x)*x^(1/2))/(2*b^2*log(F)^2) + (3*F^a*x^(3/2)*pi^(1/2)*erfc((-b*x*log(F))^(1/2)))/(4*b*log(F)*(-b*x*log(F))^(3/2))`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int F^{a+bx} x^{3/2} dx = \frac{f^a \left( -3\sqrt{\pi} \operatorname{erf} \left( \sqrt{x} \sqrt{b} \sqrt{\log(f)} \right) i + 4\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \log(f) bx - 6\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \right)}{4\sqrt{b} \sqrt{\log(f)} \log(f)^2 b^2}$$

input `int(F^(b*x+a)*x^(3/2),x)`output `(f**a*( - 3*sqrt(pi)*erf(sqrt(x)*sqrt(b)*sqrt(log(f))*i)*i + 4*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f))*log(f)*b*x - 6*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f)))/(4*sqrt(b)*sqrt(log(f))*log(f)**2*b**2)`



### 3.63 $\int F^{a+bx} \sqrt{x} dx$

Optimal result	448
Mathematica [A] (verified)	448
Rubi [A] (verified)	449
Maple [A] (verified)	450
Fricas [A] (verification not implemented)	450
Sympy [F]	451
Maxima [A] (verification not implemented)	451
Giac [A] (verification not implemented)	451
Mupad [B] (verification not implemented)	452
Reduce [B] (verification not implemented)	452

#### Optimal result

Integrand size = 13, antiderivative size = 62

$$\int F^{a+bx} \sqrt{x} dx = -\frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{2b^{3/2} \log^{3/2}(F)} + \frac{F^{a+bx} \sqrt{x}}{b \log(F)}$$

output

```
-1/2*F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))/b^(3/2)/ln(F)^(3/2)+F^(b*x+a)*x^(1/2)/b/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.48

$$\int F^{a+bx} \sqrt{x} dx = -\frac{F^a x^{3/2} \Gamma\left(\frac{3}{2}, -bx \log(F)\right)}{(-bx \log(F))^{3/2}}$$

input

```
Integrate[F^(a + b*x)*Sqrt[x], x]
```

output

```
-((F^a*x^(3/2)*Gamma[3/2, -(b*x*Log[F])])/(-(b*x*Log[F]))^(3/2))
```

**Rubi [A] (verified)**

Time = 0.39 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x} F^{a+bx} dx$$

$$\downarrow 2607$$

$$\frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int \frac{F^{a+bx}}{\sqrt{x}} dx}{2b \log(F)}$$

$$\downarrow 2611$$

$$\frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\int F^{a+bx} d\sqrt{x}}{b \log(F)}$$

$$\downarrow 2633$$

$$\frac{\sqrt{x} F^{a+bx}}{b \log(F)} - \frac{\sqrt{\pi} F^a \operatorname{erfi}\left(\sqrt{b} \sqrt{x} \sqrt{\log(F)}\right)}{2b^{3/2} \log^{3/2}(F)}$$

input `Int[F^(a + b*x)*Sqrt[x],x]`

output 
$$-1/2*(F^a*\sqrt{\pi}*\operatorname{Erfi}[\sqrt{b}*\sqrt{x}*\sqrt{\log[F]}])/(b^{3/2}*\log[F]^{3/2}) + (F^{(a + b*x)}*\sqrt{x})/(b*\log[F])$$

**Defintions of rubi rules used**

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

### Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

method	result	size
meijerg	$-\frac{F^a \left( \frac{\sqrt{x} (-b)^{\frac{3}{2}} \sqrt{\ln(F)} e^{\ln(F)bx}}{b} - (-b)^{\frac{3}{2}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{x} \sqrt{\ln(F)}}{2b^{\frac{3}{2}}} \right) \right)}{\sqrt{-b} \ln(F)^{\frac{3}{2}} b}$	66

input `int(F^(b*x+a)*x^(1/2),x,method=_RETURNVERBOSE)`

output `-F^a/(-b)^(1/2)/ln(F)^(3/2)/b*(x^(1/2)*(-b)^(3/2)*ln(F)^(1/2)/b*exp(ln(F)*b*x)-1/2*(-b)^(3/2)/b^(3/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`

### Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int F^{a+bx} \sqrt{x} dx = \frac{2 F^{bx+a} b \sqrt{x} \log(F) + \sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right)}{2 b^2 \log(F)^2}$$

input `integrate(F^(b*x+a)*x^(1/2),x, algorithm="fricas")`

output `1/2*(2*F^(b*x + a)*b*sqrt(x)*log(F) + sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x)))/(b^2*log(F)^2)`

**Sympy [F]**

$$\int F^{a+bx} \sqrt{x} dx = \int F^{a+bx} \sqrt{x} dx$$

input `integrate(F**(b*x+a)*x**(1/2),x)`

output `Integral(F**(a + b*x)*sqrt(x), x)`

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.39

$$\int F^{a+bx} \sqrt{x} dx = -\frac{F^a x^{\frac{3}{2}} \Gamma\left(\frac{3}{2}, -bx \log(F)\right)}{(-bx \log(F))^{\frac{3}{2}}}$$

input `integrate(F^(b*x+a)*x^(1/2),x, algorithm="maxima")`

output `-F^a*x^(3/2)*gamma(3/2, -b*x*log(F))/(-b*x*log(F))^(3/2)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int F^{a+bx} \sqrt{x} dx = \frac{\sqrt{\pi} F^a \operatorname{erf}\left(-\sqrt{-b \log(F)} \sqrt{x}\right)}{2 \sqrt{-b \log(F)} b \log(F)} + \frac{\sqrt{x} e^{(bx \log(F) + a \log(F))}}{b \log(F)}$$

input `integrate(F^(b*x+a)*x^(1/2),x, algorithm="giac")`

output `1/2*sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/(sqrt(-b*log(F))*b*log(F)) + sqrt(x)*e^(b*x*log(F) + a*log(F))/(b*log(F))`

**Mupad [B] (verification not implemented)**

Time = 23.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int F^{a+bx} \sqrt{x} dx = \frac{F^a F^{bx} \sqrt{x}}{b \ln(F)} + \frac{F^a \sqrt{x} \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-bx \ln(F)}\right)}{2 b \ln(F) \sqrt{-bx \ln(F)}}$$

input `int(F^(a + b*x)*x^(1/2),x)`output `(F^a*F^(b*x)*x^(1/2))/(b*log(F)) + (F^a*x^(1/2)*pi^(1/2)*erfc((-b*x*log(F))^(1/2)))/(2*b*log(F)*(-b*x*log(F))^(1/2))`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int F^{a+bx} \sqrt{x} dx = \frac{f^a \left( \sqrt{\pi} \operatorname{erf}\left(\sqrt{x} \sqrt{b} \sqrt{\log(f)} i\right) i + 2\sqrt{x} f^{bx} \sqrt{b} \sqrt{\log(f)} \right)}{2\sqrt{b} \sqrt{\log(f)} \log(f) b}$$

input `int(F^(b*x+a)*x^(1/2),x)`output `(f**a*(sqrt(pi)*erf(sqrt(x)*sqrt(b)*sqrt(log(f))*i)*i + 2*sqrt(x)*f**(b*x)*sqrt(b)*sqrt(log(f)))/(2*sqrt(b)*sqrt(log(f))*log(f)*b)`

### 3.64 $\int \frac{F^{a+bx}}{\sqrt{x}} dx$

Optimal result	453
Mathematica [A] (verified)	453
Rubi [A] (verified)	454
Maple [A] (verified)	455
Fricas [A] (verification not implemented)	455
Sympy [F]	455
Maxima [A] (verification not implemented)	456
Giac [A] (verification not implemented)	456
Mupad [B] (verification not implemented)	456
Reduce [B] (verification not implemented)	457

#### Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = \frac{F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{\sqrt{b}\sqrt{\log(F)}}$$

output  $F^a \pi^{1/2} \operatorname{erfi}(b^{1/2} x^{1/2} \ln(F)^{1/2}) / b^{1/2} / \ln(F)^{1/2}$

#### Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = -\frac{F^a \sqrt{x} \Gamma\left(\frac{1}{2}, -bx \log(F)\right)}{\sqrt{-bx \log(F)}}$$

input `Integrate[F^(a + b*x)/Sqrt[x], x]`

output  $-((F^a \operatorname{Sqrt}[x] \operatorname{Gamma}[1/2, -(b*x \operatorname{Log}[F])]) / \operatorname{Sqrt}[-(b*x \operatorname{Log}[F])])$

**Rubi [A] (verified)**

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx$$

$$\downarrow 2611$$

$$2 \int F^{a+bx} d\sqrt{x}$$

$$\downarrow 2633$$

$$\frac{\sqrt{\pi} F^a \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)}{\sqrt{b}\sqrt{\log(F)}}$$

input `Int[F^(a + b*x)/Sqrt[x],x]`

output `(F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]]])/(Sqrt[b]*Sqrt[Log[F]])`

**Defintions of rubi rules used**

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{F^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{\sqrt{b} \sqrt{\ln(F)}}$	27

input `int(F^(b*x+a)/x^(1/2),x,method=_RETURNVERBOSE)`output `F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))/b^(1/2)/ln(F)^(1/2)`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = -\frac{\sqrt{\pi} \sqrt{-b \log(F)} F^a \operatorname{erf}(\sqrt{-b \log(F)} \sqrt{x})}{b \log(F)}$$

input `integrate(F^(b*x+a)/x^(1/2),x, algorithm="fricas")`output `-sqrt(pi)*sqrt(-b*log(F))*F^a*erf(sqrt(-b*log(F))*sqrt(x))/(b*log(F))`**Sympy [F]**

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = \int \frac{F^{a+bx}}{\sqrt{x}} dx$$

input `integrate(F**(b*x+a)/x**(1/2),x)`output `Integral(F**(a + b*x)/sqrt(x), x)`



**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = \frac{\sqrt{\pi} F^a \sqrt{x} \left( \operatorname{erf} \left( \sqrt{-bx \log(F)} \right) - 1 \right)}{\sqrt{-bx \log(F)}}$$

input `integrate(F^(b*x+a)/x^(1/2),x, algorithm="maxima")`output `sqrt(pi)*F^a*sqrt(x)*(erf(sqrt(-b*x*log(F))) - 1)/sqrt(-b*x*log(F))`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = -\frac{\sqrt{\pi} F^a \operatorname{erf} \left( -\sqrt{-b \log(F)} \sqrt{x} \right)}{\sqrt{-b \log(F)}}$$

input `integrate(F^(b*x+a)/x^(1/2),x, algorithm="giac")`output `-sqrt(pi)*F^a*erf(-sqrt(-b*log(F))*sqrt(x))/sqrt(-b*log(F))`**Mupad [B] (verification not implemented)**

Time = 22.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.84

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = \frac{F^a \operatorname{erfc} \left( \sqrt{-bx \ln(F)} \right) \sqrt{-\pi bx \ln(F)}}{b \sqrt{x} \ln(F)}$$

input `int(F^(a + b*x)/x^(1/2),x)`output `(F^a*erfc((-b*x*log(F))^(1/2))*(-b*x*pi*log(F))^(1/2))/(b*x^(1/2)*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{F^{a+bx}}{\sqrt{x}} dx = -\frac{\sqrt{\pi} f^a \operatorname{erf}\left(\sqrt{x} \sqrt{b} \sqrt{\log(f)} i\right) i}{\sqrt{b} \sqrt{\log(f)}}$$

input `int(F^(b*x+a)/x^(1/2),x)`

output `( - sqrt(pi)*f**a*erf(sqrt(x)*sqrt(b)*sqrt(log(f))*i)*i)/(sqrt(b)*sqrt(log(f))`

### 3.65 $\int \frac{F^{a+bx}}{x^{3/2}} dx$

Optimal result . . . . .	458
Mathematica [A] (verified) . . . . .	458
Rubi [A] (verified) . . . . .	459
Maple [A] (verified) . . . . .	460
Fricas [A] (verification not implemented) . . . . .	460
Sympy [F] . . . . .	461
Maxima [A] (verification not implemented) . . . . .	461
Giac [F] . . . . .	461
Mupad [B] (verification not implemented) . . . . .	462
Reduce [F] . . . . .	462

#### Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = -\frac{2F^{a+bx}}{\sqrt{x}} + 2\sqrt{b}F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)\sqrt{\log(F)}$$

output `-2*F^(b*x+a)/x^(1/2)+2*b^(1/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))*ln(F)^(1/2)`

#### Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = -\frac{2F^a\left(F^{bx} - \Gamma\left(\frac{1}{2}, -bx \log(F)\right)\sqrt{-bx \log(F)}\right)}{\sqrt{x}}$$

input `Integrate[F^(a + b*x)/x^(3/2),x]`

output `(-2*F^a*(F^(b*x) - Gamma[1/2, -(b*x*Log[F])]*Sqrt[-(b*x*Log[F])])/Sqrt[x]`

**Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{F^{a+bx}}{x^{3/2}} dx \\ & \quad \downarrow \text{2608} \\ & 2b \log(F) \int \frac{F^{a+bx}}{\sqrt{x}} dx - \frac{2F^{a+bx}}{\sqrt{x}} \\ & \quad \downarrow \text{2611} \\ & 4b \log(F) \int F^{a+bx} d\sqrt{x} - \frac{2F^{a+bx}}{\sqrt{x}} \\ & \quad \downarrow \text{2633} \\ & 2\sqrt{\pi}\sqrt{b}F^a \sqrt{\log(F)} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}} \end{aligned}$$

input `Int[F^(a + b*x)/x^(3/2),x]`

output `(-2*F^(a + b*x))/Sqrt[x] + 2*Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]]]*Sqrt[Log[F]]`

**Defintions of rubi rules used**

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

### Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

method	result	size
meijerg	$-\frac{F^a (-b)^{\frac{3}{2}} \sqrt{\ln(F)} \left( -\frac{2 e^{\ln(F) b x}}{\sqrt{x} \sqrt{-b} \sqrt{\ln(F)}} + \frac{2 \sqrt{b} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{\sqrt{-b}} \right)}{b}$	64

input `int(F^(b*x+a)/x^(3/2), x, method=_RETURNVERBOSE)`

output `-F^a*(-b)^(3/2)*ln(F)^(1/2)/b*(-2/x^(1/2)/(-b)^(1/2)/ln(F)^(1/2)*exp(ln(F)*b*x)+2/(-b)^(1/2)*b^(1/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = -\frac{2 \left( \sqrt{\pi} \sqrt{-b \log(F)} F^a x \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right) + F^{bx+a} \sqrt{x} \right)}{x}$$

input `integrate(F^(b*x+a)/x^(3/2), x, algorithm="fricas")`

output `-2*(sqrt(pi)*sqrt(-b*log(F))*F^a*x*erf(sqrt(-b*log(F))*sqrt(x)) + F^(b*x + a)*sqrt(x))/x`

**Sympy [F]**

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = \int \frac{F^{a+bx}}{x^{\frac{3}{2}}} dx$$

input `integrate(F**(b*x+a)/x**(3/2),x)`

output `Integral(F**(a + b*x)/x**(3/2), x)`

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = -\frac{\sqrt{-bx \log(F)} F^a \Gamma(-\frac{1}{2}, -bx \log(F))}{\sqrt{x}}$$

input `integrate(F^(b*x+a)/x^(3/2),x, algorithm="maxima")`

output `-sqrt(-b*x*log(F))*F^a*gamma(-1/2, -b*x*log(F))/sqrt(x)`

**Giac [F]**

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = \int \frac{F^{bx+a}}{x^{\frac{3}{2}}} dx$$

input `integrate(F^(b*x+a)/x^(3/2),x, algorithm="giac")`

output `integrate(F^(b*x + a)/x^(3/2), x)`

**Mupad [B] (verification not implemented)**

Time = 22.86 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = \frac{2 F^a \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-bx \ln(F)}\right) \sqrt{-bx \ln(F)}}{\sqrt{x}} - \frac{2 F^a F^{bx}}{\sqrt{x}}$$

input `int(F^(a + b*x)/x^(3/2),x)`output `(2*F^a*pi^(1/2)*erfc((-b*x*log(F))^(1/2))*(-b*x*log(F))^(1/2))/x^(1/2) - (2*F^a*F^(b*x))/x^(1/2)`**Reduce [F]**

$$\int \frac{F^{a+bx}}{x^{3/2}} dx = f^a \left( \int \frac{f^{bx}}{\sqrt{x} x} dx \right)$$

input `int(F^(b*x+a)/x^(3/2),x)`output `f**a*int(f**(b*x)/(sqrt(x)*x),x)`

### 3.66 $\int \frac{F^{a+bx}}{x^{5/2}} dx$

Optimal result	463
Mathematica [A] (verified)	463
Rubi [A] (verified)	464
Maple [A] (verified)	465
Fricas [A] (verification not implemented)	466
Sympy [F]	466
Maxima [A] (verification not implemented)	466
Giac [F]	467
Mupad [B] (verification not implemented)	467
Reduce [F]	467

#### Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = -\frac{2F^{a+bx}}{3x^{3/2}} - \frac{4bF^{a+bx} \log(F)}{3\sqrt{x}} + \frac{4}{3}b^{3/2}F^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right)\log^{3/2}(F)$$

output 
$$-2/3 * F^{(b*x+a)} / x^{(3/2)} - 4/3 * b * F^{(b*x+a)} * \ln(F) / x^{(1/2)} + 4/3 * b^{(3/2)} * F^a * \pi^{(1/2)} * \operatorname{erfi}(b^{(1/2)} * x^{(1/2)} * \ln(F)^{(1/2)}) * \ln(F)^{(3/2)}$$

#### Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = -\frac{2F^a(2\Gamma(\frac{1}{2}, -bx \log(F))(-bx \log(F))^{3/2} + F^{bx}(1 + 2bx \log(F)))}{3x^{3/2}}$$

input `Integrate[F^(a + b*x)/x^(5/2), x]`

output 
$$(-2 * F^a * (2 * \Gamma[1/2, -(b*x * \operatorname{Log}[F])] * (-(b*x * \operatorname{Log}[F]))^{(3/2)} + F^{(b*x)} * (1 + 2 * b * x * \operatorname{Log}[F]))) / (3 * x^{(3/2)})$$



**Rubi [A] (verified)**

Time = 0.46 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{F^{a+bx}}{x^{5/2}} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{2}{3}b \log(F) \int \frac{F^{a+bx}}{x^{3/2}} dx - \frac{2F^{a+bx}}{3x^{3/2}} \\
 & \quad \downarrow \text{2608} \\
 & \frac{2}{3}b \log(F) \left( 2b \log(F) \int \frac{F^{a+bx}}{\sqrt{x}} dx - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \\
 & \quad \downarrow \text{2611} \\
 & \frac{2}{3}b \log(F) \left( 4b \log(F) \int F^{a+bx} d\sqrt{x} - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \\
 & \quad \downarrow \text{2633} \\
 & \frac{2}{3}b \log(F) \left( 2\sqrt{\pi}\sqrt{b}F^a \sqrt{\log(F)} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}}
 \end{aligned}$$

input `Int [F^(a + b*x)/x^(5/2) ,x]`

output `(-2*F^(a + b*x))/(3*x^(3/2)) + (2*b*((-2*F^(a + b*x))/Sqrt[x] + 2*Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]]]*Sqrt[Log[F]])*Log[F])/3`

## Definitions of rubi rules used

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

## Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result	size
meijerg	$F^a (-b)^{\frac{5}{2}} \ln(F)^{\frac{3}{2}} \left( -\frac{2(2\ln(F)bx+1)e^{\ln(F)bx}}{3x^{\frac{3}{2}}(-b)^{\frac{3}{2}}\ln(F)^{\frac{3}{2}}} + \frac{4b^{\frac{3}{2}}\sqrt{\pi}\operatorname{erfi}(\sqrt{b}\sqrt{x}\sqrt{\ln(F)})}{3(-b)^{\frac{3}{2}}} \right)$	72

input

```
int(F^(b*x+a)/x^(5/2), x, method=_RETURNVERBOSE)
```

output

```
-F^a*(-b)^(5/2)*ln(F)^(3/2)/b*(-2/3/x^(3/2)/(-b)^(3/2)/ln(F)^(3/2)*(2*ln(F)
)*b*x+1)*exp(ln(F)*b*x)+4/3/(-b)^(3/2)*b^(3/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/
2)*ln(F)^(1/2))
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = \frac{2 \left( 2 \sqrt{\pi} \sqrt{-b \log(F)} F^a b x^2 \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right) \log(F) + (2 b x \log(F) + 1) F^{bx+a} \sqrt{x} \right)}{3 x^2}$$

input `integrate(F^(b*x+a)/x^(5/2),x, algorithm="fricas")`

output `-2/3*(2*sqrt(pi)*sqrt(-b*log(F))*F^a*b*x^2*erf(sqrt(-b*log(F))*sqrt(x))*log(F) + (2*b*x*log(F) + 1)*F^(b*x + a)*sqrt(x))/x^2`

**Sympy [F]**

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = \int \frac{F^{a+bx}}{x^{\frac{5}{2}}} dx$$

input `integrate(F**(b*x+a)/x**(5/2),x)`

output `Integral(F**(a + b*x)/x**(5/2), x)`

**Maxima [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = -\frac{(-bx \log(F))^{\frac{3}{2}} F^a \Gamma(-\frac{3}{2}, -bx \log(F))}{x^{\frac{3}{2}}}$$

input `integrate(F^(b*x+a)/x^(5/2),x, algorithm="maxima")`

output `-(-b*x*log(F))^(3/2)*F^a*gamma(-3/2, -b*x*log(F))/x^(3/2)`

**Giac [F]**

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = \int \frac{F^{bx+a}}{x^{5/2}} dx$$

input `integrate(F^(b*x+a)/x^(5/2),x, algorithm="giac")`

output `integrate(F^(b*x + a)/x^(5/2), x)`

**Mupad [B] (verification not implemented)**

Time = 22.82 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = \frac{4 F^a b \sqrt{\pi} \operatorname{erfc}\left(\sqrt{-b x \ln(F)}\right) \ln(F) \sqrt{-b x \ln(F)}}{3 \sqrt{x}} - \frac{4 F^a F^{bx} b \ln(F)}{3 \sqrt{x}} - \frac{2 F^a F^{bx}}{3 x^{3/2}}$$

input `int(F^(a + b*x)/x^(5/2),x)`

output `(4*F^a*b*pi^(1/2)*erfc((-b*x*log(F))^(1/2))*log(F)*(-b*x*log(F))^(1/2))/(3*x^(1/2)) - (4*F^a*F^(b*x)*b*log(F))/(3*x^(1/2)) - (2*F^a*F^(b*x))/(3*x^(3/2))`

**Reduce [F]**

$$\int \frac{F^{a+bx}}{x^{5/2}} dx = f^a \left( \int \frac{f^{bx}}{\sqrt{x} x^2} dx \right)$$

input `int(F^(b*x+a)/x^(5/2),x)`

output `f**a*int(f**(b*x)/(sqrt(x)*x**2),x)`

### 3.67 $\int \frac{F^{a+bx}}{x^{7/2}} dx$

Optimal result	468
Mathematica [A] (verified)	468
Rubi [A] (verified)	469
Maple [A] (verified)	470
Fricas [A] (verification not implemented)	471
Sympy [F]	471
Maxima [A] (verification not implemented)	471
Giac [F]	472
Mupad [B] (verification not implemented)	472
Reduce [F]	472

#### Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = -\frac{2F^{a+bx}}{5x^{5/2}} - \frac{4bF^{a+bx} \log(F)}{15x^{3/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{15\sqrt{x}} + \frac{8}{15} b^{5/2} F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) \log^{5/2}(F)$$

output

```
-2/5*F^(b*x+a)/x^(5/2)-4/15*b*F^(b*x+a)*ln(F)/x^(3/2)-8/15*b^2*F^(b*x+a)*ln(F)^2/x^(1/2)+8/15*b^(5/2)*F^a*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))*ln(F)^(5/2)
```

#### Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.61

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = -\frac{2F^a \left(-4\Gamma\left(\frac{1}{2}, -bx \log(F)\right) (-bx \log(F))^{5/2} + F^{bx} (3 + 2bx \log(F) + 4b^2 x^2 \log^2(F))\right)}{15x^{5/2}}$$

input

```
Integrate[F^(a + b*x)/x^(7/2), x]
```

output

$$\frac{(-2F^a(-4\Gamma[1/2, -(b*x*\text{Log}[F])]) * (-(b*x*\text{Log}[F]))^{5/2} + F^{(b*x)} * (3 + 2*b*x*\text{Log}[F] + 4*b^2*x^2*\text{Log}[F]^2))}{(15*x^{5/2})}$$

**Rubi [A] (verified)**

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {2608, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{F^{a+bx}}{x^{7/2}} dx \\ & \quad \downarrow \text{2608} \\ & \frac{2}{5}b \log(F) \int \frac{F^{a+bx}}{x^{5/2}} dx - \frac{2F^{a+bx}}{5x^{5/2}} \\ & \quad \downarrow \text{2608} \\ & \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \int \frac{F^{a+bx}}{x^{3/2}} dx - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \\ & \quad \downarrow \text{2608} \\ & \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \left( 2b \log(F) \int \frac{F^{a+bx}}{\sqrt{x}} dx - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \\ & \quad \downarrow \text{2611} \\ & \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \left( 4b \log(F) \int F^{a+bx} d\sqrt{x} - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \\ & \quad \downarrow \text{2633} \\ & \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \left( 2\sqrt{\pi}\sqrt{b}F^a \sqrt{\log(F)} \text{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \end{aligned}$$

input

$$\text{Int}[F^{(a + b*x)}/x^{(7/2)}, x]$$

```
output (-2*F^(a + b*x))/(5*x^(5/2)) + (2*b*Log[F]*((-2*F^(a + b*x))/(3*x^(3/2)) +
(2*b*((-2*F^(a + b*x))/Sqrt[x] + 2*Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt
[x]*Sqrt[Log[F]]]*Sqrt[Log[F]])*Log[F])/3))/5
```

**Defintions of rubi rules used**

```
rule 2608 Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] && !TrueQ[$UseGamma]
```

```
rule 2611 Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

```
rule 2633 Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

method	result	size
meijerg	$F^a(-b)^{\frac{7}{2}} \ln(F)^{\frac{5}{2}} \left( -\frac{2 \left( \frac{4b^2 x^2 \ln(F)^2}{3} + \frac{2 \ln(F) b x}{3} + 1 \right) e^{\ln(F) b x}}{5x^{\frac{5}{2}} (-b)^{\frac{5}{2}} \ln(F)^{\frac{5}{2}}} + \frac{8b^{\frac{5}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{15(-b)^{\frac{5}{2}}} \right)$	84

```
input int(F^(b*x+a)/x^(7/2), x, method=_RETURNVERBOSE)
```

```
output -F^a*(-b)^(7/2)*ln(F)^(5/2)/b*(-2/5/x^(5/2)/(-b)^(5/2)/ln(F)^(5/2)*(4/3*b^
2*x^2*ln(F)^2+2/3*ln(F)*b*x+1)*exp(ln(F)*b*x)+8/15/(-b)^(5/2)*b^(5/2)*Pi^(
1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2)))
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = \frac{2 \left( 4 \sqrt{\pi} \sqrt{-b \log(F)} F^a b^2 x^3 \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right) \log(F)^2 + (4 b^2 x^2 \log(F)^2 + 2 b x \log(F) + 3) F^{bx+a} \right)}{15 x^3}$$

input `integrate(F^(b*x+a)/x^(7/2),x, algorithm="fricas")`output `-2/15*(4*sqrt(pi)*sqrt(-b*log(F))*F^a*b^2*x^3*erf(sqrt(-b*log(F))*sqrt(x))  
*log(F)^2 + (4*b^2*x^2*log(F)^2 + 2*b*x*log(F) + 3)*F^(b*x + a)*sqrt(x))/x  
^3`**Sympy [F]**

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = \int \frac{F^{a+bx}}{x^{7/2}} dx$$

input `integrate(F**(b*x+a)/x**(7/2),x)`output `Integral(F**(a + b*x)/x**(7/2), x)`**Maxima [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.24

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = -\frac{(-bx \log(F))^{\frac{5}{2}} F^a \Gamma\left(-\frac{5}{2}, -bx \log(F)\right)}{x^{\frac{5}{2}}}$$

input `integrate(F^(b*x+a)/x^(7/2),x, algorithm="maxima")`output `-(-b*x*log(F))^(5/2)*F^a*gamma(-5/2, -b*x*log(F))/x^(5/2)`



**Giac [F]**

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = \int \frac{F^{bx+a}}{x^{7/2}} dx$$

input `integrate(F^(b*x+a)/x^(7/2),x, algorithm="giac")`

output `integrate(F^(b*x + a)/x^(7/2), x)`

**Mupad [B] (verification not implemented)**

Time = 22.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = \frac{\frac{2 F^{a+bx}}{5} + \frac{4 F^{a+bx} b x \ln(F)}{15} + \frac{8 F^{a+bx} b^2 x^2 \ln(F)^2}{15} - \frac{8 F^a b^2 x^2 \operatorname{erfc}(\sqrt{-b x \ln(F)}) \ln(F)^2 \sqrt{-\pi b x \ln(F)}}{15}}{x^{5/2}}$$

input `int(F^(a + b*x)/x^(7/2),x)`

output `-((2*F^(a + b*x))/5 + (4*F^(a + b*x)*b*x*log(F))/15 + (8*F^(a + b*x)*b^2*x^2*log(F)^2)/15 - (8*F^a*b^2*x^2*erfc((-b*x*log(F))^(1/2))*log(F)^2*(-b*x*pi*log(F))^(1/2))/15)/x^(5/2)`

**Reduce [F]**

$$\int \frac{F^{a+bx}}{x^{7/2}} dx = f^a \left( \int \frac{f^{bx}}{\sqrt{x} x^3} dx \right)$$

input `int(F^(b*x+a)/x^(7/2),x)`

output `f**a*int(f**(b*x)/(sqrt(x)*x**3),x)`

### 3.68 $\int \frac{F^{a+bx}}{x^{9/2}} dx$

Optimal result	473
Mathematica [A] (verified)	473
Rubi [A] (verified)	474
Maple [A] (verified)	476
Fricas [A] (verification not implemented)	476
Sympy [F]	477
Maxima [A] (verification not implemented)	477
Giac [F]	477
Mupad [B] (verification not implemented)	478
Reduce [F]	478

#### Optimal result

Integrand size = 13, antiderivative size = 123

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = -\frac{2F^{a+bx}}{7x^{7/2}} - \frac{4bF^{a+bx} \log(F)}{35x^{5/2}} - \frac{8b^2 F^{a+bx} \log^2(F)}{105x^{3/2}} - \frac{16b^3 F^{a+bx} \log^3(F)}{105\sqrt{x}} + \frac{16}{105} b^{7/2} F^a \sqrt{\pi} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) \log^{7/2}(F)$$

output

```
-2/7*F^(b*x+a)/x^(7/2)-4/35*b*F^(b*x+a)*ln(F)/x^(5/2)-8/105*b^2*F^(b*x+a)*
ln(F)^2/x^(3/2)-16/105*b^3*F^(b*x+a)*ln(F)^3/x^(1/2)+16/105*b^(7/2)*F^a*Pi
^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))*ln(F)^(7/2)
```

#### Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = \frac{2F^a \left( 8\Gamma\left(\frac{1}{2}, -bx \log(F)\right) (-bx \log(F))^{7/2} + F^{bx} (15 + 6bx \log(F) + 4b^2 x^2 \log^2(F) + 8b^3 x^3 \log^3(F)) \right)}{105x^{7/2}}$$

input

```
Integrate[F^(a + b*x)/x^(9/2), x]
```

output

$$(-2F^a(8\Gamma[1/2, -(b*x*\text{Log}[F])]*(b*x*\text{Log}[F])^{7/2} + F^{b*x}(15 + 6*b*x*\text{Log}[F] + 4*b^2*x^2*\text{Log}[F]^2 + 8*b^3*x^3*\text{Log}[F]^3)))/(105*x^{7/2})$$

### Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$ , Rules used = {2608, 2608, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{F^{a+bx}}{x^{9/2}} dx \\ & \quad \downarrow \text{2608} \\ & \frac{2}{7}b \log(F) \int \frac{F^{a+bx}}{x^{7/2}} dx - \frac{2F^{a+bx}}{7x^{7/2}} \\ & \quad \downarrow \text{2608} \\ & \frac{2}{7}b \log(F) \left( \frac{2}{5}b \log(F) \int \frac{F^{a+bx}}{x^{5/2}} dx - \frac{2F^{a+bx}}{5x^{5/2}} \right) - \frac{2F^{a+bx}}{7x^{7/2}} \\ & \quad \downarrow \text{2608} \\ & \frac{2}{7}b \log(F) \left( \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \int \frac{F^{a+bx}}{x^{3/2}} dx - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \right) - \frac{2F^{a+bx}}{7x^{7/2}} \\ & \quad \downarrow \text{2608} \\ & \frac{2}{7}b \log(F) \left( \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \left( 2b \log(F) \int \frac{F^{a+bx}}{\sqrt{x}} dx - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \right) - \frac{2F^{a+bx}}{7x^{7/2}} \\ & \quad \downarrow \text{2611} \\ & \frac{2}{7}b \log(F) \left( \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \left( 4b \log(F) \int F^{a+bx} d\sqrt{x} - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \right) - \frac{2F^{a+bx}}{7x^{7/2}} \end{aligned}$$

↓ 2633

$$\frac{2}{7}b \log(F) \left( \frac{2}{5}b \log(F) \left( \frac{2}{3}b \log(F) \left( 2\sqrt{\pi}\sqrt{b}F^a \sqrt{\log(F)} \operatorname{erfi}\left(\sqrt{b}\sqrt{x}\sqrt{\log(F)}\right) - \frac{2F^{a+bx}}{\sqrt{x}} \right) - \frac{2F^{a+bx}}{3x^{3/2}} \right) - \frac{2F^{a+bx}}{5x^{5/2}} \right) - \frac{2F^{a+bx}}{7x^{7/2}}$$

input `Int[F^(a + b*x)/x^(9/2),x]`

output `(-2*F^(a + b*x))/(7*x^(7/2)) + (2*b*Log[F]*((-2*F^(a + b*x))/(5*x^(5/2)) + (2*b*Log[F]*((-2*F^(a + b*x))/(3*x^(3/2)) + (2*b*((-2*F^(a + b*x))/Sqrt[x] + 2*Sqrt[b]*F^a*Sqrt[Pi]*Erfi[Sqrt[b]*Sqrt[x]*Sqrt[Log[F]])*Sqrt[Log[F]])*Log[F])/3))/5))/7`

### Defintions of rubi rules used

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

**Maple [A] (verified)**

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.78

method	result	size
meijerg	$F^a (-b)^{\frac{9}{2}} \ln(F)^{\frac{7}{2}} \left( -\frac{2 \left( \frac{8b^3 x^3 \ln(F)^3}{15} + \frac{4b^2 x^2 \ln(F)^2}{15} + \frac{2 \ln(F) b x}{5} + 1 \right) e^{\ln(F) b x}}{7x^{\frac{7}{2}} (-b)^{\frac{7}{2}} \ln(F)^{\frac{7}{2}}} + \frac{16b^{\frac{7}{2}} \sqrt{\pi} \operatorname{erfi}(\sqrt{b} \sqrt{x} \sqrt{\ln(F)})}{105(-b)^{\frac{7}{2}}} \right)$	96

input `int(F^(b*x+a)/x^(9/2),x,method=_RETURNVERBOSE)`output `-F^a*(-b)^(9/2)*ln(F)^(7/2)/b*(-2/7/x^(7/2)/(-b)^(7/2)/ln(F)^(7/2)*(8/15*b^3*x^3*ln(F)^3+4/15*b^2*x^2*ln(F)^2+2/5*ln(F)*b*x+1)*exp(ln(F)*b*x)+16/105/(-b)^(7/2)*b^(7/2)*Pi^(1/2)*erfi(b^(1/2)*x^(1/2)*ln(F)^(1/2))`**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.70

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = \frac{2 \left( 8 \sqrt{\pi} \sqrt{-b \log(F)} F^a b^3 x^4 \operatorname{erf} \left( \sqrt{-b \log(F)} \sqrt{x} \right) \log(F)^3 + (8 b^3 x^3 \log(F)^3 + 4 b^2 x^2 \log(F)^2 + 6 b x \log(F) + 15) F^a \right)}{105 x^4}$$

input `integrate(F^(b*x+a)/x^(9/2),x, algorithm="fricas")`output `-2/105*(8*sqrt(pi)*sqrt(-b*log(F))*F^a*b^3*x^4*erf(sqrt(-b*log(F))*sqrt(x))*log(F)^3 + (8*b^3*x^3*log(F)^3 + 4*b^2*x^2*log(F)^2 + 6*b*x*log(F) + 15)*F^(b*x + a)*sqrt(x))/x^4`

**Sympy [F]**

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = \int \frac{F^{a+bx}}{x^{\frac{9}{2}}} dx$$

input `integrate(F**(b*x+a)/x**(9/2),x)`

output `Integral(F**(a + b*x)/x**(9/2), x)`

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.20

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = -\frac{(-bx \log(F))^{\frac{7}{2}} F^a \Gamma(-\frac{7}{2}, -bx \log(F))}{x^{\frac{7}{2}}}$$

input `integrate(F^(b*x+a)/x^(9/2),x, algorithm="maxima")`

output `-(-b*x*log(F))^(7/2)*F^a*gamma(-7/2, -b*x*log(F))/x^(7/2)`

**Giac [F]**

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = \int \frac{F^{bx+a}}{x^{\frac{9}{2}}} dx$$

input `integrate(F^(b*x+a)/x^(9/2),x, algorithm="giac")`

output `integrate(F^(b*x + a)/x^(9/2), x)`

**Mupad [B] (verification not implemented)**

Time = 22.87 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{F^{a+bx}}{x^{9/2}} dx =$$

$$-\frac{2F^{a+bx}}{7} + \frac{4F^{a+bx} b x \ln(F)}{35} + \frac{8F^{a+bx} b^2 x^2 \ln(F)^2}{105} + \frac{16F^{a+bx} b^3 x^3 \ln(F)^3}{105} - \frac{16F^a b^3 x^3 \operatorname{erfc}\left(\sqrt{-bx \ln(F)}\right) \ln(F)^3 \sqrt{-\pi b x \ln(F)}}{105 x^{7/2}}$$

input `int(F^(a + b*x)/x^(9/2),x)`output `-((2*F^(a + b*x))/7 + (4*F^(a + b*x)*b*x*log(F))/35 + (8*F^(a + b*x)*b^2*x^2*log(F)^2)/105 + (16*F^(a + b*x)*b^3*x^3*log(F)^3)/105 - (16*F^a*b^3*x^3*erfc((-b*x*log(F))^(1/2))*log(F)^3*(-b*x*pi*log(F))^(1/2))/105)/x^(7/2)`**Reduce [F]**

$$\int \frac{F^{a+bx}}{x^{9/2}} dx = f^a \left( \int \frac{f^{bx}}{\sqrt{x} x^4} dx \right)$$

input `int(F^(b*x+a)/x^(9/2),x)`output `f**a*int(f**(b*x)/(sqrt(x)*x**4),x)`

### 3.69 $\int F^{c(a+bx)}(d+ex)^{7/2} dx$

Optimal result	479
Mathematica [A] (verified)	480
Rubi [A] (verified)	480
Maple [F]	482
Fricas [A] (verification not implemented)	483
Sympy [F(-1)]	483
Maxima [F]	484
Giac [B] (verification not implemented)	484
Mupad [F(-1)]	485
Reduce [F]	486

#### Optimal result

Integrand size = 19, antiderivative size = 208

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \frac{105e^{7/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{16b^{9/2}c^{9/2}\log^{9/2}(F)} - \frac{105e^3F^{c(a+bx)}\sqrt{d+ex}}{8b^4c^4\log^4(F)} + \frac{35e^2F^{c(a+bx)}(d+ex)^{3/2}}{4b^3c^3\log^3(F)} - \frac{7eF^{c(a+bx)}(d+ex)^{5/2}}{2b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{7/2}}{bc\log(F)}$$

output

```
105/16*e^(7/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)
*ln(F)^(1/2)/e^(1/2))/b^(9/2)/c^(9/2)/ln(F)^(9/2)-105/8*e^3*F^(c*(b*x+a))*
(e*x+d)^(1/2)/b^4/c^4/ln(F)^4+35/4*e^2*F^(c*(b*x+a))*(e*x+d)^(3/2)/b^3/c^3
/ln(F)^3-7/2*e*F^(c*(b*x+a))*(e*x+d)^(5/2)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*
(e*x+d)^(7/2)/b/c/ln(F)
```



### Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.35

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \frac{e^4 F^{c(a-\frac{bd}{e})} \Gamma\left(\frac{9}{2}, -\frac{bc(d+ex)\log(F)}{e}\right) \sqrt{-\frac{bc(d+ex)\log(F)}{e}}}{b^5 c^5 \sqrt{d+ex} \log^5(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^(7/2), x]
```

output

```
(e^4 * F^(c*(a - (b*d)/e)) * Gamma[9/2, -((b*c*(d + e*x)*Log[F])/e)] * Sqrt[-((b*c*(d + e*x)*Log[F])/e)]) / (b^5 * c^5 * Sqrt[d + e*x] * Log[F]^5)
```

### Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {2607, 2607, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int (d+ex)^{7/2} F^{c(a+bx)} dx \\ & \quad \downarrow 2607 \\ & \frac{(d+ex)^{7/2} F^{c(a+bx)}}{bc \log(F)} - \frac{7e \int F^{c(a+bx)} (d+ex)^{5/2} dx}{2bc \log(F)} \\ & \quad \downarrow 2607 \\ & \frac{(d+ex)^{7/2} F^{c(a+bx)}}{bc \log(F)} - \frac{7e \left( \frac{(d+ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \int F^{c(a+bx)} (d+ex)^{3/2} dx}{2bc \log(F)} \right)}{2bc \log(F)} \\ & \quad \downarrow 2607 \\ & \frac{(d+ex)^{7/2} F^{c(a+bx)}}{bc \log(F)} - \frac{7e \left( \frac{(d+ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} \sqrt{d+ex} dx}{2bc \log(F)} \right)}{2bc \log(F)} \right)}{2bc \log(F)} \end{aligned}$$

$$\begin{array}{c}
 \downarrow 2607 \\
 \frac{(d+ex)^{7/2} F^{c(a+bx)}}{bc \log(F)} - \\
 7e \left( \frac{(d+ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{e \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{2bc \log(F)} \right)}{2bc \log(F)} \right)}{2bc \log(F)} \right) \\
 \hline
 2bc \log(F)
 \end{array}$$

$$\begin{array}{c}
 \downarrow 2611 \\
 \frac{(d+ex)^{7/2} F^{c(a+bx)}}{bc \log(F)} - \\
 7e \left( \frac{(d+ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\int F^{c\left(a-\frac{bd}{e}\right) + \frac{bc(d+ex)}{e} d\sqrt{d+ex}}{bc \log(F)} \right)}{2bc \log(F)} \right)}{2bc \log(F)} \right) \\
 \hline
 2bc \log(F)
 \end{array}$$

$$\begin{array}{c}
 \downarrow 2633 \\
 \frac{(d+ex)^{7/2} F^{c(a+bx)}}{bc \log(F)} - \\
 7e \left( \frac{(d+ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{2b^{3/2} e^{3/2} \log^{\frac{3}{2}}(F)} \right)}{2bc \log(F)} \right)}{2bc \log(F)} \right) \\
 \hline
 2bc \log(F)
 \end{array}$$

input `Int[F^(c*(a + b*x))*(d + e*x)^(7/2),x]`

output `(F^(c*(a + b*x))*(d + e*x)^(7/2))/(b*c*Log[F]) - (7*e*((F^(c*(a + b*x))*(d + e*x)^(5/2))/(b*c*Log[F]) - (5*e*((F^(c*(a + b*x))*(d + e*x)^(3/2))/(b*c*Log[F]) - (3*e*(-1/2*(Sqrt[e]*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]])/(b^(3/2)*c^(3/2)*Log[F]^(3/2)) + (F^(c*(a + b*x))*Sqrt[d + e*x])/(b*c*Log[F])))/(2*b*c*Log[F])))/(2*b*c*Log[F])))/(2*b*c*Log[F])`

### Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

### Maple [F]

$$\int F^{c(bx+a)}(ex+d)^{\frac{7}{2}} dx$$

input `int(F^(c*(b*x+a))*(e*x+d)^(7/2),x)`

output `int(F^(c*(b*x+a))*(e*x+d)^(7/2),x)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.11

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \frac{105\sqrt{\pi}\sqrt{-\frac{bc\log(F)}{e}}e^4\operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}} + 2(105bce^3\log(F) - 8(b^4c^4e^3x^3 + 3b^4c^4de^2x^2 + 3b^4c^4d^2ex + b^4c^4d^2) + 16b^5c^4d^2)F^{c(a+bx)}$$

input `integrate(F**((b*x+a)*c)*(e*x+d)**(7/2),x, algorithm="fricas")`output `-1/16*(105*sqrt(pi)*sqrt(-b*c*log(F)/e)*e^4*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/F**((b*c*d - a*c*e)/e) + 2*(105*b*c*e^3*log(F) - 8*(b^4*c^4*e^3*x^3 + 3*b^4*c^4*d*e^2*x^2 + 3*b^4*c^4*d^2*e*x + b^4*c^4*d^3)*log(F)^4 + 28*(b^3*c^3*e^3*x^2 + 2*b^3*c^3*d*e^2*x + b^3*c^3*d^2*e)*log(F)^3 - 70*(b^2*c^2*e^3*x + b^2*c^2*d*e^2)*log(F)^2)*sqrt(e*x + d)*F**(b*c*x + a*c))/(b^5*c^5*log(F)^5)`**Sympy [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \text{Timed out}$$

input `integrate(F**((b*x+a)*c)*(e*x+d)**(7/2),x)`output `Timed out`

**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \int (ex+d)^{\frac{7}{2}} F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(7/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(7/2)*F^((b*x + a)*c), x)`

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs.  $2(172) = 344$ .

Time = 0.20 (sec) , antiderivative size = 1023, normalized size of antiderivative = 4.92

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(7/2),x, algorithm="giac")`

output

```
-1/16*(16*sqrt(pi)*d^4*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*
c*d*log(F) - a*c*e*log(F))/e)/sqrt(-b*c*e*log(F)) - 32*d^3*(sqrt(pi)*(2*b*
c*d*log(F) + e)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log
(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b*c*log(F)) + 2*sqrt(e*x + d)*
e*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b*c*log(F)))
+ 24*d^2*(sqrt(pi)*(4*b^2*c^2*d^2*log(F)^2 + 4*b*c*d*e*log(F) + 3*e^2)*e*
erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F)
)/e)/(sqrt(-b*c*e*log(F))*b^2*c^2*log(F)^2) - 2*(2*(e*x + d)^(3/2)*b*c*e*log
(F) - 4*sqrt(e*x + d)*b*c*d*e*log(F) - 3*sqrt(e*x + d)*e^2)*e^(((e*x + d)
)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b^2*c^2*log(F)^2)) - 8*d*(
sqrt(pi)*(8*b^3*c^3*d^3*log(F)^3 + 12*b^2*c^2*d^2*e*log(F)^2 + 18*b*c*d*e^
2*log(F) + 15*e^3)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*
log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b^3*c^3*log(F)^3) + 2*(4*(e
*x + d)^(5/2)*b^2*c^2*e*log(F)^2 - 12*(e*x + d)^(3/2)*b^2*c^2*d*e*log(F)^2
+ 12*sqrt(e*x + d)*b^2*c^2*d^2*e*log(F)^2 - 10*(e*x + d)^(3/2)*b*c*e^2*lo
g(F) + 18*sqrt(e*x + d)*b*c*d*e^2*log(F) + 15*sqrt(e*x + d)*e^3)*e^(((e*x
+ d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b^3*c^3*log(F)^3)) + sq
rt(pi)*(16*b^4*c^4*d^4*log(F)^4 + 32*b^3*c^3*d^3*e*log(F)^3 + 72*b^2*c^2*d
^2*e^2*log(F)^2 + 120*b*c*d*e^3*log(F) + 105*e^4)*e*erf(-sqrt(-b*c*e*log(F)
))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*1...
```

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^{7/2} dx = \int F^{c(a+bx)}(d+ex)^{7/2} dx$$

input

```
int(F^(c*(a + b*x))*(d + e*x)^(7/2), x)
```

output

```
int(F^(c*(a + b*x))*(d + e*x)^(7/2), x)
```

**Reduce [F]**

$$\int F^{c(a+bx)}(d + ex)^{7/2} dx = f^{ac} \left( \left( \int f^{bcx} \sqrt{ex + d} x^3 dx \right) e^3 + 3 \left( \int f^{bcx} \sqrt{ex + d} x^2 dx \right) d e^2 + 3 \left( \int f^{bcx} \sqrt{ex + d} x dx \right) d^2 e + \left( \int f^{bcx} \sqrt{ex + d} dx \right) d^3 \right)$$

input `int(F^((b*x+a)*c)*(e*x+d)^(7/2),x)`

output `f**(a*c)*(int(f**(b*c*x)*sqrt(d + e*x)*x**3,x)*e**3 + 3*int(f**(b*c*x)*sqrt(d + e*x)*x**2,x)*d*e**2 + 3*int(f**(b*c*x)*sqrt(d + e*x)*x,x)*d**2*e + int(f**(b*c*x)*sqrt(d + e*x),x)*d**3)`

### 3.70 $\int F^{c(a+bx)}(d+ex)^{5/2} dx$

Optimal result	487
Mathematica [A] (verified)	487
Rubi [A] (verified)	488
Maple [F]	490
Fricas [A] (verification not implemented)	490
Sympy [F]	490
Maxima [F]	491
Giac [B] (verification not implemented)	491
Mupad [F(-1)]	492
Reduce [F]	492

#### Optimal result

Integrand size = 19, antiderivative size = 173

$$\int F^{c(a+bx)}(d+ex)^{5/2} dx = -\frac{15e^{5/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{8b^{7/2}c^{7/2}\log^{7/2}(F)} + \frac{15e^2F^{c(a+bx)}\sqrt{d+ex}}{4b^3c^3\log^3(F)} - \frac{5eF^{c(a+bx)}(d+ex)^{3/2}}{2b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{5/2}}{bc\log(F)}$$

output

```
-15/8*e^(5/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*
ln(F)^(1/2)/e^(1/2))/b^(7/2)/c^(7/2)/ln(F)^(7/2)+15/4*e^2*F^(c*(b*x+a))*(e
*x+d)^(1/2)/b^3/c^3/ln(F)^3-5/2*e*F^(c*(b*x+a))*(e*x+d)^(3/2)/b^2/c^2/ln(F
)^2+F^(c*(b*x+a))*(e*x+d)^(5/2)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.42

$$\int F^{c(a+bx)}(d+ex)^{5/2} dx = \frac{e^2F^{c(a-\frac{bd}{e})}\sqrt{d+ex}\Gamma\left(\frac{7}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)}{b^3c^3\log^3(F)\sqrt{-\frac{bc(d+ex)\log(F)}{e}}}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^(5/2), x]
```



output

$$\frac{(e^{2F^{c(a - (b*d)/e)}} \sqrt{d + e*x} \Gamma[7/2, -((b*c*(d + e*x)*\text{Log}[F])/e)])}{(b^3*c^3*\text{Log}[F]^3*\text{Sqrt}[-((b*c*(d + e*x)*\text{Log}[F])/e)])}$$

**Rubi [A] (verified)**

Time = 0.80 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2607, 2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{5/2} F^{c(a+bx)} dx$$

$$\downarrow 2607$$

$$\frac{(d + ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \int F^{c(a+bx)} (d + ex)^{3/2} dx}{2bc \log(F)}$$

$$\downarrow 2607$$

$$\frac{(d + ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} \sqrt{d+ex} dx}{2bc \log(F)} \right)}{2bc \log(F)}$$

$$\downarrow 2607$$

$$\frac{(d + ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{e \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{2bc \log(F)} \right)}{2bc \log(F)} \right)}{2bc \log(F)}$$

$$\downarrow 2611$$

$$\frac{(d + ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{5e \left( \frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\int F^{c(a - \frac{bd}{e}) + \frac{bc(d+ex)}{e}} d\sqrt{d+ex}}{bc \log(F)} \right)}{2bc \log(F)} \right)}{2bc \log(F)}$$

$$\downarrow 2633$$

$$\frac{5e \left( \frac{(d+ex)^{5/2} F^{c(a+bx)}}{bc \log(F)} - \frac{\frac{(d+ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c(a-\frac{bd}{e})} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{2b^{3/2}c^{3/2}\log^2(F)}\right)}{2bc \log(F)}}{2bc \log(F)} \right)}{2bc \log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x)^(5/2), x]`

output 
$$\frac{(F^{c(a+bx)}(d+ex)^{5/2})/(bc \log[F]) - (5e((F^{c(a+bx)}(d+ex)^{3/2})/(bc \log[F]) - (3e(-1/2(\sqrt{e}F^{c(a-(bd/e))}\sqrt{\pi}\operatorname{Erfi}[(\sqrt{b}\sqrt{c}\sqrt{\log[F]}\sqrt{d+ex})/\sqrt{e}])/(b^{3/2}c^{3/2}\log^2[F]) + (F^{c(a+bx)}\sqrt{d+ex})/(bc \log[F])))/(2bc \log[F])))/(2bc \log[F])}{2bc \log[F]}$$

### Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m-1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

**Maple [F]**

$$\int F^{c(bx+a)}(ex+d)^{\frac{5}{2}} dx$$

input `int(F^(c*(b*x+a))*(e*x+d)^(5/2),x)`

output `int(F^(c*(b*x+a))*(e*x+d)^(5/2),x)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}(d+ex)^{\frac{5}{2}} dx = \frac{15\sqrt{\pi}\sqrt{-\frac{bc\log(F)}{e}}e^3\operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}} + 2(15bce^2\log(F) + 4(b^3c^3e^2x^2 + 2b^3c^3dex + b^3c^3d^2)\log(F))$$

$8b^4c^4\log(F)^4$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(5/2),x, algorithm="fricas")`

output `1/8*(15*sqrt(pi)*sqrt(-b*c*log(F)/e)*e^3*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/F^((b*c*d - a*c*e)/e) + 2*(15*b*c*e^2*log(F) + 4*(b^3*c^3*e^2*x^2 + 2*b^3*c^3*d*e*x + b^3*c^3*d^2)*log(F)^3 - 10*(b^2*c^2*e^2*x + b^2*c^2*d*e)*log(F)^2)*sqrt(e*x + d)*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4)`

**Sympy [F]**

$$\int F^{c(a+bx)}(d+ex)^{\frac{5}{2}} dx = \int F^{c(a+bx)}(d+ex)^{\frac{5}{2}} dx$$

input `integrate(F**((b*x+a)*c)*(e*x+d)**(5/2),x)`

output `Integral(F**(c*(a + b*x))*(d + e*x)**(5/2), x)`

**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^{5/2} dx = \int (ex+d)^{\frac{5}{2}} F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(5/2),x, algorithm="maxima")`

output `integrate((e*x + d)^(5/2)*F^((b*x + a)*c), x)`

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. 2(141) = 282.

Time = 0.18 (sec) , antiderivative size = 643, normalized size of antiderivative = 3.72

$$\int F^{c(a+bx)}(d+ex)^{5/2} dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(5/2),x, algorithm="giac")`

output `-1/8*(8*sqrt(pi)*d^3*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/sqrt(-b*c*e*log(F)) - 12*d^2*(sqrt(pi)*(2*b*c*d*log(F) + e)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b*c*log(F)) + 2*sqrt(e*x + d)*e*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b*c*log(F))) + 6*d*(sqrt(pi)*(4*b^2*c^2*d^2*log(F)^2 + 4*b*c*d*e*log(F) + 3*e^2)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b^2*c^2*log(F)^2) - 2*(2*(e*x + d)^(3/2)*b*c*e*log(F) - 4*sqrt(e*x + d)*b*c*d*e*log(F) - 3*sqrt(e*x + d)*e^2)*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b^2*c^2*log(F)^2)) - sqrt(pi)*(8*b^3*c^3*d^3*log(F)^3 + 12*b^2*c^2*d^2*e*log(F)^2 + 18*b*c*d*e^2*log(F) + 15*e^3)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b^3*c^3*log(F)^3) - 2*(4*(e*x + d)^(5/2)*b^2*c^2*e*log(F)^2 - 12*(e*x + d)^(3/2)*b^2*c^2*d*e*log(F)^2 + 12*sqrt(e*x + d)*b^2*c^2*d^2*e*log(F)^2 - 10*(e*x + d)^(3/2)*b*c*e^2*log(F) + 18*sqrt(e*x + d)*b*c*d*e^2*log(F) + 15*sqrt(e*x + d)*e^3)*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b^3*c^3*log(F)^3))/e`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^{5/2} dx = \int F^{c(a+bx)}(d+ex)^{5/2} dx$$

input `int(F^(c*(a + b*x))*(d + e*x)^(5/2), x)`output `int(F^(c*(a + b*x))*(d + e*x)^(5/2), x)`**Reduce [F]**

$$\int F^{c(a+bx)}(d+ex)^{5/2} dx = f^{ac} \left( \left( \int f^{bcx} \sqrt{ex+d} x^2 dx \right) e^2 + 2 \left( \int f^{bcx} \sqrt{ex+d} x dx \right) de + \left( \int f^{bcx} \sqrt{ex+d} dx \right) d^2 \right)$$

input `int(F^((b*x+a)*c)*(e*x+d)^(5/2), x)`output `f**(a*c)*(int(f**(b*c*x)*sqrt(d + e*x)*x**2,x)*e**2 + 2*int(f**(b*c*x)*sqrt(d + e*x)*x,x)*d*e + int(f**(b*c*x)*sqrt(d + e*x),x)*d**2)`

### 3.71 $\int F^{c(a+bx)}(d+ex)^{3/2} dx$

Optimal result	493
Mathematica [A] (verified)	493
Rubi [A] (verified)	494
Maple [F]	495
Fricas [A] (verification not implemented)	496
Sympy [F]	496
Maxima [F]	496
Giac [B] (verification not implemented)	497
Mupad [F(-1)]	497
Reduce [F]	498

#### Optimal result

Integrand size = 19, antiderivative size = 138

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = \frac{3e^{3/2}F^{c(a-\frac{bd}{e})}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{4b^{5/2}c^{5/2}\log^{5/2}(F)} - \frac{3eF^{c(a+bx)}\sqrt{d+ex}}{2b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d+ex)^{3/2}}{bc\log(F)}$$

output

```
3/4*e^(3/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln
(F)^(1/2)/e^(1/2))/b^(5/2)/c^(5/2)/ln(F)^(5/2)-3/2*e*F^(c*(b*x+a))*(e*x+d)
^(1/2)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)^(3/2)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.46

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = -\frac{F^{c(a-\frac{bd}{e})}(d+ex)^{5/2}\Gamma\left(\frac{5}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)}{e\left(-\frac{bc(d+ex)\log(F)}{e}\right)^{5/2}}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^(3/2), x]
```

output

$$-\left(\frac{F^{c(a - (b*d)/e)}(d + e*x)^{5/2} \Gamma[5/2, -(b*c*(d + e*x)*\text{Log}[F])/e]}{e}\right) / \left(e * \left(-\frac{b*c*(d + e*x)*\text{Log}[F]}{e}\right)^{5/2}\right)$$
**Rubi [A] (verified)**

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$ , Rules used = {2607, 2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{3/2} F^{c(a+bx)} dx$$

$$\downarrow 2607$$

$$\frac{(d + ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \int F^{c(a+bx)} \sqrt{d + ex} dx}{2bc \log(F)}$$

$$\downarrow 2607$$

$$\frac{(d + ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{e \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{2bc \log(F)} \right)}{2bc \log(F)}$$

$$\downarrow 2611$$

$$\frac{(d + ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\int F^{c(a - \frac{bd}{e}) + \frac{bc(d+ex)}{e}} d\sqrt{d+ex}}{bc \log(F)} \right)}{2bc \log(F)}$$

$$\downarrow 2633$$

$$\frac{(d + ex)^{3/2} F^{c(a+bx)}}{bc \log(F)} - \frac{3e \left( \frac{\sqrt{d+ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c(a - \frac{bd}{e})} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c} \sqrt{\log(F)} \sqrt{d+ex}}{\sqrt{e}} \right)}{2b^{3/2} c^{3/2} \log^{\frac{3}{2}}(F)} \right)}{2bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)}(d + e*x)^{3/2}, x]$$

output

```
(F^(c*(a + b*x))*(d + e*x)^(3/2))/(b*c*Log[F]) - (3*e*(-1/2*(Sqrt[e]*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]])/(b^(3/2)*c^(3/2)*Log[F]^(3/2)) + (F^(c*(a + b*x))*Sqrt[d + e*x])/(b*c*Log[F]))/(2*b*c*Log[F])
```

### Defintions of rubi rules used

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2611

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Maple [F]

$$\int F^{c(bx+a)}(ex+d)^{\frac{3}{2}} dx$$

input

```
int(F^(c*(b*x+a))*(e*x+d)^(3/2),x)
```

output

```
int(F^(c*(b*x+a))*(e*x+d)^(3/2),x)
```



**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.88

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = \frac{3\sqrt{\pi}\sqrt{-\frac{bc\log(F)}{e}}e^2 \operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc\log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}} + \frac{2(3bce\log(F) - 2(b^2c^2ex + b^2c^2d)\log(F)^2)\sqrt{ex+d}F^{bcx+ac}}{4b^3c^3\log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(3/2),x, algorithm="fricas")`output `-1/4*(3*sqrt(pi)*sqrt(-b*c*log(F)/e)*e^2*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/F^((b*c*d - a*c*e)/e) + 2*(3*b*c*e*log(F) - 2*(b^2*c^2*e*x + b^2*c^2*d)*log(F)^2)*sqrt(e*x + d)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3)`**Sympy [F]**

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = \int F^{c(a+bx)}(d+ex)^{\frac{3}{2}} dx$$

input `integrate(F**((b*x+a)*c)*(e*x+d)**(3/2),x)`output `Integral(F**(c*(a + b*x))*(d + e*x)**(3/2), x)`**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = \int (ex+d)^{\frac{3}{2}}F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(3/2),x, algorithm="maxima")`output `integrate((e*x + d)^(3/2)*F^((b*x + a)*c), x)`

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 372 vs.  $2(110) = 220$ .

Time = 0.15 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.70

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = \frac{4\sqrt{\pi}d^2 e \operatorname{erf}\left(-\frac{\sqrt{-bce \log(F)}\sqrt{ex+d}}{e}\right) e^{\left(-\frac{bcd \log(F)-ace \log(F)}{e}\right)}}{\sqrt{-bce \log(F)}} - 4d \left( \frac{\sqrt{\pi}(2bcd \log(F)+e) e \operatorname{erf}\left(-\frac{\sqrt{-bce \log(F)}\sqrt{ex+d}}{e}\right) e^{\left(-\frac{bcd \log(F)-ace \log(F)}{e}\right)}}{\sqrt{-bce \log(F)}bc \log(F)} \right)$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(3/2),x, algorithm="giac")`

output `-1/4*(4*sqrt(pi)*d^2*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/sqrt(-b*c*e*log(F)) - 4*d*(sqrt(pi)*(2*b*c*d*log(F) + e)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b*c*log(F)) + 2*sqrt(e*x + d)*e*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b*c*log(F))) + sqrt(pi)*(4*b^2*c^2*d^2*log(F)^2 + 4*b*c*d*e*log(F) + 3*e^2)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b^2*c^2*log(F)^2) - 2*(2*(e*x + d)^(3/2)*b*c*e*log(F) - 4*sqrt(e*x + d)*b*c*d*e*log(F) - 3*sqrt(e*x + d)*e^2)*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b^2*c^2*log(F)^2))/e`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = \int F^{c(a+bx)}(d+ex)^{3/2} dx$$

input `int(F^(c*(a + b*x))*(d + e*x)^(3/2),x)`

output `int(F^(c*(a + b*x))*(d + e*x)^(3/2), x)`

**Reduce [F]**

$$\int F^{c(a+bx)}(d+ex)^{3/2} dx = f^{ac} \left( \left( \int f^{bcx} \sqrt{ex+d} x dx \right) e + \left( \int f^{bcx} \sqrt{ex+d} dx \right) d \right)$$

input `int(F^((b*x+a)*c)*(e*x+d)^(3/2),x)`

output `f**(a*c)*(int(f**(b*c*x)*sqrt(d + e*x)*x,x)*e + int(f**(b*c*x)*sqrt(d + e*x),x)*d)`

### 3.72 $\int F^{c(a+bx)} \sqrt{d+ex} dx$

Optimal result	499
Mathematica [A] (verified)	499
Rubi [A] (verified)	500
Maple [F]	501
Fricas [A] (verification not implemented)	501
Sympy [F]	502
Maxima [F]	502
Giac [B] (verification not implemented)	502
Mupad [F(-1)]	503
Reduce [F]	503

#### Optimal result

Integrand size = 19, antiderivative size = 105

$$\int F^{c(a+bx)} \sqrt{d+ex} dx = -\frac{\sqrt{e} F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{2b^{3/2}c^{3/2}\log^{3/2}(F)} + \frac{F^{c(a+bx)}\sqrt{d+ex}}{bc\log(F)}$$

output

```
-1/2*e^(1/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln(F)^(1/2)/e^(1/2))/b^(3/2)/c^(3/2)/ln(F)^(3/2)+F^(c*(b*x+a))*(e*x+d)^(1/2)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int F^{c(a+bx)} \sqrt{d+ex} dx = -\frac{F^{c(a-\frac{bd}{e})} (d+ex)^{3/2} \Gamma\left(\frac{3}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)}{e\left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3/2}}$$

input

```
Integrate[F^(c*(a + b*x))*Sqrt[d + e*x], x]
```

output

$$-\left(\frac{F^{c(a - (b*d)/e)}(d + e*x)^{3/2} \Gamma[3/2, -(b*c*(d + e*x)*\text{Log}[F])/e]}{e}\right) / \left(e^{-(b*c*(d + e*x)*\text{Log}[F])/e}\right)^{3/2}$$

**Rubi [A] (verified)**

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2607, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \sqrt{d + ex} F^{c(a+bx)} dx \\ & \quad \downarrow 2607 \\ & \frac{\sqrt{d + ex} F^{c(a+bx)}}{bc \log(F)} - \frac{e \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{2bc \log(F)} \\ & \quad \downarrow 2611 \\ & \frac{\sqrt{d + ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\int F^{c\left(a - \frac{bd}{e}\right) + \frac{bc(d+ex)}{e}} d\sqrt{d + ex}}{bc \log(F)} \\ & \quad \downarrow 2633 \\ & \frac{\sqrt{d + ex} F^{c(a+bx)}}{bc \log(F)} - \frac{\sqrt{\pi} \sqrt{e} F^{c\left(a - \frac{bd}{e}\right)} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c} \sqrt{\log(F)} \sqrt{d+ex}}{\sqrt{e}}\right)}{2b^{3/2} c^{3/2} \log^{3/2}(F)} \end{aligned}$$

input

$$\text{Int}[F^{c(a + b*x)} * \text{Sqrt}[d + e*x], x]$$

output

$$-1/2 * (\text{Sqrt}[e] * F^{c(a - (b*d)/e)} * \text{Sqrt}[\text{Pi}] * \operatorname{Erfi}[(\text{Sqrt}[b] * \text{Sqrt}[c] * \text{Sqrt}[d + e*x] * \text{Sqrt}[\text{Log}[F]]) / \text{Sqrt}[e]]) / (b^{3/2} * c^{3/2} * \text{Log}[F]^{3/2}) + (F^{c(a + b*x)} * \text{Sqrt}[d + e*x]) / (b*c*\text{Log}[F])$$

## Definitions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

## Maple [F]

$$\int F^{c(bx+a)} \sqrt{ex+d} dx$$

input `int(F^(c*(b*x+a))*(e*x+d)^(1/2),x)`

output `int(F^(c*(b*x+a))*(e*x+d)^(1/2),x)`

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int F^{c(a+bx)} \sqrt{d+ex} dx = \frac{2\sqrt{ex+d}F^{bcx+ac}bc \log(F) + \frac{\sqrt{\pi}\sqrt{-\frac{bc \log(F)}{e}} e \operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc \log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}}}{2b^2c^2 \log(F)^2}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(1/2),x, algorithm="fricas")`

output

```
1/2*(2*sqrt(e*x + d)*F^(b*c*x + a*c)*b*c*log(F) + sqrt(pi)*sqrt(-b*c*log(F)
)/e)*e*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/F^((b*c*d - a*c*e)/e)/(b^2*
c^2*log(F)^2)
```

**Sympy [F]**

$$\int F^{c(a+bx)}\sqrt{d+ex} dx = \int F^{c(a+bx)}\sqrt{d+ex} dx$$

input

```
integrate(F**((b*x+a)*c)*(e*x+d)**(1/2), x)
```

output

```
Integral(F**(c*(a + b*x))*sqrt(d + e*x), x)
```

**Maxima [F]**

$$\int F^{c(a+bx)}\sqrt{d+ex} dx = \int \sqrt{ex+d}F^{(bx+a)c} dx$$

input

```
integrate(F^((b*x+a)*c)*(e*x+d)^(1/2), x, algorithm="maxima")
```

output

```
integrate(sqrt(e*x + d)*F^((b*x + a)*c), x)
```

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 189 vs.  $2(81) = 162$ .

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.80

$$\int F^{c(a+bx)}\sqrt{d+ex} dx = \frac{2\sqrt{\pi}de \operatorname{erf}\left(-\frac{\sqrt{-bce \log(F)}\sqrt{ex+d}}{e}\right) e^{\left(-\frac{bcd \log(F)-ace \log(F)}{e}\right)}}{\sqrt{-bce \log(F)}} - \frac{\sqrt{\pi}(2bcd \log(F)+e) e \operatorname{erf}\left(-\frac{\sqrt{-bce \log(F)}\sqrt{ex+d}}{e}\right) e^{\left(-\frac{bcd \log(F)-ace \log(F)}{e}\right)}}{\sqrt{-bce \log(F)}bc \log(F)}$$

2 e

input `integrate(F^((b*x+a)*c)*(e*x+d)^(1/2),x, algorithm="giac")`

output `-1/2*(2*sqrt(pi)*d*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/sqrt(-b*c*e*log(F)) - sqrt(pi)*(2*b*c*d*log(F) + e)*e*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/(sqrt(-b*c*e*log(F))*b*c*log(F)) - 2*sqrt(e*x + d)*e*e^(((e*x + d)*b*c*log(F) - b*c*d*log(F) + a*c*e*log(F))/e)/(b*c*log(F)))/e`

### Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)} \sqrt{d+ex} dx = \int F^{c(a+bx)} \sqrt{d+ex} dx$$

input `int(F^(c*(a + b*x))*(d + e*x)^(1/2),x)`

output `int(F^(c*(a + b*x))*(d + e*x)^(1/2), x)`

### Reduce [F]

$$\int F^{c(a+bx)} \sqrt{d+ex} dx = f^{ac} \left( \int f^{bcx} \sqrt{ex+ddx} \right)$$

input `int(F^((b*x+a)*c)*(e*x+d)^(1/2),x)`

output `f**(a*c)*int(f**(b*c*x)*sqrt(d + e*x),x)`



### 3.73 $\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$

Optimal result	504
Mathematica [A] (verified)	504
Rubi [A] (verified)	505
Maple [F]	506
Fricas [A] (verification not implemented)	506
Sympy [F]	506
Maxima [F]	507
Giac [A] (verification not implemented)	507
Mupad [F(-1)]	507
Reduce [F]	508

#### Optimal result

Integrand size = 19, antiderivative size = 72

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = \frac{F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)}{\sqrt{b}\sqrt{c}\sqrt{e}\sqrt{\log(F)}}$$

output

```
F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln(F)^(1/2)/e^(1/2))/b^(1/2)/c^(1/2)/e^(1/2)/ln(F)^(1/2)
```

#### Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = -\frac{F^{c\left(a-\frac{bd}{e}\right)} \sqrt{d+ex} \Gamma\left(\frac{1}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)}{e \sqrt{-\frac{bc(d+ex)\log(F)}{e}}}$$

input

```
Integrate[F^(c*(a + b*x))/Sqrt[d + e*x], x]
```

output

```
-((F^(c*(a - (b*d)/e))*Sqrt[d + e*x]*Gamma[1/2, -((b*c*(d + e*x)*Log[F])/e]))/(e*Sqrt[-((b*c*(d + e*x)*Log[F])/e]))
```

**Rubi [A] (verified)**

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

↓ 2611

$$\frac{2 \int F^{c\left(a-\frac{bd}{e}\right)+\frac{bc(d+ex)}{e}} d\sqrt{d+ex}}{e}$$

↓ 2633

$$\frac{\sqrt{\pi} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{\sqrt{b}\sqrt{c}\sqrt{e}\sqrt{\log(F)}}$$

input `Int[F^(c*(a + b*x))/Sqrt[d + e*x],x]`

output `(F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]])/(Sqrt[b]*Sqrt[c]*Sqrt[e]*Sqrt[Log[F]])`

**Defintions of rubi rules used**

rule 2611 `Int[(F_)^((g_)*((e_) + (f_)*(x_)))/Sqrt[(c_) + (d_)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

**Maple [F]**

$$\int \frac{F^{c(bx+a)}}{\sqrt{ex+d}} dx$$

input `int(F^(c*(b*x+a))/(e*x+d)^(1/2),x)`

output `int(F^(c*(b*x+a))/(e*x+d)^(1/2),x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = -\frac{\sqrt{\pi} \sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}} bc \log(F)}$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(1/2),x, algorithm="fricas")`

output `-sqrt(pi)*sqrt(-b*c*log(F)/e)*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))/(F^((b*c*d - a*c*e)/e)*b*c*log(F))`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**(1/2),x)`

output `Integral(F**(c*(a + b*x))/sqrt(d + e*x), x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = \int \frac{F^{(bx+a)c}}{\sqrt{ex+d}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(1/2),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/sqrt(e*x + d), x)`

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = -\frac{\sqrt{\pi} \operatorname{erf}\left(-\frac{\sqrt{-bce \log(F)} \sqrt{ex+d}}{e}\right) e^{\left(-\frac{bcd \log(F) - ace \log(F)}{e}\right)}}{\sqrt{-bce \log(F)}}$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(1/2),x, algorithm="giac")`

output `-sqrt(pi)*erf(-sqrt(-b*c*e*log(F))*sqrt(e*x + d)/e)*e^(-(b*c*d*log(F) - a*c*e*log(F))/e)/sqrt(-b*c*e*log(F))`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^(1/2),x)`

output `int(F^(c*(a + b*x))/(d + e*x)^(1/2), x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx = f^{ac} \left( \int \frac{f^{bcx}}{\sqrt{ex+d}} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^(1/2),x)`

output `f**(a*c)*int(f**(b*c*x)/sqrt(d + e*x),x)`

### 3.74 $\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$

Optimal result	509
Mathematica [A] (verified)	509
Rubi [A] (verified)	510
Maple [F]	511
Fricas [A] (verification not implemented)	511
Sympy [F]	512
Maxima [F]	512
Giac [F]	512
Mupad [F(-1)]	513
Reduce [F]	513

#### Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = -\frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} + \frac{2\sqrt{b}\sqrt{c}F^{c\left(a-\frac{bd}{e}\right)}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right)\sqrt{\log(F)}}{e^{3/2}}$$

output

```
-2*F^(c*(b*x+a))/e/(e*x+d)^(1/2)+2*b^(1/2)*c^(1/2)*F^(c*(a-b*d/e))*Pi^(1/2)
)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln(F)^(1/2)/e^(1/2))*ln(F)^(1/2)/e^(3/2)
```

#### Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = -\frac{2\left(F^{c(a+bx)} - F^{c\left(a-\frac{bd}{e}\right)}\Gamma\left(\frac{1}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)\sqrt{-\frac{bc(d+ex)\log(F)}{e}}\right)}{e\sqrt{d+ex}}$$

input

```
Integrate[F^(c*(a + b*x))/(d + e*x)^(3/2), x]
```

output

$$\frac{(-2*(F^{c*(a + b*x)}) - F^{c*(a - (b*d)/e)}*Gamma[1/2, -((b*c*(d + e*x)*Log[F])/e)]*Sqrt[-((b*c*(d + e*x)*Log[F])/e)])/(e*Sqrt[d + e*x])$$
**Rubi [A] (verified)**

Time = 0.47 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$$

$$\downarrow 2608$$

$$\frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{e} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}$$

$$\downarrow 2611$$

$$\frac{4bc \log(F) \int F^{c\left(a - \frac{bd}{e}\right) + \frac{bc(d+ex)}{e}} d\sqrt{d+ex}}{e^2} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}$$

$$\downarrow 2633$$

$$\frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)}F^{c\left(a - \frac{bd}{e}\right)} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}$$

input

$$\text{Int}[F^{c*(a + b*x)}/(d + e*x)^{(3/2)}, x]$$

output

$$\frac{(-2*F^{c*(a + b*x)})/(e*Sqrt[d + e*x]) + (2*Sqrt[b]*Sqrt[c]*F^{c*(a - (b*d)/e)}*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]]*Sqrt[Log[F]])/e^{(3/2)}$$

## Definitions of rubi rules used

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

## Maple [F]

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{\frac{3}{2}}} dx$$

input `int(F^(c*(b*x+a))/(e*x+d)^(3/2),x)`

output `int(F^(c*(b*x+a))/(e*x+d)^(3/2),x)`

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = -\frac{2 \left( \frac{\sqrt{\pi}(ex+d)\sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d}\sqrt{-\frac{bc \log(F)}{e}}\right)}{F^{\frac{bcd-ace}{e}}} + \sqrt{ex+d}F^{bcx+ac} \right)}{e^2x + de}$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(3/2),x, algorithm="fricas")`



output

```
-2*(sqrt(pi)*(e*x + d)*sqrt(-b*c*log(F)/e)*erf(sqrt(e*x + d)*sqrt(-b*c*log
(F)/e))/F^((b*c*d - a*c*e)/e) + sqrt(e*x + d)*F^(b*c*x + a*c))/(e^2*x + d*
e)
```

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{\frac{3}{2}}} dx$$

input

```
integrate(F**((b*x+a)*c)/(e*x+d)**(3/2), x)
```

output

```
Integral(F**(c*(a + b*x))/(d + e*x)**(3/2), x)
```

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{3}{2}}} dx$$

input

```
integrate(F^((b*x+a)*c)/(e*x+d)^(3/2), x, algorithm="maxima")
```

output

```
integrate(F^((b*x + a)*c)/(e*x + d)^(3/2), x)
```

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{3}{2}}} dx$$

input

```
integrate(F^((b*x+a)*c)/(e*x+d)^(3/2), x, algorithm="giac")
```

output

```
integrate(F^((b*x + a)*c)/(e*x + d)^(3/2), x)
```

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^(3/2), x)`output `int(F^(c*(a + b*x))/(d + e*x)^(3/2), x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx = f^{ac} \left( \int \frac{f^{bcx}}{\sqrt{ex+d}d + \sqrt{ex+d}ex} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^(3/2), x)`output `f**(a*c)*int(f**(b*c*x)/(sqrt(d + e*x)*d + sqrt(d + e*x)*e*x), x)`

### 3.75 $\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$

Optimal result	514
Mathematica [A] (verified)	514
Rubi [A] (verified)	515
Maple [F]	516
Fricas [A] (verification not implemented)	517
Sympy [F]	517
Maxima [F]	518
Giac [F]	518
Mupad [F(-1)]	518
Reduce [F]	519

#### Optimal result

Integrand size = 19, antiderivative size = 130

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = -\frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{3e^2 \sqrt{d+ex}} + \frac{4b^{3/2} c^{3/2} F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right) \log^{\frac{3}{2}}(F)}{3e^{5/2}}$$

output

```
-2/3*F^(c*(b*x+a))/e/(e*x+d)^(3/2)-4/3*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^(1/2)+4/3*b^(3/2)*c^(3/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln(F)^(1/2)/e^(1/2))*ln(F)^(3/2)/e^(5/2)
```

#### Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = \frac{2\left(2eF^{c(a-\frac{bd}{e})}\Gamma\left(\frac{1}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3/2} + F^{c(a+bx)}(e + 2bc(d+ex)\log(F))\right)}{3e^2(d+ex)^{3/2}}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x)^(5/2), x]`

output  $(-2*(2*e*F^{c*(a - (b*d)/e)})*\text{Gamma}[1/2, -((b*c*(d + e*x)*\text{Log}[F])/e)]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{3/2} + F^{c*(a + b*x)}*(e + 2*b*c*(d + e*x)*\text{Log}[F]))/(3*e^2*(d + e*x)^{3/2})$

### Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$ , Rules used = {2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \\
 & \quad \downarrow \text{2608} \\
 & \frac{2bc \log(F) \left( \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{e} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \\
 & \quad \downarrow \text{2611} \\
 & \frac{2bc \log(F) \left( \frac{4bc \log(F) \int F^{c\left(a - \frac{bd}{e} + \frac{bc(d+ex)}{e} d\sqrt{d+ex}\right)}{e^2} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \\
 & \quad \downarrow \text{2633} \\
 & \frac{2bc \log(F) \left( \frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)} F^{c\left(a - \frac{bd}{e}\right)} \text{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right)}{e^{3/2}} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}}
 \end{aligned}$$

input `Int[F^(c*(a + b*x))/(d + e*x)^(5/2),x]`

output `(-2*F^(c*(a + b*x)))/(3*e*(d + e*x)^(3/2)) + (2*b*c*(-2*F^(c*(a + b*x)))/(e*Sqrt[d + e*x]) + (2*Sqrt[b]*Sqrt[c]*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]]*Sqrt[Log[F]])/e^(3/2))*Log[F])/(3*e)`

### Defintions of rubi rules used

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

### Maple [F]

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{\frac{5}{2}}} dx$$

input `int(F^(c*(b*x+a))/(e*x+d)^(5/2),x)`

output `int(F^(c*(b*x+a))/(e*x+d)^(5/2),x)`

**Fricas [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.08

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = \frac{2 \left( \frac{2\sqrt{\pi}(bce^2x^2 + 2bcde x + bcd^2) \sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right) \log(F)}{F^{\frac{bcd-ace}{e}}} + \sqrt{ex+d} (2(bce x + bcd) \log(F) + e) F^{bcx+a} \right)}{3(e^4x^2 + 2de^3x + d^2e^2)}$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(5/2),x, algorithm="fricas")`

output `-2/3*(2*sqrt(pi)*(b*c*e^2*x^2 + 2*b*c*d*e*x + b*c*d^2)*sqrt(-b*c*log(F)/e) *erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e))*log(F)/F^((b*c*d - a*c*e)/e) + sqrt(e*x + d)*(2*(b*c*e*x + b*c*d)*log(F) + e)*F^(b*c*x + a*c))/(e^4*x^2 + 2*d*e^3*x + d^2*e^2)`

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{\frac{5}{2}}} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**(5/2),x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**(5/2), x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{5}{2}}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(5/2),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^(5/2), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{5}{2}}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(5/2),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^(5/2), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^(5/2),x)`

output `int(F^(c*(a + b*x))/(d + e*x)^(5/2), x)`

**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx = f^{ac} \left( \int \frac{f^{bcx}}{\sqrt{ex+d} d^2 + 2\sqrt{ex+d} dex + \sqrt{ex+d} e^2 x^2} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^(5/2),x)`

output `f**(a*c)*int(f**(b*c*x)/(sqrt(d + e*x)*d**2 + 2*sqrt(d + e*x)*d*e*x + sqrt(d + e*x)*e**2*x**2),x)`



### 3.76 $\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$

Optimal result	520
Mathematica [A] (verified)	520
Rubi [A] (verified)	521
Maple [F]	523
Fricas [A] (verification not implemented)	523
Sympy [F]	524
Maxima [F]	524
Giac [F]	524
Mupad [F(-1)]	525
Reduce [F]	525

#### Optimal result

Integrand size = 19, antiderivative size = 165

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = -\frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{15e^2(d+ex)^{3/2}} - \frac{8b^2c^2F^{c(a+bx)} \log^2(F)}{15e^3\sqrt{d+ex}} + \frac{8b^{5/2}c^{5/2}F^{c(a-\frac{bd}{e})} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right) \log^{\frac{5}{2}}(F)}{15e^{7/2}}$$

output

```
-2/5*F^(c*(b*x+a))/e/(e*x+d)^(5/2)-4/15*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^(3/2)-8/15*b^2*c^2*F^(c*(b*x+a))*ln(F)^2/e^3/(e*x+d)^(1/2)+8/15*b^(5/2)*c^(5/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln(F)^(1/2)/e^(1/2))*ln(F)^(5/2)/e^(7/2)
```

#### Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = \frac{2\left(-3e^2F^{c(a+bx)} - 2bc(d+ex)\log(F)\right)\left(2eF^{c(a-\frac{bd}{e})}\Gamma\left(\frac{1}{2}, -\frac{bc(d+ex)\log(F)}{e}\right)\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)}{15e^3(d+ex)^{5/2}}$$

input

```
Integrate[F^(c*(a + b*x))/(d + e*x)^(7/2), x]
```

output

```
(2*(-3*e^2*F^(c*(a + b*x)) - 2*b*c*(d + e*x)*Log[F]*(2*e*F^(c*(a - (b*d)/e)))*Gamma[1/2, -((b*c*(d + e*x)*Log[F])/e)]*(-((b*c*(d + e*x)*Log[F])/e))^(3/2) + F^(c*(a + b*x))*(e + 2*b*c*(d + e*x)*Log[F]))/(15*e^3*(d + e*x)^(5/2))
```

**Rubi [A] (verified)**

Time = 0.72 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2608, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} \\
 & \quad \downarrow \text{2608} \\
 & \frac{2bc \log(F) \left( \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} \\
 & \quad \downarrow \text{2608} \\
 & \frac{2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{\sqrt{d+ex}} dx}{e} - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} \right) - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} \\
 & \quad \downarrow \text{2611}
 \end{aligned}$$

$$\frac{2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{4bc \log(F) \int F^{c(a - \frac{bd}{e}) + \frac{bc(d+ex)}{e} d\sqrt{d+ex} - 2F^{c(a+bx)}}{e\sqrt{d+ex}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{\frac{5e}{2F^{c(a+bx)}}} \frac{5e}{5e(d+ex)^{5/2}}$$

↓ 2633

$$\frac{2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)} F^{c(a - \frac{bd}{e})} \operatorname{erfi} \left( \frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}} \right) - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{\frac{5e}{2F^{c(a+bx)}}} \frac{5e}{5e(d+ex)^{5/2}}$$

input `Int [F^(c*(a + b*x))/(d + e*x)^(7/2), x]`

output `(-2*F^(c*(a + b*x)))/(5*e*(d + e*x)^(5/2)) + (2*b*c*Log[F]*((-2*F^(c*(a + b*x)))/(3*e*(d + e*x)^(3/2)) + (2*b*c*((-2*F^(c*(a + b*x)))/(e*Sqrt[d + e*x]) + (2*Sqrt[b]*Sqrt[c]*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]]*Sqrt[Log[F]])/e^(3/2))*Log[F])/(3*e)))/(5*e)`

**Defintions of rubi rules used**

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

**Maple [F]**

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{\frac{7}{2}}} dx$$

input

```
int(F^(c*(b*x+a))/(e*x+d)^(7/2),x)
```

output

```
int(F^(c*(b*x+a))/(e*x+d)^(7/2),x)
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.39

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = \frac{2 \left( \frac{4 \sqrt{\pi} (b^2 c^2 e^3 x^3 + 3 b^2 c^2 d e^2 x^2 + 3 b^2 c^2 d^2 e x + b^2 c^2 d^3) \sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf} \left( \sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}} \right) \log(F)^2}{F^{\frac{bcd-ace}{e}}} + (4 (b^2 c^2 e^2 x^2 + 2 b^2 c^2 d e x + b^2 c^2 d^2)) \log(F)^2 \right)}{15 (e^6 x^3 + 3 d e^5 x^2 + 3 d^2 e^4 x + d^3 e^3)}$$

input

```
integrate(F^((b*x+a)*c)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output

```
-2/15*(4*sqrt(pi)*(b^2*c^2*e^3*x^3 + 3*b^2*c^2*d*e^2*x^2 + 3*b^2*c^2*d^2*e
*x + b^2*c^2*d^3)*sqrt(-b*c*log(F)/e)*erf(sqrt(e*x + d)*sqrt(-b*c*log(F)/e
)))*log(F)^2/F^((b*c*d - a*c*e)/e) + (4*(b^2*c^2*e^2*x^2 + 2*b^2*c^2*d*e*x
+ b^2*c^2*d^2)*log(F)^2 + 3*e^2 + 2*(b*c*e^2*x + b*c*d*e)*log(F))*sqrt(e*x
+ d)*F^(b*c*x + a*c)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

**Sympy [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{\frac{7}{2}}} dx$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**(7/2),x)`

output `Integral(F**(c*(a + b*x))/(d + e*x)**(7/2), x)`

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{7}{2}}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(7/2),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^(7/2), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{7}{2}}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(7/2),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^(7/2), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^(7/2), x)`output `int(F^(c*(a + b*x))/(d + e*x)^(7/2), x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx = f^{ac} \left( \int \frac{f^{bcx}}{\sqrt{ex+d} d^3 + 3\sqrt{ex+d} d^2 ex + 3\sqrt{ex+d} d e^2 x^2 + \sqrt{ex+d} e^3 x^3} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^(7/2), x)`output `f**(a*c)*int(f**(b*c*x)/(sqrt(d + e*x)*d**3 + 3*sqrt(d + e*x)*d**2*e*x + 3*sqrt(d + e*x)*d*e**2*x**2 + sqrt(d + e*x)*e**3*x**3), x)`

### 3.77 $\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$

Optimal result	526
Mathematica [A] (verified)	527
Rubi [A] (verified)	527
Maple [F]	530
Fricas [A] (verification not implemented)	530
Sympy [F(-1)]	531
Maxima [F]	531
Giac [F]	531
Mupad [F(-1)]	532
Reduce [F]	532

#### Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = -\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} - \frac{4bcF^{c(a+bx)} \log(F)}{35e^2(d+ex)^{5/2}} - \frac{8b^2c^2F^{c(a+bx)} \log^2(F)}{105e^3(d+ex)^{3/2}} - \frac{16b^3c^3F^{c(a+bx)} \log^3(F)}{105e^4\sqrt{d+ex}} + \frac{16b^{7/2}c^{7/2}F^{c\left(a-\frac{bd}{e}\right)} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{d+ex}\sqrt{\log(F)}}{\sqrt{e}}\right) \log^{7/2}(F)}{105e^{9/2}}$$

output

```
-2/7*F^(c*(b*x+a))/e/(e*x+d)^(7/2)-4/35*b*c*F^(c*(b*x+a))*ln(F)/e^2/(e*x+d)^(5/2)-8/105*b^2*c^2*F^(c*(b*x+a))*ln(F)^2/e^3/(e*x+d)^(3/2)-16/105*b^3*c^3*F^(c*(b*x+a))*ln(F)^3/e^4/(e*x+d)^(1/2)+16/105*b^(7/2)*c^(7/2)*F^(c*(a-b*d/e))*Pi^(1/2)*erfi(b^(1/2)*c^(1/2)*(e*x+d)^(1/2)*ln(F)^(1/2)/e^(1/2))*ln(F)^(7/2)/e^(9/2)
```

### Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.72

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = \frac{2 \left( -15e^3 F^{c(a+bx)} + 2bc(d+ex) \log(F) \left( -3e^2 F^{c(a+bx)} - 2bc(d+ex) \log(F) \right) \left( 2e F^{c(a+bx)} - 2bc(d+ex) \log(F) \right) \right)}{105e^4(d+ex)^{7/2}}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x)^(9/2), x]`

output `(2*(-15*e^3*F^(c*(a + b*x)) + 2*b*c*(d + e*x)*Log[F]*(-3*e^2*F^(c*(a + b*x))) - 2*b*c*(d + e*x)*Log[F]*(2*e*F^(c*(a - (b*d)/e))*Gamma[1/2, -(b*c*(d + e*x)*Log[F])/e])*(-((b*c*(d + e*x)*Log[F])/e))^(3/2) + F^(c*(a + b*x))*(e + 2*b*c*(d + e*x)*Log[F])))/(105*e^4*(d + e*x)^(7/2))`

### Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {2608, 2608, 2608, 2608, 2611, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx \\ & \quad \downarrow 2608 \\ & \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^{7/2}} dx}{7e} - \frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} \\ & \quad \downarrow 2608 \\ & \frac{2bc \log(F) \left( \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^{5/2}} dx}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} \right)}{7e} - \frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}} \\ & \quad \downarrow 2608 \end{aligned}$$



$$2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{(d+ex)^{3/2}} dx}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} \right)$$


---


$$\frac{7e}{7e(d+ex)^{7/2}} - \frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}}$$

↓ 2608

$$2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{2bc \log(F) \int \frac{F^{c(a+bx)}}{e \sqrt{d+ex}} dx}{3e} - \frac{2F^{c(a+bx)}}{e \sqrt{d+ex}} \right)}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}} \right)$$


---


$$\frac{7e}{7e(d+ex)^{7/2}} - \frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}}$$

↓ 2611

$$2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{4bc \log(F) \int F^{c(a-\frac{bd}{e}) + \frac{bc(d+ex)}{e} d \sqrt{d+ex}}{e^2} - \frac{2F^{c(a+bx)}}{e \sqrt{d+ex}} \right)}{3e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)$$


---


$$\frac{7e}{7e(d+ex)^{7/2}} - \frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}}$$

↓ 2633

$$\frac{2bc \log(F) \left( \frac{2bc \log(F) \left( \frac{2\sqrt{\pi}\sqrt{b}\sqrt{c}\sqrt{\log(F)} F^{c\left(a-\frac{bd}{e}\right)} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c}\sqrt{\log(F)}\sqrt{d+ex}}{\sqrt{e}}\right) - \frac{2F^{c(a+bx)}}{e\sqrt{d+ex}}\right)}{e^{3/2}} - \frac{2F^{c(a+bx)}}{3e} \right)}{5e} - \frac{2F^{c(a+bx)}}{3e(d+ex)^{3/2}} \right)}{5e} - \frac{2F^{c(a+bx)}}{5e(d+ex)^{5/2}}$$


---


$$\frac{2F^{c(a+bx)}}{7e(d+ex)^{7/2}}$$

input `Int[F^(c*(a + b*x))/(d + e*x)^(9/2), x]`

output `(-2*F^(c*(a + b*x)))/(7*e*(d + e*x)^(7/2)) + (2*b*c*Log[F]*((-2*F^(c*(a + b*x)))/(5*e*(d + e*x)^(5/2)) + (2*b*c*Log[F]*((-2*F^(c*(a + b*x)))/(3*e*(d + e*x)^(3/2)) + (2*b*c*((-2*F^(c*(a + b*x)))/(e*Sqrt[d + e*x]) + (2*Sqrt[b]*Sqrt[c]*F^(c*(a - (b*d)/e))*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c]*Sqrt[d + e*x]*Sqrt[Log[F]])/Sqrt[e]]*Sqrt[Log[F]])/e^(3/2))*Log[F])/(3*e)))/(5*e))/(7*e)`

**Defintions of rubi rules used**

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2633

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

**Maple [F]**

$$\int \frac{F^{c(bx+a)}}{(ex+d)^{\frac{9}{2}}} dx$$

input

```
int(F^(c*(b*x+a))/(e*x+d)^(9/2),x)
```

output

```
int(F^(c*(b*x+a))/(e*x+d)^(9/2),x)
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.60

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = \frac{2 \left( 8\sqrt{\pi}(b^3c^3e^4x^4 + 4b^3c^3de^3x^3 + 6b^3c^3d^2e^2x^2 + 4b^3c^3d^3ex + b^3c^3d^4) \sqrt{-\frac{bc \log(F)}{e}} \operatorname{erf}\left(\sqrt{ex+d} \sqrt{-\frac{bc \log(F)}{e}}\right) \log(F)^3 + (8(b^3c^3e^3x^3 + \dots) \right)}{F^{\frac{bcd-ace}{e}}}$$

105 (

input

```
integrate(F^((b*x+a)*c)/(e*x+d)^(9/2),x, algorithm="fricas")
```

output

```
-2/105*(8*sqrt(pi)*(b^3*c^3*e^4*x^4 + 4*b^3*c^3*d*e^3*x^3 + 6*b^3*c^3*d^2*
e^2*x^2 + 4*b^3*c^3*d^3*e*x + b^3*c^3*d^4)*sqrt(-b*c*log(F)/e)*erf(sqrt(e*
x + d)*sqrt(-b*c*log(F)/e))*log(F)^3/F^((b*c*d - a*c*e)/e) + (8*(b^3*c^3*e
^3*x^3 + 3*b^3*c^3*d*e^2*x^2 + 3*b^3*c^3*d^2*e*x + b^3*c^3*d^3)*log(F)^3 +
15*e^3 + 4*(b^2*c^2*e^3*x^2 + 2*b^2*c^2*d*e^2*x + b^2*c^2*d^2*e)*log(F)^2
+ 6*(b*c*e^3*x + b*c*d*e^2)*log(F))*sqrt(e*x + d)*F^(b*c*x + a*c))/(e^8*x
^4 + 4*d*e^7*x^3 + 6*d^2*e^6*x^2 + 4*d^3*e^5*x + d^4*e^4)
```

**Sympy [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = \text{Timed out}$$

input `integrate(F**((b*x+a)*c)/(e*x+d)**(9/2), x)`

output Timed out

**Maxima [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{9}{2}}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(9/2), x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^(9/2), x)`

**Giac [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^{\frac{9}{2}}} dx$$

input `integrate(F^((b*x+a)*c)/(e*x+d)^(9/2), x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^(9/2), x)`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^(9/2), x)`output `int(F^(c*(a + b*x))/(d + e*x)^(9/2), x)`**Reduce [F]**

$$\int \frac{F^{c(a+bx)}}{(d+ex)^{9/2}} dx = f^{ac} \left( \int \frac{f^{bcx}}{\sqrt{ex+d} d^4 + 4\sqrt{ex+d} d^3 ex + 6\sqrt{ex+d} d^2 e^2 x^2 + 4\sqrt{ex+d} d e^3 x^3 + \sqrt{ex+d} e^4 x^4}} dx \right)$$

input `int(F^((b*x+a)*c)/(e*x+d)^(9/2), x)`output `f**(a*c)*int(f**(b*c*x)/(sqrt(d + e*x)*d**4 + 4*sqrt(d + e*x)*d**3*e*x + 6*sqrt(d + e*x)*d**2*e**2*x**2 + 4*sqrt(d + e*x)*d*e**3*x**3 + sqrt(d + e*x)*e**4*x**4), x)`

### 3.78 $\int e^{-bx} x^{13/2} dx$

Optimal result	533
Mathematica [A] (verified)	533
Rubi [A] (verified)	534
Maple [A] (verified)	542
Fricas [A] (verification not implemented)	544
Sympy [A] (verification not implemented)	544
Maxima [A] (verification not implemented)	545
Giac [A] (verification not implemented)	545
Mupad [B] (verification not implemented)	546
Reduce [F]	546

#### Optimal result

Integrand size = 12, antiderivative size = 151

$$\int e^{-bx} x^{13/2} dx = -\frac{135135e^{-bx} \sqrt{x}}{64b^7} - \frac{45045e^{-bx} x^{3/2}}{32b^6} - \frac{9009e^{-bx} x^{5/2}}{16b^5} - \frac{1287e^{-bx} x^{7/2}}{8b^4} - \frac{143e^{-bx} x^{9/2}}{4b^3} - \frac{13e^{-bx} x^{11/2}}{2b^2} - \frac{e^{-bx} x^{13/2}}{b} + \frac{135135\sqrt{\pi}\operatorname{erf}(\sqrt{b}\sqrt{x})}{128b^{15/2}}$$

output

```
-135135/64*x^(1/2)/b^7/exp(b*x)-45045/32*x^(3/2)/b^6/exp(b*x)-9009/16*x^(5/2)/b^5/exp(b*x)-1287/8*x^(7/2)/b^4/exp(b*x)-143/4*x^(9/2)/b^3/exp(b*x)-13/2*x^(11/2)/b^2/exp(b*x)-x^(13/2)/b/exp(b*x)+135135/128*Pi^(1/2)*erf(b^(1/2)*x^(1/2))/b^(15/2)
```

#### Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.16

$$\int e^{-bx} x^{13/2} dx = -\frac{\sqrt{bx}\Gamma\left(\frac{15}{2}, bx\right)}{b^8\sqrt{x}}$$

input

```
Integrate[x^(13/2)/E^(b*x), x]
```

output  $-\left(\text{Sqrt}[b*x]*\text{Gamma}[15/2, b*x]\right)/\left(b^8*\text{Sqrt}[x]\right)$

### Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$ , Rules used = {2607, 2607, 2607, 2607, 2607, 2607, 2607, 2611, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{array}{c}
 \int x^{13/2} e^{-bx} dx \\
 \downarrow 2607 \\
 \frac{13 \int e^{-bx} x^{11/2} dx}{2b} - \frac{x^{13/2} e^{-bx}}{b} \\
 \downarrow 2607 \\
 \frac{13 \left( \frac{11 \int e^{-bx} x^{9/2} dx}{2b} - \frac{x^{11/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{13/2} e^{-bx}}{b} \\
 \downarrow 2607 \\
 \frac{13 \left( \frac{11 \left( \frac{9 \int e^{-bx} x^{7/2} dx}{2b} - \frac{x^{9/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{11/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{13/2} e^{-bx}}{b} \\
 \downarrow 2607 \\
 \frac{13 \left( \frac{11 \left( \frac{9 \left( \frac{7 \int e^{-bx} x^{5/2} dx}{2b} - \frac{x^{7/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{9/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{11/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{13/2} e^{-bx}}{b} \\
 \downarrow 2607
 \end{array}$$

$$\left( \frac{\left( \frac{\left( \frac{\left( \frac{5 \int e^{-bx} x^{3/2} dx - x^{5/2} e^{-bx}}{2b} \right) - x^{7/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{9/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{11/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{13/2} \frac{e^{-bx}}{b}}{2b}$$

↓ 2607

$$\left( \frac{\left( \frac{\left( \frac{\left( \frac{3 \int e^{-bx} \sqrt{x} dx - x^{3/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{5/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{7/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{9/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{11/2} \frac{e^{-bx}}{b}}{2b} \right) - x^{13/2} \frac{e^{-bx}}{b}}{2b}$$

↓ 2607



$$\left( \frac{\int \frac{e^{-bx}}{\sqrt{x}} dx}{2b} - \frac{\sqrt{x}e^{-bx}}{b} \right)$$

$$\frac{5}{2b} \left( \frac{\int \frac{e^{-bx}}{\sqrt{x}} dx}{2b} - \frac{\sqrt{x}e^{-bx}}{b} \right) - \frac{x^{3/2}e^{-bx}}{b}$$

$$\frac{7}{2b} \left( \frac{\int \frac{e^{-bx}}{\sqrt{x}} dx}{2b} - \frac{\sqrt{x}e^{-bx}}{b} \right) - \frac{x^{5/2}e^{-bx}}{b}$$

$$\frac{9}{2b} \left( \frac{\int \frac{e^{-bx}}{\sqrt{x}} dx}{2b} - \frac{\sqrt{x}e^{-bx}}{b} \right) - \frac{x^{7/2}e^{-bx}}{b}$$

$$\frac{11}{2b} \left( \frac{\int \frac{e^{-bx}}{\sqrt{x}} dx}{2b} - \frac{\sqrt{x}e^{-bx}}{b} \right) - \frac{x^{9/2}e^{-bx}}{b}$$

$$\frac{13}{2b} \left( \frac{\int \frac{e^{-bx}}{\sqrt{x}} dx}{2b} - \frac{\sqrt{x}e^{-bx}}{b} \right) - \frac{x^{11/2}e^{-bx}}{b}$$

↓ 2611

$$\begin{aligned}
 & \left( \left( \left( \left( \left( \left( \left( \frac{3 \left( \frac{\int e^{-bx} d\sqrt{x}}{b} - \frac{\sqrt{x} e^{-bx}}{b} \right)}{2b} - \frac{x^{3/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{5/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{7/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{9/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{11/2} e^{-bx}}{b} \right)}{2b} - \frac{x^{13/2} e^{-bx}}{b} \right) \right)
 \end{aligned}$$

↓ 2634

$$\left( \left( \left( \left( \left( \left( \frac{3 \left( \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{b}\sqrt{x})}{2b^{3/2}} - \frac{\sqrt{x}e^{-bx}}{b} \right)}{2b} - \frac{x^{3/2}e^{-bx}}{b} \right)}{2b} - \frac{x^{5/2}e^{-bx}}{b} \right)}{2b} - \frac{x^{7/2}e^{-bx}}{b} \right)}{2b} - \frac{x^{9/2}e^{-bx}}{b} \right)}{2b} - \frac{x^{11/2}e^{-bx}}{b} \right) \right)$$

input `Int[x^(13/2)/E^(b*x),x]`

output `-(x^(13/2)/(b*E^(b*x))) + (13*(-(x^(11/2)/(b*E^(b*x)))) + (11*(-(x^(9/2)/(b*E^(b*x)))) + (9*(-(x^(7/2)/(b*E^(b*x)))) + (7*(-(x^(5/2)/(b*E^(b*x)))) + (5*(-(x^(3/2)/(b*E^(b*x)))) + (3*(-(Sqrt[x]/(b*E^(b*x)))) + (Sqrt[Pi]*Erf[Sqrt[b]*Sqrt[x]])/(2*b^(3/2)))/(2*b)))/(2*b)))/(2*b)))/(2*b)))/(2*b)))/(2*b)))/(2*b)`

### Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2611 `Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2634 `Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]`

**Maple [A] (verified)**

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.52

method	result
meijerg	$-\frac{\sqrt{x}\sqrt{b}\left(960b^6x^6+6240b^5x^5+34320b^4x^4+154440b^3x^3+540540b^2x^2+1351350bx+2027025\right)e^{-bx}}{960b^{\frac{15}{2}}} + \frac{135135\sqrt{\pi}\operatorname{erf}\left(\sqrt{b}\sqrt{x}\right)}{128}$ $-\frac{11x^{\frac{7}{2}}e^{-bx}}{4b} + \frac{9}{2b} \left( -7x^{\frac{5}{2}}e^{-bx} + \frac{5}{2b} \left( -5x^{\frac{3}{2}}e^{-bx} + \frac{5}{2b} \left( -3\sqrt{x}e^{-bx} + \frac{3\sqrt{\pi}}{2b} \right) \right) \right)$ $- \frac{11x^{\frac{9}{2}}e^{-bx}}{4b} + \frac{9}{2b}$
derivativedivides	$- \frac{x^{\frac{13}{2}}e^{-bx}}{b} + \frac{13x^{\frac{11}{2}}e^{-bx}}{2b} + \frac{13}{b}$ $-\frac{11x^{\frac{7}{2}}e^{-bx}}{4b} + \frac{9}{2b} \left( -7x^{\frac{5}{2}}e^{-bx} + \frac{5}{2b} \left( -5x^{\frac{3}{2}}e^{-bx} + \frac{5}{2b} \left( -3\sqrt{x}e^{-bx} + \frac{3\sqrt{\pi}}{2b} \right) \right) \right)$ $- \frac{11x^{\frac{9}{2}}e^{-bx}}{4b} + \frac{9}{2b}$



input `int(x^(13/2)/exp(b*x),x,method=_RETURNVERBOSE)`

output `1/b^(15/2)*(-1/960*x^(1/2)*b^(1/2)*(960*b^6*x^6+6240*b^5*x^5+34320*b^4*x^4+154440*b^3*x^3+540540*b^2*x^2+1351350*b*x+2027025)*exp(-b*x)+135135/128*Pi^(1/2)*erf(b^(1/2)*x^(1/2)))`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

$$\int e^{-bx} x^{13/2} dx = \frac{2(64b^7x^6 + 416b^6x^5 + 2288b^5x^4 + 10296b^4x^3 + 36036b^3x^2 + 90090b^2x + 135135b)\sqrt{x}e^{-bx} - 135135}{128b^8}$$

input `integrate(x^(13/2)/exp(b*x),x, algorithm="fricas")`

output `-1/128*(2*(64*b^7*x^6 + 416*b^6*x^5 + 2288*b^5*x^4 + 10296*b^4*x^3 + 36036*b^3*x^2 + 90090*b^2*x + 135135*b)*sqrt(x)*e^(-b*x) - 135135*sqrt(pi)*sqrt(b)*erf(sqrt(b)*sqrt(x)))/b^8`

### Sympy [A] (verification not implemented)

Time = 124.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.91

$$\int e^{-bx} x^{13/2} dx = -\frac{x^{13/2} e^{-bx}}{b} - \frac{13x^{11/2} e^{-bx}}{2b^2} - \frac{143x^{9/2} e^{-bx}}{4b^3} - \frac{1287x^{7/2} e^{-bx}}{8b^4} - \frac{9009x^{5/2} e^{-bx}}{16b^5} - \frac{45045x^{3/2} e^{-bx}}{32b^6} - \frac{135135\sqrt{x}e^{-bx}}{64b^7} + \frac{135135\sqrt{\pi} \operatorname{erf}(\sqrt{b}\sqrt{x})}{128b^{15/2}}$$

input `integrate(x**(13/2)/exp(b*x),x)`

output

```
-x**(13/2)*exp(-b*x)/b - 13*x**(11/2)*exp(-b*x)/(2*b**2) - 143*x**(9/2)*exp(-b*x)/(4*b**3) - 1287*x**(7/2)*exp(-b*x)/(8*b**4) - 9009*x**(5/2)*exp(-b*x)/(16*b**5) - 45045*x**(3/2)*exp(-b*x)/(32*b**6) - 135135*sqrt(x)*exp(-b*x)/(64*b**7) + 135135*sqrt(pi)*erf(sqrt(b)*sqrt(x))/(128*b**(15/2))
```

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.52

$$\int e^{-bx} x^{13/2} dx = \frac{\left(64 b^6 x^{\frac{13}{2}} + 416 b^5 x^{\frac{11}{2}} + 2288 b^4 x^{\frac{9}{2}} + 10296 b^3 x^{\frac{7}{2}} + 36036 b^2 x^{\frac{5}{2}} + 90090 b x^{\frac{3}{2}} + 135135 \sqrt{x}\right) e^{-bx}}{64 b^7} + \frac{135135 \sqrt{\pi} \operatorname{erf}\left(\sqrt{b} \sqrt{x}\right)}{128 b^{\frac{15}{2}}}$$

input

```
integrate(x^(13/2)/exp(b*x),x, algorithm="maxima")
```

output

```
-1/64*(64*b^6*x^(13/2) + 416*b^5*x^(11/2) + 2288*b^4*x^(9/2) + 10296*b^3*x^(7/2) + 36036*b^2*x^(5/2) + 90090*b*x^(3/2) + 135135*sqrt(x))*e^(-b*x)/b^7 + 135135/128*sqrt(pi)*erf(sqrt(b)*sqrt(x))/b^(15/2)
```

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.53

$$\int e^{-bx} x^{13/2} dx = \frac{\left(64 b^6 x^{\frac{13}{2}} + 416 b^5 x^{\frac{11}{2}} + 2288 b^4 x^{\frac{9}{2}} + 10296 b^3 x^{\frac{7}{2}} + 36036 b^2 x^{\frac{5}{2}} + 90090 b x^{\frac{3}{2}} + 135135 \sqrt{x}\right) e^{-bx}}{64 b^7} - \frac{135135 \sqrt{\pi} \operatorname{erf}\left(-\sqrt{b} \sqrt{x}\right)}{128 b^{\frac{15}{2}}}$$

input

```
integrate(x^(13/2)/exp(b*x),x, algorithm="giac")
```

output

$$-1/64*(64*b^6*x^(13/2) + 416*b^5*x^(11/2) + 2288*b^4*x^(9/2) + 10296*b^3*x^(7/2) + 36036*b^2*x^(5/2) + 90090*b*x^(3/2) + 135135*sqrt(x))*e^(-b*x)/b^7 - 135135/128*sqrt(pi)*erf(-sqrt(b)*sqrt(x))/b^(15/2)$$

**Mupad [B] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.59

$$\int e^{-bx} x^{13/2} dx = -\frac{135135 x^{13/2} \sqrt{\pi} \operatorname{erfc}(\sqrt{bx})}{128 b (bx)^{13/2}}$$

$$-\frac{x^{13/2} e^{-bx} \left( \frac{135135 \sqrt{bx}}{64} + \frac{45045 (bx)^{3/2}}{32} + \frac{9009 (bx)^{5/2}}{16} + \frac{1287 (bx)^{7/2}}{8} + \frac{143 (bx)^{9/2}}{4} + \frac{13 (bx)^{11/2}}{2} + (bx)^{13/2} \right)}{b (bx)^{13/2}}$$

input

int(x^(13/2)\*exp(-b\*x), x)

output

$$- (135135*x^(13/2)*pi^(1/2)*erfc((b*x)^(1/2)))/(128*b*(b*x)^(13/2)) - (x^(13/2)*exp(-b*x)*((135135*(b*x)^(1/2))/64 + (45045*(b*x)^(3/2))/32 + (9009*(b*x)^(5/2))/16 + (1287*(b*x)^(7/2))/8 + (143*(b*x)^(9/2))/4 + (13*(b*x)^(11/2))/2 + (b*x)^(13/2)))/(b*(b*x)^(13/2))$$

**Reduce [F]**

$$\int e^{-bx} x^{13/2} dx = \frac{135135 e^{bx} \left( \int \frac{\sqrt{x}}{e^{bx}} dx \right) - 128 \sqrt{x} b^6 x^6 - 832 \sqrt{x} b^5 x^5 - 4576 \sqrt{x} b^4 x^4 - 20592 \sqrt{x} b^3 x^3 - 72072 \sqrt{x} b^2 x^2 - 180180 \sqrt{x} b x - 270270 \sqrt{x}}{128 e^{bx} b^7}$$

input

int(x^(13/2)/exp(b\*x), x)

output

$$(135135*e**(b*x)*int(sqrt(x)/(e**(b*x)*x), x) - 128*sqrt(x)*b**6*x**6 - 832*sqrt(x)*b**5*x**5 - 4576*sqrt(x)*b**4*x**4 - 20592*sqrt(x)*b**3*x**3 - 72072*sqrt(x)*b**2*x**2 - 180180*sqrt(x)*b*x - 270270*sqrt(x))/(128*e**(b*x)*b**7)$$

### 3.79 $\int F^{c(a+bx)}(d+ex)^{4/3} dx$

Optimal result	547
Mathematica [A] (verified)	547
Rubi [A] (verified)	548
Maple [F]	549
Fricas [A] (verification not implemented)	549
Sympy [F]	549
Maxima [F]	550
Giac [F]	550
Mupad [F(-1)]	550
Reduce [F]	551

#### Optimal result

Integrand size = 19, antiderivative size = 71

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = -\frac{eF^{c(a-\frac{bd}{e})}\sqrt[3]{d+ex}\Gamma\left(\frac{7}{3}, -\frac{bc(d+ex)\log(F)}{e}\right)}{b^2c^2\log^2(F)\sqrt[3]{-\frac{bc(d+ex)\log(F)}{e}}}$$

output

```
-e*F^(c*(a-b*d/e))*(e*x+d)^(1/3)*GAMMA(7/3,-b*c*(e*x+d)*ln(F)/e)/b^2/c^2/ln(F)^2/(-b*c*(e*x+d)*ln(F)/e)^(1/3)
```

#### Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = -\frac{F^{c(a-\frac{bd}{e})}(d+ex)^{7/3}\Gamma\left(\frac{7}{3}, -\frac{bc(d+ex)\log(F)}{e}\right)}{e\left(-\frac{bc(d+ex)\log(F)}{e}\right)^{7/3}}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^(4/3), x]
```

output

$$-\left(\frac{F^{c(a - (b*d)/e)}(d + e*x)^{7/3} \Gamma\left[\frac{7}{3}, -\frac{(b*c*(d + e*x)*\text{Log}[F])}{e}\right]}{e}\right) / \left(e * \left(-\frac{(b*c*(d + e*x)*\text{Log}[F])}{e}\right)^{7/3}\right)$$
**Rubi [A] (verified)**

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{4/3} F^{c(a+bx)} dx$$

↓ 2612

$$-\frac{e^3 \sqrt[3]{d + ex} F^{c\left(a - \frac{bd}{e}\right)} \Gamma\left(\frac{7}{3}, -\frac{bc(d+ex)\log(F)}{e}\right)}{b^2 c^2 \log^2(F) \sqrt[3]{-\frac{bc \log(F)(d + ex)}{e}}}$$

input

$$\text{Int}[F^{c(a + b*x)}(d + e*x)^{4/3}, x]$$

output

$$-\left(\frac{e F^{c(a - (b*d)/e)}(d + e*x)^{1/3} \Gamma\left[\frac{7}{3}, -\frac{(b*c*(d + e*x)*\text{Log}[F])}{e}\right]}{b^2 c^2 \text{Log}[F]^2 * \left(-\frac{(b*c*(d + e*x)*\text{Log}[F])}{e}\right)^{1/3}}\right)$$
**Defintions of rubi rules used**

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d))
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

**Maple [F]**

$$\int F^{c(bx+a)}(ex+d)^{\frac{4}{3}} dx$$

input `int(F^(c*(b*x+a))*(e*x+d)^(4/3),x)`

output `int(F^(c*(b*x+a))*(e*x+d)^(4/3),x)`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.65

$$\int F^{c(a+bx)}(d+ex)^{\frac{4}{3}} dx = \frac{4 \left( -\frac{bc \log(F)}{e} \right)^{\frac{2}{3}} e^{2\Gamma\left(\frac{1}{3}, -\frac{(bcex+bcd) \log(F)}{e}\right)}}{F^{\frac{bcd-ace}{e}}} - 3(4bce \log(F) - 3(b^2c^2ex + b^2c^2d) \log(F)^2)(ex+d)^{\frac{1}{3}} F^{c(a+bx)} \log(F)^3}{9b^3c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(4/3),x, algorithm="fricas")`

output `1/9*(4*(-b*c*log(F)/e)^(2/3)*e^2*gamma(1/3, -(b*c*e*x + b*c*d)*log(F)/e)/F^((b*c*d - a*c*e)/e) - 3*(4*b*c*e*log(F) - 3*(b^2*c^2*e*x + b^2*c^2*d)*log(F)^2)*(e*x + d)^(1/3)*F^(b*c*x + a*c)/(b^3*c^3*log(F)^3)`

**Sympy [F]**

$$\int F^{c(a+bx)}(d+ex)^{\frac{4}{3}} dx = \int F^{c(a+bx)}(d+ex)^{\frac{4}{3}} dx$$

input `integrate(F**((b*x+a)*c)*(e*x+d)**(4/3),x)`

output `Integral(F**(c*(a + b*x))*(d + e*x)**(4/3), x)`

**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = \int (ex+d)^{\frac{4}{3}} F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(4/3),x, algorithm="maxima")`

output `integrate((e*x + d)^(4/3)*F^((b*x + a)*c), x)`

**Giac [F]**

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = \int (ex+d)^{\frac{4}{3}} F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^(4/3),x, algorithm="giac")`

output `integrate((e*x + d)^(4/3)*F^((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = \int F^{c(a+bx)}(d+ex)^{4/3} dx$$

input `int(F^(c*(a + b*x))*(d + e*x)^(4/3),x)`

output `int(F^(c*(a + b*x))*(d + e*x)^(4/3), x)`

**Reduce [F]**

$$\int F^{c(a+bx)}(d+ex)^{4/3} dx = f^{ac} \left( \left( \int f^{bcx}(ex+d)^{\frac{1}{3}} x dx \right) e + \left( \int f^{bcx}(ex+d)^{\frac{1}{3}} dx \right) d \right)$$

input `int(F^((b*x+a)*c)*(e*x+d)^(4/3),x)`

output `f**(a*c)*(int(f**(b*c*x)*(d + e*x)**(1/3)*x,x)*e + int(f**(b*c*x)*(d + e*x)**(1/3),x)*d)`



### 3.80 $\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx$

Optimal result	552
Mathematica [A] (verified)	552
Rubi [A] (verified)	553
Maple [F]	554
Fricas [A] (verification not implemented)	554
Sympy [F(-1)]	555
Maxima [F]	555
Giac [F]	555
Mupad [F(-1)]	556
Reduce [F]	556

#### Optimal result

Integrand size = 21, antiderivative size = 98

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = \frac{eF^{c(a-\frac{bd}{e})n-cn(a+bx)} (F^{c(a+bx)})^n \sqrt[3]{d+ex} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex)\log(F)}{e}\right)}{b^2c^2n^2 \log^2(F) \sqrt[3]{-\frac{bcn(d+ex)\log(F)}{e}}}$$

output

```
-e*F^(c*(a-b*d/e)*n-c*n*(b*x+a))*(F^(c*(b*x+a)))^n*(e*x+d)^(1/3)*GAMMA(7/3
,-b*c*n*(e*x+d)*ln(F)/e)/b^2/c^2/n^2/ln(F)^2/(-b*c*n*(e*x+d)*ln(F)/e)^(1/3
)
```

#### Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = -\frac{F^{-\frac{bcn(d+ex)}{e}} (F^{c(a+bx)})^n (d+ex)^{7/3} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex)\log(F)}{e}\right)}{e \left(-\frac{bcn(d+ex)\log(F)}{e}\right)^{7/3}}$$

input

```
Integrate[(F^(c*(a + b*x)))^n*(d + e*x)^(4/3),x]
```

output

```
-(((F^(c*(a + b*x)))^n*(d + e*x)^(7/3)*Gamma[7/3, -((b*c*n*(d + e*x)*Log[F])/e)]/e))/e*(F^(b*c*n*(d + e*x)/e)*(-(b*c*n*(d + e*x)*Log[F])/e))^(7/3))
```

**Rubi [A] (verified)**

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2613, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{4/3} (F^{c(a+bx)})^n dx$$

↓ 2613

$$F^{-cn(a+bx)} (F^{c(a+bx)})^n \int F^{cn(a+bx)} (d + ex)^{4/3} dx$$

↓ 2612

$$\frac{e^{\sqrt[3]{d + ex}} (F^{c(a+bx)})^n F^{cn(a - \frac{bd}{e}) - cn(a+bx)} \Gamma\left(\frac{7}{3}, -\frac{bcn(d+ex) \log(F)}{e}\right)}{b^2 c^2 n^2 \log^2(F) \sqrt[3]{-\frac{bcn \log(F)(d + ex)}{e}}}$$

input

```
Int[(F^(c*(a + b*x)))^n*(d + e*x)^(4/3),x]
```

output

```
-((e*F^(c*(a - (b*d)/e)*n - c*n*(a + b*x))*(F^(c*(a + b*x)))^n*(d + e*x)^(1/3)*Gamma[7/3, -((b*c*n*(d + e*x)*Log[F])/e)]/(b^2*c^2*n^2*Log[F]^2*(-((b*c*n*(d + e*x)*Log[F])/e))^(1/3)))
```

## Definitions of rubi rules used

rule 2612

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]]*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2613

```
Int[((b_.)*(F_)^((g_.)*(e_.) + (f_.)*(x_)))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)) Int[(c + d*
x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

## Maple [F]

$$\int (F^{c(bx+a)})^n (ex + d)^{\frac{4}{3}} dx$$

input

```
int((F^(c*(b*x+a)))^n*(e*x+d)^(4/3),x)
```

output

```
int((F^(c*(b*x+a)))^n*(e*x+d)^(4/3),x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.36

$$\int (F^{c(a+bx)})^n (d + ex)^{4/3} dx = \frac{4 \left( -\frac{bcn \log(F)}{e} \right)^{\frac{2}{3}} e^{2\Gamma\left(\frac{1}{3}, -\frac{(bcenx+bcdn) \log(F)}{e}\right)}}{F^{\frac{(bcd-ace)n}{e}}} - 3 (4bcen \log(F) - 3(b^2c^2en^2x + b^2c^2dn^2) \log(F)^2) (ex + d)^{1/3}}{9b^3c^3n^3 \log(F)^3}$$

input

```
integrate((F^((b*x+a)*c))^n*(e*x+d)^(4/3),x, algorithm="fricas")
```

output

```
1/9*(4*(-b*c*n*log(F)/e)^(2/3)*e^2*gamma(1/3, -(b*c*e*n*x + b*c*d*n)*log(F)/e)/F^((b*c*d - a*c*e)*n/e) - 3*(4*b*c*e*n*log(F) - 3*(b^2*c^2*e*n^2*x + b^2*c^2*d*n^2)*log(F)^2)*(e*x + d)^(1/3)*F^(b*c*n*x + a*c*n))/(b^3*c^3*n^3*log(F)^3)
```

**Sympy [F(-1)]**

Timed out.

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = \text{Timed out}$$

input

```
integrate((F**((b*x+a)*c))**n*(e*x+d)**(4/3), x)
```

output

Timed out

**Maxima [F]**

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = \int (ex+d)^{\frac{4}{3}} (F^{(bx+a)c})^n dx$$

input

```
integrate((F^((b*x+a)*c))^n*(e*x+d)^(4/3), x, algorithm="maxima")
```

output

```
integrate((e*x + d)^(4/3)*F^((b*x + a)*c*n), x)
```

**Giac [F]**

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = \int (ex+d)^{\frac{4}{3}} (F^{(bx+a)c})^n dx$$

input

```
integrate((F^((b*x+a)*c))^n*(e*x+d)^(4/3), x, algorithm="giac")
```

output

```
integrate((e*x + d)^(4/3)*(F^((b*x + a)*c))^n, x)
```

**Mupad [F(-1)]**

Timed out.

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = \int (F^{c(a+bx)})^n (d+ex)^{4/3} dx$$

input `int((F^(c*(a + b*x)))^n*(d + e*x)^(4/3), x)`

output `int((F^(c*(a + b*x)))^n*(d + e*x)^(4/3), x)`

**Reduce [F]**

$$\int (F^{c(a+bx)})^n (d+ex)^{4/3} dx = f^{acn} \left( \left( \int f^{bcnx} (ex+d)^{\frac{1}{3}} x dx \right) e + \left( \int f^{bcnx} (ex+d)^{\frac{1}{3}} dx \right) d \right)$$

input `int((F^((b*x+a)*c))^n*(e*x+d)^(4/3), x)`

output `f**(a*c*n)*(int(f**(b*c*n*x)*(d + e*x)**(1/3)*x,x)*e + int(f**(b*c*n*x)*(d + e*x)**(1/3),x)*d)`

### 3.81 $\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$

Optimal result	557
Mathematica [A] (verified)	557
Rubi [A] (verified)	558
Maple [F]	559
Fricas [F]	559
Sympy [F]	560
Maxima [F]	560
Giac [F]	561
Mupad [F(-1)]	561
Reduce [F]	562

#### Optimal result

Integrand size = 50, antiderivative size = 71

$$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$$

$$= \frac{F^{c(a-\frac{bd}{e})}((d+ex)^4)^m \Gamma\left(1+4m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-4m}}{bc \log(F)}$$

output

```
F^(c*(a-b*d/e))*((e*x+d)^4)^m*GAMMA(1+4*m,-b*c*(e*x+d)*ln(F)/e)/b/c/ln(F)/
((-b*c*(e*x+d)*ln(F)/e)^(4*m))
```

#### Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$$

$$= \frac{F^{c(a-\frac{bd}{e})}((d+ex)^4)^m \Gamma\left(1+4m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-4m}}{bc \log(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 +
e^4*x^4)^m,x]
```

output

$$\frac{(F^{c(a - (b*d)/e)})*((d + e*x)^4)^m * \text{Gamma}[1 + 4*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{(4*m)})}$$
**Rubi [A] (verified)**

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {2008, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m F^{c(a+bx)} dx$$

$$\downarrow \text{2008}$$

$$(d + ex)^{-4m} ((d + ex)^4)^m \int F^{c(a+bx)} (d + ex)^{4m} dx$$

$$\downarrow \text{2612}$$

$$\frac{((d + ex)^4)^m F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-4m} \Gamma(4m + 1, -\frac{bc(d+ex) \log(F)}{e})}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)}*(d^4 + 4*d^3*e*x + 6*d^2*e^2*x^2 + 4*d*e^3*x^3 + e^4*x^4)^m, x]$$

output

$$\frac{(F^{c(a - (b*d)/e)})*((d + e*x)^4)^m * \text{Gamma}[1 + 4*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{(4*m)})}$$

## Definitions of rubi rules used

rule 2008

```
Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Expon[Px, x])^p/(a + b*x)^(Expon[Px, x]*p) Int[u*(a + b*x)^(Expon[Px, x]*p), x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; !IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m])*Gamma[m + 1, ((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

## Maple [F]

$$\int F^{c(bx+a)} (e^4 x^4 + 4d e^3 x^3 + 6d^2 e^2 x^2 + 4d^3 e x + d^4)^m dx$$

input

```
int(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m,x)
```

output

```
int(F^(c*(b*x+a))*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m,x)
```

## Fricas [F]

$$\begin{aligned} & \int F^{c(a+bx)} (d^4 + 4d^3 e x + 6d^2 e^2 x^2 + 4d e^3 x^3 + e^4 x^4)^m dx \\ & = \int (e^4 x^4 + 4d e^3 x^3 + 6d^2 e^2 x^2 + 4d^3 e x + d^4)^m F^{(bx+a)c} dx \end{aligned}$$

input

```
integrate(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m,x, algorithm="fricas")
```



output `integral((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)^m*F^(b*c*x + a*c), x)`

### Sympy [F]

$$\int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx = \int F^{c(a+bx)}((d + ex)^4)^m dx$$

input `integrate(F**((b*x+a)*c)*(e**4*x**4+4*d*e**3*x**3+6*d**2*e**2*x**2+4*d**3*e*x+d**4)**m,x)`

output `Integral(F**(c*(a + b*x))*((d + e*x)**4)**m, x)`

### Maxima [F]

$$\begin{aligned} & \int F^{c(a+bx)}(d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx \\ &= \int (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)^m F^{(bx+a)c} dx \end{aligned}$$

input `integrate(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m,x, algorithm="maxima")`

output `integrate((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)^m*F^(b*x + a)*c), x)`

**Giac [F]**

$$\int F^{c(a+bx)} (d^4 + 4d^3 ex + 6d^2 e^2 x^2 + 4de^3 x^3 + e^4 x^4)^m dx$$

$$= \int (e^4 x^4 + 4de^3 x^3 + 6d^2 e^2 x^2 + 4d^3 ex + d^4)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m,x, algorithm="giac")`

output `integrate((e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4)^m*F((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)} (d^4 + 4d^3 ex + 6d^2 e^2 x^2 + 4de^3 x^3 + e^4 x^4)^m dx$$

$$= \int F^{c(a+bx)} (d^4 + 4d^3 ex + 6d^2 e^2 x^2 + 4de^3 x^3 + e^4 x^4)^m dx$$

input `int(F^c*(a + b*x))*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)^m,x)`

output `int(F^c*(a + b*x))*(d^4 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)^m, x)`

**Reduce [F]**

$$\int F^{c(a+bx)} (d^4 + 4d^3ex + 6d^2e^2x^2 + 4de^3x^3 + e^4x^4)^m dx$$

$$= \frac{f^{ac} \left( f^{bcx} (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)^m - 4 \left( \int \frac{f^{bcx} (e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4)^m}{ex+d} dx \right) em \right)}{\log(f)bc}$$

input

```
int(F^((b*x+a)*c)*(e^4*x^4+4*d*e^3*x^3+6*d^2*e^2*x^2+4*d^3*e*x+d^4)^m,x)
```

output

```
(f**(a*c)*(f**(b*c*x)*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4)**m - 4*int((f**(b*c*x)*(d**4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4)**m)/(d + e*x),x)*em))/(log(f)*b*c)
```

### 3.82 $\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$

Optimal result	563
Mathematica [A] (verified)	563
Rubi [A] (verified)	564
Maple [F]	565
Fricas [F]	565
Sympy [F]	566
Maxima [F]	566
Giac [F]	566
Mupad [F(-1)]	567
Reduce [F]	567

#### Optimal result

Integrand size = 39, antiderivative size = 71

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^3)^m \Gamma\left(1+3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-3m}}{bc \log(F)}$$

output `F^(c*(a-b*d/e))*((e*x+d)^3)^m*GAMMA(1+3*m, -b*c*(e*x+d)*ln(F)/e)/b/c/ln(F)/((-b*c*(e*x+d)*ln(F)/e)^(3*m))`

#### Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^3)^m \Gamma\left(1+3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-3m}}{bc \log(F)}$$

input `Integrate[F^(c*(a + b*x))*(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m,x]`

output

$$\frac{(F^{c(a - (b*d)/e)}) * ((d + e*x)^3)^m * \text{Gamma}[1 + 3*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(3*m)})}$$
**Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$ , Rules used = {2008, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m F^{c(a+bx)} dx$$

$$\downarrow \text{2008}$$

$$(d + ex)^{-3m} ((d + ex)^3)^m \int F^{c(a+bx)} (d + ex)^{3m} dx$$

$$\downarrow \text{2612}$$

$$\frac{((d + ex)^3)^m F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-3m} \Gamma\left(3m + 1, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)} * (d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m, x]$$

output

$$\frac{(F^{c(a - (b*d)/e)}) * ((d + e*x)^3)^m * \text{Gamma}[1 + 3*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(3*m)})}$$

## Definitions of rubi rules used

rule 2008

```
Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p) Int[u*(a + b*x)^(Expon[Px, x]*p),
x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:= Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

## Maple [F]

$$\int F^{c(bx+a)} (e^3 x^3 + 3d e^2 x^2 + 3d^2 e x + d^3)^m dx$$

input

```
int(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x)
```

output

```
int(F^(c*(b*x+a))*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x)
```

## Fricas [F]

$$\begin{aligned} \int F^{c(a+bx)} (d^3 + 3d^2 e x + 3d e^2 x^2 + e^3 x^3)^m dx \\ = \int (e^3 x^3 + 3d e^2 x^2 + 3d^2 e x + d^3)^m F^{(bx+a)c} dx \end{aligned}$$

input

```
integrate(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm
="fricas")
```

output

```
integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m*F^(b*c*x + a*c), x)
```

**Sympy [F]**

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx = \int F^{c(a+bx)} ((d + ex)^3)^m dx$$

input `integrate(F**((b*x+a)*c)*(e**3*x**3+3*d*e**2*x**2+3*d**2*e*x+d**3)**m,x)`

output `Integral(F**(c*(a + b*x))*((d + e*x)**3)**m, x)`

**Maxima [F]**

$$\begin{aligned} \int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx \\ = \int (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m F^{(bx+a)c} dx \end{aligned}$$

input `integrate(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm="maxima")`

output `integrate((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m * F^((b*x + a)*c), x)`

**Giac [F]**

$$\begin{aligned} \int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx \\ = \int (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m F^{(bx+a)c} dx \end{aligned}$$

input `integrate(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x, algorithm="giac")`

output `integrate((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m * F^((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$$

$$= \int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$$

input

```
int(F^(c*(a + b*x))*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)^m,x)
```

output

```
int(F^(c*(a + b*x))*(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)^m, x)
```

**Reduce [F]**

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m dx$$

$$= \frac{f^{ac} \left( f^{bcx} (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m - 3 \left( \int \frac{f^{bcx} (e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m}{ex+d} dx \right) em \right)}{\log(f) bc}$$

input

```
int(F^((b*x+a)*c)*(e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m,x)
```

output

```
(f**(a*c)*(f**(b*c*x)*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3)**m -
3*int((f**(b*c*x)*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3)**m)/(d
+ e*x),x)*e*m))/(log(f)*b*c)
```



### 3.83 $\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$

Optimal result	568
Mathematica [A] (verified)	568
Rubi [A] (verified)	569
Maple [F]	570
Fricas [F]	570
Sympy [F]	571
Maxima [F]	571
Giac [F]	571
Mupad [F(-1)]	572
Reduce [F]	572

#### Optimal result

Integrand size = 28, antiderivative size = 71

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^2)^m \Gamma\left(1+2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-2m}}{bc \log(F)}$$

output  $F^{(c*(a-b*d/e))*((e*x+d)^2)^m \text{GAMMA}(1+2*m, -b*c*(e*x+d)*\ln(F)/e)/b/c/\ln(F)/((-b*c*(e*x+d)*\ln(F)/e)^{(2*m))}$

#### Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^2)^m \Gamma\left(1+2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-2m}}{bc \log(F)}$$

input  $\text{Integrate}[F^{(c*(a + b*x))*(d^2 + 2*d*e*x + e^2*x^2)^m, x]$

output

$$\frac{(F^{c(a - (b*d)/e)})*((d + e*x)^2)^m * \text{Gamma}[1 + 2*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{(2*m)})}$$
**Rubi [A] (verified)**

Time = 0.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {2008, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d^2 + 2dex + e^2x^2)^m F^{c(a+bx)} dx$$

$$\downarrow \text{2008}$$

$$(d + ex)^{-2m} ((d + ex)^2)^m \int F^{c(a+bx)} (d + ex)^{2m} dx$$

$$\downarrow \text{2612}$$

$$\frac{((d + ex)^2)^m F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-2m} \Gamma\left(2m + 1, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)}*(d^2 + 2*d*e*x + e^2*x^2)^m, x]$$

output

$$\frac{(F^{c(a - (b*d)/e)})*((d + e*x)^2)^m * \text{Gamma}[1 + 2*m, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F]*(-((b*c*(d + e*x)*\text{Log}[F])/e))^{(2*m)})}$$

### Defintions of rubi rules used

rule 2008

```
Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p) Int[u*(a + b*x)^(Expon[Px, x]*p),
x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:= Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

### Maple [F]

$$\int F^{c(bx+a)} (e^2 x^2 + 2exd + d^2)^m dx$$

input

```
int(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m,x)
```

output

```
int(F^(c*(b*x+a))*(e^2*x^2+2*d*e*x+d^2)^m,x)
```

### Fricas [F]

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2 x^2)^m dx = \int (e^2 x^2 + 2 dex + d^2)^m F^{(bx+a)c} dx$$

input

```
integrate(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2)^m,x, algorithm="fricas")
```

output

```
integral((e^2*x^2 + 2*d*e*x + d^2)^m*F^(b*c*x + a*c), x)
```

**Sympy [F]**

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx = \int F^{c(a+bx)}((d + ex)^2)^m dx$$

input `integrate(F**((b*x+a)*c)*(e**2*x**2+2*d*e*x+d**2)**m,x)`

output `Integral(F**(c*(a + b*x))*((d + e*x)**2)**m, x)`

**Maxima [F]**

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx = \int (e^2x^2 + 2dex + d^2)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2)^m,x, algorithm="maxima")`

output `integrate((e^2*x^2 + 2*d*e*x + d^2)^m*F^((b*x + a)*c), x)`

**Giac [F]**

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^m dx = \int (e^2x^2 + 2dex + d^2)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2)^m,x, algorithm="giac")`

output `integrate((e^2*x^2 + 2*d*e*x + d^2)^m*F^((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^m dx = \int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^m dx$$

input `int(F^(c*(a + b*x))*(d^2 + e^2*x^2 + 2*d*e*x)^m,x)`

output `int(F^(c*(a + b*x))*(d^2 + e^2*x^2 + 2*d*e*x)^m, x)`

**Reduce [F]**

$$\begin{aligned} & \int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^m dx \\ &= \frac{f^{ac} \left( f^{bcx} (e^2x^2 + 2dex + d^2)^m - 2 \left( \int \frac{f^{bcx} (e^2x^2 + 2dex + d^2)^m}{ex+d} dx \right) em \right)}{\log(f) bc} \end{aligned}$$

input `int(F^((b*x+a)*c)*(e^2*x^2+2*d*e*x+d^2)^m,x)`

output `(f**(a*c)*(f**(b*c*x)*(d**2 + 2*d*e*x + e**2*x**2)**m - 2*int((f**(b*c*x)*(d**2 + 2*d*e*x + e**2*x**2)**m)/(d + e*x),x)*e*m))/(log(f)*b*c)`

### 3.84 $\int F^{c(a+bx)}(d+ex)^m dx$

Optimal result	573
Mathematica [A] (verified)	573
Rubi [A] (verified)	574
Maple [F]	575
Fricas [A] (verification not implemented)	575
Sympy [F]	575
Maxima [F]	576
Giac [F]	576
Mupad [F(-1)]	576
Reduce [F]	577

#### Optimal result

Integrand size = 17, antiderivative size = 67

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

```
output F^(c*(a-b*d/e))*(e*x+d)^m*GAMMA(1+m,-b*c*(e*x+d)*ln(F)/e)/b/c/ln(F)/((-b*c*(e*x+d)*ln(F)/e)^m)
```

#### Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

```
input Integrate[F^(c*(a + b*x))*(d + e*x)^m,x]
```

output  $(F^{c(a - (b*d)/e)} * (d + e*x)^m * \text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

### Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^m F^{c(a+bx)} dx$$

↓ 2612

$$\frac{(d + ex)^m F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \Gamma\left(m + 1, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input  $\text{Int}[F^{c(a + b*x)} * (d + e*x)^m, x]$

output  $(F^{c(a - (b*d)/e)} * (d + e*x)^m * \text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

### Defintions of rubi rules used

rule 2612  $\text{Int}[(F_)^{((g_.) * (e_.) + (f_.) * (x_))} * ((c_.) + (d_.) * (x_))^{(m_)}, x\_Symbol]$   
 $:= \text{Simp}[(-F^{(g*(e - c*(f/d)))}) * ((c + d*x)^{\text{FracPart}[m]} / (d * ((-f)*g*(\text{Log}[F]/d)))^{(\text{IntPart}[m] + 1)} * ((-f)*g*\text{Log}[F] * ((c + d*x)/d))^{\text{FracPart}[m]}) * \text{Gamma}[m + 1, ((-f)*g*(\text{Log}[F]/d) * (c + d*x)], x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& !\text{IntegerQ}[m]$

**Maple [F]**

$$\int F^{c(bx+a)}(ex+d)^m dx$$

input `int(F^(c*(b*x+a))*(e*x+d)^m,x)`

output `int(F^(c*(b*x+a))*(e*x+d)^m,x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{e^{\left(-\frac{em \log\left(-\frac{bc \log(F)}{e}\right) + (bcd - ace) \log(F)}{e}\right)} \Gamma\left(m+1, -\frac{(bcex+bcd) \log(F)}{e}\right)}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^m,x, algorithm="fricas")`

output `e^(-(e*m*log(-b*c*log(F)/e) + (b*c*d - a*c*e)*log(F))/e)*gamma(m + 1, -(b*c*e*x + b*c*d)*log(F)/e)/(b*c*log(F))`

**Sympy [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \int F^{c(a+bx)}(d+ex)^m dx$$

input `integrate(F**(c*(a + b*x))*(d + e*x)**m, x)`

output `Integral(F**(c*(a + b*x))*(d + e*x)**m, x)`



**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \int (ex+d)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^m,x, algorithm="maxima")`

output `integrate((e*x + d)^m*F^((b*x + a)*c), x)`

**Giac [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \int (ex+d)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^m,x, algorithm="giac")`

output `integrate((e*x + d)^m*F^((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^m dx = \int F^{c(a+bx)}(d+ex)^m dx$$

input `int(F^(c*(a + b*x))*(d + e*x)^m,x)`

output `int(F^(c*(a + b*x))*(d + e*x)^m, x)`

**Reduce [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{f^{ac} \left( f^{bcx} (ex+d)^m - \left( \int \frac{f^{bcx} (ex+d)^m}{ex+d} dx \right) em \right)}{\log(f) bc}$$

input `int(F^((b*x+a)*c)*(e*x+d)^m,x)`

output `(f**(a*c)*(f**(b*c*x)*(d + e*x)**m - int((f**(b*c*x)*(d + e*x)**m)/(d + e*x),x)*e*m))/(log(f)*b*c)`

### 3.85 $\int F^{c(a+bx)}(d+ex)^{-m} dx$

Optimal result	578
Mathematica [A] (verified)	578
Rubi [A] (verified)	579
Maple [F]	580
Fricas [A] (verification not implemented)	580
Sympy [F(-2)]	580
Maxima [F]	581
Giac [F]	581
Mupad [F(-1)]	581
Reduce [F]	582

#### Optimal result

Integrand size = 19, antiderivative size = 69

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^{-m}\Gamma\left(1-m, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^m}{bc\log(F)}$$

output `F^(c*(a-b*d/e))*GAMMA(1-m,-b*c*(e*x+d)*ln(F)/e)*(-b*c*(e*x+d)*ln(F)/e)^m/b/c/((e*x+d)^m)/ln(F)`

#### Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^{-m}\Gamma\left(1-m, -\frac{bc(d+ex)\log(F)}{e}\right)\left(-\frac{bc(d+ex)\log(F)}{e}\right)^m}{bc\log(F)}$$

input `Integrate[F^(c*(a + b*x))/(d + e*x)^m,x]`

output

$$(F^{c(a - (b*d)/e)}) * \text{Gamma}[1 - m, -((b*c*(d + e*x)*\text{Log}[F])/e)] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^m / (b*c*(d + e*x)^m * \text{Log}[F])$$
**Rubi [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^{-m} F^{c(a+bx)} dx$$

↓ 2612

$$\frac{(d + ex)^{-m} F^{c(a - \frac{bd}{e})} \left( -\frac{bc \log(F)(d+ex)}{e} \right)^m \Gamma\left(1 - m, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)} / (d + e*x)^m, x]$$

output

$$(F^{c(a - (b*d)/e)}) * \text{Gamma}[1 - m, -((b*c*(d + e*x)*\text{Log}[F])/e)] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^m / (b*c*(d + e*x)^m * \text{Log}[F])$$
**Defintions of rubi rules used**

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

**Maple [F]**

$$\int F^{c(bx+a)}(ex+d)^{-m} dx$$

input `int(F^(c*(b*x+a))/((e*x+d)^m),x)`

output `int(F^(c*(b*x+a))/((e*x+d)^m),x)`

**Fricas [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \frac{e^{\left(\frac{em \log\left(-\frac{bc \log(F)}{e}\right) - (bcd - ace) \log(F)}{e}\right)}}{bc \log(F)} \Gamma\left(-m + 1, -\frac{(bcex + bcd) \log(F)}{e}\right)$$

input `integrate(F^((b*x+a)*c)/((e*x+d)^m),x, algorithm="fricas")`

output `e^((e*m*log(-b*c*log(F)/e) - (b*c*d - a*c*e)*log(F))/e)*gamma(-m + 1, -(b*c*e*x + b*c*d)*log(F)/e)/(b*c*log(F))`

**Sympy [F(-2)]**

Exception generated.

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \text{Exception raised: HeuristicGCDFailed}$$

input `integrate(F**((b*x+a)*c)/((e*x+d)**m),x)`

output `Exception raised: HeuristicGCDFailed >> no luck`

**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^m} dx$$

input `integrate(F^((b*x+a)*c)/((e*x+d)^m),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^m, x)`

**Giac [F]**

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \int \frac{F^{(bx+a)c}}{(ex+d)^m} dx$$

input `integrate(F^((b*x+a)*c)/((e*x+d)^m),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e*x + d)^m, x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = \int \frac{F^{c(a+bx)}}{(d+ex)^m} dx$$

input `int(F^(c*(a + b*x))/(d + e*x)^m,x)`

output `int(F^(c*(a + b*x))/(d + e*x)^m, x)`

**Reduce [F]**

$$\int F^{c(a+bx)}(d+ex)^{-m} dx = f^{ac} \left( \int \frac{f^{bcx}}{(ex+d)^m} dx \right)$$

input `int(F^((b*x+a)*c)/((e*x+d)^m),x)`

output `f**(a*c)*int(f**(b*c*x)/(d + e*x)**m,x)`

### 3.86 $\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx$

Optimal result	583
Mathematica [A] (verified)	583
Rubi [A] (verified)	584
Maple [F]	585
Fricas [F]	585
Sympy [F]	586
Maxima [F]	586
Giac [F]	586
Mupad [F(-1)]	587
Reduce [F]	587

#### Optimal result

Integrand size = 30, antiderivative size = 73

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^2)^{-m} \Gamma\left(1-2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{2m}}{bc \log(F)}$$

output `F^(c*(a-b*d/e))*GAMMA(1-2*m,-b*c*(e*x+d)*ln(F)/e)*(-b*c*(e*x+d)*ln(F)/e)^(2*m)/b/c/(((e*x+d)^2)^m)/ln(F)`

#### Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^2)^{-m} \Gamma\left(1-2m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{2m}}{bc \log(F)}$$

input `Integrate[F^(c*(a + b*x))/(d^2 + 2*d*e*x + e^2*x^2)^m,x]`



output

$$(F^{c(a - (b*d)/e)}) * \text{Gamma}[1 - 2*m, -((b*c*(d + e*x)*\text{Log}[F])/e)] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(2*m)} / (b*c*((d + e*x)^2)^m * \text{Log}[F])$$
**Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2008, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d^2 + 2dex + e^2x^2)^{-m} F^{c(a+bx)} dx$$

$$\downarrow \text{2008}$$

$$(d + ex)^{2m} ((d + ex)^2)^{-m} \int F^{c(a+bx)} (d + ex)^{-2m} dx$$

$$\downarrow \text{2612}$$

$$\frac{((d + ex)^2)^{-m} F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{2m} \Gamma\left(1 - 2m, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)} / (d^2 + 2*d*e*x + e^2*x^2)^m, x]$$

output

$$(F^{c(a - (b*d)/e)}) * \text{Gamma}[1 - 2*m, -((b*c*(d + e*x)*\text{Log}[F])/e)] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(2*m)} / (b*c*((d + e*x)^2)^m * \text{Log}[F])$$

### Defintions of rubi rules used

rule 2008

```
Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Expon[Px, x])^p/(a + b*x)^(Expon[Px, x]*p) Int[u*(a + b*x)^(Expon[Px, x]*p), x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; !IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2612

```
Int[(F_)^((g_)*((e_)+(f_)*(x_)))*((c_)+(d_)*(x_))^(m_), x_Symbol] := Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d)^FracPart[m])*Gamma[m + 1, ((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Maple [F]

$$\int F^{c(bx+a)} (e^2x^2 + 2exd + d^2)^{-m} dx$$

input

```
int(F^(c*(b*x+a))/((e^2*x^2+2*d*e*x+d^2)^m), x)
```

output

```
int(F^(c*(b*x+a))/((e^2*x^2+2*d*e*x+d^2)^m), x)
```

### Fricas [F]

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^{-m} dx = \int \frac{F^{(bx+a)c}}{(e^2x^2 + 2dex + d^2)^m} dx$$

input

```
integrate(F^((b*x+a)*c)/((e^2*x^2+2*d*e*x+d^2)^m), x, algorithm="fricas")
```

output

```
integral(F^(b*c*x + a*c)/(e^2*x^2 + 2*d*e*x + d^2)^m, x)
```

**Sympy [F]**

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx = \int F^{c(a+bx)}((d + ex)^2)^{-m} dx$$

input `integrate(F**((b*x+a)*c)/((e**2*x**2+2*d*e*x+d**2)**m), x)`

output `Integral(F**(c*(a + b*x))/((d + e*x)**2)**m, x)`

**Maxima [F]**

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx = \int \frac{F^{(bx+a)c}}{(e^2x^2 + 2dex + d^2)^m} dx$$

input `integrate(F^((b*x+a)*c)/((e^2*x^2+2*d*e*x+d^2)^m), x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e^2*x^2 + 2*d*e*x + d^2)^m, x)`

**Giac [F]**

$$\int F^{c(a+bx)}(d^2 + 2dex + e^2x^2)^{-m} dx = \int \frac{F^{(bx+a)c}}{(e^2x^2 + 2dex + d^2)^m} dx$$

input `integrate(F^((b*x+a)*c)/((e^2*x^2+2*d*e*x+d^2)^m), x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e^2*x^2 + 2*d*e*x + d^2)^m, x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^{-m} dx = \int \frac{F^{c(a+bx)}}{(d^2 + 2dex + e^2x^2)^m} dx$$

input `int(F^(c*(a + b*x))/(d^2 + e^2*x^2 + 2*d*e*x)^m,x)`

output `int(F^(c*(a + b*x))/(d^2 + e^2*x^2 + 2*d*e*x)^m, x)`

**Reduce [F]**

$$\int F^{c(a+bx)} (d^2 + 2dex + e^2x^2)^{-m} dx = f^{ac} \left( \int \frac{f^{bcx}}{(e^2x^2 + 2dex + d^2)^m} dx \right)$$

input `int(F^((b*x+a)*c)/((e^2*x^2+2*d*e*x+d^2)^m),x)`

output `f**(a*c)*int(f**(b*c*x)/(d**2 + 2*d*e*x + e**2*x**2)**m,x)`

### 3.87 $\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$

Optimal result	588
Mathematica [A] (verified)	588
Rubi [A] (verified)	589
Maple [F]	590
Fricas [F]	590
Sympy [F]	591
Maxima [F]	591
Giac [F]	591
Mupad [F(-1)]	592
Reduce [F]	592

#### Optimal result

Integrand size = 41, antiderivative size = 73

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$$

$$= \frac{F^{c(a-\frac{bd}{e})}((d+ex)^3)^{-m} \Gamma\left(1-3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3m}}{bc \log(F)}$$

output

```
F^(c*(a-b*d/e))*GAMMA(1-3*m,-b*c*(e*x+d)*ln(F)/e)*(-b*c*(e*x+d)*ln(F)/e)^(3*m)/b/c/(((e*x+d)^3)^m)/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$$

$$= \frac{F^{c(a-\frac{bd}{e})}((d+ex)^3)^{-m} \Gamma\left(1-3m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{3m}}{bc \log(F)}$$

input

```
Integrate[F^(c*(a + b*x))/(d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m,x]
```

output

$$(F^{c(a - (b*d)/e)}) * \text{Gamma}[1 - 3*m, -((b*c*(d + e*x)*\text{Log}[F])/e)] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(3*m)}) / (b*c*((d + e*x)^3)^m * \text{Log}[F])$$
**Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$ , Rules used = {2008, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} F^{c(a+bx)} dx$$

$$\downarrow \text{2008}$$

$$(d + ex)^{3m} ((d + ex)^3)^{-m} \int F^{c(a+bx)} (d + ex)^{-3m} dx$$

$$\downarrow \text{2612}$$

$$\frac{((d + ex)^3)^{-m} F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{3m} \Gamma\left(1 - 3m, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)} / (d^3 + 3*d^2*e*x + 3*d*e^2*x^2 + e^3*x^3)^m, x]$$

output

$$(F^{c(a - (b*d)/e)}) * \text{Gamma}[1 - 3*m, -((b*c*(d + e*x)*\text{Log}[F])/e)] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^{(3*m)}) / (b*c*((d + e*x)^3)^m * \text{Log}[F])$$

## Definitions of rubi rules used

rule 2008

```
Int[(u_)*(Px_)^(p_), x_Symbol] := With[{a = Rt[Coeff[Px, x, 0], Expon[Px, x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Expon[Px, x])^p/(a + b*x)^(Expon[Px, x]*p) Int[u*(a + b*x)^(Expon[Px, x]*p), x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]] /; !IntegerQ[p] && PolyQ[Px, x] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-F^(g*(e - c*(f/d))))*(c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d))^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m])*Gamma[m + 1, ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

## Maple [F]

$$\int F^{c(bx+a)} (e^3 x^3 + 3d e^2 x^2 + 3d^2 e x + d^3)^{-m} dx$$

input

```
int(F^(c*(b*x+a))/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m), x)
```

output

```
int(F^(c*(b*x+a))/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m), x)
```

## Fricas [F]

$$\int F^{c(a+bx)} (d^3 + 3d^2 e x + 3d e^2 x^2 + e^3 x^3)^{-m} dx = \int \frac{F^{(bx+a)c}}{(e^3 x^3 + 3d e^2 x^2 + 3d^2 e x + d^3)^m} dx$$

input

```
integrate(F^((b*x+a)*c)/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m), x, algorithm="fricas")
```

output

```
integral(F^(b*c*x + a*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m, x)
```

**Sympy [F]**

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx = \int F^{c(a+bx)} ((d + ex)^3)^{-m} dx$$

input `integrate(F**((b*x+a)*c)/((e**3*x**3+3*d*e**2*x**2+3*d**2*e*x+d**3)**m),x)`

output `Integral(F**(c*(a + b*x))/((d + e*x)**3)**m, x)`

**Maxima [F]**

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx = \int \frac{F^{(bx+a)c}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m} dx$$

input `integrate(F^((b*x+a)*c)/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m),x, algorithm="maxima")`

output `integrate(F^((b*x + a)*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m, x)`

**Giac [F]**

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx = \int \frac{F^{(bx+a)c}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m} dx$$

input `integrate(F^((b*x+a)*c)/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m),x, algorithm="giac")`

output `integrate(F^((b*x + a)*c)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)^m, x)`



**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx = \int \frac{F^{c(a+bx)}}{(d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^m} dx$$

input `int(F^(c*(a + b*x))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)^m,x)`

output `int(F^(c*(a + b*x))/(d^3 + e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)^m, x)`

**Reduce [F]**

$$\int F^{c(a+bx)} (d^3 + 3d^2ex + 3de^2x^2 + e^3x^3)^{-m} dx$$

$$= f^{ac} \left( \int \frac{f^{bcx}}{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)^m} dx \right)$$

input `int(F^((b*x+a)*c)/((e^3*x^3+3*d*e^2*x^2+3*d^2*e*x+d^3)^m),x)`

output `f**(a*c)*int(f**(b*c*x)/(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3)**m,x)`

### 3.88 $\int F^{c(a+bx)}(d+ex)^m dx$

Optimal result	593
Mathematica [A] (verified)	593
Rubi [A] (verified)	594
Maple [F]	595
Fricas [A] (verification not implemented)	595
Sympy [F]	595
Maxima [F]	596
Giac [F]	596
Mupad [F(-1)]	596
Reduce [F]	597

#### Optimal result

Integrand size = 17, antiderivative size = 67

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

output

```
F^(c*(a-b*d/e))*(e*x+d)^m*GAMMA(1+m,-b*c*(e*x+d)*ln(F)/e)/b/c/ln(F)/((-b*c*(e*x+d)*ln(F)/e)^m)
```

#### Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{F^{c(a-\frac{bd}{e})}(d+ex)^m \Gamma\left(1+m, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-m}}{bc \log(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x)^m,x]
```

output  $(F^{c(a - (b*d)/e)} * (d + e*x)^m * \text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

### Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)^m F^{c(a+bx)} dx$$

$$\downarrow 2612$$

$$\frac{(d + ex)^m F^{c(a - \frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-m} \Gamma\left(m + 1, -\frac{bc(d+ex) \log(F)}{e}\right)}{bc \log(F)}$$

input  $\text{Int}[F^{c(a + b*x)} * (d + e*x)^m, x]$

output  $(F^{c(a - (b*d)/e)} * (d + e*x)^m * \text{Gamma}[1 + m, -((b*c*(d + e*x)*\text{Log}[F])/e)]) / (b*c*\text{Log}[F] * (-((b*c*(d + e*x)*\text{Log}[F])/e))^m)$

### Defintions of rubi rules used

rule 2612  $\text{Int}[(F_)^{((g_.) * (e_.) + (f_.) * (x_))} * ((c_.) + (d_.) * (x_))^{(m_)}, x\_Symbol]$   
 $:= \text{Simp}[(-F^{(g*(e - c*(f/d)))}) * ((c + d*x)^{\text{FracPart}[m]} / (d * ((-f)*g*(\text{Log}[F]/d)))^{(\text{IntPart}[m] + 1)} * ((-f)*g*\text{Log}[F] * ((c + d*x)/d))^{\text{FracPart}[m]}) * \text{Gamma}[m + 1, ((-f)*g*(\text{Log}[F]/d) * (c + d*x)], x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& !\text{IntegerQ}[m]$

**Maple [F]**

$$\int F^{c(bx+a)}(ex+d)^m dx$$

input `int(F^(c*(b*x+a))*(e*x+d)^m,x)`

output `int(F^(c*(b*x+a))*(e*x+d)^m,x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{e^{\left(-\frac{em \log\left(-\frac{bc \log(F)}{e}\right) + (bcd - ace) \log(F)}{e}\right)} \Gamma\left(m+1, -\frac{(bcex+bcd) \log(F)}{e}\right)}{bc \log(F)}$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^m,x, algorithm="fricas")`

output `e^(-(e*m*log(-b*c*log(F)/e) + (b*c*d - a*c*e)*log(F))/e)*gamma(m + 1, -(b*c*e*x + b*c*d)*log(F)/e)/(b*c*log(F))`

**Sympy [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \int F^{c(a+bx)}(d+ex)^m dx$$

input `integrate(F**(c*(a + b*x))*(d + e*x)**m, x)`

output `Integral(F**(c*(a + b*x))*(d + e*x)**m, x)`

**Maxima [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \int (ex+d)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^m,x, algorithm="maxima")`

output `integrate((e*x + d)^m * F^((b*x + a)*c), x)`

**Giac [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \int (ex+d)^m F^{(bx+a)c} dx$$

input `integrate(F^((b*x+a)*c)*(e*x+d)^m,x, algorithm="giac")`

output `integrate((e*x + d)^m * F^((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}(d+ex)^m dx = \int F^{c(a+bx)}(d+ex)^m dx$$

input `int(F^(c*(a + b*x))*(d + e*x)^m,x)`

output `int(F^(c*(a + b*x))*(d + e*x)^m, x)`

**Reduce [F]**

$$\int F^{c(a+bx)}(d+ex)^m dx = \frac{f^{ac} \left( f^{bcx} (ex+d)^m - \left( \int \frac{f^{bcx} (ex+d)^m}{ex+d} dx \right) em \right)}{\log(f) bc}$$

input `int(F^((b*x+a)*c)*(e*x+d)^m,x)`

output `(f**(a*c)*(f**(b*c*x)*(d + e*x)**m - int((f**(b*c*x)*(d + e*x)**m)/(d + e*x),x)*e*m))/(log(f)*b*c)`

### 3.89 $\int F^{c(a+bx)}((d+ex)^n)^m dx$

Optimal result	598
Mathematica [A] (verified)	598
Rubi [A] (verified)	599
Maple [F]	600
Fricas [A] (verification not implemented)	600
Sympy [F]	601
Maxima [F]	601
Giac [F]	601
Mupad [F(-1)]	602
Reduce [F]	602

#### Optimal result

Integrand size = 19, antiderivative size = 72

$$\int F^{c(a+bx)}((d+ex)^n)^m dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^n)^m \Gamma\left(1+mn, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-mn}}{bc \log(F)}$$

output `F^(c*(a-b*d/e))*((e*x+d)^n)^m*GAMMA(m*n+1, -b*c*(e*x+d)*ln(F)/e)/b/c/ln(F)/((-b*c*(e*x+d)*ln(F)/e)^(m*n))`

#### Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}((d+ex)^n)^m dx$$

$$= \frac{F^{c\left(a-\frac{bd}{e}\right)}((d+ex)^n)^m \Gamma\left(1+mn, -\frac{bc(d+ex)\log(F)}{e}\right) \left(-\frac{bc(d+ex)\log(F)}{e}\right)^{-mn}}{bc \log(F)}$$

input `Integrate[F^(c*(a + b*x))*((d + e*x)^n)^m, x]`

output

$$\frac{(F^{c(a - (b*d)/e)}) * ((d + e*x)^n)^m * \text{Gamma}[1 + m*n, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F] * -((b*c*(d + e*x)*\text{Log}[F])/e))^{(m*n)}}$$
**Rubi [A] (verified)**

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2045, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)} (d+ex)^n dx$$

$$\downarrow \text{2045}$$

$$(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-mn} \int F^{c(a+bx)} \left(\frac{ex}{d} + 1\right)^{mn} dx$$

$$\downarrow \text{2612}$$

$$\frac{(d+ex)^n F^{c(a-\frac{bd}{e})} \left(-\frac{bc \log(F)(d+ex)}{e}\right)^{-mn} \Gamma(mn+1, -\frac{bc(d+ex)\log(F)}{e})}{bc \log(F)}$$

input

$$\text{Int}[F^{c(a + b*x)} * ((d + e*x)^n)^m, x]$$

output

$$\frac{(F^{c(a - (b*d)/e)}) * ((d + e*x)^n)^m * \text{Gamma}[1 + m*n, -((b*c*(d + e*x)*\text{Log}[F])/e)]}{(b*c*\text{Log}[F] * -((b*c*(d + e*x)*\text{Log}[F])/e))^{(m*n)}}$$



## Definitions of rubi rules used

rule 2045

```
Int[(u_)*((c_)*((a_) + (b_)*(x_)^(n_))^(q_))^(p_), x_Symbol] :> Simp[Simp[
  mp[(c*(a + b*x^n)^q]^p/(1 + b*(x^n/a))^(p*q)] Int[u*(1 + b*(x^n/a))^(p*q)
  , x], x] /; FreeQ[{a, b, c, n, p, q}, x] && !GeQ[a, 0]
```

rule 2612

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*(-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m])*Gamma[m + 1,
((-f)*g*(Log[F]/d)*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

## Maple [F]

$$\int F^{c(bx+a)}((ex+d)^n)^m dx$$

input

```
int(F^(c*(b*x+a))*((e*x+d)^n)^m,x)
```

output

```
int(F^(c*(b*x+a))*((e*x+d)^n)^m,x)
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int F^{c(a+bx)}((d+ex)^n)^m dx$$

$$= \frac{e^{\left(-\frac{emn \log\left(-\frac{bc \log(F)}{e}\right) + (bcd - ace) \log(F)}{e}\right)} \Gamma\left(mn + 1, -\frac{(bcex + bcd) \log(F)}{e}\right)}{bc \log(F)}$$

input

```
integrate(F^((b*x+a)*c)*((e*x+d)^n)^m,x, algorithm="fricas")
```

output  $e^{-(e*m*n*\log(-b*c*\log(F)/e) + (b*c*d - a*c*e)*\log(F))/e}*\gamma(m*n + 1, -(b*c*e*x + b*c*d)*\log(F)/e)/(b*c*\log(F))$

### Sympy [F]

$$\int F^{c(a+bx)}((d+ex)^n)^m dx = \int F^{c(a+bx)}((d+ex)^n)^m dx$$

input `integrate(F**((b*x+a)*c)*((e*x+d)**n)**m,x)`

output `Integral(F**(c*(a + b*x))*((d + e*x)**n)**m, x)`

### Maxima [F]

$$\int F^{c(a+bx)}((d+ex)^n)^m dx = \int ((ex+d)^n)^m F^{(bx+a)c} dx$$

input `integrate(F**((b*x+a)*c)*((e*x+d)^n)^m,x, algorithm="maxima")`

output `integrate(((e*x + d)^n)^m * F**((b*x + a)*c), x)`

### Giac [F]

$$\int F^{c(a+bx)}((d+ex)^n)^m dx = \int ((ex+d)^n)^m F^{(bx+a)c} dx$$

input `integrate(F**((b*x+a)*c)*((e*x+d)^n)^m,x, algorithm="giac")`

output `integrate(((e*x + d)^n)^m * F**((b*x + a)*c), x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{c(a+bx)}((d+ex)^n)^m dx = \int F^{c(a+bx)}((d+ex)^n)^m dx$$

input `int(F^(c*(a + b*x))*((d + e*x)^n)^m, x)`output `int(F^(c*(a + b*x))*((d + e*x)^n)^m, x)`**Reduce [F]**

$$\int F^{c(a+bx)}((d+ex)^n)^m dx = \frac{f^{ac} \left( f^{bcx} (ex+d)^{mn} - \left( \int \frac{f^{bcx} (ex+d)^{mn}}{ex+d} dx \right) emn \right)}{\log(f) bc}$$

input `int(F^((b*x+a)*c))*((e*x+d)^n)^m, x)`output `(f**(a*c)*(f**(b*c*x)*(d + e*x)**(m*n) - int((f**(b*c*x)*(d + e*x)**(m*n)) / (d + e*x), x)*e*m*n)) / (log(f)*b*c)`

### 3.90 $\int F^{c(a+bx)}(d+ex) dx$

Optimal result	603
Mathematica [A] (verified)	603
Rubi [A] (verified)	604
Maple [A] (verified)	605
Fricas [A] (verification not implemented)	605
Sympy [A] (verification not implemented)	606
Maxima [A] (verification not implemented)	606
Giac [C] (verification not implemented)	606
Mupad [B] (verification not implemented)	607
Reduce [B] (verification not implemented)	608

#### Optimal result

Integrand size = 15, antiderivative size = 48

$$\int F^{c(a+bx)}(d+ex) dx = -\frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex)}{bc \log(F)}$$

output

```
-e*F^(c*(b*x+a))/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(e*x+d)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{c(a+bx)}(-e+bc(d+ex)\log(F))}{b^2c^2 \log^2(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x), x]
```

output

```
(F^(c*(a + b*x))*(-e + b*c*(d + e*x)*Log[F]))/(b^2*c^2*Log[F]^2)
```

**Rubi [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex)F^{c(a+bx)} dx$$

$$\downarrow 2607$$

$$\frac{(d + ex)F^{c(a+bx)}}{bc \log(F)} - \frac{e \int F^{c(a+bx)} dx}{bc \log(F)}$$

$$\downarrow 2624$$

$$\frac{(d + ex)F^{c(a+bx)}}{bc \log(F)} - \frac{eF^{c(a+bx)}}{b^2c^2 \log^2(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x),x]`

output `-((e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2)) + (F^(c*(a + b*x))*(d + e*x))/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

method	result	size
gospers	$\frac{(ex \ln(F)bc + bcd \ln(F) - e)F^{c(bx+a)}}{\ln(F)^2 b^2 c^2}$	38
risch	$\frac{(ex \ln(F)bc + bcd \ln(F) - e)F^{c(bx+a)}}{\ln(F)^2 b^2 c^2}$	38
orering	$\frac{(ex \ln(F)bc + bcd \ln(F) - e)F^{c(bx+a)}}{\ln(F)^2 b^2 c^2}$	38
norman	$\frac{(bcd \ln(F) - e)e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} + \frac{ex e^{c(bx+a) \ln(F)}}{\ln(F)bc}$	56
parallelrisch	$\frac{e F^{c(bx+a)} x \ln(F) bc + \ln(F) F^{c(bx+a)} bcd - F^{c(bx+a)} e}{\ln(F)^2 b^2 c^2}$	56
meijerg	$\frac{F^{ac} e \left( 1 - \frac{(-2bcx \ln(F) + 2)e^{bcx \ln(F)}}{2} \right)}{\ln(F)^2 b^2 c^2} - \frac{F^{ac} d (1 - e^{bcx \ln(F)})}{bc \ln(F)}$	68

input `int(F^(c*(b*x+a))*(e*x+d),x,method=_RETURNVERBOSE)`

output `(e*x*ln(F)*b*c+b*c*d*ln(F)-e)*F^(c*(b*x+a))/ln(F)^2/b^2/c^2`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}(d+ex) dx = \frac{((bcex + bcd) \log(F) - e)F^{bcx+ac}}{b^2 c^2 \log(F)^2}$$

input `integrate(F^((b*x+a)*c)*(e*x+d),x, algorithm="fricas")`

output `((b*c*e*x + b*c*d)*log(F) - e)*F^(b*c*x + a*c)/(b^2*c^2*log(F)^2)`

**Sympy [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int F^{c(a+bx)}(d+ex) dx = \begin{cases} \frac{F^{c(a+bx)}(bcd \log(F) + bce x \log(F) - e)}{b^2 c^2 \log(F)^2} & \text{for } b^2 c^2 \log(F)^2 \neq 0 \\ dx + \frac{ex^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(e*x+d), x)`

output `Piecewise((F**(c*(a + b*x))*(b*c*d*log(F) + b*c*e*x*log(F) - e)/(b**2*c**2*log(F)**2), Ne(b**2*c**2*log(F)**2, 0)), (d*x + e*x**2/2, True))`

**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2 c^2 \log(F)^2}$$

input `integrate(F^((b*x+a)*c)*(e*x+d), x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*e/(b^2*c^2*log(F)^2)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 898, normalized size of antiderivative = 18.71

$$\int F^{c(a+bx)}(d+ex) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(e*x+d), x, algorithm="giac")`

output

```
(2*((pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(pi*b*c*e*x*sgn(F) - pi*b*c*e*x + pi*b*c*d*sgn(F) - pi*b*c*d)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) + (pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2*(b*c*e*x*log(abs(F)) + b*c*d*log(abs(F)) - e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2*(pi*b*c*e*x*sgn(F) - pi*b*c*e*x + pi*b*c*d*sgn(F) - pi*b*c*d)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))*(b*c*e*x*log(abs(F)) + b*c*d*log(abs(F)) - e)/((pi^2*b^2*c^2*sgn(F) - pi^2*b^2*c^2 + 2*b^2*c^2*log(abs(F)))^2)^2 + 4*(pi*b^2*c^2*log(abs(F))*sgn(F) - pi*b^2*c^2*log(abs(F)))^2))*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) - 1/2*I*((pi*b*c*e*x*sgn(F) - pi*b*c*e*x - 2*I*b*c*e*x*log(abs(F)) + pi*b*c*d*sgn(F) - pi*b*c*d - 2*I*b*c*d*log(abs(F)) + 2*I*e))*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(pi^2*b^2*c^2*sgn(F) + 2*I*pi*b^2*c^2*log(abs(F))*sgn(F) - pi^2*b^2*c^2 - 2*I*pi*b^2*c^2...
```

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}(d+ex) dx = \frac{F^{a+bcx} (bcd \ln(F) - e + bce x \ln(F))}{b^2 c^2 \ln(F)^2}$$

input

```
int(F^(c*(a + b*x))*(d + e*x),x)
```

output

```
(F^(a*c + b*c*x)*(b*c*d*log(F) - e + b*c*e*x*log(F)))/(b^2*c^2*log(F)^2)
```



**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}(d+ex) dx = \frac{f^{bcx+ac}(\log(f) bcd + \log(f) bce x - e)}{\log(f)^2 b^2 c^2}$$

input `int(F^((b*x+a)*c)*(e*x+d),x)`

output `(f**(a*c + b*c*x)*(log(f)*b*c*d + log(f)*b*c*e*x - e))/(log(f)**2*b**2*c**2)`

### 3.91 $\int F^{c(a+bx)}(d + ex + fx^2) dx$

Optimal result . . . . .	609
Mathematica [A] (verified) . . . . .	609
Rubi [A] (verified) . . . . .	610
Maple [A] (verified) . . . . .	611
Fricas [A] (verification not implemented) . . . . .	611
Sympy [A] (verification not implemented) . . . . .	612
Maxima [A] (verification not implemented) . . . . .	612
Giac [C] (verification not implemented) . . . . .	613
Mupad [B] (verification not implemented) . . . . .	614
Reduce [B] (verification not implemented) . . . . .	614

#### Optimal result

Integrand size = 20, antiderivative size = 80

$$\int F^{c(a+bx)}(d+ex+fx^2) dx = \frac{2fF^{c(a+bx)}}{b^3c^3 \log^3(F)} - \frac{F^{c(a+bx)}(e+2fx)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d+ex+fx^2)}{bc \log(F)}$$

output

```
2*f*F^(c*(b*x+a))/b^3/c^3/ln(F)^3-F^(c*(b*x+a))*(2*f*x+e)/b^2/c^2/ln(F)^2+
F^(c*(b*x+a))*(f*x^2+e*x+d)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int F^{c(a+bx)}(d + ex + fx^2) dx = \frac{F^{c(a+bx)}(2f - bc(e + 2fx) \log(F) + b^2c^2(d + x(e + fx)) \log^2(F))}{b^3c^3 \log^3(F)}$$

input

```
Integrate[F^(c*(a + b*x))*(d + e*x + f*x^2),x]
```

output

```
(F^(c*(a + b*x))*(2*f - b*c*(e + 2*f*x)*Log[F] + b^2*c^2*(d + x*(e + f*x))
*Log[F]^2))/(b^3*c^3*Log[F]^3)
```

**Rubi [A] (verified)**

Time = 0.50 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex + fx^2) F^{c(a+bx)} dx$$

$$\downarrow 2626$$

$$\int \left( dF^{c(a+bx)} + exF^{c(a+bx)} + fx^2F^{c(a+bx)} \right) dx$$

$$\downarrow 2009$$

$$\frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{exF^{c(a+bx)}}{bc\log(F)} + \frac{fx^2F^{c(a+bx)}}{bc\log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x + f*x^2),x]`

output `(2*f*F^(c*(a + b*x)))/(b^3*c^3*Log[F]^3) - (e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2) - (2*f*F^(c*(a + b*x))*x)/(b^2*c^2*Log[F]^2) + (d*F^(c*(a + b*x)))/(b*c*Log[F]) + (e*F^(c*(a + b*x))*x)/(b*c*Log[F]) + (f*F^(c*(a + b*x))*x^2)/(b*c*Log[F])`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

method	result
gospers	$\frac{(f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 e x + \ln(F)^2 b^2 c^2 d - 2 \ln(F) b c f x - \ln(F) b c e + 2 f) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
risch	$\frac{(f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 e x + \ln(F)^2 b^2 c^2 d - 2 \ln(F) b c f x - \ln(F) b c e + 2 f) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
orering	$\frac{(f x^2 \ln(F)^2 b^2 c^2 + \ln(F)^2 b^2 c^2 e x + \ln(F)^2 b^2 c^2 d - 2 \ln(F) b c f x - \ln(F) b c e + 2 f) F^{c(bx+a)}}{\ln(F)^3 b^3 c^3}$
norman	$\frac{(\ln(F)^2 b^2 c^2 d - \ln(F) b c e + 2 f) e^{c(bx+a) \ln(F)}}{c^3 b^3 \ln(F)^3} + \frac{(\ln(F) b c e - 2 f) x e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} + \frac{f x^2 e^{c(bx+a) \ln(F)}}{\ln(F) b c}$
meijerg	$- \frac{F^{ac} f \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3} + \frac{F^{ac} e \left( 1 - \frac{(-2bcx \ln(F) + 2) e^{bcx \ln(F)}}{2} \right)}{\ln(F)^2 b^2 c^2} - \frac{F^{ac} d (1 - e^{bcx \ln(F)})}{bc \ln(F)}$
parallelrisch	$\frac{x^2 F^{c(bx+a)} f \ln(F)^2 b^2 c^2 + \ln(F)^2 x F^{c(bx+a)} b^2 c^2 e + \ln(F)^2 F^{c(bx+a)} b^2 c^2 d - 2 \ln(F) x F^{c(bx+a)} b c f - \ln(F) F^{c(bx+a)} b c e + 2 F^{c(bx+a)} f}{\ln(F)^3 b^3 c^3}$

input `int(F^(c*(b*x+a))*(f*x^2+e*x+d),x,method=_RETURNVERBOSE)`output 
$$(f*x^2*\ln(F)^2*b^2*c^2+\ln(F)^2*b^2*c^2*e*x+\ln(F)^2*b^2*c^2*d-2*\ln(F)*b*c*f*x-\ln(F)*b*c*e+2*f)*F^(c*(b*x+a))/\ln(F)^3/b^3/c^3$$
**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \frac{((b^2 c^2 f x^2 + b^2 c^2 e x + b^2 c^2 d) \log(F)^2 - (2 b c f x + b c e) \log(F) + 2 f) F^{bcx+ac}}{b^3 c^3 \log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d),x, algorithm="fricas")`output 
$$((b^2*c^2*f*x^2 + b^2*c^2*e*x + b^2*c^2*d)*\log(F)^2 - (2*b*c*f*x + b*c*e)*\log(F) + 2*f)*F^(b*c*x + a*c)/(b^3*c^3*\log(F)^3)$$

**Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^2c^2d\log(F)^2+b^2c^2ex\log(F)^2+b^2c^2fx^2\log(F)^2-bce\log(F)-2bcfx\log(F)+2f)}{b^3c^3\log(F)^3} & \text{for } b^3c^3\log(F)^3 \neq 0 \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(f*x**2+e*x+d),x)`output `Piecewise((F**(c*(a + b*x))*(b**2*c**2*d*log(F)**2 + b**2*c**2*e*x*log(F)**2 + b**2*c**2*f*x**2*log(F)**2 - b*c*e*log(F) - 2*b*c*f*x*log(F) + 2*f)/(b**3*c**3*log(F)**3), Ne(b**3*c**3*log(F)**3, 0)), (d*x + e*x**2/2 + f*x**3/3, True))`**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.46

$$\int F^{c(a+bx)}(d+ex+fx^2) dx = \frac{F^{bcx+ac}d}{bc\log(F)} + \frac{(F^{ac}bcx\log(F) - F^{ac})F^{bcx}e}{b^2c^2\log(F)^2} + \frac{(F^{ac}b^2c^2x^2\log(F)^2 - 2F^{ac}bcx\log(F) + 2F^{ac})F^{bcx}f}{b^3c^3\log(F)^3}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d),x, algorithm="maxima")`output `F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*f/(b^3*c^3*log(F)^3)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 2068, normalized size of antiderivative = 25.85

$$\int F^{c(a+bx)}(d+ex+fx^2) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(f*x^2+e*x+d),x, algorithm="giac")`

output

```
((pi^2*b^2*c^2*f*x^2*sgn(F) - pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*log(abs(F))^2 + pi^2*b^2*c^2*e*x*sgn(F) - pi^2*b^2*c^2*e*x + 2*b^2*c^2*e*x*log(abs(F))^2 + pi^2*b^2*c^2*d*sgn(F) - pi^2*b^2*c^2*d + 2*b^2*c^2*d*log(abs(F))^2 - 4*b*c*f*x*log(abs(F)) - 2*b*c*e*log(abs(F)) + 4*f)*(3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)/((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2) - (pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(2*pi*b^2*c^2*f*x^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*f*x^2*log(abs(F)) + 2*pi*b^2*c^2*e*x*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*e*x*log(abs(F)) + 2*pi*b^2*c^2*d*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*d*log(abs(F)) - 2*pi*b*c*f*x*sgn(F) + 2*pi*b*c*f*x - pi*b*c*e*sgn(F) + pi*b*c*e)/(pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2 + (3*pi^2*b^3*c^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*log(abs(F)) + 2*b^3*c^3*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c) + ((pi^3*b^3*c^3*sgn(F) - 3*pi*b^3*c^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3 + 3*pi*b^3*c^3*log(abs(F))^2)*(pi^2*b^2*c^2*f*x^2*sgn(F) - pi^2*b^2*c^2*f*x^2 + 2*b^2*c^2*f*x^2*log(abs(F))^2 + pi^2*b^2*c^2*e*x*sgn(F) - pi^2*b^2*c^2*e*x + 2*b^2*c^2*e*x*log(abs...
```

**Mupad [B] (verification not implemented)**

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \frac{F^{a+bcx} (fb^2c^2x^2 \ln(F)^2 + eb^2c^2x \ln(F)^2 + db^2c^2 \ln(F)^2 - 2fbcx \ln(F) - ebc \ln(F) + 2f)}{b^3c^3 \ln(F)^3}$$

input `int(F^(c*(a + b*x))*(d + e*x + f*x^2),x)`output `(F^(a*c + b*c*x)*(2*f - b*c*e*log(F) + b^2*c^2*d*log(F)^2 + b^2*c^2*f*x^2*log(F)^2 - 2*b*c*f*x*log(F) + b^2*c^2*e*x*log(F)^2))/(b^3*c^3*log(F)^3)`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}(d+ex+fx^2) dx$$

$$= \frac{f^{bcx+ac}(\log(f)^2 b^2 c^2 d + \log(f)^2 b^2 c^2 ex + \log(f)^2 b^2 c^2 f x^2 - \log(f) bce - 2 \log(f) bcf x + 2f)}{\log(f)^3 b^3 c^3}$$

input `int(F^((b*x+a)*c)*(f*x^2+e*x+d),x)`output `(f**(a*c + b*c*x)*(log(f)**2*b**2*c**2*d + log(f)**2*b**2*c**2*e*x + log(f)**2*b**2*c**2*f*x**2 - log(f)*b*c*e - 2*log(f)*b*c*f*x + 2*f))/(log(f)**3*b**3*c**3)`

### 3.92 $\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx$

Optimal result . . . . .	615
Mathematica [A] (verified) . . . . .	615
Rubi [A] (verified) . . . . .	616
Maple [A] (verified) . . . . .	617
Fricas [A] (verification not implemented) . . . . .	618
Sympy [A] (verification not implemented) . . . . .	618
Maxima [A] (verification not implemented) . . . . .	619
Giac [C] (verification not implemented) . . . . .	619
Mupad [B] (verification not implemented) . . . . .	620
Reduce [B] (verification not implemented) . . . . .	621

#### Optimal result

Integrand size = 25, antiderivative size = 118

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx = -\frac{6F^{c(a+bx)}g}{b^4c^4\log^4(F)} + \frac{2F^{c(a+bx)}(f + 3gx)}{b^3c^3\log^3(F)} - \frac{F^{c(a+bx)}(e + 2fx + 3gx^2)}{b^2c^2\log^2(F)} + \frac{F^{c(a+bx)}(d + ex + fx^2 + gx^3)}{bc\log(F)}$$

output

```
-6*F^(c*(b*x+a))*g/b^4/c^4/ln(F)^4+2*F^(c*(b*x+a))*(3*g*x+f)/b^3/c^3/ln(F)^3-F^(c*(b*x+a))*(3*g*x^2+2*f*x+e)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d)/b/c/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx = \frac{F^{c(a+bx)}(-6g + 2bc(f + 3gx)\log(F) - b^2c^2(e + x(2f + 3gx))\log^2(F) + b^3c^3(d + x(e + x(f + gx)))\log^3(F))}{b^4c^4\log^4(F)}$$



input `Integrate[F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3),x]`

output  $(F^{c(a+bx)})*(-6*g + 2*b*c*(f + 3*g*x)*\text{Log}[F] - b^2*c^2*(e + x*(2*f + 3*g*x))*\text{Log}[F]^2 + b^3*c^3*(d + x*(e + x*(f + g*x)))*\text{Log}[F]^3)/(b^4*c^4*\text{Log}[F]^4)$

### Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.94, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx$$

$$\downarrow 2626$$

$$\int \left( dF^{c(a+bx)} + exF^{c(a+bx)} + fx^2F^{c(a+bx)} + gx^3F^{c(a+bx)} \right) dx$$

$$\downarrow 2009$$

$$-\frac{6gF^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{6gxF^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fxF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{exF^{c(a+bx)}}{bc\log(F)} + \frac{fx^2F^{c(a+bx)}}{bc\log(F)} + \frac{gx^3F^{c(a+bx)}}{bc\log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3),x]`

output  $(-6*F^{c(a+bx)}*g)/(b^4*c^4*\text{Log}[F]^4) + (2*f*F^{c(a+bx)})/(b^3*c^3*\text{Log}[F]^3) + (6*F^{c(a+bx)}*g*x)/(b^3*c^3*\text{Log}[F]^3) - (e*F^{c(a+bx)})/(b^2*c^2*\text{Log}[F]^2) - (2*f*F^{c(a+bx)}*x)/(b^2*c^2*\text{Log}[F]^2) - (3*F^{c(a+bx)}*g*x^2)/(b^2*c^2*\text{Log}[F]^2) + (d*F^{c(a+bx)})/(b*c*\text{Log}[F]) + (e*F^{c(a+bx)}*x)/(b*c*\text{Log}[F]) + (f*F^{c(a+bx)}*x^2)/(b*c*\text{Log}[F]) + (F^{c(a+bx)}*g*x^3)/(b*c*\text{Log}[F])$

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2626 Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17

method	result
gospers	$\frac{(g x^3 \ln(F)^3 b^3 c^3 + \ln(F)^3 b^3 c^3 f x^2 + \ln(F)^3 b^3 c^3 e x + \ln(F)^3 b^3 c^3 d - 3 \ln(F)^2 b^2 c^2 g x^2 - 2 f x \ln(F)^2 b^2 c^2 - \ln(F)^2 b^2 c^2 e + 6 \ln(F) b^2 c^2 d - 3 \ln(F) b^2 c^2 f x - 2 b c x \ln(F) + 2 a) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
risch	$\frac{(g x^3 \ln(F)^3 b^3 c^3 + \ln(F)^3 b^3 c^3 f x^2 + \ln(F)^3 b^3 c^3 e x + \ln(F)^3 b^3 c^3 d - 3 \ln(F)^2 b^2 c^2 g x^2 - 2 f x \ln(F)^2 b^2 c^2 - \ln(F)^2 b^2 c^2 e + 6 \ln(F) b^2 c^2 d - 3 \ln(F) b^2 c^2 f x - 2 b c x \ln(F) + 2 a) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
orering	$\frac{(g x^3 \ln(F)^3 b^3 c^3 + \ln(F)^3 b^3 c^3 f x^2 + \ln(F)^3 b^3 c^3 e x + \ln(F)^3 b^3 c^3 d - 3 \ln(F)^2 b^2 c^2 g x^2 - 2 f x \ln(F)^2 b^2 c^2 - \ln(F)^2 b^2 c^2 e + 6 \ln(F) b^2 c^2 d - 3 \ln(F) b^2 c^2 f x - 2 b c x \ln(F) + 2 a) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$
norman	$\frac{(\ln(F)^3 b^3 c^3 d - \ln(F)^2 b^2 c^2 e + 2 \ln(F) b c f - 6 g) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4} + \frac{(\ln(F) b c f - 3 g) x^2 e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} + \frac{(\ln(F)^2 b^2 c^2 e - 2 \ln(F) b^2 c^2 d - 3 \ln(F) b^2 c^2 f x - 2 b c x \ln(F) + 2 a) e^{c(bx+a) \ln(F)}}{\ln(F)}$
meijerg	$\frac{F^{ac} g \left( 6 - \frac{(-4b^3 c^3 x^3 \ln(F)^3 + 12b^2 c^2 x^2 \ln(F)^2 - 24bcx \ln(F) + 24) e^{bcx \ln(F)}}{4} \right)}{\ln(F)^4 b^4 c^4} - \frac{F^{ac} f \left( 2 - \frac{(3b^2 c^2 x^2 \ln(F)^2 - 6bcx \ln(F) + 6) e^{bcx \ln(F)}}{3} \right)}{c^3 b^3 \ln(F)^3}$
parallelrisch	$\frac{x^3 F^{c(bx+a)} g \ln(F)^3 b^3 c^3 + \ln(F)^3 x^2 F^{c(bx+a)} b^3 c^3 f + \ln(F)^3 x F^{c(bx+a)} b^3 c^3 e + \ln(F)^3 F^{c(bx+a)} b^3 c^3 d - 3 \ln(F)^2 x^2 F^{c(bx+a)} b^2 c^2 g - 2 f x \ln(F)^2 b^2 c^2 - \ln(F)^2 b^2 c^2 e + 6 \ln(F) b^2 c^2 d - 3 \ln(F) b^2 c^2 f x - 2 b c x \ln(F) + 2 a) e^{c(bx+a) \ln(F)}}{\ln(F)^4 b^4 c^4}$

```
input int(F^(c*(b*x+a))*(g*x^3+f*x^2+e*x+d), x, method=_RETURNVERBOSE)
```

```
output (g*x^3*ln(F)^3*b^3*c^3+ln(F)^3*b^3*c^3*f*x^2+ln(F)^3*b^3*c^3*e*x+ln(F)^3*b^3*c^3*d-3*ln(F)^2*b^2*c^2*g*x^2-2*f*x*ln(F)^2*b^2*c^2-ln(F)^2*b^2*c^2*e+6*ln(F)*b*c*g*x+2*ln(F)*b*c*f-6*g)*F^(c*(b*x+a))/ln(F)^4/b^4/c^4
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3) dx$$

$$= \frac{((b^3c^3gx^3 + b^3c^3fx^2 + b^3c^3ex + b^3c^3d) \log(F)^3 - (3b^2c^2gx^2 + 2b^2c^2fx + b^2c^2e) \log(F)^2 + 2(3bcgx + b^2c^2d) \log(F) - 6*g) * F^{(b*c*x + a*c)}}{b^4c^4 \log(F)^4}$$

input `integrate(F^((b*x+a)*c)*(g*x^3+f*x^2+e*x+d),x, algorithm="fricas")`

output `((b^3*c^3*g*x^3 + b^3*c^3*f*x^2 + b^3*c^3*e*x + b^3*c^3*d)*log(F)^3 - (3*b^2*c^2*g*x^2 + 2*b^2*c^2*f*x + b^2*c^2*e)*log(F)^2 + 2*(3*b*c*g*x + b*c*f)*log(F) - 6*g)*F^(b*c*x + a*c)/(b^4*c^4*log(F)^4)`

**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.61

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3) dx$$

$$= \begin{cases} \frac{F^{c(a+bx)}(b^3c^3d \log(F)^3 + b^3c^3ex \log(F)^3 + b^3c^3fx^2 \log(F)^3 + b^3c^3gx^3 \log(F)^3 - b^2c^2e \log(F)^2 - 2b^2c^2fx \log(F)^2 - 3b^2c^2gx^2 \log(F)^2 + 2bcf \log(F) - 6g)}{b^4c^4 \log(F)^4} \\ dx + \frac{ex^2}{2} + \frac{fx^3}{3} + \frac{gx^4}{4} \end{cases}$$

input `integrate(F**((b*x+a)*c)*(g*x**3+f*x**2+e*x+d),x)`

output `Piecewise((F**(c*(a + b*x))*(b**3*c**3*d*log(F)**3 + b**3*c**3*e*x*log(F)**3 + b**3*c**3*f*x**2*log(F)**3 + b**3*c**3*g*x**3*log(F)**3 - b**2*c**2*e*log(F)**2 - 2*b**2*c**2*f*x*log(F)**2 - 3*b**2*c**2*g*x**2*log(F)**2 + 2*b*c*f*log(F) + 6*b*c*g*x*log(F) - 6*g)/(b**4*c**4*log(F)**4), Ne(b**4*c**4*log(F)**4, 0)), (d*x + e*x**2/2 + f*x**3/3 + g*x**4/4, True))`

**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.64

$$\begin{aligned} & \int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx \\ &= \frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2c^2 \log(F)^2} \\ &+ \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f}{b^3c^3 \log(F)^3} \\ &+ \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}g}{b^4c^4 \log(F)^4} \end{aligned}$$

input `integrate(F^((b*x+a)*c)*(g*x^3+f*x^2+e*x+d),x, algorithm="maxima")`

output `F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F) + 2*F^(a*c))*F^(b*c*x)*f/(b^3*c^3*log(F)^3) + (F^(a*c)*b^3*c^3*x^3*log(F)^3 - 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))*F^(b*c*x)*g/(b^4*c^4*log(F)^4)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 4188, normalized size of antiderivative = 35.49

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3) dx = \text{Too large to display}$$

input `integrate(F^((b*x+a)*c)*(g*x^3+f*x^2+e*x+d),x, algorithm="giac")`

output

```

-(((3*pi^2*b^3*c^3*g*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*g*x^3*log(abs
(F)) + 2*b^3*c^3*g*x^3*log(abs(F))^3 + 3*pi^2*b^3*c^3*f*x^2*log(abs(F))*sg
n(F) - 3*pi^2*b^3*c^3*f*x^2*log(abs(F)) + 2*b^3*c^3*f*x^2*log(abs(F))^3 +
3*pi^2*b^3*c^3*e*x*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*e*x*log(abs(F)) + 2
*b^3*c^3*e*x*log(abs(F))^3 + 3*pi^2*b^3*c^3*d*log(abs(F))*sgn(F) - 3*pi^2*b
^3*c^3*d*log(abs(F)) + 2*b^3*c^3*d*log(abs(F))^3 - 3*pi^2*b^2*c^2*g*x^2*sg
n(F) + 3*pi^2*b^2*c^2*g*x^2 - 6*b^2*c^2*g*x^2*log(abs(F))^2 - 2*pi^2*b^2*c
^2*f*x*sgn(F) + 2*pi^2*b^2*c^2*f*x - 4*b^2*c^2*f*x*log(abs(F))^2 - pi^2*b
^2*c^2*e*sgn(F) + pi^2*b^2*c^2*e - 2*b^2*c^2*e*log(abs(F))^2 + 12*b*c*g*x*
log(abs(F)) + 4*b*c*f*log(abs(F)) - 12*g)*(pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^
4*c^4*log(abs(F))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 -
2*b^4*c^4*log(abs(F))^4)/((pi^4*b^4*c^4*sgn(F) - 6*pi^2*b^4*c^4*log(abs(F)
))^2*sgn(F) - pi^4*b^4*c^4 + 6*pi^2*b^4*c^4*log(abs(F))^2 - 2*b^4*c^4*log(
abs(F))^4)^2 + 16*(pi^3*b^4*c^4*log(abs(F))*sgn(F) - pi*b^4*c^4*log(abs(F)
)^3*sgn(F) - pi^3*b^4*c^4*log(abs(F)) + pi*b^4*c^4*log(abs(F))^3)^2 - 4*(
pi^3*b^3*c^3*g*x^3*sgn(F) - 3*pi*b^3*c^3*g*x^3*log(abs(F))^2*sgn(F) - pi^3
*b^3*c^3*g*x^3 + 3*pi*b^3*c^3*g*x^3*log(abs(F))^2 + pi^3*b^3*c^3*f*x^2*sgn
(F) - 3*pi*b^3*c^3*f*x^2*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3*f*x^2 + 3*pi*
b^3*c^3*f*x^2*log(abs(F))^2 + pi^3*b^3*c^3*e*x*sgn(F) - 3*pi*b^3*c^3*e*x*
log(abs(F))^2*sgn(F) - pi^3*b^3*c^3*e*x + 3*pi*b^3*c^3*e*x*log(abs(F))^2...

```

### Mupad [B] (verification not implemented)

Time = 22.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3) dx$$

$$= \frac{F^{ac+bcx} (gb^3c^3x^3 \ln(F)^3 + fb^3c^3x^2 \ln(F)^3 + eb^3c^3x \ln(F)^3 + db^3c^3 \ln(F)^3 - 3gb^2c^2x^2 \ln(F)^2 - 2gb^2c^2x \ln(F) - b^2c^2 \ln(F))}{b^4c^4 \ln(F)^4}$$

input

```
int(F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3),x)
```

output

```

(F^(a*c + b*c*x)*(2*b*c*f*log(F) - 6*g + b^3*c^3*d*log(F)^3 - b^2*c^2*e*lo
g(F)^2 + b^3*c^3*f*x^2*log(F)^3 - 3*b^2*c^2*g*x^2*log(F)^2 + b^3*c^3*g*x^3
*log(F)^3 + 6*b*c*g*x*log(F) + b^3*c^3*e*x*log(F)^3 - 2*b^2*c^2*f*x*log(F)
^2))/ (b^4*c^4*log(F)^4)

```

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3) dx$$

$$= \frac{f^{bcx+ac}(\log(f)^3 b^3 c^3 d + \log(f)^3 b^3 c^3 ex + \log(f)^3 b^3 c^3 f x^2 + \log(f)^3 b^3 c^3 g x^3 - \log(f)^2 b^2 c^2 e - 2\log(f)^2 b^2 c^2 f x - 2\log(f)^2 b^2 c^2 g x^2 + 6\log(f) b^2 c^2 d + 6\log(f) b^2 c^2 ex + 6\log(f) b^2 c^2 f x^2 + 6\log(f) b^2 c^2 g x^3 - 6\log(f) b^2 c^2 d - 6\log(f) b^2 c^2 ex - 6\log(f) b^2 c^2 f x^2 - 6\log(f) b^2 c^2 g x^3)}{\log(f)^4 b^4 c^4}$$

input `int(F^((b*x+a)*c)*(g*x^3+f*x^2+e*x+d),x)`output `(f**(a*c + b*c*x)*(log(f)**3*b**3*c**3*d + log(f)**3*b**3*c**3*e*x + log(f)**3*b**3*c**3*f*x**2 + log(f)**3*b**3*c**3*g*x**3 - log(f)**2*b**2*c**2*e - 2*log(f)**2*b**2*c**2*f*x - 3*log(f)**2*b**2*c**2*g*x**2 + 2*log(f)*b*c*f + 6*log(f)*b*c*g*x - 6*g))/(log(f)**4*b**4*c**4)`

### 3.93 $\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx$

Optimal result	622
Mathematica [A] (verified)	623
Rubi [B] (verified)	623
Maple [A] (verified)	625
Fricas [A] (verification not implemented)	625
Sympy [A] (verification not implemented)	626
Maxima [A] (verification not implemented)	627
Giac [C] (verification not implemented)	627
Mupad [B] (verification not implemented)	628
Reduce [B] (verification not implemented)	629

#### Optimal result

Integrand size = 30, antiderivative size = 162

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx = \frac{24F^{c(a+bx)}h}{b^5c^5 \log^5(F)} - \frac{6F^{c(a+bx)}(g + 4hx)}{b^4c^4 \log^4(F)} + \frac{2F^{c(a+bx)}(f + 3gx + 6hx^2)}{b^3c^3 \log^3(F)} - \frac{F^{c(a+bx)}(e + 2fx + 3gx^2 + 4hx^3)}{b^2c^2 \log^2(F)} + \frac{F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4)}{bc \log(F)}$$

output

```
24*F^(c*(b*x+a))*h/b^5/c^5/ln(F)^5-6*F^(c*(b*x+a))*(4*h*x+g)/b^4/c^4/ln(F)^4+2*F^(c*(b*x+a))*(6*h*x^2+3*g*x+f)/b^3/c^3/ln(F)^3-F^(c*(b*x+a))*(4*h*x^3+3*g*x^2+2*f*x+e)/b^2/c^2/ln(F)^2+F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d)/b/c/ln(F)
```

**Mathematica [A] (verified)**

Time = 0.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.72

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx$$

$$= \frac{F^{c(a+bx)}(24h - 6bc(g + 4hx) \log(F) + 2b^2c^2(f + 3x(g + 2hx)) \log^2(F) - b^3c^3(e + x(2f + 3gx + 4hx^2)))}{b^5c^5 \log^5(F)}$$

input `Integrate[F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]`

output `(F^(c*(a + b*x))*(24*h - 6*b*c*(g + 4*h*x)*Log[F] + 2*b^2*c^2*(f + 3*x*(g + 2*h*x))*Log[F]^2 - b^3*c^3*(e + x*(2*f + 3*g*x + 4*h*x^2))*Log[F]^3 + b^4*c^4*(d + x*(e + x*(f + x*(g + h*x))))*Log[F]^4)/(b^5*c^5*Log[F]^5)`

**Rubi [B] (verified)**

Leaf count is larger than twice the leaf count of optimal. 348 vs. 2(162) = 324.

Time = 0.96 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.15, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx$$

$$\downarrow 2626$$

$$\int \left( dF^{c(a+bx)} + exF^{c(a+bx)} + fx^2F^{c(a+bx)} + gx^3F^{c(a+bx)} + hx^4F^{c(a+bx)} \right) dx$$

$$\downarrow 2009$$



$$\frac{24hF^{c(a+bx)}}{b^5c^5\log^5(F)} - \frac{6gF^{c(a+bx)}}{b^4c^4\log^4(F)} - \frac{24hx^2F^{c(a+bx)}}{b^4c^4\log^4(F)} + \frac{2fF^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{6gx^2F^{c(a+bx)}}{b^3c^3\log^3(F)} + \frac{12hx^2F^{c(a+bx)}}{b^3c^3\log^3(F)} - \frac{eF^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{2fx^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{3gx^2F^{c(a+bx)}}{b^2c^2\log^2(F)} - \frac{4hx^3F^{c(a+bx)}}{b^2c^2\log^2(F)} + \frac{dF^{c(a+bx)}}{bc\log(F)} + \frac{exF^{c(a+bx)}}{bc\log(F)} + \frac{fx^2F^{c(a+bx)}}{bc\log(F)} + \frac{gx^3F^{c(a+bx)}}{bc\log(F)} + \frac{hx^4F^{c(a+bx)}}{bc\log(F)}$$

input `Int[F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3 + h*x^4),x]`

output `(24*F^(c*(a + b*x))*h)/(b^5*c^5*Log[F]^5) - (6*F^(c*(a + b*x))*g)/(b^4*c^4*Log[F]^4) - (24*F^(c*(a + b*x))*h*x)/(b^4*c^4*Log[F]^4) + (2*f*F^(c*(a + b*x)))/(b^3*c^3*Log[F]^3) + (6*F^(c*(a + b*x))*g*x)/(b^3*c^3*Log[F]^3) + (12*F^(c*(a + b*x))*h*x^2)/(b^3*c^3*Log[F]^3) - (e*F^(c*(a + b*x)))/(b^2*c^2*Log[F]^2) - (2*f*F^(c*(a + b*x))*x)/(b^2*c^2*Log[F]^2) - (3*F^(c*(a + b*x))*g*x^2)/(b^2*c^2*Log[F]^2) - (4*F^(c*(a + b*x))*h*x^3)/(b^2*c^2*Log[F]^2) + (d*F^(c*(a + b*x)))/(b*c*Log[F]) + (e*F^(c*(a + b*x))*x)/(b*c*Log[F]) + (f*F^(c*(a + b*x))*x^2)/(b*c*Log[F]) + (F^(c*(a + b*x))*g*x^3)/(b*c*Log[F]) + (F^(c*(a + b*x))*h*x^4)/(b*c*Log[F])`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(P_x_), x_Symbol] := Int[ExpandIntegrand[F^v, P_x, x], x] /; FreeQ[F, x] && PolynomialQ[P_x, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.31

method	result
gospers	$\frac{(h x^4 \ln(F)^4 b^4 c^4 + \ln(F)^4 b^4 c^4 g x^3 + \ln(F)^4 b^4 c^4 f x^2 + \ln(F)^4 b^4 c^4 e x + \ln(F)^4 b^4 c^4 d - 4 \ln(F)^3 b^3 c^3 h x^3 - 3 g x^2 \ln(F)^3 b^3 c^3 - 2 \ln(F)^3 b^3 c^3 e x - \ln(F)^3 b^3 c^3 d) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5}$
risch	$\frac{(h x^4 \ln(F)^4 b^4 c^4 + \ln(F)^4 b^4 c^4 g x^3 + \ln(F)^4 b^4 c^4 f x^2 + \ln(F)^4 b^4 c^4 e x + \ln(F)^4 b^4 c^4 d - 4 \ln(F)^3 b^3 c^3 h x^3 - 3 g x^2 \ln(F)^3 b^3 c^3 - 2 \ln(F)^3 b^3 c^3 e x - \ln(F)^3 b^3 c^3 d) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5}$
orering	$\frac{(h x^4 \ln(F)^4 b^4 c^4 + \ln(F)^4 b^4 c^4 g x^3 + \ln(F)^4 b^4 c^4 f x^2 + \ln(F)^4 b^4 c^4 e x + \ln(F)^4 b^4 c^4 d - 4 \ln(F)^3 b^3 c^3 h x^3 - 3 g x^2 \ln(F)^3 b^3 c^3 - 2 \ln(F)^3 b^3 c^3 e x - \ln(F)^3 b^3 c^3 d) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5}$
norman	$\frac{(\ln(F)^4 b^4 c^4 d - \ln(F)^3 b^3 c^3 e + 2 f \ln(F)^2 b^2 c^2 - 6 \ln(F) b c g + 24 h) e^{c(bx+a) \ln(F)}}{\ln(F)^5 b^5 c^5} + \frac{(\ln(F) b c g - 4 h) x^3 e^{c(bx+a) \ln(F)}}{\ln(F)^2 b^2 c^2} + \frac{(f \ln(F) - 2 h) x^2 e^{c(bx+a) \ln(F)}}{\ln(F) b c}$
meijerg	$- \frac{F^{ac} h \left( 24 - \frac{(5 b^4 c^4 x^4 \ln(F)^4 - 20 b^3 c^3 x^3 \ln(F)^3 + 60 b^2 c^2 x^2 \ln(F)^2 - 120 b c x \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5} + \frac{F^{ac} g \left( 6 - \frac{(-4 b^3 c^3 x^3 \ln(F)^3 + 12 b^2 c^2 x^2 \ln(F)^2 - 12 b c x \ln(F) + 120) e^{bcx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 c^5}$
parallelrisc	$\frac{x^4 F^{c(bx+a)} h \ln(F)^4 b^4 c^4 + \ln(F)^4 x^3 F^{c(bx+a)} b^4 c^4 g + \ln(F)^4 x^2 F^{c(bx+a)} b^4 c^4 f + \ln(F)^4 x F^{c(bx+a)} b^4 c^4 e + \ln(F)^4 F^{c(bx+a)} b^4 c^4 d}{\ln(F)^5 b^5 c^5}$

input

```
int(F^(c*(b*x+a))*(h*x^4+g*x^3+f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output

```
(h*x^4*ln(F)^4*b^4*c^4+ln(F)^4*b^4*c^4*g*x^3+ln(F)^4*b^4*c^4*f*x^2+ln(F)^4*b^4*c^4*e*x+ln(F)^4*b^4*c^4*d-4*ln(F)^3*b^3*c^3*h*x^3-3*g*x^2*ln(F)^3*b^3*c^3-2*ln(F)^3*b^3*c^3*f*x-ln(F)^3*b^3*c^3*e+12*ln(F)^2*b^2*c^2*h*x^2+6*ln(F)^2*b^2*c^2*g*x+2*f*ln(F)^2*b^2*c^2-24*ln(F)*b*c*h*x-6*ln(F)*b*c*g+24*h)*F^(c*(b*x+a))/ln(F)^5/b^5/c^5
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

$$\int F^{c(a+bx)} (d + ex + fx^2 + gx^3 + hx^4) dx$$

$$= \frac{((b^4 c^4 h x^4 + b^4 c^4 g x^3 + b^4 c^4 f x^2 + b^4 c^4 e x + b^4 c^4 d) \log(F)^4 - (4 b^3 c^3 h x^3 + 3 b^3 c^3 g x^2 + 2 b^3 c^3 f x + b^3 c^3 e) \log(F)^3 + (6 b^2 c^2 h x^2 + 5 b^2 c^2 g x + 4 b^2 c^2 f) \log(F)^2 + (4 b c h x + 3 b c g + 2 b c f) \log(F) + b c h) F^{c(a+bx)}}{b^5 c^5 \log(F)^5}$$

input

```
integrate(F^((b*x+a)*c)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="fricas")
```

output

```
((b^4*c^4*h*x^4 + b^4*c^4*g*x^3 + b^4*c^4*f*x^2 + b^4*c^4*e*x + b^4*c^4*d)
*log(F)^4 - (4*b^3*c^3*h*x^3 + 3*b^3*c^3*g*x^2 + 2*b^3*c^3*f*x + b^3*c^3*e
)*log(F)^3 + 2*(6*b^2*c^2*h*x^2 + 3*b^2*c^2*g*x + b^2*c^2*f)*log(F)^2 - 6*
(4*b*c*h*x + b*c*g)*log(F) + 24*h)*F^(b*c*x + a*c)/(b^5*c^5*log(F)^5)
```

**Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.75

$$\int F^{c(a+bx)} (d + ex + fx^2 + gx^3 + hx^4) dx$$

$$= \left\{ \frac{F^{c(a+bx)} (b^4 c^4 d \log(F)^4 + b^4 c^4 e x \log(F)^4 + b^4 c^4 f x^2 \log(F)^4 + b^4 c^4 g x^3 \log(F)^4 + b^4 c^4 h x^4 \log(F)^4 - b^3 c^3 e \log(F)^3 - 2b^3 c^3 f x \log(F)^3 - 3b^3 c^3 g x^2 \log(F)^3 - 4b^3 c^3 h x^3 \log(F)^3 - 2b^2 c^2 f \log(F)^2 + 6b^2 c^2 g x \log(F)^2 + 12b^2 c^2 h x^2 \log(F)^2 - 6b^2 c^2 e \log(F) - 24b^2 c^2 f x \log(F) + 24b^2 c^2 g x^2 \log(F) + 24b^2 c^2 h x^3 \log(F) - 24b^2 c^2 e \log(F) + 24b^2 c^2 f x \log(F) + 24b^2 c^2 g x^2 \log(F) + 24b^2 c^2 h x^3 \log(F))}{b^5 c^5 \log(F)^5} \right.$$

$$\left. dx + \frac{ex^2}{2} + \frac{fx^3}{3} + \frac{gx^4}{4} + \frac{hx^5}{5} \right.$$

input

```
integrate(F**(b*x+a)*c)*(h*x**4+g*x**3+f*x**2+e*x+d), x)
```

output

```
Piecewise((F**(c*(a + b*x))*(b**4*c**4*d*log(F)**4 + b**4*c**4*e*x*log(F)*
**4 + b**4*c**4*f*x**2*log(F)**4 + b**4*c**4*g*x**3*log(F)**4 + b**4*c**4*h
*x**4*log(F)**4 - b**3*c**3*e*log(F)**3 - 2*b**3*c**3*f*x*log(F)**3 - 3*b
**3*c**3*g*x**2*log(F)**3 - 4*b**3*c**3*h*x**3*log(F)**3 + 2*b**2*c**2*f*lo
g(F)**2 + 6*b**2*c**2*g*x*log(F)**2 + 12*b**2*c**2*h*x**2*log(F)**2 - 6*b
*c*g*log(F) - 24*b*c*h*x*log(F) + 24*h)/(b**5*c**5*log(F)**5), Ne(b**5*c**5
*log(F)**5, 0)), (d*x + e*x**2/2 + f*x**3/3 + g*x**4/4 + h*x**5/5, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.80

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3+hx^4) dx = \frac{F^{bcx+ac}d}{bc \log(F)} + \frac{(F^{ac}bcx \log(F) - F^{ac})F^{bcx}e}{b^2c^2 \log(F)^2}$$

$$+ \frac{(F^{ac}b^2c^2x^2 \log(F)^2 - 2F^{ac}bcx \log(F) + 2F^{ac})F^{bcx}f}{b^3c^3 \log(F)^3}$$

$$+ \frac{(F^{ac}b^3c^3x^3 \log(F)^3 - 3F^{ac}b^2c^2x^2 \log(F)^2 + 6F^{ac}bcx \log(F) - 6F^{ac})F^{bcx}g}{b^4c^4 \log(F)^4}$$

$$+ \frac{(F^{ac}b^4c^4x^4 \log(F)^4 - 4F^{ac}b^3c^3x^3 \log(F)^3 + 12F^{ac}b^2c^2x^2 \log(F)^2 - 24F^{ac}bcx \log(F) + 24F^{ac})F^{bcx}h}{b^5c^5 \log(F)^5}$$

input

```
integrate(F^((b*x+a)*c)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="maxima")
```

output

```
F^(b*c*x + a*c)*d/(b*c*log(F)) + (F^(a*c)*b*c*x*log(F) - F^(a*c))*F^(b*c*x)
*e/(b^2*c^2*log(F)^2) + (F^(a*c)*b^2*c^2*x^2*log(F)^2 - 2*F^(a*c)*b*c*x*log(F)
+ 2*F^(a*c))*F^(b*c*x)*f/(b^3*c^3*log(F)^3) + (F^(a*c)*b^3*c^3*x^3*log(F)^3
- 3*F^(a*c)*b^2*c^2*x^2*log(F)^2 + 6*F^(a*c)*b*c*x*log(F) - 6*F^(a*c))
*F^(b*c*x)*g/(b^4*c^4*log(F)^4) + (F^(a*c)*b^4*c^4*x^4*log(F)^4 - 4*F^(a*c)
*b^3*c^3*x^3*log(F)^3 + 12*F^(a*c)*b^2*c^2*x^2*log(F)^2 - 24*F^(a*c)*
b*c*x*log(F) + 24*F^(a*c))*F^(b*c*x)*h/(b^5*c^5*log(F)^5)
```

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 7630, normalized size of antiderivative = 47.10

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3+hx^4) dx = \text{Too large to display}$$

input

```
integrate(F^((b*x+a)*c)*(h*x^4+g*x^3+f*x^2+e*x+d),x, algorithm="giac")
```

output

```

-(((4*pi^3*b^4*c^4*h*x^4*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*h*x^4*log(abs(F))
))^3*sgn(F) - 4*pi^3*b^4*c^4*h*x^4*log(abs(F)) + 4*pi*b^4*c^4*h*x^4*log(ab
s(F))^3 + 4*pi^3*b^4*c^4*g*x^3*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*g*x^3*log
(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*g*x^3*log(abs(F)) + 4*pi*b^4*c^4*g*x^3*
log(abs(F))^3 + 4*pi^3*b^4*c^4*f*x^2*log(abs(F))*sgn(F) - 4*pi*b^4*c^4*f*x
^2*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*f*x^2*log(abs(F)) + 4*pi*b^4*c^4*
f*x^2*log(abs(F))^3 + 4*pi^3*b^4*c^4*e*x*log(abs(F))*sgn(F) - 4*pi*b^4*c^4
*e*x*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*c^4*e*x*log(abs(F)) + 4*pi*b^4*c^4*
e*x*log(abs(F))^3 - 4*pi^3*b^3*c^3*h*x^3*sgn(F) + 4*pi^3*b^4*c^4*d*log(abs
(F))*sgn(F) + 12*pi*b^3*c^3*h*x^3*log(abs(F))^2*sgn(F) - 4*pi*b^4*c^4*d*lo
g(abs(F))^3*sgn(F) + 4*pi^3*b^3*c^3*h*x^3 - 4*pi^3*b^4*c^4*d*log(abs(F)) -
12*pi*b^3*c^3*h*x^3*log(abs(F))^2 + 4*pi*b^4*c^4*d*log(abs(F))^3 - 3*pi^3
*b^3*c^3*g*x^2*sgn(F) + 9*pi*b^3*c^3*g*x^2*log(abs(F))^2*sgn(F) + 3*pi^3*b
^3*c^3*g*x^2 - 9*pi*b^3*c^3*g*x^2*log(abs(F))^2 - 2*pi^3*b^3*c^3*f*x*sgn(F)
) + 6*pi*b^3*c^3*f*x*log(abs(F))^2*sgn(F) + 2*pi^3*b^3*c^3*f*x - 6*pi*b^3*
c^3*f*x*log(abs(F))^2 - pi^3*b^3*c^3*e*sgn(F) + 3*pi*b^3*c^3*e*log(abs(F))
^2*sgn(F) + pi^3*b^3*c^3*e - 3*pi*b^3*c^3*e*log(abs(F))^2 - 24*pi*b^2*c^2*
h*x^2*log(abs(F))*sgn(F) + 24*pi*b^2*c^2*h*x^2*log(abs(F)) - 12*pi*b^2*c^2
*g*x*log(abs(F))*sgn(F) + 12*pi*b^2*c^2*g*x*log(abs(F)) - 4*pi*b^2*c^2*f*l
og(abs(F))*sgn(F) + 4*pi*b^2*c^2*f*log(abs(F)) + 24*pi*b*c*h*x*sgn(F) - ...

```

### Mupad [B] (verification not implemented)

Time = 22.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.31

$$\int F^{c(a+bx)}(d+ex+fx^2+gx^3+hx^4) dx$$

$$= \frac{F^{ac+bcx} (hb^4c^4x^4 \ln(F)^4 + gb^4c^4x^3 \ln(F)^4 + fb^4c^4x^2 \ln(F)^4 + eb^4c^4x \ln(F)^4 + db^4c^4 \ln(F)^4 - 4hb^4c^4x^4 \ln(F)^3 + 4gb^4c^4x^3 \ln(F)^3 + 4fb^4c^4x^2 \ln(F)^3 + 4eb^4c^4x \ln(F)^3 + 4db^4c^4 \ln(F)^3 - 12hb^4c^4x^4 \ln(F)^2 + 12gb^4c^4x^3 \ln(F)^2 + 12fb^4c^4x^2 \ln(F)^2 + 12eb^4c^4x \ln(F)^2 + 12db^4c^4 \ln(F)^2 - 24hb^4c^4x^4 \ln(F) + 24gb^4c^4x^3 \ln(F) + 24fb^4c^4x^2 \ln(F) + 24eb^4c^4x \ln(F) + 24db^4c^4 \ln(F) - 24hb^4c^4x^4 + 24gb^4c^4x^3 + 24fb^4c^4x^2 + 24eb^4c^4x + 24db^4c^4)}{b^5c^5 \ln(F)^5}$$

input

```
int(F^(c*(a + b*x))*(d + e*x + f*x^2 + g*x^3 + h*x^4),x)
```

output

```

(F^(a*c + b*c*x)*(24*h - 6*b*c*g*log(F) + b^4*c^4*d*log(F)^4 - b^3*c^3*e*l
og(F)^3 + 2*b^2*c^2*f*log(F)^2 + b^4*c^4*f*x^2*log(F)^4 - 3*b^3*c^3*g*x^2*
log(F)^3 + b^4*c^4*g*x^3*log(F)^4 + 12*b^2*c^2*h*x^2*log(F)^2 - 4*b^3*c^3*
h*x^3*log(F)^3 + b^4*c^4*h*x^4*log(F)^4 - 24*b*c*h*x*log(F) + b^4*c^4*e*x*
log(F)^4 - 2*b^3*c^3*f*x*log(F)^3 + 6*b^2*c^2*g*x*log(F)^2))/(b^5*c^5*log(
F)^5)

```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.31

$$\int F^{c(a+bx)}(d + ex + fx^2 + gx^3 + hx^4) dx$$

$$= \frac{f^{bcx+ac}(\log(f)^4 b^4 c^4 d + \log(f)^4 b^4 c^4 ex + \log(f)^4 b^4 c^4 f x^2 + \log(f)^4 b^4 c^4 g x^3 + \log(f)^4 b^4 c^4 h x^4 - \log(f)^3}{}$$

input `int(F^((b*x+a)*c)*(h*x^4+g*x^3+f*x^2+e*x+d),x)`

output `(f**(a*c + b*c*x)*(log(f)**4*b**4*c**4*d + log(f)**4*b**4*c**4*e*x + log(f)**4*b**4*c**4*f*x**2 + log(f)**4*b**4*c**4*g*x**3 + log(f)**4*b**4*c**4*h*x**4 - log(f)**3*b**3*c**3*e - 2*log(f)**3*b**3*c**3*f*x - 3*log(f)**3*b**3*c**3*g*x**2 - 4*log(f)**3*b**3*c**3*h*x**3 + 2*log(f)**2*b**2*c**2*f + 6*log(f)**2*b**2*c**2*g*x + 12*log(f)**2*b**2*c**2*h*x**2 - 6*log(f)*b*c*g - 24*log(f)*b*c*h*x + 24*h))/(log(f)**5*b**5*c**5)`

### 3.94 $\int e^{-a-bx} x^m (a + bx)^3 dx$

Optimal result	630
Mathematica [A] (verified)	630
Rubi [A] (verified)	631
Maple [C] (verified)	632
Fricas [A] (verification not implemented)	633
Sympy [A] (verification not implemented)	633
Maxima [A] (verification not implemented)	634
Giac [F]	634
Mupad [F(-1)]	635
Reduce [F]	635

#### Optimal result

Integrand size = 21, antiderivative size = 116

$$\int e^{-a-bx} x^m (a + bx)^3 dx = -\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(1 + m, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(2 + m, bx)}{b} - \frac{3a e^{-a} x^m (bx)^{-m} \Gamma(3 + m, bx)}{b} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(4 + m, bx)}{b}$$

output

```
-a^3*x^m*GAMMA(1+m,b*x)/b/exp(a)/((b*x)^m)-3*a^2*x^m*GAMMA(2+m,b*x)/b/exp(a)/((b*x)^m)-3*a*x^m*GAMMA(3+m,b*x)/b/exp(a)/((b*x)^m)-x^m*GAMMA(4+m,b*x)/b/exp(a)/((b*x)^m)
```

#### Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int e^{-a-bx} x^m (a + bx)^3 dx = -\frac{e^{-a} x^m (bx)^{-m} (a^3 \Gamma(1 + m, bx) + 3a^2 \Gamma(2 + m, bx) + 3a \Gamma(3 + m, bx) + \Gamma(4 + m, bx))}{b}$$

input

```
Integrate[E^(-a - b*x)*x^m*(a + b*x)^3,x]
```

output

$$-\left(\frac{x^m(a^3\Gamma[1+m, bx] + 3a^2\Gamma[2+m, bx] + 3a\Gamma[3+m, bx] + \Gamma[4+m, bx])}{(bE^a(bx))^m}\right)$$

**Rubi [A] (verified)**

Time = 0.60 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m e^{-a-bx} (a+bx)^3 dx$$

$$\downarrow 2629$$

$$\int \left( a^3 x^m e^{-a-bx} + 3a^2 b x^{m+1} e^{-a-bx} + b^3 x^{m+3} e^{-a-bx} + 3ab^2 x^{m+2} e^{-a-bx} \right) dx$$

$$\downarrow 2009$$

$$\frac{a^3 e^{-a} x^m (bx)^{-m} \Gamma(m+1, bx)}{b} - \frac{3a^2 e^{-a} x^m (bx)^{-m} \Gamma(m+2, bx)}{b} - \frac{3a e^{-a} x^m (bx)^{-m} \Gamma(m+3, bx)}{b} - \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+4, bx)}{b}$$

input

$$\text{Int}[E^{-a - bx} * x^m * (a + bx)^3, x]$$

output

$$-\left(\frac{a^3 x^m \Gamma[1+m, bx]}{(bE^a(bx))^m}\right) - \frac{3a^2 x^m \Gamma[2+m, bx]}{(bE^a(bx))^m} - \frac{3a x^m \Gamma[3+m, bx]}{(bE^a(bx))^m} - \frac{x^m \Gamma[4+m, bx]}{(bE^a(bx))^m}$$



## Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.88

method	result
meijerg	$b^{-1-m}e^{-a} \left( x^m b^m (m^2 + 5m + 6) (bx)^{-\frac{m}{2}} e^{-\frac{bx}{2}} \text{WhittakerM} \left( \frac{m}{2}, \frac{m}{2} + \frac{1}{2}, bx \right) - x^m b^m (b^2 x^2 + bmx) \right)$

input `int(exp(-b*x-a)*x^m*(b*x+a)^3,x,method=_RETURNVERBOSE)`

output `b^(-1-m)*exp(-a)*(x^m*b^m*(m^2+5*m+6)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m,1/2*m+1/2,b*x)-x^m*b^m*(b^2*x^2+b*m*x+3*b*x+m^2+5*m+6)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m+1,1/2*m+1/2,b*x))+3*b^(-1-m)*exp(-a)*a*(x^m*b^m*(2+m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m,1/2*m+1/2,b*x)-x^m*b^m*(b*x+m+2)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m+1,1/2*m+1/2,b*x))+3*b^(-1-m)*exp(-a)*a^2*(x^m*b^m*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m,1/2*m+1/2,b*x)+1/(2+m)*x^m*b^m*(-2-m)*(b*x)^(-1/2*m)*exp(-1/2*b*x)*WhittakerM(1/2*m+1,1/2*m+1/2,b*x))+exp(-a-1/2*b*x)/b*a^3/(1+m)*x^m*(b*x)^(-1/2*m)*WhittakerM(1/2*m,1/2*m+1/2,b*x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09

$$\int e^{-a-bx} x^m (a+bx)^3 dx = \frac{(b^3 x^3 + (3(a+1)b^2 + b^2 m)x^2 + ((3a+5)bm + bm^2 + 3(a^2 + 2a + 2)b)x)x^m e^{(-bx-a)} + (a^3 + 3(a + b^3 x^3 + (3(a+1)b^2 + b^2 m)x^2 + ((3a+5)bm + bm^2 + 3(a^2 + 2a + 2)b)x)x^m e^{(-bx-a))}}{b}$$

input `integrate(exp(-b*x-a)*x^m*(b*x+a)^3,x, algorithm="fricas")`

output `-((b^3*x^3 + (3*(a + 1)*b^2 + b^2*m)*x^2 + ((3*a + 5)*b*m + b*m^2 + 3*(a^2 + 2*a + 2)*b)*x)*x^m*e^(-b*x - a) + (a^3 + 3*(a + 2)*m^2 + m^3 + 3*a^2 + (3*a^2 + 9*a + 11)*m + 6*a + 6)*e^(-m*log(b) - a)*gamma(m + 1, b*x))/b`

**Sympy [A] (verification not implemented)**

Time = 9.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

$$\int e^{-a-bx} x^m (a+bx)^3 dx = \left( -\frac{a^3 x^m (bx)^{-m} \Gamma(m+1, bx)}{b} - 3a^2 x^{m+1} (bx)^{-m-1} \Gamma(m+2, bx) - 3abx^{m+2} (bx)^{-m-2} \Gamma(m+3, bx) - b^2 x^{m+3} (bx)^{-m-3} \Gamma(m+4, bx) \right) e^{-a}$$

input `integrate(exp(-b*x-a)*x**m*(b*x+a)**3,x)`

output `(-a**3*x**m*uppergamma(m + 1, b*x)/(b*(b*x)**m) - 3*a**2*x**(m + 1)*(b*x)*(-m - 1)*uppergamma(m + 2, b*x) - 3*a*b*x**(m + 2)*(b*x)**(-m - 2)*uppergamma(m + 3, b*x) - b**2*x**(m + 3)*(b*x)**(-m - 3)*uppergamma(m + 4, b*x))*exp(-a)`

**Maxima [A] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.06

$$\int e^{-a-bx} x^m (a+bx)^3 dx = -(bx)^{-m-4} b^3 x^{m+4} e^{(-a)} \Gamma(m+4, bx) \\ - 3 (bx)^{-m-3} a b^2 x^{m+3} e^{(-a)} \Gamma(m+3, bx) \\ - 3 (bx)^{-m-2} a^2 b x^{m+2} e^{(-a)} \Gamma(m+2, bx) \\ - (bx)^{-m-1} a^3 x^{m+1} e^{(-a)} \Gamma(m+1, bx)$$

input `integrate(exp(-b*x-a)*x^m*(b*x+a)^3,x, algorithm="maxima")`

output `-(b*x)^(-m - 4)*b^3*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 3*(b*x)^(-m - 3)*  
a*b^2*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 3*(b*x)^(-m - 2)*a^2*b*x^(m + 2)  
) *e^(-a)*gamma(m + 2, b*x) - (b*x)^(-m - 1)*a^3*x^(m + 1)*e^(-a)*gamma(m +  
1, b*x)`

**Giac [F]**

$$\int e^{-a-bx} x^m (a+bx)^3 dx = \int (bx+a)^3 x^m e^{(-bx-a)} dx$$

input `integrate(exp(-b*x-a)*x^m*(b*x+a)^3,x, algorithm="giac")`

output `integrate((b*x + a)^3*x^m*e^(-b*x - a), x)`

**Mupad [F(-1)]**

Timed out.

$$\int e^{-a-bx} x^m (a+bx)^3 dx = \int x^m e^{-a-bx} (a+bx)^3 dx$$

input `int(x^m*exp(- a - b*x)*(a + b*x)^3,x)`output `int(x^m*exp(- a - b*x)*(a + b*x)^3, x)`**Reduce [F]**

$$\int e^{-a-bx} x^m (a+bx)^3 dx$$

$$= \frac{-x^m b^3 x^3 - x^m b^2 m x^2 - x^m b m^2 x - 3x^m a^2 b x - 3x^m a b^2 x^2 - 6x^m a b x - 5x^m b m x + 3e^{bx} \left( \int \frac{x^m}{e^{bx}} dx \right) a^2 m^2}{1}$$

input `int(exp(-b*x-a)*x^m*(b*x+a)^3,x)`output `(e**(b*x)*int(x**m/(e**(b*x)*x),x)*a**3*m + 3*e**(b*x)*int(x**m/(e**(b*x)*x),x)*a**2*m**2 + 3*e**(b*x)*int(x**m/(e**(b*x)*x),x)*a**2*m + 3*e**(b*x)*int(x**m/(e**(b*x)*x),x)*a*m**3 + 9*e**(b*x)*int(x**m/(e**(b*x)*x),x)*a*m**2 + 6*e**(b*x)*int(x**m/(e**(b*x)*x),x)*a*m + e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**4 + 6*e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**3 + 11*e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**2 + 6*e**(b*x)*int(x**m/(e**(b*x)*x),x)*m - x**m*a**3 - 3*x**m*a**2*b*x - 3*x**m*a**2*m - 3*x**m*a**2 - 3*x**m*a*b**2*x**2 - 3*x**m*a*b*m*x - 6*x**m*a*b*x - 3*x**m*a*m**2 - 9*x**m*a*m - 6*x**m*a - x**m*b**3*x**3 - x**m*b**2*m*x**2 - 3*x**m*b**2*x**2 - x**m*b*m**2*x - 5*x**m*b*m*x - 6*x**m*b*x - x**m*m**3 - 6*x**m*m**2 - 11*x**m*m - 6*x**m)/(e**(a + b*x)*b)`

### 3.95 $\int e^{-a-bx} x^3 (a + bx)^3 dx$

Optimal result . . . . .	636
Mathematica [A] (verified) . . . . .	637
Rubi [A] (verified) . . . . .	637
Maple [A] (verified) . . . . .	638
Fricas [A] (verification not implemented) . . . . .	639
Sympy [A] (verification not implemented) . . . . .	640
Maxima [A] (verification not implemented) . . . . .	640
Giac [A] (verification not implemented) . . . . .	641
Mupad [B] (verification not implemented) . . . . .	642
Reduce [B] (verification not implemented) . . . . .	642

#### Optimal result

Integrand size = 21, antiderivative size = 227

$$\int e^{-a-bx} x^3 (a + bx)^3 dx = -\frac{720e^{-a-bx}}{b^4} - \frac{e^{-a-bx} x^3 (a + bx)^3}{b} - \frac{360e^{-a-bx} (a + 2bx)}{b^4} - \frac{72e^{-a-bx} (a^2 + 5abx + 5b^2 x^2)}{b^4} - \frac{3e^{-a-bx} x^2 (a^3 + 4a^2 bx + 5ab^2 x^2 + 2b^3 x^3)}{b^2} - \frac{6e^{-a-bx} x (a^3 + 6a^2 bx + 10ab^2 x^2 + 5b^3 x^3)}{b^3} - \frac{6e^{-a-bx} (a^3 + 12a^2 bx + 30ab^2 x^2 + 20b^3 x^3)}{b^4}$$

```
output -720*exp(-b*x-a)/b^4-exp(-b*x-a)*x^3*(b*x+a)^3/b-360*exp(-b*x-a)*(2*b*x+a)
/b^4-72*exp(-b*x-a)*(5*b^2*x^2+5*a*b*x+a^2)/b^4-3*exp(-b*x-a)*x^2*(2*b^3*x
^3+5*a*b^2*x^2+4*a^2*b*x+a^3)/b^2-6*exp(-b*x-a)*x*(5*b^3*x^3+10*a*b^2*x^2+
6*a^2*b*x+a^3)/b^3-6*exp(-b*x-a)*(20*b^3*x^3+30*a*b^2*x^2+12*a^2*b*x+a^3)/
b^4
```

**Mathematica [A] (verified)**

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.53

$$\int e^{-a-bx} x^3 (a+bx)^3 dx = e^{-a-bx} \left( -\frac{6(120+60a+12a^2+a^3)}{b^4} - \frac{6(120+60a+12a^2+a^3)x}{b^3} - \frac{3(120+60a+12a^2+a^3)x^2}{b^2} - \frac{(120+60a+12a^2+a^3)x^3}{b} - 3(10+5a+a^2)x^4 - 3(2+a)bx^5 - b^2x^6 \right)$$

input `Integrate[E^(-a - b*x)*x^3*(a + b*x)^3,x]`

output `E^(-a - b*x)*((-6*(120 + 60*a + 12*a^2 + a^3))/b^4 - (6*(120 + 60*a + 12*a^2 + a^3)*x)/b^3 - (3*(120 + 60*a + 12*a^2 + a^3)*x^2)/b^2 - ((120 + 60*a + 12*a^2 + a^3)*x^3)/b - 3*(10 + 5*a + a^2)*x^4 - 3*(2 + a)*b*x^5 - b^2*x^6)`

**Rubi [A] (verified)**

Time = 1.32 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.75, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{-a-bx} (a+bx)^3 dx$$

$$\downarrow 2626$$

$$\int \left( a^3 x^3 e^{-a-bx} + 3a^2 b x^4 e^{-a-bx} + b^3 x^6 e^{-a-bx} + 3ab^2 x^5 e^{-a-bx} \right) dx$$

$$\downarrow 2009$$

$$\begin{array}{cccccc}
-\frac{6a^3e^{-a-bx}}{b^4} & -\frac{6a^3xe^{-a-bx}}{b^3} & -\frac{3a^3x^2e^{-a-bx}}{b^2} & -\frac{a^3x^3e^{-a-bx}}{b} & -\frac{72a^2e^{-a-bx}}{b^4} & -\frac{72a^2xe^{-a-bx}}{b^3} \\
\frac{36a^2x^2e^{-a-bx}}{b^2} & -\frac{3a^2x^4e^{-a-bx}}{b^3} & -\frac{12a^2x^3e^{-a-bx}}{b} & -\frac{360ae^{-a-bx}}{b^4} & -\frac{720e^{-a-bx}}{b^4} & -\frac{360axe^{-a-bx}}{b^3} \\
\frac{720xe^{-a-bx}}{b^3} & -\frac{b^2x^6e^{-a-bx}}{b^3} & -\frac{180ax^2e^{-a-bx}}{b^2} & -\frac{360x^2e^{-a-bx}}{b^2} & -\frac{3abx^5e^{-a-bx}}{b^4} & -\frac{6bx^5e^{-a-bx}}{b^3} \\
& & 15ax^4e^{-a-bx} & -\frac{30x^4e^{-a-bx}}{b} & -\frac{60ax^3e^{-a-bx}}{b} & -\frac{120x^3e^{-a-bx}}{b}
\end{array}$$

input `Int[E^(-a - b*x)*x^3*(a + b*x)^3,x]`

output  $(-720 * E^{-a - b * x}) / b^4 - (360 * a * E^{-a - b * x}) / b^4 - (72 * a^2 * E^{-a - b * x}) / b^4 - (6 * a^3 * E^{-a - b * x}) / b^4 - (720 * E^{-a - b * x} * x) / b^3 - (360 * a * E^{-a - b * x} * x) / b^3 - (72 * a^2 * E^{-a - b * x} * x) / b^3 - (6 * a^3 * E^{-a - b * x} * x) / b^3 - (360 * E^{-a - b * x} * x^2) / b^2 - (180 * a * E^{-a - b * x} * x^2) / b^2 - (36 * a^2 * E^{-a - b * x} * x^2) / b^2 - (3 * a^3 * E^{-a - b * x} * x^2) / b^2 - (120 * E^{-a - b * x} * x^3) / b - (60 * a * E^{-a - b * x} * x^3) / b - (12 * a^2 * E^{-a - b * x} * x^3) / b - (a^3 * E^{-a - b * x} * x^3) / b - 30 * E^{-a - b * x} * x^4 - 15 * a * E^{-a - b * x} * x^4 - 3 * a^2 * E^{-a - b * x} * x^4 - 6 * b * E^{-a - b * x} * x^5 - 3 * a * b * E^{-a - b * x} * x^5 - b^2 * E^{-a - b * x} * x^6$

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.78

method	result
norman	$(-3ab - 6b)x^5e^{-bx-a} + (-3a^2 - 15a - 30)x^4e^{-bx-a} - b^2x^6e^{-bx-a} - \frac{6(a^3+12a^2+60a+120)}{b^4}$
gospers	$-\frac{(b^6x^6+3b^5x^5a+3a^2b^4x^4+6b^5x^5+a^3b^3x^3+15ab^4x^4+12a^2b^3x^3+30b^4x^4+3a^3b^2x^2+60ab^3x^3+36a^2b^2x^2+120b^3x^3)}{b^4}$
risch	$-\frac{(b^6x^6+3b^5x^5a+3a^2b^4x^4+6b^5x^5+a^3b^3x^3+15ab^4x^4+12a^2b^3x^3+30b^4x^4+3a^3b^2x^2+60ab^3x^3+36a^2b^2x^2+120b^3x^3)}{b^4}$
orering	$-\frac{(b^6x^6+3b^5x^5a+3a^2b^4x^4+6b^5x^5+a^3b^3x^3+15ab^4x^4+12a^2b^3x^3+30b^4x^4+3a^3b^2x^2+60ab^3x^3+36a^2b^2x^2+120b^3x^3)}{b^4}$
meijerg	$e^{-a} \left( \frac{720 - \frac{(7b^6x^6+42b^5x^5+210b^4x^4+840b^3x^3+2520b^2x^2+5040bx+5040)e^{-bx}}{7}}{b^4} \right) + \frac{3e^{-a} \left( 120 - \frac{(6b^5x^5+30b^4x^4+120b^3x^3)}{b^4} \right)}{b^4}$
parallelrisk	$-\frac{b^6e^{-bx-a}x^6+3b^5e^{-bx-a}x^5a+6b^5e^{-bx-a}x^5+3x^4e^{-bx-a}a^2b^4+15x^4e^{-bx-a}ab^4+x^3e^{-bx-a}a^3b^3+30b^4x^4e^{-bx-a}}{b^4}$
derivativdivides	$-\frac{e^{-bx-a}(-bx-a)^6-6(-bx-a)^5e^{-bx-a}+30(-bx-a)^4e^{-bx-a}-120e^{-bx-a}(-bx-a)^3+360(-bx-a)^2e^{-bx-a}-720e^{-bx-a}}{b^4}$
default	$-\frac{e^{-bx-a}(-bx-a)^6-6(-bx-a)^5e^{-bx-a}+30(-bx-a)^4e^{-bx-a}-120e^{-bx-a}(-bx-a)^3+360(-bx-a)^2e^{-bx-a}-720e^{-bx-a}}{b^4}$
parts	$-b^2x^6e^{-bx-a} - 3e^{-bx-a}x^5ba - 3e^{-bx-a}x^4a^2 - \frac{e^{-bx-a}x^3a^3}{b} - \frac{3 \left( -\frac{2((-bx-a)^5e^{-bx-a}-5(-bx-a)^4}{b^4} \right)}{b^4}$

input `int(exp(-b*x-a)*x^3*(b*x+a)^3,x,method=_RETURNVERBOSE)`

output `(-3*a*b-6*b)*x^5*exp(-b*x-a)+(-3*a^2-15*a-30)*x^4*exp(-b*x-a)-b^2*x^6*exp(-b*x-a)-6*(a^3+12*a^2+60*a+120)/b^4*exp(-b*x-a)-6*(a^3+12*a^2+60*a+120)/b^4*3*x*exp(-b*x-a)-3*(a^3+12*a^2+60*a+120)/b^2*x^2*exp(-b*x-a)-(a^3+12*a^2+60*a+120)/b*x^3*exp(-b*x-a)`

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.53

$$\int e^{-a-bx} x^3 (a+bx)^3 dx = -\frac{(b^6x^6+3(a+2)b^5x^5+3(a^2+5a+10)b^4x^4+(a^3+12a^2+60a+120)b^3x^3+3(a^3+12a^2+60a+120)b^2x^2+3(a^3+12a^2+60a+120)bx+3(a^3+12a^2+60a+120))e^{-bx-a}}{b^4}$$

input `integrate(exp(-b*x-a)*x^3*(b*x+a)^3,x, algorithm="fricas")`



output

$$-(b^6 x^6 + 3(a+2)b^5 x^5 + 3(a^2 + 5a + 10)b^4 x^4 + (a^3 + 12a^2 + 60a + 120)b^3 x^3 + 3(a^3 + 12a^2 + 60a + 120)b^2 x^2 + 6a^3 + 6(a^3 + 12a^2 + 60a + 120)bx + 72a^2 + 360a + 720)e^{(-bx-a)}/b^4$$

**Sympy [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\int e^{-a-bx} x^3 (a+bx)^3 dx$$

$$= \left\{ \frac{(-a^3 b^3 x^3 - 3a^3 b^2 x^2 - 6a^3 b x - 6a^3 - 3a^2 b^4 x^4 - 12a^2 b^3 x^3 - 36a^2 b^2 x^2 - 72a^2 b x - 72a^2 - 3ab^5 x^5 - 15ab^4 x^4 - 60ab^3 x^3 - 180ab^2 x^2 - 360abx - 360a - b^4)}{b^4} \right.$$

$$\left. \frac{a^3 x^4}{4} + \frac{3a^2 b x^5}{5} + \frac{ab^2 x^6}{2} + \frac{b^3 x^7}{7} \right.$$

input

```
integrate(exp(-b*x-a)*x**3*(b*x+a)**3,x)
```

output

```
Piecewise((((-a**3*b**3*x**3 - 3*a**3*b**2*x**2 - 6*a**3*b*x - 6*a**3 - 3*a**2*b**4*x**4 - 12*a**2*b**3*x**3 - 36*a**2*b**2*x**2 - 72*a**2*b*x - 72*a**2 - 3*a*b**5*x**5 - 15*a*b**4*x**4 - 60*a*b**3*x**3 - 180*a*b**2*x**2 - 360*a*b*x - 360*a - b**6*x**6 - 6*b**5*x**5 - 30*b**4*x**4 - 120*b**3*x**3 - 360*b**2*x**2 - 720*b*x - 720)*exp(-a - b*x)/b**4, Ne(b**4, 0)), (a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

$$\int e^{-a-bx} x^3 (a+bx)^3 dx$$

$$= -\frac{(b^3 x^3 + 3b^2 x^2 + 6bx + 6)a^3 e^{(-bx-a)}}{b^4}$$

$$- \frac{3(b^4 x^4 + 4b^3 x^3 + 12b^2 x^2 + 24bx + 24)a^2 e^{(-bx-a)}}{b^4}$$

$$- \frac{3(b^5 x^5 + 5b^4 x^4 + 20b^3 x^3 + 60b^2 x^2 + 120bx + 120)ae^{(-bx-a)}}{b^4}$$

$$- \frac{(b^6 x^6 + 6b^5 x^5 + 30b^4 x^4 + 120b^3 x^3 + 360b^2 x^2 + 720bx + 720)e^{(-bx-a)}}{b^4}$$

input `integrate(exp(-b*x-a)*x^3*(b*x+a)^3,x, algorithm="maxima")`

output 
$$-(b^3x^3 + 3b^2x^2 + 6bx + 6)a^3e^{-bx - a}/b^4 - 3(b^4x^4 + 4b^3x^3 + 12b^2x^2 + 24bx + 24)a^2e^{-bx - a}/b^4 - 3(b^5x^5 + 5b^4x^4 + 20b^3x^3 + 60b^2x^2 + 120bx + 120)a^1e^{-bx - a}/b^4 - (b^6x^6 + 6b^5x^5 + 30b^4x^4 + 120b^3x^3 + 360b^2x^2 + 720bx + 720)e^{-bx - a}/b^4$$

### Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.89

$$\int e^{-a-bx}x^3(a+bx)^3 dx = \frac{(b^9x^6 + 3ab^8x^5 + 3a^2b^7x^4 + 6b^8x^5 + a^3b^6x^3 + 15ab^7x^4 + 12a^2b^6x^3 + 30b^7x^4 + 3a^3b^5x^2 + 60ab^6x^3 + \dots)}{\dots}$$

input `integrate(exp(-b*x-a)*x^3*(b*x+a)^3,x, algorithm="giac")`

output 
$$-(b^9x^6 + 3a^3b^6x^3 + 15a^2b^7x^4 + 12a^2b^6x^3 + 30b^7x^4 + 3a^3b^5x^2 + 60a^2b^6x^3 + 36a^2b^5x^2 + 120b^6x^3 + 6a^3b^4x + 180a^2b^5x^2 + 72a^2b^4x + 360b^5x^2 + 6a^3b^3 + 360a^2b^4x + 72a^2b^3 + 720b^4x + 360a^2b^3 + 720b^3)e^{-bx - a}/b^7$$

**Mupad [B] (verification not implemented)**

Time = 22.45 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

$$\int e^{-a-bx} x^3 (a+bx)^3 dx = -x^4 e^{-a-bx} (3a^2 + 15a + 30) - b^2 x^6 e^{-a-bx} - \frac{6e^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b^4} - 3bx^5 e^{-a-bx} (a+2) - \frac{6xe^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b^3} - \frac{x^3 e^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b} - \frac{3x^2 e^{-a-bx} (a^3 + 12a^2 + 60a + 120)}{b^2}$$

input `int(x^3*exp(- a - b*x)*(a + b*x)^3,x)`output `- x^4*exp(- a - b*x)*(15*a + 3*a^2 + 30) - b^2*x^6*exp(- a - b*x) - (6*exp(- a - b*x)*(60*a + 12*a^2 + a^3 + 120))/b^4 - 3*b*x^5*exp(- a - b*x)*(a + 2) - (6*x*exp(- a - b*x)*(60*a + 12*a^2 + a^3 + 120))/b^3 - (x^3*exp(- a - b*x)*(60*a + 12*a^2 + a^3 + 120))/b - (3*x^2*exp(- a - b*x)*(60*a + 12*a^2 + a^3 + 120))/b^2`**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.80

$$\int e^{-a-bx} x^3 (a+bx)^3 dx = \frac{-b^6 x^6 - 3a b^5 x^5 - 3a^2 b^4 x^4 - 6b^5 x^5 - a^3 b^3 x^3 - 15a b^4 x^4 - 12a^2 b^3 x^3 - 30b^4 x^4 - 3a^3 b^2 x^2 - 60a b^3 x^3 - 30a^2 b^2 x^2 - 60a^2 b x^2 - 30a^2 x^2}{e^{bx}}$$

input `int(exp(-b*x-a)*x^3*(b*x+a)^3,x)`

output

```
( - a**3*b**3*x**3 - 3*a**3*b**2*x**2 - 6*a**3*b*x - 6*a**3 - 3*a**2*b**4*  
x**4 - 12*a**2*b**3*x**3 - 36*a**2*b**2*x**2 - 72*a**2*b*x - 72*a**2 - 3*a  
*b**5*x**5 - 15*a*b**4*x**4 - 60*a*b**3*x**3 - 180*a*b**2*x**2 - 360*a*b*x  
- 360*a - b**6*x**6 - 6*b**5*x**5 - 30*b**4*x**4 - 120*b**3*x**3 - 360*b*  
*2*x**2 - 720*b*x - 720)/(e**(a + b*x)*b**4)
```

### 3.96 $\int e^{-a-bx} x^2 (a + bx)^3 dx$

Optimal result	644
Mathematica [A] (verified)	645
Rubi [A] (verified)	645
Maple [A] (verified)	647
Fricas [A] (verification not implemented)	647
Sympy [A] (verification not implemented)	648
Maxima [A] (verification not implemented)	648
Giac [A] (verification not implemented)	649
Mupad [B] (verification not implemented)	649
Reduce [B] (verification not implemented)	650

#### Optimal result

Integrand size = 21, antiderivative size = 187

$$\int e^{-a-bx} x^2 (a + bx)^3 dx = -\frac{120e^{-a-bx}}{b^3} - \frac{e^{-a-bx} x^2 (a + bx)^3}{b} - \frac{24e^{-a-bx} (3a + 5bx)}{b^3} - \frac{6e^{-a-bx} (3a^2 + 12abx + 10b^2x^2)}{b^3} - \frac{e^{-a-bx} x (2a^3 + 9a^2bx + 12ab^2x^2 + 5b^3x^3)}{b^2} - \frac{2e^{-a-bx} (a^3 + 9a^2bx + 18ab^2x^2 + 10b^3x^3)}{b^3}$$

output

```
-120*exp(-b*x-a)/b^3-exp(-b*x-a)*x^2*(b*x+a)^3/b-24*exp(-b*x-a)*(5*b*x+3*a)/b^3-6*exp(-b*x-a)*(10*b^2*x^2+12*a*b*x+3*a^2)/b^3-exp(-b*x-a)*x*(5*b^3*x^3+12*a*b^2*x^2+9*a^2*b*x+2*a^3)/b^2-2*exp(-b*x-a)*(10*b^3*x^3+18*a*b^2*x^2+9*a^2*b*x+a^3)/b^3
```

**Mathematica [A] (verified)**

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int e^{-a-bx} x^2 (a+bx)^3 dx = e^{-bx} \left( -\frac{2(60+36a+9a^2+a^3)e^{-a}}{b^3} - \frac{2(60+36a+9a^2+a^3)e^{-a}x}{b^2} - \frac{(60+36a+9a^2+a^3)e^{-a}x^2}{b} - (20+12a+3a^2)e^{-a}x^3 - (5+3a)be^{-a}x^4 - b^2e^{-a}x^5 \right)$$

input `Integrate[E^(-a - b*x)*x^2*(a + b*x)^3,x]`

output  $((-2*(60 + 36*a + 9*a^2 + a^3))/(b^3*E^a) - (2*(60 + 36*a + 9*a^2 + a^3)*x)/(b^2*E^a) - ((60 + 36*a + 9*a^2 + a^3)*x^2)/(b*E^a) - ((20 + 12*a + 3*a^2)*x^3)/E^a - ((5 + 3*a)*b*x^4)/E^a - (b^2*x^5)/E^a)/E^{(b*x)}$

**Rubi [A] (verified)**

Time = 1.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.70, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{-a-bx} (a+bx)^3 dx$$

$$\downarrow 2626$$

$$\int (a^3 x^2 e^{-a-bx} + 3a^2 b x^3 e^{-a-bx} + b^3 x^5 e^{-a-bx} + 3ab^2 x^4 e^{-a-bx}) dx$$

$$\downarrow 2009$$

$$\begin{aligned}
& -\frac{2a^3e^{-a-bx}}{b^3} - \frac{2a^3xe^{-a-bx}}{b^2} - \frac{a^3x^2e^{-a-bx}}{b} - \frac{18a^2e^{-a-bx}}{b^3} - \frac{18a^2xe^{-a-bx}}{b^2} - 3a^2x^3e^{-a-bx} - \\
& \frac{9a^2x^2e^{-a-bx}}{b} - \frac{72ae^{-a-bx}}{b^3} - \frac{120e^{-a-bx}}{b^3} - b^2x^5e^{-a-bx} - \frac{72axe^{-a-bx}}{b^2} - \frac{120xe^{-a-bx}}{b^2} - \\
& 3abx^4e^{-a-bx} - 5bx^4e^{-a-bx} - 12ax^3e^{-a-bx} - 20x^3e^{-a-bx} - \frac{36ax^2e^{-a-bx}}{b} - \frac{60x^2e^{-a-bx}}{b}
\end{aligned}$$

input `Int[E^(-a - b*x)*x^2*(a + b*x)^3,x]`

output `(-120*E^(-a - b*x))/b^3 - (72*a*E^(-a - b*x))/b^3 - (18*a^2*E^(-a - b*x))/b^3 - (2*a^3*E^(-a - b*x))/b^3 - (120*E^(-a - b*x)*x)/b^2 - (72*a*E^(-a - b*x)*x)/b^2 - (18*a^2*E^(-a - b*x)*x)/b^2 - (2*a^3*E^(-a - b*x)*x)/b^2 - (60*E^(-a - b*x)*x^2)/b - (36*a*E^(-a - b*x)*x^2)/b - (9*a^2*E^(-a - b*x)*x^2)/b - (a^3*E^(-a - b*x)*x^2)/b - 20*E^(-a - b*x)*x^3 - 12*a*E^(-a - b*x)*x^3 - 3*a^2*E^(-a - b*x)*x^3 - 5*b*E^(-a - b*x)*x^4 - 3*a*b*E^(-a - b*x)*x^4 - b^2*E^(-a - b*x)*x^5`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

method	result
gospers	$-\frac{(b^5 x^5 + 3a b^4 x^4 + 3a^2 b^3 x^3 + 5b^4 x^4 + a^3 b^2 x^2 + 12a b^3 x^3 + 9a^2 b^2 x^2 + 20b^3 x^3 + 2a^3 b x + 36a b^2 x^2 + 18a^2 b x + 60b^2 x^2 + 2a^3 + b^3)}{b^3}$
risch	$-\frac{(b^5 x^5 + 3a b^4 x^4 + 3a^2 b^3 x^3 + 5b^4 x^4 + a^3 b^2 x^2 + 12a b^3 x^3 + 9a^2 b^2 x^2 + 20b^3 x^3 + 2a^3 b x + 36a b^2 x^2 + 18a^2 b x + 60b^2 x^2 + 2a^3 + b^3)}{b^3}$
orering	$-\frac{(b^5 x^5 + 3a b^4 x^4 + 3a^2 b^3 x^3 + 5b^4 x^4 + a^3 b^2 x^2 + 12a b^3 x^3 + 9a^2 b^2 x^2 + 20b^3 x^3 + 2a^3 b x + 36a b^2 x^2 + 18a^2 b x + 60b^2 x^2 + 2a^3 + b^3)}{b^3}$
norman	$(-3ab - 5b) x^4 e^{-bx-a} + (-3a^2 - 12a - 20) x^3 e^{-bx-a} - b^2 x^5 e^{-bx-a} - \frac{2(a^3 + 9a^2 + 36a + 60)e^{-bx-a}}{b^3}$
meijerg	$\frac{e^{-a} \left( 120 - \frac{(6b^5 x^5 + 30b^4 x^4 + 120b^3 x^3 + 360b^2 x^2 + 720bx + 720)e^{-bx}}{6} \right)}{b^3} + \frac{3e^{-a} \left( 24 - \frac{(5b^4 x^4 + 20b^3 x^3 + 60b^2 x^2 + 120bx + 120)e^{-bx}}{5} \right)}{b^3}$
derivativedivides	$\frac{(-bx-a)^5 e^{-bx-a} - 5(-bx-a)^4 e^{-bx-a} + 20e^{-bx-a}(-bx-a)^3 - 60(-bx-a)^2 e^{-bx-a} + 120(-bx-a)e^{-bx-a} - 120e^{-bx-a}}{b^3}$
default	$\frac{(-bx-a)^5 e^{-bx-a} - 5(-bx-a)^4 e^{-bx-a} + 20e^{-bx-a}(-bx-a)^3 - 60(-bx-a)^2 e^{-bx-a} + 120(-bx-a)e^{-bx-a} - 120e^{-bx-a}}{b^3}$
parallelrisch	$-\frac{b^5 e^{-bx-a} x^5 + 3x^4 e^{-bx-a} a b^4 + 5b^4 x^4 e^{-bx-a} + 3x^3 e^{-bx-a} a^2 b^3 + 12a b^3 x^3 e^{-bx-a} + x^2 e^{-bx-a} a^3 b^2 + 20e^{-bx-a} x^3 b^3}{b^3}$
parts	$-b^2 x^5 e^{-bx-a} - 3e^{-bx-a} x^4 b a - 3e^{-bx-a} x^3 a^2 - \frac{e^{-bx-a} x^2 a^3}{b} - \frac{5(-bx-a)^4 e^{-bx-a} - 20e^{-bx-a}(-bx-a)}{b^3}$

input `int (exp(-b*x-a)*x^2*(b*x+a)^3,x,method=_RETURNVERBOSE)`

output 
$$-(b^5 x^5 + 3a b^4 x^4 + 3a^2 b^3 x^3 + 5b^4 x^4 + a^3 b^2 x^2 + 12a b^3 x^3 + 9a^2 b^2 x^2 + 20b^3 x^3 + 2a^3 b x + 36a b^2 x^2 + 18a^2 b x + 60b^2 x^2 + 2a^3 + b^3) \exp(-bx-a) / b^3$$

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.55

$$\int e^{-a-bx} x^2 (a + bx)^3 dx = -\frac{(b^5 x^5 + (3a + 5)b^4 x^4 + (3a^2 + 12a + 20)b^3 x^3 + (a^3 + 9a^2 + 36a + 60)b^2 x^2 + 2a^3 + 2(a^3 + 9a^2 + 36a + 60)b x + 2a^3 + b^3)e^{-bx-a}}{b^3}$$

input `integrate(exp(-b*x-a)*x^2*(b*x+a)^3,x, algorithm="fricas")`



output

$$-(b^5 x^5 + (3a + 5)b^4 x^4 + (3a^2 + 12a + 20)b^3 x^3 + (a^3 + 9a^2 + 36a + 60)b^2 x^2 + 2a^3 + 2(a^3 + 9a^2 + 36a + 60)bx + 18a^2 + 72a + 120)e^{(-bx - a)}/b^3$$

**Sympy [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.05

$$\int e^{-a-bx} x^2 (a+bx)^3 dx = \left\{ \frac{(-a^3 b^2 x^2 - 2a^3 bx - 2a^3 - 3a^2 b^3 x^3 - 9a^2 b^2 x^2 - 18a^2 bx - 18a^2 - 3ab^4 x^4 - 12ab^3 x^3 - 36ab^2 x^2 - 72abx - 72a - b^5 x^5 - 5b^4 x^4 - 20b^3 x^3 - 60b^2 x^2 - 120bx - 120)}{b^3} \right\}$$

$$= \left\{ \frac{a^3 x^3}{3} + \frac{3a^2 bx^4}{4} + \frac{3ab^2 x^5}{5} + \frac{b^3 x^6}{6} \right\}$$

input

```
integrate(exp(-b*x-a)*x**2*(b*x+a)**3,x)
```

output

```
Piecewise((((-a**3*b**2*x**2 - 2*a**3*b*x - 2*a**3 - 3*a**2*b**3*x**3 - 9*a**2*b**2*x**2 - 18*a**2*b*x - 18*a**2 - 3*a*b**4*x**4 - 12*a*b**3*x**3 - 36*a*b**2*x**2 - 72*a*b*x - 72*a - b**5*x**5 - 5*b**4*x**4 - 20*b**3*x**3 - 60*b**2*x**2 - 120*b*x - 120)*exp(-a - b*x)/b**3, Ne(b**3, 0)), (a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.88

$$\int e^{-a-bx} x^2 (a+bx)^3 dx = -\frac{(b^2 x^2 + 2bx + 2)a^3 e^{(-bx-a)}}{b^3} - \frac{3(b^3 x^3 + 3b^2 x^2 + 6bx + 6)a^2 e^{(-bx-a)}}{b^3} - \frac{3(b^4 x^4 + 4b^3 x^3 + 12b^2 x^2 + 24bx + 24)ae^{(-bx-a)}}{b^3} - \frac{(b^5 x^5 + 5b^4 x^4 + 20b^3 x^3 + 60b^2 x^2 + 120bx + 120)e^{(-bx-a)}}{b^3}$$

input

```
integrate(exp(-b*x-a)*x^2*(b*x+a)^3,x, algorithm="maxima")
```

output

$$-(b^2x^2 + 2bx + 2)a^3e^{-bx - a}/b^3 - 3(b^3x^3 + 3b^2x^2 + 6bx + 6)a^2e^{-bx - a}/b^3 - 3(b^4x^4 + 4b^3x^3 + 12b^2x^2 + 24bx + 24)ae^{-bx - a}/b^3 - (b^5x^5 + 5b^4x^4 + 20b^3x^3 + 60b^2x^2 + 120bx + 120)e^{-bx - a}/b^3$$

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.87

$$\int e^{-a-bx}x^2(a+bx)^3 dx = \frac{(b^8x^5 + 3ab^7x^4 + 3a^2b^6x^3 + 5b^7x^4 + a^3b^5x^2 + 12ab^6x^3 + 9a^2b^5x^2 + 20b^6x^3 + 2a^3b^4x + 36ab^5x^2 + 18a^2b^4x + 60b^5x^2 + 2a^3b^3 + 72a^2b^3 + 120b^4x + 72a^2b^3 + 120b^3)e^{-bx - a}}{b^6}$$

input

```
integrate(exp(-b*x-a)*x^2*(b*x+a)^3,x, algorithm="giac")
```

output

$$-(b^8x^5 + 3ab^7x^4 + 3a^2b^6x^3 + 5b^7x^4 + a^3b^5x^2 + 12a^2b^6x^3 + 9a^2b^5x^2 + 20b^6x^3 + 2a^3b^4x + 36a^2b^5x^2 + 18a^2b^4x + 60b^5x^2 + 2a^3b^3 + 72a^2b^4x + 18a^2b^3 + 120b^4x + 72a^2b^3 + 120b^3)e^{-bx - a}/b^6$$

**Mupad [B] (verification not implemented)**

Time = 22.56 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.67

$$\int e^{-a-bx}x^2(a+bx)^3 dx = -x^3 e^{-a-bx} (3a^2 + 3abx + 12a + b^2x^2 + 5bx + 20) - \frac{2e^{-a-bx}(a^3 + 9a^2 + 36a + 60)}{b^3} - \frac{2xe^{-a-bx}(a^3 + 9a^2 + 36a + 60)}{b^2} - \frac{x^2e^{-a-bx}(a^3 + 9a^2 + 36a + 60)}{b}$$

input

```
int(x^2*exp(- a - b*x)*(a + b*x)^3,x)
```

output

```
- x^3*exp(- a - b*x)*(12*a + 5*b*x + 3*a^2 + b^2*x^2 + 3*a*b*x + 20) - (2*
exp(- a - b*x)*(36*a + 9*a^2 + a^3 + 60))/b^3 - (2*x*exp(- a - b*x)*(36*a
+ 9*a^2 + a^3 + 60))/b^2 - (x^2*exp(- a - b*x)*(36*a + 9*a^2 + a^3 + 60))/
b
```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.76

$$\int e^{-a-bx} x^2 (a+bx)^3 dx$$

$$= \frac{-b^5 x^5 - 3a b^4 x^4 - 3a^2 b^3 x^3 - 5b^4 x^4 - a^3 b^2 x^2 - 12a b^3 x^3 - 9a^2 b^2 x^2 - 20b^3 x^3 - 2a^3 b x - 36a b^2 x^2 - 18a^2 b^2 x^2}{e^{bx+ab^3}}$$

input

```
int(exp(-b*x-a)*x^2*(b*x+a)^3,x)
```

output

```
( - a**3*b**2*x**2 - 2*a**3*b*x - 2*a**3 - 3*a**2*b**3*x**3 - 9*a**2*b**2*
x**2 - 18*a**2*b*x - 18*a**2 - 3*a*b**4*x**4 - 12*a*b**3*x**3 - 36*a*b**2*
x**2 - 72*a*b*x - 72*a - b**5*x**5 - 5*b**4*x**4 - 20*b**3*x**3 - 60*b**2*
x**2 - 120*b*x - 120)/(e**(a + b*x)*b**3)
```

### 3.97 $\int e^{-a-bx} x(a+bx)^3 dx$

Optimal result	651
Mathematica [A] (verified)	651
Rubi [A] (verified)	652
Maple [A] (verified)	653
Fricas [A] (verification not implemented)	654
Sympy [A] (verification not implemented)	654
Maxima [A] (verification not implemented)	655
Giac [A] (verification not implemented)	655
Mupad [B] (verification not implemented)	656
Reduce [B] (verification not implemented)	656

#### Optimal result

Integrand size = 19, antiderivative size = 137

$$\int e^{-a-bx} x(a+bx)^3 dx = -\frac{24e^{-a-bx}}{b^2} - \frac{e^{-a-bx}x(a+bx)^3}{b} - \frac{6e^{-a-bx}(3a+4bx)}{b^2} - \frac{6e^{-a-bx}(a^2+3abx+2b^2x^2)}{b^2} - \frac{e^{-a-bx}(a^3+6a^2bx+9ab^2x^2+4b^3x^3)}{b^2}$$

output

```
-24*exp(-b*x-a)/b^2-exp(-b*x-a)*x*(b*x+a)^3/b-6*exp(-b*x-a)*(4*b*x+3*a)/b^2-6*exp(-b*x-a)*(2*b^2*x^2+3*a*b*x+a^2)/b^2-exp(-b*x-a)*(4*b^3*x^3+9*a*b^2*x^2+6*a^2*b*x+a^3)/b^2
```

#### Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int e^{-a-bx} x(a+bx)^3 dx = \frac{e^{-a-bx}(-24 - 24bx - 12b^2x^2 - 4b^3x^3 - b^4x^4 - a^3(1+bx) - 3a^2(2+2bx+b^2x^2) - 3a(6+6bx+3b^2x^2))}{b^2}$$

input

```
Integrate[E^(-a - b*x)*x*(a + b*x)^3,x]
```

output

$$\frac{(E^{-a - bx}) * (-24 - 24 * b * x - 12 * b^2 * x^2 - 4 * b^3 * x^3 - b^4 * x^4 - a^3 * (1 + b * x) - 3 * a^2 * (2 + 2 * b * x + b^2 * x^2) - 3 * a * (6 + 6 * b * x + 3 * b^2 * x^2 + b^3 * x^3))}{b^2}$$

**Rubi [A] (verified)**

Time = 0.75 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.34, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x e^{-a - bx} (a + bx)^3 dx$$

$$\downarrow 2626$$

$$\int \left( \frac{e^{-a - bx} (a + bx)^4}{b} - \frac{a e^{-a - bx} (a + bx)^3}{b} \right) dx$$

$$\downarrow 2009$$

$$-\frac{e^{-a - bx} (a + bx)^4}{b^2} + \frac{a e^{-a - bx} (a + bx)^3}{b^2} - \frac{4 e^{-a - bx} (a + bx)^3}{b^2} + \frac{3 a e^{-a - bx} (a + bx)^2}{b^2} - \frac{12 e^{-a - bx} (a + bx)^2}{b^2} + \frac{6 a e^{-a - bx} (a + bx)}{b^2} - \frac{24 e^{-a - bx} (a + bx)}{b^2} + \frac{6 a e^{-a - bx}}{b^2} - \frac{24 e^{-a - bx}}{b^2}$$

input

$$\text{Int}[E^{-a - bx} * x * (a + b * x)^3, x]$$

output

$$\frac{(-24 * E^{-a - bx})}{b^2} + \frac{(6 * a * E^{-a - bx})}{b^2} - \frac{(24 * E^{-a - bx}) * (a + b * x)}{b^2} + \frac{(6 * a * E^{-a - bx}) * (a + b * x)}{b^2} - \frac{(12 * E^{-a - bx}) * (a + b * x)^2}{b^2} + \frac{(3 * a * E^{-a - bx}) * (a + b * x)^2}{b^2} - \frac{(4 * E^{-a - bx}) * (a + b * x)^3}{b^2} + \frac{(a * E^{-a - bx}) * (a + b * x)^3}{b^2} - \frac{(E^{-a - bx}) * (a + b * x)^4}{b^2}$$

Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2626 Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74

method	result
gospers	$\frac{(b^4x^4+3ab^3x^3+3a^2b^2x^2+4b^3x^3+a^3bx+9ab^2x^2+6a^2bx+12b^2x^2+a^3+18abx+6a^2+24bx+18a+24)e^{-bx-a}}{b^2}$
risch	$\frac{(b^4x^4+3ab^3x^3+3a^2b^2x^2+4b^3x^3+a^3bx+9ab^2x^2+6a^2bx+12b^2x^2+a^3+18abx+6a^2+24bx+18a+24)e^{-bx-a}}{b^2}$
orering	$\frac{(b^4x^4+3ab^3x^3+3a^2b^2x^2+4b^3x^3+a^3bx+9ab^2x^2+6a^2bx+12b^2x^2+a^3+18abx+6a^2+24bx+18a+24)e^{-bx-a}}{b^2}$
norman	$(-3ab - 4b)x^3e^{-bx-a} + (-3a^2 - 9a - 12)x^2e^{-bx-a} - b^2x^4e^{-bx-a} - \frac{(a^3+6a^2+18a+24)e^{-bx-a}}{b^2}$
meijerg	$\frac{e^{-a} \left( 24 - \frac{(5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)e^{-bx}}{5} \right)}{b^2} + \frac{3e^{-a} \left( 6 - \frac{(4b^3x^3+12b^2x^2+24bx+24)e^{-bx}}{4} \right)}{b^2} + \frac{3e^{-a}a^2 \left( 2 - \frac{(-bx-a)e^{-bx-a}}{b} \right)}{b^2}$
derivativedivides	$\frac{(-bx-a)^4e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a} + a(e^{-bx-a}(-bx-a) - 2e^{-bx-a})}{b^2}$
default	$\frac{(-bx-a)^4e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a} + a(e^{-bx-a}(-bx-a) - 2e^{-bx-a})}{b^2}$
parts	$-b^2x^4e^{-bx-a} - 3e^{-bx-a}x^3ba - 3e^{-bx-a}x^2a^2 - \frac{e^{-bx-a}xa^3}{b} - \frac{4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2e^{-bx-a}}{b^2}$
parallelrisch	$\frac{b^4x^4e^{-bx-a} + 3ab^3x^3e^{-bx-a} + 4e^{-bx-a}x^3b^3 + 3a^2b^2x^2e^{-bx-a} + 9x^2e^{-bx-a}ab^2 + a^3bx e^{-bx-a} + 12b^2e^{-bx-a}x^2 + 6x^3e^{-bx-a}}{b^2}$

```
input int(exp(-b*x-a)*x*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
output -(b^4*x^4+3*a*b^3*x^3+3*a^2*b^2*x^2+4*b^3*x^3+a^3*b*x+9*a*b^2*x^2+6*a^2*b*x+12*b^2*x^2+a^3+18*a*b*x+6*a^2+24*b*x+18*a+24)*exp(-b*x-a)/b^2
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int e^{-a-bx} x(a+bx)^3 dx = \frac{(b^4 x^4 + (3a+4)b^3 x^3 + 3(a^2+3a+4)b^2 x^2 + a^3 + (a^3+6a^2+18a+24)bx + 6a^2+18a+24)e^{(-bx-a)}}{b^2}$$

input `integrate(exp(-b*x-a)*x*(b*x+a)^3,x, algorithm="fricas")`output `-(b^4*x^4 + (3*a + 4)*b^3*x^3 + 3*(a^2 + 3*a + 4)*b^2*x^2 + a^3 + (a^3 + 6*a^2 + 18*a + 24)*b*x + 6*a^2 + 18*a + 24)*e^(-b*x - a)/b^2`**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

$$\int e^{-a-bx} x(a+bx)^3 dx = \begin{cases} \frac{(-a^3bx-a^3-3a^2b^2x^2-6a^2bx-6a^2-3ab^3x^3-9ab^2x^2-18abx-18a-b^4x^4-4b^3x^3-12b^2x^2-24bx-24)e^{-a-bx}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{a^3x^2}{2} + a^2bx^3 + \frac{3ab^2x^4}{4} + \frac{b^3x^5}{5} & \text{otherwise} \end{cases}$$

input `integrate(exp(-b*x-a)*x*(b*x+a)**3,x)`output `Piecewise((((a**3*b*x - a**3 - 3*a**2*b**2*x**2 - 6*a**2*b*x - 6*a**2 - 3*a*b**3*x**3 - 9*a*b**2*x**2 - 18*a*b*x - 18*a - b**4*x**4 - 4*b**3*x**3 - 12*b**2*x**2 - 24*b*x - 24)*exp(-a - b*x)/b**2, Ne(b**2, 0)), (a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5, True))`





**Mupad [B] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int e^{-a-bx} x(a+bx)^3 dx = -x^2 e^{-a-bx} (3a^2 + 9a + 12) - b^2 x^4 e^{-a-bx} - \frac{e^{-a-bx} (a^3 + 6a^2 + 18a + 24)}{b^2} - \frac{x e^{-a-bx} (a^3 + 6a^2 + 18a + 24)}{b} - b x^3 e^{-a-bx} (3a + 4)$$

input `int(x*exp(- a - b*x)*(a + b*x)^3,x)`output `- x^2*exp(- a - b*x)*(9*a + 3*a^2 + 12) - b^2*x^4*exp(- a - b*x) - (exp(- a - b*x)*(18*a + 6*a^2 + a^3 + 24))/b^2 - (x*exp(- a - b*x)*(18*a + 6*a^2 + a^3 + 24))/b - b*x^3*exp(- a - b*x)*(3*a + 4)`**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int e^{-a-bx} x(a+bx)^3 dx = \frac{-b^4 x^4 - 3a b^3 x^3 - 3a^2 b^2 x^2 - 4b^3 x^3 - a^3 b x - 9a b^2 x^2 - 6a^2 b x - 12b^2 x^2 - a^3 - 18abx - 6a^2 - 24bx - 18a^2}{e^{bx+ab^2}}$$

input `int(exp(-b*x-a)*x*(b*x+a)^3,x)`output `( - a**3*b*x - a**3 - 3*a**2*b**2*x**2 - 6*a**2*b*x - 6*a**2 - 3*a*b**3*x**3 - 9*a*b**2*x**2 - 18*a*b*x - 18*a - b**4*x**4 - 4*b**3*x**3 - 12*b**2*x**2 - 24*b*x - 24)/(e**(a + b*x)*b**2)`

### 3.98 $\int e^{-a-bx}(a+bx)^3 dx$

Optimal result	657
Mathematica [A] (verified)	657
Rubi [A] (verified)	658
Maple [A] (verified)	659
Fricas [A] (verification not implemented)	660
Sympy [A] (verification not implemented)	660
Maxima [A] (verification not implemented)	661
Giac [A] (verification not implemented)	661
Mupad [B] (verification not implemented)	662
Reduce [B] (verification not implemented)	662

#### Optimal result

Integrand size = 18, antiderivative size = 80

$$\int e^{-a-bx}(a+bx)^3 dx = -\frac{6e^{-a-bx}}{b} - \frac{6e^{-a-bx}(a+bx)}{b} - \frac{3e^{-a-bx}(a+bx)^2}{b} - \frac{e^{-a-bx}(a+bx)^3}{b}$$

output

```
-6*exp(-b*x-a)/b-6*exp(-b*x-a)*(b*x+a)/b-3*exp(-b*x-a)*(b*x+a)^2/b-exp(-b*x-a)*(b*x+a)^3/b
```

#### Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int e^{-a-bx}(a+bx)^3 dx = \frac{e^{-a-bx}(-6 - 6(a+bx) - 3(a+bx)^2 - (a+bx)^3)}{b}$$

input

```
Integrate[E^(-a - b*x)*(a + b*x)^3,x]
```

output

```
(E^(-a - b*x)*(-6 - 6*(a + b*x) - 3*(a + b*x)^2 - (a + b*x)^3))/b
```

**Rubi [A] (verified)**

Time = 0.52 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int e^{-a-bx}(a+bx)^3 dx \\
 & \quad \downarrow \text{2607} \\
 & 3 \int e^{-a-bx}(a+bx)^2 dx - \frac{e^{-a-bx}(a+bx)^3}{b} \\
 & \quad \downarrow \text{2607} \\
 & 3 \left( 2 \int e^{-a-bx}(a+bx) dx - \frac{e^{-a-bx}(a+bx)^2}{b} \right) - \frac{e^{-a-bx}(a+bx)^3}{b} \\
 & \quad \downarrow \text{2607} \\
 & 3 \left( 2 \left( \int e^{-a-bx} dx - \frac{e^{-a-bx}(a+bx)}{b} \right) - \frac{e^{-a-bx}(a+bx)^2}{b} \right) - \frac{e^{-a-bx}(a+bx)^3}{b} \\
 & \quad \downarrow \text{2624} \\
 & 3 \left( 2 \left( -\frac{e^{-a-bx}(a+bx)}{b} - \frac{e^{-a-bx}}{b} \right) - \frac{e^{-a-bx}(a+bx)^2}{b} \right) - \frac{e^{-a-bx}(a+bx)^3}{b}
 \end{aligned}$$

input `Int [E^(-a - b*x)*(a + b*x)^3,x]`

output `-((E^(-a - b*x)*(a + b*x)^3)/b) + 3*(-((E^(-a - b*x)*(a + b*x)^2)/b) + 2*(-(E^(-a - b*x)/b) - (E^(-a - b*x)*(a + b*x))/b))`

Defintions of rubi rules used

```
rule 2607 Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

```
rule 2624 Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

method	result
gospers	$\frac{(b^3x^3+3ab^2x^2+3a^2bx+3b^2x^2+a^3+6abx+3a^2+6bx+6a+6)e^{-bx-a}}{b}$
risch	$\frac{(b^3x^3+3ab^2x^2+3a^2bx+3b^2x^2+a^3+6abx+3a^2+6bx+6a+6)e^{-bx-a}}{b}$
oring	$\frac{(b^3x^3+3ab^2x^2+3a^2bx+3b^2x^2+a^3+6abx+3a^2+6bx+6a+6)e^{-bx-a}}{b}$
derivativdivides	$\frac{e^{-bx-a}(-bx-a)^3-3(-bx-a)^2e^{-bx-a}+6(-bx-a)e^{-bx-a}-6e^{-bx-a}}{b}$
default	$\frac{e^{-bx-a}(-bx-a)^3-3(-bx-a)^2e^{-bx-a}+6(-bx-a)e^{-bx-a}-6e^{-bx-a}}{b}$
norman	$(-3ab - 3b)x^2e^{-bx-a} + (-3a^2 - 6a - 6)xe^{-bx-a} - b^2x^3e^{-bx-a} - \frac{(a^3+3a^2+6a+6)e^{-bx-a}}{b}$
meijerg	$\frac{e^{-a} \left( 6 - \frac{(4b^3x^3+12b^2x^2+24bx+24)e^{-bx}}{4} \right)}{b} + \frac{3e^{-a}a \left( 2 - \frac{(3b^2x^2+6bx+6)e^{-bx}}{3} \right)}{b} + \frac{3e^{-a}a^2 \left( 1 - \frac{(2bx+2)e^{-bx}}{2} \right)}{b} + e^{-bx-a}$
parts	$-b^2x^3e^{-bx-a} - 3e^{-bx-a}bax^2 - 3e^{-bx-a}a^2x - \frac{e^{-bx-a}a^3}{b} - \frac{3((-bx-a)^2e^{-bx-a}-2(-bx-a)e^{-bx-a})}{b}$
parallelrisc	$-\frac{e^{-bx-a}x^3b^3+3x^2e^{-bx-a}ab^2+3b^2e^{-bx-a}x^2+3xe^{-bx-a}a^2b+6abe^{-bx-a}x+e^{-bx-a}a^3+6be^{-bx-a}x+3a^2e^{-bx-a}}{b}$

```
input int(exp(-b*x-a)*(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
output -(b^3*x^3+3*a*b^2*x^2+3*a^2*b*x+3*b^2*x^2+a^3+6*a*b*x+3*a^2+6*b*x+6*a+6)*
xp(-b*x-a)/b
```

**Fricas [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int e^{-a-bx}(a+bx)^3 dx = -\frac{(b^3x^3 + 3(a+1)b^2x^2 + a^3 + 3(a^2 + 2a + 2)bx + 3a^2 + 6a + 6)e^{-bx-a}}{b}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3,x, algorithm="fricas")`output `-(b^3*x^3 + 3*(a + 1)*b^2*x^2 + a^3 + 3*(a^2 + 2*a + 2)*b*x + 3*a^2 + 6*a + 6)*e^(-b*x - a)/b`**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int e^{-a-bx}(a+bx)^3 dx = \begin{cases} \frac{(-a^3-3a^2bx-3a^2-3ab^2x^2-6abx-6a-b^3x^3-3b^2x^2-6bx-6)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4} & \text{otherwise} \end{cases}$$

input `integrate(exp(-b*x-a)*(b*x+a)**3,x)`output `Piecewise((((-a**3 - 3*a**2*b*x - 3*a**2 - 3*a*b**2*x**2 - 6*a*b*x - 6*a - b**3*x**3 - 3*b**2*x**2 - 6*b*x - 6)*exp(-a - b*x)/b, Ne(b, 0)), (a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4, True))`

**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.29

$$\int e^{-a-bx}(a+bx)^3 dx = -\frac{3(bx+1)a^2e^{(-bx-a)}}{b} - \frac{a^3e^{(-bx-a)}}{b} - \frac{3(b^2x^2+2bx+2)ae^{(-bx-a)}}{b} - \frac{(b^3x^3+3b^2x^2+6bx+6)e^{(-bx-a)}}{b}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3,x, algorithm="maxima")`output `-3*(b*x + 1)*a^2*e^(-b*x - a)/b - a^3*e^(-b*x - a)/b - 3*(b^2*x^2 + 2*b*x + 2)*a*e^(-b*x - a)/b - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/b`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.09

$$\int e^{-a-bx}(a+bx)^3 dx = \frac{(b^6x^3 + 3ab^5x^2 + 3a^2b^4x + 3b^5x^2 + a^3b^3 + 6ab^4x + 3a^2b^3 + 6b^4x + 6ab^3 + 6b^3)e^{(-bx-a)}}{b^4}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3,x, algorithm="giac")`output `-(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + 3*b^5*x^2 + a^3*b^3 + 6*a*b^4*x + 3*a^2*b^3 + 6*b^4*x + 6*a*b^3 + 6*b^3)*e^(-b*x - a)/b^4`

**Mupad [B] (verification not implemented)**

Time = 22.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int e^{-a-bx}(a+bx)^3 dx = -x e^{-a-bx} (3a^2 + 3abx + 6a + b^2x^2 + 3bx + 6) - \frac{e^{-a-bx}(a^3 + 3a^2 + 6a + 6)}{b}$$

input `int(exp(- a - b*x)*(a + b*x)^3,x)`output `- x*exp(- a - b*x)*(6*a + 3*b*x + 3*a^2 + b^2*x^2 + 3*a*b*x + 6) - (exp(- a - b*x)*(6*a + 3*a^2 + a^3 + 6))/b`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int e^{-a-bx}(a+bx)^3 dx = \frac{-b^3x^3 - 3ab^2x^2 - 3a^2bx - 3b^2x^2 - a^3 - 6abx - 3a^2 - 6bx - 6a - 6}{e^{bx+ab}}$$

input `int(exp(-b*x-a)*(b*x+a)^3,x)`output `( - a**3 - 3*a**2*b*x - 3*a**2 - 3*a*b**2*x**2 - 6*a*b*x - 6*a - b**3*x**3 - 3*b**2*x**2 - 6*b*x - 6)/(e**(a + b*x)*b)`

### 3.99 $\int \frac{e^{-a-bx}(a+bx)^3}{x} dx$

Optimal result . . . . .	663
Mathematica [A] (verified) . . . . .	663
Rubi [A] (verified) . . . . .	664
Maple [A] (verified) . . . . .	665
Fricas [A] (verification not implemented) . . . . .	665
Sympy [A] (verification not implemented) . . . . .	666
Maxima [A] (verification not implemented) . . . . .	666
Giac [A] (verification not implemented) . . . . .	666
Mupad [B] (verification not implemented) . . . . .	667
Reduce [B] (verification not implemented) . . . . .	667

#### Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = -2e^{-a-bx} - 3ae^{-a-bx} - 3a^2e^{-a-bx} - 2be^{-a-bx}x - 3abe^{-a-bx}x - b^2e^{-a-bx}x^2 + a^3e^{-a} \text{ExpIntegralEi}(-bx)$$

output `-2*exp(-b*x-a)-3*a*exp(-b*x-a)-3*a^2*exp(-b*x-a)-2*b*exp(-b*x-a)*x-3*a*b*exp(-b*x-a)*x-b^2*exp(-b*x-a)*x^2+a^3*Ei(-b*x)/exp(a)`

#### Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = e^{-a-bx}(-2 - 3a^2 - 2bx - b^2x^2 - 3a(1 + bx) + a^3e^{bx} \text{ExpIntegralEi}(-bx))$$

input `Integrate[(E^(-a - b*x))*(a + b*x)^3/x,x]`

output `E^(-a - b*x)*(-2 - 3*a^2 - 2*b*x - b^2*x^2 - 3*a*(1 + b*x) + a^3*E^(b*x)*ExpIntegralEi[-(b*x)])`



**Rubi [A] (verified)**

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx$$

↓ 2629

$$\int \left( \frac{a^3 e^{-a-bx}}{x} + 3a^2 b e^{-a-bx} + b^3 x^2 e^{-a-bx} + 3ab^2 x e^{-a-bx} \right) dx$$

↓ 2009

$$e^{-a} a^3 \text{ExpIntegralEi}(-bx) - 3a^2 e^{-a-bx} - b^2 x^2 e^{-a-bx} - 3a e^{-a-bx} - 3abx e^{-a-bx} - 2e^{-a-bx} - 2bx e^{-a-bx}$$

input `Int[(E^(-a - b*x))*(a + b*x)^3]/x,x]`

output `-2*E^(-a - b*x) - 3*a*E^(-a - b*x) - 3*a^2*E^(-a - b*x) - 2*b*E^(-a - b*x)*x - 3*a*b*E^(-a - b*x)*x - b^2*E^(-a - b*x)*x^2 + (a^3*ExpIntegralEi[-(b*x)])]/E^a`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.05 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

method	result
meijerg	$e^{-a} \left( 2 - \frac{(3b^2x^2+6bx+6)e^{-bx}}{3} \right) + 3e^{-a}a \left( 1 - \frac{(2bx+2)e^{-bx}}{2} \right) + 3e^{-a}a^2(1 - e^{-bx}) + e^{-a}a^3(\ln(x) + \ln(b) - \ln(bx) - \text{Ei}(-bx))$
risch	$-b^2e^{-bx-a}x^2 - a^3e^{-a} \text{expIntegral}_1(bx) - 3abe^{-bx-a}x - 3a^2e^{-bx-a} - 2be^{-bx-a}x - 3a$
derivativedivides	$-a^2e^{-bx-a} + a((-bx - a)e^{-bx-a} - e^{-bx-a}) - (-bx - a)^2e^{-bx-a} + 2(-bx - a)e^{-bx-a}$
default	$-a^2e^{-bx-a} + a((-bx - a)e^{-bx-a} - e^{-bx-a}) - (-bx - a)^2e^{-bx-a} + 2(-bx - a)e^{-bx-a}$

input `int(exp(-b*x-a)*(b*x+a)^3/x,x,method=_RETURNVERBOSE)`

output `exp(-a)*(2-1/3*(3*b^2*x^2+6*b*x+6)*exp(-b*x))+3*exp(-a)*a*(1-1/2*(2*b*x+2)*exp(-b*x))+3*exp(-a)*a^2*(1-exp(-b*x))+exp(-a)*a^3*(ln(x)+ln(b)-ln(b*x)-Ei(1,b*x))`

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = a^3 \text{Ei}(-bx) e^{-a} - (b^2x^2 + (3a+2)bx + 3a^2 + 3a + 2)e^{-bx-a}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="fricas")`

output `a^3*Ei(-b*x)*e^(-a) - (b^2*x^2 + (3*a + 2)*b*x + 3*a^2 + 3*a + 2)*e^(-b*x - a)`

**Sympy [A] (verification not implemented)**

Time = 3.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = (a^3 \operatorname{Ei}(-bx) - 3a^2 e^{-bx} - 3a(bxe^{-bx} + e^{-bx}) - b^2 x^2 e^{-bx} - 2bxe^{-bx} - 2e^{-bx}) e^{-a}$$

input `integrate(exp(-b*x-a)*(b*x+a)**3/x,x)`output `(a**3*Ei(-b*x) - 3*a**2*exp(-b*x) - 3*a*(b*x*exp(-b*x) + exp(-b*x)) - b**2*x**2*exp(-b*x) - 2*b*x*exp(-b*x) - 2*exp(-b*x))*exp(-a)`**Maxima [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = a^3 \operatorname{Ei}(-bx) e^{(-a)} - 3(bx+1) a e^{(-bx-a)} - 3a^2 e^{(-bx-a)} - (b^2 x^2 + 2bx + 2) e^{(-bx-a)}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="maxima")`output `a^3*Ei(-b*x)*e^(-a) - 3*(b*x + 1)*a*e^(-b*x - a) - 3*a^2*e^(-b*x - a) - (b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = -b^2 x^2 e^{(-bx-a)} + a^3 \operatorname{Ei}(-bx) e^{(-a)} - 3abx e^{(-bx-a)} - 3a^2 e^{(-bx-a)} - 2bxe^{(-bx-a)} - 3ae^{(-bx-a)} - 2e^{(-bx-a)}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x,x, algorithm="giac")`

output

$$-b^2 x^2 e^{-bx-a} + a^3 \text{Ei}(-bx) e^{-a} - 3a b x e^{-bx-a} - 3a^2 e^{-bx-a} - 2b^2 x e^{-bx-a} - 3a e^{-bx-a} - 2e^{-bx-a}$$
**Mupad [B] (verification not implemented)**

Time = 22.87 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.68

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = -e^{-a-bx} (b^2 x^2 + 2bx + 2) - 3a^2 e^{-a-bx} - 3a e^{-a-bx} (bx + 1) - a^3 e^{-a} \text{expint}(bx)$$

input

$$\text{int}((\exp(-a-b*x)*(a+b*x)^3)/x,x)$$

output

$$-\exp(-a-b*x)*(2*b*x + b^2*x^2 + 2) - 3*a^2*\exp(-a-b*x) - 3*a*\exp(-a-b*x)*(b*x + 1) - a^3*\exp(-a)*\text{expint}(b*x)$$
**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{e^{-a-bx}(a+bx)^3}{x} dx = \frac{e^{bx} \text{ei}(-bx) a^3 - 3a^2 - 3abx - 3a - b^2 x^2 - 2bx - 2}{e^{bx+a}}$$

input

$$\text{int}(\exp(-b*x-a)*(b*x+a)^3/x,x)$$

output

$$(e^{b*x}*\text{ei}(-b*x)*a^3 - 3*a^2 - 3*a*b*x - 3*a - b^2*x^2 - 2*b*x - 2)/e^{a+b*x}$$

### 3.100 $\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx$

Optimal result . . . . .	668
Mathematica [A] (verified) . . . . .	668
Rubi [A] (verified) . . . . .	669
Maple [A] (verified) . . . . .	670
Fricas [A] (verification not implemented) . . . . .	670
Sympy [A] (verification not implemented) . . . . .	671
Maxima [A] (verification not implemented) . . . . .	671
Giac [A] (verification not implemented) . . . . .	672
Mupad [B] (verification not implemented) . . . . .	672
Reduce [B] (verification not implemented) . . . . .	673

#### Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = -be^{-a-bx} - 3abe^{-a-bx} - \frac{a^3e^{-a-bx}}{x} - b^2e^{-a-bx}x + 3a^2be^{-a} \text{ExpIntegralEi}(-bx) - a^3be^{-a} \text{ExpIntegralEi}(-bx)$$

output `-b*exp(-b*x-a)-3*a*b*exp(-b*x-a)-a^3*exp(-b*x-a)/x-b^2*exp(-b*x-a)*x+3*a^2*b*Ei(-b*x)/exp(a)-a^3*b*Ei(-b*x)/exp(a)`

#### Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = \frac{e^{-a-bx}(-a^3 - 3abx - bx(1+bx) - (-3+a)a^2be^{bx}x \text{ExpIntegralEi}(-bx))}{x}$$

input `Integrate[(E^(-a - b*x))*(a + b*x)^3/x^2,x]`

output

```
(E^(-a - b*x)*(-a^3 - 3*a*b*x - b*x*(1 + b*x) - (-3 + a)*a^2*b*E^(b*x)*x*ExpIntegralEi[-(b*x)]))/x
```

**Rubi [A] (verified)**

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx$$

↓ 2629

$$\int \left( \frac{a^3 e^{-a-bx}}{x^2} + \frac{3a^2 b e^{-a-bx}}{x} + b^3 x e^{-a-bx} + 3ab^2 e^{-a-bx} \right) dx$$

↓ 2009

$$e^{-a} a^3 (-b) \text{ExpIntegralEi}(-bx) - \frac{a^3 e^{-a-bx}}{x} + 3e^{-a} a^2 b \text{ExpIntegralEi}(-bx) - b^2 x e^{-a-bx} - 3ab e^{-a-bx} - b e^{-a-bx}$$

input

```
Int[(E^(-a - b*x)*(a + b*x)^3)/x^2,x]
```

output

```
-(b*E^(-a - b*x)) - 3*a*b*E^(-a - b*x) - (a^3*E^(-a - b*x))/x - b^2*E^(-a - b*x)*x + (3*a^2*b*ExpIntegralEi[-(b*x)])/E^a - (a^3*b*ExpIntegralEi[-(b*x)])/E^a
```

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

method	result
risch	$-3ab e^{-bx-a} - b^2 e^{-bx-a} x - b e^{-bx-a} - \frac{a^3 e^{-bx-a}}{x} + b a^3 e^{-a} \operatorname{expIntegral}_1(bx) - 3b a^2 e^{-a}$
derivativedivides	$b \left( -2a e^{-bx-a} + (-bx - a) e^{-bx-a} - e^{-bx-a} - a^3 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \operatorname{expIntegral}_1(bx) \right) \right) - 3$
default	$b \left( -2a e^{-bx-a} + (-bx - a) e^{-bx-a} - e^{-bx-a} - a^3 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \operatorname{expIntegral}_1(bx) \right) \right) - 3$
meijerg	$b e^{-a} \left( 1 - \frac{(2bx+2)e^{-bx}}{2} \right) + 3 e^{-a} b a (1 - e^{-bx}) + 3b e^{-a} a^2 (\ln(x) + \ln(b) - \ln(bx) - \operatorname{expIntegral}_1(bx))$

input `int(exp(-b*x-a)*(b*x+a)^3/x^2,x,method=_RETURNVERBOSE)`

output `-3*a*b*exp(-b*x-a)-b^2*exp(-b*x-a)*x-b*exp(-b*x-a)-a^3*exp(-b*x-a)/x+b*a^3*exp(-a)*Ei(1,b*x)-3*b*a^2*exp(-a)*Ei(1,b*x)`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = -\frac{(a^3 - 3a^2)bx \operatorname{Ei}(-bx) e^{(-a)} + (b^2x^2 + a^3 + (3a+1)bx)e^{(-bx-a)}}{x}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^2,x, algorithm="fricas")`

output

```

-((a^3 - 3*a^2)*b*x*Ei(-b*x)*e^(-a) + (b^2*x^2 + a^3 + (3*a + 1)*b*x)*e^(-
b*x - a))/x

```

**Sympy [A] (verification not implemented)**

Time = 1.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = -\frac{a^3 e^{-a} E_2(bx)}{x} + 3a^2 b e^{-a} \operatorname{Ei}(-bx)$$

$$+ 3ab^2 \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) e^{-a}$$

$$+ b^3 x \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) e^{-a}$$

$$- b^3 \left( \begin{cases} \frac{x^2}{2} & \text{for } b = 0 \\ \begin{cases} -\frac{e^{-bx}}{b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases} & \text{otherwise} \end{cases} \right) e^{-a}$$

input

```

integrate(exp(-b*x-a)*(b*x+a)**3/x**2,x)

```

output

```

-a**3*exp(-a)*expint(2, b*x)/x + 3*a**2*b*exp(-a)*Ei(-b*x) + 3*a*b**2*Piec
ewise((x, Eq(b, 0)), (-exp(-b*x)/b, True))*exp(-a) + b**3*x*Piecewise((x,
Eq(b, 0)), (-exp(-b*x)/b, True))*exp(-a) - b**3*Piecewise((x**2/2, Eq(b, 0
)), (-Piecewise((-exp(-b*x)/b, Ne(b, 0)), (x, True))/b, True))*exp(-a)

```

**Maxima [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = -a^3 b e^{(-a)} \Gamma(-1, bx) + 3 a^2 b \operatorname{Ei}(-bx) e^{(-a)}$$

$$- (bx + 1) b e^{(-bx-a)} - 3 a b e^{(-bx-a)}$$



input `integrate(exp(-b*x-a)*(b*x+a)^3/x^2,x, algorithm="maxima")`

output 
$$-a^3 b e^{-a} \gamma(-1, b x) + 3 a^2 b \operatorname{Ei}(-b x) e^{-a} - (b x + 1) b e^{-b x - a} - 3 a b e^{-b x - a}$$

### Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = \frac{a^3 b x \operatorname{Ei}(-bx) e^{-a} - 3 a^2 b x \operatorname{Ei}(-bx) e^{-a} + b^2 x^2 e^{-bx-a} + a^3 e^{-bx-a} + 3 a b x e^{-bx-a} + b x e^{-bx-a}}{x}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^2,x, algorithm="giac")`

output 
$$-(a^3 b x \operatorname{Ei}(-b x) e^{-a} - 3 a^2 b x \operatorname{Ei}(-b x) e^{-a} + b^2 x^2 e^{-b x - a} + a^3 e^{-b x - a} + 3 a b x e^{-b x - a} + b x e^{-b x - a}) / x$$

### Mupad [B] (verification not implemented)

Time = 22.66 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = a^3 b e^{-a} \left( \operatorname{expint}(b x) - \frac{e^{-b x}}{b x} \right) - 3 a b e^{-a-b x} - b e^{-a-b x} (b x + 1) - 3 a^2 b e^{-a} \operatorname{expint}(b x)$$

input `int((exp(- a - b*x)*(a + b*x)^3)/x^2,x)`

output 
$$a^3 b \exp(-a) (\operatorname{expint}(b x) - \exp(-b x) / (b x)) - 3 a b \exp(-a - b x) - b \exp(-a - b x) (b x + 1) - 3 a^2 b \exp(-a) \operatorname{expint}(b x)$$

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^2} dx = \frac{-e^{bx} \operatorname{ei}(-bx) a^3 bx + 3e^{bx} \operatorname{ei}(-bx) a^2 bx - a^3 - 3abx - b^2 x^2 - bx}{e^{bx+a} x}$$

input `int(exp(-b*x-a)*(b*x+a)^3/x^2,x)`

output `( - e**(b*x)*ei( - b*x)*a**3*b*x + 3*e**(b*x)*ei( - b*x)*a**2*b*x - a**3 - 3*a*b*x - b**2*x**2 - b*x)/(e**(a + b*x)*x)`

### 3.101 $\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$

Optimal result . . . . .	674
Mathematica [A] (verified) . . . . .	674
Rubi [A] (verified) . . . . .	675
Maple [A] (verified) . . . . .	676
Fricas [A] (verification not implemented) . . . . .	676
Sympy [A] (verification not implemented) . . . . .	677
Maxima [A] (verification not implemented) . . . . .	677
Giac [A] (verification not implemented) . . . . .	678
Mupad [B] (verification not implemented) . . . . .	678
Reduce [B] (verification not implemented) . . . . .	679

#### Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx = -b^2 e^{-a-bx} - \frac{a^3 e^{-a-bx}}{2x^2} - \frac{3a^2 b e^{-a-bx}}{x} + \frac{a^3 b e^{-a-bx}}{2x} + 3ab^2 e^{-a} \text{ExpIntegralEi}(-bx) - 3a^2 b^2 e^{-a} \text{ExpIntegralEi}(-bx) + \frac{1}{2} a^3 b^2 e^{-a} \text{ExpIntegralEi}(-bx)$$

output

```
-b^2*exp(-b*x-a)-1/2*a^3*exp(-b*x-a)/x^2-3*a^2*b*exp(-b*x-a)/x+1/2*a^3*b*exp(-b*x-a)/x+3*a*b^2*Ei(-b*x)/exp(a)-3*a^2*b^2*Ei(-b*x)/exp(a)+1/2*a^3*b^2*Ei(-b*x)/exp(a)
```

#### Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.52

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx = \frac{e^{-a-bx}(-6a^2bx - 2b^2x^2 + a^3(-1+bx)) + a(6-6a+a^2)b^2e^{bx}x^2 \text{ExpIntegralEi}(-bx)}{2x^2}$$

input

```
Integrate[(E^(-a - b*x))*(a + b*x)^3/x^3,x]
```

output

$$(E^{(-a - b*x)}*(-6*a^2*b*x - 2*b^2*x^2 + a^3*(-1 + b*x) + a*(6 - 6*a + a^2) * b^2 * E^{(b*x)} * x^2 * \text{ExpIntegralEi}[-(b*x)])) / (2*x^2)$$
**Rubi [A] (verified)**

Time = 0.64 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$$

↓ 2629

$$\int \left( \frac{a^3 e^{-a-bx}}{x^3} + \frac{3a^2 b e^{-a-bx}}{x^2} + b^3 e^{-a-bx} + \frac{3ab^2 e^{-a-bx}}{x} \right) dx$$

↓ 2009

$$\frac{1}{2} e^{-a} a^3 b^2 \text{ExpIntegralEi}(-bx) - \frac{a^3 e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{2x} - 3e^{-a} a^2 b^2 \text{ExpIntegralEi}(-bx) - \frac{3a^2 b e^{-a-bx}}{x} + 3e^{-a} a b^2 \text{ExpIntegralEi}(-bx) - b^2 e^{-a-bx}$$

input

$$\text{Int}[(E^{(-a - b*x)}*(a + b*x)^3)/x^3, x]$$

output

$$-(b^2 * E^{(-a - b*x)}) - (a^3 * E^{(-a - b*x)}) / (2 * x^2) - (3 * a^2 * b * E^{(-a - b*x)}) / x + (a^3 * b * E^{(-a - b*x)}) / (2 * x) + (3 * a * b^2 * \text{ExpIntegralEi}[-(b*x)]) / E^a - (3 * a^2 * b^2 * \text{ExpIntegralEi}[-(b*x)]) / E^a + (a^3 * b^2 * \text{ExpIntegralEi}[-(b*x)]) / (2 * E^a)$$

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

method	result
derivativedivides	$-b^2 \left( e^{-bx-a} - a^3 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \expIntegral_1(bx)}{2} \right) \right) + 3a^2 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \expIntegral_1(bx) \right)$
default	$-b^2 \left( e^{-bx-a} - a^3 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \expIntegral_1(bx)}{2} \right) \right) + 3a^2 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \expIntegral_1(bx) \right)$
risch	$-b^2 e^{-bx-a} - \frac{a^3 e^{-bx-a}}{2x^2} + \frac{a^3 b e^{-bx-a}}{2x} - \frac{b^2 a^3 e^{-a} \expIntegral_1(bx)}{2} - \frac{3a^2 b e^{-bx-a}}{x} + 3b^2 a^2 e^{-a} \expIntegral_1(bx)$
meijerg	$e^{-a} b^2 (1 - e^{-bx}) + 3b^2 e^{-a} a (\ln(x) + \ln(b) - \ln(bx) - \expIntegral_1(bx)) + 3b^2 e^{-a} a^2 \left( \frac{e^{-bx-a}}{bx} - e^{-a} \expIntegral_1(bx) \right)$

input `int(exp(-b*x-a)*(b*x+a)^3/x^3,x,method=_RETURNVERBOSE)`

output `-b^2*(exp(-b*x-a)-a^3*(-1/2*exp(-b*x-a)/b^2/x^2+1/2*exp(-b*x-a)/b/x-1/2*exp(-a)*Ei(1,b*x))+3*a^2*(exp(-b*x-a)/b/x-exp(-a)*Ei(1,b*x))+3*a*exp(-a)*Ei(1,b*x))`

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.54

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$$

$$= \frac{(a^3 - 6a^2 + 6a)b^2x^2 \text{Ei}(-bx) e^{(-a)} - (2b^2x^2 + a^3 - (a^3 - 6a^2)bx) e^{(-bx-a)}}{2x^2}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="fricas")`

output  $\frac{1}{2}*((a^3 - 6*a^2 + 6*a)*b^2*x^2*Ei(-b*x)*e^{-a} - (2*b^2*x^2 + a^3 - (a^3 - 6*a^2)*b*x)*e^{-b*x - a})/x^2$

### Sympy [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx = \left( -\frac{a^3 E_3(bx)}{x^2} - \frac{3a^2 b E_2(bx)}{x} + 3ab^2 Ei(-bx) + b^3 \left( \begin{cases} x & \text{for } b = 0 \\ -\frac{e^{-bx}}{b} & \text{otherwise} \end{cases} \right) \right) e^{-a}$$

input `integrate(exp(-b*x-a)*(b*x+a)**3/x**3,x)`

output  $(-a**3*expint(3, b*x)/x**2 - 3*a**2*b*expint(2, b*x)/x + 3*a*b**2*Ei(-b*x) + b**3*Piecewise((x, Eq(b, 0)), (-exp(-b*x)/b, True)))*exp(-a)$

### Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx = -a^3 b^2 e^{(-a)} \Gamma(-2, bx) - 3a^2 b^2 e^{(-a)} \Gamma(-1, bx) + 3ab^2 Ei(-bx) e^{(-a)} - b^2 e^{(-bx-a)}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="maxima")`

output  $-a^3*b^2*e^{-a}*gamma(-2, b*x) - 3*a^2*b^2*e^{-a}*gamma(-1, b*x) + 3*a*b^2*Ei(-b*x)*e^{-a} - b^2*e^{-b*x - a}$

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.96

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$$

$$= \frac{a^3 b^2 x^2 \operatorname{Ei}(-bx) e^{(-a)} - 6 a^2 b^2 x^2 \operatorname{Ei}(-bx) e^{(-a)} + 6 a b^2 x^2 \operatorname{Ei}(-bx) e^{(-a)} + a^3 b x e^{(-bx-a)} - 6 a^2 b x e^{(-bx-a)} - 2 a^3 e^{(-bx-a)}}{2 x^2}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^3,x, algorithm="giac")`

output `1/2*(a^3*b^2*x^2*Ei(-b*x)*e^(-a) - 6*a^2*b^2*x^2*Ei(-b*x)*e^(-a) + 6*a*b^2*x^2*Ei(-b*x)*e^(-a) + a^3*b*x*e^(-b*x - a) - 6*a^2*b*x*e^(-b*x - a) - 2*b^2*x^2*e^(-b*x - a) - a^3*e^(-b*x - a))/x^2`

**Mupad [B] (verification not implemented)**

Time = 22.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.77

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx = 3 a^2 b^2 e^{-a} \left( \operatorname{expint}(bx) - \frac{e^{-bx}}{bx} \right) - 3 a b^2 e^{-a} \operatorname{expint}(bx) - b^2 e^{-a-bx} + a^3 b^2 e^{-a} \left( e^{-bx} \left( \frac{1}{2bx} - \frac{1}{2b^2 x^2} \right) - \frac{\operatorname{expint}(bx)}{2} \right)$$

input `int((exp(- a - b*x)*(a + b*x)^3)/x^3,x)`

output `3*a^2*b^2*exp(-a)*(expint(b*x) - exp(-b*x)/(b*x)) - 3*a*b^2*exp(-a)*expint(b*x) - b^2*exp(- a - b*x) + a^3*b^2*exp(-a)*(exp(-b*x)*(1/(2*b*x) - 1/(2*b^2*x^2)) - expint(b*x)/2)`

**Reduce [B] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^3} dx$$

$$= \frac{e^{bx} \operatorname{ei}(-bx) a^3 b^2 x^2 - 6e^{bx} \operatorname{ei}(-bx) a^2 b^2 x^2 + 6e^{bx} \operatorname{ei}(-bx) a b^2 x^2 + a^3 b x - a^3 - 6a^2 b x - 2b^2 x^2}{2e^{bx+a} x^2}$$

input `int(exp(-b*x-a)*(b*x+a)^3/x^3,x)`output `(e**(b*x)*ei(-b*x)*a**3*b**2*x**2 - 6*e**(b*x)*ei(-b*x)*a**2*b**2*x**2 + 6*e**(b*x)*ei(-b*x)*a*b**2*x**2 + a**3*b*x - a**3 - 6*a**2*b*x - 2*b**2*x**2)/(2*e**(a+b*x)*x**2)`



### 3.102 $\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx$

Optimal result . . . . .	680
Mathematica [A] (verified) . . . . .	681
Rubi [A] (verified) . . . . .	681
Maple [A] (verified) . . . . .	682
Fricas [A] (verification not implemented) . . . . .	683
Sympy [A] (verification not implemented) . . . . .	683
Maxima [A] (verification not implemented) . . . . .	684
Giac [A] (verification not implemented) . . . . .	684
Mupad [B] (verification not implemented) . . . . .	685
Reduce [B] (verification not implemented) . . . . .	685

#### Optimal result

Integrand size = 21, antiderivative size = 198

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = -\frac{a^3 e^{-a-bx}}{3x^3} - \frac{3a^2 b e^{-a-bx}}{2x^2} + \frac{a^3 b e^{-a-bx}}{6x^2} - \frac{3ab^2 e^{-a-bx}}{x}$$

$$+ \frac{3a^2 b^2 e^{-a-bx}}{2x} - \frac{a^3 b^2 e^{-a-bx}}{6x} + b^3 e^{-a} \text{ExpIntegralEi}(-bx)$$

$$- 3ab^3 e^{-a} \text{ExpIntegralEi}(-bx) + \frac{3}{2} a^2 b^3 e^{-a} \text{ExpIntegralEi}(-bx)$$

$$- \frac{1}{6} a^3 b^3 e^{-a} \text{ExpIntegralEi}(-bx)$$

output

```
-1/3*a^3*exp(-b*x-a)/x^3-3/2*a^2*b*exp(-b*x-a)/x^2+1/6*a^3*b*exp(-b*x-a)/x
^2-3*a*b^2*exp(-b*x-a)/x+3/2*a^2*b^2*exp(-b*x-a)/x-1/6*a^3*b^2*exp(-b*x-a)
/x+b^3*Ei(-b*x)/exp(a)-3*a*b^3*Ei(-b*x)/exp(a)+3/2*a^2*b^3*Ei(-b*x)/exp(a)
-1/6*a^3*b^3*Ei(-b*x)/exp(a)
```

**Mathematica [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.41

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = \frac{1}{6}e^{-a} \left( -\frac{ae^{-bx}(18b^2x^2 - 9abx(-1+bx) + a^2(2-bx+b^2x^2))}{x^3} - (-6 + 18a - 9a^2 + a^3) b^3 \text{ExpIntegralEi}(-bx) \right)$$

input `Integrate[(E^(-a - b*x))*(a + b*x)^3/x^4,x]`

output `(-((a*(18*b^2*x^2 - 9*a*b*x*(-1 + b*x) + a^2*(2 - b*x + b^2*x^2)))/(E^(b*x)*x^3)) - (-6 + 18*a - 9*a^2 + a^3)*b^3*ExpIntegralEi[-(b*x)])/(6*E^a)`

**Rubi [A] (verified)**

Time = 0.81 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx \\ & \quad \downarrow \text{2629} \\ & \int \left( \frac{a^3e^{-a-bx}}{x^4} + \frac{3a^2be^{-a-bx}}{x^3} + \frac{b^3e^{-a-bx}}{x} + \frac{3ab^2e^{-a-bx}}{x^2} \right) dx \\ & \quad \downarrow \text{2009} \\ & -\frac{1}{6}e^{-a}a^3b^3 \text{ExpIntegralEi}(-bx) - \frac{a^3b^2e^{-a-bx}}{6x} - \frac{a^3e^{-a-bx}}{3x^3} + \frac{a^3be^{-a-bx}}{6x^2} + \\ & \frac{3}{2}e^{-a}a^2b^3 \text{ExpIntegralEi}(-bx) + \frac{3a^2b^2e^{-a-bx}}{2x} - \frac{3a^2be^{-a-bx}}{2x^2} - 3e^{-a}ab^3 \text{ExpIntegralEi}(-bx) + \\ & e^{-a}b^3 \text{ExpIntegralEi}(-bx) - \frac{3ab^2e^{-a-bx}}{x} \end{aligned}$$

input `Int[(E^(-a - b*x))*(a + b*x)^3/x^4,x]`

output 
$$-1/3*(a^3*E^(-a - b*x))/x^3 - (3*a^2*b*E^(-a - b*x))/(2*x^2) + (a^3*b*E^(-a - b*x))/(6*x^2) - (3*a*b^2*E^(-a - b*x))/x + (3*a^2*b^2*E^(-a - b*x))/(2*x) - (a^3*b^2*E^(-a - b*x))/(6*x) + (b^3*ExpIntegralEi[-(b*x)]/E^a - (3*a*b^3*ExpIntegralEi[-(b*x)]/E^a + (3*a^2*b^3*ExpIntegralEi[-(b*x)]/(2*E^a) - (a^3*b^3*ExpIntegralEi[-(b*x)]/(6*E^a)$$

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

method	result
derivativedivides	$b^3 \left( 3a^2 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \expIntegral_1(bx)}{2} \right) - a^3 \left( \frac{e^{-bx-a}}{3b^3x^3} - \frac{e^{-bx-a}}{6b^2x^2} + \frac{e^{-bx-a}}{6bx} - \frac{e^{-a} \expIntegral_1(bx)}{2} \right) \right)$
default	$b^3 \left( 3a^2 \left( -\frac{e^{-bx-a}}{2b^2x^2} + \frac{e^{-bx-a}}{2bx} - \frac{e^{-a} \expIntegral_1(bx)}{2} \right) - a^3 \left( \frac{e^{-bx-a}}{3b^3x^3} - \frac{e^{-bx-a}}{6b^2x^2} + \frac{e^{-bx-a}}{6bx} - \frac{e^{-a} \expIntegral_1(bx)}{2} \right) \right)$
risch	$-\frac{3a^2b e^{-bx-a}}{2x^2} + \frac{3a^2b^2 e^{-bx-a}}{2x} - \frac{3b^3a^2 e^{-a} \expIntegral_1(bx)}{2} + \frac{b^3a^3 e^{-a} \expIntegral_1(bx)}{6} - \frac{a^3b^2 e^{-bx-a}}{6x} + \dots$
meijerg	$b^3 e^{-a} (\ln(x) + \ln(b) - \ln(bx) - \expIntegral_1(bx)) + 3b^3 e^{-a} a \left( -\frac{1}{bx} + 1 - \ln(x) - \ln(b) \right)$

input `int(exp(-b*x-a)*(b*x+a)^3/x^4,x,method=_RETURNVERBOSE)`

output

```
b^3*(3*a^2*(-1/2*exp(-b*x-a)/b^2/x^2+1/2*exp(-b*x-a)/b/x-1/2*exp(-a)*Ei(1,
b*x))-a^3*(1/3*exp(-b*x-a)/b^3/x^3-1/6*exp(-b*x-a)/b^2/x^2+1/6*exp(-b*x-a)
/b/x-1/6*exp(-a)*Ei(1,b*x))-3*a*(exp(-b*x-a)/b/x-exp(-a)*Ei(1,b*x))-exp(-a)
)*Ei(1,b*x))
```

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.42

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = \frac{(a^3 - 9a^2 + 18a - 6)b^3x^3\text{Ei}(-bx)e^{-a} + ((a^3 - 9a^2 + 18a)b^2x^2 + 2a^3 - (a^3 - 9a^2)bx)e^{-bx-a}}{6x^3}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="fricas")
```

output

```
-1/6*((a^3 - 9*a^2 + 18*a - 6)*b^3*x^3*Ei(-b*x)*e^(-a) + ((a^3 - 9*a^2 + 1
8*a)*b^2*x^2 + 2*a^3 - (a^3 - 9*a^2)*b*x)*e^(-b*x - a))/x^3
```

**Sympy [A] (verification not implemented)**

Time = 1.15 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = \left( -\frac{a^3 E_4(bx)}{x^3} - \frac{3a^2 b E_3(bx)}{x^2} - \frac{3ab^2 E_2(bx)}{x} + b^3 \text{Ei}(-bx) \right) e^{-a}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)**3/x**4,x)
```

output

```
(-a**3*expint(4, b*x)/x**3 - 3*a**2*b*expint(3, b*x)/x**2 - 3*a*b**2*expin
t(2, b*x)/x + b**3*Ei(-b*x))*exp(-a)
```

**Maxima [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.32

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = -a^3 b^3 e^{(-a)} \Gamma(-3, bx) - 3 a^2 b^3 e^{(-a)} \Gamma(-2, bx) - 3 a b^3 e^{(-a)} \Gamma(-1, bx) + b^3 \text{Ei}(-bx) e^{(-a)}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="maxima")`output `-a^3*b^3*e^(-a)*gamma(-3, b*x) - 3*a^2*b^3*e^(-a)*gamma(-2, b*x) - 3*a*b^3*e^(-a)*gamma(-1, b*x) + b^3*Ei(-b*x)*e^(-a)`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.92

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = \frac{a^3 b^3 x^3 \text{Ei}(-bx) e^{(-a)} - 9 a^2 b^3 x^3 \text{Ei}(-bx) e^{(-a)} + 18 a b^3 x^3 \text{Ei}(-bx) e^{(-a)} + a^3 b^2 x^2 e^{(-bx-a)} - 6 b^3 x^3 \text{Ei}(-bx) e^{(-a)}}{6 x^3}$$

input `integrate(exp(-b*x-a)*(b*x+a)^3/x^4,x, algorithm="giac")`output `-1/6*(a^3*b^3*x^3*Ei(-b*x)*e^(-a) - 9*a^2*b^3*x^3*Ei(-b*x)*e^(-a) + 18*a*b^3*x^3*Ei(-b*x)*e^(-a) + a^3*b^2*x^2*e^(-b*x - a) - 6*b^3*x^3*Ei(-b*x)*e^(-a) - 9*a^2*b^2*x^2*e^(-b*x - a) - a^3*b*x*e^(-b*x - a) + 18*a*b^2*x^2*e^(-b*x - a) + 9*a^2*b*x*e^(-b*x - a) + 2*a^3*e^(-b*x - a))/x^3`

**Mupad [B] (verification not implemented)**

Time = 22.64 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.72

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = 3ab^3 e^{-a} \left( \operatorname{expint}(bx) - \frac{e^{-bx}}{bx} \right) - b^3 e^{-a} \operatorname{expint}(bx) + \frac{a^3 b^3 e^{-a} \operatorname{expint}(bx)}{6} + 3a^2 b^3 e^{-a} \left( e^{-bx} \left( \frac{1}{2bx} - \frac{1}{2b^2 x^2} \right) - \frac{\operatorname{expint}(bx)}{2} \right) - a^3 b^3 e^{-a-bx} \left( \frac{1}{6bx} - \frac{1}{6b^2 x^2} + \frac{1}{3b^3 x^3} \right)$$

input `int((exp(- a - b*x)*(a + b*x)^3)/x^4,x)`output `3*a*b^3*exp(-a)*(expint(b*x) - exp(-b*x)/(b*x)) - b^3*exp(-a)*expint(b*x) + (a^3*b^3*exp(-a)*expint(b*x))/6 + 3*a^2*b^3*exp(-a)*(exp(-b*x)*(1/(2*b*x) - 1/(2*b^2*x^2)) - expint(b*x)/2) - a^3*b^3*exp(- a - b*x)*(1/(6*b*x) - 1/(6*b^2*x^2) + 1/(3*b^3*x^3))`**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.72

$$\int \frac{e^{-a-bx}(a+bx)^3}{x^4} dx = \frac{-e^{bx} \operatorname{ei}(-bx) a^3 b^3 x^3 + 9e^{bx} \operatorname{ei}(-bx) a^2 b^3 x^3 - 18e^{bx} \operatorname{ei}(-bx) a b^3 x^3 + 6e^{bx} \operatorname{ei}(-bx) b^3 x^3 - a^3 b^2 x^2 + a^3 b x - 18e^{bx} \operatorname{ei}(-bx) a^2 b^2 x^2 + 9e^{bx} \operatorname{ei}(-bx) a^2 b^2 x^2 - 18e^{bx} \operatorname{ei}(-bx) a b^2 x^2 + 6e^{bx} \operatorname{ei}(-bx) b^2 x^2 - a^3 b x + 18e^{bx} \operatorname{ei}(-bx) a^2 b x - 9e^{bx} \operatorname{ei}(-bx) a^2 b x + 18e^{bx} \operatorname{ei}(-bx) a b x - 6e^{bx} \operatorname{ei}(-bx) b x + a^3}{6e^{bx+a} x^3}$$

input `int(exp(-b*x-a)*(b*x+a)^3/x^4,x)`output `( - e**(b*x)*ei( - b*x)*a**3*b**3*x**3 + 9*e**(b*x)*ei( - b*x)*a**2*b**3*x**3 - 18*e**(b*x)*ei( - b*x)*a*b**3*x**3 + 6*e**(b*x)*ei( - b*x)*b**3*x**3 - a**3*b**2*x**2 + a**3*b*x - 2*a**3 + 9*a**2*b**2*x**2 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*e**(a + b*x)*x**3)`

### 3.103 $\int F^{a+b(c+dx)} x^m (e + fx)^2 dx$

Optimal result . . . . .	686
Mathematica [A] (verified) . . . . .	687
Rubi [A] (verified) . . . . .	687
Maple [B] (verified) . . . . .	688
Fricas [A] (verification not implemented) . . . . .	689
Sympy [F] . . . . .	689
Maxima [A] (verification not implemented) . . . . .	690
Giac [F] . . . . .	690
Mupad [F(-1)] . . . . .	691
Reduce [F] . . . . .	691

#### Optimal result

Integrand size = 22, antiderivative size = 139

$$\int F^{a+b(c+dx)} x^m (e + fx)^2 dx = \frac{f^2 F^{a+bc} x^m \Gamma(3 + m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^3 d^3 \log^3(F)} - \frac{2ef F^{a+bc} x^m \Gamma(2 + m, -bdx \log(F)) (-bdx \log(F))^{-m}}{b^2 d^2 \log^2(F)} + \frac{e^2 F^{a+bc} x^m \Gamma(1 + m, -bdx \log(F)) (-bdx \log(F))^{-m}}{bd \log(F)}$$

output

```
f^2*F^(b*c+a)*x^m*GAMMA(3+m,-b*d*x*ln(F))/b^3/d^3/ln(F)^3/((-b*d*x*ln(F))^m)-2*e*f*F^(b*c+a)*x^m*GAMMA(2+m,-b*d*x*ln(F))/b^2/d^2/ln(F)^2/((-b*d*x*ln(F))^m)+e^2*F^(b*c+a)*x^m*GAMMA(1+m,-b*d*x*ln(F))/b/d/ln(F)/((-b*d*x*ln(F))^m)
```

**Mathematica [A] (verified)**

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.62

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx$$

$$= \frac{F^{a+bc} x^m (-bdx \log(F))^{-m} (f^2 \Gamma(3+m, -bdx \log(F)) + bde \log(F) (-2f \Gamma(2+m, -bdx \log(F)) + bde \Gamma(2+m, -bdx \log(F)))}{b^3 d^3 \log^3(F)}$$

input

```
Integrate[F^(a + b*(c + d*x))*x^m*(e + f*x)^2,x]
```

output

```
(F^(a + b*c)*x^m*(f^2*Gamma[3 + m, -(b*d*x*Log[F])]) + b*d*e*Log[F]*(-2*f*Gamma[2 + m, -(b*d*x*Log[F])]) + b*d*e*Gamma[1 + m, -(b*d*x*Log[F])]*Log[F])/
(b^3*d^3*Log[F]^3*(-(b*d*x*Log[F]))^m)
```

**Rubi [A] (verified)**

Time = 0.98 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (e+fx)^2 F^{a+b(c+dx)} dx$$

$$\downarrow 2629$$

$$\int \left( e^2 x^m F^{a+b(c+dx)} + 2e f x^{m+1} F^{a+b(c+dx)} + f^2 x^{m+2} F^{a+b(c+dx)} \right) dx$$

$$\downarrow 2009$$

$$\frac{f^2 x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+3, -bdx \log(F))}{b^3 d^3 \log^3(F)} -$$

$$\frac{2e f x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+2, -bdx \log(F))}{b^2 d^2 \log^2(F)} +$$

$$\frac{e^2 x^m F^{a+bc} (-bdx \log(F))^{-m} \Gamma(m+1, -bdx \log(F))}{bd \log(F)}$$



input `Int[F^(a + b*(c + d*x))*x^m*(e + f*x)^2,x]`

output 
$$\frac{(f^2 F^{a+b*c} x^m \Gamma[3+m, -(b*d*x*\text{Log}[F])])}{(b^3 d^3 \text{Log}[F]^3 (-(b*d*x*\text{Log}[F]))^m)} - \frac{(2*e*f F^{a+b*c} x^m \Gamma[2+m, -(b*d*x*\text{Log}[F])])}{(b^2 d^2 \text{Log}[F]^2 (-(b*d*x*\text{Log}[F]))^m)} + \frac{(e^2 F^{a+b*c} x^m \Gamma[1+m, -(b*d*x*\text{Log}[F])])}{(b*d \text{Log}[F] (-(b*d*x*\text{Log}[F]))^m)}$$

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs.  $2(139) = 278$ .

Time = 0.56 (sec) , antiderivative size = 433, normalized size of antiderivative = 3.12

method	result
meijerg	$-\frac{\ln(F)^{-3-m} (-bd)^{-m} F^{bc+a} f^2 \left( x^m (-bd)^m \ln(F)^m m(m^2+3m+2) \Gamma(m) (-bdx \ln(F))^{-m} - x^m (-bd)^m \ln(F)^m \left( b^2 d^2 x^2 \ln(F)^2 - \dots \right) \right)}{b^3 d^3}$

input `int(F^(a+b*(d*x+c))*x^m*(f*x+e)^2,x,method=_RETURNVERBOSE)`

output

```
-1/b^3/d^3*ln(F)^(-3-m)*(-b*d)^(-m)*F^(b*c+a)*f^2*(x^m*(-b*d)^m*ln(F)^m*(m^2+3*m+2)*GAMMA(m)*(-b*d*x*ln(F))^(-m)-x^m*(-b*d)^m*ln(F)^m*(b^2*d^2*x^2*ln(F)^2-m*b*d*x*ln(F)+m^2-2*b*d*x*ln(F)+3*m+2)*exp(b*d*x*ln(F))-x^m*(-b*d)^m*ln(F)^m*(m^2+3*m+2)*(-b*d*x*ln(F))^(-m)*GAMMA(m,-b*d*x*ln(F)))+2/b^2/d^2*ln(F)^(-2-m)*(-b*d)^(-m)*F^(b*c+a)*f*e*(x^m*(-b*d)^m*ln(F)^m*(1+m)*GAMMA(m)*(-b*d*x*ln(F))^(-m)+x^m*(-b*d)^m*ln(F)^m*(b*d*x*ln(F)-1-m)*exp(b*d*x*ln(F))-x^m*(-b*d)^m*ln(F)^m*(1+m)*(-b*d*x*ln(F))^(-m)*GAMMA(m,-b*d*x*ln(F)))-F^(b*c+a)*(-b*d)^(-m)*ln(F)^(-1-m)*e^2/b/d*(x^m*(-b*d)^m*ln(F)^m*m*GAMMA(m)*(-b*d*x*ln(F))^(-m)-x^m*(-b*d)^m*ln(F)^m*exp(b*d*x*ln(F))-x^m*(-b*d)^m*ln(F)^m*(b*d*x*ln(F))^(-m)*GAMMA(m,-b*d*x*ln(F)))
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.16

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx = \frac{((bdf^2m + 2bdf^2)x \log(F) - (b^2d^2f^2x^2 + 2b^2d^2efx) \log(F)^2) F^{bdx+bc+a} x^m - (b^2d^2e^2 \log(F)^2 + f^2m)}{b^3d^3 \log(F)}$$

input

```
integrate(F^(a+b*(d*x+c))*x^m*(f*x+e)^2,x, algorithm="fricas")
```

output

```
-(((b*d*f^2*m + 2*b*d*f^2)*x*log(F) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x)*log(F)^2)*F^(b*d*x + b*c + a)*x^m - (b^2*d^2*e^2*log(F)^2 + f^2*m^2 + 3*f^2*m + 2*f^2 - 2*(b*d*e*f*m + b*d*e*f)*log(F))*e^(-m*log(-b*d*log(F)) + (b*c + a)*log(F))*gamma(m + 1, -b*d*x*log(F)))/(b^3*d^3*log(F)^3)
```

### Sympy [F]

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx = \int F^{a+b(c+dx)} x^m (e+fx)^2 dx$$

input

```
integrate(F**(a+b*(d*x+c))*x**m*(f*x+e)**2,x)
```

output

```
Integral(F**(a + b*(c + d*x))*x**m*(e + f*x)**2, x)
```

**Maxima [A] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.88

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx = -(-bdx \log(F))^{-m-3} F^{bc+a} f^2 x^{m+3} \Gamma(m+3, -bdx \log(F)) - 2(-bdx \log(F))^{-m-2} F^{bc+a} e f x^{m+2} \Gamma(m+2, -bdx \log(F)) - (-bdx \log(F))^{-m-1} F^{bc+a} e^2 x^{m+1} \Gamma(m+1, -bdx \log(F))$$

input `integrate(F^(a+b*(d*x+c))*x^m*(f*x+e)^2,x, algorithm="maxima")`

output `-(-b*d*x*log(F))^(m+3)*F^(b*c+a)*f^2*x^(m+3)*gamma(m+3,-b*d*x*log(F)) - 2*(-b*d*x*log(F))^(m+2)*F^(b*c+a)*e*f*x^(m+2)*gamma(m+2,-b*d*x*log(F)) - (-b*d*x*log(F))^(m+1)*F^(b*c+a)*e^2*x^(m+1)*gamma(m+1,-b*d*x*log(F))`

**Giac [F]**

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx = \int (fx+e)^2 F^{(dx+c)b+a} x^m dx$$

input `integrate(F^(a+b*(d*x+c))*x^m*(f*x+e)^2,x, algorithm="giac")`

output `integrate((f*x+e)^2*F^((d*x+c)*b+a)*x^m,x)`

**Mupad [F(-1)]**

Timed out.

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx = \int F^{a+b(c+dx)} x^m (e+fx)^2 dx$$

input `int(F^(a + b*(c + d*x))*x^m*(e + f*x)^2,x)`output `int(F^(a + b*(c + d*x))*x^m*(e + f*x)^2, x)`**Reduce [F]**

$$\int F^{a+b(c+dx)} x^m (e+fx)^2 dx$$

$$= \frac{f^{bc+a} \left( x^m f^{bdx} \log(f)^2 b^2 d^2 e^2 + 2x^m f^{bdx} \log(f)^2 b^2 d^2 e f x + x^m f^{bdx} \log(f)^2 b^2 d^2 f^2 x^2 - 2x^m f^{bdx} \log(f) b d e f \right)}{\dots}$$

input `int(F^(a+b*(d*x+c))*x^m*(f*x+e)^2,x)`output `(f**(a + b*c)*(x**m*f**(b*d*x)*log(f)**2*b**2*d**2*e**2 + 2*x**m*f**(b*d*x)*log(f)**2*b**2*d**2*e*f*x + x**m*f**(b*d*x)*log(f)**2*b**2*d**2*f**2*x**2 - 2*x**m*f**(b*d*x)*log(f)*b*d*e*f*m - 2*x**m*f**(b*d*x)*log(f)*b*d*e*f - x**m*f**(b*d*x)*log(f)*b*d*f**2*m*x - 2*x**m*f**(b*d*x)*log(f)*b*d*f**2*x + x**m*f**(b*d*x)*f**2*m**2 + 3*x**m*f**(b*d*x)*f**2*m + 2*x**m*f**(b*d*x)*f**2 - int((x**m*f**(b*d*x))/x,x)*log(f)**2*b**2*d**2*e**2*m + 2*int((x**m*f**(b*d*x))/x,x)*log(f)*b*d*e*f*m**2 + 2*int((x**m*f**(b*d*x))/x,x)*log(f)*b*d*e*f*m - int((x**m*f**(b*d*x))/x,x)*f**2*m**3 - 3*int((x**m*f**(b*d*x))/x,x)*f**2*m**2 - 2*int((x**m*f**(b*d*x))/x,x)*f**2*m))/(log(f)**3*b**3*d**3)`

### 3.104 $\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$

Optimal result	692
Mathematica [A] (verified)	693
Rubi [A] (verified)	693
Maple [A] (verified)	695
Fricas [A] (verification not implemented)	695
Sympy [A] (verification not implemented)	696
Maxima [A] (verification not implemented)	697
Giac [C] (verification not implemented)	697
Mupad [B] (verification not implemented)	698
Reduce [B] (verification not implemented)	699

#### Optimal result

Integrand size = 22, antiderivative size = 219

$$\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx = -\frac{120f^2 F^{a+bc+bdx}}{b^6 d^6 \log^6(F)} + \frac{24f F^{a+bc+bdx} (2e + 5fx)}{b^5 d^5 \log^5(F)} - \frac{6F^{a+bc+bdx} (e^2 + 8efx + 10f^2 x^2)}{b^4 d^4 \log^4(F)} + \frac{2F^{a+bc+bdx} x (3e^2 + 12efx + 10f^2 x^2)}{b^3 d^3 \log^3(F)} - \frac{F^{a+bc+bdx} x^2 (3e^2 + 8efx + 5f^2 x^2)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx} x^3 (e + fx)^2}{bd \log(F)}$$

output

```
-120*f^2*F^(b*d*x+b*c+a)/b^6/d^6/ln(F)^6+24*f*F^(b*d*x+b*c+a)*(5*f*x+2*e)/
b^5/d^5/ln(F)^5-6*F^(b*d*x+b*c+a)*(10*f^2*x^2+8*e*f*x+e^2)/b^4/d^4/ln(F)^4
+2*F^(b*d*x+b*c+a)*x*(10*f^2*x^2+12*e*f*x+3*e^2)/b^3/d^3/ln(F)^3-F^(b*d*x+
b*c+a)*x^2*(5*f^2*x^2+8*e*f*x+3*e^2)/b^2/d^2/ln(F)^2+F^(b*d*x+b*c+a)*x^3*(
f*x+e)^2/b/d/ln(F)
```

**Mathematica [A] (verified)**

Time = 0.45 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.73

$$\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx$$

$$= \frac{F^{a+b(c+dx)} (-120f^2 + 24bdf(2e+5fx) \log(F) - 6b^2d^2(e^2 + 8efx + 10f^2x^2) \log^2(F) + 2b^3d^3x(3e^2 + 12efx + 10f^2x^2) \log^3(F) - b^4d^4x^2(3e^2 + 8efx + 5f^2x^2) \log^4(F) + b^5d^5x^3(e+fx)^2 \log^5(F))}{b^6d^6 \log^6(F)}$$

input `Integrate[F^(a + b*(c + d*x))*x^3*(e + f*x)^2,x]`

output `(F^(a + b*(c + d*x))*(-120*f^2 + 24*b*d*f*(2*e + 5*f*x)*Log[F] - 6*b^2*d^2*(e^2 + 8*e*f*x + 10*f^2*x^2)*Log[F]^2 + 2*b^3*d^3*x*(3*e^2 + 12*e*f*x + 10*f^2*x^2)*Log[F]^3 - b^4*d^4*x^2*(3*e^2 + 8*e*f*x + 5*f^2*x^2)*Log[F]^4 + b^5*d^5*x^3*(e + f*x)^2*Log[F]^5))/(b^6*d^6*Log[F]^6)`

**Rubi [A] (verified)**

Time = 1.63 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.89, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (e+fx)^2 F^{a+b(c+dx)} dx$$

$$\downarrow 2626$$

$$\int \left( e^2 x^3 F^{a+b(c+dx)} + 2efx^4 F^{a+b(c+dx)} + f^2 x^5 F^{a+b(c+dx)} \right) dx$$

$$\downarrow 2009$$

$$\begin{aligned}
& -\frac{120f^2F^{a+bc+bdx}}{b^6d^6\log^6(F)} + \frac{48efF^{a+bc+bdx}}{b^5d^5\log^5(F)} + \frac{120f^2xF^{a+bc+bdx}}{b^5d^5\log^5(F)} - \frac{6e^2F^{a+bc+bdx}}{b^4d^4\log^4(F)} - \frac{48efxF^{a+bc+bdx}}{b^4d^4\log^4(F)} \\
& -\frac{60f^2x^2F^{a+bc+bdx}}{b^4d^4\log^4(F)} + \frac{6e^2xF^{a+bc+bdx}}{b^3d^3\log^3(F)} + \frac{24efx^2F^{a+bc+bdx}}{b^3d^3\log^3(F)} + \frac{20f^2x^3F^{a+bc+bdx}}{b^3d^3\log^3(F)} - \frac{3e^2x^2F^{a+bc+bdx}}{b^2d^2\log^2(F)} \\
& -\frac{b^4d^4\log^4(F)}{8efx^3F^{a+bc+bdx}} + \frac{b^3d^3\log^3(F)}{5f^2x^4F^{a+bc+bdx}} + \frac{b^3d^3\log^3(F)}{e^2x^3F^{a+bc+bdx}} + \frac{b^3d^3\log^3(F)}{2efx^4F^{a+bc+bdx}} - \frac{b^2d^2\log^2(F)}{f^2x^5F^{a+bc+bdx}} \\
& -\frac{b^2d^2\log^2(F)}{b^2d^2\log^2(F)} + \frac{bd\log(F)}{bd\log(F)} + \frac{bd\log(F)}{bd\log(F)} + \frac{bd\log(F)}{bd\log(F)} + \frac{bd\log(F)}{bd\log(F)}
\end{aligned}$$

input `Int[F^(a + b*(c + d*x))*x^3*(e + f*x)^2,x]`

output  $(-120f^2F^{a+bc+bdx})/(b^6d^6\log[F]^6) + (48efF^{a+bc+bdx})/(b^5d^5\log[F]^5) + (120f^2F^{a+bc+bdx}x)/(b^5d^5\log[F]^5) - (6e^2F^{a+bc+bdx})/(b^4d^4\log[F]^4) - (48efF^{a+bc+bdx}x)/(b^4d^4\log[F]^4) - (60f^2F^{a+bc+bdx}x^2)/(b^4d^4\log[F]^4) + (6e^2F^{a+bc+bdx}x)/(b^3d^3\log[F]^3) + (24efF^{a+bc+bdx}x^2)/(b^3d^3\log[F]^3) + (20f^2F^{a+bc+bdx}x^3)/(b^3d^3\log[F]^3) - (3e^2F^{a+bc+bdx}x^2)/(b^2d^2\log[F]^2) - (8efF^{a+bc+bdx}x^3)/(b^2d^2\log[F]^2) - (5f^2F^{a+bc+bdx}x^4)/(b^2d^2\log[F]^2) + (e^2F^{a+bc+bdx}x^3)/(b*d*\log[F]) + (2efF^{a+bc+bdx}x^4)/(b*d*\log[F]) + (f^2F^{a+bc+bdx}x^5)/(b*d*\log[F])$

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.14

method	result
gospers	$(f^2 x^5 \ln(F)^5 b^5 d^5 + 2 \ln(F)^5 b^5 d^5 e f x^4 + \ln(F)^5 b^5 d^5 e^2 x^3 - 5 \ln(F)^4 b^4 d^4 f^2 x^4 - 8 \ln(F)^4 b^4 d^4 e f x^3 - 3 \ln(F)^4 b^4 d^4 e^2 x^2 + 20 \ln(F)^3 b^3 d^3 f^2 x^3 + 24 \ln(F)^3 b^3 d^3 e f x^2 + 6 \ln(F)^3 b^3 d^3 e^2 x - 60 \ln(F)^2 b^2 d^2 f^2 x^2 - 48 \ln(F)^2 b^2 d^2 e f x - 6 \ln(F)^2 b^2 d^2 e^2 + 120 \ln(F) b d f^2 x + 48 e f \ln(F) b d - 120 f^2) F^{b d x + b c + a} / \ln(F)^6 b^6 d^6$
risch	$(f^2 x^5 \ln(F)^5 b^5 d^5 + 2 \ln(F)^5 b^5 d^5 e f x^4 + \ln(F)^5 b^5 d^5 e^2 x^3 - 5 \ln(F)^4 b^4 d^4 f^2 x^4 - 8 \ln(F)^4 b^4 d^4 e f x^3 - 3 \ln(F)^4 b^4 d^4 e^2 x^2 + 20 \ln(F)^3 b^3 d^3 f^2 x^3 + 24 \ln(F)^3 b^3 d^3 e f x^2 + 6 \ln(F)^3 b^3 d^3 e^2 x - 60 \ln(F)^2 b^2 d^2 f^2 x^2 - 48 \ln(F)^2 b^2 d^2 e f x - 6 \ln(F)^2 b^2 d^2 e^2 + 120 \ln(F) b d f^2 x + 48 e f \ln(F) b d - 120 f^2) F^{b d x + b c + a} / \ln(F)^6 b^6 d^6$
orering	$(f^2 x^5 \ln(F)^5 b^5 d^5 + 2 \ln(F)^5 b^5 d^5 e f x^4 + \ln(F)^5 b^5 d^5 e^2 x^3 - 5 \ln(F)^4 b^4 d^4 f^2 x^4 - 8 \ln(F)^4 b^4 d^4 e f x^3 - 3 \ln(F)^4 b^4 d^4 e^2 x^2 + 20 \ln(F)^3 b^3 d^3 f^2 x^3 + 24 \ln(F)^3 b^3 d^3 e f x^2 + 6 \ln(F)^3 b^3 d^3 e^2 x - 60 \ln(F)^2 b^2 d^2 f^2 x^2 - 48 \ln(F)^2 b^2 d^2 e f x - 6 \ln(F)^2 b^2 d^2 e^2 + 120 \ln(F) b d f^2 x + 48 e f \ln(F) b d - 120 f^2) F^{b d x + b c + a} / \ln(F)^6 b^6 d^6$
meijerg	$F^{bc+a} f^2 \left( 120 - \frac{(-6b^5 d^5 x^5 \ln(F)^5 + 30b^4 d^4 x^4 \ln(F)^4 - 120b^3 d^3 x^3 \ln(F)^3 + 360b^2 d^2 x^2 \ln(F)^2 - 720bdx \ln(F) + 720) e^{bdx \ln(F)}}{6} \right) - 2F^{bc+a} \ln(F)^6 b^6 d^6$
norman	$\frac{f^2 x^5 e^{(a+b(dx+c)) \ln(F)}}{bd \ln(F)} + \frac{(\ln(F)^2 b^2 d^2 e^2 - 8ef \ln(F) bd + 20f^2) x^3 e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^3 b^3 d^3} + \frac{f(2 \ln(F) b d e - 5f) x^4 e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^2 b^2 d^2}$
parallelrisch	$x^5 F^{bdx+bc+a} f^2 \ln(F)^5 b^5 d^5 + 2 \ln(F)^5 x^4 F^{bdx+bc+a} b^5 d^5 e f + \ln(F)^5 x^3 F^{bdx+bc+a} b^5 d^5 e^2 - 5 \ln(F)^4 x^4 F^{bdx+bc+a} b^4 d^4 f^2 - 8 \ln(F)^4 x^3 F^{bdx+bc+a} b^4 d^4 e f - 3 \ln(F)^4 x^2 F^{bdx+bc+a} b^4 d^4 e^2 + 20 \ln(F)^3 x^3 F^{bdx+bc+a} b^3 d^3 f^2 + 24 \ln(F)^3 x^2 F^{bdx+bc+a} b^3 d^3 e f + 6 \ln(F)^3 x F^{bdx+bc+a} b^3 d^3 e^2 - 60 \ln(F)^2 x^2 F^{bdx+bc+a} b^2 d^2 f^2 - 48 \ln(F)^2 x F^{bdx+bc+a} b^2 d^2 e f - 6 \ln(F)^2 F^{bdx+bc+a} b^2 d^2 e^2 + 120 \ln(F) x F^{bdx+bc+a} b d f^2 + 48 e f \ln(F) b d - 120 f^2) F^{b d x + b c + a} / \ln(F)^6 b^6 d^6$

```
input int(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
output (f^2*x^5*ln(F)^5*b^5*d^5+2*ln(F)^5*b^5*d^5*e*f*x^4+ln(F)^5*b^5*d^5*e^2*x^3-5*ln(F)^4*b^4*d^4*f^2*x^4-8*ln(F)^4*b^4*d^4*e*f*x^3-3*ln(F)^4*b^4*d^4*e^2*x^2+20*ln(F)^3*b^3*d^3*f^2*x^3+24*ln(F)^3*b^3*d^3*e*f*x^2+6*ln(F)^3*b^3*d^3*e^2*x-60*ln(F)^2*b^2*d^2*f^2*x^2-48*ln(F)^2*b^2*d^2*e*f*x-6*ln(F)^2*b^2*d^2*e^2+120*ln(F)*b*d*f^2*x+48*e*f*ln(F)*b*d-120*f^2)*F^(b*d*x+b*c+a)/ln(F)^6/b^6/d^6
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.04

$$\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$$

$$= \frac{((b^5 d^5 f^2 x^5 + 2 b^5 d^5 e f x^4 + b^5 d^5 e^2 x^3) \log(F)^5 - (5 b^4 d^4 f^2 x^4 + 8 b^4 d^4 e f x^3 + 3 b^4 d^4 e^2 x^2) \log(F)^4 + 2 (10 b^3 d^3 f^2 x^3 + 24 b^3 d^3 e f x^2 + 6 b^3 d^3 e^2 x - 60 b^2 d^2 f^2 x^2 - 48 b^2 d^2 e f x - 6 b^2 d^2 e^2 + 120 b d f^2 x + 48 e f \ln(F) b d - 120 f^2) F^{b d x + b c + a}}{\ln(F)^6 b^6 d^6}$$

```
input integrate(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x,algorithm="fricas")
```



output

```
((b^5*d^5*f^2*x^5 + 2*b^5*d^5*e*f*x^4 + b^5*d^5*e^2*x^3)*log(F)^5 - (5*b^4*d^4*f^2*x^4 + 8*b^4*d^4*e*f*x^3 + 3*b^4*d^4*e^2*x^2)*log(F)^4 + 2*(10*b^3*d^3*f^2*x^3 + 12*b^3*d^3*e*f*x^2 + 3*b^3*d^3*e^2*x)*log(F)^3 - 6*(10*b^2*d^2*f^2*x^2 + 8*b^2*d^2*e*f*x + b^2*d^2*e^2)*log(F)^2 - 120*f^2 + 24*(5*b*d*f^2*x + 2*b*d*e*f)*log(F))*F^(b*d*x + b*c + a)/(b^6*d^6*log(F)^6)
```

### Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.47

$$\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$$

$$= \left\{ \frac{F^{a+b(c+dx)} (b^5 d^5 e^2 x^3 \log(F)^5 + 2b^5 d^5 e f x^4 \log(F)^5 + b^5 d^5 f^2 x^5 \log(F)^5 - 3b^4 d^4 e^2 x^2 \log(F)^4 - 8b^4 d^4 e f x^3 \log(F)^4 - 5b^4 d^4 f^2 x^4 \log(F)^4 + 6b^4 d^4 e^2 x \log(F)^4 - 120 f^2 + 24(5 b d f^2 x + 2 b d e f) \log(F))}{b^6 d^6 \log(F)^6}, \text{Ne}(b^6 d^6 \log(F)^6, 0) \right\}$$

$$= \left\{ \frac{e^2 x^4}{4} + \frac{2 e f x^5}{5} + \frac{f^2 x^6}{6} \right\}$$

input

```
integrate(F**(a+b*(d*x+c))*x**3*(f*x+e)**2,x)
```

output

```
Piecewise((F**(a + b*(c + d*x))*(b**5*d**5*e**2*x**3*log(F)**5 + 2*b**5*d**5*e*f*x**4*log(F)**5 + b**5*d**5*f**2*x**5*log(F)**5 - 3*b**4*d**4*e**2*x**2*log(F)**4 - 8*b**4*d**4*e*f*x**3*log(F)**4 - 5*b**4*d**4*f**2*x**4*log(F)**4 + 6*b**3*d**3*e**2*x*log(F)**3 + 24*b**3*d**3*e*f*x**2*log(F)**3 + 20*b**3*d**3*f**2*x**3*log(F)**3 - 6*b**2*d**2*e**2*log(F)**2 - 48*b**2*d**2*e*f*x*log(F)**2 - 60*b**2*d**2*f**2*x**2*log(F)**2 + 48*b*d*e*f*log(F) + 120*b*d*f**2*x*log(F) - 120*f**2)/(b**6*d**6*log(F)**6), Ne(b**6*d**6*log(F)**6, 0)), (e**2*x**4/4 + 2*e*f*x**5/5 + f**2*x**6/6, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.50

$$\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx$$

$$= \frac{(F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6 F^{bc+a} b d x \log(F) - 6 F^{bc+a}) F^{bdx} e^2}{b^4 d^4 \log(F)^4}$$

$$+ \frac{2 (F^{bc+a} b^4 d^4 x^4 \log(F)^4 - 4 F^{bc+a} b^3 d^3 x^3 \log(F)^3 + 12 F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 24 F^{bc+a} b d x \log(F) + 24 F^{bc+a}) F^{bdx} e^2}{b^5 d^5 \log(F)^5}$$

$$+ \frac{(F^{bc+a} b^5 d^5 x^5 \log(F)^5 - 5 F^{bc+a} b^4 d^4 x^4 \log(F)^4 + 20 F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 60 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 120 F^{bc+a} b d x \log(F) - 120 F^{bc+a}) F^{bdx} e^2}{b^6 d^6 \log(F)^6}$$

input `integrate(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x, algorithm="maxima")`

output `(F^(b*c + a)*b^3*d^3*x^3*log(F)^3 - 3*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 + 6*F^(b*c + a)*b*d*x*log(F) - 6*F^(b*c + a))*F^(b*d*x)*e^2/(b^4*d^4*log(F)^4) + 2*(F^(b*c + a)*b^4*d^4*x^4*log(F)^4 - 4*F^(b*c + a)*b^3*d^3*x^3*log(F)^3 + 12*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 - 24*F^(b*c + a)*b*d*x*log(F) + 24*F^(b*c + a))*F^(b*d*x)*e*f/(b^5*d^5*log(F)^5) + (F^(b*c + a)*b^5*d^5*x^5*log(F)^5 - 5*F^(b*c + a)*b^4*d^4*x^4*log(F)^4 + 20*F^(b*c + a)*b^3*d^3*x^3*log(F)^3 - 60*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 + 120*F^(b*c + a)*b*d*x*log(F) - 120*F^(b*c + a))*F^(b*d*x)*f^2/(b^6*d^6*log(F)^6)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 9584, normalized size of antiderivative = 43.76

$$\int F^{a+b(c+dx)} x^3 (e + fx)^2 dx = \text{Too large to display}$$

input `integrate(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x, algorithm="giac")`

output

```

-(((5*pi^4*b^5*d^5*f^2*x^5*log(abs(F))*sgn(F) - 10*pi^2*b^5*d^5*f^2*x^5*log(abs(F))^3*sgn(F) - 5*pi^4*b^5*d^5*f^2*x^5*log(abs(F)) + 10*pi^2*b^5*d^5*f^2*x^5*log(abs(F))^3 - 2*b^5*d^5*f^2*x^5*log(abs(F))^5 + 10*pi^4*b^5*d^5*e*f*x^4*log(abs(F))*sgn(F) - 20*pi^2*b^5*d^5*e*f*x^4*log(abs(F))^3*sgn(F) - 10*pi^4*b^5*d^5*e*f*x^4*log(abs(F)) + 20*pi^2*b^5*d^5*e*f*x^4*log(abs(F))^3 - 4*b^5*d^5*e*f*x^4*log(abs(F))^5 + 5*pi^4*b^5*d^5*e^2*x^3*log(abs(F))*sgn(F) - 10*pi^2*b^5*d^5*e^2*x^3*log(abs(F))^3*sgn(F) - 5*pi^4*b^5*d^5*e^2*x^3*log(abs(F)) + 10*pi^2*b^5*d^5*e^2*x^3*log(abs(F))^3 - 2*b^5*d^5*e^2*x^3*log(abs(F))^5 - 5*pi^4*b^4*d^4*f^2*x^4*sgn(F) + 30*pi^2*b^4*d^4*f^2*x^4*log(abs(F))^2*sgn(F) + 5*pi^4*b^4*d^4*f^2*x^4 - 30*pi^2*b^4*d^4*f^2*x^4*log(abs(F))^2 + 10*b^4*d^4*f^2*x^4*log(abs(F))^4 - 8*pi^4*b^4*d^4*e*f*x^3*sgn(F) + 48*pi^2*b^4*d^4*e*f*x^3*log(abs(F))^2*sgn(F) + 8*pi^4*b^4*d^4*e*f*x^3 - 48*pi^2*b^4*d^4*e*f*x^3*log(abs(F))^2 + 16*b^4*d^4*e*f*x^3*log(abs(F))^4 - 3*pi^4*b^4*d^4*e^2*x^2*sgn(F) + 18*pi^2*b^4*d^4*e^2*x^2*log(abs(F))^2*sgn(F) + 3*pi^4*b^4*d^4*e^2*x^2 - 18*pi^2*b^4*d^4*e^2*x^2*log(abs(F))^2 + 6*b^4*d^4*e^2*x^2*log(abs(F))^4 - 60*pi^2*b^3*d^3*f^2*x^3*log(abs(F))*sgn(F) + 60*pi^2*b^3*d^3*f^2*x^3*log(abs(F)) - 40*b^3*d^3*f^2*x^3*log(abs(F))^3 - 72*pi^2*b^3*d^3*e*f*x^2*log(abs(F))*sgn(F) + 72*pi^2*b^3*d^3*e*f*x^2*log(abs(F)) - 48*b^3*d^3*e*f*x^2*log(abs(F))^3 - 18*pi^2*b^3*d^3*e^2*x*log(abs(F))*sgn(F) + 18*pi^2*b^3*d^3*e^2*x*log(abs(F)) - 12*b^3*d^3*e^2...

```

### Mupad [B] (verification not implemented)

Time = 22.65 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.14

$$\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx$$

$$= \frac{F^{a+b(c+dx)} (b^5 d^5 e^2 x^3 \ln(F)^5 + 2 b^5 d^5 e f x^4 \ln(F)^5 + b^5 d^5 f^2 x^5 \ln(F)^5 - 3 b^4 d^4 e^2 x^2 \ln(F)^4 - 8 b^4 d^4 e f x^3 \ln(F)^4 - 12 b^3 d^3 e^2 x \ln(F)^3 - 12 b^3 d^3 e f x^2 \ln(F)^3 - 8 b^2 d^2 e^2 \ln(F)^2 - 24 b^2 d^2 e f x \ln(F)^2 - 12 b^2 d^2 f^2 x^2 \ln(F)^2 + 20 b^3 d^3 f^2 x^3 \ln(F)^3 - 5 b^4 d^4 f^2 x^4 \ln(F)^4 + b^5 d^5 f^2 x^5 \ln(F)^5 + 48 b^4 d^4 e f \ln(F)^3 - 48 b^2 d^2 e f x \ln(F)^2 + 24 b^3 d^3 e f x^2 \ln(F)^3 - 8 b^4 d^4 e f x^3 \ln(F)^4 + 2 b^5 d^5 e f x^4 \ln(F)^5)}{(b^6 d^6 \ln(F))^6}$$

input

```
int(F^(a + b*(c + d*x))*x^3*(e + f*x)^2,x)
```

output

```

(F^(a + b*c + b*d*x)*(120*b*d*f^2*x*log(F) - 6*b^2*d^2*e^2*log(F)^2 - 120*f^2 + 6*b^3*d^3*e^2*x*log(F)^3 - 3*b^4*d^4*e^2*x^2*log(F)^4 + b^5*d^5*e^2*x^3*log(F)^5 - 60*b^2*d^2*f^2*x^2*log(F)^2 + 20*b^3*d^3*f^2*x^3*log(F)^3 - 5*b^4*d^4*f^2*x^4*log(F)^4 + b^5*d^5*f^2*x^5*log(F)^5 + 48*b*d*e*f*log(F) - 48*b^2*d^2*e*f*x*log(F)^2 + 24*b^3*d^3*e*f*x^2*log(F)^3 - 8*b^4*d^4*e*f*x^3*log(F)^4 + 2*b^5*d^5*e*f*x^4*log(F)^5))/(b^6*d^6*log(F)^6)

```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.14

$$\int F^{a+b(c+dx)} x^3 (e+fx)^2 dx$$

$$= \frac{f^{bdx+bc+a} (\log(f)^5 b^5 d^5 e^2 x^3 + 2\log(f)^5 b^5 d^5 e f x^4 + \log(f)^5 b^5 d^5 f^2 x^5 - 3\log(f)^4 b^4 d^4 e^2 x^2 - 8\log(f)^4 b^4 d^4 e f x^3 - 5\log(f)^4 b^4 d^4 f^2 x^4 + 6\log(f)^3 b^3 d^3 e^2 x + 24\log(f)^3 b^3 d^3 e f x^2 + 20\log(f)^3 b^3 d^3 f^2 x^3 - 6\log(f)^2 b^2 d^2 e^2 x - 48\log(f)^2 b^2 d^2 e f x^2 + 48\log(f) b d e f x + 120\log(f) b d f^2 x - 120 f^2)}{(\log(f))^6 b^6 d^6}$$

input `int(F^(a+b*(d*x+c))*x^3*(f*x+e)^2,x)`output `(f**(a + b*c + b*d*x)*(log(f)**5*b**5*d**5*e**2*x**3 + 2*log(f)**5*b**5*d**5*e*f*x**4 + log(f)**5*b**5*d**5*f**2*x**5 - 3*log(f)**4*b**4*d**4*e**2*x**2 - 8*log(f)**4*b**4*d**4*e*f*x**3 - 5*log(f)**4*b**4*d**4*f**2*x**4 + 6*log(f)**3*b**3*d**3*e**2*x + 24*log(f)**3*b**3*d**3*e*f*x**2 + 20*log(f)**3*b**3*d**3*f**2*x**3 - 6*log(f)**2*b**2*d**2*e**2 - 48*log(f)**2*b**2*d**2*e*f*x - 60*log(f)**2*b**2*d**2*f**2*x**2 + 48*log(f)*b*d*e*f + 120*log(f)*b*d*f**2*x - 120*f**2))/(log(f)**6*b**6*d**6)`

### 3.105 $\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx$

Optimal result	700
Mathematica [A] (verified)	701
Rubi [A] (verified)	701
Maple [A] (verified)	702
Fricas [A] (verification not implemented)	703
Sympy [A] (verification not implemented)	704
Maxima [A] (verification not implemented)	704
Giac [C] (verification not implemented)	705
Mupad [B] (verification not implemented)	706
Reduce [B] (verification not implemented)	707

#### Optimal result

Integrand size = 22, antiderivative size = 170

$$\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx = \frac{24f^2 F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} - \frac{12f F^{a+bc+bdx} (e + 2fx)}{b^4 d^4 \log^4(F)} + \frac{2F^{a+bc+bdx} (e^2 + 6efx + 6f^2 x^2)}{b^3 d^3 \log^3(F)} - \frac{2F^{a+bc+bdx} x (e^2 + 3efx + 2f^2 x^2)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx} x^2 (e + fx)^2}{bd \log(F)}$$

output

```
24*f^2*F^(b*d*x+b*c+a)/b^5/d^5/ln(F)^5-12*f*F^(b*d*x+b*c+a)*(2*f*x+e)/b^4/d^4/ln(F)^4+2*F^(b*d*x+b*c+a)*(6*f^2*x^2+6*e*f*x+e^2)/b^3/d^3/ln(F)^3-2*F^(b*d*x+b*c+a)*x*(2*f^2*x^2+3*e*f*x+e^2)/b^2/d^2/ln(F)^2+F^(b*d*x+b*c+a)*x^2*(f*x+e)^2/b/d/ln(F)
```

**Mathematica [A] (verified)**

Time = 0.42 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.71

$$\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx$$

$$= \frac{F^{a+b(c+dx)} (24f^2 - 12bdf(e + 2fx) \log(F) + 2b^2 d^2 (e^2 + 6efx + 6f^2 x^2) \log^2(F) - 2b^3 d^3 x (e^2 + 3efx + 2f^2 x^2) \log^3(F) + b^4 d^4 x^2 (e + fx)^2 \log^4(F))}{b^5 d^5 \log^5(F)}$$

input

```
Integrate[F^(a + b*(c + d*x))*x^2*(e + f*x)^2,x]
```

output

```
(F^(a + b*(c + d*x))*(24*f^2 - 12*b*d*f*(e + 2*f*x)*Log[F] + 2*b^2*d^2*(e^2 + 6*e*f*x + 6*f^2*x^2)*Log[F]^2 - 2*b^3*d^3*x*(e^2 + 3*e*f*x + 2*f^2*x^2)*Log[F]^3 + b^4*d^4*x^2*(e + f*x)^2*Log[F]^4))/(b^5*d^5*Log[F]^5)
```

**Rubi [A] (verified)**

Time = 1.45 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.93, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (e + fx)^2 F^{a+b(c+dx)} dx$$

$$\downarrow 2626$$

$$\int \left( e^2 x^2 F^{a+b(c+dx)} + 2efx^3 F^{a+b(c+dx)} + f^2 x^4 F^{a+b(c+dx)} \right) dx$$

$$\downarrow 2009$$

$$\frac{24f^2 F^{a+bc+bdx}}{b^5 d^5 \log^5(F)} - \frac{12ef F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} - \frac{24f^2 x F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{2e^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{12efx F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} +$$

$$\frac{12f^2 x^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{2e^2 x F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{6efx^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4f^2 x^3 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{e^2 x^2 F^{a+bc+bdx}}{bd \log(F)} +$$

$$\frac{2efx^3 F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 x^4 F^{a+bc+bdx}}{bd \log(F)}$$

input `Int[F^(a + b*(c + d*x))*x^2*(e + f*x)^2,x]`

output `(24*f^2*F^(a + b*c + b*d*x))/(b^5*d^5*Log[F]^5) - (12*e*f*F^(a + b*c + b*d*x))/(b^4*d^4*Log[F]^4) - (24*f^2*F^(a + b*c + b*d*x)*x)/(b^4*d^4*Log[F]^4) + (2*e^2*F^(a + b*c + b*d*x))/(b^3*d^3*Log[F]^3) + (12*e*f*F^(a + b*c + b*d*x)*x)/(b^3*d^3*Log[F]^3) + (12*f^2*F^(a + b*c + b*d*x)*x^2)/(b^3*d^3*Log[F]^3) - (2*e^2*F^(a + b*c + b*d*x)*x)/(b^2*d^2*Log[F]^2) - (6*e*f*F^(a + b*c + b*d*x)*x^2)/(b^2*d^2*Log[F]^2) - (4*f^2*F^(a + b*c + b*d*x)*x^3)/(b^2*d^2*Log[F]^2) + (e^2*F^(a + b*c + b*d*x)*x^2)/(b*d*Log[F]) + (2*e*f*F^(a + b*c + b*d*x)*x^3)/(b*d*Log[F]) + (f^2*F^(a + b*c + b*d*x)*x^4)/(b*d*Log[F])`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.16

method	result
gospers	$\frac{(\ln(F)^4 b^4 d^4 f^2 x^4 + 2 \ln(F)^4 b^4 d^4 e f x^3 + \ln(F)^4 b^4 d^4 e^2 x^2 - 4 \ln(F)^3 b^3 d^3 f^2 x^3 - 6 \ln(F)^3 b^3 d^3 e f x^2 - 2 \ln(F)^3 b^3 d^3 e^2 x + 12 \ln(F)^2 b^2 d^2 e^2 x^2 - 24 \ln(F)^2 b^2 d^2 e f x + 12 \ln(F)^2 b^2 d^2 e^2 x^2 - 24 \ln(F) b d^2 f^2 x^2 + 12 \ln(F) b d^2 e f x + 2 \ln(F) b d^2 e^2 x - 24 \ln(F) b d f^2 x - 12 \ln(F) b d e f x + 24 \ln(F) b d e^2 x - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^4 d^4 e f + \ln(F)^4 x^2 F^{bdx+bc+a} b^4 d^4 e^2 - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x}{\ln(F)^5 b^5 d^5}$
risch	$\frac{(\ln(F)^4 b^4 d^4 f^2 x^4 + 2 \ln(F)^4 b^4 d^4 e f x^3 + \ln(F)^4 b^4 d^4 e^2 x^2 - 4 \ln(F)^3 b^3 d^3 f^2 x^3 - 6 \ln(F)^3 b^3 d^3 e f x^2 - 2 \ln(F)^3 b^3 d^3 e^2 x + 12 \ln(F)^2 b^2 d^2 e^2 x^2 - 24 \ln(F)^2 b^2 d^2 e f x + 12 \ln(F)^2 b^2 d^2 e^2 x^2 - 24 \ln(F) b d^2 f^2 x^2 + 12 \ln(F) b d^2 e f x + 2 \ln(F) b d^2 e^2 x - 24 \ln(F) b d f^2 x - 12 \ln(F) b d e f x + 24 \ln(F) b d e^2 x - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^4 d^4 e f + \ln(F)^4 x^2 F^{bdx+bc+a} b^4 d^4 e^2 - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x}{\ln(F)^5 b^5 d^5}$
orering	$\frac{(\ln(F)^4 b^4 d^4 f^2 x^4 + 2 \ln(F)^4 b^4 d^4 e f x^3 + \ln(F)^4 b^4 d^4 e^2 x^2 - 4 \ln(F)^3 b^3 d^3 f^2 x^3 - 6 \ln(F)^3 b^3 d^3 e f x^2 - 2 \ln(F)^3 b^3 d^3 e^2 x + 12 \ln(F)^2 b^2 d^2 e^2 x^2 - 24 \ln(F)^2 b^2 d^2 e f x + 12 \ln(F)^2 b^2 d^2 e^2 x^2 - 24 \ln(F) b d^2 f^2 x^2 + 12 \ln(F) b d^2 e f x + 2 \ln(F) b d^2 e^2 x - 24 \ln(F) b d f^2 x - 12 \ln(F) b d e f x + 24 \ln(F) b d e^2 x - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^4 d^4 e f + \ln(F)^4 x^2 F^{bdx+bc+a} b^4 d^4 e^2 - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x}{\ln(F)^5 b^5 d^5}$
meijerg	$-\frac{F^{bc+a} f^2 \left( 24 - \frac{(5b^4 d^4 x^4 \ln(F)^4 - 20b^3 d^3 x^3 \ln(F)^3 + 60b^2 d^2 x^2 \ln(F)^2 - 120bdx \ln(F) + 120) e^{bdx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 d^5} + \frac{2F^{bc+a} f e \left( 6 - \frac{(-4b^3 d^3 x^3 \ln(F)^3 + 12b^2 d^2 x^2 \ln(F)^2 - 24bdx \ln(F) + 12) e^{bdx \ln(F)}}{5} \right)}{\ln(F)^5 b^5 d^5}$
norman	$\frac{f^2 x^4 e^{(a+b(dx+c)) \ln(F)}}{bd \ln(F)} + \frac{(\ln(F)^2 b^2 d^2 e^2 - 6ef \ln(F) bd + 12f^2) x^2 e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^3 b^3 d^3} + \frac{2(\ln(F)^2 b^2 d^2 e^2 - 6ef \ln(F) bd + 12f^2) x^2 e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^5 b^5 d^5}$
parallelrisch	$\frac{\ln(F)^4 x^4 F^{bdx+bc+a} b^4 d^4 f^2 + 2 \ln(F)^4 x^3 F^{bdx+bc+a} b^4 d^4 e f + \ln(F)^4 x^2 F^{bdx+bc+a} b^4 d^4 e^2 - 4 \ln(F)^3 x^3 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F)^2 x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 - 4 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f - 2 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 f^2 x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e f x + 12 \ln(F) x^2 F^{bdx+bc+a} b^3 d^3 e^2 x}{\ln(F)^5 b^5 d^5}$

```
input int(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
output (ln(F)^4*b^4*d^4*f^2*x^4+2*ln(F)^4*b^4*d^4*e*f*x^3+ln(F)^4*b^4*d^4*e^2*x^2-4*ln(F)^3*b^3*d^3*f^2*x^3-6*ln(F)^3*b^3*d^3*e*f*x^2-2*ln(F)^3*b^3*d^3*e^2*x+12*ln(F)^2*b^2*d^2*f^2*x^2+12*ln(F)^2*b^2*d^2*e*f*x+2*ln(F)^2*b^2*d^2*e^2*x-24*ln(F)*b*d*f^2*x-12*e*f*ln(F)*b*d+24*f^2)*F^(b*d*x+b*c+a)/ln(F)^5/b^5/d^5
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.05

$$\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx = \frac{((b^4 d^4 f^2 x^4 + 2 b^4 d^4 e f x^3 + b^4 d^4 e^2 x^2) \log(F)^4 - 2(2 b^3 d^3 f^2 x^3 + 3 b^3 d^3 e f x^2 + b^3 d^3 e^2 x) \log(F)^3 + 2(6 b^2 d^2 f^2 x^2 + 12 b^2 d^2 e f x + 6 b^2 d^2 e^2 x) \log(F)^2 - 24 b^2 d^2 f^2 x \log(F) - 12 b^2 d^2 e f \log(F) + 24 b^2 d^2 f^2) F^{(b d x + b c + a)}}{b^5 d^5 \log(F)^5}$$

```
input integrate(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x, algorithm="fricas")
```



output

```
((b^4*d^4*f^2*x^4 + 2*b^4*d^4*e*f*x^3 + b^4*d^4*e^2*x^2)*log(F)^4 - 2*(2*b^3*d^3*f^2*x^3 + 3*b^3*d^3*e*f*x^2 + b^3*d^3*e^2*x)*log(F)^3 + 2*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e*f*x + b^2*d^2*e^2)*log(F)^2 + 24*f^2 - 12*(2*b*d*f^2*x + b*d*e*f)*log(F))*F^(b*d*x + b*c + a)/(b^5*d^5*log(F)^5)
```

**Sympy [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.53

$$\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx$$

$$= \frac{F^{a+b(c+dx)} (b^4 d^4 e^2 x^2 \log(F)^4 + 2b^4 d^4 e f x^3 \log(F)^4 + b^4 d^4 f^2 x^4 \log(F)^4 - 2b^3 d^3 e^2 x \log(F)^3 - 6b^3 d^3 e f x^2 \log(F)^3 - 4b^3 d^3 f^2 x^3 \log(F)^3 + 2b^2 d^2 e^2 x \log(F)^2 + 24f^2 - 12(2b d f^2 x + b d e f) \log(F)) F^{b d x + b c + a}}{b^5 d^5 \log(F)^5}$$

$$= \frac{e^2 x^3}{3} + \frac{e f x^4}{2} + \frac{f^2 x^5}{5}$$

input

```
integrate(F**(a+b*(d*x+c))*x**2*(f*x+e)**2,x)
```

output

```
Piecewise((F**(a + b*(c + d*x))*(b**4*d**4*e**2*x**2*log(F)**4 + 2*b**4*d**4*e*f*x**3*log(F)**4 + b**4*d**4*f**2*x**4*log(F)**4 - 2*b**3*d**3*e**2*x**2*log(F)**3 - 6*b**3*d**3*e*f*x**2*log(F)**3 - 4*b**3*d**3*f**2*x**3*log(F)**3 + 2*b**2*d**2*e**2*log(F)**2 + 12*b**2*d**2*e*f*x*log(F)**2 + 12*b**2*d**2*f**2*x**2*log(F)**2 - 12*b*d*e*f*log(F) - 24*b*d*f**2*x*log(F) + 24*f**2)/(b**5*d**5*log(F)**5), Ne(b**5*d**5*log(F)**5, 0)), (e**2*x**3/3 + e*f*x**4/2 + f**2*x**5/5, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.54

$$\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx$$

$$= \frac{(F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 2 F^{bc+a} b d x \log(F) + 2 F^{bc+a}) F^{bdx} e^2}{b^3 d^3 \log(F)^3}$$

$$+ \frac{2 (F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3 F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6 F^{bc+a} b d x \log(F) - 6 F^{bc+a}) F^{bdx} e f}{b^4 d^4 \log(F)^4}$$

$$+ \frac{(F^{bc+a} b^4 d^4 x^4 \log(F)^4 - 4 F^{bc+a} b^3 d^3 x^3 \log(F)^3 + 12 F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 24 F^{bc+a} b d x \log(F) + 24 F^{bc+a}) F^{bdx} e^2}{b^5 d^5 \log(F)^5}$$

input `integrate(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x, algorithm="maxima")`

output  $(F^{(b*c + a)}*b^2*d^2*x^2*\log(F)^2 - 2*F^{(b*c + a)}*b*d*x*\log(F) + 2*F^{(b*c + a)}*F^{(b*d*x)}*e^2/(b^3*d^3*\log(F)^3) + 2*(F^{(b*c + a)}*b^3*d^3*x^3*\log(F)^3 - 3*F^{(b*c + a)}*b^2*d^2*x^2*\log(F)^2 + 6*F^{(b*c + a)}*b*d*x*\log(F) - 6*F^{(b*c + a)}*F^{(b*d*x)}*e*f/(b^4*d^4*\log(F)^4) + (F^{(b*c + a)}*b^4*d^4*x^4*\log(F)^4 - 4*F^{(b*c + a)}*b^3*d^3*x^3*\log(F)^3 + 12*F^{(b*c + a)}*b^2*d^2*x^2*\log(F)^2 - 24*F^{(b*c + a)}*b*d*x*\log(F) + 24*F^{(b*c + a)}*F^{(b*d*x)}*f^2/(b^5*d^5*\log(F)^5)$

### Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 6582, normalized size of antiderivative = 38.72

$$\int F^{a+b(c+dx)} x^2 (e + fx)^2 dx = \text{Too large to display}$$

input `integrate(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x, algorithm="giac")`

output

```

-((2*(2*pi^3*b^4*d^4*f^2*x^4*log(abs(F))*sgn(F) - 2*pi*b^4*d^4*f^2*x^4*log
(abs(F))^3*sgn(F) - 2*pi^3*b^4*d^4*f^2*x^4*log(abs(F)) + 2*pi*b^4*d^4*f^2*
x^4*log(abs(F))^3 + 4*pi^3*b^4*d^4*e*f*x^3*log(abs(F))*sgn(F) - 4*pi*b^4*d
^4*e*f*x^3*log(abs(F))^3*sgn(F) - 4*pi^3*b^4*d^4*e*f*x^3*log(abs(F)) + 4*p
i*b^4*d^4*e*f*x^3*log(abs(F))^3 + 2*pi^3*b^4*d^4*e^2*x^2*log(abs(F))*sgn(F
) - 2*pi*b^4*d^4*e^2*x^2*log(abs(F))^3*sgn(F) - 2*pi^3*b^4*d^4*e^2*x^2*log
(abs(F)) + 2*pi*b^4*d^4*e^2*x^2*log(abs(F))^3 - 2*pi^3*b^3*d^3*f^2*x^3*sgn
(F) + 6*pi*b^3*d^3*f^2*x^3*log(abs(F))^2*sgn(F) + 2*pi^3*b^3*d^3*f^2*x^3 -
6*pi*b^3*d^3*f^2*x^3*log(abs(F))^2 - 3*pi^3*b^3*d^3*e*f*x^2*sgn(F) + 9*pi
*b^3*d^3*e*f*x^2*log(abs(F))^2*sgn(F) + 3*pi^3*b^3*d^3*e*f*x^2 - 9*pi*b^3*
d^3*e*f*x^2*log(abs(F))^2 - pi^3*b^3*d^3*e^2*x*sgn(F) + 3*pi*b^3*d^3*e^2*x
*log(abs(F))^2*sgn(F) + pi^3*b^3*d^3*e^2*x - 3*pi*b^3*d^3*e^2*x*log(abs(F)
)^2 - 12*pi*b^2*d^2*f^2*x^2*log(abs(F))*sgn(F) + 12*pi*b^2*d^2*f^2*x^2*log
(abs(F)) - 12*pi*b^2*d^2*e*f*x*log(abs(F))*sgn(F) + 12*pi*b^2*d^2*e*f*x*lo
g(abs(F)) - 2*pi*b^2*d^2*e^2*log(abs(F))*sgn(F) + 2*pi*b^2*d^2*e^2*log(abs
(F)) + 12*pi*b*d*f^2*x*sgn(F) - 12*pi*b*d*f^2*x + 6*pi*b*d*e*f*sgn(F) - 6*
pi*b*d*e*f)*(pi^5*b^5*d^5*sgn(F) - 10*pi^3*b^5*d^5*log(abs(F))^2*sgn(F) +
5*pi*b^5*d^5*log(abs(F))^4*sgn(F) - pi^5*b^5*d^5 + 10*pi^3*b^5*d^5*log(abs
(F))^2 - 5*pi*b^5*d^5*log(abs(F))^4)/(pi^5*b^5*d^5*sgn(F) - 10*pi^3*b^5*d
^5*log(abs(F))^2*sgn(F) + 5*pi*b^5*d^5*log(abs(F))^4*sgn(F) - pi^5*b^5*...

```

### Mupad [B] (verification not implemented)

Time = 22.52 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

$$\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx = \frac{F^{a+bc+bdx} (b^4 d^4 e^2 x^2 \ln(F)^4 + 2b^4 d^4 e f x^3 \ln(F)^4 + b^4 d^4 f^2 x^4 \ln(F)^4 - 2b^3 d^3 e^2 x \ln(F)^3 - 6b^3 d^3 e}{\dots}$$

input

```
int(F^(a + b*(c + d*x))*x^2*(e + f*x)^2,x)
```

output

```

(F^(a + b*c + b*d*x)*(24*f^2 + 2*b^2*d^2*e^2*log(F)^2 - 24*b*d*f^2*x*log(F)
) - 2*b^3*d^3*e^2*x*log(F)^3 + b^4*d^4*e^2*x^2*log(F)^4 + 12*b^2*d^2*f^2*x
^2*log(F)^2 - 4*b^3*d^3*f^2*x^3*log(F)^3 + b^4*d^4*f^2*x^4*log(F)^4 - 12*b
*d*e*f*log(F) + 12*b^2*d^2*e*f*x*log(F)^2 - 6*b^3*d^3*e*f*x^2*log(F)^3 + 2
*b^4*d^4*e*f*x^3*log(F)^4)/(b^5*d^5*log(F)^5)

```

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

$$\int F^{a+b(c+dx)} x^2 (e+fx)^2 dx$$

$$= \frac{f^{bdx+bc+a} (\log(f)^4 b^4 d^4 e^2 x^2 + 2\log(f)^4 b^4 d^4 e f x^3 + \log(f)^4 b^4 d^4 f^2 x^4 - 2\log(f)^3 b^3 d^3 e^2 x - 6\log(f)^3 b^3 d^3 e}{\log(f)^5 b^5 d^5}$$

input `int(F^(a+b*(d*x+c))*x^2*(f*x+e)^2,x)`output `(f**(a + b*c + b*d*x)*(log(f)**4*b**4*d**4*e**2*x**2 + 2*log(f)**4*b**4*d**4*e*f*x**3 + log(f)**4*b**4*d**4*f**2*x**4 - 2*log(f)**3*b**3*d**3*e**2*x - 6*log(f)**3*b**3*d**3*e*f*x**2 - 4*log(f)**3*b**3*d**3*f**2*x**3 + 2*log(f)**2*b**2*d**2*e**2 + 12*log(f)**2*b**2*d**2*e*f*x + 12*log(f)**2*b**2*d**2*f**2*x**2 - 12*log(f)*b*d*e*f - 24*log(f)*b*d*f**2*x + 24*f**2))/(log(f)**5*b**5*d**5)`

### 3.106 $\int F^{a+b(c+dx)} x(e + fx)^2 dx$

Optimal result . . . . .	708
Mathematica [A] (verified) . . . . .	708
Rubi [A] (verified) . . . . .	709
Maple [A] (verified) . . . . .	710
Fricas [A] (verification not implemented) . . . . .	711
Sympy [A] (verification not implemented) . . . . .	711
Maxima [A] (verification not implemented) . . . . .	712
Giac [C] (verification not implemented) . . . . .	712
Mupad [B] (verification not implemented) . . . . .	713
Reduce [B] (verification not implemented) . . . . .	714

#### Optimal result

Integrand size = 20, antiderivative size = 129

$$\int F^{a+b(c+dx)} x(e + fx)^2 dx = -\frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{2f F^{a+bc+bdx} (2e + 3fx)}{b^3 d^3 \log^3(F)} - \frac{F^{a+bc+bdx} (e^2 + 4efx + 3f^2 x^2)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx} x(e + fx)^2}{bd \log(F)}$$

output

```
-6*f^2*F^(b*d*x+b*c+a)/b^4/d^4/ln(F)^4+2*f*F^(b*d*x+b*c+a)*(3*f*x+2*e)/b^3/d^3/ln(F)^3-F^(b*d*x+b*c+a)*(3*f^2*x^2+4*e*f*x+e^2)/b^2/d^2/ln(F)^2+F^(b*d*x+b*c+a)*x*(f*x+e)^2/b/d/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int F^{a+b(c+dx)} x(e + fx)^2 dx = \frac{F^{a+b(c+dx)} (-6f^2 + 2bdf(2e + 3fx) \log(F) - b^2 d^2 (e^2 + 4efx + 3f^2 x^2) \log^2(F) + b^3 d^3 x(e + fx)^2 \log^3(F))}{b^4 d^4 \log^4(F)}$$

input

```
Integrate[F^(a + b*(c + d*x))*x*(e + f*x)^2,x]
```

output

$$\frac{(F^{(a + b(c + dx))} * (-6*f^2 + 2*b*d*f*(2*e + 3*f*x)*\text{Log}[F] - b^2*d^2*(e^2 + 4*e*f*x + 3*f^2*x^2)*\text{Log}[F]^2 + b^3*d^3*x*(e + f*x)^2*\text{Log}[F]^3)) / (b^4*d^4*\text{Log}[F]^4)}$$

**Rubi [A] (verified)**

Time = 1.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.88, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(e + fx)^2 F^{a+b(c+dx)} dx$$

↓ 2626

$$\int \left( e^2 x F^{a+b(c+dx)} + 2efx^2 F^{a+b(c+dx)} + f^2 x^3 F^{a+b(c+dx)} \right) dx$$

↓ 2009

$$-\frac{6f^2 F^{a+bc+bdx}}{b^4 d^4 \log^4(F)} + \frac{4ef F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} + \frac{6f^2 x F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{e^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{4efx F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} - \frac{3f^2 x^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{e^2 x F^{a+bc+bdx}}{bd \log(F)} + \frac{2efx^2 F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 x^3 F^{a+bc+bdx}}{bd \log(F)}$$

input

$$\text{Int}[F^{(a + b*(c + d*x))*x*(e + f*x)^2, x]$$

output

$$\begin{aligned} & (-6*f^2*F^{(a + b*c + b*d*x)}) / (b^4*d^4*\text{Log}[F]^4) + (4*e*f*F^{(a + b*c + b*d*x)}) / (b^3*d^3*\text{Log}[F]^3) + (6*f^2*F^{(a + b*c + b*d*x)*x}) / (b^3*d^3*\text{Log}[F]^3) \\ & - (e^2*F^{(a + b*c + b*d*x)}) / (b^2*d^2*\text{Log}[F]^2) - (4*e*f*F^{(a + b*c + b*d*x)*x}) / (b^2*d^2*\text{Log}[F]^2) - (3*f^2*F^{(a + b*c + b*d*x)*x^2}) / (b^2*d^2*\text{Log}[F]^2) \\ & + (e^2*F^{(a + b*c + b*d*x)*x}) / (b*d*\text{Log}[F]) + (2*e*f*F^{(a + b*c + b*d*x)*x^2}) / (b*d*\text{Log}[F]) + (f^2*F^{(a + b*c + b*d*x)*x^3}) / (b*d*\text{Log}[F]) \end{aligned}$$

### Defintions of rubi rules used

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2626 Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

### Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.12

method	result
gospers	$\frac{(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 + 6 \ln(F) b d f^2 x)}{\ln(F)^4 b^4 d^4}$
risch	$\frac{(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 + 6 \ln(F) b d f^2 x)}{\ln(F)^4 b^4 d^4}$
orering	$\frac{(\ln(F)^3 b^3 d^3 f^2 x^3 + 2 \ln(F)^3 b^3 d^3 e f x^2 + \ln(F)^3 b^3 d^3 e^2 x - 3 \ln(F)^2 b^2 d^2 f^2 x^2 - 4 \ln(F)^2 b^2 d^2 e f x - \ln(F)^2 b^2 d^2 e^2 + 6 \ln(F) b d f^2 x)}{\ln(F)^4 b^4 d^4}$
meijerg	$\frac{F^{bc+a} f^2 \left( 6 - \frac{(-4b^3 d^3 x^3 \ln(F)^3 + 12b^2 d^2 x^2 \ln(F)^2 - 24bdx \ln(F) + 24) e^{bdx \ln(F)}}{4} \right)}{\ln(F)^4 b^4 d^4} - \frac{2F^{bc+a} f e \left( 2 - \frac{(3b^2 d^2 x^2 \ln(F)^2 - 6bdx \ln(F) + 6) e^{bdx \ln(F)}}{3} \right)}{b^3 d^3 \ln(F)^3}$
norman	$\frac{f^2 x^3 e^{(a+b(dx+c)) \ln(F)}}{bd \ln(F)} + \frac{(\ln(F)^2 b^2 d^2 e^2 - 4ef \ln(F) bd + 6f^2) x e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^3 b^3 d^3} + \frac{f(2 \ln(F) bde - 3f) x^2 e^{(a+b(dx+c)) \ln(F)}}{\ln(F)^2 b^2 d^2}$
parallelrisch	$\frac{\ln(F)^3 x^3 F^{bdx+bc+a} b^3 d^3 f^2 + 2 \ln(F)^3 x^2 F^{bdx+bc+a} b^3 d^3 e f + \ln(F)^3 x F^{bdx+bc+a} b^3 d^3 e^2 - 3 \ln(F)^2 x^2 F^{bdx+bc+a} b^2 d^2 f^2 - 4 \ln(F)^2 x F^{bdx+bc+a} b^2 d^2 e f - \ln(F)^2 x^2 F^{bdx+bc+a} b^2 d^2 e^2 + 6 \ln(F) x F^{bdx+bc+a} b d f^2 + 6 \ln(F) x F^{bdx+bc+a} b d e f + 6 \ln(F) x F^{bdx+bc+a} b d e^2 - 6 \ln(F) x F^{bdx+bc+a} b^2 d^2 f^2 - 6 \ln(F) x F^{bdx+bc+a} b^2 d^2 e f - 6 \ln(F) x F^{bdx+bc+a} b^2 d^2 e^2 + 6 \ln(F) F^{bdx+bc+a} b^2 d^2 f^2 + 6 \ln(F) F^{bdx+bc+a} b^2 d^2 e f + 6 \ln(F) F^{bdx+bc+a} b^2 d^2 e^2 - 6 \ln(F) F^{bdx+bc+a} b^3 d^3 f^2 - 6 \ln(F) F^{bdx+bc+a} b^3 d^3 e f - 6 \ln(F) F^{bdx+bc+a} b^3 d^3 e^2}{\ln(F)^4 b^4 d^4}$

```
input int(F^(a+b*(d*x+c))*x*(f*x+e)^2,x,method=_RETURNVERBOSE)
```

```
output (ln(F)^3*b^3*d^3*f^2*x^3+2*ln(F)^3*b^3*d^3*e*f*x^2+ln(F)^3*b^3*d^3*e^2*x-3*ln(F)^2*b^2*d^2*f^2*x^2-4*ln(F)^2*b^2*d^2*e*f*x-ln(F)^2*b^2*d^2*e^2+6*ln(F)*b*d*f^2*x+4*e*f*ln(F)*b*d-6*f^2)*F^(b*d*x+b*c+a)/ln(F)^4/b^4/d^4
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int F^{a+b(c+dx)} x(e+fx)^2 dx$$

$$= \frac{((b^3 d^3 f^2 x^3 + 2 b^3 d^3 e f x^2 + b^3 d^3 e^2 x) \log(F)^3 - (3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 - 6 f^2 + 2(3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 e f x + b^2 d^2 e^2) \log(F) - 6 f^2 + 2(3 b^2 d^2 f^2 x^2 + 4 b^2 d^2 e f x + b^2 d^2 e^2))}{b^4 d^4 \log(F)^4}$$

input `integrate(F^(a+b*(d*x+c))*x*(f*x+e)^2,x, algorithm="fricas")`

output `((b^3*d^3*f^2*x^3 + 2*b^3*d^3*e*f*x^2 + b^3*d^3*e^2*x)*log(F)^3 - (3*b^2*d^2*f^2*x^2 + 4*b^2*d^2*e*f*x + b^2*d^2*e^2)*log(F)^2 - 6*f^2 + 2*(3*b*d*f^2*x + 2*b*d*e*f)*log(F))*F^(b*d*x + b*c + a)/(b^4*d^4*log(F)^4)`

**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.54

$$\int F^{a+b(c+dx)} x(e+fx)^2 dx$$

$$= \begin{cases} \frac{F^{a+b(c+dx)}(b^3 d^3 e^2 x \log(F)^3 + 2 b^3 d^3 e f x^2 \log(F)^3 + b^3 d^3 f^2 x^3 \log(F)^3 - b^2 d^2 e^2 \log(F)^2 - 4 b^2 d^2 e f x \log(F)^2 - 3 b^2 d^2 f^2 x^2 \log(F)^2 + 4 b d e f \log(F) - 6 f^2)}{b^4 d^4 \log(F)^4} \\ \frac{e^2 x^2}{2} + \frac{2 e f x^3}{3} + \frac{f^2 x^4}{4} \end{cases}$$

input `integrate(F**(a+b*(d*x+c))*x*(f*x+e)**2,x)`

output `Piecewise((F**(a + b*(c + d*x))*(b**3*d**3*e**2*x*log(F)**3 + 2*b**3*d**3*e*f*x**2*log(F)**3 + b**3*d**3*f**2*x**3*log(F)**3 - b**2*d**2*e**2*log(F)**2 - 4*b**2*d**2*e*f*x*log(F)**2 - 3*b**2*d**2*f**2*x**2*log(F)**2 + 4*b*d*e*f*log(F) + 6*b*d*f**2*x*log(F) - 6*f**2)/(b**4*d**4*log(F)**4), Ne(b**4*d**4*log(F)**4, 0)), (e**2*x**2/2 + 2*e*f*x**3/3 + f**2*x**4/4, True))`



**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.52

$$\int F^{a+b(c+dx)} x(e+fx)^2 dx = \frac{(F^{bc+a} bdx \log(F) - F^{bc+a}) F^{bdx} e^2}{b^2 d^2 \log(F)^2} + \frac{2(F^{bc+a} b^2 d^2 x^2 \log(F)^2 - 2F^{bc+a} bdx \log(F) + 2F^{bc+a}) F^{bdx} e f}{b^3 d^3 \log(F)^3} + \frac{(F^{bc+a} b^3 d^3 x^3 \log(F)^3 - 3F^{bc+a} b^2 d^2 x^2 \log(F)^2 + 6F^{bc+a} bdx \log(F) - 6F^{bc+a}) F^{bdx} f^2}{b^4 d^4 \log(F)^4}$$

input `integrate(F^(a+b*(d*x+c))*x*(f*x+e)^2,x, algorithm="maxima")`

output `(F^(b*c + a)*b*d*x*log(F) - F^(b*c + a))*F^(b*d*x)*e^2/(b^2*d^2*log(F)^2) + 2*(F^(b*c + a)*b^2*d^2*x^2*log(F)^2 - 2*F^(b*c + a)*b*d*x*log(F) + 2*F^(b*c + a))*F^(b*d*x)*e*f/(b^3*d^3*log(F)^3) + (F^(b*c + a)*b^3*d^3*x^3*log(F)^3 - 3*F^(b*c + a)*b^2*d^2*x^2*log(F)^2 + 6*F^(b*c + a)*b*d*x*log(F) - 6*F^(b*c + a))*F^(b*d*x)*f^2/(b^4*d^4*log(F)^4)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 4114, normalized size of antiderivative = 31.89

$$\int F^{a+b(c+dx)} x(e+fx)^2 dx = \text{Too large to display}$$

input `integrate(F^(a+b*(d*x+c))*x*(f*x+e)^2,x, algorithm="giac")`

output

```

-(((3*pi^2*b^3*d^3*f^2*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*f^2*x^3*log
(abs(F)) + 2*b^3*d^3*f^2*x^3*log(abs(F))^3 + 6*pi^2*b^3*d^3*e*f*x^2*log(ab
s(F))*sgn(F) - 6*pi^2*b^3*d^3*e*f*x^2*log(abs(F)) + 4*b^3*d^3*e*f*x^2*log(
abs(F))^3 + 3*pi^2*b^3*d^3*e^2*x*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*e^2*x
*log(abs(F)) + 2*b^3*d^3*e^2*x*log(abs(F))^3 - 3*pi^2*b^2*d^2*f^2*x^2*sgn(
F) + 3*pi^2*b^2*d^2*f^2*x^2 - 6*b^2*d^2*f^2*x^2*log(abs(F))^2 - 4*pi^2*b^2
*d^2*e*f*x*sgn(F) + 4*pi^2*b^2*d^2*e*f*x - 8*b^2*d^2*e*f*x*log(abs(F))^2 -
pi^2*b^2*d^2*e^2*sgn(F) + pi^2*b^2*d^2*e^2 - 2*b^2*d^2*e^2*log(abs(F))^2
+ 12*b*d*f^2*x*log(abs(F)) + 8*b*d*e*f*log(abs(F)) - 12*f^2)*(pi^4*b^4*d^4
*sgn(F) - 6*pi^2*b^4*d^4*log(abs(F))^2*sgn(F) - pi^4*b^4*d^4 + 6*pi^2*b^4*
d^4*log(abs(F))^2 - 2*b^4*d^4*log(abs(F))^4)/((pi^4*b^4*d^4*sgn(F) - 6*pi^
2*b^4*d^4*log(abs(F))^2*sgn(F) - pi^4*b^4*d^4 + 6*pi^2*b^4*d^4*log(abs(F))
^2 - 2*b^4*d^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*d^4*log(abs(F))*sgn(F) - pi
*b^4*d^4*log(abs(F))^3*sgn(F) - pi^3*b^4*d^4*log(abs(F)) + pi*b^4*d^4*log(
abs(F))^3)^2) - 4*(pi^3*b^3*d^3*f^2*x^3*sgn(F) - 3*pi*b^3*d^3*f^2*x^3*log(
abs(F))^2*sgn(F) - pi^3*b^3*d^3*f^2*x^3 + 3*pi*b^3*d^3*f^2*x^3*log(abs(F))
^2 + 2*pi^3*b^3*d^3*e*f*x^2*sgn(F) - 6*pi*b^3*d^3*e*f*x^2*log(abs(F))^2*sg
n(F) - 2*pi^3*b^3*d^3*e*f*x^2 + 6*pi*b^3*d^3*e*f*x^2*log(abs(F))^2 + pi^3*
b^3*d^3*e^2*x*sgn(F) - 3*pi*b^3*d^3*e^2*x*log(abs(F))^2*sgn(F) - pi^3*b^3*
d^3*e^2*x + 3*pi*b^3*d^3*e^2*x*log(abs(F))^2 + 6*pi*b^2*d^2*f^2*x^2*log...

```

**Mupad [B] (verification not implemented)**

Time = 23.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int F^{a+b(c+dx)} x (e + fx)^2 dx = \frac{F^{a+bc+bdx} (b^3 d^3 e^2 x \ln(F)^3 + 2 b^3 d^3 e f x^2 \ln(F)^3 + b^3 d^3 f^2 x^3 \ln(F)^3 - b^2 d^2 e^2 \ln(F)^2 - 4 b^2 d^2 e f x \ln(F) - b^2 d^2 f^2 x^2 \ln(F)^2 + 2 b^2 d^2 e f x \ln(F) + b^2 d^2 f^2 x^2 \ln(F)^2)}{b^4 d^4 \ln(F)^4}$$

input

```
int(F^(a + b*(c + d*x))*x*(e + f*x)^2,x)
```

output

```

(F^(a + b*c + b*d*x)*(6*b*d*f^2*x*log(F) - b^2*d^2*e^2*log(F)^2 - 6*f^2 +
b^3*d^3*e^2*x*log(F)^3 - 3*b^2*d^2*f^2*x^2*log(F)^2 + b^3*d^3*f^2*x^3*log(
F)^3 + 4*b*d*e*f*log(F) - 4*b^2*d^2*e*f*x*log(F)^2 + 2*b^3*d^3*e*f*x^2*log
(F)^3)/(b^4*d^4*log(F)^4)

```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int F^{a+b(c+dx)} x(e+fx)^2 dx$$

$$= \frac{f^{bdx+bc+a} (\log(f)^3 b^3 d^3 e^2 x + 2\log(f)^3 b^3 d^3 e f x^2 + \log(f)^3 b^3 d^3 f^2 x^3 - \log(f)^2 b^2 d^2 e^2 - 4\log(f)^2 b^2 d^2 e f x - 6\log(f)^2 b^2 d^2 f^2 x^2 + 6\log(f) b d e^2 + 6\log(f) b d e f x + 6\log(f) b d f^2 x^2 - 6f^2)}{\log(f)^4 b^4 d^4}$$

input `int(F^(a+b*(d*x+c))*x*(f*x+e)^2,x)`output `(f**(a + b*c + b*d*x)*(log(f)**3*b**3*d**3*e**2*x + 2*log(f)**3*b**3*d**3*e*f*x**2 + log(f)**3*b**3*d**3*f**2*x**3 - log(f)**2*b**2*d**2*e**2 - 4*log(f)**2*b**2*d**2*e*f*x - 3*log(f)**2*b**2*d**2*f**2*x**2 + 4*log(f)*b*d*e*f + 6*log(f)*b*d*f**2*x - 6*f**2))/(log(f)**4*b**4*d**4)`

### 3.107 $\int F^{a+b(c+dx)}(e+fx)^2 dx$

Optimal result	715
Mathematica [A] (verified)	715
Rubi [A] (verified)	716
Maple [A] (verified)	717
Fricas [A] (verification not implemented)	718
Sympy [A] (verification not implemented)	718
Maxima [A] (verification not implemented)	719
Giac [C] (verification not implemented)	719
Mupad [B] (verification not implemented)	720
Reduce [B] (verification not implemented)	721

#### Optimal result

Integrand size = 19, antiderivative size = 85

$$\int F^{a+b(c+dx)}(e+fx)^2 dx = \frac{2f^2 F^{a+bc+bdx}}{b^3 d^3 \log^3(F)} - \frac{2f F^{a+bc+bdx}(e+fx)}{b^2 d^2 \log^2(F)} + \frac{F^{a+bc+bdx}(e+fx)^2}{bd \log(F)}$$

output

$$\frac{2*f^2*F^{(b*d*x+b*c+a)}/b^3/d^3/\ln(F)^3-2*f*F^{(b*d*x+b*c+a)}*(f*x+e)/b^2/d^2/\ln(F)^2+F^{(b*d*x+b*c+a)}*(f*x+e)^2/b/d/\ln(F)}{}$$

#### Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int F^{a+b(c+dx)}(e+fx)^2 dx = \frac{F^{a+b(c+dx)}(2f^2 - 2bdf(e+fx)\log(F) + b^2d^2(e+fx)^2\log^2(F))}{b^3d^3\log^3(F)}$$

input

$$\text{Integrate}[F^{(a + b*(c + d*x))}*(e + f*x)^2, x]$$

output

$$\frac{(F^{(a + b*(c + d*x))}*(2*f^2 - 2*b*d*f*(e + f*x)*\text{Log}[F] + b^2*d^2*(e + f*x)^2*\text{Log}[F]^2))/(b^3*d^3*\text{Log}[F]^3)}{}$$

**Rubi [A] (verified)**

Time = 0.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$ , Rules used = {2618, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int (e + fx)^2 F^{a+b(c+dx)} dx \\
 & \quad \downarrow \text{2618} \\
 & \int (e + fx)^2 F^{a+bc+bdx} dx \\
 & \quad \downarrow \text{2607} \\
 & \frac{(e + fx)^2 F^{a+bc+bdx}}{bd \log(F)} - \frac{2f \int F^{a+bc+bdx} (e + fx) dx}{bd \log(F)} \\
 & \quad \downarrow \text{2607} \\
 & \frac{(e + fx)^2 F^{a+bc+bdx}}{bd \log(F)} - \frac{2f \left( \frac{(e+fx)F^{a+bc+bdx}}{bd \log(F)} - \frac{f \int F^{a+bc+bdx} dx}{bd \log(F)} \right)}{bd \log(F)} \\
 & \quad \downarrow \text{2624} \\
 & \frac{(e + fx)^2 F^{a+bc+bdx}}{bd \log(F)} - \frac{2f \left( \frac{(e+fx)F^{a+bc+bdx}}{bd \log(F)} - \frac{f F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} \right)}{bd \log(F)}
 \end{aligned}$$

input `Int[F^(a + b*(c + d*x))*(e + f*x)^2,x]`

output `(F^(a + b*c + b*d*x)*(e + f*x)^2)/(b*d*Log[F]) - (2*f*(-((f*F^(a + b*c + b*d*x))/(b^2*d^2*Log[F]^2)) + (F^(a + b*c + b*d*x)*(e + f*x))/(b*d*Log[F]))/(b*d*Log[F])`

Defintions of rubi rules used

```
rule 2607 Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

```
rule 2618 Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
/; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v
, x] && IntegerQ[m]
```

```
rule 2624 Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

method	result
gosper	$\frac{(\ln(F)^2 b^2 d^2 f^2 x^2 + 2 \ln(F)^2 b^2 d^2 e f x + \ln(F)^2 b^2 d^2 e^2 - 2 \ln(F) b d f^2 x - 2 e f \ln(F) b d + 2 f^2) F^{b d x + b c + a}}{b^3 d^3 \ln(F)^3}$
risch	$\frac{(\ln(F)^2 b^2 d^2 f^2 x^2 + 2 \ln(F)^2 b^2 d^2 e f x + \ln(F)^2 b^2 d^2 e^2 - 2 \ln(F) b d f^2 x - 2 e f \ln(F) b d + 2 f^2) F^{b d x + b c + a}}{b^3 d^3 \ln(F)^3}$
orering	$\frac{(\ln(F)^2 b^2 d^2 f^2 x^2 + 2 \ln(F)^2 b^2 d^2 e f x + \ln(F)^2 b^2 d^2 e^2 - 2 \ln(F) b d f^2 x - 2 e f \ln(F) b d + 2 f^2) F^{a + b(dx+c)}}{b^3 d^3 \ln(F)^3}$
norman	$\frac{(\ln(F)^2 b^2 d^2 e^2 - 2 e f \ln(F) b d + 2 f^2) e^{(a+b(dx+c)) \ln(F)}}{b^3 d^3 \ln(F)^3} + \frac{f^2 x^2 e^{(a+b(dx+c)) \ln(F)}}{b d \ln(F)} + \frac{2 f (\ln(F) b d e - f) x e^{(a+b(dx+c)) \ln(F)}}{b^2 d^2 \ln(F)^2}$
meijerg	$-\frac{F^{bc+a} f^2 \left( 2 - \frac{(3b^2 d^2 x^2 \ln(F)^2 - 6bdx \ln(F) + 6) e^{bdx \ln(F)}}{3} \right)}{b^3 d^3 \ln(F)^3} + \frac{2 F^{bc+a} f e \left( 1 - \frac{(-2bdx \ln(F) + 2) e^{bdx \ln(F)}}{2} \right)}{b^2 d^2 \ln(F)^2} - \frac{F^{bc+a} e^2 (1 - e^{\ln(F)})}{b d \ln(F)}$
parallelrisch	$\frac{\ln(F)^2 x^2 F^{bdx+bc+a} b^2 d^2 f^2 + 2 \ln(F)^2 x F^{bdx+bc+a} b^2 d^2 e f + \ln(F)^2 F^{bdx+bc+a} b^2 d^2 e^2 - 2 \ln(F) x F^{bdx+bc+a} b d f^2 - 2 \ln(F) F^{bc+a} b d e f}{b^3 d^3 \ln(F)^3}$

```
input int(F^(a+b*(d*x+c))*(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output

$$(\ln(F)^2 b^2 d^2 f^2 x^2 + 2 \ln(F)^2 b^2 d^2 e f x + \ln(F)^2 b^2 d^2 e^2 - 2 \ln(F) b^2 d^2 f^2 x - 2 e f \ln(F) b^2 d^2 f^2) F^{(b d x + b c + a)} / b^3 d^3 \ln(F)^3$$
**Fricas [A] (verification not implemented)**

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int F^{a+b(c+dx)}(e+fx)^2 dx$$

$$= \frac{((b^2 d^2 f^2 x^2 + 2 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 + 2 f^2 - 2 (b d f^2 x + b d e f) \log(F)) F^{b d x + b c + a}}{b^3 d^3 \log(F)^3}$$

input

```
integrate(F^(a+b*(d*x+c))*(f*x+e)^2,x, algorithm="fricas")
```

output

$$((b^2 d^2 f^2 x^2 + 2 b^2 d^2 e f x + b^2 d^2 e^2) \log(F)^2 + 2 f^2 - 2 (b d f^2 x + b d e f) \log(F)) F^{(b d x + b c + a)} / (b^3 d^3 \log(F)^3)$$
**Sympy [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int F^{a+b(c+dx)}(e+fx)^2 dx$$

$$= \begin{cases} \frac{F^{a+b(c+dx)}(b^2 d^2 e^2 \log(F)^2 + 2 b^2 d^2 e f x \log(F)^2 + b^2 d^2 f^2 x^2 \log(F)^2 - 2 b d e f \log(F) - 2 b d f^2 x \log(F) + 2 f^2)}{b^3 d^3 \log(F)^3} & \text{for } b^3 d^3 \log(F)^3 \neq 0 \\ e^2 x + e f x^2 + \frac{f^2 x^3}{3} & \text{otherwise} \end{cases}$$

input

```
integrate(F**(a+b*(d*x+c))*(f*x+e)**2,x)
```

output

```
Piecewise((F**(a + b*(c + d*x))*(b**2*d**2*e**2*log(F)**2 + 2*b**2*d**2*e*f*x*log(F)**2 + b**2*d**2*f**2*x**2*log(F)**2 - 2*b*d*e*f*log(F) - 2*b*d*f**2*x*log(F) + 2*f**2)/(b**3*d**3*log(F)**3), Ne(b**3*d**3*log(F)**3, 0)), (e**2*x + e*f*x**2 + f**2*x**3/3, True))
```

**Maxima [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int F^{a+b(c+dx)}(e+fx)^2 dx$$

$$= \frac{F^{bdx+bc+a}e^2}{bd \log(F)} + \frac{2(F^{bc+a}bdx \log(F) - F^{bc+a})F^{bdx}ef}{b^2d^2 \log(F)^2}$$

$$+ \frac{(F^{bc+a}b^2d^2x^2 \log(F)^2 - 2F^{bc+a}bdx \log(F) + 2F^{bc+a})F^{bdx}f^2}{b^3d^3 \log(F)^3}$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2,x, algorithm="maxima")`

output `F^(b*d*x + b*c + a)*e^2/(b*d*log(F)) + 2*(F^(b*c + a)*b*d*x*log(F) - F^(b*c + a))*F^(b*d*x)*e*f/(b^2*d^2*log(F)^2) + (F^(b*c + a)*b^2*d^2*x^2*log(F)^2 - 2*F^(b*c + a)*b*d*x*log(F) + 2*F^(b*c + a))*F^(b*d*x)*f^2/(b^3*d^3*log(F)^3)`

**Giac [C] (verification not implemented)**

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 2264, normalized size of antiderivative = 26.64

$$\int F^{a+b(c+dx)}(e+fx)^2 dx = \text{Too large to display}$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2,x, algorithm="giac")`



output

```

-((2*(pi*b^2*d^2*f^2*x^2*log(abs(F))*sgn(F) - pi*b^2*d^2*f^2*x^2*log(abs(F)
)) + 2*pi*b^2*d^2*e*f*x*log(abs(F))*sgn(F) - 2*pi*b^2*d^2*e*f*x*log(abs(F)
) + pi*b^2*d^2*e^2*log(abs(F))*sgn(F) - pi*b^2*d^2*e^2*log(abs(F)) - pi*b*
d*f^2*x*sgn(F) + pi*b*d*f^2*x - pi*b*d*e*f*sgn(F) + pi*b*d*e*f)*(pi^3*b^3*
d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d
^3*log(abs(F))^2)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^2*sgn(F)
) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*log(abs
(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)^2) - (
pi^2*b^2*d^2*f^2*x^2*sgn(F) - pi^2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*f^2*x^2*log
(abs(F))^2 + 2*pi^2*b^2*d^2*e*f*x*sgn(F) - 2*pi^2*b^2*d^2*e*f*x + 4*b^2*d^
2*e*f*x*log(abs(F))^2 + pi^2*b^2*d^2*e^2*sgn(F) - pi^2*b^2*d^2*e^2 + 2*b^2
*d^2*e^2*log(abs(F))^2 - 4*b*d*f^2*x*log(abs(F)) - 4*b*d*e*f*log(abs(F)) +
4*f^2)*(3*pi^2*b^3*d^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) +
2*b^3*d^3*log(abs(F))^3)/((pi^3*b^3*d^3*sgn(F) - 3*pi*b^3*d^3*log(abs(F))^
2*sgn(F) - pi^3*b^3*d^3 + 3*pi*b^3*d^3*log(abs(F))^2)^2 + (3*pi^2*b^3*d^3*
log(abs(F))*sgn(F) - 3*pi^2*b^3*d^3*log(abs(F)) + 2*b^3*d^3*log(abs(F))^3)
^2))*cos(-1/2*pi*b*d*x*sgn(F) + 1/2*pi*b*d*x - 1/2*pi*b*c*sgn(F) + 1/2*pi*
b*c - 1/2*pi*a*sgn(F) + 1/2*pi*a) - ((pi^2*b^2*d^2*f^2*x^2*sgn(F) - pi^2*b
^2*d^2*f^2*x^2 + 2*b^2*d^2*f^2*x^2*log(abs(F))^2 + 2*pi^2*b^2*d^2*e*f*x*sg
n(F) - 2*pi^2*b^2*d^2*e*f*x + 4*b^2*d^2*e*f*x*log(abs(F))^2 + pi^2*b^2*...

```

### Mupad [B] (verification not implemented)

Time = 22.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int F^{a+b(c+dx)}(e+fx)^2 dx$$

$$= \frac{F^{a+b(c+dx)}(b^2 d^2 e^2 \ln(F)^2 + 2b^2 d^2 e f x \ln(F)^2 + b^2 d^2 f^2 x^2 \ln(F)^2 - 2b d e f \ln(F) - 2b d f^2 x \ln(F))}{b^3 d^3 \ln(F)^3}$$

input

```
int(F^(a + b*(c + d*x))*(e + f*x)^2,x)
```

output

```

(F^(a + b*c + b*d*x)*(2*f^2 + b^2*d^2*e^2*log(F)^2 - 2*b*d*f^2*x*log(F) +
b^2*d^2*f^2*x^2*log(F)^2 - 2*b*d*e*f*log(F) + 2*b^2*d^2*e*f*x*log(F)^2))/(
b^3*d^3*log(F)^3)

```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int F^{a+b(c+dx)}(e+fx)^2 dx$$

$$= \frac{f^{bdx+bc+a}(\log(f)^2 b^2 d^2 e^2 + 2\log(f)^2 b^2 d^2 e f x + \log(f)^2 b^2 d^2 f^2 x^2 - 2\log(f) b d e f - 2\log(f) b d f^2 x + 2f^2)}{\log(f)^3 b^3 d^3}$$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2,x)`output `(f**(a + b*c + b*d*x)*(log(f)**2*b**2*d**2*e**2 + 2*log(f)**2*b**2*d**2*e*f*x + log(f)**2*b**2*d**2*f**2*x**2 - 2*log(f)*b*d*e*f - 2*log(f)*b*d*f**2*x + 2*f**2))/(log(f)**3*b**3*d**3)`

### 3.108 $\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$

Optimal result . . . . .	722
Mathematica [A] (verified) . . . . .	722
Rubi [A] (verified) . . . . .	723
Maple [A] (verified) . . . . .	724
Fricas [A] (verification not implemented) . . . . .	724
Sympy [F] . . . . .	725
Maxima [A] (verification not implemented) . . . . .	725
Giac [F] . . . . .	726
Mupad [B] (verification not implemented) . . . . .	726
Reduce [B] (verification not implemented) . . . . .	726

#### Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx = e^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + \frac{2ef F^{a+bc+bdx}}{bd \log(F)} + \frac{f^2 F^{a+bc+bdx} x}{bd \log(F)}$$

output

```
e^2*F^(b*c+a)*Ei(b*d*x*ln(F))-f^2*F^(b*d*x+b*c+a)/b^2/d^2/ln(F)^2+2*e*f*F^(b*d*x+b*c+a)/b/d/ln(F)+f^2*F^(b*d*x+b*c+a)*x/b/d/ln(F)
```

#### Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx = F^{a+bc} \left( e^2 \text{ExpIntegralEi}(bdx \log(F)) + \frac{f F^{bdx} (-f + bd(2e + fx) \log(F))}{b^2 d^2 \log^2(F)} \right)$$

input

```
Integrate[(F^(a + b*(c + d*x)))*(e + f*x)^2)/x,x]
```

output

$$F^{(a + b*c)}*(e^2*ExpIntegralEi[b*d*x*Log[F]] + (f*F^{(b*d*x)}*(-f + b*d*(2*e + f*x)*Log[F]))/(b^2*d^2*Log[F]^2))$$

**Rubi [A] (verified)**

Time = 0.79 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e + fx)^2 F^{a+b(c+dx)}}{x} dx$$

$$\downarrow 2629$$

$$\int \left( \frac{e^2 F^{a+b(c+dx)}}{x} + 2ef F^{a+b(c+dx)} + f^2 x F^{a+b(c+dx)} \right) dx$$

$$\downarrow 2009$$

$$-\frac{f^2 F^{a+bc+bdx}}{b^2 d^2 \log^2(F)} + e^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) + \frac{2ef F^{a+b(c+dx)}}{bd \log(F)} + \frac{f^2 x F^{a+bc+bdx}}{bd \log(F)}$$

input

$$\text{Int}[(F^{(a + b*(c + d*x))}*(e + f*x)^2)/x,x]$$

output

$$e^2 * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x*Log[F]] - (f^2 * F^{(a + b*c + b*d*x)}) / (b^2 * d^2 * Log[F]^2) + (2 * e * f * F^{(a + b*(c + d*x))}) / (b*d*Log[F]) + (f^2 * F^{(a + b*c + b*d*x)*x}) / (b*d*Log[F])$$

## Definitions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

method	result
meijerg	$\frac{F^{bc+a} f^2 \left( 1 - \frac{(-2bdx \ln(F) + 2)e^{bdx \ln(F)}}{2} \right)}{b^2 d^2 \ln(F)^2} - \frac{2F^{bc+a} f e^{(1-e^{bdx \ln(F)})}}{bd \ln(F)} + F^{bc+a} e^2 (\ln(x) + \ln(-bd) + \ln(\ln(F))) -$
risch	$-e^2 F^{bc} F^a \expIntegral_1(\ln(F) bc + \ln(F) a - bdx \ln(F) - (bc + a) \ln(F)) + \frac{F^{bdx} F^{bc+a} f^2 x}{\ln(F) bd} + \frac{2F}{\ln(F)}$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x,x,method=_RETURNVERBOSE)`

output 
$$\frac{1}{b^2 d^2 \ln(F)^2} F^{(b*c+a)} f^2 (1 - \frac{1}{2} (-2*b*d*x*\ln(F) + 2) * \exp(b*d*x*\ln(F))) - 2 * F^{(b*c+a)} f * e / b / d / \ln(F) * (1 - \exp(b*d*x*\ln(F))) + F^{(b*c+a)} e^2 * (\ln(x) + \ln(-b*d) + \ln(\ln(F)) - \ln(-b*d*x*\ln(F)) - \text{Ei}(1, -b*d*x*\ln(F)))$$

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{F^{a+b(c+dx)} (e + fx)^2}{x} dx$$

$$= \frac{F^{bc+a} b^2 d^2 e^2 \text{Ei}(bdx \log(F)) \log(F)^2 - (f^2 - (bdf^2 x + 2bdef) \log(F)) F^{bdx+bc+a}}{b^2 d^2 \log(F)^2}$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="fricas")`

output

```
(F^(b*c + a)*b^2*d^2*e^2*Ei(b*d*x*log(F))*log(F)^2 - (f^2 - (b*d*f^2*x + 2
*b*d*e*f)*log(F))*F^(b*d*x + b*c + a))/(b^2*d^2*log(F)^2)
```

### Sympy [F]

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx = \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$$

input

```
integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x,x)
```

output

```
Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x, x)
```

### Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx = F^{bc+a} e^2 \text{Ei}(bdx \log(F)) + \frac{2 F^{bdx+bc+a} e f}{bd \log(F)} + \frac{(F^{bc+a} bdx \log(F) - F^{bc+a}) F^{bdx} f^2}{b^2 d^2 \log(F)^2}$$

input

```
integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="maxima")
```

output

```
F^(b*c + a)*e^2*Ei(b*d*x*log(F)) + 2*F^(b*d*x + b*c + a)*e*f/(b*d*log(F))
+ (F^(b*c + a)*b*d*x*log(F) - F^(b*c + a))*F^(b*d*x)*f^2/(b^2*d^2*log(F)^2
)
```

**Giac [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx = \int \frac{(fx+e)^2 F^{(dx+c)b+a}}{x} dx$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x,x, algorithm="giac")`

output `integrate((f*x + e)^2*F^((d*x + c)*b + a)/x, x)`

**Mupad [B] (verification not implemented)**

Time = 22.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$$

$$= \frac{F^{a+bc} (b^2 d^2 e^2 \operatorname{ei}(bdx \ln(F)) \ln(F)^2 - F^{bdx} f^2 + F^{bdx} b d f^2 x \ln(F) + 2 F^{bdx} b d e f \ln(F))}{b^2 d^2 \ln(F)^2}$$

input `int((F^(a + b*(c + d*x)))*(e + f*x)^2)/x,x)`

output `(F^(a + b*c)*(b^2*d^2*e^2*ei(b*d*x*log(F))*log(F)^2 - F^(b*d*x)*f^2 + F^(b*d*x)*b*d*f^2*x*log(F) + 2*F^(b*d*x)*b*d*e*f*log(F))/(b^2*d^2*log(F)^2)`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.83

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x} dx$$

$$= \frac{f^{bc+a} (\operatorname{ei}(\log(f) bdx) \log(f)^2 b^2 d^2 e^2 + 2 f^{bdx} \log(f) b d e f + f^{bdx} \log(f) b d f^2 x - f^{bdx} f^2)}{\log(f)^2 b^2 d^2}$$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x,x)`

output

```
(f**(a + b*c)*(ei(log(f)*b*d*x)*log(f)**2*b**2*d**2*e**2 + 2*f**(b*d*x)*log(f)*b*d*e*f + f**(b*d*x)*log(f)*b*d*f**2*x - f**(b*d*x)*f**2))/(log(f)**2*b**2*d**2)
```



### 3.109 $\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$

Optimal result . . . . .	728
Mathematica [A] (verified) . . . . .	728
Rubi [A] (verified) . . . . .	729
Maple [A] (verified) . . . . .	730
Fricas [A] (verification not implemented) . . . . .	730
Sympy [F] . . . . .	731
Maxima [A] (verification not implemented) . . . . .	731
Giac [F] . . . . .	732
Mupad [B] (verification not implemented) . . . . .	732
Reduce [B] (verification not implemented) . . . . .	732

#### Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = -\frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) + \frac{f^2 F^{a+bc+bdx}}{bd \log(F)} + bde^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log(F)$$

output

```
-e^2*F^(b*d*x+b*c+a)/x+2*e*f*F^(b*c+a)*Ei(b*d*x*ln(F))+f^2*F^(b*d*x+b*c+a)/b/d/ln(F)+b*d*e^2*F^(b*c+a)*Ei(b*d*x*ln(F))*ln(F)
```

#### Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = F^{a+bc} \left( F^{bdx} \left( -\frac{e^2}{x} + \frac{f^2}{bd \log(F)} \right) + e \text{ExpIntegralEi}(bdx \log(F))(2f + bde \log(F)) \right)$$

input

```
Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^2,x]
```

output

$$F^{(a + b*c)} * (F^{(b*d*x)} * (-e^{2/x} + f^{2/(b*d*\text{Log}[F])}) + e * \text{ExpIntegralEi}[b*d*x*\text{Log}[F]]) * (2*f + b*d*e*\text{Log}[F]))$$
**Rubi [A] (verified)**

Time = 0.83 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e + fx)^2 F^{a+b(c+dx)}}{x^2} dx$$

$$\downarrow 2629$$

$$\int \left( \frac{e^2 F^{a+b(c+dx)}}{x^2} + \frac{2ef F^{a+b(c+dx)}}{x} + f^2 F^{a+b(c+dx)} \right) dx$$

$$\downarrow 2009$$

$$bde^2 \log(F) F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{e^2 F^{a+bc+bdx}}{x} + 2ef F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) + \frac{f^2 F^{a+b(c+dx)}}{bd \log(F)}$$

input

$$\text{Int}[(F^{(a + b*(c + d*x))} * (e + f*x)^2) / x^2, x]$$

output

$$-((e^2 * F^{(a + b*c + b*d*x)}) / x) + 2 * e * f * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] + (f^2 * F^{(a + b*(c + d*x))}) / (b*d*\text{Log}[F]) + b*d*e^2 * F^{(a + b*c)} * \text{ExpIntegralEi}[b*d*x*\text{Log}[F]] * \text{Log}[F]$$

**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2629 Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

**Maple [A] (verified)**

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
risch	$-\frac{\ln(F)^2 F^{bc} F^a \operatorname{expIntegral}_1(\ln(F)bc + \ln(F)a - bdx \ln(F) - (bc+a) \ln(F)) b^2 d^2 e^2 x + 2 \ln(F) F^{bc} F^a \operatorname{expIntegral}_1(\ln(F)bc + \ln(F)a - \ln(F)bdx)}{\ln(F)bdx}$
meijerg	$-\frac{F^{bc+a} f^2 (1 - e^{bdx \ln(F)})}{bd \ln(F)} + 2 F^{bc+a} f e (\ln(x) + \ln(-bd) + \ln(\ln(F)) - \ln(-bdx \ln(F)) - \operatorname{expIntegral}_1(\ln(F)bc + \ln(F)a - \ln(F)bdx))$

```
input int(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x,method=_RETURNVERBOSE)
```

```
output -1/ln(F)/b/d*(ln(F)^2*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))*b^2*d^2*e^2*x+2*ln(F)*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))*b*d*e*f*x+ln(F)*F^(b*d*x)*F^(b*c+a)*b*d*e^2-F^(b*d*x)*F^(b*c+a)*f^2*x)/x
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{F^{a+b(c+dx)}(e + fx)^2}{x^2} dx$$

$$= \frac{(b^2 d^2 e^2 x \log(F)^2 + 2 b d e f x \log(F)) F^{bc+a} \operatorname{Ei}(bdx \log(F)) - (b d e^2 \log(F) - f^2 x) F^{bdx+bc+a}}{bdx \log(F)}$$

```
input integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x, algorithm="fricas")
```

output  $((b^2 d^2 e^{2x} \log(F)^2 + 2 b d e f x \log(F)) F^{(b c + a)} \text{Ei}(b d x \log(F)) - (b d e^2 \log(F) - f^2 x) F^{(b d x + b c + a)}) / (b d x \log(F))$

### Sympy [F]

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx$$

input `integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**2,x)`

output `Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x**2, x)`

### Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = F^{bc+a} b d e^2 \Gamma(-1, -b d x \log(F)) \log(F) + 2 F^{bc+a} e f \text{Ei}(b d x \log(F)) + \frac{F^{bdx+bc+a} f^2}{b d \log(F)}$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x, algorithm="maxima")`

output `F^(b*c + a)*b*d*e^2*gamma(-1, -b*d*x*log(F))*log(F) + 2*F^(b*c + a)*e*f*Ei(b*d*x*log(F)) + F^(b*d*x + b*c + a)*f^2/(b*d*log(F))`

**Giac [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = \int \frac{(fx+e)^2 F^{(dx+c)b+a}}{x^2} dx$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x, algorithm="giac")`

output `integrate((f*x + e)^2*F^((d*x + c)*b + a)/x^2, x)`

**Mupad [B] (verification not implemented)**

Time = 22.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = 2 F^{a+bc} e f \operatorname{ei}(bdx \ln(F)) - \frac{F^{bdx} F^{a+bc} e^2}{x} + \frac{F^{a+bc+bdx} f^2}{bd \ln(F)} - F^{a+bc} b d e^2 \ln(F) \operatorname{expint}(-bdx \ln(F))$$

input `int((F^(a + b*(c + d*x))*(e + f*x)^2)/x^2,x)`

output `2*F^(a + b*c)*e*f*ei(b*d*x*log(F)) - (F^(b*d*x)*F^(a + b*c)*e^2)/x + (F^(a + b*c + b*d*x)*f^2)/(b*d*log(F)) - F^(a + b*c)*b*d*e^2*log(F)*expint(-b*d*x*log(F))`

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^2} dx = \frac{f^{bc+a} (ei(\log(f) bdx) \log(f)^2 b^2 d^2 e^2 x + 2ei(\log(f) bdx) \log(f) b d e f x - f^{bdx} \log(f) b d e^2 + f^{bdx} f^2 x)}{\log(f) b d x}$$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^2,x)`

output

```
(f**(a + b*c)*(ei(log(f)*b*d*x)*log(f)**2*b**2*d**2*e**2*x + 2*ei(log(f)*b
*d*x)*log(f)*b*d*e*f*x - f**(b*d*x)*log(f)*b*d*e**2 + f**(b*d*x)*f**2*x))/
(log(f)*b*d*x)
```

**3.110**  $\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$

Optimal result . . . . .	734
Mathematica [A] (verified) . . . . .	735
Rubi [A] (verified) . . . . .	735
Maple [A] (verified) . . . . .	736
Fricas [A] (verification not implemented) . . . . .	737
Sympy [F] . . . . .	737
Maxima [A] (verification not implemented) . . . . .	737
Giac [F] . . . . .	738
Mupad [B] (verification not implemented) . . . . .	738
Reduce [B] (verification not implemented) . . . . .	739

**Optimal result**

Integrand size = 22, antiderivative size = 136

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx = -\frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{bde^2 F^{a+bc+bdx} \log(F)}{2x} + 2bdef F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log(F) + \frac{1}{2} b^2 d^2 e^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log^2(F)$$

```
output -1/2*e^2*F^(b*d*x+b*c+a)/x^2-2*e*f*F^(b*d*x+b*c+a)/x+f^2*F^(b*c+a)*Ei(b*d*x*ln(F))-1/2*b*d*e^2*F^(b*d*x+b*c+a)*ln(F)/x+2*b*d*e*f*F^(b*c+a)*Ei(b*d*x*ln(F))*ln(F)+1/2*b^2*d^2*e^2*F^(b*c+a)*Ei(b*d*x*ln(F))*ln(F)^2
```

**Mathematica [A] (verified)**

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

$$= \frac{F^{a+bc}(-eF^{bdx}(e+4fx+bdex \log(F)) + x^2 \text{ExpIntegralEi}(bdx \log(F)) (2f^2 + 4bdef \log(F) + b^2d^2e^2 \log^2(F)))}{2x^2}$$

input

```
Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^3,x]
```

output

```
(F^(a + b*c)*(-(e*F^(b*d*x))*(e + 4*f*x + b*d*e*x*Log[F])) + x^2*ExpIntegralEi[b*d*x*Log[F]]*(2*f^2 + 4*b*d*e*f*Log[F] + b^2*d^2*e^2*Log[F]^2))/(2*x^2)
```

**Rubi [A] (verified)**

Time = 1.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 F^{a+b(c+dx)}}{x^3} dx$$

$$\downarrow \text{2629}$$

$$\int \left( \frac{e^2 F^{a+b(c+dx)}}{x^3} + \frac{2ef F^{a+b(c+dx)}}{x^2} + \frac{f^2 F^{a+b(c+dx)}}{x} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{1}{2} b^2 d^2 e^2 \log^2(F) F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{e^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 \log(F) F^{a+bc+bdx}}{2x} + 2bdef \log(F) F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{2ef F^{a+bc+bdx}}{x} + f^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F))$$



input `Int[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^3,x]`

output `-1/2*(e^2*F^(a + b*c + b*d*x))/x^2 - (2*e*f*F^(a + b*c + b*d*x))/x + f^2*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]] - (b*d*e^2*F^(a + b*c + b*d*x)*Log[F])/(2*x) + 2*b*d*e*f*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]]*Log[F] + (b^2*d^2*e^2*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^2)/2`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{\ln(F)^2 F^{bc} F^a \exp\text{Integral}_1(\ln(F)bc + \ln(F)a - bdx \ln(F) - (bc+a) \ln(F)) b^2 d^2 e^2 x^2 + 4 \ln(F) F^{bc} F^a \exp\text{Integral}_1(\ln(F)bc + \ln(F)a - bdx \ln(F))}{2}$
meijerg	$F^{bc+a} f^2 (\ln(x) + \ln(-bd) + \ln(\ln(F)) - \ln(-bdx \ln(F)) - \exp\text{Integral}_1(-bdx \ln(F))) - 2bc$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^3,x,method=_RETURNVERBOSE)`

output `-1/2*(ln(F)^2*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))*b^2*d^2*e^2*x^2+4*ln(F)*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))*b*d*e*f*x^2+F^(b*d*x)*F^(b*c+a)*b*d*e^2*x*ln(F)+2*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))*f^2*x^2+4*F^(b*d*x)*F^(b*c+a)*e*f*x+F^(b*d*x)*F^(b*c+a)*e^2)/x^2`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.65

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx = \frac{(b^2 d^2 e^2 x^2 \log(F)^2 + 4 b d e f x^2 \log(F) + 2 f^2 x^2) F^{bc+a} \text{Ei}(bdx \log(F)) - (b d e^2 x \log(F) + 4 e f x + e^2) F^{bdx}}{2 x^2}$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^3,x, algorithm="fricas")`output `1/2*((b^2*d^2*e^2*x^2*log(F)^2 + 4*b*d*e*f*x^2*log(F) + 2*f^2*x^2)*F^(b*c + a)*Ei(b*d*x*log(F)) - (b*d*e^2*x*log(F) + 4*e*f*x + e^2)*F^(b*d*x + b*c + a))/x^2`**Sympy [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx = \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

input `integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**3,x)`output `Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x**3, x)`**Maxima [A] (verification not implemented)**

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.54

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx = -F^{bc+a} b^2 d^2 e^2 \Gamma(-2, -bdx \log(F)) \log(F)^2 + 2 F^{bc+a} b d e f \Gamma(-1, -bdx \log(F)) \log(F) + F^{bc+a} f^2 \text{Ei}(bdx \log(F))$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^3,x, algorithm="maxima")`

output

```
-F^(b*c + a)*b^2*d^2*e^2*gamma(-2, -b*d*x*log(F))*log(F)^2 + 2*F^(b*c + a)
*b*d*e*f*gamma(-1, -b*d*x*log(F))*log(F) + F^(b*c + a)*f^2*Ei(b*d*x*log(F)
)
```

**Giac [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx = \int \frac{(fx+e)^2 F^{(dx+c)b+a}}{x^3} dx$$

input

```
integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^3,x, algorithm="giac")
```

output

```
integrate((f*x + e)^2*F^((d*x + c)*b + a)/x^3, x)
```

**Mupad [B] (verification not implemented)**

Time = 22.69 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx = & F^{a+bc} f^2 \operatorname{ei}(b dx \ln(F)) - \frac{2 F^{bdx} F^{a+bc} e f}{x} \\ & - F^{a+bc} b^2 d^2 e^2 \ln(F)^2 \left( \frac{\operatorname{expint}(-b dx \ln(F))}{2} \right. \\ & \left. + F^{bdx} \left( \frac{1}{2 b dx \ln(F)} + \frac{1}{2 b^2 d^2 x^2 \ln(F)^2} \right) \right) \\ & - 2 F^{a+bc} b d e f \ln(F) \operatorname{expint}(-b dx \ln(F)) \end{aligned}$$

input

```
int((F^(a + b*(c + d*x))*(e + f*x)^2)/x^3,x)
```

output

```
F^(a + b*c)*f^2*ei(b*d*x*log(F)) - (2*F^(b*d*x)*F^(a + b*c)*e*f)/x - F^(a
+ b*c)*b^2*d^2*e^2*log(F)^2*(expint(-b*d*x*log(F))/2 + F^(b*d*x)*(1/(2*b*d
*x*log(F)) + 1/(2*b^2*d^2*x^2*log(F)^2))) - 2*F^(a + b*c)*b*d*e*f*log(F)*e
xpint(-b*d*x*log(F))
```

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^3} dx$$

$$= \frac{f^{bc+a} (ei(\log(f) bdx) \log(f)^2 b^2 d^2 e^2 x^2 + 4ei(\log(f) bdx) \log(f) bde f x^2 + 2ei(\log(f) bdx) f^2 x^2 - f^{bdx} \log(f) bdx)}{2x^2}$$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^3,x)`output `(f**(a + b*c)*(ei(log(f)*b*d*x)*log(f)**2*b**2*d**2*e**2*x**2 + 4*ei(log(f)*b*d*x)*log(f)*b*d*e*f*x**2 + 2*ei(log(f)*b*d*x)*f**2*x**2 - f**(b*d*x)*log(f)*b*d*e**2*x - f**(b*d*x)*e**2 - 4*f**(b*d*x)*e*f*x))/(2*x**2)`

**3.111**  $\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$

Optimal result . . . . .	740
Mathematica [A] (verified) . . . . .	741
Rubi [A] (verified) . . . . .	741
Maple [A] (verified) . . . . .	742
Fricas [A] (verification not implemented) . . . . .	743
Sympy [F] . . . . .	743
Maxima [A] (verification not implemented) . . . . .	744
Giac [F] . . . . .	744
Mupad [B] (verification not implemented) . . . . .	745
Reduce [B] (verification not implemented) . . . . .	745

**Optimal result**

Integrand size = 22, antiderivative size = 217

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx = -\frac{e^2 F^{a+bc+bdx}}{3x^3} - \frac{ef F^{a+bc+bdx}}{x^2} - \frac{f^2 F^{a+bc+bdx}}{x} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{6x^2} - \frac{bdef F^{a+bc+bdx} \log(F)}{x} + bdf^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log(F) - \frac{b^2 d^2 e^2 F^{a+bc+bdx} \log^2(F)}{6x} + b^2 d^2 ef F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log^2(F) + \frac{1}{6} b^3 d^3 e^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log^3(F)$$

output

```
-1/3*e^2*F^(b*d*x+b*c+a)/x^3-e*f*F^(b*d*x+b*c+a)/x^2-f^2*F^(b*d*x+b*c+a)/x
-1/6*b*d*e^2*F^(b*d*x+b*c+a)*ln(F)/x^2-b*d*e*f*F^(b*d*x+b*c+a)*ln(F)/x+b*d
*f^2*F^(b*c+a)*Ei(b*d*x*ln(F))*ln(F)-1/6*b^2*d^2*e^2*F^(b*d*x+b*c+a)*ln(F)
^2/x+b^2*d^2*e*f*F^(b*c+a)*Ei(b*d*x*ln(F))*ln(F)^2+1/6*b^3*d^3*e^2*F^(b*c+
a)*Ei(b*d*x*ln(F))*ln(F)^3
```

**Mathematica [A] (verified)**

Time = 0.44 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.53

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$$

$$= \frac{F^{a+bc}(bdx^3 \text{ExpIntegralEi}(bdx \log(F)) \log(F) (6f^2 + 6bdef \log(F) + b^2d^2e^2 \log^2(F)) - F^{bdx} (2(e^2 + 3e$$

$$\frac{\phantom{F^{a+bc}(bdx^3 \text{ExpIntegralEi}(bdx \log(F)) \log(F) (6f^2 + 6bdef \log(F) + b^2d^2e^2 \log^2(F)) - F^{bdx} (2(e^2 + 3e}}{6x^3}$$

input

```
Integrate[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^4,x]
```

output

```
(F^(a + b*c)*(b*d*x^3*ExpIntegralEi[b*d*x*Log[F]]*Log[F]*(6*f^2 + 6*b*d*e*
f*Log[F] + b^2*d^2*e^2*Log[F]^2) - F^(b*d*x)*(2*(e^2 + 3*e*f*x + 3*f^2*x^2
) + b*d*e*x*(e + 6*f*x)*Log[F] + b^2*d^2*e^2*x^2*Log[F]^2)))/(6*x^3)
```

**Rubi [A] (verified)**

Time = 1.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 F^{a+b(c+dx)}}{x^4} dx$$

$$\downarrow \text{2629}$$

$$\int \left( \frac{e^2 F^{a+b(c+dx)}}{x^4} + \frac{2ef F^{a+b(c+dx)}}{x^3} + \frac{f^2 F^{a+b(c+dx)}}{x^2} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{1}{6}b^3d^3e^2 \log^3(F)F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{b^2d^2e^2 \log^2(F)F^{a+bc+bdx}}{3x^3} + \frac{b^2d^2ef \log^2(F)F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{e^2F^{a+bc+bdx}}{6x^2} - \frac{6x}{bde^2 \log(F)F^{a+bc+bdx}}}{x^2} - \frac{bdef \log(F)F^{a+bc+bdx}}{x} + \frac{bdf^2 \log(F)F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - f^2F^{a+bc+bdx}}{x}$$

input `Int[(F^(a + b*(c + d*x))*(e + f*x)^2)/x^4,x]`

output `-1/3*(e^2*F^(a + b*c + b*d*x))/x^3 - (e*f*F^(a + b*c + b*d*x))/x^2 - (f^2*F^(a + b*c + b*d*x))/x - (b*d*e^2*F^(a + b*c + b*d*x)*Log[F])/(6*x^2) - (b*d*e*f*F^(a + b*c + b*d*x)*Log[F])/x + b*d*f^2*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]]*Log[F] - (b^2*d^2*e^2*F^(a + b*c + b*d*x)*Log[F]^2)/(6*x) + b^2*d^2*e*f*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^2 + (b^3*d^3*e^2*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^3)/6`

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{F^{bc} F^a \exp\text{Integral}_1(\ln(F)bc+\ln(F)a-bdx \ln(F)-(bc+a) \ln(F)) \ln(F)^3 b^3 d^3 e^2 x^3 + 6F^{bc} F^a \exp\text{Integral}_1(\ln(F)bc+\ln(F)a-bdx \ln(F)) \ln(F)^2 b^2 d^2 e^2 x^2 + 6F^{bc} F^a \exp\text{Integral}_1(\ln(F)bc+\ln(F)a-bdx \ln(F)) \ln(F) b d e^2 x + 6F^{bc} F^a \exp\text{Integral}_1(\ln(F)bc+\ln(F)a-bdx \ln(F))}{6x^3}$
meijerg	$-bd \ln(F) F^{bc+a} f^2 \left( \frac{1}{bdx \ln(F)} + 1 - \ln(x) - \ln(-bd) - \ln(\ln(F)) - \frac{2+2bdx \ln(F)}{2bdx \ln(F)} + \frac{e^{bdx \ln(F)}}{bdx \ln(F)} + \ln(F) \right)$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x,method=_RETURNVERBOSE)`

output 
$$-1/6*(F^{(b*c)}*F^a*Ei(1,\ln(F)*b*c+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))*\ln(F)^3*b^3*d^3*e^2*x^3+6*F^{(b*c)}*F^a*Ei(1,\ln(F)*b*c+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))*\ln(F)^2*b^2*d^2*e*f*x^3+F^{(b*d*x)}*F^{(b*c+a)}*b^2*d^2*e^2*x^2*\ln(F)^2+6*F^{(b*c)}*F^a*Ei(1,\ln(F)*b*c+\ln(F)*a-b*d*x*\ln(F)-(b*c+a)*\ln(F))*\ln(F)*b*d*f^2*x^3+6*F^{(b*d*x)}*F^{(b*c+a)}*b*d*e*f*x^2*\ln(F)+F^{(b*d*x)}*F^{(b*c+a)}*b*d*e^2*x*\ln(F)+6*F^{(b*d*x)}*F^{(b*c+a)}*f^2*x^2+6*F^{(b*d*x)}*F^{(b*c+a)}*e*f*x+2*F^{(b*d*x)}*F^{(b*c+a)}*e^2)/x^3$$

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.63

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx = \frac{(b^3 d^3 e^2 x^3 \log(F)^3 + 6 b^2 d^2 e f x^3 \log(F)^2 + 6 b d f^2 x^3 \log(F)) F^{bc+a} Ei(bdx \log(F)) - (b^2 d^2 e^2 x^2 \log(F)^2 + 6 b^2 d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F)) F^{bc+a} Ei(bdx \log(F)) - (b^2 d^2 e^2 x^2 \log(F)^2 + 6 b^2 d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F) + 6 b d^2 e^2 x^2 \log(F)) F^{bc+a} Ei(bdx \log(F))}{6 x^3}$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="fricas")`

output 
$$1/6*((b^3*d^3*e^2*x^3*\log(F)^3 + 6*b^2*d^2*e*f*x^3*\log(F)^2 + 6*b*d*f^2*x^3*\log(F))*F^{(b*c + a)}*Ei(b*d*x*\log(F)) - (b^2*d^2*e^2*x^2*\log(F)^2 + 6*f^2*x^2 + 6*e*f*x + 2*e^2 + (6*b*d*e*f*x^2 + b*d*e^2*x)*\log(F))*F^{(b*d*x + b*c + a)})/x^3$$

### Sympy [F]

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx = \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$$

input `integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**4,x)`

output `Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x**4, x)`



**Maxima [A] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.39

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx = F^{bc+a} b^3 d^3 e^2 \Gamma(-3, -bdx \log(F)) \log(F)^3$$

$$- 2 F^{bc+a} b^2 d^2 e f \Gamma(-2, -bdx \log(F)) \log(F)^2$$

$$+ F^{bc+a} b d f^2 \Gamma(-1, -bdx \log(F)) \log(F)$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="maxima")`

output `F^(b*c + a)*b^3*d^3*e^2*gamma(-3, -b*d*x*log(F))*log(F)^3 - 2*F^(b*c + a)*b^2*d^2*e*f*gamma(-2, -b*d*x*log(F))*log(F)^2 + F^(b*c + a)*b*d*f^2*gamma(-1, -b*d*x*log(F))*log(F)`

**Giac [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx = \int \frac{(fx+e)^2 F^{(dx+c)b+a}}{x^4} dx$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x, algorithm="giac")`

output `integrate((f*x + e)^2*F^((d*x + c)*b + a)/x^4, x)`

### Mupad [B] (verification not implemented)

Time = 22.88 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.93

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx = -\frac{F^{b dx} F^{a+bc} f^2}{x}$$

$$- F^{a+bc} b^3 d^3 e^2 \ln(F)^3 \left( F^{b dx} \left( \frac{1}{6 b d x \ln(F)} + \frac{1}{6 b^2 d^2 x^2 \ln(F)^2} + \frac{1}{3 b^3 d^3 x^3 \ln(F)^3} + \frac{\text{expint}(-b d x \ln(F))}{6} \right) \right)$$

$$- F^{a+bc} b d f^2 \ln(F) \text{expint}(-b d x \ln(F))$$

$$- 2 F^{a+bc} b^2 d^2 e f \ln(F)^2 \left( \frac{\text{expint}(-b d x \ln(F))}{2} + F^{b dx} \left( \frac{1}{2 b d x \ln(F)} + \frac{1}{2 b^2 d^2 x^2 \ln(F)^2} \right) \right)$$

input `int((F^(a + b*(c + d*x))*(e + f*x)^2)/x^4,x)`

output

```
- (F^(b*d*x)*F^(a + b*c)*f^2)/x - F^(a + b*c)*b^3*d^3*e^2*log(F)^3*(F^(b*d*x)*(1/(6*b*d*x*log(F)) + 1/(6*b^2*d^2*x^2*log(F)^2) + 1/(3*b^3*d^3*x^3*log(F)^3)) + expint(-b*d*x*log(F))/6) - F^(a + b*c)*b*d*f^2*log(F)*expint(-b*d*x*log(F)) - 2*F^(a + b*c)*b^2*d^2*e*f*log(F)^2*(expint(-b*d*x*log(F))/2 + F^(b*d*x)*(1/(2*b*d*x*log(F)) + 1/(2*b^2*d^2*x^2*log(F)^2)))
```

### Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.80

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^4} dx$$


---


$$= \frac{f^{bc+a} (ei(\log(f) b dx) \log(f)^3 b^3 d^3 e^2 x^3 + 6ei(\log(f) b dx) \log(f)^2 b^2 d^2 e f x^3 + 6ei(\log(f) b dx) \log(f) b d f^2 x^3 + 6ei \log(f) b d x f^2 x^3 + 6ei \log(f) b d x^3 + 6ei \log(f) b d x^3 + 6ei \log(f) b d x^3 + 6ei \log(f) b d x^3)}{6 b^3 d^3 e^2 x^3 + \dots}$$

input `int(F^(a+b*(d*x+c))*(f*x+e)^2/x^4,x)`

output

```
(f**(a + b*c)*(ei(log(f)*b*d*x)*log(f)**3*b**3*d**3*e**2*x**3 + 6*ei(log(f)
)*b*d*x)*log(f)**2*b**2*d**2*e*f*x**3 + 6*ei(log(f)*b*d*x)*log(f)*b*d*f**2
*x**3 - f**(b*d*x)*log(f)**2*b**2*d**2*e**2*x**2 - f**(b*d*x)*log(f)*b*d*e
**2*x - 6*f**(b*d*x)*log(f)*b*d*e*f*x**2 - 2*f**(b*d*x)*e**2 - 6*f**(b*d*x
)*e*f*x - 6*f**(b*d*x)*f**2*x**2))/(6*x**3)
```

**3.112** 
$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

Optimal result . . . . .	747
Mathematica [A] (verified) . . . . .	748
Rubi [A] (verified) . . . . .	748
Maple [A] (verified) . . . . .	750
Fricas [A] (verification not implemented) . . . . .	750
Sympy [F] . . . . .	751
Maxima [A] (verification not implemented) . . . . .	751
Giac [F] . . . . .	752
Mupad [B] (verification not implemented) . . . . .	752
Reduce [B] (verification not implemented) . . . . .	753

**Optimal result**

Integrand size = 22, antiderivative size = 321

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx = & -\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{2ef F^{a+bc+bdx}}{3x^3} \\ & - \frac{f^2 F^{a+bc+bdx}}{2x^2} - \frac{bde^2 F^{a+bc+bdx} \log(F)}{12x^3} \\ & - \frac{bdef F^{a+bc+bdx} \log(F)}{3x^2} - \frac{bdf^2 F^{a+bc+bdx} \log(F)}{12x^3} \\ & - \frac{b^2 d^2 e^2 F^{a+bc+bdx} \log^2(F)}{24x^2} - \frac{b^2 d^2 e f F^{a+bc+bdx} \log^2(F)}{3x} \\ & + \frac{1}{2} b^2 d^2 f^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log^2(F) \\ & - \frac{b^3 d^3 e^2 F^{a+bc+bdx} \log^3(F)}{24x} \\ & + \frac{1}{3} b^3 d^3 e f F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log^3(F) \\ & + \frac{1}{24} b^4 d^4 e^2 F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) \log^4(F) \end{aligned}$$

output

$$\begin{aligned}
& -1/4*e^2*F^(b*d*x+b*c+a)/x^4-2/3*e*f*F^(b*d*x+b*c+a)/x^3-1/2*f^2*F^(b*d*x+ \\
& b*c+a)/x^2-1/12*b*d*e^2*F^(b*d*x+b*c+a)*\ln(F)/x^3-1/3*b*d*e*f*F^(b*d*x+b*c \\
& +a)*\ln(F)/x^2-1/2*b*d*f^2*F^(b*d*x+b*c+a)*\ln(F)/x-1/24*b^2*d^2*e^2*F^(b*d* \\
& x+b*c+a)*\ln(F)^2/x^2-1/3*b^2*d^2*e*f*F^(b*d*x+b*c+a)*\ln(F)^2/x+1/2*b^2*d^2 \\
& *f^2*F^(b*c+a)*Ei(b*d*x*\ln(F))*\ln(F)^2-1/24*b^3*d^3*e^2*F^(b*d*x+b*c+a)*\ln \\
& (F)^3/x+1/3*b^3*d^3*e*f*F^(b*c+a)*Ei(b*d*x*\ln(F))*\ln(F)^3+1/24*b^4*d^4*e^2 \\
& *F^(b*c+a)*Ei(b*d*x*\ln(F))*\ln(F)^4
\end{aligned}$$
**Mathematica [A] (verified)**

Time = 0.52 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.49

$$\begin{aligned}
& \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx \\
& = \frac{F^{a+bc}(b^2d^2x^4 \text{ExpIntegralEi}(bdx \log(F)) \log^2(F) (12f^2 + 8bdef \log(F) + b^2d^2e^2 \log^2(F)) - F^{bdx} (2(3e^2 \\
& \hspace{15em} 24.
\end{aligned}$$

input

$$\text{Integrate}[(F^{(a + b*(c + d*x))}*(e + f*x)^2)/x^5, x]$$

output

$$\begin{aligned}
& (F^{(a + b*c)}*(b^2*d^2*x^4*\text{ExpIntegralEi}[b*d*x*\text{Log}[F]]*\text{Log}[F]^2*(12*f^2 + 8 \\
& *b*d*e*f*\text{Log}[F] + b^2*d^2*e^2*\text{Log}[F]^2) - F^{(b*d*x)}*(2*(3*e^2 + 8*e*f*x + \\
& 6*f^2*x^2) + 2*b*d*x*(e^2 + 4*e*f*x + 6*f^2*x^2)*\text{Log}[F] + b^2*d^2*e*x^2*(e \\
& + 8*f*x)*\text{Log}[F]^2 + b^3*d^3*e^2*x^3*\text{Log}[F]^3)))/(24*x^4)
\end{aligned}$$
**Rubi [A] (verified)**Time = 1.48 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(e+fx)^2 F^{a+b(c+dx)}}{x^5} dx$$

$$\begin{array}{c}
\downarrow 2629 \\
\int \left( \frac{e^2 F^{a+b(c+dx)}}{x^5} + \frac{2ef F^{a+b(c+dx)}}{x^4} + \frac{f^2 F^{a+b(c+dx)}}{x^3} \right) dx \\
\downarrow 2009 \\
\frac{1}{24} b^4 d^4 e^2 \log^4(F) F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{b^3 d^3 e^2 \log^3(F) F^{a+bc+bdx}}{24x} + \\
\frac{1}{3} b^3 d^3 e f \log^3(F) F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \frac{b^2 d^2 e^2 \log^2(F) F^{a+bc+bdx}}{24x^2} - \\
\frac{b^2 d^2 e f \log^2(F) F^{a+bc+bdx}}{3x} + \frac{1}{2} b^2 d^2 f^2 \log^2(F) F^{a+bc} \text{ExpIntegralEi}(bdx \log(F)) - \\
\frac{e^2 F^{a+bc+bdx}}{4x^4} - \frac{bde^2 \log(F) F^{a+bc+bdx}}{2x^2} - \frac{2ef F^{a+bc+bdx}}{2x^2} - \frac{bde f \log(F) F^{a+bc+bdx}}{3x^2} - \\
\frac{12x^3}{f^2 F^{a+bc+bdx}} - \frac{3x^3}{bdf^2 \log(F) F^{a+bc+bdx}}
\end{array}$$

input

```
Int[(F^(a + b*(c + d*x)))*(e + f*x)^2]/x^5,x]
```

output

```
-1/4*(e^2*F^(a + b*c + b*d*x))/x^4 - (2*e*f*F^(a + b*c + b*d*x))/(3*x^3) -
(f^2*F^(a + b*c + b*d*x))/(2*x^2) - (b*d*e^2*F^(a + b*c + b*d*x)*Log[F])/
(12*x^3) - (b*d*e*f*F^(a + b*c + b*d*x)*Log[F])/(3*x^2) - (b*d*f^2*F^(a +
b*c + b*d*x)*Log[F])/(2*x) - (b^2*d^2*e^2*F^(a + b*c + b*d*x)*Log[F]^2)/(2
4*x^2) - (b^2*d^2*e*f*F^(a + b*c + b*d*x)*Log[F]^2)/(3*x) + (b^2*d^2*f^2*F
^(a + b*c)*ExpIntegralEi[b*d*x*Log[F]]*Log[F]^2)/2 - (b^3*d^3*e^2*F^(a + b
*c + b*d*x)*Log[F]^3)/(24*x) + (b^3*d^3*e*f*F^(a + b*c)*ExpIntegralEi[b*d*
x*Log[F]]*Log[F]^3)/3 + (b^4*d^4*e^2*F^(a + b*c)*ExpIntegralEi[b*d*x*Log[F
]]*Log[F]^4)/24
```

### Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2629

```
Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_), x_Symbol] := Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

### Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{\ln(F)^4 F^{bc} F^a \operatorname{expIntegral}_1(\ln(F)bc+\ln(F)a-bdx \ln(F)-(bc+a) \ln(F))b^4 d^4 e^2 x^4+8 \ln(F)^3 F^{bc} F^a \operatorname{expIntegral}_1(\ln(F)bc+\ln(F)a-bdx \ln(F)-(bc+a) \ln(F))}{\ln(F)^2 b^2 d^2 F^{bc+a} f^2}$
meijerg	$f^2 \left( -\frac{1}{2b^2 d^2 x^2 \ln(F)^2} - \frac{1}{bdx \ln(F)} - \frac{3}{4} + \frac{\ln(x)}{2} + \frac{\ln(-bd)}{2} + \frac{\ln(\ln(F))}{2} + \frac{9b^2 d^2 x^2 \ln(F)^2 + 12bdx \ln(F)}{12b^2 d^2 x^2 \ln(F)^2} \right)$

```
input int(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x,method=_RETURNVERBOSE)
```

```
output -1/24*(ln(F)^4*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F))
)*b^4*d^4*e^2*x^4+8*ln(F)^3*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)
)-(b*c+a)*ln(F))*b^3*d^3*e*f*x^4+F^(b*d*x)*F^(b*c+a)*b^3*d^3*e^2*x^3*ln(F)
^3+12*ln(F)^2*F^(b*c)*F^a*Ei(1,ln(F)*b*c+ln(F)*a-b*d*x*ln(F)-(b*c+a)*ln(F)
)*b^2*d^2*f^2*x^4+8*F^(b*d*x)*F^(b*c+a)*b^2*d^2*e*f*x^3*ln(F)^2+F^(b*d*x)*
F^(b*c+a)*b^2*d^2*e^2*x^2*ln(F)^2+12*F^(b*d*x)*F^(b*c+a)*b*d*f^2*x^3*ln(F)
+8*F^(b*d*x)*F^(b*c+a)*b*d*e*f*x^2*ln(F)+2*F^(b*d*x)*F^(b*c+a)*b*d*e^2*x*ln(F)
+12*F^(b*d*x)*F^(b*c+a)*f^2*x^2+16*F^(b*d*x)*F^(b*c+a)*e*f*x+6*F^(b*d*x)
)*F^(b*c+a)*e^2)/x^4
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.58

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

$$= \frac{(b^4 d^4 e^2 x^4 \log(F)^4 + 8 b^3 d^3 e f x^4 \log(F)^3 + 12 b^2 d^2 f^2 x^4 \log(F)^2) F^{bc+a} \operatorname{Ei}(bdx \log(F)) - (b^3 d^3 e^2 x^3 \log(F)^3 + 6 b^2 d^2 e f x^3 \log(F)^2 + 6 b d e^2 x^2 \log(F) + 6 e^2 x) F^{bc+a}}{x^4}$$

```
input integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x, algorithm="fricas")
```

output

```
1/24*((b^4*d^4*e^2*x^4*log(F)^4 + 8*b^3*d^3*e*f*x^4*log(F)^3 + 12*b^2*d^2*
f^2*x^4*log(F)^2)*F^(b*c + a)*Ei(b*d*x*log(F)) - (b^3*d^3*e^2*x^3*log(F)^3
+ 12*f^2*x^2 + 16*e*f*x + (8*b^2*d^2*e*f*x^3 + b^2*d^2*e^2*x^2)*log(F)^2
+ 6*e^2 + 2*(6*b*d*f^2*x^3 + 4*b*d*e*f*x^2 + b*d*e^2*x)*log(F))*F^(b*d*x +
b*c + a))/x^4
```

**Sympy [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx = \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

input

```
integrate(F**(a+b*(d*x+c))*(f*x+e)**2/x**5,x)
```

output

```
Integral(F**(a + b*(c + d*x))*(e + f*x)**2/x**5, x)
```

**Maxima [A] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.29

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx = & -F^{bc+a}b^4d^4e^2\Gamma(-4, -bdx \log(F)) \log(F)^4 \\ & + 2F^{bc+a}b^3d^3ef\Gamma(-3, -bdx \log(F)) \log(F)^3 \\ & - F^{bc+a}b^2d^2f^2\Gamma(-2, -bdx \log(F)) \log(F)^2 \end{aligned}$$

input

```
integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x, algorithm="maxima")
```

output

```
-F^(b*c + a)*b^4*d^4*e^2*gamma(-4, -b*d*x*log(F))*log(F)^4 + 2*F^(b*c + a)
*b^3*d^3*e*f*gamma(-3, -b*d*x*log(F))*log(F)^3 - F^(b*c + a)*b^2*d^2*f^2*
gamma(-2, -b*d*x*log(F))*log(F)^2
```



**Giac [F]**

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx = \int \frac{(fx+e)^2 F^{(dx+c)b+a}}{x^5} dx$$

input `integrate(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x, algorithm="giac")`

output `integrate((f*x + e)^2*F^((d*x + c)*b + a)/x^5, x)`

**Mupad [B] (verification not implemented)**

Time = 22.96 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.80

$$\begin{aligned} \int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx = & -F^{a+bc} b^2 d^2 f^2 \ln(F)^2 \left( \frac{\operatorname{expint}(-bdx \ln(F))}{2} \right. \\ & \left. + F^{bdx} \left( \frac{1}{2bdx \ln(F)} + \frac{1}{2b^2 d^2 x^2 \ln(F)^2} \right) \right) \\ & - F^{a+bc} b^4 d^4 e^2 \ln(F)^4 \left( F^{bdx} \left( \frac{1}{24bdx \ln(F)} \right. \right. \\ & \left. \left. + \frac{1}{24b^2 d^2 x^2 \ln(F)^2} + \frac{1}{12b^3 d^3 x^3 \ln(F)^3} \right. \right. \\ & \left. \left. + \frac{1}{4b^4 d^4 x^4 \ln(F)^4} \right) + \frac{\operatorname{expint}(-bdx \ln(F))}{24} \right) \\ & - 2F^{a+bc} b^3 d^3 e f \ln(F)^3 \left( F^{bdx} \left( \frac{1}{6bdx \ln(F)} \right. \right. \\ & \left. \left. + \frac{1}{6b^2 d^2 x^2 \ln(F)^2} + \frac{1}{3b^3 d^3 x^3 \ln(F)^3} \right) \right. \\ & \left. \left. + \frac{\operatorname{expint}(-bdx \ln(F))}{6} \right) \right) \end{aligned}$$

input `int((F^(a + b*(c + d*x))*(e + f*x)^2)/x^5,x)`

output

```
- F^(a + b*c)*b^2*d^2*f^2*log(F)^2*(expint(-b*d*x*log(F))/2 + F^(b*d*x)*(1
/(2*b*d*x*log(F)) + 1/(2*b^2*d^2*x^2*log(F)^2))) - F^(a + b*c)*b^4*d^4*e^2
*log(F)^4*(F^(b*d*x)*(1/(24*b*d*x*log(F)) + 1/(24*b^2*d^2*x^2*log(F)^2) +
1/(12*b^3*d^3*x^3*log(F)^3) + 1/(4*b^4*d^4*x^4*log(F)^4)) + expint(-b*d*x*
log(F))/24) - 2*F^(a + b*c)*b^3*d^3*e*f*log(F)^3*(F^(b*d*x)*(1/(6*b*d*x*lo
g(F)) + 1/(6*b^2*d^2*x^2*log(F)^2) + 1/(3*b^3*d^3*x^3*log(F)^3)) + expint(
-b*d*x*log(F))/6)
```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.76

$$\int \frac{F^{a+b(c+dx)}(e+fx)^2}{x^5} dx$$

$$= \frac{f^{bc+a} (ei(\log(f) bdx) \log(f)^4 b^4 d^4 e^2 x^4 + 8ei(\log(f) bdx) \log(f)^3 b^3 d^3 e f x^4 + 12ei(\log(f) bdx) \log(f)^2 b^2 d^2 e^2 x^4 + 8ei(\log(f) bdx) \log(f) b d^2 e^2 x^4 + 8ei(\log(f) bdx) e^2 x^4)}{24 x^4}$$

input

```
int(F^(a+b*(d*x+c))*(f*x+e)^2/x^5,x)
```

output

```
(f**(a + b*c)*(ei(log(f)*b*d*x)*log(f)**4*b**4*d**4*e**2*x**4 + 8*ei(log(f)
)*b*d*x)*log(f)**3*b**3*d**3*e*f*x**4 + 12*ei(log(f)*b*d*x)*log(f)**2*b**2
*d**2*f**2*x**4 - f**(b*d*x)*log(f)**3*b**3*d**3*e**2*x**3 - f**(b*d*x)*lo
g(f)**2*b**2*d**2*e**2*x**2 - 8*f**(b*d*x)*log(f)**2*b**2*d**2*e*f*x**3 -
2*f**(b*d*x)*log(f)*b*d*e**2*x - 8*f**(b*d*x)*log(f)*b*d*e*f*x**2 - 12*f**
(b*d*x)*log(f)*b*d*f**2*x**3 - 6*f**(b*d*x)*e**2 - 16*f**(b*d*x)*e*f*x - 1
2*f**(b*d*x)*f**2*x**2))/(24*x**4)
```

### 3.113 $\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx$

Optimal result	754
Mathematica [A] (verified)	755
Rubi [A] (verified)	756
Maple [A] (verified)	758
Fricas [A] (verification not implemented)	759
Sympy [A] (verification not implemented)	760
Maxima [A] (verification not implemented)	761
Giac [A] (verification not implemented)	762
Mupad [B] (verification not implemented)	764
Reduce [B] (verification not implemented)	765

#### Optimal result

Integrand size = 25, antiderivative size = 805

$$\begin{aligned}
 & \int e^{-a-bx}(a+bx)^4(c+dx)^3 dx \\
 = & -\frac{5040d^3e^{-a-bx}}{b^4} - \frac{e^{-a-bx}(a+bx)^4(c+dx)^3}{b} - \frac{720d^2e^{-a-bx}(3bc+4ad+7bdx)}{b^4} \\
 & - \frac{360e^{-a-bx}(d(b^2c^2+4abcd+2a^2d^2)+2bd^2(3bc+4ad)x+7b^2d^3x^2)}{b^4} \\
 & - \frac{24e^{-a-bx}(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3+15bd(b^2c^2+4abcd+2a^2d^2)x+15b^2d^2(3bc+4ad)x^2)}{b^4} \\
 & - \frac{6e^{-a-bx}(a(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3)+4b(b^3c^3+12ab^2c^2d+18a^2bcd^2+4a^3d^3)x+30b^2d^2(c+dx)^3)}{b^4} \\
 & - \frac{6e^{-a-bx}(a^2c(2b^2c^2+4abcd+a^2d^2)+a(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3)x+2b(b^3c^3+12ab^2c^2d+12a^2bcd^2+a^3d^3)x^2)}{b^4} \\
 & - \frac{e^{-a-bx}(a^3c^2(4bc+3ad)+6a^2c(2b^2c^2+4abcd+a^2d^2)x+3a(4b^3c^3+18ab^2c^2d+12a^2bcd^2+a^3d^3)x^2)}{b^3}
 \end{aligned}$$

output

```

-5040*d^3*exp(-b*x-a)/b^4-exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3/b-720*d^2*exp(-b
*x-a)*(7*b*d*x+4*a*d+3*b*c)/b^4-360*exp(-b*x-a)*(d*(2*a^2*d^2+4*a*b*c*d+b^
2*c^2)+2*b*d^2*(4*a*d+3*b*c)*x+7*b^2*d^3*x^2)/b^4-24*exp(-b*x-a)*(b^3*c^3+
12*a*b^2*c^2*d+18*a^2*b*c*d^2+4*a^3*d^3+15*b*d*(2*a^2*d^2+4*a*b*c*d+b^2*c^
2)*x+15*b^2*d^2*(4*a*d+3*b*c)*x^2+35*b^3*d^3*x^3)/b^4-6*exp(-b*x-a)*(a*(a^
3*d^3+12*a^2*b*c*d^2+18*a*b^2*c^2*d+4*b^3*c^3)+4*b*(4*a^3*d^3+18*a^2*b*c*d
^2+12*a*b^2*c^2*d+b^3*c^3)*x+30*b^2*d*(2*a^2*d^2+4*a*b*c*d+b^2*c^2)*x^2+20
*b^3*d^2*(4*a*d+3*b*c)*x^3+35*b^4*d^3*x^4)/b^4-6*exp(-b*x-a)*(a^2*c*(a^2*d
^2+4*a*b*c*d+2*b^2*c^2)+a*(a^3*d^3+12*a^2*b*c*d^2+18*a*b^2*c^2*d+4*b^3*c^3
)*x+2*b*(4*a^3*d^3+18*a^2*b*c*d^2+12*a*b^2*c^2*d+b^3*c^3)*x^2+10*b^2*d*(2*
a^2*d^2+4*a*b*c*d+b^2*c^2)*x^3+5*b^3*d^2*(4*a*d+3*b*c)*x^4+7*b^4*d^3*x^5)/
b^3-exp(-b*x-a)*(a^3*c^2*(3*a*d+4*b*c)+6*a^2*c*(a^2*d^2+4*a*b*c*d+2*b^2*c^
2)*x+3*a*(a^3*d^3+12*a^2*b*c*d^2+18*a*b^2*c^2*d+4*b^3*c^3)*x^2+4*b*(4*a^3*
d^3+18*a^2*b*c*d^2+12*a*b^2*c^2*d+b^3*c^3)*x^3+15*b^2*d*(2*a^2*d^2+4*a*b*c
*d+b^2*c^2)*x^4+6*b^3*d^2*(4*a*d+3*b*c)*x^5+7*b^4*d^3*x^6)/b^2

```

**Mathematica [A] (verified)**

Time = 7.24 (sec) , antiderivative size = 458, normalized size of antiderivative = 0.57

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx$$

$$= \frac{e^{-a-bx}(-6(840+480a+120a^2+16a^3+a^4)d^3 - b^7x^4(c+dx)^3 - b^6x^3(c+dx)^2(4(1+a)c + (7+4a)d))}{b^3}$$

input

```
Integrate[E^(-a - b*x)*(a + b*x)^4*(c + d*x)^3,x]
```

output

```
(E^(-a - b*x)*(-6*(840 + 480*a + 120*a^2 + 16*a^3 + a^4)*d^3 - b^7*x^4*(c
+ d*x)^3 - b^6*x^3*(c + d*x)^2*(4*(1 + a)*c + (7 + 4*a)*d*x) - 6*b*d^2*((3
60 + 240*a + 72*a^2 + 12*a^3 + a^4)*c + (840 + 480*a + 120*a^2 + 16*a^3 +
a^4)*d*x) - 6*b^5*x^2*(c + d*x)*((2 + 2*a + a^2)*c^2 + 2*(4 + 3*a + a^2)*c
*d*x + (7 + 4*a + a^2)*d^2*x^2) - 3*b^2*d*((120 + 96*a + 36*a^2 + 8*a^3 +
a^4)*c^2 + 2*(360 + 240*a + 72*a^2 + 12*a^3 + a^4)*c*d*x + (840 + 480*a +
120*a^2 + 16*a^3 + a^4)*d^2*x^2) - 2*b^4*x*(2*(6 + 6*a + 3*a^2 + a^3)*c^3
+ 3*(30 + 24*a + 9*a^2 + 2*a^3)*c^2*d*x + 6*(30 + 20*a + 6*a^2 + a^3)*c*d^
2*x^2 + (105 + 60*a + 15*a^2 + 2*a^3)*d^3*x^3) - b^3*((24 + 24*a + 12*a^2
+ 4*a^3 + a^4)*c^3 + 3*(120 + 96*a + 36*a^2 + 8*a^3 + a^4)*c^2*d*x + 3*(36
0 + 240*a + 72*a^2 + 12*a^3 + a^4)*c*d^2*x^2 + (840 + 480*a + 120*a^2 + 16
*a^3 + a^4)*d^3*x^3))/b^4
```

**Rubi [A] (verified)**

Time = 2.20 (sec) , antiderivative size = 754, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx$$

$$\downarrow 2626$$

$$\int \left( \frac{3d^2 e^{-a-bx}(a+bx)^6(bc-ad)}{b^3} + \frac{3de^{-a-bx}(a+bx)^5(bc-ad)^2}{b^3} + \frac{e^{-a-bx}(a+bx)^4(bc-ad)^3}{b^3} + \frac{d^3 e^{-a-bx}(a+bx)^3}{b^3} \right) dx$$

$$\downarrow 2009$$

$$\frac{3d^2e^{-a-bx}(a+bx)^6(bc-ad)}{b^4} - \frac{18d^2e^{-a-bx}(a+bx)^5(bc-ad)}{b^4} - \frac{90d^2e^{-a-bx}(a+bx)^4(bc-ad)}{b^4} - \frac{360d^2e^{-a-bx}(a+bx)^3(bc-ad)}{b^4} - \frac{1080d^2e^{-a-bx}(a+bx)^2(bc-ad)}{b^4} - \frac{2160d^2e^{-a-bx}(a+bx)(bc-ad)}{b^4} - \frac{2160d^2e^{-a-bx}(bc-ad)}{b^4} - \frac{3de^{-a-bx}(a+bx)^5(bc-ad)^2}{b^4} - \frac{e^{-a-bx}(a+bx)^4(bc-ad)^3}{b^4} - \frac{15de^{-a-bx}(a+bx)^4(bc-ad)^2}{b^4} - \frac{4e^{-a-bx}(a+bx)^3(bc-ad)^3}{b^4} - \frac{60de^{-a-bx}(a+bx)^3(bc-ad)^2}{b^4} - \frac{12e^{-a-bx}(a+bx)^2(bc-ad)^3}{b^4} - \frac{180de^{-a-bx}(a+bx)^2(bc-ad)^2}{b^4} - \frac{24e^{-a-bx}(a+bx)(bc-ad)^3}{b^4} - \frac{360de^{-a-bx}(a+bx)(bc-ad)^2}{b^4} - \frac{24e^{-a-bx}(bc-ad)^3}{b^4} - \frac{360de^{-a-bx}(bc-ad)^2}{b^4} - \frac{d^3e^{-a-bx}(a+bx)^7}{b^4} - \frac{7d^3e^{-a-bx}(a+bx)^6}{b^4} - \frac{42d^3e^{-a-bx}(a+bx)^5}{b^4} - \frac{210d^3e^{-a-bx}(a+bx)^4}{b^4} - \frac{840d^3e^{-a-bx}(a+bx)^3}{b^4} - \frac{2520d^3e^{-a-bx}(a+bx)^2}{b^4} - \frac{5040d^3e^{-a-bx}(a+bx)}{b^4} - \frac{5040d^3e^{-a-bx}}{b^4}$$

input

Int [E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^4\*(c + d\*x)^3,x]

output

(-5040\*d^3\*E<sup>^</sup>(-a - b\*x))/b^4 - (2160\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x))/b^4 - (360\*d\*(b\*c - a\*d)^2\*E<sup>^</sup>(-a - b\*x))/b^4 - (24\*(b\*c - a\*d)^3\*E<sup>^</sup>(-a - b\*x))/b^4 - (5040\*d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x))/b^4 - (2160\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x))/b^4 - (360\*d\*(b\*c - a\*d)^2\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x))/b^4 - (24\*(b\*c - a\*d)^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x))/b^4 - (2520\*d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^2)/b^4 - (1080\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^2)/b^4 - (180\*d\*(b\*c - a\*d)^2\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^2)/b^4 - (12\*(b\*c - a\*d)^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^2)/b^4 - (840\*d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^3)/b^4 - (360\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^3)/b^4 - (60\*d\*(b\*c - a\*d)^2\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^3)/b^4 - (4\*(b\*c - a\*d)^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^3)/b^4 - (210\*d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^4)/b^4 - (90\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^4)/b^4 - (15\*d\*(b\*c - a\*d)^2\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^4)/b^4 - ((b\*c - a\*d)^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^4)/b^4 - (42\*d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^5)/b^4 - (18\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^5)/b^4 - (3\*d\*(b\*c - a\*d)^2\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^5)/b^4 - (7\*d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^6)/b^4 - (3\*d^2\*(b\*c - a\*d)\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^6)/b^4 - (d^3\*E<sup>^</sup>(-a - b\*x)\*(a + b\*x)^7)/b^4

**Defintions of rubi rules used**

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

**Maple [A] (verified)**

Time = 0.15 (sec) , antiderivative size = 900, normalized size of antiderivative = 1.12

method	result
norman	$(-4a b^2 d^3 - 3c d^2 b^3 - 7b^2 d^3) x^6 e^{-bx-a} + (-4a^3 d^3 - 18a^2 b c d^2 - 12a b^2 c^2 d - b^3 c^3 - 30$
meijerg	Expression too large to display
gosper	Expression too large to display
risch	Expression too large to display
orering	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display
parallelrisch	Expression too large to display

input `int(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x,method=_RETURNVERBOSE)`

output

```
(-4*a*b^2*d^3-3*b^3*c*d^2-7*b^2*d^3)*x^6*exp(-b*x-a)+(-4*a^3*d^3-18*a^2*b*
c*d^2-12*a*b^2*c^2*d-b^3*c^3-30*a^2*d^3-60*a*b*c*d^2-15*b^2*c^2*d-120*a*d^
3-90*b*c*d^2-210*d^3)*x^4*exp(-b*x-a)-(a^4*b^3*c^3+3*a^4*b^2*c^2*d+4*a^3*b
^3*c^3+6*a^4*b*c*d^2+24*a^3*b^2*c^2*d+12*a^2*b^3*c^3+6*a^4*d^3+72*a^3*b*c*
d^2+108*a^2*b^2*c^2*d+24*a*b^3*c^3+96*a^3*d^3+432*a^2*b*c*d^2+288*a*b^2*c^
2*d+24*b^3*c^3+720*a^2*d^3+1440*a*b*c*d^2+360*b^2*c^2*d+2880*a*d^3+2160*b*
c*d^2+5040*d^3)/b^4*exp(-b*x-a)-d^3*b^3*x^7*exp(-b*x-a)-(a^4*d^3+12*a^3*b*
c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3+16*a^3*d^3+72*a^2*b*c*d^2+48*a*b^2*c^2*
d+4*b^3*c^3+120*a^2*d^3+240*a*b*c*d^2+60*b^2*c^2*d+480*a*d^3+360*b*c*d^2+8
40*d^3)/b*x^3*exp(-b*x-a)-3*(a^4*b*c*d^2+4*a^3*b^2*c^2*d+2*a^2*b^3*c^3+a^4
*d^3+12*a^3*b*c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3+16*a^3*d^3+72*a^2*b*c*d^2
+48*a*b^2*c^2*d+4*b^3*c^3+120*a^2*d^3+240*a*b*c*d^2+60*b^2*c^2*d+480*a*d^3
+360*b*c*d^2+840*d^3)/b^2*x^2*exp(-b*x-a)-(3*a^4*b^2*c^2*d+4*a^3*b^3*c^3+6
*a^4*b*c*d^2+24*a^3*b^2*c^2*d+12*a^2*b^3*c^3+6*a^4*d^3+72*a^3*b*c*d^2+108*
a^2*b^2*c^2*d+24*a*b^3*c^3+96*a^3*d^3+432*a^2*b*c*d^2+288*a*b^2*c^2*d+24*b
^3*c^3+720*a^2*d^3+1440*a*b*c*d^2+360*b^2*c^2*d+2880*a*d^3+2160*b*c*d^2+50
40*d^3)/b^3*x*exp(-b*x-a)-3*b*d*(2*a^2*d^2+4*a*b*c*d+b^2*c^2+8*a*d^2+6*b*c
*d+14*d^2)*x^5*exp(-b*x-a)
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 544, normalized size of antiderivative = 0.68

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx =$$

$$\frac{(b^7 d^3 x^7 + (a^4 + 4 a^3 + 12 a^2 + 24 a + 24) b^3 c^3 + (3 b^7 c d^2 + (4 a + 7) b^6 d^3) x^6 + 3 (a^4 + 8 a^3 + 36 a^2 + 9$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```



output

```

-(b^7*d^3*x^7 + (a^4 + 4*a^3 + 12*a^2 + 24*a + 24)*b^3*c^3 + (3*b^7*c*d^2
+ (4*a + 7)*b^6*d^3)*x^6 + 3*(a^4 + 8*a^3 + 36*a^2 + 96*a + 120)*b^2*c^2*d
+ 3*(b^7*c^2*d + 2*(2*a + 3)*b^6*c*d^2 + 2*(a^2 + 4*a + 7)*b^5*d^3)*x^5 +
6*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)*b*c*d^2 + (b^7*c^3 + 3*(4*a + 5)*
b^6*c^2*d + 6*(3*a^2 + 10*a + 15)*b^5*c*d^2 + 2*(2*a^3 + 15*a^2 + 60*a + 1
05)*b^4*d^3)*x^4 + 6*(a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*d^3 + (4*(a +
1)*b^6*c^3 + 6*(3*a^2 + 8*a + 10)*b^5*c^2*d + 12*(a^3 + 6*a^2 + 20*a + 30)
*b^4*c*d^2 + (a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b^3*d^3)*x^3 + 3*(2*(a
^2 + 2*a + 2)*b^5*c^3 + 2*(2*a^3 + 9*a^2 + 24*a + 30)*b^4*c^2*d + (a^4 + 1
2*a^3 + 72*a^2 + 240*a + 360)*b^3*c*d^2 + (a^4 + 16*a^3 + 120*a^2 + 480*a
+ 840)*b^2*d^3)*x^2 + (4*(a^3 + 3*a^2 + 6*a + 6)*b^4*c^3 + 3*(a^4 + 8*a^3
+ 36*a^2 + 96*a + 120)*b^3*c^2*d + 6*(a^4 + 12*a^3 + 72*a^2 + 240*a + 360)
*b^2*c*d^2 + 6*(a^4 + 16*a^3 + 120*a^2 + 480*a + 840)*b*d^3)*x)*e^(-b*x -
a)/b^4

```

### Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1445, normalized size of antiderivative = 1.80

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx = \text{Too large to display}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)**4*(d*x+c)**3,x)
```

output

```
Piecewise((( -a**4*b**3*c**3 - 3*a**4*b**3*c**2*d*x - 3*a**4*b**3*c*d**2*x*
**2 - a**4*b**3*d**3*x**3 - 3*a**4*b**2*c**2*d - 6*a**4*b**2*c*d**2*x - 3*a
**4*b**2*d**3*x**2 - 6*a**4*b*c*d**2 - 6*a**4*b*d**3*x - 6*a**4*d**3 - 4*a
**3*b**4*c**3*x - 12*a**3*b**4*c**2*d*x**2 - 12*a**3*b**4*c*d**2*x**3 - 4*
a**3*b**4*d**3*x**4 - 4*a**3*b**3*c**3 - 24*a**3*b**3*c**2*d*x - 36*a**3*b
**3*c*d**2*x**2 - 16*a**3*b**3*d**3*x**3 - 24*a**3*b**2*c**2*d - 72*a**3*b
**2*c*d**2*x - 48*a**3*b**2*d**3*x**2 - 72*a**3*b*c*d**2 - 96*a**3*b*d**3*
x - 96*a**3*d**3 - 6*a**2*b**5*c**3*x**2 - 18*a**2*b**5*c**2*d*x**3 - 18*a
**2*b**5*c*d**2*x**4 - 6*a**2*b**5*d**3*x**5 - 12*a**2*b**4*c**3*x - 54*a*
**2*b**4*c**2*d*x**2 - 72*a**2*b**4*c*d**2*x**3 - 30*a**2*b**4*d**3*x**4 -
12*a**2*b**3*c**3 - 108*a**2*b**3*c**2*d*x - 216*a**2*b**3*c*d**2*x**2 - 1
20*a**2*b**3*d**3*x**3 - 108*a**2*b**2*c**2*d - 432*a**2*b**2*c*d**2*x - 3
60*a**2*b**2*d**3*x**2 - 432*a**2*b*c*d**2 - 720*a**2*b*d**3*x - 720*a**2*
d**3 - 4*a*b**6*c**3*x**3 - 12*a*b**6*c**2*d*x**4 - 12*a*b**6*c*d**2*x**5
- 4*a*b**6*d**3*x**6 - 12*a*b**5*c**3*x**2 - 48*a*b**5*c**2*d*x**3 - 60*a*
b**5*c*d**2*x**4 - 24*a*b**5*d**3*x**5 - 24*a*b**4*c**3*x - 144*a*b**4*c**
2*d*x**2 - 240*a*b**4*c*d**2*x**3 - 120*a*b**4*d**3*x**4 - 24*a*b**3*c**3
- 288*a*b**3*c**2*d*x - 720*a*b**3*c*d**2*x**2 - 480*a*b**3*d**3*x**3 - 28
8*a*b**2*c**2*d - 1440*a*b**2*c*d**2*x - 1440*a*b**2*d**3*x**2 - 1440*a*b*
c*d**2 - 2880*a*b*d**3*x - 2880*a*d**3 - b**7*c**3*x**4 - 3*b**7*c**2*d...
```

### Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.11

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx = \text{Too large to display}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

output

```

-4*(b*x + 1)*a^3*c^3*e^(-b*x - a)/b - a^4*c^3*e^(-b*x - a)/b - 3*(b*x + 1)
*a^4*c^2*d*e^(-b*x - a)/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c^3*e^(-b*x - a)
/b - 12*(b^2*x^2 + 2*b*x + 2)*a^3*c^2*d*e^(-b*x - a)/b^2 - 3*(b^2*x^2 + 2*
b*x + 2)*a^4*c*d^2*e^(-b*x - a)/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*
a*c^3*e^(-b*x - a)/b - 18*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*c^2*d*e^(-
b*x - a)/b^2 - 12*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*c*d^2*e^(-b*x - a)
/b^3 - (b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^4*d^3*e^(-b*x - a)/b^4 - (b^4*x^
4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c^3*e^(-b*x - a)/b - 12*(b^4*x^
4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*c^2*d*e^(-b*x - a)/b^2 - 18*(b
^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*c*d^2*e^(-b*x - a)/b^3
- 4*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^3*d^3*e^(-b*x - a)/
b^4 - 3*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*c^
2*d*e^(-b*x - a)/b^2 - 12*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 +
120*b*x + 120)*a*c*d^2*e^(-b*x - a)/b^3 - 6*(b^5*x^5 + 5*b^4*x^4 + 20*b^3
*x^3 + 60*b^2*x^2 + 120*b*x + 120)*a^2*d^3*e^(-b*x - a)/b^4 - 3*(b^6*x^6 +
6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 360*b^2*x^2 + 720*b*x + 720)*c*d^2
*e^(-b*x - a)/b^3 - 4*(b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3 + 36
0*b^2*x^2 + 720*b*x + 720)*a*d^3*e^(-b*x - a)/b^4 - (b^7*x^7 + 7*b^6*x^6 +
42*b^5*x^5 + 210*b^4*x^4 + 840*b^3*x^3 + 2520*b^2*x^2 + 5040*b*x + 5040)*
d^3*e^(-b*x - a)/b^4

```

**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 1096, normalized size of antiderivative = 1.36

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx = \text{Too large to display}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x, algorithm="giac")
```

output

```

-(b^11*d^3*x^7 + 3*b^11*c*d^2*x^6 + 4*a*b^10*d^3*x^6 + 3*b^11*c^2*d*x^5 +
12*a*b^10*c*d^2*x^5 + 6*a^2*b^9*d^3*x^5 + 7*b^10*d^3*x^6 + b^11*c^3*x^4 +
12*a*b^10*c^2*d*x^4 + 18*a^2*b^9*c*d^2*x^4 + 4*a^3*b^8*d^3*x^4 + 18*b^10*c
*d^2*x^5 + 24*a*b^9*d^3*x^5 + 4*a*b^10*c^3*x^3 + 18*a^2*b^9*c^2*d*x^3 + 12
*a^3*b^8*c*d^2*x^3 + a^4*b^7*d^3*x^3 + 15*b^10*c^2*d*x^4 + 60*a*b^9*c*d^2*
x^4 + 30*a^2*b^8*d^3*x^4 + 42*b^9*d^3*x^5 + 6*a^2*b^9*c^3*x^2 + 12*a^3*b^8
*c^2*d*x^2 + 3*a^4*b^7*c*d^2*x^2 + 4*b^10*c^3*x^3 + 48*a*b^9*c^2*d*x^3 + 7
2*a^2*b^8*c*d^2*x^3 + 16*a^3*b^7*d^3*x^3 + 90*b^9*c*d^2*x^4 + 120*a*b^8*d^
3*x^4 + 4*a^3*b^8*c^3*x + 3*a^4*b^7*c^2*d*x + 12*a*b^9*c^3*x^2 + 54*a^2*b^
8*c^2*d*x^2 + 36*a^3*b^7*c*d^2*x^2 + 3*a^4*b^6*d^3*x^2 + 60*b^9*c^2*d*x^3
+ 240*a*b^8*c*d^2*x^3 + 120*a^2*b^7*d^3*x^3 + 210*b^8*d^3*x^4 + a^4*b^7*c^
3 + 12*a^2*b^8*c^3*x + 24*a^3*b^7*c^2*d*x + 6*a^4*b^6*c*d^2*x + 12*b^9*c^3
*x^2 + 144*a*b^8*c^2*d*x^2 + 216*a^2*b^7*c*d^2*x^2 + 48*a^3*b^6*d^3*x^2 +
360*b^8*c*d^2*x^3 + 480*a*b^7*d^3*x^3 + 4*a^3*b^7*c^3 + 3*a^4*b^6*c^2*d +
24*a*b^8*c^3*x + 108*a^2*b^7*c^2*d*x + 72*a^3*b^6*c*d^2*x + 6*a^4*b^5*d^3*
x + 180*b^8*c^2*d*x^2 + 720*a*b^7*c*d^2*x^2 + 360*a^2*b^6*d^3*x^2 + 840*b^
7*d^3*x^3 + 12*a^2*b^7*c^3 + 24*a^3*b^6*c^2*d + 6*a^4*b^5*c*d^2 + 24*b^8*c
^3*x + 288*a*b^7*c^2*d*x + 432*a^2*b^6*c*d^2*x + 96*a^3*b^5*d^3*x + 1080*b^
7*c*d^2*x^2 + 1440*a*b^6*d^3*x^2 + 24*a*b^7*c^3 + 108*a^2*b^6*c^2*d + 72*
a^3*b^5*c*d^2 + 6*a^4*b^4*d^3 + 360*b^7*c^2*d*x + 1440*a*b^6*c*d^2*x + ...

```

**Mupad [B] (verification not implemented)**

Time = 23.29 (sec) , antiderivative size = 803, normalized size of antiderivative = 1.00

$$\begin{aligned}
& \int e^{-a-bx}(a+bx)^4(c+dx)^3 dx = -x^3 e^{-a-bx} \left( b^2 (4ac^3 + 4c^3) + 360cd^2 \right. \\
& \quad + \frac{a^4 d^3 + 16a^3 d^3 + 120a^2 d^3 + 480ad^3 + 840d^3}{b} + b(18da^2c^2 + 48dca^2 + 60dc^2) \\
& \quad \left. + 72a^2cd^2 + 12a^3cd^2 + 240acd^2 \right) - x^4 e^{-a-bx} (4a^3d^3 + 18a^2bcd^2 + 30a^2d^3 \\
& \quad + 12ab^2c^2d + 60abc d^2 + 120ad^3 + b^3c^3 + 15b^2c^2d + 90bcd^2 + 210d^3) \\
& \quad - \frac{e^{-a-bx} (a^4b^3c^3 + 3a^4b^2c^2d + 6a^4bcd^2 + 6a^4d^3 + 4a^3b^3c^3 + 24a^3b^2c^2d + 72a^3bcd^2 + 96a^3d^3 + \\
& \quad - xe^{-a-bx} \left( 4c^3(a^3 + 3a^2 + 6a + 6) + \frac{6d^3(a^4 + 16a^3 + 120a^2 + 480a + 840)}{b^3} \right. \\
& \quad \left. + \frac{3c^2d(a^4 + 8a^3 + 36a^2 + 96a + 120)}{b} + \frac{6cd^2(a^4 + 12a^3 + 72a^2 + 240a + 360)}{b^2} \right) \\
& \quad - 3x^2 e^{-a-bx} (a^4bcd^2 + a^4d^3 + 4a^3b^2c^2d + 12a^3bcd^2 + 16a^3d^3 + 2a^2b^3c^3 + 18a^2b^2c^2d + 72a^2bcd^2 \\
& \quad - b^3d^3x^7 e^{-a-bx} - b^2d^2x^6 e^{-a-bx} (7d + 4ad + 3bc) \\
& \quad - 3bdx^5 e^{-a-bx} (2a^2d^2 + 4abcd + 8ad^2 + b^2c^2 + 6bcd + 14d^2)
\end{aligned}$$

input

```
int(exp(- a - b*x)*(a + b*x)^4*(c + d*x)^3,x)
```

output

```

- x^3*exp(- a - b*x)*(b^2*(4*a*c^3 + 4*c^3) + 360*c*d^2 + (480*a*d^3 + 840
*d^3 + 120*a^2*d^3 + 16*a^3*d^3 + a^4*d^3)/b + b*(60*c^2*d + 18*a^2*c^2*d
+ 48*a*c^2*d) + 72*a^2*c*d^2 + 12*a^3*c*d^2 + 240*a*c*d^2) - x^4*exp(- a -
b*x)*(120*a*d^3 + 210*d^3 + 30*a^2*d^3 + 4*a^3*d^3 + b^3*c^3 + 15*b^2*c^2
*d + 90*b*c*d^2 + 60*a*b*c*d^2 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2) - (exp(-
a - b*x)*(2880*a*d^3 + 5040*d^3 + 720*a^2*d^3 + 96*a^3*d^3 + 24*b^3*c^3 +
6*a^4*d^3 + 24*a*b^3*c^3 + 360*b^2*c^2*d + 12*a^2*b^3*c^3 + 4*a^3*b^3*c^3
+ a^4*b^3*c^3 + 2160*b*c*d^2 + 108*a^2*b^2*c^2*d + 24*a^3*b^2*c^2*d + 3*a
^4*b^2*c^2*d + 1440*a*b*c*d^2 + 288*a*b^2*c^2*d + 432*a^2*b*c*d^2 + 72*a^3
*b*c*d^2 + 6*a^4*b*c*d^2))/b^4 - x*exp(- a - b*x)*(4*c^3*(6*a + 3*a^2 + a^
3 + 6) + (6*d^3*(480*a + 120*a^2 + 16*a^3 + a^4 + 840))/b^3 + (3*c^2*d*(96
*a + 36*a^2 + 8*a^3 + a^4 + 120))/b + (6*c*d^2*(240*a + 72*a^2 + 12*a^3 +
a^4 + 360))/b^2) - (3*x^2*exp(- a - b*x)*(480*a*d^3 + 840*d^3 + 120*a^2*d^
3 + 16*a^3*d^3 + 4*b^3*c^3 + a^4*d^3 + 4*a*b^3*c^3 + 60*b^2*c^2*d + 2*a^2*
b^3*c^3 + 360*b*c*d^2 + 18*a^2*b^2*c^2*d + 4*a^3*b^2*c^2*d + 240*a*b*c*d^2
+ 48*a*b^2*c^2*d + 72*a^2*b*c*d^2 + 12*a^3*b*c*d^2 + a^4*b*c*d^2))/b^2 -
b^3*d^3*x^7*exp(- a - b*x) - b^2*d^2*x^6*exp(- a - b*x)*(7*d + 4*a*d + 3*b
*c) - 3*b*d*x^5*exp(- a - b*x)*(8*a*d^2 + 14*d^2 + 2*a^2*d^2 + b^2*c^2 + 6
*b*c*d + 4*a*b*c*d)

```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.32

$$\int e^{-a-bx}(a+bx)^4(c+dx)^3 dx = \text{Too large to display}$$

input

```
int(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^3,x)
```

output

```
( - a**4*b**3*c**3 - 3*a**4*b**3*c**2*d*x - 3*a**4*b**3*c*d**2*x**2 - a**4
*b**3*d**3*x**3 - 3*a**4*b**2*c**2*d - 6*a**4*b**2*c*d**2*x - 3*a**4*b**2*
d**3*x**2 - 6*a**4*b*c*d**2 - 6*a**4*b*d**3*x - 6*a**4*d**3 - 4*a**3*b**4*
c**3*x - 12*a**3*b**4*c**2*d*x**2 - 12*a**3*b**4*c*d**2*x**3 - 4*a**3*b**4
*d**3*x**4 - 4*a**3*b**3*c**3 - 24*a**3*b**3*c**2*d*x - 36*a**3*b**3*c*d**
2*x**2 - 16*a**3*b**3*d**3*x**3 - 24*a**3*b**2*c**2*d - 72*a**3*b**2*c*d**
2*x - 48*a**3*b**2*d**3*x**2 - 72*a**3*b*c*d**2 - 96*a**3*b*d**3*x - 96*a*
*3*d**3 - 6*a**2*b**5*c**3*x**2 - 18*a**2*b**5*c**2*d*x**3 - 18*a**2*b**5*
c*d**2*x**4 - 6*a**2*b**5*d**3*x**5 - 12*a**2*b**4*c**3*x - 54*a**2*b**4*c
**2*d*x**2 - 72*a**2*b**4*c*d**2*x**3 - 30*a**2*b**4*d**3*x**4 - 12*a**2*b
**3*c**3 - 108*a**2*b**3*c**2*d*x - 216*a**2*b**3*c*d**2*x**2 - 120*a**2*b
**3*d**3*x**3 - 108*a**2*b**2*c**2*d - 432*a**2*b**2*c*d**2*x - 360*a**2*b
**2*d**3*x**2 - 432*a**2*b*c*d**2 - 720*a**2*b*d**3*x - 720*a**2*d**3 - 4*
a*b**6*c**3*x**3 - 12*a*b**6*c**2*d*x**4 - 12*a*b**6*c*d**2*x**5 - 4*a*b**
6*d**3*x**6 - 12*a*b**5*c**3*x**2 - 48*a*b**5*c**2*d*x**3 - 60*a*b**5*c*d
**2*x**4 - 24*a*b**5*d**3*x**5 - 24*a*b**4*c**3*x - 144*a*b**4*c**2*d*x**2
- 240*a*b**4*c*d**2*x**3 - 120*a*b**4*d**3*x**4 - 24*a*b**3*c**3 - 288*a*b
**3*c**2*d*x - 720*a*b**3*c*d**2*x**2 - 480*a*b**3*d**3*x**3 - 288*a*b**2*
c**2*d - 1440*a*b**2*c*d**2*x - 1440*a*b**2*d**3*x**2 - 1440*a*b*c*d**2 -
2880*a*b*d**3*x - 2880*a*d**3 - b**7*c**3*x**4 - 3*b**7*c**2*d*x**5 - 3...
```

### 3.114 $\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$

Optimal result . . . . .	767
Mathematica [A] (verified) . . . . .	768
Rubi [A] (verified) . . . . .	769
Maple [A] (verified) . . . . .	770
Fricas [A] (verification not implemented) . . . . .	772
Sympy [A] (verification not implemented) . . . . .	772
Maxima [A] (verification not implemented) . . . . .	774
Giac [A] (verification not implemented) . . . . .	775
Mupad [B] (verification not implemented) . . . . .	776
Reduce [B] (verification not implemented) . . . . .	777

#### Optimal result

Integrand size = 25, antiderivative size = 498

$$\begin{aligned}
 & \int e^{-a-bx}(a+bx)^4(c+dx)^2 dx \\
 = & -\frac{720d^2e^{-a-bx}}{b^3} - \frac{e^{-a-bx}(a+bx)^4(c+dx)^2}{b} - \frac{240de^{-a-bx}(bc+2ad+3bdx)}{b^3} \\
 & - \frac{24e^{-a-bx}(b^2c^2+8abcd+6a^2d^2+10bd(bc+2ad)x+15b^2d^2x^2)}{b^3} \\
 & - \frac{24e^{-a-bx}(a(b^2c^2+3abcd+a^2d^2)+b(b^2c^2+8abcd+6a^2d^2)x+5b^2d(bc+2ad)x^2+5b^3d^2x^3)}{b^3} \\
 & - \frac{2e^{-a-bx}(a^2(6b^2c^2+8abcd+a^2d^2)+12ab(b^2c^2+3abcd+a^2d^2)x+6b^2(b^2c^2+8abcd+6a^2d^2)x^2+20abd^2x^3)}{b^3} \\
 & - \frac{2e^{-a-bx}(a^3c(2bc+ad)+a^2(6b^2c^2+8abcd+a^2d^2)x+6ab(b^2c^2+3abcd+a^2d^2)x^2+2b^2(b^2c^2+8abcd+a^2d^2)x^3)}{b^2}
 \end{aligned}$$



output

```
-720*d^2*exp(-b*x-a)/b^3-exp(-b*x-a)*(b*x+a)^4*(d*x+c)^2/b-240*d*exp(-b*x-a)*(3*b*d*x+2*a*d+b*c)/b^3-24*exp(-b*x-a)*(b^2*c^2+8*a*b*c*d+6*a^2*d^2+10*b*d*(2*a*d+b*c)*x+15*b^2*d^2*x^2)/b^3-24*exp(-b*x-a)*(a*(a^2*d^2+3*a*b*c*d+b^2*c^2)+b*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*x+5*b^2*d*(2*a*d+b*c)*x^2+5*b^3*d^2*x^3)/b^3-2*exp(-b*x-a)*(a^2*(a^2*d^2+8*a*b*c*d+6*b^2*c^2)+12*a*b*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x+6*b^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*x^2+20*b^3*d*(2*a*d+b*c)*x^3+15*b^4*d^2*x^4)/b^3-2*exp(-b*x-a)*(a^3*c*(a*d+2*b*c)+a^2*(a^2*d^2+8*a*b*c*d+6*b^2*c^2)*x+6*a*b*(a^2*d^2+3*a*b*c*d+b^2*c^2)*x^2+2*b^2*(6*a^2*d^2+8*a*b*c*d+b^2*c^2)*x^3+5*b^3*d*(2*a*d+b*c)*x^4+3*b^4*d^2*x^5)/b^2
```

**Mathematica [A] (verified)**

Time = 2.67 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.64

$$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$$

$$= \frac{e^{-a-bx}(-2(360+240a+72a^2+12a^3+a^4)d^2 - b^6x^4(c+dx)^2 - 2b^5x^3(c+dx)(2(1+a)c + (3+2a)dx))}{b^3}$$

input

```
Integrate[E^(-a - b*x)*(a + b*x)^4*(c + d*x)^2,x]
```

output

```
(E^(-a - b*x)*(-2*(360 + 240*a + 72*a^2 + 12*a^3 + a^4)*d^2 - b^6*x^4*(c + d*x)^2 - 2*b^5*x^3*(c + d*x)*(2*(1 + a)*c + (3 + 2*a)*d*x) - 2*b*d*((120 + 96*a + 36*a^2 + 8*a^3 + a^4)*c + (360 + 240*a + 72*a^2 + 12*a^3 + a^4)*d*x) - 2*b^4*x^2*(3*(2 + 2*a + a^2)*c^2 + 2*(10 + 8*a + 3*a^2)*c*d*x + (15 + 10*a + 3*a^2)*d^2*x^2) - 4*b^3*x*((6 + 6*a + 3*a^2 + a^3)*c^2 + (30 + 24*a + 9*a^2 + 2*a^3)*c*d*x + (30 + 20*a + 6*a^2 + a^3)*d^2*x^2) - b^2*((24 + 24*a + 12*a^2 + 4*a^3 + a^4)*c^2 + 2*(120 + 96*a + 36*a^2 + 8*a^3 + a^4)*c*d*x + (360 + 240*a + 72*a^2 + 12*a^3 + a^4)*d^2*x^2))/b^3
```

**Rubi [A] (verified)**

Time = 1.59 (sec) , antiderivative size = 495, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$$

$$\downarrow 2626$$

$$\int \left( \frac{2de^{-a-bx}(a+bx)^5(bc-ad)}{b^2} + \frac{e^{-a-bx}(a+bx)^4(bc-ad)^2}{b^2} + \frac{d^2e^{-a-bx}(a+bx)^6}{b^2} \right) dx$$

$$\downarrow 2009$$

$$\frac{2de^{-a-bx}(a+bx)^5(bc-ad)}{b^3} - \frac{e^{-a-bx}(a+bx)^4(bc-ad)^2}{b^3} - \frac{10de^{-a-bx}(a+bx)^4(bc-ad)}{b^3} -$$

$$\frac{4e^{-a-bx}(a+bx)^3(bc-ad)^2}{b^3} - \frac{40de^{-a-bx}(a+bx)^3(bc-ad)}{b^3} - \frac{12e^{-a-bx}(a+bx)^2(bc-ad)^2}{b^3} -$$

$$\frac{120de^{-a-bx}(a+bx)^2(bc-ad)}{b^3} - \frac{24e^{-a-bx}(a+bx)(bc-ad)^2}{b^3} -$$

$$\frac{240de^{-a-bx}(a+bx)(bc-ad)}{b^3} - \frac{24e^{-a-bx}(bc-ad)^2}{b^3} - \frac{240de^{-a-bx}(bc-ad)}{b^3} -$$

$$\frac{d^2e^{-a-bx}(a+bx)^6}{b^3} - \frac{6d^2e^{-a-bx}(a+bx)^5}{b^3} - \frac{30d^2e^{-a-bx}(a+bx)^4}{b^3} - \frac{120d^2e^{-a-bx}(a+bx)^3}{b^3} -$$

$$\frac{360d^2e^{-a-bx}(a+bx)^2}{b^3} - \frac{720d^2e^{-a-bx}(a+bx)}{b^3} - \frac{720d^2e^{-a-bx}}{b^3}$$

input

```
Int[E^(-a - b*x)*(a + b*x)^4*(c + d*x)^2,x]
```

output

$$\begin{aligned} & (-720*d^2*E^{(-a - b*x)})/b^3 - (240*d*(b*c - a*d)*E^{(-a - b*x)})/b^3 - (24*(b*c - a*d)^2*E^{(-a - b*x)})/b^3 - (720*d^2*E^{(-a - b*x)*(a + b*x)})/b^3 - (240*d*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)})/b^3 - (24*(b*c - a*d)^2*E^{(-a - b*x)*(a + b*x)})/b^3 - (360*d^2*E^{(-a - b*x)*(a + b*x)^2})/b^3 - (120*d*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)^2})/b^3 - (12*(b*c - a*d)^2*E^{(-a - b*x)*(a + b*x)^2})/b^3 - (120*d^2*E^{(-a - b*x)*(a + b*x)^3})/b^3 - (40*d*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)^3})/b^3 - (4*(b*c - a*d)^2*E^{(-a - b*x)*(a + b*x)^3})/b^3 - (30*d^2*E^{(-a - b*x)*(a + b*x)^4})/b^3 - (10*d*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)^4})/b^3 - ((b*c - a*d)^2*E^{(-a - b*x)*(a + b*x)^4})/b^3 - (6*d^2*E^{(-a - b*x)*(a + b*x)^5})/b^3 - (2*d*(b*c - a*d)*E^{(-a - b*x)*(a + b*x)^5})/b^3 - (d^2*E^{(-a - b*x)*(a + b*x)^6})/b^3 \end{aligned}$$

### Defintions of rubi rules used

rule 2009

$$\text{Int}[u_, x\_Symbol] \text{ :> Simp[IntSum}[u, x], x] \text{ /; SumQ}[u]$$

rule 2626

$$\text{Int}[(F\_)^{(v\_)}*(P\_), x\_Symbol] \text{ :> Int[ExpandIntegrand}[F^{v}, P, x], x] \text{ /; FreeQ}[F, x] \ \&\& \ \text{PolynomialQ}[P, x] \ \&\& \ \text{LinearQ}[v, x] \ \&\& \ !\text{TrueQ}[\$UseGamma]$$

### Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.12



**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.71

$$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx =$$

$$\frac{(b^6 d^2 x^6 + 2(b^6 cd + (2a+3)b^5 d^2)x^5 + (a^4 + 4a^3 + 12a^2 + 24a + 24)b^2 c^2 + (b^6 c^2 + 2(4a+5)b^5 cd +$$

input `integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^2,x, algorithm="fricas")`

output 
$$-(b^6 d^2 x^6 + 2(b^6 cd + (2a+3)b^5 d^2)x^5 + (a^4 + 4a^3 + 12a^2 + 24a + 24)b^2 c^2 + (b^6 c^2 + 2(4a+5)b^5 cd + 2(3a^2 + 10a + 15)b^4 d^2)x^4 + 2(a^4 + 8a^3 + 36a^2 + 96a + 120)b^2 c^2 + 4((a+1)b^5 c^2 + (3a^2 + 8a + 10)b^4 cd + (a^3 + 6a^2 + 20a + 30)b^3 d^2)x^3 + 2(a^4 + 12a^3 + 72a^2 + 240a + 360)d^2 + (6(a^2 + 2a + 2)b^4 c^2 + 4(2a^3 + 9a^2 + 24a + 30)b^3 cd + (a^4 + 12a^3 + 72a^2 + 240a + 360)b^2 d^2)x^2 + 2(2(a^3 + 3a^2 + 6a + 6)b^3 c^2 + (a^4 + 8a^3 + 36a^2 + 96a + 120)b^2 cd + (a^4 + 12a^3 + 72a^2 + 240a + 360)b d^2)x) * e^{(-b*x - a)}/b^3$$

**Sympy [A] (verification not implemented)**

Time = 0.20 (sec) , antiderivative size = 899, normalized size of antiderivative = 1.81

$$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$$

$$= \left\{ \frac{(-a^4 b^2 c^2 - 2a^4 b^2 cdx - a^4 b^2 d^2 x^2 - 2a^4 bcd - 2a^4 bd^2 x - 2a^4 d^2 - 4a^3 b^3 c^2 x - 8a^3 b^3 cdx^2 - 4a^3 b^3 d^2 x^3 - 4a^3 b^2 c^2 - 16a^3 b^2 cdx - 12a^3 b^2 d^2 x^2 - 16a^3 bcd}{a^4 c^2 x + \frac{b^4 d^2 x^7}{7} + x^6 \cdot \left( \frac{2ab^3 d^2}{3} + \frac{b^4 cd}{3} \right) + x^5 \cdot \left( \frac{6a^2 b^2 d^2}{5} + \frac{8ab^3 cd}{5} + \frac{b^4 c^2}{5} \right) + x^4 (a^3 bd^2 + 3a^2 b^2 cd + ab^3 c^2) + x^3 (a^2 b^2 d^2 + 3a b^3 cd + b^4 c^2) + x^2 (a b^3 d^2 + 3a^2 b^2 cd + ab^3 c^2) + x (a^2 b^2 d^2 + 3a b^3 cd + b^4 c^2) + a^3 b^2 d^2 + 3a^2 b^2 cd + ab^3 c^2} \right.$$

input `integrate(exp(-b*x-a)*(b*x+a)**4*(d*x+c)**2,x)`

output

```

Piecewise((( -a**4*b**2*c**2 - 2*a**4*b**2*c*d*x - a**4*b**2*d**2*x**2 - 2*
a**4*b*c*d - 2*a**4*b*d**2*x - 2*a**4*d**2 - 4*a**3*b**3*c**2*x - 8*a**3*b
**3*c*d*x**2 - 4*a**3*b**3*d**2*x**3 - 4*a**3*b**2*c**2 - 16*a**3*b**2*c*d
*x - 12*a**3*b**2*d**2*x**2 - 16*a**3*b*c*d - 24*a**3*b*d**2*x - 24*a**3*d
**2 - 6*a**2*b**4*c**2*x**2 - 12*a**2*b**4*c*d*x**3 - 6*a**2*b**4*d**2*x**
4 - 12*a**2*b**3*c**2*x - 36*a**2*b**3*c*d*x**2 - 24*a**2*b**3*d**2*x**3 -
12*a**2*b**2*c**2 - 72*a**2*b**2*c*d*x - 72*a**2*b**2*d**2*x**2 - 72*a**2
*b*c*d - 144*a**2*b*d**2*x - 144*a**2*d**2 - 4*a*b**5*c**2*x**3 - 8*a*b**5
*c*d*x**4 - 4*a*b**5*d**2*x**5 - 12*a*b**4*c**2*x**2 - 32*a*b**4*c*d*x**3
- 20*a*b**4*d**2*x**4 - 24*a*b**3*c**2*x - 96*a*b**3*c*d*x**2 - 80*a*b**3*d
**2*x**3 - 24*a*b**2*c**2 - 192*a*b**2*c*d*x - 240*a*b**2*d**2*x**2 - 192
*a*b*c*d - 480*a*b*d**2*x - 480*a*d**2 - b**6*c**2*x**4 - 2*b**6*c*d*x**5
- b**6*d**2*x**6 - 4*b**5*c**2*x**3 - 10*b**5*c*d*x**4 - 6*b**5*d**2*x**5
- 12*b**4*c**2*x**2 - 40*b**4*c*d*x**3 - 30*b**4*d**2*x**4 - 24*b**3*c**2*x
- 120*b**3*c*d*x**2 - 120*b**3*d**2*x**3 - 24*b**2*c**2 - 240*b**2*c*d*x
- 360*b**2*d**2*x**2 - 240*b*c*d - 720*b*d**2*x - 720*d**2)*exp(-a - b*x)
/b**3, Ne(b**3, 0)), (a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2
/3 + b**4*c*d/3) + x**5*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5
) + x**4*(a**3*b*d**2 + 3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3
+ 8*a**3*b*c*d/3 + 2*a**2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)...

```

**Maxima [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 599, normalized size of antiderivative = 1.20

$$\begin{aligned}
& \int e^{-a-bx}(a+bx)^4(c+dx)^2 dx \\
&= -\frac{4(bx+1)a^3c^2e^{(-bx-a)}}{b} - \frac{a^4c^2e^{(-bx-a)}}{b} - \frac{2(bx+1)a^4cde^{(-bx-a)}}{b^2} \\
&\quad - \frac{6(b^2x^2+2bx+2)a^2c^2e^{(-bx-a)}}{b} - \frac{8(b^2x^2+2bx+2)a^3cde^{(-bx-a)}}{b^2} \\
&\quad - \frac{(b^2x^2+2bx+2)a^4d^2e^{(-bx-a)}}{b^3} - \frac{4(b^3x^3+3b^2x^2+6bx+6)ac^2e^{(-bx-a)}}{b} \\
&\quad - \frac{12(b^3x^3+3b^2x^2+6bx+6)a^2cde^{(-bx-a)}}{b^2} \\
&\quad - \frac{4(b^3x^3+3b^2x^2+6bx+6)a^3d^2e^{(-bx-a)}}{b^3} \\
&\quad - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)c^2e^{(-bx-a)}}{b} \\
&\quad - \frac{8(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)acde^{(-bx-a)}}{b^2} \\
&\quad - \frac{6(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)a^2d^2e^{(-bx-a)}}{b^3} \\
&\quad - \frac{2(b^5x^5+5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)cde^{(-bx-a)}}{b^2} \\
&\quad - \frac{4(b^5x^5+5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)ad^2e^{(-bx-a)}}{b^3} \\
&\quad - \frac{(b^6x^6+6b^5x^5+30b^4x^4+120b^3x^3+360b^2x^2+720bx+720)d^2e^{(-bx-a)}}{b^3}
\end{aligned}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^2,x, algorithm="maxima")`

output

```

-4*(b*x + 1)*a^3*c^2*e^(-b*x - a)/b - a^4*c^2*e^(-b*x - a)/b - 2*(b*x + 1)
*a^4*c*d*e^(-b*x - a)/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c^2*e^(-b*x - a)/b
- 8*(b^2*x^2 + 2*b*x + 2)*a^3*c*d*e^(-b*x - a)/b^2 - (b^2*x^2 + 2*b*x + 2)
*a^4*d^2*e^(-b*x - a)/b^3 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*c^2*e^(-
-b*x - a)/b - 12*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*c*d*e^(-b*x - a)/b^
2 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^3*d^2*e^(-b*x - a)/b^3 - (b^4*x^
4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c^2*e^(-b*x - a)/b - 8*(b^4*x^4
+ 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*c*d*e^(-b*x - a)/b^2 - 6*(b^4*x^
4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a^2*d^2*e^(-b*x - a)/b^3 - 2*(b^
5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*c*d*e^(-b*x -
a)/b^2 - 4*(b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)
*a*d^2*e^(-b*x - a)/b^3 - (b^6*x^6 + 6*b^5*x^5 + 30*b^4*x^4 + 120*b^3*x^3
+ 360*b^2*x^2 + 720*b*x + 720)*d^2*e^(-b*x - a)/b^3

```

**Giac [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 674, normalized size of antiderivative = 1.35

$$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx = \frac{(b^{10}d^2x^6 + 2b^{10}cdx^5 + 4ab^9d^2x^5 + b^{10}c^2x^4 + 8ab^9cdx^4 + 6a^2b^8d^2x^4 + 6b^9d^2x^5 + 4ab^9c^2x^3 + 12a^2b^8cdx^3 + 6a^3b^7d^2x^3 + 4a^2b^8cdx^2 + 12a^3b^7cdx^2 + 6a^4b^6d^2x^2 + 4a^3b^7cdx + 6a^4b^6cdx + 6a^5b^5d^2x + 4a^4b^6cd + 6a^5b^5cd + 6a^6b^4d^2x + 4a^5b^5cd + 6a^6b^4cd + 6a^7b^3d^2x + 4a^6b^4cd + 6a^7b^3cd + 6a^8b^2d^2x + 4a^7b^3cd + 6a^8b^2cd + 6a^9b^1d^2x + 4a^8b^2cd + 6a^9b^1cd + 6a^{10}d^2x + 4a^9b^1cd + 6a^{10}cd + 6a^{11}d^2 + 4a^{10}cd + 6a^{11}c)}{b^{11}}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^2,x, algorithm="giac")
```



output

```

-(b^10*d^2*x^6 + 2*b^10*c*d*x^5 + 4*a*b^9*d^2*x^5 + b^10*c^2*x^4 + 8*a*b^9
*c*d*x^4 + 6*a^2*b^8*d^2*x^4 + 6*b^9*d^2*x^5 + 4*a*b^9*c^2*x^3 + 12*a^2*b^
8*c*d*x^3 + 4*a^3*b^7*d^2*x^3 + 10*b^9*c*d*x^4 + 20*a*b^8*d^2*x^4 + 6*a^2*
b^8*c^2*x^2 + 8*a^3*b^7*c*d*x^2 + a^4*b^6*d^2*x^2 + 4*b^9*c^2*x^3 + 32*a*b
^8*c*d*x^3 + 24*a^2*b^7*d^2*x^3 + 30*b^8*d^2*x^4 + 4*a^3*b^7*c^2*x + 2*a^4
*b^6*c*d*x + 12*a*b^8*c^2*x^2 + 36*a^2*b^7*c*d*x^2 + 12*a^3*b^6*d^2*x^2 +
40*b^8*c*d*x^3 + 80*a*b^7*d^2*x^3 + a^4*b^6*c^2 + 12*a^2*b^7*c^2*x + 16*a^
3*b^6*c*d*x + 2*a^4*b^5*d^2*x + 12*b^8*c^2*x^2 + 96*a*b^7*c*d*x^2 + 72*a^2
*b^6*d^2*x^2 + 120*b^7*d^2*x^3 + 4*a^3*b^6*c^2 + 2*a^4*b^5*c*d + 24*a*b^7*
c^2*x + 72*a^2*b^6*c*d*x + 24*a^3*b^5*d^2*x + 120*b^7*c*d*x^2 + 240*a*b^6*
d^2*x^2 + 12*a^2*b^6*c^2 + 16*a^3*b^5*c*d + 2*a^4*b^4*d^2 + 24*b^7*c^2*x +
192*a*b^6*c*d*x + 144*a^2*b^5*d^2*x + 360*b^6*d^2*x^2 + 24*a*b^6*c^2 + 72
*a^2*b^5*c*d + 24*a^3*b^4*d^2 + 240*b^6*c*d*x + 480*a*b^5*d^2*x + 24*b^6*c
^2 + 192*a*b^5*c*d + 144*a^2*b^4*d^2 + 720*b^5*d^2*x + 240*b^5*c*d + 480*a
*b^4*d^2 + 720*b^4*d^2)*e^(-b*x - a)/b^7

```

### Mupad [B] (verification not implemented)

Time = 23.02 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.08

$$\begin{aligned}
 \int e^{-a-bx}(a+bx)^4(c+dx)^2 dx &= -x^2 e^{-a-bx} \left( 120cd + b(6a^2c^2 + 12ac^2 + 12c^2) \right. \\
 &\quad \left. + \frac{a^4d^2 + 12a^3d^2 + 72a^2d^2 + 240ad^2 + 360d^2}{b} + 96acd + 36a^2cd + 8a^3cd \right) \\
 &\quad - x^3 e^{-a-bx} \left( 4a^3d^2 + 12a^2bcd + 24a^2d^2 + 4ab^2c^2 + 32abcd + 80ad^2 + 4b^2c^2 \right. \\
 &\quad \left. + 40bcd + 120d^2 \right) \\
 &\quad - \frac{e^{-a-bx} \left( a^4b^2c^2 + 2a^4bcd + 2a^4d^2 + 4a^3b^2c^2 + 16a^3bcd + 24a^3d^2 + 12a^2b^2c^2 + 72a^2bcd + 144 \right.}{b^3} \\
 &\quad \left. - b^3d^2x^6 e^{-a-bx} - bx^4 e^{-a-bx} (6a^2d^2 + 8abcd + 20ad^2 + b^2c^2 + 10bcd + 30d^2) \right. \\
 &\quad \left. - 2xe^{-a-bx} (a^4bcd + a^4d^2 + 2a^3b^2c^2 + 8a^3bcd + 12a^3d^2 + 6a^2b^2c^2 + 36a^2bcd + 72a^2d^2 + 12ab \right.}{b^2} \\
 &\quad \left. - 2b^2dx^5 e^{-a-bx} (3d + 2ad + bc) \right)
 \end{aligned}$$

input

```
int(exp(- a - b*x)*(a + b*x)^4*(c + d*x)^2,x)
```

output

```

- x^2*exp(- a - b*x)*(120*c*d + b*(12*a*c^2 + 12*c^2 + 6*a^2*c^2) + (240*a
*d^2 + 360*d^2 + 72*a^2*d^2 + 12*a^3*d^2 + a^4*d^2)/b + 96*a*c*d + 36*a^2*
c*d + 8*a^3*c*d) - x^3*exp(- a - b*x)*(80*a*d^2 + 120*d^2 + 24*a^2*d^2 + 4
*b^2*c^2 + 4*a^3*d^2 + 4*a*b^2*c^2 + 40*b*c*d + 12*a^2*b*c*d + 32*a*b*c*d)
- (exp(- a - b*x)*(480*a*d^2 + 720*d^2 + 144*a^2*d^2 + 24*b^2*c^2 + 24*a^
3*d^2 + 2*a^4*d^2 + 24*a*b^2*c^2 + 240*b*c*d + 12*a^2*b^2*c^2 + 4*a^3*b^2*
c^2 + a^4*b^2*c^2 + 72*a^2*b*c*d + 16*a^3*b*c*d + 2*a^4*b*c*d + 192*a*b*c*
d))/b^3 - b^3*d^2*x^6*exp(- a - b*x) - b*x^4*exp(- a - b*x)*(20*a*d^2 + 30
*d^2 + 6*a^2*d^2 + b^2*c^2 + 10*b*c*d + 8*a*b*c*d) - (2*x*exp(- a - b*x)*
(240*a*d^2 + 360*d^2 + 72*a^2*d^2 + 12*b^2*c^2 + 12*a^3*d^2 + a^4*d^2 + 12*
a*b^2*c^2 + 120*b*c*d + 6*a^2*b^2*c^2 + 2*a^3*b^2*c^2 + 36*a^2*b*c*d + 8*a
^3*b*c*d + a^4*b*c*d + 96*a*b*c*d))/b^2 - 2*b^2*d*x^5*exp(- a - b*x)*(3*d
+ 2*a*d + b*c)

```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.29

$$\int e^{-a-bx}(a+bx)^4(c+dx)^2 dx$$

$$= \frac{-b^6 d^2 x^6 - 4a b^5 d^2 x^5 - 2b^6 c d x^5 - 6a^2 b^4 d^2 x^4 - 8a b^5 c d x^4 - b^6 c^2 x^4 - 6b^5 d^2 x^5 - 4a^3 b^3 d^2 x^3 - 12a^2 b^4 c d x^3}{1}$$

input

```
int(exp(-b*x-a)*(b*x+a)^4*(d*x+c)^2,x)
```

output

```
( - a**4*b**2*c**2 - 2*a**4*b**2*c*d*x - a**4*b**2*d**2*x**2 - 2*a**4*b*c*
d - 2*a**4*b*d**2*x - 2*a**4*d**2 - 4*a**3*b**3*c**2*x - 8*a**3*b**3*c*d*x
**2 - 4*a**3*b**3*d**2*x**3 - 4*a**3*b**2*c**2 - 16*a**3*b**2*c*d*x - 12*a
**3*b**2*d**2*x**2 - 16*a**3*b*c*d - 24*a**3*b*d**2*x - 24*a**3*d**2 - 6*a
**2*b**4*c**2*x**2 - 12*a**2*b**4*c*d*x**3 - 6*a**2*b**4*d**2*x**4 - 12*a*
**2*b**3*c**2*x - 36*a**2*b**3*c*d*x**2 - 24*a**2*b**3*d**2*x**3 - 12*a**2*
b**2*c**2 - 72*a**2*b**2*c*d*x - 72*a**2*b**2*d**2*x**2 - 72*a**2*b*c*d -
144*a**2*b*d**2*x - 144*a**2*d**2 - 4*a*b**5*c**2*x**3 - 8*a*b**5*c*d*x**4
- 4*a*b**5*d**2*x**5 - 12*a*b**4*c**2*x**2 - 32*a*b**4*c*d*x**3 - 20*a*b*
**4*d**2*x**4 - 24*a*b**3*c**2*x - 96*a*b**3*c*d*x**2 - 80*a*b**3*d**2*x**3
- 24*a*b**2*c**2 - 192*a*b**2*c*d*x - 240*a*b**2*d**2*x**2 - 192*a*b*c*d
- 480*a*b*d**2*x - 480*a*d**2 - b**6*c**2*x**4 - 2*b**6*c*d*x**5 - b**6*d*
**2*x**6 - 4*b**5*c**2*x**3 - 10*b**5*c*d*x**4 - 6*b**5*d**2*x**5 - 12*b**4
*c**2*x**2 - 40*b**4*c*d*x**3 - 30*b**4*d**2*x**4 - 24*b**3*c**2*x - 120*b
**3*c*d*x**2 - 120*b**3*d**2*x**3 - 24*b**2*c**2 - 240*b**2*c*d*x - 360*b*
**2*d**2*x**2 - 240*b*c*d - 720*b*d**2*x - 720*d**2)/(e**(a + b*x)*b**3)
```

### 3.115 $\int e^{-a-bx}(a+bx)^4(c+dx) dx$

Optimal result . . . . .	779
Mathematica [A] (verified) . . . . .	780
Rubi [A] (verified) . . . . .	780
Maple [A] (verified) . . . . .	782
Fricas [A] (verification not implemented) . . . . .	783
Sympy [A] (verification not implemented) . . . . .	783
Maxima [A] (verification not implemented) . . . . .	784
Giac [A] (verification not implemented) . . . . .	785
Mupad [B] (verification not implemented) . . . . .	786
Reduce [B] (verification not implemented) . . . . .	786

#### Optimal result

Integrand size = 23, antiderivative size = 275

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx = -\frac{120de^{-a-bx}}{b^2} - \frac{e^{-a-bx}(a+bx)^4(c+dx)}{b} - \frac{24e^{-a-bx}(bc+4ad+5bdx)}{b^2} - \frac{12e^{-a-bx}(a(2bc+3ad)+2b(bc+4ad)x+5b^2dx^2)}{b^2} - \frac{4e^{-a-bx}(a^2(3bc+2ad)+3ab(2bc+3ad)x+3b^2(bc+4ad)x^2+5b^3dx^3)}{b^2} - \frac{e^{-a-bx}(a^3(4bc+ad)+4a^2b(3bc+2ad)x+6ab^2(2bc+3ad)x^2+4b^3(bc+4ad)x^3+5b^4dx^4)}{b^2}$$

output

```
-120*d*exp(-b*x-a)/b^2-exp(-b*x-a)*(b*x+a)^4*(d*x+c)/b-24*exp(-b*x-a)*(5*b*d*x+4*a*d+b*c)/b^2-12*exp(-b*x-a)*(a*(3*a*d+2*b*c)+2*b*(4*a*d+b*c)*x+5*b^2*d*x^2)/b^2-4*exp(-b*x-a)*(a^2*(2*a*d+3*b*c)+3*a*b*(3*a*d+2*b*c)*x+3*b^2*(4*a*d+b*c)*x^2+5*b^3*d*x^3)/b^2-exp(-b*x-a)*(a^3*(a*d+4*b*c)+4*a^2*b*(2*a*d+3*b*c)*x+6*a*b^2*(3*a*d+2*b*c)*x^2+4*b^3*(4*a*d+b*c)*x^3+5*b^4*d*x^4)/b^2
```

**Mathematica [A] (verified)**

Time = 0.94 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.69

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx$$

$$= \frac{e^{-a-bx}(-((120+96a+36a^2+8a^3+a^4)d) - b^5x^4(c+dx) - b^4x^3(4(1+a)c + (5+4a)dx) - 2b^3x^2(3(2$$

input `Integrate[E^(-a - b*x)*(a + b*x)^4*(c + d*x),x]`

output `(E^(-a - b*x)*(-(120 + 96*a + 36*a^2 + 8*a^3 + a^4)*d) - b^5*x^4*(c + d*x) - b^4*x^3*(4*(1 + a)*c + (5 + 4*a)*d*x) - 2*b^3*x^2*(3*(2 + 2*a + a^2)*c + (10 + 8*a + 3*a^2)*d*x) - 2*b^2*x*(2*(6 + 6*a + 3*a^2 + a^3)*c + (30 + 24*a + 9*a^2 + 2*a^3)*d*x) - b*((24 + 24*a + 12*a^2 + 4*a^3 + a^4)*c + (120 + 96*a + 36*a^2 + 8*a^3 + a^4)*d*x))/b^2`

**Rubi [A] (verified)**

Time = 0.97 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2626, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx$$

$$\downarrow \text{2626}$$

$$\int \left( \frac{e^{-a-bx}(a+bx)^4(bc-ad)}{b} + \frac{de^{-a-bx}(a+bx)^5}{b} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{e^{-a-bx}(a+bx)^4(bc-ad)}{b^2} - \frac{4e^{-a-bx}(a+bx)^3(bc-ad)}{b^2} - \frac{12e^{-a-bx}(a+bx)^2(bc-ad)}{b^2} - \frac{24e^{-a-bx}(a+bx)(bc-ad)}{b^2} - \frac{24e^{-a-bx}(bc-ad)}{b^2} - \frac{de^{-a-bx}(a+bx)^5}{b^2} - \frac{5de^{-a-bx}(a+bx)^4}{b^2} - \frac{20de^{-a-bx}(a+bx)^3}{b^2} - \frac{60de^{-a-bx}(a+bx)^2}{b^2} - \frac{120de^{-a-bx}(a+bx)}{b^2} - \frac{120de^{-a-bx}}{b^2}$$

input `Int[E^(-a - b*x)*(a + b*x)^4*(c + d*x), x]`

output `(-120*d*E^(-a - b*x))/b^2 - (24*(b*c - a*d)*E^(-a - b*x))/b^2 - (120*d*E^(-a - b*x)*(a + b*x))/b^2 - (24*(b*c - a*d)*E^(-a - b*x)*(a + b*x))/b^2 - (60*d*E^(-a - b*x)*(a + b*x)^2)/b^2 - (12*(b*c - a*d)*E^(-a - b*x)*(a + b*x)^2)/b^2 - (20*d*E^(-a - b*x)*(a + b*x)^3)/b^2 - (4*(b*c - a*d)*E^(-a - b*x)*(a + b*x)^3)/b^2 - (5*d*E^(-a - b*x)*(a + b*x)^4)/b^2 - ((b*c - a*d)*E^(-a - b*x)*(a + b*x)^4)/b^2 - (d*E^(-a - b*x)*(a + b*x)^5)/b^2`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2626 `Int[(F_)^(v_)*(Px_), x_Symbol] := Int[ExpandIntegrand[F^v, Px, x], x] /; FreeQ[F, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00

method	result
norman	$(-4a b^2 d - c b^3 - 5b^2 d) x^4 e^{-bx-a} + (-6a^2 b d - 4a b^2 c - 16abd - 4b^2 c - 20bd) x^3 e^{-bx-a}$
gospers	$\frac{(d b^5 x^5 + 4a b^4 d x^4 + b^5 c x^4 + 6a^2 b^3 d x^3 + 4a b^4 c x^3 + 5b^4 d x^4 + 4a^3 b^2 d x^2 + 6a^2 b^3 c x^2 + 16a b^3 d x^3 + 4b^4 c x^3 + a^4 d x b + 4a^4 d^2 x^2)}{(d b^5 x^5 + 4a b^4 d x^4 + b^5 c x^4 + 6a^2 b^3 d x^3 + 4a b^4 c x^3 + 5b^4 d x^4 + 4a^3 b^2 d x^2 + 6a^2 b^3 c x^2 + 16a b^3 d x^3 + 4b^4 c x^3 + a^4 d x b + 4a^4 d^2 x^2)}$
risch	$\frac{(d b^5 x^5 + 4a b^4 d x^4 + b^5 c x^4 + 6a^2 b^3 d x^3 + 4a b^4 c x^3 + 5b^4 d x^4 + 4a^3 b^2 d x^2 + 6a^2 b^3 c x^2 + 16a b^3 d x^3 + 4b^4 c x^3 + a^4 d x b + 4a^4 d^2 x^2)}{(d b^5 x^5 + 4a b^4 d x^4 + b^5 c x^4 + 6a^2 b^3 d x^3 + 4a b^4 c x^3 + 5b^4 d x^4 + 4a^3 b^2 d x^2 + 6a^2 b^3 c x^2 + 16a b^3 d x^3 + 4b^4 c x^3 + a^4 d x b + 4a^4 d^2 x^2)}$
orering	$\frac{(d b^5 x^5 + 4a b^4 d x^4 + b^5 c x^4 + 6a^2 b^3 d x^3 + 4a b^4 c x^3 + 5b^4 d x^4 + 4a^3 b^2 d x^2 + 6a^2 b^3 c x^2 + 16a b^3 d x^3 + 4b^4 c x^3 + a^4 d x b + 4a^4 d^2 x^2)}{(d b^5 x^5 + 4a b^4 d x^4 + b^5 c x^4 + 6a^2 b^3 d x^3 + 4a b^4 c x^3 + 5b^4 d x^4 + 4a^3 b^2 d x^2 + 6a^2 b^3 c x^2 + 16a b^3 d x^3 + 4b^4 c x^3 + a^4 d x b + 4a^4 d^2 x^2)}$
derivativedivides	$\frac{c((-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}) - \frac{d((-bx-a)^5 e^{-bx-a}}{5}}{c((-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}) - \frac{d((-bx-a)^5 e^{-bx-a}}{5}}$
default	$\frac{c((-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}) - \frac{d((-bx-a)^5 e^{-bx-a}}{5}}{c((-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}) - \frac{d((-bx-a)^5 e^{-bx-a}}{5}}$
meijerg	$\frac{e^{-a} d \left( 120 - \frac{(6b^5 x^5 + 30b^4 x^4 + 120b^3 x^3 + 360b^2 x^2 + 720bx + 720)e^{-bx}}{6} \right)}{b^2} + \frac{e^{-a} c \left( 24 - \frac{(5b^4 x^4 + 20b^3 x^3 + 60b^2 x^2 + 120bx + 120)e^{-bx}}{5} \right)}{b}$
parts	$-d b^3 x^5 e^{-bx-a} - 4e^{-bx-a} b^2 a d x^4 - e^{-bx-a} b^3 c x^4 - 6e^{-bx-a} b a^2 d x^3 - 4e^{-bx-a} b^2 a c x^3 -$
parallelrisch	$\frac{-120e^{-bx-a} d + d b^5 e^{-bx-a} x^5 + x^4 e^{-bx-a} b^5 c + 5x^4 e^{-bx-a} b^4 d + 4x^3 e^{-bx-a} b^4 c + 20x^3 e^{-bx-a} b^3 d + 12x^2 e^{-bx-a} b^3 c + 120e^{-bx-a} d}{b^2} + \frac{e^{-a} c \left( 24 - \frac{(5b^4 x^4 + 20b^3 x^3 + 60b^2 x^2 + 120bx + 120)e^{-bx}}{5} \right)}{b}$

```
input int (exp(-b*x-a)*(b*x+a)^4*(d*x+c) , x,method=_RETURNVERBOSE)
```

```
output (-4*a*b^2*d-b^3*c-5*b^2*d)*x^4*exp(-b*x-a)+(-6*a^2*b*d-4*a*b^2*c-16*a*b*d-4*b^2*c-20*b*d)*x^3*exp(-b*x-a)+(-4*a^3*d-6*a^2*b*c-18*a^2*d-12*a*b*c-48*a*d-12*b*c-60*d)*x^2*exp(-b*x-a)-(a^4*b*c+a^4*d+4*a^3*b*c+8*a^3*d+12*a^2*b*c+36*a^2*d+24*a*b*c+96*a*d+24*b*c+120*d)/b^2*exp(-b*x-a)-d*b^3*x^5*exp(-b*x-a)-(a^4*d+4*a^3*b*c+8*a^3*d+12*a^2*b*c+36*a^2*d+24*a*b*c+96*a*d+24*b*c+120*d)/b*x*exp(-b*x-a)
```

**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.72

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx = \frac{(b^5 dx^5 + (b^5 c + (4a + 5)b^4 d)x^4 + 2(2(a + 1)b^4 c + (3a^2 + 8a + 10)b^3 d)x^3 + (a^4 + 4a^3 + 12a^2 + 24a + 24)bx^2 + (2a^4 c + 8a^3 d + 12a^2 c + 24a^2 d + 24a^3 c + 24a^3 d + 24a^4 c + 24a^4 d)x + (a^4 c + 4a^3 d + 12a^2 c + 24a^2 d + 24a^3 c + 24a^3 d + 24a^4 c + 24a^4 d))e^{-bx-a}}{b^5}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c),x, algorithm="fricas")`output 
$$-(b^5 d x^5 + (b^5 c + (4 a + 5) b^4 d) x^4 + 2(2(a + 1) b^4 c + (3 a^2 + 8 a + 10) b^3 d) x^3 + (a^4 + 4 a^3 + 12 a^2 + 24 a + 24) b x^2 + 2(3(a^2 + 2 a + 2) b^3 c + (2 a^3 + 9 a^2 + 24 a + 30) b^2 d) x + (a^4 + 8 a^3 + 36 a^2 + 96 a + 120) d + (4(a^3 + 3 a^2 + 6 a + 6) b^2 c + (a^4 + 8 a^3 + 36 a^2 + 96 a + 120) b d) x) e^{-b x - a} / b^5$$
**Sympy [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.63

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx = \frac{(-a^4 bc - a^4 b d x - a^4 d - 4a^3 b^2 c x - 4a^3 b^2 d x^2 - 4a^3 b c - 8a^3 b d x - 8a^3 d - 6a^2 b^3 c x^2 - 6a^2 b^3 d x^3 - 12a^2 b^2 c x - 18a^2 b^2 d x^2 - 12a^2 b c - 36a^2 b d x - 36a^2 d - 4a^4 c - 8a^4 d - 4a^3 b^2 c x - 4a^3 b^2 d x^2 - 4a^3 b c - 8a^3 b d x - 8a^3 d - 6a^2 b^3 c x^2 - 6a^2 b^3 d x^3 - 12a^2 b^2 c x - 18a^2 b^2 d x^2 - 12a^2 b c - 36a^2 b d x - 36a^2 d)}{b^5}$$

$$= \left\{ a^4 c x + \frac{b^4 d x^6}{6} + x^5 \cdot \left( \frac{4ab^3 d}{5} + \frac{b^4 c}{5} \right) + x^4 \cdot \left( \frac{3a^2 b^2 d}{2} + ab^3 c \right) + x^3 \cdot \left( \frac{4a^3 b d}{3} + 2a^2 b^2 c \right) + x^2 \left( \frac{a^4 d}{2} + 2a^3 b c \right) \right.$$

input `integrate(exp(-b*x-a)*(b*x+a)**4*(d*x+c),x)`



output

```
Piecewise((( -a**4*b*c - a**4*b*d*x - a**4*d - 4*a**3*b**2*c*x - 4*a**3*b**2*d*x**2 - 4*a**3*b*c - 8*a**3*b*d*x - 8*a**3*d - 6*a**2*b**3*c*x**2 - 6*a**2*b**3*d*x**3 - 12*a**2*b**2*c*x - 18*a**2*b**2*d*x**2 - 12*a**2*b*c - 36*a**2*b*d*x - 36*a**2*d - 4*a*b**4*c*x**3 - 4*a*b**4*d*x**4 - 12*a*b**3*c*x**2 - 16*a*b**3*d*x**3 - 24*a*b**2*c*x - 48*a*b**2*d*x**2 - 24*a*b*c - 96*a*b*d*x - 96*a*d - b**5*c*x**4 - b**5*d*x**5 - 4*b**4*c*x**3 - 5*b**4*d*x**4 - 12*b**3*c*x**2 - 20*b**3*d*x**3 - 24*b**2*c*x - 60*b**2*d*x**2 - 24*b*c - 120*b*d*x - 120*d)*exp(-a - b*x)/b**2, Ne(b**2, 0)), (a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b**2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 + 2*a**3*b*c), True))
```

### Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.25

$$\begin{aligned}
 & \int e^{-a-bx}(a+bx)^4(c+dx) dx \\
 &= -\frac{4(bx+1)a^3ce^{(-bx-a)}}{b} - \frac{a^4ce^{(-bx-a)}}{b} - \frac{(bx+1)a^4de^{(-bx-a)}}{b^2} \\
 & - \frac{6(b^2x^2+2bx+2)a^2ce^{(-bx-a)}}{b} - \frac{4(b^2x^2+2bx+2)a^3de^{(-bx-a)}}{b^2} \\
 & - \frac{4(b^3x^3+3b^2x^2+6bx+6)ace^{(-bx-a)}}{b} - \frac{6(b^3x^3+3b^2x^2+6bx+6)a^2de^{(-bx-a)}}{b^2} \\
 & - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)ce^{(-bx-a)}}{b} \\
 & - \frac{4(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)ade^{(-bx-a)}}{b^2} \\
 & - \frac{(b^5x^5+5b^4x^4+20b^3x^3+60b^2x^2+120bx+120)de^{(-bx-a)}}{b^2}
 \end{aligned}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c),x, algorithm="maxima")
```

output

```
-4*(b*x + 1)*a^3*c*e^(-b*x - a)/b - a^4*c*e^(-b*x - a)/b - (b*x + 1)*a^4*d
*e^(-b*x - a)/b^2 - 6*(b^2*x^2 + 2*b*x + 2)*a^2*c*e^(-b*x - a)/b - 4*(b^2*
x^2 + 2*b*x + 2)*a^3*d*e^(-b*x - a)/b^2 - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x +
6)*a*c*e^(-b*x - a)/b - 6*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a^2*d*e^(-b*x
- a)/b^2 - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*c*e^(-b*x - a
)/b - 4*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*a*d*e^(-b*x - a)/
b^2 - (b^5*x^5 + 5*b^4*x^4 + 20*b^3*x^3 + 60*b^2*x^2 + 120*b*x + 120)*d*e^
(-b*x - a)/b^2
```

### Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.20

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx = \frac{(b^9 dx^5 + b^9 cx^4 + 4ab^8 dx^4 + 4ab^8 cx^3 + 6a^2 b^7 dx^3 + 5b^8 dx^4 + 6a^2 b^7 cx^2 + 4a^3 b^6 dx^2 + 4b^8 cx^3 + 16ab^8 dx^2 + 12a^2 b^7 cx + 4a^3 b^6 c)x^2 + 12a^2 b^7 dx^2 + 18a^2 b^6 dx^2 + 20b^7 dx^3 + a^4 b^5 c + 12a^2 b^6 cx + 8a^3 b^5 dx + 12b^7 cx^2 + 48a^2 b^6 dx^2 + 4a^3 b^5 c + a^4 b^4 d + 24a^2 b^6 cx + 36a^2 b^5 dx + 60b^6 dx^2 + 12a^2 b^5 c + 8a^3 b^4 d + 24b^6 cx + 96a^2 b^5 dx + 24a^2 b^5 c + 36a^2 b^4 d + 120b^5 dx + 24b^5 c + 96a^2 b^4 d + 120b^4 d)*e^{-b*x - a}/b^6$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4*(d*x+c),x, algorithm="giac")
```

output

```
-(b^9*d*x^5 + b^9*c*x^4 + 4*a*b^8*d*x^4 + 4*a*b^8*c*x^3 + 6*a^2*b^7*d*x^3
+ 5*b^8*d*x^4 + 6*a^2*b^7*c*x^2 + 4*a^3*b^6*d*x^2 + 4*b^8*c*x^3 + 16*a*b^7
*d*x^3 + 4*a^3*b^6*c*x + a^4*b^5*d*x + 12*a*b^7*c*x^2 + 18*a^2*b^6*d*x^2 +
20*b^7*d*x^3 + a^4*b^5*c + 12*a^2*b^6*c*x + 8*a^3*b^5*d*x + 12*b^7*c*x^2
+ 48*a^2*b^6*d*x^2 + 4*a^3*b^5*c + a^4*b^4*d + 24*a^2*b^6*c*x + 36*a^2*b^5*d*x
+ 60*b^6*d*x^2 + 12*a^2*b^5*c + 8*a^3*b^4*d + 24*b^6*c*x + 96*a^2*b^5*d*x +
24*a^2*b^5*c + 36*a^2*b^4*d + 120*b^5*d*x + 24*b^5*c + 96*a^2*b^4*d + 120*b^4
*d)*e^(-b*x - a)/b^6
```

**Mupad [B] (verification not implemented)**

Time = 22.89 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx =$$

$$\frac{e^{-a-bx}(120d+96ad+24bc+36a^2d+8a^3d+a^4d+24abc+12a^2bc+4a^3bc+a^4bc)}{b^2}$$

$$- x^2 e^{-a-bx}(60d+48ad+12bc+18a^2d+4a^3d+12abc+6a^2bc)$$

$$- x e^{-a-bx} \left( 24c+24ac+12a^2c+4a^3c + \frac{da^4+8da^3+36da^2+96da+120d}{b} \right)$$

$$- b^3 dx^5 e^{-a-bx} - b^2 x^4 e^{-a-bx}(5d+4ad+bc)$$

$$- 2bx^3 e^{-a-bx}(10d+8ad+2bc+3a^2d+2abc)$$

input `int(exp(- a - b*x)*(a + b*x)^4*(c + d*x),x)`output `- (exp(- a - b*x)*(120*d + 96*a*d + 24*b*c + 36*a^2*d + 8*a^3*d + a^4*d + 24*a*b*c + 12*a^2*b*c + 4*a^3*b*c + a^4*b*c))/b^2 - x^2*exp(- a - b*x)*(60*d + 48*a*d + 12*b*c + 18*a^2*d + 4*a^3*d + 12*a*b*c + 6*a^2*b*c) - x*exp(- a - b*x)*(24*c + 24*a*c + 12*a^2*c + 4*a^3*c + (120*d + 96*a*d + 36*a^2*d + 8*a^3*d + a^4*d)/b) - b^3*d*x^5*exp(- a - b*x) - b^2*x^4*exp(- a - b*x)*(5*d + 4*a*d + b*c) - 2*b*x^3*exp(- a - b*x)*(10*d + 8*a*d + 2*b*c + 3*a^2*d + 2*a*b*c)`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.09

$$\int e^{-a-bx}(a+bx)^4(c+dx) dx$$

$$= \frac{-b^5 dx^5 - 4ab^4 dx^4 - b^5 cx^4 - 6a^2 b^3 dx^3 - 4ab^4 cx^3 - 5b^4 dx^4 - 4a^3 b^2 dx^2 - 6a^2 b^3 cx^2 - 16ab^3 dx^3 - 4b^4 cx^4}{b^5}$$

input `int(exp(-b*x-a)*(b*x+a)^4*(d*x+c),x)`

output

```
( - a**4*b*c - a**4*b*d*x - a**4*d - 4*a**3*b**2*c*x - 4*a**3*b**2*d*x**2
- 4*a**3*b*c - 8*a**3*b*d*x - 8*a**3*d - 6*a**2*b**3*c*x**2 - 6*a**2*b**3*
d*x**3 - 12*a**2*b**2*c*x - 18*a**2*b**2*d*x**2 - 12*a**2*b*c - 36*a**2*b*
d*x - 36*a**2*d - 4*a*b**4*c*x**3 - 4*a*b**4*d*x**4 - 12*a*b**3*c*x**2 - 1
6*a*b**3*d*x**3 - 24*a*b**2*c*x - 48*a*b**2*d*x**2 - 24*a*b*c - 96*a*b*d*x
- 96*a*d - b**5*c*x**4 - b**5*d*x**5 - 4*b**4*c*x**3 - 5*b**4*d*x**4 - 12
*b**3*c*x**2 - 20*b**3*d*x**3 - 24*b**2*c*x - 60*b**2*d*x**2 - 24*b*c - 12
0*b*d*x - 120*d)/(e**(a + b*x)*b**2)
```

### 3.116 $\int e^{-a-bx}(a+bx)^4 dx$

Optimal result	788
Mathematica [A] (verified)	788
Rubi [A] (verified)	789
Maple [A] (verified)	790
Fricas [A] (verification not implemented)	791
Sympy [A] (verification not implemented)	792
Maxima [A] (verification not implemented)	792
Giac [A] (verification not implemented)	793
Mupad [B] (verification not implemented)	793
Reduce [B] (verification not implemented)	794

#### Optimal result

Integrand size = 18, antiderivative size = 102

$$\int e^{-a-bx}(a+bx)^4 dx = -\frac{24e^{-a-bx}}{b} - \frac{24e^{-a-bx}(a+bx)}{b} - \frac{12e^{-a-bx}(a+bx)^2}{b} - \frac{4e^{-a-bx}(a+bx)^3}{b} - \frac{e^{-a-bx}(a+bx)^4}{b}$$

output

```
-24*exp(-b*x-a)/b-24*exp(-b*x-a)*(b*x+a)/b-12*exp(-b*x-a)*(b*x+a)^2/b-4*exp(-b*x-a)*(b*x+a)^3/b-exp(-b*x-a)*(b*x+a)^4/b
```

#### Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.49

$$\int e^{-a-bx}(a+bx)^4 dx = \frac{e^{-a-bx}(-24 - 24(a+bx) - 12(a+bx)^2 - 4(a+bx)^3 - (a+bx)^4)}{b}$$

input

```
Integrate[E^(-a - b*x)*(a + b*x)^4,x]
```

output

$$(E^{(-a - b*x)*(-24 - 24*(a + b*x) - 12*(a + b*x)^2 - 4*(a + b*x)^3 - (a + b*x)^4))/b$$
**Rubi [A] (verified)**

Time = 0.65 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {2607, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int e^{-a-bx}(a+bx)^4 dx \\ & \quad \downarrow 2607 \\ & 4 \int e^{-a-bx}(a+bx)^3 dx - \frac{e^{-a-bx}(a+bx)^4}{b} \\ & \quad \downarrow 2607 \\ & 4 \left( 3 \int e^{-a-bx}(a+bx)^2 dx - \frac{e^{-a-bx}(a+bx)^3}{b} \right) - \frac{e^{-a-bx}(a+bx)^4}{b} \\ & \quad \downarrow 2607 \\ & 4 \left( 3 \left( 2 \int e^{-a-bx}(a+bx) dx - \frac{e^{-a-bx}(a+bx)^2}{b} \right) - \frac{e^{-a-bx}(a+bx)^3}{b} \right) - \frac{e^{-a-bx}(a+bx)^4}{b} \\ & \quad \downarrow 2607 \\ & 4 \left( 3 \left( 2 \left( \int e^{-a-bx} dx - \frac{e^{-a-bx}(a+bx)}{b} \right) - \frac{e^{-a-bx}(a+bx)^2}{b} \right) - \frac{e^{-a-bx}(a+bx)^3}{b} \right) - \frac{e^{-a-bx}(a+bx)^4}{b} \\ & \quad \downarrow 2624 \\ & 4 \left( 3 \left( 2 \left( -\frac{e^{-a-bx}(a+bx)}{b} - \frac{e^{-a-bx}}{b} \right) - \frac{e^{-a-bx}(a+bx)^2}{b} \right) - \frac{e^{-a-bx}(a+bx)^3}{b} \right) - \frac{e^{-a-bx}(a+bx)^4}{b} \end{aligned}$$

input `Int[E^(-a - b*x)*(a + b*x)^4,x]`

output `-((E^(-a - b*x)*(a + b*x)^4)/b) + 4*(-((E^(-a - b*x)*(a + b*x)^3)/b) + 3*(-((E^(-a - b*x)*(a + b*x)^2)/b) + 2*(-(E^(-a - b*x)/b) - (E^(-a - b*x)*(a + b*x))/b)))`

### Defintions of rubi rules used

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

### Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.97

method	result
derivativdivides	$-\frac{(-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}}{b}$
default	$-\frac{(-bx-a)^4 e^{-bx-a} - 4e^{-bx-a}(-bx-a)^3 + 12(-bx-a)^2 e^{-bx-a} - 24(-bx-a)e^{-bx-a} + 24e^{-bx-a}}{b}$
gospers	$-\frac{(b^4 x^4 + 4a b^3 x^3 + 6a^2 b^2 x^2 + 4b^3 x^3 + 4a^3 b x + 12a b^2 x^2 + a^4 + 12a^2 b x + 12b^2 x^2 + 4a^3 + 24abx + 12a^2 + 24bx + 24a + 24)e^{-bx}}{b}$
risch	$-\frac{(b^4 x^4 + 4a b^3 x^3 + 6a^2 b^2 x^2 + 4b^3 x^3 + 4a^3 b x + 12a b^2 x^2 + a^4 + 12a^2 b x + 12b^2 x^2 + 4a^3 + 24abx + 12a^2 + 24bx + 24a + 24)e^{-bx}}{b}$
orering	$-\frac{(b^4 x^4 + 4a b^3 x^3 + 6a^2 b^2 x^2 + 4b^3 x^3 + 4a^3 b x + 12a b^2 x^2 + a^4 + 12a^2 b x + 12b^2 x^2 + 4a^3 + 24abx + 12a^2 + 24bx + 24a + 24)e^{-bx}}{b}$
norman	$(-4b^2 a - 4b^2) x^3 e^{-bx-a} + (-4a^3 - 12a^2 - 24a - 24) x e^{-bx-a} - b^3 x^4 e^{-bx-a} - \frac{(a^4 + 4a^3 - 4a^2 - 4a - 4)}{b} e^{-bx-a}$
parts	$-b^3 x^4 e^{-bx-a} - 4e^{-bx-a} b^2 a x^3 - 6e^{-bx-a} b a^2 x^2 - 4e^{-bx-a} a^3 x - \frac{e^{-bx-a} a^4}{b} + \frac{4e^{-bx-a}(-bx-a)}{b}$
meijerg	$\frac{e^{-a} \left( 24 - \frac{(5b^4 x^4 + 20b^3 x^3 + 60b^2 x^2 + 120bx + 120)e^{-bx}}{5} \right)}{b} + \frac{4e^{-a} a \left( 6 - \frac{(4b^3 x^3 + 12b^2 x^2 + 24bx + 24)e^{-bx}}{4} \right)}{b} + \frac{6e^{-a} a^2 \left( 2 - \frac{e^{-bx-a}}{b} \right)}{b}$
parallelrisch	$-\frac{b^4 x^4 e^{-bx-a} + 4a b^3 x^3 e^{-bx-a} + 4e^{-bx-a} x^3 b^3 + 6a^2 b^2 x^2 e^{-bx-a} + 12x^2 e^{-bx-a} a b^2 + 4a^3 b x e^{-bx-a} + 12b^2 e^{-bx-a} x^2 + a^4 e^{-bx-a}}{b}$

```
input int(exp(-b*x-a)*(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
output -1/b*((-b*x-a)^4*exp(-b*x-a)-4*exp(-b*x-a)*(-b*x-a)^3+12*(-b*x-a)^2*exp(-b*x-a)-24*(-b*x-a)*exp(-b*x-a)+24*exp(-b*x-a))
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81

$$\int e^{-a-bx}(a+bx)^4 dx = -\frac{(b^4 x^4 + 4(a+1)b^3 x^3 + 6(a^2 + 2a + 2)b^2 x^2 + a^4 + 4a^3 + 4(a^3 + 3a^2 + 6a + 6)bx + 12a^2 + 24a + 24)e^{-bx}}{b}$$

```
input integrate(exp(-b*x-a)*(b*x+a)^4,x, algorithm="fricas")
```

```
output -(b^4*x^4 + 4*(a + 1)*b^3*x^3 + 6*(a^2 + 2*a + 2)*b^2*x^2 + a^4 + 4*a^3 + 4*(a^3 + 3*a^2 + 6*a + 6)*b*x + 12*a^2 + 24*a + 24)*e^(-b*x - a)/b
```



**Sympy [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.55

$$\int e^{-a-bx}(a+bx)^4 dx = \begin{cases} \frac{(-a^4-4a^3bx-4a^3-6a^2b^2x^2-12a^2bx-12a^2-4ab^3x^3-12ab^2x^2-24abx-24a-b^4x^4-4b^3x^3-12b^2x^2-24bx-24)e^{-a-bx}}{b} & \text{for } b \neq 0 \\ a^4x + 2a^3bx^2 + 2a^2b^2x^3 + ab^3x^4 + \frac{b^4x^5}{5} & \text{otherwise} \end{cases}$$

input `integrate(exp(-b*x-a)*(b*x+a)**4,x)`output `Piecewise((((-a**4 - 4*a**3*b*x - 4*a**3 - 6*a**2*b**2*x**2 - 12*a**2*b*x - 12*a**2 - 4*a*b**3*x**3 - 12*a*b**2*x**2 - 24*a*b*x - 24*a - b**4*x**4 - 4*b**3*x**3 - 12*b**2*x**2 - 24*b*x - 24)*exp(-a - b*x)/b, Ne(b, 0)), (a**4*x + 2*a**3*b*x**2 + 2*a**2*b**2*x**3 + a*b**3*x**4 + b**4*x**5/5, True))`**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.46

$$\int e^{-a-bx}(a+bx)^4 dx = -\frac{4(bx+1)a^3e^{(-bx-a)}}{b} - \frac{a^4e^{(-bx-a)}}{b} - \frac{6(b^2x^2+2bx+2)a^2e^{(-bx-a)}}{b} - \frac{4(b^3x^3+3b^2x^2+6bx+6)ae^{(-bx-a)}}{b} - \frac{(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)e^{(-bx-a)}}{b}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4,x, algorithm="maxima")`output `-4*(b*x + 1)*a^3*e^(-b*x - a)/b - a^4*e^(-b*x - a)/b - 6*(b^2*x^2 + 2*b*x + 2)*a^2*e^(-b*x - a)/b - 4*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*a*e^(-b*x - a)/b - (b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b`

**Giac [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.29

$$\int e^{-a-bx}(a+bx)^4 dx = \frac{(b^8 x^4 + 4 ab^7 x^3 + 6 a^2 b^6 x^2 + 4 b^7 x^3 + 4 a^3 b^5 x + 12 ab^6 x^2 + a^4 b^4 + 12 a^2 b^5 x + 12 b^6 x^2 + 4 a^3 b^4 + 24 ab^5 x + a^4 b^3 + 12 a^2 b^4 + 24 a^3 b^3 + 12 a^2 b^2 + 24 a^3 b^2 + 24 a^2 b + 24 a^3) e^{-a-bx}}{b^5}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4,x, algorithm="giac")`output `-(b^8*x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*b^7*x^3 + 4*a^3*b^5*x + 12*a*b^6*x^2 + a^4*b^4 + 12*a^2*b^5*x + 12*b^6*x^2 + 4*a^3*b^4 + 24*a*b^5*x + 12*a^2*b^4 + 24*b^5*x + 24*a*b^4 + 24*b^4)*e^(-b*x - a)/b^5`**Mupad [B] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\int e^{-a-bx}(a+bx)^4 dx = -b^3 x^4 e^{-a-bx} - x e^{-a-bx} (4 a^3 + 12 a^2 + 24 a + 24) - \frac{e^{-a-bx} (a^4 + 4 a^3 + 12 a^2 + 24 a + 24)}{b} - 6 b x^2 e^{-a-bx} (a^2 + 2 a + 2) - 4 b^2 x^3 e^{-a-bx} (a + 1)$$

input `int(exp(- a - b*x)*(a + b*x)^4,x)`output `- b^3*x^4*exp(- a - b*x) - x*exp(- a - b*x)*(24*a + 12*a^2 + 4*a^3 + 24) - (exp(- a - b*x)*(24*a + 12*a^2 + 4*a^3 + a^4 + 24))/b - 6*b*x^2*exp(- a - b*x)*(2*a + a^2 + 2) - 4*b^2*x^3*exp(- a - b*x)*(a + 1)`

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int e^{-a-bx}(a+bx)^4 dx$$

$$= \frac{-b^4 x^4 - 4ab^3 x^3 - 6a^2 b^2 x^2 - 4b^3 x^3 - 4a^3 bx - 12ab^2 x^2 - a^4 - 12a^2 bx - 12b^2 x^2 - 4a^3 - 24abx - 12a^2}{e^{bx+ab}}$$

input `int(exp(-b*x-a)*(b*x+a)^4,x)`output `( - a**4 - 4*a**3*b*x - 4*a**3 - 6*a**2*b**2*x**2 - 12*a**2*b*x - 12*a**2  
- 4*a*b**3*x**3 - 12*a*b**2*x**2 - 24*a*b*x - 24*a - b**4*x**4 - 4*b**3*x*  
*3 - 12*b**2*x**2 - 24*b*x - 24)/(e**(a + b*x)*b)`

**3.117**  $\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$

Optimal result . . . . .	795
Mathematica [A] (verified) . . . . .	796
Rubi [A] (verified) . . . . .	796
Maple [A] (verified) . . . . .	797
Fricas [A] (verification not implemented) . . . . .	798
Sympy [F] . . . . .	799
Maxima [F] . . . . .	799
Giac [B] (verification not implemented) . . . . .	800
Mupad [F(-1)] . . . . .	800
Reduce [F] . . . . .	801

**Optimal result**

Integrand size = 25, antiderivative size = 277

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx = -\frac{6e^{-a-bx}}{d} + \frac{2(bc-ad)e^{-a-bx}}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}}{d^3} + \frac{(bc-ad)^3 e^{-a-bx}}{d^4} - \frac{6e^{-a-bx}(a+bx)}{d} + \frac{2(bc-ad)e^{-a-bx}(a+bx)}{d^2} - \frac{(bc-ad)^2 e^{-a-bx}(a+bx)}{d^3} - \frac{3e^{-a-bx}(a+bx)^2}{d} + \frac{(bc-ad)e^{-a-bx}(a+bx)^2}{d^2} - \frac{e^{-a-bx}(a+bx)^3}{d} + \frac{(bc-ad)^4 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5}$$

output

```
-6*exp(-b*x-a)/d+2*(-a*d+b*c)*exp(-b*x-a)/d^2-(-a*d+b*c)^2*exp(-b*x-a)/d^3
+(-a*d+b*c)^3*exp(-b*x-a)/d^4-6*exp(-b*x-a)*(b*x+a)/d+2*(-a*d+b*c)*exp(-b*
x-a)*(b*x+a)/d^2-(-a*d+b*c)^2*exp(-b*x-a)*(b*x+a)/d^3-3*exp(-b*x-a)*(b*x+a
)^2/d+(-a*d+b*c)*exp(-b*x-a)*(b*x+a)^2/d^2-exp(-b*x-a)*(b*x+a)^3/d+(-a*d+b
*c)^4*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^5
```

**Mathematica [A] (verified)**

Time = 1.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.63

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$$

$$= \frac{e^{-a-bx} \left( -d(2(3+4a+3a^2+2a^3)d^3 + 2bd^2(-((1+2a+3a^2)c) + (3+4a+3a^2)dx) + b^2d((1+4a)c^2 \right. \right.$$

input

```
Integrate[(E^(-a - b*x))*(a + b*x)^4]/(c + d*x),x]
```

output

```
(E^(-a - b*x))*(-(d*(2*(3 + 4*a + 3*a^2 + 2*a^3)*d^3 + 2*b*d^2*(-((1 + 2*a + 3*a^2)*c) + (3 + 4*a + 3*a^2)*d*x) + b^2*d*((1 + 4*a)*c^2 - 2*(1 + 2*a)*c*d*x + (3 + 4*a)*d^2*x^2) + b^3*(-c^3 + c^2*d*x - c*d^2*x^2 + d^3*x^3))) + (b*c - a*d)^4*E^(b*(c/d + x))*ExpIntegralEi[-((b*(c + d*x))/d)))/d^5
```

**Rubi [A] (verified)**

Time = 1.00 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$$

$$\downarrow 2629$$

$$\int \left( \frac{e^{-a-bx}(ad-bc)^4}{d^4(c+dx)} - \frac{be^{-a-bx}(bc-ad)^3}{d^4} + \frac{be^{-a-bx}(a+bx)(bc-ad)^2}{d^3} - \frac{be^{-a-bx}(a+bx)^2(bc-ad)}{d^2} + \frac{be^{-a-bx}(a+bx)^3}{d} \right) dx$$

$$\downarrow 2009$$

$$\frac{e^{\frac{bc}{d}-a}(bc-ad)^4 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{e^{-a-bx}(bc-ad)^3}{d^4} - \frac{e^{-a-bx}(bc-ad)^2}{d^3} - \frac{e^{-a-bx}(a+bx)(bc-ad)^2}{d^3} + \frac{2e^{-a-bx}(bc-ad)}{d^2} + \frac{e^{-a-bx}(a+bx)^2(bc-ad)}{d^2} + \frac{2e^{-a-bx}(a+bx)(bc-ad)}{d^2} - \frac{6e^{-a-bx}}{d} - \frac{e^{-a-bx}(a+bx)^3}{d} - \frac{3e^{-a-bx}(a+bx)^2}{d} - \frac{6e^{-a-bx}(a+bx)}{d}$$

```
input Int[(E^(-a - b*x)*(a + b*x)^4)/(c + d*x),x]
```

```
output (-6*E^(-a - b*x))/d + (2*(b*c - a*d)*E^(-a - b*x))/d^2 - ((b*c - a*d)^2*E^(-a - b*x))/d^3 + ((b*c - a*d)^3*E^(-a - b*x))/d^4 - (6*E^(-a - b*x)*(a + b*x))/d + (2*(b*c - a*d)*E^(-a - b*x)*(a + b*x))/d^2 - ((b*c - a*d)^2*E^(-a - b*x)*(a + b*x))/d^3 - (3*E^(-a - b*x)*(a + b*x)^2)/d + ((b*c - a*d)*E^(-a - b*x)*(a + b*x)^2)/d^2 - (E^(-a - b*x)*(a + b*x)^3)/d + ((b*c - a*d)^4*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^5
```

**Defintions of rubi rules used**

```
rule 2009 Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
```

```
rule 2629 Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
```

**Maple [A] (verified)**

Time = 0.11 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{b a^3 e^{-bx-a}}{d} - \frac{3b^2 a^2 c e^{-bx-a}}{d^2} - \frac{b a^2 ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d} + \frac{3b^3 a c^2 e^{-bx-a}}{d^3} + \frac{2b^2 a c ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \dots$
default	$-\frac{b a^3 e^{-bx-a}}{d} - \frac{3b^2 a^2 c e^{-bx-a}}{d^2} - \frac{b a^2 ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d} + \frac{3b^3 a c^2 e^{-bx-a}}{d^3} + \frac{2b^2 a c ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \dots$
risch	$-\frac{6 e^{-bx-a}}{d} - \frac{8 a e^{-bx-a}}{d} - \frac{6 a^2 e^{-bx-a}}{d} - \frac{4 a^3 e^{-bx-a}}{d} + \frac{4 b^2 c a e^{-bx-a} x}{d^2} + \frac{4 b e^{-\frac{da-bc}{d}} \text{expIntegral}_1(bx+a-\dots)}{d^2}$

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x,method=_RETURNVERBOSE)`

output 
$$\begin{aligned} & -1/b*(b/d*a^3*\exp(-b*x-a)-3*b^2/d^2*a^2*c*\exp(-b*x-a)-b/d*a^2*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))+3*b^3/d^3*a*c^2*\exp(-b*x-a)+2*b^2/d^2*a*c*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))+b/d*a*((-b*x-a)^2*\exp(-b*x-a)-2*(-b*x-a)*\exp(-b*x-a)+2*\exp(-b*x-a))-b^4/d^4*c^3*\exp(-b*x-a)-b^3/d^3*c^2*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))-b^2/d^2*c*((-b*x-a)^2*\exp(-b*x-a)-2*(-b*x-a)*\exp(-b*x-a)+2*\exp(-b*x-a))-1/d*b*(\exp(-b*x-a)*(-b*x-a)^3-3*(-b*x-a)^2*\exp(-b*x-a)+6*(-b*x-a)*\exp(-b*x-a)-6*\exp(-b*x-a))+a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b/d^5*\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d) \end{aligned}$$

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.85

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx = \frac{(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4)\text{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(\frac{bc-ad}{d}\right)} - (b^3d^4x^3 - b^3c^3d + (4a+1)b^2c^2d^2 - 2*(3a^2+2a+1)*b*c*d^3 + 2*(2a^3+3a^2+4a+3)*d^4 - (b^3*c*d^3 - (4a+3)*b^2*d^4)*x^2 + (b^3*c^2*d^2 - 2*(2a+1)*b^2*c*d^3 + 2*(3a^2+4a+3)*b*d^4)*x)*e^{-bx-a}}{d^5}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x, algorithm="fricas")`

output 
$$\begin{aligned} & ((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\text{Ei}(-\frac{(b*d*x + b*c)}{d})*e^{\frac{(b*c - a*d)}{d}} - (b^3*d^4*x^3 - b^3*c^3*d + (4*a + 1)*b^2*c^2*d^2 - 2*(3*a^2 + 2*a + 1)*b*c*d^3 + 2*(2*a^3 + 3*a^2 + 4*a + 3)*d^4 - (b^3*c*d^3 - (4*a + 3)*b^2*d^4)*x^2 + (b^3*c^2*d^2 - 2*(2*a + 1)*b^2*c*d^3 + 2*(3*a^2 + 4*a + 3)*b*d^4)*x)*e^{-b*x - a})/d^5 \end{aligned}$$

**Sympy [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx = \left( \int \frac{a^4}{ce^{bx} + dxe^{bx}} dx + \int \frac{b^4x^4}{ce^{bx} + dxe^{bx}} dx + \int \frac{4ab^3x^3}{ce^{bx} + dxe^{bx}} dx + \int \frac{6a^2b^2x^2}{ce^{bx} + dxe^{bx}} dx + \int \frac{4a^3bx}{ce^{bx} + dxe^{bx}} dx \right) e^{-a}$$

input `integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c), x)`

output `(Integral(a**4/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(b**4*x**4/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(4*a*b**3*x**3/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(6*a**2*b**2*x**2/(c*exp(b*x) + d*x*exp(b*x)), x) + Integral(4*a**3*b*x/(c*exp(b*x) + d*x*exp(b*x)), x))*exp(-a)`

**Maxima [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx = \int \frac{(bx+a)^4 e^{(-bx-a)}}{dx+c} dx$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c), x, algorithm="maxima")`

output `-a^4*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - (b^3*d^2*x^4 + (4*a*b^2*d^2 + 3*b^2*d^2)*x^3 + (6*a^2*b*d^2 + b^2*c*d + 8*a*b*d^2 + 6*b*d^2)*x^2 + (4*a^3*d^2 - b^2*c^2 + 6*a^2*d^2 + 4*b*c*d + 4*(b*c*d + 2*d^2)*a + 6*d^2)*x)*e^(-b*x)/(d^3*x*e^a + c*d^2*e^a) + integrate((4*a^3*c*d^2 - b^2*c^3 + 6*a^2*b*c*d^2 + 4*b*c^2*d + 6*c*d^2 + 4*(b*c^2*d + 2*c*d^2)*a + (b^3*c^3 + 6*a^2*b*c*d^2 - 2*b^2*c^2*d + 6*b*c*d^2 - 4*(b^2*c^2*d - 2*b*c*d^2)*a)*x)*e^(-b*x)/(d^4*x^2*e^a + 2*c*d^3*x*e^a + c^2*d^2*e^a), x)`



**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 546 vs.  $2(266) = 532$ .

Time = 0.13 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.97

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx = \frac{b^3 d^4 x^3 e^{(-bx-a)} - b^3 c d^3 x^2 e^{(-bx-a)} + 4 a b^2 d^4 x^2 e^{(-bx-a)} - b^4 c^4 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} + 4 a b^3 c^3 d \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)}{d^5}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x, algorithm="giac")`

output `-(b^3*d^4*x^3*e^(-b*x - a) - b^3*c*d^3*x^2*e^(-b*x - a) + 4*a*b^2*d^4*x^2*e^(-b*x - a) - b^4*c^4*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 4*a*b^3*c^3*d*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 6*a^2*b^2*c^2*d^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 4*a^3*b*c*d^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - a^4*d^4*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^3*c^2*d^2*x*e^(-b*x - a) - 4*a*b^2*c*d^3*x*e^(-b*x - a) + 6*a^2*b*d^4*x*e^(-b*x - a) + 3*b^2*d^4*x^2*e^(-b*x - a) - b^3*c^3*d*e^(-b*x - a) + 4*a*b^2*c^2*d^2*e^(-b*x - a) - 6*a^2*b*c*d^3*e^(-b*x - a) + 4*a^3*d^4*e^(-b*x - a) - 2*b^2*c*d^3*x*e^(-b*x - a) + 8*a*b*d^4*x*e^(-b*x - a) + b^2*c^2*d^2*e^(-b*x - a) - 4*a*b*c*d^3*e^(-b*x - a) + 6*a^2*d^4*e^(-b*x - a) + 6*b*d^4*x*e^(-b*x - a) - 2*b*c*d^3*e^(-b*x - a) + 8*a*d^4*e^(-b*x - a) + 6*d^4*e^(-b*x - a))/d^5`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx = \int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$$

input `int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x), x)`

output `int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x), x)`

**Reduce [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{c+dx} dx$$

$$= \frac{6a^2bc d^2 - 6a^2b d^3 x - 4a b^2 c^2 d - 4a b^2 d^3 x^2 + 4abc d^2 - 8ab d^3 x - b^3 c^2 dx + b^3 c d^2 x^2 + 2b^2 c d^2 x + e^{bx} \left( \int \dots \right)}{}$$

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c),x)`

output

```
(e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d*x),x)*a**4*d**4 - 4*e**(b*x)*int(
1/(e**(b*x)*c + e**(b*x)*d*x),x)*a**3*b*c*d**3 + 6*e**(b*x)*int(1/(e**(b*x)
)*c + e**(b*x)*d*x),x)*a**2*b**2*c**2*d**2 - 4*e**(b*x)*int(1/(e**(b*x)*c
+ e**(b*x)*d*x),x)*a*b**3*c**3*d + e**(b*x)*int(1/(e**(b*x)*c + e**(b*x)*d
*x),x)*b**4*c**4 - 4*a**3*d**3 + 6*a**2*b*c*d**2 - 6*a**2*b*d**3*x - 6*a**
2*d**3 - 4*a*b**2*c**2*d + 4*a*b**2*c*d**2*x - 4*a*b**2*d**3*x**2 + 4*a*b*
c*d**2 - 8*a*b*d**3*x - 8*a*d**3 + b**3*c**3 - b**3*c**2*d*x + b**3*c*d**2
*x**2 - b**3*d**3*x**3 - b**2*c**2*d + 2*b**2*c*d**2*x - 3*b**2*d**3*x**2
+ 2*b*c*d**2 - 6*b*d**3*x - 6*d**3)/(e**(a + b*x)*d**4)
```

**3.118**  $\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$

Optimal result	802
Mathematica [A] (verified)	803
Rubi [A] (verified)	803
Maple [A] (verified)	805
Fricas [A] (verification not implemented)	805
Sympy [F]	806
Maxima [F]	807
Giac [B] (verification not implemented)	807
Mupad [F(-1)]	808
Reduce [F]	809

**Optimal result**

Integrand size = 25, antiderivative size = 258

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = -\frac{2be^{-a-bx}}{d^2} + \frac{4b(bc-ad)e^{-a-bx}}{d^3} - \frac{6b(bc-ad)^2e^{-a-bx}}{d^4} - \frac{(bc-ad)^4e^{-a-bx}}{d^5(c+dx)} - \frac{2b^2e^{-a-bx}(c+dx)}{d^3} + \frac{4b^2(bc-ad)e^{-a-bx}(c+dx)}{d^4} - \frac{b^3e^{-a-bx}(c+dx)^2}{d^4} - \frac{4b(bc-ad)^3e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5} - \frac{b(bc-ad)^4e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6}$$

output

```
-2*b*exp(-b*x-a)/d^2+4*b*(-a*d+b*c)*exp(-b*x-a)/d^3-6*b*(-a*d+b*c)^2*exp(-
b*x-a)/d^4-(-a*d+b*c)^4*exp(-b*x-a)/d^5/(d*x+c)-2*b^2*exp(-b*x-a)*(d*x+c)/
d^3+4*b^2*(-a*d+b*c)*exp(-b*x-a)*(d*x+c)/d^4-b^3*exp(-b*x-a)*(d*x+c)^2/d^4
-4*b*(-a*d+b*c)^3*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^5-b*(-a*d+b*c)^4*exp(-a
+b*c/d)*Ei(-b*(d*x+c)/d)/d^6
```

### Mathematica [A] (verified)

Time = 1.69 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.63

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$$

$$= \frac{e^{-a} \left( -\frac{de^{-bx}((bc-ad)^4 + bd(3b^2c^2 - 2(1+4a)bcd + 2(1+2a+3a^2)d^2)(c+dx) - 2b^2d^2(bc - (1+2a)d)x(c+dx) + b^3d^3x^2(c+dx))}{c+dx} \right) - b(bc - (-4 + a)d)(b^3c^2d^3 - 2b^2cd^2(c+dx) + b^3d^3x^2)}{d^6}$$

input

```
Integrate[(E^(-a - b*x)*(a + b*x)^4)/(c + d*x)^2,x]
```

output

```
((-((d*((b*c - a*d)^4 + b*d*(3*b^2*c^2 - 2*(1 + 4*a)*b*c*d + 2*(1 + 2*a + 3*a^2)*d^2)*(c + d*x) - 2*b^2*d^2*(b*c - (1 + 2*a)*d)*x*(c + d*x) + b^3*d^3*x^2*(c + d*x)))/(E^(b*x)*(c + d*x))) - b*(b*c - (-4 + a)*d)*(b*c - a*d)^3 *E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(d^6*E^a)
```

### Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$$

$$\downarrow \text{2629}$$

$$\int \left( \frac{b^4 e^{-a-bx}(c+dx)^2}{d^4} - \frac{4b^3 e^{-a-bx}(c+dx)(bc-ad)}{d^4} + \frac{6b^2 e^{-a-bx}(bc-ad)^2}{d^4} - \frac{4b e^{-a-bx}(bc-ad)^3}{d^4(c+dx)} + \frac{e^{-a-bx}(a+bx)^4}{d^4(c+dx)} \right) dx$$

$$\downarrow \text{2009}$$

$$\frac{-\frac{b^3 e^{-a-bx}(c+dx)^2}{d^4} + \frac{4b^2 e^{-a-bx}(c+dx)(bc-ad)}{d^4} - \frac{2b^2 e^{-a-bx}(c+dx)}{d^3} - \frac{be^{\frac{bc}{d}-a}(bc-ad)^4 \operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4be^{\frac{bc}{d}-a}(bc-ad)^3 \operatorname{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5}}{\frac{e^{-a-bx}(bc-ad)^4}{d^5(c+dx)} - \frac{6be^{-a-bx}(bc-ad)^2}{d^4} + \frac{4be^{-a-bx}(bc-ad)}{d^3} - \frac{2be^{-a-bx}}{d^2}}$$

input `Int[(E^(-a - b*x))*(a + b*x)^4]/(c + d*x)^2,x]`

output `(-2*b*E^(-a - b*x))/d^2 + (4*b*(b*c - a*d)*E^(-a - b*x))/d^3 - (6*b*(b*c - a*d)^2*E^(-a - b*x))/d^4 - ((b*c - a*d)^4*E^(-a - b*x))/(d^5*(c + d*x)) - (2*b^2*E^(-a - b*x)*(c + d*x))/d^3 + (4*b^2*(b*c - a*d)*E^(-a - b*x)*(c + d*x))/d^4 - (b^3*E^(-a - b*x)*(c + d*x)^2)/d^4 - (4*b*(b*c - a*d)^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^5 - (b*(b*c - a*d)^4*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^6`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_) + (e_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.57

method	result
derivativedivides	$-\frac{3b^2a^2e^{-bx-a}}{d^2} - \frac{6b^3ace^{-bx-a}}{d^3} - \frac{2b^2a((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \frac{3b^4c^2e^{-bx-a}}{d^4} + \frac{2b^3c((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + \dots$
default	$-\frac{3b^2a^2e^{-bx-a}}{d^2} - \frac{6b^3ace^{-bx-a}}{d^3} - \frac{2b^2a((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^2} + \frac{3b^4c^2e^{-bx-a}}{d^4} + \frac{2b^3c((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + \dots$
risch	$-\frac{4bae^{-bx-a}}{d^2} + \frac{2b^2ce^{-bx-a}}{d^3} - \frac{b^3e^{-bx-a}x^2}{d^2} - \frac{2b^2e^{-bx-a}x}{d^2} - \frac{4b^2e^{-bx-a}a^3c}{d^3(-bx-\frac{bc}{d})} + \frac{6b^3e^{-bx-a}a^2c^2}{d^4(-bx-\frac{bc}{d})} - \frac{4b^4e^{-bx-a}}{d^5(-bx-\frac{bc}{d})} + \dots$

```
input int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
output -1/b*(3*b^2/d^2*a^2*exp(-b*x-a)-6*b^3/d^3*a*c*exp(-b*x-a)-2*b^2/d^2*a*((-b*x-a)*exp(-b*x-a)-exp(-b*x-a))+3*b^4/d^4*c^2*exp(-b*x-a)+2*b^3/d^3*c*((-b*x-a)*exp(-b*x-a)-exp(-b*x-a))+1/d^2*b^2*((-b*x-a)^2*exp(-b*x-a)-2*(-b*x-a)*exp(-b*x-a)+2*exp(-b*x-a))+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^2/d^6*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+4/d^5*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.37

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = \frac{(b^5c^5 - 4(a-1)b^4c^4d + 6(a^2 - 2a)b^3c^3d^2 - 4(a^3 - 3a^2)b^2c^2d^3 + (a^4 - 4a^3)bcd^4 + (b^5c^4d - 4(a-1)b^4c^3d^2 - 4(a^2 - 2a)b^3c^2d^2 + (a^3 - 3a^2)bc^2d^3 + (a^4 - 4a^3)cd^4 + (b^5c^3d - 4(a-1)b^4c^2d^2 - 4(a^2 - 2a)b^3cd^2 + (a^3 - 3a^2)cd^3 + (a^4 - 4a^3)d^4))e^{-bx-a}}{(c+dx)^2}$$

```
input integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

output

```

-((b^5*c^5 - 4*(a - 1)*b^4*c^4*d + 6*(a^2 - 2*a)*b^3*c^3*d^2 - 4*(a^3 - 3*
a^2)*b^2*c^2*d^3 + (a^4 - 4*a^3)*b*c*d^4 + (b^5*c^4*d - 4*(a - 1)*b^4*c^3*
d^2 + 6*(a^2 - 2*a)*b^3*c^2*d^3 - 4*(a^3 - 3*a^2)*b^2*c*d^4 + (a^4 - 4*a^3
)*b*d^5)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + (b^3*d^5*x^3 + b^4*c^
4*d - (4*a - 3)*b^3*c^3*d^2 + a^4*d^5 + 2*(3*a^2 - 4*a - 1)*b^2*c^2*d^3 -
2*(2*a^3 - 3*a^2 - 2*a - 1)*b*c*d^4 - (b^3*c*d^4 - 2*(2*a + 1)*b^2*d^5)*x^
2 + (b^3*c^2*d^3 - 4*a*b^2*c*d^4 + 2*(3*a^2 + 2*a + 1)*b*d^5)*x)*e^(-b*x -
a))/(d^7*x + c*d^6)

```

SymPy [F]

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = & \left( \int \frac{a^4}{c^2 e^{bx} + 2cdx e^{bx} + d^2 x^2 e^{bx}} dx \right. \\
& + \int \frac{b^4 x^4}{c^2 e^{bx} + 2cdx e^{bx} + d^2 x^2 e^{bx}} dx \\
& + \int \frac{4ab^3 x^3}{c^2 e^{bx} + 2cdx e^{bx} + d^2 x^2 e^{bx}} dx \\
& + \int \frac{6a^2 b^2 x^2}{c^2 e^{bx} + 2cdx e^{bx} + d^2 x^2 e^{bx}} dx \\
& \left. + \int \frac{4a^3 bx}{c^2 e^{bx} + 2cdx e^{bx} + d^2 x^2 e^{bx}} dx \right) e^{-a}
\end{aligned}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c)**2,x)
```

output

```

(Integral(a**4/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x)
+ Integral(b**4*x**4/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*x**2*exp(b*
x)), x) + Integral(4*a*b**3*x**3/(c**2*exp(b*x) + 2*c*d*x*exp(b*x) + d**2*
x**2*exp(b*x)), x) + Integral(6*a**2*b**2*x**2/(c**2*exp(b*x) + 2*c*d*x*ex
p(b*x) + d**2*x**2*exp(b*x)), x) + Integral(4*a**3*b*x/(c**2*exp(b*x) + 2*
c*d*x*exp(b*x) + d**2*x**2*exp(b*x)), x))*exp(-a)

```

**Maxima [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = \int \frac{(bx+a)^4 e^{(-bx-a)}}{(dx+c)^2} dx$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")`

output

```
-a^4*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - (b^3*
d^2*x^4 + 2*(2*a*b^2*d^2 + b^2*d^2)*x^3 + 2*(3*a^2*b*d^2 + b^2*c*d + 2*a*b
*d^2 + b*d^2)*x^2 + 2*(2*a^3*d^2 - b^2*c^2 + 4*a*b*c*d + 2*b*c*d)*x)*e^(-b
*x)/(d^4*x^2*e^a + 2*c*d^3*x*e^a + c^2*d^2*e^a) - integrate(-2*(2*a^3*c*d^
2 - b^2*c^3 + 4*a*b*c^2*d + 2*b*c^2*d + (b^3*c^3 - 4*a*b^2*c^2*d + 6*a^2*b
*c*d^2 - 2*a^3*d^3 + b^2*c^2*d)*x)*e^(-b*x)/(d^5*x^3*e^a + 3*c*d^4*x^2*e^a
+ 3*c^2*d^3*x*e^a + c^3*d^2*e^a), x)
```

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. 2(249) = 498.

Time = 0.16 (sec) , antiderivative size = 2861, normalized size of antiderivative = 11.09

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = \text{Too large to display}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x, algorithm="giac")`



output

```

-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^6*c^4*Ei(-((d*x + c)*(b
- b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^7*c
^5*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((
b*c - a*d)/d) - 4*(d*x + c)*a*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^5*c^3*
d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b
*c - a*d)/d) - 5*a*b^6*c^4*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x
+ c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 6*(d*x + c)*a^2*(b - b*c/(d*x +
c) + a*d/(d*x + c))*b^4*c^2*d^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 10*a^2*b^5*c^3*d^2*Ei(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d)
- 4*(d*x + c)*a^3*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^3*c*d^3*Ei(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d)
) - 10*a^3*b^4*c^2*d^3*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))
+ b*c - a*d)/d)*e^((b*c - a*d)/d) + (d*x + c)*a^4*(b - b*c/(d*x + c) + a*d
/(d*x + c))*b^2*d^4*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b
*c - a*d)/d)*e^((b*c - a*d)/d) + 5*a^4*b^3*c*d^4*Ei(-((d*x + c)*(b - b*c/(
d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) - a^5*b^2*d^5*
Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c
- a*d)/d) + 4*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^5*c^3*d*Ei(
-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c...

```

### Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx$$

input

```
int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^2,x)
```

output

```
int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^2, x)
```

## Reduce [F]

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^2} dx = \text{too large to display}$$

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^2,x)`

output

```
( - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c
*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x
)*a**4*b**2*c**2*d**4 - e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**
2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e
**(b*x)*d**3*x**2),x)*a**4*b**2*c*d**5*x - e**(b*x)*int(x/(e**(b*x)*b*c**3
+ 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e
*(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**4*b*c*d**5 - e**(b*x)*int(x/(e
**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)
*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**4*b*d**6*x + 4*e
**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2
*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**
3*b**3*c**3*d**3 + 4*e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d
*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(
b*x)*d**3*x**2),x)*a**3*b**3*c**2*d**4*x + 8*e**(b*x)*int(x/(e**(b*x)*b*c*
*3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*
e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**3*b**2*c**2*d**4 + 8*e**(b*x)
)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e**(b*x)*b*c*d**2*x**2
+ e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)*d**3*x**2),x)*a**3*b**2
*c*d**5*x + 4*e**(b*x)*int(x/(e**(b*x)*b*c**3 + 2*e**(b*x)*b*c**2*d*x + e
*(b*x)*b*c*d**2*x**2 + e**(b*x)*c**2*d + 2*e**(b*x)*c*d**2*x + e**(b*x)...
```

**3.119**  $\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$

Optimal result	810
Mathematica [A] (verified)	811
Rubi [A] (verified)	811
Maple [A] (verified)	813
Fricas [A] (verification not implemented)	813
Sympy [F]	814
Maxima [F]	815
Giac [B] (verification not implemented)	815
Mupad [F(-1)]	816
Reduce [F]	817

**Optimal result**

Integrand size = 25, antiderivative size = 294

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx = -\frac{b^2 e^{-a-bx}}{d^3} + \frac{b^2(3bc-4ad)e^{-a-bx}}{d^4} - \frac{b^3 e^{-a-bx} x}{d^3}$$

$$- \frac{(bc-ad)^4 e^{-a-bx}}{2d^5(c+dx)^2} + \frac{4b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)} + \frac{b(bc-ad)^4 e^{-a-bx}}{2d^6(c+dx)}$$

$$+ \frac{6b^2(bc-ad)^2 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5}$$

$$+ \frac{4b^2(bc-ad)^3 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6}$$

$$+ \frac{b^2(bc-ad)^4 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^7}$$

output

```
-b^2*exp(-b*x-a)/d^3+b^2*(-4*a*d+3*b*c)*exp(-b*x-a)/d^4-b^3*exp(-b*x-a)*x/
d^3-1/2*(-a*d+b*c)^4*exp(-b*x-a)/d^5/(d*x+c)^2+4*b*(-a*d+b*c)^3*exp(-b*x-a)
)/d^5/(d*x+c)+1/2*b*(-a*d+b*c)^4*exp(-b*x-a)/d^6/(d*x+c)+6*b^2*(-a*d+b*c)^
2*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^5+4*b^2*(-a*d+b*c)^3*exp(-a+b*c/d)*Ei(-
b*(d*x+c)/d)/d^6+1/2*b^2*(-a*d+b*c)^4*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^7
```

### Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$$

$$= e^{-a} \left( \frac{de^{-bx}(-a^4d^5+b^5c^4(c+dx)+a^3bd^4((-4+a)c+(-8+a)dx)+b^4c^3d((7-4a)c-4(-2+a)dx)-2b^2d^3((1+4a-9a^2+2a^3)c^2+2(1+4a-6a^2+2a^3)c+2a^3)d^2)}{(c+dx)^2} \right)$$

input `Integrate[(E^(-a - b*x))*(a + b*x)^4]/(c + d*x)^3,x]`

output `((d*(-(a^4*d^5) + b^5*c^4*(c + d*x) + a^3*b*d^4*((-4 + a)*c + (-8 + a)*d*x) + b^4*c^3*d*((7 - 4*a)*c - 4*(-2 + a)*d*x) - 2*b^2*d^3*((1 + 4*a - 9*a^2 + 2*a^3)*c^2 + 2*(1 + 4*a - 6*a^2 + a^3)*c*d*x + (1 + 4*a)*d^2*x^2) + 2*b^3*d^2*((3 - 10*a + 3*a^2)*c^3 + (5 - 12*a + 3*a^2)*c^2*d*x + c*d^2*x^2 - d^3*x^3)))/(E^(b*x)*(c + d*x)^2) + b^2*(b*c - a*d)^2*(b^2*c^2 - 2*(-4 + a)*b*c*d + (12 - 8*a + a^2)*d^2)*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/ (2*d^7*E^a)`

### Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$$

↓ 2629

$$\int \left( \frac{b^4 x e^{-a-bx}}{d^3} - \frac{b^3 e^{-a-bx}(3bc - 4ad)}{d^4} + \frac{6b^2 e^{-a-bx}(bc - ad)^2}{d^4(c+dx)} - \frac{4b e^{-a-bx}(bc - ad)^3}{d^4(c+dx)^2} + \frac{e^{-a-bx}(ad - bc)^4}{d^4(c+dx)^3} \right) dx$$

↓ 2009

$$\begin{aligned}
& -\frac{b^3 x e^{-a-bx}}{d^3} + \frac{b^2 e^{\frac{bc}{d}-a} (bc-ad)^4 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{2d^7} + \\
& \frac{4b^2 e^{\frac{bc}{d}-a} (bc-ad)^3 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6} + \\
& \frac{6b^2 e^{\frac{bc}{d}-a} (bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{b^2 e^{-a-bx} (3bc-4ad)}{d^3} - \frac{b^2 e^{-a-bx}}{d^3} + \\
& \frac{b e^{-a-bx} (bc-ad)^4}{2d^6 (c+dx)} - \frac{e^{-a-bx} (bc-ad)^4}{2d^5 (c+dx)^2} + \frac{4b e^{-a-bx} (bc-ad)^3}{d^5 (c+dx)}
\end{aligned}$$

input `Int[(E^(-a - b*x))*(a + b*x)^4]/(c + d*x)^3,x]`

output `-((b^2*E^(-a - b*x))/d^3) + (b^2*(3*b*c - 4*a*d)*E^(-a - b*x))/d^4 - (b^3*E^(-a - b*x)*x)/d^3 - ((b*c - a*d)^4*E^(-a - b*x))/(2*d^5*(c + d*x)^2) + (4*b*(b*c - a*d)^3*E^(-a - b*x))/(d^5*(c + d*x)) + (b*(b*c - a*d)^4*E^(-a - b*x))/(2*d^6*(c + d*x)) + (6*b^2*(b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^5 + (4*b^2*(b*c - a*d)^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^6 + (b^2*(b*c - a*d)^4*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^7`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`

### Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.42

method	result
derivativedivides	$-\frac{3b^3 a e^{-bx-a}}{d^3} - \frac{3b^4 c e^{-bx-a}}{d^4} - \frac{b^3 ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + \frac{4(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3) b^3 \left( -\frac{e^{-bx-a}}{-bx-a + \frac{da-bc}{d}} \right)}{d^6}$
default	$-\frac{3b^3 a e^{-bx-a}}{d^3} - \frac{3b^4 c e^{-bx-a}}{d^4} - \frac{b^3 ((-bx-a)e^{-bx-a} - e^{-bx-a})}{d^3} + \frac{4(a^3 d^3 - 3a^2 bc d^2 + 3a b^2 c^2 d - b^3 c^3) b^3 \left( -\frac{e^{-bx-a}}{-bx-a + \frac{da-bc}{d}} \right)}{d^6}$
risch	Expression too large to display

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x,method=_RETURNVERBOSE)`

output 
$$-1/b*(3*b^3/d^3*a*\exp(-b*x-a)-3*b^4/d^4*c*\exp(-b*x-a)-1/d^3*b^3*((-b*x-a)*\exp(-b*x-a)-\exp(-b*x-a))+4/d^6*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^3*(-\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+6/d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b^3*\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^3/d^7*(-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*\exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*\exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))$$

### Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 550, normalized size of antiderivative = 1.87

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$$

$$= \frac{(b^6 c^6 - 4(a-2)b^5 c^5 d + 6(a^2 - 4a + 2)b^4 c^4 d^2 - 4(a^3 - 6a^2 + 6a)b^3 c^3 d^3 + (a^4 - 8a^3 + 12a^2)b^2 c^2 d^4 + \dots)}{d^6}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")`

output

```

1/2*((b^6*c^6 - 4*(a - 2)*b^5*c^5*d + 6*(a^2 - 4*a + 2)*b^4*c^4*d^2 - 4*(a
^3 - 6*a^2 + 6*a)*b^3*c^3*d^3 + (a^4 - 8*a^3 + 12*a^2)*b^2*c^2*d^4 + (b^6*c
c^4*d^2 - 4*(a - 2)*b^5*c^3*d^3 + 6*(a^2 - 4*a + 2)*b^4*c^2*d^4 - 4*(a^3 -
6*a^2 + 6*a)*b^3*c*d^5 + (a^4 - 8*a^3 + 12*a^2)*b^2*d^6)*x^2 + 2*(b^6*c^5
*d - 4*(a - 2)*b^5*c^4*d^2 + 6*(a^2 - 4*a + 2)*b^4*c^3*d^3 - 4*(a^3 - 6*a^
2 + 6*a)*b^3*c^2*d^4 + (a^4 - 8*a^3 + 12*a^2)*b^2*c*d^5)*x)*Ei(-(b*d*x + b
*c)/d)*e^((b*c - a*d)/d) - (2*b^3*d^6*x^3 - b^5*c^5*d + (4*a - 7)*b^4*c^4*
d^2 - 2*(3*a^2 - 10*a + 3)*b^3*c^3*d^3 + a^4*d^6 + 2*(2*a^3 - 9*a^2 + 4*a
+ 1)*b^2*c^2*d^4 - (a^4 - 4*a^3)*b*c*d^5 - 2*(b^3*c*d^5 - (4*a + 1)*b^2*d^
6)*x^2 - (b^5*c^4*d^2 - 4*(a - 2)*b^4*c^3*d^3 + 2*(3*a^2 - 12*a + 5)*b^3*c
^2*d^4 - 4*(a^3 - 6*a^2 + 4*a + 1)*b^2*c*d^5 + (a^4 - 8*a^3)*b*d^6)*x)*e^(-
b*x - a))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)

```

### Sympy [F]

$$\begin{aligned}
\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx = & \left( \int \frac{a^4}{c^3e^{bx} + 3c^2dxe^{bx} + 3cd^2x^2e^{bx} + d^3x^3e^{bx}} dx \right. \\
& + \int \frac{b^4x^4}{c^3e^{bx} + 3c^2dxe^{bx} + 3cd^2x^2e^{bx} + d^3x^3e^{bx}} dx \\
& + \int \frac{4ab^3x^3}{c^3e^{bx} + 3c^2dxe^{bx} + 3cd^2x^2e^{bx} + d^3x^3e^{bx}} dx \\
& + \int \frac{6a^2b^2x^2}{c^3e^{bx} + 3c^2dxe^{bx} + 3cd^2x^2e^{bx} + d^3x^3e^{bx}} dx \\
& \left. + \int \frac{4a^3bx}{c^3e^{bx} + 3c^2dxe^{bx} + 3cd^2x^2e^{bx} + d^3x^3e^{bx}} dx \right) e^{-a}
\end{aligned}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c)**3,x)
```

output

```

(Integral(a**4/(c**3*exp(b*x) + 3*c**2*d*x*exp(b*x) + 3*c*d**2*x**2*exp(b*
x) + d**3*x**3*exp(b*x)), x) + Integral(b**4*x**4/(c**3*exp(b*x) + 3*c**2*
d*x*exp(b*x) + 3*c*d**2*x**2*exp(b*x) + d**3*x**3*exp(b*x)), x) + Integral
(4*a*b**3*x**3/(c**3*exp(b*x) + 3*c**2*d*x*exp(b*x) + 3*c*d**2*x**2*exp(b*
x) + d**3*x**3*exp(b*x)), x) + Integral(6*a**2*b**2*x**2/(c**3*exp(b*x) +
3*c**2*d*x*exp(b*x) + 3*c*d**2*x**2*exp(b*x) + d**3*x**3*exp(b*x)), x) + I
ntegral(4*a**3*b*x/(c**3*exp(b*x) + 3*c**2*d*x*exp(b*x) + 3*c*d**2*x**2*ex
p(b*x) + d**3*x**3*exp(b*x)), x))*exp(-a)

```

**Maxima [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx = \int \frac{(bx+a)^4 e^{(-bx-a)}}{(dx+c)^3} dx$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")`

output `-a^4*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) - (b^3*d^2*x^4 + (4*a*b^2*d^2 + b^2*d^2)*x^3 + 3*(2*a^2*b*d^2 + b^2*c*d)*x^2 + (4*a^3*d^2 - 3*b^2*c^2 + 12*a*b*c*d - 6*a^2*d^2)*x)*e^(-b*x)/(d^5*x^3*e^a + 3*c*d^4*x^2*e^a + 3*c^2*d^3*x*e^a + c^3*d^2*e^a) - integrate(-(4*a^3*c*d^2 - 3*b^2*c^3 + 12*a*b*c^2*d - 6*a^2*c*d^2 + (3*b^3*c^3 - 8*a^3*d^3 + 12*b^2*c^2*d + 6*(3*b*c*d^2 + 2*d^3))*a^2 - 12*(b^2*c^2*d + 2*b*c*d^2)*a)*x)*e^(-b*x)/(d^6*x^4*e^a + 4*c*d^5*x^3*e^a + 6*c^2*d^4*x^2*e^a + 4*c^3*d^3*x*e^a + c^4*d^2*e^a), x)`

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs.  $2(279) = 558$ .

Time = 0.14 (sec) , antiderivative size = 1995, normalized size of antiderivative = 6.79

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx = \text{Too large to display}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x, algorithm="giac")`



output

```

1/2*(b^6*c^4*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^5*c^3*d^3
*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 6*a^2*b^4*c^2*d^4*x^2*Ei(-(b*d*
x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-
a + b*c/d) + a^4*b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 2*b^6*c
^5*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 8*a*b^5*c^4*d^2*x*Ei(-(b*d*x
+ b*c)/d)*e^(-a + b*c/d) + 12*a^2*b^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d)*e^(-a
+ b*c/d) - 8*a^3*b^3*c^2*d^4*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 2*a^
4*b^2*c*d^5*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 8*b^5*c^3*d^3*x^2*Ei(-
(b*d*x + b*c)/d)*e^(-a + b*c/d) - 24*a*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d
)*e^(-a + b*c/d) + 24*a^2*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d)
) - 8*a^3*b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^6*c^6*Ei(-(b
*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^5*c^5*d*Ei(-(b*d*x + b*c)/d)*e^(-a +
b*c/d) + 6*a^2*b^4*c^4*d^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b^
3*c^3*d^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + a^4*b^2*c^2*d^4*Ei(-(b*d*x
+ b*c)/d)*e^(-a + b*c/d) + 16*b^5*c^4*d^2*x*Ei(-(b*d*x + b*c)/d)*e^(-a +
b*c/d) - 48*a*b^4*c^3*d^3*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 48*a^2*b
^3*c^2*d^4*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 16*a^3*b^2*c*d^5*x*Ei(-
(b*d*x + b*c)/d)*e^(-a + b*c/d) + 12*b^4*c^2*d^4*x^2*Ei(-(b*d*x + b*c)/d)*
e^(-a + b*c/d) - 24*a*b^3*c*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) +
12*a^2*b^2*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^5*c^4*d^2*x*...

```

### Mupad [F(-1)]

Timed out.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx = \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx$$

input

```
int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^3,x)
```

output

```
int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^3, x)
```

## Reduce [F]

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^3} dx = \text{too large to display}$$

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^3,x)`

output `( - e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**4*b**2*c**3*d**4 - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**4*b**2*c**2*d**5*x - e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**4*b**2*c*d**6*x**2 - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**4*b*c**2*d**5 - 4*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**4*b*c*d**6*x - 2*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e**(b*x)*b*c*d**3*x**3 + 2*e**(b*x)*c**3*d + 6*e**(b*x)*c**2*d**2*x + 6*e**(b*x)*c*d**3*x**2 + 2*e**(b*x)*d**4*x**3),x)*a**4*b*d**7*x**2 + 4*e**(b*x)*int(x/(e**(b*x)*b*c**4 + 3*e**(b*x)*b*c**3*d*x + 3*e**(b*x)*b*c**2*d**2*x**2 + e...`

### 3.120 $\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$

Optimal result	818
Mathematica [A] (verified)	819
Rubi [A] (verified)	820
Maple [A] (verified)	821
Fricas [B] (verification not implemented)	822
Sympy [F]	823
Maxima [F]	824
Giac [B] (verification not implemented)	825
Mupad [F(-1)]	826
Reduce [F]	826

#### Optimal result

Integrand size = 25, antiderivative size = 396

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx = -\frac{b^3 e^{-a-bx}}{d^4} - \frac{(bc-ad)^4 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{2b(bc-ad)^3 e^{-a-bx}}{d^5(c+dx)^2}$$

$$+ \frac{b(bc-ad)^4 e^{-a-bx}}{6d^6(c+dx)^2} - \frac{6b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)}$$

$$- \frac{2b^2(bc-ad)^3 e^{-a-bx}}{d^6(c+dx)} - \frac{b^2(bc-ad)^4 e^{-a-bx}}{6d^7(c+dx)}$$

$$- \frac{4b^3(bc-ad)e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5}$$

$$- \frac{6b^3(bc-ad)^2 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6}$$

$$- \frac{2b^3(bc-ad)^3 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^7}$$

$$- \frac{b^3(bc-ad)^4 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{6d^8}$$

output

$$\begin{aligned}
& -b^3 \exp(-bx-a)/d^4 - 1/3(-a+d+bc)^4 \exp(-bx-a)/d^5 / (d*x+c)^3 + 2*b*(-a+d+bc)^3 \exp(-bx-a)/d^5 / (d*x+c)^2 + 1/6*b*(-a+d+bc)^4 \exp(-bx-a)/d^6 / (d*x+c)^2 - 6*b^2*(-a+d+bc)^2 \exp(-bx-a)/d^5 / (d*x+c) - 2*b^2*(-a+d+bc)^3 \exp(-bx-a)/d^6 / (d*x+c) - 1/6*b^2*(-a+d+bc)^4 \exp(-bx-a)/d^7 / (d*x+c) - 4*b^3*(-a+d+bc) \exp(-a+bc/d) * Ei(-b*(d*x+c)/d) / d^5 - 6*b^3*(-a+d+bc)^2 \exp(-a+bc/d) * Ei(-b*(d*x+c)/d) / d^6 - 2*b^3*(-a+d+bc)^3 \exp(-a+bc/d) * Ei(-b*(d*x+c)/d) / d^7 - 1/6*b^3*(-a+d+bc)^4 \exp(-a+bc/d) * Ei(-b*(d*x+c)/d) / d^8
\end{aligned}$$
**Mathematica [A] (verified)**

Time = 2.91 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.98

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$$

$$\begin{aligned}
& e^{-a} \left( -\frac{de^{-bx}(2a^4d^6+b^6c^4(c+dx)^2-a^3bd^5((-4+a)c+(-12+a)dx)-b^5c^3d(c+dx)((-11+4a)c+4(-3+a)dx)+a^2b^2d^4((12-8a+a^2)c^2+2(18-10a+a^2)c+3a^2))}{(c+dx)^4} \right)
\end{aligned}$$

input

$$\text{Integrate}[(E^{-a-b*x}*(a+b*x)^4)/(c+d*x)^4,x]$$

output

$$\begin{aligned}
& (-((d*(2*a^4*d^6 + b^6*c^4*(c + d*x)^2 - a^3*b*d^5*((-4 + a)*c + (-12 + a)*d*x) - b^5*c^3*d*(c + d*x)*((-11 + 4*a)*c + 4*(-3 + a)*d*x) + a^2*b^2*d^4*((12 - 8*a + a^2)*c^2 + 2*(18 - 10*a + a^2)*c*d*x + (-6 + a)^2*d^2*x^2) + 2*b^4*c^2*d^2*((13 - 16*a + 3*a^2)*c^2 + 2*(15 - 17*a + 3*a^2)*c*d*x + 3*(6 - 6*a + a^2)*d^2*x^2) + 2*b^3*d^3*((3 - 22*a + 15*a^2 - 2*a^3)*c^3 + (9 - 54*a + 33*a^2 - 4*a^3)*c^2*d*x + (9 - 36*a + 18*a^2 - 2*a^3)*c*d^2*x^2 + 3*d^3*x^3)))/(E^(b*x)*(c + d*x)^3) - b^3*(b^4*c^4 - 4*(-3 + a)*b^3*c^3*d + 6*(6 - 6*a + a^2)*b^2*c^2*d^2 - 4*(-6 + 18*a - 9*a^2 + a^3)*b*c*d^3 + a*(-24 + 36*a - 12*a^2 + a^3)*d^4)*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(6*d^8*E^a)
\end{aligned}$$

**Rubi [A] (verified)**

Time = 1.30 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$$

↓ 2629

$$\int \left( \frac{b^4 e^{-a-bx}}{d^4} - \frac{4b^3 e^{-a-bx}(bc-ad)}{d^4(c+dx)} + \frac{6b^2 e^{-a-bx}(bc-ad)^2}{d^4(c+dx)^2} - \frac{4b e^{-a-bx}(bc-ad)^3}{d^4(c+dx)^3} + \frac{e^{-a-bx}(ad-bc)^4}{d^4(c+dx)^4} \right) dx$$

↓ 2009

$$\frac{b^3 e^{\frac{bc}{d}-a}(bc-ad)^4 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{6d^8} - \frac{2b^3 e^{\frac{bc}{d}-a}(bc-ad)^3 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^7} - \frac{6b^3 e^{\frac{bc}{d}-a}(bc-ad)^2 \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6} - \frac{4b^3 e^{\frac{bc}{d}-a}(bc-ad) \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5} - \frac{b^3 e^{-a-bx}}{d^4} - \frac{b^2 e^{-a-bx}(bc-ad)^4}{6d^7(c+dx)} - \frac{2b^2 e^{-a-bx}(bc-ad)^3}{d^6(c+dx)} - \frac{6b^2 e^{-a-bx}(bc-ad)^2}{d^5(c+dx)} + \frac{be^{-a-bx}(bc-ad)^4}{6d^6(c+dx)^2} - \frac{e^{-a-bx}(bc-ad)^4}{3d^5(c+dx)^3} + \frac{2be^{-a-bx}(bc-ad)^3}{d^5(c+dx)^2}$$

input

```
Int[(E^(-a - b*x))*(a + b*x)^4/(c + d*x)^4,x]
```

output

```

-((b^3*E^(-a - b*x))/d^4) - ((b*c - a*d)^4*E^(-a - b*x))/(3*d^5*(c + d*x)^
3) + (2*b*(b*c - a*d)^3*E^(-a - b*x))/(d^5*(c + d*x)^2) + (b*(b*c - a*d)^4
*E^(-a - b*x))/(6*d^6*(c + d*x)^2) - (6*b^2*(b*c - a*d)^2*E^(-a - b*x))/(d
^5*(c + d*x)) - (2*b^2*(b*c - a*d)^3*E^(-a - b*x))/(d^6*(c + d*x)) - (b^2*
(b*c - a*d)^4*E^(-a - b*x))/(6*d^7*(c + d*x)) - (4*b^3*(b*c - a*d)*E^(-a +
(b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/d^5 - (6*b^3*(b*c - a*d)^2*E^
(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/d^6 - (2*b^3*(b*c - a*d)
^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/d^7 - (b^3*(b*c - a
*d)^4*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]/(6*d^8)
    
```

**Defintions of rubi rules used**

rule 2009

```

Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]
    
```

rule 2629

```

Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_), x_Symbol] := Int[ExpandInte
grand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[
Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]
    
```

**Maple [A] (verified)**

Time = 0.16 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.29

method	result
derivativedivides	$-\frac{b^4 e^{-bx-a}}{d^4} + \frac{6(a^2 d^2 - 2abcd + b^2 c^2) b^4 \left( -\frac{e^{-bx-a}}{-bx-a + \frac{da-bc}{d}} - e^{-\frac{da-bc}{d}} \expIntegral_1\left(bx+a - \frac{da-bc}{d}\right) \right)}{d^6} + \frac{(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a^2 b^2 c^2 d + 4a^2 b^2 c^2)}{d^8}$
default	$-\frac{b^4 e^{-bx-a}}{d^4} + \frac{6(a^2 d^2 - 2abcd + b^2 c^2) b^4 \left( -\frac{e^{-bx-a}}{-bx-a + \frac{da-bc}{d}} - e^{-\frac{da-bc}{d}} \expIntegral_1\left(bx+a - \frac{da-bc}{d}\right) \right)}{d^6} + \frac{(a^4 d^4 - 4a^3 bc d^3 + 6a^2 b^2 c^2 d^2 - 4a^2 b^2 c^2 d + 4a^2 b^2 c^2)}{d^8}$
risch	Expression too large to display

input

```

int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x,method=_RETURNVERBOSE)
    
```

output

```
-1/b*(b^4/d^4*exp(-b*x-a)+6/d^6*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b^4*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*b^4/d^8*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+4/d^5*(a*d-b*c)*b^4*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-4/d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*b^4*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))
```

### Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs.  $2(377) = 754$ .

Time = 0.08 (sec) , antiderivative size = 793, normalized size of antiderivative = 2.00

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx = \text{Too large to display}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x, algorithm="fricas")
```

output

```

-1/6*((b^7*c^7 - 4*(a - 3)*b^6*c^6*d + 6*(a^2 - 6*a + 6)*b^5*c^5*d^2 - 4*(
a^3 - 9*a^2 + 18*a - 6)*b^4*c^4*d^3 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c
^3*d^4 + (b^7*c^4*d^3 - 4*(a - 3)*b^6*c^3*d^4 + 6*(a^2 - 6*a + 6)*b^5*c^2*
d^5 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c*d^6 + (a^4 - 12*a^3 + 36*a^2 - 24*a
)*b^3*d^7)*x^3 + 3*(b^7*c^5*d^2 - 4*(a - 3)*b^6*c^4*d^3 + 6*(a^2 - 6*a + 6
)*b^5*c^3*d^4 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^2*d^5 + (a^4 - 12*a^3 + 3
6*a^2 - 24*a)*b^3*c*d^6)*x^2 + 3*(b^7*c^6*d - 4*(a - 3)*b^6*c^5*d^2 + 6*(a
^2 - 6*a + 6)*b^5*c^4*d^3 - 4*(a^3 - 9*a^2 + 18*a - 6)*b^4*c^3*d^4 + (a^4
- 12*a^3 + 36*a^2 - 24*a)*b^3*c^2*d^5)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a
*d)/d) + (b^6*c^6*d - (4*a - 11)*b^5*c^5*d^2 + 6*b^3*d^7*x^3 + 2*(3*a^2 -
16*a + 13)*b^4*c^4*d^3 - 2*(2*a^3 - 15*a^2 + 22*a - 3)*b^3*c^3*d^4 + 2*a^4
*d^7 + (a^4 - 8*a^3 + 12*a^2)*b^2*c^2*d^5 - (a^4 - 4*a^3)*b*c*d^6 + (b^6*c
^4*d^3 - 4*(a - 3)*b^5*c^3*d^4 + 6*(a^2 - 6*a + 6)*b^4*c^2*d^5 - 2*(2*a^3
- 18*a^2 + 36*a - 9)*b^3*c*d^6 + (a^4 - 12*a^3 + 36*a^2)*b^2*d^7)*x^2 + (2
*b^6*c^5*d^2 - (8*a - 23)*b^5*c^4*d^3 + 4*(3*a^2 - 17*a + 15)*b^4*c^3*d^4
- 2*(4*a^3 - 33*a^2 + 54*a - 9)*b^3*c^2*d^5 + 2*(a^4 - 10*a^3 + 18*a^2)*b^
2*c*d^6 - (a^4 - 12*a^3)*b*d^7)*x)*e^(-b*x - a))/(d^11*x^3 + 3*c*d^10*x^2
+ 3*c^2*d^9*x + c^3*d^8)

```

## SymPy [F]

$$\begin{aligned}
& \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx \\
&= \left( \int \frac{a^4}{c^4 e^{bx} + 4c^3 dx e^{bx} + 6c^2 d^2 x^2 e^{bx} + 4cd^3 x^3 e^{bx} + d^4 x^4 e^{bx}} dx \right. \\
&\quad + \int \frac{b^4 x^4}{c^4 e^{bx} + 4c^3 dx e^{bx} + 6c^2 d^2 x^2 e^{bx} + 4cd^3 x^3 e^{bx} + d^4 x^4 e^{bx}} dx \\
&\quad + \int \frac{4ab^3 x^3}{c^4 e^{bx} + 4c^3 dx e^{bx} + 6c^2 d^2 x^2 e^{bx} + 4cd^3 x^3 e^{bx} + d^4 x^4 e^{bx}} dx \\
&\quad + \int \frac{6a^2 b^2 x^2}{c^4 e^{bx} + 4c^3 dx e^{bx} + 6c^2 d^2 x^2 e^{bx} + 4cd^3 x^3 e^{bx} + d^4 x^4 e^{bx}} dx \\
&\quad \left. + \int \frac{4a^3 bx}{c^4 e^{bx} + 4c^3 dx e^{bx} + 6c^2 d^2 x^2 e^{bx} + 4cd^3 x^3 e^{bx} + d^4 x^4 e^{bx}} dx \right) e^{-a}
\end{aligned}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c)**4,x)
```



output

```
(Integral(a**4/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(b**4*x**4/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(4*a*b**3*x**3/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(6*a**2*b**2*x**2/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x) + Integral(4*a**3*b*x/(c**4*exp(b*x) + 4*c**3*d*x*exp(b*x) + 6*c**2*d**2*x**2*exp(b*x) + 4*c*d**3*x**3*exp(b*x) + d**4*x**4*exp(b*x)), x))*exp(-a)
```

**Maxima [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx = \int \frac{(bx+a)^4 e^{(-bx-a)}}{(dx+c)^4} dx$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x, algorithm="maxima")
```

output

```
-a^4*e^(-a + b*c/d)*exp_integral_e(4, (d*x + c)*b/d)/((d*x + c)^3*d) - (b^3*d^2*x^4 + 4*a*b^2*d^2*x^3 + 2*(3*a^2*b*d^2 + 2*b^2*c*d - 2*a*b*d^2)*x^2 + 4*(a^3*d^2 - b^2*c^2 - 3*a^2*d^2 - 2*b*c*d + 2*(2*b*c*d + d^2)*a)*x)*e^(-b*x)/(d^6*x^4*e^a + 4*c*d^5*x^3*e^a + 6*c^2*d^4*x^2*e^a + 4*c^3*d^3*x*e^a + c^4*d^2*e^a) - integrate(-4*(a^3*c*d^2 - b^2*c^3 - 3*a^2*c*d^2 - 2*b*c^2*d + 2*(2*b*c^2*d + c*d^2)*a + (b^3*c^3 - 3*a^3*d^3 + 7*b^2*c^2*d + 6*b*c*d^2 + 3*(2*b*c*d^2 + 3*d^3)*a^2 - 2*(2*b^2*c^2*d + 8*b*c*d^2 + 3*d^3)*a)*x)*e^(-b*x)/(d^7*x^5*e^a + 5*c*d^6*x^4*e^a + 10*c^2*d^5*x^3*e^a + 10*c^3*d^4*x^2*e^a + 5*c^4*d^3*x*e^a + c^5*d^2*e^a), x)
```

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 3178 vs.  $2(377) = 754$ .

Time = 0.15 (sec) , antiderivative size = 3178, normalized size of antiderivative = 8.03

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx = \text{Too large to display}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x, algorithm="giac")`

output

```
-1/6*(b^7*c^4*d^3*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a*b^6*c^3*d^4*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 6*a^2*b^5*c^2*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 4*a^3*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + a^4*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b^7*c^5*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a*b^6*c^4*d^3*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 18*a^2*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a^3*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*a^4*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 12*b^6*c^3*d^4*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 36*a*b^5*c^2*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*a^2*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a^3*b^3*d^7*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*b^7*c^6*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a*b^6*c^5*d^2*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 18*a^2*b^5*c^4*d^3*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 12*a^3*b^4*c^3*d^4*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 3*a^4*b^3*c^2*d^5*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*b^6*c^4*d^3*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 108*a*b^5*c^3*d^4*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 108*a^2*b^4*c^2*d^5*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 36*a^3*b^3*c*d^6*x^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*b^5*c^2*d^5*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - 72*a*b^4*c*d^6*x^3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 36*a^2*b^3*d^7*x^3*Ei(-...
```

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx = \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx$$

input `int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^4,x)`output `int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^4, x)`**Reduce [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^4} dx = \text{too large to display}$$

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^4,x)`

output

```
( - e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b
*c**3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3
*e**(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 +
12*e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**4*b**2*c**4*d**4 - 3
*e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c*
**3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e
*(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*
e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**4*b**2*c**3*d**5*x - 3*
e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**
3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**
(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*
e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**4*b**2*c**2*d**6*x**2 -
e**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**
3*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**
(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*
e**(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**4*b**2*c*d**7*x**3 - 3*e
**(b*x)*int(x/(e**(b*x)*b*c**5 + 4*e**(b*x)*b*c**4*d*x + 6*e**(b*x)*b*c**3
*d**2*x**2 + 4*e**(b*x)*b*c**2*d**3*x**3 + e**(b*x)*b*c*d**4*x**4 + 3*e**
(b*x)*c**4*d + 12*e**(b*x)*c**3*d**2*x + 18*e**(b*x)*c**2*d**3*x**2 + 12*e
*(b*x)*c*d**4*x**3 + 3*e**(b*x)*d**5*x**4),x)*a**4*b*c**3*d**5 - 9*e**(...
```

### 3.121 $\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$

Optimal result	828
Mathematica [A] (verified)	829
Rubi [A] (verified)	830
Maple [A] (verified)	832
Fricas [B] (verification not implemented)	832
Sympy [F]	833
Maxima [F]	834
Giac [B] (verification not implemented)	835
Mupad [F(-1)]	836
Reduce [F]	836

#### Optimal result

Integrand size = 25, antiderivative size = 557

$$\begin{aligned}
 \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx = & -\frac{(bc-ad)^4 e^{-a-bx}}{4d^5(c+dx)^4} + \frac{4b(bc-ad)^3 e^{-a-bx}}{3d^5(c+dx)^3} + \frac{b(bc-ad)^4 e^{-a-bx}}{12d^6(c+dx)^3} \\
 & - \frac{3b^2(bc-ad)^2 e^{-a-bx}}{d^5(c+dx)^2} - \frac{2b^2(bc-ad)^3 e^{-a-bx}}{3d^6(c+dx)^2} \\
 & - \frac{b^2(bc-ad)^4 e^{-a-bx}}{24d^7(c+dx)^2} + \frac{4b^3(bc-ad) e^{-a-bx}}{d^5(c+dx)} \\
 & + \frac{3b^3(bc-ad)^2 e^{-a-bx}}{d^6(c+dx)} + \frac{2b^3(bc-ad)^3 e^{-a-bx}}{3d^7(c+dx)} \\
 & + \frac{b^3(bc-ad)^4 e^{-a-bx}}{24d^8(c+dx)} + \frac{b^4 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5} \\
 & + \frac{4b^4(bc-ad) e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6} \\
 & + \frac{3b^4(bc-ad)^2 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^7} \\
 & + \frac{2b^4(bc-ad)^3 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{3d^8} \\
 & + \frac{b^4(bc-ad)^4 e^{-a+\frac{bc}{d}} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{24d^9}
 \end{aligned}$$

output

```

-1/4*(-a*d+b*c)^4*exp(-b*x-a)/d^5/(d*x+c)^4+4/3*b*(-a*d+b*c)^3*exp(-b*x-a)
/d^5/(d*x+c)^3+1/12*b*(-a*d+b*c)^4*exp(-b*x-a)/d^6/(d*x+c)^3-3*b^2*(-a*d+b
*c)^2*exp(-b*x-a)/d^5/(d*x+c)^2-2/3*b^2*(-a*d+b*c)^3*exp(-b*x-a)/d^6/(d*x+
c)^2-1/24*b^2*(-a*d+b*c)^4*exp(-b*x-a)/d^7/(d*x+c)^2+4*b^3*(-a*d+b*c)*exp(
-b*x-a)/d^5/(d*x+c)+3*b^3*(-a*d+b*c)^2*exp(-b*x-a)/d^6/(d*x+c)+2/3*b^3*(-a
*d+b*c)^3*exp(-b*x-a)/d^7/(d*x+c)+1/24*b^3*(-a*d+b*c)^4*exp(-b*x-a)/d^8/(d
*x+c)+b^4*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^5+4*b^4*(-a*d+b*c)*exp(-a+b*c/d
)*Ei(-b*(d*x+c)/d)/d^6+3*b^4*(-a*d+b*c)^2*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d
^7+2/3*b^4*(-a*d+b*c)^3*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^8+1/24*b^4*(-a*d+
b*c)^4*exp(-a+b*c/d)*Ei(-b*(d*x+c)/d)/d^9

```

**Mathematica [A] (verified)**

Time = 3.00 (sec) , antiderivative size = 669, normalized size of antiderivative = 1.20

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$$

$$= e^{-a} \left( \frac{de^{-bx}(-6d^3(bc-ad)^4+2bd^2(bc-(-16+a)d)(bc-ad)^3(c+dx)-b^2d(bc-ad)^2(b^2c^2-2(-8+a)bcd+(72-16a+a^2)d^2)(c+dx)^2+b^3(b^4c^4-4(c+dx)^4)}{c+dx} \right)$$

input

```
Integrate[(E^(-a - b*x))*(a + b*x)^4]/(c + d*x)^5,x]
```

output

```

((d*(-6*d^3*(b*c - a*d)^4 + 2*b*d^2*(b*c - (-16 + a)*d)*(b*c - a*d)^3*(c +
d*x) - b^2*d*(b*c - a*d)^2*(b^2*c^2 - 2*(-8 + a)*b*c*d + (72 - 16*a + a^2
)*d^2)*(c + d*x)^2 + b^3*(b^4*c^4 - 4*(-4 + a)*b^3*c^3*d + 6*(12 - 8*a + a
^2)*b^2*c^2*d^2 - 4*(-24 + 36*a - 12*a^2 + a^3)*b*c*d^3 + a*(-96 + 72*a -
16*a^2 + a^3)*d^4)*(c + d*x)^3)/(E^(b*x)*(c + d*x)^4 + b^8*c^4*E^((b*c)/
d)*ExpIntegralEi[-((b*(c + d*x))/d)] + 16*b^7*c^3*d*E^((b*c)/d)*ExpIntegra
lEi[-((b*(c + d*x))/d)] - 4*a*b^7*c^3*d*E^((b*c)/d)*ExpIntegralEi[-((b*(c
+ d*x))/d)] + 72*b^6*c^2*d^2*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]
- 48*a*b^6*c^2*d^2*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] + 6*a^2*
b^6*c^2*d^2*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] + 96*b^5*c*d^3*E
^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] - 144*a*b^5*c*d^3*E^((b*c)/d)
*ExpIntegralEi[-((b*(c + d*x))/d)] + 48*a^2*b^5*c*d^3*E^((b*c)/d)*ExpInteg
ralEi[-((b*(c + d*x))/d)] - 4*a^3*b^5*c*d^3*E^((b*c)/d)*ExpIntegralEi[-((b
*(c + d*x))/d)] + 24*b^4*d^4*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)]
- 96*a*b^4*d^4*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] + 72*a^2*b^4
*d^4*E^((b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] - 16*a^3*b^4*d^4*E^((b*
c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)] + a^4*b^4*d^4*E^((b*c)/d)*ExpInteg
ralEi[-((b*(c + d*x))/d)]/(24*d^9*E^a)

```

### Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {2629, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$$

↓ 2629

$$\int \left( \frac{b^4 e^{-a-bx}}{d^4(c+dx)} - \frac{4b^3 e^{-a-bx}(bc-ad)}{d^4(c+dx)^2} + \frac{6b^2 e^{-a-bx}(bc-ad)^2}{d^4(c+dx)^3} - \frac{4b e^{-a-bx}(bc-ad)^3}{d^4(c+dx)^4} + \frac{e^{-a-bx}(ad-bc)^4}{d^4(c+dx)^5} \right) dx$$

↓ 2009

$$\frac{b^4(bc-ad)^4 e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{24d^9} + \frac{2b^4(bc-ad)^3 e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{3d^8} +$$

$$\frac{3b^4(bc-ad)^2 e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^7} + \frac{4b^4(bc-ad) e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^6} +$$

$$\frac{b^4 e^{\frac{bc}{d}-a} \text{ExpIntegralEi}\left(-\frac{b(c+dx)}{d}\right)}{d^5} + \frac{b^3 e^{-a-bx}(bc-ad)^4}{24d^8(c+dx)} + \frac{2b^3 e^{-a-bx}(bc-ad)^3}{3d^7(c+dx)} +$$

$$\frac{3b^3 e^{-a-bx}(bc-ad)^2}{d^6(c+dx)} + \frac{4b^3 e^{-a-bx}(bc-ad)}{d^5(c+dx)} - \frac{b^2 e^{-a-bx}(bc-ad)^4}{24d^7(c+dx)^2} - \frac{3d^6(c+dx)^2}{2b^2 e^{-a-bx}(bc-ad)^3} -$$

$$\frac{3b^2 e^{-a-bx}(bc-ad)^2}{d^5(c+dx)^2} + \frac{be^{-a-bx}(bc-ad)^4}{12d^6(c+dx)^3} + \frac{4be^{-a-bx}(bc-ad)^3}{3d^5(c+dx)^3} - \frac{e^{-a-bx}(bc-ad)^4}{4d^5(c+dx)^4}$$

input `Int[(E^(-a - b*x))*(a + b*x)^4]/(c + d*x)^5,x]`

output `-1/4*((b*c - a*d)^4*E^(-a - b*x))/(d^5*(c + d*x)^4) + (4*b*(b*c - a*d)^3*E^(-a - b*x))/(3*d^5*(c + d*x)^3) + (b*(b*c - a*d)^4*E^(-a - b*x))/(12*d^6*(c + d*x)^3) - (3*b^2*(b*c - a*d)^2*E^(-a - b*x))/(d^5*(c + d*x)^2) - (2*b^2*(b*c - a*d)^3*E^(-a - b*x))/(3*d^6*(c + d*x)^2) - (b^2*(b*c - a*d)^4*E^(-a - b*x))/(24*d^7*(c + d*x)^2) + (4*b^3*(b*c - a*d)*E^(-a - b*x))/(d^5*(c + d*x)) + (3*b^3*(b*c - a*d)^2*E^(-a - b*x))/(d^6*(c + d*x)) + (2*b^3*(b*c - a*d)^3*E^(-a - b*x))/(3*d^7*(c + d*x)) + (b^3*(b*c - a*d)^4*E^(-a - b*x))/(24*d^8*(c + d*x)) + (b^4*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^5 + (4*b^4*(b*c - a*d)*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^6 + (3*b^4*(b*c - a*d)^2*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^7 + (2*b^4*(b*c - a*d)^3*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^8 + (b^4*(b*c - a*d)^4*E^(-a + (b*c)/d)*ExpIntegralEi[-((b*(c + d*x))/d)])/d^9`

### Defintions of rubi rules used

rule 2009 `Int[u_, x_Symbol] := Simp[IntSum[u, x], x] /; SumQ[u]`

rule 2629 `Int[(F_)^(v_)*(Px_)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[F^v, Px*(d + e*x)^m, x], x] /; FreeQ[{F, d, e, m}, x] && PolynomialQ[Px, x] && LinearQ[v, x] && !TrueQ[$UseGamma]`



### Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.07

method	result
derivativdivides	$-\frac{4(da-bc)b^5 \left( -\frac{e^{-bx-a}}{-bx-a+\frac{da-bc}{d}} - e^{-\frac{da-bc}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{da-bc}{d}\right) \right)}{d^6} + \frac{4(da-bc)^3 b^5 \left( -\frac{e^{-bx-a}}{3(-bx-a+\frac{da-bc}{d})^3} - \frac{e^{-\frac{da-bc}{d}}}{6(-bx-a+\frac{da-bc}{d})} \right)}{d^6}$
default	$-\frac{4(da-bc)b^5 \left( -\frac{e^{-bx-a}}{-bx-a+\frac{da-bc}{d}} - e^{-\frac{da-bc}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{da-bc}{d}\right) \right)}{d^6} + \frac{4(da-bc)^3 b^5 \left( -\frac{e^{-bx-a}}{3(-bx-a+\frac{da-bc}{d})^3} - \frac{e^{-\frac{da-bc}{d}}}{6(-bx-a+\frac{da-bc}{d})} \right)}{d^6}$
risch	Expression too large to display

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^5,x,method=_RETURNVERBOSE)`

output 
$$-1/b*(4*(a*d-b*c)/d^6*b^5*(-exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+4*(a*d-b*c)^3/d^8*b^5*(-1/3*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/6*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/6*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))-(a*d-b*c)^4/d^9*b^5*(-1/4*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^4-1/12*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^3-1/24*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/24*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/24*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))+b^5/d^5*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-6*(a*d-b*c)^2/d^7*b^5*(-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)^2-1/2*exp(-b*x-a)/(-b*x-a+(a*d-b*c)/d)-1/2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d))$$

### Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1084 vs. 2(524) = 1048.

Time = 0.10 (sec) , antiderivative size = 1084, normalized size of antiderivative = 1.95

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx = \text{Too large to display}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^5,x, algorithm="fricas")`

output

```

1/24*((b^8*c^8 - 4*(a - 4)*b^7*c^7*d + 6*(a^2 - 8*a + 12)*b^6*c^6*d^2 - 4*
(a^3 - 12*a^2 + 36*a - 24)*b^5*c^5*d^3 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 2
4)*b^4*c^4*d^4 + (b^8*c^4*d^4 - 4*(a - 4)*b^7*c^3*d^5 + 6*(a^2 - 8*a + 12)
*b^6*c^2*d^6 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c*d^7 + (a^4 - 16*a^3 + 72
*a^2 - 96*a + 24)*b^4*d^8)*x^4 + 4*(b^8*c^5*d^3 - 4*(a - 4)*b^7*c^4*d^4 +
6*(a^2 - 8*a + 12)*b^6*c^3*d^5 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^2*d^6
+ (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c*d^7)*x^3 + 6*(b^8*c^6*d^2 - 4*
(a - 4)*b^7*c^5*d^3 + 6*(a^2 - 8*a + 12)*b^6*c^4*d^4 - 4*(a^3 - 12*a^2 + 3
6*a - 24)*b^5*c^3*d^5 + (a^4 - 16*a^3 + 72*a^2 - 96*a + 24)*b^4*c^2*d^6)*x
^2 + 4*(b^8*c^7*d - 4*(a - 4)*b^7*c^6*d^2 + 6*(a^2 - 8*a + 12)*b^6*c^5*d^3
- 4*(a^3 - 12*a^2 + 36*a - 24)*b^5*c^4*d^4 + (a^4 - 16*a^3 + 72*a^2 - 96*
a + 24)*b^4*c^3*d^5)*x)*Ei(-(b*d*x + b*c)/d)*e^((b*c - a*d)/d) + (b^7*c^7*
d - (4*a - 15)*b^6*c^6*d^2 + 2*(3*a^2 - 22*a + 29)*b^5*c^5*d^3 - 2*(2*a^3
- 21*a^2 + 52*a - 25)*b^4*c^4*d^4 + (a^4 - 12*a^3 + 36*a^2 - 24*a)*b^3*c^3
*d^5 - 6*a^4*d^8 - (a^4 - 8*a^3 + 12*a^2)*b^2*c^2*d^6 + 2*(a^4 - 4*a^3)*b*
c*d^7 + (b^7*c^4*d^4 - 4*(a - 4)*b^6*c^3*d^5 + 6*(a^2 - 8*a + 12)*b^5*c^2*
d^6 - 4*(a^3 - 12*a^2 + 36*a - 24)*b^4*c*d^7 + (a^4 - 16*a^3 + 72*a^2 - 96
*a)*b^3*d^8)*x^3 + (3*b^7*c^5*d^3 - (12*a - 47)*b^6*c^4*d^4 + 2*(9*a^2 - 7
0*a + 100)*b^5*c^3*d^5 - 6*(2*a^3 - 23*a^2 + 64*a - 36)*b^4*c^2*d^6 + (3*a
^4 - 44*a^3 + 168*a^2 - 144*a)*b^3*c*d^7 - (a^4 - 16*a^3 + 72*a^2)*b^2*...

```

## Sympy [F]

$$\begin{aligned}
& \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx \\
&= \left( \int \frac{a^4}{c^5 e^{bx} + 5c^4 dx e^{bx} + 10c^3 d^2 x^2 e^{bx} + 10c^2 d^3 x^3 e^{bx} + 5cd^4 x^4 e^{bx} + d^5 x^5 e^{bx}} dx \right. \\
&\quad + \int \frac{b^4 x^4}{c^5 e^{bx} + 5c^4 dx e^{bx} + 10c^3 d^2 x^2 e^{bx} + 10c^2 d^3 x^3 e^{bx} + 5cd^4 x^4 e^{bx} + d^5 x^5 e^{bx}} dx \\
&\quad + \int \frac{4ab^3 x^3}{c^5 e^{bx} + 5c^4 dx e^{bx} + 10c^3 d^2 x^2 e^{bx} + 10c^2 d^3 x^3 e^{bx} + 5cd^4 x^4 e^{bx} + d^5 x^5 e^{bx}} dx \\
&\quad + \int \frac{6a^2 b^2 x^2}{c^5 e^{bx} + 5c^4 dx e^{bx} + 10c^3 d^2 x^2 e^{bx} + 10c^2 d^3 x^3 e^{bx} + 5cd^4 x^4 e^{bx} + d^5 x^5 e^{bx}} dx \\
&\quad \left. + \int \frac{4a^3 bx}{c^5 e^{bx} + 5c^4 dx e^{bx} + 10c^3 d^2 x^2 e^{bx} + 10c^2 d^3 x^3 e^{bx} + 5cd^4 x^4 e^{bx} + d^5 x^5 e^{bx}} dx \right) e^{-a}
\end{aligned}$$

input

```
integrate(exp(-b*x-a)*(b*x+a)**4/(d*x+c)**5,x)
```

output

```
(Integral(a**4/(c**5*exp(b*x) + 5*c**4*d*x*exp(b*x) + 10*c**3*d**2*x**2*exp(b*x) + 10*c**2*d**3*x**3*exp(b*x) + 5*c*d**4*x**4*exp(b*x) + d**5*x**5*exp(b*x)), x) + Integral(b**4*x**4/(c**5*exp(b*x) + 5*c**4*d*x*exp(b*x) + 10*c**3*d**2*x**2*exp(b*x) + 10*c**2*d**3*x**3*exp(b*x) + 5*c*d**4*x**4*exp(b*x) + d**5*x**5*exp(b*x)), x) + Integral(4*a*b**3*x**3/(c**5*exp(b*x) + 5*c**4*d*x*exp(b*x) + 10*c**3*d**2*x**2*exp(b*x) + 10*c**2*d**3*x**3*exp(b*x) + 5*c*d**4*x**4*exp(b*x) + d**5*x**5*exp(b*x)), x) + Integral(6*a**2*b**2*x**2/(c**5*exp(b*x) + 5*c**4*d*x*exp(b*x) + 10*c**3*d**2*x**2*exp(b*x) + 10*c**2*d**3*x**3*exp(b*x) + 5*c*d**4*x**4*exp(b*x) + d**5*x**5*exp(b*x)), x) + Integral(4*a**3*b*x/(c**5*exp(b*x) + 5*c**4*d*x*exp(b*x) + 10*c**3*d**2*x**2*exp(b*x) + 10*c**2*d**3*x**3*exp(b*x) + 5*c*d**4*x**4*exp(b*x) + d**5*x**5*exp(b*x)), x))*exp(-a)
```

**Maxima [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx = \int \frac{(bx+a)^4 e^{(-bx-a)}}{(dx+c)^5} dx$$

input

```
integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^5,x, algorithm="maxima")
```

output

```
-(b^3*d^2*x^4 + (4*a*b^2*d^2 - b^2*d^2)*x^3 + (6*a^2*b*d^2 + 5*b^2*c*d - 8*a*b*d^2 + 2*b*d^2)*x^2 + (4*a^3*d^2 - 5*b^2*c^2 - 18*a^2*d^2 - 20*b*c*d + 4*(5*b*c*d + 6*d^2)*a - 6*d^2)*x)*e^(-b*x)/(d^7*x^5*e^a + 5*c*d^6*x^4*e^a + 10*c^2*d^5*x^3*e^a + 10*c^3*d^4*x^2*e^a + 5*c^4*d^3*x*e^a + c^5*d^2*e^a) - a^4*e^(-a + b*c/d)*exp_integral_e(5, (d*x + c)*b/d)/((d*x + c)^4*d) - integrate(-(4*a^3*c*d^2 - 5*b^2*c^3 - 18*a^2*c*d^2 - 20*b*c^2*d - 6*c*d^2 + 4*(5*b*c^2*d + 6*c*d^2)*a + (5*b^3*c^3 - 16*a^3*d^3 + 50*b^2*c^2*d + 90*b*c*d^2 + 6*(5*b*c*d^2 + 12*d^3)*a^2 + 24*d^3 - 4*(5*b^2*c^2*d + 30*b*c*d^2 + 24*d^3)*a)*x)*e^(-b*x)/(d^8*x^6*e^a + 6*c*d^7*x^5*e^a + 15*c^2*d^6*x^4*e^a + 20*c^3*d^5*x^3*e^a + 15*c^4*d^4*x^2*e^a + 6*c^5*d^3*x*e^a + c^6*d^2*e^a), x)
```

**Giac [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 16988 vs. 2(524) = 1048.

Time = 0.24 (sec) , antiderivative size = 16988, normalized size of antiderivative = 30.50

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx = \text{Too large to display}$$

input `integrate(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^5,x, algorithm="giac")`

output

```
1/24*((d*x + c)^4*(b - b*c/(d*x + c) + a*d/(d*x + c))^4*b^9*c^4*Ei(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d)
+ 4*(d*x + c)^3*(b - b*c/(d*x + c) + a*d/(d*x + c))^3*b^10*c^5*Ei(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d)
+ 6*(d*x + c)^2*(b - b*c/(d*x + c) + a*d/(d*x + c))^2*b^11*c^6*Ei(-((d*x
+ c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d)
+ 4*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^12*c^7*Ei(-((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b
^13*c^8*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)
*e^((b*c - a*d)/d) - 4*(d*x + c)^4*a*(b - b*c/(d*x + c) + a*d/(d*x + c))^4
*b^8*c^3*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^((b*c - a*d)/d) - 20*(d*x + c)^3*a*(b - b*c/(d*x + c) + a*d/(d*x + c)
)^3*b^9*c^4*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c -
a*d)/d)*e^((b*c - a*d)/d) - 36*(d*x + c)^2*a*(b - b*c/(d*x + c) + a*d/(d*x
+ c))^2*b^10*c^5*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b
*c - a*d)/d)*e^((b*c - a*d)/d) - 28*(d*x + c)*a*(b - b*c/(d*x + c) + a*d/(
d*x + c))*b^11*c^6*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
b*c - a*d)/d)*e^((b*c - a*d)/d) - 8*a*b^12*c^7*d*Ei(-((d*x + c)*(b - b*c/(
d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 6*(d*x + c)^
4*a^2*(b - b*c/(d*x + c) + a*d/(d*x + c))^4*b^7*c^2*d^2*Ei(-((d*x + c)*...
```

**Mupad [F(-1)]**

Timed out.

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx = \int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx$$

input `int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^5,x)`output `int((exp(- a - b*x)*(a + b*x)^4)/(c + d*x)^5, x)`**Reduce [F]**

$$\int \frac{e^{-a-bx}(a+bx)^4}{(c+dx)^5} dx = \text{too large to display}$$

input `int(exp(-b*x-a)*(b*x+a)^4/(d*x+c)^5,x)`

output

```
( - e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*
b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x
**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x
+ 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c
*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a**4*b**2*c**5*d**4 - 4*e**(b*x)*int
(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2
+ 10*e**(b*x)*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b
*c*d**5*x**5 + 4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c
**3*d**3*x**2 + 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e
**(b*x)*d**6*x**5),x)*a**4*b**2*c**4*d**5*x - 6*e**(b*x)*int(x/(e**(b*x)*b
*c**6 + 5*e**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*e**(b*x)
*b*c**3*d**3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**5*x**5 +
4*e**(b*x)*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d**3*x**2
+ 40*e**(b*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x)*d**6*x
**5),x)*a**4*b**2*c**3*d**6*x**2 - 4*e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e
**(b*x)*b*c**5*d*x + 10*e**(b*x)*b*c**4*d**2*x**2 + 10*e**(b*x)*b*c**3*d**
3*x**3 + 5*e**(b*x)*b*c**2*d**4*x**4 + e**(b*x)*b*c*d**5*x**5 + 4*e**(b*x)
*c**5*d + 20*e**(b*x)*c**4*d**2*x + 40*e**(b*x)*c**3*d**3*x**2 + 40*e**(b
*x)*c**2*d**4*x**3 + 20*e**(b*x)*c*d**5*x**4 + 4*e**(b*x)*d**6*x**5),x)*a**
4*b**2*c**2*d**7*x**3 - e**(b*x)*int(x/(e**(b*x)*b*c**6 + 5*e**(b*x)*b...
```

### 3.122 $\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$

Optimal result . . . . .	838
Mathematica [A] (verified) . . . . .	838
Rubi [A] (verified) . . . . .	839
Maple [C] (warning: unable to verify) . . . . .	839
Fricas [A] (verification not implemented) . . . . .	840
Sympy [F(-1)] . . . . .	840
Maxima [A] (verification not implemented) . . . . .	841
Giac [F] . . . . .	841
Mupad [B] (verification not implemented) . . . . .	842
Reduce [B] (verification not implemented) . . . . .	842

#### Optimal result

Integrand size = 39, antiderivative size = 24

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= e F^{c(a+bx)} x^{1+m} \log^{1+n}(dx)$$

output

```
e*F^(c*(b*x+a))*x^(1+m)*ln(d*x)^(1+n)
```

#### Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= e F^{ac+bcx} x^{1+m} \log^{1+n}(dx)$$

input

```
Integrate[F^(c*(a + b*x))*x^m*Log[d*x]^n*(e + e*n + e*(1 + m + b*c*x*Log[F
])*Log[d*x]), x]
```

output

```
e*F^(a*c + b*c*x)*x^(1 + m)*Log[d*x]^(1 + n)
```

**Rubi [A] (verified)**

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2631}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \log^n(dx) F^{c(a+bx)} (e \log(dx) (bcx \log(F) + m + 1) + en + e) dx$$

$$\downarrow \text{2631}$$

$$e x^{m+1} \log^{n+1}(dx) F^{c(a+bx)}$$

input

```
Int[F^(c*(a + b*x))*x^m*Log[d*x]^n*(e + e*n + e*(1 + m + b*c*x*Log[F]))*Log[d*x], x]
```

output

```
eF^(c*(a + b*x))*x^(1 + m)*Log[d*x]^(1 + n)
```

**Defintions of rubi rules used**

rule 2631

```
Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^(v_.)*(x_.)^(m_.)*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_.))), x_Symbol] :> Simp[e*x^(m + 1)*F^v*(Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, d, e, f, g, h, m, n}, x] && LinearQ[v, x] && EqQ[e*(m + 1), f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]
```

**Maple [C] (warning: unable to verify)**

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.03 (sec) , antiderivative size = 192, normalized size of antiderivative = 8.00

$$(2ex F^{c(bx+a)} \ln(x) - ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) + ix F^{c(bx+a)} e\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 + ia$$



input `int(F^(c*(b*x+a))*x^m*ln(d*x)^n*(e+e*n+e*(1+m+b*c*x*ln(F))*ln(d*x)),x)`

output `1/2*(2*e*x*F^(c*(b*x+a))*ln(x)-I*x*F^(c*(b*x+a))*e*Pi*csgn(I*d)*csgn(I*x)*csgn(I*d*x)+I*x*F^(c*(b*x+a))*e*Pi*csgn(I*d)*csgn(I*d*x)^2+I*x*F^(c*(b*x+a))*e*Pi*csgn(I*x)*csgn(I*d*x)^2-I*x*F^(c*(b*x+a))*e*Pi*csgn(I*d*x)^3+2*x*F^(c*(b*x+a))*e*ln(d))*x^m*(ln(d)+ln(x)-1/2*I*Pi*csgn(I*d*x)*(-csgn(I*d*x)+csgn(I*d))*(-csgn(I*d*x)+csgn(I*x)))^n`

### Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= (ex \log(d) + ex \log(x)) F^{bcx+ac} x^m (\log(d) + \log(x))^n$$

input `integrate(F^((b*x+a)*c))*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)),x, algorithm="fricas")`

output `(e*x*log(d) + e*x*log(x))*F^(b*c*x + a*c)*x^m*(log(d) + log(x))^n`

### Sympy [F(-1)]

Timed out.

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx = \text{Timed out}$$

input `integrate(F**((b*x+a)*c)*x**m*ln(d*x)**n*(e+e*n+e*(1+m+b*c*x*ln(F))*ln(d*x)),x)`

output `Timed out`

**Maxima [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= (F^{ac} e x \log(d) + F^{ac} e x \log(x)) e^{(bcx \log(F) + m \log(x) + n \log(\log(d) + \log(x)))}$$

input `integrate(F^((b*x+a)*c)*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)),x, algorithm="maxima")`

output `(F^(a*c)*e*x*log(d) + F^(a*c)*e*x*log(x))*e^(b*c*x*log(F) + m*log(x) + n*log(log(d) + log(x)))`

**Giac [F]**

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= \int ((bcx \log(F) + m + 1)e \log(dx) + en + e) F^{(bx+a)c} x^m \log(dx)^n dx$$

input `integrate(F^((b*x+a)*c)*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)),x, algorithm="giac")`

output `integrate(((b*c*x*log(F) + m + 1)*e*log(d*x) + e*n + e)*F^((b*x + a)*c)*x^m*log(d*x)^n, x)`

**Mupad [B] (verification not implemented)**

Time = 22.97 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= F^{ac+bcx} e x^{m+1} \ln(dx)^{n+1}$$

input

```
int(F^(c*(a + b*x))*x^m*log(d*x)^n*(e + e*n + e*log(d*x)*(m + b*c*x*log(F)
+ 1)),x)
```

output

```
F^(a*c + b*c*x)*e*x^(m + 1)*log(d*x)^(n + 1)
```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int F^{c(a+bx)} x^m \log^n(dx) (e + en + e(1 + m + bcx \log(F)) \log(dx)) dx$$

$$= x^m f^{bcx+ac} \log(dx)^n \log(dx) ex$$

input

```
int(F^((b*x+a)*c)*x^m*log(d*x)^n*(e+e*n+e*(1+m+b*c*x*log(F))*log(d*x)),x)
```

output

```
x**m*f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e*x
```

### 3.123 $\int F^{c(a+bx)} x^2 \log^n(dx) (e+en+e(3+bcx \log(F))) \log(dx)$

Optimal result	843
Mathematica [A] (verified)	843
Rubi [A] (verified)	844
Maple [A] (verified)	844
Fricas [A] (verification not implemented)	845
Sympy [F]	845
Maxima [A] (verification not implemented)	846
Giac [F(-2)]	846
Mupad [B] (verification not implemented)	847
Reduce [B] (verification not implemented)	847

#### Optimal result

Integrand size = 38, antiderivative size = 22

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F))) \log(dx) dx = eF^{c(a+bx)} x^3 \log^{1+n}(dx)$$

output `e*F^(c*(b*x+a))*x^3*ln(d*x)^(1+n)`

#### Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F))) \log(dx) dx = eF^{ac+bcx} x^3 \log^{1+n}(dx)$$

input `Integrate[F^(c*(a + b*x))*x^2*Log[d*x]^n*(e + e*n + e*(3 + b*c*x*Log[F]))*Log[d*x], x]`

output `e*F^(a*c + b*c*x)*x^3*Log[d*x]^(1 + n)`

### Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2631}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \log^n(dx) F^{c(a+bx)} (e \log(dx) (bcx \log(F) + 3) + en + e) dx$$

↓ 2631

$$ex^3 \log^{n+1}(dx) F^{c(a+bx)}$$

input `Int[F^(c*(a + b*x))*x^2*Log[d*x]^n*(e + e*n + e*(3 + b*c*x*Log[F])*Log[d*x]),x]`

output `eF^(c*(a + b*x))*x^3*Log[d*x]^(1 + n)`

#### Defintions of rubi rules used

rule 2631 `Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^(v_.)*(x_)^(m_.)*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := Simp[e*x^(m + 1)*F^v*(Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, d, e, f, g, h, m, n}, x] && LinearQ[v, x] && EqQ[e*(m + 1), f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]`

### Maple [A] (verified)

Time = 83.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result
parallelrisch	$x^3 \ln(dx) \ln(dx)^n F^{c(bx+a)} e$
risch	$\left( -\frac{i\pi e x^3 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(id) F^{c(bx+a)}}{2} + \frac{i\pi e x^3 \operatorname{csgn}(id) \operatorname{csgn}(id)^2 F^{c(bx+a)}}{2} + \frac{i\pi e x^3 \operatorname{csgn}(ix) \operatorname{csgn}(id)^2 F^{c(bx+a)}}{2} \right)$

input `int(F^(c*(b*x+a))*x^2*ln(d*x)^n*(e+e*n+e*(3+b*c*x*ln(F))*ln(d*x)),x,method  
=_RETURNVERBOSE)`

output `x^3*ln(d*x)*ln(d*x)^n*F^(c*(b*x+a))*e`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx$$

$$= F^{bcx+ac} e x^3 \log(dx)^n \log(dx)$$

input `integrate(F^((b*x+a)*c)*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x))  
,x, algorithm="fricas")`

output `F^(b*c*x + a*c)*e*x^3*log(d*x)^n*log(d*x)`

### Sympy [F]

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx$$

$$= e \left( \int F^{ac+bcx} x^2 \log(dx)^n dx + \int F^{ac+bcx} n x^2 \log(dx)^n dx \right.$$

$$\left. + \int 3 F^{ac+bcx} x^2 \log(dx) \log(dx)^n dx + \int F^{ac+bcx} bcx^3 \log(F) \log(dx) \log(dx)^n dx \right)$$

input `integrate(F**((b*x+a)*c)*x**2*ln(d*x)**n*(e+e*n+e*(3+b*c*x*ln(F))*ln(d*x))  
,x)`

output `e*(Integral(F**(a*c + b*c*x)*x**2*log(d*x)**n, x) + Integral(F**(a*c + b*c  
*x)*n*x**2*log(d*x)**n, x) + Integral(3*F**(a*c + b*c*x)*x**2*log(d*x)*log  
(d*x)**n, x) + Integral(F**(a*c + b*c*x)*b*c*x**3*log(F)*log(d*x)*log(d*x)  
**n, x))`

**Maxima [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx$$

$$= (F^{ac} e x^3 \log(d) + F^{ac} e x^3 \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

input `integrate(F^((b*x+a)*c)*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),x, algorithm="maxima")`

output `(F^(a*c)*e*x^3*log(d) + F^(a*c)*e*x^3*log(x))*e^(b*c*x*log(F) + n*log(log(d) + log(x)))`

**Giac [F(-2)]**

Exception generated.

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx$$

$$= \text{Exception raised: RuntimeError}$$

input `integrate(F^((b*x+a)*c)*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0,2,0,0,0,2,1]%%}+%%{2,[0,2,0,0,0,1,1]%%}+%%{1,[0,2,0,0,0,0,1]%%} / %%{1,[0,3,0,0`

**Mupad [B] (verification not implemented)**

Time = 23.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx = F^{ac+bcx} e x^3 \ln(dx)^{n+1}$$

input

```
int(F^(c*(a + b*x))*x^2*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) + 3
)),x)
```

output

```
F^(a*c + b*c*x)*e*x^3*log(d*x)^(n + 1)
```

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)} x^2 \log^n(dx) (e + en + e(3 + bcx \log(F)) \log(dx)) dx = f^{bcx+ac} \log(dx)^n \log(dx) e x^3$$

input

```
int(F^((b*x+a)*c)*x^2*log(d*x)^n*(e+e*n+e*(3+b*c*x*log(F))*log(d*x)),x)
```

output

```
f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e*x**3
```



### 3.124 $\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) \log(dx)$

Optimal result	848
Mathematica [A] (verified)	848
Rubi [A] (verified)	849
Maple [A] (verified)	849
Fricas [A] (verification not implemented)	850
Sympy [F]	850
Maxima [A] (verification not implemented)	851
Giac [F(-2)]	851
Mupad [B] (verification not implemented)	852
Reduce [B] (verification not implemented)	852

#### Optimal result

Integrand size = 36, antiderivative size = 22

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = eF^{c(a+bx)} x^2 \log^{1+n}(dx)$$

output `e*F^(c*(b*x+a))*x^2*ln(d*x)^(1+n)`

#### Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = eF^{ac+bcx} x^2 \log^{1+n}(dx)$$

input `Integrate[F^(c*(a + b*x))*x*Log[d*x]^n*(e + e*n + e*(2 + b*c*x*Log[F])*Log[d*x]), x]`

output `e*F^(a*c + b*c*x)*x^2*Log[d*x]^(1 + n)`

### Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.028$ , Rules used = {2631}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \log^n(dx) F^{c(a+bx)} (e \log(dx) (bcx \log(F) + 2) + en + e) dx$$

↓ 2631

$$ex^2 \log^{n+1}(dx) F^{c(a+bx)}$$

input `Int[F^(c*(a + b*x))*x*Log[d*x]^n*(e + e*n + e*(2 + b*c*x*Log[F])*Log[d*x]),x]`

output `eF^(c*(a + b*x))*x^2*Log[d*x]^(1 + n)`

#### Defintions of rubi rules used

rule 2631 `Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^(v_.)*(x_)^(m_.)*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := Simp[e*x^(m + 1)*F^v*(Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, d, e, f, g, h, m, n}, x] && LinearQ[v, x] && EqQ[e*(m + 1), f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]`

### Maple [A] (verified)

Time = 48.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result
parallelrisch	$x^2 \ln(dx) \ln(dx)^n F^{c(bx+a)} e$
risch	$\left( -\frac{i\pi e x^2 \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(id) F^{c(bx+a)}}{2} + \frac{i\pi e x^2 \operatorname{csgn}(id) \operatorname{csgn}(id) F^{c(bx+a)}}{2} + \frac{i\pi e x^2 \operatorname{csgn}(ix) \operatorname{csgn}(id) F^{c(bx+a)}}{2} \right)$

input `int(F^(c*(b*x+a))*x*ln(d*x)^n*(e+e*n+e*(2+b*c*x*ln(F))*ln(d*x)),x,method=_RETURNVERBOSE)`

output `x^2*ln(d*x)*ln(d*x)^n*F^(c*(b*x+a))*e`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = F^{bcx+ac} e x^2 \log(dx)^n \log(dx)$$

input `integrate(F^((b*x+a)*c)*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x, algorithm="fricas")`

output `F^(b*c*x + a*c)*e*x^2*log(d*x)^n*log(d*x)`

### Sympy [F]

$$\begin{aligned} & \int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx \\ &= e \left( \int F^{ac+bcx} x \log(dx)^n dx + \int F^{ac+bcx} n x \log(dx)^n dx \right. \\ & \quad \left. + \int 2F^{ac+bcx} x \log(dx) \log(dx)^n dx + \int F^{ac+bcx} bcx^2 \log(F) \log(dx) \log(dx)^n dx \right) \end{aligned}$$

input `integrate(F**((b*x+a)*c)*x*ln(d*x)**n*(e+e*n+e*(2+b*c*x*ln(F))*ln(d*x)),x)`

output `e*(Integral(F**(a*c + b*c*x)*x*log(d*x)**n, x) + Integral(F**(a*c + b*c*x)*n*x*log(d*x)**n, x) + Integral(2*F**(a*c + b*c*x)*x*log(d*x)*log(d*x)**n, x) + Integral(F**(a*c + b*c*x)*b*c*x**2*log(F)*log(d*x)*log(d*x)**n, x))`

**Maxima [A] (verification not implemented)**

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx$$

$$= (F^{ac} e x^2 \log(d) + F^{ac} e x^2 \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

input `integrate(F^((b*x+a)*c)*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x  
, algorithm="maxima")`

output `(F^(a*c)*e*x^2*log(d) + F^(a*c)*e*x^2*log(x))*e^(b*c*x*log(F) + n*log(log(d)  
+ log(x)))`

**Giac [F(-2)]**

Exception generated.

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx$$

$$= \text{Exception raised: RuntimeError}$$

input `integrate(F^((b*x+a)*c)*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x  
, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command  
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0  
,2,0,0,0,2,1]%%}+%%{2,[0,2,0,0,0,1,1]%%}+%%{1,[0,2,0,0,0,0,1]%%} / %%  
%{1,[0,3,0,0`

**Mupad [B] (verification not implemented)**

Time = 24.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = F^{ac+bcx} e x^2 \ln(dx)^{n+1}$$

input `int(F^(c*(a + b*x))*x*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) + 2)),x)`

output `F^(a*c + b*c*x)*e*x^2*log(d*x)^(n + 1)`

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int F^{c(a+bx)} x \log^n(dx) (e + en + e(2 + bcx \log(F)) \log(dx)) dx = f^{bcx+ac} \log(dx)^n \log(dx) e x^2$$

input `int(F^((b*x+a)*c)*x*log(d*x)^n*(e+e*n+e*(2+b*c*x*log(F))*log(d*x)),x)`

output `f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e*x**2`

### 3.125 $\int F^{c(a+bx)} \log^n(dx)(e+en+e(1+bcx \log(F)) \log(dx)) \log(dx)$

Optimal result . . . . .	853
Mathematica [A] (verified) . . . . .	853
Rubi [A] (verified) . . . . .	854
Maple [A] (verified) . . . . .	854
Fricas [A] (verification not implemented) . . . . .	855
Sympy [F] . . . . .	855
Maxima [A] (verification not implemented) . . . . .	856
Giac [F(-2)] . . . . .	856
Mupad [B] (verification not implemented) . . . . .	857
Reduce [B] (verification not implemented) . . . . .	857

#### Optimal result

Integrand size = 35, antiderivative size = 20

$$\int F^{c(a+bx)} \log^n(dx)(e + en + e(1 + bcx \log(F)) \log(dx)) dx = eF^{c(a+bx)} x \log^{1+n}(dx)$$

output `e*F^(c*(b*x+a))*x*ln(d*x)^(1+n)`

#### Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} \log^n(dx)(e + en + e(1 + bcx \log(F)) \log(dx)) dx = eF^{ac+bcx} x \log^{1+n}(dx)$$

input `Integrate[F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + e*(1 + b*c*x*Log[F])*Log[d*x]), x]`

output `e*F^(a*c + b*c*x)*x*Log[d*x]^(1 + n)`

### Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2630}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \log^n(dx) F^{c(a+bx)} (e \log(dx) (bcx \log(F) + 1) + en + e) dx$$

↓ 2630

$$ex \log^{n+1}(dx) F^{c(a+bx)}$$

input `Int[F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + e*(1 + b*c*x*Log[F])*Log[d*x]),x]`

output `e*F^(c*(a + b*x))*x*Log[d*x]^(1 + n)`

#### Defintions of rubi rules used

rule 2630 `Int[Log[(d_.)*(x_)]^(n_.)*(F_)^(v_)*((e_) + Log[(d_.)*(x_)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[e*x*F^v*(Log[d*x]^(n + 1)/(n + 1)), x] /; FreeQ[{F, d, e, f, g, h, n}, x] && LinearQ[v, x] && EqQ[e, f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]`

### Maple [A] (verified)

Time = 26.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result
parallelrisch	$x \ln(dx) \ln(dx)^n F^{c(bx+a)} e$
risch	$\left( -\frac{ix F^{c(bx+a)} e \pi \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx)}{2} + \frac{ix F^{c(bx+a)} e \pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2}{2} + \frac{ix F^{c(bx+a)} e \pi \operatorname{csgn}(ix) \operatorname{csgn}(idx)}{2} \right)$

input `int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+e*(1+b*c*x*ln(F))*ln(d*x)),x,method=_RE  
TURNVERBOSE)`

output `x*ln(d*x)*ln(d*x)^n*F^(c*(b*x+a))*e`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx = F^{bcx+ac} ex \log(dx)^n \log(dx)$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x,  
algorithm="fricas")`

output `F^(b*c*x + a*c)*e*x*log(d*x)^n*log(d*x)`

### Sympy [F]

$$\begin{aligned} & \int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx \\ &= e \left( \int F^{ac+bcx} \log(dx)^n dx + \int F^{ac+bcx} n \log(dx)^n dx \right. \\ & \quad \left. + \int F^{ac+bcx} \log(dx) \log(dx)^n dx + \int F^{ac+bcx} bcx \log(F) \log(dx) \log(dx)^n dx \right) \end{aligned}$$

input `integrate(F**((b*x+a)*c)*ln(d*x)**n*(e+e*n+e*(1+b*c*x*ln(F))*ln(d*x)),x)`

output `e*(Integral(F**(a*c + b*c*x)*log(d*x)**n, x) + Integral(F**(a*c + b*c*x)*n  
*log(d*x)**n, x) + Integral(F**(a*c + b*c*x)*log(d*x)*log(d*x)**n, x) + In  
tegral(F**(a*c + b*c*x)*b*c*x*log(F)*log(d*x)*log(d*x)**n, x))`



**Maxima [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx$$

$$= (F^{ac} ex \log(d) + F^{ac} ex \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x,  
algorithm="maxima")`

output `(F^(a*c)*e*x*log(d) + F^(a*c)*e*x*log(x))*e^(b*c*x*log(F) + n*log(log(d) +  
log(x)))`

**Giac [F(-2)]**

Exception generated.

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx$$

$$= \text{Exception raised: RuntimeError}$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x,  
algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command  
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0  
,2,0,0,0,2,1]%%}+%%{2,[0,2,0,0,0,1,1]%%}+%%{1,[0,2,0,0,0,0,1]%%} / %%  
%{1,[0,3,0,0`

**Mupad [B] (verification not implemented)**

Time = 23.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx = F^{ac+bcx} e x \ln(dx)^{n+1}$$

input

```
int(F^(c*(a + b*x))*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) + 1)),x)
```

output

```
F^(a*c + b*c*x)*e*x*log(d*x)^(n + 1)
```

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int F^{c(a+bx)} \log^n(dx) (e + en + e(1 + bcx \log(F)) \log(dx)) dx = f^{bcx+ac} \log(dx)^n \log(dx) e x$$

input

```
int(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(1+b*c*x*log(F))*log(d*x)),x)
```

output

```
f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e*x
```

**3.126** 
$$\int \frac{F^{c(a+bx)} \log^n(dx)(e+en+bcex \log(F) \log(dx))}{x} dx$$

Optimal result . . . . .	858
Mathematica [A] (verified) . . . . .	858
Rubi [A] (verified) . . . . .	859
Maple [A] (verified) . . . . .	859
Fricas [A] (verification not implemented) . . . . .	860
Sympy [F] . . . . .	860
Maxima [A] (verification not implemented) . . . . .	861
Giac [F(-2)] . . . . .	861
Mupad [B] (verification not implemented) . . . . .	862
Reduce [B] (verification not implemented) . . . . .	862

**Optimal result**

Integrand size = 35, antiderivative size = 19

$$\int \frac{F^{c(a+bx)} \log^n(dx)(e + en + bcex \log(F) \log(dx))}{x} dx = eF^{c(a+bx)} \log^{1+n}(dx)$$

output `e*F^(c*(b*x+a))*ln(d*x)^(1+n)`

**Mathematica [A] (verified)**

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{F^{c(a+bx)} \log^n(dx)(e + en + bcex \log(F) \log(dx))}{x} dx = eF^{c(a+bx)} \log^{1+n}(dx)$$

input `Integrate[(F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + b*c*e*x*Log[F]*Log[d*x]))/x,x]`

output `e*F^(c*(a + b*x))*Log[d*x]^(1 + n)`

### Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$ , Rules used = {2631}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(dx) F^{c(a+bx)} (bcex \log(F) \log(dx) + en + e)}{x} dx$$

$\downarrow$  2631  
 $e \log^{n+1}(dx) F^{c(a+bx)}$

input `Int[(F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + b*c*e*x*Log[F]*Log[d*x]))/x,x]`

output `e*F^(c*(a + b*x))*Log[d*x]^(1 + n)`

#### Defintions of rubi rules used

rule 2631 `Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^(v_.)*(x_)^(m_.)*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_.))), x_Symbol] := Simp[e*x^(m + 1)*F^v*(Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, d, e, f, g, h, m, n}, x] && LinearQ[v, x] && EqQ[e*(m + 1), f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]`

### Maple [A] (verified)

Time = 17.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result
parallelrisch	$\ln(dx) \ln(dx)^n F^{c(bx+a)} e$
risch	$\left( -\frac{i\pi e \operatorname{csgn}(id) \operatorname{csgn}(ix) \operatorname{csgn}(idx) F^{c(bx+a)}}{2} + \frac{i\pi e \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} + \frac{i\pi e \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 F^{c(bx+a)}}{2} \right)$

input `int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+b*c*e*x*ln(F)*ln(d*x))/x,x,method=_RETURNERVERBOSE)`

output `ln(d*x)*ln(d*x)^n*F^(c*(b*x+a))*e`

### Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx = F^{bcx+ac} e \log(dx)^n \log(dx)$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x,algorithm="fricas")`

output `F^(b*c*x + a*c)*e*log(d*x)^n*log(d*x)`

### Sympy [F]

$$\begin{aligned} & \int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx \\ &= e \left( \int \frac{F^{ac+bcx} \log(dx)^n}{x} dx + \int \frac{F^{ac+bcx} n \log(dx)^n}{x} dx \right. \\ & \quad \left. + \int F^{ac+bcx} bc \log(F) \log(dx) \log(dx)^n dx \right) \end{aligned}$$

input `integrate(F**((b*x+a)*c)*ln(d*x)**n*(e+e*n+b*c*e*x*ln(F)*ln(d*x))/x,x)`

output `e*(Integral(F**(a*c + b*c*x)*log(d*x)**n/x, x) + Integral(F**(a*c + b*c*x)*n*log(d*x)**n/x, x) + Integral(F**(a*c + b*c*x)*b*c*log(F)*log(d*x)*log(d*x)**n, x))`

**Maxima [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx$$

$$= (F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x, algorithm="maxima")`

output `(F^(a*c)*e*log(d) + F^(a*c)*e*log(x))*e^(b*c*x*log(F) + n*log(log(d) + log(x)))`

**Giac [F(-2)]**

Exception generated.

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx = \text{Exception raised: RuntimeError}$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x, algorithm="giac")`

output `Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0,2,0,0,0,2,1]%%}+%%{2,[0,2,0,0,0,1,1]%%}+%%{1,[0,2,0,0,0,0,1]%%} / %%{1,[0,3,0,0`

**Mupad [B] (verification not implemented)**

Time = 23.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx = F^{ac+bcx} e \ln(dx)^{n+1}$$

input `int((F^(c*(a + b*x))*log(d*x)^n*(e + e*n + b*c*e*x*log(d*x)*log(F)))/x,x)`output `F^(a*c + b*c*x)*e*log(d*x)^(n + 1)`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + bcex \log(F) \log(dx))}{x} dx = f^{bcx+ac} \log(dx)^n \log(dx) e$$

input `int(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+b*c*e*x*log(F)*log(d*x))/x,x)`output `f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e`

**3.127**  $\int \frac{F^{c(a+bx)} \log^n(dx)(e+en+e(-1+bcx \log(F)) \log(dx))}{x^2} dx$

Optimal result . . . . .	863
Mathematica [A] (verified) . . . . .	863
Rubi [A] (verified) . . . . .	864
Maple [A] (verified) . . . . .	865
Fricas [A] (verification not implemented) . . . . .	865
Sympy [F] . . . . .	866
Maxima [A] (verification not implemented) . . . . .	866
Giac [F(-2)] . . . . .	867
Mupad [B] (verification not implemented) . . . . .	867
Reduce [B] (verification not implemented) . . . . .	868

**Optimal result**

Integrand size = 38, antiderivative size = 22

$$\int \frac{F^{c(a+bx)} \log^n(dx)(e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx = \frac{eF^{c(a+bx)} \log^{1+n}(dx)}{x}$$

output `e*F^(c*(b*x+a))*ln(d*x)^(1+n)/x`

**Mathematica [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{F^{c(a+bx)} \log^n(dx)(e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx = \frac{eF^{ac+bcx} \log^{1+n}(dx)}{x}$$

input `Integrate[(F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + e*(-1 + b*c*x*Log[F]))*Log[d*x])/x^2,x]`

output `(e*F^(a*c + b*c*x)*Log[d*x]^(1 + n))/x`



**Rubi [A] (verified)**

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2631}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(dx) F^{c(a+bx)} (e \log(dx) (bcx \log(F) - 1) + en + e)}{x^2} dx$$

↓ 2631

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x}$$

input

```
Int[(F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + e*(-1 + b*c*x*Log[F]))*Log[d*x])/x^2,x]
```

output

```
(e*F^(c*(a + b*x))*Log[d*x]^(1 + n))/x
```

**Defintions of rubi rules used**

rule 2631

```
Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^(v_.)*(x_.)^(m_.)*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := Simp[e*x^(m + 1)*F^v*(Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, d, e, f, g, h, m, n}, x] && LinearQ[v, x] && EqQ[e*(m + 1), f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]
```

**Maple [A] (verified)**

Time = 17.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{\ln(dx) \ln(dx)^n F^{c(bx+a)} e}{x}$
risch	$\frac{F^{c(bx+a)} e \left( 2 \ln(d) + 2 \ln(x) - i\pi \operatorname{csgn}(idx) \operatorname{csgn}(id) \operatorname{csgn}(ix) + i\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 - i\pi \operatorname{csgn}(idx)^3 \right)}{2x}$

input

```
int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+e*(-1+b*c*x*ln(F))*ln(d*x))/x^2,x,method=_RETURNVERBOSE)
```

output

$$1/x * \ln(dx) * \ln(dx)^n * F^{c(bx+a)} * e$$
**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

$$= \frac{F^{bcx+ac} e \log(dx)^n \log(dx)}{x}$$

input

```
integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2,x, algorithm="fricas")
```

output

$$F^{(b*c*x + a*c)} * e * \log(dx)^n * \log(dx) / x$$

**Sympy [F]**

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

$$= e \left( \int \frac{F^{ac+bcx} \log(dx)^n}{x^2} dx + \int \frac{F^{ac+bcx} n \log(dx)^n}{x^2} dx \right. \\ \left. + \int \left( -\frac{F^{ac+bcx} \log(dx) \log(dx)^n}{x^2} \right) dx + \int \frac{F^{ac+bcx} bc \log(F) \log(dx) \log(dx)^n}{x} dx \right)$$

input `integrate(F**((b*x+a)*c)*ln(d*x)**n*(e+e*n+e*(-1+b*c*x*ln(F))*ln(d*x))/x**2,x)`

output `e*(Integral(F**(a*c + b*c*x)*log(d*x)**n/x**2, x) + Integral(F**(a*c + b*c*x)*n*log(d*x)**n/x**2, x) + Integral(-F**(a*c + b*c*x)*log(d*x)*log(d*x)**n/x**2, x) + Integral(F**(a*c + b*c*x)*b*c*log(F)*log(d*x)*log(d*x)**n/x, x))`

**Maxima [A] (verification not implemented)**

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

$$= \frac{(F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}}{x}$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x**2,x, algorithm="maxima")`

output `(F^(a*c)*e*log(d) + F^(a*c)*e*log(x))*e^(b*c*x*log(F) + n*log(log(d) + log(x)))/x`

**Giac [F(-2)]**

Exception generated.

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

= Exception raised: RuntimeError

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2,x, algorithm="giac")`

output Exception raised: RuntimeError >> an error occurred running a Giac command :INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%{1,[0,2,0,0,0,2,1]%%}+%%{2,[0,2,0,0,0,1,1]%%}+%%{1,[0,2,0,0,0,0,1]%%} / %%{1,[0,3,0,0

**Mupad [B] (verification not implemented)**

Time = 22.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx = \frac{F^{ac+bcx} e \ln(dx)^{n+1}}{x}$$

input `int((F^(c*(a + b*x))*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) - 1)))/x^2,x)`

output `(F^(a*c + b*c*x)*e*log(d*x)^(n + 1))/x`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-1 + bcx \log(F)) \log(dx))}{x^2} dx$$

$$= \frac{f^{bcx+ac} \log(dx)^n \log(dx) e}{x}$$

input

```
int(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-1+b*c*x*log(F))*log(d*x))/x^2,x)
```

output

```
(f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e)/x
```

**3.128**  $\int \frac{F^{c(a+bx)} \log^n(dx)(e+en+e(-2+bcx \log(F)) \log(dx))}{x^3} dx$

Optimal result . . . . .	869
Mathematica [A] (verified) . . . . .	869
Rubi [A] (verified) . . . . .	870
Maple [A] (verified) . . . . .	871
Fricas [A] (verification not implemented) . . . . .	871
Sympy [F] . . . . .	872
Maxima [A] (verification not implemented) . . . . .	872
Giac [F] . . . . .	873
Mupad [B] (verification not implemented) . . . . .	873
Reduce [B] (verification not implemented) . . . . .	873

**Optimal result**

Integrand size = 38, antiderivative size = 22

$$\int \frac{F^{c(a+bx)} \log^n(dx)(e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx = \frac{eF^{c(a+bx)} \log^{1+n}(dx)}{x^2}$$

output `e*F^(c*(b*x+a))*ln(d*x)^(1+n)/x^2`

**Mathematica [A] (verified)**

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{F^{c(a+bx)} \log^n(dx)(e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx = \frac{eF^{ac+bcx} \log^{1+n}(dx)}{x^2}$$

input `Integrate[(F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + e*(-2 + b*c*x*Log[F]))*Log[d*x])/x^3,x]`

output `(e*F^(a*c + b*c*x)*Log[d*x]^(1 + n))/x^2`

**Rubi [A] (verified)**

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {2631}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^n(dx) F^{c(a+bx)} (e \log(dx) (bcx \log(F) - 2) + en + e)}{x^3} dx$$

↓ 2631

$$\frac{e \log^{n+1}(dx) F^{c(a+bx)}}{x^2}$$

input

```
Int[(F^(c*(a + b*x))*Log[d*x]^n*(e + e*n + e*(-2 + b*c*x*Log[F]))*Log[d*x])/x^3,x]
```

output

```
(e*F^(c*(a + b*x))*Log[d*x]^(1 + n))/x^2
```

**Defintions of rubi rules used**

rule 2631

```
Int[Log[(d_.)*(x_.)]^(n_.)*(F_)^(v_.)*(x_)^(m_.)*((e_) + Log[(d_.)*(x_.)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := Simp[e*x^(m + 1)*F^v*(Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, d, e, f, g, h, m, n}, x] && LinearQ[v, x] && EqQ[e*(m + 1), f*h*(n + 1)] && EqQ[g*h*(n + 1), D[v, x]*e*Log[F]] && NeQ[n, -1]
```

**Maple [A] (verified)**

Time = 17.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{\ln(dx) \ln(dx)^n F^{c(bx+a)} e}{x^2}$
risch	$\frac{F^{c(bx+a)} e (2 \ln(d) + 2 \ln(x) - i\pi \operatorname{csgn}(idx) \operatorname{csgn}(id) \operatorname{csgn}(ix) + i\pi \operatorname{csgn}(id) \operatorname{csgn}(idx)^2 + i\pi \operatorname{csgn}(ix) \operatorname{csgn}(idx)^2 - i\pi \operatorname{csgn}(idx)^3)}{2x^2}$

input

```
int(F^(c*(b*x+a))*ln(d*x)^n*(e+e*n+e*(-2+b*c*x*ln(F))*ln(d*x))/x^3,x,method=_RETURNVERBOSE)
```

output

$$1/x^2 * \ln(dx) * \ln(dx)^n * F^{c(bx+a)} * e$$
**Fricas [A] (verification not implemented)**

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

$$= \frac{F^{bcx+ac} e \log(dx)^n \log(dx)}{x^2}$$

input

```
integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3,x, algorithm="fricas")
```

output

$$F^{(b*c*x + a*c)} * e * \log(dx)^n * \log(dx) / x^2$$



**Sympy [F]**

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

$$= e \left( \int \frac{F^{ac+bcx} \log(dx)^n}{x^3} dx + \int \frac{F^{ac+bcx} n \log(dx)^n}{x^3} dx \right. \\ \left. + \int \left( -\frac{2F^{ac+bcx} \log(dx) \log(dx)^n}{x^3} \right) dx \right. \\ \left. + \int \frac{F^{ac+bcx} bc \log(F) \log(dx) \log(dx)^n}{x^2} dx \right)$$

input

```
integrate(F**((b*x+a)*c)*ln(d*x)**n*(e+e*n+e*(-2+b*c*x*ln(F))*ln(d*x))/x**3,x)
```

output

```
e*(Integral(F**(a*c + b*c*x)*log(d*x)**n/x**3, x) + Integral(F**(a*c + b*c*x)*n*log(d*x)**n/x**3, x) + Integral(-2*F**(a*c + b*c*x)*log(d*x)*log(d*x)**n/x**3, x) + Integral(F**(a*c + b*c*x)*b*c*log(F)*log(d*x)*log(d*x)**n/x**2, x))
```

**Maxima [A] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

$$= \frac{(F^{ac} e \log(d) + F^{ac} e \log(x)) e^{(bcx \log(F) + n \log(\log(d) + \log(x)))}}{x^2}$$

input

```
integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3,x, algorithm="maxima")
```

output

```
(F^(a*c)*e*log(d) + F^(a*c)*e*log(x))*e^(b*c*x*log(F) + n*log(log(d) + log(x)))/x^2
```

**Giac [F]**

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

$$= \int \frac{((bcx \log(F) - 2)e \log(dx) + en + e) F^{(bx+a)c} \log(dx)^n}{x^3} dx$$

input `integrate(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3,x, algorithm="giac")`

output `integrate(((b*c*x*log(F) - 2)*e*log(d*x) + e*n + e)*F^((b*x + a)*c)*log(d*x)^n/x^3, x)`

**Mupad [B] (verification not implemented)**

Time = 22.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx = \frac{F^{ac+bcx} e \ln(dx)^{n+1}}{x^2}$$

input `int((F^(c*(a + b*x))*log(d*x)^n*(e + e*n + e*log(d*x)*(b*c*x*log(F) - 2)))/x^3,x)`

output `(F^(a*c + b*c*x)*e*log(d*x)^(n + 1))/x^2`

**Reduce [B] (verification not implemented)**

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{F^{c(a+bx)} \log^n(dx) (e + en + e(-2 + bcx \log(F)) \log(dx))}{x^3} dx$$

$$= \frac{f^{bcx+ac} \log(dx)^n \log(dx) e}{x^2}$$

input `int(F^((b*x+a)*c)*log(d*x)^n*(e+e*n+e*(-2+b*c*x*log(F))*log(d*x))/x^3,x)`

output `(f**(a*c + b*c*x)*log(d*x)**n*log(d*x)*e)/x**2`

### 3.129 $\int \sqrt{e^{a+bx}} x^4 dx$

Optimal result . . . . .	875
Mathematica [A] (verified) . . . . .	875
Rubi [A] (verified) . . . . .	876
Maple [A] (verified) . . . . .	877
Fricas [A] (verification not implemented) . . . . .	878
Sympy [A] (verification not implemented) . . . . .	878
Maxima [A] (verification not implemented) . . . . .	879
Giac [A] (verification not implemented) . . . . .	879
Mupad [B] (verification not implemented) . . . . .	879
Reduce [B] (verification not implemented) . . . . .	880

#### Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \sqrt{e^{a+bx}} x^4 dx = \frac{768\sqrt{e^{a+bx}}}{b^5} - \frac{384\sqrt{e^{a+bx}}x}{b^4} + \frac{96\sqrt{e^{a+bx}}x^2}{b^3} - \frac{16\sqrt{e^{a+bx}}x^3}{b^2} + \frac{2\sqrt{e^{a+bx}}x^4}{b}$$

output

$768*\exp(b*x+a)^{(1/2)}/b^5-384*\exp(b*x+a)^{(1/2)}*x/b^4+96*\exp(b*x+a)^{(1/2)}*x^2/b^3-16*\exp(b*x+a)^{(1/2)}*x^3/b^2+2*\exp(b*x+a)^{(1/2)}*x^4/b$

#### Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \sqrt{e^{a+bx}} x^4 dx = \frac{2\sqrt{e^{a+bx}}(384 - 192bx + 48b^2x^2 - 8b^3x^3 + b^4x^4)}{b^5}$$

input

`Integrate[Sqrt[E^(a + b*x)]]*x^4,x]`

output

$(2*\text{Sqrt}[E^{(a + b*x)}]*(384 - 192*b*x + 48*b^2*x^2 - 8*b^3*x^3 + b^4*x^4))/b^5$

**Rubi [A] (verified)**

Time = 0.66 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2607, 2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^4 \sqrt{e^{a+bx}} dx \\
 & \quad \downarrow \text{2607} \\
 & \frac{2x^4 \sqrt{e^{a+bx}}}{b} - \frac{8 \int \sqrt{e^{a+bx}} x^3 dx}{b} \\
 & \quad \downarrow \text{2607} \\
 & \frac{2x^4 \sqrt{e^{a+bx}}}{b} - \frac{8 \left( \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \int \sqrt{e^{a+bx}} x^2 dx}{b} \right)}{b} \\
 & \quad \downarrow \text{2607} \\
 & \frac{2x^4 \sqrt{e^{a+bx}}}{b} - \frac{8 \left( \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \left( \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \int \sqrt{e^{a+bx}} x dx}{b} \right)}{b} \right)}{b} \\
 & \quad \downarrow \text{2607} \\
 & \frac{2x^4 \sqrt{e^{a+bx}}}{b} - \frac{8 \left( \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \left( \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \left( \frac{2x \sqrt{e^{a+bx}}}{b} - \frac{2 \int \sqrt{e^{a+bx}} dx}{b} \right)}{b} \right)}{b} \right)}{b} \\
 & \quad \downarrow \text{2624} \\
 & \frac{2x^4 \sqrt{e^{a+bx}}}{b} - \frac{8 \left( \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \left( \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \left( \frac{2x \sqrt{e^{a+bx}}}{b} - \frac{4 \sqrt{e^{a+bx}}}{b^2} \right)}{b} \right)}{b} \right)}{b}
 \end{aligned}$$

input `Int[Sqrt[E^(a + b*x)]*x^4,x]`

output  $(2\sqrt{E^{(a + b*x)}}*x^4)/b - (8*((2\sqrt{E^{(a + b*x)}}*x^3)/b - (6*((2\sqrt{E^{(a + b*x)}}*x^2)/b - (4*((-4\sqrt{E^{(a + b*x)}})/b^2 + (2\sqrt{E^{(a + b*x)}}*x)/b))/b))/b$

**Defintions of rubi rules used**

rule 2607 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])), x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2624 `Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /; FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (verified)**

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

method	result	size
gospers	$\frac{2(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384)\sqrt{e^{bx+a}}}{b^5}$	43
risch	$\frac{2(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384)\sqrt{e^{bx+a}}}{b^5}$	43
orering	$\frac{2(b^4x^4 - 8b^3x^3 + 48b^2x^2 - 192bx + 384)\sqrt{e^{bx+a}}}{b^5}$	43
paralelrisch	$\frac{2\sqrt{e^{bx+a}}x^4b^4 - 16\sqrt{e^{bx+a}}x^3b^3 + 96\sqrt{e^{bx+a}}x^2b^2 - 384b\sqrt{e^{bx+a}}x + 768\sqrt{e^{bx+a}}}{b^5}$	76
meijerg	$-\frac{32\sqrt{e^{bx+a}}e^{-\frac{5a}{2} - \frac{bx}{2}\frac{a}{2}} \left( 24 - \frac{\left( \frac{5b^4x^4e^{2a}}{16} - \frac{5b^3x^3e^{\frac{3a}{2}}}{2} + 15b^2x^2e^a - 60bxe^{\frac{a}{2}} + 120 \right) e^{\frac{bx}{2}\frac{a}{2}}}{5}}{b^5}$	84

input `int(exp(b*x+a)^(1/2)*x^4,x,method=_RETURNVERBOSE)`

output `2*(b^4*x^4-8*b^3*x^3+48*b^2*x^2-192*b*x+384)*exp(b*x+a)^(1/2)/b^5`

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

$$\int \sqrt{e^{a+bx}} x^4 dx = \frac{2(b^4 x^4 - 8b^3 x^3 + 48b^2 x^2 - 192bx + 384)e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b^5}$$

input `integrate(exp(b*x+a)^(1/2)*x^4,x, algorithm="fricas")`

output `2*(b^4*x^4 - 8*b^3*x^3 + 48*b^2*x^2 - 192*b*x + 384)*e^(1/2*b*x + 1/2*a)/b^5`

### Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.56

$$\int \sqrt{e^{a+bx}} x^4 dx = \begin{cases} \frac{(2b^4 x^4 - 16b^3 x^3 + 96b^2 x^2 - 384bx + 768)\sqrt{e^{a+bx}}}{b^5} & \text{for } b^5 \neq 0 \\ \frac{x^5}{5} & \text{otherwise} \end{cases}$$

input `integrate(exp(b*x+a)**(1/2)*x**4,x)`

output `Piecewise(((2*b**4*x**4 - 16*b**3*x**3 + 96*b**2*x**2 - 384*b*x + 768)*sqrt(exp(a + b*x))/b**5, Ne(b**5, 0)), (x**5/5, True))`

**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.66

$$\int \sqrt{e^{a+bx}} x^4 dx = \frac{2 \left( b^4 x^4 e^{\frac{1}{2}a} - 8 b^3 x^3 e^{\frac{1}{2}a} + 48 b^2 x^2 e^{\frac{1}{2}a} - 192 b x e^{\frac{1}{2}a} + 384 e^{\frac{1}{2}a} \right) e^{\frac{1}{2}bx}}{b^5}$$

input `integrate(exp(b*x+a)^(1/2)*x^4,x, algorithm="maxima")`output `2*(b^4*x^4*e^(1/2*a) - 8*b^3*x^3*e^(1/2*a) + 48*b^2*x^2*e^(1/2*a) - 192*b*x*e^(1/2*a) + 384*e^(1/2*a))*e^(1/2*b*x)/b^5`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

$$\int \sqrt{e^{a+bx}} x^4 dx = \frac{2(b^4 x^4 - 8 b^3 x^3 + 48 b^2 x^2 - 192 b x + 384) e^{\frac{1}{2}bx + \frac{1}{2}a}}{b^5}$$

input `integrate(exp(b*x+a)^(1/2)*x^4,x, algorithm="giac")`output `2*(b^4*x^4 - 8*b^3*x^3 + 48*b^2*x^2 - 192*b*x + 384)*e^(1/2*b*x + 1/2*a)/b^5`**Mupad [B] (verification not implemented)**

Time = 22.77 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \sqrt{e^{a+bx}} x^4 dx = \sqrt{e^{a+bx}} \left( \frac{768}{b^5} - \frac{384x}{b^4} + \frac{2x^4}{b} - \frac{16x^3}{b^2} + \frac{96x^2}{b^3} \right)$$

input `int(x^4*exp(a + b*x)^(1/2),x)`



output

```
exp(a + b*x)^(1/2)*(768/b^5 - (384*x)/b^4 + (2*x^4)/b - (16*x^3)/b^2 + (96*x^2)/b^3)
```

**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.48

$$\int \sqrt{e^{a+bx}} x^4 dx = \frac{2e^{\frac{bx}{2} + \frac{a}{2}} (b^4 x^4 - 8b^3 x^3 + 48b^2 x^2 - 192bx + 384)}{b^5}$$

input

```
int(exp(b*x+a)^(1/2)*x^4,x)
```

output

```
(2*e**((a + b*x)/2)*(b**4*x**4 - 8*b**3*x**3 + 48*b**2*x**2 - 192*b*x + 384))/b**5
```

### 3.130 $\int \sqrt{e^{a+bx}} x^3 dx$

Optimal result . . . . .	881
Mathematica [A] (verified) . . . . .	881
Rubi [A] (verified) . . . . .	882
Maple [A] (verified) . . . . .	883
Fricas [A] (verification not implemented) . . . . .	884
Sympy [A] (verification not implemented) . . . . .	884
Maxima [A] (verification not implemented) . . . . .	884
Giac [A] (verification not implemented) . . . . .	885
Mupad [B] (verification not implemented) . . . . .	885
Reduce [B] (verification not implemented) . . . . .	885

#### Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \sqrt{e^{a+bx}} x^3 dx = -\frac{96\sqrt{e^{a+bx}}}{b^4} + \frac{48\sqrt{e^{a+bx}}x}{b^3} - \frac{12\sqrt{e^{a+bx}}x^2}{b^2} + \frac{2\sqrt{e^{a+bx}}x^3}{b}$$

output

```
-96*exp(b*x+a)^(1/2)/b^4+48*exp(b*x+a)^(1/2)*x/b^3-12*exp(b*x+a)^(1/2)*x^2/b^2+2*exp(b*x+a)^(1/2)*x^3/b
```

#### Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \sqrt{e^{a+bx}} x^3 dx = \frac{2\sqrt{e^{a+bx}}(-48 + 24bx - 6b^2x^2 + b^3x^3)}{b^4}$$

input

```
Integrate[Sqrt[E^(a + b*x)]*x^3,x]
```

output

```
(2*Sqrt[E^(a + b*x)]*(-48 + 24*b*x - 6*b^2*x^2 + b^3*x^3))/b^4
```

**Rubi [A] (verified)**

Time = 0.53 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2607, 2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^3 \sqrt{e^{a+bx}} dx \\
 & \quad \downarrow 2607 \\
 & \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \int \sqrt{e^{a+bx}} x^2 dx}{b} \\
 & \quad \downarrow 2607 \\
 & \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \left( \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \int \sqrt{e^{a+bx}} x dx}{b} \right)}{b} \\
 & \quad \downarrow 2607 \\
 & \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \left( \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \left( \frac{2x \sqrt{e^{a+bx}}}{b} - \frac{2 \int \sqrt{e^{a+bx}} dx}{b} \right)}{b} \right)}{b} \\
 & \quad \downarrow 2624 \\
 & \frac{2x^3 \sqrt{e^{a+bx}}}{b} - \frac{6 \left( \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \left( \frac{2x \sqrt{e^{a+bx}}}{b} - \frac{4 \sqrt{e^{a+bx}}}{b^2} \right)}{b} \right)}{b}
 \end{aligned}$$

input `Int[Sqrt[E^(a + b*x)]*x^3,x]`

output `(2*Sqrt[E^(a + b*x)]*x^3)/b - (6*((2*Sqrt[E^(a + b*x)]*x^2)/b - (4*((-4*Sqrt[E^(a + b*x)])/b^2 + (2*Sqrt[E^(a + b*x)]*x)/b))/b)`

## Definitions of rubi rules used

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

rule 2624

```
Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

method	result	size
gosper	$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)\sqrt{e^{bx+a}}}{b^4}$	35
risch	$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)\sqrt{e^{bx+a}}}{b^4}$	35
orering	$\frac{2(b^3x^3 - 6b^2x^2 + 24bx - 48)\sqrt{e^{bx+a}}}{b^4}$	35
parallelrisc	$\frac{2\sqrt{e^{bx+a}}x^3b^3 - 12\sqrt{e^{bx+a}}x^2b^2 + 48b\sqrt{e^{bx+a}}x - 96\sqrt{e^{bx+a}}}{b^4}$	60
meijerg	$\frac{16\sqrt{e^{bx+a}}e^{-2a - \frac{bx}{2}} \left( 6 - \frac{\left( -\frac{b^3x^3}{2}e^{\frac{3a}{2}} + 3b^2x^2e^a - 12bx e^{\frac{a}{2}} + 24 \right) e^{\frac{bx}{2}}}{4} \right)}{b^4}$	72

input

```
int(exp(b*x+a)^(1/2)*x^3,x,method=_RETURNVERBOSE)
```

output

```
2*(b^3*x^3-6*b^2*x^2+24*b*x-48)*exp(b*x+a)^(1/2)/b^4
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

$$\int \sqrt{e^{a+bx}} x^3 dx = \frac{2(b^3 x^3 - 6b^2 x^2 + 24bx - 48)e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b^4}$$

input `integrate(exp(b*x+a)^(1/2)*x^3,x, algorithm="fricas")`output `2*(b^3*x^3 - 6*b^2*x^2 + 24*b*x - 48)*e^(1/2*b*x + 1/2*a)/b^4`**Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \sqrt{e^{a+bx}} x^3 dx = \begin{cases} \frac{(2b^3 x^3 - 12b^2 x^2 + 48bx - 96)\sqrt{e^{a+bx}}}{b^4} & \text{for } b^4 \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases}$$

input `integrate(exp(b*x+a)**(1/2)*x**3,x)`output `Piecewise(((2*b**3*x**3 - 12*b**2*x**2 + 48*b*x - 96)*sqrt(exp(a + b*x))/b**4, Ne(b**4, 0)), (x**4/4, True))`**Maxima [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \sqrt{e^{a+bx}} x^3 dx = \frac{2\left(b^3 x^3 e^{(\frac{1}{2}a)} - 6b^2 x^2 e^{(\frac{1}{2}a)} + 24bx e^{(\frac{1}{2}a)} - 48e^{(\frac{1}{2}a)}\right) e^{(\frac{1}{2}bx)}}{b^4}$$

input `integrate(exp(b*x+a)^(1/2)*x^3,x, algorithm="maxima")`output `2*(b^3*x^3*e^(1/2*a) - 6*b^2*x^2*e^(1/2*a) + 24*b*x*e^(1/2*a) - 48*e^(1/2*a))*e^(1/2*b*x)/b^4`

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.49

$$\int \sqrt{e^{a+bx}} x^3 dx = \frac{2(b^3 x^3 - 6b^2 x^2 + 24bx - 48)e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b^4}$$

input `integrate(exp(b*x+a)^(1/2)*x^3,x, algorithm="giac")`

output `2*(b^3*x^3 - 6*b^2*x^2 + 24*b*x - 48)*e^(1/2*b*x + 1/2*a)/b^4`

**Mupad [B] (verification not implemented)**

Time = 22.78 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int \sqrt{e^{a+bx}} x^3 dx = \sqrt{e^{a+bx}} \left( \frac{48x}{b^3} - \frac{96}{b^4} + \frac{2x^3}{b} - \frac{12x^2}{b^2} \right)$$

input `int(x^3*exp(a + b*x)^(1/2),x)`

output `exp(a + b*x)^(1/2)*((48*x)/b^3 - 96/b^4 + (2*x^3)/b - (12*x^2)/b^2)`

**Reduce [B] (verification not implemented)**

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.50

$$\int \sqrt{e^{a+bx}} x^3 dx = \frac{2e^{\frac{bx}{2} + \frac{a}{2}}(b^3 x^3 - 6b^2 x^2 + 24bx - 48)}{b^4}$$

input `int(exp(b*x+a)^(1/2)*x^3,x)`

output `(2*e**((a + b*x)/2)*(b**3*x**3 - 6*b**2*x**2 + 24*b*x - 48))/b**4`

### 3.131 $\int \sqrt{e^{a+bx}} x^2 dx$

Optimal result . . . . .	886
Mathematica [A] (verified) . . . . .	886
Rubi [A] (verified) . . . . .	887
Maple [A] (verified) . . . . .	888
Fricas [A] (verification not implemented) . . . . .	888
Sympy [A] (verification not implemented) . . . . .	889
Maxima [A] (verification not implemented) . . . . .	889
Giac [A] (verification not implemented) . . . . .	889
Mupad [B] (verification not implemented) . . . . .	890
Reduce [B] (verification not implemented) . . . . .	890

#### Optimal result

Integrand size = 15, antiderivative size = 53

$$\int \sqrt{e^{a+bx}} x^2 dx = \frac{16\sqrt{e^{a+bx}}}{b^3} - \frac{8\sqrt{e^{a+bx}}x}{b^2} + \frac{2\sqrt{e^{a+bx}}x^2}{b}$$

output  $16*\exp(b*x+a)^{(1/2)}/b^3-8*\exp(b*x+a)^{(1/2)}*x/b^2+2*\exp(b*x+a)^{(1/2)}*x^2/b$

#### Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \sqrt{e^{a+bx}} x^2 dx = \frac{2\sqrt{e^{a+bx}}(8 - 4bx + b^2x^2)}{b^3}$$

input  $\text{Integrate}[\text{Sqrt}[E^{(a + b*x)}]*x^2,x]$

output  $(2*\text{Sqrt}[E^{(a + b*x)}]*(8 - 4*b*x + b^2*x^2))/b^3$

**Rubi [A] (verified)**

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2607, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int x^2 \sqrt{e^{a+bx}} dx \\
 & \quad \downarrow \text{2607} \\
 & \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \int \sqrt{e^{a+bx}} x dx}{b} \\
 & \quad \downarrow \text{2607} \\
 & \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \left( \frac{2x \sqrt{e^{a+bx}}}{b} - \frac{2 \int \sqrt{e^{a+bx}} dx}{b} \right)}{b} \\
 & \quad \downarrow \text{2624} \\
 & \frac{2x^2 \sqrt{e^{a+bx}}}{b} - \frac{4 \left( \frac{2x \sqrt{e^{a+bx}}}{b} - \frac{4 \sqrt{e^{a+bx}}}{b^2} \right)}{b}
 \end{aligned}$$

input `Int[Sqrt[E^(a + b*x)]*x^2,x]`

output `(2*Sqrt[E^(a + b*x)]*x^2)/b - (4*((-4*Sqrt[E^(a + b*x)]))/b^2 + (2*Sqrt[E^(a + b*x)]*x)/b))/b`

**Defintions of rubi rules used**

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```



rule 2624

```
Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

method	result	size
gospers	$\frac{2(b^2x^2 - 4bx + 8)\sqrt{e^{bx+a}}}{b^3}$	27
risch	$\frac{2(b^2x^2 - 4bx + 8)\sqrt{e^{bx+a}}}{b^3}$	27
orering	$\frac{2(b^2x^2 - 4bx + 8)\sqrt{e^{bx+a}}}{b^3}$	27
parallelrisch	$\frac{2\sqrt{e^{bx+a}}x^2b^2 - 8b\sqrt{e^{bx+a}}x + 16\sqrt{e^{bx+a}}}{b^3}$	44
meijerg	$-\frac{8\sqrt{e^{bx+a}}e^{-\frac{3a}{2} - \frac{bx}{2}} \left( 2 - \frac{\left( \frac{3b^2x^2e^a}{4} - 3bx e^{\frac{a}{2}} + 6 \right) e^{\frac{bx}{2}}}{3} \right)}{b^3}$	60

input

```
int(exp(b*x+a)^(1/2)*x^2,x,method=_RETURNVERBOSE)
```

output

```
2*(b^2*x^2-4*b*x+8)*exp(b*x+a)^(1/2)/b^3
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \sqrt{e^{a+bx}} x^2 dx = \frac{2(b^2x^2 - 4bx + 8)e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b^3}$$

input

```
integrate(exp(b*x+a)^(1/2)*x^2,x, algorithm="fricas")
```

output

```
2*(b^2*x^2 - 4*b*x + 8)*e^(1/2*b*x + 1/2*a)/b^3
```

**Sympy [A] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \sqrt{e^{a+bx}} x^2 dx = \begin{cases} \frac{(2b^2x^2 - 8bx + 16)\sqrt{e^{a+bx}}}{b^3} & \text{for } b^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

input `integrate(exp(b*x+a)**(1/2)*x**2,x)`output `Piecewise(((2*b**2*x**2 - 8*b*x + 16)*sqrt(exp(a + b*x))/b**3, Ne(b**3, 0)), (x**3/3, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \sqrt{e^{a+bx}} x^2 dx = \frac{2 \left( b^2 x^2 e^{\frac{1}{2}a} - 4bx e^{\frac{1}{2}a} + 8e^{\frac{1}{2}a} \right) e^{\frac{1}{2}bx}}{b^3}$$

input `integrate(exp(b*x+a)^(1/2)*x^2,x, algorithm="maxima")`output `2*(b^2*x^2*e^(1/2*a) - 4*b*x*e^(1/2*a) + 8*e^(1/2*a))*e^(1/2*b*x)/b^3`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.51

$$\int \sqrt{e^{a+bx}} x^2 dx = \frac{2(b^2x^2 - 4bx + 8)e^{\frac{1}{2}bx + \frac{1}{2}a}}{b^3}$$

input `integrate(exp(b*x+a)^(1/2)*x^2,x, algorithm="giac")`output `2*(b^2*x^2 - 4*b*x + 8)*e^(1/2*b*x + 1/2*a)/b^3`

**Mupad [B] (verification not implemented)**

Time = 23.93 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \sqrt{e^{a+bx}} x^2 dx = \sqrt{e^{a+bx}} \left( \frac{16}{b^3} - \frac{8x}{b^2} + \frac{2x^2}{b} \right)$$

input `int(x^2*exp(a + b*x)^(1/2),x)`output `exp(a + b*x)^(1/2)*(16/b^3 - (8*x)/b^2 + (2*x^2)/b)`**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.53

$$\int \sqrt{e^{a+bx}} x^2 dx = \frac{2e^{\frac{bx}{2} + \frac{a}{2}} (b^2 x^2 - 4bx + 8)}{b^3}$$

input `int(exp(b*x+a)^(1/2)*x^2,x)`output `(2*e**((a + b*x)/2)*(b**2*x**2 - 4*b*x + 8))/b**3`

### 3.132 $\int \sqrt{e^{a+bx}} x dx$

Optimal result . . . . .	891
Mathematica [A] (verified) . . . . .	891
Rubi [A] (verified) . . . . .	892
Maple [A] (verified) . . . . .	893
Fricas [A] (verification not implemented) . . . . .	893
Sympy [A] (verification not implemented) . . . . .	894
Maxima [A] (verification not implemented) . . . . .	894
Giac [A] (verification not implemented) . . . . .	894
Mupad [B] (verification not implemented) . . . . .	895
Reduce [B] (verification not implemented) . . . . .	895

#### Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \sqrt{e^{a+bx}} x dx = -\frac{4\sqrt{e^{a+bx}}}{b^2} + \frac{2\sqrt{e^{a+bx}} x}{b}$$

output `-4*exp(b*x+a)^(1/2)/b^2+2*exp(b*x+a)^(1/2)*x/b`

#### Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \sqrt{e^{a+bx}} x dx = \frac{2\sqrt{e^{a+bx}}(-2 + bx)}{b^2}$$

input `Integrate[Sqrt[E^(a + b*x)]]*x,x]`

output `(2*Sqrt[E^(a + b*x)]*(-2 + b*x))/b^2`

**Rubi [A] (verified)**

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{e^{a+bx}} dx$$

$$\downarrow \text{2607}$$

$$\frac{2x\sqrt{e^{a+bx}}}{b} - \frac{2}{b} \int \sqrt{e^{a+bx}} dx$$

$$\downarrow \text{2624}$$

$$\frac{2x\sqrt{e^{a+bx}}}{b} - \frac{4\sqrt{e^{a+bx}}}{b^2}$$

input `Int[Sqrt[E^(a + b*x)]]*x,x`

output `(-4*Sqrt[E^(a + b*x)])/b^2 + (2*Sqrt[E^(a + b*x)]]*x)/b`

**Defintions of rubi rules used**

rule 2607

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] := Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F])) Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] && !TrueQ[$UseGamma]
```

rule 2624

```
Int[((F_)^(v_))^(n_.), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

method	result	size
gospers	$\frac{2(bx-2)\sqrt{e^{bx+a}}}{b^2}$	19
risch	$\frac{2(bx-2)\sqrt{e^{bx+a}}}{b^2}$	19
orering	$\frac{2(bx-2)\sqrt{e^{bx+a}}}{b^2}$	19
parallelrisch	$\frac{2b\sqrt{e^{bx+a}}x - 4\sqrt{e^{bx+a}}}{b^2}$	28
meijerg	$\frac{4\sqrt{e^{bx+a}}e^{-a - \frac{bx}{2}} \left( 1 - \frac{(2 - bx e^{\frac{a}{2}})e^{\frac{bx}{2}}}{2} \right)}{b^2}$	50

input `int(exp(b*x+a)^(1/2)*x,x,method=_RETURNVERBOSE)`output `2*(b*x-2)*exp(b*x+a)^(1/2)/b^2`**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \sqrt{e^{a+bx}} x dx = \frac{2(bx-2)e^{\left(\frac{1}{2}bx + \frac{1}{2}a\right)}}{b^2}$$

input `integrate(exp(b*x+a)^(1/2)*x,x, algorithm="fricas")`output `2*(b*x - 2)*e^(1/2*b*x + 1/2*a)/b^2`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{e^{a+bx}} x dx = \begin{cases} \frac{(2bx-4)\sqrt{e^{a+bx}}}{b^2} & \text{for } b^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

input `integrate(exp(b*x+a)**(1/2)*x,x)`output `Piecewise(((2*b*x - 4)*sqrt(exp(a + b*x))/b**2, Ne(b**2, 0)), (x**2/2, True))`**Maxima [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \sqrt{e^{a+bx}} x dx = \frac{2 \left( bxe^{\frac{1}{2}a} - 2e^{\frac{1}{2}a} \right) e^{\frac{1}{2}bx}}{b^2}$$

input `integrate(exp(b*x+a)^(1/2)*x,x, algorithm="maxima")`output `2*(b*x*e^(1/2*a) - 2*e^(1/2*a))*e^(1/2*b*x)/b^2`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \sqrt{e^{a+bx}} x dx = \frac{2(bx-2)e^{\frac{1}{2}bx+\frac{1}{2}a}}{b^2}$$

input `integrate(exp(b*x+a)^(1/2)*x,x, algorithm="giac")`output `2*(b*x - 2)*e^(1/2*b*x + 1/2*a)/b^2`

**Mupad [B] (verification not implemented)**

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int \sqrt{e^{a+bx}} x dx = \frac{2\sqrt{e^{a+bx}}(bx - 2)}{b^2}$$

input `int(x*exp(a + b*x)^(1/2),x)`output `(2*exp(a + b*x)^(1/2)*(b*x - 2))/b^2`**Reduce [B] (verification not implemented)**

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \sqrt{e^{a+bx}} x dx = \frac{2e^{\frac{bx}{2} + \frac{a}{2}}(bx - 2)}{b^2}$$

input `int(exp(b*x+a)^(1/2)*x,x)`output `(2*e**((a + b*x)/2)*(b*x - 2))/b**2`



### 3.133 $\int \sqrt{e^{a+bx}} dx$

Optimal result . . . . .	896
Mathematica [A] (verified) . . . . .	896
Rubi [A] (verified) . . . . .	897
Maple [A] (verified) . . . . .	898
Fricas [A] (verification not implemented) . . . . .	898
Sympy [A] (verification not implemented) . . . . .	899
Maxima [A] (verification not implemented) . . . . .	899
Giac [A] (verification not implemented) . . . . .	899
Mupad [B] (verification not implemented) . . . . .	900
Reduce [B] (verification not implemented) . . . . .	900

#### Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \sqrt{e^{a+bx}} dx = \frac{2\sqrt{e^{a+bx}}}{b}$$

output `2*exp(b*x+a)^(1/2)/b`

#### Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{e^{a+bx}} dx = \frac{2\sqrt{e^{a+bx}}}{b}$$

input `Integrate[Sqrt[E^(a + b*x)],x]`

output `(2*Sqrt[E^(a + b*x))]/b`

**Rubi [A] (verified)**

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e^{a+bx}} dx$$

$$\downarrow 2624$$

$$\frac{2\sqrt{e^{a+bx}}}{b}$$

input `Int[Sqrt[E^(a + b*x)],x]`

output `(2*Sqrt[E^(a + b*x)])/b`

**Defintions of rubi rules used**

rule 2624 `Int[((F_)^(v_))^(n_), x_Symbol] := Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;`  
`FreeQ[{F, n}, x] && LinearQ[v, x]`

**Maple [A] (verified)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
gospers	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
derivativdivides	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
default	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
risch	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
parallelrisch	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
orering	$\frac{2\sqrt{e^{bx+a}}}{b}$	14
meijerg	$-\frac{2\sqrt{e^{bx+a}} e^{-\frac{a}{2} - \frac{bx}{2}} \left(1 - e^{-\frac{bx}{2}}\right)}{b}$	40

input `int(exp(b*x+a)^(1/2), x, method=_RETURNVERBOSE)`output `2*exp(b*x+a)^(1/2)/b`**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{e^{a+bx}} dx = \frac{2 e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b}$$

input `integrate(exp(b*x+a)^(1/2), x, algorithm="fricas")`output `2*e^(1/2*b*x + 1/2*a)/b`

**Sympy [A] (verification not implemented)**

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{e^{a+bx}} dx = \begin{cases} \frac{2\sqrt{e^{a+bx}}}{b} & \text{for } b \neq 0 \\ x & \text{otherwise} \end{cases}$$

input `integrate(exp(b*x+a)**(1/2),x)`output `Piecewise((2*sqrt(exp(a + b*x))/b, Ne(b, 0)), (x, True))`**Maxima [A] (verification not implemented)**

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{e^{a+bx}} dx = \frac{2e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b}$$

input `integrate(exp(b*x+a)^(1/2),x, algorithm="maxima")`output `2*e^(1/2*b*x + 1/2*a)/b`**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{e^{a+bx}} dx = \frac{2e^{(\frac{1}{2}bx + \frac{1}{2}a)}}{b}$$

input `integrate(exp(b*x+a)^(1/2),x, algorithm="giac")`output `2*e^(1/2*b*x + 1/2*a)/b`

**Mupad [B] (verification not implemented)**

Time = 23.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \sqrt{e^{a+bx}} dx = \frac{2\sqrt{e^{a+bx}}}{b}$$

input `int(exp(a + b*x)^(1/2), x)`

output `(2*exp(a + b*x)^(1/2))/b`

**Reduce [B] (verification not implemented)**

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{e^{a+bx}} dx = \frac{2e^{\frac{bx}{2} + \frac{a}{2}}}{b}$$

input `int(exp(b*x+a)^(1/2), x)`

output `(2*e**((a + b*x)/2))/b`

### 3.134 $\int \frac{\sqrt{e^{a+bx}}}{x} dx$

Optimal result	901
Mathematica [A] (verified)	901
Rubi [A] (verified)	902
Maple [B] (verified)	903
Fricas [A] (verification not implemented)	903
Sympy [F]	903
Maxima [A] (verification not implemented)	904
Giac [A] (verification not implemented)	904
Mupad [F(-1)]	904
Reduce [F]	905

#### Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{ExpIntegralEi} \left( \frac{bx}{2} \right)$$

output `exp(b*x+a)^(1/2)*Ei(1/2*b*x)/exp(1/2*b*x)`

#### Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{ExpIntegralEi} \left( \frac{bx}{2} \right)$$

input `Integrate[Sqrt[E^(a + b*x)]/x,x]`

output `(Sqrt[E^(a + b*x)]*ExpIntegralEi[(b*x)/2])/E^((b*x)/2)`

**Rubi [A] (verified)**

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44, number of steps used = 2, number of rules used = 2,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {2613, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx$$

$$\downarrow \text{2613}$$

$$e^{\frac{1}{2}(-a-bx)} \sqrt{e^{a+bx}} \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx$$

$$\downarrow \text{2609}$$

$$e^{\frac{1}{2}(-a-bx) + \frac{a}{2}} \sqrt{e^{a+bx}} \text{ExpIntegralEi} \left( \frac{bx}{2} \right)$$

input `Int[Sqrt[E^(a + b*x)]/x,x]`

output `E^(a/2 + (-a - b*x)/2)*Sqrt[E^(a + b*x)]*ExpIntegralEi[(b*x)/2]`

**Defintions of rubi rules used**

rule 2609 `Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] :> Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]`

rule 2613 `Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] :> Simp[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)) Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]`

**Maple [B] (verified)**

Leaf count of result is larger than twice the leaf count of optimal. 56 vs.  $2(21) = 42$ .

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.11

method	result	size
meijerg	$\sqrt{e^{bx+a}} e^{-\frac{bx}{2}} \left( \ln(x) - \ln(2) + \ln(-b e^{\frac{a}{2}}) - \ln\left(-\frac{bx}{2} e^{\frac{a}{2}}\right) - \operatorname{expIntegral}_1\left(-\frac{bx}{2} e^{\frac{a}{2}}\right) \right)$	57

input `int(exp(b*x+a)^(1/2)/x,x,method=_RETURNVERBOSE)`

output `exp(b*x+a)^(1/2)*exp(-1/2*b*x*exp(1/2*a))*(ln(x)-ln(2)+ln(-b*exp(1/2*a))-ln(-1/2*b*x*exp(1/2*a))-Ei(1,-1/2*b*x*exp(1/2*a)))`

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \operatorname{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)}$$

input `integrate(exp(b*x+a)^(1/2)/x,x, algorithm="fricas")`

output `Ei(1/2*b*x)*e^(1/2*a)`

**Sympy [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \int \frac{\sqrt{e^a e^{bx}}}{x} dx$$

input `integrate(exp(b*x+a)**(1/2)/x,x)`

output `Integral(sqrt(exp(a)*exp(b*x))/x, x)`



**Maxima [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \text{Ei}\left(\frac{1}{2}bx\right) e^{\left(\frac{1}{2}a\right)}$$

input `integrate(exp(b*x+a)^(1/2)/x,x, algorithm="maxima")`output `Ei(1/2*b*x)*e^(1/2*a)`**Giac [A] (verification not implemented)**

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \text{Ei}\left(\frac{1}{2}bx\right) e^{\left(\frac{1}{2}a\right)}$$

input `integrate(exp(b*x+a)^(1/2)/x,x, algorithm="giac")`output `Ei(1/2*b*x)*e^(1/2*a)`**Mupad [F(-1)]**

Timed out.

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \int \frac{\sqrt{e^{a+bx}}}{x} dx$$

input `int(exp(a + b*x)^(1/2)/x,x)`output `int(exp(a + b*x)^(1/2)/x, x)`

**Reduce [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x} dx = \int \frac{e^{\frac{bx}{2} + \frac{a}{2}}}{x} dx$$

input `int(exp(b*x+a)^(1/2)/x,x)`

output `int(e**((a + b*x)/2)/x,x)`

### 3.135 $\int \frac{\sqrt{e^{a+bx}}}{x^2} dx$

Optimal result . . . . .	906
Mathematica [A] (verified) . . . . .	906
Rubi [A] (verified) . . . . .	907
Maple [B] (verified) . . . . .	908
Fricas [A] (verification not implemented) . . . . .	908
Sympy [F] . . . . .	909
Maxima [A] (verification not implemented) . . . . .	909
Giac [A] (verification not implemented) . . . . .	909
Mupad [F(-1)] . . . . .	910
Reduce [F] . . . . .	910

#### Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = -\frac{\sqrt{e^{a+bx}}}{x} + \frac{1}{2}be^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\text{ExpIntegralEi}\left(\frac{bx}{2}\right)$$

output `-exp(b*x+a)^(1/2)/x+1/2*b*exp(b*x+a)^(1/2)*Ei(1/2*b*x)/exp(1/2*b*x)`

#### Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \frac{e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\left(-2e^{\frac{bx}{2}} + bx\text{ExpIntegralEi}\left(\frac{bx}{2}\right)\right)}{2x}$$

input `Integrate[Sqrt[E^(a + b*x)]/x^2,x]`

output `(Sqrt[E^(a + b*x)]*(-2*E^((b*x)/2) + b*x*ExpIntegralEi[(b*x)/2]))/(2*E^((b*x)/2)*x)`

**Rubi [A] (verified)**

Time = 0.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25, number of steps used = 3, number of rules used = 3,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$ , Rules used = {2608, 2613, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned} & \int \frac{\sqrt{e^{a+bx}}}{x^2} dx \\ & \quad \downarrow \text{2608} \\ & \frac{1}{2}b \int \frac{\sqrt{e^{a+bx}}}{x} dx - \frac{\sqrt{e^{a+bx}}}{x} \\ & \quad \downarrow \text{2613} \\ & \frac{1}{2}be^{\frac{1}{2}(-a-bx)}\sqrt{e^{a+bx}} \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx - \frac{\sqrt{e^{a+bx}}}{x} \\ & \quad \downarrow \text{2609} \\ & \frac{1}{2}be^{\frac{1}{2}(-a-bx)+\frac{a}{2}}\sqrt{e^{a+bx}} \text{ExpIntegralEi}\left(\frac{bx}{2}\right) - \frac{\sqrt{e^{a+bx}}}{x} \end{aligned}$$

input `Int[Sqrt[E^(a + b*x)]/x^2,x]`

output `-(Sqrt[E^(a + b*x)]/x) + (b*E^(a/2 + (-a - b*x)/2)*Sqrt[E^(a + b*x)]*ExpIntegralEi[(b*x)/2])/2`

**Defintions of rubi rules used**

rule 2608 `Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1))), x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && IntegerQ[2*m] && !TrueQ[$UseGamma]`

rule 2609

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/((c_) + (d_)*(x_)), x_Symbol] := Simp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2613

```
Int[((b_)*(F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)) Int[(c + d*x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

### Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs.  $2(37) = 74$ .

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.42

method	result
meijerg	$\frac{\sqrt{e^{bx+a}} e^{\frac{a}{2}} - \frac{bx e^{\frac{a}{2}}}{2} b \left( \frac{2e^{-\frac{a}{2}}}{xb} + 1 - \ln(x) + \ln(2) - \ln(-b e^{\frac{a}{2}}) - \frac{e^{-\frac{a}{2}} (2 + bx e^{\frac{a}{2}})}{bx} + 2e^{-\frac{a}{2}} + \frac{bx e^{\frac{a}{2}}}{bx} + \ln\left(-\frac{bx e^{\frac{a}{2}}}{2}\right) + \expIntegral_1\left(-\frac{bx}{2}\right) \right)}{2}$

input

```
int(exp(b*x+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*exp(b*x+a)^(1/2)*exp(1/2*a-1/2*b*x*exp(1/2*a))*b*(2/x/b*exp(-1/2*a)+1
-ln(x)+ln(2)-ln(-b*exp(1/2*a))-1/b/x*exp(-1/2*a)*(2+b*x*exp(1/2*a))+2/b/x*
exp(-1/2*a+1/2*b*x*exp(1/2*a))+ln(-1/2*b*x*exp(1/2*a))+Ei(1,-1/2*b*x*exp(1
/2*a)))
```

### Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \frac{bx \operatorname{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)} - 2 e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)}}{2x}$$

input

```
integrate(exp(b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

output  $1/2*(b*x*Ei(1/2*b*x)*e^{(1/2*a)} - 2*e^{(1/2*b*x + 1/2*a)})/x$

### Sympy [F]

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \int \frac{\sqrt{e^a e^{bx}}}{x^2} dx$$

input `integrate(exp(b*x+a)**(1/2)/x**2,x)`

output `Integral(sqrt(exp(a)*exp(b*x))/x**2, x)`

### Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \frac{1}{2} b e^{(\frac{1}{2} a)} \Gamma\left(-1, -\frac{1}{2} b x\right)$$

input `integrate(exp(b*x+a)^(1/2)/x^2,x, algorithm="maxima")`

output `1/2*b*e^(1/2*a)*gamma(-1, -1/2*b*x)`

### Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \frac{bx Ei(\frac{1}{2} bx) e^{(\frac{1}{2} a)} - 2 e^{(\frac{1}{2} bx + \frac{1}{2} a)}}{2x}$$

input `integrate(exp(b*x+a)^(1/2)/x^2,x, algorithm="giac")`

output  $1/2*(b*x*Ei(1/2*b*x)*e^{(1/2*a)} - 2*e^{(1/2*b*x + 1/2*a)})/x$

**Mupad [F(-1)]**

Timed out.

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \int \frac{\sqrt{e^{a+bx}}}{x^2} dx$$

input `int(exp(a + b*x)^(1/2)/x^2,x)`output `int(exp(a + b*x)^(1/2)/x^2, x)`**Reduce [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x^2} dx = \frac{-2e^{\frac{bx}{2} + \frac{a}{2}} + \left( \int \frac{e^{\frac{bx}{2} + \frac{a}{2}}}{x} dx \right) bx}{2x}$$

input `int(exp(b*x+a)^(1/2)/x^2,x)`output `( - 2*e**((a + b*x)/2) + int(e**((a + b*x)/2)/x,x)*b*x)/(2*x)`

### 3.136 $\int \frac{\sqrt{e^{a+bx}}}{x^3} dx$

Optimal result . . . . .	911
Mathematica [A] (verified) . . . . .	911
Rubi [A] (verified) . . . . .	912
Maple [B] (verified) . . . . .	913
Fricas [A] (verification not implemented) . . . . .	914
Sympy [F] . . . . .	914
Maxima [A] (verification not implemented) . . . . .	914
Giac [A] (verification not implemented) . . . . .	915
Mupad [F(-1)] . . . . .	915
Reduce [F] . . . . .	915

#### Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = -\frac{\sqrt{e^{a+bx}}}{2x^2} - \frac{b\sqrt{e^{a+bx}}}{4x} + \frac{1}{8}b^2 e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \text{ExpIntegralEi}\left(\frac{bx}{2}\right)$$

output `-1/2*exp(b*x+a)^(1/2)/x^2-1/4*b*exp(b*x+a)^(1/2)/x+1/8*b^2*exp(b*x+a)^(1/2)*Ei(1/2*b*x)/exp(1/2*b*x)`

#### Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = \frac{e^{-\frac{bx}{2}} \sqrt{e^{a+bx}} \left( -2e^{\frac{bx}{2}} (2 + bx) + b^2 x^2 \text{ExpIntegralEi}\left(\frac{bx}{2}\right) \right)}{8x^2}$$

input `Integrate[Sqrt[E^(a + b*x)]/x^3,x]`

output `(Sqrt[E^(a + b*x)]*(-2*E^((b*x)/2)*(2 + b*x) + b^2*x^2*ExpIntegralEi[(b*x)/2]))/(8*E^((b*x)/2)*x^2)`



**Rubi [A] (verified)**

Time = 0.54 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2608, 2608, 2613, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{e^{a+bx}}}{x^3} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{1}{4}b \int \frac{\sqrt{e^{a+bx}}}{x^2} dx - \frac{\sqrt{e^{a+bx}}}{2x^2} \\
 & \quad \downarrow \text{2608} \\
 & \frac{1}{4}b \left( \frac{1}{2}b \int \frac{\sqrt{e^{a+bx}}}{x} dx - \frac{\sqrt{e^{a+bx}}}{x} \right) - \frac{\sqrt{e^{a+bx}}}{2x^2} \\
 & \quad \downarrow \text{2613} \\
 & \frac{1}{4}b \left( \frac{1}{2}be^{\frac{1}{2}(-a-bx)}\sqrt{e^{a+bx}} \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx - \frac{\sqrt{e^{a+bx}}}{x} \right) - \frac{\sqrt{e^{a+bx}}}{2x^2} \\
 & \quad \downarrow \text{2609} \\
 & \frac{1}{4}b \left( \frac{1}{2}be^{\frac{1}{2}(-a-bx)+\frac{a}{2}}\sqrt{e^{a+bx}} \text{ExpIntegralEi} \left( \frac{bx}{2} \right) - \frac{\sqrt{e^{a+bx}}}{x} \right) - \frac{\sqrt{e^{a+bx}}}{2x^2}
 \end{aligned}$$

input `Int [Sqrt [E^(a + b*x)]/x^3,x]`

output `-1/2*Sqrt[E^(a + b*x)]/x^2 + (b*(-(Sqrt[E^(a + b*x)]/x) + (b*E^(a/2 + (-a - b*x)/2)*Sqrt[E^(a + b*x)]*ExpIntegralEi[(b*x)/2]))/4`

## Definitions of rubi rules used

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2613

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_)*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] := Simp[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)) Int[(c + d*
x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(53) = 106.

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.18

method	result
meijerg	$\frac{\sqrt{e^{bx+a}} e^{a - \frac{bx}{2}} b^2 \left( -\frac{2e^{-a}}{x^2 b^2} - \frac{2e^{-\frac{a}{2}}}{xb} - \frac{3}{4} + \frac{\ln(x)}{2} - \frac{\ln(2)}{2} + \frac{\ln(-be^{\frac{a}{2}})}{2} + \frac{e^{-a} \left( \frac{9b^2 x^2 e^a}{4} + 6bx e^{\frac{a}{2}} + 6 \right)}{3b^2 x^2} - \frac{2e^{-a + \frac{bx}{2}} \left( 3 + \frac{3bx e^{\frac{a}{2}}}{2} \right)}{3b^2 x^2} \right)}{4} \ln(-$

input

```
int(exp(b*x+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output

```
1/4*exp(b*x+a)^(1/2)*exp(a-1/2*b*x*exp(1/2*a))*b^2*(-2/x^2/b^2*exp(-a)-2/x
/b*exp(-1/2*a)-3/4+1/2*ln(x)-1/2*ln(2)+1/2*ln(-b*exp(1/2*a))+1/3/b^2/x^2*e
xp(-a)*(9/4*b^2*x^2*exp(a)+6*b*x*exp(1/2*a)+6)-2/3/b^2/x^2*exp(-a+1/2*b*x*
exp(1/2*a))*(3+3/2*b*x*exp(1/2*a))-1/2*ln(-1/2*b*x*exp(1/2*a))-1/2*Ei(1,-1
/2*b*x*exp(1/2*a)))
```

**Fricas [A] (verification not implemented)**

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = \frac{b^2 x^2 \operatorname{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)} - 2(bx + 2)e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)}}{8x^2}$$

input `integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="fricas")`output `1/8*(b^2*x^2*Ei(1/2*b*x)*e^(1/2*a) - 2*(b*x + 2)*e^(1/2*b*x + 1/2*a))/x^2`**Sympy [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = \int \frac{\sqrt{e^a e^{bx}}}{x^3} dx$$

input `integrate(exp(b*x+a)**(1/2)/x**3,x)`output `Integral(sqrt(exp(a)*exp(b*x))/x**3, x)`**Maxima [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = -\frac{1}{4} b^2 e^{\left(\frac{1}{2} a\right)} \Gamma\left(-2, -\frac{1}{2} bx\right)$$

input `integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="maxima")`output `-1/4*b^2*e^(1/2*a)*gamma(-2, -1/2*b*x)`

**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = \frac{b^2 x^2 \operatorname{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)} - 2 b x e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)} - 4 e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)}}{8 x^2}$$

input `integrate(exp(b*x+a)^(1/2)/x^3,x, algorithm="giac")`

output `1/8*(b^2*x^2*Ei(1/2*b*x)*e^(1/2*a) - 2*b*x*e^(1/2*b*x + 1/2*a) - 4*e^(1/2*b*x + 1/2*a))/x^2`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = \int \frac{\sqrt{e^{a+bx}}}{x^3} dx$$

input `int(exp(a + b*x)^(1/2)/x^3,x)`

output `int(exp(a + b*x)^(1/2)/x^3, x)`

**Reduce [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x^3} dx = \frac{-2e^{\frac{bx}{2} + \frac{a}{2}} bx - 4e^{\frac{bx}{2} + \frac{a}{2}} + \left(\int \frac{e^{\frac{bx}{2} + \frac{a}{2}}}{x} dx\right) b^2 x^2}{8x^2}$$

input `int(exp(b*x+a)^(1/2)/x^3,x)`

output `( - 2*e**((a + b*x)/2)*b*x - 4*e**((a + b*x)/2) + int(e**((a + b*x)/2)/x,x )*b**2*x**2)/(8*x**2)`

### 3.137 $\int \frac{\sqrt{e^{a+bx}}}{x^4} dx$

Optimal result . . . . .	916
Mathematica [A] (verified) . . . . .	916
Rubi [A] (verified) . . . . .	917
Maple [B] (verified) . . . . .	918
Fricas [A] (verification not implemented) . . . . .	919
Sympy [F] . . . . .	919
Maxima [A] (verification not implemented) . . . . .	919
Giac [A] (verification not implemented) . . . . .	920
Mupad [F(-1)] . . . . .	920
Reduce [F] . . . . .	920

#### Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = -\frac{\sqrt{e^{a+bx}}}{3x^3} - \frac{b\sqrt{e^{a+bx}}}{12x^2} - \frac{b^2\sqrt{e^{a+bx}}}{24x} + \frac{1}{48}b^3e^{-\frac{bx}{2}}\sqrt{e^{a+bx}} \text{ExpIntegralEi}\left(\frac{bx}{2}\right)$$

```
output -1/3*exp(b*x+a)^(1/2)/x^3-1/12*b*exp(b*x+a)^(1/2)/x^2-1/24*b^2*exp(b*x+a)^(1/2)/x+1/48*b^3*exp(b*x+a)^(1/2)*Ei(1/2*b*x)/exp(1/2*b*x)
```

#### Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \frac{e^{-\frac{bx}{2}}\sqrt{e^{a+bx}}\left(-2e^{\frac{bx}{2}}(8+2bx+b^2x^2)+b^3x^3 \text{ExpIntegralEi}\left(\frac{bx}{2}\right)\right)}{48x^3}$$

```
input Integrate[Sqrt[E^(a + b*x)]/x^4,x]
```

```
output (Sqrt[E^(a + b*x)]*(-2*E^((b*x)/2)*(8 + 2*b*x + b^2*x^2) + b^3*x^3*ExpIntegralEi[(b*x)/2]))/(48*E^((b*x)/2)*x^3)
```

**Rubi [A] (verified)**

Time = 0.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2608, 2608, 2608, 2613, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\begin{aligned}
 & \int \frac{\sqrt{e^{a+bx}}}{x^4} dx \\
 & \quad \downarrow \text{2608} \\
 & \frac{1}{6}b \int \frac{\sqrt{e^{a+bx}}}{x^3} dx - \frac{\sqrt{e^{a+bx}}}{3x^3} \\
 & \quad \downarrow \text{2608} \\
 & \frac{1}{6}b \left( \frac{1}{4}b \int \frac{\sqrt{e^{a+bx}}}{x^2} dx - \frac{\sqrt{e^{a+bx}}}{2x^2} \right) - \frac{\sqrt{e^{a+bx}}}{3x^3} \\
 & \quad \downarrow \text{2608} \\
 & \frac{1}{6}b \left( \frac{1}{4}b \left( \frac{1}{2}b \int \frac{\sqrt{e^{a+bx}}}{x} dx - \frac{\sqrt{e^{a+bx}}}{x} \right) - \frac{\sqrt{e^{a+bx}}}{2x^2} \right) - \frac{\sqrt{e^{a+bx}}}{3x^3} \\
 & \quad \downarrow \text{2613} \\
 & \frac{1}{6}b \left( \frac{1}{4}b \left( \frac{1}{2}be^{\frac{1}{2}(-a-bx)}\sqrt{e^{a+bx}} \int \frac{e^{\frac{1}{2}(a+bx)}}{x} dx - \frac{\sqrt{e^{a+bx}}}{x} \right) - \frac{\sqrt{e^{a+bx}}}{2x^2} \right) - \frac{\sqrt{e^{a+bx}}}{3x^3} \\
 & \quad \downarrow \text{2609} \\
 & \frac{1}{6}b \left( \frac{1}{4}b \left( \frac{1}{2}be^{\frac{1}{2}(-a-bx)+\frac{a}{2}}\sqrt{e^{a+bx}} \text{ExpIntegralEi} \left( \frac{bx}{2} \right) - \frac{\sqrt{e^{a+bx}}}{x} \right) - \frac{\sqrt{e^{a+bx}}}{2x^2} \right) - \frac{\sqrt{e^{a+bx}}}{3x^3}
 \end{aligned}$$

input `Int[Sqrt[E^(a + b*x)]/x^4,x]`

output `-1/3*Sqrt[E^(a + b*x)]/x^3 + (b*(-1/2*Sqrt[E^(a + b*x)]/x^2 + (b*(-(Sqrt[E^(a + b*x)]/x) + (b*E^(a/2 + (-a - b*x)/2)*Sqrt[E^(a + b*x)]*ExpIntegralEi[(b*x)/2])/2))/4)/6`

## Defintions of rubi rules used

rule 2608

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_
_), x_Symbol] := Simp[(c + d*x)^(m + 1)*((b*F^(g*(e + f*x)))^n/(d*(m + 1)))
, x] - Simp[f*g*n*(Log[F]/(d*(m + 1))) Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && In
tegerQ[2*m] && !TrueQ[$UseGamma]
```

rule 2609

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2613

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_) * ((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] := Simp[(b*F^(g*(e + f*x)))^n/F^(g*n*(e + f*x)) Int[(c + d*
x)^m*F^(g*n*(e + f*x)), x], x] /; FreeQ[{F, b, c, d, e, f, g, m, n}, x]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs.  $2(69) = 138$ .

Time = 0.06 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.05

method	result
meijerg	$\frac{\sqrt{e^{bx+a}} e^{\frac{3a}{2} - \frac{bx}{2} \frac{a}{2}} b^3 \left( \frac{8e^{-\frac{3a}{2}}}{3x^3 b^3} + \frac{2e^{-a}}{x^2 b^2} + \frac{e^{-\frac{a}{2}}}{xb} + \frac{11}{36} - \frac{\ln(x)}{6} + \frac{\ln(2)}{6} - \frac{\ln\left(-be^{\frac{a}{2}}\right)}{6} - \frac{e^{-\frac{3a}{2}} \left( \frac{11b^3 x^3 e^{\frac{3a}{2}}}{4} + 9b^2 x^2 e^a + 18bx e^{\frac{a}{2}} + 24 \right)}{9b^3 x^3} \right)}{8} + \dots$

input

```
int(exp(b*x+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output

```
-1/8*exp(b*x+a)^(1/2)*exp(3/2*a-1/2*b*x*exp(1/2*a))*b^3*(8/3/x^3/b^3*exp(-
3/2*a)+2/x^2/b^2*exp(-a)+1/x/b*exp(-1/2*a)+11/36-1/6*ln(x)+1/6*ln(2)-1/6*1
n(-b*exp(1/2*a))-1/9/b^3/x^3*exp(-3/2*a)*(11/4*b^3*x^3*exp(3/2*a)+9*b^2*x^
2*exp(a)+18*b*x*exp(1/2*a)+24)+1/3/b^3/x^3*exp(-3/2*a+1/2*b*x*exp(1/2*a))*
(b^2*x^2*exp(a)+2*b*x*exp(1/2*a)+8)+1/6*ln(-1/2*b*x*exp(1/2*a))+1/6*Ei(1,-
1/2*b*x*exp(1/2*a)))
```

**Fricas [A] (verification not implemented)**

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \frac{b^3 x^3 \operatorname{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)} - 2(b^2 x^2 + 2bx + 8)e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)}}{48 x^3}$$

input `integrate(exp(b*x+a)^(1/2)/x^4,x, algorithm="fricas")`output `1/48*(b^3*x^3*Ei(1/2*b*x))*e^(1/2*a) - 2*(b^2*x^2 + 2*b*x + 8)*e^(1/2*b*x + 1/2*a))/x^3`**Sympy [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \int \frac{\sqrt{e^a e^{bx}}}{x^4} dx$$

input `integrate(exp(b*x+a)**(1/2)/x**4,x)`output `Integral(sqrt(exp(a)*exp(b*x))/x**4, x)`**Maxima [A] (verification not implemented)**

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \frac{1}{8} b^3 e^{\left(\frac{1}{2} a\right)} \Gamma\left(-3, -\frac{1}{2} bx\right)$$

input `integrate(exp(b*x+a)^(1/2)/x^4,x, algorithm="maxima")`output `1/8*b^3*e^(1/2*a)*gamma(-3, -1/2*b*x)`



**Giac [A] (verification not implemented)**

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \frac{b^3 x^3 \operatorname{Ei}\left(\frac{1}{2} bx\right) e^{\left(\frac{1}{2} a\right)} - 2 b^2 x^2 e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)} - 4 b x e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)} - 16 e^{\left(\frac{1}{2} bx + \frac{1}{2} a\right)}}{48 x^3}$$

input `integrate(exp(b*x+a)^(1/2)/x^4,x, algorithm="giac")`

output `1/48*(b^3*x^3*Ei(1/2*b*x)*e^(1/2*a) - 2*b^2*x^2*e^(1/2*b*x + 1/2*a) - 4*b*x*e^(1/2*b*x + 1/2*a) - 16*e^(1/2*b*x + 1/2*a))/x^3`

**Mupad [F(-1)]**

Timed out.

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \int \frac{\sqrt{e^{a+bx}}}{x^4} dx$$

input `int(exp(a + b*x)^(1/2)/x^4,x)`

output `int(exp(a + b*x)^(1/2)/x^4, x)`

**Reduce [F]**

$$\int \frac{\sqrt{e^{a+bx}}}{x^4} dx = \frac{-2e^{\frac{bx}{2} + \frac{a}{2}} b^2 x^2 - 4e^{\frac{bx}{2} + \frac{a}{2}} b x - 16e^{\frac{bx}{2} + \frac{a}{2}} + \left(\int \frac{e^{\frac{bx}{2} + \frac{a}{2}}}{x} dx\right) b^3 x^3}{48 x^3}$$

input `int(exp(b*x+a)^(1/2)/x^4,x)`

output `( - 2*e**((a + b*x)/2)*b**2*x**2 - 4*e**((a + b*x)/2)*b*x - 16*e**((a + b*x)/2) + int(e**((a + b*x)/2)/x,x)*b**3*x**3)/(48*x**3)`

# CHAPTER 4

## APPENDIX

4.1	Listing of Grading functions . . . . .	921
4.2	Links to plain text integration problems used in this report for each CAS .	939

### 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

#### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*                               Small rewrite of logic in main function to make it*)
(*                               match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCountOptimal},
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count
          ]
        ,(*ELSE*)
        finalresult={"C","Result contains complex when optimal does not."}
      ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
    ]
  ]
  ,(*ELSE*)(*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order "
  ,

```

```

        finalresult={"F","Contains unresolved integral."}
    ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],

```

```

  Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 5]],
  If [AppellFunctionQ [Head [expn]],
    Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 6]],
  If [Head [expn] === RootSum,
    Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 7]],
  If [Head [expn] === Integrate || Head [expn] === Int,
    Apply [Max, Append [Map [ExpnType, Apply [List, expn]], 8]],
  9]]]]]]]]]]]

```

```

ElementaryFunctionQ [func_] :=
  MemberQ [ {
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ [func_] :=
  MemberQ [ {
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ [func_] :=
  MemberQ [ {Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```

AppellFunctionQ [func_] :=
  MemberQ [ {AppellF1}, func]

```

## Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022  add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B","result has leaf size over 500,000. Avoiding possible recursion issue";
      fi;

      leaf_count_optimal := leafcount(optimal);
      ExpnType_result    := ExpnType(result);
      ExpnType_optimal   := ExpnType(optimal);

      if debug then
          print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#     is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal

```

```

# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (" ,
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf

            end if
        else #result contains complex but optimal is not
            if debug then
                print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
        # this assumes optimal do not as well. No check is needed here.
        if debug then
            print("result do not contain complex, this assumes optimal do not as well
        fi;

```

```

        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of
                            convert(leaf_count_result,string)," $ vs. $2(",
                            convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                        convert(ExpnType_result,string)," vs. order ",
                        convert(ExpnType_optimal,string),".");
    fi;
end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function

```



```

# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+'') or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9

```

```

    end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.

```

```
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc;
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#      Port of original Maple grading function by
#      Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#      added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]
```

```

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')

```

```

    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""

```

```

else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation = "Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = ""
            else:
                grade = "B"
                grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType

#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

## SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#       issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

```

```

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arcoth','arcsech','arcsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:

```

```

    if m:
        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral',
        'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
        'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
    if m:
        print ("func ", func , " is special_function")
    else:
        print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric'] #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False

```



```

if debug:
    print ("Enter is_atom, expn=",expn)

if not hasattr(expn, 'parent'):
    return False

#thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
try:
    if expn.parent() is SR:
        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().gens()

    return False

except AttributeError as error:
    print ("Exception,AttributeError in is_atom")
    print ("caught exception" , type(error).__name__ )
    return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)

```

```

    return expnType(expn.operands()[0]) #expnType(expn.args[0])
elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
    if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
else:
    return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

if debug:
    print ("Enter grade_antiderivative for sagemath")
    print ("Enter grade_antiderivative, result=",result)
    print ("Enter grade_antiderivative, optimal=",optimal)
    print ("type(anti)=", type(result))
    print ("type(optimal)=", type(optimal))

```

```

leaf_count_result = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

expnType_result = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result - 2*leaf_count_optimal)
    else:
        grade = "C"
        grade_annotation = "Result contains higher order function than in optimal. Order "+str(expnType_result - expnType_optimal)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file