Computer Algebra
Independent Integration Tests

Summer 2024

2-Exponentials/158-2.3

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**93**]. This is test number [158].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (93)	0.00 (0)
Fricas	100.00 (93)	0.00 (0)
Mathematica	91.40 (85)	8.60 (8)
Maple	87.10 (81)	12.90 (12)
Maxima	83.87 (78)	16.13 (15)
Giac	58.06 (54)	41.94 (39)
Reduce	58.06 (54)	41.94 (39)
Mupad	56.99 (53)	43.01 (40)
Sympy	53.76 (50)	46.24 (43)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

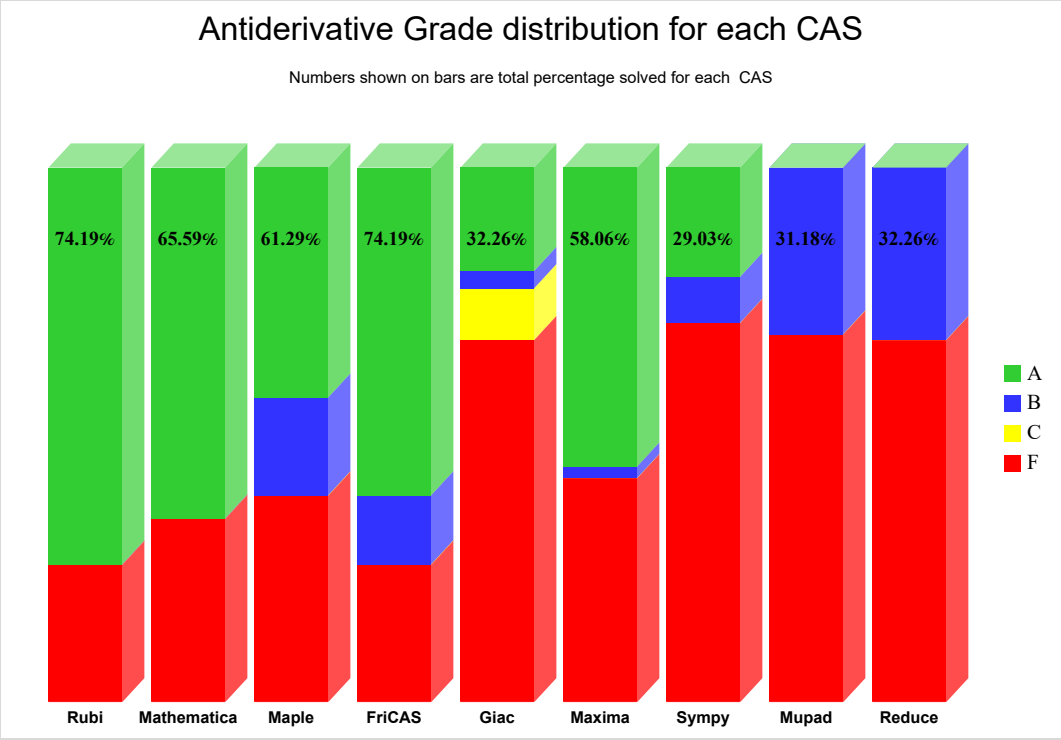
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

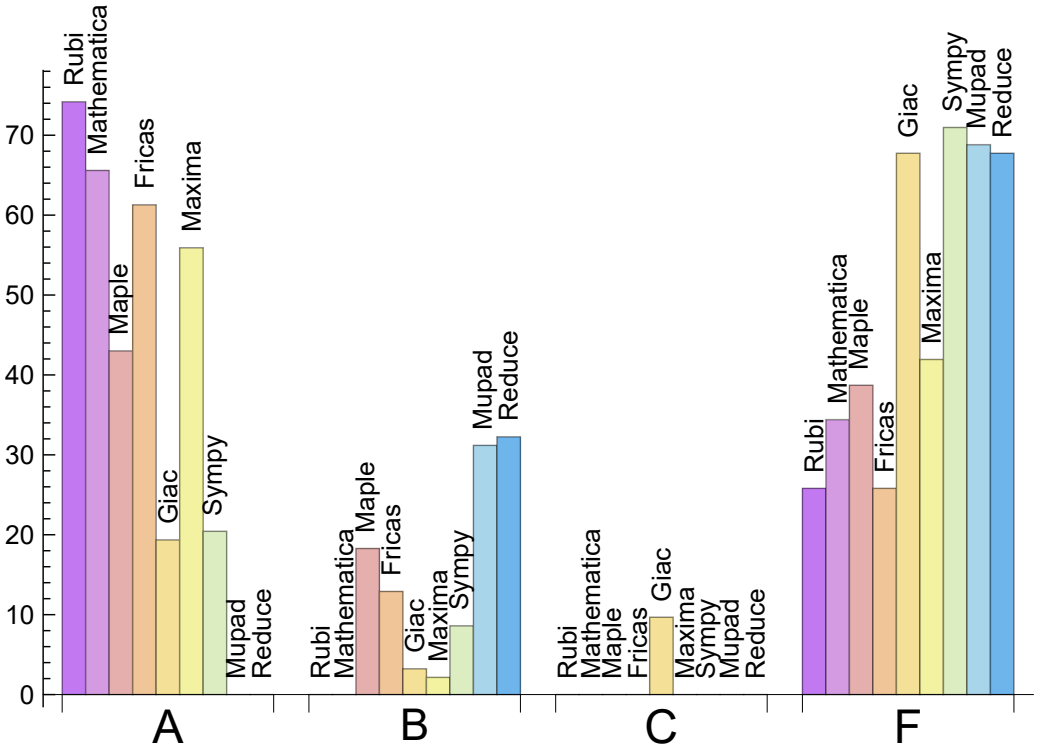
System	% A grade	% B grade	% C grade	% F grade
Rubi	74.194	0.000	0.000	25.806
Mathematica	65.591	0.000	0.000	34.409
Fricas	61.290	12.903	0.000	25.806
Maxima	55.914	2.151	0.000	41.935
Maple	43.011	18.280	0.000	38.710
Sympy	20.430	8.602	0.000	70.968
Giac	19.355	3.226	9.677	67.742
Mupad	0.000	31.183	0.000	68.817
Reduce	0.000	32.258	0.000	67.742

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Mathematica	8	100.00	0.00	0.00
Maple	12	100.00	0.00	0.00
Maxima	15	100.00	0.00	0.00
Giac	39	100.00	0.00	0.00
Reduce	39	100.00	0.00	0.00
Mupad	40	0.00	100.00	0.00
Sympy	43	83.72	16.28	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Fricas	0.08
Maple	0.16
Reduce	0.18
Giac	0.18
Mathematica	0.87
Rubi	1.05
Sympy	5.20
Mupad	20.79

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	86.15	1.14	27.00	1.08
Mathematica	89.80	0.95	67.00	0.97
Rubi	141.26	1.07	84.00	1.00
Sympy	147.02	2.02	52.00	1.09
Reduce	160.26	3.03	68.50	1.68
Maxima	179.22	2.50	94.00	1.14
Fricas	224.75	1.68	108.00	1.39
Maple	334.52	1.85	77.00	1.04
Giac	1165.43	5.16	29.00	1.08

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

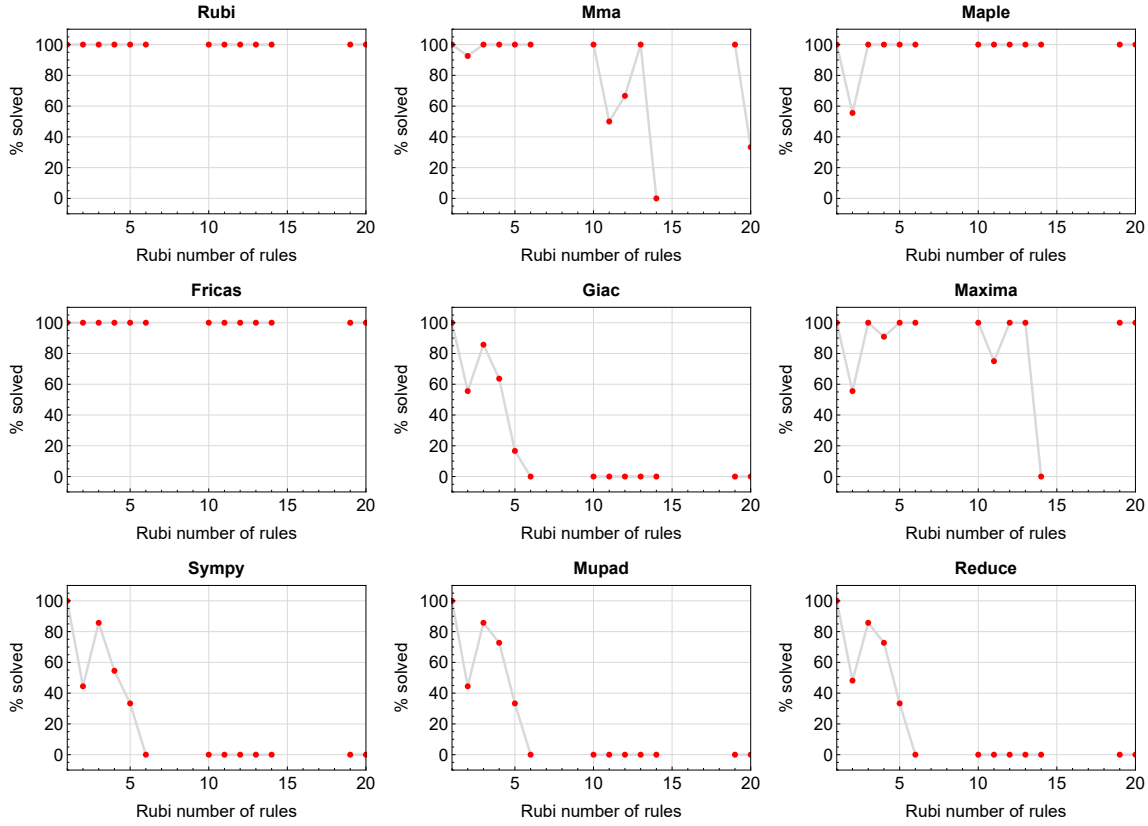


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

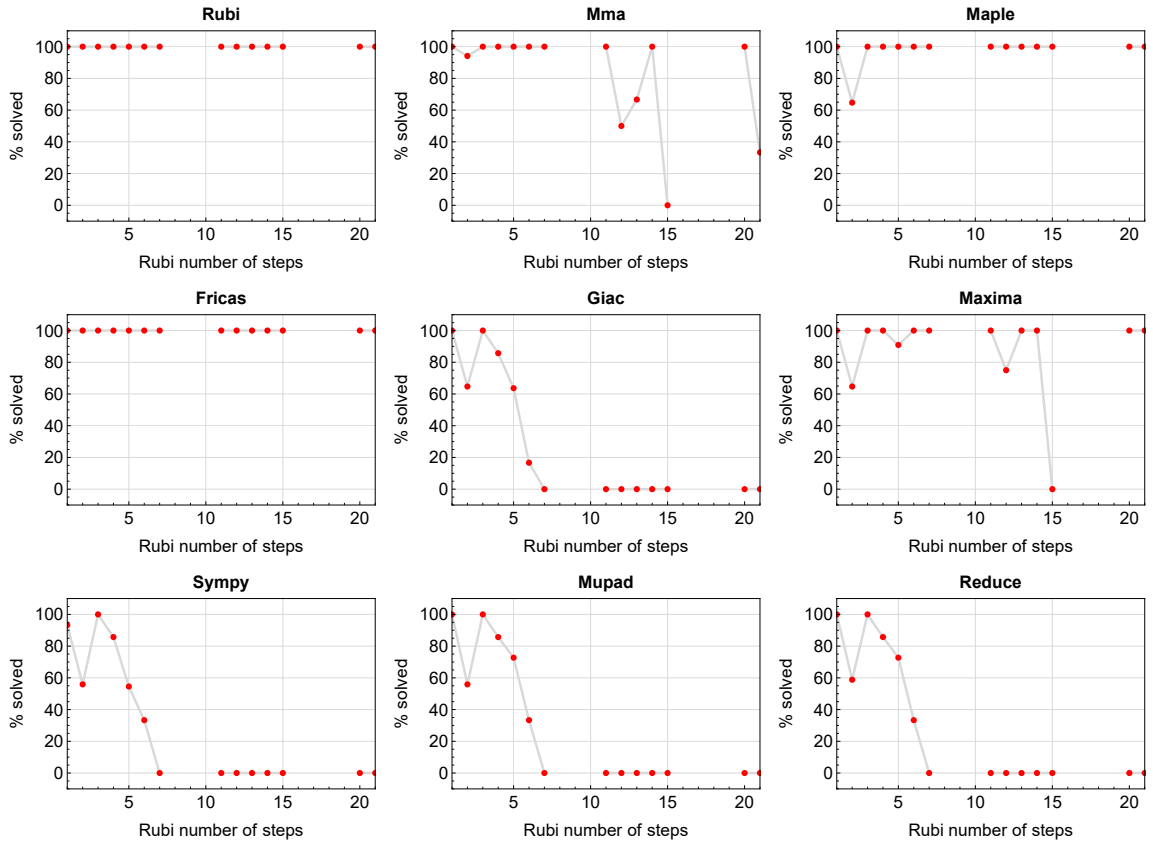


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

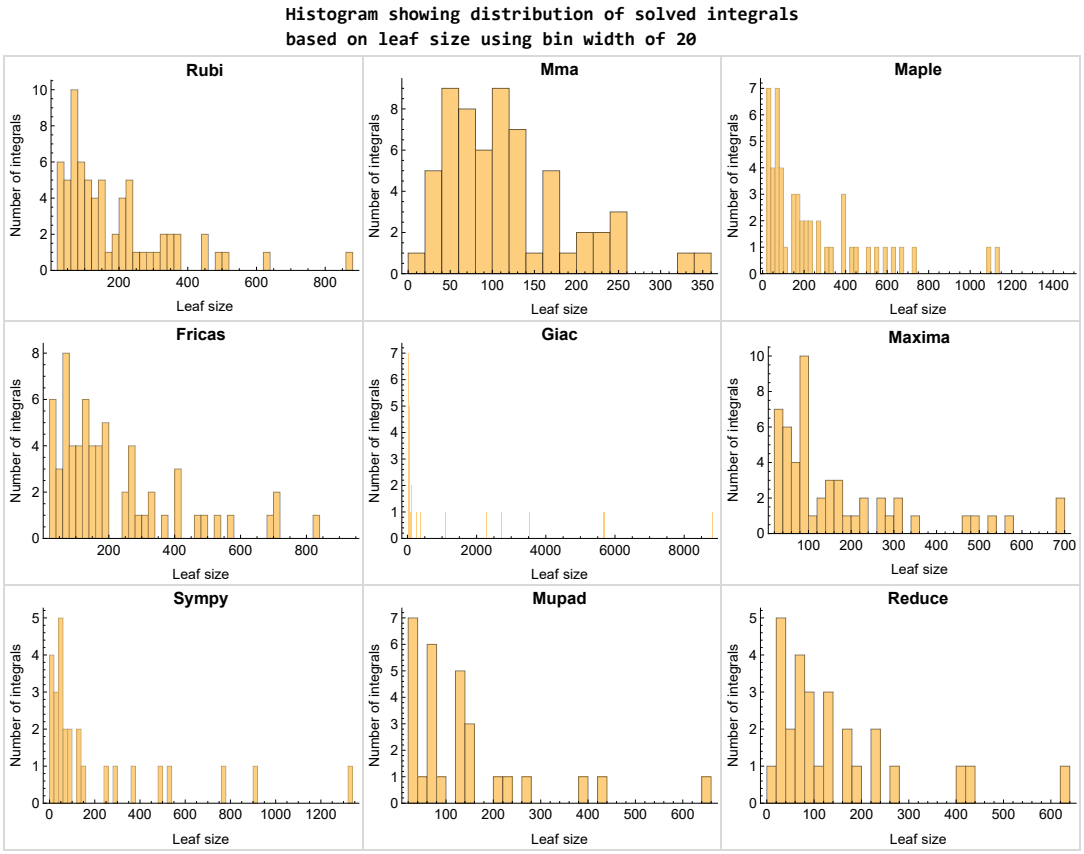


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

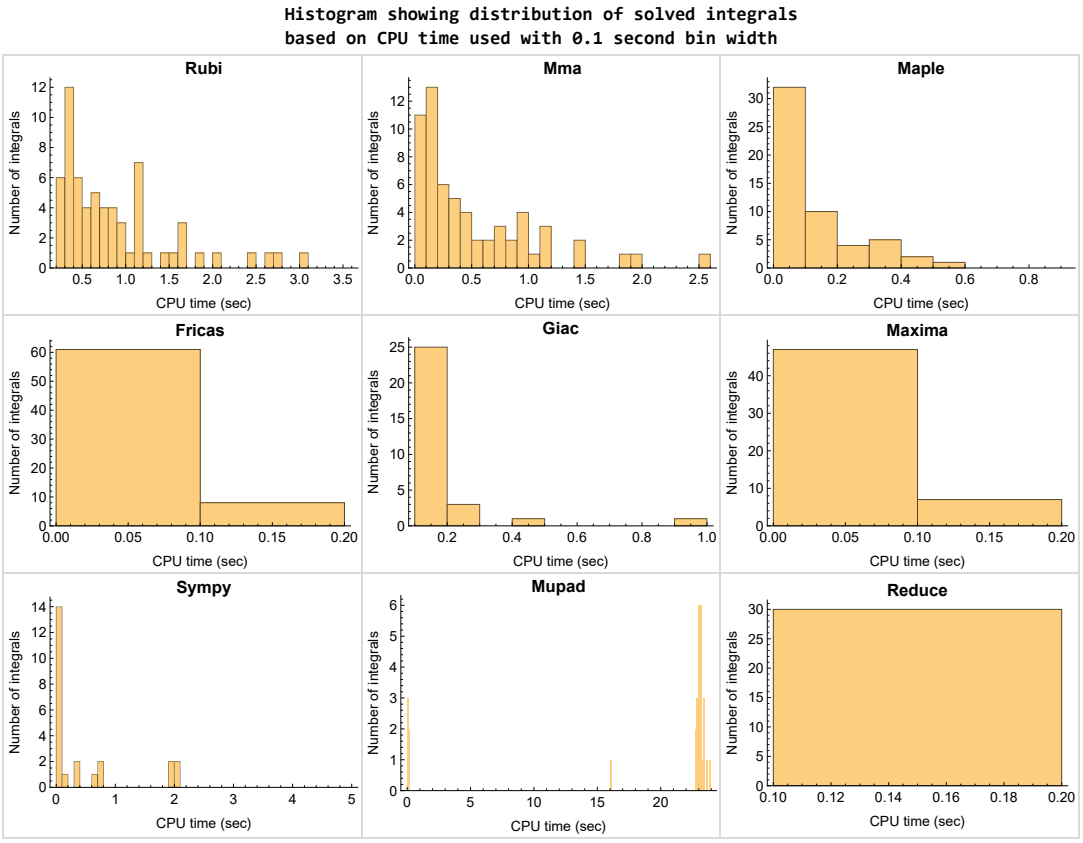


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

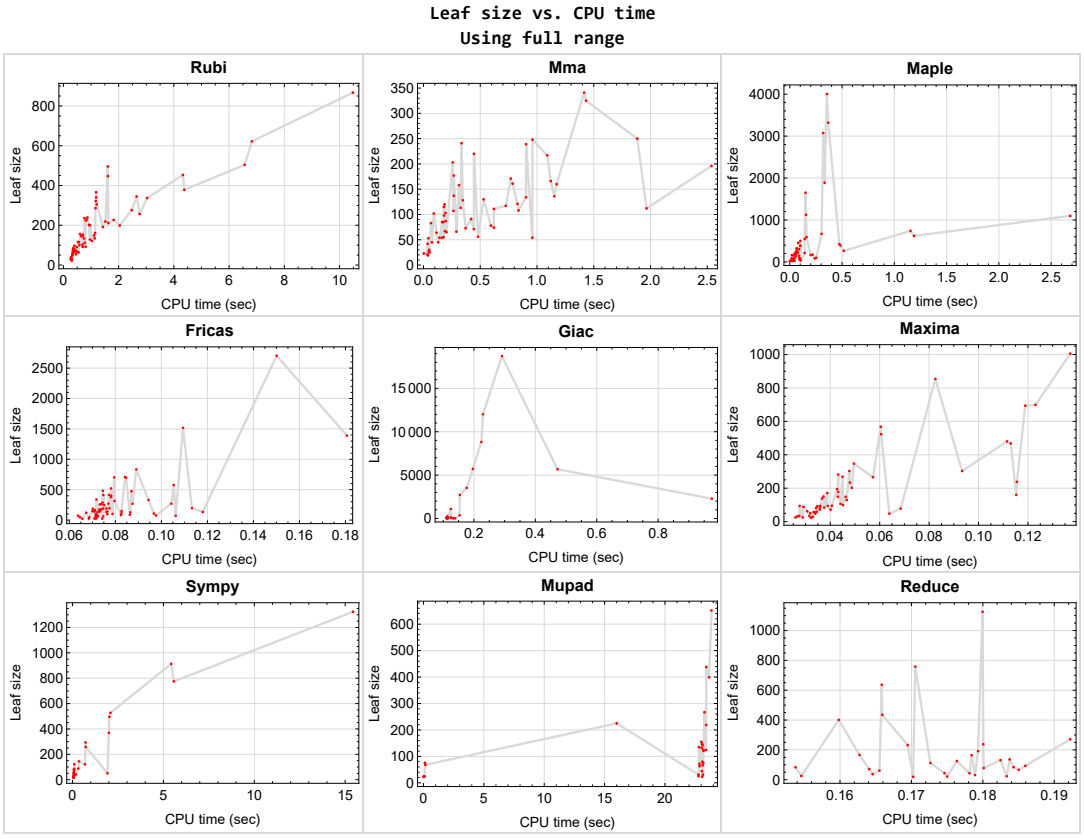


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{5, 6, 13, 14, 21, 22, 50, 51, 56, 57, 62, 63, 67, 68, 69, 73, 74, 75, 80, 81, 86, 87, 92, 93\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{35, 42\}$

Mathematica {}

Maple $\{32, 33, 35, 39, 40, 41, 42\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

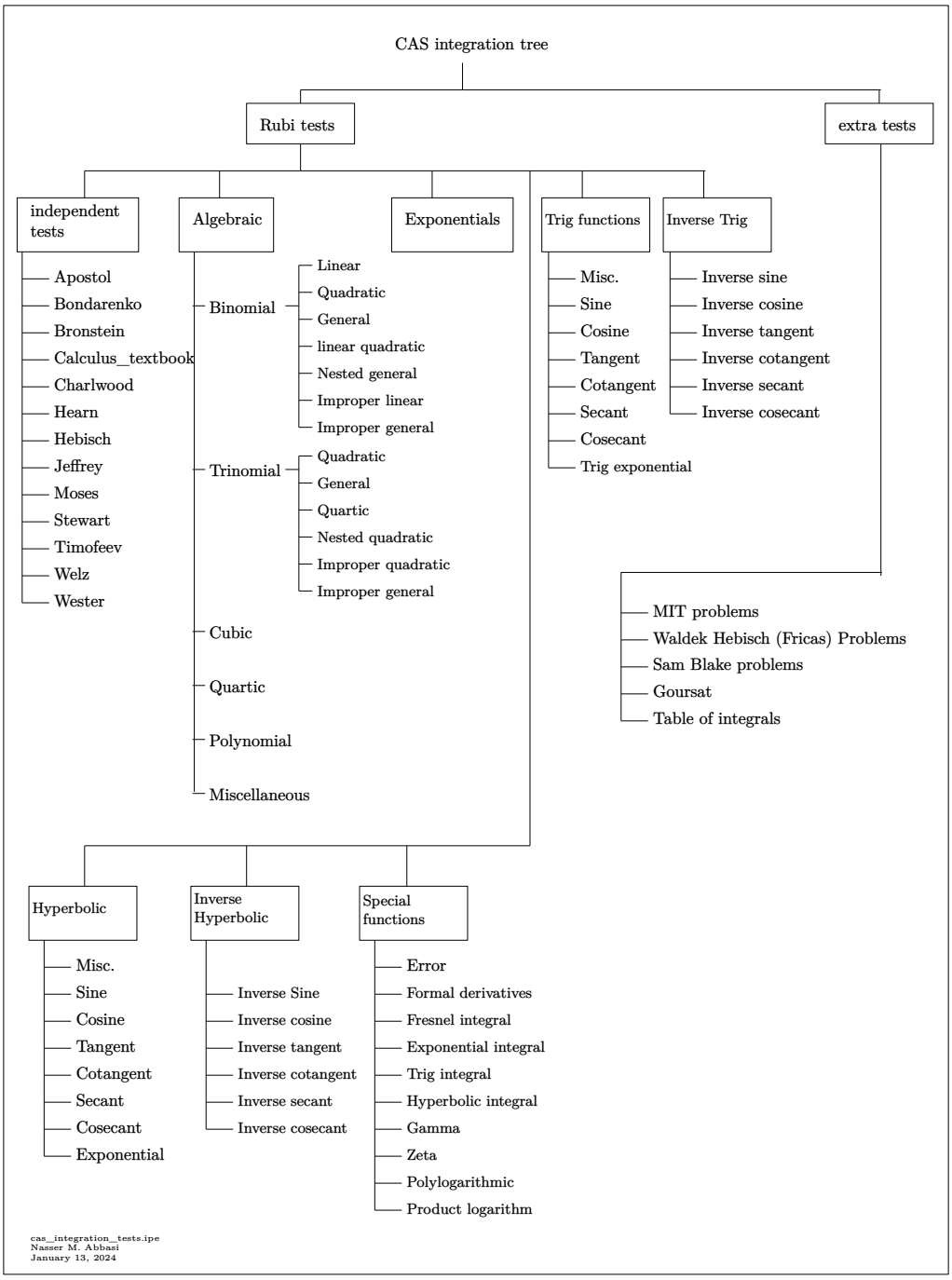
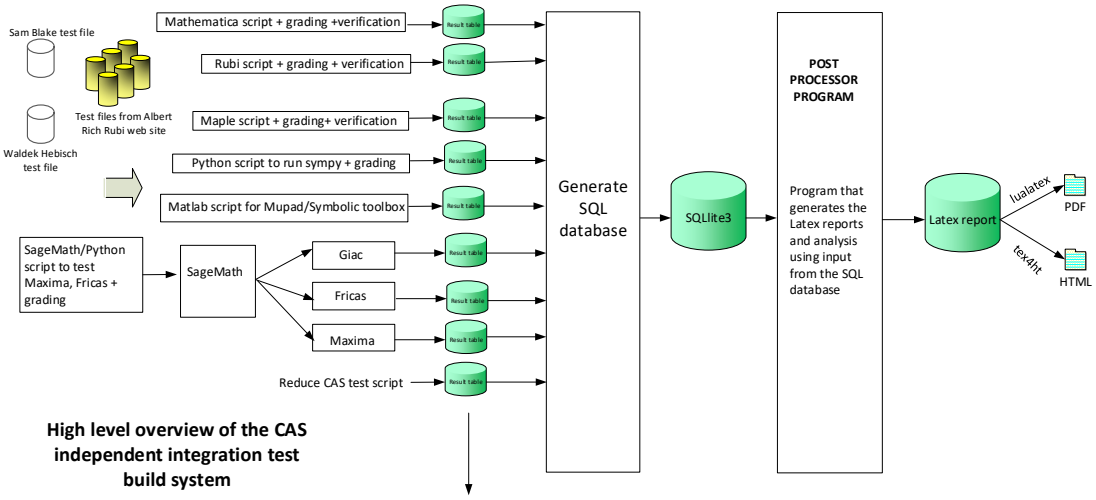


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 26
Mma 26
Maple 27
Fricas 27
Maxima 27
Giac 28
Mupad 28
Sympy 28
Reduce 29

Rubi

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 58, 59, 60, 61, 64, 65, 66, 70, 71, 72, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 55, 61, 64, 65, 66, 72, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91 }

B grade { }

C grade { }

F normal fail { 52, 53, 54, 58, 59, 60, 70, 71 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 3, 4, 7, 8, 9, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 26, 27, 28, 33, 34, 35, 40, 41, 42, 49, 55, 61, 64, 65, 66, 76, 79, 82, 84, 85, 88, 89, 90, 91 }

B grade { 2, 10, 25, 32, 39, 46, 47, 48, 52, 53, 54, 58, 59, 60, 77, 78, 83 }

C grade { }

F normal fail { 29, 30, 31, 36, 37, 38, 43, 44, 45, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 55, 61, 64, 65, 66, 70, 71, 72, 76, 77, 78, 79, 82, 83, 84, 85, 90, 91 }

B grade { 17, 18, 19, 46, 52, 53, 54, 58, 59, 60, 88, 89 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 52, 53, 55, 58, 59, 61, 64, 65, 66, 76, 77, 78, 79, 82, 83, 84, 85, 88, 89, 90, 91 }

B grade { 46, 47 }

C grade { }

F normal fail { 29, 30, 31, 36, 37, 38, 43, 44, 45, 48, 54, 60, 70, 71, 72 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 28, 35, 42, 49, 55, 61, 79, 85, 91 }

B grade { 64, 65, 66 }

C grade { 25, 26, 27, 32, 33, 34, 39, 40, 41 }

F normal fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 70, 71, 72, 76, 77, 78, 82, 83, 84, 88, 89, 90 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 55, 61, 79, 84, 85, 90, 91 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 64, 65, 66, 70, 71, 72, 76, 77, 78, 82, 83, 88, 89 }

F(-2) exception fail { }

Sympy

A grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 28, 34, 35, 41, 42, 49, 79, 84, 85, 90 }

B grade { 25, 26, 27, 32, 33, 39, 40, 91 }

C grade { }

F normal fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 55, 61, 64, 65, 66, 70, 71, 72, 76, 77, 78, 82, 83, 88, 89 }

F(-1) timedout fail { 52, 53, 54, 58, 59, 60, 75 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 4, 7, 8, 12, 15, 16, 20, 23, 24, 25, 26, 27, 28, 32, 33, 34, 35, 39, 40, 41, 42, 49, 55, 61, 64, 79, 84, 85, 90, 91 }

C grade { }

F normal fail { 1, 2, 3, 9, 10, 11, 17, 18, 19, 29, 30, 31, 36, 37, 38, 43, 44, 45, 46, 47, 48, 52, 53, 54, 58, 59, 60, 65, 66, 70, 71, 72, 76, 77, 78, 82, 83, 88, 89 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	121	102	191	94	120	0	0	19	0
N.S.	1	1.10	0.93	1.74	0.85	1.09	0.00	0.00	0.17	0.00
time (sec)	N/A	1.033	0.090	0.046	0.039	0.067	0.000	0.000	0.153	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	83	162	72	100	0	0	19	0
N.S.	1	1.10	0.99	1.93	0.86	1.19	0.00	0.00	0.23	0.00
time (sec)	N/A	0.804	0.066	0.020	0.036	0.072	0.000	0.000	0.176	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	63	53	84	48	72	0	0	17	0
N.S.	1	1.09	0.91	1.45	0.83	1.24	0.00	0.00	0.29	0.00
time (sec)	N/A	0.549	0.044	0.017	0.034	0.064	0.000	0.000	0.182	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	34	42	25	32	24	17	31	25	24
N.S.	1	1.31	1.62	0.96	1.23	0.92	0.65	1.19	0.96	0.92
time (sec)	N/A	0.274	0.036	0.015	0.027	0.071	0.068	0.132	0.155	0.090

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	17	15	18	18	18
N.S.	1	1.00	1.12	0.94	1.06	1.00	0.88	1.06	1.06	1.06
time (sec)	N/A	0.302	0.217	0.082	0.065	0.063	1.004	0.107	0.172	23.077

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	18	21	17	18	22	18
N.S.	1	1.00	1.12	0.94	1.06	1.24	1.00	1.06	1.29	1.06
time (sec)	N/A	0.301	0.349	0.072	0.064	0.064	0.828	0.110	0.161	23.085

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	37	21	24	34	23	17	33	21	23
N.S.	1	1.42	0.81	0.92	1.31	0.88	0.65	1.27	0.81	0.88
time (sec)	N/A	0.282	0.036	0.020	0.032	0.074	0.049	0.113	0.175	23.086

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	41	19	26	34	25	19	33	19	25
N.S.	1	1.46	0.68	0.93	1.21	0.89	0.68	1.18	0.68	0.89
time (sec)	N/A	0.289	0.039	0.018	0.027	0.068	0.053	0.116	0.170	0.049

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	257	158	382	195	330	0	0	37	0
N.S.	1	1.18	0.73	1.76	0.90	1.52	0.00	0.00	0.17	0.00
time (sec)	N/A	2.760	0.313	0.101	0.043	0.094	0.000	0.000	0.171	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	199	113	324	149	263	0	0	37	0
N.S.	1	1.21	0.68	1.96	0.90	1.59	0.00	0.00	0.22	0.00
time (sec)	N/A	2.040	0.328	0.066	0.043	0.074	0.000	0.000	0.184	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	134	85	154	95	176	0	0	35	0
N.S.	1	1.25	0.79	1.44	0.89	1.64	0.00	0.00	0.33	0.00
time (sec)	N/A	1.134	0.175	0.033	0.041	0.074	0.000	0.000	0.167	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	45	49	51	60	39	51	83	66
N.S.	1	1.11	0.98	1.07	1.11	1.30	0.85	1.11	1.80	1.43
time (sec)	N/A	0.328	0.074	0.023	0.032	0.075	0.061	0.110	0.154	23.122

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	53	35	70	18	37	18
N.S.	1	1.00	1.12	0.94	3.12	2.06	4.12	1.06	2.18	1.06
time (sec)	N/A	0.306	3.232	0.201	0.069	0.061	1.321	0.114	0.173	23.021

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	57	41	73	18	43	18
N.S.	1	1.00	1.12	0.94	3.35	2.41	4.29	1.06	2.53	1.06
time (sec)	N/A	0.302	1.521	0.204	0.066	0.065	1.335	0.112	0.153	23.065

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	55	45	53	55	65	39	54	71	68
N.S.	1	1.15	0.94	1.10	1.15	1.35	0.81	1.12	1.48	1.42
time (sec)	N/A	0.333	0.129	0.025	0.034	0.072	0.088	0.126	0.164	0.156

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	61	66	59	57	73	42	58	66	76
N.S.	1	1.17	1.27	1.13	1.10	1.40	0.81	1.12	1.27	1.46
time (sec)	N/A	0.332	0.290	0.023	0.033	0.072	0.095	0.115	0.185	0.120

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	504	241	548	303	702	0	0	55	0
N.S.	1	1.51	0.72	1.65	0.91	2.11	0.00	0.00	0.17	0.00
time (sec)	N/A	6.564	0.335	0.142	0.048	0.080	0.000	0.000	0.158	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	378	203	385	234	521	0	0	55	0
N.S.	1	1.56	0.84	1.58	0.96	2.14	0.00	0.00	0.23	0.00
time (sec)	N/A	4.378	0.258	0.100	0.048	0.078	0.000	0.000	0.174	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	227	120	216	149	338	0	0	53	0
N.S.	1	1.43	0.75	1.36	0.94	2.13	0.00	0.00	0.33	0.00
time (sec)	N/A	1.823	0.186	0.055	0.046	0.072	0.000	0.000	0.161	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	64	66	84	127	76	65	164	121
N.S.	1	1.03	0.93	0.96	1.22	1.84	1.10	0.94	2.38	1.75
time (sec)	N/A	0.347	0.115	0.036	0.038	0.086	0.081	0.110	0.178	23.194

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	128	53	165	18	56	18
N.S.	1	1.00	1.12	0.94	7.53	3.12	9.71	1.06	3.29	1.06
time (sec)	N/A	0.299	4.233	0.266	0.078	0.065	2.186	0.116	0.172	23.009

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	16	128	61	165	18	64	18
N.S.	1	1.00	1.12	0.94	7.53	3.59	9.71	1.06	3.76	1.06
time (sec)	N/A	0.302	1.995	0.269	0.077	0.065	1.969	0.112	0.159	23.041

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	67	71	88	132	78	71	136	124
N.S.	1	1.06	0.93	0.99	1.22	1.83	1.08	0.99	1.89	1.72
time (sec)	N/A	0.349	0.190	0.036	0.029	0.072	0.080	0.112	0.184	23.181

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	84	54	77	92	140	85	75	125	132
N.S.	1	1.08	0.69	0.99	1.18	1.79	1.09	0.96	1.60	1.69
time (sec)	N/A	0.356	0.959	0.035	0.035	0.071	0.087	0.110	0.176	23.041

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	130	392	282	267	496	5716	401	225
N.S.	1	1.00	0.85	2.56	1.84	1.75	3.24	37.36	2.62	1.47
time (sec)	N/A	0.703	0.529	0.484	0.043	0.087	2.026	0.196	0.160	16.004

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	91	163	171	166	294	2716	238	135
N.S.	1	1.00	0.79	1.42	1.49	1.44	2.56	23.62	2.07	1.17
time (sec)	N/A	0.559	0.419	0.200	0.039	0.071	0.714	0.154	0.180	22.815

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	73	84	85	87	144	1101	112	72
N.S.	1	1.00	0.95	1.09	1.10	1.13	1.87	14.30	1.45	0.94
time (sec)	N/A	0.420	0.371	0.092	0.036	0.072	0.351	0.125	0.173	22.864

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	29	37	41	31	37	31
N.S.	1	1.00	1.00	1.03	0.97	1.23	1.37	1.03	1.23	1.03
time (sec)	N/A	0.266	0.050	0.104	0.033	0.071	0.181	0.112	0.165	23.155

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	56	0	0	50	0	0	39	0
N.S.	1	1.00	0.82	0.00	0.00	0.74	0.00	0.00	0.57	0.00
time (sec)	N/A	0.496	0.481	0.000	0.000	0.071	0.000	0.000	0.196	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	78	0	0	87	0	0	93	0
N.S.	1	1.00	0.78	0.00	0.00	0.87	0.00	0.00	0.93	0.00
time (sec)	N/A	0.581	0.594	0.000	0.000	0.086	0.000	0.000	0.167	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	111	0	0	160	0	0	186	0
N.S.	1	1.00	0.76	0.00	0.00	1.09	0.00	0.00	1.27	0.00
time (sec)	N/A	0.695	0.620	0.000	0.000	0.073	0.000	0.000	0.179	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	239	739	568	482	913	12013	758	438
N.S.	1	1.00	0.74	2.30	1.76	1.50	2.84	37.31	2.35	1.36
time (sec)	N/A	1.170	0.904	1.155	0.060	0.075	5.427	0.229	0.171	23.424

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	171	424	347	287	527	5675	435	267
N.S.	1	1.00	0.72	1.77	1.45	1.20	2.21	23.74	1.82	1.12
time (sec)	N/A	0.870	0.769	0.474	0.050	0.074	2.087	0.472	0.166	23.277

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	117	174	178	139	258	2284	191	146
N.S.	1	1.00	0.75	1.12	1.14	0.89	1.65	14.64	1.22	0.94
time (sec)	N/A	0.603	0.725	0.218	0.043	0.073	0.729	0.975	0.179	23.082

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	65	60	62	61	88	74	61	75
N.S.	1	0.94	0.97	0.90	0.93	0.91	1.31	1.10	0.91	1.12
time (sec)	N/A	0.353	0.203	0.094	0.031	0.072	0.315	0.114	0.166	23.177

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	0	0	95	0	0	73	0
N.S.	1	1.00	0.81	0.00	0.00	0.71	0.00	0.00	0.54	0.00
time (sec)	N/A	0.744	0.838	0.000	0.000	0.076	0.000	0.000	0.178	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	136	0	0	171	0	0	713	0
N.S.	1	1.00	0.67	0.00	0.00	0.85	0.00	0.00	3.53	0.00
time (sec)	N/A	0.926	1.152	0.000	0.000	0.073	0.000	0.000	0.208	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	217	0	0	314	0	0	1407	0
N.S.	1	1.00	0.76	0.00	0.00	1.10	0.00	0.00	4.92	0.00
time (sec)	N/A	1.162	1.090	0.000	0.000	0.080	0.000	0.000	0.185	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	B	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	341	1097	854	708	1324	18707	1126	652
N.S.	1	1.00	0.69	2.21	1.72	1.43	2.67	37.72	2.27	1.31
time (sec)	N/A	1.609	1.415	2.683	0.083	0.084	15.432	0.291	0.180	23.853

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	248	620	523	414	775	8820	637	399
N.S.	1	1.00	0.68	1.69	1.43	1.13	2.12	24.10	1.74	1.09
time (sec)	N/A	1.183	0.960	1.188	0.061	0.077	5.571	0.224	0.166	23.661

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	161	263	269	192	369	3528	271	219
N.S.	1	1.00	0.68	1.11	1.14	0.81	1.56	14.95	1.15	0.93
time (sec)	N/A	0.760	0.785	0.516	0.045	0.077	1.999	0.176	0.192	23.412

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	103	83	87	82	94	84	121	98	84	124
N.S.	1	0.81	0.84	0.80	0.91	0.82	1.17	0.95	0.82	1.20
time (sec)	N/A	0.364	0.196	0.238	0.028	0.082	0.686	0.113	0.184	23.403

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	160	0	0	140	0	0	107	0
N.S.	1	1.00	0.80	0.00	0.00	0.70	0.00	0.00	0.54	0.00
time (sec)	N/A	0.964	1.173	0.000	0.000	0.077	0.000	0.000	0.205	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	250	0	0	259	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	1.206	1.883	0.000	0.000	0.073	0.000	0.000	0.235	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	325	0	0	475	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	1.06	0.00	0.00	0.00	0.00
time (sec)	N/A	1.611	1.432	0.000	0.000	0.087	0.000	0.000	0.237	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	219	166	3075	480	413	0	0	154	0
N.S.	1	1.14	0.86	16.02	2.50	2.15	0.00	0.00	0.80	0.00
time (sec)	N/A	1.521	1.122	0.320	0.112	0.075	0.000	0.000	0.196	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	161	121	1124	303	271	0	0	116	0
N.S.	1	1.11	0.83	7.75	2.09	1.87	0.00	0.00	0.80	0.00
time (sec)	N/A	1.149	0.828	0.155	0.093	0.077	0.000	0.000	0.180	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	103	74	456	0	147	0	0	76	0
N.S.	1	1.05	0.76	4.65	0.00	1.50	0.00	0.00	0.78	0.00
time (sec)	N/A	0.695	0.620	0.081	0.000	0.070	0.000	0.000	0.199	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	52	55	44	59	44	51	72	44	44
N.S.	1	1.30	1.38	1.10	1.48	1.10	1.28	1.80	1.10	1.10
time (sec)	N/A	0.329	0.177	0.040	0.035	0.069	1.921	0.115	0.175	23.055

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	33	22	27	43	27
N.S.	1	1.00	1.08	1.00	1.04	1.32	0.88	1.08	1.72	1.08
time (sec)	N/A	0.472	3.162	0.048	0.074	0.069	1.816	0.126	0.178	23.292

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	57	24	27	79	27
N.S.	1	1.00	1.08	1.00	1.04	2.28	0.96	1.08	3.16	1.08
time (sec)	N/A	0.458	0.661	0.063	0.084	0.068	4.033	0.557	0.185	23.309

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	453	0	3319	699	1390	0	0	537	0
N.S.	1	1.17	0.00	8.55	1.80	3.58	0.00	0.00	1.38	0.00
time (sec)	N/A	4.328	0.000	0.368	0.123	0.181	0.000	0.000	0.219	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	337	0	1650	468	834	0	0	400	0
N.S.	1	1.15	0.00	5.61	1.59	2.84	0.00	0.00	1.36	0.00
time (sec)	N/A	3.031	0.000	0.150	0.113	0.089	0.000	0.000	0.225	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	211	0	591	0	400	0	0	257	0
N.S.	1	1.10	0.00	3.09	0.00	2.09	0.00	0.00	1.35	0.00
time (sec)	N/A	1.627	0.000	0.160	0.000	0.078	0.000	0.000	0.182	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	73	107	74	94	100	0	108	131	80
N.S.	1	0.99	1.45	1.00	1.27	1.35	0.00	1.46	1.77	1.08
time (sec)	N/A	0.374	0.264	0.091	0.036	0.079	0.000	0.116	0.182	23.094

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	192	67	24	27	90	27
N.S.	1	1.00	1.08	1.00	7.68	2.68	0.96	1.08	3.60	1.08
time (sec)	N/A	0.473	1.532	0.066	0.099	0.068	3.899	0.134	0.191	23.158

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	280	108	26	27	157	27
N.S.	1	1.00	1.08	1.00	11.20	4.32	1.04	1.08	6.28	1.08
time (sec)	N/A	0.456	1.252	0.003	0.136	0.069	22.359	0.177	0.188	23.304

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	867	0	4005	1005	2704	0	0	1075	0
N.S.	1	1.46	0.00	6.74	1.69	4.55	0.00	0.00	1.81	0.00
time (sec)	N/A	10.481	0.000	0.355	0.137	0.150	0.000	0.000	0.213	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	439	622	0	1887	694	1518	0	0	793	0
N.S.	1	1.42	0.00	4.30	1.58	3.46	0.00	0.00	1.81	0.00
time (sec)	N/A	6.823	0.000	0.333	0.119	0.109	0.000	0.000	0.202	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	345	0	668	0	696	0	0	501	0
N.S.	1	1.25	0.00	2.42	0.00	2.52	0.00	0.00	1.82	0.00
time (sec)	N/A	2.651	0.000	0.304	0.000	0.085	0.000	0.000	0.201	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	97	128	96	138	190	0	132	232	142
N.S.	1	0.87	1.15	0.86	1.24	1.71	0.00	1.19	2.09	1.28
time (sec)	N/A	0.399	0.346	0.253	0.037	0.075	0.000	0.115	0.169	23.108

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	651	101	24	27	137	27
N.S.	1	1.00	1.08	1.00	26.04	4.04	0.96	1.08	5.48	1.08
time (sec)	N/A	0.454	2.033	0.005	0.135	0.097	11.343	0.151	0.188	23.200

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	807	159	26	27	235	27
N.S.	1	1.00	1.08	1.00	32.28	6.36	1.04	1.08	9.40	1.08
time (sec)	N/A	0.447	1.821	0.005	0.215	0.072	117.771	0.204	0.172	23.271

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	77	82	70	0	132	93	0
N.S.	1	1.00	1.00	1.08	1.15	0.99	0.00	1.86	1.31	0.00
time (sec)	N/A	0.424	0.444	0.100	0.034	0.106	0.000	0.125	0.186	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	134	144	160	133	0	250	159	0
N.S.	1	1.00	0.92	0.99	1.10	0.92	0.00	1.72	1.10	0.00
time (sec)	N/A	0.645	0.904	0.088	0.115	0.118	0.000	0.113	0.180	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	196	211	238	196	0	368	225	0
N.S.	1	1.00	0.88	0.94	1.06	0.88	0.00	1.64	1.00	0.00
time (sec)	N/A	0.811	2.537	0.142	0.116	0.113	0.000	0.153	0.173	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	18	18	15	18	18	18
N.S.	1	1.00	1.11	0.84	0.95	0.95	0.79	0.95	0.95	0.95
time (sec)	N/A	0.306	0.927	0.013	0.084	0.077	0.824	0.115	0.182	23.002

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	18	30	17	18	31	18
N.S.	1	1.00	1.11	0.84	0.95	1.58	0.89	0.95	1.63	0.95
time (sec)	N/A	0.309	2.694	0.006	0.127	0.064	3.969	0.169	0.164	23.064

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	18	42	17	18	44	18
N.S.	1	1.00	1.11	0.84	0.95	2.21	0.89	0.95	2.32	0.95
time (sec)	N/A	0.307	9.859	0.004	0.145	0.093	5.722	0.134	0.202	23.232

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	270	0	0	478	0
N.S.	1	1.00	0.00	0.00	0.00	0.79	0.00	0.00	1.41	0.00
time (sec)	N/A	1.187	0.000	0.000	0.000	0.104	0.000	0.000	0.185	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	0	0	0	191	0	0	325	0
N.S.	1	1.00	0.00	0.00	0.00	0.84	0.00	0.00	1.43	0.00
time (sec)	N/A	0.836	0.000	0.000	0.000	0.076	0.000	0.000	0.165	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	112	0	0	111	0	0	175	0
N.S.	1	1.00	0.97	0.00	0.00	0.96	0.00	0.00	1.51	0.00
time (sec)	N/A	0.529	1.965	0.000	0.000	0.083	0.000	0.000	0.183	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	28	22	27	28	27
N.S.	1	1.00	1.08	1.00	1.04	1.12	0.88	1.08	1.12	1.08
time (sec)	N/A	0.328	0.681	0.016	0.071	0.066	2.145	0.124	0.169	23.133

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	50	24	27	50	27
N.S.	1	1.00	1.08	1.00	1.04	2.00	0.96	1.08	2.00	1.08
time (sec)	N/A	0.324	0.800	0.007	0.071	0.070	30.812	0.137	0.210	23.062

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	27	25	26	28	0	27	28	27
N.S.	1	1.00	1.08	1.00	1.04	1.12	0.00	1.08	1.12	1.08
time (sec)	N/A	0.320	0.473	0.032	0.096	0.092	0.000	0.389	0.177	22.955

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	128	115	225	106	134	0	0	28	0
N.S.	1	1.11	1.00	1.96	0.92	1.17	0.00	0.00	0.24	0.00
time (sec)	N/A	0.961	0.179	0.079	0.044	0.076	0.000	0.000	0.194	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	91	85	194	78	108	0	0	28	0
N.S.	1	1.07	1.00	2.28	0.92	1.27	0.00	0.00	0.33	0.00
time (sec)	N/A	0.698	0.164	0.063	0.069	0.097	0.000	0.000	0.200	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	154	48	75	0	0	26	0
N.S.	1	1.00	1.00	2.85	0.89	1.39	0.00	0.00	0.48	0.00
time (sec)	N/A	0.451	0.141	0.055	0.064	0.098	0.000	0.000	0.187	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	23	23	17	24	23	23
N.S.	1	1.00	1.00	1.04	1.00	1.00	0.74	1.04	1.00	1.00
time (sec)	N/A	0.306	0.003	0.000	0.032	0.066	0.053	0.135	0.183	0.002

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	32	26	19	26	28	26
N.S.	1	1.00	1.08	1.00	1.33	1.08	0.79	1.08	1.17	1.08
time (sec)	N/A	0.361	0.446	0.007	0.087	0.064	0.764	0.141	0.162	22.674

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	38	30	20	26	32	26
N.S.	1	1.00	1.08	1.00	1.58	1.25	0.83	1.08	1.33	1.08
time (sec)	N/A	0.355	0.398	0.009	0.087	0.098	0.660	0.142	0.201	22.685

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	149	137	274	129	246	0	0	313	0
N.S.	1	1.06	0.98	1.96	0.92	1.76	0.00	0.00	2.24	0.00
time (sec)	N/A	1.109	0.267	0.066	0.047	0.075	0.000	0.000	0.199	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	103	231	99	186	0	0	194	0
N.S.	1	1.05	0.96	2.16	0.93	1.74	0.00	0.00	1.81	0.00
time (sec)	N/A	0.774	0.194	0.058	0.045	0.070	0.000	0.000	0.176	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	54	67	72	74	58	0	78	63
N.S.	1	0.99	0.78	0.97	1.04	1.07	0.84	0.00	1.13	0.91
time (sec)	N/A	0.421	0.165	0.039	0.040	0.072	0.077	0.000	0.180	22.868

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	25	26	26	31	31
N.S.	1	1.00	1.00	1.04	1.00	1.00	1.04	1.04	1.24	1.24
time (sec)	N/A	0.288	0.054	0.031	0.026	0.072	0.045	0.139	0.179	22.774

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	63	45	65	26	47	26
N.S.	1	1.00	1.08	1.00	2.62	1.88	2.71	1.08	1.96	1.08
time (sec)	N/A	0.473	0.423	0.008	0.097	0.066	0.757	0.202	0.187	22.690

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	67	51	70	26	376	26
N.S.	1	1.00	1.08	1.00	2.79	2.12	2.92	1.08	15.67	1.08
time (sec)	N/A	0.472	0.546	0.009	0.096	0.065	0.789	0.209	0.190	22.780

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	276	220	501	266	577	0	0	636	0
N.S.	1	1.06	0.84	1.92	1.02	2.21	0.00	0.00	2.44	0.00
time (sec)	N/A	2.475	0.445	0.101	0.057	0.105	0.000	0.000	0.176	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	192	177	304	202	379	0	0	397	0
N.S.	1	1.05	0.97	1.67	1.11	2.08	0.00	0.00	2.18	0.00
time (sec)	N/A	1.430	0.267	0.089	0.049	0.078	0.000	0.000	0.196	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	90	98	111	150	148	122	0	166	155
N.S.	1	0.85	0.92	1.05	1.42	1.40	1.15	0.00	1.57	1.46
time (sec)	N/A	0.485	0.180	0.055	0.037	0.083	0.099	0.000	0.163	23.014

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	25	46	53	26	43	25
N.S.	1	1.00	1.00	0.96	0.93	1.70	1.96	0.96	1.59	0.93
time (sec)	N/A	0.296	0.047	0.045	0.029	0.065	0.056	0.114	0.178	22.781

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	145	64	170	26	66	26
N.S.	1	1.00	1.08	1.00	6.04	2.67	7.08	1.08	2.75	1.08
time (sec)	N/A	0.479	1.289	0.010	0.094	0.069	2.601	0.220	0.172	22.891

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	147	72	172	26	1023	26
N.S.	1	1.00	1.08	1.00	6.12	3.00	7.17	1.08	42.62	1.08
time (sec)	N/A	0.480	1.768	0.016	0.099	0.067	2.657	0.212	0.195	22.982

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [17] had the largest ratio of [1.17646999999999990]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	6	1.10	17	0.353
2	A	6	5	1.10	17	0.294
3	A	5	4	1.09	15	0.267
4	A	5	4	1.31	13	0.308
5	N/A	1	0	1.00	17	0.000
6	N/A	1	0	1.00	17	0.000
7	A	5	4	1.42	14	0.286
8	A	5	4	1.46	16	0.250
9	A	13	12	1.18	17	0.706
10	A	12	11	1.21	17	0.647
11	A	11	10	1.25	15	0.667
12	A	4	3	1.11	13	0.231
13	N/A	1	0	1.00	17	0.000
14	N/A	1	0	1.00	17	0.000
15	A	4	3	1.15	14	0.214
16	A	4	3	1.17	16	0.188
17	A	21	20	1.51	17	1.176
18	A	20	19	1.56	17	1.118
19	A	14	13	1.43	15	0.867
20	A	4	3	1.03	13	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	N/A	1	0	1.00	17	0.000
22	N/A	1	0	1.00	17	0.000
23	A	4	3	1.06	14	0.214
24	A	4	3	1.08	16	0.188
25	A	2	2	1.00	23	0.087
26	A	2	2	1.00	23	0.087
27	A	2	2	1.00	21	0.095
28	A	1	1	1.00	15	0.067
29	A	2	2	1.00	23	0.087
30	A	2	2	1.00	23	0.087
31	A	2	2	1.00	23	0.087
32	A	2	2	1.00	25	0.080
33	A	2	2	1.00	25	0.080
34	A	2	2	1.00	23	0.087
35	A	5	4	0.94	17	0.235
36	A	2	2	1.00	25	0.080
37	A	2	2	1.00	25	0.080
38	A	2	2	1.00	25	0.080
39	A	2	2	1.00	25	0.080
40	A	2	2	1.00	25	0.080
41	A	2	2	1.00	23	0.087
42	A	5	4	0.81	17	0.235
43	A	2	2	1.00	25	0.080
44	A	2	2	1.00	25	0.080
45	A	2	2	1.00	25	0.080
46	A	7	6	1.14	25	0.240
47	A	6	5	1.11	25	0.200
48	A	5	4	1.05	23	0.174
49	A	6	5	1.30	17	0.294
50	N/A	2	0	1.00	25	0.000
51	N/A	2	0	1.00	25	0.000
52	A	13	12	1.17	25	0.480

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
53	A	12	11	1.15	25	0.440
54	A	12	11	1.10	23	0.478
55	A	5	4	0.99	17	0.235
56	N/A	2	0	1.00	25	0.000
57	N/A	2	0	1.00	25	0.000
58	A	21	20	1.46	25	0.800
59	A	21	20	1.42	25	0.800
60	A	15	14	1.25	23	0.609
61	A	5	4	0.87	17	0.235
62	N/A	2	0	1.00	25	0.000
63	N/A	2	0	1.00	25	0.000
64	A	2	2	1.00	17	0.118
65	A	2	2	1.00	19	0.105
66	A	2	2	1.00	19	0.105
67	N/A	1	0	1.00	19	0.000
68	N/A	1	0	1.00	19	0.000
69	N/A	1	0	1.00	19	0.000
70	A	2	2	1.00	25	0.080
71	A	2	2	1.00	25	0.080
72	A	2	2	1.00	23	0.087
73	N/A	1	0	1.00	25	0.000
74	N/A	1	0	1.00	25	0.000
75	N/A	1	0	1.00	25	0.000
76	A	6	5	1.11	24	0.208
77	A	5	4	1.07	24	0.167
78	A	4	3	1.00	22	0.136
79	A	3	2	1.00	21	0.095
80	N/A	1	0	1.00	24	0.000
81	N/A	1	0	1.00	24	0.000
82	A	7	6	1.06	24	0.250
83	A	6	5	1.05	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
84	A	6	5	0.99	22	0.227
85	A	3	2	1.00	21	0.095
86	N/A	2	0	1.00	24	0.000
87	N/A	2	0	1.00	24	0.000
88	A	13	12	1.06	24	0.500
89	A	12	11	1.05	24	0.458
90	A	5	4	0.85	22	0.182
91	A	3	2	1.00	21	0.095
92	N/A	2	0	1.00	24	0.000
93	N/A	2	0	1.00	24	0.000

CHAPTER 3

Listing of integrals

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3.83 $\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx$ . . . 691
3.84 $\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx$ . . . 698
3.85 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx$ . . . 704
3.86 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2x}\,dx$ . . . 709

3.87 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})^2 x^2}\,dx$ 715
3.88 $\int \frac{F^{c+dx} x^3}{(a+bF^{c+dx})^3}\,dx$ 721
3.89 $\int \frac{F^{c+dx} x^2}{(a+bF^{c+dx})^3}\,dx$ 734
3.90 $\int \frac{F^{c+dx} x}{(a+bF^{c+dx})^3}\,dx$ 744
3.91 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})^3}\,dx$ 750
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3.1 $\int \frac{x^3}{a+be^{c+dx}}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \frac{x^4}{4a} - \frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{ad^3} - \frac{6\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{ad^4}$$

output
```
1/4*x^4/a-x^3*ln(1+b*exp(d*x+c)/a)/a/d-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a/
d^2+6*x*polylog(3,-b*exp(d*x+c)/a)/a/d^3-6*polylog(4,-b*exp(d*x+c)/a)/a/d^
4
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \frac{-d^3x^3\log\left(1+\frac{ae^{-c-dx}}{b}\right) + 3d^2x^2\,\text{PolyLog}\left(2,-\frac{ae^{-c-dx}}{b}\right) + 6dx\,\text{PolyLog}\left(3,-\frac{ae^{-c-dx}}{b}\right) + 6\,\text{PolyLog}\left(4,-\right.}{ad^4}$$

input
```
Integrate[x^3/(a + b*E^(c + d*x)),x]
```

output

```
(-(d^3*x^3*Log[1 + (a*E^(-c - d*x))/b]) + 3*d^2*x^2*PolyLog[2, -((a*E^(-c
- d*x))/b)] + 6*d*x*PolyLog[3, -((a*E^(-c - d*x))/b)] + 6*PolyLog[4, -((a*
E^(-c - d*x))/b)])/(a*d^4)
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+be^{c+dx}}\,dx$$

$$\downarrow 2615$$

$$\frac{x^4}{4a} - \frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}}dx}{a}$$

$$\downarrow 2620$$

$$\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}$$

$$\downarrow 3011$$

$$\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d} - \frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}$$

$$\downarrow 7163$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d} - \frac{\int \mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)dx}{d}\right)}{d} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}$$

$$\downarrow 2720$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d} - \frac{\int e^{-c-dx}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)de^{c+dx}}{d^2}\right)}{d} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}$$

$$\downarrow 7143$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d} - \frac{\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{d^2}\right)}{d} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}$$

input
```
Int[x^3/(a + b*E^(c + d*x)),x]
```

output
```
x^4/(4*a) - (b*((x^3*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (3*(-((x^2*PolyLo
g[2, -((b*E^(c + d*x))/a)])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x))/a)])/
d - PolyLog[4, -((b*E^(c + d*x))/a)]/d^2))/d))/(b*d)))/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.74

method	result
risch	$\frac{c^3 \ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{d^4 a} - \frac{c^3 \ln\left(\mathrm{e}^{dx+c}\right)}{d^4 a} - \frac{\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)c^3}{d^4 a} + \frac{x^4}{4a} + \frac{3c^4}{4d^4 a} - \frac{6\,\mathrm{polylog}\left(4,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a\,d^4} + \frac{c^3 x}{d^3 a} - \frac{x^3}{}$
derivativedivides	$-\frac{-\frac{(dx+c)^4}{4}+(dx+c)^3\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+3(dx+c)^2\,\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)-6(dx+c)\,\mathrm{polylog}\left(3,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+6\,\mathrm{polylog}\left(4,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right.}{a}$
default	$-\frac{-\frac{(dx+c)^4}{4}+(dx+c)^3\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+3(dx+c)^2\,\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)-6(dx+c)\,\mathrm{polylog}\left(3,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)+6\,\mathrm{polylog}\left(4,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right.}{a}$

input

```
int(x^3/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d^4*c^3/a*ln(a+b*exp(d*x+c))-1/d^4*c^3/a*ln(exp(d*x+c))-1/d^4/a*ln(1+b*e
xp(d*x+c)/a)*c^3+1/4*x^4/a+3/4/d^4/a*c^4-6*polylog(4,-b*exp(d*x+c)/a)/a/d^
4+1/d^3/a*c^3*x-x^3*ln(1+b*exp(d*x+c)/a)/a/d-3*x^2*polylog(2,-b*exp(d*x+c)
/a)/a/d^2+6*x*polylog(3,-b*exp(d*x+c)/a)/a/d^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{a+be^{c+dx}}\,dx$$
$$= \frac{d^4x^4 - 12\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + 4\,c^3\log\left(be^{(dx+c)}+a\right) + 24\,dx\,\mathrm{polylog}\left(3,-\frac{be^{(dx+c)}}{a}\right) - 4\,(d^3x^3+c^3}{4\,ad^4}$$

input

```
integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(b*e^(d*x + c) + a)/a + 1) + 4*c^3*log(b*
e^(d*x + c) + a) + 24*d*x*polylog(3, -b*e^(d*x + c)/a) - 4*(d^3*x^3 + c^3)
*log((b*e^(d*x + c) + a)/a) - 24*polylog(4, -b*e^(d*x + c)/a))/(a*d^4)
```

Sympy [F]

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \int \frac{x^3}{a+be^{c}e^{dx}}\,dx$$

input
```
integrate(x**3/(a+b*exp(d*x+c)),x)
```

output
```
Integral(x**3/(a + b*exp(c)*exp(d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \frac{x^4}{4\,a} - \frac{d^3x^3\log\left(\frac{be^{(dx+c)}}{a}+1\right) + 3\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 6\,dx\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a}) + 6\,\mathrm{Li}_4(-\frac{be^{(dx+c)}}{a})}{ad^4}$$

input
```
integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*x^4/a - (d^3*x^3*log(b*e^(d*x + c)/a + 1) + 3*d^2*x^2*dilog(-b*e^(d*x
+ c)/a) - 6*d*x*polylog(3, -b*e^(d*x + c)/a) + 6*polylog(4, -b*e^(d*x + c)
/a))/(a*d^4)
```

Giac [F]

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \int \frac{x^3}{be^{(dx+c)}+a}\,dx$$

input
```
integrate(x^3/(a+b*exp(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*e^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \int \frac{x^3}{a+b\,\mathrm{e}^{c+d\,x}}\,dx$$

input
```
int(x^3/(a + b*exp(c + d*x)),x)
```

output
```
int(x^3/(a + b*exp(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3}{a+be^{c+dx}}\,dx = \int \frac{x^3}{e^{dx+c}b+a}dx$$

input
```
int(x^3/(a+b*exp(d*x+c)),x)
```

output
```
int(x**3/(e**(c + d*x)*b + a),x)
```

3.2 $\int \frac{x^2}{a+be^{c+dx}}\,dx$

Optimal result . . . 69
Mathematica [A] (verified) . . . 69
Rubi [A] (verified) . . . 70
Maple [B] (verified) . . . 72
Fricas [A] (verification not implemented) . . . 73
Sympy [F] . . . 73
Maxima [A] (verification not implemented) . . . 73
Giac [F] . . . 74
Mupad [F(-1)] . . . 74
Reduce [F] . . . 74

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \frac{x^3}{3a} - \frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2} + \frac{2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{ad^3}$$

output

```
1/3*x^3/a-x^2*ln(1+b*exp(d*x+c)/a)/a/d-2*x*polylog(2,-b*exp(d*x+c)/a)/a/d^
2+2*polylog(3,-b*exp(d*x+c)/a)/a/d^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = -\frac{x^2\log\left(1+\frac{ae^{-c-dx}}{b}\right)}{ad} + \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{-c-dx}}{b}\right)}{ad^2} + \frac{2\,\mathrm{PolyLog}\left(3,-\frac{ae^{-c-dx}}{b}\right)}{ad^3}$$

input

```
Integrate[x^2/(a + b*E^(c + d*x)),x]
```

output
```
-((x^2*Log[1 + (a*E^(-c - d*x))/b])/(a*d)) + (2*x*PolyLog[2, -((a*E^(-c -
d*x))/b)])/(a*d^2) + (2*PolyLog[3, -((a*E^(-c - d*x))/b)])/(a*d^3)
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+be^{c+dx}}\,dx$$

$$\downarrow 2615$$

$$\frac{x^3}{3a} - \frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}\,dx}{a}$$

$$\downarrow 2620$$

$$\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}$$

$$\downarrow 3011$$

$$\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)de^{c+dx}}{d^2} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}$$

$$\downarrow 7143$$

$$\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d^2} - \frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}$$

input
```
Int[x^2/(a + b*E^(c + d*x)),x]
```

output
```
x^3/(3*a) - (b*((x^2*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (2*(-((x*PolyLog[
2, -((b*E^(c + d*x))/a)])/d) + PolyLog[3, -((b*E^(c + d*x))/a)]/d^2))/(b*d
)))/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(79) = 158.

Time = 0.02 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.93

method	result
derivativedivides	$\frac{c^2\left(\frac{\ln\left(e^{dx+c}\right)}{a}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a}\right)-\frac{-\frac{(dx+c)^3}{3}+(dx+c)^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+2(dx+c)\,\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)-2\,\mathrm{polylog}\left(3,-\frac{b\,e^{dx}}{a}\right.}{a}}{d^3}$
default	$\frac{c^2\left(\frac{\ln\left(e^{dx+c}\right)}{a}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a}\right)-\frac{-\frac{(dx+c)^3}{3}+(dx+c)^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+2(dx+c)\,\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)-2\,\mathrm{polylog}\left(3,-\frac{b\,e^{dx}}{a}\right.}{a}}{d^3}$
risch	$\frac{c^2\ln\left(e^{dx+c}\right)}{d^3a}-\frac{c^2\ln\left(a+b\,e^{dx+c}\right)}{d^3a}+\frac{x^3}{3a}-\frac{c^2x}{d^2a}-\frac{2c^3}{3d^3a}-\frac{x^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{ad}+\frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c^2}{d^3a}-\frac{2x\,\mathrm{poly}}{}$

input

```
int(x^2/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d^3*(c^2*(1/a*ln(exp(d*x+c))-1/a*ln(a+b*exp(d*x+c)))-1/a*(-1/3*(d*x+c)^3
+(d*x+c)^2*ln(1+b*exp(d*x+c)/a)+2*(d*x+c)*polylog(2,-b*exp(d*x+c)/a)-2*pol
ylog(3,-b*exp(d*x+c)/a))+2*c/a*(-1/2*(d*x+c)^2+(d*x+c)*ln(1+b*exp(d*x+c)/a
)+polylog(2,-b*exp(d*x+c)/a)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{a+be^{c+dx}}\,dx$$
$$= \frac{d^3x^3 - 6\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) - 3\,c^2\log\left(be^{(dx+c)}+a\right) - 3\left(d^2x^2-c^2\right)\log\left(\frac{be^{(dx+c)}+a}{a}\right) + 6\,\mathrm{polylog}\left(3,\right.}{3\,ad^3}$$

input
```
integrate(x^2/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(d^3*x^3 - 6*d*x*dilog(-(b*e^(d*x + c) + a)/a + 1) - 3*c^2*log(b*e^(d*
x + c) + a) - 3*(d^2*x^2 - c^2)*log((b*e^(d*x + c) + a)/a) + 6*polylog(3,
-b*e^(d*x + c)/a))/(a*d^3)
```

Sympy [F]

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \int \frac{x^2}{a+be^ce^{dx}}\,dx$$

input
```
integrate(x**2/(a+b*exp(d*x+c)),x)
```

output
```
Integral(x**2/(a + b*exp(c)*exp(d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \frac{x^3}{3\,a} - \frac{d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right) + 2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})}{ad^3}$$

input
```
integrate(x^2/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*x^3/a - (d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)
/a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a*d^3)
```

Giac [F]

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \int \frac{x^2}{be^{(dx+c)}+a}\,dx$$

input
```
integrate(x^2/(a+b*exp(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*e^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \int \frac{x^2}{a+b\,\mathrm{e}^{c+d\,x}}\,dx$$

input
```
int(x^2/(a + b*exp(c + d*x)),x)
```

output
```
int(x^2/(a + b*exp(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2}{a+be^{c+dx}}\,dx = \int \frac{x^2}{e^{dx+c}b+a}dx$$

input
```
int(x^2/(a+b*exp(d*x+c)),x)
```

output
```
int(x**2/(e**(c + d*x)*b + a),x)
```

3.3 $\int \frac{x}{a+be^{c+dx}}\,dx$

Optimal result . . . 75
Mathematica [A] (verified) . . . 75
Rubi [A] (verified) . . . 76
Maple [A] (verified) . . . 77
Fricas [A] (verification not implemented) . . . 78
Sympy [F] . . . 78
Maxima [A] (verification not implemented) . . . 79
Giac [F] . . . 79
Mupad [F(-1)] . . . 79
Reduce [F] . . . 80

Optimal result

Integrand size = 15, antiderivative size = 58

$$\int \frac{x}{a+be^{c+dx}}\,dx = \frac{x^2}{2a} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{ad} - \frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{ad^2}$$

output
```
1/2*x^2/a-x*ln(1+b*exp(d*x+c)/a)/a/d-polylog(2,-b*exp(d*x+c)/a)/a/d^2
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{x}{a+be^{c+dx}}\,dx = -\frac{x\log\left(1+\frac{ae^{-c-dx}}{b}\right)}{ad} + \frac{\text{PolyLog}\left(2,-\frac{ae^{-c-dx}}{b}\right)}{ad^2}$$

input
```
Integrate[x/(a + b*E^(c + d*x)),x]
```

output
```
-((x*Log[1 + (a*E^(-c - d*x))/b])/(a*d)) + PolyLog[2, -((a*E^(-c - d*x))/b
)]/(a*d^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+be^{c+dx}}\,dx$$

$$\downarrow 2615$$

$$\frac{x^2}{2a} - \frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}$$

$$\downarrow 2620$$

$$\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{\int \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}$$

$$\downarrow 2715$$

$$\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{x^2}{2a} - \frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2} + \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}$$

input
```
Int[x/(a + b*E^(c + d*x)),x]
```

output
```
x^2/(2*a) - (b*((x*Log[1 + (b*E^(c + d*x))/a])/(b*d) + PolyLog[2, -((b*E^(
c + d*x))/a)]/(b*d^2)))/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{(dx+c)^2}{2}+(dx+c)\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)}{a}-\frac{c\ln\left(e^{dx+c}\right)}{a}+\frac{c\ln\left(a+b\,e^{dx+c}\right)}{a}}{d^2}$
default	$\frac{-\frac{(dx+c)^2}{2}+(dx+c)\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)+\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)}{a}-\frac{c\ln\left(e^{dx+c}\right)}{a}+\frac{c\ln\left(a+b\,e^{dx+c}\right)}{a}}{d^2}$
risch	$\frac{x^2}{2a}+\frac{cx}{da}+\frac{c^2}{2d^2a}-\frac{x\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{ad}-\frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c}{d^2a}-\frac{\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)}{a\,d^2}-\frac{c\ln\left(e^{dx+c}\right)}{d^2a}+\frac{c\ln(a+}{d}$

input
```
int(x/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(-1/a*(-1/2*(d*x+c)^2+(d*x+c)*ln(1+b*exp(d*x+c)/a)+polylog(2,-b*exp(
d*x+c)/a))-c/a*ln(exp(d*x+c))+c/a*ln(a+b*exp(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int \frac{x}{a+be^{c+dx}}\,dx = \frac{d^2x^2 + 2\,c\log\left(be^{(dx+c)}+a\right) - 2\,(dx+c)\log\left(\frac{be^{(dx+c)}+a}{a}\right) - 2\,\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right)}{2\,ad^2}$$

input
```
integrate(x/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(d^2*x^2 + 2*c*log(b*e^(d*x + c) + a) - 2*(d*x + c)*log((b*e^(d*x + c)
 + a)/a) - 2*dilog(-(b*e^(d*x + c) + a)/a + 1))/(a*d^2)
```

Sympy [F]

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{a+be^{c}e^{dx}}\,dx$$

input
```
integrate(x/(a+b*exp(d*x+c)),x)
```

output
```
Integral(x/(a + b*exp(c)*exp(d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{x}{a+be^{c+dx}}\,dx = \frac{x^2}{2\,a} - \frac{dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{ad^2}$$

input
```
integrate(x/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*x^2/a - (d*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a*d^
2)
```

Giac [F]

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{be^{(dx+c)}+a}\,dx$$

input
```
integrate(x/(a+b*exp(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x/(b*e^(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{a+b\,e^{c+d\,x}}\,dx$$

input
```
int(x/(a + b*exp(c + d*x)),x)
```

output
```
int(x/(a + b*exp(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x}{a+be^{c+dx}}\,dx = \int \frac{x}{e^{dx+c}b+a}dx$$

input
```
int(x/(a+b*exp(d*x+c)),x)
```

output
```
int(x/(e**(c + d*x)*b + a),x)
```

3.4 $\int \frac{1}{a+be^{c+dx}}\,dx$

Optimal result 81
Mathematica [A] (verified) 81
Rubi [A] (verified) 82
Maple [A] (verified) 83
Fricas [A] (verification not implemented) 84
Sympy [A] (verification not implemented) 84
Maxima [A] (verification not implemented) 84
Giac [A] (verification not implemented) 85
Mupad [B] (verification not implemented) 85
Reduce [B] (verification not implemented) 85

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{x}{a} - \frac{\log\left(a+be^{c+dx}\right)}{ad}$$

output

```
x/a-ln(a+b*exp(d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{\log\left(e^{c+dx}\right)}{ad} - \frac{\log\left(a^2d+abde^{c+dx}\right)}{ad}$$

input

```
Integrate[(a + b*E^(c + d*x))^(-1),x]
```

output

```
Log[E^(c + d*x)]/(a*d) - Log[a^2*d + a*b*d*E^(c + d*x)]/(a*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+be^{c+dx}}\,dx$$

$$\downarrow 2720$$

$$\frac{\int \frac{e^{-c-dx}}{a+be^{c+dx}}\,de^{c+dx}}{d}$$

$$\downarrow 47$$

$$\frac{\frac{\int e^{-c-dx}\,de^{c+dx}}{a} - \frac{b\int \frac{1}{a+be^{c+dx}}\,de^{c+dx}}{a}}{d}$$

$$\downarrow 14$$

$$\frac{\frac{\log\left(e^{c+dx}\right)}{a} - \frac{b\int \frac{1}{a+be^{c+dx}}\,de^{c+dx}}{a}}{d}$$

$$\downarrow 16$$

$$\frac{\frac{\log\left(e^{c+dx}\right)}{a} - \frac{\log\left(a+be^{c+dx}\right)}{a}}{d}$$

input
```
Int[(a + b*E^(c + d*x))^(-1),x]
```

output
```
(Log[E^(c + d*x)]/a - Log[a + b*E^(c + d*x)]/a)/d
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$-\frac{-dx+\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{da}$	25
norman	$\frac{x}{a}-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{ad}$	26
derivativedivides	$\frac{\frac{\ln\left(\mathrm{e}^{dx+c}\right)}{a}-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a}}{d}$	33
default	$\frac{\frac{\ln\left(\mathrm{e}^{dx+c}\right)}{a}-\frac{\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a}}{d}$	33
risch	$\frac{x}{a}+\frac{c}{da}-\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a}{b}\right)}{da}$	36

input
```
int(1/(a+b*exp(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(-d*x+ln(a+b*exp(d*x+c)))/d/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{dx - \log\left(be^{(dx+c)}+a\right)}{ad}$$

input
```
integrate(1/(a+b*exp(d*x+c)),x, algorithm="fricas")
```

output
```
(d*x - log(b*e^(d*x + c) + a))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{x}{a} - \frac{\log\left(\frac{a}{b}+e^{c+dx}\right)}{ad}$$

input
```
integrate(1/(a+b*exp(d*x+c)),x)
```

output
```
x/a - log(a/b + exp(c + d*x))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{dx+c}{ad} - \frac{\log\left(be^{(dx+c)}+a\right)}{ad}$$

input
```
integrate(1/(a+b*exp(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) - log(b*e^(d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{\frac{dx+c}{a} - \frac{\log\left(\left|be^{(dx+c)}+a\right|\right)}{a}}{d}$$

input
```
integrate(1/(a+b*exp(d*x+c)),x, algorithm="giac")
```

output
```
((d*x + c)/a - log(abs(b*e^(d*x + c) + a))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+be^{c+dx}}\,dx = -\frac{\ln\left(a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)-d\,x}{a\,d}$$

input
```
int(1/(a + b*exp(c + d*x)),x)
```

output
```
-(log(a + b*exp(d*x)*exp(c)) - d*x)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+be^{c+dx}}\,dx = \frac{-\log\left(e^{dx+c}b+a\right)+dx}{ad}$$

input
```
int(1/(a+b*exp(d*x+c)),x)
```

output
```
( - log(e**(c + d*x)*b + a) + d*x)/(a*d)
```

3.5 $\int \frac{1}{(a+be^{c+dx})x}\,dx$

Optimal result 86
Mathematica [N/A] 86
Rubi [N/A] 87
Maple [N/A] 87
Fricas [N/A] 88
Sympy [N/A] 88
Maxima [N/A] 89
Giac [N/A] 89
Mupad [N/A] 89
Reduce [N/A] 90

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \text{Int}\left(\frac{1}{(a+be^{c+dx})\,x}, x\right)$$

output
```
Defer(Int)(1/(a+b*exp(d*x+c))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{(a+be^{c+dx})\,x}\,dx$$

input
```
Integrate[1/((a + b*E^(c + d*x))*x),x]
```

output
```
Integrate[1/((a + b*E^(c + d*x))*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+be^{c+dx}\right)}\,dx$$
$$\downarrow 2619$$
$$\int \frac{1}{x\left(a+be^{c+dx}\right)}dx$$

input
```
Int[1/((a + b*E^(c + d*x))*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)x}dx$$

input
```
int(1/(a+b*exp(d*x+c))/x,x)
```

output
```
int(1/(a+b*exp(d*x+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+be^{c+dx}\right)x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x,x, algorithm="fricas")
```

output
```
integral(1/(b*x*e^(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+be^{c+dx}\right)x}\,dx = \int \frac{1}{x\left(a+be^{c}e^{dx}\right)}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x,x)
```

output
```
Integral(1/(x*(a + b*exp(c)*exp(d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x,x, algorithm="maxima")
```

output
```
integrate(1/((b*e^(d*x + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x,x, algorithm="giac")
```

output
```
integrate(1/((b*e^(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 23.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+be^{c+dx})\,x}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}\,dx$$

input
```
int(1/(x*(a + b*exp(c + d*x))),x)
```

output
```
int(1/(x*(a + b*exp(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)x}\,dx = \int \frac{1}{e^{dx+c}bx+ax}dx$$

input
```
int(1/(a+b*exp(d*x+c))/x,x)
```

output
```
int(1/(e**(c + d*x)*b*x + a*x),x)
```

3.6 $\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx$

Optimal result 91
Mathematica [N/A] 91
Rubi [N/A] 92
Maple [N/A] 92
Fricas [N/A] 93
Sympy [N/A] 93
Maxima [N/A] 94
Giac [N/A] 94
Mupad [N/A] 94
Reduce [N/A] 95

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)x^2}, x\right)$$

output
```
Defer(Int)(1/(a+b*exp(d*x+c))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx$$

input
```
Integrate[1/((a + b*E^(c + d*x))*x^2),x]
```

output
```
Integrate[1/((a + b*E^(c + d*x))*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(a+be^{c+dx}\right)}\,dx$$
$$\downarrow 2619$$
$$\int \frac{1}{x^2\left(a+be^{c+dx}\right)}dx$$

input
```
Int[1/((a + b*E^(c + d*x))*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)x^2}dx$$

input
```
int(1/(a+b*exp(d*x+c))/x^2,x)
```

output
```
int(1/(a+b*exp(d*x+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x^2,x, algorithm="fricas")
```

output
```
integral(1/(b*x^2*e^(d*x + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \int \frac{1}{x^2\left(a+be^{c}e^{dx}\right)}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x**2,x)
```

output
```
Integral(1/(x**2*(a + b*exp(c)*exp(d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x^2,x, algorithm="maxima")
```

output
```
integrate(1/((b*e^(d*x + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \int \frac{1}{(be^{(dx+c)}+a)x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))/x^2,x, algorithm="giac")
```

output
```
integrate(1/((b*e^(d*x + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)x^2}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{e}^{c+d\,x}\right)}\,dx$$

input
```
int(1/(x^2*(a + b*exp(c + d*x))),x)
```

output
```
int(1/(x^2*(a + b*exp(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{1}{\left(a + be^{c+dx}\right) x^2}\, dx = \int \frac{1}{e^{dx+c} b\, x^2 + a\, x^2} dx$$

input
```
int(1/(a+b*exp(d*x+c))/x^2,x)
```

output
```
int(1/(e**(c + d*x)*b*x**2 + a*x**2),x)
```

3.7 $\int \frac{1}{a+be^{c-dx}}\,dx$

Optimal result 96
Mathematica [A] (verified) 96
Rubi [A] (verified) 97
Maple [A] (verified) 98
Fricas [A] (verification not implemented) 99
Sympy [A] (verification not implemented) 99
Maxima [A] (verification not implemented) 99
Giac [A] (verification not implemented) 100
Mupad [B] (verification not implemented) 100
Reduce [B] (verification not implemented) 100

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{x}{a} + \frac{\log\left(a+be^{c-dx}\right)}{ad}$$

output
```
x/a+ln(a+b*exp(-d*x+c))/a/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\log\left(be^{c}+ae^{dx}\right)}{ad}$$

input
```
Integrate[(a + b*E^(c - d*x))^(-1),x]
```

output
```
Log[b*E^c + a*E^(d*x)]/(a*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+be^{c-dx}}\,dx$$

$$\downarrow 2720$$

$$-\frac{\int \frac{e^{dx-c}}{a+be^{c-dx}}\,de^{c-dx}}{d}$$

$$\downarrow 47$$

$$-\frac{\frac{\int e^{dx-c}de^{c-dx}}{a}-\frac{b\int \frac{1}{a+be^{c-dx}}\,de^{c-dx}}{a}}{d}$$

$$\downarrow 14$$

$$-\frac{\frac{\log\left(e^{c-dx}\right)}{a}-\frac{b\int \frac{1}{a+be^{c-dx}}\,de^{c-dx}}{a}}{d}$$

$$\downarrow 16$$

$$-\frac{\frac{\log\left(e^{c-dx}\right)}{a}-\frac{\log\left(a+be^{c-dx}\right)}{a}}{d}$$

input
```
Int[(a + b*E^(c - d*x))^(-1),x]
```

output
```
-((Log[E^(c - d*x)]/a - Log[a + b*E^(c - d*x)]/a)/d)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{dx+\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{da}$	24
norman	$\frac{x}{a}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	26
derivativedivides	$-\frac{\ln\left(\mathrm{e}^{-dx+c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	37
default	$-\frac{\ln\left(\mathrm{e}^{-dx+c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx+c}\right)}{ad}$	37
risch	$\frac{x}{a}-\frac{c}{da}+\frac{\ln\left(\mathrm{e}^{-dx+c}+\frac{a}{b}\right)}{da}$	37

input
```
int(1/(a+b*exp(-d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x+ln(a+b*exp(-d*x+c)))/d/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{dx+\log\left(be^{(-dx+c)}+a\right)}{ad}$$

input
```
integrate(1/(a+b*exp(-d*x+c)),x, algorithm="fricas")
```

output
```
(d*x + log(b*e^(-d*x + c) + a))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{x}{a}+\frac{\log\left(\frac{a}{b}+e^{c-dx}\right)}{ad}$$

input
```
integrate(1/(a+b*exp(-d*x+c)),x)
```

output
```
x/a + log(a/b + exp(c - d*x))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{dx-c}{ad}+\frac{\log\left(be^{(-dx+c)}+a\right)}{ad}$$

input
```
integrate(1/(a+b*exp(-d*x+c)),x, algorithm="maxima")
```

output
```
(d*x - c)/(a*d) + log(b*e^(-d*x + c) + a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\frac{dx-c}{a} + \frac{\log\left(\left|be^{(-dx+c)}+a\right|\right)}{a}}{d}$$

input
```
integrate(1/(a+b*exp(-d*x+c)),x, algorithm="giac")
```

output
```
((d*x - c)/a + log(abs(b*e^(-d*x + c) + a))/a)/d
```

Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^{-d\,x}\,\mathrm{e}^{c}\right)+d\,x}{a\,d}$$

input
```
int(1/(a + b*exp(c - d*x)),x)
```

output
```
(log(a + b*exp(-d*x)*exp(c)) + d*x)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+be^{c-dx}}\,dx = \frac{\log\left(e^{dx}a+e^{c}b\right)}{ad}$$

input
```
int(1/(a+b*exp(-d*x+c)),x)
```

output
```
log(e**(d*x)*a + e**c*b)/(a*d)
```

3.8 $\int \frac{1}{a+be^{-c-dx}}\,dx$

Optimal result 101
Mathematica [A] (verified) 101
Rubi [A] (verified) 102
Maple [A] (verified) 103
Fricas [A] (verification not implemented) 104
Sympy [A] (verification not implemented) 104
Maxima [A] (verification not implemented) 104
Giac [A] (verification not implemented) 105
Mupad [B] (verification not implemented) 105
Reduce [B] (verification not implemented) 105

Optimal result

Integrand size = 16, antiderivative size = 28

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{x}{a} + \frac{\log\left(a+be^{-c-dx}\right)}{ad}$$

output

```
x/a+ln(a+b*exp(-d*x-c))/a/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{\log\left(b+ae^{c+dx}\right)}{ad}$$

input

```
Integrate[(a + b*E^(-c - d*x))^(-1),x]
```

output

```
Log[b + a*E^(c + d*x)]/(a*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+be^{-c-dx}}\,dx$$

$$\downarrow 2720$$

$$-\frac{\int \frac{e^{c+dx}}{a+be^{-c-dx}}\,de^{-c-dx}}{d}$$

$$\downarrow 47$$

$$-\frac{\frac{\int e^{c+dx}\,de^{-c-dx}}{a}-\frac{b\int \frac{1}{a+be^{-c-dx}}\,de^{-c-dx}}{a}}{d}$$

$$\downarrow 14$$

$$-\frac{\frac{\log\left(e^{-c-dx}\right)}{a}-\frac{b\int \frac{1}{a+be^{-c-dx}}\,de^{-c-dx}}{a}}{d}$$

$$\downarrow 16$$

$$-\frac{\frac{\log\left(e^{-c-dx}\right)}{a}-\frac{\log\left(a+be^{-c-dx}\right)}{a}}{d}$$

input
```
Int[(a + b*E^(-c - d*x))^(-1),x]
```

output
```
-((Log[E^(-c - d*x)]/a - Log[a + b*E^(-c - d*x)]/a)/d)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{dx+\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{da}$	26
norman	$\frac{x}{a}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	28
risch	$\frac{x}{a}+\frac{c}{da}+\frac{\ln\left(\mathrm{e}^{-dx-c}+\frac{a}{b}\right)}{da}$	38
derivativedivides	$-\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	41
default	$-\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{da}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{ad}$	41

input
```
int(1/(a+b*exp(-d*x-c)),x,method=_RETURNVERBOSE)
```

output
```
(d*x+ln(a+b*exp(-d*x-c)))/d/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{dx+\log\left(be^{(-dx-c)}+a\right)}{ad}$$

input
```
integrate(1/(a+b*exp(-d*x-c)),x, algorithm="fricas")
```

output
```
(d*x + log(b*e^(-d*x - c) + a))/(a*d)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{x}{a}+\frac{\log\left(\frac{a}{b}+e^{-c-dx}\right)}{ad}$$

input
```
integrate(1/(a+b*exp(-d*x-c)),x)
```

output
```
x/a + log(a/b + exp(-c - d*x))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{dx+c}{ad}+\frac{\log\left(be^{(-dx-c)}+a\right)}{ad}$$

input
```
integrate(1/(a+b*exp(-d*x-c)),x, algorithm="maxima")
```

output
```
(d*x + c)/(a*d) + log(b*e^(-d*x - c) + a)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{\frac{dx+c}{a} + \frac{\log\left(\left|be^{(-dx-c)}+a\right|\right)}{a}}{d}$$

input
```
integrate(1/(a+b*exp(-d*x-c)),x, algorithm="giac")
```

output
```
((d*x + c)/a + log(abs(b*e^(-d*x - c) + a))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{\ln\left(a+b\,\mathrm{e}^{-c}\,\mathrm{e}^{-d\,x}\right)+d\,x}{a\,d}$$

input
```
int(1/(a + b*exp(- c - d*x)),x)
```

output
```
(log(a + b*exp(-c)*exp(-d*x)) + d*x)/(a*d)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{a+be^{-c-dx}}\,dx = \frac{\log\left(e^{dx+c}a+b\right)}{ad}$$

input
```
int(1/(a+b*exp(-d*x-c)),x)
```

output
```
log(e**(c + d*x)*a + b)/(a*d)
```

3.9 $\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx$

Optimal result . . . 106
Mathematica [A] (verified) . . . 107
Rubi [A] (verified) . . . 107
Maple [A] (verified) . . . 116
Fricas [A] (verification not implemented) . . . 117
Sympy [F] . . . 117
Maxima [A] (verification not implemented) . . . 118
Giac [F] . . . 118
Mupad [F(-1)] . . . 119
Reduce [F] . . . 119

Optimal result

Integrand size = 17, antiderivative size = 217

$$\begin{aligned}\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx = &-\frac{x^3}{a^2d}+\frac{x^3}{ad\left(a+be^{c+dx}\right)}+\frac{x^4}{4a^2}+\frac{3x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2}\\ &-\frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d}+\frac{6x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}\\ &-\frac{3x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2}-\frac{6\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}\\ &+\frac{6x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}-\frac{6\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^2d^4}\end{aligned}$$

output

```
-x^3/a^2/d+x^3/a/d/(a+b*exp(d*x+c))+1/4*x^4/a^2+3*x^2*ln(1+b*exp(d*x+c)/a)
/a^2/d^2-x^3*ln(1+b*exp(d*x+c)/a)/a^2/d+6*x*polylog(2,-b*exp(d*x+c)/a)/a^2
/d^3-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2-6*polylog(3,-b*exp(d*x+c)/a)
/a^2/d^4+6*x*polylog(3,-b*exp(d*x+c)/a)/a^2/d^3-6*polylog(4,-b*exp(d*x+c)/
a)/a^2/d^4
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx$$

$$= \frac{-\frac{4x^3}{d}+\frac{4ax^3}{ad+bde^{c+dx}}+x^4+\frac{12x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{4x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{d}-\frac{12x(-2+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d^3}+\frac{24(-1+dx)\,\mathrm{Poly}}{4a^2}$$

input
```
Integrate[x^3/(a + b*E^(c + d*x))^2,x]
```

output
```
((-4*x^3)/d + (4*a*x^3)/(a*d + b*d*E^(c + d*x)) + x^4 + (12*x^2*Log[1 + (b
*E^(c + d*x))/a])/d^2 - (4*x^3*Log[1 + (b*E^(c + d*x))/a])/d - (12*x*(-2 +
 d*x)*PolyLog[2, -((b*E^(c + d*x))/a)])/d^3 + (24*(-1 + d*x)*PolyLog[3, -(
(b*E^(c + d*x))/a)])/d^4 - (24*PolyLog[4, -((b*E^(c + d*x))/a)])/d^4)/(4*a
^2)
```

Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {2616, 2615, 2620, 2621, 2615, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{x^3}{a+be^{c+dx}}dx}{a}-\frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^4}{4a}-\frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}}dx}{a}}{a}-\frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2 \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2 \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{3\int \frac{x^2}{a+be^{c+dx}}dx}{bd} - \frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2 \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{3\left(\frac{x^3}{3a} - \frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}\right)}{bd} - \frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2 \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{3\left(\frac{x^3}{3a} - \frac{b\left(\frac{x^2 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\int x \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd} - \frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 3011$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\int x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{\left(b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\int\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\int x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}-b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)de^{c+dx}}{d^2}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 7143$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\int x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

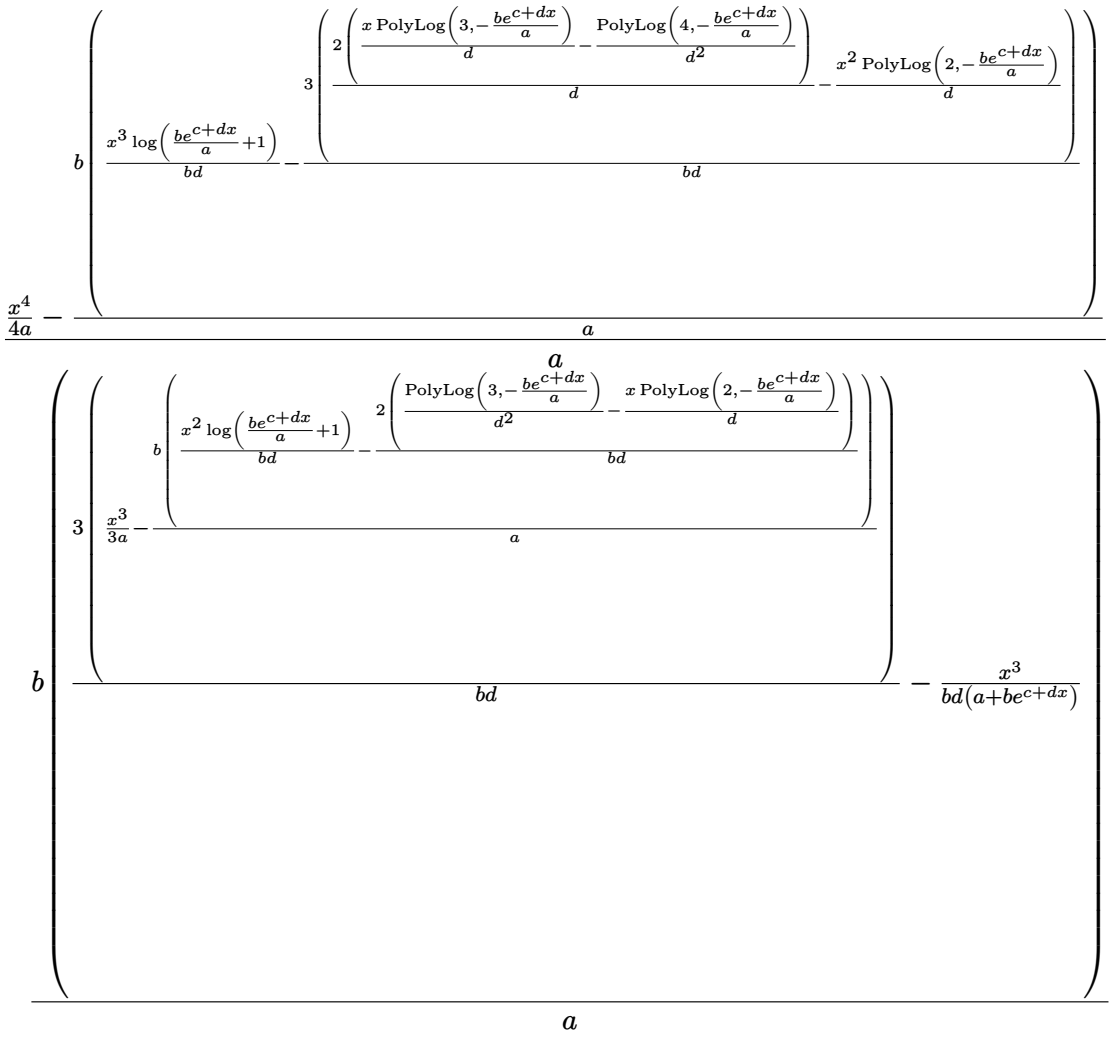
↓ 7163

$$\frac{x^4}{4a}-\frac{\left(b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d}-\frac{\int\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)dx}{d}\right)}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)\right)}{a}-\frac{\left(b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a}$$

$$\downarrow 2720$$

$$\frac{x^4}{4a}-\frac{\left(b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d}-\frac{\int e^{-c-dx}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)\right)}{a}-\frac{\left(b\left(\frac{x^3}{3a}-\frac{3\left(b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 7143$$



input
```
Int[x^3/(a + b*E^(c + d*x))^2,x]
```

output
```
-((b*(-(x^3/(b*d*(a + b*E^(c + d*x)))) + (3*(x^3/(3*a) - (b*((x^2*Log[1 +
(b*E^(c + d*x))/a])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/a)])/d)
+ PolyLog[3, -((b*E^(c + d*x))/a)]/d^2))/(b*d)))/a))/(b*d)))/a) + (x^4/(4*
a) - (b*((x^3*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (3*(-((x^2*PolyLog[2, -(
(b*E^(c + d*x))/a)])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x))/a)])/d - Pol
yLog[4, -((b*E^(c + d*x))/a)]/d^2))/d))/(b*d)))/a)/a
```

Defintions of rubi rules used

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.76

method	result
risch	$\frac{x^3}{ad\left(a+b\,\mathrm{e}^{dx+c}\right)}+\frac{3c^2\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2d^4}+\frac{3x^2\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2d^2}+\frac{6x\,\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2d^3}+\frac{c^3x}{a^2d^3}-\frac{x^3\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^2d}-\frac{3}{}$

input

```
int(x^3/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
x^3/a/d/(a+b*exp(d*x+c))+3/a^2/d^4*c^2*ln(a+b*exp(d*x+c))+3*x^2*ln(1+b*exp
(d*x+c)/a)/a^2/d^2+6*x*polylog(2,-b*exp(d*x+c)/a)/a^2/d^3+1/a^2/d^3*c^3*x-
x^3*ln(1+b*exp(d*x+c)/a)/a^2/d-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2+6*
x*polylog(3,-b*exp(d*x+c)/a)/a^2/d^3+3/a^2/d^3*c^2*x+1/a^2/d^4*c^3*ln(a+b*
exp(d*x+c))-1/a^2/d^4*c^3*ln(exp(d*x+c))+1/4*x^4/a^2-1/a^2/d^4*ln(1+b*exp(
d*x+c)/a)*c^3-x^3/a^2/d-3/a^2/d^4*ln(1+b*exp(d*x+c)/a)*c^2-3/a^2/d^4*c^2*l
n(exp(d*x+c))+3/4/a^2/d^4*c^4-6*polylog(4,-b*exp(d*x+c)/a)/a^2/d^4+2/a^2/d
^4*c^3-6*polylog(3,-b*exp(d*x+c)/a)/a^2/d^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx$$
$$= \frac{ad^4x^4 - ac^4 - 4\,ac^3 - 12\left(ad^2x^2 - 2\,adx + (bd^2x^2 - 2\,bdx)e^{(dx+c)}\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + (bd^4x^4 - 4\,bd^{}$$

input
```
integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(a*d^4*x^4 - a*c^4 - 4*a*c^3 - 12*(a*d^2*x^2 - 2*a*d*x + (b*d^2*x^2 -
2*b*d*x)*e^(d*x + c))*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b*d^4*x^4 - 4*b
*d^3*x^3 - b*c^4 - 4*b*c^3)*e^(d*x + c) + 4*(a*c^3 + 3*a*c^2 + (b*c^3 + 3*
b*c^2)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 4*(a*d^3*x^3 - 3*a*d^2*x^2 +
a*c^3 + 3*a*c^2 + (b*d^3*x^3 - 3*b*d^2*x^2 + b*c^3 + 3*b*c^2)*e^(d*x + c))
*log((b*e^(d*x + c) + a)/a) - 24*(b*e^(d*x + c) + a)*polylog(4, -b*e^(d*x
+ c)/a) + 24*(a*d*x + (b*d*x - b)*e^(d*x + c) - a)*polylog(3, -b*e^(d*x +
c)/a))/(a^2*b*d^4*e^(d*x + c) + a^3*d^4)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{x^3}{a^2d + abde^{c+dx}} + \frac{\int \left(-\frac{3x^2}{a+be^ce^{dx}}\right)\,dx + \int \frac{dx^3}{a+be^ce^{dx}}\,dx}{ad}$$

input
```
integrate(x**3/(a+b*exp(d*x+c))**2,x)
```

output
```
x**3/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(-3*x**2/(a + b*exp(c)*exp(d
*x)), x) + Integral(d*x**3/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{x^3}{abde^{(dx+c)}+a^2d} + \frac{d^4x^4-4\,d^3x^3}{4\,a^2d^4} - \frac{d^3x^3\log\left(\frac{be^{(dx+c)}}{a}+1\right)+3\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)-6\,dx\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})+6\,\mathrm{Li}_4(-\frac{be^{(dx+c)}}{a})}{a^2d^4} + \frac{3\left(d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right)+2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)-2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})\right)}{a^2d^4}$$

input
```
integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

output
```
x^3/(a*b*d*e^(d*x + c) + a^2*d) + 1/4*(d^4*x^4 - 4*d^3*x^3)/(a^2*d^4) - (d
^3*x^3*log(b*e^(d*x + c)/a + 1) + 3*d^2*x^2*dilog(-b*e^(d*x + c)/a) - 6*d*
x*polylog(3, -b*e^(d*x + c)/a) + 6*polylog(4, -b*e^(d*x + c)/a))/(a^2*d^4)
 + 3*(d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)/a) - 2
*polylog(3, -b*e^(d*x + c)/a))/(a^2*d^4)
```

Giac [F]

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}\,dx = \int \frac{x^3}{\left(be^{(dx+c)}+a\right)^2}\,dx$$

input
```
integrate(x^3/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(x^3/(b*e^(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = \int \frac{x^3}{(a+b\,\mathrm{e}^{c+d\,x})^2}\,dx$$

input
```
int(x^3/(a + b*exp(c + d*x))^2,x)
```

output
```
int(x^3/(a + b*exp(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{x^3}{(a+be^{c+dx})^2}\,dx = \int \frac{x^3}{e^{2dx+2c}b^2+2e^{dx+c}ab+a^2}dx$$

input
```
int(x^3/(a+b*exp(d*x+c))^2,x)
```

output
```
int(x**3/(e**(2*c + 2*d*x)*b**2 + 2*e**(c + d*x)*a*b + a**2),x)
```

3.10 $\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx$

Optimal result . . . 120
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Maxima [A] (verification not implemented) . . . 128
Giac [F] . . . 129
Mupad [F(-1)] . . . 129
Reduce [F] . . . 130

Optimal result

Integrand size = 17, antiderivative size = 165

$$\begin{aligned}\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx = &-\frac{x^2}{a^2 d}+\frac{x^2}{ad\left(a+be^{c+dx}\right)}+\frac{x^3}{3a^2}+\frac{2x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d^2}\\ &-\frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2d}+\frac{2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}\\ &-\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2d^2}+\frac{2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^2d^3}\end{aligned}$$

output

```
-x^2/a^2/d+x^2/a/d/(a+b*exp(d*x+c))+1/3*x^3/a^2+2*x*ln(1+b*exp(d*x+c)/a)/a
^2/d^2-x^2*ln(1+b*exp(d*x+c)/a)/a^2/d+2*polylog(2,-b*exp(d*x+c)/a)/a^2/d^3
-2*x*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2+2*polylog(3,-b*exp(d*x+c)/a)/a^2/d
^3
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx$$
$$= \frac{\frac{d^2x^2(adx+be^{c+dx}(-3+dx))}{a+be^{c+dx}} - 3dx(-2+dx)\log\left(1+\frac{be^{c+dx}}{a}\right) + (6-6dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right) + 6\,\mathrm{PolyLog}\left(}{3a^2d^3}$$

input
```
Integrate[x^2/(a + b*E^(c + d*x))^2,x]
```

output
```
((d^2*x^2*(a*d*x + b*E^(c + d*x)*(-3 + d*x)))/(a + b*E^(c + d*x)) - 3*d*x*
(-2 + d*x)*Log[1 + (b*E^(c + d*x))/a] + (6 - 6*d*x)*PolyLog[2, -((b*E^(c +
 d*x))/a)] + 6*PolyLog[3, -((b*E^(c + d*x))/a)])/(3*a^2*d^3)
```

Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx$$
$$\downarrow 2616$$
$$\frac{\int \frac{x^2}{a+be^{c+dx}}dx}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}$$
$$\downarrow 2615$$
$$\frac{\frac{x^3}{3a} - \frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}$$
$$\downarrow 2620$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\int\frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\int\frac{x}{a+be^{c+dx}}dx}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\int\frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2715$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}-b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}-b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

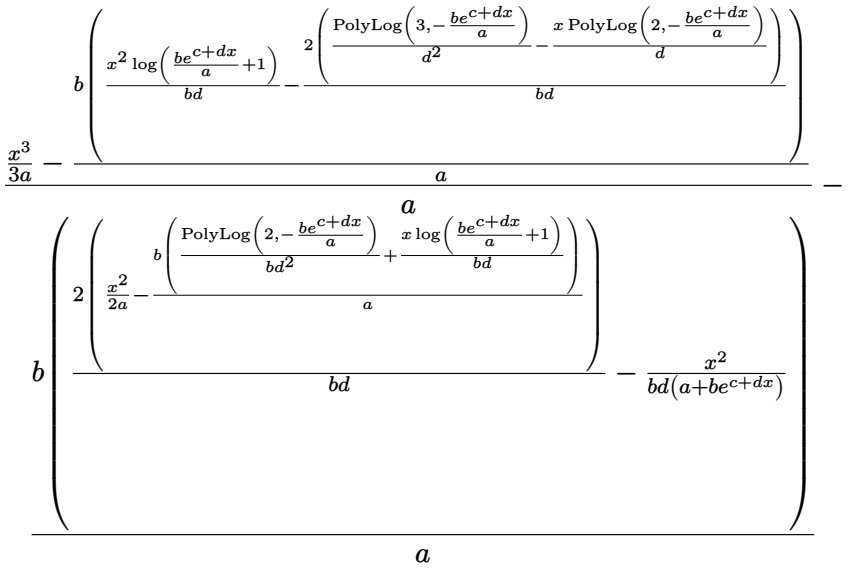
$$\downarrow 3011$$

$$\frac{x^3}{3a} - \frac{b\left(\frac{x^2 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\left(\frac{\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a} - \frac{b\left(\frac{2\left(\frac{x^2}{2a} - \frac{b\left(\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2} + \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}\right)}{bd} - \frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{x^3}{3a} - \frac{b\left(\frac{x^2 \log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\left(\frac{\int e^{-c-dx} \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)de^{c+dx}}{d^2} - \frac{x \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a} - \frac{b\left(\frac{2\left(\frac{x^2}{2a} - \frac{b\left(\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2} + \frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}\right)}{bd} - \frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 7143$$



input
```
Int[x^2/(a + b*E^(c + d*x))^2,x]
```

output
```
-((b*(-(x^2/(b*d*(a + b*E^(c + d*x)))) + (2*(x^2/(2*a) - (b*((x*Log[1 + (b
*E^(c + d*x))/a])/(b*d) + PolyLog[2, -((b*E^(c + d*x))/a)]/(b*d^2)))/a))/(
b*d)))/a) + (x^3/(3*a) - (b*((x^2*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (2*(
-((x*PolyLog[2, -((b*E^(c + d*x))/a)])/d) + PolyLog[3, -((b*E^(c + d*x))/a
)]/d^2))/(b*d)))/a)/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(157) = 314$.

Time = 0.07 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.96

method	result
risch	$\frac{x^2}{ad\left(a+b\,\mathrm{e}^{dx+c}\right)} - \frac{2cx}{a^2d^2} - \frac{c^2\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2d^3} + \frac{2c\ln\left(\mathrm{e}^{dx+c}\right)}{a^2d^3} - \frac{2c\ln\left(a+b\,\mathrm{e}^{dx+c}\right)}{a^2d^3} - \frac{c^2x}{a^2d^2} - \frac{c^2}{a^2d^3} + \frac{2\,\mathrm{polylog}\left(3,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right.}{a^2d^3}$

input

```
int(x^2/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output

```
x^2/a/d/(a+b*exp(d*x+c))-2/a^2/d^2*c*x-1/a^2/d^3*c^2*ln(a+b*exp(d*x+c))+2/
a^2/d^3*c*ln(exp(d*x+c))-2/a^2/d^3*c*ln(a+b*exp(d*x+c))-1/a^2/d^2*c^2*x-1/
a^2/d^3*c^2+2*polylog(3,-b*exp(d*x+c)/a)/a^2/d^3+1/3*x^3/a^2+2*polylog(2,-
b*exp(d*x+c)/a)/a^2/d^3+1/a^2/d^3*c^2*ln(exp(d*x+c))-2/3/a^2/d^3*c^3-x^2*l
n(1+b*exp(d*x+c)/a)/a^2/d-2*x*polylog(2,-b*exp(d*x+c)/a)/a^2/d^2+2*x*ln(1+
b*exp(d*x+c)/a)/a^2/d^2+2/a^2/d^3*ln(1+b*exp(d*x+c)/a)*c+1/a^2/d^3*ln(1+b*
exp(d*x+c)/a)*c^2-x^2/a^2/d
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.59

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx$$
$$= \frac{ad^3x^3 + ac^3 + 3\,ac^2 - 6\left(adx + (bdx-b)e^{(dx+c)} - a\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + (bd^3x^3 - 3\,bd^2x^2 + bc^3 + 3$$

input

```
integrate(x^2/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3*(a*d^3*x^3 + a*c^3 + 3*a*c^2 - 6*(a*d*x + (b*d*x - b)*e^(d*x + c) - a)
*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b*d^3*x^3 - 3*b*d^2*x^2 + b*c^3 + 3*
b*c^2)*e^(d*x + c) - 3*(a*c^2 + 2*a*c + (b*c^2 + 2*b*c)*e^(d*x + c))*log(b
*e^(d*x + c) + a) - 3*(a*d^2*x^2 - a*c^2 - 2*a*d*x - 2*a*c + (b*d^2*x^2 -
b*c^2 - 2*b*d*x - 2*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) + 6*(b*e^
(d*x + c) + a)*polylog(3, -b*e^(d*x + c)/a))/(a^2*b*d^3*e^(d*x + c) + a^3*
d^3)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{x^2}{a^2d+abde^{c+dx}} + \frac{\int \left(-\frac{2x}{a+be^ce^{dx}}\right)\,dx + \int \frac{dx^2}{a+be^ce^{dx}}\,dx}{ad}$$

input
```
integrate(x**2/(a+b*exp(d*x+c))**2,x)
```

output
```
x**2/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(-2*x/(a + b*exp(c)*exp(d*x)
), x) + Integral(d*x**2/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.90

$$\begin{aligned}\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx = {} & \frac{x^2}{abde^{(dx+c)}+a^2d} + \frac{d^3x^3-3\,d^2x^2}{3\,a^2d^3} \\ & - \frac{d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right) + 2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})}{a^2d^3} \\ & + \frac{2\left(dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^2d^3}\end{aligned}$$

input
```
integrate(x^2/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

output
```
x^2/(a*b*d*e^(d*x + c) + a^2*d) + 1/3*(d^3*x^3 - 3*d^2*x^2)/(a^2*d^3) - (d
^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*x*dilog(-b*e^(d*x + c)/a) - 2*polylo
g(3, -b*e^(d*x + c)/a))/(a^2*d^3) + 2*(d*x*log(b*e^(d*x + c)/a + 1) + dilo
g(-b*e^(d*x + c)/a))/(a^2*d^3)
```

Giac [F]

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \int \frac{x^2}{(be^{(dx+c)}+a)^2}\,dx$$

input
```
integrate(x^2/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(x^2/(b*e^(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+be^{c+dx})^2}\,dx = \int \frac{x^2}{(a+b\,\mathrm{e}^{c+d\,x})^2}\,dx$$

input
```
int(x^2/(a + b*exp(c + d*x))^2,x)
```

output
```
int(x^2/(a + b*exp(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}\,dx = \int \frac{x^2}{e^{2dx+2c}b^2+2e^{dx+c}ab+a^2}dx$$

input
```
int(x^2/(a+b*exp(d*x+c))^2,x)
```

output
```
int(x**2/(e**(2*c + 2*d*x)*b**2 + 2*e**(c + d*x)*a*b + a**2),x)
```

3.11 $\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx$

Optimal result . . . 131
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Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = -\frac{x}{a^2 d} + \frac{x}{ad\left(a+be^{c+dx}\right)} + \frac{x^2}{2a^2} + \frac{\log\left(a+be^{c+dx}\right)}{a^2 d^2} - \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^2 d} - \frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^2 d^2}$$

output

```
-x/a^2/d+x/a/d/(a+b*exp(d*x+c))+1/2*x^2/a^2+ln(a+b*exp(d*x+c))/a^2/d^2-x*l
n(1+b*exp(d*x+c)/a)/a^2/d-polylog(2,-b*exp(d*x+c)/a)/a^2/d^2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{\frac{dx\left(adx+be^{c+dx}(-2+dx)\right)}{a+be^{c+dx}} - 2(-1+dx)\log\left(1+\frac{be^{c+dx}}{a}\right) - 2\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{2a^2 d^2}$$

input

```
Integrate[x/(a + b*E^(c + d*x))^2,x]
```

output

```
((d*x*(a*d*x + b*E^(c + d*x)*(-2 + d*x)))/(a + b*E^(c + d*x)) - 2*(-1 + d*
x)*Log[1 + (b*E^(c + d*x))/a] - 2*PolyLog[2, -((b*E^(c + d*x))/a)])/(2*a^2
*d^2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {2616, 2615, 2620, 2621, 2715, 2720, 47, 14, 16, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{x}{a+be^{c+dx}}dx}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^2}{2a} - \frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{\int \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{\int \log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{\int \frac{1}{a+be^{c+dx}}dx}{bd} - \frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2715$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\int\frac{1}{a+be^{c+dx}}dx}{bd}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\int\frac{e^{-c-dx}}{a+be^{c+dx}}de^{c+dx}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 47$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\int e^{-c-dx}de^{c+dx}}{a}-\frac{b\int\frac{1}{a+be^{c+dx}}de^{c+dx}}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 14$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\frac{\log\left(e^{c+dx}\right)}{a}-\frac{b\int\frac{1}{a+be^{c+dx}}de^{c+dx}}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 16$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

input

```
Int[x/(a + b*E^(c + d*x))^2,x]
```

output
```
-((b*(-(x/(b*d*(a + b*E^(c + d*x)))) + (Log[E^(c + d*x)]/a - Log[a + b*E^(
c + d*x)]/a)/(b*d^2)))/a) + (x^2/(2*a) - (b*((x*Log[1 + (b*E^(c + d*x))/a]
)/(b*d) + PolyLog[2, -((b*E^(c + d*x))/a)]/(b*d^2)))/a)/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.44

method	result
derivativedivides	$\frac{\frac{(dx+c)^2}{2a^2}+\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^2}-\frac{b(dx+c)e^{dx+c}}{a^2\left(a+b\,e^{dx+c}\right)}-\frac{\mathrm{dilog}\left(\frac{a+b\,e^{dx+c}}{a}\right)}{a^2}-\frac{(dx+c)\ln\left(\frac{a+b\,e^{dx+c}}{a}\right)}{a^2}-c\left(\frac{\ln\left(e^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{dx+c}\right)}\right.}{d^2}$
default	$\frac{\frac{(dx+c)^2}{2a^2}+\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^2}-\frac{b(dx+c)e^{dx+c}}{a^2\left(a+b\,e^{dx+c}\right)}-\frac{\mathrm{dilog}\left(\frac{a+b\,e^{dx+c}}{a}\right)}{a^2}-\frac{(dx+c)\ln\left(\frac{a+b\,e^{dx+c}}{a}\right)}{a^2}-c\left(\frac{\ln\left(e^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{dx+c}\right)}\right.}{d^2}$
risch	$\frac{x}{ad\left(a+b\,e^{dx+c}\right)}+\frac{x^2}{2a^2}+\frac{cx}{a^2d}+\frac{c^2}{2a^2d^2}-\frac{x\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{a^2d}-\frac{\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)c}{a^2d^2}-\frac{\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)}{a^2d^2}-$

input
```
int(x/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/2*(d*x+c)^2/a^2+1/a^2*ln(a+b*exp(d*x+c))-1/a^2*b*(d*x+c)*exp(d*x+
c)/(a+b*exp(d*x+c))-1/a^2*dilog((a+b*exp(d*x+c))/a)-1/a^2*(d*x+c)*ln((a+b*
exp(d*x+c))/a)-c*(1/a^2*ln(exp(d*x+c))+1/a/(a+b*exp(d*x+c))-1/a^2*ln(a+b*e
xp(d*x+c))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.64

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx$$
$$= \frac{ad^2x^2 - ac^2 - 2\,ac - 2\left(be^{(dx+c)} + a\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + \left(bd^2x^2 - bc^2 - 2\,bdx - 2\,bc\right)e^{(dx+c)} + 2\left(ac\right.}{2\left(a^2bd^2e^{(dx+c)} + a\right.}$$

input
```
integrate(x/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(a*d^2*x^2 - a*c^2 - 2*a*c - 2*(b*e^(d*x + c) + a)*dilog(-(b*e^(d*x +
c) + a)/a + 1) + (b*d^2*x^2 - b*c^2 - 2*b*d*x - 2*b*c)*e^(d*x + c) + 2*(a*
c + (b*c + b)*e^(d*x + c) + a)*log(b*e^(d*x + c) + a) - 2*(a*d*x + a*c + (
b*d*x + b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a))/(a^2*b*d^2*e^(d*x +
c) + a^3*d^2)
```

Sympy [F]

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{x}{a^2d + abde^{c+dx}} + \frac{\int \frac{dx}{a+be^ce^{dx}}\,dx + \int \left(-\frac{1}{a+be^ce^{dx}}\right)\,dx}{ad}$$

input
```
integrate(x/(a+b*exp(d*x+c))**2,x)
```

output
```
x/(a**2*d + a*b*d*exp(c + d*x)) + (Integral(d*x/(a + b*exp(c)*exp(d*x)), x
) + Integral(-1/(a + b*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.89

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{x}{abde^{(dx+c)}+a^2d} + \frac{x^2}{2\,a^2} - \frac{x}{a^2d} - \frac{dx\log\left(\frac{be^{(dx+c)}}{a}+1\right)+\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{a^2d^2} + \frac{\log\left(be^{(dx+c)}+a\right)}{a^2d^2}$$

input
```
integrate(x/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

output
```
x/(a*b*d*e^(d*x + c) + a^2*d) + 1/2*x^2/a^2 - x/(a^2*d) - (d*x*log(b*e^(d*
x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^2*d^2) + log(b*e^(d*x + c) + a
)/(a^2*d^2)
```

Giac [F]

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = \int \frac{x}{\left(be^{(dx+c)}+a\right)^2}\,dx$$

input
```
integrate(x/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(x/(b*e^(d*x + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = \int \frac{x}{\left(a+b\,e^{c+d\,x}\right)^2}\,dx$$

input
```
int(x/(a + b*exp(c + d*x))^2,x)
```

output
```
int(x/(a + b*exp(c + d*x))^2, x)
```

Reduce [F]

$$\int \frac{x}{\left(a+be^{c+dx}\right)^2}\,dx = \int \frac{x}{e^{2dx+2c}b^2+2e^{dx+c}ab+a^2}dx$$

input
```
int(x/(a+b*exp(d*x+c))^2,x)
```

output
```
int(x/(e**(2*c + 2*d*x)*b**2 + 2*e**(c + d*x)*a*b + a**2),x)
```

3.12 $\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{1}{ad\left(a+be^{c+dx}\right)} + \frac{x}{a^2} - \frac{\log\left(a+be^{c+dx}\right)}{a^2 d}$$

output
```
1/a/d/(a+b*exp(d*x+c))+x/a^2-ln(a+b*exp(d*x+c))/a^2/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{\frac{a}{a+be^{c+dx}} + \log\left(e^{c+dx}\right) - \log\left(a+be^{c+dx}\right)}{a^2 d}$$

input
```
Integrate[(a + b*E^(c + d*x))^(-2),x]
```

output
```
(a/(a + b*E^(c + d*x)) + Log[E^(c + d*x)] - Log[a + b*E^(c + d*x)])/(a^2*d
)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+be^{c+dx})^2}\,dx$$

$$\downarrow 2720$$

$$\frac{\int \frac{e^{-c-dx}}{(a+be^{c+dx})^2}\,de^{c+dx}}{d}$$

$$\downarrow 54$$

$$\frac{\int \left(-\frac{b}{a^2(a+be^{c+dx})} - \frac{b}{a(a+be^{c+dx})^2} + \frac{e^{-c-dx}}{a^2}\right) de^{c+dx}}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(a+be^{c+dx})}{a^2} + \frac{\log(e^{c+dx})}{a^2} + \frac{1}{a(a+be^{c+dx})}}{d}$$

input
```
Int[(a + b*E^(c + d*x))^(-2),x]
```

output
```
(1/(a*(a + b*E^(c + d*x))) + Log[E^(c + d*x)]/a^2 - Log[a + b*E^(c + d*x)]
/a^2)/d
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{\frac{\ln\left(e^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{dx+c}\right)}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^2}}{d}$	49
default	$\frac{\frac{\ln\left(e^{dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{dx+c}\right)}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^2}}{d}$	49
risch	$\frac{x}{a^2}+\frac{c}{a^2d}+\frac{1}{ad\left(a+b\,e^{dx+c}\right)}-\frac{\ln\left(e^{dx+c}+\frac{a}{b}\right)}{a^2d}$	55
norman	$\frac{\frac{x}{a}+\frac{bx\,e^{dx+c}}{a^2}-\frac{b\,e^{dx+c}}{a^2d}}{a+b\,e^{dx+c}}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^2d}$	67
parallelrisch	$-\frac{-b^2e^{dx+c}xd+\ln\left(a+b\,e^{dx+c}\right)e^{dx+c}b^2-xabd+\ln\left(a+b\,e^{dx+c}\right)ab-ab}{a^2bd\left(a+b\,e^{dx+c}\right)}$	83

input
```
int(1/(a+b*exp(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(exp(d*x+c))+1/a/(a+b*exp(d*x+c))-1/a^2*ln(a+b*exp(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{bdxe^{(dx+c)} + adx - \left(be^{(dx+c)} + a\right)\log\left(be^{(dx+c)} + a\right) + a}{a^2bde^{(dx+c)} + a^3d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="fricas")
```

output
```
(b*d*x*e^(d*x + c) + a*d*x - (b*e^(d*x + c) + a)*log(b*e^(d*x + c) + a) +
a)/(a^2*b*d*e^(d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{1}{a^2d + abde^{c+dx}} + \frac{x}{a^2} - \frac{\log\left(\frac{a}{b} + e^{c+dx}\right)}{a^2d}$$

input
```
integrate(1/(a+b*exp(d*x+c))**2,x)
```

output
```
1/(a**2*d + a*b*d*exp(c + d*x)) + x/a**2 - log(a/b + exp(c + d*x))/(a**2*d
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{1}{(abe^{(dx+c)} + a^2)d} + \frac{dx+c}{a^2d} - \frac{\log\left(be^{(dx+c)} + a\right)}{a^2d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/((a*b*e^(d*x + c) + a^2)*d) + (d*x + c)/(a^2*d) - log(b*e^(d*x + c) + a)
/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(dx+c)}+a}+1\right|\right)}{a^2b}+\frac{1}{(be^{(dx+c)}+a)ab}\right)}{d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^2,x, algorithm="giac")
```

output
```
b*(log(abs(-a/(b*e^(d*x + c) + a) + 1))/(a^2*b) + 1/((b*e^(d*x + c) + a)*a
*b))/d
```

Mupad [B] (verification not implemented)

Time = 23.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx = \frac{\frac{x}{a}+\frac{b\,x\,\mathrm{e}^{c+d\,x}}{a^2}-\frac{b\,\mathrm{e}^{c+d\,x}}{a^2\,d}}{a+b\,\mathrm{e}^{c+d\,x}}-\frac{\ln\left(a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a^2\,d}$$

input
```
int(1/(a + b*exp(c + d*x))^2,x)
```

output
```
(x/a + (b*x*exp(c + d*x))/a^2 - (b*exp(c + d*x))/(a^2*d))/(a + b*exp(c + d
*x)) - log(a + b*exp(d*x)*exp(c))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.80

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2}\,dx$$
$$= \frac{-e^{dx+c}\log\left(e^{dx+c}b+a\right)b+e^{dx+c}bdx-e^{dx+c}b-\log\left(e^{dx+c}b+a\right)a+adx}{a^2d\left(e^{dx+c}b+a\right)}$$

input
```
int(1/(a+b*exp(d*x+c))^2,x)
```

output
```
( - e**(c + d*x)*log(e**(c + d*x)*b + a)*b + e**(c + d*x)*b*d*x - e**(c +
d*x)*b - log(e**(c + d*x)*b + a)*a + a*d*x)/(a**2*d*(e**(c + d*x)*b + a))
```

3.13 $\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^2 x}, x\right)$$

output
```
Defer(Int)(1/(a+b*exp(d*x+c))^2/x,x)
```

Mathematica [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx$$

input
```
Integrate[1/((a + b*E^(c + d*x))^2*x),x]
```

output
```
Integrate[1/((a + b*E^(c + d*x))^2*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+be^{c+dx}\right)^2}\,dx$$
$$\downarrow 2619$$
$$\int \frac{1}{x\left(a+be^{c+dx}\right)^2}dx$$

input
```
Int[1/((a + b*E^(c + d*x))^2*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)^2 x}dx$$

input
```
int(1/(a+b*exp(d*x+c))^2/x,x)
```

output
```
int(1/(a+b*exp(d*x+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^2/x,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x*e^(2*d*x + 2*c) + 2*a*b*x*e^(d*x + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \frac{1}{a^2dx+abdxe^{c+dx}} + \frac{\int \frac{dx}{ax^2+bx^2e^ce^{dx}}\,dx + \int \frac{1}{ax^2+bx^2e^ce^{dx}}\,dx}{ad}$$

input
```
integrate(1/(a+b*exp(d*x+c))**2/x,x)
```

output
```
1/(a**2*d*x + a*b*d*x*exp(c + d*x)) + (Integral(d*x/(a*x**2 + b*x**2*exp(c
)*exp(d*x)), x) + Integral(1/(a*x**2 + b*x**2*exp(c)*exp(d*x)), x))/(a*d)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^2/x,x, algorithm="maxima")
```

output
```
1/(a*b*d*x*e^(d*x + c) + a^2*d*x) + integrate((d*x + 1)/(a*b*d*x^2*e^(d*x
+ c) + a^2*d*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^2/x,x, algorithm="giac")
```

output
```
integrate(1/((b*e^(d*x + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 23.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{x\left(a+b\,\mathrm{e}^{c+d\,x}\right)^2}\,dx$$

input
```
int(1/(x*(a + b*exp(c + d*x))^2),x)
```

output
```
int(1/(x*(a + b*exp(c + d*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x}\,dx = \int \frac{1}{e^{2dx+2c}b^2x + 2e^{dx+c}abx + a^2x}dx$$

input
```
int(1/(a+b*exp(d*x+c))^2/x,x)
```

output
```
int(1/(e**(2*c + 2*d*x)*b**2*x + 2*e**(c + d*x)*a*b*x + a**2*x),x)
```

3.14 $\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx$

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Rubi [N/A] 151
Maple [N/A] 151
Fricas [N/A] 152
Sympy [N/A] 152
Maxima [N/A] 153
Giac [N/A] 153
Mupad [N/A] 153
Reduce [N/A] 154

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^2 x^2}, x\right)$$

output
```
Defer(Int)(1/(a+b*exp(d*x+c))^2/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx$$

input
```
Integrate[1/((a + b*E^(c + d*x))^2*x^2),x]
```

output
```
Integrate[1/((a + b*E^(c + d*x))^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a + be^{c+dx}\right)^2} dx$$
$$\downarrow 2619$$
$$\int \frac{1}{x^2 \left(a + be^{c+dx}\right)^2} dx$$

input
```
Int[1/((a + b*E^(c + d*x))^2*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a + b\,\mathrm{e}^{dx+c}\right)^2 x^2} dx$$

input
```
int(1/(a+b*exp(d*x+c))^2/x^2,x)
```

output
```
int(1/(a+b*exp(d*x+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.41

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^2/x^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*x^2*e^(2*d*x + 2*c) + 2*a*b*x^2*e^(d*x + c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.29

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \frac{1}{a^2dx^2+abdx^2e^{c+dx}} + \frac{\int \frac{dx}{ax^3+bx^3e^ce^{dx}}\,dx + \int \frac{2}{ax^3+bx^3e^ce^{dx}}\,dx}{ad}$$

input
```
integrate(1/(a+b*exp(d*x+c))**2/x**2,x)
```

output
```
1/(a**2*d*x**2 + a*b*d*x**2*exp(c + d*x)) + (Integral(d*x/(a*x**3 + b*x**3
*exp(c)*exp(d*x)), x) + Integral(2/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x))/
(a*d)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.35

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^2/x^2,x, algorithm="maxima")
```

output
```
1/(a*b*d*x^2*e^(d*x + c) + a^2*d*x^2) + integrate((d*x + 2)/(a*b*d*x^3*e^(
d*x + c) + a^2*d*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^2 x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^2/x^2,x, algorithm="giac")
```

output
```
integrate(1/((b*e^(d*x + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \int \frac{1}{x^2\left(a+b\,\mathrm{e}^{c+d\,x}\right)^2}\,dx$$

input
```
int(1/(x^2*(a + b*exp(c + d*x))^2),x)
```

output
```
int(1/(x^2*(a + b*exp(c + d*x))^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.53

$$\int \frac{1}{\left(a+be^{c+dx}\right)^2 x^2}\,dx = \int \frac{1}{e^{2dx+2c}b^2x^2 + 2e^{dx+c}ab\,x^2 + a^2x^2}dx$$

input
```
int(1/(a+b*exp(d*x+c))^2/x^2,x)
```

output
```
int(1/(e**(2*c + 2*d*x)*b**2*x**2 + 2*e**(c + d*x)*a*b*x**2 + a**2*x**2),x
)
```

3.15 $\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx$

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Rubi [A] (verified) 156
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Maxima [A] (verification not implemented) 158
Giac [A] (verification not implemented) 159
Mupad [B] (verification not implemented) 159
Reduce [B] (verification not implemented) 159

Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = -\frac{1}{ad\left(a+be^{c-dx}\right)} + \frac{x}{a^2} + \frac{\log\left(a+be^{c-dx}\right)}{a^2 d}$$

output
```
-1/a/d/(a+b*exp(-d*x+c))+x/a^2+ln(a+b*exp(-d*x+c))/a^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = \frac{\frac{be^c}{be^c+ae^{dx}} + \log\left(ad\left(be^c+ae^{dx}\right)\right)}{a^2 d}$$

input
```
Integrate[(a + b*E^(c - d*x))^(-2),x]
```

output
```
((b*E^c)/(b*E^c + a*E^(d*x)) + Log[a*d*(b*E^c + a*E^(d*x))])/(a^2*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx$$

$$\downarrow 2720$$

$$-\frac{\int \frac{e^{dx-c}}{\left(a+be^{c-dx}\right)^2}\,de^{c-dx}}{d}$$

$$\downarrow 54$$

$$-\frac{\int \left(-\frac{b}{a^2\left(a+be^{c-dx}\right)}-\frac{b}{a\left(a+be^{c-dx}\right)^2}+\frac{e^{dx-c}}{a^2}\right)de^{c-dx}}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\log\left(a+be^{c-dx}\right)}{a^2}+\frac{\log\left(e^{c-dx}\right)}{a^2}+\frac{1}{a\left(a+be^{c-dx}\right)}}{d}$$

input
```
Int[(a + b*E^(c - d*x))^(-2),x]
```

output
```
-((1/(a*(a + b*E^(c - d*x))) + Log[E^(c - d*x)]/a^2 - Log[a + b*E^(c - d*x
)]/a^2)/d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^2}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{-dx+c}\right)}}{d}$	53
default	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^2}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^2}+\frac{1}{a\left(a+b\,e^{-dx+c}\right)}}{d}$	53
risch	$\frac{x}{a^2}-\frac{c}{a^2d}-\frac{1}{ad\left(a+b\,e^{-dx+c}\right)}+\frac{\ln\left(e^{-dx+c}+\frac{a}{b}\right)}{a^2d}$	58
norman	$\frac{\frac{x}{a}+\frac{bx\,e^{-dx+c}}{a^2}-\frac{1}{da}}{a+b\,e^{-dx+c}}+\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^2d}$	62
parallelrisch	$\frac{b^2e^{-dx+c}xd+\ln\left(a+b\,e^{-dx+c}\right)e^{-dx+c}b^2+xabd+\ln\left(a+b\,e^{-dx+c}\right)ab-ab}{a^2bd\left(a+b\,e^{-dx+c}\right)}$	85

input
```
int(1/(a+b*exp(-d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/a^2*ln(exp(-d*x+c))-1/a^2*ln(a+b*exp(-d*x+c))+1/a/(a+b*exp(-d*x+c)
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = \frac{bdxe^{(-dx+c)} + adx + \left(be^{(-dx+c)} + a\right)\log\left(be^{(-dx+c)} + a\right) - a}{a^2bde^{(-dx+c)} + a^3d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="fricas")
```

output
```
(b*d*x*e^(-d*x + c) + a*d*x + (b*e^(-d*x + c) + a)*log(b*e^(-d*x + c) + a)
 - a)/(a^2*b*d*e^(-d*x + c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = -\frac{1}{a^2d + abde^{c-dx}} + \frac{x}{a^2} + \frac{\log\left(\frac{a}{b} + e^{c-dx}\right)}{a^2d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))**2,x)
```

output
```
-1/(a**2*d + a*b*d*exp(c - d*x)) + x/a**2 + log(a/b + exp(c - d*x))/(a**2*
d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = -\frac{1}{(abe^{(-dx+c)} + a^2)d} + \frac{dx - c}{a^2d} + \frac{\log\left(be^{(-dx+c)} + a\right)}{a^2d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/((a*b*e^(-d*x + c) + a^2)*d) + (d*x - c)/(a^2*d) + log(b*e^(-d*x + c) +
 a)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = -\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(-dx+c)}+a}+1\right|\right)}{a^2b}+\frac{1}{(be^{(-dx+c)}+a)ab}\right)}{d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))^2,x, algorithm="giac")
```

output
```
-b*(log(abs(-a/(b*e^(-d*x + c) + a) + 1))/(a^2*b) + 1/((b*e^(-d*x + c) + a
)*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = \frac{\frac{x}{a}+\frac{b\,x\,\mathrm{e}^{c-d\,x}}{a^2}+\frac{b\,\mathrm{e}^{c-d\,x}}{a^2\,d}}{a+b\,\mathrm{e}^{c-d\,x}}+\frac{\ln\left(a+b\,\mathrm{e}^{-d\,x}\,\mathrm{e}^{c}\right)}{a^2\,d}$$

input
```
int(1/(a + b*exp(c - d*x))^2,x)
```

output
```
(x/a + (b*x*exp(c - d*x))/a^2 + (b*exp(c - d*x))/(a^2*d))/(a + b*exp(c - d
*x)) + log(a + b*exp(-d*x)*exp(c))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.48

$$\int \frac{1}{\left(a+be^{c-dx}\right)^2}\,dx = \frac{e^{dx}\log\left(e^{dx}a+e^{c}b\right)a-e^{dx}a+e^{c}\log\left(e^{dx}a+e^{c}b\right)b}{a^2d\left(e^{dx}a+e^{c}b\right)}$$

input
```
int(1/(a+b*exp(-d*x+c))^2,x)
```

output

```
(e**(d*x)*log(e**(d*x)*a + e**c*b)*a - e**(d*x)*a + e**c*log(e**(d*x)*a +
e**c*b)*b)/(a**2*d*(e**(d*x)*a + e**c*b))
```

3.16 $\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx$

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Rubi [A] (verified) 162
Maple [A] (verified) 163
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Sympy [A] (verification not implemented) 164
Maxima [A] (verification not implemented) 164
Giac [A] (verification not implemented) 165
Mupad [B] (verification not implemented) 165
Reduce [B] (verification not implemented) 165

Optimal result

Integrand size = 16, antiderivative size = 52

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = -\frac{1}{ad\left(a+be^{-c-dx}\right)} + \frac{x}{a^2} + \frac{\log\left(a+be^{-c-dx}\right)}{a^2 d}$$

output
```
-1/a/d/(a+b*exp(-d*x-c))+x/a^2+ln(a+b*exp(-d*x-c))/a^2/d
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = \frac{b+bdx+ade^{c+dx}x+2\left(b+ae^{c+dx}\right)\operatorname{arctanh}\left(1+\frac{2ae^{c+dx}}{b}\right)}{a^2 d\left(b+ae^{c+dx}\right)}$$

input
```
Integrate[(a + b*E^(-c - d*x))^(-2),x]
```

output
```
(b + b*d*x + a*d*E^(c + d*x)*x + 2*(b + a*E^(c + d*x))*ArcTanh[1 + (2*a*E^
(c + d*x))/b])/(a^2*d*(b + a*E^(c + d*x)))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx$$

$$\downarrow 2720$$

$$-\frac{\int \frac{e^{c+dx}}{\left(a+be^{-c-dx}\right)^2}\,de^{-c-dx}}{d}$$

$$\downarrow 54$$

$$-\frac{\int \left(-\frac{b}{a^2\left(a+be^{-c-dx}\right)}-\frac{b}{a\left(a+be^{-c-dx}\right)^2}+\frac{e^{c+dx}}{a^2}\right)de^{-c-dx}}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\log\left(a+be^{-c-dx}\right)}{a^2}+\frac{\log\left(e^{-c-dx}\right)}{a^2}+\frac{1}{a\left(a+be^{-c-dx}\right)}}{d}$$

input
```
Int[(a + b*E^(-c - d*x))^(-2),x]
```

output
```
-((1/(a*(a + b*E^(-c - d*x))) + Log[E^(-c - d*x)]/a^2 - Log[a + b*E^(-c -
d*x)]/a^2)/d)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$-\frac{\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{a^2}-\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{-dx-c}\right)}}{d}$	59
default	$-\frac{\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{a^2}-\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^2}+\frac{1}{a\left(a+b\,\mathrm{e}^{-dx-c}\right)}}{d}$	59
risch	$\frac{x}{a^2}+\frac{c}{a^2d}-\frac{1}{ad\left(a+b\,\mathrm{e}^{-dx-c}\right)}+\frac{\ln\left(\mathrm{e}^{-dx-c}+\frac{a}{b}\right)}{a^2d}$	61
norman	$\frac{\frac{x}{a}+\frac{bx\,\mathrm{e}^{-dx-c}}{a^2}-\frac{1}{da}}{a+b\,\mathrm{e}^{-dx-c}}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^2d}$	68
parallelrisch	$\frac{b^2\mathrm{e}^{-dx-c}xd+\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)\mathrm{e}^{-dx-c}b^2+xabd+\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)ab-ab}{a^2bd\left(a+b\,\mathrm{e}^{-dx-c}\right)}$	95

input
```
int(1/(a+b*exp(-d*x-c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/a^2*ln(exp(-d*x-c))-1/a^2*ln(a+b*exp(-d*x-c))+1/a/(a+b*exp(-d*x-c)
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.40

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = \frac{bdxe^{(-dx-c)} + adx + \left(be^{(-dx-c)} + a\right)\log\left(be^{(-dx-c)} + a\right) - a}{a^2bde^{(-dx-c)} + a^3d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="fricas")
```

output
```
(b*d*x*e^(-d*x - c) + a*d*x + (b*e^(-d*x - c) + a)*log(b*e^(-d*x - c) + a)
 - a)/(a^2*b*d*e^(-d*x - c) + a^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = -\frac{1}{a^2d + abde^{-c-dx}} + \frac{x}{a^2} + \frac{\log\left(\frac{a}{b} + e^{-c-dx}\right)}{a^2d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))**2,x)
```

output
```
-1/(a**2*d + a*b*d*exp(-c - d*x)) + x/a**2 + log(a/b + exp(-c - d*x))/(a**
2*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = -\frac{1}{(abe^{(-dx-c)} + a^2)d} + \frac{dx+c}{a^2d} + \frac{\log\left(be^{(-dx-c)} + a\right)}{a^2d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="maxima")
```

output
```
-1/((a*b*e^(-d*x - c) + a^2)*d) + (d*x + c)/(a^2*d) + log(b*e^(-d*x - c) +
 a)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = -\frac{b\left(\frac{\log\left(\left|-\frac{a}{be^{(-dx-c)}+a}+1\right|\right)}{a^2b}+\frac{1}{\left(be^{(-dx-c)}+a\right)ab}\right)}{d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))^2,x, algorithm="giac")
```

output
```
-b*(log(abs(-a/(b*e^(-d*x - c) + a) + 1))/(a^2*b) + 1/((b*e^(-d*x - c) + a
)*a*b))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = \frac{\frac{x}{a}+\frac{b\,\mathrm{e}^{-c-d\,x}}{a^2\,d}+\frac{b\,x\,\mathrm{e}^{-c-d\,x}}{a^2}}{a+b\,\mathrm{e}^{-c-d\,x}}+\frac{\ln\left(a+b\,\mathrm{e}^{-c}\,\mathrm{e}^{-d\,x}\right)}{a^2\,d}$$

input
```
int(1/(a + b*exp(- c - d*x))^2,x)
```

output
```
(x/a + (b*exp(- c - d*x))/(a^2*d) + (b*x*exp(- c - d*x))/a^2)/(a + b*exp(-
 c - d*x)) + log(a + b*exp(-c)*exp(-d*x))/(a^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^2}\,dx = \frac{e^{dx+c}\log\left(e^{dx+c}a+b\right)a-e^{dx+c}a+\log\left(e^{dx+c}a+b\right)b}{a^2d\left(e^{dx+c}a+b\right)}$$

input
```
int(1/(a+b*exp(-d*x-c))^2,x)
```

output

```
(e**(c + d*x)*log(e**(c + d*x)*a + b)*a - e**(c + d*x)*a + log(e**(c + d*x
)*a + b)*b)/(a**2*d*(e**(c + d*x)*a + b))
```

3.17 $\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 333

$$\begin{aligned}\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx &= \frac{3x^2}{2a^3d^2}-\frac{3x^2}{2a^2d^2\left(a+be^{c+dx}\right)}-\frac{3x^3}{2a^3d}+\frac{x^3}{2ad\left(a+be^{c+dx}\right)^2}\\ &+\frac{x^3}{a^2d\left(a+be^{c+dx}\right)}+\frac{x^4}{4a^3}-\frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^3}\\ &+\frac{9x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{2a^3d^2}-\frac{x^3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d}\\ &-\frac{3\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^4}+\frac{9x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^3}\\ &-\frac{3x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2}-\frac{9\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^4}\\ &+\frac{6x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^3}-\frac{6\,\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a}\right)}{a^3d^4}\end{aligned}$$

output
```
3/2*x^2/a^3/d^2-3/2*x^2/a^2/d^2/(a+b*exp(d*x+c))-3/2*x^3/a^3/d+1/2*x^3/a/d
/(a+b*exp(d*x+c))^2+x^3/a^2/d/(a+b*exp(d*x+c))+1/4*x^4/a^3-3*x*ln(1+b*exp(
d*x+c)/a)/a^3/d^3+9/2*x^2*ln(1+b*exp(d*x+c)/a)/a^3/d^2-x^3*ln(1+b*exp(d*x+
c)/a)/a^3/d-3*polylog(2,-b*exp(d*x+c)/a)/a^3/d^4+9*x*polylog(2,-b*exp(d*x+
c)/a)/a^3/d^3-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^3/d^2-9*polylog(3,-b*exp(
d*x+c)/a)/a^3/d^4+6*x*polylog(3,-b*exp(d*x+c)/a)/a^3/d^3-6*polylog(4,-b*ex
p(d*x+c)/a)/a^3/d^4
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.72

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx$$

$$= \frac{\frac{6x^2}{d^2} - \frac{6ax^2}{d^2\left(a+be^{c+dx}\right)} - \frac{6x^3}{d} + \frac{2a^2x^3}{d\left(a+be^{c+dx}\right)^2} + \frac{4ax^3}{ad+bde^{c+dx}} + x^4 - \frac{12x\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^3} + \frac{18x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^2} - \frac{4x^3\log\left(1+\right.}{d}}{4a^3}$$

input
```
Integrate[x^3/(a + b*E^(c + d*x))^3,x]
```

output
```
((6*x^2)/d^2 - (6*a*x^2)/(d^2*(a + b*E^(c + d*x))) - (6*x^3)/d + (2*a^2*x^
3)/(d*(a + b*E^(c + d*x))^2) + (4*a*x^3)/(a*d + b*d*E^(c + d*x)) + x^4 - (
12*x*Log[1 + (b*E^(c + d*x))/a])/d^3 + (18*x^2*Log[1 + (b*E^(c + d*x))/a])
/d^2 - (4*x^3*Log[1 + (b*E^(c + d*x))/a])/d - (12*(1 - 3*d*x + d^2*x^2)*Po
lyLog[2, -((b*E^(c + d*x))/a)])/d^4 + (12*(-3 + 2*d*x)*PolyLog[3, -((b*E^(
c + d*x))/a)])/d^4 - (24*PolyLog[4, -((b*E^(c + d*x))/a)])/d^4)/(4*a^3)
```

Rubi [A] (verified)

Time = 6.56 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.51, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 1.176$, Rules used = {2616, 2616, 2615, 2620, 2621, 2615, 2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{x^3}{\left(a+be^{c+dx}\right)^2}dx}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{\int \frac{x^3}{a+be^{c+dx}}dx}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{\frac{x^4}{4a} - \frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^3}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{\frac{x^4}{4a} - \frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{3\int \frac{x^2}{a+be^{c+dx}}dx}{bd} - \frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a} - \frac{b\left(\frac{3\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}dx}{2bd} - \frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\int\frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{3\int\frac{x^2}{\left(a+be^{c+dx}\right)^2}dx}{2bd}-\frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\int\frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{3\left(\frac{\int\frac{x^2}{a+be^{c+dx}}dx}{a}-\frac{b\int\frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}\right)}{2bd}-\frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\int\frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\int\frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}}{a}-\frac{b\int\frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}\right)}{2bd}-\frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

$$b\left(\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\int\frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}\right)}{2bd}-\frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)$$

$$a$$

$$\downarrow 2621$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

$$b\left(\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\int\frac{x}{a+be^{c+dx}}dx}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}\right)}{2bd}-\frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)$$

$$\Big\downarrow 2615$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{bd}\right)}{a}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

$$b\left(\frac{3\left(\frac{\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\int\frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{2bd}-\frac{x^3}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

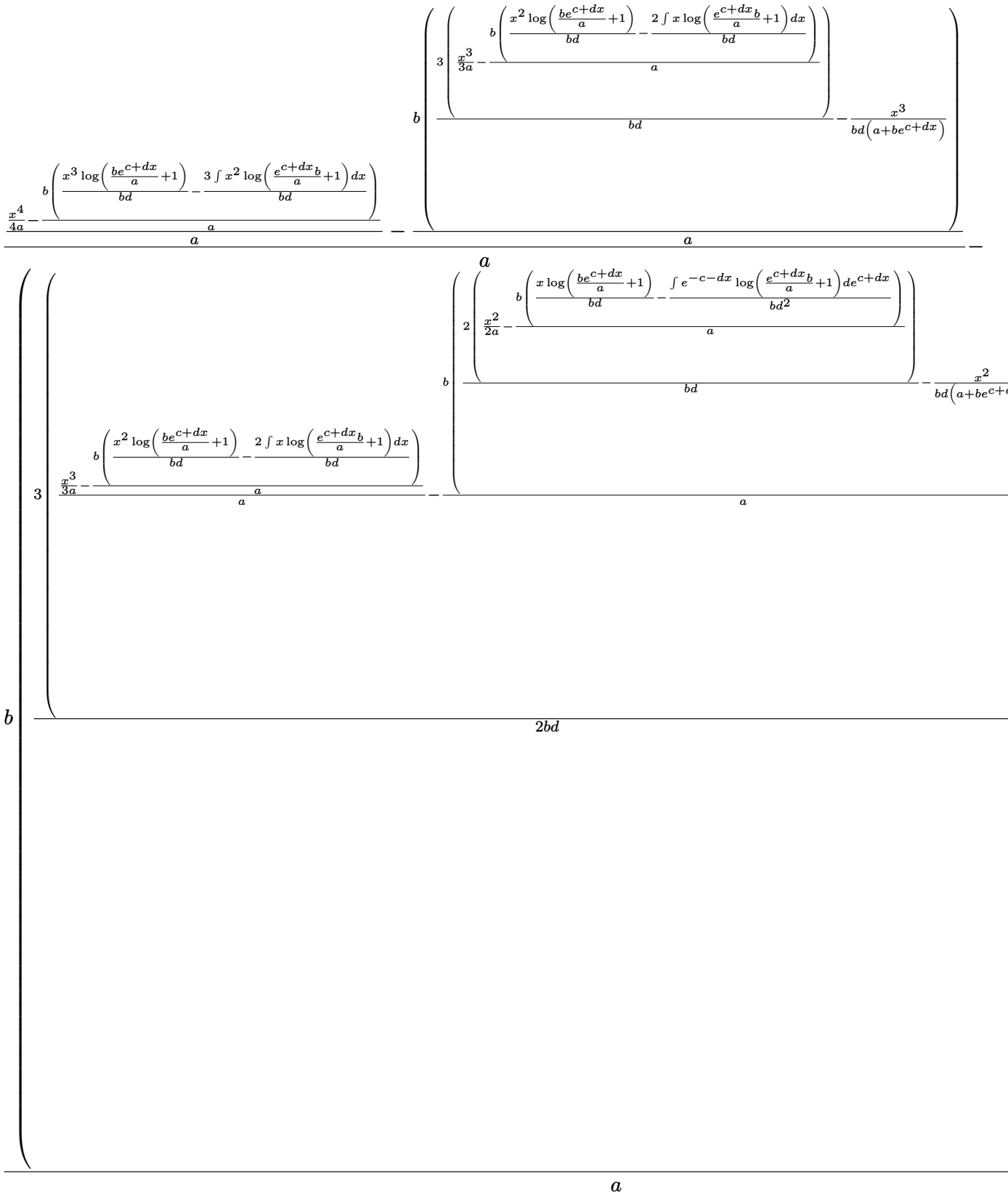
$$\downarrow 2620$$

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

$$b\left(\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}\right)}{2bd}-$$

$$\frac{a}{a}$$

2715



2838

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^3}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

$$b\left(\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{a\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}\right)}{2bd}-\right.$$

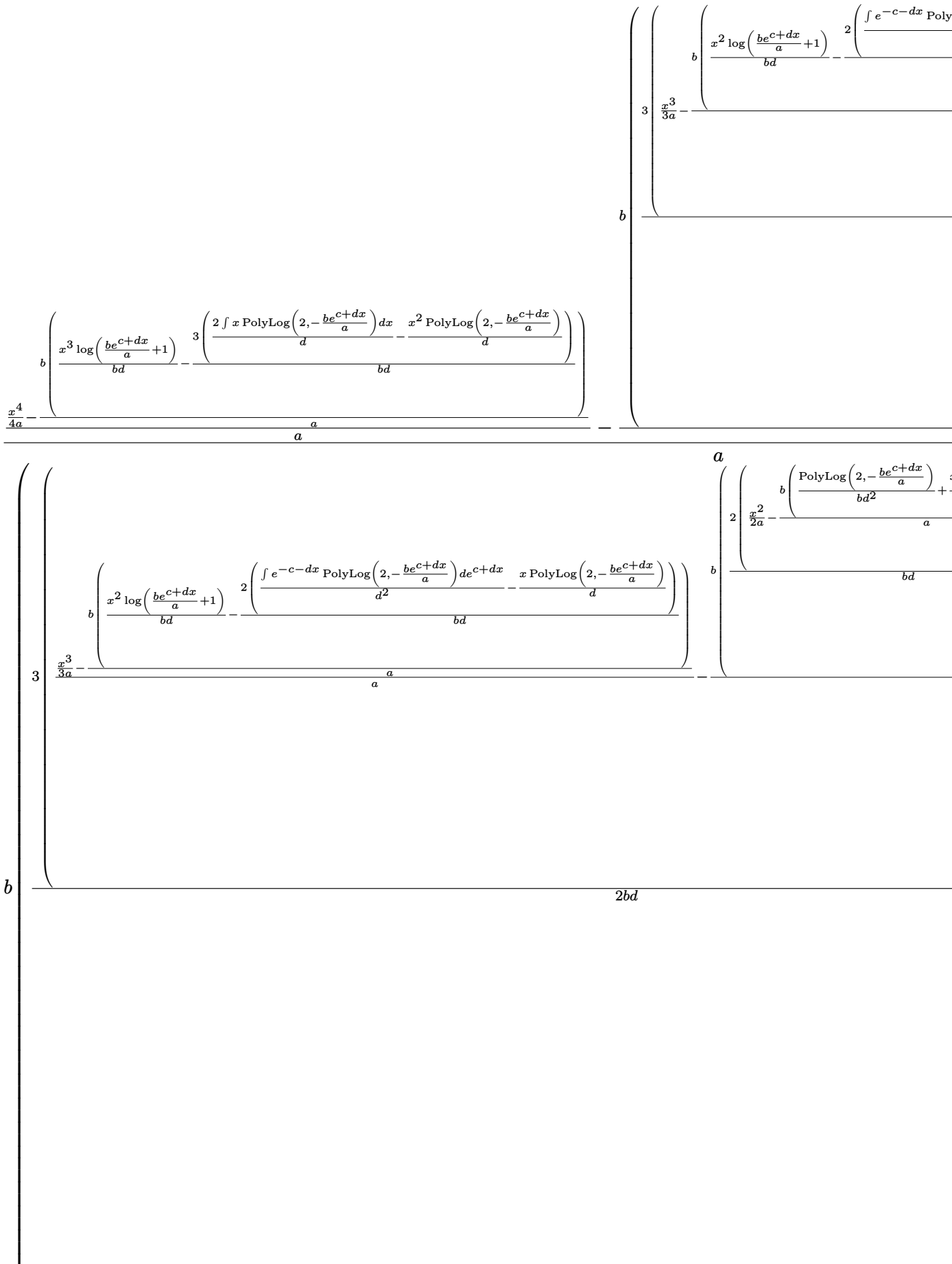
$$\left.\frac{}{a}\right)$$

3011

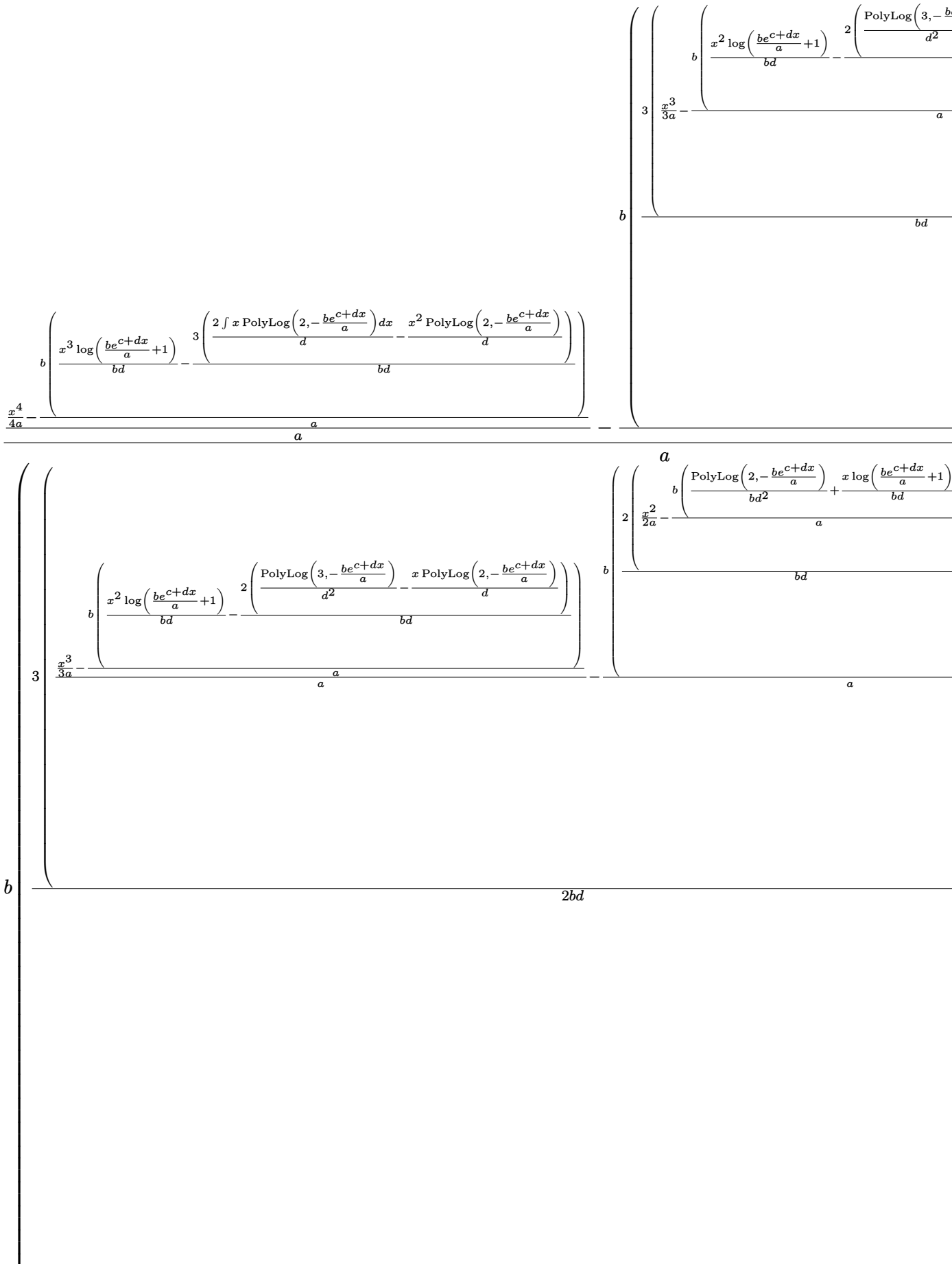
$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\int x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\int \mathrm{PolyLog}\left(2,-\right.}{d}\right.}{}\right.}{}\right.}{bd}\right.}{a}$$

$$b\left(\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\int \mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}\right.}{bd}\right.}{a}\right.}{bd}\right.}{a}\right.}{2bd}$$

2720



7143



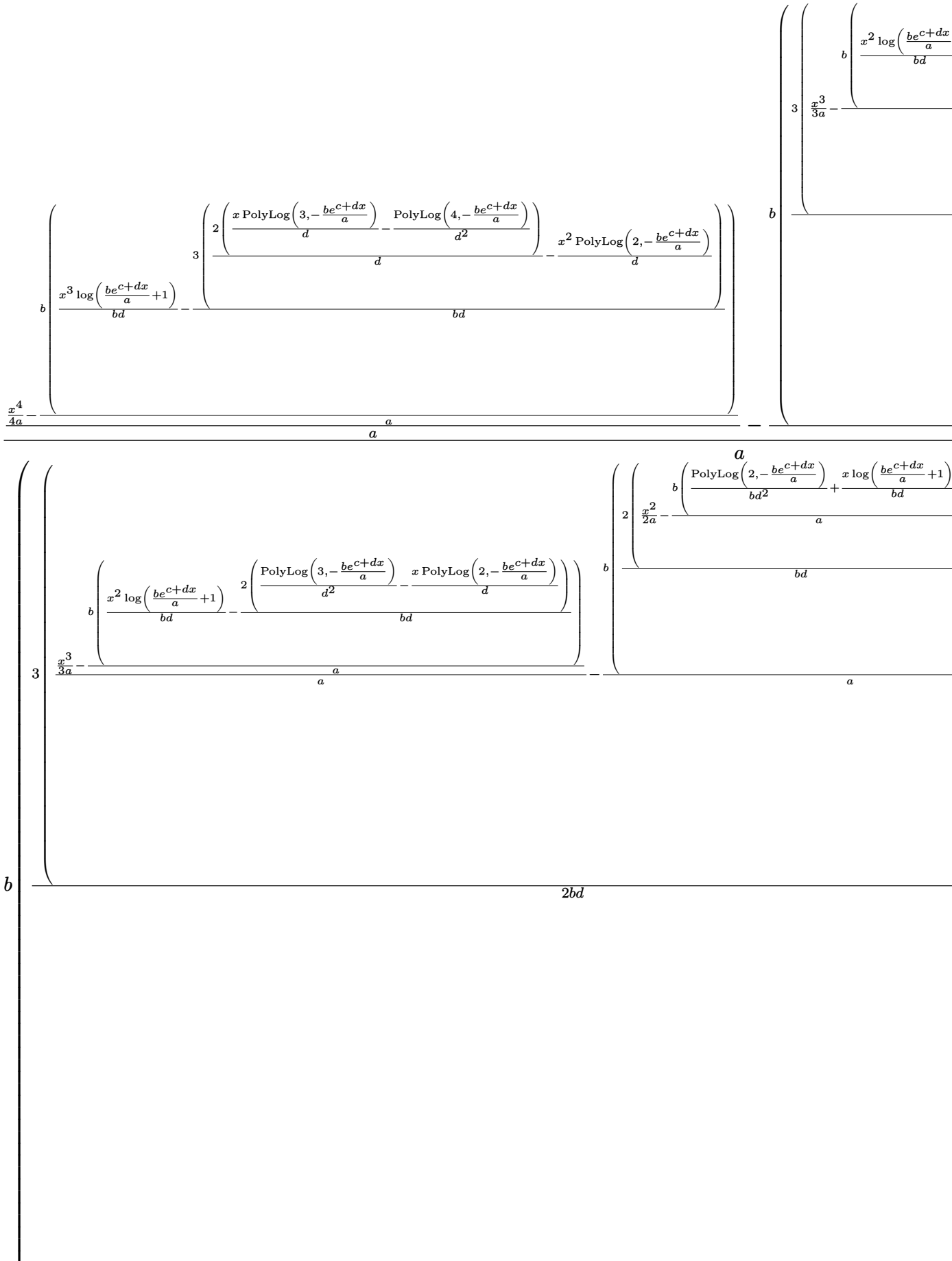
7163

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d}-\frac{\int\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)dx}{d}\right)}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(2\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}}{bd}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd^2}\right)}{a}\right)}{a}\right)}{2bd}$$

2720

$$\frac{\frac{x^4}{4a}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d}-\frac{\int e^{-c-dx}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{d^2}-\frac{x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(2\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}}{bd}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\frac{\log\left(\frac{be^{c+dx}}{a}+1\right)}{e^{c+dx}}dx}{d}\right)}{a}\right)}{a}\right)}{2bd}\right)}{a}}{b}$$

7143



input
```
Int[x^3/(a + b*E^(c + d*x))^3,x]
```

output
```
-((b*(-1/2*x^3/(b*d*(a + b*E^(c + d*x))^2) + (3*(-((b*(-(x^2/(b*d*(a + b*E
^(c + d*x)))) + (2*(x^2/(2*a) - (b*((x*Log[1 + (b*E^(c + d*x))/a])/(b*d) +
 PolyLog[2, -((b*E^(c + d*x))/a)]/(b*d^2)))/a))/(b*d)))/a) + (x^3/(3*a) -
(b*((x^2*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c
 + d*x))/a)])/d) + PolyLog[3, -((b*E^(c + d*x))/a)]/d^2))/(b*d)))/a))/(
2*b*d)))/a) + (-((b*(-(x^3/(b*d*(a + b*E^(c + d*x)))) + (3*(x^3/(3*a) - (b
*((x^2*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c +
 d*x))/a)])/d) + PolyLog[3, -((b*E^(c + d*x))/a)]/d^2))/(b*d)))/a))/(b*d))
)/a) + (x^4/(4*a) - (b*((x^3*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (3*(-((x^
2*PolyLog[2, -((b*E^(c + d*x))/a)])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x
))/a)])/d - PolyLog[4, -((b*E^(c + d*x))/a)]/d^2))/d))/(b*d)))/a)/a)/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.65

method	result
risch	$-\frac{3x\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{a^3d^3}+\frac{9x^2\ln\left(1+\frac{b\,e^{dx+c}}{a}\right)}{2a^3d^2}+\frac{9x\,\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)}{a^3d^3}-\frac{3x^2\,\mathrm{polylog}\left(2,-\frac{b\,e^{dx+c}}{a}\right)}{a^3d^2}+\frac{6x\,\mathrm{polylog}\left(3,-\frac{b\,e^{dx}}{a}\right.}$

input

```
int(x^3/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output

```
-3*x*ln(1+b*exp(d*x+c)/a)/a^3/d^3+9/2*x^2*ln(1+b*exp(d*x+c)/a)/a^3/d^2+9*x
*polylog(2,-b*exp(d*x+c)/a)/a^3/d^3-3*x^2*polylog(2,-b*exp(d*x+c)/a)/a^3/d
^2+6*x*polylog(3,-b*exp(d*x+c)/a)/a^3/d^3-9*polylog(3,-b*exp(d*x+c)/a)/a^3
/d^4+3/2*x^2/a^3/d^2+3/4/a^3/d^4*c^4+3/2/a^3/d^4*c^2+3/a^3/d^4*c^3-3/2*x^3
/a^3/d-6*polylog(4,-b*exp(d*x+c)/a)/a^3/d^4-3*polylog(2,-b*exp(d*x+c)/a)/a
^3/d^4-x^3*ln(1+b*exp(d*x+c)/a)/a^3/d+1/2*x^2*(2*x*b*d*exp(d*x+c)+3*x*d*a-
3*b*exp(d*x+c)-3*a)/d^2/a^2/(a+b*exp(d*x+c))^2+3/a^3/d^3*c*x+9/2/a^3/d^3*c
^2*x+1/a^3/d^3*c^3*x-3/a^3/d^4*ln(1+b*exp(d*x+c)/a)*c-9/2/a^3/d^4*ln(1+b*e
xp(d*x+c)/a)*c^2-1/a^3/d^4*ln(1+b*exp(d*x+c)/a)*c^3+1/4*x^4/a^3-3/a^3/d^4*
c*ln(exp(d*x+c))+3/a^3/d^4*c*ln(a+b*exp(d*x+c))-9/2/a^3/d^4*c^2*ln(exp(d*x
+c))+9/2/a^3/d^4*c^2*ln(a+b*exp(d*x+c))-1/a^3/d^4*c^3*ln(exp(d*x+c))+1/a^3
/d^4*c^3*ln(a+b*exp(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 702 vs. 2(306) = 612.

Time = 0.08 (sec) , antiderivative size = 702, normalized size of antiderivative = 2.11

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(a^2*d^4*x^4 - a^2*c^4 - 6*a^2*c^3 - 6*a^2*c^2 - 12*(a^2*d^2*x^2 - 3*a
^2*d*x + a^2 + (b^2*d^2*x^2 - 3*b^2*d*x + b^2)*e^(2*d*x + 2*c) + 2*(a*b*d^
2*x^2 - 3*a*b*d*x + a*b)*e^(d*x + c))*dilog(-(b*e^(d*x + c) + a)/a + 1) +
(b^2*d^4*x^4 - 6*b^2*d^3*x^3 - b^2*c^4 + 6*b^2*d^2*x^2 - 6*b^2*c^3 - 6*b^2
*c^2)*e^(2*d*x + 2*c) + 2*(a*b*d^4*x^4 - 4*a*b*d^3*x^3 - a*b*c^4 + 3*a*b*d
^2*x^2 - 6*a*b*c^3 - 6*a*b*c^2)*e^(d*x + c) + 2*(2*a^2*c^3 + 9*a^2*c^2 + 6
*a^2*c + (2*b^2*c^3 + 9*b^2*c^2 + 6*b^2*c)*e^(2*d*x + 2*c) + 2*(2*a*b*c^3
+ 9*a*b*c^2 + 6*a*b*c)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 2*(2*a^2*d^3*
x^3 - 9*a^2*d^2*x^2 + 2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*d*x + 6*a^2*c + (2*b^2
*d^3*x^3 - 9*b^2*d^2*x^2 + 2*b^2*c^3 + 9*b^2*c^2 + 6*b^2*d*x + 6*b^2*c)*e^
(2*d*x + 2*c) + 2*(2*a*b*d^3*x^3 - 9*a*b*d^2*x^2 + 2*a*b*c^3 + 9*a*b*c^2 +
 6*a*b*d*x + 6*a*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) - 24*(b^2*e^
(2*d*x + 2*c) + 2*a*b*e^(d*x + c) + a^2)*polylog(4, -b*e^(d*x + c)/a) + 12
*(2*a^2*d*x - 3*a^2 + (2*b^2*d*x - 3*b^2)*e^(2*d*x + 2*c) + 2*(2*a*b*d*x -
 3*a*b)*e^(d*x + c))*polylog(3, -b*e^(d*x + c)/a))/(a^3*b^2*d^4*e^(2*d*x +
 2*c) + 2*a^4*b*d^4*e^(d*x + c) + a^5*d^4)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{3adx^3 - 3ax^2 + \left(2bdx^3 - 3bx^2\right)e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \frac{6x}{a+be^ce^{dx}}\,dx + \int \left(-\frac{9dx^2}{a+be^ce^{dx}}\right)\,dx + \int \frac{2d^2x^3}{a+be^ce^{dx}}\,dx}{2a^2d^2}$$

input
```
integrate(x**3/(a+b*exp(d*x+c))**3,x)
```

output
```
(3*a*d*x**3 - 3*a*x**2 + (2*b*d*x**3 - 3*b*x**2)*exp(c + d*x))/(2*a**4*d**
2 + 4*a**3*b*d**2*exp(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Int
egral(6*x/(a + b*exp(c)*exp(d*x)), x) + Integral(-9*d*x**2/(a + b*exp(c)*e
xp(d*x)), x) + Integral(2*d**2*x**3/(a + b*exp(c)*exp(d*x)), x))/(2*a**2*d
**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.91

$$
\begin{aligned}
&\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx \\
&= \frac{3\,adx^3 - 3\,ax^2 + \left(2\,bdx^3e^{c} - 3\,bx^2e^{c}\right)e^{(dx)}}{2\left(a^2b^2d^2e^{(2\,dx+2\,c)} + 2\,a^3bd^2e^{(dx+c)} + a^4d^2\right)} + \frac{d^4x^4 - 6\,d^3x^3 + 6\,d^2x^2}{4\,a^3d^4} \\
&\quad - \frac{d^3x^3\log\left(\frac{be^{(dx+c)}}{a}+1\right) + 3\,d^2x^2\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 6\,dx\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a}) + 6\,\mathrm{Li}_4(-\frac{be^{(dx+c)}}{a})}{a^3d^4} \\
&\quad + \frac{9\left(d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right) + 2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})\right)}{2\,a^3d^4} \\
&\quad - \frac{3\left(dx\log\left(\frac{be^{(dx+c)}}{a}+1\right) + \mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^3d^4}
\end{aligned}
$$

input
```
integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(3*a*d*x^3 - 3*a*x^2 + (2*b*d*x^3*e^c - 3*b*x^2*e^c)*e^(d*x))/(a^2*b^2
*d^2*e^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + a^4*d^2) + 1/4*(d^4*x^4 -
 6*d^3*x^3 + 6*d^2*x^2)/(a^3*d^4) - (d^3*x^3*log(b*e^(d*x + c)/a + 1) + 3*
d^2*x^2*dilog(-b*e^(d*x + c)/a) - 6*d*x*polylog(3, -b*e^(d*x + c)/a) + 6*p
olylog(4, -b*e^(d*x + c)/a))/(a^3*d^4) + 9/2*(d^2*x^2*log(b*e^(d*x + c)/a
+ 1) + 2*d*x*dilog(-b*e^(d*x + c)/a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a^
3*d^4) - 3*(d*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^3*d
^4)
```

Giac [F]

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \int \frac{x^3}{\left(be^{(dx+c)}+a\right)^3}\,dx$$

input
```
integrate(x^3/(a+b*exp(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(x^3/(b*e^(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \int \frac{x^3}{\left(a+b\,\mathrm{e}^{c+d\,x}\right)^3}\,dx$$

input
```
int(x^3/(a + b*exp(c + d*x))^3,x)
```

output
```
int(x^3/(a + b*exp(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{x^3}{\left(a+be^{c+dx}\right)^3}\,dx = \int \frac{x^3}{e^{3dx+3c}b^3+3e^{2dx+2c}a\,b^2+3e^{dx+c}a^2b+a^3}dx$$

input
```
int(x^3/(a+b*exp(d*x+c))^3,x)
```

output
```
int(x**3/(e**(3*c + 3*d*x)*b**3 + 3*e**(2*c + 2*d*x)*a*b**2 + 3*e**(c + d*
x)*a**2*b + a**3),x)
```

3.18 $\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 243

$$\begin{aligned}\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx &= \frac{x}{a^3d^2}-\frac{x}{a^2d^2\left(a+be^{c+dx}\right)}-\frac{3x^2}{2a^3d}+\frac{x^2}{2ad\left(a+be^{c+dx}\right)^2}\\ &+\frac{x^2}{a^2d\left(a+be^{c+dx}\right)}+\frac{x^3}{3a^3}-\frac{\log\left(a+be^{c+dx}\right)}{a^3d^3}+\frac{3x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d^2}\\ &-\frac{x^2\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d}+\frac{3\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^3}\\ &-\frac{2x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2}+\frac{2\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a}\right)}{a^3d^3}\end{aligned}$$

output

```
x/a^3/d^2-x/a^2/d^2/(a+b*exp(d*x+c))-3/2*x^2/a^3/d+1/2*x^2/a/d/(a+b*exp(d*
x+c))^2+x^2/a^2/d/(a+b*exp(d*x+c))+1/3*x^3/a^3-ln(a+b*exp(d*x+c))/a^3/d^3+
3*x*ln(1+b*exp(d*x+c)/a)/a^3/d^2-x^2*ln(1+b*exp(d*x+c)/a)/a^3/d+3*polylog(
2,-b*exp(d*x+c)/a)/a^3/d^3-2*x*polylog(2,-b*exp(d*x+c)/a)/a^3/d^2+2*polylo
g(3,-b*exp(d*x+c)/a)/a^3/d^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx$$

$$= \frac{\frac{6x}{d^2} - \frac{6ax}{d^2\left(a+be^{c+dx}\right)} - \frac{9x^2}{d} + \frac{3a^2x^2}{d\left(a+be^{c+dx}\right)^2} + \frac{6ax^2}{ad+bde^{c+dx}} + 2x^3 - \frac{6\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^3} + \frac{18x\log\left(1+\frac{be^{c+dx}}{a}\right)}{d^2} - \frac{6x^2\log\left(1+\frac{be^{c}}{d}\right)}{d}}{6a^3}$$

input
```
Integrate[x^2/(a + b*E^(c + d*x))^3,x]
```

output
```
((6*x)/d^2 - (6*a*x)/(d^2*(a + b*E^(c + d*x))) - (9*x^2)/d + (3*a^2*x^2)/(
d*(a + b*E^(c + d*x))^2) + (6*a*x^2)/(a*d + b*d*E^(c + d*x)) + 2*x^3 - (6*
Log[1 + (b*E^(c + d*x))/a])/d^3 + (18*x*Log[1 + (b*E^(c + d*x))/a])/d^2 -
(6*x^2*Log[1 + (b*E^(c + d*x))/a])/d - (6*(-3 + 2*d*x)*PolyLog[2, -((b*E^(
c + d*x))/a)])/d^3 + (12*PolyLog[3, -((b*E^(c + d*x))/a)])/d^3)/(6*a^3)
```

Rubi [A] (verified)

Time = 4.38 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.56, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.118$, Rules used = {2616, 2616, 2615, 2620, 2621, 2615, 2616, 2615, 2620, 2621, 2715, 2720, 47, 14, 16, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{x^2}{\left(a+be^{c+dx}\right)^2}dx}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{\int \frac{x^2}{a+be^{c+dx}}dx}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{\frac{x^3}{3a} - \frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a} - \frac{b\int \frac{e^{c+dx}x^2}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{2\int \frac{x}{a+be^{c+dx}}dx}{bd} - \frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a} - \frac{b\left(\frac{\int \frac{x}{\left(a+be^{c+dx}\right)^2}dx}{bd} - \frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{\frac{x^3}{3a} - \frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd} - \frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a} - \frac{b\left(\frac{2\left(\frac{x^2}{2a} - \frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}\right)}{bd} - \frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a} - \frac{b\left(\frac{\int \frac{x}{\left(a+be^{c+dx}\right)^2}dx}{bd} - \frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\int\frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{a}{b\left(\frac{\int\frac{x}{a+be^{c+dx}}dx}{a}-\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}}{bd}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\int\frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{a}{b\left(\frac{\frac{x^2}{2a}-\frac{b\int\frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}}{a}-\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}}{bd}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{a}{b\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}}{bd}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a}-$$

$$b\left(\frac{\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{bd}-\frac{b\left(\frac{\int\frac{1}{a+be^{c+dx}}dx}{bd}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}}{a}\right)$$

$$\downarrow 2715$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a}-$$

$$b\left(\frac{\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{bd}-\frac{b\left(\frac{\int\frac{1}{a+be^{c+dx}}dx}{bd}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}}{a}\right)$$

$$\downarrow 2720$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$b\left(\frac{\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{bd}-\frac{b\left(\frac{\int\frac{e^{-c-dx}}{a+be^{c+dx}}de^{c+dx}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}}{a}\right)$$

$$\downarrow 47$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$b\left(\frac{\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{bd}-\frac{b\left(\frac{\int e^{-c-dx}de^{c+dx}}{a}-\frac{b\int\frac{1}{a+be^{c+dx}}de^{c+dx}}{bd^2}\right)}{a}-\frac{x}{bd\left(a+be^{c+dx}\right)}}{a}\right)$$

$$\downarrow 14$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$b\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{bd}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{b\int\frac{1}{a+be^{c+dx}}de^{c+dx}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)$$

$$\Big\downarrow 16$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}$$

$$b\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{bd}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)$$

$$\Big\downarrow 2838$$

$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

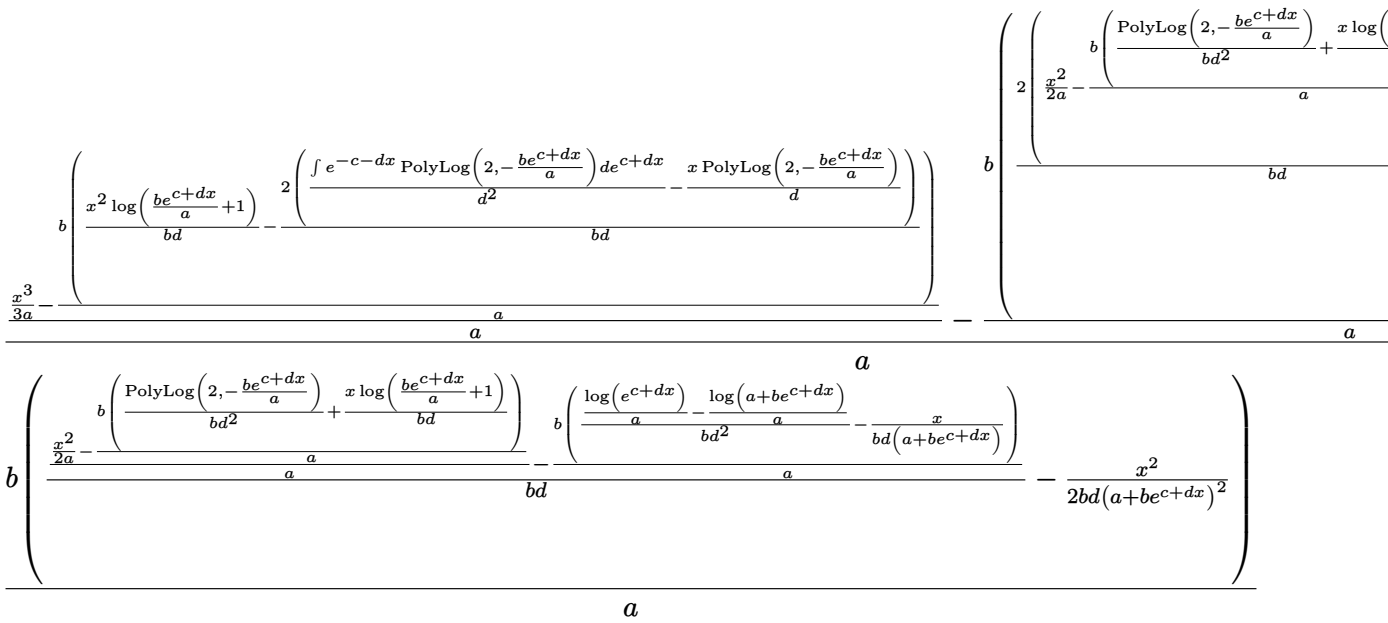
$$\frac{b\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{bd}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 3011$$

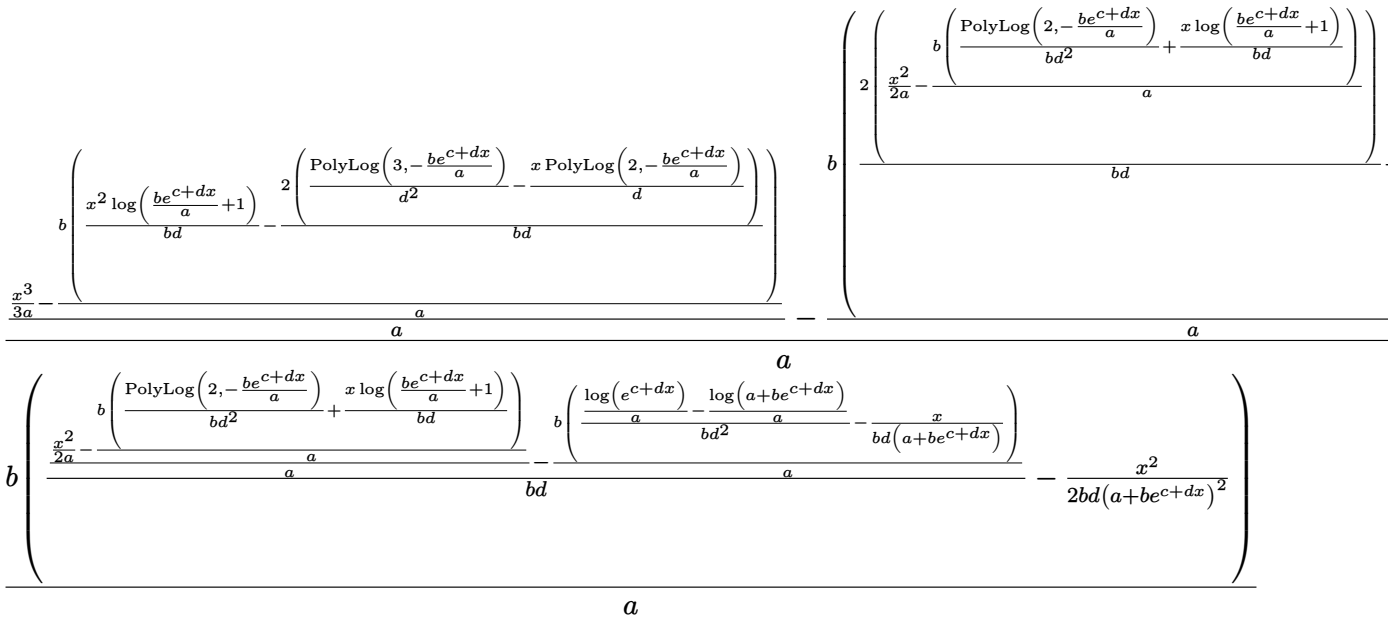
$$\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{2\left(\frac{\int\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{d}\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}\right)}{bd}-\frac{x^2}{bd\left(a+be^{c+dx}\right)}\right)}{a}-$$

$$\frac{b\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{bd}-\frac{x^2}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2720$$



$$\downarrow 7143$$



input `Int[x^2/(a + b*E^(c + d*x))^3,x]`

output
```
-((b*(-1/2*x^2/(b*d*(a + b*E^(c + d*x))^2) + (-((b*(-(x/(b*d*(a + b*E^(c +
 d*x)))) + (Log[E^(c + d*x)]/a - Log[a + b*E^(c + d*x)]/a)/(b*d^2)))/a) +
(x^2/(2*a) - (b*((x*Log[1 + (b*E^(c + d*x))/a])/(b*d) + PolyLog[2, -((b*E^
(c + d*x))/a)]/(b*d^2)))/a)/a)/(b*d)))/a) + (-((b*(-(x^2/(b*d*(a + b*E^(c
+ d*x)))) + (2*(x^2/(2*a) - (b*((x*Log[1 + (b*E^(c + d*x))/a])/(b*d) + Pol
yLog[2, -((b*E^(c + d*x))/a)]/(b*d^2)))/a))/(b*d)))/a) + (x^3/(3*a) - (b*(
(x^2*Log[1 + (b*E^(c + d*x))/a])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c + d
*x))/a)])/d) + PolyLog[3, -((b*E^(c + d*x))/a)]/d^2))/(b*d)))/a)/a)/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.58

method	result
risch	$\frac{x\left(2xbd\,\mathrm{e}^{dx+c}+3xda-2b\,\mathrm{e}^{dx+c}-2a\right)}{2d^2a^2\left(a+b\,\mathrm{e}^{dx+c}\right)^2}-\frac{c^2x}{a^3d^2}-\frac{x^2\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3d}-\frac{2x\,\mathrm{polylog}\left(2,-\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3d^2}-\frac{3cx}{a^3d^2}+\frac{3x\ln\left(1+\frac{b\,\mathrm{e}^{dx+c}}{a}\right)}{a^3d^2}$

input
```
int(x^2/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(2*x*b*d*exp(d*x+c)+3*x*d*a-2*b*exp(d*x+c)-2*a)/d^2/a^2/(a+b*exp(d*x
+c))^2-1/a^3/d^2*c^2*x-x^2*ln(1+b*exp(d*x+c)/a)/a^3/d-2*x*polylog(2,-b*exp
(d*x+c)/a)/a^3/d^2-3/a^3/d^2*c*x+3*x*ln(1+b*exp(d*x+c)/a)/a^3/d^2-2/3/a^3/
d^3*c^3+2*polylog(3,-b*exp(d*x+c)/a)/a^3/d^3-1/a^3/d^3*c^2*ln(a+b*exp(d*x+
c))+3/a^3/d^3*c*ln(exp(d*x+c))-3/a^3/d^3*c*ln(a+b*exp(d*x+c))+1/a^3/d^3*ln
(exp(d*x+c))-ln(a+b*exp(d*x+c))/a^3/d^3+1/3*x^3/a^3+1/a^3/d^3*ln(1+b*exp(d
*x+c)/a)*c^2-3/2*x^2/a^3/d+3/a^3/d^3*ln(1+b*exp(d*x+c)/a)*c+3*polylog(2,-b
*exp(d*x+c)/a)/a^3/d^3+1/a^3/d^3*c^2*ln(exp(d*x+c))-3/2/a^3/d^3*c^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. 2(226) = 452.

Time = 0.08 (sec) , antiderivative size = 521, normalized size of antiderivative = 2.14

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx$$
$$= \frac{2\,a^2d^3x^3+2\,a^2c^3+9\,a^2c^2+6\,a^2c-6\left(2\,a^2dx-3\,a^2+\left(2\,b^2dx-3\,b^2\right)e^{(2\,dx+2\,c)}+2\left(2\,abdx-3\,ab\right)e^{(dx+c)}\right.}{}$$

input
```
integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(2*a^2*d^3*x^3 + 2*a^2*c^3 + 9*a^2*c^2 + 6*a^2*c - 6*(2*a^2*d*x - 3*a^
2 + (2*b^2*d*x - 3*b^2)*e^(2*d*x + 2*c) + 2*(2*a*b*d*x - 3*a*b)*e^(d*x + c
))*dilog(-(b*e^(d*x + c) + a)/a + 1) + (2*b^2*d^3*x^3 - 9*b^2*d^2*x^2 + 2*
b^2*c^3 + 9*b^2*c^2 + 6*b^2*d*x + 6*b^2*c)*e^(2*d*x + 2*c) + 2*(2*a*b*d^3*
x^3 - 6*a*b*d^2*x^2 + 2*a*b*c^3 + 9*a*b*c^2 + 3*a*b*d*x + 6*a*b*c)*e^(d*x
+ c) - 6*(a^2*c^2 + 3*a^2*c + a^2 + (b^2*c^2 + 3*b^2*c + b^2)*e^(2*d*x + 2
*c) + 2*(a*b*c^2 + 3*a*b*c + a*b)*e^(d*x + c))*log(b*e^(d*x + c) + a) - 6*
(a^2*d^2*x^2 - a^2*c^2 - 3*a^2*d*x - 3*a^2*c + (b^2*d^2*x^2 - b^2*c^2 - 3*
b^2*d*x - 3*b^2*c)*e^(2*d*x + 2*c) + 2*(a*b*d^2*x^2 - a*b*c^2 - 3*a*b*d*x
- 3*a*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a) + 12*(b^2*e^(2*d*x + 2*
c) + 2*a*b*e^(d*x + c) + a^2)*polylog(3, -b*e^(d*x + c)/a))/(a^3*b^2*d^3*e
^(2*d*x + 2*c) + 2*a^4*b*d^3*e^(d*x + c) + a^5*d^3)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{3adx^2 - 2ax + \left(2bdx^2 - 2bx\right)e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \left(-\frac{3dx}{a+be^ce^{dx}}\right)\,dx + \int \frac{d^2x^2}{a+be^ce^{dx}}\,dx + \int \frac{1}{a+be^ce^{dx}}\,dx}{a^2d^2}$$

input
```
integrate(x**2/(a+b*exp(d*x+c))**3,x)
```

output
```
(3*a*d*x**2 - 2*a*x + (2*b*d*x**2 - 2*b*x)*exp(c + d*x))/(2*a**4*d**2 + 4*
a**3*b*d**2*exp(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integral(
-3*d*x/(a + b*exp(c)*exp(d*x)), x) + Integral(d**2*x**2/(a + b*exp(c)*exp(
d*x)), x) + Integral(1/(a + b*exp(c)*exp(d*x)), x))/(a**2*d**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.96

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \frac{3\,adx^2 - 2\,ax + 2\,(bdx^2e^c - bxe^c)e^{(dx)}}{2\,(a^2b^2d^2e^{(2\,dx+2\,c)} + 2\,a^3bd^2e^{(dx+c)} + a^4d^2)} + \frac{x}{a^3d^2} + \frac{2\,d^3x^3 - 9\,d^2x^2}{6\,a^3d^3} - \frac{d^2x^2\log\left(\frac{be^{(dx+c)}}{a}+1\right) + 2\,dx\mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right) - 2\,\mathrm{Li}_3(-\frac{be^{(dx+c)}}{a})}{a^3d^3} + \frac{3\left(dx\log\left(\frac{be^{(dx+c)}}{a}+1\right) + \mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)\right)}{a^3d^3} - \frac{\log\left(be^{(dx+c)}+a\right)}{a^3d^3}$$

input
```
integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(3*a*d*x^2 - 2*a*x + 2*(b*d*x^2*e^c - b*x*e^c)*e^(d*x))/(a^2*b^2*d^2*e
^(2*d*x + 2*c) + 2*a^3*b*d^2*e^(d*x + c) + a^4*d^2) + x/(a^3*d^2) + 1/6*(2
*d^3*x^3 - 9*d^2*x^2)/(a^3*d^3) - (d^2*x^2*log(b*e^(d*x + c)/a + 1) + 2*d*
x*dilog(-b*e^(d*x + c)/a) - 2*polylog(3, -b*e^(d*x + c)/a))/(a^3*d^3) + 3*
(d*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^3*d^3) - log(b
*e^(d*x + c) + a)/(a^3*d^3)
```

Giac [F]

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \int \frac{x^2}{(be^{(dx+c)}+a)^3}\,dx$$

input
```
integrate(x^2/(a+b*exp(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(x^2/(b*e^(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \int \frac{x^2}{(a+b\,\mathrm{e}^{c+d\,x})^3}\,dx$$

input
```
int(x^2/(a + b*exp(c + d*x))^3,x)
```

output
```
int(x^2/(a + b*exp(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{x^2}{(a+be^{c+dx})^3}\,dx = \int \frac{x^2}{e^{3dx+3c}b^3+3e^{2dx+2c}a\,b^2+3e^{dx+c}a^2b+a^3}dx$$

input
```
int(x^2/(a+b*exp(d*x+c))^3,x)
```

output
```
int(x**2/(e**(3*c + 3*d*x)*b**3 + 3*e**(2*c + 2*d*x)*a*b**2 + 3*e**(c + d*
x)*a**2*b + a**3),x)
```

3.19 $\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx$

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Optimal result

Integrand size = 15, antiderivative size = 159

$$\begin{aligned}\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx = &-\frac{1}{2a^2d^2\left(a+be^{c+dx}\right)} - \frac{3x}{2a^3d} + \frac{x}{2ad\left(a+be^{c+dx}\right)^2} \\ &+ \frac{x}{a^2d\left(a+be^{c+dx}\right)} + \frac{x^2}{2a^3} + \frac{3\log\left(a+be^{c+dx}\right)}{2a^3d^2} \\ &- \frac{x\log\left(1+\frac{be^{c+dx}}{a}\right)}{a^3d} - \frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a^3d^2}\end{aligned}$$

output

```
-1/2/a^2/d^2/(a+b*exp(d*x+c))-3/2*x/a^3/d+1/2*x/a/d/(a+b*exp(d*x+c))^2+x/a
^2/d/(a+b*exp(d*x+c))+1/2*x^2/a^3+3/2*ln(a+b*exp(d*x+c))/a^3/d^2-x*ln(1+b*
exp(d*x+c)/a)/a^3/d-polylog(2,-b*exp(d*x+c)/a)/a^3/d^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.75

$$\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{\frac{adx}{\left(a+be^{c+dx}\right)^2}+\frac{-1+2dx}{a+be^{c+dx}}+\frac{-3dx+3\log\left(1+\frac{be^{c+dx}}{a}\right)}{a}+\frac{dx\left(dx-2\log\left(1+\frac{be^{c+dx}}{a}\right)\right)-2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{a}}{2a^2d^2}$$

input
```
Integrate[x/(a + b*E^(c + d*x))^3,x]
```

output
```
((a*d*x)/(a + b*E^(c + d*x))^2 + (-1 + 2*d*x)/(a + b*E^(c + d*x)) + (-3*d*
x + 3*Log[1 + (b*E^(c + d*x))/a])/a + (d*x*(d*x - 2*Log[1 + (b*E^(c + d*x)
)/a]) - 2*PolyLog[2, -((b*E^(c + d*x))/a)])/a)/(2*a^2*d^2)
```

Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.43, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.867$, Rules used = {2616, 2616, 2615, 2620, 2621, 2715, 2720, 47, 14, 16, 54, 2009, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx$$
$$\downarrow 2616$$
$$\frac{\int \frac{x}{\left(a+be^{c+dx}\right)^2}dx}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^3}dx}{a}$$
$$\downarrow 2616$$
$$\frac{\int \frac{x}{a+be^{c+dx}}dx}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a} - \frac{b\int \frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^3}dx}{a}$$
$$\downarrow 2615$$

$$\frac{\frac{x^2}{2a}-\frac{b\int\frac{e^{c+dx}x}{a+be^{c+dx}}dx}{a}}{a}-\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a}-\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^3}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^2}dx}{a}}{a}-\frac{b\int\frac{e^{c+dx}x}{\left(a+be^{c+dx}\right)^3}dx}{a}}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a}+1\right)dx}{bd}\right)}{a}}{a}-\frac{\frac{b\left(\frac{\int\frac{1}{a+be^{c+dx}}dx}{bd}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a}-\frac{b\left(\frac{\int\frac{1}{\left(a+be^{c+dx}\right)^2}dx}{2bd}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}}{a}$$

$$\downarrow 2715$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{\frac{b\left(\frac{\int\frac{1}{a+be^{c+dx}}dx}{bd}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a}-\frac{b\left(\frac{\int\frac{1}{\left(a+be^{c+dx}\right)^2}dx}{2bd}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}}{a}$$

$$\downarrow 2720$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{\frac{b\left(\frac{\int\frac{e^{-c-dx}}{a+be^{c+dx}}de^{c+dx}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}}{a}-\frac{b\left(\frac{\int\frac{e^{-c-dx}}{\left(a+be^{c+dx}\right)^2}de^{c+dx}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}}{a}$$

$$\downarrow 47$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\int e^{-c-dx}de^{c+dx}}{a}-\frac{b\int\frac{1}{a+be^{c+dx}}de^{c+dx}}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{\int\frac{e^{-c-dx}}{\left(a+be^{c+dx}\right)^2}de^{c+dx}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 14$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{b\int\frac{1}{a+be^{c+dx}}de^{c+dx}}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{\int\frac{e^{-c-dx}}{\left(a+be^{c+dx}\right)^2}de^{c+dx}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 16$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{\int\frac{e^{-c-dx}}{\left(a+be^{c+dx}\right)^2}de^{c+dx}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 54$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{\int\left(-\frac{b}{a^2\left(a+be^{c+dx}\right)}-\frac{b}{a\left(a+be^{c+dx}\right)^2}+\frac{e^{-c-dx}}{a^2}\right)de^{c+dx}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2009$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a}+1\right)de^{c+dx}}{bd^2}\right)}{a}}{a}-\frac{b\left(\frac{\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{-\frac{\log\left(a+be^{c+dx}\right)}{a^2}+\frac{\log\left(e^{c+dx}\right)}{a^2}+\frac{1}{a\left(a+be^{c+dx}\right)}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a}+1\right)}{bd}\right)}{a}}{a}-\frac{b\left(\frac{\frac{\log\left(e^{c+dx}\right)}{a}-\frac{\log\left(a+be^{c+dx}\right)}{a}}{bd^2}-\frac{x}{bd\left(a+be^{c+dx}\right)}\right)}{a}-\frac{b\left(\frac{-\frac{\log\left(a+be^{c+dx}\right)}{a^2}+\frac{\log\left(e^{c+dx}\right)}{a^2}+\frac{1}{a\left(a+be^{c+dx}\right)}}{2bd^2}-\frac{x}{2bd\left(a+be^{c+dx}\right)^2}\right)}{a}$$

input
```
Int[x/(a + b*E^(c + d*x))^3,x]
```

output
```
-((b*(-1/2*x/(b*d*(a + b*E^(c + d*x))^2) + (1/(a*(a + b*E^(c + d*x))) + Lo
g[E^(c + d*x)]/a^2 - Log[a + b*E^(c + d*x)]/a^2)/(2*b*d^2)))/a) + (-((b*(-
(x/(b*d*(a + b*E^(c + d*x)))) + (Log[E^(c + d*x)]/a - Log[a + b*E^(c + d*x
)]/a)/(b*d^2)))/a) + (x^2/(2*a) - (b*((x*Log[1 + (b*E^(c + d*x))/a])/(b*d)
 + PolyLog[2, -((b*E^(c + d*x))/a)]/(b*d^2)))/a)/a)/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36

method	result
risch	$\frac{2xbd\,e^{dx+c}+3xda-b\,e^{dx+c}-a}{2d^2a^2\left(a+b\,e^{dx+c}\right)^2}-\frac{3\ln\left(e^{dx+c}\right)}{2d^2a^3}+\frac{3\ln\left(a+b\,e^{dx+c}\right)}{2a^3d^2}-\frac{c\ln\left(e^{dx+c}\right)}{d^2a^3}+\frac{c\ln\left(a+b\,e^{dx+c}\right)}{d^2a^3}+\frac{x^2}{2a^3}+$
derivativedivides	$\frac{\frac{(dx+c)^2}{2a^3}+\frac{3\ln\left(a+b\,e^{dx+c}\right)}{2a^3}-\frac{b(dx+c)e^{dx+c}}{a^3\left(a+b\,e^{dx+c}\right)}-\frac{1}{2a^2\left(a+b\,e^{dx+c}\right)}-\frac{b^2(dx+c)e^{2dx+2c}}{2a^3\left(a+b\,e^{dx+c}\right)^2}-\frac{b(dx+c)e^{dx+c}}{a^2\left(a+b\,e^{dx+c}\right)^2}-\frac{\mathrm{dilog}\left(\frac{a+b\,e^{dx+c}}{a}\right)}{a^3}}{d^2}$
default	$\frac{\frac{(dx+c)^2}{2a^3}+\frac{3\ln\left(a+b\,e^{dx+c}\right)}{2a^3}-\frac{b(dx+c)e^{dx+c}}{a^3\left(a+b\,e^{dx+c}\right)}-\frac{1}{2a^2\left(a+b\,e^{dx+c}\right)}-\frac{b^2(dx+c)e^{2dx+2c}}{2a^3\left(a+b\,e^{dx+c}\right)^2}-\frac{b(dx+c)e^{dx+c}}{a^2\left(a+b\,e^{dx+c}\right)^2}-\frac{\mathrm{dilog}\left(\frac{a+b\,e^{dx+c}}{a}\right)}{a^3}}{d^2}$

input
```
int(x/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*x*b*d*exp(d*x+c)+3*x*d*a-b*exp(d*x+c)-a)/d^2/a^2/(a+b*exp(d*x+c))^2
-3/2/d^2/a^3*ln(exp(d*x+c))+3/2*ln(a+b*exp(d*x+c))/a^3/d^2-1/d^2/a^3*c*ln(
exp(d*x+c))+1/d^2/a^3*c*ln(a+b*exp(d*x+c))+1/2*x^2/a^3+1/d/a^3*c*x+1/2/d^2
/a^3*c^2-x*ln(1+b*exp(d*x+c)/a)/a^3/d-1/d^2/a^3*ln(1+b*exp(d*x+c)/a)*c-pol
ylog(2,-b*exp(d*x+c)/a)/a^3/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(142) = 284.

Time = 0.07 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.13

$$\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx$$
$$= \frac{a^2d^2x^2 - a^2c^2 - 3\,a^2c - a^2 - 2\left(b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + a^2\right)\mathrm{Li}_2\left(-\frac{be^{(dx+c)}+a}{a}+1\right) + (b^2d^2x^2 - b^2c^2 - 3$$

input
```
integrate(x/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(a^2*d^2*x^2 - a^2*c^2 - 3*a^2*c - a^2 - 2*(b^2*e^(2*d*x + 2*c) + 2*a*
b*e^(d*x + c) + a^2)*dilog(-(b*e^(d*x + c) + a)/a + 1) + (b^2*d^2*x^2 - b^
2*c^2 - 3*b^2*d*x - 3*b^2*c)*e^(2*d*x + 2*c) + (2*a*b*d^2*x^2 - 2*a*b*c^2
 - 4*a*b*d*x - 6*a*b*c - a*b)*e^(d*x + c) + (2*a^2*c + 3*a^2 + (2*b^2*c + 3
*b^2)*e^(2*d*x + 2*c) + 2*(2*a*b*c + 3*a*b)*e^(d*x + c))*log(b*e^(d*x + c)
 + a) - 2*(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*e^(2*d*x + 2*c) + 2*(a*b*d*
x + a*b*c)*e^(d*x + c))*log((b*e^(d*x + c) + a)/a))/(a^3*b^2*d^2*e^(2*d*x
 + 2*c) + 2*a^4*b*d^2*e^(d*x + c) + a^5*d^2)
```

Sympy [F]

$$\int \frac{x}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{3adx - a + (2bdx - b)\,e^{c+dx}}{2a^4d^2 + 4a^3bd^2e^{c+dx} + 2a^2b^2d^2e^{2c+2dx}} + \frac{\int \frac{2dx}{a+be^ce^{dx}}\,dx + \int \left(-\frac{3}{a+be^ce^{dx}}\right)\,dx}{2a^2d}$$

input
```
integrate(x/(a+b*exp(d*x+c))**3,x)
```

output
```
(3*a*d*x - a + (2*b*d*x - b)*exp(c + d*x))/(2*a**4*d**2 + 4*a**3*b*d**2*ex
p(c + d*x) + 2*a**2*b**2*d**2*exp(2*c + 2*d*x)) + (Integral(2*d*x/(a + b*e
xp(c)*exp(d*x)), x) + Integral(-3/(a + b*exp(c)*exp(d*x)), x))/(2*a**2*d)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \frac{3\,adx + (2\,bdxe^{c} - be^{c})e^{(dx)} - a}{2\,(a^2b^2d^2e^{(2\,dx+2\,c)} + 2\,a^3bd^2e^{(dx+c)} + a^4d^2)} + \frac{x^2}{2\,a^3} - \frac{3\,x}{2\,a^3d} - \frac{dx\log\left(\frac{be^{(dx+c)}}{a}+1\right) + \mathrm{Li}_2\left(-\frac{be^{(dx+c)}}{a}\right)}{a^3d^2} + \frac{3\,\log\left(be^{(dx+c)}+a\right)}{2\,a^3d^2}$$

input
```
integrate(x/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(3*a*d*x + (2*b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*b^2*d^2*e^(2*d*x +
2*c) + 2*a^3*b*d^2*e^(d*x + c) + a^4*d^2) + 1/2*x^2/a^3 - 3/2*x/(a^3*d) -
(d*x*log(b*e^(d*x + c)/a + 1) + dilog(-b*e^(d*x + c)/a))/(a^3*d^2) + 3/2*l
og(b*e^(d*x + c) + a)/(a^3*d^2)
```

Giac [F]

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \int \frac{x}{(be^{(dx+c)}+a)^3}\,dx$$

input
```
integrate(x/(a+b*exp(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(x/(b*e^(d*x + c) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \int \frac{x}{(a+b\,\mathrm{e}^{c+d\,x})^3}\,dx$$

input
```
int(x/(a + b*exp(c + d*x))^3,x)
```

output
```
int(x/(a + b*exp(c + d*x))^3, x)
```

Reduce [F]

$$\int \frac{x}{(a+be^{c+dx})^3}\,dx = \int \frac{x}{e^{3dx+3c}b^3+3e^{2dx+2c}a\,b^2+3e^{dx+c}a^2b+a^3}dx$$

input
```
int(x/(a+b*exp(d*x+c))^3,x)
```

output
```
int(x/(e**(3*c + 3*d*x)*b**3 + 3*e**(2*c + 2*d*x)*a*b**2 + 3*e**(c + d*x)*
a**2*b + a**3),x)
```

3.20 $\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx$

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Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{1}{2ad\left(a+be^{c+dx}\right)^2} + \frac{1}{a^2d\left(a+be^{c+dx}\right)} + \frac{x}{a^3} - \frac{\log\left(a+be^{c+dx}\right)}{a^3d}$$

output
```
1/2/a/d/(a+b*exp(d*x+c))^2+1/a^2/d/(a+b*exp(d*x+c))+x/a^3-ln(a+b*exp(d*x+c
))/a^3/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{\frac{a\left(3a+2be^{c+dx}\right)}{\left(a+be^{c+dx}\right)^2} + 2\log\left(e^{c+dx}\right) - 2\log\left(a+be^{c+dx}\right)}{2a^3d}$$

input
```
Integrate[(a + b*E^(c + d*x))^(-3),x]
```

output
```
((a*(3*a + 2*b*E^(c + d*x)))/(a + b*E^(c + d*x))^2 + 2*Log[E^(c + d*x)] -
2*Log[a + b*E^(c + d*x)])/(2*a^3*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+be^{c+dx})^3}\,dx$$

$$\downarrow 2720$$

$$\frac{\int \frac{e^{-c-dx}}{(a+be^{c+dx})^3}\,de^{c+dx}}{d}$$

$$\downarrow 54$$

$$\frac{\int \left(-\frac{b}{a^3(a+be^{c+dx})} - \frac{b}{a^2(a+be^{c+dx})^2} - \frac{b}{a(a+be^{c+dx})^3} + \frac{e^{-c-dx}}{a^3}\right) de^{c+dx}}{d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(a+be^{c+dx})}{a^3} + \frac{\log(e^{c+dx})}{a^3} + \frac{1}{a^2(a+be^{c+dx})} + \frac{1}{2a(a+be^{c+dx})^2}}{d}$$

input
```
Int[(a + b*E^(c + d*x))^(-3),x]
```

output
```
(1/(2*a*(a + b*E^(c + d*x))^2) + 1/(a^2*(a + b*E^(c + d*x))) + Log[E^(c + d*x)]/a^3 - Log[a + b*E^(c + d*x)]/a^3)/d
```

Defintions of rubi rules used

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{\ln\left(e^{dx+c}\right)}{a^3}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,e^{dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{dx+c}\right)^2}}{d}$
default	$\frac{\frac{\ln\left(e^{dx+c}\right)}{a^3}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,e^{dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{dx+c}\right)^2}}{d}$
risch	$\frac{x}{a^3}+\frac{c}{a^3d}+\frac{2b\,e^{dx+c}+3a}{2d\,a^2\left(a+b\,e^{dx+c}\right)^2}-\frac{\ln\left(e^{dx+c}+\frac{a}{b}\right)}{a^3d}$
norman	$\frac{\frac{x}{a}+\frac{b^2x\,e^{2dx+2c}}{a^3}+\frac{2bx\,e^{dx+c}}{a^2}-\frac{2b\,e^{dx+c}}{a^2d}-\frac{3b^2e^{2dx+2c}}{2a^3d}}{\left(a+b\,e^{dx+c}\right)^2}-\frac{\ln\left(a+b\,e^{dx+c}\right)}{a^3d}$
parallelrisch	$-\frac{-2\,e^{2dx+2c}b^2dx+2\ln\left(a+b\,e^{dx+c}\right)e^{2dx+2c}b^2-4\,e^{dx+c}abdx+4\ln\left(a+b\,e^{dx+c}\right)e^{dx+c}ab-2a^2dx+3b^2e^{2dx+2c}+2\ln(a+$

input

```
int(1/(a+b*exp(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/a^3*ln(exp(d*x+c))-1/a^3*ln(a+b*exp(d*x+c))+1/a^2/(a+b*exp(d*x+c))+
1/2/a/(a+b*exp(d*x+c))^2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.84

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx$$
$$= \frac{2\,b^2dxe^{(2\,dx+2\,c)} + 2\,a^2dx + 3\,a^2 + 2\,(2\,abdx + ab)e^{(dx+c)} - 2\left(b^2e^{(2\,dx+2\,c)} + 2\,abe^{(dx+c)} + a^2\right)\log\left(be^{(dx+c)}\right.}{2\left(a^3b^2de^{(2\,dx+2\,c)} + 2\,a^4bde^{(dx+c)} + a^5d\right)}$$

input
```
integrate(1/(a+b*exp(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*d*x*e^(2*d*x + 2*c) + 2*a^2*d*x + 3*a^2 + 2*(2*a*b*d*x + a*b)*e
^(d*x + c) - 2*(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) + a^2)*log(b*e^(d*
x + c) + a))/(a^3*b^2*d*e^(2*d*x + 2*c) + 2*a^4*b*d*e^(d*x + c) + a^5*d)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{3a+2be^{c+dx}}{2a^4d+4a^3bde^{c+dx}+2a^2b^2de^{2c+2dx}} + \frac{x}{a^3} - \frac{\log\left(\frac{a}{b}+e^{c+dx}\right)}{a^3d}$$

input
```
integrate(1/(a+b*exp(d*x+c))**3,x)
```

output
```
(3*a + 2*b*exp(c + d*x))/(2*a**4*d + 4*a**3*b*d*exp(c + d*x) + 2*a**2*b**2
*d*exp(2*c + 2*d*x)) + x/a**3 - log(a/b + exp(c + d*x))/(a**3*d)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{2\,be^{(dx+c)}+3\,a}{2\left(a^2b^2e^{(2\,dx+2\,c)}+2\,a^3be^{(dx+c)}+a^4\right)d} + \frac{dx+c}{a^3d} - \frac{\log\left(be^{(dx+c)}+a\right)}{a^3d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*b*e^(d*x + c) + 3*a)/((a^2*b^2*e^(2*d*x + 2*c) + 2*a^3*b*e^(d*x + c
) + a^4)*d) + (d*x + c)/(a^3*d) - log(b*e^(d*x + c) + a)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{\frac{2\,(dx+c)}{a^3} - \frac{2\,\log\left(\left|be^{(dx+c)}+a\right|\right)}{a^3} + \frac{2\,abe^{(dx+c)}+3\,a^2}{\left(be^{(dx+c)}+a\right)^2 a^3}}{2\,d}$$

input
```
integrate(1/(a+b*exp(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a^3 - 2*log(abs(b*e^(d*x + c) + a))/a^3 + (2*a*b*e^(d*x +
 c) + 3*a^2)/((b*e^(d*x + c) + a)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 23.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx = \frac{\frac{x}{a} + \frac{b^2\,x\,\mathrm{e}^{2\,c+2\,d\,x}}{a^3} + \frac{2\,b\,x\,\mathrm{e}^{c+d\,x}}{a^2} - \frac{3\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{2\,a^3\,d} - \frac{2\,b\,\mathrm{e}^{c+d\,x}}{a^2\,d}}{a^2 + 2\,\mathrm{e}^{c+d\,x}\,a\,b + \mathrm{e}^{2\,c+2\,d\,x}\,b^2} - \frac{\ln\left(a+b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{a^3\,d}$$

input
```
int(1/(a + b*exp(c + d*x))^3,x)
```

output
```
(x/a + (b^2*x*exp(2*c + 2*d*x))/a^3 + (2*b*x*exp(c + d*x))/a^2 - (3*b^2*ex
p(2*c + 2*d*x))/(2*a^3*d) - (2*b*exp(c + d*x))/(a^2*d))/(a^2 + b^2*exp(2*c
 + 2*d*x) + 2*a*b*exp(c + d*x)) - log(a + b*exp(d*x)*exp(c))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.38

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3}\,dx$$
$$= \frac{-2e^{2dx+2c}\log\left(e^{dx+c}b+a\right)b^2+2e^{2dx+2c}b^2dx-e^{2dx+2c}b^2-4e^{dx+c}\log\left(e^{dx+c}b+a\right)ab+4e^{dx+c}abdx-2\log}{2a^3d\left(e^{2dx+2c}b^2+2e^{dx+c}ab+a^2\right)}$$

input
```
int(1/(a+b*exp(d*x+c))^3,x)
```

output
```
( - 2*e**(2*c + 2*d*x)*log(e**(c + d*x)*b + a)*b**2 + 2*e**(2*c + 2*d*x)*b
**2*d*x - e**(2*c + 2*d*x)*b**2 - 4*e**(c + d*x)*log(e**(c + d*x)*b + a)*a
*b + 4*e**(c + d*x)*a*b*d*x - 2*log(e**(c + d*x)*b + a)*a**2 + 2*a**2*d*x
+ 2*a**2)/(2*a**3*d*(e**(2*c + 2*d*x)*b**2 + 2*e**(c + d*x)*a*b + a**2))
```

3.21 $\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx$

Optimal result 228
Mathematica [N/A] 228
Rubi [N/A] 229
Maple [N/A] 229
Fricas [N/A] 230
Sympy [N/A] 230
Maxima [N/A] 231
Giac [N/A] 231
Mupad [N/A] 231
Reduce [N/A] 232

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^3 x}, x\right)$$

output
```
Defer(Int)(1/(a+b*exp(d*x+c))^3/x,x)
```

Mathematica [N/A]

Not integrable

Time = 4.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx$$

input
```
Integrate[1/((a + b*E^(c + d*x))^3*x),x]
```

output
```
Integrate[1/((a + b*E^(c + d*x))^3*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+be^{c+dx}\right)^3}\,dx$$
$$\downarrow 2619$$
$$\int \frac{1}{x\left(a+be^{c+dx}\right)^3}dx$$

input
```
Int[1/((a + b*E^(c + d*x))^3*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a+b\,\mathrm{e}^{dx+c}\right)^3 x}dx$$

input
```
int(1/(a+b*exp(d*x+c))^3/x,x)
```

output
```
int(1/(a+b*exp(d*x+c))^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^3/x,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x*e^(3*d*x + 3*c) + 3*a*b^2*x*e^(2*d*x + 2*c) + 3*a^2*b*x*
e^(d*x + c) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.71

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \frac{3adx + a + (2bdx + b)\,e^{c+dx}}{2a^4d^2x^2 + 4a^3bd^2x^2e^{c+dx} + 2a^2b^2d^2x^2e^{2c+2dx}} + \frac{\int \frac{3dx}{ax^3+bx^3e^ce^{dx}}\,dx + \int \frac{2d^2x^2}{ax^3+bx^3e^ce^{dx}}\,dx + \int \frac{2}{ax^3+bx^3e^ce^{dx}}\,dx}{2a^2d^2}$$

input
```
integrate(1/(a+b*exp(d*x+c))**3/x,x)
```

output
```
(3*a*d*x + a + (2*b*d*x + b)*exp(c + d*x))/(2*a**4*d**2*x**2 + 4*a**3*b*d*
*2*x**2*exp(c + d*x) + 2*a**2*b**2*d**2*x**2*exp(2*c + 2*d*x)) + (Integral
(3*d*x/(a*x**3 + b*x**3*exp(c)*exp(d*x)), x) + Integral(2*d**2*x**2/(a*x**
3 + b*x**3*exp(c)*exp(d*x)), x) + Integral(2/(a*x**3 + b*x**3*exp(c)*exp(d
*x)), x))/(2*a**2*d**2)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^3/x,x, algorithm="maxima")
```

output
```
1/2*(3*a*d*x + (2*b*d*x*e^c + b*e^c)*e^(d*x) + a)/(a^2*b^2*d^2*x^2*e^(2*d*
x + 2*c) + 2*a^3*b*d^2*x^2*e^(d*x + c) + a^4*d^2*x^2) + integrate(1/2*(2*d
^2*x^2 + 3*d*x + 2)/(a^2*b*d^2*x^3*e^(d*x + c) + a^3*d^2*x^3), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^3/x,x, algorithm="giac")
```

output
```
integrate(1/((b*e^(d*x + c) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 23.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{x\left(a+b\,\mathrm{e}^{c+d\,x}\right)^3}\,dx$$

input
```
int(1/(x*(a + b*exp(c + d*x))^3),x)
```

output
```
int(1/(x*(a + b*exp(c + d*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x}\,dx = \int \frac{1}{e^{3dx+3c}b^3x + 3e^{2dx+2c}a\,b^2x + 3e^{dx+c}a^2bx + a^3x}dx$$

input
```
int(1/(a+b*exp(d*x+c))^3/x,x)
```

output
```
int(1/(e**(3*c + 3*d*x)*b**3*x + 3*e**(2*c + 2*d*x)*a*b**2*x + 3*e**(c + d
*x)*a**2*b*x + a**3*x),x)
```

3.22 $\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx$

Optimal result 233
Mathematica [N/A] 233
Rubi [N/A] 234
Maple [N/A] 234
Fricas [N/A] 235
Sympy [N/A] 235
Maxima [N/A] 236
Giac [N/A] 236
Mupad [N/A] 236
Reduce [N/A] 237

Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \text{Int}\left(\frac{1}{\left(a+be^{c+dx}\right)^3 x^2}, x\right)$$

output
```
Defer(Int)(1/(a+b*exp(d*x+c))^3/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx$$

input
```
Integrate[1/((a + b*E^(c + d*x))^3*x^2),x]
```

output
```
Integrate[1/((a + b*E^(c + d*x))^3*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a + be^{c+dx}\right)^3} dx$$
$$\downarrow 2619$$
$$\int \frac{1}{x^2 \left(a + be^{c+dx}\right)^3} dx$$

input
```
Int[1/((a + b*E^(c + d*x))^3*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(a + b\,\mathrm{e}^{dx+c}\right)^3 x^2} dx$$

input
```
int(1/(a+b*exp(d*x+c))^3/x^2,x)
```

output
```
int(1/(a+b*exp(d*x+c))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.59

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^3/x^2,x, algorithm="fricas")
```

output
```
integral(1/(b^3*x^2*e^(3*d*x + 3*c) + 3*a*b^2*x^2*e^(2*d*x + 2*c) + 3*a^2*
b*x^2*e^(d*x + c) + a^3*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 165, normalized size of antiderivative = 9.71

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \frac{3adx+2a+\left(2bdx+2b\right)e^{c+dx}}{2a^4d^2x^3+4a^3bd^2x^3e^{c+dx}+2a^2b^2d^2x^3e^{2c+2dx}} + \frac{\int \frac{3dx}{ax^4+bx^4e^ce^{dx}}\,dx+\int \frac{d^2x^2}{ax^4+bx^4e^ce^{dx}}\,dx+\int \frac{3}{ax^4+bx^4e^ce^{dx}}\,dx}{a^2d^2}$$

input
```
integrate(1/(a+b*exp(d*x+c))**3/x**2,x)
```

output
```
(3*a*d*x + 2*a + (2*b*d*x + 2*b)*exp(c + d*x))/(2*a**4*d**2*x**3 + 4*a**3*
b*d**2*x**3*exp(c + d*x) + 2*a**2*b**2*d**2*x**3*exp(2*c + 2*d*x)) + (Inte
gral(3*d*x/(a*x**4 + b*x**4*exp(c)*exp(d*x)), x) + Integral(d**2*x**2/(a*x
**4 + b*x**4*exp(c)*exp(d*x)), x) + Integral(3/(a*x**4 + b*x**4*exp(c)*exp
(d*x)), x))/(a**2*d**2)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.53

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^3/x^2,x, algorithm="maxima")
```

output
```
1/2*(3*a*d*x + 2*(b*d*x*e^c + b*e^c)*e^(d*x) + 2*a)/(a^2*b^2*d^2*x^3*e^(2*
d*x + 2*c) + 2*a^3*b*d^2*x^3*e^(d*x + c) + a^4*d^2*x^3) + integrate((d^2*x
^2 + 3*d*x + 3)/(a^2*b*d^2*x^4*e^(d*x + c) + a^3*d^2*x^4), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{\left(be^{(dx+c)}+a\right)^3 x^2}\,dx$$

input
```
integrate(1/(a+b*exp(d*x+c))^3/x^2,x, algorithm="giac")
```

output
```
integrate(1/((b*e^(d*x + c) + a)^3*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{x^2\left(a+b\,\mathrm{e}^{c+d\,x}\right)^3}\,dx$$

input
```
int(1/(x^2*(a + b*exp(c + d*x))^3),x)
```

output
```
int(1/(x^2*(a + b*exp(c + d*x))^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.76

$$\int \frac{1}{\left(a+be^{c+dx}\right)^3 x^2}\,dx = \int \frac{1}{e^{3dx+3c}b^3x^2+3e^{2dx+2c}a\,b^2x^2+3e^{dx+c}a^2b\,x^2+a^3x^2}dx$$

input
```
int(1/(a+b*exp(d*x+c))^3/x^2,x)
```

output
```
int(1/(e**(3*c + 3*d*x)*b**3*x**2 + 3*e**(2*c + 2*d*x)*a*b**2*x**2 + 3*e**
(c + d*x)*a**2*b*x**2 + a**3*x**2),x)
```

3.23 $\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx$

Optimal result 238
Mathematica [A] (verified) 238
Rubi [A] (verified) 239
Maple [A] (verified) 240
Fricas [A] (verification not implemented) 241
Sympy [A] (verification not implemented) 241
Maxima [A] (verification not implemented) 242
Giac [A] (verification not implemented) 242
Mupad [B] (verification not implemented) 242
Reduce [B] (verification not implemented) 243

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = -\frac{1}{2ad\left(a+be^{c-dx}\right)^2} - \frac{1}{a^2d\left(a+be^{c-dx}\right)} + \frac{x}{a^3} + \frac{\log\left(a+be^{c-dx}\right)}{a^3d}$$

output

```
-1/2/a/d/(a+b*exp(-d*x+c))^2-1/a^2/d/(a+b*exp(-d*x+c))+x/a^3+ln(a+b*exp(-d
*x+c))/a^3/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = \frac{\frac{be^c\left(3be^c+4ae^{dx}\right)}{\left(be^c+ae^{dx}\right)^2} + 2\log\left(a^2d\left(be^c+ae^{dx}\right)\right)}{2a^3d}$$

input

```
Integrate[(a + b*E^(c - d*x))^(-3),x]
```

output

```
((b*E^c*(3*b*E^c + 4*a*E^(d*x)))/(b*E^c + a*E^(d*x))^2 + 2*Log[a^2*d*(b*E^
c + a*E^(d*x))])/(2*a^3*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+be^{c-dx})^3}\,dx$$

$$\downarrow 2720$$

$$-\frac{\int \frac{e^{dx-c}}{(a+be^{c-dx})^3}\,de^{c-dx}}{d}$$

$$\downarrow 54$$

$$-\frac{\int \left(-\frac{b}{a^3(a+be^{c-dx})}-\frac{b}{a^2(a+be^{c-dx})^2}-\frac{b}{a(a+be^{c-dx})^3}+\frac{e^{dx-c}}{a^3}\right)de^{c-dx}}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\log(a+be^{c-dx})}{a^3}+\frac{\log(e^{c-dx})}{a^3}+\frac{1}{a^2(a+be^{c-dx})}+\frac{1}{2a(a+be^{c-dx})^2}}{d}$$

input
```
Int[(a + b*E^(c - d*x))^(-3),x]
```

output
```
-((1/(2*a*(a + b*E^(c - d*x))^2) + 1/(a^2*(a + b*E^(c - d*x))) + Log[E^(c
- d*x)]/a^3 - Log[a + b*E^(c - d*x)]/a^3)/d)
```

Defintions of rubi rules used

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^3}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,e^{-dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{-dx+c}\right)^2}}{d}$
default	$-\frac{\frac{\ln\left(e^{-dx+c}\right)}{a^3}-\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,e^{-dx+c}\right)}+\frac{1}{2a\left(a+b\,e^{-dx+c}\right)^2}}{d}$
risch	$\frac{x}{a^3}-\frac{c}{a^3d}-\frac{2b\,e^{-dx+c}+3a}{2d\,a^2\left(a+b\,e^{-dx+c}\right)^2}+\frac{\ln\left(e^{-dx+c}+\frac{a}{b}\right)}{a^3d}$
norman	$\frac{\frac{x}{a}+\frac{b^2x\,e^{-2dx+2c}}{a^3}+\frac{2bx\,e^{-dx+c}}{a^2}+\frac{2b\,e^{-dx+c}}{a^2d}+\frac{3b^2e^{-2dx+2c}}{2a^3d}}{\left(a+b\,e^{-dx+c}\right)^2}+\frac{\ln\left(a+b\,e^{-dx+c}\right)}{a^3d}$
parallelrisch	$\frac{2\,e^{-2dx+2c}b^2dx+2\ln\left(a+b\,e^{-dx+c}\right)e^{-2dx+2c}b^2+4\,e^{-dx+c}abdx+4\ln\left(a+b\,e^{-dx+c}\right)e^{-dx+c}ab+2a^2dx+3b^2e^{-2dx+2c}+2}{2a^3d\left(a+b\,e^{-dx+c}\right)^2}$

input

```
int(1/(a+b*exp(-d*x+c))^3,x,method=_RETURNVERBOSE)
```

output

```
-1/d*(1/a^3*ln(exp(-d*x+c))-1/a^3*ln(a+b*exp(-d*x+c))+1/a^2/(a+b*exp(-d*x+
c))+1/2/a/(a+b*exp(-d*x+c))^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx$$
$$= \frac{2\,b^2dxe^{(-2\,dx+2\,c)} + 2\,a^2dx - 3\,a^2 + 2\,(2\,abdx - ab)e^{(-dx+c)} + 2\left(2\,abe^{(-dx+c)} + b^2e^{(-2\,dx+2\,c)} + a^2\right)\log\left(be^{(}\right.}{2\left(2\,a^4bde^{(-dx+c)} + a^3b^2de^{(-2\,dx+2\,c)} + a^5d\right)}$$

input
```
integrate(1/(a+b*exp(-d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*d*x*e^(-2*d*x + 2*c) + 2*a^2*d*x - 3*a^2 + 2*(2*a*b*d*x - a*b)*
e^(-d*x + c) + 2*(2*a*b*e^(-d*x + c) + b^2*e^(-2*d*x + 2*c) + a^2)*log(b*e
^(-d*x + c) + a))/(2*a^4*b*d*e^(-d*x + c) + a^3*b^2*d*e^(-2*d*x + 2*c) + a
^5*d)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = \frac{-3a-2be^{c-dx}}{2a^4d+4a^3bde^{c-dx}+2a^2b^2de^{2c-2dx}} + \frac{x}{a^3} + \frac{\log\left(\frac{a}{b}+e^{c-dx}\right)}{a^3d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))**3,x)
```

output
```
(-3*a - 2*b*exp(c - d*x))/(2*a**4*d + 4*a**3*b*d*exp(c - d*x) + 2*a**2*b**
2*d*exp(2*c - 2*d*x)) + x/a**3 + log(a/b + exp(c - d*x))/(a**3*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = -\frac{2\,be^{(-dx+c)}+3\,a}{2\left(2\,a^3be^{(-dx+c)}+a^2b^2e^{(-2\,dx+2\,c)}+a^4\right)d} + \frac{dx-c}{a^3d} + \frac{\log\left(be^{(-dx+c)}+a\right)}{a^3d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*e^(-d*x + c) + 3*a)/((2*a^3*b*e^(-d*x + c) + a^2*b^2*e^(-2*d*x +
 2*c) + a^4)*d) + (d*x - c)/(a^3*d) + log(b*e^(-d*x + c) + a)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = \frac{\frac{2\,(dx-c)}{a^3} + \frac{2\log\left(\left|be^{(-dx+c)}+a\right|\right)}{a^3} - \frac{2\,abe^{(-dx+c)}+3\,a^2}{\left(be^{(-dx+c)}+a\right)^2a^3}}{2\,d}$$

input
```
integrate(1/(a+b*exp(-d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x - c)/a^3 + 2*log(abs(b*e^(-d*x + c) + a))/a^3 - (2*a*b*e^(-d*x
 + c) + 3*a^2)/((b*e^(-d*x + c) + a)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 23.18 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx = \frac{\frac{x}{a} + \frac{b^2\,x\,\mathrm{e}^{2\,c-2\,d\,x}}{a^3} + \frac{2\,b\,x\,\mathrm{e}^{c-d\,x}}{a^2} + \frac{3\,b^2\,\mathrm{e}^{2\,c-2\,d\,x}}{2\,a^3\,d} + \frac{2\,b\,\mathrm{e}^{c-d\,x}}{a^2\,d}}{a^2+2\,\mathrm{e}^{c-d\,x}\,a\,b+\mathrm{e}^{2\,c-2\,d\,x}\,b^2} + \frac{\ln\left(a+b\,\mathrm{e}^{-d\,x}\,\mathrm{e}^{c}\right)}{a^3\,d}$$

input
```
int(1/(a + b*exp(c - d*x))^3,x)
```

output
```
(x/a + (b^2*x*exp(2*c - 2*d*x))/a^3 + (2*b*x*exp(c - d*x))/a^2 + (3*b^2*ex
p(2*c - 2*d*x))/(2*a^3*d) + (2*b*exp(c - d*x))/(a^2*d))/(a^2 + b^2*exp(2*c
 - 2*d*x) + 2*a*b*exp(c - d*x)) + log(a + b*exp(-d*x)*exp(c))/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.89

$$\int \frac{1}{\left(a+be^{c-dx}\right)^3}\,dx$$
$$= \frac{2e^{2dx}\log\left(e^{dx}a+e^{c}b\right)a^2-2e^{2dx}a^2+4e^{dx+c}\log\left(e^{dx}a+e^{c}b\right)ab+2e^{2c}\log\left(e^{dx}a+e^{c}b\right)b^2+e^{2c}b^2}{2a^3d\left(e^{2dx}a^2+2e^{dx+c}ab+e^{2c}b^2\right)}$$

input
```
int(1/(a+b*exp(-d*x+c))^3,x)
```

output
```
(2*e**(2*d*x)*log(e**(d*x)*a + e**c*b)*a**2 - 2*e**(2*d*x)*a**2 + 4*e**(c
+ d*x)*log(e**(d*x)*a + e**c*b)*a*b + 2*e**(2*c)*log(e**(d*x)*a + e**c*b)*
b**2 + e**(2*c)*b**2)/(2*a**3*d*(e**(2*d*x)*a**2 + 2*e**(c + d*x)*a*b + e*
*(2*c)*b**2))
```

3.24 $\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = -\frac{1}{2ad\left(a+be^{-c-dx}\right)^2} - \frac{1}{a^2d\left(a+be^{-c-dx}\right)} + \frac{x}{a^3} + \frac{\log\left(a+be^{-c-dx}\right)}{a^3d}$$

output

```
-1/2/a/d/(a+b*exp(-d*x-c))^2-1/a^2/d/(a+b*exp(-d*x-c))+x/a^3+ln(a+b*exp(-d
*x-c))/a^3/d
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = \frac{\frac{b\left(3b+4ae^{c+dx}\right)}{\left(b+ae^{c+dx}\right)^2} + 2\log\left(b+ae^{c+dx}\right)}{2a^3d}$$

input

```
Integrate[(a + b*E^(-c - d*x))^(-3),x]
```

output

```
((b*(3*b + 4*a*E^(c + d*x)))/(b + a*E^(c + d*x))^2 + 2*Log[b + a*E^(c + d*
x)])/(2*a^3*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+be^{-c-dx})^3}\,dx$$

$$\downarrow 2720$$

$$-\frac{\int \frac{e^{c+dx}}{(a+be^{-c-dx})^3}\,de^{-c-dx}}{d}$$

$$\downarrow 54$$

$$-\frac{\int \left(-\frac{b}{a^3(a+be^{-c-dx})}-\frac{b}{a^2(a+be^{-c-dx})^2}-\frac{b}{a(a+be^{-c-dx})^3}+\frac{e^{c+dx}}{a^3}\right)de^{-c-dx}}{d}$$

$$\downarrow 2009$$

$$-\frac{-\frac{\log(a+be^{-c-dx})}{a^3}+\frac{\log(e^{-c-dx})}{a^3}+\frac{1}{a^2(a+be^{-c-dx})}+\frac{1}{2a(a+be^{-c-dx})^2}}{d}$$

input
```
Int[(a + b*E^(-c - d*x))^(-3),x]
```

output
```
-((1/(2*a*(a + b*E^(-c - d*x))^2) + 1/(a^2*(a + b*E^(-c - d*x))) + Log[E^(
-c - d*x)]/a^3 - Log[a + b*E^(-c - d*x)]/a^3)/d)
```

Defintions of rubi rules used

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.99

method	result
risch	$\frac{x}{a^3}+\frac{c}{a^3 d}-\frac{2b\,\mathrm{e}^{-dx-c}+3a}{2d\,a^2\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}+\frac{\ln\left(\mathrm{e}^{-dx-c}+\frac{a}{b}\right)}{a^3 d}$
derivativedivides	$-\frac{\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{a^3}-\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,\mathrm{e}^{-dx-c}\right)}+\frac{1}{2a\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}}{d}$
default	$-\frac{\frac{\ln\left(\mathrm{e}^{-dx-c}\right)}{a^3}-\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^3}+\frac{1}{a^2\left(a+b\,\mathrm{e}^{-dx-c}\right)}+\frac{1}{2a\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}}{d}$
norman	$\frac{\frac{x}{a}+\frac{b^2x\,\mathrm{e}^{-2dx-2c}}{a^3}+\frac{2bx\,\mathrm{e}^{-dx-c}}{a^2}+\frac{2b\,\mathrm{e}^{-dx-c}}{a^2 d}+\frac{3b^2\mathrm{e}^{-2dx-2c}}{2a^3 d}}{\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}+\frac{\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)}{a^3 d}$
parallelrisch	$\frac{2\,\mathrm{e}^{-2dx-2c}b^2dx+2\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)\mathrm{e}^{-2dx-2c}b^2+4\,\mathrm{e}^{-dx-c}abdx+4\ln\left(a+b\,\mathrm{e}^{-dx-c}\right)\mathrm{e}^{-dx-c}ab+2a^2dx+3b^2\mathrm{e}^{-2dx-2c}+\cdots}{2a^3d\left(a+b\,\mathrm{e}^{-dx-c}\right)^2}$

input

```
int(1/(a+b*exp(-d*x-c))^3,x,method=_RETURNVERBOSE)
```

output

```
x/a^3+1/a^3/d*c-1/2*(2*b*exp(-d*x-c)+3*a)/d/a^2/(a+b*exp(-d*x-c))^2+1/a^3/
d*ln(exp(-d*x-c)+1/b*a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx$$
$$= \frac{2\,b^2dxe^{(-2\,dx-2\,c)} + 2\,a^2dx - 3\,a^2 + 2\,(2\,abdx - ab)e^{(-dx-c)} + 2\left(2\,abe^{(-dx-c)} + b^2e^{(-2\,dx-2\,c)} + a^2\right)\log\left(be^{(}\right.}{2\left(2\,a^4bde^{(-dx-c)} + a^3b^2de^{(-2\,dx-2\,c)} + a^5d\right)}$$

input
```
integrate(1/(a+b*exp(-d*x-c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*d*x*e^(-2*d*x - 2*c) + 2*a^2*d*x - 3*a^2 + 2*(2*a*b*d*x - a*b)*
e^(-d*x - c) + 2*(2*a*b*e^(-d*x - c) + b^2*e^(-2*d*x - 2*c) + a^2)*log(b*e
^(-d*x - c) + a))/(2*a^4*b*d*e^(-d*x - c) + a^3*b^2*d*e^(-2*d*x - 2*c) + a
^5*d)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = \frac{-3a - 2be^{-c-dx}}{2a^4d + 4a^3bde^{-c-dx} + 2a^2b^2de^{-2c-2dx}} + \frac{x}{a^3} + \frac{\log\left(\frac{a}{b} + e^{-c-dx}\right)}{a^3d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))**3,x)
```

output
```
(-3*a - 2*b*exp(-c - d*x))/(2*a**4*d + 4*a**3*b*d*exp(-c - d*x) + 2*a**2*b
**2*d*exp(-2*c - 2*d*x)) + x/a**3 + log(a/b + exp(-c - d*x))/(a**3*d)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = -\frac{2\,be^{(-dx-c)}+3\,a}{2\left(2\,a^3be^{(-dx-c)}+a^2b^2e^{(-2\,dx-2\,c)}+a^4\right)d} + \frac{dx+c}{a^3d} + \frac{\log\left(be^{(-dx-c)}+a\right)}{a^3d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b*e^(-d*x - c) + 3*a)/((2*a^3*b*e^(-d*x - c) + a^2*b^2*e^(-2*d*x -
 2*c) + a^4)*d) + (d*x + c)/(a^3*d) + log(b*e^(-d*x - c) + a)/(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = \frac{\frac{2\,(dx+c)}{a^3} + \frac{2\,\log\left(\left|be^{(-dx-c)}+a\right|\right)}{a^3} - \frac{2\,abe^{(-dx-c)}+3\,a^2}{\left(be^{(-dx-c)}+a\right)^2a^3}}{2\,d}$$

input
```
integrate(1/(a+b*exp(-d*x-c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(d*x + c)/a^3 + 2*log(abs(b*e^(-d*x - c) + a))/a^3 - (2*a*b*e^(-d*x
 - c) + 3*a^2)/((b*e^(-d*x - c) + a)^2*a^3))/d
```

Mupad [B] (verification not implemented)

Time = 23.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.69

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx = \frac{\frac{x}{a} + \frac{2\,b\,\mathrm{e}^{-c-d\,x}}{a^2\,d} + \frac{b^2\,x\,\mathrm{e}^{-2\,c-2\,d\,x}}{a^3} + \frac{3\,b^2\,\mathrm{e}^{-2\,c-2\,d\,x}}{2\,a^3\,d} + \frac{2\,b\,x\,\mathrm{e}^{-c-d\,x}}{a^2}}{a^2+2\,\mathrm{e}^{-c-d\,x}\,a\,b+\mathrm{e}^{-2\,c-2\,d\,x}\,b^2} + \frac{\ln\left(a+b\,\mathrm{e}^{-c}\,\mathrm{e}^{-d\,x}\right)}{a^3\,d}$$

input
```
int(1/(a + b*exp(- c - d*x))^3,x)
```

output
```
(x/a + (2*b*exp(- c - d*x))/(a^2*d) + (b^2*x*exp(- 2*c - 2*d*x))/a^3 + (3*
b^2*exp(- 2*c - 2*d*x))/(2*a^3*d) + (2*b*x*exp(- c - d*x))/a^2)/(a^2 + b^2
*exp(- 2*c - 2*d*x) + 2*a*b*exp(- c - d*x)) + log(a + b*exp(-c)*exp(-d*x))
/(a^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

$$\int \frac{1}{\left(a+be^{-c-dx}\right)^3}\,dx$$
$$= \frac{2e^{2dx+2c}\log\left(e^{dx+c}a+b\right)a^2 - 2e^{2dx+2c}a^2 + 4e^{dx+c}\log\left(e^{dx+c}a+b\right)ab + 2\log\left(e^{dx+c}a+b\right)b^2 + b^2}{2a^3d\left(e^{2dx+2c}a^2 + 2e^{dx+c}ab + b^2\right)}$$

input
```
int(1/(a+b*exp(-d*x-c))^3,x)
```

output
```
(2*e**(2*c + 2*d*x)*log(e**(c + d*x)*a + b)*a**2 - 2*e**(2*c + 2*d*x)*a**2
 + 4*e**(c + d*x)*log(e**(c + d*x)*a + b)*a*b + 2*log(e**(c + d*x)*a + b)*
b**2 + b**2)/(2*a**3*d*(e**(2*c + 2*d*x)*a**2 + 2*e**(c + d*x)*a*b + b**2)
)
```

3.25 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 153

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx &= \frac{a(c+dx)^4}{4d}-\frac{6bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}\\ &+\frac{6bd^2\left(F^{eg+fgx}\right)^n(c+dx)}{f^3g^3n^3\log^3(F)}\\ &-\frac{3bd\left(F^{eg+fgx}\right)^n(c+dx)^2}{f^2g^2n^2\log^2(F)}\\ &+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^3}{fgn\log(F)}\end{aligned}$$

output

```
1/4*a*(d*x+c)^4/d-6*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/ln(F)^4+6*b*d^2*(F
^(f*g*x+e*g))^n*(d*x+c)/f^3/g^3/n^3/ln(F)^3-3*b*d*(F^(f*g*x+e*g))^n*(d*x+c
)^2/f^2/g^2/n^2/ln(F)^2+b*(F^(f*g*x+e*g))^n*(d*x+c)^3/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.85

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = ac^3x+\frac{3}{2}ac^2dx^2+acd^2x^3+\frac{1}{4}ad^3x^4 + \frac{b\left(F^{g(e+fx)}\right)^n\left(-6d^3+6d^2fgn(c+dx)\log(F)-3df^2g^2n^2(c+dx)^2\log^2(F)+f^3g^3n^3(c+dx)^3\log^3(F)\right)}{f^4g^4n^4\log^4(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^3,x]
```

output
```
a*c^3*x + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 + (a*d^3*x^4)/4 + (b*(F^(g*(e +
f*x)))^n*(-6*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 3*d*f^2*g^2*n^2*(c + d*x
)^2*Log[F]^2 + f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(f^4*g^4*n^4*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\left(a+b\left(F^{g(e+fx)}\right)^n\right)dx$$

$$\downarrow 2614$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\left(F^{eg+fgx}\right)^n\right)dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-\frac{3bd(c+dx)^2\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b(c+dx)^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{6bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^3,x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (6*b*d^3*(F^(e*g + f*g*x))^n)/(f^4*g^4*n^4*Log[F]^
4) + (6*b*d^2*(F^(e*g + f*g*x))^n*(c + d*x))/(f^3*g^3*n^3*Log[F]^3) - (3*b
*d*(F^(e*g + f*g*x))^n*(c + d*x)^2)/(f^2*g^2*n^2*Log[F]^2) + (b*(F^(e*g +
f*g*x))^n*(c + d*x)^3)/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(151) = 302$.

Time $= 0.48$ (sec) , antiderivative size $= 392$, normalized size of antiderivative $= 2.56$

method	result
parallelrisch	$\frac{a\,d^3x^4n^4g^4f^4\ln(F)^4+4d^2acx^3n^4g^4f^4\ln(F)^4+6da\,c^2x^2n^4g^4f^4\ln(F)^4+4a\,c^3xn^4g^4f^4\ln(F)^4+4x^3\left(F^{g(fx+e)}\right)^n b\,d^3n^3g^3f^3}{}$
orering	$\frac{\left(\ln(F)^4d^4f^4g^4n^4x^5+5\ln(F)^4cd^3f^4g^4n^4x^4+10\ln(F)^4c^2d^2f^4g^4n^4x^3+10\ln(F)^4c^3df^4g^4n^4x^2+4\ln(F)^4c^4f^4g^4n^4x+3\ln(F\right.}{}$

input
```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(a*d^3*x^4*n^4*g^4*f^4*ln(F)^4+4*d^2*a*c*x^3*n^4*g^4*f^4*ln(F)^4+6*d*a
*c^2*x^2*n^4*g^4*f^4*ln(F)^4+4*a*c^3*x*n^4*g^4*f^4*ln(F)^4+4*x^3*(F^(g*(f*
x+e)))^n*b*d^3*n^3*g^3*f^3*ln(F)^3+12*ln(F)^3*x^2*(F^(g*(f*x+e)))^n*b*c*d^
2*f^3*g^3*n^3+12*ln(F)^3*x*(F^(g*(f*x+e)))^n*b*c^2*d*f^3*g^3*n^3+4*ln(F)^3
*(F^(g*(f*x+e)))^n*b*c^3*f^3*g^3*n^3-12*ln(F)^2*x^2*(F^(g*(f*x+e)))^n*b*d^
3*f^2*g^2*n^2-24*ln(F)^2*x*(F^(g*(f*x+e)))^n*b*c*d^2*f^2*g^2*n^2-12*ln(F)^
2*(F^(g*(f*x+e)))^n*b*c^2*d*f^2*g^2*n^2+24*ln(F)*x*(F^(g*(f*x+e)))^n*b*d^3
*f*g*n+24*ln(F)*(F^(g*(f*x+e)))^n*b*c*d^2*f*g*n-24*(F^(g*(f*x+e)))^n*b*d^3
)/n^4/g^4/f^4/ln(F)^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.75

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$$
$$= \frac{(ad^3f^4g^4n^4x^4 + 4\,acd^2f^4g^4n^4x^3 + 6\,ac^2df^4g^4n^4x^2 + 4\,ac^3f^4g^4n^4x)\log(F)^4 - 4\left(6\,bd^3 - (bd^3f^3g^3n^3x^3 +$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((a*d^3*f^4*g^4*n^4*x^4 + 4*a*c*d^2*f^4*g^4*n^4*x^3 + 6*a*c^2*d*f^4*g^
4*n^4*x^2 + 4*a*c^3*f^4*g^4*n^4*x)*log(F)^4 - 4*(6*b*d^3 - (b*d^3*f^3*g^3*
n^3*x^3 + 3*b*c*d^2*f^3*g^3*n^3*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + b*c^3*f^3*
g^3*n^3)*log(F)^3 + 3*(b*d^3*f^2*g^2*n^2*x^2 + 2*b*c*d^2*f^2*g^2*n^2*x + b
*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(b*d^3*f*g*n*x + b*c*d^2*f*g*n)*log(F))*F
^(f*g*n*x + e*g*n))/(f^4*g^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(151) = 302$.

Time = 2.03 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.24

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$$

$$= \begin{cases} (a+b)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \\ (a+b(F^{eg})^n)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \\ (a+b)\left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right) \\ ac^3x+\frac{3ac^2dx^2}{2}+acd^2x^3+\frac{ad^3x^4}{4}+\frac{bc^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{3bc^2dx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{3bc^2d\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{3bcd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}- \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**3,x)
```

output
```
Piecewise(((a + b)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4),
 Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)*(c**3*
x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(f, 0)), ((a + b)*(c**
3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) | Eq(g, 0) |
Eq(n, 0)), (a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 +
 b*c**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 3*b*c**2*d*x*(F**(e*g + f*g
*x))**n/(f*g*n*log(F)) - 3*b*c**2*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*
log(F)**2) + 3*b*c*d**2*x**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*b*c*
d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 6*b*c*d**2*(F**(
e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) + b*d**3*x**3*(F**(e*g + f*g*x
))**n/(f*g*n*log(F)) - 3*b*d**3*x**2*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2
*log(F)**2) + 6*b*d**3*x*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3)
- 6*b*d**3*(F**(e*g + f*g*x))**n/(f**4*g**4*n**4*log(F)**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.84

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = \frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x + \frac{F^{fgnx+egn}bc^3}{fgn\log(F)} + \frac{3\left(F^{egn}fgnx\log(F) - F^{egn}\right)F^{fgnx}bc^2d}{f^2g^2n^2\log(F)^2} + \frac{3\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{egn}fgnx\log(F) + 2\,F^{egn}\right)F^{fgnx}bcd^2}{f^3g^3n^3\log(F)^3} + \frac{\left(F^{egn}f^3g^3n^3x^3\log(F)^3 - 3\,F^{egn}f^2g^2n^2x^2\log(F)^2 + 6\,F^{egn}fgnx\log(F) - 6\,F^{egn}\right)F^{fgnx}bd^3}{f^4g^4n^4\log(F)^4}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + F^(f*g*n*x + e*g
*n)*b*c^3/(f*g*n*log(F)) + 3*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g
*n*x)*b*c^2*d/(f^2*g^2*n^2*log(F)^2) + 3*(F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)
^2 - 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*b*c*d^2/(f^3*g^
3*n^3*log(F)^3) + (F^(e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 3*F^(e*g*n)*f^2*g^
2*n^2*x^2*log(F)^2 + 6*F^(e*g*n)*f*g*n*x*log(F) - 6*F^(e*g*n))*F^(f*g*n*x)
*b*d^3/(f^4*g^4*n^4*log(F)^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 5716, normalized size of antiderivative = 37.36

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x, algorithm="giac")
```

output

```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - (((3*pi^2*b*d^3*
f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 3*pi^2*b*d^3*f^3*g^3*n^3*x^3*log(abs(
F)) + 2*b*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 9*pi^2*b*c*d^2*f^3*g^3*n^3*x
^2*log(abs(F))*sgn(F) - 9*pi^2*b*c*d^2*f^3*g^3*n^3*x^2*log(abs(F)) + 6*b*c
*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(
F))*sgn(F) - 9*pi^2*b*c^2*d*f^3*g^3*n^3*x*log(abs(F)) + 6*b*c^2*d*f^3*g^3*
n^3*x*log(abs(F))^3 + 3*pi^2*b*c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2
*b*c^3*f^3*g^3*n^3*log(abs(F)) + 2*b*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^
2*b*d^3*f^2*g^2*n^2*x^2*sgn(F) + 3*pi^2*b*d^3*f^2*g^2*n^2*x^2 - 6*b*d^3*f^
2*g^2*n^2*x^2*log(abs(F))^2 - 6*pi^2*b*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2
*b*c*d^2*f^2*g^2*n^2*x - 12*b*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b
*c^2*d*f^2*g^2*n^2*sgn(F) + 3*pi^2*b*c^2*d*f^2*g^2*n^2 - 6*b*c^2*d*f^2*g^2
*n^2*log(abs(F))^2 + 12*b*d^3*f*g*n*x*log(abs(F)) + 12*b*c*d^2*f*g*n*log(a
bs(F)) - 12*b*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F
))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*
g^4*n^4*log(abs(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(
abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2
*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) -
pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^
4*g^4*n^4*log(abs(F))^3)^2) - 4*(pi^3*b*d^3*f^3*g^3*n^3*x^3*sgn(F) - 3*...
```

Mupad [B] (verification not implemented)

Time = 16.00 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.47

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx = \frac{a\,d^3\,x^4}{4} - \left(F^{f\,g\,x}\,F^{e\,g}\right)^n \left(\frac{b\left(-c^3\,f^3\,g^3\,n^3\ln(F)^3 + 3\,c^2\,d\,f^2\,g^2\,n^2\ln(F)^2 - 6\,c\,d^2\,f\,g\,n\ln(F) + 6\,d^3\right)}{f^4\,g^4\,n^4\ln(F)^4} - \frac{b\,d^3\,x^3}{f\,g\,n\ln(F)} - \frac{3\,b\,d\,x\left(c^2\,f^2\,g^2\,n^2\ln(F)^2 - 2\,c\,d\,f\,g\,n\ln(F) + 2\,d^2\right)}{f^3\,g^3\,n^3\ln(F)^3} + \frac{3\,b\,d^2\,x^2\left(d - c\,f\,g\,n\ln(F)\right)}{f^2\,g^2\,n^2\ln(F)^2}\right) + a\,c^3\,x + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3$$

input

```
int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^3,x)
```

output
```
(a*d^3*x^4)/4 - (F^(f*g*x)*F^(e*g))^n*((b*(6*d^3 - c^3*f^3*g^3*n^3*log(F)^
3 - 6*c*d^2*f*g*n*log(F) + 3*c^2*d*f^2*g^2*n^2*log(F)^2))/(f^4*g^4*n^4*log
(F)^4) - (b*d^3*x^3)/(f*g*n*log(F)) - (3*b*d*x*(2*d^2 + c^2*f^2*g^2*n^2*lo
g(F)^2 - 2*c*d*f*g*n*log(F)))/(f^3*g^3*n^3*log(F)^3) + (3*b*d^2*x^2*(d - c
*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + a*c^3*x + (3*a*c^2*d*x^2)/2 + a*
c*d^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.62

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^3\,dx$$
$$= \frac{4f^{fgnx+egn}\log(f)^3\,b\,c^3f^3g^3n^3 + 12f^{fgnx+egn}\log(f)^3\,b\,c^2d\,f^3g^3n^3x + 12f^{fgnx+egn}\log(f)^3\,bc\,d^2f^3g^3n^3x^2 + 4$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^3,x)
```

output
```
(4*f**(e*g*n + f*g*n*x)*log(f)**3*b*c**3*f**3*g**3*n**3 + 12*f**(e*g*n + f
*g*n*x)*log(f)**3*b*c**2*d*f**3*g**3*n**3*x + 12*f**(e*g*n + f*g*n*x)*log(
f)**3*b*c*d**2*f**3*g**3*n**3*x**2 + 4*f**(e*g*n + f*g*n*x)*log(f)**3*b*d*
*3*f**3*g**3*n**3*x**3 - 12*f**(e*g*n + f*g*n*x)*log(f)**2*b*c**2*d*f**2*g
**2*n**2 - 24*f**(e*g*n + f*g*n*x)*log(f)**2*b*c*d**2*f**2*g**2*n**2*x - 1
2*f**(e*g*n + f*g*n*x)*log(f)**2*b*d**3*f**2*g**2*n**2*x**2 + 24*f**(e*g*n
 + f*g*n*x)*log(f)*b*c*d**2*f*g*n + 24*f**(e*g*n + f*g*n*x)*log(f)*b*d**3*
f*g*n*x - 24*f**(e*g*n + f*g*n*x)*b*d**3 + 4*log(f)**4*a*c**3*f**4*g**4*n*
*4*x + 6*log(f)**4*a*c**2*d*f**4*g**4*n**4*x**2 + 4*log(f)**4*a*c*d**2*f**
4*g**4*n**4*x**3 + log(f)**4*a*d**3*f**4*g**4*n**4*x**4)/(4*log(f)**4*f**4
*g**4*n**4)
```

3.26 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx = \frac{a(c+dx)^3}{3d}+\frac{2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} - \frac{2bd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)}+\frac{b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)}$$

output
```
1/3*a*(d*x+c)^3/d+2*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/ln(F)^3-2*b*d*(F^(
f*g*x+e*g))^n*(d*x+c)/f^2/g^2/n^2/ln(F)^2+b*(F^(f*g*x+e*g))^n*(d*x+c)^2/f/
g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\begin{aligned}&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx \\ &= ac^2x+acdx^2+\frac{1}{3}ad^2x^3 \\ &\quad+\frac{b\left(F^{g(e+fx)}\right)^n\left(2d^2-2dfgn(c+dx)\log(F)+f^2g^2n^2(c+dx)^2\log^2(F)\right)}{f^3g^3n^3\log^3(F)}\end{aligned}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2,x]
```

output
```
a*c^2*x + a*c*d*x^2 + (a*d^2*x^3)/3 + (b*(F^(g*(e + f*x)))^n*(2*d^2 - 2*d*
f*g*n*(c + d*x)*Log[F] + f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(f^3*g^3*n^3*L
og[F]^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \left(a+b\left(F^{g(e+fx)}\right)^n\right) dx$$

$$\downarrow 2614$$

$$\int \left(a(c+dx)^2+b(c+dx)^2\left(F^{eg+fgx}\right)^n\right) dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b(c+dx)^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2,x]
```

output
```
(a*(c + d*x)^3)/(3*d) + (2*b*d^2*(F^(e*g + f*g*x))^n)/(f^3*g^3*n^3*Log[F]^
3) - (2*b*d*(F^(e*g + f*g*x))^n*(c + d*x))/(f^2*g^2*n^2*Log[F]^2) + (b*(F^
(e*g + f*g*x))^n*(c + d*x)^2)/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.42

method	result
norman	$a\,c^2x+acd\,x^2+\frac{b\left(\ln(F)^2c^2f^2g^2n^2-2\ln(F)cdfgn+2d^2\right)\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{\ln(F)^3f^3g^3n^3}+\frac{b\,d^2x^2\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)}+\frac{d^2}{}$
parallelrisch	$\frac{d^2a\,x^3\ln(F)^3f^3g^3n^3+3acd\,x^2\ln(F)^3f^3g^3n^3+3a\,c^2x\ln(F)^3f^3g^3n^3+3x^2\left(F^{g(fx+e)}\right)^nb\,d^2\ln(F)^2f^2g^2n^2+6\ln(F)^2x(F^{g(fx}}{3\ln($
orering	$\frac{\left(\ln(F)^3d^3f^3g^3n^3x^4+4\ln(F)^3c\,d^2f^3g^3n^3x^3+6\ln(F)^3c^2d\,f^3g^3n^3x^2+3\ln(F)^3c^3f^3g^3n^3x+2\ln(F)^2d^3f^2g^2n^2x^3+6\ln(F)^2c}{3}$

input

```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output

```
a*c^2*x+a*c*d*x^2+b*(ln(F)^2*c^2*f^2*g^2*n^2-2*ln(F)*c*d*f*g*n+2*d^2)/ln(F
)^3/f^3/g^3/n^3*exp(n*ln(exp(g*(f*x+e)*ln(F))))+1/n/g/f/ln(F)*b*d^2*x^2*ex
p(n*ln(exp(g*(f*x+e)*ln(F))))+1/3*d^2*a*x^3+2*b*d*(ln(F)*c*f*g*n-d)/ln(F)^
2/f^2/g^2/n^2*x*exp(n*ln(exp(g*(f*x+e)*ln(F))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$= \frac{(ad^2f^3g^3n^3x^3 + 3\,acdf^3g^3n^3x^2 + 3\,ac^2f^3g^3n^3x)\log(F)^3 + 3\left(2\,bd^2 + (bd^2f^2g^2n^2x^2 + 2\,bcdf^2g^2n^2x + bc^2\right.}{3\,f^3g^3n^3\log(F)^3}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/3*((a*d^2*f^3*g^3*n^3*x^3 + 3*a*c*d*f^3*g^3*n^3*x^2 + 3*a*c^2*f^3*g^3*n^
3*x)*log(F)^3 + 3*(2*b*d^2 + (b*d^2*f^2*g^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*
x + b*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))*
F^(f*g*n*x + e*g*n))/(f^3*g^3*n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(110) = 220.

Time = 0.71 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.56

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$= \begin{cases} (a+b)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \\ (a + b(F^{eg})^n)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \\ (a+b)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \\ ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)} + \frac{2bcdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)} - \frac{2bcd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2} + \frac{bd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)} - \frac{2bd^2x\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2} \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**2,x)
```

output
```
Piecewise(((a + b)*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) & Eq(f, 0)
& Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)*(c**2*x + c*d*x**2 + d**2*x
**3/3), Eq(f, 0)), ((a + b)*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) |
Eq(g, 0) | Eq(n, 0)), (a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*(F**
(e*g + f*g*x))**n/(f*g*n*log(F)) + 2*b*c*d*x*(F**(e*g + f*g*x))**n/(f*g*n*
log(F)) - 2*b*c*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + b*d**
2*x**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 2*b*d**2*x*(F**(e*g + f*g*x)
)**n/(f**2*g**2*n**2*log(F)**2) + 2*b*d**2*(F**(e*g + f*g*x))**n/(f**3*g**
3*n**3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$= \frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{F^{fgnx+egn}bc^2}{fgn\log(F)} + \frac{2\left(F^{egn}fgnx\log(F) - F^{egn}\right)F^{fgnx}bcd}{f^2g^2n^2\log(F)^2}$$
$$+ \frac{\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{egn}fgnx\log(F) + 2\,F^{egn}\right)F^{fgnx}bd^2}{f^3g^3n^3\log(F)^3}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + F^(f*g*n*x + e*g*n)*b*c^2/(f*g*n*log
(F)) + 2*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*b*c*d/(f^2*g^2
*n^2*log(F)^2) + (F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(e*g*n)*f*g*n*x
*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*b*d^2/(f^3*g^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 2716, normalized size of antiderivative = 23.62

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x - ((2*(pi*b*d^2*f^2*g^2*n^2*x^2*log(ab
s(F))*sgn(F) - pi*b*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 2*pi*b*c*d*f^2*g^2*n
^2*x*log(abs(F))*sgn(F) - 2*pi*b*c*d*f^2*g^2*n^2*x*log(abs(F)) + pi*b*c^2*
f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*b*c^2*f^2*g^2*n^2*log(abs(F)) - pi*b*d
^2*f*g*n*x*sgn(F) + pi*b*d^2*f*g*n*x - pi*b*c*d*f*g*n*sgn(F) + pi*b*c*d*f*
g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi
^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F)
 - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3
*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^
3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)^2) - (pi^2*b*d^2*f^2*
g^2*n^2*x^2*sgn(F) - pi^2*b*d^2*f^2*g^2*n^2*x^2 + 2*b*d^2*f^2*g^2*n^2*x^2*
log(abs(F))^2 + 2*pi^2*b*c*d*f^2*g^2*n^2*x*sgn(F) - 2*pi^2*b*c*d*f^2*g^2*n
^2*x + 4*b*c*d*f^2*g^2*n^2*x*log(abs(F))^2 + pi^2*b*c^2*f^2*g^2*n^2*sgn(F)
 - pi^2*b*c^2*f^2*g^2*n^2 + 2*b*c^2*f^2*g^2*n^2*log(abs(F))^2 - 4*b*d^2*f*
g*n*x*log(abs(F)) - 4*b*c*d*f*g*n*log(abs(F)) + 4*b*d^2)*(3*pi^2*f^3*g^3*n
^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log
(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn
(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g
^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3
*log(abs(F))^3)^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*p...
```

Mupad [B] (verification not implemented)

Time = 22.81 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$= \left(F^{f\,g\,x}\,F^{e\,g}\right)^n \left(\frac{b\left(c^2\,f^2\,g^2\,n^2\ln(F)^2 - 2\,c\,d\,f\,g\,n\ln(F) + 2\,d^2\right)}{f^3\,g^3\,n^3\ln(F)^3} + \frac{b\,d^2\,x^2}{f\,g\,n\ln(F)} - \frac{2\,b\,d\,x\,(d - c\,f\,g\,n\ln(F))}{f^2\,g^2\,n^2\ln(F)^2}\right) + \frac{a\,d^2\,x^3}{3} + a\,c^2\,x + a\,c\,d\,x^2$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2,x)
```

output
```
(F^(f*g*x)*F^(e*g))^n*((b*(2*d^2 + c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*
log(F)))/(f^3*g^3*n^3*log(F)^3) + (b*d^2*x^2)/(f*g*n*log(F)) - (2*b*d*x*(d
 - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + (a*d^2*x^3)/3 + a*c^2*x + a*
c*d*x^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.07

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2\,dx$$
$$= \frac{3f^{fgnx+egn}\log(f)^2\,b\,c^2f^2g^2n^2+6f^{fgnx+egn}\log(f)^2\,bcd\,f^2g^2n^2x+3f^{fgnx+egn}\log(f)^2\,b\,d^2f^2g^2n^2x^2-6f^{fgr}}{}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^2,x)
```

output
```
(3*f**(e*g*n + f*g*n*x)*log(f)**2*b*c**2*f**2*g**2*n**2 + 6*f**(e*g*n + f*
g*n*x)*log(f)**2*b*c*d*f**2*g**2*n**2*x + 3*f**(e*g*n + f*g*n*x)*log(f)**2
*b*d**2*f**2*g**2*n**2*x**2 - 6*f**(e*g*n + f*g*n*x)*log(f)*b*c*d*f*g*n -
6*f**(e*g*n + f*g*n*x)*log(f)*b*d**2*f*g*n*x + 6*f**(e*g*n + f*g*n*x)*b*d*
*2 + 3*log(f)**3*a*c**2*f**3*g**3*n**3*x + 3*log(f)**3*a*c*d*f**3*g**3*n**
3*x**2 + log(f)**3*a*d**2*f**3*g**3*n**3*x**3)/(3*log(f)**3*f**3*g**3*n**3
)
```

3.27 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx$

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Mupad [B] (verification not implemented) . . . 270
Reduce [B] (verification not implemented) . . . 270

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = \frac{a(c+dx)^2}{2d} - \frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} + \frac{b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)}$$

output
```
1/2*a*(d*x+c)^2/d-b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/ln(F)^2+b*(F^(f*g*x+e*
g))^n*(d*x+c)/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.95

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = \frac{1}{2}ax(2c+dx) - \frac{bd\left(F^{g(e+fx)}\right)^n}{f^2g^2n^2\log^2(F)} + \frac{b\left(F^{g(e+fx)}\right)^n(c+dx)}{fgn\log(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x),x]
```

output
```
(a*x*(2*c + d*x))/2 - (b*d*(F^(g*(e + f*x)))^n)/(f^2*g^2*n^2*Log[F]^2) + (
b*(F^(g*(e + f*x)))^n*(c + d*x))/(f*g*n*Log[F])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx$$

$$\downarrow 2614$$

$$\int \left(a(c+dx)+b(c+dx)\left(F^{eg+fgx}\right)^n\right)dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*Log[F]^2) +
 (b*(F^(e*g + f*g*x))^n*(c + d*x))/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

method	result
norman	$acx + \frac{b(\ln(F)cfgn-d)e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{n^2g^2f^2\ln(F)^2} + \frac{bdx\,e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)} + \frac{da\,x^2}{2}$
parallelrisch	$\frac{da\,x^2n^2g^2f^2\ln(F)^2+2acx\,n^2g^2f^2\ln(F)^2+2\left(F^{g(fx+e)}\right)^n\ln(F)bdfgnx+2\left(F^{g(fx+e)}\right)^n\ln(F)bcfgn-2\left(F^{g(fx+e)}\right)^nbd}{2n^2g^2f^2\ln(F)^2}$
orering	$\frac{\left(\ln(F)^2d^2f^2g^2n^2x^3+3\ln(F)^2cd\,f^2g^2n^2x^2+2\ln(F)^2c^2f^2g^2n^2x+\ln(F)d^2fgn\,x^2+4\ln(F)cdfgnx+2\ln(F)c^2fgn-4d^2x-2c\right.}{2n^2g^2f^2\ln(F)^2(dx+c)}$

input
```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*c*x+b*(ln(F)*c*f*g*n-d)/n^2/g^2/f^2/ln(F)^2*exp(n*ln(exp(g*(f*x+e)*ln(F)
)))+1/n/g/f/ln(F)*b*d*x*exp(n*ln(exp(g*(f*x+e)*ln(F))))+1/2*d*a*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx$$
$$= \frac{(adf^2g^2n^2x^2 + 2\,acf^2g^2n^2x)\log(F)^2 - 2\,(bd - (bdfgnx + bcfgn)\log(F))F^{fgnx+egn}}{2\,f^2g^2n^2\log(F)^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((a*d*f^2*g^2*n^2*x^2 + 2*a*c*f^2*g^2*n^2*x)*log(F)^2 - 2*(b*d - (b*d*
f*g*n*x + b*c*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^2*g^2*n^2*log(F)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(68) = 136.

Time = 0.35 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.87

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx$$

$$= \begin{cases} (a+b)\left(cx + \frac{dx^2}{2}\right) & \text{for } F = 1 \wedge f = 0 \wedge g = 0 \wedge n = 0 \\ (a + b(F^{eg})^n)\left(cx + \frac{dx^2}{2}\right) & \text{for } f = 0 \\ (a+b)\left(cx + \frac{dx^2}{2}\right) & \text{for } F = 1 \vee g = 0 \vee n = 0 \\ acx + \frac{adx^2}{2} + \frac{bc\left(F^{eg+fgx}\right)^n}{fgn\log(F)} + \frac{bdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)} - \frac{bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c),x)
```

output
```
Piecewise(((a + b)*(c*x + d*x**2/2), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n
, 0)), ((a + b*(F**(e*g))**n)*(c*x + d*x**2/2), Eq(f, 0)), ((a + b)*(c*x +
 d*x**2/2), Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), (a*c*x + a*d*x**2/2 + b*c*(F*
*(e*g + f*g*x))**n/(f*g*n*log(F)) + b*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log
(F)) - b*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = \frac{1}{2}adx^2 + acx + \frac{F^{fgnx+egn}bc}{fgn\log(F)} + \frac{(F^{egn}fgnx\log(F) - F^{egn})F^{fgnx}bd}{f^2g^2n^2\log(F)^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + F^(f*g*n*x + e*g*n)*b*c/(f*g*n*log(F)) + (F^(e*g*n)*
f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*b*d/(f^2*g^2*n^2*log(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 1101, normalized size of antiderivative = 14.30

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + (2*((pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*
n^2*log(abs(F)))*(pi*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x + pi*b*c*f*g*n*sg
n(F) - pi*b*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*
g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g
^2*n^2*log(abs(F)))^2) + (pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f
^2*g^2*n^2*log(abs(F))^2)*(b*d*f*g*n*x*log(abs(F)) + b*c*f*g*n*log(abs(F))
 - b*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(a
bs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(ab
s(F)))^2))*cos(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(
F) + 1/2*pi*e*g*n) + ((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*
g^2*n^2*log(abs(F))^2)*(pi*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x + pi*b*c*f*
g*n*sgn(F) - pi*b*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 +
2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi
*f^2*g^2*n^2*log(abs(F)))^2) - 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f
^2*g^2*n^2*log(abs(F)))*(b*d*f*g*n*x*log(abs(F)) + b*c*f*g*n*log(abs(F)) -
 b*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs
(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(
F)))^2))*sin(-1/2*pi*f*g*n*x*sgn(F) + 1/2*pi*f*g*n*x - 1/2*pi*e*g*n*sgn(F)
 + 1/2*pi*e*g*n))*e^(f*g*n*x*log(abs(F)) + e*g*n*log(abs(F))) - 1/2*I*((pi
*b*d*f*g*n*x*sgn(F) - pi*b*d*f*g*n*x - 2*I*b*d*f*g*n*x*log(abs(F)) + pi...
```

Mupad [B] (verification not implemented)

Time = 22.86 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx = a\,c\,x - \left(\frac{b\,(d - c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\,\ln(F)^2} - \frac{b\,d\,x}{f\,g\,n\,\ln(F)}\right)\left(F^{f\,g\,x}\,F^{e\,g}\right)^n + \frac{a\,d\,x^2}{2}$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x),x)
```

output
```
a*c*x - ((b*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2) - (b*d*x)/(f*g*n*
log(F)))*(F^(f*g*x)*F^(e*g))^n + (a*d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.45

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)\,dx$$
$$= \frac{2f^{fgnx+egn}\log(f)\,bcfgn + 2f^{fgnx+egn}\log(f)\,bdfgnx - 2f^{fgnx+egn}bd + 2\log(f)^2\,ac\,f^2g^2n^2x + \log(f)^2\,ad\,f}{2\log(f)^2\,f^2g^2n^2}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c),x)
```

output
```
(2*f**(e*g*n + f*g*n*x)*log(f)*b*c*f*g*n + 2*f**(e*g*n + f*g*n*x)*log(f)*b
*d*f*g*n*x - 2*f**(e*g*n + f*g*n*x)*b*d + 2*log(f)**2*a*c*f**2*g**2*n**2*x
 + log(f)**2*a*d*f**2*g**2*n**2*x**2)/(2*log(f)**2*f**2*g**2*n**2)
```

3.28 $\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx$

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Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx = ax + \frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

output
```
a*x+b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx = ax + \frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

input
```
Integrate[a + b*(F^(g*(e + f*x)))^n,x]
```

output
```
a*x + (b*(F^(g*(e + f*x)))^n)/(f*g*n*Log[F])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx$$

$$\downarrow 2009$$

$$ax + \frac{b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

input
```
Int[a + b*(F^(g*(e + f*x)))^n,x]
```

output
```
a*x + (b*(F^(g*(e + f*x)))^n)/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result	size
default	$ax+\frac{b\left(F^{g(fx+e)}\right)^{n}}{fgn\ln(F)}$	31
parallelrisch	$ax+\frac{b\left(F^{g(fx+e)}\right)^{n}}{fgn\ln(F)}$	31
parts	$ax+\frac{b\left(F^{g(fx+e)}\right)^{n}}{fgn\ln(F)}$	31
norman	$ax+\frac{b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)}$	34
derivativedivides	$\frac{b\left(F^{g(fx+e)}\right)^{n}+a\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{gf\ln(F)n}$	43
orering	$\frac{(xngf\ln(F)+1)\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)}{ngf\ln(F)}-xb\left(F^{g(fx+e)}\right)^{n}$	55

input
```
int(a+b*(F^(g*(f*x+e)))^n,x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a+b\left(F^{g(e+fx)}\right)^{n}\right)\,dx=\frac{afgnx\log\left(F\right)+F^{fgnx+egn}b}{fgn\log\left(F\right)}$$

input
```
integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="fricas")
```

output
```
(a*f*g*n*x*log(F) + F^(f*g*n*x + e*g*n)*b)/(f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx = ax+b\left(\begin{cases} x & \text{for } F=1 \wedge f=0 \wedge g=0 \wedge n=0 \\ x\left(F^{eg}\right)^n & \text{for } f=0 \\ x & \text{for } F=1 \vee g=0 \vee n=0 \\ \frac{\left(F^{eg+fgx}\right)^n}{fgn\log(F)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*(F**(g*(f*x+e)))**n,x)
```

output
```
a*x + b*Piecewise((x, Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), (x*(F**(
e*g))**n, Eq(f, 0)), (x, Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), ((F**(e*g + f*g*
x))**n/(f*g*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)\,dx = ax+\frac{F^{(fx+e)gn}b}{fgn\log(F)}$$

input
```
integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="maxima")
```

output
```
a*x + F^((f*x + e)*g*n)*b/(f*g*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx = ax + \frac{F^{fgnx+egn} b}{fgn \log(F)}$$

input
```
integrate(a+b*(F^(g*(f*x+e)))^n,x, algorithm="giac")
```

output
```
a*x + F^(f*g*n*x + e*g*n)*b/(f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx = a\,x + \frac{b\left(F^{e\,g+f\,g\,x}\right)^n}{f\,g\,n\,\ln(F)}$$

input
```
int(a + b*(F^(g*(e + f*x)))^n,x)
```

output
```
a*x + (b*(F^(e*g + f*g*x))^n)/(f*g*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)\, dx = \frac{f^{fgnx+egn} b + \log(f)\, afgnx}{\log(f)\, fgn}$$

input
```
int(a+b*(F^(g*(f*x+e)))^n,x)
```

output
```
(f**(e*g*n + f*g*n*x)*b + log(f)*a*f*g*n*x)/(log(f)*f*g*n)
```

3.29 $\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 68

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} + \frac{a\log(c+dx)}{d}$$

output

```
b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*Ei(f*g*n*(d*x+c)*ln(F)/d
)/d+a*ln(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \frac{bF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + a\log(c+dx)}{d}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x),x]
```

output
```
((b*(F^(g*(e + f*x)))^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/F^((f*g
*n*(c + d*x))/d) + a*Log[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\left(F^{g(e+fx)}\right)^n}{c + dx}\,dx$$

$$\downarrow 2614$$

$$\int \left(\frac{a}{c + dx} + \frac{b\left(F^{eg+fgx}\right)^n}{c + dx}\right)dx$$

$$\downarrow 2009$$

$$\frac{a\log(c + dx)}{d} + \frac{b\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x),x]
```

output
```
(b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*ExpIntegralEi
[(f*g*n*(c + d*x)*Log[F])/d])/d + (a*Log[c + d*x])/d
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{a+b\left(F^{g(fx+e)}\right)^n}{dx+c}dx$$

input

```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

output

```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \frac{F^{\frac{(de-cf)gn}{d}}\,b\mathrm{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)+a\log\left(dx+c\right)}{d}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="fricas")
```

output

```
(F^((d*e - c*f)*g*n/d)*b*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d) + a*log(d*x +
c))/d
```

Sympy [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \int \frac{a+b\left(F^{eg+fgx}\right)^n}{c+dx}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)/(d*x+c),x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)/(c + d*x), x)
```

Maxima [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{dx+c}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="maxima")
```

output
```
F^(e*g*n)*b*integrate(F^(f*g*n*x)/(d*x + c), x) + a*log(d*x + c)/d
```

Giac [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{dx+c}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \int \frac{a+b\left(F^{g\,(e+f\,x)}\right)^n}{c+d\,x}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x),x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{c+dx}\,dx = \frac{f^{egn}\left(\int \frac{f^{fgnx}}{dx+c}dx\right) bd + \log(dx+c)\,a}{d}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

output
```
(f**(e*g*n)*int(f**(f*g*n*x)/(c + d*x),x)*b*d + log(c + d*x)*a)/d
```

3.30 $\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx$

Optimal result . . . 281
Mathematica [A] (verified) . . . 281
Rubi [A] (verified) . . . 282
Maple [F] . . . 283
Fricas [A] (verification not implemented) . . . 283
Sympy [F] . . . 284
Maxima [F] . . . 284
Giac [F] . . . 284
Mupad [F(-1)] . . . 285
Reduce [F] . . . 285

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)} - \frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)} + \frac{bfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

output

```
-a/d/(d*x+c)-b*(F^(f*g*x+e*g))^n/d/(d*x+c)+b*f*F^((e-c*f/d)*g*n-g*n*(f*x+e
))*(F^(f*g*x+e*g))^n*g*n*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \frac{-\frac{d\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{c+dx} + bfF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2,x]
```

output
```
(-((d*(a + b*(F^(g*(e + f*x)))^n))/(c + d*x)) + (b*f*(F^(g*(e + f*x)))^n*g
*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((f*g*n*(c + d*x))/
d))/d^2
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx$$

$$\downarrow 2614$$

$$\int \left(\frac{a}{(c+dx)^2} + \frac{b\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a}{d(c+dx)} + \frac{bfgn\log(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{b\left(F^{eg+fgx}\right)^n}{d(c+dx)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) - (b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) + (b*f*F^((e -
(c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g*n*ExpIntegralEi[(f*g*n
*(c + d*x)*Log[F])/d]*Log[F])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{a+b\left(F^{g(fx+e)}\right)^n}{(dx+c)^2}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \frac{(bdfgnx+bcfgn)F^{\frac{(de-cf)gn}{d}}\mathrm{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)\log(F)-F^{fgnx+egn}bd-ad}{d^3x+cd^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b*d*f*g*n*x + b*c*f*g*n)*F^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*
log(F)/d)*log(F) - F^(f*g*n*x + e*g*n)*b*d - a*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{a+b\left(F^{eg+fgx}\right)^n}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)/(d*x+c)**2,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
F^(e*g*n)*b*integrate(F^(f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) - a/(d^2*x
 + c*d)
```

Giac [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \int \frac{a+b\left(F^{g\,(e+f\,x)}\right)^n}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^2}\,dx = \frac{f^{egn}\left(\int \frac{f^{fgnx}}{d^2x^2+2cdx+c^2}dx\right) b\,c^2 + f^{egn}\left(\int \frac{f^{fgnx}}{d^2x^2+2cdx+c^2}dx\right) bcdx + ax}{c\,(dx+c)}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

output
```
(f**(e*g*n)*int(f**(f*g*n*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c**2 + f**(
e*g*n)*int(f**(f*g*n*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c*d*x + a*x)/(c*
(c + d*x))
```

3.31 $\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx$

Optimal result . . . 286
Mathematica [A] (verified) . . . 287
Rubi [A] (verified) . . . 287
Maple [F] . . . 288
Fricas [A] (verification not implemented) . . . 289
Sympy [F] . . . 289
Maxima [F] . . . 289
Giac [F] . . . 290
Mupad [F(-1)] . . . 290
Reduce [F] . . . 290

Optimal result

Integrand size = 23, antiderivative size = 147

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx$$
$$= -\frac{a}{2d(c+dx)^2} - \frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)}$$
$$+ \frac{bf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}$$

output

```
-1/2*a/d/(d*x+c)^2-1/2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-1/2*b*f*(F^(f*g*x+e
*g))^n*g*n*ln(F)/d^2/(d*x+c)+1/2*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f
*g*x+e*g))^n*g^2*n^2*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.76

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = -\frac{ad^2 - bf^2F^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n g^2n^2(c+dx)^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F) + bd\left(F^{g(e+fx)}\right.}{2d^3(c+dx)^2}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x]
```

output
```
-1/2*(a*d^2 - (b*f^2*(F^(g*(e + f*x)))^n*g^2*n^2*(c + d*x)^2*ExpIntegralEi
[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((f*g*n*(c + d*x))/d) + b*d*(F^(g
*(e + f*x)))^n*(d + f*g*n*(c + d*x)*Log[F]))/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx$$

$$\downarrow 2614$$

$$\int \left(\frac{a}{(c+dx)^3} + \frac{b\left(F^{eg+fgx}\right)^n}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{2d^3}-\frac{bfgn\log(F)\left(F^{eg+fgx}\right)^n}{2d^2(c+dx)}-\frac{b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (b*(F^(e*g + f*g*x))^n)/(2*d*(c + d*x)^2) - (b*f*
(F^(e*g + f*g*x))^n*g*n*Log[F])/(2*d^2*(c + d*x)) + (b*f^2*F^((e - (c*f)/d
)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*g^2*n^2*ExpIntegralEi[(f*g*n*(c
 + d*x)*Log[F])/d]*Log[F]^2)/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{a+b\left(F^{g(fx+e)}\right)^n}{(dx+c)^3}\,dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.09

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx$$
$$= \frac{(bd^2f^2g^2n^2x^2+2\,bcdf^2g^2n^2x+bc^2f^2g^2n^2)F^{\frac{(de-cf)gn}{d}}\,\mathrm{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right)\log(F)^2-ad^2-(bd^2+(bd^2f}{2\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*((b*d^2*f^2*g^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*x + b*c^2*f^2*g^2*n^2)*F
^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)^2 - a*d^2 -
 (b*d^2 + (b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(d^5*
x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = \int \frac{a+b\left(F^{eg+fgx}\right)^n}{(c+dx)^3}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)/(d*x+c)**3,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{(dx+c)^3}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
F^(e*g*n)*b*integrate(F^(f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3
), x) - 1/2*a/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = \int \frac{\left(F^{(fx+e)g}\right)^n b+a}{(dx+c)^3}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx = \int \frac{a+b\left(F^{g\,(e+f\,x)}\right)^n}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\left(F^{g(e+fx)}\right)^n}{(c+dx)^3}\,dx$$
$$= \frac{2f^{egn}\left(\int \frac{f^{fgnx}}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx\right)bc^2d + 4f^{egn}\left(\int \frac{f^{fgnx}}{d^3x^3+3cd^2x^2+3c^2dx+c^3}dx\right)bcd^2x + 2f^{egn}\left(\int \frac{f^{fgnx}}{d^3x^3+3cd^2x^2+3c}\right.}{2d\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^3,x)
```

output

```
(2*f**(e*g*n)*int(f**(f*g*n*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3),x)*b*c**2*d + 4*f**(e*g*n)*int(f**(f*g*n*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*b*c*d**2*x + 2*f**(e*g*n)*int(f**(f*g*n*x)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b*d**3*x**2 - a)/(2*d*(c**2
+ 2*c*d*x + d**2*x**2))
```

3.32 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx$

Optimal result . . . 292
Mathematica [A] (verified) . . . 293
Rubi [A] (verified) . . . 293
Maple [B] (warning: unable to verify) . . . 295
Fricas [A] (verification not implemented) . . . 295
Sympy [B] (verification not implemented) . . . 296
Maxima [A] (verification not implemented) . . . 298
Giac [C] (verification not implemented) . . . 299
Mupad [B] (verification not implemented) . . . 300
Reduce [B] (verification not implemented) . . . 301

Optimal result

Integrand size = 25, antiderivative size = 322

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = {} & \frac{a^2(c+dx)^4}{4d} - \frac{12abd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} \\ & - \frac{3b^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)} + \frac{12abd^2\left(F^{eg+fgx}\right)^n(c+dx)}{f^3g^3n^3\log^3(F)} \\ & + \frac{3b^2d^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{4f^3g^3n^3\log^3(F)} \\ & - \frac{6abd\left(F^{eg+fgx}\right)^n(c+dx)^2}{f^2g^2n^2\log^2(F)} \\ & - \frac{3b^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)} \\ & + \frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^3}{fgn\log(F)} \\ & + \frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)}\end{aligned}$$

output
```
1/4*a^2*(d*x+c)^4/d-12*a*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/ln(F)^4-3/8*b
^2*d^3*(F^(f*g*x+e*g))^(2*n)/f^4/g^4/n^4/ln(F)^4+12*a*b*d^2*(F^(f*g*x+e*g)
)^n*(d*x+c)/f^3/g^3/n^3/ln(F)^3+3/4*b^2*d^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/
f^3/g^3/n^3/ln(F)^3-6*a*b*d*(F^(f*g*x+e*g))^n*(d*x+c)^2/f^2/g^2/n^2/ln(F)^
2-3/4*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2+2*a*b*(F^(
f*g*x+e*g))^n*(d*x+c)^3/f/g/n/ln(F)+1/2*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^
3/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.74

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = a^2c^3x + \frac{3}{2}a^2c^2dx^2 + a^2cd^2x^3 + \frac{1}{4}a^2d^3x^4$$
$$+\frac{2ab\left(F^{g(e+fx)}\right)^n\left(-6d^3+6d^2fgn(c+dx)\log(F)-3df^2g^2n^2(c+dx)^2\log^2(F)+f^3g^3n^3(c+dx)^3\log^3(\right.}{f^4g^4n^4\log^4(F)}$$
$$+\frac{b^2\left(F^{g(e+fx)}\right)^{2n}\left(-3d^3+6d^2fgn(c+dx)\log(F)-6df^2g^2n^2(c+dx)^2\log^2(F)+4f^3g^3n^3(c+dx)^3\log^3(\right.}{8f^4g^4n^4\log^4(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^3,x]
```

output
```
a^2*c^3*x + (3*a^2*c^2*d*x^2)/2 + a^2*c*d^2*x^3 + (a^2*d^3*x^4)/4 + (2*a*b
*(F^(g*(e + f*x)))^n*(-6*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 3*d*f^2*g^2*
n^2*(c + d*x)^2*Log[F]^2 + f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(f^4*g^4*n^4
*Log[F]^4) + (b^2*(F^(g*(e + f*x)))^(2*n)*(-3*d^3 + 6*d^2*f*g*n*(c + d*x)*
Log[F] - 6*d*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2 + 4*f^3*g^3*n^3*(c + d*x)^3*
Log[F]^3))/(8*f^4*g^4*n^4*Log[F]^4)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx$$

$$\downarrow 2614$$

$$\int \left(a^2(c+dx)^3 + 2ab(c+dx)^3 \left(F^{eg+fgx}\right)^n + b^2(c+dx)^3 \left(F^{eg+fgx}\right)^{2n}\right) dx$$

$$\downarrow 2009$$

$$\frac{a^2(c+dx)^4}{4d} + \frac{12abd^2(c+dx)\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} - \frac{6abd(c+dx)^2\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)} - \frac{12abd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} + \frac{3b^2d^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)} - \frac{3b^2d(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} + \frac{b^2(c+dx)^3\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)} - \frac{3b^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^3,x]
```

output
```
(a^2*(c + d*x)^4)/(4*d) - (12*a*b*d^3*(F^(e*g + f*g*x))^n)/(f^4*g^4*n^4*Lo
g[F]^4) - (3*b^2*d^3*(F^(e*g + f*g*x))^(2*n))/(8*f^4*g^4*n^4*Log[F]^4) + (
12*a*b*d^2*(F^(e*g + f*g*x))^n*(c + d*x))/(f^3*g^3*n^3*Log[F]^3) + (3*b^2*
d^2*(F^(e*g + f*g*x))^(2*n)*(c + d*x))/(4*f^3*g^3*n^3*Log[F]^3) - (6*a*b*d
*(F^(e*g + f*g*x))^n*(c + d*x)^2)/(f^2*g^2*n^2*Log[F]^2) - (3*b^2*d*(F^(e*
g + f*g*x))^(2*n)*(c + d*x)^2)/(4*f^2*g^2*n^2*Log[F]^2) + (2*a*b*(F^(e*g +
 f*g*x))^n*(c + d*x)^3)/(f*g*n*Log[F]) + (b^2*(F^(e*g + f*g*x))^(2*n)*(c +
 d*x)^3)/(2*f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(312) = 624.

Time = 1.16 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.30

method	result
parallelrisch	$\frac{8a^2d^2cx^3n^4g^4f^4\ln(F)^4-96\ln(F)^2x\left(F^{g(fx+e)}\right)^nabcd^2f^2g^2n^2+48\ln(F)^3x^2\left(F^{g(fx+e)}\right)^nabcd^2f^3g^3n^3+48\ln(F)^3x\left(F^{g(fx}\right.}{}$
orering	Expression too large to display

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(8*a^2*d^2*c*x^3*n^4*g^4*f^4*ln(F)^4-96*ln(F)^2*x*(F^(g*(f*x+e)))^n*a*
b*c*d^2*f^2*g^2*n^2+48*ln(F)^3*x^2*(F^(g*(f*x+e)))^n*a*b*c*d^2*f^3*g^3*n^3
+48*ln(F)^3*x*(F^(g*(f*x+e)))^n*a*b*c^2*d*f^3*g^3*n^3+12*a^2*d*c^2*x^2*n^4
*g^4*f^4*ln(F)^4+4*b^2*d^3*((F^(g*(f*x+e)))^n)^2*x^3*n^3*g^3*f^3*ln(F)^3+1
6*ln(F)^3*(F^(g*(f*x+e)))^n*a*b*c^3*f^3*g^3*n^3-6*ln(F)^2*x^2*((F^(g*(f*x+
e)))^n)^2*b^2*d^3*f^2*g^2*n^2-6*ln(F)^2*((F^(g*(f*x+e)))^n)^2*b^2*c^2*d*f^
2*g^2*n^2+6*ln(F)*x*((F^(g*(f*x+e)))^n)^2*b^2*d^3*f*g*n+6*ln(F)*((F^(g*(f*
x+e)))^n)^2*b^2*c*d^2*f*g*n+2*a^2*d^3*x^4*n^4*g^4*f^4*ln(F)^4+8*a^2*c^3*x*
n^4*g^4*f^4*ln(F)^4+4*ln(F)^3*((F^(g*(f*x+e)))^n)^2*b^2*c^3*f^3*g^3*n^3-96
*(F^(g*(f*x+e)))^n*a*b*d^3+12*ln(F)^3*x*((F^(g*(f*x+e)))^n)^2*b^2*c^2*d*f^
3*g^3*n^3-48*ln(F)^2*x^2*(F^(g*(f*x+e)))^n*a*b*d^3*f^2*g^2*n^2-12*ln(F)^2*
x*((F^(g*(f*x+e)))^n)^2*b^2*c*d^2*f^2*g^2*n^2-48*ln(F)^2*(F^(g*(f*x+e)))^n
*a*b*c^2*d*f^2*g^2*n^2+96*ln(F)*x*(F^(g*(f*x+e)))^n*a*b*d^3*f*g*n+96*ln(F)
*(F^(g*(f*x+e)))^n*a*b*c*d^2*f*g*n+16*a*b*d^3*(F^(g*(f*x+e)))^n*x^3*n^3*g^
3*f^3*ln(F)^3+12*ln(F)^3*x^2*((F^(g*(f*x+e)))^n)^2*b^2*c*d^2*f^3*g^3*n^3-3
*((F^(g*(f*x+e)))^n)^2*b^2*d^3)/n^4/g^4/f^4/ln(F)^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.50

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx$$
$$= \frac{2\left(a^2d^3f^4g^4n^4x^4+4\,a^2cd^2f^4g^4n^4x^3+6\,a^2c^2df^4g^4n^4x^2+4\,a^2c^3f^4g^4n^4x\right)\log(F)^4-\left(3\,b^2d^3-4\,(b^2d^3f^3g^3$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(2*(a^2*d^3*f^4*g^4*n^4*x^4 + 4*a^2*c*d^2*f^4*g^4*n^4*x^3 + 6*a^2*c^2*
d*f^4*g^4*n^4*x^2 + 4*a^2*c^3*f^4*g^4*n^4*x)*log(F)^4 - (3*b^2*d^3 - 4*(b^
2*d^3*f^3*g^3*n^3*x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*
n^3*x + b^2*c^3*f^3*g^3*n^3)*log(F)^3 + 6*(b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2
*c*d^2*f^2*g^2*n^2*x + b^2*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(b^2*d^3*f*g*n*
x + b^2*c*d^2*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) - 16*(6*a*b*d^3 - (a*
b*d^3*f^3*g^3*n^3*x^3 + 3*a*b*c*d^2*f^3*g^3*n^3*x^2 + 3*a*b*c^2*d*f^3*g^3*
n^3*x + a*b*c^3*f^3*g^3*n^3)*log(F)^3 + 3*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b
*c*d^2*f^2*g^2*n^2*x + a*b*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(a*b*d^3*f*g*n*
x + a*b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^4*g^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 913 vs. $2(323) = 646$.

Time = 5.43 (sec) , antiderivative size = 913, normalized size of antiderivative = 2.84

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**3,x)
```

output

```
Piecewise(((a + b)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/
4), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**2*
(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(f, 0)), ((a + b
)**2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) | Eq
(g, 0) | Eq(n, 0)), (a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3
 + a**2*d**3*x**4/4 + 2*a*b*c**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 6*
a*b*c**2*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*a*b*c**2*d*(F**(e*g
+ f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 6*a*b*c*d**2*x**2*(F**(e*g + f*g
*x))**n/(f*g*n*log(F)) - 12*a*b*c*d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*
n**2*log(F)**2) + 12*a*b*c*d**2*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(
F)**3) + 2*a*b*d**3*x**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*a*b*d**3
*x**2*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 12*a*b*d**3*x*(F*
*(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) - 12*a*b*d**3*(F**(e*g + f*g
*x))**n/(f**4*g**4*n**4*log(F)**4) + b**2*c**3*(F**(e*g + f*g*x))**(2*n)/(
2*f*g*n*log(F)) + 3*b**2*c**2*d*x*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F
)) - 3*b**2*c**2*d*(F**(e*g + f*g*x))**(2*n)/(4*f**2*g**2*n**2*log(F)**2)
+ 3*b**2*c*d**2*x**2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 3*b**2*c
*d**2*x*(F**(e*g + f*g*x))**(2*n)/(2*f**2*g**2*n**2*log(F)**2) + 3*b**2*c*
d**2*(F**(e*g + f*g*x))**(2*n)/(4*f**3*g**3*n**3*log(F)**3) + b**2*d**3*x*
*3*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 3*b**2*d**3*x**2*(F**(e...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.76

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = \frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + a^2c^3x$$
$$+ \frac{2\,F^{fgnx+egn}abc^3}{fgn\log(F)} + \frac{F^{2\,fgnx+2\,egn}b^2c^3}{2\,fgn\log(F)} + \frac{6\left(F^{egn}fgnx\log(F) - F^{egn}\right)F^{fgnx}abc^2d}{f^2g^2n^2\log(F)^2}$$
$$+ \frac{3\left(2\,F^{2\,egn}fgnx\log(F) - F^{2\,egn}\right)F^{2\,fgnx}b^2c^2d}{4\,f^2g^2n^2\log(F)^2}$$
$$+ \frac{6\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{egn}fgnx\log(F) + 2\,F^{egn}\right)F^{fgnx}abcd^2}{f^3g^3n^3\log(F)^3}$$
$$+ \frac{3\left(2\,F^{2\,egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{2\,egn}fgnx\log(F) + F^{2\,egn}\right)F^{2\,fgnx}b^2cd^2}{4\,f^3g^3n^3\log(F)^3}$$
$$+ \frac{2\left(F^{egn}f^3g^3n^3x^3\log(F)^3 - 3\,F^{egn}f^2g^2n^2x^2\log(F)^2 + 6\,F^{egn}fgnx\log(F) - 6\,F^{egn}\right)F^{fgnx}abd^3}{f^4g^4n^4\log(F)^4}$$
$$+ \frac{\left(4\,F^{2\,egn}f^3g^3n^3x^3\log(F)^3 - 6\,F^{2\,egn}f^2g^2n^2x^2\log(F)^2 + 6\,F^{2\,egn}fgnx\log(F) - 3\,F^{2\,egn}\right)F^{2\,fgnx}b^2d^3}{8\,f^4g^4n^4\log(F)^4}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x + 2*F^(f*g
*n*x + e*g*n)*a*b*c^3/(f*g*n*log(F)) + 1/2*F^(2*f*g*n*x + 2*e*g*n)*b^2*c^3
/(f*g*n*log(F)) + 6*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*a*b
*c^2*d/(f^2*g^2*n^2*log(F)^2) + 3/4*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e
*g*n))*F^(2*f*g*n*x)*b^2*c^2*d/(f^2*g^2*n^2*log(F)^2) + 6*(F^(e*g*n)*f^2*g
^2*n^2*x^2*log(F)^2 - 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x
)*a*b*c*d^2/(f^3*g^3*n^3*log(F)^3) + 3/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^2*lo
g(F)^2 - 2*F^(2*e*g*n)*f*g*n*x*log(F) + F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*c*d
^2/(f^3*g^3*n^3*log(F)^3) + 2*(F^(e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 3*F^(e
*g*n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(e*g*n)*f*g*n*x*log(F) - 6*F^(e*g*n))
*F^(f*g*n*x)*a*b*d^3/(f^4*g^4*n^4*log(F)^4) + 1/8*(4*F^(2*e*g*n)*f^3*g^3*n
^3*x^3*log(F)^3 - 6*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(2*e*g*n)*f
*g*n*x*log(F) - 3*F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*d^3/(f^4*g^4*n^4*log(F)^4
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 12013, normalized size of antiderivative = 37.31

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = \text{Too large to display}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x, algorithm="giac")
```

output

```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x - 1/4*(((6
*pi^2*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 6*pi^2*b^2*d^3*f^3*g^3*
n^3*x^3*log(abs(F)) + 4*b^2*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 18*pi^2*b^
2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 18*pi^2*b^2*c*d^2*f^3*g^3*n^3
*x^2*log(abs(F)) + 12*b^2*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 18*pi^2*b^
2*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 18*pi^2*b^2*c^2*d*f^3*g^3*n^3*x
*log(abs(F)) + 12*b^2*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 6*pi^2*b^2*c^3*f
^3*g^3*n^3*log(abs(F))*sgn(F) - 6*pi^2*b^2*c^3*f^3*g^3*n^3*log(abs(F)) + 4
*b^2*c^3*f^3*g^3*n^3*log(abs(F))^3 - 3*pi^2*b^2*d^3*f^2*g^2*n^2*x^2*sgn(F)
 + 3*pi^2*b^2*d^3*f^2*g^2*n^2*x^2 - 6*b^2*d^3*f^2*g^2*n^2*x^2*log(abs(F))^
2 - 6*pi^2*b^2*c*d^2*f^2*g^2*n^2*x*sgn(F) + 6*pi^2*b^2*c*d^2*f^2*g^2*n^2*x
 - 12*b^2*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 3*pi^2*b^2*c^2*d*f^2*g^2*n^2
*sgn(F) + 3*pi^2*b^2*c^2*d*f^2*g^2*n^2 - 6*b^2*c^2*d*f^2*g^2*n^2*log(abs(F
))^2 + 6*b^2*d^3*f*g*n*x*log(abs(F)) + 6*b^2*c*d^2*f*g*n*log(abs(F)) - 3*b
^2*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*sgn(F)
 - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^4*log
(abs(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs(F))^2*s
gn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^4*g^4*n^
4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F) - pi*f^4*g^4*
n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + pi*f^4*g^4*n^...
```

Mupad [B] (verification not implemented)

Time = 23.42 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.36

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx = a^2 c^3 x$$
$$- \left(F^{f g x} F^{e g}\right)^n \left(\frac{2ab\left(-c^3 f^3 g^3 n^3 \ln(F)^3 + 3c^2 d f^2 g^2 n^2 \ln(F)^2 - 6cd^2 f g n \ln(F) + 6d^3\right)}{f^4 g^4 n^4 \ln(F)^4} - \frac{2abd^3 x^3}{f g n \ln(F)} - \frac{6abdx\left(c^2 f^2 g^2 n^2 \ln(F)^2 - 2cdfgn\ln(F) + 2d^2\right)}{f^3 g^3 n^3 \ln(F)^3} + \frac{6abd^2 x^2 (d - cfgn\ln(F))}{f^2 g^2 n^2 \ln(F)^2}\right)$$
$$- \left(F^{f g x} F^{e g}\right)^{2n} \left(\frac{b^2\left(-4c^3 f^3 g^3 n^3 \ln(F)^3 + 6c^2 d f^2 g^2 n^2 \ln(F)^2 - 6cd^2 f g n \ln(F) + 3d^3\right)}{8 f^4 g^4 n^4 \ln(F)^4} - \frac{b^2 d^3 x^3}{2 f g n \ln(F)} - \frac{3b^2 dx\left(2c^2 f^2 g^2 n^2 \ln(F)^2 - 2cdfgn\ln(F) + d^2\right)}{4 f^3 g^3 n^3 \ln(F)^3} + \frac{3b^2 d^2 x^2 (d - 2cfgn\ln(F))}{4 f^2 g^2 n^2 \ln(F)^2}\right) + \frac{a^2 d^3 x^4}{4} + \frac{3a^2 c^2 d x^2}{2} + a^2 c d^2 x^3$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^3,x)
```

output
```
a^2*c^3*x - (F^(f*g*x)*F^(e*g))^n*((2*a*b*(6*d^3 - c^3*f^3*g^3*n^3*log(F)^
3 - 6*c*d^2*f*g*n*log(F) + 3*c^2*d*f^2*g^2*n^2*log(F)^2))/(f^4*g^4*n^4*log
(F)^4) - (2*a*b*d^3*x^3)/(f*g*n*log(F)) - (6*a*b*d*x*(2*d^2 + c^2*f^2*g^2*
n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/(f^3*g^3*n^3*log(F)^3) + (6*a*b*d^2*x^
2*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) - (F^(f*g*x)*F^(e*g))^(2*n
)*((b^2*(3*d^3 - 4*c^3*f^3*g^3*n^3*log(F)^3 - 6*c*d^2*f*g*n*log(F) + 6*c^2
*d*f^2*g^2*n^2*log(F)^2))/(8*f^4*g^4*n^4*log(F)^4) - (b^2*d^3*x^3)/(2*f*g*
n*log(F)) - (3*b^2*d*x*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log
(F)))/(4*f^3*g^3*n^3*log(F)^3) + (3*b^2*d^2*x^2*(d - 2*c*f*g*n*log(F)))/(4
*f^2*g^2*n^2*log(F)^2)) + (a^2*d^3*x^4)/4 + (3*a^2*c^2*d*x^2)/2 + a^2*c*d^
2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.35

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^3\,dx$$
$$= \frac{4f^{2fgnx+2egn}\log(f)^3 b^2c^3f^3g^3n^3 + 12f^{2fgnx+2egn}\log(f)^3 b^2c^2d\,f^3g^3n^3x + 12f^{2fgnx+2egn}\log(f)^3 b^2c\,d^2f^3g^3n$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^3,x)
```

output
```
(4*f**(2*e*g*n + 2*f*g*n*x)*log(f)**3*b**2*c**3*f**3*g**3*n**3 + 12*f**(2*
e*g*n + 2*f*g*n*x)*log(f)**3*b**2*c**2*d*f**3*g**3*n**3*x + 12*f**(2*e*g*n
 + 2*f*g*n*x)*log(f)**3*b**2*c*d**2*f**3*g**3*n**3*x**2 + 4*f**(2*e*g*n +
2*f*g*n*x)*log(f)**3*b**2*d**3*f**3*g**3*n**3*x**3 - 6*f**(2*e*g*n + 2*f*g
*n*x)*log(f)**2*b**2*c**2*d*f**2*g**2*n**2 - 12*f**(2*e*g*n + 2*f*g*n*x)*l
og(f)**2*b**2*c*d**2*f**2*g**2*n**2*x - 6*f**(2*e*g*n + 2*f*g*n*x)*log(f)*
*2*b**2*d**3*f**2*g**2*n**2*x**2 + 6*f**(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*
c*d**2*f*g*n + 6*f**(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*d**3*f*g*n*x - 3*f**
(2*e*g*n + 2*f*g*n*x)*b**2*d**3 + 16*f**(e*g*n + f*g*n*x)*log(f)**3*a*b*c*
*3*f**3*g**3*n**3 + 48*f**(e*g*n + f*g*n*x)*log(f)**3*a*b*c**2*d*f**3*g**3
*n**3*x + 48*f**(e*g*n + f*g*n*x)*log(f)**3*a*b*c*d**2*f**3*g**3*n**3*x**2
 + 16*f**(e*g*n + f*g*n*x)*log(f)**3*a*b*d**3*f**3*g**3*n**3*x**3 - 48*f**
(e*g*n + f*g*n*x)*log(f)**2*a*b*c**2*d*f**2*g**2*n**2 - 96*f**(e*g*n + f*g
*n*x)*log(f)**2*a*b*c*d**2*f**2*g**2*n**2*x - 48*f**(e*g*n + f*g*n*x)*log(
f)**2*a*b*d**3*f**2*g**2*n**2*x**2 + 96*f**(e*g*n + f*g*n*x)*log(f)*a*b*c*
d**2*f*g*n + 96*f**(e*g*n + f*g*n*x)*log(f)*a*b*d**3*f*g*n*x - 96*f**(e*g*
n + f*g*n*x)*a*b*d**3 + 8*log(f)**4*a**2*c**3*f**4*g**4*n**4*x + 12*log(f)
**4*a**2*c**2*d*f**4*g**4*n**4*x**2 + 8*log(f)**4*a**2*c*d**2*f**4*g**4*n*
*4*x**3 + 2*log(f)**4*a**2*d**3*f**4*g**4*n**4*x**4)/(8*log(f)**4*f**4*g**
4*n**4)
```

3.33 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 239

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx = {} & \frac{a^2(c+dx)^3}{3d}+\frac{4abd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} \\ & +\frac{b^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}-\frac{4abd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)} \\ & -\frac{b^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2f^2g^2n^2\log^2(F)} \\ & +\frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)} \\ & +\frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)}\end{aligned}$$

output

```
1/3*a^2*(d*x+c)^3/d+4*a*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/ln(F)^3+1/4*b^
2*d^2*(F^(f*g*x+e*g))^(2*n)/f^3/g^3/n^3/ln(F)^3-4*a*b*d*(F^(f*g*x+e*g))^n*
(d*x+c)/f^2/g^2/n^2/ln(F)^2-1/2*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^2/g^
2/n^2/ln(F)^2+2*a*b*(F^(f*g*x+e*g))^n*(d*x+c)^2/f/g/n/ln(F)+1/2*b^2*(F^(f*
g*x+e*g))^(2*n)*(d*x+c)^2/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.72

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx \\
&= a^2c^2x + a^2cdx^2 + \frac{1}{3}a^2d^2x^3 \\
&\quad + \frac{2ab\left(F^{g(e+fx)}\right)^n \left(2d^2 - 2dfgn(c+dx)\log(F) + f^2g^2n^2(c+dx)^2\log^2(F)\right)}{f^3g^3n^3\log^3(F)} \\
&\quad + \frac{b^2\left(F^{g(e+fx)}\right)^{2n} \left(d^2 - 2dfgn(c+dx)\log(F) + 2f^2g^2n^2(c+dx)^2\log^2(F)\right)}{4f^3g^3n^3\log^3(F)}
\end{aligned}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2,x]
```

output
```
a^2*c^2*x + a^2*c*d*x^2 + (a^2*d^2*x^3)/3 + (2*a*b*(F^(g*(e + f*x)))^n*(2*
d^2 - 2*d*f*g*n*(c + d*x)*Log[F] + f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(f^3
*g^3*n^3*Log[F]^3) + (b^2*(F^(g*(e + f*x)))^(2*n)*(d^2 - 2*d*f*g*n*(c + d*
x)*Log[F] + 2*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(4*f^3*g^3*n^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx$$

$$\downarrow \text{2614}$$

$$\int \left(a^2(c+dx)^2 + 2ab(c+dx)^2\left(F^{eg+fgx}\right)^n + b^2(c+dx)^2\left(F^{eg+fgx}\right)^{2n}\right) dx$$

$$\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d} - \frac{4abd(c+dx)\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} + \frac{2ab(c+dx)^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)} + \frac{4abd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} - \frac{b^2d(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2f^2g^2n^2\log^2(F)} + \frac{b^2(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)} + \frac{b^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2,x]
```

output
```
(a^2*(c + d*x)^3)/(3*d) + (4*a*b*d^2*(F^(e*g + f*g*x))^n)/(f^3*g^3*n^3*Log
[F]^3) + (b^2*d^2*(F^(e*g + f*g*x))^(2*n))/(4*f^3*g^3*n^3*Log[F]^3) - (4*a
*b*d*(F^(e*g + f*g*x))^n*(c + d*x))/(f^2*g^2*n^2*Log[F]^2) - (b^2*d*(F^(e*
g + f*g*x))^(2*n)*(c + d*x))/(2*f^2*g^2*n^2*Log[F]^2) + (2*a*b*(F^(e*g + f
*g*x))^n*(c + d*x)^2)/(f*g*n*Log[F]) + (b^2*(F^(e*g + f*g*x))^(2*n)*(c + d
*x)^2)/(2*f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.77

method	result
parallelrisch	$\frac{4a^2d^2x^3n^3g^3f^3\ln(F)^3+12a^2dcx^2n^3g^3f^3\ln(F)^3+12a^2c^2xn^3g^3f^3\ln(F)^3+6x^2\left(F^{g(fx+e)}\right)^{2n}b^2d^2n^2g^2f^2\ln(F)^2+24x^2(F^g$
orering	$\frac{\left(4\ln(F)^4d^4f^4g^4n^4x^5+20\ln(F)^4cd^3f^4g^4n^4x^4+40\ln(F)^4c^2d^2f^4g^4n^4x^3+36\ln(F)^4c^3df^4g^4n^4x^2+12\ln(F)^4c^4f^4g^4n^4x+1\right.}{}$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*a^2*d^2*x^3*n^3*g^3*f^3*ln(F)^3+12*a^2*d*c*x^2*n^3*g^3*f^3*ln(F)^3
+12*a^2*c^2*x*n^3*g^3*f^3*ln(F)^3+6*x^2*((F^(g*(f*x+e)))^n)^2*b^2*d^2*n^2*
g^2*f^2*ln(F)^2+24*x^2*(F^(g*(f*x+e)))^n*a*b*d^2*n^2*g^2*f^2*ln(F)^2+12*ln
(F)^2*x*((F^(g*(f*x+e)))^n)^2*b^2*c*d*f^2*g^2*n^2+48*ln(F)^2*x*(F^(g*(f*x+
e)))^n*a*b*c*d*f^2*g^2*n^2+6*ln(F)^2*((F^(g*(f*x+e)))^n)^2*b^2*c^2*f^2*g^2
*n^2+24*ln(F)^2*(F^(g*(f*x+e)))^n*a*b*c^2*f^2*g^2*n^2-6*ln(F)*x*((F^(g*(f*
x+e)))^n)^2*b^2*d^2*f*g*n-48*ln(F)*x*(F^(g*(f*x+e)))^n*a*b*d^2*f*g*n-6*ln(
F)*((F^(g*(f*x+e)))^n)^2*b^2*c*d*f*g*n-48*ln(F)*(F^(g*(f*x+e)))^n*a*b*c*d*
f*g*n+3*((F^(g*(f*x+e)))^n)^2*b^2*d^2+48*(F^(g*(f*x+e)))^n*a*b*d^2)/n^3/g^
3/f^3/ln(F)^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.20

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx$$
$$= \frac{4\left(a^2d^2f^3g^3n^3x^3 + 3\,a^2cdf^3g^3n^3x^2 + 3\,a^2c^2f^3g^3n^3x\right)\log(F)^3 + 3\left(b^2d^2 + 2\left(b^2d^2f^2g^2n^2x^2 + 2\,b^2cdf^2g^2n^2\right.\right.}{}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/12*(4*(a^2*d^2*f^3*g^3*n^3*x^3 + 3*a^2*c*d*f^3*g^3*n^3*x^2 + 3*a^2*c^2*f
^3*g^3*n^3*x)*log(F)^3 + 3*(b^2*d^2 + 2*(b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2*c
*d*f^2*g^2*n^2*x + b^2*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(b^2*d^2*f*g*n*x + b^
2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 24*(2*a*b*d^2 + (a*b*d^2*f^
2*g^2*n^2*x^2 + 2*a*b*c*d*f^2*g^2*n^2*x + a*b*c^2*f^2*g^2*n^2)*log(F)^2 -
2*(a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^3*g^3*
n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(231) = 462.

Time = 2.09 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.21

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx$$

$$= \begin{cases} (a+b)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ (a+b(F^{eg})^n)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ (a+b)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \\ a^2c^2x+a^2cdx^2+\frac{a^2d^2x^3}{3}+\frac{2abc^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{4abcdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{4abcd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{2abd^2x^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{4abd^2x}{f^2g^2} \end{cases}$$

input

```
integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**2,x)
```

output

```
Piecewise(((a + b)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) & Eq(f,
0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**2*(c**2*x + c*d*x**2 +
d**2*x**3/3), Eq(f, 0)), ((a + b)**2*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq
(F, 1) | Eq(g, 0) | Eq(n, 0)), (a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x*
*3/3 + 2*a*b*c**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 4*a*b*c*d*x*(F**(
e*g + f*g*x))**n/(f*g*n*log(F)) - 4*a*b*c*d*(F**(e*g + f*g*x))**n/(f**2*g*
*2*n**2*log(F)**2) + 2*a*b*d**2*x**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F))
- 4*a*b*d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 4*a*b*d*
*2*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) + b**2*c**2*(F**(e*g +
 f*g*x))**(2*n)/(2*f*g*n*log(F)) + b**2*c*d*x*(F**(e*g + f*g*x))**(2*n)/(f
*g*n*log(F)) - b**2*c*d*(F**(e*g + f*g*x))**(2*n)/(2*f**2*g**2*n**2*log(F)
**2) + b**2*d**2*x**2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - b**2*d*
*2*x*(F**(e*g + f*g*x))**(2*n)/(2*f**2*g**2*n**2*log(F)**2) + b**2*d**2*(F
**(e*g + f*g*x))**(2*n)/(4*f**3*g**3*n**3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.45

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx \\
&= \frac{1}{3}a^2d^2x^3 + a^2cdx^2 + a^2c^2x + \frac{2\,F^{fgnx+egn}abc^2}{fgn\log(F)} \\
&\quad + \frac{F^{2\,fgnx+2\,egn}b^2c^2}{2\,fgn\log(F)} + \frac{4\left(F^{egn}fgnx\log(F) - F^{egn}\right)F^{fgnx}abcd}{f^2g^2n^2\log(F)^2} \\
&\quad + \frac{\left(2\,F^{2\,egn}fgnx\log(F) - F^{2\,egn}\right)F^{2\,fgnx}b^2cd}{2\,f^2g^2n^2\log(F)^2} \\
&\quad + \frac{2\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{egn}fgnx\log(F) + 2\,F^{egn}\right)F^{fgnx}abd^2}{f^3g^3n^3\log(F)^3} \\
&\quad + \frac{\left(2\,F^{2\,egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{2\,egn}fgnx\log(F) + F^{2\,egn}\right)F^{2\,fgnx}b^2d^2}{4\,f^3g^3n^3\log(F)^3}
\end{aligned}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + a^2*c^2*x + 2*F^(f*g*n*x + e*g*n)*a*b*c^2/
(f*g*n*log(F)) + 1/2*F^(2*f*g*n*x + 2*e*g*n)*b^2*c^2/(f*g*n*log(F)) + 4*(F
^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*a*b*c*d/(f^2*g^2*n^2*log(
F)^2) + 1/2*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)*b^2
*c*d/(f^2*g^2*n^2*log(F)^2) + 2*(F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^
(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*a*b*d^2/(f^3*g^3*n^3*log
(F)^3) + 1/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(2*e*g*n)*f*g*n
*x*log(F) + F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*d^2/(f^3*g^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 5675, normalized size of antiderivative = 23.74

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x, algorithm="giac")
```

output

```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + a^2*c^2*x - 1/2*(((2*pi*b^2*d^2*f^2*g^2*n^
2*x^2*log(abs(F))*sgn(F) - 2*pi*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 4*pi
*b^2*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 4*pi*b^2*c*d*f^2*g^2*n^2*x*log
(abs(F)) + 2*pi*b^2*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 2*pi*b^2*c^2*f^2*
g^2*n^2*log(abs(F)) - pi*b^2*d^2*f*g*n*x*sgn(F) + pi*b^2*d^2*f*g*n*x - pi*
b^2*c*d*f*g*n*sgn(F) + pi*b^2*c*d*f*g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f
^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(
abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(
F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^
3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*
log(abs(F))^3)^2) - (pi^2*b^2*d^2*f^2*g^2*n^2*x^2*sgn(F) - pi^2*b^2*d^2*f^
2*g^2*n^2*x^2 + 2*b^2*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 2*pi^2*b^2*c*d*f
^2*g^2*n^2*x*sgn(F) - 2*pi^2*b^2*c*d*f^2*g^2*n^2*x + 4*b^2*c*d*f^2*g^2*n^2
*x*log(abs(F))^2 + pi^2*b^2*c^2*f^2*g^2*n^2*sgn(F) - pi^2*b^2*c^2*f^2*g^2*
n^2 + 2*b^2*c^2*f^2*g^2*n^2*log(abs(F))^2 - 2*b^2*d^2*f*g*n*x*log(abs(F))
- 2*b^2*c*d*f*g*n*log(abs(F)) + b^2*d^2)*(3*pi^2*f^3*g^3*n^3*log(abs(F))*s
gn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^3)/((pi
^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g
^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3*g^3*n^3*log(abs(F
))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3*log(abs(F))^...
```

Mupad [B] (verification not implemented)

Time = 23.28 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.12

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx$$

$$= \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2n}\left(\frac{b^2\left(2\,c^2\,f^2\,g^2\,n^2\ln(F)^2 - 2\,c\,d\,f\,g\,n\,\ln(F) + d^2\right)}{4\,f^3\,g^3\,n^3\ln(F)^3} + \frac{b^2\,d^2\,x^2}{2\,f\,g\,n\,\ln(F)} - \frac{b^2\,d\,x\,(d - 2\,c\,f\,g\,n\,\ln(F))}{2\,f^2\,g^2\,n^2\ln(F)^2}\right)$$

$$+ \left(F^{f\,g\,x}\,F^{e\,g}\right)^n\left(\frac{2\,a\,b\left(c^2\,f^2\,g^2\,n^2\ln(F)^2 - 2\,c\,d\,f\,g\,n\,\ln(F) + 2\,d^2\right)}{f^3\,g^3\,n^3\ln(F)^3} + \frac{2\,a\,b\,d^2\,x^2}{f\,g\,n\,\ln(F)} - \frac{4\,a\,b\,d\,x\,(d - c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\ln(F)^2}\right) + a^2\,c^2\,x + \frac{a^2\,d^2\,x^3}{3} + a^2\,c\,d\,x^2$$

input

```
int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2,x)
```

output
```
(F^(f*g*x)*F^(e*g))^(2*n)*((b^2*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*
f*g*n*log(F)))/(4*f^3*g^3*n^3*log(F)^3) + (b^2*d^2*x^2)/(2*f*g*n*log(F)) -
 (b^2*d*x*(d - 2*c*f*g*n*log(F)))/(2*f^2*g^2*n^2*log(F)^2)) + (F^(f*g*x)*F
^(e*g))^n*((2*a*b*(2*d^2 + c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))
/(f^3*g^3*n^3*log(F)^3) + (2*a*b*d^2*x^2)/(f*g*n*log(F)) - (4*a*b*d*x*(d -
 c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + a^2*c^2*x + (a^2*d^2*x^3)/3 +
a^2*c*d*x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.82

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2\,dx$$
$$= \frac{6f^{2fgnx+2egn}\log(f)^2 b^2c^2f^2g^2n^2 + 12f^{2fgnx+2egn}\log(f)^2 b^2cd\,f^2g^2n^2x + 6f^{2fgnx+2egn}\log(f)^2 b^2d^2f^2g^2n^2x^2}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^2,x)
```

output
```
(6*f**(2*e*g*n + 2*f*g*n*x)*log(f)**2*b**2*c**2*f**2*g**2*n**2 + 12*f**(2*
e*g*n + 2*f*g*n*x)*log(f)**2*b**2*c*d*f**2*g**2*n**2*x + 6*f**(2*e*g*n + 2
*f*g*n*x)*log(f)**2*b**2*d**2*f**2*g**2*n**2*x**2 - 6*f**(2*e*g*n + 2*f*g*
n*x)*log(f)*b**2*c*d*f*g*n - 6*f**(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*d**2*f
*g*n*x + 3*f**(2*e*g*n + 2*f*g*n*x)*b**2*d**2 + 24*f**(e*g*n + f*g*n*x)*lo
g(f)**2*a*b*c**2*f**2*g**2*n**2 + 48*f**(e*g*n + f*g*n*x)*log(f)**2*a*b*c*
d*f**2*g**2*n**2*x + 24*f**(e*g*n + f*g*n*x)*log(f)**2*a*b*d**2*f**2*g**2*
n**2*x**2 - 48*f**(e*g*n + f*g*n*x)*log(f)*a*b*c*d*f*g*n - 48*f**(e*g*n +
f*g*n*x)*log(f)*a*b*d**2*f*g*n*x + 48*f**(e*g*n + f*g*n*x)*a*b*d**2 + 12*l
og(f)**3*a**2*c**2*f**3*g**3*n**3*x + 12*log(f)**3*a**2*c*d*f**3*g**3*n**3
*x**2 + 4*log(f)**3*a**2*d**2*f**3*g**3*n**3*x**3)/(12*log(f)**3*f**3*g**3
*n**3)
```

3.34 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \frac{a^2(c+dx)^2}{2d} - \frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} - \frac{b^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} + \frac{2ab\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)}$$

output
```
1/2*a^2*(d*x+c)^2/d-2*a*b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/ln(F)^2-1/4*b^2*
d*(F^(f*g*x+e*g))^(2*n)/f^2/g^2/n^2/ln(F)^2+2*a*b*(F^(f*g*x+e*g))^n*(d*x+c
)/f/g/n/ln(F)+1/2*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \frac{-bd\left(F^{g(e+fx)}\right)^n\left(8a+b\left(F^{g(e+fx)}\right)^n\right)+2bf\left(F^{g(e+fx)}\right)^n\left(4a+b\left(F^{g(e+fx)}\right)^n\right)gn(c+dx)\log(F)+2a^2f^2g^2}{4f^2g^2n^2\log^2(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x),x]
```

output
```
(-(b*d*(F^(g*(e + f*x)))^n*(8*a + b*(F^(g*(e + f*x)))^n)) + 2*b*f*(F^(g*(e
 + f*x)))^n*(4*a + b*(F^(g*(e + f*x)))^n)*g*n*(c + d*x)*Log[F] + 2*a^2*f^2
*g^2*n^2*x*(2*c + d*x)*Log[F]^2)/(4*f^2*g^2*n^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx$$

$$\downarrow 2614$$

$$\int \left(a^2(c+dx)+2ab(c+dx)\left(F^{eg+fgx}\right)^n+b^2(c+dx)\left(F^{eg+fgx}\right)^{2n}\right) dx$$

$$\downarrow 2009$$

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{b^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{b^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x),x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) - (2*a*b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*Log[F
]^2) - (b^2*d*(F^(e*g + f*g*x))^(2*n))/(4*f^2*g^2*n^2*Log[F]^2) + (2*a*b*(
F^(e*g + f*g*x))^n*(c + d*x))/(f*g*n*Log[F]) + (b^2*(F^(e*g + f*g*x))^(2*n
)*(c + d*x))/(2*f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.12

method	result
norman	$a^2cx+\frac{a^2dx^2}{2}+\frac{b^2(2\ln(F)cfgn-d)e^{2n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{4n^2g^2f^2\ln(F)^2}+\frac{2ab(\ln(F)cfgn-d)e^{n\ln\left(e^{g(fx+e)\ln(F)}\right)}}{n^2g^2f^2\ln(F)^2}+\frac{b^2dxe^{2n\ln(e}}{2ngf}$
parallelrisch	$\frac{2a^2dx^2n^2g^2f^2\ln(F)^2+4a^2cxn^2g^2f^2\ln(F)^2+2\ln(F)\left(F^{g(fx+e)}\right)^{2n}b^2dfgnx+8\ln(F)\left(F^{g(fx+e)}\right)^nabdfgnx+2\ln(F)(F^{g(fx+e)}}{4n^2g^2f^2\ln(F)^2}$
orering	$\frac{\left(2\ln(F)^3d^3f^3g^3n^3x^4+8\ln(F)^3cd^2f^3g^3n^3x^3+10\ln(F)^3c^2df^3g^3n^3x^2+4\ln(F)^3c^3f^3g^3n^3x+3\ln(F)^2d^3f^2g^2n^2x^3+15\ln(F}{}$

input

```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x,method=_RETURNVERBOSE)
```

output

```
a^2*c*x+1/2*a^2*d*x^2+1/4*b^2*(2*ln(F)*c*f*g*n-d)/n^2/g^2/f^2/ln(F)^2*exp(
n*ln(exp(g*(f*x+e)*ln(F))))^2+2*a*b*(ln(F)*c*f*g*n-d)/n^2/g^2/f^2/ln(F)^2*
exp(n*ln(exp(g*(f*x+e)*ln(F))))+1/2/n/g/f/ln(F)*b^2*d*x*exp(n*ln(exp(g*(f*
x+e)*ln(F))))^2+2/n/g/f/ln(F)*a*b*d*x*exp(n*ln(exp(g*(f*x+e)*ln(F))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$$
$$= \frac{2\left(a^2df^2g^2n^2x^2+2\,a^2cf^2g^2n^2x\right)\log(F)^2-\left(b^2d-2\left(b^2dfgnx+b^2cfgn\right)\log(F)\right)F^{2\,fgnx+2\,egn}-8\left(abd-\right.}{4\,f^2g^2n^2\log(F)^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(2*(a^2*d*f^2*g^2*n^2*x^2 + 2*a^2*c*f^2*g^2*n^2*x)*log(F)^2 - (b^2*d -
 2*(b^2*d*f*g*n*x + b^2*c*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) - 8*(a*b*
d - (a*b*d*f*g*n*x + a*b*c*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^2*g^2*n^
2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.65

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$$
$$= \begin{cases} (a+b)^2\left(cx+\frac{dx^2}{2}\right) \\ (a+b(F^{eg})^n)^2\left(cx+\frac{dx^2}{2}\right) \\ (a+b)^2\left(cx+\frac{dx^2}{2}\right) \\ a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{2abdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{2abd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{b^2c\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}+\frac{b^2dx\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{b^2d(F^e} \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c),x)
```

output
```
Piecewise(((a + b)**2*(c*x + d*x**2/2), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & E
q(n, 0)), ((a + b*(F**(e*g))**n)**2*(c*x + d*x**2/2), Eq(f, 0)), ((a + b)*
*2*(c*x + d*x**2/2), Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), (a**2*c*x + a**2*d*x
**2/2 + 2*a*b*c*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 2*a*b*d*x*(F**(e*g
+ f*g*x))**n/(f*g*n*log(F)) - 2*a*b*d*(F**(e*g + f*g*x))**n/(f**2*g**2*n**
2*log(F)**2) + b**2*c*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + b**2*d*
x*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - b**2*d*(F**(e*g + f*g*x))**
(2*n)/(4*f**2*g**2*n**2*log(F)**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \frac{1}{2}a^2dx^2 + a^2cx + \frac{2\,F^{fgnx+egn}abc}{fgn\log(F)} + \frac{F^{2\,fgnx+2\,egn}b^2c}{2\,fgn\log(F)} + \frac{2\,(F^{egn}fgnx\log(F) - F^{egn})F^{fgnx}abd}{f^2g^2n^2\log(F)^2} + \frac{(2\,F^{2\,egn}fgnx\log(F) - F^{2\,egn})F^{2\,fgnx}b^2d}{4\,f^2g^2n^2\log(F)^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + a^2*c*x + 2*F^(f*g*n*x + e*g*n)*a*b*c/(f*g*n*log(F)) + 1/2
*F^(2*f*g*n*x + 2*e*g*n)*b^2*c/(f*g*n*log(F)) + 2*(F^(e*g*n)*f*g*n*x*log(F
) - F^(e*g*n))*F^(f*g*n*x)*a*b*d/(f^2*g^2*n^2*log(F)^2) + 1/4*(2*F^(2*e*g*
n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)*b^2*d/(f^2*g^2*n^2*log(F)^2
)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 2284, normalized size of antiderivative = 14.64

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x, algorithm="giac")
```

output
```
1/2*a^2*d*x^2 + a^2*c*x + 1/2*((2*(pi*b^2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*
n*x + pi*b^2*c*f*g*n*sgn(F) - pi*b^2*c*f*g*n)*(pi*f^2*g^2*n^2*log(abs(F))*
sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*
g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*s
gn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) + (pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f
^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(2*b^2*d*f*g*n*x*log(abs(F)) + 2
*b^2*c*f*g*n*log(abs(F)) - b^2*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2
*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(
F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*cos(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x -
 pi*e*g*n*sgn(F) + pi*e*g*n) + ((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^
2 + 2*f^2*g^2*n^2*log(abs(F))^2)*(pi*b^2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*n
*x + pi*b^2*c*f*g*n*sgn(F) - pi*b^2*c*f*g*n)/((pi^2*f^2*g^2*n^2*sgn(F) - p
i^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(a
bs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) - 2*(pi*f^2*g^2*n^2*log(abs
(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))*(2*b^2*d*f*g*n*x*log(abs(F)) + 2
*b^2*c*f*g*n*log(abs(F)) - b^2*d)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2
*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(
F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*sin(-pi*f*g*n*x*sgn(F) + pi*f*g*n*x -
 pi*e*g*n*sgn(F) + pi*e*g*n))*e^(2*f*g*n*x*log(abs(F)) + 2*e*g*n*log(abs(F
))) - 1/4*I*((pi*b^2*d*f*g*n*x*sgn(F) - pi*b^2*d*f*g*n*x - 2*I*b^2*d*f*...
```

Mupad [B] (verification not implemented)

Time = 23.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx = \frac{a^2\,d\,x^2}{2} - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{b^2\,(d-2\,c\,f\,g\,n\,\ln(F))}{4\,f^2\,g^2\,n^2\,\ln(F)^2} - \frac{b^2\,d\,x}{2\,f\,g\,n\,\ln(F)}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^n\left(\frac{2\,a\,b\,(d-c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\,\ln(F)^2} - \frac{2\,a\,b\,d\,x}{f\,g\,n\,\ln(F)}\right) + a^2\,c\,x$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x),x)
```

output
```
(a^2*d*x^2)/2 - (F^(f*g*x)*F^(e*g))^(2*n)*((b^2*(d - 2*c*f*g*n*log(F)))/(4
*f^2*g^2*n^2*log(F)^2) - (b^2*d*x)/(2*f*g*n*log(F))) - (F^(f*g*x)*F^(e*g))
^n*((2*a*b*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2) - (2*a*b*d*x)/(f*g
*n*log(F))) + a^2*c*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)\,dx$$
$$= \frac{2f^{2fgnx+2egn}\log(f)\,b^2cfgn + 2f^{2fgnx+2egn}\log(f)\,b^2dfgnx - f^{2fgnx+2egn}b^2d + 8f^{fgnx+egn}\log(f)\,abcfgn +}{4\log(f)^2 f^2g^2}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c),x)
```

output
```
(2*f**(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*c*f*g*n + 2*f**(2*e*g*n + 2*f*g*n*
x)*log(f)*b**2*d*f*g*n*x - f**(2*e*g*n + 2*f*g*n*x)*b**2*d + 8*f**(e*g*n +
 f*g*n*x)*log(f)*a*b*c*f*g*n + 8*f**(e*g*n + f*g*n*x)*log(f)*a*b*d*f*g*n*x
 - 8*f**(e*g*n + f*g*n*x)*a*b*d + 4*log(f)**2*a**2*c*f**2*g**2*n**2*x + 2*
log(f)**2*a**2*d*f**2*g**2*n**2*x**2)/(4*log(f)**2*f**2*g**2*n**2)
```

3.35 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx$

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Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = a^2x + \frac{2ab\left(F^{g(e+fx)}\right)^n}{fgn\log(F)} + \frac{b^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}$$

output
```
a^2*x+2*a*b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)+1/2*b^2*(F^(g*(f*x+e)))^(2*n)/f/
g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = \frac{b\left(F^{g(e+fx)}\right)^n\left(4a+b\left(F^{g(e+fx)}\right)^n\right)+2a^2\log\left(\left(F^{g(e+fx)}\right)^n\right)}{2fgn\log(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
(b*(F^(g*(e + f*x)))^n*(4*a + b*(F^(g*(e + f*x)))^n) + 2*a^2*Log[(F^(g*(e
+ f*x)))^n])/(2*f*g*n*Log[F])
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 dx$$

$$\downarrow 2720$$

$$\frac{\int F^{-g(e+fx)}\left(b\left(F^{g(e+fx)}\right)^n + a\right)^2 dF^{g(e+fx)}}{fg\log(F)}$$

$$\downarrow 798$$

$$\frac{\int F^{-g(e+fx)}\left(b\left(F^{g(e+fx)}\right)^n + a\right)^2 d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 49$$

$$\frac{\int \left(b^2\left(F^{g(e+fx)}\right)^n + a^2F^{-g(e+fx)} + 2ab\right) d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 2009$$

$$\frac{a^2\log\left(\left(F^{g(e+fx)}\right)^n\right) + 2ab\left(F^{g(e+fx)}\right)^n + \frac{1}{2}b^2F^{2g(e+fx)}}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
((b^2*F^(2*g*(e + f*x)))/2 + 2*a*b*(F^(g*(e + f*x)))^n + a^2*Log[(F^(g*(e
+ f*x)))^n])/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n  Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{2a^2x\ln(F)fgn+\left(F^{g(fx+e)}\right)^{2n}b^2+4\left(F^{g(fx+e)}\right)^n ab}{2\ln(F)fgn}$
derivativedivides	$\frac{\frac{\left(F^{g(fx+e)}\right)^{2n}b^2}{2}+2\left(F^{g(fx+e)}\right)^n ab+a^2\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{gf\ln(F)n}$
default	$\frac{\frac{\left(F^{g(fx+e)}\right)^{2n}b^2}{2}+2\left(F^{g(fx+e)}\right)^n ab+a^2\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{gf\ln(F)n}$
norman	$a^2x+\frac{b^2\mathrm{e}^{2n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{2ngf\ln(F)}+\frac{2ab\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)}$
orering	$\frac{(2xngf\ln(F)+3)\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}{2\ln(F)fgn}-\frac{(3xngf\ln(F)+1)\left(a+b\left(F^{g(fx+e)}\right)^n\right)b\left(F^{g(fx+e)}\right)^n}{\ln(F)fgn}+\frac{x\left(2b^2\left(F^{g(fx+e)}\right)\right.}{}$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*a^2*x*ln(F)*f*g*n+b^2*((F^(g*(f*x+e)))^n)^2+4*(F^(g*(f*x+e)))^n*a*b
)/ln(F)/f/g/n
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 dx = \frac{2\,a^2 f g n x \log(F) + 4\,F^{fgnx+egn} a b + F^{2\,fgnx+2\,egn} b^2}{2\,f g n \log(F)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*f*g*n*x*log(F) + 4*F^(f*g*n*x + e*g*n)*a*b + F^(2*f*g*n*x + 2*e
*g*n)*b^2)/(f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 dx = \begin{cases} x(a+b)^2 & \text{for } F = 1 \wedge f = 0 \wedge g = 0 \wedge n = 0 \\ x\left(a + b\left(F^{eg}\right)^n\right)^2 & \text{for } f = 0 \\ x(a+b)^2 & \text{for } F = 1 \vee g = 0 \vee n = 0 \\ a^2 x + \frac{2ab\left(F^{eg+fgx}\right)^n}{fgn\log(F)} + \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2,x)
```

output
```
Piecewise((x*(a + b)**2, Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), (x*(a
 + b*(F**(e*g))**n)**2, Eq(f, 0)), (x*(a + b)**2, Eq(F, 1) | Eq(g, 0) | Eq
(n, 0)), (a**2*x + 2*a*b*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + b**2*(F**(
e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = a^2x + \frac{2\,F^{(fx+e)gn}ab}{fgn\log(F)} + \frac{F^{2\,(fx+e)gn}b^2}{2\,fgn\log(F)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

output
```
a^2*x + 2*F^((f*x + e)*g*n)*a*b/(f*g*n*log(F)) + 1/2*F^(2*(f*x + e)*g*n)*b
^2/(f*g*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = \frac{4\,F^{fgnx}F^{egn}ab + F^{2\,fgnx}F^{2\,egn}b^2 + 2\,a^2\log\left(|F|^{fgnx}|F|^{egn}\right)}{2\,fgn\log(F)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

output
```
1/2*(4*F^(f*g*n*x)*F^(e*g*n)*a*b + F^(2*f*g*n*x)*F^(2*e*g*n)*b^2 + 2*a^2*l
og(abs(F)^(f*g*n*x)*abs(F)^(e*g*n)))/(f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx = \frac{\frac{b^2\left(F^{e\,g+f\,g\,x}\right)^{2\,n}}{2} + 2\,a\,b\left(F^{e\,g+f\,g\,x}\right)^n}{f\,g\,n\,\ln(F)} + \frac{a^2\,\ln\left(F^{g\,(e+f\,x)}\right)}{f\,g\,\ln(F)}$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2,x)
```

output
```
((b^2*(F^(e*g + f*g*x))^(2*n))/2 + 2*a*b*(F^(e*g + f*g*x))^n)/(f*g*n*log(F
)) + (a^2*log(F^(g*(e + f*x))))/(f*g*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 dx = \frac{f^{2fgnx+2egn}b^2 + 4f^{fgnx+egn}ab + 2\log(f)\,a^2 fgnx}{2\log(f)\,fgn}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output
```
(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 4*f**(e*g*n + f*g*n*x)*a*b + 2*log(f)*a**
2*f*g*n*x)/(2*log(f)*f*g*n)
```

3.36 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 134

$$\begin{aligned}
&\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx \\
&= \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} \\
&\quad + \frac{b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} \operatorname{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} \\
&\quad + \frac{a^2\log(c+dx)}{d}
\end{aligned}$$

output

```
2*a*b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*Ei(f*g*n*(d*x+c)*ln(
F)/d)/d+b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*Ei(2*f
*g*n*(d*x+c)*ln(F)/d)/d+a^2*ln(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx$$
$$= \frac{2abF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n \text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + b^2F^{-\frac{2fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^{2n}\text{ExpIntegralEi}\left($$
$$d$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x),x]
```

output
```
((2*a*b*(F^(g*(e + f*x)))^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/F^(
(f*g*n*(c + d*x))/d) + (b^2*(F^(g*(e + f*x)))^(2*n)*ExpIntegralEi[(2*f*g*n
*(c + d*x)*Log[F])/d])/F^((2*f*g*n*(c + d*x))/d) + a^2*Log[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx$$
$$\downarrow 2614$$
$$\int \left(\frac{a^2}{c+dx} + \frac{2ab\left(F^{eg+fgx}\right)^n}{c+dx} + \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{c+dx}\right)dx$$
$$\downarrow 2009$$

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x),x]
```

output
```
(2*a*b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*ExpIntegr
alEi[(f*g*n*(c + d*x)*Log[F])/d])/d + (b^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*
(e + f*x))*(F^(e*g + f*g*x))^(2*n)*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F]
)/d])/d + (a^2*Log[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int\frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}{dx+c}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx$$
$$= \frac{F^{\frac{2(de-cf)gn}{d}} b^2 \mathrm{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right) + 2F^{\frac{(de-cf)gn}{d}} ab\,\mathrm{Ei}\left(\frac{(dfgnx+cfgn)\log(F)}{d}\right) + a^2\log(dx+c)}{d}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="fricas")
```

output
```
(F^(2*(d*e - c*f)*g*n/d)*b^2*Ei(2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 2*F^((
d*e - c*f)*g*n/d)*a*b*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d) + a^2*log(d*x + c
))/d
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}{c+dx}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c),x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**2/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{dx+c}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="maxima")
```

output
```
F^(2*e*g*n)*b^2*integrate(F^(2*f*g*n*x)/(d*x + c), x) + 2*F^(e*g*n)*a*b*in
tegrate(F^(f*g*n*x)/(d*x + c), x) + a^2*log(d*x + c)/d
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{dx+c}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^2/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}{c+d\,x}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x),x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx}\,dx = \frac{f^{2egn}\left(\int \frac{f^{2fgnx}}{dx+c}dx\right) b^2 d + 2f^{egn}\left(\int \frac{f^{fgnx}}{dx+c}dx\right) abd + \log(dx+c)\,a^2}{d}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

output

```
(f**(2*e*g*n)*int(f**(2*f*g*n*x)/(c + d*x),x)*b**2*d + 2*f**(e*g*n)*int(f*
*(f*g*n*x)/(c + d*x),x)*a*b*d + log(c + d*x)*a**2)/d
```

3.37 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 202

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$$
$$= -\frac{a^2}{d(c+dx)} - \frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}$$
$$+ \frac{2abfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$
$$+ \frac{2b^2fF^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} gn\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

output

```
-a^2/d/(d*x+c)-2*a*b*(F^(f*g*x+e*g))^n/d/(d*x+c)-b^2*(F^(f*g*x+e*g))^(2*n)
/d/(d*x+c)+2*a*b*f*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g*n*Ei(
f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2+2*b^2*f*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))
*(F^(f*g*x+e*g))^(2*n)*g*n*Ei(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.67

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$$
$$= \frac{-\frac{d\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{c+dx} + 2abfF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F) + 2b^2fF^{-\frac{2f}{}}}{d^2}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2,x]
```

output
```
(-((d*(a + b*(F^(g*(e + f*x)))^n)^2)/(c + d*x)) + (2*a*b*f*(F^(g*(e + f*x)
))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((f*g*n*(c +
d*x))/d) + (2*b^2*f*(F^(g*(e + f*x)))^(2*n)*g*n*ExpIntegralEi[(2*f*g*n*(c
+ d*x)*Log[F])/d]*Log[F])/F^((2*f*g*n*(c + d*x))/d))/d^2
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx$$
$$\downarrow 2614$$
$$\int \left(\frac{a^2}{(c+dx)^2} + \frac{2ab\left(F^{eg+fgx}\right)^n}{(c+dx)^2} + \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}\right)dx$$
$$\downarrow 2009$$

$$\frac{-\frac{a^2}{d(c+dx)} + \frac{2abfgn\log(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{2ab\left(F^{eg+fgx}\right)^n}{d(c+dx)} + \frac{2b^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^2} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}}{}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2,x]
```

output
```
-(a^2/(d*(c + d*x))) - (2*a*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) - (b^2*(F
^(e*g + f*g*x))^(2*n))/(d*(c + d*x)) + (2*a*b*f*F^((e - (c*f)/d)*g*n - g*n
*(e + f*x))*(F^(e*g + f*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])
/d]*Log[F])/d^2 + (2*b^2*f*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e
*g + f*g*x))^(2*n)*g*n*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])
/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}{(dx+c)^2}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}dx =$$
$$-\frac{2\,F^{fgnx+egn}abd+F^{2\,fgnx+2\,egn}b^2d-2\left(b^2dfgnx+b^2cfgn\right)F^{\frac{2\,(de-cf)gn}{d}}\mathrm{Ei}\left(\frac{2\,(dfgnx+cfgn)\log(F)}{d}\right)\log(F)-}{d^3x+cd^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(2*F^(f*g*n*x + e*g*n)*a*b*d + F^(2*f*g*n*x + 2*e*g*n)*b^2*d - 2*(b^2*d*f
*g*n*x + b^2*c*f*g*n)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n)*l
og(F)/d)*log(F) - 2*(a*b*d*f*g*n*x + a*b*c*f*g*n)*F^((d*e - c*f)*g*n/d)*Ei
((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F) + a^2*d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}{(c+dx)^2}dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**2/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
F^(2*e*g*n)*b^2*integrate(F^(2*f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) + 2*
F^(e*g*n)*a*b*integrate(F^(f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) - a^2/(d
^2*x + c*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^2/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

output
```
(f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**3*f*g*n + 2*log(f)*c**2*d*f*g*
n*x + log(f)*c*d**2*f*g*n*x**2 - c**2*d - 2*c*d**2*x - d**3*x**2),x)*log(f
)*b**2*c**3*f*g*n + f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**3*f*g*n + 2
*log(f)*c**2*d*f*g*n*x + log(f)*c*d**2*f*g*n*x**2 - c**2*d - 2*c*d**2*x -
d**3*x**2),x)*log(f)*b**2*c**2*d*f*g*n*x - f**(2*e*g*n)*int(f**(2*f*g*n*x)
/(log(f)*c**3*f*g*n + 2*log(f)*c**2*d*f*g*n*x + log(f)*c*d**2*f*g*n*x**2 -
 c**2*d - 2*c*d**2*x - d**3*x**2),x)*b**2*c**2*d - f**(2*e*g*n)*int(f**(2*
f*g*n*x)/(log(f)*c**3*f*g*n + 2*log(f)*c**2*d*f*g*n*x + log(f)*c*d**2*f*g*
n*x**2 - c**2*d - 2*c*d**2*x - d**3*x**2),x)*b**2*c*d**2*x + 4*f**(e*g*n)*
int(f**(f*g*n*x)/(2*log(f)*c**3*f*g*n + 4*log(f)*c**2*d*f*g*n*x + 2*log(f)
*c*d**2*f*g*n*x**2 - c**2*d - 2*c*d**2*x - d**3*x**2),x)*log(f)*a*b*c**3*f
*g*n + 4*f**(e*g*n)*int(f**(f*g*n*x)/(2*log(f)*c**3*f*g*n + 4*log(f)*c**2*
d*f*g*n*x + 2*log(f)*c*d**2*f*g*n*x**2 - c**2*d - 2*c*d**2*x - d**3*x**2),
x)*log(f)*a*b*c**2*d*f*g*n*x - 2*f**(e*g*n)*int(f**(f*g*n*x)/(2*log(f)*c**
3*f*g*n + 4*log(f)*c**2*d*f*g*n*x + 2*log(f)*c*d**2*f*g*n*x**2 - c**2*d -
2*c*d**2*x - d**3*x**2),x)*a*b*c**2*d - 2*f**(e*g*n)*int(f**(f*g*n*x)/(2*l
og(f)*c**3*f*g*n + 4*log(f)*c**2*d*f*g*n*x + 2*log(f)*c*d**2*f*g*n*x**2 -
c**2*d - 2*c*d**2*x - d**3*x**2),x)*a*b*c*d**2*x + a**2*x)/(c*(c + d*x))
```

3.38 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx$

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Maxima [F] . . . 339
Giac [F] . . . 339
Mupad [F(-1)] . . . 340
Reduce [F] . . . 340

Optimal result

Integrand size = 25, antiderivative size = 286

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = -\frac{a^2}{2d(c+dx)^2} - \frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2} - \frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{abf\left(F^{eg+fgx}\right)^n gn\log(F)}{d^2(c+dx)} - \frac{b^2 f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} + \frac{abf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3} + \frac{2b^2f^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3}$$

output

```
-1/2*a^2/d/(d*x+c)^2-a*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-1/2*b^2*(F^(f*g*x+e
*g))^(2*n)/d/(d*x+c)^2-a*b*f*(F^(f*g*x+e*g))^n*g*n*ln(F)/d^2/(d*x+c)-b^2*f
*(F^(f*g*x+e*g))^(2*n)*g*n*ln(F)/d^2/(d*x+c)+a*b*f^2*F^((e-c*f/d)*g*n-g*n*
(f*x+e))*(F^(f*g*x+e*g))^n*g^2*n^2*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3+2
*b^2*f^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g^2*n^2*E
i(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3
```

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx =$$

$$-\frac{a^2d^2-2abf^2F^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n g^2n^2(c+dx)^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)-4b^2f^2F^{-}}{}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3,x]
```

output
```
-1/2*(a^2*d^2 - (2*a*b*f^2*(F^(g*(e + f*x)))^n*g^2*n^2*(c + d*x)^2*ExpInte
gralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((f*g*n*(c + d*x))/d) - (4*
b^2*f^2*(F^(g*(e + f*x)))^(2*n)*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(2*f*g*n
*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((2*f*g*n*(c + d*x))/d) + 2*a*b*d*(F^(g*
(e + f*x)))^n*(d + f*g*n*(c + d*x)*Log[F]) + b^2*d*(F^(g*(e + f*x)))^(2*n)
*(d + 2*f*g*n*(c + d*x)*Log[F]))/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx$$

$$\downarrow 2614$$

$$\int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\left(F^{eg+fgx}\right)^n}{(c+dx)^3}+\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^3}-\frac{abfgn\log(F)\left(F^{eg+fgx}\right)^n}{d^2(c+dx)}-\frac{ab\left(F^{eg+fgx}\right)^n}{d(c+dx)^2}+\frac{2b^2f^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^{2n}F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^3}-\frac{b^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n}}{d^2(c+dx)}-\frac{b^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3,x]
```

output
```
-1/2*a^2/(d*(c + d*x)^2) - (a*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)^2) - (b^
2*(F^(e*g + f*g*x))^(2*n))/(2*d*(c + d*x)^2) - (a*b*f*(F^(e*g + f*g*x))^n*
g*n*Log[F])/(d^2*(c + d*x)) - (b^2*f*(F^(e*g + f*g*x))^(2*n)*g*n*Log[F])/(
d^2*(c + d*x)) + (a*b*f^2*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g +
f*g*x))^n*g^2*n^2*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/d^3
+ (2*b^2*f^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(
2*n)*g^2*n^2*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}{(dx+c)^3}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx$$
$$= \frac{4\left(b^2d^2f^2g^2n^2x^2+2\,b^2cdf^2g^2n^2x+b^2c^2f^2g^2n^2\right)F^{\frac{2\,(de-cf)gn}{d}}\,\mathrm{Ei}\left(\frac{2\,(dfgnx+cfgn)\log(F)}{d}\right)\log(F)^2+2\,(abd^2f^2g^2n$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(4*(b^2*d^2*f^2*g^2*n^2*x^2 + 2*b^2*c*d*f^2*g^2*n^2*x + b^2*c^2*f^2*g^
2*n^2)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)
^2 + 2*(a*b*d^2*f^2*g^2*n^2*x^2 + 2*a*b*c*d*f^2*g^2*n^2*x + a*b*c^2*f^2*g^
2*n^2)*F^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F)^2 -
 a^2*d^2 - (b^2*d^2 + 2*(b^2*d^2*f*g*n*x + b^2*c*d*f*g*n)*log(F))*F^(2*f*g
*n*x + 2*e*g*n) - 2*(a*b*d^2 + (a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*log(F))*F
^(f*g*n*x + e*g*n))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}{(c+dx)^3}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c)**3,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**2/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{(dx+c)^3}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
F^(2*e*g*n)*b^2*integrate(F^(2*f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x) + 2*F^(e*g*n)*a*b*integrate(F^(f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2
 + 3*c^2*d*x + c^3), x) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}{(dx+c)^3}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^2/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^3,x)
```

output

```
(2*f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**4*f*g*n + 3*log(f)*c**3*d*f*
g*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**3*f*g*n*x**3 - 2*c**3*
d - 6*c**2*d**2*x - 6*c*d**3*x**2 - 2*d**4*x**3),x)*log(f)*b**2*c**3*d*f*g
*n + 4*f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**4*f*g*n + 3*log(f)*c**3*
d*f*g*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**3*f*g*n*x**3 - 2*c
**3*d - 6*c**2*d**2*x - 6*c*d**3*x**2 - 2*d**4*x**3),x)*log(f)*b**2*c**2*d
**2*f*g*n*x + 2*f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**4*f*g*n + 3*log
(f)*c**3*d*f*g*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**3*f*g*n*x
**3 - 2*c**3*d - 6*c**2*d**2*x - 6*c*d**3*x**2 - 2*d**4*x**3),x)*log(f)*b*
*2*c*d**3*f*g*n*x**2 - 4*f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**4*f*g*
n + 3*log(f)*c**3*d*f*g*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**
3*f*g*n*x**3 - 2*c**3*d - 6*c**2*d**2*x - 6*c*d**3*x**2 - 2*d**4*x**3),x)*
b**2*c**2*d**2 - 8*f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**4*f*g*n + 3*
log(f)*c**3*d*f*g*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**3*f*g*
n*x**3 - 2*c**3*d - 6*c**2*d**2*x - 6*c*d**3*x**2 - 2*d**4*x**3),x)*b**2*c
*d**3*x - 4*f**(2*e*g*n)*int(f**(2*f*g*n*x)/(log(f)*c**4*f*g*n + 3*log(f)*
c**3*d*f*g*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**3*f*g*n*x**3
- 2*c**3*d - 6*c**2*d**2*x - 6*c*d**3*x**2 - 2*d**4*x**3),x)*b**2*d**4*x**
2 + 4*f**(e*g*n)*int(f**(f*g*n*x)/(log(f)*c**4*f*g*n + 3*log(f)*c**3*d*f*g
*n*x + 3*log(f)*c**2*d**2*f*g*n*x**2 + log(f)*c*d**3*f*g*n*x**3 - c**3*...
```

3.39 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx$

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Reduce [B] (verification not implemented) 352

Optimal result

Integrand size = 25, antiderivative size = 496

$$
\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx \\
&= \frac{a^3(c+dx)^4}{4d} - \frac{18a^2bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} - \frac{9ab^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)} - \frac{2b^3d^3\left(F^{eg+fgx}\right)^{3n}}{27f^4g^4n^4\log^4(F)} \\
&\quad + \frac{18a^2bd^2\left(F^{eg+fgx}\right)^n(c+dx)}{f^3g^3n^3\log^3(F)} + \frac{9ab^2d^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{4f^3g^3n^3\log^3(F)} \\
&\quad + \frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}(c+dx)}{9f^3g^3n^3\log^3(F)} - \frac{9a^2bd\left(F^{eg+fgx}\right)^n(c+dx)^2}{f^2g^2n^2\log^2(F)} \\
&\quad - \frac{9ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{4f^2g^2n^2\log^2(F)} - \frac{b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3f^2g^2n^2\log^2(F)} \\
&\quad + \frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)^3}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^3}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^3}{3fgn\log(F)}
\end{aligned}
$$

output

```
1/4*a^3*(d*x+c)^4/d-18*a^2*b*d^3*(F^(f*g*x+e*g))^n/f^4/g^4/n^4/ln(F)^4-9/8
*a*b^2*d^3*(F^(f*g*x+e*g))^(2*n)/f^4/g^4/n^4/ln(F)^4-2/27*b^3*d^3*(F^(f*g*
x+e*g))^(3*n)/f^4/g^4/n^4/ln(F)^4+18*a^2*b*d^2*(F^(f*g*x+e*g))^n*(d*x+c)/f
^3/g^3/n^3/ln(F)^3+9/4*a*b^2*d^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^3/g^3/n^3
/ln(F)^3+2/9*b^3*d^2*(F^(f*g*x+e*g))^(3*n)*(d*x+c)/f^3/g^3/n^3/ln(F)^3-9*a
^2*b*d*(F^(f*g*x+e*g))^n*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2-9/4*a*b^2*d*(F^(f*g
*x+e*g))^(2*n)*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2-1/3*b^3*d*(F^(f*g*x+e*g))^(3*
n)*(d*x+c)^2/f^2/g^2/n^2/ln(F)^2+3*a^2*b*(F^(f*g*x+e*g))^n*(d*x+c)^3/f/g/n
/ln(F)+3/2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^3/f/g/n/ln(F)+1/3*b^3*(F^(f
*g*x+e*g))^(3*n)*(d*x+c)^3/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 1.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.69

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = a^3c^3x + \frac{3}{2}a^3c^2dx^2 + a^3cd^2x^3 + \frac{1}{4}a^3d^3x^4$$
$$+\frac{3a^2b\left(F^{g(e+fx)}\right)^n\left(-6d^3+6d^2fgn(c+dx)\log(F)-3df^2g^2n^2(c+dx)^2\log^2(F)+f^3g^3n^3(c+dx)^3\log^3\right.}{f^4g^4n^4\log^4(F)}$$
$$+\frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}\left(-3d^3+6d^2fgn(c+dx)\log(F)-6df^2g^2n^2(c+dx)^2\log^2(F)+4f^3g^3n^3(c+dx)^3\log\right.}{8f^4g^4n^4\log^4(F)}$$
$$+\frac{b^3\left(F^{g(e+fx)}\right)^{3n}\left(-2d^3+6d^2fgn(c+dx)\log(F)-9df^2g^2n^2(c+dx)^2\log^2(F)+9f^3g^3n^3(c+dx)^3\log^3\right.}{27f^4g^4n^4\log^4(F)}$$

input

```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^3,x]
```

output

```
a^3*c^3*x + (3*a^3*c^2*d*x^2)/2 + a^3*c*d^2*x^3 + (a^3*d^3*x^4)/4 + (3*a^2
*b*(F^(g*(e + f*x)))^n*(-6*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 3*d*f^2*g^
2*n^2*(c + d*x)^2*Log[F]^2 + f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(f^4*g^4*n
^4*Log[F]^4) + (3*a*b^2*(F^(g*(e + f*x)))^(2*n)*(-3*d^3 + 6*d^2*f*g*n*(c +
 d*x)*Log[F] - 6*d*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2 + 4*f^3*g^3*n^3*(c + d
*x)^3*Log[F]^3))/(8*f^4*g^4*n^4*Log[F]^4) + (b^3*(F^(g*(e + f*x)))^(3*n)*(
-2*d^3 + 6*d^2*f*g*n*(c + d*x)*Log[F] - 9*d*f^2*g^2*n^2*(c + d*x)^2*Log[F]
^2 + 9*f^3*g^3*n^3*(c + d*x)^3*Log[F]^3))/(27*f^4*g^4*n^4*Log[F]^4)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$

$$\downarrow 2614$$

$$\int \left(a^3(c+dx)^3 + 3a^2b(c+dx)^3 \left(F^{eg+fgx}\right)^n + 3ab^2(c+dx)^3 \left(F^{eg+fgx}\right)^{2n} + b^3(c+dx)^3 \left(F^{eg+fgx}\right)^{3n}\right) dx$$

$$\downarrow 2009$$

$$\frac{a^3(c+dx)^4}{4d} + \frac{18a^2bd^2(c+dx)\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} - \frac{9a^2bd(c+dx)^2\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} + \frac{3a^2b(c+dx)^3\left(F^{eg+fgx}\right)^n}{fgn\log(F)} - \frac{18a^2bd^3\left(F^{eg+fgx}\right)^n}{f^4g^4n^4\log^4(F)} + \frac{9ab^2d^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)} - \frac{9ab^2d(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} + \frac{3ab^2(c+dx)^3\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)} - \frac{9ab^2d^3\left(F^{eg+fgx}\right)^{2n}}{8f^4g^4n^4\log^4(F)} + \frac{2b^3d^2(c+dx)\left(F^{eg+fgx}\right)^{3n}}{9f^3g^3n^3\log^3(F)} - \frac{b^3d(c+dx)^2\left(F^{eg+fgx}\right)^{3n}}{3f^2g^2n^2\log^2(F)} + \frac{b^3(c+dx)^3\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)} - \frac{2b^3d^3\left(F^{eg+fgx}\right)^{3n}}{27f^4g^4n^4\log^4(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^3,x]
```

output
```
(a^3*(c + d*x)^4)/(4*d) - (18*a^2*b*d^3*(F^(e*g + f*g*x))^n)/(f^4*g^4*n^4*
Log[F]^4) - (9*a*b^2*d^3*(F^(e*g + f*g*x))^(2*n))/(8*f^4*g^4*n^4*Log[F]^4)
 - (2*b^3*d^3*(F^(e*g + f*g*x))^(3*n))/(27*f^4*g^4*n^4*Log[F]^4) + (18*a^2
*b*d^2*(F^(e*g + f*g*x))^n*(c + d*x))/(f^3*g^3*n^3*Log[F]^3) + (9*a*b^2*d^
2*(F^(e*g + f*g*x))^(2*n)*(c + d*x))/(4*f^3*g^3*n^3*Log[F]^3) + (2*b^3*d^2
*(F^(e*g + f*g*x))^(3*n)*(c + d*x))/(9*f^3*g^3*n^3*Log[F]^3) - (9*a^2*b*d*
(F^(e*g + f*g*x))^n*(c + d*x)^2)/(f^2*g^2*n^2*Log[F]^2) - (9*a*b^2*d*(F^(e
*g + f*g*x))^(2*n)*(c + d*x)^2)/(4*f^2*g^2*n^2*Log[F]^2) - (b^3*d*(F^(e*g
+ f*g*x))^(3*n)*(c + d*x)^2)/(3*f^2*g^2*n^2*Log[F]^2) + (3*a^2*b*(F^(e*g +
 f*g*x))^n*(c + d*x)^3)/(f*g*n*Log[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*
(c + d*x)^3)/(2*f*g*n*Log[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x)^3)/
(3*f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(478) = 956$.

Time = 2.68 (sec) , antiderivative size = 1097, normalized size of antiderivative = 2.21

method	result	size
parallelrisch	Expression too large to display	1097
orering	Expression too large to display	2381

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output

```
1/216*(-486*ln(F)^2*x^2*((F^(g*(f*x+e)))^n)^2*a*b^2*d^3*f^2*g^2*n^2-144*ln
(F)^2*x*((F^(g*(f*x+e)))^n)^3*b^3*c*d^2*f^2*g^2*n^2-1944*ln(F)^2*x^2*(F^(g
*(f*x+e)))^n*a^2*b*d^3*f^2*g^2*n^2-486*ln(F)^2*((F^(g*(f*x+e)))^n)^2*a*b^2
*c^2*d*f^2*g^2*n^2-16*((F^(g*(f*x+e)))^n)^3*b^3*d^3+324*a^3*d*c^2*x^2*n^4*
g^4*f^4*ln(F)^4+72*d^3*b^3*((F^(g*(f*x+e)))^n)^3*x^3*n^3*g^3*f^3*ln(F)^3+3
24*ln(F)^3*((F^(g*(f*x+e)))^n)^2*a*b^2*c^3*f^3*g^3*n^3+648*ln(F)^3*(F^(g*(
f*x+e)))^n*a^2*b*c^3*f^3*g^3*n^3-72*ln(F)^2*x^2*((F^(g*(f*x+e)))^n)^3*b^3*
d^3*f^2*g^2*n^2-72*ln(F)^2*((F^(g*(f*x+e)))^n)^3*b^3*c^2*d*f^2*g^2*n^2+48*
ln(F)*x*((F^(g*(f*x+e)))^n)^3*b^3*d^3*f*g*n+48*ln(F)*((F^(g*(f*x+e)))^n)^3
*b^3*c*d^2*f*g*n-1944*ln(F)^2*(F^(g*(f*x+e)))^n*a^2*b*c^2*d*f^2*g^2*n^2+48
6*ln(F)*x*((F^(g*(f*x+e)))^n)^2*a*b^2*d^3*f*g*n+3888*ln(F)*x*(F^(g*(f*x+e)
))^n*a^2*b*d^3*f*g*n+486*ln(F)*((F^(g*(f*x+e)))^n)^2*a*b^2*c*d^2*f*g*n+388
8*ln(F)*(F^(g*(f*x+e)))^n*a^2*b*c*d^2*f*g*n+324*a*b^2*d^3*((F^(g*(f*x+e)))
^n)^2*x^3*n^3*g^3*f^3*ln(F)^3+216*ln(F)^3*x^2*((F^(g*(f*x+e)))^n)^3*b^3*c*
d^2*f^3*g^3*n^3+648*a^2*b*d^3*(F^(g*(f*x+e)))^n*x^3*n^3*g^3*f^3*ln(F)^3+21
6*ln(F)^3*x*((F^(g*(f*x+e)))^n)^3*b^3*c^2*d*f^3*g^3*n^3+972*ln(F)^3*x^2*((
F^(g*(f*x+e)))^n)^2*a*b^2*c*d^2*f^3*g^3*n^3+1944*ln(F)^3*x^2*(F^(g*(f*x+e)
))^n*a^2*b*c*d^2*f^3*g^3*n^3+972*ln(F)^3*x*((F^(g*(f*x+e)))^n)^2*a*b^2*c^2
*d*f^3*g^3*n^3+1944*ln(F)^3*x*(F^(g*(f*x+e)))^n*a^2*b*c^2*d*f^3*g^3*n^3-97
2*ln(F)^2*x*((F^(g*(f*x+e)))^n)^2*a*b^2*c*d^2*f^2*g^2*n^2-3888*ln(F)^2*...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 708, normalized size of antiderivative = 1.43

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx$$
$$= \frac{54\left(a^3d^3f^4g^4n^4x^4 + 4\,a^3cd^2f^4g^4n^4x^3 + 6\,a^3c^2df^4g^4n^4x^2 + 4\,a^3c^3f^4g^4n^4x\right)\log(F)^4 - 8\left(2\,b^3d^3 - 9\left(b^3d^3f\right.\right.}{}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="fricas")
```

output

```
1/216*(54*(a^3*d^3*f^4*g^4*n^4*x^4 + 4*a^3*c*d^2*f^4*g^4*n^4*x^3 + 6*a^3*c
^2*d*f^4*g^4*n^4*x^2 + 4*a^3*c^3*f^4*g^4*n^4*x)*log(F)^4 - 8*(2*b^3*d^3 -
9*(b^3*d^3*f^3*g^3*n^3*x^3 + 3*b^3*c*d^2*f^3*g^3*n^3*x^2 + 3*b^3*c^2*d*f^3
*g^3*n^3*x + b^3*c^3*f^3*g^3*n^3)*log(F)^3 + 9*(b^3*d^3*f^2*g^2*n^2*x^2 +
2*b^3*c*d^2*f^2*g^2*n^2*x + b^3*c^2*d*f^2*g^2*n^2)*log(F)^2 - 6*(b^3*d^3*f
*g*n*x + b^3*c*d^2*f*g*n)*log(F))*F^(3*f*g*n*x + 3*e*g*n) - 81*(3*a*b^2*d^
3 - 4*(a*b^2*d^3*f^3*g^3*n^3*x^3 + 3*a*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*a*b^2
*c^2*d*f^3*g^3*n^3*x + a*b^2*c^3*f^3*g^3*n^3)*log(F)^3 + 6*(a*b^2*d^3*f^2*
g^2*n^2*x^2 + 2*a*b^2*c*d^2*f^2*g^2*n^2*x + a*b^2*c^2*d*f^2*g^2*n^2)*log(F
)^2 - 6*(a*b^2*d^3*f*g*n*x + a*b^2*c*d^2*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e
*g*n) - 648*(6*a^2*b*d^3 - (a^2*b*d^3*f^3*g^3*n^3*x^3 + 3*a^2*b*c*d^2*f^3*
g^3*n^3*x^2 + 3*a^2*b*c^2*d*f^3*g^3*n^3*x + a^2*b*c^3*f^3*g^3*n^3)*log(F)^
3 + 3*(a^2*b*d^3*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d^2*f^2*g^2*n^2*x + a^2*b*c^2
*d*f^2*g^2*n^2)*log(F)^2 - 6*(a^2*b*d^3*f*g*n*x + a^2*b*c*d^2*f*g*n)*log(F
))*F^(f*g*n*x + e*g*n))/(f^4*g^4*n^4*log(F)^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(498) = 996$.

Time = 15.43 (sec) , antiderivative size = 1324, normalized size of antiderivative = 2.67

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = \text{Too large to display}$$

input

```
integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c)**3,x)
```

output

```
Piecewise(((a + b)**3*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/
4), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**3*
(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(f, 0)), ((a + b
)**3*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(F, 1) | Eq
(g, 0) | Eq(n, 0)), (a**3*c**3*x + 3*a**3*c**2*d*x**2/2 + a**3*c*d**2*x**3
 + a**3*d**3*x**4/4 + 3*a**2*b*c**3*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) +
 9*a**2*b*c**2*d*x*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 9*a**2*b*c**2*d*
(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)**2) + 9*a**2*b*c*d**2*x**2*(F
**(e*g + f*g*x))**n/(f*g*n*log(F)) - 18*a**2*b*c*d**2*x*(F**(e*g + f*g*x))
**n/(f**2*g**2*n**2*log(F)**2) + 18*a**2*b*c*d**2*(F**(e*g + f*g*x))**n/(f
**3*g**3*n**3*log(F)**3) + 3*a**2*b*d**3*x**3*(F**(e*g + f*g*x))**n/(f*g*n
*log(F)) - 9*a**2*b*d**3*x**2*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)
**2) + 18*a**2*b*d**3*x*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) -
 18*a**2*b*d**3*(F**(e*g + f*g*x))**n/(f**4*g**4*n**4*log(F)**4) + 3*a*b**
2*c**3*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + 9*a*b**2*c**2*d*x*(F**
(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 9*a*b**2*c**2*d*(F**(e*g + f*g*x)
)**(2*n)/(4*f**2*g**2*n**2*log(F)**2) + 9*a*b**2*c*d**2*x**2*(F**(e*g + f*
g*x))**(2*n)/(2*f*g*n*log(F)) - 9*a*b**2*c*d**2*x*(F**(e*g + f*g*x))**(2*n
)/(2*f**2*g**2*n**2*log(F)**2) + 9*a*b**2*c*d**2*(F**(e*g + f*g*x))**(2*n)
/(4*f**3*g**3*n**3*log(F)**3) + 3*a*b**2*d**3*x**3*(F**(e*g + f*g*x))**...
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 854, normalized size of antiderivative = 1.72

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = \text{Too large to display}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="maxima")
```

output

```
1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*x + 3*F^(f*g
*n*x + e*g*n)*a^2*b*c^3/(f*g*n*log(F)) + 3/2*F^(2*f*g*n*x + 2*e*g*n)*a*b^2
*c^3/(f*g*n*log(F)) + 1/3*F^(3*f*g*n*x + 3*e*g*n)*b^3*c^3/(f*g*n*log(F)) +
 9*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f*g*n*x)*a^2*b*c^2*d/(f^2*g^2
*n^2*log(F)^2) + 9/4*(2*F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g
*n*x)*a*b^2*c^2*d/(f^2*g^2*n^2*log(F)^2) + 1/3*(3*F^(3*e*g*n)*f*g*n*x*log(
F) - F^(3*e*g*n))*F^(3*f*g*n*x)*b^3*c^2*d/(f^2*g^2*n^2*log(F)^2) + 9*(F^(e
*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))
*F^(f*g*n*x)*a^2*b*c*d^2/(f^3*g^3*n^3*log(F)^3) + 9/4*(2*F^(2*e*g*n)*f^2*g
^2*n^2*x^2*log(F)^2 - 2*F^(2*e*g*n)*f*g*n*x*log(F) + F^(2*e*g*n))*F^(2*f*g
*n*x)*a*b^2*c*d^2/(f^3*g^3*n^3*log(F)^3) + 1/9*(9*F^(3*e*g*n)*f^2*g^2*n^2*
x^2*log(F)^2 - 6*F^(3*e*g*n)*f*g*n*x*log(F) + 2*F^(3*e*g*n))*F^(3*f*g*n*x)
*b^3*c*d^2/(f^3*g^3*n^3*log(F)^3) + 3*(F^(e*g*n)*f^3*g^3*n^3*x^3*log(F)^3
- 3*F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(e*g*n)*f*g*n*x*log(F) - 6*F^
(e*g*n))*F^(f*g*n*x)*a^2*b*d^3/(f^4*g^4*n^4*log(F)^4) + 3/8*(4*F^(2*e*g*n)
*f^3*g^3*n^3*x^3*log(F)^3 - 6*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(
2*e*g*n)*f*g*n*x*log(F) - 3*F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*d^3/(f^4*g^4*
n^4*log(F)^4) + 1/27*(9*F^(3*e*g*n)*f^3*g^3*n^3*x^3*log(F)^3 - 9*F^(3*e*g*
n)*f^2*g^2*n^2*x^2*log(F)^2 + 6*F^(3*e*g*n)*f*g*n*x*log(F) - 2*F^(3*e*g*n)
)*F^(3*f*g*n*x)*b^3*d^3/(f^4*g^4*n^4*log(F)^4)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 18707, normalized size of antiderivative = 37.72

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = \text{Too large to display}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x, algorithm="giac")
```

output

```
1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*x - 1/27*(((
27*pi^2*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))*sgn(F) - 27*pi^2*b^3*d^3*f^3*g
^3*n^3*x^3*log(abs(F)) + 18*b^3*d^3*f^3*g^3*n^3*x^3*log(abs(F))^3 + 81*pi^
2*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))*sgn(F) - 81*pi^2*b^3*c*d^2*f^3*g^3
*n^3*x^2*log(abs(F)) + 54*b^3*c*d^2*f^3*g^3*n^3*x^2*log(abs(F))^3 + 81*pi^
2*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))*sgn(F) - 81*pi^2*b^3*c^2*d*f^3*g^3*n
^3*x*log(abs(F)) + 54*b^3*c^2*d*f^3*g^3*n^3*x*log(abs(F))^3 + 27*pi^2*b^3*
c^3*f^3*g^3*n^3*log(abs(F))*sgn(F) - 27*pi^2*b^3*c^3*f^3*g^3*n^3*log(abs(F
)) + 18*b^3*c^3*f^3*g^3*n^3*log(abs(F))^3 - 9*pi^2*b^3*d^3*f^2*g^2*n^2*x^2
*sgn(F) + 9*pi^2*b^3*d^3*f^2*g^2*n^2*x^2 - 18*b^3*d^3*f^2*g^2*n^2*x^2*log(
abs(F))^2 - 18*pi^2*b^3*c*d^2*f^2*g^2*n^2*x*sgn(F) + 18*pi^2*b^3*c*d^2*f^2
*g^2*n^2*x - 36*b^3*c*d^2*f^2*g^2*n^2*x*log(abs(F))^2 - 9*pi^2*b^3*c^2*d*f
^2*g^2*n^2*sgn(F) + 9*pi^2*b^3*c^2*d*f^2*g^2*n^2 - 18*b^3*c^2*d*f^2*g^2*n^
2*log(abs(F))^2 + 12*b^3*d^3*f*g*n*x*log(abs(F)) + 12*b^3*c*d^2*f*g*n*log(
abs(F)) - 4*b^3*d^3)*(pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*log(abs
(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 - 2*f^
4*g^4*n^4*log(abs(F))^4)/((pi^4*f^4*g^4*n^4*sgn(F) - 6*pi^2*f^4*g^4*n^4*lo
g(abs(F))^2*sgn(F) - pi^4*f^4*g^4*n^4 + 6*pi^2*f^4*g^4*n^4*log(abs(F))^2 -
 2*f^4*g^4*n^4*log(abs(F))^4)^2 + 16*(pi^3*f^4*g^4*n^4*log(abs(F))*sgn(F)
- pi*f^4*g^4*n^4*log(abs(F))^3*sgn(F) - pi^3*f^4*g^4*n^4*log(abs(F)) + ...
```

Mupad [B] (verification not implemented)

Time = 23.85 (sec) , antiderivative size = 652, normalized size of antiderivative = 1.31

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = a^3 c^3 x$$
$$-\left(F^{f g x} F^{e g}\right)^n \left(\frac{3a^2 b\left(-c^3 f^3 g^3 n^3 \ln(F)^3 + 3c^2 d f^2 g^2 n^2 \ln(F)^2 - 6c d^2 f g n \ln(F) + 6d^3\right)}{f^4 g^4 n^4 \ln(F)^4} - \frac{3a^2 b d^3 x^3}{f g n \ln(F)} - \frac{9a^2 b d x\left(c^2 f^2 g^2 n^2 \ln(F)^2 - 2c d f g n \ln(F) + 2d^2\right)}{f^3 g^3 n^3 \ln(F)^3} + \frac{9a^2 b d^2 x^2 (d - c f g n \ln(F))}{f^2 g^2 n^2 \ln(F)^2}\right)$$
$$-\left(F^{f g x} F^{e g}\right)^{2n} \left(\frac{3a b^2\left(-4c^3 f^3 g^3 n^3 \ln(F)^3 + 6c^2 d f^2 g^2 n^2 \ln(F)^2 - 6c d^2 f g n \ln(F) + 3d^3\right)}{8 f^4 g^4 n^4 \ln(F)^4} - \frac{3a b^2 d^3 x^3}{2 f g n \ln(F)} - \frac{9a b^2 d x\left(2c^2 f^2 g^2 n^2 \ln(F)^2 - 2c d f g n \ln(F) + d^2\right)}{4 f^3 g^3 n^3 \ln(F)^3} + \frac{9a b^2 d^2 x^2 (d - 2c f g n \ln(F))}{4 f^2 g^2 n^2 \ln(F)^2}\right)$$
$$-\left(F^{f g x} F^{e g}\right)^{3n} \left(\frac{b^3\left(-9c^3 f^3 g^3 n^3 \ln(F)^3 + 9c^2 d f^2 g^2 n^2 \ln(F)^2 - 6c d^2 f g n \ln(F) + 2d^3\right)}{27 f^4 g^4 n^4 \ln(F)^4} - \frac{b^3 d^3 x^3}{3 f g n \ln(F)} - \frac{b^3 d x\left(9c^2 f^2 g^2 n^2 \ln(F)^2 - 6c d f g n \ln(F) + 2d^2\right)}{9 f^3 g^3 n^3 \ln(F)^3} + \frac{b^3 d^2 x^2 (d - 3c f g n \ln(F))}{3 f^2 g^2 n^2 \ln(F)^2}\right) + \frac{a^3 d^3 x^4}{4} + \frac{3a^3 c^2 d x^2}{2} + a^3 c d^2 x^3$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^3,x)
```

output

```
a^3*c^3*x - (F^(f*g*x)*F^(e*g))^n*((3*a^2*b*(6*d^3 - c^3*f^3*g^3*n^3*log(F
)^3 - 6*c*d^2*f*g*n*log(F) + 3*c^2*d*f^2*g^2*n^2*log(F)^2))/(f^4*g^4*n^4*l
og(F)^4) - (3*a^2*b*d^3*x^3)/(f*g*n*log(F)) - (9*a^2*b*d*x*(2*d^2 + c^2*f^
2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/(f^3*g^3*n^3*log(F)^3) + (9*a^2*
b*d^2*x^2*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) - (F^(f*g*x)*F^(e*
g))^(2*n)*((3*a*b^2*(3*d^3 - 4*c^3*f^3*g^3*n^3*log(F)^3 - 6*c*d^2*f*g*n*lo
g(F) + 6*c^2*d*f^2*g^2*n^2*log(F)^2))/(8*f^4*g^4*n^4*log(F)^4) - (3*a*b^2*
d^3*x^3)/(2*f*g*n*log(F)) - (9*a*b^2*d*x*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2
 - 2*c*d*f*g*n*log(F)))/(4*f^3*g^3*n^3*log(F)^3) + (9*a*b^2*d^2*x^2*(d - 2
*c*f*g*n*log(F)))/(4*f^2*g^2*n^2*log(F)^2)) - (F^(f*g*x)*F^(e*g))^(3*n)*((
b^3*(2*d^3 - 9*c^3*f^3*g^3*n^3*log(F)^3 - 6*c*d^2*f*g*n*log(F) + 9*c^2*d*f
^2*g^2*n^2*log(F)^2))/(27*f^4*g^4*n^4*log(F)^4) - (b^3*d^3*x^3)/(3*f*g*n*l
og(F)) - (b^3*d*x*(2*d^2 + 9*c^2*f^2*g^2*n^2*log(F)^2 - 6*c*d*f*g*n*log(F)
))/(9*f^3*g^3*n^3*log(F)^3) + (b^3*d^2*x^2*(d - 3*c*f*g*n*log(F)))/(3*f^2*
g^2*n^2*log(F)^2)) + (a^3*d^3*x^4)/4 + (3*a^3*c^2*d*x^2)/2 + a^3*c*d^2*x^3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1126, normalized size of antiderivative = 2.27

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^3\,dx = \text{Too large to display}$$

input

```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^3,x)
```

output

```
(72*f**(3*e*g*n + 3*f*g*n*x)*log(f)**3*b**3*c**3*f**3*g**3*n**3 + 216*f**(
3*e*g*n + 3*f*g*n*x)*log(f)**3*b**3*c**2*d*f**3*g**3*n**3*x + 216*f**(3*e*
g*n + 3*f*g*n*x)*log(f)**3*b**3*c*d**2*f**3*g**3*n**3*x**2 + 72*f**(3*e*g*
n + 3*f*g*n*x)*log(f)**3*b**3*d**3*f**3*g**3*n**3*x**3 - 72*f**(3*e*g*n +
3*f*g*n*x)*log(f)**2*b**3*c**2*d*f**2*g**2*n**2 - 144*f**(3*e*g*n + 3*f*g*
n*x)*log(f)**2*b**3*c*d**2*f**2*g**2*n**2*x - 72*f**(3*e*g*n + 3*f*g*n*x)*
log(f)**2*b**3*d**3*f**2*g**2*n**2*x**2 + 48*f**(3*e*g*n + 3*f*g*n*x)*log(
f)*b**3*c*d**2*f*g*n + 48*f**(3*e*g*n + 3*f*g*n*x)*log(f)*b**3*d**3*f*g*n*
x - 16*f**(3*e*g*n + 3*f*g*n*x)*b**3*d**3 + 324*f**(2*e*g*n + 2*f*g*n*x)*l
og(f)**3*a*b**2*c**3*f**3*g**3*n**3 + 972*f**(2*e*g*n + 2*f*g*n*x)*log(f)*
*3*a*b**2*c**2*d*f**3*g**3*n**3*x + 972*f**(2*e*g*n + 2*f*g*n*x)*log(f)**3
*a*b**2*c*d**2*f**3*g**3*n**3*x**2 + 324*f**(2*e*g*n + 2*f*g*n*x)*log(f)**
3*a*b**2*d**3*f**3*g**3*n**3*x**3 - 486*f**(2*e*g*n + 2*f*g*n*x)*log(f)**2
*a*b**2*c**2*d*f**2*g**2*n**2 - 972*f**(2*e*g*n + 2*f*g*n*x)*log(f)**2*a*b
**2*c*d**2*f**2*g**2*n**2*x - 486*f**(2*e*g*n + 2*f*g*n*x)*log(f)**2*a*b**
2*d**3*f**2*g**2*n**2*x**2 + 486*f**(2*e*g*n + 2*f*g*n*x)*log(f)*a*b**2*c*
d**2*f*g*n + 486*f**(2*e*g*n + 2*f*g*n*x)*log(f)*a*b**2*d**3*f*g*n*x - 243
*f**(2*e*g*n + 2*f*g*n*x)*a*b**2*d**3 + 648*f**(e*g*n + f*g*n*x)*log(f)**3
*a**2*b*c**3*f**3*g**3*n**3 + 1944*f**(e*g*n + f*g*n*x)*log(f)**3*a**2*b*c
**2*d*f**3*g**3*n**3*x + 1944*f**(e*g*n + f*g*n*x)*log(f)**3*a**2*b*c*d...
```

3.40 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 366

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx \\
&= \frac{a^3(c+dx)^3}{3d} + \frac{6a^2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)} + \frac{3ab^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)} + \frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}}{27f^3g^3n^3\log^3(F)} \\
&\quad - \frac{6a^2bd\left(F^{eg+fgx}\right)^n(c+dx)}{f^2g^2n^2\log^2(F)} - \frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2f^2g^2n^2\log^2(F)} \\
&\quad - \frac{2b^3d\left(F^{eg+fgx}\right)^{3n}(c+dx)}{9f^2g^2n^2\log^2(F)} + \frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)^2}{fgn\log(F)} \\
&\quad + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)^2}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)^2}{3fgn\log(F)}
\end{aligned}$$

output

```
1/3*a^3*(d*x+c)^3/d+6*a^2*b*d^2*(F^(f*g*x+e*g))^n/f^3/g^3/n^3/ln(F)^3+3/4*
a*b^2*d^2*(F^(f*g*x+e*g))^(2*n)/f^3/g^3/n^3/ln(F)^3+2/27*b^3*d^2*(F^(f*g*x
+e*g))^(3*n)/f^3/g^3/n^3/ln(F)^3-6*a^2*b*d*(F^(f*g*x+e*g))^n*(d*x+c)/f^2/g
^2/n^2/ln(F)^2-3/2*a*b^2*d*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f^2/g^2/n^2/ln(F)
^2-2/9*b^3*d*(F^(f*g*x+e*g))^(3*n)*(d*x+c)/f^2/g^2/n^2/ln(F)^2+3*a^2*b*(F^
(f*g*x+e*g))^n*(d*x+c)^2/f/g/n/ln(F)+3/2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+
c)^2/f/g/n/ln(F)+1/3*b^3*(F^(f*g*x+e*g))^(3*n)*(d*x+c)^2/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.68

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx \\
&= a^3c^2x + a^3cdx^2 + \frac{1}{3}a^3d^2x^3 \\
&\quad + \frac{3a^2b\left(F^{g(e+fx)}\right)^n\left(2d^2 - 2dfgn(c+dx)\log(F) + f^2g^2n^2(c+dx)^2\log^2(F)\right)}{f^3g^3n^3\log^3(F)} \\
&\quad + \frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}\left(d^2 - 2dfgn(c+dx)\log(F) + 2f^2g^2n^2(c+dx)^2\log^2(F)\right)}{4f^3g^3n^3\log^3(F)} \\
&\quad + \frac{b^3\left(F^{g(e+fx)}\right)^{3n}\left(2d^2 - 6dfgn(c+dx)\log(F) + 9f^2g^2n^2(c+dx)^2\log^2(F)\right)}{27f^3g^3n^3\log^3(F)}
\end{aligned}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2,x]
```

output
```
a^3*c^2*x + a^3*c*d*x^2 + (a^3*d^2*x^3)/3 + (3*a^2*b*(F^(g*(e + f*x)))^n*(
2*d^2 - 2*d*f*g*n*(c + d*x)*Log[F] + f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(f
^3*g^3*n^3*Log[F]^3) + (3*a*b^2*(F^(g*(e + f*x)))^(2*n)*(d^2 - 2*d*f*g*n*(
c + d*x)*Log[F] + 2*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(4*f^3*g^3*n^3*Log[
F]^3) + (b^3*(F^(g*(e + f*x)))^(3*n)*(2*d^2 - 6*d*f*g*n*(c + d*x)*Log[F] +
 9*f^2*g^2*n^2*(c + d*x)^2*Log[F]^2))/(27*f^3*g^3*n^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$

$$\downarrow \text{2614}$$

$$\int \left(a^3(c+dx)^2+3a^2b(c+dx)^2\left(F^{eg+fgx}\right)^n+3ab^2(c+dx)^2\left(F^{eg+fgx}\right)^{2n}+b^3(c+dx)^2\left(F^{eg+fgx}\right)^{3n}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^3}{3d}-\frac{6a^2bd(c+dx)\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{3a^2b(c+dx)^2\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{6a^2bd^2\left(F^{eg+fgx}\right)^n}{f^3g^3n^3\log^3(F)}-$$
$$\frac{3ab^2d(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2f^2g^2n^2\log^2(F)}+\frac{3ab^2(c+dx)^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}+\frac{3ab^2d^2\left(F^{eg+fgx}\right)^{2n}}{4f^3g^3n^3\log^3(F)}-$$
$$\frac{2b^3d(c+dx)\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)}+\frac{b^3(c+dx)^2\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)}+\frac{2b^3d^2\left(F^{eg+fgx}\right)^{3n}}{27f^3g^3n^3\log^3(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2,x]
```

output
```
(a^3*(c + d*x)^3)/(3*d) + (6*a^2*b*d^2*(F^(e*g + f*g*x))^n)/(f^3*g^3*n^3*L
og[F]^3) + (3*a*b^2*d^2*(F^(e*g + f*g*x))^(2*n))/(4*f^3*g^3*n^3*Log[F]^3)
+ (2*b^3*d^2*(F^(e*g + f*g*x))^(3*n))/(27*f^3*g^3*n^3*Log[F]^3) - (6*a^2*b
*d*(F^(e*g + f*g*x))^n*(c + d*x))/(f^2*g^2*n^2*Log[F]^2) - (3*a*b^2*d*(F^(
e*g + f*g*x))^(2*n)*(c + d*x))/(2*f^2*g^2*n^2*Log[F]^2) - (2*b^3*d*(F^(e*g
 + f*g*x))^(3*n)*(c + d*x))/(9*f^2*g^2*n^2*Log[F]^2) + (3*a^2*b*(F^(e*g +
f*g*x))^n*(c + d*x)^2)/(f*g*n*Log[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*(
c + d*x)^2)/(2*f*g*n*Log[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x)^2)/(
3*f*g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 1.19 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.69

method	result
parallelrisch	$\frac{36a^3d^2x^3n^3g^3f^3\ln(F)^3+108a^3dcx^2n^3g^3f^3\ln(F)^3+108a^3c^2xn^3g^3f^3\ln(F)^3+36d^2b^3\left(F^{g(fx+e)}\right)^{3n}x^2n^2g^2f^2\ln(F)^2+162$
orering	Expression too large to display

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/108*(36*a^3*d^2*x^3*n^3*g^3*f^3*ln(F)^3+108*a^3*d*c*x^2*n^3*g^3*f^3*ln(F
)^3+108*a^3*c^2*x*n^3*g^3*f^3*ln(F)^3+36*d^2*b^3*((F^(g*(f*x+e)))^n)^3*x^2
*n^2*g^2*f^2*ln(F)^2+162*a*b^2*d^2*((F^(g*(f*x+e)))^n)^2*x^2*n^2*g^2*f^2*l
n(F)^2+72*ln(F)^2*x*((F^(g*(f*x+e)))^n)^3*b^3*c*d*f^2*g^2*n^2+324*a^2*b*d^
2*(F^(g*(f*x+e)))^n*x^2*n^2*g^2*f^2*ln(F)^2+324*ln(F)^2*x*((F^(g*(f*x+e)))
^n)^2*a*b^2*c*d*f^2*g^2*n^2+36*ln(F)^2*((F^(g*(f*x+e)))^n)^3*b^3*c^2*f^2*g
^2*n^2+648*ln(F)^2*x*(F^(g*(f*x+e)))^n*a^2*b*c*d*f^2*g^2*n^2+162*ln(F)^2*(
(F^(g*(f*x+e)))^n)^2*a*b^2*c^2*f^2*g^2*n^2+324*ln(F)^2*(F^(g*(f*x+e)))^n*a
^2*b*c^2*f^2*g^2*n^2-24*ln(F)*x*((F^(g*(f*x+e)))^n)^3*b^3*d^2*f*g*n-162*ln
(F)*x*((F^(g*(f*x+e)))^n)^2*a*b^2*d^2*f*g*n-24*ln(F)*((F^(g*(f*x+e)))^n)^3
*b^3*c*d*f*g*n-648*ln(F)*x*(F^(g*(f*x+e)))^n*a^2*b*d^2*f*g*n-162*ln(F)*((F
^(g*(f*x+e)))^n)^2*a*b^2*c*d*f*g*n-648*ln(F)*(F^(g*(f*x+e)))^n*a^2*b*c*d*f
*g*n+8*((F^(g*(f*x+e)))^n)^3*b^3*d^2+81*((F^(g*(f*x+e)))^n)^2*a*b^2*d^2+64
8*(F^(g*(f*x+e)))^n*a^2*b*d^2)/n^3/g^3/f^3/ln(F)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.13

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$$
$$= \frac{36\left(a^3d^2f^3g^3n^3x^3+3\,a^3cdf^3g^3n^3x^2+3\,a^3c^2f^3g^3n^3x\right)\log(F)^3+4\left(2\,b^3d^2+9\left(b^3d^2f^2g^2n^2x^2+2\,b^3cdf^2g^2\right.\right.}{}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x, algorithm="fricas")
```

output

```
1/108*(36*(a^3*d^2*f^3*g^3*n^3*x^3 + 3*a^3*c*d*f^3*g^3*n^3*x^2 + 3*a^3*c^2
*f^3*g^3*n^3*x)*log(F)^3 + 4*(2*b^3*d^2 + 9*(b^3*d^2*f^2*g^2*n^2*x^2 + 2*b
^3*c*d*f^2*g^2*n^2*x + b^3*c^2*f^2*g^2*n^2)*log(F)^2 - 6*(b^3*d^2*f*g*n*x
+ b^3*c*d*f*g*n)*log(F))*F^(3*f*g*n*x + 3*e*g*n) + 81*(a*b^2*d^2 + 2*(a*b^
2*d^2*f^2*g^2*n^2*x^2 + 2*a*b^2*c*d*f^2*g^2*n^2*x + a*b^2*c^2*f^2*g^2*n^2)
*log(F)^2 - 2*(a*b^2*d^2*f*g*n*x + a*b^2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x +
 2*e*g*n) + 324*(2*a^2*b*d^2 + (a^2*b*d^2*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d*f^
2*g^2*n^2*x + a^2*b*c^2*f^2*g^2*n^2)*log(F)^2 - 2*(a^2*b*d^2*f*g*n*x + a^2
*b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(f^3*g^3*n^3*log(F)^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(362) = 724$.

Time = 5.57 (sec) , antiderivative size = 775, normalized size of antiderivative = 2.12

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx = \text{Too large to display}$$

input

```
integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c)**2,x)
```

output

```
Piecewise(((a + b)**3*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(F, 1) & Eq(f,
0) & Eq(g, 0) & Eq(n, 0)), ((a + b*(F**(e*g))**n)**3*(c**2*x + c*d*x**2 +
d**2*x**3/3), Eq(f, 0)), ((a + b)**3*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq
(F, 1) | Eq(g, 0) | Eq(n, 0)), (a**3*c**2*x + a**3*c*d*x**2 + a**3*d**2*x*
*3/3 + 3*a**2*b*c**2*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 6*a**2*b*c*d*x
*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) - 6*a**2*b*c*d*(F**(e*g + f*g*x))**n
/(f**2*g**2*n**2*log(F)**2) + 3*a**2*b*d**2*x**2*(F**(e*g + f*g*x))**n/(f*
g*n*log(F)) - 6*a**2*b*d**2*x*(F**(e*g + f*g*x))**n/(f**2*g**2*n**2*log(F)
**2) + 6*a**2*b*d**2*(F**(e*g + f*g*x))**n/(f**3*g**3*n**3*log(F)**3) + 3*
a*b**2*c**2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + 3*a*b**2*c*d*x*(F
**(e*g + f*g*x))**(2*n)/(f*g*n*log(F)) - 3*a*b**2*c*d*(F**(e*g + f*g*x))**
(2*n)/(2*f**2*g**2*n**2*log(F)**2) + 3*a*b**2*d**2*x**2*(F**(e*g + f*g*x))
**(2*n)/(2*f*g*n*log(F)) - 3*a*b**2*d**2*x*(F**(e*g + f*g*x))**(2*n)/(2*f*
*2*g**2*n**2*log(F)**2) + 3*a*b**2*d**2*(F**(e*g + f*g*x))**(2*n)/(4*f**3*
g**3*n**3*log(F)**3) + b**3*c**2*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)
) + 2*b**3*c*d*x*(F**(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)) - 2*b**3*c*d*(
F**(e*g + f*g*x))**(3*n)/(9*f**2*g**2*n**2*log(F)**2) + b**3*d**2*x**2*(F*
*(e*g + f*g*x))**(3*n)/(3*f*g*n*log(F)) - 2*b**3*d**2*x*(F**(e*g + f*g*x))
**(3*n)/(9*f**2*g**2*n**2*log(F)**2) + 2*b**3*d**2*(F**(e*g + f*g*x))**(3*
n)/(27*f**3*g**3*n**3*log(F)**3), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 523, normalized size of antiderivative = 1.43

$$\begin{aligned}
&\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx \\
&= \frac{1}{3}a^3d^2x^3 + a^3cdx^2 + a^3c^2x + \frac{3\,F^{fgnx+egn}a^2bc^2}{fgn\log(F)} + \frac{3\,F^{2\,fgnx+2\,egn}ab^2c^2}{2\,fgn\log(F)} \\
&\quad + \frac{F^{3\,fgnx+3\,egn}b^3c^2}{3\,fgn\log(F)} + \frac{6\,(F^{egn}fgnx\log(F) - F^{egn})F^{fgnx}a^2bcd}{f^2g^2n^2\log(F)^2} \\
&\quad + \frac{3\,(2\,F^{2\,egn}fgnx\log(F) - F^{2\,egn})F^{2\,fgnx}ab^2cd}{2\,f^2g^2n^2\log(F)^2} \\
&\quad + \frac{2\,(3\,F^{3\,egn}fgnx\log(F) - F^{3\,egn})F^{3\,fgnx}b^3cd}{9\,f^2g^2n^2\log(F)^2} \\
&\quad + \frac{3\left(F^{egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{egn}fgnx\log(F) + 2\,F^{egn}\right)F^{fgnx}a^2bd^2}{f^3g^3n^3\log(F)^3} \\
&\quad + \frac{3\left(2\,F^{2\,egn}f^2g^2n^2x^2\log(F)^2 - 2\,F^{2\,egn}fgnx\log(F) + F^{2\,egn}\right)F^{2\,fgnx}ab^2d^2}{4\,f^3g^3n^3\log(F)^3} \\
&\quad + \frac{\left(9\,F^{3\,egn}f^2g^2n^2x^2\log(F)^2 - 6\,F^{3\,egn}fgnx\log(F) + 2\,F^{3\,egn}\right)F^{3\,fgnx}b^3d^2}{27\,f^3g^3n^3\log(F)^3}
\end{aligned}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + a^3*c^2*x + 3*F^(f*g*n*x + e*g*n)*a^2*b*c^
2/(f*g*n*log(F)) + 3/2*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*c^2/(f*g*n*log(F)) +
1/3*F^(3*f*g*n*x + 3*e*g*n)*b^3*c^2/(f*g*n*log(F)) + 6*(F^(e*g*n)*f*g*n*x*
log(F) - F^(e*g*n))*F^(f*g*n*x)*a^2*b*c*d/(f^2*g^2*n^2*log(F)^2) + 3/2*(2*
F^(2*e*g*n)*f*g*n*x*log(F) - F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*c*d/(f^2*g^2
*n^2*log(F)^2) + 2/9*(3*F^(3*e*g*n)*f*g*n*x*log(F) - F^(3*e*g*n))*F^(3*f*g
*n*x)*b^3*c*d/(f^2*g^2*n^2*log(F)^2) + 3*(F^(e*g*n)*f^2*g^2*n^2*x^2*log(F)
^2 - 2*F^(e*g*n)*f*g*n*x*log(F) + 2*F^(e*g*n))*F^(f*g*n*x)*a^2*b*d^2/(f^3*
g^3*n^3*log(F)^3) + 3/4*(2*F^(2*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 2*F^(2*e
*g*n)*f*g*n*x*log(F) + F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*d^2/(f^3*g^3*n^3*l
og(F)^3) + 1/27*(9*F^(3*e*g*n)*f^2*g^2*n^2*x^2*log(F)^2 - 6*F^(3*e*g*n)*f*
g*n*x*log(F) + 2*F^(3*e*g*n))*F^(3*f*g*n*x)*b^3*d^2/(f^3*g^3*n^3*log(F)^3)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 8820, normalized size of antiderivative = 24.10

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx = \text{Too large to display}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x, algorithm="giac")
```

output

```
1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + a^3*c^2*x - 1/27*((6*(3*pi*b^3*d^2*f^2*g^2
*n^2*x^2*log(abs(F))*sgn(F) - 3*pi*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F)) + 6
*pi*b^3*c*d*f^2*g^2*n^2*x*log(abs(F))*sgn(F) - 6*pi*b^3*c*d*f^2*g^2*n^2*x*
log(abs(F)) + 3*pi*b^3*c^2*f^2*g^2*n^2*log(abs(F))*sgn(F) - 3*pi*b^3*c^2*f
^2*g^2*n^2*log(abs(F)) - pi*b^3*d^2*f*g*n*x*sgn(F) + pi*b^3*d^2*f*g*n*x -
pi*b^3*c*d*f*g*n*sgn(F) + pi*b^3*c*d*f*g*n)*(pi^3*f^3*g^3*n^3*sgn(F) - 3*p
i*f^3*g^3*n^3*log(abs(F))^2*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*l
og(abs(F))^2)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2*s
gn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f^3
*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n
^3*log(abs(F))^3)^2) - (9*pi^2*b^3*d^2*f^2*g^2*n^2*x^2*sgn(F) - 9*pi^2*b^3
*d^2*f^2*g^2*n^2*x^2 + 18*b^3*d^2*f^2*g^2*n^2*x^2*log(abs(F))^2 + 18*pi^2*
b^3*c*d*f^2*g^2*n^2*x*sgn(F) - 18*pi^2*b^3*c*d*f^2*g^2*n^2*x + 36*b^3*c*d*
f^2*g^2*n^2*x*log(abs(F))^2 + 9*pi^2*b^3*c^2*f^2*g^2*n^2*sgn(F) - 9*pi^2*b
^3*c^2*f^2*g^2*n^2 + 18*b^3*c^2*f^2*g^2*n^2*log(abs(F))^2 - 12*b^3*d^2*f*g
*n*x*log(abs(F)) - 12*b^3*c*d*f*g*n*log(abs(F)) + 4*b^3*d^2)*(3*pi^2*f^3*g
^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*g^3*n^3
*log(abs(F))^3)/((pi^3*f^3*g^3*n^3*sgn(F) - 3*pi*f^3*g^3*n^3*log(abs(F))^2
*sgn(F) - pi^3*f^3*g^3*n^3 + 3*pi*f^3*g^3*n^3*log(abs(F))^2)^2 + (3*pi^2*f
^3*g^3*n^3*log(abs(F))*sgn(F) - 3*pi^2*f^3*g^3*n^3*log(abs(F)) + 2*f^3*...
```

Mupad [B] (verification not implemented)

Time = 23.66 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.09

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$$

$$= \left(F^{f\,g\,x}\,F^{e\,g}\right)^{3\,n}\left(\frac{b^3\left(9\,c^2\,f^2\,g^2\,n^2\ln(F)^2 - 6\,c\,d\,f\,g\,n\,\ln(F) + 2\,d^2\right)}{27\,f^3\,g^3\,n^3\ln(F)^3} + \frac{b^3\,d^2\,x^2}{3\,f\,g\,n\,\ln(F)} - \frac{2\,b^3\,d\,x\,(d - 3\,c\,f\,g\,n\,\ln(F))}{9\,f^2\,g^2\,n^2\ln(F)^2}\right)$$

$$+ \left(F^{f\,g\,x}\,F^{e\,g}\right)^{n}\left(\frac{3\,a^2\,b\left(c^2\,f^2\,g^2\,n^2\ln(F)^2 - 2\,c\,d\,f\,g\,n\,\ln(F) + 2\,d^2\right)}{f^3\,g^3\,n^3\ln(F)^3} + \frac{3\,a^2\,b\,d^2\,x^2}{f\,g\,n\,\ln(F)} - \frac{6\,a^2\,b\,d\,x\,(d - c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\ln(F)^2}\right)$$

$$+ \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{3\,a\,b^2\left(2\,c^2\,f^2\,g^2\,n^2\ln(F)^2 - 2\,c\,d\,f\,g\,n\,\ln(F) + d^2\right)}{4\,f^3\,g^3\,n^3\ln(F)^3} + \frac{3\,a\,b^2\,d^2\,x^2}{2\,f\,g\,n\,\ln(F)} - \frac{3\,a\,b^2\,d\,x\,(d - 2\,c\,f\,g\,n\,\ln(F))}{2\,f^2\,g^2\,n^2\ln(F)^2}\right) + a^3\,c^2\,x + \frac{a^3\,d^2\,x^3}{3} + a^3\,c\,d\,x^2$$

input

```
int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2,x)
```

output

```
(F^(f*g*x)*F^(e*g))^(3*n)*((b^3*(2*d^2 + 9*c^2*f^2*g^2*n^2*log(F)^2 - 6*c*
d*f*g*n*log(F)))/(27*f^3*g^3*n^3*log(F)^3) + (b^3*d^2*x^2)/(3*f*g*n*log(F)
) - (2*b^3*d*x*(d - 3*c*f*g*n*log(F)))/(9*f^2*g^2*n^2*log(F)^2)) + (F^(f*g
*x)*F^(e*g))^n*((3*a^2*b*(2*d^2 + c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*l
og(F)))/(f^3*g^3*n^3*log(F)^3) + (3*a^2*b*d^2*x^2)/(f*g*n*log(F)) - (6*a^2
*b*d*x*(d - c*f*g*n*log(F)))/(f^2*g^2*n^2*log(F)^2)) + (F^(f*g*x)*F^(e*g))
^(2*n)*((3*a*b^2*(d^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 - 2*c*d*f*g*n*log(F)))/
(4*f^3*g^3*n^3*log(F)^3) + (3*a*b^2*d^2*x^2)/(2*f*g*n*log(F)) - (3*a*b^2*d
*x*(d - 2*c*f*g*n*log(F)))/(2*f^2*g^2*n^2*log(F)^2)) + a^3*c^2*x + (a^3*d^
2*x^3)/3 + a^3*c*d*x^2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.74

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2\,dx$$
$$= \frac{36f^{3fgnx+3egn}\log(f)^2 b^3c^2f^2g^2n^2 + 72f^{3fgnx+3egn}\log(f)^2 b^3cd\,f^2g^2n^2x + 36f^{3fgnx+3egn}\log(f)^2 b^3d^2f^2g^2n^2}{}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^2,x)
```

output
```
(36*f**(3*e*g*n + 3*f*g*n*x)*log(f)**2*b**3*c**2*f**2*g**2*n**2 + 72*f**(3
*e*g*n + 3*f*g*n*x)*log(f)**2*b**3*c*d*f**2*g**2*n**2*x + 36*f**(3*e*g*n +
 3*f*g*n*x)*log(f)**2*b**3*d**2*f**2*g**2*n**2*x**2 - 24*f**(3*e*g*n + 3*f
*g*n*x)*log(f)*b**3*c*d*f*g*n - 24*f**(3*e*g*n + 3*f*g*n*x)*log(f)*b**3*d*
*2*f*g*n*x + 8*f**(3*e*g*n + 3*f*g*n*x)*b**3*d**2 + 162*f**(2*e*g*n + 2*f*
g*n*x)*log(f)**2*a*b**2*c**2*f**2*g**2*n**2 + 324*f**(2*e*g*n + 2*f*g*n*x)
*log(f)**2*a*b**2*c*d*f**2*g**2*n**2*x + 162*f**(2*e*g*n + 2*f*g*n*x)*log(
f)**2*a*b**2*d**2*f**2*g**2*n**2*x**2 - 162*f**(2*e*g*n + 2*f*g*n*x)*log(f
)*a*b**2*c*d*f*g*n - 162*f**(2*e*g*n + 2*f*g*n*x)*log(f)*a*b**2*d**2*f*g*n
*x + 81*f**(2*e*g*n + 2*f*g*n*x)*a*b**2*d**2 + 324*f**(e*g*n + f*g*n*x)*lo
g(f)**2*a**2*b*c**2*f**2*g**2*n**2 + 648*f**(e*g*n + f*g*n*x)*log(f)**2*a*
*2*b*c*d*f**2*g**2*n**2*x + 324*f**(e*g*n + f*g*n*x)*log(f)**2*a**2*b*d**2
*f**2*g**2*n**2*x**2 - 648*f**(e*g*n + f*g*n*x)*log(f)*a**2*b*c*d*f*g*n -
648*f**(e*g*n + f*g*n*x)*log(f)*a**2*b*d**2*f*g*n*x + 648*f**(e*g*n + f*g*
n*x)*a**2*b*d**2 + 108*log(f)**3*a**3*c**2*f**3*g**3*n**3*x + 108*log(f)**
3*a**3*c*d*f**3*g**3*n**3*x**2 + 36*log(f)**3*a**3*d**2*f**3*g**3*n**3*x**
3)/(108*log(f)**3*f**3*g**3*n**3)
```

3.41 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 236

$$\begin{aligned}\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx = {} & \frac{a^3(c+dx)^2}{2d} - \frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)} - \frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)} \\ & - \frac{b^3d\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)} + \frac{3a^2b\left(F^{eg+fgx}\right)^n(c+dx)}{fgn\log(F)} \\ & + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}(c+dx)}{2fgn\log(F)} \\ & + \frac{b^3\left(F^{eg+fgx}\right)^{3n}(c+dx)}{3fgn\log(F)}\end{aligned}$$

output

```
1/2*a^3*(d*x+c)^2/d-3*a^2*b*d*(F^(f*g*x+e*g))^n/f^2/g^2/n^2/ln(F)^2-3/4*a*
b^2*d*(F^(f*g*x+e*g))^(2*n)/f^2/g^2/n^2/ln(F)^2-1/9*b^3*d*(F^(f*g*x+e*g))^
(3*n)/f^2/g^2/n^2/ln(F)^2+3*a^2*b*(F^(f*g*x+e*g))^n*(d*x+c)/f/g/n/ln(F)+3/
2*a*b^2*(F^(f*g*x+e*g))^(2*n)*(d*x+c)/f/g/n/ln(F)+1/3*b^3*(F^(f*g*x+e*g))^
(3*n)*(d*x+c)/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$$
$$= \frac{-bd\left(F^{g(e+fx)}\right)^n \left(108a^2+27ab\left(F^{g(e+fx)}\right)^n+4b^2\left(F^{g(e+fx)}\right)^{2n}\right)+6bf\left(F^{g(e+fx)}\right)^n \left(18a^2+9ab\left(F^{g(e+fx)}\right)\right.}{36f^2g^2n^2\log^2(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x),x]
```

output
```
(-(b*d*(F^(g*(e + f*x)))^n*(108*a^2 + 27*a*b*(F^(g*(e + f*x)))^n + 4*b^2*(
F^(g*(e + f*x)))^(2*n))) + 6*b*f*(F^(g*(e + f*x)))^n*(18*a^2 + 9*a*b*(F^(g
*(e + f*x)))^n + 2*b^2*(F^(g*(e + f*x)))^(2*n))*g*n*(c + d*x)*Log[F] + 18*
a^3*f^2*g^2*n^2*x*(2*c + d*x)*Log[F]^2)/(36*f^2*g^2*n^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$
$$\downarrow \text{2614}$$
$$\int \left(a^3(c+dx)+3a^2b(c+dx)\left(F^{eg+fgx}\right)^n+3ab^2(c+dx)\left(F^{eg+fgx}\right)^{2n}+b^3(c+dx)\left(F^{eg+fgx}\right)^{3n}\right) dx$$
$$\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log^2(F)}+\frac{3ab^2(c+dx)\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}-\frac{3ab^2d\left(F^{eg+fgx}\right)^{2n}}{4f^2g^2n^2\log^2(F)}+\frac{b^3(c+dx)\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)}-\frac{b^3d\left(F^{eg+fgx}\right)^{3n}}{9f^2g^2n^2\log^2(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x),x]
```

output
```
(a^3*(c + d*x)^2)/(2*d) - (3*a^2*b*d*(F^(e*g + f*g*x))^n)/(f^2*g^2*n^2*Log
[F]^2) - (3*a*b^2*d*(F^(e*g + f*g*x))^(2*n))/(4*f^2*g^2*n^2*Log[F]^2) - (b
^3*d*(F^(e*g + f*g*x))^(3*n))/(9*f^2*g^2*n^2*Log[F]^2) + (3*a^2*b*(F^(e*g
+ f*g*x))^n*(c + d*x))/(f*g*n*Log[F]) + (3*a*b^2*(F^(e*g + f*g*x))^(2*n)*(
c + d*x))/(2*f*g*n*Log[F]) + (b^3*(F^(e*g + f*g*x))^(3*n)*(c + d*x))/(3*f*
g*n*Log[F])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{18a^3dx^2n^2g^2f^2\ln(F)^2+36a^3cxn^2g^2f^2\ln(F)^2+12x\left(F^{g(fx+e)}\right)^{3n}b^3dngf\ln(F)+54x\left(F^{g(fx+e)}\right)^{2n}ab^2dngf\ln(F)+12\ln(F$
orering	Expression too large to display

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/36*(18*a^3*d*x^2*n^2*g^2*f^2*ln(F)^2+36*a^3*c*x*n^2*g^2*f^2*ln(F)^2+12*x
*((F^(g*(f*x+e)))^n)^3*b^3*d*n*g*f*ln(F)+54*x*((F^(g*(f*x+e)))^n)^2*a*b^2*
d*n*g*f*ln(F)+12*ln(F)*((F^(g*(f*x+e)))^n)^3*b^3*c*f*g*n+108*x*(F^(g*(f*x+
e)))^n*a^2*b*d*n*g*f*ln(F)+54*ln(F)*((F^(g*(f*x+e)))^n)^2*a*b^2*c*f*g*n+10
8*ln(F)*(F^(g*(f*x+e)))^n*a^2*b*c*f*g*n-4*((F^(g*(f*x+e)))^n)^3*b^3*d-27*(
(F^(g*(f*x+e)))^n)^2*a*b^2*d-108*(F^(g*(f*x+e)))^n*a^2*b*d)/n^2/g^2/f^2/ln
(F)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$$
$$= \frac{18\left(a^3df^2g^2n^2x^2+2\,a^3cf^2g^2n^2x\right)\log(F)^2-4\left(b^3d-3\left(b^3dfgnx+b^3cfgn\right)\log(F)\right)F^{3\,fgnx+3\,egn}-27\,(ab}{36\,f^2g^2n}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x, algorithm="fricas")
```

output
```
1/36*(18*(a^3*d*f^2*g^2*n^2*x^2 + 2*a^3*c*f^2*g^2*n^2*x)*log(F)^2 - 4*(b^3
*d - 3*(b^3*d*f*g*n*x + b^3*c*f*g*n)*log(F))*F^(3*f*g*n*x + 3*e*g*n) - 27*
(a*b^2*d - 2*(a*b^2*d*f*g*n*x + a*b^2*c*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*
g*n) - 108*(a^2*b*d - (a^2*b*d*f*g*n*x + a^2*b*c*f*g*n)*log(F))*F^(f*g*n*x
 + e*g*n))/(f^2*g^2*n^2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.56

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx$$
$$= \begin{cases} (a+b)^3\left(cx+\frac{dx^2}{2}\right) \\ \left(a+b(F^{eg})^n\right)^3\left(cx+\frac{dx^2}{2}\right) \\ (a+b)^3\left(cx+\frac{dx^2}{2}\right) \\ a^3cx+\frac{a^3dx^2}{2}+\frac{3a^2bc\left(F^{eg+fgx}\right)^n}{fgn\log(F)}+\frac{3a^2bdx\left(F^{eg+fgx}\right)^n}{fgn\log(F)}-\frac{3a^2bd\left(F^{eg+fgx}\right)^n}{f^2g^2n^2\log(F)^2}+\frac{3ab^2c\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}+\frac{3ab^2dx\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)}- \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c),x)
```

output
```
Piecewise(((a + b)**3*(c*x + d*x**2/2), Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & E
q(n, 0)), ((a + b*(F**(e*g))**n)**3*(c*x + d*x**2/2), Eq(f, 0)), ((a + b)*
*3*(c*x + d*x**2/2), Eq(F, 1) | Eq(g, 0) | Eq(n, 0)), (a**3*c*x + a**3*d*x
**2/2 + 3*a**2*b*c*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 3*a**2*b*d*x*(F*
*(e*g + f*g*x))**n/(f*g*n*log(F)) - 3*a**2*b*d*(F**(e*g + f*g*x))**n/(f**2
*g**2*n**2*log(F)**2) + 3*a*b**2*c*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(
F)) + 3*a*b**2*d*x*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) - 3*a*b**2*d
*(F**(e*g + f*g*x))**(2*n)/(4*f**2*g**2*n**2*log(F)**2) + b**3*c*(F**(e*g
+ f*g*x))**(3*n)/(3*f*g*n*log(F)) + b**3*d*x*(F**(e*g + f*g*x))**(3*n)/(3*
f*g*n*log(F)) - b**3*d*(F**(e*g + f*g*x))**(3*n)/(9*f**2*g**2*n**2*log(F)*
*2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.14

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx = \frac{1}{2}a^3dx^2 + a^3cx + \frac{3F^{fgnx+egn}a^2bc}{fgn\log(F)} + \frac{3F^{2fgnx+2egn}ab^2c}{2fgn\log(F)} + \frac{F^{3fgnx+3egn}b^3c}{3fgn\log(F)} + \frac{3(F^{egn}fgnx\log(F) - F^{egn})F^{fgnx}a^2bd}{f^2g^2n^2\log(F)^2} + \frac{3(2F^{2egn}fgnx\log(F) - F^{2egn})F^{2fgnx}ab^2d}{4f^2g^2n^2\log(F)^2} + \frac{(3F^{3egn}fgnx\log(F) - F^{3egn})F^{3fgnx}b^3d}{9f^2g^2n^2\log(F)^2}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x, algorithm="maxima")
```

output

```
1/2*a^3*d*x^2 + a^3*c*x + 3*F^(f*g*n*x + e*g*n)*a^2*b*c/(f*g*n*log(F)) + 3
/2*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*c/(f*g*n*log(F)) + 1/3*F^(3*f*g*n*x + 3*e
*g*n)*b^3*c/(f*g*n*log(F)) + 3*(F^(e*g*n)*f*g*n*x*log(F) - F^(e*g*n))*F^(f
*g*n*x)*a^2*b*d/(f^2*g^2*n^2*log(F)^2) + 3/4*(2*F^(2*e*g*n)*f*g*n*x*log(F)
 - F^(2*e*g*n))*F^(2*f*g*n*x)*a*b^2*d/(f^2*g^2*n^2*log(F)^2) + 1/9*(3*F^(3
*e*g*n)*f*g*n*x*log(F) - F^(3*e*g*n))*F^(3*f*g*n*x)*b^3*d/(f^2*g^2*n^2*log
(F)^2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 3528, normalized size of antiderivative = 14.95

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c + dx)\, dx = \text{Too large to display}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x, algorithm="giac")
```

output

```
1/2*a^3*d*x^2 + a^3*c*x + 1/9*(2*((3*b^3*d*f*g*n*x*log(abs(F)) + 3*b^3*c*f
*g*n*log(abs(F)) - b^3*d)*(pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*
f^2*g^2*n^2*log(abs(F))^2)/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 +
2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi
*f^2*g^2*n^2*log(abs(F)))^2) + 3*(pi*b^3*d*f*g*n*x*sgn(F) - pi*b^3*d*f*g*n
*x + pi*b^3*c*f*g*n*sgn(F) - pi*b^3*c*f*g*n)*(pi*f^2*g^2*n^2*log(abs(F))*s
gn(F) - pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*sgn(F) - pi^2*f^2*g
^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*n^2*log(abs(F))*sg
n(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*cos(-3/2*pi*f*g*n*x*sgn(F) + 3/2*pi
*f*g*n*x - 3/2*pi*e*g*n*sgn(F) + 3/2*pi*e*g*n) + (3*(pi*b^3*d*f*g*n*x*sgn(
F) - pi*b^3*d*f*g*n*x + pi*b^3*c*f*g*n*sgn(F) - pi*b^3*c*f*g*n)*(pi^2*f^2*
g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)/((pi^2*f^
2*g^2*n^2*sgn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(
pi*f^2*g^2*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2) - 4*(3*
b^3*d*f*g*n*x*log(abs(F)) + 3*b^3*c*f*g*n*log(abs(F)) - b^3*d)*(pi*f^2*g^2
*n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))/((pi^2*f^2*g^2*n^2*s
gn(F) - pi^2*f^2*g^2*n^2 + 2*f^2*g^2*n^2*log(abs(F))^2)^2 + 4*(pi*f^2*g^2*
n^2*log(abs(F))*sgn(F) - pi*f^2*g^2*n^2*log(abs(F)))^2))*sin(-3/2*pi*f*g*n
*x*sgn(F) + 3/2*pi*f*g*n*x - 3/2*pi*e*g*n*sgn(F) + 3/2*pi*e*g*n))*e^(3*f*g
*n*x*log(abs(F)) + 3*e*g*n*log(abs(F))) - 1/18*I*((3*pi*b^3*d*f*g*n*x*s...
```

Mupad [B] (verification not implemented)

Time = 23.41 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.93

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)\,dx = \frac{a^3\,d\,x^2}{2} - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}\left(\frac{3\,a\,b^2\,(d-2\,c\,f\,g\,n\,\ln(F))}{4\,f^2\,g^2\,n^2\,\ln(F)^2} - \frac{3\,a\,b^2\,d\,x}{2\,f\,g\,n\,\ln(F)}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{3\,n}\left(\frac{b^3\,(d-3\,c\,f\,g\,n\,\ln(F))}{9\,f^2\,g^2\,n^2\,\ln(F)^2} - \frac{b^3\,d\,x}{3\,f\,g\,n\,\ln(F)}\right) - \left(F^{f\,g\,x}\,F^{e\,g}\right)^{n}\left(\frac{3\,a^2\,b\,(d-c\,f\,g\,n\,\ln(F))}{f^2\,g^2\,n^2\,\ln(F)^2} - \frac{3\,a^2\,b\,d\,x}{f\,g\,n\,\ln(F)}\right) + a^3\,c\,x$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x),x)
```

output
```
(a^3*d*x^2)/2 - (F^(f*g*x)*F^(e*g))^(2*n)*((3*a*b^2*(d - 2*c*f*g*n*log(F))
)/(4*f^2*g^2*n^2*log(F)^2) - (3*a*b^2*d*x)/(2*f*g*n*log(F))) - (F^(f*g*x)*
F^(e*g))^(3*n)*((b^3*(d - 3*c*f*g*n*log(F)))/(9*f^2*g^2*n^2*log(F)^2) - (b
^3*d*x)/(3*f*g*n*log(F))) - (F^(f*g*x)*F^(e*g))^n*((3*a^2*b*(d - c*f*g*n*l
og(F)))/(f^2*g^2*n^2*log(F)^2) - (3*a^2*b*d*x)/(f*g*n*log(F))) + a^3*c*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.15

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 (c + dx)\, dx$$

$$= \frac{12 f^{3fgnx+3egn} \log(f)\, b^3 cfgn + 12 f^{3fgnx+3egn} \log(f)\, b^3 dfgnx - 4 f^{3fgnx+3egn} b^3 d + 54 f^{2fgnx+2egn} \log(f)\, a\, b^2}{}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c),x)
```

output
```
(12*f**(3*e*g*n + 3*f*g*n*x)*log(f)*b**3*c*f*g*n + 12*f**(3*e*g*n + 3*f*g*
n*x)*log(f)*b**3*d*f*g*n*x - 4*f**(3*e*g*n + 3*f*g*n*x)*b**3*d + 54*f**(2*
e*g*n + 2*f*g*n*x)*log(f)*a*b**2*c*f*g*n + 54*f**(2*e*g*n + 2*f*g*n*x)*log
(f)*a*b**2*d*f*g*n*x - 27*f**(2*e*g*n + 2*f*g*n*x)*a*b**2*d + 108*f**(e*g*
n + f*g*n*x)*log(f)*a**2*b*c*f*g*n + 108*f**(e*g*n + f*g*n*x)*log(f)*a**2*
b*d*f*g*n*x - 108*f**(e*g*n + f*g*n*x)*a**2*b*d + 36*log(f)**2*a**3*c*f**2
*g**2*n**2*x + 18*log(f)**2*a**3*d*f**2*g**2*n**2*x**2)/(36*log(f)**2*f**2
*g**2*n**2)
```

3.42 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx$

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Optimal result

Integrand size = 17, antiderivative size = 103

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx = a^3x+\frac{3a^2b\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}+\frac{3ab^2\left(F^{g(e+fx)}\right)^{2n}}{2fgn\log(F)}+\frac{b^3\left(F^{g(e+fx)}\right)^{3n}}{3fgn\log(F)}$$

output
```
a^3*x+3*a^2*b*(F^(g*(f*x+e)))^n/f/g/n/ln(F)+3/2*a*b^2*(F^(g*(f*x+e)))^(2*n
)/f/g/n/ln(F)+1/3*b^3*(F^(g*(f*x+e)))^(3*n)/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx = \frac{b\left(F^{g(e+fx)}\right)^n\left(18a^2+9ab\left(F^{g(e+fx)}\right)^n+2b^2\left(F^{g(e+fx)}\right)^{2n}\right)+6a^3\log\left(\left(F^{g(e+fx)}\right)^n\right)}{6fgn\log(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
(b*(F^(g*(e + f*x)))^n*(18*a^2 + 9*a*b*(F^(g*(e + f*x)))^n + 2*b^2*(F^(g*(
e + f*x)))^(2*n)) + 6*a^3*Log[(F^(g*(e + f*x)))^n])/(6*f*g*n*Log[F])
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$

$$\downarrow 2720$$

$$\frac{\int F^{-g(e+fx)}\left(b\left(F^{g(e+fx)}\right)^n + a\right)^3 dF^{g(e+fx)}}{fg\log(F)}$$

$$\downarrow 798$$

$$\frac{\int F^{-g(e+fx)}\left(b\left(F^{g(e+fx)}\right)^n + a\right)^3 d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 49$$

$$\frac{\int \left(3ab^2\left(F^{g(e+fx)}\right)^n + a^3F^{-g(e+fx)} + b^3F^{2g(e+fx)} + 3a^2b\right) d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 2009$$

$$\frac{a^3\log\left(\left(F^{g(e+fx)}\right)^n\right) + 3a^2b\left(F^{g(e+fx)}\right)^n + \frac{3}{2}ab^2F^{2g(e+fx)} + \frac{1}{3}b^3F^{3g(e+fx)}}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
((3*a*b^2*F^(2*g*(e + f*x)))/2 + (b^3*F^(3*g*(e + f*x)))/3 + 3*a^2*b*(F^(g
*(e + f*x)))^n + a^3*Log[(F^(g*(e + f*x)))^n])/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n  Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{6a^3x\ln(F)fgn+2\left(F^{g(fx+e)}\right)^{3n}b^3+9\left(F^{g(fx+e)}\right)^{2n}ab^2+18\left(F^{g(fx+e)}\right)^{n}a^2b}{6\ln(F)fgn}$
derivativedivides	$\frac{\frac{\left(F^{g(fx+e)}\right)^{3n}b^3}{3}+\frac{3\left(F^{g(fx+e)}\right)^{2n}ab^2}{2}+3\left(F^{g(fx+e)}\right)^{n}a^2b+a^3\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{gf\ln(F)n}$
default	$\frac{\frac{\left(F^{g(fx+e)}\right)^{3n}b^3}{3}+\frac{3\left(F^{g(fx+e)}\right)^{2n}ab^2}{2}+3\left(F^{g(fx+e)}\right)^{n}a^2b+a^3\ln\left(\left(F^{g(fx+e)}\right)^{n}\right)}{gf\ln(F)n}$
norman	$a^3x+\frac{b^3\mathrm{e}^{3n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{3ngf\ln(F)}+\frac{3ab^2\mathrm{e}^{2n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{2ngf\ln(F)}+\frac{3a^2b\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{ngf\ln(F)}$
orering	$\frac{(6xngf\ln(F)+11)\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^3}{6\ln(F)fgn}-\frac{(11xngf\ln(F)+6)\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^2b\left(F^{g(fx+e)}\right)^{n}}{2\ln(F)fgn}+\frac{(6xngf\ln(F)-$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(6*a^3*x*ln(F)*f*g*n+2*b^3*((F^(g*(f*x+e)))^n)^3+9*a*b^2*((F^(g*(f*x+e
)))^n)^2+18*(F^(g*(f*x+e)))^n*a^2*b)/ln(F)/f/g/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$
$$= \frac{6\,a^3 fgnx \log(F) + 18\,F^{fgnx+egn} a^2 b + 9\,F^{2\,fgnx+2\,egn} ab^2 + 2\,F^{3\,fgnx+3\,egn} b^3}{6\,fgn\log(F)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

output
```
1/6*(6*a^3*f*g*n*x*log(F) + 18*F^(f*g*n*x + e*g*n)*a^2*b + 9*F^(2*f*g*n*x
+ 2*e*g*n)*a*b^2 + 2*F^(3*f*g*n*x + 3*e*g*n)*b^3)/(f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.17

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$
$$= \begin{cases} x(a+b)^3 & \text{for } F = 1 \wedge f = 0 \wedge g = 0 \wedge n = 0 \\ x(a + b(F^{eg})^n)^3 & \text{for } f = 0 \\ x(a+b)^3 & \text{for } F = 1 \vee g = 0 \vee n = 0 \\ a^3 x + \frac{3a^2 b\left(F^{eg+fgx}\right)^n}{fgn\log(F)} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2fgn\log(F)} + \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{3fgn\log(F)} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**3,x)
```

output
```
Piecewise((x*(a + b)**3, Eq(F, 1) & Eq(f, 0) & Eq(g, 0) & Eq(n, 0)), (x*(a
 + b*(F**(e*g))**n)**3, Eq(f, 0)), (x*(a + b)**3, Eq(F, 1) | Eq(g, 0) | Eq
(n, 0)), (a**3*x + 3*a**2*b*(F**(e*g + f*g*x))**n/(f*g*n*log(F)) + 3*a*b**
2*(F**(e*g + f*g*x))**(2*n)/(2*f*g*n*log(F)) + b**3*(F**(e*g + f*g*x))**(3
*n)/(3*f*g*n*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.91

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 \, dx = a^3 x + \frac{3\, F^{(fx+e)gn} a^2 b}{fgn \log(F)} + \frac{3\, F^{2\,(fx+e)gn} a b^2}{2\, fgn \log(F)} + \frac{F^{3\,(fx+e)gn} b^3}{3\, fgn \log(F)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

output
```
a^3*x + 3*F^((f*x + e)*g*n)*a^2*b/(f*g*n*log(F)) + 3/2*F^(2*(f*x + e)*g*n)
*a*b^2/(f*g*n*log(F)) + 1/3*F^(3*(f*x + e)*g*n)*b^3/(f*g*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.95

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 \, dx$$
$$= \frac{18\, F^{fgnx} F^{egn} a^2 b + 9\, F^{2\,fgnx} F^{2\,egn} a b^2 + 2\, F^{3\,fgnx} F^{3\,egn} b^3 + 6\, a^3 \log\left(|F|^{fgnx} |F|^{egn}\right)}{6\, fgn \log(F)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

output
```
1/6*(18*F^(f*g*n*x)*F^(e*g*n)*a^2*b + 9*F^(2*f*g*n*x)*F^(2*e*g*n)*a*b^2 +
2*F^(3*f*g*n*x)*F^(3*e*g*n)*b^3 + 6*a^3*log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n
)))/(f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.40 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.20

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 dx = \frac{a^3 \ln\left(F^{fgx}\right)}{fg \ln(F)} + \frac{b^3 \left(F^{fgx} F^{eg}\right)^{3n}}{3fgn \ln(F)} + \frac{3a^2 b \left(F^{fgx} F^{eg}\right)^n}{fgn \ln(F)} + \frac{3ab^2 \left(F^{fgx} F^{eg}\right)^{2n}}{2fgn \ln(F)}$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3,x)
```

output
```
(a^3*log(F^(f*g*x)))/(f*g*log(F)) + (b^3*(F^(f*g*x)*F^(e*g))^(3*n))/(3*f*g
*n*log(F)) + (3*a^2*b*(F^(f*g*x)*F^(e*g))^n)/(f*g*n*log(F)) + (3*a*b^2*(F^
(f*g*x)*F^(e*g))^(2*n))/(2*f*g*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.82

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^3 dx = \frac{2f^{3fgnx+3egn} b^3 + 9f^{2fgnx+2egn} a\, b^2 + 18 f^{fgnx+egn} a^2 b + 6\log(f)\, a^3 fgnx}{6\log(f)\, fgn}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3,x)
```

output
```
(2*f**(3*e*g*n + 3*f*g*n*x)*b**3 + 9*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 18*
f**(e*g*n + f*g*n*x)*a**2*b + 6*log(f)*a**3*f*g*n*x)/(6*log(f)*f*g*n)
```

3.43 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 200

$$\begin{aligned}
&\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx \\
&= \frac{3a^2bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n \operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d} \\
&\quad + \frac{3ab^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} \operatorname{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d} \\
&\quad + \frac{b^3F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} \operatorname{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d} \\
&\quad + \frac{a^3\log(c+dx)}{d}
\end{aligned}$$

output

```
3*a^2*b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*Ei(f*g*n*(d*x+c)*l
n(F)/d)/d+3*a*b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*
Ei(2*f*g*n*(d*x+c)*ln(F)/d)/d+b^3*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^(f*
g*x+e*g))^(3*n)*Ei(3*f*g*n*(d*x+c)*ln(F)/d)/d+a^3*ln(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.80

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$$
$$= \frac{3a^2bF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n \text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right) + 3ab^2F^{-\frac{2fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^{2n}\text{ExpIntegralE}}{d}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x),x]
```

output
```
((3*a^2*b*(F^(g*(e + f*x)))^n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d])/F
^((f*g*n*(c + d*x))/d) + (3*a*b^2*(F^(g*(e + f*x)))^(2*n)*ExpIntegralEi[(2
*f*g*n*(c + d*x)*Log[F])/d])/F^((2*f*g*n*(c + d*x))/d) + (b^3*(F^(g*(e + f
*x)))^(3*n)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d])/F^((3*f*g*n*(c +
d*x))/d) + a^3*Log[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$$
$$\downarrow 2614$$
$$\int \left(\frac{a^3}{c+dx} + \frac{3a^2b\left(F^{eg+fgx}\right)^n}{c+dx} + \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{c+dx} + \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{c+dx}\right)dx$$
$$\downarrow 2009$$

$$\frac{a^3\log(c+dx)}{d}+\frac{3a^2b\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\operatorname{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\operatorname{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d}+\frac{b^3\left(F^{eg+fgx}\right)^{3n} F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)}\operatorname{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x),x]
```

output
```
(3*a^2*b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*ExpInte
gralEi[(f*g*n*(c + d*x)*Log[F])/d])/d + (3*a*b^2*F^(2*(e - (c*f)/d)*g*n -
2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*ExpIntegralEi[(2*f*g*n*(c + d*x)*
Log[F])/d])/d + (b^3*F^(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f
*g*x))^(3*n)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d])/d + (a^3*Log[c +
 d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int\frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3}{dx+c}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$$
$$= \frac{F^{\frac{3(de-cf)gn}{d}} b^3 \mathrm{Ei}\left(\frac{3(dfgnx+cfgn)\log(F)}{d}\right) + 3F^{\frac{2(de-cf)gn}{d}} ab^2 \mathrm{Ei}\left(\frac{2(dfgnx+cfgn)\log(F)}{d}\right) + 3F^{\frac{(de-cf)gn}{d}} a^2 b \mathrm{Ei}\left(\frac{(dfgnx+}{d}\right.$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="fricas")
```

output
```
(F^(3*(d*e - c*f)*g*n/d)*b^3*Ei(3*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 3*F^(2
*(d*e - c*f)*g*n/d)*a*b^2*Ei(2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 3*F^((d*e
 - c*f)*g*n/d)*a^2*b*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d) + a^3*log(d*x + c)
)/d
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}{c+dx}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c),x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**3/(c + d*x), x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{dx+c}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="maxima")
```

output
```
F^(3*e*g*n)*b^3*integrate(F^(3*f*g*n*x)/(d*x + c), x) + 3*F^(2*e*g*n)*a*b^
2*integrate(F^(2*f*g*n*x)/(d*x + c), x) + 3*F^(e*g*n)*a^2*b*integrate(F^(f
*g*n*x)/(d*x + c), x) + a^3*log(d*x + c)/d
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{dx+c}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^3/(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}{c+d\,x}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x),x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{c+dx}\,dx$$
$$= \frac{f^{3egn}\left(\int \frac{f^{3fgnx}}{dx+c}dx\right) b^3 d + 3f^{2egn}\left(\int \frac{f^{2fgnx}}{dx+c}dx\right) a\,b^2 d + 3f^{egn}\left(\int \frac{f^{fgnx}}{dx+c}dx\right) a^2 b d + \log(dx+c)\,a^3}{d}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

output
```
(f**(3*e*g*n)*int(f**(3*f*g*n*x)/(c + d*x),x)*b**3*d + 3*f**(2*e*g*n)*int(
f**(2*f*g*n*x)/(c + d*x),x)*a*b**2*d + 3*f**(e*g*n)*int(f**(f*g*n*x)/(c +
d*x),x)*a**2*b*d + log(c + d*x)*a**3)/d
```

3.44 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx$

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Sympy [F] . . . 388
Maxima [F] . . . 388
Giac [F] . . . 388
Mupad [F(-1)] . . . 389
Reduce [F] . . . 389

Optimal result

Integrand size = 25, antiderivative size = 305

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx$$
$$= -\frac{a^3}{d(c+dx)} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}$$
$$+ \frac{3a^2bfF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n gn\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$
$$+ \frac{6ab^2fF^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} gn\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$
$$+ \frac{3b^3fF^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} gn\,\text{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)\log(F)}{d^2}$$

output

```
-a^3/d/(d*x+c)-3*a^2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)-3*a*b^2*(F^(f*g*x+e*g))
^(2*n)/d/(d*x+c)-b^3*(F^(f*g*x+e*g))^(3*n)/d/(d*x+c)+3*a^2*b*f*F^((e-c*f/d
)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g*n*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d
^2+6*a*b^2*f*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g*n*E
i(2*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2+3*b^3*f*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x
+e))*(F^(f*g*x+e*g))^(3*n)*g*n*Ei(3*f*g*n*(d*x+c)*ln(F)/d)*ln(F)/d^2
```

Mathematica [A] (verified)

Time = 1.88 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx =$$

$$-\frac{a^3d+3a^2bd\left(F^{g(e+fx)}\right)^n+3ab^2d\left(F^{g(e+fx)}\right)^{2n}+b^3d\left(F^{g(e+fx)}\right)^{3n}-3a^2bfF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n gn(c+$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2,x]
```

output
```
-((a^3*d + 3*a^2*b*d*(F^(g*(e + f*x)))^n + 3*a*b^2*d*(F^(g*(e + f*x)))^(2*
n) + b^3*d*(F^(g*(e + f*x)))^(3*n) - (3*a^2*b*f*(F^(g*(e + f*x)))^n*g*n*(c
 + d*x)*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((f*g*n*(c + d
*x))/d) - (6*a*b^2*f*(F^(g*(e + f*x)))^(2*n)*g*n*(c + d*x)*ExpIntegralEi[(
2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/F^((2*f*g*n*(c + d*x))/d) - (3*b^3*f*
(F^(g*(e + f*x)))^(3*n)*g*n*(c + d*x)*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log
[F])/d]*Log[F])/F^((3*f*g*n*(c + d*x))/d))/(d^2*(c + d*x)))
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx$$

$$\downarrow 2614$$

$$\int \left(\frac{a^3}{(c+dx)^2}+\frac{3a^2b\left(F^{eg+fgx}\right)^n}{(c+dx)^2}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^2}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^3}{d(c+dx)}+\frac{3a^2bfgn\log(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{d^2}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{d(c+dx)}+\frac{6ab^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n}F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^2}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{d(c+dx)}+\frac{3b^3fgn\log(F)\left(F^{eg+fgx}\right)^{3n}F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{d^2}-\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{d(c+dx)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2,x]
```

output
```
-(a^3/(d*(c + d*x))) - (3*a^2*b*(F^(e*g + f*g*x))^n)/(d*(c + d*x)) - (3*a*
b^2*(F^(e*g + f*g*x))^(2*n))/(d*(c + d*x)) - (b^3*(F^(e*g + f*g*x))^(3*n))
/(d*(c + d*x)) + (3*a^2*b*f*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g
+ f*g*x))^n*g*n*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (6
*a*b^2*f*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)
*g*n*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2 + (3*b^3*f*F^
(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*g*n*ExpInt
egralEi[(3*f*g*n*(c + d*x)*Log[F])/d]*Log[F])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3}{(dx+c)^2}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx =$$
$$-\frac{3\,F^{fgnx+egn}a^2bd+3\,F^{2\,fgnx+2\,egn}ab^2d+F^{3\,fgnx+3\,egn}b^3d+a^3d-3\,(b^3dfgnx+b^3cfgn)F^{\frac{3\,(de-cf)gn}{d}}\mathrm{Ei}\left(\frac{3}{}\right.}{}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(3*F^(f*g*n*x + e*g*n)*a^2*b*d + 3*F^(2*f*g*n*x + 2*e*g*n)*a*b^2*d + F^(3
*f*g*n*x + 3*e*g*n)*b^3*d + a^3*d - 3*(b^3*d*f*g*n*x + b^3*c*f*g*n)*F^(3*(
d*e - c*f)*g*n/d)*Ei(3*(d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F) - 6*(a*b^2*d
*f*g*n*x + a*b^2*c*f*g*n)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*
n)*log(F)/d)*log(F) - 3*(a^2*b*d*f*g*n*x + a^2*b*c*f*g*n)*F^((d*e - c*f)*g
*n/d)*Ei((d*f*g*n*x + c*f*g*n)*log(F)/d)*log(F))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c)**2,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**3/(c + d*x)**2, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
F^(3*e*g*n)*b^3*integrate(F^(3*f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) + 3*
F^(2*e*g*n)*a*b^2*integrate(F^(2*f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) +
3*F^(e*g*n)*a^2*b*integrate(F^(f*g*n*x)/(d^2*x^2 + 2*c*d*x + c^2), x) - a^
3/(d^2*x + c*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^3/(d*x + c)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^2}\,dx = \text{too large to display}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

output

```
(2*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(2*log(f)**2*c**4*f**2*g**2*n**2 + 4*lo
g(f)**2*c**3*d*f**2*g**2*n**2*x + 2*log(f)**2*c**2*d**2*f**2*g**2*n**2*x**
2 - 3*log(f)*c**3*d*f*g*n - 6*log(f)*c**2*d**2*f*g*n*x - 3*log(f)*c*d**3*f
*g*n*x**2 + c**2*d**2 + 2*c*d**3*x + d**4*x**2),x)*log(f)**2*b**3*c**4*f**
2*g**2*n**2 + 2*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(2*log(f)**2*c**4*f**2*g**
2*n**2 + 4*log(f)**2*c**3*d*f**2*g**2*n**2*x + 2*log(f)**2*c**2*d**2*f**2*
g**2*n**2*x**2 - 3*log(f)*c**3*d*f*g*n - 6*log(f)*c**2*d**2*f*g*n*x - 3*lo
g(f)*c*d**3*f*g*n*x**2 + c**2*d**2 + 2*c*d**3*x + d**4*x**2),x)*log(f)**2*
b**3*c**3*d*f**2*g**2*n**2*x - 3*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(2*log(f)
**2*c**4*f**2*g**2*n**2 + 4*log(f)**2*c**3*d*f**2*g**2*n**2*x + 2*log(f)**
2*c**2*d**2*f**2*g**2*n**2*x**2 - 3*log(f)*c**3*d*f*g*n - 6*log(f)*c**2*d*
*2*f*g*n*x - 3*log(f)*c*d**3*f*g*n*x**2 + c**2*d**2 + 2*c*d**3*x + d**4*x*
*2),x)*log(f)*b**3*c**3*d*f*g*n - 3*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(2*log
(f)**2*c**4*f**2*g**2*n**2 + 4*log(f)**2*c**3*d*f**2*g**2*n**2*x + 2*log(f
)**2*c**2*d**2*f**2*g**2*n**2*x**2 - 3*log(f)*c**3*d*f*g*n - 6*log(f)*c**2
*d**2*f*g*n*x - 3*log(f)*c*d**3*f*g*n*x**2 + c**2*d**2 + 2*c*d**3*x + d**4
*x**2),x)*log(f)*b**3*c**2*d**2*f*g*n*x + f**(3*e*g*n)*int(f**(3*f*g*n*x)/
(2*log(f)**2*c**4*f**2*g**2*n**2 + 4*log(f)**2*c**3*d*f**2*g**2*n**2*x + 2
*log(f)**2*c**2*d**2*f**2*g**2*n**2*x**2 - 3*log(f)*c**3*d*f*g*n - 6*log(f
)*c**2*d**2*f*g*n*x - 3*log(f)*c*d**3*f*g*n*x**2 + c**2*d**2 + 2*c*d**3...
```

3.45 $\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 447

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = -\frac{a^3}{2d(c+dx)^2} - \frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2} - \frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2} - \frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2} - \frac{3a^2bf\left(F^{eg+fgx}\right)^n gn\log(F)}{2d^2(c+dx)} - \frac{3ab^2f\left(F^{eg+fgx}\right)^{2n} gn\log(F)}{d^2(c+dx)} - \frac{3b^3f\left(F^{eg+fgx}\right)^{3n} gn\log(F)}{2d^2(c+dx)} + \frac{3a^2bf^2F^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3} + \frac{6ab^2f^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n} g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{d^3} + \frac{9b^3f^2F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n} g^2n^2\,\mathrm{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)\log^2(F)}{2d^3}$$

output

```
-1/2*a^3/d/(d*x+c)^2-3/2*a^2*b*(F^(f*g*x+e*g))^n/d/(d*x+c)^2-3/2*a*b^2*(F^
(f*g*x+e*g))^(2*n)/d/(d*x+c)^2-1/2*b^3*(F^(f*g*x+e*g))^(3*n)/d/(d*x+c)^2-3
/2*a^2*b*f*(F^(f*g*x+e*g))^n*g*n*ln(F)/d^2/(d*x+c)-3*a*b^2*f*(F^(f*g*x+e*g
))^(2*n)*g*n*ln(F)/d^2/(d*x+c)-3/2*b^3*f*(F^(f*g*x+e*g))^(3*n)*g*n*ln(F)/d
^2/(d*x+c)+3/2*a^2*b*f^2*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*g
^2*n^2*Ei(f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3+6*a*b^2*f^2*F^(2*(e-c*f/d)*g*
n-2*g*n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*g^2*n^2*Ei(2*f*g*n*(d*x+c)*ln(F)/d)
*ln(F)^2/d^3+9/2*b^3*f^2*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(F^(f*g*x+e*g))
^(3*n)*g^2*n^2*Ei(3*f*g*n*(d*x+c)*ln(F)/d)*ln(F)^2/d^3
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.73

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx =$$

$$-\frac{a^3d^2-3a^2bf^2F^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n g^2n^2(c+dx)^2\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)\log^2(F)-12ab^2f^2$$

input

```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3,x]
```

output

```
-1/2*(a^3*d^2 - (3*a^2*b*f^2*(F^(g*(e + f*x)))^n*g^2*n^2*(c + d*x)^2*ExpIn
tegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((f*g*n*(c + d*x))/d) - (
12*a*b^2*f^2*(F^(g*(e + f*x)))^(2*n)*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(2*
f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/F^((2*f*g*n*(c + d*x))/d) - (9*b^3*f^
2*(F^(g*(e + f*x)))^(3*n)*g^2*n^2*(c + d*x)^2*ExpIntegralEi[(3*f*g*n*(c +
d*x)*Log[F])/d]*Log[F]^2)/F^((3*f*g*n*(c + d*x))/d) + 3*a^2*b*d*(F^(g*(e +
 f*x)))^n*(d + f*g*n*(c + d*x)*Log[F]) + 3*a*b^2*d*(F^(g*(e + f*x)))^(2*n)
*(d + 2*f*g*n*(c + d*x)*Log[F]) + b^3*d*(F^(g*(e + f*x)))^(3*n)*(d + 3*f*g
*n*(c + d*x)*Log[F]))/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx$$

$$\downarrow 2614$$

$$\int \left(\frac{a^3}{(c+dx)^3}+\frac{3a^2b\left(F^{eg+fgx}\right)^n}{(c+dx)^3}+\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{(c+dx)^3}+\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{(c+dx)^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{a^3}{2d(c+dx)^2}+\frac{3a^2bf^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{fgn(c+dx)\log(F)}{d}\right)}{2d^3}-\frac{3a^2bfgn\log(F)\left(F^{eg+fgx}\right)^n}{2d^2(c+dx)}-\frac{3a^2b\left(F^{eg+fgx}\right)^n}{2d(c+dx)^2}+\frac{6ab^2f^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^{2n}F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{2fgn(c+dx)\log(F)}{d}\right)}{d^3}-\frac{3ab^2fgn\log(F)\left(F^{eg+fgx}\right)^{2n}}{d^2(c+dx)}-\frac{3ab^2\left(F^{eg+fgx}\right)^{2n}}{2d(c+dx)^2}+\frac{9b^3f^2g^2n^2\log^2(F)\left(F^{eg+fgx}\right)^{3n}F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)}\,\text{ExpIntegralEi}\left(\frac{3fgn(c+dx)\log(F)}{d}\right)}{2d^3}-\frac{3b^3fgn\log(F)\left(F^{eg+fgx}\right)^{3n}}{2d^2(c+dx)}-\frac{b^3\left(F^{eg+fgx}\right)^{3n}}{2d(c+dx)^2}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3,x]
```

output
```
-1/2*a^3/(d*(c + d*x)^2) - (3*a^2*b*(F^(e*g + f*g*x))^n)/(2*d*(c + d*x)^2)
 - (3*a*b^2*(F^(e*g + f*g*x))^(2*n))/(2*d*(c + d*x)^2) - (b^3*(F^(e*g + f*
g*x))^(3*n))/(2*d*(c + d*x)^2) - (3*a^2*b*f*(F^(e*g + f*g*x))^n*g*n*Log[F]
)/(2*d^2*(c + d*x)) - (3*a*b^2*f*(F^(e*g + f*g*x))^(2*n)*g*n*Log[F])/(d^2*
(c + d*x)) - (3*b^3*f*(F^(e*g + f*g*x))^(3*n)*g*n*Log[F])/(2*d^2*(c + d*x)
) + (3*a^2*b*f^2*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n
*g^2*n^2*ExpIntegralEi[(f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/(2*d^3) + (6*
a*b^2*f^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n
)*g^2*n^2*ExpIntegralEi[(2*f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/d^3 + (9*b
^3*f^2*F^(3*(e - (c*f)/d)*g*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*g
^2*n^2*ExpIntegralEi[(3*f*g*n*(c + d*x)*Log[F])/d]*Log[F]^2)/(2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \frac{\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^{3}}{(dx+c)^{3}}dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.06

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx =$$

$$-\frac{a^3d^2-9\left(b^3d^2f^2g^2n^2x^2+2\,b^3cdf^2g^2n^2x+b^3c^2f^2g^2n^2\right)F^{\frac{3\,(de-cf)gn}{d}}\mathrm{Ei}\left(\frac{3\,(dfgnx+cfgn)\log(F)}{d}\right)\log(F)^2-12$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(a^3*d^2 - 9*(b^3*d^2*f^2*g^2*n^2*x^2 + 2*b^3*c*d*f^2*g^2*n^2*x + b^3
*c^2*f^2*g^2*n^2)*F^(3*(d*e - c*f)*g*n/d)*Ei(3*(d*f*g*n*x + c*f*g*n)*log(F
)/d)*log(F)^2 - 12*(a*b^2*d^2*f^2*g^2*n^2*x^2 + 2*a*b^2*c*d*f^2*g^2*n^2*x
+ a*b^2*c^2*f^2*g^2*n^2)*F^(2*(d*e - c*f)*g*n/d)*Ei(2*(d*f*g*n*x + c*f*g*n
)*log(F)/d)*log(F)^2 - 3*(a^2*b*d^2*f^2*g^2*n^2*x^2 + 2*a^2*b*c*d*f^2*g^2*
n^2*x + a^2*b*c^2*f^2*g^2*n^2)*F^((d*e - c*f)*g*n/d)*Ei((d*f*g*n*x + c*f*g
*n)*log(F)/d)*log(F)^2 + (b^3*d^2 + 3*(b^3*d^2*f*g*n*x + b^3*c*d*f*g*n)*lo
g(F))*F^(3*f*g*n*x + 3*e*g*n) + 3*(a*b^2*d^2 + 2*(a*b^2*d^2*f*g*n*x + a*b^
2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 3*(a^2*b*d^2 + (a^2*b*d^2*f
*g*n*x + a^2*b*c*d*f*g*n)*log(F))*F^(f*g*n*x + e*g*n))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = \int \frac{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}{(c+dx)^3}\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c)**3,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**3/(c + d*x)**3, x)
```

Maxima [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{(dx+c)^3}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
F^(3*e*g*n)*b^3*integrate(F^(3*f*g*n*x)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x) + 3*F^(2*e*g*n)*a*b^2*integrate(F^(2*f*g*n*x)/(d^3*x^3 + 3*c*d
^2*x^2 + 3*c^2*d*x + c^3), x) + 3*F^(e*g*n)*a^2*b*integrate(F^(f*g*n*x)/(d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3), x) - 1/2*a^3/(d^3*x^2 + 2*c*d^2*x
 + c^2*d)
```

Giac [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = \int \frac{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}{(dx+c)^3}\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^3/(d*x + c)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = \int \frac{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^3/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}{(c+dx)^3}\,dx = \text{too large to display}$$

input

```
int((a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^3,x)
```

output

```
(2*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(log(f)**2*c**5*f**2*g**2*n**2 + 3*log(
f)**2*c**4*d*f**2*g**2*n**2*x + 3*log(f)**2*c**3*d**2*f**2*g**2*n**2*x**2
+ log(f)**2*c**2*d**3*f**2*g**2*n**2*x**3 - 3*log(f)*c**4*d*f*g*n - 9*log(
f)*c**3*d**2*f*g*n*x - 9*log(f)*c**2*d**3*f*g*n*x**2 - 3*log(f)*c*d**4*f*g
*n*x**3 + 2*c**3*d**2 + 6*c**2*d**3*x + 6*c*d**4*x**2 + 2*d**5*x**3),x)*lo
g(f)**2*b**3*c**4*d*f**2*g**2*n**2 + 4*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(lo
g(f)**2*c**5*f**2*g**2*n**2 + 3*log(f)**2*c**4*d*f**2*g**2*n**2*x + 3*log(
f)**2*c**3*d**2*f**2*g**2*n**2*x**2 + log(f)**2*c**2*d**3*f**2*g**2*n**2*x
**3 - 3*log(f)*c**4*d*f*g*n - 9*log(f)*c**3*d**2*f*g*n*x - 9*log(f)*c**2*d
**3*f*g*n*x**2 - 3*log(f)*c*d**4*f*g*n*x**3 + 2*c**3*d**2 + 6*c**2*d**3*x
+ 6*c*d**4*x**2 + 2*d**5*x**3),x)*log(f)**2*b**3*c**3*d**2*f**2*g**2*n**2*
x + 2*f**(3*e*g*n)*int(f**(3*f*g*n*x)/(log(f)**2*c**5*f**2*g**2*n**2 + 3*l
og(f)**2*c**4*d*f**2*g**2*n**2*x + 3*log(f)**2*c**3*d**2*f**2*g**2*n**2*x*
*2 + log(f)**2*c**2*d**3*f**2*g**2*n**2*x**3 - 3*log(f)*c**4*d*f*g*n - 9*l
og(f)*c**3*d**2*f*g*n*x - 9*log(f)*c**2*d**3*f*g*n*x**2 - 3*log(f)*c*d**4*
f*g*n*x**3 + 2*c**3*d**2 + 6*c**2*d**3*x + 6*c*d**4*x**2 + 2*d**5*x**3),x)
*log(f)**2*b**3*c**2*d**3*f**2*g**2*n**2*x**2 - 6*f**(3*e*g*n)*int(f**(3*f
*g*n*x)/(log(f)**2*c**5*f**2*g**2*n**2 + 3*log(f)**2*c**4*d*f**2*g**2*n**2
*x + 3*log(f)**2*c**3*d**2*f**2*g**2*n**2*x**2 + log(f)**2*c**2*d**3*f**2*
g**2*n**2*x**3 - 3*log(f)*c**4*d*f*g*n - 9*log(f)*c**3*d**2*f*g*n*x - 9...
```

3.46 $\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 192

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{(c+dx)^4}{4ad} - \frac{(c+dx)^3 \log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)} - \frac{6d^3\,\mathrm{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^4g^4n^4\log^4(F)}$$

output

```
1/4*(d*x+c)^4/a/d-(d*x+c)^3*ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/ln(F)-3*d*
(d*x+c)^2*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2+6*d^2*(d
*x+c)*polylog(3,-b*(F^(g*(f*x+e)))^n/a)/a/f^3/g^3/n^3/ln(F)^3-6*d^3*polylo
g(4,-b*(F^(g*(f*x+e)))^n/a)/a/f^4/g^4/n^4/ln(F)^4
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$= \frac{-(c+dx)^3 \log\left(1+\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right) + \frac{3d\left(f^2g^2n^2(c+dx)^2\log^2(F)\,\text{PolyLog}\left(2,-\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)+2d\left(fgn(c+dx)\log(F)\,\text{Poly}\right.\right.}{f^3g^3n^3\log^3(F)}}{afgn\log(F)}$$

input
```
Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
(-((c + d*x)^3*Log[1 + a/(b*(F^(g*(e + f*x)))^n)]) + (3*d*(f^2*g^2*n^2*(c
+ d*x)^2*Log[F]^2*PolyLog[2, -(a/(b*(F^(g*(e + f*x)))^n))] + 2*d*(f*g*n*(c
 + d*x)*Log[F]*PolyLog[3, -(a/(b*(F^(g*(e + f*x)))^n))] + d*PolyLog[4, -(a
/(b*(F^(g*(e + f*x)))^n))])))/(f^3*g^3*n^3*Log[F]^3))/(a*f*g*n*Log[F])
```

Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$\downarrow 2615$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a}$$

$$\downarrow 2620$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}$$

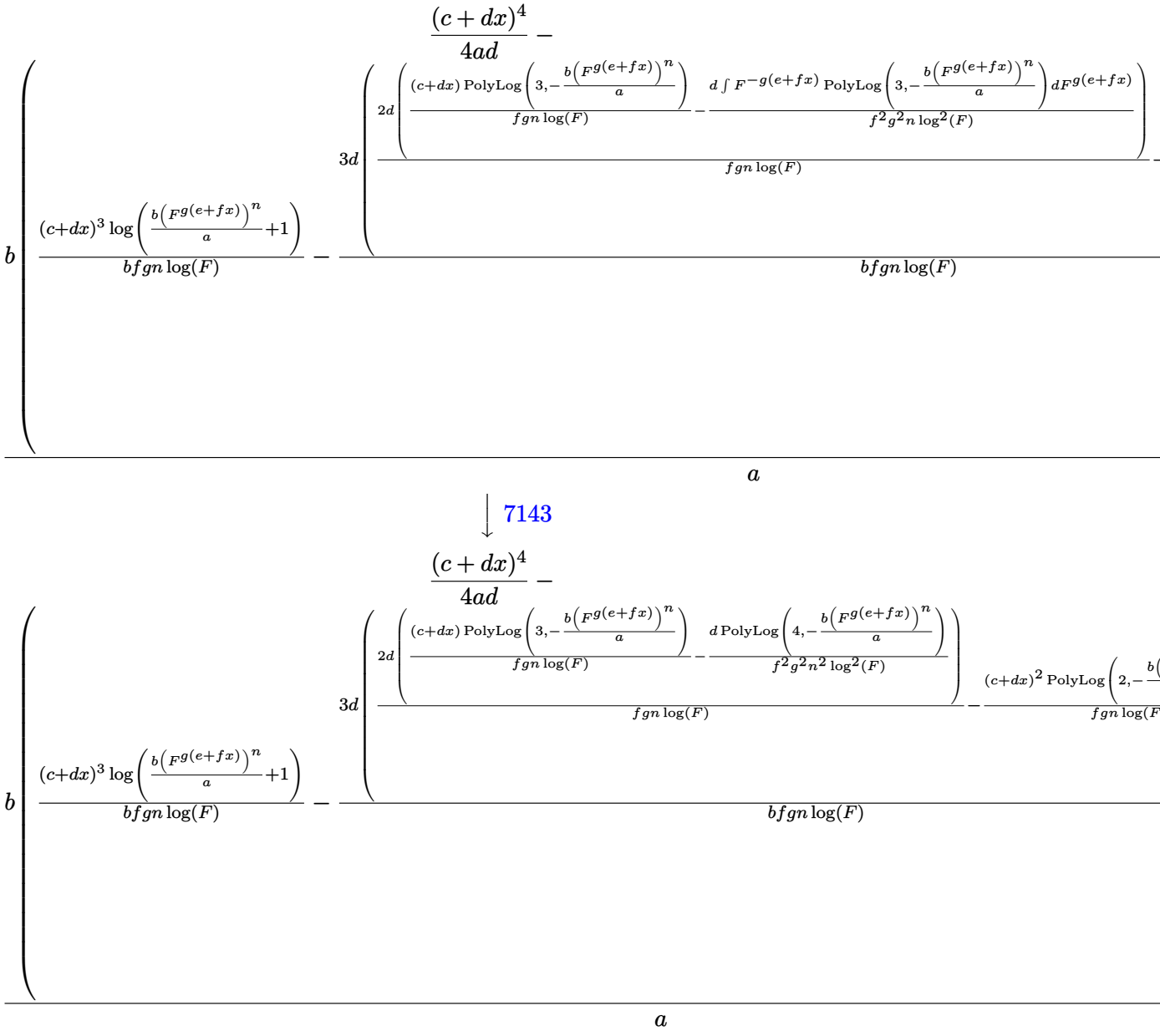
$$\downarrow 3011$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)} - \frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}$$

$$\downarrow 7163$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)} - \frac{d\int\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}\right)}{fgn\log(F)} - \frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,\right.}{fgn\log}\right.}{bfgn\log(F)}\right)}{a}$$

$$\downarrow 2720$$



$$\downarrow 7143$$

input
```
Int[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
(c + d*x)^4/(4*a*d) - (b*(((c + d*x)^3*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])
/(b*f*g*n*Log[F]) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*(F^(g*(e + f*x)))
^n)/a)])/(f*g*n*Log[F])) + (2*d*(((c + d*x)*PolyLog[3, -((b*(F^(g*(e + f*x
)))^n)/a)])/(f*g*n*Log[F]) - (d*PolyLog[4, -((b*(F^(g*(e + f*x)))^n)/a)])/
(f^2*g^2*n^2*Log[F]^2)))/(f*g*n*Log[F])))/(b*f*g*n*Log[F])))/a
```

Defintions of rubi rules used

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3074 vs. 2(190) = 380.

Time = 0.32 (sec) , antiderivative size = 3075, normalized size of antiderivative = 16.02

method	result	size
risch	Expression too large to display	3075

input
```
int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
```

output
```
-1/n/g^4/f^4/ln(F)^4*d^3*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^3/a*ln(F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/n/g^4/f^4/ln(F)^4*d^3*(ln(F^(g*(f*
x+e)))-g*(f*x+e)*ln(F))^3/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*
b+a)-1/n/g/f/ln(F)*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n
/a)*x^3-3/n^2/g^2/f^2/ln(F)^2*d^3/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*
(F^(g*(f*x+e)))^n/a)*x^2-1/n/g^4/f^4/ln(F)^4*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-
n*g*f*x)*(F^(g*(f*x+e)))^n/a)*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^3-3/n^2/
g^2/f^2/ln(F)^2*c^2*d/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e
)))^n/a)-1/n/g/f^4/ln(F)*d^3/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+
e)))^n/a)*e^3+3/g/f/ln(F)*c^2*d/a*x*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))-3/
g^2/f^2/ln(F)^2*c*d^2/a*x*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2+3/g^2/f^3/
ln(F)^2*d^3/a*x*e*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2-3/n/g/f^2/ln(F)*c^
2*d/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*e-3/n/g^2/f^2/l
n(F)^2*c^2*d/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*(ln(F^
(g*(f*x+e)))-g*(f*x+e)*ln(F))-6/g/f^2/ln(F)*c*d^2/a*x*e*(ln(F^(g*(f*x+e)))
-g*(f*x+e)*ln(F))-3/n/g/f/ln(F)*c*d^2/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F
^(g*(f*x+e)))^n/a)*x^2+3/n/g/f^3/ln(F)*c*d^2/a*ln(1+b*F^(n*g*f*x)*F^(-n*g*
f*x)*(F^(g*(f*x+e)))^n/a)*e^2+3/n/g^3/f^3/ln(F)^3*c*d^2/a*ln(1+b*F^(n*g*f*
x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*(ln(F^(g*(f*x+e)))-g*(f*x+e)*ln(F))^2
-6/n^2/g^2/f^2/ln(F)^2*c*d^2/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(189) = 378$.

Time $= 0.07$ (sec) , antiderivative size $= 413$, normalized size of antiderivative $= 2.15$

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$= \frac{4\left(d^3e^3 - 3\,cd^2e^2f + 3\,c^2def^2 - c^3f^3\right)g^3n^3\log\left(F^{fgnx+egn}b + a\right)\log\left(F\right)^3 + \left(d^3f^4g^4n^4x^4 + 4\,cd^2f^4g^4n^4x^3\right.}{}$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

output
```
1/4*(4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*g^3*n^3*log(F^(
f*g*n*x + e*g*n)*b + a)*log(F)^3 + (d^3*f^4*g^4*n^4*x^4 + 4*c*d^2*f^4*g^4*
n^4*x^3 + 6*c^2*d*f^4*g^4*n^4*x^2 + 4*c^3*f^4*g^4*n^4*x)*log(F)^4 - 4*(d^3
*f^3*g^3*n^3*x^3 + 3*c*d^2*f^3*g^3*n^3*x^2 + 3*c^2*d*f^3*g^3*n^3*x + (d^3*
e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*g^3*n^3)*log(F)^3*log((F^(f*g*n*x + e
*g*n)*b + a)/a) - 12*(d^3*f^2*g^2*n^2*x^2 + 2*c*d^2*f^2*g^2*n^2*x + c^2*d*
f^2*g^2*n^2)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1)*log(F)^2 - 24*d^3*p
olylog(4, -F^(f*g*n*x + e*g*n)*b/a) + 24*(d^3*f*g*n*x + c*d^2*f*g*n)*log(F
)*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(a*f^4*g^4*n^4*log(F)^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+dx)^3}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*(F**(g*(f*x+e)))**n),x)
```

output
```
Integral((c + d*x)**3/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(189) = 378$.

Time $= 0.11$ (sec) , antiderivative size $= 480$, normalized size of antiderivative $= 2.50$

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = c^3\left(\frac{fgnx+egn}{afgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{afgn\log(F)}\right)$$
$$- \frac{3\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)c^2d}{af^2g^2n^2\log(F)^2}$$
$$- \frac{3\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)cd}{af^3g^3n^3\log(F)^3}$$
$$- \frac{\left(f^3g^3n^3x^3\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^3+3\,f^2g^2n^2x^2\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)^2-6\,fgnx\log(F)\mathrm{Li}_3\right.}{af^4g^4n^4\log(F)^4}$$
$$+ \frac{d^3f^4g^4n^4x^4\log(F)^4+4\,cd^2f^4g^4n^4x^3\log(F)^4+6\,c^2df^4g^4n^4x^2\log(F)^4}{4\,af^4g^4n^4\log(F)^4}$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

output
```
c^3*((f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n
*log(F))) - 3*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-
F^(f*g*n*x)*F^(e*g*n)*b/a))*c^2*d/(a*f^2*g^2*n^2*log(F)^2) - 3*(f^2*g^2*n^
2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*
g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*c
*d^2/(a*f^3*g^3*n^3*log(F)^3) - (f^3*g^3*n^3*x^3*log(F^(f*g*n*x)*F^(e*g*n)
*b/a + 1)*log(F)^3 + 3*f^2*g^2*n^2*x^2*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*l
og(F)^2 - 6*f*g*n*x*log(F)*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a) + 6*poly
log(4, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^3/(a*f^4*g^4*n^4*log(F)^4) + 1/4*(d^
3*f^4*g^4*n^4*x^4*log(F)^4 + 4*c*d^2*f^4*g^4*n^4*x^3*log(F)^4 + 6*c^2*d*f^
4*g^4*n^4*x^2*log(F)^4)/(a*f^4*g^4*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^3}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+d\,x)^3}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

input
```
int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n),x)
```

output
```
int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$= \frac{\left(\int \frac{x^3}{f^{fgnx+egn}b+a}dx\right)\log(f)\,a\,d^3fgn + 3\left(\int \frac{x^2}{f^{fgnx+egn}b+a}dx\right)\log(f)\,ac\,d^2fgn + 3\left(\int \frac{x}{f^{fgnx+egn}b+a}dx\right)\log(f)\,c}{\log(f)\,afgn}$$

input
```
int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n),x)
```

output
```
(int(x**3/(f**(e*g*n + f*g*n*x)*b + a),x)*log(f)*a*d**3*f*g*n + 3*int(x**2
/(f**(e*g*n + f*g*n*x)*b + a),x)*log(f)*a*c*d**2*f*g*n + 3*int(x/(f**(e*g*
n + f*g*n*x)*b + a),x)*log(f)*a*c**2*d*f*g*n - log(f**(e*g*n + f*g*n*x)*b
+ a)*c**3 + log(f)*c**3*f*g*n*x)/(log(f)*a*f*g*n)
```

3.47 $\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 145

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{(c+dx)^3}{3ad} - \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)} + \frac{2d^2\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^3g^3n^3\log^3(F)}$$

output

```
1/3*(d*x+c)^3/a/d-(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/ln(F)-2*d*
(d*x+c)*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2+2*d^2*poly
log(3,-b*(F^(g*(f*x+e)))^n/a)/a/f^3/g^3/n^3/ln(F)^3
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$= \frac{-f^2g^2n^2(c+dx)^2\log^2(F)\log\left(1+\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)+2dfgn(c+dx)\log(F)\,\text{PolyLog}\left(2,-\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)}{af^3g^3n^3\log^3(F)}$$

input
```
Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
(-(f^2*g^2*n^2*(c + d*x)^2*Log[F]^2*Log[1 + a/(b*(F^(g*(e + f*x)))^n)]) +
2*d*f*g*n*(c + d*x)*Log[F]*PolyLog[2, -(a/(b*(F^(g*(e + f*x)))^n))] + 2*d^
2*PolyLog[3, -(a/(b*(F^(g*(e + f*x)))^n))])/(a*f^3*g^3*n^3*Log[F]^3)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$\downarrow 2615$$
$$\frac{(c+dx)^3}{3ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}$$
$$\downarrow 2620$$
$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}$$

$$\downarrow 3011$$

$$\frac{(c+dx)^3}{3ad} - \frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{2d\left(\frac{d\int \mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)} - \frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{(c+dx)^3}{3ad} - \frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{2d\left(\frac{d\int F^{-g(e+fx)}\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)} - \frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}$$

$$\downarrow 7143$$

$$\frac{(c+dx)^3}{3ad} - \frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)} - \frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}$$

input

```
Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n),x]
```

output

```
(c + d*x)^3/(3*a*d) - (b*(((c + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])
/(b*f*g*n*Log[F]) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n
)/a)])/(f*g*n*Log[F])) + (d*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2
*g^2*n^2*Log[F]^2)))/(b*f*g*n*Log[F])))/a
```

Defintions of rubi rules used

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. 2(143) = 286.

Time = 0.16 (sec) , antiderivative size = 1124, normalized size of antiderivative = 7.75

method	result	size
risch	Expression too large to display	1124

input
```
int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
```

output
```
-2/ln(F)^2/f^2/g^2/n*d^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*
ln(F^(g*(f*x+e)))*x+2/ln(F)^2/f^2/g^2/n*d^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g
*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))*x+1/ln(F)^2/f^2/g^2*c*d/a*ln(F^(g
*(f*x+e)))^2-2/ln(F)^2/f^2/g^2/n^2*c*d/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*
f*x)*(F^(g*(f*x+e)))^n/a)+1/ln(F)^3/f^3/g^3/n*d^2/a*ln(F^(g*(f*x+e)))^2*ln
(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+2/ln(F)/f/g/n*c*d/a*ln(
F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-2/ln(F)^2/f^2/g^2/n*c*d/a*ln
(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-2/ln(F)/f/g
/n*c*d/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+2/ln(F)^2/f^
2/g^2/n*c*d/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(
f*x+e)))+1/ln(F)^2/f^2/g^2*d^2/a*ln(F^(g*(f*x+e)))^2*x-2/ln(F)^2/f^2/g^2/n
^2*d^2/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x+1/ln
(F)/f/g/n*c^2/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-1/ln(F)/f/g
/n*c^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-2/3/ln(F)^3/f^
3/g^3*d^2/a*ln(F^(g*(f*x+e)))^3+2/ln(F)^3/f^3/g^3/n^3*d^2/a*polylog(3,-b*F
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/ln(F)/f/g/n*d^2/a*ln(F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2+1/ln(F)^3/f^3/g^3/n*d^2/a*ln(F^(
n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2-1/ln(F)/f/g/n
*d^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2-1/ln(F)^3/f^
3/g^3/n*d^2/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(...
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.87

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx =$$

$$-\frac{3\left(d^2e^2-2\,cdef+c^2f^2\right)g^2n^2\log\left(F^{fgnx+egn}b+a\right)\log\left(F\right)^2-\left(d^2f^3g^3n^3x^3+3\,cdf^3g^3n^3x^2+3\,c^2f^3g^3n\right.}{}$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

output
```
-1/3*(3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*g^2*n^2*log(F^(f*g*n*x + e*g*n)*b
+ a)*log(F)^2 - (d^2*f^3*g^3*n^3*x^3 + 3*c*d*f^3*g^3*n^3*x^2 + 3*c^2*f^3*g
^3*n^3*x)*log(F)^3 + 3*(d^2*f^2*g^2*n^2*x^2 + 2*c*d*f^2*g^2*n^2*x - (d^2*e
^2 - 2*c*d*e*f)*g^2*n^2)*log(F)^2*log((F^(f*g*n*x + e*g*n)*b + a)/a) + 6*(
d^2*f*g*n*x + c*d*f*g*n)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1)*log(F)
- 6*d^2*polylog(3, -F^(f*g*n*x + e*g*n)*b/a))/(a*f^3*g^3*n^3*log(F)^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+dx)^2}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*(F**(g*(f*x+e)))**n),x)
```

output
```
Integral((c + d*x)**2/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(142) = 284.

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.09

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = c^2\left(\frac{fgnx+egn}{afgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{afgn\log(F)}\right) - \frac{2\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)cd}{af^2g^2n^2\log(F)^2} - \frac{\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3(-\frac{F^{fgnx}F^{egn}b}{a})\right)d^2}{af^3g^3n^3\log(F)^3} + \frac{d^2f^3g^3n^3x^3\log(F)^3+3\,cdf^3g^3n^3x^2\log(F)^3}{3\,af^3g^3n^3\log(F)^3}$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

output
```
c^2*((f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n
*log(F))) - 2*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-
F^(f*g*n*x)*F^(e*g*n)*b/a))*c*d/(a*f^2*g^2*n^2*log(F)^2) - (f^2*g^2*n^2*x^
2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*
x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^2/(
a*f^3*g^3*n^3*log(F)^3) + 1/3*(d^2*f^3*g^3*n^3*x^3*log(F)^3 + 3*c*d*f^3*g^
3*n^3*x^2*log(F)^3)/(a*f^3*g^3*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^2}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+d\,x)^2}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

input
```
int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n),x)
```

output
```
int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$= \frac{\left(\int \frac{x^2}{f^{fgnx+egn}b+a}dx\right)\log(f)\,a\,d^2fgn + 2\left(\int \frac{x}{f^{fgnx+egn}b+a}dx\right)\log(f)\,acdfgn - \log\left(f^{fgnx+egn}b+a\right)c^2 + \log(}{\log(f)\,afgn}$$

input
```
int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n),x)
```

output
```
(int(x**2/(f**(e*g*n + f*g*n*x)*b + a),x)*log(f)*a*d**2*f*g*n + 2*int(x/(f
**(e*g*n + f*g*n*x)*b + a),x)*log(f)*a*c*d*f*g*n - log(f**(e*g*n + f*g*n*x
)*b + a)*c**2 + log(f)*c**2*f*g*n*x)/(log(f)*a*f*g*n)
```

3.48 $\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{(c+dx)^2}{2ad} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{afgn\log(F)} - \frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{af^2g^2n^2\log^2(F)}$$

output

```
1/2*(d*x+c)^2/a/d-(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a/f/g/n/ln(F)-d*poly
log(2,-b*(F^(g*(f*x+e)))^n/a)/a/f^2/g^2/n^2/ln(F)^2
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{-fgn(c+dx)\log(F)\log\left(1+\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right) + d\,\mathrm{PolyLog}\left(2,-\frac{a\left(F^{g(e+fx)}\right)^{-n}}{b}\right)}{af^2g^2n^2\log^2(F)}$$

input
```
Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
(-(f*g*n*(c + d*x)*Log[F]*Log[1 + a/(b*(F^(g*(e + f*x)))^n)]) + d*PolyLog[
2, -(a/(b*(F^(g*(e + f*x)))^n))])/(a*f^2*g^2*n^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$\downarrow 2615$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n (c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a}$$

$$\downarrow 2620$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}$$

$$\downarrow 2715$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} + \frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}$$

input
```
Int[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
(c + d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(
b*f*g*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(b*f^2*g^2*
n^2*Log[F]^2)))/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. 2(96) = 192.

Time = 0.08 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.65

method	result
risch	$\frac{c\ln\left(F^{ngfx}F^{-ngfx}\left(F^{g(fx+e)}\right)^n\right)}{\ln(F)fgna}-\frac{c\ln\left(\left(F^{g(fx+e)}\right)^nF^{-ngfx}F^{ngfx}b+a\right)}{\ln(F)fgna}+\frac{d\ln\left(F^{g(fx+e)}\right)^2}{2\ln(F)^2f^2g^2a}-\frac{d\ln\left(F^{g(fx+e)}\right)\ln\left(1+\frac{bF^n}{\ln(F)^2}\right.}$

input
```
int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/f/g/n*c/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-1/ln(F)/f
/g/n*c/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/2/ln(F)^2/f^
2/g^2*d/a*ln(F^(g*(f*x+e)))^2-1/ln(F)^2/f^2/g^2/n*d/a*ln(F^(g*(f*x+e)))*ln
(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F)^2/f^2/g^2/n^2*d
/a*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/ln(F)/f/g/
n*d/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-1/ln(F)^2/f^2/g^2/n
*d/a*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-1/ln
(F)/f/g/n*d/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+1/ln(F)
^2/f^2/g^2/n*d/a*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(
g*(f*x+e)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$=\frac{2\left(de-cf\right)gn\log\left(F^{fgnx+egn}b+a\right)\log\left(F\right)+\left(df^2g^2n^2x^2+2\,cf^2g^2n^2x\right)\log\left(F\right)^2-2\left(dfgnx+degn\right)\log}{2\,af^2g^2n^2\log\left(F\right)^2}$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

output
```
1/2*(2*(d*e - c*f)*g*n*log(F^(f*g*n*x + e*g*n)*b + a)*log(F) + (d*f^2*g^2*
n^2*x^2 + 2*c*f^2*g^2*n^2*x)*log(F)^2 - 2*(d*f*g*n*x + d*e*g*n)*log(F)*log
((F^(f*g*n*x + e*g*n)*b + a)/a) - 2*d*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a
 + 1))/(a*f^2*g^2*n^2*log(F)^2)
```

Sympy [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{c+dx}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n),x)
```

output
```
Integral((c + d*x)/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{dx+c}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

output
```
c*((f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n*l
og(F))) + d*integrate(x/(F^(f*g*n*x)*F^(e*g*n)*b + a), x)
```

Giac [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{dx+c}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

output
```
integrate((d*x + c)/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{c+d\,x}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

input
```
int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n),x)
```

output
```
int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n), x)
```

Reduce [F]

$$\int \frac{c+dx}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$
$$= \frac{\left(\int \frac{x}{f^{fgnx+egn}b+a}dx\right)\log(f)\,adfgn - \log\left(f^{fgnx+egn}b+a\right)c + \log(f)\,cfgnx}{\log(f)\,afgn}$$

input
```
int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n),x)
```

output
```
(int(x/(f**(e*g*n + f*g*n*x)*b + a),x)*log(f)*a*d*f*g*n - log(f**(e*g*n +
f*g*n*x)*b + a)*c + log(f)*c*f*g*n*x)/(log(f)*a*f*g*n)
```

3.49 $\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$

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Mupad [B] (verification not implemented) . . . 426
Reduce [B] (verification not implemented) . . . 426

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{x}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{afgn\log(F)}$$

output
```
x/a-ln(a+b*(F^(g*(f*x+e)))^n)/a/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{\log\left(\left(F^{g(e+fx)}\right)^n\right) - \log\left(af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)\right)}{afgn\log(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-1),x]
```

output
```
(Log[(F^(g*(e + f*x)))^n] - Log[a*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F]
])/(a*f*g*n*Log[F])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2720, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$\downarrow 2720$$

$$\frac{\int \frac{F^{-g(e+fx)}}{b\left(F^{g(e+fx)}\right)^n+a} dF^{g(e+fx)}}{fg\log(F)}$$

$$\downarrow 798$$

$$\frac{\int \frac{F^{-g(e+fx)}}{b\left(F^{g(e+fx)}\right)^n+a} d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 47$$

$$\frac{\frac{\int F^{-g(e+fx)} d\left(F^{g(e+fx)}\right)^n}{a} - \frac{b\int \frac{1}{b\left(F^{g(e+fx)}\right)^n+a} d\left(F^{g(e+fx)}\right)^n}{a}}{fgn\log(F)}$$

$$\downarrow 14$$

$$\frac{\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a} - \frac{b\int \frac{1}{b\left(F^{g(e+fx)}\right)^n+a} d\left(F^{g(e+fx)}\right)^n}{a}}{fgn\log(F)}$$

$$\downarrow 16$$

$$\frac{\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^(-1),x]
```

output
```
(Log[(F^(g*(e + f*x)))^n]/a - Log[a + b*(F^(g*(e + f*x)))^n]/a)/(f*g*n*Log
[F])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

method	result	size
norman	$\frac{x}{a} - \frac{\ln\left(a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}\right)}{\ln(F)afgn}$	44
parallelrisch	$\frac{xngf\ln(F)-\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{\ln(F)afgn}$	44
derivativedivides	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a}}{gf\ln(F)n}$	53
default	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a}}{gf\ln(F)n}$	53
risch	$\frac{\ln\left(F^{g(fx+e)}\right)}{\ln(F)afg} - \frac{\ln\left(\left(F^{g(fx+e)}\right)^n+\frac{a}{b}\right)}{\ln(F)afgn}$	62

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n),x,method=_RETURNVERBOSE)
```

output
```
x/a-1/ln(F)/a/f/g/n*ln(a+b*exp(n*ln(exp(g*(f*x+e)*ln(F)))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{fgnx\log(F) - \log\left(F^{fgnx+egn}b+a\right)}{afgn\log(F)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

output
```
(f*g*n*x*log(F) - log(F^(f*g*n*x + e*g*n)*b + a))/(a*f*g*n*log(F))
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{2\operatorname{atan}\left(\frac{2\left(\frac{a}{2b}+\left(F^{g(e+fx)}\right)^n\right)}{\sqrt{-\frac{a^2}{b^2}}}\right)}{bfgn\sqrt{-\frac{a^2}{b^2}}\log\left(F\right)}$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n),x)
```

output
```
2*atan(2*(a/(2*b) + (F**(g*(e + f*x)))**n)/sqrt(-a**2/b**2))/(b*f*g*n*sqrt
(-a**2/b**2)*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{fgnx+egn}{afgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{afgn\log\left(F\right)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

output
```
(f*g*n*x + e*g*n)/(a*f*g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a*f*g*n*log(
F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.80

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{\log\left(|F|^{fgnx}|F|^{egn}\right)}{afgn\log\left(F\right)} - \frac{\log\left(\left|F^{fgnx}F^{egn}b+a\right|\right)}{afgn\log\left(F\right)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

output
```
log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))/(a*f*g*n*log(F)) - log(abs(F^(f*g*n*x
)*F^(e*g*n)*b + a))/(a*f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = -\frac{\ln\left(a+b\left(F^{e\,g+f\,g\,x}\right)^n\right)-f\,g\,n\,x\,\ln(F)}{a\,f\,g\,n\,\ln(F)}$$

input
```
int(1/(a + b*(F^(g*(e + f*x)))^n),x)
```

output
```
-(log(a + b*(F^(e*g + f*g*x))^n) - f*g*n*x*log(F))/(a*f*g*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10

$$\int \frac{1}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \frac{-\log\left(f^{fgnx+egn}b+a\right)+\log(f)\,fgnx}{\log(f)\,afgn}$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n),x)
```

output
```
( - log(f**(e*g*n + f*g*n*x)*b + a) + log(f)*f*g*n*x)/(log(f)*a*f*g*n)
```

3.50 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx$

Optimal result 427
Mathematica [N/A] 427
Rubi [N/A] 428
Maple [N/A] 429
Fricas [N/A] 429
Sympy [N/A] 429
Maxima [N/A] 430
Giac [N/A] 430
Mupad [N/A] 431
Reduce [N/A] 431

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)},x\right)$$

output
```
Defer(Int)(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx$$

input
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)),x]
```

output
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\,dx$$

$$\downarrow 2618$$

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{eg+fgx}\right)^n\right)}\,dx$$

$$\downarrow 2619$$

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{eg+fgx}\right)^n\right)}\,dx$$

input
```
Int[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)(dx+c)}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

output
```
int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + (b*d*x + b*c)*(F^(f*g*x + e*g))^n + a*c), x)
```

Sympy [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)}\,dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)/(d*x+c),x)
```

output
```
Integral(1/((a + b*(F**(e*g + f*g*x))**n)*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(1/((F^((f*x + e)*g*n)*b + a)*(d*x + c)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(1/(((F^((f*x + e)*g))^n*b + a)*(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 23.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx$$

input
```
int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)),x)
```

output
```
int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)}\,dx = \int \frac{1}{f^{fgnx+egn}bc+f^{fgnx+egn}bdx+ac+adx}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c),x)
```

output
```
int(1/(f**(e*g*n + f*g*n*x)*b*c + f**(e*g*n + f*g*n*x)*b*d*x + a*c + a*d*x
),x)
```

3.51 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$

Optimal result . . . 432
Mathematica [N/A] . . . 432
Rubi [N/A] . . . 433
Maple [N/A] . . . 434
Fricas [N/A] . . . 434
Sympy [N/A] . . . 434
Maxima [N/A] . . . 435
Giac [N/A] . . . 435
Mupad [N/A] . . . 436
Reduce [N/A] . . . 436

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}, x\right)$$

output
```
Defer(Int)(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$$

input
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2),x]
```

output
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\,dx$$

$$\downarrow 2618$$

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)}\,dx$$

$$\downarrow 2619$$

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)}\,dx$$

input
```
Int[1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)(dx+c)^2}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

output
```
int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^2}\,dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)/(d*x+c)**2,x)
```

output
```
Integral(1/((a + b*(F**(e*g + f*g*x))**n)*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(1/((F^((f*x + e)*g*n)*b + a)*(d*x + c)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(((F^((f*x + e)*g))^n*b + a)*(d*x + c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$$

input
```
int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2),x)
```

output
```
int(1/((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.16

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^2}\,dx$$
$$= \int \frac{1}{f^{fgnx+egn}b\,c^2+2f^{fgnx+egn}bcdx+f^{fgnx+egn}b\,d^2x^2+a\,c^2+2acdx+a\,d^2x^2}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)/(d*x+c)^2,x)
```

output
```
int(1/(f**(e*g*n + f*g*n*x)*b*c**2 + 2*f**(e*g*n + f*g*n*x)*b*c*d*x + f**(
e*g*n + f*g*n*x)*b*d**2*x**2 + a*c**2 + 2*a*c*d*x + a*d**2*x**2),x)
```

3.52 $\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

Optimal result 438
Mathematica [F] 439
Rubi [A] (verified) 439
Maple [B] (verified) 450
Fricas [B] (verification not implemented) 451
Sympy [F(-1)] 452
Maxima [A] (verification not implemented) 452
Giac [F] 453
Mupad [F(-1)] 453
Reduce [F] 453

Optimal result

Integrand size = 25, antiderivative size = 388

$$\begin{aligned}\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx &= \frac{(c+dx)^4}{4a^2d} - \frac{(c+dx)^3}{a^2fgn\log(F)} \\ &+ \frac{(c+dx)^3}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\ &+ \frac{3d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} \\ &- \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} \\ &+ \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} \\ &- \frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} \\ &- \frac{6d^3\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)} \\ &+ \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} \\ &- \frac{6d^3\,\mathrm{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^4g^4n^4\log^4(F)}\end{aligned}$$

output

```
1/4*(d*x+c)^4/a^2/d-(d*x+c)^3/a^2/f/g/n/ln(F)+(d*x+c)^3/a/f/(a+b*(F^(g*(f*
x+e)))^n)/g/n/ln(F)+3*d*(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/
n^2/ln(F)^2-(d*x+c)^3*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/ln(F)+6*d^2*(d
*x+c)*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3-3*d*(d*x+c
)^2*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/ln(F)^2-6*d^3*polylo
g(3,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^4/g^4/n^4/ln(F)^4+6*d^2*(d*x+c)*polylog(
3,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3-6*d^3*polylog(4,-b*(F^(g
*(f*x+e)))^n/a)/a^2/f^4/g^4/n^4/ln(F)^4
```

Mathematica [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Rubi [A] (verified)

Time = 4.33 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.480$, Rules used = {2616, 2615, 2620, 2621, 2615, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{(c+dx)^3}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^4}{4ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dx}{a}$$

$$\downarrow 2620$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a} - \frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}$$

$$\downarrow 2621$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a} - \frac{b\left(\frac{3d\int\frac{(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)} - \frac{(c+dx)^3}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a} - \frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad} - \frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)} - \frac{(c+dx)^3}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\left(b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^3}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 3011$$

$$\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-$$

$$\frac{b\left(3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{}{bfg}$$

$$a$$

$$\downarrow 2720$$

$$\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-$$

$$\frac{b\left(3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int F^{-g(e+fx)}\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)}\right)}{a}$$

↓ 7143

$$\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-$$

$$b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\,\mathrm{lo}}\right)$$

$$a$$

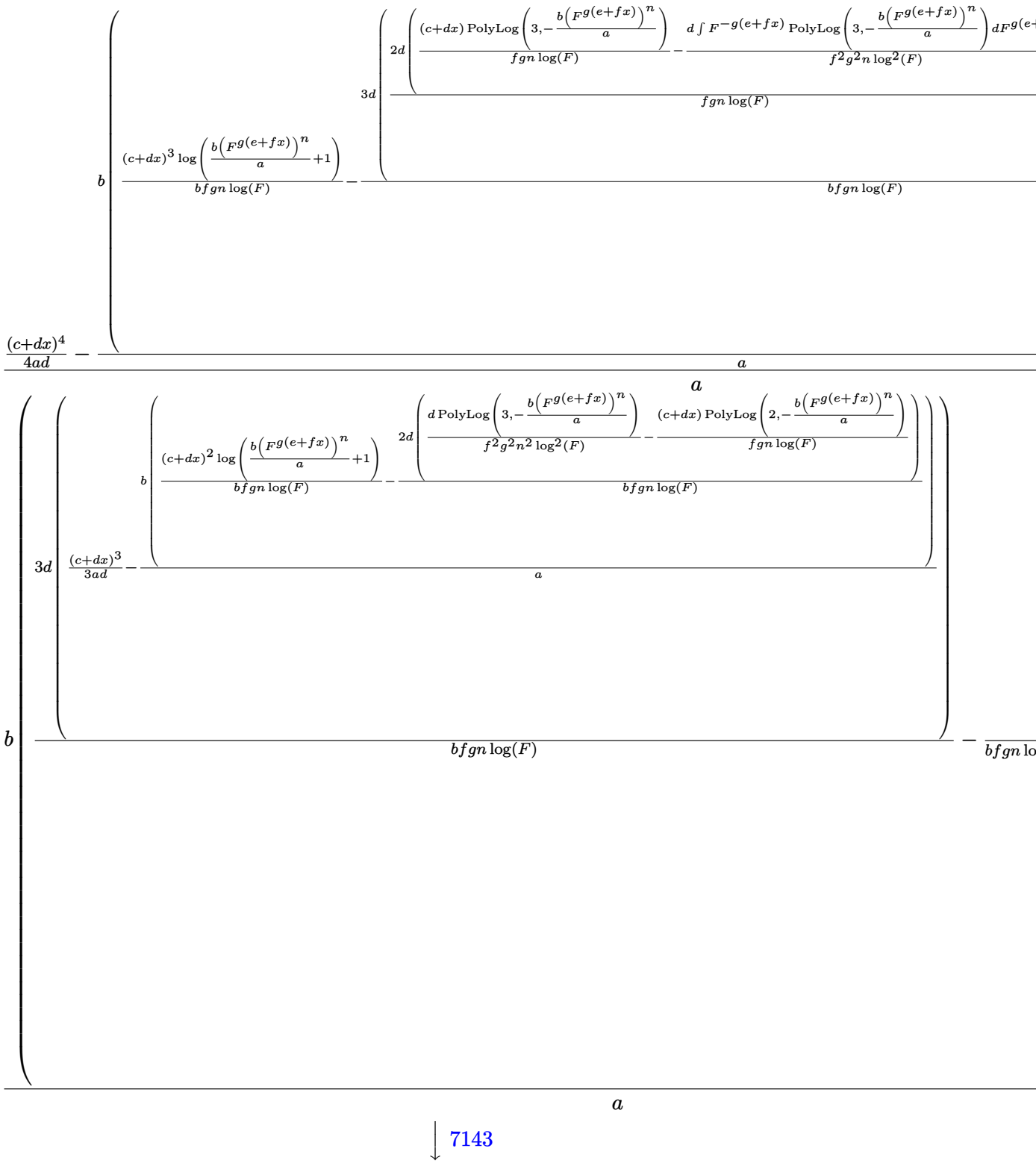
$$\downarrow\ 7163$$

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)} - \frac{d\int \mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}\right)}{fgn\log(F)} - \frac{(c+dx)^2\,\mathrm{Poly}}{bfgn\log(F)}\right)}{a}}{a}$$

$$b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad} - \frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)} - \frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)} - \frac{}{bfgn\,\mathrm{lo}}\right)$$

$$\frac{}{a}$$

$$\downarrow\ 2720$$



$\downarrow$ 7143

$$\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3 \log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)} - \frac{d\,\mathrm{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)}\right)}{fgn\log(F)} - \frac{(c+dx)^2\,\mathrm{PolyLog}}{f}\right)}{bfgn\log(F)}\right)}{a}$$

$$b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad} - \frac{b\left(\frac{(c+dx)^2 \log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)} - \frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)} - \frac{}{bfgn\log}\right)$$

$$\frac{}{a}$$

input
```
Int[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
-((b*(-((c + d*x)^3/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (3*d*(
(c + d*x)^3/(3*a*d) - (b*(((c + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])
/(b*f*g*n*Log[F]) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n
)/a)])/(f*g*n*Log[F])) + (d*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2
*g^2*n^2*Log[F]^2)))/(b*f*g*n*Log[F])))/a))/(b*f*g*n*Log[F])))/a) + ((c +
d*x)^4/(4*a*d) - (b*(((c + d*x)^3*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f
*g*n*Log[F]) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a
)])/(f*g*n*Log[F])) + (2*d*(((c + d*x)*PolyLog[3, -((b*(F^(g*(e + f*x)))^n
)/a)])/(f*g*n*Log[F]) - (d*PolyLog[4, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2*
g^2*n^2*Log[F]^2)))/(f*g*n*Log[F])))/(b*f*g*n*Log[F])))/a)/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3318 vs. 2(386) = 772.

Time = 0.37 (sec) , antiderivative size = 3319, normalized size of antiderivative = 8.55

method	result	size
risch	Expression too large to display	3319

input
```
int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
-6/n^2/g^3/f^3/ln(F)^3/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*
f*x)*b+a)*ln(F^(g*(f*x+e)))-6/n^2/g^2/f^2/ln(F)^2/a^2*c*d^2*ln(F^(n*g*f*x)
*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x+6/n^2/g^3/f^3/ln(F)^3/a^2*c*d^2*ln(F^(n
*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-3/n/g^2/f^2/ln(F
)^2/a^2*c^2*d*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g
*(f*x+e)))+3/n/g/f/ln(F)/a^2*c^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+
e)))^n)*x-3/n/g^2/f^2/ln(F)^2/a^2*c^2*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*
(f*x+e)))^n)*ln(F^(g*(f*x+e)))-3/n/g/f/ln(F)/a^2*c^2*d*ln((F^(g*(f*x+e)))^
n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+3/n/g^2/f^2/ln(F)^2/a^2*c^2*d*ln((F^(g*(
f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+3/n/g/f/ln(F)/a
^2*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x^2+3/n/g^3/f^3/ln
(F)^3/a^2*c*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*
x+e)))^2-3/n/g/f/ln(F)/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*
f*x)*b+a)*x^2-3/n/g^3/f^3/ln(F)^3/a^2*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f
*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2-3/n/g^2/f^2/ln(F)^2/a^2*d^3*ln(F^
(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x^2+3/n/g^3/f^
3/ln(F)^3/a^2*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(
f*x+e)))^2*x+3/n/g^2/f^2/ln(F)^2/a^2*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)
*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))*x^2-3/n/g^3/f^3/ln(F)^3/a^2*d^3*ln((F^
(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2*x-3/n/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1390 vs. $2(384) = 768$.

Time = 0.18 (sec) , antiderivative size = 1390, normalized size of antiderivative = 3.58

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

output

```
-1/4*(4*(a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*g^3*n^
3*log(F)^3 - (a*d^3*f^4*g^4*n^4*x^4 + 4*a*c*d^2*f^4*g^4*n^4*x^3 + 6*a*c^2*
d*f^4*g^4*n^4*x^2 + 4*a*c^3*f^4*g^4*n^4*x - (a*d^3*e^4 - 4*a*c*d^2*e^3*f +
 6*a*c^2*d*e^2*f^2 - 4*a*c^3*e*f^3)*g^4*n^4)*log(F)^4 - ((b*d^3*f^4*g^4*n^
4*x^4 + 4*b*c*d^2*f^4*g^4*n^4*x^3 + 6*b*c^2*d*f^4*g^4*n^4*x^2 + 4*b*c^3*f^
4*g^4*n^4*x - (b*d^3*e^4 - 4*b*c*d^2*e^3*f + 6*b*c^2*d*e^2*f^2 - 4*b*c^3*e
*f^3)*g^4*n^4)*log(F)^4 - 4*(b*d^3*f^3*g^3*n^3*x^3 + 3*b*c*d^2*f^3*g^3*n^3
*x^2 + 3*b*c^2*d*f^3*g^3*n^3*x + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*
e*f^2)*g^3*n^3)*log(F)^3)*F^(f*g*n*x + e*g*n) + 12*((a*d^3*f^2*g^2*n^2*x^2
 + 2*a*c*d^2*f^2*g^2*n^2*x + a*c^2*d*f^2*g^2*n^2)*log(F)^2 + ((b*d^3*f^2*g
^2*n^2*x^2 + 2*b*c*d^2*f^2*g^2*n^2*x + b*c^2*d*f^2*g^2*n^2)*log(F)^2 - 2*(
b*d^3*f*g*n*x + b*c*d^2*f*g*n)*log(F))*F^(f*g*n*x + e*g*n) - 2*(a*d^3*f*g*
n*x + a*c*d^2*f*g*n)*log(F))*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 4
*((a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*g^3*n^3*log(
F)^3 + 3*(a*d^3*e^2 - 2*a*c*d^2*e*f + a*c^2*d*f^2)*g^2*n^2*log(F)^2 + ((b*
d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*g^3*n^3*log(F)^3
+ 3*(b*d^3*e^2 - 2*b*c*d^2*e*f + b*c^2*d*f^2)*g^2*n^2*log(F)^2)*F^(f*g*n*x
 + e*g*n))*log(F^(f*g*n*x + e*g*n)*b + a) + 4*((a*d^3*f^3*g^3*n^3*x^3 + 3*
a*c*d^2*f^3*g^3*n^3*x^2 + 3*a*c^2*d*f^3*g^3*n^3*x + (a*d^3*e^3 - 3*a*c*d^2
*e^2*f + 3*a*c^2*d*e*f^2)*g^3*n^3)*log(F)^3 - 3*(a*d^3*f^2*g^2*n^2*x^2 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

output
```
c^3*((f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a*b + a^2)*f*
g*n*log(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))) + (d^3*x^
3 + 3*c*d^2*x^2 + 3*c^2*d*x)/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2
*f*g*n*log(F)) - 3*c^2*d*x/(a^2*f*g*n*log(F)) + 3*c^2*d*log(F^(f*g*n*x)*F^
(e*g*n)*b + a)/(a^2*f^2*g^2*n^2*log(F)^2) - 3*(c^2*d*f*g*n*log(F) - 2*c*d^
2)*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^(f*g*n*x)
*F^(e*g*n)*b/a))/(a^2*f^3*g^3*n^3*log(F)^3) - (f^3*g^3*n^3*x^3*log(F^(f*g*
n*x)*F^(e*g*n)*b/a + 1)*log(F)^3 + 3*f^2*g^2*n^2*x^2*dilog(-F^(f*g*n*x)*F^
(e*g*n)*b/a)*log(F)^2 - 6*f*g*n*x*log(F)*polylog(3, -F^(f*g*n*x)*F^(e*g*n)
*b/a) + 6*polylog(4, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^3/(a^2*f^4*g^4*n^4*log
(F)^4) - 3*(f^2*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 +
2*f*g*n*x*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*
n*x)*F^(e*g*n)*b/a))*(c*d^2*f*g*n*log(F) - d^3)/(a^2*f^4*g^4*n^4*log(F)^4)
 + 1/4*(d^3*f^4*g^4*n^4*x^4*log(F)^4 + 4*(c*d^2*f*g*n*log(F) - d^3)*f^3*g^
3*n^3*x^3*log(F)^3 + 6*(c^2*d*f^2*g^2*n^2*log(F)^2 - 2*c*d^2*f*g*n*log(F))
*f^2*g^2*n^2*x^2*log(F)^2)/(a^2*f^4*g^4*n^4*log(F)^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(dx+c)^3}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+d\,x)^3}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

input
```
int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2,x)
```

output
```
int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$= \frac{f^{fgnx+egn}\left(\int \frac{x^3}{f^{2fgnx+2egn}b^2+2f^{fgnx+egn}ab+a^2}dx\right)\log(f)\,a^2b\,d^3fgn+3f^{fgnx+egn}\left(\int \frac{x^2}{f^{2fgnx+2egn}b^2+2f^{fgnx+egn}ab+a^2}\right.}$$

input
```
int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output

```
(f**(e*g*n + f*g*n*x)*int(x**3/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*
n + f*g*n*x)*a*b + a**2),x)*log(f)*a**2*b*d**3*f*g*n + 3*f**(e*g*n + f*g*n
*x)*int(x**2/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b +
 a**2),x)*log(f)*a**2*b*c*d**2*f*g*n + 3*f**(e*g*n + f*g*n*x)*int(x/(f**(2
*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b + a**2),x)*log(f)*a*
*2*b*c**2*d*f*g*n - f**(e*g*n + f*g*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*b
*c**3 + f**(e*g*n + f*g*n*x)*log(f)*b*c**3*f*g*n*x - f**(e*g*n + f*g*n*x)*
b*c**3 + int(x**3/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*
a*b + a**2),x)*log(f)*a**3*d**3*f*g*n + 3*int(x**2/(f**(2*e*g*n + 2*f*g*n*
x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b + a**2),x)*log(f)*a**3*c*d**2*f*g*n +
 3*int(x/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b + a**
2),x)*log(f)*a**3*c**2*d*f*g*n - log(f**(e*g*n + f*g*n*x)*b + a)*a*c**3 +
log(f)*a*c**3*f*g*n*x)/(log(f)*a**2*f*g*n*(f**(e*g*n + f*g*n*x)*b + a))
```

3.53 $\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 294

$$\begin{aligned}\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx &= \frac{(c+dx)^3}{3a^2d} - \frac{(c+dx)^2}{a^2fgn\log(F)} \\ &+ \frac{(c+dx)^2}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\ &+ \frac{2d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} \\ &- \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} \\ &+ \frac{2d^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)} \\ &- \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)} \\ &+ \frac{2d^2\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^3g^3n^3\log^3(F)}\end{aligned}$$

output
```
1/3*(d*x+c)^3/a^2/d-(d*x+c)^2/a^2/f/g/n/ln(F)+(d*x+c)^2/a/f/(a+b*(F^(g*(f*
x+e)))^n)/g/n/ln(F)+2*d*(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^
2/ln(F)^2-(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/ln(F)+2*d^2*poly
log(2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3-2*d*(d*x+c)*polylog(
2,-b*(F^(g*(f*x+e)))^n/a)/a^2/f^2/g^2/n^2/ln(F)^2+2*d^2*polylog(3,-b*(F^(g
*(f*x+e)))^n/a)/a^2/f^3/g^3/n^3/ln(F)^3
```

Mathematica [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2d\int\frac{c+dx}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-}{a}$$

$$b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)$$

$$\Big\downarrow 2715$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-}{a}$$

$$b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)$$

$$\Big\downarrow 2838$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 3011$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int F^{-g(e+fx)}\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}$$

$$b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)$$

$$\frac{a}{a}$$

$$\downarrow 7143$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

input
```
Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
-((b*(-((c + d*x)^2/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (2*d*(
(c + d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(
b*f*g*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(b*f^2*g^2*
n^2*Log[F]^2)))/a))/(b*f*g*n*Log[F])))/a) + ((c + d*x)^3/(3*a*d) - (b*(((c
 + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]) - (2*d*(-((
(c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(f*g*n*Log[F])) + (d*P
olyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2*g^2*n^2*Log[F]^2)))/(b*f*g*n
*Log[F])))/a)/a
```

Defintions of rubi rules used

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. $2(292) = 584$.

Time = 0.15 (sec) , antiderivative size = 1650, normalized size of antiderivative = 5.61

method	result	size
risch	Expression too large to display	1650

input
```
int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
```

output

```
-2/a^2/ln(F)^3/f^3/g^3/n^2*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*
x)*b+a)*ln(F^(g*(f*x+e)))-2/a^2/ln(F)^2/f^2/g^2/n^2*d^2*polylog(2,-b*F^(n*
g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*x-2/a^2/ln(F)^2/f^2/g^2/n^2*c*d*p
olylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/a^2/ln(F)^3/f^
3/g^3/n*d^2*ln(F^(g*(f*x+e)))^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x
+e)))^n/a)-2/a^2/ln(F)^2/f^2/g^2/n^2*c*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g
*(f*x+e)))^n)+2/a^2/ln(F)^2/f^2/g^2/n^2*c*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f
*x)*F^(n*g*f*x)*b+a)+1/a^2/ln(F)/f/g/n*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^
(g*(f*x+e)))^n)*x^2+1/a^2/ln(F)^3/f^3/g^3/n*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x
)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))^2-1/a^2/ln(F)/f/g/n*d^2*ln((F^(g*(f
*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x^2-1/a^2/ln(F)^3/f^3/g^3/n*d^2*ln
((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+2/a^2
/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)
*(F^(g*(f*x+e)))^n/a)-2/a^2/ln(F)^2/f^2/g^2/n^2*d^2*ln(F^(n*g*f*x)*F^(-n*g
*f*x)*(F^(g*(f*x+e)))^n)*x+2/a^2/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(n*g*f*x)*F^
(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))+2/a^2/ln(F)^2/f^2/g^2/n^2*
d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+1/a^2/ln(F)^2/f^2
/g^2*d^2*ln(F^(g*(f*x+e)))^2*x+1/a^2/ln(F)^2/f^2/g^2*c*d*ln(F^(g*(f*x+e)))
^2+1/a^2/ln(F)/f/g/n*c^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-1/
a^2/ln(F)/f/g/n*c^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 834 vs. $2(290) = 580$.

Time = 0.09 (sec) , antiderivative size = 834, normalized size of antiderivative = 2.84

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

output

```
1/3*(3*(a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*g^2*n^2*log(F)^2 + (a*d^2*f^3
*g^3*n^3*x^3 + 3*a*c*d*f^3*g^3*n^3*x^2 + 3*a*c^2*f^3*g^3*n^3*x + (a*d^2*e^
3 - 3*a*c*d*e^2*f + 3*a*c^2*e*f^2)*g^3*n^3)*log(F)^3 + ((b*d^2*f^3*g^3*n^3
*x^3 + 3*b*c*d*f^3*g^3*n^3*x^2 + 3*b*c^2*f^3*g^3*n^3*x + (b*d^2*e^3 - 3*b*
c*d*e^2*f + 3*b*c^2*e*f^2)*g^3*n^3)*log(F)^3 - 3*(b*d^2*f^2*g^2*n^2*x^2 +
2*b*c*d*f^2*g^2*n^2*x - (b*d^2*e^2 - 2*b*c*d*e*f)*g^2*n^2)*log(F)^2)*F^(f*
g*n*x + e*g*n) + 6*(a*d^2 + (b*d^2 - (b*d^2*f*g*n*x + b*c*d*f*g*n)*log(F))
*F^(f*g*n*x + e*g*n) - (a*d^2*f*g*n*x + a*c*d*f*g*n)*log(F))*dilog(-(F^(f*
g*n*x + e*g*n)*b + a)/a + 1) - 3*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*g^
2*n^2*log(F)^2 + 2*(a*d^2*e - a*c*d*f)*g*n*log(F) + ((b*d^2*e^2 - 2*b*c*d*
e*f + b*c^2*f^2)*g^2*n^2*log(F)^2 + 2*(b*d^2*e - b*c*d*f)*g*n*log(F))*F^(f
*g*n*x + e*g*n))*log(F^(f*g*n*x + e*g*n)*b + a) - 3*((a*d^2*f^2*g^2*n^2*x^
2 + 2*a*c*d*f^2*g^2*n^2*x - (a*d^2*e^2 - 2*a*c*d*e*f)*g^2*n^2)*log(F)^2 +
((b*d^2*f^2*g^2*n^2*x^2 + 2*b*c*d*f^2*g^2*n^2*x - (b*d^2*e^2 - 2*b*c*d*e*f
)*g^2*n^2)*log(F)^2 - 2*(b*d^2*f*g*n*x + b*d^2*e*g*n)*log(F))*F^(f*g*n*x +
 e*g*n) - 2*(a*d^2*f*g*n*x + a*d^2*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)
*b + a)/a) + 6*(F^(f*g*n*x + e*g*n)*b*d^2 + a*d^2)*polylog(3, -F^(f*g*n*x
+ e*g*n)*b/a))/(F^(f*g*n*x + e*g*n)*a^2*b*f^3*g^3*n^3*log(F)^3 + a^3*f^3*g
^3*n^3*log(F)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

input

```
integrate((d*x+c)**2/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 468, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$= c^2\left(\frac{fgnx+egn}{a^2fgn}+\frac{1}{(F^{fgnx+egn}ab+a^2)fgn\log(F)}-\frac{\log\left(F^{fgnx+egn}b+a\right)}{a^2fgn\log(F)}\right)$$
$$+\frac{d^2x^2+2\,cdx}{F^{fgnx}F^{egn}abfgn\log(F)+a^2fgn\log(F)}$$
$$-\frac{2\,cdx}{a^2fgn\log(F)}+\frac{2\,cd\log\left(F^{fgnx}F^{egn}b+a\right)}{a^2f^2g^2n^2\log(F)^2}$$
$$-\frac{\left(f^2g^2n^2x^2\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)^2+2\,fgnx\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\log(F)-2\,\mathrm{Li}_3(-\frac{F^{fgnx}F^{egn}b}{a})\right)d^2}{a^2f^3g^3n^3\log(F)^3}$$
$$-\frac{2\left(cdfgn\log(F)-d^2\right)\left(fgnx\log\left(\frac{F^{fgnx}F^{egn}b}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{fgnx}F^{egn}b}{a}\right)\right)}{a^2f^3g^3n^3\log(F)^3}$$
$$+\frac{d^2f^3g^3n^3x^3\log(F)^3+3\left(cdfgn\log(F)-d^2\right)f^2g^2n^2x^2\log(F)^2}{3\,a^2f^3g^3n^3\log(F)^3}$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

output
```
c^2*((f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a*b + a^2)*f*
g*n*log(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))) + (d^2*x^
2 + 2*c*d*x)/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*g*n*log(F)) -
 2*c*d*x/(a^2*f*g*n*log(F)) + 2*c*d*log(F^(f*g*n*x)*F^(e*g*n)*b + a)/(a^2*
f^2*g^2*n^2*log(F)^2) - (f^2*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1
)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylo
g(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^2/(a^2*f^3*g^3*n^3*log(F)^3) - 2*(c*d*
f*g*n*log(F) - d^2)*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) + d
ilog(-F^(f*g*n*x)*F^(e*g*n)*b/a))/(a^2*f^3*g^3*n^3*log(F)^3) + 1/3*(d^2*f^
3*g^3*n^3*x^3*log(F)^3 + 3*(c*d*f*g*n*log(F) - d^2)*f^2*g^2*n^2*x^2*log(F)
^2)/(a^2*f^3*g^3*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(dx+c)^2}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+d\,x)^2}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2,x)
```

output
```
int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$= \frac{f^{fgnx+egn}\left(\int \frac{x^2}{f^{2fgnx+2egn}b^2+2f^{fgnx+egn}ab+a^2}dx\right)\log(f)\,a^2b\,d^2fgn+2f^{fgnx+egn}\left(\int \frac{x}{f^{2fgnx+2egn}b^2+2f^{fgnx+egn}ab+a^2}\right.}$$

input
```
int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output

```
(f**(e*g*n + f*g*n*x)*int(x**2/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*
n + f*g*n*x)*a*b + a**2),x)*log(f)*a**2*b*d**2*f*g*n + 2*f**(e*g*n + f*g*n
*x)*int(x/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b + a*
*2),x)*log(f)*a**2*b*c*d*f*g*n - f**(e*g*n + f*g*n*x)*log(f**(e*g*n + f*g*
n*x)*b + a)*b*c**2 + f**(e*g*n + f*g*n*x)*log(f)*b*c**2*f*g*n*x - f**(e*g*
n + f*g*n*x)*b*c**2 + int(x**2/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*
n + f*g*n*x)*a*b + a**2),x)*log(f)*a**3*d**2*f*g*n + 2*int(x/(f**(2*e*g*n
+ 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b + a**2),x)*log(f)*a**3*c*d*
f*g*n - log(f**(e*g*n + f*g*n*x)*b + a)*a*c**2 + log(f)*a*c**2*f*g*n*x)/(l
og(f)*a**2*f*g*n*(f**(e*g*n + f*g*n*x)*b + a))
```

3.54 $\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{(c+dx)^2}{2a^2d} - \frac{dx}{a^2fgn\log(F)} + \frac{c+dx}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} + \frac{d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2f^2g^2n^2\log^2(F)} - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2fgn\log(F)} - \frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^2f^2g^2n^2\log^2(F)}$$

output

```
1/2*(d*x+c)^2/a^2/d-d*x/a^2/f/g/n/ln(F)+(d*x+c)/a/f/(a+b*(F^(g*(f*x+e)))^n
)/g/n/ln(F)+d*ln(a+b*(F^(g*(f*x+e)))^n)/a^2/f^2/g^2/n^2/ln(F)^2-(d*x+c)*ln
(1+b*(F^(g*(f*x+e)))^n/a)/a^2/f/g/n/ln(F)-d*polylog(2,-b*(F^(g*(f*x+e)))^n
/a)/a^2/f^2/g^2/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {2616, 2615, 2620, 2621, 2715, 2720, 798, 47, 14, 16, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{c+dx}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^2}{2ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}\,dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dx}{a}$$

$$\downarrow 2621$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a} - \frac{b\left(\frac{d\int \frac{1}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2715$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a} - \frac{b\left(\frac{d\int \frac{1}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a} - \frac{b\left(\frac{d\int \frac{F^{-g(e+fx)}}{b\left(F^{g(e+fx)}\right)^n+a}dF^{g(e+fx)}}{bf^2g^2n\log^2(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 798$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int \left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a} - \frac{b\left(\frac{d\int \frac{F^{-g(e+fx)}}{b\left(F^{g(e+fx)}\right)^n+a}d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 47$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a} -$$

$$\frac{b\left(\frac{d\left(\frac{\int F^{-g(e+fx)}d\left(F^{g(e+fx)}\right)^n}{a} - \frac{b\int\frac{1}{b\left(F^{g(e+fx)}\right)^n+a}d\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 14$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a} -$$

$$\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a} - \frac{b\int\frac{1}{b\left(F^{g(e+fx)}\right)^n+a}d\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 16$$

$$\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a} -$$

$$\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{\frac{(c+dx)^2}{2ad} - \frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} + \frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}}{a} - \frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)} - \frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}}{a}$$

input
```
Int[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
-((b*(-((c + d*x)/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (d*(Log[
(F^(g*(e + f*x)))^n]/a - Log[a + b*(F^(g*(e + f*x)))^n]/a))/(b*f^2*g^2*n^2
*Log[F]^2)))/a) + ((c + d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g*(
e + f*x)))^n)/a])/(b*f*g*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^
n)/a)])/(b*f^2*g^2*n^2*Log[F]^2)))/a)/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(189) = 378.

Time = 0.16 (sec) , antiderivative size = 591, normalized size of antiderivative = 3.09

method	result
risch	$\frac{dx+c}{af\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)gn\ln(F)}+\frac{d\ln\left(F^{g(fx+e)}\right)^{2}}{2a^{2}g^{2}f^{2}\ln(F)^{2}}-\frac{d\ln\left(F^{g(fx+e)}\right)\ln\left(1+\frac{b\,F^{ngfx}F^{-ngfx}\left(F^{g(fx+e)}\right)^{n}}{a}\right)}{a^{2}n\,g^{2}f^{2}\ln(F)^{2}}-\frac{d\,\mathrm{polylog}\left(2,-$

input

```
int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
```

output

```
(d*x+c)/a/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)+1/2/a^2/g^2/f^2/ln(F)^2*d*ln
(F^(g*(f*x+e)))^2-1/a^2/n/g^2/f^2/ln(F)^2*d*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*
g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/a^2/n^2/g^2/f^2/ln(F)^2*d*polyl
og(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/a^2/n/g/f/ln(F)*c*
ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-1/a^2/n/g/f/ln(F)*c*ln((F^(
g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)-1/a^2/n^2/g^2/f^2/ln(F)^2*d*ln
(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+1/a^2/n^2/g^2/f^2/ln(F)^2*d*l
n((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/a^2/n/g/f/ln(F)*d*ln(F
^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-1/a^2/n/g^2/f^2/ln(F)^2*d*ln(
F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-1/a^2/n/g/f/
ln(F)*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x+1/a^2/n/g^2/f
^2/ln(F)^2*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f
*x+e)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(188) = 376.

Time = 0.08 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx =$$

$$-\frac{2\,(ade-acf)gn\log(F)-(adf^2g^2n^2x^2+2\,acf^2g^2n^2x-(ade^2-2\,acef)g^2n^2)\log(F)^2-((bdf^2g^2n^2x$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a*d*e - a*c*f)*g*n*log(F) - (a*d*f^2*g^2*n^2*x^2 + 2*a*c*f^2*g^2*
n^2*x - (a*d*e^2 - 2*a*c*e*f)*g^2*n^2)*log(F)^2 - ((b*d*f^2*g^2*n^2*x^2 +
2*b*c*f^2*g^2*n^2*x - (b*d*e^2 - 2*b*c*e*f)*g^2*n^2)*log(F)^2 - 2*(b*d*f*g
*n*x + b*d*e*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 2*(F^(f*g*n*x + e*g*n)*b*d
 + a*d)*dilog(-(F^(f*g*n*x + e*g*n)*b + a)/a + 1) - 2*((a*d*e - a*c*f)*g*n
*log(F) + ((b*d*e - b*c*f)*g*n*log(F) + b*d)*F^(f*g*n*x + e*g*n) + a*d)*lo
g(F^(f*g*n*x + e*g*n)*b + a) + 2*((b*d*f*g*n*x + b*d*e*g*n)*F^(f*g*n*x + e
*g*n)*log(F) + (a*d*f*g*n*x + a*d*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*
b + a)/a))/(F^(f*g*n*x + e*g*n)*a^2*b*f^2*g^2*n^2*log(F)^2 + a^3*f^2*g^2*n
^2*log(F)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

output
```
d*(x/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*g*n*log(F)) + integra
te((f*g*n*x*log(F) - 1)/(F^(f*g*n*x)*F^(e*g*n)*a*b*f*g*n*log(F) + a^2*f*g*
n*log(F)), x)) + c*((f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n
)*a*b + a^2)*f*g*n*log(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log
(F)))
```

Giac [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{c+d\,x}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

input
```
int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2,x)
```

output
```
int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

Reduce [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$= \frac{f^{fgnx+egn}\left(\int \frac{x}{f^{2fgnx+2egn}b^2+2f^{fgnx+egn}ab+a^2}dx\right)\log(f)\,a^2bdfgn - f^{fgnx+egn}\log\left(f^{fgnx+egn}b+a\right)bc + f^{fgnx+e}}{\log(}$$

input
```
int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output
```
(f**(e*g*n + f*g*n*x)*int(x/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n +
 f*g*n*x)*a*b + a**2),x)*log(f)*a**2*b*d*f*g*n - f**(e*g*n + f*g*n*x)*log(
f**(e*g*n + f*g*n*x)*b + a)*b*c + f**(e*g*n + f*g*n*x)*log(f)*b*c*f*g*n*x
- f**(e*g*n + f*g*n*x)*b*c + int(x/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(
e*g*n + f*g*n*x)*a*b + a**2),x)*log(f)*a**3*d*f*g*n - log(f**(e*g*n + f*g*
n*x)*b + a)*a*c + log(f)*a*c*f*g*n*x)/(log(f)*a**2*f*g*n*(f**(e*g*n + f*g*
n*x)*b + a))
```

3.55 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{x}{a^2} + \frac{1}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2fgn\log(F)}$$

output
```
x/a^2+1/a/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)-ln(a+b*(F^(g*(f*x+e)))^n)/a^
2/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{\frac{1}{af\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn} + \frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^2fgn} - \frac{\log\left(a^3fgn\log(F)+a^2bf\left(F^{g(e+fx)}\right)^n gn\log(F)\right)}{a^2fgn}}{\log(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-2),x]
```

output
```
(1/(a*f*(a + b*(F^(g*(e + f*x)))^n)*g*n) + Log[(F^(g*(e + f*x)))^n]/(a^2*f
*g*n) - Log[a^3*f*g*n*Log[F] + a^2*b*f*(F^(g*(e + f*x)))^n*g*n*Log[F]]/(a^
2*f*g*n))/Log[F]
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$\downarrow 2720$$

$$\frac{\int \frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,dF^{g(e+fx)}}{fg\log(F)}$$

$$\downarrow 798$$

$$\frac{\int \frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\,d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{F^{-g(e+fx)}}{a^2}-\frac{b}{a^2\left(b\left(F^{g(e+fx)}\right)^n+a\right)}-\frac{b}{a\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\right)d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2}+\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^2}+\frac{1}{a\left(a+b\left(F^{g(e+fx)}\right)^n\right)}}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^(-2),x]
```

output
```
(1/(a*(a + b*(F^(g*(e + f*x)))^n)) + Log[(F^(g*(e + f*x)))^n]/a^2 - Log[a
+ b*(F^(g*(e + f*x)))^n]/a^2)/(f*g*n*Log[F])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a^2}+\frac{1}{a\left(a+b\left(F^{g(fx+e)}\right)^n\right)}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a^2}}{gf\ln(F)n}$
default	$\frac{-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a^2}+\frac{1}{a\left(a+b\left(F^{g(fx+e)}\right)^n\right)}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a^2}}{gf\ln(F)n}$
risch	$\frac{\ln\left(F^{g(fx+e)}\right)}{\ln(F)fg\,a^2}+\frac{1}{af\left(a+b\left(F^{g(fx+e)}\right)^n\right)gn\ln(F)}-\frac{\ln\left(\left(F^{g(fx+e)}\right)^n+\frac{a}{b}\right)}{\ln(F)fgn\,a^2}$
parallelrisch	$\frac{b^2\left(F^{g(fx+e)}\right)^n x\ln(F)fgn+x\ln(F)abfgn-\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)\left(F^{g(fx+e)}\right)^n b^2-\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)ab+ab}{\ln(F)a^2bfgn\left(a+b\left(F^{g(fx+e)}\right)^n\right)}$
norman	$\frac{-\frac{b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{\ln(F)fgn\,a^2}+\frac{b\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{a^2\ln(F)fg}+\frac{\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}{\ln(F)afg}}{a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}-\frac{\ln\left(a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(}\right.\right.}{\ln(F)fgn\,a^2}$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2,x,method=_RETURNVERBOSE)
```

output
```
1/g/f/ln(F)/n*(-1/a^2*ln(a+b*(F^(g*(f*x+e)))^n)+1/a/(a+b*(F^(g*(f*x+e)))^n
)+1/a^2*ln((F^(g*(f*x+e)))^n))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$=\frac{F^{fgnx+egn}bfgnx\log(F)+afgnx\log(F)-\left(F^{fgnx+egn}b+a\right)\log\left(F^{fgnx+egn}b+a\right)+a}{F^{fgnx+egn}a^2bfgn\log(F)+a^3fgn\log(F)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

output
```
(F^(f*g*n*x + e*g*n)*b*f*g*n*x*log(F) + a*f*g*n*x*log(F) - (F^(f*g*n*x + e
*g*n)*b + a)*log(F^(f*g*n*x + e*g*n)*b + a) + a)/(F^(f*g*n*x + e*g*n)*a^2*
b*f*g*n*log(F) + a^3*f*g*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

output
```
Integral((a + b*(F**(g*(e + f*x)))**n)**(-2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{fgnx+egn}{a^2fgn} + \frac{1}{(F^{fgnx+egn}ab+a^2)fgn\log(F)} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{a^2fgn\log(F)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

output
```
(f*g*n*x + e*g*n)/(a^2*f*g*n) + 1/((F^(f*g*n*x + e*g*n)*a*b + a^2)*f*g*n*l
og(F)) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^2*f*g*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.46

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{\log\left(|F|^{fgnx}|F|^{egn}\right)}{a^2fgn\log(F)} - \frac{\log\left(\left|F^{fgnx}F^{egn}b+a\right|\right)}{a^2fgn\log(F)} + \frac{1}{(F^{fgnx}F^{egn}b+a)afgn\log(F)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

output
```
log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))/(a^2*f*g*n*log(F)) - log(abs(F^(f*g*n
*x)*F^(e*g*n)*b + a))/(a^2*f*g*n*log(F)) + 1/((F^(f*g*n*x)*F^(e*g*n)*b + a
)*a*f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \frac{x}{a^2} + \frac{1}{a\,f\,g\,n\,\ln(F)\,\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)} - \frac{\ln\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)}{a^2\,f\,g\,n\,\ln(F)}$$

input
```
int(1/(a + b*(F^(g*(e + f*x)))^n)^2,x)
```

output
```
x/a^2 + 1/(a*f*g*n*log(F)*(a + b*(F^(f*g*x)*F^(e*g))^n)) - log(a + b*(F^(f
*g*x)*F^(e*g))^n)/(a^2*f*g*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.77

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$= \frac{-f^{fgnx+egn}\log\left(f^{fgnx+egn}b+a\right)b + f^{fgnx+egn}\log(f)\,bfgnx - f^{fgnx+egn}b - \log\left(f^{fgnx+egn}b+a\right)a + \log(f}{\log(f)\,a^2 fgn\left(f^{fgnx+egn}b+a\right)}$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output
```
( - f**(e*g*n + f*g*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*b + f**(e*g*n + f
*g*n*x)*log(f)*b*f*g*n*x - f**(e*g*n + f*g*n*x)*b - log(f**(e*g*n + f*g*n*
x)*b + a)*a + log(f)*a*f*g*n*x)/(log(f)*a**2*f*g*n*(f**(e*g*n + f*g*n*x)*b
 + a))
```

3.56 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx$

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Rubi [N/A] . . . 486
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Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)},x\right)$$

output
```
Defer(Int)(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx$$

input
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)),x]
```

output
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$\downarrow 2618$$

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}\,dx$$

$$\downarrow 2619$$

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}\,dx$$

input
```
Int[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2 (dx+c)} dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

output
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)} dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)} dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*(F^(f*g*x + e*g))^(2*n) +
2*(a*b*d*x + a*b*c)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)} dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)} dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c),x)
```

output
```
Integral(1/((a + b*(F**(e*g + f*g*x))**n)**2*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 192, normalized size of antiderivative = 7.68

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2(dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/(a^2*d*f*g*n*x*log(F) + a^2*c*f*g*n*log(F) + (F^(e*g*n)*a*b*d*f*g*n*x*lo
g(F) + F^(e*g*n)*a*b*c*f*g*n*log(F))*F^(f*g*n*x)) + integrate((d*f*g*n*x*l
og(F) + c*f*g*n*log(F) + d)/(a^2*d^2*f*g*n*x^2*log(F) + 2*a^2*c*d*f*g*n*x*
log(F) + a^2*c^2*f*g*n*log(F) + (F^(e*g*n)*a*b*d^2*f*g*n*x^2*log(F) + 2*F^
(e*g*n)*a*b*c*d*f*g*n*x*log(F) + F^(e*g*n)*a*b*c^2*f*g*n*log(F))*F^(f*g*n*
x)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2(dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(1/(((F^((f*x + e)*g))^n*b + a)^2*(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 23.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx$$

input
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)),x)
```

output
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.60

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2(c+dx)}\,dx$$
$$= \int \frac{1}{f^{2fgnx+2egn}b^2c + f^{2fgnx+2egn}b^2dx + 2f^{fgnx+egn}abc + 2f^{fgnx+egn}abdx + a^2c + a^2dx}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c),x)
```

output
```
int(1/(f**(2*e*g*n + 2*f*g*n*x)*b**2*c + f**(2*e*g*n + 2*f*g*n*x)*b**2*d*x
 + 2*f**(e*g*n + f*g*n*x)*a*b*c + 2*f**(e*g*n + f*g*n*x)*a*b*d*x + a**2*c
+ a**2*d*x),x)
```

3.57 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx$

Optimal result 490
Mathematica [N/A] 490
Rubi [N/A] 491
Maple [N/A] 492
Fricas [N/A] 492
Sympy [N/A] 492
Maxima [N/A] 493
Giac [N/A] 493
Mupad [N/A] 494
Reduce [N/A] 494

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}, x\right)$$

output
```
Defer(Int)(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx$$

input
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2),x]
```

output
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$
$$\downarrow 2618$$
$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}dx$$
$$\downarrow 2619$$
$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}dx$$

input
```
Int[1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2 (dx+c)^2}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

output
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.32

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*(F^(f*g*x + e*g))^(2*n) + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b
*c^2)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 22.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)^2}\,dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)**2/(d*x+c)**2,x)
```

output
```
Integral(1/((a + b*(F**(e*g + f*g*x))**n)**2*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 11.20

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/(a^2*d^2*f*g*n*x^2*log(F) + 2*a^2*c*d*f*g*n*x*log(F) + a^2*c^2*f*g*n*log
(F) + (F^(e*g*n)*a*b*d^2*f*g*n*x^2*log(F) + 2*F^(e*g*n)*a*b*c*d*f*g*n*x*lo
g(F) + F^(e*g*n)*a*b*c^2*f*g*n*log(F))*F^(f*g*n*x)) + integrate((d*f*g*n*x
*log(F) + c*f*g*n*log(F) + 2*d)/(a^2*d^3*f*g*n*x^3*log(F) + 3*a^2*c*d^2*f*
g*n*x^2*log(F) + 3*a^2*c^2*d*f*g*n*x*log(F) + a^2*c^3*f*g*n*log(F) + (F^(e
*g*n)*a*b*d^3*f*g*n*x^3*log(F) + 3*F^(e*g*n)*a*b*c*d^2*f*g*n*x^2*log(F) +
3*F^(e*g*n)*a*b*c^2*d*f*g*n*x*log(F) + F^(e*g*n)*a*b*c^3*f*g*n*log(F))*F^(
f*g*n*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(((F^((f*x + e)*g))^n*b + a)^2*(d*x + c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2 (c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 6.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^2}\,dx$$
$$= \int \frac{1}{f^{2fgnx+2egn}b^2c^2 + 2f^{2fgnx+2egn}b^2cdx + f^{2fgnx+2egn}b^2d^2x^2 + 2f^{fgnx+egn}ab\,c^2 + 4f^{fgnx+egn}abcdx + 2f^{fg}}$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^2/(d*x+c)^2,x)
```

output
```
int(1/(f**(2*e*g*n + 2*f*g*n*x)*b**2*c**2 + 2*f**(2*e*g*n + 2*f*g*n*x)*b**
2*c*d*x + f**(2*e*g*n + 2*f*g*n*x)*b**2*d**2*x**2 + 2*f**(e*g*n + f*g*n*x)
*a*b*c**2 + 4*f**(e*g*n + f*g*n*x)*a*b*c*d*x + 2*f**(e*g*n + f*g*n*x)*a*b*
d**2*x**2 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2),x)
```

3.58 $\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$

Optimal result 496
Mathematica [F] 497
Rubi [A] (verified) 497
Maple [B] (verified) 523
Fricas [B] (verification not implemented) 524
Sympy [F(-1)] 525
Maxima [A] (verification not implemented) 526
Giac [F] 526
Mupad [F(-1)] 527
Reduce [F] 527

Optimal result

Integrand size = 25, antiderivative size = 594

$$\begin{aligned}\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx &= \frac{(c+dx)^4}{4a^3d} + \frac{3d(c+dx)^2}{2a^3f^2g^2n^2\log^2(F)} \\ &- \frac{3d(c+dx)^2}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\ &- \frac{3(c+dx)^3}{2a^3fgn\log(F)} + \frac{(c+dx)^3}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\ &+ \frac{(c+dx)^3}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\ &- \frac{3d^2(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\ &+ \frac{9d(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{2a^3f^2g^2n^2\log^2(F)} \\ &- \frac{(c+dx)^3\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\ &- \frac{3d^3\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} \\ &+ \frac{9d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\ &- \frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\ &- \frac{9d^3\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)} \\ &+ \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\ &- \frac{6d^3\,\mathrm{PolyLog}\left(4,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^4g^4n^4\log^4(F)}\end{aligned}$$

output
```
1/4*(d*x+c)^4/a^3/d+3/2*d*(d*x+c)^2/a^3/f^2/g^2/n^2/ln(F)^2-3/2*d*(d*x+c)^
2/a^2/f^2/(a+b*(F^(g*(f*x+e)))^n)/g^2/n^2/ln(F)^2-3/2*(d*x+c)^3/a^3/f/g/n/
ln(F)+1/2*(d*x+c)^3/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+(d*x+c)^3/a^2/
f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)-3*d^2*(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n
/a)/a^3/f^3/g^3/n^3/ln(F)^3+9/2*d*(d*x+c)^2*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^
3/f^2/g^2/n^2/ln(F)^2-(d*x+c)^3*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f/g/n/ln(F
)-3*d^3*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^4/g^4/n^4/ln(F)^4+9*d^2*(d
*x+c)*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^3/g^3/n^3/ln(F)^3-3*d*(d*x+c
)^2*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^2/g^2/n^2/ln(F)^2-9*d^3*polylo
g(3,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^4/g^4/n^4/ln(F)^4+6*d^2*(d*x+c)*polylog(
3,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^3/g^3/n^3/ln(F)^3-6*d^3*polylog(4,-b*(F^(g
*(f*x+e)))^n/a)/a^3/f^4/g^4/n^4/ln(F)^4
```

Mathematica [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

input
```
Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
Integrate[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3, x]
```

Rubi [A] (verified)

Time = 10.48 (sec) , antiderivative size = 867, normalized size of antiderivative = 1.46, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2616, 2616, 2615, 2620, 2621, 2615, 2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2616$$

$$\frac{\int \frac{(c+dx)^3}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^4}{4ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^3}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{(c+dx)^4}{4ad} - \frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)} - \frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a} - \frac{b\left(\frac{3d\int \frac{(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)} - \frac{(c+dx)^3}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}}{a} - \frac{b\left(\frac{3d\int \frac{(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{2bfgn\log(F)} - \frac{(c+dx)^3}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\lo}\right.}{a}$$

$$\left.\frac{b\left(\frac{3d\int\frac{(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{2bfgn\log(F)}-\frac{(c+dx)^3}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}\right)$$

$$\downarrow 2616$$

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\lo}\right.}{a}$$

$$\left.b\left(\frac{3d\left(\frac{\int\frac{(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}\right)}{2bfgn\log(F)}-\frac{(c+dx)^3}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)$$

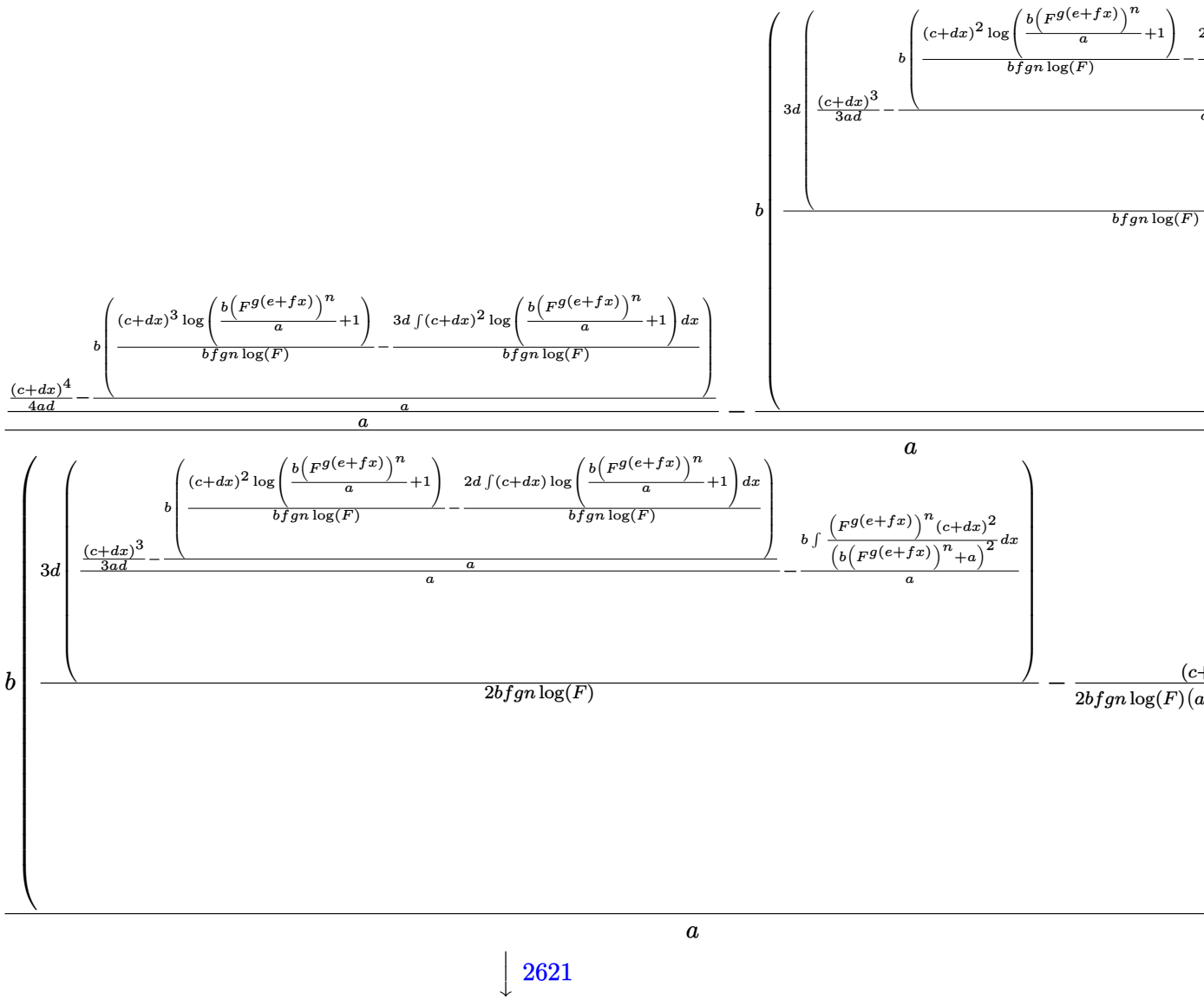
$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\lo}\right)}{a}$$

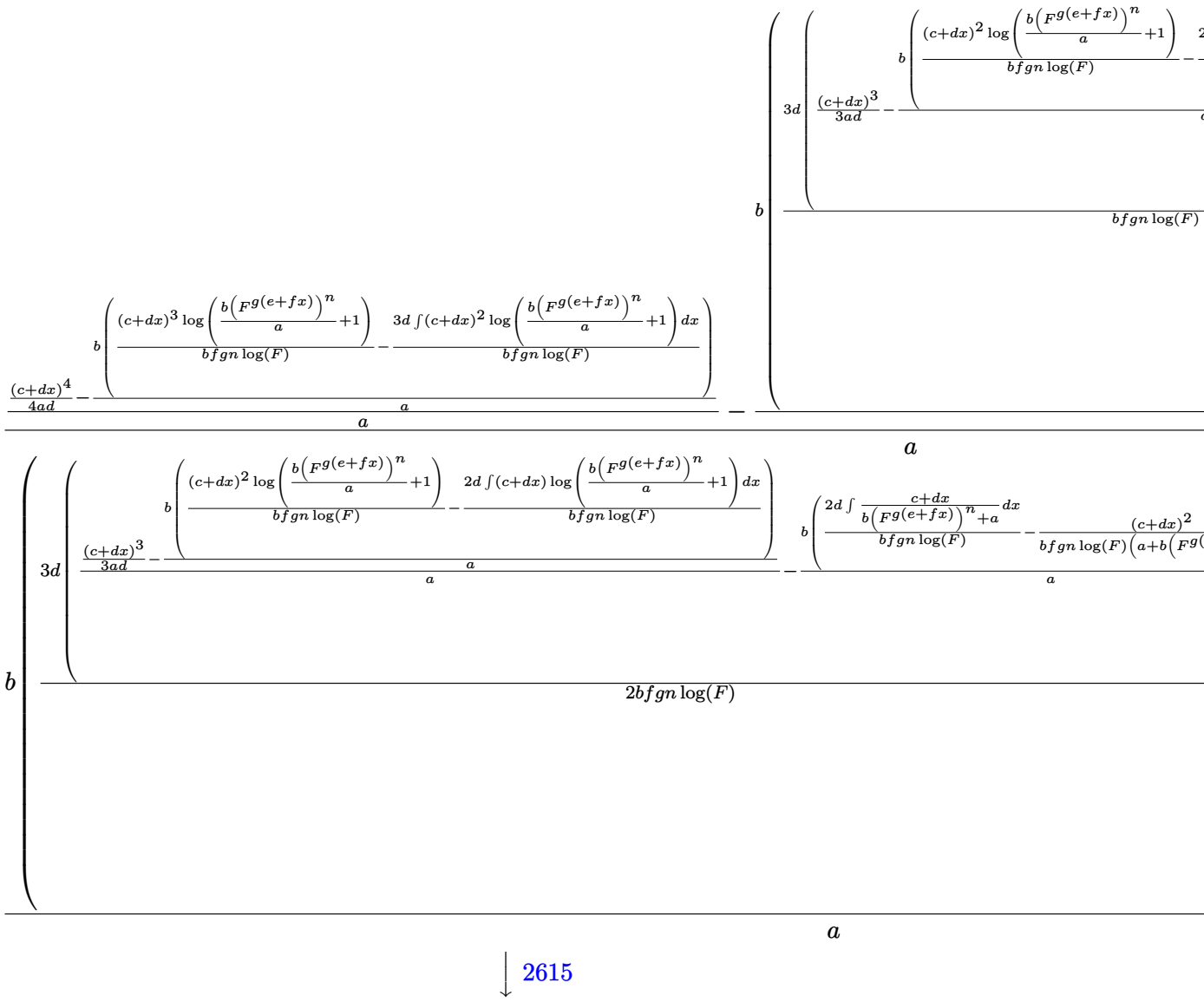
$$b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}\right)}{2bfgn\log(F)}-\frac{(c+dx)^3}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)$$

$$a$$

$$\downarrow 2620$$



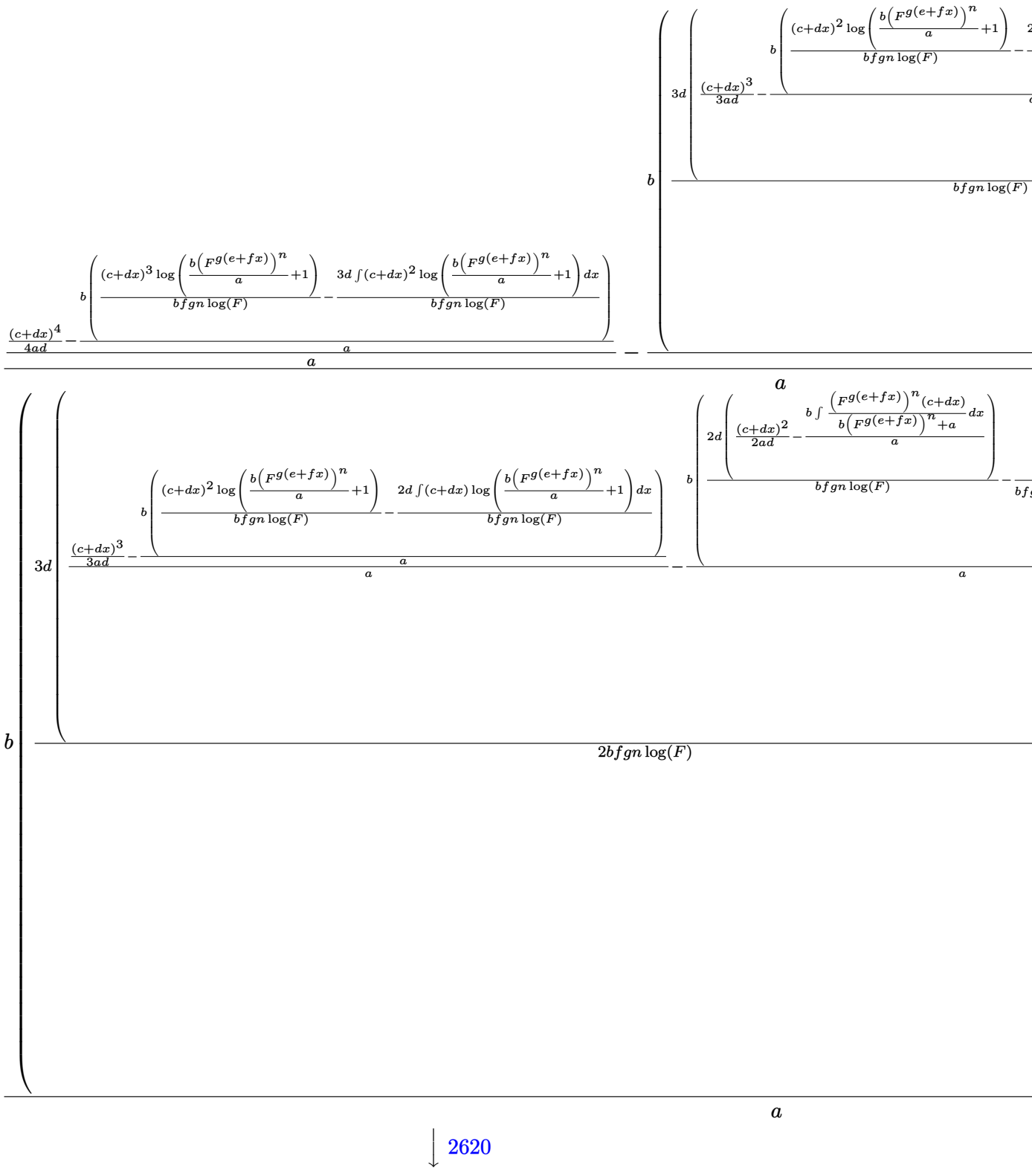
$\downarrow$ 2621



$$\frac{\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2\cdots}{\cdots}\right)}{\cdots}\right)}{bfgn\log(F)}\cdots\right)}{\cdots}}{a}$$

$$\frac{b\left(\frac{3d\left(\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2d\int\frac{c+dx}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(\cdots}\right.\right.}\cdots\right)}{a}\cdots\right)}{2bfgn\log(F)}\cdots\right)}{a}$$

$\downarrow$ 2615



2620

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)}{a}-\frac{b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)}{a}-\frac{b\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)}{2bfgn\log(F)}}{a}}{a}$$

2715

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}}{a}}{a}$$

2838

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\int(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}}{a}}{a}$$

3011

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(3d\left(\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(2d\left(\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\left(\frac{d\int\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{\frac{c+dx}{ad}-\frac{b\left(\frac{\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\left(\frac{d\int\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{c+dx}dx}{fgn\log(F)}-\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{\frac{\log(c+dx)}{ad}-\frac{b\left(\frac{\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\left(\frac{d\int\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{c+dx}dx}{fgn\log(F)}-\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{\frac{1}{ad(c+dx)}-\frac{b\left(\frac{\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\left(\frac{d\int\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{c+dx}dx}{fgn\log(F)}-\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{\frac{1}{2ad(c+dx)^2}-\frac{b\left(\frac{\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\left(\frac{d\int\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{c+dx}dx}{fgn\log(F)}-\frac{\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}}{a}}{a}}{a}}{a}}{a}}{a}$$

2720

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{3d\left(\frac{3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int F^{-g(e+fx)}\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx}\right.\right.\right.

7143

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\int(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(3d\left(\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{a}\right)}{a}$$

7163

$$\frac{\frac{(c+dx)^4}{4ad}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}-\frac{d\int\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dx}{fgn\log(F)}\right)}{fgn\log(F)}-\frac{(c+dx)^2\,\mathrm{PolyLo}}{f}\right)}{bfgn\log(F)}\right)}{a}}{a}$$

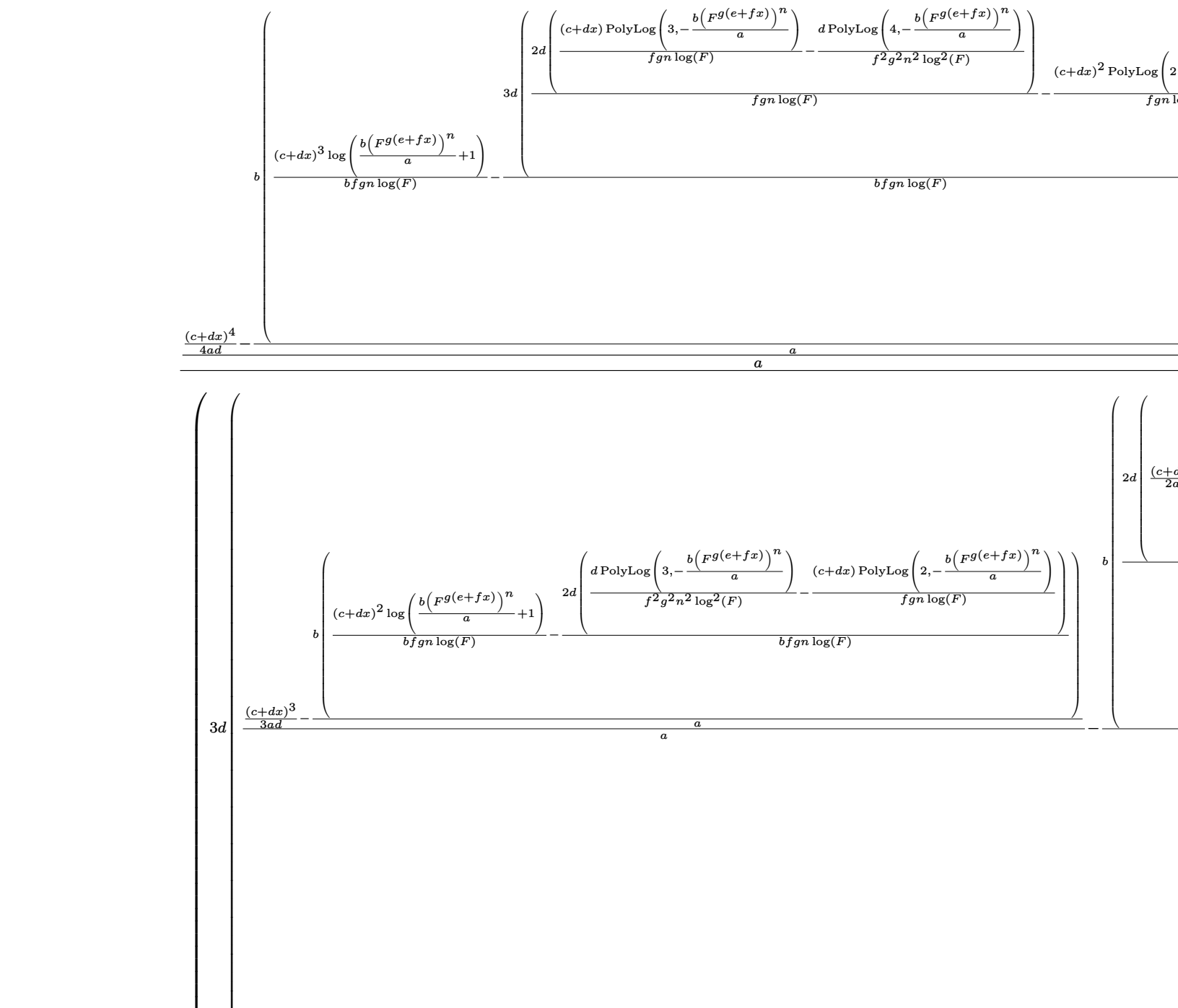
$$3d\left(\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-\frac{b\left(\frac{2d\left(\frac{(c+d}{2a}\right.\right.}{}\right.\right.\right.

2720

$$\frac{(c+dx)^4}{4ad}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}-\frac{d\int F^{-g(e+fx)}\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)}\right)}{fgn\log(F)}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}-\frac{d\int F^{-g(e+fx)}\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}$$

$$3d\left(\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{f^2g^2n^2\log^2(F)}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}-\frac{b\left(\frac{2d\left(\frac{(c+d}{2a}\right.\right.}{}\right.$$

7143



input
```
Int[(c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
-((b*(-1/2*(c + d*x)^3/(b*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*n*Log[F]) + (3
*d*(-((b*(-((c + d*x)^2/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (2
*d*((c + d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a
])/(b*f*g*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(b*f^2*
g^2*n^2*Log[F]^2)))/a))/(b*f*g*n*Log[F])))/a) + ((c + d*x)^3/(3*a*d) - (b*
(((c + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]) - (2*d*
(-(((c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(f*g*n*Log[F])) +
(d*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2*g^2*n^2*Log[F]^2)))/(b*f
*g*n*Log[F])))/a)/a))/(2*b*f*g*n*Log[F])))/a) + (-((b*(-((c + d*x)^3/(b*f*
(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (3*d*((c + d*x)^3/(3*a*d) - (b*
(((c + d*x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]) - (2*d*
(-(((c + d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(f*g*n*Log[F])) +
(d*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2*g^2*n^2*Log[F]^2)))/(b*f
*g*n*Log[F])))/a))/(b*f*g*n*Log[F])))/a) + ((c + d*x)^4/(4*a*d) - (b*(((c
+ d*x)^3*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]) - (3*d*(-(((
c + d*x)^2*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(f*g*n*Log[F])) + (2*
d*(((c + d*x)*PolyLog[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f*g*n*Log[F]) - (
d*PolyLog[4, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2*g^2*n^2*Log[F]^2)))/(f*g*
n*Log[F])))/(b*f*g*n*Log[F])))/a)/a)/a
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))  Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4004 vs. $2(582) = 1164$.

Time = 0.36 (sec) , antiderivative size = 4005, normalized size of antiderivative = 6.74

method	result	size
risch	Expression too large to display	4005

input
```
int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

output

```
3/2/n^2/g^4/f^4/ln(F)^4/a^3*d^3*ln(F^(g*(f*x+e)))^2+3/n/g^4/f^4/ln(F)^4/a^
3*d^3*ln(F^(g*(f*x+e)))^3-1/n/g/f/ln(F)/a^3*c^3*ln((F^(g*(f*x+e)))^n*F^(-n
*g*f*x)*F^(n*g*f*x)*b+a)-9/n^4/g^4/f^4/ln(F)^4/a^3*d^3*polylog(3,-b*F^(n*g
*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/n/g/f/ln(F)/a^3*c^3*ln(F^(n*g*f*
x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)-3/n^4/g^4/f^4/ln(F)^4/a^3*d^3*polylog(2
,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-6/n^4/g^4/f^4/ln(F)^4/a^
3*d^3*polylog(4,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+3/2/g^2/f
^2/ln(F)^2/a^3*d*c^2*ln(F^(g*(f*x+e)))^2+3/2/g^2/f^2/ln(F)^2/a^3*d^3*ln(F^
(g*(f*x+e)))^2*x^2-2/g^3/f^3/ln(F)^3/a^3*d^3*ln(F^(g*(f*x+e)))^3*x+9/n^2/g
^2/f^2/ln(F)^2/a^3*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a
)*x-9/n^2/g^3/f^3/ln(F)^3/a^3*c*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n
*g*f*x)*b+a)*ln(F^(g*(f*x+e)))+9/n^2/g^3/f^3/ln(F)^3/a^3*d^2*c*ln(1+b*F^(n
*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^(g*(f*x+e)))+9/n^2/g^3/f^3/
ln(F)^3/a^3*d^3*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)*ln(F^
(g*(f*x+e)))*x+9/n^2/g^3/f^3/ln(F)^3/a^3*d^3*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(
F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))*x-9/n^2/g^3/f^3/ln(F)^3/a^3*d^3*ln((F^
(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))*x+3/n/g^2/f
^2/ln(F)^2/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F
^(g*(f*x+e)))*x^2-3/n/g^3/f^3/ln(F)^3/a^3*d^3*ln((F^(g*(f*x+e)))^n*F^(-n*g
*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2*x-3/n/g^2/f^2/ln(F)^2/a^3*d^...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2704 vs. $2(579) = 1158$.

Time $= 0.15$ (sec) , antiderivative size $= 2704$, normalized size of antiderivative $= 4.55$

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

output

```
-1/4*(6*(a^2*d^3*e^3 - 3*a^2*c*d^2*e^2*f + 3*a^2*c^2*d*e*f^2 - a^2*c^3*f^3
)*g^3*n^3*log(F)^3 + 6*(a^2*d^3*e^2 - 2*a^2*c*d^2*e*f + a^2*c^2*d*f^2)*g^2
*n^2*log(F)^2 - (a^2*d^3*f^4*g^4*n^4*x^4 + 4*a^2*c*d^2*f^4*g^4*n^4*x^3 + 6
*a^2*c^2*d*f^4*g^4*n^4*x^2 + 4*a^2*c^3*f^4*g^4*n^4*x - (a^2*d^3*e^4 - 4*a^
2*c*d^2*e^3*f + 6*a^2*c^2*d*e^2*f^2 - 4*a^2*c^3*e*f^3)*g^4*n^4)*log(F)^4 -
 ((b^2*d^3*f^4*g^4*n^4*x^4 + 4*b^2*c*d^2*f^4*g^4*n^4*x^3 + 6*b^2*c^2*d*f^4
*g^4*n^4*x^2 + 4*b^2*c^3*f^4*g^4*n^4*x - (b^2*d^3*e^4 - 4*b^2*c*d^2*e^3*f
+ 6*b^2*c^2*d*e^2*f^2 - 4*b^2*c^3*e*f^3)*g^4*n^4)*log(F)^4 - 6*(b^2*d^3*f^
3*g^3*n^3*x^3 + 3*b^2*c*d^2*f^3*g^3*n^3*x^2 + 3*b^2*c^2*d*f^3*g^3*n^3*x +
(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2)*g^3*n^3)*log(F)^3 +
6*(b^2*d^3*f^2*g^2*n^2*x^2 + 2*b^2*c*d^2*f^2*g^2*n^2*x - (b^2*d^3*e^2 - 2*
b^2*c*d^2*e*f)*g^2*n^2)*log(F)^2)*F^(2*f*g*n*x + 2*e*g*n) - 2*((a*b*d^3*f^
4*g^4*n^4*x^4 + 4*a*b*c*d^2*f^4*g^4*n^4*x^3 + 6*a*b*c^2*d*f^4*g^4*n^4*x^2
+ 4*a*b*c^3*f^4*g^4*n^4*x - (a*b*d^3*e^4 - 4*a*b*c*d^2*e^3*f + 6*a*b*c^2*d
*e^2*f^2 - 4*a*b*c^3*e*f^3)*g^4*n^4)*log(F)^4 - 2*(2*a*b*d^3*f^3*g^3*n^3*x
^3 + 6*a*b*c*d^2*f^3*g^3*n^3*x^2 + 6*a*b*c^2*d*f^3*g^3*n^3*x + (3*a*b*d^3*
e^3 - 9*a*b*c*d^2*e^2*f + 9*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*g^3*n^3)*log(F)
^3 + 3*(a*b*d^3*f^2*g^2*n^2*x^2 + 2*a*b*c*d^2*f^2*g^2*n^2*x - (2*a*b*d^3*e
^2 - 4*a*b*c*d^2*e*f + a*b*c^2*d*f^2)*g^2*n^2)*log(F)^2)*F^(f*g*n*x + e*g*
n) + 12*(a^2*d^3 + (a^2*d^3*f^2*g^2*n^2*x^2 + 2*a^2*c*d^2*f^2*g^2*n^2*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Timed out}$$

input

```
integrate((d*x+c)**3/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

output
```
1/2*c^3*((2*F^(f*g*n*x + e*g*n)*b + 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F
^(2*f*g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*g*n*log(F)) + 2*(f*g*n*x + e*g*n)/
(a^3*f*g*n) - 2*log(F^(f*g*n*x + e*g*n)*b + a)/(a^3*f*g*n*log(F))) + 1/2*(
3*a*d^3*f*g*n*x^3*log(F) - 3*a*c^2*d + 3*(3*a*c*d^2*f*g*n*log(F) - a*d^3)*
x^2 + (2*F^(e*g*n)*b*d^3*f*g*n*x^3*log(F) - 3*F^(e*g*n)*b*c^2*d + 3*(2*F^(
e*g*n)*b*c*d^2*f*g*n*log(F) - F^(e*g*n)*b*d^3)*x^2 + 6*(F^(e*g*n)*b*c^2*d*
f*g*n*log(F) - F^(e*g*n)*b*c*d^2)*x)*F^(f*g*n*x) + 3*(3*a*c^2*d*f*g*n*log(
F) - 2*a*c*d^2)*x)/(2*F^(f*g*n*x)*F^(e*g*n)*a^3*b*f^2*g^2*n^2*log(F)^2 + F
^(2*f*g*n*x)*F^(2*e*g*n)*a^2*b^2*f^2*g^2*n^2*log(F)^2 + a^4*f^2*g^2*n^2*lo
g(F)^2) - 3/2*(3*c^2*d*f*g*n*log(F) - 2*c*d^2)*x/(a^3*f^2*g^2*n^2*log(F)^2
) + 3/2*(3*c^2*d*f*g*n*log(F) - 2*c*d^2)*log(F^(f*g*n*x)*F^(e*g*n)*b + a)/
(a^3*f^3*g^3*n^3*log(F)^3) - (f^3*g^3*n^3*x^3*log(F^(f*g*n*x)*F^(e*g*n)*b/
a + 1)*log(F)^3 + 3*f^2*g^2*n^2*x^2*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(
F)^2 - 6*f*g*n*x*log(F)*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*b/a) + 6*polylog
(4, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^3/(a^3*f^4*g^4*n^4*log(F)^4) - 3/2*(f^2
*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F)^2 + 2*f*g*n*x*dilog
(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylog(3, -F^(f*g*n*x)*F^(e*g*n)*
b/a))*(2*c*d^2*f*g*n*log(F) - 3*d^3)/(a^3*f^4*g^4*n^4*log(F)^4) - 3*(c^2*d
*f^2*g^2*n^2*log(F)^2 - 3*c*d^2*f*g*n*log(F) + d^3)*(f*g*n*x*log(F^(f*g*n*
x)*F^(e*g*n)*b/a + 1)*log(F) + dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a))/(a^3*...
```

Giac [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(dx+c)^3}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/((F^((f*x + e)*g))^n*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+d\,x)^3}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}\,dx$$

input
```
int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3,x)
```

output
```
int((c + d*x)^3/(a + b*(F^(g*(e + f*x)))^n)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3/(a+b*(F^(g*(f*x+e)))^n)^3,x)
```

output

```
(2*f**(2*e*g*n + 2*f*g*n*x)*int(x**3/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f*
*(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*l
og(f)*a**3*b**2*d**3*f*g*n + 6*f**(2*e*g*n + 2*f*g*n*x)*int(x**2/(f**(3*e*
g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n +
f*g*n*x)*a**2*b + a**3),x)*log(f)*a**3*b**2*c*d**2*f*g*n + 6*f**(2*e*g*n +
 2*f*g*n*x)*int(x/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n
*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**3*b**2*c**
2*d*f*g*n - 2*f**(2*e*g*n + 2*f*g*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*b**
2*c**3 + 2*f**(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*c**3*f*g*n*x - f**(2*e*g*n
 + 2*f*g*n*x)*b**2*c**3 + 4*f**(e*g*n + f*g*n*x)*int(x**3/(f**(3*e*g*n + 3
*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x
)*a**2*b + a**3),x)*log(f)*a**4*b*d**3*f*g*n + 12*f**(e*g*n + f*g*n*x)*int
(x**2/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 +
 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**4*b*c*d**2*f*g*n + 12*
f**(e*g*n + f*g*n*x)*int(x/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n
+ 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**4
*b*c**2*d*f*g*n - 4*f**(e*g*n + f*g*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*a
*b*c**3 + 4*f**(e*g*n + f*g*n*x)*log(f)*a*b*c**3*f*g*n*x + 2*int(x**3/(f**
(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g
*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**5*d**3*f*g*n + 6*int(x**2/(f*...
```

3.59 $\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$

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Optimal result

Integrand size = 25, antiderivative size = 439

$$\begin{aligned}\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx &= \frac{(c+dx)^3}{3a^3d} + \frac{d^2x}{a^3f^2g^2n^2\log^2(F)} \\ &- \frac{d(c+dx)}{a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\ &- \frac{3(c+dx)^2}{2a^3fgn\log(F)} + \frac{(c+dx)^2}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\ &+ \frac{(c+dx)^2}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\ &- \frac{d^2\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3f^3g^3n^3\log^3(F)} \\ &+ \frac{3d(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\ &- \frac{(c+dx)^2\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\ &+ \frac{3d^2\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)} \\ &- \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)} \\ &+ \frac{2d^2\,\mathrm{PolyLog}\left(3,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^3g^3n^3\log^3(F)}\end{aligned}$$

output

```
1/3*(d*x+c)^3/a^3/d+d^2*x/a^3/f^2/g^2/n^2/ln(F)^2-d*(d*x+c)/a^2/f^2/(a+b*(
F^(g*(f*x+e)))^n)/g^2/n^2/ln(F)^2-3/2*(d*x+c)^2/a^3/f/g/n/ln(F)+1/2*(d*x+c
)^2/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+(d*x+c)^2/a^2/f/(a+b*(F^(g*(f*
x+e)))^n)/g/n/ln(F)-d^2*ln(a+b*(F^(g*(f*x+e)))^n)/a^3/f^3/g^3/n^3/ln(F)^3+
3*d*(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f^2/g^2/n^2/ln(F)^2-(d*x+c)^2*
ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f/g/n/ln(F)+3*d^2*polylog(2,-b*(F^(g*(f*x+
e)))^n/a)/a^3/f^3/g^3/n^3/ln(F)^3-2*d*(d*x+c)*polylog(2,-b*(F^(g*(f*x+e)))
^n/a)/a^3/f^2/g^2/n^2/ln(F)^2+2*d^2*polylog(3,-b*(F^(g*(f*x+e)))^n/a)/a^3/
f^3/g^3/n^3/ln(F)^3
```

Mathematica [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

input
```
Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
Integrate[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3, x]
```

Rubi [A] (verified)

Time = 6.82 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.42, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {2616, 2616, 2615, 2620, 2621, 2615, 2616, 2615, 2620, 2621, 2715, 2720, 798, 47, 14, 16, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{\int \frac{(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{\frac{(c+dx)^3}{3ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2620$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)^2}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2621$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\frac{b\left(\frac{2d\int\frac{c+dx}{b\left(F^{g(e+fx)}\right)^n+a}dx}{bfgn\log(F)}-\frac{(c+dx)^2}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}-\frac{b\left(\frac{d\int\frac{c+dx}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{bfgn\log(F)}-\frac{(c+dx)^2}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2615$$

$$\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}-\frac{b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\log(}\right)}{a}-\frac{b\left(\frac{d\int\frac{c+dx}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{bfgn\log(F)}-\frac{(c+dx)^2}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\log(}\right.}{a}$$

$$\left.\frac{b\left(\frac{d\left(\frac{\int\frac{c+dx}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}\right)$$

$$\downarrow 2615$$

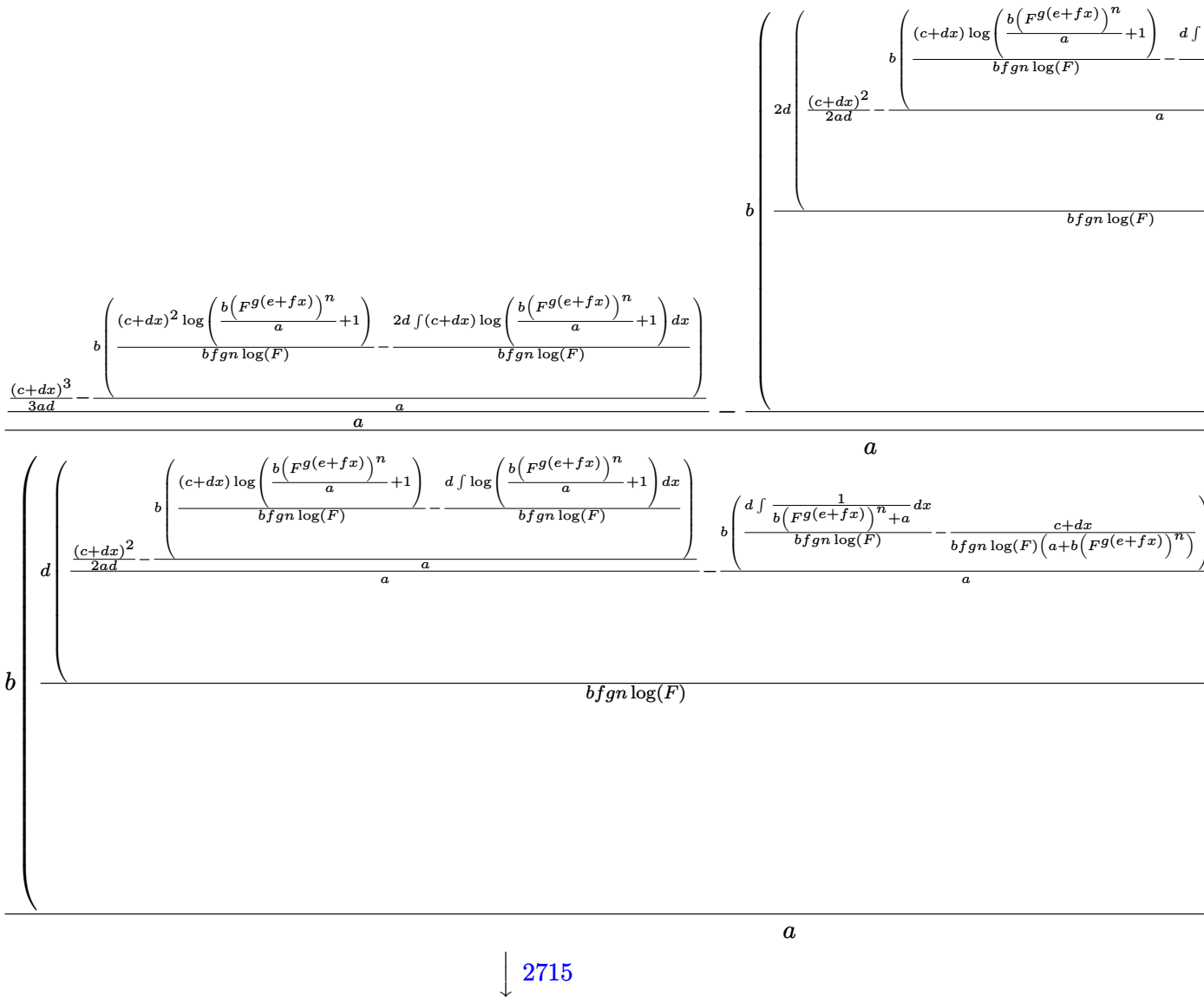
$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}\right)}{bfgn\log(F)}-\frac{}{bfgn\log(}\right.}{a}$$

$$\left.\frac{b\left(\frac{d\left(\frac{(c+dx)^2}{2ad}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}\right)}{bfgn\log(F)}-\frac{(c+dx)^2}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}\right)$$

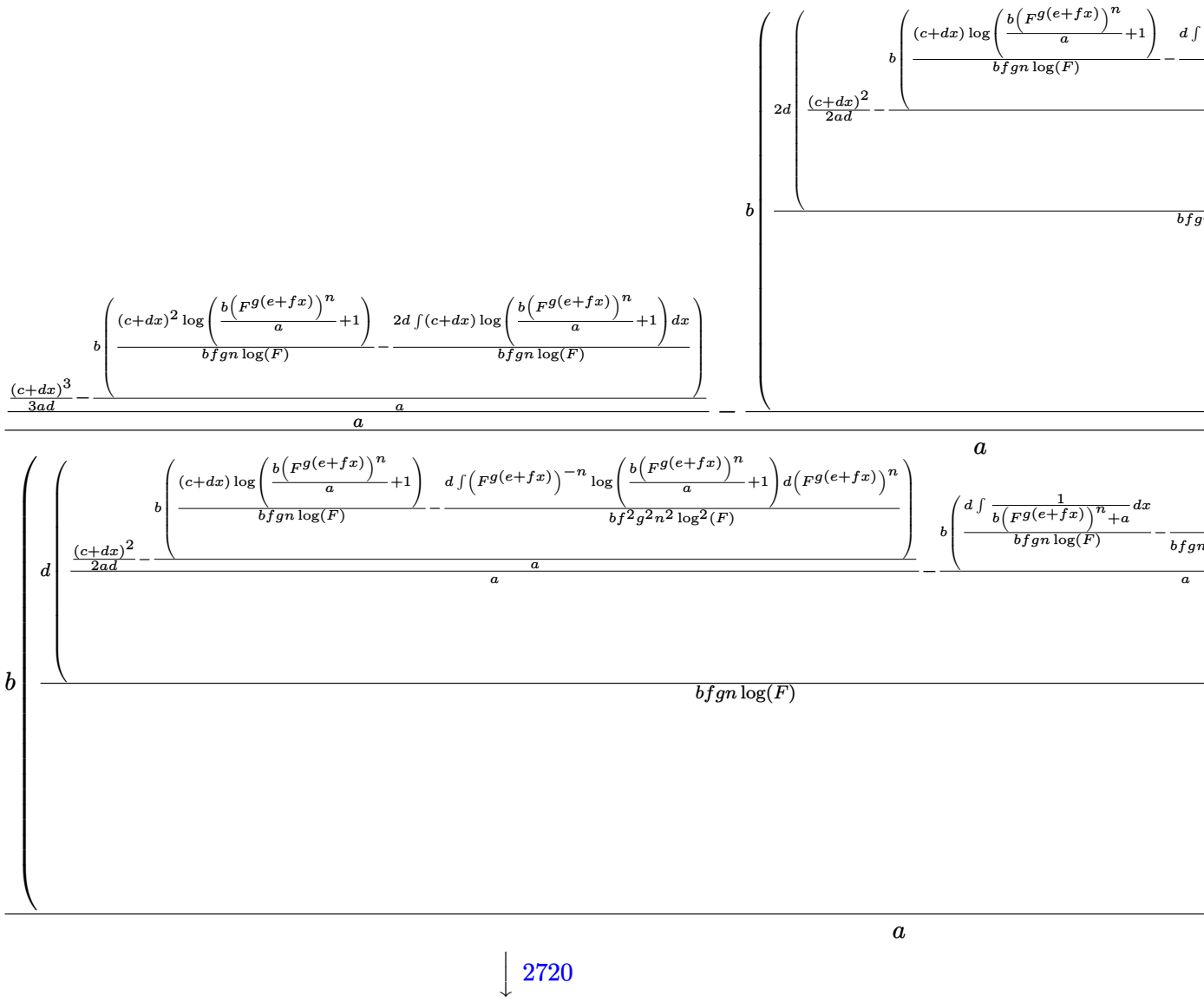
$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}\right)}{bfgn\log(F)}\right)}{a}-\frac{(c+dx)^2}{2bfgn\log(F)(a+b(F^{g(e+}$$

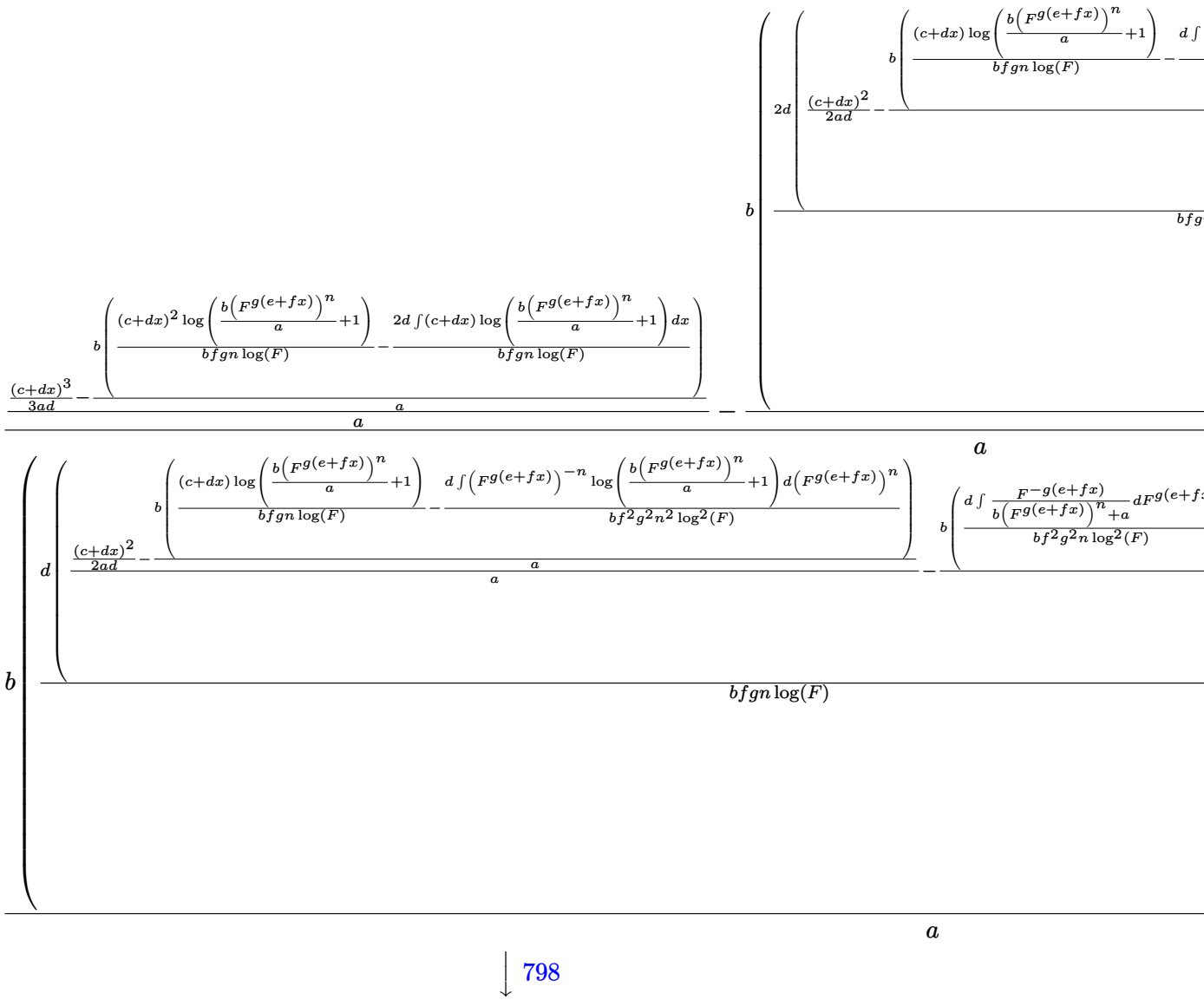
$$\downarrow 2621$$



$\downarrow$ 2715



↓ 2720

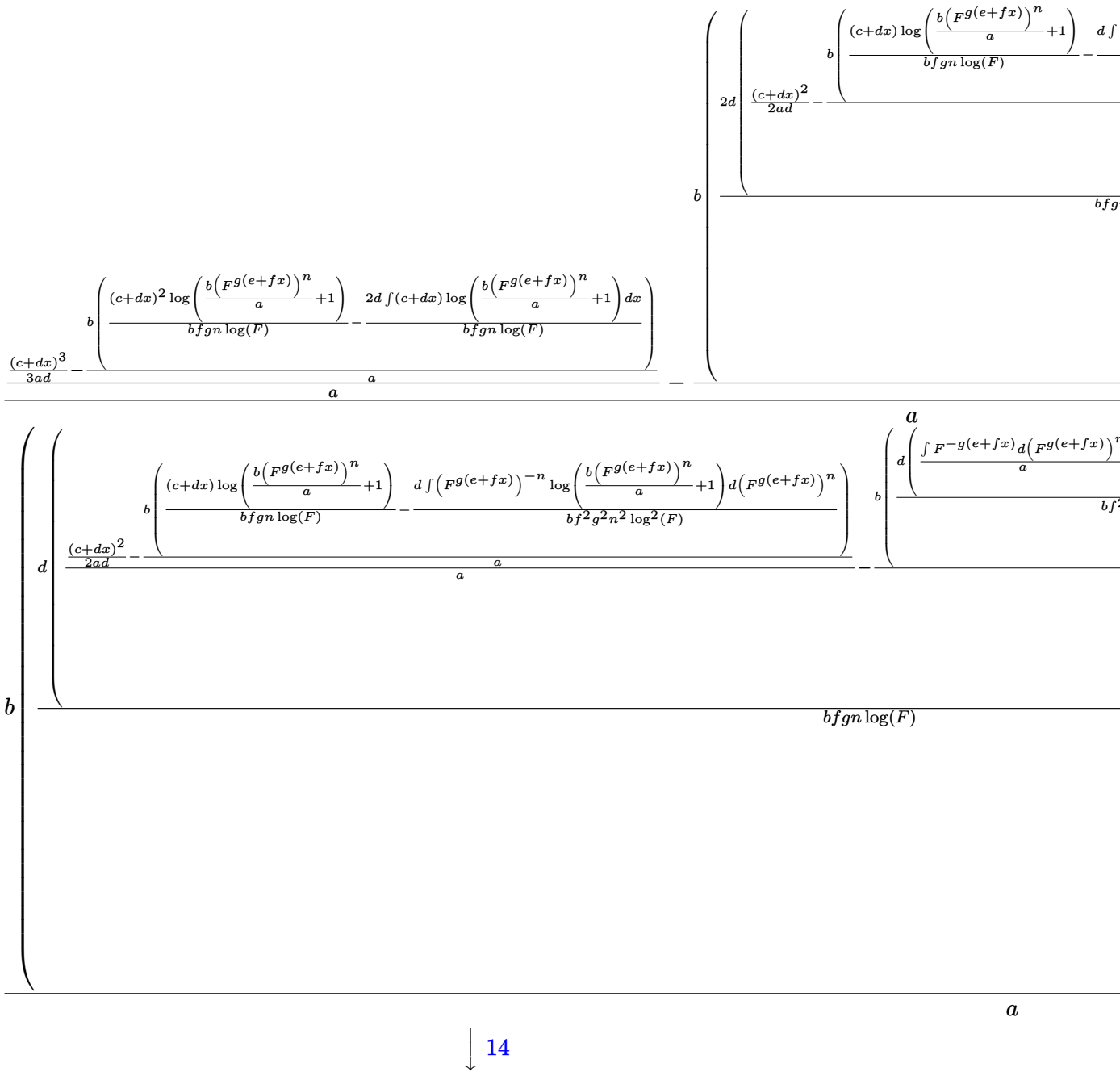


798

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int}{}\right.\right.\right.}{bfg}}{a}$$

$$b\left(\frac{d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)}{bfgn\log(F)}-\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{b\left(F^{g(e+fx)}\right)^n+a}d\left(F^{g(e+}\right.\right.}{bf^2g^2n^2\log^2(F)}}{a}$$

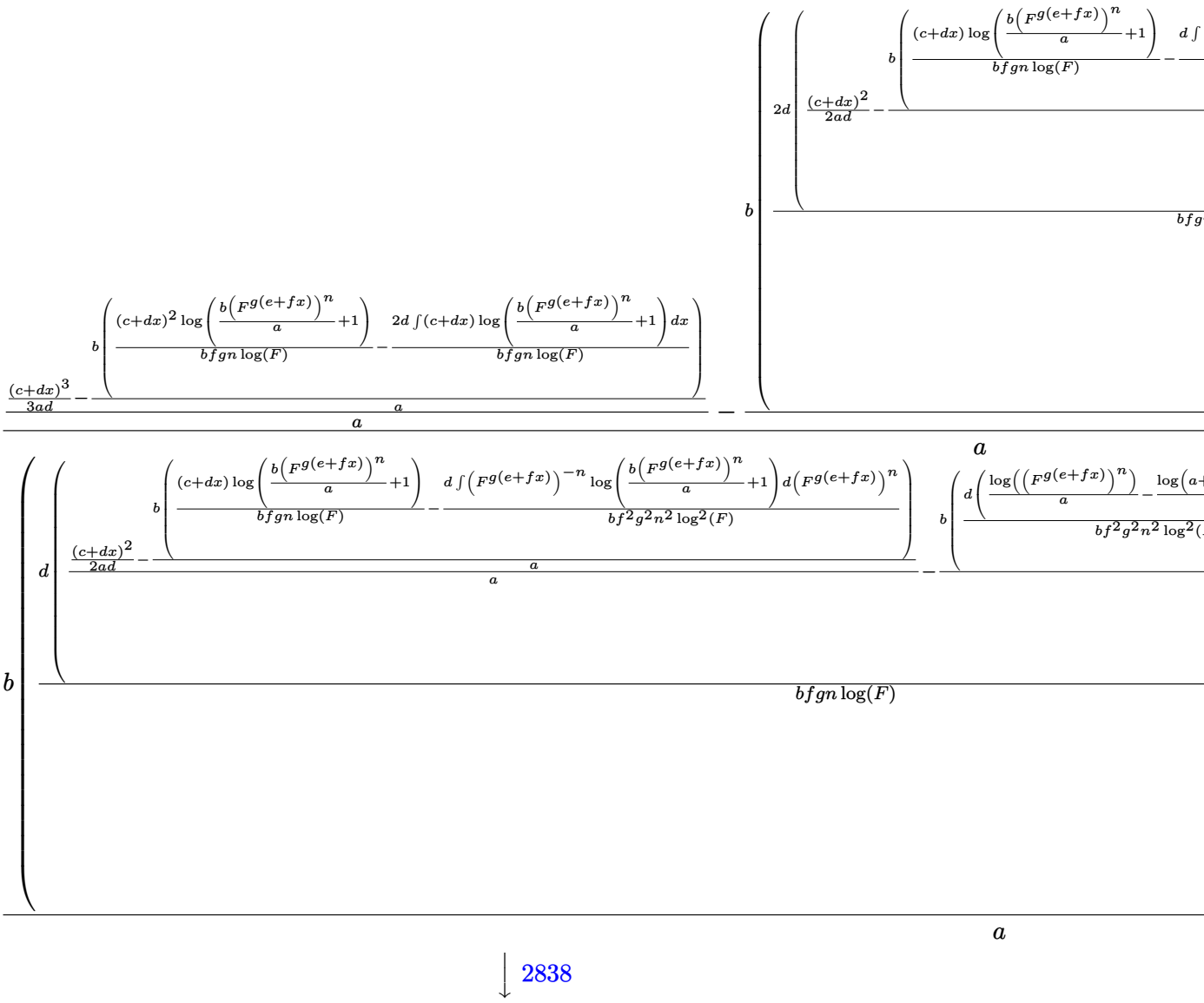
47



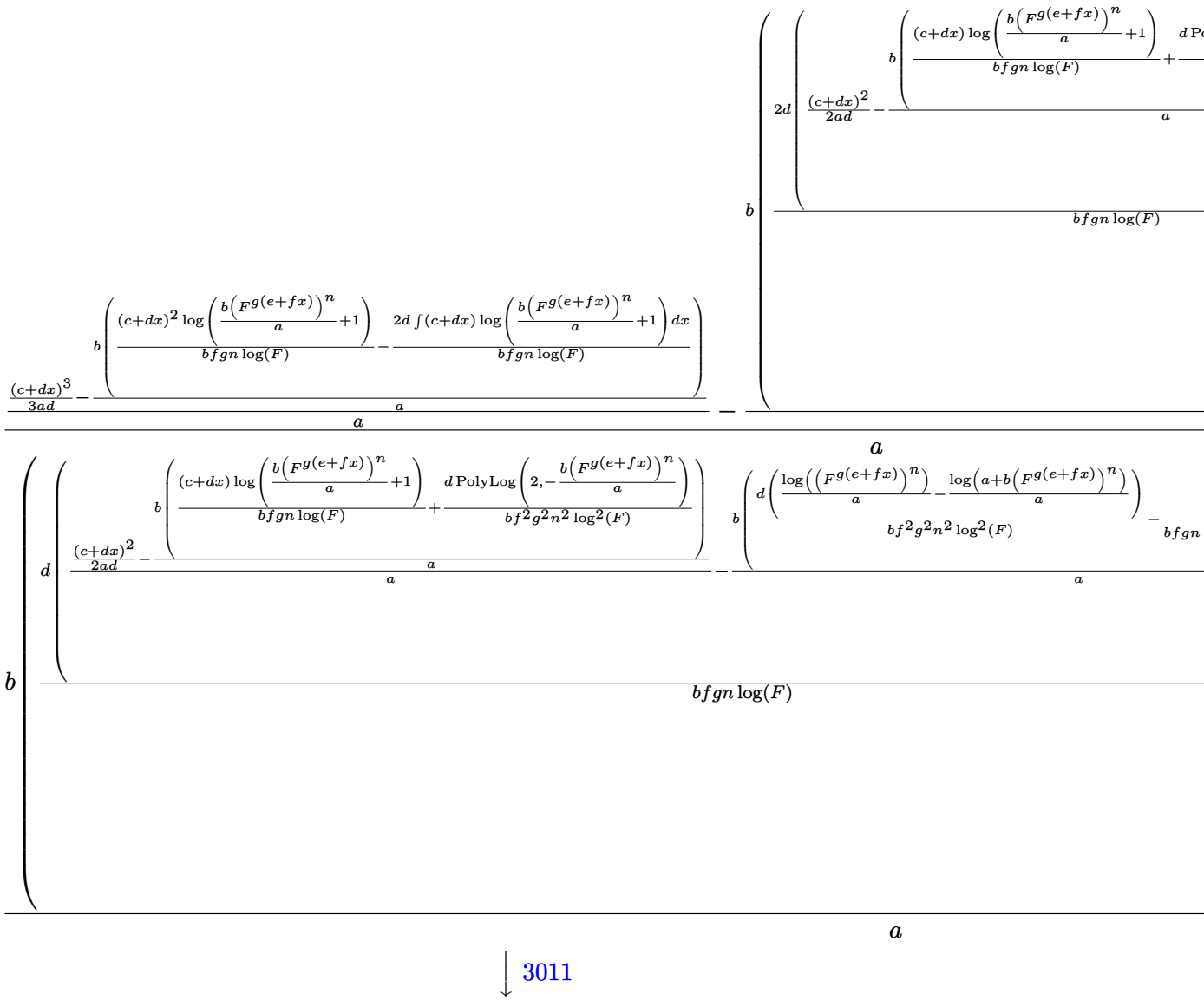
↓ 14

$$\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}\right)d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{b\int\frac{1}{b\left(\right.}}{bf^2g^2n}\right.}{bfgn\log(F)}\right.}{a}$$

16



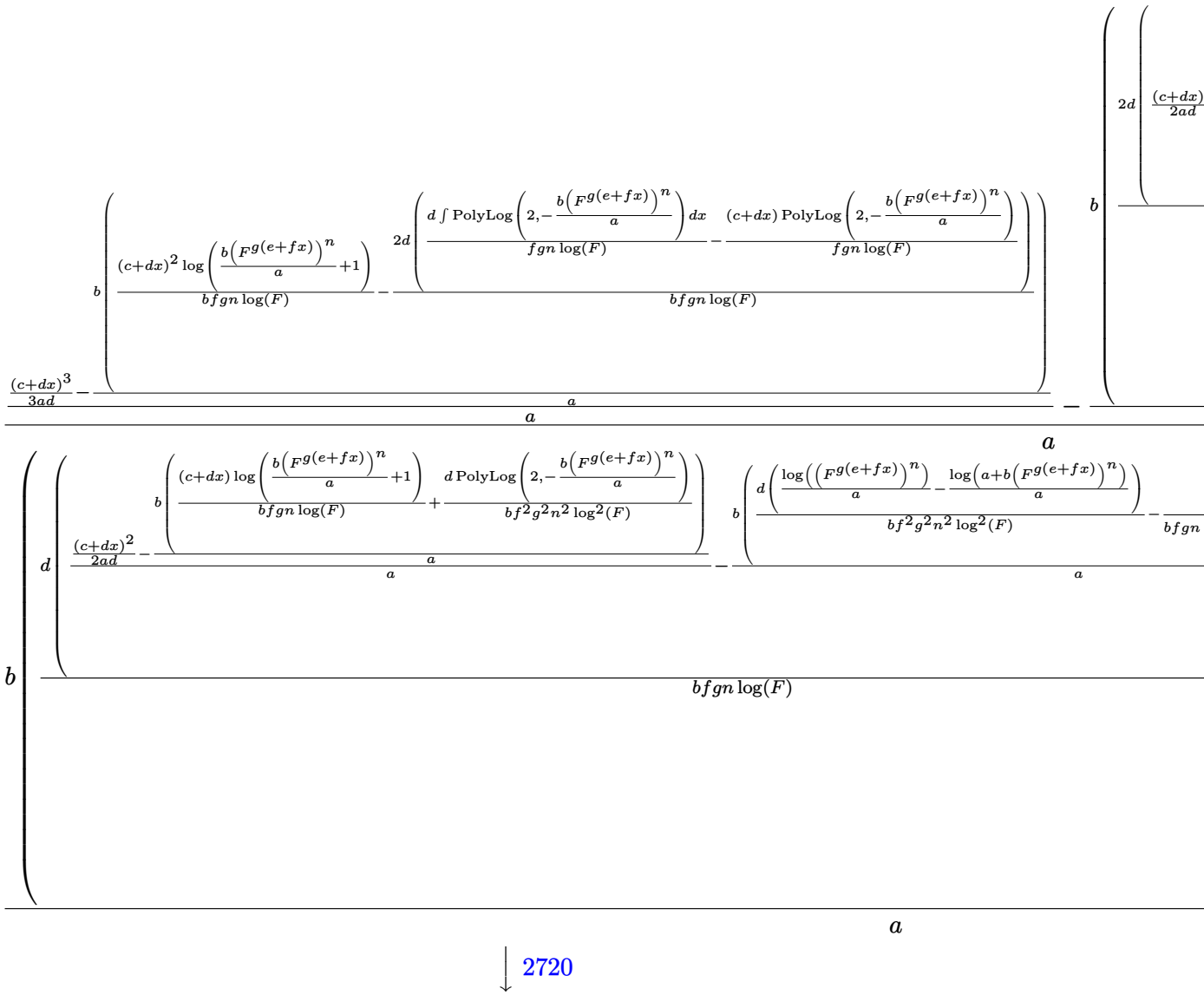
↓ 2838



$$\frac{\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\int(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2d\left(\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{Po}}{}\right.}{a}\right.}{bfgn\log(F)}\right.}{}}{a}$$

$$\frac{b\left(\frac{d\left(\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)}-\frac{}{bfgn}\right.}{a}\right.}{bfgn\log(F)}\right.}{a}$$

$\downarrow$ 3011

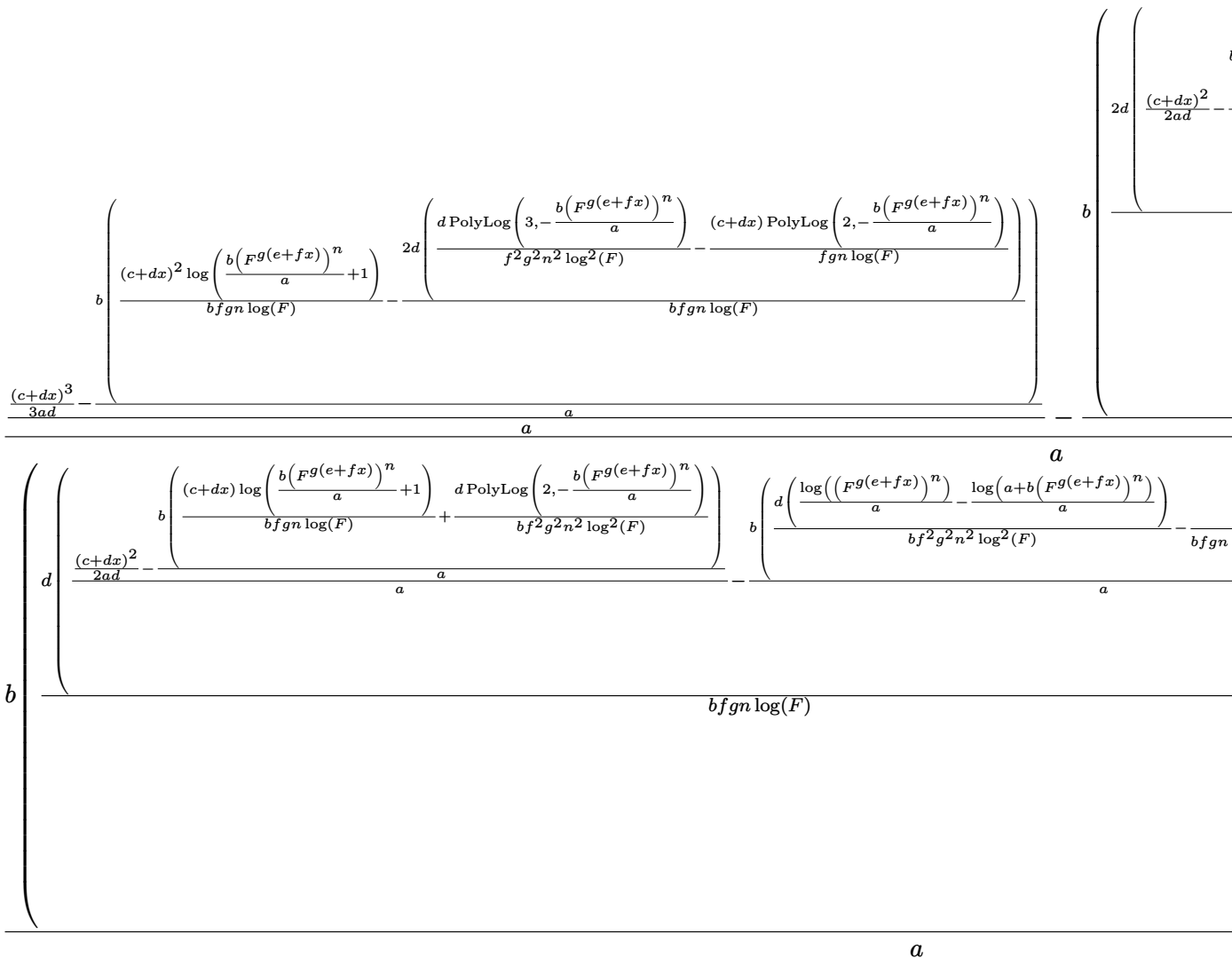


↓ 2720

$$\frac{\frac{\frac{(c+dx)^3}{3ad}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{2d\left(\frac{d\int F^{-g(e+fx)}\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)dF^{g(e+fx)}}{f^2g^2n\log^2(F)}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{fgn\log(F)}\right)}{bfgn\log(F)}\right)}{a}}{a}}{a}$$

$$\frac{b\left(\frac{d\left(\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\operatorname{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)}-\frac{}{bfgn}\right.}{a}\right.}{bfgn\log(F)}\right.}{a}$$

$\downarrow$ 7143



input
```
Int[(c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
-((b*(-1/2*(c + d*x)^2/(b*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*n*Log[F]) + (d
*(-((b*(-((c + d*x)/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (d*(Lo
g[(F^(g*(e + f*x)))^n]/a - Log[a + b*(F^(g*(e + f*x)))^n]/a))/(b*f^2*g^2*n
^2*Log[F]^2)))/a) + ((c + d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g
*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x))
)^n)/a)])/(b*f^2*g^2*n^2*Log[F]^2)))/a)/a))/(b*f*g*n*Log[F])))/a) + (-((b*
(-((c + d*x)^2/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (2*d*((c +
d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g
*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(b*f^2*g^2*n^2*L
og[F]^2)))/a))/(b*f*g*n*Log[F])))/a) + ((c + d*x)^3/(3*a*d) - (b*(((c + d*
x)^2*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]) - (2*d*(-(((c +
d*x)*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(f*g*n*Log[F])) + (d*PolyLo
g[3, -((b*(F^(g*(e + f*x)))^n)/a)])/(f^2*g^2*n^2*Log[F]^2)))/(b*f*g*n*Log[
F])))/a)/a)/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1886 vs. $2(433) = 866$.

Time = 0.33 (sec) , antiderivative size = 1887, normalized size of antiderivative = 4.30

method	result	size
risch	Expression too large to display	1887

input

```
int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

output

```
1/a^3/ln(F)^2/f^2/g^2*d^2*ln(F^(g*(f*x+e)))^2*x-3/2/a^3/ln(F)^3/f^3/g^3/n*
d^2*ln(F^(g*(f*x+e)))^2+3/a^3/ln(F)^3/f^3/g^3/n^3*d^2*polylog(2,-b*F^(n*g*
f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/a^3/ln(F)/f/g/n*c^2*ln(F^(n*g*f*x
)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+3/a^3/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(n*g*
f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e)))-3/a^3/ln(F)^3/f^3/g
^3/n^2*d^2*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x
+e)))+1/a^3/ln(F)^3/f^3/g^3/n*d^2*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e
)))^n)*ln(F^(g*(f*x+e)))^2-1/a^3/ln(F)^3/f^3/g^3/n*d^2*ln((F^(g*(f*x+e)))^
n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*ln(F^(g*(f*x+e)))^2+1/a^3/ln(F)^3/f^3/g^3/
n*d^2*ln(F^(g*(f*x+e)))^2*ln(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^
n/a)+3/a^3/ln(F)^3/f^3/g^3/n^2*d^2*ln(F^(g*(f*x+e)))*ln(1+b*F^(n*g*f*x)*F^
(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-3/a^3/ln(F)^2/f^2/g^2/n^2*c*d*ln(F^(n*g*f*
x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+3/a^3/ln(F)^2/f^2/g^2/n^2*c*d*ln((F^(g*
(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/2*(2*ln(F)*b*d^2*f*g*n*x^2*(F^
(g*(f*x+e)))^n+3*ln(F)*a*d^2*f*g*n*x^2+4*ln(F)*b*c*d*f*g*n*x*(F^(g*(f*x+e)
))^n+6*ln(F)*a*c*d*f*g*n*x+2*ln(F)*b*c^2*f*g*n*(F^(g*(f*x+e)))^n+3*ln(F)*a
*c^2*f*g*n-2*b*d^2*x*(F^(g*(f*x+e)))^n-2*a*d^2*x-2*b*c*d*(F^(g*(f*x+e)))^n
-2*a*c*d)/n^2/g^2/f^2/ln(F)^2/a^2/(a+b*(F^(g*(f*x+e)))^n)^2-2/3/a^3/ln(F)^
3/f^3/g^3*d^2*ln(F^(g*(f*x+e)))^3-3/a^3/ln(F)^2/f^2/g^2/n^2*d^2*ln(F^(n*g*
f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x+3/a^3/ln(F)^2/f^2/g^2/n^2*d^2*ln...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1518 vs. $2(431) = 862$.

Time = 0.11 (sec) , antiderivative size = 1518, normalized size of antiderivative = 3.46

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

output

```
1/6*(9*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*g^2*n^2*log(F)^2 + 6*(a
^2*d^2*e - a^2*c*d*f)*g*n*log(F) + 2*(a^2*d^2*f^3*g^3*n^3*x^3 + 3*a^2*c*d*
f^3*g^3*n^3*x^2 + 3*a^2*c^2*f^3*g^3*n^3*x + (a^2*d^2*e^3 - 3*a^2*c*d*e^2*f
 + 3*a^2*c^2*e*f^2)*g^3*n^3)*log(F)^3 + (2*(b^2*d^2*f^3*g^3*n^3*x^3 + 3*b^
2*c*d*f^3*g^3*n^3*x^2 + 3*b^2*c^2*f^3*g^3*n^3*x + (b^2*d^2*e^3 - 3*b^2*c*d
*e^2*f + 3*b^2*c^2*e*f^2)*g^3*n^3)*log(F)^3 - 9*(b^2*d^2*f^2*g^2*n^2*x^2 +
 2*b^2*c*d*f^2*g^2*n^2*x - (b^2*d^2*e^2 - 2*b^2*c*d*e*f)*g^2*n^2)*log(F)^2
 + 6*(b^2*d^2*f*g*n*x + b^2*d^2*e*g*n)*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2
*(2*(a*b*d^2*f^3*g^3*n^3*x^3 + 3*a*b*c*d*f^3*g^3*n^3*x^2 + 3*a*b*c^2*f^3*g
^3*n^3*x + (a*b*d^2*e^3 - 3*a*b*c*d*e^2*f + 3*a*b*c^2*e*f^2)*g^3*n^3)*log(
F)^3 - 3*(2*a*b*d^2*f^2*g^2*n^2*x^2 + 4*a*b*c*d*f^2*g^2*n^2*x - (3*a*b*d^2
*e^2 - 6*a*b*c*d*e*f + a*b*c^2*f^2)*g^2*n^2)*log(F)^2 + 3*(a*b*d^2*f*g*n*x
 + (2*a*b*d^2*e - a*b*c*d*f)*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 6*(3*a^2*d
^2 + (3*b^2*d^2 - 2*(b^2*d^2*f*g*n*x + b^2*c*d*f*g*n)*log(F))*F^(2*f*g*n*x
 + 2*e*g*n) + 2*(3*a*b*d^2 - 2*(a*b*d^2*f*g*n*x + a*b*c*d*f*g*n)*log(F))*F
^(f*g*n*x + e*g*n) - 2*(a^2*d^2*f*g*n*x + a^2*c*d*f*g*n)*log(F))*dilog(-(F
^(f*g*n*x + e*g*n)*b + a)/a + 1) - 6*((a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c
^2*f^2)*g^2*n^2*log(F)^2 + a^2*d^2 + 3*(a^2*d^2*e - a^2*c*d*f)*g*n*log(F)
+ ((b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*g^2*n^2*log(F)^2 + b^2*d^2
+ 3*(b^2*d^2*e - b^2*c*d*f)*g*n*log(F))*F^(2*f*g*n*x + 2*e*g*n) + 2*((a...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Timed out}$$

input

```
integrate((d*x+c)**2/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

output

```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

output
```
1/2*c^2*((2*F^(f*g*n*x + e*g*n)*b + 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F
^(2*f*g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*g*n*log(F)) + 2*(f*g*n*x + e*g*n)/
(a^3*f*g*n) - 2*log(F^(f*g*n*x + e*g*n)*b + a)/(a^3*f*g*n*log(F))) + 1/2*(
3*a*d^2*f*g*n*x^2*log(F) - 2*a*c*d + 2*(F^(e*g*n)*b*d^2*f*g*n*x^2*log(F) -
 F^(e*g*n)*b*c*d + (2*F^(e*g*n)*b*c*d*f*g*n*log(F) - F^(e*g*n)*b*d^2)*x)*F
^(f*g*n*x) + 2*(3*a*c*d*f*g*n*log(F) - a*d^2)*x)/(2*F^(f*g*n*x)*F^(e*g*n)*
a^3*b*f^2*g^2*n^2*log(F)^2 + F^(2*f*g*n*x)*F^(2*e*g*n)*a^2*b^2*f^2*g^2*n^2
*log(F)^2 + a^4*f^2*g^2*n^2*log(F)^2) - (3*c*d*f*g*n*log(F) - d^2)*x/(a^3*
f^2*g^2*n^2*log(F)^2) - (f^2*g^2*n^2*x^2*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1
)*log(F)^2 + 2*f*g*n*x*dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a)*log(F) - 2*polylo
g(3, -F^(f*g*n*x)*F^(e*g*n)*b/a))*d^2/(a^3*f^3*g^3*n^3*log(F)^3) - (2*c*d*
f*g*n*log(F) - 3*d^2)*(f*g*n*x*log(F^(f*g*n*x)*F^(e*g*n)*b/a + 1)*log(F) +
 dilog(-F^(f*g*n*x)*F^(e*g*n)*b/a))/(a^3*f^3*g^3*n^3*log(F)^3) + (3*c*d*f*
g*n*log(F) - d^2)*log(F^(f*g*n*x)*F^(e*g*n)*b + a)/(a^3*f^3*g^3*n^3*log(F)
^3) + 1/6*(2*d^2*f^3*g^3*n^3*x^3*log(F)^3 + 3*(2*c*d*f*g*n*log(F) - 3*d^2)
*f^2*g^2*n^2*x^2*log(F)^2)/(a^3*f^3*g^3*n^3*log(F)^3)
```

Giac [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(dx+c)^2}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/((F^((f*x + e)*g))^n*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{(c+d\,x)^2}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}\,dx$$

input
```
int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3,x)
```

output
```
int((c + d*x)^2/(a + b*(F^(g*(e + f*x)))^n)^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$= \frac{2f^{2fgnx+2egn}\left(\int \frac{x^2}{f^{3fgnx+3egn}b^3+3f^{2fgnx+2egn}a\,b^2+3f^{fgnx+egn}a^2b+a^3}dx\right)\log(f)\,a^3b^2d^2fgn+4f^{2fgnx+2egn}\left(\int \frac{}{f^{3fgnx-}}\right.}$$

input
```
int((d*x+c)^2/(a+b*(F^(g*(f*x+e)))^n)^3,x)
```

output

```
(2*f**(2*e*g*n + 2*f*g*n*x)*int(x**2/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f*
*(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*l
og(f)*a**3*b**2*d**2*f*g*n + 4*f**(2*e*g*n + 2*f*g*n*x)*int(x/(f**(3*e*g*n
 + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g
*n*x)*a**2*b + a**3),x)*log(f)*a**3*b**2*c*d*f*g*n - 2*f**(2*e*g*n + 2*f*g
*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*b**2*c**2 + 2*f**(2*e*g*n + 2*f*g*n*
x)*log(f)*b**2*c**2*f*g*n*x - f**(2*e*g*n + 2*f*g*n*x)*b**2*c**2 + 4*f**(e
*g*n + f*g*n*x)*int(x**2/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n +
2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**4*b
*d**2*f*g*n + 8*f**(e*g*n + f*g*n*x)*int(x/(f**(3*e*g*n + 3*f*g*n*x)*b**3
+ 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3
),x)*log(f)*a**4*b*c*d*f*g*n - 4*f**(e*g*n + f*g*n*x)*log(f**(e*g*n + f*g*
n*x)*b + a)*a*b*c**2 + 4*f**(e*g*n + f*g*n*x)*log(f)*a*b*c**2*f*g*n*x + 2*
int(x**2/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**
2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**5*d**2*f*g*n + 4*in
t(x/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3
*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**5*c*d*f*g*n - 2*log(f**(
e*g*n + f*g*n*x)*b + a)*a**2*c**2 + 2*log(f)*a**2*c**2*f*g*n*x + 2*a**2*c*
*2)/(2*log(f)*a**3*f*g*n*(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*
g*n*x)*a*b + a**2))
```

3.60 $\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$

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Optimal result

Integrand size = 23, antiderivative size = 276

$$\begin{aligned}\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx &= \frac{(c+dx)^2}{2a^3d} - \frac{d}{2a^2f^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)g^2n^2\log^2(F)} \\ &\quad - \frac{3dx}{2a^3fgn\log(F)} + \frac{c+dx}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2gn\log(F)} \\ &\quad + \frac{c+dx}{a^2f\left(a+b\left(F^{g(e+fx)}\right)^n\right)gn\log(F)} \\ &\quad + \frac{3d\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{2a^3f^2g^2n^2\log^2(F)} \\ &\quad - \frac{(c+dx)\log\left(1+\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3fgn\log(F)} \\ &\quad - \frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{a^3f^2g^2n^2\log^2(F)}\end{aligned}$$

output

```
1/2*(d*x+c)^2/a^3/d-1/2*d/a^2/f^2/(a+b*(F^(g*(f*x+e)))^n)/g^2/n^2/ln(F)^2-
3/2*d*x/a^3/f/g/n/ln(F)+1/2*(d*x+c)/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F
)+(d*x+c)/a^2/f/(a+b*(F^(g*(f*x+e)))^n)/g/n/ln(F)+3/2*d*ln(a+b*(F^(g*(f*x+
e)))^n)/a^3/f^2/g^2/n^2/ln(F)^2-(d*x+c)*ln(1+b*(F^(g*(f*x+e)))^n/a)/a^3/f/
g/n/ln(F)-d*polylog(2,-b*(F^(g*(f*x+e)))^n/a)/a^3/f^2/g^2/n^2/ln(F)^2
```

Mathematica [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

input
```
Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output
```
Integrate[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3, x]
```

Rubi [A] (verified)

Time = 2.65 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.25, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {2616, 2616, 2615, 2620, 2621, 2715, 2720, 798, 47, 14, 16, 54, 2009, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$\downarrow 2616$$

$$\frac{\int \frac{c+dx}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2616$$

$$\frac{\frac{\int \frac{c+dx}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2615$$

$$\frac{\frac{(c+dx)^2}{2ad} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{b\left(F^{g(e+fx)}\right)^n+a}dx}{a}}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a} - \frac{b\int \frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2620$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{a}-\frac{b\int\frac{\left(F^{g(e+fx)}\right)^n(c+dx)}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}dx}{a}$$

$$\downarrow 2621$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)dx}{bfgn\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{d\int\frac{1}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)}dx}{bfgn\log(F)}-\frac{c+dx}{bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}-\frac{b\left(\frac{d\int\frac{1}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{2bfgn\log(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2715$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{d\int\frac{1}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)}dx}{bfgn\log(F)}-\frac{}{}\right.}{a}$$

$$\frac{b\left(\frac{d\int\frac{1}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dx}{2bfgn\log(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2720$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)}dF^{g(e+fx)}}{bf^2g^2n\log^2(F)}-\right.}{a}$$

$$\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}dF^{g(e+fx)}}{2bf^2g^2n\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 798$$

$$\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}-\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{b\left(F^{g(e+fx)}\right)^n+a}d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right.}{a}$$

$$\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}d\left(F^{g(e+fx)}\right)^n}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 47$$

$$\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}-\frac{b\left(d\left(\frac{\int F^{-g(e+fx)}d\left(F^{g(e+fx)}\right)^n}{a}-\frac{b\int}{}\right.\right.}{bf^2g^2n^2}$$

$$\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}d\left(F^{g(e+fx)}\right)^n}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 14$$

$$\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}-\frac{b\left(d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{b\int}{b\left(F^{g(e+}\right.\right.}{bf^2g^2n^2\log^2}$$

$$\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}d\left(F^{g(e+fx)}\right)^n}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 16$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}-\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)}-\frac{a}{b\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\frac{b\left(\frac{d\int\frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}d\left(F^{g(e+fx)}\right)^n}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 54$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}-\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)}-\frac{a}{b\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\frac{b\left(\frac{d\int\left(\frac{F^{-g(e+fx)}}{a^2}-\frac{b}{a^2\left(b\left(F^{g(e+fx)}\right)^n+a\right)}-\frac{b}{a\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}\right)d\left(F^{g(e+fx)}\right)^n}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2009$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}-\frac{d\int\left(F^{g(e+fx)}\right)^{-n}\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)d\left(F^{g(e+fx)}\right)^n}{bf^2g^2n^2\log^2(F)}\right)}{a}-\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)}-\frac{a}{b\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{a}$$

$$\frac{b\left(\frac{d\left(-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2}+\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^2}+\frac{1}{a\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

$$\downarrow 2838$$

$$\frac{\frac{(c+dx)^2}{2ad}-\frac{b\left(\frac{(c+dx)\log\left(\frac{b\left(F^{g(e+fx)}\right)^n}{a}+1\right)}{bfgn\log(F)}+\frac{d\,\mathrm{PolyLog}\left(2,-\frac{b\left(F^{g(e+fx)}\right)^n}{a}\right)}{bf^2g^2n^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{d\left(\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a}-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a}\right)}{bf^2g^2n^2\log^2(F)}-\frac{}{bfgn\log(F}\right)}{a}$$

$$\frac{b\left(\frac{d\left(-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^2}+\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^2}+\frac{1}{a\left(a+b\left(F^{g(e+fx)}\right)^n\right)}\right)}{2bf^2g^2n^2\log^2(F)}-\frac{c+dx}{2bfgn\log(F)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\right)}{a}$$

input

```
Int[(c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3,x]
```

output

```
-((b*(-1/2*(c + d*x)/(b*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*n*Log[F]) + (d*(
1/(a*(a + b*(F^(g*(e + f*x)))^n)) + Log[(F^(g*(e + f*x)))^n]/a^2 - Log[a +
 b*(F^(g*(e + f*x)))^n]/a^2))/(2*b*f^2*g^2*n^2*Log[F]^2)))/a) + (-((b*(-((
c + d*x)/(b*f*(a + b*(F^(g*(e + f*x)))^n)*g*n*Log[F])) + (d*(Log[(F^(g*(e
+ f*x)))^n]/a - Log[a + b*(F^(g*(e + f*x)))^n]/a))/(b*f^2*g^2*n^2*Log[F]^2
)))/a) + ((c + d*x)^2/(2*a*d) - (b*(((c + d*x)*Log[1 + (b*(F^(g*(e + f*x))
)^n)/a])/(b*f*g*n*Log[F]) + (d*PolyLog[2, -((b*(F^(g*(e + f*x)))^n)/a)])/(
b*f^2*g^2*n^2*Log[F]^2)))/a)/a)/a
```

Defintions of rubi rules used

rule 14

```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16

```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2615

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 667 vs. $2(266) = 532$.

Time = 0.30 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.42

method	result
risch	$\frac{2\left(F^{g(fx+e)}\right)^n \ln(F) bdfgnx+3\ln(F)adfgnx+2\left(F^{g(fx+e)}\right)^n \ln(F)bcfgn+3c\ln(F)afgn-\left(F^{g(fx+e)}\right)^n bd-da}{2n^2g^2f^2\ln(F)^2a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2} - \frac{3d\ln\left(F^{ngfx}F^{-}\right.}{2\ln(F)^2}$

input
```
int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

output

```
1/2*(2*(F^(g*(f*x+e)))^n*ln(F)*b*d*f*g*n*x+3*ln(F)*a*d*f*g*n*x+2*(F^(g*(f*
x+e)))^n*ln(F)*b*c*f*g*n+3*c*ln(F)*a*f*g*n-(F^(g*(f*x+e)))^n*b*d-d*a)/n^2/
g^2/f^2/ln(F)^2/a^2/(a+b*(F^(g*(f*x+e)))^n)^2-3/2/ln(F)^2/f^2/g^2/n^2/a^3*
d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)+3/2/ln(F)^2/f^2/g^2/n^2/a
^3*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)+1/2/ln(F)^2/f^2/g^
2/a^3*d*ln(F^(g*(f*x+e)))^2-1/ln(F)^2/f^2/g^2/n/a^3*d*ln(F^(g*(f*x+e)))*ln
(1+b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)-1/ln(F)^2/f^2/g^2/n^2/a
^3*d*polylog(2,-b*F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n/a)+1/ln(F)/f/
g/n/a^3*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*x-1/ln(F)^2/f^2/g
^2/n/a^3*d*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(g*(f*x+e)))^n)*ln(F^(g*(f*x+e))
)-1/ln(F)/f/g/n/a^3*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b+a)*x
+1/ln(F)^2/f^2/g^2/n/a^3*d*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*g*f*x)*b
+a)*ln(F^(g*(f*x+e)))+1/ln(F)/f/g/n/a^3*c*ln(F^(n*g*f*x)*F^(-n*g*f*x)*(F^(
g*(f*x+e)))^n)-1/ln(F)/f/g/n/a^3*c*ln((F^(g*(f*x+e)))^n*F^(-n*g*f*x)*F^(n*
g*f*x)*b+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(265) = 530$.

Time $= 0.08$ (sec) , antiderivative size $= 696$, normalized size of antiderivative $= 2.52$

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx =$$

$$-\frac{3\left(a^2de-a^2cf\right)gn\log\left(F\right)+a^2d-\left(a^2df^2g^2n^2x^2+2\,a^2cf^2g^2n^2x-\left(a^2de^2-2\,a^2cef\right)g^2n^2\right)\log\left(F\right)^2-}{}$$

input

```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

output
```
-1/2*(3*(a^2*d*e - a^2*c*f)*g*n*log(F) + a^2*d - (a^2*d*f^2*g^2*n^2*x^2 +
2*a^2*c*f^2*g^2*n^2*x - (a^2*d*e^2 - 2*a^2*c*e*f)*g^2*n^2)*log(F)^2 - ((b^
2*d*f^2*g^2*n^2*x^2 + 2*b^2*c*f^2*g^2*n^2*x - (b^2*d*e^2 - 2*b^2*c*e*f)*g^
2*n^2)*log(F)^2 - 3*(b^2*d*f*g*n*x + b^2*d*e*g*n)*log(F))*F^(2*f*g*n*x + 2
*e*g*n) + (a*b*d - 2*(a*b*d*f^2*g^2*n^2*x^2 + 2*a*b*c*f^2*g^2*n^2*x - (a*b
*d*e^2 - 2*a*b*c*e*f)*g^2*n^2)*log(F)^2 + 2*(2*a*b*d*f*g*n*x + (3*a*b*d*e
- a*b*c*f)*g*n)*log(F))*F^(f*g*n*x + e*g*n) + 2*(2*F^(f*g*n*x + e*g*n)*a*b
*d + F^(2*f*g*n*x + 2*e*g*n)*b^2*d + a^2*d)*dilog(-(F^(f*g*n*x + e*g*n)*b
+ a)/a + 1) - (2*(a^2*d*e - a^2*c*f)*g*n*log(F) + 3*a^2*d + (2*(b^2*d*e -
b^2*c*f)*g*n*log(F) + 3*b^2*d)*F^(2*f*g*n*x + 2*e*g*n) + 2*(2*(a*b*d*e - a
*b*c*f)*g*n*log(F) + 3*a*b*d)*F^(f*g*n*x + e*g*n))*log(F^(f*g*n*x + e*g*n)
*b + a) + 2*((b^2*d*f*g*n*x + b^2*d*e*g*n)*F^(2*f*g*n*x + 2*e*g*n)*log(F)
+ 2*(a*b*d*f*g*n*x + a*b*d*e*g*n)*F^(f*g*n*x + e*g*n)*log(F) + (a^2*d*f*g*
n*x + a^2*d*e*g*n)*log(F))*log((F^(f*g*n*x + e*g*n)*b + a)/a))/(2*F^(f*g*n
*x + e*g*n)*a^4*b*f^2*g^2*n^2*log(F)^2 + F^(2*f*g*n*x + 2*e*g*n)*a^3*b^2*f
^2*g^2*n^2*log(F)^2 + a^5*f^2*g^2*n^2*log(F)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx}{\left(a + b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{c + dx}{\left(a + b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{dx + c}{\left(\left(F^{(fx+e)g}\right)^n b + a\right)^3}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

output
```
1/2*d*((3*a*f*g*n*x*log(F) + (2*F^(e*g*n)*b*f*g*n*x*log(F) - F^(e*g*n)*b)*
F^(f*g*n*x) - a)/(2*F^(f*g*n*x)*F^(e*g*n)*a^3*b*f^2*g^2*n^2*log(F)^2 + F^(
2*f*g*n*x)*F^(2*e*g*n)*a^2*b^2*f^2*g^2*n^2*log(F)^2 + a^4*f^2*g^2*n^2*log(
F)^2) + 2*integrate(1/2*(2*f*g*n*x*log(F) - 3)/(F^(f*g*n*x)*F^(e*g*n)*a^2*
b*f*g*n*log(F) + a^3*f*g*n*log(F)), x)) + 1/2*c*((2*F^(f*g*n*x + e*g*n)*b
+ 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F^(2*f*g*n*x + 2*e*g*n)*a^2*b^2 + a
^4)*f*g*n*log(F)) + 2*(f*g*n*x + e*g*n)/(a^3*f*g*n) - 2*log(F^(f*g*n*x + e
*g*n)*b + a)/(a^3*f*g*n*log(F)))
```

Giac [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{dx+c}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3}\,dx$$

input
```
integrate((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)/((F^((f*x + e)*g))^n*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \int \frac{c+d\,x}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3}\,dx$$

input
```
int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3,x)
```

output
```
int((c + d*x)/(a + b*(F^(g*(e + f*x)))^n)^3, x)
```

Reduce [F]

$$\int \frac{c+dx}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$= \frac{2f^{2fgnx+2egn}\left(\int \frac{x}{f^{3fgnx+3egn}b^3+3f^{2fgnx+2egn}ab^2+3f^{fgnx+egn}a^2b+a^3}dx\right)\log(f)\,a^3b^2dfgn-2f^{2fgnx+2egn}\log\left(f^{fgnx+}\right.}$$

input
```
int((d*x+c)/(a+b*(F^(g*(f*x+e)))^n)^3,x)
```

output
```
(2*f**(2*e*g*n + 2*f*g*n*x)*int(x/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2
*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(
f)*a**3*b**2*d*f*g*n - 2*f**(2*e*g*n + 2*f*g*n*x)*log(f**(e*g*n + f*g*n*x)
*b + a)*b**2*c + 2*f**(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*c*f*g*n*x - f**(2*
e*g*n + 2*f*g*n*x)*b**2*c + 4*f**(e*g*n + f*g*n*x)*int(x/(f**(3*e*g*n + 3*
f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2 + 3*f**(e*g*n + f*g*n*x)
*a**2*b + a**3),x)*log(f)*a**4*b*d*f*g*n - 4*f**(e*g*n + f*g*n*x)*log(f**(
e*g*n + f*g*n*x)*b + a)*a*b*c + 4*f**(e*g*n + f*g*n*x)*log(f)*a*b*c*f*g*n*
x + 2*int(x/(f**(3*e*g*n + 3*f*g*n*x)*b**3 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*
b**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b + a**3),x)*log(f)*a**5*d*f*g*n - 2*lo
g(f**(e*g*n + f*g*n*x)*b + a)*a**2*c + 2*log(f)*a**2*c*f*g*n*x + 2*a**2*c)
/(2*log(f)*a**3*f*g*n*(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n
*x)*a*b + a**2))
```

3.61 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$

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Optimal result

Integrand size = 17, antiderivative size = 111

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{x}{a^3} + \frac{1}{2af\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn\log(F)} + \frac{1}{a^2 f\left(a+b\left(F^{g(e+fx)}\right)^n\right) gn\log(F)} - \frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3 fgn\log(F)}$$

output

```
x/a^3+1/2/a/f/(a+b*(F^(g*(f*x+e)))^n)^2/g/n/ln(F)+1/a^2/f/(a+b*(F^(g*(f*x+
e)))^n)/g/n/ln(F)-ln(a+b*(F^(g*(f*x+e)))^n)/a^3/f/g/n/ln(F)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{\frac{3a+2b\left(F^{g(e+fx)}\right)^n}{2a^2 f\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 gn} + \frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^3 fgn} - \frac{\log\left(a^4 fgn\log(F)+a^3 bf\left(F^{g(e+fx)}\right)^n gn\log(F)\right)}{a^3 fgn}}{\log(F)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^(-3),x]
```

output
```
((3*a + 2*b*(F^(g*(e + f*x)))^n)/(2*a^2*f*(a + b*(F^(g*(e + f*x)))^n)^2*g*
n) + Log[(F^(g*(e + f*x)))^n]/(a^3*f*g*n) - Log[a^4*f*g*n*Log[F] + a^3*b*f
*(F^(g*(e + f*x)))^n*g*n*Log[F]]/(a^3*f*g*n))/Log[F]
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2720, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$\downarrow 2720$$

$$\frac{\int \frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}\,dF^{g(e+fx)}}{fg\log(F)}$$

$$\downarrow 798$$

$$\frac{\int \frac{F^{-g(e+fx)}}{\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}\,d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{F^{-g(e+fx)}}{a^3}-\frac{b}{a^3\left(b\left(F^{g(e+fx)}\right)^n+a\right)}-\frac{b}{a^2\left(b\left(F^{g(e+fx)}\right)^n+a\right)^2}-\frac{b}{a\left(b\left(F^{g(e+fx)}\right)^n+a\right)^3}\right)d\left(F^{g(e+fx)}\right)^n}{fgn\log(F)}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log\left(a+b\left(F^{g(e+fx)}\right)^n\right)}{a^3}+\frac{\log\left(\left(F^{g(e+fx)}\right)^n\right)}{a^3}+\frac{1}{a^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)}+\frac{1}{2a\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^(-3),x]
```

output
```
(1/(2*a*(a + b*(F^(g*(e + f*x)))^n)^2) + 1/(a^2*(a + b*(F^(g*(e + f*x)))^n
)) + Log[(F^(g*(e + f*x)))^n]/a^3 - Log[a + b*(F^(g*(e + f*x)))^n]/a^3)/(f
*g*n*Log[F])
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{1}{2a\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a^3}+\frac{1}{a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a^3}}{gf\ln(F)n}$
default	$\frac{\frac{1}{2a\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}-\frac{\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)}{a^3}+\frac{1}{a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)}+\frac{\ln\left(\left(F^{g(fx+e)}\right)^n\right)}{a^3}}{gf\ln(F)n}$
risch	$\frac{\ln\left(F^{g(fx+e)}\right)}{\ln(F)a^3fg}+\frac{2b\left(F^{g(fx+e)}\right)^n+3a}{2\ln(F)fgn\,a^2\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}-\frac{\ln\left(\left(F^{g(fx+e)}\right)^n+\frac{a}{b}\right)}{\ln(F)a^3fgn}$
norman	$\frac{\frac{b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{\ln(F)fgn\,a^2}+\frac{b^2x\,\mathrm{e}^{2n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{a^3}+\frac{x}{a}+\frac{2bx\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}}{a^2}+\frac{3}{2\ln(F)afgn}}{\left(a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx+e)\ln(F)}\right)}\right)^2}-\frac{\ln\left(a+b\,\mathrm{e}^{n\ln\left(\mathrm{e}^{g(fx}\right.\right.}{\ln(F)a^3f}$
parallelrisch	$\frac{2b^4\left(F^{g(fx+e)}\right)^{2n}x\ln(F)fgn+4b^3\left(F^{g(fx+e)}\right)^n x\ln(F)afgn+2x\ln(F)a^2b^2fgn-2\ln\left(a+b\left(F^{g(fx+e)}\right)^n\right)\left(F^{g(fx+e)}\right)^{2}}{2\ln(F)a^3b^2fgn\left(a+b\left(F^{g}\right.\right.}$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3,x,method=_RETURNVERBOSE)
```

output
```
1/g/f/ln(F)/n*(1/2/a/(a+b*(F^(g*(f*x+e)))^n)^2-1/a^3*ln(a+b*(F^(g*(f*x+e)))^n)+1/a^2/(a+b*(F^(g*(f*x+e)))^n)+1/a^3*ln((F^(g*(f*x+e)))^n))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.71

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$
$$=\frac{2\,F^{2\,fgnx+2\,egn}b^2fgnx\log(F)+2\,a^2fgnx\log(F)+2\,(2\,abfgnx\log(F)+ab)F^{fgnx+egn}+3\,a^2-2\,(2\,F^{fg}}{2\,(2\,F^{fgnx+egn}a^4bfgn\log(F)+F^{2\,fgnx+2\,egn}a^3b^2fgn\log(F)+a^{}}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="fricas")
```

output
```
1/2*(2*F^(2*f*g*n*x + 2*e*g*n)*b^2*f*g*n*x*log(F) + 2*a^2*f*g*n*x*log(F) +
 2*(2*a*b*f*g*n*x*log(F) + a*b)*F^(f*g*n*x + e*g*n) + 3*a^2 - 2*(2*F^(f*g*
n*x + e*g*n)*a*b + F^(2*f*g*n*x + 2*e*g*n)*b^2 + a^2)*log(F^(f*g*n*x + e*g
*n)*b + a))/(2*F^(f*g*n*x + e*g*n)*a^4*b*f*g*n*log(F) + F^(2*f*g*n*x + 2*e
*g*n)*a^3*b^2*f*g*n*log(F) + a^5*f*g*n*log(F))
```

Sympy [F]

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}}\,dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)**3,x)
```

output
```
Integral((a + b*(F**(g*(e + f*x)))**n)**(-3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}}\,dx = \frac{2\,F^{fgnx+egn}b+3\,a}{2\left(2\,F^{fgnx+egn}a^{3}b+F^{2\,fgnx+2\,egn}a^{2}b^{2}+a^{4}\right)fgn\log\left(F\right)} + \frac{fgnx+egn}{a^{3}fgn} - \frac{\log\left(F^{fgnx+egn}b+a\right)}{a^{3}fgn\log\left(F\right)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="maxima")
```

output
```
1/2*(2*F^(f*g*n*x + e*g*n)*b + 3*a)/((2*F^(f*g*n*x + e*g*n)*a^3*b + F^(2*f
*g*n*x + 2*e*g*n)*a^2*b^2 + a^4)*f*g*n*log(F)) + (f*g*n*x + e*g*n)/(a^3*f*
g*n) - log(F^(f*g*n*x + e*g*n)*b + a)/(a^3*f*g*n*log(F))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{\log\left(|F|^{fgnx}|F|^{egn}\right)}{a^3 fgn\log(F)} - \frac{\log\left(\left|F^{fgnx}F^{egn}b+a\right|\right)}{a^3 fgn\log(F)} + \frac{2\,F^{fgnx}F^{egn}ab+3\,a^2}{2\left(F^{fgnx}F^{egn}b+a\right)^2 a^3 fgn\log(F)}$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3,x, algorithm="giac")
```

output
```
log(abs(F)^(f*g*n*x)*abs(F)^(e*g*n))/(a^3*f*g*n*log(F)) - log(abs(F^(f*g*n
*x)*F^(e*g*n)*b + a))/(a^3*f*g*n*log(F)) + 1/2*(2*F^(f*g*n*x)*F^(e*g*n)*a*
b + 3*a^2)/((F^(f*g*n*x)*F^(e*g*n)*b + a)^2*a^3*f*g*n*log(F))
```

Mupad [B] (verification not implemented)

Time = 23.11 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx = \frac{x}{a^3} + \frac{1}{2\,a\,f\,g\,n\,\ln(F)\left(a^2+b^2\left(F^{f\,g\,x}\,F^{e\,g}\right)^{2\,n}+2\,a\,b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)} + \frac{1}{a^2\,f\,g\,n\,\ln(F)\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)} - \frac{\ln\left(a+b\left(F^{f\,g\,x}\,F^{e\,g}\right)^n\right)}{a^3\,f\,g\,n\,\ln(F)}$$

input
```
int(1/(a + b*(F^(g*(e + f*x)))^n)^3,x)
```

output
```
x/a^3 + 1/(2*a*f*g*n*log(F)*(a^2 + b^2*(F^(f*g*x)*F^(e*g))^(2*n) + 2*a*b*(
F^(f*g*x)*F^(e*g))^n)) + 1/(a^2*f*g*n*log(F)*(a + b*(F^(f*g*x)*F^(e*g))^n)
) - log(a + b*(F^(f*g*x)*F^(e*g))^n)/(a^3*f*g*n*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.09

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}}\,dx$$

$$= \frac{-2f^{2fgnx+2egn}\log\left(f^{fgnx+egn}b+a\right)b^{2}+2f^{2fgnx+2egn}\log(f)\,b^{2}fgnx-f^{2fgnx+2egn}b^{2}-4f^{fgnx+egn}\log\left(f^{fgn}\right.}{2\log(f)\,a^{3}fgn\left(f^{2fgnx+2egn}b^{2}\right.}$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3,x)
```

output
```
( - 2*f**(2*e*g*n + 2*f*g*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*b**2 + 2*f*
*(2*e*g*n + 2*f*g*n*x)*log(f)*b**2*f*g*n*x - f**(2*e*g*n + 2*f*g*n*x)*b**2
 - 4*f**(e*g*n + f*g*n*x)*log(f**(e*g*n + f*g*n*x)*b + a)*a*b + 4*f**(e*g*
n + f*g*n*x)*log(f)*a*b*f*g*n*x - 2*log(f**(e*g*n + f*g*n*x)*b + a)*a**2 +
 2*log(f)*a**2*f*g*n*x + 2*a**2)/(2*log(f)*a**3*f*g*n*(f**(2*e*g*n + 2*f*g
*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a*b + a**2))
```

3.62 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx$

Optimal result . . . 573
Mathematica [N/A] . . . 573
Rubi [N/A] . . . 574
Maple [N/A] . . . 575
Fricas [N/A] . . . 575
Sympy [N/A] . . . 575
Maxima [N/A] . . . 576
Giac [N/A] . . . 577
Mupad [N/A] . . . 577
Reduce [N/A] . . . 577

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}, x\right)$$

output
```
Defer(Int)(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx$$

input
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)),x]
```

output
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)), x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$\downarrow 2618$$

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}\,dx$$

$$\downarrow 2619$$

$$\int \frac{1}{(c+dx)\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}\,dx$$

input
```
Int[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^3 (dx+c)} dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

output
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)} dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)} dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(1/(a^3*d*x + a^3*c + (b^3*d*x + b^3*c)*(F^(f*g*x + e*g))^(3*n) +
3*(a*b^2*d*x + a*b^2*c)*(F^(f*g*x + e*g))^(2*n) + 3*(a^2*b*d*x + a^2*b*c)*
(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 11.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)} dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3 (c+dx)} dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c),x)
```

output
```
Integral(1/((a + b*(F**(e*g + f*g*x))**n)**3*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 651, normalized size of antiderivative = 26.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(3*a*d*f*g*n*x*log(F) + 3*a*c*f*g*n*log(F) + (2*F^(e*g*n)*b*d*f*g*n*x*
log(F) + 2*F^(e*g*n)*b*c*f*g*n*log(F) + F^(e*g*n)*b*d)*F^(f*g*n*x) + a*d)/
(a^4*d^2*f^2*g^2*n^2*x^2*log(F)^2 + 2*a^4*c*d*f^2*g^2*n^2*x*log(F)^2 + a^4
*c^2*f^2*g^2*n^2*log(F)^2 + (F^(2*e*g*n)*a^2*b^2*d^2*f^2*g^2*n^2*x^2*log(F
)^2 + 2*F^(2*e*g*n)*a^2*b^2*c*d*f^2*g^2*n^2*x*log(F)^2 + F^(2*e*g*n)*a^2*b
^2*c^2*f^2*g^2*n^2*log(F)^2)*F^(2*f*g*n*x) + 2*(F^(e*g*n)*a^3*b*d^2*f^2*g^
2*n^2*x^2*log(F)^2 + 2*F^(e*g*n)*a^3*b*c*d*f^2*g^2*n^2*x*log(F)^2 + F^(e*g
*n)*a^3*b*c^2*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)) + integrate(1/2*(2*d^2*f^
2*g^2*n^2*x^2*log(F)^2 + 2*c^2*f^2*g^2*n^2*log(F)^2 + 3*c*d*f*g*n*log(F) +
 2*d^2 + (4*c*d*f^2*g^2*n^2*log(F)^2 + 3*d^2*f*g*n*log(F))*x)/(a^3*d^3*f^2
*g^2*n^2*x^3*log(F)^2 + 3*a^3*c*d^2*f^2*g^2*n^2*x^2*log(F)^2 + 3*a^3*c^2*d
*f^2*g^2*n^2*x*log(F)^2 + a^3*c^3*f^2*g^2*n^2*log(F)^2 + (F^(e*g*n)*a^2*b*
d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*F^(e*g*n)*a^2*b*c*d^2*f^2*g^2*n^2*x^2*log
(F)^2 + 3*F^(e*g*n)*a^2*b*c^2*d*f^2*g^2*n^2*x*log(F)^2 + F^(e*g*n)*a^2*b*c
^3*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3(dx+c)}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(1/(((F^((f*x + e)*g))^n*b + a)^3*(d*x + c)), x)
```

Mupad [N/A]

Not integrable

Time = 23.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx = \int \frac{1}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)),x)
```

output
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 5.48

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3(c+dx)}\,dx$$
$$= \int \frac{1}{f^{3fgnx+3egn}b^3c + f^{3fgnx+3egn}b^3dx + 3f^{2fgnx+2egn}a\,b^2c + 3f^{2fgnx+2egn}a\,b^2dx + 3f^{fgnx+egn}a^2bc + 3f^{fgnx}$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c),x)
```

output
```
int(1/(f**(3*e*g*n + 3*f*g*n*x)*b**3*c + f**(3*e*g*n + 3*f*g*n*x)*b**3*d*x
 + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2*c + 3*f**(2*e*g*n + 2*f*g*n*x)*a*b**2
*d*x + 3*f**(e*g*n + f*g*n*x)*a**2*b*c + 3*f**(e*g*n + f*g*n*x)*a**2*b*d*x
 + a**3*c + a**3*d*x),x)
```

3.63 $\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx$

Optimal result . . . 579
Mathematica [N/A] . . . 579
Rubi [N/A] . . . 580
Maple [N/A] . . . 581
Fricas [N/A] . . . 581
Sympy [N/A] . . . 582
Maxima [N/A] . . . 582
Giac [N/A] . . . 583
Mupad [N/A] . . . 584
Reduce [N/A] . . . 584

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \text{Int}\left(\frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}, x\right)$$

output
```
Defer(Int)(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx$$

input
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2),x]
```

output
```
Integrate[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2618, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3}\,dx$$

$$\downarrow 2618$$

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}dx$$

$$\downarrow 2619$$

$$\int \frac{1}{(c+dx)^2\left(a+b\left(F^{eg+fgx}\right)^n\right)^3}dx$$

input
```
Int[1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2618
```
Int[((a_.) + (b_.)*((F_)^((g_.)*(v_)))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Int[(c + d*x)^m*(a + b*(F^(g*ExpandToSum[v, x]))^n)^p, x]
 /; FreeQ[{F, a, b, c, d, g, n, p}, x] && LinearQ[v, x] &&  !LinearMatchQ[v
, x] && IntegerQ[m]
```

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+b\left(F^{g(fx+e)}\right)^{n}\right)^{3}(dx+c)^{2}}dx$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

output
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 159, normalized size of antiderivative = 6.36

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}(c+dx)^{2}}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^{n}b+a\right)^{3}(dx+c)^{2}}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(1/(a^3*d^2*x^2 + 2*a^3*c*d*x + a^3*c^2 + (b^3*d^2*x^2 + 2*b^3*c*d
*x + b^3*c^2)*(F^(f*g*x + e*g))^(3*n) + 3*(a*b^2*d^2*x^2 + 2*a*b^2*c*d*x +
 a*b^2*c^2)*(F^(f*g*x + e*g))^(2*n) + 3*(a^2*b*d^2*x^2 + 2*a^2*b*c*d*x + a
^2*b*c^2)*(F^(f*g*x + e*g))^n), x)
```

Sympy [N/A]

Not integrable

Time = 117.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^3 (c+dx)^2}\,dx$$

input
```
integrate(1/(a+b*(F**(g*(f*x+e)))**n)**3/(d*x+c)**2,x)
```

output
```
Integral(1/((a + b*(F**(e*g + f*g*x))**n)**3*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 807, normalized size of antiderivative = 32.28

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)^2}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(3*a*d*f*g*n*x*log(F) + 3*a*c*f*g*n*log(F) + 2*(F^(e*g*n)*b*d*f*g*n*x*
log(F) + F^(e*g*n)*b*c*f*g*n*log(F) + F^(e*g*n)*b*d)*F^(f*g*n*x) + 2*a*d)/
(a^4*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*a^4*c*d^2*f^2*g^2*n^2*x^2*log(F)^2 +
 3*a^4*c^2*d*f^2*g^2*n^2*x*log(F)^2 + a^4*c^3*f^2*g^2*n^2*log(F)^2 + (F^(2
*e*g*n)*a^2*b^2*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*F^(2*e*g*n)*a^2*b^2*c*d^2
*f^2*g^2*n^2*x^2*log(F)^2 + 3*F^(2*e*g*n)*a^2*b^2*c^2*d*f^2*g^2*n^2*x*log(
F)^2 + F^(2*e*g*n)*a^2*b^2*c^3*f^2*g^2*n^2*log(F)^2)*F^(2*f*g*n*x) + 2*(F^
(e*g*n)*a^3*b*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 3*F^(e*g*n)*a^3*b*c*d^2*f^2*g
^2*n^2*x^2*log(F)^2 + 3*F^(e*g*n)*a^3*b*c^2*d*f^2*g^2*n^2*x*log(F)^2 + F^(
e*g*n)*a^3*b*c^3*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)) + integrate((d^2*f^2*g
^2*n^2*x^2*log(F)^2 + c^2*f^2*g^2*n^2*log(F)^2 + 3*c*d*f*g*n*log(F) + 3*d^
2 + (2*c*d*f^2*g^2*n^2*log(F)^2 + 3*d^2*f*g*n*log(F))*x)/(a^3*d^4*f^2*g^2*
n^2*x^4*log(F)^2 + 4*a^3*c*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 6*a^3*c^2*d^2*f^
2*g^2*n^2*x^2*log(F)^2 + 4*a^3*c^3*d*f^2*g^2*n^2*x*log(F)^2 + a^3*c^4*f^2*
g^2*n^2*log(F)^2 + (F^(e*g*n)*a^2*b*d^4*f^2*g^2*n^2*x^4*log(F)^2 + 4*F^(e*
g*n)*a^2*b*c*d^3*f^2*g^2*n^2*x^3*log(F)^2 + 6*F^(e*g*n)*a^2*b*c^2*d^2*f^2*
g^2*n^2*x^2*log(F)^2 + 4*F^(e*g*n)*a^2*b*c^3*d*f^2*g^2*n^2*x*log(F)^2 + F^
(e*g*n)*a^2*b*c^4*f^2*g^2*n^2*log(F)^2)*F^(f*g*n*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^{n}\right)^{3}(c+dx)^{2}}\,dx = \int \frac{1}{\left(\left(F^{(fx+e)g}\right)^{n}b+a\right)^{3}(dx+c)^{2}}\,dx$$

input
```
integrate(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(1/(((F^((f*x + e)*g))^n*b + a)^3*(d*x + c)^2), x)
```

Mupad [N/A]

Not integrable

Time = 23.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx = \int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx$$

input
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2),x)
```

output
```
int(1/((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 9.40

$$\int \frac{1}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^2}\,dx$$
$$= \int \frac{}{f^{3fgnx+3egn}b^3c^2 + 2f^{3fgnx+3egn}b^3cdx + f^{3fgnx+3egn}b^3d^2x^2 + 3f^{2fgnx+2egn}a\,b^2c^2 + 6f^{2fgnx+2egn}a\,b^2cdx -$$

input
```
int(1/(a+b*(F^(g*(f*x+e)))^n)^3/(d*x+c)^2,x)
```

output
```
int(1/(f**(3*e*g*n + 3*f*g*n*x)*b**3*c**2 + 2*f**(3*e*g*n + 3*f*g*n*x)*b**
3*c*d*x + f**(3*e*g*n + 3*f*g*n*x)*b**3*d**2*x**2 + 3*f**(2*e*g*n + 2*f*g*
n*x)*a*b**2*c**2 + 6*f**(2*e*g*n + 2*f*g*n*x)*a*b**2*c*d*x + 3*f**(2*e*g*n
 + 2*f*g*n*x)*a*b**2*d**2*x**2 + 3*f**(e*g*n + f*g*n*x)*a**2*b*c**2 + 6*f*
*(e*g*n + f*g*n*x)*a**2*b*c*d*x + 3*f**(e*g*n + f*g*n*x)*a**2*b*d**2*x**2
+ a**3*c**2 + 2*a**3*c*d*x + a**3*d**2*x**2),x)
```

3.64 $\int (a+be^x)\sqrt{c+dx}\,dx$

Optimal result . . . 585
Mathematica [A] (verified) . . . 585
Rubi [A] (verified) . . . 586
Maple [A] (verified) . . . 587
Fricas [A] (verification not implemented) . . . 587
Sympy [F] . . . 588
Maxima [A] (verification not implemented) . . . 588
Giac [B] (verification not implemented) . . . 588
Mupad [F(-1)] . . . 589
Reduce [B] (verification not implemented) . . . 589

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int (a+be^x)\sqrt{c+dx}\,dx = be^x\sqrt{c+dx} + \frac{2a(c+dx)^{3/2}}{3d} - \frac{1}{2}b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)$$

output

```
b*exp(x)*(d*x+c)^(1/2)+2/3*a*(d*x+c)^(3/2)/d-1/2*b*d^(1/2)*Pi^(1/2)*erfi((
d*x+c)^(1/2)/d^(1/2))/exp(c/d)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int (a+be^x)\sqrt{c+dx}\,dx = be^x\sqrt{c+dx} + \frac{2a(c+dx)^{3/2}}{3d} - \frac{1}{2}b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)$$

input

```
Integrate[(a + b*E^x)*Sqrt[c + d*x],x]
```

output

```
b*E^x*Sqrt[c + d*x] + (2*a*(c + d*x)^(3/2))/(3*d) - (b*Sqrt[d]*Sqrt[Pi]*Er
fi[Sqrt[c + d*x]/Sqrt[d]])/(2*E^(c/d))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + be^x)\sqrt{c + dx}\,dx$$

$$\downarrow 2614$$

$$\int \left(a\sqrt{c + dx} + be^x\sqrt{c + dx}\right) dx$$

$$\downarrow 2009$$

$$\frac{2a(c + dx)^{3/2}}{3d} - \frac{1}{2}\sqrt{\pi}b\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c + dx}}{\sqrt{d}}\right) + be^x\sqrt{c + dx}$$

input
```
Int[(a + b*E^x)*Sqrt[c + d*x],x]
```

output
```
b*E^x*Sqrt[c + d*x] + (2*a*(c + d*x)^(3/2))/(3*d) - (b*Sqrt[d]*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/(2*E^(c/d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\frac{\frac{2a(dx+c)^{\frac{3}{2}}}{3}+2b\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right)}{d}$	77
default	$\frac{\frac{2a(dx+c)^{\frac{3}{2}}}{3}+2b\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right)}{d}$	77
parts	$\frac{2a(dx+c)^{\frac{3}{2}}}{3d}+\frac{2b\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+c}\right)}{4\sqrt{-\frac{1}{d}}}\right)}{d}$	79

input
```
int((a+b*exp(x))*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/3*a*(d*x+c)^(3/2)+b/exp(c/d)*(1/2*(d*x+c)^(1/2)*exp(1/d*(d*x+c))*d-
1/4*d*Pi^(1/2)/(-1/d)^(1/2)*erf((-1/d)^(1/2)*(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int (a+be^x)\sqrt{c+dx}\,dx$$
$$= \frac{3\sqrt{\pi}bd^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}+2\left(2\,adx+3\,bde^x+2\,ac\right)\sqrt{dx+c}}{6\,d}$$

input
```
integrate((a+b*exp(x))*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/6*(3*sqrt(pi)*b*d^2*sqrt(-1/d)*erf(sqrt(d*x + c)*sqrt(-1/d))*e^(-c/d) +
2*(2*a*d*x + 3*b*d*e^x + 2*a*c)*sqrt(d*x + c))/d
```

Sympy [F]

$$\int (a + be^x)\sqrt{c + dx}\,dx = \int (a + be^x)\sqrt{c + dx}\,dx$$

input
```
integrate((a+b*exp(x))*(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*exp(x))*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int (a + be^x)\sqrt{c + dx}\,dx$$
$$= \frac{4\,(dx + c)^{\frac{3}{2}}a - 3\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx + c}de^{\left(\frac{dx+c}{d} - \frac{c}{d}\right)}\right)b}{6\,d}$$

input
```
integrate((a+b*exp(x))*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(4*(d*x + c)^(3/2)*a - 3*(sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-1/d))*e^(
-c/d)/sqrt(-1/d) - 2*sqrt(d*x + c)*d*e^((d*x + c)/d - c/d))*b)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(53) = 106.

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

$$\int (a + be^x)\sqrt{c + dx}\,dx =$$
$$-\frac{\frac{6\sqrt{\pi}bcd\,\mathrm{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} - 12\sqrt{dx + c}ac - 4\left((dx + c)^{\frac{3}{2}} - 3\sqrt{dx + c}c\right)a - 3\left(\frac{\sqrt{\pi}(2\,c+d)d\,\mathrm{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-}\right.}{d}\right.}{\sqrt{-d}}\right.}{6\,d}$$

input
```
integrate((a+b*exp(x))*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(6*sqrt(pi)*b*c*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) -
12*sqrt(d*x + c)*a*c - 4*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a - 3*(sqrt
(pi)*(2*c + d)*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) + 2*sqrt
(d*x + c)*d*e^x)*b)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+be^x)\sqrt{c+dx}\,dx = \int (a+b\,\mathrm{e}^x)\,\sqrt{c+d\,x}\,dx$$

input
```
int((a + b*exp(x))*(c + d*x)^(1/2),x)
```

output
```
int((a + b*exp(x))*(c + d*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\int (a+be^x)\sqrt{c+dx}\,dx = \frac{3\sqrt{\pi}\sqrt{d}\,\mathrm{erf}\left(\frac{\sqrt{dx+c}\,i}{\sqrt{d}}\right)bdi + 6e^{\frac{dx+c}{d}}\sqrt{dx+c}\,bd + 4e^{\frac{c}{d}}\sqrt{dx+c}\,ac + 4e^{\frac{c}{d}}\sqrt{dx+c}\,adx}{6e^{\frac{c}{d}}d}$$

input
```
int((a+b*exp(x))*(d*x+c)^(1/2),x)
```

output
```
(3*sqrt(pi)*sqrt(d)*erf((sqrt(c + d*x)*i)/sqrt(d))*b*d*i + 6*e**((c + d*x)
/d)*sqrt(c + d*x)*b*d + 4*e**(c/d)*sqrt(c + d*x)*a*c + 4*e**(c/d)*sqrt(c +
 d*x)*a*d*x)/(6*e**(c/d)*d)
```

3.65 $\int (a+be^x)^2 \sqrt{c+dx}\, dx$

Optimal result . . . 590
Mathematica [A] (verified) . . . 590
Rubi [A] (verified) . . . 591
Maple [A] (verified) . . . 592
Fricas [A] (verification not implemented) . . . 593
Sympy [F] . . . 593
Maxima [A] (verification not implemented) . . . 593
Giac [B] (verification not implemented) . . . 594
Mupad [F(-1)] . . . 595
Reduce [F] . . . 595

Optimal result

Integrand size = 19, antiderivative size = 145

$$\int (a+be^x)^2 \sqrt{c+dx}\, dx = 2abe^x\sqrt{c+dx} + \frac{1}{2}b^2e^{2x}\sqrt{c+dx} + \frac{2a^2(c+dx)^{3/2}}{3d} - ab\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) - \frac{1}{4}b^2\sqrt{d}e^{-\frac{2c}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)$$

output

```
2*a*b*exp(x)*(d*x+c)^(1/2)+1/2*b^2*exp(2*x)*(d*x+c)^(1/2)+2/3*a^2*(d*x+c)^
(3/2)/d-a*b*d^(1/2)*Pi^(1/2)*erfi((d*x+c)^(1/2)/d^(1/2))/exp(c/d)-1/8*b^2*
d^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*(d*x+c)^(1/2)/d^(1/2))/exp(2*c/d)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int (a+be^x)^2 \sqrt{c+dx}\, dx = -ab\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{4\sqrt{c+dx}(12abde^x + 3b^2de^{2x} + 4a^2(c+dx)) - 3b^2d^{3/2}e^{-\frac{2c}{d}}\sqrt{2\pi}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)}{24d}$$

input
```
Integrate[(a + b*E^x)^2*Sqrt[c + d*x],x]
```

output
```
-((a*b*Sqrt[d]*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/E^(c/d)) + (4*Sqrt[c
+ d*x]*(12*a*b*d*E^x + 3*b^2*d*E^(2*x) + 4*a^2*(c + d*x)) - (3*b^2*d^(3/2)
*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/Sqrt[d]])/E^((2*c)/d))/(24*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^x)^2\sqrt{c+dx}\,dx$$

$$\downarrow 2614$$

$$\int \left(a^2\sqrt{c+dx}+2abe^x\sqrt{c+dx}+b^2e^{2x}\sqrt{c+dx}\right)dx$$

$$\downarrow 2009$$

$$\frac{2a^2(c+dx)^{3/2}}{3d}-\sqrt{\pi}ab\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)+2abe^x\sqrt{c+dx}-\frac{1}{4}\sqrt{\frac{\pi}{2}}b^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right)+\frac{1}{2}b^2e^{2x}\sqrt{c+dx}$$

input
```
Int[(a + b*E^x)^2*Sqrt[c + d*x],x]
```

output
```
2*a*b*E^x*Sqrt[c + d*x] + (b^2*E^(2*x)*Sqrt[c + d*x])/2 + (2*a^2*(c + d*x)
^(3/2))/(3*d) - (a*b*Sqrt[d]*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/E^(c/d)
 - (b^2*Sqrt[d]*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/Sqrt[d]])/(4*E^((2
*c)/d))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{2a^2(dx+c)^{\frac{3}{2}}}{3}+2b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}}\right)+4ab\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+}\right.\right.}{4\sqrt{-\frac{1}{d}}}$
default	$\frac{\frac{2a^2(dx+c)^{\frac{3}{2}}}{3}+2b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}}\right)+4ab\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\sqrt{dx+}\right.\right.}{4\sqrt{-\frac{1}{d}}}$
parts	$\frac{2a^2(dx+c)^{\frac{3}{2}}}{3d}+\frac{2b^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\sqrt{dx+c}\right)}{8\sqrt{-\frac{2}{d}}}\right)}{d}+\frac{4ab\,\mathrm{e}^{-\frac{c}{d}}\left(\frac{\sqrt{dx+c}\,\mathrm{e}^{\frac{dx+c}{d}}d}{2}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{1}{d}}\right.\right.}{4\sqrt{-\frac{1}{c}}}$

input

```
int((a+b*exp(x))^2*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/d*(1/3*a^2*(d*x+c)^(3/2)+b^2/exp(c/d)^2*(1/4*d*(d*x+c)^(1/2)*exp(2/d*(d*
x+c))-1/8*d*Pi^(1/2)/(-2/d)^(1/2)*erf((-2/d)^(1/2)*(d*x+c)^(1/2)))+2*a*b/e
xp(c/d)*(1/2*(d*x+c)^(1/2)*exp(1/d*(d*x+c))*d-1/4*d*Pi^(1/2)/(-1/d)^(1/2)*
erf((-1/d)^(1/2)*(d*x+c)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92

$$\int (a+be^x)^2\sqrt{c+dx}\,dx$$
$$= \frac{3\sqrt{2}\sqrt{\pi}b^2d^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)} + 24\sqrt{\pi}abd^2\sqrt{-\frac{1}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)} + 4\left(4a^2\right.}{24\,d}$$

input
```
integrate((a+b*exp(x))^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(3*sqrt(2)*sqrt(pi)*b^2*d^2*sqrt(-1/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt
(-1/d))*e^(-2*c/d) + 24*sqrt(pi)*a*b*d^2*sqrt(-1/d)*erf(sqrt(d*x + c)*sqrt
(-1/d))*e^(-c/d) + 4*(4*a^2*d*x + 3*b^2*d*e^(2*x) + 12*a*b*d*e^x + 4*a^2*c
)*sqrt(d*x + c))/d
```

Sympy [F]

$$\int (a+be^x)^2\sqrt{c+dx}\,dx = \int (a+be^x)^2\sqrt{c+dx}\,dx$$

input
```
integrate((a+b*exp(x))**2*(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*exp(x))**2*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int (a+be^x)^2\sqrt{c+dx}\,dx$$
$$= \frac{16\,(dx+c)^{\frac{3}{2}}a^2 - 24\left(\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx+c}de^{\left(\frac{dx+c}{d}-\frac{c}{d}\right)}\right)ab - 3\left(\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(\right.}\right.}{\sqrt{-\frac{1}{d}}}}{24\,d}$$

input
```
integrate((a+b*exp(x))^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(16*(d*x + c)^(3/2)*a^2 - 24*(sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-1/d)
)*e^(-c/d)/sqrt(-1/d) - 2*sqrt(d*x + c)*d*e^((d*x + c)/d - c/d))*a*b - 3*(
sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-1/d))*e^(-2*c/d)/sqrt(-
1/d) - 4*sqrt(d*x + c)*d*e^(2*(d*x + c)/d - 2*c/d))*b^2)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(110) = 220$.

Time $= 0.11$ (sec) , antiderivative size $= 250$, normalized size of antiderivative $= 1.72$

$$\int (a + be^x)^2 \sqrt{c + dx}\, dx =$$

$$-\frac{\frac{12\sqrt{2}\sqrt{\pi}b^2cd\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-d}} + \frac{48\sqrt{\pi}abcd\operatorname{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}} - 48\sqrt{dx+c}a^2c - 16\left((dx+c)^{\frac{3}{2}} - 3\right.}{}$$

input
```
integrate((a+b*exp(x))^2*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/24*(12*sqrt(2)*sqrt(pi)*b^2*c*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d)*
e^(-2*c/d)/sqrt(-d) + 48*sqrt(pi)*a*b*c*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e
^(-c/d)/sqrt(-d) - 48*sqrt(d*x + c)*a^2*c - 16*((d*x + c)^(3/2) - 3*sqrt(d
*x + c)*c)*a^2 - 24*(sqrt(pi)*(2*c + d)*d*erf(-sqrt(d*x + c)*sqrt(-d)/d)*e
^(-c/d)/sqrt(-d) + 2*sqrt(d*x + c)*d*e^x)*a*b - 3*(sqrt(2)*sqrt(pi)*(4*c +
 d)*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d)*e^(-2*c/d)/sqrt(-d) + 4*sqrt(
d*x + c)*d*e^(2*x))*b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+be^x)^2\sqrt{c+dx}\,dx = \int (a+b\,\mathrm{e}^x)^2\sqrt{c+d\,x}\,dx$$

input
```
int((a + b*exp(x))^2*(c + d*x)^(1/2),x)
```

output
```
int((a + b*exp(x))^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int (a+be^x)^2\sqrt{c+dx}\,dx$$
$$= \frac{12\sqrt{\pi}\,\sqrt{d}\,\mathrm{erf}\left(\frac{\sqrt{dx+c}\,i}{\sqrt{d}}\right)abdi + 6e^{\frac{2dx+c}{d}}\sqrt{dx+c}\,b^2d + 24e^{\frac{dx+c}{d}}\sqrt{dx+c}\,abd + 8e^{\frac{c}{d}}\sqrt{dx+c}\,a^2c + 8e^{\frac{c}{d}}\sqrt{dx+}}{12e^{\frac{c}{d}}d}$$

input
```
int((a+b*exp(x))^2*(d*x+c)^(1/2),x)
```

output
```
(12*sqrt(pi)*sqrt(d)*erf((sqrt(c + d*x)*i)/sqrt(d))*a*b*d*i + 6*e**((c + 2
*d*x)/d)*sqrt(c + d*x)*b**2*d + 24*e**((c + d*x)/d)*sqrt(c + d*x)*a*b*d +
8*e**(c/d)*sqrt(c + d*x)*a**2*c + 8*e**(c/d)*sqrt(c + d*x)*a**2*d*x - 3*e*
*(c/d)*int((e**(2*x)*sqrt(c + d*x))/(c + d*x),x)*b**2*d**2)/(12*e**(c/d)*d
)
```

3.66 $\int (a+be^x)^3 \sqrt{c+dx}\, dx$

Optimal result . . . 596
Mathematica [A] (verified) . . . 597
Rubi [A] (verified) . . . 597
Maple [A] (verified) . . . 599
Fricas [A] (verification not implemented) . . . 599
Sympy [F] . . . 600
Maxima [A] (verification not implemented) . . . 600
Giac [B] (verification not implemented) . . . 601
Mupad [F(-1)] . . . 601
Reduce [F] . . . 602

Optimal result

Integrand size = 19, antiderivative size = 224

$$\begin{aligned}\int (a+be^x)^3 \sqrt{c+dx}\, dx &= 3a^2be^x\sqrt{c+dx} + \frac{3}{2}ab^2e^{2x}\sqrt{c+dx} + \frac{1}{3}b^3e^{3x}\sqrt{c+dx} \\ &\quad + \frac{2a^3(c+dx)^{3/2}}{3d} - \frac{3}{2}a^2b\sqrt{d}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) \\ &\quad - \frac{3}{4}ab^2\sqrt{d}e^{-\frac{2c}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) \\ &\quad - \frac{1}{6}b^3\sqrt{d}e^{-\frac{3c}{d}}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{c+dx}}{\sqrt{d}}\right)\end{aligned}$$

output

```
3*a^2*b*exp(x)*(d*x+c)^(1/2)+3/2*a*b^2*exp(2*x)*(d*x+c)^(1/2)+1/3*b^3*exp(
3*x)*(d*x+c)^(1/2)+2/3*a^3*(d*x+c)^(3/2)/d-3/2*a^2*b*d^(1/2)*Pi^(1/2)*erfi
((d*x+c)^(1/2)/d^(1/2))/exp(c/d)-3/8*a*b^2*d^(1/2)*2^(1/2)*Pi^(1/2)*erfi(2
^(1/2)*(d*x+c)^(1/2)/d^(1/2))/exp(2*c/d)-1/18*b^3*d^(1/2)*3^(1/2)*Pi^(1/2)
*erfi(3^(1/2)*(d*x+c)^(1/2)/d^(1/2))/exp(3*c/d)
```

Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int (a+be^x)^3\sqrt{c+dx}\,dx = -\frac{-12\sqrt{c+dx}(18a^2bde^x+9ab^2de^{2x}+2b^3de^{3x}+4a^3(c+dx))+108a^2bd^{3/2}e^{-\frac{c}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right)+27ab^2}{72d}$$

input
```
Integrate[(a + b*E^x)^3*Sqrt[c + d*x],x]
```

output
```
-1/72*(-12*Sqrt[c + d*x]*(18*a^2*b*d*E^x + 9*a*b^2*d*E^(2*x) + 2*b^3*d*E^(
3*x) + 4*a^3*(c + d*x)) + (108*a^2*b*d^(3/2)*Sqrt[Pi]*Erfi[Sqrt[c + d*x]/S
qrt[d]])/E^(c/d) + (27*a*b^2*d^(3/2)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[c + d*x
])/Sqrt[d]])/E^((2*c)/d) + (4*b^3*d^(3/2)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[c
+ d*x])/Sqrt[d]])/E^((3*c)/d))/d
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+be^x)^3\sqrt{c+dx}\,dx$$

$$\downarrow \text{2614}$$

$$\int \left(a^3\sqrt{c+dx}+3a^2be^x\sqrt{c+dx}+3ab^2e^{2x}\sqrt{c+dx}+b^3e^{3x}\sqrt{c+dx}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2a^3(c+dx)^{3/2}}{3d} - \frac{3}{2}\sqrt{\pi}a^2b\sqrt{d}e^{-\frac{c}{d}}\operatorname{erfi}\left(\frac{\sqrt{c+dx}}{\sqrt{d}}\right) + 3a^2be^x\sqrt{c+dx} - \frac{3}{4}\sqrt{\frac{\pi}{2}}ab^2\sqrt{d}e^{-\frac{2c}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{3}{2}ab^2e^{2x}\sqrt{c+dx} - \frac{1}{6}\sqrt{\frac{\pi}{3}}b^3\sqrt{d}e^{-\frac{3c}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{c+dx}}{\sqrt{d}}\right) + \frac{1}{3}b^3e^{3x}\sqrt{c+dx}$$

input

```
Int[(a + b*E^x)^3*Sqrt[c + d*x],x]
```

output

```
3*a^2*b*E^x*Sqrt[c + d*x] + (3*a*b^2*E^(2*x)*Sqrt[c + d*x])/2 + (b^3*E^(3*
x)*Sqrt[c + d*x])/3 + (2*a^3*(c + d*x)^(3/2))/(3*d) - (3*a^2*b*Sqrt[d]*Sqr
t[Pi]*Erfi[Sqrt[c + d*x]/Sqrt[d]])/(2*E^(c/d)) - (3*a*b^2*Sqrt[d]*Sqrt[Pi/
2]*Erfi[(Sqrt[2]*Sqrt[c + d*x])/Sqrt[d]])/(4*E^((2*c)/d)) - (b^3*Sqrt[d]*S
qrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[c + d*x])/Sqrt[d]])/(6*E^((3*c)/d))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\frac{2a^3(dx+c)^{\frac{3}{2}}}{3}+2b^3\mathrm{e}^{-\frac{3c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{3dx+3c}{d}}}{6}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{3}{d}}\sqrt{dx+c}\right)}{12\sqrt{-\frac{3}{d}}}\right)+6ab^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\sqrt{}\right.\right.}{8\sqrt{-\frac{2}{d}}}}{d}$
default	$\frac{\frac{2a^3(dx+c)^{\frac{3}{2}}}{3}+2b^3\mathrm{e}^{-\frac{3c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{3dx+3c}{d}}}{6}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{3}{d}}\sqrt{dx+c}\right)}{12\sqrt{-\frac{3}{d}}}\right)+6ab^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{2}{d}}\sqrt{}\right.\right.}{8\sqrt{-\frac{2}{d}}}}{d}$
parts	$\frac{2a^3(dx+c)^{\frac{3}{2}}}{3d}+\frac{2b^3\mathrm{e}^{-\frac{3c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{3dx+3c}{d}}}{6}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-\frac{3}{d}}\sqrt{dx+c}\right)}{12\sqrt{-\frac{3}{d}}}\right)}{d}+\frac{6ab^2\mathrm{e}^{-\frac{2c}{d}}\left(\frac{d\sqrt{dx+c}\,\mathrm{e}^{\frac{2dx+2c}{d}}}{4}-\frac{d\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{}\right.\right.}{8}}{d}$

input
```
int((a+b*exp(x))^3*(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/3*a^3*(d*x+c)^(3/2)+b^3/exp(c/d)^3*(1/6*d*(d*x+c)^(1/2)*exp(3/d*(d*
x+c))-1/12*d*Pi^(1/2)/(-3/d)^(1/2)*erf((-3/d)^(1/2)*(d*x+c)^(1/2)))+3*a*b^
2/exp(c/d)^2*(1/4*d*(d*x+c)^(1/2)*exp(2/d*(d*x+c))-1/8*d*Pi^(1/2)/(-2/d)^(
1/2)*erf((-2/d)^(1/2)*(d*x+c)^(1/2)))+3*a^2*b/exp(c/d)*(1/2*(d*x+c)^(1/2)*
exp(1/d*(d*x+c))*d-1/4*d*Pi^(1/2)/(-1/d)^(1/2)*erf((-1/d)^(1/2)*(d*x+c)^(1
/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.88

$$\int (a+be^x)^3\sqrt{c+dx}\,dx$$
$$=\frac{27\sqrt{2}\sqrt{\pi}ab^2d^2\sqrt{-\frac{1}{d}}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{2c}{d}\right)}+4\sqrt{3}\sqrt{\pi}b^3d^2\sqrt{-\frac{1}{d}}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right)e^{\left(-\frac{3c}{d}\right)}-}$$

input
```
integrate((a+b*exp(x))^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/72*(27*sqrt(2)*sqrt(pi)*a*b^2*d^2*sqrt(-1/d)*erf(sqrt(2)*sqrt(d*x + c)*s
qrt(-1/d))*e^(-2*c/d) + 4*sqrt(3)*sqrt(pi)*b^3*d^2*sqrt(-1/d)*erf(sqrt(3)*
sqrt(d*x + c)*sqrt(-1/d))*e^(-3*c/d) + 108*sqrt(pi)*a^2*b*d^2*sqrt(-1/d)*e
rf(sqrt(d*x + c)*sqrt(-1/d))*e^(-c/d) + 12*(4*a^3*d*x + 2*b^3*d*e^(3*x) +
9*a*b^2*d*e^(2*x) + 18*a^2*b*d*e^x + 4*a^3*c)*sqrt(d*x + c))/d
```

Sympy [F]

$$\int (a + be^x)^3 \sqrt{c + dx}\, dx = \int (a + be^x)^3 \sqrt{c + dx}\, dx$$

input
```
integrate((a+b*exp(x))**3*(d*x+c)**(1/2),x)
```

output
```
Integral((a + b*exp(x))**3*sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.06

$$\int (a + be^x)^3 \sqrt{c + dx}\, dx$$

$$= \frac{48\,(dx + c)^{\frac{3}{2}} a^3 - 108 \left(\frac{\sqrt{\pi} d \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right) e^{\left(-\frac{c}{d}\right)}}{\sqrt{-\frac{1}{d}}} - 2\sqrt{dx + c} d e^{\left(\frac{dx+c}{d} - \frac{c}{d}\right)} \right) a^2 b - 27 \left(\frac{\sqrt{2}\sqrt{\pi} d \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{1}{d}}\right.\right.}{\sqrt{-\frac{1}{d}}}$$

input
```
integrate((a+b*exp(x))^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/72*(48*(d*x + c)^(3/2)*a^3 - 108*(sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-1/d
))*e^(-c/d)/sqrt(-1/d) - 2*sqrt(d*x + c)*d*e^((d*x + c)/d - c/d))*a^2*b -
27*(sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-1/d))*e^(-2*c/d)/sq
rt(-1/d) - 4*sqrt(d*x + c)*d*e^(2*(d*x + c)/d - 2*c/d))*a*b^2 - 4*(sqrt(3)
*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-1/d))*e^(-3*c/d)/sqrt(-1/d) -
6*sqrt(d*x + c)*d*e^(3*(d*x + c)/d - 3*c/d))*b^3)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(168) = 336.

Time = 0.15 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.64

$$\int (a+be^x)^3\sqrt{c+dx}\,dx =$$

$$-\frac{108\sqrt{2}\sqrt{\pi}ab^2cd\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{2c}{d}\right)}}{\sqrt{-d}}+\frac{24\sqrt{3}\sqrt{\pi}b^3cd\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{3c}{d}\right)}}{\sqrt{-d}}+\frac{216\sqrt{\pi}a^2bcd\,\mathrm{erf}\left(-\frac{\sqrt{dx+c}\sqrt{-d}}{d}\right)e^{\left(-\frac{c}{d}\right)}}{\sqrt{-d}}$$

input
```
integrate((a+b*exp(x))^3*(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/72*(108*sqrt(2)*sqrt(pi)*a*b^2*c*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/
d)*e^(-2*c/d)/sqrt(-d) + 24*sqrt(3)*sqrt(pi)*b^3*c*d*erf(-sqrt(3)*sqrt(d*x
 + c)*sqrt(-d)/d)*e^(-3*c/d)/sqrt(-d) + 216*sqrt(pi)*a^2*b*c*d*erf(-sqrt(d
*x + c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) - 144*sqrt(d*x + c)*a^3*c - 48*((d*x
 + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3 - 108*(sqrt(pi)*(2*c + d)*d*erf(-sqrt
(d*x + c)*sqrt(-d)/d)*e^(-c/d)/sqrt(-d) + 2*sqrt(d*x + c)*d*e^x)*a^2*b - 2
7*(sqrt(2)*sqrt(pi)*(4*c + d)*d*erf(-sqrt(2)*sqrt(d*x + c)*sqrt(-d)/d)*e^(
-2*c/d)/sqrt(-d) + 4*sqrt(d*x + c)*d*e^(2*x))*a*b^2 - 4*(sqrt(3)*sqrt(pi)*
(6*c + d)*d*erf(-sqrt(3)*sqrt(d*x + c)*sqrt(-d)/d)*e^(-3*c/d)/sqrt(-d) + 6
*sqrt(d*x + c)*d*e^(3*x))*b^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int (a+be^x)^3\sqrt{c+dx}\,dx = \int (a+b\,\mathrm{e}^x)^3\sqrt{c+d\,x}\,dx$$

input
```
int((a + b*exp(x))^3*(c + d*x)^(1/2),x)
```

output
```
int((a + b*exp(x))^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int (a+be^x)^3\sqrt{c+dx}\,dx$$

$$=\frac{18\sqrt{\pi}\sqrt{d}\,\mathrm{erf}\left(\frac{\sqrt{dx+c}\,i}{\sqrt{d}}\right)a^2bdi+4e^{\frac{3dx+c}{d}}\sqrt{dx+c}\,b^3d+18e^{\frac{2dx+c}{d}}\sqrt{dx+c}\,ab^2d+36e^{\frac{dx+c}{d}}\sqrt{dx+c}\,a^2bd+8e}{12e^{\frac{c}{d}}d}$$

input

```
int((a+b*exp(x))^3*(d*x+c)^(1/2),x)
```

output

```
(18*sqrt(pi)*sqrt(d)*erf((sqrt(c + d*x)*i)/sqrt(d))*a**2*b*d*i + 4*e**((c
+ 3*d*x)/d)*sqrt(c + d*x)*b**3*d + 18*e**((c + 2*d*x)/d)*sqrt(c + d*x)*a*b
**2*d + 36*e**((c + d*x)/d)*sqrt(c + d*x)*a**2*b*d + 8*e**(c/d)*sqrt(c + d
*x)*a**3*c + 8*e**(c/d)*sqrt(c + d*x)*a**3*d*x - 2*e**(c/d)*int((e**(3*x)*
sqrt(c + d*x))/(c + d*x),x)*b**3*d**2 - 9*e**(c/d)*int((e**(2*x)*sqrt(c +
d*x))/(c + d*x),x)*a*b**2*d**2)/(12*e**(c/d)*d)
```

3.67 $\int \frac{\sqrt{c+dx}}{a+be^x}\,dx$

Optimal result 603
Mathematica [N/A] 603
Rubi [N/A] 604
Maple [N/A] 604
Fricas [N/A] 605
Sympy [N/A] 605
Maxima [N/A] 606
Giac [N/A] 606
Mupad [N/A] 606
Reduce [N/A] 607

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \text{Int}\left(\frac{\sqrt{c+dx}}{a+be^x}, x\right)$$

output
```
Defer(Int)((d*x+c)^(1/2)/(a+b*exp(x)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{c+dx}}{a+be^x}\,dx$$

input
```
Integrate[Sqrt[c + d*x]/(a + b*E^x),x]
```

output
```
Integrate[Sqrt[c + d*x]/(a + b*E^x), x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx$$
$$\downarrow 2619$$
$$\int \frac{\sqrt{c+dx}}{a+be^x}dx$$

input
```
Int[Sqrt[c + d*x]/(a + b*E^x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{dx+c}}{a+b\,e^x}dx$$

input
```
int((d*x+c)^(1/2)/(a+b*exp(x)),x)
```

output
```
int((d*x+c)^(1/2)/(a+b*exp(x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{dx+c}}{be^x+a}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x)),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)/(b*e^x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{c+dx}}{a+be^x}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(a+b*exp(x)),x)
```

output
```
Integral(sqrt(c + d*x)/(a + b*exp(x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{dx+c}}{be^x+a}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(b*e^x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{dx+c}}{be^x+a}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(b*e^x + a), x)
```

Mupad [N/A]

Not integrable

Time = 23.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{c+d\,x}}{a+b\,\mathrm{e}^x}\,dx$$

input
```
int((c + d*x)^(1/2)/(a + b*exp(x)),x)
```

output
```
int((c + d*x)^(1/2)/(a + b*exp(x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{a+be^x}\,dx = \int \frac{\sqrt{dx+c}}{e^x b+a}dx$$

input
```
int((d*x+c)^(1/2)/(a+b*exp(x)),x)
```

output
```
int(sqrt(c + d*x)/(e**x*b + a),x)
```

3.68 $\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx$

Optimal result 608
Mathematica [N/A] 608
Rubi [N/A] 609
Maple [N/A] 609
Fricas [N/A] 610
Sympy [N/A] 610
Maxima [N/A] 611
Giac [N/A] 611
Mupad [N/A] 611
Reduce [N/A] 612

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \text{Int}\left(\frac{\sqrt{c+dx}}{(a+be^x)^2}, x\right)$$

output
```
Defer(Int)((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx$$

input
```
Integrate[Sqrt[c + d*x]/(a + b*E^x)^2,x]
```

output
```
Integrate[Sqrt[c + d*x]/(a + b*E^x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx$$
$$\downarrow 2619$$
$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}dx$$

input
```
Int[Sqrt[c + d*x]/(a + b*E^x)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{dx+c}}{(a+b\,\mathrm{e}^x)^2}dx$$

input
```
int((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
```

output
```
int((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^2}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x))^2,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)/(b^2*e^(2*x) + 2*a*b*e^x + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(a+b*exp(x))**2,x)
```

output
```
Integral(sqrt(c + d*x)/(a + b*exp(x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^2}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x))^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(b*e^x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^2}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(b*e^x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 23.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{c+d\,x}}{(a+b\,e^x)^2}\,dx$$

input
```
int((c + d*x)^(1/2)/(a + b*exp(x))^2,x)
```

output
```
int((c + d*x)^(1/2)/(a + b*exp(x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^2}\,dx = \int \frac{\sqrt{dx+c}}{e^{2x}b^2+2e^x ab+a^2}dx$$

input
```
int((d*x+c)^(1/2)/(a+b*exp(x))^2,x)
```

output
```
int(sqrt(c + d*x)/(e**(2*x)*b**2 + 2*e**x*a*b + a**2),x)
```

3.69 $\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$

Optimal result 613
Mathematica [N/A] 613
Rubi [N/A] 614
Maple [N/A] 614
Fricas [N/A] 615
Sympy [N/A] 615
Maxima [N/A] 616
Giac [N/A] 616
Mupad [N/A] 616
Reduce [N/A] 617

Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \text{Int}\left(\frac{\sqrt{c+dx}}{(a+be^x)^3}, x\right)$$

output
```
Defer(Int)((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
```

Mathematica [N/A]

Not integrable

Time = 9.86 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$

input
```
Integrate[Sqrt[c + d*x]/(a + b*E^x)^3,x]
```

output
```
Integrate[Sqrt[c + d*x]/(a + b*E^x)^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$
$$\downarrow 2619$$
$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}dx$$

input
```
Int[Sqrt[c + d*x]/(a + b*E^x)^3,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{dx+c}}{(a+b\,\mathrm{e}^x)^3}dx$$

input
```
int((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
```

output
```
int((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^3}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x))^3,x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)/(b^3*e^(3*x) + 3*a*b^2*e^(2*x) + 3*a^2*b*e^x + a^3)
, x)
```

Sympy [N/A]

Not integrable

Time = 5.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx$$

input
```
integrate((d*x+c)**(1/2)/(a+b*exp(x))**3,x)
```

output
```
Integral(sqrt(c + d*x)/(a + b*exp(x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^3}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x))^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)/(b*e^x + a)^3, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{(be^x+a)^3}\,dx$$

input
```
integrate((d*x+c)^(1/2)/(a+b*exp(x))^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)/(b*e^x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 23.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{c+d\,x}}{(a+b\,e^x)^3}\,dx$$

input
```
int((c + d*x)^(1/2)/(a + b*exp(x))^3,x)
```

output
```
int((c + d*x)^(1/2)/(a + b*exp(x))^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{c+dx}}{(a+be^x)^3}\,dx = \int \frac{\sqrt{dx+c}}{e^{3x}b^3+3e^{2x}a\,b^2+3e^x a^2 b+a^3}\,dx$$

input
```
int((d*x+c)^(1/2)/(a+b*exp(x))^3,x)
```

output
```
int(sqrt(c + d*x)/(e**(3*x)*b**3 + 3*e**(2*x)*a*b**2 + 3*e**x*a**2*b + a**
3),x)
```

3.70 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx$

Optimal result . . . 618
Mathematica [F] . . . 619
Rubi [A] (verified) . . . 619
Maple [F] . . . 620
Fricas [A] (verification not implemented) . . . 621
Sympy [F] . . . 621
Maxima [F] . . . 622
Giac [F] . . . 622
Mupad [F(-1)] . . . 622
Reduce [F] . . . 623

Optimal result

Integrand size = 25, antiderivative size = 340

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \frac{a^3(c+dx)^{1+m}}{d(1+m)}$$
$$+\frac{3^{-1-m}b^3F^{3\left(e-\frac{cf}{d}\right)gn-3gn(e+fx)}\left(F^{eg+fgx}\right)^{3n}(c+dx)^m\Gamma\left(1+m,-\frac{3fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$
$$+\frac{3\,2^{-1-m}ab^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\Gamma\left(1+m,-\frac{2fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$
$$+\frac{3a^2bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^{n}(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

output

```
a^3*(d*x+c)^(1+m)/d/(1+m)+3^(-1-m)*b^3*F^(3*(e-c*f/d)*g*n-3*g*n*(f*x+e))*(
F^(f*g*x+e*g))^(3*n)*(d*x+c)^m*GAMMA(1+m,-3*f*g*n*(d*x+c)*ln(F)/d)/f/g/n/l
n(F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)+3*2^(-1-m)*a*b^2*F^(2*(e-c*f/d)*g*n-2*g*
n*(f*x+e))*(F^(f*g*x+e*g))^(2*n)*(d*x+c)^m*GAMMA(1+m,-2*f*g*n*(d*x+c)*ln(F
)/d)/f/g/n/ln(F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)+3*a^2*b*F^((e-c*f/d)*g*n-g*n
*(f*x+e))*(F^(f*g*x+e*g))^n*(d*x+c)^m*GAMMA(1+m,-f*g*n*(d*x+c)*ln(F)/d)/f/
g/n/ln(F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)
```

Mathematica [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m,x]
```

output
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m, x]
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 dx$$

$$\downarrow 2614$$

$$\int \left(a^3(c+dx)^m + 3a^2b(c+dx)^m \left(F^{eg+fgx}\right)^n + 3ab^2(c+dx)^m \left(F^{eg+fgx}\right)^{2n} + b^3(c+dx)^m \left(F^{eg+fgx}\right)^{3n}\right) dx$$

$$\downarrow 2009$$

$$\frac{a^3(c+dx)^{m+1}}{d(m+1)} + \frac{3a^2b(c+dx)^m \left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)} + \frac{3ab^2 2^{-m-1}(c+dx)^m \left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{2fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)} + \frac{b^3 3^{-m-1}(c+dx)^m \left(F^{eg+fgx}\right)^{3n} F^{3gn\left(e-\frac{cf}{d}\right)-3gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{3fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m,x]
```

output
```
(a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3^(-1 - m)*b^3*F^(3*(e - (c*f)/d)*g
*n - 3*g*n*(e + f*x))*(F^(e*g + f*g*x))^(3*n)*(c + d*x)^m*Gamma[1 + m, (-3
*f*g*n*(c + d*x)*Log[F])/d])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))
^m) + (3*2^(-1 - m)*a*b^2*F^(2*(e - (c*f)/d)*g*n - 2*g*n*(e + f*x))*(F^(e*
g + f*g*x))^(2*n)*(c + d*x)^m*Gamma[1 + m, (-2*f*g*n*(c + d*x)*Log[F])/d])
/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m) + (3*a^2*b*F^((e - (c*f)
/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*(c + d*x)^m*Gamma[1 + m, -((f
*g*n*(c + d*x)*Log[F])/d)])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^
m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \left(a + b\left(F^{g(fx+e)}\right)^n\right)^3 (dx + c)^m \, dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.79

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx$$

$$= \frac{18\,(a^2bdm+a^2bd)e^{\left(\frac{(de-cf)gn\log(F)-dm\log\left(-\frac{fgn\log(F)}{d}\right)}{d}\right)}\Gamma\left(m+1,-\frac{(dfgnx+cfgn)\log(F)}{d}\right)+9\,(ab^2dm+ab^2d)e^{\left(}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x, algorithm="fricas")
```

output
```
1/6*(18*(a^2*b*d*m + a^2*b*d)*e^(((d*e - c*f)*g*n*log(F) - d*m*log(-f*g*n*
log(F)/d))/d)*gamma(m + 1, -(d*f*g*n*x + c*f*g*n)*log(F)/d) + 9*(a*b^2*d*m
 + a*b^2*d)*e^((2*(d*e - c*f)*g*n*log(F) - d*m*log(-2*f*g*n*log(F)/d))/d)*
gamma(m + 1, -2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 2*(b^3*d*m + b^3*d)*e^((
3*(d*e - c*f)*g*n*log(F) - d*m*log(-3*f*g*n*log(F)/d))/d)*gamma(m + 1, -3*
(d*f*g*n*x + c*f*g*n)*log(F)/d) + 6*(a^3*d*f*g*n*x + a^3*c*f*g*n)*(d*x + c
)^m*log(F))/((d*f*g*m + d*f*g)*n*log(F))
```

Sympy [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(a+b\left(F^{eg+fgx}\right)^n\right)^3 (c+dx)^m\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**3*(d*x+c)**m,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**3*(c + d*x)**m, x)
```

Maxima [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)^m\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^3/(d*(m + 1)) + integrate((3*F^(f*g*n*x)*F^(e*g*n)*a^2
*b + 3*F^(2*f*g*n*x)*F^(2*e*g*n)*a*b^2 + F^(3*f*g*n*x)*F^(3*e*g*n)*b^3)*(d
*x + c)^m, x)
```

Giac [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^3 (dx+c)^m\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx = \int \left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^3 (c+d\,x)^m\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^3 (c+dx)^m\,dx$$

$$= \frac{2f^{3fgnx+3egn}(dx+c)^m b^3 dm + 2f^{3fgnx+3egn}(dx+c)^m b^3 d + 9f^{2fgnx+2egn}(dx+c)^m a\,b^2 dm + 9f^{2fgnx+2egn}$$

input

```
int((a+b*(F^(g*(f*x+e)))^n)^3*(d*x+c)^m,x)
```

output

```
(2*f**(3*e*g*n + 3*f*g*n*x)*(c + d*x)**m*b**3*d*m + 2*f**(3*e*g*n + 3*f*g*
n*x)*(c + d*x)**m*b**3*d + 9*f**(2*e*g*n + 2*f*g*n*x)*(c + d*x)**m*a*b**2*
d*m + 9*f**(2*e*g*n + 2*f*g*n*x)*(c + d*x)**m*a*b**2*d + 18*f**(e*g*n + f*
g*n*x)*(c + d*x)**m*a**2*b*d*m + 18*f**(e*g*n + f*g*n*x)*(c + d*x)**m*a**2
*b*d + 6*(c + d*x)**m*log(f)*a**3*c*f*g*n + 6*(c + d*x)**m*log(f)*a**3*d*f
*g*n*x - 2*f**(3*e*g*n)*int((f**(3*f*g*n*x)*(c + d*x)**m)/(c + d*x),x)*b**
3*d**2*m**2 - 2*f**(3*e*g*n)*int((f**(3*f*g*n*x)*(c + d*x)**m)/(c + d*x),x
)*b**3*d**2*m - 9*f**(2*e*g*n)*int((f**(2*f*g*n*x)*(c + d*x)**m)/(c + d*x)
,x)*a*b**2*d**2*m**2 - 9*f**(2*e*g*n)*int((f**(2*f*g*n*x)*(c + d*x)**m)/(c
 + d*x),x)*a*b**2*d**2*m - 18*f**(e*g*n)*int((f**(f*g*n*x)*(c + d*x)**m)/(
c + d*x),x)*a**2*b*d**2*m**2 - 18*f**(e*g*n)*int((f**(f*g*n*x)*(c + d*x)**
m)/(c + d*x),x)*a**2*b*d**2*m)/(6*log(f)*d*f*g*n*(m + 1))
```

3.71 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx$

Optimal result . . . 624
Mathematica [F] . . . 625
Rubi [A] (verified) . . . 625
Maple [F] . . . 626
Fricas [A] (verification not implemented) . . . 626
Sympy [F] . . . 627
Maxima [F] . . . 627
Giac [F] . . . 628
Mupad [F(-1)] . . . 628
Reduce [F] . . . 628

Optimal result

Integrand size = 25, antiderivative size = 228

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{2^{-1-m}b^2F^{2\left(e-\frac{cf}{d}\right)gn-2gn(e+fx)}\left(F^{eg+fgx}\right)^{2n}(c+dx)^m\Gamma\left(1+m,-\frac{2fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)} + \frac{2abF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^{n}(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

output

```
a^2*(d*x+c)^(1+m)/d/(1+m)+2^(-1-m)*b^2*F^(2*(e-c*f/d)*g*n-2*g*n*(f*x+e))*(
F^(f*g*x+e*g))^(2*n)*(d*x+c)^m*GAMMA(1+m,-2*f*g*n*(d*x+c)*ln(F)/d)/f/g/n/l
n(F)/((-f*g*n*(d*x+c)*ln(F)/d)^m)+2*a*b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(
f*g*x+e*g))^n*(d*x+c)^m*GAMMA(1+m,-f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln(F)/((-f
*g*n*(d*x+c)*ln(F)/d)^m)
```

Mathematica [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m,x]
```

output
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m, x]
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 dx$$

$$\downarrow 2614$$

$$\int \left(a^2(c+dx)^m + 2ab(c+dx)^m \left(F^{eg+fgx}\right)^n + b^2(c+dx)^m \left(F^{eg+fgx}\right)^{2n}\right) dx$$

$$\downarrow 2009$$

$$\frac{a^2(c+dx)^{m+1}}{d(m+1)} + \frac{2ab(c+dx)^m \left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)} + \frac{b^2 2^{-m-1}(c+dx)^m \left(F^{eg+fgx}\right)^{2n} F^{2gn\left(e-\frac{cf}{d}\right)-2gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{2fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m,x]
```

output
```
(a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (2^(-1 - m)*b^2*F^(2*(e - (c*f)/d)*g
*n - 2*g*n*(e + f*x))*(F^(e*g + f*g*x))^(2*n)*(c + d*x)^m*Gamma[1 + m, (-2
*f*g*n*(c + d*x)*Log[F])/d])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))
^m) + (2*a*b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x))*(F^(e*g + f*g*x))^n*(c
+ d*x)^m*Gamma[1 + m, -((f*g*n*(c + d*x)*Log[F])/d)])/(f*g*n*Log[F]*(-((f*
g*n*(c + d*x)*Log[F])/d))^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \left(a + b\left(F^{g(fx+e)}\right)^n\right)^2 (dx+c)^m \, dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.84

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m \, dx$$

$$= \frac{4\,(abdm + abd)e^{\left(\frac{(de-cf)gn\log(F) - dm\log\left(-\frac{fgn\log(F)}{d}\right)}{d}\right)}\,\Gamma\left(m+1, -\frac{(dfgnx+cfgn)\log(F)}{d}\right) + (b^2dm + b^2d)e^{\left(\frac{2\,(de-cf)g}{d}\right.}}{2\,(dfgm + dfg)}$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x, algorithm="fricas")
```

output
```
1/2*(4*(a*b*d*m + a*b*d)*e^(((d*e - c*f)*g*n*log(F) - d*m*log(-f*g*n*log(F
)/d))/d)*gamma(m + 1, -(d*f*g*n*x + c*f*g*n)*log(F)/d) + (b^2*d*m + b^2*d)
*e^((2*(d*e - c*f)*g*n*log(F) - d*m*log(-2*f*g*n*log(F)/d))/d)*gamma(m + 1
, -2*(d*f*g*n*x + c*f*g*n)*log(F)/d) + 2*(a^2*d*f*g*n*x + a^2*c*f*g*n)*(d*
x + c)^m*log(F))/((d*f*g*m + d*f*g)*n*log(F))
```

Sympy [F]

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m \, dx = \int \left(a + b\left(F^{eg+fgx}\right)^n\right)^2 (c+dx)^m \, dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**2*(d*x+c)**m,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)**2*(c + d*x)**m, x)
```

Maxima [F]

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m \, dx = \int \left(\left(F^{(fx+e)g}\right)^n b + a\right)^2 (dx+c)^m \, dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate((2*F^(f*g*n*x)*F^(e*g*n)*a*b
 + F^(2*f*g*n*x)*F^(2*e*g*n)*b^2)*(d*x + c)^m, x)
```

Giac [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)^2 (dx+c)^m\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+d\,x)^m\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^2 (c+dx)^m\,dx$$
$$= \frac{f^{2fgnx+2egn}(dx+c)^m b^2 dm + f^{2fgnx+2egn}(dx+c)^m b^2 d + 4f^{fgnx+egn}(dx+c)^m abdm + 4f^{fgnx+egn}(dx+}{}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^2*(d*x+c)^m,x)
```

output

```
(f**(2*e*g*n + 2*f*g*n*x)*(c + d*x)**m*b**2*d*m + f**(2*e*g*n + 2*f*g*n*x)
*(c + d*x)**m*b**2*d + 4*f**(e*g*n + f*g*n*x)*(c + d*x)**m*a*b*d*m + 4*f**
(e*g*n + f*g*n*x)*(c + d*x)**m*a*b*d + 2*(c + d*x)**m*log(f)*a**2*c*f*g*n
+ 2*(c + d*x)**m*log(f)*a**2*d*f*g*n*x - f**(2*e*g*n)*int((f**(2*f*g*n*x)*
(c + d*x)**m)/(c + d*x),x)*b**2*d**2*m**2 - f**(2*e*g*n)*int((f**(2*f*g*n*
x)*(c + d*x)**m)/(c + d*x),x)*b**2*d**2*m - 4*f**(e*g*n)*int((f**(f*g*n*x)
*(c + d*x)**m)/(c + d*x),x)*a*b*d**2*m**2 - 4*f**(e*g*n)*int((f**(f*g*n*x)
*(c + d*x)**m)/(c + d*x),x)*a*b*d**2*m)/(2*log(f)*d*f*g*n*(m + 1))
```

3.72 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$

Optimal result . . . 630
Mathematica [A] (verified) . . . 630
Rubi [A] (verified) . . . 631
Maple [F] . . . 632
Fricas [A] (verification not implemented) . . . 632
Sympy [F] . . . 633
Maxima [F] . . . 633
Giac [F] . . . 633
Mupad [F(-1)] . . . 634
Reduce [F] . . . 634

Optimal result

Integrand size = 23, antiderivative size = 116

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx = \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{bF^{\left(e-\frac{cf}{d}\right)gn-gn(e+fx)}\left(F^{eg+fgx}\right)^n(c+dx)^m\Gamma\left(1+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\left(-\frac{fgn(c+dx)\log(F)}{d}\right)^{-m}}{fgn\log(F)}$$

output

```
a*(d*x+c)^(1+m)/d/(1+m)+b*F^((e-c*f/d)*g*n-g*n*(f*x+e))*(F^(f*g*x+e*g))^n*
(d*x+c)^m*GAMMA(1+m,-f*g*n*(d*x+c)*ln(F)/d)/f/g/n/ln(F)/((-f*g*n*(d*x+c)*l
n(F)/d)^m)
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx = \frac{(c+dx)^m\left(\left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx) - bF^{-\frac{fgn(c+dx)}{d}}\left(F^{g(e+fx)}\right)^n(c+dx)\Gamma\left(2+m,-\frac{fgn(c+dx)\log(F)}{d}\right)\right.}{d(1+m)}$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m,x]
```

output
```
((c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)*(c + d*x) - (b*(F^(g*(e + f*x)))
^n*(c + d*x)*Gamma[2 + m, -((f*g*n*(c + d*x)*Log[F])/d)]*(-((f*g*n*(c + d*
x)*Log[F])/d))^(-1 - m))/F^((f*g*n*(c + d*x))/d)))/(d*(1 + m))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2614, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m \left(a+b\left(F^{g(e+fx)}\right)^n\right) dx$$

$$\downarrow 2614$$

$$\int \left(a(c+dx)^m + b(c+dx)^m \left(F^{eg+fgx}\right)^n\right) dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^{m+1}}{d(m+1)} + \frac{b(c+dx)^m \left(F^{eg+fgx}\right)^n F^{gn\left(e-\frac{cf}{d}\right)-gn(e+fx)} \left(-\frac{fgn\log(F)(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{fgn(c+dx)\log(F)}{d}\right)}{fgn\log(F)}$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m,x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*F^((e - (c*f)/d)*g*n - g*n*(e + f*x
))*(F^(e*g + f*g*x))^n*(c + d*x)^m*Gamma[1 + m, -((f*g*n*(c + d*x)*Log[F])
/d)])/(f*g*n*Log[F]*(-((f*g*n*(c + d*x)*Log[F])/d))^m)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2614

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [F]

$$\int \left(a + b\left(F^{g(fx+e)}\right)^n\right)(dx+c)^m\,dx$$

input

```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x)
```

output

```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$$

$$= \frac{(bdm+bd)e^{\left(\frac{(de-cf)gn\log(F)-dm\log\left(-\frac{fgn\log(F)}{d}\right)}{d}\right)}\Gamma\left(m+1,-\frac{(dfgnx+cfgn)\log(F)}{d}\right)+(adfgnx+acfgn)(dx+c)^{\prime}}{(dfgm+dfg)n\log(F)}$$

input

```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x, algorithm="fricas")
```

output

```
((b*d*m + b*d)*e^(((d*e - c*f)*g*n*log(F) - d*m*log(-f*g*n*log(F)/d))/d)*g
amma(m + 1, -(d*f*g*n*x + c*f*g*n)*log(F)/d) + (a*d*f*g*n*x + a*c*f*g*n)*(
d*x + c)^m*log(F))/((d*f*g*m + d*f*g)*n*log(F))
```

Sympy [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx = \int \left(a+b\left(F^{eg+fgx}\right)^n\right)(c+dx)^m\,dx$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)*(d*x+c)**m,x)
```

output
```
Integral((a + b*(F**(e*g + f*g*x))**n)*(c + d*x)**m, x)
```

Maxima [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^m\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x, algorithm="maxima")
```

output
```
F^(e*g*n)*b*integrate(e^(f*g*n*x*log(F) + m*log(d*x + c)), x) + (d*x + c)^
(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx = \int \left(\left(F^{(fx+e)g}\right)^n b+a\right)(dx+c)^m\,dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)*(c + d*x)^m, x)
```

Reduce [F]

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)(c+dx)^m\,dx$$
$$= \frac{f^{fgnx+egn}(dx+c)^m\,bdm + f^{fgnx+egn}(dx+c)^m\,bd + (dx+c)^m\log(f)\,acfgn + (dx+c)^m\log(f)\,adfgnx}{\log(f)\,dfgn\,(m+1)}$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)*(d*x+c)^m,x)
```

output
```
(f**(e*g*n + f*g*n*x)*(c + d*x)**m*b*d*m + f**(e*g*n + f*g*n*x)*(c + d*x)*
*m*b*d + (c + d*x)**m*log(f)*a*c*f*g*n + (c + d*x)**m*log(f)*a*d*f*g*n*x -
 f**(e*g*n)*int((f**(f*g*n*x)*(c + d*x)**m)/(c + d*x),x)*b*d**2*m**2 - f**
(e*g*n)*int((f**(f*g*n*x)*(c + d*x)**m)/(c + d*x),x)*b*d**2*m)/(log(f)*d*f
*g*n*(m + 1))
```

3.73 $\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$

Optimal result . . . 635
Mathematica [N/A] . . . 635
Rubi [N/A] . . . 636
Maple [N/A] . . . 636
Fricas [N/A] . . . 637
Sympy [N/A] . . . 637
Maxima [N/A] . . . 638
Giac [N/A] . . . 638
Mupad [N/A] . . . 638
Reduce [N/A] . . . 639

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \text{Int}\left(\frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}, x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x)
```

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx$$

$$\downarrow 2619$$

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}dx$$

input
```
Int[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+b\left(F^{g(fx+e)}\right)^n}dx$$

input
```
int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x)
```

output
```
int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^m}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/((F^(f*g*x + e*g))^n*b + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+dx)^m}{a+b\left(F^{eg+fgx}\right)^n}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*(F**(g*(f*x+e)))**n),x)
```

output
```
Integral((c + d*x)**m/(a + b*(F**(e*g + f*g*x))**n), x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^m}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(F^((f*x + e)*g*n)*b + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^m}{\left(F^{(fx+e)g}\right)^n b+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/((F^((f*x + e)*g))^n*b + a), x)
```

Mupad [N/A]

Not integrable

Time = 23.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(c+d\,x)^m}{a+b\left(F^{g\,(e+f\,x)}\right)^n}\,dx$$

input
```
int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n),x)
```

output
```
int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^m}{a+b\left(F^{g(e+fx)}\right)^n}\,dx = \int \frac{(dx+c)^m}{f^{fgnx+egn}b+a}dx$$

input
```
int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n),x)
```

output
```
int((c + d*x)**m/(f**(e*g*n + f*g*n*x)*b + a),x)
```

3.74 $\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$

Optimal result 640
Mathematica [N/A] 640
Rubi [N/A] 641
Maple [N/A] 641
Fricas [N/A] 642
Sympy [N/A] 642
Maxima [N/A] 643
Giac [N/A] 643
Mupad [N/A] 643
Reduce [N/A] 644

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \text{Int}\left(\frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}, x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx$$

$$\downarrow 2619$$

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}dx$$

input
```
Int[(c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{\left(a+b\left(F^{g(fx+e)}\right)^n\right)^2}dx$$

input
```
int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output
```
int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(dx+c)^m}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(2*(F^(f*g*x + e*g))^n*a*b + (F^(f*g*x + e*g))^(2*n)*
b^2 + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 30.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+dx)^m}{\left(a+b\left(F^{eg+fgx}\right)^n\right)^2}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*(F**(g*(f*x+e)))**n)**2,x)
```

output
```
Integral((c + d*x)**m/(a + b*(F**(e*g + f*g*x))**n)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(dx+c)^m}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(F^((f*x + e)*g*n)*b + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(dx+c)^m}{\left(\left(F^{(fx+e)g}\right)^n b+a\right)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/((F^((f*x + e)*g))^n*b + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 23.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(c+d\,x)^m}{\left(a+b\left(F^{g\,(e+f\,x)}\right)^n\right)^2}\,dx$$

input
```
int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2,x)
```

output
```
int((c + d*x)^m/(a + b*(F^(g*(e + f*x)))^n)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{(c+dx)^m}{\left(a+b\left(F^{g(e+fx)}\right)^n\right)^2}\,dx = \int \frac{(dx+c)^m}{f^{2fgnx+2egn}b^2+2f^{fgnx+egn}ab+a^2}dx$$

input
```
int((d*x+c)^m/(a+b*(F^(g*(f*x+e)))^n)^2,x)
```

output
```
int((c + d*x)**m/(f**(2*e*g*n + 2*f*g*n*x)*b**2 + 2*f**(e*g*n + f*g*n*x)*a
*b + a**2),x)
```

3.75 $\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx$

Optimal result . . . 645
Mathematica [N/A] . . . 645
Rubi [N/A] . . . 646
Maple [N/A] . . . 646
Fricas [N/A] . . . 647
Sympy [F(-1)] . . . 647
Maxima [N/A] . . . 647
Giac [N/A] . . . 648
Mupad [N/A] . . . 648
Reduce [N/A] . . . 649

Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx = \text{Int}\left(\left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m, x\right)$$

output
```
Defer(Int)((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x)
```

Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx = \int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx$$

input
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m,x]
```

output
```
Integrate[(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m, x]
```

Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p dx$$

$$\downarrow 2619$$

$$\int (c+dx)^m \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p dx$$

input
```
Int[(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m,x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(a+b\left(F^{g(fx+e)}\right)^n\right)^p (dx+c)^m \, dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x)
```

output
```
int((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m \, dx = \int \left(\left(F^{(fx+e)g}\right)^n b + a\right)^p (dx+c)^m \, dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x, algorithm="fricas")
```

output
```
integral(((F^(f*g*x + e*g))^n*b + a)^p*(d*x + c)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m \, dx = \text{Timed out}$$

input
```
integrate((a+b*(F**(g*(f*x+e)))**n)**p*(d*x+c)**m,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m \, dx = \int \left(\left(F^{(fx+e)g}\right)^n b + a\right)^p (dx+c)^m \, dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x, algorithm="maxima")
```

output
```
integrate((F^((f*x + e)*g*n)*b + a)^p*(d*x + c)^m, x)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m \, dx = \int \left(\left(F^{(fx+e)g}\right)^n b + a\right)^p (dx+c)^m \, dx$$

input
```
integrate((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x, algorithm="giac")
```

output
```
integrate(((F^((f*x + e)*g))^n*b + a)^p*(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 22.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \left(a + b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m \, dx = \int \left(a + b\left(F^{g\,(e+f\,x)}\right)^n\right)^p (c+d\,x)^m \, dx$$

input
```
int((a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m,x)
```

output
```
int((a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \left(a+b\left(F^{g(e+fx)}\right)^n\right)^p (c+dx)^m\,dx = \int (dx+c)^m \left(f^{fgnx+egn}b+a\right)^p dx$$

input
```
int((a+b*(F^(g*(f*x+e)))^n)^p*(d*x+c)^m,x)
```

output
```
int((c + d*x)**m*(f**(e*g*n + f*g*n*x)*b + a)**p,x)
```

3.76 $\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx$

Optimal result . . . 650
Mathematica [A] (verified) . . . 650
Rubi [A] (verified) . . . 651
Maple [A] (verified) . . . 653
Fricas [A] (verification not implemented) . . . 654
Sympy [F] . . . 654
Maxima [A] (verification not implemented) . . . 654
Giac [F] . . . 655
Mupad [F(-1)] . . . 655
Reduce [F] . . . 656

Optimal result

Integrand size = 24, antiderivative size = 115

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{x^3\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6\,\mathrm{PolyLog}\left(4,-\frac{bF^{c+dx}}{a}\right)}{bd^4\log^4(F)}$$

output

```
x^3*ln(1+b*F^(d*x+c)/a)/b/d/ln(F)+3*x^2*polylog(2,-b*F^(d*x+c)/a)/b/d^2/ln
(F)^2-6*x*polylog(3,-b*F^(d*x+c)/a)/b/d^3/ln(F)^3+6*polylog(4,-b*F^(d*x+c)
/a)/b/d^4/ln(F)^4
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \frac{x^3\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{3x^2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{6x\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)} + \frac{6\,\mathrm{PolyLog}\left(4,-\frac{bF^{c+dx}}{a}\right)}{bd^4\log^4(F)}$$

input
```
Integrate[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x)),x]
```

output
```
(x^3*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (3*x^2*PolyLog[2, -((b*F^(
c + d*x))/a)])/(b*d^2*Log[F]^2) - (6*x*PolyLog[3, -((b*F^(c + d*x))/a)])/(
b*d^3*Log[F]^3) + (6*PolyLog[4, -((b*F^(c + d*x))/a)])/(b*d^4*Log[F]^4)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 F^{c+dx}}{a + bF^{c+dx}}\,dx$$

$$\downarrow 2620$$

$$\frac{x^3 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{3\int x^2 \log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}$$

$$\downarrow 3011$$

$$\frac{x^3 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{3\left(\frac{2\int x\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)dx}{d\log(F)} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

$$\downarrow 7163$$

$$\frac{x^3 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)} - \frac{\int \mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)dx}{d\log(F)}\right)}{d\log(F)} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

$$\downarrow 2720$$

$$\frac{x^3 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)} - \frac{\int F^{-c-dx}\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)dF^{c+dx}}{d^2\log^2(F)}\right)}{d\log(F)} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

$$\downarrow 7143$$

$$\frac{x^3 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)} - \frac{\mathrm{PolyLog}\left(4,-\frac{bF^{c+dx}}{a}\right)}{d^2\log^2(F)}\right)}{d\log(F)} - \frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

input
```
Int[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x)),x]
```

output
```
(x^3*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) - (3*(-((x^2*PolyLog[2, -((b
*F^(c + d*x))/a)])/(d*Log[F])) + (2*((x*PolyLog[3, -((b*F^(c + d*x))/a)])/
(d*Log[F]) - PolyLog[4, -((b*F^(c + d*x))/a)]/(d^2*Log[F]^2)))/(d*Log[F]))
)/(b*d*Log[F])
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.96

method	result
risch	$\frac{\ln\left(1+\frac{b F^{c} F^{dx}}{a}\right) c^{3}}{d^{4} \ln(F) b}+\frac{c^{3} \ln\left(F^{c} F^{dx}\right)}{d^{4} \ln(F) b}-\frac{c^{3} \ln\left(F^{dx} F^{c} b+a\right)}{d^{4} \ln(F) b}-\frac{c^{3} x}{d^{3} b}+\frac{\ln\left(1+\frac{b F^{c} F^{dx}}{a}\right) x^{3}}{d \ln(F) b}+\frac{3 \operatorname{polylog}\left(2,-\frac{b F^{c} F^{dx}}{a}\right) x^{2}}{d^{2} \ln(F)^{2} b}-\frac{6 \mathrm{p}$

input

```
int(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

output

```
1/d^4/ln(F)/b*ln(1+b*F^c*F^(d*x)/a)*c^3+1/d^4/ln(F)/b*c^3*ln(F^c*F^(d*x))-
1/d^4/ln(F)/b*c^3*ln(F^(d*x)*F^c*b+a)-1/d^3/b*c^3*x+1/d/ln(F)/b*ln(1+b*F^c
*F^(d*x)/a)*x^3+3/d^2/ln(F)^2/b*polylog(2,-b*F^c*F^(d*x)/a)*x^2-6/d^3/ln(F
)^3/b*polylog(3,-b*F^c*F^(d*x)/a)*x+6/d^4/ln(F)^4/b*polylog(4,-b*F^c*F^(d*
x)/a)-3/4/d^4/b*c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx$$
$$= \frac{3\,d^2x^2\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)\log(F)^2 - c^3\log\left(F^{dx+c}b+a\right)\log(F)^3 + \left(d^3x^3+c^3\right)\log(F)^3\log\left(\frac{F^{dx+c}b+a}{a}\right)}{bd^4\log(F)^4}$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

output
```
(3*d^2*x^2*dilog(-(F^(d*x + c)*b + a)/a + 1)*log(F)^2 - c^3*log(F^(d*x + c
)*b + a)*log(F)^3 + (d^3*x^3 + c^3)*log(F)^3*log((F^(d*x + c)*b + a)/a) -
6*d*x*log(F)*polylog(3, -F^(d*x + c)*b/a) + 6*polylog(4, -F^(d*x + c)*b/a)
)/(b*d^4*log(F)^4)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+dx}x^3}{F^{c+dx}b+a}\,dx$$

input
```
integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c)),x)
```

output
```
Integral(F**(c + d*x)*x**3/(F**(c + d*x)*b + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx$$
$$= \frac{d^3x^3\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F)^3 + 3\,d^2x^2\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log(F)^2 - 6\,dx\log(F)\,\mathrm{Li}_3(-\frac{F^{dx}F^cb}{a}) + 6\,\mathrm{Li}_4(-\frac{F^{dx}}{a}$$
$$bd^4\log(F)^4$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

output
```
(d^3*x^3*log(F^(d*x)*F^c*b/a + 1)*log(F)^3 + 3*d^2*x^2*dilog(-F^(d*x)*F^c*
b/a)*log(F)^2 - 6*d*x*log(F)*polylog(3, -F^(d*x)*F^c*b/a) + 6*polylog(4, -
F^(d*x)*F^c*b/a))/(b*d^4*log(F)^4)
```

Giac [F]

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \int \frac{F^{dx+c}x^3}{F^{dx+c}b+a}\,dx$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x^3/(F^(d*x + c)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+d\,x}\,x^3}{a+F^{c+d\,x}\,b}\,dx$$

input
```
int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b),x)
```

output
```
int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b), x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x^3}{a+bF^{c+dx}}\,dx = f^c\left(\int \frac{f^{dx}x^3}{f^{dx+c}b+a}dx\right)$$

input
```
int(F^(d*x+c)*x^3/(a+b*F^(d*x+c)),x)
```

output
```
f**c*int((f**(d*x)*x**3)/(f**(c + d*x)*b + a),x)
```

3.77 $\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx$

Optimal result . . . 657
Mathematica [A] (verified) . . . 657
Rubi [A] (verified) . . . 658
Maple [B] (verified) . . . 660
Fricas [A] (verification not implemented) . . . 660
Sympy [F] . . . 661
Maxima [A] (verification not implemented) . . . 661
Giac [F] . . . 661
Mupad [F(-1)] . . . 662
Reduce [F] . . . 662

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{2\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)}$$

output

```
x^2*ln(1+b*F^(d*x+c)/a)/b/d/ln(F)+2*x*polylog(2,-b*F^(d*x+c)/a)/b/d^2/ln(F
)^2-2*polylog(3,-b*F^(d*x+c)/a)/b/d^3/ln(F)^3
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{2x\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} - \frac{2\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{bd^3\log^3(F)}$$

input
```
Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x)),x]
```

output
```
(x^2*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + (2*x*PolyLog[2, -((b*F^(c
+ d*x))/a)])/(b*d^2*Log[F]^2) - (2*PolyLog[3, -((b*F^(c + d*x))/a)])/(b*d^
3*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 F^{c+dx}}{a + bF^{c+dx}}\,dx$$

$$\downarrow 2620$$

$$\frac{x^2 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{2\int x \log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}$$

$$\downarrow 3011$$

$$\frac{x^2 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)dx}{d\log(F)} - \frac{x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

$$\downarrow 2720$$

$$\frac{x^2 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{2\left(\frac{\int F^{-c-dx}\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)dF^{c+dx}}{d^2\log^2(F)} - \frac{x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

$$\downarrow 7143$$

$$\frac{x^2 \log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{d^2\log^2(F)} - \frac{x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}$$

input
```
Int[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x)),x]
```

output
```
(x^2*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) - (2*(-((x*PolyLog[2, -((b*F
^(c + d*x))/a)])/(d*Log[F])) + PolyLog[3, -((b*F^(c + d*x))/a)]/(d^2*Log[F
]^2)))/(b*d*Log[F])
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(85) = 170.

Time = 0.06 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.28

method	result
risch	$\frac{c^2 x}{d^2 b}+\frac{2c^3}{3d^3 b}+\frac{\ln\left(1+\frac{b F^c F^{dx}}{a}\right)x^2}{d\ln(F)b}-\frac{\ln\left(1+\frac{b F^c F^{dx}}{a}\right)c^2}{d^3\ln(F)b}+\frac{2\,\mathrm{polylog}\left(2,-\frac{b F^c F^{dx}}{a}\right)x}{d^2\ln(F)^2 b}-\frac{2\,\mathrm{polylog}\left(3,-\frac{b F^c F^{dx}}{a}\right)}{d^3\ln(F)^3 b}-\frac{c^2\ln(F$

input
```
int(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d^2/b*c^2*x+2/3/d^3/b*c^3+1/d/ln(F)/b*ln(1+b*F^c*F^(d*x)/a)*x^2-1/d^3/ln
(F)/b*ln(1+b*F^c*F^(d*x)/a)*c^2+2/d^2/ln(F)^2/b*polylog(2,-b*F^c*F^(d*x)/a
)*x-2/d^3/ln(F)^3/b*polylog(3,-b*F^c*F^(d*x)/a)-1/d^3/ln(F)/b*c^2*ln(F^c*F
^(d*x))+1/d^3/ln(F)/b*c^2*ln(F^(d*x)*F^c*b+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.27

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx$$
$$=\frac{c^2\log\left(F^{dx+c}b+a\right)\log(F)^2+2\,dx\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)\log(F)+\left(d^2x^2-c^2\right)\log(F)^2\log\left(\frac{F^{dx+c}b+a}{a}\right)-2}{bd^3\log(F)^3}$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

output
```
(c^2*log(F^(d*x + c)*b + a)*log(F)^2 + 2*d*x*dilog(-(F^(d*x + c)*b + a)/a
+ 1)*log(F) + (d^2*x^2 - c^2)*log(F)^2*log((F^(d*x + c)*b + a)/a) - 2*poly
log(3, -F^(d*x + c)*b/a))/(b*d^3*log(F)^3)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+dx}x^2}{F^{c+dx}b+a}\,dx$$

input
```
integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c)),x)
```

output
```
Integral(F**(c + d*x)*x**2/(F**(c + d*x)*b + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \frac{d^2x^2\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F)^2 + 2\,dx\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log(F) - 2\,\mathrm{Li}_3(-\frac{F^{dx}F^cb}{a})}{bd^3\log(F)^3}$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

output
```
(d^2*x^2*log(F^(d*x)*F^c*b/a + 1)*log(F)^2 + 2*d*x*dilog(-F^(d*x)*F^c*b/a)
*log(F) - 2*polylog(3, -F^(d*x)*F^c*b/a))/(b*d^3*log(F)^3)
```

Giac [F]

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \int \frac{F^{dx+c}x^2}{F^{dx+c}b+a}\,dx$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x^2/(F^(d*x + c)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+d\,x}\,x^2}{a+F^{c+d\,x}\,b}\,dx$$

input
```
int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b),x)
```

output
```
int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b), x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x^2}{a+bF^{c+dx}}\,dx = f^c\left(\int \frac{f^{dx}x^2}{f^{dx+c}b+a}dx\right)$$

input
```
int(F^(d*x+c)*x^2/(a+b*F^(d*x+c)),x)
```

output
```
f**c*int((f**(d*x)*x**2)/(f**(c + d*x)*b + a),x)
```

3.78 $\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx$

Optimal result . . . 663
Mathematica [A] (verified) . . . 663
Rubi [A] (verified) . . . 664
Maple [B] (verified) . . . 665
Fricas [A] (verification not implemented) . . . 666
Sympy [F] . . . 666
Maxima [A] (verification not implemented) . . . 666
Giac [F] . . . 667
Mupad [F(-1)] . . . 667
Reduce [F] . . . 667

Optimal result

Integrand size = 22, antiderivative size = 54

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}$$

output
```
x*ln(1+b*F^(d*x+c)/a)/b/d/ln(F)+polylog(2,-b*F^(d*x+c)/a)/b/d^2/ln(F)^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{bd\log(F)} + \frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}$$

input
```
Integrate[(F^(c + d*x)*x)/(a + b*F^(c + d*x)),x]
```

output
```
(x*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + PolyLog[2, -((b*F^(c + d*x))
/a)]/(b*d^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xF^{c+dx}}{a+bF^{c+dx}}\,dx$$

$$\downarrow 2620$$

$$\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int \log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}$$

$$\downarrow 2715$$

$$\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}$$

$$\downarrow 2838$$

$$\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)} + \frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}$$

input
```
Int[(F^(c + d*x)*x)/(a + b*F^(c + d*x)),x]
```

output
```
(x*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[F]) + PolyLog[2, -((b*F^(c + d*x))
/a)]/(b*d^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(54) = 108$.

Time = 0.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.85

method	result	si
risch	$-\frac{cx}{db}-\frac{c^2}{2d^2b}+\frac{\ln\left(1+\frac{b\,F^{c}F^{dx}}{a}\right)x}{d\ln(F)b}+\frac{\ln\left(1+\frac{b\,F^{c}F^{dx}}{a}\right)c}{d^2\ln(F)b}+\frac{\mathrm{polylog}\left(2,-\frac{b\,F^{c}F^{dx}}{a}\right)}{d^2\ln(F)^2b}+\frac{c\ln\left(F^{c}F^{dx}\right)}{d^2\ln(F)b}-\frac{c\ln\left(F^{dx}F^{c}b+a\right)}{d^2\ln(F)b}$	1

input
```
int(F^(d*x+c)*x/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/d/b*c*x-1/2/d^2/b*c^2+1/d/ln(F)/b*ln(1+b*F^c*F^(d*x)/a)*x+1/d^2/ln(F)/b
*ln(1+b*F^c*F^(d*x)/a)*c+1/d^2/ln(F)^2/b*polylog(2,-b*F^c*F^(d*x)/a)+1/d^2
/ln(F)/b*c*ln(F^c*F^(d*x))-1/d^2/ln(F)/b*c*ln(F^(d*x)*F^c*b+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.39

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx$$
$$= -\frac{c\log\left(F^{dx+c}b+a\right)\log\left(F\right)-\left(dx+c\right)\log\left(F\right)\log\left(\frac{F^{dx+c}b+a}{a}\right)-\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)}{bd^2\log\left(F\right)^2}$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

output
```
-(c*log(F^(d*x + c)*b + a)*log(F) - (d*x + c)*log(F)*log((F^(d*x + c)*b + a)/a) - dilog(-(F^(d*x + c)*b + a)/a + 1))/(b*d^2*log(F)^2)
```

Sympy [F]

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+dx}x}{F^{c+dx}b+a}\,dx$$

input
```
integrate(F**(d*x+c)*x/(a+b*F**(d*x+c)),x)
```

output
```
Integral(F**(c + d*x)*x/(F**(c + d*x)*b + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \frac{dx\log\left(\frac{F^{dx}F^{c}b}{a}+1\right)\log\left(F\right)+\mathrm{Li}_2\left(-\frac{F^{dx}F^{c}b}{a}\right)}{bd^2\log\left(F\right)^2}$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x*log(F^(d*x)*F^c*b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(b*d^2*log
(F)^2)
```

Giac [F]

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \int \frac{F^{dx+c}x}{F^{dx+c}b+a}\,dx$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x/(F^(d*x + c)*b + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = \int \frac{F^{c+d\,x}\,x}{a+F^{c+d\,x}\,b}\,dx$$

input
```
int((F^(c + d*x)*x)/(a + F^(c + d*x)*b),x)
```

output
```
int((F^(c + d*x)*x)/(a + F^(c + d*x)*b), x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x}{a+bF^{c+dx}}\,dx = f^c\left(\int \frac{f^{dx}x}{f^{dx+c}b+a}dx\right)$$

input
```
int(F^(d*x+c)*x/(a+b*F^(d*x+c)),x)
```

output
```
f**c*int((f**(d*x)*x)/(f**(c + d*x)*b + a),x)
```

3.79 $\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx$

Optimal result . . . 668
Mathematica [A] (verified) . . . 668
Rubi [A] (verified) . . . 669
Maple [A] (verified) . . . 670
Fricas [A] (verification not implemented) . . . 670
Sympy [A] (verification not implemented) . . . 671
Maxima [A] (verification not implemented) . . . 671
Giac [A] (verification not implemented) . . . 671
Mupad [B] (verification not implemented) . . . 672
Reduce [B] (verification not implemented) . . . 672

Optimal result

Integrand size = 21, antiderivative size = 23

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

output
```
ln(a+b*F^(d*x+c))/b/d/ln(F)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

input
```
Integrate[F^(c + d*x)/(a + b*F^(c + d*x)),x]
```

output
```
Log[a + b*F^(c + d*x)]/(b*d*Log[F])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2676, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx$$
$$\downarrow 2676$$
$$\frac{\int \frac{1}{bF^{c+dx}+a}dF^{c+dx}}{d\log(F)}$$
$$\downarrow 16$$
$$\frac{\log\left(a+bF^{c+dx}\right)}{bd\log(F)}$$

input
```
Int[F^(c + d*x)/(a + b*F^(c + d*x)),x]
```

output
```
Log[a + b*F^(c + d*x)]/(b*d*Log[F])
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{\ln\left(a+b\,F^{dx+c}\right)}{bd\ln(F)}$	24
default	$\frac{\ln\left(a+b\,F^{dx+c}\right)}{bd\ln(F)}$	24
parallelrisch	$\frac{\ln\left(a+b\,F^{dx+c}\right)}{bd\ln(F)}$	24
norman	$\frac{\ln\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}{bd\ln(F)}$	26
risch	$-\frac{c}{bd}+\frac{\ln\left(F^{dx+c}+\frac{a}{b}\right)}{bd\ln(F)}$	36

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*F^(d*x+c))/b/d/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{dx+c}b+a\right)}{bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="fricas")
```

output
```
log(F^(d*x + c)*b + a)/(b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{bd\log\left(F\right)}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c)),x)
```

output
```
log(F**(c + d*x) + a/b)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(F^{dx+c}b+a\right)}{bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="maxima")
```

output
```
log(F^(d*x + c)*b + a)/(b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(\left|F^{dx+c}b+a\right|\right)}{bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(F^(d*x + c)*b + a))/(b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\ln\left(a+F^{c+d\,x}\,b\right)}{b\,d\,\ln\left(F\right)}$$

input
```
int(F^(c + d*x)/(a + F^(c + d*x)*b),x)
```

output
```
log(a + F^(c + d*x)*b)/(b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{a+bF^{c+dx}}\,dx = \frac{\log\left(f^{dx+c}b+a\right)}{\log\left(f\right)bd}$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c)),x)
```

output
```
log(f**(c + d*x)*b + a)/(log(f)*b*d)
```

3.80 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})x}\,dx$

Optimal result 673
Mathematica [N/A] 673
Rubi [N/A] 674
Maple [N/A] 675
Fricas [N/A] 675
Sympy [N/A] 675
Maxima [N/A] 676
Giac [N/A] 676
Mupad [N/A] 677
Reduce [N/A] 677

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \text{Int}\left(\frac{F^{c+dx}}{(a+bF^{c+dx})\,x}, x\right)$$

output
```
Defer(Int)(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
```

Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx$$

input
```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x),x]
```

output
```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2622}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{x\left(a+bF^{c+dx}\right)}\,dx$$
$$\downarrow 2622$$
$$\int \frac{F^{c+dx}}{x\left(a+bF^{c+dx}\right)}dx$$

input
```
Int[F^(c + d*x)/((a + b*F^(c + d*x))*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2622
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Unintegrable[(F^(g*(e + f*x)))^n*(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m
, x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})\,x}dx$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
```

output
```
int(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x,x, algorithm="fricas")
```

output
```
integral(F^(d*x + c)/(F^(d*x + c)*b*x + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{c+dx}}{x\,(F^{c+dx}b+a)}\,dx$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))/x,x)
```

output
```
Integral(F**(c + d*x)/(x*(F**(c + d*x)*b + a)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x,x, algorithm="maxima")
```

output
```
-a*integrate(1/(F^(d*x)*F^c*b^2*x + a*b*x), x) + log(x)/b
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)/((F^(d*x + c)*b + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 22.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \int \frac{F^{c+d\,x}}{x\,(a+F^{c+d\,x}\,b)}\,dx$$

input
```
int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)),x)
```

output
```
int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x}\,dx = \frac{-\left(\int \frac{1}{f^{dx+c}bx+ax}dx\right)a+\log(x)}{b}$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))/x,x)
```

output
```
( - int(1/(f**(c + d*x)*b*x + a*x),x)*a + log(x))/b
```

3.81 $\int \frac{F^{c+dx}}{(a+bF^{c+dx})x^2}\,dx$

Optimal result 678
Mathematica [N/A] 678
Rubi [N/A] 679
Maple [N/A] 680
Fricas [N/A] 680
Sympy [N/A] 680
Maxima [N/A] 681
Giac [N/A] 681
Mupad [N/A] 682
Reduce [N/A] 682

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \text{Int}\left(\frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}, x\right)$$

output
```
Defer(Int)(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx$$

input
```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x^2),x]
```

output
```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2622}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{x^2\left(a+bF^{c+dx}\right)}\,dx$$
$$\downarrow 2622$$
$$\int \frac{F^{c+dx}}{x^2\left(a+bF^{c+dx}\right)}dx$$

input
```
Int[F^(c + d*x)/((a + b*F^(c + d*x))*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2622
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Unintegrable[(F^(g*(e + f*x)))^n*(a + b*(F^(g*(e + f*x)))^n)^p*(c + d*x)^m
, x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n, p}, x]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{(a+b\,F^{dx+c})\,x^2}dx$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
```

output
```
int(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x, algorithm="fricas")
```

output
```
integral(F^(d*x + c)/(F^(d*x + c)*b*x^2 + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{c+dx}}{x^2\,(F^{c+dx}b+a)}\,dx$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))/x**2,x)
```

output
```
Integral(F**(c + d*x)/(x**2*(F**(c + d*x)*b + a)), x)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)x^2}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x, algorithm="maxima")
```

output
```
-a*integrate(1/(F^(d*x)*F^c*b^2*x^2 + a*b*x^2), x) - 1/(b*x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)x^2}\,dx = \int \frac{F^{dx+c}}{(F^{dx+c}b+a)x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)/((F^(d*x + c)*b + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 22.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = \int \frac{F^{c+d\,x}}{x^2\,(a+F^{c+d\,x}\,b)}\,dx$$

input
```
int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)),x)
```

output
```
int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{F^{c+dx}}{(a+bF^{c+dx})\,x^2}\,dx = f^c\left(\int \frac{f^{dx}}{f^{dx+c}b\,x^2+a\,x^2}dx\right)$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))/x^2,x)
```

output
```
f**c*int(f**(d*x)/(f**(c + d*x)*b*x**2 + a*x**2),x)
```

3.82 $\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx$

Optimal result . . . 683
Mathematica [A] (verified) . . . 684
Rubi [A] (verified) . . . 684
Maple [A] (verified) . . . 687
Fricas [A] (verification not implemented) . . . 688
Sympy [F] . . . 688
Maxima [A] (verification not implemented) . . . 689
Giac [F] . . . 689
Mupad [F(-1)] . . . 690
Reduce [F] . . . 690

Optimal result

Integrand size = 24, antiderivative size = 140

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{x^3}{abd\log(F)} - \frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{6x\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)} + \frac{6\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{abd^4\log^4(F)}$$

output

```
x^3/a/b/d/ln(F)-x^3/b/d/(a+b*F^(d*x+c))/ln(F)-3*x^2*ln(1+b*F^(d*x+c)/a)/a/
b/d^2/ln(F)^2-6*x*polylog(2,-b*F^(d*x+c)/a)/a/b/d^3/ln(F)^3+6*polylog(3,-b
*F^(d*x+c)/a)/a/b/d^4/ln(F)^4
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx$$
$$= -\frac{x^3}{bd\left(a+bF^{c+dx}\right)\log(F)}$$
$$+\frac{3\left(\frac{x^3}{3a}-\frac{x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{ad\log(F)}-\frac{2x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{ad^2\log^2(F)}+\frac{2\,\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{ad^3\log^3(F)}\right)}{bd\log(F)}$$

input
```
Integrate[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^2,x]
```

output
```
-(x^3/(b*d*(a + b*F^(c + d*x))*Log[F])) + (3*(x^3/(3*a) - (x^2*Log[1 + (b*
F^(c + d*x))/a])/(a*d*Log[F]) - (2*x*PolyLog[2, -((b*F^(c + d*x))/a)])/(a*
d^2*Log[F]^2) + (2*PolyLog[3, -((b*F^(c + d*x))/a)])/(a*d^3*Log[F]^3)))/(b
*d*Log[F])
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2621, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx$$
$$\downarrow 2621$$
$$\frac{3\int \frac{x^2}{bF^{c+dx}+a}dx}{bd\log(F)}-\frac{x^3}{bd\log(F)\left(a+bF^{c+dx}\right)}$$
$$\downarrow 2615$$

$$\frac{3\left(\frac{x^3}{3a}-\frac{b\int\frac{F^{c+dx}x^2}{bF^{c+dx}+a}dx}{a}\right)}{bd\log(F)}-\frac{x^3}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2620$$

$$\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^3}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 3011$$

$$\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\left(\frac{\int\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)dx}{d\log(F)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^3}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2720$$

$$\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\left(\frac{\int F^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)dF^{c+dx}}{d^2\log^2(F)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^3}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 7143$$

$$\frac{3\left(\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{d^2\log^2(F)}-\frac{x\,\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^3}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

input
```
Int[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^2,x]
```

output
```
-(x^3/(b*d*(a + b*F^(c + d*x))*Log[F])) + (3*(x^3/(3*a) - (b*((x^2*Log[1 +
 (b*F^(c + d*x))/a])/(b*d*Log[F]) - (2*(-((x*PolyLog[2, -((b*F^(c + d*x))/
a)])/(d*Log[F])) + PolyLog[3, -((b*F^(c + d*x))/a)]/(d^2*Log[F]^2)))/(b*d*
Log[F])))/a))/(b*d*Log[F])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{x^3}{bd(a+b\,F^{dx+c})\ln(F)}+\frac{x^3}{abd\ln(F)}-\frac{3c^2x}{b\,d^3\ln(F)a}-\frac{2c^3}{b\,d^4\ln(F)a}-\frac{3\ln\left(1+\frac{b\,F^cF^{dx}}{a}\right)x^2}{b\,d^2\ln(F)^2a}+\frac{3\ln\left(1+\frac{b\,F^cF^{dx}}{a}\right)c^2}{b\,d^4\ln(F)^2a}-\frac{6\,\mathrm{poly}}{}$

input
```
int(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-x^3/b/d/(a+b*F^(d*x+c))/ln(F)+x^3/a/b/d/ln(F)-3/b/d^3/ln(F)/a*c^2*x-2/b/d
^4/ln(F)/a*c^3-3/b/d^2/ln(F)^2/a*ln(1+b*F^c*F^(d*x)/a)*x^2+3/b/d^4/ln(F)^2
/a*ln(1+b*F^c*F^(d*x)/a)*c^2-6/b/d^3/ln(F)^3/a*polylog(2,-b*F^c*F^(d*x)/a)
*x+6/b/d^4/ln(F)^4/a*polylog(3,-b*F^c*F^(d*x)/a)+3/b/d^4/ln(F)^2*c^2/a*ln(
F^c*F^(d*x))-3/b/d^4/ln(F)^2*c^2/a*ln(F^(d*x)*F^c*b+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.76

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx$$
$$= \frac{ac^3\log\left(F\right)^3 + \left(bd^3x^3 + bc^3\right)F^{dx+c}\log\left(F\right)^3 - 6\left(F^{dx+c}bdx\log\left(F\right) + adx\log\left(F\right)\right)\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right) - 3$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
```

output
```
(a*c^3*log(F)^3 + (b*d^3*x^3 + b*c^3)*F^(d*x + c)*log(F)^3 - 6*(F^(d*x + c
)*b*d*x*log(F) + a*d*x*log(F))*dilog(-(F^(d*x + c)*b + a)/a + 1) - 3*(F^(d
*x + c)*b*c^2*log(F)^2 + a*c^2*log(F)^2)*log(F^(d*x + c)*b + a) - 3*((b*d^
2*x^2 - b*c^2)*F^(d*x + c)*log(F)^2 + (a*d^2*x^2 - a*c^2)*log(F)^2)*log((F
^(d*x + c)*b + a)/a) + 6*(F^(d*x + c)*b + a)*polylog(3, -F^(d*x + c)*b/a))
/(F^(d*x + c)*a*b^2*d^4*log(F)^4 + a^2*b*d^4*log(F)^4)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{x^3}{F^{c+dx}b^2d\log\left(F\right) + abd\log\left(F\right)} + \frac{3\int \frac{x^2}{a+be^{c\log\left(F\right)}e^{dx\log\left(F\right)}}\,dx}{bd\log\left(F\right)}$$

input
```
integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c))**2,x)
```

output
```
-x**3/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F)) + 3*Integral(x**2/(a + b
*exp(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx$$
$$= -\frac{x^3}{F^{dx}F^c b^2 d\log(F)+abd\log(F)} + \frac{x^3}{abd\log(F)}$$
$$- \frac{3\left(d^2x^2\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F)^2 + 2\,dx\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\log(F) - 2\,\mathrm{Li}_3(-\frac{F^{dx}F^cb}{a})\right)}{abd^4\log(F)^4}$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
```

output
```
-x^3/(F^(d*x)*F^c*b^2*d*log(F) + a*b*d*log(F)) + x^3/(a*b*d*log(F)) - 3*(d
^2*x^2*log(F^(d*x)*F^c*b/a + 1)*log(F)^2 + 2*d*x*dilog(-F^(d*x)*F^c*b/a)*l
og(F) - 2*polylog(3, -F^(d*x)*F^c*b/a))/(a*b*d^4*log(F)^4)
```

Giac [F]

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx = \int \frac{F^{dx+c}x^3}{(F^{dx+c}b+a)^2}\,dx$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x^3/(F^(d*x + c)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx = \int \frac{F^{c+d\,x}\,x^3}{(a+F^{c+d\,x}\,b)^2}\,dx$$

input
```
int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^2,x)
```

output
```
int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^2, x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x^3}{(a+bF^{c+dx})^2}\,dx$$
$$= \frac{3f^{dx+c}\left(\int \frac{x^2}{f^{2dx+2c}b^2+2f^{dx+c}ab+a^2}dx\right)\log(f)^3 a^2 b\,d^3 + 6f^{dx+c}\left(\int \frac{x}{f^{2dx+2c}b^2+2f^{dx+c}ab+a^2}dx\right)\log(f)^2 a^2 b\,d^2 - 6f^{}$$

input
```
int(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^2,x)
```

output
```
(3*f**(c + d*x)*int(x**2/(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**
2),x)*log(f)**3*a**2*b*d**3 + 6*f**(c + d*x)*int(x/(f**(2*c + 2*d*x)*b**2
 + 2*f**(c + d*x)*a*b + a**2),x)*log(f)**2*a**2*b*d**2 - 6*f**(c + d*x)*log
(f**(c + d*x)*b + a)*b + 6*f**(c + d*x)*log(f)*b*d*x + 3*int(x**2/(f**(2*c
 + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**2),x)*log(f)**3*a**3*d**3 + 6*int
(x/(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**2),x)*log(f)**2*a**3*d
**2 - 6*log(f**(c + d*x)*b + a)*a - log(f)**3*a*d**3*x**3 - 3*log(f)**2*a*
d**2*x**2)/(log(f)**4*a*b*d**4*(f**(c + d*x)*b + a))
```

3.83 $\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx$

Optimal result . . . 691
Mathematica [A] (verified) . . . 691
Rubi [A] (verified) . . . 692
Maple [B] (verified) . . . 694
Fricas [A] (verification not implemented) . . . 694
Sympy [F] . . . 695
Maxima [A] (verification not implemented) . . . 695
Giac [F] . . . 696
Mupad [F(-1)] . . . 696
Reduce [F] . . . 696

Optimal result

Integrand size = 24, antiderivative size = 107

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{x^2}{abd\log(F)} - \frac{x^2}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{2x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{abd^2\log^2(F)} - \frac{2\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\log^3(F)}$$

output

```
x^2/a/b/d/ln(F)-x^2/b/d/(a+b*F^(d*x+c))/ln(F)-2*x*ln(1+b*F^(d*x+c)/a)/a/b/
d^2/ln(F)^2-2*polylog(2,-b*F^(d*x+c)/a)/a/b/d^3/ln(F)^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{dx\log(F)\left(bdF^{c+dx}x\log(F) - 2\left(a+bF^{c+dx}\right)\log\left(1+\frac{bF^{c+dx}}{a}\right)\right) - 2\left(a+bF^{c+dx}\right)\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{abd^3\left(a+bF^{c+dx}\right)\log^3(F)}$$

input

```
Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^2,x]
```

output

```
(d*x*Log[F]*(b*d*F^(c + d*x)*x*Log[F] - 2*(a + b*F^(c + d*x))*Log[1 + (b*F
^(c + d*x))/a]) - 2*(a + b*F^(c + d*x))*PolyLog[2, -((b*F^(c + d*x))/a)])/
(a*b*d^3*(a + b*F^(c + d*x))*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2621, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx$$

$$\downarrow 2621$$

$$\frac{2\int \frac{x}{bF^{c+dx}+a}dx}{bd\log(F)} - \frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2615$$

$$\frac{2\left(\frac{x^2}{2a} - \frac{b\int \frac{F^{c+dx}x}{bF^{c+dx}+a}dx}{a}\right)}{bd\log(F)} - \frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2620$$

$$\frac{2\left(\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int \log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}\right)}{bd\log(F)} - \frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2715$$

$$\frac{2\left(\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}\right)}{bd\log(F)} - \frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2838$$

$$\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}+\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

input
```
Int[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^2,x]
```

output
```
-(x^2/(b*d*(a + b*F^(c + d*x))*Log[F])) + (2*(x^2/(2*a) - (b*((x*Log[1 + (
b*F^(c + d*x))/a])/(b*d*Log[F]) + PolyLog[2, -((b*F^(c + d*x))/a)]/(b*d^2*
Log[F]^2)))/a))/(b*d*Log[F])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(107) = 214.

Time = 0.06 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.16

method	result
risch	$-\frac{x^2}{bd(a+b\,F^{dx+c})\ln(F)}+\frac{x^2}{abd\ln(F)}+\frac{2cx}{b\,d^2\ln(F)a}+\frac{c^2}{b\,d^3\ln(F)a}-\frac{2\ln\left(1+\frac{b\,F^c\,F^{dx}}{a}\right)x}{b\,d^2\ln(F)^2a}-\frac{2\ln\left(1+\frac{b\,F^c\,F^{dx}}{a}\right)c}{b\,d^3\ln(F)^2a}-\frac{2\,\mathrm{polylo}}{b\,d}$

input
```
int(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-x^2/b/d/(a+b*F^(d*x+c))/ln(F)+x^2/a/b/d/ln(F)+2/b/d^2/ln(F)/a*c*x+1/b/d^3
/ln(F)/a*c^2-2/b/d^2/ln(F)^2/a*ln(1+b*F^c*F^(d*x)/a)*x-2/b/d^3/ln(F)^2/a*l
n(1+b*F^c*F^(d*x)/a)*c-2/b/d^3/ln(F)^3/a*polylog(2,-b*F^c*F^(d*x)/a)-2/b/d
^3/ln(F)^2*c/a*ln(F^c*F^(d*x))+2/b/d^3/ln(F)^2*c/a*ln(F^(d*x)*F^c*b+a)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.74

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx =$$
$$-\frac{ac^2\log(F)^2-(bd^2x^2-bc^2)F^{dx+c}\log(F)^2+2\left(F^{dx+c}b+a\right)\mathrm{Li}_2\left(-\frac{F^{dx+c}b+a}{a}+1\right)-2\left(F^{dx+c}bc\log(F)\right.}{F^{dx+c}ab^2d^3\log(F)^3}$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(a*c^2*log(F)^2 - (b*d^2*x^2 - b*c^2)*F^(d*x + c)*log(F)^2 + 2*(F^(d*x +
c)*b + a)*dilog(-(F^(d*x + c)*b + a)/a + 1) - 2*(F^(d*x + c)*b*c*log(F) +
a*c*log(F))*log(F^(d*x + c)*b + a) + 2*((b*d*x + b*c)*F^(d*x + c)*log(F) +
 (a*d*x + a*c)*log(F))*log((F^(d*x + c)*b + a)/a))/(F^(d*x + c)*a*b^2*d^3*
log(F)^3 + a^2*b*d^3*log(F)^3)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{x^2}{F^{c+dx}b^2d\log(F)+abd\log(F)} + \frac{2\int \frac{x}{a+be^{c\log(F)}e^{dx\log(F)}}\,dx}{bd\log(F)}$$

input
```
integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c))**2,x)
```

output
```
-x**2/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F)) + 2*Integral(x/(a + b*ex
p(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{x^2}{F^{dx}F^cb^2d\log(F)+abd\log(F)} + \frac{x^2}{abd\log(F)} - \frac{2\left(dx\log\left(\frac{F^{dx}F^cb}{a}+1\right)\log(F)+\mathrm{Li}_2\left(-\frac{F^{dx}F^cb}{a}\right)\right)}{abd^3\log(F)^3}$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
```

output
```
-x^2/(F^(d*x)*F^c*b^2*d*log(F) + a*b*d*log(F)) + x^2/(a*b*d*log(F)) - 2*(d
*x*log(F^(d*x)*F^c*b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(a*b*d^3*log
(F)^3)
```

Giac [F]

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = \int \frac{F^{dx+c}x^2}{\left(F^{dx+c}b+a\right)^2}\,dx$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x^2/(F^(d*x + c)*b + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx = \int \frac{F^{c+d\,x}\,x^2}{\left(a+F^{c+d\,x}\,b\right)^2}\,dx$$

input
```
int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^2,x)
```

output
```
int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^2, x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^2}\,dx$$
$$= \frac{2f^{dx+c}\left(\int \frac{x}{f^{2dx+2c}b^2+2f^{dx+c}ab+a^2}dx\right)\log(f)^2\,a^2b\,d^2 - 2f^{dx+c}\log\left(f^{dx+c}b+a\right)b + 2f^{dx+c}\log(f)\,bdx + 2\left(\int \frac{}{f^2}\right.}{\log(f)^3\,ab\,d^3\left(f^{dx+c}b+a\right)}$$

input
```
int(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^2,x)
```

output

```
(2*f**(c + d*x)*int(x/(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**2),
x)*log(f)**2*a**2*b*d**2 - 2*f**(c + d*x)*log(f**(c + d*x)*b + a)*b + 2*f*
*(c + d*x)*log(f)*b*d*x + 2*int(x/(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*
a*b + a**2),x)*log(f)**2*a**3*d**2 - 2*log(f**(c + d*x)*b + a)*a - log(f)*
*2*a*d**2*x**2)/(log(f)**3*a*b*d**3*(f**(c + d*x)*b + a))
```

3.84 $\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx$

Optimal result . . . 698
Mathematica [A] (verified) . . . 698
Rubi [A] (verified) . . . 699
Maple [A] (verified) . . . 700
Fricas [A] (verification not implemented) . . . 701
Sympy [A] (verification not implemented) . . . 701
Maxima [A] (verification not implemented) . . . 702
Giac [F] . . . 702
Mupad [B] (verification not implemented) . . . 702
Reduce [B] (verification not implemented) . . . 703

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{x}{abd\log(F)} - \frac{x}{bd\left(a+bF^{c+dx}\right)\log(F)} - \frac{\log\left(a+bF^{c+dx}\right)}{abd^2\log^2(F)}$$

output
```
x/a/b/d/ln(F)-x/b/d/(a+b*F^(d*x+c))/ln(F)-ln(a+b*F^(d*x+c))/a/b/d^2/ln(F)^
2
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{\frac{dF^{c+dx}x\log(F)}{a+bF^{c+dx}} - \frac{\log\left(a+bF^{c+dx}\right)}{b}}{ad^2\log^2(F)}$$

input
```
Integrate[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^2,x]
```

output
```
((d*F^(c + d*x)*x*Log[F])/(a + b*F^(c + d*x)) - Log[a + b*F^(c + d*x)]/b)/
(a*d^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2621, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xF^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx$$

$$\downarrow 2621$$

$$\frac{\int \frac{1}{bF^{c+dx}+a}dx}{bd\log(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2720$$

$$\frac{\int \frac{F^{-c-dx}}{bF^{c+dx}+a}dF^{c+dx}}{bd^2\log^2(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 47$$

$$\frac{\frac{\int F^{-c-dx}dF^{c+dx}}{a} - \frac{b\int \frac{1}{bF^{c+dx}+a}dF^{c+dx}}{a}}{bd^2\log^2(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 14$$

$$\frac{\frac{\log\left(F^{c+dx}\right)}{a} - \frac{b\int \frac{1}{bF^{c+dx}+a}dF^{c+dx}}{a}}{bd^2\log^2(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 16$$

$$\frac{\frac{\log\left(F^{c+dx}\right)}{a} - \frac{\log\left(a+bF^{c+dx}\right)}{a}}{bd^2\log^2(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

input
```
Int[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^2,x]
```

output
```
-(x/(b*d*(a + b*F^(c + d*x))*Log[F])) + (Log[F^(c + d*x)]/a - Log[a + b*F^
(c + d*x)]/a)/(b*d^2*Log[F]^2)
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result	size
norman	$\frac{x\,\mathrm{e}^{(dx+c)\ln(F)}}{\ln(F)ad\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}-\frac{\ln\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}{\ln(F)^2bd^2a}$	67
parallelrisch	$\frac{bdx\,F^{dx+c}\ln(F)-\ln\left(a+b\,F^{dx+c}\right)F^{dx+c}b-\ln\left(a+b\,F^{dx+c}\right)a}{\ln(F)^2abd^2\left(a+b\,F^{dx+c}\right)}$	79
risch	$\frac{x}{abd\ln(F)}+\frac{c}{\ln(F)bd^2a}-\frac{x}{bd\left(a+b\,F^{dx+c}\right)\ln(F)}-\frac{\ln\left(F^{dx+c}+\frac{a}{b}\right)}{\ln(F)^2bd^2a}$	87

input
```
int(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/ln(F)/a/d*x*exp((d*x+c)*ln(F))/(a+b*exp((d*x+c)*ln(F)))-1/ln(F)^2/b/d^2/
a*ln(a+b*exp((d*x+c)*ln(F)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{F^{dx+c}bdx\log\left(F\right) - \left(F^{dx+c}b+a\right)\log\left(F^{dx+c}b+a\right)}{F^{dx+c}ab^2d^2\log\left(F\right)^2 + a^2bd^2\log\left(F\right)^2}$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
```

output
```
(F^(d*x + c)*b*d*x*log(F) - (F^(d*x + c)*b + a)*log(F^(d*x + c)*b + a))/(F
^(d*x + c)*a*b^2*d^2*log(F)^2 + a^2*b*d^2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{x}{F^{c+dx}b^2d\log\left(F\right)+abd\log\left(F\right)} + \frac{x}{abd\log\left(F\right)} - \frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{abd^2\log\left(F\right)^2}$$

input
```
integrate(F**(d*x+c)*x/(a+b*F**(d*x+c))**2,x)
```

output
```
-x/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F)) + x/(a*b*d*log(F)) - log(F*
*(c + d*x) + a/b)/(a*b*d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{F^{dx}F^{c}x}{F^{dx}F^{c}abd\log\left(F\right)+a^2d\log\left(F\right)} - \frac{\log\left(\frac{F^{dx}F^{c}b+a}{F^{c}b}\right)}{abd^2\log\left(F\right)^2}$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
```

output
```
F^(d*x)*F^c*x/(F^(d*x)*F^c*a*b*d*log(F) + a^2*d*log(F)) - log((F^(d*x)*F^c
*b + a)/(F^c*b))/(a*b*d^2*log(F)^2)
```

Giac [F]

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = \int \frac{F^{dx+c}x}{\left(F^{dx+c}b+a\right)^2}\,dx$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x/(F^(d*x + c)*b + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 22.87 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{F^{c}\,F^{d\,x}\,x}{a\,d\,\ln\left(F\right)\,\left(a+F^{c}\,F^{d\,x}\,b\right)} - \frac{\ln\left(a+F^{c}\,F^{d\,x}\,b\right)}{a\,b\,d^2\,\ln\left(F\right)^2}$$

input
```
int((F^(c + d*x)*x)/(a + F^(c + d*x)*b)^2,x)
```

output
```
(F^c*F^(d*x)*x)/(a*d*log(F)*(a + F^c*F^(d*x)*b)) - log(a + F^c*F^(d*x)*b)/
(a*b*d^2*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^2}\,dx = \frac{-f^{dx+c}\log\left(f^{dx+c}b+a\right)b + f^{dx+c}\log(f)\,bdx - \log\left(f^{dx+c}b+a\right)a}{\log(f)^2\,ab\,d^2\left(f^{dx+c}b+a\right)}$$

input

```
int(F^(d*x+c)*x/(a+b*F^(d*x+c))^2,x)
```

output

```
( - f**(c + d*x)*log(f**(c + d*x)*b + a)*b + f**(c + d*x)*log(f)*b*d*x - l
og(f**(c + d*x)*b + a)*a)/(log(f)**2*a*b*d**2*(f**(c + d*x)*b + a))
```

3.85 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx$

Optimal result 704
Mathematica [A] (verified) 704
Rubi [A] (verified) 705
Maple [A] (verified) 706
Fricas [A] (verification not implemented) 706
Sympy [A] (verification not implemented) 707
Maxima [A] (verification not implemented) 707
Giac [A] (verification not implemented) 707
Mupad [B] (verification not implemented) 708
Reduce [B] (verification not implemented) 708

Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{bd\left(a+bF^{c+dx}\right)\log(F)}$$

output
```
-1/b/d/(a+b*F^(d*x+c))/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{bd\left(a+bF^{c+dx}\right)\log(F)}$$

input
```
Integrate[F^(c + d*x)/(a + b*F^(c + d*x))^2,x]
```

output
```
-(1/(b*d*(a + b*F^(c + d*x))*Log[F]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx$$
$$\downarrow 2676$$
$$\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^2}\,dF^{c+dx}}{d\log(F)}$$
$$\downarrow 17$$
$$-\frac{1}{bd\log(F)\left(a+bF^{c+dx}\right)}$$

input
```
Int[F^(c + d*x)/(a + b*F^(c + d*x))^2,x]
```

output
```
-(1/(b*d*(a + b*F^(c + d*x))*Log[F]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln(F)}$	26
default	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln(F)}$	26
risch	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln(F)}$	26
parallelrisch	$-\frac{1}{bd\left(a+b\,F^{dx+c}\right)\ln(F)}$	26
norman	$\frac{\mathrm{e}^{(dx+c)\ln(F)}}{\ln(F)ad\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)}$	36

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b/d/(a+b*F^(d*x+c))/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{F^{dx+c}b^2d\log\left(F\right)+abd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/(F^(d*x + c)*b^2*d*log(F) + a*b*d*log(F))
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{F^{c+dx}b^2d\log\left(F\right)+abd\log\left(F\right)}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))**2,x)
```

output
```
-1/(F**(c + d*x)*b**2*d*log(F) + a*b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{(F^{dx+c}b+a)bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/((F^(d*x + c)*b + a)*b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = -\frac{1}{(F^{dx}F^{c}b+a)bd\log\left(F\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/((F^(d*x)*F^c*b + a)*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 22.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{F^{c+d\,x}}{a^2\,d\,\ln\left(F\right)+F^{c+d\,x}\,a\,b\,d\,\ln\left(F\right)}$$

input
```
int(F^(c + d*x)/(a + F^(c + d*x)*b)^2,x)
```

output
```
F^(c + d*x)/(a^2*d*log(F) + F^(c + d*x)*a*b*d*log(F))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2}\,dx = \frac{f^{dx+c}}{\log\left(f\right)ad\left(f^{dx+c}b+a\right)}$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2,x)
```

output
```
f**(c + d*x)/(log(f)*a*d*(f**(c + d*x)*b + a))
```

3.86 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx$

Optimal result . . . 709
Mathematica [N/A] . . . 709
Rubi [N/A] . . . 710
Maple [N/A] . . . 711
Fricas [N/A] . . . 711
Sympy [N/A] . . . 712
Maxima [N/A] . . . 712
Giac [N/A] . . . 713
Mupad [N/A] . . . 713
Reduce [N/A] . . . 713

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = -\frac{1}{bd\left(a+bF^{c+dx}\right)x\log(F)} - \frac{\operatorname{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)x^2},x\right)}{bd\log(F)}$$

output

```
-1/b/d/(a+b*F^(d*x+c))/x/ln(F)-Defer(Int)(1/(a+b*F^(d*x+c))/x^2,x)/b/d/ln(
F)
```

Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = \int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx$$

input

```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x),x]
```

output

```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2621, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{x\left(a+bF^{c+dx}\right)^2}\,dx$$

$$\downarrow 2621$$

$$-\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)x^2}\,dx}{bd\log(F)} - \frac{1}{bdx\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2619$$

$$-\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)x^2}\,dx}{bd\log(F)} - \frac{1}{bdx\log(F)\left(a+bF^{c+dx}\right)}$$

input
```
Int[F^(c + d*x)/((a + b*F^(c + d*x))^2*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{\left(a+b\,F^{dx+c}\right)^2 x}dx$$

input

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x)
```

output

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x}\,dx$$

input

```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x, algorithm="fricas")
```

output

```
integral(F^(d*x + c)/(2*F^(d*x + c)*a*b*x + F^(2*d*x + 2*c)*b^2*x + a^2*x)
, x)
```

Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = -\frac{1}{F^{c+dx}b^2dx\log(F)+abdx\log(F)} - \frac{\int \frac{1}{ax^2+bx^2e^{c\log(F)}e^{dx\log(F)}}\,dx}{bd\log(F)}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))**2/x,x)
```

output
```
-1/(F**(c + d*x)*b**2*d*x*log(F) + a*b*d*x*log(F)) - Integral(1/(a*x**2 +
b*x**2*exp(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x, algorithm="maxima")
```

output
```
-1/(F^(d*x)*F^c*b^2*d*x*log(F) + a*b*d*x*log(F)) - integrate(1/(F^(d*x)*F^
c*b^2*d*x^2*log(F) + a*b*d*x^2*log(F)), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 22.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = \int \frac{F^{c+d\,x}}{x\left(a+F^{c+d\,x}\,b\right)^2}\,dx$$

input
```
int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^2),x)
```

output
```
int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x}\,dx = f^c\left(\int \frac{f^{dx}}{f^{2dx+2c}b^2x+2f^{dx+c}abx+a^2x}dx\right)$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x,x)
```

output

```
f**c*int(f**(d*x)/(f**(2*c + 2*d*x)*b**2*x + 2*f**(c + d*x)*a*b*x + a**2*x
),x)
```

3.87 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx$

Optimal result . . . 715
Mathematica [N/A] . . . 715
Rubi [N/A] . . . 716
Maple [N/A] . . . 717
Fricas [N/A] . . . 717
Sympy [N/A] . . . 718
Maxima [N/A] . . . 718
Giac [N/A] . . . 719
Mupad [N/A] . . . 719
Reduce [N/A] . . . 719

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = -\frac{1}{bd\left(a+bF^{c+dx}\right)x^2\log(F)} - \frac{2\text{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)x^3},x\right)}{bd\log(F)}$$

output
```
-1/b/d/(a+b*F^(d*x+c))/x^2/ln(F)-2*Defer(Int)(1/(a+b*F^(d*x+c))/x^3,x)/b/d
/ln(F)
```

Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = \int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx$$

input
```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x^2),x]
```

output
```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2621, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{x^2\left(a+bF^{c+dx}\right)^2}\,dx$$

$$\downarrow 2621$$

$$-\frac{2\int \frac{1}{\left(bF^{c+dx}+a\right)x^3}\,dx}{bd\log(F)} - \frac{1}{bdx^2\log(F)\left(a+bF^{c+dx}\right)}$$

$$\downarrow 2619$$

$$-\frac{2\int \frac{1}{\left(bF^{c+dx}+a\right)x^3}\,dx}{bd\log(F)} - \frac{1}{bdx^2\log(F)\left(a+bF^{c+dx}\right)}$$

input
```
Int[F^(c + d*x)/((a + b*F^(c + d*x))^2*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{\left(a+b\,F^{dx+c}\right)^2 x^2}dx$$

input

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x)
```

output

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x^2}\,dx$$

input

```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x, algorithm="fricas")
```

output

```
integral(F^(d*x + c)/(2*F^(d*x + c)*a*b*x^2 + F^(2*d*x + 2*c)*b^2*x^2 + a^
2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = -\frac{1}{F^{c+dx}b^2dx^2\log(F)+abdx^2\log(F)} - \frac{2\int \frac{1}{ax^3+bx^3e^{c\log(F)}e^{dx\log(F)}}\,dx}{bd\log(F)}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))**2/x**2,x)
```

output
```
-1/(F**(c + d*x)*b**2*d*x**2*log(F) + a*b*d*x**2*log(F)) - 2*Integral(1/(a
*x**3 + b*x**3*exp(c*log(F))*exp(d*x*log(F))), x)/(b*d*log(F))
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x, algorithm="maxima")
```

output
```
-1/(F^(d*x)*F^c*b^2*d*x^2*log(F) + a*b*d*x^2*log(F)) - 2*integrate(1/(F^(d
*x)*F^c*b^2*d*x^3*log(F) + a*b*d*x^3*log(F)), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^2 x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 22.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx = \int \frac{F^{c+d\,x}}{x^2\left(a+F^{c+d\,x}\,b\right)^2}\,dx$$

input
```
int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^2),x)
```

output
```
int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 376, normalized size of antiderivative = 15.67

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^2 x^2}\,dx$$
$$= \frac{f^c\left(f^{dx+2c}\left(\int \frac{f^{2dx}}{f^{2dx+2c}b^2x^2+2f^{dx+c}ab\,x^2+a^2x^2}dx\right)b^2x+2f^{dx+c}\left(\int \frac{f^{dx}}{f^{2dx+2c}b^2x^2+2f^{dx+c}ab\,x^2+a^2x^2}dx\right)abx-f^{dx+c}\left(\int$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^2/x^2,x)
```

output
```
(f**c*(f**(2*c + d*x)*int(f**(2*d*x)/(f**(2*c + 2*d*x)*b**2*x**2 + 2*f**(c
 + d*x)*a*b*x**2 + a**2*x**2),x)*b**2*x + 2*f**(c + d*x)*int(f**(d*x)/(f**
(2*c + 2*d*x)*b**2*x**2 + 2*f**(c + d*x)*a*b*x**2 + a**2*x**2),x)*a*b*x -
f**(c + d*x)*int(f**(d*x)/(f**(2*c + 2*d*x)*b**2*x + 2*f**(c + d*x)*a*b*x
+ a**2*x),x)*log(f)*a*b*d*x + f**(d*x) + f**c*int(f**(2*d*x)/(f**(2*c + 2*
d*x)*b**2*x**2 + 2*f**(c + d*x)*a*b*x**2 + a**2*x**2),x)*a*b*x + 2*int(f**
(d*x)/(f**(2*c + 2*d*x)*b**2*x**2 + 2*f**(c + d*x)*a*b*x**2 + a**2*x**2),x
)*a**2*x - int(f**(d*x)/(f**(2*c + 2*d*x)*b**2*x + 2*f**(c + d*x)*a*b*x +
a**2*x),x)*log(f)*a**2*d*x))/(a*x*(f**(c + d*x)*b + a))
```

3.88 $\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx$

Optimal result . . . 721
Mathematica [A] (verified) . . . 722
Rubi [A] (verified) . . . 722
Maple [A] (verified) . . . 730
Fricas [B] (verification not implemented) . . . 730
Sympy [F] . . . 731
Maxima [A] (verification not implemented) . . . 732
Giac [F] . . . 732
Mupad [F(-1)] . . . 733
Reduce [F] . . . 733

Optimal result

Integrand size = 24, antiderivative size = 261

$$\begin{aligned}\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx = &-\frac{3x^2}{2a^2bd^2\log^2(F)}+\frac{3x^2}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)}+\frac{x^3}{2a^2bd\log(F)}\\ &-\frac{x^3}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}+\frac{3x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}\\ &-\frac{3x^2\log\left(1+\frac{bF^{c+dx}}{a}\right)}{2a^2bd^2\log^2(F)}+\frac{3\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)}\\ &-\frac{3x\,\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}+\frac{3\,\mathrm{PolyLog}\left(3,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^4\log^4(F)}\end{aligned}$$

output

```
-3/2*x^2/a^2/b/d^2/ln(F)^2+3/2*x^2/a/b/d^2/(a+b*F^(d*x+c))/ln(F)^2+1/2*x^3
/a^2/b/d/ln(F)-1/2*x^3/b/d/(a+b*F^(d*x+c))^2/ln(F)+3*x*ln(1+b*F^(d*x+c)/a)
/a^2/b/d^3/ln(F)^3-3/2*x^2*ln(1+b*F^(d*x+c)/a)/a^2/b/d^2/ln(F)^2+3*polylog
(2,-b*F^(d*x+c)/a)/a^2/b/d^4/ln(F)^4-3*x*polylog(2,-b*F^(d*x+c)/a)/a^2/b/d
^3/ln(F)^3+3*polylog(3,-b*F^(d*x+c)/a)/a^2/b/d^4/ln(F)^4
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.84

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$= \frac{dx\log(F)\left(bd^2F^{c+dx}\left(2a+bF^{c+dx}\right)x^2\log^2(F)+6\left(a+bF^{c+dx}\right)^2\log\left(1+\frac{bF^{c+dx}}{a}\right)-3d\left(a+bF^{c+dx}\right)x\log\right.}{}$$

input
```
Integrate[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^3,x]
```

output
```
(d*x*Log[F]*(b*d^2*F^(c + d*x)*(2*a + b*F^(c + d*x))*x^2*Log[F]^2 + 6*(a +
 b*F^(c + d*x))^2*Log[1 + (b*F^(c + d*x))/a] - 3*d*(a + b*F^(c + d*x))*x*L
og[F]*(b*F^(c + d*x) + (a + b*F^(c + d*x))*Log[1 + (b*F^(c + d*x))/a])) -
6*(a + b*F^(c + d*x))^2*(-1 + d*x*Log[F])*PolyLog[2, -((b*F^(c + d*x))/a)]
 + 6*(a + b*F^(c + d*x))^2*PolyLog[3, -((b*F^(c + d*x))/a)])/(2*a^2*b*d^4*
(a + b*F^(c + d*x))^2*Log[F]^4)
```

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {2621, 2616, 2615, 2620, 2621, 2615, 2620, 2715, 2838, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$\downarrow 2621$$
$$\frac{3\int \frac{x^2}{\left(bF^{c+dx}+a\right)^2}dx}{2bd\log(F)} - \frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$
$$\downarrow 2616$$

$$\frac{3\left(\frac{\int \frac{x^2}{bF^{c+dx}+a}dx}{a}-\frac{b\int \frac{F^{c+dx}x^2}{\left(bF^{c+dx}+a\right)^2}dx}{a}\right)}{2bd\log(F)}-\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2615$$

$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\int \frac{F^{c+dx}x^2}{bF^{c+dx}+a}dx}{a}}{a}-\frac{b\int \frac{F^{c+dx}x^2}{\left(bF^{c+dx}+a\right)^2}dx}{a}\right)}{2bd\log(F)}-\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2620$$

$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a}-\frac{b\int \frac{F^{c+dx}x^2}{\left(bF^{c+dx}+a\right)^2}dx}{a}\right)}{2bd\log(F)}-\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2621$$

$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2\int \frac{x}{bF^{c+dx}+a}dx}{bd\log(F)}-\frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}\right)}{2bd\log(F)}-\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2615$$

$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\int\frac{F^{c+dx}x}{bF^{c+dx}+a}dx}{a}\right)}{bd\log(F)}-\frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}\right)}{2bd\log(F)}-\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2620$$

$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{\int\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}\right)}{2bd\log(F)}-\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2715$$

$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}\right)}{bd\log(F)}-\frac{}{bd\log(F)}\right.}{a}\right.}{2bd\log(F)}$$

$$\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2838$$

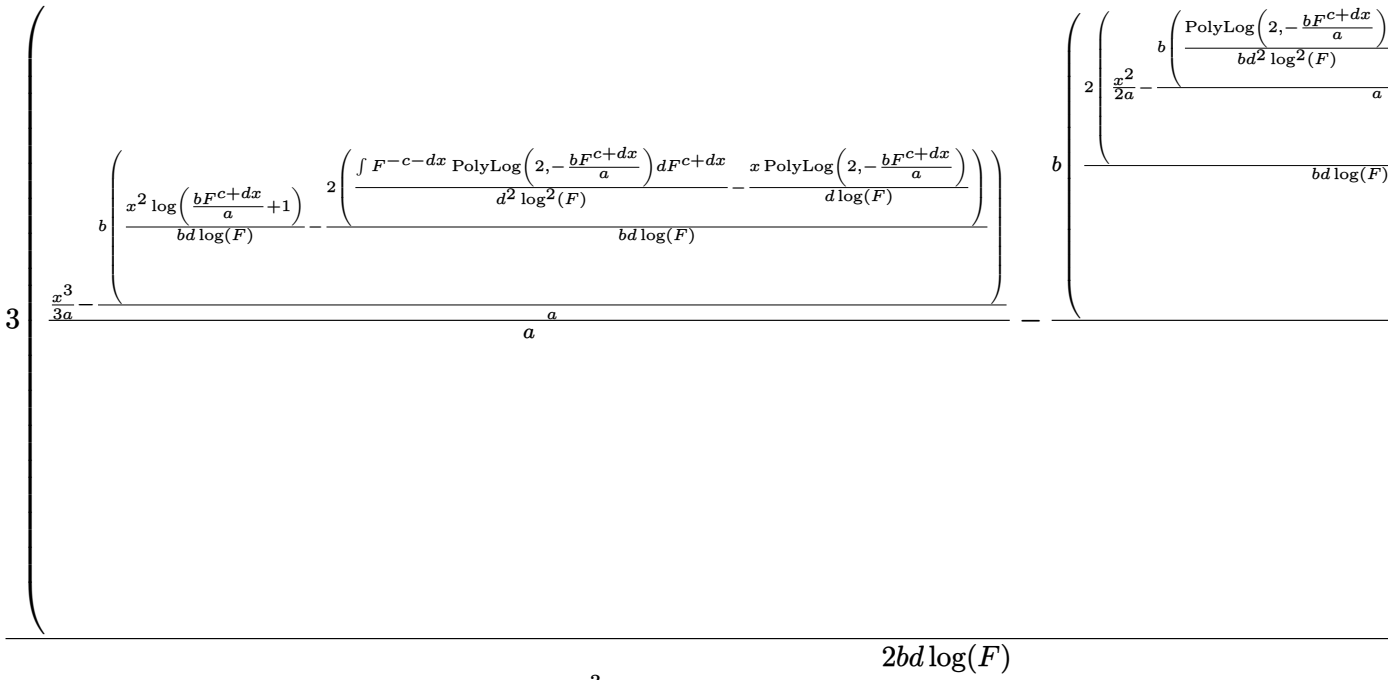
$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\int x\log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{x^2}{2a}-\frac{b\left(\frac{\mathrm{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}+\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}\right)}{a}\right)}{bd\log(F)}-\frac{x^2}{bd\log(F)\left(a+bF^{c+dx}\right)}\right.}{a}\right.}{2bd\log(F)}$$

$$\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 3011$$

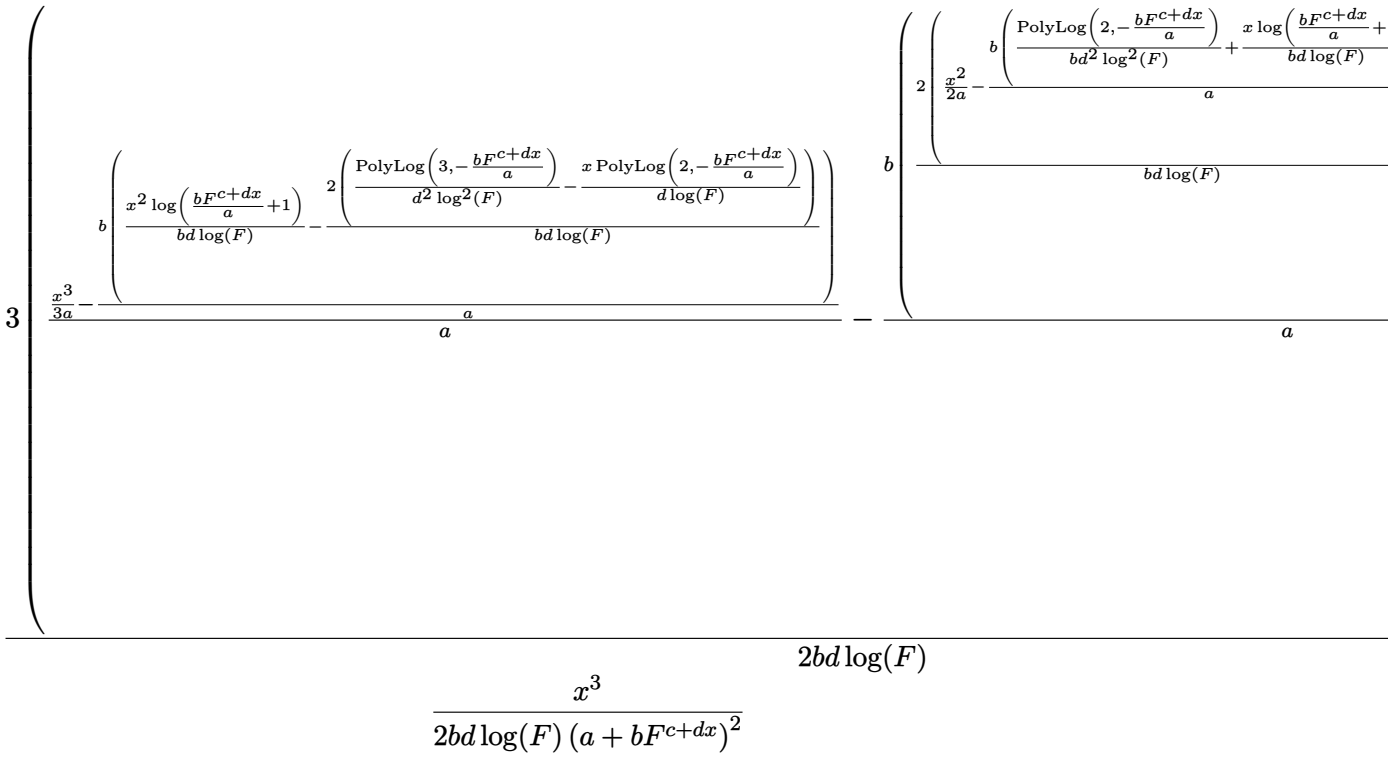
$$\frac{3\left(\frac{\frac{x^3}{3a}-\frac{b\left(\frac{x^2\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{2\left(\frac{\int\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)dx}{d\log(F)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{d\log(F)}\right)}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{2\left(\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}+\frac{x\log\left(\frac{bF^{c+d}}{a}\right.}{bd\log(F)}\right)}{a}}{bd\log(F)}\right)}{a}\right)}{2bd\log(F)}}{\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}}$$

$$\downarrow 2720$$



$$\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 7143$$



$$\frac{x^3}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

input
```
Int[(F^(c + d*x)*x^3)/(a + b*F^(c + d*x))^3,x]
```

output
```
-1/2*x^3/(b*d*(a + b*F^(c + d*x))^2*Log[F]) + (3*(-((b*(-(x^2/(b*d*(a + b*
F^(c + d*x))*Log[F])) + (2*(x^2/(2*a) - (b*((x*Log[1 + (b*F^(c + d*x))/a])
/(b*d*Log[F]) + PolyLog[2, -((b*F^(c + d*x))/a)]/(b*d^2*Log[F]^2)))/a))/(b
*d*Log[F])))/a) + (x^3/(3*a) - (b*((x^2*Log[1 + (b*F^(c + d*x))/a])/(b*d*L
og[F]) - (2*(-((x*PolyLog[2, -((b*F^(c + d*x))/a)])/(d*Log[F])) + PolyLog[
3, -((b*F^(c + d*x))/a)]/(d^2*Log[F]^2)))/(b*d*Log[F])))/a)/a))/(2*b*d*Log
[F])
```

Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))  Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{x^2(\ln(F)adx-3bF^{dx+c}-3a)}{2\ln(F)^2d^2b(a+bF^{dx+c})^2a}-\frac{3c^2x}{2ba^2d^3\ln(F)}-\frac{3\ln\left(1+\frac{bF^cF^{dx}}{a}\right)x^2}{2ba^2d^2\ln(F)^2}-\frac{3\,\mathrm{polylog}\left(2,-\frac{bF^cF^{dx}}{a}\right)x}{ba^2d^3\ln(F)^3}-\frac{3cx}{ba^2d^3\ln(F)^2}+\frac{3}{}$

input
```
int(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*(ln(F)*a*d*x-3*b*F^(d*x+c)-3*a)/ln(F)^2/d^2/b/(a+b*F^(d*x+c))^2/a
-3/2/b/a^2/d^3/ln(F)*c^2*x-3/2/b/a^2/d^2/ln(F)^2*ln(1+b*F^c*F^(d*x)/a)*x^2
-3/b/a^2/d^3/ln(F)^3*polylog(2,-b*F^c*F^(d*x)/a)*x-3/b/a^2/d^3/ln(F)^2*c*x
+3/b/a^2/d^3/ln(F)^3*ln(1+b*F^c*F^(d*x)/a)*x+3/b/a^2/d^4/ln(F)^3*ln(1+b*F^
c*F^(d*x)/a)*c+3/b/a^2/d^4/ln(F)^4*polylog(2,-b*F^c*F^(d*x)/a)-1/b/a^2/d^4
/ln(F)*c^3-3/2/b/a^2/d^4/ln(F)^2*c^2+3/b/a^2/d^4/ln(F)^4*polylog(3,-b*F^c*
F^(d*x)/a)+3/b/a^2/d^4/ln(F)^3*c*ln(F^c*F^(d*x))-3/b/a^2/d^4/ln(F)^3*c*ln(
F^(d*x)*F^c*b+a)+1/2*x^3/a^2/b/d/ln(F)+3/2/b/a^2/d^4/ln(F)^2*ln(1+b*F^c*F^
(d*x)/a)*c^2+3/2/b/a^2/d^4/ln(F)^2*c^2*ln(F^c*F^(d*x))-3/2/b/a^2/d^4/ln(F)
^2*c^2*ln(F^(d*x)*F^c*b+a)-3/2*x^2/a^2/b/d^2/ln(F)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(249) = 498.

Time = 0.11 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.21

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx$$

$$= \frac{a^2c^3\log(F)^3+3\,a^2c^2\log(F)^2+\left(\left(b^2d^3x^3+b^2c^3\right)\log(F)^3-3\left(b^2d^2x^2-b^2c^2\right)\log(F)^2\right)F^{2\,dx+2\,c}+\left(2\,(ab}{}$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

output

```
1/2*(a^2*c^3*log(F)^3 + 3*a^2*c^2*log(F)^2 + ((b^2*d^3*x^3 + b^2*c^3)*log(
F)^3 - 3*(b^2*d^2*x^2 - b^2*c^2)*log(F)^2)*F^(2*d*x + 2*c) + (2*(a*b*d^3*x
^3 + a*b*c^3)*log(F)^3 - 3*(a*b*d^2*x^2 - 2*a*b*c^2)*log(F)^2)*F^(d*x + c)
 - 6*(a^2*d*x*log(F) + (b^2*d*x*log(F) - b^2)*F^(2*d*x + 2*c) + 2*(a*b*d*x
*log(F) - a*b)*F^(d*x + c) - a^2)*dilog(-(F^(d*x + c)*b + a)/a + 1) - 3*(a
^2*c^2*log(F)^2 + 2*a^2*c*log(F) + (b^2*c^2*log(F)^2 + 2*b^2*c*log(F))*F^(
2*d*x + 2*c) + 2*(a*b*c^2*log(F)^2 + 2*a*b*c*log(F))*F^(d*x + c))*log(F^(d
*x + c)*b + a) - 3*((a^2*d^2*x^2 - a^2*c^2)*log(F)^2 + ((b^2*d^2*x^2 - b^2
*c^2)*log(F)^2 - 2*(b^2*d*x + b^2*c)*log(F))*F^(2*d*x + 2*c) + 2*((a*b*d^2
*x^2 - a*b*c^2)*log(F)^2 - 2*(a*b*d*x + a*b*c)*log(F))*F^(d*x + c) - 2*(a^
2*d*x + a^2*c)*log(F))*log((F^(d*x + c)*b + a)/a) + 6*(2*F^(d*x + c)*a*b +
 F^(2*d*x + 2*c)*b^2 + a^2)*polylog(3, -F^(d*x + c)*b/a))/(2*F^(d*x + c)*a
^3*b^2*d^4*log(F)^4 + F^(2*d*x + 2*c)*a^2*b^3*d^4*log(F)^4 + a^4*b*d^4*log
(F)^4)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx = \frac{3F^{c+dx}bx^2 - adx^3\log(F) + 3ax^2}{4F^{c+dx}a^2b^2d^2\log(F)^2 + 2F^{2c+2dx}ab^3d^2\log(F)^2 + 2a^3bd^2\log(F)^2} + \frac{3\left(\int\left(-\frac{2x}{a+be^{c\log(F)}e^{dx\log(F)}}\right)dx + \int\frac{dx^2\log(F)}{a+be^{c\log(F)}e^{dx\log(F)}}\,dx\right)}{2abd^2\log(F)^2}$$

input

```
integrate(F**(d*x+c)*x**3/(a+b*F**(d*x+c))**3,x)
```

output

```
(3*F**(c + d*x)*b*x**2 - a*d*x**3*log(F) + 3*a*x**2)/(4*F**(c + d*x)*a**2*
b**2*d**2*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*
d**2*log(F)**2) + 3*(Integral(-2*x/(a + b*exp(c*log(F))*exp(d*x*log(F))),
x) + Integral(d*x**2*log(F)/(a + b*exp(c*log(F))*exp(d*x*log(F))), x))/(2*
a*b*d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.02

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$= -\frac{adx^3\log(F) - 3\,F^{dx}F^{c}bx^2 - 3\,ax^2}{2\left(2\,F^{dx}F^{c}a^2b^2d^2\log(F)^2 + F^{2\,dx}F^{2\,c}ab^3d^2\log(F)^2 + a^3bd^2\log(F)^2\right)}$$
$$+ \frac{d^3x^3\log(F)^3 - 3\,d^2x^2\log(F)^2}{2\,a^2bd^4\log(F)^4}$$
$$- \frac{3\left(d^2x^2\log\left(\frac{F^{dx}F^{c}b}{a}+1\right)\log(F)^2 + 2\,dx\mathrm{Li}_2\left(-\frac{F^{dx}F^{c}b}{a}\right)\log(F) - 2\,\mathrm{Li}_3(-\frac{F^{dx}F^{c}b}{a})\right)}{2\,a^2bd^4\log(F)^4}$$
$$+ \frac{3\left(dx\log\left(\frac{F^{dx}F^{c}b}{a}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{dx}F^{c}b}{a}\right)\right)}{a^2bd^4\log(F)^4}$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(a*d*x^3*log(F) - 3*F^(d*x)*F^c*b*x^2 - 3*a*x^2)/(2*F^(d*x)*F^c*a^2*b
^2*d^2*log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*log(F)^2 + a^3*b*d^2*log(F)^
2) + 1/2*(d^3*x^3*log(F)^3 - 3*d^2*x^2*log(F)^2)/(a^2*b*d^4*log(F)^4) - 3/
2*(d^2*x^2*log(F^(d*x)*F^c*b/a + 1)*log(F)^2 + 2*d*x*dilog(-F^(d*x)*F^c*b/
a)*log(F) - 2*polylog(3, -F^(d*x)*F^c*b/a))/(a^2*b*d^4*log(F)^4) + 3*(d*x*
log(F^(d*x)*F^c*b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(a^2*b*d^4*log(
F)^4)
```

Giac [F]

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx = \int \frac{F^{dx+c}x^3}{\left(F^{dx+c}b+a\right)^3}\,dx$$

input
```
integrate(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x^3/(F^(d*x + c)*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx = \int \frac{F^{c+d\,x}\,x^3}{\left(a+F^{c+d\,x}\,b\right)^3}\,dx$$

input
```
int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^3,x)
```

output
```
int((F^(c + d*x)*x^3)/(a + F^(c + d*x)*b)^3, x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x^3}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$= \frac{12f^{2dx+2c}\left(\int \frac{x^2}{f^{3dx+3c}b^3+3f^{2dx+2c}a\,b^2+3f^{dx+c}a^2b+a^3}dx\right)\log(f)^3\,a^3b^2d^3+12f^{2dx+2c}\left(\int \frac{x}{f^{3dx+3c}b^3+3f^{2dx+2c}a\,b^2+3f^{dx+c}}\right.}{}$$

input
```
int(F^(d*x+c)*x^3/(a+b*F^(d*x+c))^3,x)
```

output
```
(12*f**(2*c + 2*d*x)*int(x**2/(f**(3*c + 3*d*x)*b**3 + 3*f**(2*c + 2*d*x)*
a*b**2 + 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**3*a**3*b**2*d**3 + 12*f*
*(2*c + 2*d*x)*int(x/(f**(3*c + 3*d*x)*b**3 + 3*f**(2*c + 2*d*x)*a*b**2 +
3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**2*a**3*b**2*d**2 - 6*f**(2*c + 2*
d*x)*log(f**(c + d*x)*b + a)*b**2 + 6*f**(2*c + 2*d*x)*log(f)*b**2*d*x - 3
*f**(2*c + 2*d*x)*b**2 + 24*f**(c + d*x)*int(x**2/(f**(3*c + 3*d*x)*b**3 +
 3*f**(2*c + 2*d*x)*a*b**2 + 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**3*a*
*4*b*d**3 + 24*f**(c + d*x)*int(x/(f**(3*c + 3*d*x)*b**3 + 3*f**(2*c + 2*d
*x)*a*b**2 + 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**2*a**4*b*d**2 - 12*f
**(c + d*x)*log(f**(c + d*x)*b + a)*a*b + 12*f**(c + d*x)*log(f)*a*b*d*x +
 12*int(x**2/(f**(3*c + 3*d*x)*b**3 + 3*f**(2*c + 2*d*x)*a*b**2 + 3*f**(c
+ d*x)*a**2*b + a**3),x)*log(f)**3*a**5*d**3 + 12*int(x/(f**(3*c + 3*d*x)*
b**3 + 3*f**(2*c + 2*d*x)*a*b**2 + 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)
**2*a**5*d**2 - 6*log(f**(c + d*x)*b + a)*a**2 - 4*log(f)**3*a**2*d**3*x**
3 - 6*log(f)**2*a**2*d**2*x**2 + 3*a**2)/(8*log(f)**4*a**2*b*d**4*(f**(2*c
 + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**2))
```

3.89 $\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx$

Optimal result . . . 734
Mathematica [A] (verified) . . . 735
Rubi [A] (verified) . . . 735
Maple [A] (verified) . . . 739
Fricas [B] (verification not implemented) . . . 740
Sympy [F] . . . 741
Maxima [A] (verification not implemented) . . . 741
Giac [F] . . . 742
Mupad [F(-1)] . . . 742
Reduce [F] . . . 743

Optimal result

Integrand size = 24, antiderivative size = 182

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{x}{a^2bd^2\log^2(F)} + \frac{x}{abd^2\left(a+bF^{c+dx}\right)\log^2(F)} + \frac{x^2}{2a^2bd\log(F)} - \frac{x^2}{2bd\left(a+bF^{c+dx}\right)^2\log(F)} + \frac{\log\left(a+bF^{c+dx}\right)}{a^2bd^3\log^3(F)} - \frac{x\log\left(1+\frac{bF^{c+dx}}{a}\right)}{a^2bd^2\log^2(F)} - \frac{\text{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{a^2bd^3\log^3(F)}$$

output

```
-x/a^2/b/d^2/ln(F)^2+x/a/b/d^2/(a+b*F^(d*x+c))/ln(F)^2+1/2*x^2/a^2/b/d/ln(
F)-1/2*x^2/b/d/(a+b*F^(d*x+c))^2/ln(F)+ln(a+b*F^(d*x+c))/a^2/b/d^3/ln(F)^3
-x*ln(1+b*F^(d*x+c)/a)/a^2/b/d^2/ln(F)^2-polylog(2,-b*F^(d*x+c)/a)/a^2/b/d
^3/ln(F)^3
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$= \frac{bd^2F^{c+dx}\left(2a+bF^{c+dx}\right)x^2\log^2(F)+2\left(a+bF^{c+dx}\right)^2\log\left(1+\frac{bF^{c+dx}}{a}\right)-2d\left(a+bF^{c+dx}\right)x\log(F)\left(bF^{c+}\right.}{2a^2bd^3\left(a+bF^{c+dx}\right)^2\log^3(F}$$

input
```
Integrate[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^3,x]
```

output
```
(b*d^2*F^(c + d*x)*(2*a + b*F^(c + d*x))*x^2*Log[F]^2 + 2*(a + b*F^(c + d*
x))^2*Log[1 + (b*F^(c + d*x))/a] - 2*d*(a + b*F^(c + d*x))*x*Log[F]*(b*F^(
c + d*x) + (a + b*F^(c + d*x))*Log[1 + (b*F^(c + d*x))/a]) - 2*(a + b*F^(c
 + d*x))^2*PolyLog[2, -((b*F^(c + d*x))/a)])/(2*a^2*b*d^3*(a + b*F^(c + d*
x))^2*Log[F]^3)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {2621, 2616, 2615, 2620, 2621, 2715, 2720, 47, 14, 16, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$\downarrow 2621$$
$$\frac{\int \frac{x}{\left(bF^{c+dx}+a\right)^2}dx}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$
$$\downarrow 2616$$

$$\frac{\frac{\int \frac{x}{bF^{c+dx}+a}dx}{a} - \frac{b\int \frac{F^{c+dx}x}{\left(bF^{c+dx}+a\right)^2}dx}{a}}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2615$$

$$\frac{\frac{\frac{x^2}{2a} - \frac{b\int \frac{F^{c+dx}x}{bF^{c+dx}+a}dx}{a}}{a} - \frac{b\int \frac{F^{c+dx}x}{\left(bF^{c+dx}+a\right)^2}dx}{a}}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2620$$

$$\frac{\frac{\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int \log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a} - \frac{b\int \frac{F^{c+dx}x}{\left(bF^{c+dx}+a\right)^2}dx}{a}}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2621$$

$$\frac{\frac{\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int \log\left(\frac{bF^{c+dx}}{a}+1\right)dx}{bd\log(F)}\right)}{a}}{a} - \frac{b\left(\frac{\int \frac{1}{bF^{c+dx}+a}dx}{bd\log(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2715$$

$$\frac{\frac{\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}}{a} - \frac{b\left(\frac{\int \frac{1}{bF^{c+dx}+a}dx}{bd\log(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2720$$

$$\frac{\frac{\frac{x^2}{2a} - \frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)} - \frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}}{a} - \frac{b\left(\frac{\int \frac{F^{-c-dx}}{bF^{c+dx}+a}dF^{c+dx}}{bd^2\log^2(F)} - \frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)} - \frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 47$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{\int F^{-c-dx}dF^{c+dx}}{a}-\frac{b\int\frac{1}{bF^{c+dx}+a}dF^{c+dx}}{a}}{bd^2\log^2(F)}-\frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)}-$$
$$\frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 14$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(F^{c+dx}\right)}{a}-\frac{b\int\frac{1}{bF^{c+dx}+a}dF^{c+dx}}{a}}{bd^2\log^2(F)}-\frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)}-$$
$$\frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 16$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}-\frac{\int F^{-c-dx}\log\left(\frac{bF^{c+dx}}{a}+1\right)dF^{c+dx}}{bd^2\log^2(F)}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(F^{c+dx}\right)}{a}-\frac{\log\left(a+bF^{c+dx}\right)}{a}}{bd^2\log^2(F)}-\frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)}-$$
$$\frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2838$$

$$\frac{\frac{x^2}{2a}-\frac{b\left(\frac{\operatorname{PolyLog}\left(2,-\frac{bF^{c+dx}}{a}\right)}{bd^2\log^2(F)}+\frac{x\log\left(\frac{bF^{c+dx}}{a}+1\right)}{bd\log(F)}\right)}{a}}{a}-\frac{b\left(\frac{\log\left(F^{c+dx}\right)}{a}-\frac{\log\left(a+bF^{c+dx}\right)}{a}}{bd^2\log^2(F)}-\frac{x}{bd\log(F)\left(a+bF^{c+dx}\right)}\right)}{a}}{bd\log(F)}-$$
$$\frac{x^2}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

input
```
Int[(F^(c + d*x)*x^2)/(a + b*F^(c + d*x))^3,x]
```

output
```
-1/2*x^2/(b*d*(a + b*F^(c + d*x))^2*Log[F]) + (-((b*(-(x/(b*d*(a + b*F^(c
+ d*x))*Log[F])) + (Log[F^(c + d*x)]/a - Log[a + b*F^(c + d*x)]/a)/(b*d^2*
Log[F]^2)))/a) + (x^2/(2*a) - (b*((x*Log[1 + (b*F^(c + d*x))/a])/(b*d*Log[
F]) + PolyLog[2, -((b*F^(c + d*x))/a)]/(b*d^2*Log[F]^2)))/a)/a)/(b*d*Log[F
])
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2616
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[1/a   Int[(c + d*x)^m*(a + b*(F^(g*(e
+ f*x)))^n)^(p + 1), x], x] - Simp[b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^
n*(a + b*(F^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n
}, x] && ILtQ[p, 0] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))  Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])  Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
  Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{x\left(\ln(F)adx-2b\,F^{dx+c}-2a\right)}{2\ln(F)^2d^2b\left(a+b\,F^{dx+c}\right)^2a}+\frac{x^2}{2a^2bd\ln(F)}+\frac{cx}{b\,a^2d^2\ln(F)}+\frac{c^2}{2b\,a^2d^3\ln(F)}-\frac{\ln\left(1+\frac{b\,F^cF^{dx}}{a}\right)x}{b\,a^2d^2\ln(F)^2}-\frac{\ln\left(1+\frac{b\,F^cF^{dx}}{a}\right)c}{b\,a^2d^3\ln(F)^2}-$

input
```
int(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*(ln(F)*a*d*x-2*b*F^(d*x+c)-2*a)/ln(F)^2/d^2/b/(a+b*F^(d*x+c))^2/a+1
/2*x^2/a^2/b/d/ln(F)+1/b/a^2/d^2/ln(F)*c*x+1/2/b/a^2/d^3/ln(F)*c^2-1/b/a^2
/d^2/ln(F)^2*ln(1+b*F^c*F^(d*x)/a)*x-1/b/a^2/d^3/ln(F)^2*ln(1+b*F^c*F^(d*x
)/a)*c-1/b/a^2/d^3/ln(F)^3*polylog(2,-b*F^c*F^(d*x)/a)-1/b/a^2/d^3/ln(F)^3
*ln(F^c*F^(d*x))+1/b/a^2/d^3/ln(F)^3*ln(F^(d*x)*F^c*b+a)-1/b/a^2/d^3/ln(F)
^2*c*ln(F^c*F^(d*x))+1/b/a^2/d^3/ln(F)^2*c*ln(F^(d*x)*F^c*b+a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(177) = 354$.

Time $= 0.08$ (sec) , antiderivative size $= 379$, normalized size of antiderivative $= 2.08$

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx =$$

$$-\frac{a^2c^2\log\left(F\right)^2+2\,a^2c\log\left(F\right)-\left(\left(b^2d^2x^2-b^2c^2\right)\log\left(F\right)^2-2\left(b^2dx+b^2c\right)\log\left(F\right)\right)F^{2\,dx+2\,c}-2\left((abd^2x$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*c^2*log(F)^2 + 2*a^2*c*log(F) - ((b^2*d^2*x^2 - b^2*c^2)*log(F)^
2 - 2*(b^2*d*x + b^2*c)*log(F))*F^(2*d*x + 2*c) - 2*((a*b*d^2*x^2 - a*b*c^
2)*log(F)^2 - (a*b*d*x + 2*a*b*c)*log(F))*F^(d*x + c) + 2*(2*F^(d*x + c)*a
*b + F^(2*d*x + 2*c)*b^2 + a^2)*dilog(-(F^(d*x + c)*b + a)/a + 1) - 2*(a^2
*c*log(F) + (b^2*c*log(F) + b^2)*F^(2*d*x + 2*c) + 2*(a*b*c*log(F) + a*b)*
F^(d*x + c) + a^2)*log(F^(d*x + c)*b + a) + 2*((b^2*d*x + b^2*c)*F^(2*d*x
+ 2*c)*log(F) + 2*(a*b*d*x + a*b*c)*F^(d*x + c)*log(F) + (a^2*d*x + a^2*c)
*log(F))*log((F^(d*x + c)*b + a)/a))/(2*F^(d*x + c)*a^3*b^2*d^3*log(F)^3 +
 F^(2*d*x + 2*c)*a^2*b^3*d^3*log(F)^3 + a^4*b*d^3*log(F)^3)
```

Sympy [F]

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx = \frac{2F^{c+dx}bx - adx^2\log(F) + 2ax}{4F^{c+dx}a^2b^2d^2\log(F)^2 + 2F^{2c+2dx}ab^3d^2\log(F)^2 + 2a^3bd^2\log(F)^2} + \frac{\int \frac{dx\log(F)}{a+be^{c\log(F)}e^{dx\log(F)}}\,dx + \int\left(-\frac{1}{a+be^{c\log(F)}e^{dx\log(F)}}\right)dx}{abd^2\log(F)^2}$$

input
```
integrate(F**(d*x+c)*x**2/(a+b*F**(d*x+c))**3,x)
```

output
```
(2*F**(c + d*x)*b*x - a*d*x**2*log(F) + 2*a*x)/(4*F**(c + d*x)*a**2*b**2*d
**2*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*d**2*l
og(F)**2) + (Integral(d*x*log(F)/(a + b*exp(c*log(F))*exp(d*x*log(F))), x)
 + Integral(-1/(a + b*exp(c*log(F))*exp(d*x*log(F))), x))/(a*b*d**2*log(F)
**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx &= -\frac{adx^2\log(F) - 2\,F^{dx}F^{c}bx - 2\,ax}{2\left(2\,F^{dx}F^{c}a^2b^2d^2\log(F)^2 + F^{2\,dx}F^{2\,c}ab^3d^2\log(F)^2 + a^3bd^2\log(F)^2\right)} \\ &\quad + \frac{x^2}{2\,a^2bd\log(F)} - \frac{x}{a^2bd^2\log(F)^2} \\ &\quad - \frac{dx\log\left(\frac{F^{dx}F^{c}b}{a}+1\right)\log(F) + \mathrm{Li}_2\left(-\frac{F^{dx}F^{c}b}{a}\right)}{a^2bd^3\log(F)^3} + \frac{\log\left(F^{dx}F^{c}b+a\right)}{a^2bd^3\log(F)^3}\end{aligned}$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(a*d*x^2*log(F) - 2*F^(d*x)*F^c*b*x - 2*a*x)/(2*F^(d*x)*F^c*a^2*b^2*d
^2*log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*log(F)^2 + a^3*b*d^2*log(F)^2) +
 1/2*x^2/(a^2*b*d*log(F)) - x/(a^2*b*d^2*log(F)^2) - (d*x*log(F^(d*x)*F^c*
b/a + 1)*log(F) + dilog(-F^(d*x)*F^c*b/a))/(a^2*b*d^3*log(F)^3) + log(F^(d
*x)*F^c*b + a)/(a^2*b*d^3*log(F)^3)
```

Giac [F]

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx = \int \frac{F^{dx+c}x^2}{\left(F^{dx+c}b+a\right)^3}\,dx$$

input
```
integrate(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x^2/(F^(d*x + c)*b + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx = \int \frac{F^{c+d\,x}\,x^2}{\left(a+F^{c+d\,x}\,b\right)^3}\,dx$$

input
```
int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^3,x)
```

output
```
int((F^(c + d*x)*x^2)/(a + F^(c + d*x)*b)^3, x)
```

Reduce [F]

$$\int \frac{F^{c+dx}x^2}{\left(a+bF^{c+dx}\right)^3}\,dx$$

$$= \frac{4f^{2dx+2c}\left(\int \frac{x}{f^{3dx+3c}b^3+3f^{2dx+2c}ab^2+3f^{dx+c}a^2b+a^3}dx\right)\log(f)^2a^3b^2d^2 - 2f^{2dx+2c}\log\left(f^{dx+c}b+a\right)b^2 + 2f^{2dx+2c}\text{lo}}{}$$

input

```
int(F^(d*x+c)*x^2/(a+b*F^(d*x+c))^3,x)
```

output

```
(4*f**(2*c + 2*d*x)*int(x/(f**(3*c + 3*d*x)*b**3 + 3*f**(2*c + 2*d*x)*a*b*
*2 + 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**2*a**3*b**2*d**2 - 2*f**(2*c
 + 2*d*x)*log(f**(c + d*x)*b + a)*b**2 + 2*f**(2*c + 2*d*x)*log(f)*b**2*d*
x - f**(2*c + 2*d*x)*b**2 + 8*f**(c + d*x)*int(x/(f**(3*c + 3*d*x)*b**3 +
3*f**(2*c + 2*d*x)*a*b**2 + 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**2*a**
4*b*d**2 - 4*f**(c + d*x)*log(f**(c + d*x)*b + a)*a*b + 4*f**(c + d*x)*log
(f)*a*b*d*x + 4*int(x/(f**(3*c + 3*d*x)*b**3 + 3*f**(2*c + 2*d*x)*a*b**2 +
 3*f**(c + d*x)*a**2*b + a**3),x)*log(f)**2*a**5*d**2 - 2*log(f**(c + d*x)
*b + a)*a**2 - 2*log(f)**2*a**2*d**2*x**2 + a**2)/(4*log(f)**3*a**2*b*d**3
*(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**2))
```

3.90 $\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx$

Optimal result . . . 744
Mathematica [A] (verified) . . . 744
Rubi [A] (verified) . . . 745
Maple [A] (verified) . . . 746
Fricas [A] (verification not implemented) . . . 747
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Maxima [A] (verification not implemented) . . . 748
Giac [F] . . . 748
Mupad [B] (verification not implemented) . . . 749
Reduce [B] (verification not implemented) . . . 749

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx = \frac{1}{2abd^2\left(a+bF^{c+dx}\right)\log^2(F)} + \frac{x}{2a^2bd\log(F)} - \frac{x}{2bd\left(a+bF^{c+dx}\right)^2\log(F)} - \frac{\log\left(a+bF^{c+dx}\right)}{2a^2bd^2\log^2(F)}$$

output
```
1/2/a/b/d^2/(a+b*F^(d*x+c))/ln(F)^2+1/2*x/a^2/b/d/ln(F)-1/2*x/b/d/(a+b*F^(
d*x+c))^2/ln(F)-1/2*ln(a+b*F^(d*x+c))/a^2/b/d^2/ln(F)^2
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx = \frac{bdF^{c+dx}\left(2a+bF^{c+dx}\right)x\log(F) - \left(a+bF^{c+dx}\right)\left(-a+\left(a+bF^{c+dx}\right)\log\left(a+bF^{c+dx}\right)\right)}{2a^2bd^2\left(a+bF^{c+dx}\right)^2\log^2(F)}$$

input
```
Integrate[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^3,x]
```

output
```
(b*d*F^(c + d*x)*(2*a + b*F^(c + d*x))*x*Log[F] - (a + b*F^(c + d*x))*(-a
+ (a + b*F^(c + d*x))*Log[a + b*F^(c + d*x)]))/(2*a^2*b*d^2*(a + b*F^(c +
d*x))^2*Log[F]^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2621, 2720, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{xF^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx$$

$$\downarrow 2621$$

$$\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^2}dx}{2bd\log(F)} - \frac{x}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2720$$

$$\frac{\int \frac{F^{-c-dx}}{\left(bF^{c+dx}+a\right)^2}dF^{c+dx}}{2bd^2\log^2(F)} - \frac{x}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 54$$

$$\frac{\int \left(\frac{F^{-c-dx}}{a^2} - \frac{b}{a^2\left(bF^{c+dx}+a\right)} - \frac{b}{a\left(bF^{c+dx}+a\right)^2}\right)dF^{c+dx}}{2bd^2\log^2(F)} - \frac{x}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log\left(a+bF^{c+dx}\right)}{a^2} + \frac{\log\left(F^{c+dx}\right)}{a^2} + \frac{1}{a\left(a+bF^{c+dx}\right)}}{2bd^2\log^2(F)} - \frac{x}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

input
```
Int[(F^(c + d*x)*x)/(a + b*F^(c + d*x))^3,x]
```

output
```
-1/2*x/(b*d*(a + b*F^(c + d*x))^2*Log[F]) + (1/(a*(a + b*F^(c + d*x))) + L
og[F^(c + d*x)]/a^2 - Log[a + b*F^(c + d*x)]/a^2)/(2*b*d^2*Log[F]^2)
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2621
```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05

method	result
risch	$\frac{x}{2a^2bd\ln(F)}+\frac{c}{2a^2bd^2\ln(F)}-\frac{\ln(F)adx-bF^{dx+c}-a}{2\ln(F)^2ad^2b\left(a+bF^{dx+c}\right)^2}-\frac{\ln\left(F^{dx+c}+\frac{a}{b}\right)}{2a^2bd^2\ln(F)^2}$
norman	$\frac{\frac{e^{(dx+c)\ln(F)}}{2\ln(F)^2ad^2}+\frac{xe^{(dx+c)\ln(F)}}{\ln(F)ad}+\frac{bxe^{(2dx+2c)\ln(F)}}{2\ln(F)a^2d}+\frac{1}{2\ln(F)^2bd^2}}{\left(a+be^{(dx+c)\ln(F)}\right)^2}-\frac{\ln\left(a+be^{(dx+c)\ln(F)}\right)}{2a^2bd^2\ln(F)^2}$
parallelrisch	$\frac{b^3F^{2dx+2c}x\ln(F)d+2xF^{dx+c}\ln(F)ab^2d-\ln\left(a+bF^{dx+c}\right)F^{2dx+2c}b^3-2\ln\left(a+bF^{dx+c}\right)F^{dx+c}ab^2-\ln\left(a+bF^{dx+c}\right)a^2b+F^{c}$
	$2\ln(F)^2a^2b^2d^2\left(a+bF^{dx+c}\right)^2$

input
```
int(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a^2/b/d/ln(F)+1/2/a^2/b/d^2/ln(F)*c-1/2*(ln(F)*a*d*x-b*F^(d*x+c)-a)/
ln(F)^2/a/d^2/b/(a+b*F^(d*x+c))^2-1/2/a^2/b/d^2/ln(F)^2*ln(F^(d*x+c)+1/b*a
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.40

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx$$
$$= \frac{F^{2\,dx+2\,c}b^2dx\log\left(F\right)+\left(2\,abdx\log\left(F\right)+ab\right)F^{dx+c}+a^2-\left(2\,F^{dx+c}ab+F^{2\,dx+2\,c}b^2+a^2\right)\log\left(F^{dx+c}b+a\right.}{2\left(2\,F^{dx+c}a^3b^2d^2\log\left(F\right)^2+F^{2\,dx+2\,c}a^2b^3d^2\log\left(F\right)^2+a^4bd^2\log\left(F\right)^2\right)}$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(F^(2*d*x + 2*c)*b^2*d*x*log(F) + (2*a*b*d*x*log(F) + a*b)*F^(d*x + c)
 + a^2 - (2*F^(d*x + c)*a*b + F^(2*d*x + 2*c)*b^2 + a^2)*log(F^(d*x + c)*b
 + a))/(2*F^(d*x + c)*a^3*b^2*d^2*log(F)^2 + F^(2*d*x + 2*c)*a^2*b^3*d^2*l
og(F)^2 + a^4*b*d^2*log(F)^2)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.15

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx = \frac{F^{c+dx}b-adx\log\left(F\right)+a}{4F^{c+dx}a^2b^2d^2\log\left(F\right)^2+2F^{2c+2dx}ab^3d^2\log\left(F\right)^2+2a^3bd^2\log\left(F\right)^2} + \frac{x}{2a^2bd\log\left(F\right)}-\frac{\log\left(F^{c+dx}+\frac{a}{b}\right)}{2a^2bd^2\log\left(F\right)^2}$$

input
```
integrate(F**(d*x+c)*x/(a+b*F**(d*x+c))**3,x)
```

output
```
(F**(c + d*x)*b - a*d*x*log(F) + a)/(4*F**(c + d*x)*a**2*b**2*d**2*log(F)*
*2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*log(F)**2 + 2*a**3*b*d**2*log(F)**2) +
 x/(2*a**2*b*d*log(F)) - log(F**(c + d*x) + a/b)/(2*a**2*b*d**2*log(F)**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.42

$$\begin{aligned}&\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx \\ &= \frac{F^{2\,dx}F^{2\,c}b^2dx\log\left(F\right)+\left(2\,F^{c}abdx\log\left(F\right)+F^{c}ab\right)F^{dx}+a^2}{2\left(2\,F^{dx}F^{c}a^3b^2d^2\log\left(F\right)^2+F^{2\,dx}F^{2\,c}a^2b^3d^2\log\left(F\right)^2+a^4bd^2\log\left(F\right)^2\right)} \\ &\quad -\frac{\log\left(\frac{F^{dx}F^{c}b+a}{F^{c}b}\right)}{2\,a^2bd^2\log\left(F\right)^2}\end{aligned}$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(F^(2*d*x)*F^(2*c)*b^2*d*x*log(F) + (2*F^c*a*b*d*x*log(F) + F^c*a*b)*F
^(d*x) + a^2)/(2*F^(d*x)*F^c*a^3*b^2*d^2*log(F)^2 + F^(2*d*x)*F^(2*c)*a^2*
b^3*d^2*log(F)^2 + a^4*b*d^2*log(F)^2) - 1/2*log((F^(d*x)*F^c*b + a)/(F^c*
b))/(a^2*b*d^2*log(F)^2)
```

Giac [F]

$$\int \frac{F^{c+dx}x}{\left(a+bF^{c+dx}\right)^3}\,dx = \int \frac{F^{dx+c}x}{\left(F^{dx+c}b+a\right)^3}\,dx$$

input
```
integrate(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)*x/(F^(d*x + c)*b + a)^3, x)
```

Mupad [B] (verification not implemented)

Time = 23.01 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.46

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3}\,dx = -\frac{\frac{F^c F^{dx}}{2ad^2\ln(F)^2} - \frac{F^c F^{dx}x}{ad\ln(F)} + \frac{F^{2c}F^{2dx}b}{2a^2d^2\ln(F)^2} - \frac{F^{2c}F^{2dx}bx}{2a^2d\ln(F)}}{a^2 + F^{2c}F^{2dx}b^2 + 2F^cF^{dx}ab} - \frac{\ln\left(a+F^cF^{dx}b\right)}{2a^2bd^2\ln(F)^2}$$

input
```
int((F^(c + d*x)*x)/(a + F^(c + d*x)*b)^3,x)
```

output
```
- ((F^c*F^(d*x))/(2*a*d^2*log(F)^2) - (F^c*F^(d*x)*x)/(a*d*log(F)) + (F^(2
*c)*F^(2*d*x)*b)/(2*a^2*d^2*log(F)^2) - (F^(2*c)*F^(2*d*x)*b*x)/(2*a^2*d*l
og(F)))/(a^2 + F^(2*c)*F^(2*d*x)*b^2 + 2*F^c*F^(d*x)*a*b) - log(a + F^c*F^
(d*x)*b)/(2*a^2*b*d^2*log(F)^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.57

$$\int \frac{F^{c+dx}x}{(a+bF^{c+dx})^3}\,dx$$
$$= \frac{-2f^{2dx+2c}\log\left(f^{dx+c}b+a\right)b^2 + 2f^{2dx+2c}\log(f)\,b^2dx - f^{2dx+2c}b^2 - 4f^{dx+c}\log\left(f^{dx+c}b+a\right)ab + 4f^{dx+c}\log}{4\log(f)^2a^2bd^2\left(f^{2dx+2c}b^2 + 2f^{dx+c}ab + a^2\right)}$$

input
```
int(F^(d*x+c)*x/(a+b*F^(d*x+c))^3,x)
```

output
```
( - 2*f**(2*c + 2*d*x)*log(f**(c + d*x)*b + a)*b**2 + 2*f**(2*c + 2*d*x)*l
og(f)*b**2*d*x - f**(2*c + 2*d*x)*b**2 - 4*f**(c + d*x)*log(f**(c + d*x)*b
 + a)*a*b + 4*f**(c + d*x)*log(f)*a*b*d*x - 2*log(f**(c + d*x)*b + a)*a**2
 + a**2)/(4*log(f)**2*a**2*b*d**2*(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*
a*b + a**2))
```

3.91 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx$

Optimal result 750
Mathematica [A] (verified) 750
Rubi [A] (verified) 751
Maple [A] (verified) 752
Fricas [A] (verification not implemented) 752
Sympy [B] (verification not implemented) 753
Maxima [A] (verification not implemented) 753
Giac [A] (verification not implemented) 753
Mupad [B] (verification not implemented) 754
Reduce [B] (verification not implemented) 754

Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}$$

output
```
-1/2/b/d/(a+b*F^(d*x+c))^2/ln(F)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2\log(F)}$$

input
```
Integrate[F^(c + d*x)/(a + b*F^(c + d*x))^3,x]
```

output
```
-1/2*1/(b*d*(a + b*F^(c + d*x))^2*Log[F])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2676, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx$$

$$\downarrow 2676$$

$$\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^3}\,dF^{c+dx}}{d\log(F)}$$

$$\downarrow 17$$

$$-\frac{1}{2bd\log(F)\left(a+bF^{c+dx}\right)^2}$$

input
```
Int[F^(c + d*x)/(a + b*F^(c + d*x))^3,x]
```

output
```
-1/2*1/(b*d*(a + b*F^(c + d*x))^2*Log[F])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2676
```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Simp[1/(d*e*n*Log[F])   Sub
st[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d
, e, n, p}, x]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
default	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
risch	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
parallelrisch	$-\frac{1}{2bd\left(a+b\,F^{dx+c}\right)^2\ln(F)}$	26
norman	$-\frac{1}{2bd\ln(F)\left(a+b\,\mathrm{e}^{(dx+c)\ln(F)}\right)^2}$	28

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/d/(a+b*F^(d*x+c))^2/ln(F)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2\left(2\,F^{dx+c}ab^2d\log\left(F\right)+F^{2\,dx+2\,c}b^3d\log\left(F\right)+a^2bd\log\left(F\right)\right)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2/(2*F^(d*x + c)*a*b^2*d*log(F) + F^(2*d*x + 2*c)*b^3*d*log(F) + a^2*b*
d*log(F))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.96

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{4F^{c+dx}ab^2d\log(F) + 2F^{2c+2dx}b^3d\log(F) + 2a^2bd\log(F)}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))**3,x)
```

output
```
-1/(4*F**(c + d*x)*a*b**2*d*log(F) + 2*F**(2*c + 2*d*x)*b**3*d*log(F) + 2*
a**2*b*d*log(F))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2\left(F^{dx+c}b+a\right)^2bd\log(F)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2/((F^(d*x + c)*b + a)^2*b*d*log(F))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2\left(F^{dx}F^cb+a\right)^2bd\log(F)}$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2/((F^(d*x)*F^c*b + a)^2*b*d*log(F))
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2\,b\,d\,\ln\left(F\right)\,\left(a+F^{c+d\,x}\,b\right)^2}$$

input
```
int(F^(c + d*x)/(a + F^(c + d*x)*b)^3,x)
```

output
```
-1/(2*b*d*log(F)*(a + F^(c + d*x)*b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.59

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3}\,dx = -\frac{1}{2\log\left(f\right)bd\left(f^{2dx+2c}b^2+2f^{dx+c}ab+a^2\right)}$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3,x)
```

output
```
( - 1)/(2*log(f)*b*d*(f**(2*c + 2*d*x)*b**2 + 2*f**(c + d*x)*a*b + a**2))
```

3.92 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx$

Optimal result . . . 755
Mathematica [N/A] . . . 755
Rubi [N/A] . . . 756
Maple [N/A] . . . 757
Fricas [N/A] . . . 757
Sympy [N/A] . . . 758
Maxima [N/A] . . . 758
Giac [N/A] . . . 759
Mupad [N/A] . . . 759
Reduce [N/A] . . . 759

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2 x\log(F)} - \frac{\operatorname{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)^2 x^2},x\right)}{2bd\log(F)}$$

output

```
-1/2/b/d/(a+b*F^(d*x+c))^2/x/ln(F)-1/2*Defer(Int)(1/(a+b*F^(d*x+c))^2/x^2,
x)/b/d/ln(F)
```

Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx$$

input

```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x),x]
```

output

```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2621, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{x\left(a+bF^{c+dx}\right)^3}\,dx$$

$$\downarrow 2621$$

$$-\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^2 x^2}dx}{2bd\log(F)} - \frac{1}{2bdx\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2619$$

$$-\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^2 x^2}dx}{2bd\log(F)} - \frac{1}{2bdx\log(F)\left(a+bF^{c+dx}\right)^2}$$

input
```
Int[F^(c + d*x)/((a + b*F^(c + d*x))^3*x),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{\left(a+b\,F^{dx+c}\right)^3 x}dx$$

input

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x)
```

output

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x}\,dx$$

input

```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x, algorithm="fricas")
```

output

```
integral(F^(d*x + c)/(3*F^(d*x + c)*a^2*b*x + 3*F^(2*d*x + 2*c)*a*b^2*x +
F^(3*d*x + 3*c)*b^3*x + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 170, normalized size of antiderivative = 7.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx$$
$$= \frac{-F^{c+dx}b - adx\log(F) - a}{4F^{c+dx}a^2b^2d^2x^2\log(F)^2 + 2F^{2c+2dx}ab^3d^2x^2\log(F)^2 + 2a^3bd^2x^2\log(F)^2}$$
$$- \frac{\int \frac{dx\log(F)}{ax^3+bx^3e^{c\log(F)}e^{dx\log(F)}}\,dx + \int \frac{2}{ax^3+bx^3e^{c\log(F)}e^{dx\log(F)}}\,dx}{2abd^2\log(F)^2}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))**3/x,x)
```

output
```
(-F**(c + d*x)*b - a*d*x*log(F) - a)/(4*F**(c + d*x)*a**2*b**2*d**2*x**2*l
og(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*x**2*log(F)**2 + 2*a**3*b*d**2*x
**2*log(F)**2) - (Integral(d*x*log(F)/(a*x**3 + b*x**3*exp(c*log(F))*exp(d
*x*log(F))), x) + Integral(2/(a*x**3 + b*x**3*exp(c*log(F))*exp(d*x*log(F)
)), x))/(2*a*b*d**2*log(F)**2)
```

Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.04

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x, algorithm="maxima")
```

output
```
-1/2*(a*d*x*log(F) + F^(d*x)*F^c*b + a)/(2*F^(d*x)*F^c*a^2*b^2*d^2*x^2*log
(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*x^2*log(F)^2 + a^3*b*d^2*x^2*log(F)^2)
 - integrate(1/2*(d*x*log(F) + 2)/(F^(d*x)*F^c*a*b^2*d^2*x^3*log(F)^2 + a^
2*b*d^2*x^3*log(F)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 22.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = \int \frac{F^{c+d\,x}}{x\left(a+F^{c+d\,x}\,b\right)^3}\,dx$$

input
```
int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^3),x)
```

output
```
int(F^(c + d*x)/(x*(a + F^(c + d*x)*b)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x}\,dx = f^c\left(\int \frac{f^{dx}}{f^{3dx+3c}b^3x+3f^{2dx+2c}a\,b^2x+3f^{dx+c}a^2bx+a^3x}dx\right)$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x,x)
```

output

```
f**c*int(f**(d*x)/(f**(3*c + 3*d*x)*b**3*x + 3*f**(2*c + 2*d*x)*a*b**2*x +
 3*f**(c + d*x)*a**2*b*x + a**3*x),x)
```

3.93 $\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx$

Optimal result . . . 761
Mathematica [N/A] . . . 761
Rubi [N/A] . . . 762
Maple [N/A] . . . 763
Fricas [N/A] . . . 763
Sympy [N/A] . . . 764
Maxima [N/A] . . . 764
Giac [N/A] . . . 765
Mupad [N/A] . . . 765
Reduce [N/A] . . . 765

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx = -\frac{1}{2bd\left(a+bF^{c+dx}\right)^2 x^2 \log(F)} - \frac{\text{Int}\left(\frac{1}{\left(a+bF^{c+dx}\right)^2 x^3}, x\right)}{bd\log(F)}$$

output

```
-1/2/b/d/(a+b*F^(d*x+c))^2/x^2/ln(F)-Defer(Int)(1/(a+b*F^(d*x+c))^2/x^3,x)
/b/d/ln(F)
```

Mathematica [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx = \int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx$$

input

```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x^2),x]
```

output

```
Integrate[F^(c + d*x)/((a + b*F^(c + d*x))^3*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {2621, 2619}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{F^{c+dx}}{x^2\left(a+bF^{c+dx}\right)^3}\,dx$$

$$\downarrow 2621$$

$$-\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^2 x^3}\,dx}{bd\log(F)} - \frac{1}{2bdx^2\log(F)\left(a+bF^{c+dx}\right)^2}$$

$$\downarrow 2619$$

$$-\frac{\int \frac{1}{\left(bF^{c+dx}+a\right)^2 x^3}\,dx}{bd\log(F)} - \frac{1}{2bdx^2\log(F)\left(a+bF^{c+dx}\right)^2}$$

input
```
Int[F^(c + d*x)/((a + b*F^(c + d*x))^3*x^2),x]
```

output
```
$Aborted
```

Defintions of rubi rules used

rule 2619
```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Unintegrable[(a + b*(F^(g*(e + f*x)))^n)^p
*(c + d*x)^m, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x]
```

rule 2621

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[(c + d*x)^m*((a + b*(F^(g*(e + f*x)))^n)^(p + 1)/(b*f*g*n*(p + 1)*Log
[F])), x] - Simp[d*(m/(b*f*g*n*(p + 1)*Log[F]))   Int[(c + d*x)^(m - 1)*(a
+ b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g,
m, n, p}, x] && NeQ[p, -1]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{F^{dx+c}}{\left(a+b\,F^{dx+c}\right)^3 x^2} dx$$

input

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x)
```

output

```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\, dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x^2}\, dx$$

input

```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x, algorithm="fricas")
```

output

```
integral(F^(d*x + c)/(3*F^(d*x + c)*a^2*b*x^2 + 3*F^(2*d*x + 2*c)*a*b^2*x^
2 + F^(3*d*x + 3*c)*b^3*x^2 + a^3*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 172, normalized size of antiderivative = 7.17

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx$$
$$= \frac{-2F^{c+dx}b - adx\log(F) - 2a}{4F^{c+dx}a^2b^2d^2x^3\log(F)^2 + 2F^{2c+2dx}ab^3d^2x^3\log(F)^2 + 2a^3bd^2x^3\log(F)^2}$$
$$- \frac{\int \frac{dx\log(F)}{ax^4+bx^4e^{c\log(F)}e^{dx\log(F)}}\,dx + \int \frac{3}{ax^4+bx^4e^{c\log(F)}e^{dx\log(F)}}\,dx}{abd^2\log(F)^2}$$

input
```
integrate(F**(d*x+c)/(a+b*F**(d*x+c))**3/x**2,x)
```

output
```
(-2*F**(c + d*x)*b - a*d*x*log(F) - 2*a)/(4*F**(c + d*x)*a**2*b**2*d**2*x*
*3*log(F)**2 + 2*F**(2*c + 2*d*x)*a*b**3*d**2*x**3*log(F)**2 + 2*a**3*b*d*
*2*x**3*log(F)**2) - (Integral(d*x*log(F)/(a*x**4 + b*x**4*exp(c*log(F))*e
xp(d*x*log(F))), x) + Integral(3/(a*x**4 + b*x**4*exp(c*log(F))*exp(d*x*lo
g(F))), x))/(a*b*d**2*log(F)**2)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 6.12

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x, algorithm="maxima")
```

output
```
-1/2*(a*d*x*log(F) + 2*F^(d*x)*F^c*b + 2*a)/(2*F^(d*x)*F^c*a^2*b^2*d^2*x^3
*log(F)^2 + F^(2*d*x)*F^(2*c)*a*b^3*d^2*x^3*log(F)^2 + a^3*b*d^2*x^3*log(F
)^2) - integrate((d*x*log(F) + 3)/(F^(d*x)*F^c*a*b^2*d^2*x^4*log(F)^2 + a^
2*b*d^2*x^4*log(F)^2), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx = \int \frac{F^{dx+c}}{\left(F^{dx+c}b+a\right)^3 x^2}\,dx$$

input
```
integrate(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x, algorithm="giac")
```

output
```
integrate(F^(d*x + c)/((F^(d*x + c)*b + a)^3*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 22.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx = \int \frac{F^{c+d\,x}}{x^2\left(a+F^{c+d\,x}\,b\right)^3}\,dx$$

input
```
int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^3),x)
```

output
```
int(F^(c + d*x)/(x^2*(a + F^(c + d*x)*b)^3), x)
```

Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 1023, normalized size of antiderivative = 42.62

$$\int \frac{F^{c+dx}}{\left(a+bF^{c+dx}\right)^3 x^2}\,dx = \text{Too large to display}$$

input
```
int(F^(d*x+c)/(a+b*F^(d*x+c))^3/x^2,x)
```

output

```
(f**c*(2*f**(3*c + 2*d*x)*int(f**(2*d*x)/(f**(3*c + 3*d*x)*b**3*x**2 + 3*f
**(2*c + 2*d*x)*a*b**2*x**2 + 3*f**(c + d*x)*a**2*b*x**2 + a**3*x**2),x)*b
**3*x + 2*f**(3*c + 2*d*x)*int(f**(2*d*x)/(f**(3*c + 3*d*x)*b**3*x + 3*f**
(2*c + 2*d*x)*a*b**2*x + 3*f**(c + d*x)*a**2*b*x + a**3*x),x)*log(f)*b**3*
d*x + 3*f**(2*c + 2*d*x)*int(f**(d*x)/(f**(3*c + 3*d*x)*b**3*x**2 + 3*f**(
2*c + 2*d*x)*a*b**2*x**2 + 3*f**(c + d*x)*a**2*b*x**2 + a**3*x**2),x)*a*b*
*2*x - 2*f**(2*c + 2*d*x)*int(f**(d*x)/(f**(3*c + 3*d*x)*b**3*x + 3*f**(2*
c + 2*d*x)*a*b**2*x + 3*f**(c + d*x)*a**2*b*x + a**3*x),x)*log(f)*a*b**2*d
*x + 4*f**(2*c + d*x)*int(f**(2*d*x)/(f**(3*c + 3*d*x)*b**3*x**2 + 3*f**(2
*c + 2*d*x)*a*b**2*x**2 + 3*f**(c + d*x)*a**2*b*x**2 + a**3*x**2),x)*a*b**
2*x + 4*f**(2*c + d*x)*int(f**(2*d*x)/(f**(3*c + 3*d*x)*b**3*x + 3*f**(2*c
 + 2*d*x)*a*b**2*x + 3*f**(c + d*x)*a**2*b*x + a**3*x),x)*log(f)*a*b**2*d*
x + 6*f**(c + d*x)*int(f**(d*x)/(f**(3*c + 3*d*x)*b**3*x**2 + 3*f**(2*c +
2*d*x)*a*b**2*x**2 + 3*f**(c + d*x)*a**2*b*x**2 + a**3*x**2),x)*a**2*b*x -
 4*f**(c + d*x)*int(f**(d*x)/(f**(3*c + 3*d*x)*b**3*x + 3*f**(2*c + 2*d*x)
*a*b**2*x + 3*f**(c + d*x)*a**2*b*x + a**3*x),x)*log(f)*a**2*b*d*x + 2*f**
(d*x) + 2*f**c*int(f**(2*d*x)/(f**(3*c + 3*d*x)*b**3*x**2 + 3*f**(2*c + 2*
d*x)*a*b**2*x**2 + 3*f**(c + d*x)*a**2*b*x**2 + a**3*x**2),x)*a**2*b*x + 2
*f**c*int(f**(2*d*x)/(f**(3*c + 3*d*x)*b**3*x + 3*f**(2*c + 2*d*x)*a*b**2*
x + 3*f**(c + d*x)*a**2*b*x + a**3*x),x)*log(f)*a**2*b*d*x + 3*int(f**(...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 767
4.2 Links to plain text integration problems used in this report for each CAS . 785

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if  not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file