Computer Algebra
Independent Integration Tests

Summer 2024

2-Exponentials/157-2.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**124**]. This is test number [157].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	96.77 (120)	3.23 (4)
Fricas	91.13 (113)	8.87 (11)
Mathematica	90.32 (112)	9.68 (12)
Maple	76.61 (95)	23.39 (29)
Maxima	72.58 (90)	27.42 (34)
Giac	64.52 (80)	35.48 (44)
Mupad	54.03 (67)	45.97 (57)
Reduce	52.42 (65)	47.58 (59)
Sympy	50.00 (62)	50.00 (62)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

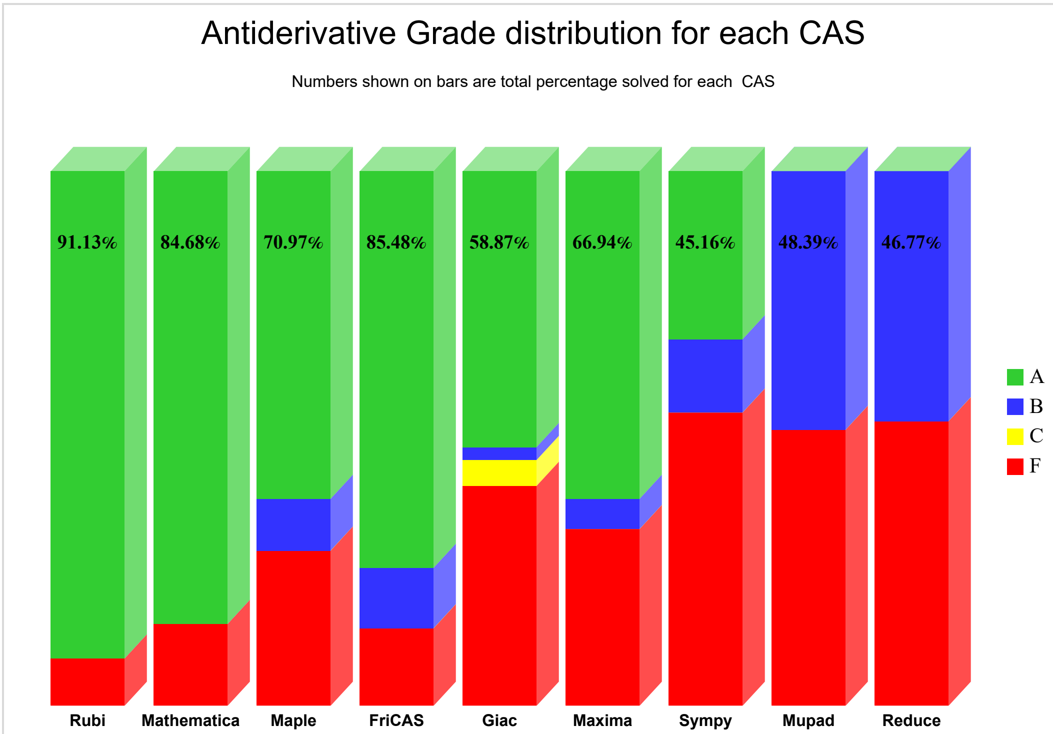
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

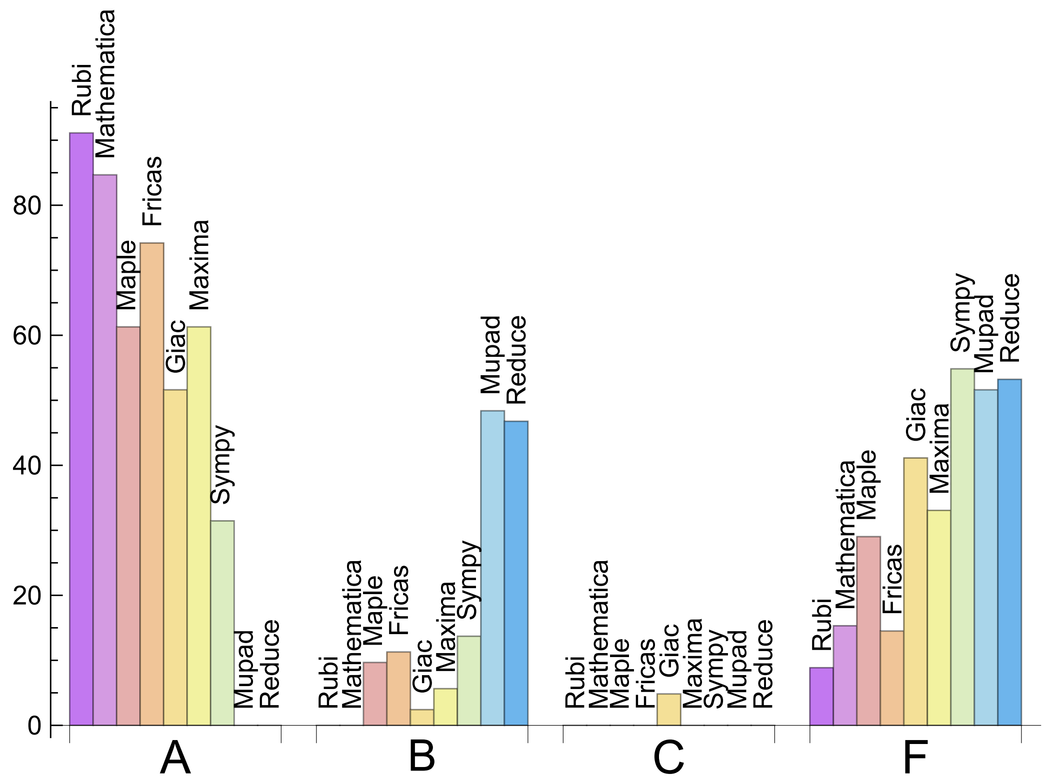
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.129	0.000	0.000	8.871
Mathematica	84.677	0.000	0.000	15.323
Fricas	74.194	11.290	0.000	14.516
Maple	61.290	9.677	0.000	29.032
Maxima	61.290	5.645	0.000	33.065
Giac	51.613	2.419	4.839	41.129
Sympy	31.452	13.710	0.000	54.839
Mupad	0.000	48.387	0.000	51.613
Reduce	0.000	46.774	0.000	53.226

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	4	100.00	0.00	0.00
Fricas	11	36.36	0.00	63.64
Mathematica	12	100.00	0.00	0.00
Maple	29	100.00	0.00	0.00
Maxima	34	100.00	0.00	0.00
Giac	44	100.00	0.00	0.00
Mupad	57	0.00	100.00	0.00
Reduce	59	100.00	0.00	0.00
Sympy	62	79.03	20.97	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.08
Maple	0.11
Giac	0.13
Reduce	0.18
Mathematica	0.35
Rubi	0.61
Sympy	2.10
Mupad	18.44

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	57.24	1.10	31.00	1.03
Reduce	79.65	1.48	44.00	1.23
Mathematica	89.77	0.99	66.00	0.98
Sympy	91.02	1.64	48.50	1.23
Maxima	99.61	1.37	59.00	1.03
Rubi	107.98	1.00	71.00	1.00
Fricas	168.28	1.55	90.00	1.20
Maple	184.48	1.48	51.00	1.00
Giac	425.14	2.87	35.50	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

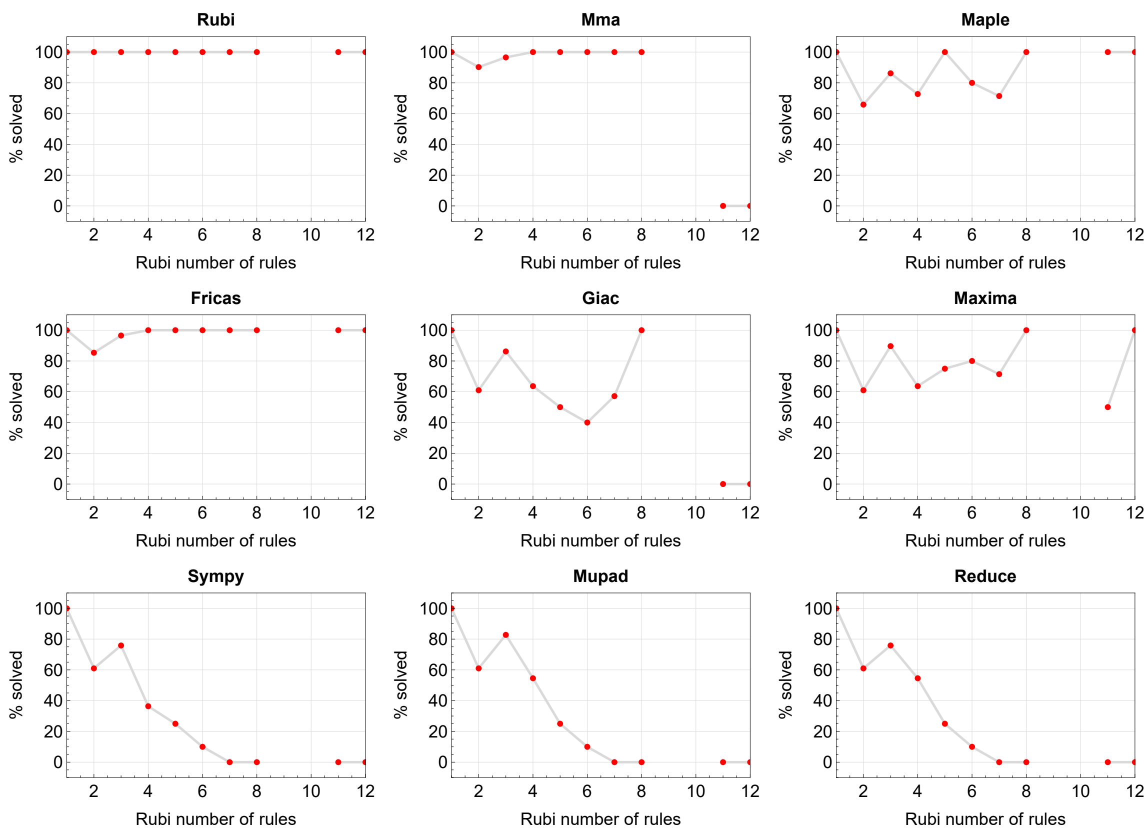


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

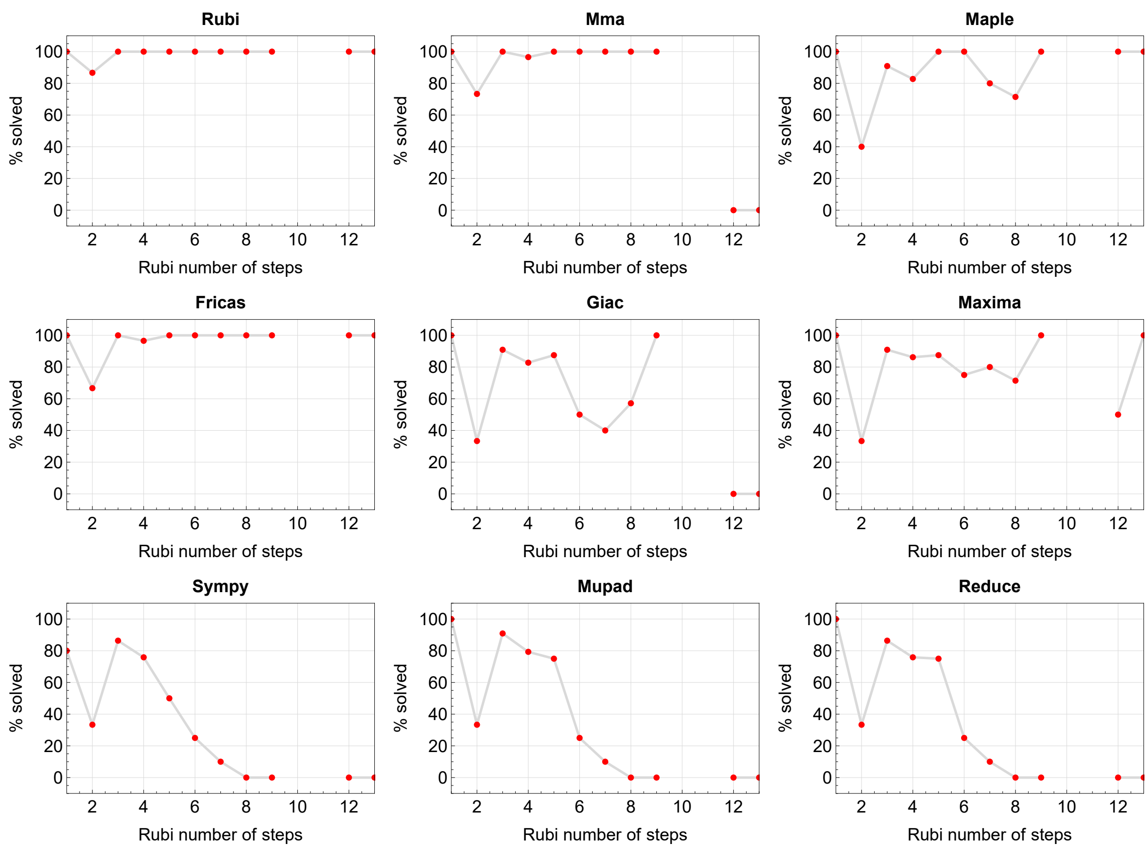


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

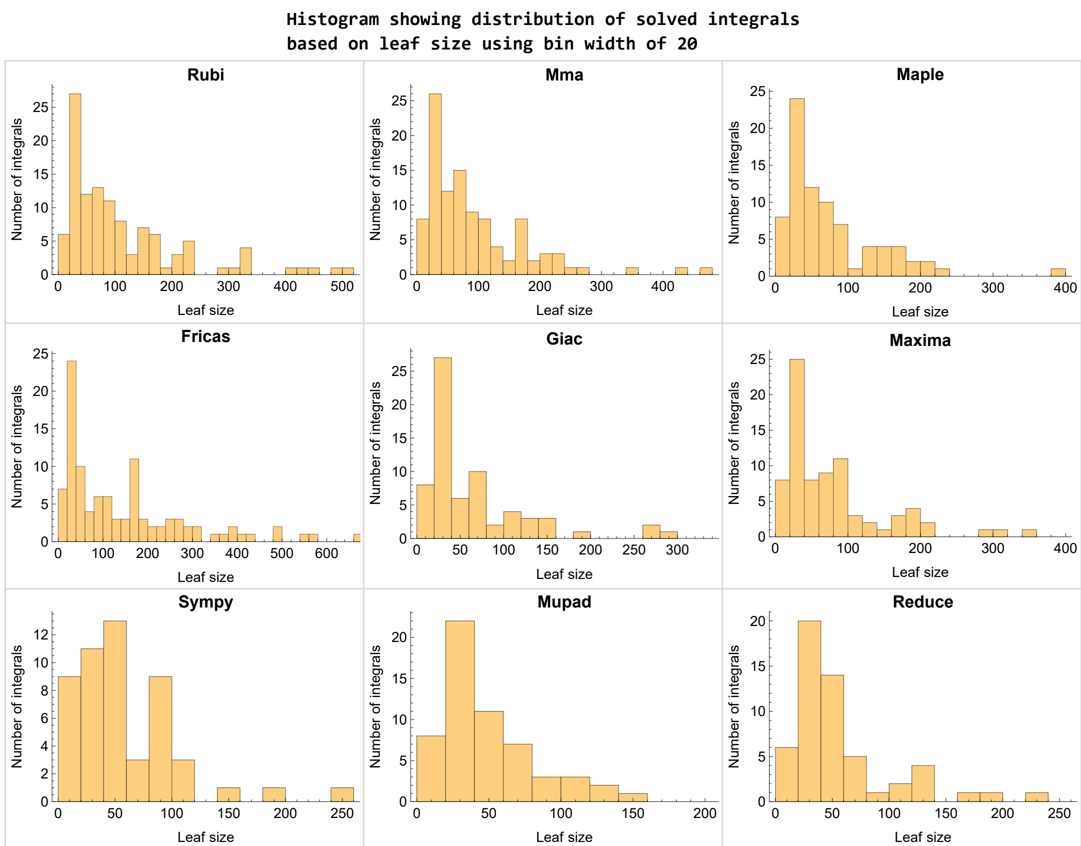


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

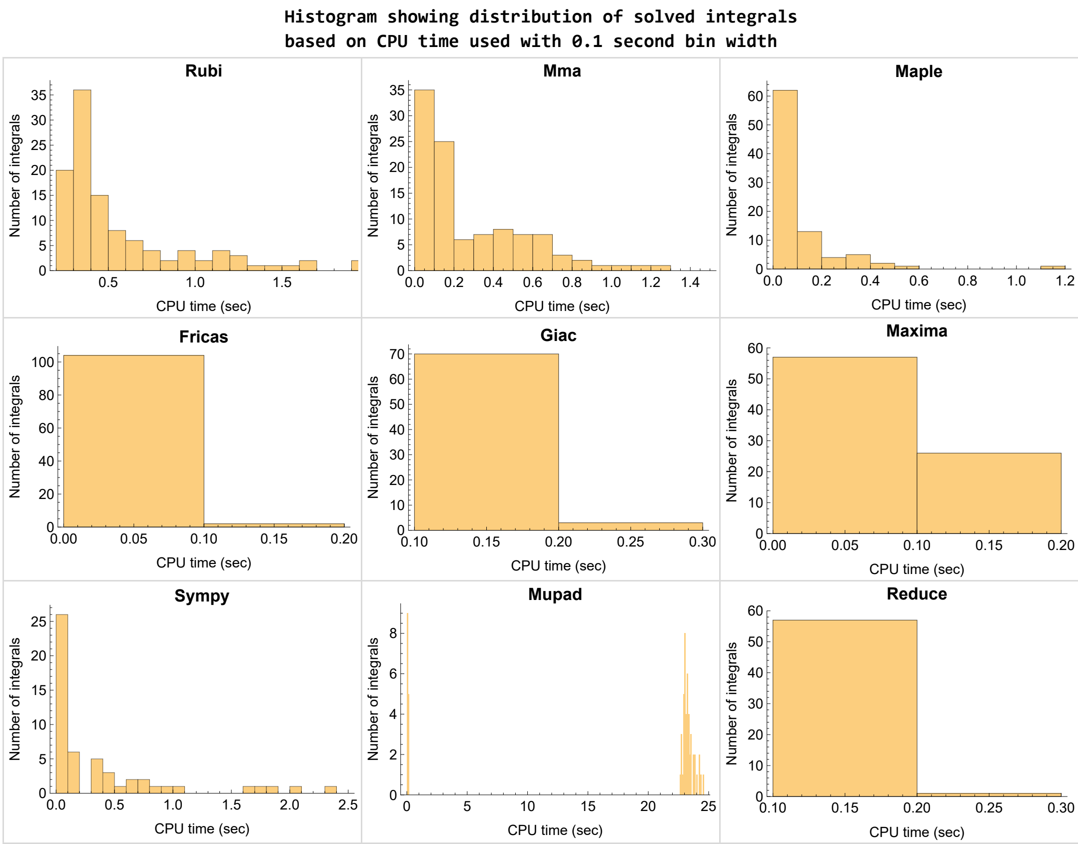


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

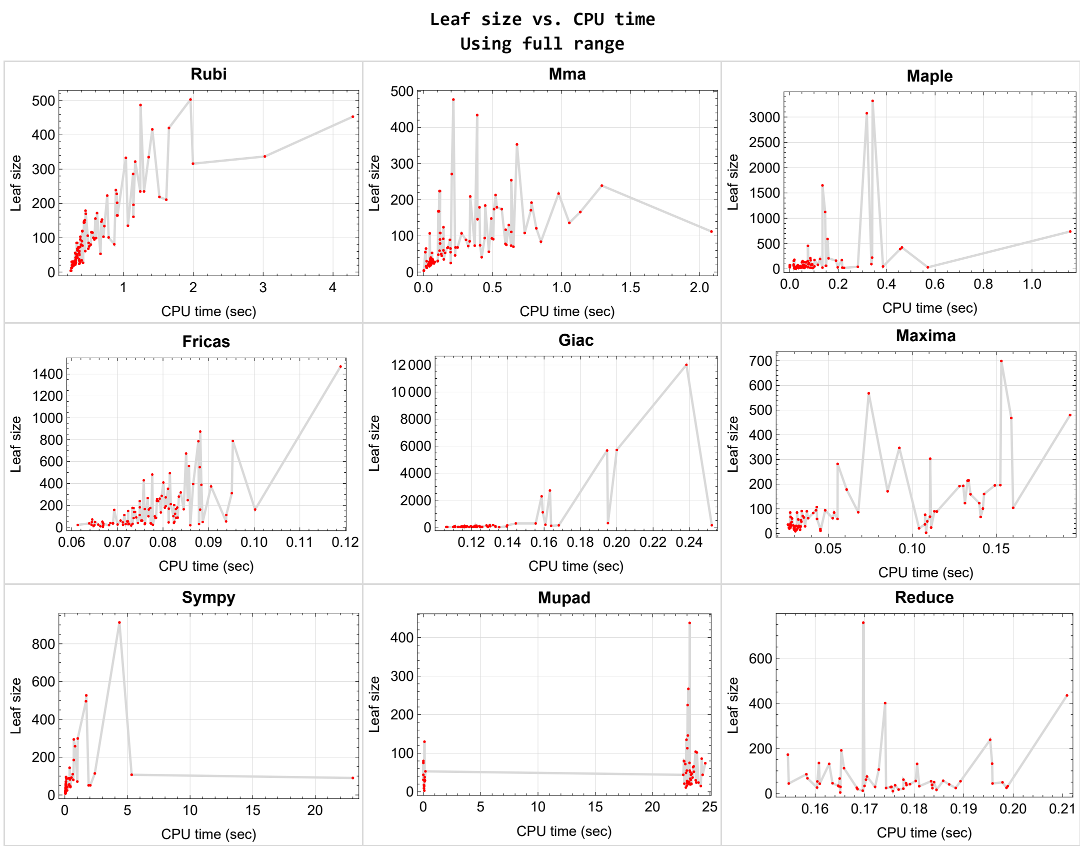


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{37, 38, 43, 44, 52, 55, 56\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{1, 29\}$

Mathematica $\{\}$

Maple $\{1, 26, 27, 29, 84, 85\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
  r"""
  Return the tree size of this expression.
  """
  if expr not in SR:
    # deal with lists, tuples, vectors
    return 1 + sum(tree_size(a) for a in expr)
  expr = SR(expr)
  x, aa = expr.operator(), expr.operands()
  if x is None:
    return 1
  else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

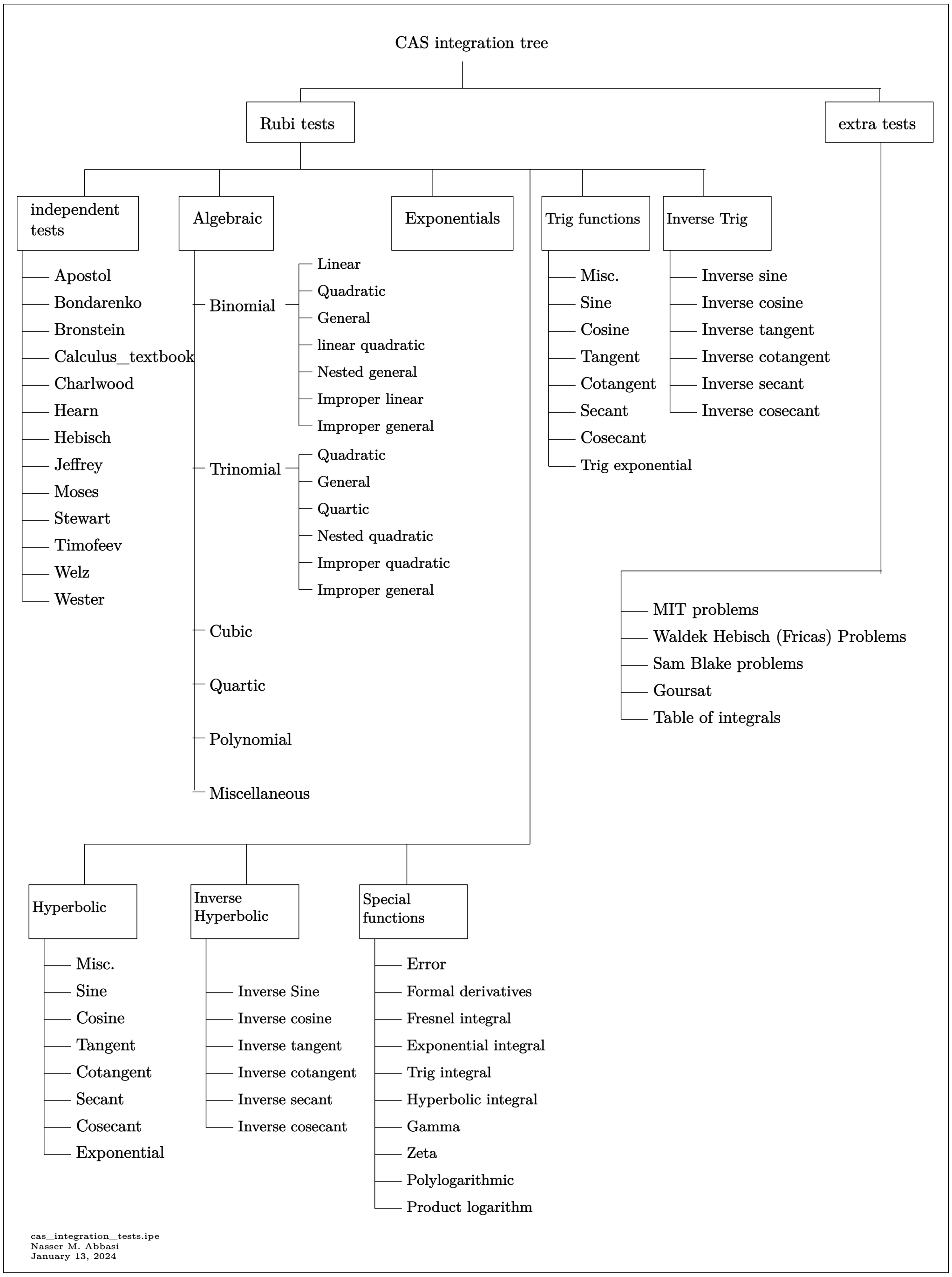
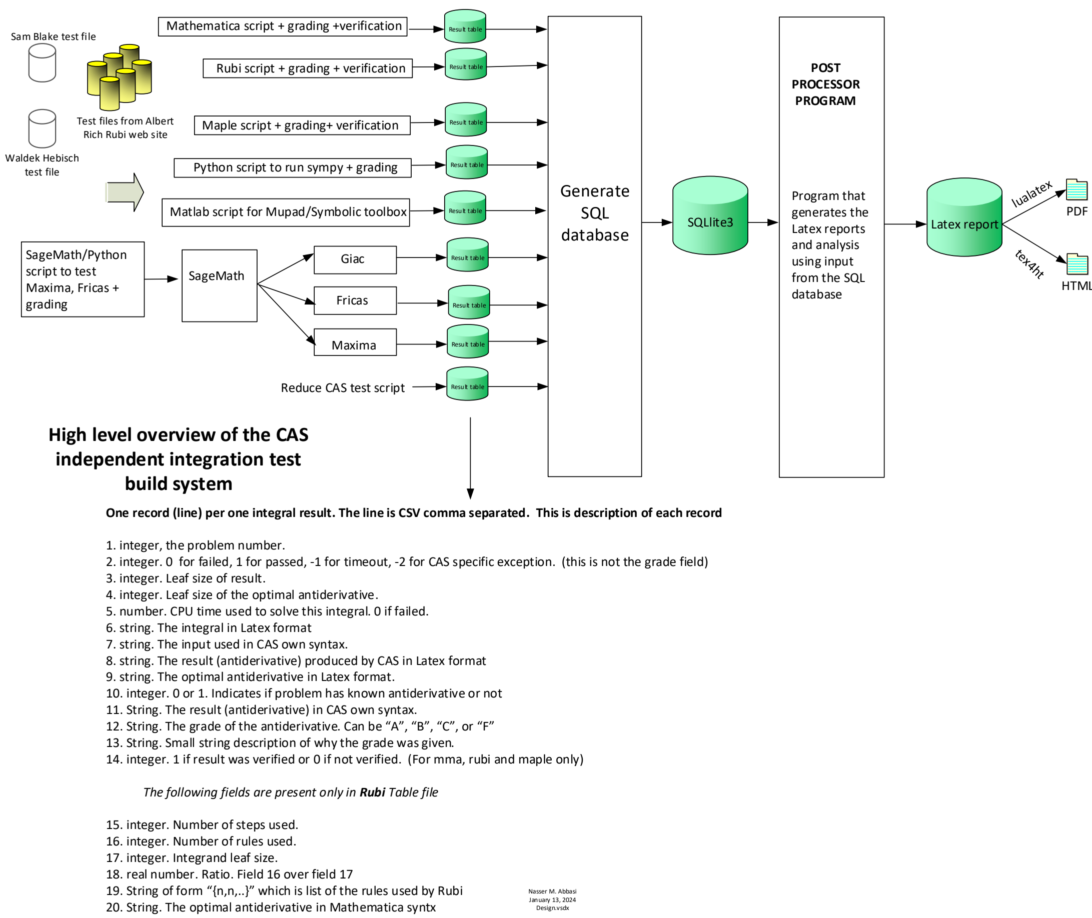


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 49, 50, 51, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

B grade { }

C grade { }

F normal fail { 45, 46, 47, 48 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 42, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

B grade { }

C grade { }

F normal fail { 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 53, 73 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 20, 21, 22, 27, 28, 29, 36, 42, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 92, 94, 95, 96, 97, 98, 100, 103, 104, 107, 108, 110, 111, 114, 115, 116, 117, 118, 119 }

B grade { 19, 26, 33, 34, 35, 39, 40, 41, 89, 91, 93, 99 }

C grade { }

F normal fail { 18, 23, 24, 25, 30, 31, 32, 45, 46, 47, 48, 49, 50, 51, 53, 54, 73, 101, 102, 105, 106, 109, 112, 113, 120, 121, 122, 123, 124 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 42, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 118, 121 }

B grade { 17, 33, 39, 40, 41, 76, 80, 95, 104, 108, 109, 117, 119, 120 }

C grade { }

F normal fail { 18, 122, 123, 124 }

F(-1) timedout fail { }

F(-2) exception fail { 45, 46, 47, 48, 49, 50, 51 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 26, 27, 28, 29, 36, 39, 40, 42, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 98, 99, 103, 107, 110, 114, 115, 116, 117, 118, 121 }

B grade { 33, 34, 76, 77, 80, 81, 95 }

C grade { }

F normal fail { 18, 23, 24, 25, 30, 31, 32, 35, 41, 45, 46, 47, 48, 49, 50, 51, 53, 54, 100, 101, 102, 104, 105, 106, 108, 109, 111, 112, 113, 119, 120, 122, 123, 124 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 22, 29, 36, 42, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 103, 107, 110, 114, 118, 121 }

B grade { 15, 16, 95 }

C grade { 19, 20, 21, 26, 27, 28 }

F normal fail { 18, 23, 24, 25, 30, 31, 32, 33, 34, 35, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 53, 54, 96, 97, 98, 100, 101, 102, 104, 105, 106, 108, 109, 111, 112, 113, 115, 116, 117, 119, 120, 122, 123, 124 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 19, 20, 21, 22, 26, 27, 28, 29, 36, 42, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 103, 107, 110, 114, 115, 118, 121 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 30, 31, 32, 33, 34, 35, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 53, 54, 73, 96, 97, 98, 100, 101, 102, 104, 105, 106, 108, 109, 111, 112, 113, 116, 117, 119, 120, 122, 123, 124 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 22, 28, 29, 36, 57, 58, 59, 60, 64, 65, 66, 74, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 99, 103, 107, 110, 114, 115, 118 }

B grade { 19, 20, 21, 26, 27, 61, 62, 63, 67, 68, 69, 70, 72, 76, 80, 94, 95 }

C grade { }

F normal fail { 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, 18, 23, 24, 25, 30, 31, 32, 33, 34, 35, 42, 47, 48, 49, 53, 54, 71, 96, 97, 98, 100, 101, 102, 104, 105, 106, 108, 109, 111, 112, 113, 116, 117, 119, 120, 121, 123, 124 }

F(-1) timedout fail { 9, 10, 17, 39, 40, 41, 45, 46, 50, 51, 52, 73, 122 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 19, 20, 21, 22, 26, 27, 28, 29, 36, 42, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 103, 107, 110, 114, 115, 118 }

C grade { }

F normal fail { 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 30, 31, 32, 33, 34, 35, 39, 40, 41, 45, 46, 47, 48, 49, 50, 51, 53, 54, 73, 82, 96, 97, 98, 100, 101, 102, 104, 105, 106, 108, 109, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 124 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	65	60	62	61	88	74	61	75
N.S.	1	0.94	0.97	0.90	0.93	0.91	1.31	1.10	0.91	1.12
time (sec)	N/A	0.343	0.196	0.150	0.038	0.071	0.376	0.120	0.178	23.216

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	29	37	41	31	37	31
N.S.	1	1.00	1.00	1.03	0.97	1.23	1.37	1.03	1.23	1.03
time (sec)	N/A	0.273	0.049	0.034	0.034	0.077	0.189	0.116	0.184	23.330

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	52	55	44	59	44	51	72	44	44
N.S.	1	1.30	1.38	1.10	1.48	1.10	1.28	1.80	1.10	1.10
time (sec)	N/A	0.328	0.196	0.042	0.043	0.071	2.031	0.117	0.155	22.671

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	73	107	74	94	100	0	108	131	80
N.S.	1	0.99	1.45	1.00	1.27	1.35	0.00	1.46	1.77	1.08
time (sec)	N/A	0.380	0.276	0.092	0.043	0.078	0.000	0.116	0.181	22.731

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	95	89	82	123	203	0	90	95	0
N.S.	1	0.81	0.75	0.69	1.04	1.72	0.00	0.76	0.81	0.00
time (sec)	N/A	0.385	0.308	0.086	0.131	0.082	0.000	0.124	0.171	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	69	68	62	101	162	0	66	19	0
N.S.	1	0.85	0.84	0.77	1.25	2.00	0.00	0.81	0.23	0.00
time (sec)	N/A	0.354	0.247	0.066	0.142	0.077	0.000	0.116	0.163	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	67	123	0	45	21	0
N.S.	1	1.00	1.00	0.89	1.46	2.67	0.00	0.98	0.46	0.00
time (sec)	N/A	0.337	0.229	0.065	0.141	0.083	0.000	0.122	0.180	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	72	72	65	104	247	0	87	60	0
N.S.	1	0.86	0.86	0.77	1.24	2.94	0.00	1.04	0.71	0.00
time (sec)	N/A	0.356	0.324	0.016	0.160	0.085	0.000	0.133	0.174	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	103	94	87	123	395	0	107	82	0
N.S.	1	0.84	0.76	0.71	1.00	3.21	0.00	0.87	0.67	0.00
time (sec)	N/A	0.385	0.443	0.016	0.140	0.087	0.000	0.164	0.184	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	170	213	176	215	180	0	185	100	0
N.S.	1	0.87	1.09	0.90	1.10	0.92	0.00	0.94	0.51	0.00
time (sec)	N/A	0.463	0.522	0.102	0.134	0.083	0.000	0.161	0.206	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	145	179	154	192	182	0	158	20	0
N.S.	1	0.91	1.12	0.96	1.20	1.14	0.00	0.99	0.12	0.00
time (sec)	N/A	0.444	0.532	0.074	0.128	0.075	0.000	0.132	0.164	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	144	179	154	193	161	0	157	20	0
N.S.	1	0.91	1.13	0.97	1.21	1.01	0.00	0.99	0.13	0.00
time (sec)	N/A	0.436	0.406	0.053	0.130	0.088	0.000	0.130	0.179	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	119	148	129	159	373	0	138	20	0
N.S.	1	0.96	1.19	1.04	1.28	3.01	0.00	1.11	0.16	0.00
time (sec)	N/A	0.414	0.491	0.067	0.135	0.091	0.000	0.140	0.164	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	120	146	130	160	170	0	137	20	0
N.S.	1	0.96	1.17	1.04	1.28	1.36	0.00	1.10	0.16	0.00
time (sec)	N/A	0.413	0.391	0.050	0.143	0.081	0.000	0.168	0.213	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	148	174	159	195	560	0	278	57	0
N.S.	1	0.92	1.08	0.99	1.21	3.48	0.00	1.73	0.35	0.00
time (sec)	N/A	0.438	0.511	0.017	0.149	0.086	0.000	0.155	0.175	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	151	174	159	196	273	0	278	57	0
N.S.	1	0.92	1.06	0.97	1.20	1.66	0.00	1.70	0.35	0.00
time (sec)	N/A	0.448	0.567	0.017	0.152	0.081	0.000	0.144	0.163	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	179	192	181	214	789	0	298	97	0
N.S.	1	0.90	0.96	0.90	1.07	3.94	0.00	1.49	0.48	0.00
time (sec)	N/A	0.458	0.782	0.019	0.133	0.095	0.000	0.195	0.180	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	68	0	0	0	0	0	20	0
N.S.	1	0.99	0.99	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.341	0.232	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	130	392	282	267	496	5716	401	225
N.S.	1	1.00	0.85	2.56	1.84	1.75	3.24	37.36	2.62	1.47
time (sec)	N/A	0.695	0.618	0.456	0.056	0.077	1.686	0.200	0.174	23.053

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	163	171	166	294	2716	238	135
N.S.	1	1.00	0.79	1.42	1.49	1.44	2.56	23.62	2.07	1.17
time (sec)	N/A	0.559	0.505	0.191	0.085	0.076	0.714	0.163	0.195	22.954

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	73	84	85	87	144	1101	112	72
N.S.	1	1.00	0.95	1.09	1.10	1.13	1.87	14.30	1.45	0.94
time (sec)	N/A	0.415	0.371	0.089	0.053	0.083	0.378	0.159	0.166	22.852

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	29	37	41	31	37	31
N.S.	1	1.00	1.00	1.03	0.97	1.23	1.37	1.03	1.23	1.03
time (sec)	N/A	0.261	0.023	0.000	0.036	0.069	0.198	0.115	0.178	0.002

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	0	0	53	0	0	39	0
N.S.	1	1.00	0.82	0.00	0.00	0.78	0.00	0.00	0.57	0.00
time (sec)	N/A	0.499	0.474	0.000	0.000	0.072	0.000	0.000	0.161	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	90	0	0	93	0
N.S.	1	1.00	0.78	0.00	0.00	0.90	0.00	0.00	0.93	0.00
time (sec)	N/A	0.571	0.594	0.000	0.000	0.074	0.000	0.000	0.175	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	111	0	0	162	0	0	186	0
N.S.	1	1.00	0.76	0.00	0.00	1.10	0.00	0.00	1.27	0.00
time (sec)	N/A	0.686	0.636	0.000	0.000	0.100	0.000	0.000	0.179	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	239	739	568	482	913	12013	758	438
N.S.	1	1.00	0.74	2.30	1.76	1.50	2.84	37.31	2.35	1.36
time (sec)	N/A	1.170	1.292	1.158	0.074	0.078	4.359	0.238	0.170	23.231

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	171	424	347	287	527	5675	435	267
N.S.	1	1.00	0.72	1.77	1.45	1.20	2.21	23.74	1.82	1.12
time (sec)	N/A	0.893	0.778	0.463	0.092	0.080	1.710	0.195	0.211	23.118

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	117	174	178	139	258	2284	191	146
N.S.	1	1.00	0.75	1.12	1.14	0.89	1.65	14.64	1.22	0.94
time (sec)	N/A	0.600	0.595	0.215	0.061	0.074	0.813	0.159	0.165	23.073

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	65	60	62	61	88	74	61	75
N.S.	1	0.94	0.97	0.90	0.93	0.91	1.31	1.10	0.91	1.12
time (sec)	N/A	0.346	0.018	0.000	0.053	0.075	0.305	0.122	0.170	0.002

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	0	0	101	0	0	73	0
N.S.	1	1.00	0.81	0.00	0.00	0.75	0.00	0.00	0.54	0.00
time (sec)	N/A	0.728	0.733	0.000	0.000	0.082	0.000	0.000	0.184	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	136	0	0	177	0	0	713	0
N.S.	1	1.00	0.67	0.00	0.00	0.88	0.00	0.00	3.53	0.00
time (sec)	N/A	0.911	1.056	0.000	0.000	0.073	0.000	0.000	0.178	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	217	0	0	318	0	0	1407	0
N.S.	1	1.00	0.76	0.00	0.00	1.11	0.00	0.00	4.92	0.00
time (sec)	N/A	1.139	0.979	0.000	0.000	0.084	0.000	0.000	0.241	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	219	166	3075	480	429	0	0	154	0
N.S.	1	1.14	0.86	16.02	2.50	2.23	0.00	0.00	0.80	0.00
time (sec)	N/A	1.516	1.136	0.318	0.194	0.076	0.000	0.000	0.200	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	161	121	1124	303	280	0	0	116	0
N.S.	1	1.11	0.83	7.75	2.09	1.93	0.00	0.00	0.80	0.00
time (sec)	N/A	1.144	0.817	0.146	0.111	0.083	0.000	0.000	0.214	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	74	456	0	149	0	0	76	0
N.S.	1	1.05	0.76	4.65	0.00	1.52	0.00	0.00	0.78	0.00
time (sec)	N/A	0.715	0.410	0.075	0.000	0.072	0.000	0.000	0.203	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	52	55	44	59	44	51	72	44	44
N.S.	1	1.30	1.38	1.10	1.48	1.10	1.28	1.80	1.10	1.10
time (sec)	N/A	0.321	0.013	0.000	0.056	0.081	1.897	0.111	0.196	0.002

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	33	22	27	43	27
N.S.	1	1.00	1.08	1.00	1.04	1.32	0.88	1.08	1.72	1.08
time (sec)	N/A	0.458	2.811	0.043	0.097	0.073	1.837	0.122	0.194	22.683

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	57	24	27	79	27
N.S.	1	1.00	1.08	1.00	1.04	2.28	0.96	1.08	3.16	1.08
time (sec)	N/A	0.451	0.622	0.060	0.134	0.072	4.092	0.159	0.196	22.647

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	453	0	3319	699	1469	0	0	537	0
N.S.	1	1.17	0.00	8.55	1.80	3.79	0.00	0.00	1.38	0.00
time (sec)	N/A	4.283	0.000	0.342	0.153	0.119	0.000	0.000	0.230	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	337	0	1650	468	875	0	0	400	0
N.S.	1	1.15	0.00	5.61	1.59	2.98	0.00	0.00	1.36	0.00
time (sec)	N/A	3.024	0.000	0.135	0.159	0.088	0.000	0.000	0.198	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	211	0	591	0	409	0	0	257	0
N.S.	1	1.10	0.00	3.09	0.00	2.14	0.00	0.00	1.35	0.00
time (sec)	N/A	1.614	0.000	0.156	0.000	0.080	0.000	0.000	0.190	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	73	107	74	94	100	0	108	131	80
N.S.	1	0.99	1.45	1.00	1.27	1.35	0.00	1.46	1.77	1.08
time (sec)	N/A	0.373	0.045	0.000	0.048	0.077	0.000	0.123	0.163	0.002

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	192	67	24	27	90	27
N.S.	1	1.00	1.08	1.00	7.68	2.68	0.96	1.08	3.60	1.08
time (sec)	N/A	0.457	1.328	0.063	0.139	0.073	4.199	0.131	0.198	23.861

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	280	108	26	27	157	27
N.S.	1	1.00	1.08	1.00	11.20	4.32	1.04	1.08	6.28	1.08
time (sec)	N/A	0.438	1.254	0.004	0.235	0.069	25.442	0.182	0.197	23.605

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	535	0	0	0	0	0	0	0	495	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.297	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	451	0	0	0	0	0	0	0	364	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.81	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.251	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	367	0	0	0	0	0	0	0	45	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.185	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	F	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	0	0	0	0	0	0	0	49	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.184	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	0	0	0	0	0	0	126	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	1.362	0.000	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	421	420	0	0	0	0	0	0	170	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00
time (sec)	N/A	1.652	0.000	0.000	0.000	0.000	0.000	0.000	0.227	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	505	503	0	0	0	0	0	0	214	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42	0.00
time (sec)	N/A	1.961	0.000	0.000	0.000	0.000	0.000	0.000	0.238	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	28	0	27	28	27
N.S.	1	1.00	1.08	1.00	1.04	1.12	0.00	1.08	1.12	1.08
time (sec)	N/A	0.323	0.501	0.029	0.131	0.082	0.000	0.379	0.182	23.037

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	0	0	0	193	0	0	325	0
N.S.	1	1.00	0.00	0.00	0.00	0.85	0.00	0.00	1.43	0.00
time (sec)	N/A	0.905	0.000	0.000	0.000	0.081	0.000	0.000	0.184	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	0	0	112	0	0	175	0
N.S.	1	1.00	0.97	0.00	0.00	0.97	0.00	0.00	1.51	0.00
time (sec)	N/A	0.543	2.086	0.000	0.000	0.072	0.000	0.000	0.190	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	28	22	27	28	27
N.S.	1	1.00	1.08	1.00	1.04	1.12	0.88	1.08	1.12	1.08
time (sec)	N/A	0.326	0.707	0.013	0.080	0.067	3.123	0.125	0.169	22.831

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	50	24	27	50	27
N.S.	1	1.00	1.08	1.00	1.04	2.00	0.96	1.08	2.00	1.08
time (sec)	N/A	0.324	0.851	0.008	0.106	0.070	37.454	0.137	0.214	22.934

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	8	9	9	8	9	10	9
N.S.	1	1.00	1.00	0.67	0.75	0.75	0.67	0.75	0.83	0.75
time (sec)	N/A	0.257	0.024	0.023	0.030	0.066	0.036	0.116	0.176	0.051

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	11	11	8	12	12	11
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.67	1.00	1.00	0.92
time (sec)	N/A	0.263	0.023	0.018	0.045	0.065	0.047	0.125	0.170	22.900

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	22	19	23	24	22
N.S.	1	1.00	1.00	0.96	0.92	0.92	0.79	0.96	1.00	0.92
time (sec)	N/A	0.369	0.039	0.035	0.030	0.071	0.073	0.110	0.188	23.077

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	18	18	14	19	19	18
N.S.	1	1.00	1.00	1.00	0.95	0.95	0.74	1.00	1.00	0.95
time (sec)	N/A	0.290	0.033	0.019	0.045	0.086	0.053	0.115	0.169	0.052

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	20	19	22	56	19	25	19
N.S.	1	1.00	0.95	1.00	0.95	1.10	2.80	0.95	1.25	0.95
time (sec)	N/A	0.265	0.053	0.058	0.030	0.078	0.350	0.119	0.199	23.289

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	30	36	114	30	41	44
N.S.	1	1.00	0.97	0.97	0.94	1.12	3.56	0.94	1.28	1.38
time (sec)	N/A	0.372	0.104	0.198	0.030	0.076	2.392	0.117	0.179	23.343

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	27	26	33	107	26	36	26
N.S.	1	1.00	0.96	1.00	0.96	1.22	3.96	0.96	1.33	0.96
time (sec)	N/A	0.299	0.070	0.049	0.030	0.077	5.333	0.110	0.176	23.066

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	16	16	12	17	16	16
N.S.	1	1.00	1.00	1.06	1.00	1.00	0.75	1.06	1.00	1.00
time (sec)	N/A	0.272	0.043	0.026	0.031	0.073	0.052	0.107	0.185	22.983

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	33	28	28	24	30	28	28
N.S.	1	1.00	1.00	1.18	1.00	1.00	0.86	1.07	1.00	1.00
time (sec)	N/A	0.381	0.081	0.047	0.027	0.072	0.178	0.115	0.175	23.007

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	17	24	23	23
N.S.	1	1.00	1.00	1.04	1.00	1.00	0.74	1.04	1.00	1.00
time (sec)	N/A	0.300	0.058	0.025	0.027	0.064	0.055	0.116	0.184	23.010

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	25	24	28	82	24	29	24
N.S.	1	1.00	1.00	1.04	1.00	1.17	3.42	1.00	1.21	1.00
time (sec)	N/A	0.275	0.076	0.084	0.032	0.076	0.412	0.111	0.175	23.067

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	35	48	36	50	71	36	45	55
N.S.	1	1.00	0.97	1.33	1.00	1.39	1.97	1.00	1.25	1.53
time (sec)	N/A	0.368	0.147	0.385	0.026	0.083	0.996	0.122	0.178	22.986

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	30	32	31	39	299	31	40	52
N.S.	1	1.00	0.97	1.03	1.00	1.26	9.65	1.00	1.29	1.68
time (sec)	N/A	0.299	0.103	0.075	0.031	0.074	1.028	0.125	0.187	23.599

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	24	29	80	24	32	24
N.S.	1	1.00	1.00	1.00	0.96	1.16	3.20	0.96	1.28	0.96
time (sec)	N/A	0.291	0.197	0.135	0.027	0.072	0.675	0.114	0.165	23.806

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	36	31	24	29	0	24	32	24
N.S.	1	1.00	0.97	0.84	0.65	0.78	0.00	0.65	0.86	0.65
time (sec)	N/A	0.368	0.065	0.570	0.027	0.088	0.000	0.120	0.199	24.002

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	42	40	53	90	42	56	74
N.S.	1	1.00	1.00	1.02	0.98	1.29	2.20	1.02	1.37	1.80
time (sec)	N/A	0.333	0.421	0.281	0.028	0.094	23.000	0.115	0.186	24.595

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	A	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	0	0	86	88	0	143	47	0
N.S.	1	1.00	0.00	0.00	1.08	1.10	0.00	1.79	0.59	0.00
time (sec)	N/A	0.534	0.000	0.000	0.068	0.083	0.000	0.252	0.171	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	19	20	21	21	20
N.S.	1	1.00	1.00	0.95	0.91	0.86	0.91	0.95	0.95	0.91
time (sec)	N/A	0.314	0.036	0.030	0.027	0.068	0.056	0.106	0.178	23.386

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	26	28	31	24	26	43	27
N.S.	1	1.00	0.96	0.96	1.04	1.15	0.89	0.96	1.59	1.00
time (sec)	N/A	0.321	0.062	0.062	0.032	0.073	0.057	0.123	0.161	0.109

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	26	18	61	35	37	20	32	29
N.S.	1	1.00	1.24	0.86	2.90	1.67	1.76	0.95	1.52	1.38
time (sec)	N/A	0.274	0.049	0.096	0.028	0.064	0.068	0.110	0.181	0.142

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	21	85	47	51	20	50	53
N.S.	1	1.00	0.76	0.62	2.50	1.38	1.50	0.59	1.47	1.56
time (sec)	N/A	0.321	0.050	0.224	0.031	0.064	0.072	0.110	0.184	0.181

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	28	26	25	24	27	26	26	24
N.S.	1	0.97	0.90	0.84	0.81	0.77	0.87	0.84	0.84	0.77
time (sec)	N/A	0.315	0.041	0.030	0.033	0.066	0.066	0.113	0.160	23.182

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	35	33	32	34	38	32	32	54	34
N.S.	1	0.95	0.89	0.86	0.92	1.03	0.86	0.86	1.46	0.92
time (sec)	N/A	0.318	0.068	0.059	0.028	0.067	0.062	0.133	0.183	23.279

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	30	20	67	39	41	24	34	31
N.S.	1	1.00	1.30	0.87	2.91	1.70	1.78	1.04	1.48	1.35
time (sec)	N/A	0.280	0.055	0.090	0.028	0.068	0.058	0.126	0.165	0.119

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	30	25	91	51	54	24	54	55
N.S.	1	1.11	0.79	0.66	2.39	1.34	1.42	0.63	1.42	1.45
time (sec)	N/A	0.316	0.057	0.217	0.034	0.067	0.068	0.123	0.189	23.237

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	44	31	27	32	25	46	32	19	26
N.S.	1	1.05	0.74	0.64	0.76	0.60	1.10	0.76	0.45	0.62
time (sec)	N/A	0.329	0.066	0.020	0.027	0.070	0.441	0.115	0.182	23.174

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	22	16	16	15	21	15	17	24	15
N.S.	1	1.38	1.00	1.00	0.94	1.31	0.94	1.06	1.50	0.94
time (sec)	N/A	0.288	0.019	0.036	0.033	0.065	0.039	0.107	0.174	0.073

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	32	35	36	32	30	34	39	29	38	30
N.S.	1	1.09	1.12	1.00	0.94	1.06	1.22	0.91	1.19	0.94
time (sec)	N/A	0.315	0.050	0.056	0.027	0.069	0.076	0.129	0.184	23.755

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	53	49	47	48	71	43	53	44
N.S.	1	1.00	1.02	0.94	0.90	0.92	1.37	0.83	1.02	0.85
time (sec)	N/A	0.334	0.062	0.102	0.028	0.089	0.086	0.117	0.160	24.362

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	40	42	32	39	34	39	45	38
N.S.	1	1.08	1.00	1.05	0.80	0.98	0.85	0.98	1.12	0.95
time (sec)	N/A	0.343	0.053	0.027	0.029	0.082	0.066	0.121	0.164	0.115

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	59	57	84	61	60	106	86
N.S.	1	1.00	0.97	0.97	0.93	1.38	1.00	0.98	1.74	1.41
time (sec)	N/A	0.371	0.116	0.042	0.032	0.074	0.078	0.117	0.173	24.278

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	80	79	73	85	140	95	77	172	104
N.S.	1	0.96	0.95	0.88	1.02	1.69	1.14	0.93	2.07	1.25
time (sec)	N/A	0.396	0.120	0.054	0.027	0.074	0.097	0.122	0.154	23.749

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	55	50	91	37	179	51	45	55	64
N.S.	1	1.10	1.00	1.82	0.74	3.58	1.02	0.90	1.10	1.28
time (sec)	N/A	0.368	0.129	0.093	0.108	0.080	0.340	0.115	0.180	23.531

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	47	32	32	42	35	33	37
N.S.	1	1.00	1.00	1.38	0.94	0.94	1.24	1.03	0.97	1.09
time (sec)	N/A	0.405	0.056	0.050	0.028	0.073	0.441	0.129	0.170	23.484

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	70	67	171	68	166	110	69	67	66
N.S.	1	0.80	0.76	1.94	0.77	1.89	1.25	0.78	0.76	0.75
time (sec)	N/A	0.356	0.173	0.086	0.111	0.083	0.626	0.130	0.158	23.583

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	50	73	76	65	53	92	58	49	47
N.S.	1	0.82	1.20	1.25	1.07	0.87	1.51	0.95	0.80	0.77
time (sec)	N/A	0.371	0.635	0.059	0.035	0.072	0.528	0.115	0.198	23.419

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	90	85	212	89	211	185	106	85	102
N.S.	1	0.71	0.67	1.67	0.70	1.66	1.46	0.83	0.67	0.80
time (sec)	N/A	0.407	0.334	0.085	0.115	0.082	0.709	0.122	0.158	23.859

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	3	15	3	4	3
N.S.	1	1.00	1.00	1.00	0.75	0.75	3.75	0.75	1.00	0.75
time (sec)	N/A	0.247	0.002	0.025	0.108	0.067	0.044	0.135	0.165	0.084

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	15	15	15	16	17	15
N.S.	1	1.00	1.00	1.00	3.75	3.75	3.75	4.00	4.25	3.75
time (sec)	N/A	0.253	0.003	0.023	0.030	0.067	0.046	0.115	0.177	24.206

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	45	34	31	31	0	0	18	0
N.S.	1	1.00	1.67	1.26	1.15	1.15	0.00	0.00	0.67	0.00
time (sec)	N/A	0.423	0.132	0.023	0.033	0.067	0.000	0.000	0.161	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	53	60	51	48	48	0	0	20	0
N.S.	1	1.32	1.50	1.28	1.20	1.20	0.00	0.00	0.50	0.00
time (sec)	N/A	0.673	0.145	0.029	0.034	0.065	0.000	0.000	0.183	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	81	89	74	71	71	0	0	20	0
N.S.	1	1.17	1.29	1.07	1.03	1.03	0.00	0.00	0.29	0.00
time (sec)	N/A	0.870	0.192	0.033	0.035	0.064	0.000	0.000	0.179	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	53	21	86	24	21	29	21
N.S.	1	1.00	1.00	1.77	0.70	2.87	0.80	0.70	0.97	0.70
time (sec)	N/A	0.289	0.054	0.055	0.104	0.078	0.081	0.111	0.165	23.377

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	99	108	134	0	112	0	0	18	0
N.S.	1	0.90	0.98	1.22	0.00	1.02	0.00	0.00	0.16	0.00
time (sec)	N/A	0.609	0.122	0.057	0.000	0.075	0.000	0.000	0.170	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	165	168	0	0	176	0	0	20	0
N.S.	1	0.90	0.91	0.00	0.00	0.96	0.00	0.00	0.11	0.00
time (sec)	N/A	0.910	0.112	0.000	0.000	0.077	0.000	0.000	0.155	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	235	224	0	0	239	0	0	20	0
N.S.	1	0.88	0.84	0.00	0.00	0.89	0.00	0.00	0.07	0.00
time (sec)	N/A	1.296	0.116	0.000	0.000	0.075	0.000	0.000	0.179	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	56	53	82	49	164	53	49	75	49
N.S.	1	0.95	0.90	1.39	0.83	2.78	0.90	0.83	1.27	0.83
time (sec)	N/A	0.310	0.156	0.116	0.109	0.085	0.110	0.117	0.170	22.912

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	271	195	0	311	0	0	31	0
N.S.	1	1.00	1.58	1.13	0.00	1.81	0.00	0.00	0.18	0.00
time (sec)	N/A	0.623	0.205	0.125	0.000	0.095	0.000	0.000	0.157	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	477	0	0	388	0	0	33	0
N.S.	1	1.00	1.43	0.00	0.00	1.17	0.00	0.00	0.10	0.00
time (sec)	N/A	1.036	0.217	0.000	0.000	0.088	0.000	0.000	0.201	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	501	487	434	0	0	549	0	0	33	0
N.S.	1	0.97	0.87	0.00	0.00	1.10	0.00	0.00	0.07	0.00
time (sec)	N/A	1.246	0.389	0.000	0.000	0.088	0.000	0.000	0.164	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	68	94	76	258	85	61	135	79
N.S.	1	1.01	0.81	1.12	0.90	3.07	1.01	0.73	1.61	0.94
time (sec)	N/A	0.331	0.178	0.337	0.108	0.079	0.134	0.139	0.161	22.759

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	184	223	0	494	0	0	44	0
N.S.	1	1.00	0.83	1.00	0.00	2.22	0.00	0.00	0.20	0.00
time (sec)	N/A	0.769	0.447	0.340	0.000	0.082	0.000	0.000	0.191	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	420	416	353	0	0	786	0	0	46	0
N.S.	1	0.99	0.84	0.00	0.00	1.87	0.00	0.00	0.11	0.00
time (sec)	N/A	1.414	0.677	0.000	0.000	0.088	0.000	0.000	0.154	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	22	24	86	26	21	29	21
N.S.	1	1.00	1.00	0.73	0.80	2.87	0.87	0.70	0.97	0.70
time (sec)	N/A	0.275	0.050	0.043	0.111	0.078	0.085	0.126	0.172	22.769

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	99	108	134	0	112	0	0	18	0
N.S.	1	0.90	0.98	1.22	0.00	1.02	0.00	0.00	0.16	0.00
time (sec)	N/A	0.577	0.120	0.059	0.000	0.094	0.000	0.000	0.178	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	184	165	168	0	0	176	0	0	20	0
N.S.	1	0.90	0.91	0.00	0.00	0.96	0.00	0.00	0.11	0.00
time (sec)	N/A	0.916	0.107	0.000	0.000	0.074	0.000	0.000	0.160	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	235	224	0	0	239	0	0	20	0
N.S.	1	0.88	0.84	0.00	0.00	0.89	0.00	0.00	0.07	0.00
time (sec)	N/A	1.242	0.120	0.000	0.000	0.079	0.000	0.000	0.177	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	23	21	23	21	22	20	25	20
N.S.	1	1.00	1.05	0.95	1.05	0.95	1.00	0.91	1.14	0.91
time (sec)	N/A	0.279	0.038	0.044	0.027	0.061	0.054	0.114	0.168	23.185

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	48	56	54	61	54	0	66	51
N.S.	1	0.97	0.76	0.89	0.86	0.97	0.86	0.00	1.05	0.81
time (sec)	N/A	0.455	0.107	0.058	0.034	0.075	0.090	0.000	0.165	23.290

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	101	90	91	83	159	0	0	156	0
N.S.	1	1.03	0.92	0.93	0.85	1.62	0.00	0.00	1.59	0.00
time (sec)	N/A	0.789	0.120	0.076	0.041	0.069	0.000	0.000	0.174	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	135	124	119	107	241	0	0	250	0
N.S.	1	1.05	0.97	0.93	0.84	1.88	0.00	0.00	1.95	0.00
time (sec)	N/A	1.065	0.145	0.079	0.043	0.079	0.000	0.000	0.159	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	85	70	62	90	261	87	66	132	113
N.S.	1	0.98	0.80	0.71	1.03	3.00	1.00	0.76	1.52	1.30
time (sec)	N/A	0.340	0.651	0.149	0.113	0.079	0.130	0.118	0.196	23.028

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	209	209	0	352	0	0	319	0
N.S.	1	1.00	1.07	1.07	0.00	1.80	0.00	0.00	1.63	0.00
time (sec)	N/A	1.145	0.339	0.160	0.000	0.081	0.000	0.000	0.176	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	254	0	0	674	0	0	517	0
N.S.	1	1.00	0.80	0.00	0.00	2.13	0.00	0.00	1.64	0.00
time (sec)	N/A	1.996	0.636	0.000	0.000	0.085	0.000	0.000	0.168	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	93	0	90	135	0	130	32	130
N.S.	1	1.00	0.98	0.00	0.95	1.42	0.00	1.37	0.34	1.37
time (sec)	N/A	0.585	0.495	0.000	0.037	0.080	0.000	0.128	0.176	0.098

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	92	0	0	0	0	0	31	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.29	0.00
time (sec)	N/A	0.490	0.143	0.000	0.000	0.000	0.000	0.000	0.155	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	0	0	0	58	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.77	0.00
time (sec)	N/A	0.512	0.600	0.000	0.000	0.000	0.000	0.000	0.164	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	84	0	0	0	0	0	70	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.82	0.00
time (sec)	N/A	0.503	0.851	0.000	0.000	0.000	0.000	0.000	0.176	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [39] had the largest ratio of [.4799999999999999982]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.94	17	0.235
2	A	1	1	1.00	15	0.067
3	A	6	5	1.30	17	0.294
4	A	5	4	0.99	17	0.235
5	A	7	6	0.81	19	0.316
6	A	6	5	0.85	19	0.263
7	A	5	4	1.00	19	0.211
8	A	6	5	0.86	19	0.263
9	A	7	6	0.84	19	0.316
10	A	9	8	0.87	19	0.421
11	A	8	7	0.91	19	0.368
12	A	8	7	0.91	19	0.368
13	A	7	6	0.96	19	0.316
14	A	7	6	0.96	19	0.316
15	A	8	7	0.92	19	0.368
16	A	8	7	0.92	19	0.368
17	A	9	8	0.90	19	0.421
18	A	4	3	0.99	17	0.176
19	A	2	2	1.00	23	0.087
20	A	2	2	1.00	23	0.087
21	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	1	1	1.00	15	0.067
23	A	2	2	1.00	23	0.087
24	A	2	2	1.00	23	0.087
25	A	2	2	1.00	23	0.087
26	A	2	2	1.00	25	0.080
27	A	2	2	1.00	25	0.080
28	A	2	2	1.00	23	0.087
29	A	5	4	0.94	17	0.235
30	A	2	2	1.00	25	0.080
31	A	2	2	1.00	25	0.080
32	A	2	2	1.00	25	0.080
33	A	7	6	1.14	25	0.240
34	A	6	5	1.11	25	0.200
35	A	5	4	1.05	23	0.174
36	A	6	5	1.30	17	0.294
37	N/A	2	0	1.00	25	0.000
38	N/A	2	0	1.00	25	0.000
39	A	13	12	1.17	25	0.480
40	A	12	11	1.15	25	0.440
41	A	12	11	1.10	23	0.478
42	A	5	4	0.99	17	0.235
43	N/A	2	0	1.00	25	0.000
44	N/A	2	0	1.00	25	0.000
45	F	0	0	N/A	0.000	N/A
46	F	0	0	N/A	0.000	N/A
47	F	0	0	N/A	0.000	N/A
48	F	0	0	N/A	0.000	N/A
49	A	2	2	1.00	25	0.080
50	A	2	2	1.00	25	0.080
51	A	2	2	1.00	25	0.080
52	N/A	1	0	1.00	25	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	2	2	1.00	25	0.080
54	A	2	2	1.00	23	0.087
55	N/A	1	0	1.00	25	0.000
56	N/A	1	0	1.00	25	0.000
57	A	3	2	1.00	13	0.154
58	A	3	2	1.00	13	0.154
59	A	4	3	1.00	19	0.158
60	A	3	2	1.00	21	0.095
61	A	3	2	1.00	13	0.154
62	A	4	3	1.00	19	0.158
63	A	3	2	1.00	21	0.095
64	A	3	2	1.00	13	0.154
65	A	4	3	1.00	19	0.158
66	A	3	2	1.00	21	0.095
67	A	3	2	1.00	13	0.154
68	A	4	3	1.00	19	0.158
69	A	3	2	1.00	21	0.095
70	A	3	2	1.00	17	0.118
71	A	4	3	1.00	17	0.176
72	A	3	2	1.00	29	0.069
73	A	4	3	1.00	44	0.068
74	A	4	3	1.00	15	0.200
75	A	4	3	1.00	15	0.200
76	A	3	2	1.00	15	0.133
77	A	4	3	1.00	15	0.200
78	A	4	3	0.97	17	0.176
79	A	4	3	0.95	17	0.176
80	A	3	2	1.00	17	0.118
81	A	4	3	1.11	17	0.176
82	A	4	3	1.05	19	0.158
83	A	4	3	1.38	16	0.188
84	A	4	3	1.09	18	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
85	A	4	3	1.00	18	0.167
86	A	4	3	1.08	18	0.167
87	A	4	3	1.00	18	0.167
88	A	4	3	0.96	18	0.167
89	A	3	2	1.10	22	0.091
90	A	4	3	1.00	23	0.130
91	A	4	3	0.80	23	0.130
92	A	4	3	0.82	23	0.130
93	A	4	3	0.71	23	0.130
94	A	3	2	1.00	13	0.154
95	A	3	2	1.00	15	0.133
96	A	4	3	1.00	16	0.188
97	A	6	5	1.32	18	0.278
98	A	7	6	1.17	18	0.333
99	A	3	2	1.00	15	0.133
100	A	6	5	0.90	16	0.312
101	A	7	6	0.90	18	0.333
102	A	8	7	0.88	18	0.389
103	A	4	3	0.95	15	0.200
104	A	2	2	1.00	16	0.125
105	A	4	4	1.00	18	0.222
106	A	4	4	0.97	18	0.222
107	A	5	4	1.01	15	0.267
108	A	2	2	1.00	16	0.125
109	A	4	4	0.99	18	0.222
110	A	3	2	1.00	15	0.133
111	A	6	5	0.90	17	0.294
112	A	7	6	0.90	19	0.316
113	A	8	7	0.88	19	0.368
114	A	3	2	1.00	15	0.133
115	A	7	6	0.97	17	0.353
116	A	7	6	1.03	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
117	A	8	7	1.05	19	0.368
118	A	5	4	0.98	15	0.267
119	A	3	3	1.00	17	0.176
120	A	3	3	1.00	19	0.158
121	A	3	3	1.00	25	0.120
122	A	2	2	1.00	25	0.080
123	A	2	2	1.00	34	0.059
124	A	2	2	1.00	34	0.059

CHAPTER 3

Listing of integrals

3.1 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = d^2x+\frac{2de\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}+\frac{e^2\left(F^{c(a+bx)}\right)^{2n}}{2bcn\log(F)}$$

output
```
d^2*x+2*d*e*(F^(c*(b*x+a)))^n/b/c/n/ln(F)+1/2*e^2*(F^(c*(b*x+a)))^(2*n)/b/
c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{e\left(F^{c(a+bx)}\right)^n\left(4d+e\left(F^{c(a+bx)}\right)^n\right)+2d^2\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2bcn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
(e*(F^(c*(a + b*x)))^n*(4*d + e*(F^(c*(a + b*x)))^n) + 2*d^2*Log[(F^(c*(a
+ b*x)))^n])/(2*b*c*n*Log[F])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2720, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(e^2\left(F^{c(a+bx)}\right)^n+d^2F^{-c(a+bx)}+2de\right) d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\log\left(\left(F^{c(a+bx)}\right)^n\right)+2de\left(F^{c(a+bx)}\right)^n+\frac{1}{2}e^2F^{2c(a+bx)}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
((e^2*F^(2*c*(a + b*x)))/2 + 2*d*e*(F^(c*(a + b*x)))^n + d^2*Log[(F^(c*(a
+ b*x)))^n])/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (warning: unable to verify)

Time = 0.15 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{2d^2x\ln(F)bcn+e^2\left(F^{c(bx+a)}\right)^{2n}+4de\left(F^{c(bx+a)}\right)^{n}}{2\ln(F)bcn}$
derivativedivides	$\frac{\frac{e^2\left(F^{c(bx+a)}\right)^{2n}}{2}+2de\left(F^{c(bx+a)}\right)^{n}+d^2\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{\ln(F)bcn}$
default	$\frac{\frac{e^2\left(F^{c(bx+a)}\right)^{2n}}{2}+2de\left(F^{c(bx+a)}\right)^{n}+d^2\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{\ln(F)bcn}$
norman	$d^2x+\frac{e^2\mathrm{e}^{2n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{2\ln(F)bcn}+\frac{2de\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{\ln(F)bcn}$
orering	$\frac{(2\ln(F)bcnx+3)\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^2}{2\ln(F)bcn}-\frac{(3\ln(F)bcnx+1)\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)e\left(F^{c(bx+a)}\right)^{n}}{\ln(F)bcn}+\frac{x\left(2e^2\left(F^{c(bx+a)}\right)^{2r}\right.}{}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*d^2*x*ln(F)*b*c*n+e^2*((F^(c*(b*x+a)))^n)^2+4*d*e*(F^(c*(b*x+a)))^n
)/ln(F)/b/c/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{2\,bcd^2nx\log\left(F\right)+4\,F^{bcnx+acn}de+F^{2\,bcnx+2\,acn}e^2}{2\,bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*c*d^2*n*x*log(F) + 4*F^(b*c*n*x + a*c*n)*d*e + F^(2*b*c*n*x + 2*a
*c*n)*e^2)/(b*c*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \begin{cases} x(d+e)^2 & \text{for } F=1\wedge b=0\wedge c=0\wedge n=0 \\ x\left(d+e(F^{ac})^n\right)^2 & \text{for } b=0 \\ x(d+e)^2 & \text{for } F=1\vee c=0\vee n=0 \\ d^2x+\frac{2de\left(F^{ac+bcx}\right)^n}{bcn\log\left(F\right)}+\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{2bcn\log\left(F\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Piecewise((x*(d + e)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*(d
 + e*(F**(a*c))**n)**2, Eq(b, 0)), (x*(d + e)**2, Eq(F, 1) | Eq(c, 0) | Eq
(n, 0)), (d**2*x + 2*d*e*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + e**2*(F**(
a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = d^2x + \frac{2F^{(bx+a)cn}de}{bcn\log(F)} + \frac{F^{2(bx+a)cn}e^2}{2bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
d^2*x + 2*F^((b*x + a)*c*n)*d*e/(b*c*n*log(F)) + 1/2*F^(2*(b*x + a)*c*n)*e
^2/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{4F^{bcnx}F^{acn}de + F^{2bcnx}F^{2acn}e^2 + 2d^2\log\left(|F|^{bcnx}|F|^{acn}\right)}{2bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
1/2*(4*F^(b*c*n*x)*F^(a*c*n)*d*e + F^(2*b*c*n*x)*F^(2*a*c*n)*e^2 + 2*d^2*l
og(abs(F)^(b*c*n*x)*abs(F)^(a*c*n)))/(b*c*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{d^2\ln\left(F^{c(a+bx)}\right)}{bc\ln(F)} + \frac{\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{2} + 2de\left(F^{ac+bcx}\right)^n}{bcn\ln(F)}$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
(d^2*log(F^(c*(a + b*x))))/(b*c*log(F)) + ((e^2*(F^(a*c + b*c*x))^(2*n))/2
 + 2*d*e*(F^(a*c + b*c*x))^n)/(b*c*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{f^{2bcnx+2acn}e^2+4f^{bcnx+acn}de+2\log(f)\,bc\,d^2nx}{2\log(f)\,bcn}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 4*f**(a*c*n + b*c*n*x)*d*e + 2*log(f)*b*c
*d**2*n*x)/(2*log(f)*b*c*n)
```

3.2 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right) dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right) dx = dx+\frac{e\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

output
```
d*x+e*(F^(c*(b*x+a)))^n/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right) dx = dx+\frac{e\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

input
```
Integrate[d + e*(F^(c*(a + b*x)))^n,x]
```

output
```
d*x + (e*(F^(c*(a + b*x)))^n)/(b*c*n*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$\frac{e\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}+dx$$

input
```
Int[d + e*(F^(c*(a + b*x)))^n,x]
```

output
```
d*x + (e*(F^(c*(a + b*x)))^n)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$dx+\frac{e\left(F^{c(bx+a)}\right)^{n}}{bcn\ln\left(F\right)}$	31
parallelrisch	$dx+\frac{e\left(F^{c(bx+a)}\right)^{n}}{bcn\ln\left(F\right)}$	31
parts	$dx+\frac{e\left(F^{c(bx+a)}\right)^{n}}{bcn\ln\left(F\right)}$	31
norman	$dx+\frac{e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}{\ln\left(F\right)bcn}$	34
derivativedivides	$\frac{e\left(F^{c(bx+a)}\right)^{n}+d\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{\ln\left(F\right)bcn}$	43
orering	$\frac{\left(\ln\left(F\right)bcnx+1\right)\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{\ln\left(F\right)bcn}-xe\left(F^{c(bx+a)}\right)^{n}$	55

input
```
int(d+e*(F^(c*(b*x+a)))^n,x,method=_RETURNVERBOSE)
```

output
```
d*x+e*(F^(c*(b*x+a)))^n/b/c/n/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)\,dx=\frac{bcdnx\log\left(F\right)+F^{bcnx+acn}e}{bcn\log\left(F\right)}$$

input
```
integrate(d+e*(F^((b*x+a)*c))^n,x, algorithm="fricas")
```

output
```
(b*c*d*n*x*log(F) + F^(b*c*n*x + a*c*n)*e)/(b*c*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx=dx+e\left(\begin{cases} x & \text{for } F=1\wedge b=0\wedge c=0\wedge n=0\\ x\left(F^{ac}\right)^n & \text{for } b=0\\ x & \text{for } F=1\vee c=0\vee n=0\\ \frac{\left(F^{ac+bcx}\right)^n}{bcn\log\left(F\right)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(d+e*(F**((b*x+a)*c))**n,x)
```

output
```
d*x + e*Piecewise((x, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*(F**(
a*c))**n, Eq(b, 0)), (x, Eq(F, 1) | Eq(c, 0) | Eq(n, 0)), ((F**(a*c + b*c*
x))**n/(b*c*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx=dx+\frac{F^{(bx+a)cn}e}{bcn\log\left(F\right)}$$

input
```
integrate(d+e*(F^((b*x+a)*c))^n,x, algorithm="maxima")
```

output
```
d*x + F^((b*x + a)*c*n)*e/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(d + e\left(F^{c(a+bx)}\right)^n\right)\,dx = dx + \frac{F^{bcnx+acn}e}{bcn\log(F)}$$

input
```
integrate(d+e*(F^((b*x+a)*c))^n,x, algorithm="giac")
```

output
```
d*x + F^(b*c*n*x + a*c*n)*e/(b*c*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(d + e\left(F^{c(a+bx)}\right)^n\right)\,dx = d\,x + \frac{e\,\left(F^{a\,c+b\,c\,x}\right)^n}{b\,c\,n\,\ln(F)}$$

input
```
int(d + e*(F^(c*(a + b*x)))^n,x)
```

output
```
d*x + (e*(F^(a*c + b*c*x))^n)/(b*c*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(d + e\left(F^{c(a+bx)}\right)^n\right)\,dx = \frac{f^{bcnx+acn}e + \log(f)\,bcdnx}{\log(f)\,bcn}$$

input
```
int(d+e*(F^((b*x+a)*c))^n,x)
```

output
```
(f**(a*c*n + b*c*n*x)*e + log(f)*b*c*d*n*x)/(log(f)*b*c*n)
```

3.3 $\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{x}{d}-\frac{\log\left(d+e\left(F^{c(a+bx)}\right)^n\right)}{bcdn\log(F)}$$

output
```
x/d-ln(d+e*(F^(c*(b*x+a)))^n)/b/c/d/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)-\log\left(bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)\right)}{bcdn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-1),x]
```

output
```
(Log[(F^(c*(a + b*x)))^n] - Log[b*c*d*(d + e*(F^(c*(a + b*x)))^n)*n*Log[F]
])/(b*c*d*n*Log[F])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2720, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e\left(F^{c(a+bx)}\right)^n+d}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{e\left(F^{c(a+bx)}\right)^n+d}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{\int F^{-c(a+bx)}d\left(F^{c(a+bx)}\right)^n}{d}-\frac{e\int \frac{1}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{d}}{bcn\log(F)}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d}-\frac{e\int \frac{1}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{d}}{bcn\log(F)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d}-\frac{\log\left(e\left(F^{c(a+bx)}\right)^n+d\right)}{d}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-1),x]
```

output
```
(Log[(F^(c*(a + b*x)))^n]/d - Log[d + e*(F^(c*(a + b*x)))^n]/d)/(b*c*n*Log
[F])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{x}{d}-\frac{\ln\left(d+e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}\right)}{\ln(F)bcdn}$	44
parallelrisch	$\frac{\ln(F)bcnx-\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{\ln(F)bcdn}$	44
derivativedivides	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d}}{\ln(F)bcn}$	53
default	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d}}{\ln(F)bcn}$	53
risch	$\frac{\ln\left(F^{c(bx+a)}\right)}{\ln(F)bcd}-\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}+\frac{d}{e}\right)}{\ln(F)bcdn}$	62

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
x/d-1/ln(F)/b/c/d/n*ln(d+e*exp(n*ln(exp(c*(b*x+a)*ln(F)))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{bcnx\log(F)-\log\left(F^{bcnx+acn}e+d\right)}{bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
(b*c*n*x*log(F) - log(F^(b*c*n*x + a*c*n)*e + d))/(b*c*d*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{2\operatorname{atan}\left(\frac{2\left(\frac{d}{2e}+\left(F^{c(a+bx)}\right)^n\right)}{\sqrt{-\frac{d^2}{e^2}}}\right)}{bcen\sqrt{-\frac{d^2}{e^2}}\log(F)}$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n),x)
```

output
```
2*atan(2*(d/(2*e) + (F**(c*(a + b*x)))**n)/sqrt(-d**2/e**2))/(b*c*e*n*sqrt
(-d**2/e**2)*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{bcnx+acn}{bcdn}-\frac{\log\left(F^{bcnx+acn}e+d\right)}{bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
(b*c*n*x + a*c*n)/(b*c*d*n) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d*n*log(
F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{\log\left(|F|^{bcnx}|F|^{acn}\right)}{bcdn\log(F)}-\frac{\log\left(\left|F^{bcnx}F^{acn}e+d\right|\right)}{bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
log(abs(F)^(b*c*n*x)*abs(F)^(a*c*n))/(b*c*d*n*log(F)) - log(abs(F^(b*c*n*x
)*F^(a*c*n)*e + d))/(b*c*d*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 22.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = -\frac{\ln\left(d+e\left(F^{a\,c+b\,c\,x}\right)^n\right)-b\,c\,n\,x\,\ln\left(F\right)}{b\,c\,d\,n\,\ln\left(F\right)}$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
-(log(d + e*(F^(a*c + b*c*x))^n) - b*c*n*x*log(F))/(b*c*d*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{-\log\left(f^{bcnx+acn}e+d\right)+\log(f)\,bcnx}{\log\left(f\right)bcdn}$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n),x)
```

output
```
( - log(f**(a*c*n + b*c*n*x)*e + d) + log(f)*b*c*n*x)/(log(f)*b*c*d*n)
```

3.4 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{x}{d^2}+\frac{1}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)}-\frac{\log\left(d+e\left(F^{c(a+bx)}\right)^n\right)}{bcd^2n\log(F)}$$

output
```
x/d^2+1/b/c/d/(d+e*(F^(c*(b*x+a)))^n)/n/ln(F)-ln(d+e*(F^(c*(b*x+a)))^n)/b/
c/d^2/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{\frac{1}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n}+\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{bcd^2n}-\frac{\log\left(bcd^3n\log(F)+bcd^2e\left(F^{c(a+bx)}\right)^n n\log(F)\right)}{bcd^2n}}{\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-2),x]
```

output
```
(1/(b*c*d*(d + e*(F^(c*(a + b*x)))^n)*n) + Log[(F^(c*(a + b*x)))^n]/(b*c*d
^2*n) - Log[b*c*d^3*n*Log[F] + b*c*d^2*e*(F^(c*(a + b*x)))^n*n*Log[F]]/(b*
c*d^2*n))/Log[F]
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{F^{-c(a+bx)}}{d^2}-\frac{e}{d^2\left(e\left(F^{c(a+bx)}\right)^n+d\right)}-\frac{e}{d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}\right)d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log\left(e\left(F^{c(a+bx)}\right)^n+d\right)}{d^2}+\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d^2}+\frac{1}{d\left(e\left(F^{c(a+bx)}\right)^n+d\right)}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-2),x]
```

output
```
(1/(d*(d + e*(F^(c*(a + b*x)))^n)) + Log[(F^(c*(a + b*x)))^n]/d^2 - Log[d
+ e*(F^(c*(a + b*x)))^n]/d^2)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}+\frac{1}{d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}}{\ln(F)bcn}$
default	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}+\frac{1}{d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}}{\ln(F)bcn}$
risch	$\frac{\ln\left(F^{c(bx+a)}\right)}{\ln(F)bc\,d^{2}}+\frac{1}{bcd\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)n\ln(F)}-\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}+\frac{d}{e}\right)}{\ln(F)bcn\,d^{2}}$
parallelrisch	$\frac{e^{2}\left(F^{c(bx+a)}\right)^{n}x\ln(F)bcn+x\ln(F)bcden-\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)\left(F^{c(bx+a)}\right)^{n}e^{2}-\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)de+de}{\ln(F)bc\,d^{2}en\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}$
norman	$\frac{-\frac{e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{\ln(F)bcn\,d^{2}}+\frac{e\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{d^{2}\ln(F)bc}+\frac{\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}{\ln(F)bcd}}{d+e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}-\frac{\ln\left(d+e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}\right)}{\ln(F)bcn\,d^{2}}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(-1/d^2*ln(d+e*(F^(c*(b*x+a)))^n)+1/d/(d+e*(F^(c*(b*x+a)))^n
)+1/d^2*ln((F^(c*(b*x+a)))^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}}\,dx = \frac{F^{bcnx+acn}bcenx\log(F)+bcdnx\log(F)-\left(F^{bcnx+acn}e+d\right)\log\left(F^{bcnx+acn}e+d\right)+d}{F^{bcnx+acn}bcd^{2}en\log(F)+bcd^{3}n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
(F^(b*c*n*x + a*c*n)*b*c*e*n*x*log(F) + b*c*d*n*x*log(F) - (F^(b*c*n*x + a
*c*n)*e + d)*log(F^(b*c*n*x + a*c*n)*e + d) + d)/(F^(b*c*n*x + a*c*n)*b*c*
d^2*e*n*log(F) + b*c*d^3*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{bcnx+acn}{bcd^2n} + \frac{1}{(F^{bcnx+acn}de+d^2)bcn\log(F)} - \frac{\log\left(F^{bcnx+acn}e+d\right)}{bcd^2n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
(b*c*n*x + a*c*n)/(b*c*d^2*n) + 1/((F^(b*c*n*x + a*c*n)*d*e + d^2)*b*c*n*l
og(F)) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d^2*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{\log\left(|F|^{bcnx}|F|^{acn}\right)}{bcd^2n\log(F)} - \frac{\log\left(\left|F^{bcnx}F^{acn}e+d\right|\right)}{bcd^2n\log(F)} + \frac{1}{(F^{bcnx}F^{acn}e+d)bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
log(abs(F)^(b*c*n*x)*abs(F)^(a*c*n))/(b*c*d^2*n*log(F)) - log(abs(F^(b*c*n
*x)*F^(a*c*n)*e + d))/(b*c*d^2*n*log(F)) + 1/((F^(b*c*n*x)*F^(a*c*n)*e + d
)*b*c*d*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 22.73 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{x}{d^2}+\frac{1}{b\,c\,d\,n\,\ln\left(F\right)\,\left(d+e\left(F^{b\,c\,x}\,F^{a\,c}\right)^n\right)} - \frac{\ln\left(d+e\left(F^{b\,c\,x}\,F^{a\,c}\right)^n\right)}{b\,c\,d^2\,n\,\ln\left(F\right)}$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
x/d^2 + 1/(b*c*d*n*log(F)*(d + e*(F^(b*c*x)*F^(a*c))^n)) - log(d + e*(F^(b
*c*x)*F^(a*c))^n)/(b*c*d^2*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{-f^{bcnx+acn}\log\left(f^{bcnx+acn}e+d\right)e+f^{bcnx+acn}\log(f)\,bcenx-f^{bcnx+acn}e-\log\left(f^{bcnx+acn}e+d\right)d+\log(f)\,b}{\log\left(f\right)bc\,d^2n\left(f^{bcnx+acn}e+d\right)}$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
( - f**(a*c*n + b*c*n*x)*log(f**(a*c*n + b*c*n*x)*e + d)*e + f**(a*c*n + b
*c*n*x)*log(f)*b*c*e*n*x - f**(a*c*n + b*c*n*x)*e - log(f**(a*c*n + b*c*n*
x)*e + d)*d + log(f)*b*c*d*n*x)/(log(f)*b*c*d**2*n*(f**(a*c*n + b*c*n*x)*e
 + d))
```

3.5 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}\,dx=\frac{2d\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{bcn\log(F)}+\frac{2\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}{3bcn\log(F)}-\frac{2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcn\log(F)}$$

output
```
2*d*(d+e*(F^(c*(b*x+a)))^n)^(1/2)/b/c/n/ln(F)+2/3*(d+e*(F^(c*(b*x+a)))^n)^
(3/2)/b/c/n/ln(F)-2*d^(3/2)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))
/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}\,dx=\frac{2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\left(4d+e\left(F^{c(a+bx)}\right)^n\right)-6d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{3bcn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(3/2),x]
```

output
```
(2*Sqrt[d + e*(F^(c*(a + b*x)))^n]*(4*d + e*(F^(c*(a + b*x)))^n) - 6*d^(3/
2)*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n/Sqrt[d]])/(3*b*c*n*Log[F])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2720, 798, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{60}$$

$$\frac{d\int F^{-c(a+bx)}\sqrt{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n+\frac{2}{3}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}{bcn\log(F)}$$

$$\Big\downarrow \text{60}$$

$$\frac{d\left(d\int \frac{F^{-c(a+bx)}}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n+2\sqrt{e\left(F^{c(a+bx)}\right)^n+d}\right)+\frac{2}{3}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}{bcn\log(F)}$$

$$\Big\downarrow \text{73}$$

$$\frac{d\left(\frac{2d\int \frac{1}{\frac{F^{2c(a+bx)}}{e}-\frac{d}{e}}d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{e}+2\sqrt{e\left(F^{c(a+bx)}\right)^n+d}\right)+\frac{2}{3}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}{bcn\log(F)}$$

$$\Big\downarrow \; 221$$

$$\frac{d\left(2\sqrt{e\left(F^{c(a+bx)}\right)^n+d}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(3/2),x]
```

output
```
((2*(d + e*(F^(c*(a + b*x)))^n)^(3/2))/3 + d*(2*Sqrt[d + e*(F^(c*(a + b*x)
))^n] - 2*Sqrt[d]*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]]))/(b*c*
n*Log[F])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{\frac{2\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{3}{2}}}{3}+2d\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}-2d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{\ln\left(F\right)bcn}$	82
default	$\frac{\frac{2\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{3}{2}}}{3}+2d\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}-2d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{\ln\left(F\right)bcn}$	82
risch	$\frac{2\left(e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}+4d\right)\sqrt{d+e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}}{3ncb\ln\left(F\right)}-\frac{2d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{d+e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}}{\sqrt{d}}\right)}{\ln\left(F\right)bcn}$	100

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(2/3*(d+e*(F^(c*(b*x+a)))^n)^(3/2)+2*d*(d+e*(F^(c*(b*x+a)))^
n)^(1/2)-2*d^(3/2)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.72

$$\int \left(d + e\left(F^{c(a+bx)}\right)^{n}\right)^{3/2} dx = \left[\frac{3\,d^{\frac{3}{2}}\log\left(\frac{F^{bcnx+acn}e-2\,\sqrt{F^{bcnx+acn}e+d}\sqrt{d}+2\,d}{F^{bcnx+acn}}\right)+2\,\left(F^{bcnx+acn}e+4\,d\right)\sqrt{F^{bcnx+acn}e+d}}{3\,bcn\log\left(F\right)}\right.$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*d^(3/2)*log((F^(b*c*n*x + a*c*n)*e - 2*sqrt(F^(b*c*n*x + a*c*n)*e
+ d)*sqrt(d) + 2*d)/F^(b*c*n*x + a*c*n)) + 2*(F^(b*c*n*x + a*c*n)*e + 4*d)
*sqrt(F^(b*c*n*x + a*c*n)*e + d))/(b*c*n*log(F)), 2/3*(3*sqrt(-d)*d*arctan
(sqrt(-d)/sqrt(F^(b*c*n*x + a*c*n)*e + d)) + (F^(b*c*n*x + a*c*n)*e + 4*d)
*sqrt(F^(b*c*n*x + a*c*n)*e + d))/(b*c*n*log(F))]
```

Sympy [F]

$$\int \left(d + e\left(F^{c(a+bx)}\right)^{n}\right)^{3/2} dx = \int \left(d + e\left(F^{c(a+bx)}\right)^{n}\right)^{\frac{3}{2}} dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(3/2),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}\,dx = \frac{d^{\frac{3}{2}}\log\left(\frac{\sqrt{F^{bcnx+acn}e+d}-\sqrt{d}}{\sqrt{F^{bcnx+acn}e+d}+\sqrt{d}}\right)}{bcn\log(F)} + \frac{2\left(\left(F^{bcnx+acn}e+d\right)^{\frac{3}{2}}+3\sqrt{F^{bcnx+acn}e+d}d\right)}{3\,bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="maxima")
```

output
```
d^(3/2)*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x +
 a*c*n)*e + d) + sqrt(d)))/(b*c*n*log(F)) + 2/3*((F^(b*c*n*x + a*c*n)*e +
d)^(3/2) + 3*sqrt(F^(b*c*n*x + a*c*n)*e + d)*d)/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}\,dx = \frac{2\left(\frac{3\,d^2\arctan\left(\frac{\sqrt{F^{bcnx+acn}e+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\left(F^{bcnx+acn}e+d\right)^{\frac{3}{2}}+3\sqrt{F^{bcnx+acn}e+d}d\right)}{3\,bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="giac")
```

output
```
2/3*(3*d^2*arctan(sqrt(F^(b*c*n*x + a*c*n)*e + d)/sqrt(-d))/sqrt(-d) + (F^
(b*c*n*x + a*c*n)*e + d)^(3/2) + 3*sqrt(F^(b*c*n*x + a*c*n)*e + d)*d)/(b*c
*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2} dx = \int \left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{3/2} dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^(3/2),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^(3/2), x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2} dx = \frac{2f^{bcnx+acn}\sqrt{f^{bcnx+acn}e+d}\,e+2\sqrt{f^{bcnx+acn}e+d}\,d+3\left(\int \sqrt{f^{bcnx+acn}e+d}dx\right)\log}{3\log(f)\,bcn}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(3/2),x)
```

output
```
(2*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)*e + 2*sqrt(f**(a*
c*n + b*c*n*x)*e + d)*d + 3*int(sqrt(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)
*b*c*d*n)/(3*log(f)*b*c*n)
```

3.6 $\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{bcn\log(F)}-\frac{2\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcn\log(F)}$$

output
```
2*(d+e*(F^(c*(b*x+a)))^n)^(1/2)/b/c/n/ln(F)-2*d^(1/2)*arctanh((d+e*(F^(c*(
b*x+a)))^n)^(1/2)/d^(1/2))/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{2\left(\sqrt{d+e\left(F^{c(a+bx)}\right)^n}-\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\right)}{bcn\log(F)}$$

input
```
Integrate[Sqrt[d + e*(F^(c*(a + b*x)))^n],x]
```

output
```
(2*(Sqrt[d + e*(F^(c*(a + b*x)))^n] - Sqrt[d]*ArcTanh[Sqrt[d + e*(F^(c*(a
+ b*x)))^n]/Sqrt[d]]))/(b*c*n*Log[F])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2720, 798, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{e \left(F^{c(a+bx)}\right)^n+d}\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int F^{-c(a+bx)}\sqrt{e \left(F^{c(a+bx)}\right)^n+d}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \; 798$$

$$\frac{\int F^{-c(a+bx)}\sqrt{e \left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \; 60$$

$$\frac{d\int \frac{F^{-c(a+bx)}}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n+2\sqrt{e \left(F^{c(a+bx)}\right)^n+d}}{bcn\log(F)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{2d\int \frac{1}{\frac{F^{2c(a+bx)}}{e}-\frac{d}{e}}d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{e}+2\sqrt{e \left(F^{c(a+bx)}\right)^n+d}}{bcn\log(F)}$$

$$\Big\downarrow \; 221$$

$$\frac{2\sqrt{e \left(F^{c(a+bx)}\right)^n+d}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcn\log(F)}$$

input
```
Int[Sqrt[d + e*(F^(c*(a + b*x)))^n],x]
```

output
```
(2*Sqrt[d + e*(F^(c*(a + b*x)))^n] - 2*Sqrt[d]*ArcTanh[Sqrt[d + e*(F^(c*(a
 + b*x)))^n]/Sqrt[d]])/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{2\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}-2\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{\ln(F)bcn}$	62
default	$\frac{2\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}-2\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{\ln(F)bcn}$	62
risch	$\frac{2\sqrt{d+e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}}{ncb\ln(F)}-\frac{2\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{d+e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}}{\sqrt{d}}\right)}{\ln(F)bcn}$	80

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(2*(d+e*(F^(c*(b*x+a)))^n)^(1/2)-2*d^(1/2)*arctanh((d+e*(F^(
c*(b*x+a)))^n)^(1/2)/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.00

$$\int\sqrt{d+e\left(F^{c(a+bx)}\right)^{n}}\,dx = \left[\frac{\sqrt{d}\log\left(\frac{F^{bcnx+acn}e-2\sqrt{F^{bcnx+acn}e+d}\sqrt{d}+2d}{F^{bcnx+acn}}\right)+2\sqrt{F^{bcnx+acn}e+d}}{bcn\log(F)}, \frac{2\left(\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{F^{bcnx+acn}e+d}}\right)+\sqrt{F^{b}\right.}{bcn\log(F)}\right.$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(d)*log((F^(b*c*n*x + a*c*n)*e - 2*sqrt(F^(b*c*n*x + a*c*n)*e + d)*s
qrt(d) + 2*d)/F^(b*c*n*x + a*c*n)) + 2*sqrt(F^(b*c*n*x + a*c*n)*e + d))/(b
*c*n*log(F)), 2*(sqrt(-d)*arctan(sqrt(-d)/sqrt(F^(b*c*n*x + a*c*n)*e + d))
 + sqrt(F^(b*c*n*x + a*c*n)*e + d))/(b*c*n*log(F))]
```

Sympy [F]

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(1/2),x)
```

output
```
Integral(sqrt(d + e*(F**(c*(a + b*x)))**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{\sqrt{d}\log\left(\frac{\sqrt{F^{bcnx+acn}e+d}-\sqrt{d}}{\sqrt{F^{bcnx+acn}e+d}+\sqrt{d}}\right)}{bcn\log(F)}+\frac{2\sqrt{F^{bcnx+acn}e+d}}{bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(d)*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x +
 a*c*n)*e + d) + sqrt(d)))/(b*c*n*log(F)) + 2*sqrt(F^(b*c*n*x + a*c*n)*e +
 d)/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{2\left(\frac{d\arctan\left(\frac{\sqrt{F^{bcnx+acn}e+d}}{\sqrt{-d}}\right)}{\sqrt{-d}}+\sqrt{F^{bcnx+acn}e+d}\right)}{bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="giac")
```

output
```
2*(d*arctan(sqrt(F^(b*c*n*x + a*c*n)*e + d)/sqrt(-d))/sqrt(-d) + sqrt(F^(b
*c*n*x + a*c*n)*e + d))/(b*c*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d + e\left(F^{c(a+bx)}\right)^n}\,dx = \int \sqrt{d + e\left(F^{c\,(a+b\,x)}\right)^n}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^(1/2),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d + e\left(F^{c(a+bx)}\right)^n}\,dx = \int \sqrt{f^{bcnx+acn}e + d}dx$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(1/2),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d),x)
```

3.7 $\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bc\sqrt{d}n\log(F)}$$

output
```
-2*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c/d^(1/2)/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=-\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bc\sqrt{d}n\log(F)}$$

input
```
Integrate[1/Sqrt[d + e*(F^(c*(a + b*x)))^n],x]
```

output
```
(-2*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(b*c*Sqrt[d]*n*Log[F
])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {2720, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2\int \frac{1}{\frac{F^{2c(a+bx)}}{e}-\frac{d}{e}}d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{bcen\log(F)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bc\sqrt{d}n\log(F)}$$

input
```
Int[1/Sqrt[d + e*(F^(c*(a + b*x)))^n],x]
```

output
```
(-2*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(b*c*Sqrt[d]*n*Log[F
])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{bc\sqrt{d}\,n\ln\left(F\right)}$	41
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{bc\sqrt{d}\,n\ln\left(F\right)}$	41

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c/d^(1/2)/n/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \left[\frac{\log\left(\frac{F^{bcnx+acn}e-2\sqrt{F^{bcnx+acn}e+d}\sqrt{d}+2\,d}{F^{bcnx+acn}}\right)}{bc\sqrt{d}n\log(F)}, \frac{2\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{F^{bcnx+acn}e+d}}\right)}{bcdn\log(F)}\right]$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="fricas")
```

output
```
[log((F^(b*c*n*x + a*c*n)*e - 2*sqrt(F^(b*c*n*x + a*c*n)*e + d)*sqrt(d) +
2*d)/F^(b*c*n*x + a*c*n))/(b*c*sqrt(d)*n*log(F)), 2*sqrt(-d)*arctan(sqrt(-
d)/sqrt(F^(b*c*n*x + a*c*n)*e + d))/(b*c*d*n*log(F))]
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(1/2),x)
```

output
```
Integral(1/sqrt(d + e*(F**(c*(a + b*x)))**n), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \frac{\log\left(\frac{\sqrt{F^{bcnx+acn}e+d}-\sqrt{d}}{\sqrt{F^{bcnx+acn}e+d}+\sqrt{d}}\right)}{bc\sqrt{d}n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="maxima")
```

output
```
log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x + a*c*n)*
e + d) + sqrt(d)))/(b*c*sqrt(d)*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \frac{2\arctan\left(\frac{\sqrt{F^{bcnx+acn}e+d}}{\sqrt{-d}}\right)}{bc\sqrt{-d}n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(F^(b*c*n*x + a*c*n)*e + d)/sqrt(-d))/(b*c*sqrt(-d)*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \int \frac{1}{\sqrt{d+e\left(F^{c\,(a+b\,x)}\right)^n}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(1/2),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d + e\left(F^{c(a+bx)}\right)^n}}\,dx = \int \frac{1}{\sqrt{f^{bcnx+acn}e + d}}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(1/2),x)
```

output
```
int(1/sqrt(f**(a*c*n + b*c*n*x)*e + d),x)
```

3.8 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\frac{2}{bcd\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcd^{3/2}n\log(F)}$$

output
```
2/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(1/2)/n/ln(F)-2*arctanh((d+e*(F^(c*(b*x+a)
))^n)^(1/2)/d^(1/2))/b/c/d^(3/2)/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\frac{\frac{2}{d\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{d^{3/2}}}{bcn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-3/2),x]
```

output
```
(2/(d*Sqrt[d + e*(F^(c*(a + b*x)))^n]) - (2*ArcTanh[Sqrt[d + e*(F^(c*(a +
b*x)))^n]/Sqrt[d]])/d^(3/2))/(b*c*n*Log[F])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {2720, 798, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{F^{-c(a+bx)}}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{2}{d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}}{bcn\log(F)}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{2\int \frac{1}{\frac{F^{2c(a+bx)}}{e}-\frac{d}{e}}d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{de}+\frac{2}{d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}}{bcn\log(F)}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2}{d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{d^{3/2}}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-3/2),x]
```

output
```
(2/(d*Sqrt[d + e*(F^(c*(a + b*x)))^n]) - (2*ArcTanh[Sqrt[d + e*(F^(c*(a +
b*x)))^n]/Sqrt[d]])/d^(3/2))/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{d\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}}{\ln(F)bcn}$	65
default	$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}}+\frac{2}{d\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}}{\ln(F)bcn}$	65

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(-2/d^(3/2)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))+2
/d/(d+e*(F^(c*(b*x+a)))^n)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{3/2}}\,dx = \left[\frac{\left(F^{bcnx+acn}\sqrt{d}e+d^{\frac{3}{2}}\right)\log\left(\frac{F^{bcnx+acn}e-2\sqrt{F^{bcnx+acn}e+d}\sqrt{d}+2\,d}{F^{bcnx+acn}}\right)+2\sqrt{F^{bcnx+ac}}}{F^{bcnx+acn}bcd^{2}en\log\left(F\right)+bcd^{3}n\log\left(F\right)}\right.$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="fricas")
```

output
```
[((F^(b*c*n*x + a*c*n)*sqrt(d)*e + d^(3/2))*log((F^(b*c*n*x + a*c*n)*e - 2
*sqrt(F^(b*c*n*x + a*c*n)*e + d)*sqrt(d) + 2*d)/F^(b*c*n*x + a*c*n)) + 2*s
qrt(F^(b*c*n*x + a*c*n)*e + d)*d)/(F^(b*c*n*x + a*c*n)*b*c*d^2*e*n*log(F)
+ b*c*d^3*n*log(F)), 2*((F^(b*c*n*x + a*c*n)*sqrt(-d)*e + sqrt(-d)*d)*arct
an(sqrt(-d)/sqrt(F^(b*c*n*x + a*c*n)*e + d)) + sqrt(F^(b*c*n*x + a*c*n)*e
+ d)*d)/(F^(b*c*n*x + a*c*n)*b*c*d^2*e*n*log(F) + b*c*d^3*n*log(F))]
```

Sympy [F]

$$\int \frac{1}{\left(d + e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\, dx = \int \frac{1}{\left(d + e\left(F^{c(a+bx)}\right)^n\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(3/2),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(d + e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\, dx = \frac{\log\left(\frac{\sqrt{F^{bcnx+acn}e+d}-\sqrt{d}}{\sqrt{F^{bcnx+acn}e+d}+\sqrt{d}}\right)}{bcd^{\frac{3}{2}}n\log(F)} + \frac{2}{\sqrt{F^{bcnx+acn}e+d}bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="maxima")
```

output
```
log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x + a*c*n)*
e + d) + sqrt(d)))/(b*c*d^(3/2)*n*log(F)) + 2/(sqrt(F^(b*c*n*x + a*c*n)*e
+ d)*b*c*d*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{F^{bcnx}F^{acn}e+d}}{\sqrt{-d}}\right)}{bc\sqrt{-d}dn\log\left(F\right)} + \frac{2}{\sqrt{F^{bcnx}F^{acn}e+d}bcdn\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d)/sqrt(-d))/(b*c*sqrt(-d)*d*n*log
(F)) + 2/(sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d)*b*c*d*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx = \int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{3/2}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(3/2),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx = \int \frac{\sqrt{f^{bcnx+acn}e+d}}{f^{2bcnx+2acn}e^{2}+2f^{bcnx+acn}de+d^{2}}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(3/2),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f*
*(a*c*n + b*c*n*x)*d*e + d**2),x)
```

3.9 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\frac{2}{3bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}n\log(F)}+\frac{2}{bcd^2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcd^{5/2}n\log(F)}$$

output
```
2/3/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(3/2)/n/ln(F)+2/b/c/d^2/(d+e*(F^(c*(b*x+
a)))^n)^(1/2)/n/ln(F)-2*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c
/d^(5/2)/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\frac{2\left(\frac{\sqrt{d}\left(4d+3e\left(F^{c(a+bx)}\right)^n\right)}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}-3\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\right)}{3bcd^{5/2}n\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-5/2),x]
```

output
```
(2*((Sqrt[d]*(4*d + 3*e*(F^(c*(a + b*x)))^n))/(d + e*(F^(c*(a + b*x)))^n)^
(3/2) - 3*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]]))/(3*b*c*d^(5/2
)*n*Log[F])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2720, 798, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{2}{3d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}}{bcn\log(F)}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\frac{\int \frac{F^{-c(a+bx)}}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{2}{d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}}{d}+\frac{2}{3d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}}{bcn\log(F)}$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{2\int \frac{1}{\frac{F^{2c(a+bx)}}{e}-\frac{d}{e}}d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{de}+\frac{2}{d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}}{d}+\frac{2}{3d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}}{bcn\log(F)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{2}{d\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-5/2),x]
```

output
```
(2/(3*d*(d + e*(F^(c*(a + b*x)))^n)^(3/2)) + (2/(d*Sqrt[d + e*(F^(c*(a + b
*x)))^n]) - (2*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/d^(3/2))/
d)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{2}{d^2\sqrt{d+e\left(F^{c(bx+a)}\right)^n}}+\frac{2}{3d\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^n}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}}{\ln(F)bcn}$	87
default	$\frac{\frac{2}{d^2\sqrt{d+e\left(F^{c(bx+a)}\right)^n}}+\frac{2}{3d\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{3}{2}}}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(bx+a)}\right)^n}}{\sqrt{d}}\right)}{d^{\frac{5}{2}}}}{\ln(F)bcn}$	87

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(2/d^2/(d+e*(F^(c*(b*x+a)))^n)^(1/2)+2/3/d/(d+e*(F^(c*(b*x+a
)))^n)^(3/2)-2/d^(5/2)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 395, normalized size of antiderivative = 3.21

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \left[\frac{3\left(2\,F^{bcnx+acn}d^{\frac{3}{2}}e+F^{2\,bcnx+2\,acn}\sqrt{d}e^2+d^{\frac{5}{2}}\right)\log\left(\frac{F^{bcnx+acn}e-2\,\sqrt{F^{bcnx+acn}e+d}}{F^{bcnx+acn}}\right.}{3\,(2\,F^{bcnx+acn}bcd^4en\log\left(F\right)+F^{2\,bcnx+2\,acn}bcd^3e^2}\right.$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(2*F^(b*c*n*x + a*c*n)*d^(3/2)*e + F^(2*b*c*n*x + 2*a*c*n)*sqrt(d)
*e^2 + d^(5/2))*log((F^(b*c*n*x + a*c*n)*e - 2*sqrt(F^(b*c*n*x + a*c*n)*e
+ d)*sqrt(d) + 2*d)/F^(b*c*n*x + a*c*n)) + 2*(3*F^(b*c*n*x + a*c*n)*d*e +
4*d^2)*sqrt(F^(b*c*n*x + a*c*n)*e + d))/(2*F^(b*c*n*x + a*c*n)*b*c*d^4*e*n
*log(F) + F^(2*b*c*n*x + 2*a*c*n)*b*c*d^3*e^2*n*log(F) + b*c*d^5*n*log(F))
, 2/3*(3*(2*F^(b*c*n*x + a*c*n)*sqrt(-d)*d*e + F^(2*b*c*n*x + 2*a*c*n)*sqr
t(-d)*e^2 + sqrt(-d)*d^2)*arctan(sqrt(-d)/sqrt(F^(b*c*n*x + a*c*n)*e + d))
 + (3*F^(b*c*n*x + a*c*n)*d*e + 4*d^2)*sqrt(F^(b*c*n*x + a*c*n)*e + d))/(2
*F^(b*c*n*x + a*c*n)*b*c*d^4*e*n*log(F) + F^(2*b*c*n*x + 2*a*c*n)*b*c*d^3*
e^2*n*log(F) + b*c*d^5*n*log(F))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \frac{\log\left(\frac{\sqrt{F^{bcnx+acn}e+d}-\sqrt{d}}{\sqrt{F^{bcnx+acn}e+d}+\sqrt{d}}\right)}{bcd^{\frac{5}{2}}n\log(F)} + \frac{2\left(3\,F^{bcnx+acn}e+4\,d\right)}{3\left(F^{bcnx+acn}e+d\right)^{\frac{3}{2}}bcd^2n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(5/2),x, algorithm="maxima")
```

output
```
log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x + a*c*n)*
e + d) + sqrt(d)))/(b*c*d^(5/2)*n*log(F)) + 2/3*(3*F^(b*c*n*x + a*c*n)*e +
 4*d)/((F^(b*c*n*x + a*c*n)*e + d)^(3/2)*b*c*d^2*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{F^{bcnx}F^{acn}e+d}}{\sqrt{-d}}\right)}{bc\sqrt{-d}d^2n\log(F)} + \frac{2\left(3\,F^{bcnx}F^{acn}e+4\,d\right)}{3\left(F^{bcnx}F^{acn}e+d\right)^{\frac{3}{2}}bcd^2n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(5/2),x, algorithm="giac")
```

output
```
2*arctan(sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d)/sqrt(-d))/(b*c*sqrt(-d)*d^2*n*l
og(F)) + 2/3*(3*F^(b*c*n*x)*F^(a*c*n)*e + 4*d)/((F^(b*c*n*x)*F^(a*c*n)*e +
 d)^(3/2)*b*c*d^2*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{5/2}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(5/2),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\int \frac{\sqrt{f^{bcnx+acn}e+d}}{f^{3bcnx+3acn}e^3+3f^{2bcnx+2acn}d\,e^2+3f^{bcnx+acn}d^2e+d^3}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(5/2),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(3*a*c*n + 3*b*c*n*x)*e**3 + 3*f*
*(2*a*c*n + 2*b*c*n*x)*d*e**2 + 3*f**(a*c*n + b*c*n*x)*d**2*e + d**3),x)
```

3.10 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 196

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}\,dx = -\frac{1}{2}d^{4/3}x+\frac{3d\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{bcn\log(F)} + \frac{3\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}{4bcn\log(F)} - \frac{\sqrt{3}d^{4/3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcn\log(F)} + \frac{3d^{4/3}\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcn\log(F)}$$

output
```
-1/2*d^(4/3)*x+3*d*(d+e*(F^(c*(b*x+a)))^n)^(1/3)/b/c/n/ln(F)+3/4*(d+e*(F^(
c*(b*x+a)))^n)^(4/3)/b/c/n/ln(F)-3^(1/2)*d^(4/3)*arctan(1/3*(d^(1/3)+2*(d+
e*(F^(c*(b*x+a)))^n)^(1/3))*3^(1/2)/d^(1/3))/b/c/n/ln(F)+3/2*d^(4/3)*ln(d^
(1/3)-(d+e*(F^(c*(b*x+a)))^n)^(1/3))/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3} dx = \frac{15d\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}+3e\left(F^{c(a+bx)}\right)^n\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}-4\sqrt{3}d^{4/3}\arctan\left(\frac{1+}{}\right.}{}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(4/3),x]
```

output
```
(15*d*(d + e*(F^(c*(a + b*x)))^n)^(1/3) + 3*e*(F^(c*(a + b*x)))^n*(d + e*(
F^(c*(a + b*x)))^n)^(1/3) - 4*Sqrt[3]*d^(4/3)*ArcTan[(1 + (2*(d + e*(F^(c*
(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]] + 4*d^(4/3)*Log[d^(1/3) - (d + e*(
F^(c*(a + b*x)))^n)^(1/3)] - 2*d^(4/3)*Log[d^(2/3) + d^(1/3)*(d + e*(F^(c*
(a + b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^n)^(2/3)])/(4*b*c*n*Log[F]
)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2720, 798, 60, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3} dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3} dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \; 798$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \; 60$$

$$\frac{d\int F^{-c(a+bx)}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n+\frac{3}{4}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}{bcn\log(F)}$$

$$\Big\downarrow \; 60$$

$$\frac{d\left(d\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}d\left(F^{c(a+bx)}\right)^n+3\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)+\frac{3}{4}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}{bcn\log(F)}$$

$$\Big\downarrow \; 69$$

$$\frac{d\left(d\left(-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2d^{2/3}}-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a\ldots}}{2\sqrt[3]{d}}\ldots\right.\right.}{bcn\log(F)}$$

$$\Big\downarrow \; 16$$

$$\frac{d\left(d\left(-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\ldots\right.}{\ldots}\right.\right.}{bcn\log(F)}$$

$$\Big\downarrow \; 1082$$

$$\frac{d\left(d\left(\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}\right)+3\sqrt[3]{e\ldots}\right.}{bcn\log(F)}$$

$$\Big\downarrow \; 217$$

$$\frac{d\left(d\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}\right)+3\sqrt[3]{e\left(F^{c(a\cdots}\right.}\right.}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(4/3),x]
```

output
```
((3*(d + e*(F^(c*(a + b*x)))^n)^(4/3))/4 + d*(3*(d + e*(F^(c*(a + b*x)))^n
)^(1/3) + d*(-((Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/
d^(1/3))/Sqrt[3]])/d^(2/3)) - Log[(F^(c*(a + b*x)))^n]/(2*d^(2/3)) + (3*Lo
g[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)])/(2*d^(2/3)))))/(b*c*n*Log[
F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{3\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{4}{3}}}{4}+3d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+3\left(\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{3d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+\right.\right.}{6d^{\frac{2}{3}}}\right.}{\ln(F)bcn}$
default	$\frac{\frac{3\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{4}{3}}}{4}+3d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+3\left(\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{3d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+\right.\right.}{6d^{\frac{2}{3}}}\right.}{\ln(F)bcn}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(3/4*(d+e*(F^(c*(b*x+a)))^n)^(4/3)+3*d*(d+e*(F^(c*(b*x+a)))^
n)^(1/3)+3*(1/3/d^(2/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/6/d^(2
/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3)
+d^(2/3))-1/3/d^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^(c*(b*
x+a)))^n)^(1/3)+1)))*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.92

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}\,dx =$$

$$-\frac{4\sqrt{3}d^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{2}{3}}+\sqrt{3}d}{3d}\right)+2d^{\frac{4}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{4\,bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(4/3),x, algorithm="fricas")
```

output
```
-1/4*(4*sqrt(3)*d^(4/3)*arctan(1/3*(2*sqrt(3)*(F^(b*c*n*x + a*c*n)*e + d)^
(1/3)*d^(2/3) + sqrt(3)*d)/d) + 2*d^(4/3)*log((F^(b*c*n*x + a*c*n)*e + d)^
(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) - 4*d^(4/3)*l
og((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)) - 3*(F^(b*c*n*x + a*c*n)*e
 + 5*d)*(F^(b*c*n*x + a*c*n)*e + d)^(1/3))/(b*c*n*log(F))
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}\,dx = \text{Timed out}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(4/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}\,dx = -\frac{\sqrt{3}d^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcn\log(F)} - \frac{d^{\frac{4}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcn\log(F)} + \frac{d^{\frac{4}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcn\log(F)} + \frac{3\left(\left(F^{bcnx+acn}e+d\right)^{\frac{4}{3}}+4\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d\right)}{4\,bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(4/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*d^(4/3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) +
 d^(1/3))/d^(1/3))/(b*c*n*log(F)) - 1/2*d^(4/3)*log((F^(b*c*n*x + a*c*n)*e
 + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*n*
log(F)) + d^(4/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*n*
log(F)) + 3/4*((F^(b*c*n*x + a*c*n)*e + d)^(4/3) + 4*(F^(b*c*n*x + a*c*n)*
e + d)^(1/3)*d)/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}\,dx = -\frac{4\sqrt{3}d^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)+2\,d^{\frac{4}{3}}\log\left(\left(F^{bcnx}F^{acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{4\,bcn\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(4/3),x, algorithm="giac")
```

output
```
-1/4*(4*sqrt(3)*d^(4/3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x)*F^(a*c*n)*e + d
)^(1/3) + d^(1/3))/d^(1/3)) + 2*d^(4/3)*log((F^(b*c*n*x)*F^(a*c*n)*e + d)^
(2/3) + (F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) - 4*d^(4/3)
*log(abs((F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3) - d^(1/3))) - 3*(F^(b*c*n*x)*
F^(a*c*n)*e + d)^(4/3) - 12*(F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3)*d)/(b*c*n*
log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3} dx = \int \left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{4/3} dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^(4/3),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^(4/3), x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3} dx = \frac{3f^{bcnx+acn}\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}e+15\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}d+4\left(\int \frac{1}{\left(f^{bcnx+acn}e+d\right)^{\frac{2}{3}}}dx\right)\log \ldots}{4\log\left(f\right)bcn}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(4/3),x)
```

output
```
(3*f**(a*c*n + b*c*n*x)*(f**(a*c*n + b*c*n*x)*e + d)**(1/3)*e + 15*(f**(a*
c*n + b*c*n*x)*e + d)**(1/3)*d + 4*int((f**(a*c*n + b*c*n*x)*e + d)**(1/3)
/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d**2*n)/(4*log(f)*b*c*n)
```

3.11 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}\,dx=-\frac{1}{2}d^{2/3}x+\frac{3\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}{2bcn\log(F)}+\frac{\sqrt{3}d^{2/3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcn\log(F)}+\frac{3d^{2/3}\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcn\log(F)}$$

output
```
-1/2*d^(2/3)*x+3/2*(d+e*(F^(c*(b*x+a)))^n)^(2/3)/b/c/n/ln(F)+3^(1/2)*d^(2/
3)*arctan(1/3*(d^(1/3)+2*(d+e*(F^(c*(b*x+a)))^n)^(1/3))*3^(1/2)/d^(1/3))/b
/c/n/ln(F)+3/2*d^(2/3)*ln(d^(1/3)-(d+e*(F^(c*(b*x+a)))^n)^(1/3))/b/c/n/ln(
F)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.12

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3} dx = \frac{3\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}+2\sqrt{3}d^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2d^{2/3}\log\left(\right.}{}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(2/3),x]
```

output
```
(3*(d + e*(F^(c*(a + b*x)))^n)^(2/3) + 2*Sqrt[3]*d^(2/3)*ArcTan[(1 + (2*(d
 + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]] + 2*d^(2/3)*Log[d^(1/3)
 - (d + e*(F^(c*(a + b*x)))^n)^(1/3)] - d^(2/3)*Log[d^(2/3) + d^(1/3)*(d +
 e*(F^(c*(a + b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^n)^(2/3)])/(2*b*c
*n*Log[F])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2720, 798, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3} dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \; 798$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$\Big\downarrow$ 60

$$\frac{d\int \frac{F^{-c(a+bx)}}{\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n+\frac{3}{2}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}{bcn\log(F)}$$

$\Big\downarrow$ 67

$$\frac{d\left(\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right.}}{2\sqrt[3]{d}}\right.}{bcn\log(F)}$$

$\Big\downarrow$ 16

$$\frac{d\left(\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a\right.\right.}}{2\sqrt[3]{}}\right.}{bcn\log(F)}$$

$\Big\downarrow$ 1082

$$\frac{d\left(-\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}\right)+\frac{3}{2}\left(e\left(F\right.\right.}{bcn\log(F)}$$

$\Big\downarrow$ 217

$$\frac{d\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}\right)+\frac{3}{2}\left(e\left(F^{c(a+bx)}\right)^n\right.}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(2/3),x]
```

output
```
((3*(d + e*(F^(c*(a + b*x)))^n)^(2/3))/2 + d*((Sqrt[3]*ArcTan[(1 + (2*(d +
 e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]])/d^(1/3) - Log[(F^(c*(a +
 b*x)))^n]/(2*d^(1/3)) + (3*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3
)])/(2*d^(1/3))))/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{3\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{3d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{6d^{\frac{1}{3}}}+\right.}{\ln(F)bcn}$
default	$\frac{\frac{3\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}}{2}+3\left(\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{3d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{6d^{\frac{1}{3}}}+\right.}{\ln(F)bcn}$
risch	$\frac{3\left(d+e\,e^{n\ln\left(e^{c(bx+a)\ln(F)}\right)}\right)^{\frac{2}{3}}}{2ncb\ln(F)}+\frac{d\left(\frac{\ln\left(\left(d+e\,e^{n\ln\left(e^{c(bx+a)\ln(F)}\right)}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\,e^{n\ln\left(e^{c(bx+a)\ln(F)}\right)}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\right.}{2d}\right.}{}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(3/2*(d+e*(F^(c*(b*x+a)))^n)^(2/3)+3*(1/3/d^(1/3)*ln((d+e*(F
^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/6/d^(1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/
3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3)+d^(2/3))+1/3*3^(1/2)/d^(1/3)*arct
an(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3)+1)))*d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3} dx = \frac{2\sqrt{3}(d^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}d+2\sqrt{3}(d^2)^{\frac{1}{3}}(F^{bcnx+acn}e+d)^{\frac{1}{3}}}{3d}\right) - (d^2)^{\frac{1}{3}} \log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}d \cdots\right.}{\cdots}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(2/3),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(3)*(d^2)^(1/3)*arctan(1/3*(sqrt(3)*d + 2*sqrt(3)*(d^2)^(1/3)*(
F^(b*c*n*x + a*c*n)*e + d)^(1/3))/d) - (d^2)^(1/3)*log((F^(b*c*n*x + a*c*n
)*e + d)^(2/3)*d + (d^2)^(1/3)*d + (d^2)^(2/3)*(F^(b*c*n*x + a*c*n)*e + d)
^(1/3)) + 2*(d^2)^(1/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d - (d^2)^(2
/3)) + 3*(F^(b*c*n*x + a*c*n)*e + d)^(2/3))/(b*c*n*log(F))
```

Sympy [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3} dx = \int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{\frac{2}{3}} dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(2/3),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.20

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}\,dx = \frac{\sqrt{3}d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcn\log\left(F\right)} - \frac{d^{\frac{2}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcn\log\left(F\right)} + \frac{d^{\frac{2}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcn\log\left(F\right)} + \frac{3\,\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}}{2\,bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(2/3),x, algorithm="maxima")
```

output
```
sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) +
d^(1/3))/d^(1/3))/(b*c*n*log(F)) - 1/2*d^(2/3)*log((F^(b*c*n*x + a*c*n)*e
+ d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*n*l
og(F)) + d^(2/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*n*l
og(F)) + 3/2*(F^(b*c*n*x + a*c*n)*e + d)^(2/3)/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.99

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}\,dx = \frac{2\,\sqrt{3}d^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right) - d^{\frac{2}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+\cdots\right.\right.}{2\,bcn\,l\cdots}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(2/3),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(
1/3) + d^(1/3))/d^(1/3)) - d^(2/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(2/3) +
 (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) + 2*d^(2/3)*log(abs(
(F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))) + 3*(F^(b*c*n*x + a*c*n)*e +
 d)^(2/3))/(b*c*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3} dx = \int \left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{2/3} dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^(2/3),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^(2/3), x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3} dx = \int \left(f^{bcnx+acn}e+d\right)^{\frac{2}{3}} dx$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(2/3),x)
```

output
```
int((f**(a*c*n + b*c*n*x)*e + d)**(2/3),x)
```

3.12 $\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 159

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx = -\frac{1}{2}\sqrt[3]{d}x + \frac{3\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{bcn\log(F)} - \frac{\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcn\log(F)} + \frac{3\sqrt[3]{d}\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcn\log(F)}$$

output
```
-1/2*d^(1/3)*x+3*(d+e*(F^(c*(b*x+a)))^n)^(1/3)/b/c/n/ln(F)-3^(1/2)*d^(1/3)
*arctan(1/3*(d^(1/3)+2*(d+e*(F^(c*(b*x+a)))^n)^(1/3))*3^(1/2)/d^(1/3))/b/c
/n/ln(F)+3/2*d^(1/3)*ln(d^(1/3)-(d+e*(F^(c*(b*x+a)))^n)^(1/3))/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx$$

$$= \frac{6\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n} - 2\sqrt{3}\sqrt[3]{d}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2\sqrt[3]{d}\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right.}{2bcn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(1/3),x]
```

output
```
(6*(d + e*(F^(c*(a + b*x)))^n)^(1/3) - 2*Sqrt[3]*d^(1/3)*ArcTan[(1 + (2*(d
 + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]] + 2*d^(1/3)*Log[d^(1/3)
 - (d + e*(F^(c*(a + b*x)))^n)^(1/3)] - d^(1/3)*Log[d^(2/3) + d^(1/3)*(d +
 e*(F^(c*(a + b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^n)^(2/3)])/(2*b*c
*n*Log[F])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.91, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2720, 798, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int F^{-c(a+bx)}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\,dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int F^{-c(a+bx)}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\,d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{d\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}d\left(F^{c(a+bx)}\right)^n+3\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{bcn\log(F)}$$

$$\Big\downarrow \text{ 69}$$

$$\frac{d\left(-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2d^{2/3}}-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx}\cdots}{2\sqrt[3]{d}}\cdots\right.}{bcn\log(F)\cdots}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{d\left(-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(}\cdots\right.\right.}{2d^{\cdots}}\cdots\right.}{bcn\log(F)\cdots}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{d\left(\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}\right)+3\sqrt[3]{e\left(F^{c}\cdots\right.}}{bcn\log(F)\cdots}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{d\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}\right)+3\sqrt[3]{e\left(F^{c(a+bx}\right.}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(1/3),x]
```

output
```
(3*(d + e*(F^(c*(a + b*x)))^n)^(1/3) + d*(-((Sqrt[3]*ArcTan[(1 + (2*(d + e
*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]])/d^(2/3)) - Log[(F^(c*(a +
b*x)))^n]/(2*d^(2/3)) + (3*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)
])/(2*d^(2/3))))/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{3\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+3\Bigg(\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{3d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{6d^{\frac{2}{3}}}-}{\ln(F)bcn}$
default	$\frac{3\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+3\Bigg(\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{3d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{6d^{\frac{2}{3}}}-}{\ln(F)bcn}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(3*(d+e*(F^(c*(b*x+a)))^n)^(1/3)+3*(1/3/d^(2/3)*ln((d+e*(F^(
c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/6/d^(2/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)
+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3)+d^(2/3))-1/3/d^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3)+1)))*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^{n}}\,dx =$$

$$-\frac{2\sqrt{3}d^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{2}{3}}+\sqrt{3}d}{3\,d}\right)+d^{\frac{1}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*d^(1/3)*arctan(1/3*(2*sqrt(3)*(F^(b*c*n*x + a*c*n)*e + d)^
(1/3)*d^(2/3) + sqrt(3)*d)/d) + d^(1/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(2
/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) - 2*d^(1/3)*log
((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)) - 6*(F^(b*c*n*x + a*c*n)*e +
 d)^(1/3))/(b*c*n*log(F))
```

Sympy [F]

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(1/3),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.21

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx=-\frac{\sqrt{3}d^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcn\log\left(F\right)}-\frac{d^{\frac{1}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcn\log\left(F\right)}+\frac{d^{\frac{1}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcn\log\left(F\right)}+\frac{3\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}}{bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*d^(1/3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) +
 d^(1/3))/d^(1/3))/(b*c*n*log(F)) - 1/2*d^(1/3)*log((F^(b*c*n*x + a*c*n)*e
 + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*n*
log(F)) + d^(1/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*n*
log(F)) + 3*(F^(b*c*n*x + a*c*n)*e + d)^(1/3)/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx = -\frac{2\sqrt{3}d^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)+d^{\frac{1}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/3),x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(3)*d^(1/3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^
(1/3) + d^(1/3))/d^(1/3)) + d^(1/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(2/3)
+ (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) - 2*d^(1/3)*log(abs
((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))) - 6*(F^(b*c*n*x + a*c*n)*e
+ d)^(1/3))/(b*c*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{1/3}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^(1/3),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}dx$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(1/3),x)
```

output
```
int((f**(a*c*n + b*c*n*x)*e + d)**(1/3),x)
```

3.13 $\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=-\frac{x}{2\sqrt[3]{d}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bc\sqrt[3]{d}n\log(F)}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bc\sqrt[3]{d}n\log(F)}$$

output
```
-1/2*x/d^(1/3)+3^(1/2)*arctan(1/3*(d^(1/3)+2*(d+e*(F^(c*(b*x+a)))^n)^(1/3)
)*3^(1/2)/d^(1/3))/b/c/d^(1/3)/n/ln(F)+3/2*ln(d^(1/3)-(d+e*(F^(c*(b*x+a)))
^n)^(1/3))/b/c/d^(1/3)/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$$

$$= \frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)-\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{d+e\left(\right.\right.}{2bc\sqrt[3]{d}n\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-1/3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt
[3]] + 2*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)] - Log[d^(2/3) +
d^(1/3)*(d + e*(F^(c*(a + b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^n)^(2
/3)])/(2*b*c*d^(1/3)*n*Log[F])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2720, 798, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{67}$$

$$\frac{\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+}}{2\sqrt[3]{d}}}{bcn\log(F)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx}\right.\right.}{2\sqrt[3]{d}}}{bcn\log(F)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{-\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{bcn\log(F)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-1/3),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[
3]])/d^(1/3) - Log[(F^(c*(a + b*x)))^n]/(2*d^(1/3)) + (3*Log[d^(1/3) - (d
+ e*(F^(c*(a + b*x)))^n)^(1/3)])/(2*d^(1/3)))/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b}\right.\right.}{d^{\frac{1}{3}}}\right.}{3}\right.}{d^{\frac{1}{3}}}}{\ln(F)bcn}$
default	$\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b}\right.\right.}{d^{\frac{1}{3}}}\right.}{3}\right.}{d^{\frac{1}{3}}}}{\ln(F)bcn}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(1/d^(1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/2/d^(
1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3
)+d^(2/3))+3^(1/2)/d^(1/3)*arctan(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^(c*(b*x+a
)))^n)^(1/3)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 3.01

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^{n}}}\,dx$$

$$= \left[\frac{\sqrt{3}d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}d^{\frac{2}{3}}\sqrt{-\frac{1}{d^{\frac{2}{3}}}}-\sqrt{3}d^{\frac{4}{3}}\sqrt{-\frac{1}{d^{\frac{2}{3}}}}+2F^{bcnx+acn}e-\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}\left(\sqrt{3}d\sqrt{-\frac{1}{d^{\frac{2}{3}}}}+3d^{\frac{2}{3}}\right)+3d}{F^{bcnx+acn}}\right)}{2\,bcdn\log}\right.$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(1/3),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*d*sqrt(-1/d^(2/3))*log((2*sqrt(3)*(F^(b*c*n*x + a*c*n)*e + d
)^(2/3)*d^(2/3)*sqrt(-1/d^(2/3)) - sqrt(3)*d^(4/3)*sqrt(-1/d^(2/3)) + 2*F^
(b*c*n*x + a*c*n)*e - (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*(sqrt(3)*d*sqrt(-1
/d^(2/3)) + 3*d^(2/3)) + 3*d)/F^(b*c*n*x + a*c*n)) - d^(2/3)*log((F^(b*c*n
*x + a*c*n)*e + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(
2/3)) + 2*d^(2/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)))/(b*c*d
*n*log(F)), 1/2*(2*sqrt(3)*d^(2/3)*arctan(1/3*sqrt(3) + 2/3*sqrt(3)*(F^(b*
c*n*x + a*c*n)*e + d)^(1/3)/d^(1/3)) - d^(2/3)*log((F^(b*c*n*x + a*c*n)*e
+ d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) + 2*d^(2
/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)))/(b*c*d*n*log(F))]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(1/3),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-1/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcd^{\frac{1}{3}}n\log\left(F\right)}-\frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcd^{\frac{1}{3}}n\log\left(F\right)}+\frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcd^{\frac{1}{3}}n\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(1/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) + d^(1/3))
/d^(1/3))/(b*c*d^(1/3)*n*log(F)) - 1/2*log((F^(b*c*n*x + a*c*n)*e + d)^(2/
3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*d^(1/3)*n*l
og(F)) + log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*d^(1/3)*n*l
og(F))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \frac{\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{d^{\frac{1}{3}}} - \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{d^{\frac{1}{3}}} + \frac{2\log\left(\left|\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right|\right)}{d^{\frac{1}{3}}}}{2\,bcn\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(1/3),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) + d
^(1/3))/d^(1/3))/d^(1/3) - log((F^(b*c*n*x + a*c*n)*e + d)^(2/3) + (F^(b*c
*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/d^(1/3) + 2*log(abs((F^(b*c*
n*x + a*c*n)*e + d)^(1/3) - d^(1/3)))/d^(1/3))/(b*c*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx = \int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{1/3}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(1/3),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(1/3), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{1}{\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(1/3),x)
```

output
```
int(1/(f**(a*c*n + b*c*n*x)*e + d)**(1/3),x)
```

3.14 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx = -\frac{x}{2d^{2/3}} - \frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcd^{2/3}n\log(F)} + \frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcd^{2/3}n\log(F)}$$

output
```
-1/2*x/d^(2/3)-3^(1/2)*arctan(1/3*(d^(1/3)+2*(d+e*(F^(c*(b*x+a)))^n)^(1/3)
)*3^(1/2)/d^(1/3))/b/c/d^(2/3)/n/ln(F)+3/2*ln(d^(1/3)-(d+e*(F^(c*(b*x+a)))
^n)^(1/3))/b/c/d^(2/3)/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)+\log\left(d^{2/3}+\sqrt[3]{d}\sqrt[3]{d+e\left(F\right.}\right.}{2bcd^{2/3}n\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-2/3),x]
```

output
```
-1/2*(2*Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))
/Sqrt[3]] - 2*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)] + Log[d^(2/
3) + d^(1/3)*(d + e*(F^(c*(a + b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^
n)^(2/3)])/(b*c*d^(2/3)*n*Log[F])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2720, 798, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$\Big\downarrow$ 69

$$\frac{-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2d^{2/3}}-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n}}{2\sqrt[3]{d}}}{bcn\log(F)}$$

$\Big\downarrow$ 16

$$\frac{-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx}\right.\right.}{2d^{2/3}}}{bcn\log(F)}$$

$\Big\downarrow$ 1082

$$\frac{\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}}{bcn\log(F)}$$

$\Big\downarrow$ 217

$$\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-2/3),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqr
t[3]])/d^(2/3)) - Log[(F^(c*(a + b*x)))^n]/(2*d^(2/3)) + (3*Log[d^(1/3) -
(d + e*(F^(c*(a + b*x)))^n)^(1/3)])/(2*d^(2/3)))/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b}\right.\right.}{d^{\frac{1}{3}}}\right.}{3}\right.}{d^{\frac{2}{3}}}}{\ln(F)bcn}$
default	$\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b}\right.\right.}{d^{\frac{1}{3}}}\right.}{3}\right.}{d^{\frac{2}{3}}}}{\ln(F)bcn}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(1/d^(2/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/2/d^(
2/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(1/3
)+d^(2/3))-1/d^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^(c*(b*x
+a)))^n)^(1/3)+1)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2/3}}\,dx =$$

$$-\frac{2\sqrt{3}(d^2)^{\frac{1}{6}}d\arctan\left(\frac{\sqrt{3}\sqrt{d^2}d+2\sqrt{3}(d^2)^{\frac{5}{6}}\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}}{3d^2}\right)+(d^2)^{\frac{2}{3}}\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}d+(d^2)^{\frac{1}{3}}d+(d^2)^{\frac{2}{3}}\right(}{2bcd^2n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(2/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*(d^2)^(1/6)*d*arctan(1/3*(sqrt(3)*sqrt(d^2)*d + 2*sqrt(3)*
(d^2)^(5/6)*(F^(b*c*n*x + a*c*n)*e + d)^(1/3))/d^2) + (d^2)^(2/3)*log((F^(
b*c*n*x + a*c*n)*e + d)^(2/3)*d + (d^2)^(1/3)*d + (d^2)^(2/3)*(F^(b*c*n*x
+ a*c*n)*e + d)^(1/3)) - 2*(d^2)^(2/3)*log((F^(b*c*n*x + a*c*n)*e + d)^(1/
3)*d - (d^2)^(2/3)))/(b*c*d^2*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx=\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(2/3),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-2/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcd^{\frac{2}{3}}n\log\left(F\right)}-\frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcd^{\frac{2}{3}}n\log\left(F\right)}+\frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcd^{\frac{2}{3}}n\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(2/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) + d^(1/3)
)/d^(1/3))/(b*c*d^(2/3)*n*log(F)) - 1/2*log((F^(b*c*n*x + a*c*n)*e + d)^(2
/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*d^(2/3)*n*
log(F)) + log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*d^(2/3)*n*
log(F))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx =$$

$$-\frac{\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{d^{\frac{2}{3}}}+\frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{d^{\frac{2}{3}}}-\frac{2\log\left(\left|\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right|\right)}{d^{\frac{2}{3}}}}{2\,bcn\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(2/3),x, algorithm="giac")
```

output
```
-1/2*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) +
d^(1/3))/d^(1/3))/d^(2/3) + log((F^(b*c*n*x + a*c*n)*e + d)^(2/3) + (F^(b*
c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/d^(2/3) - 2*log(abs((F^(b*c
*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)))/d^(2/3))/(b*c*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx = \int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{2/3}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(2/3),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(2/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}}\,dx = \int \frac{1}{\left(f^{bcnx+acn}e+d\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(2/3),x)
```

output
```
int(1/(f**(a*c*n + b*c*n*x)*e + d)**(2/3),x)
```

3.15 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 161

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx = -\frac{x}{2d^{4/3}}+\frac{3}{bcd\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcd^{4/3}n\log(F)} + \frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcd^{4/3}n\log(F)}$$

output

```
-1/2*x/d^(4/3)+3/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(1/3)/n/ln(F)+3^(1/2)*arcta
n(1/3*(d^(1/3)+2*(d+e*(F^(c*(b*x+a)))^n)^(1/3))*3^(1/2)/d^(1/3))/b/c/d^(4/
3)/n/ln(F)+3/2*ln(d^(1/3)-(d+e*(F^(c*(b*x+a)))^n)^(1/3))/b/c/d^(4/3)/n/ln(
F)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx = \frac{\frac{6\sqrt[3]{d}}{\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}+2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2\log\left(\sqrt[3]{d}\right.}{2b}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-4/3),x]
```

output
```
((6*d^(1/3))/(d + e*(F^(c*(a + b*x)))^n)^(1/3) + 2*Sqrt[3]*ArcTan[(1 + (2*
(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]] + 2*Log[d^(1/3) - (d
+ e*(F^(c*(a + b*x)))^n)^(1/3)] - Log[d^(2/3) + d^(1/3)*(d + e*(F^(c*(a +
b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^n)^(2/3)])/(2*b*c*d^(4/3)*n*Log
[F])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2720, 798, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}dx$$

$$\Big\downarrow \ 2720$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \ 798$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$\downarrow$ 61

$$\frac{\frac{\int \frac{F^{-c(a+bx)}}{\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{3}{d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}}{bcn\log(F)}$$

$\downarrow$ 67

$$\frac{\frac{\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2\sqrt[3]{d}}}{d}}{bcn\log(F)}$$

$\downarrow$ 16

$$\frac{\frac{\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{d}+}{bcn\log(F)}$$

$\downarrow$ 1082

$$\frac{\frac{-\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{d}+\frac{3}{d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n}}}{bcn\log(F)}$$

$\downarrow$ 217

$$\frac{\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{d}+\frac{3}{d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-4/3),x]
```

output
```
(3/(d*(d + e*(F^(c*(a + b*x)))^n)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(d + e
*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]])/d^(1/3) - Log[(F^(c*(a + b
*x)))^n]/(2*d^(1/3)) + (3*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)]
)/(2*d^(1/3)))/d)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b\cdots}\right.\right.}{d^{\frac{1}{3}}}\cdots\right)}{3}\right)}{d^{\frac{1}{3}}}}{d}\cdots}{\ln(F)bcn}$
default	$\frac{\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^n\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{1}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b\cdots}\right.\right.}{d^{\frac{1}{3}}}\cdots\right)}{3}\right)}{d^{\frac{1}{3}}}}{d}\cdots}{\ln(F)bcn}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(3*(1/3/d^(1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/
6/d^(1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)
^(1/3)+d^(2/3))+1/3*3^(1/2)/d^(1/3)*arctan(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^
(c*(b*x+a)))^n)^(1/3)+1)))/d+3/d/(d+e*(F^(c*(b*x+a)))^n)^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 560, normalized size of antiderivative = 3.48

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx = \left[\frac{\sqrt{3}\left(F^{bcnx+acn}de+d^2\right)\sqrt{-\frac{1}{d^{\frac{2}{3}}}}\log\left(\frac{2\sqrt{3}\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}d^{\frac{2}{3}}\sqrt{-\frac{1}{d^{\frac{2}{3}}}}-\sqrt{3}d^{\frac{4}{3}}\sqrt{-\frac{1}{d^{\frac{2}{3}}}}+2\cdots}{F\cdots}\right.\cdots}{\cdots}\right.$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(4/3),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*(F^(b*c*n*x + a*c*n)*d*e + d^2)*sqrt(-1/d^(2/3))*log((2*sqrt
(3)*(F^(b*c*n*x + a*c*n)*e + d)^(2/3)*d^(2/3)*sqrt(-1/d^(2/3)) - sqrt(3)*d
^(4/3)*sqrt(-1/d^(2/3)) + 2*F^(b*c*n*x + a*c*n)*e - (F^(b*c*n*x + a*c*n)*e
 + d)^(1/3)*(sqrt(3)*d*sqrt(-1/d^(2/3)) + 3*d^(2/3)) + 3*d)/F^(b*c*n*x + a
*c*n)) - (F^(b*c*n*x + a*c*n)*d^(2/3)*e + d^(5/3))*log((F^(b*c*n*x + a*c*n
)*e + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)) + 2*
(F^(b*c*n*x + a*c*n)*d^(2/3)*e + d^(5/3))*log((F^(b*c*n*x + a*c*n)*e + d)^
(1/3) - d^(1/3)) + 6*(F^(b*c*n*x + a*c*n)*e + d)^(2/3)*d)/(F^(b*c*n*x + a*
c*n)*b*c*d^2*e*n*log(F) + b*c*d^3*n*log(F)), 1/2*(2*sqrt(3)*(F^(b*c*n*x +
a*c*n)*d*e + d^2)*arctan(1/3*sqrt(3) + 2/3*sqrt(3)*(F^(b*c*n*x + a*c*n)*e
+ d)^(1/3)/d^(1/3))/d^(1/3) - (F^(b*c*n*x + a*c*n)*d^(2/3)*e + d^(5/3))*lo
g((F^(b*c*n*x + a*c*n)*e + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^
(1/3) + d^(2/3)) + 2*(F^(b*c*n*x + a*c*n)*d^(2/3)*e + d^(5/3))*log((F^(b*c
*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)) + 6*(F^(b*c*n*x + a*c*n)*e + d)^(2/3
)*d)/(F^(b*c*n*x + a*c*n)*b*c*d^2*e*n*log(F) + b*c*d^3*n*log(F))]
```

Sympy [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx = \int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(4/3),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-4/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcd^{\frac{4}{3}}n\log\left(F\right)} - \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcd^{\frac{4}{3}}n\log\left(F\right)} + \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcd^{\frac{4}{3}}n\log\left(F\right)} + \frac{3}{\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}bcdn\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(4/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) + d^(1/3))
/d^(1/3))/(b*c*d^(4/3)*n*log(F)) - 1/2*log((F^(b*c*n*x + a*c*n)*e + d)^(2/
3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*d^(4/3)*n*l
og(F)) + log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*d^(4/3)*n*l
og(F)) + 3/((F^(b*c*n*x + a*c*n)*e + d)^(1/3)*b*c*d*n*log(F))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(136) = 272.

Time = 0.16 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.73

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx = \frac{\left(\pi-\pi\mathrm{sgn}(F)-2\sqrt{3}\log\left(|F|\right)\right)\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{\pi^2bcd^{\frac{4}{3}}n\mathrm{sgn}\left(F\right)-\pi^2bcd^{\frac{4}{3}}n-2\,bcd^{\frac{4}{3}}n\log\left(|F|\right)^2} - \frac{\left(\sqrt{3}\pi\mathrm{sgn}(F)-\sqrt{3}\pi-2\,\log\left(|F|\right)\right)\log\left(\left(F^{bcnx}F^{acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\left(\pi^2bcd^{\frac{4}{3}}n\mathrm{sgn}\left(F\right)-\pi^2bcd^{\frac{4}{3}}n-2\,bcd^{\frac{4}{3}}n\log\left(|F|\right)^2\right)} + \frac{\log\left(\left|\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right|\right)}{bcd^{\frac{4}{3}}n\log\left(F\right)} + \frac{3}{\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}bcdn\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(4/3),x, algorithm="giac")
```

output
```
(pi - pi*sgn(F) - 2*sqrt(3)*log(abs(F)))*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x
)*F^(a*c*n)*e + d)^(1/3) + d^(1/3))/d^(1/3))/(pi^2*b*c*d^(4/3)*n*sgn(F) -
pi^2*b*c*d^(4/3)*n - 2*b*c*d^(4/3)*n*log(abs(F))^2) - 1/2*(sqrt(3)*pi*sgn(
F) - sqrt(3)*pi - 2*log(abs(F)))*log((F^(b*c*n*x)*F^(a*c*n)*e + d)^(2/3) +
 (F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(pi^2*b*c*d^(4/3)*
n*sgn(F) - pi^2*b*c*d^(4/3)*n - 2*b*c*d^(4/3)*n*log(abs(F))^2) + log(abs((
F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3) - d^(1/3)))/(b*c*d^(4/3)*n*log(F)) + 3/
((F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3)*b*c*d*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx=\int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{4/3}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(4/3),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(4/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}\,dx=\int \frac{1}{f^{bcnx+acn}\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}e+\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}d}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(4/3),x)
```

output
```
int(1/(f**(a*c*n + b*c*n*x)*(f**(a*c*n + b*c*n*x)*e + d)**(1/3)*e + (f**(a
*c*n + b*c*n*x)*e + d)**(1/3)*d),x)
```

3.16 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 164

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx=-\frac{x}{2d^{5/3}}+\frac{3}{2bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}n\log(F)}-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcd^{5/3}n\log(F)}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcd^{5/3}n\log(F)}$$

output
```
-1/2*x/d^(5/3)+3/2/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(2/3)/n/ln(F)-3^(1/2)*arc
tan(1/3*(d^(1/3)+2*(d+e*(F^(c*(b*x+a)))^n)^(1/3))*3^(1/2)/d^(1/3))/b/c/d^(
5/3)/n/ln(F)+3/2*ln(d^(1/3)-(d+e*(F^(c*(b*x+a)))^n)^(1/3))/b/c/d^(5/3)/n/l
n(F)
```

Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx = \frac{\frac{3d^{2/3}}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{2/3}} - 2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 2\log\left(\sqrt[3]{d} - \cdots\right.}{2bc\cdots}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-5/3),x]
```

output
```
((3*d^(2/3))/(d + e*(F^(c*(a + b*x)))^n)^(2/3) - 2*Sqrt[3]*ArcTan[(1 + (2*
(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]] + 2*Log[d^(1/3) - (d
+ e*(F^(c*(a + b*x)))^n)^(1/3)] - Log[d^(2/3) + d^(1/3)*(d + e*(F^(c*(a +
b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)))^n)^(2/3)])/(2*b*c*d^(5/3)*n*Log
[F])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2720, 798, 61, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/3}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/3}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/3}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{61}$$

$$\frac{\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{3}{2d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}}{bcn\log(F)}$$

$$\Big\downarrow \text{69}$$

$$\frac{\frac{-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2d^{2/3}}-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+}}{2\sqrt[3]{d}}}{d}}{bcn\log(F)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{-\frac{3\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}}{d}}{bcn\log(F)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}}{d}+\frac{3}{2d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}}{bcn\log(F)}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{d^{2/3}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2d^{2/3}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2d^{2/3}}}{d}+\frac{3}{2d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{2/3}}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-5/3),x]
```

output
```
(3/(2*d*(d + e*(F^(c*(a + b*x)))^n)^(2/3)) + (-((Sqrt[3]*ArcTan[(1 + (2*(d
 + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3))/Sqrt[3]])/d^(2/3)) - Log[(F^(c*(
a + b*x)))^n]/(2*d^(2/3)) + (3*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(
1/3)])/(2*d^(2/3)))/d)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.97

method	result
derivativedivides	$\frac{\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b\cdots}\right.\right.}{d^{\frac{1}{3}}}\cdots\right)}{3}\right)}{d^{\frac{2}{3}}}\cdots}{d}\cdots}{\ln(F)bcn}$
default	$\frac{\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2d^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(d+e\left(F^{c(b\cdots}\right.\right.}{d^{\frac{1}{3}}}\cdots\right)}{3}\right)}{d^{\frac{2}{3}}}\cdots}{d}\cdots}{\ln(F)bcn}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(3*(1/3/d^(2/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/
6/d^(2/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)
^(1/3)+d^(2/3))-1/3/d^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^
(c*(b*x+a)))^n)^(1/3)+1)))/d+3/2/d/(d+e*(F^(c*(b*x+a)))^n)^(2/3))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.66

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx = -\frac{2\sqrt{3}\left(F^{bcnx+acn}de+d^2\right)\left(d^2\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\sqrt{d^2}d+2\sqrt{3}\left(d^2\right)^{\frac{5}{6}}\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}}{3d^2}\right)-3\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^2+\left(F^{bcn}\right.}{}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(5/3),x, algorithm="fricas")
```

output
```
-1/2*(2*sqrt(3)*(F^(b*c*n*x + a*c*n)*d*e + d^2)*(d^2)^(1/6)*arctan(1/3*(sq
rt(3)*sqrt(d^2)*d + 2*sqrt(3)*(d^2)^(5/6)*(F^(b*c*n*x + a*c*n)*e + d)^(1/3
))/d^2) - 3*(F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^2 + (F^(b*c*n*x + a*c*n)*(
d^2)^(2/3)*e + (d^2)^(2/3)*d)*log((F^(b*c*n*x + a*c*n)*e + d)^(2/3)*d + (d
^2)^(1/3)*d + (d^2)^(2/3)*(F^(b*c*n*x + a*c*n)*e + d)^(1/3)) - 2*(F^(b*c*n
*x + a*c*n)*(d^2)^(2/3)*e + (d^2)^(2/3)*d)*log((F^(b*c*n*x + a*c*n)*e + d)
^(1/3)*d - (d^2)^(2/3)))/(F^(b*c*n*x + a*c*n)*b*c*d^3*e*n*log(F) + b*c*d^4
*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx = \int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(5/3),x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-5/3), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcd^{\frac{5}{3}}n\log(F)} - \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcd^{\frac{5}{3}}n\log(F)} + \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcd^{\frac{5}{3}}n\log(F)} + \frac{3}{2\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(5/3),x, algorithm="maxima")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) + d^(1/3)
)/d^(1/3))/(b*c*d^(5/3)*n*log(F)) - 1/2*log((F^(b*c*n*x + a*c*n)*e + d)^(2
/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*d^(5/3)*n*
log(F)) + log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*d^(5/3)*n*
log(F)) + 3/2/((F^(b*c*n*x + a*c*n)*e + d)^(2/3)*b*c*d*n*log(F))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(137) = 274.

Time = 0.14 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.70

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx = \frac{\left(\pi-\pi\operatorname{sgn}(F)+2\sqrt{3}\log(|F|)\right)\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{\pi^2bcd^{\frac{5}{3}}n\operatorname{sgn}(F)-\pi^2bcd^{\frac{5}{3}}n-2\,bcd^{\frac{5}{3}}n\log(|F|)^2} + \frac{\left(\sqrt{3}\pi\operatorname{sgn}(F)-\sqrt{3}\pi+2\log(|F|)\right)\log\left(\left(F^{bcnx}F^{acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\left(\pi^2bcd^{\frac{5}{3}}n\operatorname{sgn}(F)-\pi^2bcd^{\frac{5}{3}}n-2\,bcd^{\frac{5}{3}}n\log(|F|)^2\right)} + \frac{\log\left(\left|\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right|\right)}{bcd^{\frac{5}{3}}n\log(F)} + \frac{3}{2\left(F^{bcnx}F^{acn}e+d\right)^{\frac{2}{3}}bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(5/3),x, algorithm="giac")
```

output
```
(pi - pi*sgn(F) + 2*sqrt(3)*log(abs(F)))*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x
)*F^(a*c*n)*e + d)^(1/3) + d^(1/3))/d^(1/3))/(pi^2*b*c*d^(5/3)*n*sgn(F) -
pi^2*b*c*d^(5/3)*n - 2*b*c*d^(5/3)*n*log(abs(F))^2) + 1/2*(sqrt(3)*pi*sgn(
F) - sqrt(3)*pi + 2*log(abs(F)))*log((F^(b*c*n*x)*F^(a*c*n)*e + d)^(2/3) +
 (F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(pi^2*b*c*d^(5/3)*
n*sgn(F) - pi^2*b*c*d^(5/3)*n - 2*b*c*d^(5/3)*n*log(abs(F))^2) + log(abs((
F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3) - d^(1/3)))/(b*c*d^(5/3)*n*log(F)) + 3/
2/((F^(b*c*n*x)*F^(a*c*n)*e + d)^(2/3)*b*c*d*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx=\int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{5/3}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(5/3),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(5/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/3}}\,dx=\int \frac{1}{f^{bcnx+acn}\left(f^{bcnx+acn}e+d\right)^{\frac{2}{3}}e+\left(f^{bcnx+acn}e+d\right)^{\frac{2}{3}}d}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(5/3),x)
```

output
```
int(1/(f**(a*c*n + b*c*n*x)*(f**(a*c*n + b*c*n*x)*e + d)**(2/3)*e + (f**(a
*c*n + b*c*n*x)*e + d)**(2/3)*d),x)
```

3.17 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = -\frac{x}{2d^{7/3}} + \frac{3}{4bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}n\log(F)} + \frac{3}{bcd^2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{d}+2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{3}\sqrt[3]{d}}\right)}{bcd^{7/3}n\log(F)} + \frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}\right)}{2bcd^{7/3}n\log(F)}$$

output

```
-1/2*x/d^(7/3)+3/4/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(4/3)/n/ln(F)+3/b/c/d^2/(
d+e*(F^(c*(b*x+a)))^n)^(1/3)/n/ln(F)+3^(1/2)*arctan(1/3*(d^(1/3)+2*(d+e*(F
^(c*(b*x+a)))^n)^(1/3))*3^(1/2)/d^(1/3))/b/c/d^(7/3)/n/ln(F)+3/2*ln(d^(1/3
)-(d+e*(F^(c*(b*x+a)))^n)^(1/3))/b/c/d^(7/3)/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \frac{\frac{3\sqrt[3]{d}\left(5d+4e\left(F^{c(a+bx)}\right)^n\right)}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{4/3}}+4\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt[3]{d}}}{\sqrt{3}}\right)+4\log\left(\sqrt[3]{d} \cdots\right.}{4 \cdots}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-7/3),x]
```

output
```
((3*d^(1/3)*(5*d + 4*e*(F^(c*(a + b*x)))^n))/(d + e*(F^(c*(a + b*x)))^n)^(
4/3) + 4*Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/d^(1/3)
)/Sqrt[3]] + 4*Log[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)] - 2*Log[d^
(2/3) + d^(1/3)*(d + e*(F^(c*(a + b*x)))^n)^(1/3) + (d + e*(F^(c*(a + b*x)
))^n)^(2/3)])/(4*b*c*d^(7/3)*n*Log[F])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2720, 798, 61, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{7/3}}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{7/3}}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{7/3}}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$\Big\downarrow$ 61

$$\frac{\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{3}{4d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}}{bcn\log(F)}$$

$\Big\downarrow$ 61

$$\frac{\frac{\frac{\int \frac{F^{-c(a+bx)}}{\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\left(F^{c(a+bx)}\right)^n}{d}+\frac{3}{d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}}{d}+\frac{3}{4d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{4/3}}}{bcn\log(F)}$$

$\Big\downarrow$ 67

$$\frac{\frac{\frac{\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}-\frac{3\int \frac{1}{\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{2\sqrt[3]{d}}}{d}}{d}}{bcn\log(F)}$$

$\Big\downarrow$ 16

$$\frac{\frac{\frac{\frac{3}{2}\int \frac{1}{F^{2c(a+bx)}+d^{2/3}+\sqrt[3]{d}\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{d}+}{d}}{bcn\log(F)}$$

$\Big\downarrow$ 1082

$$\frac{\frac{-\frac{3\int \frac{1}{-F^{2c(a+bx)}-3}d\left(\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{d}+\frac{3}{d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+}}}{d}}{bcn\log(F)}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt[3]{d}}+1}{\sqrt{3}}\right)}{\sqrt[3]{d}}+\frac{3\log\left(\sqrt[3]{d}-\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}\right)}{2\sqrt[3]{d}}-\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2\sqrt[3]{d}}}{d}+\frac{3}{d\sqrt[3]{e\left(F^{c(a+bx)}\right)^n+d}}}{d}+\frac{}{4d(e}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-7/3),x]
```

output
```
(3/(4*d*(d + e*(F^(c*(a + b*x)))^n)^(4/3)) + (3/(d*(d + e*(F^(c*(a + b*x))
)^n)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(d + e*(F^(c*(a + b*x)))^n)^(1/3))/
d^(1/3))/Sqrt[3]])/d^(1/3) - Log[(F^(c*(a + b*x)))^n]/(2*d^(1/3)) + (3*Log
[d^(1/3) - (d + e*(F^(c*(a + b*x)))^n)^(1/3)])/(2*d^(1/3)))/d)/d)/(b*c*n*L
og[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{3}{d^{2}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}}+\frac{3}{4d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{4}{3}}}+\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(\right.\right.\right.}{2d^{\frac{1}{3}}}}{d}}{\ln(F)bcn}$
default	$\frac{\frac{3}{d^{2}\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}}+\frac{3}{4d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{4}{3}}}+\frac{\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{d^{\frac{1}{3}}}-\frac{\ln\left(\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{2}{3}}+d^{\frac{1}{3}}\left(d+e\left(\right.\right.\right.}{2d^{\frac{1}{3}}}}{d}}{\ln(F)bcn}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(3/d^2/(d+e*(F^(c*(b*x+a)))^n)^(1/3)+3/4/d/(d+e*(F^(c*(b*x+a
)))^n)^(4/3)+3*(1/3/d^(1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(1/3)-d^(1/3))-1/6/
d^(1/3)*ln((d+e*(F^(c*(b*x+a)))^n)^(2/3)+d^(1/3)*(d+e*(F^(c*(b*x+a)))^n)^(
1/3)+d^(2/3))+1/3*3^(1/2)/d^(1/3)*arctan(1/3*3^(1/2)*(2/d^(1/3)*(d+e*(F^(c
*(b*x+a)))^n)^(1/3)+1)))/d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(171) = 342$.

Time = 0.10 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.94

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(7/3),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(3)*(2*F^(b*c*n*x + a*c*n)*d^2*e + F^(2*b*c*n*x + 2*a*c*n)*d*e
^2 + d^3)*sqrt(-1/d^(2/3))*log((2*sqrt(3)*(F^(b*c*n*x + a*c*n)*e + d)^(2/3
)*d^(2/3)*sqrt(-1/d^(2/3)) - sqrt(3)*d^(4/3)*sqrt(-1/d^(2/3)) + 2*F^(b*c*n
*x + a*c*n)*e - (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*(sqrt(3)*d*sqrt(-1/d^(2/
3)) + 3*d^(2/3)) + 3*d)/F^(b*c*n*x + a*c*n)) - 2*(2*F^(b*c*n*x + a*c*n)*d^
(5/3)*e + F^(2*b*c*n*x + 2*a*c*n)*d^(2/3)*e^2 + d^(8/3))*log((F^(b*c*n*x +
 a*c*n)*e + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)
) + 4*(2*F^(b*c*n*x + a*c*n)*d^(5/3)*e + F^(2*b*c*n*x + 2*a*c*n)*d^(2/3)*e
^2 + d^(8/3))*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)) + 3*(4*F^(b
*c*n*x + a*c*n)*d*e + 5*d^2)*(F^(b*c*n*x + a*c*n)*e + d)^(2/3))/(2*F^(b*c*
n*x + a*c*n)*b*c*d^4*e*n*log(F) + F^(2*b*c*n*x + 2*a*c*n)*b*c*d^3*e^2*n*lo
g(F) + b*c*d^5*n*log(F)), 1/4*(4*sqrt(3)*(2*F^(b*c*n*x + a*c*n)*d^2*e + F^
(2*b*c*n*x + 2*a*c*n)*d*e^2 + d^3)*arctan(1/3*sqrt(3) + 2/3*sqrt(3)*(F^(b*
c*n*x + a*c*n)*e + d)^(1/3)/d^(1/3))/d^(1/3) - 2*(2*F^(b*c*n*x + a*c*n)*d^
(5/3)*e + F^(2*b*c*n*x + 2*a*c*n)*d^(2/3)*e^2 + d^(8/3))*log((F^(b*c*n*x +
 a*c*n)*e + d)^(2/3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3)
) + 4*(2*F^(b*c*n*x + a*c*n)*d^(5/3)*e + F^(2*b*c*n*x + 2*a*c*n)*d^(2/3)*e
^2 + d^(8/3))*log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3)) + 3*(4*F^(b
*c*n*x + a*c*n)*d*e + 5*d^2)*(F^(b*c*n*x + a*c*n)*e + d)^(2/3))/(2*F^(b*c*
n*x + a*c*n)*b*c*d^4*e*n*log(F) + F^(2*b*c*n*x + 2*a*c*n)*b*c*d^3*e^2*n...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**(7/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.07

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{bcd^{\frac{7}{3}}n\log\left(F\right)} - \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\,bcd^{\frac{7}{3}}n\log\left(F\right)} + \frac{\log\left(\left(F^{bcnx+acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right)}{bcd^{\frac{7}{3}}n\log\left(F\right)} + \frac{3\left(4\,F^{bcnx+acn}e+5\,d\right)}{4\left(F^{bcnx+acn}e+d\right)^{\frac{4}{3}}bcd^2n\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(7/3),x, algorithm="maxima")
```

output
```
sqrt(3)*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x + a*c*n)*e + d)^(1/3) + d^(1/3))
/d^(1/3))/(b*c*d^(7/3)*n*log(F)) - 1/2*log((F^(b*c*n*x + a*c*n)*e + d)^(2/
3) + (F^(b*c*n*x + a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(b*c*d^(7/3)*n*l
og(F)) + log((F^(b*c*n*x + a*c*n)*e + d)^(1/3) - d^(1/3))/(b*c*d^(7/3)*n*l
og(F)) + 3/4*(4*F^(b*c*n*x + a*c*n)*e + 5*d)/((F^(b*c*n*x + a*c*n)*e + d)^
(4/3)*b*c*d^2*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \frac{\left(\pi-\pi\mathrm{sgn}(F)-2\sqrt{3}\log\left(|F|\right)\right)\arctan\left(\frac{\sqrt{3}\left(2\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}+d^{\frac{1}{3}}\right)}{3\,d^{\frac{1}{3}}}\right)}{\pi^2bcd^{\frac{7}{3}}n\mathrm{sgn}\left(F\right)-\pi^2bcd^{\frac{7}{3}}n-2\,bcd^{\frac{7}{3}}n\log\left(|F|\right)^2} - \frac{\left(\sqrt{3}\pi\mathrm{sgn}(F)-\sqrt{3}\pi-2\,\log\left(|F|\right)\right)\log\left(\left(F^{bcnx}F^{acn}e+d\right)^{\frac{2}{3}}+\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}d^{\frac{1}{3}}+d^{\frac{2}{3}}\right)}{2\left(\pi^2bcd^{\frac{7}{3}}n\mathrm{sgn}\left(F\right)-\pi^2bcd^{\frac{7}{3}}n-2\,bcd^{\frac{7}{3}}n\log\left(|F|\right)^2\right)} + \frac{\log\left(\left|\left(F^{bcnx}F^{acn}e+d\right)^{\frac{1}{3}}-d^{\frac{1}{3}}\right|\right)}{bcd^{\frac{7}{3}}n\log\left(F\right)} + \frac{3\left(4\,F^{bcnx}F^{acn}e+5\,d\right)}{4\left(F^{bcnx}F^{acn}e+d\right)^{\frac{4}{3}}bcd^2n\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^(7/3),x, algorithm="giac")
```

output
```
(pi - pi*sgn(F) - 2*sqrt(3)*log(abs(F)))*arctan(1/3*sqrt(3)*(2*(F^(b*c*n*x
)*F^(a*c*n)*e + d)^(1/3) + d^(1/3))/d^(1/3))/(pi^2*b*c*d^(7/3)*n*sgn(F) -
pi^2*b*c*d^(7/3)*n - 2*b*c*d^(7/3)*n*log(abs(F))^2) - 1/2*(sqrt(3)*pi*sgn(
F) - sqrt(3)*pi - 2*log(abs(F)))*log((F^(b*c*n*x)*F^(a*c*n)*e + d)^(2/3) +
 (F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3)*d^(1/3) + d^(2/3))/(pi^2*b*c*d^(7/3)*
n*sgn(F) - pi^2*b*c*d^(7/3)*n - 2*b*c*d^(7/3)*n*log(abs(F))^2) + log(abs((
F^(b*c*n*x)*F^(a*c*n)*e + d)^(1/3) - d^(1/3)))/(b*c*d^(7/3)*n*log(F)) + 3/
4*(4*F^(b*c*n*x)*F^(a*c*n)*e + 5*d)/((F^(b*c*n*x)*F^(a*c*n)*e + d)^(4/3)*b
*c*d^2*n*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \int \frac{1}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{7/3}}\,dx$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(7/3),x)
```

output
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^(7/3), x)
```

Reduce [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/3}}\,dx = \int \frac{1}{f^{2bcnx+2acn}\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}e^2+2f^{bcnx+acn}\left(f^{bcnx+acn}e+d\right)^{\frac{1}{3}}de+\left(f^{bcnx}\right.}$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^(7/3),x)
```

output
```
int(1/(f**(2*a*c*n + 2*b*c*n*x)*(f**(a*c*n + b*c*n*x)*e + d)**(1/3)*e**2 +
 2*f**(a*c*n + b*c*n*x)*(f**(a*c*n + b*c*n*x)*e + d)**(1/3)*d*e + (f**(a*c
*n + b*c*n*x)*e + d)**(1/3)*d**2),x)
```

3.18 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx = -\frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,\frac{d+e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcdn(1+p)\log(F)}$$

output
```
-(d+e*(F^(c*(b*x+a)))^n)^(p+1)*hypergeom([1, p+1],[2+p],(d+e*(F^(c*(b*x+a)
))^n)/d)/b/c/d/n/(p+1)/ln(F)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx = -\frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{1+p}\text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcdn(1+p)\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^p,x]
```

output
```
-(((d + e*(F^(c*(a + b*x)))^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (e*(F^(c*(a + b*x)))^n)/d])/(b*c*d*n*(1 + p)*Log[F]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2720, 798, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right)^p\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^p dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{ 798}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^p d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{p+1}\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcdn(p+1)\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^p,x]
```

output
```
-(((d + e*(F^(c*(a + b*x)))^n)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p,
1 + (e*(F^(c*(a + b*x)))^n)/d])/(b*c*d*n*(1 + p)*Log[F]))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [F]

$$\int \left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{p}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^p,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^p,x)
```

Fricas [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{p}\,dx=\int \left(\left(F^{(bx+a)c}\right)^{n}e+d\right)^{p}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^p,x, algorithm="fricas")
```

output
```
integral(((F^(b*c*x + a*c))^n*e + d)^p, x)
```

Sympy [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx=\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**p,x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**p, x)
```

Maxima [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^p\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^p,x, algorithm="maxima")
```

output
```
integrate((F^((b*x + a)*c*n)*e + d)^p, x)
```

Giac [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^p\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^p,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx=\int \left(d+e\,{\left(F^{c\,\left(a+b\,x\right)}\right)}^n\right)^p\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^p,x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^p, x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p\,dx=\int \left(f^{bcnx+acn}e+d\right)^p\,dx$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^p,x)
```

output
```
int((f**(a*c*n + b*c*n*x)*e + d)**p,x)
```

3.19 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 153

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx=\frac{d(f+gx)^4}{4g}-\frac{6e\left(F^{ac+bcx}\right)^n g^3}{b^4c^4n^4\log^4(F)}+\frac{6e\left(F^{ac+bcx}\right)^n g^2(f+gx)}{b^3c^3n^3\log^3(F)}-\frac{3e\left(F^{ac+bcx}\right)^n g(f+gx)^2}{b^2c^2n^2\log^2(F)}+\frac{e\left(F^{ac+bcx}\right)^n (f+gx)^3}{bcn\log(F)}$$

output
```
1/4*d*(g*x+f)^4/g-6*e*(F^(b*c*x+a*c))^n*g^3/b^4/c^4/n^4/ln(F)^4+6*e*(F^(b*
c*x+a*c))^n*g^2*(g*x+f)/b^3/c^3/n^3/ln(F)^3-3*e*(F^(b*c*x+a*c))^n*g*(g*x+f
)^2/b^2/c^2/n^2/ln(F)^2+e*(F^(b*c*x+a*c))^n*(g*x+f)^3/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.85

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx = df^3x+\frac{3}{2}df^2gx^2+dfg^2x^3+\frac{1}{4}dg^3x^4 + \frac{e\left(F^{c(a+bx)}\right)^n\left(-6g^3+6bcg^2n(f+gx)\log(F)-3b^2c^2gn^2(f+gx)^2\log^2(F)+b^3c^3n^3(f+gx)^3\log^3(F)\right)}{b^4c^4n^4\log^4(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^3,x]
```

output
```
d*f^3*x + (3*d*f^2*g*x^2)/2 + d*f*g^2*x^3 + (d*g^3*x^4)/4 + (e*(F^(c*(a +
b*x)))^n*(-6*g^3 + 6*b*c*g^2*n*(f + g*x)*Log[F] - 3*b^2*c^2*g*n^2*(f + g*x
)^2*Log[F]^2 + b^3*c^3*n^3*(f + g*x)^3*Log[F]^3))/(b^4*c^4*n^4*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3\left(e\left(F^{c(a+bx)}\right)^n+d\right)dx$$

$$\Big\downarrow \; 2614$$

$$\int \left(e(f+gx)^3\left(F^{ac+bcx}\right)^n+d(f+gx)^3\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{6eg^3\left(F^{ac+bcx}\right)^n}{b^4c^4n^4\log^4(F)}+\frac{6eg^2(f+gx)\left(F^{ac+bcx}\right)^n}{b^3c^3n^3\log^3(F)}-\frac{3eg(f+gx)^2\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log^2(F)}+\frac{e(f+gx)^3\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{d(f+gx)^4}{4g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^3,x]
```

output
```
(d*(f + g*x)^4)/(4*g) - (6*e*(F^(a*c + b*c*x))^n*g^3)/(b^4*c^4*n^4*Log[F]^
4) + (6*e*(F^(a*c + b*c*x))^n*g^2*(f + g*x))/(b^3*c^3*n^3*Log[F]^3) - (3*e
*(F^(a*c + b*c*x))^n*g*(f + g*x)^2)/(b^2*c^2*n^2*Log[F]^2) + (e*(F^(a*c +
b*c*x))^n*(f + g*x)^3)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(151) = 302$.

Time = 0.46 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.56

method	result
parallelrisch	$\frac{d\,g^3x^4n^4c^4b^4\ln(F)^4+4d\,g^2f\,x^3n^4c^4b^4\ln(F)^4+6dg\,f^2x^2n^4c^4b^4\ln(F)^4+4d\,f^3x\,n^4c^4b^4\ln(F)^4+4x^3\left(F^{c(bx+a)}\right)^n e\,g^3n^3c^3b^3 \ln}{}$
orering	$\frac{\left(\ln(F)^4b^4c^4g^4n^4x^5+5\ln(F)^4b^4c^4f\,g^3n^4x^4+10\ln(F)^4b^4c^4f^2g^2n^4x^3+10\ln(F)^4b^4c^4f^3g\,n^4x^2+4\ln(F)^4b^4c^4f^4n^4x+3\ln(F)\right.}{}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)*(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(d*g^3*x^4*n^4*c^4*b^4*ln(F)^4+4*d*g^2*f*x^3*n^4*c^4*b^4*ln(F)^4+6*d*g
*f^2*x^2*n^4*c^4*b^4*ln(F)^4+4*d*f^3*x*n^4*c^4*b^4*ln(F)^4+4*x^3*(F^(c*(b*
x+a)))^n*e*g^3*n^3*c^3*b^3*ln(F)^3+12*ln(F)^3*x^2*(F^(c*(b*x+a)))^n*b^3*c^
3*e*f*g^2*n^3+12*ln(F)^3*x*(F^(c*(b*x+a)))^n*b^3*c^3*e*f^2*g*n^3+4*ln(F)^3
*(F^(c*(b*x+a)))^n*b^3*c^3*e*f^3*n^3-12*ln(F)^2*x^2*(F^(c*(b*x+a)))^n*b^2*
c^2*e*g^3*n^2-24*ln(F)^2*x*(F^(c*(b*x+a)))^n*b^2*c^2*e*f*g^2*n^2-12*ln(F)^
2*(F^(c*(b*x+a)))^n*b^2*c^2*e*f^2*g*n^2+24*ln(F)*x*(F^(c*(b*x+a)))^n*b*c*e
*g^3*n+24*ln(F)*(F^(c*(b*x+a)))^n*b*c*e*f*g^2*n-24*(F^(c*(b*x+a)))^n*e*g^3
)/n^4/c^4/b^4/ln(F)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.75

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx$$
$$= \frac{\left(b^4c^4dg^3n^4x^4+4\,b^4c^4dfg^2n^4x^3+6\,b^4c^4df^2gn^4x^2+4\,b^4c^4df^3n^4x\right)\log\left(F\right)^4-4\left(6\,eg^3-\left(b^3c^3eg^3n^3x^3+3\right.\right.}{}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^3,x, algorithm="fricas")
```

output
```
1/4*((b^4*c^4*d*g^3*n^4*x^4 + 4*b^4*c^4*d*f*g^2*n^4*x^3 + 6*b^4*c^4*d*f^2*
g*n^4*x^2 + 4*b^4*c^4*d*f^3*n^4*x)*log(F)^4 - 4*(6*e*g^3 - (b^3*c^3*e*g^3*
n^3*x^3 + 3*b^3*c^3*e*f*g^2*n^3*x^2 + 3*b^3*c^3*e*f^2*g*n^3*x + b^3*c^3*e*
f^3*n^3)*log(F)^3 + 3*(b^2*c^2*e*g^3*n^2*x^2 + 2*b^2*c^2*e*f*g^2*n^2*x + b
^2*c^2*e*f^2*g*n^2)*log(F)^2 - 6*(b*c*e*g^3*n*x + b*c*e*f*g^2*n)*log(F))*F
^(b*c*n*x + a*c*n))/(b^4*c^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(151) = 302$.

Time = 1.69 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.24

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx$$
$$=\begin{cases}(d+e)\left(f^3x+\frac{3f^2gx^2}{2}+fg^2x^3+\frac{g^3x^4}{4}\right)\\ (d+e(F^{ac})^n)\left(f^3x+\frac{3f^2gx^2}{2}+fg^2x^3+\frac{g^3x^4}{4}\right)\\ (d+e)\left(f^3x+\frac{3f^2gx^2}{2}+fg^2x^3+\frac{g^3x^4}{4}\right)\\ df^3x+\frac{3df^2gx^2}{2}+dfg^2x^3+\frac{dg^3x^4}{4}+\frac{ef^3(F^{ac+bcx})^n}{bcn\log(F)}+\frac{3ef^2gx(F^{ac+bcx})^n}{bcn\log(F)}+\frac{3efg^2x^2(F^{ac+bcx})^n}{bcn\log(F)}+\frac{eg^3x^3(F^{ac+bcx})^n}{bcn\log(F)}-\end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)*(g*x+f)**3,x)
```

output
```
Piecewise(((d + e)*(f**3*x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4),
 Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), ((d + e*(F**(a*c))**n)*(f**3*
x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4), Eq(b, 0)), ((d + e)*(f**
3*x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4), Eq(F, 1) | Eq(c, 0) |
Eq(n, 0)), (d*f**3*x + 3*d*f**2*g*x**2/2 + d*f*g**2*x**3 + d*g**3*x**4/4 +
 e*f**3*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + 3*e*f**2*g*x*(F**(a*c + b*c
*x))**n/(b*c*n*log(F)) + 3*e*f*g**2*x**2*(F**(a*c + b*c*x))**n/(b*c*n*log(
F)) + e*g**3*x**3*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) - 3*e*f**2*g*(F**(a
*c + b*c*x))**n/(b**2*c**2*n**2*log(F)**2) - 6*e*f*g**2*x*(F**(a*c + b*c*x
))**n/(b**2*c**2*n**2*log(F)**2) - 3*e*g**3*x**2*(F**(a*c + b*c*x))**n/(b*
*2*c**2*n**2*log(F)**2) + 6*e*f*g**2*(F**(a*c + b*c*x))**n/(b**3*c**3*n**3
*log(F)**3) + 6*e*g**3*x*(F**(a*c + b*c*x))**n/(b**3*c**3*n**3*log(F)**3)
- 6*e*g**3*(F**(a*c + b*c*x))**n/(b**4*c**4*n**4*log(F)**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.84

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx = \frac{1}{4}\,dg^3x^4 + dfg^2x^3 + \frac{3}{2}\,df^2gx^2 + df^3x + \frac{F^{bcnx+acn}ef^3}{bcn\log\left(F\right)} + \frac{3\left(F^{acn}bcnx\log\left(F\right) - F^{acn}\right)F^{bcnx}ef^2g}{b^2c^2n^2\log\left(F\right)^2} + \frac{3\left(F^{acn}b^2c^2n^2x^2\log\left(F\right)^2 - 2\,F^{acn}bcnx\log\left(F\right) + 2\,F^{acn}\right)F^{bcnx}efg^2}{b^3c^3n^3\log\left(F\right)^3} + \frac{\left(F^{acn}b^3c^3n^3x^3\log\left(F\right)^3 - 3\,F^{acn}b^2c^2n^2x^2\log\left(F\right)^2 + 6\,F^{acn}bcnx\log\left(F\right) - 6\,F^{acn}\right)F^{bcnx}eg^3}{b^4c^4n^4\log\left(F\right)^4}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^3,x, algorithm="maxima")
```

output
```
1/4*d*g^3*x^4 + d*f*g^2*x^3 + 3/2*d*f^2*g*x^2 + d*f^3*x + F^(b*c*n*x + a*c
*n)*e*f^3/(b*c*n*log(F)) + 3*(F^(a*c*n)*b*c*n*x*log(F) - F^(a*c*n))*F^(b*c
*n*x)*e*f^2*g/(b^2*c^2*n^2*log(F)^2) + 3*(F^(a*c*n)*b^2*c^2*n^2*x^2*log(F)
^2 - 2*F^(a*c*n)*b*c*n*x*log(F) + 2*F^(a*c*n))*F^(b*c*n*x)*e*f*g^2/(b^3*c^
3*n^3*log(F)^3) + (F^(a*c*n)*b^3*c^3*n^3*x^3*log(F)^3 - 3*F^(a*c*n)*b^2*c^
2*n^2*x^2*log(F)^2 + 6*F^(a*c*n)*b*c*n*x*log(F) - 6*F^(a*c*n))*F^(b*c*n*x)
*e*g^3/(b^4*c^4*n^4*log(F)^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 5716, normalized size of antiderivative = 37.36

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx = \text{Too large to display}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^3,x, algorithm="giac")
```

output
```
1/4*d*g^3*x^4 + d*f*g^2*x^3 + 3/2*d*f^2*g*x^2 + d*f^3*x - (((3*pi^2*b^3*c^
3*e*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*e*g^3*n^3*x^3*log(abs(
F)) + 2*b^3*c^3*e*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*b^3*c^3*e*f*g^2*n^3*x
^2*log(abs(F))*sgn(F) - 9*pi^2*b^3*c^3*e*f*g^2*n^3*x^2*log(abs(F)) + 6*b^3
*c^3*e*f*g^2*n^3*x^2*log(abs(F))^3 + 9*pi^2*b^3*c^3*e*f^2*g*n^3*x*log(abs(
F))*sgn(F) - 9*pi^2*b^3*c^3*e*f^2*g*n^3*x*log(abs(F)) + 6*b^3*c^3*e*f^2*g*
n^3*x*log(abs(F))^3 + 3*pi^2*b^3*c^3*e*f^3*n^3*log(abs(F))*sgn(F) - 3*pi^2
*b^3*c^3*e*f^3*n^3*log(abs(F)) + 2*b^3*c^3*e*f^3*n^3*log(abs(F))^3 - 3*pi^
2*b^2*c^2*e*g^3*n^2*x^2*sgn(F) + 3*pi^2*b^2*c^2*e*g^3*n^2*x^2 - 6*b^2*c^2*
e*g^3*n^2*x^2*log(abs(F))^2 - 6*pi^2*b^2*c^2*e*f*g^2*n^2*x*sgn(F) + 6*pi^2
*b^2*c^2*e*f*g^2*n^2*x - 12*b^2*c^2*e*f*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b
^2*c^2*e*f^2*g*n^2*sgn(F) + 3*pi^2*b^2*c^2*e*f^2*g*n^2 - 6*b^2*c^2*e*f^2*g
*n^2*log(abs(F))^2 + 12*b*c*e*g^3*n*x*log(abs(F)) + 12*b*c*e*f*g^2*n*log(a
bs(F)) - 12*e*g^3)*(pi^4*b^4*c^4*n^4*sgn(F) - 6*pi^2*b^4*c^4*n^4*log(abs(F
))^2*sgn(F) - pi^4*b^4*c^4*n^4 + 6*pi^2*b^4*c^4*n^4*log(abs(F))^2 - 2*b^4*
c^4*n^4*log(abs(F))^4)/((pi^4*b^4*c^4*n^4*sgn(F) - 6*pi^2*b^4*c^4*n^4*log(
abs(F))^2*sgn(F) - pi^4*b^4*c^4*n^4 + 6*pi^2*b^4*c^4*n^4*log(abs(F))^2 - 2
*b^4*c^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*n^4*log(abs(F))*sgn(F) -
pi*b^4*c^4*n^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*n^4*log(abs(F)) + pi*b^
4*c^4*n^4*log(abs(F))^3)^2) - 4*(pi^3*b^3*c^3*e*g^3*n^3*x^3*sgn(F) - 3*...
```

Mupad [B] (verification not implemented)

Time = 23.05 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.47

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx = d\,f^3\,x - \left(F^{b\,c\,x}\,F^{a\,c}\right)^n\left(\frac{e\left(-b^3\,c^3\,f^3\,n^3\ln\left(F\right)^3+3\,b^2\,c^2\,f^2\,g\,n^2\ln\left(F\right)^2-6\,b\,c\,f\,g^2\,n\,\ln\left(F\right)+6\,g^3\right)}{b^4\,c^4\,n^4\ln\left(F\right)^4} - \frac{e\,g^3\,x^3}{b\,c\,n\,\ln\left(F\right)} - \frac{3\,e\,g\,x\left(b^2\,c^2\,f^2\,n^2\ln\left(F\right)^2-2\,b\,c\,f\,g\,n\,\ln\left(F\right)+2\,g^2\right)}{b^3\,c^3\,n^3\ln\left(F\right)^3} + \frac{3\,e\,g^2\,x^2\left(g-b\,c\,f\,n\,\ln\left(F\right)\right)}{b^2\,c^2\,n^2\ln\left(F\right)^2}\right) + \frac{d\,g^3\,x^4}{4} + \frac{3\,d\,f^2\,g\,x^2}{2} + d\,f\,g^2\,x^3$$

input
```
int((f + g*x)^3*(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
d*f^3*x - (F^(b*c*x)*F^(a*c))^n*((e*(6*g^3 - b^3*c^3*f^3*n^3*log(F)^3 - 6*
b*c*f*g^2*n*log(F) + 3*b^2*c^2*f^2*g*n^2*log(F)^2))/(b^4*c^4*n^4*log(F)^4)
 - (e*g^3*x^3)/(b*c*n*log(F)) - (3*e*g*x*(2*g^2 + b^2*c^2*f^2*n^2*log(F)^2
 - 2*b*c*f*g*n*log(F)))/(b^3*c^3*n^3*log(F)^3) + (3*e*g^2*x^2*(g - b*c*f*n
*log(F)))/(b^2*c^2*n^2*log(F)^2)) + (d*g^3*x^4)/4 + (3*d*f^2*g*x^2)/2 + d*
f*g^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.62

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^3\,dx$$
$$= \frac{4f^{bcnx+acn}\log(f)^3\,b^3c^3e\,f^3n^3+12f^{bcnx+acn}\log(f)^3\,b^3c^3e\,f^2g\,n^3x+12f^{bcnx+acn}\log(f)^3\,b^3c^3ef\,g^2n^3x^2+4f}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^3,x)
```

output
```
(4*f**(a*c*n + b*c*n*x)*log(f)**3*b**3*c**3*e*f**3*n**3 + 12*f**(a*c*n + b
*c*n*x)*log(f)**3*b**3*c**3*e*f**2*g*n**3*x + 12*f**(a*c*n + b*c*n*x)*log(
f)**3*b**3*c**3*e*f*g**2*n**3*x**2 + 4*f**(a*c*n + b*c*n*x)*log(f)**3*b**3
*c**3*e*g**3*n**3*x**3 - 12*f**(a*c*n + b*c*n*x)*log(f)**2*b**2*c**2*e*f**
2*g*n**2 - 24*f**(a*c*n + b*c*n*x)*log(f)**2*b**2*c**2*e*f*g**2*n**2*x - 1
2*f**(a*c*n + b*c*n*x)*log(f)**2*b**2*c**2*e*g**3*n**2*x**2 + 24*f**(a*c*n
 + b*c*n*x)*log(f)*b*c*e*f*g**2*n + 24*f**(a*c*n + b*c*n*x)*log(f)*b*c*e*g
**3*n*x - 24*f**(a*c*n + b*c*n*x)*e*g**3 + 4*log(f)**4*b**4*c**4*d*f**3*n*
*4*x + 6*log(f)**4*b**4*c**4*d*f**2*g*n**4*x**2 + 4*log(f)**4*b**4*c**4*d*
f*g**2*n**4*x**3 + log(f)**4*b**4*c**4*d*g**3*n**4*x**4)/(4*log(f)**4*b**4
*c**4*n**4)
```

3.20 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx=\frac{d(f+gx)^3}{3g}+\frac{2e\left(F^{ac+bcx}\right)^n g^2}{b^3c^3n^3\log^3(F)}-\frac{2e\left(F^{ac+bcx}\right)^n g(f+gx)}{b^2c^2n^2\log^2(F)}+\frac{e\left(F^{ac+bcx}\right)^n(f+gx)^2}{bcn\log(F)}$$

output
```
1/3*d*(g*x+f)^3/g+2*e*(F^(b*c*x+a*c))^n*g^2/b^3/c^3/n^3/ln(F)^3-2*e*(F^(b*
c*x+a*c))^n*g*(g*x+f)/b^2/c^2/n^2/ln(F)^2+e*(F^(b*c*x+a*c))^n*(g*x+f)^2/b/
c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx = df^2x+dfgx^2+\frac{1}{3}dg^2x^3+\frac{e\left(F^{c(a+bx)}\right)^n\left(2g^2-2bcgn(f+gx)\log(F)+b^2c^2n^2(f+gx)^2\log^2(F)\right)}{b^3c^3n^3\log^3(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^2,x]
```

output
```
d*f^2*x + d*f*g*x^2 + (d*g^2*x^3)/3 + (e*(F^(c*(a + b*x)))^n*(2*g^2 - 2*b*
c*g*n*(f + g*x)*Log[F] + b^2*c^2*n^2*(f + g*x)^2*Log[F]^2))/(b^3*c^3*n^3*L
og[F]^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2 \left(e\left(F^{c(a+bx)}\right)^n+d\right) dx$$

$$\Big\downarrow \text{ 2614}$$

$$\int \left(e(f+gx)^2 \left(F^{ac+bcx}\right)^n+d(f+gx)^2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2eg^2\left(F^{ac+bcx}\right)^n}{b^3c^3n^3\log^3(F)}-\frac{2eg(f+gx)\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log^2(F)}+\frac{e(f+gx)^2\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{d(f+gx)^3}{3g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^2,x]
```

output
```
(d*(f + g*x)^3)/(3*g) + (2*e*(F^(a*c + b*c*x))^n*g^2)/(b^3*c^3*n^3*Log[F]^
3) - (2*e*(F^(a*c + b*c*x))^n*g*(f + g*x))/(b^2*c^2*n^2*Log[F]^2) + (e*(F^
(a*c + b*c*x))^n*(f + g*x)^2)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

method	result
norman	$d\,f^2 x + df g\,x^2 + \frac{e\left(\ln(F)^2 b^2 c^2 f^2 n^2 - 2\ln(F) bcfgn + 2g^2\right) \mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{\ln(F)^3 b^3 c^3 n^3} + \frac{e\,g^2 x^2 \mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{\ln(F) bcn} + \frac{d\,g}{}$
parallelrisch	$\frac{d\,g^2 x^3 \ln(F)^3 b^3 c^3 n^3 + 3dfg\,x^2 \ln(F)^3 b^3 c^3 n^3 + 3d\,f^2 x \ln(F)^3 b^3 c^3 n^3 + 3x^2 \left(F^{c(bx+a)}\right)^n e\,g^2 \ln(F)^2 b^2 c^2 n^2 + 6\ln(F)^2 x \left(F^{c(bx+a}\right.}{3\ln(F}$
orering	$\frac{\left(\ln(F)^3 b^3 c^3 g^3 n^3 x^4 + 4\ln(F)^3 b^3 c^3 f\,g^2 n^3 x^3 + 6\ln(F)^3 b^3 c^3 f^2 g\,n^3 x^2 + 3\ln(F)^3 b^3 c^3 f^3 n^3 x + 2\ln(F)^2 b^2 c^2 g^3 n^2 x^3 + 6\ln(F)^2 b^2 c\right.}{3\ln}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)*(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
d*f^2*x+d*f*g*x^2+e*(ln(F)^2*b^2*c^2*f^2*n^2-2*ln(F)*b*c*f*g*n+2*g^2)/ln(F
)^3/b^3/c^3/n^3*exp(n*ln(exp(c*(b*x+a)*ln(F))))+1/ln(F)/b/c/n*e*g^2*x^2*ex
p(n*ln(exp(c*(b*x+a)*ln(F))))+1/3*d*g^2*x^3+2*e*g*(ln(F)*b*c*f*n-g)/ln(F)^
2/b^2/c^2/n^2*x*exp(n*ln(exp(c*(b*x+a)*ln(F))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx$$
$$=\frac{(b^3c^3dg^2n^3x^3+3\,b^3c^3dfgn^3x^2+3\,b^3c^3df^2n^3x)\log(F)^3+3\left(2\,eg^2+(b^2c^2eg^2n^2x^2+2\,b^2c^2efgn^2x+b^2c^2e\right.}{3\,b^3c^3n^3\log(F)^3}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^2,x, algorithm="fricas")
```

output
```
1/3*((b^3*c^3*d*g^2*n^3*x^3 + 3*b^3*c^3*d*f*g*n^3*x^2 + 3*b^3*c^3*d*f^2*n^
3*x)*log(F)^3 + 3*(2*e*g^2 + (b^2*c^2*e*g^2*n^2*x^2 + 2*b^2*c^2*e*f*g*n^2*
x + b^2*c^2*e*f^2*n^2)*log(F)^2 - 2*(b*c*e*g^2*n*x + b*c*e*f*g*n)*log(F))*
F^(b*c*n*x + a*c*n))/(b^3*c^3*n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(110) = 220.

Time = 0.71 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.56

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx$$
$$=\begin{cases}(d+e)\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right)\\ \left(d+e(F^{ac})^n\right)\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right)\\ (d+e)\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right)\\ df^2x+dfgx^2+\frac{dg^2x^3}{3}+\frac{ef^2\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{2efgx\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{eg^2x^2\left(F^{ac+bcx}\right)^n}{bcn\log(F)}-\frac{2efg\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log(F)^2}-\frac{2eg^2x\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log(F)^2}\end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)*(g*x+f)**2,x)
```

output
```
Piecewise(((d + e)*(f**2*x + f*g*x**2 + g**2*x**3/3), Eq(F, 1) & Eq(b, 0)
& Eq(c, 0) & Eq(n, 0)), ((d + e*(F**(a*c))**n)*(f**2*x + f*g*x**2 + g**2*x
**3/3), Eq(b, 0)), ((d + e)*(f**2*x + f*g*x**2 + g**2*x**3/3), Eq(F, 1) |
Eq(c, 0) | Eq(n, 0)), (d*f**2*x + d*f*g*x**2 + d*g**2*x**3/3 + e*f**2*(F**
(a*c + b*c*x))**n/(b*c*n*log(F)) + 2*e*f*g*x*(F**(a*c + b*c*x))**n/(b*c*n*
log(F)) + e*g**2*x**2*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) - 2*e*f*g*(F**(
a*c + b*c*x))**n/(b**2*c**2*n**2*log(F)**2) - 2*e*g**2*x*(F**(a*c + b*c*x)
)**n/(b**2*c**2*n**2*log(F)**2) + 2*e*g**2*(F**(a*c + b*c*x))**n/(b**3*c**
3*n**3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx = \frac{1}{3}\,dg^2x^3+dfgx^2+df^2x+\frac{F^{bcnx+acn}ef^2}{bcn\log\left(F\right)}+\frac{2\left(F^{acn}bcnx\log\left(F\right)-F^{acn}\right)F^{bcnx}efg}{b^2c^2n^2\log\left(F\right)^2} + \frac{\left(F^{acn}b^2c^2n^2x^2\log\left(F\right)^2-2\,F^{acn}bcnx\log\left(F\right)+2\,F^{acn}\right)F^{bcnx}eg^2}{b^3c^3n^3\log\left(F\right)^3}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^2,x, algorithm="maxima")
```

output
```
1/3*d*g^2*x^3 + d*f*g*x^2 + d*f^2*x + F^(b*c*n*x + a*c*n)*e*f^2/(b*c*n*log
(F)) + 2*(F^(a*c*n)*b*c*n*x*log(F) - F^(a*c*n))*F^(b*c*n*x)*e*f*g/(b^2*c^2
*n^2*log(F)^2) + (F^(a*c*n)*b^2*c^2*n^2*x^2*log(F)^2 - 2*F^(a*c*n)*b*c*n*x
*log(F) + 2*F^(a*c*n))*F^(b*c*n*x)*e*g^2/(b^3*c^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 2716, normalized size of antiderivative = 23.62

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx = \text{Too large to display}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^2,x, algorithm="giac")
```

output
```
1/3*d*g^2*x^3 + d*f*g*x^2 + d*f^2*x - ((2*(pi*b^2*c^2*e*g^2*n^2*x^2*log(ab
s(F))*sgn(F) - pi*b^2*c^2*e*g^2*n^2*x^2*log(abs(F)) + 2*pi*b^2*c^2*e*f*g*n
^2*x*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*e*f*g*n^2*x*log(abs(F)) + pi*b^2*c^
2*e*f^2*n^2*log(abs(F))*sgn(F) - pi*b^2*c^2*e*f^2*n^2*log(abs(F)) - pi*b*c
*e*g^2*n*x*sgn(F) + pi*b*c*e*g^2*n*x - pi*b*c*e*f*g*n*sgn(F) + pi*b*c*e*f*
g*n)*(pi^3*b^3*c^3*n^3*sgn(F) - 3*pi*b^3*c^3*n^3*log(abs(F))^2*sgn(F) - pi
^3*b^3*c^3*n^3 + 3*pi*b^3*c^3*n^3*log(abs(F))^2)/((pi^3*b^3*c^3*n^3*sgn(F)
 - 3*pi*b^3*c^3*n^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3*n^3 + 3*pi*b^3*c^3
*n^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*b^
3*c^3*n^3*log(abs(F)) + 2*b^3*c^3*n^3*log(abs(F))^3)^2) - (pi^2*b^2*c^2*e*
g^2*n^2*x^2*sgn(F) - pi^2*b^2*c^2*e*g^2*n^2*x^2 + 2*b^2*c^2*e*g^2*n^2*x^2*
log(abs(F))^2 + 2*pi^2*b^2*c^2*e*f*g*n^2*x*sgn(F) - 2*pi^2*b^2*c^2*e*f*g*n
^2*x + 4*b^2*c^2*e*f*g*n^2*x*log(abs(F))^2 + pi^2*b^2*c^2*e*f^2*n^2*sgn(F)
 - pi^2*b^2*c^2*e*f^2*n^2 + 2*b^2*c^2*e*f^2*n^2*log(abs(F))^2 - 4*b*c*e*g^
2*n*x*log(abs(F)) - 4*b*c*e*f*g*n*log(abs(F)) + 4*e*g^2)*(3*pi^2*b^3*c^3*n
^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*n^3*log(abs(F)) + 2*b^3*c^3*n^3*log
(abs(F))^3)/((pi^3*b^3*c^3*n^3*sgn(F) - 3*pi*b^3*c^3*n^3*log(abs(F))^2*sgn
(F) - pi^3*b^3*c^3*n^3 + 3*pi*b^3*c^3*n^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c
^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*n^3*log(abs(F)) + 2*b^3*c^3*n^3
*log(abs(F))^3)^2))*cos(-1/2*pi*b*c*n*x*sgn(F) + 1/2*pi*b*c*n*x - 1/2*p...
```

Mupad [B] (verification not implemented)

Time = 22.95 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx = \left(F^{b\,c\,x}\,F^{a\,c}\right)^n\left(\frac{e\left(b^2\,c^2\,f^2\,n^2\ln\left(F\right)^2-2\,b\,c\,f\,g\,n\,\ln\left(F\right)+2\,g^2\right)}{b^3\,c^3\,n^3\ln\left(F\right)^3}+\frac{e\,g^2\,x^2}{b\,c\,n\,\ln\left(F\right)}-\frac{2\,e\,g\,x\,\left(g-b\,c\,f\,n\,\ln\left(F\right)\right)}{b^2\,c^2\,n^2\ln\left(F\right)^2}\right)+d\,f^2\,x+\frac{d\,g^2\,x^3}{3}+d\,f\,g\,x^2$$

input
```
int((f + g*x)^2*(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
(F^(b*c*x)*F^(a*c))^n*((e*(2*g^2 + b^2*c^2*f^2*n^2*log(F)^2 - 2*b*c*f*g*n*
log(F)))/(b^3*c^3*n^3*log(F)^3) + (e*g^2*x^2)/(b*c*n*log(F)) - (2*e*g*x*(g
 - b*c*f*n*log(F)))/(b^2*c^2*n^2*log(F)^2)) + d*f^2*x + (d*g^2*x^3)/3 + d*
f*g*x^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.07

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2\,dx$$
$$= \frac{3f^{bcnx+acn}\log(f)^2\,b^2c^2e\,f^2n^2+6f^{bcnx+acn}\log(f)^2\,b^2c^2efg\,n^2x+3f^{bcnx+acn}\log(f)^2\,b^2c^2e\,g^2n^2x^2-6f^{bcnx+}}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^2,x)
```

output
```
(3*f**(a*c*n + b*c*n*x)*log(f)**2*b**2*c**2*e*f**2*n**2 + 6*f**(a*c*n + b*
c*n*x)*log(f)**2*b**2*c**2*e*f*g*n**2*x + 3*f**(a*c*n + b*c*n*x)*log(f)**2
*b**2*c**2*e*g**2*n**2*x**2 - 6*f**(a*c*n + b*c*n*x)*log(f)*b*c*e*f*g*n -
6*f**(a*c*n + b*c*n*x)*log(f)*b*c*e*g**2*n*x + 6*f**(a*c*n + b*c*n*x)*e*g*
*2 + 3*log(f)**3*b**3*c**3*d*f**2*n**3*x + 3*log(f)**3*b**3*c**3*d*f*g*n**
3*x**2 + log(f)**3*b**3*c**3*d*g**2*n**3*x**3)/(3*log(f)**3*b**3*c**3*n**3
)
```

3.21 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)\,dx=\frac{d(f+gx)^2}{2g}-\frac{e\left(F^{ac+bcx}\right)^n g}{b^2c^2n^2\log^2(F)}+\frac{e\left(F^{ac+bcx}\right)^n(f+gx)}{bcn\log(F)}$$

output
```
1/2*d*(g*x+f)^2/g-e*(F^(b*c*x+a*c))^n*g/b^2/c^2/n^2/ln(F)^2+e*(F^(b*c*x+a*
c))^n*(g*x+f)/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)\,dx=\frac{1}{2}dx(2f+gx)-\frac{e\left(F^{c(a+bx)}\right)^n g}{b^2c^2n^2\log^2(F)}+\frac{e\left(F^{c(a+bx)}\right)^n(f+gx)}{bcn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x),x]
```

output
```
(d*x*(2*f + g*x))/2 - (e*(F^(c*(a + b*x)))^n*g)/(b^2*c^2*n^2*Log[F]^2) + (
e*(F^(c*(a + b*x)))^n*(f + g*x))/(b*c*n*Log[F])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(e\left(F^{c(a+bx)}\right)^n+d\right)dx$$

$$\Big\downarrow \ 2614$$

$$\int \left(e(f+gx)\left(F^{ac+bcx}\right)^n+d(f+gx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{eg\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log^2(F)}+\frac{e(f+gx)\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{d(f+gx)^2}{2g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x),x]
```

output
```
(d*(f + g*x)^2)/(2*g) - (e*(F^(a*c + b*c*x))^n*g)/(b^2*c^2*n^2*Log[F]^2) +
 (e*(F^(a*c + b*c*x))^n*(f + g*x))/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

method	result
norman	$dfx+\frac{e(\ln(F)bcfn-g)\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{\ln(F)^{2}b^{2}c^{2}n^{2}}+\frac{egx\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}}{\ln(F)bcn}+\frac{dg\,x^{2}}{2}$
parallelrisch	$\frac{dg\,x^{2}\ln(F)^{2}b^{2}c^{2}n^{2}+2dfx\ln(F)^{2}b^{2}c^{2}n^{2}+2\left(F^{c(bx+a)}\right)^{n}\ln(F)bcegnx+2\left(F^{c(bx+a)}\right)^{n}\ln(F)bcefn-2\left(F^{c(bx+a)}\right)^{n}eg}{2\ln(F)^{2}b^{2}c^{2}n^{2}}$
orering	$\frac{\left(\ln(F)^{2}b^{2}c^{2}g^{2}n^{2}x^{3}+3\ln(F)^{2}b^{2}c^{2}fg\,n^{2}x^{2}+2\ln(F)^{2}b^{2}c^{2}f^{2}n^{2}x+\ln(F)bc\,g^{2}n\,x^{2}+4\ln(F)bcfgnx+2\ln(F)bc\,f^{2}n-4g^{2}x-2gf\right.}{2\ln(F)^{2}b^{2}c^{2}n^{2}(gx+f)}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)*(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
d*f*x+e*(ln(F)*b*c*f*n-g)/ln(F)^2/b^2/c^2/n^2*exp(n*ln(exp(c*(b*x+a)*ln(F)
)))+1/ln(F)/b/c/n*e*g*x*exp(n*ln(exp(c*(b*x+a)*ln(F))))+1/2*d*g*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)(f+gx)\,dx$$
$$=\frac{(b^{2}c^{2}dgn^{2}x^{2}+2\,b^{2}c^{2}dfn^{2}x)\log\left(F\right)^{2}-2\left(eg-(bcegnx+bcefn)\log\left(F\right)\right)F^{bcnx+acn}}{2\,b^{2}c^{2}n^{2}\log\left(F\right)^{2}}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f),x, algorithm="fricas")
```

output
```
1/2*((b^2*c^2*d*g*n^2*x^2 + 2*b^2*c^2*d*f*n^2*x)*log(F)^2 - 2*(e*g - (b*c*
e*g*n*x + b*c*e*f*n)*log(F))*F^(b*c*n*x + a*c*n))/(b^2*c^2*n^2*log(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(68) = 136$.

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.87

$$\int \left(d + e\left(F^{c(a+bx)}\right)^n\right)(f + gx)\,dx$$
$$= \begin{cases} (d+e)\left(fx + \frac{gx^2}{2}\right) & \text{for } F = 1 \wedge b = 0 \wedge c = 0 \wedge n = 0 \\ (d + e(F^{ac})^n)\left(fx + \frac{gx^2}{2}\right) & \text{for } b = 0 \\ (d+e)\left(fx + \frac{gx^2}{2}\right) & \text{for } F = 1 \vee c = 0 \vee n = 0 \\ dfx + \frac{dgx^2}{2} + \frac{ef\left(F^{ac+bcx}\right)^n}{bcn\log(F)} + \frac{egx\left(F^{ac+bcx}\right)^n}{bcn\log(F)} - \frac{eg\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log(F)^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)*(g*x+f),x)
```

output
```
Piecewise(((d + e)*(f*x + g*x**2/2), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n
, 0)), ((d + e*(F**(a*c))**n)*(f*x + g*x**2/2), Eq(b, 0)), ((d + e)*(f*x +
 g*x**2/2), Eq(F, 1) | Eq(c, 0) | Eq(n, 0)), (d*f*x + d*g*x**2/2 + e*f*(F*
*(a*c + b*c*x))**n/(b*c*n*log(F)) + e*g*x*(F**(a*c + b*c*x))**n/(b*c*n*log
(F)) - e*g*(F**(a*c + b*c*x))**n/(b**2*c**2*n**2*log(F)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \left(d + e\left(F^{c(a+bx)}\right)^n\right)(f + gx)\,dx = \frac{1}{2}dgx^2 + dfx + \frac{F^{bcnx+acn}ef}{bcn\log(F)} + \frac{(F^{acn}bcnx\log(F) - F^{acn})F^{bcnx}eg}{b^2c^2n^2\log(F)^2}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f),x, algorithm="maxima")
```

output
```
1/2*d*g*x^2 + d*f*x + F^(b*c*n*x + a*c*n)*e*f/(b*c*n*log(F)) + (F^(a*c*n)*
b*c*n*x*log(F) - F^(a*c*n))*F^(b*c*n*x)*e*g/(b^2*c^2*n^2*log(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 1101, normalized size of antiderivative = 14.30

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)\,dx = \text{Too large to display}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f),x, algorithm="giac")
```

output
```
1/2*d*g*x^2 + d*f*x + (2*((pi*b^2*c^2*n^2*log(abs(F))*sgn(F) - pi*b^2*c^2*
n^2*log(abs(F)))*(pi*b*c*e*g*n*x*sgn(F) - pi*b*c*e*g*n*x + pi*b*c*e*f*n*sg
n(F) - pi*b*c*e*f*n)/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^2 + 2*b^2*
c^2*n^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(F) - pi*b^2*c
^2*n^2*log(abs(F)))^2) + (pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^2 + 2*b
^2*c^2*n^2*log(abs(F))^2)*(b*c*e*g*n*x*log(abs(F)) + b*c*e*f*n*log(abs(F))
 - e*g)/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^2 + 2*b^2*c^2*n^2*log(a
bs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(F) - pi*b^2*c^2*n^2*log(ab
s(F)))^2))*cos(-1/2*pi*b*c*n*x*sgn(F) + 1/2*pi*b*c*n*x - 1/2*pi*a*c*n*sgn(
F) + 1/2*pi*a*c*n) + ((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^2 + 2*b^2*
c^2*n^2*log(abs(F))^2)*(pi*b*c*e*g*n*x*sgn(F) - pi*b*c*e*g*n*x + pi*b*c*e*
f*n*sgn(F) - pi*b*c*e*f*n)/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^2 +
2*b^2*c^2*n^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(F) - pi
*b^2*c^2*n^2*log(abs(F)))^2) - 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(F) - pi*b
^2*c^2*n^2*log(abs(F)))*(b*c*e*g*n*x*log(abs(F)) + b*c*e*f*n*log(abs(F)) -
 e*g)/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^2 + 2*b^2*c^2*n^2*log(abs
(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(F) - pi*b^2*c^2*n^2*log(abs(
F)))^2))*sin(-1/2*pi*b*c*n*x*sgn(F) + 1/2*pi*b*c*n*x - 1/2*pi*a*c*n*sgn(F)
 + 1/2*pi*a*c*n))*e^(b*c*n*x*log(abs(F)) + a*c*n*log(abs(F))) - 1/2*I*((pi
*b*c*e*g*n*x*sgn(F) - pi*b*c*e*g*n*x - 2*I*b*c*e*g*n*x*log(abs(F)) + pi...
```

Mupad [B] (verification not implemented)

Time = 22.85 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)\,dx=\frac{d\,g\,x^2}{2}-\left(F^{b\,c\,x}\,F^{a\,c}\right)^n\left(\frac{e\,(g-b\,c\,f\,n\,\ln(F))}{b^2\,c^2\,n^2\,\ln(F)^2}-\frac{e\,g\,x}{b\,c\,n\,\ln(F)}\right)+d\,f\,x$$

input
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
(d*g*x^2)/2 - (F^(b*c*x)*F^(a*c))^n*((e*(g - b*c*f*n*log(F)))/(b^2*c^2*n^2
*log(F)^2) - (e*g*x)/(b*c*n*log(F))) + d*f*x
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)\,dx$$
$$=\frac{2f^{bcnx+acn}\log(f)\,bcefn+2f^{bcnx+acn}\log(f)\,bcegnx-2f^{bcnx+acn}eg+2\log(f)^2\,b^2c^2df\,n^2x+\log(f)^2\,b^2c^2dg}{2\log(f)^2\,b^2c^2n^2}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)*(g*x+f),x)
```

output
```
(2*f**(a*c*n + b*c*n*x)*log(f)*b*c*e*f*n + 2*f**(a*c*n + b*c*n*x)*log(f)*b
*c*e*g*n*x - 2*f**(a*c*n + b*c*n*x)*e*g + 2*log(f)**2*b**2*c**2*d*f*n**2*x
 + log(f)**2*b**2*c**2*d*g*n**2*x**2)/(2*log(f)**2*b**2*c**2*n**2)
```

3.22 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx=dx+\frac{e\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

output
```
d*x+e*(F^(c*(b*x+a)))^n/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx=dx+\frac{e\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

input
```
Integrate[d + e*(F^(c*(a + b*x)))^n,x]
```

output
```
d*x + (e*(F^(c*(a + b*x)))^n)/(b*c*n*Log[F])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}+dx$$

input
```
Int[d + e*(F^(c*(a + b*x)))^n,x]
```

output
```
d*x + (e*(F^(c*(a + b*x)))^n)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$dx+\frac{e\left(F^{c(bx+a)}\right)^{n}}{bcn\ln\left(F\right)}$	31
parallelrisch	$dx+\frac{e\left(F^{c(bx+a)}\right)^{n}}{bcn\ln\left(F\right)}$	31
parts	$dx+\frac{e\left(F^{c(bx+a)}\right)^{n}}{bcn\ln\left(F\right)}$	31
norman	$dx+\frac{e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}{\ln\left(F\right)bcn}$	34
derivativedivides	$\frac{e\left(F^{c(bx+a)}\right)^{n}+d\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{\ln\left(F\right)bcn}$	43
orering	$\frac{\left(\ln\left(F\right)bcnx+1\right)\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{\ln\left(F\right)bcn}-xe\left(F^{c(bx+a)}\right)^{n}$	55

input
```
int(d+e*(F^(c*(b*x+a)))^n,x,method=_RETURNVERBOSE)
```

output
```
d*x+e*(F^(c*(b*x+a)))^n/b/c/n/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)\,dx=\frac{bcdnx\log\left(F\right)+F^{bcnx+acn}e}{bcn\log\left(F\right)}$$

input
```
integrate(d+e*(F^((b*x+a)*c))^n,x, algorithm="fricas")
```

output
```
(b*c*d*n*x*log(F) + F^(b*c*n*x + a*c*n)*e)/(b*c*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx=dx+e\left(\begin{cases} x & \text{for } F=1\wedge b=0\wedge c=0\wedge n=0\\ x\left(F^{ac}\right)^n & \text{for } b=0\\ x & \text{for } F=1\vee c=0\vee n=0\\ \frac{\left(F^{ac+bcx}\right)^n}{bcn\log\left(F\right)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(d+e*(F**((b*x+a)*c))**n,x)
```

output
```
d*x + e*Piecewise((x, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*(F**(
a*c))**n, Eq(b, 0)), (x, Eq(F, 1) | Eq(c, 0) | Eq(n, 0)), ((F**(a*c + b*c*
x))**n/(b*c*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)\,dx=dx+\frac{F^{(bx+a)cn}e}{bcn\log\left(F\right)}$$

input
```
integrate(d+e*(F^((b*x+a)*c))^n,x, algorithm="maxima")
```

output
```
d*x + F^((b*x + a)*c*n)*e/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right) dx = dx + \frac{F^{bcnx+acn}e}{bcn\log(F)}$$

input
```
integrate(d+e*(F^((b*x+a)*c))^n,x, algorithm="giac")
```

output
```
d*x + F^(b*c*n*x + a*c*n)*e/(b*c*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right) dx = d\,x + \frac{e\left(F^{a\,c+b\,c\,x}\right)^n}{b\,c\,n\,\ln(F)}$$

input
```
int(d + e*(F^(c*(a + b*x)))^n,x)
```

output
```
d*x + (e*(F^(a*c + b*c*x))^n)/(b*c*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right) dx = \frac{f^{bcnx+acn}e+\log(f)\,bcdnx}{\log(f)\,bcn}$$

input
```
int(d+e*(F^((b*x+a)*c))^n,x)
```

output
```
(f**(a*c*n + b*c*n*x)*e + log(f)*b*c*d*n*x)/(log(f)*b*c*n)
```

3.23 $\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx} dx$

Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx} dx = \frac{eF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n \text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g} + \frac{d\log(f+gx)}{g}$$

output
```
e*F^(c*(a-b*f/g)*n-c*n*(b*x+a))*(F^(b*c*x+a*c))^n*Ei(b*c*n*(g*x+f)*ln(F)/g
)/g+d*ln(g*x+f)/g
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx} dx = \frac{eF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n \text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right) + d\log(f+gx)}{g}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)/(f + g*x),x]
```

output
```
((e*(F^(c*(a + b*x)))^n*ExpIntegralEi[(b*c*n*(f + g*x)*Log[F])/g])/F^((b*c
*n*(f + g*x))/g) + d*Log[f + g*x])/g
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\left(F^{c(a+bx)}\right)^n+d}{f+gx}dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(\frac{e\left(F^{ac+bcx}\right)^n}{f+gx}+\frac{d}{f+gx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\left(F^{ac+bcx}\right)^n F^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g}+\frac{d\log(f+gx)}{g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)/(f + g*x),x]
```

output
```
(e*F^(c*(a - (b*f)/g)*n - c*n*(a + b*x))*(F^(a*c + b*c*x))^n*ExpIntegralEi
[(b*c*n*(f + g*x)*Log[F])/g])/g + (d*Log[f + g*x])/g
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{d + e\left(F^{c(bx+a)}\right)^{n}}{gx + f}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)/(g*x+f),x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)/(g*x+f),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{d + e\left(F^{c(a+bx)}\right)^{n}}{f + gx}\, dx = \frac{d\log\left(gx+f\right) + \frac{e\mathrm{Ei}\left(\frac{(bcgnx+bcfn)\log(F)}{g}\right)}{F^{\frac{(bcf-acg)n}{g}}}}{g}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f),x, algorithm="fricas")
```

output
```
(d*log(g*x + f) + e*Ei((b*c*g*n*x + b*c*f*n)*log(F)/g)/F^((b*c*f - a*c*g)*
n/g))/g
```

Sympy [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx}\,dx=\int \frac{d+e\left(F^{ac+bcx}\right)^n}{f+gx}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)/(g*x+f),x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)/(f + g*x), x)
```

Maxima [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx}\,dx=\int \frac{\left(F^{(bx+a)c}\right)^n e+d}{gx+f}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f),x, algorithm="maxima")
```

output
```
F^(a*c*n)*e*integrate(F^(b*c*n*x)/(g*x + f), x) + d*log(g*x + f)/g
```

Giac [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx}\,dx=\int \frac{\left(F^{(bx+a)c}\right)^n e+d}{gx+f}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f),x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx}\,dx=\int \frac{d+e\,\left(F^{c\,(a+b\,x)}\right)^n}{f+g\,x}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)/(f + g*x),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)/(f + g*x), x)
```

Reduce [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{f+gx}\,dx=\frac{f^{acn}\left(\int \frac{f^{bcnx}}{gx+f}dx\right)eg+\log(gx+f)\,d}{g}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)/(g*x+f),x)
```

output
```
(f**(a*c*n)*int(f**(b*c*n*x)/(f + g*x),x)*e*g + log(f + g*x)*d)/g
```

3.24 $\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx = -\frac{d}{g(f+gx)}-\frac{e\left(F^{ac+bcx}\right)^n}{g(f+gx)} + \frac{bceF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n n\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log(F)}{g^2}$$

output
```
-d/g/(g*x+f)-e*(F^(b*c*x+a*c))^n/g/(g*x+f)+b*c*e*F^(c*(a-b*f/g)*n-c*n*(b*x
+a))*(F^(b*c*x+a*c))^n*n*Ei(b*c*n*(g*x+f)*ln(F)/g)*ln(F)/g^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx = \frac{-\frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)g}{f+gx}+bceF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n n\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log(F)}{g^2}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^2,x]
```

output
```
(-(((d + e*(F^(c*(a + b*x)))^n)*g)/(f + g*x)) + (b*c*e*(F^(c*(a + b*x)))^n
*n*ExpIntegralEi[(b*c*n*(f + g*x)*Log[F])/g]*Log[F])/F^((b*c*n*(f + g*x))/
g))/g^2
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\left(F^{c(a+bx)}\right)^n+d}{(f+gx)^2}dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(\frac{e\left(F^{ac+bcx}\right)^n}{(f+gx)^2}+\frac{d}{(f+gx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bcen\log(F)\left(F^{ac+bcx}\right)^n F^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g^2}-\frac{e\left(F^{ac+bcx}\right)^n}{g(f+gx)}-\frac{d}{g(f+gx)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^2,x]
```

output
```
-(d/(g*(f + g*x))) - (e*(F^(a*c + b*c*x))^n)/(g*(f + g*x)) + (b*c*e*F^(c*(
a - (b*f)/g)*n - c*n*(a + b*x))*(F^(a*c + b*c*x))^n*n*ExpIntegralEi[(b*c*n
*(f + g*x)*Log[F])/g]*Log[F])/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{d + e\left(F^{c(bx+a)}\right)^{n}}{\left(gx + f\right)^{2}}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^2,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{d + e\left(F^{c(a+bx)}\right)^{n}}{(f + gx)^{2}}\,dx = -\frac{F^{bcnx+acn}eg + dg - \frac{(bcegnx+bcefn)\mathrm{Ei}\left(\frac{(bcgnx+bcfn)\log(F)}{g}\right)\log(F)}{F^{\frac{(bcf-acg)n}{g}}}}{g^{3}x + fg^{2}}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
-(F^(b*c*n*x + a*c*n)*e*g + d*g - (b*c*e*g*n*x + b*c*e*f*n)*Ei((b*c*g*n*x
+ b*c*f*n)*log(F)/g)*log(F)/F^((b*c*f - a*c*g)*n/g))/(g^3*x + f*g^2)
```

Sympy [F]

$$\int \frac{d + e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx = \int \frac{d + e\left(F^{ac+bcx}\right)^n}{\left(f+gx\right)^2}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)/(g*x+f)**2,x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)/(f + g*x)**2, x)
```

Maxima [F]

$$\int \frac{d + e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx = \int \frac{\left(F^{(bx+a)c}\right)^n e + d}{\left(gx+f\right)^2}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
F^(a*c*n)*e*integrate(F^(b*c*n*x)/(g^2*x^2 + 2*f*g*x + f^2), x) - d/(g^2*x
 + f*g)
```

Giac [F]

$$\int \frac{d + e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx = \int \frac{\left(F^{(bx+a)c}\right)^n e + d}{\left(gx+f\right)^2}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx=\int \frac{d+e\,\left(F^{c\,(a+b\,x)}\right)^n}{\left(f+g\,x\right)^2}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^2,x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^2}\,dx
=\frac{f^{acn}\left(\int \frac{f^{bcnx}}{g^2x^2+2fgx+f^2}dx\right)e\,f^2+f^{acn}\left(\int \frac{f^{bcnx}}{g^2x^2+2fgx+f^2}dx\right)efgx+dx}{f\,(gx+f)}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x)
```

output
```
(f**(a*c*n)*int(f**(b*c*n*x)/(f**2 + 2*f*g*x + g**2*x**2),x)*e*f**2 + f**(
a*c*n)*int(f**(b*c*n*x)/(f**2 + 2*f*g*x + g**2*x**2),x)*e*f*g*x + d*x)/(f*
(f + g*x))
```

3.25 $\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx = -\frac{d}{2g(f+gx)^2}-\frac{e\left(F^{ac+bcx}\right)^n}{2g(f+gx)^2}-\frac{bce\left(F^{ac+bcx}\right)^n n\log(F)}{2g^2(f+gx)} + \frac{b^2c^2eF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n n^2\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log^2(F)}{2g^3}$$

output
```
-1/2*d/g/(g*x+f)^2-1/2*e*(F^(b*c*x+a*c))^n/g/(g*x+f)^2-1/2*b*c*e*(F^(b*c*x
+a*c))^n*n*ln(F)/g^2/(g*x+f)+1/2*b^2*c^2*e*F^(c*(a-b*f/g)*n-c*n*(b*x+a))*(
F^(b*c*x+a*c))^n*n^2*Ei(b*c*n*(g*x+f)*ln(F)/g)*ln(F)^2/g^3
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx = -\frac{dg^2-b^2c^2eF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n n^2(f+gx)^2\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log^2(F)+e\left(F^{c(a+bx)}\right)^n}{2g^3(f+gx)^2}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^3,x]
```

output
```
-1/2*(d*g^2 - (b^2*c^2*e*(F^(c*(a + b*x)))^n*n^2*(f + g*x)^2*ExpIntegralEi
[(b*c*n*(f + g*x)*Log[F])/g]*Log[F]^2)/F^((b*c*n*(f + g*x))/g) + e*(F^(c*(
a + b*x)))^n*g*(g + b*c*n*(f + g*x)*Log[F]))/(g^3*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e\left(F^{c(a+bx)}\right)^n+d}{(f+gx)^3}\,dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(\frac{e\left(F^{ac+bcx}\right)^n}{(f+gx)^3}+\frac{d}{(f+gx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2c^2en^2\log^2(F)\left(F^{ac+bcx}\right)^n F^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{2g^3}-\frac{bcen\log(F)\left(F^{ac+bcx}\right)^n}{2g^2(f+gx)}-\frac{e\left(F^{ac+bcx}\right)^n}{2g(f+gx)^2}-\frac{d}{2g(f+gx)^2}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^3,x]
```

output
```
-1/2*d/(g*(f + g*x)^2) - (e*(F^(a*c + b*c*x))^n)/(2*g*(f + g*x)^2) - (b*c*
e*(F^(a*c + b*c*x))^n*n*Log[F])/(2*g^2*(f + g*x)) + (b^2*c^2*e*F^(c*(a - (
b*f)/g)*n - c*n*(a + b*x))*(F^(a*c + b*c*x))^n*n^2*ExpIntegralEi[(b*c*n*(f
 + g*x)*Log[F])/g]*Log[F]^2)/(2*g^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{d+e\left(F^{c(bx+a)}\right)^{n}}{\left(gx+f\right)^{3}}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^3,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.10

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^{n}}{(f+gx)^{3}}\,dx = -\frac{dg^{2} - \frac{\left(b^{2}c^{2}eg^{2}n^{2}x^{2}+2\,b^{2}c^{2}efgn^{2}x+b^{2}c^{2}ef^{2}n^{2}\right)\mathrm{Ei}\left(\frac{(bcgnx+bcfn)\log(F)}{g}\right)\log(F)^{2}}{F^{\frac{(bcf-acg)n}{g}}} + \left(eg^{2} + \left(bceg^{2}nx + bcefgn\right)\log\left(F\right)\right)F}{2\left(g^{5}x^{2} + 2\,fg^{4}x + f^{2}g^{3}\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^3,x, algorithm="fricas")
```

output
```
-1/2*(d*g^2 - (b^2*c^2*e*g^2*n^2*x^2 + 2*b^2*c^2*e*f*g*n^2*x + b^2*c^2*e*f
^2*n^2)*Ei((b*c*g*n*x + b*c*f*n)*log(F)/g)*log(F)^2/F^((b*c*f - a*c*g)*n/g
) + (e*g^2 + (b*c*e*g^2*n*x + b*c*e*f*g*n)*log(F))*F^(b*c*n*x + a*c*n))/(g
^5*x^2 + 2*f*g^4*x + f^2*g^3)
```

Sympy [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx = \int \frac{d+e\left(F^{ac+bcx}\right)^n}{(f+gx)^3}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)/(g*x+f)**3,x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)/(f + g*x)**3, x)
```

Maxima [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx = \int \frac{\left(F^{(bx+a)c}\right)^n e+d}{(gx+f)^3}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^3,x, algorithm="maxima")
```

output
```
F^(a*c*n)*e*integrate(F^(b*c*n*x)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3
), x) - 1/2*d/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Giac [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx=\int \frac{\left(F^{(bx+a)c}\right)^n e+d}{(gx+f)^3}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx=\int \frac{d+e\left(F^{c\,(a+b\,x)}\right)^n}{(f+g\,x)^3}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^3,x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{d+e\left(F^{c(a+bx)}\right)^n}{(f+gx)^3}\,dx$$
$$=\frac{2f^{acn}\left(\int \frac{f^{bcnx}}{g^3x^3+3f\,g^2x^2+3f^2gx+f^3}dx\right)e\,f^2g+4f^{acn}\left(\int \frac{f^{bcnx}}{g^3x^3+3f\,g^2x^2+3f^2gx+f^3}dx\right)ef\,g^2x+2f^{acn}\left(\int \frac{f^{bcn}}{g^3x^3+3f\,g^2x^2}\right.}{2g\left(g^2x^2+2fgx+f^2\right)}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)/(g*x+f)^3,x)
```

output
```
(2*f**(a*c*n)*int(f**(b*c*n*x)/(f**3 + 3*f**2*g*x + 3*f*g**2*x**2 + g**3*x
**3),x)*e*f**2*g + 4*f**(a*c*n)*int(f**(b*c*n*x)/(f**3 + 3*f**2*g*x + 3*f*
g**2*x**2 + g**3*x**3),x)*e*f*g**2*x + 2*f**(a*c*n)*int(f**(b*c*n*x)/(f**3
 + 3*f**2*g*x + 3*f*g**2*x**2 + g**3*x**3),x)*e*g**3*x**2 - d)/(2*g*(f**2
+ 2*f*g*x + g**2*x**2))
```

3.26 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx$

Optimal result

Integrand size = 25, antiderivative size = 322

$$\begin{aligned}\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx &= \frac{d^2(f+gx)^4}{4g}-\frac{12de\left(F^{ac+bcx}\right)^n g^3}{b^4c^4n^4\log^4(F)}\\
&-\frac{3e^2\left(F^{ac+bcx}\right)^{2n} g^3}{8b^4c^4n^4\log^4(F)}\\
&+\frac{12de\left(F^{ac+bcx}\right)^n g^2(f+gx)}{b^3c^3n^3\log^3(F)}\\
&+\frac{3e^2\left(F^{ac+bcx}\right)^{2n} g^2(f+gx)}{4b^3c^3n^3\log^3(F)}\\
&-\frac{6de\left(F^{ac+bcx}\right)^n g(f+gx)^2}{b^2c^2n^2\log^2(F)}\\
&-\frac{3e^2\left(F^{ac+bcx}\right)^{2n} g(f+gx)^2}{4b^2c^2n^2\log^2(F)}\\
&+\frac{2de\left(F^{ac+bcx}\right)^n (f+gx)^3}{bcn\log(F)}\\
&+\frac{e^2\left(F^{ac+bcx}\right)^{2n} (f+gx)^3}{2bcn\log(F)}\end{aligned}$$

output
```
1/4*d^2*(g*x+f)^4/g-12*d*e*(F^(b*c*x+a*c))^n*g^3/b^4/c^4/n^4/ln(F)^4-3/8*e
^2*(F^(b*c*x+a*c))^(2*n)*g^3/b^4/c^4/n^4/ln(F)^4+12*d*e*(F^(b*c*x+a*c))^n*
g^2*(g*x+f)/b^3/c^3/n^3/ln(F)^3+3/4*e^2*(F^(b*c*x+a*c))^(2*n)*g^2*(g*x+f)/
b^3/c^3/n^3/ln(F)^3-6*d*e*(F^(b*c*x+a*c))^n*g*(g*x+f)^2/b^2/c^2/n^2/ln(F)^
2-3/4*e^2*(F^(b*c*x+a*c))^(2*n)*g*(g*x+f)^2/b^2/c^2/n^2/ln(F)^2+2*d*e*(F^(
b*c*x+a*c))^n*(g*x+f)^3/b/c/n/ln(F)+1/2*e^2*(F^(b*c*x+a*c))^(2*n)*(g*x+f)^
3/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.74

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx = d^2f^3x+\frac{3}{2}d^2f^2gx^2+d^2fg^2x^3+\frac{1}{4}d^2g^3x^4$$
$$+\frac{2de\left(F^{c(a+bx)}\right)^n\left(-6g^3+6bcg^2n(f+gx)\log(F)-3b^2c^2gn^2(f+gx)^2\log^2(F)+b^3c^3n^3(f+gx)^3\log^3(\right.}{b^4c^4n^4\log^4(F)}$$
$$+\frac{e^2\left(F^{c(a+bx)}\right)^{2n}\left(-3g^3+6bcg^2n(f+gx)\log(F)-6b^2c^2gn^2(f+gx)^2\log^2(F)+4b^3c^3n^3(f+gx)^3\log^3(\right.}{8b^4c^4n^4\log^4(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^3,x]
```

output
```
d^2*f^3*x + (3*d^2*f^2*g*x^2)/2 + d^2*f*g^2*x^3 + (d^2*g^3*x^4)/4 + (2*d*e
*(F^(c*(a + b*x)))^n*(-6*g^3 + 6*b*c*g^2*n*(f + g*x)*Log[F] - 3*b^2*c^2*g*
n^2*(f + g*x)^2*Log[F]^2 + b^3*c^3*n^3*(f + g*x)^3*Log[F]^3))/(b^4*c^4*n^4
*Log[F]^4) + (e^2*(F^(c*(a + b*x)))^(2*n)*(-3*g^3 + 6*b*c*g^2*n*(f + g*x)*
Log[F] - 6*b^2*c^2*g*n^2*(f + g*x)^2*Log[F]^2 + 4*b^3*c^3*n^3*(f + g*x)^3*
Log[F]^3))/(8*b^4*c^4*n^4*Log[F]^4)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^3 \left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(2de(f+gx)^3 \left(F^{ac+bcx}\right)^n+e^2(f+gx)^3 \left(F^{ac+bcx}\right)^{2n}+d^2(f+gx)^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{12deg^3 \left(F^{ac+bcx}\right)^n}{b^4c^4n^4\log^4(F)}-\frac{3e^2g^3 \left(F^{ac+bcx}\right)^{2n}}{8b^4c^4n^4\log^4(F)}+\frac{12deg^2(f+gx) \left(F^{ac+bcx}\right)^n}{b^3c^3n^3\log^3(F)}+\frac{3e^2g^2(f+gx) \left(F^{ac+bcx}\right)^{2n}}{4b^3c^3n^3\log^3(F)}-\frac{6deg(f+gx)^2 \left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log^2(F)}-\frac{3e^2g(f+gx)^2 \left(F^{ac+bcx}\right)^{2n}}{4b^2c^2n^2\log^2(F)}+\frac{2de(f+gx)^3 \left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{e^2(f+gx)^3 \left(F^{ac+bcx}\right)^{2n}}{2bcn\log(F)}+\frac{d^2(f+gx)^4}{4g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^3,x]
```

output
```
(d^2*(f + g*x)^4)/(4*g) - (12*d*e*(F^(a*c + b*c*x))^n*g^3)/(b^4*c^4*n^4*Lo
g[F]^4) - (3*e^2*(F^(a*c + b*c*x))^(2*n)*g^3)/(8*b^4*c^4*n^4*Log[F]^4) + (
12*d*e*(F^(a*c + b*c*x))^n*g^2*(f + g*x))/(b^3*c^3*n^3*Log[F]^3) + (3*e^2*
(F^(a*c + b*c*x))^(2*n)*g^2*(f + g*x))/(4*b^3*c^3*n^3*Log[F]^3) - (6*d*e*(
F^(a*c + b*c*x))^n*g*(f + g*x)^2)/(b^2*c^2*n^2*Log[F]^2) - (3*e^2*(F^(a*c
+ b*c*x))^(2*n)*g*(f + g*x)^2)/(4*b^2*c^2*n^2*Log[F]^2) + (2*d*e*(F^(a*c +
 b*c*x))^n*(f + g*x)^3)/(b*c*n*Log[F]) + (e^2*(F^(a*c + b*c*x))^(2*n)*(f +
 g*x)^3)/(2*b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(312) = 624$.

Time = 1.16 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.30

method	result
parallelrisch	$\frac{16de\,g^{3}\left(F^{c(bx+a)}\right)^{n}x^{3}\ln(F)^{3}b^{3}c^{3}n^{3}+12\ln(F)^{3}x^{2}\left(F^{c(bx+a)}\right)^{2n}b^{3}c^{3}e^{2}f\,g^{2}n^{3}+12\ln(F)^{3}x\left(F^{c(bx+a)}\right)^{2n}b^{3}c^{3}e^{2}f^{2}g\,n^{3}-48\ln}{}$
orering	Expression too large to display

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2*(g*x+f)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(16*d*e*g^3*(F^(c*(b*x+a)))^n*x^3*ln(F)^3*b^3*c^3*n^3+12*ln(F)^3*x^2*(
(F^(c*(b*x+a)))^n)^2*b^3*c^3*e^2*f*g^2*n^3+12*ln(F)^3*x*((F^(c*(b*x+a)))^n
)^2*b^3*c^3*e^2*f^2*g*n^3-48*ln(F)^2*x^2*(F^(c*(b*x+a)))^n*b^2*c^2*d*e*g^3
*n^2-12*ln(F)^2*x*((F^(c*(b*x+a)))^n)^2*b^2*c^2*e^2*f*g^2*n^2-48*ln(F)^2*(
F^(c*(b*x+a)))^n*b^2*c^2*d*e*f^2*g*n^2+96*ln(F)*x*(F^(c*(b*x+a)))^n*b*c*d*
e*g^3*n+96*ln(F)*(F^(c*(b*x+a)))^n*b*c*d*e*f*g^2*n+48*ln(F)^3*x^2*(F^(c*(b
*x+a)))^n*b^3*c^3*d*e*f*g^2*n^3+48*ln(F)^3*x*(F^(c*(b*x+a)))^n*b^3*c^3*d*e
*f^2*g*n^3-96*ln(F)^2*x*(F^(c*(b*x+a)))^n*b^2*c^2*d*e*f*g^2*n^2+2*d^2*g^3*
x^4*ln(F)^4*b^4*c^4*n^4+8*d^2*f^3*x*ln(F)^4*b^4*c^4*n^4+4*ln(F)^3*((F^(c*(
b*x+a)))^n)^2*b^3*c^3*e^2*f^3*n^3+12*d^2*g*f^2*x^2*ln(F)^4*b^4*c^4*n^4+4*e
^2*g^3*((F^(c*(b*x+a)))^n)^2*x^3*ln(F)^3*b^3*c^3*n^3+16*ln(F)^3*(F^(c*(b*x
+a)))^n*b^3*c^3*d*e*f^3*n^3-96*(F^(c*(b*x+a)))^n*d*e*g^3+8*d^2*g^2*f*x^3*l
n(F)^4*b^4*c^4*n^4-6*ln(F)^2*x^2*((F^(c*(b*x+a)))^n)^2*b^2*c^2*e^2*g^3*n^2
-6*ln(F)^2*((F^(c*(b*x+a)))^n)^2*b^2*c^2*e^2*f^2*g*n^2+6*ln(F)*x*((F^(c*(b
*x+a)))^n)^2*b*c*e^2*g^3*n+6*ln(F)*((F^(c*(b*x+a)))^n)^2*b*c*e^2*f*g^2*n-3
*((F^(c*(b*x+a)))^n)^2*e^2*g^3)/ln(F)^4/b^4/c^4/n^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.50

$$\int \left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}(f+gx)^{3}\,dx$$
$$=\frac{2\left(b^{4}c^{4}d^{2}g^{3}n^{4}x^{4}+4\,b^{4}c^{4}d^{2}fg^{2}n^{4}x^{3}+6\,b^{4}c^{4}d^{2}f^{2}gn^{4}x^{2}+4\,b^{4}c^{4}d^{2}f^{3}n^{4}x\right)\log\left(F\right)^{4}-\left(3\,e^{2}g^{3}-4\left(b^{3}c^{3}e^{2}g^{3}n\right.\right.}{}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^3,x, algorithm="fricas")
```

output
```
1/8*(2*(b^4*c^4*d^2*g^3*n^4*x^4 + 4*b^4*c^4*d^2*f*g^2*n^4*x^3 + 6*b^4*c^4*
d^2*f^2*g*n^4*x^2 + 4*b^4*c^4*d^2*f^3*n^4*x)*log(F)^4 - (3*e^2*g^3 - 4*(b^
3*c^3*e^2*g^3*n^3*x^3 + 3*b^3*c^3*e^2*f*g^2*n^3*x^2 + 3*b^3*c^3*e^2*f^2*g*
n^3*x + b^3*c^3*e^2*f^3*n^3)*log(F)^3 + 6*(b^2*c^2*e^2*g^3*n^2*x^2 + 2*b^2
*c^2*e^2*f*g^2*n^2*x + b^2*c^2*e^2*f^2*g*n^2)*log(F)^2 - 6*(b*c*e^2*g^3*n*
x + b*c*e^2*f*g^2*n)*log(F))*F^(2*b*c*n*x + 2*a*c*n) - 16*(6*d*e*g^3 - (b^
3*c^3*d*e*g^3*n^3*x^3 + 3*b^3*c^3*d*e*f*g^2*n^3*x^2 + 3*b^3*c^3*d*e*f^2*g*
n^3*x + b^3*c^3*d*e*f^3*n^3)*log(F)^3 + 3*(b^2*c^2*d*e*g^3*n^2*x^2 + 2*b^2
*c^2*d*e*f*g^2*n^2*x + b^2*c^2*d*e*f^2*g*n^2)*log(F)^2 - 6*(b*c*d*e*g^3*n*
x + b*c*d*e*f*g^2*n)*log(F))*F^(b*c*n*x + a*c*n))/(b^4*c^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(323) = 646$.

Time = 4.36 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.84

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx = \text{Too large to display}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2*(g*x+f)**3,x)
```

output

```
Piecewise(((d + e)**2*(f**3*x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/
4), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), ((d + e*(F**(a*c))**n)**2*
(f**3*x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4), Eq(b, 0)), ((d + e
)**2*(f**3*x + 3*f**2*g*x**2/2 + f*g**2*x**3 + g**3*x**4/4), Eq(F, 1) | Eq
(c, 0) | Eq(n, 0)), (d**2*f**3*x + 3*d**2*f**2*g*x**2/2 + d**2*f*g**2*x**3
 + d**2*g**3*x**4/4 + 2*d*e*f**3*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + 6*
d*e*f**2*g*x*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + 6*d*e*f*g**2*x**2*(F**
(a*c + b*c*x))**n/(b*c*n*log(F)) + 2*d*e*g**3*x**3*(F**(a*c + b*c*x))**n/(
b*c*n*log(F)) + e**2*f**3*(F**(a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)) + 3*e
**2*f**2*g*x*(F**(a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)) + 3*e**2*f*g**2*x*
*2*(F**(a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)) + e**2*g**3*x**3*(F**(a*c +
b*c*x))**(2*n)/(2*b*c*n*log(F)) - 6*d*e*f**2*g*(F**(a*c + b*c*x))**n/(b**2
*c**2*n**2*log(F)**2) - 12*d*e*f*g**2*x*(F**(a*c + b*c*x))**n/(b**2*c**2*n
**2*log(F)**2) - 6*d*e*g**3*x**2*(F**(a*c + b*c*x))**n/(b**2*c**2*n**2*log
(F)**2) - 3*e**2*f**2*g*(F**(a*c + b*c*x))**(2*n)/(4*b**2*c**2*n**2*log(F)
**2) - 3*e**2*f*g**2*x*(F**(a*c + b*c*x))**(2*n)/(2*b**2*c**2*n**2*log(F)*
*2) - 3*e**2*g**3*x**2*(F**(a*c + b*c*x))**(2*n)/(4*b**2*c**2*n**2*log(F)*
*2) + 12*d*e*f*g**2*(F**(a*c + b*c*x))**n/(b**3*c**3*n**3*log(F)**3) + 12*
d*e*g**3*x*(F**(a*c + b*c*x))**n/(b**3*c**3*n**3*log(F)**3) + 3*e**2*f*g**
2*(F**(a*c + b*c*x))**(2*n)/(4*b**3*c**3*n**3*log(F)**3) + 3*e**2*g**3*...
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.76

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx = \frac{1}{4}d^2g^3x^4 + d^2fg^2x^3 + \frac{3}{2}d^2f^2gx^2 + d^2f^3x + \frac{2F^{bcnx+acn}def^3}{bcn\log(F)} + \frac{F^{2bcnx+2acn}e^2f^3}{2bcn\log(F)} + \frac{6(F^{acn}bcnx\log(F) - F^{acn})F^{bcnx}def^2g}{b^2c^2n^2\log(F)^2} + \frac{3(2F^{2acn}bcnx\log(F) - F^{2acn})F^{2bcnx}e^2f^2g}{4b^2c^2n^2\log(F)^2} + \frac{6\left(F^{acn}b^2c^2n^2x^2\log(F)^2 - 2F^{acn}bcnx\log(F) + 2F^{acn}\right)F^{bcnx}defg^2}{b^3c^3n^3\log(F)^3} + \frac{3\left(2F^{2acn}b^2c^2n^2x^2\log(F)^2 - 2F^{2acn}bcnx\log(F) + F^{2acn}\right)F^{2bcnx}e^2fg^2}{4b^3c^3n^3\log(F)^3} + \frac{2\left(F^{acn}b^3c^3n^3x^3\log(F)^3 - 3F^{acn}b^2c^2n^2x^2\log(F)^2 + 6F^{acn}bcnx\log(F) - 6F^{acn}\right)F^{bcnx}deg^3}{b^4c^4n^4\log(F)^4} + \frac{\left(4F^{2acn}b^3c^3n^3x^3\log(F)^3 - 6F^{2acn}b^2c^2n^2x^2\log(F)^2 + 6F^{2acn}bcnx\log(F) - 3F^{2acn}\right)F^{2bcnx}e^2g^3}{8b^4c^4n^4\log(F)^4}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^3,x, algorithm="maxima")
```

output
```
1/4*d^2*g^3*x^4 + d^2*f*g^2*x^3 + 3/2*d^2*f^2*g*x^2 + d^2*f^3*x + 2*F^(b*c
*n*x + a*c*n)*d*e*f^3/(b*c*n*log(F)) + 1/2*F^(2*b*c*n*x + 2*a*c*n)*e^2*f^3
/(b*c*n*log(F)) + 6*(F^(a*c*n)*b*c*n*x*log(F) - F^(a*c*n))*F^(b*c*n*x)*d*e
*f^2*g/(b^2*c^2*n^2*log(F)^2) + 3/4*(2*F^(2*a*c*n)*b*c*n*x*log(F) - F^(2*a
*c*n))*F^(2*b*c*n*x)*e^2*f^2*g/(b^2*c^2*n^2*log(F)^2) + 6*(F^(a*c*n)*b^2*c
^2*n^2*x^2*log(F)^2 - 2*F^(a*c*n)*b*c*n*x*log(F) + 2*F^(a*c*n))*F^(b*c*n*x
)*d*e*f*g^2/(b^3*c^3*n^3*log(F)^3) + 3/4*(2*F^(2*a*c*n)*b^2*c^2*n^2*x^2*lo
g(F)^2 - 2*F^(2*a*c*n)*b*c*n*x*log(F) + F^(2*a*c*n))*F^(2*b*c*n*x)*e^2*f*g
^2/(b^3*c^3*n^3*log(F)^3) + 2*(F^(a*c*n)*b^3*c^3*n^3*x^3*log(F)^3 - 3*F^(a
*c*n)*b^2*c^2*n^2*x^2*log(F)^2 + 6*F^(a*c*n)*b*c*n*x*log(F) - 6*F^(a*c*n))
*F^(b*c*n*x)*d*e*g^3/(b^4*c^4*n^4*log(F)^4) + 1/8*(4*F^(2*a*c*n)*b^3*c^3*n
^3*x^3*log(F)^3 - 6*F^(2*a*c*n)*b^2*c^2*n^2*x^2*log(F)^2 + 6*F^(2*a*c*n)*b
*c*n*x*log(F) - 3*F^(2*a*c*n))*F^(2*b*c*n*x)*e^2*g^3/(b^4*c^4*n^4*log(F)^4
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 12013, normalized size of antiderivative = 37.31

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx = \text{Too large to display}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^3,x, algorithm="giac")
```

output
```
1/4*d^2*g^3*x^4 + d^2*f*g^2*x^3 + 3/2*d^2*f^2*g*x^2 + d^2*f^3*x - 1/4*(((6
*pi^2*b^3*c^3*e^2*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi^2*b^3*c^3*e^2*g^3*
n^3*x^3*log(abs(F)) + 4*b^3*c^3*e^2*g^3*n^3*x^3*log(abs(F))^3 + 18*pi^2*b^
3*c^3*e^2*f*g^2*n^3*x^2*log(abs(F))*sgn(F) - 18*pi^2*b^3*c^3*e^2*f*g^2*n^3
*x^2*log(abs(F)) + 12*b^3*c^3*e^2*f*g^2*n^3*x^2*log(abs(F))^3 + 18*pi^2*b^
3*c^3*e^2*f^2*g*n^3*x*log(abs(F))*sgn(F) - 18*pi^2*b^3*c^3*e^2*f^2*g*n^3*x
*log(abs(F)) + 12*b^3*c^3*e^2*f^2*g*n^3*x*log(abs(F))^3 + 6*pi^2*b^3*c^3*e
^2*f^3*n^3*log(abs(F))*sgn(F) - 6*pi^2*b^3*c^3*e^2*f^3*n^3*log(abs(F)) + 4
*b^3*c^3*e^2*f^3*n^3*log(abs(F))^3 - 3*pi^2*b^2*c^2*e^2*g^3*n^2*x^2*sgn(F)
 + 3*pi^2*b^2*c^2*e^2*g^3*n^2*x^2 - 6*b^2*c^2*e^2*g^3*n^2*x^2*log(abs(F))^
2 - 6*pi^2*b^2*c^2*e^2*f*g^2*n^2*x*sgn(F) + 6*pi^2*b^2*c^2*e^2*f*g^2*n^2*x
 - 12*b^2*c^2*e^2*f*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b^2*c^2*e^2*f^2*g*n^2
*sgn(F) + 3*pi^2*b^2*c^2*e^2*f^2*g*n^2 - 6*b^2*c^2*e^2*f^2*g*n^2*log(abs(F
))^2 + 6*b*c*e^2*g^3*n*x*log(abs(F)) + 6*b*c*e^2*f*g^2*n*log(abs(F)) - 3*e
^2*g^3)*(pi^4*b^4*c^4*n^4*sgn(F) - 6*pi^2*b^4*c^4*n^4*log(abs(F))^2*sgn(F)
 - pi^4*b^4*c^4*n^4 + 6*pi^2*b^4*c^4*n^4*log(abs(F))^2 - 2*b^4*c^4*n^4*log
(abs(F))^4)/((pi^4*b^4*c^4*n^4*sgn(F) - 6*pi^2*b^4*c^4*n^4*log(abs(F))^2*s
gn(F) - pi^4*b^4*c^4*n^4 + 6*pi^2*b^4*c^4*n^4*log(abs(F))^2 - 2*b^4*c^4*n^
4*log(abs(F))^4)^2 + 16*(pi^3*b^4*c^4*n^4*log(abs(F))*sgn(F) - pi*b^4*c^4*
n^4*log(abs(F))^3*sgn(F) - pi^3*b^4*c^4*n^4*log(abs(F)) + pi*b^4*c^4*n^...
```

Mupad [B] (verification not implemented)

Time = 23.23 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.36

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx = d^2 f^3 x - \left(F^{b\,c\,x}\,F^{a\,c}\right)^n \left(\frac{2\,d\,e\,\left(-b^3\,c^3\,f^3\,n^3\ln\left(F\right)^3+3\,b^2\,c^2\,f^2\,g\,n^2\ln\left(F\right)^2-6\,b\,c\,f\,g^2\,n\,\ln\left(F\right)+6\,g^3\right)}{b^4\,c^4\,n^4\ln\left(F\right)^4} - \frac{2\,d\,e\,g^3\,x^3}{b\,c\,n\,\ln\left(F\right)} - \frac{6\,d\,e\,g\,x\,\left(b^2\,c^2\,f^2\,n^2\ln\left(F\right)^2-2\,b\,c\,f\,g\,n\,\ln\left(F\right)+2\,g^2\right)}{b^3\,c^3\,n^3\ln\left(F\right)^3} + \frac{6\,d\,e\,g^2\,x^2\,\left(g-b\,c\,f\,n\,\ln\left(F\right)\right)}{b^2\,c^2\,n^2\ln\left(F\right)^2}\right) - \left(F^{b\,c\,x}\,F^{a\,c}\right)^{2\,n} \left(\frac{e^2\,\left(-4\,b^3\,c^3\,f^3\,n^3\ln\left(F\right)^3+6\,b^2\,c^2\,f^2\,g\,n^2\ln\left(F\right)^2-6\,b\,c\,f\,g^2\,n\,\ln\left(F\right)+3\,g^3\right)}{8\,b^4\,c^4\,n^4\ln\left(F\right)^4} - \frac{e^2\,g^3\,x^3}{2\,b\,c\,n\,\ln\left(F\right)} - \frac{3\,e^2\,g\,x\,\left(2\,b^2\,c^2\,f^2\,n^2\ln\left(F\right)^2-2\,b\,c\,f\,g\,n\,\ln\left(F\right)+g^2\right)}{4\,b^3\,c^3\,n^3\ln\left(F\right)^3} + \frac{3\,e^2\,g^2\,x^2\,\left(g-2\,b\,c\,f\,n\,\ln\left(F\right)\right)}{4\,b^2\,c^2\,n^2\ln\left(F\right)^2}\right) + \frac{d^2\,g^3\,x^4}{4} + \frac{3\,d^2\,f^2\,g\,x^2}{2} + d^2\,f\,g^2\,x^3$$

input

```
int((f + g*x)^3*(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output

```
d^2*f^3*x - (F^(b*c*x)*F^(a*c))^n*((2*d*e*(6*g^3 - b^3*c^3*f^3*n^3*log(F)^
3 - 6*b*c*f*g^2*n*log(F) + 3*b^2*c^2*f^2*g*n^2*log(F)^2))/(b^4*c^4*n^4*log
(F)^4) - (2*d*e*g^3*x^3)/(b*c*n*log(F)) - (6*d*e*g*x*(2*g^2 + b^2*c^2*f^2*
n^2*log(F)^2 - 2*b*c*f*g*n*log(F)))/(b^3*c^3*n^3*log(F)^3) + (6*d*e*g^2*x^
2*(g - b*c*f*n*log(F)))/(b^2*c^2*n^2*log(F)^2)) - (F^(b*c*x)*F^(a*c))^(2*n
)*((e^2*(3*g^3 - 4*b^3*c^3*f^3*n^3*log(F)^3 - 6*b*c*f*g^2*n*log(F) + 6*b^2
*c^2*f^2*g*n^2*log(F)^2))/(8*b^4*c^4*n^4*log(F)^4) - (e^2*g^3*x^3)/(2*b*c*
n*log(F)) - (3*e^2*g*x*(g^2 + 2*b^2*c^2*f^2*n^2*log(F)^2 - 2*b*c*f*g*n*log
(F)))/(4*b^3*c^3*n^3*log(F)^3) + (3*e^2*g^2*x^2*(g - 2*b*c*f*n*log(F)))/(4
*b^2*c^2*n^2*log(F)^2)) + (d^2*g^3*x^4)/4 + (3*d^2*f^2*g*x^2)/2 + d^2*f*g^
2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.35

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^3\,dx$$
$$= \frac{4f^{2bcnx+2acn}\log(f)^3 b^3c^3e^2f^3n^3 + 12f^{2bcnx+2acn}\log(f)^3 b^3c^3e^2f^2g\,n^3x + 12f^{2bcnx+2acn}\log(f)^3 b^3c^3e^2f\,g^2n^3}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^3,x)
```

output
```
(4*f**(2*a*c*n + 2*b*c*n*x)*log(f)**3*b**3*c**3*e**2*f**3*n**3 + 12*f**(2*
a*c*n + 2*b*c*n*x)*log(f)**3*b**3*c**3*e**2*f**2*g*n**3*x + 12*f**(2*a*c*n
 + 2*b*c*n*x)*log(f)**3*b**3*c**3*e**2*f*g**2*n**3*x**2 + 4*f**(2*a*c*n +
2*b*c*n*x)*log(f)**3*b**3*c**3*e**2*g**3*n**3*x**3 - 6*f**(2*a*c*n + 2*b*c
*n*x)*log(f)**2*b**2*c**2*e**2*f**2*g*n**2 - 12*f**(2*a*c*n + 2*b*c*n*x)*l
og(f)**2*b**2*c**2*e**2*f*g**2*n**2*x - 6*f**(2*a*c*n + 2*b*c*n*x)*log(f)*
*2*b**2*c**2*e**2*g**3*n**2*x**2 + 6*f**(2*a*c*n + 2*b*c*n*x)*log(f)*b*c*e
**2*f*g**2*n + 6*f**(2*a*c*n + 2*b*c*n*x)*log(f)*b*c*e**2*g**3*n*x - 3*f**
(2*a*c*n + 2*b*c*n*x)*e**2*g**3 + 16*f**(a*c*n + b*c*n*x)*log(f)**3*b**3*c
**3*d*e*f**3*n**3 + 48*f**(a*c*n + b*c*n*x)*log(f)**3*b**3*c**3*d*e*f**2*g
*n**3*x + 48*f**(a*c*n + b*c*n*x)*log(f)**3*b**3*c**3*d*e*f*g**2*n**3*x**2
 + 16*f**(a*c*n + b*c*n*x)*log(f)**3*b**3*c**3*d*e*g**3*n**3*x**3 - 48*f**
(a*c*n + b*c*n*x)*log(f)**2*b**2*c**2*d*e*f**2*g*n**2 - 96*f**(a*c*n + b*c
*n*x)*log(f)**2*b**2*c**2*d*e*f*g**2*n**2*x - 48*f**(a*c*n + b*c*n*x)*log(
f)**2*b**2*c**2*d*e*g**3*n**2*x**2 + 96*f**(a*c*n + b*c*n*x)*log(f)*b*c*d*
e*f*g**2*n + 96*f**(a*c*n + b*c*n*x)*log(f)*b*c*d*e*g**3*n*x - 96*f**(a*c*
n + b*c*n*x)*d*e*g**3 + 8*log(f)**4*b**4*c**4*d**2*f**3*n**4*x + 12*log(f)
**4*b**4*c**4*d**2*f**2*g*n**4*x**2 + 8*log(f)**4*b**4*c**4*d**2*f*g**2*n*
*4*x**3 + 2*log(f)**4*b**4*c**4*d**2*g**3*n**4*x**4)/(8*log(f)**4*b**4*c**
4*n**4)
```

3.27 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2\,dx$

Optimal result

Integrand size = 25, antiderivative size = 239

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2\,dx = \frac{d^2(f+gx)^3}{3g}+\frac{4de\left(F^{ac+bcx}\right)^n g^2}{b^3c^3n^3\log^3(F)} + \frac{e^2\left(F^{ac+bcx}\right)^{2n} g^2}{4b^3c^3n^3\log^3(F)} - \frac{4de\left(F^{ac+bcx}\right)^n g(f+gx)}{b^2c^2n^2\log^2(F)} - \frac{e^2\left(F^{ac+bcx}\right)^{2n} g(f+gx)}{2b^2c^2n^2\log^2(F)} + \frac{2de\left(F^{ac+bcx}\right)^n (f+gx)^2}{bcn\log(F)} + \frac{e^2\left(F^{ac+bcx}\right)^{2n} (f+gx)^2}{2bcn\log(F)}$$

output
```
1/3*d^2*(g*x+f)^3/g+4*d*e*(F^(b*c*x+a*c))^n*g^2/b^3/c^3/n^3/ln(F)^3+1/4*e^
2*(F^(b*c*x+a*c))^(2*n)*g^2/b^3/c^3/n^3/ln(F)^3-4*d*e*(F^(b*c*x+a*c))^n*g*
(g*x+f)/b^2/c^2/n^2/ln(F)^2-1/2*e^2*(F^(b*c*x+a*c))^(2*n)*g*(g*x+f)/b^2/c^
2/n^2/ln(F)^2+2*d*e*(F^(b*c*x+a*c))^n*(g*x+f)^2/b/c/n/ln(F)+1/2*e^2*(F^(b*
c*x+a*c))^(2*n)*(g*x+f)^2/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2\,dx$$
$$= d^2f^2x + d^2fgx^2 + \frac{1}{3}d^2g^2x^3 + \frac{2de\left(F^{c(a+bx)}\right)^n \left(2g^2-2bcgn(f+gx)\log(F)+b^2c^2n^2(f+gx)^2\log^2(F)\right)}{b^3c^3n^3\log^3(F)} + \frac{e^2\left(F^{c(a+bx)}\right)^{2n}\left(g^2-2bcgn(f+gx)\log(F)+2b^2c^2n^2(f+gx)^2\log^2(F)\right)}{4b^3c^3n^3\log^3(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^2,x]
```

output
```
d^2*f^2*x + d^2*f*g*x^2 + (d^2*g^2*x^3)/3 + (2*d*e*(F^(c*(a + b*x)))^n*(2*
g^2 - 2*b*c*g*n*(f + g*x)*Log[F] + b^2*c^2*n^2*(f + g*x)^2*Log[F]^2))/(b^3
*c^3*n^3*Log[F]^3) + (e^2*(F^(c*(a + b*x)))^(2*n)*(g^2 - 2*b*c*g*n*(f + g*
x)*Log[F] + 2*b^2*c^2*n^2*(f + g*x)^2*Log[F]^2))/(4*b^3*c^3*n^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^2\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(2de(f+gx)^2\left(F^{ac+bcx}\right)^n+e^2(f+gx)^2\left(F^{ac+bcx}\right)^{2n}+d^2(f+gx)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4deg^2\left(F^{ac+bcx}\right)^n}{b^3c^3n^3\log^3(F)}+\frac{e^2g^2\left(F^{ac+bcx}\right)^{2n}}{4b^3c^3n^3\log^3(F)}-\frac{4deg(f+gx)\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log^2(F)}-\frac{e^2g(f+gx)\left(F^{ac+bcx}\right)^{2n}}{2b^2c^2n^2\log^2(F)}+\frac{2de(f+gx)^2\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{e^2(f+gx)^2\left(F^{ac+bcx}\right)^{2n}}{2bcn\log(F)}+\frac{d^2(f+gx)^3}{3g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^2,x]
```

output
```
(d^2*(f + g*x)^3)/(3*g) + (4*d*e*(F^(a*c + b*c*x))^n*g^2)/(b^3*c^3*n^3*Log
[F]^3) + (e^2*(F^(a*c + b*c*x))^(2*n)*g^2)/(4*b^3*c^3*n^3*Log[F]^3) - (4*d
*e*(F^(a*c + b*c*x))^n*g*(f + g*x))/(b^2*c^2*n^2*Log[F]^2) - (e^2*(F^(a*c
+ b*c*x))^(2*n)*g*(f + g*x))/(2*b^2*c^2*n^2*Log[F]^2) + (2*d*e*(F^(a*c + b
*c*x))^n*(f + g*x)^2)/(b*c*n*Log[F]) + (e^2*(F^(a*c + b*c*x))^(2*n)*(f + g
*x)^2)/(2*b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.77

method	result
parallelrisch	$\frac{4d^2g^2x^3\ln(F)^3b^3c^3n^3+12d^2gf\,x^2\ln(F)^3b^3c^3n^3+12d^2f^2x\ln(F)^3b^3c^3n^3+6x^2\left(F^{c(bx+a)}\right)^{2n}e^2g^2\ln(F)^2b^2c^2n^2+24x^2\left(F^{c(b}\right.}{}$
orering	$\frac{\left(4\ln(F)^4b^4c^4g^4n^4x^5+20\ln(F)^4b^4c^4f\,g^3n^4x^4+40\ln(F)^4b^4c^4f^2g^2n^4x^3+36\ln(F)^4b^4c^4f^3g\,n^4x^2+12\ln(F)^4b^4c^4f^4n^4x+12\right.}{}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2*(g*x+f)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*d^2*g^2*x^3*ln(F)^3*b^3*c^3*n^3+12*d^2*g*f*x^2*ln(F)^3*b^3*c^3*n^3
+12*d^2*f^2*x*ln(F)^3*b^3*c^3*n^3+6*x^2*((F^(c*(b*x+a)))^n)^2*e^2*g^2*ln(F
)^2*b^2*c^2*n^2+24*x^2*(F^(c*(b*x+a)))^n*d*e*g^2*ln(F)^2*b^2*c^2*n^2+12*ln
(F)^2*x*((F^(c*(b*x+a)))^n)^2*b^2*c^2*e^2*f*g*n^2+48*ln(F)^2*x*(F^(c*(b*x+
a)))^n*b^2*c^2*d*e*f*g*n^2+6*ln(F)^2*((F^(c*(b*x+a)))^n)^2*b^2*c^2*e^2*f^2
*n^2+24*ln(F)^2*(F^(c*(b*x+a)))^n*b^2*c^2*d*e*f^2*n^2-6*ln(F)*x*((F^(c*(b*
x+a)))^n)^2*b*c*e^2*g^2*n-48*ln(F)*x*(F^(c*(b*x+a)))^n*b*c*d*e*g^2*n-6*ln(
F)*((F^(c*(b*x+a)))^n)^2*b*c*e^2*f*g*n-48*ln(F)*(F^(c*(b*x+a)))^n*b*c*d*e*
f*g*n+3*((F^(c*(b*x+a)))^n)^2*e^2*g^2+48*(F^(c*(b*x+a)))^n*d*e*g^2)/ln(F)^
3/b^3/c^3/n^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.20

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)^2\,dx = \frac{4\left(b^3c^3d^2g^2n^3x^3+3\,b^3c^3d^2fgn^3x^2+3\,b^3c^3d^2f^2n^3x\right)\log\left(F\right)^3+3\left(e^2g^2+2\left(b^2c^2e^2g^2n^2x^2+2\,b^2c^2e^2fgn^2x\right.\right.}{}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^2,x, algorithm="fricas")
```

output
```
1/12*(4*(b^3*c^3*d^2*g^2*n^3*x^3 + 3*b^3*c^3*d^2*f*g*n^3*x^2 + 3*b^3*c^3*d
^2*f^2*n^3*x)*log(F)^3 + 3*(e^2*g^2 + 2*(b^2*c^2*e^2*g^2*n^2*x^2 + 2*b^2*c
^2*e^2*f*g*n^2*x + b^2*c^2*e^2*f^2*n^2)*log(F)^2 - 2*(b*c*e^2*g^2*n*x + b*
c*e^2*f*g*n)*log(F))*F^(2*b*c*n*x + 2*a*c*n) + 24*(2*d*e*g^2 + (b^2*c^2*d*
e*g^2*n^2*x^2 + 2*b^2*c^2*d*e*f*g*n^2*x + b^2*c^2*d*e*f^2*n^2)*log(F)^2 -
2*(b*c*d*e*g^2*n*x + b*c*d*e*f*g*n)*log(F))*F^(b*c*n*x + a*c*n))/(b^3*c^3*
n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(231) = 462$.

Time = 1.71 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.21

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2\,dx$$
$$= \begin{cases} (d+e)^2\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right) \\ (d+e(F^{ac})^n)^2\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right) \\ (d+e)^2\left(f^2x+fgx^2+\frac{g^2x^3}{3}\right) \\ d^2f^2x+d^2fgx^2+\frac{d^2g^2x^3}{3}+\frac{2def^2(F^{ac+bcx})^n}{bcn\log(F)}+\frac{4defgx(F^{ac+bcx})^n}{bcn\log(F)}+\frac{2deg^2x^2(F^{ac+bcx})^n}{bcn\log(F)}+\frac{e^2f^2(F^{ac+bcx})^{2n}}{2bcn\log(F)}+\frac{e^2fgx(}{bcn} \end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2*(g*x+f)**2,x)
```

output
```
Piecewise(((d + e)**2*(f**2*x + f*g*x**2 + g**2*x**3/3), Eq(F, 1) & Eq(b,
0) & Eq(c, 0) & Eq(n, 0)), ((d + e*(F**(a*c))**n)**2*(f**2*x + f*g*x**2 +
g**2*x**3/3), Eq(b, 0)), ((d + e)**2*(f**2*x + f*g*x**2 + g**2*x**3/3), Eq
(F, 1) | Eq(c, 0) | Eq(n, 0)), (d**2*f**2*x + d**2*f*g*x**2 + d**2*g**2*x*
*3/3 + 2*d*e*f**2*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + 4*d*e*f*g*x*(F**(
a*c + b*c*x))**n/(b*c*n*log(F)) + 2*d*e*g**2*x**2*(F**(a*c + b*c*x))**n/(b
*c*n*log(F)) + e**2*f**2*(F**(a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)) + e**2
*f*g*x*(F**(a*c + b*c*x))**(2*n)/(b*c*n*log(F)) + e**2*g**2*x**2*(F**(a*c
+ b*c*x))**(2*n)/(2*b*c*n*log(F)) - 4*d*e*f*g*(F**(a*c + b*c*x))**n/(b**2*
c**2*n**2*log(F)**2) - 4*d*e*g**2*x*(F**(a*c + b*c*x))**n/(b**2*c**2*n**2*
log(F)**2) - e**2*f*g*(F**(a*c + b*c*x))**(2*n)/(2*b**2*c**2*n**2*log(F)**
2) - e**2*g**2*x*(F**(a*c + b*c*x))**(2*n)/(2*b**2*c**2*n**2*log(F)**2) +
4*d*e*g**2*(F**(a*c + b*c*x))**n/(b**3*c**3*n**3*log(F)**3) + e**2*g**2*(F
**(a*c + b*c*x))**(2*n)/(4*b**3*c**3*n**3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.45

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2 \, dx$$
$$= \frac{1}{3}d^2g^2x^3 + d^2fgx^2 + d^2f^2x + \frac{2F^{bcnx+acn}def^2}{bcn\log(F)} + \frac{F^{2bcnx+2acn}e^2f^2}{2bcn\log(F)} + \frac{4(F^{acn}bcnx\log(F) - F^{acn})F^{bcnx}defg}{b^2c^2n^2\log(F)^2} + \frac{(2F^{2acn}bcnx\log(F) - F^{2acn})F^{2bcnx}e^2fg}{2b^2c^2n^2\log(F)^2} + \frac{2\left(F^{acn}b^2c^2n^2x^2\log(F)^2 - 2F^{acn}bcnx\log(F) + 2F^{acn}\right)F^{bcnx}deg^2}{b^3c^3n^3\log(F)^3} + \frac{\left(2F^{2acn}b^2c^2n^2x^2\log(F)^2 - 2F^{2acn}bcnx\log(F) + F^{2acn}\right)F^{2bcnx}e^2g^2}{4b^3c^3n^3\log(F)^3}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^2,x, algorithm="maxima")
```

output
```
1/3*d^2*g^2*x^3 + d^2*f*g*x^2 + d^2*f^2*x + 2*F^(b*c*n*x + a*c*n)*d*e*f^2/
(b*c*n*log(F)) + 1/2*F^(2*b*c*n*x + 2*a*c*n)*e^2*f^2/(b*c*n*log(F)) + 4*(F
^(a*c*n)*b*c*n*x*log(F) - F^(a*c*n))*F^(b*c*n*x)*d*e*f*g/(b^2*c^2*n^2*log(
F)^2) + 1/2*(2*F^(2*a*c*n)*b*c*n*x*log(F) - F^(2*a*c*n))*F^(2*b*c*n*x)*e^2
*f*g/(b^2*c^2*n^2*log(F)^2) + 2*(F^(a*c*n)*b^2*c^2*n^2*x^2*log(F)^2 - 2*F^
(a*c*n)*b*c*n*x*log(F) + 2*F^(a*c*n))*F^(b*c*n*x)*d*e*g^2/(b^3*c^3*n^3*log
(F)^3) + 1/4*(2*F^(2*a*c*n)*b^2*c^2*n^2*x^2*log(F)^2 - 2*F^(2*a*c*n)*b*c*n
*x*log(F) + F^(2*a*c*n))*F^(2*b*c*n*x)*e^2*g^2/(b^3*c^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 5675, normalized size of antiderivative = 23.74

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2 \, dx = \text{Too large to display}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^2,x, algorithm="giac")
```

output
```
1/3*d^2*g^2*x^3 + d^2*f*g*x^2 + d^2*f^2*x - 1/2*(((2*pi*b^2*c^2*e^2*g^2*n^
2*x^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*e^2*g^2*n^2*x^2*log(abs(F)) + 4*pi
*b^2*c^2*e^2*f*g*n^2*x*log(abs(F))*sgn(F) - 4*pi*b^2*c^2*e^2*f*g*n^2*x*log
(abs(F)) + 2*pi*b^2*c^2*e^2*f^2*n^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*e^2*
f^2*n^2*log(abs(F)) - pi*b*c*e^2*g^2*n*x*sgn(F) + pi*b*c*e^2*g^2*n*x - pi*
b*c*e^2*f*g*n*sgn(F) + pi*b*c*e^2*f*g*n)*(pi^3*b^3*c^3*n^3*sgn(F) - 3*pi*b
^3*c^3*n^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c^3*n^3 + 3*pi*b^3*c^3*n^3*log(
abs(F))^2)/((pi^3*b^3*c^3*n^3*sgn(F) - 3*pi*b^3*c^3*n^3*log(abs(F))^2*sgn(
F) - pi^3*b^3*c^3*n^3 + 3*pi*b^3*c^3*n^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^
3*n^3*log(abs(F))*sgn(F) - 3*pi^2*b^3*c^3*n^3*log(abs(F)) + 2*b^3*c^3*n^3*
log(abs(F))^3)^2) - (pi^2*b^2*c^2*e^2*g^2*n^2*x^2*sgn(F) - pi^2*b^2*c^2*e^
2*g^2*n^2*x^2 + 2*b^2*c^2*e^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*b^2*c^2*e
^2*f*g*n^2*x*sgn(F) - 2*pi^2*b^2*c^2*e^2*f*g*n^2*x + 4*b^2*c^2*e^2*f*g*n^2
*x*log(abs(F))^2 + pi^2*b^2*c^2*e^2*f^2*n^2*sgn(F) - pi^2*b^2*c^2*e^2*f^2*
n^2 + 2*b^2*c^2*e^2*f^2*n^2*log(abs(F))^2 - 2*b*c*e^2*g^2*n*x*log(abs(F))
- 2*b*c*e^2*f*g*n*log(abs(F)) + e^2*g^2)*(3*pi^2*b^3*c^3*n^3*log(abs(F))*s
gn(F) - 3*pi^2*b^3*c^3*n^3*log(abs(F)) + 2*b^3*c^3*n^3*log(abs(F))^3)/((pi
^3*b^3*c^3*n^3*sgn(F) - 3*pi*b^3*c^3*n^3*log(abs(F))^2*sgn(F) - pi^3*b^3*c
^3*n^3 + 3*pi*b^3*c^3*n^3*log(abs(F))^2)^2 + (3*pi^2*b^3*c^3*n^3*log(abs(F
))*sgn(F) - 3*pi^2*b^3*c^3*n^3*log(abs(F)) + 2*b^3*c^3*n^3*log(abs(F))^...
```

Mupad [B] (verification not implemented)

Time = 23.12 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.12

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2\,dx$$
$$= \left(F^{b\,c\,x}\,F^{a\,c}\right)^{2\,n}\left(\frac{e^2\,\left(2\,b^2\,c^2\,f^2\,n^2\ln\left(F\right)^2-2\,b\,c\,f\,g\,n\,\ln\left(F\right)+g^2\right)}{4\,b^3\,c^3\,n^3\ln\left(F\right)^3}+\frac{e^2\,g^2\,x^2}{2\,b\,c\,n\,\ln\left(F\right)}-\frac{e^2\,g\,x\,\left(g-2\,b\,c\,f\,n\,\ln\left(F\right)\right)}{2\,b^2\,c^2\,n^2\ln\left(F\right)^2}\right)+\left(F^{b\,c\,x}\,F^{a\,c}\right)^{n}\left(\frac{2\,d\,e\,\left(b^2\,c^2\,f^2\,n^2\ln\left(F\right)^2-2\,b\,c\,f\,g\,n\,\ln\left(F\right)+2\,g^2\right)}{b^3\,c^3\,n^3\ln\left(F\right)^3}+\frac{2\,d\,e\,g^2\,x^2}{b\,c\,n\,\ln\left(F\right)}-\frac{4\,d\,e\,g\,x\,\left(g-b\,c\,f\,n\,\ln\left(F\right)\right)}{b^2\,c^2\,n^2\ln\left(F\right)^2}\right)+d^2\,f^2\,x+\frac{d^2\,g^2\,x^3}{3}+d^2\,f\,g\,x^2$$

input
```
int((f + g*x)^2*(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
(F^(b*c*x)*F^(a*c))^(2*n)*((e^2*(g^2 + 2*b^2*c^2*f^2*n^2*log(F)^2 - 2*b*c*
f*g*n*log(F)))/(4*b^3*c^3*n^3*log(F)^3) + (e^2*g^2*x^2)/(2*b*c*n*log(F)) -
 (e^2*g*x*(g - 2*b*c*f*n*log(F)))/(2*b^2*c^2*n^2*log(F)^2)) + (F^(b*c*x)*F
^(a*c))^n*((2*d*e*(2*g^2 + b^2*c^2*f^2*n^2*log(F)^2 - 2*b*c*f*g*n*log(F)))
/(b^3*c^3*n^3*log(F)^3) + (2*d*e*g^2*x^2)/(b*c*n*log(F)) - (4*d*e*g*x*(g -
 b*c*f*n*log(F)))/(b^2*c^2*n^2*log(F)^2)) + d^2*f^2*x + (d^2*g^2*x^3)/3 +
d^2*f*g*x^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.82

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2\,dx$$
$$= \frac{6f^{2bcnx+2acn}\log(f)^2\,b^2c^2e^2f^2n^2+12f^{2bcnx+2acn}\log(f)^2\,b^2c^2e^2fg\,n^2x+6f^{2bcnx+2acn}\log(f)^2\,b^2c^2e^2g^2n^2x^2 -}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^2,x)
```

output
```
(6*f**(2*a*c*n + 2*b*c*n*x)*log(f)**2*b**2*c**2*e**2*f**2*n**2 + 12*f**(2*
a*c*n + 2*b*c*n*x)*log(f)**2*b**2*c**2*e**2*f*g*n**2*x + 6*f**(2*a*c*n + 2
*b*c*n*x)*log(f)**2*b**2*c**2*e**2*g**2*n**2*x**2 - 6*f**(2*a*c*n + 2*b*c*
n*x)*log(f)*b*c*e**2*f*g*n - 6*f**(2*a*c*n + 2*b*c*n*x)*log(f)*b*c*e**2*g*
*2*n*x + 3*f**(2*a*c*n + 2*b*c*n*x)*e**2*g**2 + 24*f**(a*c*n + b*c*n*x)*lo
g(f)**2*b**2*c**2*d*e*f**2*n**2 + 48*f**(a*c*n + b*c*n*x)*log(f)**2*b**2*c
**2*d*e*f*g*n**2*x + 24*f**(a*c*n + b*c*n*x)*log(f)**2*b**2*c**2*d*e*g**2*
n**2*x**2 - 48*f**(a*c*n + b*c*n*x)*log(f)*b*c*d*e*f*g*n - 48*f**(a*c*n +
b*c*n*x)*log(f)*b*c*d*e*g**2*n*x + 48*f**(a*c*n + b*c*n*x)*d*e*g**2 + 12*l
og(f)**3*b**3*c**3*d**2*f**2*n**3*x + 12*log(f)**3*b**3*c**3*d**2*f*g*n**3
*x**2 + 4*log(f)**3*b**3*c**3*d**2*g**2*n**3*x**3)/(12*log(f)**3*b**3*c**3
*n**3)
```

3.28 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx &= \frac{d^2(f+gx)^2}{2g}-\frac{2de\left(F^{ac+bcx}\right)^n g}{b^2c^2n^2\log^2(F)} \\ &\quad -\frac{e^2\left(F^{ac+bcx}\right)^{2n} g}{4b^2c^2n^2\log^2(F)}+\frac{2de\left(F^{ac+bcx}\right)^n (f+gx)}{bcn\log(F)} \\ &\quad +\frac{e^2\left(F^{ac+bcx}\right)^{2n} (f+gx)}{2bcn\log(F)}\end{aligned}$$

output
```
1/2*d^2*(g*x+f)^2/g-2*d*e*(F^(b*c*x+a*c))^n*g/b^2/c^2/n^2/ln(F)^2-1/4*e^2*
(F^(b*c*x+a*c))^(2*n)*g/b^2/c^2/n^2/ln(F)^2+2*d*e*(F^(b*c*x+a*c))^n*(g*x+f
)/b/c/n/ln(F)+1/2*e^2*(F^(b*c*x+a*c))^(2*n)*(g*x+f)/b/c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\begin{aligned}&\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx \\ &= \frac{-e\left(F^{c(a+bx)}\right)^n \left(8d+e\left(F^{c(a+bx)}\right)^n\right) g+2bce\left(F^{c(a+bx)}\right)^n \left(4d+e\left(F^{c(a+bx)}\right)^n\right) n(f+gx)\log(F)+2b^2c^2d^2n}{4b^2c^2n^2\log^2(F)}\end{aligned}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x),x]
```

output
```
(-(e*(F^(c*(a + b*x)))^n*(8*d + e*(F^(c*(a + b*x)))^n)*g) + 2*b*c*e*(F^(c*
(a + b*x)))^n*(4*d + e*(F^(c*(a + b*x)))^n)*n*(f + g*x)*Log[F] + 2*b^2*c^2
*d^2*n^2*x*(2*f + g*x)*Log[F]^2)/(4*b^2*c^2*n^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(2de(f+gx)\left(F^{ac+bcx}\right)^n+e^2(f+gx)\left(F^{ac+bcx}\right)^{2n}+d^2(f+gx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2deg\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log^2(F)}-\frac{e^2g\left(F^{ac+bcx}\right)^{2n}}{4b^2c^2n^2\log^2(F)}+\frac{2de(f+gx)\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{e^2(f+gx)\left(F^{ac+bcx}\right)^{2n}}{2bcn\log(F)}+\frac{d^2(f+gx)^2}{2g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x),x]
```

output
```
(d^2*(f + g*x)^2)/(2*g) - (2*d*e*(F^(a*c + b*c*x))^n*g)/(b^2*c^2*n^2*Log[F
]^2) - (e^2*(F^(a*c + b*c*x))^(2*n)*g)/(4*b^2*c^2*n^2*Log[F]^2) + (2*d*e*(
F^(a*c + b*c*x))^n*(f + g*x))/(b*c*n*Log[F]) + (e^2*(F^(a*c + b*c*x))^(2*n
)*(f + g*x))/(2*b*c*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12

method	result
norman	$d^2 f x+\frac{d^2 g\,x^2}{2}+\frac{e^2 (2 \ln (F) b c f n-g) {\mathrm e}^{2 n \ln \left({\mathrm e}^{c (b x+a) \ln (F)}\right)}}{4 \ln (F)^2 b^2 c^2 n^2}+\frac{2 d e (\ln (F) b c f n-g) {\mathrm e}^{n \ln \left({\mathrm e}^{c (b x+a) \ln (F)}\right)}}{\ln (F)^2 b^2 c^2 n^2}+\frac{e^2 g x\,{\mathrm e}^{2 n \ln \left({\mathrm e}\right.}}{2 \ln (}$
parallelrisch	$\frac{2 d^2 g\,x^2 \ln (F)^2 b^2 c^2 n^2+4 d^2 f x \ln (F)^2 b^2 c^2 n^2+2 \ln (F) \left(F^{c (b x+a)}\right)^{2 n} b c\,e^2 g n x+8 \ln (F) \left(F^{c (b x+a)}\right)^{n} b c d e g n x+2 \ln (F) \left(F^{c (b x+a}\right.}{4 \ln (F)^2 b^2 c^2 n^2}$
orering	$\left(2 \ln (F)^3 b^3 c^3 g^3 n^3 x^4+8 \ln (F)^3 b^3 c^3 f\,g^2 n^3 x^3+10 \ln (F)^3 b^3 c^3 f^2 g\,n^3 x^2+4 \ln (F)^3 b^3 c^3 f^3 n^3 x+3 \ln (F)^2 b^2 c^2 g^3 n^2 x^3+15 \ln (F)^2\right.$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2*(g*x+f),x,method=_RETURNVERBOSE)
```

output
```
d^2*f*x+1/2*d^2*g*x^2+1/4*e^2*(2*ln(F)*b*c*f*n-g)/ln(F)^2/b^2/c^2/n^2*exp(
n*ln(exp(c*(b*x+a)*ln(F))))^2+2*d*e*(ln(F)*b*c*f*n-g)/ln(F)^2/b^2/c^2/n^2*
exp(n*ln(exp(c*(b*x+a)*ln(F))))+1/2/ln(F)/b/c/n*e^2*g*x*exp(n*ln(exp(c*(b*
x+a)*ln(F))))^2+2/ln(F)/b/c/n*d*e*g*x*exp(n*ln(exp(c*(b*x+a)*ln(F))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx = \frac{2\left(b^2c^2d^2gn^2x^2+2\,b^2c^2d^2fn^2x\right)\log(F)^2-\left(e^2g-2\left(bce^2gnx+bce^2fn\right)\log(F)\right)F^{2\,bcnx+2\,acn}-8\left(deg-\left(\right.\right.}{4\,b^2c^2n^2\log(F)^2}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f),x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*c^2*d^2*g*n^2*x^2 + 2*b^2*c^2*d^2*f*n^2*x)*log(F)^2 - (e^2*g -
 2*(b*c*e^2*g*n*x + b*c*e^2*f*n)*log(F))*F^(2*b*c*n*x + 2*a*c*n) - 8*(d*e*
g - (b*c*d*e*g*n*x + b*c*d*e*f*n)*log(F))*F^(b*c*n*x + a*c*n))/(b^2*c^2*n^
2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.65

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx = \begin{cases} (d+e)^2\left(fx+\frac{gx^2}{2}\right) \\ \left(d+e(F^{ac})^n\right)^2\left(fx+\frac{gx^2}{2}\right) \\ (d+e)^2\left(fx+\frac{gx^2}{2}\right) \\ d^2fx+\frac{d^2gx^2}{2}+\frac{2def\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{2degx\left(F^{ac+bcx}\right)^n}{bcn\log(F)}+\frac{e^2f\left(F^{ac+bcx}\right)^{2n}}{2bcn\log(F)}+\frac{e^2gx\left(F^{ac+bcx}\right)^{2n}}{2bcn\log(F)}-\frac{2deg\left(F^{ac+bcx}\right)^n}{b^2c^2n^2\log(F)^2}-\frac{e^2g\left(F^{ac}\right.}{4b^2c^2n^2} \end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2*(g*x+f),x)
```

output
```
Piecewise(((d + e)**2*(f*x + g*x**2/2), Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & E
q(n, 0)), ((d + e*(F**(a*c))**n)**2*(f*x + g*x**2/2), Eq(b, 0)), ((d + e)*
*2*(f*x + g*x**2/2), Eq(F, 1) | Eq(c, 0) | Eq(n, 0)), (d**2*f*x + d**2*g*x
**2/2 + 2*d*e*f*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + 2*d*e*g*x*(F**(a*c
+ b*c*x))**n/(b*c*n*log(F)) + e**2*f*(F**(a*c + b*c*x))**(2*n)/(2*b*c*n*lo
g(F)) + e**2*g*x*(F**(a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)) - 2*d*e*g*(F**
(a*c + b*c*x))**n/(b**2*c**2*n**2*log(F)**2) - e**2*g*(F**(a*c + b*c*x))**
(2*n)/(4*b**2*c**2*n**2*log(F)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)\,dx = \frac{1}{2}d^2gx^2+d^2fx+\frac{2\,F^{bcnx+acn}def}{bcn\log(F)}+\frac{F^{2\,bcnx+2\,acn}e^2f}{2\,bcn\log(F)} + \frac{2\left(F^{acn}bcnx\log(F)-F^{acn}\right)F^{bcnx}deg}{b^2c^2n^2\log(F)^2} + \frac{\left(2\,F^{2\,acn}bcnx\log(F)-F^{2\,acn}\right)F^{2\,bcnx}e^2g}{4\,b^2c^2n^2\log(F)^2}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f),x, algorithm="maxima")
```

output
```
1/2*d^2*g*x^2 + d^2*f*x + 2*F^(b*c*n*x + a*c*n)*d*e*f/(b*c*n*log(F)) + 1/2
*F^(2*b*c*n*x + 2*a*c*n)*e^2*f/(b*c*n*log(F)) + 2*(F^(a*c*n)*b*c*n*x*log(F
) - F^(a*c*n))*F^(b*c*n*x)*d*e*g/(b^2*c^2*n^2*log(F)^2) + 1/4*(2*F^(2*a*c*
n)*b*c*n*x*log(F) - F^(2*a*c*n))*F^(2*b*c*n*x)*e^2*g/(b^2*c^2*n^2*log(F)^2
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 2284, normalized size of antiderivative = 14.64

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)\,dx = \text{Too large to display}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f),x, algorithm="giac")
```

output
```
1/2*d^2*g*x^2 + d^2*f*x + 1/2*((2*(pi*b*c*e^2*g*n*x*sgn(F) - pi*b*c*e^2*g*
n*x + pi*b*c*e^2*f*n*sgn(F) - pi*b*c*e^2*f*n)*(pi*b^2*c^2*n^2*log(abs(F))*
sgn(F) - pi*b^2*c^2*n^2*log(abs(F)))/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*
c^2*n^2 + 2*b^2*c^2*n^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*s
gn(F) - pi*b^2*c^2*n^2*log(abs(F)))^2) + (pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b
^2*c^2*n^2 + 2*b^2*c^2*n^2*log(abs(F))^2)*(2*b*c*e^2*g*n*x*log(abs(F)) + 2
*b*c*e^2*f*n*log(abs(F)) - e^2*g)/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2
*n^2 + 2*b^2*c^2*n^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(
F) - pi*b^2*c^2*n^2*log(abs(F)))^2))*cos(-pi*b*c*n*x*sgn(F) + pi*b*c*n*x -
 pi*a*c*n*sgn(F) + pi*a*c*n) + ((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2*n^
2 + 2*b^2*c^2*n^2*log(abs(F))^2)*(pi*b*c*e^2*g*n*x*sgn(F) - pi*b*c*e^2*g*n
*x + pi*b*c*e^2*f*n*sgn(F) - pi*b*c*e^2*f*n)/((pi^2*b^2*c^2*n^2*sgn(F) - p
i^2*b^2*c^2*n^2 + 2*b^2*c^2*n^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(a
bs(F))*sgn(F) - pi*b^2*c^2*n^2*log(abs(F)))^2) - 2*(pi*b^2*c^2*n^2*log(abs
(F))*sgn(F) - pi*b^2*c^2*n^2*log(abs(F)))*(2*b*c*e^2*g*n*x*log(abs(F)) + 2
*b*c*e^2*f*n*log(abs(F)) - e^2*g)/((pi^2*b^2*c^2*n^2*sgn(F) - pi^2*b^2*c^2
*n^2 + 2*b^2*c^2*n^2*log(abs(F))^2)^2 + 4*(pi*b^2*c^2*n^2*log(abs(F))*sgn(
F) - pi*b^2*c^2*n^2*log(abs(F)))^2))*sin(-pi*b*c*n*x*sgn(F) + pi*b*c*n*x -
 pi*a*c*n*sgn(F) + pi*a*c*n))*e^(2*b*c*n*x*log(abs(F)) + 2*a*c*n*log(abs(F
))) - 1/4*I*((pi*b*c*e^2*g*n*x*sgn(F) - pi*b*c*e^2*g*n*x - 2*I*b*c*e^2*...
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx = d^2 f\,x - \left(F^{b\,c\,x}\,F^{a\,c}\right)^{2\,n}\left(\frac{e^2\,(g-2\,b\,c\,f\,n\,\ln(F))}{4\,b^2\,c^2\,n^2\,\ln(F)^2} - \frac{e^2\,g\,x}{2\,b\,c\,n\,\ln(F)}\right) - \left(F^{b\,c\,x}\,F^{a\,c}\right)^{n}\left(\frac{2\,d\,e\,(g-b\,c\,f\,n\,\ln(F))}{b^2\,c^2\,n^2\,\ln(F)^2} - \frac{2\,d\,e\,g\,x}{b\,c\,n\,\ln(F)}\right) + \frac{d^2\,g\,x^2}{2}$$

input
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
d^2*f*x - (F^(b*c*x)*F^(a*c))^(2*n)*((e^2*(g - 2*b*c*f*n*log(F)))/(4*b^2*c
^2*n^2*log(F)^2) - (e^2*g*x)/(2*b*c*n*log(F))) - (F^(b*c*x)*F^(a*c))^n*((2
*d*e*(g - b*c*f*n*log(F)))/(b^2*c^2*n^2*log(F)^2) - (2*d*e*g*x)/(b*c*n*log
(F))) + (d^2*g*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)\,dx$$
$$= \frac{2f^{2bcnx+2acn}\log(f)\,bc\,e^2fn + 2f^{2bcnx+2acn}\log(f)\,bc\,e^2gnx - f^{2bcnx+2acn}e^2g + 8f^{bcnx+acn}\log(f)\,bcdefn + 8}{4\log(f)^2\,b^2c^2n^2}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f),x)
```

output
```
(2*f**(2*a*c*n + 2*b*c*n*x)*log(f)*b*c*e**2*f*n + 2*f**(2*a*c*n + 2*b*c*n*
x)*log(f)*b*c*e**2*g*n*x - f**(2*a*c*n + 2*b*c*n*x)*e**2*g + 8*f**(a*c*n +
 b*c*n*x)*log(f)*b*c*d*e*f*n + 8*f**(a*c*n + b*c*n*x)*log(f)*b*c*d*e*g*n*x
 - 8*f**(a*c*n + b*c*n*x)*d*e*g + 4*log(f)**2*b**2*c**2*d**2*f*n**2*x + 2*
log(f)**2*b**2*c**2*d**2*g*n**2*x**2)/(4*log(f)**2*b**2*c**2*n**2)
```

3.29 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = d^2x+\frac{2de\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}+\frac{e^2\left(F^{c(a+bx)}\right)^{2n}}{2bcn\log(F)}$$

output
```
d^2*x+2*d*e*(F^(c*(b*x+a)))^n/b/c/n/ln(F)+1/2*e^2*(F^(c*(b*x+a)))^(2*n)/b/
c/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{e\left(F^{c(a+bx)}\right)^n\left(4d+e\left(F^{c(a+bx)}\right)^n\right)+2d^2\log\left(\left(F^{c(a+bx)}\right)^n\right)}{2bcn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
(e*(F^(c*(a + b*x)))^n*(4*d + e*(F^(c*(a + b*x)))^n) + 2*d^2*Log[(F^(c*(a
+ b*x)))^n])/(2*b*c*n*Log[F])
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2720, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int F^{-c(a+bx)}\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2 d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(e^2\left(F^{c(a+bx)}\right)^n+d^2F^{-c(a+bx)}+2de\right) d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\log\left(\left(F^{c(a+bx)}\right)^n\right)+2de\left(F^{c(a+bx)}\right)^n+\frac{1}{2}e^2F^{2c(a+bx)}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
((e^2*F^(2*c*(a + b*x)))/2 + 2*d*e*(F^(c*(a + b*x)))^n + d^2*Log[(F^(c*(a
+ b*x)))^n])/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (warning: unable to verify)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{2 d^{2} x \ln \left(F \right) b c n +e^{2} \left(F^{c \left(b x +a \right)}\right)^{2 n}+4 d e \left(F^{c \left(b x +a \right)}\right)^{n}}{2 \ln \left(F \right) b c n}$
derivativedivides	$\frac{\frac{e^{2} \left(F^{c \left(b x +a \right)}\right)^{2 n}}{2}+2 d e \left(F^{c \left(b x +a \right)}\right)^{n}+d^{2} \ln \left(\left(F^{c \left(b x +a \right)}\right)^{n}\right)}{\ln \left(F \right) b c n}$
default	$\frac{\frac{e^{2} \left(F^{c \left(b x +a \right)}\right)^{2 n}}{2}+2 d e \left(F^{c \left(b x +a \right)}\right)^{n}+d^{2} \ln \left(\left(F^{c \left(b x +a \right)}\right)^{n}\right)}{\ln \left(F \right) b c n}$
norman	$d^{2} x +\frac{e^{2} {\mathrm e}^{2 n \ln \left({\mathrm e}^{c \left(b x +a \right) \ln \left(F \right)}\right)}}{2 \ln \left(F \right) b c n}+\frac{2 d e \,{\mathrm e}^{n \ln \left({\mathrm e}^{c \left(b x +a \right) \ln \left(F \right)}\right)}}{\ln \left(F \right) b c n}$
orering	$\frac{\left(2 \ln \left(F \right) b c n x +3\right) \left(d +e \left(F^{c \left(b x +a \right)}\right)^{n}\right)^{2}}{2 \ln \left(F \right) b c n}-\frac{\left(3 \ln \left(F \right) b c n x +1\right) \left(d +e \left(F^{c \left(b x +a \right)}\right)^{n}\right) e \left(F^{c \left(b x +a \right)}\right)^{n}}{\ln \left(F \right) b c n}+\frac{x \left(2 e^{2} \left(F^{c \left(b x +a \right)}\right)^{2 n}\right.}{}$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*d^2*x*ln(F)*b*c*n+e^2*((F^(c*(b*x+a)))^n)^2+4*d*e*(F^(c*(b*x+a)))^n
)/ln(F)/b/c/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{2\,bcd^2nx\log\left(F\right)+4\,F^{bcnx+acn}de+F^{2\,bcnx+2\,acn}e^2}{2\,bcn\log\left(F\right)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*c*d^2*n*x*log(F) + 4*F^(b*c*n*x + a*c*n)*d*e + F^(2*b*c*n*x + 2*a
*c*n)*e^2)/(b*c*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \begin{cases} x(d+e)^2 & \text{for } F=1\wedge b=0\wedge c=0\wedge n=0 \\ x\left(d+e\left(F^{ac}\right)^n\right)^2 & \text{for } b=0 \\ x(d+e)^2 & \text{for } F=1\vee c=0\vee n=0 \\ d^2x+\frac{2de\left(F^{ac+bcx}\right)^n}{bcn\log\left(F\right)}+\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{2bcn\log\left(F\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Piecewise((x*(d + e)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*(d
 + e*(F**(a*c))**n)**2, Eq(b, 0)), (x*(d + e)**2, Eq(F, 1) | Eq(c, 0) | Eq
(n, 0)), (d**2*x + 2*d*e*(F**(a*c + b*c*x))**n/(b*c*n*log(F)) + e**2*(F**(
a*c + b*c*x))**(2*n)/(2*b*c*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = d^2x + \frac{2\,F^{(bx+a)cn}de}{bcn\log{(F)}} + \frac{F^{2\,(bx+a)cn}e^2}{2\,bcn\log{(F)}}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
d^2*x + 2*F^((b*x + a)*c*n)*d*e/(b*c*n*log(F)) + 1/2*F^(2*(b*x + a)*c*n)*e
^2/(b*c*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{4\,F^{bcnx}F^{acn}de + F^{2\,bcnx}F^{2\,acn}e^2 + 2\,d^2\log\left(|F|^{bcnx}|F|^{acn}\right)}{2\,bcn\log{(F)}}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
1/2*(4*F^(b*c*n*x)*F^(a*c*n)*d*e + F^(2*b*c*n*x)*F^(2*a*c*n)*e^2 + 2*d^2*l
og(abs(F)^(b*c*n*x)*abs(F)^(a*c*n)))/(b*c*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 dx = \frac{d^2\,\ln\left(F^{c\,(a+b\,x)}\right)}{b\,c\,\ln{(F)}} + \frac{\frac{e^2\,\left(F^{a\,c+b\,c\,x}\right)^{2\,n}}{2} + 2\,d\,e\,\left(F^{a\,c+b\,c\,x}\right)^n}{b\,c\,n\,\ln{(F)}}$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
(d^2*log(F^(c*(a + b*x))))/(b*c*log(F)) + ((e^2*(F^(a*c + b*c*x))^(2*n))/2
 + 2*d*e*(F^(a*c + b*c*x))^n)/(b*c*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2\,dx=\frac{f^{2bcnx+2acn}e^2+4f^{bcnx+acn}de+2\log(f)\,bc\,d^2nx}{2\log\left(f\right)bcn}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 4*f**(a*c*n + b*c*n*x)*d*e + 2*log(f)*b*c
*d**2*n*x)/(2*log(f)*b*c*n)
```

3.30 $\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx} dx$

Optimal result

Integrand size = 25, antiderivative size = 134

$$\begin{aligned}\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx} dx &= \frac{2deF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n \text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g} \\ &\quad + \frac{e^2F^{2c\left(a-\frac{bf}{g}\right)n-2cn(a+bx)}\left(F^{ac+bcx}\right)^{2n} \text{ExpIntegralEi}\left(\frac{2bcn(f+gx)\log(F)}{g}\right)}{g} \\ &\quad + \frac{d^2\log(f+gx)}{g}\end{aligned}$$

output
```
2*d*e*F^(c*(a-b*f/g)*n-c*n*(b*x+a))*(F^(b*c*x+a*c))^n*Ei(b*c*n*(g*x+f)*ln(
F)/g)/g+e^2*F^(2*c*(a-b*f/g)*n-2*c*n*(b*x+a))*(F^(b*c*x+a*c))^(2*n)*Ei(2*b
*c*n*(g*x+f)*ln(F)/g)/g+d^2*ln(g*x+f)/g
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx = \frac{2deF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n \text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)+e^2F^{-\frac{2bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^{2n} \text{ExpIntegralEi}\left(\frac{2\right.}{}}{g}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x),x]
```

output
```
((2*d*e*(F^(c*(a + b*x)))^n*ExpIntegralEi[(b*c*n*(f + g*x)*Log[F])/g])/F^(
(b*c*n*(f + g*x))/g) + (e^2*(F^(c*(a + b*x)))^(2*n)*ExpIntegralEi[(2*b*c*n
*(f + g*x)*Log[F])/g])/F^((2*b*c*n*(f + g*x))/g) + d^2*Log[f + g*x])/g
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}{f+gx}\,dx$$
$$\Big\downarrow \text{ 2614}$$
$$\int \left(\frac{2de\left(F^{ac+bcx}\right)^n}{f+gx}+\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{f+gx}+\frac{d^2}{f+gx}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2de\left(F^{ac+bcx}\right)^{n}F^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g}+\frac{e^{2}\left(F^{ac+bcx}\right)^{2n}F^{2cn\left(a-\frac{bf}{g}\right)-2cn(a+bx)}\,\text{ExpIntegralEi}\left(\frac{2bcn(f+gx)\log(F)}{g}\right)}{g}+\frac{d^{2}\log(f+gx)}{g}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x),x]
```

output
```
(2*d*e*F^(c*(a - (b*f)/g)*n - c*n*(a + b*x))*(F^(a*c + b*c*x))^n*ExpIntegr
alEi[(b*c*n*(f + g*x)*Log[F])/g])/g + (e^2*F^(2*c*(a - (b*f)/g)*n - 2*c*n*
(a + b*x))*(F^(a*c + b*c*x))^(2*n)*ExpIntegralEi[(2*b*c*n*(f + g*x)*Log[F]
)/g])/g + (d^2*Log[f + g*x])/g
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{2}}{gx+f}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f),x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.75

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx = \frac{d^2\log\left(gx+f\right)+\frac{e^2\mathrm{Ei}\left(\frac{2\,(bcgnx+bcfn)\log(F)}{g}\right)}{F^{\frac{2\,(bcf-acg)n}{g}}}+\frac{2\,de\mathrm{Ei}\left(\frac{(bcgnx+bcfn)\log(F)}{g}\right)}{F^{\frac{(bcf-acg)n}{g}}}}{g}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x, algorithm="fricas")
```

output
```
(d^2*log(g*x + f) + e^2*Ei(2*(b*c*g*n*x + b*c*f*n)*log(F)/g)/F^(2*(b*c*f -
 a*c*g)*n/g) + 2*d*e*Ei((b*c*g*n*x + b*c*f*n)*log(F)/g)/F^((b*c*f - a*c*g)
*n/g))/g
```

Sympy [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx = \int \frac{\left(d+e\left(F^{ac+bcx}\right)^n\right)^2}{f+gx}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2/(g*x+f),x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)**2/(f + g*x), x)
```

Maxima [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx = \int \frac{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}{gx+f}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x, algorithm="maxima")
```

output
```
F^(2*a*c*n)*e^2*integrate(F^(2*b*c*n*x)/(g*x + f), x) + 2*F^(a*c*n)*d*e*in
tegrate(F^(b*c*n*x)/(g*x + f), x) + d^2*log(g*x + f)/g
```

Giac [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx=\int \frac{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}{gx+f}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^2/(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx=\int \frac{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2}{f+g\,x}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x),x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x), x)
```

Reduce [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{f+gx}\,dx=\frac{f^{2acn}\left(\int \frac{f^{2bcnx}}{gx+f}dx\right)e^2g+2f^{acn}\left(\int \frac{f^{bcnx}}{gx+f}dx\right)deg+\log(gx+f)\,d^2}{g}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x)
```

output
```
(f**(2*a*c*n)*int(f**(2*b*c*n*x)/(f + g*x),x)*e**2*g + 2*f**(a*c*n)*int(f*
*(b*c*n*x)/(f + g*x),x)*d*e*g + log(f + g*x)*d**2)/g
```

3.31 $\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx = -\frac{d^2}{g(f+gx)}-\frac{2de\left(F^{ac+bcx}\right)^n}{g(f+gx)}-\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{g(f+gx)} + \frac{2bcdeF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n n\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log(F)}{g^2} + \frac{2bce^2F^{2c\left(a-\frac{bf}{g}\right)n-2cn(a+bx)}\left(F^{ac+bcx}\right)^{2n} n\,\text{ExpIntegralEi}\left(\frac{2bcn(f+gx)\log(F)}{g}\right)\log(F)}{g^2}$$

output
```
-d^2/g/(g*x+f)-2*d*e*(F^(b*c*x+a*c))^n/g/(g*x+f)-e^2*(F^(b*c*x+a*c))^(2*n)
/g/(g*x+f)+2*b*c*d*e*F^(c*(a-b*f/g)*n-c*n*(b*x+a))*(F^(b*c*x+a*c))^n*n*Ei(
b*c*n*(g*x+f)*ln(F)/g)*ln(F)/g^2+2*b*c*e^2*F^(2*c*(a-b*f/g)*n-2*c*n*(b*x+a
))*(F^(b*c*x+a*c))^(2*n)*n*Ei(2*b*c*n*(g*x+f)*ln(F)/g)*ln(F)/g^2
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx$$
$$= \frac{-\frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 g}{f+gx} + 2bcdeF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n n\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log(F) + 2bce^2F^{-\frac{2bcn}{}}}{g^2}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^2,x]
```

output
```
(-(((d + e*(F^(c*(a + b*x)))^n)^2*g)/(f + g*x)) + (2*b*c*d*e*(F^(c*(a + b*
x)))^n*n*ExpIntegralEi[(b*c*n*(f + g*x)*Log[F])/g]*Log[F])/F^((b*c*n*(f +
g*x))/g) + (2*b*c*e^2*(F^(c*(a + b*x)))^(2*n)*n*ExpIntegralEi[(2*b*c*n*(f
+ g*x)*Log[F])/g]*Log[F])/F^((2*b*c*n*(f + g*x))/g))/g^2
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}{(f+gx)^2}\,dx$$
$$\Big\downarrow \text{ 2614}$$
$$\int \left(\frac{2de\left(F^{ac+bcx}\right)^n}{(f+gx)^2}+\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{(f+gx)^2}+\frac{d^2}{(f+gx)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{2bcden\log(F)\left(F^{ac+bcx}\right)^{n}F^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g^2}-\frac{2de\left(F^{ac+bcx}\right)^{n}}{g(f+gx)}+\frac{2bce^2n\log(F)\left(F^{ac+bcx}\right)^{2n}F^{2cn\left(a-\frac{bf}{g}\right)-2cn(a+bx)}\text{ExpIntegralEi}\left(\frac{2bcn(f+gx)\log(F)}{g}\right)}{g^2}-\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{g(f+gx)}-\frac{d^2}{g(f+gx)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^2,x]
```

output
```
-(d^2/(g*(f + g*x))) - (2*d*e*(F^(a*c + b*c*x))^n)/(g*(f + g*x)) - (e^2*(F
^(a*c + b*c*x))^(2*n))/(g*(f + g*x)) + (2*b*c*d*e*F^(c*(a - (b*f)/g)*n - c
*n*(a + b*x))*(F^(a*c + b*c*x))^n*n*ExpIntegralEi[(b*c*n*(f + g*x)*Log[F])
/g]*Log[F])/g^2 + (2*b*c*e^2*F^(2*c*(a - (b*f)/g)*n - 2*c*n*(a + b*x))*(F^
(a*c + b*c*x))^(2*n)*n*ExpIntegralEi[(2*b*c*n*(f + g*x)*Log[F])/g]*Log[F])
/g^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{2}}{\left(gx+f\right)^{2}}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^2,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.88

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}}{(f+gx)^{2}}\,dx = -\frac{2\,F^{bcnx+acn}deg+F^{2\,bcnx+2\,acn}e^{2}g+d^{2}g-\frac{2\,\left(bce^{2}gnx+bce^{2}fn\right)\mathrm{Ei}\left(\frac{2\,(bcgnx+bcfn)\log(F)}{g}\right)\log(F)}{F^{\frac{2\,(bcf-acg)n}{g}}}-\frac{2\,(bcdegnx+bcdefn)\mathrm{Ei}\cdots}{F^{\cdots}}}{g^{3}x+fg^{2}}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
-(2*F^(b*c*n*x + a*c*n)*d*e*g + F^(2*b*c*n*x + 2*a*c*n)*e^2*g + d^2*g - 2*
(b*c*e^2*g*n*x + b*c*e^2*f*n)*Ei(2*(b*c*g*n*x + b*c*f*n)*log(F)/g)*log(F)/
F^(2*(b*c*f - a*c*g)*n/g) - 2*(b*c*d*e*g*n*x + b*c*d*e*f*n)*Ei((b*c*g*n*x
+ b*c*f*n)*log(F)/g)*log(F)/F^((b*c*f - a*c*g)*n/g))/(g^3*x + f*g^2)
```

Sympy [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}}{(f+gx)^{2}}\,dx = \int \frac{\left(d+e\left(F^{ac+bcx}\right)^{n}\right)^{2}}{\left(f+gx\right)^{2}}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2/(g*x+f)**2,x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)**2/(f + g*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
F^(2*a*c*n)*e^2*integrate(F^(2*b*c*n*x)/(g^2*x^2 + 2*f*g*x + f^2), x) + 2*
F^(a*c*n)*d*e*integrate(F^(b*c*n*x)/(g^2*x^2 + 2*f*g*x + f^2), x) - d^2/(g
^2*x + f*g)
```

Giac [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}{(gx+f)^2}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^2/(g*x + f)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx=\int \frac{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2}{(f+g\,x)^2}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^2,x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^2}\,dx = \text{Too large to display}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x)
```

output
```
(f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**3*n + 2*log(f)*b*c*f**2*g*
n*x + log(f)*b*c*f*g**2*n*x**2 - f**2*g - 2*f*g**2*x - g**3*x**2),x)*log(f
)*b*c*e**2*f**3*n + f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**3*n + 2
*log(f)*b*c*f**2*g*n*x + log(f)*b*c*f*g**2*n*x**2 - f**2*g - 2*f*g**2*x -
g**3*x**2),x)*log(f)*b*c*e**2*f**2*g*n*x - f**(2*a*c*n)*int(f**(2*b*c*n*x)
/(log(f)*b*c*f**3*n + 2*log(f)*b*c*f**2*g*n*x + log(f)*b*c*f*g**2*n*x**2 -
 f**2*g - 2*f*g**2*x - g**3*x**2),x)*e**2*f**2*g - f**(2*a*c*n)*int(f**(2*
b*c*n*x)/(log(f)*b*c*f**3*n + 2*log(f)*b*c*f**2*g*n*x + log(f)*b*c*f*g**2*
n*x**2 - f**2*g - 2*f*g**2*x - g**3*x**2),x)*e**2*f*g**2*x + 4*f**(a*c*n)*
int(f**(b*c*n*x)/(2*log(f)*b*c*f**3*n + 4*log(f)*b*c*f**2*g*n*x + 2*log(f)
*b*c*f*g**2*n*x**2 - f**2*g - 2*f*g**2*x - g**3*x**2),x)*log(f)*b*c*d*e*f*
*3*n + 4*f**(a*c*n)*int(f**(b*c*n*x)/(2*log(f)*b*c*f**3*n + 4*log(f)*b*c*f
**2*g*n*x + 2*log(f)*b*c*f*g**2*n*x**2 - f**2*g - 2*f*g**2*x - g**3*x**2),
x)*log(f)*b*c*d*e*f**2*g*n*x - 2*f**(a*c*n)*int(f**(b*c*n*x)/(2*log(f)*b*c
*f**3*n + 4*log(f)*b*c*f**2*g*n*x + 2*log(f)*b*c*f*g**2*n*x**2 - f**2*g -
2*f*g**2*x - g**3*x**2),x)*d*e*f**2*g - 2*f**(a*c*n)*int(f**(b*c*n*x)/(2*l
og(f)*b*c*f**3*n + 4*log(f)*b*c*f**2*g*n*x + 2*log(f)*b*c*f*g**2*n*x**2 -
f**2*g - 2*f*g**2*x - g**3*x**2),x)*d*e*f*g**2*x + d**2*x)/(f*(f + g*x))
```

3.32 $\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 286

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx = -\frac{d^2}{2g(f+gx)^2}-\frac{de\left(F^{ac+bcx}\right)^n}{g(f+gx)^2}-\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{2g(f+gx)^2} - \frac{bcde\left(F^{ac+bcx}\right)^n n\log(F)}{g^2(f+gx)}-\frac{bce^2\left(F^{ac+bcx}\right)^{2n} n\log(F)}{g^2(f+gx)} + \frac{b^2c^2deF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n n^2\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log^2(F)}{g^3} + \frac{2b^2c^2e^2F^{2c\left(a-\frac{bf}{g}\right)n-2cn(a+bx)}\left(F^{ac+bcx}\right)^{2n} n^2\,\text{ExpIntegralEi}\left(\frac{2bcn(f+gx)\log(F)}{g}\right)\log^2(F)}{g^3}$$

output
```
-1/2*d^2/g/(g*x+f)^2-d*e*(F^(b*c*x+a*c))^n/g/(g*x+f)^2-1/2*e^2*(F^(b*c*x+a
*c))^(2*n)/g/(g*x+f)^2-b*c*d*e*(F^(b*c*x+a*c))^n*n*ln(F)/g^2/(g*x+f)-b*c*e
^2*(F^(b*c*x+a*c))^(2*n)*n*ln(F)/g^2/(g*x+f)+b^2*c^2*d*e*F^(c*(a-b*f/g)*n-
c*n*(b*x+a))*(F^(b*c*x+a*c))^n*n^2*Ei(b*c*n*(g*x+f)*ln(F)/g)*ln(F)^2/g^3+2
*b^2*c^2*e^2*F^(2*c*(a-b*f/g)*n-2*c*n*(b*x+a))*(F^(b*c*x+a*c))^(2*n)*n^2*E
i(2*b*c*n*(g*x+f)*ln(F)/g)*ln(F)^2/g^3
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx = -\frac{d^2g^2 - 2b^2c^2deF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n n^2(f+gx)^2\,\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)\log^2(F) - 4b^2c^2e^2F}{}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^3,x]
```

output
```
-1/2*(d^2*g^2 - (2*b^2*c^2*d*e*(F^(c*(a + b*x)))^n*n^2*(f + g*x)^2*ExpInte
gralEi[(b*c*n*(f + g*x)*Log[F])/g]*Log[F]^2)/F^((b*c*n*(f + g*x))/g) - (4*
b^2*c^2*e^2*(F^(c*(a + b*x)))^(2*n)*n^2*(f + g*x)^2*ExpIntegralEi[(2*b*c*n
*(f + g*x)*Log[F])/g]*Log[F]^2)/F^((2*b*c*n*(f + g*x))/g) + 2*d*e*(F^(c*(a
 + b*x)))^n*g*(g + b*c*n*(f + g*x)*Log[F]) + e^2*(F^(c*(a + b*x)))^(2*n)*g
*(g + 2*b*c*n*(f + g*x)*Log[F]))/(g^3*(f + g*x)^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}{(f+gx)^3}dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(\frac{2de\left(F^{ac+bcx}\right)^n}{(f+gx)^3}+\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{(f+gx)^3}+\frac{d^2}{(f+gx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2c^2den^2\log^2(F)\left(F^{ac+bcx}\right)^nF^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\text{ExpIntegralEi}\left(\frac{bcn(f+gx)\log(F)}{g}\right)}{g^3}+\frac{2b^2c^2e^2n^2\log^2(F)\left(F^{ac+bcx}\right)^{2n}F^{2cn\left(a-\frac{bf}{g}\right)-2cn(a+bx)}\text{ExpIntegralEi}\left(\frac{2bcn(f+gx)\log(F)}{g}\right)}{g^3}-\frac{bcden\log(F)\left(F^{ac+bcx}\right)^n}{g^2(f+gx)}-\frac{de\left(F^{ac+bcx}\right)^n}{g(f+gx)^2}-\frac{bce^2n\log(F)\left(F^{ac+bcx}\right)^{2n}}{g^2(f+gx)}-\frac{e^2\left(F^{ac+bcx}\right)^{2n}}{2g(f+gx)^2}-\frac{d^2}{2g(f+gx)^2}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^3,x]
```

output
```
-1/2*d^2/(g*(f + g*x)^2) - (d*e*(F^(a*c + b*c*x))^n)/(g*(f + g*x)^2) - (e^
2*(F^(a*c + b*c*x))^(2*n))/(2*g*(f + g*x)^2) - (b*c*d*e*(F^(a*c + b*c*x))^
n*n*Log[F])/(g^2*(f + g*x)) - (b*c*e^2*(F^(a*c + b*c*x))^(2*n)*n*Log[F])/(
g^2*(f + g*x)) + (b^2*c^2*d*e*F^(c*(a - (b*f)/g)*n - c*n*(a + b*x))*(F^(a*
c + b*c*x))^n*n^2*ExpIntegralEi[(b*c*n*(f + g*x)*Log[F])/g]*Log[F]^2)/g^3
+ (2*b^2*c^2*e^2*F^(2*c*(a - (b*f)/g)*n - 2*c*n*(a + b*x))*(F^(a*c + b*c*x
))^(2*n)*n^2*ExpIntegralEi[(2*b*c*n*(f + g*x)*Log[F])/g]*Log[F]^2)/g^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{2}}{\left(gx+f\right)^{3}}dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^3,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.11

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}}{(f+gx)^{3}}\,dx = -\frac{d^{2}g^{2}-\frac{4\,(b^{2}c^{2}e^{2}g^{2}n^{2}x^{2}+2\,b^{2}c^{2}e^{2}fgn^{2}x+b^{2}c^{2}e^{2}f^{2}n^{2})\mathrm{Ei}\left(\frac{2\,(bcgnx+bcfn)\log(F)}{g}\right)\log(F)^{2}}{F^{\frac{2\,(bcf-acg)n}{g}}}-\frac{2\,(b^{2}c^{2}deg^{2}n^{2}x^{2}+2\,b^{2}c^{2}defgn^{2}x+b^{2}c^{2}de}{F^{\frac{(bcf-a}{g}}}}{}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^2*g^2 - 4*(b^2*c^2*e^2*g^2*n^2*x^2 + 2*b^2*c^2*e^2*f*g*n^2*x + b^2
*c^2*e^2*f^2*n^2)*Ei(2*(b*c*g*n*x + b*c*f*n)*log(F)/g)*log(F)^2/F^(2*(b*c*
f - a*c*g)*n/g) - 2*(b^2*c^2*d*e*g^2*n^2*x^2 + 2*b^2*c^2*d*e*f*g*n^2*x + b
^2*c^2*d*e*f^2*n^2)*Ei((b*c*g*n*x + b*c*f*n)*log(F)/g)*log(F)^2/F^((b*c*f
- a*c*g)*n/g) + (e^2*g^2 + 2*(b*c*e^2*g^2*n*x + b*c*e^2*f*g*n)*log(F))*F^(
2*b*c*n*x + 2*a*c*n) + 2*(d*e*g^2 + (b*c*d*e*g^2*n*x + b*c*d*e*f*g*n)*log(
F))*F^(b*c*n*x + a*c*n))/(g^5*x^2 + 2*f*g^4*x + f^2*g^3)
```

Sympy [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(d+e\left(F^{ac+bcx}\right)^n\right)^2}{\left(f+gx\right)^3}\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2/(g*x+f)**3,x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)**2/(f + g*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}{\left(gx+f\right)^3}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^3,x, algorithm="maxima")
```

output
```
F^(2*a*c*n)*e^2*integrate(F^(2*b*c*n*x)/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x
 + f^3), x) + 2*F^(a*c*n)*d*e*integrate(F^(b*c*n*x)/(g^3*x^3 + 3*f*g^2*x^2
 + 3*f^2*g*x + f^3), x) - 1/2*d^2/(g^3*x^2 + 2*f*g^2*x + f^2*g)
```

Giac [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}{\left(gx+f\right)^3}\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^3,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^2/(g*x + f)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx=\int \frac{\left(d+e\,\left(F^{c\,(a+b\,x)}\right)^n\right)^2}{\left(f+g\,x\right)^3}\,dx$$

input
```
int((d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^3,x)
```

output
```
int((d + e*(F^(c*(a + b*x)))^n)^2/(f + g*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}{(f+gx)^3}\,dx=\text{Too large to display}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^3,x)
```

output

```
(2*f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**4*n + 3*log(f)*b*c*f**3*
g*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f*g**3*n*x**3 - 2*f**3*
g - 6*f**2*g**2*x - 6*f*g**3*x**2 - 2*g**4*x**3),x)*log(f)*b*c*e**2*f**3*g
*n + 4*f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**4*n + 3*log(f)*b*c*f
**3*g*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f*g**3*n*x**3 - 2*f
**3*g - 6*f**2*g**2*x - 6*f*g**3*x**2 - 2*g**4*x**3),x)*log(f)*b*c*e**2*f*
*2*g**2*n*x + 2*f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**4*n + 3*log
(f)*b*c*f**3*g*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f*g**3*n*x
**3 - 2*f**3*g - 6*f**2*g**2*x - 6*f*g**3*x**2 - 2*g**4*x**3),x)*log(f)*b*
c*e**2*f*g**3*n*x**2 - 4*f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**4*
n + 3*log(f)*b*c*f**3*g*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f
*g**3*n*x**3 - 2*f**3*g - 6*f**2*g**2*x - 6*f*g**3*x**2 - 2*g**4*x**3),x)*
e**2*f**2*g**2 - 8*f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**4*n + 3*
log(f)*b*c*f**3*g*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f*g**3*
n*x**3 - 2*f**3*g - 6*f**2*g**2*x - 6*f*g**3*x**2 - 2*g**4*x**3),x)*e**2*f
*g**3*x - 4*f**(2*a*c*n)*int(f**(2*b*c*n*x)/(log(f)*b*c*f**4*n + 3*log(f)*
b*c*f**3*g*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f*g**3*n*x**3
- 2*f**3*g - 6*f**2*g**2*x - 6*f*g**3*x**2 - 2*g**4*x**3),x)*e**2*g**4*x**
2 + 4*f**(a*c*n)*int(f**(b*c*n*x)/(log(f)*b*c*f**4*n + 3*log(f)*b*c*f**3*g
*n*x + 3*log(f)*b*c*f**2*g**2*n*x**2 + log(f)*b*c*f*g**3*n*x**3 - f**3*...
```

3.33 $\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{(f+gx)^4}{4dg} - \frac{(f+gx)^3\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcdn\log(F)} - \frac{3g(f+gx)^2\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2dn^2\log^2(F)} + \frac{6g^2(f+gx)\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3dn^3\log^3(F)} - \frac{6g^3\operatorname{PolyLog}\left(4,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4dn^4\log^4(F)}$$

output
```
1/4*(g*x+f)^4/d/g-(g*x+f)^3*ln(1+e*(F^(c*(b*x+a)))^n/d)/b/c/d/n/ln(F)-3*g*
(g*x+f)^2*polylog(2,-e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d/n^2/ln(F)^2+6*g^2*(g
*x+f)*polylog(3,-e*(F^(c*(b*x+a)))^n/d)/b^3/c^3/d/n^3/ln(F)^3-6*g^3*polylo
g(4,-e*(F^(c*(b*x+a)))^n/d)/b^4/c^4/d/n^4/ln(F)^4
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$$
$$= \frac{-(f+gx)^3\log\left(1+\frac{d\left(F^{c(a+bx)}\right)^{-n}}{e}\right)+\frac{3g\left(b^2c^2n^2(f+gx)^2\log^2(F)\operatorname{PolyLog}\left(2,-\frac{d\left(F^{c(a+bx)}\right)^{-n}}{e}\right)+2g\left(bcn(f+gx)\log(F)\operatorname{PolyI}\right.\right.}{b^3c^3n^3\log^3(F)}}{bcdn\log(F)}$$

input
```
Integrate[(f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
(-((f + g*x)^3*Log[1 + d/(e*(F^(c*(a + b*x)))^n)]) + (3*g*(b^2*c^2*n^2*(f
+ g*x)^2*Log[F]^2*PolyLog[2, -(d/(e*(F^(c*(a + b*x)))^n))] + 2*g*(b*c*n*(f
 + g*x)*Log[F]*PolyLog[3, -(d/(e*(F^(c*(a + b*x)))^n))] + g*PolyLog[4, -(d
/(e*(F^(c*(a + b*x)))^n))])))/(b^3*c^3*n^3*Log[F]^3))/(b*c*d*n*Log[F])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{e\left(F^{c(a+bx)}\right)^n+d}\,dx$$
$$\Big\downarrow \text{2615}$$
$$\frac{(f+gx)^4}{4dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^3}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}$$
$$\Big\downarrow \text{2620}$$

$$\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\int(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}$$

$\Big\downarrow$ 3011

$$\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\int(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)^2\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}$$

$\Big\downarrow$ 7163

$$\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}-\frac{g\int\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}\right)}{bcn\log(F)}-\frac{(f+gx)^2\operatorname{PolyLog}\left(2 \dots\right.}{bcn\log \dots}\right.}{bcen\log(F)}\right.}{d}$$

$\Big\downarrow$ 2720

$$\frac{(f+gx)^4}{4dg} - \frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)} - \frac{3g\left(\frac{2g\left(\frac{(f+gx)\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)} - \frac{g\int F^{-c(a+bx)}\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dF^{c(a+bx)}}{b^2c^2n\log^2(F)}\right)}{bcn\log(F)} - \cdots}{bcen\log(F)}\right)}{d}$$

$\downarrow$ 7143

$$\frac{(f+gx)^4}{4dg} - \frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)} - \frac{3g\left(\frac{2g\left(\frac{(f+gx)\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)} - \frac{g\text{PolyLog}\left(4,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}\right)}{bcn\log(F)} - \frac{(f+gx)^2\text{PolyLog}\left(2,-\frac{e \cdots}{\cdots}\right)}{bcn\log(F \cdots}\right)}{bcen\log(F)}\right)}{d}$$

input
```
Int[(f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
(f + g*x)^4/(4*d*g) - (e*(((f + g*x)^3*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])
/(b*c*e*n*Log[F]) - (3*g*(-(((f + g*x)^2*PolyLog[2, -((e*(F^(c*(a + b*x)))
^n)/d)])/(b*c*n*Log[F])) + (2*g*(((f + g*x)*PolyLog[3, -((e*(F^(c*(a + b*x
)))^n)/d)])/(b*c*n*Log[F]) - (g*PolyLog[4, -((e*(F^(c*(a + b*x)))^n)/d)])/
(b^2*c^2*n^2*Log[F]^2)))/(b*c*n*Log[F])))/(b*c*e*n*Log[F])))/d
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3074 vs. $2(190) = 380$.

Time = 0.32 (sec) , antiderivative size = 3075, normalized size of antiderivative = 16.02

method	result	size
risch	Expression too large to display	3075

input
```
int((g*x+f)^3/(d+e*(F^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
3/4/c^4/b^4/ln(F)^4*g^3/d*ln(F^(c*(b*x+a)))^4-3/n/c^3/b^4/ln(F)^3*g^3*a*(l
n(F^(c*(b*x+a)))-c*(b*x+a)*ln(F))^2/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b
*x+a)))^n)-1/n/c/b/ln(F)*g^3/d*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+
a)))^n/d)*x^3+1/c^3/b^3/ln(F)^3*g^3/d*x*(ln(F^(c*(b*x+a)))-c*(b*x+a)*ln(F)
)^3+3/c/b^3/ln(F)*g^3/d*x*a^2*(ln(F^(c*(b*x+a)))-c*(b*x+a)*ln(F))+3/c^2/b^
3/ln(F)^2*g^3/d*x*a*(ln(F^(c*(b*x+a)))-c*(b*x+a)*ln(F))^2-3/c^2/b^2/ln(F)^
2*f*g^2/d*x*(ln(F^(c*(b*x+a)))-c*(b*x+a)*ln(F))^2-3/n^2/c^2/b^2/ln(F)^2*g^
3/d*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)*x^2+6/n^3/c
^3/b^3/ln(F)^3*g^3/d*polylog(3,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))
^n/d)*x-1/n/c/b^4/ln(F)*g^3/d*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a
)))^n/d)*a^3+1/n/c^4/b^4/ln(F)^4*g^3*(ln(F^(c*(b*x+a)))-c*(b*x+a)*ln(F))^3
/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)+6/n^3/c^3/b^3/ln(F)^
3*f*g^2/d*polylog(3,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)-1/n/c
/b^4/ln(F)*g^3*a^3/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)+1/n/c/
b^4/ln(F)*g^3*a^3/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)-3/c
/b/ln(F)*f^2*g/d*ln(F^(c*(b*x+a)))*x-6/c/b/ln(F)*f*g^2/d*ln(F^(c*(b*x+a)))
*x^2-2/c^3/b^3/ln(F)^3*f*g^2/d*ln(F^(c*(b*x+a)))^3+3/2/c^2/b^2/ln(F)^2*f^2
*g/d*ln(F^(c*(b*x+a)))^2-6/n^4/c^4/b^4/ln(F)^4*g^3/d*polylog(4,-e*F^(n*c*b
*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)+1/n/c/b/ln(F)*f^3/d*ln(F^(n*c*b*x)*F
^(-n*c*b*x)*(F^(c*(b*x+a)))^n)-1/n/c/b/ln(F)*f^3/d*ln((F^(c*(b*x+a)))^n...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(189) = 378$.

Time = 0.08 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.23

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = -\frac{4\,(b^3c^3f^3 - 3\,ab^2c^3f^2g + 3\,a^2bc^3fg^2 - a^3c^3g^3)n^3\log\left(F^{bcnx+acn}e+d\right)\log\left(F\right)^3 - (b^4c^4g^3n^4x^4 + 4\,b^4c^4f}{}$$

input
```
integrate((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-1/4*(4*(b^3*c^3*f^3 - 3*a*b^2*c^3*f^2*g + 3*a^2*b*c^3*f*g^2 - a^3*c^3*g^3
)*n^3*log(F^(b*c*n*x + a*c*n)*e + d)*log(F)^3 - (b^4*c^4*g^3*n^4*x^4 + 4*b
^4*c^4*f*g^2*n^4*x^3 + 6*b^4*c^4*f^2*g*n^4*x^2 + 4*b^4*c^4*f^3*n^4*x)*log(
F)^4 + 4*(b^3*c^3*g^3*n^3*x^3 + 3*b^3*c^3*f*g^2*n^3*x^2 + 3*b^3*c^3*f^2*g*
n^3*x + (3*a*b^2*c^3*f^2*g - 3*a^2*b*c^3*f*g^2 + a^3*c^3*g^3)*n^3)*log(F)^
3*log((F^(b*c*n*x + a*c*n)*e + d)/d) + 12*(b^2*c^2*g^3*n^2*x^2 + 2*b^2*c^2
*f*g^2*n^2*x + b^2*c^2*f^2*g*n^2)*dilog(-(F^(b*c*n*x + a*c*n)*e + d)/d + 1
)*log(F)^2 + 24*g^3*polylog(4, -F^(b*c*n*x + a*c*n)*e/d) - 24*(b*c*g^3*n*x
 + b*c*f*g^2*n)*log(F)*polylog(3, -F^(b*c*n*x + a*c*n)*e/d))/(b^4*c^4*d*n^
4*log(F)^4)
```

Sympy [F]

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(f+gx)^3}{d+e\left(F^{ac+bcx}\right)^n}\,dx$$

input
```
integrate((g*x+f)**3/(d+e*(F**((b*x+a)*c))**n),x)
```

output
```
Integral((f + g*x)**3/(d + e*(F**(a*c + b*c*x))**n), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(189) = 378$.

Time = 0.19 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.50

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = f^3\left(\frac{bcnx+acn}{bcdn} - \frac{\log\left(F^{bcnx+acn}e+d\right)}{bcdn\log(F)}\right) - \frac{3\left(bcnx\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\right)f^2g}{b^2c^2dn^2\log(F)^2} - \frac{3\left(b^2c^2n^2x^2\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F)^2 + 2\,bcnx\mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\log(F) - 2\,\mathrm{Li}_3\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\right)fg^2}{b^3c^3dn^3\log(F)^3} - \frac{\left(b^3c^3n^3x^3\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F)^3 + 3\,b^2c^2n^2x^2\mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\log(F)^2 - 6\,bcnx\log(F)\,\mathrm{Li}_3(-\ldots}{b^4c^4dn^4\log(F)^4} + \frac{b^4c^4g^3n^4x^4\log(F)^4 + 4\,b^4c^4fg^2n^4x^3\log(F)^4 + 6\,b^4c^4f^2gn^4x^2\log(F)^4}{4\,b^4c^4dn^4\log(F)^4}$$

input
```
integrate((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
f^3*((b*c*n*x + a*c*n)/(b*c*d*n) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d*n
*log(F))) - 3*(b*c*n*x*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F) + dilog(-
F^(b*c*n*x)*F^(a*c*n)*e/d))*f^2*g/(b^2*c^2*d*n^2*log(F)^2) - 3*(b^2*c^2*n^
2*x^2*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F)^2 + 2*b*c*n*x*dilog(-F^(b*
c*n*x)*F^(a*c*n)*e/d)*log(F) - 2*polylog(3, -F^(b*c*n*x)*F^(a*c*n)*e/d))*f
*g^2/(b^3*c^3*d*n^3*log(F)^3) - (b^3*c^3*n^3*x^3*log(F^(b*c*n*x)*F^(a*c*n)
*e/d + 1)*log(F)^3 + 3*b^2*c^2*n^2*x^2*dilog(-F^(b*c*n*x)*F^(a*c*n)*e/d)*l
og(F)^2 - 6*b*c*n*x*log(F)*polylog(3, -F^(b*c*n*x)*F^(a*c*n)*e/d) + 6*poly
log(4, -F^(b*c*n*x)*F^(a*c*n)*e/d))*g^3/(b^4*c^4*d*n^4*log(F)^4) + 1/4*(b^
4*c^4*g^3*n^4*x^4*log(F)^4 + 4*b^4*c^4*f*g^2*n^4*x^3*log(F)^4 + 6*b^4*c^4*
f^2*g*n^4*x^2*log(F)^4)/(b^4*c^4*d*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(gx+f)^3}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate((g*x + f)^3/((F^((b*x + a)*c))^n*e + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(f+g\,x)^3}{d+e\left(F^{c\,(a+b\,x)}\right)^n}\,dx$$

input
```
int((f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
int((f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int \frac{(f+gx)^3}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{\left(\int \frac{x^3}{f^{bcnx+acn}e+d}dx\right)\log(f)\,bcd\,g^3n+3\left(\int \frac{x^2}{f^{bcnx+acn}e+d}dx\right)\log(f)\,bcdf\,g^2n+3\left(\int \frac{x}{f^{bcnx+acn}e+d}dx\right)\log(f)\,bcd}{\log(f)\,bcdn}$$

input
```
int((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n),x)
```

output
```
(int(x**3/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d*g**3*n + 3*int(x**2
/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d*f*g**2*n + 3*int(x/(f**(a*c*
n + b*c*n*x)*e + d),x)*log(f)*b*c*d*f**2*g*n - log(f**(a*c*n + b*c*n*x)*e
+ d)*f**3 + log(f)*b*c*f**3*n*x)/(log(f)*b*c*d*n)
```

3.34 $\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{(f+gx)^3}{3dg} - \frac{(f+gx)^2\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcdn\log(F)} - \frac{2g(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2dn^2\log^2(F)} + \frac{2g^2\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3dn^3\log^3(F)}$$

output

```
1/3*(g*x+f)^3/d/g-(g*x+f)^2*ln(1+e*(F^(c*(b*x+a)))^n/d)/b/c/d/n/ln(F)-2*g*
(g*x+f)*polylog(2,-e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d/n^2/ln(F)^2+2*g^2*poly
log(3,-e*(F^(c*(b*x+a)))^n/d)/b^3/c^3/d/n^3/ln(F)^3
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$$
$$=\frac{-b^2c^2n^2(f+gx)^2\log^2(F)\log\left(1+\frac{d\left(F^{c(a+bx)}\right)^{-n}}{e}\right)+2bcgn(f+gx)\log(F)\operatorname{PolyLog}\left(2,-\frac{d\left(F^{c(a+bx)}\right)^{-n}}{e}\right)}{b^3c^3dn^3\log^3(F)}$$

input
```
Integrate[(f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
(-(b^2*c^2*n^2*(f + g*x)^2*Log[F]^2*Log[1 + d/(e*(F^(c*(a + b*x)))^n)]) +
2*b*c*g*n*(f + g*x)*Log[F]*PolyLog[2, -(d/(e*(F^(c*(a + b*x)))^n))] + 2*g^
2*PolyLog[3, -(d/(e*(F^(c*(a + b*x)))^n))])/(b^3*c^3*d*n^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{e\left(F^{c(a+bx)}\right)^n+d}\,dx$$

$$\Big\downarrow \ 2615$$

$$\frac{(f+gx)^3}{3dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^2}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}$$

$$\Big\downarrow \ 2620$$

$$\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int (f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\int \text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\int F^{-c(a+bx)}\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dF^{c(a+bx)}}{b^2c^2n\log^2(F)}-\frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\,\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}-\frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}$$

input
```
Int[(f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
(f + g*x)^3/(3*d*g) - (e*(((f + g*x)^2*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])
/(b*c*e*n*Log[F]) - (2*g*(-(((f + g*x)*PolyLog[2, -((e*(F^(c*(a + b*x)))^n
)/d)])/(b*c*n*Log[F])) + (g*PolyLog[3, -((e*(F^(c*(a + b*x)))^n)/d)])/(b^2
*c^2*n^2*Log[F]^2)))/(b*c*e*n*Log[F])))/d
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. $2(143) = 286$.

Time = 0.15 (sec) , antiderivative size = 1124, normalized size of antiderivative = 7.75

method	result	size
risch	Expression too large to display	1124

input
```
int((g*x+f)^2/(d+e*(F^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)^2/b^2/c^2*f*g/d*ln(F^(c*(b*x+a)))^2-2/ln(F)^2/b^2/c^2/n^2*f*g/d*po
lylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)+1/ln(F)^3/b^3/c^3
/n*g^2/d*ln(F^(c*(b*x+a)))^2*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)
))^n/d)+2/ln(F)^2/b^2/c^2/n*g^2/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c
*b*x)*e+d)*ln(F^(c*(b*x+a)))*x-2/ln(F)^2/b^2/c^2/n*g^2/d*ln(F^(n*c*b*x)*F^
(-n*c*b*x)*(F^(c*(b*x+a)))^n)*ln(F^(c*(b*x+a)))*x-2/ln(F)/b/c/n*f*g/d*ln((
F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*x+2/ln(F)^2/b^2/c^2/n*f*g/d
*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*ln(F^(c*(b*x+a)))+2/ln
(F)/b/c/n*f*g/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*x-2/ln(F)^2
/b^2/c^2/n*f*g/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*ln(F^(c*(b
*x+a)))+1/ln(F)^2/b^2/c^2*g^2/d*ln(F^(c*(b*x+a)))^2*x-2/ln(F)^2/b^2/c^2/n^
2*g^2/d*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)*x-1/ln(
F)/b/c/n*f^2/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)+1/ln(F)/
b/c/n*f^2/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)-2/ln(F)^2/b^2/c
^2/n*f*g/d*ln(F^(c*(b*x+a)))*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)
))^n/d)-2/ln(F)^2/b^2/c^2/n*g^2/d*ln(F^(c*(b*x+a)))*ln(1+e*F^(n*c*b*x)*F^(
-n*c*b*x)*(F^(c*(b*x+a)))^n/d)*x-2/3/ln(F)^3/b^3/c^3*g^2/d*ln(F^(c*(b*x+a)
))^3+2/ln(F)^3/b^3/c^3/n^3*g^2/d*polylog(3,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^
(c*(b*x+a)))^n/d)-1/ln(F)/b/c/n*g^2/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^
(n*c*b*x)*e+d)*x^2-1/ln(F)^3/b^3/c^3/n*g^2/d*ln((F^(c*(b*x+a)))^n*F^(-n...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.93

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = -\frac{3\,(b^2c^2f^2-2\,abc^2fg+a^2c^2g^2)n^2\log\left(F^{bcnx+acn}e+d\right)\log\left(F\right)^2-(b^3c^3g^2n^3x^3+3\,b^3c^3fgn^3x^2+3\,b^3c^3}{}$$

input
```
integrate((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-1/3*(3*(b^2*c^2*f^2 - 2*a*b*c^2*f*g + a^2*c^2*g^2)*n^2*log(F^(b*c*n*x + a
*c*n)*e + d)*log(F)^2 - (b^3*c^3*g^2*n^3*x^3 + 3*b^3*c^3*f*g*n^3*x^2 + 3*b
^3*c^3*f^2*n^3*x)*log(F)^3 + 3*(b^2*c^2*g^2*n^2*x^2 + 2*b^2*c^2*f*g*n^2*x
+ (2*a*b*c^2*f*g - a^2*c^2*g^2)*n^2)*log(F)^2*log((F^(b*c*n*x + a*c*n)*e +
 d)/d) + 6*(b*c*g^2*n*x + b*c*f*g*n)*dilog(-(F^(b*c*n*x + a*c*n)*e + d)/d
+ 1)*log(F) - 6*g^2*polylog(3, -F^(b*c*n*x + a*c*n)*e/d))/(b^3*c^3*d*n^3*l
og(F)^3)
```

Sympy [F]

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(f+gx)^2}{d+e\left(F^{ac+bcx}\right)^n}\,dx$$

input
```
integrate((g*x+f)**2/(d+e*(F**((b*x+a)*c))**n),x)
```

output
```
Integral((f + g*x)**2/(d + e*(F**(a*c + b*c*x))**n), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(142) = 284$.

Time = 0.11 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.09

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = f^2\left(\frac{bcnx+acn}{bcdn} - \frac{\log\left(F^{bcnx+acn}e+d\right)}{bcdn\log(F)}\right) - \frac{2\left(bcnx\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\right)fg}{b^2c^2dn^2\log(F)^2} - \frac{\left(b^2c^2n^2x^2\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F)^2 + 2\,bcnx\mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\log(F) - 2\,\mathrm{Li}_3\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\right)g^2}{b^3c^3dn^3\log(F)^3} + \frac{b^3c^3g^2n^3x^3\log(F)^3 + 3\,b^3c^3fgn^3x^2\log(F)^3}{3\,b^3c^3dn^3\log(F)^3}$$

input
```
integrate((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
f^2*((b*c*n*x + a*c*n)/(b*c*d*n) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d*n
*log(F))) - 2*(b*c*n*x*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F) + dilog(-
F^(b*c*n*x)*F^(a*c*n)*e/d))*f*g/(b^2*c^2*d*n^2*log(F)^2) - (b^2*c^2*n^2*x^
2*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F)^2 + 2*b*c*n*x*dilog(-F^(b*c*n*
x)*F^(a*c*n)*e/d)*log(F) - 2*polylog(3, -F^(b*c*n*x)*F^(a*c*n)*e/d))*g^2/(
b^3*c^3*d*n^3*log(F)^3) + 1/3*(b^3*c^3*g^2*n^3*x^3*log(F)^3 + 3*b^3*c^3*f*
g*n^3*x^2*log(F)^3)/(b^3*c^3*d*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(gx+f)^2}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/((F^((b*x + a)*c))^n*e + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(f+g\,x)^2}{d+e\left(F^{c\,(a+b\,x)}\right)^n}\,dx$$

input
```
int((f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
int((f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int \frac{(f+gx)^2}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{\left(\int \frac{x^2}{f^{bcnx+acn}e+d}dx\right)\log(f)\,bcd\,g^2n + 2\left(\int \frac{x}{f^{bcnx+acn}e+d}dx\right)\log(f)\,bcdf\,gn - \log\left(f^{bcnx+acn}e+d\right)f^2 + \log(f}{\log(f)\,bcdn}$$

input
```
int((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n),x)
```

output
```
(int(x**2/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d*g**2*n + 2*int(x/(f
**(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d*f*g*n - log(f**(a*c*n + b*c*n*x
)*e + d)*f**2 + log(f)*b*c*f**2*n*x)/(log(f)*b*c*d*n)
```

3.35 $\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{(f+gx)^2}{2dg}-\frac{(f+gx)\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcdn\log(F)}-\frac{g\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2dn^2\log^2(F)}$$

output
```
1/2*(g*x+f)^2/d/g-(g*x+f)*ln(1+e*(F^(c*(b*x+a)))^n/d)/b/c/d/n/ln(F)-g*poly
log(2,-e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d/n^2/ln(F)^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{-bcn(f+gx)\log(F)\log\left(1+\frac{d\left(F^{c(a+bx)}\right)^{-n}}{e}\right)+g\,\mathrm{PolyLog}\left(2,-\frac{d\left(F^{c(a+bx)}\right)^{-n}}{e}\right)}{b^2c^2dn^2\log^2(F)}$$

input
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
(-(b*c*n*(f + g*x)*Log[F]*Log[1 + d/(e*(F^(c*(a + b*x)))^n)]) + g*PolyLog[
2, -(d/(e*(F^(c*(a + b*x)))^n))])/(b^2*c^2*d*n^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{e\left(F^{c(a+bx)}\right)^n+d}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{(f+gx)^2}{2dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int \log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}$$

$$\Big\downarrow \text{2715}$$

$$\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int \left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(f+gx)^2}{2dg} - \frac{e\left(\frac{g\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)} + \frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}\right)}{d}$$

input
```
Int[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
(f + g*x)^2/(2*d*g) - (e*(((f + g*x)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(
b*c*e*n*Log[F]) + (g*PolyLog[2, -((e*(F^(c*(a + b*x)))^n)/d)])/(b^2*c^2*e*
n^2*Log[F]^2)))/d
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(96) = 192$.

Time = 0.08 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.65

method	result
risch	$-\frac{f\ln\left(\left(F^{c(bx+a)}\right)^{n} F^{-ncbx} F^{ncbx} e+d\right)}{\ln(F) bcnd}+\frac{f\ln\left(F^{ncbx} F^{-ncbx} \left(F^{c(bx+a)}\right)^{n}\right)}{\ln(F) bcnd}+\frac{g\ln\left(F^{c(bx+a)}\right)^{2}}{2\ln(F)^{2} b^{2} c^{2} d}-\frac{g\ln\left(F^{c(bx+a)}\right)\ln\left(1+\frac{e\,F^{n}}{}\right.}{\ln(F)^{2}}$

input
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
-1/ln(F)/b/c/n*f/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)+1/ln
(F)/b/c/n*f/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)+1/2/ln(F)^2/b
^2/c^2*g/d*ln(F^(c*(b*x+a)))^2-1/ln(F)^2/b^2/c^2/n*g/d*ln(F^(c*(b*x+a)))*l
n(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)-1/ln(F)^2/b^2/c^2/n^2*
g/d*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)-1/ln(F)/b/c
/n*g/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*x+1/ln(F)^2/b^2/
c^2/n*g/d*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*ln(F^(c*(b*x+
a)))+1/ln(F)/b/c/n*g/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*x-1/
ln(F)^2/b^2/c^2/n*g/d*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*ln(F^
(c*(b*x+a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.52

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^{n}}\,dx =$$
$$-\frac{2\,(bcf-acg)n\log\left(F^{bcnx+acn}e+d\right)\log(F)-\left(b^{2}c^{2}gn^{2}x^{2}+2\,b^{2}c^{2}fn^{2}x\right)\log(F)^{2}+2\,(bcgnx+acgn)\,\mathrm{l}}{2\,b^{2}c^{2}dn^{2}\log(F)^{2}}$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
-1/2*(2*(b*c*f - a*c*g)*n*log(F^(b*c*n*x + a*c*n)*e + d)*log(F) - (b^2*c^2
*g*n^2*x^2 + 2*b^2*c^2*f*n^2*x)*log(F)^2 + 2*(b*c*g*n*x + a*c*g*n)*log(F)*
log((F^(b*c*n*x + a*c*n)*e + d)/d) + 2*g*dilog(-(F^(b*c*n*x + a*c*n)*e + d
)/d + 1))/(b^2*c^2*d*n^2*log(F)^2)
```

Sympy [F]

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{f+gx}{d+e\left(F^{ac+bcx}\right)^n}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F**((b*x+a)*c))**n),x)
```

output
```
Integral((f + g*x)/(d + e*(F**(a*c + b*c*x))**n), x)
```

Maxima [F]

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{gx+f}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
f*((b*c*n*x + a*c*n)/(b*c*d*n) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d*n*l
og(F))) + g*integrate(x/(F^(b*c*n*x)*F^(a*c*n)*e + d), x)
```

Giac [F]

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{gx+f}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate((g*x + f)/((F^((b*x + a)*c))^n*e + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{f+g\,x}{d+e\left(F^{c\,(a+b\,x)}\right)^n}\,dx$$

input
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int \frac{f+gx}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{\left(\int \frac{x}{f^{bcnx+acn}e+d}dx\right)\log(f)\,bcdgn-\log\left(f^{bcnx+acn}e+d\right)f+\log(f)\,bcfnx}{\log(f)\,bcdn}$$

input
```
int((g*x+f)/(d+e*(F^((b*x+a)*c))^n),x)
```

output
```
(int(x/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d*g*n - log(f**(a*c*n +
b*c*n*x)*e + d)*f + log(f)*b*c*f*n*x)/(log(f)*b*c*d*n)
```

3.36 $\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{x}{d}-\frac{\log\left(d+e\left(F^{c(a+bx)}\right)^n\right)}{bcdn\log(F)}$$

output
```
x/d-ln(d+e*(F^(c*(b*x+a)))^n)/b/c/d/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)-\log\left(bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)\right)}{bcdn\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-1),x]
```

output
```
(Log[(F^(c*(a + b*x)))^n] - Log[b*c*d*(d + e*(F^(c*(a + b*x)))^n)*n*Log[F]
])/(b*c*d*n*Log[F])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2720, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{e\left(F^{c(a+bx)}\right)^n+d}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{e\left(F^{c(a+bx)}\right)^n+d}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int F^{-c(a+bx)}d\left(F^{c(a+bx)}\right)^n}{d}-\frac{e\int \frac{1}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{d}}{bcn\log(F)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d}-\frac{e\int \frac{1}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{d}}{bcn\log(F)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d}-\frac{\log\left(e\left(F^{c(a+bx)}\right)^n+d\right)}{d}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-1),x]
```

output
```
(Log[(F^(c*(a + b*x)))^n]/d - Log[d + e*(F^(c*(a + b*x)))^n]/d)/(b*c*n*Log
[F])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{x}{d}-\frac{\ln\left(d+e\,\mathrm{e}^{n\ln\left(\mathrm{e}^{c(bx+a)\ln(F)}\right)}\right)}{\ln(F)bcdn}$	44
parallelrisch	$\frac{\ln(F)bcnx-\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{\ln(F)bcdn}$	44
derivativedivides	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d}}{\ln(F)bcn}$	53
default	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d}}{\ln(F)bcn}$	53
risch	$\frac{\ln\left(F^{c(bx+a)}\right)}{\ln(F)bcd}-\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}+\frac{d}{e}\right)}{\ln(F)bcdn}$	62

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n),x,method=_RETURNVERBOSE)
```

output
```
x/d-1/ln(F)/b/c/d/n*ln(d+e*exp(n*ln(exp(c*(b*x+a)*ln(F)))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\frac{bcnx\log\left(F\right)-\log\left(F^{bcnx+acn}e+d\right)}{bcdn\log\left(F\right)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
(b*c*n*x*log(F) - log(F^(b*c*n*x + a*c*n)*e + d))/(b*c*d*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{2\operatorname{atan}\left(\frac{2\left(\frac{d}{2e}+\left(F^{c(a+bx)}\right)^n\right)}{\sqrt{-\frac{d^2}{e^2}}}\right)}{bcen\sqrt{-\frac{d^2}{e^2}}\log(F)}$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n),x)
```

output
```
2*atan(2*(d/(2*e) + (F**(c*(a + b*x)))**n)/sqrt(-d**2/e**2))/(b*c*e*n*sqrt
(-d**2/e**2)*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{bcnx+acn}{bcdn} - \frac{\log\left(F^{bcnx+acn}e+d\right)}{bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
(b*c*n*x + a*c*n)/(b*c*d*n) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d*n*log(
F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{\log\left(|F|^{bcnx}|F|^{acn}\right)}{bcdn\log(F)} - \frac{\log\left(\left|F^{bcnx}F^{acn}e+d\right|\right)}{bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
log(abs(F)^(b*c*n*x)*abs(F)^(a*c*n))/(b*c*d*n*log(F)) - log(abs(F^(b*c*n*x
)*F^(a*c*n)*e + d))/(b*c*d*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = -\frac{\ln\left(d+e\left(F^{a\,c+b\,c\,x}\right)^n\right)-b\,c\,n\,x\,\ln\left(F\right)}{b\,c\,d\,n\,\ln\left(F\right)}$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
-(log(d + e*(F^(a*c + b*c*x))^n) - b*c*n*x*log(F))/(b*c*d*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \frac{-\log\left(f^{bcnx+acn}e+d\right)+\log(f)\,bcnx}{\log\left(f\right)bcdn}$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n),x)
```

output
```
( - log(f**(a*c*n + b*c*n*x)*e + d) + log(f)*b*c*n*x)/(log(f)*b*c*d*n)
```

3.37 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx = \text{Int}\left(\frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)},x\right)$$

output
```
Defer(Int)(1/(d+e*(F^(c*(b*x+a)))^n)/(g*x+f),x)
```

Mathematica [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx = \int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx$$

input
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)*(f + g*x)),x]
```

output
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)*(f + g*x)), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}dx$$
$$\Big\downarrow \text{2618}$$
$$\int \frac{1}{(f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)}dx$$
$$\Big\downarrow \text{2619}$$
$$\int \frac{1}{(f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)}dx$$

input
```
Int[1/((d + e*(F^(c*(a + b*x)))^n)*(f + g*x)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)(gx+f)}dx$$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)/(g*x+f),x)
```

output
```
int(1/(d+e*(F^(c*(b*x+a)))^n)/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)(f+gx)}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^{n}e+d\right)(gx+f)}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f),x, algorithm="fricas")
```

output
```
integral(1/(d*g*x + (e*g*x + e*f)*(F^(b*c*x + a*c))^n + d*f), x)
```

Sympy [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)(f+gx)}\,dx=\int \frac{1}{\left(d+e\left(F^{ac+bcx}\right)^{n}\right)(f+gx)}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)/(g*x+f),x)
```

output
```
Integral(1/((d + e*(F**(a*c + b*c*x))**n)*(f + g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)(gx+f)}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f),x, algorithm="maxima")
```

output
```
integrate(1/((F^((b*x + a)*c*n)*e + d)*(g*x + f)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)(gx+f)}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f),x, algorithm="giac")
```

output
```
integrate(1/(((F^((b*x + a)*c))^n*e + d)*(g*x + f)), x)
```

Mupad [N/A]

Not integrable

Time = 22.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx=\int \frac{1}{(f+g\,x)\,\left(d+e\,{\left(F^{c\,(a+b\,x)}\right)}^n\right)}\,dx$$

input
```
int(1/((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)),x)
```

output
```
int(1/((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)}\,dx=\int \frac{1}{f^{bcnx+acn}ef+f^{bcnx+acn}egx+df+dgx}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f),x)
```

output
```
int(1/(f**(a*c*n + b*c*n*x)*e*f + f**(a*c*n + b*c*n*x)*e*g*x + d*f + d*g*x
),x)
```

3.38 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx=\text{Int}\left(\frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2},x\right)$$

output
```
Defer(Int)(1/(d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx=\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx$$

input
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^2),x]
```

output
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2 \left(e \left(F^{c(a+bx)}\right)^n+d\right)}dx$$
$$\Big\downarrow \text{2618}$$
$$\int \frac{1}{(f+gx)^2 \left(e \left(F^{ac+bcx}\right)^n+d\right)}dx$$
$$\Big\downarrow \text{2619}$$
$$\int \frac{1}{(f+gx)^2 \left(e \left(F^{ac+bcx}\right)^n+d\right)}dx$$

input
```
Int[1/((d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)\left(gx+f\right)^{2}}dx$$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^2,x)
```

output
```
int(1/(d+e*(F^(c*(b*x+a)))^n)/(g*x+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)(f+gx)^{2}}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^{n}e+d\right)\left(gx+f\right)^{2}}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral(1/(d*g^2*x^2 + 2*d*f*g*x + d*f^2 + (e*g^2*x^2 + 2*e*f*g*x + e*f^2
)*(F^(b*c*x + a*c))^n), x)
```

Sympy [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)(f+gx)^{2}}\,dx=\int \frac{1}{\left(d+e\left(F^{ac+bcx}\right)^{n}\right)\left(f+gx\right)^{2}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)/(g*x+f)**2,x)
```

output
```
Integral(1/((d + e*(F**(a*c + b*c*x))**n)*(f + g*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)(gx+f)^2}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x, algorithm="maxima")
```

output
```
integrate(1/((F^((b*x + a)*c*n)*e + d)*(g*x + f)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)(gx+f)^2}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate(1/(((F^((b*x + a)*c))^n*e + d)*(g*x + f)^2), x)
```

Mupad [N/A]

Not integrable

Time = 22.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,\left(d+e\,{\left(F^{c\,\left(a+b\,x\right)}\right)}^n\right)} \,d x$$

input
```
int(1/((f + g*x)^2*(d + e*(F^(c*(a + b*x)))^n)),x)
```

output
```
int(1/((f + g*x)^2*(d + e*(F^(c*(a + b*x)))^n)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^2}\,dx$$
$$=\int \frac{1}{f^{bcnx+acn}e\,f^2+2f^{bcnx+acn}efgx+f^{bcnx+acn}e\,g^2x^2+d\,f^2+2dfgx+d\,g^2x^2}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)/(g*x+f)^2,x)
```

output
```
int(1/(f**(a*c*n + b*c*n*x)*e*f**2 + 2*f**(a*c*n + b*c*n*x)*e*f*g*x + f**(
a*c*n + b*c*n*x)*e*g**2*x**2 + d*f**2 + 2*d*f*g*x + d*g**2*x**2),x)
```

3.39 $\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 388

$$
\begin{aligned}
\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx =& \frac{(f+gx)^4}{4d^2g}-\frac{(f+gx)^3}{bcd^2n\log(F)}+\frac{(f+gx)^3}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)} \\
&+\frac{3g(f+gx)^2\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2d^2n^2\log^2(F)} \\
&-\frac{(f+gx)^3\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcd^2n\log(F)} \\
&+\frac{6g^2(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3d^2n^3\log^3(F)} \\
&-\frac{3g(f+gx)^2\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2d^2n^2\log^2(F)} \\
&-\frac{6g^3\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4d^2n^4\log^4(F)} \\
&+\frac{6g^2(f+gx)\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3d^2n^3\log^3(F)} \\
&-\frac{6g^3\operatorname{PolyLog}\left(4,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^4c^4d^2n^4\log^4(F)}
\end{aligned}
$$

output
```
1/4*(g*x+f)^4/d^2/g-(g*x+f)^3/b/c/d^2/n/ln(F)+(g*x+f)^3/b/c/d/(d+e*(F^(c*(
b*x+a)))^n)/n/ln(F)+3*g*(g*x+f)^2*ln(1+e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d^2/
n^2/ln(F)^2-(g*x+f)^3*ln(1+e*(F^(c*(b*x+a)))^n/d)/b/c/d^2/n/ln(F)+6*g^2*(g
*x+f)*polylog(2,-e*(F^(c*(b*x+a)))^n/d)/b^3/c^3/d^2/n^3/ln(F)^3-3*g*(g*x+f
)^2*polylog(2,-e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d^2/n^2/ln(F)^2-6*g^3*polylo
g(3,-e*(F^(c*(b*x+a)))^n/d)/b^4/c^4/d^2/n^4/ln(F)^4+6*g^2*(g*x+f)*polylog(
3,-e*(F^(c*(b*x+a)))^n/d)/b^3/c^3/d^2/n^3/ln(F)^3-6*g^3*polylog(4,-e*(F^(c
*(b*x+a)))^n/d)/b^4/c^4/d^2/n^4/ln(F)^4
```

Mathematica [F]

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
Integrate[(f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n)^2, x]
```

Rubi [A] (verified)

Time = 4.28 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2616, 2615, 2620, 2621, 2615, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

$$\Big\downarrow \text{ 2616}$$

$$\frac{\int \frac{(f+gx)^3}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^3}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2615}$$

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n (f+gx)^3}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n (f+gx)^3}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\int (f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n (f+gx)^3}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2621}$$

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\int (f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{3g\int \frac{(f+gx)^2}{e\left(F^{c(a+bx)}\right)^n+d}dx}{bcen\log(F)}-\frac{(f+gx)^3}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{2615}$$

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\int (f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n (f+gx)^2}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^3}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\int(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^3}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\int(f+gx)\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)^2\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}}{d}-$$

$$\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\int\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{\cdots}{bce\cdots}\right.}{d}$$

$\downarrow$ 2720

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\int(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)^2\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}}{d}-$$

$$\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\int F^{-c(a+bx)}\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dF^{c(a+bx)}}{b^2c^2n\log^2(F)}-\frac{(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\cdots\right.}{bcen\log(F)}\cdots\right.}{d}\cdots\right.}{bcen\log(F)}\cdots\right.}{d}\cdots$$

↓ 7143

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\int(f+gx)\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)^2\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}}{d}-$$

$$\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\,\mathrm{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}-\frac{(f+gx)\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{\cdots}{bcen\,\mathrm{lo}\cdots}\right.}{d}$$

↓ 7163

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}-\frac{g\int\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}\right)}{bcn\log(F)}-\frac{(f+gx)^2\operatorname{Pol}\cdots}{\cdots}\cdots}{bcen\log(F)}\cdots}{d}\cdots}{d}$$

$$\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}-\frac{(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{\cdots}{bcen\operatorname{lo}\cdots}\cdots}{d}$$

$\downarrow$ 2720

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}-\frac{g\int F^{-c(a+bx)}\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dF^{c(a-}\cdots}{b^2c^2n\log^2(F)}\cdots\right)}{bcn\log(F)}\cdots\right)}{bcen\log(F)}\cdots\right)}{d}\cdots}{d}$$

$$\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}-\frac{(f+gx)\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{\cdots}{bcen\log\cdots}\cdots\right)}{d}$$

↓ 7143

$$\frac{\frac{(f+gx)^4}{4dg}-\frac{e\left(\frac{(f+gx)^3\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{3g\left(\frac{2g\left(\frac{(f+gx)\,\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}-\frac{g\,\text{PolyLog}\left(4,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}\right)}{bcn\log(F)}-\frac{(f+gx)^2\,\text{PolyLo}\cdots}{b\cdots}\cdots\right)}{bcen\log(F)}\cdots\right)}{d}\cdots}{d}$$

$$\frac{e\left(\frac{3g\left(\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\,\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}-\frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{\cdots}{bcen\,\text{lo}\cdots}\cdots\right)}{d}$$

input
```
Int[(f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
-((e*(-((f + g*x)^3/(b*c*e*(d + e*(F^(c*(a + b*x)))^n)*n*Log[F])) + (3*g*(
(f + g*x)^3/(3*d*g) - (e*(((f + g*x)^2*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])
/(b*c*e*n*Log[F]) - (2*g*(-(((f + g*x)*PolyLog[2, -((e*(F^(c*(a + b*x)))^n
)/d)])/(b*c*n*Log[F])) + (g*PolyLog[3, -((e*(F^(c*(a + b*x)))^n)/d)])/(b^2
*c^2*n^2*Log[F]^2)))/(b*c*e*n*Log[F])))/d))/(b*c*e*n*Log[F])))/d) + ((f +
g*x)^4/(4*d*g) - (e*(((f + g*x)^3*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(b*c
*e*n*Log[F]) - (3*g*(-(((f + g*x)^2*PolyLog[2, -((e*(F^(c*(a + b*x)))^n)/d
)])/(b*c*n*Log[F])) + (2*g*(((f + g*x)*PolyLog[3, -((e*(F^(c*(a + b*x)))^n
)/d)])/(b*c*n*Log[F]) - (g*PolyLog[4, -((e*(F^(c*(a + b*x)))^n)/d)])/(b^2*
c^2*n^2*Log[F]^2)))/(b*c*n*Log[F])))/(b*c*e*n*Log[F])))/d)/d
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3318 vs. $2(386) = 772$.

Time = 0.34 (sec) , antiderivative size = 3319, normalized size of antiderivative = 8.55

method	result	size
risch	Expression too large to display	3319

input
```
int((g*x+f)^3/(d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/n/c/b/ln(F)/d*(g^3*x^3+3*f*g^2*x^2+3*f^2*g*x+f^3)/(d+e*(F^(c*(b*x+a)))^n
)+3/n^2/c^2/b^2/ln(F)^2/d^2*f^2*g*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c
*b*x)*e+d)-3/n^2/c^2/b^2/ln(F)^2/d^2*g^3*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*
b*x)*(F^(c*(b*x+a)))^n/d)*x^2+6/n^3/c^3/b^3/ln(F)^3/d^2*g^3*polylog(3,-e*F
^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)*x+3/n^2/c^2/b^2/ln(F)^2/d^2*g
^3*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*x^2+6/n^3/c^3/b^3/ln
(F)^3/d^2*g^3*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)*x
+6/n^3/c^3/b^3/ln(F)^3/d^2*f*g^2*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^
(c*(b*x+a)))^n/d)+3/n^2/c^4/b^4/ln(F)^4/d^2*g^3*ln((F^(c*(b*x+a)))^n*F^(-n
*c*b*x)*F^(n*c*b*x)*e+d)*ln(F^(c*(b*x+a)))^2-3/n^2/c^2/b^2/ln(F)^2/d^2*g^3
*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*x^2-3/n^2/c^4/b^4/ln(F)^4/
d^2*g^3*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*ln(F^(c*(b*x+a)))^2
-3/n/c^3/b^3/ln(F)^3/d^2*f*g^2*ln(F^(c*(b*x+a)))^2-1/n/c/b/ln(F)/d^2*g^3*l
n((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*x^3+1/n/c^4/b^4/ln(F)^4/
d^2*g^3*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*ln(F^(c*(b*x+a)
))^3+1/n/c/b/ln(F)/d^2*g^3*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*
x^3-1/n/c^4/b^4/ln(F)^4/d^2*g^3*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a))
)^n)*ln(F^(c*(b*x+a)))^3+3/c^2/b^2/ln(F)^2/d^2*f*g^2*ln(F^(c*(b*x+a)))^2*x
-1/n/c^4/b^4/ln(F)^4/d^2*g^3*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)
))^n/d)*ln(F^(c*(b*x+a)))^3-3/n/c^3/b^3/ln(F)^3/d^2*g^3*ln(F^(c*(b*x+a)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. $2(384) = 768$.

Time = 0.12 (sec) , antiderivative size = 1469, normalized size of antiderivative = 3.79

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
1/4*(4*(b^3*c^3*d*f^3 - 3*a*b^2*c^3*d*f^2*g + 3*a^2*b*c^3*d*f*g^2 - a^3*c^
3*d*g^3)*n^3*log(F)^3 + (b^4*c^4*d*g^3*n^4*x^4 + 4*b^4*c^4*d*f*g^2*n^4*x^3
 + 6*b^4*c^4*d*f^2*g*n^4*x^2 + 4*b^4*c^4*d*f^3*n^4*x + (4*a*b^3*c^4*d*f^3
- 6*a^2*b^2*c^4*d*f^2*g + 4*a^3*b*c^4*d*f*g^2 - a^4*c^4*d*g^3)*n^4)*log(F)
^4 + ((b^4*c^4*e*g^3*n^4*x^4 + 4*b^4*c^4*e*f*g^2*n^4*x^3 + 6*b^4*c^4*e*f^2
*g*n^4*x^2 + 4*b^4*c^4*e*f^3*n^4*x + (4*a*b^3*c^4*e*f^3 - 6*a^2*b^2*c^4*e*
f^2*g + 4*a^3*b*c^4*e*f*g^2 - a^4*c^4*e*g^3)*n^4)*log(F)^4 - 4*(b^3*c^3*e*
g^3*n^3*x^3 + 3*b^3*c^3*e*f*g^2*n^3*x^2 + 3*b^3*c^3*e*f^2*g*n^3*x + (3*a*b
^2*c^3*e*f^2*g - 3*a^2*b*c^3*e*f*g^2 + a^3*c^3*e*g^3)*n^3)*log(F)^3)*F^(b*
c*n*x + a*c*n) - 12*((b^2*c^2*d*g^3*n^2*x^2 + 2*b^2*c^2*d*f*g^2*n^2*x + b^
2*c^2*d*f^2*g*n^2)*log(F)^2 + ((b^2*c^2*e*g^3*n^2*x^2 + 2*b^2*c^2*e*f*g^2*
n^2*x + b^2*c^2*e*f^2*g*n^2)*log(F)^2 - 2*(b*c*e*g^3*n*x + b*c*e*f*g^2*n)*
log(F))*F^(b*c*n*x + a*c*n) - 2*(b*c*d*g^3*n*x + b*c*d*f*g^2*n)*log(F))*di
log(-(F^(b*c*n*x + a*c*n)*e + d)/d + 1) - 4*((b^3*c^3*d*f^3 - 3*a*b^2*c^3*
d*f^2*g + 3*a^2*b*c^3*d*f*g^2 - a^3*c^3*d*g^3)*n^3*log(F)^3 - 3*(b^2*c^2*d
*f^2*g - 2*a*b*c^2*d*f*g^2 + a^2*c^2*d*g^3)*n^2*log(F)^2 + ((b^3*c^3*e*f^3
 - 3*a*b^2*c^3*e*f^2*g + 3*a^2*b*c^3*e*f*g^2 - a^3*c^3*e*g^3)*n^3*log(F)^3
 - 3*(b^2*c^2*e*f^2*g - 2*a*b*c^2*e*f*g^2 + a^2*c^2*e*g^3)*n^2*log(F)^2)*F
^(b*c*n*x + a*c*n))*log(F^(b*c*n*x + a*c*n)*e + d) - 4*((b^3*c^3*d*g^3*n^3
*x^3 + 3*b^3*c^3*d*f*g^2*n^3*x^2 + 3*b^3*c^3*d*f^2*g*n^3*x + (3*a*b^2*c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3/(d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.80

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
f^3*((b*c*n*x + a*c*n)/(b*c*d^2*n) + 1/((F^(b*c*n*x + a*c*n)*d*e + d^2)*b*
c*n*log(F)) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d^2*n*log(F))) + (g^3*x^
3 + 3*f*g^2*x^2 + 3*f^2*g*x)/(F^(b*c*n*x)*F^(a*c*n)*b*c*d*e*n*log(F) + b*c
*d^2*n*log(F)) - 3*f^2*g*x/(b*c*d^2*n*log(F)) + 3*f^2*g*log(F^(b*c*n*x)*F^
(a*c*n)*e + d)/(b^2*c^2*d^2*n^2*log(F)^2) - 3*(b*c*f^2*g*n*log(F) - 2*f*g^
2)*(b*c*n*x*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F) + dilog(-F^(b*c*n*x)
*F^(a*c*n)*e/d))/(b^3*c^3*d^2*n^3*log(F)^3) - (b^3*c^3*n^3*x^3*log(F^(b*c*
n*x)*F^(a*c*n)*e/d + 1)*log(F)^3 + 3*b^2*c^2*n^2*x^2*dilog(-F^(b*c*n*x)*F^
(a*c*n)*e/d)*log(F)^2 - 6*b*c*n*x*log(F)*polylog(3, -F^(b*c*n*x)*F^(a*c*n)
*e/d) + 6*polylog(4, -F^(b*c*n*x)*F^(a*c*n)*e/d))*g^3/(b^4*c^4*d^2*n^4*log
(F)^4) - 3*(b^2*c^2*n^2*x^2*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F)^2 +
2*b*c*n*x*dilog(-F^(b*c*n*x)*F^(a*c*n)*e/d)*log(F) - 2*polylog(3, -F^(b*c*
n*x)*F^(a*c*n)*e/d))*(b*c*f*g^2*n*log(F) - g^3)/(b^4*c^4*d^2*n^4*log(F)^4)
 + 1/4*(b^4*c^4*g^3*n^4*x^4*log(F)^4 + 4*(b*c*f*g^2*n*log(F) - g^3)*b^3*c^
3*n^3*x^3*log(F)^3 + 6*(b^2*c^2*f^2*g*n^2*log(F)^2 - 2*b*c*f*g^2*n*log(F))
*b^2*c^2*n^2*x^2*log(F)^2)/(b^4*c^4*d^2*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(gx+f)^3}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^3/((F^((b*x + a)*c))^n*e + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(f+g\,x)^3}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2}\,dx$$

input
```
int((f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
int((f + g*x)^3/(d + e*(F^(c*(a + b*x)))^n)^2, x)
```

Reduce [F]

$$\int \frac{(f+gx)^3}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$
$$=\frac{f^{bcnx+acn}\left(\int \frac{x^3}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right)\log(f)\,bc\,d^2e\,g^3n+3f^{bcnx+acn}\left(\int \frac{x^2}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right.}{}$$

input
```
int((g*x+f)^3/(d+e*(F^((b*x+a)*c))^n)^2,x)
```

output

```
(f**(a*c*n + b*c*n*x)*int(x**3/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*
n + b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**2*e*g**3*n + 3*f**(a*c*n + b*c*n
*x)*int(x**2/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e +
 d**2),x)*log(f)*b*c*d**2*e*f*g**2*n + 3*f**(a*c*n + b*c*n*x)*int(x/(f**(2
*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e + d**2),x)*log(f)*b*
c*d**2*e*f**2*g*n - f**(a*c*n + b*c*n*x)*log(f**(a*c*n + b*c*n*x)*e + d)*e
*f**3 + f**(a*c*n + b*c*n*x)*log(f)*b*c*e*f**3*n*x - f**(a*c*n + b*c*n*x)*
e*f**3 + int(x**3/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*
d*e + d**2),x)*log(f)*b*c*d**3*g**3*n + 3*int(x**2/(f**(2*a*c*n + 2*b*c*n*
x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**3*f*g**2*n +
 3*int(x/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e + d**
2),x)*log(f)*b*c*d**3*f**2*g*n - log(f**(a*c*n + b*c*n*x)*e + d)*d*f**3 +
log(f)*b*c*d*f**3*n*x)/(log(f)*b*c*d**2*n*(f**(a*c*n + b*c*n*x)*e + d))
```

3.40 $\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\frac{(f+gx)^3}{3d^2g}-\frac{(f+gx)^2}{bcd^2n\log(F)}+\frac{(f+gx)^2}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)}+\frac{2g(f+gx)\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2d^2n^2\log^2(F)}-\frac{(f+gx)^2\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcd^2n\log(F)}+\frac{2g^2\operatorname{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3d^2n^3\log^3(F)}-\frac{2g(f+gx)\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2d^2n^2\log^2(F)}+\frac{2g^2\operatorname{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^3c^3d^2n^3\log^3(F)}$$

output
```
1/3*(g*x+f)^3/d^2/g-(g*x+f)^2/b/c/d^2/n/ln(F)+(g*x+f)^2/b/c/d/(d+e*(F^(c*(
b*x+a)))^n)/n/ln(F)+2*g*(g*x+f)*ln(1+e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d^2/n^
2/ln(F)^2-(g*x+f)^2*ln(1+e*(F^(c*(b*x+a)))^n/d)/b/c/d^2/n/ln(F)+2*g^2*poly
log(2,-e*(F^(c*(b*x+a)))^n/d)/b^3/c^3/d^2/n^3/ln(F)^3-2*g*(g*x+f)*polylog(
2,-e*(F^(c*(b*x+a)))^n/d)/b^2/c^2/d^2/n^2/ln(F)^2+2*g^2*polylog(3,-e*(F^(c
*(b*x+a)))^n/d)/b^3/c^3/d^2/n^3/ln(F)^3
```

Mathematica [F]

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
Integrate[(f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n)^2, x]
```

Rubi [A] (verified)

Time = 3.02 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

$$\Big\downarrow \text{2616}$$

$$\frac{\int \frac{(f+gx)^2}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^2}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2615}$$

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^2}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^2}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$\big\downarrow$ 2620

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int (f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)^2}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$\big\downarrow$ 2621

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int (f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\int \frac{f+gx}{e\left(F^{c(a+bx)}\right)^n+d}dx}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$\big\downarrow$ 2615

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int (f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$\big\downarrow$ 2620

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n\right.}\right.}{d}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\int(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{g\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)}+\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$\downarrow$ 3011

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\int\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dx}{bcn\log(F)}-\frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{g\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)}+\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{(f+gx)^3}{3dg} - \frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)} - \frac{2g\left(\frac{g\int F^{-c(a+bx)}\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)dF^{c(a+bx)}}{b^2c^2n\log^2(F)} - \frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}}{bcen\log(F)}}{d}}{d}$$

$$\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg} - \frac{e\left(\frac{g\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)} + \frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)} - \frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{(f+gx)^3}{3dg}-\frac{e\left(\frac{(f+gx)^2\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{2g\left(\frac{g\,\text{PolyLog}\left(3,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2n^2\log^2(F)}-\frac{(f+gx)\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcn\log(F)}\right)}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{2g\left(\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{g\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)}+\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}\right)}{d}\right)}{bcen\log(F)}-\frac{(f+gx)^2}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

input
```
Int[(f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
-((e*(-((f + g*x)^2/(b*c*e*(d + e*(F^(c*(a + b*x)))^n)*n*Log[F])) + (2*g*(
(f + g*x)^2/(2*d*g) - (e*((((f + g*x)*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(
b*c*e*n*Log[F]) + (g*PolyLog[2, -((e*(F^(c*(a + b*x)))^n)/d)])/(b^2*c^2*e*
n^2*Log[F]^2)))/d))/(b*c*e*n*Log[F])))/d) + ((f + g*x)^3/(3*d*g) - (e*(((f
 + g*x)^2*Log[1 + (e*(F^(c*(a + b*x)))^n)/d])/(b*c*e*n*Log[F]) - (2*g*(-((
(f + g*x)*PolyLog[2, -((e*(F^(c*(a + b*x)))^n)/d)])/(b*c*n*Log[F])) + (g*P
olyLog[3, -((e*(F^(c*(a + b*x)))^n)/d)])/(b^2*c^2*n^2*Log[F]^2)))/(b*c*e*n
*Log[F])))/d)/d
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. $2(292) = 584$.

Time = 0.14 (sec) , antiderivative size = 1650, normalized size of antiderivative = 5.61

method	result	size
risch	Expression too large to display	1650

input
```
int((g*x+f)^2/(d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/d^2/ln(F)^2/b^2/c^2/n^2*g*f*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))
^n)-2/d^2/ln(F)^3/b^3/c^3/n^2*g^2*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c
*b*x)*e+d)*ln(F^(c*(b*x+a)))-2/3/d^2/ln(F)^3/b^3/c^3*g^2*ln(F^(c*(b*x+a)))
^3+1/n/c/b/ln(F)/d*(g^2*x^2+2*f*g*x+f^2)/(d+e*(F^(c*(b*x+a)))^n)-1/d^2/ln(
F)/b/c/n*g^2*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*x^2-1/d^2/
ln(F)^3/b^3/c^3/n*g^2*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*l
n(F^(c*(b*x+a)))^2+1/d^2/ln(F)/b/c/n*g^2*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c
*(b*x+a)))^n)*x^2+1/d^2/ln(F)^3/b^3/c^3/n*g^2*ln(F^(n*c*b*x)*F^(-n*c*b*x)*
(F^(c*(b*x+a)))^n)*ln(F^(c*(b*x+a)))^2+1/d^2/ln(F)^3/b^3/c^3/n*g^2*ln(F^(c
*(b*x+a)))^2*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)+2/d^2/ln
(F)^3/b^3/c^3/n^2*g^2*ln(F^(c*(b*x+a)))*ln(1+e*F^(n*c*b*x)*F^(-n*c*b*x)*(F
^(c*(b*x+a)))^n/d)-2/d^2/ln(F)^2/b^2/c^2/n^2*g*f*polylog(2,-e*F^(n*c*b*x)*
F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)+2/d^2/ln(F)^2/b^2/c^2/n^2*g*f*ln((F^(c*(
b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)-2/d^2/ln(F)^2/b^2/c^2/n^2*g^2*ln(
F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*x+2/d^2/ln(F)^3/b^3/c^3/n^2*g^
2*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*ln(F^(c*(b*x+a)))-2/d^2/l
n(F)^2/b^2/c^2/n^2*g^2*polylog(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)
))^n/d)*x+2/d^2/ln(F)^2/b^2/c^2/n^2*g^2*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*
F^(n*c*b*x)*e+d)*x-2/d^2/ln(F)^2/b^2/c^2/n*g*f*ln(F^(c*(b*x+a)))*ln(1+e*F^
(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)+2/d^2/ln(F)^2/b^2/c^2/n*g*f...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(290) = 580$.

Time = 0.09 (sec) , antiderivative size = 875, normalized size of antiderivative = 2.98

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
1/3*(3*(b^2*c^2*d*f^2 - 2*a*b*c^2*d*f*g + a^2*c^2*d*g^2)*n^2*log(F)^2 + (b
^3*c^3*d*g^2*n^3*x^3 + 3*b^3*c^3*d*f*g*n^3*x^2 + 3*b^3*c^3*d*f^2*n^3*x + (
3*a*b^2*c^3*d*f^2 - 3*a^2*b*c^3*d*f*g + a^3*c^3*d*g^2)*n^3)*log(F)^3 + ((b
^3*c^3*e*g^2*n^3*x^3 + 3*b^3*c^3*e*f*g*n^3*x^2 + 3*b^3*c^3*e*f^2*n^3*x + (
3*a*b^2*c^3*e*f^2 - 3*a^2*b*c^3*e*f*g + a^3*c^3*e*g^2)*n^3)*log(F)^3 - 3*(
b^2*c^2*e*g^2*n^2*x^2 + 2*b^2*c^2*e*f*g*n^2*x + (2*a*b*c^2*e*f*g - a^2*c^2
*e*g^2)*n^2)*log(F)^2)*F^(b*c*n*x + a*c*n) + 6*(d*g^2 + (e*g^2 - (b*c*e*g^
2*n*x + b*c*e*f*g*n)*log(F))*F^(b*c*n*x + a*c*n) - (b*c*d*g^2*n*x + b*c*d*
f*g*n)*log(F))*dilog(-(F^(b*c*n*x + a*c*n)*e + d)/d + 1) - 3*((b^2*c^2*d*f
^2 - 2*a*b*c^2*d*f*g + a^2*c^2*d*g^2)*n^2*log(F)^2 - 2*(b*c*d*f*g - a*c*d*
g^2)*n*log(F) + ((b^2*c^2*e*f^2 - 2*a*b*c^2*e*f*g + a^2*c^2*e*g^2)*n^2*log
(F)^2 - 2*(b*c*e*f*g - a*c*e*g^2)*n*log(F))*F^(b*c*n*x + a*c*n))*log(F^(b*
c*n*x + a*c*n)*e + d) - 3*((b^2*c^2*d*g^2*n^2*x^2 + 2*b^2*c^2*d*f*g*n^2*x
+ (2*a*b*c^2*d*f*g - a^2*c^2*d*g^2)*n^2)*log(F)^2 + ((b^2*c^2*e*g^2*n^2*x^
2 + 2*b^2*c^2*e*f*g*n^2*x + (2*a*b*c^2*e*f*g - a^2*c^2*e*g^2)*n^2)*log(F)^
2 - 2*(b*c*e*g^2*n*x + a*c*e*g^2*n)*log(F))*F^(b*c*n*x + a*c*n) - 2*(b*c*d
*g^2*n*x + a*c*d*g^2*n)*log(F))*log((F^(b*c*n*x + a*c*n)*e + d)/d) + 6*(F^
(b*c*n*x + a*c*n)*e*g^2 + d*g^2)*polylog(3, -F^(b*c*n*x + a*c*n)*e/d))/(F^
(b*c*n*x + a*c*n)*b^3*c^3*d^2*e*n^3*log(F)^3 + b^3*c^3*d^3*n^3*log(F)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)**2/(d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.59

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$
$$= f^2\left(\frac{bcnx+acn}{bcd^2n}+\frac{1}{(F^{bcnx+acn}de+d^2)bcn\log(F)}-\frac{\log\left(F^{bcnx+acn}e+d\right)}{bcd^2n\log(F)}\right)$$
$$+\frac{g^2x^2+2fgx}{F^{bcnx}F^{acn}bcden\log(F)+bcd^2n\log(F)}-\frac{2fgx}{bcd^2n\log(F)}+\frac{2fg\log\left(F^{bcnx}F^{acn}e+d\right)}{b^2c^2d^2n^2\log(F)^2}$$
$$-\frac{\left(b^2c^2n^2x^2\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F)^2+2\,bcnx\mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\log(F)-2\,\mathrm{Li}_3\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\right)g^2}{b^3c^3d^2n^3\log(F)^3}$$
$$-\frac{2\left(bcfgn\log(F)-g^2\right)\left(bcnx\log\left(\frac{F^{bcnx}F^{acn}e}{d}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{bcnx}F^{acn}e}{d}\right)\right)}{b^3c^3d^2n^3\log(F)^3}$$
$$+\frac{b^3c^3g^2n^3x^3\log(F)^3+3\left(bcfgn\log(F)-g^2\right)b^2c^2n^2x^2\log(F)^2}{3\,b^3c^3d^2n^3\log(F)^3}$$

input
```
integrate((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
f^2*((b*c*n*x + a*c*n)/(b*c*d^2*n) + 1/((F^(b*c*n*x + a*c*n)*d*e + d^2)*b*
c*n*log(F)) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d^2*n*log(F))) + (g^2*x^
2 + 2*f*g*x)/(F^(b*c*n*x)*F^(a*c*n)*b*c*d*e*n*log(F) + b*c*d^2*n*log(F)) -
 2*f*g*x/(b*c*d^2*n*log(F)) + 2*f*g*log(F^(b*c*n*x)*F^(a*c*n)*e + d)/(b^2*
c^2*d^2*n^2*log(F)^2) - (b^2*c^2*n^2*x^2*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1
)*log(F)^2 + 2*b*c*n*x*dilog(-F^(b*c*n*x)*F^(a*c*n)*e/d)*log(F) - 2*polylo
g(3, -F^(b*c*n*x)*F^(a*c*n)*e/d))*g^2/(b^3*c^3*d^2*n^3*log(F)^3) - 2*(b*c*
f*g*n*log(F) - g^2)*(b*c*n*x*log(F^(b*c*n*x)*F^(a*c*n)*e/d + 1)*log(F) + d
ilog(-F^(b*c*n*x)*F^(a*c*n)*e/d))/(b^3*c^3*d^2*n^3*log(F)^3) + 1/3*(b^3*c^
3*g^2*n^3*x^3*log(F)^3 + 3*(b*c*f*g*n*log(F) - g^2)*b^2*c^2*n^2*x^2*log(F)
^2)/(b^3*c^3*d^2*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(gx+f)^2}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^2/((F^((b*x + a)*c))^n*e + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(f+g\,x)^2}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2}\,dx$$

input
```
int((f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
int((f + g*x)^2/(d + e*(F^(c*(a + b*x)))^n)^2, x)
```

Reduce [F]

$$\int \frac{(f+gx)^2}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$
$$=\frac{f^{bcnx+acn}\left(\int \frac{x^2}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right)\log(f)\,bc\,d^2e\,g^2n+2f^{bcnx+acn}\left(\int \frac{x}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right.}{}$$

input
```
int((g*x+f)^2/(d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
(f**(a*c*n + b*c*n*x)*int(x**2/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*
n + b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**2*e*g**2*n + 2*f**(a*c*n + b*c*n
*x)*int(x/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e + d*
*2),x)*log(f)*b*c*d**2*e*f*g*n - f**(a*c*n + b*c*n*x)*log(f**(a*c*n + b*c*
n*x)*e + d)*e*f**2 + f**(a*c*n + b*c*n*x)*log(f)*b*c*e*f**2*n*x - f**(a*c*
n + b*c*n*x)*e*f**2 + int(x**2/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*
n + b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**3*g**2*n + 2*int(x/(f**(2*a*c*n
+ 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**3*
f*g*n - log(f**(a*c*n + b*c*n*x)*e + d)*d*f**2 + log(f)*b*c*d*f**2*n*x)/(l
og(f)*b*c*d**2*n*(f**(a*c*n + b*c*n*x)*e + d))
```

3.41 $\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\frac{(f+gx)^2}{2d^2g}-\frac{gx}{bcd^2n\log(F)}+\frac{f+gx}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)}+\frac{g\log\left(d+e\left(F^{c(a+bx)}\right)^n\right)}{b^2c^2d^2n^2\log^2(F)}-\frac{(f+gx)\log\left(1+\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{bcd^2n\log(F)}-\frac{g\,\text{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2d^2n^2\log^2(F)}$$

output
```
1/2*(g*x+f)^2/d^2/g-g*x/b/c/d^2/n/ln(F)+(g*x+f)/b/c/d/(d+e*(F^(c*(b*x+a)))
^n)/n/ln(F)+g*ln(d+e*(F^(c*(b*x+a)))^n)/b^2/c^2/d^2/n^2/ln(F)^2-(g*x+f)*ln
(1+e*(F^(c*(b*x+a)))^n/d)/b/c/d^2/n/ln(F)-g*polylog(2,-e*(F^(c*(b*x+a)))^n
/d)/b^2/c^2/d^2/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^2, x]
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {2616, 2615, 2620, 2621, 2715, 2720, 798, 47, 14, 16, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

$$\Big\downarrow \text{2616}$$

$$\frac{\int \frac{f+gx}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2615}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)}{e\left(F^{c(a+bx)}\right)^n+d}dx}{d}}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int \log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\int \frac{\left(F^{c(a+bx)}\right)^n(f+gx)}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx}{d}$$

$$\Big\downarrow \text{2621}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)dx}{bcen\log(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\int\frac{1}{e\left(F^{c(a+bx)}\right)^n+d}dx}{bcen\log(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\int\frac{1}{e\left(F^{c(a+bx)}\right)^n+d}dx}{bcen\log(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\int\frac{F^{-c(a+bx)}}{e\left(F^{c(a+bx)}\right)^n+d}dF^{c(a+bx)}}{b^2c^2en\log^2(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{798}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\int\frac{F^{-c(a+bx)}}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\left(\frac{\int F^{-c(a+bx)}d\left(F^{c(a+bx)}\right)^n}{d}-\frac{e\int\frac{1}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$\Big\downarrow$ 14

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\left(\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d}-\frac{e\int\frac{1}{e\left(F^{c(a+bx)}\right)^n+d}d\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$\Big\downarrow$ 16

$$\frac{\frac{(f+gx)^2}{2dg}-\frac{e\left(\frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}-\frac{g\int\left(F^{c(a+bx)}\right)^{-n}\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)d\left(F^{c(a+bx)}\right)^n}{b^2c^2en^2\log^2(F)}\right)}{d}}{d}-\frac{e\left(\frac{g\left(\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d}-\frac{\log\left(e\left(F^{c(a+bx)}\right)^n+d\right)}{d}\right)}{b^2c^2en^2\log^2(F)}-\frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

$\Big\downarrow$ 2838

$$\frac{\frac{(f+gx)^2}{2dg} - \frac{e\left(\frac{g\,\mathrm{PolyLog}\left(2,-\frac{e\left(F^{c(a+bx)}\right)^n}{d}\right)}{b^2c^2en^2\log^2(F)} + \frac{(f+gx)\log\left(\frac{e\left(F^{c(a+bx)}\right)^n}{d}+1\right)}{bcen\log(F)}\right)}{d}}{d} - \frac{e\left(\frac{g\left(\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d} - \frac{\log\left(e\left(F^{c(a+bx)}\right)^n+d\right)}{d}\right)}{b^2c^2en^2\log^2(F)} - \frac{f+gx}{bcen\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)}\right)}{d}$$

input
```
Int[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
-((e*(-((f + g*x)/(b*c*e*(d + e*(F^(c*(a + b*x)))^n)*n*Log[F])) + (g*(Log[
(F^(c*(a + b*x)))^n]/d - Log[d + e*(F^(c*(a + b*x)))^n]/d))/(b^2*c^2*e*n^2
*Log[F]^2)))/d) + ((f + g*x)^2/(2*d*g) - (e*(((f + g*x)*Log[1 + (e*(F^(c*(
a + b*x)))^n)/d])/(b*c*e*n*Log[F]) + (g*PolyLog[2, -((e*(F^(c*(a + b*x)))^
n)/d)])/(b^2*c^2*e*n^2*Log[F]^2)))/d)/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(189) = 378$.

Time $= 0.16$ (sec) , antiderivative size $= 591$, normalized size of antiderivative $= 3.09$

method	result
risch	$\frac{gx+f}{bcd\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)n\ln(F)} + \frac{g\ln\left(F^{c(bx+a)}\right)^{2}}{2d^{2}c^{2}b^{2}\ln(F)^{2}} - \frac{g\ln\left(F^{c(bx+a)}\right)\ln\left(1+\frac{e\,F^{ncbx}F^{-ncbx}\left(F^{c(bx+a)}\right)^{n}}{d}\right)}{d^{2}n\,c^{2}b^{2}\ln(F)^{2}} -$

input
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
(g*x+f)/b/c/d/(d+e*(F^(c*(b*x+a)))^n)/n/ln(F)+1/2/d^2/c^2/b^2/ln(F)^2*g*ln
(F^(c*(b*x+a)))^2-1/d^2/n/c^2/b^2/ln(F)^2*g*ln(F^(c*(b*x+a)))*ln(1+e*F^(n*
c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)-1/d^2/n^2/c^2/b^2/ln(F)^2*g*polyl
og(2,-e*F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n/d)-1/d^2/n/c/b/ln(F)*f*
ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)+1/d^2/n/c/b/ln(F)*f*ln(
F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)+1/d^2/n^2/c^2/b^2/ln(F)^2*g*ln
((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)-1/d^2/n^2/c^2/b^2/ln(F)^2
*g*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)-1/d^2/n/c/b/ln(F)*g*ln((
F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*x+1/d^2/n/c^2/b^2/ln(F)^2*g
*ln((F^(c*(b*x+a)))^n*F^(-n*c*b*x)*F^(n*c*b*x)*e+d)*ln(F^(c*(b*x+a)))+1/d^
2/n/c/b/ln(F)*g*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*x-1/d^2/n/c
^2/b^2/ln(F)^2*g*ln(F^(n*c*b*x)*F^(-n*c*b*x)*(F^(c*(b*x+a)))^n)*ln(F^(c*(b
*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. 2(188) = 376.

Time = 0.08 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.14

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$
$$=\frac{2\,(bcdf-acdg)n\log\left(F\right)+\left(b^2c^2dgn^2x^2+2\,b^2c^2dfn^2x+\left(2\,abc^2df-a^2c^2dg\right)n^2\right)\log\left(F\right)^2+\left(\left(b^2c^2egn^2x^2\right.\right.}{}$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(b*c*d*f - a*c*d*g)*n*log(F) + (b^2*c^2*d*g*n^2*x^2 + 2*b^2*c^2*d*f
*n^2*x + (2*a*b*c^2*d*f - a^2*c^2*d*g)*n^2)*log(F)^2 + ((b^2*c^2*e*g*n^2*x
^2 + 2*b^2*c^2*e*f*n^2*x + (2*a*b*c^2*e*f - a^2*c^2*e*g)*n^2)*log(F)^2 - 2
*(b*c*e*g*n*x + a*c*e*g*n)*log(F))*F^(b*c*n*x + a*c*n) - 2*(F^(b*c*n*x + a
*c*n)*e*g + d*g)*dilog(-(F^(b*c*n*x + a*c*n)*e + d)/d + 1) - 2*((b*c*d*f -
 a*c*d*g)*n*log(F) + ((b*c*e*f - a*c*e*g)*n*log(F) - e*g)*F^(b*c*n*x + a*c
*n) - d*g)*log(F^(b*c*n*x + a*c*n)*e + d) - 2*((b*c*e*g*n*x + a*c*e*g*n)*F
^(b*c*n*x + a*c*n)*log(F) + (b*c*d*g*n*x + a*c*d*g*n)*log(F))*log((F^(b*c*
n*x + a*c*n)*e + d)/d))/(F^(b*c*n*x + a*c*n)*b^2*c^2*d^2*e*n^2*log(F)^2 +
b^2*c^2*d^3*n^2*log(F)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)/(d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
g*(x/(F^(b*c*n*x)*F^(a*c*n)*b*c*d*e*n*log(F) + b*c*d^2*n*log(F)) + integra
te((b*c*n*x*log(F) - 1)/(F^(b*c*n*x)*F^(a*c*n)*b*c*d*e*n*log(F) + b*c*d^2*
n*log(F)), x)) + f*((b*c*n*x + a*c*n)/(b*c*d^2*n) + 1/((F^(b*c*n*x + a*c*n
)*d*e + d^2)*b*c*n*log(F)) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d^2*n*log
(F)))
```

Giac [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)/((F^((b*x + a)*c))^n*e + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{f+g\,x}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2}\,dx$$

input
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^2, x)
```

Reduce [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$
$$= \frac{f^{bcnx+acn}\left(\int \frac{x}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right)\log(f)\,bc\,d^2egn - f^{bcnx+acn}\log\left(f^{bcnx+acn}e+d\right)ef + f^{bcnx+acn}}{\log\left(f\right.}$$

input
```
int((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
(f**(a*c*n + b*c*n*x)*int(x/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n +
 b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**2*e*g*n - f**(a*c*n + b*c*n*x)*log(
f**(a*c*n + b*c*n*x)*e + d)*e*f + f**(a*c*n + b*c*n*x)*log(f)*b*c*e*f*n*x
- f**(a*c*n + b*c*n*x)*e*f + int(x/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(
a*c*n + b*c*n*x)*d*e + d**2),x)*log(f)*b*c*d**3*g*n - log(f**(a*c*n + b*c*
n*x)*e + d)*d*f + log(f)*b*c*d*f*n*x)/(log(f)*b*c*d**2*n*(f**(a*c*n + b*c*
n*x)*e + d))
```

3.42 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} \, dx = \frac{x}{d^2}+\frac{1}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n\log(F)}-\frac{\log\left(d+e\left(F^{c(a+bx)}\right)^n\right)}{bcd^2n\log(F)}$$

output
```
x/d^2+1/b/c/d/(d+e*(F^(c*(b*x+a)))^n)/n/ln(F)-ln(d+e*(F^(c*(b*x+a)))^n)/b/
c/d^2/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} \, dx = \frac{\frac{1}{bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)n}+\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{bcd^2n}-\frac{\log\left(bcd^3n\log(F)+bcd^2e\left(F^{c(a+bx)}\right)^n n\log(F)\right)}{bcd^2n}}{\log(F)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(-2),x]
```

output
```
(1/(b*c*d*(d + e*(F^(c*(a + b*x)))^n)*n) + Log[(F^(c*(a + b*x)))^n]/(b*c*d
^2*n) - Log[b*c*d^3*n*Log[F] + b*c*d^2*e*(F^(c*(a + b*x)))^n*n*Log[F]]/(b*
c*d^2*n))/Log[F]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dF^{c(a+bx)}}{bc\log(F)}$$

$$\Big\downarrow \text{798}$$

$$\frac{\int \frac{F^{-c(a+bx)}}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{F^{-c(a+bx)}}{d^2}-\frac{e}{d^2\left(e\left(F^{c(a+bx)}\right)^n+d\right)}-\frac{e}{d\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}\right)d\left(F^{c(a+bx)}\right)^n}{bcn\log(F)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log\left(e\left(F^{c(a+bx)}\right)^n+d\right)}{d^2}+\frac{\log\left(\left(F^{c(a+bx)}\right)^n\right)}{d^2}+\frac{1}{d\left(e\left(F^{c(a+bx)}\right)^n+d\right)}}{bcn\log(F)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(-2),x]
```

output
```
(1/(d*(d + e*(F^(c*(a + b*x)))^n)) + Log[(F^(c*(a + b*x)))^n]/d^2 - Log[d
+ e*(F^(c*(a + b*x)))^n]/d^2)/(b*c*n*Log[F])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}+\frac{1}{d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}}{\ln(F)bcn}$
default	$\frac{-\frac{\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}+\frac{1}{d\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}+\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}\right)}{d^{2}}}{\ln(F)bcn}$
risch	$\frac{\ln\left(F^{c(bx+a)}\right)}{\ln(F)bc\,d^{2}}+\frac{1}{bcd\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)n\ln(F)}-\frac{\ln\left(\left(F^{c(bx+a)}\right)^{n}+\frac{d}{e}\right)}{\ln(F)bcn\,d^{2}}$
parallelrisch	$\frac{e^{2}\left(F^{c(bx+a)}\right)^{n}x\ln(F)bcn+x\ln(F)bcden-\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)\left(F^{c(bx+a)}\right)^{n}e^{2}-\ln\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)de+de}{\ln(F)bc\,d^{2}en\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)}$
norman	$\frac{-\frac{e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}{\ln(F)bcn\,d^{2}}+\frac{e\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right){\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}{d^{2}\ln(F)bc}+\frac{\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}{\ln(F)bcd}}{d+e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F)}\right)}}-\frac{\ln\left(d+e\,{\mathrm e}^{n\ln\left({\mathrm e}^{c(bx+a)\ln(F}\right.\right.}{\ln(F)bcn\,d^{2}}$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/b/c/n*(-1/d^2*ln(d+e*(F^(c*(b*x+a)))^n)+1/d/(d+e*(F^(c*(b*x+a)))^n
)+1/d^2*ln((F^(c*(b*x+a)))^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{F^{bcnx+acn}bcenx\log(F)+bcdnx\log(F)-\left(F^{bcnx+acn}e+d\right)\log\left(F^{bcnx+acn}e+d\right)+d}{F^{bcnx+acn}bcd^2en\log(F)+bcd^3n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
(F^(b*c*n*x + a*c*n)*b*c*e*n*x*log(F) + b*c*d*n*x*log(F) - (F^(b*c*n*x + a
*c*n)*e + d)*log(F^(b*c*n*x + a*c*n)*e + d) + d)/(F^(b*c*n*x + a*c*n)*b*c*
d^2*e*n*log(F) + b*c*d^3*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Integral((d + e*(F**(c*(a + b*x)))**n)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\frac{bcnx+acn}{bcd^2n}+\frac{1}{(F^{bcnx+acn}de+d^2)bcn\log(F)}-\frac{\log\left(F^{bcnx+acn}e+d\right)}{bcd^2n\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
(b*c*n*x + a*c*n)/(b*c*d^2*n) + 1/((F^(b*c*n*x + a*c*n)*d*e + d^2)*b*c*n*l
og(F)) - log(F^(b*c*n*x + a*c*n)*e + d)/(b*c*d^2*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\frac{\log\left(|F|^{bcnx}|F|^{acn}\right)}{bcd^2n\log(F)}-\frac{\log\left(\left|F^{bcnx}F^{acn}e+d\right|\right)}{bcd^2n\log(F)}+\frac{1}{(F^{bcnx}F^{acn}e+d)bcdn\log(F)}$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
log(abs(F)^(b*c*n*x)*abs(F)^(a*c*n))/(b*c*d^2*n*log(F)) - log(abs(F^(b*c*n
*x)*F^(a*c*n)*e + d))/(b*c*d^2*n*log(F)) + 1/((F^(b*c*n*x)*F^(a*c*n)*e + d
)*b*c*d*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{x}{d^2}+\frac{1}{b\,c\,d\,n\,\ln\left(F\right)\,\left(d+e\left(F^{b\,c\,x}\,F^{a\,c}\right)^n\right)} - \frac{\ln\left(d+e\left(F^{b\,c\,x}\,F^{a\,c}\right)^n\right)}{b\,c\,d^2\,n\,\ln\left(F\right)}$$

input
```
int(1/(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
x/d^2 + 1/(b*c*d*n*log(F)*(d + e*(F^(b*c*x)*F^(a*c))^n)) - log(d + e*(F^(b
*c*x)*F^(a*c))^n)/(b*c*d^2*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \frac{-f^{bcnx+acn}\log\left(f^{bcnx+acn}e+d\right)e+f^{bcnx+acn}\log(f)\,bcenx-f^{bcnx+acn}e-\log\left(f^{bcnx+acn}e+d\right)d+\log(f)\,b}{\log\left(f\right)bc\,d^2n\left(f^{bcnx+acn}e+d\right)}$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
( - f**(a*c*n + b*c*n*x)*log(f**(a*c*n + b*c*n*x)*e + d)*e + f**(a*c*n + b
*c*n*x)*log(f)*b*c*e*n*x - f**(a*c*n + b*c*n*x)*e - log(f**(a*c*n + b*c*n*
x)*e + d)*d + log(f)*b*c*d*n*x)/(log(f)*b*c*d**2*n*(f**(a*c*n + b*c*n*x)*e
 + d))
```

3.43 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx=\operatorname{Int}\left(\frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)},x\right)$$

output
```
Defer(Int)(1/(d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f),x)
```

Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx=\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx$$

input
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)),x]
```

output
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$
$$\Big\downarrow \text{2618}$$
$$\int \frac{1}{(f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)^2}dx$$
$$\Big\downarrow \text{2619}$$
$$\int \frac{1}{(f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)^2}dx$$

input
```
Int[1/((d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{2}(gx+f)}dx$$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f),x)
```

output
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}(f+gx)}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^{n}e+d\right)^{2}(gx+f)}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x, algorithm="fricas")
```

output
```
integral(1/(d^2*g*x + d^2*f + (e^2*g*x + e^2*f)*(F^(b*c*x + a*c))^(2*n) +
2*(d*e*g*x + d*e*f)*(F^(b*c*x + a*c))^n), x)
```

Sympy [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}(f+gx)}\,dx=\int \frac{1}{\left(d+e\left(F^{ac+bcx}\right)^{n}\right)^{2}(f+gx)}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**2/(g*x+f),x)
```

output
```
Integral(1/((d + e*(F**(a*c + b*c*x))**n)**2*(f + g*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 192, normalized size of antiderivative = 7.68

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2(gx+f)}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x, algorithm="maxima")
```

output
```
1/(b*c*d^2*g*n*x*log(F) + b*c*d^2*f*n*log(F) + (F^(a*c*n)*b*c*d*e*g*n*x*lo
g(F) + F^(a*c*n)*b*c*d*e*f*n*log(F))*F^(b*c*n*x)) + integrate((b*c*g*n*x*l
og(F) + b*c*f*n*log(F) + g)/(b*c*d^2*g^2*n*x^2*log(F) + 2*b*c*d^2*f*g*n*x*
log(F) + b*c*d^2*f^2*n*log(F) + (F^(a*c*n)*b*c*d*e*g^2*n*x^2*log(F) + 2*F^
(a*c*n)*b*c*d*e*f*g*n*x*log(F) + F^(a*c*n)*b*c*d*e*f^2*n*log(F))*F^(b*c*n*
x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2(gx+f)}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x, algorithm="giac")
```

output
```
integrate(1/(((F^((b*x + a)*c))^n*e + d)^2*(g*x + f)), x)
```

Mupad [N/A]

Not integrable

Time = 23.86 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx=\int \frac{1}{(f+g\,x)\,\left(d+e\,\left(F^{c\,(a+b\,x)}\right)^n\right)^2}\,dx$$

input
```
int(1/((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^2),x)
```

output
```
int(1/((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)}\,dx$$
$$=\int \frac{1}{f^{2bcnx+2acn}e^2f+f^{2bcnx+2acn}e^2gx+2f^{bcnx+acn}def+2f^{bcnx+acn}degx+d^2f+d^2gx}dx$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f),x)
```

output
```
int(1/(f**(2*a*c*n + 2*b*c*n*x)*e**2*f + f**(2*a*c*n + 2*b*c*n*x)*e**2*g*x
 + 2*f**(a*c*n + b*c*n*x)*d*e*f + 2*f**(a*c*n + b*c*n*x)*d*e*g*x + d**2*f
+ d**2*g*x),x)
```

3.44 $\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2} dx = \text{Int}\left(\frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2}, x\right)$$

output
```
Defer(Int)(1/(d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2} dx = \int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^2} dx$$

input
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^2),x]
```

output
```
Integrate[1/((d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)^2\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$
$$\Big\downarrow \text{ 2618}$$
$$\int \frac{1}{(f+gx)^2\left(e\left(F^{ac+bcx}\right)^n+d\right)^2}dx$$
$$\Big\downarrow \text{ 2619}$$
$$\int \frac{1}{(f+gx)^2\left(e\left(F^{ac+bcx}\right)^n+d\right)^2}dx$$

input
```
Int[1/((d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{2}\left(gx+f\right)^{2}}dx$$

input
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^2,x)
```

output
```
int(1/(d+e*(F^(c*(b*x+a)))^n)^2/(g*x+f)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.32

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}(f+gx)^{2}}\,dx=\int \frac{1}{\left(\left(F^{(bx+a)c}\right)^{n}e+d\right)^{2}\left(gx+f\right)^{2}}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x, algorithm="fricas")
```

output
```
integral(1/(d^2*g^2*x^2 + 2*d^2*f*g*x + d^2*f^2 + (e^2*g^2*x^2 + 2*e^2*f*g
*x + e^2*f^2)*(F^(b*c*x + a*c))^(2*n) + 2*(d*e*g^2*x^2 + 2*d*e*f*g*x + d*e
*f^2)*(F^(b*c*x + a*c))^n), x)
```

Sympy [N/A]

Not integrable

Time = 25.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}(f+gx)^{2}}\,dx=\int \frac{1}{\left(d+e\left(F^{ac+bcx}\right)^{n}\right)^{2}\left(f+gx\right)^{2}}\,dx$$

input
```
integrate(1/(d+e*(F**((b*x+a)*c))**n)**2/(g*x+f)**2,x)
```

output
```
Integral(1/((d + e*(F**(a*c + b*c*x))**n)**2*(f + g*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 280, normalized size of antiderivative = 11.20

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)^2}\,dx = \int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2(gx+f)^2}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x, algorithm="maxima")
```

output
```
1/(b*c*d^2*g^2*n*x^2*log(F) + 2*b*c*d^2*f*g*n*x*log(F) + b*c*d^2*f^2*n*log
(F) + (F^(a*c*n)*b*c*d*e*g^2*n*x^2*log(F) + 2*F^(a*c*n)*b*c*d*e*f*g*n*x*lo
g(F) + F^(a*c*n)*b*c*d*e*f^2*n*log(F))*F^(b*c*n*x)) + integrate((b*c*g*n*x
*log(F) + b*c*f*n*log(F) + 2*g)/(b*c*d^2*g^3*n*x^3*log(F) + 3*b*c*d^2*f*g^
2*n*x^2*log(F) + 3*b*c*d^2*f^2*g*n*x*log(F) + b*c*d^2*f^3*n*log(F) + (F^(a
*c*n)*b*c*d*e*g^3*n*x^3*log(F) + 3*F^(a*c*n)*b*c*d*e*f*g^2*n*x^2*log(F) +
3*F^(a*c*n)*b*c*d*e*f^2*g*n*x*log(F) + F^(a*c*n)*b*c*d*e*f^3*n*log(F))*F^(
b*c*n*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)^2}\,dx = \int \frac{1}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2(gx+f)^2}\,dx$$

input
```
integrate(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x, algorithm="giac")
```

output
```
integrate(1/(((F^((b*x + a)*c))^n*e + d)^2*(g*x + f)^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.61 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)^2}\,dx=\int \frac{1}{{\left(f+g\,x\right)}^2\,{\left(d+e\,{\left(F^{c\,\left(a+b\,x\right)}\right)}^n\right)}^2} \,d x$$

input
```
int(1/((f + g*x)^2*(d + e*(F^(c*(a + b*x)))^n)^2),x)
```

output
```
int(1/((f + g*x)^2*(d + e*(F^(c*(a + b*x)))^n)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 6.28

$$\int \frac{1}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2(f+gx)^2}\,dx$$
$$=\int \frac{1}{f^{2bcnx+2acn}e^2f^2+2f^{2bcnx+2acn}e^2fgx+f^{2bcnx+2acn}e^2g^2x^2+2f^{bcnx+acn}de\,f^2+4f^{bcnx+acn}defgx+2f^{bc}}$$

input
```
int(1/(d+e*(F^((b*x+a)*c))^n)^2/(g*x+f)^2,x)
```

output
```
int(1/(f**(2*a*c*n + 2*b*c*n*x)*e**2*f**2 + 2*f**(2*a*c*n + 2*b*c*n*x)*e**
2*f*g*x + f**(2*a*c*n + 2*b*c*n*x)*e**2*g**2*x**2 + 2*f**(a*c*n + b*c*n*x)
*d*e*f**2 + 4*f**(a*c*n + b*c*n*x)*d*e*f*g*x + 2*f**(a*c*n + b*c*n*x)*d*e*
g**2*x**2 + d**2*f**2 + 2*d**2*f*g*x + d**2*g**2*x**2),x)
```

3.45 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2} (f+gx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 535

$$
\begin{aligned}
\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2} (f+gx)\,dx =& -\frac{92d^2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,g}{15b^2c^2n^2\log^2(F)} \\
&- \frac{32d\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2} g}{45b^2c^2n^2\log^2(F)} - \frac{4\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2} g}{25b^2c^2n^2\log^2(F)} \\
&+ \frac{92d^{5/2}g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{15b^2c^2n^2\log^2(F)} + \frac{2d^{5/2}g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2n^2\log^2(F)} \\
&+ \frac{2d^2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)}{bcn\log(F)} + \frac{2d\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)}{3bcn\log(F)} \\
&+ \frac{2\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}(f+gx)}{5bcn\log(F)} - \frac{2d^{5/2}(f+gx)\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcn\log(F)} \\
&- \frac{4d^{5/2}g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2n^2\log^2(F)} \\
&- \frac{2d^{5/2}g\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2n^2\log^2(F)}
\end{aligned}
$$

output
```
-92/15*d^2*(d+e*(F^(c*(b*x+a)))^n)^(1/2)*g/b^2/c^2/n^2/ln(F)^2-32/45*d*(d+
e*(F^(c*(b*x+a)))^n)^(3/2)*g/b^2/c^2/n^2/ln(F)^2-4/25*(d+e*(F^(c*(b*x+a)))
^n)^(5/2)*g/b^2/c^2/n^2/ln(F)^2+92/15*d^(5/2)*g*arctanh((d+e*(F^(c*(b*x+a)
))^n)^(1/2)/d^(1/2))/b^2/c^2/n^2/ln(F)^2+2*d^(5/2)*g*arctanh((d+e*(F^(c*(b
*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/n^2/ln(F)^2+2*d^2*(d+e*(F^(c*(b*x+a)))
^n)^(1/2)*(g*x+f)/b/c/n/ln(F)+2/3*d*(d+e*(F^(c*(b*x+a)))^n)^(3/2)*(g*x+f)/
b/c/n/ln(F)+2/5*(d+e*(F^(c*(b*x+a)))^n)^(5/2)*(g*x+f)/b/c/n/ln(F)-2*d^(5/2
)*(g*x+f)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c/n/ln(F)-4*d^(
5/2)*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2
)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/n^2/ln(F)^2-2*d^(5/2)*g*polylog(
2,1-2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/n^2/ln(F)^2
```

Mathematica [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}(f+gx)\,dx=\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}(f+gx)\,dx$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(5/2)*(f + g*x),x]
```

output
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(5/2)*(f + g*x), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}dx$$

$$\Big\downarrow \text{ 2618}$$

$$\int (f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)^{5/2}dx$$

$$\Big\downarrow \text{ 2619}$$

$$\int (f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)^{5/2}dx$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(5/2)*(f + g*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [F]

$$\int \left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{5}{2}}(gx+f)\,dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(5/2)*(g*x+f),x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^(5/2)*(g*x+f),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{5/2}(f+gx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(5/2)*(g*x+f),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2} (f+gx)\, dx = \text{Timed out}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(5/2)*(g*x+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2} (f+gx)\, dx = \int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{5}{2}} (gx+f)\, dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(5/2)*(g*x+f),x, algorithm="maxima")
```

output
```
1/15*f*(15*d^(5/2)*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F
^(b*c*n*x + a*c*n)*e + d) + sqrt(d)))/(b*c*n*log(F)) + 2*(3*(F^(b*c*n*x +
a*c*n)*e + d)^(5/2) + 5*(F^(b*c*n*x + a*c*n)*e + d)^(3/2)*d + 15*sqrt(F^(b
*c*n*x + a*c*n)*e + d)*d^2)/(b*c*n*log(F))) + g*integrate((2*F^(b*c*n*x)*F
^(a*c*n)*d*e*x + F^(2*b*c*n*x)*F^(2*a*c*n)*e^2*x + d^2*x)*sqrt(F^(b*c*n*x)
*F^(a*c*n)*e + d), x)
```

Giac [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}(f+gx)\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{5}{2}}(gx+f)\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(5/2)*(g*x+f),x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^(5/2)*(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}(f+gx)\,dx=\int (f+g\,x)\,\left(d+e\,\left(F^{c\,(a+b\,x)}\right)^n\right)^{5/2}\,dx$$

input
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^(5/2),x)
```

output
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^(5/2), x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}(f+gx)\,dx=\frac{90f^{2bcnx+2acn}\sqrt{f^{bcnx+acn}e+d}\,\log(f)\,bc\,e^2fn+90f^{2bcnx+2acn}\sqrt{f^{bcnx+acn}e+d}\,\log(f)\,bc\,e^2gnx-}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(5/2)*(g*x+f),x)
```

output
```
(90*f**(2*a*c*n + 2*b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)*log(f)*b*c*e
**2*f*n + 90*f**(2*a*c*n + 2*b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)*log
(f)*b*c*e**2*g*n*x - 36*f**(2*a*c*n + 2*b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)
*e + d)*e**2*g + 330*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)
*log(f)*b*c*d*e*f*n + 330*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e
 + d)*log(f)*b*c*d*e*g*n*x - 232*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c
*n*x)*e + d)*d*e*g + 690*sqrt(f**(a*c*n + b*c*n*x)*e + d)*log(f)*b*c*d**2*
f*n + 690*sqrt(f**(a*c*n + b*c*n*x)*e + d)*log(f)*b*c*d**2*g*n*x - 1576*sq
rt(f**(a*c*n + b*c*n*x)*e + d)*d**2*g + 225*int(sqrt(f**(a*c*n + b*c*n*x)*
e + d)/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)**2*b**2*c**2*d**3*f*n**2 - 6
90*int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(a*c*n + b*c*n*x)*e + d),x)*lo
g(f)*b*c*d**3*g*n + 225*int((sqrt(f**(a*c*n + b*c*n*x)*e + d)*x)/(f**(a*c*
n + b*c*n*x)*e + d),x)*log(f)**2*b**2*c**2*d**3*g*n**2)/(225*log(f)**2*b**
2*c**2*n**2)
```

3.46 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2} (f+gx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 451

$$\begin{aligned}
\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2} (f+gx)\,dx &= -\frac{16d\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,g}{3b^2c^2n^2\log^2(F)} \\
&-\frac{4\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2} g}{9b^2c^2n^2\log^2(F)}+\frac{16d^{3/2}g\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{3b^2c^2n^2\log^2(F)} \\
&+\frac{2d^{3/2}g\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2n^2\log^2(F)}+\frac{2d\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,(f+gx)}{bcn\log(F)} \\
&+\frac{2\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)}{3bcn\log(F)}-\frac{2d^{3/2}(f+gx)\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcn\log(F)} \\
&-\frac{4d^{3/2}g\,\mathrm{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2n^2\log^2(F)} \\
&-\frac{2d^{3/2}g\,\mathrm{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2n^2\log^2(F)}
\end{aligned}$$

output
```
-16/3*d*(d+e*(F^(c*(b*x+a)))^n)^(1/2)*g/b^2/c^2/n^2/ln(F)^2-4/9*(d+e*(F^(c
*(b*x+a)))^n)^(3/2)*g/b^2/c^2/n^2/ln(F)^2+16/3*d^(3/2)*g*arctanh((d+e*(F^(
c*(b*x+a)))^n)^(1/2)/d^(1/2))/b^2/c^2/n^2/ln(F)^2+2*d^(3/2)*g*arctanh((d+e
*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/n^2/ln(F)^2+2*d*(d+e*(F^(c*(b
*x+a)))^n)^(1/2)*(g*x+f)/b/c/n/ln(F)+2/3*(d+e*(F^(c*(b*x+a)))^n)^(3/2)*(g*
x+f)/b/c/n/ln(F)-2*d^(3/2)*(g*x+f)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d
^(1/2))/b/c/n/ln(F)-4*d^(3/2)*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1
/2))*ln(2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/n^2/ln(
F)^2-2*d^(3/2)*g*polylog(2,1-2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1
/2)))/b^2/c^2/n^2/ln(F)^2
```

Mathematica [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx=\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(3/2)*(f + g*x),x]
```

output
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^(3/2)*(f + g*x), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}dx$$
$$\Big\downarrow \text{ 2618}$$
$$\int (f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)^{3/2}dx$$
$$\Big\downarrow \text{ 2619}$$
$$\int (f+gx)\left(e\left(F^{ac+bcx}\right)^n+d\right)^{3/2}dx$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^(3/2)*(f + g*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] && !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [F]

$$\int \left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{3}{2}}(gx+f)\,dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(3/2)*(g*x+f),x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^(3/2)*(g*x+f),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{3/2}(f+gx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(3/2)*(g*x+f),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx=\text{Timed out}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(3/2)*(g*x+f),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{3}{2}}(gx+f)\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(3/2)*(g*x+f),x, algorithm="maxima")
```

output
```
1/3*f*(3*d^(3/2)*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(
b*c*n*x + a*c*n)*e + d) + sqrt(d)))/(b*c*n*log(F)) + 2*((F^(b*c*n*x + a*c*
n)*e + d)^(3/2) + 3*sqrt(F^(b*c*n*x + a*c*n)*e + d)*d)/(b*c*n*log(F))) + g
*integrate((F^(b*c*n*x)*F^(a*c*n)*e*x + d*x)*sqrt(F^(b*c*n*x)*F^(a*c*n)*e
+ d), x)
```

Giac [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{3}{2}}(gx+f)\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(3/2)*(g*x+f),x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^(3/2)*(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx=\int (f+g\,x)\,\left(d+e\,{\left(F^{c\,(a+b\,x)}\right)}^n\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^(3/2),x)
```

output
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^(3/2), x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}(f+gx)\,dx=\frac{6f^{bcnx+acn}\sqrt{f^{bcnx+acn}e+d}\log(f)\,bcefn+6f^{bcnx+acn}\sqrt{f^{bcnx+acn}e+d}\log(f)\,bcegnx-4f^{bcnx+ac}}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(3/2)*(g*x+f),x)
```

output

```
(6*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)*log(f)*b*c*e*f*n
+ 6*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)*log(f)*b*c*e*g*n
*x - 4*f**(a*c*n + b*c*n*x)*sqrt(f**(a*c*n + b*c*n*x)*e + d)*e*g + 24*sqrt
(f**(a*c*n + b*c*n*x)*e + d)*log(f)*b*c*d*f*n + 24*sqrt(f**(a*c*n + b*c*n*
x)*e + d)*log(f)*b*c*d*g*n*x - 52*sqrt(f**(a*c*n + b*c*n*x)*e + d)*d*g + 9
*int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(a*c*n + b*c*n*x)*e + d),x)*log(
f)**2*b**2*c**2*d**2*f*n**2 - 24*int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**
(a*c*n + b*c*n*x)*e + d),x)*log(f)*b*c*d**2*g*n + 9*int((sqrt(f**(a*c*n +
b*c*n*x)*e + d)*x)/(f**(a*c*n + b*c*n*x)*e + d),x)*log(f)**2*b**2*c**2*d**
2*g*n**2)/(9*log(f)**2*b**2*c**2*n**2)
```

3.47 $\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx$

Optimal result

Integrand size = 25, antiderivative size = 367

$$
\begin{aligned}
&\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx \\
&= -\frac{4\sqrt{d+e\left(F^{c(a+bx)}\right)^n}g}{b^2c^2n^2\log^2(F)} + \frac{4\sqrt{d}g\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{b^2c^2n^2\log^2(F)} \\
&\quad + \frac{2\sqrt{d}g\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2n^2\log^2(F)} + \frac{2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)}{bcn\log(F)} \\
&\quad - \frac{2\sqrt{d}(f+gx)\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcn\log(F)} \\
&\quad - \frac{4\sqrt{d}g\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2n^2\log^2(F)} \\
&\quad - \frac{2\sqrt{d}g\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2n^2\log^2(F)}
\end{aligned}
$$

output
```
-4*(d+e*(F^(c*(b*x+a)))^n)^(1/2)*g/b^2/c^2/n^2/ln(F)^2+4*d^(1/2)*g*arctanh
((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b^2/c^2/n^2/ln(F)^2+2*d^(1/2)*g*ar
ctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/n^2/ln(F)^2+2*(d+e*
(F^(c*(b*x+a)))^n)^(1/2)*(g*x+f)/b/c/n/ln(F)-2*d^(1/2)*(g*x+f)*arctanh((d+
e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c/n/ln(F)-4*d^(1/2)*g*arctanh((d+e*(
F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a))
)^n)^(1/2)))/b^2/c^2/n^2/ln(F)^2-2*d^(1/2)*g*polylog(2,1-2*d^(1/2)/(d^(1/2
)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/n^2/ln(F)^2
```

Mathematica [F]

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx=\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx$$

input
```
Integrate[Sqrt[d + e*(F^(c*(a + b*x)))^n]*(f + g*x),x]
```

output
```
Integrate[Sqrt[d + e*(F^(c*(a + b*x)))^n]*(f + g*x), x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}\,dx$$

$$\Big\downarrow \; 2618$$

$$\int (f+gx)\sqrt{e\left(F^{ac+bcx}\right)^n+d}dx$$

$$\Big\downarrow \; 2619$$

$$\int (f+gx)\sqrt{e\left(F^{ac+bcx}\right)^n+d}dx$$

input
```
Int[Sqrt[d + e*(F^(c*(a + b*x)))^n]*(f + g*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [F]

$$\int \sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}\,(gx+f)\,dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^(1/2)*(g*x+f),x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^(1/2)*(g*x+f),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^{n}}(f+gx)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/2)*(g*x+f),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx=\int \sqrt{d+e\left(F^{ac+bcx}\right)^n}(f+gx)\;dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**(1/2)*(g*x+f),x)
```

output
```
Integral(sqrt(d + e*(F**(a*c + b*c*x))**n)*(f + g*x), x)
```

Maxima [F]

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx=\int \sqrt{\left(F^{(bx+a)c}\right)^n e+d}(gx+f)\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/2)*(g*x+f),x, algorithm="maxima")
```

output
```
f*(sqrt(d)*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*
x + a*c*n)*e + d) + sqrt(d)))/(b*c*n*log(F)) + 2*sqrt(F^(b*c*n*x + a*c*n)*
e + d)/(b*c*n*log(F))) + g*integrate(sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d)*x,
x)
```

Giac [F]

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx=\int \sqrt{\left(F^{(bx+a)c}\right)^n e+d}(gx+f)\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^(1/2)*(g*x+f),x, algorithm="giac")
```

output
```
integrate(sqrt((F^((b*x + a)*c))^n*e + d)*(g*x + f), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx=\int (f+g\,x)\,\sqrt{d+e\,{\left(F^{c\,(a+b\,x)}\right)}^n}\,dx$$

input
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^(1/2),x)
```

output
```
int((f + g*x)*(d + e*(F^(c*(a + b*x)))^n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+e\left(F^{c(a+bx)}\right)^n}(f+gx)\,dx=\left(\int \sqrt{f^{bcnx+acn}e+d}dx\right)f+\left(\int \sqrt{f^{bcnx+acn}e+d}\,xdx\right)g$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^(1/2)*(g*x+f),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d),x)*f + int(sqrt(f**(a*c*n + b*c*n*x)*
e + d)*x,x)*g
```

3.48 $\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 246

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\frac{2g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2\sqrt{d}n^2\log^2(F)}-\frac{2(f+gx)\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bc\sqrt{d}n\log(F)}-\frac{4g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2\sqrt{d}n^2\log^2(F)}-\frac{2g\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2\sqrt{d}n^2\log^2(F)}$$

output
```
2*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/d^(1/2)/n^2/l
n(F)^2-2*(g*x+f)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c/d^(1/2
)/n/ln(F)-4*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))*ln(2*d^(1/2)/
(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/d^(1/2)/n^2/ln(F)^2-2*g*p
olylog(2,1-2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/d^(1
/2)/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx$$

input
```
Integrate[(f + g*x)/Sqrt[d + e*(F^(c*(a + b*x)))^n],x]
```

output
```
Integrate[(f + g*x)/Sqrt[d + e*(F^(c*(a + b*x)))^n], x]
```

Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}\,dx$$

$$\Big\downarrow \text{ 2618}$$

$$\int \frac{f+gx}{\sqrt{e\left(F^{ac+bcx}\right)^n+d}}dx$$

$$\Big\downarrow \text{ 2619}$$

$$\int \frac{f+gx}{\sqrt{e\left(F^{ac+bcx}\right)^n+d}}dx$$

input
```
Int[(f + g*x)/Sqrt[d + e*(F^(c*(a + b*x)))^n],x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [F]

$$\int \frac{gx+f}{\sqrt{d+e\left(F^{c(bx+a)}\right)^{n}}}dx$$

input
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(1/2),x)
```

output
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^{n}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{f+gx}{\sqrt{d+e\left(F^{ac+bcx}\right)^n}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F**((b*x+a)*c))**n)**(1/2),x)
```

output
```
Integral((f + g*x)/sqrt(d + e*(F**(a*c + b*c*x))**n), x)
```

Maxima [F]

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{gx+f}{\sqrt{\left(F^{(bx+a)c}\right)^n e+d}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="maxima")
```

output
```
g*integrate(x/sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d), x) + f*log((sqrt(F^(b*c*n
*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x + a*c*n)*e + d) + sqrt(d)))
/(b*c*sqrt(d)*n*log(F))
```

Giac [F]

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{gx+f}{\sqrt{\left(F^{(bx+a)c}\right)^n e+d}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(1/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/sqrt((F^((b*x + a)*c))^n*e + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\int \frac{f+g\,x}{\sqrt{d+e\left(F^{c\,(a+b\,x)}\right)^n}}\,dx$$

input
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(1/2),x)
```

output
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(1/2), x)
```

Reduce [F]

$$\int \frac{f+gx}{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\,dx=\left(\int \frac{x}{\sqrt{f^{bcnx+acn}e+d}}dx\right)g+\left(\int \frac{1}{\sqrt{f^{bcnx+acn}e+d}}dx\right)f$$

input
```
int((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(1/2),x)
```

output
```
int(x/sqrt(f**(a*c*n + b*c*n*x)*e + d),x)*g + int(1/sqrt(f**(a*c*n + b*c*n
*x)*e + d),x)*f
```

3.49 $\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 335

$$\begin{aligned}
\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx &= \frac{4g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{b^2c^2d^{3/2}n^2\log^2(F)} \\
&+ \frac{2g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2d^{3/2}n^2\log^2(F)} + \frac{2(f+gx)}{bcd\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)} \\
&- \frac{2(f+gx)\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcd^{3/2}n\log(F)} \\
&- \frac{4g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2d^{3/2}n^2\log^2(F)} \\
&- \frac{2g\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2d^{3/2}n^2\log^2(F)}
\end{aligned}$$

output
```
4*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b^2/c^2/d^(3/2)/n^2/ln(
F)^2+2*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/d^(3/2)/
n^2/ln(F)^2+2*(g*x+f)/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(1/2)/n/ln(F)-2*(g*x+f
)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c/d^(3/2)/n/ln(F)-4*g*a
rctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(d+e*(
F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/d^(3/2)/n^2/ln(F)^2-2*g*polylog(2,1-2*d^
(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/d^(3/2)/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx$$

input
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(3/2),x]
```

output
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(3/2), x]
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 2617$$

$$-g\int \left(\frac{2}{bcd\sqrt{e\left(F^{c(a+bx)}\right)^n+d}\,n\log(F)}-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{3/2}n\log(F)}\right)dx-\frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{3/2}n\log(F)}+\frac{2(f+gx)}{bcdn\log(F)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}$$

$$\Big\downarrow \text{2009}$$

$$-g\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)^2}{b^2c^2d^{3/2}n^2\log^2(F)}+\frac{4\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}\right)}{b^2c^2d^{3/2}n^2\log^2(F)}-\frac{4\text{arctanh}\left(\frac{\sqrt{e}\cdots}{}\right)}{b^2c^2d^{3/2}n\cdots}\right.$$

$$-\frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{3/2}n\log(F)}+\frac{2(f+gx)}{bcdn\log(F)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}$$

input
```
Int[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(3/2),x]
```

output
```
(2*(f + g*x))/(b*c*d*Sqrt[d + e*(F^(c*(a + b*x)))^n]*n*Log[F]) - (2*(f + g
*x)*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(b*c*d^(3/2)*n*Log[F
]) - g*((-4*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(b^2*c^2*d^(
3/2)*n^2*Log[F]^2) - (2*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]]^2
)/(b^2*c^2*d^(3/2)*n^2*Log[F]^2) + (4*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))
^n]/Sqrt[d]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*(F^(c*(a + b*x)))^n])])
/(b^2*c^2*d^(3/2)*n^2*Log[F]^2) + (2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] -
 Sqrt[d + e*(F^(c*(a + b*x)))^n])])/(b^2*c^2*d^(3/2)*n^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2617
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(a + b*(F^(g*(e + f*x)))^
n)^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{gx+f}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{3}{2}}}dx$$

input
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(3/2),x)
```

output
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\int \frac{f+gx}{\left(d+e\left(F^{ac+bcx}\right)^n\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F**((b*x+a)*c))**n)**(3/2),x)
```

output
```
Integral((f + g*x)/(d + e*(F**(a*c + b*c*x))**n)**(3/2), x)
```

Maxima [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="maxima")
```

output
```
f*(log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x + a*c*
n)*e + d) + sqrt(d)))/(b*c*d^(3/2)*n*log(F)) + 2/(sqrt(F^(b*c*n*x + a*c*n)
*e + d)*b*c*d*n*log(F))) + g*integrate(x/(F^(b*c*n*x)*F^(a*c*n)*e + d)^(3/
2), x)
```

Giac [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(3/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/((F^((b*x + a)*c))^n*e + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\int \frac{f+g\,x}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{3/2}}\,dx$$

input
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(3/2),x)
```

output
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(3/2), x)
```

Reduce [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}}\,dx=\left(\int \frac{\sqrt{f^{bcnx+acn}e+d}}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right)f+\left(\int \frac{\sqrt{f^{bcnx+acn}e+d}\,x}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx\right)g$$

input
```
int((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(3/2),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f*
*(a*c*n + b*c*n*x)*d*e + d**2),x)*f + int((sqrt(f**(a*c*n + b*c*n*x)*e + d
)*x)/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d*e + d**2),x
)*g
```

3.50 $\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 421

$$\begin{aligned}\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx &= -\frac{4g}{3b^2c^2d^2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n^2\log^2(F)} \\ &+ \frac{16g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{3b^2c^2d^{5/2}n^2\log^2(F)} + \frac{2g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2d^{5/2}n^2\log^2(F)} \\ &+ \frac{2(f+gx)}{3bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}n\log(F)} + \frac{2(f+gx)}{bcd^2\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)} \\ &- \frac{2(f+gx)\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcd^{5/2}n\log(F)} \\ &- \frac{4g\operatorname{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2d^{5/2}n^2\log^2(F)} \\ &- \frac{2g\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2d^{5/2}n^2\log^2(F)}\end{aligned}$$

output
```
-4/3*g/b^2/c^2/d^2/(d+e*(F^(c*(b*x+a)))^n)^(1/2)/n^2/ln(F)^2+16/3*g*arctan
h((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b^2/c^2/d^(5/2)/n^2/ln(F)^2+2*g*a
rctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/d^(5/2)/n^2/ln(F)^
2+2/3*(g*x+f)/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(3/2)/n/ln(F)+2*(g*x+f)/b/c/d^
2/(d+e*(F^(c*(b*x+a)))^n)^(1/2)/n/ln(F)-2*(g*x+f)*arctanh((d+e*(F^(c*(b*x+
a)))^n)^(1/2)/d^(1/2))/b/c/d^(5/2)/n/ln(F)-4*g*arctanh((d+e*(F^(c*(b*x+a))
)^n)^(1/2)/d^(1/2))*ln(2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/
b^2/c^2/d^(5/2)/n^2/ln(F)^2-2*g*polylog(2,1-2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*
(b*x+a)))^n)^(1/2)))/b^2/c^2/d^(5/2)/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx$$

input
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(5/2),x]
```

output
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(5/2), x]
```

Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}}dx$$

$$\Big\downarrow \; 2617$$

$$-g\int \left(-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{5/2}n\log(F)}+\frac{2}{bcd^2\sqrt{e\left(F^{c(a+bx)}\right)^n+d}n\log(F)}+\frac{2}{3bcd\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}n\log(F\right.$$

$$\frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{5/2}n\log(F)}+\frac{2(f+gx)}{bcd^2n\log(F)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}+$$

$$\frac{2(f+gx)}{3bcdn\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-g\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)^2}{b^2c^2d^{5/2}n^2\log^2(F)}+\frac{4\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}\right)}{b^2c^2d^{5/2}n^2\log^2(F)}-\frac{16\text{arctanh}\left(\sqrt{\right.}}{3b^2c^2d^{5/2}}\right.$$

$$\frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{5/2}n\log(F)}+\frac{2(f+gx)}{bcd^2n\log(F)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}+$$

$$\frac{2(f+gx)}{3bcdn\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}$$

input

```
Int[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(5/2),x]
```

output

```
(2*(f + g*x))/(3*b*c*d*(d + e*(F^(c*(a + b*x)))^n)^(3/2)*n*Log[F]) + (2*(f
 + g*x))/(b*c*d^2*Sqrt[d + e*(F^(c*(a + b*x)))^n]*n*Log[F]) - (2*(f + g*x)
*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(b*c*d^(5/2)*n*Log[F])
- g*(4/(3*b^2*c^2*d^2*Sqrt[d + e*(F^(c*(a + b*x)))^n]*n^2*Log[F]^2) - (16*
ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(3*b^2*c^2*d^(5/2)*n^2*L
og[F]^2) - (2*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]]^2)/(b^2*c^2
*d^(5/2)*n^2*Log[F]^2) + (4*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d
]]*Log[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*(F^(c*(a + b*x)))^n])])/(b^2*c^2*
d^(5/2)*n^2*Log[F]^2) + (2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d +
e*(F^(c*(a + b*x)))^n])])/(b^2*c^2*d^(5/2)*n^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2617
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(a + b*(F^(g*(e + f*x)))^
n)^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{gx+f}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{5}{2}}}dx$$

input
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(5/2),x)
```

output
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)/(d+e*(F**((b*x+a)*c))**n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(5/2),x, algorithm="maxima")
```

output
```
1/3*f*(3*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*x
+ a*c*n)*e + d) + sqrt(d)))/(b*c*d^(5/2)*n*log(F)) + 2*(3*F^(b*c*n*x + a*c
*n)*e + 4*d)/((F^(b*c*n*x + a*c*n)*e + d)^(3/2)*b*c*d^2*n*log(F))) + g*int
egrate(x/((2*F^(b*c*n*x)*F^(a*c*n)*d*e + F^(2*b*c*n*x)*F^(2*a*c*n)*e^2 + d
^2)*sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d)), x)
```

Giac [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx = \int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(5/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/((F^((b*x + a)*c))^n*e + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\int \frac{f+g\,x}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^{5/2}}\,dx$$

input
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(5/2),x)
```

output
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(5/2), x)
```

Reduce [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}}\,dx=\left(\int \frac{\sqrt{f^{bcnx+acn}e+d}}{f^{3bcnx+3acn}e^3+3f^{2bcnx+2acn}d\,e^2+3f^{bcnx+acn}d^2e+d^3}dx\right)f+\left(\int \frac{\sqrt{f^{bcnx+acn}e+d}\,x}{f^{3bcnx+3acn}e^3+3f^{2bcnx+2acn}d\,e^2+3f^{bcnx+acn}d^2e+d^3}dx\right)g$$

input
```
int((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(5/2),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(3*a*c*n + 3*b*c*n*x)*e**3 + 3*f*
*(2*a*c*n + 2*b*c*n*x)*d*e**2 + 3*f**(a*c*n + b*c*n*x)*d**2*e + d**3),x)*f
 + int((sqrt(f**(a*c*n + b*c*n*x)*e + d)*x)/(f**(3*a*c*n + 3*b*c*n*x)*e**3
 + 3*f**(2*a*c*n + 2*b*c*n*x)*d*e**2 + 3*f**(a*c*n + b*c*n*x)*d**2*e + d**
3),x)*g
```

3.51 $\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 505

$$
\begin{aligned}
\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx &= -\frac{4g}{15b^2c^2d^2\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}n^2\log^2(F)} \\
&- \frac{32g}{15b^2c^2d^3\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n^2\log^2(F)} \\
&+ \frac{92g\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{15b^2c^2d^{7/2}n^2\log^2(F)} + \frac{2g\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)^2}{b^2c^2d^{7/2}n^2\log^2(F)} \\
&+ \frac{2(f+gx)}{5bcd\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{5/2}n\log(F)} + \frac{2(f+gx)}{3bcd^2\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{3/2}n\log(F)} \\
&+ \frac{2(f+gx)}{bcd^3\sqrt{d+e\left(F^{c(a+bx)}\right)^n}\,n\log(F)} - \frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)}{bcd^{7/2}n\log(F)} \\
&- \frac{4g\,\text{arctanh}\left(\frac{\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2d^{7/2}n^2\log^2(F)} \\
&- \frac{2g\,\text{PolyLog}\left(2,1-\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{d+e\left(F^{c(a+bx)}\right)^n}}\right)}{b^2c^2d^{7/2}n^2\log^2(F)}
\end{aligned}
$$

output
```
-4/15*g/b^2/c^2/d^2/(d+e*(F^(c*(b*x+a)))^n)^(3/2)/n^2/ln(F)^2-32/15*g/b^2/
c^2/d^3/(d+e*(F^(c*(b*x+a)))^n)^(1/2)/n^2/ln(F)^2+92/15*g*arctanh((d+e*(F^
(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b^2/c^2/d^(7/2)/n^2/ln(F)^2+2*g*arctanh((d+
e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))^2/b^2/c^2/d^(7/2)/n^2/ln(F)^2+2/5*(g*x
+f)/b/c/d/(d+e*(F^(c*(b*x+a)))^n)^(5/2)/n/ln(F)+2/3*(g*x+f)/b/c/d^2/(d+e*(
F^(c*(b*x+a)))^n)^(3/2)/n/ln(F)+2*(g*x+f)/b/c/d^3/(d+e*(F^(c*(b*x+a)))^n)^
(1/2)/n/ln(F)-2*(g*x+f)*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))/b/c
/d^(7/2)/n/ln(F)-4*g*arctanh((d+e*(F^(c*(b*x+a)))^n)^(1/2)/d^(1/2))*ln(2*d
^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c^2/d^(7/2)/n^2/ln(F)^
2-2*g*polylog(2,1-2*d^(1/2)/(d^(1/2)-(d+e*(F^(c*(b*x+a)))^n)^(1/2)))/b^2/c
^2/d^(7/2)/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx=\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx$$

input
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(7/2),x]
```

output
```
Integrate[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(7/2), x]
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2617, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f+gx}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{7/2}}\,dx$$

$$\Big\downarrow \text{2617}$$

$$-g\int\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{7/2}n\log(F)}+\frac{2}{bcd^3\sqrt{e\left(F^{c(a+bx)}\right)^n+d}n\log(F)}+\frac{2}{3bcd^2\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}n\log(\right.$$

$$\frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{7/2}n\log(F)}+\frac{2(f+gx)}{bcd^3n\log(F)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}+\frac{2(f+gx)}{3bcd^2n\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}+\frac{2(f+gx)}{5bcdn\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}}$$

$\downarrow$ 2009

$$-g\left(-\frac{2\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)^2}{b^2c^2d^{7/2}n^2\log^2(F)}+\frac{4\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)\log\left(\frac{2\sqrt{d}}{\sqrt{d}-\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}\right)}{b^2c^2d^{7/2}n^2\log^2(F)}-\frac{92\text{arctanh}\left(\sqrt{\right.}{15b^2c^2d^{7/}}\right.$$

$$\frac{2(f+gx)\text{arctanh}\left(\frac{\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}{\sqrt{d}}\right)}{bcd^{7/2}n\log(F)}+\frac{2(f+gx)}{bcd^3n\log(F)\sqrt{e\left(F^{c(a+bx)}\right)^n+d}}+\frac{2(f+gx)}{3bcd^2n\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{3/2}}+\frac{2(f+gx)}{5bcdn\log(F)\left(e\left(F^{c(a+bx)}\right)^n+d\right)^{5/2}}$$

input
```
Int[(f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(7/2),x]
```

output
```
(2*(f + g*x))/(5*b*c*d*(d + e*(F^(c*(a + b*x)))^n)^(5/2)*n*Log[F]) + (2*(f
 + g*x))/(3*b*c*d^2*(d + e*(F^(c*(a + b*x)))^n)^(3/2)*n*Log[F]) + (2*(f +
g*x))/(b*c*d^3*Sqrt[d + e*(F^(c*(a + b*x)))^n]*n*Log[F]) - (2*(f + g*x)*Ar
cTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(b*c*d^(7/2)*n*Log[F]) - g
*(4/(15*b^2*c^2*d^2*(d + e*(F^(c*(a + b*x)))^n)^(3/2)*n^2*Log[F]^2) + 32/(
15*b^2*c^2*d^3*Sqrt[d + e*(F^(c*(a + b*x)))^n]*n^2*Log[F]^2) - (92*ArcTanh
[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]])/(15*b^2*c^2*d^(7/2)*n^2*Log[F]^
2) - (2*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]]^2)/(b^2*c^2*d^(7/
2)*n^2*Log[F]^2) + (4*ArcTanh[Sqrt[d + e*(F^(c*(a + b*x)))^n]/Sqrt[d]]*Log
[(2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*(F^(c*(a + b*x)))^n])])/(b^2*c^2*d^(7/2
)*n^2*Log[F]^2) + (2*PolyLog[2, 1 - (2*Sqrt[d])/(Sqrt[d] - Sqrt[d + e*(F^(
c*(a + b*x)))^n])])/(b^2*c^2*d^(7/2)*n^2*Log[F]^2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2617
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> With[{u = IntHide[(a + b*(F^(g*(e + f*x)))^
n)^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0] && LtQ[p, -1]
```

Maple [F]

$$\int \frac{gx+f}{\left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{\frac{7}{2}}}dx$$

input
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(7/2),x)
```

output
```
int((g*x+f)/(d+e*(F^(c*(b*x+a)))^n)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)/(d+e*(F**((b*x+a)*c))**n)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx = \int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(7/2),x, algorithm="maxima")
```

output
```
1/15*f*(15*log((sqrt(F^(b*c*n*x + a*c*n)*e + d) - sqrt(d))/(sqrt(F^(b*c*n*
x + a*c*n)*e + d) + sqrt(d)))/(b*c*d^(7/2)*n*log(F)) + 2*(15*(F^(b*c*n*x +
 a*c*n)*e + d)^2 + 5*(F^(b*c*n*x + a*c*n)*e + d)*d + 3*d^2)/((F^(b*c*n*x +
 a*c*n)*e + d)^(5/2)*b*c*d^3*n*log(F))) + g*integrate(x/((3*F^(b*c*n*x)*F^
(a*c*n)*d^2*e + 3*F^(2*b*c*n*x)*F^(2*a*c*n)*d*e^2 + F^(3*b*c*n*x)*F^(3*a*c
*n)*e^3 + d^3)*sqrt(F^(b*c*n*x)*F^(a*c*n)*e + d)), x)
```

Giac [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx=\int \frac{gx+f}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(7/2),x, algorithm="giac")
```

output
```
integrate((g*x + f)/((F^((b*x + a)*c))^n*e + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx=\int \frac{f+g\,x}{\left(d+e\,\left(F^{c\,(a+b\,x)}\right)^n\right)^{7/2}}\,dx$$

input
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(7/2),x)
```

output
```
int((f + g*x)/(d + e*(F^(c*(a + b*x)))^n)^(7/2), x)
```

Reduce [F]

$$\int \frac{f+gx}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^{7/2}}\,dx = \left(\int \frac{\sqrt{f^{bcnx+acn}e+d}}{f^{4bcnx+4acn}e^4+4f^{3bcnx+3acn}d\,e^3+6f^{2bcnx+2acn}d^2e^2+4f^{bcnx+acn}d^3e+d^4}dx\right)f + \left(\int \frac{\sqrt{f^{bcnx+acn}e+d}\,x}{f^{4bcnx+4acn}e^4+4f^{3bcnx+3acn}d\,e^3+6f^{2bcnx+2acn}d^2e^2+4f^{bcnx+acn}d^3e+d^4}dx\right)g$$

input
```
int((g*x+f)/(d+e*(F^((b*x+a)*c))^n)^(7/2),x)
```

output
```
int(sqrt(f**(a*c*n + b*c*n*x)*e + d)/(f**(4*a*c*n + 4*b*c*n*x)*e**4 + 4*f*
*(3*a*c*n + 3*b*c*n*x)*d*e**3 + 6*f**(2*a*c*n + 2*b*c*n*x)*d**2*e**2 + 4*f
**(a*c*n + b*c*n*x)*d**3*e + d**4),x)*f + int((sqrt(f**(a*c*n + b*c*n*x)*e
 + d)*x)/(f**(4*a*c*n + 4*b*c*n*x)*e**4 + 4*f**(3*a*c*n + 3*b*c*n*x)*d*e**
3 + 6*f**(2*a*c*n + 2*b*c*n*x)*d**2*e**2 + 4*f**(a*c*n + b*c*n*x)*d**3*e +
 d**4),x)*g
```

3.52 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx = \text{Int}\left(\left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m, x\right)$$

output
```
Defer(Int)((d+e*(F^(c*(b*x+a)))^n)^p*(g*x+f)^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx = \int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^p*(f + g*x)^m,x]
```

output
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^p*(f + g*x)^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m \left(e\left(F^{c(a+bx)}\right)^n+d\right)^p dx$$

$$\Big\downarrow \text{2619}$$

$$\int (f+gx)^m \left(e\left(F^{c(a+bx)}\right)^n+d\right)^p dx$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^p*(f + g*x)^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(d+e\left(F^{c(bx+a)}\right)^n\right)^p (gx+f)^m\, dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^p*(g*x+f)^m,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^p*(g*x+f)^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx = \int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^p (gx+f)^m \, dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^p*(g*x+f)^m,x, algorithm="fricas")
```

output
```
integral(((F^(b*c*x + a*c))^n*e + d)^p*(g*x + f)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx = \text{Timed out}$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**p*(g*x+f)**m,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m \, dx = \int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^p (gx+f)^m \, dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^p*(g*x+f)^m,x, algorithm="maxima")
```

output
```
integrate((F^((b*x + a)*c*n)*e + d)^p*(g*x + f)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^p (gx+f)^m\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^p*(g*x+f)^m,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^p*(g*x + f)^m, x)
```

Mupad [N/A]

Not integrable

Time = 23.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m\,dx=\int (f+g\,x)^m \left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^p\,dx$$

input
```
int((f + g*x)^m*(d + e*(F^(c*(a + b*x)))^n)^p,x)
```

output
```
int((f + g*x)^m*(d + e*(F^(c*(a + b*x)))^n)^p, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^p (f+gx)^m\,dx = \int (gx+f)^m \left(f^{bcnx+acn}e+d\right)^p\,dx$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^p*(g*x+f)^m,x)
```

output
```
int((f + g*x)**m*(f**(a*c*n + b*c*n*x)*e + d)**p,x)
```

3.53 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx$

Optimal result

Integrand size = 25, antiderivative size = 228

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx = \frac{d^2(f+gx)^{1+m}}{g(1+m)} + \frac{2^{-1-m}e^2F^{2c\left(a-\frac{bf}{g}\right)n-2cn(a+bx)}\left(F^{ac+bcx}\right)^{2n}(f+gx)^m\Gamma\left(1+m,-\frac{2bcn(f+gx)\log(F)}{g}\right)\left(-\frac{bcn(f+gx)\log(F)}{g}\right)^{-m}}{bcn\log(F)} + \frac{2deF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n(f+gx)^m\Gamma\left(1+m,-\frac{bcn(f+gx)\log(F)}{g}\right)\left(-\frac{bcn(f+gx)\log(F)}{g}\right)^{-m}}{bcn\log(F)}$$

output
```
d^2*(g*x+f)^(1+m)/g/(1+m)+2^(-1-m)*e^2*F^(2*c*(a-b*f/g)*n-2*c*n*(b*x+a))*(
F^(b*c*x+a*c))^(2*n)*(g*x+f)^m*GAMMA(1+m,-2*b*c*n*(g*x+f)*ln(F)/g)/b/c/n/l
n(F)/((-b*c*n*(g*x+f)*ln(F)/g)^m)+2*d*e*F^(c*(a-b*f/g)*n-c*n*(b*x+a))*(F^(
b*c*x+a*c))^n*(g*x+f)^m*GAMMA(1+m,-b*c*n*(g*x+f)*ln(F)/g)/b/c/n/ln(F)/((-b
*c*n*(g*x+f)*ln(F)/g)^m)
```

Mathematica [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx=\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^m,x]
```

output
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^m, x]
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2dx$$
$$\Big\downarrow \text{2614}$$
$$\int \left(2de(f+gx)^m\left(F^{ac+bcx}\right)^n+e^2(f+gx)^m\left(F^{ac+bcx}\right)^{2n}+d^2(f+gx)^m\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2de(f+gx)^m\left(F^{ac+bcx}\right)^nF^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)}\left(-\frac{bcn\log(F)(f+gx)}{g}\right)^{-m}\Gamma\left(m+1,-\frac{bcn(f+gx)\log(F)}{g}\right)}{bcn\log(F)}+\frac{e^2 2^{-m-1}(f+gx)^m\left(F^{ac+bcx}\right)^{2n}F^{2cn\left(a-\frac{bf}{g}\right)-2cn(a+bx)}\left(-\frac{bcn\log(F)(f+gx)}{g}\right)^{-m}\Gamma\left(m+1,-\frac{2bcn(f+gx)\log(F)}{g}\right)}{bcn\log(F)}+\frac{d^2(f+gx)^{m+1}}{g(m+1)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)^2*(f + g*x)^m,x]
```

output
```
(d^2*(f + g*x)^(1 + m))/(g*(1 + m)) + (2^(-1 - m)*e^2*F^(2*c*(a - (b*f)/g)
*n - 2*c*n*(a + b*x))*(F^(a*c + b*c*x))^(2*n)*(f + g*x)^m*Gamma[1 + m, (-2
*b*c*n*(f + g*x)*Log[F])/g])/(b*c*n*Log[F]*(-((b*c*n*(f + g*x)*Log[F])/g))
^m) + (2*d*e*F^(c*(a - (b*f)/g)*n - c*n*(a + b*x))*(F^(a*c + b*c*x))^n*(f
+ g*x)^m*Gamma[1 + m, -((b*c*n*(f + g*x)*Log[F])/g)])/(b*c*n*Log[F]*(-((b*
c*n*(f + g*x)*Log[F])/g))^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \left(d+e\left(F^{c(bx+a)}\right)^{n}\right)^{2}(gx+f)^{m}\,dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)^2*(g*x+f)^m,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)^2*(g*x+f)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.85

$$\int \left(d+e\left(F^{c(a+bx)}\right)^{n}\right)^{2}(f+gx)^{m}\,dx = \frac{4\,(degm+deg)e^{\left(-\frac{gm\log\left(-\frac{bcn\log(F)}{g}\right)+(bcf-acg)n\log(F)}{g}\right)}\Gamma\left(m+1,-\frac{(bcgnx+bcfn)\log(F)}{g}\right)+(e^{2}gm+e^{2}g)e^{\left(-\frac{gm\log}{}\right.}}{2\,(bcgm+bcg}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^m,x, algorithm="fricas")
```

output
```
1/2*(4*(d*e*g*m + d*e*g)*e^(-(g*m*log(-b*c*n*log(F)/g) + (b*c*f - a*c*g)*n
*log(F))/g)*gamma(m + 1, -(b*c*g*n*x + b*c*f*n)*log(F)/g) + (e^2*g*m + e^2
*g)*e^(-(g*m*log(-2*b*c*n*log(F)/g) + 2*(b*c*f - a*c*g)*n*log(F))/g)*gamma
(m + 1, -2*(b*c*g*n*x + b*c*f*n)*log(F)/g) + 2*(b*c*d^2*g*n*x + b*c*d^2*f*
n)*(g*x + f)^m*log(F))/((b*c*g*m + b*c*g)*n*log(F))
```

Sympy [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx = \int \left(d+e\left(F^{ac+bcx}\right)^n\right)^2 (f+gx)^m\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)**2*(g*x+f)**m,x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)**2*(f + g*x)**m, x)
```

Maxima [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx = \int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^2 (gx+f)^m\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^m,x, algorithm="maxima")
```

output
```
(g*x + f)^(m + 1)*d^2/(g*(m + 1)) + integrate((2*F^(b*c*n*x)*F^(a*c*n)*d*e
 + F^(2*b*c*n*x)*F^(2*a*c*n)*e^2)*(g*x + f)^m, x)
```

Giac [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx = \int \left(\left(F^{(bx+a)c}\right)^n e+d\right)^2 (gx+f)^m\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^m,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)^2*(g*x + f)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx = \int (f+g\,x)^m \left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2\,dx$$

input
```
int((f + g*x)^m*(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
int((f + g*x)^m*(d + e*(F^(c*(a + b*x)))^n)^2, x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)^2 (f+gx)^m\,dx$$
$$= \frac{f^{2bcnx+2acn}(gx+f)^m e^2gm + f^{2bcnx+2acn}(gx+f)^m e^2g + 4f^{bcnx+acn}(gx+f)^m degm + 4f^{bcnx+acn}(gx+}{}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)^2*(g*x+f)^m,x)
```

output
```
(f**(2*a*c*n + 2*b*c*n*x)*(f + g*x)**m*e**2*g*m + f**(2*a*c*n + 2*b*c*n*x)
*(f + g*x)**m*e**2*g + 4*f**(a*c*n + b*c*n*x)*(f + g*x)**m*d*e*g*m + 4*f**
(a*c*n + b*c*n*x)*(f + g*x)**m*d*e*g + 2*(f + g*x)**m*log(f)*b*c*d**2*f*n
+ 2*(f + g*x)**m*log(f)*b*c*d**2*g*n*x - f**(2*a*c*n)*int((f**(2*b*c*n*x)*
(f + g*x)**m)/(f + g*x),x)*e**2*g**2*m**2 - f**(2*a*c*n)*int((f**(2*b*c*n*
x)*(f + g*x)**m)/(f + g*x),x)*e**2*g**2*m - 4*f**(a*c*n)*int((f**(b*c*n*x)
*(f + g*x)**m)/(f + g*x),x)*d*e*g**2*m**2 - 4*f**(a*c*n)*int((f**(b*c*n*x)
*(f + g*x)**m)/(f + g*x),x)*d*e*g**2*m)/(2*log(f)*b*c*g*n*(m + 1))
```

3.54 $\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx$

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx=\frac{d(f+gx)^{1+m}}{g(1+m)}+\frac{eF^{c\left(a-\frac{bf}{g}\right)n-cn(a+bx)}\left(F^{ac+bcx}\right)^n(f+gx)^m\Gamma\left(1+m,-\frac{bcn(f+gx)\log(F)}{g}\right)\left(-\frac{bcn(f+gx)\log(F)}{g}\right)^{-m}}{bcn\log(F)}$$

output
```
d*(g*x+f)^(1+m)/g/(1+m)+e*F^(c*(a-b*f/g)*n-c*n*(b*x+a))*(F^(b*c*x+a*c))^n*
(g*x+f)^m*GAMMA(1+m,-b*c*n*(g*x+f)*ln(F)/g)/b/c/n/ln(F)/((-b*c*n*(g*x+f)*l
n(F)/g)^m)
```

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx=\frac{(f+gx)^m\left(\left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)-eF^{-\frac{bcn(f+gx)}{g}}\left(F^{c(a+bx)}\right)^n(f+gx)\Gamma\left(2+m,-\frac{bcn(f+gx)\log(F)}{g}\right)\right.}{g(1+m)}$$

input
```
Integrate[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^m,x]
```

output
```
((f + g*x)^m*((d + e*(F^(c*(a + b*x)))^n)*(f + g*x) - (e*(F^(c*(a + b*x)))
^n*(f + g*x)*Gamma[2 + m, -((b*c*n*(f + g*x)*Log[F])/g)]*(-((b*c*n*(f + g*
x)*Log[F])/g))^(-1 - m))/F^((b*c*n*(f + g*x))/g)))/(g*(1 + m))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (f+gx)^m \left(e\left(F^{c(a+bx)}\right)^n+d\right) dx$$

$$\Big\downarrow \text{2614}$$

$$\int \left(e(f+gx)^m \left(F^{ac+bcx}\right)^n+d(f+gx)^m\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e(f+gx)^m \left(F^{ac+bcx}\right)^n F^{cn\left(a-\frac{bf}{g}\right)-cn(a+bx)} \left(-\frac{bcn\log(F)(f+gx)}{g}\right)^{-m} \Gamma\left(m+1,-\frac{bcn(f+gx)\log(F)}{g}\right)}{bcn\log(F)}+\frac{d(f+gx)^{m+1}}{g(m+1)}$$

input
```
Int[(d + e*(F^(c*(a + b*x)))^n)*(f + g*x)^m,x]
```

output
```
(d*(f + g*x)^(1 + m))/(g*(1 + m)) + (e*F^(c*(a - (b*f)/g)*n - c*n*(a + b*x
))*(F^(a*c + b*c*x))^n*(f + g*x)^m*Gamma[1 + m, -((b*c*n*(f + g*x)*Log[F])
/g)])/(b*c*n*Log[F]*(-((b*c*n*(f + g*x)*Log[F])/g))^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \left(d+e\left(F^{c(bx+a)}\right)^{n}\right)(gx+f)^{m}\,dx$$

input
```
int((d+e*(F^(c*(b*x+a)))^n)*(g*x+f)^m,x)
```

output
```
int((d+e*(F^(c*(b*x+a)))^n)*(g*x+f)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \left(d+e\left(F^{c(a+bx)}\right)^{n}\right)(f+gx)^{m}\,dx = \frac{(egm+eg)e^{\left(-\frac{gm\log\left(-\frac{bcn\log(F)}{g}\right)+(bcf-acg)n\log(F)}{g}\right)}\Gamma\left(m+1,-\frac{(bcgnx+bcfn)\log(F)}{g}\right)+(bcdgnx+bcdfn)(gx+f}{(bcgm+bcg)n\log(F)}$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^m,x, algorithm="fricas")
```

output
```
((e*g*m + e*g)*e^(-(g*m*log(-b*c*n*log(F)/g) + (b*c*f - a*c*g)*n*log(F))/g
)*gamma(m + 1, -(b*c*g*n*x + b*c*f*n)*log(F)/g) + (b*c*d*g*n*x + b*c*d*f*n
)*(g*x + f)^m*log(F))/((b*c*g*m + b*c*g)*n*log(F))
```

Sympy [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx=\int \left(d+e\left(F^{ac+bcx}\right)^n\right)(f+gx)^m\,dx$$

input
```
integrate((d+e*(F**((b*x+a)*c))**n)*(g*x+f)**m,x)
```

output
```
Integral((d + e*(F**(a*c + b*c*x))**n)*(f + g*x)**m, x)
```

Maxima [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)(gx+f)^m\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^m,x, algorithm="maxima")
```

output
```
F^(a*c*n)*e*integrate(e^(b*c*n*x*log(F) + m*log(g*x + f)), x) + (g*x + f)^
(m + 1)*d/(g*(m + 1))
```

Giac [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx=\int \left(\left(F^{(bx+a)c}\right)^n e+d\right)(gx+f)^m\,dx$$

input
```
integrate((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^m,x, algorithm="giac")
```

output
```
integrate(((F^((b*x + a)*c))^n*e + d)*(g*x + f)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx=\int (f+g\,x)^m\,\left(d+e\,\left(F^{c\,(a+b\,x)}\right)^n\right)\,dx$$

input
```
int((f + g*x)^m*(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
int((f + g*x)^m*(d + e*(F^(c*(a + b*x)))^n), x)
```

Reduce [F]

$$\int \left(d+e\left(F^{c(a+bx)}\right)^n\right)(f+gx)^m\,dx$$
$$=\frac{f^{bcnx+acn}(gx+f)^m\,egm+f^{bcnx+acn}(gx+f)^m\,eg+(gx+f)^m\log(f)\,bcdfn+(gx+f)^m\log(f)\,bcdgnx}{\log(f)\,bcgn\,(m+1)}$$

input
```
int((d+e*(F^((b*x+a)*c))^n)*(g*x+f)^m,x)
```

output
```
(f**(a*c*n + b*c*n*x)*(f + g*x)**m*e*g*m + f**(a*c*n + b*c*n*x)*(f + g*x)*
*m*e*g + (f + g*x)**m*log(f)*b*c*d*f*n + (f + g*x)**m*log(f)*b*c*d*g*n*x -
 f**(a*c*n)*int((f**(b*c*n*x)*(f + g*x)**m)/(f + g*x),x)*e*g**2*m**2 - f**
(a*c*n)*int((f**(b*c*n*x)*(f + g*x)**m)/(f + g*x),x)*e*g**2*m)/(log(f)*b*c
*g*n*(m + 1))
```

3.55 $\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n} \, dx = \text{Int}\left(\frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}, x\right)$$

output
```
Defer(Int)((g*x+f)^m/(d+e*(F^(c*(b*x+a)))^n),x)
```

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n} \, dx = \int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n} \, dx$$

input
```
Integrate[(f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
Integrate[(f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^m}{e\left(F^{c(a+bx)}\right)^n+d}dx$$
$$\Big\downarrow \; 2619$$
$$\int \frac{(f+gx)^m}{e\left(F^{c(a+bx)}\right)^n+d}dx$$

input
```
Int[(f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^m}{d+e\left(F^{c(bx+a)}\right)^n}dx$$

input
```
int((g*x+f)^m/(d+e*(F^(c*(b*x+a)))^n),x)
```

output
```
int((g*x+f)^m/(d+e*(F^(c*(b*x+a)))^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{(gx+f)^m}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n),x, algorithm="fricas")
```

output
```
integral((g*x + f)^m/((F^(b*c*x + a*c))^n*e + d), x)
```

Sympy [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{(f+gx)^m}{d+e\left(F^{ac+bcx}\right)^n}\,dx$$

input
```
integrate((g*x+f)**m/(d+e*(F**((b*x+a)*c))**n),x)
```

output
```
Integral((f + g*x)**m/(d + e*(F**(a*c + b*c*x))**n), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(gx+f)^m}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n),x, algorithm="maxima")
```

output
```
integrate((g*x + f)^m/(F^((b*x + a)*c*n)*e + d), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(gx+f)^m}{\left(F^{(bx+a)c}\right)^n e+d}\,dx$$

input
```
integrate((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n),x, algorithm="giac")
```

output
```
integrate((g*x + f)^m/((F^((b*x + a)*c))^n*e + d), x)
```

Mupad [N/A]

Not integrable

Time = 22.83 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}\,dx = \int \frac{(f+g\,x)^m}{d+e\left(F^{c\,(a+b\,x)}\right)^n}\,dx$$

input
```
int((f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n),x)
```

output
```
int((f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(f+gx)^m}{d+e\left(F^{c(a+bx)}\right)^n}\,dx=\int \frac{(gx+f)^m}{f^{bcnx+acn}e+d}dx$$

input
```
int((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n),x)
```

output
```
int((f + g*x)**m/(f**(a*c*n + b*c*n*x)*e + d),x)
```

3.56 $\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} dx = \text{Int}\left(\frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}, x\right)$$

output
```
Defer(Int)((g*x+f)^m/(d+e*(F^(c*(b*x+a)))^n)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} dx = \int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2} dx$$

input
```
Integrate[(f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
Integrate[(f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^m}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

$$\Big\downarrow \text{2619}$$

$$\int \frac{(f+gx)^m}{\left(e\left(F^{c(a+bx)}\right)^n+d\right)^2}dx$$

input
```
Int[(f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(gx+f)^m}{\left(d+e\left(F^{c(bx+a)}\right)^n\right)^2}dx$$

input
```
int((g*x+f)^m/(d+e*(F^(c*(b*x+a)))^n)^2,x)
```

output
```
int((g*x+f)^m/(d+e*(F^(c*(b*x+a)))^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(gx+f)^m}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="fricas")
```

output
```
integral((g*x + f)^m/(2*(F^(b*c*x + a*c))^n*d*e + (F^(b*c*x + a*c))^(2*n)*
e^2 + d^2), x)
```

Sympy [N/A]

Not integrable

Time = 37.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx=\int \frac{(f+gx)^m}{\left(d+e\left(F^{ac+bcx}\right)^n\right)^2}\,dx$$

input
```
integrate((g*x+f)**m/(d+e*(F**((b*x+a)*c))**n)**2,x)
```

output
```
Integral((f + g*x)**m/(d + e*(F**(a*c + b*c*x))**n)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \int \frac{(gx+f)^m}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="maxima")
```

output
```
integrate((g*x + f)^m/(F^((b*x + a)*c*n)*e + d)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \int \frac{(gx+f)^m}{\left(\left(F^{(bx+a)c}\right)^n e+d\right)^2}\,dx$$

input
```
integrate((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n)^2,x, algorithm="giac")
```

output
```
integrate((g*x + f)^m/((F^((b*x + a)*c))^n*e + d)^2, x)
```

Mupad [N/A]

Not integrable

Time = 22.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \int \frac{(f+g\,x)^m}{\left(d+e\left(F^{c\,(a+b\,x)}\right)^n\right)^2}\,dx$$

input
```
int((f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n)^2,x)
```

output
```
int((f + g*x)^m/(d + e*(F^(c*(a + b*x)))^n)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{(f+gx)^m}{\left(d+e\left(F^{c(a+bx)}\right)^n\right)^2}\,dx = \int \frac{(gx+f)^m}{f^{2bcnx+2acn}e^2+2f^{bcnx+acn}de+d^2}dx$$

input
```
int((g*x+f)^m/(d+e*(F^((b*x+a)*c))^n)^2,x)
```

output
```
int((f + g*x)**m/(f**(2*a*c*n + 2*b*c*n*x)*e**2 + 2*f**(a*c*n + b*c*n*x)*d
*e + d**2),x)
```

3.57 $\int \frac{e^x}{4+6e^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^x}{4+6e^x} \, dx = \frac{1}{6} \log (2+3e^x)$$

output
```
1/6*ln(2+3*exp(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{4+6e^x} \, dx = \frac{1}{6} \log (4+6e^x)$$

input
```
Integrate[E^x/(4 + 6*E^x),x]
```

output
```
Log[4 + 6*E^x]/6
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{6e^x+4}dx$$
$$\Big\downarrow \text{2676}$$
$$\int \frac{1}{6e^x+4}de^x$$
$$\Big\downarrow \text{16}$$
$$\frac{1}{6}\log\left(3e^x+2\right)$$

input
```
Int[E^x/(4 + 6*E^x),x]
```

output
```
Log[2 + 3*E^x]/6
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

method	result	size
risch	$\frac{\ln\left(\frac{2}{3}+e^{x}\right)}{6}$	8
parallelrisch	$\frac{\ln\left(\frac{2}{3}+e^{x}\right)}{6}$	8
derivativedivides	$\frac{\ln(2+3\,e^{x})}{6}$	10
default	$\frac{\ln(2+3\,e^{x})}{6}$	10
norman	$\frac{\ln(4+6\,e^{x})}{6}$	10

input
```
int(exp(x)/(4+6*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(2/3+exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{4+6e^x}\,dx = \frac{1}{6}\,\log\left(3\,e^{x}+2\right)$$

input
```
integrate(exp(x)/(4+6*exp(x)),x, algorithm="fricas")
```

output
```
1/6*log(3*e^x + 2)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{4+6e^x}\,dx = \frac{\log\left(e^x+\frac{2}{3}\right)}{6}$$

input
```
integrate(exp(x)/(4+6*exp(x)),x)
```

output
```
log(exp(x) + 2/3)/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{4+6e^x}\,dx = \frac{1}{6}\,\log\left(3\,e^x+2\right)$$

input
```
integrate(exp(x)/(4+6*exp(x)),x, algorithm="maxima")
```

output
```
1/6*log(3*e^x + 2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{4+6e^x}\,dx = \frac{1}{6}\,\log\left(3\,e^x+2\right)$$

input
```
integrate(exp(x)/(4+6*exp(x)),x, algorithm="giac")
```

output
```
1/6*log(3*e^x + 2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{4+6e^x}\,dx = \frac{\ln\left(3\,\mathrm{e}^x+2\right)}{6}$$

input
```
int(exp(x)/(6*exp(x) + 4),x)
```

output
```
log(3*exp(x) + 2)/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{e^x}{4+6e^x}\,dx = \frac{\log(3e^x+2)}{6}$$

input
```
int(exp(x)/(4+6*exp(x)),x)
```

output
```
log(3*e**x + 2)/6
```

3.58 $\int \frac{e^x}{a+be^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{e^x}{a+be^x} \, dx = \frac{\log(a+be^x)}{b}$$

output
```
ln(a+b*exp(x))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{a+be^x} \, dx = \frac{\log(a+be^x)}{b}$$

input
```
Integrate[E^x/(a + b*E^x),x]
```

output
```
Log[a + b*E^x]/b
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{a+be^x} \, dx$$
$$\Big\downarrow \text{2676}$$
$$\int \frac{1}{a+be^x}de^x$$
$$\Big\downarrow \text{16}$$
$$\frac{\log(a+be^x)}{b}$$

input
```
Int[E^x/(a + b*E^x),x]
```

output
```
Log[a + b*E^x]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln(a+b\,\mathrm{e}^{x})}{b}$	12
default	$\frac{\ln(a+b\,\mathrm{e}^{x})}{b}$	12
norman	$\frac{\ln(a+b\,\mathrm{e}^{x})}{b}$	12
parallelrisch	$\frac{\ln(a+b\,\mathrm{e}^{x})}{b}$	12
risch	$\frac{\ln\left(\mathrm{e}^{x}+\frac{a}{b}\right)}{b}$	14

input
```
int(exp(x)/(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*exp(x))/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^x}{a+be^x}\,dx=\frac{\log\left(be^x+a\right)}{b}$$

input
```
integrate(exp(x)/(a+b*exp(x)),x, algorithm="fricas")
```

output
```
log(b*e^x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{e^x}{a+be^x}\,dx=\frac{\log\left(\frac{a}{b}+e^x\right)}{b}$$

input
```
integrate(exp(x)/(a+b*exp(x)),x)
```

output
```
log(a/b + exp(x))/b
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^x}{a+be^x}\,dx=\frac{\log\left(be^x+a\right)}{b}$$

input
```
integrate(exp(x)/(a+b*exp(x)),x, algorithm="maxima")
```

output
```
log(b*e^x + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{a+be^x}\,dx=\frac{\log\left(|be^x+a|\right)}{b}$$

input
```
integrate(exp(x)/(a+b*exp(x)),x, algorithm="giac")
```

output
```
log(abs(b*e^x + a))/b
```

Mupad [B] (verification not implemented)

Time = 22.90 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{e^x}{a+be^x}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^x\right)}{b}$$

input
```
int(exp(x)/(a + b*exp(x)),x)
```

output
```
log(a + b*exp(x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{a+be^x}\,dx = \frac{\log(e^x b+a)}{b}$$

input
```
int(exp(x)/(a+b*exp(x)),x)
```

output
```
log(e**x*b + a)/b
```

3.59 $\int \frac{e^{dx}}{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 24

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx=\frac{e^{-c}\log\left(a+be^{c+dx}\right)}{bd}$$

output
```
ln(a+b*exp(d*x+c))/b/d/exp(c)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx=\frac{e^{-c}\log\left(a+be^{c+dx}\right)}{bd}$$

input
```
Integrate[E^(d*x)/(a + b*E^(c + d*x)),x]
```

output
```
Log[a + b*E^(c + d*x)]/(b*d*E^c)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2677, 2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{dx}}{a+be^{c+dx}}dx$$
$$\Big\downarrow \text{2677}$$
$$e^{-c}\int \frac{e^{c+dx}}{a+be^{c+dx}}dx$$
$$\Big\downarrow \text{2676}$$
$$\frac{e^{-c}\int \frac{1}{a+be^{c+dx}}de^{c+dx}}{d}$$
$$\Big\downarrow \text{16}$$
$$\frac{e^{-c}\log\left(a+be^{c+dx}\right)}{bd}$$

input
```
Int[E^(d*x)/(a + b*E^(c + d*x)),x]
```

output
```
Log[a + b*E^(c + d*x)]/(b*d*E^c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln\left(a+b\,{\mathrm e}^{d x} {\mathrm e}^{c}\right) {\mathrm e}^{-c}}{d b}$	23
norman	$\frac{\ln\left(a+b\,{\mathrm e}^{d x} {\mathrm e}^{c}\right) {\mathrm e}^{-c}}{d b}$	23
risch	$\frac{{\mathrm e}^{-c} \ln\left({\mathrm e}^{d x}+\frac{{\mathrm e}^{-c} a}{b}\right)}{b d}$	27

input
```
int(exp(d*x)/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*ln(a+b*exp(d*x)*exp(c))/exp(c)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx=\frac{e^{(-c)}\log\left(be^{(dx+c)}+a\right)}{bd}$$

input
```
integrate(exp(d*x)/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

output
```
e^(-c)*log(b*e^(d*x + c) + a)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx=\frac{e^{-c}\log\left(\frac{ae^{-c}}{b}+e^{dx}\right)}{bd}$$

input
```
integrate(exp(d*x)/(a+b*exp(d*x+c)),x)
```

output
```
exp(-c)*log(a*exp(-c)/b + exp(d*x))/(b*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx=\frac{e^{(-c)}\log\left(be^{(dx+c)}+a\right)}{bd}$$

input
```
integrate(exp(d*x)/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

output
```
e^(-c)*log(b*e^(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx=\frac{e^{(-c)}\log\left(\left|be^{(dx+c)}+a\right|\right)}{bd}$$

input
```
integrate(exp(d*x)/(a+b*exp(d*x+c)),x, algorithm="giac")
```

output
```
e^(-c)*log(abs(b*e^(d*x + c) + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^{c+d\,x}\right)\,\mathrm{e}^{-c}}{b\,d}$$

input
```
int(exp(d*x)/(a + b*exp(c + d*x)),x)
```

output
```
(log(a + b*exp(c + d*x))*exp(-c))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{e^{dx}}{a+be^{c+dx}}\,dx = \frac{\log\left(e^{dx+c}b+a\right)}{e^{c}bd}$$

input
```
int(exp(d*x)/(a+b*exp(d*x+c)),x)
```

output
```
log(e**(c + d*x)*b + a)/(e**c*b*d)
```

3.60 $\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 19

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(a+be^{c+dx}\right)}{bd}$$

output
```
ln(a+b*exp(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(a+be^{c+dx}\right)}{bd}$$

input
```
Integrate[E^(c + d*x)/(a + b*E^(c + d*x)),x]
```

output
```
Log[a + b*E^(c + d*x)]/(b*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}dx$$

$$\Big\downarrow \text{ 2676}$$

$$\frac{\int \frac{1}{a+be^{c+dx}}de^{c+dx}}{d}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log\left(a+be^{c+dx}\right)}{bd}$$

input
```
Int[E^(c + d*x)/(a + b*E^(c + d*x)),x]
```

output
```
Log[a + b*E^(c + d*x)]/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln\left(a+b\,{\mathrm e}^{d x+c}\right)}{b d}$	19
default	$\frac{\ln\left(a+b\,{\mathrm e}^{d x+c}\right)}{b d}$	19
norman	$\frac{\ln\left(a+b\,{\mathrm e}^{d x+c}\right)}{b d}$	19
parallelrisch	$\frac{\ln\left(a+b\,{\mathrm e}^{d x+c}\right)}{b d}$	19
risch	$-\frac{c}{b d}+\frac{\ln\left({\mathrm e}^{d x+c}+\frac{a}{b}\right)}{b d}$	31

input
```
int(exp(d*x+c)/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*exp(d*x+c))/b/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(be^{(dx+c)}+a\right)}{bd}$$

input
```
integrate(exp(d*x+c)/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

output
```
log(b*e^(d*x + c) + a)/(b*d)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(\frac{a}{b}+e^{c+dx}\right)}{bd}$$

input
```
integrate(exp(d*x+c)/(a+b*exp(d*x+c)),x)
```

output
```
log(a/b + exp(c + d*x))/(b*d)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(be^{(dx+c)}+a\right)}{bd}$$

input
```
integrate(exp(d*x+c)/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*e^(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(\left|be^{(dx+c)}+a\right|\right)}{bd}$$

input
```
integrate(exp(d*x+c)/(a+b*exp(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*e^(d*x + c) + a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\ln\left(a+b\,\mathrm{e}^{c+d\,x}\right)}{b\,d}$$

input
```
int(exp(c + d*x)/(a + b*exp(c + d*x)),x)
```

output
```
log(a + b*exp(c + d*x))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{e^{c+dx}}{a+be^{c+dx}}\,dx=\frac{\log\left(e^{dx+c}b+a\right)}{bd}$$

input
```
int(exp(d*x+c)/(a+b*exp(d*x+c)),x)
```

output
```
log(e**(c + d*x)*b + a)/(b*d)
```

3.61 $\int e^x(a+be^x)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int e^x(a+be^x)^p \, dx = \frac{(a+be^x)^{1+p}}{b(1+p)}$$

output
```
(a+b*exp(x))^(p+1)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^x(a+be^x)^p \, dx = \frac{(a+be^x)^{1+p}}{b+bp}$$

input
```
Integrate[E^x*(a + b*E^x)^p,x]
```

output
```
(a + b*E^x)^(1 + p)/(b + b*p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x (a+be^x)^p \, dx$$

$$\Big\downarrow \text{2676}$$

$$\int (a+be^x)^p de^x$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+be^x)^{p+1}}{b(p+1)}$$

input
```
Int[E^x*(a + b*E^x)^p,x]
```

output
```
(a + b*E^x)^(1 + p)/(b*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{(a+b\,\mathrm{e}^{x})^{p+1}}{b(p+1)}$	20
default	$\frac{(a+b\,\mathrm{e}^{x})^{p+1}}{b(p+1)}$	20
risch	$\frac{(a+b\,\mathrm{e}^{x})(a+b\,\mathrm{e}^{x})^{p}}{b(p+1)}$	24
parallelrisch	$\frac{\mathrm{e}^{x}(a+b\,\mathrm{e}^{x})^{p}b+(a+b\,\mathrm{e}^{x})^{p}a}{b(p+1)}$	33
norman	$\frac{\mathrm{e}^{x}\mathrm{e}^{p\ln(a+b\,\mathrm{e}^{x})}}{p+1}+\frac{a\,\mathrm{e}^{p\ln(a+b\,\mathrm{e}^{x})}}{b(p+1)}$	40

input
```
int(exp(x)*(a+b*exp(x))^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*exp(x))^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int e^x(a+be^x)^p\,dx=\frac{(be^x+a)(be^x+a)^p}{bp+b}$$

input
```
integrate(exp(x)*(a+b*exp(x))^p,x, algorithm="fricas")
```

output
```
(b*e^x + a)*(b*e^x + a)^p/(b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(14) = 28$.

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.80

$$\int e^x(a+be^x)^p\,dx = \begin{cases} \frac{e^x}{a} & \text{for } b=0 \wedge p=-1 \\ a^pe^x & \text{for } b=0 \\ \frac{\log\left(\frac{a}{b}+e^x\right)}{b} & \text{for } p=-1 \\ \frac{a(a+be^x)^p}{bp+b} + \frac{b(a+be^x)^pe^x}{bp+b} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)*(a+b*exp(x))**p,x)
```

output
```
Piecewise((exp(x)/a, Eq(b, 0) & Eq(p, -1)), (a**p*exp(x), Eq(b, 0)), (log(
a/b + exp(x))/b, Eq(p, -1)), (a*(a + b*exp(x))**p/(b*p + b) + b*(a + b*exp
(x))**p*exp(x)/(b*p + b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^x(a+be^x)^p\,dx = \frac{(be^x+a)^{p+1}}{b(p+1)}$$

input
```
integrate(exp(x)*(a+b*exp(x))^p,x, algorithm="maxima")
```

output
```
(b*e^x + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^x(a+be^x)^p\,dx=\frac{(be^x+a)^{p+1}}{b(p+1)}$$

input
```
integrate(exp(x)*(a+b*exp(x))^p,x, algorithm="giac")
```

output
```
(b*e^x + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 23.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^x(a+be^x)^p\,dx=\frac{(a+b\,\mathrm{e}^x)^{p+1}}{b\,(p+1)}$$

input
```
int(exp(x)*(a + b*exp(x))^p,x)
```

output
```
(a + b*exp(x))^(p + 1)/(b*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int e^x(a+be^x)^p\,dx=\frac{(e^xb+a)^p\,(e^xb+a)}{b\,(p+1)}$$

input
```
int(exp(x)*(a+b*exp(x))^p,x)
```

output
```
((e**x*b + a)**p*(e**x*b + a))/(b*(p + 1))
```

3.62 $\int e^{dx}\left(a+be^{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 32

$$\int e^{dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{e^{-c}\left(a+be^{c+dx}\right)^{1+p}}{bd(1+p)}$$

output
```
(a+b*exp(d*x+c))^(p+1)/b/d/exp(c)/(p+1)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int e^{dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{e^{-c}\left(a+be^{c+dx}\right)^{1+p}}{bd+bdp}$$

input
```
Integrate[E^(d*x)*(a + b*E^(c + d*x))^p,x]
```

output
```
(a + b*E^(c + d*x))^(1 + p)/(E^c*(b*d + b*d*p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2677, 2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{dx}\left(a+be^{c+dx}\right)^p dx$$

$$\Big\downarrow \text{ 2677}$$

$$e^{-c}\int e^{c+dx}\left(a+be^{c+dx}\right)^p dx$$

$$\Big\downarrow \text{ 2676}$$

$$\frac{e^{-c}\int \left(a+be^{c+dx}\right)^p de^{c+dx}}{d}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{e^{-c}\left(a+be^{c+dx}\right)^{p+1}}{bd(p+1)}$$

input
```
Int[E^(d*x)*(a + b*E^(c + d*x))^p,x]
```

output
```
(a + b*E^(c + d*x))^(1 + p)/(b*d*E^c*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\left(a+b\,\mathrm{e}^{dx}\mathrm{e}^{c}\right)^{p+1}\mathrm{e}^{-c}}{db(p+1)}$	31
risch	$\frac{\left(a+b\,\mathrm{e}^{dx+c}\right)\mathrm{e}^{-c}\left(a+b\,\mathrm{e}^{dx+c}\right)^{p}}{bd(p+1)}$	39
norman	$\frac{\mathrm{e}^{dx}\mathrm{e}^{p\ln\left(a+b\,\mathrm{e}^{dx}\mathrm{e}^{c}\right)}}{d(p+1)}+\frac{\mathrm{e}^{-c}a\,\mathrm{e}^{p\ln\left(a+b\,\mathrm{e}^{dx}\mathrm{e}^{c}\right)}}{bd(p+1)}$	60

input
```
int(exp(d*x)*(a+b*exp(d*x+c))^p,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a+b*exp(d*x)*exp(c))^(p+1)/exp(c)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int e^{dx}\left(a+be^{c+dx}\right)^{p}\,dx=\frac{\left(be^{(dx)}+ae^{(-c)}\right)\left(be^{(dx+c)}+a\right)^{p}}{bdp+bd}$$

input
```
integrate(exp(d*x)*(a+b*exp(d*x+c))^p,x, algorithm="fricas")
```

output
```
(b*e^(d*x) + a*e^(-c))*(b*e^(d*x + c) + a)^p/(b*d*p + b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.

Time = 2.39 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

$$\int e^{dx}\left(a+be^{c+dx}\right)^p\,dx=\begin{cases}\frac{x}{a} & \text{for } b=0\wedge d=0\wedge p=-1\\ \frac{a^pe^{dx}}{d} & \text{for } b=0\\ x(a+be^c)^p & \text{for } d=0\\ \frac{e^{-c}\log\left(\frac{a}{b}+e^ce^{dx}\right)}{bd} & \text{for } p=-1\\ \frac{a(a+be^ce^{dx})^p}{bdpe^c+bde^c}+\frac{b(a+be^ce^{dx})^pe^ce^{dx}}{bdpe^c+bde^c} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(d*x)*(a+b*exp(d*x+c))**p,x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(d, 0) & Eq(p, -1)), (a**p*exp(d*x)/d, Eq(b,
0)), (x*(a + b*exp(c))**p, Eq(d, 0)), (exp(-c)*log(a/b + exp(c)*exp(d*x))/
(b*d), Eq(p, -1)), (a*(a + b*exp(c)*exp(d*x))**p/(b*d*p*exp(c) + b*d*exp(c
)) + b*(a + b*exp(c)*exp(d*x))**p*exp(c)*exp(d*x)/(b*d*p*exp(c) + b*d*exp(
c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int e^{dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(be^{(dx+c)}+a\right)^{p+1}e^{(-c)}}{bd(p+1)}$$

input
```
integrate(exp(d*x)*(a+b*exp(d*x+c))^p,x, algorithm="maxima")
```

output
```
(b*e^(d*x + c) + a)^(p + 1)*e^(-c)/(b*d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int e^{dx} \left(a + b e^{c+dx}\right)^p \, dx = \frac{\left(b e^{(dx+c)} + a\right)^{p+1} e^{(-c)}}{bd(p+1)}$$

input
```
integrate(exp(d*x)*(a+b*exp(d*x+c))^p,x, algorithm="giac")
```

output
```
(b*e^(d*x + c) + a)^(p + 1)*e^(-c)/(b*d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 23.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int e^{dx} \left(a + b e^{c+dx}\right)^p \, dx = \left(a + b \, \mathrm{e}^{c+d\,x}\right)^p \left(\frac{\mathrm{e}^{d\,x}}{d\,(p+1)} + \frac{a\,\mathrm{e}^{-c}}{b\,d\,(p+1)}\right)$$

input
```
int(exp(d*x)*(a + b*exp(c + d*x))^p,x)
```

output
```
(a + b*exp(c + d*x))^p*(exp(d*x)/(d*(p + 1)) + (a*exp(-c))/(b*d*(p + 1)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int e^{dx} \left(a + b e^{c+dx}\right)^p \, dx = \frac{\left(e^{dx+c} b + a\right)^p \left(e^{dx+c} b + a\right)}{e^c b d\,(p+1)}$$

input
```
int(exp(d*x)*(a+b*exp(d*x+c))^p,x)
```

output
```
((e**(c + d*x)*b + a)**p*(e**(c + d*x)*b + a))/(e**c*b*d*(p + 1))
```

3.63 $\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(a+be^{c+dx}\right)^{1+p}}{bd(1+p)}$$

output
```
(a+b*exp(d*x+c))^(p+1)/b/d/(p+1)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(a+be^{c+dx}\right)^{1+p}}{bd+bdp}$$

input
```
Integrate[E^(c + d*x)*(a + b*E^(c + d*x))^p,x]
```

output
```
(a + b*E^(c + d*x))^(1 + p)/(b*d + b*d*p)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{\int \left(a+be^{c+dx}\right)^p de^{c+dx}}{d}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+be^{c+dx}\right)^{p+1}}{bd(p+1)}$$

input
```
Int[E^(c + d*x)*(a + b*E^(c + d*x))^p,x]
```

output
```
(a + b*E^(c + d*x))^(1 + p)/(b*d*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{(a+b\,\mathrm{e}^{dx+c})^{p+1}}{bd(p+1)}$	27
default	$\frac{(a+b\,\mathrm{e}^{dx+c})^{p+1}}{bd(p+1)}$	27
risch	$\frac{(a+b\,\mathrm{e}^{dx+c})(a+b\,\mathrm{e}^{dx+c})^{p}}{bd(p+1)}$	35
parallelrisch	$\frac{\mathrm{e}^{dx+c}(a+b\,\mathrm{e}^{dx+c})^{p}b+(a+b\,\mathrm{e}^{dx+c})^{p}a}{bd(p+1)}$	48
norman	$\frac{\mathrm{e}^{dx+c}\mathrm{e}^{p\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}}{d(p+1)}+\frac{a\,\mathrm{e}^{p\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}}{bd(p+1)}$	58

input
```
int(exp(d*x+c)*(a+b*exp(d*x+c))^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*exp(d*x+c))^(p+1)/b/d/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.22

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(be^{(dx+c)}+a\right)\left(be^{(dx+c)}+a\right)^p}{bdp+bd}$$

input
```
integrate(exp(d*x+c)*(a+b*exp(d*x+c))^p,x, algorithm="fricas")
```

output
```
(b*e^(d*x + c) + a)*(b*e^(d*x + c) + a)^p/(b*d*p + b*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(19) = 38$.

Time = 5.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.96

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\begin{cases}\frac{xe^c}{a} & \text{for } b=0\wedge d=0\wedge p=-1\\ \frac{a^pe^ce^{dx}}{d} & \text{for } b=0\\ x(a+be^c)^p\,e^c & \text{for } d=0\\ \frac{\log\left(\frac{ae^{-c}}{b}+e^{dx}\right)}{bd} & \text{for } p=-1\\ \frac{a(a+be^ce^{dx})^p}{bdp+bd}+\frac{b(a+be^ce^{dx})^pe^ce^{dx}}{bdp+bd} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(d*x+c)*(a+b*exp(d*x+c))**p,x)
```

output
```
Piecewise((x*exp(c)/a, Eq(b, 0) & Eq(d, 0) & Eq(p, -1)), (a**p*exp(c)*exp(
d*x)/d, Eq(b, 0)), (x*(a + b*exp(c))**p*exp(c), Eq(d, 0)), (log(a*exp(-c)/
b + exp(d*x))/(b*d), Eq(p, -1)), (a*(a + b*exp(c)*exp(d*x))**p/(b*d*p + b*
d) + b*(a + b*exp(c)*exp(d*x))**p*exp(c)*exp(d*x)/(b*d*p + b*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(be^{(dx+c)}+a\right)^{p+1}}{bd(p+1)}$$

input
```
integrate(exp(d*x+c)*(a+b*exp(d*x+c))^p,x, algorithm="maxima")
```

output
```
(b*e^(d*x + c) + a)^(p + 1)/(b*d*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(be^{(dx+c)}+a\right)^{p+1}}{bd(p+1)}$$

input
```
integrate(exp(d*x+c)*(a+b*exp(d*x+c))^p,x, algorithm="giac")
```

output
```
(b*e^(d*x + c) + a)^(p + 1)/(b*d*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(a+b\,\mathrm{e}^{c+d\,x}\right)^{p+1}}{b\,d\,\left(p+1\right)}$$

input
```
int(exp(c + d*x)*(a + b*exp(c + d*x))^p,x)
```

output
```
(a + b*exp(c + d*x))^(p + 1)/(b*d*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int e^{c+dx}\left(a+be^{c+dx}\right)^p\,dx=\frac{\left(e^{dx+c}b+a\right)^p\left(e^{dx+c}b+a\right)}{bd\left(p+1\right)}$$

input
```
int(exp(d*x+c)*(a+b*exp(d*x+c))^p,x)
```

output
```
((e**(c + d*x)*b + a)**p*(e**(c + d*x)*b + a))/(b*d*(p + 1))
```

3.64 $\int \frac{F^x}{a+bF^x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{F^x}{a+bF^x} \, dx = \frac{\log(a+bF^x)}{b\log(F)}$$

output
```
ln(a+b*F^x)/b/ln(F)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{F^x}{a+bF^x} \, dx = \frac{\log(a+bF^x)}{b\log(F)}$$

input
```
Integrate[F^x/(a + b*F^x),x]
```

output
```
Log[a + b*F^x]/(b*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^x}{a+bF^x} \, dx$$

$$\Big\downarrow \; 2676$$

$$\frac{\int \frac{1}{bF^x+a}dF^x}{\log(F)}$$

$$\Big\downarrow \; 16$$

$$\frac{\log\left(a+bF^x\right)}{b\log(F)}$$

input
```
Int[F^x/(a + b*F^x),x]
```

output
```
Log[a + b*F^x]/(b*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\,F^{x})}{b\ln(F)}$	17
default	$\frac{\ln(a+b\,F^{x})}{b\ln(F)}$	17
parallelrisch	$\frac{\ln(a+b\,F^{x})}{b\ln(F)}$	17
norman	$\frac{\ln\left(a+b\,\mathrm{e}^{x\ln(F)}\right)}{b\ln(F)}$	19
risch	$\frac{\ln\left(F^{x}+\frac{a}{b}\right)}{b\ln(F)}$	19

input
```
int(F^x/(a+b*F^x),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*F^x)/b/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{F^x}{a+bF^x}\,dx = \frac{\log\left(F^x b+a\right)}{b\log\left(F\right)}$$

input
```
integrate(F^x/(a+b*F^x),x, algorithm="fricas")
```

output
```
log(F^x*b + a)/(b*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{F^x}{a+bF^x}\,dx = \frac{\log\left(F^x+\frac{a}{b}\right)}{b\log(F)}$$

input
```
integrate(F**x/(a+b*F**x),x)
```

output
```
log(F**x + a/b)/(b*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{F^x}{a+bF^x}\,dx = \frac{\log(F^x b+a)}{b\log(F)}$$

input
```
integrate(F^x/(a+b*F^x),x, algorithm="maxima")
```

output
```
log(F^x*b + a)/(b*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{F^x}{a+bF^x}\,dx = \frac{\log(|F^x b+a|)}{b\log(F)}$$

input
```
integrate(F^x/(a+b*F^x),x, algorithm="giac")
```

output
```
log(abs(F^x*b + a))/(b*log(F))
```

Mupad [B] (verification not implemented)

Time = 22.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{F^x}{a + bF^x} \, dx = \frac{\ln (a + F^x \, b)}{b \, \ln (F)}$$

input
```
int(F^x/(a + F^x*b),x)
```

output
```
log(a + F^x*b)/(b*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{F^x}{a + bF^x} \, dx = \frac{\log(f^x b + a)}{\log (f) \, b}$$

input
```
int(F^x/(a+b*F^x),x)
```

output
```
log(f**x*b + a)/(log(f)*b)
```

3.65 $\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{F^{-c}\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

output
```
ln(a+b*F^(d*x+c))/b/d/(F^c)/ln(F)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{F^{-c}\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

input
```
Integrate[F^(d*x)/(a + b*F^(c + d*x)),x]
```

output
```
Log[a + b*F^(c + d*x)]/(b*d*F^c*Log[F])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2677, 2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{dx}}{a+bF^{c+dx}}dx$$
$$\Big\downarrow \text{2677}$$
$$F^{-c}\int \frac{F^{c+dx}}{bF^{c+dx}+a}dx$$
$$\Big\downarrow \text{2676}$$
$$\frac{F^{-c}\int \frac{1}{bF^{c+dx}+a}dF^{c+dx}}{d\log(F)}$$
$$\Big\downarrow \text{16}$$
$$\frac{F^{-c}\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

input
```
Int[F^(d*x)/(a + b*F^(c + d*x)),x]
```

output
```
Log[a + b*F^(c + d*x)]/(b*d*F^c*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

method	result	size
norman	$\frac{F^{-c}\ln\left(a+b\,\mathrm{e}^{\ln(F)c}\mathrm{e}^{dx\ln(F)}\right)}{b\ln(F)d}$	33
risch	$\frac{F^{-c}\ln\left(F^{dx}+\frac{F^{-c}a}{b}\right)}{\ln(F)db}$	34

input
```
int(F^(d*x)/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/(F^c)/b/ln(F)/d*ln(a+b*exp(ln(F)*c)*exp(d*x*ln(F)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(F^{dx+c}b+a\right)}{F^{c}bd\log\left(F\right)}$$

input
```
integrate(F^(d*x)/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

output
```
log(F^(d*x + c)*b + a)/(F^c*b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{e^{-c\log(F)}\log\left(F^{c+dx}+\frac{a}{b}\right)}{bd\log(F)}$$

input
```
integrate(F**(d*x)/(a+b*F**(d*x+c)),x)
```

output
```
exp(-c*log(F))*log(F**(c + d*x) + a/b)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(F^{dx+c}b+a\right)}{F^{c}bd\log(F)}$$

input
```
integrate(F^(d*x)/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

output
```
log(F^(d*x + c)*b + a)/(F^c*b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(\left|F^{dx}F^{c}b+a\right|\right)}{F^{c}bd\log(F)}$$

input
```
integrate(F^(d*x)/(a+b*F^(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(F^(d*x)*F^c*b + a))/(F^c*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{\ln\left(a+F^{c+d\,x}\,b\right)}{F^c\,b\,d\,\ln\left(F\right)}$$

input
```
int(F^(d*x)/(a + F^(c + d*x)*b),x)
```

output
```
log(a + F^(c + d*x)*b)/(F^c*b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(f^{dx+c}b+a\right)}{f^c\log\left(f\right)bd}$$

input
```
int(F^(d*x)/(a+b*F^(d*x+c)),x)
```

output
```
log(f**(c + d*x)*b + a)/(f**c*log(f)*b*d)
```

3.66 $\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

output
```
ln(a+b*F^(d*x+c))/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx=\frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

input
```
Integrate[F^(c + d*x)/(a + b*F^(c + d*x)),x]
```

output
```
Log[a + b*F^(c + d*x)]/(b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}dx$$

$$\Big\downarrow \text{ 2676}$$

$$\frac{\int \frac{1}{bF^{c+dx}+a}dF^{c+dx}}{d\log(F)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

input
```
Int[F^(c + d*x)/(a + b*F^(c + d*x)),x]
```

output
```
Log[a + b*F^(c + d*x)]/(b*d*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\ln\left(a+b\,F^{d x+c}\right)}{b d \ln\left(F\right)}$	24
default	$\frac{\ln\left(a+b\,F^{d x+c}\right)}{b d \ln\left(F\right)}$	24
parallelrisch	$\frac{\ln\left(a+b\,F^{d x+c}\right)}{b d \ln\left(F\right)}$	24
norman	$\frac{\ln\left(a+b\,{\mathrm e}^{\left(d x+c\right) \ln\left(F\right)}\right)}{b d \ln\left(F\right)}$	26
risch	$-\frac{c}{b d}+\frac{\ln\left(F^{d x+c}+\frac{a}{b}\right)}{b d \ln\left(F\right)}$	36

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*F^(d*x+c))/b/d/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{dx+c}b+a\right)}{bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

output
```
log(F^(d*x + c)*b + a)/(b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{bd\log\left(F\right)}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c)),x)
```

output
```
log(F**(c + d*x) + a/b)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{dx+c}b+a\right)}{bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

output
```
log(F^(d*x + c)*b + a)/(b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(\left|F^{dx+c}b+a\right|\right)}{bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(F^(d*x + c)*b + a))/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\ln\left(a+F^{c+d\,x}\,b\right)}{b\,d\,\ln\left(F\right)}$$

input
```
int(F^(c + d*x)/(a + F^(c + d*x)*b),x)
```

output
```
log(a + F^(c + d*x)*b)/(b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(f^{dx+c}b+a\right)}{\log\left(f\right)bd}$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c)),x)
```

output
```
log(f**(c + d*x)*b + a)/(log(f)*b*d)
```

3.67 $\int F^x(a+bF^x)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int F^x(a+bF^x)^p \, dx = \frac{(a+bF^x)^{1+p}}{b(1+p)\log(F)}$$

output
```
(a+b*F^x)^(p+1)/b/(p+1)/ln(F)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int F^x(a+bF^x)^p \, dx = \frac{(a+bF^x)^{1+p}}{b\log(F)+bp\log(F)}$$

input
```
Integrate[F^x*(a + b*F^x)^p,x]
```

output
```
(a + b*F^x)^(1 + p)/(b*Log[F] + b*p*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^x (a+bF^x)^p \, dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{\int (bF^x+a)^p \, dF^x}{\log(F)}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+bF^x)^{p+1}}{b(p+1)\log(F)}$$

input
```
Int[F^x*(a + b*F^x)^p,x]
```

output
```
(a + b*F^x)^(1 + p)/(b*(1 + p)*Log[F])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{(a+b\,F^{x})^{p+1}}{b(p+1)\ln(F)}$	25
default	$\frac{(a+b\,F^{x})^{p+1}}{b(p+1)\ln(F)}$	25
risch	$\frac{(a+b\,F^{x})(a+b\,F^{x})^{p}}{\ln(F)b(p+1)}$	30
parallelrisch	$\frac{F^{x}(a+b\,F^{x})^{p}b+(a+b\,F^{x})^{p}a}{\ln(F)b(p+1)}$	40
norman	$\frac{\mathrm{e}^{x\ln(F)}\mathrm{e}^{p\ln\left(a+b\,\mathrm{e}^{x\ln(F)}\right)}}{\ln(F)(p+1)}+\frac{a\,\mathrm{e}^{p\ln\left(a+b\,\mathrm{e}^{x\ln(F)}\right)}}{\ln(F)b(p+1)}$	57

input
```
int(F^x*(a+b*F^x)^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*F^x)^(p+1)/b/(p+1)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int F^{x}(a+bF^{x})^{p}\,dx=\frac{(F^{x}b+a)(F^{x}b+a)^{p}}{(bp+b)\log(F)}$$

input
```
integrate(F^x*(a+b*F^x)^p,x, algorithm="fricas")
```

output
```
(F^x*b + a)*(F^x*b + a)^p/((b*p + b)*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(17) = 34$.

Time = 0.41 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.42

$$\int F^x(a+bF^x)^p\,dx = \begin{cases} \frac{x}{a} & \text{for } F = 1 \wedge b = 0 \wedge p = -1 \\ x(a+b)^p & \text{for } F = 1 \\ \frac{F^x a^p}{\log(F)} & \text{for } b = 0 \\ \frac{\log\left(F^x + \frac{a}{b}\right)}{b\log(F)} & \text{for } p = -1 \\ \frac{F^x b(F^x b+a)^p}{bp\log(F)+b\log(F)} + \frac{a(F^x b+a)^p}{bp\log(F)+b\log(F)} & \text{otherwise} \end{cases}$$

input
```
integrate(F**x*(a+b*F**x)**p,x)
```

output
```
Piecewise((x/a, Eq(F, 1) & Eq(b, 0) & Eq(p, -1)), (x*(a + b)**p, Eq(F, 1))
, (F**x*a**p/log(F), Eq(b, 0)), (log(F**x + a/b)/(b*log(F)), Eq(p, -1)), (
F**x*b*(F**x*b + a)**p/(b*p*log(F) + b*log(F)) + a*(F**x*b + a)**p/(b*p*lo
g(F) + b*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int F^x(a+bF^x)^p\,dx = \frac{(F^x b+a)^{p+1}}{b(p+1)\log(F)}$$

input
```
integrate(F^x*(a+b*F^x)^p,x, algorithm="maxima")
```

output
```
(F^x*b + a)^(p + 1)/(b*(p + 1)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int F^x(a+bF^x)^p\,dx=\frac{(F^xb+a)^{p+1}}{b(p+1)\log(F)}$$

input
```
integrate(F^x*(a+b*F^x)^p,x, algorithm="giac")
```

output
```
(F^x*b + a)^(p + 1)/(b*(p + 1)*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int F^x(a+bF^x)^p\,dx=\frac{(a+F^x\,b)^{p+1}}{b\,\ln(F)\,(p+1)}$$

input
```
int(F^x*(a + F^x*b)^p,x)
```

output
```
(a + F^x*b)^(p + 1)/(b*log(F)*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int F^x(a+bF^x)^p\,dx=\frac{(f^xb+a)^p\,(f^xb+a)}{\log(f)\,b\,(p+1)}$$

input
```
int(F^x*(a+b*F^x)^p,x)
```

output
```
((f**x*b + a)**p*(f**x*b + a))/(log(f)*b*(p + 1))
```

3.68 $\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 19, antiderivative size = 36

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{F^{-c}\left(a+bF^{c+dx}\right)^{1+p}}{bd(1+p)\log(F)}$$

output
```
(a+b*F^(d*x+c))^(p+1)/b/d/(F^c)/(p+1)/ln(F)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{F^{-c}\left(a+bF^{c+dx}\right)^{1+p}}{bd\log(F)+bdp\log(F)}$$

input
```
Integrate[F^(d*x)*(a + b*F^(c + d*x))^p,x]
```

output
```
(a + b*F^(c + d*x))^(1 + p)/(F^c*(b*d*Log[F] + b*d*p*Log[F]))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2677, 2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx$$

$$\Big\downarrow \text{2677}$$

$$F^{-c}\int F^{c+dx}\left(bF^{c+dx}+a\right)^p\,dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{F^{-c}\int \left(bF^{c+dx}+a\right)^p\,dF^{c+dx}}{d\log(F)}$$

$$\Big\downarrow \text{17}$$

$$\frac{F^{-c}\left(a+bF^{c+dx}\right)^{p+1}}{bd(p+1)\log(F)}$$

input
```
Int[F^(d*x)*(a + b*F^(c + d*x))^p,x]
```

output
```
(a + b*F^(c + d*x))^(1 + p)/(b*d*F^c*(1 + p)*Log[F])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

method	result	size
risch	$\frac{\left(F^{dx} F^{c} b+a\right) F^{-c} \left(F^{dx} F^{c} b+a\right)^{p}}{b \left(p+1\right) \ln \left(F\right) d}$	48
norman	$\frac{{\mathrm e}^{d x \ln \left(F\right)} {\mathrm e}^{p \ln \left(a+b \,{\mathrm e}^{\ln \left(F\right) c} {\mathrm e}^{d x \ln \left(F\right)}\right)}}{\ln \left(F\right) d \left(p+1\right)}+\frac{F^{-c} a \,{\mathrm e}^{p \ln \left(a+b \,{\mathrm e}^{\ln \left(F\right) c} {\mathrm e}^{d x \ln \left(F\right)}\right)}}{\ln \left(F\right) b d \left(p+1\right)}$	81

input
```
int(F^(d*x)*(a+b*F^(d*x+c))^p,x,method=_RETURNVERBOSE)
```

output
```
(F^(d*x)*F^c*b+a)/(F^c)/b/(p+1)/ln(F)/d*(F^(d*x)*F^c*b+a)^p
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.39

$$\int F^{dx} \left(a+bF^{c+dx}\right)^p \, dx=\frac{{\left(F^{d x + c} b + a\right)}^{p} {\left(\frac{F^{d x + c} b}{F^{c}} + \frac{a}{F^{c}}\right)}}{{\left(b d p + b d\right)} \log\left(F\right)}$$

input
```
integrate(F^(d*x)*(a+b*F^(d*x+c))^p,x, algorithm="fricas")
```

output
```
(F^(d*x + c)*b + a)^p*(F^(d*x + c)*b/F^c + a/F^c)/((b*d*p + b*d)*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 1.00 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx = \begin{cases} x(F^cb+a)^p & \text{for } d=0 \\ x(a+b)^p & \text{for } \log(F)=0 \\ \frac{\begin{cases} F^{dx}a^p & \text{for } F^c=0 \vee b=0 \\ \frac{F^{-c}\left(\begin{cases} \frac{(F^cF^{dx}b+a)^{p+1}}{p+1} & \text{for } p\neq -1 \\ \log\left(F^cF^{dx}b+a\right) & \text{otherwise} \end{cases}\right)}{b} & \text{otherwise} \end{cases}}{d\log(F)} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(d*x)*(a+b*F**(d*x+c))**p,x)
```

output
```
Piecewise((x*(F**c*b + a)**p, Eq(d, 0)), (x*(a + b)**p, Eq(log(F), 0)), (P
iecewise((F**(d*x)*a**p, Eq(b, 0) | Eq(F**c, 0)), (Piecewise(((F**c*F**(d*
x)*b + a)**(p + 1)/(p + 1), Ne(p, -1)), (log(F**c*F**(d*x)*b + a), True))/
(F**c*b), True))/(d*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx = \frac{\left(F^{dx+c}b+a\right)^{p+1}}{F^cbd(p+1)\log(F)}$$

input
```
integrate(F^(d*x)*(a+b*F^(d*x+c))^p,x, algorithm="maxima")
```

output
```
(F^(d*x + c)*b + a)^(p + 1)/(F^c*b*d*(p + 1)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(F^{dx+c}b+a\right)^{p+1}}{F^cbd(p+1)\log\left(F\right)}$$

input
```
integrate(F^(d*x)*(a+b*F^(d*x+c))^p,x, algorithm="giac")
```

output
```
(F^(d*x + c)*b + a)^(p + 1)/(F^c*b*d*(p + 1)*log(F))
```

Mupad [B] (verification not implemented)

Time = 22.99 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.53

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx=\left(a+F^{c+d\,x}\,b\right)^p\left(\frac{F^{d\,x}}{d\,\ln\left(F\right)\,\left(p+1\right)}+\frac{a}{F^c\,b\,d\,\ln\left(F\right)\,\left(p+1\right)}\right)$$

input
```
int(F^(d*x)*(a + F^(c + d*x)*b)^p,x)
```

output
```
(a + F^(c + d*x)*b)^p*(F^(d*x)/(d*log(F)*(p + 1)) + a/(F^c*b*d*log(F)*(p +
 1)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int F^{dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(f^{dx+c}b+a\right)^p\left(f^{dx+c}b+a\right)}{f^c\log\left(f\right)bd\left(p+1\right)}$$

input
```
int(F^(d*x)*(a+b*F^(d*x+c))^p,x)
```

output
```
((f**(c + d*x)*b + a)**p*(f**(c + d*x)*b + a))/(f**c*log(f)*b*d*(p + 1))
```

3.69 $\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(a+bF^{c+dx}\right)^{1+p}}{bd(1+p)\log(F)}$$

output
```
(a+b*F^(d*x+c))^(p+1)/b/d/(p+1)/ln(F)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(a+bF^{c+dx}\right)^{1+p}}{bd\log(F)+bdp\log(F)}$$

input
```
Integrate[F^(c + d*x)*(a + b*F^(c + d*x))^p,x]
```

output
```
(a + b*F^(c + d*x))^(1 + p)/(b*d*Log[F] + b*d*p*Log[F])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{\int \left(bF^{c+dx}+a\right)^p dF^{c+dx}}{d\log(F)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+bF^{c+dx}\right)^{p+1}}{bd(p+1)\log(F)}$$

input
```
Int[F^(c + d*x)*(a + b*F^(c + d*x))^p,x]
```

output
```
(a + b*F^(c + d*x))^(1 + p)/(b*d*(1 + p)*Log[F])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{(a+b\,F^{dx+c})^{p+1}}{bd(p+1)\ln(F)}$	32
default	$\frac{(a+b\,F^{dx+c})^{p+1}}{bd(p+1)\ln(F)}$	32
risch	$\frac{(a+b\,F^{dx+c})(a+b\,F^{dx+c})^{p}}{\ln(F)bd(p+1)}$	41
parallelrisch	$\frac{F^{dx+c}(a+b\,F^{dx+c})^{p}b+(a+b\,F^{dx+c})^{p}a}{\ln(F)bd(p+1)}$	55
norman	$\frac{e^{(dx+c)\ln(F)}e^{p\ln\left(a+b\,e^{(dx+c)\ln(F)}\right)}}{\ln(F)d(p+1)}+\frac{a\,e^{p\ln\left(a+b\,e^{(dx+c)\ln(F)}\right)}}{bd\ln(F)(p+1)}$	75

input
```
int(F^(d*x+c)*(a+b*F^(d*x+c))^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*F^(d*x+c))^(p+1)/b/d/(p+1)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(F^{dx+c}b+a\right)\left(F^{dx+c}b+a\right)^p}{(bdp+bd)\log(F)}$$

input
```
integrate(F^(d*x+c)*(a+b*F^(d*x+c))^p,x, algorithm="fricas")
```

output
```
(F^(d*x + c)*b + a)*(F^(d*x + c)*b + a)^p/((b*d*p + b*d)*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(22) = 44$.

Time = 1.03 (sec) , antiderivative size = 299, normalized size of antiderivative = 9.65

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx$$

$$= \begin{cases} \tilde{\infty}x \\ x(a+b)^p \\ \frac{0^pF^{c+dx}}{d\log(F)} \\ \frac{F^{c+dx}a^p}{d\log(F)} \\ F^cx(F^cb+a)^p \\ \frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{bd\log(F)} \\ \frac{2F^{c+dx}ab\left(F^{c+dx}b+a\right)^p}{F^{c+dx}b^2dp\log(F)+F^{c+dx}b^2d\log(F)+abdp\log(F)+abd\log(F)} + \frac{F^{2c+2dx}b^2\left(F^{c+dx}b+a\right)^p}{F^{c+dx}b^2dp\log(F)+F^{c+dx}b^2d\log(F)+abdp\log(F)+abd\log(F)} + \frac{F}{} \end{cases}$$

input
```
integrate(F**(d*x+c)*(a+b*F**(d*x+c))**p,x)
```

output
```
Piecewise((zoo*x, Eq(F, 1) & Eq(a, 0) & Eq(b, 0) & Eq(d, 0) & Eq(p, -1)),
(x*(a + b)**p, Eq(F, 1)), (0**p*F**(c + d*x)/(d*log(F)), Eq(a, -F**(c + d*
x)*b)), (F**(c + d*x)*a**p/(d*log(F)), Eq(b, 0)), (F**c*x*(F**c*b + a)**p,
 Eq(d, 0)), (log(F**(c + d*x) + a/b)/(b*d*log(F)), Eq(p, -1)), (2*F**(c +
d*x)*a*b*(F**(c + d*x)*b + a)**p/(F**(c + d*x)*b**2*d*p*log(F) + F**(c + d
*x)*b**2*d*log(F) + a*b*d*p*log(F) + a*b*d*log(F)) + F**(2*c + 2*d*x)*b**2
*(F**(c + d*x)*b + a)**p/(F**(c + d*x)*b**2*d*p*log(F) + F**(c + d*x)*b**2
*d*log(F) + a*b*d*p*log(F) + a*b*d*log(F)) + a**2*(F**(c + d*x)*b + a)**p/
(F**(c + d*x)*b**2*d*p*log(F) + F**(c + d*x)*b**2*d*log(F) + a*b*d*p*log(F
) + a*b*d*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(F^{dx+c}b+a\right)^{p+1}}{bd(p+1)\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)*(a+b*F^(d*x+c))^p,x, algorithm="maxima")
```

output
```
(F^(d*x + c)*b + a)^(p + 1)/(b*d*(p + 1)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx=\frac{\left(F^{dx+c}b+a\right)^{p+1}}{bd(p+1)\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)*(a+b*F^(d*x+c))^p,x, algorithm="giac")
```

output
```
(F^(d*x + c)*b + a)^(p + 1)/(b*d*(p + 1)*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^p\,dx=\left(a+F^{c+d\,x}\,b\right)^p\,\left(\frac{F^{c+d\,x}}{d\,\ln\left(F\right)\,\left(p+1\right)}+\frac{a}{b\,d\,\ln\left(F\right)\,\left(p+1\right)}\right)$$

input
```
int(F^(c + d*x)*(a + F^(c + d*x)*b)^p,x)
```

output
```
(a + F^(c + d*x)*b)^p*(F^(c + d*x)/(d*log(F)*(p + 1)) + a/(b*d*log(F)*(p +
 1)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int F^{c+dx}\left(a+bF^{c+dx}\right)^{p}\,dx=\frac{\left(f^{dx+c}b+a\right)^{p}\left(f^{dx+c}b+a\right)}{\log\left(f\right)bd\left(p+1\right)}$$

input
```
int(F^(d*x+c)*(a+b*F^(d*x+c))^p,x)
```

output
```
((f**(c + d*x)*b + a)**p*(f**(c + d*x)*b + a))/(log(f)*b*d*(p + 1))
```

3.70 $\int (e^x)^n (a+b(e^x)^n)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 25

$$\int (e^x)^n (a+b(e^x)^n)^p \, dx = \frac{(a+b(e^x)^n)^{1+p}}{bn(1+p)}$$

output
```
(a+b*exp(x)^n)^(p+1)/b/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (e^x)^n (a+b(e^x)^n)^p \, dx = \frac{(a+b(e^x)^n)^{1+p}}{bn(1+p)}$$

input
```
Integrate[(E^x)^n*(a + b*(E^x)^n)^p,x]
```

output
```
(a + b*(E^x)^n)^(1 + p)/(b*n*(1 + p))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e^x)^n (a+b(e^x)^n)^p \, dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{\int (b(e^x)^n+a)^p d(e^x)^n}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+b(e^x)^n)^{p+1}}{bn(p+1)}$$

input
```
Int[(E^x)^n*(a + b*(E^x)^n)^p,x]
```

output
```
(a + b*(E^x)^n)^(1 + p)/(b*n*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{(a+b(e^x)^n)^{p+1}}{bn(p+1)}$	25
default	$\frac{(a+b(e^x)^n)^{p+1}}{bn(p+1)}$	25
risch	$\frac{(a+b\,e^{xn})(a+b\,e^{xn})^p}{bn(p+1)}$	31
parallelrisch	$\frac{(e^x)^n(a+b(e^x)^n)^p b+(a+b(e^x)^n)^p a}{bn(p+1)}$	42
norman	$\frac{e^{xn}e^{p\ln(a+b\,e^{xn})}}{n(p+1)}+\frac{a\,e^{p\ln(a+b\,e^{xn})}}{bn(p+1)}$	52

input
```
int(exp(x)^n*(a+b*exp(x)^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*exp(x)^n)^(p+1)/b/n/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int (e^x)^n\,(a+b(e^x)^n)^p\,dx=\frac{\left(be^{(nx)}+a\right)\left(be^{(nx)}+a\right)^p}{bnp+bn}$$

input
```
integrate(exp(x)^n*(a+b*exp(x)^n)^p,x, algorithm="fricas")
```

output
```
(b*e^(n*x) + a)*(b*e^(n*x) + a)^p/(b*n*p + b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(17) = 34$.

Time = 0.67 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.20

$$\int (e^x)^n (a + b(e^x)^n)^p\,dx = \begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge n = 0 \wedge p = -1 \\ \frac{a^p(e^x)^n}{n} & \text{for } b = 0 \\ x(a+b)^p & \text{for } n = 0 \\ \frac{\log\left(\frac{a}{b} + (e^x)^n\right)}{bn} & \text{for } p = -1 \\ \frac{a(a+b(e^x)^n)^p}{bnp+bn} + \frac{b(a+b(e^x)^n)^p(e^x)^n}{bnp+bn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)**n*(a+b*exp(x)**n)**p,x)
```

output
```
Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(p, -1)), (a**p*exp(x)**n/n, Eq(b,
 0)), (x*(a + b)**p, Eq(n, 0)), (log(a/b + exp(x)**n)/(b*n), Eq(p, -1)), (
a*(a + b*exp(x)**n)**p/(b*n*p + b*n) + b*(a + b*exp(x)**n)**p*exp(x)**n/(b
*n*p + b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (e^x)^n (a + b(e^x)^n)^p\,dx = \frac{\left(be^{(nx)} + a\right)^{p+1}}{bn(p+1)}$$

input
```
integrate(exp(x)^n*(a+b*exp(x)^n)^p,x, algorithm="maxima")
```

output
```
(b*e^(n*x) + a)^(p + 1)/(b*n*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (e^x)^n (a + b(e^x)^n)^p \, dx = \frac{\left(be^{(nx)} + a\right)^{p+1}}{bn(p+1)}$$

input
```
integrate(exp(x)^n*(a+b*exp(x)^n)^p,x, algorithm="giac")
```

output
```
(b*e^(n*x) + a)^(p + 1)/(b*n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 23.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (e^x)^n (a + b(e^x)^n)^p \, dx = \frac{(a + b\,e^{n\,x})^{p+1}}{b\,n\,(p+1)}$$

input
```
int(exp(x)^n*(a + b*exp(x)^n)^p,x)
```

output
```
(a + b*exp(n*x))^(p + 1)/(b*n*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int (e^x)^n (a + b(e^x)^n)^p \, dx = \frac{(e^{nx}b + a)^p (e^{nx}b + a)}{bn\,(p+1)}$$

input
```
int(exp(x)^n*(a+b*exp(x)^n)^p,x)
```

output
```
((e**(n*x)*b + a)**p*(e**(n*x)*b + a))/(b*n*(p + 1))
```

3.71 $\int e^{nx}(a+b(e^x)^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int e^{nx}(a+b(e^x)^n)^p\,dx=\frac{e^{nx}(e^x)^{-n}\left(a+b(e^x)^n\right)^{1+p}}{bn(1+p)}$$

output
```
exp(n*x)*(a+b*exp(x)^n)^(p+1)/b/(exp(x)^n)/n/(p+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int e^{nx}(a+b(e^x)^n)^p\,dx=\frac{e^{nx}(e^x)^{-n}\left(a+b(e^x)^n\right)^{1+p}}{bn+bnp}$$

input
```
Integrate[E^(n*x)*(a + b*(E^x)^n)^p,x]
```

output
```
(E^(n*x)*(a + b*(E^x)^n)^(1 + p))/((E^x)^n*(b*n + b*n*p))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2677, 2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{nx}(a+b(e^x)^n)^p\,dx$$

$$\Big\downarrow \text{2677}$$

$$e^{nx}(e^x)^{-n}\int (e^x)^n\left(b(e^x)^n+a\right)^p dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{e^{nx}(e^x)^{-n}\int \left(b(e^x)^n+a\right)^p d(e^x)^n}{n}$$

$$\Big\downarrow \text{17}$$

$$\frac{e^{nx}(e^x)^{-n}\left(a+b(e^x)^n\right)^{p+1}}{bn(p+1)}$$

input
```
Int[E^(n*x)*(a + b*(E^x)^n)^p,x]
```

output
```
(E^(n*x)*(a + b*(E^x)^n)^(1 + p))/(b*(E^x)^n*n*(1 + p))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{(a+b\,\mathrm{e}^{x n})(a+b\,\mathrm{e}^{x n})^{p}}{b n(p+1)}$	31
norman	$\frac{\mathrm{e}^{x n}\mathrm{e}^{p\ln(a+b\,\mathrm{e}^{x n})}}{n(p+1)}+\frac{a\,\mathrm{e}^{p\ln(a+b\,\mathrm{e}^{x n})}}{b n(p+1)}$	52

input
```
int(exp(x*n)*(a+b*exp(x)^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*exp(x*n))/b/n/(p+1)*(a+b*exp(x*n))^p
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{nx}(a+b(e^x)^n)^p\,dx=\frac{\left(be^{(nx)}+a\right)\left(be^{(nx)}+a\right)^p}{bnp+bn}$$

input
```
integrate(exp(n*x)*(a+b*exp(x)^n)^p,x, algorithm="fricas")
```

output
```
(b*e^(n*x) + a)*(b*e^(n*x) + a)^p/(b*n*p + b*n)
```

Sympy [F]

$$\int e^{nx}(a+b(e^x)^n)^p\,dx = \int (a+b(e^x)^n)^p\,e^{nx}\,dx$$

input
```
integrate(exp(n*x)*(a+b*exp(x)**n)**p,x)
```

output
```
Integral((a + b*exp(x)**n)**p*exp(n*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int e^{nx}(a+b(e^x)^n)^p\,dx = \frac{\left(be^{(nx)}+a\right)^{p+1}}{bn(p+1)}$$

input
```
integrate(exp(n*x)*(a+b*exp(x)^n)^p,x, algorithm="maxima")
```

output
```
(b*e^(n*x) + a)^(p + 1)/(b*n*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int e^{nx}(a+b(e^x)^n)^p\,dx = \frac{\left(be^{(nx)}+a\right)^{p+1}}{bn(p+1)}$$

input
```
integrate(exp(n*x)*(a+b*exp(x)^n)^p,x, algorithm="giac")
```

output
```
(b*e^(n*x) + a)^(p + 1)/(b*n*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 24.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int e^{nx}(a+b(e^x)^n)^p\,dx=\frac{(a+b\,\mathrm{e}^{n\,x})^{p+1}}{b\,n\,(p+1)}$$

input
```
int(exp(n*x)*(a + b*exp(x)^n)^p,x)
```

output
```
(a + b*exp(n*x))^(p + 1)/(b*n*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int e^{nx}(a+b(e^x)^n)^p\,dx=\frac{(e^{nx}b+a)^p\,(e^{nx}b+a)}{bn\,(p+1)}$$

input
```
int(exp(n*x)*(a+b*exp(x)^n)^p,x)
```

output
```
((e**(n*x)*b + a)**p*(e**(n*x)*b + a))/(b*n*(p + 1))
```

3.72 $\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \, dx$

Optimal result

Integrand size = 29, antiderivative size = 41

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \, dx = \frac{\left(a+b\left(F^{e(c+dx)}\right)^n\right)^{1+p}}{bden(1+p)\log(F)}$$

output
```
(a+b*(F^(e*(d*x+c)))^n)^(p+1)/b/d/e/n/(p+1)/ln(F)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \, dx = \frac{\left(a+b\left(F^{e(c+dx)}\right)^n\right)^{1+p}}{bden(1+p)\log(F)}$$

input
```
Integrate[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,x]
```

output
```
(a + b*(F^(e*(c + d*x)))^n)^(1 + p)/(b*d*e*n*(1 + p)*Log[F])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{\int \left(b\left(F^{e(c+dx)}\right)^n+a\right)^p d\left(F^{e(c+dx)}\right)^n}{den\log(F)}$$

$$\Big\downarrow \text{17}$$

$$\frac{\left(a+b\left(F^{e(c+dx)}\right)^n\right)^{p+1}}{bden(p+1)\log(F)}$$

input
```
Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,x]
```

output
```
(a + b*(F^(e*(c + d*x)))^n)^(1 + p)/(b*d*e*n*(1 + p)*Log[F])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{\left(a+b\left(F^{e(dx+c)}\right)^{n}\right)^{p+1}}{bden(p+1)\ln(F)}$	42
default	$\frac{\left(a+b\left(F^{e(dx+c)}\right)^{n}\right)^{p+1}}{bden(p+1)\ln(F)}$	42
risch	$\frac{\left(a+b\left(F^{e(dx+c)}\right)^{n}\right)\left(a+b\left(F^{e(dx+c)}\right)^{n}\right)^{p}}{b(p+1)\ln(F)edn}$	55
parallelrisch	$\frac{\left(F^{e(dx+c)}\right)^{n}\left(a+b\left(F^{e(dx+c)}\right)^{n}\right)^{p}b+\left(a+b\left(F^{e(dx+c)}\right)^{n}\right)^{p}a}{b(p+1)\ln(F)edn}$	73

input
```
int((F^(e*(d*x+c)))^n*(a+b*(F^(e*(d*x+c)))^n)^p,x,method=_RETURNVERBOSE)
```

output
```
(a+b*(F^(e*(d*x+c)))^n)^(p+1)/b/d/e/n/(p+1)/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.29

$$\int \left(F^{e(c+dx)}\right)^{n}\left(a+b\left(F^{e(c+dx)}\right)^{n}\right)^{p}\,dx=\frac{\left(F^{denx+cen}b+a\right)\left(F^{denx+cen}b+a\right)^{p}}{(bdenp+bden)\log\left(F\right)}$$

input
```
integrate((F^(e*(d*x+c)))^n*(a+b*(F^(e*(d*x+c)))^n)^p,x, algorithm="fricas
")
```

output
```
(F^(d*e*n*x + c*e*n)*b + a)*(F^(d*e*n*x + c*e*n)*b + a)^p/((b*d*e*n*p + b*
d*e*n)*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(29) = 58$.

Time = 23.00 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.20

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \,dx = \begin{cases} x(a+b(F^{ce})^n)^p \left(F^{ce}\right)^n & \text{for}\ d=0 \\ x(a+b)^p & \text{for}\ e=0 \vee n=0 \vee \log\left(F\right)=0 \\ \frac{\begin{cases} a^p\left(F^{e(c+dx)}\right)^n & \text{for}\ b=0 \\ \frac{\begin{cases} \frac{\left(a+b\left(F^{e(c+dx)}\right)^n\right)^{p+1}}{p+1} & \text{for}\ p\neq -1 \\ \log\left(a+b\left(F^{e(c+dx)}\right)^n\right) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}}{den\log\left(F\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((F**(e*(d*x+c)))**n*(a+b*(F**(e*(d*x+c)))**n)**p,x)
```

output
```
Piecewise((x*(a + b*(F**(c*e))**n)**p*(F**(c*e))**n, Eq(d, 0)), (x*(a + b)
**p, Eq(e, 0) | Eq(n, 0) | Eq(log(F), 0)), (Piecewise((a**p*(F**(e*(c + d*
x)))**n, Eq(b, 0)), (Piecewise(((a + b*(F**(e*(c + d*x)))**n)**(p + 1)/(p
+ 1), Ne(p, -1)), (log(a + b*(F**(e*(c + d*x)))**n), True))/b, True))/(d*e
*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \,dx = \frac{\left(F^{(dx+c)en}b+a\right)^{p+1}}{bden(p+1)\log\left(F\right)}$$

input
```
integrate((F^(e*(d*x+c)))^n*(a+b*(F^(e*(d*x+c)))^n)^p,x, algorithm="maxima
")
```

output
```
(F^((d*x + c)*e*n)*b + a)^(p + 1)/(b*d*e*n*(p + 1)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p dx = \frac{\left(F^{denx+cen}b+a\right)^{p+1}}{bden(p+1)\log\left(F\right)}$$

input
```
integrate((F^(e*(d*x+c)))^n*(a+b*(F^(e*(d*x+c)))^n)^p,x, algorithm="giac")
```

output
```
(F^(d*e*n*x + c*e*n)*b + a)^(p + 1)/(b*d*e*n*(p + 1)*log(F))
```

Mupad [B] (verification not implemented)

Time = 24.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.80

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p dx = \left(\frac{\left(F^{c\,e+d\,e\,x}\right)^n}{d\,e\,n\,\ln\left(F\right)\,\left(p+1\right)} + \frac{a}{b\,d\,e\,n\,\ln\left(F\right)\,\left(p+1\right)}\right)\,\left(a+b\,\left(F^{c\,e+d\,e\,x}\right)^n\right)^p$$

input
```
int((F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,x)
```

output
```
((F^(c*e + d*e*x))^n/(d*e*n*log(F)*(p + 1)) + a/(b*d*e*n*log(F)*(p + 1)))*
(a + b*(F^(c*e + d*e*x))^n)^p
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.37

$$\int \left(F^{e(c+dx)}\right)^n \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p dx = \frac{\left(f^{denx+cen}b+a\right)^p \left(f^{denx+cen}b+a\right)}{\log\left(f\right)bden\left(p+1\right)}$$

input
```
int((F^(e*(d*x+c)))^n*(a+b*(F^(e*(d*x+c)))^n)^p,x)
```

output
```
((f**(c*e*n + d*e*n*x)*b + a)**p*(f**(c*e*n + d*e*n*x)*b + a))/(log(f)*b*d
*e*n*(p + 1))
```

3.73 $\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx$

Optimal result

Integrand size = 44, antiderivative size = 80

$$\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx = \frac{\left(F^{e(c+dx)}\right)^{-n}\left(a+b\left(F^{e(c+dx)}\right)^n\right)^{1+p}\left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}}{bden(1+p)\log(F)}$$

output
```
(a+b*(F^(e*(d*x+c)))^n)^(p+1)*(G^(h*(g*x+f)))^(d*e*n*ln(F)/g/h/ln(G))/b/d/
e/((F^(e*(d*x+c)))^n)/n/(p+1)/ln(F)
```

Mathematica [F]

$$\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx = \int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx$$

input
```
Integrate[(a + b*(F^(e*(c + d*x)))^n)^p*(G^(h*(f + g*x)))^((d*e*n*Log[F])/
(g*h*Log[G])),x]
```

output
```
Integrate[(a + b*(F^(e*(c + d*x)))^n)^p*(G^(h*(f + g*x)))^((d*e*n*Log[F])/
(g*h*Log[G])), x]
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$, Rules used = {2677, 2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}dx$$

$$\Big\downarrow \ 2677$$

$$\left(F^{e(c+dx)}\right)^{-n}\left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\int \left(F^{e(c+dx)}\right)^n\left(b\left(F^{e(c+dx)}\right)^n+a\right)^pdx$$

$$\Big\downarrow \ 2676$$

$$\frac{\left(F^{e(c+dx)}\right)^{-n}\left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\int \left(b\left(F^{e(c+dx)}\right)^n+a\right)^pd\left(F^{e(c+dx)}\right)^n}{den\log(F)}$$

$$\Big\downarrow \ 17$$

$$\frac{\left(F^{e(c+dx)}\right)^{-n}\left(a+b\left(F^{e(c+dx)}\right)^n\right)^{p+1}\left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}}{bden(p+1)\log(F)}$$

input
```
Int[(a + b*(F^(e*(c + d*x)))^n)^p*(G^(h*(f + g*x)))^((d*e*n*Log[F])/(g*h*L
og[G])),x]
```

output
```
((a + b*(F^(e*(c + d*x)))^n)^(1 + p)*(G^(h*(f + g*x)))^((d*e*n*Log[F])/(g*
h*Log[G])))/(b*d*e*(F^(e*(c + d*x)))^n*n*(1 + p)*Log[F])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [F]

$$\int \left(a+b\left(F^{e(dx+c)}\right)^{n}\right)^{p}\left(G^{h(gx+f)}\right)^{\frac{den\ln(F)}{gh\ln(G)}}dx$$

input
```
int((a+b*(F^(e*(d*x+c)))^n)^p*(G^(h*(g*x+f)))^(d*e*n*ln(F)/g/h/ln(G)),x)
```

output
```
int((a+b*(F^(e*(d*x+c)))^n)^p*(G^(h*(g*x+f)))^(d*e*n*ln(F)/g/h/ln(G)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int \left(a+b\left(F^{e(c+dx)}\right)^{n}\right)^{p}\left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}dx = \frac{\left(F^{denx+cen}F^{\frac{(def-ceg)n}{g}}b+F^{\frac{(def-ceg)n}{g}}a\right)\left(F^{denx+cen}b+a\right)^{p}}{(bdenp+bden)\log(F)}$$

input
```
integrate((a+b*(F^(e*(d*x+c)))^n)^p*(G^(h*(g*x+f)))^(d*e*n*log(F)/g/h/log(
G)),x, algorithm="fricas")
```

output
```
(F^(d*e*n*x + c*e*n)*F^((d*e*f - c*e*g)*n/g)*b + F^((d*e*f - c*e*g)*n/g)*a
)*(F^(d*e*n*x + c*e*n)*b + a)^p/((b*d*e*n*p + b*d*e*n)*log(F))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*(F**(e*(d*x+c)))**n)**p*(G**(h*(g*x+f)))**(d*e*n*ln(F)/g/h/
ln(G)),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int \left(a + b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx = \frac{\left(F^{denx}F^{cen+\frac{defn}{g}}b + F^{\frac{defn}{g}}a\right)\left(F^{denx}F^{cen}b + a\right)^p}{F^{cen}bden(p+1)\log(F)}$$

input
```
integrate((a+b*(F^(e*(d*x+c)))^n)^p*(G^(h*(g*x+f)))^(d*e*n*log(F)/g/h/log(
G)),x, algorithm="maxima")
```

output
```
(F^(d*e*n*x)*F^(c*e*n + d*e*f*n/g)*b + F^(d*e*f*n/g)*a)*(F^(d*e*n*x)*F^(c*
e*n)*b + a)^p/(F^(c*e*n)*b*d*e*n*(p + 1)*log(F))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.79

$$\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx = \frac{F^{\frac{defn}{g}}be^{\left(2\,denx\log(F)+cen\log(F)+p\log\left(be^{(denx\log(F)+cen\log(F))}+a\right)\right)}+F^{\frac{defn}{g}}ae^{\left(denx\log(F)+p\log\left(be^{(denx\log(F)+cen\log(F))}+a\right)\right)}}{bdenpe^{(denx\log(F)+cen\log(F))}\log(F)+bdene^{(denx\log(F)+cen\log(F))}\log(F)}$$

input
```
integrate((a+b*(F^(e*(d*x+c)))^n)^p*(G^(h*(g*x+f)))^(d*e*n*log(F)/g/h/log(
G)),x, algorithm="giac")
```

output
```
(F^(d*e*f*n/g)*b*e^(2*d*e*n*x*log(F) + c*e*n*log(F) + p*log(b*e^(d*e*n*x*l
og(F) + c*e*n*log(F)) + a)) + F^(d*e*f*n/g)*a*e^(d*e*n*x*log(F) + p*log(b*
e^(d*e*n*x*log(F) + c*e*n*log(F)) + a)))/(b*d*e*n*p*e^(d*e*n*x*log(F) + c*
e*n*log(F))*log(F) + b*d*e*n*e^(d*e*n*x*log(F) + c*e*n*log(F))*log(F))
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}}\,dx = \int \left(G^{h\,(f+g\,x)}\right)^{\frac{d\,e\,n\,\ln(F)}{g\,h\,\ln(G)}}\left(a+b\left(F^{e\,(c+d\,x)}\right)^n\right)^p\,dx$$

input
```
int((G^(h*(f + g*x)))^((d*e*n*log(F))/(g*h*log(G)))*(a + b*(F^(e*(c + d*x)
))^n)^p,x)
```

output
```
int((G^(h*(f + g*x)))^((d*e*n*log(F))/(g*h*log(G)))*(a + b*(F^(e*(c + d*x)
))^n)^p, x)
```

Reduce [F]

$$\int \left(a+b\left(F^{e(c+dx)}\right)^n\right)^p \left(G^{h(f+gx)}\right)^{\frac{den\log(F)}{gh\log(G)}} dx = \int g^{\frac{\log(f)defn+\log(f)degnx}{\log(g)g}} \left(f^{denx+cen}b+a\right)^p dx$$

input
```
int((a+b*(F^(e*(d*x+c)))^n)^p*(G^(h*(g*x+f)))^(d*e*n*log(F)/g/h/log(G)),x)
```

output
```
int(g**((log(f)*d*e*f*n + log(f)*d*e*g*n*x)/(log(g)*g))*(f**(c*e*n + d*e*n
*x)*b + a)**p,x)
```

3.74 $\int \frac{e^{2x}}{a+be^x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{e^{2x}}{a+be^x}\,dx = \frac{e^x}{b} - \frac{a\log(a+be^x)}{b^2}$$

output
```
exp(x)/b-a*ln(a+b*exp(x))/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{a+be^x}\,dx = \frac{e^x}{b} - \frac{a\log(a+be^x)}{b^2}$$

input
```
Integrate[E^(2*x)/(a + b*E^x),x]
```

output
```
E^x/b - (a*Log[a + b*E^x])/b^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{a+be^x} dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^x}{a+be^x} de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{b}-\frac{a}{b(a+be^x)}\right) de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^x}{b}-\frac{a\log(a+be^x)}{b^2}$$

input
```
Int[E^(2*x)/(a + b*E^x),x]
```

output
```
E^x/b - (a*Log[a + b*E^x])/b^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\mathrm{e}^{x}}{b}-\frac{a \ln \left(a+b \,\mathrm{e}^{x}\right)}{b^{2}}$	21
norman	$\frac{\mathrm{e}^{x}}{b}-\frac{a \ln \left(a+b \,\mathrm{e}^{x}\right)}{b^{2}}$	21
risch	$\frac{\mathrm{e}^{x}}{b}-\frac{a \ln \left(\mathrm{e}^{x}+\frac{a}{b}\right)}{b^{2}}$	23

input
```
int(exp(2*x)/(a+b*exp(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(x)/b-a*ln(a+b*exp(x))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{e^{2x}}{a+be^x}\,dx=\frac{be^x-a\log\left(be^x+a\right)}{b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x)),x, algorithm="fricas")
```

output
```
(b*e^x - a*log(b*e^x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{a+be^x}\,dx = -\frac{a\log\left(\frac{a}{b}+e^x\right)}{b^2} + \begin{cases} \frac{e^x}{b} & \text{for } b\neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(2*x)/(a+b*exp(x)),x)
```

output
```
-a*log(a/b + exp(x))/b**2 + Piecewise((exp(x)/b, Ne(b, 0)), (x/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{a+be^x}\,dx = \frac{e^x}{b} - \frac{a\log\left(be^x+a\right)}{b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x)),x, algorithm="maxima")
```

output
```
e^x/b - a*log(b*e^x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{e^{2x}}{a+be^x}\,dx = \frac{e^x}{b} - \frac{a\log\left(|be^x+a|\right)}{b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x)),x, algorithm="giac")
```

output
```
e^x/b - a*log(abs(b*e^x + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 23.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{e^{2x}}{a+be^x}\,dx = -\frac{a\,\ln\left(a+b\,\mathrm{e}^x\right)-b\,\mathrm{e}^x}{b^2}$$

input
```
int(exp(2*x)/(a + b*exp(x)),x)
```

output
```
-(a*log(a + b*exp(x)) - b*exp(x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{e^{2x}}{a+be^x}\,dx = \frac{e^x b-\log(e^x b+a)\,a}{b^2}$$

input
```
int(exp(2*x)/(a+b*exp(x)),x)
```

output
```
(e**x*b - log(e**x*b + a)*a)/b**2
```

3.75 $\int \frac{e^{2x}}{(a+be^x)^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{e^{2x}}{(a+be^x)^2} dx = \frac{a}{b^2(a+be^x)} + \frac{\log(a+be^x)}{b^2}$$

output
```
a/b^2/(a+b*exp(x))+ln(a+b*exp(x))/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{e^{2x}}{(a+be^x)^2} dx = \frac{\frac{a}{a+be^x} + \log(b(a+be^x))}{b^2}$$

input
```
Integrate[E^(2*x)/(a + b*E^x)^2,x]
```

output
```
(a/(a + b*E^x) + Log[b*(a + b*E^x)])/b^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{(a+be^x)^2}dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^x}{(a+be^x)^2}de^x$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{b(a+be^x)}-\frac{a}{b(a+be^x)^2}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{a}{b^2(a+be^x)}+\frac{\log(a+be^x)}{b^2}$$

input
```
Int[E^(2*x)/(a + b*E^x)^2,x]
```

output
```
a/(b^2*(a + b*E^x)) + Log[a + b*E^x]/b^2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{a}{b^{2}\left(a+b\,{\mathrm e}^{x}\right)}+\frac{\ln\left(a+b\,{\mathrm e}^{x}\right)}{b^{2}}$	26
norman	$\frac{a}{b^{2}\left(a+b\,{\mathrm e}^{x}\right)}+\frac{\ln\left(a+b\,{\mathrm e}^{x}\right)}{b^{2}}$	26
risch	$\frac{a}{b^{2}\left(a+b\,{\mathrm e}^{x}\right)}+\frac{\ln\left({\mathrm e}^{x}+\frac{a}{b}\right)}{b^{2}}$	28

input
```
int(exp(2*x)/(a+b*exp(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a/b^2/(a+b*exp(x))+ln(a+b*exp(x))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{e^{2x}}{(a+be^x)^2}\,dx=\frac{(be^x+a)\log\left(be^x+a\right)+a}{b^3e^x+ab^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^2,x, algorithm="fricas")
```

output
```
((b*e^x + a)*log(b*e^x + a) + a)/(b^3*e^x + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{e^{2x}}{(a+be^x)^2}\,dx = \frac{a}{ab^2+b^3e^x} + \frac{\log\left(\frac{a}{b}+e^x\right)}{b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))**2,x)
```

output
```
a/(a*b**2 + b**3*exp(x)) + log(a/b + exp(x))/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{e^{2x}}{(a+be^x)^2}\,dx = \frac{a}{b^3e^x+ab^2} + \frac{\log(be^x+a)}{b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^2,x, algorithm="maxima")
```

output
```
a/(b^3*e^x + a*b^2) + log(b*e^x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{e^{2x}}{(a+be^x)^2}\,dx = \frac{\log(|be^x+a|)}{b^2} + \frac{a}{(be^x+a)b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^2,x, algorithm="giac")
```

output
```
log(abs(b*e^x + a))/b^2 + a/((b*e^x + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{e^{2x}}{(a+be^x)^2}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^x\right)}{b^2} - \frac{\mathrm{e}^x}{b\,\left(a+b\,\mathrm{e}^x\right)}$$

input
```
int(exp(2*x)/(a + b*exp(x))^2,x)
```

output
```
log(a + b*exp(x))/b^2 - exp(x)/(b*(a + b*exp(x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{e^{2x}}{(a+be^x)^2}\,dx = \frac{e^x \log(e^x b+a)\,b - e^x b + \log(e^x b+a)\,a}{b^2\,(e^x b+a)}$$

input
```
int(exp(2*x)/(a+b*exp(x))^2,x)
```

output
```
(e**x*log(e**x*b + a)*b - e**x*b + log(e**x*b + a)*a)/(b**2*(e**x*b + a))
```

3.76 $\int \frac{e^{2x}}{(a+be^x)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{e^{2x}}{(a+be^x)^3} \, dx = \frac{e^{2x}}{2a(a+be^x)^2}$$

output
```
1/2*exp(2*x)/a/(a+b*exp(x))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{e^{2x}}{(a+be^x)^3} \, dx = \frac{-a-2be^x}{2b^2(a+be^x)^2}$$

input
```
Integrate[E^(2*x)/(a + b*E^x)^3,x]
```

output
```
(-a - 2*b*E^x)/(2*b^2*(a + b*E^x)^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2678, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{(a+be^x)^3} \, dx$$
$$\Big\downarrow \; 2678$$
$$\int \frac{e^x}{(a+be^x)^3} de^x$$
$$\Big\downarrow \; 48$$
$$\frac{e^{2x}}{2a(a+be^x)^2}$$

input
```
Int[E^(2*x)/(a + b*E^x)^3,x]
```

output
```
E^(2*x)/(2*a*(a + b*E^x)^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$\frac{e^{2x}}{2a(a+b\,e^{x})^{2}}$	18
risch	$-\frac{2b\,e^{x}+a}{2b^{2}(a+b\,e^{x})^{2}}$	21
norman	$\frac{-\frac{e^{x}}{b}-\frac{a}{2b^{2}}}{(a+b\,e^{x})^{2}}$	24
default	$-\frac{1}{b^{2}(a+b\,e^{x})}+\frac{a}{2b^{2}(a+b\,e^{x})^{2}}$	29

input
```
int(exp(2*x)/(a+b*exp(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(2*x)/a/(a+b*exp(x))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{e^{2x}}{(a+be^x)^3}\,dx = -\frac{2\,be^x+a}{2\,(b^4e^{(2\,x)}+2\,ab^3e^x+a^2b^2)}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b*e^x + a)/(b^4*e^(2*x) + 2*a*b^3*e^x + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{e^{2x}}{(a+be^x)^3}\,dx = \frac{-a-2be^x}{2a^2b^2+4ab^3e^x+2b^4e^{2x}}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))**3,x)
```

output
```
(-a - 2*b*exp(x))/(2*a**2*b**2 + 4*a*b**3*exp(x) + 2*b**4*exp(2*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.90

$$\int \frac{e^{2x}}{(a+be^x)^3}\,dx = -\frac{be^x}{b^4e^{(2x)}+2ab^3e^x+a^2b^2} - \frac{a}{2\left(b^4e^{(2x)}+2ab^3e^x+a^2b^2\right)}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^3,x, algorithm="maxima")
```

output
```
-b*e^x/(b^4*e^(2*x) + 2*a*b^3*e^x + a^2*b^2) - 1/2*a/(b^4*e^(2*x) + 2*a*b^
3*e^x + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{e^{2x}}{(a+be^x)^3}\,dx = -\frac{2be^x+a}{2(be^x+a)^2b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^3,x, algorithm="giac")
```

output
```
-1/2*(2*b*e^x + a)/((b*e^x + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{e^{2x}}{(a+be^x)^3}\,dx = \frac{e^{2\,x}}{2\,a\,\left(a^2+2\,e^x\,a\,b+e^{2\,x}\,b^2\right)}$$

input
```
int(exp(2*x)/(a + b*exp(x))^3,x)
```

output
```
exp(2*x)/(2*a*(b^2*exp(2*x) + a^2 + 2*a*b*exp(x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.52

$$\int \frac{e^{2x}}{(a+be^x)^3}\,dx = \frac{e^{2x}}{2a\,(e^{2x}b^2+2e^xab+a^2)}$$

input
```
int(exp(2*x)/(a+b*exp(x))^3,x)
```

output
```
e**(2*x)/(2*a*(e**(2*x)*b**2 + 2*e**x*a*b + a**2))
```

3.77 $\int \frac{e^{2x}}{(a+be^x)^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{e^{2x}}{(a+be^x)^4} dx = \frac{a}{3b^2(a+be^x)^3} - \frac{1}{2b^2(a+be^x)^2}$$

output
```
1/3*a/b^2/(a+b*exp(x))^3-1/2/b^2/(a+b*exp(x))^2
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{e^{2x}}{(a+be^x)^4} dx = \frac{-a-3be^x}{6b^2(a+be^x)^3}$$

input
```
Integrate[E^(2*x)/(a + b*E^x)^4,x]
```

output
```
(-a - 3*b*E^x)/(6*b^2*(a + b*E^x)^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{2x}}{(a+be^x)^4} \, dx$$

$$\Big\downarrow \text{2678}$$

$$\int \frac{e^x}{(a+be^x)^4}de^x$$

$$\Big\downarrow \text{53}$$

$$\int \left(\frac{1}{b\,(a+be^x)^3}-\frac{a}{b\,(a+be^x)^4}\right)de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{a}{3b^2\,(a+be^x)^3}-\frac{1}{2b^2\,(a+be^x)^2}$$

input
```
Int[E^(2*x)/(a + b*E^x)^4,x]
```

output
```
a/(3*b^2*(a + b*E^x)^3) - 1/(2*b^2*(a + b*E^x)^2)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{3b\,e^{x}+a}{6b^{2}\left(a+b\,e^{x}\right)^{3}}$	21
norman	$\frac{-\frac{a}{6b^{2}}-\frac{e^{x}}{2b}}{\left(a+b\,e^{x}\right)^{3}}$	24
default	$\frac{a}{3b^{2}\left(a+b\,e^{x}\right)^{3}}-\frac{1}{2b^{2}\left(a+b\,e^{x}\right)^{2}}$	29
parallelrisch	$\frac{b^{4}e^{x}e^{2x}+3\,e^{2x}b^{3}a}{6b^{3}a^{2}\left(a+b\,e^{x}\right)^{3}}$	38

input
```
int(exp(2*x)/(a+b*exp(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*b*exp(x)+a)/b^2/(a+b*exp(x))^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int \frac{e^{2x}}{(a+be^x)^4}\,dx=-\frac{3\,be^{x}+a}{6\,\left(b^{5}e^{(3\,x)}+3\,ab^{4}e^{(2\,x)}+3\,a^{2}b^{3}e^{x}+a^{3}b^{2}\right)}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^4,x, algorithm="fricas")
```

output
```
-1/6*(3*b*e^x + a)/(b^5*e^(3*x) + 3*a*b^4*e^(2*x) + 3*a^2*b^3*e^x + a^3*b^
2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.50

$$\int \frac{e^{2x}}{(a+be^x)^4}\,dx = \frac{-a-3be^x}{6a^3b^2+18a^2b^3e^x+18ab^4e^{2x}+6b^5e^{3x}}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))**4,x)
```

output
```
(-a - 3*b*exp(x))/(6*a**3*b**2 + 18*a**2*b**3*exp(x) + 18*a*b**4*exp(2*x)
+ 6*b**5*exp(3*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(28) = 56$.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.50

$$\int \frac{e^{2x}}{(a+be^x)^4}\,dx = -\frac{be^x}{2\left(b^5e^{(3x)}+3ab^4e^{(2x)}+3a^2b^3e^x+a^3b^2\right)} - \frac{a}{6\left(b^5e^{(3x)}+3ab^4e^{(2x)}+3a^2b^3e^x+a^3b^2\right)}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^4,x, algorithm="maxima")
```

output
```
-1/2*b*e^x/(b^5*e^(3*x) + 3*a*b^4*e^(2*x) + 3*a^2*b^3*e^x + a^3*b^2) - 1/6
*a/(b^5*e^(3*x) + 3*a*b^4*e^(2*x) + 3*a^2*b^3*e^x + a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{e^{2x}}{(a+be^x)^4}\,dx = -\frac{3\,be^x+a}{6\,(be^x+a)^3b^2}$$

input
```
integrate(exp(2*x)/(a+b*exp(x))^4,x, algorithm="giac")
```

output
```
-1/6*(3*b*e^x + a)/((b*e^x + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{e^{2x}}{(a+be^x)^4}\,dx = \frac{\frac{e^{2\,x}}{2\,a}+\frac{b\,e^{3\,x}}{6\,a^2}}{a^3+3\,e^x\,a^2\,b+3\,e^{2\,x}\,a\,b^2+e^{3\,x}\,b^3}$$

input
```
int(exp(2*x)/(a + b*exp(x))^4,x)
```

output
```
(exp(2*x)/(2*a) + (b*exp(3*x))/(6*a^2))/(b^3*exp(3*x) + a^3 + 3*a^2*b*exp(
x) + 3*a*b^2*exp(2*x))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{e^{2x}}{(a+be^x)^4}\,dx = \frac{-3e^x b-a}{6b^2\,(e^{3x}b^3+3e^{2x}a\,b^2+3e^x a^2 b+a^3)}$$

input
```
int(exp(2*x)/(a+b*exp(x))^4,x)
```

output
```
( - 3*e**x*b - a)/(6*b**2*(e**(3*x)*b**3 + 3*e**(2*x)*a*b**2 + 3*e**x*a**2
*b + a**3))
```

3.78 $\int \frac{e^{4x}}{a+be^{2x}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 31

$$\int \frac{e^{4x}}{a+be^{2x}} \, dx = \frac{e^{2x}}{2b} - \frac{a \log \left(a+be^{2x}\right)}{2b^2}$$

output
```
1/2*exp(2*x)/b-1/2*a*ln(a+b*exp(2*x))/b^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{e^{4x}}{a+be^{2x}} \, dx = \frac{be^{2x} - a \log \left(a+be^{2x}\right)}{2b^2}$$

input
```
Integrate[E^(4*x)/(a + b*E^(2*x)),x]
```

output
```
(b*E^(2*x) - a*Log[a + b*E^(2*x)])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{1}{2}\int \frac{e^{2x}}{a+be^{2x}}de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{1}{b}-\frac{a}{b\left(a+be^{2x}\right)}\right)de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{e^{2x}}{b}-\frac{a\log\left(a+be^{2x}\right)}{b^2}\right)$$

input
```
Int[E^(4*x)/(a + b*E^(2*x)),x]
```

output
```
(E^(2*x)/b - (a*Log[a + b*E^(2*x)])/b^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{{\mathrm e}^{2 x}}{2 b}-\frac{a \ln \left(a +b \,{\mathrm e}^{2 x}\right)}{2 b^{2}}$	26
norman	$\frac{{\mathrm e}^{2 x}}{2 b}-\frac{a \ln \left(a +b \,{\mathrm e}^{2 x}\right)}{2 b^{2}}$	26
risch	$\frac{{\mathrm e}^{2 x}}{2 b}-\frac{a \ln \left({\mathrm e}^{2 x}+\frac{a}{b}\right)}{2 b^{2}}$	28

input
```
int(exp(4*x)/(a+b*exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(x)^2/b-1/2/b^2*a*ln(a+b*exp(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx=\frac{be^{(2\,x)}-a\log\left(be^{(2\,x)}+a\right)}{2\,b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*(b*e^(2*x) - a*log(b*e^(2*x) + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx = -\frac{a\log\left(\frac{a}{b}+e^{2x}\right)}{2b^2} + \begin{cases} \frac{e^{2x}}{2b} & \text{for } b \neq 0 \\ \frac{x}{b} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x)),x)
```

output
```
-a*log(a/b + exp(2*x))/(2*b**2) + Piecewise((exp(2*x)/(2*b), Ne(b, 0)), (x
/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx = \frac{e^{(2x)}}{2b} - \frac{a\log\left(be^{(2x)}+a\right)}{2b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*e^(2*x)/b - 1/2*a*log(b*e^(2*x) + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx = \frac{e^{(2x)}}{2b} - \frac{a\log\left(\left|be^{(2x)}+a\right|\right)}{2b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x)),x, algorithm="giac")
```

output
```
1/2*e^(2*x)/b - 1/2*a*log(abs(b*e^(2*x) + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 23.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx = -\frac{a\,\ln\left(a+b\,\mathrm{e}^{2\,x}\right)-b\,\mathrm{e}^{2\,x}}{2\,b^2}$$

input
```
int(exp(4*x)/(a + b*exp(2*x)),x)
```

output
```
-(a*log(a + b*exp(2*x)) - b*exp(2*x))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{e^{4x}}{a+be^{2x}}\,dx = \frac{e^{2x}b-\log(e^{2x}b+a)\,a}{2b^2}$$

input
```
int(exp(4*x)/(a+b*exp(2*x)),x)
```

output
```
(e**(2*x)*b - log(e**(2*x)*b + a)*a)/(2*b**2)
```

3.79 $\int \frac{e^{4x}}{\left(a+be^{2x}\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{e^{4x}}{\left(a+be^{2x}\right)^2}\,dx=\frac{a}{2b^2\left(a+be^{2x}\right)}+\frac{\log\left(a+be^{2x}\right)}{2b^2}$$

output
```
1/2*a/b^2/(a+b*exp(2*x))+1/2*ln(a+b*exp(2*x))/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{e^{4x}}{\left(a+be^{2x}\right)^2}\,dx=\frac{\frac{a}{a+be^{2x}}+\log\left(b\left(a+be^{2x}\right)\right)}{2b^2}$$

input
```
Integrate[E^(4*x)/(a + b*E^(2*x))^2,x]
```

output
```
(a/(a + b*E^(2*x)) + Log[b*(a + b*E^(2*x))])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{(a+be^{2x})^2}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{1}{2}\int \frac{e^{2x}}{(a+be^{2x})^2}de^{2x}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{1}{b(a+be^{2x})}-\frac{a}{b(a+be^{2x})^2}\right)de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a}{b^2(a+be^{2x})}+\frac{\log(a+be^{2x})}{b^2}\right)$$

input
```
Int[E^(4*x)/(a + b*E^(2*x))^2,x]
```

output
```
(a/(b^2*(a + b*E^(2*x))) + Log[a + b*E^(2*x)]/b^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{a}{2b^{2}\left(a+b\,{\mathrm e}^{2x}\right)}+\frac{\ln\left(a+b\,{\mathrm e}^{2x}\right)}{2b^{2}}$	32
norman	$\frac{a}{2b^{2}\left(a+b\,{\mathrm e}^{2x}\right)}+\frac{\ln\left(a+b\,{\mathrm e}^{2x}\right)}{2b^{2}}$	32
risch	$\frac{a}{2b^{2}\left(a+b\,{\mathrm e}^{2x}\right)}+\frac{\ln\left({\mathrm e}^{2x}+\frac{a}{b}\right)}{2b^{2}}$	34

input
```
int(exp(4*x)/(a+b*exp(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*a/(a+b*exp(x)^2)+1/2/b^2*ln(a+b*exp(x)^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{e^{4x}}{\left(a+be^{2x}\right)^2}\,dx = \frac{\left(be^{(2\,x)}+a\right)\log\left(be^{(2\,x)}+a\right)+a}{2\left(b^3e^{(2\,x)}+ab^2\right)}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^2,x, algorithm="fricas")
```

output
```
1/2*((b*e^(2*x) + a)*log(b*e^(2*x) + a) + a)/(b^3*e^(2*x) + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{e^{4x}}{(a+be^{2x})^2}\,dx = \frac{a}{2ab^2+2b^3e^{2x}} + \frac{\log\left(\frac{a}{b}+e^{2x}\right)}{2b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))**2,x)
```

output
```
a/(2*a*b**2 + 2*b**3*exp(2*x)) + log(a/b + exp(2*x))/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{e^{4x}}{(a+be^{2x})^2}\,dx = \frac{a}{2\,(b^3e^{(2\,x)}+ab^2)} + \frac{\log\left(be^{(2\,x)}+a\right)}{2\,b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^2,x, algorithm="maxima")
```

output
```
1/2*a/(b^3*e^(2*x) + a*b^2) + 1/2*log(b*e^(2*x) + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{e^{4x}}{(a+be^{2x})^2}\,dx = \frac{\log\left(\left|be^{(2\,x)}+a\right|\right)}{2\,b^2} + \frac{a}{2\,(be^{(2\,x)}+a)b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^2,x, algorithm="giac")
```

output
```
1/2*log(abs(b*e^(2*x) + a))/b^2 + 1/2*a/((b*e^(2*x) + a)*b^2)
```

Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{e^{4x}}{(a+be^{2x})^2}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^{2\,x}\right)}{2\,b^2} - \frac{\mathrm{e}^{2\,x}}{2\,b\,\left(a+b\,\mathrm{e}^{2\,x}\right)}$$

input
```
int(exp(4*x)/(a + b*exp(2*x))^2,x)
```

output
```
log(a + b*exp(2*x))/(2*b^2) - exp(2*x)/(2*b*(a + b*exp(2*x)))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.46

$$\int \frac{e^{4x}}{(a+be^{2x})^2}\,dx = \frac{e^{2x}\log(e^{2x}b+a)\,b - e^{2x}b + \log(e^{2x}b+a)\,a}{2b^2\,(e^{2x}b+a)}$$

input
```
int(exp(4*x)/(a+b*exp(2*x))^2,x)
```

output
```
(e**(2*x)*log(e**(2*x)*b + a)*b - e**(2*x)*b + log(e**(2*x)*b + a)*a)/(2*b
**2*(e**(2*x)*b + a))
```

3.80 $\int \frac{e^{4x}}{(a+be^{2x})^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{e^{4x}}{(a+be^{2x})^3} dx = \frac{e^{4x}}{4a(a+be^{2x})^2}$$

output
```
1/4*exp(4*x)/a/(a+b*exp(2*x))^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30

$$\int \frac{e^{4x}}{(a+be^{2x})^3} dx = \frac{-a-2be^{2x}}{4b^2(a+be^{2x})^2}$$

input
```
Integrate[E^(4*x)/(a + b*E^(2*x))^3,x]
```

output
```
(-a - 2*b*E^(2*x))/(4*b^2*(a + b*E^(2*x))^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2678, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{(a+be^{2x})^3}dx$$
$$\Big\downarrow \text{ 2678}$$
$$\frac{1}{2}\int \frac{e^{2x}}{(a+be^{2x})^3}de^{2x}$$
$$\Big\downarrow \text{ 48}$$
$$\frac{e^{4x}}{4a(a+be^{2x})^2}$$

input
```
Int[E^(4*x)/(a + b*E^(2*x))^3,x]
```

output
```
E^(4*x)/(4*a*(a + b*E^(2*x))^2)
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
parallelrisch	$\frac{e^{4x}}{4a(a+b\,e^{2x})^2}$	20
risch	$-\frac{2b\,e^{2x}+a}{4b^2(a+b\,e^{2x})^2}$	25
norman	$\frac{-\frac{a}{4b^2}-\frac{e^{2x}}{2b}}{(a+b\,e^{2x})^2}$	28
default	$-\frac{1}{2b^2(a+b\,e^{2x})}+\frac{a}{4b^2(a+b\,e^{2x})^2}$	33

input
```
int(exp(4*x)/(a+b*exp(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(4*x)/a/(a+b*exp(2*x))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{e^{4x}}{(a+be^{2x})^3}\,dx = -\frac{2\,be^{(2\,x)}+a}{4\,(b^4e^{(4\,x)}+2\,ab^3e^{(2\,x)}+a^2b^2)}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*e^(2*x) + a)/(b^4*e^(4*x) + 2*a*b^3*e^(2*x) + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{e^{4x}}{(a+be^{2x})^3}\,dx = \frac{-a-2be^{2x}}{4a^2b^2+8ab^3e^{2x}+4b^4e^{4x}}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))**3,x)
```

output
```
(-a - 2*b*exp(2*x))/(4*a**2*b**2 + 8*a*b**3*exp(2*x) + 4*b**4*exp(4*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{e^{4x}}{(a+be^{2x})^3}\,dx = -\frac{be^{(2x)}}{2\left(b^4e^{(4x)}+2\,ab^3e^{(2x)}+a^2b^2\right)} - \frac{a}{4\left(b^4e^{(4x)}+2\,ab^3e^{(2x)}+a^2b^2\right)}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^3,x, algorithm="maxima")
```

output
```
-1/2*b*e^(2*x)/(b^4*e^(4*x) + 2*a*b^3*e^(2*x) + a^2*b^2) - 1/4*a/(b^4*e^(4
*x) + 2*a*b^3*e^(2*x) + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{e^{4x}}{(a+be^{2x})^3}\,dx = -\frac{2\,be^{(2x)}+a}{4\left(be^{(2x)}+a\right)^2b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^3,x, algorithm="giac")
```

output
```
-1/4*(2*b*e^(2*x) + a)/((b*e^(2*x) + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{e^{4x}}{(a+be^{2x})^3}\,dx = \frac{e^{4\,x}}{4\,a\,\left(a^2+2\,e^{2\,x}\,a\,b+e^{4\,x}\,b^2\right)}$$

input
```
int(exp(4*x)/(a + b*exp(2*x))^3,x)
```

output
```
exp(4*x)/(4*a*(b^2*exp(4*x) + a^2 + 2*a*b*exp(2*x)))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.48

$$\int \frac{e^{4x}}{(a+be^{2x})^3}\,dx = \frac{e^{4x}}{4a\left(e^{4x}b^2+2e^{2x}ab+a^2\right)}$$

input
```
int(exp(4*x)/(a+b*exp(2*x))^3,x)
```

output
```
e**(4*x)/(4*a*(e**(4*x)*b**2 + 2*e**(2*x)*a*b + a**2))
```

3.81 $\int \frac{e^{4x}}{(a+be^{2x})^4} dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx=\frac{a}{6b^2(a+be^{2x})^3}-\frac{1}{4b^2(a+be^{2x})^2}$$

output
```
1/6*a/b^2/(a+b*exp(2*x))^3-1/4/b^2/(a+b*exp(2*x))^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx=\frac{-a-3be^{2x}}{12b^2(a+be^{2x})^3}$$

input
```
Integrate[E^(4*x)/(a + b*E^(2*x))^4,x]
```

output
```
(-a - 3*b*E^(2*x))/(12*b^2*(a + b*E^(2*x))^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{(a+be^{2x})^4}dx$$

$$\Big\downarrow \ 2678$$

$$\frac{1}{2}\int \frac{e^{2x}}{(a+be^{2x})^4}de^{2x}$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{1}{b(a+be^{2x})^3}-\frac{a}{b(a+be^{2x})^4}\right)de^{2x}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a}{3b^2(a+be^{2x})^3}-\frac{1}{2b^2(a+be^{2x})^2}\right)$$

input
```
Int[E^(4*x)/(a + b*E^(2*x))^4,x]
```

output
```
(a/(3*b^2*(a + b*E^(2*x))^3) - 1/(2*b^2*(a + b*E^(2*x))^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{3b\,e^{2x}+a}{12b^{2}\left(a+b\,e^{2x}\right)^{3}}$	25
norman	$\frac{-\frac{a}{12b^{2}}-\frac{e^{2x}}{4b}}{\left(a+b\,e^{2x}\right)^{3}}$	28
default	$\frac{a}{6b^{2}\left(a+b\,e^{2x}\right)^{3}}-\frac{1}{4b^{2}\left(a+b\,e^{2x}\right)^{2}}$	33
parallelrisch	$\frac{b^{4}e^{2x}e^{4x}+3\,e^{4x}b^{3}a}{12b^{3}a^{2}\left(a+b\,e^{2x}\right)^{3}}$	42

input
```
int(exp(4*x)/(a+b*exp(2*x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*b*exp(2*x)+a)/b^2/(a+b*exp(2*x))^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx=-\frac{3\,be^{(2\,x)}+a}{12\,(b^5e^{(6\,x)}+3\,ab^4e^{(4\,x)}+3\,a^2b^3e^{(2\,x)}+a^3b^2)}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^4,x, algorithm="fricas")
```

output
```
-1/12*(3*b*e^(2*x) + a)/(b^5*e^(6*x) + 3*a*b^4*e^(4*x) + 3*a^2*b^3*e^(2*x)
 + a^3*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx = \frac{-a-3be^{2x}}{12a^3b^2+36a^2b^3e^{2x}+36ab^4e^{4x}+12b^5e^{6x}}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))**4,x)
```

output
```
(-a - 3*b*exp(2*x))/(12*a**3*b**2 + 36*a**2*b**3*exp(2*x) + 36*a*b**4*exp(
4*x) + 12*b**5*exp(6*x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(32) = 64.

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.39

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx = -\frac{be^{(2x)}}{4\left(b^5e^{(6x)}+3ab^4e^{(4x)}+3a^2b^3e^{(2x)}+a^3b^2\right)} - \frac{a}{12\left(b^5e^{(6x)}+3ab^4e^{(4x)}+3a^2b^3e^{(2x)}+a^3b^2\right)}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^4,x, algorithm="maxima")
```

output
```
-1/4*b*e^(2*x)/(b^5*e^(6*x) + 3*a*b^4*e^(4*x) + 3*a^2*b^3*e^(2*x) + a^3*b^
2) - 1/12*a/(b^5*e^(6*x) + 3*a*b^4*e^(4*x) + 3*a^2*b^3*e^(2*x) + a^3*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx = -\frac{3\,be^{(2\,x)}+a}{12\,(be^{(2\,x)}+a)^3b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^4,x, algorithm="giac")
```

output
```
-1/12*(3*b*e^(2*x) + a)/((b*e^(2*x) + a)^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 23.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx = \frac{\frac{\mathrm{e}^{4\,x}}{4\,a}+\frac{b\,\mathrm{e}^{6\,x}}{12\,a^2}}{a^3+3\,\mathrm{e}^{2\,x}\,a^2\,b+3\,\mathrm{e}^{4\,x}\,a\,b^2+\mathrm{e}^{6\,x}\,b^3}$$

input
```
int(exp(4*x)/(a + b*exp(2*x))^4,x)
```

output
```
(exp(4*x)/(4*a) + (b*exp(6*x))/(12*a^2))/(b^3*exp(6*x) + a^3 + 3*a^2*b*exp
(2*x) + 3*a*b^2*exp(4*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.42

$$\int \frac{e^{4x}}{(a+be^{2x})^4}\,dx = \frac{-3e^{2x}b-a}{12b^2\left(e^{6x}b^3+3e^{4x}a\,b^2+3e^{2x}a^2b+a^3\right)}$$

input
```
int(exp(4*x)/(a+b*exp(2*x))^4,x)
```

output
```
( - 3*e**(2*x)*b - a)/(12*b**2*(e**(6*x)*b**3 + 3*e**(4*x)*a*b**2 + 3*e**(
2*x)*a**2*b + a**3))
```

3.82 $\int \frac{e^{4x}}{(a+be^{2x})^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{e^{4x}}{(a+be^{2x})^{2/3}} \, dx = -\frac{3a\sqrt[3]{a+be^{2x}}}{2b^2} + \frac{3(a+be^{2x})^{4/3}}{8b^2}$$

output
```
-3/2*a*(a+b*exp(2*x))^(1/3)/b^2+3/8*(a+b*exp(2*x))^(4/3)/b^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{e^{4x}}{(a+be^{2x})^{2/3}} \, dx = \frac{3(-3a+be^{2x})\sqrt[3]{a+be^{2x}}}{8b^2}$$

input
```
Integrate[E^(4*x)/(a + b*E^(2*x))^(2/3),x]
```

output
```
(3*(-3*a + b*E^(2*x))*(a + b*E^(2*x))^(1/3))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2678, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{4x}}{(a+be^{2x})^{2/3}}\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{1}{2}\int \frac{e^{2x}}{(a+be^{2x})^{2/3}}de^{2x}$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{\sqrt[3]{a+be^{2x}}}{b}-\frac{a}{b\,(a+be^{2x})^{2/3}}\right)de^{2x}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3\left(a+be^{2x}\right)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+be^{2x}}}{b^2}\right)$$

input
```
Int[E^(4*x)/(a + b*E^(2*x))^(2/3),x]
```

output
```
((-3*a*(a + b*E^(2*x))^(1/3))/b^2 + (3*(a + b*E^(2*x))^(4/3))/(4*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{3\left(a+b\,{\mathrm e}^{2 x}\right)^{\frac{1}{3}} \left(-b\,{\mathrm e}^{2 x}+3 a \right)}{8 b^{2}}$	27

input
```
int(exp(4*x)/(a+b*exp(2*x))^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*(a+b*exp(2*x))^(1/3)*(-b*exp(2*x)+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.60

$$\int \frac{e^{4x}}{\left(a+be^{2x}\right)^{2/3}}\,dx=\frac{3\,\left(be^{\left(2\,x\right)}+a\right)^{\frac{1}{3}}\left(be^{\left(2\,x\right)}-3\,a\right)}{8\,b^{2}}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^(2/3),x, algorithm="fricas")
```

output
```
3/8*(b*e^(2*x) + a)^(1/3)*(b*e^(2*x) - 3*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10

$$\int \frac{e^{4x}}{(a+be^{2x})^{2/3}}\,dx = \frac{\begin{cases} \frac{3\left(-a\sqrt[3]{a+be^{2x}}+\frac{(a+be^{2x})^{\frac{4}{3}}}{4}\right)}{b^2} & \text{for } b\neq 0 \\ \frac{e^{4x}}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))**(2/3),x)
```

output
```
Piecewise((3*(-a*(a + b*exp(2*x))**(1/3) + (a + b*exp(2*x))**(4/3)/4)/b**2
, Ne(b, 0)), (exp(4*x)/(2*a**(2/3)), True))/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{e^{4x}}{(a+be^{2x})^{2/3}}\,dx = \frac{3\left(be^{(2x)}+a\right)^{\frac{4}{3}}}{8b^2} - \frac{3\left(be^{(2x)}+a\right)^{\frac{1}{3}}a}{2b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^(2/3),x, algorithm="maxima")
```

output
```
3/8*(b*e^(2*x) + a)^(4/3)/b^2 - 3/2*(b*e^(2*x) + a)^(1/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{e^{4x}}{(a+be^{2x})^{2/3}}\,dx = \frac{3\left(be^{(2x)}+a\right)^{\frac{4}{3}}}{8b^2} - \frac{3\left(be^{(2x)}+a\right)^{\frac{1}{3}}a}{2b^2}$$

input
```
integrate(exp(4*x)/(a+b*exp(2*x))^(2/3),x, algorithm="giac")
```

output
```
3/8*(b*e^(2*x) + a)^(4/3)/b^2 - 3/2*(b*e^(2*x) + a)^(1/3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 23.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{e^{4x}}{\left(a+be^{2x}\right)^{2/3}}\,dx = -\frac{3\,\left(3\,a-b\,\mathrm{e}^{2\,x}\right)\,{\left(a+b\,\mathrm{e}^{2\,x}\right)}^{1/3}}{8\,b^2}$$

input
```
int(exp(4*x)/(a + b*exp(2*x))^(2/3),x)
```

output
```
-(3*(3*a - b*exp(2*x))*(a + b*exp(2*x))^(1/3))/(8*b^2)
```

Reduce [F]

$$\int \frac{e^{4x}}{\left(a+be^{2x}\right)^{2/3}}\,dx = \int \frac{e^{4x}}{\left(e^{2x}b+a\right)^{\frac{2}{3}}}dx$$

input
```
int(exp(4*x)/(a+b*exp(2*x))^(2/3),x)
```

output
```
int(e**(4*x)/(e**(2*x)*b + a)**(2/3),x)
```

3.83 $\int e^{-nx}(a + be^{nx}) \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int e^{-nx}(a + be^{nx}) \, dx = -\frac{ae^{-nx}}{n} + bx$$

output
```
-a/exp(n*x)/n+b*x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int e^{-nx}(a + be^{nx}) \, dx = -\frac{ae^{-nx}}{n} + bx$$

input
```
Integrate[(a + b*E^(n*x))/E^(n*x),x]
```

output
```
-(a/(E^(n*x)*n)) + b*x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-nx}(a+be^{nx})\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int e^{-2nx}(a+be^{nx})\,de^{nx}}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(e^{-2nx}a+be^{-nx}\right)de^{nx}}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\log\left(e^{nx}\right)-ae^{-nx}}{n}$$

input
```
Int[(a + b*E^(n*x))/E^(n*x),x]
```

output
```
(-(a/E^(n*x)) + b*Log[E^(n*x)])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{a\,{\mathrm e}^{-x n}}{n}+b x$	16
parts	$-\frac{a\,{\mathrm e}^{-x n}}{n}+b x$	17
derivativedivides	$\frac{b \ln\left({\mathrm e}^{x n}\right)-a\,{\mathrm e}^{-x n}}{n}$	22
default	$\frac{b \ln\left({\mathrm e}^{x n}\right)-a\,{\mathrm e}^{-x n}}{n}$	22
norman	$\left(b x\,{\mathrm e}^{x n}-\frac{a}{n}\right) {\mathrm e}^{-x n}$	22
parallelrisch	$\frac{\left(x\,{\mathrm e}^{x n} b n-a\right) {\mathrm e}^{-x n}}{n}$	23
orering	$\frac{\left(x n-1\right) \left(a+b\,{\mathrm e}^{x n}\right) {\mathrm e}^{-x n}}{n}+\frac{x \left(b n-\left(a+b\,{\mathrm e}^{x n}\right) {\mathrm e}^{-x n} n\right)}{n}$	51

input
```
int((a+b*exp(x*n))/exp(x*n),x,method=_RETURNVERBOSE)
```

output
```
-a*exp(-x*n)/n+b*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int e^{-nx}(a+be^{nx})\,dx=\frac{\left(bnxe^{(nx)}-a\right)e^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))/exp(n*x),x, algorithm="fricas")
```

output
```
(b*n*x*e^(n*x) - a)*e^(-n*x)/n
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int e^{-nx}(a+be^{nx})\,dx = bx + \begin{cases} -\frac{ae^{-nx}}{n} & \text{for } n \neq 0 \\ ax & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*exp(n*x))/exp(n*x),x)
```

output
```
b*x + Piecewise((-a*exp(-n*x)/n, Ne(n, 0)), (a*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int e^{-nx}(a+be^{nx})\,dx = bx - \frac{ae^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))/exp(n*x),x, algorithm="maxima")
```

output
```
b*x - a*e^(-n*x)/n
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int e^{-nx}(a+be^{nx})\,dx = \frac{bnx - ae^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))/exp(n*x),x, algorithm="giac")
```

output
```
(b*n*x - a*e^(-n*x))/n
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int e^{-nx}(a + be^{nx})\ dx = b\,x - \frac{a\,\mathrm{e}^{-n\,x}}{n}$$

input
```
int(exp(-n*x)*(a + b*exp(n*x)),x)
```

output
```
b*x - (a*exp(-n*x))/n
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int e^{-nx}(a + be^{nx})\ dx = \frac{e^{nx}bnx - a}{e^{nx}n}$$

input
```
int((a+b*exp(n*x))/exp(n*x),x)
```

output
```
(e**(n*x)*b*n*x - a)/(e**(n*x)*n)
```

3.84 $\int e^{-nx}(a+be^{nx})^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 32

$$\int e^{-nx}(a+be^{nx})^2\,dx=-\frac{a^2e^{-nx}}{n}+\frac{b^2e^{nx}}{n}+2abx$$

output
```
-a^2/exp(n*x)/n+b^2*exp(n*x)/n+2*a*b*x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int e^{-nx}(a+be^{nx})^2\,dx=\frac{-a^2e^{-nx}+b^2e^{nx}-2ab\log\left(e^{-nx}\right)}{n}$$

input
```
Integrate[(a + b*E^(n*x))^2/E^(n*x),x]
```

output
```
(-(a^2/E^(n*x)) + b^2*E^(n*x) - 2*a*b*Log[E^(-(n*x))])/n
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-nx}(a+be^{nx})^2\,dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{\int e^{-2nx}(a+be^{nx})^2\,de^{nx}}{n}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(e^{-2nx}a^2+2be^{-nx}a+b^2\right)de^{nx}}{n}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(-e^{-nx})+2ab\log\left(e^{nx}\right)+b^2e^{nx}}{n}$$

input
```
Int[(a + b*E^(n*x))^2/E^(n*x),x]
```

output
```
(-(a^2/E^(n*x)) + b^2*E^(n*x) + 2*a*b*Log[E^(n*x)])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{a^{2} {\mathrm e}^{-x n}}{n}+\frac{b^{2} {\mathrm e}^{x n}}{n}+2 a b x$
parts	$-\frac{a^{2} {\mathrm e}^{-x n}}{n}+\frac{b^{2} {\mathrm e}^{x n}}{n}+2 a b x$
derivativedivides	$\frac{{\mathrm e}^{x n} b^{2}+2 a b \ln\left({\mathrm e}^{x n}\right)-a^{2} {\mathrm e}^{-x n}}{n}$
default	$\frac{{\mathrm e}^{x n} b^{2}+2 a b \ln\left({\mathrm e}^{x n}\right)-a^{2} {\mathrm e}^{-x n}}{n}$
parallelrisch	$\frac{\left(2 x \,{\mathrm e}^{x n} a b n +{\mathrm e}^{2 x n} b^{2}-a^{2}\right) {\mathrm e}^{-x n}}{n}$
norman	$\left(\frac{b^{2} {\mathrm e}^{2 x n}}{n}-\frac{a^{2}}{n}+2 a b x \,{\mathrm e}^{x n}\right) {\mathrm e}^{-x n}$
orering	$x \left(a +b \,{\mathrm e}^{x n}\right)^{2} {\mathrm e}^{-x n}+\frac{2 \left(a +b \,{\mathrm e}^{x n}\right) b n -\left(a +b \,{\mathrm e}^{x n}\right)^{2} {\mathrm e}^{-x n} n}{n^{2}}-\frac{x \left(2 b^{2} n^{2} {\mathrm e}^{x n}-2 \left(a +b \,{\mathrm e}^{x n}\right) n^{2} b +\left(a +b \,{\mathrm e}^{x n}\right)^{2} {\mathrm e}^{-x n} n^{2}\right)}{n^{2}}$

input
```
int((a+b*exp(x*n))^2/exp(x*n),x,method=_RETURNVERBOSE)
```

output
```
-a^2/exp(x*n)/n+b^2*exp(x*n)/n+2*a*b*x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int e^{-nx}(a+be^{nx})^2\,dx=\frac{\left(2\,abnxe^{(nx)}+b^2e^{(2\,nx)}-a^2\right)e^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))^2/exp(n*x),x, algorithm="fricas")
```

output
```
(2*a*b*n*x*e^(n*x) + b^2*e^(2*n*x) - a^2)*e^(-n*x)/n
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int e^{-nx}(a+be^{nx})^2\,dx = 2abx + \begin{cases} \frac{-a^2ne^{-nx}+b^2ne^{nx}}{n^2} & \text{for } n^2 \neq 0 \\ x(a^2+b^2) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*exp(n*x))**2/exp(n*x),x)
```

output
```
2*a*b*x + Piecewise(((-a**2*n*exp(-n*x) + b**2*n*exp(n*x))/n**2, Ne(n**2,
0)), (x*(a**2 + b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int e^{-nx}(a+be^{nx})^2\,dx = 2\,abx + \frac{b^2e^{(nx)}}{n} - \frac{a^2e^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))^2/exp(n*x),x, algorithm="maxima")
```

output
```
2*a*b*x + b^2*e^(n*x)/n - a^2*e^(-n*x)/n
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.91

$$\int e^{-nx}(a+be^{nx})^2\,dx = \frac{2\,abnx + b^2e^{(nx)} - a^2e^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))^2/exp(n*x),x, algorithm="giac")
```

output
```
(2*a*b*n*x + b^2*e^(n*x) - a^2*e^(-n*x))/n
```

Mupad [B] (verification not implemented)

Time = 23.76 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int e^{-nx}(a+be^{nx})^2\,dx = 2\,a\,b\,x - \frac{\mathrm{e}^{-n\,x}\,\left(a^2 - b^2\,\mathrm{e}^{2\,n\,x}\right)}{n}$$

input
```
int(exp(-n*x)*(a + b*exp(n*x))^2,x)
```

output
```
2*a*b*x - (exp(-n*x)*(a^2 - b^2*exp(2*n*x)))/n
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int e^{-nx}(a+be^{nx})^2\,dx = \frac{e^{2nx}b^2 + 2e^{nx}abnx - a^2}{e^{nx}n}$$

input
```
int((a+b*exp(n*x))^2/exp(n*x),x)
```

output
```
(e**(2*n*x)*b**2 + 2*e**(n*x)*a*b*n*x - a**2)/(e**(n*x)*n)
```

3.85 $\int e^{-nx}(a+be^{nx})^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 52

$$\int e^{-nx}(a+be^{nx})^3\,dx=-\frac{a^3e^{-nx}}{n}+\frac{3ab^2e^{nx}}{n}+\frac{b^3e^{2nx}}{2n}+3a^2bx$$

output
```
-a^3/exp(n*x)/n+3*a*b^2*exp(n*x)/n+1/2*b^3*exp(2*n*x)/n+3*a^2*b*x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int e^{-nx}(a+be^{nx})^3\,dx=\frac{-2a^3e^{-nx}+6ab^2e^{nx}+b^3e^{2nx}-6a^2b\log\left(e^{-nx}\right)}{2n}$$

input
```
Integrate[(a + b*E^(n*x))^3/E^(n*x),x]
```

output
```
((-2*a^3)/E^(n*x) + 6*a*b^2*E^(n*x) + b^3*E^(2*n*x) - 6*a^2*b*Log[E^(-(n*x
))])/(2*n)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-nx}(a+be^{nx})^3\,dx$$

$$\Big\downarrow \text{ 2678}$$

$$\frac{\int e^{-2nx}(a+be^{nx})^3\,de^{nx}}{n}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(e^{-2nx}a^3+3be^{-nx}a^2+3b^2a+b^3e^{nx}\right)de^{nx}}{n}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3(-e^{-nx})+3a^2b\log\left(e^{nx}\right)+3ab^2e^{nx}+\frac{1}{2}b^3e^{2nx}}{n}$$

input
```
Int[(a + b*E^(n*x))^3/E^(n*x),x]
```

output
```
(-(a^3/E^(n*x)) + 3*a*b^2*E^(n*x) + (b^3*E^(2*n*x))/2 + 3*a^2*b*Log[E^(n*x
)])/n
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{b^3 e^{2xn}}{2}+3\,e^{xn} a\,b^2-a^3 e^{-xn}+3a^2 b\ln\left(e^{xn}\right)}{n}$
default	$\frac{\frac{b^3 e^{2xn}}{2}+3\,e^{xn} a\,b^2-a^3 e^{-xn}+3a^2 b\ln\left(e^{xn}\right)}{n}$
risch	$3a^2bx+\frac{b^3 e^{2xn}}{2n}+\frac{3a\,b^2 e^{xn}}{n}-\frac{a^3 e^{-xn}}{n}$
parts	$3a^2bx+\frac{b^3 e^{2xn}}{2n}+\frac{3a\,b^2 e^{xn}}{n}-\frac{a^3 e^{-xn}}{n}$
parallelrisch	$\frac{\left(6x\,e^{xn} a^2 bn+e^{3xn} b^3+6\,e^{2xn} a\,b^2-2a^3\right) e^{-xn}}{2n}$
norman	$\left(-\frac{a^3}{n}+\frac{b^3 e^{3xn}}{2n}+\frac{3a\,b^2 e^{2xn}}{n}+3a^2bx\,e^{xn}\right) e^{-xn}$
orering	$\frac{(2xn+1)(a+b\,e^{xn})^3 e^{-xn}}{2n}-\frac{(xn-2)\left(3(a+b\,e^{xn})^2 bn-(a+b\,e^{xn})^3 e^{-xn} n\right)}{2n^2}-\frac{(2xn+1)\left(6(a+b\,e^{xn})b^2n^2e^{xn}-3(a+b\right.}{2n^3}$

input
```
int((a+b*exp(x*n))^3/exp(x*n),x,method=_RETURNVERBOSE)
```

output
```
1/n*(1/2*b^3*exp(x*n)^2+3*exp(x*n)*a*b^2-a^3/exp(x*n)+3*a^2*b*ln(exp(x*n))
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int e^{-nx}(a+be^{nx})^3\,dx = \frac{\left(6\,a^2bnxe^{(nx)}+b^3e^{(3\,nx)}+6\,ab^2e^{(2\,nx)}-2\,a^3\right)e^{(-nx)}}{2\,n}$$

input
```
integrate((a+b*exp(n*x))^3/exp(n*x),x, algorithm="fricas")
```

output
```
1/2*(6*a^2*b*n*x*e^(n*x) + b^3*e^(3*n*x) + 6*a*b^2*e^(2*n*x) - 2*a^3)*e^(-
n*x)/n
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.37

$$\int e^{-nx}(a+be^{nx})^3\,dx = 3a^2bx + \begin{cases} \frac{-2a^3n^2e^{-nx}+6ab^2n^2e^{nx}+b^3n^2e^{2nx}}{2n^3} & \text{for } n^3\neq 0 \\ x(a^3+3ab^2+b^3) & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*exp(n*x))**3/exp(n*x),x)
```

output
```
3*a**2*b*x + Piecewise(((-2*a**3*n**2*exp(-n*x) + 6*a*b**2*n**2*exp(n*x) +
 b**3*n**2*exp(2*n*x))/(2*n**3), Ne(n**3, 0)), (x*(a**3 + 3*a*b**2 + b**3)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.90

$$\int e^{-nx}(a+be^{nx})^3\,dx = 3\,a^2bx + \frac{b^3e^{(2\,nx)}}{2\,n} + \frac{3\,ab^2e^{(nx)}}{n} - \frac{a^3e^{(-nx)}}{n}$$

input
```
integrate((a+b*exp(n*x))^3/exp(n*x),x, algorithm="maxima")
```

output
```
3*a^2*b*x + 1/2*b^3*e^(2*n*x)/n + 3*a*b^2*e^(n*x)/n - a^3*e^(-n*x)/n
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int e^{-nx}(a+be^{nx})^3\,dx=\frac{6\,a^2bnx+b^3e^{(2\,nx)}+6\,ab^2e^{(nx)}-2\,a^3e^{(-nx)}}{2\,n}$$

input
```
integrate((a+b*exp(n*x))^3/exp(n*x),x, algorithm="giac")
```

output
```
1/2*(6*a^2*b*n*x + b^3*e^(2*n*x) + 6*a*b^2*e^(n*x) - 2*a^3*e^(-n*x))/n
```

Mupad [B] (verification not implemented)

Time = 24.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int e^{-nx}(a+be^{nx})^3\,dx=\frac{\mathrm{e}^{-n\,x}\,\left(-2\,a^3+6\,\mathrm{e}^{2\,n\,x}\,a\,b^2+\mathrm{e}^{3\,n\,x}\,b^3\right)}{2\,n}+3\,a^2\,b\,x$$

input
```
int(exp(-n*x)*(a + b*exp(n*x))^3,x)
```

output
```
(exp(-n*x)*(b^3*exp(3*n*x) - 2*a^3 + 6*a*b^2*exp(2*n*x)))/(2*n) + 3*a^2*b*
x
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int e^{-nx}(a+be^{nx})^3\,dx=\frac{e^{3nx}b^3+6e^{2nx}a\,b^2+6e^{nx}a^2bnx-2a^3}{2e^{nx}n}$$

input
```
int((a+b*exp(n*x))^3/exp(n*x),x)
```

output
```
(e**(3*n*x)*b**3 + 6*e**(2*n*x)*a*b**2 + 6*e**(n*x)*a**2*b*n*x - 2*a**3)/(
2*e**(n*x)*n)
```

3.86 $\int \frac{e^{-nx}}{a+be^{nx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = -\frac{e^{-nx}}{an} - \frac{bx}{a^2} + \frac{b\log\left(a+be^{nx}\right)}{a^2n}$$

output
```
-1/a/exp(n*x)/n-b*x/a^2+b*ln(a+b*exp(n*x))/a^2/n
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = -\frac{e^{-nx}}{an} + \frac{b\log\left(abn+a^2e^{-nx}n\right)}{a^2n}$$

input
```
Integrate[1/(E^(n*x)*(a + b*E^(n*x))),x]
```

output
```
-(1/(a*E^(n*x)*n)) + (b*Log[a*b*n + (a^2*n)/E^(n*x)])/(a^2*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx$$

$$\Big\downarrow \ 2678$$

$$\frac{\int \frac{e^{-2nx}}{a+be^{nx}}de^{nx}}{n}$$

$$\Big\downarrow \ 54$$

$$\frac{\int \left(\frac{b^2}{a^2(a+be^{nx})}-\frac{e^{-nx}b}{a^2}+\frac{e^{-2nx}}{a}\right)de^{nx}}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b\log(e^{nx})}{a^2}+\frac{b\log(a+be^{nx})}{a^2}-\frac{e^{-nx}}{a}}{n}$$

input
```
Int[1/(E^(n*x)*(a + b*E^(n*x))),x]
```

output
```
(-(1/(a*E^(n*x))) - (b*Log[E^(n*x)])/a^2 + (b*Log[a + b*E^(n*x)])/a^2)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{-\frac{e^{-xn}}{a}-\frac{b\ln\left(e^{xn}\right)}{a^2}+\frac{b\ln\left(a+b\,e^{xn}\right)}{a^2}}{n}$	42
default	$\frac{-\frac{e^{-xn}}{a}-\frac{b\ln\left(e^{xn}\right)}{a^2}+\frac{b\ln\left(a+b\,e^{xn}\right)}{a^2}}{n}$	42
risch	$-\frac{e^{-xn}}{an}-\frac{bx}{a^2}+\frac{b\ln\left(e^{xn}+\frac{a}{b}\right)}{a^2n}$	42
parallelrisch	$\frac{\left(-x\,e^{xn}bn+b\ln\left(a+b\,e^{xn}\right)e^{xn}-a\right)e^{-xn}}{a^2n}$	42
norman	$\left(-\frac{1}{an}-\frac{bx\,e^{xn}}{a^2}\right)e^{-xn}+\frac{b\ln\left(a+b\,e^{xn}\right)}{a^2n}$	46

input
```
int(1/exp(x*n)/(a+b*exp(x*n)),x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/a/exp(x*n)-b/a^2*ln(exp(x*n))+b/a^2*ln(a+b*exp(x*n)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = -\frac{\left(bnxe^{(nx)}-be^{(nx)}\log\left(be^{(nx)}+a\right)+a\right)e^{(-nx)}}{a^2n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x)),x, algorithm="fricas")
```

output
```
-(b*n*x*e^(n*x) - b*e^(n*x)*log(b*e^(n*x) + a) + a)*e^(-n*x)/(a^2*n)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = \begin{cases} -\frac{e^{-nx}}{an} & \text{for } an \neq 0 \\ \frac{x}{a} & \text{otherwise} \end{cases} + \frac{b\log\left(e^{-nx}+\frac{b}{a}\right)}{a^2n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x)),x)
```

output
```
Piecewise((-exp(-n*x)/(a*n), Ne(a*n, 0)), (x/a, True)) + b*log(exp(-n*x) +
 b/a)/(a**2*n)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = -\frac{e^{(-nx)}}{an} + \frac{b\log\left(ae^{(-nx)}+b\right)}{a^2n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x)),x, algorithm="maxima")
```

output
```
-e^(-n*x)/(a*n) + b*log(a*e^(-n*x) + b)/(a^2*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = -\frac{bx}{a^2} - \frac{e^{(-nx)}}{an} + \frac{b\log\left(\left|be^{(nx)}+a\right|\right)}{a^2n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x)),x, algorithm="giac")
```

output
```
-b*x/a^2 - e^(-n*x)/(a*n) + b*log(abs(b*e^(n*x) + a))/(a^2*n)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = \frac{b\,\ln\left(a+b\,\mathrm{e}^{n\,x}\right)}{a^2\,n}-\frac{b\,x}{a^2}-\frac{\mathrm{e}^{-n\,x}}{a\,n}$$

input
```
int(exp(-n*x)/(a + b*exp(n*x)),x)
```

output
```
(b*log(a + b*exp(n*x)))/(a^2*n) - (b*x)/a^2 - exp(-n*x)/(a*n)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{e^{-nx}}{a+be^{nx}}\,dx = \frac{e^{nx}\log(e^{nx}b+a)\,b-e^{nx}bnx-a}{e^{nx}a^2n}$$

input
```
int(1/exp(n*x)/(a+b*exp(n*x)),x)
```

output
```
(e**(n*x)*log(e**(n*x)*b + a)*b - e**(n*x)*b*n*x - a)/(e**(n*x)*a**2*n)
```

3.87 $\int \frac{e^{-nx}}{(a+be^{nx})^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 61

$$\int \frac{e^{-nx}}{(a+be^{nx})^2} dx = -\frac{e^{-nx}}{a^2n} - \frac{b}{a^2(a+be^{nx})n} - \frac{2bx}{a^3} + \frac{2b\log(a+be^{nx})}{a^3n}$$

output
```
-1/a^2/exp(n*x)/n-b/a^2/(a+b*exp(n*x))/n-2*b*x/a^3+2*b*ln(a+b*exp(n*x))/a^
3/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{e^{-nx}}{(a+be^{nx})^2} dx = \frac{-\frac{ab+a^2e^{-nx}-b^2e^{nx}}{a+be^{nx}} + 2b\log(b+ae^{-nx})}{a^3n}$$

input
```
Integrate[1/(E^(n*x)*(a + b*E^(n*x))^2),x]
```

output
```
(-((a*b + a^2/E^(n*x) - b^2*E^(n*x))/(a + b*E^(n*x))) + 2*b*Log[b + a/E^(n
*x)])/(a^3*n)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}dx$$
$$\Big\downarrow \text{ 2678}$$
$$\frac{\int \frac{e^{-2nx}}{(a+be^{nx})^2}de^{nx}}{n}$$
$$\Big\downarrow \text{ 54}$$
$$\frac{\int \left(\frac{2b^2}{a^3(a+be^{nx})}+\frac{b^2}{a^2(a+be^{nx})^2}-\frac{2e^{-nx}b}{a^3}+\frac{e^{-2nx}}{a^2}\right)de^{nx}}{n}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{-\frac{2b\log(e^{nx})}{a^3}+\frac{2b\log(a+be^{nx})}{a^3}-\frac{b}{a^2(a+be^{nx})}-\frac{e^{-nx}}{a^2}}{n}$$

input
```
Int[1/(E^(n*x)*(a + b*E^(n*x))^2),x]
```

output
```
(-(1/(a^2*E^(n*x))) - b/(a^2*(a + b*E^(n*x))) - (2*b*Log[E^(n*x)])/a^3 + (
2*b*Log[a + b*E^(n*x)])/a^3)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{{\mathrm e}^{-x n}}{a^{2}}-\frac{2 b \ln \left({\mathrm e}^{x n}\right)}{a^{3}}-\frac{b}{a^{2} \left(a +b \,{\mathrm e}^{x n}\right)}+\frac{2 b \ln \left(a +b \,{\mathrm e}^{x n}\right)}{a^{3}}}{n}$	59
default	$\frac{-\frac{{\mathrm e}^{-x n}}{a^{2}}-\frac{2 b \ln \left({\mathrm e}^{x n}\right)}{a^{3}}-\frac{b}{a^{2} \left(a +b \,{\mathrm e}^{x n}\right)}+\frac{2 b \ln \left(a +b \,{\mathrm e}^{x n}\right)}{a^{3}}}{n}$	59
risch	$-\frac{{\mathrm e}^{-x n}}{a^{2} n}-\frac{2 b x}{a^{3}}-\frac{b}{a^{2} \left(a +b \,{\mathrm e}^{x n}\right) n}+\frac{2 b \ln \left({\mathrm e}^{x n}+\frac{a}{b}\right)}{a^{3} n}$	62
norman	$\frac{\left(-\frac{2 b \,{\mathrm e}^{x n}}{a^{2} n}-\frac{1}{a n}-\frac{2 b x \,{\mathrm e}^{x n}}{a^{2}}-\frac{2 b^{2} x \,{\mathrm e}^{2 x n}}{a^{3}}\right) {\mathrm e}^{-x n}}{a +b \,{\mathrm e}^{x n}}+\frac{2 b \ln \left(a +b \,{\mathrm e}^{x n}\right)}{a^{3} n}$	85
parallelrisch	$\frac{\left(-2 x \,{\mathrm e}^{2 x n} b^{2} n +2 \ln \left(a +b \,{\mathrm e}^{x n}\right) {\mathrm e}^{2 x n} b^{2}-2 x \,{\mathrm e}^{x n} a b n +2 \ln \left(a +b \,{\mathrm e}^{x n}\right) {\mathrm e}^{x n} a b +2 \,{\mathrm e}^{2 x n} b^{2}-a^{2}\right) {\mathrm e}^{-x n}}{a^{3} n \left(a +b \,{\mathrm e}^{x n}\right)}$	101

input
```
int(1/exp(x*n)/(a+b*exp(x*n))^2,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/a^2/exp(x*n)-2/a^3*b*ln(exp(x*n))-b/a^2/(a+b*exp(x*n))+2/a^3*b*ln(
a+b*exp(x*n)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.38

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}\,dx = -\frac{2\,b^2nxe^{(2\,nx)}+a^2+2\,(abnx+ab)e^{(nx)}-2\left(b^2e^{(2\,nx)}+abe^{(nx)}\right)\log\left(be^{(nx)}+a\right)}{a^3bne^{(2\,nx)}+a^4ne^{(nx)}}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))^2,x, algorithm="fricas")
```

output
```
-(2*b^2*n*x*e^(2*n*x) + a^2 + 2*(a*b*n*x + a*b)*e^(n*x) - 2*(b^2*e^(2*n*x)
 + a*b*e^(n*x))*log(b*e^(n*x) + a))/(a^3*b*n*e^(2*n*x) + a^4*n*e^(n*x))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}\,dx = \frac{b^2}{a^4ne^{-nx}+a^3bn} + \begin{cases} -\frac{e^{-nx}}{a^2n} & \text{for } a^2n \neq 0 \\ \frac{x}{a^2} & \text{otherwise} \end{cases} + \frac{2b\log\left(e^{-nx}+\frac{b}{a}\right)}{a^3n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))**2,x)
```

output
```
b**2/(a**4*n*exp(-n*x) + a**3*b*n) + Piecewise((-exp(-n*x)/(a**2*n), Ne(a*
*2*n, 0)), (x/a**2, True)) + 2*b*log(exp(-n*x) + b/a)/(a**3*n)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}\,dx = \frac{b^2}{(a^4e^{(-nx)}+a^3b)n} - \frac{e^{(-nx)}}{a^2n} + \frac{2\,b\log\left(ae^{(-nx)}+b\right)}{a^3n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))^2,x, algorithm="maxima")
```

output
```
b^2/((a^4*e^(-n*x) + a^3*b)*n) - e^(-n*x)/(a^2*n) + 2*b*log(a*e^(-n*x) + b
)/(a^3*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}\,dx = -\frac{2\,bx}{a^3} + \frac{2\,b\log\left(\left|be^{(nx)}+a\right|\right)}{a^3n} - \frac{2\,be^{(nx)}+a}{(be^{(2\,nx)}+ae^{(nx)})a^2n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))^2,x, algorithm="giac")
```

output
```
-2*b*x/a^3 + 2*b*log(abs(b*e^(n*x) + a))/(a^3*n) - (2*b*e^(n*x) + a)/((b*e
^(2*n*x) + a*e^(n*x))*a^2*n)
```

Mupad [B] (verification not implemented)

Time = 24.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}\,dx = \frac{2\,b\,\ln\left(a+b\,e^{n\,x}\right)}{a^3\,n} - \frac{\frac{1}{a\,n}+\frac{2\,b^2\,x\,e^{2\,n\,x}}{a^3}-\frac{2\,b^2\,e^{2\,n\,x}}{a^3\,n}+\frac{2\,b\,x\,e^{n\,x}}{a^2}}{a\,e^{n\,x}+b\,e^{2\,n\,x}}$$

input
```
int(exp(-n*x)/(a + b*exp(n*x))^2,x)
```

output
```
(2*b*log(a + b*exp(n*x)))/(a^3*n) - (1/(a*n) + (2*b^2*x*exp(2*n*x))/a^3 -
(2*b^2*exp(2*n*x))/(a^3*n) + (2*b*x*exp(n*x))/a^2)/(a*exp(n*x) + b*exp(2*n
*x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.74

$$\int \frac{e^{-nx}}{(a+be^{nx})^2}\,dx = \frac{2e^{2nx}\log(e^{nx}b+a)\,b^2-2e^{2nx}b^2nx+2e^{2nx}b^2+2e^{nx}\log(e^{nx}b+a)\,ab-2e^{nx}abnx-a^2}{e^{nx}a^3n\,(e^{nx}b+a)}$$

input
```
int(1/exp(n*x)/(a+b*exp(n*x))^2,x)
```

output
```
(2*e**(2*n*x)*log(e**(n*x)*b + a)*b**2 - 2*e**(2*n*x)*b**2*n*x + 2*e**(2*n
*x)*b**2 + 2*e**(n*x)*log(e**(n*x)*b + a)*a*b - 2*e**(n*x)*a*b*n*x - a**2)
/(e**(n*x)*a**3*n*(e**(n*x)*b + a))
```

3.88 $\int \frac{e^{-nx}}{(a+be^{nx})^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{e^{-nx}}{(a+be^{nx})^3} dx = -\frac{e^{-nx}}{a^3n} - \frac{b}{2a^2(a+be^{nx})^2 n} - \frac{2b}{a^3(a+be^{nx})n} - \frac{3bx}{a^4} + \frac{3b\log(a+be^{nx})}{a^4n}$$

output
```
-1/a^3/exp(n*x)/n-1/2*b/a^2/(a+b*exp(n*x))^2/n-2*b/a^3/(a+b*exp(n*x))/n-3*
b*x/a^4+3*b*ln(a+b*exp(n*x))/a^4/n
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{e^{-nx}}{(a+be^{nx})^3} dx = \frac{\frac{5b^3-2a^3e^{-3nx}-4a^2be^{-2nx}+4ab^2e^{-nx}}{(b+ae^{-nx})^2} + 6b\log(b+ae^{-nx})}{2a^4n}$$

input
```
Integrate[1/(E^(n*x)*(a + b*E^(n*x))^3),x]
```

output
```
((5*b^3 - (2*a^3)/E^(3*n*x) - (4*a^2*b)/E^(2*n*x) + (4*a*b^2)/E^(n*x))/(b
+ a/E^(n*x))^2 + 6*b*Log[b + a/E^(n*x)])/(2*a^4*n)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2678, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx$$

$$\Big\downarrow \ 2678$$

$$\frac{\int \frac{e^{-2nx}}{(a+be^{nx})^3}de^{nx}}{n}$$

$$\Big\downarrow \ 54$$

$$\frac{\int \left(\frac{3b^2}{a^4(a+be^{nx})}+\frac{2b^2}{a^3(a+be^{nx})^2}+\frac{b^2}{a^2(a+be^{nx})^3}-\frac{3e^{-nx}b}{a^4}+\frac{e^{-2nx}}{a^3}\right)de^{nx}}{n}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{3b\log(e^{nx})}{a^4}+\frac{3b\log(a+be^{nx})}{a^4}-\frac{2b}{a^3(a+be^{nx})}-\frac{e^{-nx}}{a^3}-\frac{b}{2a^2(a+be^{nx})^2}}{n}$$

input
```
Int[1/(E^(n*x)*(a + b*E^(n*x))^3),x]
```

output
```
(-(1/(a^3*E^(n*x))) - b/(2*a^2*(a + b*E^(n*x))^2) - (2*b)/(a^3*(a + b*E^(n
*x))) - (3*b*Log[E^(n*x)])/a^4 + (3*b*Log[a + b*E^(n*x)])/a^4)/n
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{e^{-xn}}{a^{3}n}-\frac{3bx}{a^{4}}-\frac{b(4b\,e^{xn}+5a)}{2a^{3}n(a+b\,e^{xn})^{2}}+\frac{3b\ln\left(e^{xn}+\frac{a}{b}\right)}{a^{4}n}$
derivativedivides	$\frac{-\frac{e^{-xn}}{a^{3}}-\frac{3b\ln(e^{xn})}{a^{4}}-\frac{b}{2a^{2}(a+b\,e^{xn})^{2}}+\frac{3b\ln(a+b\,e^{xn})}{a^{4}}-\frac{2b}{a^{3}(a+b\,e^{xn})}}{n}$
default	$\frac{-\frac{e^{-xn}}{a^{3}}-\frac{3b\ln(e^{xn})}{a^{4}}-\frac{b}{2a^{2}(a+b\,e^{xn})^{2}}+\frac{3b\ln(a+b\,e^{xn})}{a^{4}}-\frac{2b}{a^{3}(a+b\,e^{xn})}}{n}$
norman	$\frac{\left(-\frac{1}{an}-\frac{3bx\,e^{xn}}{a^{2}}-\frac{6b^{2}x\,e^{2xn}}{a^{3}}-\frac{3b^{3}x\,e^{3xn}}{a^{4}}+\frac{6b^{2}e^{2xn}}{a^{3}n}+\frac{9b^{3}e^{3xn}}{2a^{4}n}\right)e^{-xn}}{(a+b\,e^{xn})^{2}}+\frac{3b\ln(a+b\,e^{xn})}{a^{4}n}$
parallelrisch	$\frac{(-6x\,e^{3xn}b^{3}n+6\ln(a+b\,e^{xn})e^{3xn}b^{3}-12x\,e^{2xn}a\,b^{2}n+12\ln(a+b\,e^{xn})e^{2xn}a\,b^{2}-6x\,e^{xn}a^{2}bn+9\,e^{3xn}b^{3}+6\ln(a+b\,e^{xn})e}{2a^{4}n(a+b\,e^{xn})^{2}}$

input
```
int(1/exp(x*n)/(a+b*exp(x*n))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3/exp(x*n)/n-3*b*x/a^4-1/2*b*(4*b*exp(x*n)+5*a)/a^3/n/(a+b*exp(x*n))^
2+3*b/a^4/n*ln(exp(x*n)+1/b*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.69

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx = -\frac{6\,b^{3}nxe^{(3\,nx)}+2\,a^{3}+6\,(2\,ab^{2}nx+ab^{2})e^{(2\,nx)}+3\,(2\,a^{2}bnx+3\,a^{2}b)e^{(nx)}-6\,\left(b^{3}e^{(3\,nx)}+2\,ab^{2}e^{(2\,nx)}+a\right.}{2\,(a^{4}b^{2}ne^{(3\,nx)}+2\,a^{5}bne^{(2\,nx)}+a^{6}ne^{(nx)})}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))^3,x, algorithm="fricas")
```

output
```
-1/2*(6*b^3*n*x*e^(3*n*x) + 2*a^3 + 6*(2*a*b^2*n*x + a*b^2)*e^(2*n*x) + 3*
(2*a^2*b*n*x + 3*a^2*b)*e^(n*x) - 6*(b^3*e^(3*n*x) + 2*a*b^2*e^(2*n*x) + a
^2*b*e^(n*x))*log(b*e^(n*x) + a))/(a^4*b^2*n*e^(3*n*x) + 2*a^5*b*n*e^(2*n*
x) + a^6*n*e^(n*x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.14

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx = \frac{6ab^2e^{-nx}+5b^3}{2a^6ne^{-2nx}+4a^5bne^{-nx}+2a^4b^2n} + \begin{cases} -\frac{e^{-nx}}{a^3n} & \text{for } a^3n\neq 0 \\ \frac{x}{a^3} & \text{otherwise} \end{cases} + \frac{3b\log\left(e^{-nx}+\frac{b}{a}\right)}{a^4n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))**3,x)
```

output
```
(6*a*b**2*exp(-n*x) + 5*b**3)/(2*a**6*n*exp(-2*n*x) + 4*a**5*b*n*exp(-n*x)
 + 2*a**4*b**2*n) + Piecewise((-exp(-n*x)/(a**3*n), Ne(a**3*n, 0)), (x/a**
3, True)) + 3*b*log(exp(-n*x) + b/a)/(a**4*n)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx = \frac{6\,ab^2e^{(-nx)}+5\,b^3}{2\,(2\,a^5be^{(-nx)}+a^6e^{(-2\,nx)}+a^4b^2)n} - \frac{e^{(-nx)}}{a^3n} + \frac{3\,b\log\left(ae^{(-nx)}+b\right)}{a^4n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))^3,x, algorithm="maxima")
```

output
```
1/2*(6*a*b^2*e^(-n*x) + 5*b^3)/((2*a^5*b*e^(-n*x) + a^6*e^(-2*n*x) + a^4*b
^2)*n) - e^(-n*x)/(a^3*n) + 3*b*log(a*e^(-n*x) + b)/(a^4*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx = -\frac{3\,bx}{a^4}+\frac{3\,b\log\left(\left|be^{(nx)}+a\right|\right)}{a^4n}-\frac{\left(6\,ab^2e^{(2\,nx)}+9\,a^2be^{(nx)}+2\,a^3\right)e^{(-nx)}}{2\left(be^{(nx)}+a\right)^2a^4n}$$

input
```
integrate(1/exp(n*x)/(a+b*exp(n*x))^3,x, algorithm="giac")
```

output
```
-3*b*x/a^4 + 3*b*log(abs(b*e^(n*x) + a))/(a^4*n) - 1/2*(6*a*b^2*e^(2*n*x)
+ 9*a^2*b*e^(n*x) + 2*a^3)*e^(-n*x)/((b*e^(n*x) + a)^2*a^4*n)
```

Mupad [B] (verification not implemented)

Time = 23.75 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx = \frac{\frac{6\,b^2\,\mathrm{e}^{2\,n\,x}}{a^3\,n}-\frac{1}{a\,n}+\frac{9\,b^3\,\mathrm{e}^{3\,n\,x}}{2\,a^4\,n}}{\mathrm{e}^{n\,x}\,a^2+2\,\mathrm{e}^{2\,n\,x}\,a\,b+\mathrm{e}^{3\,n\,x}\,b^2}-\frac{3\,b\,\ln\left(\mathrm{e}^{n\,x}\right)}{a^4\,n}+\frac{3\,b\,\ln\left(a+b\,\mathrm{e}^{n\,x}\right)}{a^4\,n}$$

input
```
int(exp(-n*x)/(a + b*exp(n*x))^3,x)
```

output
```
((6*b^2*exp(2*n*x))/(a^3*n) - 1/(a*n) + (9*b^3*exp(3*n*x))/(2*a^4*n))/(a^2
*exp(n*x) + b^2*exp(3*n*x) + 2*a*b*exp(2*n*x)) - (3*b*log(exp(n*x)))/(a^4*
n) + (3*b*log(a + b*exp(n*x)))/(a^4*n)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.07

$$\int \frac{e^{-nx}}{(a+be^{nx})^3}\,dx$$
$$= \frac{6e^{3nx}\log(e^{nx}b+a)\,b^3-6e^{3nx}b^3nx+3e^{3nx}b^3+12e^{2nx}\log(e^{nx}b+a)\,a\,b^2-12e^{2nx}a\,b^2nx+6e^{nx}\log(e^{nx}b+}{2e^{nx}a^4n\left(e^{2nx}b^2+2e^{nx}ab+a^2\right)}$$

input
```
int(1/exp(n*x)/(a+b*exp(n*x))^3,x)
```

output
```
(6*e**(3*n*x)*log(e**(n*x)*b + a)*b**3 - 6*e**(3*n*x)*b**3*n*x + 3*e**(3*n
*x)*b**3 + 12*e**(2*n*x)*log(e**(n*x)*b + a)*a*b**2 - 12*e**(2*n*x)*a*b**2
*n*x + 6*e**(n*x)*log(e**(n*x)*b + a)*a**2*b - 6*e**(n*x)*a**2*b*n*x - 6*e
**(n*x)*a**2*b - 2*a**3)/(2*e**(n*x)*a**4*n*(e**(2*n*x)*b**2 + 2*e**(n*x)*
a*b + a**2))
```

3.89 $\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-\frac{e}{2}}\arctan\left(\frac{\sqrt{d}f^{\frac{e}{2}+bx}}{\sqrt{c}}\right)}{b\sqrt{c}\sqrt{d}\log(f)}$$

output
```
f^(a-1/2*e)*arctan(d^(1/2)*f^(1/2*e+b*x)/c^(1/2))/b/c^(1/2)/d^(1/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-\frac{e}{2}}\arctan\left(\frac{\sqrt{d}f^{\frac{e}{2}+bx}}{\sqrt{c}}\right)}{b\sqrt{c}\sqrt{d}\log(f)}$$

input
```
Integrate[f^(a + b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - e/2)*ArcTan[(Sqrt[d]*f^(e/2 + b*x))/Sqrt[c]])/(b*Sqrt[c]*Sqrt[d]*L
og[f])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2679, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+bx}}{df^{2bx+e}+c}dx$$

$$\Big\downarrow \; 2679$$

$$\frac{\int \frac{1}{df^{e+2bx}+c}df^{a+bx}}{b\log(f)}$$

$$\Big\downarrow \; 218$$

$$\frac{f^{a-\frac{e}{2}}\arctan\left(\frac{\sqrt{d}f^{\frac{1}{2}(e-2a)+a+bx}}{\sqrt{c}}\right)}{b\sqrt{c}\sqrt{d}\log(f)}$$

input
```
Int[f^(a + b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - e/2)*ArcTan[(Sqrt[d]*f^(a + (-2*a + e)/2 + b*x))/Sqrt[c]])/(b*Sqrt
[c]*Sqrt[d]*Log[f])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.82

method	result	size
risch	$-\frac{f^{a} \ln\left(f^{bx+a}-\frac{f^{a} c}{\sqrt{-f^{e} c d}}\right)}{2\sqrt{-f^{e} c d}\, b \ln\left(f\right)}+\frac{f^{a} \ln\left(f^{bx+a}+\frac{f^{a} c}{\sqrt{-f^{e} c d}}\right)}{2\sqrt{-f^{e} c d}\, b \ln\left(f\right)}$	91

input
```
int(f^(b*x+a)/(c+d*f^(2*b*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-f^e*c*d)^(1/2)*f^a/b/ln(f)*ln(f^(b*x+a)-1/(-f^e*c*d)^(1/2)*f^a*c)+1
/2/(-f^e*c*d)^(1/2)*f^a/b/ln(f)*ln(f^(b*x+a)+1/(-f^e*c*d)^(1/2)*f^a*c)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 3.58

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx = \left[-\frac{\sqrt{-cdf^{-2\,a+e}}\log\left(\frac{df^{2\,bx+2\,a}f^{-2\,a+e}-2\,\sqrt{-cdf^{-2\,a+e}}f^{bx+a}-c}{df^{2\,bx+2\,a}f^{-2\,a+e}+c}\right)}{2\,bcdf^{-2\,a+e}\log\left(f\right)},\right.$$
$$\left.-\frac{\sqrt{cdf^{-2\,a+e}}\arctan\left(\frac{\sqrt{cdf^{-2\,a+e}}}{df^{bx+a}f^{-2\,a+e}}\right)}{bcdf^{-2\,a+e}\log\left(f\right)}\right]$$

input
```
integrate(f^(b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-c*d*f^(-2*a + e))*log((d*f^(2*b*x + 2*a)*f^(-2*a + e) - 2*sqrt
(-c*d*f^(-2*a + e))*f^(b*x + a) - c)/(d*f^(2*b*x + 2*a)*f^(-2*a + e) + c))
/(b*c*d*f^(-2*a + e)*log(f)), -sqrt(c*d*f^(-2*a + e))*arctan(sqrt(c*d*f^(-
2*a + e))/(d*f^(b*x + a)*f^(-2*a + e)))/(b*c*d*f^(-2*a + e)*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx = \operatorname{RootSum}\left(4z^2b^2cde^{e\log(f)}\log(f)^2+e^{2a\log(f)},\left(i\mapsto i\log\left(2ibc\log(f)+f^{a+bx}\right)\right)\right)$$

input
```
integrate(f**(b*x+a)/(c+d*f**(2*b*x+e)),x)
```

output
```
RootSum(4*_z**2*b**2*c*d*exp(e*log(f))*log(f)**2 + exp(2*a*log(f)), Lambda
(_i, _i*log(2*_i*b*c*log(f) + f**(a + b*x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.74

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx = \frac{f^a\arctan\left(\frac{df^{bx+e}}{\sqrt{cdf^e}}\right)}{\sqrt{cdf^e}b\log(f)}$$

input
```
integrate(f^(b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="maxima")
```

output
```
f^a*arctan(d*f^(b*x + e)/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx = \frac{f^{2\,a}\arctan\left(\frac{df^{bx}f^{e}}{\sqrt{cdf^{e}}}\right)}{\sqrt{cdf^{e}}bf^{a}\log\left(f\right)}$$

input
```
integrate(f^(b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="giac")
```

output
```
f^(2*a)*arctan(d*f^(b*x)*f^e/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*b*f^a*log(f))
```

Mupad [B] (verification not implemented)

Time = 23.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.28

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx = \frac{\mathrm{atan}\left(\frac{f^{a+b\,x}\,\sqrt{b^2\,c\,d\,f^e\,\ln(f)^2}}{b\,c\,\ln(f)\,\sqrt{f^{2\,a}}}\right)\,\sqrt{f^{2\,a}}}{\sqrt{b^2\,c\,d\,f^e\,\ln\left(f\right)^2}}$$

input
```
int(f^(a + b*x)/(c + d*f^(e + 2*b*x)),x)
```

output
```
(atan((f^(a + b*x)*(b^2*c*d*f^e*log(f)^2)^(1/2))/(b*c*log(f)*(f^(2*a))^(1/
2)))*(f^(2*a))^(1/2))/(b^2*c*d*f^e*log(f)^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int \frac{f^{a+bx}}{c+df^{e+2bx}}\,dx = \frac{f^{a+\frac{e}{2}}\sqrt{d}\,\sqrt{c}\,atan\left(\frac{f^{bx+e}d}{f^{\frac{e}{2}}\sqrt{d}\,\sqrt{c}}\right)}{f^{e}\log\left(f\right)bcd}$$

input
```
int(f^(b*x+a)/(c+d*f^(2*b*x+e)),x)
```

output
```
(f**((2*a + e)/2)*sqrt(d)*sqrt(c)*atan((f**(b*x + e)*d)/(f**(e/2)*sqrt(d)*
sqrt(c))))/(f**e*log(f)*b*c*d)
```

3.90 $\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 34

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-e}\log\left(c+df^{e+2bx}\right)}{2bd\log(f)}$$

output
```
1/2*f^(a-e)*ln(c+d*f^(2*b*x+e))/b/d/ln(f)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-e}\log\left(c+df^{e+2bx}\right)}{2bd\log(f)}$$

input
```
Integrate[f^(a + 2*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - e)*Log[c + d*f^(e + 2*b*x)])/(2*b*d*Log[f])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2677, 2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+2bx}}{df^{2bx+e}+c}dx$$

$$\Big\downarrow \text{2677}$$

$$f^{a-e}\int \frac{f^{e+2bx}}{df^{e+2bx}+c}dx$$

$$\Big\downarrow \text{2676}$$

$$\frac{f^{a-e}\int \frac{1}{df^{e+2bx}+c}df^{e+2bx}}{2b\log(f)}$$

$$\Big\downarrow \text{16}$$

$$\frac{f^{a-e}\log\left(df^{2bx+e}+c\right)}{2bd\log(f)}$$

input
```
Int[f^(a + 2*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - e)*Log[c + d*f^(e + 2*b*x)])/(2*b*d*Log[f])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

rule 2677
```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Simp[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n   Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p,
x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Lo
g[F], g*h*m*Log[G]]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

method	result	size
norman	$\frac{f^{-e} f^{a} \ln\left(c+d\,{e}^{-\ln(f) a+\ln(f) e} {e}^{(2 b x+a) \ln(f)}\right)}{2 d \ln(f) b}$	47
risch	$-\frac{f^{a} f^{-e} a}{2 b d}+\frac{f^{a} f^{-e} \ln\left(f^{2 b x+a}+\frac{c f^{a} f^{-e}}{d}\right)}{2 \ln(f) b d}$	62

input
```
int(f^(2*b*x+a)/(c+d*f^(2*b*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/(f^e)/d/ln(f)/b*f^a*ln(c+d*exp(-ln(f)*a+ln(f)*e)*exp((2*b*x+a)*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-e}\log\left(df^{2\,bx+e}+c\right)}{2\,bd\log\left(f\right)}$$

input
```
integrate(f^(2*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")
```

output
```
1/2*f^(a - e)*log(d*f^(2*b*x + e) + c)/(b*d*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx=\frac{e^{(a-e)\log{(f)}}\log{\left(\frac{ce^{a\log{(f)}}e^{-e\log{(f)}}}{d}+f^{a+2bx}\right)}}{2bd\log{(f)}}$$

input
```
integrate(f**(2*b*x+a)/(c+d*f**(2*b*x+e)),x)
```

output
```
exp((a - e)*log(f))*log(c*exp(a*log(f))*exp(-e*log(f))/d + f**(a + 2*b*x))
/(2*b*d*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-e}\log{\left(df^{2\,bx+e}+c\right)}}{2\,bd\log{(f)}}$$

input
```
integrate(f^(2*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="maxima")
```

output
```
1/2*f^(a - e)*log(d*f^(2*b*x + e) + c)/(b*d*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx = \frac{f^a \log\left(\left|df^{2\,bx}f^e + c\right|\right)}{2\,bdf^e \log\left(f\right)}$$

input
```
integrate(f^(2*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="giac")
```

output
```
1/2*f^a*log(abs(d*f^(2*b*x)*f^e + c))/(b*d*f^e*log(f))
```

Mupad [B] (verification not implemented)

Time = 23.48 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx = \frac{f^{a-e}\,\ln\left(d\,f^{a+e+2\,b\,x}+c\,f^a\right)}{2\,b\,d\,\ln\left(f\right)}$$

input
```
int(f^(a + 2*b*x)/(c + d*f^(e + 2*b*x)),x)
```

output
```
(f^(a - e)*log(d*f^(a + e + 2*b*x) + c*f^a))/(2*b*d*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{f^{a+2bx}}{c+df^{e+2bx}}\,dx = \frac{f^a \log\left(f^{2bx+e}d+c\right)}{2f^e \log\left(f\right)bd}$$

input
```
int(f^(2*b*x+a)/(c+d*f^(2*b*x+e)),x)
```

output
```
(f**a*log(f**(2*b*x + e)*d + c))/(2*f**e*log(f)*b*d)
```

3.91 $\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 88

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx=\frac{f^{\frac{1}{2}(2a-3e)+\frac{1}{2}(e+2bx)}}{bd\log(f)}-\frac{\sqrt{c}f^{a-\frac{3e}{2}}\arctan\left(\frac{\sqrt{d}f^{\frac{1}{2}(e+2bx)}}{\sqrt{c}}\right)}{bd^{3/2}\log(f)}$$

output
```
f^(b*x+a-e)/b/d/ln(f)-c^(1/2)*f^(a-3/2*e)*arctan(d^(1/2)*f^(1/2*e+b*x)/c^(
1/2))/b/d^(3/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx=\frac{\frac{f^{a-e+bx}}{d}-\frac{\sqrt{c}f^{a-\frac{3e}{2}}\arctan\left(\frac{\sqrt{d}f^{\frac{e}{2}+bx}}{\sqrt{c}}\right)}{d^{3/2}}}{b\log(f)}$$

input
```
Integrate[f^(a + 3*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - e + b*x)/d - (Sqrt[c]*f^(a - (3*e)/2)*ArcTan[(Sqrt[d]*f^(e/2 + b*x
))/Sqrt[c]])/d^(3/2))/(b*Log[f])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2678, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+3bx}}{df^{2bx+e}+c}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{f^{a-\frac{3e}{2}}\int \frac{f^{e+2bx}}{df^{e+2bx}+c}df^{\frac{1}{2}(e+2bx)}}{b\log(f)}$$

$$\Big\downarrow \text{262}$$

$$\frac{f^{a-\frac{3e}{2}}\left(\frac{f^{\frac{1}{2}(2bx+e)}}{d}-\frac{c\int \frac{1}{df^{e+2bx}+c}df^{\frac{1}{2}(e+2bx)}}{d}\right)}{b\log(f)}$$

$$\Big\downarrow \text{218}$$

$$\frac{f^{a-\frac{3e}{2}}\left(\frac{f^{\frac{1}{2}(2bx+e)}}{d}-\frac{\sqrt{c}\arctan\left(\frac{\sqrt{d}f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right)}{d^{3/2}}\right)}{b\log(f)}$$

input
```
Int[f^(a + 3*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - (3*e)/2)*(f^((e + 2*b*x)/2)/d - (Sqrt[c]*ArcTan[(Sqrt[d]*f^((e + 2
*b*x)/2))/Sqrt[c]])/d^(3/2)))/(b*Log[f])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(61) = 122$.

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.94

method	result	size
risch	$\frac{f^{-e} f^{\frac{2a}{3}} f^{bx+\frac{a}{3}}}{d \ln(f) b}+\frac{\sqrt{-cd}\, f^{a} f^{-\frac{3e}{2}} \ln\left(f^{bx+\frac{a}{3}}-\frac{\sqrt{-cd}\, f^{\frac{a}{3}} f^{-\frac{e}{2}}}{d}\right)}{2 d^{2} b \ln(f)}-\frac{\sqrt{-cd}\, f^{a} f^{-\frac{3e}{2}} \ln\left(f^{bx+\frac{a}{3}}+\frac{\sqrt{-cd}\, f^{\frac{a}{3}} f^{-\frac{e}{2}}}{d}\right)}{2 d^{2} b \ln(f)}$	171

input
```
int(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/(f^(1/2*e))^2/(f^(-1/3*a))^2/d/ln(f)/b*f^(b*x+1/3*a)+1/2/d^2*(-c*d)^(1/2
)/b/(f^(-1/3*a))^3/(f^(1/2*e))^3/ln(f)*ln(f^(b*x+1/3*a)-1/d*(-c*d)^(1/2)/(
f^(-1/3*a))/(f^(1/2*e)))-1/2/d^2*(-c*d)^(1/2)/b/(f^(-1/3*a))^3/(f^(1/2*e))
^3/ln(f)*ln(f^(b*x+1/3*a)+1/d*(-c*d)^(1/2)/(f^(-1/3*a))/(f^(1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.89

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx = \left[\frac{f^{a-\frac{3}{2}e}\sqrt{-\frac{c}{d}}\log\left(-\frac{2\,df^{bx+\frac{1}{2}e}\sqrt{-\frac{c}{d}}-df^{2\,bx+e}+c}{df^{2\,bx+e}+c}\right)+2\,f^{bx+\frac{1}{2}e}f^{a-\frac{3}{2}e}}{2\,bd\log(f)},\ -\frac{f^{a-\frac{3}{2}e}\sqrt{\frac{c}{d}}\arctan\left(\frac{df^{bx+\frac{1}{2}e}\sqrt{\frac{c}{d}}}{c}\right)-f^{bx+\frac{1}{2}e}f^{a-\frac{3}{2}e}}{bd\log(f)}\right]$$

input
```
integrate(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")
```

output
```
[1/2*(f^(a - 3/2*e)*sqrt(-c/d)*log(-(2*d*f^(b*x + 1/2*e)*sqrt(-c/d) - d*f^
(2*b*x + e) + c)/(d*f^(2*b*x + e) + c)) + 2*f^(b*x + 1/2*e)*f^(a - 3/2*e))
/(b*d*log(f)), -(f^(a - 3/2*e)*sqrt(c/d)*arctan(d*f^(b*x + 1/2*e)*sqrt(c/d
)/c) - f^(b*x + 1/2*e)*f^(a - 3/2*e))/(b*d*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx$$
$$= \operatorname{RootSum}\left(4z^2b^2d^3e^{3e\log(f)}\log(f)^2+ce^{2a\log(f)},\left(i\mapsto i\log\left(-2ibde^{-\frac{2a\log(f)}{3}}e^{e\log(f)}\log(f)+e^{\frac{(a+3bx)\log(f)}{3}}\right.\right.\right.$$
$$+\frac{\left(\begin{cases}x & \text{for } b=0\vee f=1\\ \frac{e^{bx\log(f)}}{b\log(f)} & \text{otherwise}\end{cases}\right)e^{a\log(f)}e^{-e\log(f)}}{d}$$

input
```
integrate(f**(3*b*x+a)/(c+d*f**(2*b*x+e)),x)
```

output
```
RootSum(4*_z**2*b**2*d**3*exp(3*e*log(f))*log(f)**2 + c*exp(2*a*log(f)), L
ambda(_i, _i*log(-2*_i*b*d*exp(-2*a*log(f)/3)*exp(e*log(f))*log(f) + exp((
a + 3*b*x)*log(f)/3)))) + Piecewise((x, Eq(b, 0) | Eq(f, 1)), (exp(b*x*log
(f))/(b*log(f)), True))*exp(a*log(f))*exp(-e*log(f))/d
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx = -\frac{cf^{a-e}\arctan\left(\frac{df^{bx+e}}{\sqrt{cdf^e}}\right)}{\sqrt{cdf^e}bd\log(f)} + \frac{f^{bx+a-e}}{bd\log(f)}$$

input
```
integrate(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="maxima")
```

output
```
-c*f^(a - e)*arctan(d*f^(b*x + e)/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*b*d*log(f)
) + f^(b*x + a - e)/(b*d*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx = -\frac{f^a\left(\frac{c\arctan\left(\frac{df^{bx}f^e}{\sqrt{cdf^e}}\right)}{\sqrt{cdf^e}df^e\log(f)} - \frac{f^{bx}}{df^e\log(f)}\right)}{b}$$

input
```
integrate(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="giac")
```

output
```
-f^a*(c*arctan(d*f^(b*x)*f^e/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*d*f^e*log(f)) -
 f^(b*x)/(d*f^e*log(f)))/b
```

Mupad [B] (verification not implemented)

Time = 23.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx = -\frac{f^a\,\mathrm{e}^{-\frac{3\,e\,\ln(f)}{2}}\,\left(c\,\mathrm{atan}\left(\frac{d\,f^{b\,x}\,\mathrm{e}^{\frac{e\,\ln(f)}{2}}}{\sqrt{c\,d}}\right)-f^{b\,x}\,\mathrm{e}^{\frac{e\,\ln(f)}{2}}\,\sqrt{c\,d}\right)}{b\,d\,\ln\left(f\right)\,\sqrt{c\,d}}$$

input
```
int(f^(a + 3*b*x)/(c + d*f^(e + 2*b*x)),x)
```

output
```
-(f^a*exp(-(3*e*log(f))/2)*(c*atan((d*f^(b*x)*exp((e*log(f))/2))/(c*d)^(1/
2)) - f^(b*x)*exp((e*log(f))/2)*(c*d)^(1/2)))/(b*d*log(f)*(c*d)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \frac{f^{a+3bx}}{c+df^{e+2bx}}\,dx = \frac{f^a\left(-f^{\frac{e}{2}}\sqrt{d}\,\sqrt{c}\,atan\left(\frac{f^{bx+e}d}{f^{\frac{e}{2}}\sqrt{d}\,\sqrt{c}}\right)+f^{bx+e}d\right)}{f^{2e}\log\left(f\right)b\,d^2}$$

input
```
int(f^(3*b*x+a)/(c+d*f^(2*b*x+e)),x)
```

output
```
(f**a*( - f**(e/2)*sqrt(d)*sqrt(c)*atan((f**(b*x + e)*d)/(f**(e/2)*sqrt(d)
*sqrt(c))) + f**(b*x + e)*d))/(f**(2*e)*log(f)*b*d**2)
```

3.92 $\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx=\frac{f^{a-e+2bx}}{2bd\log(f)}-\frac{cf^{a-2e}\log\left(c+df^{e+2bx}\right)}{2bd^2\log(f)}$$

output
```
1/2*f^(2*b*x+a-e)/b/d/ln(f)-1/2*c*f^(a-2*e)*ln(c+d*f^(2*b*x+e))/b/d^2/ln(f
)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.20

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx =\frac{f^{a-2e}\left(df^{e+2bx}-2bcx\log(f)-c\log\left(c+df^{e+2bx}\right)+c\log\left(bd^3f^{e+2bx}\log(f)\right)\right)}{2bd^2\log(f)}$$

input
```
Integrate[f^(a + 4*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - 2*e)*(d*f^(e + 2*b*x) - 2*b*c*x*Log[f] - c*Log[c + d*f^(e + 2*b*x)
] + c*Log[b*d^3*f^(e + 2*b*x)*Log[f]]))/(2*b*d^2*Log[f])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2678, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+4bx}}{df^{2bx+e}+c}dx$$

$$\Big\downarrow \text{ 2678}$$

$$\frac{f^{a-2e}\int \frac{f^{e+2bx}}{df^{e+2bx}+c}df^{e+2bx}}{2b\log(f)}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{f^{a-2e}\int \left(\frac{1}{d}-\frac{c}{d(df^{e+2bx}+c)}\right)df^{e+2bx}}{2b\log(f)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{f^{a-2e}\left(\frac{f^{2bx+e}}{d}-\frac{c\log\left(df^{2bx+e}+c\right)}{d^2}\right)}{2b\log(f)}$$

input
```
Int[f^(a + 4*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - 2*e)*(f^(e + 2*b*x)/d - (c*Log[c + d*f^(e + 2*b*x)])/d^2))/(2*b*Lo
g[f])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.25

method	result	size
norman	$\frac{f^{-2e} f^{a} {\mathrm e}^{(2bx+e)\ln(f)}}{2\ln(f) b d}-\frac{c\, f^{-2e} f^{a} \ln\left(c+d\,{\mathrm e}^{(2bx+e)\ln(f)}\right)}{2\ln(f) b\, d^{2}}$	76
risch	$\frac{f^{a} f^{-2e} f^{2bx+e}}{2\ln(f) b d}+\frac{f^{a} f^{-2e} c e}{2b\, d^{2}}-\frac{f^{a} f^{-2e} c \ln\left(f^{2bx+e}+\frac{c}{d}\right)}{2\ln(f) b\, d^{2}}$	96

input
```
int(f^(4*b*x+a)/(c+d*f^(2*b*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2/(f^e)^2/ln(f)/b/d*(f^(1/2*a))^2*exp((2*b*x+e)*ln(f))-1/2/ln(f)/b/d^2*c
/(f^e)^2*(f^(1/2*a))^2*ln(c+d*exp((2*b*x+e)*ln(f)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx=\frac{df^{2\,bx+e}f^{a-2\,e}-cf^{a-2\,e}\log\left(df^{2\,bx+e}+c\right)}{2\,bd^2\log\left(f\right)}$$

input
```
integrate(f^(4*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")
```

output
```
1/2*(d*f^(2*b*x + e)*f^(a - 2*e) - c*f^(a - 2*e)*log(d*f^(2*b*x + e) + c))
/(b*d^2*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx=\frac{\left(\begin{cases} x & \text{for}\: b=0 \vee f=1 \\ \frac{e^{2bx\log\left(f\right)}}{2b\log\left(f\right)} & \text{otherwise}\end{cases}\right)e^{a\log\left(f\right)}e^{-e\log\left(f\right)}}{d}-\frac{ce^{(a-2e)\log\left(f\right)}\log\left(\frac{ce^{\frac{a\log\left(f\right)}{2}}e^{-e\log\left(f\right)}}{d}+\sqrt{e^{(a+4bx)\log\left(f\right)}}\right)}{2bd^2\log\left(f\right)}$$

input
```
integrate(f**(4*b*x+a)/(c+d*f**(2*b*x+e)),x)
```

output
```
Piecewise((x, Eq(b, 0) | Eq(f, 1)), (exp(2*b*x*log(f))/(2*b*log(f)), True)
)*exp(a*log(f))*exp(-e*log(f))/d - c*exp((a - 2*e)*log(f))*log(c*exp(a*log
(f)/2)*exp(-e*log(f))/d + sqrt(exp((a + 4*b*x)*log(f))))/(2*b*d**2*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx = -\frac{cf^{a-2\,e}\log\left(df^{2\,bx+e}+c\right)}{2\,bd^2\log\left(f\right)} + \frac{\left(df^{2\,bx+e}+c\right)f^{a-2\,e}}{2\,bd^2\log\left(f\right)}$$

input
```
integrate(f^(4*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="maxima")
```

output
```
-1/2*c*f^(a - 2*e)*log(d*f^(2*b*x + e) + c)/(b*d^2*log(f)) + 1/2*(d*f^(2*b
*x + e) + c)*f^(a - 2*e)/(b*d^2*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx = \frac{f^a\left(\frac{f^{2\,bx}f^e}{d\log(f)} - \frac{c\log\left(\left|df^{2\,bx}f^e+c\right|\right)}{d^2\log(f)}\right)}{2\,bf^{2\,e}}$$

input
```
integrate(f^(4*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="giac")
```

output
```
1/2*f^a*(f^(2*b*x)*f^e/(d*log(f)) - c*log(abs(d*f^(2*b*x)*f^e + c))/(d^2*l
og(f)))/(b*f^(2*e))
```

Mupad [B] (verification not implemented)

Time = 23.42 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx = -\frac{f^{a-2\,e}\left(\frac{c\,\ln\left(c+d\,f^{e+2\,b\,x}\right)}{2} - \frac{d\,f^{e+2\,b\,x}}{2}\right)}{b\,d^2\,\ln\left(f\right)}$$

input
```
int(f^(a + 4*b*x)/(c + d*f^(e + 2*b*x)),x)
```

output
```
-(f^(a - 2*e)*((c*log(c + d*f^(e + 2*b*x)))/2 - (d*f^(e + 2*b*x))/2))/(b*d
^2*log(f))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.80

$$\int \frac{f^{a+4bx}}{c+df^{e+2bx}}\,dx = \frac{f^{a}\left(f^{2bx+e}d-\log\left(f^{2bx+e}d+c\right)c\right)}{2f^{2e}\log\left(f\right)b\,d^{2}}$$

input
```
int(f^(4*b*x+a)/(c+d*f^(2*b*x+e)),x)
```

output
```
(f**a*(f**(2*b*x + e)*d - log(f**(2*b*x + e)*d + c)*c))/(2*f**(2*e)*log(f)
*b*d**2)
```

3.93 $\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx = -\frac{cf^{\frac{1}{2}(2a-5e)+\frac{1}{2}(e+2bx)}}{bd^2\log(f)}+\frac{f^{\frac{1}{2}(2a-5e)+\frac{3}{2}(e+2bx)}}{3bd\log(f)}+\frac{c^{3/2}f^{a-\frac{5e}{2}}\arctan\left(\frac{\sqrt{d}f^{\frac{1}{2}(e+2bx)}}{\sqrt{c}}\right)}{bd^{5/2}\log(f)}$$

output
```
-c*f^(b*x+a-2*e)/b/d^2/ln(f)+1/3*f^(3*b*x+a-e)/b/d/ln(f)+c^(3/2)*f^(a-5/2*
e)*arctan(d^(1/2)*f^(1/2*e+b*x)/c^(1/2))/b/d^(5/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx = \frac{f^a\left(-\frac{cf^{-2e+bx}}{d^2}+\frac{f^{-e+3bx}}{3d}+\frac{c^{3/2}f^{-5e/2}\arctan\left(\frac{\sqrt{d}f^{\frac{e}{2}+bx}}{\sqrt{c}}\right)}{d^{5/2}}\right)}{b\log(f)}$$

input
```
Integrate[f^(a + 5*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^a*(-((c*f^(-2*e + b*x))/d^2) + f^(-e + 3*b*x)/(3*d) + (c^(3/2)*ArcTan[(
Sqrt[d]*f^(e/2 + b*x))/Sqrt[c]])/(d^(5/2)*f^((5*e)/2))))/(b*Log[f])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2678, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^{a+5bx}}{df^{2bx+e}+c}dx$$

$$\Big\downarrow \text{2678}$$

$$\frac{f^{a-\frac{5e}{2}}\int \frac{f^{2(e+2bx)}}{df^{e+2bx}+c}df^{\frac{1}{2}(e+2bx)}}{b\log(f)}$$

$$\Big\downarrow \text{254}$$

$$\frac{f^{a-\frac{5e}{2}}\int \left(\frac{f^{e+2bx}}{d}+\frac{c^2}{d^2\left(df^{e+2bx}+c\right)}-\frac{c}{d^2}\right)df^{\frac{1}{2}(e+2bx)}}{b\log(f)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{f^{a-\frac{5e}{2}}\left(\frac{c^{3/2}\arctan\left(\frac{\sqrt{d}f^{\frac{1}{2}(2bx+e)}}{\sqrt{c}}\right)}{d^{5/2}}-\frac{cf^{\frac{1}{2}(2bx+e)}}{d^2}+\frac{f^{\frac{3}{2}(2bx+e)}}{3d}\right)}{b\log(f)}$$

input
```
Int[f^(a + 5*b*x)/(c + d*f^(e + 2*b*x)),x]
```

output
```
(f^(a - (5*e)/2)*(-((c*f^((e + 2*b*x)/2))/d^2) + f^((3*(e + 2*b*x))/2)/(3*
d) + (c^(3/2)*ArcTan[(Sqrt[d]*f^((e + 2*b*x)/2))/Sqrt[c]])/d^(5/2)))/(b*Lo
g[f])
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2678
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[g*h*(Log[G]/(d*e*Log
[F]))]}, Simp[Denominator[m]*(G^(f*h - c*g*(h/d))/(d*e*Log[F]))   Subst[Int
[x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^(e*((c + d*x)/De
nominator[m]))], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
 e, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.67

method	result
risch	$\frac{f^{-e} f^{\frac{2a}{5}} f^{3bx+\frac{3a}{5}}}{3d\ln(f)b} - \frac{c\, f^{-2e} f^{\frac{4a}{5}} f^{bx+\frac{a}{5}}}{d^2\ln(f)b} + \frac{\sqrt{-cd}\, c\, f^{a} f^{-\frac{5e}{2}} \ln\left(f^{bx+\frac{a}{5}} + \frac{\sqrt{-cd}\, f^{\frac{a}{5}} f^{-\frac{e}{2}}}{d}\right)}{2d^3 b\ln(f)} - \frac{\sqrt{-cd}\, c\, f^{a} f^{-\frac{5e}{2}} \ln\left(f^{bx+\frac{a}{5}} - \frac{\sqrt{-cd}\, f}{}\right.}{2d^3 b\ln(f)}$

input
```
int(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/3/(f^(1/2*e))^2/(f^(-1/5*a))^2/d/ln(f)/b*(f^(b*x+1/5*a))^3-c/(f^(1/2*e))
^4/(f^(-1/5*a))^4/d^2/ln(f)/b*f^(b*x+1/5*a)+1/2/d^3*(-c*d)^(1/2)*c/b/(f^(-
1/5*a))^5/(f^(1/2*e))^5/ln(f)*ln(f^(b*x+1/5*a)+1/d*(-c*d)^(1/2)/(f^(-1/5*a
))/(f^(1/2*e)))-1/2/d^3*(-c*d)^(1/2)*c/b/(f^(-1/5*a))^5/(f^(1/2*e))^5/ln(f
)*ln(f^(b*x+1/5*a)-1/d*(-c*d)^(1/2)/(f^(-1/5*a))/(f^(1/2*e)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.66

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx = \left[\frac{3\,cf^{a-\frac{5}{2}\,e}\sqrt{-\frac{c}{d}}\log\left(\frac{2\,df^{bx+\frac{1}{2}\,e}\sqrt{-\frac{c}{d}}+df^{2\,bx+e}-c}{df^{2\,bx+e}+c}\right)+2\,df^{3\,bx+\frac{3}{2}\,e}f^{a-\frac{5}{2}\,e}-6\,cf^{bx+\frac{1}{2}\,e}f^{a-\frac{5}{2}\,e}}{6\,bd^2\log\left(f\right)}, \frac{3\,cf^{a-\frac{5}{2}\,e}\sqrt{\frac{c}{d}}\arctan\right.$$

input
```
integrate(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="fricas")
```

output
```
[1/6*(3*c*f^(a - 5/2*e)*sqrt(-c/d)*log((2*d*f^(b*x + 1/2*e)*sqrt(-c/d) + d
*f^(2*b*x + e) - c)/(d*f^(2*b*x + e) + c)) + 2*d*f^(3*b*x + 3/2*e)*f^(a -
5/2*e) - 6*c*f^(b*x + 1/2*e)*f^(a - 5/2*e))/(b*d^2*log(f)), 1/3*(3*c*f^(a
- 5/2*e)*sqrt(c/d)*arctan(d*f^(b*x + 1/2*e)*sqrt(c/d)/c) + d*f^(3*b*x + 3/
2*e)*f^(a - 5/2*e) - 3*c*f^(b*x + 1/2*e)*f^(a - 5/2*e))/(b*d^2*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.46

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx = \operatorname{RootSum}\left(4z^2b^2d^5e^{5e\log(f)}\log(f)^2+c^3e^{2a\log(f)}, \left(i\mapsto i\log\left(\frac{2ibd^2e^{-\frac{4a\log(f)}{5}}e^{2e\log(f)}\log(f)}{c}+e^{\frac{(a+5bx)\log}{5}}\right.\right.\right.$$
$$+\frac{\left(\begin{cases} x(-c+d) & \text{for } b=0\wedge f=1 \\ x\left(-ce^{a\log(f)}+de^{a\log(f)}e^{e\log(f)}\right) & \text{for } b=0 \\ x(-c+d) & \text{for } f=1 \\ -\frac{ce^{a\log(f)}e^{bx\log(f)}}{b\log(f)}+\frac{de^{a\log(f)}e^{e\log(f)}e^{3bx\log(f)}}{3b\log(f)} & \text{otherwise}\end{cases}\right)e^{-2e\log(f)}}{d^2}$$

input
```
integrate(f**(5*b*x+a)/(c+d*f**(2*b*x+e)),x)
```

output
```
RootSum(4*_z**2*b**2*d**5*exp(5*e*log(f))*log(f)**2 + c**3*exp(2*a*log(f))
, Lambda(_i, _i*log(2*_i*b*d**2*exp(-4*a*log(f)/5)*exp(2*e*log(f))*log(f)/
c + exp((a + 5*b*x)*log(f)/5)))) + Piecewise((x*(-c + d), Eq(b, 0) & Eq(f,
 1)), (x*(-c*exp(a*log(f)) + d*exp(a*log(f))*exp(e*log(f))), Eq(b, 0)), (x
*(-c + d), Eq(f, 1)), (-c*exp(a*log(f))*exp(b*x*log(f))/(b*log(f)) + d*exp
(a*log(f))*exp(e*log(f))*exp(3*b*x*log(f))/(3*b*log(f)), True))*exp(-2*e*l
og(f))/d**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx = \frac{c^2f^{a-2e}\arctan\left(\frac{df^{bx+e}}{\sqrt{cdf^e}}\right)}{\sqrt{cdf^e}bd^2\log(f)} + \frac{df^{3bx+a+e}-3cf^{bx+a}}{3bd^2f^{2e}\log(f)}$$

input
```
integrate(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="maxima")
```

output
```
c^2*f^(a - 2*e)*arctan(d*f^(b*x + e)/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*b*d^2*l
og(f)) + 1/3*(d*f^(3*b*x + a + e) - 3*c*f^(b*x + a))/(b*d^2*f^(2*e)*log(f)
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx = \frac{f^a\left(\frac{3c^2\arctan\left(\frac{df^{bx}f^e}{\sqrt{cdf^e}}\right)}{\sqrt{cdf^e}d^2f^{2e}\log(f)} + \frac{d^2f^{3bx}f^{2e}\log(f)^2-3cdf^{bx}f^e\log(f)^2}{d^3f^{3e}\log(f)^3}\right)}{3b}$$

input
```
integrate(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x, algorithm="giac")
```

output
```
1/3*f^a*(3*c^2*arctan(d*f^(b*x)*f^e/sqrt(c*d*f^e))/(sqrt(c*d*f^e)*d^2*f^(2
*e)*log(f)) + (d^2*f^(3*b*x)*f^(2*e)*log(f)^2 - 3*c*d*f^(b*x)*f^e*log(f)^2
)/(d^3*f^(3*e)*log(f)^3))/b
```

Mupad [B] (verification not implemented)

Time = 23.86 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx=\frac{f^a\,f^{3\,b\,x}}{3\,b\,d\,f^e\,\ln\left(f\right)}-\frac{c\,f^a\,f^{b\,x}}{b\,d^2\,f^{2\,e}\,\ln\left(f\right)}+\frac{c^2\,f^a\,\mathrm{e}^{-\frac{5\,e\,\ln\left(f\right)}{2}}\,\mathrm{atan}\left(\frac{d\,f^{b\,x}\,\mathrm{e}^{\frac{e\,\ln\left(f\right)}{2}}}{\sqrt{c\,d}}\right)}{b\,d^2\,\ln\left(f\right)\,\sqrt{c\,d}}$$

input
```
int(f^(a + 5*b*x)/(c + d*f^(e + 2*b*x)),x)
```

output
```
(f^a*f^(3*b*x))/(3*b*d*f^e*log(f)) - (c*f^a*f^(b*x))/(b*d^2*f^(2*e)*log(f)
) + (c^2*f^a*exp(-(5*e*log(f))/2)*atan((d*f^(b*x)*exp((e*log(f))/2))/(c*d)
^(1/2)))/(b*d^2*log(f)*(c*d)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.67

$$\int \frac{f^{a+5bx}}{c+df^{e+2bx}}\,dx=\frac{f^a\left(3f^{\frac{e}{2}}\sqrt{d}\,\sqrt{c}\,atan\left(\frac{f^{bx+e}d}{f^{\frac{e}{2}}\sqrt{d}\,\sqrt{c}}\right)c+f^{3bx+2e}d^2-3f^{bx+e}cd\right)}{3f^{3e}\log\left(f\right)b\,d^3}$$

input
```
int(f^(5*b*x+a)/(c+d*f^(2*b*x+e)),x)
```

output
```
(f**a*(3*f**(e/2)*sqrt(d)*sqrt(c)*atan((f**(b*x + e)*d)/(f**(e/2)*sqrt(d)*
sqrt(c)))*c + f**(3*b*x + 2*e)*d**2 - 3*f**(b*x + e)*c*d))/(3*f**(3*e)*log
(f)*b*d**3)
```

3.94 $\int \frac{e^x}{1+e^{2x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 4

$$\int \frac{e^x}{1+e^{2x}} \, dx = \arctan \left(e^x\right)$$

output
```
arctan(exp(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^{2x}} \, dx = \arctan \left(e^x\right)$$

input
```
Integrate[E^x/(1 + E^(2*x)),x]
```

output
```
ArcTan[E^x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2679, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{e^{2x}+1}\,dx$$
$$\Big\downarrow \text{2679}$$
$$\int \frac{1}{e^{2x}+1}de^x$$
$$\Big\downarrow \text{216}$$
$$\arctan\left(e^x\right)$$

input
```
Int[E^x/(1 + E^(2*x)),x]
```

output
```
ArcTan[E^x]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\arctan\left(\mathrm{e}^{x}\right)$	4
risch	$\frac{i\ln(\mathrm{e}^{x}+i)}{2}-\frac{i\ln(\mathrm{e}^{x}-i)}{2}$	20

input
```
int(exp(x)/(1+exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
arctan(exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^{x}}{1+e^{2x}}\,dx = \arctan\left(e^{x}\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
```

output
```
arctan(e^x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^{x}}{1+e^{2x}}\,dx = \operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x)
```

output
```
RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
```

output
```
arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \arctan\left(e^x\right)$$

input
```
integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
```

output
```
arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int \frac{e^x}{1+e^{2x}}\,dx = \operatorname{atan}(\mathrm{e}^x)$$

input
```
int(exp(x)/(exp(2*x) + 1),x)
```

output
```
atan(exp(x))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1+e^{2x}}\,dx = atan(e^x)$$

input
```
int(exp(x)/(1+exp(2*x)),x)
```

output
```
atan(e**x)
```

3.95 $\int \frac{e^x}{1-e^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 4

$$\int \frac{e^x}{1-e^{2x}}\,dx = \operatorname{arctanh}(e^x)$$

output
```
arctanh(exp(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{e^x}{1-e^{2x}}\,dx = \operatorname{arctanh}(e^x)$$

input
```
Integrate[E^x/(1 - E^(2*x)),x]
```

output
```
ArcTanh[E^x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2679, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x}{1-e^{2x}} \, dx$$
$$\Big\downarrow \; 2679$$
$$\int \frac{1}{1-e^{2x}} de^x$$
$$\Big\downarrow \; 219$$
$$\text{arctanh}(e^x)$$

input
```
Int[E^x/(1 - E^(2*x)),x]
```

output
```
ArcTanh[E^x]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
default	$\operatorname{arctanh}(e^x)$	4
norman	$-\frac{\ln(e^x-1)}{2}+\frac{\ln(e^x+1)}{2}$	16
risch	$-\frac{\ln(e^x-1)}{2}+\frac{\ln(e^x+1)}{2}$	16

input
```
int(exp(x)/(1-exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
arctanh(exp(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = \frac{1}{2}\log\left(e^x+1\right) - \frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(1-exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*log(e^x + 1) - 1/2*log(e^x - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = -\frac{\log\left(e^x-1\right)}{2} + \frac{\log\left(e^x+1\right)}{2}$$

input
```
integrate(exp(x)/(1-exp(2*x)),x)
```

output
```
-log(exp(x) - 1)/2 + log(exp(x) + 1)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(3) = 6$.

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = \frac{1}{2}\log\left(e^x+1\right) - \frac{1}{2}\log\left(e^x-1\right)$$

input
```
integrate(exp(x)/(1-exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*log(e^x + 1) - 1/2*log(e^x - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(3) = 6$.

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 4.00

$$\int \frac{e^x}{1-e^{2x}}\,dx = \frac{1}{2}\log\left(e^x+1\right) - \frac{1}{2}\log\left(|e^x-1|\right)$$

input
```
integrate(exp(x)/(1-exp(2*x)),x, algorithm="giac")
```

output
```
1/2*log(e^x + 1) - 1/2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 24.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int \frac{e^x}{1-e^{2x}}\,dx = \frac{\ln\left(\mathrm{e}^x+1\right)}{2} - \frac{\ln\left(\mathrm{e}^x-1\right)}{2}$$

input
```
int(-exp(x)/(exp(2*x) - 1),x)
```

output
```
log(exp(x) + 1)/2 - log(exp(x) - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 4.25

$$\int \frac{e^x}{1-e^{2x}}\,dx = -\frac{\log(e^x-1)}{2} + \frac{\log(e^x+1)}{2}$$

input
```
int(exp(x)/(1-exp(2*x)),x)
```

output
```
( - log(e**x - 1) + log(e**x + 1))/2
```

3.96 $\int \frac{e^x x}{1-e^{2x}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 27

$$\int \frac{e^x x}{1-e^{2x}} \, dx = x\text{arctanh}(e^x) + \frac{\text{PolyLog}(2,-e^x)}{2} - \frac{\text{PolyLog}(2,e^x)}{2}$$

output
```
x*arctanh(exp(x))+1/2*polylog(2,-exp(x))-1/2*polylog(2,exp(x))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \frac{e^x x}{1-e^{2x}} \, dx = -\frac{1}{2}x\log(1-e^x) + \frac{1}{2}x\log(1+e^x) + \frac{\text{PolyLog}(2,-e^x)}{2} - \frac{\text{PolyLog}(2,e^x)}{2}$$

input
```
Integrate[(E^x*x)/(1 - E^(2*x)),x]
```

output
```
-1/2*(x*Log[1 - E^x]) + (x*Log[1 + E^x])/2 + PolyLog[2, -E^x]/2 - PolyLog[
2, E^x]/2
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2675, 2720, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x x}{1-e^{2x}} \, dx$$

$$\Big\downarrow \text{2675}$$

$$x\text{arctanh}(e^x) - \int \text{arctanh}(e^x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$x\text{arctanh}(e^x) - \int e^{-x}\text{arctanh}(e^x) \, de^x$$

$$\Big\downarrow \text{6446}$$

$$x\text{arctanh}(e^x) + \frac{\text{PolyLog}(2,-e^x)}{2} - \frac{\text{PolyLog}(2,e^x)}{2}$$

input
```
Int[(E^x*x)/(1 - E^(2*x)),x]
```

output
```
x*ArcTanh[E^x] + PolyLog[2, -E^x]/2 - PolyLog[2, E^x]/2
```

Defintions of rubi rules used

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

method	result	size
default	$\frac{x\ln(e^{x}+1)}{2}+\frac{\text{polylog}(2,-e^{x})}{2}-\frac{x\ln(1-e^{x})}{2}-\frac{\text{polylog}(2,e^{x})}{2}$	34
risch	$\frac{x\ln(e^{x}+1)}{2}+\frac{\text{polylog}(2,-e^{x})}{2}-\frac{x\ln(1-e^{x})}{2}-\frac{\text{polylog}(2,e^{x})}{2}$	34

input
```
int(exp(x)*x/(1-exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*ln(exp(x)+1)+1/2*polylog(2,-exp(x))-1/2*x*ln(1-exp(x))-1/2*polylog(2
,exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{e^x x}{1-e^{2x}}\,dx = \frac{1}{2}\,x\log\left(e^{x}+1\right)-\frac{1}{2}\,x\log\left(-e^{x}+1\right)+\frac{1}{2}\,\mathrm{Li}_2(-e^{x})-\frac{1}{2}\,\mathrm{Li}_2(e^{x})$$

input
```
integrate(exp(x)*x/(1-exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*x*log(e^x + 1) - 1/2*x*log(-e^x + 1) + 1/2*dilog(-e^x) - 1/2*dilog(e^x
)
```

Sympy [F]

$$\int \frac{e^x x}{1 - e^{2x}}\, dx = -\int \frac{x e^x}{e^{2x} - 1}\, dx$$

input
```
integrate(exp(x)*x/(1-exp(2*x)),x)
```

output
```
-Integral(x*exp(x)/(exp(2*x) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{e^x x}{1 - e^{2x}}\, dx = \frac{1}{2}\, x \log\left(e^x + 1\right) - \frac{1}{2}\, x \log\left(-e^x + 1\right) + \frac{1}{2}\, \mathrm{Li}_2(-e^x) - \frac{1}{2}\, \mathrm{Li}_2(e^x)$$

input
```
integrate(exp(x)*x/(1-exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*x*log(e^x + 1) - 1/2*x*log(-e^x + 1) + 1/2*dilog(-e^x) - 1/2*dilog(e^x
)
```

Giac [F]

$$\int \frac{e^x x}{1 - e^{2x}}\, dx = \int -\frac{x e^x}{e^{(2\,x)} - 1}\, dx$$

input
```
integrate(exp(x)*x/(1-exp(2*x)),x, algorithm="giac")
```

output
```
integrate(-x*e^x/(e^(2*x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x x}{1-e^{2x}}\,dx = -\int \frac{x\,\mathrm{e}^x}{\mathrm{e}^{2\,x}-1}\,dx$$

input
```
int(-(x*exp(x))/(exp(2*x) - 1),x)
```

output
```
-int((x*exp(x))/(exp(2*x) - 1), x)
```

Reduce [F]

$$\int \frac{e^x x}{1-e^{2x}}\,dx = -\left(\int \frac{e^x x}{e^{2x}-1}dx\right)$$

input
```
int(exp(x)*x/(1-exp(2*x)),x)
```

output
```
- int((e**x*x)/(e**(2*x) - 1),x)
```

3.97 $\int \frac{e^x x^2}{1-e^{2x}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{e^x x^2}{1-e^{2x}} \, dx = x^2 \text{arctanh}(e^x) + x \,\text{PolyLog}(2,-e^x) - x \,\text{PolyLog}(2,e^x) - \text{PolyLog}(3,-e^x) + \text{PolyLog}(3,e^x)$$

output
```
x^2*arctanh(exp(x))+x*polylog(2,-exp(x))-x*polylog(2,exp(x))-polylog(3,-ex
p(x))+polylog(3,exp(x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.50

$$\int \frac{e^x x^2}{1-e^{2x}} \, dx = -\frac{1}{2} x^2 \log(1-e^x) + \frac{1}{2} x^2 \log(1+e^x) + x \,\text{PolyLog}(2,-e^x) - x \,\text{PolyLog}(2,e^x) - \text{PolyLog}(3,-e^x) + \text{PolyLog}(3,e^x)$$

input
```
Integrate[(E^x*x^2)/(1 - E^(2*x)),x]
```

output
```
-1/2*(x^2*Log[1 - E^x]) + (x^2*Log[1 + E^x])/2 + x*PolyLog[2, -E^x] - x*Po
lyLog[2, E^x] - PolyLog[3, -E^x] + PolyLog[3, E^x]
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {2675, 6767, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x x^2}{1-e^{2x}} \, dx$$

$$\Big\downarrow \text{2675}$$

$$x^2\text{arctanh}(e^x) - 2\int x\text{arctanh}(e^x)\,dx$$

$$\Big\downarrow \text{6767}$$

$$x^2\text{arctanh}(e^x) - 2\left(\frac{1}{2}\int x\log(1+e^x)\,dx - \frac{1}{2}\int x\log(1-e^x)\,dx\right)$$

$$\Big\downarrow \text{3011}$$

$$x^2\text{arctanh}(e^x) - 2\left(\frac{1}{2}\left(\int \text{PolyLog}(2,-e^x)\,dx - x\,\text{PolyLog}(2,-e^x)\right) + \frac{1}{2}\left(x\,\text{PolyLog}(2,e^x) - \int \text{PolyLog}(2,e^x)\,dx\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$x^2\text{arctanh}(e^x) - 2\left(\frac{1}{2}\left(\int e^{-x}\,\text{PolyLog}(2,-e^x)\,de^x - x\,\text{PolyLog}(2,-e^x)\right) + \frac{1}{2}\left(x\,\text{PolyLog}(2,e^x) - \int e^{-x}\,\text{PolyLog}(2,e^x)\,de^x\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$x^2\text{arctanh}(e^x) - 2\left(\frac{1}{2}(\text{PolyLog}(3,-e^x) - x\,\text{PolyLog}(2,-e^x)) + \frac{1}{2}(x\,\text{PolyLog}(2,e^x) - \text{PolyLog}(3,e^x))\right)$$

input
```
Int[(E^x*x^2)/(1 - E^(2*x)),x]
```

output
```
x^2*ArcTanh[E^x] - 2*((-(x*PolyLog[2, -E^x]) + PolyLog[3, -E^x])/2 + (x*Po
lyLog[2, E^x] - PolyLog[3, E^x])/2)
```

Defintions of rubi rules used

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

method	result
default	$\frac{x^2\ln(e^x+1)}{2} + x\,\mathrm{polylog}\left(2,-e^x\right) - \mathrm{polylog}\left(3,-e^x\right) - \frac{x^2\ln(1-e^x)}{2} - x\,\mathrm{polylog}\left(2,e^x\right) + \mathrm{polylog}\left(3,e\right.$
risch	$\frac{x^2\ln(e^x+1)}{2} + x\,\mathrm{polylog}\left(2,-e^x\right) - \mathrm{polylog}\left(3,-e^x\right) - \frac{x^2\ln(1-e^x)}{2} - x\,\mathrm{polylog}\left(2,e^x\right) + \mathrm{polylog}\left(3,e\right.$

input
```
int(exp(x)*x^2/(1-exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*ln(exp(x)+1)+x*polylog(2,-exp(x))-polylog(3,-exp(x))-1/2*x^2*ln(1-
exp(x))-x*polylog(2,exp(x))+polylog(3,exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{e^x x^2}{1-e^{2x}}\,dx = \frac{1}{2}\,x^2\log\left(e^x+1\right) - \frac{1}{2}\,x^2\log\left(-e^x+1\right) + x\mathrm{Li}_2(-e^x) - x\mathrm{Li}_2(e^x) - \mathrm{polylog}(3,-e^x) + \mathrm{polylog}(3,e^x)$$

input
```
integrate(exp(x)*x^2/(1-exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*x^2*log(e^x + 1) - 1/2*x^2*log(-e^x + 1) + x*dilog(-e^x) - x*dilog(e^x
) - polylog(3, -e^x) + polylog(3, e^x)
```

Sympy [F]

$$\int \frac{e^x x^2}{1-e^{2x}}\,dx = -\int \frac{x^2e^x}{e^{2x}-1}\,dx$$

input
```
integrate(exp(x)*x**2/(1-exp(2*x)),x)
```

output
```
-Integral(x**2*exp(x)/(exp(2*x) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{e^x x^2}{1-e^{2x}} \, dx = \frac{1}{2} x^2 \log \left(e^x+1\right)-\frac{1}{2} x^2 \log \left(-e^x+1\right) + x \mathrm{Li}_2(-e^x) - x \mathrm{Li}_2(e^x) - \mathrm{Li}_3(-e^x) + \mathrm{Li}_3(e^x)$$

input
```
integrate(exp(x)*x^2/(1-exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*x^2*log(e^x + 1) - 1/2*x^2*log(-e^x + 1) + x*dilog(-e^x) - x*dilog(e^x
) - polylog(3, -e^x) + polylog(3, e^x)
```

Giac [F]

$$\int \frac{e^x x^2}{1-e^{2x}} \, dx = \int -\frac{x^2 e^x}{e^{(2\,x)}-1} \, dx$$

input
```
integrate(exp(x)*x^2/(1-exp(2*x)),x, algorithm="giac")
```

output
```
integrate(-x^2*e^x/(e^(2*x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x x^2}{1-e^{2x}} \, dx = -\int \frac{x^2\,\mathrm{e}^x}{\mathrm{e}^{2\,x}-1} \, dx$$

input
```
int(-(x^2*exp(x))/(exp(2*x) - 1),x)
```

output
```
-int((x^2*exp(x))/(exp(2*x) - 1), x)
```

Reduce [F]

$$\int \frac{e^x x^2}{1-e^{2x}}\,dx = -\left(\int \frac{e^x x^2}{e^{2x}-1}dx\right)$$

input
```
int(exp(x)*x^2/(1-exp(2*x)),x)
```

output
```
- int((e**x*x**2)/(e**(2*x) - 1),x)
```

3.98 $\int \frac{e^x x^3}{1-e^{2x}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 69

$$\int \frac{e^x x^3}{1-e^{2x}} \, dx = x^3 \operatorname{arctanh}(e^x) + \frac{3}{2} x^2 \operatorname{PolyLog}(2,-e^x) - \frac{3}{2} x^2 \operatorname{PolyLog}(2,e^x) - 3x \operatorname{PolyLog}(3,-e^x) + 3x \operatorname{PolyLog}(3,e^x) + 3 \operatorname{PolyLog}(4,-e^x) - 3 \operatorname{PolyLog}(4,e^x)$$

output
```
x^3*arctanh(exp(x))+3/2*x^2*polylog(2,-exp(x))-3/2*x^2*polylog(2,exp(x))-3
*x*polylog(3,-exp(x))+3*x*polylog(3,exp(x))+3*polylog(4,-exp(x))-3*polylog
(4,exp(x))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.29

$$\int \frac{e^x x^3}{1-e^{2x}} \, dx = -\frac{1}{2} x^3 \log(1-e^x) + \frac{1}{2} x^3 \log(1+e^x) + \frac{3}{2} x^2 \operatorname{PolyLog}(2,-e^x) - \frac{3}{2} x^2 \operatorname{PolyLog}(2,e^x) - 3x \operatorname{PolyLog}(3,-e^x) + 3x \operatorname{PolyLog}(3,e^x) + 3 \operatorname{PolyLog}(4,-e^x) - 3 \operatorname{PolyLog}(4,e^x)$$

input
```
Integrate[(E^x*x^3)/(1 - E^(2*x)),x]
```

output
```
-1/2*(x^3*Log[1 - E^x]) + (x^3*Log[1 + E^x])/2 + (3*x^2*PolyLog[2, -E^x])/
2 - (3*x^2*PolyLog[2, E^x])/2 - 3*x*PolyLog[3, -E^x] + 3*x*PolyLog[3, E^x]
 + 3*PolyLog[4, -E^x] - 3*PolyLog[4, E^x]
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2675, 6767, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^x x^3}{1-e^{2x}} \, dx$$

$$\Big\downarrow \text{2675}$$

$$x^3 \text{arctanh}(e^x) - 3\int x^2 \text{arctanh}(e^x) \, dx$$

$$\Big\downarrow \text{6767}$$

$$x^3 \text{arctanh}(e^x) - 3\left(\frac{1}{2}\int x^2 \log(1+e^x) \, dx - \frac{1}{2}\int x^2 \log(1-e^x) \, dx\right)$$

$$\Big\downarrow \text{3011}$$

$$x^3 \text{arctanh}(e^x) - 3\left(\frac{1}{2}\left(2\int x \,\text{PolyLog}(2,-e^x) \, dx - x^2 \,\text{PolyLog}(2,-e^x)\right) + \frac{1}{2}\left(x^2 \,\text{PolyLog}(2,e^x) - 2\int x \,\text{PolyLog}(2,e^x) \, dx\right)\right)$$

$$\Big\downarrow \text{7163}$$

$$x^3 \text{arctanh}(e^x) - 3\left(\frac{1}{2}\left(2\left(x \,\text{PolyLog}(3,-e^x) - \int \text{PolyLog}(3,-e^x) \, dx\right) - x^2 \,\text{PolyLog}(2,-e^x)\right) + \frac{1}{2}\left(x^2 \,\text{PolyLog}(2,e^x) - 2\left(x \,\text{P}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$x^3 \text{arctanh}(e^x) - 3\left(\frac{1}{2}\left(2\left(x \,\text{PolyLog}(3,-e^x) - \int e^{-x} \,\text{PolyLog}(3,-e^x) \, de^x\right) - x^2 \,\text{PolyLog}(2,-e^x)\right) + \frac{1}{2}\left(x^2 \,\text{PolyLog}(2,e^x) - 2\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$x^3\text{arctanh}(e^x) - 3\left(\frac{1}{2}(2(x\,\text{PolyLog}(3,-e^x) - \text{PolyLog}(4,-e^x)) - x^2\,\text{PolyLog}(2,-e^x)) + \frac{1}{2}(x^2\,\text{PolyLog}(2,e^x) - 2(x\,\text{PolyLog}(3\right.$$

input
```
Int[(E^x*x^3)/(1 - E^(2*x)),x]
```

output
```
x^3*ArcTanh[E^x] - 3*((-(x^2*PolyLog[2, -E^x]) + 2*(x*PolyLog[3, -E^x] - P
olyLog[4, -E^x]))/2 + (x^2*PolyLog[2, E^x] - 2*(x*PolyLog[3, E^x] - PolyLo
g[4, E^x]))/2)
```

Defintions of rubi rules used

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6767
```
Int[ArcTanh[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol]
:> Simp[1/2   Int[x^m*Log[1 + a + b*f^(c + d*x)], x], x] - Simp[1/2   Int[x
^m*Log[1 - a - b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] && IGtQ[
m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result
default	$\frac{x^3 \ln(e^x+1)}{2}+\frac{3x^2 \operatorname{polylog}(2,-e^x)}{2}-3x \operatorname{polylog}(3,-e^x)+3 \operatorname{polylog}(4,-e^x)-\frac{x^3 \ln(1-e^x)}{2}-\frac{3x^2 \operatorname{polylog}(2,}{2}$
risch	$\frac{x^3 \ln(e^x+1)}{2}+\frac{3x^2 \operatorname{polylog}(2,-e^x)}{2}-3x \operatorname{polylog}(3,-e^x)+3 \operatorname{polylog}(4,-e^x)-\frac{x^3 \ln(1-e^x)}{2}-\frac{3x^2 \operatorname{polylog}(2,}{2}$

input
```
int(exp(x)*x^3/(1-exp(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^3*ln(exp(x)+1)+3/2*x^2*polylog(2,-exp(x))-3*x*polylog(3,-exp(x))+3*p
olylog(4,-exp(x))-1/2*x^3*ln(1-exp(x))-3/2*x^2*polylog(2,exp(x))+3*x*polyl
og(3,exp(x))-3*polylog(4,exp(x))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{e^x x^3}{1-e^{2x}}\,dx = \frac{1}{2}\,x^3\log\left(e^x+1\right)-\frac{1}{2}\,x^3\log\left(-e^x+1\right)+\frac{3}{2}\,x^2\mathrm{Li}_2(-e^x) - \frac{3}{2}\,x^2\mathrm{Li}_2(e^x)-3\,x\mathrm{polylog}(3,-e^x)+3\,x\mathrm{polylog}(3,e^x) + 3\,\mathrm{polylog}(4,-e^x)-3\,\mathrm{polylog}(4,e^x)$$

input
```
integrate(exp(x)*x^3/(1-exp(2*x)),x, algorithm="fricas")
```

output
```
1/2*x^3*log(e^x + 1) - 1/2*x^3*log(-e^x + 1) + 3/2*x^2*dilog(-e^x) - 3/2*x
^2*dilog(e^x) - 3*x*polylog(3, -e^x) + 3*x*polylog(3, e^x) + 3*polylog(4,
-e^x) - 3*polylog(4, e^x)
```

Sympy [F]

$$\int \frac{e^x x^3}{1-e^{2x}}\,dx = -\int \frac{x^3 e^x}{e^{2x}-1}\,dx$$

input
```
integrate(exp(x)*x**3/(1-exp(2*x)),x)
```

output
```
-Integral(x**3*exp(x)/(exp(2*x) - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{e^x x^3}{1-e^{2x}}\,dx = \frac{1}{2}\,x^3 \log\left(e^x+1\right) - \frac{1}{2}\,x^3 \log\left(-e^x+1\right) + \frac{3}{2}\,x^2 \mathrm{Li}_2(-e^x) - \frac{3}{2}\,x^2 \mathrm{Li}_2(e^x) - 3\,x\mathrm{Li}_3(-e^x) + 3\,x\mathrm{Li}_3(e^x) + 3\,\mathrm{Li}_4(-e^x) - 3\,\mathrm{Li}_4(e^x)$$

input
```
integrate(exp(x)*x^3/(1-exp(2*x)),x, algorithm="maxima")
```

output
```
1/2*x^3*log(e^x + 1) - 1/2*x^3*log(-e^x + 1) + 3/2*x^2*dilog(-e^x) - 3/2*x
^2*dilog(e^x) - 3*x*polylog(3, -e^x) + 3*x*polylog(3, e^x) + 3*polylog(4,
-e^x) - 3*polylog(4, e^x)
```

Giac [F]

$$\int \frac{e^x x^3}{1-e^{2x}}\,dx = \int -\frac{x^3 e^x}{e^{(2x)}-1}\,dx$$

input
```
integrate(exp(x)*x^3/(1-exp(2*x)),x, algorithm="giac")
```

output
```
integrate(-x^3*e^x/(e^(2*x) - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^x x^3}{1-e^{2x}}\,dx = -\int \frac{x^3\,\mathrm{e}^x}{\mathrm{e}^{2x}-1}\,dx$$

input
```
int(-(x^3*exp(x))/(exp(2*x) - 1),x)
```

output
```
-int((x^3*exp(x))/(exp(2*x) - 1), x)
```

Reduce [F]

$$\int \frac{e^x x^3}{1-e^{2x}}\,dx = -\left(\int \frac{e^x x^3}{e^{2x}-1}dx\right)$$

input
```
int(exp(x)*x^3/(1-exp(2*x)),x)
```

output
```
- int((e**x*x**3)/(e**(2*x) - 1),x)
```

3.99 $\int \frac{f^x}{a+bf^{2x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{f^x}{a+bf^{2x}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

output
```
arctan(b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{f^x}{a+bf^{2x}}\,dx=\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Integrate[f^x/(a + b*f^(2*x)),x]
```

output
```
ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[f])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2679, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^x}{a+bf^{2x}}\,dx$$

$$\Big\downarrow \text{2679}$$

$$\frac{\int \frac{1}{bf^{2x}+a}df^x}{\log(f)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Int[f^x/(a + b*f^(2*x)),x]
```

output
```
ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*Log[f])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

method	result	size
risch	$-\frac{\ln\left(f^{x}-\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln\left(f\right)}+\frac{\ln\left(f^{x}+\frac{a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln\left(f\right)}$	53

input
```
int(f^x/(a+b*f^(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a*b)^(1/2)/ln(f)*ln(f^x-1/(-a*b)^(1/2)*a)+1/2/(-a*b)^(1/2)/ln(f)*ln
(f^x+1/(-a*b)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.87

$$\int \frac{f^x}{a+bf^{2x}}\,dx=\left[-\frac{\sqrt{-ab}\log\left(\frac{bf^{2\,x}-2\,\sqrt{-ab}f^{x}-a}{bf^{2\,x}+a}\right)}{2\,ab\log\left(f\right)},-\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{bf^{x}}\right)}{ab\log\left(f\right)}\right]$$

input
```
integrate(f^x/(a+b*f^(2*x)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*f^(2*x) - 2*sqrt(-a*b)*f^x - a)/(b*f^(2*x) + a))/(
a*b*log(f)), -sqrt(a*b)*arctan(sqrt(a*b)/(b*f^x))/(a*b*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{f^x}{a+bf^{2x}}\,dx = \frac{\operatorname{RootSum}\left(4z^2ab+1,\left(i\mapsto i\log\left(2ia+f^x\right)\right)\right)}{\log\left(f\right)}$$

input
```
integrate(f**x/(a+b*f**(2*x)),x)
```

output
```
RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(2*_i*a + f**x)))/log(f)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{f^x}{a+bf^{2x}}\,dx = \frac{\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{\sqrt{ab}\log\left(f\right)}$$

input
```
integrate(f^x/(a+b*f^(2*x)),x, algorithm="maxima")
```

output
```
arctan(b*f^x/sqrt(a*b))/(sqrt(a*b)*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{f^x}{a+bf^{2x}}\,dx = \frac{\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{\sqrt{ab}\log\left(f\right)}$$

input
```
integrate(f^x/(a+b*f^(2*x)),x, algorithm="giac")
```

output
```
arctan(b*f^x/sqrt(a*b))/(sqrt(a*b)*log(f))
```

Mupad [B] (verification not implemented)

Time = 23.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{f^x}{a+bf^{2x}}\,dx=\frac{\operatorname{atan}\left(\frac{b\,f^x}{\sqrt{a\,b}}\right)}{\ln\left(f\right)\,\sqrt{a\,b}}$$

input
```
int(f^x/(a + b*f^(2*x)),x)
```

output
```
atan((b*f^x)/(a*b)^(1/2))/(log(f)*(a*b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{f^x}{a+bf^{2x}}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x b}{\sqrt{b}\,\sqrt{a}}\right)}{\log\left(f\right)ab}$$

input
```
int(f^x/(a+b*f^(2*x)),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((f**x*b)/(sqrt(b)*sqrt(a))))/(log(f)*a*b)
```

3.100 $\int \frac{f^x x}{a+bf^{2x}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int \frac{f^x x}{a+bf^{2x}}\,dx=\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)}+\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)}$$

output
```
x*arctan(b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)-1/2*I*polylog(2,-I*b^(
1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+1/2*I*polylog(2,I*b^(1/2)*f^x/a^
(1/2))/a^(1/2)/b^(1/2)/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{f^x x}{a+bf^{2x}}\,dx$$
$$=\frac{i\left(x\log(f)\left(\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)-\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)\right)-\mathrm{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)+\mathrm{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)\right)}{2\sqrt{a}\sqrt{b}\log^2(f)}$$

input
```
Integrate[(f^x*x)/(a + b*f^(2*x)),x]
```

output
```
((I/2)*(x*Log[f]*(Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - Log[1 + (I*Sqrt[b]*f^
x)/Sqrt[a]]) - PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + PolyLog[2, (I*Sqrt
[b]*f^x)/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2675, 27, 2720, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xf^x}{a+bf^{2x}}\,dx$$

$$\Big\downarrow \text{2675}$$

$$\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\int \frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{\int \arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{\int f^{-x}\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)df^x}{\sqrt{a}\sqrt{b}\log^2(f)}$$

$$\Big\downarrow \text{5355}$$

$$\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{\frac{1}{2}i\int f^{-x}\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)df^x-\frac{1}{2}i\int f^{-x}\log\left(\frac{i\sqrt{b}f^x}{\sqrt{a}}+1\right)df^x}{\sqrt{a}\sqrt{b}\log^2(f)}$$

$$\Big\downarrow \text{2838}$$

$$\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)}$$

input
```
Int[(f^x*x)/(a + b*f^(2*x)),x]
```

output
```
(x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Log[f]) - ((I/2)*PolyLo
g[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] - (I/2)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a
]])/(Sqrt[a]*Sqrt[b]*Log[f]^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

method	result	size
risch	$\frac{x\ln\left(\frac{-b\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)\sqrt{-ab}}-\frac{x\ln\left(\frac{b\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)\sqrt{-ab}}+\frac{\operatorname{dilog}\left(\frac{-b\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)^{2}\sqrt{-ab}}-\frac{\operatorname{dilog}\left(\frac{b\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)^{2}\sqrt{-ab}}$	134

input
```
int(f^x*x/(a+b*f^(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/ln(f)*x/(-a*b)^(1/2)*ln((-b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2/ln(f)*
x/(-a*b)^(1/2)*ln((b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2/ln(f)^2/(-a*b)^(1
/2)*dilog((-b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2/ln(f)^2/(-a*b)^(1/2)*dil
og((b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{f^x x}{a+bf^{2x}}\,dx = -\frac{x\sqrt{-\frac{b}{a}}\log\left(f^x\sqrt{-\frac{b}{a}}+1\right)\log(f)-x\sqrt{-\frac{b}{a}}\log\left(-f^x\sqrt{-\frac{b}{a}}+1\right)\log(f)-\sqrt{-\frac{b}{a}}\mathrm{Li}_2\left(f^x\sqrt{-\frac{b}{a}}\right)+\sqrt{-}}{2\,b\log(f)^2}$$

input
```
integrate(f^x*x/(a+b*f^(2*x)),x, algorithm="fricas")
```

output
```
-1/2*(x*sqrt(-b/a)*log(f^x*sqrt(-b/a) + 1)*log(f) - x*sqrt(-b/a)*log(-f^x*
sqrt(-b/a) + 1)*log(f) - sqrt(-b/a)*dilog(f^x*sqrt(-b/a)) + sqrt(-b/a)*dil
og(-f^x*sqrt(-b/a)))/(b*log(f)^2)
```

Sympy [F]

$$\int \frac{f^x x}{a + bf^{2x}} \, dx = \int \frac{f^x x}{a + bf^{2x}} \, dx$$

input
```
integrate(f**x*x/(a+b*f**(2*x)),x)
```

output
```
Integral(f**x*x/(a + b*f**(2*x)), x)
```

Maxima [F]

$$\int \frac{f^x x}{a + bf^{2x}} \, dx = \int \frac{f^x x}{bf^{2x} + a} \, dx$$

input
```
integrate(f^x*x/(a+b*f^(2*x)),x, algorithm="maxima")
```

output
```
integrate(f^x*x/(b*f^(2*x) + a), x)
```

Giac [F]

$$\int \frac{f^x x}{a + bf^{2x}} \, dx = \int \frac{f^x x}{bf^{2x} + a} \, dx$$

input
```
integrate(f^x*x/(a+b*f^(2*x)),x, algorithm="giac")
```

output
```
integrate(f^x*x/(b*f^(2*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x}{a+bf^{2x}}\,dx = \int \frac{f^x\,x}{a+b\,f^{2\,x}}\,dx$$

input
```
int((f^x*x)/(a + b*f^(2*x)),x)
```

output
```
int((f^x*x)/(a + b*f^(2*x)), x)
```

Reduce [F]

$$\int \frac{f^x x}{a+bf^{2x}}\,dx = \int \frac{f^x x}{f^{2x}b+a}dx$$

input
```
int(f^x*x/(a+b*f^(2*x)),x)
```

output
```
int((f**x*x)/(f**(2*x)*b + a),x)
```

3.101 $\int \frac{f^x x^2}{a+bf^{2x}} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 184

$$\int \frac{f^x x^2}{a+bf^{2x}} \, dx = \frac{x^2 \arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)}$$

output
```
x^2*arctan(b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)-I*x*polylog(2,-I*b^(
1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+I*x*polylog(2,I*b^(1/2)*f^x/a^(1
/2))/a^(1/2)/b^(1/2)/ln(f)^2+I*polylog(3,-I*b^(1/2)*f^x/a^(1/2))/a^(1/2)/b
^(1/2)/ln(f)^3-I*polylog(3,I*b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

$$\int \frac{f^x x^2}{a+bf^{2x}} \, dx = \frac{i\left(x^2 \log^2(f) \log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right) - x^2 \log^2(f) \log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right) - 2x\log(f) \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right) + 2x\log(f)\ \ldots}{2\sqrt{a}\sqrt{b}\log^3(f)}$$

input
```
Integrate[(f^x*x^2)/(a + b*f^(2*x)),x]
```

output
```
((I/2)*(x^2*Log[f]^2*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - x^2*Log[f]^2*Log[1
 + (I*Sqrt[b]*f^x)/Sqrt[a]] - 2*x*Log[f]*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqr
t[a]] + 2*x*Log[f]*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]] + 2*PolyLog[3, ((-I
)*Sqrt[b]*f^x)/Sqrt[a]] - 2*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]]))/(Sqrt[a]
*Sqrt[b]*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2675, 27, 5666, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 f^x}{a+bf^{2x}}\,dx$$

$$\Big\downarrow \text{2675}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-2\int \frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\int x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \text{5666}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\int x\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx-\frac{1}{2}i\int x\log\left(\frac{i\sqrt{b}f^x}{\sqrt{a}}+1\right)dx\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \text{3011}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\left(\frac{\int \operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\log(f)}-\frac{x\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{\int \operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\log(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\left(\frac{\int f^{-x}\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)df^x}{\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{\int f^{-x}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)df^x}{\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\left(\frac{\operatorname{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{\operatorname{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Int[(f^x*x^2)/(a + b*f^(2*x)),x]
```

output
```
(x^2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Log[f]) - (2*((-1/2*I
)*(-((x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/Log[f]) + PolyLog[3, ((-I)
*Sqrt[b]*f^x)/Sqrt[a]]/Log[f]^2) + (I/2)*(-((x*PolyLog[2, (I*Sqrt[b]*f^x)/
Sqrt[a]])/Log[f]) + PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]]/Log[f]^2)))/(Sqrt[
a]*Sqrt[b]*Log[f])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{f^x x^2}{a + b\, f^{2x}} dx$$

input
```
int(f^x*x^2/(a+b*f^(2*x)),x)
```

output
```
int(f^x*x^2/(a+b*f^(2*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{f^x x^2}{a + b f^{2x}} \, dx =$$

$$-\frac{x^2\sqrt{-\frac{b}{a}}\log\left(f^x\sqrt{-\frac{b}{a}}+1\right)\log(f)^2 - x^2\sqrt{-\frac{b}{a}}\log\left(-f^x\sqrt{-\frac{b}{a}}+1\right)\log(f)^2 - 2\,x\sqrt{-\frac{b}{a}}\mathrm{Li}_2\left(f^x\sqrt{-\frac{b}{a}}\right)}{2\,b}$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x)),x, algorithm="fricas")
```

output
```
-1/2*(x^2*sqrt(-b/a)*log(f^x*sqrt(-b/a) + 1)*log(f)^2 - x^2*sqrt(-b/a)*log
(-f^x*sqrt(-b/a) + 1)*log(f)^2 - 2*x*sqrt(-b/a)*dilog(f^x*sqrt(-b/a))*log(
f) + 2*x*sqrt(-b/a)*dilog(-f^x*sqrt(-b/a))*log(f) + 2*sqrt(-b/a)*polylog(3
, f^x*sqrt(-b/a)) - 2*sqrt(-b/a)*polylog(3, -f^x*sqrt(-b/a)))/(b*log(f)^3)
```

Sympy [F]

$$\int \frac{f^x x^2}{a + b f^{2x}} \, dx = \int \frac{f^x x^2}{a + b f^{2x}} \, dx$$

input
```
integrate(f**x*x**2/(a+b*f**(2*x)),x)
```

output
```
Integral(f**x*x**2/(a + b*f**(2*x)), x)
```

Maxima [F]

$$\int \frac{f^x x^2}{a + bf^{2x}} \, dx = \int \frac{f^x x^2}{bf^{2x} + a} \, dx$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x)),x, algorithm="maxima")
```

output
```
integrate(f^x*x^2/(b*f^(2*x) + a), x)
```

Giac [F]

$$\int \frac{f^x x^2}{a + bf^{2x}} \, dx = \int \frac{f^x x^2}{bf^{2x} + a} \, dx$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x)),x, algorithm="giac")
```

output
```
integrate(f^x*x^2/(b*f^(2*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x^2}{a + bf^{2x}} \, dx = \int \frac{f^x x^2}{a + b f^{2x}} \, dx$$

input
```
int((f^x*x^2)/(a + b*f^(2*x)),x)
```

output
```
int((f^x*x^2)/(a + b*f^(2*x)), x)
```

Reduce [F]

$$\int \frac{f^x x^2}{a+bf^{2x}}\,dx=\int \frac{f^x x^2}{f^{2x}b+a}dx$$

input
```
int(f^x*x^2/(a+b*f^(2*x)),x)
```

output
```
int((f**x*x**2)/(f**(2*x)*b + a),x)
```

3.102 $\int \frac{f^x x^3}{a+bf^{2x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 268

$$\int \frac{f^x x^3}{a+bf^{2x}}\,dx = \frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3ix^2\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix^2\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} + \frac{3ix\operatorname{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix\operatorname{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3i\operatorname{PolyLog}\left(4,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i\operatorname{PolyLog}\left(4,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}$$

output

```
x^3*arctan(b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)-3/2*I*x^2*polylog(2,
-I*b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+3/2*I*x^2*polylog(2,I*b^(1
/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+3*I*x*polylog(3,-I*b^(1/2)*f^x/a^
(1/2))/a^(1/2)/b^(1/2)/ln(f)^3-3*I*x*polylog(3,I*b^(1/2)*f^x/a^(1/2))/a^(1
/2)/b^(1/2)/ln(f)^3-3*I*polylog(4,-I*b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/
ln(f)^4+3*I*polylog(4,I*b^(1/2)*f^x/a^(1/2))/a^(1/2)/b^(1/2)/ln(f)^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.84

$$\int \frac{f^x x^3}{a+bf^{2x}}\,dx$$
$$= \frac{i\left(x^3\log^3(f)\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)-x^3\log^3(f)\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)-3x^2\log^2(f)\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)+3x^2\log^2}{}$$

input
```
Integrate[(f^x*x^3)/(a + b*f^(2*x)),x]
```

output
```
((I/2)*(x^3*Log[f]^3*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - x^3*Log[f]^3*Log[1
 + (I*Sqrt[b]*f^x)/Sqrt[a]] - 3*x^2*Log[f]^2*PolyLog[2, ((-I)*Sqrt[b]*f^x)
/Sqrt[a]] + 3*x^2*Log[f]^2*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]] + 6*x*Log[f
]*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] - 6*x*Log[f]*PolyLog[3, (I*Sqrt[b
]*f^x)/Sqrt[a]] - 6*PolyLog[4, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + 6*PolyLog[4,
(I*Sqrt[b]*f^x)/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]*Log[f]^4)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {2675, 27, 5666, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 f^x}{a+bf^{2x}}\,dx$$
$$\Big\downarrow \; 2675$$
$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-3\int \frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}dx$$
$$\Big\downarrow \; 27$$

$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\int x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\sqrt{a}\sqrt{b}\log(f)}$$

$\big\downarrow$ 5666

$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\int x^2\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx-\frac{1}{2}i\int x^2\log\left(\frac{i\sqrt{b}f^x}{\sqrt{a}}+1\right)dx\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$\big\downarrow$ 3011

$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$\big\downarrow$ 7163

$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}-\frac{\int \text{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)dx}{\log(f)}\right)}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}-\frac{\int \text{PolyLog}\left(3,-\frac{i\sqrt{b}\cdots}{\sqrt{\cdots}}\right.}{\log(f)}\right.}{\log(f)}\right.\right.}{\sqrt{a}\sqrt{b}\log(f)}$$

$\big\downarrow$ 2720

$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}-\frac{\int f^{-x}\,\text{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)df^x}{\log^2(f)}\right)}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}-\frac{\int f^{-x}\,\text{PolyLo}\cdots}{\cdots}\right.}{\log(f)}\right.\right.}{\sqrt{a}\sqrt{b}\log(f)}$$

$\big\downarrow$ 7143

$$\frac{x^3 \arctan\left(\frac{\sqrt{b} f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3\left(\frac{1}{2} i \left(\frac{2\left(\frac{x \operatorname{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)} - \frac{\operatorname{PolyLog}\left(4,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log^2(f)}\right)}{\log(f)} - \frac{x^2 \operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)}\right) - \frac{1}{2} i \left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log(f)} - \frac{\operatorname{PolyLog}\left(4,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\log^2(f)}\right.}{\log(f)} \cdots\right.\right.}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Int[(f^x*x^3)/(a + b*f^(2*x)),x]
```

output
```
(x^3*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Log[f]) - (3*((-1/2*I
)*(-((x^2*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/Log[f]) + (2*((x*PolyLog
[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/Log[f] - PolyLog[4, ((-I)*Sqrt[b]*f^x)/Sq
rt[a]]/Log[f]^2))/Log[f]) + (I/2)*(-((x^2*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[
a]])/Log[f]) + (2*((x*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]])/Log[f] - PolyLo
g[4, (I*Sqrt[b]*f^x)/Sqrt[a]]/Log[f]^2))/Log[f])))/(Sqrt[a]*Sqrt[b]*Log[f]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{f^x x^3}{a + b f^{2x}} dx$$

input
```
int(f^x*x^3/(a+b*f^(2*x)),x)
```

output
```
int(f^x*x^3/(a+b*f^(2*x)),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.89

$$\int \frac{f^x x^3}{a + b f^{2x}}\,dx =$$
$$-\frac{x^3\sqrt{-\frac{b}{a}}\log\left(f^x\sqrt{-\frac{b}{a}}+1\right)\log(f)^3 - x^3\sqrt{-\frac{b}{a}}\log\left(-f^x\sqrt{-\frac{b}{a}}+1\right)\log(f)^3 - 3\,x^2\sqrt{-\frac{b}{a}}\mathrm{Li}_2\left(f^x\sqrt{-\frac{b}{a}}\right)$$

input
```
integrate(f^x*x^3/(a+b*f^(2*x)),x, algorithm="fricas")
```

output
```
-1/2*(x^3*sqrt(-b/a)*log(f^x*sqrt(-b/a) + 1)*log(f)^3 - x^3*sqrt(-b/a)*log
(-f^x*sqrt(-b/a) + 1)*log(f)^3 - 3*x^2*sqrt(-b/a)*dilog(f^x*sqrt(-b/a))*lo
g(f)^2 + 3*x^2*sqrt(-b/a)*dilog(-f^x*sqrt(-b/a))*log(f)^2 + 6*x*sqrt(-b/a)
*log(f)*polylog(3, f^x*sqrt(-b/a)) - 6*x*sqrt(-b/a)*log(f)*polylog(3, -f^x
*sqrt(-b/a)) - 6*sqrt(-b/a)*polylog(4, f^x*sqrt(-b/a)) + 6*sqrt(-b/a)*poly
log(4, -f^x*sqrt(-b/a)))/(b*log(f)^4)
```

Sympy [F]

$$\int \frac{f^x x^3}{a + b f^{2x}}\,dx = \int \frac{f^x x^3}{a + b f^{2x}}\,dx$$

input
```
integrate(f**x*x**3/(a+b*f**(2*x)),x)
```

output
```
Integral(f**x*x**3/(a + b*f**(2*x)), x)
```

Maxima [F]

$$\int \frac{f^x x^3}{a + b f^{2x}}\, dx = \int \frac{f^x x^3}{b f^{2\,x} + a}\, dx$$

input
```
integrate(f^x*x^3/(a+b*f^(2*x)),x, algorithm="maxima")
```

output
```
integrate(f^x*x^3/(b*f^(2*x) + a), x)
```

Giac [F]

$$\int \frac{f^x x^3}{a + b f^{2x}}\, dx = \int \frac{f^x x^3}{b f^{2\,x} + a}\, dx$$

input
```
integrate(f^x*x^3/(a+b*f^(2*x)),x, algorithm="giac")
```

output
```
integrate(f^x*x^3/(b*f^(2*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x^3}{a + b f^{2x}}\, dx = \int \frac{f^x\, x^3}{a + b\, f^{2\,x}}\, dx$$

input
```
int((f^x*x^3)/(a + b*f^(2*x)),x)
```

output
```
int((f^x*x^3)/(a + b*f^(2*x)), x)
```

Reduce [F]

$$\int \frac{f^x x^3}{a + bf^{2x}} \, dx = \int \frac{f^x x^3}{f^{2x}b + a} dx$$

input
```
int(f^x*x^3/(a+b*f^(2*x)),x)
```

output
```
int((f**x*x**3)/(f**(2*x)*b + a),x)
```

3.103 $\int \frac{f^x}{\left(a+bf^{2x}\right)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{f^x}{\left(a+bf^{2x}\right)^2}\,dx=\frac{f^x}{2a\left(a+bf^{2x}\right)\log(f)}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}$$

output
```
1/2*f^x/a/(a+b*f^(2*x))/ln(f)+1/2*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1
/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{f^x}{\left(a+bf^{2x}\right)^2}\,dx=\frac{\frac{f^x}{a^2+abf^{2x}}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}}{2\log(f)}$$

input
```
Integrate[f^x/(a + b*f^(2*x))^2,x]
```

output
```
(f^x/(a^2 + a*b*f^(2*x)) + ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(a^(3/2)*Sqrt[b])
)/(2*Log[f])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2679, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx$$

$$\Big\downarrow \text{ 2679}$$

$$\frac{\int \frac{1}{(bf^{2x}+a)^2}df^x}{\log(f)}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{\int \frac{1}{bf^{2x}+a}df^x}{2a}+\frac{f^x}{2a(a+bf^{2x})}}{\log(f)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{f^x}{2a(a+bf^{2x})}}{\log(f)}$$

input
```
Int[f^x/(a + b*f^(2*x))^2,x]
```

output
```
(f^x/(2*a*(a + b*f^(2*x))) + ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[b]))/Log[f]
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

method	result	size
risch	$\frac{f^{x}}{2a\left(a+b\,f^{2x}\right)\ln\left(f\right)}-\frac{\ln\left(f^{x}-\frac{a}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,a\ln\left(f\right)}+\frac{\ln\left(f^{x}+\frac{a}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,a\ln\left(f\right)}$	82

input
```
int(f^x/(a+b*f^(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/ln(f)/a*f^x/(a+b*(f^x)^2)-1/4/(-a*b)^(1/2)/a/ln(f)*ln(f^x-1/(-a*b)^(1/
2)*a)+1/4/(-a*b)^(1/2)/a/ln(f)*ln(f^x+1/(-a*b)^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.78

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx$$
$$= \left[\frac{2\,abf^x - \left(\sqrt{-ab}bf^{2\,x} + \sqrt{-ab}a\right)\log\left(\frac{bf^{2\,x}-2\,\sqrt{-ab}f^x-a}{bf^{2\,x}+a}\right)}{4\,(a^2b^2f^{2\,x}\log(f) + a^3b\log(f))}, \frac{abf^x - \left(\sqrt{ab}bf^{2\,x} + \sqrt{ab}a\right)\arctan\left(\frac{\sqrt{ab}}{bf^x}\right)}{2\,(a^2b^2f^{2\,x}\log(f) + a^3b\log(f))}\right]$$

input
```
integrate(f^x/(a+b*f^(2*x))^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*f^x - (sqrt(-a*b)*b*f^(2*x) + sqrt(-a*b)*a)*log((b*f^(2*x) - 2
*sqrt(-a*b)*f^x - a)/(b*f^(2*x) + a)))/(a^2*b^2*f^(2*x)*log(f) + a^3*b*log
(f)), 1/2*(a*b*f^x - (sqrt(a*b)*b*f^(2*x) + sqrt(a*b)*a)*arctan(sqrt(a*b)/
(b*f^x)))/(a^2*b^2*f^(2*x)*log(f) + a^3*b*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx = \frac{f^x}{2a^2\log(f) + 2abf^{2x}\log(f)} + \frac{\mathrm{RootSum}\left(16z^2a^3b+1, \left(i \mapsto i\log\left(4ia^2+f^x\right)\right)\right)}{\log(f)}$$

input
```
integrate(f**x/(a+b*f**(2*x))**2,x)
```

output
```
f**x/(2*a**2*log(f) + 2*a*b*f**(2*x)*log(f)) + RootSum(16*_z**2*a**3*b + 1
, Lambda(_i, _i*log(4*_i*a**2 + f**x)))/log(f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx = \frac{f^x}{2\,(abf^{2\,x}+a^2)\log(f)} + \frac{\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a\log(f)}$$

input
```
integrate(f^x/(a+b*f^(2*x))^2,x, algorithm="maxima")
```

output
```
1/2*f^x/((a*b*f^(2*x) + a^2)*log(f)) + 1/2*arctan(b*f^x/sqrt(a*b))/(sqrt(a
*b)*a*log(f))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx = \frac{\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a\log(f)} + \frac{f^x}{2\,(bf^{2\,x}+a)a\log(f)}$$

input
```
integrate(f^x/(a+b*f^(2*x))^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*f^x/sqrt(a*b))/(sqrt(a*b)*a*log(f)) + 1/2*f^x/((b*f^(2*x) + a
)*a*log(f))
```

Mupad [B] (verification not implemented)

Time = 22.91 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx = \frac{f^x}{2\,a\,\ln(f)\,\left(a+b\,f^{2\,x}\right)} + \frac{\mathrm{atan}\left(\frac{b\,f^x}{\sqrt{a\,b}}\right)}{2\,a\,\ln(f)\,\sqrt{a\,b}}$$

input
```
int(f^x/(a + b*f^(2*x))^2,x)
```

output
```
f^x/(2*a*log(f)*(a + b*f^(2*x))) + atan((b*f^x)/(a*b)^(1/2))/(2*a*log(f)*(
a*b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{f^x}{(a+bf^{2x})^2}\,dx = \frac{f^{2x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x b}{\sqrt{b}\,\sqrt{a}}\right)b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x b}{\sqrt{b}\,\sqrt{a}}\right)a+f^x ab}{2\log(f)\,a^2 b\,(f^{2x}b+a)}$$

input
```
int(f^x/(a+b*f^(2*x))^2,x)
```

output
```
(f**(2*x)*sqrt(b)*sqrt(a)*atan((f**x*b)/(sqrt(b)*sqrt(a)))*b + sqrt(b)*sqr
t(a)*atan((f**x*b)/(sqrt(b)*sqrt(a)))*a + f**x*a*b)/(2*log(f)*a**2*b*(f**(
2*x)*b + a))
```

3.104 $\int \frac{f^x x}{\left(a+bf^{2x}\right)^2} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{f^x x}{\left(a+bf^{2x}\right)^2} \, dx = -\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{f^x x}{2a\left(a+bf^{2x}\right)\log(f)} + \frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)}$$

output

```
-1/2*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2+1/2*f^x*x/a/(a+b*
f^(2*x))/ln(f)+1/2*x*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)-1/4
*I*polylog(2,-I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2+1/4*I*polylog
(2,I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.58

$$\int \frac{f^x x}{(a+bf^{2x})^2}\,dx = -\frac{\left(1+\frac{bf^{2x}}{a}\right)\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}\,(a+bf^{2x})\log^2(f)} + \frac{f^x x}{2a\,(a+bf^{2x})\log(f)} + \frac{\frac{\frac{ix^2}{2\sqrt{a}}-\frac{ix\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log(f)}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^2(f)}}{2\sqrt{b}}+\frac{-\frac{ix^2}{2\sqrt{a}}+\frac{ix\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log(f)}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^2(f)}}{2\sqrt{b}}}{2a}$$

input
```
Integrate[(f^x*x)/(a + b*f^(2*x))^2,x]
```

output
```
-1/2*((1 + (b*f^(2*x))/a)*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*
(a + b*f^(2*x))*Log[f]^2) + (f^x*x)/(2*a*(a + b*f^(2*x))*Log[f]) + ((((I/2
)*x^2)/Sqrt[a] - (I*x*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]) -
 (I*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2))/(2*Sqrt[b]
) + (((-1/2*I)*x^2)/Sqrt[a] + (I*x*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt
[a]*Log[f]) + (I*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2))/
(2*Sqrt[b]))/(2*a)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xf^x}{(a+bf^{2x})^2}\,dx$$

$$\Big\downarrow \; 2675$$

$$-\int \left(\frac{f^x}{2a\left(bf^{2x}+a\right)\log(f)}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{xf^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)}+\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)}+\frac{xf^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

input
```
Int[(f^x*x)/(a + b*f^(2*x))^2,x]
```

output
```
-1/2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(a^(3/2)*Sqrt[b]*Log[f]^2) + (f^x*x)/(2
*a*(a + b*f^(2*x))*Log[f]) + (x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(2*a^(3/2)*
Sqrt[b]*Log[f]) - ((I/4)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^(3/2)*
Sqrt[b]*Log[f]^2) + ((I/4)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(a^(3/2)*S
qrt[b]*Log[f]^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13

method	result
risch	$\frac{f^x x}{2a(a+b\,f^{2x})\ln(f)}+\frac{x\ln\left(\frac{-b\,f^x+\sqrt{-ab}}{\sqrt{-ab}}\right)}{4a\ln(f)\sqrt{-ab}}-\frac{x\ln\left(\frac{b\,f^x+\sqrt{-ab}}{\sqrt{-ab}}\right)}{4a\ln(f)\sqrt{-ab}}+\frac{\text{dilog}\left(\frac{-b\,f^x+\sqrt{-ab}}{\sqrt{-ab}}\right)}{4a\ln(f)^2\sqrt{-ab}}-\frac{\text{dilog}\left(\frac{b\,f^x+\sqrt{-ab}}{\sqrt{-ab}}\right)}{4a\ln(f)^2\sqrt{-ab}}-\frac{\arctan\left(\frac{b\,f^x}{\sqrt{ab}}\right)}{2a\ln(f)^2\sqrt{\ldots}}$

input
```
int(f^x*x/(a+b*f^(2*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/ln(f)/a*f^x*x/(a+b*(f^x)^2)+1/4/a/ln(f)*x/(-a*b)^(1/2)*ln((-b*f^x+(-a*
b)^(1/2))/(-a*b)^(1/2))-1/4/a/ln(f)*x/(-a*b)^(1/2)*ln((b*f^x+(-a*b)^(1/2))
/(-a*b)^(1/2))+1/4/a/ln(f)^2/(-a*b)^(1/2)*dilog((-b*f^x+(-a*b)^(1/2))/(-a*
b)^(1/2))-1/4/a/ln(f)^2/(-a*b)^(1/2)*dilog((b*f^x+(-a*b)^(1/2))/(-a*b)^(1/
2))-1/2/a/ln(f)^2/(a*b)^(1/2)*arctan(b*f^x/(a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(120) = 240$.

Time = 0.10 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.81

$$\int \frac{f^x x}{(a + b f^{2x})^2} \, dx = \frac{2 b f^x x \log(f) + \left(b f^{2x} \sqrt{-\frac{b}{a}} + a \sqrt{-\frac{b}{a}}\right) \mathrm{Li}_2\left(f^x \sqrt{-\frac{b}{a}}\right) - \left(b f^{2x} \sqrt{-\frac{b}{a}} + a \sqrt{-\frac{b}{a}}\right) \mathrm{Li}_2\left(-f^x \sqrt{-\frac{b}{a}}\right) - \left(b f^{2}\right.}{}$$

input
```
integrate(f^x*x/(a+b*f^(2*x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*f^x*x*log(f) + (b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*dilog(f^x*sq
rt(-b/a)) - (b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) -
 (b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*log(2*b*f^x + 2*a*sqrt(-b/a)) + (b*
f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*log(2*b*f^x - 2*a*sqrt(-b/a)) - (b*f^(2
*x)*x*sqrt(-b/a)*log(f) + a*x*sqrt(-b/a)*log(f))*log(f^x*sqrt(-b/a) + 1) +
 (b*f^(2*x)*x*sqrt(-b/a)*log(f) + a*x*sqrt(-b/a)*log(f))*log(-f^x*sqrt(-b/
a) + 1))/(a*b^2*f^(2*x)*log(f)^2 + a^2*b*log(f)^2)
```

Sympy [F]

$$\int \frac{f^x x}{(a+bf^{2x})^2}\,dx = \frac{f^x x}{2a^2\log(f)+2abf^{2x}\log(f)} + \frac{\int \left(-\frac{f^x}{a+bf^{2x}}\right)dx + \int \frac{f^x x\log(f)}{a+bf^{2x}}\,dx}{2a\log(f)}$$

input
```
integrate(f**x*x/(a+b*f**(2*x))**2,x)
```

output
```
f**x*x/(2*a**2*log(f) + 2*a*b*f**(2*x)*log(f)) + (Integral(-f**x/(a + b*f*
*(2*x)), x) + Integral(f**x*x*log(f)/(a + b*f**(2*x)), x))/(2*a*log(f))
```

Maxima [F]

$$\int \frac{f^x x}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x}{(bf^{2x}+a)^2}\,dx$$

input
```
integrate(f^x*x/(a+b*f^(2*x))^2,x, algorithm="maxima")
```

output
```
1/2*f^x*x/(a*b*f^(2*x)*log(f) + a^2*log(f)) + integrate(1/2*(x*log(f) - 1)
*f^x/(a*b*f^(2*x)*log(f) + a^2*log(f)), x)
```

Giac [F]

$$\int \frac{f^x x}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x}{(bf^{2x}+a)^2}\,dx$$

input
```
integrate(f^x*x/(a+b*f^(2*x))^2,x, algorithm="giac")
```

output
```
integrate(f^x*x/(b*f^(2*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x}{(a+bf^{2x})^2}\,dx = \int \frac{f^x\,x}{\left(a+b\,f^{2\,x}\right)^2}\,dx$$

input
```
int((f^x*x)/(a + b*f^(2*x))^2,x)
```

output
```
int((f^x*x)/(a + b*f^(2*x))^2, x)
```

Reduce [F]

$$\int \frac{f^x x}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x}{f^{4x}b^2+2f^{2x}ab+a^2}dx$$

input
```
int(f^x*x/(a+b*f^(2*x))^2,x)
```

output
```
int((f**x*x)/(f**(4*x)*b**2 + 2*f**(2*x)*a*b + a**2),x)
```

3.105 $\int \frac{f^x x^2}{(a+bf^{2x})^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 333

$$\int \frac{f^x x^2}{(a+bf^{2x})^2} \, dx = -\frac{x \arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log^2(f)} + \frac{f^x x^2}{2a(a+bf^{2x})\log(f)} + \frac{x^2 \arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} + \frac{i \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} - \frac{ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} - \frac{i \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} - \frac{i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}$$

output
```
-x*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2+1/2*f^x*x^2/a/(a+b*
f^(2*x))/ln(f)+1/2*x^2*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)+1
/2*I*polylog(2,-I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^3-1/2*I*x*pol
ylog(2,-I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2-1/2*I*polylog(2,I*b
^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^3+1/2*I*x*polylog(2,I*b^(1/2)*f^
x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2+1/2*I*polylog(3,-I*b^(1/2)*f^x/a^(1/2))
/a^(3/2)/b^(1/2)/ln(f)^3-1/2*I*polylog(3,I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^
(1/2)/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.43

$$\int \frac{f^x x^2}{(a+bf^{2x})^2}\,dx = \frac{f^x x^2}{2a\,(a+bf^{2x})\log(f)} - \frac{\frac{\frac{ix^2}{2\sqrt{a}} - \frac{ix\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log(f)} - \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^2(f)}}{2\sqrt{b}} + \frac{-\frac{ix^2}{2\sqrt{a}} + \frac{ix\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log(f)} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^2(f)}}{2\sqrt{b}}}{a\log(f)} + \frac{\frac{\frac{ix^3}{3\sqrt{a}} - \frac{ix^2\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log(f)} - \frac{2ix\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^2(f)} + \frac{2i\,\text{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^3(f)}}{2\sqrt{b}} + \frac{-\frac{ix^3}{3\sqrt{a}} + \frac{ix^2\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log(f)} + \frac{2ix\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\log^2(f)} - 2i\,\text{PolyL}}{2\sqrt{b}}}{2a}$$

input
```
Integrate[(f^x*x^2)/(a + b*f^(2*x))^2,x]
```

output
```
(f^x*x^2)/(2*a*(a + b*f^(2*x))*Log[f]) - (((((I/2)*x^2)/Sqrt[a] - (I*x*Log[
1 + (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]) - (I*PolyLog[2, ((-I)*Sqrt[
b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2))/(2*Sqrt[b]) + (((-1/2*I)*x^2)/Sqrt[a
] + (I*x*Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]) + (I*PolyLog[2
, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^2))/(2*Sqrt[b]))/(a*Log[f]) +
((((I/3)*x^3)/Sqrt[a] - (I*x^2*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*
Log[f]) - ((2*I)*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]
^2) + ((2*I)*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log[f]^3))/(
2*Sqrt[b]) + (((-1/3*I)*x^3)/Sqrt[a] + (I*x^2*Log[1 - (I*Sqrt[b]*f^x)/Sqrt
[a]])/(Sqrt[a]*Log[f]) + ((2*I)*x*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sq
rt[a]*Log[f]^2) - ((2*I)*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Log
[f]^3))/(2*Sqrt[b]))/(2*a)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2675, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 f^x}{\left(a+bf^{2x}\right)^2}dx$$

$$\Big\downarrow \ 2675$$

$$-2\int \frac{1}{2}x\left(\frac{f^x}{a\left(bf^{2x}+a\right)\log(f)}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^2f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \ 27$$

$$-\int x\left(\frac{f^x}{a\left(bf^{2x}+a\right)\log(f)}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^2f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \ 2010$$

$$-\int \left(\frac{xf^x}{a\left(bf^{2x}+a\right)\log(f)}+\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^2f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}-\frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log^2(f)}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}+\frac{i\,\text{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}-\frac{i\,\text{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)}-\frac{ix\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)}+\frac{ix\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)}+\frac{x^2f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

input
```
Int[(f^x*x^2)/(a + b*f^(2*x))^2,x]
```

output
```
-((x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Log[f]^2)) + (f^x*x^2
)/(2*a*(a + b*f^(2*x))*Log[f]) + (x^2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(2*a^
(3/2)*Sqrt[b]*Log[f]) + ((I/2)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^
(3/2)*Sqrt[b]*Log[f]^3) - ((I/2)*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])
/(a^(3/2)*Sqrt[b]*Log[f]^2) - ((I/2)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])/
(a^(3/2)*Sqrt[b]*Log[f]^3) + ((I/2)*x*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])
/(a^(3/2)*Sqrt[b]*Log[f]^2) + ((I/2)*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]
])/(a^(3/2)*Sqrt[b]*Log[f]^3) - ((I/2)*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]]
)/(a^(3/2)*Sqrt[b]*Log[f]^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Maple [F]

$$\int \frac{f^x x^2}{(a+b f^{2x})^2}dx$$

input
```
int(f^x*x^2/(a+b*f^(2*x))^2,x)
```

output
```
int(f^x*x^2/(a+b*f^(2*x))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.17

$$\int \frac{f^x x^2}{(a+bf^{2x})^2}\,dx = \frac{2\,bf^x x^2 \log(f)^2 + 2\left((bx\log(f)-b)f^{2\,x}\sqrt{-\frac{b}{a}} + (ax\log(f)-a)\sqrt{-\frac{b}{a}}\right)\mathrm{Li}_2\left(f^x\sqrt{-\frac{b}{a}}\right) - 2\left((bx\log(f)\right.}{}$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*f^x*x^2*log(f)^2 + 2*((b*x*log(f) - b)*f^(2*x)*sqrt(-b/a) + (a*x*
log(f) - a)*sqrt(-b/a))*dilog(f^x*sqrt(-b/a)) - 2*((b*x*log(f) - b)*f^(2*x
)*sqrt(-b/a) + (a*x*log(f) - a)*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) - ((b*x
^2*log(f)^2 - 2*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (a*x^2*log(f)^2 - 2*a*x*l
og(f))*sqrt(-b/a))*log(f^x*sqrt(-b/a) + 1) + ((b*x^2*log(f)^2 - 2*b*x*log(
f))*f^(2*x)*sqrt(-b/a) + (a*x^2*log(f)^2 - 2*a*x*log(f))*sqrt(-b/a))*log(-
f^x*sqrt(-b/a) + 1) - 2*(b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*polylog(3, f
^x*sqrt(-b/a)) + 2*(b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*polylog(3, -f^x*s
qrt(-b/a)))/(a*b^2*f^(2*x)*log(f)^3 + a^2*b*log(f)^3)
```

Sympy [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^2}\,dx = \frac{f^x x^2}{2a^2\log(f) + 2abf^{2x}\log(f)} + \frac{\int \left(-\frac{2f^x x}{a+bf^{2x}}\right)dx + \int \frac{f^x x^2 \log(f)}{a+bf^{2x}}\,dx}{2a\log(f)}$$

input
```
integrate(f**x*x**2/(a+b*f**(2*x))**2,x)
```

output
```
f**x*x**2/(2*a**2*log(f) + 2*a*b*f**(2*x)*log(f)) + (Integral(-2*f**x*x/(a
 + b*f**(2*x)), x) + Integral(f**x*x**2*log(f)/(a + b*f**(2*x)), x))/(2*a*
log(f))
```

Maxima [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^2} \, dx = \int \frac{f^x x^2}{(bf^{2x}+a)^2} \, dx$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x))^2,x, algorithm="maxima")
```

output
```
1/2*f^x*x^2/(a*b*f^(2*x)*log(f) + a^2*log(f)) + integrate(1/2*(x^2*log(f)
- 2*x)*f^x/(a*b*f^(2*x)*log(f) + a^2*log(f)), x)
```

Giac [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^2} \, dx = \int \frac{f^x x^2}{(bf^{2x}+a)^2} \, dx$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x))^2,x, algorithm="giac")
```

output
```
integrate(f^x*x^2/(b*f^(2*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x^2}{(a+bf^{2x})^2} \, dx = \int \frac{f^x x^2}{(a+b f^{2x})^2} \, dx$$

input
```
int((f^x*x^2)/(a + b*f^(2*x))^2,x)
```

output
```
int((f^x*x^2)/(a + b*f^(2*x))^2, x)
```

Reduce [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x^2}{f^{4x}b^2+2f^{2x}ab+a^2}dx$$

input
```
int(f^x*x^2/(a+b*f^(2*x))^2,x)
```

output
```
int((f**x*x**2)/(f**(4*x)*b**2 + 2*f**(2*x)*a*b + a**2),x)
```

3.106 $\int \frac{f^x x^3}{(a+bf^{2x})^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 501

$$
\begin{aligned}
\int \frac{f^x x^3}{(a+bf^{2x})^2} \, dx = & -\frac{3x^2 \arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)} + \frac{f^x x^3}{2a(a+bf^{2x})\log(f)} + \frac{x^3 \arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)} \\
& + \frac{3ix \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} - \frac{3ix^2 \operatorname{PolyLog}\left(2, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} \\
& - \frac{3ix \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} + \frac{3ix^2 \operatorname{PolyLog}\left(2, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{3/2}\sqrt{b}\log^2(f)} \\
& - \frac{3i \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^4(f)} + \frac{3ix \operatorname{PolyLog}\left(3, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} \\
& + \frac{3i \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^4(f)} - \frac{3ix \operatorname{PolyLog}\left(3, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^3(f)} \\
& - \frac{3i \operatorname{PolyLog}\left(4, -\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^4(f)} + \frac{3i \operatorname{PolyLog}\left(4, \frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^4(f)}
\end{aligned}
$$

output
```
-3/2*x^2*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2+1/2*f^x*x^3/a
/(a+b*f^(2*x))/ln(f)+1/2*x^3*arctan(b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/l
n(f)+3/2*I*x*polylog(2,-I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^3-3/4
*I*x^2*polylog(2,-I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2-3/2*I*x*p
olylog(2,I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^3+3/4*I*x^2*polylog(
2,I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^2-3/2*I*polylog(3,-I*b^(1/2
)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^4+3/2*I*x*polylog(3,-I*b^(1/2)*f^x/a^
(1/2))/a^(3/2)/b^(1/2)/ln(f)^3+3/2*I*polylog(3,I*b^(1/2)*f^x/a^(1/2))/a^(3
/2)/b^(1/2)/ln(f)^4-3/2*I*x*polylog(3,I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/
2)/ln(f)^3-3/2*I*polylog(4,-I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^4
+3/2*I*polylog(4,I*b^(1/2)*f^x/a^(1/2))/a^(3/2)/b^(1/2)/ln(f)^4
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 434, normalized size of antiderivative = 0.87

$$\int \frac{f^x x^3}{(a+bf^{2x})^2}\,dx = \frac{\frac{2\sqrt{a}f^x x^3\log^3(f)}{a+bf^{2x}} - \frac{3ix^2\log^2(f)\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{ix^3\log^3(f)\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{3ix^2\log^2(f)\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{b}} - \frac{ix^3\log^3(f)\log\left(1+\frac{i\sqrt{b}f}{\sqrt{a}}\right)}{\sqrt{b}}}{}$$

input
```
Integrate[(f^x*x^3)/(a + b*f^(2*x))^2,x]
```

output
```
((2*Sqrt[a]*f^x*x^3*Log[f]^3)/(a + b*f^(2*x)) - ((3*I)*x^2*Log[f]^2*Log[1
 - (I*Sqrt[b]*f^x)/Sqrt[a]])/Sqrt[b] + (I*x^3*Log[f]^3*Log[1 - (I*Sqrt[b]*f
^x)/Sqrt[a]])/Sqrt[b] + ((3*I)*x^2*Log[f]^2*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a
]])/Sqrt[b] - (I*x^3*Log[f]^3*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]])/Sqrt[b] -
((3*I)*x*Log[f]*(-2 + x*Log[f])*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/Sq
rt[b] + ((3*I)*x*Log[f]*(-2 + x*Log[f])*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]
])/Sqrt[b] - ((6*I)*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/Sqrt[b] + ((6*
I)*x*Log[f]*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/Sqrt[b] + ((6*I)*PolyL
og[3, (I*Sqrt[b]*f^x)/Sqrt[a]])/Sqrt[b] - ((6*I)*x*Log[f]*PolyLog[3, (I*Sq
rt[b]*f^x)/Sqrt[a]])/Sqrt[b] - ((6*I)*PolyLog[4, ((-I)*Sqrt[b]*f^x)/Sqrt[a
]])/Sqrt[b] + ((6*I)*PolyLog[4, (I*Sqrt[b]*f^x)/Sqrt[a]])/Sqrt[b])/(4*a^(3
/2)*Log[f]^4)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 487, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2675, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 f^x}{(a+bf^{2x})^2} \, dx$$

$$\Big\downarrow \text{ 2675}$$

$$-3\int \frac{1}{2}x^2\left(\frac{f^x}{a\left(bf^{2x}+a\right)\log(f)}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^3f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{2}\int x^2\left(\frac{f^x}{a\left(bf^{2x}+a\right)\log(f)}+\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^3f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \text{ 2010}$$

$$-\frac{3}{2}\int \left(\frac{x^2f^x}{a\left(bf^{2x}+a\right)\log(f)}+\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log(f)}\right)dx+\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^3f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{3}{2}\left(\frac{x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log^2(f)}+\frac{ix^2\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)}-\frac{ix^2\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log^2(f)}+\frac{i\,\text{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\log^4(f)}-\frac{i\,\text{Poly}}{a^3}\right.$$
$$\frac{x^3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}\log(f)}+\frac{x^3f^x}{2a\log(f)\left(a+bf^{2x}\right)}$$

input
```
Int[(f^x*x^3)/(a + b*f^(2*x))^2,x]
```

output
```
(f^x*x^3)/(2*a*(a + b*f^(2*x))*Log[f]) + (x^3*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]
])/(2*a^(3/2)*Sqrt[b]*Log[f]) - (3*((x^2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(a
^(3/2)*Sqrt[b]*Log[f]^2) - (I*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a
^(3/2)*Sqrt[b]*Log[f]^3) + ((I/2)*x^2*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a
]])/(a^(3/2)*Sqrt[b]*Log[f]^2) + (I*x*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a]])
/(a^(3/2)*Sqrt[b]*Log[f]^3) - ((I/2)*x^2*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[a
]])/(a^(3/2)*Sqrt[b]*Log[f]^2) + (I*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]]
)/(a^(3/2)*Sqrt[b]*Log[f]^4) - (I*x*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]]
)/(a^(3/2)*Sqrt[b]*Log[f]^3) - (I*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]])/(a^
(3/2)*Sqrt[b]*Log[f]^4) + (I*x*PolyLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]])/(a^(3/
2)*Sqrt[b]*Log[f]^3) + (I*PolyLog[4, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a^(3/2)
*Sqrt[b]*Log[f]^4) - (I*PolyLog[4, (I*Sqrt[b]*f^x)/Sqrt[a]])/(a^(3/2)*Sqrt
[b]*Log[f]^4)))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Maple [F]

$$\int \frac{f^x x^3}{(a + b\, f^{2x})^2} dx$$

input
```
int(f^x*x^3/(a+b*f^(2*x))^2,x)
```

output
```
int(f^x*x^3/(a+b*f^(2*x))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.10

$$\int \frac{f^x x^3}{(a + bf^{2x})^2}\, dx = \frac{2\, bf^x x^3 \log(f)^3 + 3\left(\left(bx^2 \log(f)^2 - 2\, bx \log(f)\right) f^{2\,x} \sqrt{-\frac{b}{a}} + \left(ax^2 \log(f)^2 - 2\, ax \log(f)\right) \sqrt{-\frac{b}{a}}\right) \mathrm{Li}_2\left(f^x\right.}{}$$

input
```
integrate(f^x*x^3/(a+b*f^(2*x))^2,x, algorithm="fricas")
```

output
```
1/4*(2*b*f^x*x^3*log(f)^3 + 3*((b*x^2*log(f)^2 - 2*b*x*log(f))*f^(2*x)*sqr
t(-b/a) + (a*x^2*log(f)^2 - 2*a*x*log(f))*sqrt(-b/a))*dilog(f^x*sqrt(-b/a)
) - 3*((b*x^2*log(f)^2 - 2*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (a*x^2*log(f)^
2 - 2*a*x*log(f))*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) - ((b*x^3*log(f)^3 -
3*b*x^2*log(f)^2)*f^(2*x)*sqrt(-b/a) + (a*x^3*log(f)^3 - 3*a*x^2*log(f)^2)
*sqrt(-b/a))*log(f^x*sqrt(-b/a) + 1) + ((b*x^3*log(f)^3 - 3*b*x^2*log(f)^2
)*f^(2*x)*sqrt(-b/a) + (a*x^3*log(f)^3 - 3*a*x^2*log(f)^2)*sqrt(-b/a))*log
(-f^x*sqrt(-b/a) + 1) + 6*(b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*polylog(4,
 f^x*sqrt(-b/a)) - 6*(b*f^(2*x)*sqrt(-b/a) + a*sqrt(-b/a))*polylog(4, -f^x
*sqrt(-b/a)) - 6*((b*x*log(f) - b)*f^(2*x)*sqrt(-b/a) + (a*x*log(f) - a)*s
qrt(-b/a))*polylog(3, f^x*sqrt(-b/a)) + 6*((b*x*log(f) - b)*f^(2*x)*sqrt(-
b/a) + (a*x*log(f) - a)*sqrt(-b/a))*polylog(3, -f^x*sqrt(-b/a)))/(a*b^2*f^
(2*x)*log(f)^4 + a^2*b*log(f)^4)
```

Sympy [F]

$$\int \frac{f^x x^3}{(a+bf^{2x})^2}\,dx = \frac{f^x x^3}{2a^2\log(f)+2abf^{2x}\log(f)} + \frac{\int \left(-\frac{3f^x x^2}{a+bf^{2x}}\right)dx + \int \frac{f^x x^3 \log(f)}{a+bf^{2x}}\,dx}{2a\log(f)}$$

input
```
integrate(f**x*x**3/(a+b*f**(2*x))**2,x)
```

output
```
f**x*x**3/(2*a**2*log(f) + 2*a*b*f**(2*x)*log(f)) + (Integral(-3*f**x*x**2
/(a + b*f**(2*x)), x) + Integral(f**x*x**3*log(f)/(a + b*f**(2*x)), x))/(2
*a*log(f))
```

Maxima [F]

$$\int \frac{f^x x^3}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x^3}{(bf^{2x}+a)^2}\,dx$$

input
```
integrate(f^x*x^3/(a+b*f^(2*x))^2,x, algorithm="maxima")
```

output
```
1/2*f^x*x^3/(a*b*f^(2*x)*log(f) + a^2*log(f)) + integrate(1/2*(x^3*log(f)
- 3*x^2)*f^x/(a*b*f^(2*x)*log(f) + a^2*log(f)), x)
```

Giac [F]

$$\int \frac{f^x x^3}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x^3}{(bf^{2x}+a)^2}\,dx$$

input
```
integrate(f^x*x^3/(a+b*f^(2*x))^2,x, algorithm="giac")
```

output
```
integrate(f^x*x^3/(b*f^(2*x) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x^3}{(a+bf^{2x})^2}\,dx = \int \frac{f^x\,x^3}{\left(a+b\,f^{2\,x}\right)^2}\,dx$$

input
```
int((f^x*x^3)/(a + b*f^(2*x))^2,x)
```

output
```
int((f^x*x^3)/(a + b*f^(2*x))^2, x)
```

Reduce [F]

$$\int \frac{f^x x^3}{(a+bf^{2x})^2}\,dx = \int \frac{f^x x^3}{f^{4x}b^2+2f^{2x}ab+a^2}dx$$

input
```
int(f^x*x^3/(a+b*f^(2*x))^2,x)
```

output
```
int((f**x*x**3)/(f**(4*x)*b**2 + 2*f**(2*x)*a*b + a**2),x)
```

3.107 $\int \frac{f^x}{(a+bf^{2x})^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{f^x}{(a+bf^{2x})^3} dx = \frac{f^x}{4a(a+bf^{2x})^2\log(f)} + \frac{3f^x}{8a^2(a+bf^{2x})\log(f)} + \frac{3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}$$

output
```
1/4*f^x/a/(a+b*f^(2*x))^2/ln(f)+3/8*f^x/a^2/(a+b*f^(2*x))/ln(f)+3/8*arctan
(b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{f^x}{(a+bf^{2x})^3} dx = \frac{\frac{5af^x+3bf^{3x}}{8a^2(a+bf^{2x})^2} + \frac{3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}}{\log(f)}$$

input
```
Integrate[f^x/(a + b*f^(2*x))^3,x]
```

output
```
((5*a*f^x + 3*b*f^(3*x))/(8*a^2*(a + b*f^(2*x))^2) + (3*ArcTan[(Sqrt[b]*f^
x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]))/Log[f]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2679, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{f^x}{(a+bf^{2x})^3}dx$$

$$\Big\downarrow \text{ 2679}$$

$$\frac{\int \frac{1}{(bf^{2x}+a)^3}df^x}{\log(f)}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\int \frac{1}{(bf^{2x}+a)^2}df^x}{4a}+\frac{f^x}{4a(a+bf^{2x})^2}}{\log(f)}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{bf^{2x}+a}df^x}{2a}+\frac{f^x}{2a(a+bf^{2x})}\right)}{4a}+\frac{f^x}{4a(a+bf^{2x})^2}}{\log(f)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{f^x}{2a(a+bf^{2x})}\right)}{4a}+\frac{f^x}{4a(a+bf^{2x})^2}}{\log(f)}$$

input
```
Int[f^x/(a + b*f^(2*x))^3,x]
```

output
```
(f^x/(4*a*(a + b*f^(2*x))^2) + (3*(f^x/(2*a*(a + b*f^(2*x))) + ArcTan[(Sqr
t[b]*f^x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/Log[f]
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{f^{x}\left(3 b\,f^{2 x}+5 a\right)}{8 \ln\left(f\right) a^{2} \left(a+b\,f^{2 x}\right)^{2}}-\frac{3 \ln\left(f^{x}-\frac{a}{\sqrt{-a b}}\right)}{16 \sqrt{-a b}\, a^{2} \ln\left(f\right)}+\frac{3 \ln\left(f^{x}+\frac{a}{\sqrt{-a b}}\right)}{16 \sqrt{-a b}\, a^{2} \ln\left(f\right)}$	94

input
```
int(f^x/(a+b*f^(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^x*(3*b*(f^x)^2+5*a)/ln(f)/a^2/(a+b*(f^x)^2)^2-3/16/(-a*b)^(1/2)/a^2/
ln(f)*ln(f^x-1/(-a*b)^(1/2)*a)+3/16/(-a*b)^(1/2)/a^2/ln(f)*ln(f^x+1/(-a*b)
^(1/2)*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.07

$$\int \frac{f^x}{(a+bf^{2x})^3}\,dx = \left[\frac{6\,ab^2f^{3\,x}+10\,a^2bf^x-3\left(\sqrt{-ab}b^2f^{4\,x}+2\,\sqrt{-ab}abf^{2\,x}+\sqrt{-ab}a^2\right)\log\left(\frac{bf^{2\,x}-2\,\sqrt{-ab}f^x-a}{bf^{2\,x}+a}\right)}{16\left(a^3b^3f^{4\,x}\log\left(f\right)+2\,a^4b^2f^{2\,x}\log\left(f\right)+a^5b\log\left(f\right)\right)}, \frac{3\,ab^2f^{3\,x}+5}{8}\right.$$

input
```
integrate(f^x/(a+b*f^(2*x))^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*f^(3*x) + 10*a^2*b*f^x - 3*(sqrt(-a*b)*b^2*f^(4*x) + 2*sqrt
(-a*b)*a*b*f^(2*x) + sqrt(-a*b)*a^2)*log((b*f^(2*x) - 2*sqrt(-a*b)*f^x - a
)/(b*f^(2*x) + a)))/(a^3*b^3*f^(4*x)*log(f) + 2*a^4*b^2*f^(2*x)*log(f) + a
^5*b*log(f)), 1/8*(3*a*b^2*f^(3*x) + 5*a^2*b*f^x - 3*(sqrt(a*b)*b^2*f^(4*x
) + 2*sqrt(a*b)*a*b*f^(2*x) + sqrt(a*b)*a^2)*arctan(sqrt(a*b)/(b*f^x)))/(a
^3*b^3*f^(4*x)*log(f) + 2*a^4*b^2*f^(2*x)*log(f) + a^5*b*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01

$$\int \frac{f^x}{(a+bf^{2x})^3}\,dx = \frac{5af^x+3bf^{3x}}{8a^4\log(f)+16a^3bf^{2x}\log(f)+8a^2b^2f^{4x}\log(f)} + \frac{\operatorname{RootSum}\left(256z^2a^5b+9,\left(i\mapsto i\log\left(\frac{16ia^3}{3}+f^x\right)\right)\right)}{\log(f)}$$

input
```
integrate(f**x/(a+b*f**(2*x))**3,x)
```

output
```
(5*a*f**x + 3*b*f**(3*x))/(8*a**4*log(f) + 16*a**3*b*f**(2*x)*log(f) + 8*a
**2*b**2*f**(4*x)*log(f)) + RootSum(256*_z**2*a**5*b + 9, Lambda(_i, _i*lo
g(16*_i*a**3/3 + f**x)))/log(f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{f^x}{(a+bf^{2x})^3}\,dx = \frac{3\,bf^{3\,x}+5\,af^x}{8\,(a^2b^2f^{4\,x}+2\,a^3bf^{2\,x}+a^4)\log(f)}+\frac{3\,\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2\log(f)}$$

input
```
integrate(f^x/(a+b*f^(2*x))^3,x, algorithm="maxima")
```

output
```
1/8*(3*b*f^(3*x) + 5*a*f^x)/((a^2*b^2*f^(4*x) + 2*a^3*b*f^(2*x) + a^4)*log
(f)) + 3/8*arctan(b*f^x/sqrt(a*b))/(sqrt(a*b)*a^2*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{f^x}{(a+bf^{2x})^3}\,dx = \frac{3\,\arctan\left(\frac{bf^x}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2\log(f)}+\frac{3\,bf^{3\,x}+5\,af^x}{8\,(bf^{2\,x}+a)^2a^2\log(f)}$$

input
```
integrate(f^x/(a+b*f^(2*x))^3,x, algorithm="giac")
```

output
```
3/8*arctan(b*f^x/sqrt(a*b))/(sqrt(a*b)*a^2*log(f)) + 1/8*(3*b*f^(3*x) + 5*
a*f^x)/((b*f^(2*x) + a)^2*a^2*log(f))
```

Mupad [B] (verification not implemented)

Time = 22.76 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

$$\int \frac{f^x}{(a+bf^{2x})^3}\,dx = \frac{\frac{5\,f^x}{8\,a\,\ln(f)}+\frac{3\,b\,f^{3\,x}}{8\,a^2\,\ln(f)}}{b^2\,f^{4\,x}+a^2+2\,a\,b\,f^{2\,x}}+\frac{3\,\mathrm{atan}\left(\frac{b\,f^x}{\sqrt{a\,b}}\right)}{8\,a^2\,\ln(f)\,\sqrt{a\,b}}$$

input
```
int(f^x/(a + b*f^(2*x))^3,x)
```

output
```
((5*f^x)/(8*a*log(f)) + (3*b*f^(3*x))/(8*a^2*log(f)))/(b^2*f^(4*x) + a^2 +
 2*a*b*f^(2*x)) + (3*atan((b*f^x)/(a*b)^(1/2)))/(8*a^2*log(f)*(a*b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.61

$$\int \frac{f^x}{(a+bf^{2x})^3}\,dx$$
$$= \frac{3f^{4x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x b}{\sqrt{b}\,\sqrt{a}}\right)b^2+6f^{2x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x b}{\sqrt{b}\,\sqrt{a}}\right)ab+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x b}{\sqrt{b}\,\sqrt{a}}\right)a^2+3f^{3x}a\,b^2+5f^x a^2 b}{8\log(f)\,a^3 b\,(f^{4x}b^2+2f^{2x}ab+a^2)}$$

input
```
int(f^x/(a+b*f^(2*x))^3,x)
```

output
```
(3*f**(4*x)*sqrt(b)*sqrt(a)*atan((f**x*b)/(sqrt(b)*sqrt(a)))*b**2 + 6*f**(
2*x)*sqrt(b)*sqrt(a)*atan((f**x*b)/(sqrt(b)*sqrt(a)))*a*b + 3*sqrt(b)*sqrt
(a)*atan((f**x*b)/(sqrt(b)*sqrt(a)))*a**2 + 3*f**(3*x)*a*b**2 + 5*f**x*a**
2*b)/(8*log(f)*a**3*b*(f**(4*x)*b**2 + 2*f**(2*x)*a*b + a**2))
```

3.108 $\int \frac{f^x x}{\left(a+bf^{2x}\right)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 223

$$\int \frac{f^x x}{\left(a+bf^{2x}\right)^3}\,dx = -\frac{f^x}{8a^2\left(a+bf^{2x}\right)\log^2(f)} - \frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}\log^2(f)} + \frac{f^x x}{4a\left(a+bf^{2x}\right)^2\log(f)} + \frac{3f^x x}{8a^2\left(a+bf^{2x}\right)\log(f)} + \frac{3x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)} - \frac{3i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)} + \frac{3i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)}$$

output

```
-1/8*f^x/a^2/(a+b*f^(2*x))/ln(f)^2-1/2*arctan(b^(1/2)*f^x/a^(1/2))/a^(5/2)
/b^(1/2)/ln(f)^2+1/4*f^x*x/a/(a+b*f^(2*x))^2/ln(f)+3/8*f^x*x/a^2/(a+b*f^(2
*x))/ln(f)+3/8*x*arctan(b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)-3/16*I*
polylog(2,-I*b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^2+3/16*I*polylog(2
,I*b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.83

$$\int \frac{f^x x}{(a+bf^{2x})^3}\,dx$$
$$= \frac{-\frac{16\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{8af^x x\log(f)}{(a+bf^{2x})^2} + \frac{4f^x(-1+3x\log(f))}{a+bf^{2x}} + \frac{6i\left(x\log(f)\left(\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)-\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)\right)-\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)+\text{Pol}}{\sqrt{a}\sqrt{b}}}{32a^2\log^2(f)}$$

input
```
Integrate[(f^x*x)/(a + b*f^(2*x))^3,x]
```

output
```
((-16*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (8*a*f^x*x*Log[f]
)/(a + b*f^(2*x))^2 + (4*f^x*(-1 + 3*x*Log[f]))/(a + b*f^(2*x)) + ((6*I)*(
x*Log[f]*(Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - Log[1 + (I*Sqrt[b]*f^x)/Sqrt[
a]]) - PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + PolyLog[2, (I*Sqrt[b]*f^x)
/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]))/(32*a^2*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2675, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xf^x}{(a+bf^{2x})^3}\,dx$$
$$\Big\downarrow \text{2675}$$
$$-\int \left(\frac{3f^x}{8a^2(bf^{2x}+a)\log(f)} + \frac{f^x}{4a(bf^{2x}+a)^2\log(f)} + \frac{3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}\right)dx + \frac{3x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)} + \frac{3xf^x}{8a^2\log(f)(a+bf^{2x})} + \frac{xf^x}{4a\log(f)(a+bf^{2x})^2}$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}\log^2(f)}+\frac{3x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}-\frac{3i\,\text{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)}+\frac{3i\,\text{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{16a^{5/2}\sqrt{b}\log^2(f)}-\frac{f^x}{8a^2\log^2(f)\left(a+bf^{2x}\right)}+\frac{3xf^x}{8a^2\log(f)\left(a+bf^{2x}\right)}+\frac{xf^x}{4a\log(f)\left(a+bf^{2x}\right)^2}$$

input
```
Int[(f^x*x)/(a + b*f^(2*x))^3,x]
```

output
```
-1/8*f^x/(a^2*(a + b*f^(2*x))*Log[f]^2) - ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(2
*a^(5/2)*Sqrt[b]*Log[f]^2) + (f^x*x)/(4*a*(a + b*f^(2*x))^2*Log[f]) + (3*f
^x*x)/(8*a^2*(a + b*f^(2*x))*Log[f]) + (3*x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])
/(8*a^(5/2)*Sqrt[b]*Log[f]) - (((3*I)/16)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sq
rt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2) + (((3*I)/16)*PolyLog[2, (I*Sqrt[b]*f^x
)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00

method	result
risch	$\frac{f^{x}\left(3xb\,f^{2x}\ln\left(f\right)+5\ln\left(f\right)ax-b\,f^{2x}-a\right)}{8\ln\left(f\right)^{2}a^{2}\left(a+b\,f^{2x}\right)^{2}}-\frac{\arctan\left(\frac{b\,f^{x}}{\sqrt{ab}}\right)}{2a^{2}\ln\left(f\right)^{2}\sqrt{ab}}+\frac{3x\ln\left(\frac{-b\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{16a^{2}\ln\left(f\right)\sqrt{-ab}}-\frac{3x\ln\left(\frac{b\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{16a^{2}\ln\left(f\right)\sqrt{-ab}}+\frac{3\,\text{dilog}\left(\frac{-b\,f^{x}+\sqrt{-}}{\sqrt{-ab}}\right.}{16a^{2}\ln\left(f\right)^{2}\sqrt{-}}$

input
```
int(f^x*x/(a+b*f^(2*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^x*(3*ln(f)*b*x*(f^x)^2+5*ln(f)*a*x-b*(f^x)^2-a)/ln(f)^2/a^2/(a+b*(f^
x)^2)^2-1/2/a^2/ln(f)^2/(a*b)^(1/2)*arctan(b*f^x/(a*b)^(1/2))+3/16/a^2/ln(
f)*x/(-a*b)^(1/2)*ln((-b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-3/16/a^2/ln(f)*x/
(-a*b)^(1/2)*ln((b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))+3/16/a^2/ln(f)^2/(-a*b)
^(1/2)*dilog((-b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-3/16/a^2/ln(f)^2/(-a*b)^(
1/2)*dilog((b*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(167) = 334$.

Time = 0.08 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.22

$$\int \frac{f^x x}{(a+bf^{2x})^3} \, dx$$
$$= \frac{2\left(3\,b^2x\log\left(f\right)-b^2\right)f^{3\,x}+2\left(5\,abx\log\left(f\right)-ab\right)f^x+3\left(b^2f^{4\,x}\sqrt{-\frac{b}{a}}+2\,abf^{2\,x}\sqrt{-\frac{b}{a}}+a^2\sqrt{-\frac{b}{a}}\right)\mathrm{Li}_2\left(f^x\right.}{}$$

input
```
integrate(f^x*x/(a+b*f^(2*x))^3,x, algorithm="fricas")
```

output
```
1/16*(2*(3*b^2*x*log(f) - b^2)*f^(3*x) + 2*(5*a*b*x*log(f) - a*b)*f^x + 3*
(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*dilog
(f^x*sqrt(-b/a)) - 3*(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) +
a^2*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) - 4*(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b
*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*log(2*b*f^x + 2*a*sqrt(-b/a)) + 4*(b
^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*log(2*b
*f^x - 2*a*sqrt(-b/a)) - 3*(b^2*f^(4*x)*x*sqrt(-b/a)*log(f) + 2*a*b*f^(2*x
)*x*sqrt(-b/a)*log(f) + a^2*x*sqrt(-b/a)*log(f))*log(f^x*sqrt(-b/a) + 1) +
 3*(b^2*f^(4*x)*x*sqrt(-b/a)*log(f) + 2*a*b*f^(2*x)*x*sqrt(-b/a)*log(f) +
a^2*x*sqrt(-b/a)*log(f))*log(-f^x*sqrt(-b/a) + 1))/(a^2*b^3*f^(4*x)*log(f)
^2 + 2*a^3*b^2*f^(2*x)*log(f)^2 + a^4*b*log(f)^2)
```

Sympy [F]

$$\int \frac{f^x x}{(a+bf^{2x})^3}\,dx = \frac{f^{3x}(3bx\log(f)-b)+f^x(5ax\log(f)-a)}{8a^4\log(f)^2+16a^3bf^{2x}\log(f)^2+8a^2b^2f^{4x}\log(f)^2} + \frac{\int\left(-\frac{4f^x}{a+bf^{2x}}\right)dx+\int\frac{3f^x x\log(f)}{a+bf^{2x}}\,dx}{8a^2\log(f)}$$

input
```
integrate(f**x*x/(a+b*f**(2*x))**3,x)
```

output
```
(f**(3*x)*(3*b*x*log(f) - b) + f**x*(5*a*x*log(f) - a))/(8*a**4*log(f)**2
+ 16*a**3*b*f**(2*x)*log(f)**2 + 8*a**2*b**2*f**(4*x)*log(f)**2) + (Integr
al(-4*f**x/(a + b*f**(2*x)), x) + Integral(3*f**x*x*log(f)/(a + b*f**(2*x)
), x))/(8*a**2*log(f))
```

Maxima [F]

$$\int \frac{f^x x}{(a+bf^{2x})^3}\,dx = \int \frac{f^x x}{(bf^{2x}+a)^3}\,dx$$

input
```
integrate(f^x*x/(a+b*f^(2*x))^3,x, algorithm="maxima")
```

output
```
1/8*((3*b*x*log(f) - b)*f^(3*x) + (5*a*x*log(f) - a)*f^x)/(a^2*b^2*f^(4*x)
*log(f)^2 + 2*a^3*b*f^(2*x)*log(f)^2 + a^4*log(f)^2) + integrate(1/8*(3*x*
log(f) - 4)*f^x/(a^2*b*f^(2*x)*log(f) + a^3*log(f)), x)
```

Giac [F]

$$\int \frac{f^x x}{(a+bf^{2x})^3}\,dx = \int \frac{f^x x}{(bf^{2x}+a)^3}\,dx$$

input
```
integrate(f^x*x/(a+b*f^(2*x))^3,x, algorithm="giac")
```

output
```
integrate(f^x*x/(b*f^(2*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x}{(a+bf^{2x})^3}\,dx = \int \frac{f^x\,x}{(a+b\,f^{2\,x})^3}\,dx$$

input
```
int((f^x*x)/(a + b*f^(2*x))^3,x)
```

output
```
int((f^x*x)/(a + b*f^(2*x))^3, x)
```

Reduce [F]

$$\int \frac{f^x x}{(a+bf^{2x})^3}\,dx = \int \frac{f^x x}{f^{6x}b^3+3f^{4x}a\,b^2+3f^{2x}a^2b+a^3}dx$$

input
```
int(f^x*x/(a+b*f^(2*x))^3,x)
```

output
```
int((f**x*x)/(f**(6*x)*b**3 + 3*f**(4*x)*a*b**2 + 3*f**(2*x)*a**2*b + a**3
),x)
```

3.109 $\int \frac{f^x x^2}{\left(a+bf^{2x}\right)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 420

$$\begin{aligned}\int \frac{f^x x^2}{\left(a+bf^{2x}\right)^3}\,dx = {} & \frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}\log^3(f)} - \frac{f^x x}{4a^2\left(a+bf^{2x}\right)\log^2(f)} - \frac{x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log^2(f)} \\ & + \frac{f^x x^2}{4a\left(a+bf^{2x}\right)^2\log(f)} + \frac{3f^x x^2}{8a^2\left(a+bf^{2x}\right)\log(f)} + \frac{3x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)} \\ & + \frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}\log^3(f)} - \frac{3ix\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log^2(f)} \\ & - \frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}\log^3(f)} + \frac{3ix\operatorname{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log^2(f)} \\ & + \frac{3i\operatorname{PolyLog}\left(3,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log^3(f)} - \frac{3i\operatorname{PolyLog}\left(3,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log^3(f)}\end{aligned}$$

output
```
1/4*arctan(b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^3-1/4*f^x*x/a^2/(a+b
*f^(2*x))/ln(f)^2-x*arctan(b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^2+1/
4*f^x*x^2/a/(a+b*f^(2*x))^2/ln(f)+3/8*f^x*x^2/a^2/(a+b*f^(2*x))/ln(f)+3/8*
x^2*arctan(b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)+1/2*I*polylog(2,-I*b
^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^3-3/8*I*x*polylog(2,-I*b^(1/2)*f
^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^2-1/2*I*polylog(2,I*b^(1/2)*f^x/a^(1/2))
/a^(5/2)/b^(1/2)/ln(f)^3+3/8*I*x*polylog(2,I*b^(1/2)*f^x/a^(1/2))/a^(5/2)/
b^(1/2)/ln(f)^2+3/8*I*polylog(3,-I*b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln
(f)^3-3/8*I*polylog(3,I*b^(1/2)*f^x/a^(1/2))/a^(5/2)/b^(1/2)/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.84

$$\int \frac{f^x x^2}{\left(a+bf^{2x}\right)^3}\,dx = \frac{\frac{4\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} + \frac{4af^x x^2\log^2(f)}{\left(a+bf^{2x}\right)^2} + \frac{2f^x x\log(f)(-2+3x\log(f))}{a+bf^{2x}} - \frac{8i\left(x\log(f)\left(\log\left(1-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)-\log\left(1+\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)\right)-\operatorname{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right.}{\sqrt{a}\sqrt{b}}}{}$$

input
```
Integrate[(f^x*x^2)/(a + b*f^(2*x))^3,x]
```

output
```
((4*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (4*a*f^x*x^2*Log[f]
^2)/(a + b*f^(2*x))^2 + (2*f^x*x*Log[f]*(-2 + 3*x*Log[f]))/(a + b*f^(2*x))
 - ((8*I)*(x*Log[f]*(Log[1 - (I*Sqrt[b]*f^x)/Sqrt[a]] - Log[1 + (I*Sqrt[b]
*f^x)/Sqrt[a]]) - PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + PolyLog[2, (I*S
qrt[b]*f^x)/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]) + ((3*I)*(x^2*Log[f]^2*Log[1 - (I
*Sqrt[b]*f^x)/Sqrt[a]] - x^2*Log[f]^2*Log[1 + (I*Sqrt[b]*f^x)/Sqrt[a]] - 2
*x*Log[f]*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] + 2*x*Log[f]*PolyLog[2, (
I*Sqrt[b]*f^x)/Sqrt[a]] + 2*PolyLog[3, ((-I)*Sqrt[b]*f^x)/Sqrt[a]] - 2*Pol
yLog[3, (I*Sqrt[b]*f^x)/Sqrt[a]]))/(Sqrt[a]*Sqrt[b]))/(16*a^2*Log[f]^3)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 416, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2675, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 f^x}{(a+bf^{2x})^3} \, dx$$

$$\Big\downarrow \text{2675}$$

$$-2\int \frac{1}{8}x\left(\frac{3f^x}{a^2(bf^{2x}+a)\log(f)}+\frac{2f^x}{a(bf^{2x}+a)^2\log(f)}+\frac{3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log(f)}\right)dx+\frac{3x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}+\frac{3x^2f^x}{8a^2\log(f)(a+bf^{2x})}+\frac{x^2f^x}{4a\log(f)(a+bf^{2x})^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}\int x\left(\frac{3f^x}{a^2(bf^{2x}+a)\log(f)}+\frac{2f^x}{a(bf^{2x}+a)^2\log(f)}+\frac{3\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log(f)}\right)dx+\frac{3x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}+\frac{3x^2f^x}{8a^2\log(f)(a+bf^{2x})}+\frac{x^2f^x}{4a\log(f)(a+bf^{2x})^2}$$

$$\Big\downarrow \text{2010}$$

$$-\frac{1}{4}\int \left(\frac{3xf^x}{a^2(bf^{2x}+a)\log(f)}+\frac{2xf^x}{a(bf^{2x}+a)^2\log(f)}+\frac{3x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log(f)}\right)dx+\frac{3x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}+\frac{3x^2f^x}{8a^2\log(f)(a+bf^{2x})}+\frac{x^2f^x}{4a\log(f)(a+bf^{2x})^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3x^2\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\log(f)}+\frac{3x^2f^x}{8a^2\log(f)\left(a+bf^{2x}\right)}+$$

$$\frac{1}{4}\left(\frac{\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log^3(f)}-\frac{4x\arctan\left(\frac{\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log^2(f)}+\frac{2i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log^3(f)}-\frac{2i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{b}f^x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}\log^3(f)}+\frac{3i\,\mathrm{PolyLog}\left(3,-\cdots\right.}{2a^{5/2}\sqrt{b}\log^3\cdots}\right.$$

$$\frac{x^2f^x}{4a\log(f)\left(a+bf^{2x}\right)^2}$$

input
```
Int[(f^x*x^2)/(a + b*f^(2*x))^3,x]
```

output
```
(f^x*x^2)/(4*a*(a + b*f^(2*x))^2*Log[f]) + (3*f^x*x^2)/(8*a^2*(a + b*f^(2*
x))*Log[f]) + (3*x^2*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*Log
[f]) + (ArcTan[(Sqrt[b]*f^x)/Sqrt[a]]/(a^(5/2)*Sqrt[b]*Log[f]^3) - (f^x*x)
/(a^2*(a + b*f^(2*x))*Log[f]^2) - (4*x*ArcTan[(Sqrt[b]*f^x)/Sqrt[a]])/(a^(
5/2)*Sqrt[b]*Log[f]^2) + ((2*I)*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt[a]])/(a
^(5/2)*Sqrt[b]*Log[f]^3) - (((3*I)/2)*x*PolyLog[2, ((-I)*Sqrt[b]*f^x)/Sqrt
[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2) - ((2*I)*PolyLog[2, (I*Sqrt[b]*f^x)/Sqrt[
a]])/(a^(5/2)*Sqrt[b]*Log[f]^3) + (((3*I)/2)*x*PolyLog[2, (I*Sqrt[b]*f^x)/
Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^2) + (((3*I)/2)*PolyLog[3, ((-I)*Sqrt[b]
*f^x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^3) - (((3*I)/2)*PolyLog[3, (I*Sqrt
[b]*f^x)/Sqrt[a]])/(a^(5/2)*Sqrt[b]*Log[f]^3))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2675
```
Int[(F_)^((e_.)*((c_.) + (d_.)*(x_)))*((a_.) + (b_.)*(F_)^(v_))^(p_)*(x_)^(
m_.), x_Symbol] :> With[{u = IntHide[F^(e*(c + d*x))*(a + b*F^v)^p, x]}, Si
mp[x^m   u, x] - Simp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d,
 e}, x] && EqQ[v, 2*e*(c + d*x)] && GtQ[m, 0] && ILtQ[p, 0]
```

Maple [F]

$$\int \frac{f^x x^2}{(a + b\, f^{2x})^3} dx$$

input
```
int(f^x*x^2/(a+b*f^(2*x))^3,x)
```

output
```
int(f^x*x^2/(a+b*f^(2*x))^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. 2(298) = 596.

Time = 0.09 (sec) , antiderivative size = 786, normalized size of antiderivative = 1.87

$$\int \frac{f^x x^2}{(a + b f^{2x})^3} dx = \text{Too large to display}$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x))^3,x, algorithm="fricas")
```

output
```
1/16*(2*(3*b^2*x^2*log(f)^2 - 2*b^2*x*log(f))*f^(3*x) + 2*(5*a*b*x^2*log(f
)^2 - 2*a*b*x*log(f))*f^x + 2*((3*b^2*x*log(f) - 4*b^2)*f^(4*x)*sqrt(-b/a)
 + 2*(3*a*b*x*log(f) - 4*a*b)*f^(2*x)*sqrt(-b/a) + (3*a^2*x*log(f) - 4*a^2
)*sqrt(-b/a))*dilog(f^x*sqrt(-b/a)) - 2*((3*b^2*x*log(f) - 4*b^2)*f^(4*x)*
sqrt(-b/a) + 2*(3*a*b*x*log(f) - 4*a*b)*f^(2*x)*sqrt(-b/a) + (3*a^2*x*log(
f) - 4*a^2)*sqrt(-b/a))*dilog(-f^x*sqrt(-b/a)) + 2*(b^2*f^(4*x)*sqrt(-b/a)
 + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*log(2*b*f^x + 2*a*sqrt(-b/a)
) - 2*(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))
*log(2*b*f^x - 2*a*sqrt(-b/a)) - ((3*b^2*x^2*log(f)^2 - 8*b^2*x*log(f))*f^
(4*x)*sqrt(-b/a) + 2*(3*a*b*x^2*log(f)^2 - 8*a*b*x*log(f))*f^(2*x)*sqrt(-b
/a) + (3*a^2*x^2*log(f)^2 - 8*a^2*x*log(f))*sqrt(-b/a))*log(f^x*sqrt(-b/a)
 + 1) + ((3*b^2*x^2*log(f)^2 - 8*b^2*x*log(f))*f^(4*x)*sqrt(-b/a) + 2*(3*a
*b*x^2*log(f)^2 - 8*a*b*x*log(f))*f^(2*x)*sqrt(-b/a) + (3*a^2*x^2*log(f)^2
 - 8*a^2*x*log(f))*sqrt(-b/a))*log(-f^x*sqrt(-b/a) + 1) - 6*(b^2*f^(4*x)*s
qrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(-b/a))*polylog(3, f^x*sqrt
(-b/a)) + 6*(b^2*f^(4*x)*sqrt(-b/a) + 2*a*b*f^(2*x)*sqrt(-b/a) + a^2*sqrt(
-b/a))*polylog(3, -f^x*sqrt(-b/a)))/(a^2*b^3*f^(4*x)*log(f)^3 + 2*a^3*b^2*
f^(2*x)*log(f)^3 + a^4*b*log(f)^3)
```

Sympy [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^3}\,dx = \frac{f^{3x}(3bx^2\log(f)-2bx)+f^x(5ax^2\log(f)-2ax)}{8a^4\log(f)^2+16a^3bf^{2x}\log(f)^2+8a^2b^2f^{4x}\log(f)^2} + \frac{\int \frac{2f^x}{a+bf^{2x}}\,dx + \int \left(-\frac{8f^x x\log(f)}{a+bf^{2x}}\right)dx + \int \frac{3f^x x^2\log(f)^2}{a+bf^{2x}}\,dx}{8a^2\log(f)^2}$$

input
```
integrate(f**x*x**2/(a+b*f**(2*x))**3,x)
```

output
```
(f**(3*x)*(3*b*x**2*log(f) - 2*b*x) + f**x*(5*a*x**2*log(f) - 2*a*x))/(8*a
**4*log(f)**2 + 16*a**3*b*f**(2*x)*log(f)**2 + 8*a**2*b**2*f**(4*x)*log(f)
**2) + (Integral(2*f**x/(a + b*f**(2*x)), x) + Integral(-8*f**x*x*log(f)/(
a + b*f**(2*x)), x) + Integral(3*f**x*x**2*log(f)**2/(a + b*f**(2*x)), x))
/(8*a**2*log(f)**2)
```

Maxima [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^3} dx = \int \frac{f^x x^2}{(bf^{2x}+a)^3} dx$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x))^3,x, algorithm="maxima")
```

output
```
1/8*((3*b*x^2*log(f) - 2*b*x)*f^(3*x) + (5*a*x^2*log(f) - 2*a*x)*f^x)/(a^2
*b^2*f^(4*x)*log(f)^2 + 2*a^3*b*f^(2*x)*log(f)^2 + a^4*log(f)^2) + integra
te(1/8*(3*x^2*log(f)^2 - 8*x*log(f) + 2)*f^x/(a^2*b*f^(2*x)*log(f)^2 + a^3
*log(f)^2), x)
```

Giac [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^3} dx = \int \frac{f^x x^2}{(bf^{2x}+a)^3} dx$$

input
```
integrate(f^x*x^2/(a+b*f^(2*x))^3,x, algorithm="giac")
```

output
```
integrate(f^x*x^2/(b*f^(2*x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{f^x x^2}{(a+bf^{2x})^3} dx = \int \frac{f^x x^2}{(a+b f^{2x})^3} dx$$

input
```
int((f^x*x^2)/(a + b*f^(2*x))^3,x)
```

output
```
int((f^x*x^2)/(a + b*f^(2*x))^3, x)
```

Reduce [F]

$$\int \frac{f^x x^2}{(a+bf^{2x})^3}\,dx = \int \frac{f^x x^2}{f^{6x}b^3 + 3f^{4x}a\,b^2 + 3f^{2x}a^2b + a^3}dx$$

input
```
int(f^x*x^2/(a+b*f^(2*x))^3,x)
```

output
```
int((f**x*x**2)/(f**(6*x)*b**3 + 3*f**(4*x)*a*b**2 + 3*f**(2*x)*a**2*b + a
**3),x)
```

3.110 $\int \frac{1}{bf^{-x}+af^x} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{1}{bf^{-x}+af^x} dx = \frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

output
```
arctan(a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{1}{bf^{-x}+af^x} dx = \frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Integrate[(b/f^x + a*f^x)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*f^x)/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*Log[f])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2720, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{af^x+bf^{-x}}dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{1}{af^{2x}+b}df^x}{\log(f)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Int[(b/f^x + a*f^x)^(-1),x]
```

output
```
ArcTan[(Sqrt[a]*f^x)/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*Log[f])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{a f^x}{\sqrt{ab}}\right)}{\ln(f)\sqrt{ab}}$	22
default	$\frac{\arctan\left(\frac{a f^x}{\sqrt{ab}}\right)}{\ln(f)\sqrt{ab}}$	22
risch	$-\frac{\ln\left(f^x-\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln(f)}+\frac{\ln\left(f^x+\frac{b}{\sqrt{-ab}}\right)}{2\sqrt{-ab}\,\ln(f)}$	53

input
```
int(1/(b/(f^x)+a*f^x),x,method=_RETURNVERBOSE)
```

output
```
1/ln(f)/(a*b)^(1/2)*arctan(a*f^x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.87

$$\int \frac{1}{bf^{-x}+af^x}\,dx=\left[-\frac{\sqrt{-ab}\log\left(\frac{af^{2x}-2\sqrt{-ab}f^x-b}{af^{2x}+b}\right)}{2\,ab\log(f)},-\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{af^x}\right)}{ab\log(f)}\right]$$

input
```
integrate(1/(b/(f^x)+a*f^x),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((a*f^(2*x) - 2*sqrt(-a*b)*f^x - b)/(a*f^(2*x) + b))/(
a*b*log(f)), -sqrt(a*b)*arctan(sqrt(a*b)/(a*f^x))/(a*b*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{bf^{-x} + af^{x}}\,dx = \frac{\operatorname{RootSum}\left(4z^{2}ab + 1, \left(i \mapsto i \log\left(-2ia + f^{-x}\right)\right)\right)}{\log\left(f\right)}$$

input
```
integrate(1/(b/(f**x)+a*f**x),x)
```

output
```
RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(-2*_i*a + f**(-x))))/log(f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{1}{bf^{-x} + af^{x}}\,dx = -\frac{\arctan\left(\frac{b}{\sqrt{ab}f^{x}}\right)}{\sqrt{ab}\log\left(f\right)}$$

input
```
integrate(1/(b/(f^x)+a*f^x),x, algorithm="maxima")
```

output
```
-arctan(b/(sqrt(a*b)*f^x))/(sqrt(a*b)*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{1}{bf^{-x} + af^{x}}\,dx = \frac{\arctan\left(\frac{af^{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\log\left(f\right)}$$

input
```
integrate(1/(b/(f^x)+a*f^x),x, algorithm="giac")
```

output
```
arctan(a*f^x/sqrt(a*b))/(sqrt(a*b)*log(f))
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.70

$$\int \frac{1}{bf^{-x} + af^x} \, dx = \frac{\operatorname{atan}\left(\frac{a\,f^x}{\sqrt{a\,b}}\right)}{\ln\left(f\right)\,\sqrt{a\,b}}$$

input
```
int(1/(b/f^x + a*f^x),x)
```

output
```
atan((a*f^x)/(a*b)^(1/2))/(log(f)*(a*b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{1}{bf^{-x} + af^x} \, dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)}{\log\left(f\right)ab}$$

input
```
int(1/(b/(f^x)+a*f^x),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a))))/(log(f)*a*b)
```

3.111 $\int \frac{x}{bf^{-x}+af^x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{x}{bf^{-x}+af^x}\,dx=\frac{x\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)}+\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)}$$

output
```
x*arctan(a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)-1/2*I*polylog(2,-I*a^(
1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+1/2*I*polylog(2,I*a^(1/2)*f^x/b^
(1/2))/a^(1/2)/b^(1/2)/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{x}{bf^{-x}+af^x}\,dx =\frac{i\left(x\log(f)\left(\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-\log\left(1+\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)\right)-\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)+\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)\right)}{2\sqrt{a}\sqrt{b}\log^2(f)}$$

input
```
Integrate[x/(b/f^x + a*f^x),x]
```

output
```
((I/2)*(x*Log[f]*(Log[1 - (I*Sqrt[a]*f^x)/Sqrt[b]] - Log[1 + (I*Sqrt[a]*f^
x)/Sqrt[b]]) - PolyLog[2, ((-I)*Sqrt[a]*f^x)/Sqrt[b]] + PolyLog[2, (I*Sqrt
[a]*f^x)/Sqrt[b]]))/(Sqrt[a]*Sqrt[b]*Log[f]^2)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2696, 27, 2720, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{af^x + bf^{-x}} \, dx$$

$$\Big\downarrow \text{ 2696}$$

$$\frac{x \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \int \frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{\int \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{x \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{\int f^{-x} \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)df^x}{\sqrt{a}\sqrt{b}\log^2(f)}$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{x \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{\frac{1}{2}i\int f^{-x}\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)df^x - \frac{1}{2}i\int f^{-x}\log\left(\frac{i\sqrt{a}f^x}{\sqrt{b}}+1\right)df^x}{\sqrt{a}\sqrt{b}\log^2(f)}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{x \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{\frac{1}{2}i\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)}$$

input
```
Int[x/(b/f^x + a*f^x),x]
```

output
```
(x*ArcTan[(Sqrt[a]*f^x)/Sqrt[b]])/(Sqrt[a]*Sqrt[b]*Log[f]) - ((I/2)*PolyLo
g[2, ((-I)*Sqrt[a]*f^x)/Sqrt[b]] - (I/2)*PolyLog[2, (I*Sqrt[a]*f^x)/Sqrt[b
]])/(Sqrt[a]*Sqrt[b]*Log[f]^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2696
```
Int[(x_)^(m_.)/((b_.)*(F_)^(v_) + (a_.)*(F_)^((c_.) + (d_.)*(x_))), x_Symbo
l] :> With[{u = IntHide[1/(a*F^(c + d*x) + b*F^v), x]}, Simp[x^m*u, x] - Si
mp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d}, x] && EqQ[v, -(c
+ d*x)] && GtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22

method	result	size
risch	$\frac{x\ln\left(\frac{-a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)\sqrt{-ab}}-\frac{x\ln\left(\frac{a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)\sqrt{-ab}}+\frac{\operatorname{dilog}\left(\frac{-a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)^{2}\sqrt{-ab}}-\frac{\operatorname{dilog}\left(\frac{a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{2\ln(f)^{2}\sqrt{-ab}}$	134

input
```
int(x/(b/(f^x)+a*f^x),x,method=_RETURNVERBOSE)
```

output
```
1/2/ln(f)*x/(-a*b)^(1/2)*ln((-a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2/ln(f)*
x/(-a*b)^(1/2)*ln((a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/2/ln(f)^2/(-a*b)^(1
/2)*dilog((-a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/2/ln(f)^2/(-a*b)^(1/2)*dil
og((a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.02

$$\int \frac{x}{bf^{-x}+af^{x}}\,dx = -\frac{x\sqrt{-\frac{a}{b}}\log\left(f^{x}\sqrt{-\frac{a}{b}}+1\right)\log\left(f\right)-x\sqrt{-\frac{a}{b}}\log\left(-f^{x}\sqrt{-\frac{a}{b}}+1\right)\log\left(f\right)-\sqrt{-\frac{a}{b}}\mathrm{Li}_{2}\left(f^{x}\sqrt{-\frac{a}{b}}\right)+\sqrt{-\frac{a}{b}}\mathrm{I}}{2\,a\log\left(f\right)^{2}}$$

input
```
integrate(x/(b/(f^x)+a*f^x),x, algorithm="fricas")
```

output
```
-1/2*(x*sqrt(-a/b)*log(f^x*sqrt(-a/b) + 1)*log(f) - x*sqrt(-a/b)*log(-f^x*
sqrt(-a/b) + 1)*log(f) - sqrt(-a/b)*dilog(f^x*sqrt(-a/b)) + sqrt(-a/b)*dil
og(-f^x*sqrt(-a/b)))/(a*log(f)^2)
```

Sympy [F]

$$\int \frac{x}{bf^{-x} + af^x}\,dx = \int \frac{f^x x}{af^{2x} + b}\,dx$$

input
```
integrate(x/(b/(f**x)+a*f**x),x)
```

output
```
Integral(f**x*x/(a*f**(2*x) + b), x)
```

Maxima [F]

$$\int \frac{x}{bf^{-x} + af^x}\,dx = \int \frac{x}{af^x + \frac{b}{f^x}}\,dx$$

input
```
integrate(x/(b/(f^x)+a*f^x),x, algorithm="maxima")
```

output
```
integrate(x/(a*f^x + b/f^x), x)
```

Giac [F]

$$\int \frac{x}{bf^{-x} + af^x}\,dx = \int \frac{x}{af^x + \frac{b}{f^x}}\,dx$$

input
```
integrate(x/(b/(f^x)+a*f^x),x, algorithm="giac")
```

output
```
integrate(x/(a*f^x + b/f^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{bf^{-x} + af^x} \, dx = \int \frac{x}{\frac{b}{f^x} + a\, f^x} \, dx$$

input
```
int(x/(b/f^x + a*f^x),x)
```

output
```
int(x/(b/f^x + a*f^x), x)
```

Reduce [F]

$$\int \frac{x}{bf^{-x} + af^x} \, dx = \int \frac{f^x x}{f^{2x}a + b} dx$$

input
```
int(x/(b/(f^x)+a*f^x),x)
```

output
```
int((f**x*x)/(f**(2*x)*a + b),x)
```

3.112 $\int \frac{x^2}{bf^{-x}+af^x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{x^2}{bf^{-x}+af^x}\,dx=\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{ix\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)}+\frac{ix\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^2(f)}+\frac{i\,\text{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)}-\frac{i\,\text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)}$$

output
```
x^2*arctan(a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)-I*x*polylog(2,-I*a^(
1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+I*x*polylog(2,I*a^(1/2)*f^x/b^(1
/2))/a^(1/2)/b^(1/2)/ln(f)^2+I*polylog(3,-I*a^(1/2)*f^x/b^(1/2))/a^(1/2)/b
^(1/2)/ln(f)^3-I*polylog(3,I*a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{bf^{-x}+af^x}\,dx = \frac{i\left(x^2\log^2(f)\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-x^2\log^2(f)\log\left(1+\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-2x\log(f)\operatorname{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)+2x\log(f)\right.}{2\sqrt{a}\sqrt{b}\log^3(f)}$$

input
```
Integrate[x^2/(b/f^x + a*f^x),x]
```

output
```
((I/2)*(x^2*Log[f]^2*Log[1 - (I*Sqrt[a]*f^x)/Sqrt[b]] - x^2*Log[f]^2*Log[1
 + (I*Sqrt[a]*f^x)/Sqrt[b]] - 2*x*Log[f]*PolyLog[2, ((-I)*Sqrt[a]*f^x)/Sqr
t[b]] + 2*x*Log[f]*PolyLog[2, (I*Sqrt[a]*f^x)/Sqrt[b]] + 2*PolyLog[3, ((-I
)*Sqrt[a]*f^x)/Sqrt[b]] - 2*PolyLog[3, (I*Sqrt[a]*f^x)/Sqrt[b]]))/(Sqrt[a]
*Sqrt[b]*Log[f]^3)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2696, 27, 5666, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{af^x+bf^{-x}}\,dx$$

$$\Big\downarrow \text{2696}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-2\int \frac{x\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\int x\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \ 5666$$

$$\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\int x\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx-\frac{1}{2}i\int x\log\left(\frac{i\sqrt{a}f^x}{\sqrt{b}}+1\right)dx\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \ 3011$$

$$\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\left(\frac{\int \text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\log(f)}-\frac{x\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{\int \text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \ 2720$$

$$\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\left(\frac{\int f^{-x}\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)df^x}{\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{\int f^{-x}\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)df^x}{\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$$\Big\downarrow \ 7143$$

$$\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{2\left(\frac{1}{2}i\left(\frac{\text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{\text{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Int[x^2/(b/f^x + a*f^x),x]
```

output
```
(x^2*ArcTan[(Sqrt[a]*f^x)/Sqrt[b]])/(Sqrt[a]*Sqrt[b]*Log[f]) - (2*((-1/2*I
)*(-((x*PolyLog[2, ((-I)*Sqrt[a]*f^x)/Sqrt[b]])/Log[f]) + PolyLog[3, ((-I)
*Sqrt[a]*f^x)/Sqrt[b]]/Log[f]^2) + (I/2)*(-((x*PolyLog[2, (I*Sqrt[a]*f^x)/
Sqrt[b]])/Log[f]) + PolyLog[3, (I*Sqrt[a]*f^x)/Sqrt[b]]/Log[f]^2)))/(Sqrt[
a]*Sqrt[b]*Log[f])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2696
```
Int[(x_)^(m_.)/((b_.)*(F_)^(v_) + (a_.)*(F_)^((c_.) + (d_.)*(x_))), x_Symbo
l] :> With[{u = IntHide[1/(a*F^(c + d*x) + b*F^v), x]}, Simp[x^m*u, x] - Si
mp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d}, x] && EqQ[v, -(c
+ d*x)] && GtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{b\,f^{-x}+a\,f^{x}}dx$$

input
```
int(x^2/(b/(f^x)+a*f^x),x)
```

output
```
int(x^2/(b/(f^x)+a*f^x),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{bf^{-x}+af^x}\,dx = -\frac{x^2\sqrt{-\frac{a}{b}}\log\left(f^x\sqrt{-\frac{a}{b}}+1\right)\log\left(f\right)^2 - x^2\sqrt{-\frac{a}{b}}\log\left(-f^x\sqrt{-\frac{a}{b}}+1\right)\log\left(f\right)^2 - 2\,x\sqrt{-\frac{a}{b}}\mathrm{Li}_2\left(f^x\sqrt{-\frac{a}{b}}\right)\log}{2\,a\log}$$

input
```
integrate(x^2/(b/(f^x)+a*f^x),x, algorithm="fricas")
```

output
```
-1/2*(x^2*sqrt(-a/b)*log(f^x*sqrt(-a/b) + 1)*log(f)^2 - x^2*sqrt(-a/b)*log
(-f^x*sqrt(-a/b) + 1)*log(f)^2 - 2*x*sqrt(-a/b)*dilog(f^x*sqrt(-a/b))*log(
f) + 2*x*sqrt(-a/b)*dilog(-f^x*sqrt(-a/b))*log(f) + 2*sqrt(-a/b)*polylog(3
, f^x*sqrt(-a/b)) - 2*sqrt(-a/b)*polylog(3, -f^x*sqrt(-a/b)))/(a*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{bf^{-x}+af^x}\,dx = \int \frac{f^x x^2}{af^{2x}+b}\,dx$$

input
```
integrate(x**2/(b/(f**x)+a*f**x),x)
```

output
```
Integral(f**x*x**2/(a*f**(2*x) + b), x)
```

Maxima [F]

$$\int \frac{x^2}{bf^{-x} + af^x}\,dx = \int \frac{x^2}{af^x + \frac{b}{f^x}}\,dx$$

input
```
integrate(x^2/(b/(f^x)+a*f^x),x, algorithm="maxima")
```

output
```
integrate(x^2/(a*f^x + b/f^x), x)
```

Giac [F]

$$\int \frac{x^2}{bf^{-x} + af^x}\,dx = \int \frac{x^2}{af^x + \frac{b}{f^x}}\,dx$$

input
```
integrate(x^2/(b/(f^x)+a*f^x),x, algorithm="giac")
```

output
```
integrate(x^2/(a*f^x + b/f^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{bf^{-x} + af^x}\,dx = \int \frac{x^2}{\frac{b}{f^x} + a\,f^x}\,dx$$

input
```
int(x^2/(b/f^x + a*f^x),x)
```

output
```
int(x^2/(b/f^x + a*f^x), x)
```

Reduce [F]

$$\int \frac{x^2}{bf^{-x} + af^x}\,dx = \int \frac{f^x x^2}{f^{2x}a + b}dx$$

input
```
int(x^2/(b/(f^x)+a*f^x),x)
```

output
```
int((f**x*x**2)/(f**(2*x)*a + b),x)
```

3.113 $\int \frac{x^3}{bf^{-x}+af^x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\begin{aligned}\int \frac{x^3}{bf^{-x}+af^x}\,dx = {} & \frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3ix^2\operatorname{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} \\ & + \frac{3ix^2\operatorname{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}\log^2(f)} \\ & + \frac{3ix\operatorname{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} - \frac{3ix\operatorname{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^3(f)} \\ & - \frac{3i\operatorname{PolyLog}\left(4,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)} + \frac{3i\operatorname{PolyLog}\left(4,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log^4(f)}\end{aligned}$$

output
```
x^3*arctan(a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)-3/2*I*x^2*polylog(2,
-I*a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+3/2*I*x^2*polylog(2,I*a^(1
/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)^2+3*I*x*polylog(3,-I*a^(1/2)*f^x/b^
(1/2))/a^(1/2)/b^(1/2)/ln(f)^3-3*I*x*polylog(3,I*a^(1/2)*f^x/b^(1/2))/a^(1
/2)/b^(1/2)/ln(f)^3-3*I*polylog(4,-I*a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/
ln(f)^4+3*I*polylog(4,I*a^(1/2)*f^x/b^(1/2))/a^(1/2)/b^(1/2)/ln(f)^4
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{bf^{-x}+af^x}\,dx$$
$$= \frac{i\left(x^3\log^3(f)\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-x^3\log^3(f)\log\left(1+\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-3x^2\log^2(f)\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)+3x^2\log^2\right.}{}$$

input
```
Integrate[x^3/(b/f^x + a*f^x),x]
```

output
```
((I/2)*(x^3*Log[f]^3*Log[1 - (I*Sqrt[a]*f^x)/Sqrt[b]] - x^3*Log[f]^3*Log[1
 + (I*Sqrt[a]*f^x)/Sqrt[b]] - 3*x^2*Log[f]^2*PolyLog[2, ((-I)*Sqrt[a]*f^x)
/Sqrt[b]] + 3*x^2*Log[f]^2*PolyLog[2, (I*Sqrt[a]*f^x)/Sqrt[b]] + 6*x*Log[f
]*PolyLog[3, ((-I)*Sqrt[a]*f^x)/Sqrt[b]] - 6*x*Log[f]*PolyLog[3, (I*Sqrt[a
]*f^x)/Sqrt[b]] - 6*PolyLog[4, ((-I)*Sqrt[a]*f^x)/Sqrt[b]] + 6*PolyLog[4,
(I*Sqrt[a]*f^x)/Sqrt[b]]))/(Sqrt[a]*Sqrt[b]*Log[f]^4)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2696, 27, 5666, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{af^x+bf^{-x}}\,dx$$
$$\Big\downarrow \text{2696}$$
$$\frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-3\int \frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}dx$$
$$\Big\downarrow \text{27}$$

$$\frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\int x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\sqrt{a}\sqrt{b}\log(f)}$$

$\Big\downarrow$ 5666

$$\frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\int x^2\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx-\frac{1}{2}i\int x^2\log\left(\frac{i\sqrt{a}f^x}{\sqrt{b}}+1\right)dx\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$\Big\downarrow$ 3011

$$\frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)\right)}{\sqrt{a}\sqrt{b}\log(f)}$$

$\Big\downarrow$ 7163

$$\frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}-\frac{\int \text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)dx}{\log(f)}\right)}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}-\frac{\int \text{PolyLog}\left(3,-\frac{i\sqrt{}}{}\right.}{\log(f)}\right.}{\log(f)}\right.}{\sqrt{a}\sqrt{b}\log(f)}$$

$\Big\downarrow$ 2720

$$\frac{x^3\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)}-\frac{3\left(\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}-\frac{\int f^{-x}\,\text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)df^x}{\log^2(f)}\right)}{\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right)-\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}-\frac{\int f^{-x}\,\text{PolyL}}{}\right.}{\log(f)}\right.}{\sqrt{a}\sqrt{b}\log(f)}$$

$\Big\downarrow$ 7143

$$\frac{x^3 \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}\log(f)} - \frac{3\left(\frac{1}{2}i\left(\frac{2\left(\frac{x \operatorname{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)} - \frac{\operatorname{PolyLog}\left(4,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log^2(f)}\right)}{\log(f)} - \frac{x^2\operatorname{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)}\right) - \frac{1}{2}i\left(\frac{2\left(\frac{x\operatorname{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log(f)} - \frac{\operatorname{PolyLog}\left(4,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{\log^2(f)}\right.}{\log(f)}\right.\right.}{\sqrt{a}\sqrt{b}\log(f)}$$

input
```
Int[x^3/(b/f^x + a*f^x),x]
```

output
```
(x^3*ArcTan[(Sqrt[a]*f^x)/Sqrt[b]])/(Sqrt[a]*Sqrt[b]*Log[f]) - (3*((-1/2*I
)*(-((x^2*PolyLog[2, ((-I)*Sqrt[a]*f^x)/Sqrt[b]])/Log[f]) + (2*((x*PolyLog
[3, ((-I)*Sqrt[a]*f^x)/Sqrt[b]])/Log[f] - PolyLog[4, ((-I)*Sqrt[a]*f^x)/Sq
rt[b]]/Log[f]^2))/Log[f]) + (I/2)*(-((x^2*PolyLog[2, (I*Sqrt[a]*f^x)/Sqrt[
b]])/Log[f]) + (2*((x*PolyLog[3, (I*Sqrt[a]*f^x)/Sqrt[b]])/Log[f] - PolyLo
g[4, (I*Sqrt[a]*f^x)/Sqrt[b]]/Log[f]^2))/Log[f])))/(Sqrt[a]*Sqrt[b]*Log[f]
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2696
```
Int[(x_)^(m_.)/((b_.)*(F_)^(v_) + (a_.)*(F_)^((c_.) + (d_.)*(x_))), x_Symbo
l] :> With[{u = IntHide[1/(a*F^(c + d*x) + b*F^v), x]}, Simp[x^m*u, x] - Si
mp[m   Int[x^(m - 1)*u, x], x]] /; FreeQ[{F, a, b, c, d}, x] && EqQ[v, -(c
+ d*x)] && GtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{b\,f^{-x}+a\,f^{x}}dx$$

input
```
int(x^3/(b/(f^x)+a*f^x),x)
```

output
```
int(x^3/(b/(f^x)+a*f^x),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{bf^{-x}+af^x}\,dx = -\frac{x^3\sqrt{-\frac{a}{b}}\log\left(f^x\sqrt{-\frac{a}{b}}+1\right)\log(f)^3 - x^3\sqrt{-\frac{a}{b}}\log\left(-f^x\sqrt{-\frac{a}{b}}+1\right)\log(f)^3 - 3\,x^2\sqrt{-\frac{a}{b}}\mathrm{Li}_2\left(f^x\sqrt{-\frac{a}{b}}\right)\,\mathrm{lo}}{}$$

input
```
integrate(x^3/(b/(f^x)+a*f^x),x, algorithm="fricas")
```

output
```
-1/2*(x^3*sqrt(-a/b)*log(f^x*sqrt(-a/b) + 1)*log(f)^3 - x^3*sqrt(-a/b)*log
(-f^x*sqrt(-a/b) + 1)*log(f)^3 - 3*x^2*sqrt(-a/b)*dilog(f^x*sqrt(-a/b))*lo
g(f)^2 + 3*x^2*sqrt(-a/b)*dilog(-f^x*sqrt(-a/b))*log(f)^2 + 6*x*sqrt(-a/b)
*log(f)*polylog(3, f^x*sqrt(-a/b)) - 6*x*sqrt(-a/b)*log(f)*polylog(3, -f^x
*sqrt(-a/b)) - 6*sqrt(-a/b)*polylog(4, f^x*sqrt(-a/b)) + 6*sqrt(-a/b)*poly
log(4, -f^x*sqrt(-a/b)))/(a*log(f)^4)
```

Sympy [F]

$$\int \frac{x^3}{bf^{-x}+af^x}\,dx = \int \frac{f^x x^3}{af^{2x}+b}\,dx$$

input
```
integrate(x**3/(b/(f**x)+a*f**x),x)
```

output
```
Integral(f**x*x**3/(a*f**(2*x) + b), x)
```

Maxima [F]

$$\int \frac{x^3}{bf^{-x} + af^x}\, dx = \int \frac{x^3}{af^x + \frac{b}{f^x}}\, dx$$

input
```
integrate(x^3/(b/(f^x)+a*f^x),x, algorithm="maxima")
```

output
```
integrate(x^3/(a*f^x + b/f^x), x)
```

Giac [F]

$$\int \frac{x^3}{bf^{-x} + af^x}\, dx = \int \frac{x^3}{af^x + \frac{b}{f^x}}\, dx$$

input
```
integrate(x^3/(b/(f^x)+a*f^x),x, algorithm="giac")
```

output
```
integrate(x^3/(a*f^x + b/f^x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{bf^{-x} + af^x}\, dx = \int \frac{x^3}{\frac{b}{f^x} + a\, f^x}\, dx$$

input
```
int(x^3/(b/f^x + a*f^x),x)
```

output
```
int(x^3/(b/f^x + a*f^x), x)
```

Reduce [F]

$$\int \frac{x^3}{bf^{-x} + af^x}\,dx = \int \frac{f^x x^3}{f^{2x}a + b}dx$$

input
```
int(x^3/(b/(f^x)+a*f^x),x)
```

output
```
int((f**x*x**3)/(f**(2*x)*a + b),x)
```

3.114 $\int \frac{1}{(bf^{-x}+af^x)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 22

$$\int \frac{1}{(bf^{-x}+af^x)^2}\,dx = -\frac{1}{2a\,(b+af^{2x})\log(f)}$$

output
```
-1/2/a/(b+a*f^(2*x))/ln(f)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{(bf^{-x}+af^x)^2}\,dx = -\frac{1}{2ab\log(f)+2a^2f^{2x}\log(f)}$$

input
```
Integrate[(b/f^x + a*f^x)^(-2),x]
```

output
```
-(2*a*b*Log[f] + 2*a^2*f^(2*x)*Log[f])^(-1)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2720, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(af^x+bf^{-x})^2}dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{f^x}{(af^{2x}+b)^2}df^x}{\log(f)}$$
$$\Big\downarrow \text{241}$$
$$-\frac{1}{2a\log(f)\,(af^{2x}+b)}$$

input
```
Int[(b/f^x + a*f^x)^(-2),x]
```

output
```
-1/2*1/(a*(b + a*f^(2*x))*Log[f])
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$-\frac{1}{2a(b+a\,f^{2x})\ln(f)}$	21
default	$-\frac{1}{2a(b+a\,f^{2x})\ln(f)}$	21
risch	$-\frac{1}{2a(b+a\,f^{2x})\ln(f)}$	21
parallelrisch	$-\frac{1}{2a(b+a\,f^{2x})\ln(f)}$	21
norman	$-\frac{1}{2\ln(f)a\left(a\,\mathrm{e}^{2x\ln(f)}+b\right)}$	23

input
```
int(1/(b/(f^x)+a*f^x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/ln(f)/a/((f^x)^2*a+b)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int \frac{1}{(bf^{-x}+af^x)^2}\,dx = -\frac{1}{2\,(a^2f^{2\,x}\log{(f)}+ab\log{(f)})}$$

input
```
integrate(1/(b/(f^x)+a*f^x)^2,x, algorithm="fricas")
```

output
```
-1/2/(a^2*f^(2*x)*log(f) + a*b*log(f))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bf^{-x} + af^x)^2} \, dx = \frac{1}{2ab \log{(f)} + 2b^2 f^{-2x} \log{(f)}}$$

input
```
integrate(1/(b/(f**x)+a*f**x)**2,x)
```

output
```
1/(2*a*b*log(f) + 2*b**2*log(f)/f**(2*x))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{(bf^{-x} + af^x)^2} \, dx = \frac{1}{2 \left(ab + \frac{b^2}{f^{2x}}\right) \log{(f)}}$$

input
```
integrate(1/(b/(f^x)+a*f^x)^2,x, algorithm="maxima")
```

output
```
1/2/((a*b + b^2/f^(2*x))*log(f))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(bf^{-x} + af^x)^2} \, dx = -\frac{1}{2 (af^{2x} + b)a \log{(f)}}$$

input
```
integrate(1/(b/(f^x)+a*f^x)^2,x, algorithm="giac")
```

output
```
-1/2/((a*f^(2*x) + b)*a*log(f))
```

Mupad [B] (verification not implemented)

Time = 23.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(bf^{-x} + af^x)^2} \, dx = -\frac{1}{2\,a\,\ln(f)\,\left(b + a\,f^{2\,x}\right)}$$

input
```
int(1/(b/f^x + a*f^x)^2,x)
```

output
```
-1/(2*a*log(f)*(b + a*f^(2*x)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{(bf^{-x} + af^x)^2} \, dx = \frac{f^{2x}}{2\log(f)\,b\,(f^{2x}a + b)}$$

input
```
int(1/(b/(f^x)+a*f^x)^2,x)
```

output
```
f**(2*x)/(2*log(f)*b*(f**(2*x)*a + b))
```

3.115 $\int \frac{x}{(bf^{-x}+af^x)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{x}{(bf^{-x}+af^x)^2}\,dx = \frac{x}{2ab\log(f)} - \frac{x}{2a\left(b+af^{2x}\right)\log(f)} - \frac{\log\left(b+af^{2x}\right)}{4ab\log^2(f)}$$

output
```
1/2*x/a/b/ln(f)-1/2*x/a/(b+a*f^(2*x))/ln(f)-1/4*ln(b+a*f^(2*x))/a/b/ln(f)^
2
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.76

$$\int \frac{x}{(bf^{-x}+af^x)^2}\,dx = \frac{\frac{2f^{2x}x\log(f)}{b+af^{2x}} - \frac{\log(b+af^{2x})}{a}}{4b\log^2(f)}$$

input
```
Integrate[x/(b/f^x + a*f^x)^2,x]
```

output
```
((2*f^(2*x)*x*Log[f])/(b + a*f^(2*x)) - Log[b + a*f^(2*x)]/a)/(4*b*Log[f]^
2)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2721, 2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(af^x + bf^{-x})^2} \, dx$$

$$\Big\downarrow \text{2721}$$

$$\int \frac{xf^{2x}}{(af^{2x} + b)^2} dx$$

$$\Big\downarrow \text{2621}$$

$$\frac{\int \frac{1}{af^{2x}+b} dx}{2a\log(f)} - \frac{x}{2a\log(f)\left(af^{2x} + b\right)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{f^{-2x}}{af^{2x}+b} df^{2x}}{4a\log^2(f)} - \frac{x}{2a\log(f)\left(af^{2x} + b\right)}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int f^{-2x} df^{2x}}{b} - \frac{a\int \frac{1}{af^{2x}+b} df^{2x}}{b}}{4a\log^2(f)} - \frac{x}{2a\log(f)\left(af^{2x} + b\right)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(f^{2x})}{b} - \frac{a\int \frac{1}{af^{2x}+b} df^{2x}}{b}}{4a\log^2(f)} - \frac{x}{2a\log(f)\left(af^{2x} + b\right)}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(f^{2x})}{b} - \frac{\log(af^{2x}+b)}{b}}{4a\log^2(f)} - \frac{x}{2a\log(f)\left(af^{2x} + b\right)}$$

input
```
Int[x/(b/f^x + a*f^x)^2,x]
```

output
```
-1/2*x/(a*(b + a*f^(2*x))*Log[f]) + (Log[f^(2*x)]/b - Log[b + a*f^(2*x)]/b
)/(4*a*Log[f]^2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
norman	$\frac{x\,\mathrm{e}^{2x\ln(f)}}{2b\ln(f)\left(a\,\mathrm{e}^{2x\ln(f)}+b\right)}-\frac{\ln\left(a\,\mathrm{e}^{2x\ln(f)}+b\right)}{4\ln(f)^2ab}$	56
risch	$\frac{x}{2ab\ln(f)}-\frac{x}{2a(b+a\,f^{2x})\ln(f)}-\frac{\ln\left(f^{2x}+\frac{b}{a}\right)}{4\ln(f)^2ab}$	60
parallelrisch	$\frac{2f^{2x}\ln(f)ax-\ln(b+a\,f^{2x})f^{2x}a-\ln(b+a\,f^{2x})b}{4\ln(f)^2ab(b+a\,f^{2x})}$	67

input
```
int(x/(b/(f^x)+a*f^x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/ln(f)*x*exp(x*ln(f))^2/(a*exp(x*ln(f))^2+b)-1/4/ln(f)^2/a/b*ln(a*exp
(x*ln(f))^2+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{x}{(bf^{-x}+af^x)^2}\,dx=\frac{2\,af^{2\,x}x\log{(f)}-(af^{2\,x}+b)\log{(af^{2\,x}+b)}}{4\left(a^2bf^{2\,x}\log{(f)}^2+ab^2\log{(f)}^2\right)}$$

input
```
integrate(x/(b/(f^x)+a*f^x)^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*f^(2*x)*x*log(f) - (a*f^(2*x) + b)*log(a*f^(2*x) + b))/(a^2*b*f^(
2*x)*log(f)^2 + a*b^2*log(f)^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{x}{(bf^{-x} + af^x)^2}\, dx = \frac{x}{2ab\log(f) + 2b^2f^{-2x}\log(f)} - \frac{x}{2ab\log(f)} - \frac{\log\left(\frac{a}{b} + f^{-2x}\right)}{4ab\log(f)^2}$$

input
```
integrate(x/(b/(f**x)+a*f**x)**2,x)
```

output
```
x/(2*a*b*log(f) + 2*b**2*log(f)/f**(2*x)) - x/(2*a*b*log(f)) - log(a/b + f
**(-2*x))/(4*a*b*log(f)**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{x}{(bf^{-x} + af^x)^2}\, dx = \frac{f^{2x}x}{2\,(abf^{2x}\log(f) + b^2\log(f))} - \frac{\log\left(\frac{af^{2x}+b}{a}\right)}{4\,ab\log(f)^2}$$

input
```
integrate(x/(b/(f^x)+a*f^x)^2,x, algorithm="maxima")
```

output
```
1/2*f^(2*x)*x/(a*b*f^(2*x)*log(f) + b^2*log(f)) - 1/4*log((a*f^(2*x) + b)/
a)/(a*b*log(f)^2)
```

Giac [F]

$$\int \frac{x}{(bf^{-x} + af^x)^2}\, dx = \int \frac{x}{\left(af^x + \frac{b}{f^x}\right)^2}\, dx$$

input
```
integrate(x/(b/(f^x)+a*f^x)^2,x, algorithm="giac")
```

output
```
integrate(x/(a*f^x + b/f^x)^2, x)
```

Mupad [B] (verification not implemented)

Time = 23.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{x}{(bf^{-x}+af^x)^2}\,dx = \frac{f^{2x}\,x}{2\,(b^2\,\ln(f)+a\,b\,f^{2x}\,\ln(f))} - \frac{\ln(b+a\,f^{2x})}{4\,a\,b\ln(f)^2}$$

input
```
int(x/(b/f^x + a*f^x)^2,x)
```

output
```
(f^(2*x)*x)/(2*(b^2*log(f) + a*b*f^(2*x)*log(f))) - log(b + a*f^(2*x))/(4*
a*b*log(f)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{x}{(bf^{-x}+af^x)^2}\,dx = \frac{-f^{2x}\log(f^{2x}a+b)\,a+2f^{2x}\log(f)\,ax-\log(f^{2x}a+b)\,b}{4\log(f)^2\,ab\,(f^{2x}a+b)}$$

input
```
int(x/(b/(f^x)+a*f^x)^2,x)
```

output
```
( - f**(2*x)*log(f**(2*x)*a + b)*a + 2*f**(2*x)*log(f)*a*x - log(f**(2*x)*
a + b)*b)/(4*log(f)**2*a*b*(f**(2*x)*a + b))
```

3.116 $\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx = \frac{x^2}{2ab\log(f)} - \frac{x^2}{2a(b+af^{2x})\log(f)} - \frac{x\log\left(1+\frac{af^{2x}}{b}\right)}{2ab\log^2(f)} - \frac{\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{4ab\log^3(f)}$$

output
```
1/2*x^2/a/b/ln(f)-1/2*x^2/a/(b+a*f^(2*x))/ln(f)-1/2*x*ln(1+a*f^(2*x)/b)/a/
b/ln(f)^2-1/4*polylog(2,-a*f^(2*x)/b)/a/b/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx = \frac{2x\log(f)\left(af^{2x}x\log(f)-(b+af^{2x})\log\left(1+\frac{af^{2x}}{b}\right)\right)-(b+af^{2x})\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{4ab(b+af^{2x})\log^3(f)}$$

input
```
Integrate[x^2/(b/f^x + a*f^x)^2,x]
```

output
```
(2*x*Log[f]*(a*f^(2*x)*x*Log[f] - (b + a*f^(2*x))*Log[1 + (a*f^(2*x))/b])
- (b + a*f^(2*x))*PolyLog[2, -((a*f^(2*x))/b)])/(4*a*b*(b + a*f^(2*x))*Log
[f]^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2721, 2621, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(af^x+bf^{-x})^2}dx$$

$$\Big\downarrow \text{2721}$$

$$\int \frac{x^2f^{2x}}{(af^{2x}+b)^2}dx$$

$$\Big\downarrow \text{2621}$$

$$\frac{\int \frac{x}{af^{2x}+b}dx}{a\log(f)}-\frac{x^2}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \text{2615}$$

$$\frac{\frac{x^2}{2b}-\frac{a\int \frac{f^{2x}x}{af^{2x}+b}dx}{b}}{a\log(f)}-\frac{x^2}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{x^2}{2b}-\frac{a\left(\frac{x\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}-\frac{\int \log\left(\frac{af^{2x}}{b}+1\right)dx}{2a\log(f)}\right)}{b}}{a\log(f)}-\frac{x^2}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{x^2}{2b}-\frac{a\left(\frac{x\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}-\frac{\int f^{-2x}\log\left(\frac{af^{2x}}{b}+1\right)df^{2x}}{4a\log^2(f)}\right)}{b}}{a\log(f)}-\frac{x^2}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{x^2}{2b}-\frac{a\left(\frac{\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{4a\log^2(f)}+\frac{x\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}\right)}{b}}{a\log(f)}-\frac{x^2}{2a\log(f)\left(af^{2x}+b\right)}$$

input
```
Int[x^2/(b/f^x + a*f^x)^2,x]
```

output
```
-1/2*x^2/(a*(b + a*f^(2*x))*Log[f]) + (x^2/(2*b) - (a*((x*Log[1 + (a*f^(2*
x))/b])/(2*a*Log[f]) + PolyLog[2, -((a*f^(2*x))/b)]/(4*a*Log[f]^2)))/b)/(a
*Log[f])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{x^2}{2ab\ln(f)}-\frac{x^2}{2a(b+a\,f^{2x})\ln(f)}-\frac{x\ln\left(1+\frac{a\,f^{2x}}{b}\right)}{2ab\ln(f)^2}-\frac{\operatorname{polylog}\left(2,-\frac{a\,f^{2x}}{b}\right)}{4ab\ln(f)^3}$	91

input
```
int(x^2/(b/(f^x)+a*f^x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/ln(f)*x^2/a/((f^x)^2*a+b)+1/2*x^2/a/b/ln(f)-1/2*x*ln(1+a*f^(2*x)/b)/a
/b/ln(f)^2-1/4*polylog(2,-a*f^(2*x)/b)/a/b/ln(f)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx$$
$$=\frac{af^{2x}x^2\log(f)^2-(af^{2x}+b)\mathrm{Li}_2\left(f^x\sqrt{-\frac{a}{b}}\right)-(af^{2x}+b)\mathrm{Li}_2\left(-f^x\sqrt{-\frac{a}{b}}\right)-(af^{2x}x\log(f)+bx\log(f))\log}{2\left(a^2bf^{2x}\log(f)^3+ab^2\log(f)^3\right)}$$

input
```
integrate(x^2/(b/(f^x)+a*f^x)^2,x, algorithm="fricas")
```

output
```
1/2*(a*f^(2*x)*x^2*log(f)^2 - (a*f^(2*x) + b)*dilog(f^x*sqrt(-a/b)) - (a*f
^(2*x) + b)*dilog(-f^x*sqrt(-a/b)) - (a*f^(2*x)*x*log(f) + b*x*log(f))*log
(f^x*sqrt(-a/b) + 1) - (a*f^(2*x)*x*log(f) + b*x*log(f))*log(-f^x*sqrt(-a/
b) + 1))/(a^2*b*f^(2*x)*log(f)^3 + a*b^2*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{(bf^{-x} + af^x)^2}\, dx = \frac{x^2}{2ab\log(f) + 2b^2f^{-2x}\log(f)} - \frac{\int \frac{f^{2x}x}{af^{2x}+b}\, dx}{b\log(f)}$$

input
```
integrate(x**2/(b/(f**x)+a*f**x)**2,x)
```

output
```
x**2/(2*a*b*log(f) + 2*b**2*log(f)/f**(2*x)) - Integral(f**(2*x)*x/(a*f**(
2*x) + b), x)/(b*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(bf^{-x} + af^x)^2}\, dx = -\frac{x^2}{2\,(a^2f^{2\,x}\log(f) + ab\log(f))} + \frac{x^2}{2\,ab\log(f)} - \frac{2\,x\log\left(\frac{af^{2\,x}}{b} + 1\right)\log(f) + \mathrm{Li}_2\left(-\frac{af^{2\,x}}{b}\right)}{4\,ab\log(f)^3}$$

input
```
integrate(x^2/(b/(f^x)+a*f^x)^2,x, algorithm="maxima")
```

output
```
-1/2*x^2/(a^2*f^(2*x)*log(f) + a*b*log(f)) + 1/2*x^2/(a*b*log(f)) - 1/4*(2
*x*log(a*f^(2*x)/b + 1)*log(f) + dilog(-a*f^(2*x)/b))/(a*b*log(f)^3)
```

Giac [F]

$$\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx=\int \frac{x^2}{\left(af^x+\frac{b}{f^x}\right)^2}\,dx$$

input
```
integrate(x^2/(b/(f^x)+a*f^x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/(a*f^x + b/f^x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx=\int \frac{x^2}{\left(\frac{b}{f^x}+a\,f^x\right)^2}\,dx$$

input
```
int(x^2/(b/f^x + a*f^x)^2,x)
```

output
```
int(x^2/(b/f^x + a*f^x)^2, x)
```

Reduce [F]

$$\int \frac{x^2}{(bf^{-x}+af^x)^2}\,dx$$
$$=\frac{4f^{2x}\left(\int \frac{x}{f^{4x}a^2+2f^{2x}ab+b^2}dx\right)\log(f)^2\,a\,b^2-f^{2x}\log(f^{2x}a+b)\,a+2f^{2x}\log(f)\,ax+4\left(\int \frac{x}{f^{4x}a^2+2f^{2x}ab+b^2}dx\right)\mathrm{lo}}{4\log(f)^3\,ab\,(f^{2x}a+b)}$$

input
```
int(x^2/(b/(f^x)+a*f^x)^2,x)
```

output
```
(4*f**(2*x)*int(x/(f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2),x)*log(f)**2*a*b
**2 - f**(2*x)*log(f**(2*x)*a + b)*a + 2*f**(2*x)*log(f)*a*x + 4*int(x/(f*
*(4*x)*a**2 + 2*f**(2*x)*a*b + b**2),x)*log(f)**2*b**3 - log(f**(2*x)*a +
b)*b - 2*log(f)**2*b*x**2)/(4*log(f)**3*a*b*(f**(2*x)*a + b))
```

3.117 $\int \frac{x^3}{\left(bf^{-x}+af^x\right)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 128

$$\int \frac{x^3}{\left(bf^{-x}+af^x\right)^2} \, dx=\frac{x^3}{2ab\log(f)}-\frac{x^3}{2a\left(b+af^{2x}\right)\log(f)}-\frac{3x^2\log\left(1+\frac{af^{2x}}{b}\right)}{4ab\log^2(f)}-\frac{3x\operatorname{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{4ab\log^3(f)}+\frac{3\operatorname{PolyLog}\left(3,-\frac{af^{2x}}{b}\right)}{8ab\log^4(f)}$$

output
```
1/2*x^3/a/b/ln(f)-1/2*x^3/a/(b+a*f^(2*x))/ln(f)-3/4*x^2*ln(1+a*f^(2*x)/b)/
a/b/ln(f)^2-3/4*x*polylog(2,-a*f^(2*x)/b)/a/b/ln(f)^3+3/8*polylog(3,-a*f^(
2*x)/b)/a/b/ln(f)^4
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{\left(bf^{-x}+af^x\right)^2} \, dx=-\frac{x^3}{2a\left(b+af^{2x}\right)\log(f)}+\frac{3\left(\frac{x^3}{3b}-\frac{x^2\log\left(1+\frac{af^{2x}}{b}\right)}{2b\log(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{2b\log^2(f)}+\frac{\operatorname{PolyLog}\left(3,-\frac{af^{2x}}{b}\right)}{4b\log^3(f)}\right)}{2a\log(f)}$$

input
```
Integrate[x^3/(b/f^x + a*f^x)^2,x]
```

output
```
-1/2*x^3/(a*(b + a*f^(2*x))*Log[f]) + (3*(x^3/(3*b) - (x^2*Log[1 + (a*f^(2
*x))/b])/(2*b*Log[f]) - (x*PolyLog[2, -((a*f^(2*x))/b)])/(2*b*Log[f]^2) +
PolyLog[3, -((a*f^(2*x))/b)]/(4*b*Log[f]^3)))/(2*a*Log[f])
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2721, 2621, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(af^x+bf^{-x})^2}dx$$

$$\Big\downarrow \text{2721}$$

$$\int \frac{x^3f^{2x}}{(af^{2x}+b)^2}dx$$

$$\Big\downarrow \text{2621}$$

$$\frac{3\int \frac{x^2}{af^{2x}+b}dx}{2a\log(f)}-\frac{x^3}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \text{2615}$$

$$\frac{3\left(\frac{x^3}{3b}-\frac{a\int \frac{f^{2x}x^2}{af^{2x}+b}dx}{b}\right)}{2a\log(f)}-\frac{x^3}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \text{2620}$$

$$\frac{3\left(\frac{x^3}{3b}-\frac{a\left(\frac{x^2\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}-\frac{\int x\log\left(\frac{af^{2x}}{b}+1\right)dx}{a\log(f)}\right)}{b}\right)}{2a\log(f)}-\frac{x^3}{2a\log(f)(af^{2x}+b)}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3\left(\frac{x^3}{3b}-\frac{a\left(\frac{x^2\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)dx}{2\log(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{2\log(f)}}{a\log(f)}\right)}{b}\right)}{2a\log(f)}-\frac{x^3}{2a\log(f)\left(af^{2x}+b\right)}$$

$$\Big\downarrow \text{2720}$$

$$\frac{3\left(\frac{x^3}{3b}-\frac{a\left(\frac{x^2\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}-\frac{\frac{\int f^{-2x}\,\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)df^{2x}}{4\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{2\log(f)}}{a\log(f)}\right)}{b}\right)}{2a\log(f)}-\frac{x^3}{2a\log(f)\left(af^{2x}+b\right)}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3\left(\frac{x^3}{3b}-\frac{a\left(\frac{x^2\log\left(\frac{af^{2x}}{b}+1\right)}{2a\log(f)}-\frac{\frac{\text{PolyLog}\left(3,-\frac{af^{2x}}{b}\right)}{4\log^2(f)}-\frac{x\,\text{PolyLog}\left(2,-\frac{af^{2x}}{b}\right)}{2\log(f)}}{a\log(f)}\right)}{b}\right)}{2a\log(f)}-\frac{x^3}{2a\log(f)\left(af^{2x}+b\right)}$$

input
```
Int[x^3/(b/f^x + a*f^x)^2,x]
```

output
```
-1/2*x^3/(a*(b + a*f^(2*x))*Log[f]) + (3*(x^3/(3*b) - (a*((x^2*Log[1 + (a*
f^(2*x))/b])/(2*a*Log[f]) - (-1/2*(x*PolyLog[2, -((a*f^(2*x))/b)])/Log[f]
+ PolyLog[3, -((a*f^(2*x))/b)]/(4*Log[f]^2))/(a*Log[f])))/b))/(2*a*Log[f])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{x^{3}}{2ab\ln(f)}-\frac{x^{3}}{2a(b+a\,f^{2x})\ln(f)}-\frac{3x^{2}\ln\left(1+\frac{a\,f^{2x}}{b}\right)}{4ab\ln(f)^{2}}-\frac{3x\,\mathrm{polylog}\left(2,-\frac{a\,f^{2x}}{b}\right)}{4ab\ln(f)^{3}}+\frac{3\,\mathrm{polylog}\left(3,-\frac{a\,f^{2x}}{b}\right)}{8ab\ln(f)^{4}}$	119

input
```
int(x^3/(b/(f^x)+a*f^x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/ln(f)*x^3/a/((f^x)^2*a+b)+1/2*x^3/a/b/ln(f)-3/4*x^2*ln(1+a*f^(2*x)/b)
/a/b/ln(f)^2-3/4*x*polylog(2,-a*f^(2*x)/b)/a/b/ln(f)^3+3/8*polylog(3,-a*f^
(2*x)/b)/a/b/ln(f)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(117) = 234.

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.88

$$\int \frac{x^{3}}{\left(bf^{-x}+af^{x}\right)^{2}}\,dx$$
$$= \frac{2\,af^{2\,x}x^{3}\log\left(f\right)^{3}-6\left(af^{2\,x}x\log\left(f\right)+bx\log\left(f\right)\right)\mathrm{Li}_{2}\left(f^{x}\sqrt{-\frac{a}{b}}\right)-6\left(af^{2\,x}x\log\left(f\right)+bx\log\left(f\right)\right)\mathrm{Li}_{2}\left(-f^{x}\right.}{}$$

input
```
integrate(x^3/(b/(f^x)+a*f^x)^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*f^(2*x)*x^3*log(f)^3 - 6*(a*f^(2*x)*x*log(f) + b*x*log(f))*dilog(
f^x*sqrt(-a/b)) - 6*(a*f^(2*x)*x*log(f) + b*x*log(f))*dilog(-f^x*sqrt(-a/b
)) - 3*(a*f^(2*x)*x^2*log(f)^2 + b*x^2*log(f)^2)*log(f^x*sqrt(-a/b) + 1) -
 3*(a*f^(2*x)*x^2*log(f)^2 + b*x^2*log(f)^2)*log(-f^x*sqrt(-a/b) + 1) + 6*
(a*f^(2*x) + b)*polylog(3, f^x*sqrt(-a/b)) + 6*(a*f^(2*x) + b)*polylog(3,
-f^x*sqrt(-a/b)))/(a^2*b*f^(2*x)*log(f)^4 + a*b^2*log(f)^4)
```

Sympy [F]

$$\int \frac{x^3}{(bf^{-x}+af^x)^2}\,dx=\frac{x^3}{2ab\log(f)+2b^2f^{-2x}\log(f)}-\frac{3\int\frac{f^{2x}x^2}{af^{2x}+b}\,dx}{2b\log(f)}$$

input
```
integrate(x**3/(b/(f**x)+a*f**x)**2,x)
```

output
```
x**3/(2*a*b*log(f) + 2*b**2*log(f)/f**(2*x)) - 3*Integral(f**(2*x)*x**2/(a
*f**(2*x) + b), x)/(2*b*log(f))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\begin{aligned}\int \frac{x^3}{(bf^{-x}+af^x)^2}\,dx\\ &=-\frac{x^3}{2\,(a^2f^{2\,x}\log(f)+ab\log(f))}+\frac{x^3}{2\,ab\log(f)}\\ &\quad-\frac{3\left(2\,x^2\log\left(\frac{af^{2\,x}}{b}+1\right)\log(f)^2+2\,x\mathrm{Li}_2\left(-\frac{af^{2\,x}}{b}\right)\log(f)-\mathrm{Li}_3(-\frac{af^{2\,x}}{b})\right)}{8\,ab\log(f)^4}\end{aligned}$$

input
```
integrate(x^3/(b/(f^x)+a*f^x)^2,x, algorithm="maxima")
```

output
```
-1/2*x^3/(a^2*f^(2*x)*log(f) + a*b*log(f)) + 1/2*x^3/(a*b*log(f)) - 3/8*(2
*x^2*log(a*f^(2*x)/b + 1)*log(f)^2 + 2*x*dilog(-a*f^(2*x)/b)*log(f) - poly
log(3, -a*f^(2*x)/b))/(a*b*log(f)^4)
```

Giac [F]

$$\int \frac{x^3}{(bf^{-x}+af^x)^2}\,dx=\int \frac{x^3}{\left(af^x+\frac{b}{f^x}\right)^2}\,dx$$

input
```
integrate(x^3/(b/(f^x)+a*f^x)^2,x, algorithm="giac")
```

output
```
integrate(x^3/(a*f^x + b/f^x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(bf^{-x}+af^x)^2}\,dx=\int \frac{x^3}{\left(\frac{b}{f^x}+a\,f^x\right)^2}\,dx$$

input
```
int(x^3/(b/f^x + a*f^x)^2,x)
```

output
```
int(x^3/(b/f^x + a*f^x)^2, x)
```

Reduce [F]

$$\int \frac{x^3}{(bf^{-x}+af^x)^2}\,dx$$
$$=\frac{12f^{2x}\left(\int \frac{x^2}{f^{4x}a^2+2f^{2x}ab+b^2}dx\right)\log(f)^3\,a\,b^2+12f^{2x}\left(\int \frac{x}{f^{4x}a^2+2f^{2x}ab+b^2}dx\right)\log(f)^2\,a\,b^2-3f^{2x}\log(f^{2x}a+b)\,a}{}$$

input
```
int(x^3/(b/(f^x)+a*f^x)^2,x)
```

output
```
(12*f**(2*x)*int(x**2/(f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2),x)*log(f)**3
*a*b**2 + 12*f**(2*x)*int(x/(f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2),x)*log
(f)**2*a*b**2 - 3*f**(2*x)*log(f**(2*x)*a + b)*a + 6*f**(2*x)*log(f)*a*x +
 12*int(x**2/(f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2),x)*log(f)**3*b**3 + 1
2*int(x/(f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2),x)*log(f)**2*b**3 - 3*log(
f**(2*x)*a + b)*b - 4*log(f)**3*b*x**3 - 6*log(f)**2*b*x**2)/(8*log(f)**4*
a*b*(f**(2*x)*a + b))
```

3.118 $\int \frac{1}{(bf^{-x}+af^x)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = -\frac{f^x}{4a\,(b+af^{2x})^2\log(f)}+\frac{f^x}{8ab\,(b+af^{2x})\log(f)}+\frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)}$$

output
```
-1/4*f^x/a/(b+a*f^(2*x))^2/ln(f)+1/8*f^x/a/b/(b+a*f^(2*x))/ln(f)+1/8*arcta
n(a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}f^x(-b+af^{2x})}{(b+af^{2x})^2}+\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)}$$

input
```
Integrate[(b/f^x + a*f^x)^(-3),x]
```

output
```
((Sqrt[a]*Sqrt[b]*f^x*(-b + a*f^(2*x)))/(b + a*f^(2*x))^2 + ArcTan[(Sqrt[a
]*f^x)/Sqrt[b]])/(8*a^(3/2)*b^(3/2)*Log[f])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2720, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(af^x+bf^{-x})^3}\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{f^{2x}}{(af^{2x}+b)^3}df^x}{\log(f)}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\int \frac{1}{\left(af^{2x}+b\right)^2}df^x}{4a}-\frac{f^x}{4a(af^{2x}+b)^2}}{\log(f)}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\frac{\int \frac{1}{af^{2x}+b}df^x}{2b}+\frac{f^x}{2b\left(af^{2x}+b\right)}}{4a}-\frac{f^x}{4a(af^{2x}+b)^2}}{\log(f)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{2\sqrt{a}b^{3/2}}+\frac{f^x}{2b\left(af^{2x}+b\right)}}{4a}-\frac{f^x}{4a(af^{2x}+b)^2}}{\log(f)}$$

input
```
Int[(b/f^x + a*f^x)^(-3),x]
```

output
```
(-1/4*f^x/(a*(b + a*f^(2*x))^2) + (f^x/(2*b*(b + a*f^(2*x))) + ArcTan[(Sqr
t[a]*f^x)/Sqrt[b]]/(2*Sqrt[a]*b^(3/2)))/(4*a))/Log[f]
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{\frac{f^{3x}}{8b}-\frac{f^{x}}{8a}}{\left(b+a\,f^{2x}\right)^{2}}+\frac{\arctan\left(\frac{a\,f^{x}}{\sqrt{ab}}\right)}{8ab\sqrt{ab}}}{\ln\left(f\right)}$	62
default	$\frac{\frac{\frac{f^{3x}}{8b}-\frac{f^{x}}{8a}}{\left(b+a\,f^{2x}\right)^{2}}+\frac{\arctan\left(\frac{a\,f^{x}}{\sqrt{ab}}\right)}{8ab\sqrt{ab}}}{\ln\left(f\right)}$	62
risch	$\frac{f^{x}\left(-b+a\,f^{2x}\right)}{8\ln\left(f\right)a\left(b+a\,f^{2x}\right)^{2}b}-\frac{\ln\left(f^{x}-\frac{b}{\sqrt{-ab}}\right)}{16\sqrt{-ab}\,ba\ln\left(f\right)}+\frac{\ln\left(f^{x}+\frac{b}{\sqrt{-ab}}\right)}{16\sqrt{-ab}\,ba\ln\left(f\right)}$	102

input
```
int(1/(b/(f^x)+a*f^x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/ln(f)*((1/8/b*(f^x)^3-1/8/a*f^x)/((f^x)^2*a+b)^2+1/8/a/b/(a*b)^(1/2)*arc
tan(a*f^x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.00

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = \left[\frac{2\,a^2bf^{3\,x}-2\,ab^2f^x-\left(\sqrt{-ab}a^2f^{4\,x}+2\,\sqrt{-ab}abf^{2\,x}+\sqrt{-ab}b^2\right)\log\left(\frac{af^{2\,x}-2\,\sqrt{-ab}f^x-b}{af^{2\,x}+b}\right)}{16\left(a^4b^2f^{4\,x}\log\left(f\right)+2\,a^3b^3f^{2\,x}\log\left(f\right)+a^2b^4\log\left(f\right)\right)}, \frac{a^2bf^{3\,x}-ab^2f^x}{8\left(a^4b^2\right.}\right.$$

input
```
integrate(1/(b/(f^x)+a*f^x)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*a^2*b*f^(3*x) - 2*a*b^2*f^x - (sqrt(-a*b)*a^2*f^(4*x) + 2*sqrt(-a
*b)*a*b*f^(2*x) + sqrt(-a*b)*b^2)*log((a*f^(2*x) - 2*sqrt(-a*b)*f^x - b)/(
a*f^(2*x) + b)))/(a^4*b^2*f^(4*x)*log(f) + 2*a^3*b^3*f^(2*x)*log(f) + a^2*
b^4*log(f)), 1/8*(a^2*b*f^(3*x) - a*b^2*f^x - (sqrt(a*b)*a^2*f^(4*x) + 2*s
qrt(a*b)*a*b*f^(2*x) + sqrt(a*b)*b^2)*arctan(sqrt(a*b)/(a*f^x)))/(a^4*b^2*
f^(4*x)*log(f) + 2*a^3*b^3*f^(2*x)*log(f) + a^2*b^4*log(f))]
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = \frac{af^{-x}-bf^{-3x}}{8a^3b\log\left(f\right)+16a^2b^2f^{-2x}\log\left(f\right)+8ab^3f^{-4x}\log\left(f\right)} + \frac{\operatorname{RootSum}\left(256z^2a^3b^3+1,\left(i\mapsto i\log\left(-16ia^2b+f^{-x}\right)\right)\right)}{\log\left(f\right)}$$

input
```
integrate(1/(b/(f**x)+a*f**x)**3,x)
```

output
```
(a/f**x - b/f**(3*x))/(8*a**3*b*log(f) + 16*a**2*b**2*log(f)/f**(2*x) + 8*
a*b**3*log(f)/f**(4*x)) + RootSum(256*_z**2*a**3*b**3 + 1, Lambda(_i, _i*l
og(-16*_i*a**2*b + f**(-x))))/log(f)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = -\frac{\frac{b}{f^{3x}}-\frac{a}{f^x}}{8\left(a^3b+\frac{ab^3}{f^{4x}}+\frac{2a^2b^2}{f^{2x}}\right)\log(f)}-\frac{\arctan\left(\frac{b}{\sqrt{ab}f^x}\right)}{8\sqrt{ab}ab\log(f)}$$

input
```
integrate(1/(b/(f^x)+a*f^x)^3,x, algorithm="maxima")
```

output
```
-1/8*(b/f^(3*x) - a/f^x)/((a^3*b + a*b^3/f^(4*x) + 2*a^2*b^2/f^(2*x))*log(
f)) - 1/8*arctan(b/(sqrt(a*b)*f^x))/(sqrt(a*b)*a*b*log(f))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.76

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = \frac{\arctan\left(\frac{af^x}{\sqrt{ab}}\right)}{8\sqrt{ab}ab\log(f)}+\frac{af^{3x}-bf^x}{8(af^{2x}+b)^2ab\log(f)}$$

input
```
integrate(1/(b/(f^x)+a*f^x)^3,x, algorithm="giac")
```

output
```
1/8*arctan(a*f^x/sqrt(a*b))/(sqrt(a*b)*a*b*log(f)) + 1/8*(a*f^(3*x) - b*f^
x)/((a*f^(2*x) + b)^2*a*b*log(f))
```

Mupad [B] (verification not implemented)

Time = 23.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.30

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = \frac{f^x}{8\,(a\,b^2\,\ln(f)+a^2\,b\,f^{2\,x}\,\ln(f))} - \frac{f^x}{4\,(a\,b^2\,\ln(f)+a^3\,f^{4\,x}\,\ln(f)+2\,a^2\,b\,f^{2\,x}\,\ln(f))} + \frac{\mathrm{atan}\left(\frac{f^x\,\sqrt{a^3\,b^3\,\ln(f)^2}}{a\,b^2\,\ln(f)}\right)}{8\,\sqrt{a^3\,b^3\,\ln(f)^2}}$$

input
```
int(1/(b/f^x + a*f^x)^3,x)
```

output
```
f^x/(8*(a*b^2*log(f) + a^2*b*f^(2*x)*log(f))) - f^x/(4*(a*b^2*log(f) + a^3
*f^(4*x)*log(f) + 2*a^2*b*f^(2*x)*log(f))) + atan((f^x*(a^3*b^3*log(f)^2)^
(1/2))/(a*b^2*log(f)))/(8*(a^3*b^3*log(f)^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.52

$$\int \frac{1}{(bf^{-x}+af^x)^3}\,dx = \frac{f^{4x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)a^2+2f^{2x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)b^2+f^{3x}a^2b-f^x a\,b^2}{8\log(f)\,a^2b^2\,(f^{4x}a^2+2f^{2x}ab+b^2)}$$

input
```
int(1/(b/(f^x)+a*f^x)^3,x)
```

output
```
(f**(4*x)*sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a)))*a**2 + 2*f**(2*
x)*sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*
atan((f**x*a)/(sqrt(b)*sqrt(a)))*b**2 + f**(3*x)*a**2*b - f**x*a*b**2)/(8*
log(f)*a**2*b**2*(f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2))
```

3.119 $\int \frac{x}{(bf^{-x}+af^x)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 196

$$\begin{aligned}\int \frac{x}{(bf^{-x}+af^x)^3}\,dx =& \frac{f^x}{8ab\,(b+af^{2x})\log^2(f)} - \frac{f^x x}{4a\,(b+af^{2x})^2\log(f)} \\ &+ \frac{f^x x}{8ab\,(b+af^{2x})\log(f)} + \frac{x\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)} \\ &- \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)} + \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)}\end{aligned}$$

output
```
1/8*f^x/a/b/(b+a*f^(2*x))/ln(f)^2-1/4*f^x*x/a/(b+a*f^(2*x))^2/ln(f)+1/8*f^
x*x/a/b/(b+a*f^(2*x))/ln(f)+1/8*x*arctan(a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3
/2)/ln(f)-1/16*I*polylog(2,-I*a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^2
+1/16*I*polylog(2,I*a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07

$$\int \frac{x}{(bf^{-x}+af^x)^3}\,dx$$
$$=\frac{\frac{2\sqrt{a}f^x}{b^2+abf^{2x}}-\frac{4\sqrt{a}f^x x\log(f)}{(b+af^{2x})^2}+\frac{2\sqrt{a}f^x x\log(f)}{b^2+abf^{2x}}+\frac{ix\log(f)\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{ix\log(f)\log\left(1+\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}}+\frac{i\,\mathrm{P}}{}}{16a^{3/2}\log^2(f)}$$

input
```
Integrate[x/(b/f^x + a*f^x)^3,x]
```

output
```
((2*Sqrt[a]*f^x)/(b^2 + a*b*f^(2*x)) - (4*Sqrt[a]*f^x*x*Log[f])/(b + a*f^(
2*x))^2 + (2*Sqrt[a]*f^x*x*Log[f])/(b^2 + a*b*f^(2*x)) + (I*x*Log[f]*Log[1
 - (I*Sqrt[a]*f^x)/Sqrt[b]])/b^(3/2) - (I*x*Log[f]*Log[1 + (I*Sqrt[a]*f^x)
/Sqrt[b]])/b^(3/2) - (I*PolyLog[2, ((-I)*Sqrt[a]*f^x)/Sqrt[b]])/b^(3/2) +
(I*PolyLog[2, (I*Sqrt[a]*f^x)/Sqrt[b]])/b^(3/2))/(16*a^(3/2)*Log[f]^2)
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2721, 2684, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(af^x+bf^{-x})^3}\,dx$$
$$\Big\downarrow \ 2721$$
$$\int \frac{xf^{3x}}{(af^{2x}+b)^3}dx$$
$$\Big\downarrow \ 2684$$
$$\int \left(\frac{xf^x}{a\,(af^{2x}+b)^2}-\frac{bxf^x}{a\,(af^{2x}+b)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)}-\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)}+\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{16a^{3/2}b^{3/2}\log^2(f)}+\frac{f^x}{8ab\log^2(f)\left(af^{2x}+b\right)}+\frac{xf^x}{8ab\log(f)\left(af^{2x}+b\right)}-\frac{xf^x}{4a\log(f)\left(af^{2x}+b\right)^2}$$

input
```
Int[x/(b/f^x + a*f^x)^3,x]
```

output
```
f^x/(8*a*b*(b + a*f^(2*x))*Log[f]^2) - (f^x*x)/(4*a*(b + a*f^(2*x))^2*Log[
f]) + (f^x*x)/(8*a*b*(b + a*f^(2*x))*Log[f]) + (x*ArcTan[(Sqrt[a]*f^x)/Sqr
t[b]])/(8*a^(3/2)*b^(3/2)*Log[f]) - ((I/16)*PolyLog[2, ((-I)*Sqrt[a]*f^x)/
Sqrt[b]])/(a^(3/2)*b^(3/2)*Log[f]^2) + ((I/16)*PolyLog[2, (I*Sqrt[a]*f^x)/
Sqrt[b]])/(a^(3/2)*b^(3/2)*Log[f]^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2684
```
Int[((a_.) + (b_.)*(F_)^(u_))^(p_.)*((c_.) + (d_.)*(F_)^(v_))^(q_.)*((e_.)
+ (f_.)*(x_))^(m_.), x_Symbol] :> With[{w = ExpandIntegrand[(e + f*x)^m, (a
 + b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
 d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07

method	result
risch	$\frac{f^{x}\left(f^{2x}\ln(f)ax-\ln(f)bx+a\,f^{2x}+b\right)}{8\ln(f)^{2}a(b+a\,f^{2x})^{2}b}+\frac{x\ln\left(\frac{-a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{16\ln(f)ab\sqrt{-ab}}-\frac{x\ln\left(\frac{a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{16\ln(f)ab\sqrt{-ab}}+\frac{\operatorname{dilog}\left(\frac{-a\,f^{x}+\sqrt{-ab}}{\sqrt{-ab}}\right)}{16\ln(f)^{2}ab\sqrt{-ab}}-\frac{\operatorname{dilog}\left(\frac{a\,f^{x}+\sqrt{-a}}{\sqrt{-ab}}\right)}{16\ln(f)^{2}ab\sqrt{-a}}$

input
```
int(x/(b/(f^x)+a*f^x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^x*((f^x)^2*ln(f)*a*x-ln(f)*b*x+(f^x)^2*a+b)/ln(f)^2/a/((f^x)^2*a+b)^
2/b+1/16/ln(f)/a/b*x/(-a*b)^(1/2)*ln((-a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-1
/16/ln(f)/a/b*x/(-a*b)^(1/2)*ln((a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))+1/16/ln
(f)^2/a/b/(-a*b)^(1/2)*dilog((-a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))-1/16/ln(f
)^2/a/b/(-a*b)^(1/2)*dilog((a*f^x+(-a*b)^(1/2))/(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(150) = 300.

Time = 0.08 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.80

$$\int \frac{x}{(bf^{-x}+af^{x})^{3}}\,dx$$
$$=\frac{2\left(a^{2}x\log(f)+a^{2}\right)f^{3x}-2\left(abx\log(f)-ab\right)f^{x}+\left(a^{2}f^{4x}\sqrt{-\frac{a}{b}}+2\,abf^{2x}\sqrt{-\frac{a}{b}}+b^{2}\sqrt{-\frac{a}{b}}\right)\operatorname{Li}_{2}\left(f^{x}\sqrt{-\frac{a}{b}}\right)}{}$$

input
```
integrate(x/(b/(f^x)+a*f^x)^3,x, algorithm="fricas")
```

output
```
1/16*(2*(a^2*x*log(f) + a^2)*f^(3*x) - 2*(a*b*x*log(f) - a*b)*f^x + (a^2*f
^(4*x)*sqrt(-a/b) + 2*a*b*f^(2*x)*sqrt(-a/b) + b^2*sqrt(-a/b))*dilog(f^x*s
qrt(-a/b)) - (a^2*f^(4*x)*sqrt(-a/b) + 2*a*b*f^(2*x)*sqrt(-a/b) + b^2*sqrt
(-a/b))*dilog(-f^x*sqrt(-a/b)) - (a^2*f^(4*x)*x*sqrt(-a/b)*log(f) + 2*a*b*
f^(2*x)*x*sqrt(-a/b)*log(f) + b^2*x*sqrt(-a/b)*log(f))*log(f^x*sqrt(-a/b)
+ 1) + (a^2*f^(4*x)*x*sqrt(-a/b)*log(f) + 2*a*b*f^(2*x)*x*sqrt(-a/b)*log(f
) + b^2*x*sqrt(-a/b)*log(f))*log(-f^x*sqrt(-a/b) + 1))/(a^4*b*f^(4*x)*log(
f)^2 + 2*a^3*b^2*f^(2*x)*log(f)^2 + a^2*b^3*log(f)^2)
```

Sympy [F]

$$\int \frac{x}{(bf^{-x}+af^x)^3}\,dx = \frac{f^{-x}(ax\log(f)+a)+f^{-3x}(-bx\log(f)+b)}{8a^3b\log(f)^2+16a^2b^2f^{-2x}\log(f)^2+8ab^3f^{-4x}\log(f)^2} + \frac{\int \frac{f^x x}{af^{2x}+b}\,dx}{8ab}$$

input
```
integrate(x/(b/(f**x)+a*f**x)**3,x)
```

output
```
((a*x*log(f) + a)/f**x + (-b*x*log(f) + b)/f**(3*x))/(8*a**3*b*log(f)**2 +
 16*a**2*b**2*log(f)**2/f**(2*x) + 8*a*b**3*log(f)**2/f**(4*x)) + Integral
(f**x*x/(a*f**(2*x) + b), x)/(8*a*b)
```

Maxima [F]

$$\int \frac{x}{(bf^{-x}+af^x)^3}\,dx = \int \frac{x}{\left(af^x+\frac{b}{f^x}\right)^3}\,dx$$

input
```
integrate(x/(b/(f^x)+a*f^x)^3,x, algorithm="maxima")
```

output
```
1/8*((a*x*log(f) + a)*f^(3*x) - (b*x*log(f) - b)*f^x)/(a^3*b*f^(4*x)*log(f
)^2 + 2*a^2*b^2*f^(2*x)*log(f)^2 + a*b^3*log(f)^2) + integrate(1/8*f^x*x/(
a^2*b*f^(2*x) + a*b^2), x)
```

Giac [F]

$$\int \frac{x}{(bf^{-x}+af^x)^3}\,dx = \int \frac{x}{\left(af^x+\frac{b}{f^x}\right)^3}\,dx$$

input
```
integrate(x/(b/(f^x)+a*f^x)^3,x, algorithm="giac")
```

output
```
integrate(x/(a*f^x + b/f^x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(bf^{-x}+af^x)^3}\,dx=\int \frac{x}{\left(\frac{b}{f^x}+a\,f^x\right)^3}\,dx$$

input
```
int(x/(b/f^x + a*f^x)^3,x)
```

output
```
int(x/(b/f^x + a*f^x)^3, x)
```

Reduce [F]

$$\int \frac{x}{(bf^{-x}+af^x)^3}\,dx$$
$$=\frac{f^{4x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)a^2+2f^{2x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)b^2+2f^{4x}\left(\int \frac{f^x}{f^{6x}a^3+3f^{4x}a^2b}\right.}{}$$

input
```
int(x/(b/(f^x)+a*f^x)^3,x)
```

output
```
(f**(4*x)*sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a)))*a**2 + 2*f**(2*
x)*sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*
atan((f**x*a)/(sqrt(b)*sqrt(a)))*b**2 + 2*f**(4*x)*int((f**x*x)/(f**(6*x)*
a**3 + 3*f**(4*x)*a**2*b + 3*f**(2*x)*a*b**2 + b**3),x)*log(f)**2*a**3*b**
3 + f**(3*x)*a**2*b + 4*f**(2*x)*int((f**x*x)/(f**(6*x)*a**3 + 3*f**(4*x)*
a**2*b + 3*f**(2*x)*a*b**2 + b**3),x)*log(f)**2*a**2*b**4 - 2*f**x*log(f)*
a*b**2*x + f**x*a*b**2 + 2*int((f**x*x)/(f**(6*x)*a**3 + 3*f**(4*x)*a**2*b
 + 3*f**(2*x)*a*b**2 + b**3),x)*log(f)**2*a*b**5)/(6*log(f)**2*a**2*b**2*(
f**(4*x)*a**2 + 2*f**(2*x)*a*b + b**2))
```

3.120 $\int \frac{x^2}{(bf^{-x}+af^x)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 316

$$\begin{aligned}\int \frac{x^2}{(bf^{-x}+af^x)^3}\,dx = & -\frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}\log^3(f)}+\frac{f^x x}{4ab\left(b+af^{2x}\right)\log^2(f)} \\ & -\frac{f^x x^2}{4a\left(b+af^{2x}\right)^2\log(f)}+\frac{f^x x^2}{8ab\left(b+af^{2x}\right)\log(f)} \\ & +\frac{x^2\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)}-\frac{ix\operatorname{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)} \\ & +\frac{ix\operatorname{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)} \\ & +\frac{i\operatorname{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)}-\frac{i\operatorname{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)}\end{aligned}$$

output
```
-1/4*arctan(a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^3+1/4*f^x*x/a/b/(b+
a*f^(2*x))/ln(f)^2-1/4*f^x*x^2/a/(b+a*f^(2*x))^2/ln(f)+1/8*f^x*x^2/a/b/(b+
a*f^(2*x))/ln(f)+1/8*x^2*arctan(a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)
-1/8*I*x*polylog(2,-I*a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^2+1/8*I*x
*polylog(2,I*a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^2+1/8*I*polylog(3,
-I*a^(1/2)*f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^3-1/8*I*polylog(3,I*a^(1/2)*
f^x/b^(1/2))/a^(3/2)/b^(3/2)/ln(f)^3
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\left(bf^{-x}+af^x\right)^3}\,dx$$
$$= \frac{-\frac{12\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{b^{3/2}} - \frac{12\sqrt{a}f^x x^2\log^2(f)}{(b+af^{2x})^2} + \frac{6\sqrt{a}f^x x\log(f)(2+x\log(f))}{b(b+af^{2x})} + \frac{3i\left(x^2\log^2(f)\log\left(1-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-x^2\log^2(f)\log\left(1+\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)-2x}{}}{48a^{3/2}\log^3(f)}$$

input
```
Integrate[x^2/(b/f^x + a*f^x)^3,x]
```

output
```
((-12*ArcTan[(Sqrt[a]*f^x)/Sqrt[b]])/b^(3/2) - (12*Sqrt[a]*f^x*x^2*Log[f]^
2)/(b + a*f^(2*x))^2 + (6*Sqrt[a]*f^x*x*Log[f]*(2 + x*Log[f]))/(b*(b + a*f
^(2*x))) + ((3*I)*(x^2*Log[f]^2*Log[1 - (I*Sqrt[a]*f^x)/Sqrt[b]] - x^2*Log
[f]^2*Log[1 + (I*Sqrt[a]*f^x)/Sqrt[b]] - 2*x*Log[f]*PolyLog[2, ((-I)*Sqrt[
a]*f^x)/Sqrt[b]] + 2*x*Log[f]*PolyLog[2, (I*Sqrt[a]*f^x)/Sqrt[b]] + 2*Poly
Log[3, ((-I)*Sqrt[a]*f^x)/Sqrt[b]] - 2*PolyLog[3, (I*Sqrt[a]*f^x)/Sqrt[b]]
))/b^(3/2))/(48*a^(3/2)*Log[f]^3)
```

Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2721, 2684, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(af^x+bf^{-x}\right)^3}\,dx$$
$$\Big\downarrow \; 2721$$
$$\int \frac{x^2 f^{3x}}{\left(af^{2x}+b\right)^3}dx$$
$$\Big\downarrow \; 2684$$

$$\int \left(\frac{x^2 f^x}{a\left(af^{2x}+b\right)^2}-\frac{bx^2 f^x}{a\left(af^{2x}+b\right)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^2 \arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log(f)}-\frac{\arctan\left(\frac{\sqrt{a}f^x}{\sqrt{b}}\right)}{4a^{3/2}b^{3/2}\log^3(f)}+\frac{i\,\text{PolyLog}\left(3,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)}-\frac{i\,\text{PolyLog}\left(3,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^3(f)}-\frac{ix\,\text{PolyLog}\left(2,-\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)}+\frac{ix\,\text{PolyLog}\left(2,\frac{i\sqrt{a}f^x}{\sqrt{b}}\right)}{8a^{3/2}b^{3/2}\log^2(f)}+\frac{x^2 f^x}{8ab\log(f)\left(af^{2x}+b\right)}-\frac{x^2 f^x}{4a\log(f)\left(af^{2x}+b\right)^2}+\frac{xf^x}{4ab\log^2(f)\left(af^{2x}+b\right)}$$

input
```
Int[x^2/(b/f^x + a*f^x)^3,x]
```

output
```
-1/4*ArcTan[(Sqrt[a]*f^x)/Sqrt[b]]/(a^(3/2)*b^(3/2)*Log[f]^3) + (f^x*x)/(4
*a*b*(b + a*f^(2*x))*Log[f]^2) - (f^x*x^2)/(4*a*(b + a*f^(2*x))^2*Log[f])
+ (f^x*x^2)/(8*a*b*(b + a*f^(2*x))*Log[f]) + (x^2*ArcTan[(Sqrt[a]*f^x)/Sqr
t[b]])/(8*a^(3/2)*b^(3/2)*Log[f]) - ((I/8)*x*PolyLog[2, ((-I)*Sqrt[a]*f^x)
/Sqrt[b]])/(a^(3/2)*b^(3/2)*Log[f]^2) + ((I/8)*x*PolyLog[2, (I*Sqrt[a]*f^x
)/Sqrt[b]])/(a^(3/2)*b^(3/2)*Log[f]^2) + ((I/8)*PolyLog[3, ((-I)*Sqrt[a]*f
^x)/Sqrt[b]])/(a^(3/2)*b^(3/2)*Log[f]^3) - ((I/8)*PolyLog[3, (I*Sqrt[a]*f^
x)/Sqrt[b]])/(a^(3/2)*b^(3/2)*Log[f]^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2684
```
Int[((a_.) + (b_.)*(F_)^(u_))^(p_.)*((c_.) + (d_.)*(F_)^(v_))^(q_.)*((e_.)
+ (f_.)*(x_))^(m_.), x_Symbol] :> With[{w = ExpandIntegrand[(e + f*x)^m, (a
 + b*F^u)^p*(c + d*F^v)^q, x]}, Int[w, x] /; SumQ[w]] /; FreeQ[{F, a, b, c,
 d, e, f, m}, x] && IntegersQ[p, q] && LinearQ[{u, v}, x] && RationalQ[Simp
lify[u/v]]
```

rule 2721
```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,f^{-x}+a\,f^{x}\right)^3}dx$$

input
```
int(x^2/(b/(f^x)+a*f^x)^3,x)
```

output
```
int(x^2/(b/(f^x)+a*f^x)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(232) = 464$.

Time = 0.09 (sec) , antiderivative size = 674, normalized size of antiderivative = 2.13

$$\int \frac{x^2}{\left(bf^{-x}+af^{x}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(b/(f^x)+a*f^x)^3,x, algorithm="fricas")
```

output
```
1/16*(2*(a^2*x^2*log(f)^2 + 2*a^2*x*log(f))*f^(3*x) - 2*(a*b*x^2*log(f)^2
- 2*a*b*x*log(f))*f^x + 2*(a^2*f^(4*x)*x*sqrt(-a/b)*log(f) + 2*a*b*f^(2*x)
*x*sqrt(-a/b)*log(f) + b^2*x*sqrt(-a/b)*log(f))*dilog(f^x*sqrt(-a/b)) - 2*
(a^2*f^(4*x)*x*sqrt(-a/b)*log(f) + 2*a*b*f^(2*x)*x*sqrt(-a/b)*log(f) + b^2
*x*sqrt(-a/b)*log(f))*dilog(-f^x*sqrt(-a/b)) - 2*(a^2*f^(4*x)*sqrt(-a/b) +
 2*a*b*f^(2*x)*sqrt(-a/b) + b^2*sqrt(-a/b))*log(2*a*f^x + 2*b*sqrt(-a/b))
+ 2*(a^2*f^(4*x)*sqrt(-a/b) + 2*a*b*f^(2*x)*sqrt(-a/b) + b^2*sqrt(-a/b))*l
og(2*a*f^x - 2*b*sqrt(-a/b)) - (a^2*f^(4*x)*x^2*sqrt(-a/b)*log(f)^2 + 2*a*
b*f^(2*x)*x^2*sqrt(-a/b)*log(f)^2 + b^2*x^2*sqrt(-a/b)*log(f)^2)*log(f^x*s
qrt(-a/b) + 1) + (a^2*f^(4*x)*x^2*sqrt(-a/b)*log(f)^2 + 2*a*b*f^(2*x)*x^2*
sqrt(-a/b)*log(f)^2 + b^2*x^2*sqrt(-a/b)*log(f)^2)*log(-f^x*sqrt(-a/b) + 1
) - 2*(a^2*f^(4*x)*sqrt(-a/b) + 2*a*b*f^(2*x)*sqrt(-a/b) + b^2*sqrt(-a/b))
*polylog(3, f^x*sqrt(-a/b)) + 2*(a^2*f^(4*x)*sqrt(-a/b) + 2*a*b*f^(2*x)*sq
rt(-a/b) + b^2*sqrt(-a/b))*polylog(3, -f^x*sqrt(-a/b)))/(a^4*b*f^(4*x)*log
(f)^3 + 2*a^3*b^2*f^(2*x)*log(f)^3 + a^2*b^3*log(f)^3)
```

Sympy [F]

$$\int \frac{x^2}{(bf^{-x} + af^x)^3}\,dx = \frac{f^{-x}(ax^2\log(f) + 2ax) + f^{-3x}(-bx^2\log(f) + 2bx)}{8a^3b\log(f)^2 + 16a^2b^2f^{-2x}\log(f)^2 + 8ab^3f^{-4x}\log(f)^2} + \frac{\int\left(-\frac{2f^x}{af^{2x}+b}\right)dx + \int\frac{f^xx^2\log(f)^2}{af^{2x}+b}\,dx}{8ab\log(f)^2}$$

input
```
integrate(x**2/(b/(f**x)+a*f**x)**3,x)
```

output
```
((a*x**2*log(f) + 2*a*x)/f**x + (-b*x**2*log(f) + 2*b*x)/f**(3*x))/(8*a**3
*b*log(f)**2 + 16*a**2*b**2*log(f)**2/f**(2*x) + 8*a*b**3*log(f)**2/f**(4*
x)) + (Integral(-2*f**x/(a*f**(2*x) + b), x) + Integral(f**x*x**2*log(f)**
2/(a*f**(2*x) + b), x))/(8*a*b*log(f)**2)
```

Maxima [F]

$$\int \frac{x^2}{(bf^{-x}+af^x)^3}\,dx = \int \frac{x^2}{\left(af^x+\frac{b}{f^x}\right)^3}\,dx$$

input
```
integrate(x^2/(b/(f^x)+a*f^x)^3,x, algorithm="maxima")
```

output
```
1/8*((a*x^2*log(f) + 2*a*x)*f^(3*x) - (b*x^2*log(f) - 2*b*x)*f^x)/(a^3*b*f
^(4*x)*log(f)^2 + 2*a^2*b^2*f^(2*x)*log(f)^2 + a*b^3*log(f)^2) + integrate
(1/8*(x^2*log(f)^2 - 2)*f^x/(a^2*b*f^(2*x)*log(f)^2 + a*b^2*log(f)^2), x)
```

Giac [F]

$$\int \frac{x^2}{(bf^{-x}+af^x)^3}\,dx = \int \frac{x^2}{\left(af^x+\frac{b}{f^x}\right)^3}\,dx$$

input
```
integrate(x^2/(b/(f^x)+a*f^x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/(a*f^x + b/f^x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(bf^{-x}+af^x)^3}\,dx = \int \frac{x^2}{\left(\frac{b}{f^x}+a\,f^x\right)^3}\,dx$$

input
```
int(x^2/(b/f^x + a*f^x)^3,x)
```

output
```
int(x^2/(b/f^x + a*f^x)^3, x)
```

Reduce [F]

$$\int \frac{x^2}{(bf^{-x}+af^x)^3}\,dx$$
$$= \frac{f^{4x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)a^2 + 2f^{2x}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)ab + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{f^x a}{\sqrt{b}\,\sqrt{a}}\right)b^2 + 3f^{4x}\left(\int \frac{f^x x}{f^{6x}a^3+3f^{4x}a^2b}\right.}{}$$

input
```
int(x^2/(b/(f^x)+a*f^x)^3,x)
```

output
```
(f**(4*x)*sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a)))*a**2 + 2*f**(2*
x)*sqrt(b)*sqrt(a)*atan((f**x*a)/(sqrt(b)*sqrt(a)))*a*b + sqrt(b)*sqrt(a)*
atan((f**x*a)/(sqrt(b)*sqrt(a)))*b**2 + 3*f**(4*x)*int((f**x*x**2)/(f**(6*
x)*a**3 + 3*f**(4*x)*a**2*b + 3*f**(2*x)*a*b**2 + b**3),x)*log(f)**3*a**3*
b**3 + 8*f**(4*x)*int((f**x*x)/(f**(6*x)*a**3 + 3*f**(4*x)*a**2*b + 3*f**(
2*x)*a*b**2 + b**3),x)*log(f)**2*a**3*b**3 + f**(3*x)*a**2*b + 6*f**(2*x)*
int((f**x*x**2)/(f**(6*x)*a**3 + 3*f**(4*x)*a**2*b + 3*f**(2*x)*a*b**2 + b
**3),x)*log(f)**3*a**2*b**4 + 16*f**(2*x)*int((f**x*x)/(f**(6*x)*a**3 + 3*
f**(4*x)*a**2*b + 3*f**(2*x)*a*b**2 + b**3),x)*log(f)**2*a**2*b**4 - 3*f**
x*log(f)**2*a*b**2*x**2 - 2*f**x*log(f)*a*b**2*x + f**x*a*b**2 + 3*int((f*
*x*x**2)/(f**(6*x)*a**3 + 3*f**(4*x)*a**2*b + 3*f**(2*x)*a*b**2 + b**3),x)
*log(f)**3*a*b**5 + 8*int((f**x*x)/(f**(6*x)*a**3 + 3*f**(4*x)*a**2*b + 3*
f**(2*x)*a*b**2 + b**3),x)*log(f)**2*a*b**5)/(9*log(f)**3*a**2*b**2*(f**(4
*x)*a**2 + 2*f**(2*x)*a*b + b**2))
```

3.121 $\int f^{a+bx+cx^2} g^{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 95

$$\int f^{a+bx+cx^2} g^{d+ex+fx^2}\,dx=\frac{e^{-\frac{(b\log(f)+e\log(g))^2}{4(c\log(f)+f\log(g))}}f^a g^d\sqrt{\pi}\,\text{erfi}\left(\frac{b\log(f)+e\log(g)+2x(c\log(f)+f\log(g))}{2\sqrt{c\log(f)+f\log(g)}}\right)}{2\sqrt{c\log(f)+f\log(g)}}$$

output
```
1/2*f^a*g^d*Pi^(1/2)*erfi(1/2*(b*ln(f)+e*ln(g)+2*x*(c*ln(f)+f*ln(g)))/(c*l
n(f)+f*ln(g))^(1/2))/exp((b*ln(f)+e*ln(g))^2/(4*c*ln(f)+4*f*ln(g)))/(c*ln(
f)+f*ln(g))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int f^{a+bx+cx^2} g^{d+ex+fx^2}\,dx=\frac{e^{-\frac{(b\log(f)+e\log(g))^2}{4(c\log(f)+f\log(g))}}f^a g^d\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\log(f)+(e+2fx)\log(g)}{2\sqrt{c\log(f)+f\log(g)}}\right)}{2\sqrt{c\log(f)+f\log(g)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*g^(d + e*x + f*x^2),x]
```

output
```
(f^a*g^d*Sqrt[Pi]*Erfi[((b + 2*c*x)*Log[f] + (e + 2*f*x)*Log[g])/(2*Sqrt[c
*Log[f] + f*Log[g]])])/(2*E^((b*Log[f] + e*Log[g])^2/(4*(c*Log[f] + f*Log[
g])))*Sqrt[c*Log[f] + f*Log[g]])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2725, 2664, 2633}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2} g^{d+ex+fx^2} \, dx$$

$$\Big\downarrow \text{ 2725}$$

$$\int \exp\left(a\log(f) + x(b\log(f) + e\log(g)) + x^2(c\log(f) + f\log(g)) + d\log(g)\right) dx$$

$$\Big\downarrow \text{ 2664}$$

$$f^a g^d \exp\left(-\frac{(b\log(f) + e\log(g))^2}{4(c\log(f) + f\log(g))}\right) \int \exp\left(\frac{(b\log(f) + e\log(g) + 2x(c\log(f) + f\log(g)))^2}{4(c\log(f) + f\log(g))}\right) dx$$

$$\Big\downarrow \text{ 2633}$$

$$\frac{\sqrt{\pi} f^a g^d \exp\left(-\frac{(b\log(f)+e\log(g))^2}{4(c\log(f)+f\log(g))}\right) \text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f\log(g))+e\log(g)}{2\sqrt{c\log(f)+f\log(g)}}\right)}{2\sqrt{c\log(f) + f\log(g)}}$$

input
```
Int[f^(a + b*x + c*x^2)*g^(d + e*x + f*x^2),x]
```

output
```
(f^a*g^d*Sqrt[Pi]*Erfi[(b*Log[f] + e*Log[g] + 2*x*(c*Log[f] + f*Log[g]))/(
2*Sqrt[c*Log[f] + f*Log[g]])])/(2*E^((b*Log[f] + e*Log[g])^2/(4*(c*Log[f]
+ f*Log[g])))*Sqrt[c*Log[f] + f*Log[g]])
```

Defintions of rubi rules used

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2664
```
Int[(F_)^((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[F^(a - b^2/
(4*c))   Int[F^((b + 2*c*x)^2/(4*c)), x], x] /; FreeQ[{F, a, b, c}, x]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Maple [F]

$$\int f^{c\,x^{2}+b x +a} g^{f\,x^{2}+e x +d}dx$$

input
```
int(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d),x)
```

output
```
int(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.42

$$\int f^{a+bx+cx^2} g^{d+ex+fx^2}\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c\log\left(f\right)-f\log\left(g\right)}\,\mathrm{erf}\left(\frac{\left(\left(2\,cx+b\right)\log(f)+\left(2\,fx+e\right)\log(g)\right)\sqrt{-c\log(f)-f\log(g)}}{2\left(c\log(f)+f\log(g)\right)}\right)e^{\left(-\frac{\left(b^2-4\,ac\right)\log(f)^2-2\left(2\,cd-be+2\,a\right.}{4\left(c\log(f\right.}\right.}}{2\left(c\log\left(f\right)+f\log\left(g\right)\right)}$$

input
```
integrate(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/2*sqrt(pi)*sqrt(-c*log(f) - f*log(g))*erf(1/2*((2*c*x + b)*log(f) + (2*
f*x + e)*log(g))*sqrt(-c*log(f) - f*log(g))/(c*log(f) + f*log(g)))*e^(-1/4
*((b^2 - 4*a*c)*log(f)^2 - 2*(2*c*d - b*e + 2*a*f)*log(f)*log(g) + (e^2 -
4*d*f)*log(g)^2)/(c*log(f) + f*log(g)))/(c*log(f) + f*log(g))
```

Sympy [F]

$$\int f^{a+bx+cx^2}g^{d+ex+fx^2}\,dx=\int f^{a+bx+cx^2}g^{d+ex+fx^2}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*g**(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*g**(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\int f^{a+bx+cx^2}g^{d+ex+fx^2}\,dx = \frac{\sqrt{\pi}f^a g^d \operatorname{erf}\left(\sqrt{-c\log(f)-f\log(g)}x-\frac{b\log(f)+e\log(g)}{2\sqrt{-c\log(f)-f\log(g)}}\right)e^{\left(-\frac{(b\log(f)+e\log(g))^2}{4(c\log(f)+f\log(g))}\right)}}{2\sqrt{-c\log(f)-f\log(g)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/2*sqrt(pi)*f^a*g^d*erf(sqrt(-c*log(f) - f*log(g))*x - 1/2*(b*log(f) + e*
log(g))/sqrt(-c*log(f) - f*log(g)))*e^(-1/4*(b*log(f) + e*log(g))^2/(c*log
(f) + f*log(g)))/sqrt(-c*log(f) - f*log(g))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int f^{a+bx+cx^2}g^{d+ex+fx^2}\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f\log(g)}\left(2x+\frac{b\log(f)+e\log(g)}{c\log(f)+f\log(g)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)\log(g)+2be\log(f)\log(g}{4(c\log(f)+f\log(g}}}{2\sqrt{-c\log(f)-f\log(g)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/2*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f*log(g))*(2*x + (b*log(f) + e*log
(g))/(c*log(f) + f*log(g))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*
d*log(f)*log(g) + 2*b*e*log(f)*log(g) - 4*a*f*log(f)*log(g) + e^2*log(g)^2
 - 4*d*f*log(g)^2)/(c*log(f) + f*log(g)))/sqrt(-c*log(f) - f*log(g))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int f^{a+bx+cx^2}g^{d+ex+fx^2}\,dx =$$

$$-\frac{f^a\,g^d\,\sqrt{\pi}\,\mathrm{e}^{-\frac{b^2\ln(f)^2}{4\,(c\,\ln(f)+f\,\ln(g))}-\frac{e^2\ln(g)^2}{4\,(c\,\ln(f)+f\,\ln(g))}-\frac{b\,e\,\ln(f)\,\ln(g)}{2\,(c\,\ln(f)+f\,\ln(g))}}\,\mathrm{erf}\left(\frac{x\,(c\,\ln(f)+f\,\ln(g))\,2\mathrm{i}+b\,\ln(f)\,1\mathrm{i}+e\,\ln(g)\,1\mathrm{i}}{2\,\sqrt{c\,\ln(f)+f\,\ln(g)}}\right)\,1\mathrm{i}}{2\,\sqrt{c\,\ln(f)+f\,\ln(g)}}$$

input
```
int(f^(a + b*x + c*x^2)*g^(d + e*x + f*x^2),x)
```

output
```
-(f^a*g^d*pi^(1/2)*exp(- (b^2*log(f)^2)/(4*(c*log(f) + f*log(g))) - (e^2*l
og(g)^2)/(4*(c*log(f) + f*log(g))) - (b*e*log(f)*log(g))/(2*(c*log(f) + f*
log(g))))*erf((x*(c*log(f) + f*log(g))*2i + b*log(f)*1i + e*log(g)*1i)/(2*
(c*log(f) + f*log(g))^(1/2)))*1i)/(2*(c*log(f) + f*log(g))^(1/2))
```

Reduce [F]

$$\int f^{a+bx+cx^2}g^{d+ex+fx^2}\,dx = g^d f^a\left(\int g^{f\,x^2+ex}f^{c\,x^2+bx}dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*g^(f*x^2+e*x+d),x)
```

output
```
g**d*f**a*int(g**(e*x + f*x**2)*f**(b*x + c*x**2),x)
```

3.122 $\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx$

Optimal result

Integrand size = 25, antiderivative size = 106

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx = \frac{F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\left(1+\frac{bG^{h(f+gx)}}{a}\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{de\log(F)}{gh\log(G)},1+\frac{de\log(F)}{gh\log(G)},-\frac{bG^{h(f+gx)}}{a}\right)}{de\log(F)}$$

output
```
F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n*hypergeom([-n, d*e*ln(F)/g/h/ln(G)],[1
+d*e*ln(F)/g/h/ln(G)],-b*G^(h*(g*x+f))/a)/d/e/((1+b*G^(h*(g*x+f))/a)^n)/ln
(F)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx = \frac{F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^{1+n}\text{Hypergeometric2F1}\left(1,1+n+\frac{de\log(F)}{gh\log(G)},1+\frac{de\log(F)}{gh\log(G)},-\frac{bG^{h(f+gx)}}{a}\right)}{ade\log(F)}$$

input
```
Integrate[F^(e*(c + d*x))*(a + b*G^(h*(f + g*x)))^n,x]
```

output
```
(F^(e*(c + d*x))*(a + b*G^(h*(f + g*x)))^(1 + n)*Hypergeometric2F1[1, 1 +
n + (d*e*Log[F])/(g*h*Log[G]), 1 + (d*e*Log[F])/(g*h*Log[G]), -((b*G^(h*(f
 + g*x)))/a)])/(a*d*e*Log[F])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2682, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx$$

$$\Big\downarrow \text{2682}$$

$$\left(a+bG^{h(f+gx)}\right)^n\left(\frac{bG^{h(f+gx)}}{a}+1\right)^{-n}\int F^{e(c+dx)}\left(\frac{bG^{h(f+gx)}}{a}+1\right)^n dx$$

$$\Big\downarrow \text{2681}$$

$$\frac{F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\left(\frac{bG^{h(f+gx)}}{a}+1\right)^{-n}\text{Hypergeometric2F1}\left(-n,\frac{de\log(F)}{gh\log(G)},\frac{de\log(F)}{gh\log(G)}+1,-\frac{bG^{h(f+gx)}}{a}\right)}{de\log(F)}$$

input
```
Int[F^(e*(c + d*x))*(a + b*G^(h*(f + g*x)))^n,x]
```

output
```
(F^(e*(c + d*x))*(a + b*G^(h*(f + g*x)))^n*Hypergeometric2F1[-n, (d*e*Log[
F])/(g*h*Log[G]), 1 + (d*e*Log[F])/(g*h*Log[G]), -((b*G^(h*(f + g*x)))/a)]
)/(d*e*(1 + (b*G^(h*(f + g*x)))/a)^n*Log[F])
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2682
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a + b*F^(e*(c + d*x)))^p/(1 + (b/a)*F^
(e*(c + d*x)))^p   Int[G^(h*(f + g*x))*(1 + (b/a)*F^(e*(c + d*x)))^p, x], x
] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x] &&  !(ILtQ[p, 0] || GtQ[a,
 0])
```

Maple [F]

$$\int F^{e(dx+c)} \left(a+b\, G^{h(gx+f)}\right)^{n} dx$$

input
```
int(F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n,x)
```

output
```
int(F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n,x)
```

Fricas [F]

$$\int F^{e(c+dx)} \left(a+bG^{h(f+gx)}\right)^{n} \, dx = \int \left(G^{(gx+f)h}b+a\right)^{n} F^{(dx+c)e} \, dx$$

input
```
integrate(F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n,x, algorithm="fricas")
```

output
```
integral((G^(g*h*x + f*h)*b + a)^n*F^(d*e*x + c*e), x)
```

Sympy [F(-1)]

Timed out.

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx=\text{Timed out}$$

input
```
integrate(F**(e*(d*x+c))*(a+b*G**(h*(g*x+f)))**n,x)
```

output
```
Timed out
```

Maxima [F]

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx=\int \left(G^{(gx+f)h}b+a\right)^nF^{(dx+c)e}\,dx$$

input
```
integrate(F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n,x, algorithm="maxima")
```

output
```
integrate((G^((g*x + f)*h)*b + a)^n*F^((d*x + c)*e), x)
```

Giac [F]

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx=\int \left(G^{(gx+f)h}b+a\right)^nF^{(dx+c)e}\,dx$$

input
```
integrate(F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n,x, algorithm="giac")
```

output
```
integrate((G^((g*x + f)*h)*b + a)^n*F^((d*x + c)*e), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx=\int F^{e\,(c+d\,x)}\,\left(a+G^{h\,(f+g\,x)}\,b\right)^n\,dx$$

input
```
int(F^(e*(c + d*x))*(a + G^(h*(f + g*x))*b)^n,x)
```

output
```
int(F^(e*(c + d*x))*(a + G^(h*(f + g*x))*b)^n, x)
```

Reduce [F]

$$\int F^{e(c+dx)}\left(a+bG^{h(f+gx)}\right)^n\,dx=f^{ce}\left(\int f^{dex}\left(g^{ghx+fh}b+a\right)^n dx\right)$$

input
```
int(F^(e*(d*x+c))*(a+b*G^(h*(g*x+f)))^n,x)
```

output
```
f**(c*e)*int(f**(d*e*x)*(g**(f*h + g*h*x)*b + a)**n,x)
```

3.123 $\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 75

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \frac{H^{t(r+sx)}\,\text{Hypergeometric2F1}\left(1,-\frac{st\log(H)}{de\log(F)},1-\frac{st\log(H)}{de\log(F)},-\frac{aF^{-e(c+dx)}}{b}\right)}{bst\log(H)}$$

output
```
H^(t*(s*x+r))*hypergeom([1, -s*t*ln(H)/d/e/ln(F)],[1-s*t*ln(H)/d/e/ln(F)],
-a/b/(F^(e*(d*x+c))))/b/s/t/ln(H)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = -\frac{H^{t(r+sx)}\left(-1+\text{Hypergeometric2F1}\left(1,\frac{st\log(H)}{de\log(F)},1+\frac{st\log(H)}{de\log(F)},-\frac{bF^{e(c+dx)}}{a}\right)\right)}{bst\log(H)}$$

input
```
Integrate[(F^(e*(c + d*x))*H^(t*(r + s*x)))/(a + b*F^(e*(c + d*x))),x]
```

output
```
-((H^(t*(r + s*x))*(-1 + Hypergeometric2F1[1, (s*t*Log[H])/(d*e*Log[F]), 1
 + (s*t*Log[H])/(d*e*Log[F]), -((b*F^(e*(c + d*x)))/a)]))/(b*s*t*Log[H]))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2686, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}dx$$

$$\Big\downarrow \text{2686}$$

$$\int \frac{H^{t(r+sx)}}{aF^{-e(c+dx)}+b}dx$$

$$\Big\downarrow \text{2681}$$

$$\frac{H^{t(r+sx)}\,\text{Hypergeometric2F1}\left(1,-\frac{st\log(H)}{de\log(F)},1-\frac{st\log(H)}{de\log(F)},-\frac{aF^{-e(c+dx)}}{b}\right)}{bst\log(H)}$$

input
```
Int[(F^(e*(c + d*x))*H^(t*(r + s*x)))/(a + b*F^(e*(c + d*x))),x]
```

output
```
(H^(t*(r + s*x))*Hypergeometric2F1[1, -((s*t*Log[H])/(d*e*Log[F])), 1 - (s
*t*Log[H])/(d*e*Log[F]), -(a/(b*F^(e*(c + d*x))))])/(b*s*t*Log[H])
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2686
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^((
f - c*(g/d))*h)   Int[H^(t*(r + s*x))*(b + a/F^(e*(c + d*x)))^p, x], x] /;
FreeQ[{F, G, H, a, b, c, d, e, f, g, h, r, s, t}, x] && EqQ[d*e*p*Log[F] +
g*h*Log[G], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{F^{e(dx+c)} H^{t(sx+r)}}{a+b\,F^{e(dx+c)}}dx$$

input
```
int(F^(e*(d*x+c))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x)
```

output
```
int(F^(e*(d*x+c))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x)
```

Fricas [F]

$$\int \frac{F^{e(c+dx)} H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{(dx+c)e} H^{(sx+r)t}}{F^{(dx+c)e}b+a}\,dx$$

input
```
integrate(F^(e*(d*x+c))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x, algorithm="fr
icas")
```

output
```
integral(F^(d*e*x + c*e)*H^(s*t*x + r*t)/(F^(d*e*x + c*e)*b + a), x)
```

Sympy [F]

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{e(c+dx)}H^{t(r+sx)}}{F^{ce+dex}b+a}\,dx$$

input
```
integrate(F**(e*(d*x+c))*H**(t*(s*x+r))/(a+b*F**(e*(d*x+c))),x)
```

output
```
Integral(F**(e*(c + d*x))*H**(t*(r + s*x))/(F**(c*e + d*e*x)*b + a), x)
```

Maxima [F]

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{(dx+c)e}H^{(sx+r)t}}{F^{(dx+c)e}b+a}\,dx$$

input
```
integrate(F^(e*(d*x+c))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x, algorithm="ma
xima")
```

output
```
-H^(r*t)*a^2*d*e*integrate(H^(s*t*x)/(a^2*b*d*e*log(F) - a^2*b*s*t*log(H)
+ (F^(2*c*e)*b^3*d*e*log(F) - F^(2*c*e)*b^3*s*t*log(H))*F^(2*d*e*x) + 2*(F
^(c*e)*a*b^2*d*e*log(F) - F^(c*e)*a*b^2*s*t*log(H))*F^(d*e*x)), x)*log(F)
+ (H^(r*t)*a*d*e*log(F) + (F^(c*e)*H^(r*t)*b*d*e*log(F) - F^(c*e)*H^(r*t)*
b*s*t*log(H))*F^(d*e*x))*H^(s*t*x)/(a*b*d*e*s*t*log(F)*log(H) - a*b*s^2*t^
2*log(H)^2 + (F^(c*e)*b^2*d*e*s*t*log(F)*log(H) - F^(c*e)*b^2*s^2*t^2*log(
H)^2)*F^(d*e*x))
```

Giac [F]

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{(dx+c)e}H^{(sx+r)t}}{F^{(dx+c)e}b+a}\,dx$$

input
```
integrate(F^(e*(d*x+c))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x, algorithm="gi
ac")
```

output
```
integrate(F^((d*x + c)*e)*H^((s*x + r)*t)/(F^((d*x + c)*e)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{e\,(c+d\,x)}\,H^{t\,(r+s\,x)}}{a+F^{e\,(c+d\,x)}\,b}\,dx$$

input
```
int((F^(e*(c + d*x))*H^(t*(r + s*x)))/(a + F^(e*(c + d*x))*b),x)
```

output
```
int((F^(e*(c + d*x))*H^(t*(r + s*x)))/(a + F^(e*(c + d*x))*b), x)
```

Reduce [F]

$$\int \frac{F^{e(c+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \frac{h^{rt}\left(h^{stx}-\left(\int \frac{h^{stx}}{f^{dex+ce}b+a}dx\right)\log(h)\,ast\right)}{\log\left(h\right)bst}$$

input
```
int(F^(e*(d*x+c))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x)
```

output
```
(h**(r*t)*(h**(s*t*x) - int(h**(s*t*x)/(f**(c*e + d*e*x)*b + a),x)*log(h)*
a*s*t))/(log(h)*b*s*t)
```

3.124 $\int \frac{F^{e(f+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 85

$$\int \frac{F^{e(f+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \frac{F^{-e(c-f)}H^{t(r+sx)}\,\text{Hypergeometric2F1}\left(1,-\frac{st\log(H)}{de\log(F)},1-\frac{st\log(H)}{de\log(F)},-\frac{aF^{-e(c+dx)}}{b}\right)}{bst\log(H)}$$

output
```
H^(t*(s*x+r))*hypergeom([1, -s*t*ln(H)/d/e/ln(F)],[1-s*t*ln(H)/d/e/ln(F)],
-a/b/(F^(e*(d*x+c))))/b/(F^(e*(c-f)))/s/t/ln(H)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{F^{e(f+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = -\frac{F^{e(-c+f)}H^{t(r+sx)}\left(-1+\text{Hypergeometric2F1}\left(1,\frac{st\log(H)}{de\log(F)},1+\frac{st\log(H)}{de\log(F)},-\frac{bF^{e(c+dx)}}{a}\right)\right)}{bst\log(H)}$$

input
```
Integrate[(F^(e*(f + d*x))*H^(t*(r + s*x)))/(a + b*F^(e*(c + d*x))),x]
```

output
```
-((F^(e*(-c + f))*H^(t*(r + s*x))*(-1 + Hypergeometric2F1[1, (s*t*Log[H])/
(d*e*Log[F]), 1 + (s*t*Log[H])/(d*e*Log[F]), -((b*F^(e*(c + d*x)))/a)]))/(
b*s*t*Log[H]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2686, 2681}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{e(dx+f)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}dx$$

$$\Big\downarrow \text{2686}$$

$$F^{-e(c-f)}\int \frac{H^{t(r+sx)}}{aF^{-e(c+dx)}+b}dx$$

$$\Big\downarrow \text{2681}$$

$$\frac{F^{-e(c-f)}H^{t(r+sx)}\,\text{Hypergeometric2F1}\left(1,-\frac{st\log(H)}{de\log(F)},1-\frac{st\log(H)}{de\log(F)},-\frac{aF^{-e(c+dx)}}{b}\right)}{bst\log(H)}$$

input
```
Int[(F^(e*(f + d*x))*H^(t*(r + s*x)))/(a + b*F^(e*(c + d*x))),x]
```

output
```
(H^(t*(r + s*x))*Hypergeometric2F1[1, -((s*t*Log[H])/(d*e*Log[F])), 1 - (s
*t*Log[H])/(d*e*Log[F]), -(a/(b*F^(e*(c + d*x))))])/(b*F^(e*(c - f))*s*t*L
og[H])
```

Defintions of rubi rules used

rule 2681
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[a^p*(G^(h*(f + g*x))/(g*h*Log[G]))*Hype
rgeometric2F1[-p, g*h*(Log[G]/(d*e*Log[F])), g*h*(Log[G]/(d*e*Log[F])) + 1,
 Simplify[(-b/a)*F^(e*(c + d*x))]], x] /; FreeQ[{F, G, a, b, c, d, e, f, g,
 h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

rule 2686
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_)))*(H_)^((t_.)*((r_.) + (s_.)*(x_))), x_Symbol] :> Simp[G^((
f - c*(g/d))*h)   Int[H^(t*(r + s*x))*(b + a/F^(e*(c + d*x)))^p, x], x] /;
FreeQ[{F, G, H, a, b, c, d, e, f, g, h, r, s, t}, x] && EqQ[d*e*p*Log[F] +
g*h*Log[G], 0] && IntegerQ[p]
```

Maple [F]

$$\int \frac{F^{e(dx+f)} H^{t(sx+r)}}{a+b\,F^{e(dx+c)}}dx$$

input
```
int(F^(e*(d*x+f))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x)
```

output
```
int(F^(e*(d*x+f))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x)
```

Fricas [F]

$$\int \frac{F^{e(f+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{(dx+f)e}H^{(sx+r)t}}{F^{(dx+c)e}b+a}\,dx$$

input
```
integrate(F^(e*(d*x+f))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x, algorithm="fr
icas")
```

output
```
integral(F^(d*e*x + e*f)*H^(s*t*x + r*t)/(F^(d*e*x + c*e)*b + a), x)
```

Sympy [F]

$$\int \frac{F^{e(f+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{e(dx+f)}H^{t(r+sx)}}{F^{ce+dex}b+a}\,dx$$

input
```
integrate(F**(e*(d*x+f))*H**(t*(s*x+r))/(a+b*F**(e*(d*x+c))),x)
```

output
```
Integral(F**(e*(d*x + f))*H**(t*(r + s*x))/(F**(c*e + d*e*x)*b + a), x)
```

Maxima [F]

$$\int \frac{F^{e(f+dx)}H^{t(r+sx)}}{a+bF^{e(c+dx)}}\,dx = \int \frac{F^{(dx+f)e}H^{(sx+r)t}}{F^{(dx+c)e}b+a}\,dx$$

input
```
integrate(F^(e*(d*x+f))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x, algorithm="ma
xima")
```

output
```
-F^(e*f)*H^(r*t)*a^2*d*e*integrate(H^(s*t*x)/(F^(c*e)*a^2*b*d*e*log(F) - F
^(c*e)*a^2*b*s*t*log(H) + (F^(3*c*e)*b^3*d*e*log(F) - F^(3*c*e)*b^3*s*t*lo
g(H))*F^(2*d*e*x) + 2*(F^(2*c*e)*a*b^2*d*e*log(F) - F^(2*c*e)*a*b^2*s*t*lo
g(H))*F^(d*e*x)), x)*log(F) + (F^(e*f)*H^(r*t)*a*d*e*log(F) + (F^(c*e + e*
f)*H^(r*t)*b*d*e*log(F) - F^(c*e + e*f)*H^(r*t)*b*s*t*log(H))*F^(d*e*x))*H
^(s*t*x)/(F^(c*e)*a*b*d*e*s*t*log(F)*log(H) - F^(c*e)*a*b*s^2*t^2*log(H)^2
 + (F^(2*c*e)*b^2*d*e*s*t*log(F)*log(H) - F^(2*c*e)*b^2*s^2*t^2*log(H)^2)*
F^(d*e*x))
```

Giac [F]

$$\int \frac{F^{e(f+dx)} H^{t(r+sx)}}{a + bF^{e(c+dx)}} \, dx = \int \frac{F^{(dx+f)e} H^{(sx+r)t}}{F^{(dx+c)e} b + a} \, dx$$

input
```
integrate(F^(e*(d*x+f))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x, algorithm="gi
ac")
```

output
```
integrate(F^((d*x + f)*e)*H^((s*x + r)*t)/(F^((d*x + c)*e)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{e(f+dx)} H^{t(r+sx)}}{a + bF^{e(c+dx)}} \, dx = \int \frac{F^{e(f+dx)} H^{t(r+sx)}}{a + F^{e(c+dx)} b} \, dx$$

input
```
int((F^(e*(f + d*x))*H^(t*(r + s*x)))/(a + F^(e*(c + d*x))*b),x)
```

output
```
int((F^(e*(f + d*x))*H^(t*(r + s*x)))/(a + F^(e*(c + d*x))*b), x)
```

Reduce [F]

$$\int \frac{F^{e(f+dx)} H^{t(r+sx)}}{a + bF^{e(c+dx)}} \, dx = \frac{h^{rt} f^{ef} \left(h^{stx} - \left(\int \frac{h^{stx}}{f^{dex+ce}b+a} dx\right) \log(h)\, ast\right)}{f^{ce} \log(h)\, bst}$$

input
```
int(F^(e*(d*x+f))*H^(t*(s*x+r))/(a+b*F^(e*(d*x+c))),x)
```

output
```
(h**(r*t)*f**(e*f)*(h**(s*t*x) - int(h**(s*t*x)/(f**(c*e + d*e*x)*b + a),x
)*log(h)*a*s*t))/(f**(c*e)*log(h)*b*s*t)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
          finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```python
    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file